PEARSON mathematics

S.B.

9

AUSTRALIAN CURRICULUM

Pearson Australia

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Fibonacci poem

The poem on the inside front cover is called a Fib. The sequence of syllables in the lines follows a Fibonacci sequence: 1, 1, 2, 3, 5, 8.

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PEARSON mathematics

Student Book

The student book has been designed for maximum student engagement while providing teachers, particularly those lacking a strong mathematical background, with everything they need to teach for deeper understanding. It has been structured according to the latest research on how students learn mathematics and on how to avoid common misunderstandings, making it easy for teachers to cater for mixed abilities and ask good questions.

Bridging Workbook

The *Pearson Mathematics 7 Bridging Workbook* provides:

- support for Year 7 transition
- write-in skills revision for each student book section
- confidence-boosting content, particularly for weaker students
- language scaffolding for low-literacy students
- extra NAPLAN support.

Homework Program

The homework program covers systematic cumulative revision of basic skills and class material.

Teacher Companion

The teacher companion makes lesson preparation easy by combining full-colour textbook pages with teaching strategies, ideas for class activities and fully worked solutions in a series of chapter booklets.

AUSTRALIAN CURRICULUM

ALWAYS LEARNING

Pearson Reader

Much more than an e-book, **Pearson Reader** is an interactive online version of your student book linked to rich media sources, specifically developed for Mathematics. It supports you with ready-made activities and teaching tools, including whole-class and individual games and videos that will enliven new topics. Integrated throughout are lessons for independent learning that are designed to keep students engaged while developing their conceptual understanding of topics covered. Planning and navigation are a breeze with a resource summary page and a host of interactive whiteboard tools to use in whole-class presentations.

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Because wherever learning flourishes, so do people.

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USING PEARSON mathematics

Bringing the heart and soul of the Australian Mathematics Curriculum to life

Mathspace

Our Global village

Facts about our global village

There are over 6.9 billion people on earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

Race

- 0.61 Asian
- $\frac{12}{100}$ European
- 0.08 North American
- 0.05 South American & Caribbean
- 0.13 African
- $\frac{1}{100}$ Oceania (this includes Australia)

Conditions

- 0.43 live without basic sanitation
- $\frac{1}{3}$ don't have access to clean, safe drinking water
- 0.13 are hungry and/or malnourished

Wealth

- 0.06 of the population own 59% of the entire wealth
- $\frac{1}{5}$ of the population have 75% of the income
- 0.53 of the population live on \$2.50 or less per day

Education and technology

- 0.14 can't read
- 0.07 have a secondary education
- 0.12 have a computer
- 0.03 have the Internet

How rich are you?

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage*.

$1.2 - 2.7 + 3.5 - 1.25 =$

How many people?

Currently there are approximately 6 900 000 000 people living on earth.

3 Using the facts you now know about our world, *estimate* how many people:

(a) Don't have access to clean water

(b) Struggle to live on \$2.50 or less per day

(c) Have a computer

(d) Don't have a secondary school education

I want to help

There are many charities out there that are trying to create more equality in the world—Oxfam is one of these. Charities often rely on the generosity of everyday people. Here is a pie chart of the age groups of Oxfam volunteers.

Volunteers by age group

18 – 25, 26 – 30, 31 – 40, 41 – 50, 51 – 65, 65+

43%, 28%, 12%, 8%, 7%, 2%

Oxfam Annual Report 07-08

4 Using the pie chart above, find out the following:

(a) What percentage of volunteers are between 18 and 30?

(b) Why do you think most volunteers are in this age range?

(c) One of the challenges charities face is the need to pay for fundraising and administration costs. If Oxfam spends 24.4% of their income on fundraising and 8.9% on administration costs, what percentage of their income is available to go directly to community aid projects?

In the future

5 Imagine it's the year 2055. How do you hope the 'Global Village' percentages have changed? Write down your predictions and ideas.

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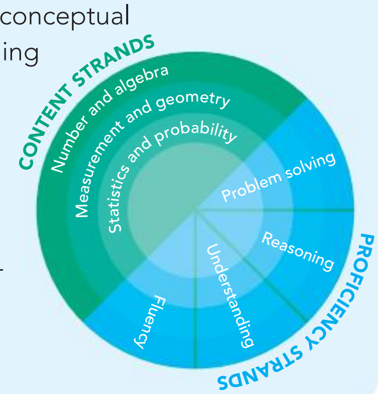
The Australian Curriculum: Mathematics aims to ensure that students:

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens.*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in* Number and Algebra, Measurement and Geometry, *and* Statistics and Probability.
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

Pearson Mathematics shares these aims.

Pearson Mathematics is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills, in addition to addressing the Australian Curriculum's General capabilities and Cross-curriculum priorities.



Making teachers' lives easier

Differentiation

Pearson Mathematics has been written and designed with the needs of the full ability spectrum of students in Australian classrooms in mind.

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9(a), Q10, Q13, Q14, Q15(a), Q18	Q1(b), Q2(b), Q3(b), Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q14, Q15, Q17, Q18	Q4, Q5, Q6, Q7(g)–(l), Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q19

Navigator—Three optional graded pathways through *every* Exercise.

Recall—Each chapter begins with a review of assumed and necessary knowledge, with Recall worksheets available for each skill for those students needing extra revision.

Challenge—Every chapter has a full page challenge section for earlier finishers and those students who need to be extended.

Scaffolded Exercises—All Exercises are split into the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has Open-ended questions to encourage students' creative thinking and ability to communicate mathematics effectively.

Regular revision and reinforcement

Pearson Mathematics has a broad range of cumulative and chapter-based revision.

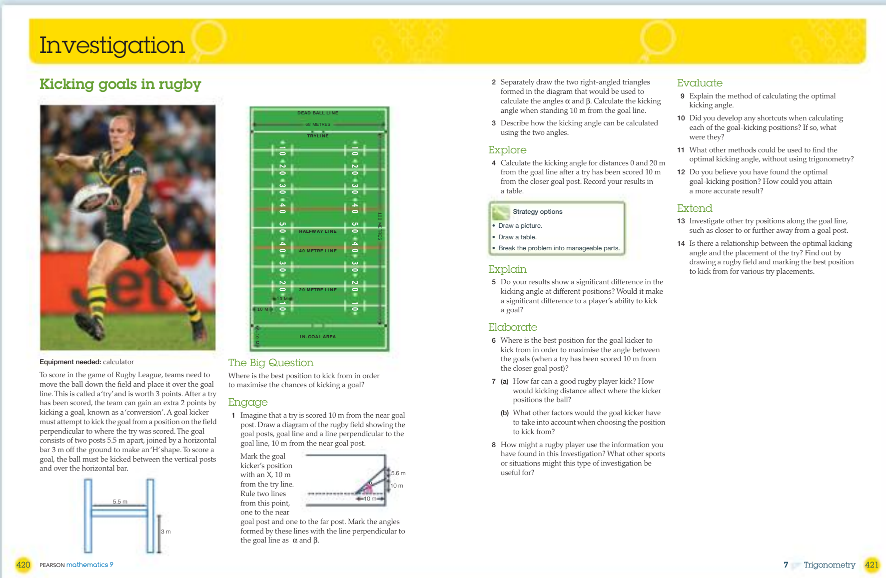
NAPLAN practice—An end-of-chapter section that helps students prepare for their NAPLAN tests

Half-time—A mid-chapter review

Chapter review—An end-of-chapter review

Mixed review—Cumulative revision for every second chapter

Encouraging inquiry and problem solving



The Australian Mathematics Curriculum focuses on students developing problem-solving skills, in particular the ability to employ mathematical strategies to solve problems efficiently and to be able to respond to unfamiliar situations. **Pearson Mathematics** is designed to develop these skills.

Problem solving toolbox

12 key problem-solving strategies are included inside the front cover of the Student Book and these strategy options are referenced throughout investigations and problem solving.

Outside the Square: Problem solving

These are located throughout every chapter and give students ample opportunity to develop their problem-solving skills and logical reasoning.

Outside the Square
Problem solving

Investigations

Investigation

These are scaffolded to be accessible to all learners and structured following a 5e+ format.

Technology Explorations

Technology Exploration

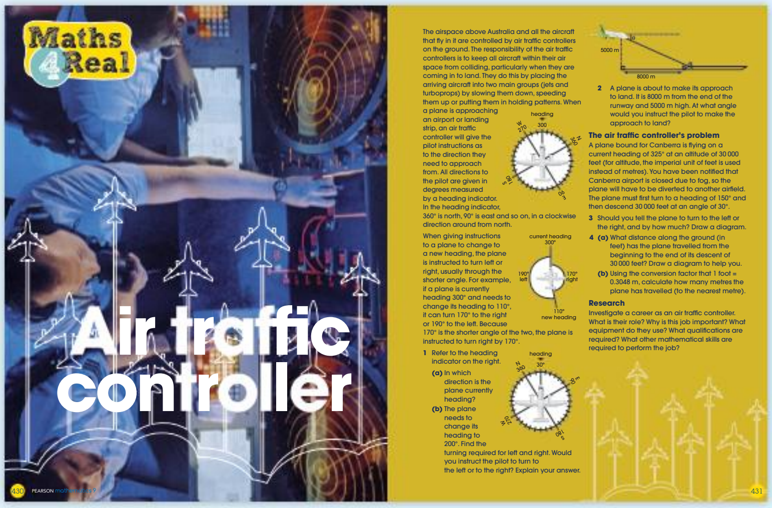
Pearson Mathematics understands the importance the Australian Curriculum places on students developing ICT literacy. Technology Explorations help develop students' deeper understanding of the mathematical concepts covered.

Make a table
Act it out
Look for a pattern
Solve a simpler probl
Draw a picture
Test all possible combinations
Work backwards
Bre

Engaging students

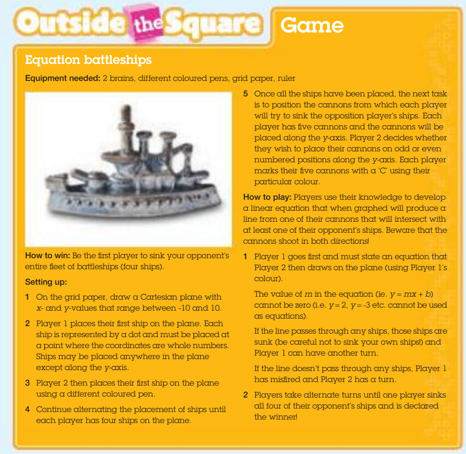
Pearson Mathematics has been designed to capture students' interest, with the incorporation of Mathspaces, Maths 4 Reals, Outside the Squares and Home Pages.

Maths 4 Reals



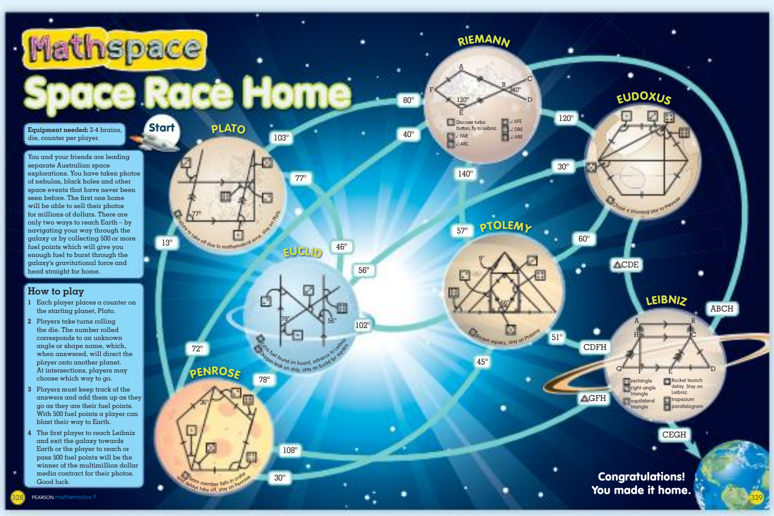
Scenarios that help students make connections with mathematics in the real world.

Outside the Squares



Small activities for early finishers involving problem solving, puzzles and topic-related games.

Mathspaces



Skill consolidation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games, solve the riddle and find the clue style tasks.

Home Pages

Anticipate the 'Why learn this?' question and provide a motivating entry into a new topic, including discussion-provoking Forum questions.

Seek an exception Have I seen a similar problem? Guess and check Draw a picture Make a model Break problem into manageable parts

1


THE BEST JOB IN THE WORLD.
Islands of the Great Barrier Reef, Queensland Australia.
Position Vacant: Island Caretaker
Salary: AUD$150,000 6-month contract
Responsibilities: • Clean the pool • Feed the fish • Collect the mail • Explore and report back
Applications close: 22 February 2009 Interviews: 4 May 2009 Announcement made: 8 May 2009
Work begins: 1 July 2009
ANYONE CAN APPLY.
www.islandreefjob.com
QUEENSLAND

Financial mathematics

1

'The best job in the world!' In January 2009, Tourism Queensland ran a world-wide advertising campaign with this phrase as the headline. They received 34 000 applications!

The position of 'Island Caretaker' was created to promote the islands of the Great Barrier Reef to the world. The list of 'duties' for the successful applicant to undertake included swimming, snorkelling, exploring and keeping an online photo diary and blog of their experiences. Anyone wishing to apply for the position had to make a 60-second video that showcased their skills and creativity. After sifting through the thousands of videos that were submitted, a shortlist of 16 applicants was formed. After the interview process, Englishman Ben Southall was announced as the winner of the 6-month contract worth $150 000. Ben's enthusiastic blogging convinced many that the ad's headline was not an exaggeration!.

Forum

What is your idea of 'The best job in the world'? Give some reasons for your answer. Is it more important to have a well-paying job or to do something you enjoy? If you had to make a 60-second video as a job application, what would you include in your 60 seconds?

Why learn this?

How do you decide whether that discounted pair of jeans really is a bargain? How much money is the store making by selling them to you? How can you work out whether you have been paid the right amount of money for the work you have done? What is tax? How is it calculated? What is interest? How is it paid? What is credit? How does it work? The answers to these questions, and an understanding of the mathematics involved in earning, borrowing and spending money, are essential for us, as consumers, to avoid being ripped off and to live within our means.

After completing this chapter you will be able to:

- perform percentage calculations
- calculate profit, loss, discount, mark up, selling price and cost price
- calculate incomes earned under a variety of payment systems
- understand some of the employment conditions laid out by awards and agreements
- perform simple interest calculations
- describe different methods of payment (such as credit cards), and perform calculations associated with them
- calculate the cost of using resources such as electricity, gas and water
- calculate the cost of using a mobile phone.

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R1.1

1 Convert the following fractions to decimals.

(a) $\frac{17}{100}$ (b) $\frac{57}{1000}$ (c) $\frac{321}{100\,000}$

PP Worksheet R1.2

2 Convert the following fractions to decimals. If the decimal is a recurring decimal, use the appropriate notation.

(a) $\frac{16}{25}$ (b) $\frac{9}{8}$ (c) $\frac{23}{40}$ (d) $\frac{1}{6}$ (e) $\frac{10}{11}$ (f) $\frac{5}{7}$

PP Worksheet R1.3

3 Convert the following decimals to fractions in simplest form.

(a) 0.79 (b) 0.003 (c) 0.014 (d) 1.065 (e) 10.140 (f) 0.0902

PP Worksheet R1.4

4 Evaluate the following.

(a) $5.8 + 2.9$ (b) $12.7 - 3.6$ (c) 5.2×1.5 (d) $6.2 \div 0.03$

PP Worksheet R1.5

5 Convert the following fractions and decimals to percentages. (Round your answers to two decimal places, if necessary.)

(a) $\frac{14}{20}$ (b) 0.91 (c) 0.032 (d) $\frac{23}{28}$

PP Worksheet R1.6

6 Write the following percentages as fractions in simplest form.

(a) 85% (b) 22% (c) 6.7% (d) $\frac{1}{2}$%

PP Worksheet R1.7

7 Write the following percentages in decimal form.

(a) 49% (b) 30% (c) 2.5% (d) 370%

PP Worksheet R1.8

8 Calculate the following:

(a) $\frac{5}{6}$ of 90 m (b) $\frac{3}{4}$ of 105 kg (c) $\frac{11}{10}$ of $25 600

PP Worksheet R1.9

9 Find:

(a) 5% of 20 m (b) 10% of 63.5 km (c) 22% of 18.1 cm

Key Words

award	earnings	net income	salary
buying on terms	EFTPOS	overtime	simple interest
cheque	gross income	PAYG	superannuation
commission	income	piece work	tax
credit card	income tax	principal	tax deduction
debit card	interest	profit	tax return
deferred payment	lay-by	rate of interest	taxable income
discount	loss	retainer	wages

Percentages review

1.1

Good skills in working with percentages, fractions and decimals are very important for the many daily calculations and estimations we make as consumers. It is also important to know the fraction, decimal and percentage equivalents of some common values, such as the ones given below:

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	$0.\dot{3}$	$0.\dot{6}$	0.1	0.2	0.05	0.01
Percentage	50%	25%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%	5%	1%

Finding a percentage of an amount

You will recall that 'per cent' means 'out of 100' or 'for every hundred'. Percentages are simply fractions with a denominator of 100; for example, $40\% = \frac{40}{100} = \frac{2}{5}$.

If you know some common fraction and percentage equivalents and how to find them, you will be able to perform many percentage calculations mentally. This is demonstrated in Method 1 of the Worked Example below. Method 2 involves writing the percentage as a fraction or decimal and multiplying by the amount. This is a good 'pen and paper' method.

Worked Example 1

WE1

Calculate the percentages of the given amounts:

(a) 35% of \$62 **(b)** 27.5% of \$300

Method 1: Use basic percentages

Thinking	Working
(a) 1 Break down the percentage amount into basic percentages that are easy to calculate, such as 25%, 10% or 5%.	(a) $35\% = 30\% + 5\%$ $= 3 \times 10\% + 5\%$
2 Calculate each of these basic percentages. (Here, we find 10% by dividing by 10, and 5% by halving 10%.)	10% of 62 $= 62 \div 10$ $= 6.2$ 5% of 62 $= (10\% \text{ of } 62) \div 2$ $= 6.2 \div 2$ $= 3.1$
3 Multiply and/or add the values of each of the basic percentages to find the required percentage value.	$3 \times 10\% + 5\%$ $= (3 \times 6.2) + 3.1$ $= 18.6 + 3.1$ $= 21.7$
4 State the answer.	35% of \$62 = \$21.70

Thinking	Working
(b) 1 Break down the percentage amount into basic percentages that are easy to calculate.	(b) $27.5\% = 25\% + 2.5\%$
2 Calculate each of these basic percentages. (Here, we find 25% by dividing by 4 and 2.5% by dividing 25% by 10.)	25% of 300 $= 300 \div 4$ $= 300 \div 2 \div 2$ $= 150 \div 2$ $= 75$ $2.5\% = 25\% \div 10$ $= 75 \div 10$ $= 7.5$
3 Multiply and/or add the values of each of the basic percentages to find the required value.	$25\% + 2.5\%$ $= 75 + 7.5$ $= 82.5$
4 State the answer.	27.5% of $\$300 = \$82.50$

Method 2: Fraction or decimal multiplication

Thinking	Working
(a) 1 Use the fact that 'of' is equivalent to '×' to convert the sentence into a mathematical one.	(a) 35% of 62 $= 35\% \times 62$
2 Convert the percentage to a fraction or decimal by dividing by 100.	$= \frac{35}{100} \times 62$ or 0.35×62
3 Perform the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, we divide by common factors of 5 and then 2.)	$\frac{\cancel{35}^{7}}{{}_{20}\cancel{100}} \times \frac{62}{1}$ $= \frac{7}{{}_{10}\cancel{20}} \times \frac{\cancel{62}^{31}}{1}$ $= \frac{7}{10} \times \frac{31}{1}$ $= \frac{217}{10}$ $= 21.7$
4 State the answer.	35% of $\$62 = \$21.70$
(b) 1 Use the fact that 'of' is equivalent to '×' to convert the sentence into a mathematical one.	(b) 27.5% of 300 $= 27.5\% \times 300$
2 Convert the percentage to a fraction or decimal by dividing by 100.	$= \frac{27.5}{100} \times 300$ or 0.275×300
3 Perform the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, we divide by a common factor of 100.)	$= \frac{27.5}{1\cancel{00}} \times 3\cancel{00}$ $= 27.5 \times 3$ $= 82.5$
4 State the answer.	27.5% of $\$300 = \$82.50$

Increasing or decreasing by a given percentage

A percentage can also represent a change in a quantity. We can calculate a percentage increase or decrease in a simple 'one-step' calculation if we use the fact that the original amount represents 100%.

Worked Example 2

WE2

Increase or decrease the following amounts by the given percentages. If necessary, round decimal answers to two decimal places.

(a) Increase \$420 by 65% **(b)** Decrease 87 kg by 13.6%

Thinking	Working
(a) 1 Add the percentage increase to 100%. Write this new percentage as a decimal.	(a) $(100 + 65)\% = 165\%$ $= 1.65$
2 This decimal represents a scale factor. Multiply it by the original amount to find the new amount.	$1.65 \times 420 = 693$
3 State the answer.	\$420 increased by 65% is \$693.
(b) 1 Subtract the percentage decrease from 100%. Write this new percentage as a decimal.	(b) $(100 - 13.6)\% = 86.4\%$ $= 0.864$
2 This decimal represents a scale factor. Multiply it by the original amount to find the new amount.	$0.864 \times 87 = 75.168$
3 State the answer.	87 kg decreased by 13.6% is 75.17 kg.

Writing one amount as a percentage of another

To write one amount as a percentage of another:

1. Make sure both amounts are the same type, or measured in the same units. Convert units if necessary.
2. Write one amount as a fraction of the other.
3. Convert this fraction into a percentage by multiplying by 100%.

Worked Example 3

WE3

Round your answers to the following questions to one decimal place.

(a) Tyler kicked 13 goals from 18 shots at goal. Calculate his kicking accuracy as a percentage.

(b) A packet of 'Chocky' biscuits is on sale for \$2.35, a saving of 57c. Write the saving as a percentage of the original price.

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Write the number as a fraction of the total. Multiply this fraction by 100%.	(a) $\frac{13}{18} \times \frac{100}{1}\%$

	2	Simplify the multiplication by cancelling common factors first.	$= \frac{13}{\cancel{18}_{9}} \times \frac{\cancel{100}^{50}}{1}$ $= \frac{650}{9}$ $= 72\frac{2}{9}\%$ or $72.\dot{2}\%$
	3	Write the answer, rounded to the specified number of decimal places, in a sentence.	Tyler's kicking accuracy is 72.2% (1 d.p.).
(b)	1	Find the original price by adding the saving back on to the sale price.	(b) $\$2.35 + \$0.57 = \$2.92$
	2	Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars). Multiply this fraction by 100%.	$\frac{0.57}{2.92} \times \frac{100}{1}\%$ $= \frac{57}{2.92}\%$ $= 19.520\,54\ldots\%$
	3	Write the answer, rounded to the specified number of decimal places, in a sentence.	The saving is 19.5% of the original price (1 d.p.).

Method 2: Convert to a decimal, then multiply

Thinking			Working
(a)	1	Write the number as a fraction of the total. Convert this fraction to a decimal by performing the division.	(a) $\frac{13}{18} = 13 \div 18 = 0.7222\ldots$
	2	Multiply the decimal by 100%.	$0.7222\ldots \times 100\% = 72.2222\ldots\%$
	3	Write your answer, rounded to the specified number of decimal places, in a sentence.	Tyler's kicking accuracy is 72.2% (1 d.p.).
(b)	1	Find the original price by adding the saving back on to the sale price.	(b) $\$2.35 + \$0.57 = \$2.92$
	2	Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars).	$\frac{0.57}{2.92}$
	3	Convert this fraction to a decimal by performing the division.	$\frac{0.57}{2.92} = 0.57 \div 2.92 = 0.1952\ldots$
	4	Multiply the decimal by 100%.	$0.1952\ldots \times 100\% = 19.52\ldots\%$
	5	Write your answer, rounded to the specified number of decimal places, in a sentence.	The saving is 19.5% of the original price (1 d.p.).

Rounding errors

Most financial applications of percentages use decimals instead of fractions. For practical purposes, we often use the rounded approximation of a recurring decimal in a calculation. For example, $\frac{5}{6}$ expressed as a decimal and rounded to one decimal place is 0.8. If we rounded it to two decimal places, we would use 0.83.

As soon as you use a rounded value, you introduce error into your answer. To minimise this error you should round off as late as possible in a calculation or use fractions for as long as you can before converting to a decimal and rounding off. If you do give a rounded answer you should state its accuracy (e.g. if you have rounded to two decimal places, write 'correct to 2 d.p.').

Fractions and decimals on the calculator

If you are working with decimals on your calculator, keep all the decimal places in the calculator display and only round off your final answer. Scientific calculators have a key that enables you to convert between fractions, decimals and mixed numbers. It may look like this **S⇔D** or this **$a^b/_c$**.

To enter the fraction $\frac{2}{9}$, for example, you would press: [fraction key], then enter 2 and 9, using the cursor [cursor key] to move between numerator and denominator.
To convert to a decimal, press **S⇔D**.
To convert back to a fraction, press **S⇔D** again.

To enter a mixed number such as $3\frac{2}{9}$, press **SHIFT** [fraction key] and use the cursor to enter the numbers. Pressing **S⇔D** will switch between the mixed number and decimal form. Pressing **SHIFT** **S⇔D** will convert the improper fraction back to a mixed number.

1.1 Percentages review

Navigator

Q1 Columns 1 & 2, Q2 (a) & (b), Q3, Q4 (a), Q5, Q6, Q7, Q10, Q15, Q16, Q17	Q1 Columns 2 & 3, Q2 (c) & (d), Q3 (a)–(f), Q4, Q5, Q7, Q8, Q10, Q11, Q12, Q14, Q15, Q16, Q17	Q1 Column 3, Q2 (c) & (d), Q3 (e)–(h), Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17

Answers page 598

Fluency

1 Calculate the percentages of the given amounts. If necessary, round decimal answers to two decimal places.

WE1

(a) 25% of \$40 (b) 45% of \$50 (c) 30% of \$150

(d) 11% of 60 L (e) 66% of 400 m (f) 77% of 35 kg

(g) 9% of 20 kg (h) 59% of 74 kg (i) 88% of 60 L

(j) 0.5% of \$3500 (k) 12.5% of 1500 m (l) 7.2% of 15 L

(m) $5\frac{3}{4}$% of \$17 000 (n) $6\frac{1}{5}$% of \$90 (o) $\frac{2}{3}$% of \$200 000

WE2

2 Increase or decrease the following amounts by the given percentages. If necessary, round decimal answers to two decimal places.

(a) Increase 240 by:

(i) 50% (ii) 35% (iii) 8% (iv) 140%

(b) Decrease 5700 by:

(i) 20% (ii) 75% (iii) 46% (iv) 98%

(c) Increase \$64.15 by:

(i) 10% (ii) 260% (iii) 7.5% (iv) $6\frac{3}{4}$%

(d) Decrease 9.8 seconds by:

(i) 50% (ii) 13% (iii) 6.3% (iv) $3\frac{1}{3}$%

WE3

3 Round your answers to the following questions to one decimal place.

(a) Ash kicked nine goals from 13 shots at goal. Calculate Ash's percentage kicking accuracy.

(b) A jar of 'Nuttie's' peanut butter is on sale for \$3.18, a saving of 64 cents. Write the saving as a percentage of the original price.

(c) Maryam had 22 shots at the netball goals and got 14 in. Calculate Maryam's percentage shooting accuracy.

(d) A box of 'Crunchy-Bix' cereal has been reduced by \$1.02 to \$4.37. Calculate the percentage saving based on the original cost price.

(e) Raf got 76 of 114 serves into the tennis court. Calculate the percentage accuracy of Raf's serve.

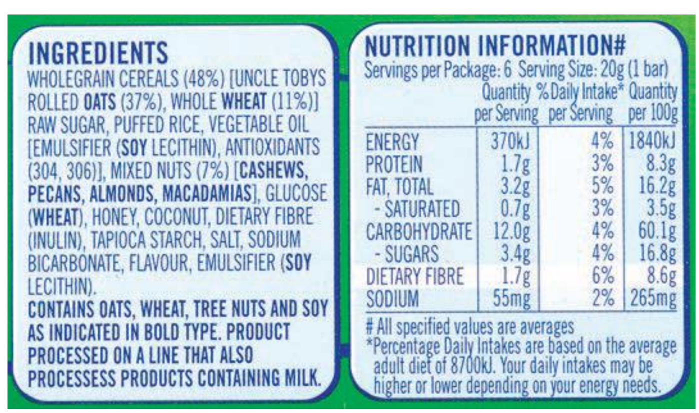
(f) A tin of 'Beanz' baked beans is on sale for \$1.17, a saving of 32 cents. Write the saving as a percentage of the original price.

(g) Brett bowled a cricket ball at a set of stumps 54 times. He hit the stumps 41 times. Calculate Brett's stump-hitting accuracy as a percentage of the number of balls bowled.

(h) A tub of 'Yo-Cal' yoghurt has been reduced by 74 cents to \$2.11. Calculate the percentage saving based on the original cost price.

Understanding

4 This is the nutritional label from a packet of muesli bars. The 'serving size' is one bar, which has a mass of 20 g.



INGREDIENTS

WHOLEGRAIN CEREALS (48%) [UNCLE TOBYS ROLLED **OATS** (37%), WHOLE **WHEAT** (11%)] RAW SUGAR, PUFFED RICE, VEGETABLE OIL [EMULSIFIER (**SOY** LECITHIN), ANTIOXIDANTS (304, 306)], MIXED NUTS (7%) [**CASHEWS, PECANS, ALMONDS, MACADAMIAS**], GLUCOSE (**WHEAT**), HONEY, COCONUT, DIETARY FIBRE (INULIN), TAPIOCA STARCH, SALT, SODIUM BICARBONATE, FLAVOUR, EMULSIFIER (**SOY** LECITHIN).

CONTAINS OATS, WHEAT, TREE NUTS AND SOY AS INDICATED IN BOLD TYPE. PRODUCT PROCESSED ON A LINE THAT ALSO PROCESSESS PRODUCTS CONTAINING MILK.

NUTRITION INFORMATION#

Servings per Package: 6 Serving Size: 20g (1 bar)

	Quantity per Serving	%Daily Intake* per Serving	Quantity per 100g
ENERGY	370kJ	4%	1840kJ
PROTEIN	1.7g	3%	8.3g
FAT, TOTAL	3.2g	5%	16.2g
- SATURATED	0.7g	3%	3.5g
CARBOHYDRATE	12.0g	4%	60.1g
- SUGARS	3.4g	4%	16.8g
DIETARY FIBRE	1.7g	6%	8.6g
SODIUM	55mg	2%	265mg

All specified values are averages
*Percentage Daily Intakes are based on the average adult diet of 8700kJ. Your daily intakes may be higher or lower depending on your energy needs.

(a) Use the information in the 'Quantity per serving' column to calculate the percentage mass of the muesli bar that is:

(i) carbohydrate

(ii) protein

(iii) fat (total).

(b) Sugar is a type of carbohydrate.

(i) What percentage of the mass of a muesli bar is sugar?

(ii) What percentage of the carbohydrate mass is sugar?

Answer to one demical place.

5 The label on a bottle of tropical fruit juice says:

Pineapple juice:	78%
Orange juice:	15.5%
Passionfruit juice:	4.5%
Guava juice:	2%

There is 2.5 L of juice in the bottle. Calculate the volume of each type of juice, rounding answers to the nearest mL if necessary.

6 Approximately 65% of the mass of an adult human is water. Akira weighs 58 kg. How much of Akira's mass is water, to the nearest 100 g?

7 Stuart earns $1023.87 a week working as a plumber. He receives a weekly pay increase of 3.5%.

(a) Calculate Stuart's new weekly wage.

(b) How much more is Stuart now earning per week?

8 Charlotte is a runner whose personal best time for her favourite race is 3 min 34 seconds.

(a) Charlotte wants to decrease her time by 4%. How many seconds is this?

(b) At the state titles, Charlotte lowered her time by 6.9 seconds. What percentage improvement is this, correct to two decimal places?

9 There are two male and seven female gorillas in a zoo enclosure.

(a) What fraction of the gorillas are female?

(b) What percentage of the gorillas are male?

(c) One extra female is added to the enclosure. Recalculate your answers to **(a)** and **(b)**.

10 Copy and complete the following table.

	Common fraction	Decimal fraction	Percentage
(a)	$\frac{1}{2}$		
(b)		0.45	
(c)			72%
(d)	$\frac{5}{4}$		
(e)		0.56	
(f)			12.5%

11 Approximately $\frac{7}{8}$ of an iceberg lies under water. The mass of the iceberg is 150 000 tonnes.

(a) What mass of the iceberg is under water?

(b) What percentage of the whole mass is this?

12 At the end of the first week of trading on the share market, shares in Jaydeep's company had increased by 17.5%. The shares were worth $3.40 each at the beginning of the week.

(a) What is the value of a share now?

(b) At the end of the second week, the share price had decreased by 9% compared with its price at the beginning of that week. Calculate the value of one share at the end of the second week.

(c) Calculate the overall increase or decrease at the end of the 2 weeks as a percentage of the original price of $3.40, correct to two decimal places.

Reasoning

13 In her Drama class, $\frac{3}{5}$ of Anita's overall mark is allocated to the practical exam and the remaining $\frac{2}{5}$ is allocated to the written exam. Anita scored 82% in her practical exam and 76% in the written exam. Calculate Anita's overall percentage for the subject, correct to 1 decimal place.

14 The local newspaper had the following headline: 'Prime Minister's approval rating 67%'. The article stated that the newspaper had surveyed all 180 000 of its readers, of which 14% had responded. Of those who responded, 67% said that they thought the Prime Minister was doing a good job. How many people actually said this?

15 **(a)** **(i)** Increase 100 by 20%

(ii) Now decrease the answer obtained by 20%

(b) **(i)** Increase 300 by 30%

(ii) Now decrease the answer obtained by 30%

(c) Explain why you do not arrive back at the starting number after an increase and decrease by the same percentage.

Open-ended

16 Write three whole numbers that when increased by 140% give an answer between 100 and 200.

17 Trent is looking to buy a new iPod. In one store, the iPod was on sale for $258, an advertised saving of $31. In another store, the same model iPod had a marked price of $299, but a sale sticker next to it said 'take 25% off the marked price'. Trent decided that $31 sounded like a better saving than 25% and bought the iPod from the first store.

(a) Was this the right choice in terms of saving money? Explain why or why not.

(b) Describe a simple calculation Trent could have done while standing in the shop that could have helped him make a more effective decision.

Outside the Square — Problem solving

Age-old dilemmas

1 My mother is three times my age.

My sister is 75% of my age and 15% of my grandmother's age.

My father is 40, 4 years older than my mother.

Find my age, my grandmother's age and my sister's age.

2 Harry, Larry and Garry are three brothers who all go to secondary school.

Harry and Larry are an exact number of years apart. They have their birthdays on the same day. Garry's birthday is on a different day.

Harry is 80% of Larry's age.

Garry is 110% of Larry's age.

How old are each of the brothers?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Buying and selling

1.2

Discounts

A **discount** is a reduction in the marked price of an item. Discounts can be offered for a variety of reasons:

- A store may be overstocked with certain items and might wish to clear them.
- Frequent customers are often given special discounts.
- Reductions can be made for bulk purchases (e.g. 'Buy 9, get the 10th free!') to encourage high sales for a particular item.
- A store might have a loyalty system, where a discount is given for a certain number of purchases.

Worked Example 4

WE4

Calculate the sale price of a $46 shirt discounted by 25%.

Method 1: Find and subtract the discount amount

Thinking	Working
1 Find the dollar value of the percentage discount. (Here, we can find 25% by halving twice, because 25% = $\frac{1}{4}$.)	50% of \$46 = 46 ÷ 2 = 23 25% of \$46 = 23 ÷ 2 = \$11.50
2 Subtract the value of the discount from the marked price to find the sale price.	Sale price = 46 − 11.50 = \$34.50

Method 2: Find the remaining percentage

Thinking	Working
1 Calculate the remaining percentage of the price by subtracting the discount percentage from 100%	100% − 25% = 75%
2 Multiply the remaining percentage by the marked price to find the sale price.	Sale price = 75% of \$46 = 0.75 × 46 = \$34.50

Profit and loss

Retail businesses manufacture products, or purchase them from suppliers. The cost of buying or manufacturing the goods (including things such as the cost of materials and transporting the goods) is called the cost price. In order to make a **profit**, the retailer needs to sell their product to consumers (the people buying the product) at a higher price than the cost price. This is called the selling price. The amount added onto the cost price is called the mark up. If the selling price is less than the cost of the item and associated costs, then a **loss** is made.

Profit or loss can be expressed as a percentage of the cost price or selling price.

$$\% \text{ Profit} = \frac{\text{profit in \$}}{CP} \times 100\% \qquad \text{or} \qquad \frac{\text{profit in \$}}{SP} \times 100\%$$

where CP = cost price, SP = selling price

To find the % Loss, substitute 'Loss in $' into the numerators of the above formulas.

Worked Example 5

WE 5

A home theatre system costs an electrical store $510. It sells the system for $965. Calculate, correct to one decimal place, the percentage profit made as a percentage of:

(a) the cost price **(b)** the selling price.

Thinking	Working
(a) **1** Calculate the dollar value of the profit.	(a) Profit = $SP - CP$ = \$(965 − 510) = \$455
2 To calculate percentage profit, write the profit as a fraction of the cost price and multiply by 100%.	$\% \text{ Profit} = \frac{\text{profit}}{CP} \times 100\%$ $= \frac{455}{510} \times 100\%$ = 89.2% (1 d.p.)
(b) **1** Calculate the dollar value of the profit.	(b) Profit = \$455
2 To calculate percentage profit, write the profit as a fraction of the selling price and multiply by 100%.	$\% \text{ Profit} = \frac{\text{profit}}{SP} \times 100\%$ $= \frac{455}{965} \times 100\%$ = 47.2% (1 d.p.)

Assuming a profit is made, the percentage profit based on the cost price will always be greater than the percentage profit based on the selling price. Can you explain why?

Profit and loss can also be applied to investments. People purchase land, property or goods (such as antiques or memorabilia) in the hope that they will increase in value. If the value of the investment is more than the cost of purchasing and maintaining it, then a profit is made. Buying shares is a way of investing in a business. If the business is successful, investors receive some of the profits (in proportion to the number of shares they own). The value of their shares also increases, which means they can also be sold for profit.

Mark up

The 'mark up' on a product is the amount a business has added to the cost of the product, before placing it out for sale. It can vary from business to business and product to product. A retailer may feel that they can sell more of a particular item at a lower mark up. There may be a company policy that requires that certain products make a certain percentage profit and so a particular mark up must be added. For example, a furniture store may require a 65% profit on all sofas sold.

We use the mark up to find the selling price from the cost price. The cost price represents 100%. If we want a profit of 65%, the selling price must be 100% + 65%, or 165% of the cost price. To find 165% of the cost price, we multiply by 1.65.

Worked Example 6

WE6

A furniture store has a policy of selling their chairs and sofas with a mark up on the cost price of 65%. What would a sofa that cost the store $300 be sold for?

Thinking	Working
1 Identify the mark up and add it onto 100%. Express as a decimal.	$65\% + 100\% = 165\%$ $= 1.65$
2 Multiply by the cost price to find the selling price.	Selling price $= 1.65 \times \$300$ $= \$495$

Calculating the original price

As customers in a store we see only the final selling price of goods after the mark up has been added. If we knew the value of the mark up, we could calculate the cost price of the goods by simply reversing the process used to find the selling price. In the above Worked Example, we *multiplied the cost price* of a sofa of $300 by 1.65 to find the selling price of $495. If we *divide the selling price* by 1.65, we arrive back at the cost price of $300.

$$\text{Cost Price } (CP) \underset{\div(100\% + \text{mark up})}{\overset{\times(100\% + \text{mark up})}{\rightleftarrows}} \text{Selling Price } (SP)$$

Worked Example 7

WE7

A jewellery store has a policy of marking up the cost price of their goods by 75%. Calculate the cost price of a necklace that was sold for $56.

Thinking	Working
1 Identify the mark up and add it to 100%. Express as a decimal.	$100\% + 75\% = 175\%$ $= 1.75$
2 Summarise the problem by writing an equation.	175% of $CP = \$56$
3 Replace the percentage with its decimal value, and 'of' with a multiplication sign.	$1.75 \times CP = \$56$
4 Solve the equation by dividing both sides by the decimal value of the percentage.	$CP = \$56 \div 1.75$ $= \$32$

We can also use this method to determine the original marked price of discounted items. We subtract the discount percentage from 100% and convert this value to a decimal. We then divide the discounted sale price (*SP*) to find the original marked price (*MP*).

For discounts:

$$\text{Marked Price } (MP) \underset{\div(100\% - \text{discount})}{\overset{\times(100\% - \text{discount})}{\rightleftarrows}} \text{Sale Price } (SP)$$

Worked Example 8 WE8

A smartphone has a sale price of $539 after a discount of 22.5% is applied. What was the original marked price of the smartphone? Answer to the nearest dollar.

Thinking	Working
1 Identify the discount and subtract it from 100%. Express as a decimal.	$100\% - 22.5\% = 77.5\%$ $= 0.775$
2 Summarise the question by writing an equation.	77.5% of $MP = \$539$
3 Replace the percentage with its decimal value, and 'of' with a multiplication sign.	$0.775 \times MP = \$539$
4 Solve the equation by dividing both sides by the decimal value of the percentage.	$MP = \$539 \div 0.775$ $= \$695$ (nearest \$)

1.2 Buying and selling

Navigator

Answers page 598

Q1, Q2, Q3, Q4, Q5, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q19, Q22	Q1 (c) & (d), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14, Q15, Q16, Q18, Q19, Q20, Q22, Q23 (a)	Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23

Fluency

WE4

1 Calculate the sale price of the following items:

(a) a $24 pair of shorts discounted by 15%

(b) a $95 pair of shoes discounted by 25%

(c) a $63.99 skirt discounted by 40%

(d) a $419.95 suit discounted by 37.5%.

WE5

2 (a) A camera costs a camera shop $360. The shop sells the camera for $504. Calculate, correct to one decimal place, the profit as a percentage of:

(i) the cost price (ii) the selling price.

(b) An old car costs a used car saleman $4200. He sells the car for $4050. Calculate, correct to one decimal place, the loss as a percentage of:

(i) the cost price (ii) the selling price.

WE6

3 A clothing store has a policy of selling jeans and T-shirts with a mark up on the cost price of 92%. Calculate the selling price of the following items, rounding your answer to the nearest 5 cents:

(a) a pair of jeans, cost price $39.30 (b) a T-shirt, cost price $8.75.

WE7

4 A hardware store has a policy of marking up the cost price of their goods by 85%. Calculate the cost price of the following items, rounding your answer to the nearest cent:

(a) a packet of nails, selling price $3.70 (b) a cordless drill, selling price $51.00.

5 (a) A tennis racquet has a sale price of $149 after a discount of 15% has been applied. What was the original marked price of the racquet? Answer to the nearest dollar.

WE8

(b) A plasma TV has a sale price of $2497 after a discount of 7.5% has been applied. What was the original marked price of the TV? Answer to the nearest dollar.

6 If a table cost $486 and was subsequently sold for $350, the percentage profit or loss, based on the cost price, was:

A $\frac{(486-350)}{486} \times 100\%$ loss

B $\frac{(486-350)}{350} \times 100\%$ loss

C $\frac{(486-350)}{350} \times 100\%$ profit

D $\frac{(486-350)}{486} \times 100\%$ profit

7 If a laptop is sold for $960 at a 114% profit, the cost of the laptop was:

A $\$960 \times 2.14 \times 960$

B $\$960 \times 2.14$

C $\$960 \times 1.14$

D $\frac{\$960}{2.14}$

8 If a desk is for sale for $140 after a 25% discount, it means that it was originally marked at:

A $105 B $175 C $186.67 D $245

Understanding

9 Calculate the selling price for the following cost prices and percentage profits (on cost price).

(a) $CP = \$48$, percentage profit = 45%

(b) $CP = \$60$, percentage profit = 60%

10 A builder buys $840 worth of timber to build an extension. If he is allowed a 12.5% discount, how much does he pay?

11 A hairdresser gets a discount of 24% on hair products bought in bulk. How much is saved on $370 worth of products?

12 A store purchases a bulk order of 120 watches for $960.

(a) Calculate the profit made if the watches are sold individually for $19.95 each.

(b) Calculate the total profit as a percentage of the cost price.

13 A store purchased a bulk order of pyjamas cheaply for a sale. If a customer paid $11 for them at the sale, what were they marked at prior to the sale if they were reduced by 30%?

14 In Australia, a Goods and Services Tax (GST) of 10% is added to the price of all products and services purchased by consumers (with the exception of some basic grocery items). GST is added after the product has been marked up by the retailer. Mei is a clothing store owner who has just purchased a large number of T-shirts for $6.49 each. She adds a mark up of 180%, then the 10% GST.

(a) What is the final selling price of the T-shirts? (Answer to the nearest cent.)

(b) What amount of the final price is GST?

15 Most of the time the marked prices we see on goods in shops have the 10% GST already added to the price. Find the original, pre-GST, price of:

(a) a DVD with a marked price of $29.90

(b) a vacuum cleaner with a marked price of $349.

16 A mechanic buys a car at an auction for $3120. The car is then repaired at a cost of $430. It is resprayed and has its panels straightened for $840. The car is then advertised and sold for $9800. Calculate the percentage profit on the selling price if the cost of advertising was $80.

17 Louise bought shares in the following companies through a share broker (a person who buys and sells shares on behalf of others). The share broker charges a fee of 2% of the value of the shares bought and sold.

150 shares in Oro Mining, bought for $39.05 each
200 shares in Beacon Bank, bought for $19.86 each
250 shares in JeansEast, bought for $6.14 each

(a) Calculate the share broker's fee.

(b) Including the fee, how much did Louise pay for her shares in total?

Louise instructs her share broker to sell her shares once they reach the following prices:

Oro Mining	$42
Beacon Bank	$21.50
JeansEast	$8

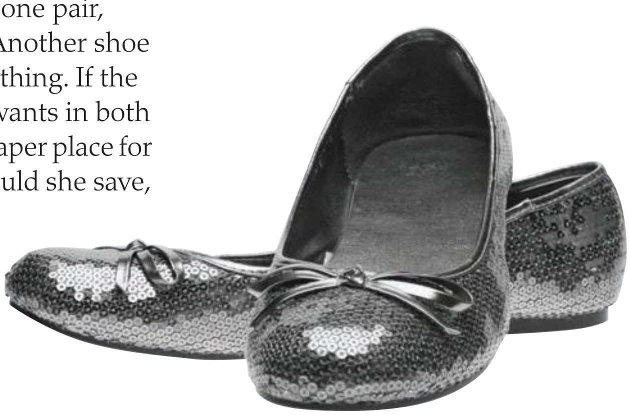
Her share broker does so and subtracts his fee.

(c) Calculate Louise's profit from the sale of the shares.

(d) Calculate Louise's profit as a percentage of the cost of her shares, including her share broker's fee. State your answer correct to two decimal places.

Reasoning

18 Evie's favourite shoe shop has a 'buy one pair, get the second pair half-price' offer. Another shoe shop nearby is offering 20% off everything. If the regular retail price of the shoes Evie wants in both stores is $49.95, which store is the cheaper place for Evie to buy two pairs? How much would she save, compared with the other store?



19 A furniture store purchases a dining table and chairs for \$750. It marks up this price by 45% to obtain its selling price.

(a) What is the selling price of the dining set? State your answer correct to the nearest dollar.

(b) The furniture is not selling, so the selling price is discounted by 45%. Calculate the sale price to the nearest dollar.

(c) The mark up on the cost price was 45%. Why doesn't a 45% discount on the selling price return the price to the original cost price?

(d) What percentage saving will they be able to advertise if they take the price back to the cost price? State your answer correct to the nearest whole number.

20 A stallholder at a market buys 20 kg of peanuts for \$65, 5 kg of hazelnuts for \$30 and 1 kg of cashews for \$13. He mixes them together and puts them into plastic bags each containing 200 g of mixed nuts and sells each bag for \$1.00. If the plastic bags cost 2 cents each, what is the percentage profit on the selling price of the nuts?

21 If you wanted to be able to advertise a discount of 30% on goods that cost you \$5000 and still make a profit of 10%, at what price would you initially advertise them? Give your answer to the nearest dollar.

Open-ended

22 (a) An outdoor furniture setting has a marked price of \$999. State three different percentage discounts, correct to one decimal place, that would make the selling price between \$750 and \$800. Show your calculations.

(b) A lounge suite is sold for \$2450. State three different percentage discounts, correct to one decimal place, that could have been applied to the lounge suite if you know the marked price was somewhere between \$3000 and \$3200. Also state the marked price for your discount. Show the calculations that allow you to state your answer.

23 Using an example, explain why taking an additional 20% off an item already discounted by 30% is not the same as one single overall discount of 50%.

Outside the Square — Problem solving

Patrick's prized painting

Patrick is an art lover who bought a painting for \$6000. A year later, Patrick has some financial difficulties and has to sell his painting for \$7000. As soon as he is able to, he buys the painting back again. Unfortunately he has to pay \$8000 to get it back. A year later Patrick moves house. He no longer has room for the painting, so he sells it for \$9000.

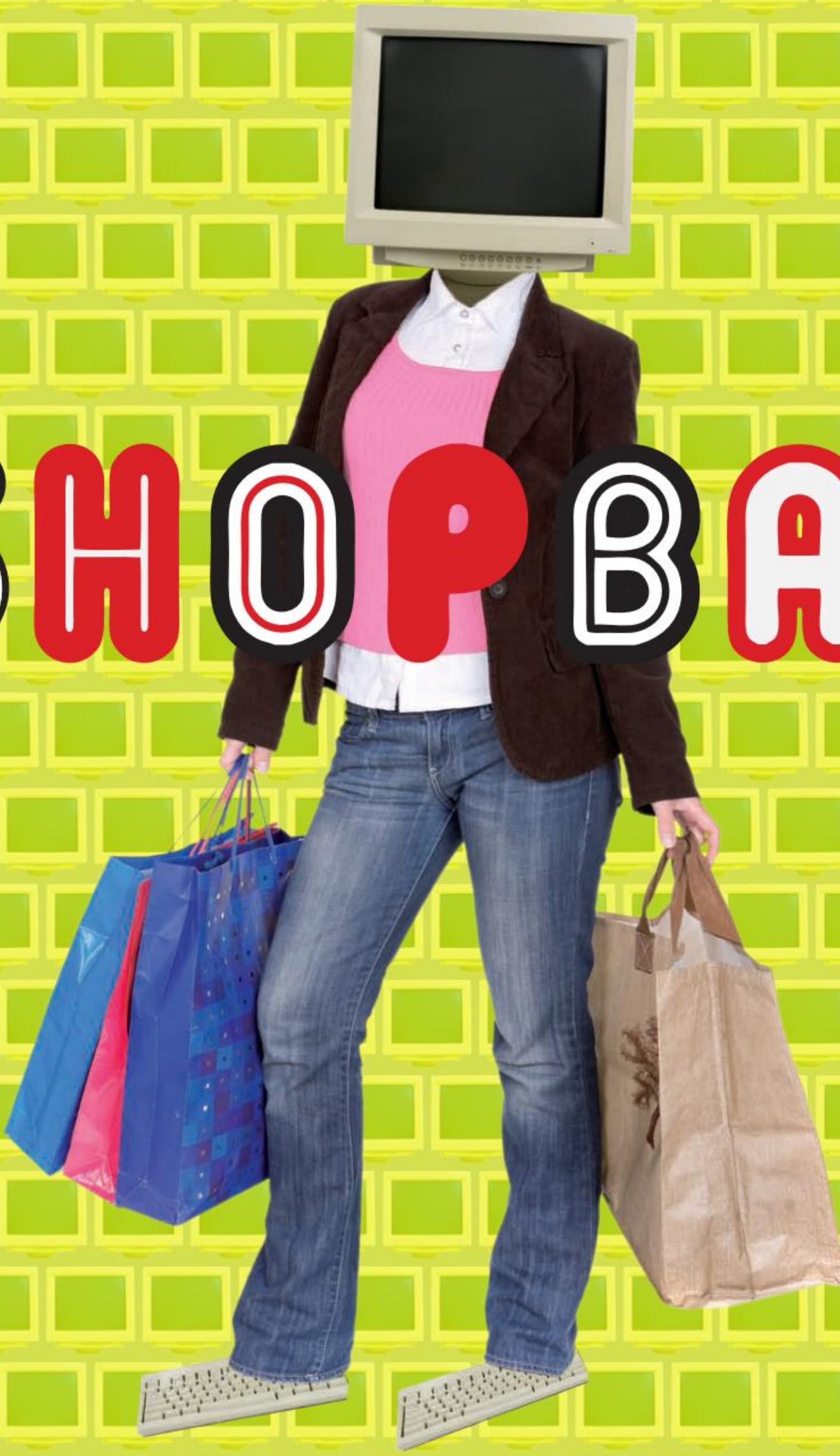
How much profit or loss does Patrick make?

Strategy options

- Act it out.
- Draw a diagram.
- Break problem down into manageable parts.

Mathspace

SHOPBAY



Greatest gain

'ShopBay' is an online auction site where people can place goods to sell. The seller has two options when listing their item for sale:

- put a 'starting bid' price and allow people to place higher bids for the item before a specified 'sale time' (say, 9 p.m. Thursday) or
- select a 'buy-it-now' option with a fixed price.

Penny decided to sell some things online on 'ShopBay'.

1 Which option, A or B, gives her more profit for each of the following items?

Item	Option A	Option B
Yoga mat	Starting price \$10. The price increases 50% by the sale time	Sell for a buy-it-now price of \$17
Guitar	Starting price \$200. The price increases 70% by the sale time	Sell for a buy-it-now price of \$332
Blue wig	Starting price \$24. The price increases 84% by the sale time	Sell for a buy-it-now price of \$45
Shoes	Starting price \$60. The price increases 20% by the sale time	Sell for a buy-it-now price of \$70
Antique teapot	Starting price \$330. The price increases 44% by the sale time	Sell for a buy-it-now price of \$450
iPod	Starting price \$50. The price increases 200% by the sale time	Sell for a buy-it-now price of \$150

True or false?

2 Which of the following statements are true and which are false?

(a) 'An easy way to calculate 30% is to calculate 10% and multiply it by 3.'

(b) 'To find 100% of something you multiply by 100.'

(c) 'An easy way to calculate 25% is to halve and then halve again.'

(d) 'To find 50% of something you multiply by $\frac{1}{50}$.'

(e) 'To find 150% you multiply by 2.5.'

(f) 'To find 1% you find 10% and divide by 5.'

(g) '20% is the same as a $\frac{1}{5}$.'

(h) 'To reduce something by 30% you multiply by 0.7.'

(i) 'To find 30% of something you divide by 3.'

3 For the statements that were false, rewrite them so they are correct.

Payment maze

At the end of your shopping on ShopBay the total cost of the items in your online shopping cart is \$150.

There are a few different discounts and costs that must be added.

Find a way through the Payment Maze so that you pay the least amount at the checkout at the end.

1.3 Earning an income

The payment, or **income**, we receive for working in a particular job can be calculated in several different ways depending on the type of work done and the agreed working conditions.

Awards

An **award** is a document that sets out the minimum conditions of employment for employees in a certain occupation or industry. For example, there are awards for architects and airline pilots, or for people working in the fast food industry or hair and beauty industry.

Important information contained in Awards includes:

- descriptions of the different job classifications covered under the award
- the basic hourly, weekly or annual rate of pay for each of the different job classifications
- standard hours of work—a full time job is usually around 38 hours per week
- extra amounts paid for extra or unusual hours worked (often called 'loading' or 'penalty rates')
- recreation leave—at least 4 weeks a year of paid recreation leave (i.e. time off work for a holiday) for full-time employees
- other types of paid and unpaid leave, such as sick leave, carer's leave or parental leave
- other working conditions, such as rest and meal breaks, allowances, training and dispute resolution.

Wage requirements for employers

Employees must be paid at least the minimum rate in the award for their industry—it can be more, but it can't be less. Many awards have special conditions for junior employees (those under 21 years of age).

Agreements and contracts

An agreement is like an award created specifically for the employees of a particular company. An agreement must be at least equal to the corresponding award.

Some people, often in a specialised or managerial position, sign an individual, negotiated contract before beginning their employment.

Wages

Wages are paid to people in many different job areas, including retail (shop assistants, packers), hospitality (waiting staff, kitchen staff) and trades (electricians, plumbers, hairdressers). Wages are based on a standard hourly rate of pay that applies to an agreed number of 'ordinary' hours, worked between set times. Extra work, or work outside of ordinary hours (such as evenings, weekends or public holidays), is paid as **overtime** at a higher hourly rate.

Worked Example 9

WE 9

Sim is employed at a photographic studio. Sim's job is covered by an award that states that he must be paid a standard hourly rate of $15.80 for a standard 38-hour Monday to Friday week. On Saturday he is paid time-and-a-half (1.5 times the standard hourly rate) for the first 3 hours and double time (twice the standard rate) for any additional hours thereafter. One particular week, Sim worked 4.5 hours on Saturday in addition to his standard hours. How much money did he earn?

Thinking	Working
1 Determine the number of hours worked at each of the rates. (The first 3 hours of a Saturday are paid time-and-a-half so the remaining 1.5 hours must be paid double time.)	Standard hours: 38 Time-and-a-half: 3 hours Double time: 1.5 hours
2 Multiply each number of hours by the appropriate hourly rate. (Multiply the standard rate by 1.5 for time-and-a-half and by 2 for double time.)	Standard hours: $38 \times 15.8 = \$600.40$ Time-and-a-half: $3 \times 15.8 \times 1.5 = \$71.10$ Double time: $1.5 \times 15.8 \times 2 = \$47.40$
3 Add up the individual amounts to find the total wage.	Total $= \$718.90$

Salary

People in professions, such as teachers, lawyers, engineers, scientists and accountants, are usually paid a **salary**—a set amount paid every fortnight or month. Other people who may also be on salaries are usually permanent employees of a company. It is rare for people on a salary to receive overtime payments for extra hours worked; occasionally, they may receive some time off instead.

Commission

A **commission** is usually paid to salespeople, such as real estate agents, car salespeople, or the sales representatives of manufacturing companies. The commission paid is either a percentage of the value of the sales achieved by the salesperson or a set amount for every product sold. Salespeople are also paid a **retainer** (a regular amount) per week or fortnight, to which any commission is added.

Worked Example 10

WE 10

Lina works as a real estate agent. She receives a retainer of $450 a fortnight, plus a commission of 2.3% of the value of the property she sells. How much does Lina earn in a fortnight in which she sells a house for $340 000?

Thinking	Working
1 Let C be the value of the commission. Find C by calculating the stated percentage of the sale amount.	$C = 2.3\%$ of $\$340\,000$ $= 0.023 \times 340\,000$ $= \$7820$
2 Add the commission to the retainer to calculate Lina's total income.	Total income $= 7820 + 450$ $= \$8270$

Piece work

Piece work applies to workers in some manufacturing industries, such as the clothing, footwear and textiles industry, and other jobs, such as fruit picking and sheep shearing. Workers are paid per item produced (e.g. for each pair of jeans made or each bucket of grapes picked) rather than for the number of hours worked.

Worked Example 11

WE 11

Emma is employed sewing dresses. She is paid at the rate of $7.50 per dress and she can complete 42 dresses each week. Calculate her weekly income.

Thinking	Working
Multiply the number of items produced (42) by the rate ($7.50).	Income $= 42 \times 7.5$ $= \$315$

Types of income

Wages:	Hourly rate of pay. Extra work may be paid as overtime at penalty rates (time-and-a-half, double time).
Salary:	Weekly, monthly or annual rate of pay. Overtime not usually paid; time off may be given instead.
Commission:	Retainer or flat fee plus percentage of the value of sales made, or a certain amount for every sale.
Piece work:	Payment per item produced.

1.3 Earning an income

Navigator

Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q15, Q19	Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q15, Q17, Q18, Q19	Q1, Q2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q18, Q19

Answers page 599

Fluency

1 Jake works in a clothing store. Under his award he is paid \$14.70 per hour for a standard 38 hour week, time-and-a-half for the first 2 hours on Saturday and double time for any hours after that. How much will he receive if he works a standard week, and then 5 hours on Saturday? WE9

2 Brian works as a secondhand car salesperson. He is paid a retainer of \$180 per week plus a commission of 1.5% of the value of his sales. In a week in which he sells a Commodore for \$28 000 and a Cruze for \$13 000, how much will Brian be paid? WE10

3 A semitrailer driver is paid \$12.75 per tonne of timber delivered to a wood-chipping mill. If he delivers 120 tonnes to the mill, how much will he be paid? WE11

4 A casual worker earns \$10.75 per hour stacking shelves in a supermarket. If she works 18.5 hours in 5 days she will be paid:

A 18.5×10.75 **B** $18.5 \times 10.75 \times 5$ **C** $18.5 \times \frac{10.75}{5}$ **D** 18.5×5

5 If a retail assistant, paid at the rate of \$9.80 per hour, works 7 hours standard plus 3 hours at time-and-a-half and 1 hour double time, the wages for that week will be:

A $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 19.6)$ **B** $\$(7 \times 9.8 + 3 \times 14.7 + 1 \times 19.6)$

C $\$(7 \times 9.8 + 3 \times 9.8 + 1 \times 9.8)$ **D** $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 4.9)$

6 A salesperson is paid a \$130 retainer per week plus 2.5% of sales. If \$20 000 worth of goods are sold during one week, the income will be:

A $\$2.5 \times 20\,000$ **B** $\$130 + (2.5 \times 20\,000)$

C $\$130 + 0.025 \times 20\,000$ **D** $\$0.025 \times 20\,000$

7 The following time sheet summarises the hours worked by four employees. Calculate the total income of each employee for the week if the standard rate is $16.40 per hour.

	Standard hours	Time-and-a-half	Double time
Adam	35	2	0
Britta	25	4	4
Con	30	0	4
Deng	20	2	2

Understanding

8 A real estate agent is paid commission on the sale of houses at the following rate:

2% on the first $150 000
1.5% on the next $100 000
1% on the rest

How much would the seller have to pay in commission if a house was sold for:

(a) $280 000 **(b)** $340 000 **(c)** $495 000 **(d)** $600 000?

9 Mike is a shearer who earns up to $450 a day. If he is paid $3.90 for every sheep he shears, what is the maximum number of sheep he is likely to shear in one day?

10 Aleesha is 18 and works part time at a fast food outlet. The adult weekly wage for her position is $656 for a standard week consisting of 38 hours. Aleesha receives 80% of the adult hourly rate.

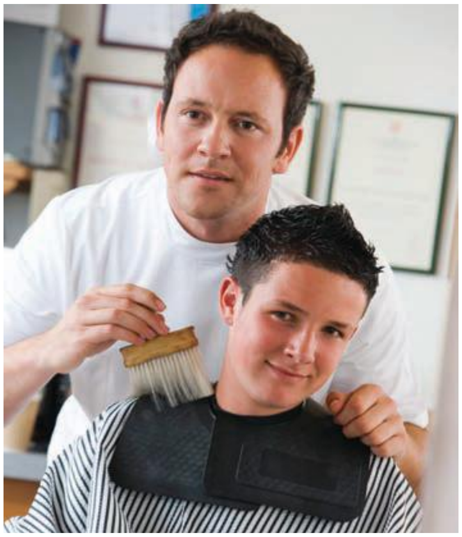
(a) Calculate Aleesha's hourly rate.

Aleesha works from 4 p.m. to 8 p.m., Wednesday to Sunday. Her award states that after 6 p.m. on weekdays, she must be paid an extra 10% of her hourly rate. An extra 25% is paid for all hours worked on Saturday, and an extra 75% for all hours worked on a Sunday.

(b) Calculate Aleesha's weekly wage.

Hint
Calculate the hourly rate of a full-time person first.

11 Marco works as a part-time hairdresser. His award states that a full-time person in his position must be paid a minimum wage of $650 for an ordinary 38-hour week, plus an extra 33% of the hourly rate for any hours worked on a Saturday. Calculate Marco's weekly wage if he works 8 hours a day for 4 weekdays and 5 hours on Saturday.



12 Hayley is a telemarketer with an insurance company. She is paid a retainer of $275 a week, plus $50 commission for every customer she signs up on a new insurance policy. One week, Hayley sells five $149 policies and three $179 policies.

(a) Calculate Hayley's earnings for the week.

(b) Calculate Hayley's commission as a percentage of the value of the products she sold. Give your answer to one decimal place.

13 Erica sells cars and receives a weekly retainer of $150 plus 0.75% of the value of the cars she sells. In a particular week Erica earns $1275. What was the value of the cars Erica sold that week?

14 Patrick is paid $28 per hour for the first 38 hours worked in a week. He is then paid time-and-a-half for the next 6 hours worked and double time for anything beyond that. How many hours did Patrick work in a week in which he earned:

(a) $1190 (b) $1372 (c) $1652?

(d) How many hours overtime would need to be worked so that his total overtime payments were at least as much as his standard hours payment?

Reasoning

15 Another difference between earning wages and earning a salary is that some types of payments are additional to wage earnings, but are already incorporated into a salary.

(a) Hong gets paid a basic weekly wage of $450. Her leave loading, added onto her pay for holidays, is 17.5% of her basic weekly wage.

(i) Calculate her leave loading per holiday week.

(ii) What pay will she receive during a 2-week holiday period?

(b) Barry's income is $1250 per fortnight. If he receives leave loading of 17.5%, what pay will he receive for a 4-week holiday period?

16 When employed as a salesman, Nabil was given a choice as to how he would be paid.
Option A: A retainer of $250 per week plus 2.5% of his sales.
Option B: No retainer but 3.5% of his sales.

(a) Nabil needs to earn at least $1250 per week. What value of sales does he need to make under each payment option to meet this requirement? Give answers correct to the nearest dollar.

(b) What value of sales will give Nabil the same pay regardless of the option chosen?

(c) So, when would it be better for Nabil to choose Option A?

17 Lucy is the manager of a toy store. She earns a salary of $48 700, which is paid fortnightly.

(a) If there are 26 fortnights in a year, calculate Lucy's fortnightly pay to the nearest cent.

(b) If Lucy works a 40-hour week, what is her equivalent hourly rate?

(c) During the mid-year stocktake, Lucy worked an extra 2 hours each weekday. She was not paid overtime. What was Lucy's equivalent hourly rate for that week?

Open-ended

18 Malik is an 18-year-old first-year apprentice electrician. His industry award states that first-year apprentices should be paid 40% of the adult hourly rate of $16.78, rising to 52% in the second year, 70% in the third year and 82% in the fourth and final year. Malik works 8 hours a day, 4 days a week. He goes to TAFE on the fifth day (he isn't paid for this). He also gets a tool allowance in his pay of $17 a week. Malik is wondering if his apprenticeship is worth it. He thinks that he could earn more working the same number of hours in a fast food outlet. The award for that industry states that 18-year-old employees earn 70% of the minimum weekly adult wage of $600 for a 38 hour week, rising by 10% every year until they turn 21 and are on the full adult rate.

(a) Do some calculations to decide whether Malik would earn more at a fast food outlet than on his apprenticeship, either now or in the future.

(b) What would you advise Malik to do? (Some additional information to consider: fully qualified electricians can earn between $40 000 and $80 000 a year.)

19 Tegan has a casual position in a local supermarket. As a casual, Tegan is not entitled to leave and other benefits, but she is paid an extra 25% of the regular hourly rate of $15.80 an hour. She gets a further 10% per hour on Saturdays and Sundays.

Tegan has weekly living expenses (food, rent, transport etc.) of $300.

Give two or three different combinations of weekday and weekend hours that Tegan could work in order to earn $300 a week.

Outside the Square — Problem solving

Careless Carly

Carly is given a savings account by her parents with some money already deposited into it. Carly is trying to avoid withdrawing money from the account but is not very good at sticking to her plan, withdrawing $\$m$ every week. Soon she realises she has already withdrawn half the available money from the account. The next week Carly withdraws another $\$m$ and realises the ratio of withdrawn money to money remaining is now 3 : 2.

What is the ratio of withdrawn money to money remaining when Carly withdraws another $\$m$?

Strategy options

- Guess, check and improve.
- Test all possible combinations.

Investigation

Working out the roster

Equipment required: 1–2 brains, 1A Timetable template

PP Blackline Master

Marie is the supervisor of the toy department in a large store. She has to organise the weekly roster for her department. The store's opening hours are:

Mon–Thurs: 9 a.m. – 6 p.m.
Fri: 9 a.m. – 9 p.m.
Sat: 9 a.m. – 5 p.m.
Sun: 10 a.m. – 4 p.m.

Employees must be at work half an hour either side of the opening and closing times, to set up and pack up the store.

Marie is the only full-time employee—she works regular hours of 8.30 a.m.–5 p.m., Mon–Fri. Marie's job classification under the Retail Employees Award is Level 4.

Marie has one permanent part-time member on her team—Nicki, a Level 3 employee who works the regular hours of 8.30 a.m.–12.30 p.m., Tues–Sat.

Marie has four casual staff on her team. They are:

Name	Age	Job classification	Availability
Jian	20	Level 2 casual	Any time Tues–Thurs, Sat and Sun
Hamish	19	Level 2 casual	After 12 p.m. on Mon, Wed and Fri, all day Sat
Sally	18	Level 1 casual	Any time Mon and Tues, and a maximum of 8 hours on the weekend
David	17	Level 1 casual	After 4 p.m. weekdays, after 12 p.m. on weekends

When Marie writes her roster, she needs to take into account the following conditions:

- two staff must be rostered on at all times, one of whom must be a supervisor
- if Marie is not working, the other supervisors are Nicki, Jian or Hamish
- no employee can be asked to work more than 9 hours at a time
- the minimum number of hours an employee must be paid for is 3 (even if they work less than this)
- neither Marie nor Nicki wish to work overtime (i.e. outside their regular hours)
- the retail award gives the minimum weekly wages for each of the levels of staff as follows:

Classification	Per week $
Retail Employee Level 1	600.00
Retail Employee Level 2	615.00
Retail Employee Level 3	625.00
Retail Employee Level 4	637.60

- staff under 21 are paid a percentage of the adult rate according to the following table:

Age	% of adult weekly rate of pay
17	60
18	70
19	80
20	90

The Big Question

Marie needs to design a roster that takes into account her staff's availability and the above conditions, and costs the company as little as possible in wages. What could Marie's roster for the week look like?

Engage

1 Use the tables above to calculate the standard hourly rate for Marie, Nicki, Jian, Hamish, Sally and David.

Explore

2 **(a)** Use the timetable template to allocate hours to each staff member.

(b) Once you have all the hours covered, see if you can shuffle or juggle the hours of each staff member to bring down the total wages that the store will need to pay.

Strategy options

- Draw up a table.
- Test all possible combinations.
- Guess and check.
- Break problem down into manageable parts.

Explain

3 (a) Does your roster take everyone's availability into account?

(b) Were any days harder to staff than others?

Elaborate

4 Is your roster the 'cheapest possible' roster? How confident are you of your answer?

5 Which staff member was most useful in 'plugging gaps' in the roster?

6 Which staff member was most difficult to place in the roster?

Evaluate

7 Summarise the main steps or stages you went through in completing your roster. Was there a particular order in which you placed your staff into the roster?

8 Which of Marie's list of conditions did you have the most difficulty remembering or ensuring it was met?

9 Why do you think there is a condition about the minimum number of hours for which a person can be paid?

Extend

10 Use the award information on minimum wages and junior rates to calculate the actual wages of each staff member and the total wage bill for the store for the week of your roster.

Tax and other pay deductions

1.4

What is tax?

Tax is money collected by the government from Australian residents. It is used to fund public services, such as hospitals, schools, the police and defence forces and transport (road and train networks).

Income tax

The Federal Government collects **income tax** by having employers withhold a certain amount of income from each employee every time they are paid (usually fortnightly or monthly). This is known as the Pay As You Go (**PAYG**) system. The amount of tax withheld depends on your income. The table below shows how income tax was calculated in the financial year 2011–12. The incomes given are annual incomes. Each row on the table shows the tax payable on incomes within a certain range. The ranges shown on each row of the table are often called 'tax brackets'.

Taxable income	Tax on this income
\$0–\$6000	Nil
\$6 001–\$37 000	15c for each \$1 over \$6000
\$37 001–\$80 000	\$4650 plus 30c for each \$1 over \$37 000
\$80 001–\$180 000	\$17 550 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 550 plus 45c for each \$1 over \$180 000

We can see from the table that no tax is paid if the annual income is \$6000 or less. This is known as the tax-free threshold. Before tax, your income is referred to as **gross income**. The amount remaining after tax and other deductions are paid is known as **net income** (sometimes called 'take home pay').

Superannuation

Superannuation (also known as 'super') is a way of saving money for retirement. Employers are required to regularly pay a certain percentage of their employee's **earnings** into an account or fund. This payment is made in addition to the employee's salary. The money can then be accessed when a person retires, (provided they are over a certain age), or in an extreme situation such as death, illness or disability. Employees can also make voluntary contributions to their super in addition to those made by their employer.

Other pay deductions

Apart from income tax, other deductions from an employee's pay must be agreed to in writing by the employee. Unless it forms part of the employee's award or contract of employment, it is illegal for an employer to deduct money for things such as uniforms or tools without the employee's consent.

Net income = gross income − (tax + voluntary super contributions + any authorised deductions)

Worked Example 12

WE 12

Calculate **(a)** the annual income tax and **(b)** the annual net income for an annual gross income of \$68 000. Assume no other pay deductions are made.

Thinking	Working
(a) 1 Use the tax table to find the tax bracket that this annual gross income falls into.	(a) Tax = \$4650 + 30c for each \$1 over \$37 000
2 Subtract the highest figure of the previous tax bracket (\$37 000 in this case) from the annual gross income.	$68\,000 - 37\,000 = 31\,000$
3 Calculate the stated percentage of the remaining amount (here it is 30%, which is equivalent to 30c in every \$1).	$0.3 \times 31\,000 = 9300$
4 Add the amount calculated to the stated fixed amount (\$4650) to find the annual income tax.	Annual tax payable = $9300 + 4650$ = \$13 950
(b) Subtract the annual income tax from the annual gross income to find the annual net income.	(b) $68\,000 - 13\,950$ Annual net income = \$54 050

Worked Example 13

WE 13

Calculate **(a)** the tax that is deducted each pay and **(b)** the net income each pay for a gross income of \$980 paid fortnightly. Assume no other pay deductions are made. (Assume there are 26 fortnights in a year.)

Thinking	Working
(a) 1 Calculate the annual gross income.	(a) $\$980 \times 26 = \$25\,480$
2 Use the tax table to find the tax bracket that this annual gross income falls into.	Tax = 15c for each \$1 over \$6000
3 Subtract the tax-free threshold (\$6000).	$25\,480 - 6000 = 19\,480$
4 Calculate the stated percentage of the remaining amount (here it is 15%, which is equivalent to 15c in every \$1) to find the tax payable.	Annual tax payable = 15% of 19 480 $= 0.15 \times 19\,480$ = \$2922
5 Divide the annual amount of tax by 26 to find the fortnightly tax. Round to the nearest cent.	Fortnightly tax = $2922 \div 26$ = \$112.38
(b) Subtract the fortnightly tax from fortnightly gross income to find net income for the fortnight.	(b) Fortnightly net income = $980 - 112.38$ = \$867.62

Tax deductions

Your **taxable income** is the part of your income that the government believes you should pay tax on. A **tax deduction** reduces the amount of income on which you have to pay tax. If you spend money on something that helps you to earn your income, you may be entitled to claim that cost as a tax deduction. These work-related expenses vary from person to person and job to job, but may include things such as tools, equipment and protective clothing necessary for your job, membership fees for professional associations or unions, or fees for attending training courses. If your work requires you to use your car or phone, you may be able to claim some of the costs of these. Other tax deductions include donations to recognised charities.

Taxable income = gross income − tax deductions

Tax returns

At the end of the financial year, most people complete a **tax return**. This is a document used to assess whether the total amount of tax paid over the year is correct. It takes into account all of a person's income (earned through work or investments) and deductions they can claim, such as work-related expenses.

Depending on their tax return, a person may receive a tax refund or may have to pay more tax.

Worked Example 14

WE14

Carrie is an engineer whose annual gross salary is $95 000. During the year she had work-related expenses totalling $1200, paid membership fees to Engineers Australia of $255 and donated $650 to the Red Cross.

(a) Calculate Carrie's taxable income.

(b) Calculate how much income tax Carrie should have paid using the table on page 31.

Thinking	Working
(a) Subtract allowable deductions from gross income to find the taxable income.	(a) Taxable income = 95 000 − (1200 + 255 + 650) = 95 000 − 2105 = $92 895
(b) 1 Use the tax table to find the tax bracket that this income falls into.	(b) Tax payable is $17 550 + 37 cents for each $1 over $80 000.
2 Subtract the highest figure of the previous tax bracket ($80 000 in this case) from the taxable income.	92 895 − 80 000 = 12 895
3 Calculate the stated percentage of the remaining amount (here it is 37%, which is equivalent to 37c in every $1).	37% of $12 895 = 0.37 × 12 895 = 4771.15
4 Add the amount calculated to the stated fixed amount ($17 550) to find the total tax payable.	Tax payable = 4771.15 + 17 550 = $22 321.15

1.4 Tax and other pay deductions

Navigator

Answers page 599

Q1, Q2 (a)–(d), Q3, Q4, Q5, Q6, Q7, Q10, Q12	Q1 (a)–(f), Q2 (c)–(f), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12	Q1(e)–(h), Q2 (c)–(f), Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE12

1 Calculate **(i)** the annual income tax and **(ii)** the annual net income for the following annual gross incomes. Assume no other pay deductions are made.

(a) \$45 674 **(b)** \$77 365 **(c)** \$125 000 **(d)** \$29 576

(e) \$14 033 **(f)** \$200 000 **(g)** \$59 960 **(h)** \$5976

WE13

2 Calculate **(i)** the tax deducted each pay and **(ii)** the net income each pay for the following gross incomes. Assume no other pay deductions are made and that there are 12 months, 26 fortnights and 52 weeks in a year.

(a) \$725, paid fortnightly **(b)** \$4630, paid monthly

(c) \$1262.39, paid fortnightly **(d)** \$176, paid weekly

(e) \$8635.24, paid monthly **(f)** \$537.04, paid weekly

WE14

3 Rosanna is a nurse who receives an annual gross income of \$58 700.

(a) What is her taxable income if she donates \$200 a year to Amnesty International, has work expenses totalling \$575 and pays \$160 for her Nurses Federation membership.

(b) Calculate how much income tax Rosanna should pay annually using the tax table on page 31.

4 The tax payable on \$56 740 is closest to:

A \$5922 **B** \$10 572 **C** \$10 600 **D** \$17 022

Understanding

5 Karl's fortnightly gross income is \$1856. There are 26 fortnights in a year.

(a) Calculate Karl's annual gross income.

(b) Use your answer to **(a)** to calculate Karl's fortnightly PAYG tax, using the tax table on page 31.

(c) Find Karl's fortnightly 'net' pay (take-home pay).

(d) On top of his pay, Karl's employer must pay superannuation at a rate of 9% of his gross income. Calculate the employer's fortnightly super contribution.

SAARBORDE Pty Ltd	PAY PERIOD 06/09/2011–20/09/2011

Employee Name	Employee Number
Karl	000758

Description	Hours	Rate	Amount
Standard	38	Basic	\$1856·00
TOTAL GROSS PAY			\$1856·00

Description	Amount
PAYG Tax	
NET PAY	

Description	Amount
Employer super contribution	

6 Leanne earns a gross annual income of $67 200 and is paid monthly. Every month her employer also calculates 9% of her gross income for the month and pays it into her superannuation fund.

(a) How much is paid into Leanne's fund each month?

Leanne decides to make a voluntary super contribution of $80 each pay. She asks that this payment be deducted from her gross income.

(b) Including Leanne's voluntary contribution, how much is paid into her super account by the end of the year?

(c) Calculate Leanne's monthly net income by deducting her tax and her voluntary super contribution.

7 Adila has two permanent part-time jobs. In her first job, she works 15 hours a week at an hourly rate of $18.29 per hour. In her second job, she works 12 hours a week at an hourly rate of $16.43 an hour. For her second job, Adila bought some protective clothing that cost $125. Her union fees cost her $97 a year. Assume there are 52 weeks in a year.

(a) Calculate Adila's annual gross income.

(b) Calculate Adila's annual taxable income.

(c) Calculate the tax payable on Adila's taxable income.

8 At the beginning of the financial year Justin started in a new position with an annual gross salary of $54 500. There are 52 weeks in a year.

(a) How much should Justin pay per year in tax?

(b) How much is Justin's weekly gross income?

(c) How much per week does Justin get withheld as PAYG tax?

Justin decides to leave his job after 34 weeks in the position, and travel around Australia without working for the rest of the financial year. Before leaving Justin had earned $35 635 and paid $6473 in PAYG tax.

(d) Use the table to calculate how much tax Justin should pay on the amount he earned in the financial year.

(e) How much tax should Justin be refunded? Round your answer to the nearest dollar.

Reasoning

9 Brad and Jennifer are married. At the moment Brad has a taxable income of $80 000 and Jennifer does not do paid work.

(a) How much tax does Brad pay?

(b) What percentage (correct to the nearest whole number) of his taxable income does this represent?

Brad and Jennifer decide they would be better off if they both worked part-time. They are able to obtain positions that give each of them a taxable income of $40 000.

(c) What is their combined tax contribution?

(d) What percentage, correct to the nearest whole number, of their combined taxable income does this represent?

(e) How much better off are Brad and Jennifer under this scheme?

(f) Why is this different to Brad's contribution when he was the sole earner?

10 The five rows in the tax table on page 31 are sometimes called tax brackets. If you change to a higher-paid job you may move from one tax bracket to the next. The following pairs of incomes represent the 'border' between two tax brackets. Calculate the difference in the tax paid on them.

(a) \$37 000 and \$37 001 (b) \$80 000 and \$80 001 (c) \$180 000 and \$180 001

(d) Use your results to explain why the amounts added to each of the tax rates given in the table are \$4650, \$17 550 and \$54 550.

Open-ended

11 Do you think the tax table is fairly designed? Give a reason for your answer.

12 Why do you think the government has made it compulsory for employers to pay superannuation for each of their employees?

Outside the Square Game

Master of the Market

Equipment required: 2 brains, 2 dice

How to play: To get ready to play, each player should draw up the following table in their book:

Company	No. of shares bought	$ Invested	Dice roll	$ Profit/loss	Running total

Each player begins with \$3000 to invest in the sharemarket. The profit or loss you make on that investment is determined by the roll of a die. On your turn, choose one of the three companies below, decide how many shares you will buy (between 10 and 100) and calculate the value of your investment (the cost of buying the shares). Fill in the first three columns of your table. Roll the dice to see how well your investment performs. Take the number rolled and use the Investment Outcomes table on the right to calculate your profit or loss and fill in the fourth and fifth columns in the table above. Add or subtract the profit or loss to \$3000 to keep a running total in the sixth column. You may invest money you won in a previous turn on your next turn if you choose.

How to win: The winner is the first player to double their money.

Investment outcomes:

Dice roll	Profit or loss
2	100% loss
3	Lose everything except \$50
4	75% loss
5	50% loss
6	25% loss
7	No profit or loss
8	25% profit
9	50% profit
10	75% profit
11	100% profit
12	200% profit

Companies:
Company A \$5 per share Company B \$10 per share Company C \$15 per share

Half-time 1

1 (a) Giaan had 15 shots at the basketball ring from the free throw line. She got 11 in. Calculate Giaan's percentage shooting accuracy, correct to one decimal place. **Ex.1.1**

(b) A box of 100 'T42' tea bags is on sale for \$5.38, a saving of 91 cents. Calculate the value of the saving as a percentage of the original price, correct to one decimal place.

2 Ying is working as a grape picker in a vineyard. He is paid \$3.85 for each bucket of grapes picked. How much does Ying earn if he picks 17 buckets in one day? **Ex.1.3**

3 Find the selling price of the following items: **Ex.1.2**

(a) coffee table, cost price \$48, mark up 65%

(b) recliner chair, cost price \$197, mark up 40%

(c) bookcase, marked price \$67, discount 17.5%

(d) chest of drawers, marked price \$299, discount $33\frac{1}{3}\%$

4 Lucas wants to buy a new skateboard. In shop A it has a marked price of \$170 and a sticker that says 'take \$40 off'. Shop B has the same skateboard with a marked price of \$165 and a sticker that says 'take 25% off'. Decide which skateboard is cheaper. **Ex.1.1**

5 Sheri is a salesperson for a publishing company. She is paid a \$365 weekly retainer as well as 1.7% of the value of the sales she makes. One week she made the following sales: \$10 980, \$4372, \$23 065, \$17 834 and \$31 178. Calculate Sheri's earnings for the week. **Ex.1.3**

6 Mo bought an old desk for \$155. He bought some special polish for \$47.50, polished it up and then sold the desk for \$260. **Ex.1.2**

(a) Calculate Mo's profit as a percentage of the cost price (taking into account all costs), correct to one decimal place.

(b) Calculate Mo's profit as a percentage of the selling price, correct to one decimal place.

7 Jasminder's annual taxable income is \$47 530, which is paid fortnightly. Use the tax table on page 31 to calculate: **Ex.1.4**

(a) her annual income tax

(b) her net fortnightly pay (assume there are 26 fortnights in the year).

8 Naomi has been out shopping for bargains. She has bought a pair of jeans for \$65 that had been reduced by 10%, a jacket for \$47 that had been reduced by 35% and a scarf for \$11 that had been reduced by 20%. Calculate the original price on each of the items and find out how much Naomi saved in total. **Ex.1.2**

9 Franco works as a waiter in a restaurant. He is paid an hourly rate of \$18.95, with time-and-a-half for any hours worked on Saturday or after 10 p.m. on weekdays and double time for any hours worked on Sunday. **Ex.1.3**

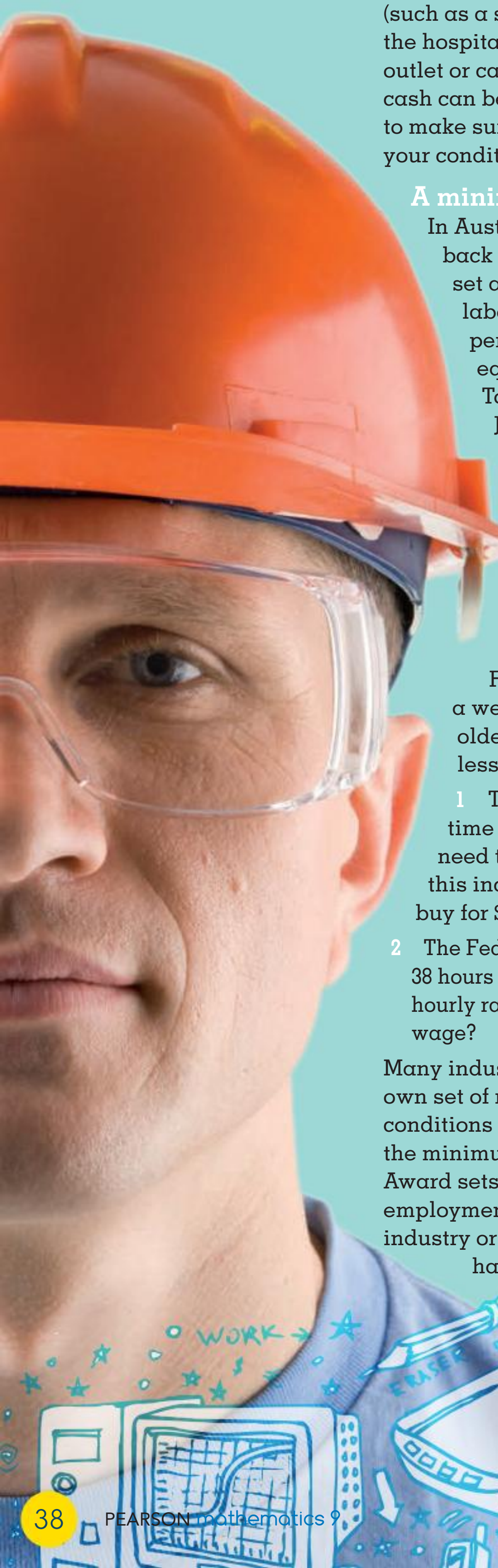
Here is Franco's time sheet for the week:

Tuesday	5 p.m.–10 p.m.
Wednesday	5 p.m.–10 p.m.
Thursday	5 p.m.–midnight
Friday	5 p.m.–midnight
Saturday	noon–4 p.m.
Sunday	noon–4 p.m.

Calculate Franco's wage for the week.

Maths 4 Real

A FAIR



Many students seek a part time job while still at school—often in the retail sector (such as a supermarket or smaller shop) or the hospitality industry (such as a fast food outlet or café). The prospect of earning some cash can be quite exciting, but it is important to make sure that the wage you are paid and your conditions of employment are fair.

A minimum wage

In Australia, the idea of a fair wage goes back to 1907, when Sir Justice Higgins set a minimum wage for unskilled labourers of 2 pounds, 2 shillings per week. In today's money this is equivalent to approximately $4.20. To set the minimum weekly wage, Justice Higgins considered the cost of living - the amount an average worker paid for food, shelter and clothing. Many Australian workers received a pay rise as the result of this judgement. In July 2011, the Federal Minimum Wage (set by the Government agency Fair Work Australia) was $589.30 a week. No adult worker (21 years or older) employed full-time can earn less than this amount.

1 The general increase in prices over time is known as inflation. Wages also need to be increased to keep up with this increase in prices. What could you buy for $4.20 today?

2 The Federal Minimum Wage assumes that 38 hours are worked in the week. What is the hourly rate paid to someone on the minimum wage?

Many industries and employers have their own set of rules for the pay and working conditions of their staff (often better than the minimum wage), known as 'Awards'. An Award sets out the minimum conditions of employment for employees in a particular industry or trade, for example retail or hairdressing.

Awards

Important information contained in Awards includes:

- descriptions of the different job classifications covered under the award
- the minimum weekly wage for each of the different job classifications
- the rates paid to junior employees (people under 21 years of age), apprentices and trainees
- extra amounts (often called 'loading' or 'penalty rates') paid for overtime
- special conditions for casual employees.

Here is some information taken from the General Industry Retail Award 2010. This award covers people who work in places such as supermarkets, department stores and clothing and furniture shops.

Minimum weekly wages (for a 38-hour week)

Classification	Per week
Retail Employee Level 1	$647.30
Retail Employee Level 2	$662.80
Retail Employee Level 3	$673.10
Retail Employee Level 4	$686.20

Junior rates

Junior employees are paid the following percentage of the minimum wages given above:

Age	% of weekly rate of pay
Under 16 years of age	45
16 years of age	50
17 years of age	60
18 years of age	70
19 years of age	80
20 years of age	90

WAGE

Overtime

Below are the overtime conditions for employees under the General Industrial Retail Award 2010:

Time worked	Conditions
Evening work Monday to Friday	An additional 25% will be paid for hours worked after 6 p.m. This does not apply to casuals.
Saturday	An additional 25% will be paid for hours worked on a Saturday between 7 a.m. and 9 p.m. This does not apply to casuals.
Sunday	An additional 100% will be paid for all hours worked on a Sunday.
Public holidays	An additional 150% will be paid for all hours worked on a public holiday.

Casual employees

- Casual employees are paid an additional 25% of the hourly rate for full-time employees.
- Casual employees are not paid overtime for evening work or Saturday work, but must be paid an additional 100% of their hourly rate for work performed on Sunday between 7 a.m. and 6 p.m.

3 Sabrina is 15 and has just started work as a Retail Employee Level 1 on a casual basis.

(a) Calculate Sabrina's hourly rate by first calculating the hourly rate of an adult in her position.

(b) Sabrina works from 4 p.m. to 7 p.m. on Tuesday and Friday, and from noon to 5 p.m. on Saturday. When Sabrina works for 5 hours or more, she must take an unpaid meal break of 30 minutes. (Depending on the hours worked, she also gets one or two 10-minute rest breaks, but these are paid.)

Calculate Sabrina's pay for the week.

4 Adam is 19 and has a full-time job with the same company as a Retail Employee Level 2.

(a) Calculate Adam's hourly rate.

(b) Adam works from 8 a.m. to 4.00 p.m. Monday to Friday, with a 30-minute unpaid meal break every day (plus two 10-minute paid breaks). Calculate Adam's weekly pay.

5 Here is Sabrina's work roster for the Christmas period:

24 Dec (Mon)	26 Dec (Wed)	27 Dec (Thurs)	28 Dec (Fri)	29 Dec (Sat)	30 Dec (Sun)
11 a.m. –7 p.m.	10 a.m. –4 p.m.	8 a.m. –2 p.m.	9 a.m. –5 p.m.	9 a.m. –5 p.m.	9 a.m. –5 p.m.

The 26th of December is a public holiday, and the store is closed on Christmas Day (25 December).

Calculate Sabrina's pay for this time period, remembering that she must take a 30-minute unpaid meal break if she works 5 hours or more. (It may help to calculate her hourly rate for each day first.)

6 Imagine that you got a casual job at the same level as Sabrina. (Alternatively, find the award that covers a job you are interested in.) Write down the days and times you would like to work, taking into account school, study time, sport and other commitments.

(a) Calculate your hourly rate.

(b) Calculate your weekly pay.

Research

Find out more about the rights of young people in the workplace. Areas you could include are safety, discrimination, unpaid 'trial work', joining a union and termination ('getting the sack'). Use your research to make a poster, presentation or short film.

1.5 Simple interest

What is interest?

Whenever money is borrowed, the borrower (an individual, organisation or community) pays the lender (a bank or similar financial institution) for the use of that money. This payment is known as **interest**. Interest is paid on top of the repayment of the amount borrowed (called the **principal**). It is charged as a 'percentage rate'—a percentage of the amount borrowed, calculated at regular time periods.

Whenever money is invested (deposited into an account) with a bank, the bank pays the investor interest because it uses the investor's money to conduct business.

Simple interest

There are different ways in which interest can be calculated. **Simple interest** is calculated on the initial amount borrowed or invested, regardless of how that amount changes over the time period of the loan or investment. In the future, you will learn about another type of interest that is calculated using the balance of the account; that is, the amount saved or owing at any particular point during the period of the loan/investment.

The amount of interest paid depends on:

- the initial amount of money borrowed or invested, known as the principal, P
- the annual percentage **rate of interest** that is charged or paid, R
- the time period, in years, for which the money is borrowed or invested, T.

Simple interest is the product of these three variables: $I = PRT$

The annual percentage rate, R, is often called the rate 'per annum', which means 'for every year' in Latin, and is abbreviated as p.a. We can express this rate in decimal form by dividing the percentge amount by 100.

Simple interest formula

$$I = PRT$$

Where I = simple interest (in dollars)
P = principal (in dollars)
R = interest rate per annum as a percentage
T = time (in years)

Worked Example 15

WE15

Calculate the simple interest paid on a loan of \$5000 at 16.4% p.a. interest over four years.

Thinking	Working
1 Write the formula for calculating simple interest.	$I = PRT$
2 Identify the values for P (the amount borrowed), R (the percentage rate in decimal form) and T (time in years).	$P = \$5000$ $R = 16.4\% = 0.164$ $T = 4$
3 Substitute the values into the formula and evaluate.	$I = 5000 \times 0.164 \times 4$ $= \$3280$

Interest is not always paid yearly

Interest can be calculated and paid quarterly, monthly or even daily. In these cases we need to express T as a fraction of a year when working with an annual rate.

In the following Worked Example, interest is paid quarterly, so T is expressed as a fraction with a denominator of 4. In Worked Example 17, interest is paid daily or monthly, so T is a fraction of 365 or 12.

Worked Example 16

WE16

Barbara borrows $24 000 to set up a business. She repaid it over 5 years and 3 months, after paying 18.5% p.a. in simple interest, payable quarterly.

(a) How much interest did she pay quarterly?

(b) What was the total amount of money repaid?

Thinking	Working
(a) 1 Write the simple interest formula.	(a) $I = PRT$
2 Identify P, R, and T. As interest is paid quarterly, T is expressed as $\frac{3}{12}$, or $\frac{1}{4}$ of a year.	$P = \$24\,000$ $R = 18.5\% = 0.185$ $T = \frac{3}{12} = \frac{1}{4}$
3 Substitute into the formula and calculate the simple interest.	$I = \$24\,000 \times 0.185 \times \frac{1}{4}$ $= \$1110$ per quarter
(b) 1 Identify the number of quarterly periods taken to repay the debt.	(b) $T = 5$ years 3 months $= 5\frac{1}{4}$ years $= \frac{21}{4}$ years $= 21$ quarters
2 Multiply the interest paid per quarter by the number of quarters.	$I = \$1110 \times 21$ $= \$23\,310$
3 Add the interest to the principal to find the total amount repaid.	Total Repaid $= P + I$ $= \$24\,000 + \$23\,310$ $= \$47\,310$

When calculating daily interest, it is important to carefully count the number of days the balance remains at a certain amount. The balance changes on the day a withdrawal or deposit is made. Make sure the number of days used adds up to the total number of days in the month.

Worked Example 17

WE17

Craig's bank statement for September is shown:

If the account pays 6% interest, calculate the interest accrued in September if interest is calculated:

(a) daily, to four decimal places

(b) on the minimum monthly balance.

Date	Transaction		Balance
Sept 1			\$490.15
Sept 6	Deposit	\$180	\$670.15
Sept 19	Withdrawal	\$400	\$270.15

Thinking	Working
(a) 1 Write the formula.	(a) $I = PRT$
2 Identify P, the daily balances.	$P = \$490.15, \$670.15, \$270.15$
3 Identify the rate R.	$R = 6\% = 0.06$
4 Find T for each balance by counting the number of days the balance remained at that amount, and writing it as a fraction of 365.	$T = \frac{5}{365}, \frac{13}{365}, \frac{12}{365}$
5 Substitute into the formula the respective values of P, R and T and calculate correct to four decimal places.	$I = \$490.15 \times 0.06 \times \frac{5}{365}$ $= \$0.4029$ $I = \$670.15 \times 0.06 \times \frac{13}{365}$ $= \$1.4321$ $I = \$270.15 \times 0.06 \times \frac{12}{365}$ $= \$0.5329$
6 Calculate the total interest by adding the individual amounts.	$I = \$0.4029 + \$1.4321 + \$0.5329$ $= \$2.3679$
7 Round to two decimal places (the nearest cent).	$= \$2.37$
(b) 1 Write the formula.	(b) $I = PRT$
2 Identify the balances.	$\$490.15, \$670.15, \$270.15$
3 Identify the smallest balance for the month.	$P = \$270.15$
4 Identify R and T, where T is the number of days in the month.	$R = 6 \div 100 = 0.06$ $T = \frac{30}{365}$
5 Substitute into the formula.	$= \$270.15 \times 0.06 \times \frac{30}{365}$
6 Calculate to two decimal places.	$= \$1.33$

Finding *P*, *R* or *T*

If we know three of the four variables in the simple interest formula, we can calculate the fourth.

Worked Example 18

WE18

Allan put \$12 500 into a cash management account. He received \$13 740.15 when he withdrew all of the money 9 months later. What was the average rate of return, p.a., on his capital (correct to one decimal place)?

Thinking	Working
1 Write the formula.	$I = PRT$
2 Calculate the interest earned by subtracting the principal from the total.	$I = \$13\,740.15 - \$12\,500$ $= \$1240.15$
3 Identify P, R (unknown) and T.	$P = 12\,500$ $R = ?$ $T = \frac{9}{12} = \frac{3}{4}$
4 Substitute into the formula.	$1240.15 = 12\,500 \times R \times \frac{3}{4}$
5 Simplify as much as possible.	$1240.15 = 9375R$
6 Solve the equation for R by dividing on both sides.	$\frac{1240.15}{9375} = R$
7 Convert to a percentage correct to one decimal place.	$R = 0.13228$ $r = 13.2\%$

1.5 Simple interest

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q11, Q12, Q13, Q17, Q19, Q21, Q23	Q1, Q2, Q3, Q4, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q16, Q17, Q19, Q21, Q22, Q23	Q2,Q3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q20, Q21, Q22, Q23

Answers page 600

Fluency

1 (a) Calculate the simple interest payable on a loan of \$45 000 at a rate of 12% p.a. for 3 years. WE15

(b) Calculate the simple interest payable on a loan of \$3500 at an annual rate of 7% for 4 years.

(c) Calculate the simple interest payable on a loan of \$11 450 at 8.2% p.a. for 5 years.

(d Calculate the simple interest payable on a loan of \$8500 at 5.5% p.a. for 6 months.

2 Chloe borrowed \$15 000 to buy a new car. She repaid it over $3\frac{1}{2}$ years, at a simple interest rate of 12.5% p.a., payable every 6 months. WE16

(a) How much interest did Chloe pay every 6 months?

(b) What was the total amount of money repaid?

3 (a) A statement for an account offering 6.9% p.a. simple interest is shown for the month of July. WE17

How much interest is payable for July if interest is calculated:

(i) daily, to four decimal places

(ii) on the minimum monthly balance?

Date	Transaction	Balance
July 1		\$350
July 5	Withdrawal \$170	\$180
July 22	Deposit \$450	\$630

(b) A statement for an account offering 7.8% p.a. is shown for the month of May.

How much interest is payable for May if interest is calculated:

(i) daily, to four decimal places

(ii) on the minimum monthly balance?

Date	Transaction		Balance
May 1			$65.20
May 16	Deposit	$530	$595.20
May 22	Withdrawal	$260	$335.20
May 27	Withdrawal	$300	$35.20

WE18

4 After investing $5000 into a fixed deposit account for 2 years for a trip to London, Inika received $5890. What percentage return p.a. did she get?

5 Calculate the simple interest in the following cases:

(a) $P = \$440, R = 0.08, T = 5$ years

(b) $P = \$600, R = 0.09, T = 7$ years

(c) $P = \$369, R = 0.12, T = 3$ months

(c) $P = \$3800, r = 16.8\%, T = 20$ days

6 The simple interest on $870 for 4 years at 15% p.a. is:

A $870 \times 15 \times 4$

B $870 \times 0.15 \times 4$

C $870 \times 1.15 \times 4$

D $870 \times 0.15 \times 4 + 870$

7 What is the total amount which is paid back on a loan of $8150 over 3 years 6 months at 18.5% p.a.?

8 Calculate R, then write as a percentage (p.a.) correct to one decimal place when:

(a) $P = \$245, T = 5$ years, $I = \$110$

(b) $P = \$420, T = 2.25$ years, $I = \$132$

9 Calculate T (in years) correct to one decimal place when:

(a) $P = \$245, R = 0.06, I = \$116$

(b) $P = \$120, R = 0.07, I = \$32$

10 Calculate P (in dollars) when:

(a) $T = 3.5$ years, $R = 0.05, I = \$35$

(b) $T = 2$ years, $R = 0.08, I = \$216$

Understanding

11 Claudia, to save for a trip to Thailand, puts her tax return cheque of $950 into an account paying 8% p.a. simple interest. How long (in years, correct to one decimal place) will her money take to accumulate to $1300?

12 How much will be paid out to Sol, who puts $1400 into a bank account paying 6% simple interest p.a., if he withdraws the total amount after 240 days?

13 Victor agrees to lend Esme $1500 so that she can go on a holiday to Fiji. They agree that Esme will make quarterly payments, with simple interest to be paid at a rate of 12% p.a. As well as the principal, Esme will pay the debt in full in 5 years. Calculate Esme's quarterly payments.

14 Manny borrows $6100 from his mother to buy a car. He agrees to pay 11% simple interest each year without paying off the principal. After 2 years, in addition to the interest, he pays $2500 off his principal, reducing the principal for the rest of the loan. How much in total did he pay his mother if he pays off the loan in another 1.5 years?

15 **(a)** Calculate the interest payable on an account in the month of April if it pays 5.8% interest p.a., and it has a balance of $155 for 3 days, $820 for 19 days, and $580 for the rest of the month, if the interest is paid on:

(i) minimum monthly balance

(ii) daily balance

(b) How much more interest is paid under the conditions in part **(ii)** than in part **(i)**?

16 After investing $6000 into a savings account paying 7% interest p.a., Ian received $6238. For how many days (correct to the nearest day) was the money invested?

17 What amount of money must be invested at 9% p.a. in order to earn $540 in interest in 3 months?

18 Tula repays her father a total of $350 in interest to cover a debt for a motorbike which she incurred 15 months ago, agreeing to pay 4% p.a. simple interest. How much did she borrow?

Reasoning

19 Consider the statement shown at right for a savings account.

Date	Transaction		Balance
Jul 1			$975.00
17	Deposit	$197	
28	Withdrawal	$641	
Aug 4	Deposit	$299	
19	Deposit	$862	
29	Withdrawal	$176	
Sep 6	Deposit	$194	
28	Withdrawal	$800	
Oct 16	Deposit	$426	
27	Withdrawal	$500	
31	Balance		

(a) Copy and complete the statement for the 4-month period shown.

(b) State the minimum balance for each month.

(c) Calculate the interest owed by the bank if the interest rate is 2.75% p.a. and it is paid on the minimum monthly balance.

Late in each month there has been a withdrawal. Assume that these withdrawals were delayed until the first day of the next month.

(d) What are the minimum monthly balances now?

(e) How much interest is now earned?

(f) What percentage change has there been in the amount earned?

(g) In light of the calculations you have done, what advice would you offer people or businesses who want to earn as much interest as possible on their savings?

20 Phillip borrows $15 000 from his mother to help pay for his new car. He needs to pay back the money within 3 years and pay 5% simple interest for the period of the loan. He intends to make monthly repayments to his mother.

(a) How much interest will Phillip need to pay?

(b) How much in total does Phillip owe his mother?

(c) How much is each monthly repayment?

(d) Assuming $\frac{1}{3}$ of the interest is repaid each year and $\frac{1}{3}$ of the principal is repaid each year, what percentage of each payment is interest?

(e) How much interest does Phillip still need to repay after the first 12 months?

(f) If the interest calculation was done again based on the principal still owing after 12 months, how much interest would be owing?

(g) Why is there a difference in the answers to parts **(e)** and **(f)**?

Open-ended

21 Hung earns \$640 on an investment of \$8000. Give three possible simple interest rates as percentages per annum, and their corresponding time periods, that give interest of \$640 on an investment of \$8000.

22 Michael the miser is sure that the bank has made a mistake on the interest it added to his account for the month.

Interest is calculated daily at a rate of 3.5% p.a. Here is Michael's statement:

Date	Transaction	Balance
1 June	Opening balance	\$578.12
9 June	\$250.03 deposit	\$828.15
21 June	\$185 withdrawal	\$643.15
30 June	Closing balance	\$643.15

Here are Michael's calculations:

$P = \$578.12, \$828.15, \$643.15$

$R = 0.035$

$T = \frac{9}{365}, \frac{13}{365}, \frac{10}{365}$

$I = (\$578.12 \times 0.035 \times \frac{9}{365}) + (\$828.15 \times 0.035 \times \frac{13}{365}) + (\$643.15 \times 0.035 \times \frac{10}{365})$

$= \$0.4989 + 1.0324 + \$0.6167$

$= \$2.15$

The amount of interest given on his bank statement is \$2.01.

Has the bank made a mistake? Or has Michael? Explain the difference in the two amounts of interest.

23 'Neither a borrower nor a lender be' is a line from Shakespeare's famous play *Hamlet*. Do you think this is good advice? Give some reasons for your answer.

Using iteration to investigate savings plans

PP Technology

Versions of this Exploration for other technologies are available in Pearson Reader.

Equipment required: 1 brain, 1 computer with Excel or a spreadsheet on a CAS calculator

In mathematics, iteration occurs when the output from a mathematical rule becomes the input to the rule the next time the rule is processed.

Consider the rule of doubling a number and taking away 10.

If the first input is 20 then the rule becomes $2 \times 20 - 10$ and the output is 30. The output now becomes the next input. That is, we now apply the rule to 30.

The second output is 50, as $2 \times 30 - 10 = 50$. Each time we go through the process we say an iteration has been performed. The first iteration gave a result of 30 and the second iteration gave a result of 50.

Excel allows us to use iteration in a very simple manner. Simply enter the start value in A1, the rule in A2 and then copy the formula down as far as you like. The spreadsheet below shows the first six iterations of the rule. Cell A2 is highlighted so you can see the formula. We say that 30 is the result of the first iteration, but note it is in row 2.

A2 | fx =A1*2-10

	A	B	C	D	E
1	20				
2	30				
3	50				
4	90				
5	170				
6	330				
7	650				
8					

Suppose you are able to save $1200 each year towards a major holiday in 10 years' time. You have to choose the best plan from the following options.

1 Find the value of the fourth, tenth and twentieth iterations. You will have to extend the table beyond the size shown below.

	Payment*	Interest rate
Plan A: Regular Saver	$100 per month	9% p.a. paid monthly
Plan B: Periodical Saver	$600 per 6 months	10% p.a. paid 6-monthly
Plan C: Yearly Lump Sum	$1200 per year	9% p.a. paid yearly

* All payments commence at the beginning of the payment period.

Investigating Plan A: Regular Saver

Calculate the monthly rate of interest. Dividing 9% p.a. by 12 gives a rate of 0.75% per month or 0.0075 per month as a decimal rate.

The amount saved at the beginning of the second month is calculated as follows:

$$\begin{aligned} \text{The initial deposit} &= \$100 \\ \text{Interest on deposit} &= 0.0075 \times 100 = \$0.75 \\ \text{The second deposit} &= \$100 \\ \text{Total} &= \$(100 + 0.75 + 100) \\ &= \$200.75 \end{aligned}$$

The amount saved at the beginning of the third month is calculated as follows:

$$\begin{aligned} \text{Previous balance} &= \$200.75 \\ \text{Interest on balance} &= 0.0075 \times 200.75 = \$1.51 \\ \text{The third deposit} &= \$100 \\ \text{Total} &= \$(200.75 + 1.51 + 100) \\ &= \$302.26 \end{aligned}$$

To perform these calculations using Excel, enter an initial amount of $100 into B1 and the formula =B1+(0.09/12)*B1+100 into B2.

If you have entered this correctly you should see a result of 200.75 as the amount in the account at the beginning of the second month. You might like to fix this cell to show two decimal places only, so it matches our money system.

B2 | fx =B1+(0.09/12)*B1+100

A	B	C	D	E	F
20	100				
30	200.75				
50					

2 Explain what each part of the formula =B1+(0.09/12)*B1+100 does.

3 Copy the formula into cell B3. This shows the amount in the account at the beginning of the third month. Does it match the value calculated earlier? What has happened to the formula?

4 Extend the table to find the amount saved at the beginning of each month for months 3–13 inclusive of the savings plan. (Hint: The end of the 12th month occurs at the beginning of the 13th month.)

5 How much is saved at the end of 1 year (not including payment of \$100 at the beginning of month 13)?

Investigating Plan B: Periodical Saver

To calculate the 6-monthly rate of interest, divide 10% p.a. by 2. This gives a rate of 5% per 6 months, or 0.05 per month as a decimal rate.

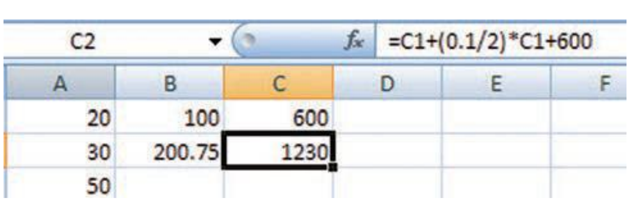
At the beginning of the second 6-month period the amount saved is calculated as:

The initial deposit = \$600
Interest on deposit = 0.05 × 600 = \$30
The second deposit = \$600
Total = \$(600 + 30 + 600)
= \$1230

Copy and complete the calculation below for the beginning of the third 6-month period:

Previous balance = \$ _____
Interest on balance = 0.0 _____ × 1230 = \$_____. 50
The third deposit = \$600
Total = \$(1230 + _____. 50 + 600)
= \$_____.50

We will enter this plan in column C of our spreadsheet. Cell C2 has been highlighted to show you the formula to use.



6 Explain what each part of the formula =C1+(0.1/2)*C1+600 does.

7 Copy the formula to cell C3. Does this give you the same value you calculated earlier for the amount in the account at the beginning of the third 6-month period?

8 How much is saved at the end of one year if the payment of \$600 (at the beginning of the second year) is not included?

Investigating Plan C: Yearly Lump Sum

9 Find the amount saved after one year. Do not include the payment made at the start of the second year.

Comparing the plans

10 Write a brief report that compares the plans and the amount saved in each plan after one year. Which plan would you recommend? How practical are the plans?

Payment methods

1.6

There are a number of ways that you can pay for goods if you do not have actual cash with you.

- **Debit card**

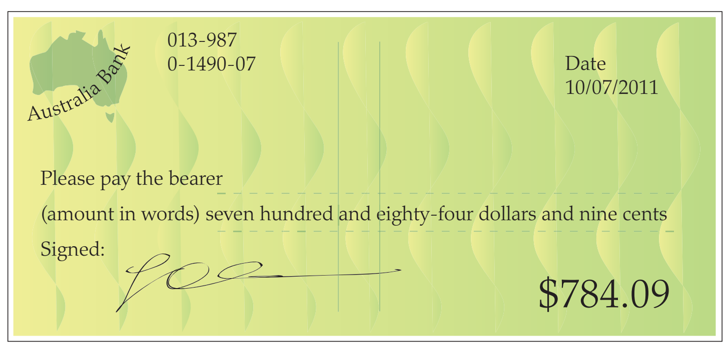
  Debit cards allow money for your purchase to be transferred electronically from your bank account to the store's account. This is done through the Electronic Funds Transfer at Point of Sale (**EFTPOS**) system. The money must be present in your bank account in order for it to be transferred and the bank may charge a small fee for each EFTPOS transaction. Credit card companies such as VISA and MasterCard provide debit cards that, in addition to being used for EFTPOS, can also be used overseas and online.

- **Credit card**

  Credit cards look like debit cards and are used in a similar way, but there is one important difference. When you use a credit card, the card provider (usually a bank) pays the store for you and sends you a bill at the end of the month. If you do not pay the bill in full, a high rate of interest is charged.

- **Cheque**

  Cheques have been used as a substitute for cash for a long time, but are being used less and less as electronic banking has become more common. To write a cheque you enter the name of the person or company being paid, the amount in words, and your signature. The cheque must then be deposited at a bank. Many banks charge a cheque handling fee, and the money takes several days to become available.



- **Lay-by**

  The store holds the item for you while you make payments. You receive the item after it has been completely paid for. No interest has to be paid for this form of payment.

- **Deferred payment**

  This is usually used to describe the payment method for repaying university or college fees that have been accumulated by using HELP – the Higher Education Loan Program. Payment begins once the borrower (the student) begins earning above a certain amount set by the government. Repayments are calculated as a percentage of the borrower's income.

- **Buying on terms** (sometimes called 'buy now, pay later')

  Large purchases, such as furniture, computers or cars, can be bought in this way. You take possession of the goods then pay them off. Many stores offer an 'interest-free period' where you pay a small account-keeping fee and a monthly repayment. However, if you have not fully paid for the item by the end of this period, a very high interest rate is charged (often between 20% and 30%).

Fees and charges

In addition to charging for EFTPOS transactions, banks may also charge monthly account-keeping fees and/or fees for withdrawing cash from ATMs (Automatic Teller Machines), particularly if the ATM belongs to another bank, or for paying bills online.

Worked Example 19

WE19

Jemima's bank charges her a $2 account-keeping fee every month, plus the following charges:

- 50c for a withdrawal from the bank's ATMs
- $2.00 for a withdrawal from another bank's ATMs
- 75c for EFTPOS transactions
- 50c for transferring money online

In one month, Jemima makes two withdrawals from her own bank's ATM, three withdrawals from another bank's ATM, seven EFTPOS transactions and two online transfers.

Calculate the total amount of Jemima's bank charges for the month.

Thinking	Working
1 Go through the list and calculate each fee separately.	Bank ATM: 2 × $0.50 = $1.00 Other bank ATM: 3 × $2.00 = $6.00 EFTPOS: 7 × $0.75 = $5.25 Online transfers: 2 × $0.50 = $1.00 Account keeping fee: $2.00
2 Find the total and write your answer in a sentence.	Total fees = $15.25 Jemima's bank will charge her $15.25 for the month.

Using a credit card

Using a credit card is an extremely popular method of payment. In 2009 there were approximately 16 million credit cards in use in Australia. When you use a credit card to purchase goods, the bank that issued the card to you pays the cost, then sends you a bill, called a statement, for that amount (plus any other credit purchases) at the end of the month. Most cards have an 'interest free' period of up to 55 days. This period begins on the first day of the statement period and ends on the date when payment is due. If you pay the full amount of the bill by the due date, you will not be charged interest. However, if you do not pay in full

by the due date you will be charged interest, even if you pay the minimum repayment amount stated on the bill. The interest rate is stated as an Annual Percentage Rate (APR). This varies from card to card, but is usually somewhere between 15% and 20%. Some credit cards do not have an interest-free period and calculate interest from the day the purchase is made. Credit card interest is calculated using the method outlined below.

Calculating credit card interest:

- The balance (the amount owing) at the end of each day of the statement period is determined. These daily balances are then added up and divided by the number of days in the statement period to get an 'Average Daily Balance', or ADB.
- A daily interest rate is calculated by dividing the annual rate by 365.
- The ADB is multiplied by the daily interest rate, then by the number of days in the statement period, to give the amount of interest for the period.
- This interest will then be charged on the next statement if the amount is not paid in full by the due date.

Worked Example 20

WE20

Ali has a credit card with an Annual Percentage Rate (APR) of 18.4% interest and an interest-free period of 55 days. He has just received a statement covering the period from 24 May to 23 June. If Ali's average daily balance (ADB) for this period is \$2184.36, calculate the interest that will be payable if he does not pay the account in full by the due date.

Thinking	Working
1 Find the daily interest rate by dividing the annual rate by 365. Express this as a decimal.	$R = 18.4\%$ APR $\div 365$ $= 0.0504\%$ $= 0.000\,504$
2 Calculate the number of days in the statement period, being sure to include the days at the beginning and end of the period.	$T =$ 24 May to 23 June $= 31$ days
3 Define P as the Average Daily Balance (ADB) for the period. Substitute the values for P, R and T into the interest formula and evaluate. Round your answers to the nearest cent.	$P = \$2184.36$ $I = PRT$ $= \$2184.36 \times 0.000\,504 \times 31$ $= \$34.13$

Buying on terms

Furniture and electrical stores offer arrangements where customers can take the goods and pay for them over a period of time. 'Interest-free' terms allow customers a period (usually 6–12 months) during which no interest is charged. During this period, a small monthly repayment, plus a monthly account fee, is paid. An 'establishment fee' is paid at the beginning of the period when the account is set up. The remaining balance of the account must be paid at the end of the interest-free period to avoid interest being charged from that point on.

'Buy now, pay later' terms are similar, except that no monthly repayments are required during the interest free period, although you may choose to make them to avoid a big payment at the end of the period. The interest on any outstanding amount at the end of the interest-free period is calculated in the same way as credit card interest.

Worked Example 21

WE21

Ravi wants to purchase a new lounge suite. He has seen one advertised for $2799, but he does not have the money. The store offers Ravi a 12-month interest-free payment plan where he must pay a minimum monthly repayment of 3% of the cost, a $25 up-front application fee and a $3.95 monthly account-keeping fee.

(a) How much will Ravi pay in fees over the 12 months?

(b) If he only makes the minimum monthly repayments, how much will Ravi have paid off the cost of the lounge suite after 12 months?

(c) If Ravi does not pay the remainder of the cost at the end of the 12 months, he will be charged 29% p.a. interest until it is all paid off. Calculate the interest that will be charged after the 13th month. (Assume there are 30 days in the month.)

Thinking	Working
(a) Multiply the monthly account-keeping fee by 12, then add the application fee.	(a) $(\$3.95 \times 12) + \$25 = \$47.40 + \$25$ $= \$72.40$
(b) Find the minimum monthly repayment, then multiply it by 12 months.	(b) 3% of $\$2799 = 0.03 \times 2799$ $= \$83.97$ per month $\$83.97 \times 12 = \$1007.64$ after 1 year
(c) **1** Find the outstanding balance—the amount still owing after 12 minimum monthly repayments. Define this as *P*.	(c) $P = \$2799 - \$1007.64$ $= \$1791.36$
2 Find the daily interest rate by dividing the annual rate by 365 then converting to a decimal.	$R = 29\% \div 365$ $= 0.079\,452\%$ $= 0.000\,795$
3 Define *T*.	$T = 30$ days
4 Find the interest by substituting the values into the interest formula, then evaluating.	$I = PRT$ $= \$1791.36 \times 0.000\,795 \times 30$ $= \$42.70$

1.6 Payment methods

Navigator

Answers page 600

Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q10, Q11, Q12	Q1(a)–(b), Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12	Q1(c)–(d), Q2, Q3, Q4, Q5, Q6, Q9, Q10, Q11, Q12

Fluency

WE19

1 Federation Bank charges customers the following fees on their 'Easy Saver' account:

- a $3 monthly account-keeping fee
- 70c for withdrawals from the bank's ATMs
- $1.50 for withdrawals from other banks' ATMs
- 70c for EFTPOS transactions
- 50c for online transfers

Calculate how much the bank will charge the following customers in a month if the following transactions are made:

(a) five ATM withdrawals (one from a different bank), four EFTPOS payments, three online transfers

(b) six EFTPOS payments, four ATM withdrawals (two from the bank's ATMs), five bills paid online

(c) two withdrawals from the bank's ATMs, eight EFTPOS payments

(d) four withdrawals from other banks' machines, six online transfers.

2 Shannon has a credit card with an annual interest rate of 16.99% and an interest-free period of 55 days. She has just received a statement covering the period from 15 September to 14 October. If Shannon's average daily balance (ADB) for this period is $3205.48, calculate the interest that will be payable if she does not pay the account in full by the due date. WE20

3 John wants to buy a home entertainment system priced at $3299 but doesn't have the money. The store offers him a 6-month interest-free plan with a minimum monthly repayment of 5% of the cost, a $20 up-front application fee and a monthly account-keeping fee of $3.50. WE21

(a) How much will John pay in fees over 6 months?

(b) How much will John have paid off the cost of the TV after 6 months if he only makes the minimum monthly repayments?

(c) If John does not pay the remainder of the cost at the end of 6 months, he will be charged 24% p.a. interest until it is paid off. Calculate the interest that will be charged after the seventh month. (Assume there are 30 days in the month.)

4 Jackson wanted to buy a new telescope for $109.95, but did not have enough money, so he placed it on lay-by. He paid a 10% deposit and needs to make payments of $15 a week.

(a) How long will it take Jackson to pay off his telescope?

(b) How much will his last repayment be?

Understanding

5 Farouk is 18 years old and has just obtained a credit card that has an interest rate of 17.35% and an interest-free period of 55 days. His statement after his first month is shown.

St Geoff Bank

Account Number: 8675 0032 6701 2130

Statement Period: 03/07/11 to 02/08/11

Date	Purchase	Amount	Balance
	Opening balance		$0.00
5 July	Shoes	$59.00	$59.00
11 July	Petrol	$72.30	$131.30
17 July	Concert ticket	$124.80	$256.10
22 July	Phone and internet bill	$89.95	$346.05
29 July	Groceries	$147.32	$493.37
1 August	Ski jacket and goggles	$239.86	$ 733.23
	Closing balance		$733.23

Hint

If no purchases have been made on a particular day, the closing balance is the same as the previous day's balance.

(a) How many days are in the statement period?

(b) Calculate the average daily balance by finding the balance at the end of each day, adding them up and dividing by the number of days in the statement period. Remember to include the days at the beginning and end of the statement period (shown on the bill).

(c) Use your answer to **(b)** to calculate the interest that Farouk will be charged if he does not pay the balance in full by the due date.

6 Yolanda has bought a new spa bath worth $4300 using a 'buy now, pay later' 24-month payment plan. The plan requires her to pay an up-front establishment fee of $25 and a monthly account-keeping fee of $2.95. Yolanda does not have to make any monthly repayments, although she can do so if she wants to. Yolanda must pay the full amount by the end of the 24-month period or she will pay 30% p.a. interest every month on whatever is owing until it is all paid off.

(a) Calculate Yolanda's fees for the 24 months.

(b) If Yolanda does not pay anything off the spa in the 24-month period, how much interest will be added to her account by the end of the 25th month? (Assume there are 30 days in the month.)

(c) Aside from the account-keeping fees, what voluntary monthly repayment amount should Yolanda make in order to have the spa paid off by the end of the 24 months?

7 Poh and Dzung are trying to decide which method they should use to pay their rent of $950 each month. They can withdraw cash from their account using an ATM, then go into a bank to deposit it into their landlord's account. Alternatively, they can post a cheque directly to their landlord's workplace. Poh's bank charges $6 to process a cheque.

(a) The ATM will only allow withdrawals of up to $400 each time, at a cost of 70c per withdrawal. How much will it cost to withdraw the $950 from the ATM?

(b) Which method of payment would you recommend to Poh and Dzung?

Reasoning

8 Edwin is leasing a work car using a hire purchase agreement. He pays a monthly amount of $810 to the finance company for the use of the car over a 5-year period. This monthly amount includes the cost of the car and interest. At the end of the 5 years, Edwin is the owner of the car.

(a) Calculate the total amount Edwin pays for the car over 5 years.

(b) If the initial value of the car is $39 995, how much has Edwin paid in interest?

(c) Using the value of the car as the principal, calculate the equivalent rate p.a. of simple interest that Edwin has paid correct to one decimal place.

9 Shelby has used her credit card to buy furniture for her new house. This month, her credit card statement is for the period from 1 March to 31 March. The total owing on the credit card is $8691.57. Shelby is unable to pay her bill, so interest and a $9 late payment fee are added to the balance. The interest rate of the card is 20.99% p.a.

(a) Calculate the interest that is added if the ADB over the statement period was $6982.45.

(b) Calculate the opening balance on Shelby's account at the beginning of April.

(c) During April, Shelby makes two purchases: a plasma screen TV for $1299 on the 14th and a new lounge suite for $2499 on the 21st. Calculate the balance on her account on 30 April.

(d) If Shelby again doesn't pay her bill, how much interest will be calculated on April's balance? (You will need to calculate the ADB for April.)

(e) How much in interest and fees will have been charged to Shelby's account after 2 months of no payments?

(f) Use your answers from **(a)–(e)** to explain why credit card debt can become a problem for some people.

Open-ended

10 Jack and Jill would like to purchase some furniture for $1999. They have two payment options. Option 1 is lay-by the furniture by placing a 10% deposit up front and make payments of $160 a month until it is paid off. Option 2 is to use a 'buy now, pay later' arrangement of an upfront fee of $25, a monthly account fee of $3, and no other compulsory repayments for 12 months, after which an interest rate of 29% p.a. is applied.

(a) If they choose to lay-by, calculate how long it will be before Jack and Jill will be able to take home their furniture.

(b) How much extra will they pay if they choose to 'buy now, pay later', even if no interest is applied?

(c) Which payment plan would you choose? Give at least one reason for your answer.

(d) (i) Why might people choose 'buy now, pay later' over lay-by, even though it may mean paying extra money?

(ii) Why might people opt to lay-by instead?

11 Kezia's bank charges her 50c for a withdrawal from the bank's ATMs, $2.00 for withdrawing from another bank's ATMs, 75c every time she uses EFTPOS and 50c every time she uses the internet to transfer money or pay bills online.

Kezia's bank statements show that she is losing between $10 and $20 a month in fees and charges.

(a) Give some combinations of fees that have a sum between $10 and $20.

(b) Give Kezia a couple of pieces of advice to help her lower this amount.

12 List some of the advantages and disadvantages of using a debit card instead of a credit card.

1.7 The cost of resources

Electricity, gas and water are resources, often referred to as utilities, that are supplied to households. The members of the household are billed according to their consumption of the resource (that is, you pay for the amount you use).

Tariffs

A tariff is the price charged per unit of electricity, gas or water used. Often, a more expensive tariff is applied after a certain number of units have been used. A cheaper tariff may be applied to low use or use during 'off peak' periods (say between 11 p.m. and 7 a.m., or from November until June). A tariff for the supply of the utility, often known as the service charge, also appears on the bill.

Electricity

Electricity is supplied to individual houses by a network of power stations, substations and cables known as the National Grid. Each house connected to the National Grid has a meter that displays the amount of electrical energy consumed by that household. Companies that sell electricity take regular readings of the meter and bill the members of the household for the energy they have used.

To understand how the electrical bill for a household is calculated, we need to consider how energy consumption is measured.

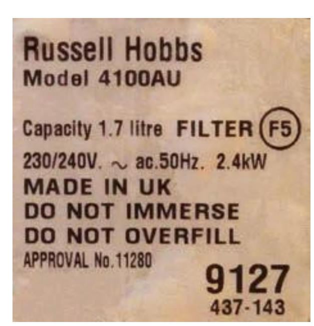
The basic unit of energy is the joule (J). Different household appliances use electrical energy at different rates. The rate at which an appliance uses energy is called its power, which is measured in watts (W).

A 1-watt appliance uses 1 joule of energy in 1 second: 1 W = 1 J/s

1000 watts is equal to 1 kilowatt (kW): 1000 W = 1 kW

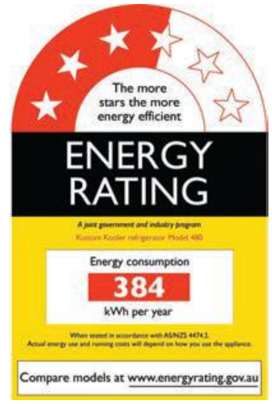
A 1000-watt appliance uses energy twice as fast as a 500-watt appliance.

The 'power rating' of an appliance tells you how many watts of power the appliance uses. It can usually be found on a label on the appliance itself. The label shown is from an electric kettle. The power rating of the kettle is 2.4 kW, or 2400 W.



Another label found on many appliances is the 'Energy Rating' label. The label uses a 'star' rating system to rate how energy efficient the appliance is (1 star = not very efficient; 6 stars = extremely efficient). The label also tells you how much energy the appliance will consume in 1 year if used in the normal way.

To find the amount of electrical energy used by an appliance, we multiply its power rating by the length of time (in hours; h) that the appliance has been in use. This gives us the unit of energy used by the supplier to calculate your bill, the kilowatt-hour (kWh).



Energy consumption of an appliance (kWh) = Power of the appliance (kW) × time (h)

The daily running cost of an appliance is calculated by multiplying its energy consumption by the tariff:

Daily running cost (\$) = energy consumption (kWh) × tariff (\$/kWh)

Worked Example 22

WE22

For these household appliances, calculate **(i)** the daily energy consumption and **(ii)** the daily running cost, given that the tariff is 18.5 cents/kWh.

(a) a 75-W light bulb switched on for 12 h a day

(b) a 4300-W clothes dryer, in use for 45 min a day.

Thinking	Working
(a) **(i)** **1** Convert the power of the appliance to kW. State the time in hours.	75 W = 0.075 kW Time = 12 h
2 Multiply the power by the number of hours the appliance is in use to find the daily energy consumption in kWh.	0.075 × 12 = 0.9 kWh consumed daily
(ii) Multiply the daily consumption by the tariff to find the daily running cost. Convert to dollars and round to the nearest cent.	0.9 × 18.5 = 16.65c = \$0.1665 = \$0.17 per day
(b) **(i)** **1** Convert the power of the appliance to kW. Convert the time to hours.	4300 W = 4.3 kW Time = $\frac{45}{60}$ h = 0.75 h
2 Multiply the power by the number of hours the appliance is in use to find the daily energy consumption in kWh.	4.3 × 0.75 = 3.225 kWh
(ii) Multiply the daily consumption by the tariff to find the daily running cost. Convert to dollars and round to the nearest cent.	3.225 × 18.5 = 59.6625c = \$0.596 625 = \$0.60 per day

Gas

Most households in Australia have gas supplied to them. Gas can be used for heating, cooking and hot water services. The volume of gas consumed (in cubic metres; m^3) is measured by a gas meter.

To calculate the cost of the energy produced by burning gas, the gas supplier must convert the volume of gas burnt to an amount of energy in megajoules (MJ; 1 MJ = 1 000 000 J) in the following way:

> Energy used (MJ) = volume of gas (m^3) × heating value (MJ released by burning 1 m^3 of gas) × correction factor (to adjust for differences in pressure)

origin

Origin Energy (Vic) Pty Ltd
A.B.N. 11 086 013 283

Enquiries & moving address	13 24 61
Emergencies or Leaking Gas (24 hr)	1800 676 300

www.originenergy.com.au

Gas Account **Tax Invoice**

MR AND MRS A B SAMPLE
11 SAMPLE ST
SAMPLETOWN SAMPLE 1234

Customer Number	0000 1111 22
Invoice Number	123-4567
New Charges (Details over)	$190.94
Due Date	13/07/07
Total Owing	$190.94

Tax Invoice
Payment Summary Issued on 27/06/07 Page 1 of 2

Previous invoice amounts	We received thank you	Current credits	New charges	Amount due
$34.55	$34.55 CR	$0.00	$190.94	$190.94

Property Location
11 SAMPLE ST SAMPLETOWN 1234

Reading Date	20/06/07 = 62 days
Approx Next Reading	17/08/07
Last Bill Reading	19/04/07

Page 2 of 2

Meter Readings	Present	Previous	Metered Volume	Correction Factor	Corrected Volume	Heating Value	MJ's Used
	430	61	369	1.0272	379.0368	38.23	14,491

Gas Charges	Tariff (Domestic) 616 - GreenEarth Gas			
PEAK: 10-07-2009 to 08-09-2009 (Supply Charge - 60 Days)				26.26
	First	6,000 MJ's @	1.3629 c per MJ	
	Next	3,000 MJ's @	0.9474 c per MJ	
	Balance	5,491 MJ's @	0.6761 c per MJ	
GST Charge (for this invoice)				
			Total New Charges	

Worked Example 23

WE23

Look at the bill, issued by Origin Energy for the gas supplied to a domestic household.

(a) What is the billing period (how many days of usage are covered by this bill)?

(b) What volume of gas was used in this period?

(c) How many megajoules (MJ) of energy was this volume equivalent to?

(d) Calculate the total cost of the gas used over this period.

(e) Calculate the total bill (including GST).

Thinking	Working
(a) Find the information on the bill. (Here the number of billing days is next to the Reading Date on the right-hand side.) Check the billing period is correct by finding the number of days from the Last Bill Reading date to the current Reading Date.	(a) Billing period = 62 days (19 April to 20 June)
(b) The volume of gas used is the difference between the present meter reading and the previous meter reading. (It is shown on this bill next to the Present and Previous readings as Metered Volume.)	(b) Volume of gas used = 369 m^3 (= 430 − 61)
(c) Find the information on the bill. (On this bill, the number of MJ is shown at the end of the Meter Readings row of numbers.) The number of MJ can be calculated by multiplying the Metered Volume, the Correction Factor and the Heating Value together.	(c) MJ used = 14 491 $(= 369 \times 1.0272 \times 38.23)$
(d) Multiply the tariffs by the number of MJs used. (Here, there are three tariffs, one for the first 6000 MJ used, one for the next 3000 MJ and one for the balance.)	(d) Cost of gas usage $= (6000 \times 1.3629) + (3000 \times 0.9474)$ $+ (5491 \times 0.6761)$ = 14 732.0651 c = \$147.32
(e) Add the cost of gas usage calculated in **(d)** to the gas supply charge. Calculate 10% GST on this total and add it on.	(e) Total bill = gas usage + supply charge + GST = \$147.32 + \$26.26 + GST = \$173.58 + (10% of \$173.58) = \$173.58 + \$17.36 = \$190.94

Water

The water usage of households is also measured by a meter. Households are billed according to the number of kilolitres (1 kL = 1000 L) they have used. Water bills also contain charges for supplying water to the house and for carrying sewage away from the house.

1.7 The cost of resources

Navigator

Answers page 601

Q1, Q2, Q3, Q4, Q5, Q7, Q9, Q10, Q12, Q13	Q1 (d)–(h), Q2, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13	Q1 (e)–(h), Q2, Q4, Q6, Q7, Q8, Q10, Q11, Q12, Q13

Fluency

WE22

1 For the following household appliances, calculate **(i)** the daily energy consumption, and **(ii)** the daily running costs for a peak period tariff of 18.5 cents/kWh.

(a) A 270-W television switched on for 5 h a day

(b) A 950-W dishwasher, on for 1 cycle of 48 min a day

(c) A 1950W electric kettle used for 13 min a day

(d) A 200-W computer (including the monitor) switched on for $2\frac{1}{2}$ h a day

(e) A 2400-W heater, used for 8 h a day

(f) A 1600-W refrigerator, on all day

(g) A 1700-W electric oven, on for 45 min a day

(h) A 1500-W air conditioner, on for $13\frac{1}{2}$ h a day

red™ we're living energy

Gas account

Enquiries 131 806
8am - 8.30pm Monday - Friday / 9am - 5.30pm Saturday (AEST)
redenergy.com.au
enquiries@redenergy.com.au

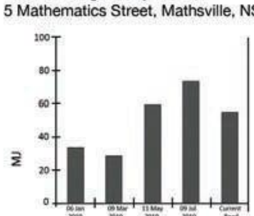
Red Energy Pty Ltd - ABN 60 107 479 372

Tax Invoice/Statement/Adjustment Note

Faults & Emergencies 24 hours	1800 676 300 ENVESTRA
Customer No.	652682
Due Date	28 SEP 10
Total Due	

Mr John Citizen
5 Mathematics St
Mathsville NSW 2222

Your average daily use at:
5 Mathematics Street, Mathsville, NSW 2222



Average daily cost for this account (incl. discount): $1.17
Average daily usage for this account: 54.95 MJ

ISSUE DATE	10 SEP 10
TRANSACTIONS SINCE PREVIOUS ACCOUNT	
Previous Invoice Amount	$87.43
Pay On Time™ Discount (incl GST of $0.56)	$6.12 Cr
Payment Received - Thank You	$81.31 Cr
Balance Brought Forward	$0.00
CURRENT TRANSACTIONS	
Gas Charges	
TOTAL CURRENT TRANSACTIONS	
TOTAL AMOUNT DUE	

Current Transactions

Gas Charges — Charges based on actual read

Your Plan Peak Economy — MIRN 53214193124
From 10 July 2010 to 07 September 2010 (60 days)

Tariff Description	Meter Number	Previous Reading	Current Reading	Base Usage	Heating Value	Pressure Factor	Usage MJ	Rate c/MJ	Charges
Peak	6145FY:1	5525	5609	84	38.8266	1.0109	3297		
Peak Step1							3180	1.330	
Peak Step2							117	1.165	
Total Peak							3297		
Service to Property Charge							60 days	42.000 c/day	$25.20
GST									
Total Gas Charges									

WE23

2 From the gas bill shown, issued by Red Energy for the gas supplied to a domestic household:

(a) What is the billing period (how many days of usage are covered by this bill)?

(b) What volume of gas was used in this period?

(c) How many megajoules (MJ) of energy was this volume equivalent to?

(d) What was the total cost of the gas used over this period? (Find the cost of the gas used for each tariff—Peak Step 1 and Peak Step 2—then add them together.)

(e) Calculate the total bill, including GST.

3 Estimate, then calculate, which of the following pairs of appliances uses more kWh of energy:

(a) a 40-W games console in use for 80 min or a 180-W computer (PC and monitor) in use for 20 min

(b) an 820-W dishwasher completing a 45 min cycle or a 1100-W washing machine (using hot water) completing a half hour cycle.

Understanding

4 Here is a copy of the Simpson family's water bill.

Blue Water ABN 00 000 000 000

Mountain Water
simple clean fresh

Mr & Mrs Simpson
18 High Street
Highton Vic 3997

Quarterly Account

Enquiries	13 0000
Faults (24 hrs)	13 0000
Account Number	**a000 000**
Invoice Number	**000 0000 0000**
Total Due	**$267.35**
Due Date	**27 January 2010**

Your financial institution will be debited on this due date.
Please see over.

Tax Invoice Issues 06 Jan 2010

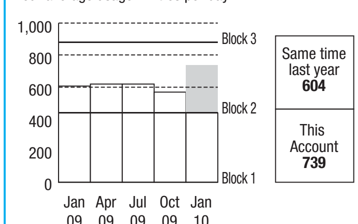
Account Summary

18 High St, Highton
Property Number 0000 0000, LOT 00 PLAN 00000

Product/Service	Description	Amount
Water Usage	05 Oct 09 to 05 Jan 10	
	Block 1 @$1.2532 per kilolitre $50.73	
	Block 2 @$1.4702 per kilolitre $40.46	$91.19
Sewage Disposal	05 Oct 09 to 05 Jan 10	$81.26
Service Charges	01 Jan 10 to 31 Mar 10	$77.99
Waterways and Drainage Charge on behalf of Melbourne Water		$16.91
TOTAL (GST does not apply)		$267.35

Compare Your Usage

Your average usage in litres per day



Account Details

Water Usage from 05/10/2009 to 05/01/2010

Meter Number	Current Reading		Last Reading		Usage
MA632611	3,533kL	–	3,465kL	=	68kL

In 92 days you used 68.000 kilolitres, equalling 739 litres per day.
One kilolitre (kL) equals 1,000 litres.

Usage*			Price $/kL		Amount
BLOCK 1	40,480	·	1.2532	=	$50.73
BLOCK 2	27,520	·	1.4702	=	$40.46
Total	68,000				$91.19

*Rising block tariffs are adjusted according to the days in your meter reading period, and applied on a daily basis.

Sewage Disposal from 05/10/2009 to 05/01/2010.
For the disposal and treatment of sewage from your property. It is based on your water usage and adjusted for seasonal variations.

Usage		Seasonal Factor		Seasonal Volume		Discharge Factor		Sewage Volume
68,000kL	·	0.8779	=	59.695	·	0.900	=	53.725kL

Sewage Volume		Price $/kL		Amount
53.725	·	1.5126	=	$81.26

Look closely at the bill to find the following:

(a) Besides water usage, what other charges appear on the bill?

(b) What amount has been charged for actual water usage?

Hint
Look in the 'Account Details' section.

(c) How many days in the billing period?

(d) How many kL were used in this period?

(e) There are four people in the Simpson family. On average, how many litres per day did each one use during this period? Give your answer to the nearest litre.

5 In the Patel household, five loads of washing are done each week. Each load is dried in a 4500-W drier. One cycle of the drier takes 50 min.

(a) How much energy (in kWh) would they save each week if they dried their washing on a clothes line instead of using the dryer?

(b) How much money would they save if their electricity tariff was 19.8 c/kWh?

6 Gianni has bought six 15-W energy-efficient compact fluorescent light bulbs (CFL) to replace six 75-W inefficient incandescent globes in his house. (One 15-W CFL provides the same level of light as one 75-W incandescent globe.) He estimates that each globe is on for approximately 3 h a day. Calculate:

(a) how much energy Gianni will save each day

(b) how much money he will save in 1 year by replacing six light bulbs if his electricity tariff is 19 cents/kWh.

7 For this question you will need to refer back to the Simpson family's water bill in Question **4**. The family measure the flow rate of their showerhead and find it delivers 16 litres of water every minute (16 L/min). They now install a new efficient showerhead that delivers 9 L/min.

(a) How much water will they save per shower if their average shower time is 7 min?

(b) If each of the four members of the family has one shower a day, how many kL will the family save over the period of their water bill? Answer to the nearest kL.

8 When TVs and DVD players are switched on and off with a remote control they go into 'stand-by' mode. Typically, a TV in stand-by mode uses 10 W of power and a DVD player uses 7 W. Nerida estimates that her TV is in stand-by mode for 20 hours a day and the DVD player for 22 hours a day.

(a) Calculate the daily energy consumption (in kWh) of the TV and DVD player in stand-by mode.

(b) Use your answer from **(a)** to calculate the daily cost of leaving the TV and DVD in stand-by mode if the tariff is 19 cents/kWh.

(c) Calculate the yearly saving Nerida could make on her energy bill if she turned both the TV and DVD completely off instead of turning them to stand-by mode.

Reasoning

9 In the water bill shown in Question **4**, two tariffs (Block 1 and Block 2) are used to calculate the water usage charge.

(a) What is the price difference between the two tariffs?

(b) Suggest a reason why the second tariff is applied after a certain number of kL have been used.

10 Utility companies often provide a graph on the bill that enables people to track their consumption over time. The graph that follows appeared on an electricity bill. It gives the average daily usage (in kWh) for each billing period of the last year as a bar graph. The line graph shows the total greenhouse gas emissions (in the form of carbon dioxide, CO_2) for each billing period. From the graph:

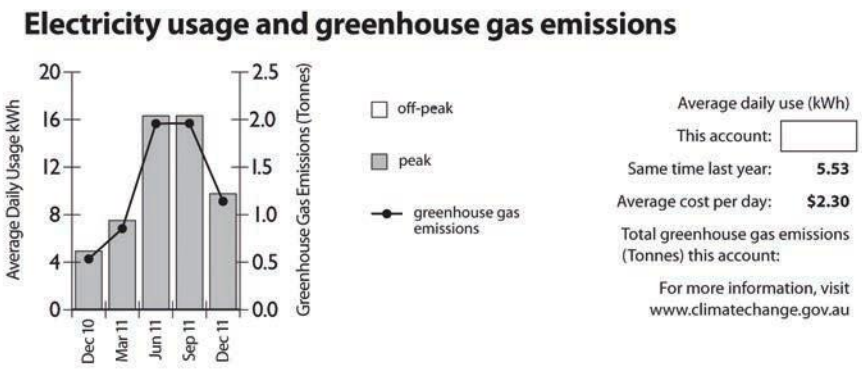
(a) Estimate the average daily use for the most recent billing period (Dec 11).

(b) Estimate the number of tonnes of CO_2 emitted during this period.

(c) If the billing period was 90 days, how many kg of CO_2 were emitted, on average, each day?

(d) What does the shape of the graph tell you about electricity consumption by this household over the year shown?

(e) Suggest a reason to explain the shape of the graph.



Open-ended

11 From the list below, choose two of the following appliances that you use on a daily basis. Estimate the number of hours a day that you use each of the appliances. Calculate how much money you would save in 1 year if you halved the amount of time you spent using each of the appliances. Assume a tariff of 19 cents/kWh.

Appliance	Typical power rating
Games console	25 W
Computer (PC and monitor)	190 W
Stereo	307 W
Television (LCD)	270 W
Television (plasma)	311 W

Ratings taken from Origin Home Energy Project site.

12 Make a realistic estimate of the average time it takes you to have a shower.

(a) If your showerhead delivers 15 L/min, how many litres are you using per shower?

(b) How many kL is this a year (assuming you have one shower every day)?

(c) If your water supplier charges $1.26/kL, how much do your showers cost in 1 year?

(d) Calculate how many kL and how much money you would save in 1 year if you reduced your daily shower time by 2 min.

13 (a) Make a list of the different ways in which gas can be used in a home.

(b) Considering your answer to part **(a)**, give a reason to explain the shape of the graph on the gas bill in Question **1**.

1.8 Communication costs

Mobile phones and the internet have become essential communication tools for many people, especially young people. Most Australian households also have a regular phone (known as a 'fixed line' or 'landline') and access to the internet.

Calculating the cost of calling, messaging and downloading has become a complex process, with many phone and internet companies offering hundreds of different arrangements for mobile and home phone use.

Payment for mobile phone use falls into one of two categories:

- 'Prepaid', where you purchase a certain amount of credit, say \$20, and 'charge' your phone with this credit. When you make a call, the cost of the call is deducted from the credit. Once you run out of credit, you must purchase more credit before you can make more calls.
- 'Post-paid', where you enter into a contract with the phone company to use their network at certain call rates. You are billed every month for your phone usage. The length of the contract is usually 12 or 24 months. Most people with a post-paid arrangement have a 'cap plan' (see 'Different types of charges', below).

Different types of charges

- Call rate

  The amount you pay per minute to make a phone call. Different types of calls, such as local, national and international calls, as well as calls to mobiles, often have different call rates.
- Flagfall

  A one-off charge, sometimes called a connection fee, charged as soon as a call is connected.
- Call cost

  Calculated by multiplying the call rate by the number of minutes the call was made for (rounded up to the nearest minute), then adding the flagfall (if there is one).
- SMS

  An abbreviation of 'Short Message Service', known more commonly as a 'text message' or just a 'text'.
- MMS

  An abbreviation of 'Multimedia Message Service'. This is similar to SMS, but is used for sending pictures, videos or music files to another mobile.
- Cap plan

  The user pays a set amount per month that covers all calls and messages (and often data) up to a certain, higher value. For example, a \$29 cap may give the user the equivalent of \$150 worth of calls, messages and data. If this amount is exceeded, the calls, messages and excess data are charged individually, sometimes at higher than usual rates. A plan like this means undertaking a contract with the phone company. If you break the contract, extra fees will be charged (see 'Early exit fee').

- Minimum cost/minimum spend

  The minimum amount that will need to be paid over the period of a contract (usually 12 or 24 months). Under a cap plan, the minimum spend is the monthly cap amount multiplied by the number of months in the contract. It does not include payment for the phone itself.

- Early exit fee/break fee

  The amount you will be expected to pay if you want to end a phone contract before the end date. Often it will mean at least paying the rest of the minimum spend. Frequently you will need to pay to 'unlock' your phone from the phone company's network.

1.8 Communication costs

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q9, Q11, Q12	Q1 (c)–(f), Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q12	Q1 (e) & (f), Q2 (b) & (d), Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12

Answers page 601

Fluency

1 Calculate the minimum spend that would be required on:

(a) a \$49 mobile phone plan over 12 months

(b) a \$80 mobile phone plan over 24 months

(c) a \$98 home phone and internet plan over 24 months

(d) a \$19 mobile phone plan over 24 months

(e) a \$40 mobile phone plan over 12 months

(f) a \$198 home phone, internet and TV plan over 24 months.

2 Rafal has a 24-month phone plan with a minimum monthly spend of \$19. His contract states that the early exit fee is the minimum monthly spend multiplied by the number of months left on his contract, plus an extra \$250 if he leaves within the first 6 months. Calculate Rafal's exit fee if he exits:

(a) after 2 months   **(b)** after 5 months

(c) after 1 year   **(d)** after a year and a half.

3 Alice puts \$20 of prepaid credit onto her phone. The prepaid call rate of her phone company is 88 c/min, plus a flagfall of 35c. Text messages are 28c each.

(a) Calculate the cost of a 3-min phone call.

(b) Calculate how much credit Alice has left after making four 3-min phone calls and sending six text messages.

Understanding

4 Jian has recently moved to Australia to study and wants to be able to call his family in China regularly on his mobile. One mobile phone network is offering the following special international plan:

- international calls (landline or mobile): 3 c/min, plus a flagfall of 20c
- national calls (calls within Australia, either landline or mobile): 40 c/min, plus a flagfall of 25c
- SMS or MMS (national or international): 20c
- voicemail deposit: free
- voicemail retrieval: 40 c/min

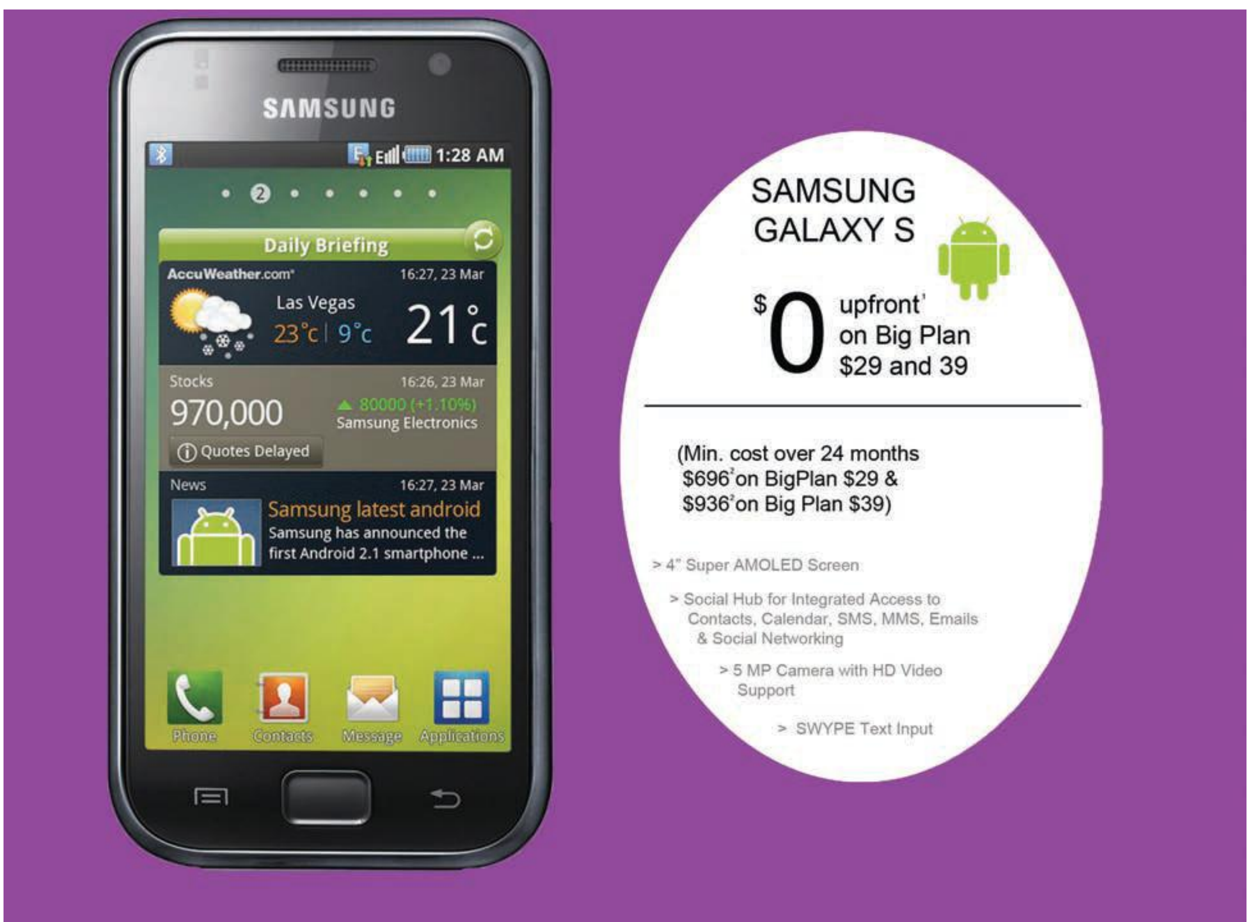
Jian estimates that he will phone his family in China for half an hour, once a week. He thinks that he will make about six 5-minute national calls, send six SMS or MMS messages and check his voicemail for 1 minute, each day.

Calculate Jian's weekly phone bill.

5 The advertisement for this smartphone states that there are no upfront costs if a \$29/month or \$39/month cap plan is purchased. The minimum costs over the 24-month period of both plans are given in brackets. However, this cost does not include the cost of the phone. Read the fine print at the bottom of the ad to answer the following questions.

(a) Find the cost of this particular model of phone, and calculate how much extra you would be paying per month in order to purchase the phone.

(b) Calculate how much you will pay if you choose to leave after 5 months of a \$29/month cap plan contract. Assume you have already paid all the monthly costs up to this point.



The Fine Print: Big Plan 29 and 39 offer availa ble only on selected mobiles. [1]24-month connection required. The Samsung Galaxy S is \$720, HTC Aria is \$648, Sony Ericsson Xperia X10 is \$1008, Samsung Wave is \$643, and the LG Optimus Black is \$648. Each month the cost of the phone/24 months is paid off, which includes your monthly contribution, if any. [2]Leave anytime, just pay all outstanding charges + balance of the phone + \$250 if you leave within 1st 6 months. [3]Plan credit may be used for standard national and international calls, text, MMS, and 1300 numbers (excluding ZMA customer service) only. Standard charges apply if plan credit is exceeded and for non-included call and messaging types. [4]Included data has one-month expiry. Additional usage charged at 0.2c per KB. Plan credit cannot be used for data content changes. [5]Any unused plan credit rolls over to the following month only and is used once the following months credit is exhausted. Data does not rollover.

6 (a) Holly chose the $29/month cap plan from the table below. Use the table to find:

(i) the minimum cost for 24 months

(ii) the value of all the calls and messages included under the monthly cap amount

(iii) how much data (internet) is included each month.

(b) Calculate the cost of a 10-minute phone call and three SMS messages on this plan.

(c) Now read 'the important bits' underneath the table.

(i) How much extra would Holly pay if she used 40 MB of data more than the amount allowed under the cap?

(ii) What does the statement 'Any unused monthly data cannot be rolled over' mean?

(d) Below is a table of call rates for prepaid mobile phone plans from the same phone company. Calculate the cost of a 10-minute phone call and three SMSs.

Rates for your plan

Prepaid Mobile rates (excludes $2 Days)

Standard National Voice Calls (charged per min increments)	89c per minute
Flagfall	39c
Standard National SMS	29c (160 characters max)
Standard National MMS	50c (160 characters)
National Video Calling	50c per 30 seconds + 35c flagfall (Turbo Cap Plus) $1 per minute + 35c flagfall (Crew Cap) 50c per minute + 35c flagfall (Long Expiry Cap)
Zoo Browsing + Mobile Internet	$2 per 1MB
VoiceMail retrieval (charged per 30 second increments)	89c per minute + 39c flagfall
VoiceMail deposit	FREE

"I want great value for talk, text and web"

What's included	$19	$29	$49	$59	$79
Included value	$70	$180	$550	$750	$900
Included mobile internet data value (excluding BlackBerry® – See page 14 for details)	100MB	200MB	1.5GB	2GB	3GB
Mobile Access within Australia to	UNLIMITED				
Unlimited standard national SMS to Australian mobiles	N/A	N/A	N/A	✔	✔
Unlimited standard national MMS to Australian mobiles	N/A	N/A	N/A	N/A	✔
Calls to Optus	'yes' for 5. Included 5-minute national voice calls (up to 1,000 minutes per month) to Australian GSM mobiles on the same account, 24/7. Standard rates apply after first 5 minutes.				
Smart safe storage	500MB FREE				
Minimum total cost over 12/24 months	$228/$456	$348/$696	$588/$1,176	$708/$1,416	$948/$1,896

Cap rates	$19	$29	$49	$59	$79
Call rate (per min*)	90c				
Flagfall	35c				
Standard national SMS (per message)	25c			Unlimited	
Voicemail	$0 deposit and 90c/min retrieval		$0 deposit and 30c/30 sec retrieval		

The important bits.

1.Cancellation fees apply for all plans. 2. Data usage will be counted in kilobytes, and includes both uploads and downloads. Any unused monthly data cannot be rolled over. Excludes data used while roaming internationally. **Excess Usage Data charges:** 50c per MB or part thereof on the $19/$29 Optus Cap; 25c per MB or part thereof on the $49/$59/$79 Optus Caps and Timeless Extreme Plans. **3. Unlimited Mobile Access to Facebook®, Twitter®, LinkedIn, Myspace, eBay, Foursquare within Australia:** unlimited mobile access to these services within Australia only. Use of these services is separate and does not count towards your included Mobile Internet Data Value. These features are only available with compatible handsets.

Reasoning

7 Kwok has a mobile phone plan that charges him 85 cents/min plus a 29 cent flagfall for calls (to other mobiles and to landlines). At home, Kwok's family has a landline phone, but Kwok often uses his mobile instead. The call rates on the landline are as follows:

- calls to landline numbers: 30 cents (regardless of length)
- calls to mobiles: 80 cents/min (no flagfall)

Calculate how much cheaper it is for Kwok to use the landline phone instead of his mobile to make a 10-minute call:

(a) to a local landline **(b)** to a mobile.

8 Johanna buys prepaid credit for her mobile. She is finding that she is constantly 'topping up' the credit every few days. Johanna is charged the following rates by her phone company:

- calls (to mobiles or landlines): 78 c/min, plus a flagfall of 39c
- texts (SMS): 22c each

Johanna estimates that, on average, she makes four phone calls (each call is approximately 5 minutes long) and sends about 10 text messages every day.

(a) Calculate the cost of one of Johanna's 5-min phone calls.

(b) Calculate Johanna's daily phone bill.

(c) Calculate Johanna's monthly phone bill (assume that there are 30 days in a month).

Johanna decides to go on a $79 cap plan for her mobile, which gives her $600 worth of calls and texts every month. Once she exceeds this amount, calls are billed at a rate of 80 c/min, plus a flagfall of 35c per call, and text messages are 25c each.

(d) If Johanna maintains her current phone habits and stays underneath the cap, how much money will she save **(i)** every month and **(ii)** every year?

(e) If Johanna has reached the cap limit of $600, what would her average daily phone use cost her under the rates of this plan?

Open-ended

9 Athena's phone company charges 82 c/min per standard call, plus a flagfall of 30c. Ordinary text messages (SMS) cost 27c each, whereas picture text messages (MMS) cost 50c. Athena has $20 of credit on her phone.

How many calls and texts could Athena make before running completely out of credit?

Give three different combinations, making sure that you include calls of different time lengths.

10 **(a)** Compare your answers to Questions **6 (b)** and **(d)**. Is the difference in the call and SMS costs between the plan and prepaid rates significant?

(b) List some reasons why people may choose a prepaid method of using their phone instead of a cap plan.

11 Consider your own mobile phone use (if you don't have a mobile phone, consider someone you know).

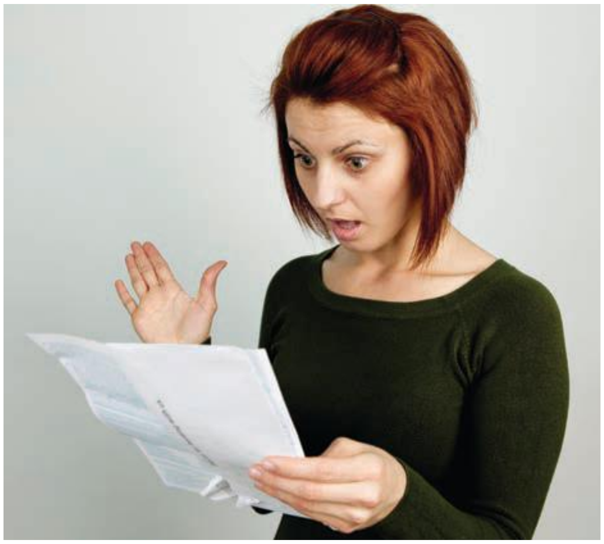
(a) Approximately how many calls do you make per day, and how many minutes per call?

(b) How many SMS and/or MMS messages would you send?

(c) If you know the call rates for your phone, use them to calculate the cost of your daily phone usage. If you don't know them, use a rate of 80 cents/min and a flagfall of 35 cents for calls and 25 cents per SMS or MMS.

12 (a) Some people find it difficult to estimate how much data they are using when using their phone to browse the internet or download a song or app. Find out what a data allowance of 200 MB a month is equivalent to in terms of time online or downloaded files.

(b) In recent years, many people have experienced 'bill shock', an unexpectedly high phone bill, often due to a high cost of data for internet usage. Should phone companies warn people when they are nearing their data limit for the month?



Outside the Square Puzzle

Sudoku

Fill in the empty squares on the grid so that every row and column and each of the mini 3 × 3 grids contains the numbers 1 to 9 once each.

	2			3	6			
3						1	5	7
	8	9			5	6		
2	9		8	4				5
	3		9		1		7	
6				5	2		1	8
		6	7			4	9	
4	7	2						3
			5	8			2	

Challenge 1

1 Find five consecutive numbers such that the sum of the first and the fifth is 1708.

2 If 7^{2048} is divided by 100, what is the remainder?

3 Which numbers less than 100 have 5 as their smallest prime factor?

4 If m and n are different whole numbers, what is the smallest value of m for which $m^3 = n^4$ is possible?

5 A palindromic number reads the same backwards as it does forwards. There was one palindromic date in 2002, 20/02/2002 (20 February 2002). In the USA, dates are written with the month number first, then the day number, then the year. For example, 15 April 2003 is written as 04/15/2003.

(a) Write down three palindromic dates in Australia for the 21st century.

(b) Write down three palindromic dates in the USA for the 21st century.

6 Copy and complete the pattern below:

0, 3, 9, 18, 30, _____, _____, _____

7 Copy and fill in the missing numbers if the difference between each term in the sequence is the same:

$\frac{1}{12}$, _____, _____, $\frac{1}{7}$

8 If it takes 12 students 30 min to clean up 100 dishes after lunch on a school camp, how long will it take eight students to clean up 90 dishes?

9 What percentage is m of n?

A $\frac{n}{m}$ B $\frac{100n}{m}$ C $\frac{m}{n}$ D $\frac{100m}{n}$

10 When 1000^{2010} is written as a numeral, the number of digits written down is

A 2010 B 2011 C 6030 D 6031

11 Successive discounts of 10%, 20% and $33\frac{1}{3}$ % are equivalent to a single discount of

A 48% B 50% C 52% D $63\frac{1}{3}$ %

Questions **12** and **13** may have more than one answer. List all the letters corresponding to correct answers.

12 Which of the following will simplify to 160?

A $\dfrac{40}{\frac{1}{4}}$ B $1000 \div 160\,000$ C 80×20 D $3.2 \div 0.2$

13 Which of the following will simplify to 5?

A $\sqrt{13^2 - 12^2}$ B 2.5^2 C $29 - 4 \times 6$ D $40 \div 4 \times 2$

Chapter review 1

D.I.Y. Summary

Key Words

award	earnings	net income	salary
buying on terms	EFTPOS	overtime	simple interest
cheque	gross income	PAYG	superannuation
commission	income	piece work	tax
credit card	income tax	principal	tax deduction
debit card	interest	profit	tax return
deferred payment	lay-by	rate of interest	taxable income
discount	loss	retainer	wages

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1. Simple interest is calculated by multiplying the ___________, the ___________ (in decimal form) and the time period.
2. Before tax and other deductions are made, a person's earnings are referred to as ___________.
3. When a business sells a product for more than it was bought or made for, a ___________ has been made.
4. ___________ is a payment based on a percentage of sales made.
5. Work-related expenses and donations to recognised charities are both examples of ___________s.
6. When a person works ___________ they should be paid at a rate higher than their normal rate of pay.
7. What does EFTPOS stand for?
8. Explain the difference between salary and wages.
9. What are some of the extra costs associated with buying on terms?
10. The set of rules outlining working conditions for employees, including wages, overtime and leave, is called an ___________.
11. ___________ is an amount of money paid into an account or fund to be accessed at retirement.
12. What is a retainer?
13. Explain the differences between gross income, net income and taxable income.
14. Explain the difference between 'buying on terms' and using 'lay-by'.

Fluency

1. Calculate the following: Ex. 1.1
   (a) 130% of $47.85 (b) 0.5% of $52 000
2. (a) Ami is a hockey goalkeeper. In one game she stopped 13 of 16 shots on goal. Calculate Ami's 'stopping' percentage. Ex. 1.1

(b) A box of soft-centred chocolates is on sale for \$7.34, a saving of \$2.45. Calculate the dollar saving as a percentage saving, correct to one decimal place.

3 Calculate the selling price on the following items. Give answers correct to the nearest cent where necessary. Ex 1.2

(a) A \$75 shirt discounted by 30%   **(b)** A \$29.99 DVD discounted by $12\frac{1}{2}$ %

4 A timber dining table cost \$800 to construct and was sold for \$1250. The percentage profit, based on cost price, is closest to: Ex 1.2

A 36%   **B** 43.75%   **C** 56.25%   **D** 64%

5 The Goods and Services Tax (GST) is a 10% tax added to the price of most retail items. Ex 1.2

(a) Reprice the following so that the prices include GST:

(i) a \$5.60 bottle of shampoo   **(ii)** a \$37.35 hairdryer

(b) The retail prices of the following items have GST included. Calculate the 'pre-GST' price of:

(i) a \$129 calculator   **(ii)** a \$38.95 computer game

6 Andrea works as a casual in a supermarket. She earns \$9.45 per hour for hours worked between 7 a.m. and 7 p.m., time-and-a-half for any hours worked after 7 p.m. or on Saturday and double time for any hours on Sunday. Ex 1.3

Calculate Andrea's wage for the following week.

Monday	Wednesday	Friday	Saturday	Sunday
8 a.m.–4 p.m.	8 a.m.–4 p.m.	1 p.m.–10 p.m.	10 a.m.–3 p.m.	10 a.m.–3 p.m.

7 Nazim is picking grapes in a vineyard. He is paid \$4.75 for every bucket of grapes picked. How much can Nazim earn in a 5-day week if he picks an average of 35 buckets in a day? Ex 1.3

8 How much income tax is payable on the following taxable incomes? (Use the tax table on p. 31.) Ex 1.4

(a) \$5900   **(b)** \$47 890   **(c)** \$62 780

9 Evan borrows \$7000 to purchase a car. He will pay 8.5% simple interest over a period of 4 years. Ex 1.5

(a) How much interest is payable?

(b) How much will Evan pay back altogether?

(c) If Evan makes monthly repayments, how much will these repayments need to be, to ensure he pays off both the principal and the interest in the 4 years?

10 Olivia has a credit card with an annual interest rate of 17.3%. The card has an interest-free period of 55 days. She has just received a statement covering the period from 12 June to 11 of July. The average daily balance owing on the card over this period was \$4509.82. Calculate the interest that will be payable if Olivia does not pay the full amount owing by the end of the interest free period. Ex 1.6

11 For the following household appliances, calculate **(i)** the daily energy consumption, and **(ii)** the daily running cost, if the tariff is 19.8 c/kWh. Ex 1.7

(a) A 215-W computer on for 9 hours a day.

(b) A 1750-W refrigerator on all day.

(c) A 900-W microwave oven used for 15 minutes a day.

12 Chen has a mobile phone plan that charges him 89c/min plus a flagfall of 30c to make calls. It costs 25c to send an SMS. Chen estimates that he makes three 5-min calls per day, one 10-min call and sends six SMS messages. Calculate the cost of Chen's daily phone usage.

Ex. 1.8

Understanding

13 Peter is a salesperson for an audiovisual company. He is paid a weekly retainer of $250 plus 3.5% of the value of his sales. One week, Peter sold products worth $4500, $7890, $2630 and $6100. Calculate his income for the week.

14 Calculate the percentage rate of simple interest paid if a $5000 investment over $6\frac{1}{2}$ years gave a return of $7730.

15 A shop purchases crystal vases for $75 each.

(a) At what price should they be sold if there is an 85% mark up?

(b) The vases are not selling well so they are discounted by 10%. What is their new selling price?

(c) What percentage mark up does the discounted price represent on the cost price?

16 Effie works as a cleaner and is paid $19.50 per hour for a standard 38-hour week. If she works overtime she is paid time-and-a-half for the first 2 hours of overtime each day and double time for any hours after that. In one week Effie worked 3 hours overtime on Tuesday, 1 hour overtime on Wednesday and 4 hours overtime on Friday. Find her wage for the week.

17 Jenny has a gross weekly income of $860. She donates $750 per year to registered charities, and has work-related expenses totalling $690 annually.

(a) What is Jenny's annual gross income, assuming there are 52 weeks in a year?

(b) What is Jenny's taxable income?

(c) How much should Jenny pay in tax per year?

18 Natalie purchased an $80 necklace with lay-by. She had to pay a 20% deposit to begin the lay-by.

(a) How much was the initial payment?

(b) If Natalie paid $15 per week, how many weeks did she take to pay off the necklace?

19 A monthly statement for a bank account offering 7% p.a. has the entries shown for the month of June below.

(a) How much interest is payable for June if interest is calculated:

(i) daily?

(ii) on the minimum monthly balance?

(b) By how much is **(i)** greater than **(ii)**?

Date	Transaction	Balance
June 1	Opening balance	$100
June 14	Deposit $285	$385
June 21	Withdrawal $207	$178

20 Andrew has purchased a $7000 boat on terms, with an interest-free period of 24 months. He must pay a $25 establishment fee, a monthly account fee of $3.50 and a minimum monthly repayment of $60.

(a) **(i)** How much will Andrew have paid off the cost of the boat after 24 months if he only makes the minimum repayments?

(ii) How much will still be owing?

(iii) How much will Andrew have paid in fees?

(b) If Andrew wants to have the boat paid in full by the end of the interest-free period, how much should his monthly repayments be?

(c) If Andrew has not paid the amount owing in full at the end of 24 months, an interest rate of 29% p.a. is charged. How much interest will be charged on the amount calculated in part **(a) (ii)** after the 25th month? (Assume 30 days in a month.)

Reasoning

21 Freya is 19 and has a part time position at a supermarket. Her hourly rate for weekdays is $15.43, with time-and-a-half paid on Saturdays, and double time on Sundays.

Tanya is also 19 and works in the same supermarket. Because she is a casual employee, her conditions are different to Freya's. Tanya gets an extra 25% on the hourly rate on weekdays. On weekends she gets a further 10% on top of her casual rate.

Both Freya and Tanya work for 18 hours during the week, 4 hours on Saturday and 4 hours on Sunday.

(a) Who earned more money?

(b) Why do casual workers receive an extra 25% on top of their hourly rate?

22 Annika works in a jewellery store. Her manager has asked her to place a mark up of 78% on the cost price of all pairs of earrings. These earrings were later discounted by 20% during a mid-year sale. Find the overall profit or loss made by the store on a pair of earrings sold in the sale if the initial cost price of the earrings was $46.60.

23 The Vu family have a 4500-W clothes dryer that takes 1 hour to complete a drying cycle. On average, the dryer completes three cycles a week. The Vu's electricity tariff is 19.5 c/kWh. Calculate the amount of money the family could save in a year if they stopped using their dryer.

24 Rachel buys prepaid credit for her phone. She is finding that she is constantly 'topping up the credit' every few days. Rachel is charged the following rates by her phone company:

- calls (to mobiles or landlines): 80c/min, plus a flagfall of 35c
- texts (SMS): 29c each
- data (using the internet): $2 per MB

Rachel estimates that in a typical day she makes three 5-min calls, sends six SMS messages and spends approximately 15 min browsing the internet on her phone, which uses about 2 MB of data.

(a) Calculate the cost of Rachel's monthly phone bill, given her typical daily usage above. Assume there are 30 days in a month.

(b) Rachel is thinking of moving to a $59 cap plan for 24 months, which would give her $600 worth of calls, texts and data each month. If Rachel went on the plan and stayed under the monthly cap, how much would she save in the 24-month period compared with using prepaid credit?

NAPLAN practice 1

Numeracy: Non-calculator

1 Julie bought 36 cans of soft drink for a party. 75% of the cans are empty at the end of the party. How many are still full?

A 4 B 9 C 16 D 27

2 Ali has 240 DVDs in his collection. Two-thirds of the DVDs are animated movies. Of the rest, 20% are action movies. How many action movies does Ali have in his collection?

A 16 B 48 C 80 D 160

3 The GST is a 10% tax added to the price of many retail items. A toaster costs $33.50 before the addition of GST. Which calculation will give the new price of the toaster that includes the GST?

A $33.50 + 10 B $33.50 × 1.1 C $33.50 + 0.1 D $33.50 + 1.1

4 Emilio works part time in a take away food shop. He gets $15.80 an hour, except on Saturdays, when he gets one-and-a-half times his hourly rate for the first 3 hours and double his hourly rate for any hours worked after that.

One week, Emilio works 15 hours between Monday and Friday and 5 hours on Saturday. Which calculation will give Emilio's total wage for the week?

A 15.8×20 B $(15.8 \times 15) + (15.8 \times 1.5 \times 5)$

C $(15.8 \times 15) + (15.8 \times 1.5 \times 3) + (15.8 \times 2 \times 2)$ D $(15.8 \times 15) + (17.3 \times 3) + (17.8 \times 2)$

5 Raelene is going to invest $5000 at 6% simple interest for 2 years. If she invested her money for twice as long, the amount of interest she would earn on her investment would:

A decrease slightly B double C halve D increase slightly.

Numeracy: Calculator allowed

6 Rohan bought a new phone at a 35% discount. If the phone was originally priced at $99, how much did Rohan pay for the phone?

A $64 B $34.65 C $64.35 D $134

7 Of the 2153 books stored on the shelves of the local library; 847 are out on loan. What percentage of the library books are out on loan?

A 28.23% B 39.34% C 71.76% D 254.54%

8 Tyson purchases a $1579 home theatre system on the following terms: 20% deposit up front, with monthly repayments of $85 for the next 2 years. How much more money is paid using this method of payment than paying the full price up front?

9 Yang earns a yearly salary of $57 300. The income tax rules that apply to Yang state that she must pay $4650 plus 30c for every dollar over $37 000. Calculate Yang's annual income tax bill.

10 A large chain of clothing stores has a policy of making 85% profit on all jeans and trousers sold. How much would a pair of jeans with a marked retail price of $79.95 have cost the store? (Answer to the nearest cent.)

2

Pythagoras' Theorem

2

Did Pythagoras 'steal' the theorem that bears his name? One of the world's most famous mathematical statements is not named after its discoverer.

Several ancient civilisations, including the Babylonians, Chinese and Egyptians, knew about the special relationship between the sides of a right-angled triangle. Pythagoras of Samos (approx. 580–496 BC) was a Greek philosopher who probably learnt about it from a group of Egyptian engineers known as the 'rope stretchers'. They used a circle of rope with 12 knots tied at regular intervals around the circle. Pegging the rope to the ground at intervals of 3, 4 and 5 knots produced a right-angled triangle, which was used to ensure that building foundations or walls were constructed accurately. So, why does Pythagoras get all the credit? He was a notable mathematician who headed a secretive cult known as the Brotherhood, which was devoted to the study of mathematics, with their motto being 'all is number'. Although he did not discover the theorem, it is possible that Pythagoras was the first to produce a proof of the theorem using geometry. The photograph on the opposite page is of a statue of Pythagoras on the island of Samos, in Greece.

Forum

Why do you think Pythagoras and his followers held secret meetings and kept their discoveries to themselves?

Think of some real-life objects that can consist of, or be described by, right-angled triangles.

Why learn this?

Pythagoras' Theorem is a useful mathematical tool applied by builders, carpenters, engineers and surveyors to determine the straight-line distance between two points or to check whether an angle is a right angle. It is also used in the audio-visual industry to determine the height and width of TV and projection screens.

After completing this chapter you will be able to:

- use Pythagoras' Theorem to determine whether a triangle is right-angled
- use Pythagoras' Theorem to find side lengths in right-angled triangles
- apply Pythagoras' Theorem to practical situations
- complete simple calculations using surds
- express irrational answers in exact (surd) form
- identify Pythagorean triples.

Recall 2

Equipment required: calculator for Questions **2–4** and **6**.

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R2.1

1 Round each of the following numbers to two decimal places.

(a) 45.789 **(b)** 12.2311 **(c)** 4.549 567 835 6

Worksheet R2.2

2 Use your calculator to find the following, correct to two decimal places where appropriate.

(a) 12^2 **(b)** 55^2 **(c)** 37.5^2

(d) $3^2 + 4^2$ **(e)** $9^2 + 13^2$ **(f)** $2.5^2 + 7.1^2$

Worksheet R2.3

3 Use your calculator to find the exact values of each of the following.

(a) $\sqrt{81}$ **(b)** $\sqrt{169}$ **(c)** $\sqrt{256}$

Worksheet R2.4

4 Use your calculator to find the following. Round your answers to two decimal places.

(a) $\sqrt{65}$ **(b)** $\sqrt{658}$ **(c)** $\sqrt{321.45}$

Worksheet R2.5

5 Solve the following equations.

(a) $x + 5 = 11$ **(b)** $144 + x = 225$ **(c)** $45 - x = 12$

Worksheet R2.6

6 Solve the following for positive values of x. Where necessary, express your answer as a decimal correct to two decimal places.

(a) $x^2 = 36$ **(b)** $x^2 = 39$ **(c)** $x^2 = 325$

(d) $x^2 + 6 = 70$ **(e)** $x^2 - 20 = 101$ **(f)** $\frac{x^2}{4} = 325$

Key Words

converse	proof	rational approximation	theorem
hypotenuse	Pythagoras' Theorem	right-angled triangle	
irrational number	Pythagorean triple	surd	

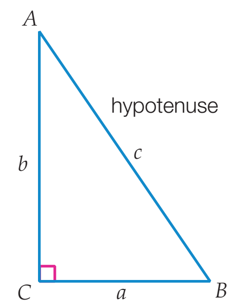
Pythagoras' Theorem and right-angled triangles

2.1

A triangle that contains a 90° angle (a right angle) is called a **right-angled triangle**. The longest side of a right-angled triangle, which is opposite the right angle, is called the **hypotenuse**.

When labelling triangles, we use capital letters for the vertices and lower case letters for the side lengths.

So, triangle ABC on the right, which can be written as ΔABC, has vertices A, B and C with sides a, b and c opposite the vertices that have the same letter. It is usual to label the hypotenuse as side c and the two shorter sides as a and b. (It does not matter which side is a and which is b.)

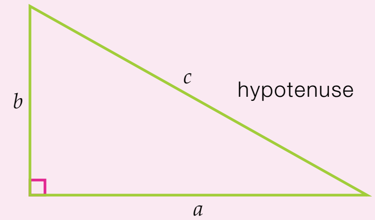


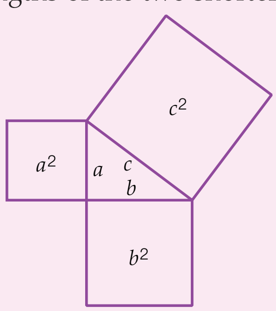
What is a theorem?

A **theorem** is a mathematical statement that can be shown to be true. A **proof** is a step-by-step argument that demonstrates the truth of a mathematical theorem.

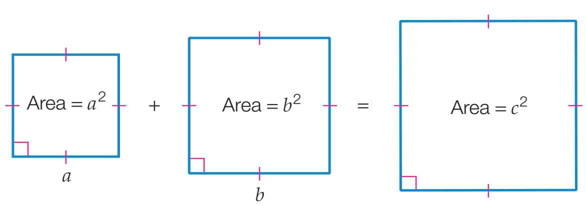
Pythagoras' Theorem: For any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

$c^2 = a^2 + b^2$



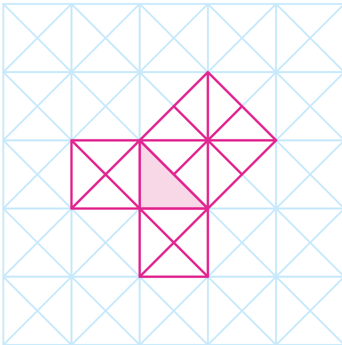


The area of a square of side $a = a^2$. In a right-angled triangle, the areas of the squares drawn onto the two shorter sides add to give the area of the square drawn onto the hypotenuse.



Demonstrating Pythagoras' Theorem

We can visually demonstrate the theorem as shown. If we take an isosceles right-angled triangle and place it onto square grid paper that has been divided into small, identical, triangular tiles, the combined area of the squares on the two shorter sides is eight small triangles, the same number of triangles covered by the square on the hypotenuse.



This shows only that the theorem works for one particular type of right-angled triangle. For the theorem to be shown as true, we must prove that it works for all right-angled triangles.

Proofs of Pythagoras' Theorem by dissection

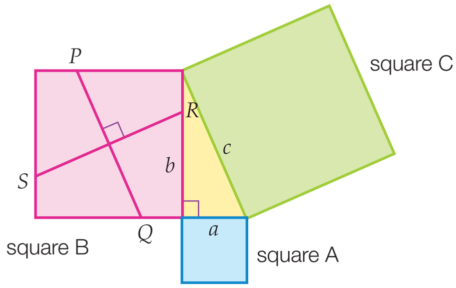
Mathematicians have established at least 85 different methods of proving that Pythagoras' Theorem is true for every right-angled triangle. In 1830, a proof using areas was published by the mathematician Henry Perigal, who had the diagram of the proof carved on his gravestone!

Demonstrating Pythagoras' Theorem using areas

Step 1 On a piece of grid paper, draw any right-angled triangle, and label the sides a, b and c.

Step 2 Construct a square on each side of the triangle as shown below. Label the squares A, B and C to correspond to the sides a, b and c.

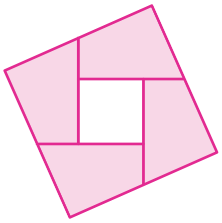
Step 3 Find the centre of square B. Rule a line PQ through the centre, so that it is parallel to the hypotenuse of the triangle.



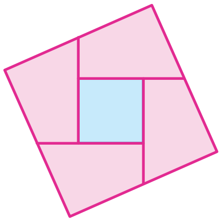
Step 4 Rule a second line, RS, perpendicular (at right angles) to PQ that passes through the centre of the square.

Step 5 Cut square B into its four pieces. Then, cut out square A and square C.

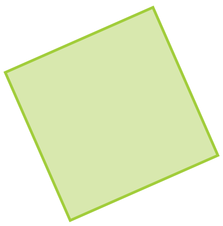
Step 6 Rearrange the four pieces of square B as shown.



Step 7 Place square A in the centre. It should fit exactly. The total area of this square = area of square A + area of square B $= a^2 + b^2$



Step 8 Compare the area of this square to the area of square C, c^2. The two squares should be identical in area, demonstrating that $c^2 = a^2 + b^2$.



The converse of Pythagoras' Theorem

A **converse** is a reversal of a proved theorem.

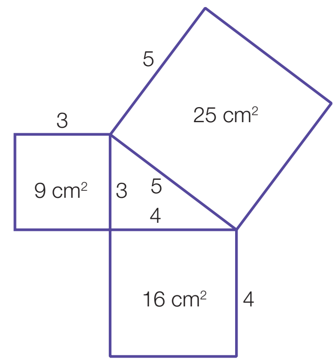
Because Pythagoras' Theorem is only true for right-angled triangles, the converse of Pythagoras' Theorem can be used to check whether or not a triangle is right-angled. That is, if the sum of the squares of the two shorter sides is equal to the square of the longer side, the triangle is right-angled.

Consider a triangle of sides 3 cm, 4 cm, and 5 cm.

We can show this is a right-angled triangle using the converse of Pythagoras' Theorem.

Here, $a = 3$, $b = 4$ and $c = 5$.

Now: $c^2 = 5^2$
$= 25$ and
$a^2 + b^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$



As $c^2 = a^2 + b^2$ holds for this triangle, we can say that this is a right-angled triangle.

> If Pythagoras' Theorem holds, then the triangle must be a right-angled triangle. This statement is known as the 'converse' of Pythagoras' Theorem.

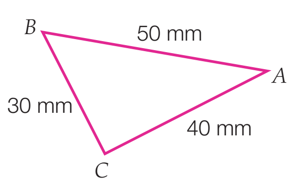
Builders use the converse of Pythagoras' Theorem to make sure building frames and walls are right-angled. They call it 'the 3-4-5 principle'—can you see why?

Worked Example 1

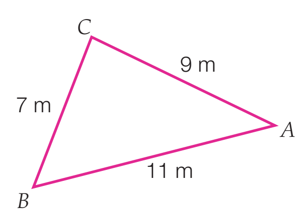
WE1

Use the converse of Pythagoras' Theorem to determine whether each of the following triangles is a right-angled triangle. If it is, state which angle is the right angle.

(a)



(b)



Thinking	Working
(a) 1 Identify the longest side as c and the other two sides as a and b.	(a) $a = 30$ mm, $b = 40$ mm, $c = 50$ mm
2 Evaluate the left-hand side of Pythagoras' Theorem, c^2.	$c^2 = 50^2$ $= 2500$
3 Evaluate the right-hand side of Pythagoras' Theorem, $a^2 + b^2$.	$a^2 + b^2 = 30^2 + 40^2$ $= 900 + 1600$ $= 2500$
4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled.	$c^2 = a^2 + b^2$, so ΔABC is a right-angled triangle.
5 The right angle is the angle opposite the hypotenuse, c.	$\angle ACB$ is the right angle.

(b) 1 Identify the longest side as c and the other two sides as a and b.

(b) $a = 7$ m, $b = 9$ m, $c = 11$ m

2 Evaluate c^2.

$c^2 = 11^2$
$= 121$

3 Evaluate $a^2 + b^2$.

$a^2 + b^2 = 7^2 + 9^2$
$= 49 + 81$
$= 130$

4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled. (Here, the triangle is not right angled, so we use the symbol ≠, which means 'is not equal to').

$c^2 \neq a^2 + b^2$, so ΔABC is not a right-angled triangle.

2.1 Pythagoras' Theorem and right-angled triangles

Navigator

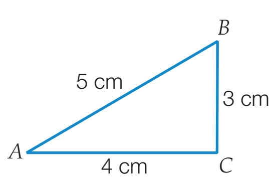
Answers page 603

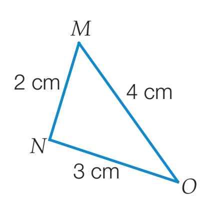
Q1 Column 1, Q2, Q3, Q4, Q5 Column 1, Q7 (a), (b), (d), Q9, Q10, Q11	Q1 Column 2, Q2, Q3, Q4, Q5 Column 2, Q6, Q7 (a), (c), (e), Q8, Q9, Q10, Q11	Q1 Column 3, Q2, Q4, Q5 Column 2, Q6, Q7 (b), (e), Q8, Q9, Q10, Q11, Q12

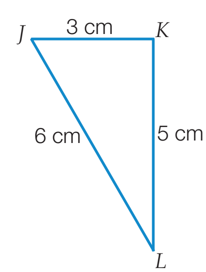
Fluency

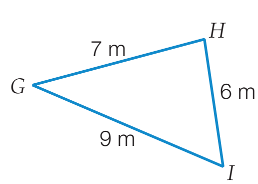
WE1

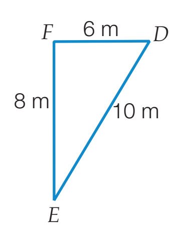
1 Use the converse of Pythagoras' Theorem to determine whether each of the following triangles is a right-angled triangle. If it is, state which angle is the right angle.

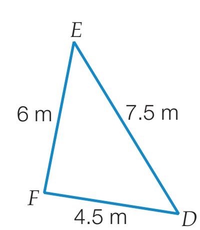
(a)


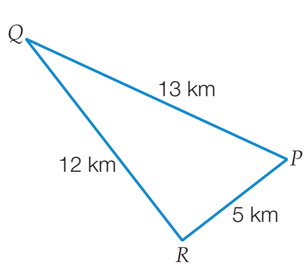
(b)


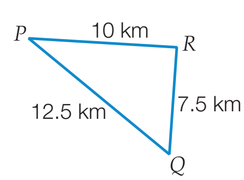
(c)


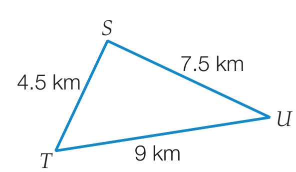
(d)


(e)


(f)


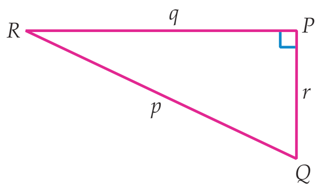
(g)


(h)


(i)


2 For each of the following triangles, select the correct statement of Pythagoras' Theorem from those provided.

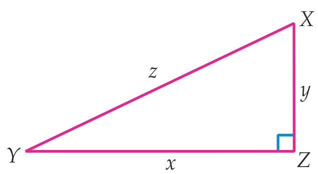
(a)



A $p^2 = q^2 + r^2$ C $r^2 = p^2 + q^2$

B $q^2 = p^2 + r^2$ D $q^2 - p^2 = r^2$

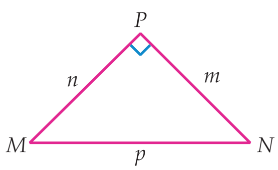
(b)



A $z^2 + y^2 = x^2$ C $x^2 - y^2 = z^2$

B $x^2 + z^2 = y^2$ D $z^2 = x^2 + y^2$

(c)

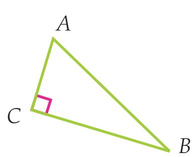


A $n^2 + p^2 = m^2$ C $n^2 - m^2 = p^2$

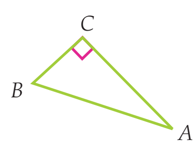
B $p^2 + m^2 = n^2$ D $n^2 + m^2 = p^2$

3 For each of the following right-angled triangles, identify which side length (AB, BC or AC) is the hypotenuse, and which angle is the right angle ($\angle ABC$, $\angle ACB$ or $\angle CAB$).

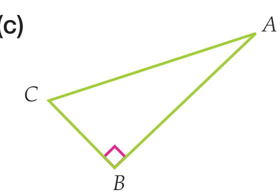
(a)



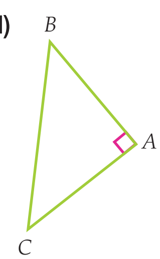
(b)



(c)

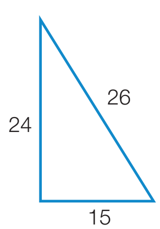


(d)

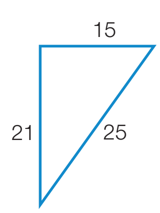


4 Which one of these triangles is a right-angled triangle?

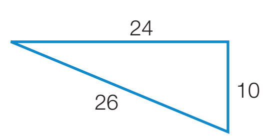
A



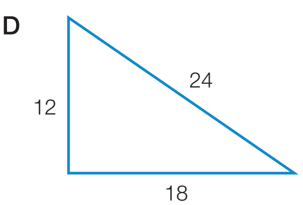
B



C



D



Understanding

5 Find which of these triangles with the following side lengths are right-angled triangles.

(a) 9 mm, 12 mm, 15 mm

(b) 2 cm, 4 cm, 6 cm

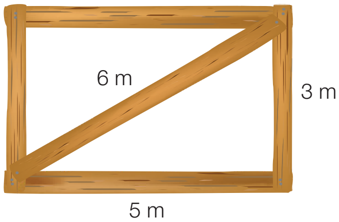
(c) 2 m, 2 m, 4 m

(d) 7 km, 24 km, 25 km

(e) 18 mm, 24 mm, 30 mm

(f) 5 cm, 12 cm, 13 cm

6 Jack has constructed part of a building frame. He needs to check that it is 'square'; that is, that the corners are right angles. The frame is 5 m wide and 3 m high. Jack has measured the diagonal to be 6 m.



(a) Use the converse of Pythagoras' Theorem to check whether the frame is 'square'.

(b) To make the frame 'square', should Jack increase or decrease the length of the diagonal?

7 For each of the following, construct right-angled triangles using the side lengths given in the table.

(a) Find the length of the hypotenuse, c, by measurement, then complete the rest of the table.

(b) Which columns of the table would you check to see if Pythagoras' Theorem was true?

(c) What would you look for in those columns?

a	b	c	a^2	b^2	a^2+b^2	c^2
3 cm	4 cm					
12 cm	16 cm					
60 mm	80 mm					
5 cm	12 cm					

Reasoning

8 Pythagoras' Theorem holds for the right-angled triangle whose sides are of length 5 cm, 12 cm and 13 cm.

(a) Triple each side length. Does Pythagoras' Theorem still hold?

(b) Halve each original side length. Does Pythagoras' Theorem still hold?

(c) What conclusion can you make from your answers to **(a)** and **(b)**?

9 Both a triangle whose sides are of length 3 units, 4 units and 5 units and a triangle whose sides are of lenth 5 units, 12 units and 13 units are right-angled triangles. Find another two right-angled triangles from the following list and justify your selection.

A 9 units, 12 units, 15 units

B 4 units, 5 units, 6 units

C 10 units, 24 units, 26 units

D 7 units, 14 units, 15 units

Open-ended

10 Draw a right-angled isosceles triangle, measure the three sides and show that Pythagoras' Theorem holds.

11 You have a 40 m long piece of rope with which to create a large right-angled triangle on the school oval. Determine the lengths of the sides of three different triangles that you can create. You do not have to use the whole length of the rope.

12 Show how you would explain Pythagoras' Theorem to a friend.

Outside the Square — Problem solving

Olives and pots

Pythagoras loved the challenge of solving problems. One of his favourite problems involved olives and clay pots.

If nine olives are placed in each pot, two pots will remain empty.

If six olives are placed in each pot, there will be three olives left over.

How many olives and clay pots are there?

Strategy options

- Draw a diagram.
- Make a model.
- Guess and check.
- Test all possible combinations.

Demonstrating Pythagoras' Theorem

Versions of this Exploration for other technologies are available in Pearson Reader.

Equipment required: 1 brain, 1 computer with GeoGebra

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. We will refer to these by number, counting from the left. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. (The arrow turns red when you hover the cursor over it.) If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen. (Some versions of GeoGebra have different names for the tools; however, the icons remain the same.)

1 The GeoGebra screen is divided into two panels. The thinner panel on the left is called the 'Algebra View'. The panel on the right is called the 'Graphics View'. Click on the View menu and deselect 'Axes' to get rid of the Cartesian Plane. (Or, right click in the 'Graphics View' and deselect 'Axes'.)

2 Click on the Options menu. Select 'Point Capturing' and 'On (Grid)'.

3 From the Options menu, select 'Labeling' and 'New Points Only'.

4 If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided.

Creating a right-angled triangle

5 From the third icon, select the 'Interval Between Two Points' tool . Draw an interval *AB* by clicking two points in the Graphics View.

6 From the fourth icon, select the 'Perpendicular Line' tool . Create a perpendicular line through *A* by clicking on the interval *AB* and then on Point *A*.

7 From the third icon, select the 'Interval Between Two Points' tool . Create an interval *AC* by clicking on Point *A* and then anywhere along the perpendicular line you just created.

8 Still using the 'Interval' tool, join Points *B* and *C* with an interval.

9 Hide the perpendicular line by right clicking on the line outside the interval *AC* and deselecting 'Show Object'.

10 From the fifth icon, select the 'Regular Polygon' tool .

11 Click on Points *B* and *C*, heading in a clockwise direction. Enter '4' under 'Points' in the box that appears on screen. If the square that appears overlaps the triangle, undo and reselect your points in the opposite order.

12 Repeat Step **11** for the other two sides of the triangle (*AB* and *AC*). If the resulting figure is too big to all be seen, rolling the mouse will make it smaller.

Analysing your construction

13 Look at the Algebra View on the left of the screen. You will see poly1, poly2 and poly3 at the bottom of a list of points and lines. Each one will have a number associated with it. This number is the area of the square. Poly1 is the area of the square on the hypotenuse because you created this square first.

14 In the Input bar (at the bottom of the screen), type: sum=poly2+poly3. Press 'Enter'.

15 What do you notice about the values of poly1 and sum?

16 To label poly1 as the area of the square on the hypotenuse, select the 'Insert Text' tool  from the 10th icon.

17 Click in the Graphics View where you would like the text to appear and type the following into the box that appears: "The square on the hypotenuse = " + poly1. (You must type the inverted commas, as shown.) Click OK.

18 With the 'Insert Text' tool still selected, click on a new position in the Graphics View. Type the following: 'The sum of the squares on the other two sides = " + sum'. Click 'OK'.

19 Using the 'Select' tool under the first icon, , you can click on the text boxes and move them anywhere you like in the Graphics View. You can also edit colours and text sizes by right clicking on each and selecting 'Object Properties'.

20 Using the 'Select' tool, click and drag either Point *A* or Point *B*, changing the lengths of the sides of the triangle.

21 What do you notice about the values of 'The square on the hypotenuse' and 'The sum of the squares on the other two sides'?

22 Write a statement linking the 'The area of the square on the hypotenuse' and 'The sum of the areas of the squares on the other two sides'.

Replacing the squares on the sides with triangles

23 Right click inside each of the squares about the triangle and select 'Delete'.

24 From the fifth icon, select the 'Regular Polygon' tool .

25 Click on Points *B* and *C*, heading in a clockwise direction. Enter '3' under 'Points' in the box that appears on the screen. An equilateral triangle (a three-sided regular polygon) will appear, with *BC* as one of its sides. If the triangle you create goes over the original triangle, undo and reselect your points in the opposite order.

26 Repeat Step **25** for the other two sides of the right-angled triangle (*AB* and *AC*).

27 Poly1, poly2 and poly3 should appear in the Algebra View. This time they are the areas of the equilateral triangles.

28 In the Input bar (at the bottom of the screen), type: sum=poly2+poly3. Press 'Enter'.

29 What do you notice about the values of poly1 and sum in the Algebra View?

30 Select the 'Insert Text' tool  from the 10th icon.

31 Click in the Graphics View where you would like the text to appear and type the following into the box that appears: "The triangle on the hypotenuse = " + poly1. (You must type the inverted commas, as shown.) Click 'OK'.

32 With the 'Insert Text' tool still selected, click on a new position in the Graphics View. Type in the following: "The sum of the triangles on the other two sides = " + sum. Click 'OK' when you are done.

33 Use the 'Select' tool , to click on the text boxes and move them anywhere you like in the Graphics View. You can also edit colours and text sizes by right clicking on each and selecting 'Object Properties'.

34 Using the 'Select' tool, click and drag either Point *A* or Point *B*, changing the lengths of the sides of the triangle.

35 What do you notice about 'The triangle on the hypotenuse' and 'The sum of the triangles on the other two sides'?

36 Write a statement linking the 'The area of the triangle on the hypotenuse' and 'The sum of the areas of the triangles on the other two sides'.

Replacing the sides with semicircles

37 Right click inside the triangles on the sides of the triangle and select 'Delete'.

38 From the sixth icon, select the 'Semicircle Through Two Points' tool .

39 Starting with the interval *BC* (the hypotenuse), click on pairs of points in a clockwise manner to create semicircles on each of the three sides of the triangle. If the semicircle you create overlaps the original triangle, undo and reselect your points in the opposite order.

40 From the second icon, select the 'New Point' tool . Starting with the arc (semicircle) on the hypotenuse, place a point on each of the three arcs.

41 From the sixth icon, select the 'Circumcircular Sector Through Three points' tool . Click on the three points that make up the semicircle on the hypotenuse, starting with Point *C*, then the new Point *D* and then Point *B* (endpoint, arc, endpoint). Repeat for the other arcs about the triangle.

42 Notice that the areas of the semicircles are represented in the Algebra View by the letters h, k and p, where h is the area on the hypotenuse.

43 In the Input bar, calculate the sum of the smaller two areas by typing: sum=k+p. Press 'Enter'.

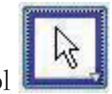
44 What do you notice about the values of h and sum in the Algebra View?

45 Select the 'Insert Text' tool 

from the 10th icon.

46 Click in the Graphics View where you would like the text to appear and type the following into the box that appears: "The semicircle on the hypotenuse = " + h. (You must type the inverted commas, as shown.) Click 'OK'.

47 With the 'Insert Text' tool still selected, click on a new position in the Graphics View. Type, or copy and paste, the following: "The sum of the semicircles on the other two sides = " + sum. Click 'OK'.

48 From the first icon, click on the 'Select' tool . Then, by clicking and dragging either Point *A* or Point *B*, change the lengths of the sides of the triangle.

49 What do you notice about 'The area of the semicircle on the hypotenuse' and 'The sum of the areas of the semicircles on the other two sides'?

50 Write a statement linking the 'The semicircle on the hypotenuse' and 'The sum of the semicircles on the other two sides'.

Conclusion

51 You have constructed squares, equilateral triangles and semicircles on the sides of a right-angled triangle. What conclusion can you make about the sum of the areas on the shorter sides of the triangle? Make a general statement that links all the ideas into one. What feature is the important linking feature of the three examples we have constructed?

Taking it further

Can this process be repeated with regular pentagons or hexagons on each side? Check this for yourself using GeoGebra and the techniques described above.

Frank and Ernest

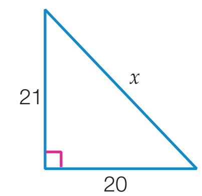
2.2 Finding the length of the hypotenuse

If we know the lengths of the two shorter sides of a right-angled triangle, we can use Pythagoras' Theorem to find the length of the hypotenuse.

Worked Example 2

WE2

Calculate the length of the hypotenuse in the following triangle.



Thinking	Working
1 State Pythagoras' Theorem and define the side lengths a, b, and c, letting c be the unknown length of the hypotenuse, x.	$c^2 = a^2 + b^2$ $a = 20$, $b = 21$, $c = x$
2 Substitute the values for a, b and c into Pythagoras' Theorem.	$x^2 = 20^2 + 21^2$
3 Simplify the equation.	$x^2 = 400 + 441$ $= 841$
4 Solve for x by finding the square root of both sides of the equation.	$x = \sqrt{841}$ $x = 29$ units

Surds and decimal approximations

The length of the hypotenuse may not always be a whole number. It may have an exact decimal value, such as $c = \sqrt{6.25}$, which gives an exact answer of $c = 2.5$. However, sometimes the length cannot be evaluated exactly by a decimal equivalent and is written as an approximation.

Consider a right-angled triangle with shorter side lengths of 1 m and 2 m:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 1^2 + 2^2 \\ &= 1 + 4 \\ &= 5 \\ c &= \sqrt{5} \text{ m} \end{aligned}$$

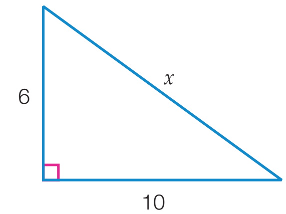
If we evaluate $\sqrt{5}$ on the calculator, it is shown as 2.236 067 977. This is a decimal approximation. There is no exact decimal equivalent because the number does not terminate (it has an infinite number of decimal places) and it has no repeating pattern. These types of numbers are called **irrational numbers**. Irrational numbers such as $\sqrt{2}$, $\sqrt{5}$, $\sqrt{20}$ and $\sqrt{35}$ are known as **surds**. The only way to write these numbers accurately is in surd form; that is, as $\sqrt{2}$ or $\sqrt{5}$. However, this is not always practical when we are working with the side lengths of triangles. Depending on the type of answer and the degree of accuracy required, a calculator can be used to find a **rational approximation**. We will show this by using the 'approximately equal to' sign, ≈, then writing it equal to a decimal number correct to the required number of decimal places. At other times we may leave the answer in surd form (also known as 'exact form').

For example, $c = \sqrt{5}$ m
$\approx 2.236\,067\,977$
$= 2.24$ m (correct to two decimal places)

Worked Example 3

WE3

Find the length of the hypotenuse in the following right-angled triangle. Give your answer in exact form (surd form).



Thinking	Working
1 State Pythagoras' Theorem, and define the side lengths.	$c^2 = a^2 + b^2$ $a = 6$, $b = 10$, $c = x$
2 Substitute the values for a, b and c into Pythagoras' Theorem.	$x^2 = 6^2 + 10^2$
3 Simplify the equation.	$x^2 = 36 + 100$ $= 136$
4 Solve for x by finding the square root of both sides of the equation. Leave the answer as a square root (surd form).	$x = \sqrt{136}$

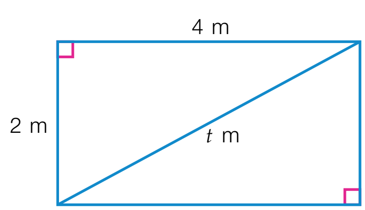
Pythagoras' Theorem may be applied to other shapes provided they contain right-angled triangles. Sometimes, it is necessary to identify the required right-angled triangle by carefully adding a line or lines to the diagram.

Worked Example 4

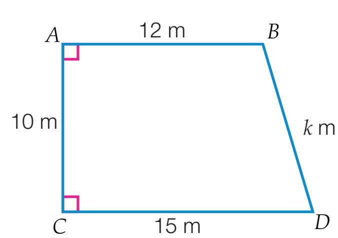
WE4

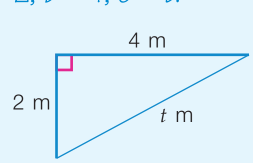
Use Pythagoras' Theorem to find the value of the unknown length in each of the following diagrams. Give your answer as a rational approximation, correct to two decimal places, if necessary.

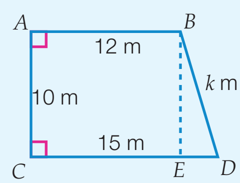
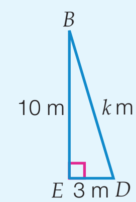
(a)



(b)



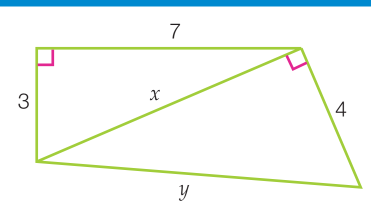
Thinking	Working
(a) 1 Identify a right-angled triangle and define the side lengths.	(a) Let $a = 2$, $b = 4$, $c = t$. 

	Thinking	Working
2	State Pythagoras' Theorem, then subtitute in the values.	$c^2 = a^2 + b^2$ $t^2 = 2^2 + 4^2$
3	Simplify the equation.	$= 4 + 16$ $= 20$
4	Solve for t by finding the square root of both sides. Use a calculator to find its rational approximation and write it correct to two decimal places, with the correct units.	$t = \sqrt{20}$ $\approx 4.472\,135\,955$ $t = 4.47\text{ m (2 d.p.)}$
(b) 1	Draw a line from point B down to the line CD to form a right-angled triangle BED.	(b) 
2	Redraw the right-angled triangle and define the side lengths.	$c = k$, $a = 10$, $b = ED$ $= 15 - 12$ $= 3$ 
3	State Pythagoras' Theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $k^2 = 10^2 + 3^2$
4	Simplify the equation.	$= 100 + 9$ $= 109$
5	Solve for k by finding the square root of both sides of the equation. Use a calculator to find its rational approximation and write it correct to two decimal places, with the correct units.	$k = \sqrt{109}$ $\approx 10.440\,306\,51$ $k = 10.44\text{ m (2 d.p.)}$

Worked Example 5

WE5

Find the unknown lengths in the following diagram. Give your answers in exact form (surd form).



	Thinking	Working
1	Identify the right-angled triangle where two of the three sides are known and define the side lengths.	$a = 3$ $b = 7$ $c = x$

2 State Pythagoras' Theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $x^2 = 3^2 + 7^2$
3 Simplify the equation.	$= 9 + 49$ $= 58$
4 Solve for x by finding the square root of both sides. Leave your answer as a square root (surd form).	$x = \sqrt{58}$
5 Identify the second right-angled triangle and define the side lengths using your answer for x.	$a = x = \sqrt{58}$ $b = 4$ $c = y$
6 State Pythagoras' Theorem, then substitute in the values. Use the exact surd value determined previously.	$c^2 = a^2 + b^2$ $y^2 = (\sqrt{58})^2 + (4)^2$ $y^2 = 58 + 16$
7 Simplify the equation.	$= 74$
8 Solve for y by finding the square root of both sides. Leave your answer as a square root (surd form).	$y = \sqrt{74}$

2.2 Finding the length of the hypotenuse

Navigator

Q1, Q2, Q3 (a)–(f), Q4, Q5, Q8, Q10 (a) & (b), Q12, Q14, Q15, Q17	Q1, Q2, Q3 (d)–(i), Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q15, Q16, Q17	Q1 (d)–(f), Q2, Q3 (g)–(l), Q4, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Answers page 603

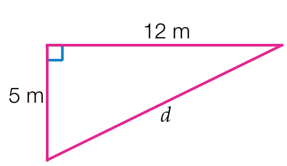
Equipment required: calculator

Fluency

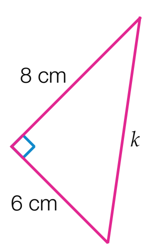
1 Calculate the length of the hypotenuse in the following triangles.

WE2

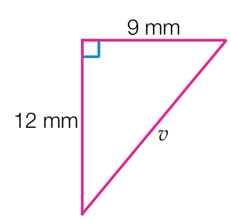
(a)



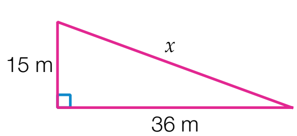
(b)



(c)



(d)



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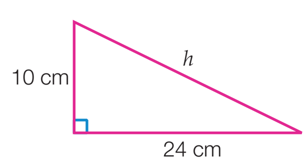
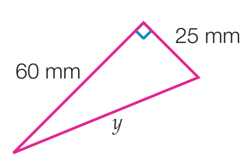
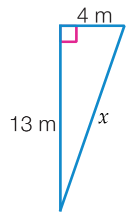


(f)



WE3

2 Find the length of the hypotenuse in the following right-angled triangles. Give your answer in exact form (surd form).

(a)


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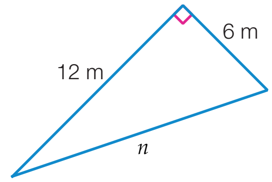

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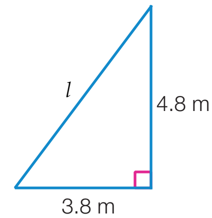

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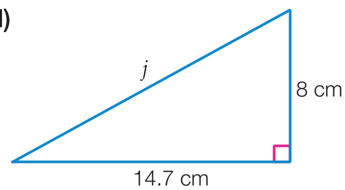

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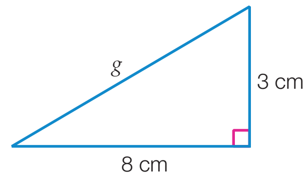
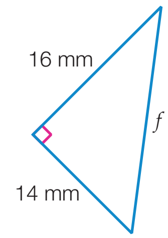
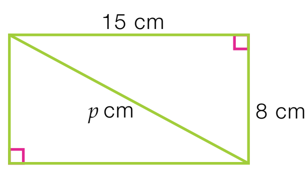
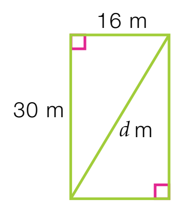


(f)


WE4

3 Use Pythagoras' Theorem to find the value of the unknown length in each of the following diagrams. Give your answers as rational approximations, correct to two decimal places if necessary.

(a)


(b)


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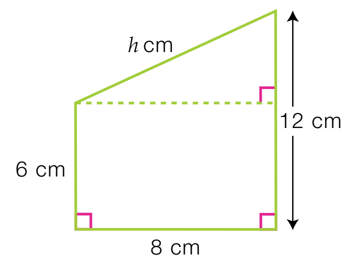
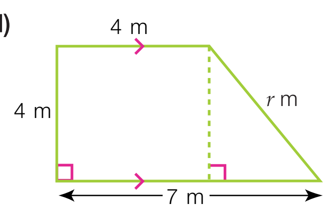
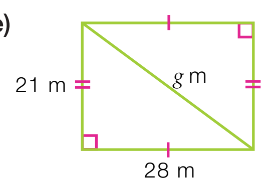
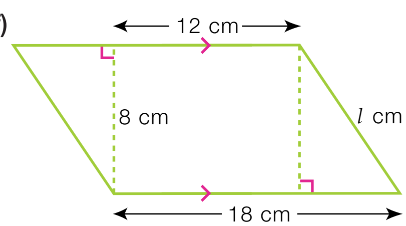
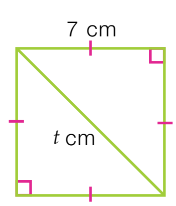


(d)


(e)


(f)


(g)


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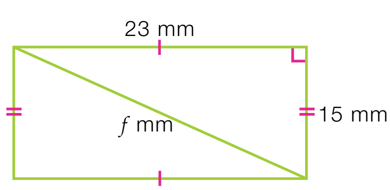
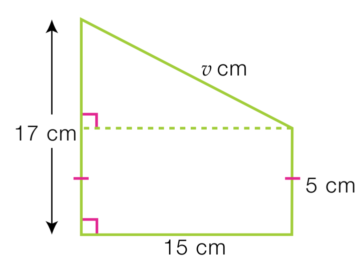
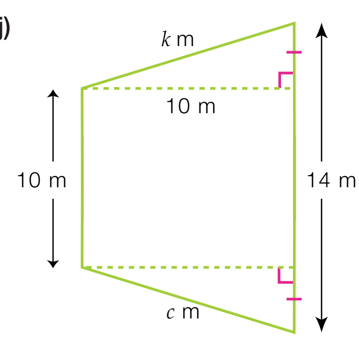
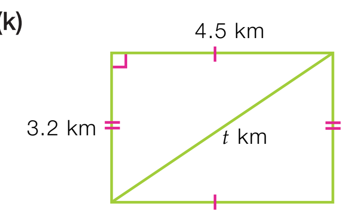
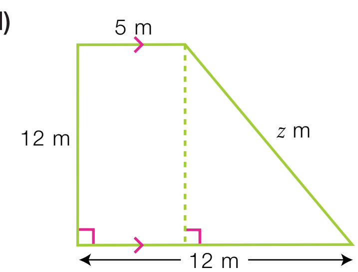


(i)


(j)


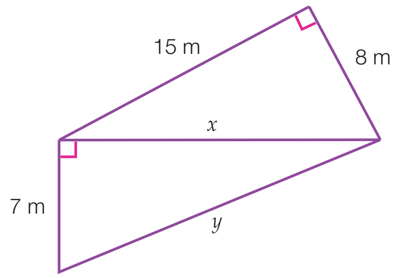
(k)


(l)


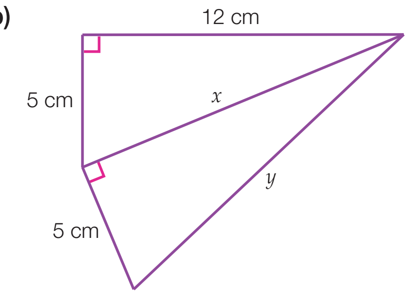
4 Find the unknown lengths in the following diagrams. Give your answers in exact form (surd form).

WE5

(a)



(b)



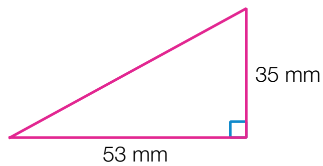
5 The length of the hypotenuse in this right-angled triangle, correct to two decimal places, is:

A 39.80 mm

B 63.51 mm

C 88.00 mm

D 4034.00 mm



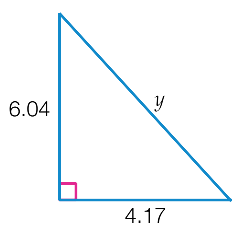
6 The value of y in the following diagram is closest to:

A 7.33

B 7.34

C 10.21

D 53.87



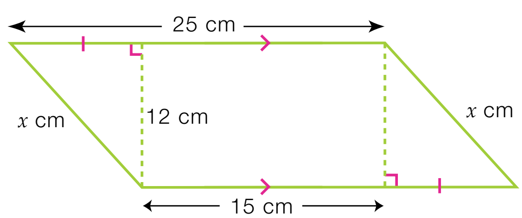
7 The value of x in the diagram is:

A $\sqrt{244}$ cm

B $\sqrt{288}$ cm

C $\sqrt{369}$ cm

D $\sqrt{769}$ cm



Understanding

8 A ladder is leaning against a built framework and just reaches the roof, a height of 4.27 m above the ground. The foot of the ladder is 1.41 m from the base of the framework.

(a) Draw a diagram of a right-angled triangle that represents the ladder, the ground and the framework. Label the diagram with the known measurements, and label the unknown side x.

(b) Use Pythagoras' Theorem to calculate the length of the ladder, correct to two decimal places.

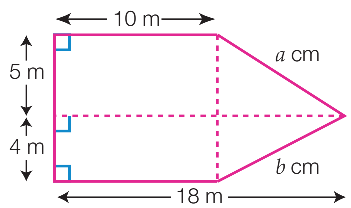
9 Corporal Vince is in charge of designing a ropes course for an army training camp. In the final part of the course, recruits must slide down a rope connected from the top of a tower to a point on the ground 35 m from the base of the tower. The top of the tower is 14 m high and at right angles to the ground.

(a) Draw a diagram of a right-angled triangle that shows the tower, the rope and the ground. Label the diagram with the known measurements and label the unknown side x.

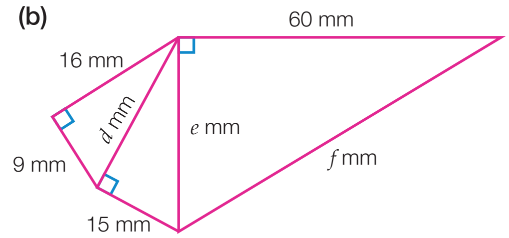
(b) Use Pythagoras' Theorem to calculate the length of rope Corporal Vince needs for this part of the course, correct to the nearest metre.

10 Find the exact value of the variables in the following diagrams. For any surd values, also give the answer correct to two decimal places.

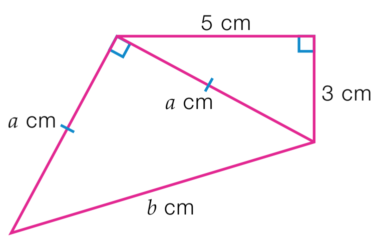
(a)



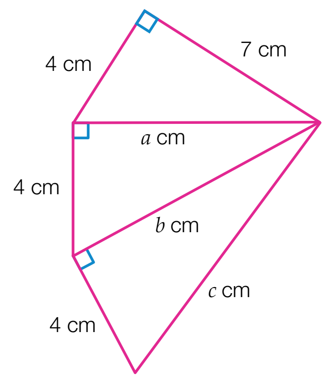
(b)



(c)



(d)



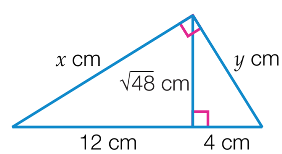
11 The values of x and y in the diagram are closest to:

A $x = 9.80$ cm, $y = 8$ cm

B $x = 9.80$ cm, $y = 5.66$ cm

C $x = 13.86$ cm, $y = 8$ cm

D $x = 13.86$ cm, $y = 5.66$ cm



Reasoning

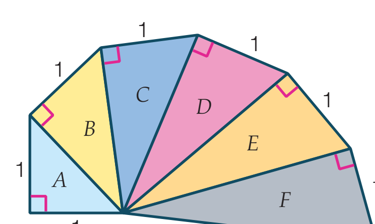
12 (a) Find the length of the hypotenuse of a right-angled isosceles triangle with a side length of 10 cm. Now divide the length of the hypotenuse by the length of the shorter side. Compare your answer to the value of $\sqrt{2}$, rounded to four decimal places.

(b) Find the length of the hypotenuse for several other right-angled isosceles triangles. Then, divide the length of the hypotenuse by the length of the shorter side. What do you notice about the result?

13 A Pythagorean spiral has the shape and side lengths shown.

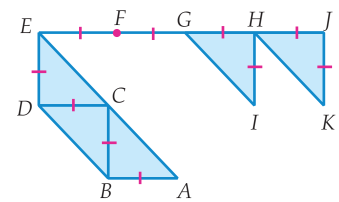
(a) Use Pythagoras' Theorem to calculate the values of the hypotenuses of A, B, C, D and E as exact values and correct to two decimal places where necessary.

(b) More triangles are added until the letter Z is used so that the side lengths are $A, B, C, D, E, F, G, \ldots, X, Y, Z$. How many side lengths cannot be written as exact decimal values?



14 (a) Use the letters shown to describe the shortest path possible to go from point A to point K if you can only follow the straight lines shown.

(b) If each shorter side of a triangle is 1 cm in length, what is the total length of the path described in **(a)**? State your answer as an exact value.



15 Alex wants to buy a new flat screen TV. He has allocated a rectangular space on his lounge wall that is 98 cm in length and 54 cm in width. At the TV store, Alex learns that TVs come in the following screen sizes: 55 cm, 66 cm, 81 cm, 94 cm, 106 cm, 116 cm and 127 cm. He also learns that these 'screen sizes' are the diagonal distance from one corner of the screen to the other.

Which is the largest screen size that could fit in the space on Alex's wall?

Open-ended

16 A swimming pool is 50 m long and 25 m wide. Anastasia is trying to swim a length of the pool, starting from a corner at the shallow end. However, she has difficulty swimming in a straight line, and, instead, heads diagonally across the pool, usually ending up somewhere between the two corners at the other end of the pool. Draw a diagram and calculate four possible distances, correct to two decimal places, that Anastasia may actually end up swimming when completing a lap of the 50 m pool.

17 The city council needs to build a wheelchair ramp from the ground to the top of a step 30 cm off the ground.

(a) Design three different ramps by first choosing the horizontal length, then calculating the sloping ramp length.

(b) Sketch your three ramps roughly to scale.

(c) Which ramp design would you choose if the ramp were to be used everyday?

Outside the Square Game

Perfect square root bingo

Equipment required: 2 brains, 2 dice

How to win:
Cross three numbers in a row off your grid. Play the best of three games.

How to play:
Each player should draw up a 3×3 grid and fill each grid space with any one of the following square roots (used only once):

$\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{49}$, $\sqrt{64}$, $\sqrt{81}$, $\sqrt{100}$, $\sqrt{121}$, $\sqrt{144}$

Roll both dice. The sum of the dice is the number both players can cross off their board.

Sample Roll: 2 and a 3 are rolled $2 + 3 = 5$, so you could cross $\sqrt{25}$ off your bingo board.

The aim is to get 3 in a row.

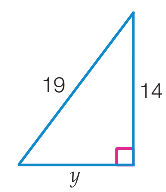
Consider (the following before your next game): Which square root is likely to come up more times than the others? Where would be a good location on your board to place this number?

2.3 Finding the length of a shorter side

The length of one of the shorter sides of a right-angled triangle can be found if we are given the lengths of the hypotenuse and the other side.

Worked Example 6

Find the value of the unknown shorter side in this right-angled triangle. Give your answer as an exact value.

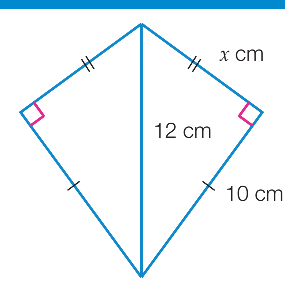


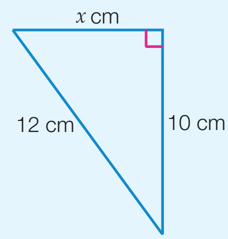
Thinking	Working
1 State Pythagoras' Theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = y$, $b = 14$, $c = 19$
2 Substitute the values into Pythagoras' Theorem.	$19^2 = y^2 + 14^2$
3 Rewrite so that the unknown is isolated on one side of the equation. (Here, 14^2 has been subtracted from both sides.)	$y^2 = 19^2 - 14^2$
4 Simplify the equation.	$= 361 - 196$ $= 165$
5 Solve for y by finding the square root of both sides of the equation. Leave your answer as a surd (exact form).	$y = \sqrt{165}$

It may be necessary to draw a right-angled triangle from a diagram in order to apply Pythagoras' Theorem.

Worked Example 7

Find the value of the variable, correct to two decimal places, in the following diagram.



Thinking	Working
1 Draw a diagram of the appropriate right-angled triangle.	
2 State Pythagoras' Theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = 10$, $b = x$, $c = 12$
3 Substitute the values into Pythagoras' Theorem.	$12^2 = 10^2 + x^2$
4 Rewrite so that the unknown is isolated on one side of the equation. (Here, 10^2 has been subtracted from both sides).	$x^2 = 12^2 - 10^2$
5 Simplify the equation.	$= 144 - 100$ $= 44$
6 Solve for x by finding the square root of both sides of the equation. Use a calculator to find its rational approximation and write it correct to two decimal places.	$x = \sqrt{44}$ ≈ 6.6332495 $x = 6.63$ m (2 d.p.)

2.3 Finding the length of a shorter side

Navigator

Q1 (a)–(i), Q2 (a)–(c), Q3 (a), Q4, Q5, Q6, Q8	Q1 (d)–(l), Q2, Q3, Q4, Q5, Q6, Q7, Q8	Q1 (g)–(l), Q2, Q3 (b), Q4, Q5, Q6, Q7, Q8, Q9

Answers page 604

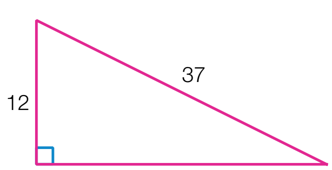
Equipment required: calculator

Fluency

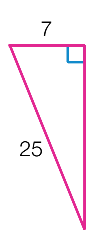
WE6

1 Find the value of the unknown shorter side in these right-angled triangles. Give your answer as an exact value.

(a)



(b)



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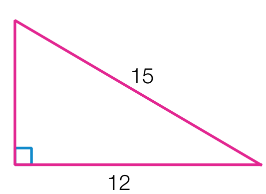
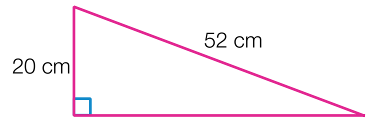
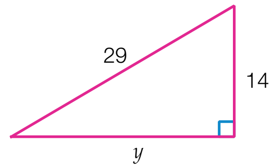
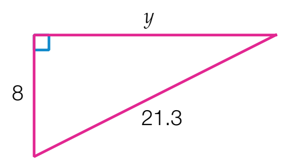


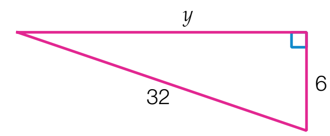
(d)


(e)


(f)


(g)


(h)


(i)


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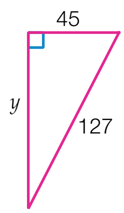
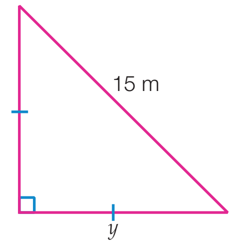
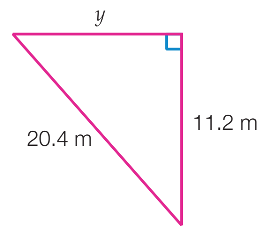
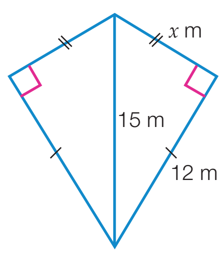


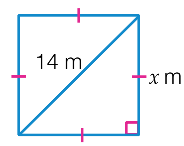
(k)


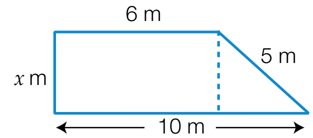
(l)


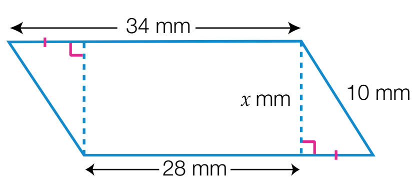
WE7

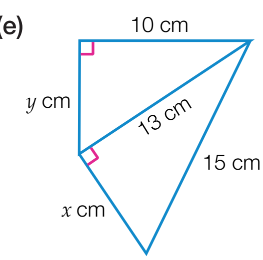
2 Find the value of the variable, correct to two decimal places, in each of the following diagrams.

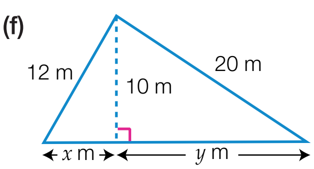
(a)


(b)


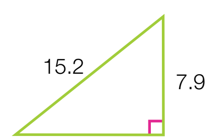
(c)


(d)


(e)


(f)


3 (a) The value of x in this right-angled triangle is closest to:



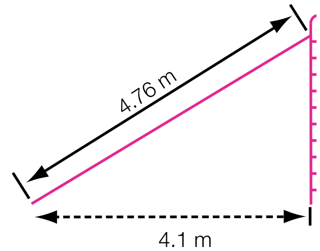
A 7.30 B 12.99
C 17.13 D 23.10

(b) The hypotenuse of a right-angled triangle has a length of 55 cm. One of the shorter sides has a length of 53 cm. The length of the third side is closest to:

A 2 cm B 14.6 cm C 14.7 cm D 216 cm

Understanding

4 A playground slide has a length of 4.76 m, while it covers a horizontal distance of 4.1 m on the ground. What is the height of the slide, correct to two decimal places?



5 A 'guy rope' is attached to the top of a tent pole and pegged into the ground so that the rope is tight. If the rope is 3.8 m long, and the pole is 2.9 m long, how far from the base of the pole should the rope be pegged if it is to be tight?

Reasoning

6 Find the lengths of the shorter sides, correct to two decimal places, if the length of the hypotenuse of a right-angled isosceles triangle is:

(a) 20 cm (b) 12 m (c) 100 cm

7 Matilda uses a piece of rope 12 m long to form an equilateral triangle.

(a) How long are the sides of the triangle?

(b) Find the height of the triangle, correct to two decimal places.

Open-ended

8 Safety regulations for ladders state that the distance that a ladder can be placed from a wall is no closer than one-quarter of its length and no further than half its length. If a ladder is 3.2 m long, give three heights that it can reach up the wall.

9 Find two sets of possible lengths of the shorter sides, correct to two decimal places, if the length of the hypotenuse of a right-angled scalene triangle is as given.

(a) 20 cm (b) 35 m

Outside the Square Puzzle

Roots, but no plants

The square root of a number multiplied by itself gives the number. The cube root of a number is multiplied by itself twice to give the number. The fourth root is multiplied by itself three times.

"I know a number of which the cube root is ten times the fourth root", said Sue.

"Not possible!" said Alec.

"Is too!" said Sue.

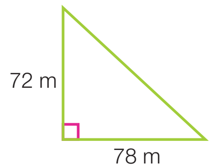
Who was right?

Half-time 2

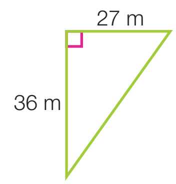
Ex.2.2

1 For each of the following triangles, calculate the length of the hypotenuse (round your answers to two decimal places where necessary).

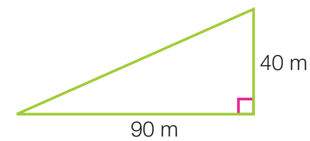
(a)



(b)



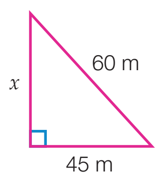
(c)



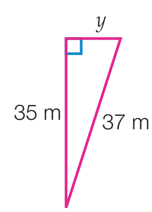
Ex.2.3

2 For each of the following triangles, calculate the length of the unknown side (round to two decimal places where necessary).

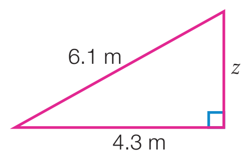
(a)



(b)



(c)



Ex.2.1

3 Use Pythagoras' Theorem to show that a triangle with side lengths 10, 24 and 26 must be right-angled.

Ex.2.2

4 Sharon is 20 metres due south of a target. She throws a ball which stops 16 metres due east of the target.

(a) Draw a diagram that shows this information.

(b) Calculate, correct to two decimal places, how far the ball travelled.

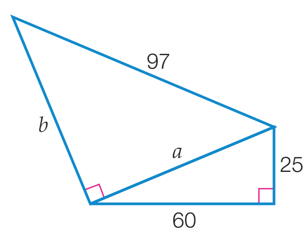
Ex.2.1

5 Decide if each triangle with the given side lengths is right angled.

(a) (16, 63, 65) (b) (48, 55, 73) (c) (13, 84, 86)

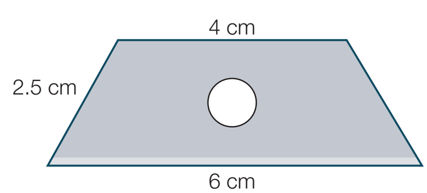
Ex.2.2

6 Calculate the exact values of a and b.



Ex.2.2

7 A safety blade is a symmetrical shape with the dimensions shown. Calculate the height of the blade (round your answer to two decimal places).



Ex.2.3

8 A ladder 4 m long is placed against a wall so that the foot of the ladder is 1.5 m away from the wall.

(a) Draw a diagram that shows this information.

(b) Calculate how far up the wall the ladder will reach. Give your answer correct to two decimal places.

Investigation

Estimating with Pythagoras

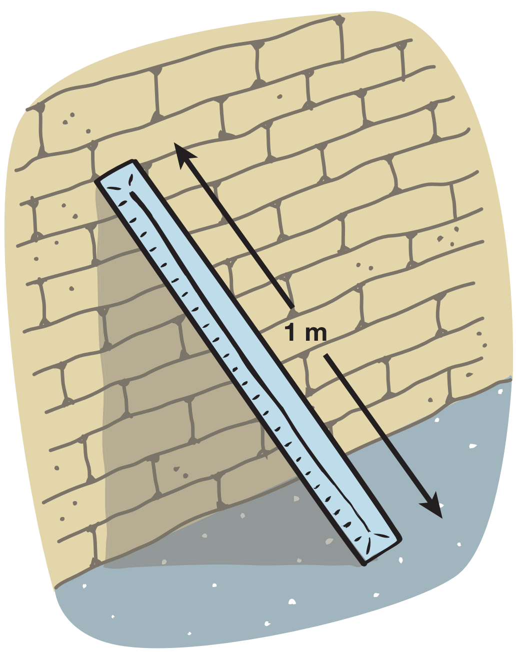
Equipment required: 1 brain, 2 × 1-metre rulers, a wall

The Big Question

How does the height of a right-angled triangle change as we change the length of the base?

Engage

We can create a right-angled triangle by leaning a metre ruler up against a wall.



If we know the base length of this triangle (how far the end of the ruler is from the wall), we can use Pythagoras' Theorem to calculate its height.

Rewriting $a^2 + b^2 = c^2$ as $a^2 = c^2 - b^2$ (subtracting b^2 from both sides of the equation) puts the theorem into a more useful form for these sorts of calculations.

1 Place the ruler against the wall so that the base length of the triangle is 20 cm. Write down an estimation for the height of the triangle. Calculate the height of the triangle (remember, the length of the hypotenuse is 100 cm). Use the other metre ruler to measure the height. Compare your estimated, calculated and measured heights.

Explore

2 Create a series of five or six different triangles by placing the ruler lower down against the wall, lengthening the base. For each triangle, estimate, calculate and measure the height of the triangle. Make sure you know the length of the base of each triangle you create.

Strategy options

- Make a table.
- Act it out.
- Look for a pattern.

Explain

3 For each triangle, compare your estimated heights with the calculated heights. How close were they? Calculate your average error by finding the average of the difference between the estimated and the measured heights. Were you better at estimating taller or shorter heights?

4 Now, compare your measured heights with your calculations. Are they exactly the same? How could you explain any differences between them?

Elaborate

5 Can you see any pattern or connection between the increasing base length and the decreasing height? Were you expecting to see a pattern?

6 Draw a line graph of the height for each base length. What does your graph show?

7 Use your answers to Questions **5** and **6** to answer the Big Question.

Evaluate

8 Consider how you went about solving this problem and the methods you used. Could you have approached it differently? How?

Extend

9 Using a tape measure or a piece of string for the hypotenuse, keep the base of the triangle constant (say 50 cm) and compare how the length of the hypotenuse changes as the height of the triangle is increased.

2.4 Applications of Pythagoras' Theorem

Pythagoras' Theorem is a powerful mathematical tool, having uses in geometrical proofs as well as in many practical situations involving right-angled triangles.

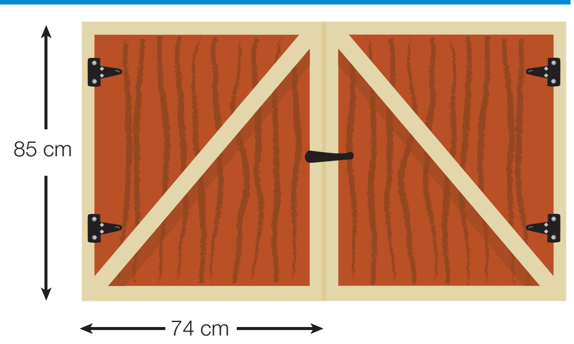
Follow these steps to solve a problem with Pythagoras' Theorem:

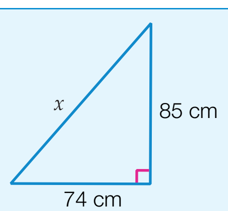
1. Draw a diagram that includes a right-angled triangle to represent the situation described in the problem.
2. Label the diagram with the right angle and the measurements given in the problem.
3. Identify the hypotenuse.
4. Label the unknown measurements that you need to find with variables.
5. If necessary, re-draw a simpler right-angled triangle showing all the information.
6. Use Pythagoras' Theorem to find the unknown length, to the required accuracy.
7. State your answer to the question in a sentence, including appropriate units.

Worked Example 8

WE8

What is the length of one of the diagonal supports of the frame shown? State your answer correct to two decimal places.



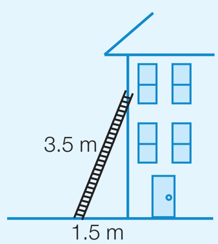
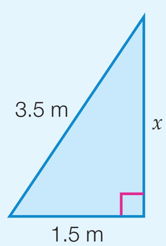
Thinking	Working
1 Draw a diagram of the appropriate triangle. Label the right angle, the given measurements and the unknown side.	

2 Use Pythagoras' Theorem to find the value of the variable.	$c^2 = a^2 + b^2$ $x^2 = 74^2 + 85^2$ $= 5476 + 7225$ $= 12\,701$ $x = \sqrt{12\,701}$ $\approx 112.698\,713\,4$ $= 112.70$ (2 d.p.)
3 State the answer correct to two decimal places.	The support is approximately 112.70 cm long.

Worked Example 9

WE9

A ladder 3.5 m long is leaning against a vertical wall with the base 1.5 m from the bottom of the wall on horizontal ground. How high up the wall does the ladder reach? State your answer correct to two decimal places.

Thinking	Working
1 Draw a labelled diagram with all measurements indicated.	
2 Draw a diagram of the appropriate triangle.	
3 Use Pythagoras' Theorem to find the value of the variable.	$c^2 = a^2 + b^2$ Let $a = 1.5$, $b = x$, $c = 3.5$ $3.5^2 = 1.5^2 + x^2$ $x^2 = 3.5^2 - 1.5^2$ $= 12.25 - 2.25$ $= 10$ $x = \sqrt{10}$ $\approx 3.162\,277\,66$ $x = 3.16$ (2 d.p.)
4 State the answer.	The ladder will reach 3.16 m up the wall.

2.4 Applications of Pythagoras' Theorem

Navigator

Answers page 604

Q1, Q2, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q18	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q17, Q18	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18, Q19

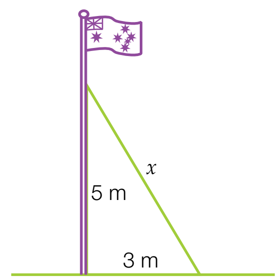
Equipment required: calculator

Where necessary, state your answers correct to two decimal places.

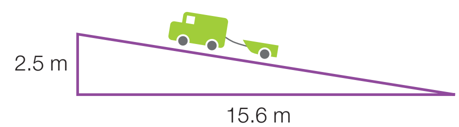
Fluency

WE8

1 **(a)** A support wire is attached 5 m up a flagpole. The other end of the wire is attached to the ground 3 m from the base of the flagpole. What is the length of the wire?

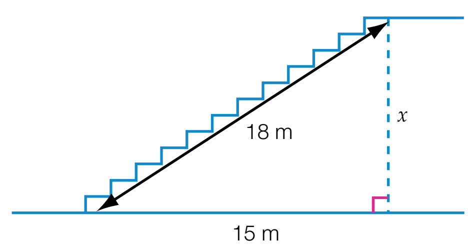


(b) A boat ramp is to be built that will cover a horizontal distance of 15.6 metres while descending a vertical distance of 2.5 metres. What is the length of the ramp?



WE9

2 The escalator at a department store is 18 m long. When you ride on the escalator you move across 15 m. Calculate the height through which you travel from the lower level to the higher level.



3 **(a)** A leaf rake 1.8 m long is leaning against a vertical post. The distance from the bottom of the post to the bottom of the rake is 0.6 m. How high up the post will the top of the rake be?

(b) A rope is tied to a vertical tree trunk 2 m from the ground and is attached to the ground at a point 1.2 m from the base of the tree. How long is the segment of rope between the tree and the ground?

4 The top of a 6.5 m long ladder is resting against a wall at a point that is 4.2 m above the ground. What is the approximate distance of the base of the ladder from the base of the wall?

A 2.46 m **B** 2.99 m **C** 3.24 m **D** 4.96 m

5 A slide is 5 m long and the vertical ladder is 3.6 m. How far is the base of the ladder from the bottom of the slide?

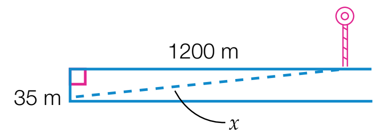
6 Andrew is standing at the corner of a rectangular park, 35 m long and 28 m wide. He has decided to cross diagonally rather than go around the outside.

(a) How far does Andrew travel when crossing the park?

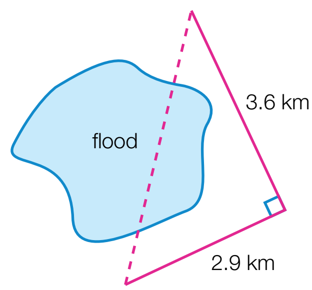
(b) How much distance does he save by not going around the outside?

Hint
Draw a diagram for Questions **5** and **6** to represent the given information.

7 Kensington race course has races that are run over a straight 1200 m course. One horse starts near the outside fence and runs in a straight line towards the finishing post on the inside fence. If the track is 35 m wide, how much further does this horse run than a horse that runs in a straight line along the inside fence?



8 During a heavy rainstorm, the road that Allie normally takes to travel home from school has become flooded. She must take a detour as shown in the diagram. How much further does she now have to travel?



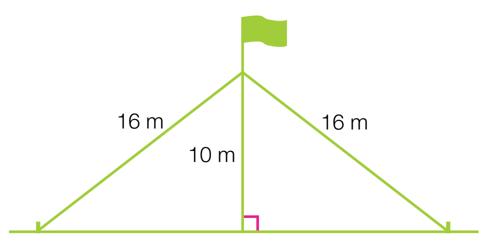
9 Edmond has attached a 17 m long support wire to a radio tower, 2 m from the top of the tower. If the other end of the wire is attached to the ground 7.5 m from the base of the tower, the total height of the tower is:

A 13.26 m B 15.00 m C 15.26 m D 17.26 m

Understanding

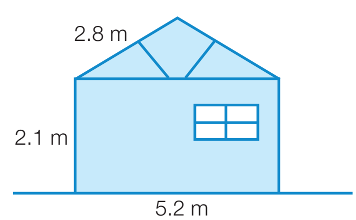
10 A flagpole, 10 m tall, is supported by two wires, each of length 16 m which are fastened to pegs in the ground. How far apart are the pegs?

A 6.25 m B 12.49 m
C 16.97 m D 24.98 m



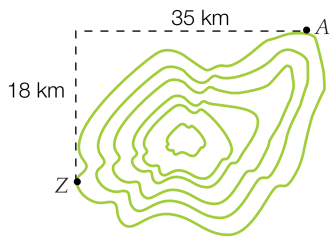
11 The length of the hypotenuse on an isosceles right-angled triangle is 16 cm. What is the length of the other two sides?

12 The end view of a garden shed is shown in the diagram. The shed is 5.2 m wide, the sides are 2.1 m high and the sloping part of the roof is 2.8 m long. Calculate the overall height of the shed.



13 The Australian flag flying at Parliament House in Canberra is 6.4 m wide and 12.8 metres long. It has been suggested that nylon tape be used in the lining along each diagonal and along each of its four sides to prevent creasing of the material. If the nylon tape costs $15 per metre, what total cost will be involved? State your answer to the nearest cent.

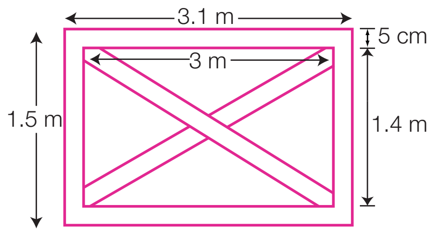
14 The roads corporation needs to build a new road between the two towns of Alderton and Zincton. Unfortunately, there is a mountainous region between the towns. It costs $2500 per kilometre to build a road around the mountains and $4000 per kilometre to build a road through a tunnel in the mountains. Which is the cheaper way to build the required road?



15 The rectangular frame for a wall of a new building is 7.5 m high and 18 m long. It has one diagonal support. If the frame, including the diagonal support, is constructed from steel girders that come in 10 m lengths, find the cost of the frame if the girders cost $230 each.

Reasoning

16 Angelo wants to build a rectangular gate from timber. The gate is to be 1.5 m high and 3.1 m wide. He has also decided to put in two diagonal cross supports to give the gate more strength. The timber he will use is 5 cm wide.

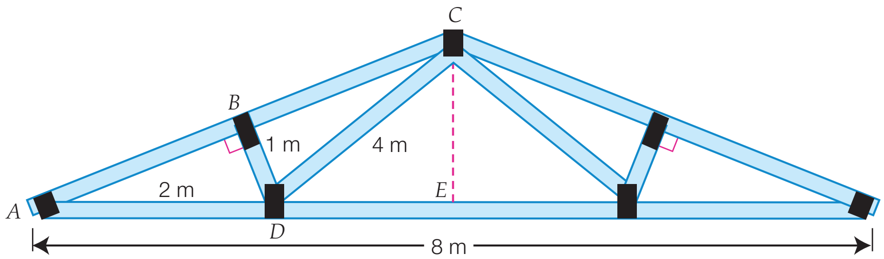


(a) What length of timber is required for the external rectangle?

(b) What length of timber is required for each of the diagonals?

(c) What is the total length of wood he will need?

17 A symmetrical timber roof frame has been constructed with the dimensions shown below.



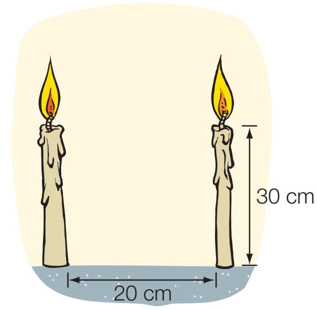
(a) What is the length of section AB?

(b) What is the length of section BC?

(c) Use your answers to **(a)** and **(b)** to find the total length of timber used.

Open-ended

18 Two candles, each initially of length 30 cm, are 20 cm apart. The thinner candle burns at a rate of 1 cm every 8 minutes and the other candle burns at a rate of 1 cm every 10 minutes. Find two possible lengths for the distance between the top of the two candles if the candles will burn for up to 3 hours.



19 Joel, a surveyor, is marking out the boundary of a block of land for a new house. From a peg in the ground he walks x metres north, $2x$ metres east, $4x$ metres south and $8x$ metres west. To finish, Joel then walks back to the peg.

(a) Choose a reasonable value for x and draw the shape of the block of land.

(b) How far did Joel walk altogether?

Outside the Square — Puzzle

Pythagoras and the missing fig

Plato, a friend of Pythagoras, wanted to give him some of his 100 figs. This is how Plato did it.

Figs given to Pythagoras	Figs remaining
50	100 – 50 = 50
25	50 – 25 = 25
10	25 – 10 = 15
8	15 – 8 = 7
5	7 – 5 = 2
2	2 – 2 = 0

The total number of figs Pythagoras received was 50 + 25 + 10 + 8 + 5 + 2 = 100.

The total number of figs remaining is 50 + 25 + 15 + 7 + 2 + 0 = 99.

Can you explain why the two answers are not the same?

Maths 4 Real

Pythagoras and plasma screens

Flat-screen televisions, such as plasma, LCD and LED screens, have become extremely popular in the last decade. The high-definition picture, excellent colour and, more recently, 3D capabilities have had people racing to trade in their old TVs, with some even opting for complete home theatre systems. Manufacturers are now producing TVs in ever-increasing sizes as the demand grows.

Screen size

Even though they represent the latest technology, the screen size of a flat-screen TV is given in the old imperial unit of inches ("). A range of screen sizes is given below.

22"	26"	32"	37"	42"	54"	58"	65"	70"

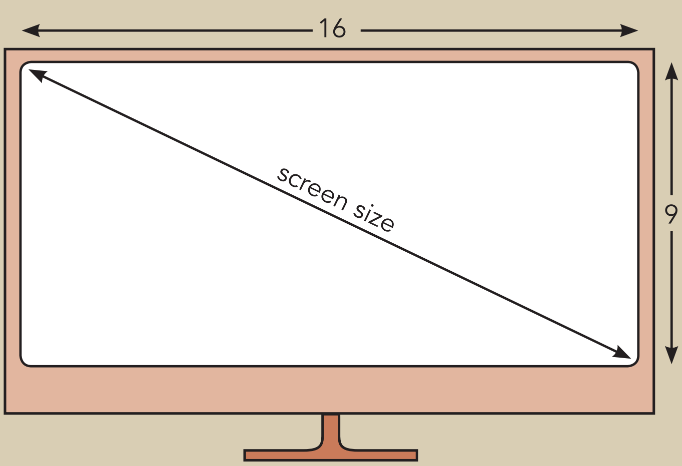
To convert these measurements in inches to centimetres, we multiply by 2.54.

1 inch = 2.54 cm

Just what length does this 'screen size' measurement refer to though? It is not the length or the width of the screen, but the diagonal distance across the screen from one corner to the other. This diagonal cuts the screen into two right-angled triangles, with the screen size measurement as the hypotenuse.

Aspect ratio

The aspect ratio of a screen is the ratio of the screen's length to its width. Plasma TVs have a screen aspect ratio of 16 : 9. We can convert this ratio to a scale factor by dividing: 16 ÷ 9 = 1.78 (2 d.p.). This means that the length of the screen is 1.78 times the width. Old fashioned TV screens, and some computer screens, have an aspect ratio of 4 : 3, a scale factor of 1.33 (2 d.p.).



The customer's problem

1 Tom wants to buy a new plasma TV. He has marked a space on his wall for it that measures 1.4 m in length and 1.1 m in width. When he arrives at the TV store, he learns that screen sizes are measured diagonally in inches and that the lengths and widths of the screens are always in a ratio of 16 : 9.

Tom wants the biggest screen possible, but he now has no idea how big that is!

Help Tom by following these steps:

(a) Write the ratio of the length to the width of the wall space in simplest form. Convert your ratio to a scale factor by dividing the length by the width.

(b) The ratio 16 : 9 written as a decimal scale factor is 1.78 (to 2 d.p.).

(i) Is the scale factor you calculated in **(a)** greater or less than 1.78?

(ii) To make your answer to **(a)** equal 1.78, should you decrease the length or the width of the wall space?

(c) Find a length and a width that give a ratio of 16 : 9 and make maximum use of Tom's wall space.

(d) Use Pythagoras' Theorem to determine the size of the plasma screen with the length and width you determined in **(c)**. Convert your answer to inches. What size screen should Tom buy?

The technician's problem

2 Sam works for a company that installs home theatre systems. He has a customer who wants a TV with a 70" screen. Sam needs to work out the length and width of the screen, so that he knows how much wall space is needed to hang it on. Sam uses the following method:

How to find the length and width of a TV screen

Step 1: Because the aspect ratio is 16 : 9, imagine a TV that has a length of 16" and a width of 9". Use Pythagoras' Theorem to find the diagonal screen size of this TV.

Step 2: Take the screen size of the TV the customer wants. Divide it by your answer to **(a)**. This gives a scale factor of how many times larger the customer's TV is than the 16" × 9" TV you considered in **(a)**.

Step 3: To get the length of the customer's TV, multiply the scale factor calculated in **(b)** by 16. To get the width, multiply it by 9.

Step 4: Convert lengths in inches to cm by multiplying by 2.54.

(a) The wall space the customer wants to hang the TV on is 1.3 m long and 1 m high. Follow the steps in Sam's method to determine whether a 70" TV will fit in this space.

(b) Will the more common screen size of 42" fit?

The biggest plasma screen in the world!

3 The biggest plasma screen ever produced was 150". How long and wide was it? Give your answer in metres.

Research

Find out about the aspect ratios that have been used to film movies and TV shows. What problems are encountered when a movie shot in one aspect ratio is shown on a screen with a different ratio? How are these problems solved?

2.5 Pythagorean triples

A group of three positive whole numbers (a, b, c) that satisfy Pythagoras' Theorem is called a **Pythagorean triple**. Some common Pythagorean triples are shown in the table below.

(3, 4, 5)	(8, 15, 17)	(12, 35, 37)
(5, 12, 13)	(9, 40, 41)	(16, 63, 65)
(7, 24, 25)	(11, 60, 61)	(20, 21, 29)

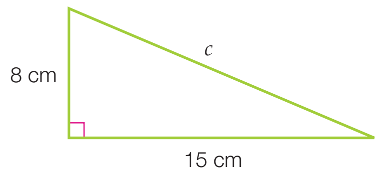
Multiples of a Pythagorean triple also satisfy Pythagoras' Theorem. For example, if we multiply each number in (3, 4, 5) by 2, we have (6, 8, 10) and $6^2 + 8^2 = 10^2$. Dividing Pythagorean triples by a number also gives a set of numbers that satisfy Pythagoras' Theorem. For example, dividing (7, 24, 25) by 2 gives (3.5, 12, 12.5) and $3.5^2 + 12^2 = 12.5^2$. The numbers in a Pythagorean triple need to be exact; it is not good enough for them to be rounded off to the required value.

Worked Example 10

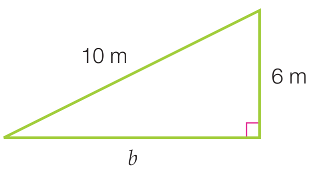
WE10

Using your knowledge of common Pythagorean triples, state the value of the unknown side in each of the following triangles.

(a)



(b)



Thinking			Working
(a)	**1**	Check if the values of the shorter sides match a common triple, (a, b, c).	(a) $a = 8$, $b = 15$, $c = ?$
	2	State the answer.	$c = 17$ cm
(b)	**1**	Check if the values of the hypotenuse and one of the shorter sides match a common triple, (a, b, c).	(b) $a = 6$ and $c = 10$ $= 3 \times 2$ $= 5 \times 2$ (6, ?, 10) is a multiple ($\times$ 2) of the common triple (3, 4, 5).
	2	Find the missing number in the triple by multiplying the common triple number by the same multiple.	$b = 4 \times 2$
	3	State the answer.	$b = 8$ m

2.5 Pythagorean triples

Navigator

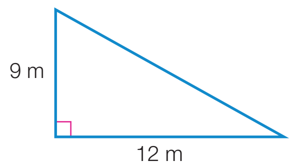
Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q11, Q12	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3, Q4, Q5 (a)–(d), Q6, Q7, Q9, Q10, Q11, Q12	Q1 Column 3, Q2 Columns 2 & 3, Q3, Q4, Q5 (e)–(h), Q6, Q8, Q9, Q10, Q11, Q12

Answers page 605

Fluency

WE10

1 Using your knowledge of Pythagorean triples, find the value of the missing sides in each of the following.

(a)


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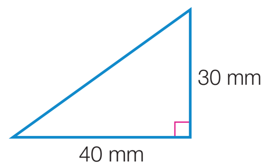

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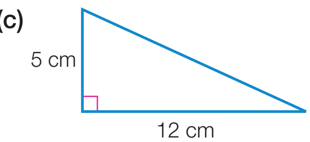
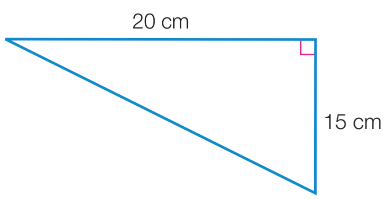
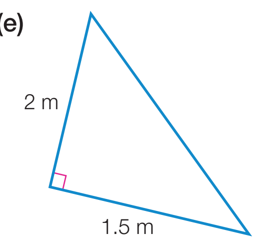
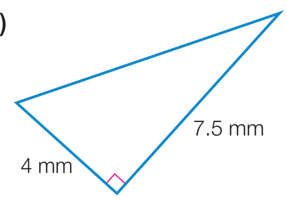
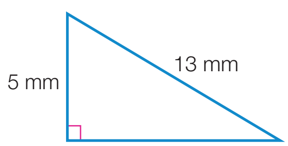


(d)


(e)


(f)


(g)


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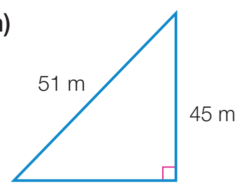
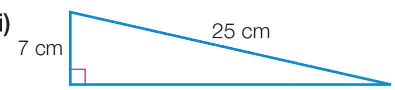


(i)


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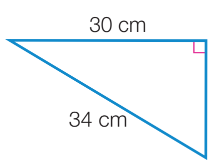
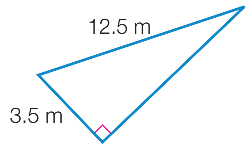
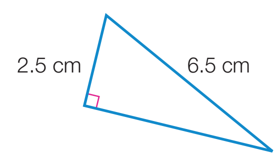


(k)


(l)


2 Find the value of c, using known Pythagorean triples.

(a) $a = 10, b = 24$ (b) $a = 15, b = 36$ (c) $a = 9, b = 12$
(d) $a = 27, b = 36$ (e) $a = 18, b = 80$ (f) $a = 50, b = 120$
(g) $a = 25, b = 60$ (h) $a = 500, b = 1200$ (i) $a = 80, b = 84$

3 If $a = 9$, $b = 40$ and $c = 41$ is a Pythagorean triple, which of the following is also a Pythagorean triple?

A (36, 160, 165) B (18, 80, 82) C (55, 240, 246) D (90, 405, 410)

4 If (5.6, 9, 10.6) satisfies Pythagoras' Theorem, another set of numbers that also satisfies Pythagoras' Theorem is:

A (65, 90, 106) B (10.2, 18, 1.06) C (14, 22.5, 26.5) D (14, 2.25, 20)

Understanding

5 Determine which of the following are Pythagorean triples.

(a) (6, 8, 10) (b) (24, 45, 51) (c) (14, 48, 50) (d) (25, 60, 80)
(e) (10, 16, 28) (f) (20, 48, 52) (g) (10, 18, 22) (h) (12, 50, 53)

6 Toula has just baked a cake in the shape of a rectangle. She wants to divide the cake into two right-angled triangles by cutting along a diagonal of the cake. Toula is then going to place a cake ribbon around the outside of each of the two triangular pieces of cake. The length of the cake is 32 cm and its width is 24 cm. Use a Pythagorean triple to determine exactly how much cake ribbon she will need to buy.

7 (a) Complete the table, using only odd values for m.

(b) What do you notice about the numbers in each row of the table?

(c) Beginning with 'Take any odd number', describe the process used to generate the Pythagorean triples in the table.

m	$\frac{m^2-1}{2}$	$\frac{m^2+1}{2}$
a	b	c
3	4	5
5	12	13
7		
9		
11		

8 (a) Complete the table, using consecutive odd numbers for x and y.

(b) What do you notice about the numbers in the last three columns?

(c) Beginning with 'Take any two consecutive odd numbers', describe the process used to generate the Pythagorean triples in the table.

x	y	$x+y$	xy	$\sqrt{a^2+b^2}$
		a	b	c
1	3	4	3	5
3	5	8	15	17
5	7			
7	9			
9	11			

Reasoning

9 Explain how the Pythagorean triple (160, 384, 416) can be obtained from another Pythagorean triple.

10 Derek believes he has found a rule for Pythagorean triples. He says 'In each Pythagorean triple with no common factor, it appears that there is one even and two odd numbers, one number with a factor of 5 and at least one prime number'.

3, 4, 5	8, 15, 17	20, 21, 29
5, 12, 13	12, 35, 37	28, 45, 53
7, 24, 25	16, 63, 65	36, 77, 85
9, 40, 41	20, 99, 101	44, 117, 125
11, 60, 61	24, 143, 145	52, 165, 173
13, 84, 85	28, 195, 197	

From the given sets of Pythagorean triple:

(a) Find at least four triples that satisfy Derek's rule.

(b) Find at least one triple that does not satisfy Derek's rule.

(c) If you could modify Derek's rule to make it more accurate, how would you change it?

(d) Can you find the pattern in each column and then write down the next triple in each column?

Open-ended

11 Choose one of the Pythagorean triples from the list on p. 112 and list its multiples (a, b, c) that have a, b and c less than 300.

12 **(a)** Stefan told Gabriella that for a particular Pythagorean triple (a, b, c), a *is* a multiple of 4 and b is a multiple of 5. What is one Pythagorean triple that satisfies this information?

(b) Write down two multiples of the answers to **(a)**.

Outside the Square — Problem solving

How old was Pythagoras?

We are not certain of Pythagoras' exact age, but use the clues about the Pythagorean triple (a, b, c), to obtain an estimate of how long he lived:

- c is Pythagoras' age, a value less than 100
- c is bigger than b and their difference is a cube number
- c is bigger than a and their difference is a square number
- a is a square number
- b is bigger than a and their difference is 41
- the digits of b are the same and are both odd numbers.

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Mathspace

The Pythagoreans

Pythagoras was known to have had a secret society of followers called the 'Pythagoreans'. These were a group of men and women dedicated to ethical living and studying the truth of the universe through mathematics. Legend has it that the outer circle, called the 'akousmatikoi' or 'the listeners', were not allowed to see Pythagoras and that he spoke from behind a veil, whereas the inner circle called themselves the 'mathematikoi' or 'mathematicians'.

ΡΣΤΥΦΧΨΩ ΑΒΓΔΕΖΗΘ

Truth, or False?

There are many strange myths surrounding the Pythagoreans. To find which myths have been attributed to the Pythagoreans you need to complete the following tasks.

Once you have completed the tasks your MYTH CODE answers will tell you which of the myths in the 'Myth List' are actually true.

Myth Breaker #1

The Pythagoreans saw odd numbers as masculine and even numbers as feminine.

Write the following square roots as whole numbers and say whether, according to the Pythagoreans, the answers are masculine or feminine.

1 $\sqrt{9}$

2 $\sqrt{16}$

3 $\sqrt{25}$

4 $\sqrt{81}$

5 $\sqrt{36}$

6 $\sqrt{121}$

7 $\sqrt{64}$

8 $\sqrt{100}$

9 $\sqrt{144}$

10 $\sqrt{169}$

11 $\sqrt{225}$

MYTH CODE: The first three feminine numbers in your answers

Myth Breaker #2

Pythagoras' Theorem can be stated in several ways. Half of the equations below are correct statements of Pythagoras' Theorem. Find which equations are correct.

1 $c^2 = a^2 + b^2$

2 $b^2 + a^2 = c^2$

3 $c^2 = b^2 - a^2$

4 $b^2 = c^2 - a^2$

5 $a^2 = b^2 - c^2$

6 $b^2 = a^2 - c^2$

7 $a^2 = c^2 - b^2$

8 $c^2 = a^2 - b^2$

MYTH CODE: The question number of the first correct rearrangement

Myth Breaker #3

Pythagorean triples are groups of three numbers (a, b, c) that satisfy Pythagoras' Theorem. Which of the following groups of numbers are Pythagorean triples?

1 (1, 2, 3)

2 (3, 4, 5)

3 (5, 12, 13)

4 (5, 9, 11)

5 (7, 24, 25)

6 (15, 18, 22)

7 (8, 15, 17)

8 (16, 63, 65)

MYTH CODE: The number of groups of three that were not a Pythagorean triple

Myth Breaker #4

Solve the following equations.

$d = \sqrt{3^2 + 4^2}$

$c = \sqrt{3^2 + 5^2}$

$x^2 = 5^2 + 12^2$

$p^2 = 4^2 + 65$

$6 = q^2 + 4$

$5 = 41 - a^2$

$3 = 52 - b^2$

$1 = 101 - m^2$

MYTH CODE: The numerical value of x, p and m

Myth list—which of these are true?

Myth 1 If you broke their rule of silence you were punished by death

Myth 2 They avoided eating tomatoes

Myth 3 Pythagoreans swore by the number 10 rather than by the gods

Myth 4 They avoided eating beans

Myth 5 One Pythagorean would only eat porridge in which an even number of oats had been used

Myth 6 The inner circle lived at the school

Myth 7 Each inner circle member was referred to by a number rather than a first name

Myth 8 The inner circle owned no personal possessions

Myth 9 They were mostly vegetarian

Myth 10 Pythagoreans discovered square numbers

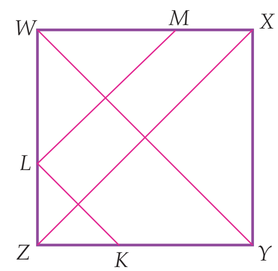
Myth 11 If you broke their rule of silence you were locked in a room and had to count whole numbers from 1 to 1 000 000 aloud

Myth 12 They believed eating breakfast on an even numbered day was bad for men's health

Myth 13 One Pythagorean discovered irrational numbers; the idea disturbed Pythagoras so much the man was executed

Challenge 2

1 *WXYZ* is a square of side 3 units. *K* is a point on *YZ*, *L* is a point on *ZW* and *M* is a point on *WX*. $KL \parallel YW$ and $LM \parallel ZX$. Let $ZK = a$ and $WL = b$ units.



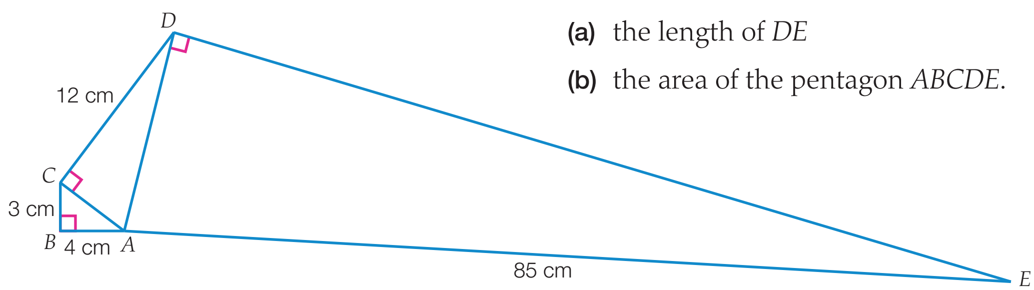
(a) Write an equation for b in terms of a and rearrange to find $a + b$.

(b) What is the exact length of KL in terms of a?

(c) What is the exact length of LM in terms of b?

(d) What is the exact length of $KL + LM$?

2 Triangles *ABC*, *ACD* and *ADE* have a right angle at *B*, *C* and *D*, respectively. $AB = 4$ cm, $BC = 3$ cm and $CD = 12$ cm.



If $AE = 85$ cm, find:

(a) the length of *DE*

(b) the area of the pentagon *ABCDE*.

3 Write a whole number whose square root is between 7 and 8.

4 What is the smallest number that, when multiplied by 2016, gives a square number?

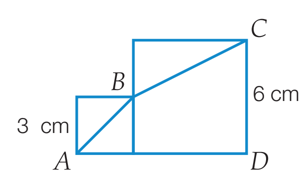
5 If m, n are integers and $m > n \geq 1$, then $m^2 - n^2$, $2mn$, $m^2 + n^2$ are a set of Pythagorean triples; that is, a set of three numbers that obey Pythagoras' Theorem.

(a) Show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$.

(b) Find the Pythagorean triples, with no common factors, whose smallest member is:

(i) 3 **(ii)** 5 **(iii)** 7 **(iv)** 8 **(v)** 9

6 A square of side length 3 cm is placed next to a square of side length 6 cm as shown. The perimeter of the quadrilateral *ABCD* is:



A $3\sqrt{2}$ cm **B** $3\sqrt{5}$ cm

C $6\sqrt{2}$ cm **D** $3(5 + \sqrt{2} + \sqrt{5})$ cm

Chapter review 2

D.I.Y. Summary

Key Words

converse	proof	rational approximation	theorem
hypotenuse	Pythagoras' Theorem	right-angled triangle	
irrational number	Pythagorean triple	surd	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The longest side of a right-angled triangle is called the __________.

2 For any __________, the square of the length of the __________ is equal to the sum of the squares of the lengths of the two shorter sides.

3 The equation $c^2 = a^2 + b^2$ is known as __________.

4 A __________ is a logical, step-by-step argument that demonstrates the truth of a mathematical statement.

5 A mathematical statement that has been proven true is known as a __________.

6 When a number such as $\sqrt{84}$ is written as a rounded decimal, we say this is a __________.

7 An irrational number written in exact form, such as $\sqrt{5}$, is known as a __________.

8 A group of three whole numbers that satisfy Pythagoras' Theorem, such as (3, 4, 5), is called a __________.

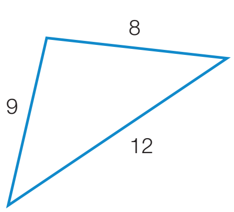
9 Explain what is meant by the 'converse of Pythagoras' Theorem'.

10 Explain, with examples, what we mean by 'irrational number'.

Fluency

1 Use the converse of Pythagoras' Theorem to determine whether the following triangles are right-angled. **Ex. 2.1**

(a)



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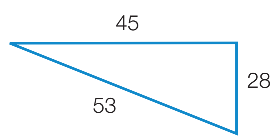

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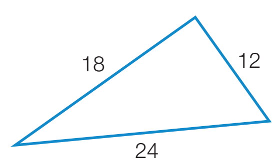
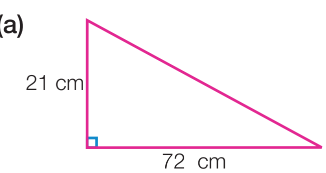


2 Find the length of the hypotenuse in the following right-angled triangles. Write your answers in exact form. **Ex. 2.2**

(a)



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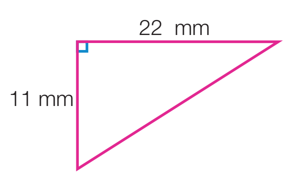

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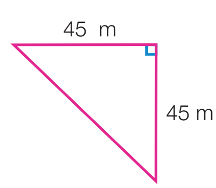
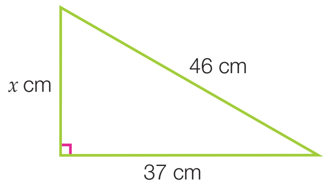


3 Use Pythagoras' Theorem to find the value of x. Write your answer correct to two decimal places if necessary. **Ex 2.3**

(a)



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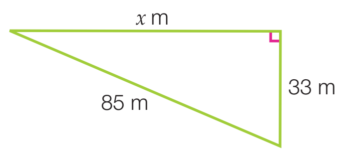

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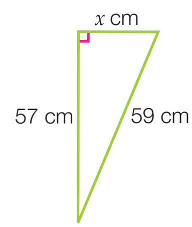
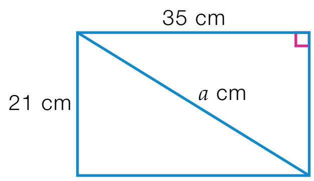
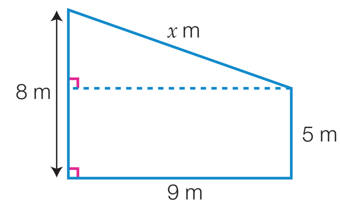


4 Use Pythagoras' Theorem to find the value of the unknown in each of the following diagrams. Write your answer correct to two decimal places if necessary. **Ex 2.2, 2.3**

(a)



(b)



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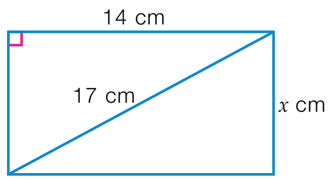

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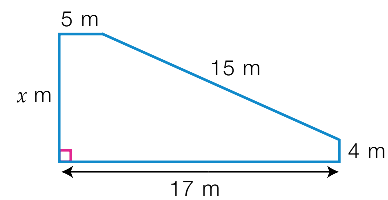


5 The Leaning Tower of Pisa was originally 150 m tall, but is leaning over so that a stone dropped from the top lands 5 m from the base. How far vertically does the stone fall? **Ex 2.4**

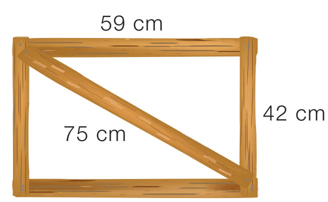
A 149.92 m **B** 150.00 m **C** 159.98 m **D** 162.00 m

6 Find the value of c in each of the following Pythagorean triples. **Ex 2.5**

(a) $a = 12, b = 16, c = ?$ (b) $a = 45, b = 200, c = ?$ (c) $a = 132, b = 385, c = ?$

Understanding

7 Simon is making the base for a dog's kennel. He put in a diagonal brace for extra support.

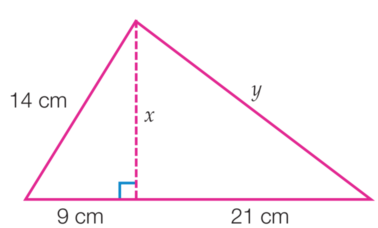


(a) From the measurements shown, has Simon built the base 'square'?

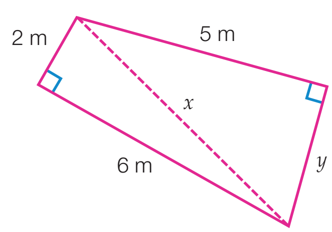
(b) What would be a reasonable measurement for the diagonal for Simon to consider the base to be 'square'?

8 Find the exact value of the variables in the following diagrams.

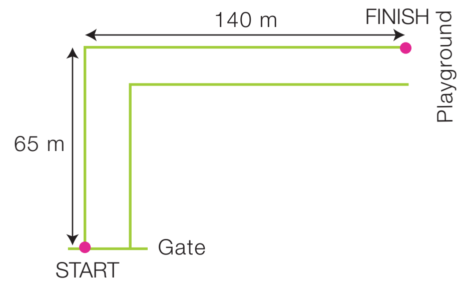
(a)



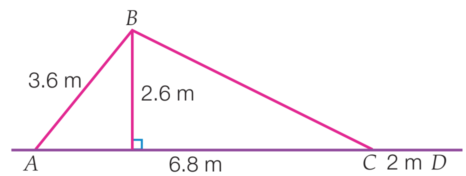
(b)



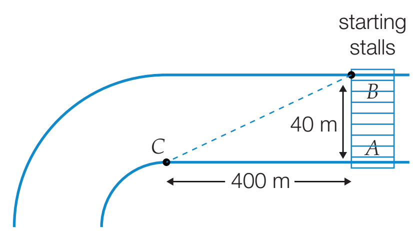
9 Ming and Tong decided to have a race from the gate of the park to the playground. Ming has decided to run along the left edge of the path and can do so at a speed of 7 metres per second. However, Tong decides to go directly from the gate to the playground, but this is very sandy and he can only run at 5 metres per second. Who wins the race to the playground?



10 A playground slide is 2.6 m high. The ladder is 3.6 m long. The distance from the bottom of the slide, C, to the base of the ladder, A, is 6.8 m. The end of the slide, CD, is 2 m. Find the total length of the slide from B to D.



11 Before each horse race, a barrier draw occurs to determine each horse's starting position. Closest to the rails is the more desired position and is represented by point A on the diagram shown. The least preferred position is near the outside edge of the track (point B). It can be assumed that all jockeys aim to take their horses in a direct line to the beginning of the first corner (point C).



(a) How much further does the horse at point B travel than the horse at point A?

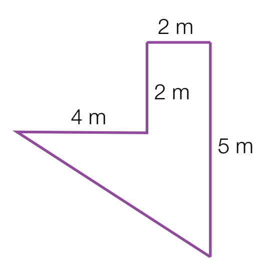
(b) Is this difference significant? How might this affect the race?

Reasoning

12 State whether or not each of the following is a Pythagorean triple.

(a) (120, 350, 370) **(b)** (16, 30, 35) **(c)** (77, 420, 427)

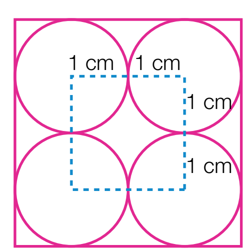
13 Daniel and Courtney are going to make a garden in an area in the backyard of their new house. They have measured the space and it is shown in the diagram.



(a) Calculate the perimeter of the garden and find the number of 2 m long sleepers they will need to form the perimeter of the garden.

(b) If 2 m long sleepers cost $35 each, how much will it cost to place sleepers all around the garden bed?

14 Four steel discs, each with radius 1 cm, are placed in a square frame as shown.



(a) Find the length of the sides of this frame.

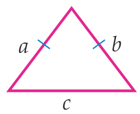
(b) In order to brace the frame, a diagonal support is to be added to the frame. Find the length of this support.

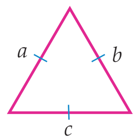
(c) Find the length of material needed to build the frame, including the diagonal brace.

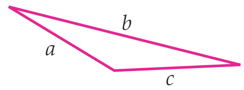
NAPLAN practice 2

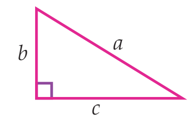
Numeracy: Non-calculator

1 Which triangle will satisfy Pythagoras' Theorem?

A


B


C


D


2 Choose the side-lengths that form a right-angled triangle.

A (20, 21, 23) **B** (20, 48, 52) **C** (6, 7, 8) **D** (5, 7, 10)

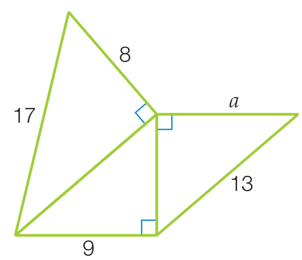
3 Which Pythagorean triple can be obtained from (36, 105, 111)?

A (12, 35, 37) **B** (36, 110, 333) **C** (24, 52.5, 55.5) **D** (72, 110, 222)

4 What is the value of the variable a?

A 5 **B** 7

C 9 **D** 11



Numeracy: Calculator allowed

5 The two shorter sides of a right-angled triangle are of length 14 mm and 48 mm, respectively. The length of the hypotenuse is:

A 25 mm **B** 34 mm **C** 46 mm **D** 50 mm

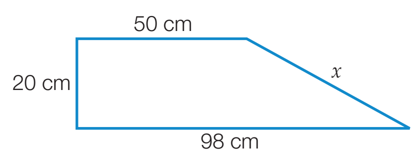
6 The hypotenuse of a right-angled triangle is 8.70 cm and one of its shorter sides is of length 6.30 cm. The length of the other shorter side is:

A 6.00 cm **B** 6.52 cm **C** 7.14 cm **D** 8.96 cm

7 What is the value of the variable in the diagram shown on the right?

A 42 cm **B** 48 cm

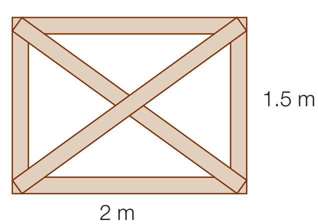
C 52 cm **D** 64 cm



8 Jemima wants to build a rectangular timber gate of width 2 m and height 1.5 m with two diagonal supports. What is the minimum total length of timber required to build the frame of the gate?

A 6 m **B** 7 m

C 9.5 m **D** 12 m



Mixed review A

Fluency

1 After investing $8000 into a savings account paying 6.5% interest p.a., Andrew had $8390 in his account. For how many months (correct to the nearest month) was it invested? Ex. 1.5

2 Calculate the following: Ex. 1.1

(a) 12.5% of 450 m (b) 38% of 208 kg (c) $5\frac{1}{2}$% of $3500

3 A triangle has side lengths of 51 cm, 140 cm and 141 cm. Determine if the triangle is right-angled. Ex. 2.1

4 Convert the following to percentages, rounding answers to one decimal place: Ex. 1.1

(a) 34 goals out of 47 shots

(b) four blond-haired students out of a class of 23

(c) nine rotten apples in a crate containing 140.

5 (a) A plasma screen TV priced at $1649 is discounted by 15%. Find the new price of the TV. Ex. 1.2

(b) A smartphone has been discounted down to a sale price of $699. If a 10% discount was applied, what was the original price of the phone? Answer to the nearest dollar.

6 Use the following table to calculate how much income tax should be paid on the taxable incomes below. Ex. 1.4

Taxable income	Tax on this income
$1–$6000	Nil
$6001–$37 000	15 cents for each $1 over $6000
$37 001–$80 000	$4650 + 30 cents for each $1 over $37 000
$80 001–$180 000	$17 550 + 37 cents for each $1 over $80 000
$180 001 and above	$54 550 + 45 cents for each $1 over $180 000

(a) $5500 (b) $14 000 (c) $54 000

7 Marc works part-time at the cinema. He receives a basic hourly rate of $16.82 for any hours worked between 9 a.m. and 9 p.m. He receives time-and-a-half for any hours worked after 9 p.m. and any hours on Saturday. He receives double time for any hours on a Sunday. Calculate Marc's wage for the hours worked in the following week. **Ex 1.3**

Tuesday	Wednesday	Thursday	Saturday	Sunday
11 a.m.–9 p.m.	2 p.m.–10 p.m.	3:30 p.m.–11:30 p.m.	12 p.m.–6 p.m.	2 p.m.– 9 p.m.

8 For each of the following household items, calculate: **Ex 1.7**

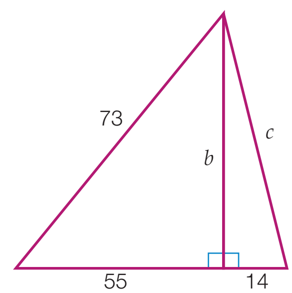
- **(i)** the weekly energy consumption, in kWh
- **(ii)** the weekly running cost, given an electricity tariff of 19.2 cents/kWh.

(a) A 100-W TV, in use for 4.5 hours per day.

(b) A 75-W fridge, in use the whole week.

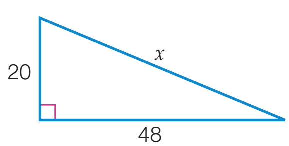
9 Kazim is a real estate agent. He is paid a $650 retainer per week, plus 2.5% of the value of any houses he sells. Calculate Kazim's income for a week in which he sold a house for $415 000. **Ex 1.3**

10 Find the values of b and c. **Ex 2.2, 2.3**

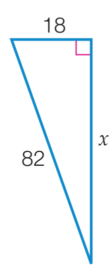


11 Find the value of the unknown side in each triangle. Give your answer in exact form. **Ex 2.2, 2.3**

(a)



(b)



Understanding

12 Mustafa has 8 shots at the basketball ring and gets 7 in. His friend Adam has 11 shots and gets 9 in. Calculate the number of goals as a percentage of the successful shots for each boy to determine who is the more accurate shooter.

13 Richard inspects old buildings to see whether they need support to keep them standing straight up. He believes that an old house is beginning to lean, so he stands at a point on the ground 3 m from the base of a wall, which is 2.3 m high. He measures the distance from this point to the top of the wall and finds that it is 3.5 m.

(a) Draw a diagram of a triangle to represent the situation, labelling it with all known information.

(b) Use Pythagoras' Theorem to determine whether the wall is at 90° to the ground.

14 'Snooker' Joe hits the cue ball. The ball travels 16 cm, strikes the side, rebounds at 90° to its original direction and then travels a further 63 cm before coming to rest. What is the distance between the start and finish position of the ball?

15 After investing $5000 in a savings account for 8 months, Michael received $5347. What was the rate of interest, assuming a constant rate of simple interest?

16 An aeroplane leaves Melbourne airport and flies 37 km due east. It then changes direction and flies 684 km due north to land at Waratah airport. What is the direct distance between the two airports?

Reasoning

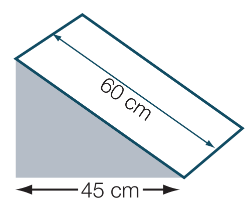
17 A department store buys a large crate of DVDs for $4.70 each. It places a 250% mark up on them and puts them out for sale. Later on, the store discounted the marked price of the DVDs by 23%.

(a) What was the original selling price of the DVDs?

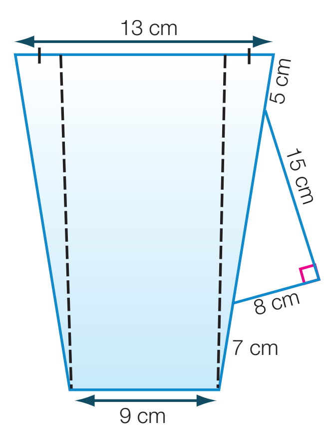
(b) What was the discounted price, rounded to the nearest 5 cents?

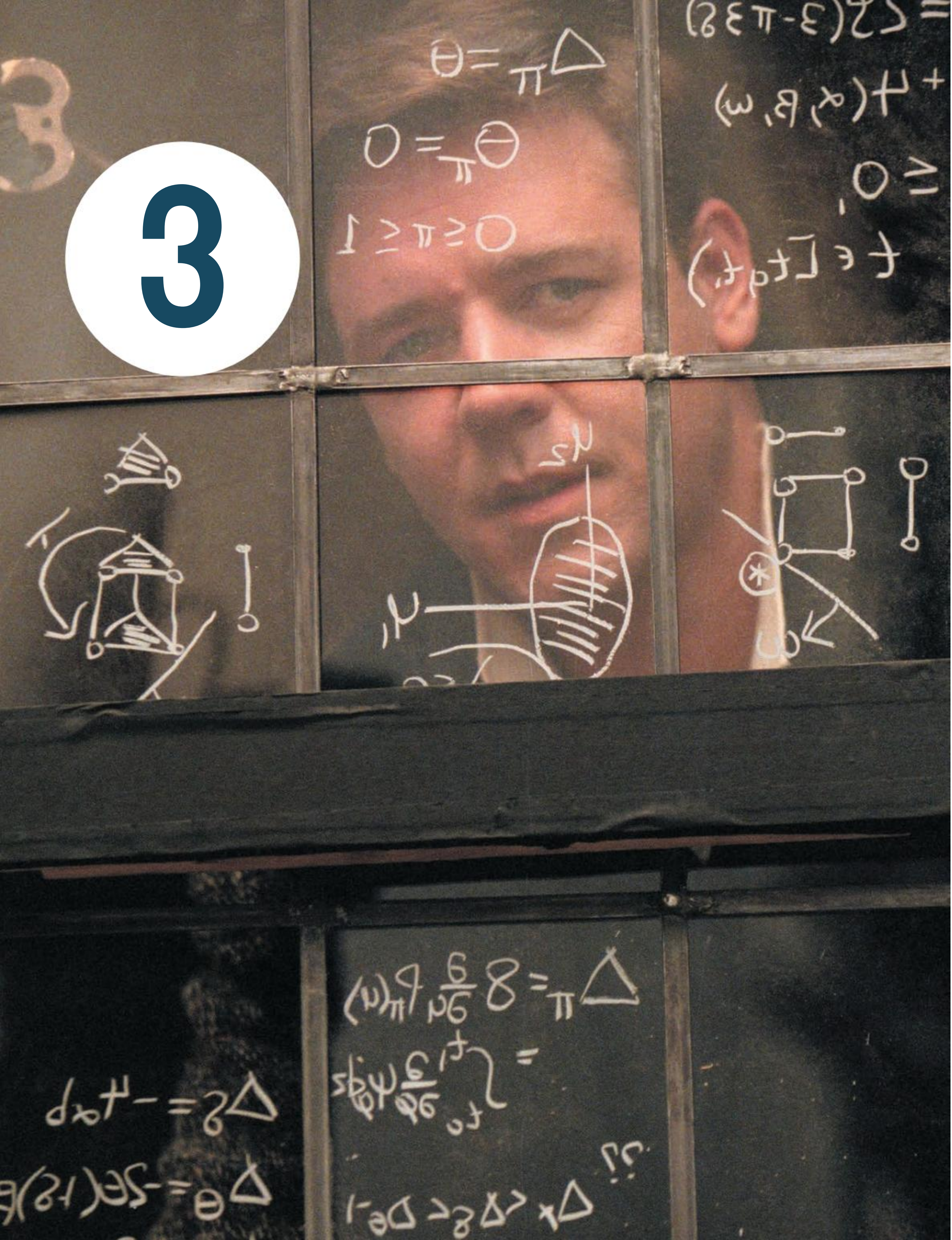
(c) What percentage mark-up does the new, discounted price represent, correct to the nearest whole number?

18 A ramp has the dimensions shown. It is placed against a raised platform that is 35 cm high. Does the ramp provide smooth access to the platform?



19 The cross-section of a water jug can be approximated by the shape shown. What is its height, correct to two decimal places?





3

Algebra

3

The game of algebra

What do strategy games like noughts and crosses have to do with winning the Nobel Prize in Economics?

Game theory is the mathematical analysis of strategies and interactions between people in competitive situations. One of the central components of game theory is the Nash equilibrium. This is named after John Nash, the mathematician who is the subject of the movie *A Beautiful Mind*, starring Russell Crowe. In a Nash equilibrium, no player in the 'game' can improve their strategy even if they know the strategies of the other players.

Mathematicians, and especially economists, analyse Nash equilibria using, in part, algebraic techniques to find solutions to complex equations. These are just slightly more advanced versions of the techniques you will study in this chapter.

At least eight recipients of the Nobel Prize in Economics, including John Nash, have been rewarded for their work involving game theory. There is no Nobel Prize in Mathematics!

Forum

If you go first in noughts and crosses you should never lose. Do you know how to ensure this? What does this have to do with the Nash equilibrium?

Why learn this?

Algebra gives us a symbolic way to describe and generalise patterns that occur in the real world. Algebraic techniques allow us to manipulate algebraic expressions into different forms to allow us to compare these patterns. This chapter on algebraic techniques will provide you with skills you can use later in the graphing and solving of equations. Understanding and interpreting these equations help us investigate the past, explain the present and predict the future.

After completing this chapter you will be able to:

- use the index laws and properties to simplify expressions, including operating with negative powers
- complete operations involving scientific notation and significant figures
- rearrange formulas to make different variables the subject
- expand algebraic expressions
- factorise algebraic expressions.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from the Pearson Reader.

Worksheet R3.1

1 Simplify the following expressions.

(a) $3x + 4x + 7x$ (b) $5y + 7y - 4y$ (c) $3k^2 + 5k^2 - 7k^2$

Worksheet R3.2

2 Simplify the following expressions.

(a) $5y + 6h - 2h - 3y$ (b) $6x^2 + 5y^3 - 2x + 3y^3$

Worksheet R3.3

3 Simplify the following expressions.

(a) $3x \times 4y$ (b) $5k \times 4k$ (c) $-5g \times 6g \times 2g$

Worksheet R3.4

4 Simplify, writing your answer in index form.

(a) $2^4 \times 2^9$ (b) $6^8 \div 6^2$ (c) $(5^2)^2$ (d) $(8.6325 \times 5.15)^0$

Worksheet R3.5

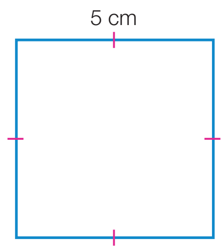
5 Expand:

(a) $\left(\frac{4^2}{5^3}\right)^3$ (b) $(2^4 \times 5^2)^4$ (c) $\left(\frac{7^2 \times 3^4}{4^5}\right)^2$

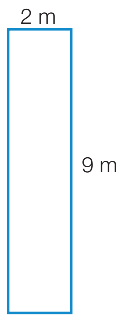
Worksheet R3.6

6 Find **(i)** the perimeter and **(ii)** the area of each of the following.

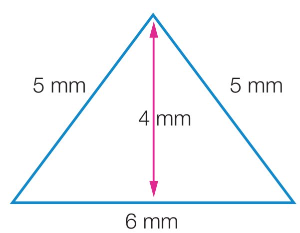
(a)



(b)



(c)



Worksheet R3.7

7 Expand the following expressions.

(a) $4(x + 2)$ (b) $6(x - 3)$ (c) $-2(x + 4)$

Worksheet R3.8

8 Factorise the following expressions.

(a) $6x - 12$ (b) $10x - 25$ (c) $-3x + 15$

Key Words

base	expand	highest common factor	reciprocal
binomial	expanded form	index	scientific notation
difference of two squares	expression	index form	significant figures
distributive law	factorise	perfect square	variable
equation	grouping in pairs	quadratic trinomial	

Introducing index laws using variables

3.1

Index form

Index form is a way of expressing repeated factors in a concise way.

We can write a number as the product of its prime factors.

$81 = 3 \times 3 \times 3 \times 3$

This is called **expanded form**.

We can use indices to write the number in **index** form. An index is also called an exponent or power.

number → $81 = 3^4$ ← index

base

We say that 3 to the power of 4 is 81.

81	=	$3 \times 3 \times 3 \times 3$	=	3^4
		Expanded form		Index form

Index laws

In Year 8, we found rules we could use when numbers were written in index form:

$2^4 \times 2^5 = 2^{4+5}$ $= 2^9$	When multiplying numbers in index form that have the same **base**, we keep the base and add the indices.
$\frac{3^6}{3^4} = 3^{6-4}$ $= 3^2$	When dividing numbers in index form that have the same base, we keep the base and subtract the indices.
$(4^2)^5 = 4^{2\times 5}$ $= 4^{10}$	When raising a number in index form to a power, we keep the base and multiply the indices.
$(2 \times 5)^3 = 2^3 \times 5^3$	When the product of factors in brackets is raised to a power, each factor in the brackets is raised to the same power.
$\left(\frac{7}{3}\right)^4 = \frac{7^4}{3^4}$	When the quotient of factors is raised to a power, each factor in the brackets is raised to the same power.

We also found the following property:

$2^0 = 1$ $4975^0 = 1$ $6.347^0 = 1$ $\left(\frac{5}{7}\right)^0 = 1$	Any number, except 0, raised to the zero index is equal to 1 (0^0 is undefined).

We will now explore these laws using **variables**.

Multiplication with indices

If $a^2 = a \times a$ and $a^3 = a \times a \times a$, then $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$.

Notice that $2 + 3 = 5$.

We can write:

$$\begin{aligned} &a^2 \times a^3 \\ &= a^{2+3} \\ &= a^5 \end{aligned}$$

Here is another example.

$$\begin{aligned} b^4 \times b^2 \times b &= (b \times b \times b \times b) \times (b \times b) \times (b) \\ &= b^7 \end{aligned}$$

Therefore, $b^4 \times b^2 \times b^1 = b^7$.

Notice that $4 + 2 + 1 = 7$.

Remember that $b = b^1$.

We can write:

$$\begin{aligned} &b^4 \times b^2 \times b^2 \\ &= b^{4+2+1} \\ &= b^7 \end{aligned}$$

What do a^m and a^n mean?

$a^2 = a \times a$. There are two factors of base a in the multiplication string.

$a^3 = a \times a \times a$. There are three factors of base a in the multiplication string.

$a^m = \underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a}$. There are m factors of base a in the multiplication string.

$$\begin{aligned} \text{so, } a^m \times a^n &= \underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a} \times \underbrace{a \times a \times \ldots \times a}_{n \text{ factors of } a} \\ &= \underbrace{a \times a \times \ldots \times a}_{m + n \text{ factors of } a} \\ &= a^{m+n} \end{aligned}$$

There are $m + n$ factors of base a in the multiplication string.

When multiplying variables in index form that have the same base, we keep the base and add the indices.

$a^m \times a^n = a^{m+n}$

Worked Example 1

WE1

Simplify each of the following, leaving your answers in index form.

(a) $7^3 \times 7^8$ **(b)** $c^3 \times c^6$ **(c)** $4h^5 \times 3h^9$

Thinking	Working
(a) 1 Are the bases the same? (Yes.)	(a) $7^3 \times 7^8$
2 Apply the index law for multiplication—keep the base and add the indices.	$= 7^{3+8}$ $= 7^{11}$
(b) 1 Are the bases the same? (Yes.)	(b) $c^3 \times c^6$
2 Apply the index law for multiplication—keep the base and add the indices.	$= c^{3+6}$ $= c^9$
(c) 1 Insert the multiplication signs. Group the coefficients (the numbers) and the variables.	(c) $4h^5 \times 3h^9$ $= 4 \times 3 \times h^5 \times h^9$
2 Simplify by writing the product of the coefficients and applying the index law for multiplication to the variables.	$= 12h^{5+9}$ $= 12h^{14}$

Division with indices

As we did with multiplication, we can identify a pattern for the division of numbers in index form. To calculate $a^5 \div a^3$, we can write the division as a fraction: $\dfrac{a^5}{a^3}$.

We can then write each power in expanded form: $\dfrac{a \times a \times a \times a \times a}{a \times a \times a}$.

By cancelling the common factor of a^3, we obtain $\dfrac{a \times a \times \cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1}{{}_1\cancel{a} \times {}_1\cancel{a} \times {}_1\cancel{a}}$.

This simplifies to $\dfrac{a^2 \times 1}{1} = a^2$.

So $\dfrac{a^5}{a^3} = a^2$. Notice that $5 - 3 = 2$.

We can write:

$$\frac{a^5}{a^3}$$
$$= a^{5-3}$$
$$= a^2$$

When dividing variables in index form that have the same base, we keep the base and subtract the indices:

$$a^m \div a^n = \frac{a^m}{a^n}$$
$$= a^{m-n}$$

Worked Example 2

WE 2

Simplify each of the following, leaving your answers in index form.

(a) $\dfrac{11^9}{11^5}$ **(b)** $\dfrac{d^7}{d^3}$ **(c)** $20c^8e^7 \div 4c^3e^2$

Thinking	Working
(a) 1 Are the bases the same? (Yes.)	(a) $\dfrac{11^9}{11^5}$
2 Apply the index law for division—keep the base and subtract the indices.	$= 11^{9-5}$ $= 11^4$
(b) 1 Are the bases the same? (Yes.)	(b) $\dfrac{d^7}{d^3}$
2 Apply the index law for division—keep the base and subtract the indices.	$= d^{7-3}$ $= d^4$
(c) 1 Write the expression in fraction form.	(c) $\dfrac{20c^8e^7}{4c^3e^2}$
2 Simplify the coefficients (the numbers 20 and 4) by cancelling common factors.	$= \dfrac{{}^{5}\cancel{20}c^{8-3}e^{7-2}}{{}_{1}\cancel{4}}$
3 Apply the index law for division to powers that have the same base.	$= 5c^{8-3}e^{7-2}$
(Simplify $\dfrac{c^8}{c^3}$ and $\dfrac{e^7}{e^2}$.)	$= 5c^5e^5$

Power of a power

What does $(m^5)^4$ mean? $(m^5)^4 = m^5 \times m^5 \times m^5 \times m^5$

$= m^{5+5+5+5}$

$= m^{4\times 5}$ or $m^{5\times 4}$

$= m^{20}$

Alternatively, $(m^5)^4 = (m \times m \times m \times m \times m) \times (m \times m \times m \times m \times m) \times (m \times m \times m \times m \times m)$

$\times (m \times m \times m \times m \times m)$

$= m^{20}$

So $(m^5)^4 = m^{5\times 4}$

$= m^{20}$

When raising a variable in index form to a power, keep the base and multiply the indices.

$$(a^m)^n = a^{m\times n}$$

This rule is known as the 'power of a power' law.

Worked Example 3

WE3

Simplify each of the following, leaving your answers in index form.

(a) $(6^3)^4$ (b) $(2^3)^2 \times (2^4)^3$ (c) $(5^3)^4 \div (5^5)^2$

(d) $(r^3)^5$ (e) $(p^3)^4 \times (p^5)^2$ (f) $(m^5)^3 \div (m^2)^6$

Thinking	Working
(a) Apply the power of a power law—keep the base and multiply the indices.	(a) $(6^3)^4$ $= 6^{3 \times 4}$ $= 6^{12}$
(b) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(b) $(2^3)^2 \times (2^4)^3$ $= 2^{3 \times 2} \times 2^{4 \times 3}$ $= 2^6 \times 2^{12}$
2 To multiply, apply the index law for multiplication—keep the base and add the indices.	$= 2^{6+12}$ $= 2^{18}$
(c) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(c) $(5^3)^4 \div (5^5)^2$ $= 5^{3 \times 4} \div 5^{5 \times 2}$ $= 5^{12} \div 5^{10}$
2 To divide, apply the index law for division—keep the base and subtract the indices.	$= 5^{12-10}$ $= 5^2$
(d) Apply the power of a power law—keep the base and multiply the indices.	(d) $(r^3)^5$ $= r^{3 \times 5}$ $= r^{15}$
(e) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(e) $(p^3)^4 \times (p^5)^2$ $= p^{3 \times 4} \times p^{5 \times 2}$ $= p^{12} \times p^{10}$
2 To multiply, apply the index law for multiplication—keep the base and add the indices.	$= p^{12+10}$ $= p^{22}$
(f) 1 Write the expression in fraction form.	(f) $\dfrac{(m^5)^3}{(m^2)^6}$
2 Apply the power of a power law to each term—keep the base and multiply the indices.	$= \dfrac{m^{5 \times 3}}{m^{2 \times 6}}$ $= \dfrac{m^{15}}{m^{12}}$
3 To divide, apply the index law for division—keep the base and subtract the indices.	$= m^{15-12}$ $= m^3$

3.1 Introducing index laws using variables

Navigator

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Fluency

WE1

1 Simplify each of the following, leaving your answers in index form.

(a) $2^4 \times 2^5$ (b) $3^7 \times 3^4$ (c) $5^{11} \times 5^7$

(d) $a^3 \times a^4$ (e) $b^6 \times b^5$ (f) $5c^3 \times c^2$

(g) $3d^3 \times 2d^2$ (h) $4b^4 \times 10b^4$ (i) $7b^2 \times 4b^2$

(j) $4a^5 \times 3a$ (k) $7a^3 \times 2a$ (l) $20a^3 \times 10a^4$

(m) $6a^4b^2 \times 4a^5$ (n) $9a^3b^3 \times 3ab^2$ (o) $5b^4c^5 \times 6b^3c^2$

(p) $-5a^4b^6 \times 7b^2$ (q) $-9e^5f^5 \times -4ef$ (r) $3a^5b^8 \times -8ab^2$

WE2

2 Simplify each of the following, leaving your answers in index form.

(a) $3^8 \div 3^5$ (b) $4^7 \div 4^2$ (c) $9^5 \div 9^3$

(d) $m^5 \div m^2$ (e) $4b^5 \div b^3$ (f) $5g^7 \div g^3$

(g) $\frac{n^6}{n^4}$ (h) $\frac{14b^7}{7b^5}$ (i) $\frac{22c^6}{11c^2}$

(j) $\frac{6a^5}{3a^4}$ (k) $\frac{12a^7}{8a^3}$ (l) $\frac{14e^4f^5}{10ef^2}$

(m) $10a^7b^7 \div 5a^4b^3$ (n) $-24a^5b^3 \div 21a^2b$ (o) $144a^6b^8 \div -12a^5b^7$

WE3

3 Simplify each of the following, leaving your answers in index form.

(a) $(2^3)^4$ (b) $(3^2)^5$ (c) $(5^4)^2$

(d) $(3^2)^5 \times (3^4)^2$ (e) $(4^3)^2 \times (4^5)^3$ (f) $(7^2)^5 \times (7^4)^2$

(g) $(5^2)^8 \div (5^3)^2$ (h) $(8^5)^8 \div (8^2)^3$ (i) $(11^4)^8 \div (11^3)^9$

(j) $(b^7)^3$ (k) $(k^8)^4$ (l) $(a^6)^2$

(m) $y^5 \times (y^3)^4$ (n) $(k^3)^8 \times k^3$ (o) $(m^6)^3 \times m^2$

(p) $(g^2)^5 \times (g^4)^2$ (q) $(y^2)^4 \times (y^5)^3$ (r) $(t^3)^3 \times (t^2)^6$

(s) $(p^3)^4 \div (p^5)^2$ (t) $(n^7)^2 \div (n^2)^4$ (u) $(b^8)^3 \div (b^3)^7$

4 (a) $-4r^3 \times -r^2$ can be simplified to give:

A $4r^1$ B $-4r^5$ C $4r^5$ D $4r^6$

(b) The product of $-abc$, $-5a$ and $-2a$ is:

A $-10a^3bc$ B $-7a^2bc$ C $10abc$ D $10a^2bc$

Understanding

5 Copy and complete each of the following by writing the correct index number in the box.

(a) $\dfrac{x^7 \times x^9}{x^{12}}$

$= \dfrac{x^{\square}}{x^{12}}$ (Add indices)

$= x^{\square}$ (Subtract indices)

(b) $\dfrac{6y^4 \times y^6}{y^2 \times y^5}$

$= \dfrac{6y^{\square}}{y^{\square}}$ (Add indices)

$= 6y^{\square}$ (Subtract indices)

6 Simplify each of the following by completing any multiplication of variables first.

(a) $\dfrac{x^8 \times x^7}{x^6}$ (b) $\dfrac{t^7}{t^2 \times 3t^4}$ (c) $\dfrac{c^6 \times c^9}{c^5 \times c^3}$

(d) $\dfrac{j^4 \times j^8}{j^2 \times j^5}$ (e) $\dfrac{4a^7 \times 3a^4}{a^3}$ (f) $\dfrac{8q \times 3q^6}{12q^4}$

7 (a) $\dfrac{f^8 \times f^6}{f^{10}}$ simplifies to:

A f^4 B f^6 C f^{48} D f^{12}

(b) $\dfrac{15x^9 \times 3x^6}{9x^{10} \times x^4}$ simplifies to:

A $5x^9$ B $9x$ C $5x$ D $9x^9$

(c) $-36k^4n^3 \div 9n^2k^3$ can be simplified to:

A $4k^4n$ B $-4k^7n^6$ C $-4kn$ D $45k^{12}n^6$

(d) $\dfrac{42x^5y^7z^6}{14x^4y^2z^3}$ simplifies to:

A $3xy^5z^2$ B $28xy^5z^3$ C $3xy^5z^3$ D $3x^9y^9z^9$

8 (a) How many groups of $3a^2$ will divide into $15a^4$?

(b) Use your answer to **(a)** to rewrite this as a multiplication that has $15a^4$ as the product.

9 (a) $\dfrac{a^8 \times b^5}{a^2 \times a^6}$ simplifies to:

A $a^{16}b^5$ B $a^{10}b^{11}$ C b^5 D b^{11}

(b) $\dfrac{3c^7 \times 8d^4}{12c^3 \times 4c^4}$ simplifies to:

A $2d^4$ B $\dfrac{d^4}{2}$ C $\dfrac{c^4d^4}{24}$ D $\dfrac{d^4}{24}$

10 Simplify each of the following.

(a) $(m^5)^2 \div m^7$ (b) $(p^6)^4 \div p^{10}$ (c) $\dfrac{(k^4)^3}{k^9}$ (d) $\dfrac{(n^8)^2}{n^6}$

11 (a) $(k^2)^3 \times (k^5)^2$ simplifies to:

A k^{16} B k^{12} C k^{60} D k^{15}

(b) $(b^5)^4 \div (b^3)^5$ simplifies to:

A b^4 B b C b^5 D b^{12}

Reasoning

Hint
If the bases on both sides of the equation are the same, the expression for the powers must be equal.

12 Solve for x in the following equations.

(a) $5^{x+2} = 5^{2x-1}$ (b) $3^{x+1} = 3$ (c) $2^{2x+4} = 1$ (d) $4(7^x + 7^x) = 56$

13 When Josh started his part-time job, he decided to save some of his wages on a weekly basis. He was paid $50.00 a week, so he decided to start by saving 5c the first week, 10c the next week, 20c the third week, continuing to double the amount he saved each week. He was shocked to find that, by the 11th week, he could not afford to keep up his plan.

(a) Copy the following table and complete the second column to show how much Josh saved each week for the first 5 weeks.

Week	Amount saved each week (c)	Amount saved each week (c) in index form
1		
2		
3		
4		
5		

(b) Complete the third column in this table by writing these amounts in index form.

(c) Write a rule in index form for finding the amount Josh saved in the nth week.

(d) Use your rule to calculate how much Josh was supposed to save in the 11th week.

(e) How much would Josh need to earn each week if he was to keep up his saving plan for 6 months (26 weeks)?

Hint
Look for a pattern. what values do you get if 1 is subtracted from a power of 2?

Open-ended

14 Find three different expressions for ■ and ▲ in terms of a and b to make this a true statement:

■ × ▲ = $-3a^2b^3$

15 Write three possible divisions involving indices that will give an answer of $5a^2b^3$.

16 Create three index number division expressions and simplify them using any of $24a^2b^3$, $3ab$, $48a^5b^6$, $22a^2b^2$ or 2×3^2a. Check your answers by giving the divisions to a friend to simplify.

17 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

18 The index laws are sometimes used incorrectly. Below is a list of equations that 'look' reasonable, but are actually incorrect.

In each case, explain the mistake that has been made and give the correct answer in its place. Explain why your answer is correct.

(a) $x^2x^4 = x^8$ (b) $\dfrac{x^9}{x^3} = x^3$ (c) $(x^2)^4 = x^{2^4} = x^{16}$

Outside the Square

Problem solving

Square cubes

Equipment required: 1 brain, calculator

'I have been studying the cube numbers. You know, some of them are also square numbers!' It was smart Alec, bragging again.

'You're kidding!' said Sue. 'A number can't be a square and a cube.'

'Yes it can' said Alec. 'Like, 64 is both 8 squared and 4 cubed.'

If Alec is right there may be more.

1 Can you find any other 2-digit numbers that are a square and a cube?

2 With the help of your calculator, how many can you find under 1000?

Strategy options

- Test all possible combinations.
- Look for a pattern.
- Break into manageable parts.

Maths 4 Real

The digital world comes in byte-sized bits

If you have ever used a computer, mobile phone, or media player such as an iPod or iPad, you have probably heard of terms such as 'bit' and 'byte', or more likely 'kilobyte' (kB), 'megabyte' (MB) or 'gigabyte' (GB). These words are used to describe the capacity of these devices to store documents, web pages, text messages, photos, music or videos in the form of electronic data.

1 Depending on which devices you can access right now, try to find out the following:

(a) Approximately how big is a document file (such as Microsoft® Word) that you might create as part of your schoolwork (e.g. an essay)?

(b) What is the file size of an average text message?

(c) What is the file size of a 3-minute piece of music?

(d) What is the capacity of the hard drive of your laptop or home computer?

What's a 'bit'?

The word '**bit**' is an abbreviation of the words '**b**inary dig**it**'. The binary digits are the digits 1 and 0, which are put together in different ways to make up binary numbers. Binary numbers are used by computers to carry information. We are more familiar with decimal numbers, which use the digits 0–9. The decimal number system is called 'base-10', because the place values of the digits increase by a power of 10 as you move to the left. We could take a number such as 1101 and write it in a place value table as:

100 000 (10^5)	10 000 (10^4)	1000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)
		1	1	0	1

However, the binary number system is a 'base-2' system, which means that each place value is a power of 2 greater than the next place value to the right. The number 1101 written in a base-2 place value table looks like this:

256 (2^8)	128 (2^7)	64 (2^6)	32 (2^5)	16 (2^4)	8 (2^3)	4 (2^2)	2 (2^1)	1 (2^0)
					1	1	0	1

To find the value of the binary number 1101 as a decimal number, we write it in expanded form, then evaluate:

$$
\begin{aligned}
& 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
&= 8 \quad + 4 \quad + 0 \quad + 1 \\
&= 13
\end{aligned}
$$

2 Write the decimal number values of these binary numbers by writing them in expanded form first. (Note: the subscript 2 indicates a binary, or base 2, number.)

(a) $(101)_2$ **(b)** $(1011)_2$

(c) $(1111)_2$ **(d)** $(100111)_2$

(e) $(101111)_2$ **(f)** $(1001001)_2$

What's a 'byte'?

Bits are usually packaged into sets of 8. A set of 8 bits is called a byte, similar to the way that 12 eggs are called a dozen. With 8 bits in a byte, you can represent numbers in the range 0–255.

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0 =	0	0	0	0	0	0	0	0
1 =	0	0	0	0	0	0	0	1
2 =	0	0	0	0	0	0	1	0
3 =	0	0	0	0	0	0	1	1
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
254 =	1	1	1	1	1	1	1	0
255 =	1	1	1	1	1	1	1	1

Note that unlike the base-10 place value table, empty place-value columns are filled with zeroes so that the byte always has 8 bits.

3 Write the following decimal numbers in 8-bit binary form.

(a) 3 **(b)** 7 **(c)** 19 **(d)** 34 **(e)** 154 **(g)** 204

So how do all these 1s and 0s make words and music?

Each binary number from 0 to 255, written in 8-digit 'byte form', corresponds to a specific letter or symbol or a formatting command (such as a space between words), which is then displayed by your computer. To turn data in digital (binary) form into music, players such as iPods have a Digital to Analogue Converter (DAC) that changes the digital data into a varying electrical signal, which produces different sounds as it passes through the speaker (or headphones).

I have an '8-gig' iPod. What does this mean?

We are used to seeing prefixes such as 'kilo' and 'mega' in front of measurement units such as 'litre'. We know that 'kilo' means 'one thousand' and 'mega' means one million. In the past 10 years, the storage capacities of computers have grown so much that bigger prefixes, such as 'giga' (1 billion) and 'tera' (1 thousand billion) are needed. In base-2 binary, it is impossible to get exact values of these base-10 numbers, so the closest base-2 index number is used.

Prefix	Symbol	Decimal value	Binary value
kilo	K	10^3 (1000)	2^{10} = 1024
mega	M	10^6 (1 000 000)	2^{20} = 1 048 576
giga	G	10^9 (1 000 000 000)	2^{30} = 1 073 741 824
tera	T	10^{12} (1 000 000 000 000)	2^{40} = 1 099 511 627 776
peta	P	10^{15} (1 000 000 000 000 000)	2^{50} = 1 125 899 906 842 624
exa	E	10^{18} (1 000 000 000 000 000 000)	2^{60} = 1 152 921 504 606 846 976

An '8-gig' mp3 player means that the player can store 8 gigabytes (GB) of data, or 8×2^{30} (8 589 934 592) bytes.

4 The decimal values of the prefixes increase by a factor of 10^3 (1000) as you move down the table. What do the binary values of the prefixes increase by?

5 **(a)** The average size of the music file of a song bought online is 6 MB. How many bytes is that?

(b) How many 6 MB files could be stored on an 8-GB iPod, assuming all 8 GB are available?

(c) If the average size of a video file is 700 MB, find two different combinations of numbers of music and video files that could be stored on a 16-GB iPod (assuming all 16 GB are available)?

Research

6 Find out how to add or multiply binary numbers.

7 The sizes of music and video files can vary greatly. Make a list of all the factors that can affect file size.

8 Investigate digital photography. Find out the sizes of photo files taken with a digital camera and what can affect them. Find out how digital images can be stored and the numbers of photos that can be stored on different devices or in different formats.

3.2 More index laws and index properties

Expanding products

Consider expanding $(2 \times a)^3$.

$$\begin{aligned}(2 \times a)^3 &= (2 \times a) \times (2 \times a) \times (2 \times a)\\ &= \underbrace{2 \times 2 \times 2}_{\text{3 factors of 2}} \times \underbrace{a \times a \times a}_{\text{3 factors of } a}\\ &= 2^3 \times a^3\end{aligned}$$

(grouping the coefficients together and the variables together)

So $(2 \times a)^3 = 2^3 \times a^3$

Each factor in the brackets has been raised to the power of 3.

What do we get when we **expand** $(kp)^4$?

$$\begin{aligned}(kp)^4 &= kp \times kp \times kp \times kp\\ &= \underbrace{k \times k \times k \times k}_{\text{4 factors of } k} \times \underbrace{p \times p \times p \times p}_{\text{4 factors of } p}\\ &= k^4 \times p^4\\ &= k^4p^4\end{aligned}$$

(grouping each type of variable together)

So $(kp)^4 = k^4p^4$

Each factor in the brackets has been raised to the power of 4.

If a product of factors in brackets is to be raised to a power, each factor in the brackets is raised to that power.

$$(ab)^m = a^m b^m$$

Worked Example 4

WE4

Use an index law to expand each of the following, leaving your answers in index form.

(a) $(3 \times 5)^4$ **(b)** $(ab)^5$ **(c)** $(5p^4q)^3$

Thinking	Working
(a) Apply the index law for expanding products—raise both variables to the given power.	(a) $(3 \times 5)^4$ $= 3^4 \times 5^4$
(b) 1 Apply the index law for expanding products—raise both variables to the given power.	(b) $(ab)^5$ $= a^5 \times b^5$
2 Write the answer in the simplest way without the multiplication sign.	$= a^5b^5$

Thinking	Working
(c) **1** Apply an index law—raise each of the factors in the product (5, p^4 and q) to the power of 3.	(c) $(5p^4q)^3$ $= 5^3 \times (p^4)^3 \times q^3$
2 Apply other index laws to simplify the terms. (Here, $(p^4)^3$ simplifies to p^{12}.)	$= 5^3 \times p^{4\times 3} \times q^3$ $= 5^3 \times p^{12} \times q^3$
3 Write the answer in the simplest way, remembering the order convention.	$= 5^3p^{12}q^3$

Expanding quotients

How can we expand $\left(\frac{m}{n}\right)^5$?

$$\left(\frac{m}{n}\right)^5 = \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n}$$

$$= \frac{m \times m \times m \times m \times m}{n \times n \times n \times n \times n} \begin{array}{l} \leftarrow \text{5 factors of } m \\ \leftarrow \text{5 factors of } n \end{array}$$

$$= \frac{m^5}{n^5}$$

So $$\left(\frac{m}{n}\right)^5 = \frac{m^5}{n^5}$$

Each variable in the brackets has been raised to the power of 5. The rule is:

If a quotient in brackets is to be raised to a power, each number or variable in the brackets is raised to that power.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Worked Example 5

WE5

Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{4}{3}\right)^5$ **(b)** $\left(\frac{p}{q}\right)^7$ **(c)** $\left(\frac{2a}{c^4}\right)^3$

Thinking	Working
(a) Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(a) $\left(\frac{4}{3}\right)^5$ $= \frac{4^5}{3^5}$
(b) Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(b) $\left(\frac{p}{q}\right)^7$ $= \frac{p^7}{q^7}$

(c) 1 Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(c) $\left(\frac{2a}{c^4}\right)^3$ $= \frac{(2a)^3}{(c^4)^3}$
2 Raise all parts of each term to the power. (Here, both 2 and a are raised to the power of 3.)	$= \frac{2^3 \times a^3}{c^{4 \times 3}}$
3 Apply any other index laws to simplify. (Here, the power of a power index law is used to simplify $c^{4 \times 3}$.)	$= \frac{2^3 a^3}{c^{12}}$

We can apply the inverse of these laws to simplify **expressions**.

Worked Example 6

WE6

Simplify each of the following, leaving your answers in index form.

(a) $2^3 \times 6^3$ **(b)** $5^4 \times h^4$ **(c)** $m^2 \times n^2$

Thinking	Working
(a) Apply the inverse of the index law for expanding products to simplify. Do not evaluate.	(a) $2^3 \times 6^3$ $= (2 \times 6)^3$ $= 12^3$
(b) Apply the inverse of the index law for expanding products to simplify.	(b) $5^4 \times h^4$ $= (5h)^4$
(c) Apply the inverse of the index law for expanding products to simplify.	(c) $m^2 \times n^2$ $= (mn)^2$

Worked Example 7

WE7

Simplify each of the following, leaving your answers in index form.

(a) $\frac{16^3}{4^3}$ **(b)** $\frac{t^4}{2^4}$ **(c)** $\frac{c^8}{d^8}$

Thinking	Working
(a) Apply the inverse of the index law for expanding quotients to simplify. Do not evaluate.	(a) $\frac{16^3}{4^3}$ $= \left(\frac{16}{4}\right)^3$ $= 4^3$
(b) Apply the inverse of the index law for expanding products to simplify.	(b) $\frac{t^4}{2^4}$ $= \left(\frac{t}{2}\right)^4$

(c) Apply the inverse of the index law for expanding products to simplify.

(c) $\dfrac{c^8}{d^8}$

$= \left(\dfrac{c}{d}\right)^8$

The zero index

If $2^0 = 1$, $45\,678^0 = 1$ and $0.000\,123^0 = 1$, does this work for any number a? That is, does $a^0 = 1$ also?

To simplify $a^m \div a^m$, we can subtract indices (second index law).

$$\begin{aligned} a^m \div a^m &= a^{m-m} \\ &= a^0 \end{aligned}$$

Alternatively, treating it as a fraction division:

$$\begin{aligned} a^m \div a^m &= \frac{a^m}{a^m} \\ &= \frac{a \times a \times a \times \ldots \times a}{a \times a \times a \times \ldots \times a} \\ &= 1 \end{aligned}$$

Therefore, $a^0 = 1$

A variable or variable product raised to the power of zero equals 1. The only exception is 0^0, which is undefined.

Worked Example 8

WE8

Simplify each of the following.

(a) $4p^0$ **(b)** $\dfrac{2y^4 \times y^3}{y^5 \times y^2}$ **(c)** $\dfrac{m^3n^6}{m^3n}$

Thinking	Working
(a) Replace the variable to the zero index with 1 and simplify.	(a) $4p^0$ $= 4 \times p^0$ $= 4 \times 1 = 4$
(b) 1 First, simplify the numerator and the denominator of the fraction by applying the appropriate index law.	(b) $\dfrac{2y^4 \times y^3}{y^5 \times y^2}$ $= \dfrac{2y^{4+3}}{y^{5+2}}$ $= \dfrac{2y^7}{y^7}$
2 Apply the index law for dividing with indices.	$= 2y^{7-7}$ $= 2y^0$
3 Replace the variable to the zero index with 1 and simplify.	$= 2 \times 1$ $= 2$

(c) **1** When dividing numbers with the same base, subtract the indices.

(c) $\dfrac{m^3n^6}{m^3n}$

$= m^{3-3}n^{6-1}$

$= m^0n^5$

2 Replace the variable to the zero index with 1.

$= 1 \times n^5$

$= n^5$

Negative indices

Another property we can find involves negative indices. What does 2^{-6} mean?

If we consider $2^6 = 64$, $2^5 = 32$, $2^4 = 16$, $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ and $2^0 = 1$, we see that the indices on the left-hand side of each equation are one less than the previous one and that the numbers on the right-hand side of the equations are half the previous one. To continue this pattern:

$$2^{-1} = \frac{1}{2},\ 2^{-2} = \frac{1}{4} = \frac{1}{2^2},\ 2^{-3} = \frac{1}{8} = \frac{1}{2^3}$$

So, $2^{-6} = \dfrac{1}{2^6}$

$8^{-5} = \dfrac{1}{8^5}$	Any number written with a negative power can be written as 1 divided by the number raised to the positive power of the same magnitude.

We will now consider two ways to simplify $a^2 \div a^6$.

To simplify $a^2 \div a^6$, we can subtract indices.

$$\begin{aligned} a^2 \div a^6 &= a^{2-6} \\ &= a^{-4} \end{aligned}$$

Alternatively, we can expand the numerator and denominator and cancel common factors.

$$\begin{aligned} \frac{a^2}{a^6} &= \frac{{}^1\cancel{a} \times {}^1\cancel{a}}{{}_1\cancel{a} \times {}_1\cancel{a} \times a \times a \times a \times a} && a \neq 0 \\ &= \frac{1}{a \times a \times a \times a} && a \neq 0 \\ &= \frac{1}{a^4} && a \neq 0 \end{aligned}$$

This shows that the expressions a^{-4} and $\dfrac{1}{a^4}$ are equivalent for all values of a, except $a = 0$, giving us the following index property:

$a^{-m} = \dfrac{1}{a^m},\ a \neq 0$	A variable raised to a negative index is equal to 1 divided by the same variable raised to the positive index.

We say that a^{-4} or $\dfrac{1}{a^4}$ is the **reciprocal** of a^4. Reciprocals always multiply to give a product of 1.

$$\begin{aligned} a^4 \times a^{-4} &= a^0 \\ &= 1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} a^4 \times \frac{1}{a^4} &= \frac{a^4}{a^4} \\ &= 1 \end{aligned}$$

Worked Example 9

WE 9

Simplify the following, writing your answer with positive indices.

(a) 3^{-2} (b) $4p^{-3}$ (c) $6x^{-5}y^2$

Thinking	Working
(a) Rewrite the expression using the negative index property. This changes the negative power to a positive power.	(a) $3^{-2} = \frac{1}{3^2}$
(b) Rewrite the expression using the negative index property. This changes the negative power to a positive power by using the reciprocal. The number (4) is not affected because the power is only applied to the variable (p), so it stays in the numerator.	(b) $4p^{-3} = 4 \times p^{-3}$ $= 4 \times \frac{1}{p^3}$ $= \frac{4}{p^3}$
(c) Rewrite the expression using the negative index property. This changes the negative index to a positive index. The number (6) and one of the variables (y^2) are not affected, so they stay in the numerator.	(c) $6x^{-5}y^2 = \frac{6y^2}{x^5}$

Index laws for powers with the same base

The following laws are true if $a, b \neq 0$ (a and b cannot be zero). We will only consider integer values for m and n here.

Multiplication of powers	$a^m \times a^n = a^{m+n}$	When multiplying numbers with the same base, keep the base and add the indices.
Division of powers	$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	When dividing numbers with the same base, keep the base and subtract the indices.
Power of a power	$(a^m)^n = a^{mn}$	When raising a number in index form to a power, keep the base and multiply the indices.
Expanding products	$(ab)^m = a^m b^m$	When removing brackets around a product that has been raised to a power, raise each number to that power.
Expanding quotients	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	When removing brackets around a quotient that has been raised to a power, raise each number to that power.

Index properties

The zero index	$a^0 = 1\ (a \neq 0)$	Any variable or product of variables raised to the power of 0 is equal to 1. Note that 0^0 is undefined.
Negative indices	$a^{-m} = \dfrac{1}{a^m}\ (a \neq 0)$	A variable raised to a negative power is equal to 1 divided by the same variable raised to the positive power.

3.2 More index laws and index properties

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Fluency

WE4

1 Use an index law to expand each of the following, leaving your answers in index form.

(a) $(6 \times 5)^2$ (b) $(11 \times 3)^4$ (c) $(12 \times 9)^5$

(d) $(4x)^2$ (e) $(p^3q)^2$ (f) $(a^4f)^3$

(g) $(2p)^5$ (h) $(p^3q^5)^7$ (i) $(c^2d^7)^4$

(j) $(bc)^3$ (k) $(6a^4)^3$ (l) $(9p^5q^4)^2$

(m) $(7m^3n^6)^2$ (n) $(2ab^2)^3$ (o) $(3ef^4)^4$

WE5

2 Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{12}{5}\right)^2$ (b) $\left(\frac{7}{3}\right)^4$ (c) $\left(\frac{9}{2}\right)^8$

(d) $\left(\frac{y}{x}\right)^3$ (e) $\left(\frac{d}{e}\right)^5$ (f) $\left(\frac{m^3}{n}\right)^4$

(g) $\left(\frac{w^4}{z}\right)^2$ (h) $\left(\frac{r^5}{s}\right)^4$ (i) $\left(\frac{d^3}{g}\right)^2$

(j) $\left(\frac{3}{w^6}\right)^4$ (k) $\left(\frac{8}{m^2}\right)^3$ (l) $\left(\frac{2}{r^4}\right)^3$

(m) $\left(\frac{h^3}{g^2}\right)^5$ (n) $\left(\frac{a^3}{c^5}\right)^4$ (o) $\left(\frac{t^4}{v^3}\right)^3$

(p) $\left(\frac{5a}{b}\right)^3$ (q) $\left(\frac{c^4}{2x}\right)^6$ (r) $\left(\frac{f^6}{6g}\right)^3$

3 Simplify each of the following, leaving your answers in index form.

WE6

(a) $3^2 \times 4^2$ (b) $4^6 \times 5^6$ (c) $8^4 \times 7^4$
(d) $4^8 \times h^8$ (e) $3^4 \times p^4$ (f) $7^9 \times r^9$
(g) $a^5 \times m^5$ (h) $k^7 \times t^7$ (i) $p^8 \times n^8$

4 Simplify each of the following, leaving your answers in index form.

WE7

(a) $\frac{15^4}{3^4}$ (b) $\frac{24^3}{6^3}$ (c) $\frac{132^5}{11^5}$
(d) $\frac{r^5}{3^5}$ (e) $\frac{n^7}{6^7}$ (f) $\frac{g^9}{4^9}$
(g) $\frac{x^6}{y^6}$ (h) $\frac{k^{11}}{n^{11}}$ (i) $\frac{g^{10}}{h^{10}}$

5 Simplify each of the following.

WE8

(a) k^0 (b) $7g^0$ (c) $7 + g^0$
(d) a^0b^2 (e) $3m^0n^0$ (f) $c^3d^0e^0$
(g) $-3a^4b^0$ (h) $-8g^{11}h^0$ (i) $-(4pr)^0$
(j) $y^4 \div y^4$ (k) $3x^7 \div x^7$ (l) $24m^6 \div 8m^6$
(m) $\frac{3x^4z}{z}$ (n) $\frac{h^5k^8}{h^5}$ (o) $\frac{8m^3n^4}{16m^3}$
(p) $\frac{a^5b^4}{a^2b^4}$ (q) $\frac{6h^4r^3}{2h^4r^3}$ (r) $\frac{9b^4d^3}{6b^4d^3}$
(s) $\frac{12e^3f^2}{16e^3}$ (t) $\frac{15r^2s^9}{27r^2}$ (u) $\frac{25k^4p^2}{15k^4}$

Hint
Remember that any number (except 0) divided by itself equals one.

6 Simplify the following, writing your answers in index form.

WE9

(a) 4^{-2} (b) 2^{-3} (c) 5^{-4}
(d) 3^{-3} (e) 5^{-2} (f) 4^{-3}
(g) x^{-5} (h) y^{-3} (i) b^{-7}
(j) $3e^{-3}$ (k) $7x^{-5}$ (l) $3g^{-4}$
(m) $8p^{-5}q^{-2}$ (n) $2x^{-3}y^{-5}$ (o) $12a^{-3}b^{-9}$
(p) $3p^3q^{-1}$ (q) $5x^2y^{-7}$ (r) $7a^{-5}b^2$

Understanding

7 (a) $(2k)^6$ simplifies to:

A $2k^6$ B $12k$ C $64k^6$ D $12k^6$

(b) $(3a^4)^3$ simplifies to:

A $27a^{12}$ B $9a^{12}$ C $9a^7$ D $27a^7$

(c) $\left(\frac{3}{g}\right)^4$ simplifies to:

A $\frac{12}{g^4}$ B $\frac{81}{g^4}$ C $\frac{12}{g}$ D $\frac{81}{g}$

(d) $\left(\frac{k^8}{7}\right)^2$ simplifies to:

A $\frac{k^{10}}{7}$ B $\frac{k^{16}}{49}$ C $\frac{k^{16}}{7}$ D $\frac{k^{10}}{49}$

8 Simplify each of the following by first applying the index law for expanding products, then completing the multiplication, applying other index laws if necessary.

(a) $(xy)^2 \times (xy)^4$ (b) $(gh)^7 \times (gh)^2$ (c) $(3x)^3 \times (4x)^2$

(d) $(4y)^2 \times (5xy)^2$ (e) $(cd)^3 \times (c^3d^2)^3$ (f) $(x^2y^3)^2 \times (xy)^2$

9 Simplify each of the following by first applying the index law for expanding quotients, completing the multiplication, then simplifying if possible.

(a) $\left(\frac{f}{g}\right)^3 \times \left(\frac{f}{g}\right)^2$ (b) $\left(\frac{p}{q}\right)^6 \times \left(\frac{p}{q}\right)^4$ (c) $\left(\frac{d^6}{q^4}\right)^2 \times \left(\frac{d^5}{q^2}\right)^3$

(d) $\left(\frac{a^4}{b^2}\right)^2 \times \left(\frac{b}{a}\right)^6$ (e) $\left(\frac{2a}{b}\right)^2 \times \left(\frac{a}{b}\right)^3$ (f) $\left(\frac{3x^2}{y}\right)^3 \times \left(\frac{2x}{y}\right)^2$

10 Simplify each of the following by applying the expanding products and the powers of powers index laws, then the index law for division.

(a) $\frac{(w^4z^3)^3}{(w^5z^2)^2}$ (b) $\frac{(c^7e^5)^2}{(c^4e^2)^3}$ (c) $\frac{(pq)^{12}}{(p^2q^3)^3}$ (d) $\frac{(vw)^9}{(v^3w^2)^2}$

11 Simplify each of the following.

(a) $(x^3)^0$ (b) $(d^5)^0$ (c) $(k^0)^4$ (d) $(p^0)^2$

(e) $(m^3)^4 \times (m^2)^0$ (f) $(f^5)^4 \times (f^3)^0$ (g) $(t^0)^3 \times t^4$ (h) $((q^4)^5 \times (q^5)^4)^0$

(i) $\frac{x^6 \times x^5}{x^{11}}$ (j) $\frac{3k^2}{k \times 5k}$ (k) $\frac{4a^3}{a^2 \times 7a}$ (l) $\frac{n^4 \times n^3}{n^5 \times n^2}$

(m) $\frac{2b^6 \times 5c^8}{c^7 \times 4c}$ (n) $\frac{d^6 \times e^2}{3e \times 2e}$ (o) $\frac{c^4 \times d}{2c^2 \times 5c^2}$ (p) $\frac{p^7 \times q^5}{15p^2 \times p^5}$

12 Use the negative index property to write the following with positive indices, then evaluate.

(a) 4^{-2} (b) 3^{-3} (c) 5^{-4}

(d) $\frac{1}{2^{-1}}$ (e) $\frac{1}{4^{-3}}$ (f) $\frac{1}{3^{-2}}$

(g) 4×3^{-2} (h) 3×4^{-3} (i) 2×5^{-2}

(j) $-\frac{7 \times 3^{-2}}{3}$ (k) $-\frac{6 \times 2^{-4}}{2^{-3}}$ (l) $-\frac{3 \times 3^{-4}}{3^{-6}}$

13 Express each of the following with positive indices.

(a) $\frac{1}{d^{-2}}$ (b) $\frac{1}{k^{-5}}$ (c) $\frac{3}{a^{-1}}$

(d) $\frac{x^3}{y^{-6}}$ (e) $\frac{p^7}{q^{-2}}$ (f) $\frac{m^{-4}}{n^{-4}}$

(g) $\left(\frac{a^2}{b^3}\right)^{-4}$ (h) $\left(\frac{m^4}{n^5}\right)^{-2}$ (i) $\left(\frac{x^{-5}}{y^2}\right)^{-3}$

14 (a) a^{-4} is the same as:

A $-a^4$ B $-4a$ C $\frac{1}{a^4}$ D $\frac{1}{a^{-4}}$

(b) $\frac{1}{2^{-4}}$ is the same as:

A 16 B 8 C $\frac{1}{16}$ D -16

(c) $\frac{1}{y^2}$ is the same as:

A $-y^2$ B y^{-2} C $-y^{-2}$ D $2y$

(d) $\frac{1}{8}$ is the same as:

A 2^3 B 2^{-3} C 3^{-2} D $\frac{1}{2^{-3}}$

Reasoning

15 Kellie and Eva are given the following question as part of their indices homework.

Simplify $\left(\frac{4x^2}{2x}\right)^3$.

Both girls had the correct answer, but Eva finished much faster. 'How did you do it so fast?' Kellie asked Eva.

Explain why Eva's method took fewer steps.

Here is Kellie's working:

$\left(\frac{4x^2}{2x}\right)^3$

$= \frac{4^3 \times x^{2\times 3}}{2^3 \times x^3}$

$= \frac{64x^6}{8x^3}$

$= 8x^3$

Here is Eva's working:

$\left(\frac{4x^2}{2x}\right)^3$

$= (2x)^3$

$= 8x^3$

16 By rewriting the expression if required and then using index laws, simplify each of the following using the smallest possible base.

(a) $\frac{9^{2n}}{9^n}$ (b) $2^n \times 4^{n+1}$ (c) $\frac{125}{5^n} \times \frac{5^{2n}}{25}$

17 (a) Using index notation, write the numbers 125, 25, 5 and 1, using base 5.

(b) Using index notation, write the next five numbers using the pattern in part **(a)**.

(c) Write down the whole sequence of nine numbers in this pattern without using index notation.

18 (a) If I divide a number by its square, what is the result?

(b) If a room has one side length of $12a$ m and an area of 84 m^2, what is the length of the other side of the room?

19 A model for the number of planets, N, created in a section of a galaxy is given by the formula $N = \left(\frac{n^2T^3p}{b}\right)^2$. In the formula:

n is the number of suns
T is the temperature of the surrounding space
p is the number of planets already present
b is the number of black holes.

(a) Write the formula without brackets.

(b) (i) How many new planets does the model predict if $T = 1$, $n = 1000$ and $b = p$?

(ii) If the temperature of the surrounding space is a negative number, will we obtain a positive or negative answer? Explain your reasoning.

20 The intensity of sound is measured in watts per square metre. The level of sound (loudness) is measured in decibels. The threshold of human hearing is 10^{-12} watts/m^2 and is defined as a level of sound of 0 decibels (dB). All other decibel levels are calculated by comparing the sound intensity with the threshold of human hearing. To do this we follow a simple rule.

1 Divide the intensity measured in watts/m^2 by the sound intensity of the threshold of human hearing.

For example, normal conversation is measured at 10^{-6} watts/m^2.

$$\frac{10^{-6}}{10^{-12}} = 10^{-6-(-12)}$$
$$= 10^{6}$$

This means that normal conversation is 1 million times as loud as the threshold of human hearing.

2 Multiply the index by 10. This is the intensity level in decibels.

Intensity level = $10 \times 6 = 60$ dB

To convert from decibels to find the sound intensity, we reverse the process.

For example, the sound level for a quiet library = 40 dB.

To find the sound intensity,

1 Divide the number of decibels by 10.

$$\frac{40}{10} = 4$$

2 Use this number as an index 10 and multiply by the threshold for human hearing.

The sound intensity of a quiet library $= 10^{4} \times 10^{-12}$
$= 10^{-8}$ watts/m^2

(a) Copy and complete the following table.

Source	Sound level (loudness; dB)	Sound intensity (watts/m^2)
Jet aircraft, 50 m away		100
Threshold of pain		10
Chainsaw, 1 m distance	110	
Disco, 1 m from speaker		0.01
Busy Traffic	80	
Vacuum cleaner, 1 m distance	70	
Average home		0.000 000 1
Rustling of leaves in the distance	10	

(b) If a whisper is 20 dB, how many times as loud is normal conversation?

(c) If the intensity of a rock concert is 10^0 watts/m^2 and the level of intensity of the threshold of discomfort is 120 dB, what comment could you make comparing these two measures?

(d) Which is louder, thunder at a sound level of 110 dB or a loud TV at a sound intensity of 10^{-4} watts/m^2?

Open-ended

21 (a) Write at least two expressions involving indices and multiplication that, when simplified, equal $36m^4n^8$.

(b) Write at least two expressions involving indices and division that can be simplified to $\frac{4m^4}{5n^6}$.

22 Write three different terms that $27x^3y^7$ could be divided by so that no x appears in the answer.

23 Write a question that could be simplified to $\frac{3x^2}{y^3}$ by using:

(a) any two index laws with an expression that has only positive indices

(b) two index laws with an expression that has at least one negative index

(c) any three index laws with an expression that has only positive indices

(d) three index laws with an expression that has at least one negative index.

Show, step by step, how your question can be simplified to obtain the final answer.

24 Jodie made four mistakes when simplifying the following index question. Find them, explain to Jodie what she did incorrectly and redo the question to get the correct answer. Jodie's working is shown below:

$$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{6x^4y^6}{2x^{-2}y^5}$$
$$= 4x^2y$$

Outside the Square

Puzzle

What is a 'googolplex'?

A 'googol' is 10^{100}. If you wrote out this number it would be a 1 followed by 100 zeroes. A googol is greater than the number of atoms in the observable universe, which has been estimated to be between 10^{79} and 10^{81}.

So, how big is a googolplex?

To find out, complete the following.

1 Match the value of the number with its equivalent index form.

2 Replace the numbers in the code with its corresponding letter.

Answer key:
12-13-3 12-11 12-1-13 7-11-10-13-2 11-6
9-11-11-9-11-8

1	10
2	10 000 000 000
3	1
4	10 000 000
5	100 000 000 000
6	1 000 000 000
7	100
8	100 000
9	10 000
10	1 000 000
11	100 000 000
12	1 000 000 000 000
13	1000

E	10^3
F	10^9
G	10^4
H	10^1
L	10^5
N	10^0
O	10^8
P	10^2
R	10^{10}
T	10^{12}
U	10^{11}
W	10^6
D	10^7

Scientific notation and significant figures

3.3

Using powers of 10

We often see numbers written as a product of a power of 10, such as when writing the distance from Earth to another planet or the size of an atom. Remember that 'power of 10' is a common way of saying 10 raised to a power.

When multiplying by powers of 10, we describe the process as moving the decimal point. In reality, the decimal point does not move. It is the digits that move a number of places to the left or right.

When multiplying by a power of 10 with a positive index, move the decimal point to the right by the same number of places as the value of the index. This makes the number bigger. Zeros are inserted as the values for any places not filled by other digits.

When multiplying by a power with a negative index, move the decimal point to the left by the same number of places as the value of the index after the negative sign. This makes the number smaller. Zeros are inserted as the values for any places not filled by other digits.

This is the same as dividing by the reciprocal power.

$$\begin{aligned} 6.5 \times 10^{-4} &= \frac{6.5}{10^4} \\ &= \frac{6.5}{10\,000} \\ &= 0.000\,65 \end{aligned}$$

Worked Example 10

WE10

Find the value of each of the following.

(a) 4.35×10^6 **(b)** 1.78×10^{-3}

Thinking	Working
(a) Because the index of 10 is positive, move the decimal point to the right by the same number of places as the index of 10. This makes the number bigger. Insert zeros as the values for any places not filled by other digits. (Move the decimal six places to the right and insert four zeros.) $4.350\,000$	(a) $4.35 \times 10^6 = 4\,350\,000$

Thinking	Working
(b) Because the index of 10 is negative, we are dividing by the reciprocal power (divide by 10^3). Move the decimal point to the left by the same number of places as the index of 10 after the negative sign. This makes the number smaller. Insert zeros as the values for any places not filled by other digits. (Move the decimal point three places to the left and insert two zeros.) 0.001 78	(b) $1.78 \times 10^{-3} = 0.00178$

Scientific notation

Scientists use **scientific notation** (also known as standard form) to make it easier to calculate with very large or very small numbers. To write a number in scientific notation, we place the decimal point after the first non-zero digit and then multiply by a power of 10 so that the product is the original number. The power of 10 can have a positive index (e.g. 10^4) or a negative index (e.g. 10^{-3}).

A number is in scientific notation if it is written as the product of a number, a, and a power of 10, where 'a' is any number greater than or equal to 1 but less than 10.

$a \times 10^b$ where $1 \leq a < 10$ and b is an integer

Consider the following examples.

The average distance from the Earth to the Sun is approximately 149 600 000 000 m. To write this in scientific notation, place a decimal point after the 1 and multiply by 10^{11} because we have moved the decimal point 11 places to the left.

Therefore, $149\,600\,000\,000 = 1.496 \times 10^{11}$.

The mass of a particular atom may be 0.000 000 000 000 000 001 67 g. Place the decimal point after the 1 and multiply by 10^{-18} because we have moved the decimal point 18 places to the right.

Therefore $0.000\,000\,000\,000\,000\,001\,67 = 1.67 \times 10^{-18}$ g.

Worked Example 11

WE11

Express each of the following in scientific notation.

(a) 5800 **(b)** 0.000 603 4

Thinking	Working
(a) Place the decimal point after the first non-zero digit and multiply by a power of 10 so that the product is the original number. Count the number of places right or left that the decimal has moved. Because it has moved to the left (three places to the left to get 5.8), we multiply by a positive power (10^3). The zeros at the end do not need to be written. 5800	(a) $5800 = 5.8 \times 10^3$

(b) Place the decimal point after the first non-zero digit and multiply by a power of 10 so that the product is the original number. Count the number of places right or left that the decimal has moved. Because it has moved to the right (four places to the right to get 6.034), we multiply by a negative power (10^{-4}). The zeros in front of the first digit do not need to be written.

0.000 603 4

Do not round off the decimal part of a number in scientific form unless it is requested.

(b) $0.000\,603\,4 = 6.034 \times 10^{-4}$

Scientific notation on a calculator

When the result of an arithmetic calculation is larger or smaller than the display on a calculator allows, it is written in scientific notation. However, it looks a little different from the way we would write it.

For example:

$4\,589\,765\,125 \times 849\,235\,713 = 3.897\,792\,458 \times 10^{18}$

On a calculator, this result may be written 3.897792458E18

where E18 represents $\times 10^{18}$.

Similarly, 3.7792311E-5 would represent $3.779\,231\,1 \times 10^{-5}$.

Significant figures

Significant figures are used in measurement because all measurements are approximate. All measurements are only as accurate as our measuring devices will allow. Using significant figures is just another way of rounding. The number of significant figures indicates how accurate the measurements are.

Rounding using significant figures

1. Write the number in scientific notation: $23\,467.8 = 2.346\,78 \times 10^4$
2. Decide how many significant figures you require.

$2.346\,78 \times 10^4$ $\approx 2.3 \times 10^4$ to two significant figures
$\approx 2.35 \times 10^4$ to three significant figures
$\approx 2.347 \times 10^4$ to four significant figures

Remember to check the next digit after the one you are rounding to, to decide whether to round up or down.

Rules for counting and writing significant figures

Counting significant figures

- All non-zero digits are significant. The first significant figure in a number is the first non-zero digit:

  2.346 78 has six significant figures

- Zeroes between non-zero digits or at the end of a decimal number are significant:

  1**0** 2**0**3 has five significant figures

  0.03**0** has two significant figures

- Zeroes at the end of an integer or at the beginning of a decimal number are not significant.

  1 350 000 has three significant figures

  0.000 013 5 also has three significant figures

Writing significant figures

- Zeroes appearing at the end of a decimal number after rounding are significant figures and must be retained to give the required number of significant figures.

  7.999 is written as 8.00 to three significant figures

- Zeroes sometimes need to be added.

  7.2 is written as 7.20 to three significant figures

Worked Example 12

WE12

Round each of the following to the number of significant figures specified.

(a) 50 842 to three significant figures **(b)** 0.007 497 to three significant figures

Thinking	Working
(a) Write the number in standard form. Count from the left the number of significant figures required (three significant figures). Remember to check the fourth digit and round the third digit up if it is 5 or greater.	(a) $50\,842 = 5.0842 \times 10^4$ $\approx 5.08 \times 10^4$ or $50\,800$
(b) Write the number in standard form. Count from the left the number of significant figures required. Remember to round correctly. The zero at the end is a significant figure and must be retained.	(b) $0.007\,497 = 7.497 \times 10^{-3}$ $\approx 7.50 \times 10^{-3}$ or $0.007\,50$

3.3 Scientific notation and significant figures

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Fluency

WE10

1 Find the value of each of the following.

(a) 9.8×10^3 **(b)** 6.5×10^7 **(c)** 3.25×10^6 **(d)** 4.2071×10^5

(e) 2×10^{-3} **(f)** 5×10^{-5} **(g)** 7×10^{-8} **(h)** 5.032×10^{-2}

(i) 8.26×10^{-3} **(j)** 4.18×10^{-5} **(k)** 7.14×10^{-4} **(l)** 3.55×10^{-6}

WE11

2 Express each of the following in scientific notation.

(a) 2560 (b) 90 020 (c) 614 000

(d) 190 000 (e) 8 050 000 (f) 56 000 000

(g) 0.305 (h) 0.03 (i) 0.0004

(j) 0.002 01 (k) 0.000 072 466 (l) 0.000 000 532 18

WE12

3 Round each of the following to the number of significant figures indicated in the brackets.

(a) 394 024 (2) (b) 0.054 02 (3) (c) 674.5 (2)

(d) 0.003 (3) (e) 1 679 000 (2) (f) 0.899 5 (3)

4 To how many significant figures have each of the following been written?

(a) 825 (b) 63.73 (c) 145 000

(d) 0.0034 (e) 3.008 (f) 0.0040

5 (a) A fly's wing flap is 3 milliseconds. Express this time in seconds using scientific notation.

(b) The time for light to travel once around the Equator is 134 milliseconds. Express this time in seconds using scientific notation.

(c) The average length of a blink is 300–400 milliseconds. Express this range of times in seconds using scientific notation.

6 Express each quantity described below using scientific notation.

(a) The distance around the Earth's equator is 40 075 km.

(b) The thickness of a soap bubble is about 0.000 000 1 m.

(c) A 10-cent coin is about 0.0015 m thick.

(d) Pluto is approximately 5900 million km from the Sun.

(e) The poliomyelitis virus has a diameter of 0.000 000 3 m.

(f) The melting point of gold is 1063°C.

7 Express each quantity described below as a number.

(a) Radio waves travel at about 3×10^8 m/s.

(b) In a hydrogen atom, the average distance between the proton and the electron is 1.5×10^{-15} m.

(c) The thickness of fine paper is about 1.2×10^{-4} m.

(d) The aluminium in a soft drink can contains around 4×10^{24} atoms.

(e) This distance from the Sun to Jupiter is approximately 7.78×10^8 km.

(f) The temperature at the centre of the Sun is about 1.5×10^7°C.

Understanding

8 Find the largest number of the following:

A 1.325×10^5 B 2.853×10^2 C 9.851×10^{-2} D 6.312×10^4

9 Use a calculator to evaluate each expression. Express your answers in scientific notation.

(a) $2.4 \times 10^6 \times 5.9 \times 10^4$ (b) $7.34 \times 10^3 \times 2.9 \times 10^{-6}$

(c) $7.86 \times 10^3 \div (2.62 \times 10^{-8})$ (d) $4.38 \times 10^9 \div (2.19 \times 10^{-3})$

(e) $6.2 \times 10^{-4} - 1.34 \times 10^{-5}$ (f) $4.6 \times 10^{-10} - 3.01 \times 10^{-11}$

(g) $2(4.3 \times 10^3 - 6.9 \times 10^7)$ (h) $5(9.1 \times 10^5 - 2.7 \times 10^9)$

Hint

A negative power in the denominator can be written in the numerator with a positive power.

(i) $\dfrac{3.21 \times 10^3 \times 4.19 \times 10^4}{5 \times 10^6}$

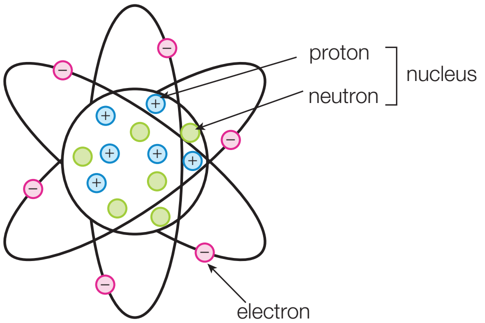
(j) $\dfrac{5.92 \times 10^4 \times 8.01 \times 10^{12}}{3 \times 10^5}$

(k) $\dfrac{3(2.5 \times 10^6 + 4.9 \times 10^7)}{6.5 \times 10^{-4}}$

(l) $\dfrac{4(7.3 \times 10^9 + 2.1 \times 10^4)}{8.6 \times 10^{-5}}$

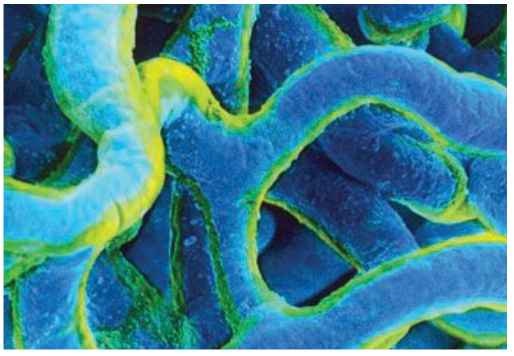
10 Express each quantity described below using scientific notation.

(a) The diameter of the nucleus of an atom is about 0.000 000 000 000 01 m.



(b) The highest Sherpa village on the slopes of Mount Everest is about 4000 m above sea level.

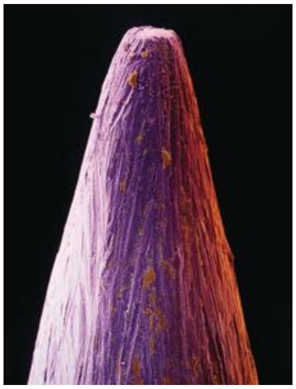
(c) This micrograph shows blood capillaries in skeletal muscle. These fine capillaries, which carry blood to the body's tissues, are about 0.000 02 m in diameter.



(d) The filament of a light bulb reaches a temperature of 2730°C.

(e) The number of athletes who attended the Sydney 2000 Olympic Games was 10 651.

(f) This micrograph shows bacteria on the point of a household pin. About 100 million million atoms could fit on the point of a pin.



(g) The Sun is approximately 150 000 000 km from Earth. The temperature at the surface of the Sun is about 6000°C.

(h) Australia has the highest rate of skin cancer in the world. More than 140 000 cases of skin cancer are reported each year in Australia.

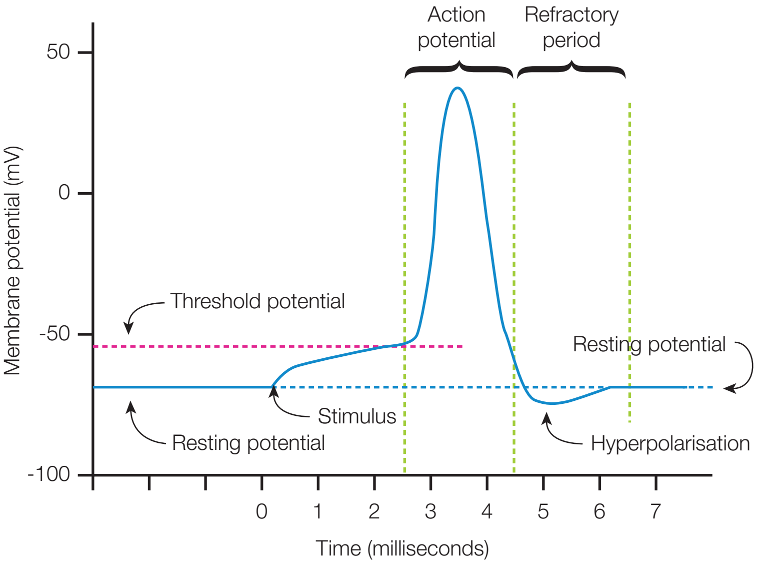
11 The nervous system controls movement in the body. Electrical impulses travel through nerves to the muscles causing them to contract. Here is a graph showing the time it takes for a nerve to transmit an impulse. Use the graph to answer the following questions, expressing all times in seconds using scientific notation.

(a) How long does it takes for a nerve to return to resting potential after it has been stimulated?

(b) How long does it takes for a nerve to reach hyperpolarisation after it has reached the threshold potential?

(c) How long does the action potential last?

(d) How long does the refractory period last?



Events that characterise the transmission of a nerve impulse.

12 (a) A personal computer in 1982 required approximately 1000 nanoseconds (1 microsecond) to perform an operation. Modern computers require between 0.3 and 1 nanosecond to perform an operation. How much faster are modern computers than the computers used in 1982? Express this range in seconds using scientific notation.

(b) The RAM of a personal computer in 1982 had a response time of 200 nanoseconds. The RAM of modern computers has a response time of between 5 and 10 nanoseconds. How much faster is the response time of the RAM in modern computers compared with computers in 1982? Express this range in seconds using scientific notation.

Hint
Express numbers in scientific notation before you compare them.

13

Timescale	Event
4570 mya	Planet Earth first formed
4440 mya	Earth cooled enough to form oceans and the atmosphere
3800 mya	First life on Earth: single-celled bacteria (microbial mat)
2400 mya	First photosynthesis by plants to produce oxygen
1700–1200 mya	Multicellular plants and fungi (sea dwellers)
850–630 mya	Ice age 'snowball Earth'—extinction event
580 mya	Multicellular animals: *Charnia* (filter feeders like sponges; sea dwellers)
550 mya	First creatures capable of movement: trilobites (first arthropod)
530 mya	First predators with mouth, teeth and stomach: echinoderms (*Arkarua*)
490–476 mya	Plants and invertebrates on land: arthropods (insects, giant dragonfly)
370 mya	Vertebrate animals on land
251 mya	Extinction event
230 mya	Dinosaurs
150 mya	First birds (type of dinosaur)
130–90 mya	Flowering plants and social insects (bees, termites, ants)
65 mya	Extinction of the dinosaurs (except birds)
6 mya	Humans (upright apes)
200 000	Modern humans

Note: mya means millions of years ago.

Above is a time line of the development of life on Earth. Use this time line to answer the following questions. Write all answers in scientific form.

(a) What is the age of the Earth?

(b) How long after the Earth cooled and the atmosphere developed did the first life on Earth appear?

(c) The ability of single-celled bacteria to create oxygen changed the Earth's atmosphere and allowed multicellular plants to develop. When did photosynthesis begin?

(d) For how long was the Earth completely covered in snow and ice (snowball Earth)?

(e) What was the time span over which multicellular plants and fungi developed?

(f) Remains of the first predator, an echinoderm, have been found in South Australia and named *Arkarua*. When did this animal first evolve?

(g) How long after invertebrates first moved out of the water and onto land did land vertebrates evolve?

(h) For how long was the Earth inhabited by dinosaurs?

(i) How many years before the first humans appeared on Earth did the first birds evolve?

(j) Extinction events are events that have extinguished almost all life on Earth. Recent extinction events are thought to have been due to volcanic eruptions or collisions with meteorites producing dust storms that blocked out the Sun, causing plants to die and disrupting the entire food chain. When were the last two extinction events?

(k) What is the time span from the formation of the Earth to the beginning of modern humans?

14 Light travels at a speed of about 3×10^8 m/s.

(a) Given that speed = $\dfrac{\text{distance}}{\text{time}}$, how far does light travel in:

(i) 2 minutes **(ii)** 6 weeks **(iii)** 1 year (365.25 days)? (This is called a *light year*.)

Express your answers in scientific form and in metres.

(b) If the mean distance of the Earth from the Sun is 1.496×10^8 km, approximately how long does light from the Sun take to reach Earth? Express your answer in minutes, correct to one decimal place.

15 Neil Armstrong sent messages back from the Moon on 20 July 1969. Radio waves travel at about 3.0×10^8 m/s. If the Moon is 384 000 km away from Earth, how long did it take, in seconds, for a message to travel from the Moon to Earth?

(Speed $= \dfrac{\text{distance}}{\text{time}}$)

16 The thickness of a sheet of paper is approximately 2×10^{-4} m and the thickness of a sheet of cellophane is approximately 2×10^{-3} m.

(a) Which is thinner?

(b) Find the difference in thickness between the two sheets.

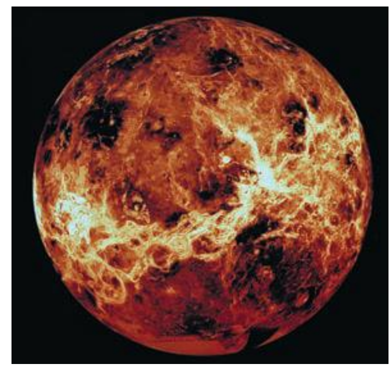
Reasoning

17 Two bacteria cultures, one containing 6.2×10^{11} cells and the other 3.5×10^9 cells, are combined in the one flask.

(a) How many cells are there in the flask?

(b) If the bacteria population doubles in size every 12 hours, how many cells will there be after 3 days?

(c) Find the value of your answer to **(b)**.



18 The planet Venus is approximately 108 million km from the Sun.

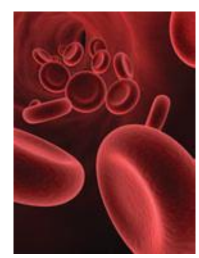
(a) Express this quantity in scientific notation.

(b) Write this distance in metres using scientific rotation.

(c) If the orbit of Venus around the Sun is assumed to be circular, find the distance, in kilometres, covered by Venus in one orbit. Answer in scientific notation.

(d) Earth is about 1.5×10^{11} m from the Sun. Find the closest distance, in metres, between Venus and Earth. Answer in scientific notation.

Open-ended



19 Estimate the diameter of a hair from your head and write your answer in scientific notation.

20 Estimate the number of red blood cells in your body and write your answer in scientific notation. (Children typically have 3.8–5.5 million red blood cells per microlitre of blood. Children weighing 40–60 kg would have 2.7–4.5 L of blood.)

Outside the Square — Problem solving

Stairway to the Moon

A standard piece of paper is approximately 0.1 mm thick.

If you cut it in half and place the two halves on top of each other, you now have a 'stack' of two pieces 0.2 mm thick. If you now cut the stack in half and place one half on top of the other, you have four pieces 0.4 mm thick.

The Moon is 363 100 km from the Earth at its closest point. How many 'cut 'n' stacks' will you need to do to have a pile of paper that reaches all the way to the Moon? (Assuming, of course, you are able to cut the stack as many times as you like.)

Strategy options

- Act it out.
- Make a table.
- Look for a pattern.

Mathspace

Equipment required: a handful of counters, 1 calculator

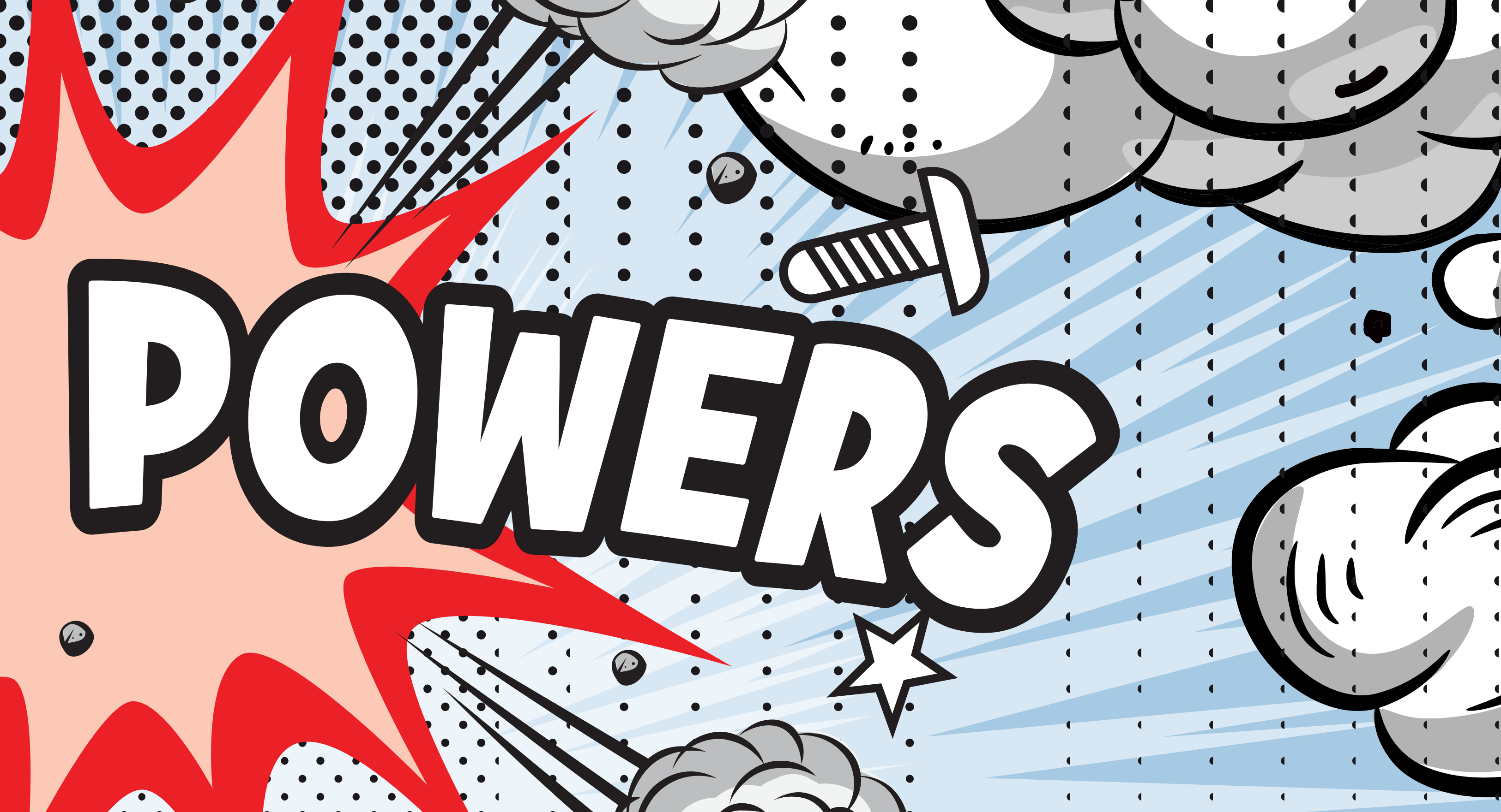
Power maze

The aim is to work your way through the maze. It will test how well you know the index laws. The squares that you can travel on have an index law correctly applied. You can not move diagonally or onto squares that contain an incorrect equation. Use your counters to keep track of your position. On which letter (A–J) does your path through the maze finish?

START

$3 \times 3 = 3^2$	$5 \times 5 = 2^5$	$\frac{y^{12}}{y^3} = y^4$	$3^{22} \div 3^{11} = 3^2$	$5^0 = 5$	$6^3 = 6 \times 6 \times 6$
$6 \times 6 \times 6 = 6^3$	$3^{-3} = \frac{1}{27}$	$\frac{x^6}{x^3} = x^3$	$\frac{q^{50}}{q^{20}} = q^{30}$	when $x = 3$, $4x^0 = 12$	$t \times t \times t \times t = 4^t$
$4^3 = 3 \times 3 \times 3 \times 3$	$4^{-4} = -16$	$\frac{w^{10}}{w^2} = w^{12}$	when $x = 2$, $3x^0 = 3$	$(m \times h^2)^2 = mh^4$	$(y^0 + x^0)^2 = 1$
$2^{-3} = -6$	$2^{100} = 200$	$p^3 \times p^4 = p^{12}$	$k^7 \times k^3 = k^{10}$	$r^3 \times r^3 = r^6$	$(n^{12})^4 = n^{48}$
$2^0 = 0$	when $w = 2$, $-5w^3 = 40$	$r^3 \times w^2 = (rw)^5$	$p^3 \times p^3 = p^3$	$f^3 \times g^2 = (fg)^5$	$(7fd^3)^0 = 7fd^3$
$(tv^3)^2 = tv^5$	$r^5 \times r^4 = r^9$	$8s^4 - 5s^4 = 3s^4$	$\frac{c^3b^2}{b^2} = c^3$	$(x^2)^3 = x^5$	$(-2)^3 = -8$
$\frac{44n^{24}}{4n^6} = 11n^4$	$(2d^7 \times 5d)^0 = 1$	$2y^2 + 2y^3 = 4y^5$	$(t^2)^{-6} = \frac{1}{t^{12}}$	$(md^2)^0 = 1$	$(a^{13}z)^2 = a^{26}z^2$
$(tv^4)^5 = t^5v^{20}$	$\frac{h^9}{h^9} = 1$	$z^0 = 0$	$5e^4 - 2e^2 = 3e^2$	$(3t^2)^3 = 27t^5$	$(3j^3 \times 5k)^0 = 15j^3k$
$\frac{20n^7}{10n^2} = 2n^5$	$(m^3)^2 = m^5$	$(2g)^2 = 4g$	$3d^2 + 3d^2 = 6d^4$	$\frac{d^4g^3h^{10}}{d^4g^3h^{10}} = 0$	$(-5)^{-2} = \frac{1}{25}$
when $m = 3$, $-3m^2 = -27$	$-40j^5 \div 10j^4 = -4j$	$(x^5)^3 = x^{15}$	$2m^3 \times m^5 = 2m^8$	$(q^3d^4)^2 = q^6d^8$	$-7n^2 \times 6n^9 = -42n^{11}$

POWERS



	$\dfrac{mq^{26}}{q^{13}} = mq^{13}$	$(pq^3)^2 = p^3q^5$	$u^3 \div u^{-1} = u^{-3}$	$3 \times 3 \times 3 \times 3 = 4^3$	A
	$(3d^7 \times 4r)^0 = 1$	$(3e^2)^4 = 81e^8$	when $x = 5$, $-2x^2 = -50$	$(a^0)^3 = a^3$	B
	$(5m^4 + 3m^4)^0 = 8$	$8^{-1} = 0$	$f^3 \times g^2 = f^3g^2$	$\dfrac{h^9}{h^2} = h^7$	C
	$3s^4 + 3s^4 = 6s^4$	$\dfrac{y^5}{x^3} = y^2$	$4 \times s^2 = (4s)^2$	$\dfrac{v^{10}}{v^2} = v^5$	D
	$(k^3 \times m^2 \times 3)^0 = 1$	$2^6 = 6 \times 6$	$(-4r^3)^2 = 16r^6$	$(-2k)^3 = -8k^3$	E
	$(-3)^2 = 9$	$r^5 \times r^4 = r^{20}$	$-20j^4 \div -4j^4 = 5$	$6b^2 + 4a^2 = 10a^2b^2$	F
	$(2a^2 \times 3d)^3 = 18a^6d^3$	$(n^1)^1 = n^2$	$(2 + n^0)^3 = 27$	$(k^2 \times 2k^3)^2 = 4k^7$	G
	$(-y^3)^2 = -y^6$	$11^{-1} = \dfrac{1}{11}$	$(-2)^5 = -32$	$7q^4 - 7q^3 = 7q$	H
	$(3p)^2 = 9p^2$	$\dfrac{a^2b^3c^4}{ab^3c^4} = a$	$\left(\dfrac{1}{2}x\right)^2 = \dfrac{1}{2}x^2$	$(3b)^2 \times r^2 = 9br^2$	I
	$(a^2g^2)^3 = a^5g^5$	$d^2 \times d^2 = d^4$	$v^3 \times g^2 \times 3^0 = 1$	$4^0 = 1$	J

3.4 Rearranging formulas

A **formula** is an **equation** showing the relationship between two or more variables or unknowns. It is written with one variable on the left-hand side of the equation and all other variables on the right hand side. The variable on the left-hand side is called the subject of the formula.

When the variable required is not the subject of the formula, we need to rearrange the equation. The formula for finding the velocity, v, of an object after accelerating over a distance d is $v^2 = u^2 + 2ad$. We can rearrange this to make u, a or d the subject; that is, we can rearrange the formula to the form $u = \dots$, $a = \dots$ or $d = \dots$.

We need to use the correct order of operations to rearrange the formula correctly.

Worked Example 13

WE13

The formula for finding the velocity, v, of an object after accelerating over a distance d is $v^2 = u^2 + 2ad$.

(a) Rearrange the formula to make a the subject.

(b) Find a when $v = 5$, $u = 1$ and $d = 2$.

Thinking	Working
(a) 1 Identify the first operation that needs to be performed on the equation and perform that operation. (Subtract u^2 from both sides.)	(a) $v^2 = u^2 + 2ad$ $v^2 - u^2 = u^2 + 2ad - u^2$ $v^2 - u^2 = 2ad$
2 Identify the first operation that needs to be performed on the equation and perform that operation. (Divide both sides by $2d$.)	$\dfrac{v^2 - u^2}{2d} = \dfrac{2ad}{2d}$ $\dfrac{v^2 - u^2}{2d} = a$
3 Rewrite the formula with a as the subject. By convention, the subject is written on the left-hand side of the equals sign.	$a = \dfrac{v^2 - u^2}{2d}$
(b) 1 Substitute the known values into the rearranged formula.	(b) $a = \dfrac{v^2 - u^2}{2d}$ $a = \dfrac{5^2 - 1^2}{2 \times 2}$
2 Simplify to find the value of a.	$a = \dfrac{25 - 1}{4}$ $a = \dfrac{24}{4}$ $a = 6$

Alternatively, the values could be substituted into the original formula and the equation solved in the usual way.

Sometimes before the required variable can become the subject of the formula, the equations need to be undone with operations such as squaring both sides or taking the square root or reciprocal of both sides.

Worked Example 14

WE14

Rearrange the formula in each of the following to make the variable in brackets the subject of the formula

(a) $A = \pi r^2 \quad (r)$ **(b)** $T = 2\pi\sqrt{\frac{l}{g}} \quad (l)$

Thinking	Working
(a) 1 Write the equation.	(a) $A = \pi r^2$
2 Identify the first inverse operation to perform on the equation and perform that operation (÷ π).	$\frac{A}{\pi} = r^2$
3 Identify the next inverse operation that needs to be performed so that the required variable is the subject of the formula (take the square root of both sides). Decide whether both the positive and negative values are appropriate. (In this case, the radius is a positive length, so reject the negative solution.)	$r = \sqrt{\frac{A}{\pi}}$
(b) 1 Write the equation.	(b) $T = 2\pi\sqrt{\frac{l}{g}}$
2 Identify the first inverse operation to perform on the equation and perform that operation (÷ 2π).	$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$
3 Identify the next inverse operation to perform on the equation and perform that operation (square both sides of the equation).	$\frac{T^2}{(2\pi)^2} = \frac{l}{g}$
4 Repeat this procedure until the required variable is the subject of the formula (× g).	$l = \frac{T^2 g}{4\pi^2}$

3.4 Rearranging formulas

Navigator

Answers page 609

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q10, Q13	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13	Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: calculator for Questions **5**, **6** and **7**

Fluency

WE13

1 **(a)** **(i)** Rearrange the formula to make k the subject if $P = mk$.

(ii) Find k when $P = 32$ and $m = -4$.

(b) **(i)** Rearrange the formula to make b the subject if $y = mx + b$.

(ii) Find b when $y = 9$, $m = 5$ and $x = 2$.

(c) **(i)** Rearrange the formula to make h the subject if $C = ah - g$.

(ii) Find h when $C = 19$, $a = -24$ and $g = -7$.

(d) **(i)** Rearrange the formula to make x the subject if $a = mx + c$.

(ii) Find x when $a = -21$, $m = 4$ and $c = 3$.

(e) **(i)** Rearrange the formula to make p the subject if $k = \dfrac{mp - 2}{n}$.

(ii) Find p when $k = 42$, $m = -4$ and $n = 5$.

(f) **(i)** Rearrange the formula to make w the subject if $u = \dfrac{w}{v} - xy$.

(ii) Find w when $u = 6$, $x = -3$, $y = 7$ and $v = -2$.

WE14

2 Rearrange the formula in each of the following to make the variable in brackets the subject of the formula. (Assume all variables are positive.)

(a) $V = \pi r^2 h \quad (r)$ **(b)** $E = \dfrac{1}{2}mv^2 \quad (v)$ **(c)** $F = \dfrac{dh^2}{k} \quad (h)$

(d) $k = 7\sqrt{\dfrac{m}{n}} \quad (m)$ **(e)** $d = 2\sqrt{\dfrac{b}{c}} \quad (b)$ **(f)** $m = x\sqrt{\dfrac{y}{z}} \quad (y)$

3 Rearrange each of the following to make x the subject.

(a) $x + y = z$ **(b)** $x - 3a = 2b$ **(c)** $kx - m = n$

(d) $dx + c = f$ **(e)** $rt - x = p$ **(f)** $vy - x = -w$

(g) $\dfrac{2x + y}{p} = k$ **(h)** $\dfrac{t + rx}{w} = m$ **(i)** $\dfrac{mp + px}{m} = n$

(j) $cdx + e = gh$ **(k)** $axp - k = mn$ **(l)** $s - tux = vw$

(m) $b + \dfrac{xd}{y} = f$ **(n)** $\dfrac{mx}{n} + kp = t$ **(o)** $ab - \dfrac{dx}{c} - f = o$

4 The formula for finding the velocity, v, of a moving object is $v^2 = u^2 + 2as$. A correct rearrangement of the formula would be:

A $u^2 = v^2 + \dfrac{2as}{2}$ **B** $s = \dfrac{v^2 - u^2}{2a}$

C $a = \dfrac{2s}{v^2 - u^2}$ **D** $a = \dfrac{u^2 - v^2}{2s}$

Understanding

5 The formula for finding the velocity, v m/s of an object starting with a velocity of u m/s and undergoing a constant acceleration of a m/s^2 over time t s is $v = u + at$.

(a) Rearrange the formula to make t the subject.

(b) Find the value of t when:

(i) $a = 14$ m/s^2, $u = 20$ m/s and $v = 76$ m/s

(ii) $a = 2.4$ m/s^2, $u = 5.7$ m/s and $v = 12.9$ m/s

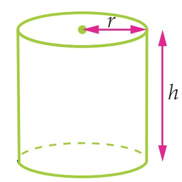
6 The circumference (C) of a circle is related to the radius (r) by the formula $C = 2\pi r$. Use the calculator value for π and give your answer correct to two decimal places.

(a) Rearrange the formula to make r the subject.

(b) Find the value of r when:

(i) $C = 628$ **(ii)** $C = 16.4$

7 The formula for the volume (V) of a cylinder is $V = \pi r^2 h$. Rearrange the formula to find h when $r = 5$ and $V = 157$. Use the calculator value for π and give your answer correct to two decimal places.



8 The perimeter of a rectangle is given by $P = 2(l + w)$, where l and w represent the length and width of the rectangle, respectively.

Rearrange the formula to make w the subject. Find the width of the rectangle if the length is 23 m and the perimeter is 84 m.

Reasoning

9 The kinetic energy (E) of an object, in joules, is found using the formula $E = \frac{1}{2}mv^2$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Determine E when $m = 2$ kg and $v = 1.2$ m/s.

(b) Determine m when $E = 162$ joules and $v = 9$ m/s.

(c) Determine v when $E = 1.96 \times 10^6$ joules and $m = 8$ kg.

(d) Explain why there are two answers for part **(c)** and what each answer means.

10 The density (D) of a substance is related to its mass (M) and its volume (V) by the formula $D = \frac{M}{V}$.

(a) Find the difference in the mass of two objects made of the same material with a density of 7 g/cm^3 if one has a volume of 12 cm^3 and the other has a volume of 15 cm^3.

(b) A flotation device is to be made of a material on the outside with a density of 1.8 g/cm^3 and a mass of 2.4 kg. It is filled inside with a material of density 0.9 g/cm^3 and a mass of 7.8 kg. Will the device float on sea water given that the density of sea water is 1.025 g/cm^3?

Hint
Find the total mass and the total volume.

11 The area of a trapezium is given by the formula $A = \frac{(a+b)h}{2}$.

(a) Rearrange the formula to make b the subject.

(b) Use this rearranged formula to find the value of b if $a = 9$, $h = 6$ and $A = 66$.

(c) We could substitute these values for a, h and A into the original formula and solve the equation $66 = \frac{(9+b) \times 6}{2}$ to find the value of b. Explain why would we rearrange the formula to find the value of b.

12 Consider this formula:

$$S = \frac{n}{2}[2a + (n-1)d]$$

Expand all the brackets in this expression and use this form to explain why it is difficult to make n the subject.

Open-ended

13 Ruth has been rearranging the formula for the area of a trapezium: $A = \frac{(a+b)h}{2}$. She has produced the following working while trying to make a the subject. Where has she made a mistake? Now rearrange the formula to make b the subject.

$$A = \frac{h(a+b)}{2}$$

$$2A = h(a+b)$$

$$\frac{2A}{h} = a+b$$

$$a = b - \frac{2A}{h}$$

Outside the Square

Puzzle

In the prime of his life

Theo found that the product of his dog's age, his mother's age and his own (all in years) is 8533. Theo is older than his dog. How old is Theo?

Half-time 3

1 Simplify each of the following, expressing your answer with positive indices. Ex.3.1

(a) $3x^3y \times 4x^4y$ (b) $\dfrac{8x^3y^5}{2xy^2}$ (c) $\dfrac{3m^4n^2 \times 2m^5n^3}{8m^3n^4}$

2 Rearrange the formula $z = 2xy - 3y$ to make x the subject and then find the value of x when $z = 7$ and $y = 5$. Ex.3.4

3 Simplify each of the following, expressing your answer with positive indices. Ex.3.2

(a) $(3x^4y^5)^2$ (b) $\left(\dfrac{a^4b^3}{6ab^4}\right)^2$ (c) $\dfrac{5^3x^4y^6}{3x^3y^2} \times \dfrac{x^6y}{x^7y^5}$

4 Express each quantity described below in scientific notation. Ex.3.3

(a) The Moon is 384 000 km away.

(b) 500 mL of water is lost through the skin daily.

5 To how many significant figures have each of the following been written? Ex.3.3

(a) 36 050 000 (b) 0.000 000 852 (c) 0.0830

6 Simplify the following, leaving your answer in index form. Ex.3.1

(a) $\dfrac{(2^3)^4 \times (5^6)^2}{(2^5) \times (5^2)^4}$ (b) $\dfrac{(3^2 \times 5^4)^2}{25 \times 27}$ (c) $\dfrac{8^3 \times 9^2}{6^4}$

7 Express each quantity described below as a number. Ex.3.3

(a) The number of bacteria that will fit on a pin head is approximately 1.1×10^{14}.

(b) The nucleus of a leaf cell is about 4.4×10^{-6} m in diameter.

8 Simplify each of the following, expressing your answer with positive indices: Ex.3.2

(a) $\dfrac{(p^2q^3)^{-1} \times p^5q^{-2}}{p^3q^2}$ (b) $\dfrac{(c^{-3}d^{-4})^2 \times c^2d^6}{c^5d^3}$

(c) $\dfrac{4(a^2b)^3 \times 3a^{-5}b^{-2}}{6ab}$ (d) $\dfrac{6(mn^3)^2 \times 4m^{-4}n^{-1}}{3m^2n}$

9 Express each of the following with positive indices. Ex.3.2

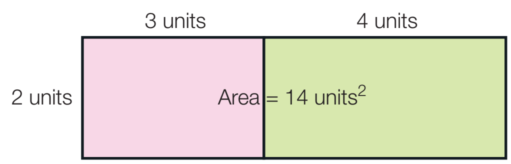
(a) x^{-5} (b) $3p^{-4}$ (c) $5m^{-3}n$ (d) $6p^{-4}q^{-1}r^2$

10 Write the following numbers correct to three significant figures: Ex.3.3

(a) 0.2999 (b) 80 (c) 62 078

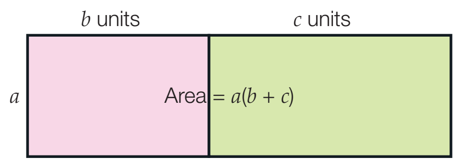
3.5 Expanding the brackets

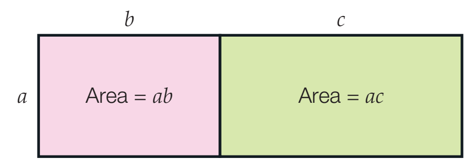
Brackets are used to change the order of operations, allowing addition or subtraction to be done before multiplication or division. We can write the expression without brackets if we multiply every term inside the brackets by the number outside the brackets. This is called expanding the brackets.



$$2(3+4) = 2 \times 7 = 14 \quad \text{and} \quad 2(3+4) = 2 \times 3 + 2 \times 4 = 6 + 8 = 14$$

If we replace numbers with variables, we can rewrite the expression without the brackets the same way.





$$a(b+c) \quad = \quad ab \quad + \quad ac$$

This is known as the **distributive law**. Every term inside the brackets is multiplied by the term outside the brackets.

Distributive law

$a(b+c) = ab + ac$

Worked Example 15

WE15

Expand:

(a) $5(p+4)$ **(b)** $2m(4m-3n)$

Thinking	Working
(a) Multiply all terms inside the brackets by the term in front of the brackets. (Here $5(p+4)$.)	(a) $5(p+4) = 5 \times p + 5 \times 4$ $= 5p + 20$
(b) Multiply all terms inside the brackets by the term in front of the brackets. (Here $2m(4m-3n)$.)	(b) $2m(4m-3n) = 2m \times 4m + 2m \times -3n$ $= 8m^2 - 6mn$

Worked Example 16

WE16

Expand and simplify:

(a) $4a(a - 3) - 2(a + 5)$ **(b)** $3(2 - b) - (4 - b)$

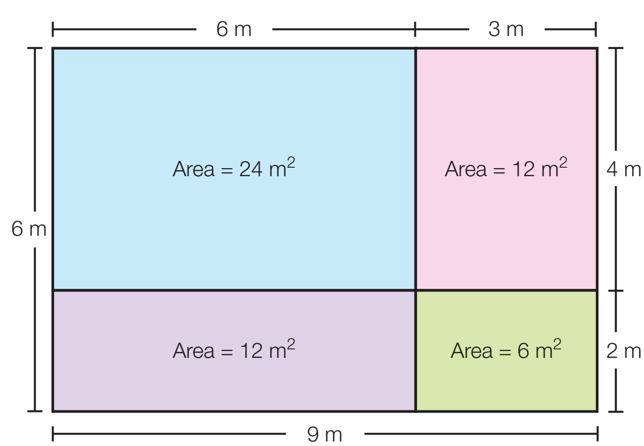
Thinking	Working
(a) **1** Expand both brackets by multiplying by the term in the front of the brackets. (Here we multiply the first bracket by $4a$ and the second bracket by -2.)	(a) $4a(a - 3) - 2(a + 5)$ $= 4a \times a + 4a \times (-3) + (-2) \times a + (-2) \times 5$ $= 4a^2 - 12a - 2a - 10$
2 Simplify by collecting like terms.	$= 4a^2 - 14a - 10$
(b) **1** Expand both brackets by multiplying by the term in the front of the brackets. (Here we multiply the first bracket by 3 and the second bracket by -1. Remember that the product of two negatives is positive: $-1 \times -b = b$.)	(b) $3(2 - b) - (4 - b)$ $= 3(2 - b) - 1(4 - b)$ $= 3 \times 2 + 3 \times (-b) + (-1) \times 4 + (-1) \times (-b)$ $= 6 - 3b - 4 + b$
2 Simplify by collecting like terms.	$= 2 - 2b$

Binomial products

A **binomial** expansion is the result of multiplying two terms in one set of brackets by two terms in another set of brackets.

$(a + b)(c + d) = ac + ad + bc + bd$.

Consider a room 6 metres long and 4 metres wide. If it is extended by 3 metres in length and 2 metres in width, find the new area, as in the diagram below.



The total area can be found in two ways:

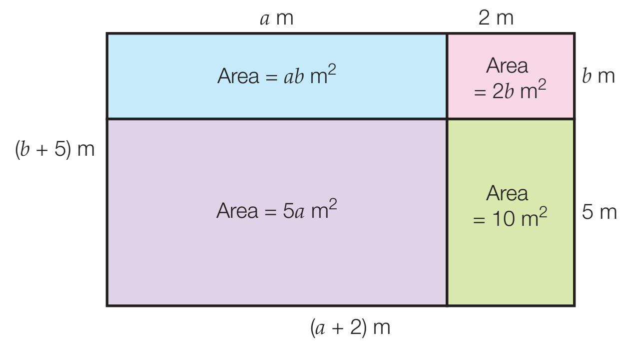
$(6 + 3)(4 + 2) = 9 \times 6$
$= 54\text{ m}^2$

or

$6 \times 4 + 6 \times 2 + 3 \times 4 + 3 \times 2 = 24 + 12 + 12 + 6$
$= 54\text{ m}^2$

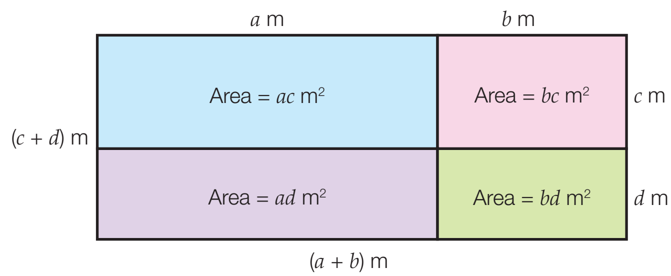
If a room a metres long and b metres wide has been extended by 2 metres in length and 5 metres in width, we can find the new area.

$$\begin{aligned}\text{Area } &= (a+2)(b+5)\\ &= a\times b + a\times 5 + 2\times b + 2\times 5\\ &= ab + 5a + 2b + 10 \text{ m}^2\end{aligned}$$



If a room a metres long and c metres wide has been extended by b metres in length and d metres in width, we can find the new area using the following diagram:

$$\begin{aligned}\text{Area } &= (a+b)(c+d)\\ &= a\times c + a\times d + b\times c + b\times d\\ &= ac + ad + bc + bd \text{ m}^2\end{aligned}$$



Binomial expansion

$(a+b)(c+d) = ac + ad + bc + bd$

Every term in the second bracket is multiplied by every term in the first bracket.

Worked Example 17

WE 17

Expand each of the following expressions.

(a) $(c+4)(d+2)$ **(b)** $(2m-4)(3n-5)$ **(c)** $(2p-3r)(3t+4)$

Thinking	Working
(a) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(c+4)(d+2)$.)	(a) $(c+4)(d+2)$ $= c\times(d+2) + 4\times(d+2)$ $= c\times d + c\times 2 + 4\times d + 4\times 2$
2 Write without multiplication signs and perform any possible calculations.	$= cd + 2c + 4d + 8$
(b) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(2m-4)(3n-5)$.)	(b) $(2m-4)(3n-5)$ $= 2m\times(3n-5) + (-4)\times(3n-5)$ $= 2m\times 3n + 2m\times(-5) + (-4)\times 3n + (-4)\times(-5)$
2 Write without multiplication signs and perform any possible calculations.	$= 6mn - 10m - 12n + 20$
(c) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(2p-3r)(3t+4)$.)	(c) $(2p-3r)(3t+4)$ $= 2p\times(3t+4) + (-3r)\times(3t+4)$ $= 2p\times 3t + 2p\times 4 + (-3r)\times 3t + (-3r)\times 4$

2	Write without multiplication signs and perform any possible calculations.	$= 6pt + 8p - 9rt - 12r$

Binomial expansion can be completed using FOIL.

First: Multiply the first terms in each bracket.
Outside: Multiply the outside terms in each bracket.
Inside: Multiply the inside terms in each bracket.
Last: Mutiply the last terms in each bracket.

$$(a + b)(c + d)$$

Often after expansion, like terms can be collected and the expression simplified.

We are sometimes left with three terms called a **quadratic trinomial**. This is demonstrated in the following worked example.

Worked Example 18

WE18

Expand each of the following expressions. Simplify where possible.

(a) $(x - 2)\ (x + 5)$ **(b)** $(t - 3)(t - 7)$ **(c)** $(4x + 3)(5x - 2)$

Thinking		Working
(a) 1	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(x - 2)(x + 5)$.)	(a) $(x - 2)(x + 5)$ $= x \times x + x \times 5 + (-2) \times x + (-2) \times 5$
2	Write without multiplication signs and perform any possible calculations.	$= x^2 + 5x - 2x - 10$
3	Simplify by collecting like terms.	$= x^2 + 3x - 10$
(b) 1	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(t - 3)(t - 7)$.)	(b) $(t - 3)(t - 7)$ $= t \times t + t \times (-7) + (-3) \times t + (-3) \times (-7)$
2	Write without multiplication signs and perform any possible calculations. Notice that the product of two negative numbers is a positive number ($-3 \times -7 = +21$).	$= t^2 - 7t - 3t + 21$
3	Simplify by collecting like terms.	$= t^2 - 10t + 21$
(c) 1	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(4x + 3)(5x - 2)$.)	(c) $(4x + 3)(5x - 2)$ $= 4x \times 5x + 4x \times (-2) + 3 \times 5x + 3 \times (-2)$

2 Write without multiplication signs and perform any possible calculations. Notice that the product of a positive and negative number is a negative number (+3 × -2 = -6). $= 20x^2 - 8x + 15x - 6$

3 Simplify by collecting like terms. $= 20x^2 + 7x - 6$

3.5 Expanding the brackets

Navigator

Answers page 609

Q1, Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q6 Column 1, Q7, Q8, Q9, Q10, Q14, Q15	Q1 Column 2, Q2 Column 1, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6 Column 2, Q7, Q8, Q9, Q10, Q14, Q16	Q1 Column 3, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16

Fluency

WE15

1 Expand each of the following expressions.

(a) $2(x+5)$ (b) $4(a+9)$ (c) $8(b+3)$

(d) $8(x-2)$ (e) $9(x-2)$ (f) $3(k-4)$

(g) $-7(x+1)$ (h) $-4(d+8)$ (i) $-6(x+3)$

(j) $-9(c-6)$ (k) $-2(x-5)$ (l) $-5(x-6)$

(m) $x(x+3)$ (n) $m(m+5)$ (o) $x(x+9)$

(p) $-x(x-8)$ (q) $-x(x-4)$ (r) $-n(n-9)$

(s) $3x(x+2)$ (t) $2u(u-6)$ (u) $5x(x-1)$

(v) $-2x(x+7)$ (w) $-7q(q+1)$ (x) $-9b(b-6)$

Hint
Remember $-(x+2)$ really means $-1(x+2)$.

WE16

2 Expand and simplify each expression.

(a) $3(x-2)+2(x+4)$ (b) $2(y+1)+3(y-2)$

(c) $4(2x-3)-(6x-1)$ (d) $3(4a+5)-(7a+2)$

(e) $a(2a+3)-2(3a-5)$ (f) $3x(x+2)-4(x-1)$

(g) $7y(y-3)-5(y+2)$ (h) $6k(2k+3)-3k(k-3)$

WE17

3 Expand each of the following.

(a) $(x+6)(y+5)$ (b) $(x+4)(y+7)$ (c) $(c+d)(e+3)$

(d) $(r+5)(t-8)$ (e) $(a+2)(b-1)$ (f) $(p+3)(q-2)$

(g) $(a-3)(y-4)$ (h) $(m-2)(n-6)$ (i) $(k-5)(m-4)$

(j) $(4p+q)(m+n)$ (k) $(7x+y)(a+k)$ (l) $(3p+2r)(4s+t)$

(m) $(2a+1)(b-3)$ (n) $(3x-1)(x+2)$ (o) $(3m+2)(n-1)$

(p) $(2a-3b)(5m-3n)$ (q) $(5c-2d)(3x-4y)$ (r) $(7p-2q)(2r-3t)$

WE18

4 Expand each of the following and simplify.

(a) $(a+3)(a+2)$ (b) $(c+5)(c+3)$ (c) $(y+7)(y+6)$

(d) $(x-3)(x+1)$ (e) $(y-2)(y+9)$ (f) $(a-7)(a+12)$

(g) $(b-2)(b-1)$ (h) $(x-4)(x-3)$ (i) $(p-8)(p-7)$

(j) $(3m + 1)(2m + 3)$ (k) $(4d + 5)(2d + 3)$ (l) $(3k + 5)(7k + 4)$

(m) $(4y - 3)(2y + 1)$ (n) $(5k + 3)(2k - 7)$ (o) $(2a + 5)(4a - 3)$

(p) $(6a - 5)(2a - 1)$ (q) $(4p - 3)(3p - 5)$ (r) $(3r - 2)(8r - 7)$

5 Expand each of the following and simplify.

(a) $(x + y)(x + y)$ (b) $(m + n)(m + n)$ (c) $(a + b)(a + b)$

(d) $(p - r)^2$ (e) $(c - d)^2$ (f) $(g - h)^2$

(g) $(2c + 3d)(2c + 3d)$ (h) $(3m + 2n)(3m + 2n)$ (i) $(5a + 3b)(5a + 3b)$

(j) $(5x - 4y)(5x - 4y)$ (k) $(2j - 3k)(2j - 3k)$ (l) $(7c - 5d)^2$

6 Expand each of the following and simplify.

(a) $(m + n)(m - n)$ (b) $(p + q)(p - q)$ (c) $(a - b)(a + b)$

(d) $(3a + 2b)(3a - 2b)$ (e) $(4x - 3y)(4x + 3y)$ (f) $(7m - 3n)(7m + 3n)$

7 When simplified, $5m(-3 + 2n) - n(10m - 1)$ is:

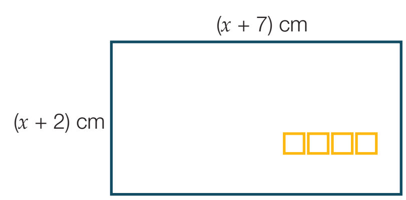
A $n - 15m$ **B** $10n - 15m - 1$ **C** $-15m - 20mn + n$ **D** $15m + n$

8 $(3x - 2)(3x + 2)$ is the same as:

A $6x^2 + 4$ **B** $9x^2 - 12x - 4$ **C** $9x^2 + 6x - 4$ **D** $9x^2 - 4$

Understanding

9 Consider this envelope.



(a) Write down the expression (in factorised form) for the area of this envelope.

(b) Use your answer to find the area when $x = 10$.

(c) What value would x represent if the area of the envelope is 104 cm^2? Try substituting different values for x until you find the appropriate one.

(d) Expand the expression you obtained for the area in part **(a)**.

(e) Check that the expanded expression is correct by calculating the area when $x = 10$ and comparing with the answer obtained in part **(b)**.

10 A vegetable bed, 15 m long and 3 m wide, is to be extended by x metres in both length and width.

(a) Write expressions for each of the following.

(i) The new length of the garden bed.

(ii) The new width of the garden bed.

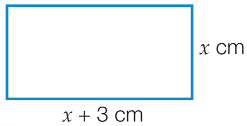
(iii) The new area of the garden bed as the product of factors.

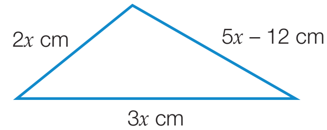
(iv) The new area of the garden bed as the expanded product of factors.

(b) Compare the area of the new garden bed with the area of the original garden bed. By how much has the area increased?

(c) Use your answer from part **(b)** to calculate how much extra area of garden bed is available if both the length and width are increased by 2 m.

11 The sides of a rectangle and a triangle have the lengths shown below.





(a) Write down the expression for the area of the rectangle using the given lengths.

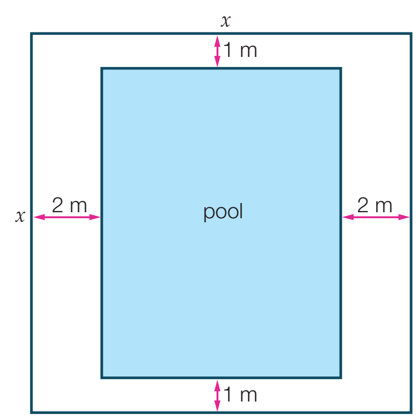
(b) Write down the expression for the perimeter of the triangle using the given lengths.

(c) Find two values for x so that the area of the rectangle is the same as the perimeter of the triangle.

Reasoning

12 There are a number of ways to check that $x(x + 2) = x^2 + 2x$ is a true statement. Choose two of these methods and show clearly how you are able to check that $x(x + 2) = x^2 + 2x$ is true. Can you use these methods to show that $a(b + c) = ab + ac$? If so, show how.

13 A backyard, square in shape, is to have an in-ground rectangular swimming pool constructed in it. A path around the pool is to be laid as shown in the diagram.



(a) Write expressions for each of the following:

(i) the total area of the back yard

(ii) the width of the swimming pool

(iii) the length of the swimming pool

(iv) the area of the swimming pool.

(b) Using your answers to parts **(i)** and **(iv)**, write an expression for the area of the path. State your answer in simplest expanded form.

Open-ended

14 Choose two variables to represent the length and width of a rectangle. Increase each dimension by a fixed amount (e.g. increase the length by 2 units and the width by 3 units). Find the area of the new rectangle.

15 Three students gave the following incorrect answers when expanding $-3(5x - 2)$:

(a) $-15x - 2$ (b) $5x + 6$ (c) $-8x + 5$

Explain the errors made by each student. What is the correct answer?

16 Three students obtained the following incorrect answers when expanding $(x - 3)(x + 4)$:

(a) $x^2 + 7x + 12$ (b) $2x - 12$ (c) $x^2 + x - 7$

Explain what errors were made by each student. What is the correct answer?

Outside the Square — Problem solving

Divisibility dilemma?

The number 123 456 is divisible by 2, 3 and 4. Find the smallest number that is divisible by all the digits from 2 to 9 (inclusive) if it begins with the digits 123 456.

Strategy options

- Test all possible combinations.
- Guess, check and improve.
- Break problem into manageable parts.

Technology Exploration GeoGebra

Equipment required: 1 brain, a computer with a GeoGebra program

Versions of this exploration for other technologies are available on Pearson Reader.

Increasing profit

Mr Slew is employed by a company as an engineer to find a new production process. After some research, he creates a new process for the manufacture of calculators. We will use a spreadsheet, graphs and algebra to explore whether Mr Slew's new process makes the production process cheaper, and therefore increases the company's profits.

It is assumed that every calculator made in a day will be sold.

1 A company that makes calculators has found that the cost, \$$C$, of producing x calculators per day is given by the rule $C = -0.01(x + 300)(x - 2300)$, $x \leq 1500$. At present, the company can produce a maximum of 1500 calculators per day. It has also found that the revenue (income for the company), \$$R$, from the sale of x calculators is given by the rule $R = -0.01x(x - 3000)$. What cost is incurred each day if no calculators are produced?

2 Open a GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen.

Go to Options, Labeling and select 'No new objects'.

From the menu, select 'View' and from the drop-down menu select 'Spreadsheet view'. Enter Number of calculators (x)' in cell A1, Cost C($) in B1 and Revenue R($) in C1. (Double clicking in the cell will allow you to edit your input.)

Enter values from 0 to 1500 in increments of 100 in column A, and the formulas (-0.01(A2+300)(A2-2300)) in B2 and (-0.01A2(A2-3000)) in C2. Remember, an asterisk (*) or a space must be used to show multiplication. Drag the formulas down to fill in the values for C($) and R($) for all your values of x, then write down:

(a) how much it would cost to produce the following number of calculators per day

(i) 100 **(ii)** 500 **(iii)** 1000 **(iv)** 1500

(b) how much revenue the company receives if it sells the following number of calculators per day

(i) 100 **(ii)** 500 **(iii)** 1000 **(iv)** 1500

The following screenshot shows the first few values you should obtain if you have entered your formulas correctly.

	A	B	C
1	Number of calculators (x)	Cost C($)	Revenue R($)
2	0	6900	0
3	100	8800	2900
4	200	10500	5600
5	300	12000	8100

3 **(a)** Enter a profit title, Profit P($), in D1 and, using the fact that profit equals revenue minus cost, a formula in D2 to calculate the profit from the sale of x calculators.

(b) Show **(i)** by using the spreadsheet and **(ii)** by using algebra that this profit formula is $P(\$) = 10x - 6900$.

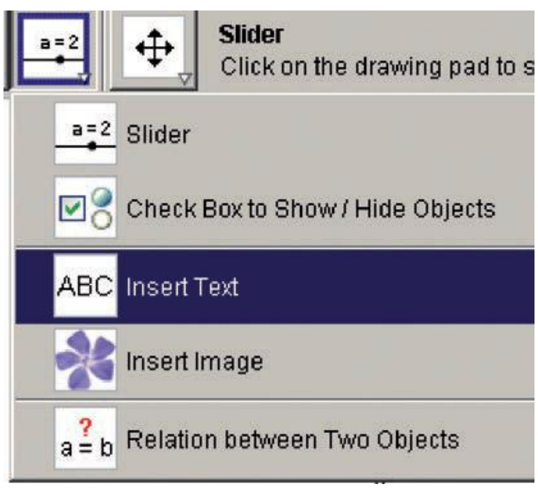
4 Produce a graph of $P = 10x - 6900$ for x-values between 0 and 1500. To do this, we need to first set up the axes and the range of values for both the x- and y-axes. From the menu, select Options and Drawing Pad. For the x-axis, input distance as 200, select x from the label list and input -200 as the min and 2100 as the max.

For the y-axis, input 1000 for the distance, y as the label, -9000 for the min and 16 000 for the max.

Input the formula in the Input bar at the bottom of your screen.

Use $f(x)$=Function[10x-6900,0,1500] to restrict the graph to the required values.

Use the text box tool to label your graph $P(\$) = 10x - 6900$.



5 What will the selling price of a calculator be if 1500 calculators are made each day?

6 What profit will be made on each calculator if 1500 calculators are made and sold each day?

7 Mr Slew recommends that the company introduces a new process to increase daily production to 2000 calculators by buying some new equipment. The cost of producing x calculators is now: $D(\$) = -0.01(x + 400)(x - 2200),\ x \leq 2000$.

8 Enter the new cost title $D(\$)$ into E1 and the new profit title $\$N$ into F1. Enter the new cost formula into to E2 and the new profit formula into F2 to calculate the profit $N(\$)$ using Mr Slew's new process.

9 Use algebra to simplify this rule.

10 Produce a graph of this rule using the same values of x as in Question **2**. (You will need to input this rule with a new name, for example $g(x)$.)

11 What does the point of intersection of the two graphs tell you?

12 As the manager of the company, would you introduce this new process? If so, how many calculators would you need to produce and sell each day to make the changeover to the new process worthwhile (i.e. to improve your profits)? Use your graphs or algebra to justify your decision.

13 Do the graphs help you understand what is happening to the profit in both the old and new processes? Explain why or why not.

Taking it further

14 Use your spreadsheet and a cost equation in the form of $F(x) = -0.01(x + a)(x + b)$ to find a profit equation. Choose various values for a and b to find the maximum profit you could make on the manufacture and sale of 1500 calculators.

Expanding special products

3.6

Did you notice anything interesting about your answers to Questions **5** and **6** in the last section (page 173)? Was there a pattern that could be used to expand using rules? All the parts of Question **5** were the square of sums or differences of two terms. All the parts of Question **6** were the product of the sum and difference of two terms.

Here we will investigate the expansion of $(a + b)(c + d)$ for two special products.

Perfect squares

When we square a sum or a difference of two terms, we call this a **perfect square**. We expand perfect squares in exactly the same way as other binomial products.

Square of a sum

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

or

Square of a difference

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

This gives us a rule for expanding perfect squares

$(a$	$+$	$b)^2$	$=$	a^2	$+$	$2ab$	$+$	b^2
↑		↑		↑		↑		↑
First term (a)		Second term (b)		First term squared (a^2)		Twice the product of the two terms $(2 \times a \times b)$		Second term squared (b^2)
↓		↓		↓		↓		↓
$(a$	$-$	$b)^2$	$=$	a^2	$-$	$2ab$	$+$	b^2

Perfect square rules

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Note that the only difference in these two rules is the sign of the middle term and that this sign is the same as the sign inside the brackets.

Worked Example 19

WE19

Expand each of the following expressions using a perfect square rule:

(a) $(p + 4)^2$ **(b)** $(d - 3)^2$ **(c)** $(2f + 5)^2$ **(d)** $(3m - 2n)^2$

Thinking	Working
(a) Use the rule $(a + b)^2 = a^2 + 2ab + b^2$. (Substitute $a = p$ and $b = 4$.)	(a) $(p+4)^2 = p^2 + 2 \times p \times 4 + 16$ $= p^2 + 8p + 16$
(b) Use the rule $(a - b)^2 = a^2 - 2ab + b^2$. (Substitute $a = d$ and $b = 3$.)	(b) $(d-3)^2 = d^2 - 2 \times d \times 3 + 9$ $= d^2 - 6d + 9$
(c) Use the rule $(a + b)^2 = a^2 + 2ab + b^2$. (Substitute $a = 2f$ and $b = 5$.)	(c) $(2f+5)^2 = (2f)^2 + 2 \times 2f \times 5 + 25$ $= 4f^2 + 20f + 25$

(d) Use the rule $(a-b)^2 = a^2 - 2ab + b^2$. (Substitute $a = 3m$ and $b = 2n$.)	(d) $(3m-2n)^2$ $= (3m)^2 - 2 \times 3m \times 2n + 2n \times 2n$ $= 9m^2 - 12mn + 4n^2$

Difference of two squares

When we multiply the sum of two terms by the difference of those terms, we see that the terms inside the brackets are the same but the signs are opposite. This gives us another useful rule.

$$\begin{aligned}(a+b)(a-b) &= a^2 - ab + ab + b^2\\ &= a^2 - b^2\end{aligned}$$

This rule is often referred to as DOTS (**difference of two squares**).

Difference of two squares rule

$(a+b)(a-b) = a^2 - b^2$

Worked Example 20

WE20

Use the difference of two squares rule to expand the following:

(a) $(x+3)(x-3)$ **(b)** $(5-k)(5+k)$ **(c)** $(3x+2)(3x-2)$ **(d)** $(4m-5n)(4m+5n)$

Thinking	Working
(a) Use the rule $(a+b)(a-b) = a^2 - b^2$. (Substitute $a = x$ and $b = 3$.)	(a) $(x+3)(x-3) = x^2 - 3^2 = x^2 - 9$
(b) Use the rule $(a+b)(a-b) = (a-b)(a+b)$ $= a^2 - b^2$. (Substitute $a = 5$ and $b = k$.)	(b) $(5-k)(5+k) = 5^2 - k^2 = 25 - k^2$
(c) Use the rule $(a+b)(a-b) = a^2 - b^2$. (Substitute $a = 3x$ and $b = 2$.)	(c) $(3x+2)(3x-2) = (3x)^2 - (2)^2 = 9x^2 - 4$
(d) Use the rule $(a-b)(a+-b) = a^2 - b^2$. (Substitute $a = 4m$ and $b = 5n$.)	(d) $(4m-5n)(4m+5n) = (4m)^2 - (5n)^2$ $= 16m^2 - 25n^2$

3.6 Expanding special products

Navigator

Answers page 610

Q1 Column 1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9 Column 1, Q11 (a), Q12	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9 Columns 1&2, Q11, Q12	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE19

1 Expand each of the following expressions using a perfect square rule.

(a) $(d-1)^2$ **(b)** $(m+5)^2$ **(c)** $(k+2)^2$

(d) $(p-3)^2$ **(e)** $(x-8)^2$ **(f)** $(y-12)^2$

(g) $(m+n)^2$ **(h)** $(w-k)^2$ **(i)** $(6-c)^2$

(j) $(y+6)(y+6)$ (k) $(x+9)(x+9)$ (l) $(k-7)(k-7)$

(m) $(4y+3)^2$ (n) $(2w+9)^2$ (o) $(3x+5)^2$

(p) $(6k-1)^2$ (q) $(7-2a)^2$ (r) $(3-8d)^2$

(s) $(1-4c)(1-4c)$ (t) $(6+7y)(6+7y)$ (u) $(4g-h)(4g-h)$

(v) $(3a+d)^2$ (w) $(2a-3b)^2$ (x) $(5m+8n)^2$

2 Use the difference of two squares rule to expand the following. WE20

(a) $(c-5)(c+5)$ (b) $(a+4)(a-4)$ (c) $(k+8)(k-8)$

(d) $(9+x)(9-x)$ (e) $(3-y)(3+y)$ (f) $(1-k)(1+k)$

(g) $(m-n)(m+n)$ (h) $(x-y)(x+y)$ (i) $(c+d)(c-d)$

(j) $(v+w)(v-w)$ (k) $(p+q)(p-q)$ (l) $(h-g)(h+g)$

(m) $(2m-1)(2m+1)$ (n) $(3x+4)(3x-4)$ (o) $(6b-7)(6b+7)$

(p) $(1+5y)(1-5y)$ (q) $(8-5a)(8+5a)$ (r) $(3-2d)(3+2d)$

(s) $(4a+5c)(4a-5c)$ (t) $(3m-2n)(3m+2n)$ (u) $(2x-7y)(2x+7y)$

(v) $(4+ab)(4-ab)$ (w) $(3-cd)(3+cd)$ (x) $(5+jk)(5-jk)$

3 $(a+5)^2$ is the same as:

A $a^2+5a+10$ B $a^2+10a+25$ C $a^2+10a+10$ D a^2+25

4 $(2k-7)^2$ is the same as:

A $2k^2-28k+49$ B $4k^2+28k+49$ C $4k^2-49$ D $4k^2-28k+49$

5 When expanded, $(2-4x)(2+4x)$ is:

A $4-16x^2$ B $4+8x$ C $4-8x^2$ D $4-8x$

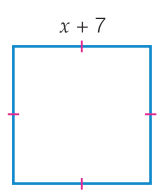
6 When expanded, $(ab+3cd)(ab-3cd)$ is:

A $ab-9cd$ B $a^2b^2+9c^2d^2$ C $a^2b^2-9c^2d^2$ D $a^2b^2+6c^2d^2$

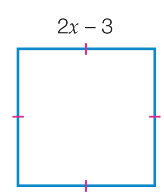
Understanding

7 (a) Find the area, in terms of x, of each of these squares. Expand your expressions.

(i)



(ii)



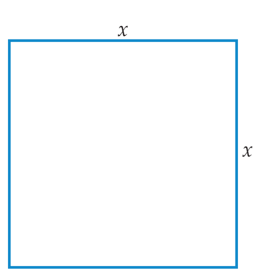
(b) Calculate the area of each when $x = 4$ cm.

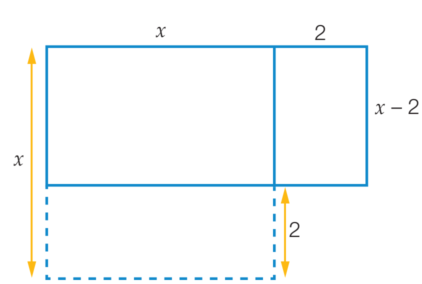
8 A square playing field is to be changed to a rectangular shape by adding 2 m to the length and subtracting 2 m from the width.

(a) Write an expression for the area of the square playing field.

(b) Write an expression for the area of the new rectangular playing field.

(c) Which playing field has the larger area? How much more area is available?





Reasoning

9 Because 16 can be written as $(10 + 6)$ or $(20 - 4)$, 16^2 can be written as $(10 + 6)^2$ or $(20 - 4)^2$. Evaluate the following, without a calculator, using your knowledge of special products.

(a) 23^2 **(b)** 49^2 **(c)** 112^2

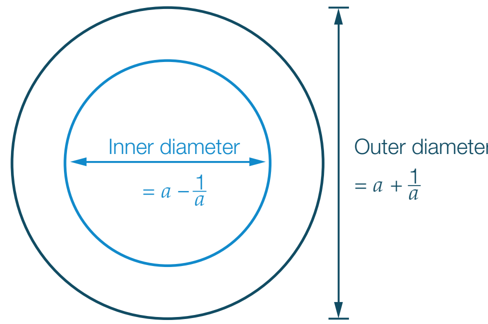
(d) 3.8^2 **(e)** 6.01^2 **(f)** 11.96^2

(g) 102×98 **(h)** 2.3×1.7 **(i)** 6.04×5.96

10 A steel washer has the dimensions shown.

(a) Find the area of one side of the washer by using your knowledge of the expansion of special products.

(b) What can you say about the area of the washer for any value of a where $a > 0$?



11 **(a)** Find the shaded area using:

Area of a trapezium $= \frac{1}{2}(a + b)h$

(b) Find the shaded area using:

Area of a triangle $= \frac{1}{2}bh$

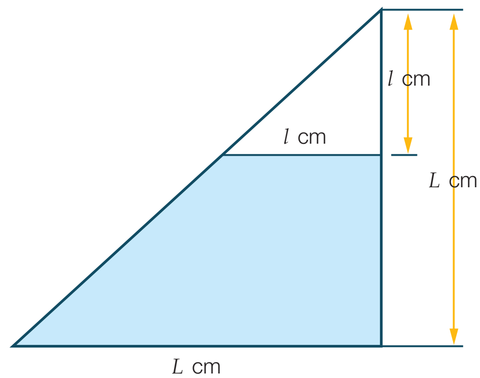
(c) Write an equation using part **(a)** and part **(b)**.

(d) If $L = 5l$, find the value of the shaded area:

(i) using your answer to part **(a)**

(ii) using your answer to part **(b)**.

(e) Show that the shaded area is 96% of the area of the large triangle when $L = 5l$.



Open-ended

12 Find three values for a and b so that $a^2 - b^2$ produces a square number.

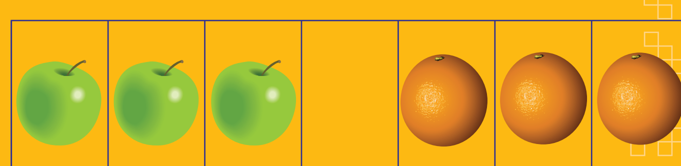
Outside the Square

Problem solving

Oranges and apples

The aim is to end up with the oranges and apples in opposite positions. You can make two types of moves:

- You can slide a piece of fruit one square into an empty space
- You can hop a piece of fruit over a different type of fruit



1 What is the minimum number of moves needed so that the oranges are on the left and the apples are on the right?

2 If there were four or five pieces of each fruit, how many moves would it take?

Strategy options

- Solve a simpler problem.
- Act it out.
- Look for a pattern.

Factorising using common factors

3.7

The opposite or inverse of expanding is called factorising.

Expanding	Factorising
$a(b + c) = ab + ac$	$ab + ac = a(b + c)$

There are many different ways to **factorise** algebraic expressions, but in this section we are going to find all the factors that are common to all terms in the expression. This is the **highest common factor** (HCF).

Factorising using the HCF

- Place the HCF outside brackets.
- Divide each term by the HCF; this quotient goes inside the brackets.
- If the HCF is not taken out as a factor, the expression is not fully factorised (e.g. $4x + 8 = 2(2x + 4)$ is not fully factorised).
- A negative sign may be taken out as well as the HCF; be careful to change signs inside the brackets (e.g. $-5x + 10 = -5(x - 2)$).

This procedure is called 'taking out the highest common factor'. The HCF may be any positive number, a variable or the product of a number and one or more variables. Once an expression is factorised, no matter what method has been used, it will have brackets in it.

Worked Example 21

WE 21

Factorise each of the following:

(a) $5x + 20$ **(b)** $y^2 - y$ **(c)** $-9y - 6z + 3$

(d) $3a^2bc - acd$ **(e)** $-8mn^2 - 12m^2n$ **(f)** $a(b + 3) + 2(b + 3)$

Thinking	Working
(a) Find the HCF of both terms (HCF = 5). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(a) $5x + 20 = 5(x + 4)$
(b) Find the HCF of both terms (HCF = y). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(b) $y^2 - y = y(y - 1)$
(c) Find the HCF of both terms (HCF = 3). Write it down and follow with brackets. Take out the negative sign also. Divide each term by the factor outside the brackets and write the quotient inside the brackets. Remember to change the signs inside the brackets.	(c) $-9y - 6z + 3 = -3(3y + 2z - 1)$

(d) Find the HCF of both terms (HCF = ac). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(d) $3a^2bc - acd = ac(3ab - d)$
(e) Find the HCF of both terms (HCF = $-4mn$) Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets. Remember to change the signs inside the brackets.	(e) $-8mn^2 - 12m^2n = -4mn(2n + 3m)$
(f) Find the HCF of both terms (HCF = $(b + 3)$). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(f) $a(b + 3) + 2(b + 3) = (b + 3)(a + 2)$

3.7 Factorising using common factors

Navigator

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Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q9, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 1, Q5, Q6, Q8, Q9, Q10, Q12, Q13	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13

Fluency

WE21

1 Factorise each of the following.

(a) $3x + 9$ (b) $12y + 24$ (c) $6 + 42a$

(d) $2k - 6$ (e) $4m - 20$ (f) $8b - 4$

(g) $7x - xy$ (h) $4s - 9st$ (i) $3a - ab$

(j) $15y + 5w + 10x$ (k) $2a + 4b - 8c$ (l) $3x - 15y + 9$

(m) $3ac + 2abc$ (n) $5xy^2 + 2x^2y$ (o) $9f^2gh - 12fg^2$

(p) $6gh^2 + 18g^2h$ (q) $16x^2y - 40xy^2$ (r) $18ab - 42a^2b$

(s) $-4m - 20$ (t) $-7d - 49$ (u) $-12r - 60$

(v) $x(a + 3) + 5(a + 3)$ (w) $m(n - 2) + 9(n - 2)$ (x) $3(x + 5) + y(x + 5)$

2 Factorise each of the following.

(a) $10x + 25$ (b) $16y + 36$ (c) $14a - 35$

(d) $12d - 18$ (e) $18 - 27m$ (f) $36 - 15k$

(g) $24abc - 10b + 7bc$ (h) $5km - 3kn + 2ghk$ (i) $c + 4bc + 2ac$

(j) $x^2 + 3x$ (k) $y^2 + 6y$ (l) $5k + k^2$

(m) $8m - 3m^2$ (n) $4p^2 - 7p$ (o) $9a^2 - 5a$

(p) $6d - 9d^2$ (q) $22g - 14g^2$ (r) $24a - 32a^2$

3 Factorise each of the following by taking out the highest negative common factor.

(a) $-2m - 12$ (b) $-4k - 24$ (c) $-5h - 25$

(d) $-x^2 - x$ (e) $-y^2 - 3y$ (f) $-p^2 - 4p$

(g) $-16a + 4$ (h) $-21b + 7$ (i) $-30r + 6$

(j) $-24p + 40$ (k) $-18w + 16$ (l) $-45k - 63$

(m) $-15x^2 - 36x$ (n) $-14y^2 - 49y$ (o) $-20r^2 - 44r$

(p) $-8d - 4f - 6g$ (q) $-15a - 10b - 5c$ (r) $-14c - 28d - 7f$

(s) $-12x^2 - 16x + 20xy + 4xz$ (t) $-2km + 4km^2 - 6k - 8k^2$ (u) $-10b^2 + 4bc - 8bc^2 + 16ab$

4 Factorise each of the following.

(a) $4(y - 1) + w(y - 1)$ (b) $5(m - 3) + n(m - 3)$ (c) $5(p - 2) + r(p - 2)$

(d) $p(q + 5) - 2(q + 5)$ (e) $y(2x + 3) - (2x + 3)$ (f) $a(3p + 5) - (3p + 5)$

(g) $d(f - 1) - 6(f - 1)$ (h) $5d(d - 2) - 4(d - 2)$ (i) $3m(m - 6) - 7(m - 6)$

5 When factorised, the expression $3ab^2c - 6abc + 3bc$ is:

A $3bc(ab - 2a)$ B $3abc(b - 2 + 3a)$

C $3bc(ab - 2a + 1)$ D $6bc(ab - a + 2bc)$

6 When fully factorised, the expression $-2x(3m - 4) + 5y(3m - 4)$ is:

A $(3m - 4)(5y - 2x)$ B $(5y - 4)(3m - 2x)$

C $(-3m + 4)(-2x + 5y)$ D $(-2x - 4)(3m + 5y)$

Understanding

7 (a) Write the expression for 'Eight times the product of two numbers added to twelve times one of the numbers'.

(b) Factorise the expression in **(a)**.

8 A rectangular pizza is of length d cm and width $(c + 2)$ cm. Another rectangular pizza of the same width has a length of 5 cm. The two pizzas are joined to make a longer pizza.

(a) Write down the area of the new shape in factorised form.

(b) What is the relationship between c and d if the pizza is square?

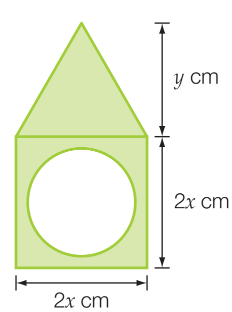
9 A floor-to-ceiling window of width x m is to be installed in a wall 5 m long and $(y + 3)$ m high. The remaining part of the wall is to be plastered.

(a) Find the length of the wall to be plastered after the window is installed.

(b) Find the area of the wall to be plastered after the window is installed.

Reasoning

10 A wood template is to be made for the front of a letter box with the dimensions shown.



(a) Write an expression that would show the area of timber used in the construction before the circular opening is cut out.

(b) Express this answer in a factorised form.

(c) If the diameter of the circular cut out needs to be 2 cm less than the width, find the area of the timber cut out for the opening.

(d) Write this in expanded form.

(e) Show that the total area of timber that will be used is $(4 - \pi)x^2 + (y + 2\pi)x - \pi$ cm^2.

(f) What area of timber would be wasted if the template was cut from a rectangular piece of timber with the smallest possible dimensions?

11 A lasagne is baked in a tray $(3p + 2)$ cm long and $(2p - 1)$ cm wide. A slice, the full width of the tray and $(r + 2)$ long is taken from the lasagne.

(a) Find the area of the lasagne, before the slice was removed, in factorised form.

(b) Find the area of the slice taken from the lasagne in factorised form.

(c) Use your answers to parts **(a)** and **(b)** to find the area remaining after the slice has been removed. Write your answer in simplest factorised form.

(d) How could you find the area in part **(c)** more simply? Show your working.

Open-ended

12 Make up three different expressions whose highest common factor is $3x$.

13 In a test, students were asked 'Factorise the expression $-5x + 10$'. Three students wrote these answers; one answer was correct, the other two answers were incorrect. Identify the correct answer and explain the mistakes the other two students made.

A $-5(x + 2)$　　B $-5(x + 10)$　　C $-5(x - 2)$

Outside the Square — Game

Matching expressions

Equipment required: 2 brains, 20 pieces of paper cut into rectangles the size of playing cards

How to win: Match the most expressions and get more cards than your opponent

How to play:

1. Each player writes out five expressions in expanded form, one on each card, and their equivalent factored form on another five cards.
2. All 20 cards are mixed up and laid out face down. This is now a game of memory.
3. Player 1 turns two cards over. Players have 10 seconds to work out whether the cards are a match.
4. If they are a match, Player 1 takes the cards and has another go, otherwise it is Player 2's turn.
5. After all the cards have been matched and claimed by a player, the game ends and the player with the most cards is the winner.

Investigation

Mr Gershwin's piano

Mr Gershwin wants to build a small performance theatre with a stage for piano recitals. The theatre cannot be longer than 16 metres, but must be able to seat at least 108 people in the main seating area, with extra space allowed outside the seating area to accommodate wheelchairs.

The theatre in which the stage is to be built is rectangular with the following specifications:

- Its width, W, in metres correct to one decimal place is given by the expression $W = 4x - 4$.
- Its length, L, in metres correct to one decimal place, is twice the theatre's width.

The Big Question

Mr Gershwin wants to know the dimensions of the smallest and largest theatre he can build with the conditions given above. Can you help him?

Engage

1 **(a)** Find L and W for $x = 2$.

(b) Draw a diagram of this theatre showing the dimensions given in part **(a)**.

The stage is also rectangular and is to be built across the width of the theatre so that the length of the stage, in metres, a, is equal to the width of the theatre. The width of the stage, in metres, b, is half its length.

(c) Find the value of a and b for $x = 2$.

(d) Draw the stage in your diagram, showing the dimensions given in part **(c)**.

Explore

The audience is to be seated in rows in a rectangular space so that there is a 2-metre clearance from the front of the stage and from the back of the room and with a 1-metre aisle either side.

2 (a) Draw the seating area in your diagram showing the clearances.

(b) Find the length in metres, c, and width in metres, d, of the audience space for $x = 2$.

(c) Add the dimensions of your seating area to your diagram.

(d) If a space of 50 cm × 50 cm is allowed for each seat, how many seats could fit into this space?

(e) Choose three other values for x and calculate the number of people that could be seated for each of these values. Do not use integer values only.

Strategy options

- Make a table.
- Draw a diagram.
- Guess and check.
- Break the problem into manageable parts.

Explain

3 (a) Explain why values for $x \leq 1$ cannot be used for the length and width of the theatre.

(b) What did you find happened to the number of people that could be seated when you changed the values of x?

(i) Explain why the smallest value for x is $1\frac{3}{8}$ m if there is to be any seating at all.

(ii) How many seats can fit if $x = 1\frac{3}{8}$ m?

Elaborate

4 (a) By substituting different values for x, find the dimensions of the smallest theatre that Mr Gershwin can build so that 108 people can be seated in the seating area. Use your results from **2 (e)** to help you.

(b) What is the largest value that x can be so that the length of the theatre does not exceed 16 metres?

(c) Hence, determine the range of values for x that Mr Gershwin could use to build his theatre.

Evaluate

5 (a) Is it sensible to change the size of the stage as the size of the theatre changes? Give a reason for your answer.

(b) Do you think the relationship used here is a good one? Give a reason for your answer.

Extend

Mr Gershwin also wants to know how much carpet he would need to cover the floor space not occupied by the stage or the seating area. You need to do the following calculations to find the area of floor to be covered with carpet.

6 (a) Find W and L in terms of x and factorise your answer.

(b) Find the area of the theatre in terms of x. Write your answer first in factorised form and then expand it.

(c) Find a and b in terms of x in factorised form.

(d) Find the area of the stage in terms of x. Write your answer first in factorised form and then expand it.

(e) Find c and d in terms of x.

(f) Find the seating area in terms of x. Write your answer first in factorised form and then expand it.

(g) Find the area in terms of x of the floor space of the theatre occupied by both the stage and the seating area.

(h) Use the area equations found in parts **6 (b)**, **(d)** and **(f)** to show that the area, in terms of x, of the remaining floor space in the theatre is equal to $(28x - 36)$ m^2.

(i) Show how this area could be found using an easier method.

Factorising by grouping in pairs

3.8

If there are four terms in an expression and there is no common factor for all four terms, we can sometimes group the expression in pairs of terms so that there is a common factor in each pair. If this gives us another common factor in brackets, we can factorise further. This technique is called **grouping in pairs**. Sometimes you may have to consider which two terms to pair together.

Worked Example 22

WE22

Factorise the following:

(a) $ab + 5b + 3a + 15$ **(b)** $3x + 12 - xy - 4y$

Thinking	Working
(a) 1 Factorise by taking a common factor from the first pair of terms (b) and a different common factor from the second pair of terms (3).	(a) $ab + 5b + 3a + 15$ $= b(a + 5) + 3(a + 5)$
2 Now we have a new common factor. (Here $(a + 5)$.) Write this factor in front of another set of brackets. Divide each product by this factor and write the quotient in the second set of brackets.	$= (a + 5)(b + 3)$
(b) 1 Factorise by taking a common factor from the first pair of terms (3) and a different common factor from the second pair of terms ($-y$). When taking out a negative common factor, change the signs inside the brackets.	(b) $3x + 12 - xy - 4y$ $= 3(x + 4) - y(x + 4)$
2 Now we have a new common factor. (Here $(x + 4)$.) Take out this common factor to complete the factorisation.	$= (x + 4)(3 - y)$

Worked Example 23

WE23

Factorise $df + 6g - 6f - dg$

Thinking	Working
1 The first two terms (df and $6g$) do not have a common factor. So, rewrite the expression to pair terms such that each pair of terms does have a common factor.	$df + 6g - 6f - dg$ $= df - 6f - dg + 6g$

2 Now factorise the first pair of terms by taking out a common factor (f) and take a different common factor from the second pair ($-g$). Change signs if necessary.	$= f(d-6) - g(d-6)$
3 Take out the remaining common factor to complete the factorisation. (Here $(d-6)$.)	$= (d-6)(f-g)$

In Questions **4**, **5** and **6** of Exercise 3.5 (page 173), we expanded some products that gave us quadratic expressions when we simplified. We explored expanding some special products in Exercise 3.6 and these also gave us quadratic expressions. Because factorising and expanding are opposite operations, we can use them to factorise quadratics. We will use our knowledge of factorising by grouping in pairs to show how this process works.

Worked Example 24

WE24

For each of the following expressions, **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 5x + 6x + 30$ **(b)** $p^2 - 4p - 4p + 16$ **(c)** $a^2 - 12a + 12a - 144$

Thinking	Working
(a) (i) 1 Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(a) (i) $x^2 + 5x + 6x + 30$ $= x(x+5) + 6(x+5)$
2 Factorise by taking out the new common factor.	$= (x+5)(x+6)$
(ii) Add the two middle terms because they are like terms.	(ii) $x^2 + 5x + 6x + 30$ $= x^2 + 11x + 30$
(iii) These answers must be equal to each other because they are both equal to the expression.	(iii) $x^2 + 11x + 30 = (x+5)(x+6)$
(iv) 1 Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(iv) $5 \times 6 = 30$
2 State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.

(b) (i) 1 Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(b) (i) $p^2 - 4p - 4p + 16$ $= p(p - 4) - 4(p - 4)$
2 Factorise by taking out the new common factor.	$= (p - 4)(p - 4)$
3 Write as a perfect square.	$= (p - 4)^2$
(ii) Add the two middle terms because they are like terms.	(ii) $p^2 - 4p - 4p + 16$ $= p^2 - 8p + 16$
(iii) These answers must be equal to each other because they are both equal to the expression.	(iii) $(p - 4)^2 = p^2 - 8p + 16$
(iv) 1 Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(iv) $-4 \times -4 = 16$
2 State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.
(c) (i) 1 Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(c) (i) $a^2 - 12a + 12a - 144$ $= a(a - 12) + 12(a - 12)$
2 Factorise by taking out the new common factor.	$= (a - 12)(a + 12)$
(ii) Add the two middle terms because they are like terms.	(ii) $a^2 - 12a + 12a - 144$ $= a^2 - 144$
(iii) These answers must be equal to each other because they are both equal to the expression.	(iii) $a^2 - 144 = (a - 12)(a + 12)$
(c) (i) 1 Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(c) (i) $-12 \times +12 = -144$
2 State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.

3.8 Factorising by grouping in pairs

Navigator

Answers page 611

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q11	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q11, Q12	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE22

1 Factorise the following.

(a) $xy + 4y + 3x + 12$ (b) $pq + q + 5p + 5$ (c) $mn + 2n + 9m + 18$

(d) $ab + bf + ad + df$ (e) $pq + qt + pr + rt$ (f) $km + 3nk + 6m + 18n$

(g) $np - 7n + p - 7$ (h) $ab - 5b + a - 5$ (i) $mn - 2n + 7m - 14$

(j) $ab - 6b - 4a + 24$ (k) $x^2 + 2x - 8x - 16$ (l) $y^2 + 4y - 6y - 24$

WE23

2 Factorise the following.

(a) $2p + 3k + kp + 6$ (b) $mn + pq + np + mq$ (c) $cd + 12 + 6c + 2d$

(d) $ad - hk - dk + ah$ (e) $xy - 8 - 4y + 2x$ (f) $eg - 3 - g + 3e$

(g) $bc - 1 + c - b$ (h) $xy - 3 + 3x - y$ (i) $mk - 3n + 3m - nk$

WE24

3 For each of the following expressons **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 7x + 2x + 14$ (b) $j^2 + 3j + 5j + 15$ (c) $z^2 + z + 8z + 8$

(d) $x^2 + 2x - 8x - 16$ (e) $y^2 + 4y - 6y - 24$ (f) $x^2 + 3x - 4x - 12$

(g) $a^2 - 4a + 5a - 20$ (h) $k^2 - 2k + 5k - 10$ (i) $m^2 - 7m + 4m - 28$

(j) $y^2 - 3y - 7y + 21$ (k) $c^2 - 9c - 5c + 45$ (l) $d^2 - 8d - 12d + 96$

(m) $m^2 + 3m + 3m + 9$ (n) $r^2 + 6r + 6r + 36$ (o) $k^2 + 11k + 11k + 121$

(p) $p^2 - 5p - 5p + 25$ (q) $d^2 - 7d - 7d + 49$ (r) $n^2 - 8n - 8n + 64$

(s) $c^2 - 3c + 3c - 9$ (t) $t^2 - 2t + 2t - 4$ (u) $b^2 - 9b + 9b - 81$

4 By taking out the common factor from $k(j - 1) + (j - 1)$ we obtain:

A $(j - 1)(k + 1)$ **B** $(j + 1)(k - 1)$ **C** $(j - 1)(k)$ **D** $(j + 1)(k + 1)$

Understanding

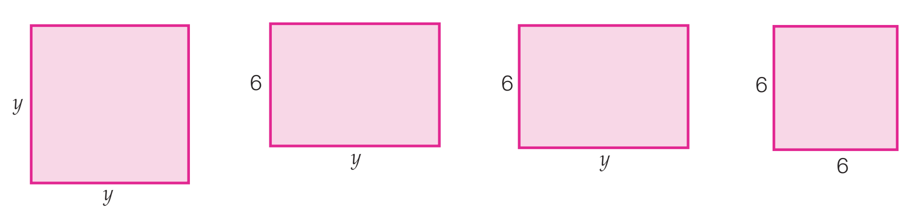
5 Square a number, x, subtract four times x, add 12 and subtract three times x.

(a) Write down the expression for the information given.

(b) Simply the expression in **(a)**.

(c) Factorise the expression in **(a)**.

6



(a) Write down the expression for the total area of the four shapes above.

(b) Factorise the expression in **(a)**.

(c) Draw a diagram to show how the four shapes can fit together to make a square (include dimensions on your diagram).

Reasoning

7 (a) Factorise $ab + ac + ad + 7b + 7c + 7d$ by grouping in threes. That is, take out a common factor from three terms at a time.

(b) Show that, if $b = a$ and $c + d = 7$, the expression is a perfect square.

8 (a) Rewrite $7x$ using two terms so that the expression $x^2 + 7x + 10$ can be factorised by grouping in pairs.

(b) Now factorise your new expression.

9 (a) Factorise $35y^2 - 21y$.

(b) If this represents the area of a rectangle in cm^2 and the factors represent the width, write down the dimensions of the rectangle.

(c) If an area of $15y - 9$ cm^2 is added to the area in part **(a)**, find the combined area in factorised form.

(d) Write down the dimensions of the new rectangle if the factors in part **(c)** represent the length and width.

10 A trapezium has an area of $\frac{3x^2 + 2xy - 6x - 4y}{2}$ m^2 where x and y are both positive numbers and $x > 2$.

(a) Write the area in factorised form.

(b) If the smaller factor represents the height of the trapezium, what is the sum of the lengths of the two parallel non-equal sides?

Open-ended

11 (a) Find two terms that, when added to $x^2 + ax$, can be factorised by grouping in pairs.

(b) Factorise your expression in **(a)**.

12 (a) Draw a rectangle of length l and width w. Extend the length by a certain amount and the width by a different amount. Use your diagram to write the area of the rectangle in two different ways.

(b) Draw a square of side s. Extend one pair of opposite sides by a certain amount and the pair of opposite sides by a different amount. Use your diagram to write the area of the new rectangle in two different ways.

(c) Draw a square of side s and extend the sides by the same amount to form a new square. Use your diagram to find the area of the new square in two different ways.

Outside the Square Puzzle

Trouble in variable paradise

Some variables were messaging online; the following is an excerpt of their conversation…

x: I am like soooo smaller than u *y*

y: Lol, yeah but at least I'm not negative

x: *cries* u might be greater than me, but at least I'm not square like u

y: Aw, don't be cut *x*, together we make one, anyway although you're negative, you're a perfect cube.

x: Luv u *y*

y: cul8r *x*

What are the values of *x* and *y*?

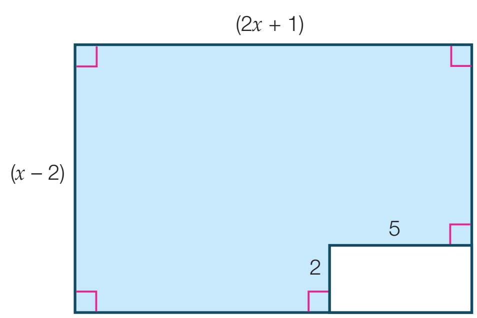
Challenge 3

1 Write two numbers that have a sum of 27 and a product of 72.

2 Bridget once said, 'The day before yesterday I was 15 and next year I will be 18'. When is Bridget's birthday and on what day did she say this?

3 How many perfect cubes are there between 2000 and 10 000?

4 Part of a rectangle is shaded as shown in the diagram.



(Diagram not to scale)

Which expression gives the area of the shaded part?

A $2x^2 - x - 8$ **B** $2x^2 - x - 12$ **C** $2x^2 - 3x - 8$ **D** $2x^2 - 3x - 12$

5 If ♦ represents a digit, find the value of ♦ if $[3(170 + \blacklozenge)]^2 = 272\,48\blacklozenge$.

6 a, b and c are positive whole numbers. Show that $(a - b)(b - c)(c - a)$ is divisible by 2.

7 Is the number $3^{404} + 4^{495}$ divisible by 5? Show why or why not.

8 Steven thinks of four different numbers. The sums Steven gets by pairing the numbers are 6, 7, 9, 9, 11 and 12. What are the four numbers?

9 A formula used in the science of optics is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

(a) Make f the subject of the formula.

(b) Make v the subject of the formula.

10 (a) Show that the difference between any two-digit number and the number formed by reversing the digits is divisible by 9. (*Hint*: let the two-digit number be $10a + b$).

(b) The difference between any three-digit number and the number formed by reversing its digits is obtained. By what number is it divisible?

(c) The difference between any four-digit number and the number formed by reversing its digits is obtained. By what number is it divisible?

Chapter review 3

D.I.Y. Summary

Key Words

base	expand	highest common factor	reciprocal
binomial	expanded form	index	scientific notation
difference of two squares	expression	index form	significant figures
distributive law	factorise	perfect square	variable
equation	grouping in pairs	quadratic trinomial	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 An expression of the form $x^2 - y^2$ is called the ___________.

2 If an expression is factorised to obtain the product of two brackets that are exactly the same, it is called a ___________.

3 To ___________ an expression is to remove brackets. The reverse operation is to ___________ the expression and put it into brackets.

4 If we write a number in ___________, then it has a number between 1 and 10 multiplied by a positive or negative power of 10.

Fluency

1 Simplify each of the following. Ex.3.1

(a) $a^3b^5 \times a^2b^2$ (b) $m^4n^7 \times m^3n$ (c) $\dfrac{p^4q^6}{p^3q^9}$

(d) $\dfrac{(x^5)^2 \times x^8}{(x^4)^4}$ (e) $\dfrac{(x^2)^3 \times (y^4)^2}{xy^3 \times x^5}$ (f) $\dfrac{c^2d^4}{c^4d^2} \times \dfrac{c^6d}{(cd)^2}$

2 $(2x^3y^2)^3$ is the same as: Ex.3.2

A $2x^3y^6$ B $8x^3y^6$ C $2x^9y^6$ D $8x^9y^6$

3 Express each of the following with positive integers. Ex.3.2

(a) $-5m^{-1}$ (b) a^2b^{-6} (c) $x^{-2}y^{-3}$ (d) $a^{-4}b^{-9}$

4 Write each of the following in scientific notation. Ex.3.3

(a) 0.000 000 000 000 002 mm (b) 0.000 15 kg

(c) 320 000 000 L (d) 480 000 000 000 000 km

5 Write each of the following as a basic numeral. Ex.3.3

(a) 5.4×10^7 (b) 8.273×10^{-4}

(c) 1.4×10^{-5} (d) 7.162×10^3

6 Rearrange each of the following to make x the subject. Ex.3.4

(a) $z = \dfrac{3 - xy}{2}$ (b) $x + 2a = 4x$

(c) $s = xt + 8t^2$ (d) $\dfrac{2 + 4z}{3x} = 10$

7 Expanded and simplified, $2c(3b - 4) - 3b(-5 + 2c)$ is: Ex 3.5

A $15b - 8c$ **B** $21b + 8c$ **C** $12bc - 12c$ **D** $7b + 14c$

8 Expand: Ex 3.5

(a) $4(x + 7)$ **(b)** $x(x - 3)$ **(c)** $-5x(2x - 1)$

9 **(a)** Expand and simplify: Ex 3.5

(i) $4(2x - 3) + 5(x + 1)$ **(ii)** $3x(6x + 5) - 2(3 - 2x)$

(b) Find the value of each expression when x has the value 4.

10 Expand and simplify: Ex 3.5

(a) $(x + 8)(x + 2)$ **(b)** $(2a + 5)(3a - 4)$

11 Expand and simplify: Ex 3.6

(a) $(x + 12)^2$ **(b)** $(5c - 2d)^2$ **(c)** $(4x - 3)(4x + 3)$

12 Factorise: Ex 3.7

(a) $8a + 12$ **(b)** $-15k - 20k^2$

(c) $5p^2q - 3pq + 2pq^2 - pqr$ **(d)** $-24a - 6ab - 72$

13 Factorise by grouping in pairs. Ex 3.8

(a) $ab + 4b + 3a + 12$ **(b)** $ab - 2b + a - 2$

(c) $ab - 6a - 5b + 30$ **(d)** $ab - 7 + 7b - a$

Understanding

14 Express the following in scientific form.

(a) A 10-cent coin is about 0.0015 m thick.

(b) Pluto is approximately 5900 million km from the Sun.

15 Express each of the following as a basic numeral.

(a) The distance from the Sun to Jupiter is approximately 7.78×10^8 km.

(b) The temperature at the centre of the Sun is believed to be about 1.5×10^7 °C.

16 The length of a greeting card is 20 cm and the width is $(5 + x)$ cm.

(a) Draw a diagram of the card.

(b) Write an expression for the perimeter.

(c) Factorise the expression obtained in part **(b)**.

(d) Write an expression for the area (in factorised form).

(e) Expand the expression obtained in part **(d)**.

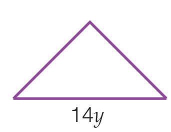
(f) Check that the two expressions obtained for the area are equivalent by substituting a value for x.

17 If the area of a rectangular postage stamp is $4x^2 + 12x$, what is the width of the stamp if the height is $2x$?

18 The mass of an object can be determined by finding the product of its density and its volume. If the density can be represented by $(m - 2)$ and the volume by $(k + 8)$, write an expression for the mass.

Suggest possible values that m and k could take to give a mass of 48.

19 Find the height of this triangular support structure if the base is $14y$ and the area enclosed by the beams is $35y^2 - 21y$.



Reasoning

20 The heart pumps 7000 L of blood daily.

(a) Express this quantity in scientific notation.

(b) How many litres of blood are pumped by the heart:

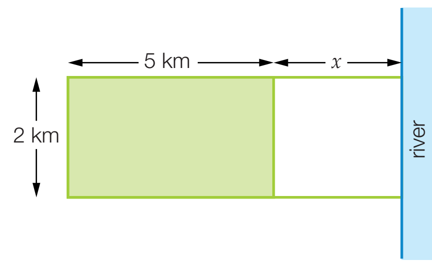
(i) each week?

(ii) each second, to one decimal place?

(iii) in 3 years (assuming no leap years)?

(c) Would this measurement be exact and the same for each person?

21 The Greenwood family had a rectangular block of land 2 km by 5 km for grazing sheep, as seen in the diagram. They then bought the land adjoining this leading down to the river. Depending on the time of year and the weather conditions, the width of the river can vary, so the land available for grazing sheep changes.



(a) Write an expression for the total area available for grazing sheep at any particular time. (Assume the land remains rectangular.)

(b) Suppose there are $\frac{1}{2}x$ sheep per km^2 of land. How many sheep are there altogether? Write your answer in expanded form.

(c) Fourteen sheep are moved to another location.

(i) Write the expression for the number of sheep remaining.

(ii) Expand $(x + 7)(x - 2)$. How is this related to the expression in **(i)**?

(iii) Explain what happens when $x = 2$ km.

NAPLAN practice 3

Numeracy: Calculator allowed

1 Which of the following numbers is the largest?

A 3.2×10^{18} **B** 9.6×10^{3} **C** 3.2×10^{-18} **D** 9.6×10^{-3}

2 What is the value of $7(x - 9)$, when $x = 15$?

3 Factorise the expression $14x + 21$.

Numeracy: Non-calculator

4 Which one of the following expressions is equivalent to $3(5x - 2)$?

A $15x - 1$ **B** $15x - 6$ **C** $15x + 1$ **D** $15x + 6$

5 A rule for y in terms of x is $y = 12 - 3x$. Write an equivalent rule for x in terms of y.

6 Which one of the following expressions is a fully factorised form of $9xy + 12y$?

A $3(3xy + 4y)$ **B** $9y(x + 2)$ **C** $y(9x + 12)$ **D** $3y(3x + 4)$

7 When the expression $-3x^2y + 6xy$ is fully factorised, the result is:

A $3xy(2 - x)$ **B** $-3xy(2 - x)$ **C** $-3xy(x - y)$ **D** $-3xy(x + y)$

4

Measurement 4

The 'Gimli Glider'. A commercial aircraft runs completely out of fuel mid-flight. How could this happen?

On 23 July 1983, Air Canada Flight 143 was about halfway through its journey from Montreal to Edmonton when alarms began to sound in the cockpit. No more fuel meant that both engines shut down and the pilots lost power to nearly all their instruments. Fortunately, the pilot, Captain Bob Pearson, was also an experienced glider pilot. He glided the plane to an old, disused airforce base at Gimli, where he managed to land it without causing any deaths or injuries to the 69 people aboard, or to the people on the ground racing go-karts on the old airstrip.

How did this near tragedy happen? At the time of the incident, Canada was in the middle of converting from the old imperial system of measurement to the new metric system. The amount of fuel required was calculated to be 22 300 *kilograms*. However, an error made by the ground and flight crews meant that incorrect conversion factors were used and the plane ended up with 22 300 *pounds* of fuel, or just over 10 000 kilograms, less than half the required amount.

Forum

When did Australia change from the imperial to the metric system of measurement?

Do any countries still use the imperial system?

Make a list of all the things (such as signs, labels or objects) that would have needed to be changed when the measurement system was changed.

Why learn this?

The ability to accurately estimate and calculate length, surface area and volume is a valuable skill both at home and in the workplace. In the kitchen, cooked food cools quicker when it has a larger surface area. In the bedroom, the volume of your wardrobe is the space available to store all your clothes. In the garden, you may lay a path around the perimeter of a pond or pool. Tradespeople, such as painters, carpenters, hairdressers, chefs and mechanics, require good measurement skills to be successful. Other professionals who require good measurement skills include surveyors, interior designers and engineers.

After completing this chapter you will be able to:

- calculate the perimeter of shapes with straight and curved edges
- calculate the area of composite shapes made up of triangles, quadrilaterals, circles and parts of circles
- calculate the surface area of prisms and cylinders
- calculate the volume and capacity of prisms and cylinders
- convert units of length, area, volume and capacity.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R4.1

1 Calculate the following.

(a) 7.21×10 **(b)** 18.21×1000

(c) $1571.23 \div 100$ **(d)** $0.74 \div 10\,000$

Worksheet R4.2

2 Copy and complete the following conversions.

(a) 4.2 cm = ______ mm **(b)** 3.9 m = ______ cm

(c) 7.1 km = ______ m **(d)** 450 mm = ______ m

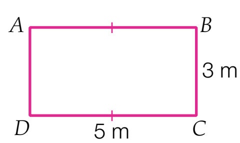
(e) 18.6 cm = ______ m **(f)** 5060 m = ______ km

(g) 1100 cm = ______ km **(h)** 3720 mm = ______ km

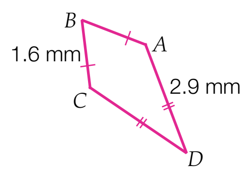
Worksheet R4.3

3 What is the length of *AB* in each of the following diagrams? Where necessary, give your answers to one decimal place.

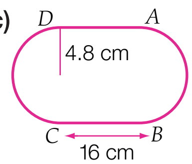
(a)



(b)



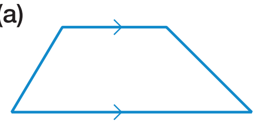
(c)



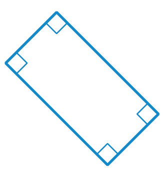
Worksheet R4.4

4 Match the shape to its name.

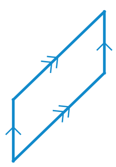
(a)



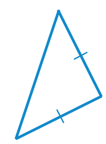
(b)



(c)



(d)



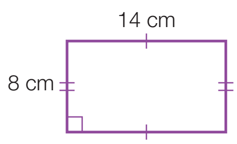
A rectangle **B** isosceles triangle

C parallelogram **D** trapezium

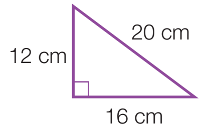
Worksheet R4.5

5 Calculate the perimeter and area of each of these shapes to the nearest whole number:

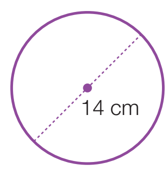
(a)



(b)



(c)



Key Words

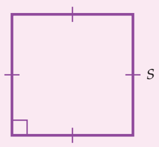
arc	cross-section	prism	volume
area	net	sector	
capacity	perimeter	subtended	
circumference	polyhedron	surface area	

4.1 Perimeter

The **perimeter** of a shape is the total length of its boundary, measured in units such as cm or km. In practical situations we can measure the individual lengths and add them together. If we have a diagram or a well-defined shape, we can use shortcuts (or formulas) where we have some lengths that are equal. Equal lengths are shown by markings such as | and ||.

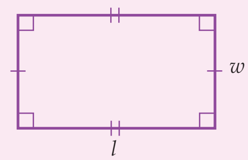
Perimeter formulas

Square



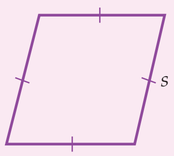
$P = 4s$

Rectangle



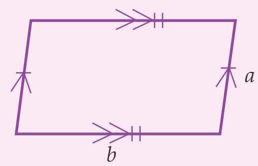
$P = 2l + 2w$ or $P = 2(l + w)$

Rhombus



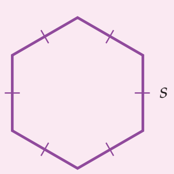
$P = 4s$

Parallelogram



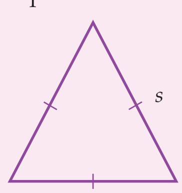
$P = 2a + 2b$ or $P = 2(a + b)$

Regular hexagon



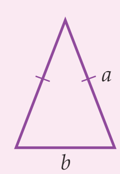
$P = 6s$

Equilateral triangle



$P = 3s$

Isosceles triangle

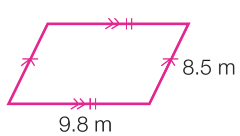


$P = 2a + b$

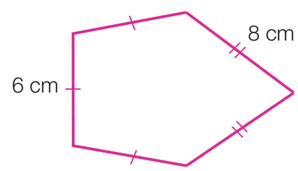
Worked Example 1

Find the perimeter of these shapes.

(a)



(b)

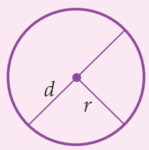


Thinking	Working
(a) 1 Identify the shape and write down the formula.	(a) $P = 2(a + b)$
2 Substitute in the values and evaluate.	$P = 2(8.5 + 9.8)$ $= 2 \times 18.3$
3 Write the answer with the given unit of length.	$P = 36.6$ m

(b)	1	Write out the sum of the lengths.	(b) $P = 8 + 8 + 6 + 6 + 6$
	2	Simplify if possible.	$= 2 \times 8 + 3 \times 6$
	3	Write the answer with the given unit of length.	$= 34$ cm

Circumference of a circle

You should recall that the perimeter of a circle is called the **circumference**. The formula used to calculate the circumference of a circle is $C = 2\pi r$, where r is the radius, or $C = \pi d$, where d is the diameter. The plural of radius is radii.



Circumference of a circle

$C = \pi d$ or $C = 2\pi r$
where d = diameter and r = radius.

If we know the circumference of a circle, we can use it to find the radius or diameter.

Worked Example 2

WE2

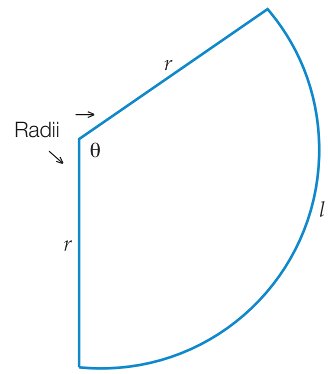
(a) Find the diameter of a circle with a circumference of 8.5 cm, correct to one decimal place.

(b) Find the radius of a circle with a circumference of 40 mm, correct to one decimal place.

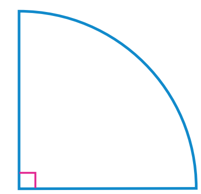
Thinking			Working
(a)	1	Write the circumference formula.	(a) $C = \pi d$
	2	Rearrange the formula to make d the subject, by dividing both sides by π.	$\frac{C}{\pi} = d$
	3	Substitute the value for C.	$\frac{8.5}{\pi} = d$
	4	Calculate, rounding your answer to the specified number of decimal places. Remember to include the unit.	$d = 2.705\ldots$ $d = 2.7$ cm (1 d.p.)
(b)	1	Write the circumference formula.	(b) $C = 2\pi r$
	2	Rearrange the formula to make r the subject, by dividing by 2π.	$\frac{C}{2\pi} = r$
	3	Substitute the value for C.	$\frac{40}{2\pi} = r$
	4	Calculate, rounding your answer to the specified number of decimal places.	$r = 6.366\ldots$ $r = 6.4$ mm (1 d.p.)

Arc length and perimeter of a sector

A **sector** of a circle is the area contained within two radii and the arc that joins them. An **arc** is part of the circumference of a circle. The distance along the arc of a sector (l) is in proportion to the angle **subtended** by (standing on) the arc at the centre of the circle.



To calculate the length of the arc, we multiply the entire circumference by the fraction represented by the arc. For example, an arc subtending an angle of 90° has a length that is one-quarter the circumference of the circle, because $\frac{90}{360} = \frac{1}{4}$. The arc length is found by calculating $\frac{1}{4} \times 2\pi r$ (which can also be written as $\frac{2\pi r}{4}$ or $\frac{\pi r}{2}$).



Similarly, an arc subtending an angle of 60° has a length that is one-sixth the circumference, because $\frac{60}{360} = \frac{1}{6}$.

An arc subtending an angle of 130° has a length that is $\frac{130}{360}$ or $\frac{13}{36}$ of the circumference.

The arc length l can be found by using the formula:

$$l = \frac{\theta}{360} \times 2\pi r$$

where θ (the Greek letter theta) is the angle subtended by the arc at the centre of the circle.

To find the perimeter of a sector of a circle, we need to add twice the radius to the length of the arc.

Arc length of a sector

$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle subtended at the centre.

Perimeter of a sector

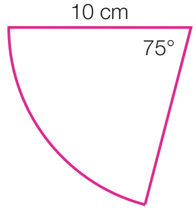
$P = l + 2r$

Worked Example 3

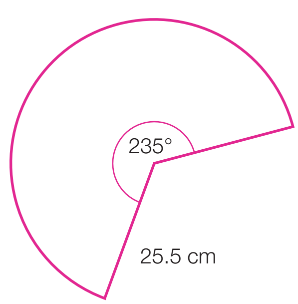
WE3

Calculate **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

(a)



(b)



Thinking	Working
(a) **(i)** **1** Write the appropriate formula.	(a) (i) $l = \frac{\theta}{360} \times 2\pi r$
2 Write down the known values.	$r = 10$ cm, $\theta = 75°$
3 Substitute values into the formula.	$l = \frac{75}{360} \times 2 \times \pi \times 10$
4 Simplify the expression by cancelling common factors and/or multiplying. (Here, we cancel common factors of 20 then 3).	$= \frac{75}{\cancel{360}_{18}} \times \cancel{20}^{1} \times \pi$ $= \frac{\cancel{75}^{25}\pi}{\cancel{18}_{6}}$ $= \frac{25\pi}{6}$
5 Evaluate, writing your answer with the specified accuracy.	$= 13$ cm (nearest whole number)
(ii) **1** Write the appropriate formula.	(ii) $P = l + 2r$
2 Write down the known values. (Use the exact value of *l*, rounding at the end of the calculation.)	$r = 10$ cm, $l = \frac{25\pi}{6}$ cm
3 Substitute values and calculate.	$P = 2 \times 10 + \frac{25\pi}{6}$ $= 33$ cm (nearest whole number)
(b) **(i)** **1** Write the appropriate formula.	(b) (i) $l = \frac{\theta}{360} \times 2\pi r$
2 Write down the known values.	$r = 25.5$ cm, $\theta = 235°$
3 Substitute values into the formula.	$l = \frac{235}{360} \times 2 \times \pi \times 25.5$
4 Simplify the expression by cancelling obvious common factors and/or multiplying. (Here, we cancel a common factor of 2.)	$= \frac{235}{\cancel{360}_{180}} \times \cancel{2}^{1} \times \pi \times 25.5$ $= \frac{5992.5\pi}{180}$
5 Evaluate, writing your answer with the specified accuracy.	$= 105$ cm (nearest whole number)
(ii) **1** Write the appropriate formula.	(ii) $P = l + 2r$
2 Write down the known values. (Use the exact value of *l*, rounding at the end of the calculation.)	$r = 25.5$ cm, $l = \frac{5992.5\pi}{180}$ cm
3 Substitute values and calculate.	$P = 2 \times 25.5 + \frac{5992.5\pi}{180}$ $= 156$ cm (nearest whole number)

4.1 Perimeter

Navigator

Q1, Q2, Q3, Q4, Q5 (a)–(e), Q6, Q7, Q10, Q12	Q1, Q2, Q3 (a)–(d), Q5, Q6, Q7, Q8, Q10, Q12, Q13	Q1 Column 2, Q2 (a)(i)–(ii), Q2 (b)(i)–(ii), Q3 (d)–(f), Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14

Answers page 614

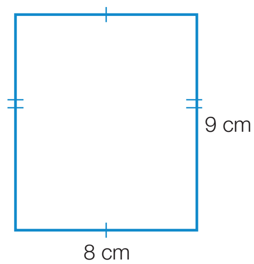
Equipment required: tape measure or ruler for Question **12**

Fluency

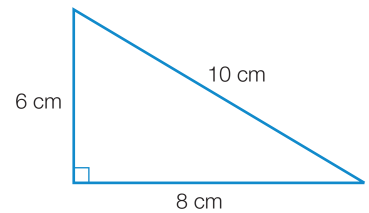
1 Find the perimeters of the following shapes using the measurements given.

WE1

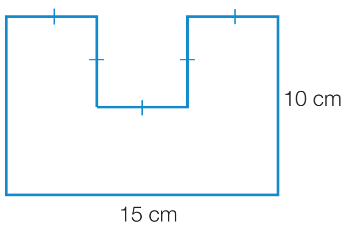
(a)



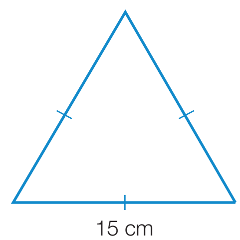
(b)



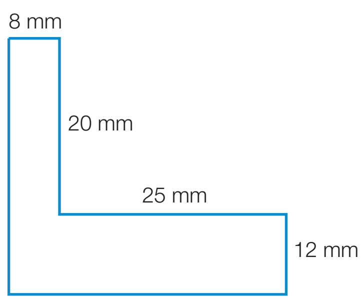
(c)



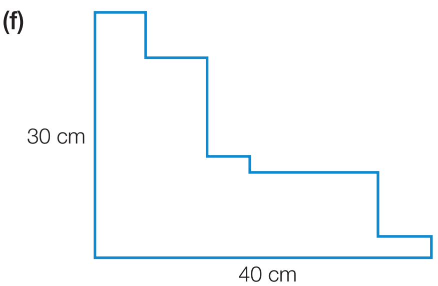
(d)



(e)



(f)



2 **(a)** Find, correct to one decimal place, the diameter of a circle with a circumference of:

WE2

(i) 15 m **(ii)** 32 m **(iii)** 141.39 mm **(iv)** 188.52 mm

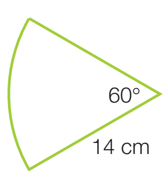
(b) Find, correct to one decimal place, the radius of a circle with a circumference of:

(i) 93 km **(ii)** 71 km **(iii)** 804.248 cm **(iv)** 1.2566 cm

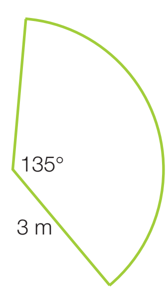
3 Find **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

WE3

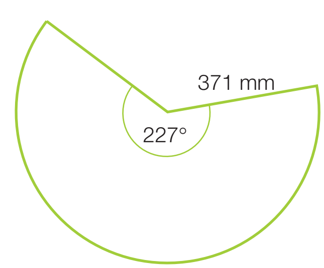
(a)



(b)



(c)



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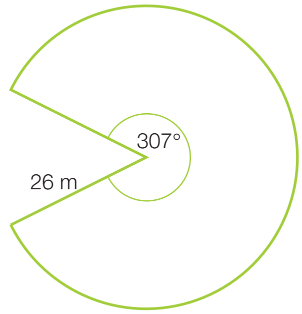

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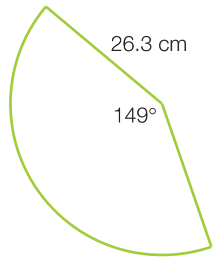

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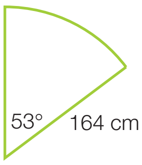
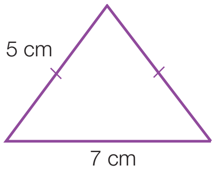


Understanding

4 Choose the correct answer.

The perimeter of the triangle at right is:



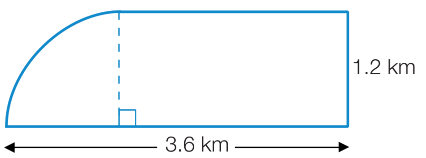
A 12 cm **B** 17 cm

C 19 cm **D** 35 cm

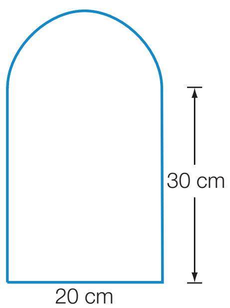
5 Find the perimeters of these shapes, giving your answers to one decimal place where necessary.

> **Warning**
> Dotted lines are there only to indicate measurements. Make sure you don't include them in your perimeter calculation.

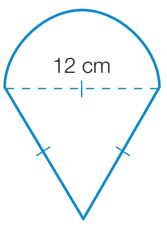
(a)



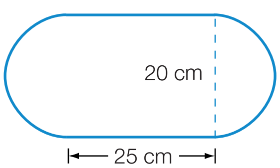
(b)



(c)



(d)



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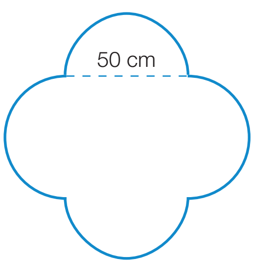

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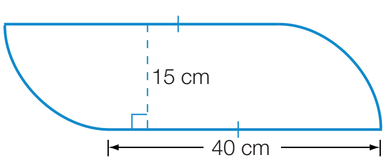
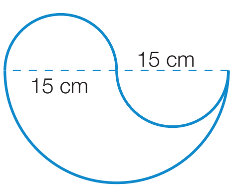


(g)



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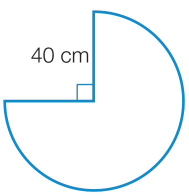
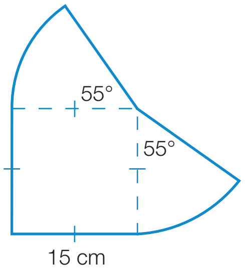


(i)



6 Find the diameter of a CD if its circumference is 37.7 cm. Write your answer correct to one decimal place.

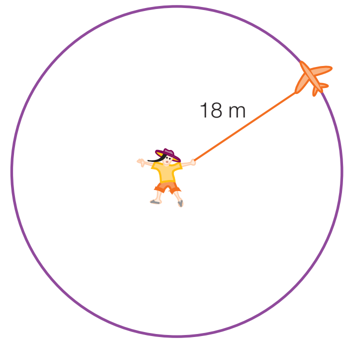
7 Find the circumference of Earth if the diameter, to the nearest 100 km, is 12 800 km. Give your answer correct to the nearest 100 km.

8 Andrea is flying her model aeroplane on a wire 18 m long, as shown.

(a) How far does it fly in one circuit? (Answer to the nearest metre.)

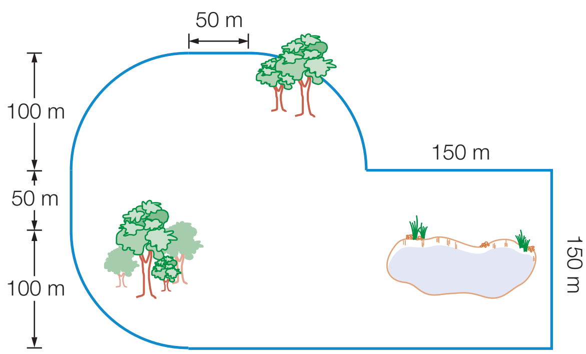
(b) How long will it take to complete one circuit if it is flying at a speed of 5 metres per second? (Answer to the nearest tenth of a second.)

(c) How long would the wire need to be if the distance for one circuit was 200 m? (Answer to the nearest centimetre.)



9 Every night Nicole walks her dog twice around the park. The plan of the park is shown below.

How far does she walk in km? (Answer to the nearest metre.)

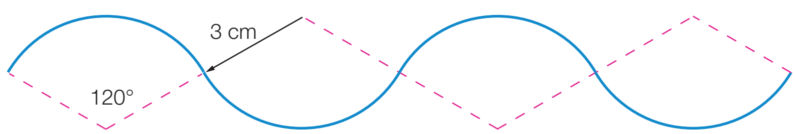


Hint
Drawing a diagram can help you 'picture' a problem.

Reasoning

10 If a plane travels halfway round the world at an altitude of 11 300 m, what extra distance, in km, will it fly compared with the distance measured at the surface of the Earth? (The radius of the Earth is approximately 6400 km.) Round your answer to one decimal place.

11 A sheet of corrugated iron has corrugations that are based on the arc of a circle of radius 3 cm and a central angle of 120°. Four corrugations are shown below.



How many corrugations are formed when a flat sheet of steel 1 m wide is bent to form a sheet of corrugated iron?

Open-ended

12 Estimate first, then use a tape measure or ruler to measure the perimeter of:

(a) the front cover of this book

(b) the top of your desk

(c) the whiteboard in your room

(d) the floor of the classroom.

13 Find whole number diameters of three different circles whose circumferences are between 50 cm and 90 cm.

14 Sketch and label a shape with a boundary containing straight as well as curved sections whose perimeter is between 30 and 40 cm.

Maths 4 Real

Veterinary maths

The job of a vet involves treating and performing surgery on sick animals. De-sexing and vaccinations are the two most common procedures. Dogs are the most common animals dealt with, along with wombats, koalas, lizards, horses, cows, cats, sheep, birds, pigs and rats.

When an animal has become dehydrated the amount of water to be replaced (the deficit) is estimated as a percentage of the body weight.

For example, a 35-kg calf with 10% dehydration has a water deficit of 3.5 kg (10% of 35 kg).

1 Find the deficit for an 80-kg Great Dane that is 5% dehydrated.

Each litre of water has a mass of 1 kg. For the calf, 3.5 kg of water is the same as 3.5 L, or 3500 mL.

2 Convert the deficit found in **1** to the number of mL needed for the Great Dane.

Animals that are severely dehydrated require additional fluid of 60 mL per day for each kilogram of body mass. For the calf, this would be:
60 mL × 35 = 2100 mL of additional fluid per day.

3 Find the additional fluid needed per day for the Great Dane.

The total amount of fluid in the first day of treatment is the total of the deficit and the additional fluid. For the calf, this would be 3500 mL + 2100 mL = 5600 mL.

4 Find the total amount of fluid needed in the first day for the Great Dane.

The fluid is given via an intravenous drip, for which 10 drops = 1 mL. For the calf, the number of drops needed in a day is 10 drops × 5600 = 56 000 drops.

5 Find the number of drops needed in a day for the Great Dane.

The drip needs to be set at a rate involving seconds.

The number of seconds in a day is:
24 × 60 × 60 = 86 400 s.

The flow rate for the calf would be:

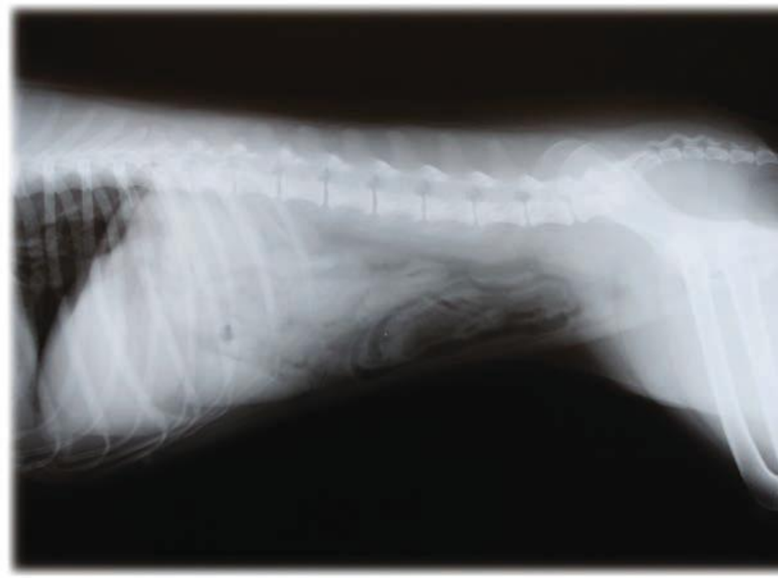
$$\frac{56\,000 \text{ drops}}{86\,400\ s} = 0.65 \text{ drops per second.}$$

6 Find the number of drops per second for the Great Dane.

7 Repeat **1** to **6** for a 12-kg Kelpie that is 10% dehydrated.

Research

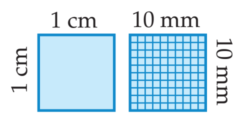
- Find out some of the common causes of dehydration in animals.
- When a dehydrated cat is brought into a surgery, water is sometimes injected directly under the skin, where it is quickly absorbed. Find out why a drip is used in some cases and not others.



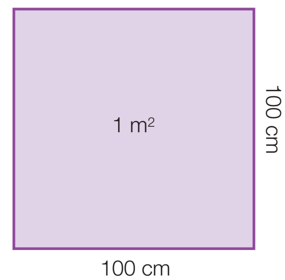
4.2 Area

Converting units of area

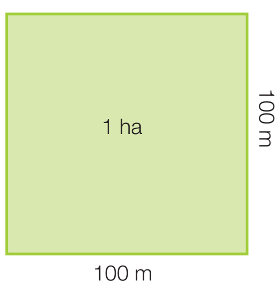
Area is the amount of surface within a boundary. Area is measured in 'square' units; that is, square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), hectares (ha) or square kilometres (km^2). It is important to remember that converting between units of area is *not* the same as converting between units of length. Because the units are square units, the conversion factor is the square of the conversion factor for length. (The conversion factor is the number we multiply or divide by when converting one unit to another.)



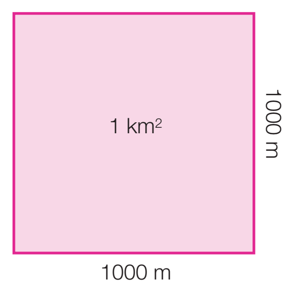
$$1\text{ cm}^2 = 10\text{ mm} \times 10\text{ mm}$$
$$= 100\text{ mm}^2$$



$$1\text{ m}^2 = 100\text{ cm} \times 100\text{ cm}$$
$$= 10\,000\text{ cm}^2$$

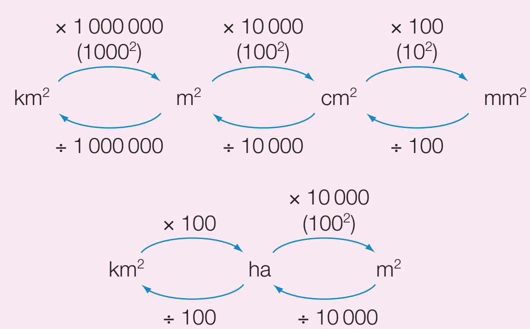


$$1\text{ ha} = 100\text{ m} \times 100\text{ m}$$
$$= 10\,000\text{ m}^2$$



$$1\text{ km}^2 = 1000\text{ m} \times 1000\text{ m}$$
$$= 1\,000\,000\text{ m}^2$$

Converting units of area:



A hectare (ha) is a square 100 metres by 100 metres (10 000 m^2). It is a handy unit for measuring land, such as the area of a farm or park.

Worked Example 4

WE4

Copy and complete the following conversions.

(a) 17.4 ha = _____ m^2 **(b)** 20 000 m^2 = _____ km^2

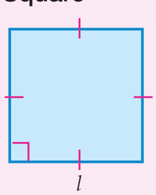
Thinking	Working
(a) We are converting from a larger unit to a smaller unit, so we need to multiply. There are 100 m^2 in every ha, so multiply by 100^2 to convert ha to m^2.	(a) 17.4×100^2 $= 174\,000$ m^2
(b) We are converting from a smaller unit to a larger unit, so we need to divide. There are 1000 m^2 in every km^2, so divide by 1000^2.	(b) $20\,000 \div 1000^2$ $= 0.02$ km^2

If you are asked to calculate the area of a shape in different units to the ones given, it is often easier to convert the dimensions of the shape into the required units of length first, then calculate the area.

Area formulas

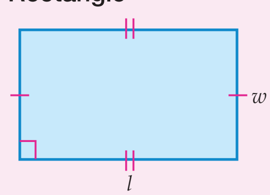
You should be familiar with the formulas for the following shapes.

Square



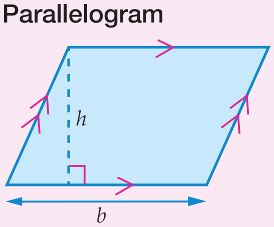
$A = l^2$

Rectangle



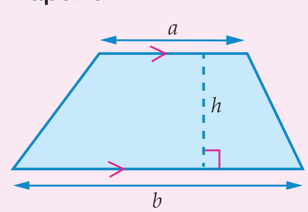
$A = lw$

Parallelogram



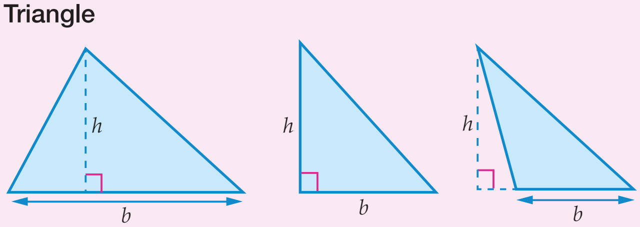
$A = bh$

Trapezium



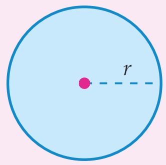
$A = \dfrac{(a+b)h}{2}$

Triangle



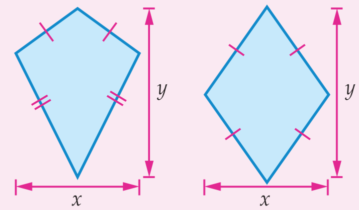
$A = \dfrac{bh}{2}$

Circle



$A = \pi r^2$

Kite and rhombus



$A = \frac{xy}{2}$

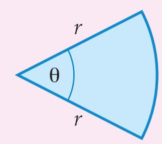
Area of a sector

The area of a sector is proportional to the angle of the sector at the centre of the circle. For example, a semicircle subtends an angle of 180° at the centre of the circle and its area is half the area of the full circle, because $\frac{180°}{360°} = \frac{1}{2}$.

Area of a sector

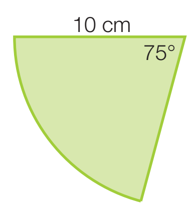
$$A = \frac{\theta}{360} \times \pi r^2$$

where θ is the angle subtended at the centre.



Worked Example 5

Find the area of this sector, correct to the nearest cm^2:



Thinking	Working
1 Write the appropriate formula.	$A = \frac{\theta}{360} \times \pi r^2$
2 Write down the known values.	$r = 10$ cm, $\theta = 75°$
3 Substitute values.	$A = \frac{75}{360} \times \pi \times 10^2$
4 Simplify the expression by cancelling common factors and/or multiplying. (Here we divide by a common factor of 10, then 3.)	$= \frac{\cancelto{25}{75}}{\cancelto{12}{360}} \times \pi \times 10 \times \cancelto{1}{10}$ $= \frac{250\pi}{12}$
5 Calculate, rounding appropriately.	$= 65$ cm^2 (nearest whole number)

Composite shapes

Composite shapes are made up of different combinations of basic shapes. We can find the area of composite shapes by breaking them up into simpler shapes, calculating the individual areas of the simpler shapes and adding the areas together. For some shapes we can find the area remaining by subtracting the area of a piece that is not included from the area of the overall shape.

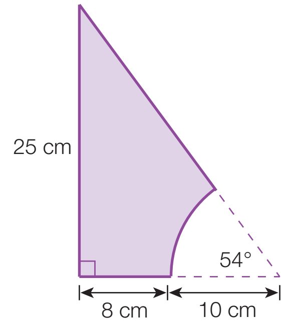
Worked Example 6

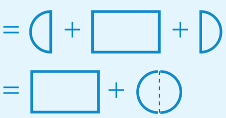
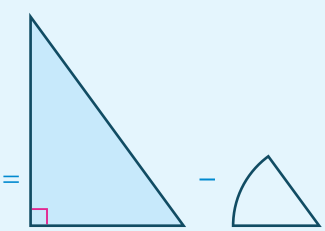
WE 6

Find the shaded area of each shape. Write your answers to the nearest whole number.

(a)

(b)



Thinking	Working
(a) 1 Break the composite shape into parts, combining parts to simplify if convenient.	(a) Area = 
2 Write the appropriate area formula for each shape and define the variables.	$A = lw + \pi r^2$, where $l = 15$ cm, $w = 8$ cm and $r = 4$ cm
3 Substitute the given measurements and simplify the expression.	$= 15 \times 8 + \pi \times 4^2$ $= 120 + 16\pi$
4 Evaluate, rounding your answer as necessary and writing it with the correct units.	$= 170$ cm^2 (nearest whole number)
(b) 1 Break the composite shape into parts.	(b) Area = 
2 Write the appropriate area formula for each shape and define the variables.	$A = \frac{bh}{2} - \left(\frac{\theta}{360} \times \pi r^2\right)$, where $b = 18$ cm, $r = 10$ cm, $\theta = 54°$ and $h = 25$ cm
3 Substitute the given measurements and simplify the expression by cancelling obvious factors. (Here we cancel common factors of 2 and 10.)	$= \frac{\overset{9}{\cancel{18}} \times 25}{\cancel{2}} - \left(\frac{\overset{27}{\cancel{54}}}{\underset{18}{\cancel{360}}} \times \pi \times 10^{\cancel{2}\,1}\right)$ $= 225 - \left(\frac{270}{18}\pi\right)$
4 Evaluate, rounding your answer as necessary and writing it with the correct units.	$= 178$ cm^2 (nearest whole number)

Rounding and accuracy

You should note in the above Worked Examples that calculations are simplified before the actual calculation is performed. (Simplifying involves cancelling common factors and/or performing simple whole number multiplications.) The final answer that results is only rounded once, at the very end. This ensures that the most accurate final answer is obtained. Using rounded answers as you work through a problem means you accumulate, or build up, a small amount of error each time, which results in an inaccurate final answer. In order to use your calculator efficiently for these calculations, you will need to apply the correct order of operations, making use of the bracket keys ((and)) and the answer key (ANS), which recalls the previous answer.

4.2 Area

Navigator

Answers page 614

Q1 Column 1, Q2, Q3 (a)–(f), Q4, Q5, Q6, Q7, Q8, Q10, Q13, Q15, Q16, Q19, Q22	Q1 Column 2, Q2, Q3 (d)–(i), Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q21, Q22	Q1 Column 2, Q2 (d)–(f), Q3 (g)–(l), Q5, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21

Fluency

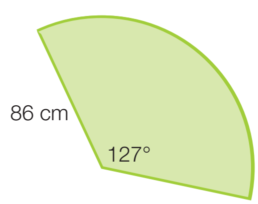
WE4

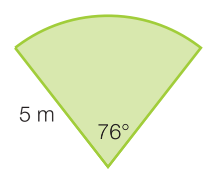
1 Copy and complete the following conversions.

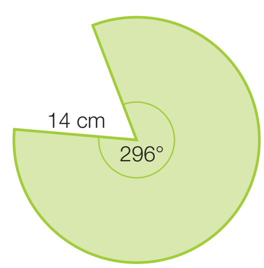
(a) $3\text{ m}^2 = \_\_\_\_\text{ cm}^2$
(b) $0.172\text{ m}^2 = \_\_\_\_\text{ cm}^2$
(c) $72.5\text{ cm}^2 = \_\_\_\_\text{ mm}^2$
(d) $0.6\text{ cm}^2 = \_\_\_\_\text{ mm}^2$
(e) $9\text{ ha} = \_\_\_\_\text{ m}^2$
(f) $4.07\text{ ha} = \_\_\_\_\text{ m}^2$
(g) $35\text{ m}^2 = \_\_\_\_\text{ ha}$
(h) $695\text{ mm}^2 = \_\_\_\_\text{ cm}^2$
(i) $127\text{ cm}^2 = \_\_\_\_\text{ m}^2$
(j) $835\text{ cm}^2 = \_\_\_\_\text{ m}^2$
(k) $43.8\text{ ha} = \_\_\_\_\text{ m}^2$
(l) $6850\text{ m}^2 = \_\_\_\_\text{ ha}$
(m) $290\text{ cm}^2 = \_\_\_\_\text{ m}^2$
(n) $39.6\text{ cm}^2 = \_\_\_\_\text{ mm}^2$
(o) $15\,000\,000\text{ m}^2 = \_\_\_\_\text{ km}^2$
(p) $10\,080\text{ ha} = \_\_\_\_\text{ km}^2$

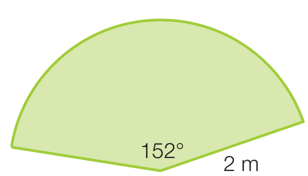
WE5

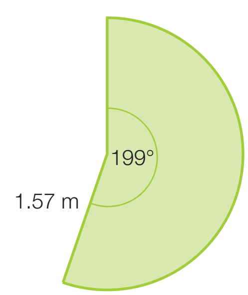
2 Calculate the area of each of these sectors correct to one decimal place.

(a)


(b)


(c)


(d)


(e)


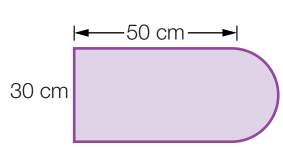
(f)

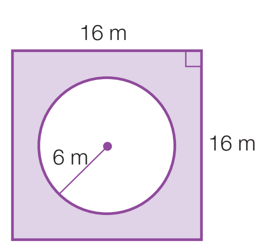

Warning

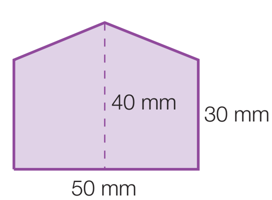
Do not round any answers until *after* you have added or subtracted the areas. Otherwise your result may be less accurate.

WE6

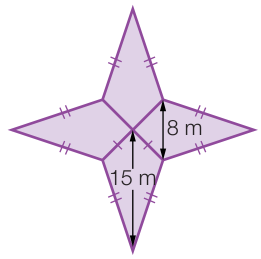
3 Find the shaded area of each of the following shapes. Write your answers correct to the nearest whole number.

(a)


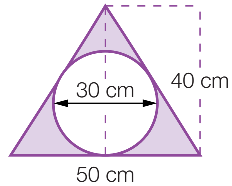
(b)


(c)


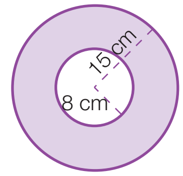
(d)



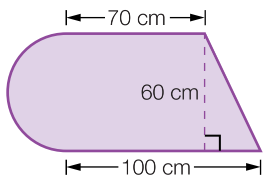
(e)



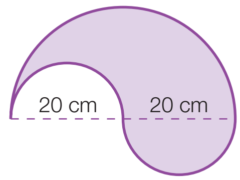
(f)



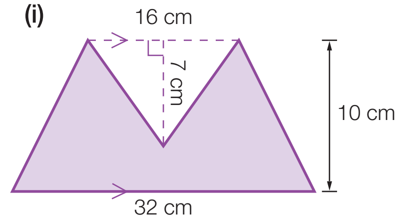
(g)



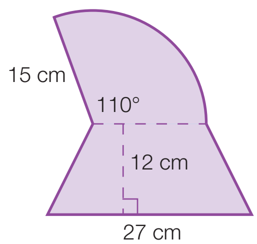
(h)



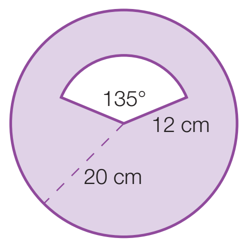
(i)



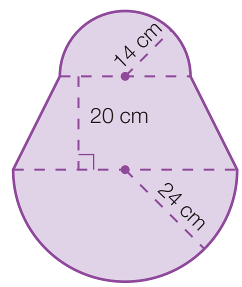
(j)



(k)



(l)

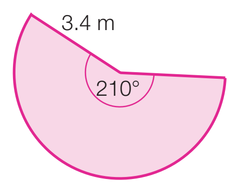


4 Choose the correct answer.

The area of the sector shown on the right is closest to:

A $12.5\ m^2$ B $21.1\ m^2$

C $21.2\ m^2$ D $36.3\ m^2$

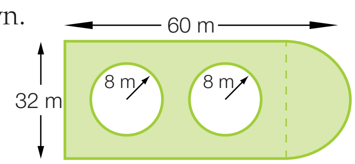


Understanding

5 A garden lawn has two ponds within its boundary, as shown.

(a) Find the area of the lawn.

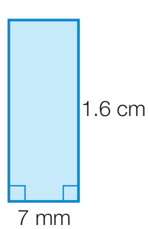
(b) How much seed is needed to grass the lawn if 1 kg of seed covers $8\ m^2$?



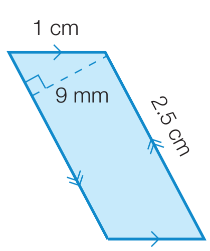
6 Find the area of each shape, correct to two decimal places where necessary, in:

(i) square millimetres (ii) square centimetres.

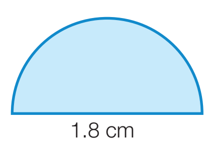
(a)



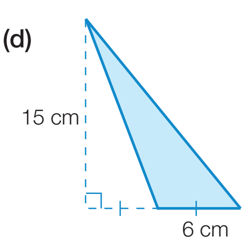
(b)



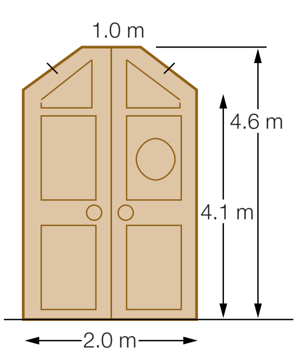
(c)



(d)



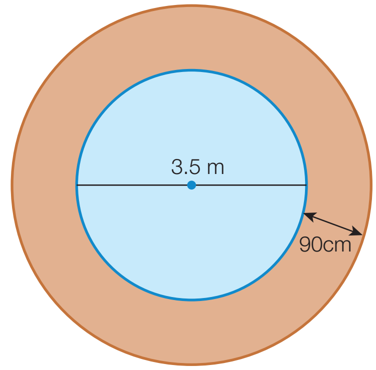
7 The doors at a local church are shaped as shown. What is the exact area of each pair of doors?



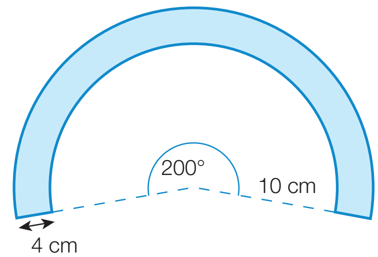
8 An annulus is the area between two circles that have the same centre but different radii. A circular pond 3.5 m in diameter is surrounded by a path 90 cm wide. The shape of the path is an annulus. The path is to be paved with small rectangular bricks that are 20 cm long and 10 cm wide.

(a) Calculate the area of the path.

(b) Calculate the number of bricks required to pave the path.



9 Nicola has made a collar from a sector of an annulus, as shown in the diagram. What area of fabric has been used for the collar? Give your answer correct to one decimal place.



10 Circular plastic drink coasters 8 cm wide are manufactured by feeding a rectangular strip of plastic 9 cm wide into a machine that cuts out the coasters. The machine starts 1 cm from one end of the strip and leaves 1 cm between coasters and 1 cm on the other end of the strip. For this machine, answer the following questions:

(a) How long would a strip of plastic have to be to cut out 50 coasters?

(b) What would be the total area of wasted plastic (to the nearest cm^2)?

(c) What percentage of the strip is used for making coasters? (Give your answer to the nearest whole number.)

Hint

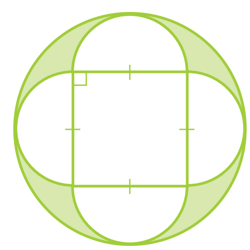
How many of the radii of the smaller circle make up the side length of the square.

11 An architect is designing a decorative circular courtyard 22 m in diameter. The shaded area is to be a garden and the rest will be pavement.

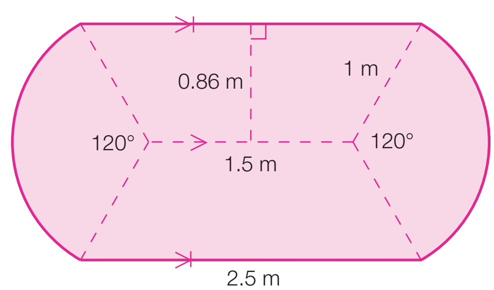
(a) Find the area of the pavement, correct to one decimal place.

(b) Find the area of the garden, correct to one decimal place.

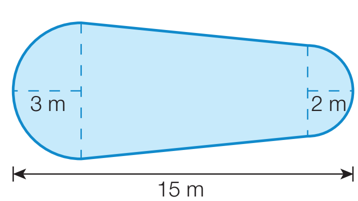
(c) What percentage of the courtyard will be pavement? (Give your answer to the nearest whole number.)



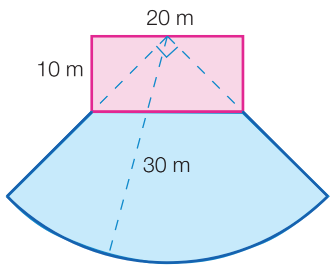
12 Norbert's dining table is the shape shown. Calculate its area, giving your answer correct to two decimal places.



13 Jennifer has a rectangular lawn at the back of her house that measures 20 m by 18 m. She is putting in a pond with the shape shown in the diagram. What area of lawn will she have left if the grass is to go all the way to the edge of the pond? (Give your answer correct to one decimal place.)



14 A new theatre is to be built. The design of the theatre includes a rectangular stage area with a portion of a quadrant of a circle for the audience to be seated in, as shown in the diagram on the next page. Calculate the total area of the theatre to the nearest whole number.



Reasoning

15 How many rectangular house-blocks 16 m × 20 m would fit in a 15 ha subdivision for a housing estate, assuming 20% of the area of the subdivision is used for roads?

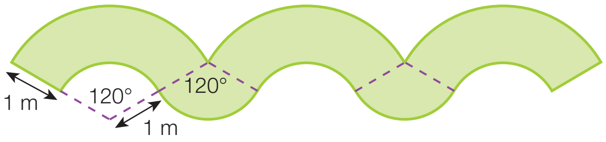
16 A square has diagonals that are 20 cm long.

(a) Remembering that the diagonals of a square bisect each other at right angles, find the area of the square.

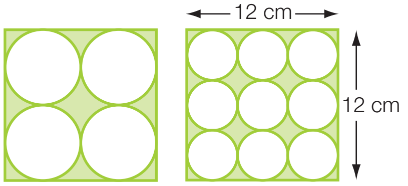
(b) Calculate the length of the sides of the square from the area you found, correct to two decimal places.

17 The area of a sector of a circle is 1000 cm^2. The radius of the sector is 30 cm. Calculate the size of the angle at the centre of the sector, correct to the nearest degree.

18 Part of the front of a shop awning consists of three sectors of an annulus and two sectors of a circle, as shown in the diagram. Calculate the total area of this section. (Give your answer correct to one decimal place.)



19 Touching circles are drawn inside two squares with side length 12 cm. Four identical circles (called a 4-Pak) just fit into the first square and nine (called a 9-Pak) fit into the second.



(a) What is the radius of each circle in:
(i) the 4-Pak **(ii)** the 9-Pak?

(b) Which of the shaded background areas is bigger?

20 A trapezium with an area of 900 cm^2 has parallel sides 40 cm apart. If one of these sides is twice as long as the other, the lengths of the parallel sides would be:

A 20 cm, 10 cm **B** 24 cm, 12 cm **C** 30 cm, 15 cm **D** 40 cm, 20 cm

Open-ended

21 Steve's Pizza shop sells pizzas that are 14 cm in radius. He decides to stop making them circular and instead make them rectangular. He uses approximately the same amount of ingredients and makes them the same thickness. Assuming that the length and width of the rectangular pizzas are in whole centimetres, write down at least three pairs of dimensions that are practical.

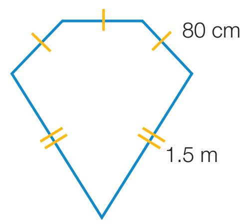
22 The perimeter of a rectangle is 60 cm. Calculate two possible areas for the rectangle.

Half-time 4

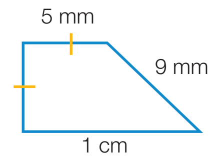
Ex. 4.1

1 Find the perimeter of each shape, in the smaller units if two are given, rounding your answer to the nearest whole number.

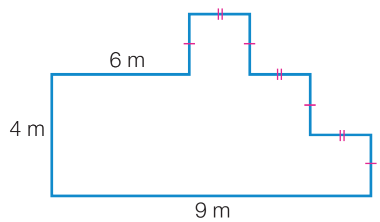
(a)



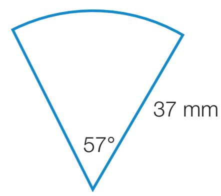
(b)



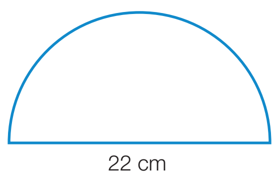
(c)



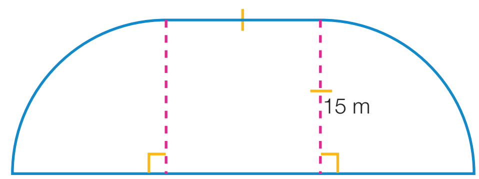
(d)



(e)



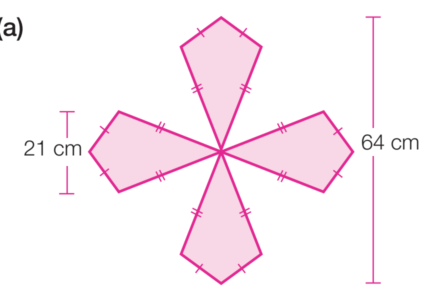
(f)



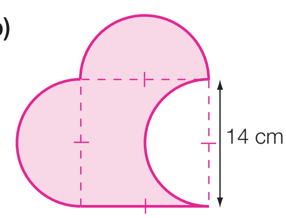
Ex. 4.2

2 Find the shaded area, correct to the nearest whole number.

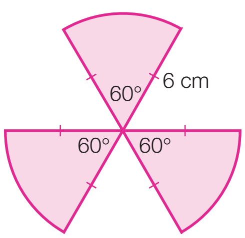
(a)



(b)



(c)



Ex. 4.1

3 Calculate the distance along the surface of Earth (to the nearest 100 km) from the Equator to the South Pole given that the diameter of Earth is approximately 12 800 km.

Ex. 4.2

4 A trapezium has an area of 600 cm^2. Its parallel sides are 15 and 25 cm long. Find the distance between the parallel sides.

Ex. 4.2

5 Copy and complete the following conversions.

(a) 4 cm^2 = _____ mm^2 (b) 7.5 m^2 = _____ cm^2

(c) 0.26 km^2 = _____ m^2 (d) 19.8 ha = _____ m^2

(e) 3.05 km^2 = _____ ha (f) 274 cm^2 = _____ m^2

Ex. 4.2

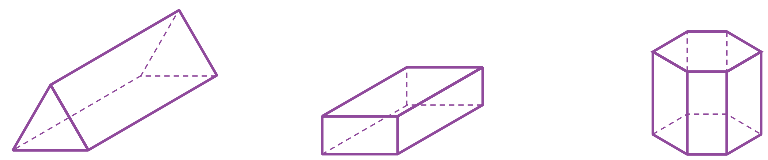
6 The perimeter of a right-angled triangle is 90 cm. Two of the sides are 15 and 39 cm long. Find the area of the triangle.

4.3 Surface area

A **polyhedron** is a solid with flat faces and straight edges. The **surface area** (SA) of a polyhedron is the total area of its faces.

To find the surface area of a polyhedron, find the area of each face, then add them together.

A **prism** is a three-dimensional object that has a uniform **cross-section** in the shape of a polygon. Prisms are named according to the cross-sectional shape. For example:

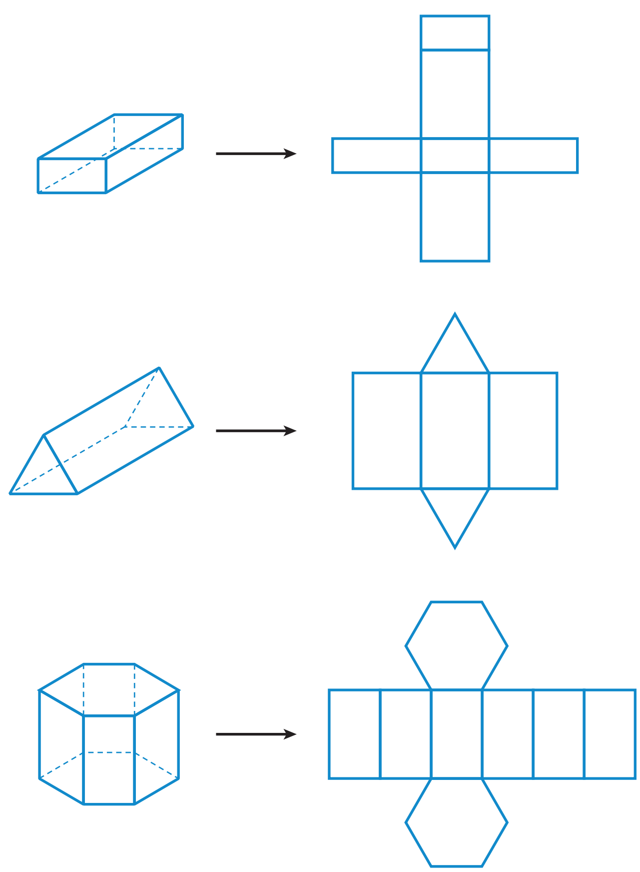


triangular prism — rectangular prism — hexagonal prism

Nets

To calculate surface area, it is helpful to consider the **net** of the object. The net is a two-dimensional (2D) representation of the three-dimensional (3D) prism. If we imagine the object as a box, we would create the net by cutting along the edges and laying the box out flat.

3D prism — 2D net



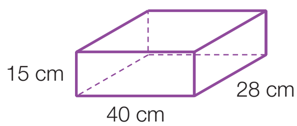
Drawing the net of a prism helps us identify each of the faces and find the area of each one. Nets of prisms are explored in more detail in Chapter 6, Solids and nets, Section 6.7 (page 375).

Worked Example 7

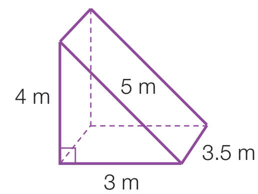
WE7

Draw the nets of the following polyhedra and use them to calculate their surface area.

(a)



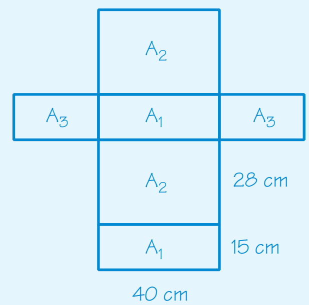
(b)



Thinking / Working

(a) 1 To draw the net, imagine cutting along the edges of the prism and ‘flattening’ it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.

(a)



2 Write a formula that shows the surface area (SA) as the total of the individual areas.

$SA = 2A_1 + 2A_2 + 2A_3$

$SA = 2l_1w_1 + 2l_2w_2 + 2l_3w_3$

3 Substitute the values of the different dimensions and evaluate each different area.

$= 2 \times (40 \times 15) + 2 \times (40 \times 28) + 2 \times (28 \times 15)$

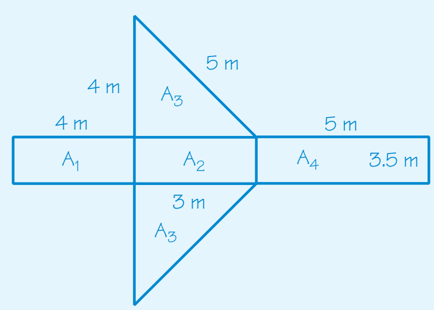
$= 1200 + 2240 + 840$

4 Find the total area, writing your answer with area units.

$= 4280\ \text{cm}^2$

(b) 1 To draw the net, imagine cutting along the edges of the prism and ‘flattening’ it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.

(b)



2 Write a formula that shows the surface area (SA) as the total of the individual areas.

$SA = A_1 + A_2 + 2A_3 + A_4$

$= l_1w_1 + l_2w_2 + 2\frac{bh}{2} + l_3w_3$

3 Substitute the values of the different dimensions and evaluate each different area.

$= (3.5 \times 4) + (3 \times 3.5) + 2 \times \left(\frac{3 \times 4}{2}\right) + (5 \times 3.5)$

$= 14 + 10.5 + 12 + 17.5$

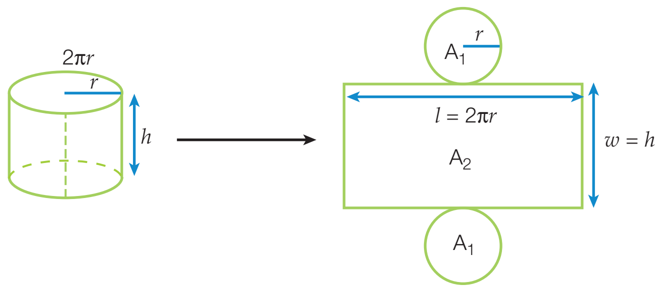
4 Find the total area, writing your answer with area units.

$= 54\ \text{cm}^2$

Note that in the above Worked Example, subscripts are used to indicate the dimensions of a shape if there is more than one of a particular type of shape. For example, the area of the rectangle labelled A_1 is written as l_1w_1, whereas that of rectangle A_2 is written as l_2w_2.

Cylinders

Although a cylinder is not strictly a prism (because its surfaces are not polygons), we can find the surface area of a cylinder in a similar way to a prism; that is, by drawing the net and calculating the area of each surface. A cylinder has three faces: a circle on each end and the curved surface in between them. If we cut this curved surface from top to bottom and flatten it out, we get a rectangle with a length equal to the circumference of the end circles and a width equal to the height of the cylinder.



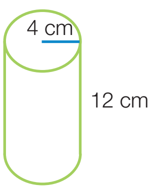
To find the surface area, we need to calculate and then add the areas of these three surfaces:

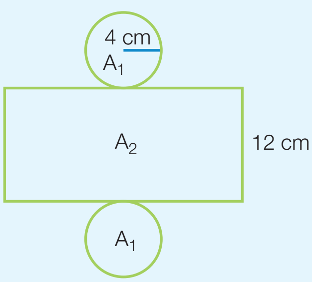
$$
\begin{aligned}
SA &= 2 \times A_1 + A_2 \\
&= 2 \times \pi r^2 + l \times w \\
&= 2\pi r^2 + 2\pi rh
\end{aligned}
$$

Worked Example 8

WE8

Find the surface area of this cylinder, correct to one decimal place.



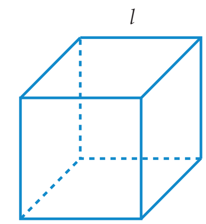
Thinking	Working
1 Draw the net of the cylinder. Label the two identical surfaces A_1 and the flattened curved surface A_2.	
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$SA = 2A_1 + A_2$ $= 2\pi r^2 + 2\pi rh$
3 Substitute the values of the dimensions and evaluate each different area.	$= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 12$ $= 32\pi + 96\pi$ $= 128\pi$
4 Find the total area, writing your answer to the specified number of decimal places, and with the correct units of area.	$= 402.1\text{ cm}^2$ (1 d.p.)

Formulas for the surface area of prisms and regular solids

Although we can determine the surface area of a prism by drawing its net and calculating the area of each surface, it is sometimes useful to create and use a formula for some common types of prisms.

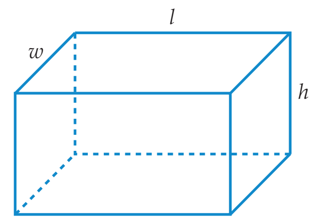
The net of a cube consists of six identical squares.

The surface area of a cube $= 6l^2$, where l is the side length



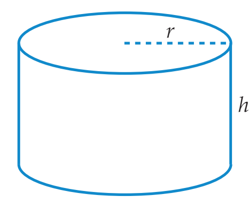
The net of a rectangular prism consists of three pairs of identical rectangles.

The surface area of a rectangular prism $= 2lw + 2lh + 2wh$
$= 2(lw + lh + wh)$, where l, w and h are the three side lengths



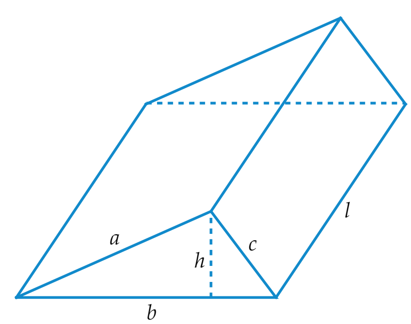
The net of a cylinder consists of two identical circles and a rectangle that has the circumference of the circular end as one of its dimensions.

The surface area of a cylinder $= 2\pi r^2 + 2\pi rh$, where r is the radius of the circular face and h is the height of the cylinder.



For other prisms and regular solids, it is best to just use the net because there are too many variables to make a formula worthwhile.

For example, the formula for a triangular prism would require us to find $2 \times \frac{1}{2} \times bh + cl + al + bl$, which would be just as easy to do from the net itself.



4.3 Surface area

Navigator

Answers page 615

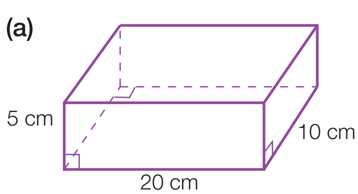
Q1 (a)–(f), Q2, Q3 (b), Q4, Q5, Q6, Q8, Q10, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q10, Q12, Q13	Q1 (a), (c), (e)–(i), Q2, Q3, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

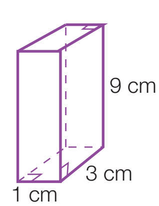
WE7

1 Draw nets of the following polyhedra and use them to calculate their surface area.

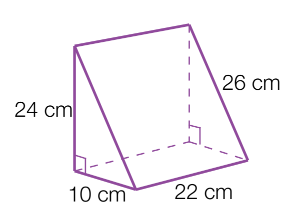
(a)



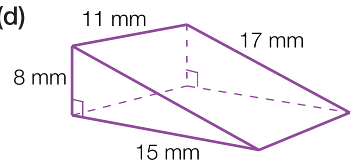
(b)



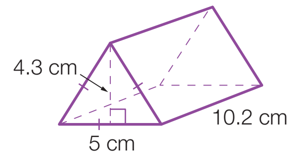
(c)



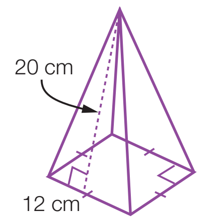
(d)



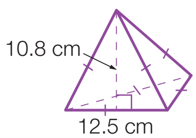
(e)



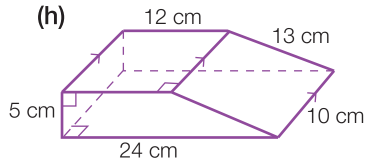
(f)



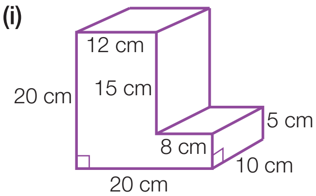
(g)



(h)

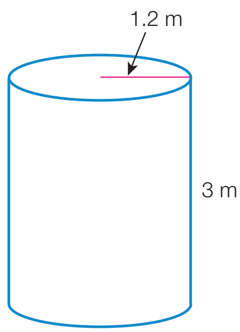


(i)

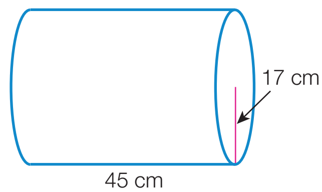


2 Find the surface area of the following cylinders, correct to one decimal place. WE8

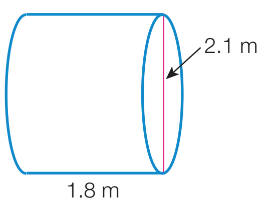
(a)



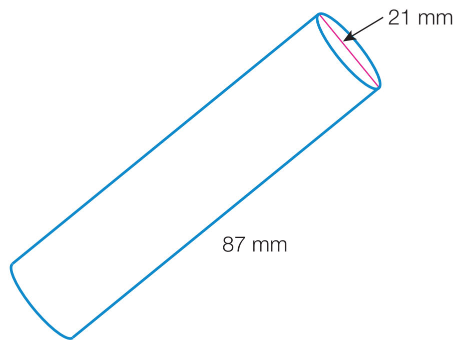
(b)



(c)



(d)



3 **(a)** The surface area of a cube with edge length 10 cm is:

A 200 cm^2 **B** 400 cm^2 **C** 600 cm^2 **D** 800 cm^2

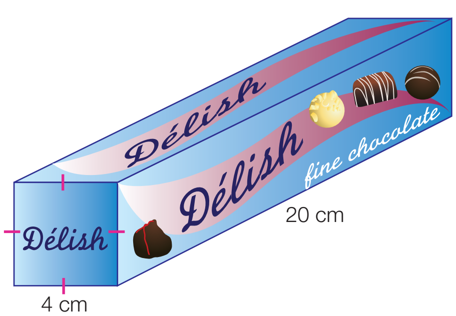
(b) The surface area of a rectangular box with dimensions 40 cm, 30 cm and 20 cm is:

A 1800 cm^2 **B** 2400 cm^2 **C** 2600 cm^2 **D** 5200 cm^2

Understanding

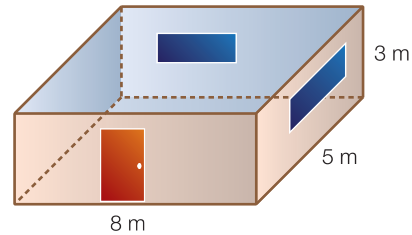
4 Joshua has to design a box for a new brand of chocolate. Assuming there is no overlap, which of these designs uses less cardboard?

Type 1



Type 2

5 Leonardo has been hired to paint the walls and ceiling of a room, as shown in the diagram. The room has dimensions 8 m × 5 m × 3 m and it has two windows, each 2 m × 1.5 m, and a door 0.9 m × 2.3 m. Find the area to be painted (excluding the windows and door).

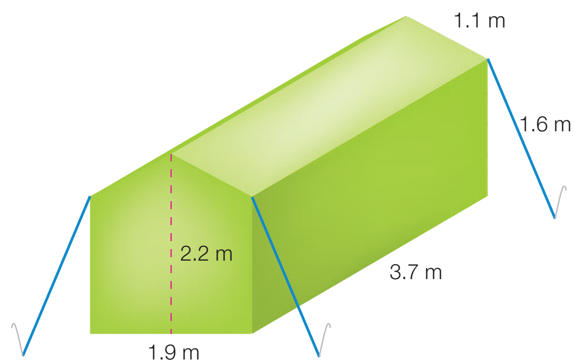


6 Paper labels are made to wrap around the curved surface of cans of food. A 1 cm overlap is allowed at the end so the label can be glued down. Calculate the area of paper required to label each of the following cans, to the nearest square centimetre.

(a)

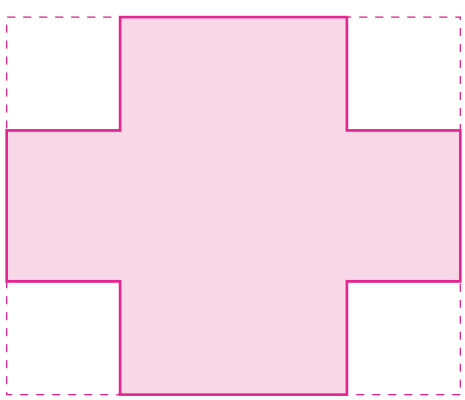
(b)

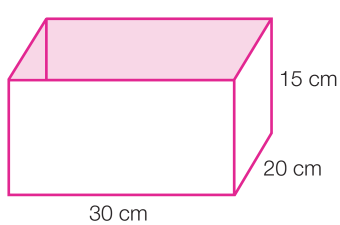
7 Calculate, to the nearest whole number, the area of the material used to make this tent, including the floor. (Assume that there is no overlap at the edges.)



Reasoning

8 The rectangular piece of cardboard on the left has four squares cut from the corners to make an open box whose dimensions are 30 cm × 20 cm × 15 cm.





(a) What was the length and width of the original piece of cardboard?

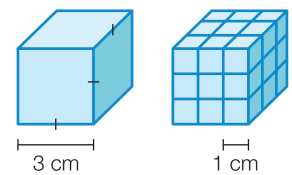
(b) What percentage of the original piece of cardboard is wasted in making the box?

9 Chewing your food is an important step in the digestive process. Chewing breaks down a large piece of food, making it easier to digest. Demonstrate your calculations using a cube to model a piece of food.

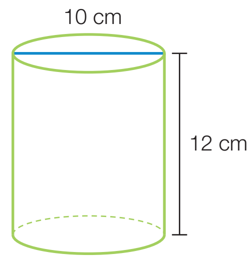
(a) Calculate the surface area of this 3 cm cube.

(b) Imagine this cube is chewed into smaller cubes each with a side length of 1 cm. Calculate the surface area of these smaller cubes.

(c) By how many times has the surface area of the original cube increased?



10 (a) Draw the net of this cylinder, labelling the measurements to the nearest whole number.



(b) The area of each circle on the ends of the cylinder can be found by the calculation:

A $\pi \times 10^2$ B $\pi \times 5^2$

C $2 \times \pi \times 10$ D $\pi \times 10$

(c) The width (shorter side) of the rectangular part of the net is:

A 5 cm B 10 cm C 12 cm D 24 cm

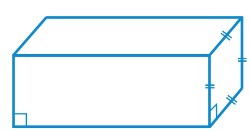
(d) The length of the rectangle can be found by the calculation:

A $\pi \times 10^2$ B $\pi \times 5^2$ C $\pi \times 10$ D $2 \times \pi \times 10$

Open-ended

11 What is the effect on the surface area of a cube if you double the length of its edges? Demonstrate your answer with two suitable cubes, one with an edge length that is double that of the other cube.

12 A rectangular box has a surface area of 960 cm^2. It has a square end, as shown. Each side length is a whole number of centimetres. Find two different sets of dimensions that satisfy these conditions.



Outside the Square — Problem solving

Icosahedron

An icosahedron is one of the five Platonic solids. Each face of a Platonic solid is a regular polygon in which all faces are identical.

A particular icosahedron has a surface of 20 equilateral triangles of edge length 20 cm.

Find the surface area of the icosahedron. Round your answer to one decimal place.

Strategy options

- Draw a diagram.
- Have you seen a similar problem?
- Break the problem down into manageable parts.

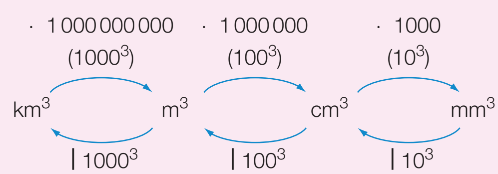
4.4 Volume and capacity

The **volume** of a solid is the total amount of space that the solid occupies. It is measured in cubic units such as cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).

$$1\text{ cm}^3 = 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm} = 10^3\text{ mm}^3 = 1000\text{ mm}^3$$

$$1\text{ m}^3 = 1\text{ m} \times 1\text{ m} \times 1\text{ m} = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} = 100^3\text{ cm}^3 = 1\,000\,000\text{ cm}^3$$

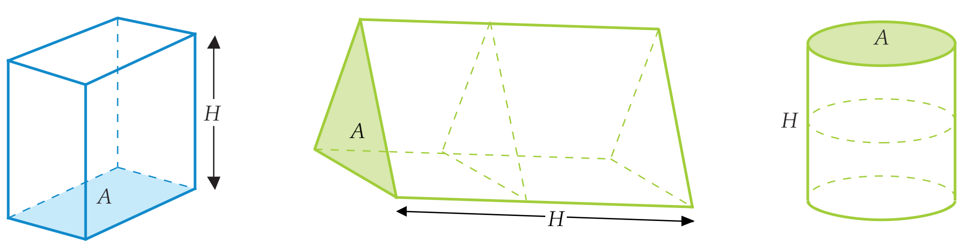
Because units of volume are cubic units, the conversion factor for converting from one unit to another is the cube of the conversion factor for length.



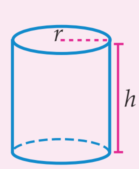
The formula $V = AH$ gives the volume of a prism, where A is the area of the base and H is the perpendicular height.

You have identified the base correctly if cutting parallel to it would leave the prism with a new 'base' identical in shape and size to the original base. In each of these diagrams, the base is shaded. (For the rectangular prism, all faces may be used as the base.)

It is important to note that the base is not necessarily the face that the prism is 'sitting on'. It is the face that has a 'uniform cross-section'.



Note: Although a cylinder is not strictly a prism, it does have a uniform cross-section and so its volume can be found in the same way. The base is the circular face which has area $A = \pi r^2$. Multiplying the area of the base by the height of the cylinder gives $V = \pi r^2 H$.



Volume of prism

$V = AH$ where
A = area of the base
H = perpendicular height of the prism

Volume of cylinder

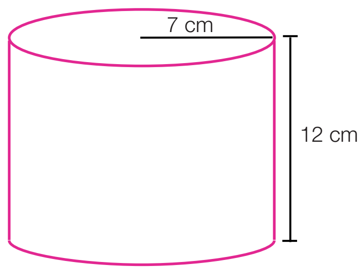
$V = \pi r^2 H$

Worked Example 9

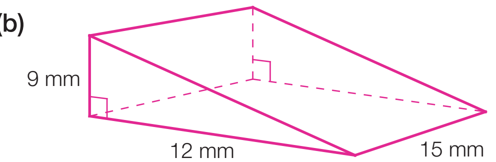
WE9

Find the volume of these solids to the nearest whole number.

(a)



(b)



Thinking	Working
(a) 1 Write the general formula for volume.	(a) $V = AH$
2 Substitute the specific formula for the area of the base ($A = \pi r^2$).	$= \pi r^2 \times H$
3 Substitute the measurements.	$= \pi \times 7^2 \times 12$
4 Calculate the volume to the nearest cm^3.	$= 1847\ cm^3$ (nearest whole number)
(b) 1 Write the general formula for volume.	(b) $V = AH$
2 Substitute the specific formula for the area of the base ($A = \frac{bh}{2}$).	$= \frac{bh}{2} \times H$
3 Substitute the measurements	$= \frac{12 \times 9}{2} \times 15$
4 Calculate the volume in mm^3.	$= 810\ mm^3$

Capacity

Capacity refers to the maximum amount that a container can hold, usually measured in millilitres (mL), litres (L), kilolitres (kL) or megalitres (ML). We often refer to the volume of liquids and gases in units of capacity. A 10 cm × 10 cm × 10 cm cube has a capacity of 1 L.

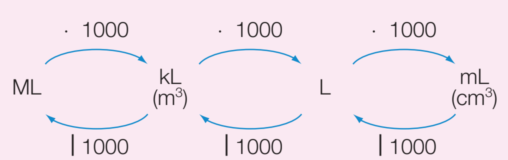
Converting units of capacity:

1 mL = 1 cm^3

1 L = 1000 mL = 1000 cm^3

1 kL = 1000 L = 1 m^3

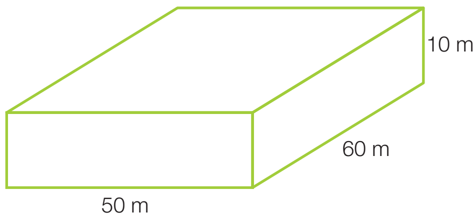
1 ML = 1000 kL



Worked Example 10 WE10

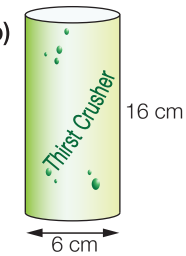
Find the capacity of each of the following in the units specified, rounded to the nearest whole number.

(a)



Water tank (in ML)

(b)



Can of soft drink (in mL)

Thinking	Working
(a) 1 Calculate the volume.	(a) $V = A \times H$ $= 60 \times 50 \times 10$ $= 30\,000 \text{ m}^3$
2 Convert to equivalent units of capacity.	$\therefore$ Capacity $= 30\,000$ kL
3 Convert to the required units.	$= 30$ ML
(b) 1 Calculate the volume.	(b) $V = \pi r^2 \times H$, where $r = \frac{6}{2} = 3$ cm $= \pi \times 3^2 \times 16$ $= 452 \text{ cm}^3$ (nearest whole number)
2 Convert to equivalent units of capacity.	$\therefore$ Capacity $= 452$ mL

Note: In each case we assume that either the internal dimensions are given or that the material of the container is so thin that we can disregard the thickness.

4.4 Volume and capacity

Navigator

Answers page 616

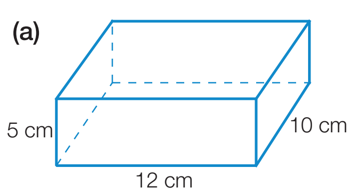
Q1, Q2, Q3, Q4, Q5 (a), Q6, Q7, Q11, Q12, Q13	Q1, Q2, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13	Q1 (a),(c),(e), Q2, Q4, Q5 (b)–(d), Q6, Q7, Q8, Q9 Q10, Q11, Q12

Fluency

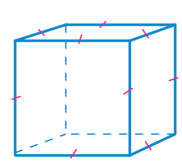
WE9

1 Find the volume of these solids to the nearest whole number.

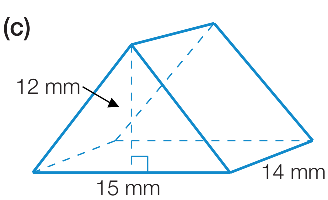
(a)



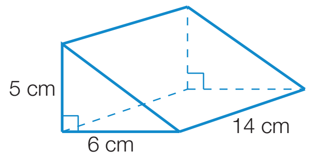
(b)



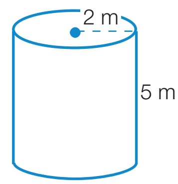
(c)



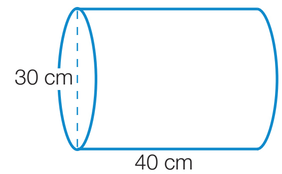
(d)



(e)

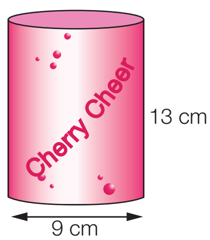


(f)



2 Find the capacity of each of the following in the units specified. WE10

(a)

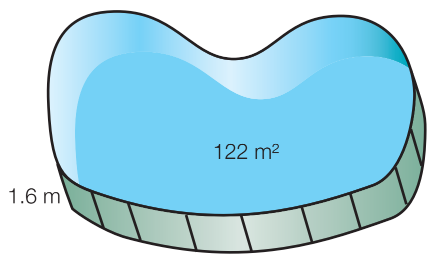


Cylindrical can of soft drink (in mL, to the nearest whole number)

(b)

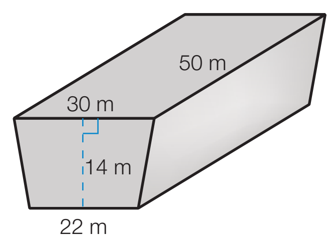
Square-based milk carton (in L, to one decimal place)

(c)



A kidney-shaped swimming pool (in kL, to the nearest whole number)

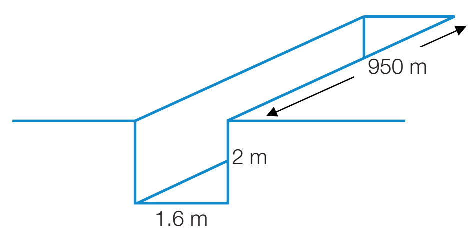
(d)



A storage tank on a ship (in ML, to one decimal place)

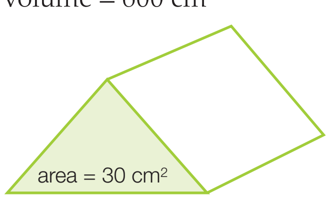
Understanding

3 The council is digging a trench to lay some new pipe. Find the volume of soil that will have to be removed.

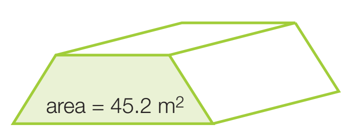


4 Find the height of these prisms.

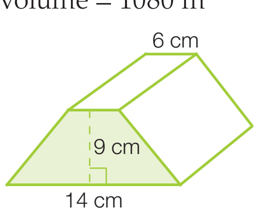
(a) volume = 600 cm^3



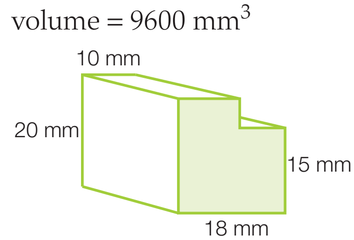
(b) volume = 560.48 cm^3



(c) volume = 1080 m^3

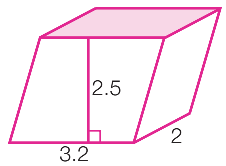


(d) volume = 9600 mm^3



5 All the lengths in the diagrams below are in metres. The volume of each solid is:

(a)



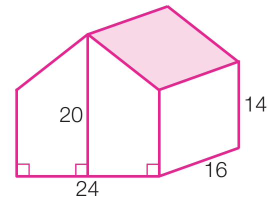
A 8 m^3

B 11.4 m^3

C 15.4 m^3

D 16 m^3

(b)



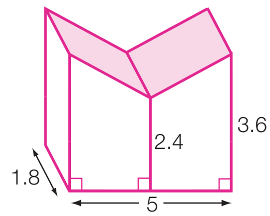
A 1320 m^3

B 1408 m^3

C 6528 m^3

D 7680 m^3

(c)



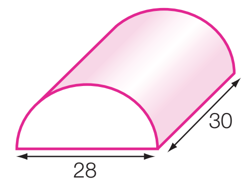
A 16 m^3

B 27 m^3

C 54 m^3

D 86.4 m^3

(d)



A 9236 m^3

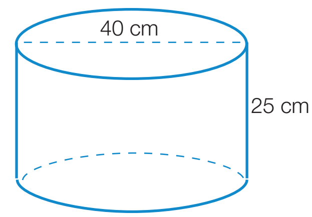
B $11\,760 \text{ m}^3$

C $36\,945 \text{ m}^3$

D $79\,168 \text{ m}^3$

Reasoning

6 Jenny is making a fruit punch for a party. She is using the cylindrical bowl shown and will fill it to 5 cm from the top. If each of the party cups holds 250 mL, find the number of drinks that can be served from the bowl.



7 A cylindrical water tank has a capacity of 8 kL and a height of 2 m.

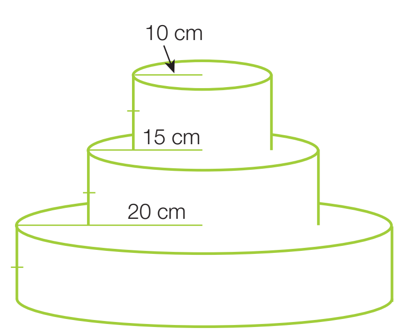
(a) Find its radius to the nearest cm.

(b) Water starts leaking out of a full tank at the rate of 25 L/day. How many days will it be before the tank is half empty?

8 Find the volume left after a wedge of 52° has been cut from a cheese wheel of diameter 30 cm and height 10 cm. Write your answer to the nearest cm^3.

9 (a) Find the volume of a three-tier cake if each tier is 7 cm high and the radii are 20 cm, 15 cm and 10 cm. Give your answer correct to the nearest cm^3.

(b) The baker decides to use the same amount of ingredients to make a cake with a square base of side length 40 cm. How high will it be, correct to the nearest cm?

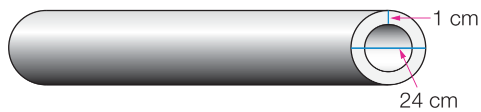


10 A steel pipe has a diameter of 24 cm and is 1 cm thick. It is made in lengths of 2 m.

(a) What is the capacity of the pipe (to the nearest mL)?

(b) How many cubic centimetres of steel are in the length of pipe (to the nearest cm^3)?

(c) How many kilolitres of water, correct to one decimal place, would pass through one length of pipe in an hour if the water is flowing at a rate of 2 metres per second?



Open-ended

11 Give an example of a substance that we might measure in:

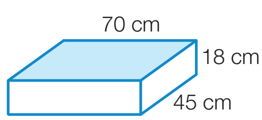
(a) mL **(b)** L **(c)** kL

12 Write down the dimensions of any rectangular prism that has a capacity of each of the following:

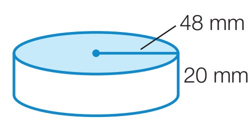
(a) 1 mL **(b)** 1 L **(c)** 1 kL **(d)** 1 ML

13 Sabanth was set the following two questions for homework.

(i) Calculate the volume of this prism, in m^3.



(ii) Calculate the volume of this cylinder, in cm^3.



Here is his workng:

$$V = lwh$$
$$= 70 \times 45 \times 18$$
$$= 56\,700 \text{ cm}^3$$
$$= 567 \text{ m}^3$$

$$V = \pi r^2 h$$
$$= \pi \times 48^2 \times 20$$
$$= 144\,764.6 \text{ mm}^3 \text{ (1 d.p.)}$$
$$= 14\,476.5 \text{ cm}^3 \text{ (1 d.p.)}$$

(a) Identify the error that Sabanth has made.

(b) Give Sabanth a tip to remember, so he can avoid similar errors in the future.

Outside the Square — Problem solving

Gold, set, match

The following statement has appeared on several 'weird facts' websites:

'A lump of pure gold that is only as big as a matchbox can be flattened into a sheet about the size of a tennis court.'

A matchbox is 5.5 cm long, 3.5 cm wide and 1.5 cm high. A tennis court is 23.77 m long and 10.97 m wide.

1 If the 'fact' is true, and a matchbox-sized lump of gold was flattened out into a sheet to cover a tennis court, how thick would the sheet of gold be? Give your answer in mm.

2 Considering the size of your answer, do you think the 'fact' could actually be true?

Strategy options

- Draw a diagram.
- Make a model.
- Break the problem down into manageable parts.

Mathspace

ORIGAMI SPORTS SHIRT

How to make an origami shirt

Equipment required: half a sheet of A4 paper

1 Make a centre crease lengthwise by folding your paper in half and unfolding again.

2 Fold the edges towards the middle crease.

3 Fold out two of the corners, to approximately a 60° angle (this will make the sleeves)

4 Turn the shirt over and fold the bottom edge up 1 cm (this will make the collar)

5 Turn the shirt over again and fold the corners into the centre (this will make the neck of the collar)

6 Turn the shirt over and fold the lower edge up until it is under the collar.

Questions

1 **(a)** Find the perimeter of your shirt (in cm).

(b) What shapes could you use to find the area of the shirt?

(c) Write down the formulas for the area of the shapes you listed in part **(b)**.

(d) Find the area of your shirt (in cm^2).

(e) What percentage of the area of your original piece of paper does the T-shirt take up?

Footy facts: dehydration

Footballers lose a lot of liquid through sweat. This can cause dehydration, which can affect performance and, if severe enough, be life threatening.

2 If a footballer loses an average of 23 mL in one minute while training, how many litres per hour (L/h) is this?

3 Use your answer to Question **2** to calculate how many litres of fluid a player will have lost (to the nearest litre) if he trained for 3.5 hours?

4 Studies have shown that footballers should consume 130% of the fluid they lose. Using your answer to Question **3**, how many litres of water should the footballer consume to replace the fluid lost while training? Answer to one decimal place.

5 In order to consume that amount of water, how many L/h should the player drink, on average, when training over 3.5 hours?

6 Using your answer to Question **5**, how many mL per minute would this be?

7 If a player has a cylindrical drink bottle that is 9 cm in diameter and 15 cm high, what is its capacity in litres? (State your answer to the nearest litre.) Will this be enough to cover the player's hourly liquid needs?

Investigation

The humble milk carton

Equipment required: 1 brain, 1 empty milk carton, cardboard, scissors, glue, hot water (optional)

In this environmentally conscious age, manufacturers are always looking for the most efficient and economical ways of packaging food and drink.

The Big Question

Does it take more cardboard to make two separate 1-L milk cartons or one 2-L carton?

Engage

1 Make a 3D pencil diagram of your milk carton.

2 Find the dimensions (length, width and height) of the part of the milk carton that holds the milk (you may need to make some assumptions). Do these dimensions give a volume of 1 L? Label them on the diagram and use a calculation to show how these dimensions give a volume of 1 L.

Explore

3 Carefully open out the carton along its seams. (If you fill it with hot water, the glue will soften and make it easier to gently take apart.) Make a scale drawing of the net of the carton.

4 Find the area of cardboard required to make the carton.

5 Find the minimum area of cardboard required to make a 2-L carton. Show your calculations.

Strategy options

- Guess and check.
- Look for a pattern.
- Test all possible combinations.

Explain

6 Make a scale drawing of the net of your 2-L carton, labelling all the measurements necessary to make the carton. Show, with calculations, how these dimensions give a volume of 2 L.

Elaborate

7 (a) Use your answers to Questions **4** and **5** to answer the Big Question.

(b) Does it take twice as much cardboard to make a carton with twice the volume?

(c) Which carton is the more efficient form of packaging?

Evaluate

8 Describe the process you used to find the area of cardboard needed to make a 2-L carton. Could you have gone about solving the problem in a different way?

9 Explain any assumptions you have made in your calculations of area and volume for both the 1- and 2-L cartons.

10 Is the design of your 2-L carton practical? Can the carton be handled easily? Could it fit inside a fridge door?

11 List the advantages and disadvantages of selling milk in 2-L instead of 1-L cartons.

Extend

12 Construct, out of cardboard, a prototype of your new 2-L carton.

13 Investigate whether it takes more aluminium to make two 375-mL soft drink cans or one 750-mL can.

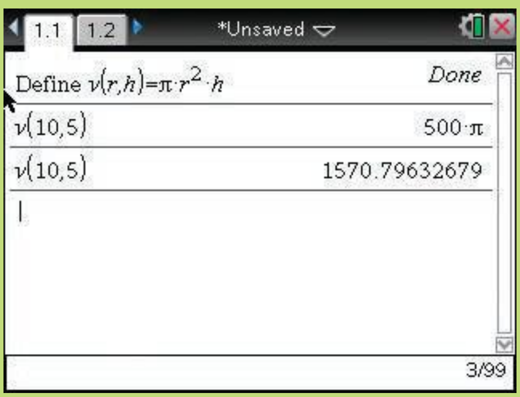
The properties of cylinders

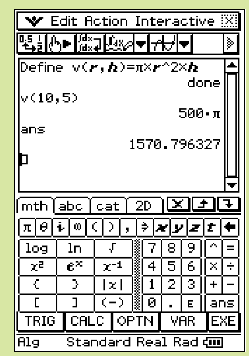
Versions of this Exploration for other technologies are available in Pearson Reader.

Technology

One of the great features of a CAS is that it enables us to define a function. In this Technology Exploration we will define a function to find the volume of a cylinder and then input various values for the radius and the height.

TI-Nspire CAS	ClassPad
Press menu > Actions > Define and then fill in the rest of the definition as shown. Press enter (or enter) when you are finished. Then, to find the volume for $r = 10$ and $h = 5$ we simply enter $v(10,5)$ and press enter (or enter). To get the approximate answer press ctrl enter (or ctrl enter).	Select 'Main' from the menu. Tap Action > Command > Define and fill in the rest of the definition as shown on the screen. The v needs to come from the abc menu, whereas the r and h need to come from the VAR tab in the mth menu. Tap EXE when you are finished. Now, to find the volume for $r = 10$ and $h = 5$ we simply enter v(10,5) and tap EXE. To switch between answers in exact form and decimal form, tap 'Standard' at the bottom of the screen and then EXE.





1 Copy the table shown below into your workbook. Use your CAS to find the volumes of the cylinders in the table.

Cylinder	Radius	Height	Volume
1	5	10	
2	10	10	
3	20	10	
4	2	10	
5	6	10	
6	18	10	
7	4	10	
8	16	10	
9	64	10	

For each of the questions below, round your answer to the nearest whole number.

2 In all the cylinders in Question **1**, the height has been kept constant at 10 units. In the first three cylinders the radius doubles, in the next three cylinders the radius triples, and in the last three cylinders the radius quadruples.

(a) What do the mathematical terms *double*, *triple* and *quadruple* mean?

(b) Use the values for volume you obtained using your CAS to calculate the ratios $\frac{V_3}{V_2}$ and $\frac{V_2}{V_1}$.

Complete the statement: Doubling the radius of a cylinder increases its volume by a factor of __________.

(c) Calculate the ratios $\frac{V_6}{V_5}$ and $\frac{V_5}{V_4}$. Complete the statement: Multiplying the radius of a cylinder by 3 increases its volume by a factor of __________.

(d) Calculate the ratios $\frac{V_9}{V_8}$ and $\frac{V_8}{V_7}$. Complete the statement: Multiplying the radius of a cylinder by 4 increases its volume by a factor of ____________.

(e) Can you generalise the pattern above? If we multiply the radius of a cylinder by a number k, where k is greater than 1, how many times is the volume increased?

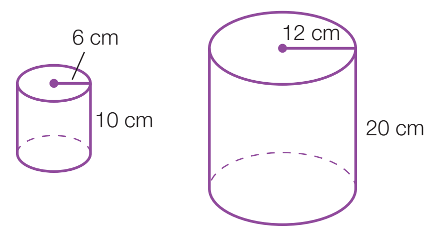
Taking it further

3 Use cardboard to make models of two cylinders of the same height but one with a radius twice that of the other. Fill the smaller cylinder with sand and use this cylinder to fill the larger cylinder. How many loads of the smaller cylinder are needed to fill the larger cylinder? How does this compare with the answer to Question **2(b)**?

4 Use algebra to prove the statements you wrote for parts **(b)** to **(e)** of Question **2**. Your teacher will assist you with this.

5 Write and test a CAS function to calculate the volume of a rectangular prism.

6 Investigate similar cylinders. Similar cylinders have corresponding radii and heights in the same ratio. For example, consider the cylinders shown. The radii of the small and large cylinders are 6 cm and 12 cm, respectively, whereas the heights are 10 cm and 20 cm, respectively .



(a) Verify that the ratios of the radii and heights are the same (i.e. show that $\frac{r_2}{r_1} = \frac{h_2}{h_1}$).

(b) Calculate the ratio $V_2 : V_1$. What is the relationship between the ratios of the corresponding sides and the ratio of the volumes of the cylinders?

(c) Repeat parts **(a)** and **(b)** for other pairs of similar cylinders.

(d) Use algebra to generalise the pattern.

Challenge 4

1 The volume of a cube is 19 683 cm^3. What is its surface area?

2 How many seconds will have you lived by the time you turn 15? (Assume three leap years and the same time of day as your birth.)

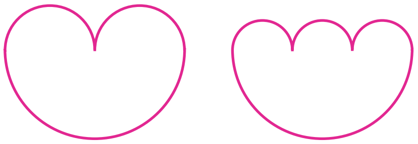
3 In a game of soccer, 11 players are on the field and two reserves are interchanged as needed. The game lasts 40 minutes. If each of the 13 members of the team had equal time on the field, how long was each of them off the field (to the nearest second)?

4 Bernard clicked his fingers once. One minute later he clicked them again. Two minutes later he clicked them again, then again 4 minutes later and 8 minutes later. If he continues this pattern, how many times will Bernard have clicked his fingers in one day?

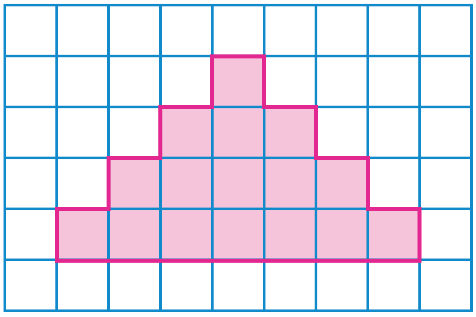
5 How can the contents of a 720-mL jug of cordial be divided evenly between two containers using three unmarked jugs that hold 150 mL, 330 mL and 390 mL?

6 A cube has its side lengths doubled. What happens to its volume? What happens to its surface area?

7 These diagrams are made up of semicircles, with the largest semicircle in each diagram having a diameter of 24 cm. Express the ratio 'perimeter of first figure : perimeter of second figure', in simplest form.

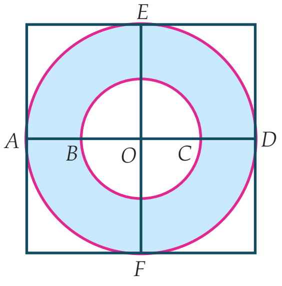


8 The area of the figure on the right is 64 cm^2. What is its perimeter?

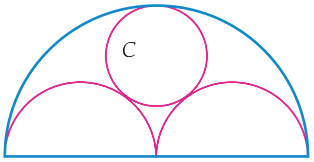


9 A, E, D and F are the midpoints of the sides of the square. $AD \perp EF$, AD and EF intersect at O and B is the midpoint of AO. The circles have radii OB and OA, respectively. The fraction of the square covered by the shaded area between the circles is:

A $\frac{3\pi}{16}$ **B** $\frac{\pi}{4}$ **C** $\frac{\pi}{16}$ **D** $\frac{1}{2}$



10 Two semicircles of radius 1 cm are drawn on the diameter of a semicircle of radius 2 cm. A circle C touches all three semicircles, as shown in the diagram. Calculate the radius of the circle C.



Chapter review 4

D.I.Y. Summary

Key Words

arc	cross-section	prism	volume
area	net	sector	
capacity	perimeter	subtended	
circumference	polyhedron	surface area	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The volume of a __________ or cylinder is found by multiplying the area of the base by the height.

2 The __________ of a container is usually measured in millilitres or litres, where $1\text{L} = 1000\ \text{cm}^3$.

3 The word __________ refers to the total distance along the boundary of a figure.

4 The perimeter of a circle is called the __________.

5 __________ is the amount of surface within a boundary. To convert between units, square the conversion factor for length.

6 __________ is the amount of space occupied by a solid. To convert between units, cube the conversion factor for length.

7 An __________ is part of the circumference of a circle.

8 To find the __________ of a regular polyhedron, multiply the area of identical faces by the number of times each face appears.

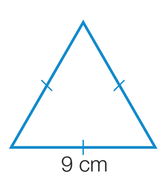
9 A __________ of a circle is the area between two radii and the arc that joins them.

10 A __________ is a solid in which every face is a polygon.

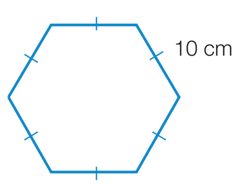
Fluency

1 Find the perimeters of these shapes. **Ex. 4.1**

(a)



(b)



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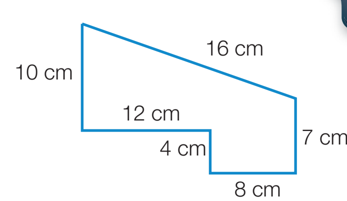
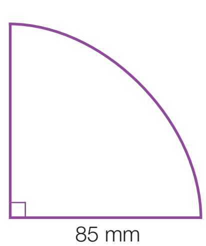
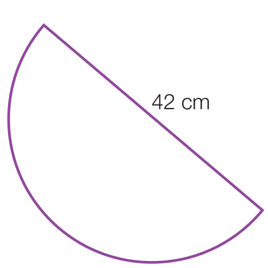


2 Calculate **(i)** the arc length and **(ii)** the perimeter of each sector to the same number of decimal places as the length in each question. **Ex. 4.1**

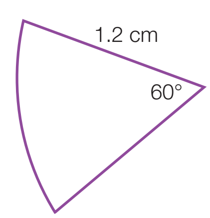
(a)



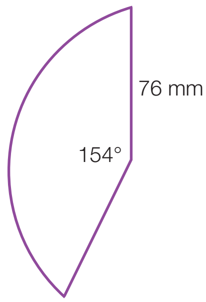
(b)



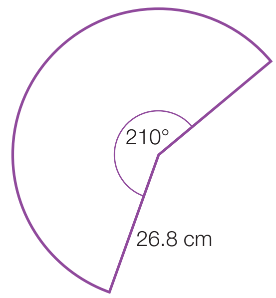
(c)



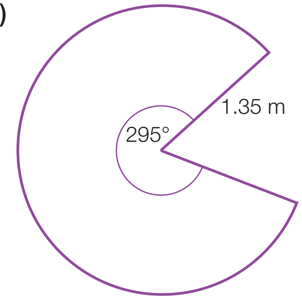
(d)



(e)



(f)



3 Copy and complete the following area conversions. **Ex 4.2**

(a) $4.2\ \text{cm}^2 = \_\_\_\_\_\_\_\_\_\_\ \text{mm}^2$ (b) $20\,000\ \text{m}^2 = \_\_\_\_\_\_\_\_\_\_\ \text{ha}$

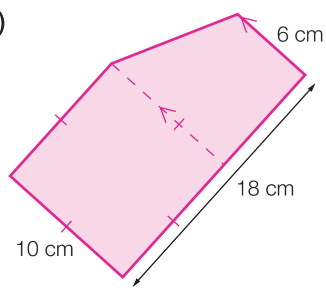
(c) $682\,500\ \text{cm}^2 = \_\_\_\_\_\_\_\_\_\_\ \text{m}^2$ (d) $528\ \text{ha} = \_\_\_\_\_\_\_\_\_\_\ \text{km}^2$

(e) $80.7\ \text{ha} = \_\_\_\_\_\_\_\_\_\_\ \text{m}^2$ (f) $0.009\ \text{m}^2 = \_\_\_\_\_\_\_\_\_\_\ \text{cm}^2$

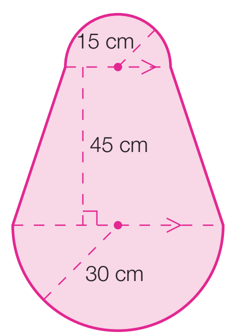
4 Calculate the area of each of the sectors in Question **2**, giving your answers to the same number of decimal places as given for the length. **Ex 4.2**

5 Find the shaded areas of the following composite shapes. **Ex 4.2**

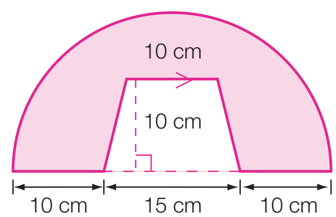
(a)



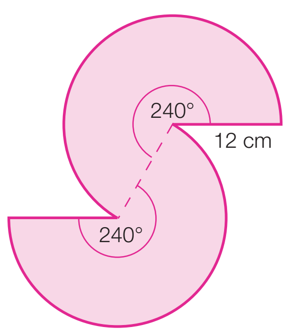
(b)



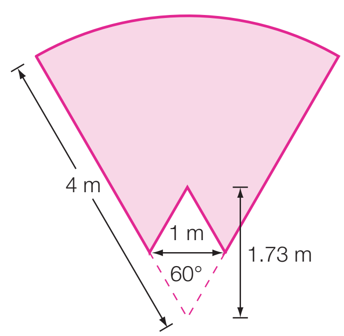
(c)



(d)

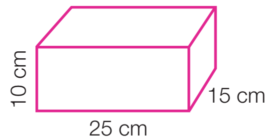


(e)

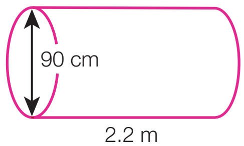


6 Find the surface area of each of these solids, correct to two decimal places where necessary. **Ex 4.3**

(a)

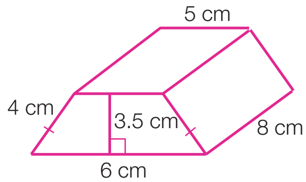


(b)

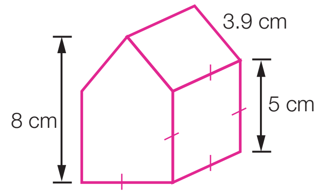


(Answer in m^2)

(c)



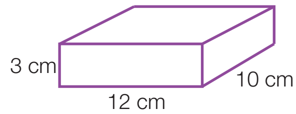
(d)



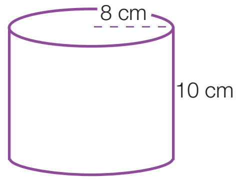
Ex 4.4

7 Find the volume of the following figures to the nearest whole number.

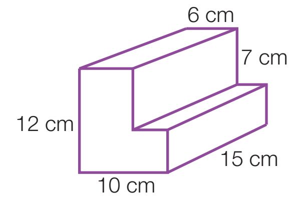
(a)



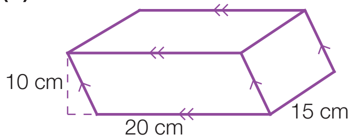
(b)



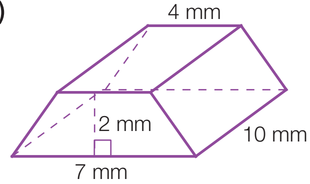
(c)



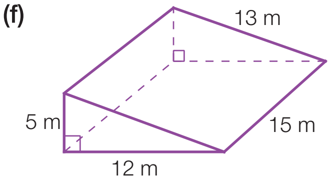
(d)



(e)



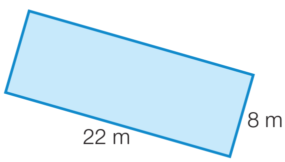
(f)



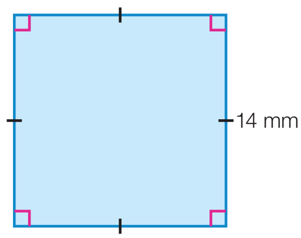
Understanding

8 Find the area of each figure below in the units indicated in brackets.

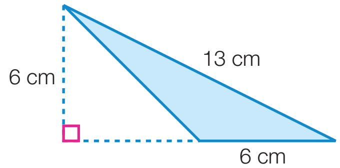
(a) (ha)



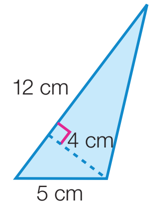
(b) (cm^2)



(c) (mm^2)

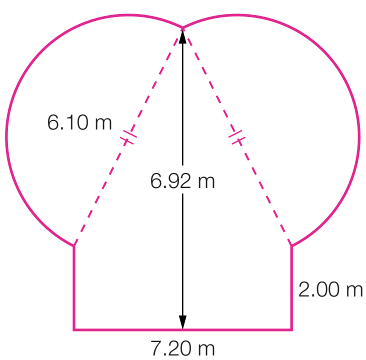


(d) (m^2)

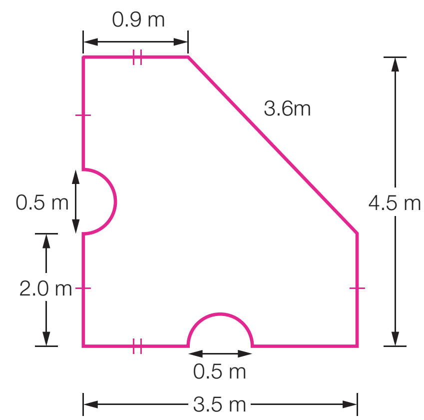


9 Find the area of the following shapes. (Give your answers correct to two decimal places.)

(a)



(b)

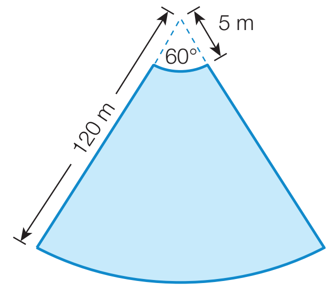


10 A farmer planted a crop in a field 300 m long and 500 m wide. One month later, seven 'crop circles', each with a diameter of 21 m, were found one morning, flattening the crop in the circle. What area of crop remained? (Give your answer to the nearest m^2.)

11 Find the capacity of a cylindrical soft drink can that is 5.5 cm wide and 13 cm high. (Give your answer in mL, correct to two decimal places.)

12 A cube has a volume of 512 cm^3. Find its surface area.

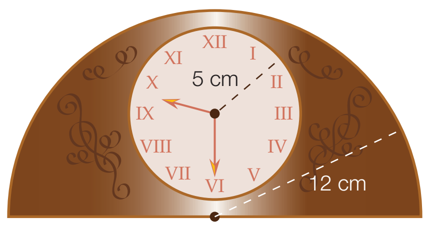
13 A snowball-throwing court has the measurements shown.



(a) Calculate the area that the thrower has to land the snowball in, to the nearest square metre.

(b) Find the perimeter of the shaded area, to the nearest metre.

14 An antique clock has a circular face of radius 5 cm on a semicircular background of radius 12 cm. The area around the clock face is decorated. Find the size of the decorated area to the nearest cm^2.



15 A cylinder is to be made from a piece of sheet metal that is 30 cm square. If the height and diameter of the cylinder are to be 8 cm each, what area of metal is wasted? Write your answer to the nearest whole cm^2.

Reasoning

16 **(a)** The inside surface of a rectangular swimming pool is to be tiled. If the pool is 12 m long, 5.5 m wide and 2.2 m deep, what is the tiled area?

(b) If the tiles are 12 cm long and 8 cm wide, how many will be needed (to the nearest 100)?

(c) How many litres will the pool hold when it is full?

17 A cylindrical paint tin has a radius of 11 cm and a capacity of 12.16 L.

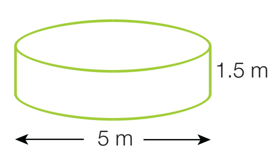
(a) What is the volume of paint in cm^3?

(b) What is the height of the tin to the nearest cm?

(c) If the paint just fills a rectangular paint tray that is 33 cm long and 22 cm wide, how deep is the tray? (Round your answer to two decimal places.)

18 A cylindrical container has a radius of 7 cm and a height of 10 cm. What must be the height of a rectangular container 7 cm long and 10 cm wide if the two containers are to have the same volume? (Answer to the nearest centimetre.)

19 **(a)** How many metric cups (250 mL) of water would it take to fill a water tank of diameter 5 m and depth 1.5 m?



(b) How long will the tank take to fill if a hose delivers 2 cups per second?

20 The hour hand of a clock is 4.5 cm long and the minute hand is 6 cm.

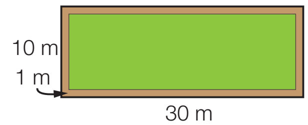
(a) To the nearest cm, how much further does the tip of the minute hand travel in the course of the day (24 hours) than the hour hand?

(b) What is the speed of each tip in centimetres per minute (cm/min)? Write your answer correct to two decimal places.

NAPLAN practice 4

Numeracy: Non-calculator

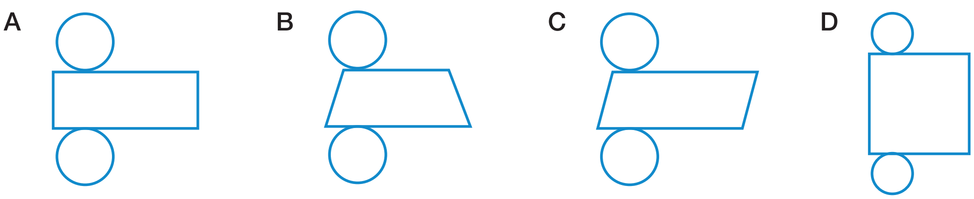
1 A 30 m × 10 m garden is bordered on the inside by a 1 m wide path. The area of the path in m^2 is:



A 39 **B** 72 **C** 76 **D** 80

2 A regular tetrahedron is a solid with four equilateral triangles as faces. Find the surface area of a regular tetrahedron where the base of the triangular face is 6 cm and the height is 5.2 cm.

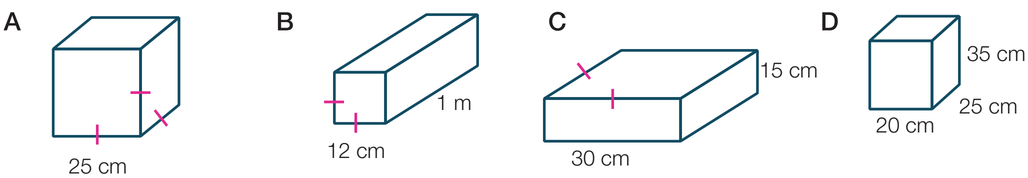
3 The net that could NOT form a cylinder is:



4 A cube has a surface area of 294 cm^2. How long is an edge of the cube (in cm)?

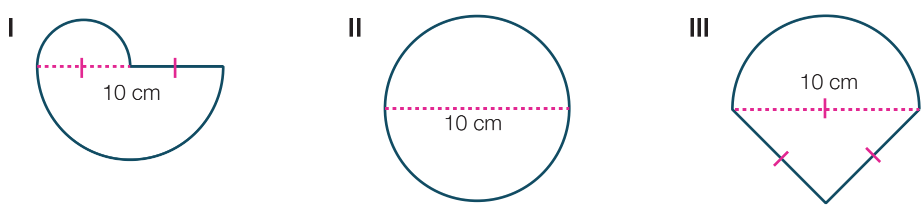
Numeracy: Calculator allowed

5 The shape with the biggest volume is:



6 The perimeters of the shapes below, in order from smallest to largest, are:

A I, II, III **B** II, III, I **C** I, III, II **D** III, II, I



7 One full turn of the wheels on this car moves the car forward 195 cm. What is the diameter of the wheels? (Answer to the nearest cm.)

8 A cylindrical can full of water has a diameter of 50 cm and a height of 50 cm. How many smaller cylinders with a diameter and height of 5 cm can it fill?

A 10 **B** 100 **C** 1000 **D** 10 000

Mixed review B

Fluency

1 Calculate: Ex.1.1

(a) 85% of 250 (b) 20% of \$0.40 (c) 90% of 80

2 Simplify: Ex.3.1

(a) $10g \times 4h \times 2gh^2$ (b) $-12k \times 3k^3$ (c) $9ef \times 6e^2 \times 2g$

3 Expand these expressions and simplify where possible: Ex.3.5

(a) $-8(x + 2)$ (b) $5y(6 - 11y)$ (c) $9(p + 7) + (3p - 5)$

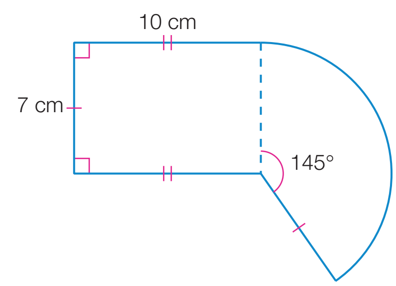
4 Factorise: Ex.3.7

(a) $4m - 28$ (b) $7ed^2 + 35d$ (c) $8xy^2 - 2y$

5 Find the percentage that: Ex.1.1

(a) 25 is of 175 (b) 90 is of 120 (c) 45 is of 360 (d) 100 is of 40

6 Find the area of the following composite shape. Ex.4.2
Write your answer to the nearest whole number.



7 Complete these conversions. Ex.4.2

(a) $20\text{ mm}^2 = \_\_\_\_\_\text{ cm}^2$ (b) $23.67\text{ km}^2 = \_\_\_\_\_\text{ m}^2$

(c) $0.007\text{ m}^2 = \_\_\_\_\_\text{ cm}^2$ (d) $14.5\text{ ha}^2 = \_\_\_\_\_\text{ m}^2$

8 Simplify the following expressions. Ex.3.1

(a) $x^3 \times x^5$ (b) $x^8 \div x^4$ (c) $\dfrac{x^3 \times x^6}{x^5}$

9 A casual worker earns \$9.70 per hour for a standard 8-hour day. He is paid time-and-a-half for the first two hours of overtime on any day and double time for any hours after that. How much will he earn if on one day he works: Ex.1.3

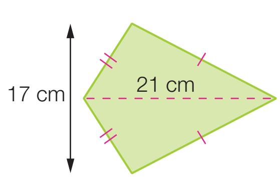
(a) 5 hours (b) 8 hours (c) 9 hours (d) 12 hours

10 Simplify each of the following: Ex.3.2

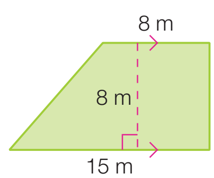
(a) z^0 (b) $5z^0$ (c) $(5z^0)$ (d) $3z^0 + 2z^0$

11 Find the shaded area of each of these shapes, correct to one decimal place where necessary. Ex.4.2

(a)



(b)



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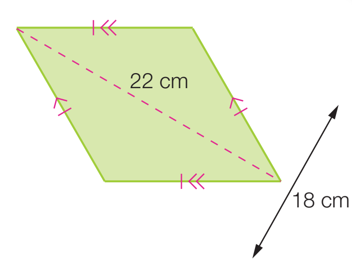

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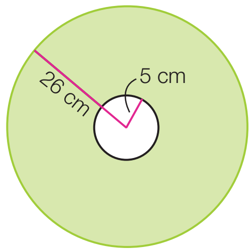
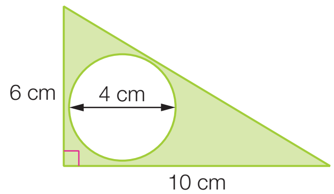
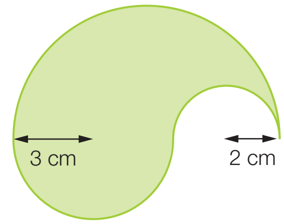


(e)



(f)



12 Use the following income tax table to find out how much income tax should be paid on the taxable incomes below. **Ex 1.4**

Taxable income	Tax on this income
\$0–\$6000	Nil
\$6001–\$37 000	15c for each \$1 over \$6000
\$37 001–\$80 000	\$4650 plus 30c for each \$1 over \$37 000
\$80 001–\$180 000	\$17 550 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 550 plus 45c for each \$1 over \$180 000

(a) \$6300 **(b)** \$18 600 **(c)** \$47 650 **(d)** \$82 700

13 Expand these expressions and simplify where possible. **Ex 3.5**

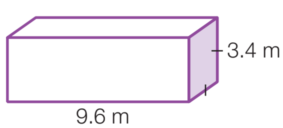
(a) $(x-3)(x+7)$ **(b)** $(x+6)^2$ **(c)** $(2x-5)^2$ **(d)** $(x+8)(x-8)$

14 Write the following in scientific notation. **Ex 3.3**

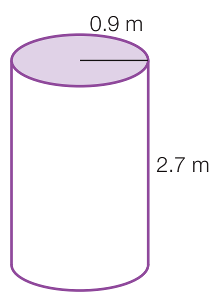
(a) 45 000 000 **(b)** 5 270 000 000 **(c)** 0.0003 **(d)** 0.000 000 298

15 Calculate the **(i)** surface area and **(ii)** volume of the following solids. State answers correct to two decimal places. **Ex 4.3, 4.4**

(a)



(b)



16 **(a)** Find the volume of a cylinder with a base diameter of 24 cm and a height of 16 cm, correct to one decimal place. **Ex 4.4**

(b) What is the capacity of this cylinder, correct to one decimal place?

17 Simplify: **Ex 3.1**

(a) $4gh^4 \times 3g^3$ **(b)** $12a^2b^6 \times 2ab^3 \times 5a^4b^2$ **(c)** $\frac{5x^5y^6}{15x^4y}$

18 After investing \$8000 in a savings account paying 6.8% interest p.a., Justine received \$8321.93. For how many days, correct to the nearest day, was her money invested? **Ex 1.5**

Understanding

19 A department store has a policy that a profit of 87% must be achieved on all furniture sold.

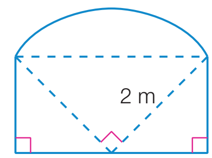
(a) What must a lounge suite, bought by the store for \$300, sell for? (Answer to the nearest \$10.)

(b) A dining table has a marked price of \$899. How much did the store purchase it for? (Answer to the nearest \$1.)

20 Simplify:

(a) $\dfrac{3m^5n^6 \times 4mn^2}{2m^2n^5 \times 3m^4n^3}$ **(b)** $\dfrac{5p^7q^4}{3p^3q} \div \dfrac{10p^5q^2}{9p^6q^6}$ **(c)** $\dfrac{5(x^3)^4y^6}{(x^2y)^2} \times \left(\dfrac{xy^3}{x^2y^2}\right)^3$

21 Find the area of the following shape.
Write your answer correct to one decimal place.



22 Kinetic energy is the energy (E) of a moving object, measured in joules (J). It is calculated by the formula $E = \dfrac{mv^2}{2}$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Rearrange the formula for kinetic energy to make mass the subject.

(b) Use your rearranged formula to find the mass of a bowling ball that has a kinetic energy of 32.4 J and a velocity of 3.5 m/s. Round your answer to one decimal place.

23 Mario is paid a retainer of $250 per week plus a commission of 9.5% on the value of sales. Calculate the income he would earn in one week if he sold goods to the value of:

(a) $1200 **(b)** $2500 **(c)** $3750 **(d)** $4210

24 **(a)** A cylindrical water tank has a capacity of 5000 L. If it has a height of 2.5 m find its base radius.

(b) If it was emptied at a rate of 225 litres per day, how many days would it take for the tank to be less than half full?

Reasoning

25 What happens to the total surface area of a rectangular prism if you multiply each dimension by two?

26 When employed as a sales assistant, Bettina was given a choice as to how she would be paid.

Option A: A retainer of $150 per week plus 5% of sales.

Option B: No retainer but 7.5% of sales.

(a) How much would Bettina need to sell to earn at least $950 per week under each of the options?

(b) What value of sales gives the same result regardless of the option chosen?

(c) When would it be better for Bettina to choose Option A?

27 Six circular discs of radius 5 cm fit snugly into the bottom of a rectangular box. Find the percentage of empty space at the bottom of the box, correct to two decimal places, if the discs are arranged in:

(a) one row of six **(b)** two rows of three.

(c) Which box is the more efficient container?

5

5

Linear relationships

Roads on the up and up. How steep can a road be before cars and trucks are no longer able to travel along it safely?

When civil engineers design roads they need to ensure the roads are both safe to travel on and cost-effective to build. The photograph of the Trollstigen (The Troll Ladder), a mountain road in Norway, shows that the civil engineers decided to make a long, winding road to overcome the steepness of the mountain, thereby allowing a wide variety of vehicles to use the road. However, this road would have cost a lot more money to build than a steep road that went straight up the slope. If civil engineers are designing a road for 4WD access only, the maximum gradient that the road can have is 1 : 5 or 20%. Conversely, if they are designing a major arterial road that will be used by all types of vehicles, the gradient must be less than 1 : 12 or 8%.

Forum

What effect would the road surface have on the maximum gradient?

What do you think a gradient of 1 : 5 means?

The safety of steep roads must be considered from both an 'uphill' and 'downhill' perspective. Why?

Why learn this?

Linear relationships are used by people in many different professions. Stockbrokers use linear relationships to model the performance of a particular share to decide when to buy or sell. Doctors use linear relationships to decide whether someone is a healthy weight based on their height (body mass index). Scientists use linear relationships to model the data collected from an experiment to hypothesise on possible relationships between two variables.

After completing this chapter you will be able to:

- solve linear equations algebraically
- solve problems using linear equations
- find the distance between two points
- find the midpoint of two points
- represent linear relationships as algebraic expressions and graphs
- determine the gradient or slope of linear relationships
- sketch linear equations.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 If $P = 2k + 5$, find P when:

(a) $k = 0$ **(b)** $k = 3$ **(c)** $k = -1$ **(d)** $k = -9$

If $C = 9 - 4d$, find C when:

(e) $d = 1$ **(f)** $d = -2$ **(g)** $d = 0$ **(h)** $d = 4$

2 Plot these coordinate points on a Cartesian plane and label them.

(a) $A(1, 2)$ **(b)** $B(3, 2)$ **(c)** $C(0, 3)$ **(d)** $D(0, -4)$
(e) $E(-2, 0)$ **(f)** $F(3, 0)$ **(g)** $G(-3, -1)$ **(h)** $H(-4, 2)$

3 Complete a table of values for x and y to plot each of the following graphs.

x	-2	-1	0	1	2
y					

(a) $y = x + 4$ **(b)** $y = x - 2$ **(c)** $y = 2x$

4 Solve the following equations to find the value of the variable.

(a) $x + 1 = 7$ **(b)** $3 + y = 18$ **(c)** $a + 4 = -6$
(d) $b - 5 = 16$ **(e)** $c - 12 = 0$ **(f)** $d - 6 = -20$

5 Solve the following equations to find the value of the variable.

(a) $2g = 38$ **(b)** $0.5x = 21$ **(c)** $-9y = -54$
(d) $\frac{x}{7} = 5$ **(e)** $\frac{b}{4} = -3$ **(f)** $\frac{k}{-9} = -4$

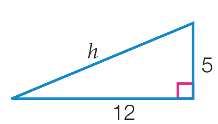
6 Expand the brackets and simplify.

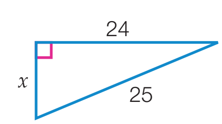
(a) $-3(5x + 2)$ **(b)** $2(7 - 3x) + 6$ **(c)** $3x + 1 + 6(2x + 7)$

7 Find the rule connecting x and y in the following table of values.

x	0	1	2	3	4	5
y	6	4	2	0	-2	-4

8 Use Pythagoras' Theorem to find the unknown sides in the following triangles.

(a)


(b)


Key Words

coordinates	gradient	inverse operation	origin	slope
dependent variable	gradient–intercept form	linear equation	parallel	solve
distance	independent variable	linear graph	perpendicular	straight line
equivalence	intercept	linear relationship	rise	undefined
general form	interval	midpoint	run	

Solving linear equations

5.1

Linear relationships are common in everyday life. A bath filling with water from a tap or the cost of electricity supplied to your home are examples of linear relationships.

A linear relationship between two variables exists if a uniform change in one variable, the **independent variable**, causes a uniform change in the other, the **dependent variable**.

A linear relationship produces a **linear graph** (**straight line**) and is represented by the equation $y = mx + b$. Graphing linear relationships will be covered later in this chapter and will require skills in solving linear relationships.

A **linear equation** is a linear relationship where the dependent variable has a specific value. We **solve** linear equations to find the value of the independent variable. A linear equation has only one solution.

There are many methods that can be used to solve an equation. Each of these methods uses the mathematical property of **equivalence**. Equivalence means that the left-hand side of the equation is equal to the right-hand side. This means you must perform the same operation on both sides to maintain the equality or balance of the equation.

In this section we will use **inverse operations** to deconstruct (undo) the equations in the opposite order on both sides of the equation to the way in which they were constructed. We need to use our knowledge of the order of operations that have been used to build our equations so that we can deconstruct the equations in the correct order. Sometimes it helps to reorder the equation to better identify the sequence of operations.

Worked Example 1

WE1

Solve the following linear equations. Check your solutions by substitution.

(a) $3x - 5 = -11$ **(b)** $7 - 2x = 19$

Thinking	Working
(a) 1 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we add 5 then divide by 3.)	(a) $3x - 5 = -11$ $3x - 5 + 5 = -11 + 5$ $3x = -6$ $\frac{3x}{3} = \frac{-6}{3}$ $x = -2$
2 Check your solution by substitution.	Check: LHS $= 3 \times -2 - 5$ $= -6 - 5$ $= -11$ $=$ RHS
(b) 1 Reorder the equation, taking care to keep the sign in front of each term correct.	(b) $7 - 2x = 19$ $-2x + 7 = 19$

2 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we subtract 7 then divide by -2.)	$-2x + 7 - 7 = 19 - 7$ $-2x = 12$ $\frac{-2x}{-2} = \frac{12}{-2}$ $x = -6$
3 Check your solution by substitution.	Check: LHS $= 7 - 2 \times -6$ $= 7 + 12$ $= 19$ $=$ RHS

Worked Example 2

WE2

Solve the following linear equations. Check your solutions by substitution.

(a) $\frac{2x}{5} - 1 = 7$ **(b)** $\frac{2x + 4}{3} = 4$

Thinking	Working
(a) 1 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we add 1, multiply by 5 and then divide by 2.)	(a) $\frac{2x}{5} - 1 = 7$ $\frac{2x}{5} - 1 + 1 = 7 + 1$ $\frac{2x}{5} = 8$ $\frac{2x}{5} \times 5 = 8 \times 5$ $2x = 40$ $\frac{2x}{2} = \frac{40}{2}$ $x = 20$
2 Check your solution by substitution.	Check: LHS $= \frac{2 \times 20}{5} - 1$ $= \frac{40}{5} - 1$ $= 8 - 1$ $= 7$ $=$ RHS
(b) 1 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we multiply by 3, subtract 4 and then divide by 2.)	(b) $\frac{2x + 4}{3} = 4$ $\frac{3 \times (2x + 4)}{3} = 4 \times 3$ $2x + 4 = 12$ $2x + 4 - 4 = 12 - 4$ $2x = 8$ $\frac{2x}{2} = \frac{8}{2}$ $x = 4$

2 Check your solution by substitution.	Check: LHS $= \dfrac{2 \times 4 + 4}{3}$ $= \dfrac{8 + 4}{3}$ $= \dfrac{12}{3}$ $= 4$ $=$ RHS

Worked Example 3

WE3

Solve the linear equation $5x - 2 = 3x + 8$. Check your solution by substitution.

Thinking	Working
1 Add or subtract terms containing the variable on both sides of the equation so that the variable term appears only on one side of the equation. (Subtract $3x$ from both sides in order to keep the coefficient of x positive.)	$5x - 2 = 3x + 8$ $5x - 3x - 2 = 3x - 3x + 8$ $2x - 2 = 8$
2 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we add 2 then divide by 2.)	$2x - 2 + 2 = 8 + 2$ $2x = 10$ $\dfrac{2x}{2} = \dfrac{10}{2}$ $x = 5$
3 Check your solution by substitution.	Check: LHS $= 5 \times 5 - 2$ $= 25 - 2$ $= 23$ RHS $= 3 \times 5 + 8$ $= 15 + 8$ $= 23$ LHS = RHS

Worked Example 4

WE4

Solve the linear equation $3(x + 4) = 2(1 - x)$. Check your solution by substitution.

Thinking	Working
1 Expand the brackets.	$3(x + 4) = 2(1 - x)$ $3x + 12 = 2 - 2x$
2 Add or subtract terms containing the variable on both sides of the equation so that the variable appears only on one side of the equation. (Add $2x$ to both sides in order to keep the coefficient of x positive.)	$3x + 2x + 12 = 2 - 2x + 2x$ $5x + 12 = 2$

3 Deconstruct the equation by performing the inverse operations in the opposite order on both sides of the equation. (Here we subtract 12 then divide by 5.)	$5x + 12 - 12 = 2 - 12$ $5x = -10$ $\frac{5x}{5} = \frac{-10}{5}$ $x = -2$
4 Check your solution by substitution.	Check: LHS $= 3(-2 + 4)$ $= 6$ RHS $= 2(1 - -2)$ $= 6$ LHS = RHS

Equations containing fractions on both sides

To solve an equation containing a fraction on both sides:

- Express each term in the equation as a fraction with the same denominator.
- Multiply both sides of the equation by that denominator.

Worked Example 5

WE 5

Solve the linear equation $\frac{2a-1}{5} = \frac{a+7}{4}$. Check your solution by substitution.

Thinking	Working
1 Express both sides of the equation with a common denominator. (Here the common denominator is 20, so $\frac{1}{5}$ is written as $\frac{4}{20}$ and $\frac{1}{4}$ is written as $\frac{5}{20}$.)	$\frac{2a-1}{5} = \frac{a+7}{4}$ $\frac{4(2a-1)}{20} = \frac{5(a+7)}{20}$
2 Eliminate the fractions by multiplying both sides by the common denominator.	$\frac{4(2a-1)}{20} \times 20 = \frac{5(a+7)}{20} \times 20$ $4(2a-1) = 5(a+7)$
3 Expand the brackets.	$8a - 4 = 5a + 35$
4 Add or subtract terms containing the variable on both sides of the equation so that the variable appears only on one side of the equation. (Subtract $5a$ from both sides in order to keep the coefficient of a positive.)	$8a - 5a - 4 = 5a - 5a + 35$ $3a - 4 = 35$
5 Deconstruct the equation by performing the inverse operations in the opposite order. (Here we add 4 then divide by 3.)	$3a - 4 + 4 = 35 + 4$ $3a = 39$ $\frac{3a}{3} = \frac{39}{3}$ $a = 13$

6 Check your solution by substitution.

$$\text{Check: LHS} = \frac{2 \times 13 - 1}{5} = \frac{26 - 1}{5} = \frac{25}{5} = 5$$

$$\text{RHS} = \frac{13 + 7}{4} = \frac{20}{4} = 5$$

$$\text{LHS} = \text{RHS}$$

Steps shown in pink may be discarded when the deconstruction process is fully understood.

To solve an equation:

- If fractions are involved, express each term with a common denominator and then eliminate the fractions by multiplying each term on both sides by the common denominator.
- If brackets are involved, expand the brackets.
- If there is more than one term containing the unknown, collect those terms together.
- Identify the operations used to construct the equation and use inverse operations to deconstruct the equation in the opposite order to the way it was constructed.

5.1 Solving linear equations

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q6 Column 1, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q13 Column 1, Q14, Q17 (a), Q18, Q20	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12 Column 2, Q13 Column 2, Q14, Q15, Q17 (a) & (b), Q18, Q20, Q21	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5 Column 2, Q6 Column 3, Q7, Q8, Q9, Q10, Q12 Column 3, Q13 Column 3, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22

Answers page 618

Fluency

1 Solve the following linear equations. Check your solutions by substitution.

WE1

(a) $2h + 9 = 5$ (b) $5b + 3 = 28$ (c) $6x + 7 = 1$

(d) $6r - 3 = 2$ (e) $3f - 2 = -14$ (f) $5k - 3 = -1$

(g) $12 = 7x + 6$ (h) $-5 = 2x + 13$ (i) $-2 = 9y - 2$

(j) $-3b + 14 = -4$ (k) $-2a - 4 = 0$ (l) $-5b - 6 = 9$

(m) $-7 - p = 8$ (n) $15 - 4e = -1$ (o) $9 - 4g = -23$

(p) $4 = 17 - 5y$ (q) $-3 = 8 - 4p$ (r) $-5 = 11 - 8y$

WE2

2 Solve these linear equations. Check your solutions by substitution.

(a) $\frac{2x}{3} + 9 = 13$ (b) $\frac{3x}{7} + 5 = 8$ (c) $\frac{5x}{3} - 8 = 7$

(d) $4 - \frac{2x}{3} = 10$ (e) $8 - \frac{3x}{4} = -1$ (f) $12 - \frac{3x}{2} = 3$

(g) $\frac{2x+3}{11} = 1$ (h) $\frac{3x+1}{5} = -1$ (i) $\frac{7x+4}{6} = 3$

(j) $\frac{1-7x}{4} = -5$ (k) $\frac{2-3x}{5} = 1$ (l) $\frac{9-4x}{3} = -5$

(m) $\frac{4x-6}{7} = -2$ (n) $\frac{7x-4}{5} = -12$ (o) $\frac{3x-1}{8} = -2$

WE3

3 Solve the following linear equations. Check your solutions by substitution.

(a) $7c + 2 = 6c - 5$ (b) $4b + 1 = 2b - 15$ (c) $9f + 5 = 5f - 3$

(d) $5d - 4 = 2d + 17$ (e) $5e - 3 = 2e + 9$ (f) $3a - 5 = a + 7$

(g) $3g + 12 = 5 + 2g$ (h) $2h - 7 = 13 + h$ (i) $4x - 3 = 6 + x$

(j) $k + 6 = 9 - k$ (k) $10j + 1 = 25 - 2j$ (l) $3y - 22 = -4 - 3y$

(m) $2m + 5 = 4m - 3$ (n) $5 + 4n = 7n - 1$ (o) $7x - 4 = 9x - 6$

(p) $2 - y = 3 - 2y$ (q) $19 - p = 16 - 2p$ (r) $6 + 3x = 21 - 2x$

(s) $15a - 17 = 16a - 19$ (t) $14 - 3x = 5 - 2x$ (u) $30 - p = 5p + 6$

WE4

4 Solve the following linear equations. Check your solutions by substitution.

(a) $3(x-2) = 2(x+1)$ (b) $7(x+4) = 3(x-4)$ (c) $4(x-5) = 5(3x+7)$

(d) $5(2x+3) = 9(x+1)$ (e) $5(2x+3) = 2(3x+1)$ (f) $4(3x-2) = 3(2x+9)$

(g) $2(4x+7) = 3(5-x)$ (h) $3(5x+4) = 4(2-3x)$ (i) $4(3x-11) = 5(29-3x)$

WE5

5 Solve the following linear equations. Check your solutions by substitution.

(a) $\frac{b+4}{3} = \frac{b-8}{5}$ (b) $\frac{a-3}{2} = \frac{a+6}{3}$

(c) $\frac{c+7}{4} = \frac{c+2}{2}$ (d) $\frac{d+5}{3} = \frac{d-1}{9}$

(e) $\frac{2x-3}{4} = \frac{x+6}{3}$ (f) $\frac{3y-2}{7} = \frac{y+6}{4}$

(g) $\frac{5g-1}{4} = \frac{7-2g}{5}$ (h) $\frac{7k-3}{4} = \frac{13-5k}{8}$

(i) $\frac{4-3m}{2} = \frac{1-5m}{3}$ (j) $\frac{2-3j}{5} = \frac{3-4j}{6}$

(k) $\frac{8x+3}{2} = 3x - 4$ (l) $\frac{6p+4}{11} = p + 9$

(m) $\frac{2(3x-5)}{3} = \frac{3(5x+1)}{4}$ (n) $\frac{4(2y+3)}{5} = \frac{5(2-y)}{2}$

6 Solve these equations. Check your solutions by substitution.

(a) $\frac{x}{2} - 3 = 0$ (b) $\frac{x}{8} - 6 = -5$ (c) $\frac{x}{9} + 3 = 3$

(d) $3 - \frac{x}{4} = 12$ (e) $5 - \frac{x}{7} = 11$ (f) $-4 - \frac{x}{3} = -8$

(g) $\frac{4x}{5} = 3$ (h) $\frac{7x}{13} = 0$ (i) $\frac{2x}{7} = -4$

(j) $\frac{x+3}{2} = 9$ (k) $\frac{x+7}{3} = 5$ (l) $\frac{x+5}{9} = 3$

(m) $\frac{x-6}{2} = 8$ (n) $\frac{x-9}{8} = 6$ (o) $\frac{x-4}{3} = 17$

7 (a) To obtain a from $2a + 7$ you would need to:

A add 7 then divide by 2
B subtract 7 then divide by 2
C divide by 2 then subtract 7
D multiply by 2 then add 7

(b) To obtain p from $5p - 4$ you would need to:

A add 4 then multiply by 5
B add 4 then divide by 5
C divide by 5 then add 4
D multiply by 5 then add 4

(c) To obtain x from $-6x - 11$ you would need to:

A add 11 then multiply by -6
B multiply by -6 then add 11
C add 11 then divide by -6
D subtract 11 then divide by -6

(d) To obtain n from $1 - 4n$ you would need to:

A add 4 then multiply by 1
B add 1 then divide by -4
C subtract 1 then divide by -4
D subtract 1 then multiply by -4

8 (a) The equation $2z + 1 = 11$ has the solution:

A $z = 4$ B $z = 5$ C $z = 6$ D $z = -5$

(b) The equation $4x - 5 = 3$ has the solution:

A $x = 3$ B $x = -1$ C $x = 2$ D $x = 0$

(c) The equation $7 - 3q = 22$ has the solution:

A $q = 4$ B $q = -3$ C $q = 5$ D $q = -5$

(d) The equation $5 - 8n = -11$ has the solution:

A $n = 2$ B $n = -3$ C $n = 0$ D $n = -2$

(e) The equation $\frac{r}{3} - 4 = 2$ has the solution:

A $r = 10$ B $r = -6$ C $r = 9$ D $r = 18$

(f) The equation $\frac{a+3}{4} = 6$ has the solution:

A $a = 5$ B $a = 7$ C $a = 21$ D $a = 27$

9 (a) The linear equation $\frac{2a}{5} - 3 = 1$ has the solution:

A $a = -10$ B $a = -5$ C $a = 4$ D $a = 10$

(b) The linear equation $\frac{4y}{3} + 7 = -1$ has the solution:

A $y = -9$ B $y = -6$ C $y = 6$ D $y = 9$

10 (a) $9x + 4 = 2x - 3$ has the solution:

A $x = 1$ B $x = -1$ C $x = 2$ D $x = 4$

(b) $2y + 7 = -8 - 3y$ has the solution:

A $y = 2$ B $y = 1$ C $y = -2$ D $y = -3$

(c) $4(3k-2)=5(k+4)$ has the solution:

A $k=3$ B $k=-2$ C $k=1$ D $k=4$

(d) $\frac{p-2}{2}=\frac{p+1}{3}$ has the solution:

A $p=8$ B $p=5$ C $p=-4$ D $p=0$

Understanding

11 Write an equation for each of the following and then solve the equation.

(a) 7 is added to x to give a result of 19.

(b) 5 is subtracted from y to give a result of 3.

(c) Three times a certain number is 14.

(d) One-quarter of a certain number is -5.

(e) Three more than two times a number is 13.

(f) 7 less than four times a number is -12.

12 Solve these equations. (First simplify each equation.)

(a) $3x+2x+7=32$ (b) $2x+x-4=-10$ (c) $6x+3x-4=14$

(d) $3(x+5)+8=-4$ (e) $2(x+3)+1=-5$ (f) $5(x-2)-3=7$

(g) $3(7-x)-6=0$ (h) $8(2-x)+1=9$ (i) $-4(3-x)+8=-12$

(j) $-2(1-4x)-3x=-2$ (k) $-7(5-2x)-6x=13$ (l) $-(8-5x)+4x=-26$

13 Solve these linear equations.

(a) $\frac{4(2x+3)}{7}=1$ (b) $\frac{3(4x+1)}{5}=-4$ (c) $\frac{5(3x+2)}{2}=1$

(d) $\frac{2(x-1)}{3}=5$ (e) $\frac{4(3x-2)}{5}=-1$ (f) $\frac{7(2x+1)}{2}=-2$

(g) $\frac{6(1-4x)}{15}=\frac{2}{3}$ (h) $\frac{3(2-5x)}{8}=-\frac{1}{2}$ (i) $\frac{3(7-2x)}{16}=\frac{1}{4}$

14 Subtracting 11 from four times a certain number gives the same result as subtracting two times the number from 7. Write an equation and solve it to find the unknown number.

15 A certain number when added to 36 gives the same result as adding 6 to this number and doubling the result. Write an equation and solve it to find the unknown number.

16 Half of the result of three times a certain number added to one is the same as one-fifth of the result when 4 is subtracted from this certain number. Write an equation and solve it to find the unknown number.

Reasoning

17 Consecutive numbers, such as 6, 7 and 8, can be represented algebraically by n, $n+1$ and $n+2$, because to get from one number to the next consecutive number we simply add 1. Similarly, three consecutive odd numbers can be represented algebraically by n, $n+2$ and $n+4$ where n is an odd number. Using this information, write each of the following as an equation and solve to find the unknown numbers.

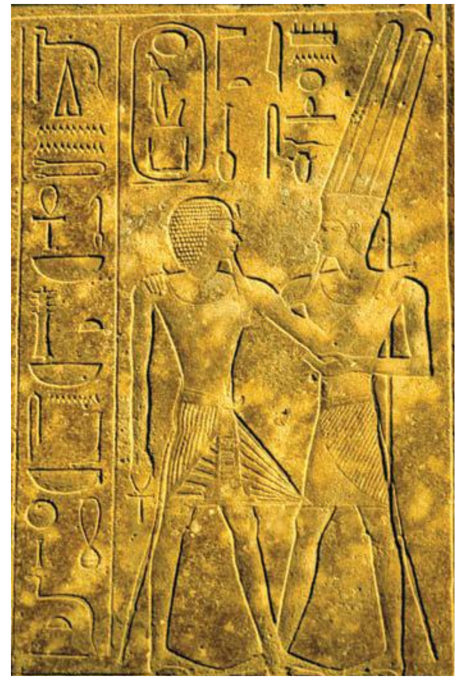
> **Hint**
> n does not have to be the first number. Three consecutive numbers could be $n-1$, n and $n+1$

(a) Find the three consecutive odd numbers that sum to 195.

(b) Find the four consecutive even numbers that sum to 204.

(c) Find the six consecutive multiples of seven that sum to 609.

18 The ancient Egyptians solved equations using a method of guessing and checking. We call this method the 'rule of false position'.



For example, consider the equation $x + \frac{x}{9} = 80$, which the ancient Egyptians would have said as 'a number added to one-ninth of itself is 80'.

First, choose a possible value for x that is divisible by 9; for example, 9. Substitute this into the equation.

$$9 + \frac{9}{9} = 9 + 1 = 10$$

Because the answer we want is eight times the answer we found, try a number eight times the value of the number we tried.

$$9 \times 8 = 72$$

$$72 + \frac{72}{9} = 72 + 8 = 80$$

So the solution to the equation 'a number added to one-ninth of itself is 80' is 72.

For the following questions, form an equation, then use the rule of false position to solve it.

(a) A number added to one-sixth of itself is 28.

(b) A number added to one-quarter of itself is 45.

(c) A number added to one-eighth of itself is 99.

Open-ended

19 On a photocopied test some numbers are not clear. One equation reads $2x + 7 =$ but the number on the right-hand side is unreadable. If we know that x is a whole negative number, give three examples of what the number on the right-hand side could be.

20 Use at least three mathematical operations to make up three different equations that have a solution of 4.

21 Make up three equations similar to those in Question **3**, where the solution is a positive whole number.

Outside the Square — Problem solving

Burning down

Two different sized candles are lit. Each candle burns at a different rate and one is 3 cm longer (or taller) than the other.

The longer candle was lit at 3.30 p.m. and the shorter one at 5 p.m.

At 7.30 pm they were both the same length.

The longer candle burned out at 9.30 p.m., whereas the shorter candle burned out at 9 p.m.

How long was each candle originally?

Strategy options

- Make a model.
- Draw a diagram.
- Guess, check and improve.
- Break into manageable parts.

Mathspace

A day out

You've got a fun day planned with your friends: movies, shopping, lunch and more. Will it turn out to be a good day or will you be too embarrassed to show your face at school tomorrow?

Equipment required: 2–4 brains, 1 die, a counter for each player

How to win: The aim of the game is to be the first to return home with some 'reputation' points left or to be the only one still left in the game.

START

You lose your wallet

$2x - 1 = \bigstar$

Solve for x and deduct this value from your points.

You tap your 'friend' on the shoulder; the person turns around and it's a stranger

$\dfrac{x}{\bigstar} + 4 = 6$

Solve for x and deduct this value from your points.

You wave at a friend

$\dfrac{3 - x}{\bigstar} = 1$

Solve for x. If your answer is *positive*, your friend smiles and waves. Take the *positive* path. If your answer is *zero*, your friend ignores you. Wait for the next turn and roll again. If your answer is *negative*, your friend frowns at you. Take the *negative* path.

+

−

Your friend takes an embarrassing photo of you and posts it online

$\dfrac{2(x + \bigstar)}{3} = 10$

Solve for x and deduct this value from your points.

You spill your milkshake onto yourself

$\dfrac{12 + \bigstar \times x}{7} = x$

Solve for x and deduct this value from your points.

You receive a text from home asking why a $500 phone bill has arrived

$3y - 2y + \bigstar = 9$

Solve for y and deduct this value from your points.

+

A phone beeps with a message received

$\dfrac{x + 5}{3} = \dfrac{9 - \bigstar}{3}$

Solve for x. If your answer is *positive* or *zero*, it's your phone. Take the *positive* path. If your answer is *negative*, it's someone else's phone. Take the *negative* path.

−

You lose your new mobile phone

$-36r + 2 + 3 \times \bigstar = -33r - 4$

Solve for r and deduct this value from your points.

How to play: *Every player begins with 50 'reputation' points; if you lose all your points you are out. Each player begins on START. Decide who will go first. You do not need to roll to move position; each player moves from event to event along the path.*

At each event, roll the die, substitute the value rolled for the star $\bigstar$ in the equation and deduct the result from your reputation points.

At certain points where the path branches, your result will determine in which direction you must next move. Follow the instructions in the 'event' box. If you calculate a decimal answer, round it to the nearest whole number. When you are asked to roll a die twice, a represents the number rolled first and b the number rolled second.

You go to the hairdresser and end up with an 80s 'mullet'

$\frac{5 \times \bigstar - z}{2} = \frac{7 \times \bigstar + 1}{4}$

Solve for z and deduct this value from your points.

You're told you have to let your pesky younger sibling join you for the rest of the day

$9 - 5x = 4 \times \bigstar - 7y + 9$

Solve for y and deduct this value from your points.

You pass the person you have a crush on in the shopping centre, but realise too late you have snot hanging from your nose

$3p + 2 = 5p - 5 \times \bigstar$

Solve for p and deduct this value from your points.

Your favourite shop has closed down

$2 - q = 2 \times \bigstar - 2q$

Solve for q and deduct this value from your points.

Your dad says you can't stay at your friend's house because you haven't cleaned your room

$3(f - 2) = 3 \times \bigstar + 3$

Solve for f and deduct this value from your points.

Your mum comes up and kisses you on the cheek in front of your friends

$15n + 1 = 17n - \bigstar$

Solve for n and deduct this value from your points.

You watch a movie. Did you enjoy it?

Roll the die twice and substitute the values into the expression $(a - b)^2$. (a = first roll, b = second roll). If the total is more than 10, the movie was awful. Move left. If the total is less than 10, the movie was great! Move right.

LEFT

RIGHT

All the toilets are 'out of order'

$\frac{5(x + 4)}{3} = \frac{14 \times \bigstar + 28}{3}$

Solve for x and deduct this value from your points.

Waiting for the train

Roll the die twice and substitute the values into the expression $2b - a^2$. (a = first roll, b = second roll)

If the total is positive, your train has arrived. Go home.

If the total is zero or negative, you'll need to continue to wait for the train and try again on your next move.

HOME

5.2 Solving problems using linear equations

Many problems can be solved by setting up and solving an equation.

To solve a problem with an equation:

- Determine what the unknown(s) you need to find are and represent each with a variable.
- Form an equation, using the key words in the statement of the problem.
- Solve the equation using inverse operations.
- Answer the question in words.
- Check that your answers make sense and satisfy all conditions of the problem.

Worked Example 6

WE6

Tim is twice Sally's age. How old are Tim and Sally now if, in 10 years time, the sum of their ages will be 89? Use s to represent Sally's age.

Thinking	Working
1 Represent one unknown with the prescribed variable.	Sally's present age $= s$
2 Identify the other unknown and decide what the relationship between the two unknown values is. Write the required expression.	Tim's present age $= 2s$
3 Decide what operation needs to be performed on each unknown value and write the required expression.	In 10 years time: Sally's age $= s + 10$ Tim's age $= 2s + 10$
4 Use the information in the question to write an equation.	$(s + 10) + (2s + 10) = 89$
5 Simplify the equation.	$3s + 20 = 89$
6 Use inverse operations in the opposite order to solve the equation.	$3s = 69$ $s = 23$
7 Use the solution to the equation to find the values of all variables.	$2s = 2 \times 23 = 46$
8 Answer the question in words.	Sally is 23 years old and Tim is 46 years old.
9 Check that your answers make sense and satisfy all conditions of the problem.	In 10 years time, they will be 33 and 56 years old, respectively, $33 + 56 = 89$. Our answers solve the problem.

5.2 Solving problems using linear equations

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q14, Q15	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15	Q1, Q2, Q3,Q4,Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Answers page 619

Fluency

1 Kate's father is three times her age. How old are Kate and her father now if, in 4 years time, the sum of their ages is 68? Use K to represent Kate's age:

WE6

2 Andrew has x coins in his pocket. He tells Lori that she has five fewer coins than he has. If they have 17 coins between them, how many coins do they each have?

3 Phil has \$$p$ in his piggy bank and Bill has \$$b$ in his. If Phil has three times the amount of money Bill has in his piggy bank and they save \$5 each per week for the next 10 weeks, the total amount of money in both piggy banks will be \$200. Which of the following equations represents this situation?

A $4p + 100 = 200$ B $4b + 100 = 200$

C $3b + 100 = 200$ D $3b + 1p + 100 = 150$

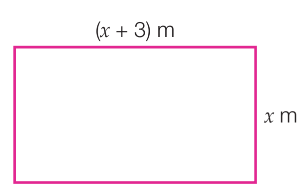
4 Choose the correct answer.

If $a = b + 2cx$, then:

A $x = \dfrac{a+b}{2c}$ B $x = \dfrac{a-b}{2c}$ C $x = \dfrac{a}{2c} - b$ D $x = a - \dfrac{2c}{b}$

Understanding

5 If the perimeter of this rectangular block of land is 46 m, find the length and width of the block.



6 Three students take different routes home from school. Mark needs to walk twice as far as Annette, whereas Sue needs to walk four times as far as Annette. If the three cover a total distance of 2.1 km (or 2100 m), how far does each need to travel to get home from school?

7 Ben bought five ice-cream cones and received \$1.75 change from his \$10 note. How much did each ice-cream cone cost?

8 Peter is 7 years older than Sharon and John is twice as old as Peter. Find how old Peter is if the mean (arithmetic average) of their ages is 19.

9 Joseph has 24 cousins. He knows that he has six more female cousins than male cousins. How many female cousins does he have?

10 In 3 years' time Joanne will be twice as old as her daughter. If Joanne is 45 years old now, how old is her daughter?

Reasoning

11 A triangle has angles of $(2x + 11)°$, $(3x - 2)°$ and $4x°$. Find each angle.

12 An isosceles triangle has a perimeter of 45 cm. Its equal sides each measure 6 cm longer than the base. How long is the base of the triangle?

> **Hint**
> Let one of the ages be represented by a variable and express the other ages in terms of this variable.

13 Gabby is twice as old as Kate and Kate is 3 years older than Joe. Brett is four times as old as Joe. If the sum of the ages of Kate and Brett is 63, find how old each person is.

14 A stock market analyst was investigating the trends of two independent stocks and was surprised to see that both stocks were at the same unit value after a 12-month period. He knew that both stocks had started at the same value but had undergone different trends over the past year. One stock had risen to three times its original unit value then lost \$8 a unit before finally finishing at $\frac{3}{4}$ of this reduced unit value. The other stock added \$3 a unit to its original unit value before crashing to $\frac{1}{5}$ of its improved unit value. It then showed a late improvement by increasing the unit value to seven times the reduced unit value. Express this information as an equation and solve it to find the original unit value of the stocks.

Open-ended

15 The sum of three consecutive whole numbers is a number between 70 and 100. Give three possible starting numbers for the group of three consecutive whole numbers.

Outside the Square — Problem solving

Fractions in fractions in fractions

The fraction $\frac{2008}{1998}$ may be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$ where a, b, c and d are all positive integers.

Can you find the value of d?

> **Hint**
> - Write the fraction $\frac{2008}{1998}$ as a mixed number. This should help you find the value of a.
> - $\frac{x}{y} = \cfrac{1}{\frac{y}{x}}$ may be useful.

> **Strategy options**
> - Break into manageable parts.
> - Solve a simpler problem.
> - Work backwards.

Coordinate geometry

5.3

Coordinate geometry uses the axes on a Cartesian plane to locate points, lines, curves and shapes and find relationships between them.

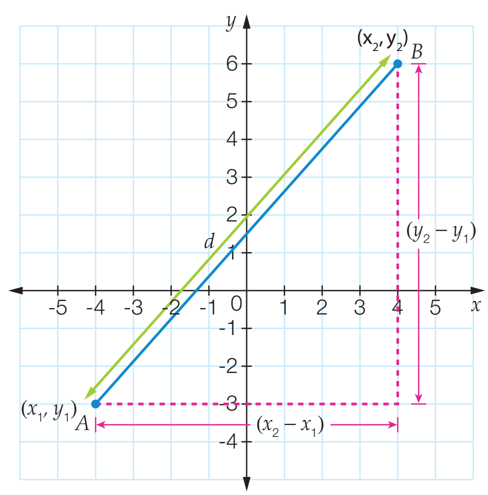
- The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.
- The point of intersection of these axes is called the **origin**, which has the coorrdinates (0, 0).
- All points can be described by an ordered pair known as **coordinates** (x, y) as measured from the origin.
- An **interval** is the line segment formed when two points are joined.

We use coordinate geometry to find relationships between points, such as the distance between two points, the midpoint of two points, the gradient and equation of lines and other curves, as well as the point of intersection of curves and lines. Coordinate geometry allows us to use algebraic and graphical techniques to solve geometric problems. In this section we will use coordinate geometry to explore linear relationships.

The distance formula

To find the distance between two points, we use the Cartesian coordinate system because the horizontal and vertical axes are at right angles to each other.

In the diagram below we have two points (x_1, y_1) and (x_2, y_2). To find the distance between these two points, we connect them with a straight line. If the two points are $A(x_1, y_1)$ and $B(x_2, y_2)$, the horizontal distance is given by $(x_2 - x_1)$, the difference in the x-coordinates and the vertical distance is given by $(y_2 - y_1)$, the difference in the y-coordinates.



- The difference between the x-coordinates $(x_2 - x_1)$ gives the length of one side of a right-angled triangle.
- The difference between the y-coordinates $(y_2 - y_1)$ gives the length of another side.
- These two lines form two of the sides of a right-angled triangle, with the distance between the two points, d, the length of the hypotenuse.
- We use Pythagoras' Theorem to find the distance between these two points.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Worked Example 7

WE7

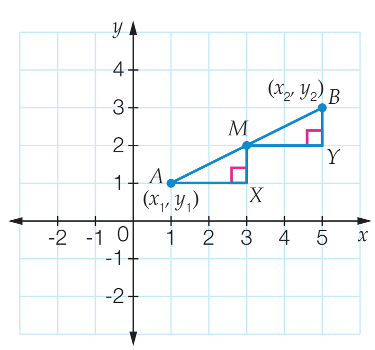
Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form.

(a) (-4, 6) and (2, -2) **(b)** (-2, 1) and (2, -4) **(c)** (3, 5) and (5, 2)

Thinking	Working
(a) 1 Identify the two points.	(a) Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -4$, $x_2 = 2$, $y_1 = 6$, $y_2 = -2$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -4)^2 + (-2 - 6)^2}$ $= \sqrt{6^2 + (-8)^2}$ $= \sqrt{36 + 64}$ $= \sqrt{100}$
3 Evaluate and include units.	$= 10$ units
(b) 1 Identify the two points.	(b) Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (2, -4)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -2$, $x_2 = 2$, $y_1 = 1$ and $y_2 = -4$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -2)^2 + (-4 - 1)^2}$ $= \sqrt{4^2 + (-5)^2}$
3 Evaluate, leaving the answer in exact surd form and including units.	$= \sqrt{41}$ units
(c) 1 Identify the two points.	(c) Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (5, 2)$
2 Write the distance formula and substitute in the relevant value. (Here $x_1 = 3$, $x_2 = 5$, $y_1 = 5$ and $y_2 = 1$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 3)^2 + (2 - 5)^2}$ $= \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{4 + 9}$
3 Evaluate by simplifying the surd value, leaving the answer in exact surd form and including units.	$= \sqrt{13}$ units

The midpoint formula

The midpoint of an interval is half-way between the two given coordinates that define that interval. The coordinates of the midpoint between two points on a Cartesian plane are found by using the average of each of the x and y coordinates. This formula can be derived using congruent triangle properties because ΔAXM and ΔMYB are congruent using SAS congruency.



The **midpoint**, $M(x, y)$, between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Worked Example 8

WE8

Find the coordinates of the midpoint between (-4, 6) and (2, -2).

Thinking	Working
1 Identify the two points.	Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$
2 Write the midpoint formula and substitute in the relevant values.	$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M(x, y) = \left(\frac{-4 + 2}{2}, \frac{6 + -2}{2}\right)$
3 Evaluate.	$M(x, y) = \left(\frac{-2}{2}, \frac{4}{2}\right)$ $M(x, y) = (-1, 2)$

The distance, d, between any two Cartesian coordinates (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The midpoint $M(x, y)$ between two points (x_1, y_1) and (x_2, y_2) is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

5.3 Coordinate geometry

Navigator

Q1 Column 1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q14	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q14	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Answers page 619

Fluency

1 Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form. WE7

(a) (2, 5) and (5, 9) (b) (-4, 2) and (8, -3) (c) (-4, 13) and (4, -2)

(d) (-6, 34) and (4, 10) (e) (-4, 5) and (4, -1) (f) (-11, -4) and (13, 3)

(g) (2, -5) and (3, -1) (h) (-1, 3) and (5, 2) (i) (4, -3) and (-2, 2)

(j) (-3, -5) and (-1, -7) (k) (3, -4) and (-1, 2) (l) (2, -7) and (6, 1)

2 Find the coordinates of the midpoints between the following Cartesian points. WE8

(a) (-3, 4) and (0, 0) (b) (-3, 5) and (1, 3) (c) (-9, -11) and (7, -3)

(d) (-1, 1) and (2, 3) (e) (-5, 8) and (3, 7) (f) (-2, -4) and (5, 3)

(g) (1, 2) and (6, 7) (h) (-2, -5) and (7, 0) (i) (0, 0) and (9, 11)

3 The distance between (-4, -5) and (4, 1) is:

A 4 **B** 5 **C** 10 **D** 9

4 A line *AB* is drawn from Point *A* at (-2, 10) to Point *B* (4, 1). Point *C* lies on the line *AB* half-way between Points *A* and *B*. Find the coordinates of Point *C*.

Understanding

5 **(a)** The midpoint of *CD* is (6, 5). If the coordinates of Point *C* are (3, 8), find the coordinates of Point *D*.

(b) The midpoint of *EF* is (-1, -2). If the coordinates of Point *E* are (-7, 6), find the coordinates of Point *F*.

(c) The midpoint of *AB* is (2, -4). If the coordinates of Point *A* are (-5, -8), what are the coordinates of Point *B*?

6 Use the distance formula and Pythagoras' Theorem to show that the points *A*(-3, -2), *B*(1, 1) and *C*(-2, 5) form an isosceles right-angled triangle.

7 A triangle has the vertices (0, 5), (3, 3) and (1, 0).

(a) Find the length of each side, correct to two decimal places.

(b) What type of triangle is it?

Reasoning

Hint

Plot the known points on a Cartesian plane.

8 The proposed course for a yacht race has been mapped out with buoys positioned at *A*(-3, -4), *B*(-1, 3) and *C*(9, 8), but the position of the last buoy *D*(*x*, *y*) is yet to be finalised. The race will begin at Position *O*(0, 0), proceed through buoys *A*, *B*, *C* and *D* and finish back at the starting point *O*.

(a) If the coordinates indicate distances measured in kilometres, what is the distance from the starting position *O* to the first buoy *A*?

(b) The judge's boat must be placed in the middle of buoys *A* and *C*. What are the coordinates of the judge's boat?

(c) If the judge's boat is also in the middle of buoys *B* and *D*, as with *A* and *C*, what are the coordinates (*x*, *y*) of buoy *D*?

(d) What is the greatest distance that the yachts must travel between any two successive buoys?

(e) What geometric shape is formed by the buoys *ABCD*?

9 A quadrilateral has the vertices *A*(0, -2), *B*(-2, -6), *C*(-6, -8) and *D*(-4, -4).

(a) Find the midpoints of the diagonals.

(b) Find the length of each side.

(c) Find the length of each diagonal.

(d) What type of quadrilateral is it?

10 Prove and explain how the points *A*(3, 5), *B*(5, 1), *C*(-1, -2) and *D*(-3, 2) are the vertices of a rectangle.

11 If *E*(1, 6), *F*(-4, 3) and *G*(-1, -2) are the vertices of a square, find the missing vertex.

12 Points *A*(-6, 3), *B*(2, 5), *C*(0, -1) and *D*(-2, 1) are joined to form a quadrilateral. The midpoints of *AB*, *BC*, *CD*, and *DA* are joined to form another quadrilateral *PQRS*. Show that *PQRS* is a parallelogram.

Open-ended

13 Choose three points that would form an equilateral triangle when joined in order and justify your choice.

14 Choose four points that would form a kite when joined in order and justify your choice.

Outside the Square Puzzle

Hitori

Copy the following grid of numbers.

Each individual number should never appear more than once in any row or column.

The aim is to shade out numbers that are not needed.

1	4	3	3	1	2
1	2	2	4	6	3
2	6	1	3	5	2
3	2	6	4	2	1
4	3	2	6	5	1
6	3	4	4	3	2

The cells that you shade cannot be beside each other vertically or horizontally (diagonally is OK).

3	4	2
6	2	3

This shading is correct.

3	4	2
6	2	3

This shading is incorrect.

The cells that are left unshaded must form a single area. That is, you must be able to move from any unshaded cell to the next unshaded cell horizontally or vertically.

5.4 Plotting linear graphs

Linear relationships can be plotted on a Cartesian plane using x- and y-axes and a suitable scale. We use the relationship to construct a table of values, plot these points on our axes and join them with a straight line.

Worked Example 9

WE 9

For each of the following table of values:

(i) plot a graph to represent the information

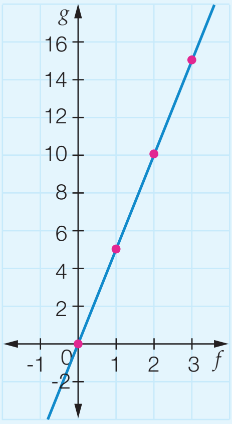
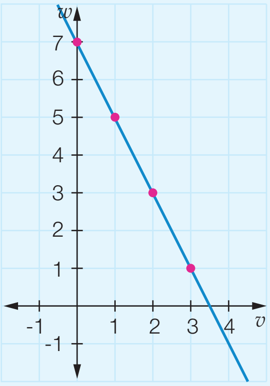
(ii) write an equation to represent the linear relationship.

(a)

f	0	1	2	3
g	0	5	10	15

(b)

v	0	1	2	3
w	7	5	3	1

Thinking	Working
(a) (i) Rule up a set of Cartesian axes, choose a suitable scale for each axis, then number and label them. Plot the points from the table and join them with a ruler to form a straight line.	(a) (i) 
(ii) Can the dependent variable be obtained from the independent variable with one operation (+, −, ×, ÷)? If the answer is yes, determine the operation and write the equation.	(ii) $g = 5f$
(b) (i) Rule up a set of Cartesian axes, choose a suitable scale for each axis, then number and label them. Plot the points from the table and join them with a ruler to form a straight line.	(b) (i) 

(ii) 1 Can the dependent variable be obtained from the independent variable with one operation (+, –, ×, ÷)? If the answer is no, find the increase in the dependent variable (w) as the independent variable (v) increases by 1. A decrease is a negative increase (-2). This is the coefficient of our independent variable in the equation.

(ii) -2v is in our equation

2 Decide what needs to be added or subtracted to obtain the equation.

7 needs to be added.

3 Write the equation.

$w = -2v + 7$

Worked Example 10

WE10

Use the equations given for each of the following to:

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $M = 9n$

n	-3	0	3	6
M				

(b) $L = 5k - 8$

k	-4	0	4	8
L				

Thinking

Working

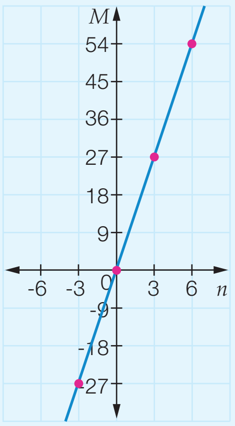
(a) (i) Use the rule to fill in the table.

(a) (i)

n	-3	0	3	6
M	-27	0	27	54

(ii) Draw and relabel the x- and y-axes as n and M. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line.

(ii)



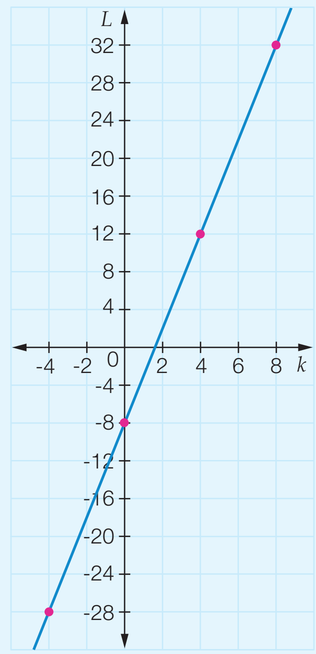
(b) **(i)** Use the rule to fill in the table.

(b) (i)

k	-4	0	4	8
L	-28	-8	12	32

(ii) Draw and relabel the x- and y-axes as k and L. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line.

(ii)



Worked Example 11

WE 11

Lisa, a travel agent working for an airline, receives a base wage of \$160 each week and a bonus of \$40 for each flight ticket sold.

No. of tickets sold in a week (n)	0	1	2	3	4	5
Weekly wage (W)						

(a) Copy and complete the table above.

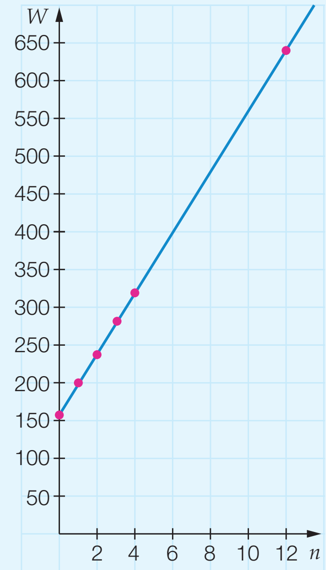
(b) Plot a graph to show the information contained in the table.

(c) Write an equation to represent the information.

(d) If Lisa sold 12 tickets one week, how much did she earn that week?

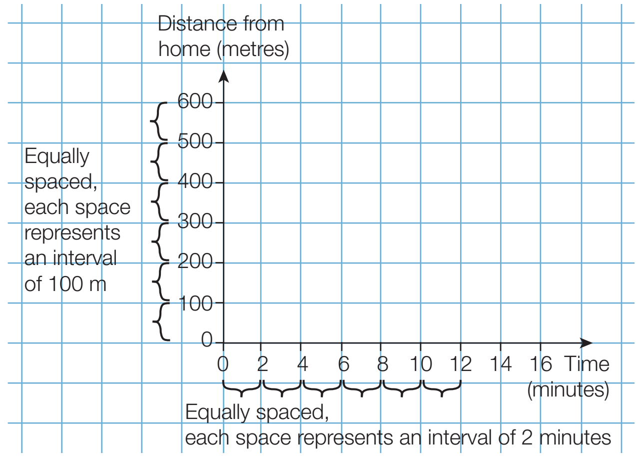
Thinking	Working
(a) Use the information to complete the table of values.	(a)

No. of tickets sold per week (n)	0	1	2	3	4	5
Weekly wage (W)	160	200	240	280	320	360

(b) Draw and relabel the x- and y-axes as n and W. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line. Note that this is a first quadrant graph only.	(b) 
(c) 1 By how much does the dependent variable (bonus) increase for each increase in the independent variable (no. of tickets sold)? (Here it is: \$40.) This is the coefficient of the independent variable in the equation.	(c) $40n$ is in our equation
2 What needs to be added or subtracted?	160 needs to be added to our equation
3 Write the equation.	$W = 40n + 160$
(d) 1 Substitute the given value into the equation and evaluate. Alternatively, read off the value from the graph using 12 as the x-coordinate, then moving vertically to meet the line, and then horizontally to the y-axis to find the y-coordinate.	$W = 40n + 160$ $W = 40 \times 12 + 160$ $= 480 + 160$ $= 640$
2 Write your answer.	Lisa would earn \$640.

Plotting graphs accurately

Use graph paper. Each axis should have a clearly marked scale and a label. The scale may or may not be the same for each axis. On an axis, the scale divisions should be equally spaced with intervals representing equal amounts. An example is shown opposite.



Linear relationships can be represented both graphically (using graphs) and algebraically (using equations).
A linear relationship is a relationship between two variables that, when graphed, produces a straight line.

5.4 Plotting linear graphs

Navigator

Answers page 619

Q1, Q2 Column 1, Q3, Q4, Q5, Q7, Q9, Q10	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: graph or grid paper

Fluency

WE9

1 For each of the following tables of values:

(i) plot a graph to represent the information

(ii) write an equation to represent the linear relationship.

(a)

a	0	1	2	3
b	0	2	4	6

(b)

c	0	3	6	9
d	5	2	-1	-4

WE10

2 Use the equation given for each of the following to:

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $A = 4L$

L	0	10	20	30
A				

(b) $Q = 5M$

M	0	2	4	6
Q				

(c) $J = 2G - 12$

G	0	4	8	12
J				

(d) $d = -6e + 5$

e	0	3	6	9
d				

(e) $B = \frac{3R}{5}$

R	-20	0	20	40
B				

(f) $C = \frac{5F}{6} - 4$

F	-6	0	6	12
C				

3 Eddy, a television salesperson, receives a base wage of \$80 each week and a bonus of \$100 for each television he sells. WE11

Number of televisions sold	0	1	2	3	4	5
Weekly wage (dollars)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table. Use the horizontal axis for the number of televisions sold and the vertical axis for weekly wage (dollars).

(c) Is the relationship linear?

(d) Write an equation to repesent the information.

(e) If Eddy sold eight televisions in one week, how much did he earn that week?

4 (a) In the linear relationship $V = 3W - 14$ when $W = 7$ the value of V will be:

A -14 B -8 C -2 D 7

(b) In the linear relationship $P = 5R$, P has a value of -30 when R is equal to:

A -10 B -6 C 6 D 10

(c) In the linear relationship $C = \frac{5}{9}F - 32$, the value of C when F is equal to 63 is:

A -32 B -3 C 3 D 18

Understanding

5 A no-name brand detergent can be bought wholesale for 95c a litre. To supply information to customers quickly, the wholesaler has prepared a graph representing this information.

(a) Copy and complete this table.

Number of litres	0	1	2	3	4	5	6	7
Cost (dollars)								

(b) Plot the cost of the detergent against the number of litres of detergent bought. This is the wholesaler's graph. Use the horizontal axis for the number of litres and the vertical axis for the cost (dollars).

(c) Is the relationship linear? How can you tell?

(d) From your graph, how much would it cost to buy:

(i) 3 L of detergent

(ii) 2.4 L of detergent

(iii) 5.7 L of detergent?

(e) From your graph, how many litres of detergent (to the nearest tenth of a litre) could be bought for:

(i) \$3.80 (ii) \$2.50 (iii) \$5.20?

(f) Write down a relationship connecting the cost in dollars, C, to the number of litres of detergent, d, bought.

6 Steve is cycling at a constant speed of 185 metres per minute.

(a) Copy and complete the following table.

Time (minutes)	0	1	2	3	4	5	6	7
Distance cycled (metres)								

(b) Plot the distance against the time. Use the horizontal axis for time (minutes) and the vertical axis for distance cycled (metres).

(c) From the graph, how far does Steve cycle in 2.7 minutes?

(d) How long does it take Steve to cycle 700 metres?

(e) Extend your graph to find how long it takes Steve to cycle 2 km.

(f) Write down the rule (or equation) connecting the distance cycled to the time taken.

(g) If Steve continued cycling, what distance would he have covered in 40 minutes?

Reasoning

7 Phuong works at the stock exchange buying and selling shares for different international and local companies on behalf of her client. She needs to be able to convert quickly from one currency to another, and uses either a calculator or a linear graph to obtain her information.

Hint
What is $A1.00 equivalent to in $US?

(a) If $20 in Australian currency converts to $21 in American (or US) currency, draw up a graph showing the relationship between the two currencies. Plot Australian currency on the horizontal axis and American currency on the vertical axis.

Use the graph to convert from one currency to the other for each of these amounts (round off to the nearest 10c).

(b) $A7.00 to $US

(c) $A17.60 to $US

(d) $US5.50 to $A

(e) $US9.60 to $A

8 Wally, Sarah and Cleo are collecting money for an upcoming charity walk. Wally is asking for $1 for each kilometre he walks. Sarah, on the other hand, is asking for a $4 donation plus $0.50 per kilometre walked. Finally, Cleo is asking for a donation of $8 regardless of how many kilometres she walks.

(a) Draw up and complete a table of values to show the amount donated for each kilometre walked for Wally, Sarah and Cleo for distances from 0 to 10 kilometres.

(b) Use your table of values to plot graphs showing the amount donated for each kilometre walked for Wally, Sarah and Cleo. You should plot each graph on the same set of axes. Plot distance on the horizontal axis and the amount donated on the vertical axis.

(c) From the graph and table of values, identify the distance at which all three collect the same amount of money.

(d) Identify the distances for which:

(i) Wally raises the most money

(ii) Sarah raises the most money

(iii) Cleo raises the most money.

9 Frank and Bruno are two emerging football players about to sign contracts with FC Mathematica. In the first year, Frank earns \$120 000, which will increase by \$60 000 for each successive year that he stays with the club. Bruno earns \$80 000 in his first year, which increases by \$80 000 for each successive year that he stays with the club. Explain in which years Frank will earn more than Bruno and in which years Bruno will earn more than Frank, assuming each stays with the club.

Open-ended

10 Draw three different linear relationships that show the same amount of distance travelled over different periods of time. Explain what happens to the graph as the time increases.

Outside the Square — Problem solving

How many edges?

Equipment required: 1–2 brains, scissors, ruler and several pieces of A4 paper

Instructions: Divide an A4 rectangular sheet of paper into two pieces using a single straight cut. Then divide one of those pieces into two, also using a single straight cut. What is the maximum number of edges that the resulting three pieces can have in total? What would the minimum number of edges of the three pieces be?

Strategy options

- Act it out.
- Look for a pattern.
- Test all combinations.

Technology Exploration GeoGebra

Equipment required: 1 brain, 1 computer with GeoGebra

Versions of this Exploration for other technologies are available in Pearson Reader.

Investigating straight lines

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen.

1 Click on the Options menu. Select 'Point Capturing' and 'On (Grid)'.

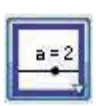
2 Click on the View menu and select 'Grid'.

3 If a larger font is required, click on the Options menu and select 'Font Size'. Choose an appropriate size from the list provided.

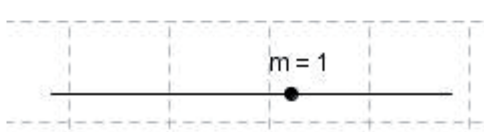
Creating a line using the equation $y = mx + b$

All straight lines can be represented by the equation $y = mx + b$.

We will now create a line and explore how the graph changes when we change the values of m and b.

4 Select the 'Slider' tool 

. Click on the right-hand side of the 'Graphics View', the space where the drawings appear. A pop-up box will appear. Under 'Name', type m. Change 'Increment' to 1. Click on the 'Slider' tab and change 'Width' to 200. Click 'Apply'. A line like this will appear on the screen:



5 To construct a second slider, repeat Step **3**, naming the second slider b.

6 In the 'Input' bar at the bottom of the screen, type *y=m*x+b* (the * is needed to tell the program to multiply the values of m and x). (Alternatively, you can leave a space between m and x.) Press Enter and a straight line should appear on the screen.

You will also notice that an equation will appear in the 'Algebra View' (on the left-hand side of the screen).

7 Click on the 'Select' tool . Drag the point on slider b to the right (increasing the value of b). Notice that as the line changes, the equation also changes (look under Dependent Objects).

8 Can you predict what will happen to the line if you drag the slider to the left (decreasing the value of b)? Try it.

9 Now move the point on slider m to the right (increasing the value of m). Take note of how the line changes on the screen as well as how the equation changes in the 'Algebra View' (look under Dependent Objects).

10 Can you predict what will happen to the line if you drag the slider to the left (decreasing the value of m)? Try it.

11 Try different combinations of m and b and compare them to the equation in the 'Algebra View'.

As you can see, the values of m and b affect the graph in different ways.

12 Use the following words to assist you to complete these sentences in your work book.

Steepness point y-axis gradient y-intercept y-coordinate

(a) Changing m changes the __________ or __________ of the line.

(b) Changing __________ changes the __________. This is the __________ of the __________ on the __________.

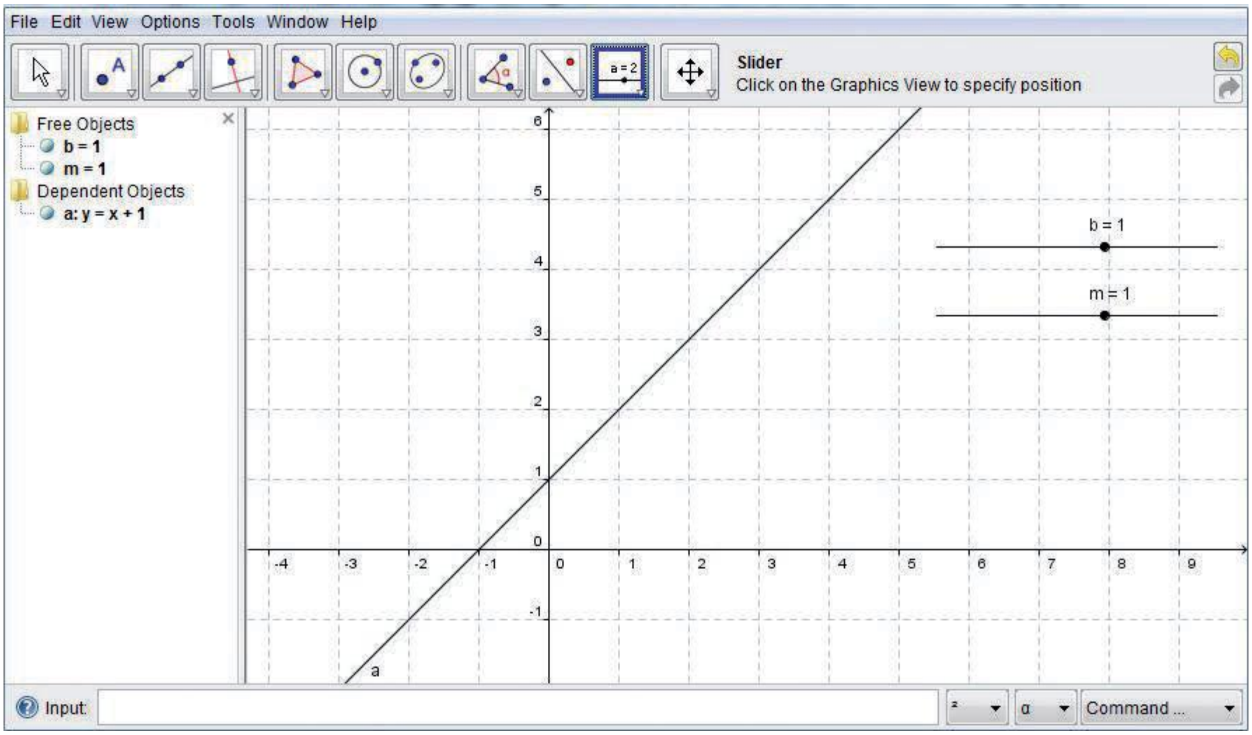
13 **(a)** What restrictions do we place on the value of b for the line to pass through the origin?

(b) What restrictions do we place on the value of m if we want the line to slope up to the right?

(c) What restrictions do we place on the value of m if we want the line to slope down to the right?

(d) What restrictions do we place on the value of m if the line is horizontal?

Technology Exploration GeoGebra

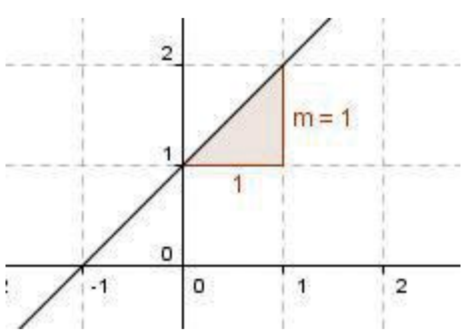


Investigating the gradient

The gradient, or slope, of a line can be described as the $\frac{\text{rise}}{\text{run}}$ (or the rise divided by the run).

14 Set sliders m and b equal to 1.

15 Click on the arrow at the bottom of the eighth tool from the left and select the 'Slope' tool . Click on the line. A right-angled triangle will appear like this, showing the rise and the run. It will confirm the value of m shown on the slider.



16 Click on the 'Select' tool . Change the value of the y-intercept by moving the point on the b slider. Does this affect the gradient of the line?

17 Now change the value of the gradient by moving the point on slider m and answer the questions below.

For different values of the gradient (m):

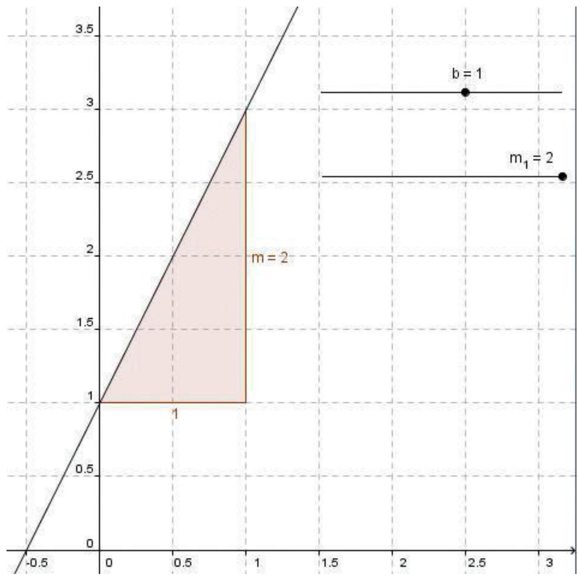
(a) If you divide the height of the triangle (rise) by the width of the triangle (run), what value do you get?

(b) What happens to the 'height' of the triangle when the gradient is negative?

(c) What happens to the height of the triangle when the gradient is zero?

18 We will now look at decimal (or fractional) values of the gradient. Right click on slider m (it is easiest if you right click on the point) and select 'Object Properties'. On the 'Slider' tab, change 'min' to -2, 'max' to 2 and 'increment' to 0.5. Click 'Close'.

19 Click on the arrow on the last tool and select the 'Zoom In' tool. Place your cursor over the y-intercept and click several times until you have axes that show increments of 0.5 (if you go too far you will have to use the 'Zoom Out' tool). Alternatively, move the cursor close to the y-intercept and use the scroll wheel on the mouse to zoom in and out (much easier).



20 Change the value of the gradient by moving the point on slider m.

21 If you divide the height of the triangle (rise) by the width of the triangle (run), what value do you get? (Hint: count grid squares rather than using actual lengths.)

22 Use the following words from the list to complete these statements.

less greater steep gradual

(a) A tall triangle is formed if the rise is ____________ than the run. This gives a line with a ____________ slope.

(b) A flatter triangle is formed if the rise is ____________ than the run. This gives a line with a ____________ slope.

Half-time 5

1 Decide which, if any, of the following tables represent linear relationships. Ex. 5.4

(a)

x	-1	0	1	2
y	-5	-3	-1	1

(b)

x	-2	0	2	4
y	5	2	-1	-3

(c)

x	0	1	2	3
y	0	1	1	2

(d)

x	-4	0	4	8
y	0	1	2	3

2 Find the midpoint of the line joining (6, 2) and (3, 8). Ex. 5.3

3 Solve the following linear equations. Ex. 5.1

(a) $2x - 5 = 13$ **(b)** $8 - 3y = -7$ **(c)** $\frac{3x}{4} - 2 = 10$

(d) $\frac{4x + 1}{5} = -7$ **(e)** $3x - 2 = 5x - 8$ **(f)** $5(x - 1) = 2(8 - x)$

(g) $4(3p - 5) = 5(7 - 2p)$ **(h)** $\frac{a + 11}{3} = \frac{3 - a}{4}$ **(i)** $\frac{3m - 5}{2} = \frac{2m + 25}{3}$

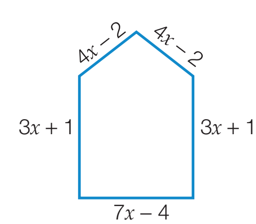
4 Find the distance between the points (-1, 7) and (4, -5). Ex. 5.3

5 Kelvin and Rob are training to be weightlifters. Rob weighs 6 kg more than Kelvin. Kelvin can lift one and a half times his weight less 6 kg. Rob can lift a quarter more than his weight. They found they can both lift the same weight. How much do Kelvin and Rob each weigh? Ex. 5.2

6 The equation $3x - 4 = 2(x - 5)$ has the solution: Ex. 5.1

A $x = -2$ **B** $x = -1$ **C** $x = -6$ **D** $x = 6$

7 The total perimeter of this shape is 36 m. Find x. Ex. 5.2



8 Use the equation $P = 3 - 4R$ to: Ex. 5.4

(a) copy and complete the following table of values

R	0	2	4	6
P				

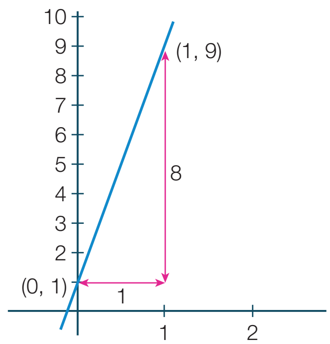
(b) plot the graph of the relationship.

5.5 Gradient

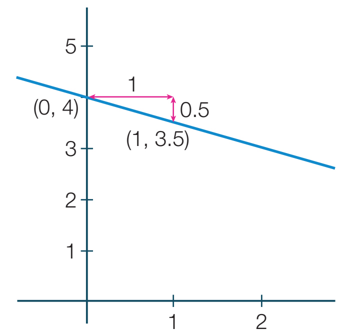
The measure of a line's steepness is called the **gradient** or **slope** of the line. The gradient of a line describes the change in the y values as the x value increases by 1.

For example:

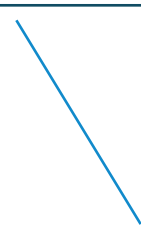
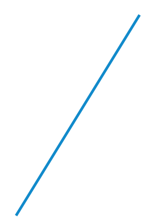
- a gradient of 8 means that as the x value increases by 1, the y value increases by 8.



- a gradient of $-\frac{1}{2}$ means that as the x value increases by 1, the y value decreases by $\frac{1}{2}$.

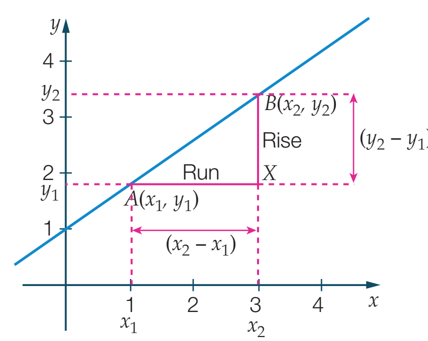


The size, or magnitude, of the gradient tells us how steep the line is, whereas the sign of the gradient (positive or negative) tells us whether the line slopes up or down as we move from left to right.

Line				
Magnitude of gradient	Small	Large	Large	Small
Sign of gradient	Negative	Negative	Positive	Positive

The gradient of a line can be found by considering the ratio of the vertical **rise** to the horizontal **run** between any two points on the line.

To find the run, start at Point A and move right horizontally (parallel to the x-axis) until you reach Point X, which is in line vertically with Point B. This change in x-value is the run. To find the rise, start at Point X and move straight up or down parallel to the y-axis until you reach Point B. This change in the y-value is called the rise.



$$\text{Gradient } AB = \frac{\text{vertical rise}}{\text{horizontal run}}$$

We use the letter m to represent the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

To keep the denominator always positive, we choose (x_1, y_1) to be the point on the left and (x_2, y_2) to be the point on the right. If (x_2, y_2) is lower than the first, you move right first then down. This means that the rise will be a negative number, which means the gradient of the line will also be negative.

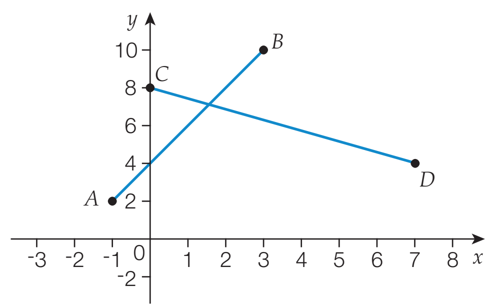
Worked Example 12

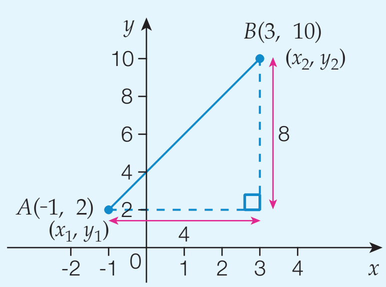
WE12

Find the gradient of:

(a) the line AB

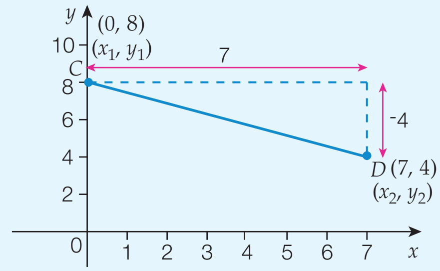
(b) the line CD.



Thinking	Working
(a) **1** Use the graph to identify the coordinates of (x_1, y_1) (Here $A = (-1, 2)$) and (x_2, y_2) (Here $B = (3, 10)$).	(a) 
2 Write out the rule for finding the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
3 Substitute values into the rule and evaluate (x_1, = -1, $x_2 = 3$, $y_1 = 2$ and $y_2 = 10$).	$= \frac{10 - 2}{3 - -1}$ $= \frac{8}{4}$ $= 2$

(b) 1 Use the graph to identify the coordinates of (x_1, y_1) (Here $C = (0, 8)$) and (x_2, y_2) (Here $D = (7, 4)$).

(b)



2 Write out the rule for the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

3 Substitute values into the rule and evaluate ($x_1, = 0, x_2 = 7, y_1 = 8$ and $y_2 = 4$).

$$= \frac{4 - 8}{7 - 0}$$
$$= \frac{-4}{7}$$
$$= -\frac{4}{7}$$

Special cases for the gradient

There are two special cases for the gradient that we will now consider. We will demonstrate how we use the rule to find the gradient of these special cases.

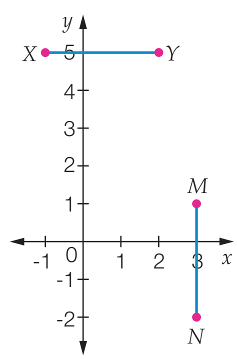
Worked Example 13

WE13

Find the gradients of:

(a) the line segment *XY*

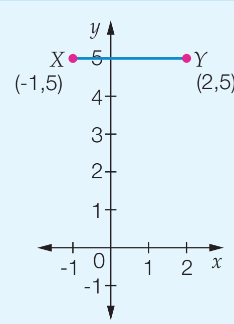
(b) the line segment *MN*.

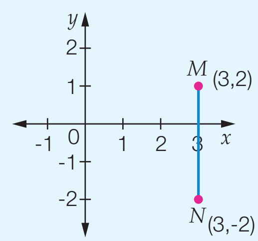


Thinking	Working

(a) 1 Use the graph to identify the coordinates of (x_1, y_1) (Here $X = (-1, 5)$) and (x_2, y_2) (Here $Y = (2, 5)$).

(a)



	2 Write the rule to find the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
	3 Substitute values into the rule and evaluate ($x_1 = -1$, $x_2 = 2$, $y_1 = 5$ and $y_2 = 5$).	$= \frac{5-5}{2-1}$ $= \frac{0}{1}$ $= 0$ The gradient is equal to zero.
(b)	**1** Use the graph to identify the coordinates of (x_1, y_1) (Here $M = (3, 1)$) and (x_2, y_2) (Here $N = (3, -2)$).	(b) 
	2 Write the rule to find the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
	3 Substitute values into the rule and evaluate ($x_1 = 3$, $x_2 = 3$, $y_1 = 1$ and $y_2 = -2$).	$= \frac{-2-1}{3-3}$ $= \frac{-3}{0}$ This cannot be evaluated, so the gradient is undefined.

From Worked Example **13 (a)**, we can see that a horizontal line has a gradient of zero. In this Worked Example, we used the rule for finding the gradient and the points (1, 5) and (2, 5). If we used any two points that had the same y-coordinate, the line segment would be horizontal, the numerator in the gradient rule would always be 0 and the gradient would be zero.

From Worked Example **13 (b)**, we can see that a vertical line has an undefined gradient. In this Worked Example, we used the rule for finding the gradient and the points (3, 2) and (3, -1). The division cannot be evaluated, so we say that it is undefined. If we used any two points that had the same x-coordinate, the line segment would be vertical, the denominator in the gradient rule would always be zero and the gradient would be **undefined**.

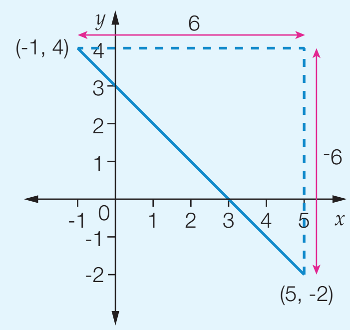
The gradient of any horizontal line is zero.

The gradient of any vertical line is undefined.

Worked Example 14

WE 14

Find the gradient of the line joining (5, -2) and (-1, 4).

Thinking	Working
1 Draw a graph showing the line joining the points. Identify (x_1, y_1) and (x_2, y_2). (Here, $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (5, -2)$.)	
2 Write out the rule for finding the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
3 Identify the values for x_1, y_1, x_2 and y_2 and substitute them into the rule and evaluate ($x_1 = 5$, $y_1 = -2$, $x_2 = -1$ and $y_2 = 4$).	$m = \frac{-2 - 4}{5 - -1}$ $= \frac{-6}{6}$ $= -1$

5.5 Gradient

Navigator

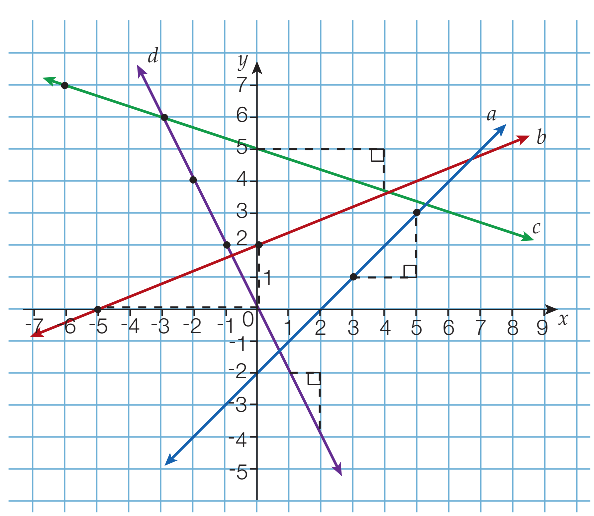
Answers page 621

Q1, Q2, Q3 Column 1, Q4, Q5, Q6, Q7, Q10, Q11, Q15	Q1,Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q15	Q1,Q2, Q3 Column 3, Q4, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15

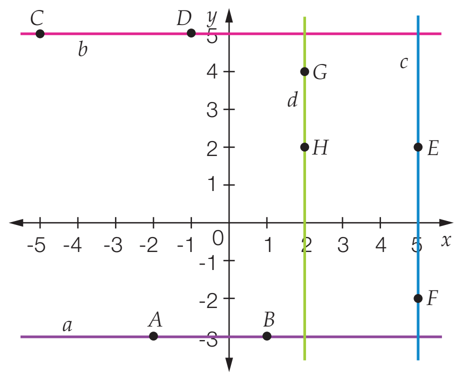
Fluency

WE 12

1 Find the gradient of each of the lines *a*–*d* using the points and triangles indicated.



2 Find the gradient of each of the lines *a*–*d* using the points on the lines as indicated. WE13



3 Find the gradient of the line joining each of the following pairs of points. WE14

(a) (2, 3) and (4, 7) **(b)** (1, 5) and (3, 11) **(c)** (5, 8) and (6, 4)

(d) (7, 0) and (-1, 8) **(e)** (0, -2) and (3, 4) **(f)** (1, -4) and (3, -2)

(g) (-7, 3) and (5, -4) **(h)** (-6, 2) and (-2, -2) **(i)** (-1, -3) and (-4, -2)

(j) (2, -1) and (-7, -1) **(k)** (4, 2) and (-3, 2) **(l)** (-3, -5) and (1, -5)

(m) (-1, -5) and (-1, 5) **(n)** (3, -4) and (3, 8) **(o)** (-7, 2) and (-7, 5)

4 Use your answers to Questions **1** and **2** to answer the following.

(a) If a line is sloping up to the right, does it have a positive or a negative gradient?

(b) If a line is sloping down to the right, does it have a positive or a negative gradient?

(c) What is the gradient of a horizontal line?

(d) What is the gradient of a vertical line?

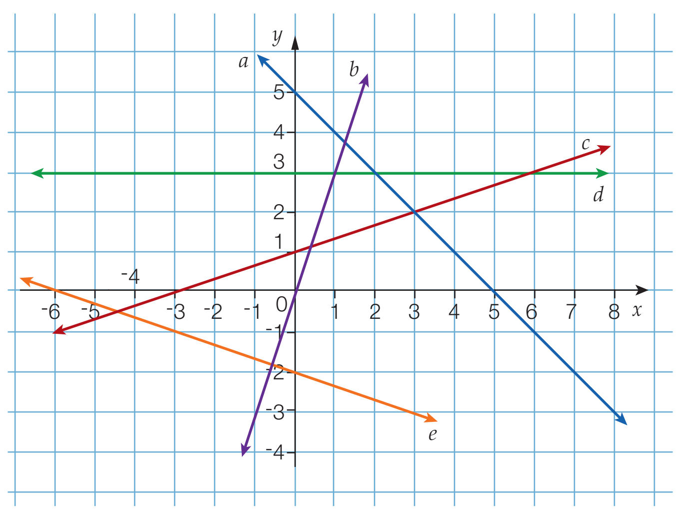
5 For each line *a*–*e* decide whether the gradient is:

A positive

B negative

C zero

D undefined



6 **(a)** Find the gradient of each of the lines in Question **5**.

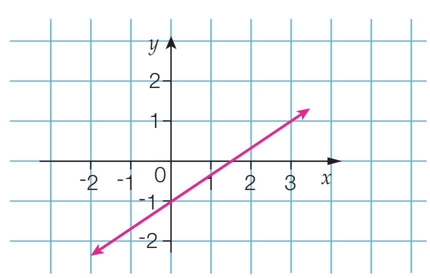
(b) Which line has the steepest positive gradient?

(c) Which line has the steepest negative gradient?

7 **(a)** The gradient of this line is:

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$

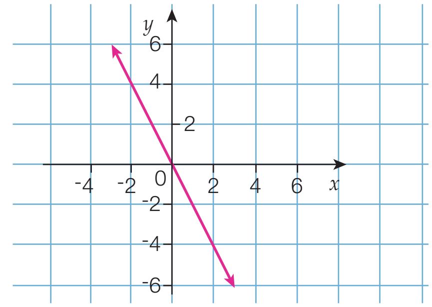
C $\frac{2}{3}$ **D** $\frac{3}{2}$



(b) The gradient of this line is:

A -2   **B** $-\frac{1}{2}$

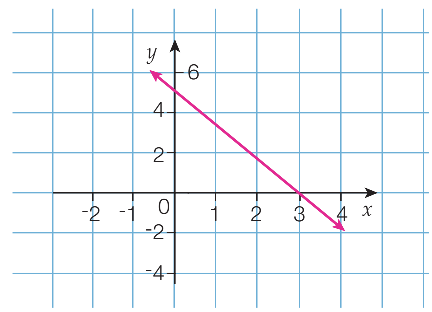
C $\frac{1}{2}$ **D** 2



(c) The gradient of this line is:

A $-\frac{5}{3}$ **B** $-\frac{3}{5}$

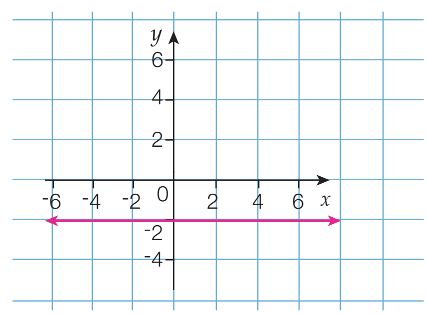
C $\frac{3}{5}$ **D** $\frac{5}{3}$



(d) The gradient of this line is:

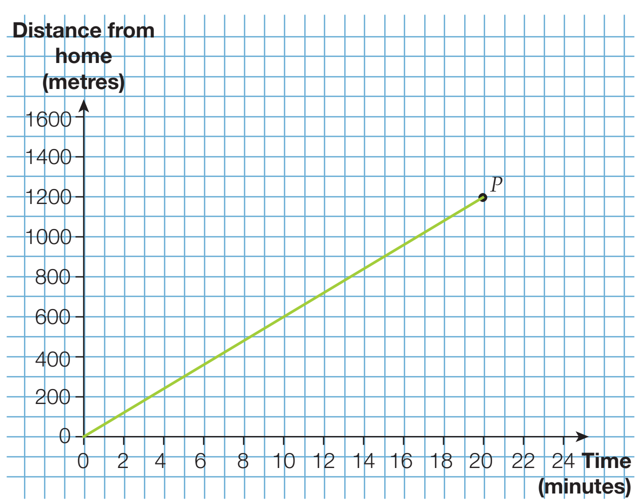
A -2   **B** -1

C 0   **D** 2



Understanding

8 Thanh decided to walk to school rather than take the bus. This graph shows his distance from home at any particular time on the walk. The school is located at Point *P*.



(a) How far is the school from Thanh's home?

(b) How long did it take Thanh to walk to school?

(c) How far did Thanh walk every minute?

(d) What is the gradient of this graph?

(e) What was Thanh's speed in m/min on the walk?

(f) How far from home was Thanh after walking for:

 (i) 6 minutes

 (ii) 12 minutes

 (iii) 18 minutes

 (iv) t minutes?

(g) Write down the equation to find the distance from home, D (in metres), after Thanh has walked t minutes.

9 Anna drove her car at a constant speed for 30 seconds, covering a distance of 450 m.

(a) Represent this information on a distance–time graph. (The distance travelled should be shown on the vertical axis and the time taken on the horizontal axis.)

(b) At what speed was Anna travelling?

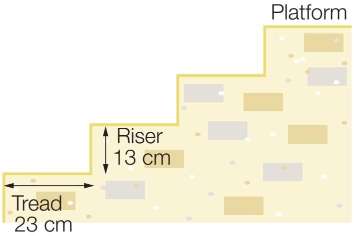
(c) What is the gradient of your graph?

(d) Write down an equation connecting the distance travelled to the time taken.

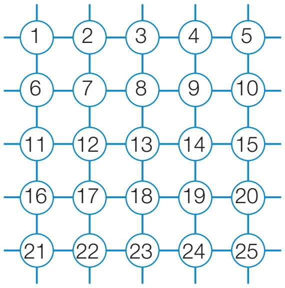
(e) Using your equation, calculate how far Anna would have travelled after 53 seconds if she maintained the same speed.

Reasoning

10 A standard step is 13 cm high (also called the riser) and has a width of 23 cm (also called the tread). A wheelchair ramp is to be constructed to reach a platform that is four steps above ground level. If wheelchair access ramps have a gradient of 1:12, (this means a rise of 1 for a run of 12) find how far from the bottom of the stairs the ramp would need to start if it is to reach the top of the platform.



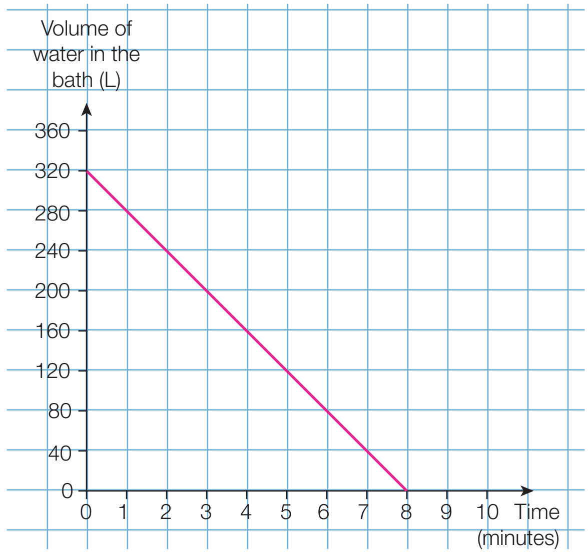
11 Using the 5×5 grid labelled with the numbers 1–25, show how many lines with different gradients can be drawn between any two of these points.



Hint

The gradients can be whole numbers, fractions, decimals positive and negative.

12 Sylvia is draining a bathtub of water. The graph shows the volume of water (in litres) in the bath after a number of minutes.



(a) How much water did the bath contain originally?

(b) How long did it take for the water to drain from the bath completely?

(c) What is the gradient of this graph?

(d) At what rate was the water being drained away? (That is, how many litres of water drained away each minute?)

(e) What does the negative represent in your answer to part **(c)**?

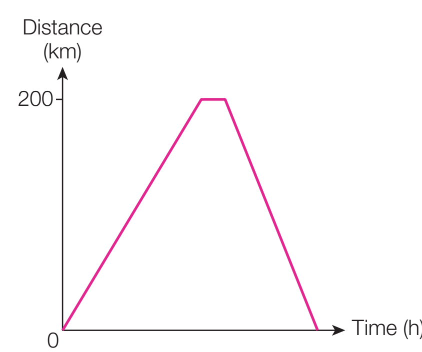
(f) How much water was left in the bath after:

(i) 4 minutes **(ii)** 6 minutes **(iii)** 2 minutes **(iv)** t minutes?

(g) Write down the equation for the volume of water left in the bath, V (in litres), after t minutes.

(h) Why is this graph only drawn in the first quadrant?

13 Monique works for a courier company and had to make a long-distance delivery one very rainy day. The following graph represents her journey. The rain had cleared for Monique's return journey, so she could travel twice as fast as she did on her way to make the delivery. It took her 4 hours and 10 minutes to make the delivery.



(a) At what speed was Monique travelling on her way to make the delivery?

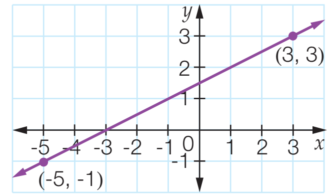
(b) At what speed was Monique travelling on her return journey?

(c) How long did the entire trip take if Monique stopped for 45 minutes for lunch?

Open-ended

14 The gradient of the line joining two points A and B is $-\frac{2}{3}$. If A is the point (1, 6), write three possible coordinates for B.

15 Kate and Sally were looking at a diagram the teacher had drawn on the whiteboard and the question written underneath, 'What is the gradient of this line?'



Kate wrote $m = \frac{\text{rise}}{\text{run}}$

$= \frac{3 - -1}{3 - -5}$

$= \frac{4}{8}$

$= \frac{1}{2}$

Sally wrote $m = \frac{\text{rise}}{\text{run}}$

$= \frac{3 - 1}{3 - 5}$

$= \frac{2}{-2}$

$= -1$

The teacher tells Kate she is correct. Explain Sally's mistake.

Outside the Square — Problem solving

Lost in the desert

An aeroplane has crashed in the desert at a point that could be labelled (-15, 12) on a Cartesian plane. A search party sets out by 4WD from the point (18, 3) and travels in a straight line passing through the point (3, 9). A light plane sets out from point (3, -15) and travels in a straight line passing over the point (-3, -6).

Strategy options

- Draw a diagram.
- Break into manageable parts.

1 Will either of the search parties pass the point where the aeroplane lies if they continue on these paths? Explain how you obtained your answer.

2 If either (or both) of the original routes does not pass the point where the aeroplane lies, give a different point (but with the same *x*-coordinate) that the searchers should pass through (or over) to be on course to locate the aeroplane.

Technology Exploration CAS

Linear relationships

Versions of this Exploration for other technologies are available in Pearson Reader.

All linear (straight line) graphs have the form $y = mx + b$ where m and b are numbers (e.g. $y = 3x + 5$ and $y = 2x - 7$). What happens to these graphs when m is changed or b is changed? Are there any patterns that we can find? We will use a CAS calculator to explore the effects m and b have on a linear graph.

Exploring the effect b has on a linear graph

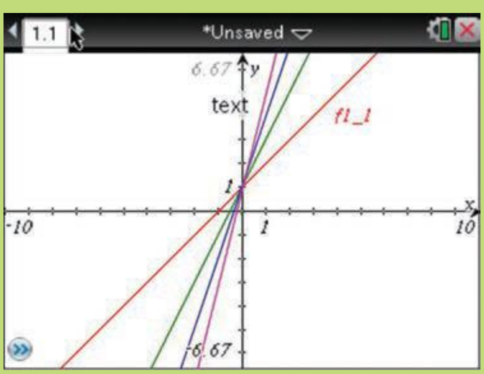
First, we will draw graphs and change the value of m but keep the value of b constant: $b = 1$.

$y = x + 1$ $\quad$ $y = 2x + 1$ $\quad$ $y = 3x + 1$ $\quad$ $y = 4x + 1$

Using the TI-Nspire CAS

To draw our graphs we will use the Graphs & Geometry screen. Press (home) > (graphs) and enter the expression $f1(x) = \{1,2,3,4\}\times x+1$.

The curly brackets are entered using (ctrl) (or (ctrl)) > ( ) (or ( )). Before you draw your graphs, you should check the window settings. Press (menu) > Window/Zoom > Zoom – Standard. When you are finished, press (enter) (or (enter)) and the graphs appear. In the screen shot you can see below, a 1 is attached to one of the graphs. If you use the central pad to select the first graph, a hand will appear with graph f1_1. This is the graph for the function associated with the first value in the curly brackets, which in this case is $y = x + 1$. If you select the other graphs, the other equations will be displayed.

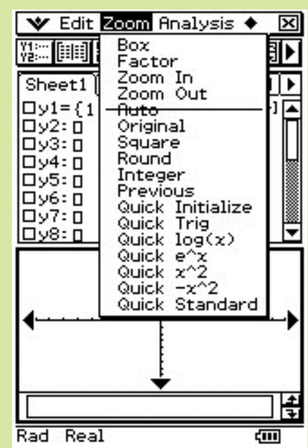


The graphs have different gradients, but all pass through the y-axis at 1.

Using the Casio ClassPad

To draw our graphs we will use the Graph and Tab screen from the main menu, so tap (Graph&Tab).

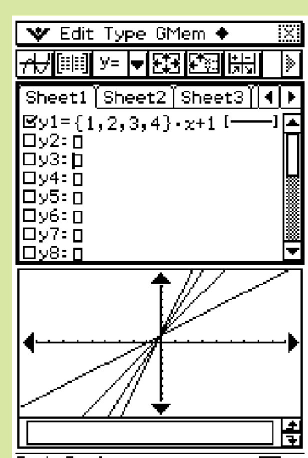
Before we enter our graphs we need to set our window. Tap on the graph half of the screen and tap Zoom>Quick Standard.



Now tap on the equation part of the screen and enter $y1=\{1,2,3,4\}\times x+1$.

Press (Keyboard) to access the curly brackets. When you have entered the equation, be sure to check the box. Then tap (graph) to see the graphs.

The graphs have different gradients, but all pass through the y-axis at 1.



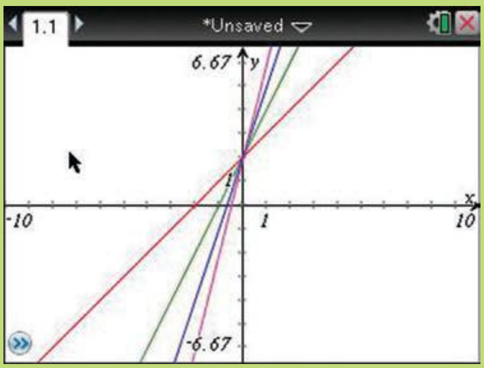
Technology Exploration CAS

What happens to these graphs when we change the value of b to 2?

$y = x + 2$ $\quad$ $y = 2x + 2$ $\quad$ $y = 3x + 2$ $\quad$ $y = 4x + 2$

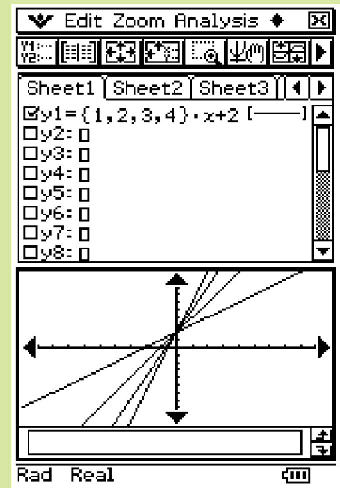
Using the TI-Nspire CAS

Edit the equation to change the value of b to 2. To do this we use the ▲ and ▼ arrows of the central control doughnut and the del (or clear) to erase the 1.



Using the Casio ClassPad

Edit the equation to change the value of b to 2. To do this, tap on the 1 at the end of the $y1$ equation to highlight it and make it a 2. Tap the graph icon to see the new graphs.



These lines appear to be the same as the previous ones except that they pass through the y-axis at 2 this time. The value of b seems to be the value of the y-intercept.

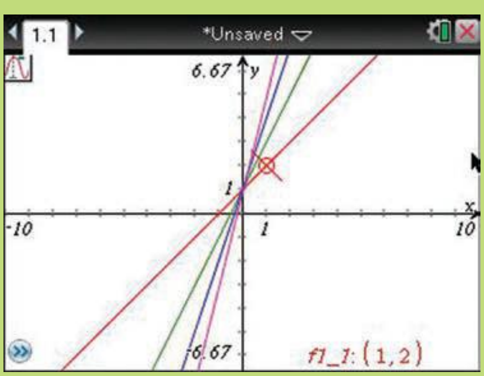
1 Test this theory now by changing the value of b to several other numbers: positive and negative, integers and fractions. Record your results.

Exploring the effect m has on a linear graph

To explore how changing the value of m affects the graph that is drawn, we will return to the original graphs (i.e. where $b = 1$).

Using the TI-Nspire CAS

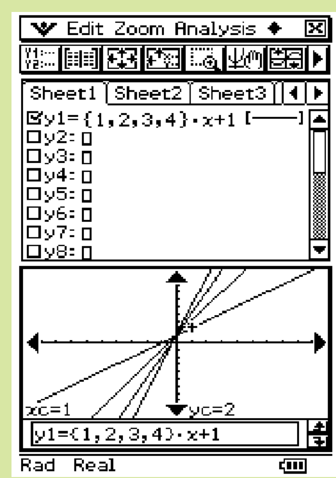
Press menu > Trace > Graph Trace > enter (or enter). Then, to indicate you want to know the coordinate pair for $x = 1$, press 1 > enter (or 1 > enter).



In the bottom right-hand corner you can see the coordinate pair given for the first function. You can use the ▲ and ▼ parts of the central doughnut to find the coordinate pair for the other functions. If you do this you should find that the steepest line contains the coordinate pair (1, 5).

Using the Casio ClassPad

Tap Analysis > Trace then press 1. A dialogue box will appear with 1 as the x-value. Tap OK to confirm your choice. The coordinate pair (1, 2) can be read off the display at the bottom of the graph screen. You can see that this is a point on the shallowest graph of the four graphs on display. Now press the ▲ part of the four-button central doughnut. This takes you to the steepest of the graphs, and the coordinates are now (1, 5).



Technology Exploration CAS

Go through this process again to find another two coordinate pairs, say (3, 4) for the shallowest graph and (3, 13) for the steepest graph. Now find the gradients of the two lines.

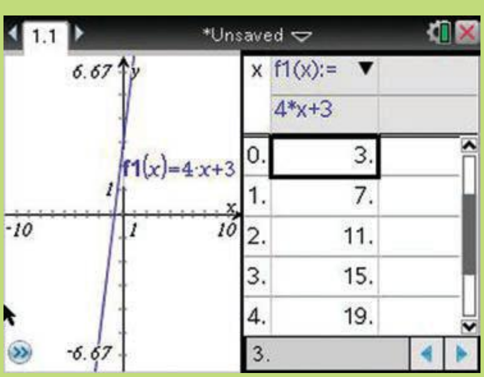
	Points	Gradient	Equation $y = mx + b$
Shallowest line:	(1, 2), (3, 4)	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{4-2}{3-1}$ $= \frac{2}{2}$ $= 1$	$y = x + 1$ $m = 1$
Steepest line:	(1, 5), (3, 13)	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{13-5}{3-1}$ $= \frac{8}{2}$ $= 4$	$y = 4x + 1$ $m = 4$

The value of m is the same as the gradient for both lines. The value of m seems to represent the gradient.

2 Test the theory by investigating $y = 4x + 3$. If our theories are correct then the graph should pass through the y-axis at $y = 3$ and have a gradient of 4. We will check this by finding two coordinate pairs, calculating the gradient and checking the y-value associated with $x = 0$.

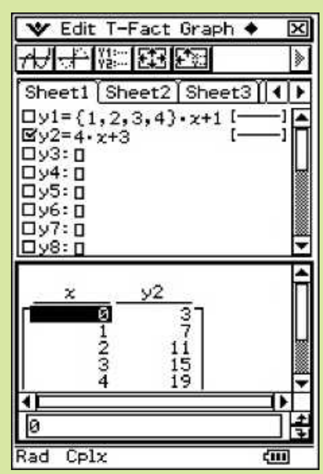
Using the TI-Nspire CAS

This time we will use the table feature to get the coordinate pairs but first we want to delete the existing equations. To do this press (menu) > Tools > Delete all > (enter) (or (enter)). Make $f1(x) = 4x + 3$ and press (enter) (or (enter)). To see the table press (menu) > View > (enter) (or (enter)) or Show Table.



Using the Casio ClassPad

This time we will use the table feature to get the coordinate pairs but we do not want to use $y1$ as its format interferes with the table. Click off the ✓ next to $y1$ and then tap (table icon) to see the table.



We will use (1, 7) and (4, 19). This gives a gradient of $\frac{19-7}{4-1} = \frac{12}{3} = 4$ as predicted, and (0, 3) confirms the y-intercept as the b-value.

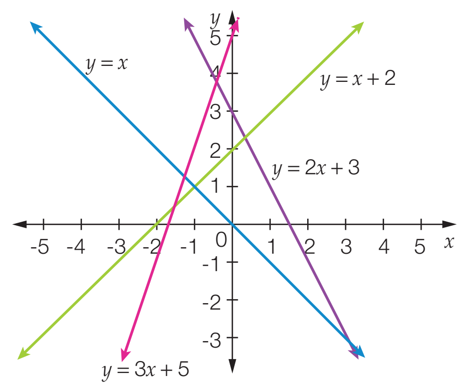
3 Now try at least six other equations, including some with negative values for a and b, and see whether the graphs and tables confirm your predictions.

Sketching linear graphs using the gradient and *y*-intercept

5.6

When we plot points that satisfy equations in the form $y = mx + b$, we find that they lie in a straight line. The graphs of these equations are linear.

If the magnitude of m increases and b stays constant in $y = mx + b$, the gradient increases. If the magnitude of m decreases and b stays constant in $y = mx + b$, the gradient decreases.



Using gradient $= \frac{\text{rise}}{\text{run}}$, we can show that the coefficient of x is the gradient m.

When the value of b increases and m stays constant in $y = mx + b$, the gradient stays the same but the y-**intercept** increases. If the value of b decreases and m stays constant in $y = mx + b$, the y-intercept decreases. We can show that the value of the constant term b is the value of the y-intercept.

We use a linear graph to show linear relationship between two variables. These linear relationships can be written as linear equations in two different forms, as described below.

The **gradient–intercept** form:

$y = mx + b$, where m is the gradient and b is the y-intercept

The **general form**:

$ax + by + c = 0$, where a, b and c are constants and a and $b \neq 0$

In this equation, the gradient $m = \frac{-a}{b}$ and the y-intercept $= \frac{-c}{b}$ can be found by rearranging the equation to make y the subject.

Finding the *y*-intercept and gradient

Before we can use linear equations to sketch a graph, we need to be able to identify the gradient and the y-intercept from the equation.

Worked Example 15

WE15

State the gradient and coordinates of the y-intercept of the graphs of each of the following linear equations:

(a) $y = -3x + 2$ **(b)** $y = x - 1$ **(c)** $2y = 5x - 4$ **(d)** $y + 2x = 6$

Thinking		Working
(a)	1 Ensure that the equation is written in the form $y = mx + b$.	(a) $y = -3x + 2$
	2 The coefficient of x is the gradient m.	gradient = -3
	3 The constant term is the y-value of the y-intercept.	y-intercept: (0, 2)
(b)	1 Ensure that the equation is written in the form $y = mx + b$.	(b) $y = x - 1$ $= 1x + (-1)$
	2 The coefficient of x is the gradient m.	gradient = 1
	3 The constant term is the y-value of the y-intercept.	y-intercept: (0, -1)
(c)	1 Ensure that the equation is written in the form $y = mx + b$.	(c) $2y = 5x - 4$ $y = \frac{5}{2}x - 2$
	2 The coefficient of x is the gradient m.	gradient = $\frac{5}{2}$
	3 The constant term is the y-value of the y-intercept.	y-intercept: (0, -2)
(d)	1 Ensure that the equation is written in the form $y = mx + b$.	(d) $y + 2x = 6$ $y = 6 - 2x$ $= -2x + 6$
	2 The coefficient of x is the gradient m.	gradient = -2
	3 The constant term is the y-value of the y-intercept.	y-intercept: (0, 6)

Sketching linear graphs that pass through the origin

$y = mx$

Graphs that pass through the origin are in the form $y = mx$. The value of b equals zero so that the origin is the y-intercept. We use the origin (0, 0) as one point on the graph and the gradient to find a second point.

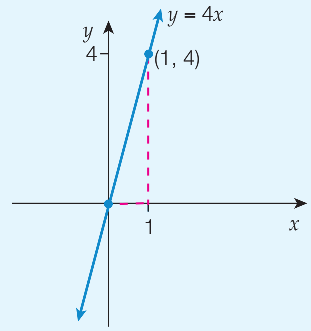
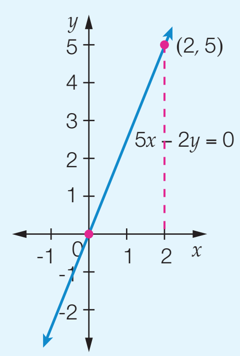
The gradient m is the coefficient of x: $m = \frac{\text{rise}}{\text{run}}$.

Worked Example 16 WE16

Sketch the graph of the relationships.

(a) $y = 4x$ **(b)** $5x - 2y = 0$

Thinking		Working
(a)	1 Find the y-intercept.	(a) y-intercept: When $x = 0$, $y = 4 \times 0$ $y = 0$ y-intercept is (0, 0)

	2 State the gradient as a fraction so that the rise and run can be identified.	$m = 4$ $= \frac{4}{1}$ rise = 4; run = 1
	3 Draw and label a set of axes. 4 Mark the origin as the first point on the graph. 5 Use the gradient to mark a second point. (Here there is a rise of 4 and a run of 1 to reach the point (1, 4). 6 Rule a line through these two points. 7 Label the graph.	
(b)	1 Find the y-intercept.	(b) y-intercept: When $x = 0$, $-2y = 0$ $y = 0$ y-intercept is $(0, 0)$
	2 Rearrange the equation so that it is in the form $y = mx$.	$5x - 2y = 0$ $5x - 5x - 2y = 0 - 5x$ $-2y = -5x$ $\frac{-2y}{-2} = \frac{-5x}{-2}$ $y = \frac{5}{2}x$
	3 State the gradient and use it to identify the rise and the run.	$m = \frac{5}{2}$ rise = 5; run = 2
	4 Draw and label a set of axes. 5 Mark the origin as the first point on the graph. 6 Use the gradient to mark a second point. (Here there is a rise of 5 for a run of 2 to reach the point (2, 5). 7 Rule a line through these two points. 8 Label the graph.	

The equations $y = 4x$ and $5x - 2y = 0$ are linear graphs for which the y-intercept is the origin.

Graphing using the y-intercept and gradient

We no longer need to plot a number of points to graph linear relationships. We can use our knowledge of gradients and intercepts to sketch the graphs. Because we only need two points to define a line, we can use the y-intercept as one point on our graph and the gradient, m, to locate a second point. Once two points are known, a straight line can be drawn through them.

Sketching other linear graphs

$y = mx + b, b \neq 0$

If the y-intercept is not the origin, we need to identify the y-intercept from the equation before we can use the gradient to find a second point.

Using the equation $y = mx + b$ and substituting $x = 0$, we obtain $y = b$.

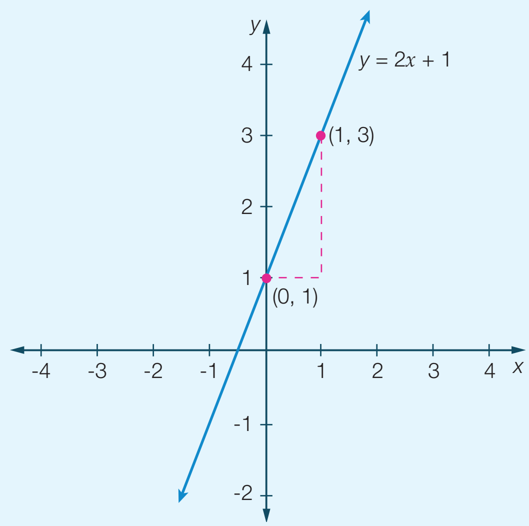
$x = 0$ is the x-coordinate of a point on the y-axis, so b is the y value at the point at which the graph crosses the y-axis. This is the y-intercept.

Worked Example 17

WE17

Use the y-intercept and the gradient to sketch the graph of $y = 2x + 1$.

Thinking	Working
1 State the y-intercept	y-intercept: $(0, 1)$
2 State the gradient as a fraction so that the rise and run can be easily identified.	Gradient $= 2$ $= \frac{2}{1}$ $= \frac{\text{rise}}{\text{run}}$
3 Draw and label a set of axes.	
4 Mark the y-intercept.	
5 Start at the y-intercept (here $(0, 1)$)and use the gradient to mark a second point (move horizontally the value of the run and move vertically the value of the rise). (Here the rise is 2 and the run is 1 to reach the point $(1, 3)$.)	
6 Rule a line through the two points continuing in both directions.	
7 Ensure that the coordinates of the two points used to sketch the line are clear and that the line is labelled with its equation.	



Sometimes the equation is not in a form where we can read off the gradient and y-intercept. We need to rearrange the equation first to make y the subject.

Worked Example 18

WE18

Use the y-intercept and the gradient to sketch the graph of $2x + 3y = 6$.

Thinking	Working
1 Rearrange the equation so that it is in the form $y = mx + b$.	$2x + 3y = 6$ $3y = 6 - 2x$ $y = \frac{6 - 2x}{3}$ $= 2 - \frac{2x}{3}$ $= -\frac{2x}{3} + 2$

2 State the y-intercept.

y-intercept: (0, 2)

3 State the gradient as a fraction so that the rise and run can be easily identified.

$$\text{Gradient} = -\frac{2}{3}$$
$$= \frac{\text{rise}}{\text{run}}$$

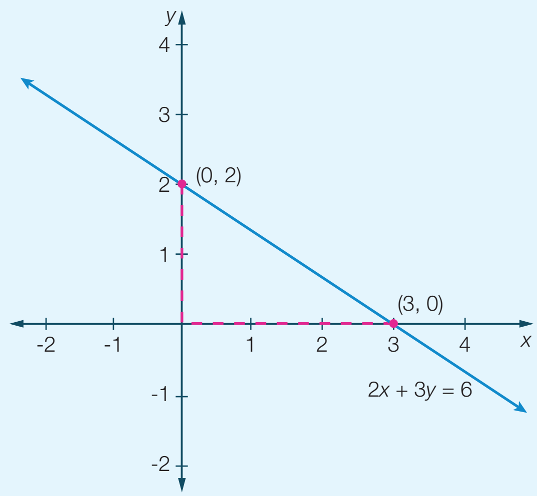
4 Draw and label a set of axes.

5 Mark the y-intercept.

6 Start at the y-intercept (here (0, 2)) and use the gradient to mark a second point (move horizontally the value of the run and move vertically the value of the rise (Here the rise is -2 and the run is 3 to reach the point (3, 0)).

7 Rule a line through the two points continuing in both directions.

8 Ensure that the coordinates of the two points used to sketch the line are clear and that the line is labelled with its equation.



Parallel and perpendicular lines

Two distinct lines in the same plane with the same gradient are **parallel** lines. The converse of this statement is also true: Parallel lines have the same gradient.

$y = 2x - 4$ and $y = 2x + 5$ are parallel because they both have a gradient of 2.

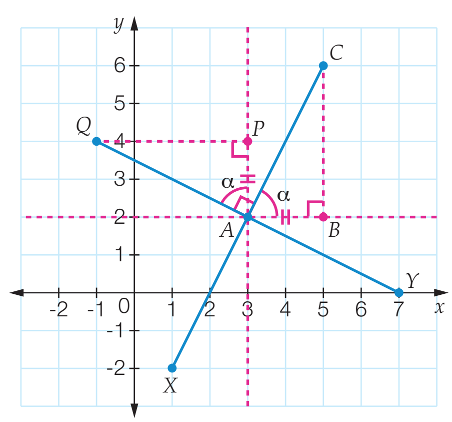
Perpendicular lines

The gradients of **perpendicular** lines, which are lines that intersect at right angles or 90°, also have an important relationship.

Consider the two lines XC and YQ drawn at right angles and intersecting at A.

Draw lines parallel to the axes through A.

Draw two right-angled triangles ABC and APQ such that $AB = AP$.



$\angle QAP = \angle CAB$ (both equal to $90° - \angle CAP$)

$\angle QPA = \angle CBA = 90°$ (given)

$\therefore \Delta APQ \equiv \Delta ABC$ (AAS congruency)

$\therefore PQ = BC$ (equal in length but opposite in direction)

$$\text{Let } m_1 = \text{gradient of } AC = \frac{BC}{AB}$$

$$\text{and } m_2 = \text{gradient of } QA = \frac{PA}{QP} = \frac{AP}{PQ}$$

Hint

Direction is important: $QP = -PQ$

$$m_1 \times m_2 = \frac{BC}{AB} \times \frac{AP}{PQ}$$
$$= \frac{{}^{1}\cancel{BC}}{{}_{1}\cancel{AB}} \times \frac{{}^{1}\cancel{AB}}{{}_{-1}\text{-}\cancel{BC}}$$
$$= -1$$

This shows that the products of the gradients of two perpendicular lines is equal to -1. Conversely it can be shown that if the products of the gradients of two lines $m_1m_2 = -1$, then the lines are perpendicular.

Worked Example 19

WE 19

Find the gradient of a line parallel to each of the following:

(a) $y = 2x - 5$ **(b)** $2x + 3y = 6$

Thinking	Working
(a) 1 Because y is the subject of the equation, identify the coefficient of x as the gradient in the given equation.	(a) $y = 2x - 5$ $m = 2$
2 Because parallel lines have the same gradient, this is the gradient of any parallel line.	Any parallel line will have a gradient of 2.
(b) 1 Rearrange the equation to make y the subject.	(b) $2x + 3y = 6$ $2x - 2x + 3y = 6 - 2x$ $\frac{3y}{3} = -\frac{2}{3}x + \frac{6}{3}$ $y = -\frac{2}{3}x + 2$
2 Because y is the subject of the equation, identify the coefficient of x as the gradient in the given equation.	$m_1 = -\frac{2}{3}$
3 Because parallel lines have the same gradient, this is the gradient of any parallel line.	Any parallel line will have a gradient of $-\frac{2}{3}$.

Worked Example 20

WE 20

Find the gradient of a line perpendicular to each of the following:

(a) $y = 4x + 3$ **(b)** $3x - 5y = 10$

Thinking	Working
(a) 1 Identify the gradient in the given equation.	(a) $y = 4x + 3$

	2 Because the product of the gradients of perpendicular lines = -1, the gradient of a perpendicular line = $-\frac{1}{m_1}$.	$m_1 = 4$ $m_2 = -\frac{1}{m_1}$ $= -\frac{1}{4}$
(b)	**1** Rearrange the equation to make y the subject.	(b) $3x - 5y = 10$ $3x - 3x - 5y = 10 - 3x$ $-5y = -3x + 10$ $\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{10}{-5}$ $y = \frac{3}{5}x - 2$
	2 Identify the gradient in the given equation.	$m_1 = \frac{3}{5}$
	3 Because the product of the gradients of perpendicular lines = -1, the gradient of a perpendicular line = $-\frac{1}{m_1}$.	$m_2 = -\frac{1}{m_1}$ $= -\frac{5}{3}$

5.6 Sketching linear graphs using gradient and the *y*-intercept

Navigator

Q1 Column 1, Q2 Column 2, Q3 Column 3, Q4 Column 1, Q5, Q6 Column 1, Q7 Column 1, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q18, Q20	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6 Column 2, Q7 Column 2, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q17, Q18, Q20	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6 Column 3, Q7 Column 3, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

Answers page 622

Fluency

1 State the gradient and the coordinates of the y-intercept of the graphs for each of the following linear equations. WE15

(a) $y = x + 3$ **(b)** $y = 2x + 1$ **(c)** $y = 7x + 3$

(d) $y = 3x - 2$ **(e)** $y = x - 4$ **(f)** $y = -4x + 3$

(g) $y = -x + 4$ **(h)** $y = 6 - 2x$ **(i)** $y = -5 + 2x$

2 Sketch the graphs of the following. WE16

(a) $y = 2x$ **(b)** $y = 3x$ **(c)** $y = -x$

(d) $y = -2x$ **(e)** $y = \frac{2}{3}x$ **(f)** $y = \frac{4}{5}x$

(g) $4x + y = 0$ **(h)** $6x + y = 0$ **(i)** $4x + 3y = 0$

WE17 3 Use the y-intercept and gradient to sketch the graph of each of the equations in Question **1**.

WE18 4 Use the y-intercept and gradient to sketch each of the following.

(a) $2x + y = 4$ **(b)** $-x + y = -3$ **(c)** $-3x + y = -2$

(d) $4x - y = 1$ **(e)** $2x - y = -3$ **(f)** $3x + 2y = 4$

(g) $2x + 3y = 3$ **(h)** $2x - 5y = 10$ **(i)** $7x + 3y - 6 = 0$

WE19 5 Find the gradient of lines parallel to each of the following.

(a) $y = 3x + 5$ **(b)** $y = 7 - 4x$ **(c)** $2y = 6x - 5$

(d) $6x - 2y = 3$ **(e)** $2x + 5y = 10$ **(f)** $4x - 3y = 8$

WE20 6 Find the gradient of lines perpendicular to each of the equations given in Question **5**.

7 Write down the equation of the straight line given:

(a) $m = 3$ and $b = 4$ **(b)** $m = 1$ and $b = 5$ **(c)** $m = 2$ and $b = 0$

(d) gradient is 4 and y-intercept is 0 **(e)** gradient is -5 and y-intercept is 1 **(f)** gradient is 3 and y-intercept is -4

8 **(a)** The equation of a linear graph with y-intercept 3 and gradient of -2 is:

A $y = 2x + 3$ **B** $2x - y = 3$ **C** $3x + 6 = 2$ **D** $2x + y = 3$

(b) The equation of a linear graph with gradient $\frac{1}{2}$ and y-intercept 1 is:

A $y = 2x + 1$ **B** $y = x + 2$ **C** $x - 2y = -2$ **D** $x + 2y = -2$

9 When using the y-intercept and gradient method of graphing, we first mark in the y-intercept and then use the gradient to locate a second coordinate point.

(a) A line has a gradient of $\frac{3}{4}$. From the y-intercept, we move:

A 3 units right and 4 units up **B** 4 units right and 3 units up

C 3 units right and 4 units down **D** 5 units right and 3 units down

(b) A line has a gradient of $-\frac{2}{5}$. From the y-intercept, we move:

A 2 units right and 5 units down **B** 5 units right and 2 units down

C 2 units left and 5 units up **D** 5 units left and 2 units down

(c) A line has a gradient of 5. From the y-intercept, we move:

A 1 unit right and 5 units up **B** 5 units right and 5 units up

C 1 unit right and 5 units down **D** 5 units right and 5 units down

(d) A line has a gradient of -3. From the y-intercept, we move:

A 3 units right and 3 units up **B** 1 unit right and 3 units down

C 0 units right and 3 units down **D** 1 unit right and 3 units up

10 **(a)** A coordinate point that lies on the line $y = x + 7$ is:

A (2, 10) **B** (-3, -4) **C** (0, -7) **D** (-4, 3)

(b) A coordinate point that lies on the line $3x + 2y = 11$ is:

A (5, -2) **B** (2, 3) **C** (-3, -1) **D** (4, 0)

(c) A coordinate point that lies on the line $3y - x + 6 = 0$ is:

A (0, 2) **B** (2, 0) **C** (3, -1) **D** (-3, 1)

11 Find the gradient of the line perpendicular to:

(a) a line with gradient 2

(b) a line with gradient 0

(c) the line passing through (4, 5) and (3, -2)

(d) the line passing through (-2, 6) and (3, 8).

12 Match the following.

(a) A line parallel to the line joining the points (2, 3) and (4, 7)	**A** A line joining (-6, 2) and (-2, -2)
(b) A line perpendicular to the line joining the points (2, 3) and (4, 7)	**B** A line joining (0, -2) and (3, 4)
(c) A line parallel to the line joining the points (7, 0) and (-1, 8)	**C** A line joining (-3, -5) and (1, -5)
(d) A line perpendicular to the line joining the points (7, 0) and (-1, 8)	**D** A line joining (7, 3) and (5, 4)
(e) A line perpendicular to the line joining the points (-1, -3) and (-4, -2)	**E** A line joining (-1, -5) and (-1, 5)
(f) A line parallel to the line joining the points (4, 2) and (-3, 2)	**F** A line joining (1, -4) and (3, -2)
(g) A line parallel to the line joining the points (3, -4) and (3, 8)	**G** A line joining (1, 5) and (3, 11)

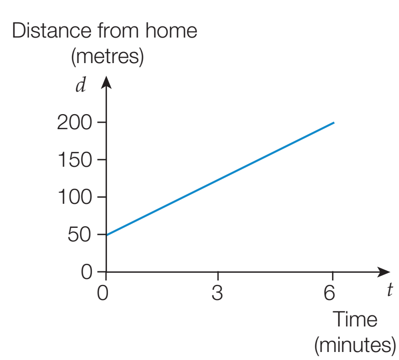
13 What is special about the following lines?

(a) Lines parallel to the line joining (4, 2) and (-3, 2).

(b) Lines parallel to the line joining (3, -4) and (3, 8).

Understanding

14 Consider this distance–time graph for Cassia as she walks to the shops.



(a) Write down the intercept with the vertical axis.

(b) Find the gradient of this graph.

(c) Use this information to write an equation linking the distance from home (d) to the time (t).

(d) At what speed was she walking? (Refer to the equation.)

(e) If Cassia continued walking at the same speed, how far from home would she be after 20 minutes?

15 A distance–time graph is represented by the equation $d = 30t + 20$, where d is the distance from Geelong in kilometres and t is the time in hours.

(a) Write down the gradient of the line.

(b) What is the speed of travel?

(c) How far from Geelong did the journey start?

(d) If the journey had started 90 km from Geelong and a constant speed of 100 km/h was maintained, what equation could now be written to represent the relationship between d and t?

Reasoning

16 An oil-tanker is being filled with crude oil. The oil-tanker initially contained 10 000 tonnes of oil and, over an 8-hour period, it received a further 96 000 tonnes of crude oil. Three full storage tanks, each of 48 000 tonnes capacity, were being used to transfer the crude oil to the tanker. Each tank was being emptied simultaneously at the same rate of 4000 tonnes per hour.

(a) Sketch a graph showing the amount of crude oil (in tonnes) in the oil-tanker during the 8-hour period.

(b) Find the gradient of this graph.

(c) Use this information to write an equation relating the amount of crude oil (m) in the tanker to the time (t).

(d) Sketch a graph showing the amount of crude oil (m) left in one of the storage tanks over the 8-hour period as oil is pumped out of it.

(e) Write an equation relating the amount of crude oil (m) left in one of the storage tanks to the time (t).

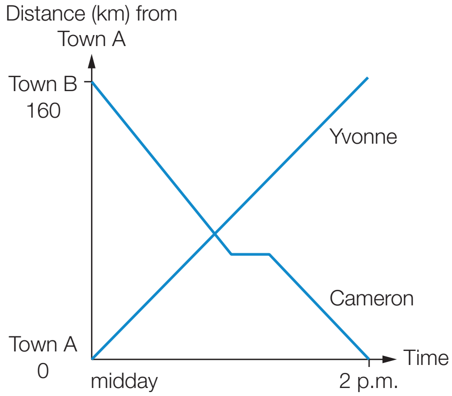
(f) How much crude oil was left in each of the storage tanks after the 8-hour period?

(g) If the pumps from one of the storage tanks were under repair, how long would it take to transfer the same amount of crude oil from the remaining storage tanks?

17 This graph shows the journeys two friends made between Towns A and B, leaving at midday and arriving at their respective destinations 2 hours later.

Yvonne travels at a constant speed straight from Town A to Town B. Cameron travels at a constant speed for the first hour. He rests for 15 minutes then continues to Town A at the same speed that Yvonne is travelling.

Find how far each of them was from Town A after travelling for half an hour.



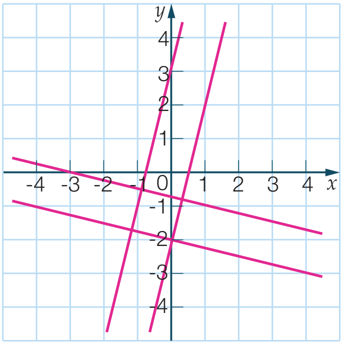
Open-ended

18 State at least three different coordinate points that lie on each of the following lines.

(a) $y = x - 4$ (b) $2x - 5y = -10$ (c) $4y - x + 2 = 0$

19 When the equations $y = 4x + 3$, $y = 4x - 2$, $y = -\frac{1}{4}x - 2$ and $y = -\frac{1}{4}x - \frac{3}{4}$ are plotted on a Cartesian plane, the area enclosed by the points of their intersection is a square.

Can you find another set of sloping lines that enclose a square?



Hint

Remember the properties of parallel and perpendicular lines.

20 (a) Write an equation for a line parallel to $y = 4x - 3$.

(b) Write an equation for a line perpendicular to $y = 4x - 3$.

Outside the Square Game

Equation battleships

Equipment required: 2 brains, different coloured pens, grid paper, ruler

How to win: Be the first player to sink your opponent's entire fleet of battleships (four ships).

Setting up:

1 On the grid paper, draw a Cartesian plane with x- and y-values that range between -10 and 10.

2 Player 1 places their first ship on the plane. Each ship is represented by a dot and must be placed at a point where the coordinates are whole numbers. Ships may be placed anywhere in the plane except along the y-axis.

3 Player 2 then places their first ship on the plane using a different coloured pen.

4 Continue alternating the placement of ships until each player has four ships on the plane.

5 Once all the ships have been placed, the next task is to position the cannons from which each player will try to sink the opposition player's ships. Each player has five cannons and the cannons will be placed along the y-axis. Player 2 decides whether they wish to place their cannons on odd or even numbered positions along the y-axis. Each player marks their five cannons with a 'C' using their particular colour.

How to play: Players use their knowledge to develop a linear equation that when graphed will produce a line from one of their cannons that will intersect with at least one of their opponent's ships. Beware that the cannons shoot in both directions!

1 Player 1 goes first and must state an equation that Player 2 then draws on the plane (using Player 1's colour).

The value of m in the equation (i.e. $y = mx + b$) cannot be zero (i.e. $y = 2$, $y = -3$ etc. cannot be used as equations).

If the line passes through any ships, those ships are sunk (be careful not to sink your own ships!) and Player 1 can have another turn. If you sink your own ship, the second turn is forfeited.

If the line doesn't pass through any ships, Player 1 has misfired and Player 2 has a turn.

2 Players take alternate turns until one player sinks all four of their opponent's ships and is declared the winner!

Engineering and linear relationships

Linear relationships are used in many different engineering disciplines. They are used by civil engineers to work out the gradient of new roads. Engineers use linear relationships to work out the total number of beams required to build a bridge. They can also be used to determine how long the traffic lights need to stay green to ensure minimal traffic congestion for a certain number of cars travelling through an intersection.

Traffic congestion

Civil engineers use algebra to devise ways to improve the flow of vehicles at busy intersections and on highways.

To try to decrease the occurrence of traffic gridlock, we need to know the maximum number of cars that could pass through the intersection in a given amount of time if the traffic moved freely. We can represent this number with the variable n. n is dependent on the velocity of the cars (v), the length of each car (l) and the distance between cars (b). To develop a mathematical model, mathematicians and engineers often simplify the situation by making assumptions. In this case we will assume all cars are the same length and that all cars are the same distance apart. We will also assume that there is only one lane of traffic moving forward in each direction and that no right turns are allowed.

1 Draw a diagram to show a set of cars waiting at lights, marking b and l carefully on your diagram.

2 Write an equation that models the length of a queue of cars waiting at traffic lights in terms of n, l and b. Use d to represent the length of the queue that starts at the bumper of the first car and ends with the rear bumper of the last car.

3 Write an equation that models the velocity (v) of the queue of cars entering an intersection in terms of n, l and b. (Remember that velocity = $\frac{\text{distance}}{\text{time}}$ and that the distance will be the length of the queue d that will get through the lights while they are on green, starting with the front bumper of the first car and ending with the back bumper of the last car.)

4 Using $v = 16$ m/s, $l = 5$ m and $b = 3$ m, find n in terms of t, where t represents time.

5 If the lights are on green for 15 seconds (i.e. $t = 15$), what is the maximum number of cars that could enter the intersection in one direction?

6 **(a)** Traffic does not usually travel through lights at the speed limit. If cars have been stopped at a set of traffic lights, it takes a few seconds for them to accelerate to the speed limit, so a slower average speed is more realistic. Using $v = 10$ m/s (36 km/h), $l = 5$ m and $b = 3$ m, find n in terms of t.

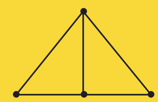
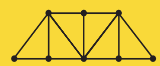
(b) If the lights are on green for 15 seconds, recalculate the maximum number of cars that could enter the intersection in one direction using this slower speed.

7 Not all traffic lights stay green for the same amount of time. On a major road the green light stays on longer than it does on a minor road. If the lights were on green for a further 5 seconds, how many more cars would enter the intersection? ($v = 10$ m/s, $l = 5$ m and $b = 3$ m.)

8 There is a traffic jam at an intersection and the velocity through the intersection has slowed. What is the maximum number of cars that can pass through the lights in one direction in 15 seconds given that the velocity is now only 5 m/s and $b = 2$ m?

Beams on a bridge

Beams of steel on a bridge meet at a joint. A truss is a type of structure constructed by beams and joints to form small to large triangles. Because triangles are the strongest type of polygon, trusses are used in many constructions, the most common being bridges and roofs. Examine the table on the right, which outlines the number of members (m) and joints (j) for particular truss designs. A member is the part of a beam between joints.

Diagram	No. members (m)	No. joints (j)
	5	4
	13	8
		

1 **(a)** Copy and complete the table above.

(b) Use the information given in the table to draw a graph of the number of joints for an increasing number of members.

(c) Use the gradient and y-intercept of the graph to write an equation that shows the relationship between the number of members (m) and the corresponding number of joints (j).

2 Use your equation to find how many joints are needed in a truss with 29 members.

3 Use your equation to find how many members are in a truss with 29 joints.

4 What restriction needs to be placed on the values of m so that the trusses can be made correctly? Can m have any value (i.e. $m = 6$)?

5.7 Sketching linear graphs using the x- and y-intercepts

When sketching a linear graph, two points on the line are needed to draw the correct graph. In the previous section we saw how the y-intercept can be used as one point and how a second point can be found using the gradient.

When equations are given in the general form $ax + by + c = 0$, it is often easier to use another method for sketching the graph that does not involve rearranging the equation.

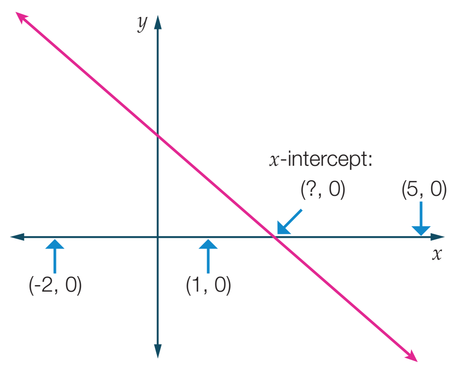
Graphing using x- and y-intercepts

The x-intercept

The x-intercept is the point where the line crosses the x-axis.

The y-coordinate for any point on the x-axis is always zero.

The x-coordinate of the x-intercept can be found by substituting $y = 0$ into the equation.

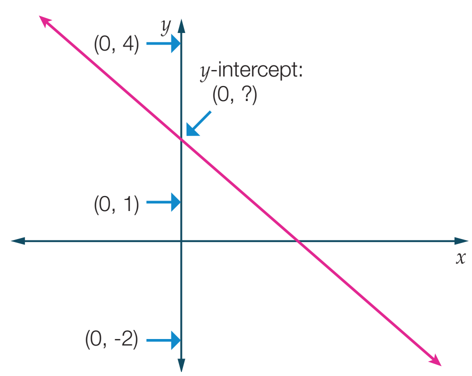


The y-intercept

The y-intercept is the point where the line crosses the y-axis.

The x-coordinate for any point on the y-axis is always zero.

The y-coordinate of the y-intercept can be found by substituting $x = 0$ into the equation.



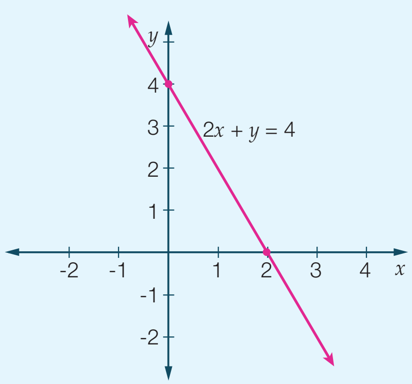
To find the x-intercept, substitute $y = 0$ into the equation.

To find the y-intercept, substitute $x = 0$ into the equation.

Worked Example 21

WE21

Use the x- and y-intercepts to sketch the graph of $2x + y = 4$.

Thinking	Working
1 Calculate the x-intercept by substituting $y = 0$ and solving the equation.	$2x + y = 4$ When $y = 0$: $2x + 0 = 4$ $2x = 4$ $x = 2$ $\therefore$ x-intercept $= (2, 0)$
2 Calculate the y-intercept by substituting $x = 0$.	When $x = 0$: $2 \times 0 + y = 4$ $y = 4$ $\therefore$ y-intercept $= (0, 4)$
3 Draw and label a set of axes. **4** Mark the x- and y-intercepts. **5** Rule a line through the two points continuing in both directions. **6** Ensure that the intercepts are clearly labelled and that the line is labelled with its equation.	

We can also use this method with linear equations in the form of $y = mx + b$.

Worked Example 22

WE22

Use the x- and y-intercepts to sketch the graph of $y = 6x - 2$.

Thinking	Working
1 Calculate the x-intercept by substituting $y = 0$ and solving the equation.	$y = 6x - 2$ When $y = 0$: $0 = 6x - 2$ $6x = 2$ $x = \frac{2}{6}$ $= \frac{1}{3}$ $\therefore$ x-intercept $= \left(\frac{1}{3}, 0\right)$

2 Calculate the y-intercept by substituting $x = 0$

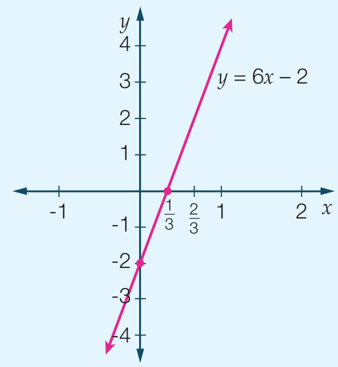
When $x = 0$:
$$y = 6 \times 0 - 2$$
$$= {}^{-}2$$
y-intercept $= (0, {}^{-}2)$

3 Draw and label a set of axes.

4 Mark the x- and y-intercepts.

5 Rule a line through the two points continuing in both directions.

6 Ensure that the intercepts are clearly labelled and that the line is labelled with its equation.



Note: when the y-intercept is the origin (e.g. $y = 4x$), this method cannot be used.

5.7 Sketching linear graphs using the x- and y-intercepts

Navigator

Answers page 625

Q1 Column 1, Q2 Column 1, Q3, Q4, Q6, Q7	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q7	Q1 Column 3, Q2 Column Q3, Q4, Q5, Q6, Q7

Fluency

WE21

1 Use the x- and y-intercepts to sketch each of the following.

(a) $x + 2y = 4$ (b) $3x + y = 6$ (c) $3x - y = 6$
(d) $x - y = 3$ (e) $4x - 2y = 12$ (f) $2x - 5y = 20$
(g) $2x + 5y = 10$ (h) $4x + 3y = 12$ (i) $3x + 4y = 2$

WE22

2 Use the x- and y-intercepts to sketch each of the following.

(a) $y = 2x - 8$ (b) $y = 3x - 9$ (c) $y = x + 5$
(d) $y = x + 2$ (e) $y = 6 - 3x$ (f) $y = 2 - 4x$
(g) $2y = 3x + 4$ (h) $5y = 2x - 5$ (i) $3y = 6 - 2x$

3 (a) The coordinates of two points that lie on the line $2x + 3y = 6$ are:

A (2, 0) and (0, 3) B (-2, 0) and (0, -3)
C (3, 0) and (0, 2) D (-3, 0) and (0, -2)

(b) The coordinates of two points that lie on the line $3y = 2x - 6$ are:

A (-3, 0) and (0, 2) B (3, 0) and (0, -2)
C (-3, 0) and (0, -2) D (2, 0) and (0, -3)

Understanding

4 Todd has seen a Blu-ray player for \$175 that he wishes to buy. The store will allow him to pay it off in weekly instalments of \$25.

(a) How much would Todd still owe the store:

(i) after 1 week (ii) after 3 weeks (iii) after n weeks?

(b) Write an equation showing the relationship between the amount (A) that Todd owes the store and the number of weeks (n) he has been paying off the Blu-ray player.

(c) (i) Find A when $n = 0$.

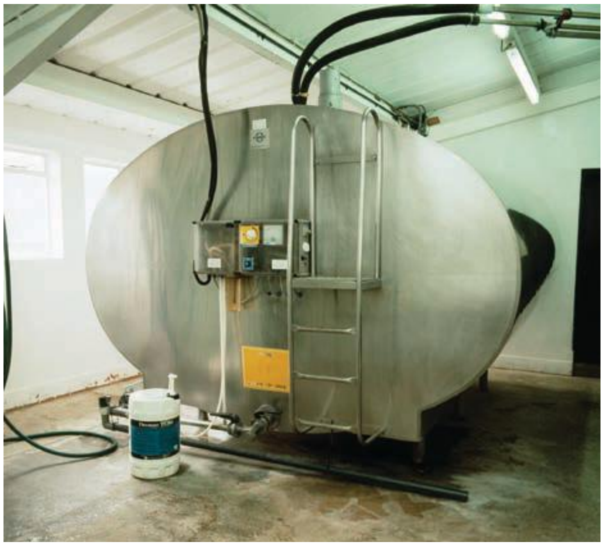
(ii) What does this mean?

(d) (i) Find n when $A = 0$.

(ii) What does this mean?

(e) Sketch the graph of this relationship: write A on the vertical axis and n on the horizontal axis.

5 A milk vat is being drained so that regular cleaning and maintenance can take place. The volume, V, of milk remaining in the vat after a given time, t, is given by the relationship $V = 630 - 15t$, where V is in litres and t is in minutes.



(a) How much milk is left in the vat after 5 minutes?

(b) How much milk did the vat originally contain?

(c) How long will it take for the vat to be completely empty?

(d) Use this information to graph this relationship on a number plane.

Reasoning

6 Chantal has borrowed \$7200 from her parents to buy a car. She agrees to repay the amount, plus an extra \$300 in interest, to her parents at the rate of \$300 per month.

(a) What is the total amount she owes to her parents?

(b) How long will it take to pay off the loan?

(c) Sketch a graph of this relationship. (Put amount owing (\$A) on the vertical axis and time (t) in months on the horizontal axis.)

(d) Write an equation describing the amount Chantal owes her parents (\$A) after t months.

(e) How much money does Chantal owe after 6 months?

(f) How much money has Chantal repaid after 14 months?

Open-ended

7 Find three different pairs of x- and y-intercepts when the gradient of the graph is equal to 3.

5.8 Vertical and horizontal graphs

Other linear relationships can be of the form:

$y = b$ where b is a constant

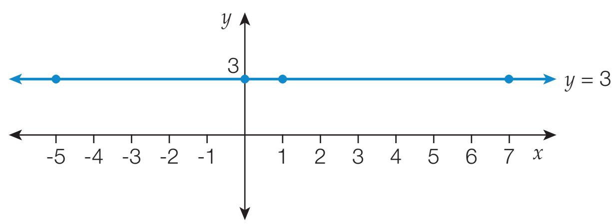
or $x = a$ where a is a constant

Graphing equations in the form of $y = b$

Consider the equation $y = 3$. All coordinate points on the line will have a y-value of 3, no matter what value of x is given. Some coordinate points that lie on this line are (1, 3), (-5, 3), (0, 3) and (7, 3).

The sketch of the equation $y = 3$ is a straight line parallel to the x-axis.

$y = 3$ can be written in gradient–intercept form as $y = 0x + 3$, a straight line with zero gradient and a y-intercept of 3.

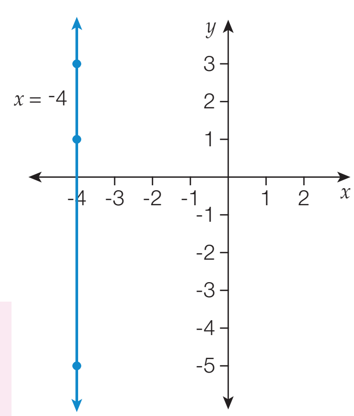


Graphing equations in the form of $x = a$

Similarly, in the equation $x = -4$, x will always have the value -4, regardless of the value of y used. Some coordinate points that lie on this line are (-4, 3), (-4, 0), (-4, -5) and (-4, 1).

The sketch of the equation $x = -4$ is a straight line parallel to the y-axis.

$x = 4$ cannot be written in gradient–intercept form because the gradient is undefined.



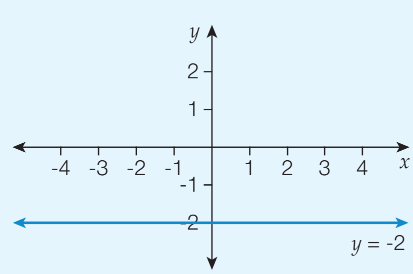
> Linear graphs of the form:
>
> $y = b$ are straight lines parallel to the x-axis (i.e. horizontal lines)
>
> $x = a$ are straight lines parallel to the y-axis (i.e. vertical lines)

Worked Example 23

WE23

Sketch the graphs.

(a) $y = -2$ **(b)** $x = 5$

Thinking	Working
(a) 1 The equation is of the form $y = b$, so it is a horizontal line passing through b on the y-axis ($b = -2$). **2** Ensure that the axes and any intercepts are labelled and that the line is labelled with its equation.	(a) 

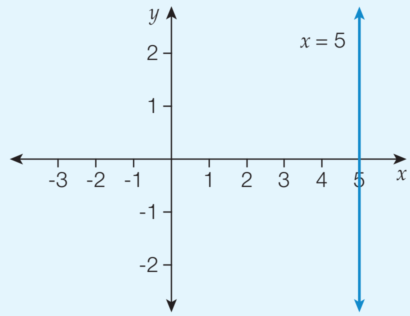
Hint

The scales on the x- and y-axes do not need to be the same.

(b) 1 The equation is of the form $x = a$, so it is a vertical line passing through a on the x-axis ($a = 5$).

2 Ensure that the axes and any intercepts are labelled and that the line is labelled with its equation.

(b)



5.8 Vertical and horizontal graphs

Navigator

Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 Column 1, Q6, Q7, Q8	Q1 Column 2, Q2, Q3, Q4, Q5, Column 2, Q6, Q7, Q8	Q1 Column 3, Q2, Q3, Q4 Column 3, Q5 Column 3, Q6, Q7, Q8, Q9

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Fluency

WE23

1 Sketch the graphs of the following, showing any intercepts on the axes.

(a) $y = 2$ (b) $y = 4$ (c) $x = -3$

(d) $x = -1$ (e) $x = 6$ (f) $x = \frac{5}{2}$

(g) $y = -3$ (h) $x = -2$ (i) $y = -\frac{3}{4}$

(j) $x = 4$ (k) $y = -\frac{7}{2}$ (l) $y = 5$

2 Write down the equation of the straight line given:

(a) $m = 0$ and $b = -6$ (b) $m = 0$ and $b = 2$

(c) gradient 0 and y-intercept $\frac{2}{3}$ (d) gradient 0 and y-intercept $\frac{3}{4}$

3 (a) A coordinate point that *does not* lie on the line $y = 5$ is:

A (0, 5) B (-1, -5) C (-5, 5) D (5, 5)

(b) A coordinate point that *does not* lie on the line $x = 1$ is:

A (1, 0) B (-1, 1) C (1, -1) D (1, 11)

(c) A coordinate point that *does not* lie on the line $y = -8$ is:

A (8, -8) B (-8, -8) C (-8, 0) D (0, -8)

4 Sketch each pair of linear graphs on the same number plane.
What coordinate point is common to both graphs in each pair?

(a) $x = 2$ and $y = 8$ (b) $y = 4$ and $x = -3$ (c) $y = 0$ and $x = -5$

(d) $y = 2$ and $x = 3$ (e) $y = -5$ and $x = 10$ (f) $x = 0$ and $y = 7$

Understanding

5 Sketch the graphs of the following linear relationships using the most appropriate method, showing any relevant points.

(a) $2y = -6$	(b) $y = -6$	(c) $2x = 8$
(d) $x + 1 = 0$	(e) $x + 4 = 0$	(f) $y - 3 = 0$
(g) $5x = 4$	(h) $3y = 2$	(i) $2y = 5$
(j) $2y = 7$	(k) $3x = -5$	(l) $3y = -4$
(m) $5x = 0$	(n) $6x = 0$	(o) $7y = 0$
(p) $3y - 6 = 0$	(q) $2y - 8 = 0$	(r) $4x + 12 = 0$
(s) $4y - 3 = 0$	(t) $3x - 8 = 0$	(u) $2x + 7 = 0$

Reasoning

6 Five streets form the boundary of an undeveloped outer suburban block of land. Taking the southernmost point of the block as the coordinate point (0, 0), each of the roads can be plotted on a number plane. The graphs that represent the roads are $y = 2x$, $x = 3$, $y = 10$, $x + 2 = 0$ and $2y + 3x = 0$.

The local council plans to subdivide this land by putting in new roads along the lines $x = 0$, $y = -5x$ and $3y - 10x = 0$.

(a) Sketch each of the linear graphs that represent the boundary roads on the same number plane.

(b) From your sketch, determine the coordinate points that are common to each adjacent pair of graphs.

(c) Sketch each of the linear graphs that represent the new roads.

(d) Will an old shed located at the coordinate point (-1, 5) be affected by the new road?

Open-ended

7 Write three equations whose graphs are parallel to the x-axis. Sketch the graphs for each.

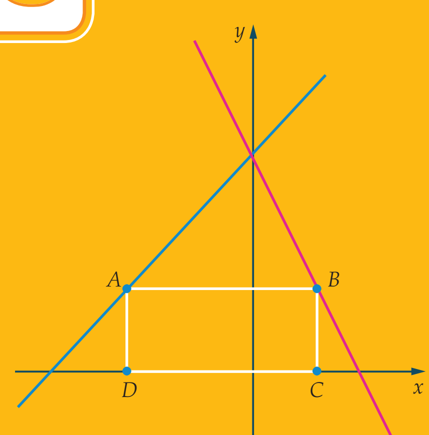
8 Write three equations whose graphs are parallel to the y-axis. Sketch the graphs for each.

9 Draw three different pairs of linear graphs on the same number plane. Each pair of graphs must have a graph parallel to the x-axis and the other parallel to the y-axis. From these graphs, determine the coordinate point which is common to each pair.

Outside the Square — Problem solving

Area of a rectangle

In the diagram, $ABCD$ is a rectangle with A on the line $y = x + 10$, B on the line $y = -2x + 10$ and C and D on the x-axis. If $AD = 4$, what is the area of rectangle $ABCD$?



Strategy options

- Solve a simpler problem.
- Break into manageable parts.

Investigation

A fun park dilemma

Equipment required: 2–4 brains, 1 calculator

Joanne and Adam are on holiday and have decided to visit a Fun Park attraction. They find that they have two options:

Option 1: A \$50 entry fee, which includes unlimited rides.

Option 2: A \$5 entry fee, which includes one free ride, and then pay \$6.00 per additional ride.

The park has 10 different rides available.

Adam started to write down the costs of going on an increasing number of rides: \$5, \$11, \$17, \$23, \$29… and then stopped because he saw a pattern emerging. He thought for a while and then wrote down an equation. 'If you tell me how many rides you want to go on, I can tell you straight away which is the better option and what it will cost,' he said to Joanne.

The Big Question

What equations did Adam use to choose the best payment option—the all-inclusive 'flat' \$50 fee or the \$5 + one free ride plus \$6 for every extra ride?

Engage

The numbers 5, 11, 17, 23 form a pattern called an 'arithmetic sequence' because there is a common difference, found by subtraction, between each term.

1 What is the common difference (d) for the terms of the above pattern?

2 It is easy to continue number patterns such as the one above because the difference between two successive terms is always the same. Use the common difference to continue the pattern to find the:

(a) 5th term **(b)** 10th term **(c)** 15th term.

3 Use this pattern to write down how much it would cost if Joanne paid the \$5 entry fee and went on:

(a) one ride **(b)** five rides **(c)** ten rides.

Explore

4 An arithmetic sequence is dependent on the first term, a, and the common difference, d. Copy and complete the following table for an arithmetic sequence: T_1 is used to denote the first term in the sequence.

T_1 (term 1)	T_2 (term 2)	T_3	T_4	T_5
$= T_1$	$= T_1 + d$	$= T_2 + d$ $= T_1 + 2d$		
$= a$	$= a + d$	=	=	=

5 Using the completed table, write an equation for the 5th term in the sequence in terms of a and d.

$T_5 =$

6 Check this equation by substituting $a = 5$ and the value for d that you obtained in Question **1** to find the value of the 5th term and compare it with your answer to Question **2(a)**.

7 Write similar equations for the 10th and 15th terms.

8 Use those equations to find the 10th and 15th terms.

Strategy options

- Break the problem into managable parts.
- Look for a pattern.
- Test all possible combinations.

Explain

9 Use your equation to calculate how much money Joanne would save if:

(a) She paid the \$50 entry fee and went on 10 rides instead of paying the \$5 entry and \$6 per additional ride.

(b) She paid the \$5 entry fee and went on six rides instead of paying the \$50 entry fee with unlimited rides.

10 By using equations or graphs, show Joanne how she can work out the best entry fee to pay for the number of rides she wants to go on.

11 What did you notice about the coefficient of d for each term in the table?

12 Explain how you could use the pattern you have found to find a rule for T_n, the nth term.

Elaborate

13 Write an equation to find the nth term of an arithmetic sequence.

14 Use your equation for the nth term of an arithmetic sequence to find:

(a) the 50th term of the sequence 5, 7, 9, 11…

(b) the 100th term of the sequence 2, 7, 12, 17…

(c) the 1000th term of the sequence -4, -2, 0, 2…

(d) the 1 000 000th term of the sequence 8, 14, 20, 26…

15 Find the first term of the sequence whose:

(a) 101st term is 209 and the common difference is 2

(b) 500th term is 1496 and the common difference is 5

(c) 768th term is 932 and the common difference is -4.

Evaluate

16 Comparing the methods used in **Engage** and **Explore**, which was the easier to use to find the 5th term?

17 Comparing the methods used in **Engage** and **Explore**, which was the easier to use to find the 15th term?

18 Which method would you use to find a term in an arithmetic sequence?

19 Where would knowledge of arithmetic sequences be used?

Extension

20 In 2004, Ben Green and Terrence Tao, two young mathematicians, found exciting new results in Number Theory, which is the theoretical study of numbers. They proved that within lists of prime numbers there are actually groups of numbers that form an arithmetic progression, but you have to examine them closely. These arithmetic progressions can be short or very long!

An example of a short sequence is that of the prime numbers 3, 7 and 11. They have a common difference of 4, but the sequence ends at 11, because 11 + 4 = 15, which is not a prime number.

Try to identify at least three more different arithmetic sequences in the following list of prime numbers (the smallest number of terms is three and the sequences can be as long as you want):

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Challenge 5

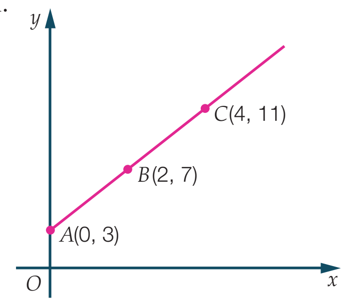
1 It takes Vinny 30 minutes to walk around the block, but Jesse can ride around on his bike in 15 minutes. If Vinny starts walking at 5:35 p.m. and Jesse follows at 5:48 p.m., when will Jesse catch up to Vinny?

2 Jakki is travelling through country Victoria and is trying to decide on her mode of travel for the next 5 days. If she hires a bicycle for 2 days and a car for the other 3 days she will pay \$150. If she hires a bicycle for 4 days and a car for the remaining 1 day she will pay \$90. What is the cost of hiring a bicycle for one day?

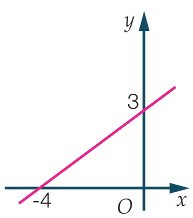
3 Nina uses the following table to produce a straight line graph.

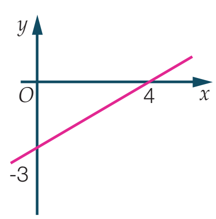
x	0	2	4
y	3	7	11

The point $D(7, w)$ also lies on the line. Find the value of w.



4 Which line has a gradient of $-\frac{3}{4}$? Diagrams are not to scale.

A


B


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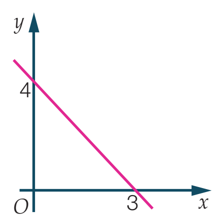
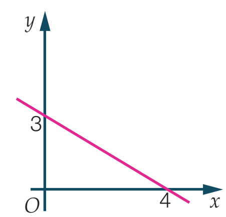


D


5 (a) The square $ABCD$ is to be plotted on a number plane. Given $A(1, 1)$, $B(3, 2)$ and $C(2, 4)$, find the coordinates of D.

(b) Write down the gradients of AB and BC. What do you notice about your answers?

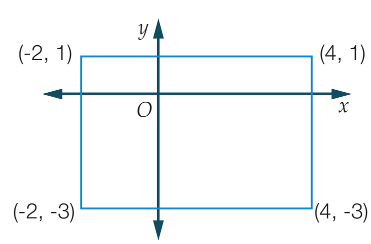
(c) Write down the gradients of AD and DC. What do you notice about your answers?

(d) What can you say about the gradients of the parallel sides of the square?

(e) What can you say about the gradients of the perpendicular sides of the square?

6 The square $KLMN$ is to be constructed on the side KL. Given $K(-2, -1)$ and $L(1, 3)$, find two pairs of values for the coordinates of M and N.

7 The diagonals of this rectangle intersect at a point. What are the coordinates of this point?



8 Solve each equation for x.

(a) $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}; x \neq b$

(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2; a \neq 0, b \neq 0$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}; x \neq \pm c$

Chapter review 5

D.I.Y. Summary

Key Words

coordinates	gradient	inverse operation	origin	slope
dependent variable	gradient–intercept form	linear equation	parallel	solve
distance	independent variable	linear graph	perpendicular	straight line
equivalence	intercept	linear relationship	rise	undefined
general form	interval	midpoint	run	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 The __________ or __________ is a measure of a line's steepness.

2 The __________ can be found by dividing the vertical rise by the horizontal __________ between two points.

3 The graph of a linear relationship is called a __________.

4 To solve an equation, we perform the __________ in the reverse order.

5 A __________ has only one solution.

6 The point where a linear graph cuts the y-axis is called the y-__________.

Fluency

1 Solve the following equations. **Ex. 5.1**

(a) $3x - 7 = -22$ (b) $7x + 4 = -3$ (c) $\frac{x}{5} + 9 = 0$

(d) $5 - \frac{x}{6} = -7$ (e) $6(x - 2) + 7 = -5$ (f) $4(2x - 5) = 0$

2 Solve these equations. **Ex. 5.1**

(a) $\frac{4x}{3} - 20 = -8$ (b) $\frac{2x - 3}{5} = 7$

(c) $\frac{3 - 9x}{2} = -4$ (d) $\frac{4(7x + 1)}{11} = 8$

3 Solve these equations. **Ex. 5.1**

(a) $4x - 5 = 8 - 2x$ (b) $8x + 5 = 3(2x - 11)$ (c) $5x + 18 = 3x$

(d) $2x = 9 - 7x$ (e) $\frac{1 - 4x}{8} = \frac{3x - 7}{4}$ (f) $\frac{2x - 5}{3} = \frac{x + 7}{4}$

4 (a) The linear equation $\frac{x}{8} - 5 = 4$ has the solution: **Ex. 5.1**

A $x = 8$ B $x = 72$ C $x = 37$ D $x = 13$

(b) The linear equation $6x + 21 = 4x + 1$ has the solution:

A $x = 2$ B $x = 10$ C $x = -10$ D $x = -2$

5 Pete keeps pet mice. He has quite a lot of mice. He has just been given four more. His friend Mal also keeps pet mice, but he has 15 fewer than Pete has now. **Ex.5.2**

(a) Write an expression to show how many pet mice Pete has now.

(b) Write an expression to show how many pet mice Mal has now.

(c) If Mal has 10 mice now, use an equation to find out how many mice Pete had originally.

6 For the points $P(-3, 5)$ and $Q(6, 11)$: **Ex.5.3**

(a) find the exact length PQ

(b) find the midpoint of PQ.

7 A 68 cm screen television set is on sale for \$1850. Terry arranges to pay \$500 as a deposit and then weekly payments of \$90. **Ex.5.4**

(a) Copy and complete this table.

Number of weeks	0	1	2	3	4	5	6
Amount paid (dollars)							

(b) Plot the amount paid for the television set against the number of weeks. Extend your graph to show the amount paid up to 20 weeks. (Plot number of weeks along the horizontal axis.)

(c) From your graph, how much would Terry have paid after 11 weeks?

(d) How long would it take Terry to pay for the television set in full?

(e) Write down a relationship connecting the amount paid in dollars, A, to the number of weeks, n.

(f) Use this relationship to calculate the amount paid after 9 weeks. Check to see whether you obtained the same result from your graph.

8 Find the gradient of the line joining (3, 2) and (4, 7). **Ex.5.5**

9 (a) The gradient of the line joining the points (-5, 3) and (2, -4) is: **Ex.5.5**

A -1 B $\frac{1}{7}$ C 1 D $\frac{7}{3}$

(b) The gradient of the line joining the points (-2, 3) and (3, 3) is:

A 0 B $\frac{1}{5}$ C 1 D 5

10 Plot the graph of $y = 2x + 4$ by first completing a table of values for x and y. Find the gradient, x-intercept and y-intercept for this line. **Ex.5.6**

11 Write down four points that lie on each of the following lines and draw the graph. **Ex.5.6**

(a) $y = 0$ (b) $x = -2$ (c) $y = 4$ (d) $x = 0$

12 Find the y-intercept and gradient for each of the following, and sketch the graph of each. **Ex.5.7**

(a) $y = 2x - 4$ (b) $2y = 3x - 4$ (c) $x + y = 5$

(d) $3x + 2y = 4$ (e) $y = 3x$ (f) $y = -2x$

13 Find the x- and y-intercepts for each of the following and hence sketch graphs of each, showing the intercepts on the axes. **Ex.5.7**

(a) $4x - y = 4$ (b) $2x + 5y = 20$ (c) $y - 3x = 6$ (d) $2x + y = 5$

14 **(a)** The y-intercept of the graph of the linear equation $y + 2x = 3$ is:

A -3 **B** -2 **C** 2 **D** 3

Ex. 5.6, 5.7

(b) The x-intercept of the graph of the linear equation $3x + 5y = 45$ is:

A 3 **B** 5 **C** 15 **D** 45

(c) The gradient of the graph of the linear equation $7x - 4y = 28$ is:

A $-\frac{7}{4}$ **B** $-\frac{4}{7}$ **C** $\frac{4}{7}$ **D** $\frac{7}{4}$

15 Sketch these graphs on the one number plane:

Ex. 5.8

(a) $y = -5$ **(b)** $y = 3$ **(c)** $x = -2$

Understanding

16 Polly walked at a constant speed for 5 minutes, covering 350 metres.

(a) To construct a distance–time graph, what two points can you use? (Do not draw the graph.)

(b) What is the gradient between these two points?

(c) What is Polly's average speed during these 5 minutes?

(d) Write an equation relating distance covered, d metres, to time taken, t minutes.

(e) Use your equation to find the distance Polly would travel in 8 minutes if she maintained her walking speed.

17 **(a)** The sum of three consecutive whole numbers is 66. What are the numbers?

(b) The sum of four consecutive even whole numbers is 100. What are the numbers?

18 The length of a rectangular paddock is 7 m less than twice the width. If the perimeter is 58 m, write an equation and solve it to find the length and width of the paddock.

Reasoning

19 Two lines are given by $kx + 6y - 5 = 0$ and $3x + 2y + 4 = 0$. Find the value of k such that the lines are parallel.

20 A bicycle manufacturing company needs to make a certain component to increase its range of products. $3000 needs to be spent on set-up costs and each unit that is produced costs an additional $5.75.

(a) Construct a cost function that describes the total cost of producing x components.

(b) Find the cost of producing 500 components.

(c) How many components could be produced on a budget of $9000?

(d) Draw a graph of the cost function to find the cost of 1000 components.

21 An electrician charges a fixed fee plus an hourly rate. A job that takes 2 hours to complete costs $280 and a job that takes 3.5 hours to complete costs $437.50.

(a) Find the linear model to express the charge $C for t hours of work.

(b) Find the amount she earns if she spends 8 hours on one job.

(c) Find the time taken to complete the job if the charge was $306.25.

22 Mrs Thomas, who works in Bendigo, has arranged to meet her husband at home in exactly 1 hour. Mr Thomas works closer to home but on the same route. He leaves work at the same time as Mrs Thomas but needs to make a stop along the way, as seen in the graph, however he still arrives home on time.

(a) Copy this graph onto graph paper.

(b) How far from Bendigo did they live?

(c) How fast was Mr Thomas travelling in the first 15 minutes?

(d) How long did he stop on his trip home?

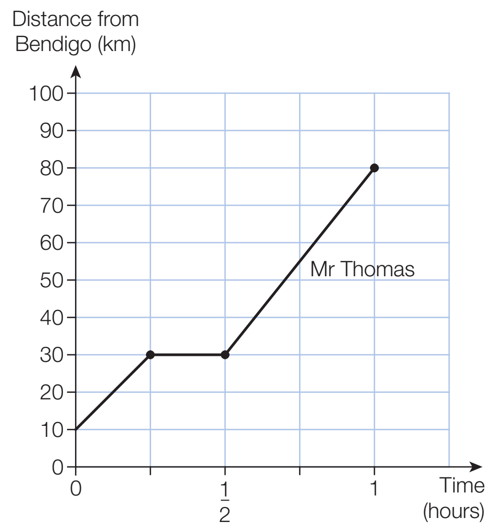
(e) How fast did he travel in the last leg of his trip?

(f) How far from Bendigo did Mr Thomas work?

(g) If Mrs Thomas drives at a constant speed from Bendigo, rule in the line on your graph that represents her travel.

(h) At what speed does Mrs Thomas need to travel to arrive home 1 hour later?

(i) Write down a linear equation that represents her travel, giving the distance from Bendigo at any particular time.



(j) At what time does Mrs Thomas pass Mr Thomas on the way home? (Use your graph.)

(k) At what distance from home does she pass him?

NAPLAN practice 5

Numeracy: Non-calculator

1 Which expression is equivalent to $5 - 7x$?

A $7x - 5$ B $-7x - 5$ C $-7x + 5$ D $7x + 5$

2 What is the gradient of the straight line connecting the points (-3, -4) and (-1, 8)?

A 8 B 4 C 2 D 6

3 What is the value of b in the equation $5b - 4 = 2b + 17$?

4 If $y = -x + 4$ and $y = 2x - 8$, which point lies on both lines?

A (-3, 7) B (4, 0) C (1, -6) D (3, 1)

5 Which of these points lies on the straight line joining (4, 4) and (20, 12)?

A (5, 5) B (6, 6) C (10, 7) D (14, 8)

Numeracy: Calculator allowed

6 What number comes next in the following sequence?

25, 50, 75, 100…

7 $y = 2x - 1$
$y = 3x + 2$

Which value of x satisfies both these equations?

A $x = -3$ B $x = -1$ C $x = 1$ D $x = 3$

8 Find the value of x that makes the equation $\frac{3x - 5}{2} = 5$ true.

9 The sum of two consecutive numbers is 55. What are the numbers?

A 33 and 22 B 22 and 23 C 27 and 28 D 30 and 25

10 An isosceles triangle has two equal sides and a third side that is 15 cm longer. Find the length of the longest side if the perimeter is 84 cm.


6
AVATAR

Geometric reasoning

6

'Rendered polygons to realistic animation'

The characters in animated movies look almost lifelike in their movements and expressions. It is hard to believe that they are created using only straight lines.

Today, most animated films are computer generated, with 60 images shown every second to create the illusion of movement. Three-dimensional graphics are created using a transparent polygon mesh mapped onto a two-dimensional computer screen. In the final stages of production, each polygon is broken into small triangles and 'rendered', a process that adds colour, shading and lighting, creating 'smooth' curves and circular shapes. Once rendered, movement points called avars (animation variables) are placed on the image, allowing the character's features to move, although only slightly in each frame. It is hard to believe that the characters, the scenery and all the images in animated movies are made with the simplest polygon of all, the triangle.

Forum

Do you believe animated movies have become as good as they can be? What improvements would you like to see in animated film? What are the advantages of animating a film over using real actors? If animations become more and more realistic, will there be a need for 'real' actors?

Why learn this?

Understanding geometry improves the ability to produce new and more complex shapes, create innovative designs and build amazing, unusual and original inventions. Technology, engineering, architecture and the building industry all rely heavily on a sound knowledge of geometric principles to visualise, design and build things that are strong, efficient and practical.

After completing this chapter you will be able to:

- apply known properties of geometric figures, including angle properties associated with parallel lines, triangles and quadrilaterals, to find other angles
- use congruence rules for triangles to prove results for other figures
- know and use the properties of the quadrilateral family
- use geometric reasoning to prove results about geometric figures
- calculate the angle sum of polygons and find the angles in regular polygons
- enlarge or reduce two-dimensional figures using projections and scale factors
- identify when a pair of triangles are similar
- use similar triangles to solve mathematical and practical problems
- draw nets for different polyhedra.

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R8.1

1 Draw angles of the following sizes, using a protractor and a rule.

(a) 48° **(b)** 127° **(c)** 235° **(d)** 315°

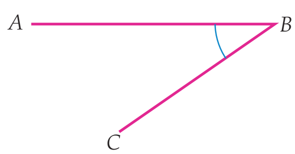
Worksheet R8.2

2 Name the type of each angle drawn in Question **1** using a word from this list: acute; right; obtuse; straight; reflex.

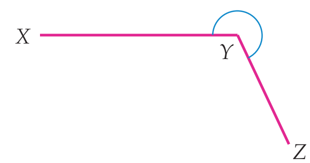
Worksheet R8.3

3 Use a protractor to measure the following angles.

(a)

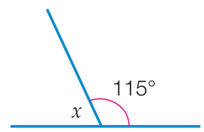


(b)



Worksheet R8.4

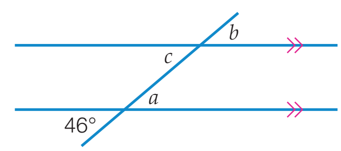
4 Choose the correct value for x.



A 45° **B** 245° **C** 115° **D** 65°

Worksheet R8.5

5 Find the value of the variables in this diagram.



Worksheet R8.6

6 Solve for x.

(a) $x + 7 = 180$ **(b)** $2x + 40 + 3x = 360$ **(c)** $x + 20 + 5x - 80 = 360$

7 Use the scale and measure this picture to determine the actual measurements, in metres, of:

(a) the length of the car

(b) the height of the car.

Scale 100 : 1

8 Write the following as unit ratios. Round answers to two decimal places if necessary.

(a) 300:50 **(b)** 14:4 **(c)** 1400:64 **(d)** 258 000:376

9 **(a)** The population of Adelaide is approximately 9.4 times the population of Darwin. If the population of Darwin is 125 000, what is the population of Adelaide?

(b) A baker's basic biscuit recipe requires 2.5 times as much flour as sugar. If 10 cups of flour are used in one batch of biscuits, how much sugar should be used?

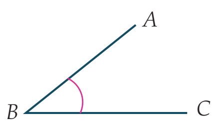
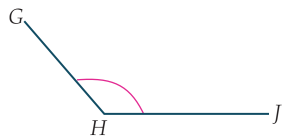
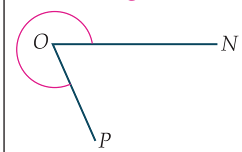
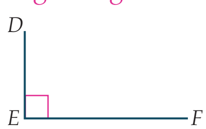
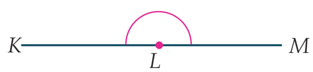
Key Words

alternate angles	edge	polygon	similar
centre of enlargement	enlargement	polyhedron	similar triangles
co-interior angles	exterior angle	prism	solid
congruent	faces	reduction	vertex
congruent triangles	interior angle	regular polyhedron	
corresponding angles	net	scale factor	

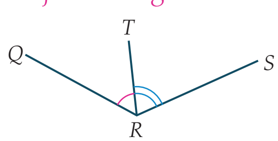
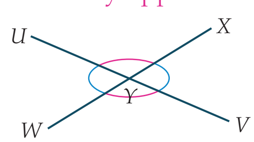
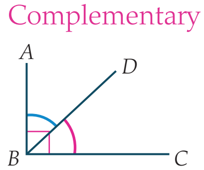
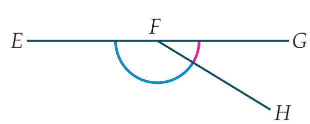
Angle review

6.1

Angles at a point

Acute angle:  Less than 90° $0 < \angle ABC < 90°$	Obtuse angle:  Greater than 90° but less than 180° $90° < \angle GHJ < 180°$	Reflex angle:  Greater than 180° but less than 360° $180° < \angle NOP < 360°$
Right angle:  Exactly 90° $\angle DEF = 90°$	Straight angle:  Exactly 180° $\angle KLM = 180°$	Revolution or angle of revolution:  Exactly 360°

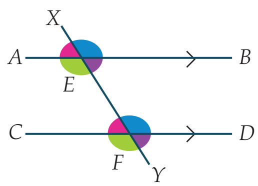
Pairs of angles

Adjacent angles:  Angles that have a common vertex and a common arm. R is a common vertex. TR is a side common to both $\angle QRT$ and $\angle TRS$. $\angle QRT$ and $\angle TRS$ are a pair of adjacent angles.	Vertically opposite angles:  A pair of angles formed when two straight lines intersect. They are on opposite sides of the vertex. Vertically opposite angles are equal. The line segments UV and WX intersect at Y. $\angle UYW$ and $\angle XYV$ are a pair of vertically opposite angles. $\angle UYW = \angle XYV$ $\angle UYX$ and $\angle WYV$ are another pair of vertically opposite angles. $\angle UYX = \angle WYV$
Complementary angles:  A pair of angles whose sum is 90°. $\angle ABD$ and $\angle DBC$ are a pair of complementary angles. $\angle ABD + \angle DBC = 90°$	Supplementary angles:  A pair of angles whose sum is 180°. $\angle EFH$ and $\angle GFH$ are a pair of supplementary angles. $\angle EFH + \angle GFH = 180°$

Angles on parallel lines

$AB \parallel CD$, the transversal XY cuts AB at E and CD at F.

Corresponding angles:

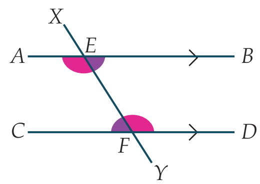


When a transversal cuts a pair of parallel lines, four pairs of equal **corresponding angles** are formed.

The pairs of equal corresponding angles are:

$\angle AEX = \angle CFE$
$\angle BEX = \angle DFE$
$\angle AEF = \angle CFY$
$\angle BEF = \angle DFY$

Alternate angles:

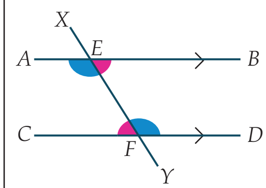


When a transversal cuts a pair of parallel lines, two pairs of equal **alternate angles** are formed.

The pairs of equal alternate angles are:

$\angle AEF = \angle DFE$
$\angle BEF = \angle CFE$

Co-interior (allied) angles:



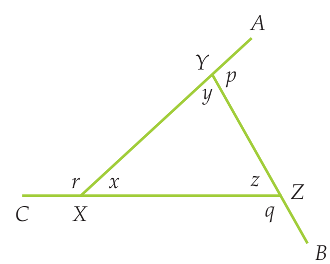
When a transversal cuts a pair of parallel lines, two pairs of supplementary **co-interior angles** are formed.

The pairs of supplementary co-interior angles are:

$\angle AEF + \angle CFE = 180°$
$\angle BEF + \angle DFE = 180°$

Exterior angle of a triangle

This is the angle formed when any side is extended. The **exterior angle** of a triangle is equal to the sum of the opposite **interior angles**.



In this diagram,

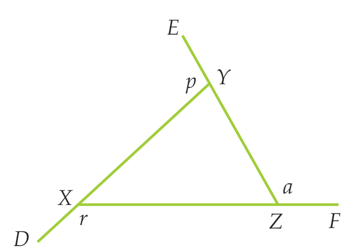
XY is extended to A to form the exterior angle $\angle AYZ$
$\angle AYZ = \angle YXZ + \angle XZY$
$p = x + z$

YZ is extended to B to form the exterior angle $\angle BZX$
$\angle BZX = \angle ZXY + \angle XYZ$
$q = x + y$

ZX is extended to C to form the exterior angle $\angle CXY$
$\angle CXY = \angle XYZ + \angle YZX$
$r = y + z$

Note that:

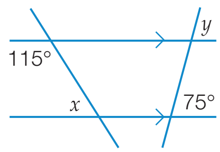
- YX can be extended to D to form the exterior angle $\angle DXZ$, where $\angle DXZ = \angle CXY$
- ZY can be extended to E to form the exterior angle $\angle EYX$, where $\angle EYX = \angle AYB$
- XZ can be extended to F to form the exterior angle $\angle FZY$, where $\angle FZY = \angle BZX$.



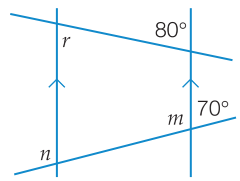
Worked Example 1

Find the value of the variables in each diagram. Give reasons for your answer.

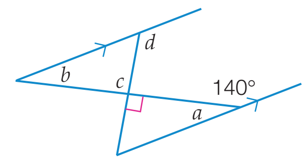
(a)



(b)



(c)



Thinking	Working
(a) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after your answer.	(a) $x = 180° - 115°$ $= 65°$ (co-interior angles between parallel lines) $y = 75°$ (corresponding angles on parallel lines)
(b) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after each answer.	(b) $m = 180° - 70°$ (straight angle) $= 110°$ $n = m$ $= 110°$ (corresponding angles on parallel lines) $r = 80°$ (alternate angles on parallel lines)
(c) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after each answer.	(c) $a = 180° - 140°$ (straight angle) $= 40°$ $b = a$ $= 40°$ (alternate angles on parallel lines) $c = 90°$ (vertically opposite) $d = b + c$ $d = 40° + 90°$ $= 130°$ (exterior angle of a triangle)

6.1 Angle review

Navigator

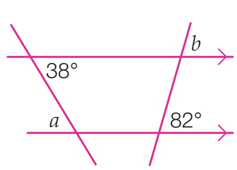
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Answers page 635

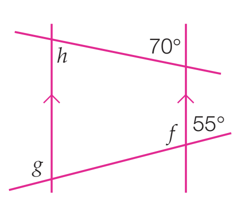
Fluency

1 Find the value of the variables in each diagram. Give reasons for your answer.

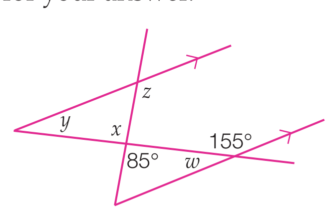
(a)



(b)

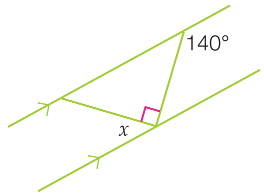


(c)

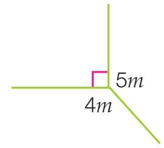


2 Find the value of the variables in each diagram. Give reasons for your answer.

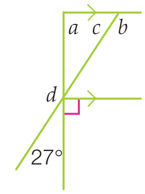
(a)



(b)

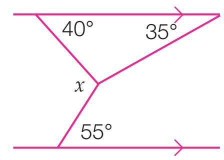


(c)



3 In the diagram, the value of x is:

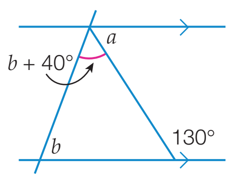
A 75 **B** 95 **C** 125 **D** 140



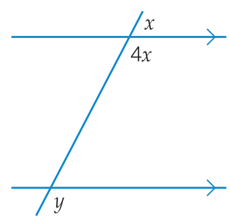
Understanding

4 Find the value of the variables in each diagram. Give reasons for your answer.

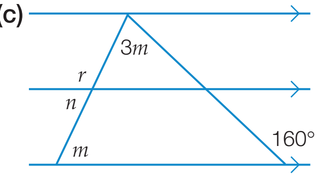
(a)



(b)



(c)



5 Write True or False for each of the following statements:

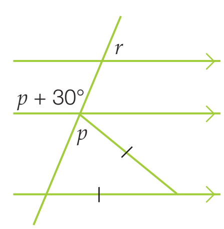
(a) Alternate angles are always equal.

(b) Co-interior angles are always supplementary.

(c) If the sum of a pair of adjacent angles is greater than 180°, then one of the angles must be an obtuse angle.

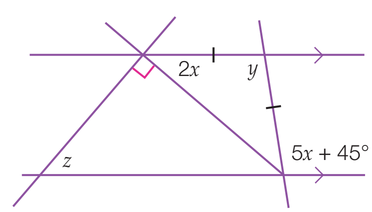
(d) The exterior angle of a triangle is always an obtuse angle.

6 Find the value of the variables in the diagram. Give reasons for your answer.



Reasoning

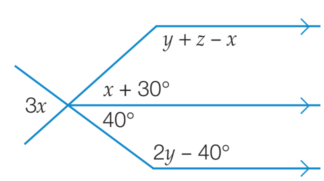
7 Find the values of x, y and z, giving reasons for your answer.



8 (a) Refer to the diagram on the right. Form an equation and solve it to find x. Give reasons for your answer.

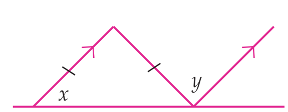
(b) Form an equation and solve it to find y. Give reasons for your answer.

(c) Hence, form an equation and solve it to find z.



Open-ended

9 Choose three possible values for x. Show that $2x + y = 180°$ and use this to find y for each of your x-values.



10 **(a)** Write down a possible pair of values for x and y.

(b) Write down a possible pair of values for m and n.

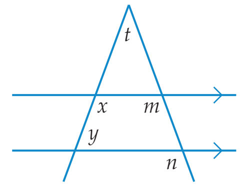
(c) Write an equation to find t in terms of n and y.

(d) Rewrite this equation to find t in terms of m and x.

(e) Use the equation in part **(c)** to find the value of t for your values of n and y.

(f) Use the equation from part **(d)** to find the value of t for your values of x and m.

(g) Compare your answers for parts **(e)** and **(f)**. Were they the same?



Outside the Square Game

Space invaders

Equipment required: 2 brains, 1 die, 3 counters per player

How to win: The first player to place all their 'satellites' in a line is the winner.

How to play:

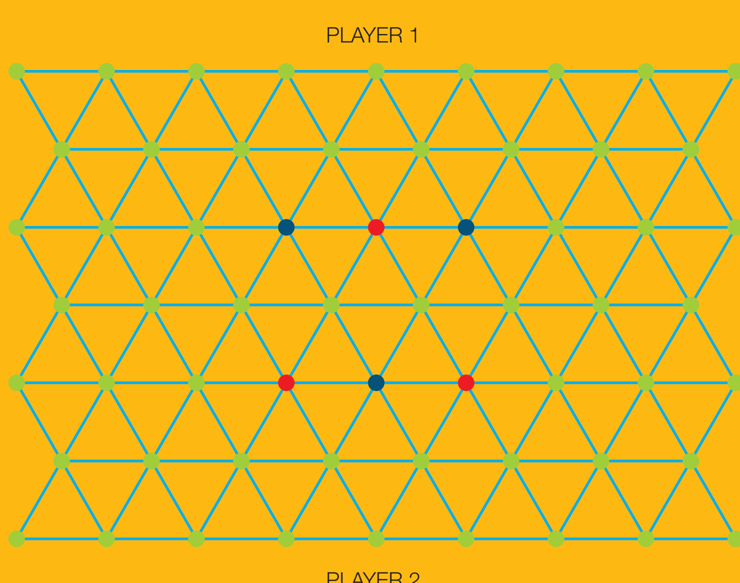
1 Players sit on opposite sides of he board. Each player has three counters, which represent satellites. Each counter must have a mark on its edge, which indicates which way the satellite is facing. Players place one satellite on each of the red or blue starting circles, facing to the right.

2 Players take turns rolling the die. The number rolled is multiplied by 60°. This angle is the size of the rotation that one of the player's satellites can rotate, either clockwise or anticlockwise. Once rotated, the satellite can move any number of spaces in the direction that it is now facing. Only one satellite is rotated and moved each turn.

3 A player can bump an opponent's satellite and send it flying to the far end of the 'galaxy', which means it is removed to the edge of the game board in the direction of the 'bump'. The player who bumps an opponent takes the position of the opponent's satellite before it was bumped. A satellite cannot be bumped if it is already on the edge of the 'galaxy'. After being bumped, the satellite is placed facing the same direction of the bump. If the number rolled does not give a rotation that allows the satellite to move, the satellite must remain on the edge of the galaxy.

4 No two satellites can occupy the same position at any one time.

5 Play continues until one player has all satellites in the same line (they do not have to be next to each other).



Mathspace

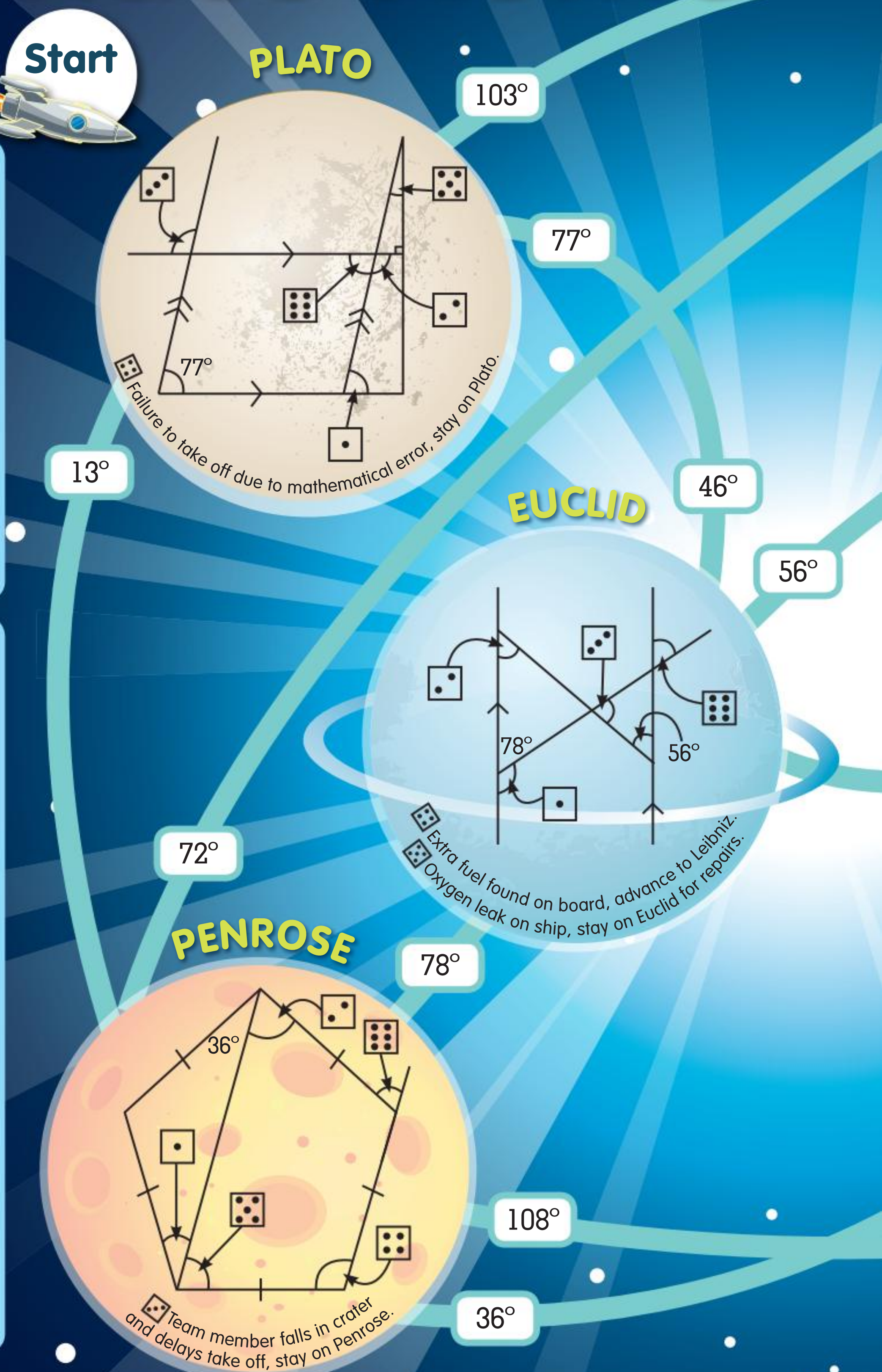
Space Race Home

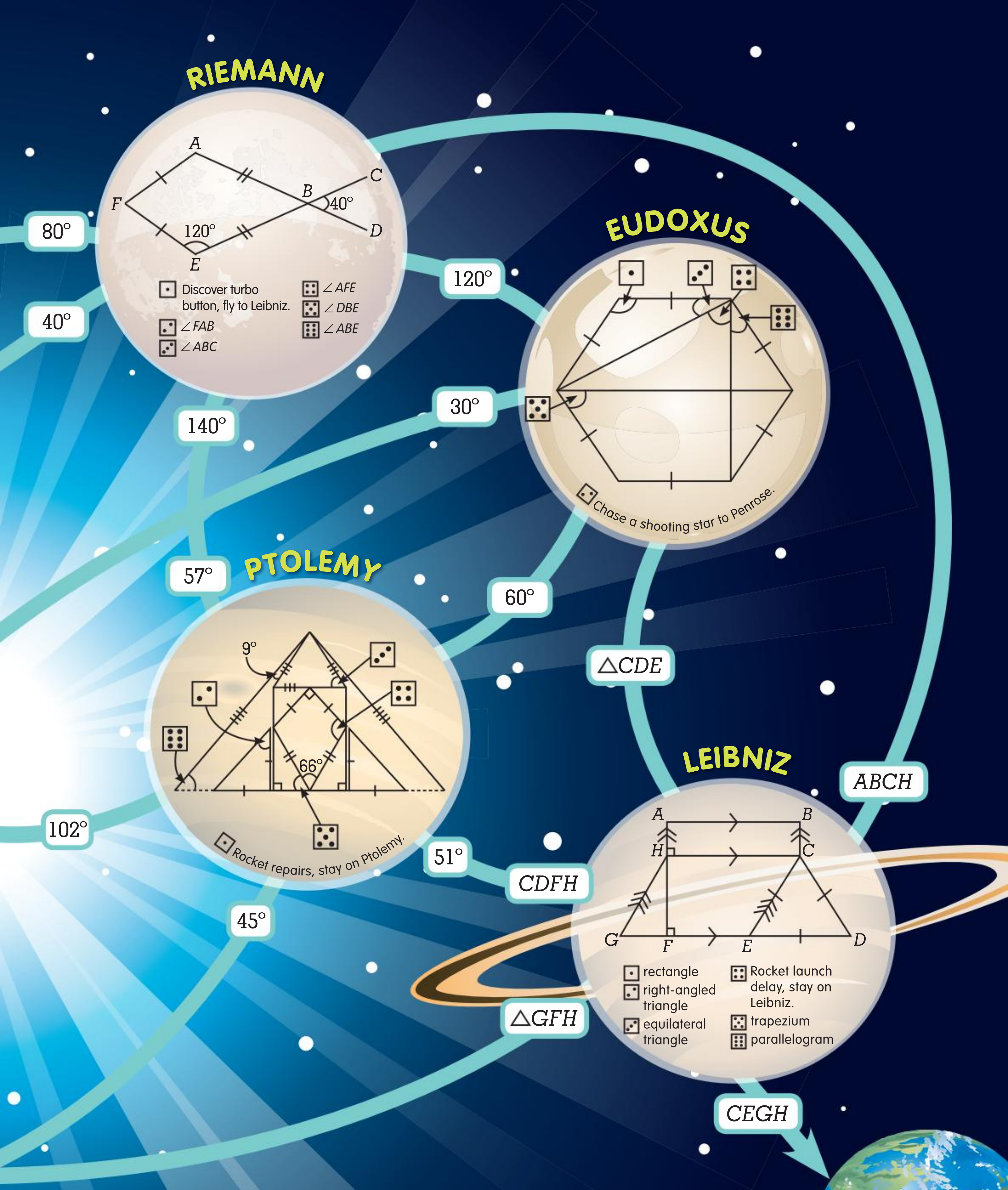
Equipment required: 2–4 brains, die, counter per player

You and your friends are leading separate Australian space explorations. You have taken photos of nebulas, black holes and other space events that have never been seen before. The first one home will be able to sell their photos for millions of dollars. There are only two ways to reach Earth—by navigating your way through the galaxy or by collecting 500 or more fuel points, which will give you enough fuel to burst through the galaxy's gravitational force and head straight for home.

How to play

1. Each player places a counter on the starting planet, Plato.
2. Players take turns rolling the die. The number rolled corresponds to an unknown angle or shape name, which, when answered, will direct the player onto another planet. At intersections, players may choose which way to go.
3. Players must keep track of the answers and add them up as they go as they are their fuel points. With 500 fuel points a player can blast their way straight back to Earth.
4. The first player to reach Leibniz and exit the galaxy towards Earth or the first player to reach or pass 500 fuel points will be the winner of the multimillion dollar media contract for their photos. Good luck.





Congratulations!
You made it home.

6.2 Triangles and congruency

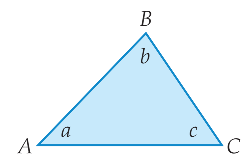
Triangles

Triangle:

The sum of the internal angles of a triangle is 180°.

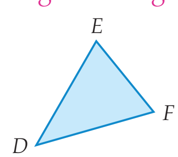
$a + b + c = 180°$

$\angle CAB + \angle ABC + \angle BCA = 180°$



Angle-named triangles

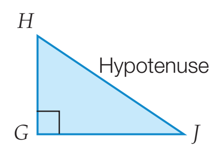
Acute-angled triangle:



All angles are acute.

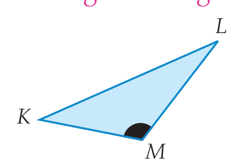
$\angle DEF < 90°$
$\angle EFD < 90°$
$\angle FDE < 90°$

Right-angled triangle:



One angle equals 90°.
$\angle HGJ = 90°$
The side opposite the right angle is called the hypotenuse.
HJ is the hypotenuse.

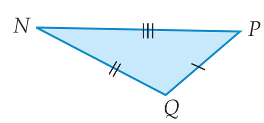
Obtuse-angled triangle:



One angle is obtuse.
$\angle KML > 90°$
The other two angles are acute.
$\angle LKM < 90°$
$\angle KML < 90°$

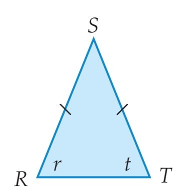
Side-named triangles

Scalene triangle:



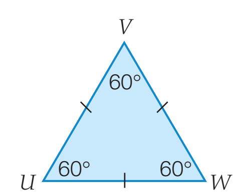
All sides and angles are unequal. NP, PQ and QN are all different lengths.

Isosceles triangle:



One pair of sides is equal in length. The angles opposite the equal sides are also equal in size. $RS = TS$
$\angle SRT = \angle STR$ or $r = t$

Equilateral triangle:

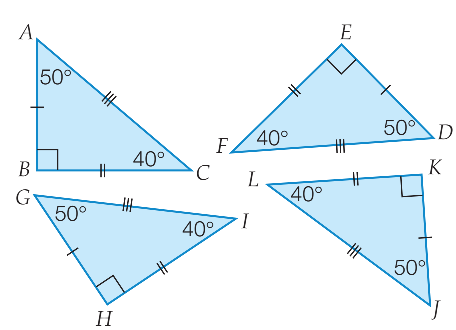


All sides are equal in length.
$UV = VW = WU$
All angles are equal to 60°.
$\angle UVW = \angle VWU = \angle WUV = 60°$

Congruent triangles

Two triangles are **congruent** if they are identical in all respects. This means that the matching sides are the same length and the matching angles are the same size.

The triangles in the diagram are all identical (congruent). Each triangle has sides marked with the same length marking and the angles opposite sides of equal length are equal.

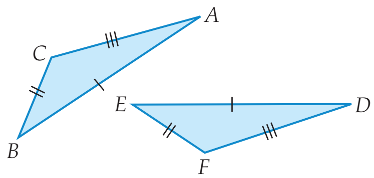


We write that $\Delta ABC \equiv \Delta DEF \equiv \Delta GHI \equiv \Delta JKL$,
where $\equiv$ means 'is congruent to'.

There are four tests for **congruent triangles** that were introduced in Year 8.

Tests for congruent triangles

1 Side Side Side (SSS):

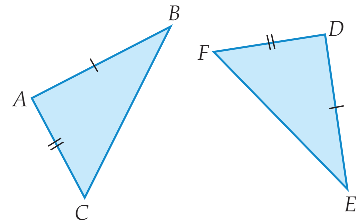


Two triangles are congruent if the matching sides have equal length.

$AB = DE$ (side)
$BC = EF$ (side)
$CA = FD$ (side)

$\Delta ABC \equiv \Delta DEF$ (SSS)

2 Side Angle Side (SAS):

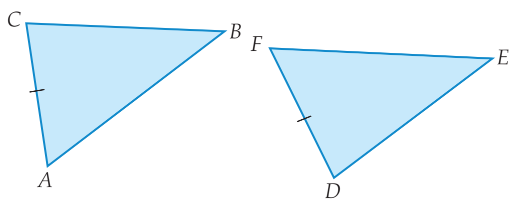


Two triangles are congruent if two matching sides have equal lengths and the angles included by these sides are equal.

$AB = DE$ (side)
$\angle CAB = \angle FDE$ (angle)
$AC = DF$ (side)

$\Delta ABC \equiv \Delta DEF$ (SAS)

3 Angle Angle Side (AAS):



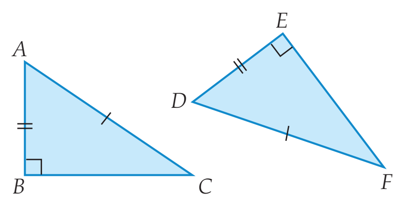
Two triangles are congruent if two matching angles are equal and a matching side is equal in length.

$\angle CAB = \angle DFE$ (angle)
$\angle CBA = \angle DEF$ (angle)
$AC = DF$ (side)

$\Delta ABC \equiv \Delta FED$ (AAS)

Note that the matching side does not need to be included between the two equal angles.

4 Right-angle Hypotenuse Side (RHS):



Two right-angled triangles are congruent if the hypotenuse and one pair of matching sides are equal in length.

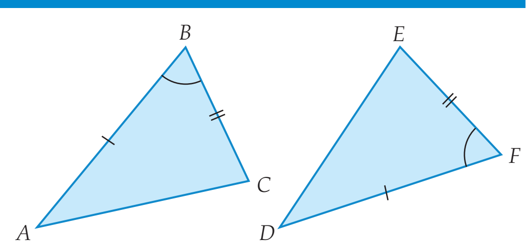
$\angle ABC = \angle DEF = 90°$ (right angle)
$AC = DF$ (hypotenuse)
$AB = DE$ (side)

$\Delta ABC \equiv \Delta DEF$ (RHS)

Worked Example 2

WE2

Show that the triangles *ABC* and *DEF* are congruent, stating the congruency test used.

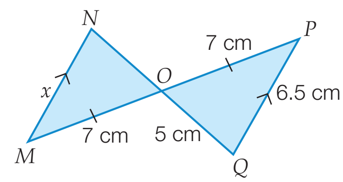


Thinking	Working
1 List the three equalities given.	$AB = DF$ (given) $\angle ABC = \angle DFE$ (given) $BC = FE$ (given)
2 Give your answer.	$\Delta ABC \equiv \Delta DFE$ (SAS)

Worked Example 3

WE3

Prove that $\Delta MNO \equiv \Delta PQO$, and hence find the value of x.



Thinking	Working
1 Name the two triangles to be used.	ΔMNO and ΔPQO
2 List three equalities about the triangles, with reasons.	$\angle MNO = \angle PQO$ (alternate angles, $MN \parallel QP$) $\angle MON = \angle POQ$ (vertically opposite angles) $MO = PO = 7$ cm (given)
3 Write down the pair of congruent triangles and the test used.	$\Delta MNO \equiv \Delta PQO$ (AAS)
4 Identify pairs of matching sides.	$MN = PQ$ and $NO = OQ$ (matching sides in congruent triangles)
5 Find x.	$x = 6.5$ cm

Worked Example 4

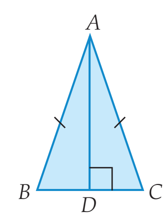
WE4

(a) ΔABC is a triangle with $AB = AC$. AD is perpendicular to BC. Prove that $\angle ABC = \angle ACB$.

(b) What type of triangle is ΔABC? Explain your answer.

(c) What triangle property have you just proved?

(d) Show that $BD = DC$.



Thinking	Working
(a) 1 Name the two triangles that you will try to prove congruent.	(a) ΔABD and ΔACD.
2 List three sets of equalities, with reasons, that apply to the two triangles.	$\angle ADB = \angle ADC = 90°$ (given) $AB = AC$ (given) AD is a common side
3 Write down the pair of congruent triangles and the test used.	$\Delta ABD \equiv \Delta ACD$ (RHS)
4 State the result that you are proving.	$\angle ABC = \angle ACB$ (matching angles in congruent triangles)
(b) Use your knowledge of types of triangles or congruency.	(b) ΔABC is an isosceles triangle because $AB = AC$.
(c) State the answer as a sentence.	(c) The base angles (angles opposite equal sides) of an isosceles triangle are equal.
(d) Use congruency to explain why the sides are equal.	(d) Matching sides of the congruent triangles ABD and ACD are equal. $\therefore BD = CD$

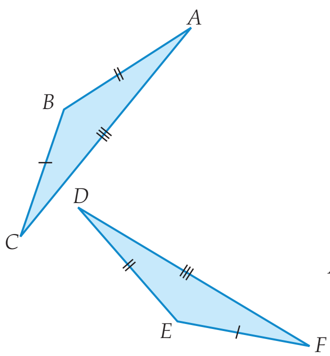
6.2 Triangles and congruency

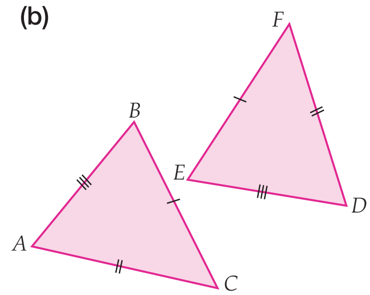
Navigator

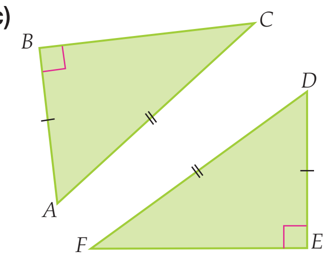
Q1 Columns 1 & 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 1, Q9, Q11 (a), (b), Q16	Q1 Columns 1 & 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q8 Column 2, Q9, Q11 (a)–(d), Q12, Q14, Q16	Q1 Columns 2 & 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 2, Q9, Q10, Q11, Q12, Q13, Q14, Q15	Answers page 636

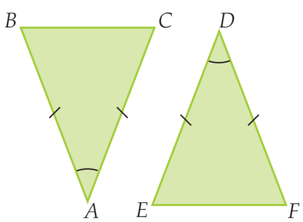
Fluency

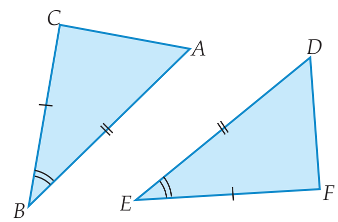
1 Show that each of the following pairs of triangles are congruent, stating the congruency test used in each. WE2

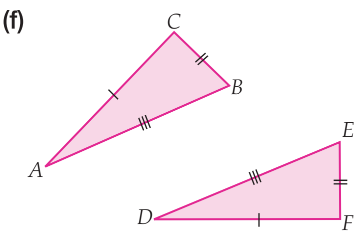
(a)


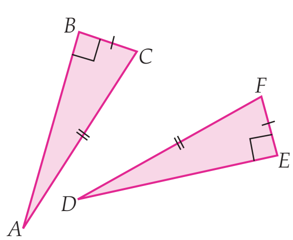
(b)


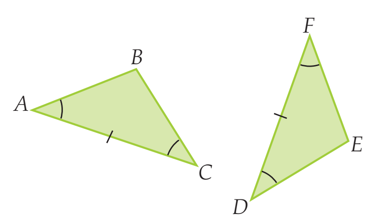
(c)


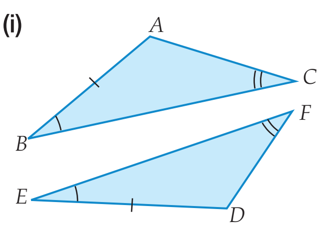
(d)


(e)


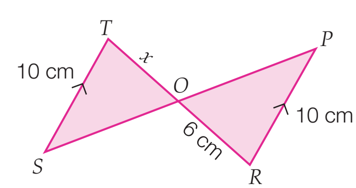
(f)


(g)


(h)


(i)


2 Prove that $\Delta STO \equiv \Delta POR$ and hence find the value of x. WE3

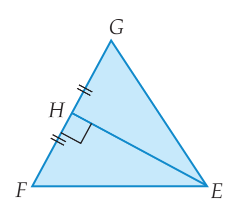


3 (a) In ΔEFG, $FH = HG$ and EH is perpendicular to FG. Prove that $\angle EFH = \angle EGH$. WE4

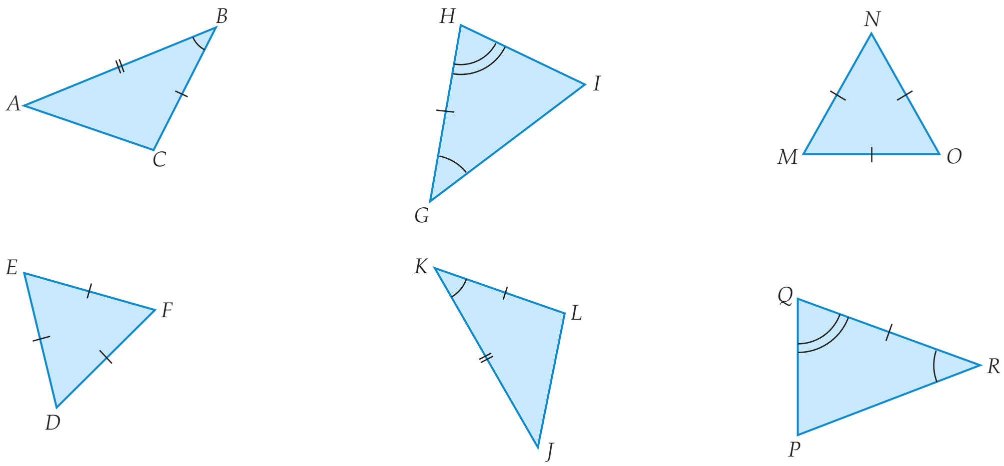
(b) What type of triangle is ΔEFG? Explain your answer.

(c) What triangle property have you just proved?

(d) Show that $EF = EG$.



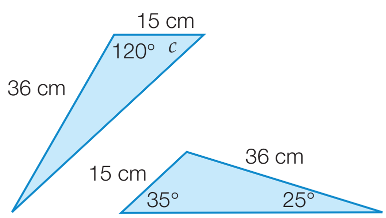
4 Find pairs of congruent triangles among the following diagrams. State the test used to show that they are congruent. Letter names must be in the correct order.



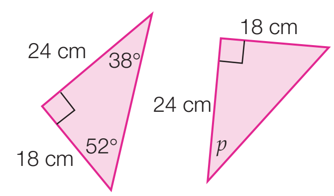
5 **(a)** State the test used to prove the following pairs of triangles are congruent.

(b) Use congruency to find the value of the unknown in each pair of triangles.

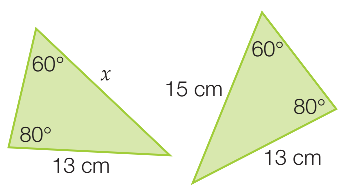
(i)



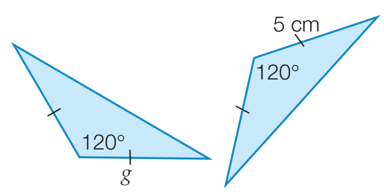
(ii)



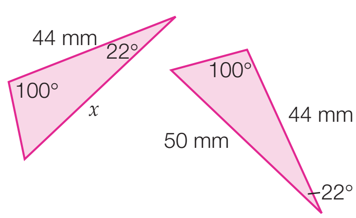
(iii)



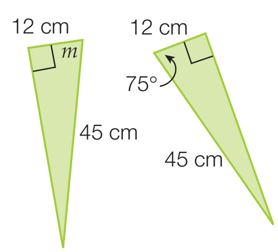
(iv)



(v)



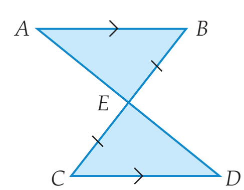
(vi)



Understanding

6 Explain why two triangles with three equal angles may not necessarily be congruent. Show this by drawing diagrams.

7 Complete the following proof by copying it and writing in the missing steps.



$AB \parallel CD$. AD and BC intersect at E so that $BE = EC$.
Prove that $AB = CD$.

Proof:

In Δs ABE and DCE,

∠________ = ∠________ (alternate angles, $AB \parallel CD$)

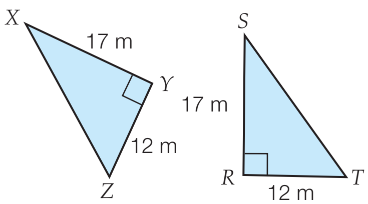
________ = ________ (given)

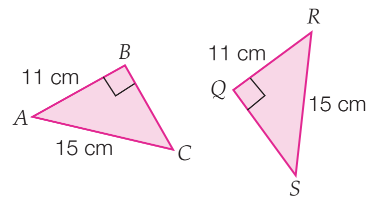
∠________ = ∠________ (vertically opposite angles)

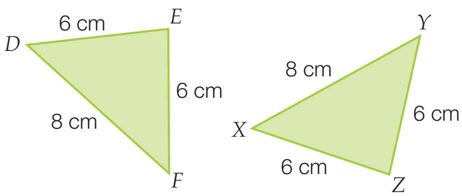
∴ Δ________ ≡ Δ________ (AAS congruency)

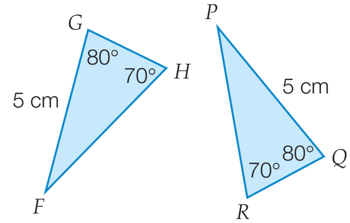
∴ ________ = ________ (matching sides of congruent triangles)

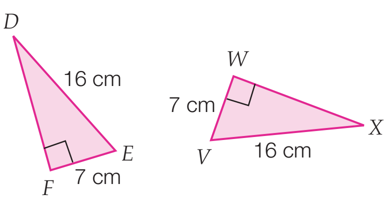
8 Prove that each of the following pairs of triangles is congruent, giving reasons for your answer.

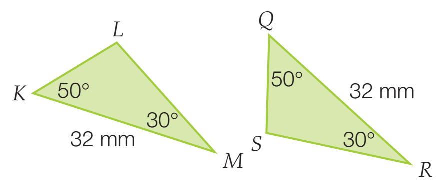
(a)


(b)


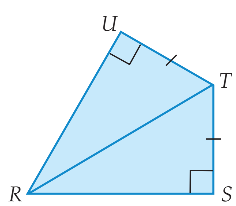
(c)


(d)


(e)


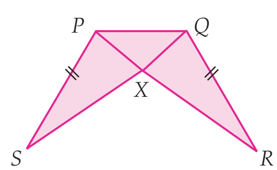
(f)


9 Prove that $\Delta RUT \equiv \Delta RST$.

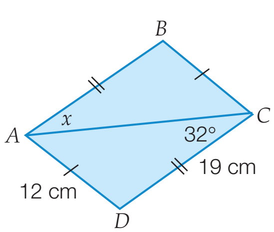


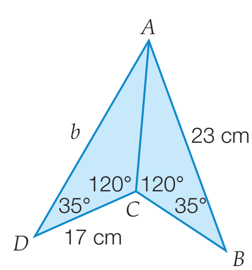
Reasoning

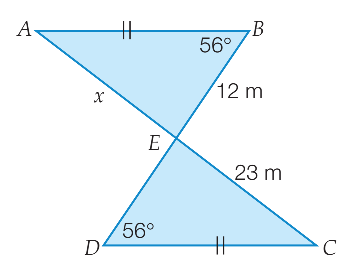
10 If $SP = RQ$ and $\angle SPQ = \angle RQP$, prove that $\Delta SPQ \equiv \Delta RQP$.

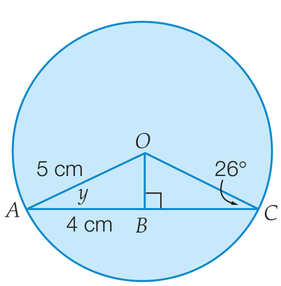


11 Prove that the following pairs of triangles are congruent and hence find the value of the variables.

(a)


(b)


(c)


(d)


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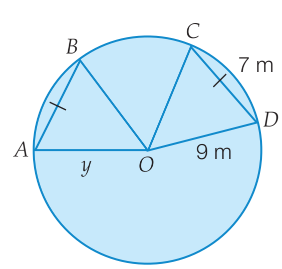

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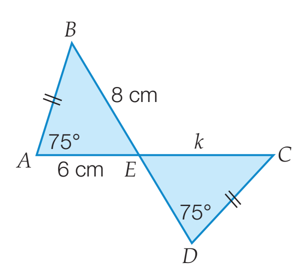
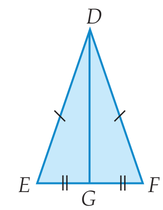


12 ΔDEF is an isosceles triangle with $DE = DF$. G is the midpoint of EF.

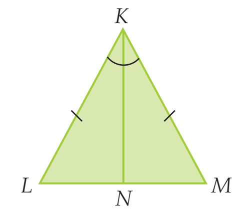
(a) Show that $DG \perp EF$.

(b) If ΔDEF is equilateral, prove that $\angle EDG = \angle FDG = 30°$.

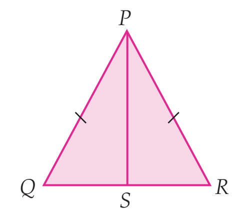


13 ΔKLM is an equilateral triangle with $KL = LM = MK$. N is a point on LM such that $\angle LKN = \angle MKN$.

Show that KN is the perpendicular bisector of LN (i.e. $LN = NM$ and $KN \perp LM$).



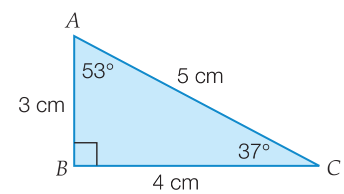
14 ΔPQR is an equilateral triangle with $PQ = QR = RP$. S is a point on QR that is joined to P. Explain why this is not enough information to prove that $QS = SR$.



Open-ended

15 Draw four pairs of congruent triangles and put just enough information on each diagram to show congruence. Each pair should represent a different test. Write the name of the test that applies to each pair of triangles.

16 ΔABC is right angled at B. The lengths of its sides and the size of its angles are shown on the figure. Draw four triangles with enough information marked on them so that each one is congruent with ΔABC using each of the congruence tests.



Outside the Square

Puzzle

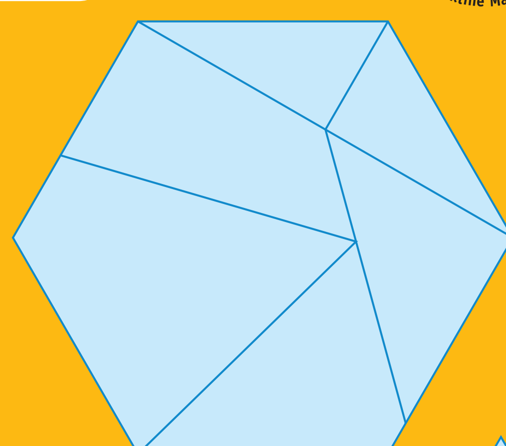
Blackline Master

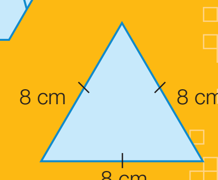
Hexagon jigsaw

Equipment required: Hexagon Jigsaw template (available on Pearson Reader), paper, scissors

By cutting a regular hexagon into pieces, it can be transformed into other polygons. Photocopy or trace the hexagon shown and cut it out. Divide the hexagon into pieces by cutting along the lines. Rearrange the pieces into an equilateral triangle with side lengths of 8 cm. Some pieces may need to be reflected to make the transformation.

Can a hexagon be cut in such a way that it can be rearranged into other polygons? Try transforming a hexagon into a rectangle, square, parallelogram and irregular quadrilateral.





6.3 Special quadrilaterals and their properties

Polygons

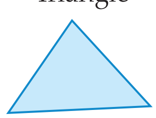
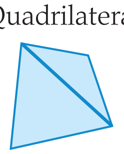
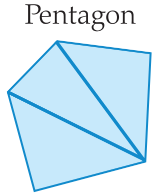
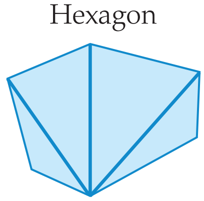
A **polygon** is a plane figure with many straight sides, so a circle is not a polygon.

A regular polygon has all its sides and interior angles equal. An equilateral triangle is the only regular three-sided polygon and a square is the only regular four-sided polygon. No other regular polygons have special names (e.g. a regular hexagon is a six-sided polygon with all sides equal).

Angle sum of a polygon

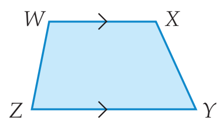
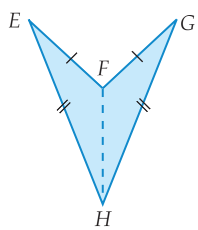
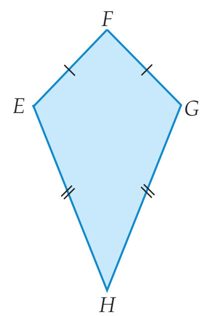
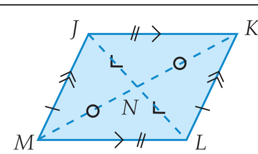
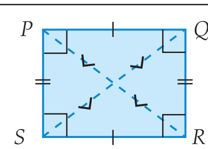
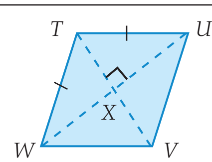
If we divide a polygon into triangles we can see that there are always two fewer triangles than there are sides. This gives us a useful rule.

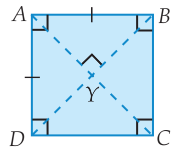
The sum of the interior angles of a polygon with n sides = $(n-2)\times 180°$.

Shape	n	Angle sum using triangles	Angle sum using $(n-2)\times 180°$
Triangle 	3	$1\times 180° = 180°$	$(n-2)\times 180°$ $=(3-2)\times 180°$ $=1\times 180°$ $=180°$
Quadrilateral 	4	$2\times 180° = 360°$	$(n-2)\times 180°$ $=(4-2)\times 180°$ $=2\times 180°$ $=360°$
Pentagon 	5	$3\times 180° = 540°$	$(n-2)\times 180°$ $=(5-2)\times 180°$ $=3\times 180°$ $=540°$
Hexagon 	6	$4\times 180° = 720°$	$(n-2)\times 180°$ $=(6-2)\times 180°$ $=4\times 180°$ $=720°$

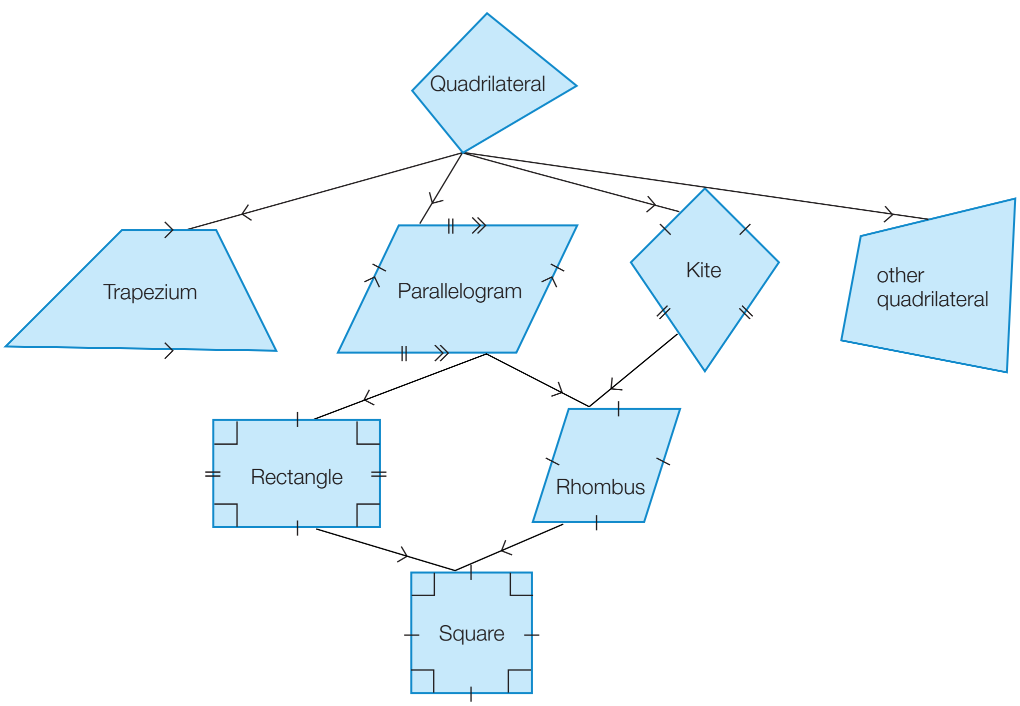
Definitions and properties

A definition is the minimum amount of information needed to describe a figure exactly. A property is an attribute or characteristic of a figure than can be proved to be always true after it has been defined. In this section, we will use congruence to prove the properties of special quadrilaterals.

Shapes and definitions	Properties
Trapezium: A trapezium is a quadrilateral with only one pair of opposite sides parallel.  *WXYZ* is a trapezium in which $WX \parallel ZY$.	A trapezium has no special properties.
Kite: A kite is a quadrilateral with two pairs of adjacent sides equal. *EFGH* is a kite in which $EF = FG$, $EH = HG$.  Concave kite  Convex kite	• The pair of angles between the unequal sides are equal: $\angle FEH = \angle FGH$ • The angles between the equal sides are bisected by the diagonal through them. • The diagonal through the equal angles is an axis of symmetry. • The diagonals are perpendicular: $FH \perp EG$ • The diagonal that passes through the equal angles is bisected by the axis of symmetry.
Parallelogram: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.  *JKLM* is a parallelogram in which $JK \parallel ML$, $JM \parallel KL$.	• Both pairs of opposite sides are equal: $JK = ML$, $JM = KL$ • Both pairs of opposite angles are equal: $\angle MJK = \angle MLK$, $\angle JML = \angle JKL$ • The diagonals bisect each other $JN = NL$, $MN = NK$
Rectangle: A rectangle is a parallelogram with a right angle.  *PQRS* is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus: A rhombus is a parallelogram with two adjacent sides that are equal.  *TUVW* is a rhombus in which $TU = TW$ (or a kite with all sides equal).	A rhombus has all the properties of a parallelogram, plus: • all the sides are equal: $TU = TW = WV = UV$ • the diagonals bisect the angles of the rhombus: $\angle WTV = \angle UTV$, $\angle WVT = \angle UVT$, $\angle TWU = \angle VWU$, $\angle TUW = \angle VUW$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$, $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$

Shapes and definitions	Properties
Square:  A square is a rectangle with one pair of adjacent sides equal. *ABCD* is a square in which $AB = AD$. or A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90°$.	A square has all the properties of a rectangle, and all the properties of a rhombus: • all the sides are equal: $AB = AD = DC = BC$ • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90°$ • the two diagonals and the two lines joining the midpoints are axes of symmetry • the diagonals are equal: $AC = BD$

Pictorial summary of the quadrilateral family

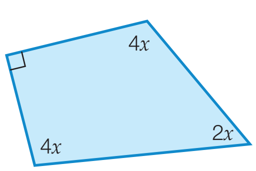


Worked Example 5

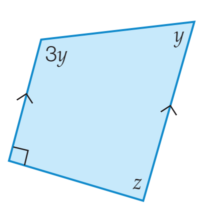
WE5

Find the value of the variables in each figure.

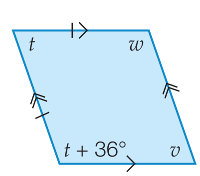
(a)



(b)



(c)

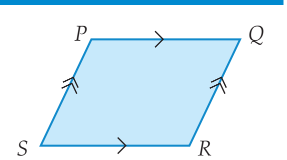


Thinking	Working
(a) 1 Identify the shape.	(a) This is a quadrilateral.
2 Form an equation with a reason.	$2x + 4x + 4x + 90° = 360°$
3 Solve the equation.	$10x + 90° = 360°$ $10x = 270°$ $x = 27°$
(b) 1 Identify the shape.	(b) This is a trapezium.
2 Form an equation with a reason.	$y + 3y = 180°$ (co-interior angles between parallel lines)
3 Solve the equation.	$4y = 180°$ $y = 45°$ $z + 90° = 180°$ (co-interior angles between parallel lines) $z = 90°$
(c) 1 Identify the shape.	(c) The figure is a parallelogram with one pair of adjacent sides equal; hence, it is a rhombus.
2 Form an equation with a reason.	$t + t + 36° = 180°$ (co-interior angles are equal)
3 Solve the equation.	$2t + 36° = 180°$ $2t = 144°$ $t = 72°$
4 Repeat for the other angles.	$v = 72°$ (opposite angles of a rhombus are equal) $w = 108°$ (interior angles between parallel lines)
5 List all answers.	$t = 72°$, $v = 72°$, $w = 108°$

Worked Example 6

WE6

PQRS is a parallelogram is which $PQ \parallel SR$ and $PS \parallel QR$.

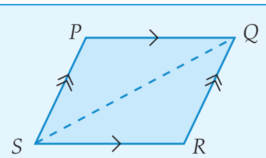


(a) Use congruence to prove that $\angle SPQ = \angle QRS$.

(b) Prove that $PQ = SR$ and $PS = QR$.

(c) Prove that $\angle PSR = \angle RQP$.

(d) What properties of a parallelogram have you proved?

Thinking	Working
(a) 1 Copy the diagram and join *SQ*.	(a) 
2 Name the two triangles that you will try to prove congruent.	ΔSPQ and ΔQRS

3 List three sets of equalities, with reasons, that apply to the two triangles.	$\angle PQS = \angle QSR$ (alternate angles, $PQ \parallel SR$) $\angle PSQ = \angle SQR$ (alternate angles, $PS \parallel QR$) QS is a common side
4 Write down the pair of congruent triangles and the test used.	$\Delta SPQ \equiv \Delta QRS$ (AAS)
5 Identify appropriate pairs of matching angles for the required result.	$\angle SPQ = \angle QRS$ (matching angles in congruent triangles)
(b) Identify appropriate pairs of matching sides.	(b) $PQ = SR$ (matching sides of congruent triangles) $PS = QR$ (matching sides of congruent triangles)
(c) Explain that the same method used in part **(a)** can prove congruency if the other diagonal is drawn.	If the diagonal PR is drawn, then the same steps as in (a) can be used to prove that $\Delta PSR \equiv \Delta RQP$. $\angle PSR = \angle RQP$ (matching angles in congruent triangles)
(d) State the properties.	(d) The opposite angles of a parallelogram are equal. The opposite sides of a parallelogram are equal.

Worked Example 7

WE7

$ABCD$ is a rhombus in which $AB \parallel DC$ and $AD \parallel BC$. The diagonals AC and BD intersect at E.

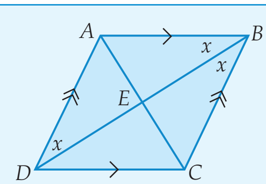
(a) Prove that $\angle ABD = \angle CBD$.

(b) Prove that $\Delta ABE \equiv \Delta CBE$.

(c) Prove that $\angle AEB = \angle CEB = 90°$.

(d) Prove that $AE = CE$.

(e) What properties of a rhombus have you proved?

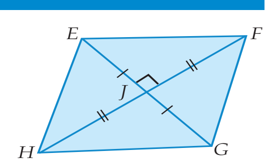
Thinking	Working
(a) 1 Draw a diagram and let $\angle ABE = x$.	(a) 

2 Explain what you marked on the diagram.	$\angle CDB = \angle ABD = x$ (alternate angles, $AB \parallel DC$) $BC = CD$ (adjacent sides of a rhombus) ΔBCD is isosceles $\angle CBD = \angle CDB = x$ (base angles of an isosceles triangle)
3 State the result you have shown.	$\angle ABD = \angle CBD$ (both equal x)
(b) 1 Name the two triangles you will try to prove congruent.	(b) Consider ΔABE and ΔCBE.
2 List three sets of equalities, with reasons, that apply to the two triangles.	$AB = BC$ (adjacent sides of a rhombus) $\angle ABE = \angle CBE$ (proved in (a)) BE is a common side
4 Write down the pair of congruent triangles and the test used.	$\Delta ABE \equiv \Delta CBE$ (SAS)
(c) 1 Identify appropriate pairs of matching angles for the required result.	(c) $\angle AEB = \angle CEB$ (matching angles in congruent triangles)
2 State what else you know about these angles.	$\angle AEB + \angle CEB = 180°$ ($\angle AEC = 180°$)
3 Make a conclusion.	$\angle AEB = 180°$ $\angle AEB = 90°$ $\angle AEB = \angle CEB = 90°$
(d) Identify a pair of matching sides.	(d) $AE = CE$ (matching sides in congruent triangles)
(e) Use the results from parts **(a)**, **(c)** and **(d)** to state the properties of a rhombus	(e) The diagonals bisect the angles of a rhombus. The diagonals of a rhombus intersect at right angles. One diagonal bisects the other diagonal.

Worked Example 8

WE8

The diagonals of the quadrilateral *EFGH* bisect each other at right angles at *J*. Prove that *EFGH* is a rhombus.



Thinking	Working
1 List the information given in the question.	$EJ = GJ$, $HJ = FJ$, $\angle FJG = \angle HJG = \angle HJE = \angle FJE = 90°$

2	Use the definition of the shape to identify what needs to be proved. (A rhombus is a parallelogram with adjacent sides equal.)	In the quadrilateral the pairs of opposite sides must be parallel and a pair of adjacent sides must be equal.
3	Identify the triangles you will use to prove congruency.	Consider ΔEJF and ΔGJH
4	List three equalities, with reasons, from these triangles.	$EJ = GJ$ (given) $\angle EJF = \angle GJH$ (both $90°$) $FJ = HJ$ (given)
5	Write down the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJH$ (SAS)
6	Identify a pair of matching angles.	$\angle EFJ = \angle GHJ$ (matching angles in congruent triangles)
7	Use you knowledge of angles on parallel lines to identify a pair of parallel lines (alternate angles).	$\angle EFH$ and $\angle GHF$ are a pair of equal alternate angles contained between EF and HG. $\therefore EF \parallel HG$
8	Identify another pair of triangles to be proved congruent and use the same methods as in Steps **4**–**7** identify another pair of parallel lines (alternate angles).	Similarly, $\Delta EJH \equiv \Delta GJF$ $\angle EHF = \angle HFG$. $\angle EHF$ and $\angle HFG$ are a pair of equal alternate angles contained between EF and HG. $\therefore EH \parallel FG$
9	Identify a third pair of triangles to be proved congruent.	Consider ΔEJF and ΔGJF.
10	List three equalities, with reasons, from these triangles.	$EJ = GJ$ (given) $\angle EJF = \angle GJF$ (both $90°$) JF is a common side
11	Write down the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJF$ (SAS)
12	Identify a pair of matching sides.	$EF = GF$ (matching sides in congruent triangles)
13	Write down the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJH$ (SAS)
14	Summarise what has been shown.	$EF = GF$ are a pair of equal adjacent sides of $EFGH$ and $EF \parallel HG$, $EH \parallel FG$ gives both pairs of opposite sides parallel. Hence, $EFGH$ is a parallelogram with a pair of adjacent sides equal, so $EFGH$ is a rhombus.

Use the definitions of each type of quadrilateral in your proofs.

6.3 Special quadrilaterals and their properties

Navigator

Answers page 637

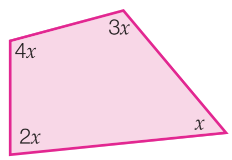
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q16, Q17, Q18, Q25	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q18, Q19, Q21, Q23, Q25	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19, Q20, Q21, Q22, Q23, Q24

Fluency

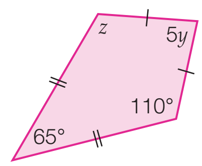
WE5

1 Find the value of the variables in each figure.

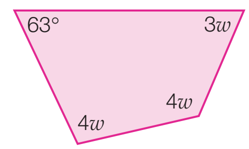
(a)



(b)

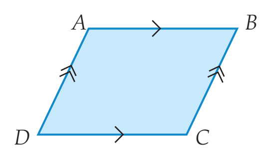


(c)



WE6

2 *ABCD* is a parallelogram is which $AB \parallel DC$ and $AD \parallel BC$.



(a) Use congruence to prove that $\angle DAB = \angle BCD$.

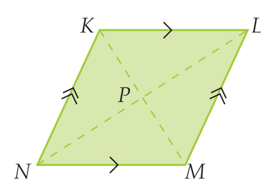
(b) Prove that $AB = DC$ and that $AD = BC$.

(c) Prove that $\angle ADC = \angle CBA$.

(d) What properties of a parallelogram have you proved?

WE7

3 *KLMN* is a rhombus in which $KL \parallel NM$ and $KN \parallel LM$. The diagonals *KM* and *LN* intersect at *P*.



(a) Prove that $\angle KLN = \angle MLN$.

(b) Prove that $\Delta KLP \equiv \Delta MLP$.

(c) Prove that $\angle KPL = \angle MPL = 90°$.

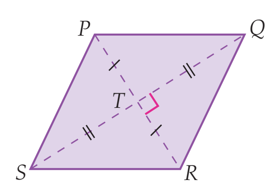
(d) Prove that $KP = MP$.

(e) What properties of a rhombus have you proved?

WE8

4 The diagonals of the quadrilateral *PQRS* bisect each other at right angles at *T*.

Prove that *PQRS* is a rhombus.

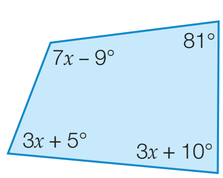


5 The size of each of the angles in a regular pentagon is:

A 60° **B** 90° **C** 108° **D** 120°

6 Find the value of the variables in each figure.

(a)



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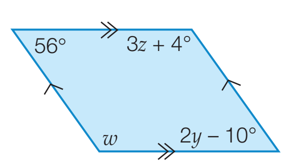

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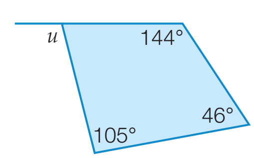


Understanding

7 Write True or False for each statement.

(a) A trapezium has one pair of opposite sides equal and parallel.

(b) A square is a special rectangle.

(c) The diagonals of a kite are equal in length.

(d) The diagonals of a rhombus bisect each other at right angles.

8 In which regular polygon is each interior angle equal to 120°?

9 Which statement is True?

A A kite has one pair of opposite angles equal.

B A rectangle is a special kite.

C A rhombus is a rectangle.

D A rhombus is a special trapezium.

10 Which statement is False?

A All parallelograms have two pairs of parallel sides.

B All trapeziums have one pair of parallel sides of equal length.

C All kites have a pair of equal angles.

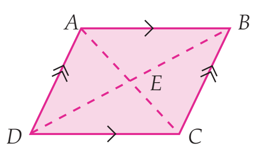
D All rectangles have a right angle.

11 (a) What is the sum of the angles of an octagon?

(b) What is the size of each interior angle of a regular octagon?

Understanding

12 *ABCD* is a parallelogram whose diagonals intersect at *E*.
Using triangles *ADC* and *CBA*, prove that $\angle ADC = \angle CBA$.



(a) Copy and complete this proof by putting in the missing reasons.

Proof:

In Δs *ADC* and *CBA*,
$\angle ACD = \angle CAB$ (_______ $AB \parallel DC$)
$\angle DAC = \angle BCA$ (_______ AD___BC)
AC is a _______
$\therefore\ \Delta ADC \equiv \Delta CBA$ (_______)
$\therefore\ \angle ADC = \angle CBA$ (_______)

(b) Show that $\angle DAB = \angle BCD$.

13 In Question **12** you have proved that $\Delta ADC \equiv \Delta CBA$. Using this result:

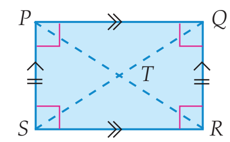
(a) show that the opposite angles of a parallelogram are equal.

(b) show that the opposite sides of a parallelogram are equal.

(c) if the diagonals intersect at Point E, prove that $\Delta AEB \equiv \Delta CED$.

(d) show that the diagonals of a parallelogram bisect each other.

14 *PQRS* is a rectangle in which $PS = QR$ and PR and QS intersect at T. Prove that $\Delta PRS \equiv \Delta QSR$.



(a) Copy and complete this proof by filling in the missing steps.

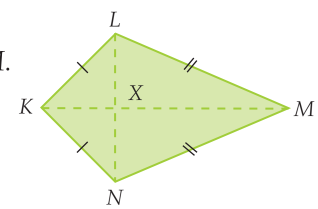
Proof:

In Δs *PRS* and *QSR*:
$\angle$_______ $=$ $\angle$_______ $= 90°$ (given)
$\angle$_______ $=$ $\angle$_______ (alternate angles, $PS \parallel QR$)
_______ $=$ _______ (given)
$\therefore$ Δ_______ $\equiv$ Δ_______ (AAS)

(b) Hence prove that $PR = QS$.

(c) What special property of a rectangle did you prove in **(b)**?

15 *KLMN* is a kite. *KM* and *LN* intersect at *X*.



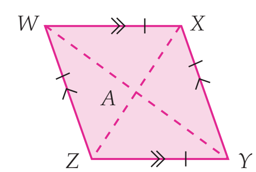
(a) Use triangles *KLM* and *KNM* to prove that $\angle KLM = \angle KNM$.

(b) Prove that $\Delta LKX \equiv \Delta NKX$.

(c) Show that $\angle LXK = \angle NXK = 90°$.

(d) What properties of a kite have you proved?

16 *WXYZ* is a rhombus as shown in the diagram. *WY* and *XZ* intersect at *A*.

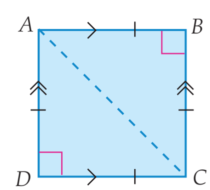


(a) Prove that $\angle WYX = \angle WYZ$.

(b) Explain why $\angle WZX = \angle YZX$.

(c) What property of a rhombus have you proved?

17 *ABCD* is a square as shown in the diagram. *AC* is a diagonal.

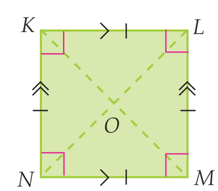


(a) Prove that $\angle ACD = \angle ACB$.

(b) Prove that $\angle ACD = 45°$.

(c) What properties of a square have you proved?

18 *KLMN* is a square as shown. *KM* and *LN* intersect at *O*.



(a) Prove that $KM = NL$.

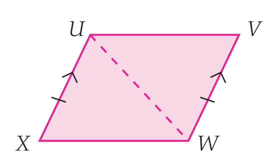
(b) Prove that $KO = OM$ and that $NO = OL$.

(c) Prove that $\angle KOL = 90°$

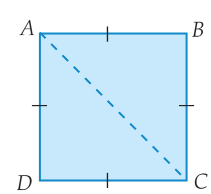
(d) What properties of a square have you proved?

Reasoning

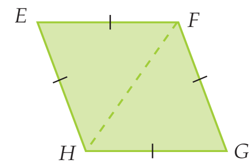
19 *UVWX* is a quadrilateral in which $UX \parallel VW$ and $UX = VW$. Prove that *UVWX* is a parallelogram.



20 *ABCD* is a quadrilateral in which all the sides are equal and $\angle ABC = 90°$. Prove that *ABCD* is a square.



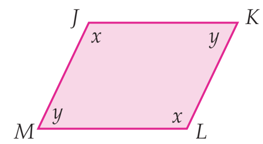
21 *EFGH* is a quadrilateral in which all the sides are equal. Prove that *EFGH* is a rhombus.



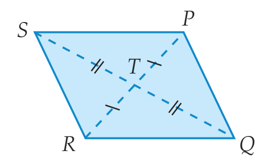
22 *JKLM* is a quadrilateral in which both pairs of opposite angles are equal; that is, $\angle MJK = \angle MLK$ and $\angle JML = \angle JKL$.

(a) Prove that $\angle MJK + \angle JML = 180°$.

(b) Hence prove that *JKLM* is a parallelogram.



23 *SPQR* is a quadrilateral in which the diagonals bisect each other at *T*, as shown. Prove that *SPQR* is a parallelogram.



Open-ended

24 (a) Simon knows that the statement 'All squares have equal sides' is true. He wonders whether the converse (opposite) statement 'all shapes with four equal sides are squares' is true. Explain to him whether it is true or not.

(b) Select another true statement about a special quadrilateral. Write the converse statement and explain whether it is true or not.

25 Explain why a square is a rectangle but a rectangle is not necessarily a square.

Outside the Square Puzzle

Matchstick quadrilaterals

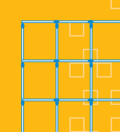
Equipment required: matches (optional)

1 With five matchsticks, three quadrilaterals can be made, as shown.

(a) With six matchsticks, make six quadrilaterals.

(b) With six matches, make nine quadrilaterals

2 Which six matches should you remove, without changing the position of the others, so that only three squares are left?

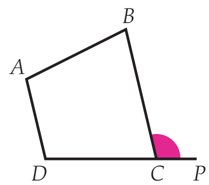


Investigation

Exterior angle sum of polygons

Equipment required: ruler, protractor

An exterior angle is formed if we extend one side of a polygon out from a vertex (corner). An exterior angle will always be less than 180°. In the diagram, side *DC* has been extended from *C* out to *P*. The angle *PCB* is one of the exterior angles of the polygon.

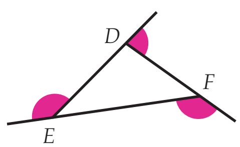


The Big Question

Is the sum of the exterior angles of a polygon always the same for all polygons? If so, why?

Engage

1 Draw a triangle and extend each of the sides to form exterior angles in a 'Catherine wheel' arrangement as shown in the diagram. Your diagram should be at least half a page in size.

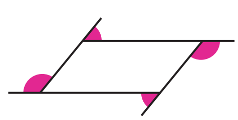


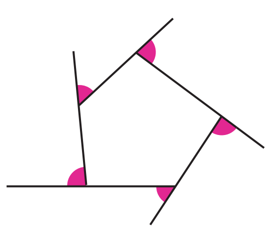
Measure each of the exterior angles and add them up.

2 Draw a different triangle. Make an exterior angle on each vertex and measure them using a protractor. What is the total sum of the exterior angles?

Explore

3 Find the sum of the exterior angles of a parallelogram, kite, pentagon and hexagon by drawing the shape and extending each side past the vertex, as shown. Measure the exterior angles formed in each shape and add them together.





Strategy options

- Draw a diagram.
- Look for a pattern.

Explain

4 On a sheet of looseleaf paper, draw any convex polygon (a polygon with all interior angles less than 180°). Mark the exterior angles of this polygon and cut the angles out. Place all vertices together. What do you notice?

5 Write a sentence describing what you have found regarding the sum of exterior angles of convex polygons.

6 Use a diagram to explain why this property does not apply to concave polygons (polygons with one or more interior angles greater than 180°).

Elaborate

7 By exploring the sum of the exterior angles of triangles, quadrilaterals, pentagons and hexagons, find a rule for calculating the sum of the exterior angles of convex polygons.

Sum of the exterior angles of a convex polygon =

____

8 Use your rule to find the sum of the exterior angles of a regular icosagon (20-sided polygon)? (A regular polygon has all sides of equal length.)

9 Answer the Big Question by summarising your findings, clearly identifying the types of polygons for which this rule works.

Evaluate

10 Do you believe your findings are correct for polygons of any number of sides? Justify your answer.

11 Do you believe that the number and variety of shapes you studied is enough to give a good indication that the rule you discovered is correct? How could you make your investigation of the sum of exterior angles in polygons more thorough?

Extend

12 **(a)** Find a rule to calculate the size of each exterior angle of any regular polygon where n is the number of sides of the polygon.

Hint

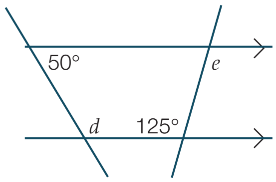
Remember that an exterior angle of a triangle is equal to the sum of the two opposite interior angles.

(b) Use this rule to show that the size of each interior angle of any regular polygon is $\frac{180(n-2)}{n}$, where n is the number of sides of the polygon.

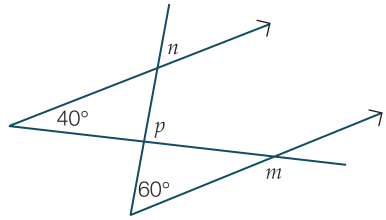
Half-time 6

1 Find the value of the variables in each figure. **Ex. 6.1**

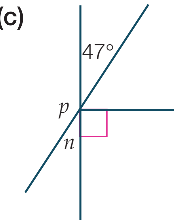
(a)



(b)

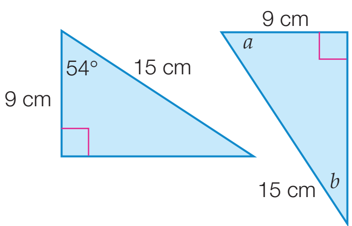


(c)

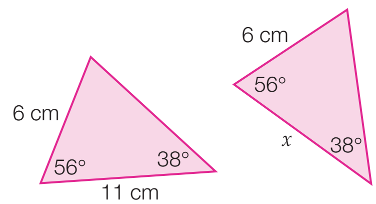


2 Find the value of the variables in each pair of congruent triangles. (State the congruency test used.) **Ex. 6.2**

(a)



(b)



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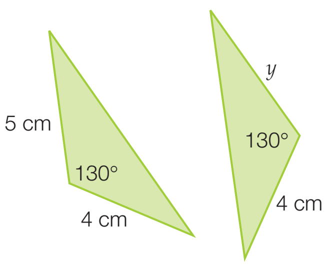
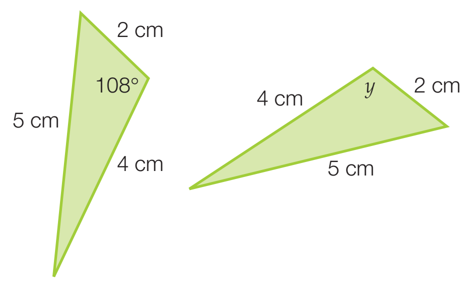
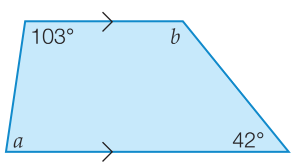


(d)

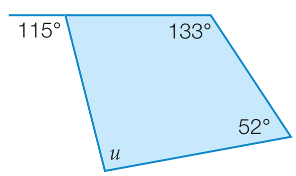


3 Find the value of the variables in each figure. **Ex. 6.3**

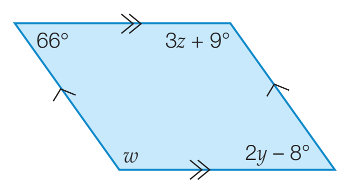
(a)



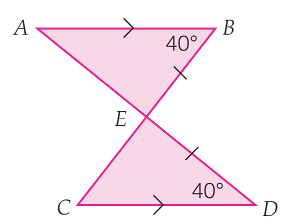
(b)



(c)



4 Complete the following proof by copying it and writing in the missing steps. **Ex. 6.2**



AD and BC intersect at E so that $BE = ED$. If $\angle ABC = \angle CDA = 40°$, prove that $AB = CD$.

Proof:

In Δs ABE and CDE,

_________ = _________ = 40° (given)

_________ = _________ (given)

_________ = _________ (vertically opposite angles)

$\therefore$ Δ_________ $\equiv$ Δ_________ (ASA)

$\therefore$ _________ = _________ (matching sides of congruent triangles)

Ex.6.1

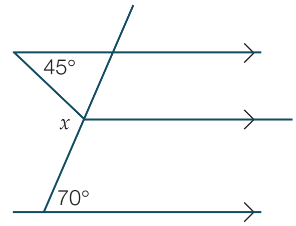
5 The value of x in the diagram is:

A 45°

B 65°

C 70°

D 115°



Ex.6.3

6 Which statements are true and which statements are false? Justify your answer.

(a) All rectangles are squares.

(b) All parallelograms are quadrilaterals.

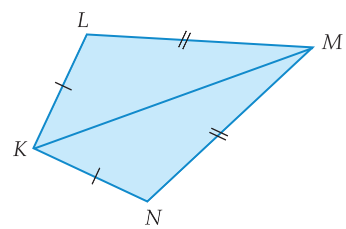
(c) All squares have equal diagonals.

(d) The diagonals of a parallelogram bisect each other at right angles.

Ex.6.2

7 Prove that $\Delta KLM \equiv \Delta KNM$.

List all the pairs of equal angles.



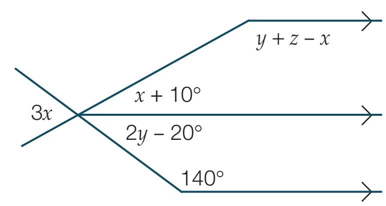
Ex.6.3

8 The diagonals of a quadrilateral are equal. Draw all the possible different quadrilaterals, naming each type.

Ex.6.1

9 Find the value of the variables in the diagram.

Give reasons for your answers.

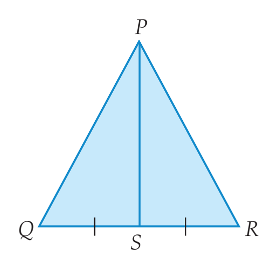


Ex.6.2

10 In ΔPQR, S is a point on QR such that $QS = SR$.

Explain why this is not enough information to prove that ΔQPS is congruent to ΔRPS.

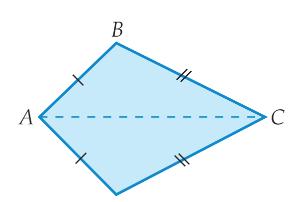
Write down what additional information you would need to prove $\Delta QPS \equiv \Delta RPS$.



Ex.6.2, 6.3

11 (a) $ABCD$ is a kite with $AB = AD$ and $BC = DC$.
Construct the diagonal AC and then use congruency to show that $\angle ABC = \angle ADC$.

(b) What property of a kite have you proved?



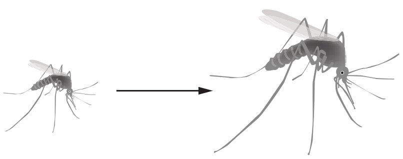
Enlarging and reducing

6.4

Maps, scale models and building plans are all based on scales. When drawing a map or constructing a model to scale, all measurements must be increased or decreased by the same multiplier; called the **scale factor**.

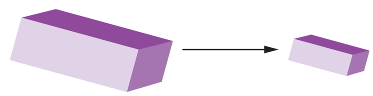
When we have both the original shape and the enlarged (or reduced) shape, we can easily find the scale factor by measuring the length of the corresponding sides of each shape.

Enlargement



A scale factor of 3 means the drawing is three times as big as the original. Every length on the enlarged shape is three times the length of the matching side on the original shape.

Reduction



A scale factor of $\frac{1}{2}$ means that the drawing is half the size of the original. Every length on the reduced shape is half the length of the matching side on the original shape.

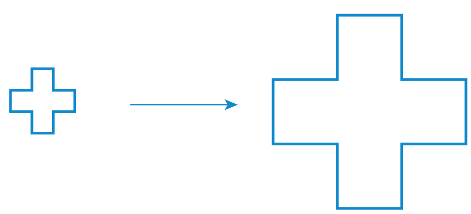
If the scale factor is greater than 1, the resulting figure is an enlargement.

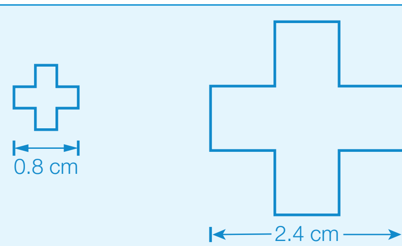
If the scale factor is less than 1, the resulting figure is a reduction.

Worked Example 9

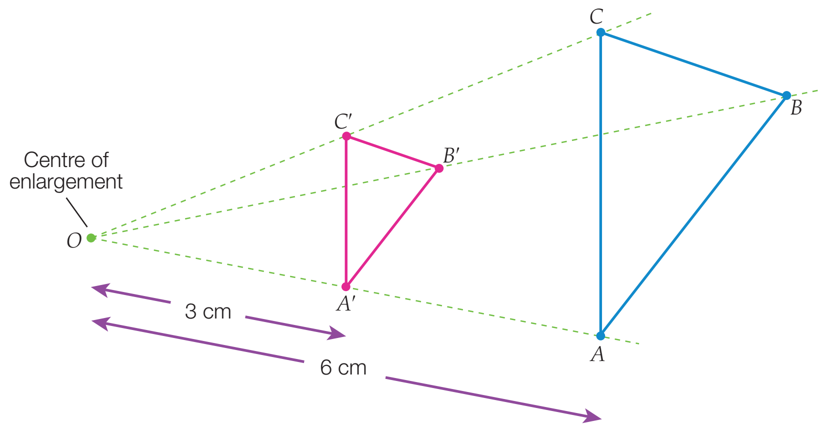
WE9

What scale factor has been used in this enlargement?



Thinking	Working
1 Choose matching sides on both shapes and measure each length.	
2 Determine the ratio of the enlarged measured length to the original measured length. The new length is always written first.	Enlarged length : original length 2.4 : 0.8 = 3 : 1
3 Calculate the scale factor.	Scale factor = 3

To enlarge or reduce a shape, we first need to choose a point of reference, known as the **centre of enlargement** (called O). We measure the distance from O to each of the points on the original shape and multiply that distance by the scale factor to determine the distance from O to the matching corners of the enlarged or reduced shape.



The original triangle ABC has been reduced by a scale factor of $\frac{1}{2}$ to produce triangle $A'B'C'$.

Note that the points on the new enlarged or reduced shape are labelled with the same letter as the matching point on the original shape, but with a dash (′) after the point. For example Point A on the original triangle ABC becomes Point A' on the reduced triangle $A'B'C'$. A' is called A dash or A prime.

Scale factor and ratio

If we measure the side lengths of the reduced triangle and compare them with the matching side lengths of the original triangle ABC, we find that each pair of sides is in the same ratio:

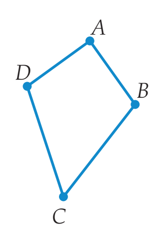
$A'B' : AB$	$A'C' : AC$	$B'C' : BC$
$= 2 : 4$	$= 2 : 4$	$= 1.3 : 2.6$
$= 1 : 2$	$= 1 : 2$	$= 2 : 4$
$= \frac{1}{2} : 1$	$= \frac{1}{2} : 1$	$= \frac{1}{2} : 1$

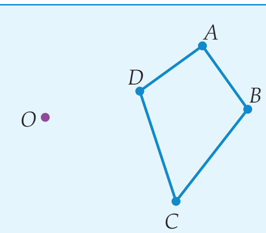
Written as a fraction, this ratio is the same as the scale factor, $\frac{1}{2}$.

Worked Example 10

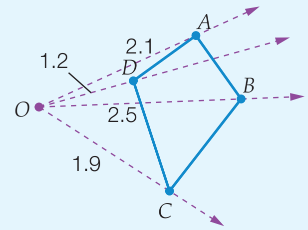
WE10

Enlarge the following shape by a scale factor of 2.

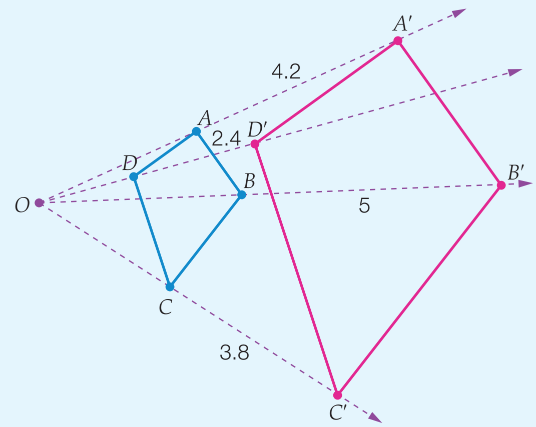


Thinking	Working
1 Select a centre of enlargement and label this point O.	

2 Rule lines from Point O through each vertex of the shape. Measure the distance from O to each of the vertices.
(Here $OA = 2.1$ cm, $OB = 2.5$ cm, $OC = 1.9$ cm and $OD = 1.2$ cm.)



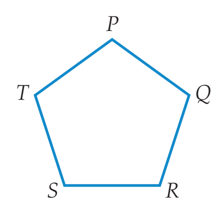
3 Multiply each distance you measured by the scale factor. Use a ruler to locate each of these new distances on the rays drawn from Point O and mark them as points. Rule lines to join these new points, which form the enlarged shape.
(Here $OA' = 4.2$ cm, $OB' = 5$ cm, $OC' = 3.8$ cm and $OD' = 2.4$ cm.)

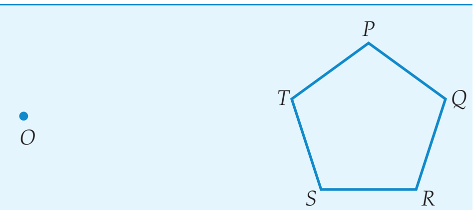
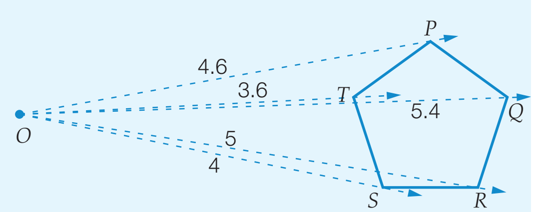
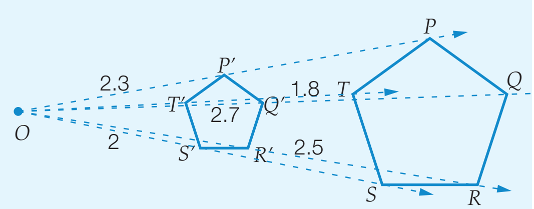


Worked Example 11

WE11

Reduce the following shape by a scale factor of $\frac{1}{2}$.



Thinking	Working
1 Select a centre of enlargement and label this Point O.	
2 Rule lines from Point O through each vertex of the shape. Measure the distance from O to each of the vertices.	
3 Multiply each distance by the scale factor. Locate each of these new distances on the rays drawn from Point O and mark them as points. Rule lines to join these new points, which form the reduced shape.	

6.4 Enlarging and reducing

Navigator

Answers page 640

Q1, Q2 (a), Q3 (a), Q4, Q5, Q7, Q9, Q10, Q12, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14	Q1, Q2 (a), Q3 (a), Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

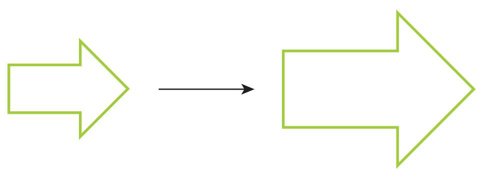
Equipment required: protractor for Question **8**, ruler for all

Fluency

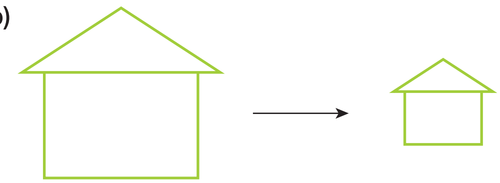
WE9

1 What scale factors have been used in each of the following enlargements or reductions?

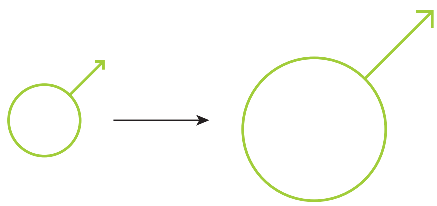
(a)



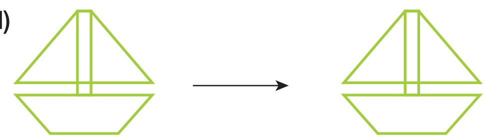
(b)



(c)



(d)

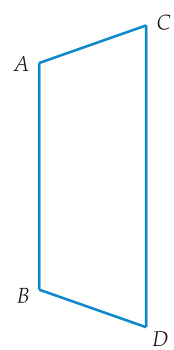


WE10

2 **(a)** Enlarge this shape by a scale factor of:

(i) 3

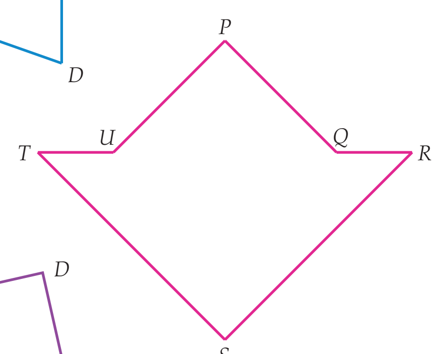
(ii) 1.5



(b) Enlarge this shape by a scale factor of:

(i) 4

(ii) $2\frac{1}{4}$

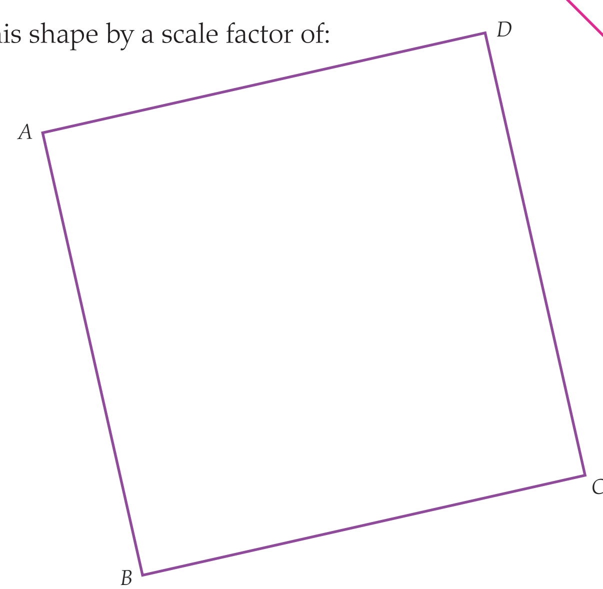


WE11

3 **(a)** Reduce this shape by a scale factor of:

(i) $\frac{1}{2}$

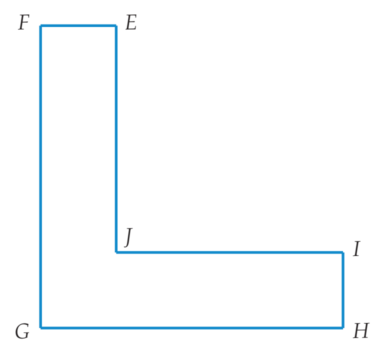
(ii) $\frac{1}{3}$



(b) Reduce this shape by a scale factor of:

(i) $\frac{3}{4}$

(ii) 0.6



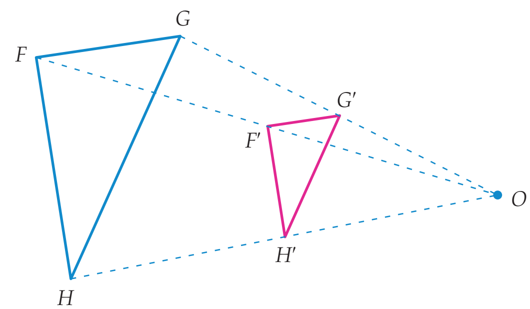
4 (a) ΔFGH has been reduced to $\Delta F'G'H'$. Measure each side of both triangles and find the following ratios.

(i) $F'G' : FG$

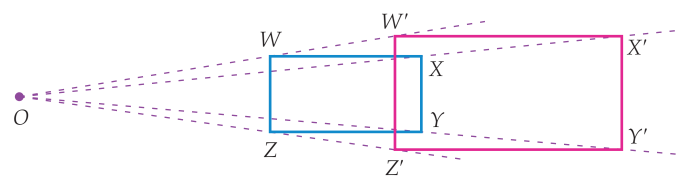
(ii) $F'H' : FH$

(iii) $G'H' : GH$

(b) What is the scale factor?



5 (a) Rectangle $WXYZ$ has been enlarged to rectangle $W'X'Y'Z'$. Measure each side of both rectangles and find the following ratios.



(i) $W'X' : WX$ (ii) $X'Y' : XY$ (iii) $Y'Z' : YZ$ (iv) $Z'W' : ZW$

(b) What is the scale factor?

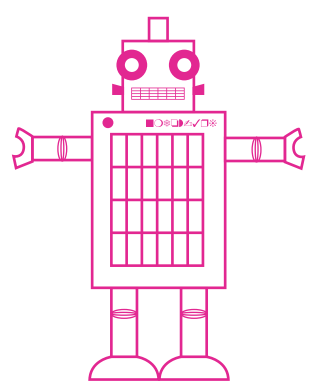
6 A scale factor of $\frac{1}{4}$ has been used to draw this toy robot. The actual height of the robot is:

A 1.2 cm

B 18 cm

C 19.2 cm

D 20 cm



Understanding

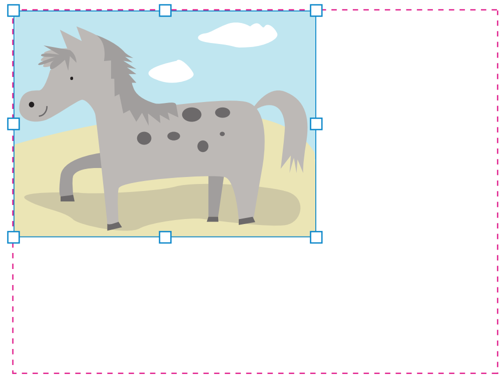
7 Rectangle $ABCD$ has dimensions of 2 cm × 3 cm and is to be enlarged by a scale factor of 3. Draw this rectangle and use a centre of enlargement to produce the enlarged shape.

8 A regular pentagon $JKLMN$ has a side length of 8 cm. Accurately construct this pentagon using a ruler and a protractor and enlarge it by a scale factor of $1\frac{1}{4}$ using a centre of enlargment, O. (Choose any reasonable point as point O.)

Hint

The internal angle of a regular pentagon is 108°.

9 Kylee is using her laptop to collect some images for a project. She has one rectangular image that measures 4.8 cm by 6.2 cm. By clicking and dragging her mouse on a corner of the image, Kylee enlarges it so that it now measures 16.8 cm by 21.7 cm.

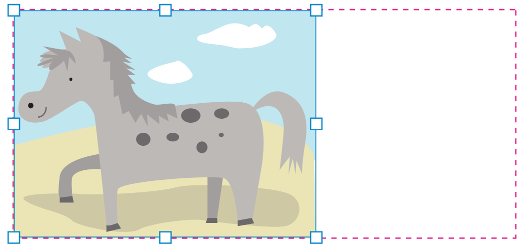


(a) What is the scale factor of this enlargement?

Later, Kylee finds that the image is now too big. She clicks and drags her mouse to reduce the image to 7.2 cm by 9.3 cm.

(b) What is the scale factor of this reduction of the enlarged image used in part **(a)**?

(c) Overall, has the original image been enlarged or reduced? Find the scale factor for this overall change.



(d) Kylee takes a copy of her original image and clicks and drags the middle of one of the sides instead of the corner.

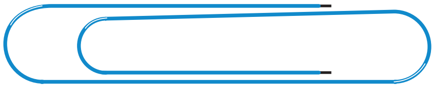
She finds that the length of the image changes from 4.8 cm to 10.2 cm, but the width remains the same as before.

Is this image a true enlargement of the original image? Explain why or why not.

Reasoning

10 The area of a square is 36 cm^2. What is the area of the square after the dimensions of the square have been reduced by a scale factor of $\frac{1}{3}$?

11 The picture on the right shows a scale drawing of a paperclip after it has been enlarged by a scale factor of 2. To make a scale drawing of the paperclip that is 4.9 cm long, what scale factor would need to be applied to the real paperclip?

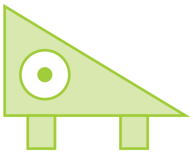


Open-ended

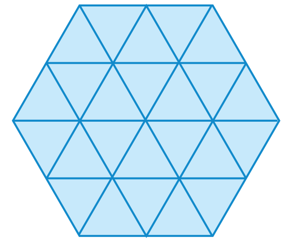
12 Draw a polygon with more than three sides. Reduce the shape by a scale factor of $\frac{1}{2}$. Enlarge the original shape by a scale factor of 3.

13 The area of a rectangle is 72 cm^2 after its length and width are enlarged by a scale factor of 3. What could the dimensions of the original rectangle be?

14 For the picture on the right, draw three scale drawings, stating the scale factor used each time.



15 Gina and Conan were playing a game using this hexagonal board. Gina noticed that the board consisted of three different sized equilateral triangles. She then commented, 'There are four small triangles in the medium sized triangle, so the scale factor must be 4, and there are nine small triangles in the large triangle so the scale factor must be 9'. Conan didn't agree.



(a) Draw the three different sized triangles.

(b) Complete scale drawings of the small triangle with a scale factor of 4 and 9.

(c) What may Conan's answer be? How could he explain this answer to Gina?

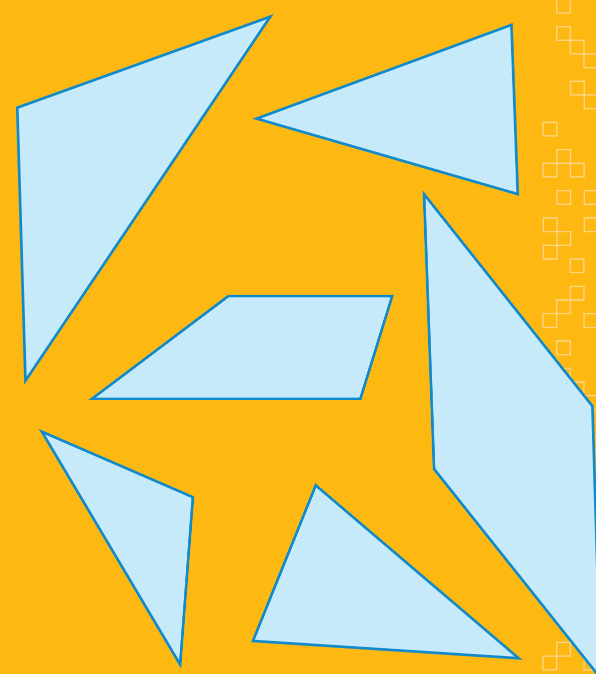
Outside the Square Puzzle

PP Blackline Master

What a star!

Equipment required: Star template (available on Pearson Reader), scissors

Copy these shapes, cut them out and rearrange them to form a perfect five-pointed star.



Technology Exploration GeoGebra

Equipment required: 1 brain, 1 computer with GeoGebra, workbook and a pen

Versions of this Exploration for other technologies are available in Pearson Reader.

Enlarging or reducing a figure

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. We will refer to these by number, counting from the left. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen. (Note that some versions of GeoGebra have different names for tools; however, the icons remain the same.)

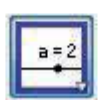
The GeoGebra screen is divided into two panels. The thinner panel on the left is called the 'Algebra View'. The wider, main panel is called the 'Graphics View'.

1 Click on the Options menu. Select 'Point Capturing' and 'On (Grid)'.

2 From the View menu, deselect 'Axis'. (Or, you can right click in the 'Graphics View' and deselect 'Axis' but select 'Grid'.)

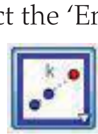
3 If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided.

Changing the size of a figure by a given scale factor

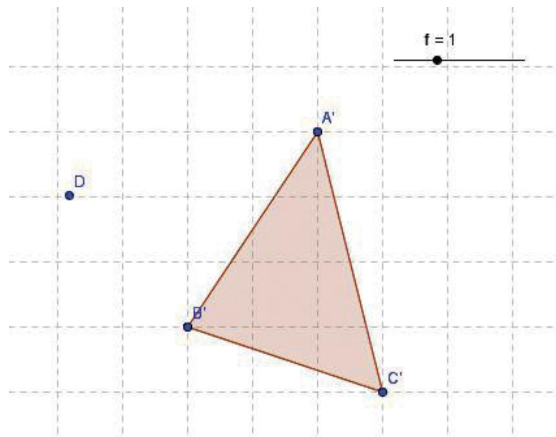
4 From the 10th icon, select the 'Slider' tool 

. Click on the 'Graphics View' where you would like the slider to appear. In the box that appears, name the slider '*f*', set 'min' to 0, 'max' to 3 and 'Increment' to 0.25. Click 'Apply'.

5 From the fifth icon, select the 'Polygon' tool . Construct a triangle by clicking on any three points in the 'Graphics View' and then click back on the starting point.

6 From the ninth icon, select the 'Enlarge Object from Point by Factor' tool . Click inside the triangle and then click to the side of the triangle where you would like the point to appear. When the dialogue box appears, enter '*f*' for the number value.

7 Your screen should look similar to this.



8 Use the 'Move' tool  from the first icon to move your points to adjust your picture.

9 With the 'Move' tool still selected, drag the point on the slider.

(a) What happens when you make the value of *f* greater than 1?

(b) What happens when you make the value of *f* less than 1?

10 From the third icon, select the 'Ray Through Two Points' tool . By clicking *D* first, draw rays through *A*, *B* and *C*.

What do you notice about the rays? (What other points do they pass through?)

11 Move any of the points *A*, *B*, *C* or *D* about the Graphics View using the 'Move' tool . What do you notice about the points *A*′, *B*′ and *C*′?

Investigating the lengths associated with enlargements

The lengths of the sides of the triangle are displayed in the 'Algebra View' (the left-hand side of the screen), with c representing the length of side AB in the original figure and c' representing the length of the matching side $A'B'$ in the enlargement.

12 In the Input bar type: cxfactor=f*c. This will create a new result in the 'Algebra View' called 'cxfactor' and press Enter. This is the length of c multiplied by the scale factor shown on the slider.

13 Repeat Step **12** to create: bxfactor=f*b.

14 Repeat Step **12** to create: axfactor=f*a.

What do you notice about the values for:

(a) axfactor and length a'?

(b) bxfactor and length b'?

(c) cxfactorand length c'?

15 Using the 'Move' tool, drag any of the points A, B, C or D around the screen.

What do you notice about the values for:

(a) axfactor and length a'?

(b) bxfactor and length b'?

(c) cxfactor and length c'?

16 Change the value of the factor by moving the slider.

What do you notice about the values for:

(a) axfactor and length a'?

(b) bxfactor and length b'?

(c) cxfactor and length c'?

17 From the eighth icon, select the 'Distance or Length' tool . Click on Point D and then on Point A to measure the interval AD. Repeat the process for intervals DB, DC, DA', DB' and DC'. Click on the 'Move' tool to drag measurement labels away from overlapping points or sides.

18 In the input bar, type: distanceDAxfactor=distanceDA*f.

19 Repeat Step **18** for distance DB and distance DC.

What do you notice about the values of:

(a) distanceDAxfactor and the distance DA'?

(b) distanceDBxfactor and the distance DB'?

(c) distanceDCxfactor and the distance DC'?

20 Using the 'Move' tool drag any of the points A, B, C or D about.

What do you notice about the values of:

(a) distanceDAxfactor and the distance DA'?

(b) distanceDBxfactor and the distance DB'?

(c) distanceDCxfactor and the distance DC'?

21 Change the value of the scale factor by moving the slider.

What do you notice about the values of:

(a) distanceDAxfactor and the distance DA'?

(b) distanceDBxfactor and the distance DB'?

(c) distanceDCxfactor and the distance DC'?

Investigating the areas of similar figures

The areas of the two triangles are given in the 'Algebra View' as poly1 and poly1'.

22 In the Input bar type: poly1xfactor=poly1*f. What do you notice about the values of poly1xfactor and poly1'?

23 Consider: We measure length in linear units, such as cm, m, and km. We measure area in square units, such as cm^2 , m^2, and km^2.

In the input bar type: poly1xfactorsquared=poly1*f*f

What do you notice about the values of poly1xfactorsquared and poly1'?

Taking it further

Click on the 'File' menu, then on 'New Window'.

24 Repeat Step **4** to create a slider.

25 Repeat Step **5** to create any shape using the 'Polygon' tool.

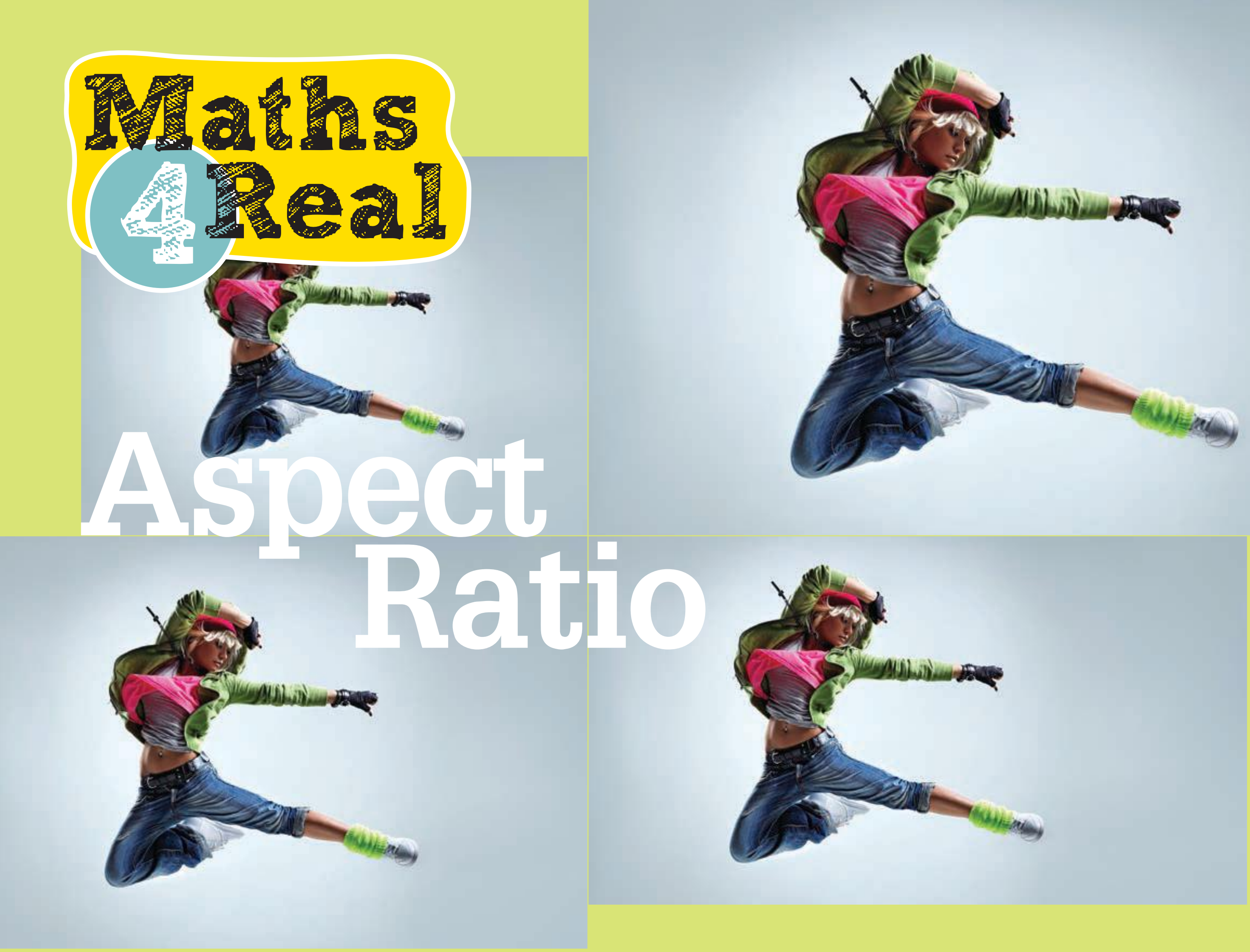
26 Create an enlargement as described in Steps **6–11**.

27 Investigate how the lengths of the sides of your shape and its area are related to the lengths of the sides and area of the enlargement.

28 Write a conclusion to summarise your findings.

Maths 4 Real

Aspect Ratio



The 'aspect ratio' of an image produced by a camera, computer screen or projector is the ratio of the image's length, **L** (horizontally), to its width, **W** (vertically). Images taken with many digital cameras have an aspect ratio **L** : **W** of 4 : 3, although some cameras also produce images with aspect ratios of 3 : 2 and 16 : 9.

4 : 3 (Digital)

3 : 2 (35mm film)

16 : 9

If we calculate $\frac{L}{W}$, we get a scale factor that tells us how many times the length is longer than the width. We multiply the width by this scale factor to get the length.

1 (a) Write the ratios of 4 : 3, 3 : 2 and 16 : 9 as scale factors. (Write recurring decimals in exact decimal form.)

(b) (i) What would the aspect ratio and scale factor be for a square image?

(ii) Which of the three aspect ratios given here gives a 'squarer' image?

When you come to print your photos, there is a wide variety of photo paper sizes available. Common ones are shown in the table below. The old imperial unit of inches (") is still used to describe the lengths and widths.

Photo paper sizes

Length (inches)	6	7	8	10	14	15	15	18
Width (inches)	4	5	6	8	11	8	10	12

An image ratio of 3 : 2 is useful because the standard photo size of 6"× 4" is in the same ratio. This is because traditional film cameras shot images in this ratio, before digital cameras were in use.

When digital cameras became popular, 4 : 3 was used as the aspect ratio because computer screens at the time had the same aspect ratio.

2 (a) Which paper sizes listed in the table provide the best 'fit' for an image that has an aspect ratio of 3 : 2?

(b) Which paper sizes provide the best 'fit' for an image that has an aspect ratio of 4 : 3?

Give reasons for your choices.

Problems arise when we want to print an image shot in one aspect ratio onto paper with a different aspect ratio. Imagine that we want to print a 4 : 3 photo onto 6" × 4" photographic paper.

To increase the length of the smaller image from 4 to 6, we need to multiply by 1.5: $4 \times 1.5 = 6$. However, multiplying the width of the smaller image by 1.5 doesn't give us the width of the larger one: $3 \times 1.5 = 4.5$, not 4.

There are two solutions to this problem.

Cropping

'Crop' (cut off) the extra 0.5" of width, meaning that some of the image is actually lost from the top and bottom of the shot.

OR

Enlarging

Enlarge the image to fill the paper widthways. This means multiplying by $1.\dot{3}$, instead of 1.5. $3 \times 1.\dot{3} = 4$. This leaves the length a little bit short: $4 \times 1.\dot{3} = 5.\dot{3}$, not 6. This results in two small thin white bars appearing on either side of the image.

3 Imagine that you work in a photographic printing business.

(a) A customer has brought in some images taken in a 3 : 2 format and asked for them to be enlarged onto 15" × 8" paper. The customer would prefer that the photos be cropped rather than have bars appear around the edges. How much of the image will be lost in the cropping process?

(b) Another customer has brought in some images taken in a 4 : 3 format and asked for them to be enlarged onto 18" × 12" paper. She does not want the images cropped.

(i) How wide will the white bars on the sides of the image be?

(ii) Recommend a very similar photo paper size to the customer that requires no cropping of the image.

How big can I make my photos?

Inside a digital camera are millions of 'pixels', tiny, light-sensitive squares that are put together like a jigsaw or mosaic to make up the image. A pixel is the smallest element of a digital image. A megapixel is equal to one million pixels. A camera with a larger number of megapixels means that more pixels are packed into a small space. The resulting image has a higher resolution because it is made up of more detailed information. This means bigger enlargements can be made before the image loses its resolution and begins to look 'pixellated', the effect where individual pixels become visible.

Professional photographers print at a resolution of 300 'pixels per inch' (ppi). This is the number of pixels that fit into a length of one inch of the image. To find the number of megapixels required for an enlargement:

- Multiply the length of the desired enlargement by 300 ppi
- Multiply the width of the desired enlargement by 300 ppi
- Multiply these two answers together
- Divide by 1 million.

4 Ellie wants to buy a new camera. She thinks that the largest print size she will make is 12"× 8".

How many megapixels will she require in the camera to get an adequate resolution of 200 ppi?

Research

Find out the old and new aspect ratios for TV and film images. Describe what can be done to show a film shot in one aspect ratio on a screen with a different aspect ratio.

6.5 Similarity and similar triangles

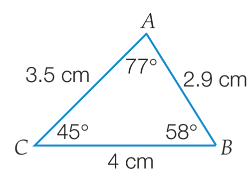
Similar figures have the same shape, but not necessarily the same size. It is important to distinguish between similar figures and congruent figures. Congruent figures have the same shape and are *exactly* the same size.

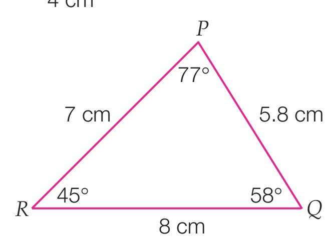
The photos below are similar. (We can see that the length and width of the smaller frame have both been multiplied by the same scale factor of 1.5 to produce the length and width of the larger frame.)


16 cm
10 cm


24 cm
15 cm

The triangles shown here are similar. Matching sides AB and PQ are highlighted.





The ratio of matching sides can be shown as:

$$\frac{PQ}{AB} = \frac{5.8}{2.9}$$
$$= 2$$

$$\frac{QR}{BC} = \frac{8}{4}$$
$$= 2$$

$$\frac{PR}{AC} = \frac{7}{3.5}$$
$$= 2$$

Matching angles are equal and matching sides have equal ratios.

The above information shows us that ΔABC *is similar to* ΔPQR.

In similar figures:

- all pairs of matching sides are in the same ratio *and*
- all pairs of matching angles are equal.

To show that two shapes are similar, we use the '~' symbol or the '|||' symbol.

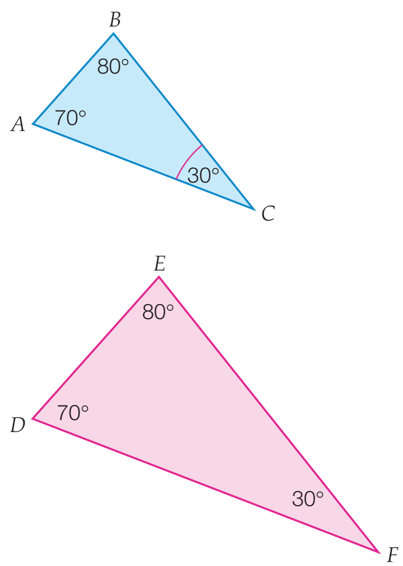
For example, triangle ABC is similar to triangle PQR is written as:

$\Delta ABC \sim \Delta PQR$ or $\Delta ABC\ |||\ \Delta PQR$

It is not necessary to know about all sides and angles to determine that two triangles are **similar triangles**. There are several tests we can apply that give us sufficient information to classify triangles as similar. The following tests can be used to show that the two triangles are similar. The test that is used depends on the information provided:

Tests for similar triangles

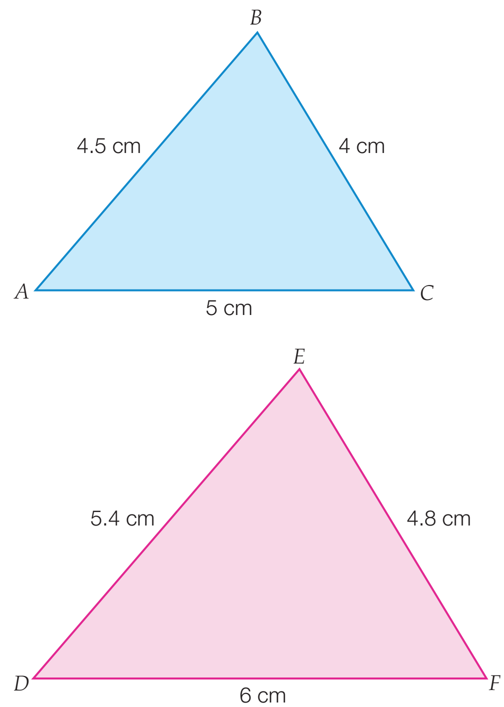
1 Angle, Angle, Angle (AAA):



$\Delta ABC \sim \Delta DEF$ (AAA)

Three pairs of angles are of the same magnitude. (It is only necessary to know the values of two of the pairs because the angle sum of a triangle is 180° and so we can calculate the value of the third pair.)

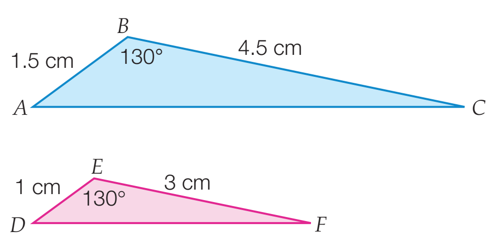
2 Side, Side, Side (SSS):



$\Delta ABC \sim \Delta DEF$ (SSS)

Three pairs of matching sides are in the same ratio.

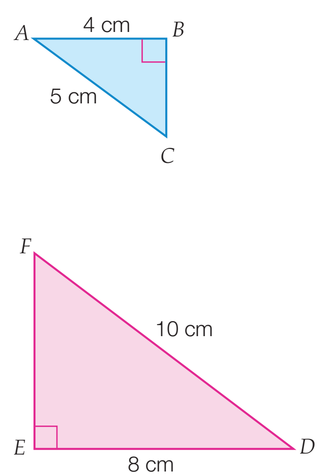
3 Side, Angle, Side (SAS):



$\Delta ABC \sim \Delta DEF$ (SAS)

Two pairs of matching side have lengths in the same ratio and the included angles are equal.

4 Right angle, Hypotenuse, Side (RHS):



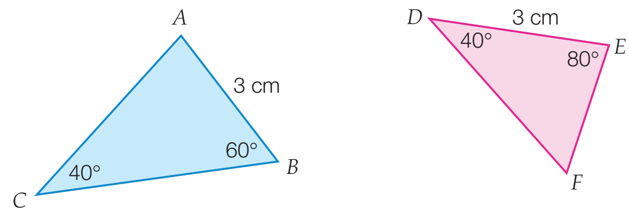
$\Delta ABC \sim \Delta DEF$ (RHS)

The hypotenuse and one side of a right-angled triangle are in the same ratio as the hypotenuse and one side of another right-angled triangle.

Worked Example 12

WE12

Use a similarity test to show that triangles *ABC* and *DEF* are similar.



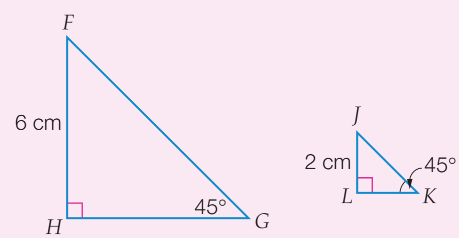
Thinking	Working
1 State the sides and angles that can be used to justify similarity.	$\angle ACB = \angle EDF = 40°$ $\angle ABC = \angle EFD = 60°$ $\angle CAB = \angle DEF = 80°$
2 Formally state that the triangles are similar, taking care to state matching vertices in the same order. Write the test that was used to show similarity in brackets as a justification of your answer.	$\therefore \Delta ABC \sim \Delta EFD$ (AAA)

Similarity and scale factor

For two similar figures, the scale factor is the ratio of matching sides.

$$\frac{JL}{FH} = \frac{2}{6} = \frac{1}{3}$$

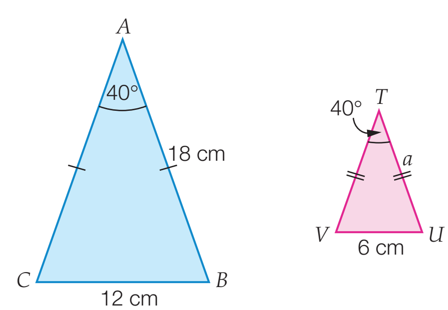
$\therefore$ ΔJKL is one-third the size of ΔFGH or ΔFGH is three times the size of ΔJKL.



Worked Example 13

WE13

Given that the following triangles are similar, find the value of *a*.



Method 1: Use the scale factor

Thinking	Working
1 Identify the scale factor using the matching sides for which values are given.	Scale factor $= \frac{UV}{BC}$ $= \frac{6}{12}$ $= \frac{1}{2}$

2 Find the unknown side length by multiplying its matching side length by the scale factor	$a = 18 \times \frac{1}{2}$ $= \frac{18}{1} \times \frac{1}{2}$ $= 9$ cm $a = 9$ cm

Method 2: Set up an equation

Thinking	Working
1 Use the fact that the ratios of the matching sides are equal to form an equation.	$\frac{TU}{AB} = \frac{UV}{BC}$ $\frac{a}{18} = \frac{6}{12}$
2 Solve the equation.	$a = \frac{6}{12} \times 18$ $= \frac{1}{2} \times \frac{18}{1}$ $= \frac{18}{2}$ $a = 9$ cm

6.5 Similarity and similar triangles

Navigator

Q1 (a), (b), Q2 (a), (c), Q3, Q4, Q5, Q6 (a), (b), Q7, Q8, Q11, Q12, Q13	Q1 (b), (c), Q2 (a), (b), Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13	Q1 (c), (d), Q2 (b), (d), Q4, Q5, Q6 c), (d), Q7, Q8, Q9, Q10, Q11, Q12, Q13

Answers page 641

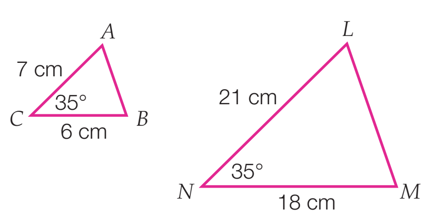
Equipment required: protractor for Question **9**

Fluency

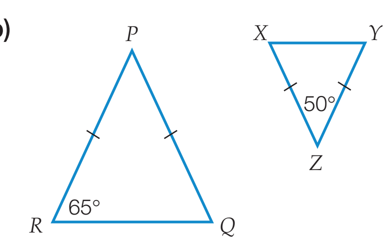
1 Use a similarity test to show why the following pairs of triangles are similar.

WE12

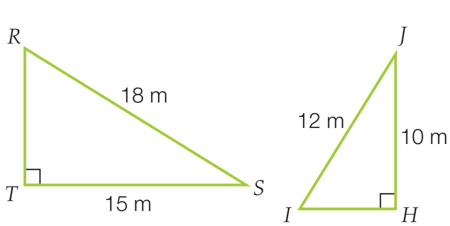
(a)



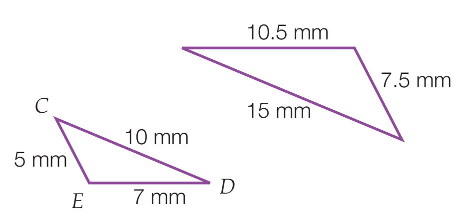
(b)



(c)



(d)



Hint

Remember to use the correct mathematical symbols:

$\angle$ means 'angle'

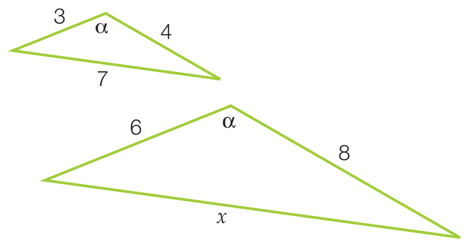
Δ means 'triangle'

~ means 'is similar to'.

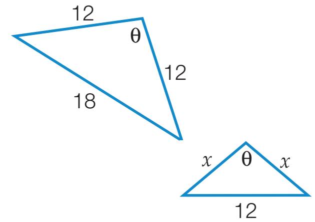
WE13

2 Find the value of x in each of the following pairs of similar triangles.

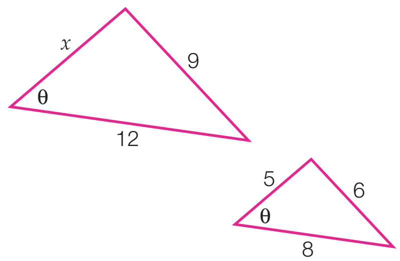
(a)



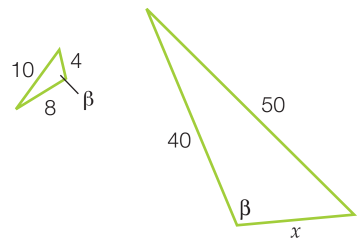
(b)



(c)



(d)



3 Given triangle ABC is similar to triangle EFG, then:

(a) the matching side to AB is

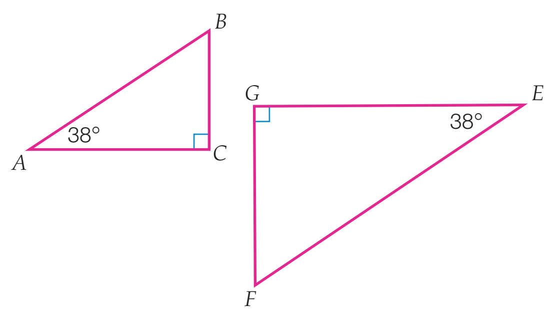
A FG **B** EF

C BC **D** EG

(b) the matching angle to $\angle EFG$ is

A $\angle CAB$ **B** $\angle ACB$

C $\angle ABC$ **D** $\angle BAC$



4 Given triangle HJK is similar to triangle LMN, then:

(a) the matching side to LM is

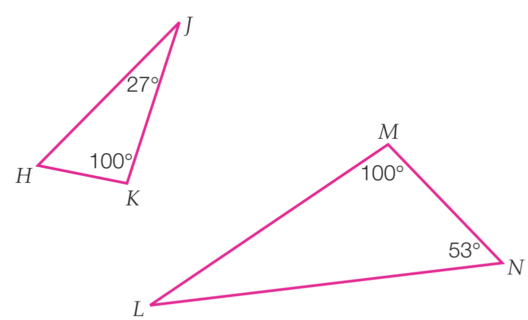
A HJ **B** MN

C JK **D** LN

(b) the matching angle to $\angle JHK$ is

A $\angle LMN$ **B** $\angle NLM$

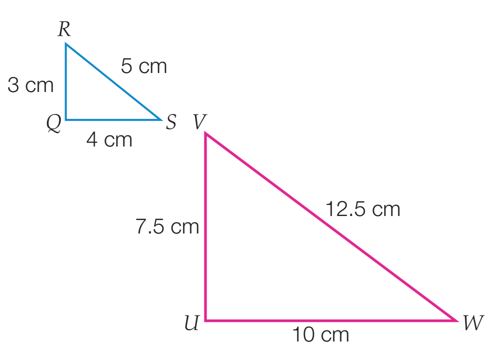
C $\angle LNM$ **D** $\angle MLN$



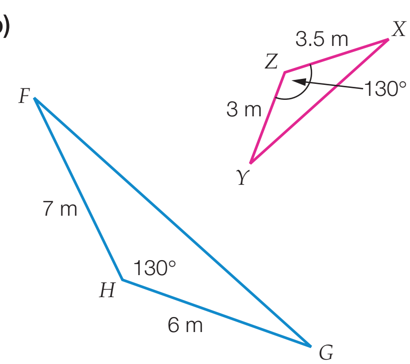
Understanding

5 Find the scale factor for the following pairs of similar triangles. (The blue triangles are the originals.)

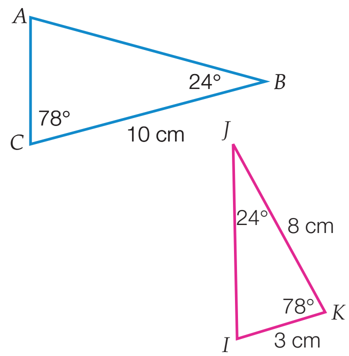
(a)



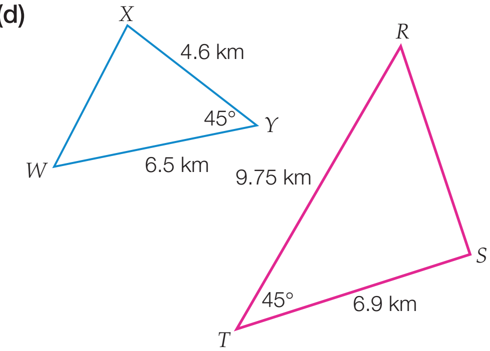
(b)



(c)

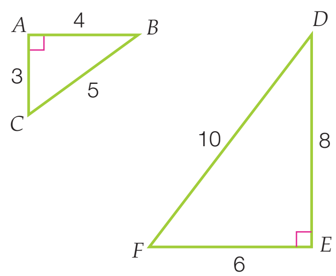


(d)

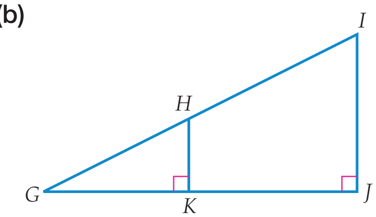


6 Show that these pairs of triangles are similar.

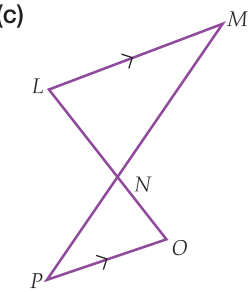
(a)



(b)



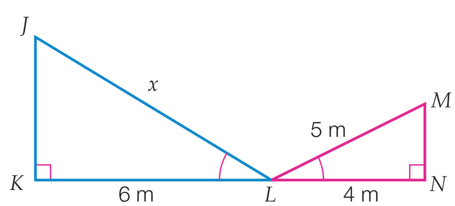
(c)



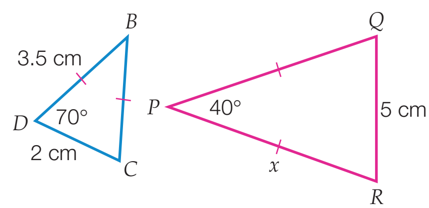
7 The following pairs of triangles are similar. (The blue triangles are the originals.) For each pair:

(i) state the scale factor **(ii)** find the value of the variable.

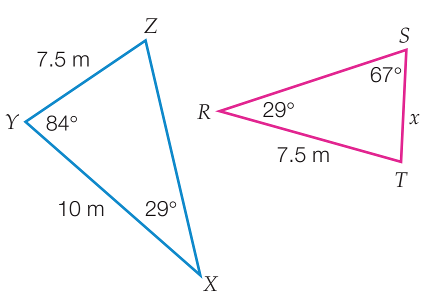
(a)



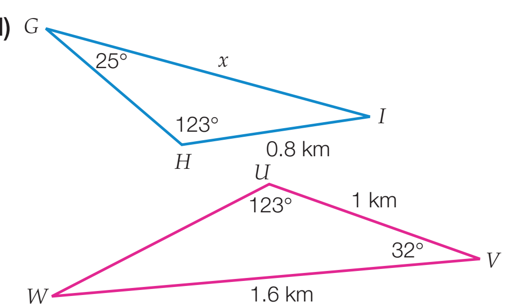
(b)



(c)



(d)



8 A 4-m ladder is leaning against a wall. The base of the ladder is 2 m from the wall. A second 5-m ladder is leaning against a wall. The base of this ladder is 3 m from the wall.

(a) Draw a diagram showing the triangle formed by the wall, the first ladder and the ground.

(b) Draw a diagram showing the triangle formed by the wall, the second ladder and the ground.

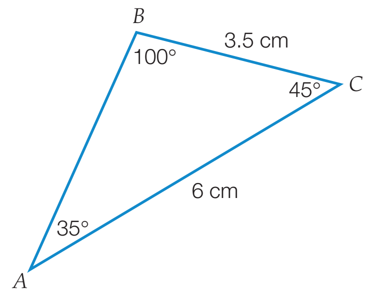
(c) Are the two triangles drawn in **(a)** and **(b)** similar? Justify your answer.

Reasoning

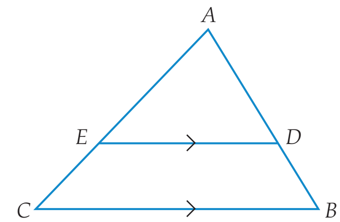
9 ΔABC and ΔPQR are similar triangles. ΔABC is shown here. $\angle PRQ = 45°$, $\angle RQP = 100°$ and $PR = 9$ cm. Use this information to:

(a) determine the scale factor

(b) construct ΔPQR using a protractor and ruler using the same scale.



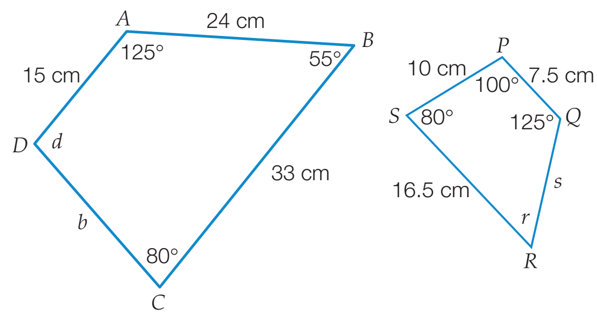
10 ΔABC and ΔADE are similar. Explain why any triangle divided with a line parallel to any side will produce two similar triangles.



11 Other two-dimensional shapes (and three-dimensional shapes) can be classed as similar. The quadrilaterals shown are similar.

(a) How can you prove this?

(b) Find the size of any unknown sides or angles in either quadrilateral.



Open-ended

12 Draw a pair of similar triangles that have angles of 30°, 60° and 90°. State the scale factor.

13 On a maths test, one of Geordie's justifications for the similarity of two triangles was:

$$\angle GHI = \angle UVT = 38°$$

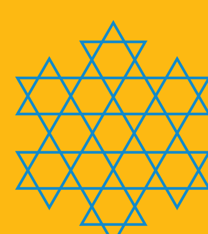
(a) Draw two similar triangles that match this justification.

(b) On the triangles drawn in **(a)**, state the minimum amount of information necessary (angle magnitudes and/or side lengths) to be able to justify that the two triangles are similar.

Outside the Square Puzzle

How many triangles?

What is the total number of triangles in this diagram?



Solving problems using similar triangles

6.6

Many questions and problems can be solved by first identifying that two triangles are similar and then using the scale factor to find the length of any unknown sides. Unknown angles can be found by identifying matching angles.

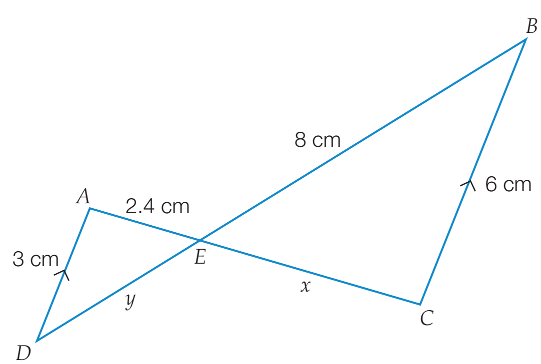
In order to identify similar triangles in a problem, you may need to recall angle facts relating to parallel lines being crossed by a transversal.

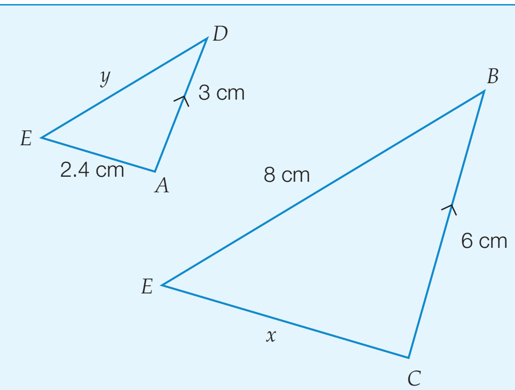
If two triangles are connected or overlapping, it is often useful to draw the two triangles separately and change the orientation of one so they both have the same orientation.

Worked Example 14

WE14

Show that the two triangles are similar and then find the values of x and y.



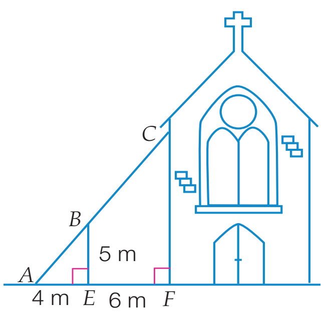
Thinking	Working
1 Redraw the triangles separately. Keep the orientation of one triangle the same and change the orientation of the other so they both have the same orientation. Identifying known equal angles and sides helps to draw the diagrams correctly.	
2 Use a similarity test to show that the triangles are similar. Write the abbreviation for the test used (AAA in this case) after the similarity statement.	$\angle AED = \angle BEC$ (vertically opposite angles) $\angle DAE = \angle BCE$ (alternate angles) $\angle ADE = \angle CBE$ (alternate angles) $\therefore \Delta ADE \sim \Delta BCE$ (AAA)

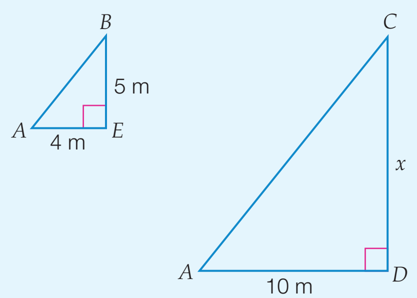
3 Identify the scale factor using the two values of the matching sides.	$\text{Scale factor} = \frac{BC}{DA}$ $= \frac{6}{3}$ $= 2$
4 Use the scale factor to determine the lengths of the unknown sides.	$\frac{x}{EA} = 2$ $x = EA \times 2$ $= 2.4 \times 2$ $= 4.8$ cm $\frac{EB}{y} = 2$ $y = \frac{EB}{2}$ $= \frac{8}{2}$ $= 4$ cm
5 State the answer.	$x = 4.8$ cm and $y = 4$ cm

Worked Example 15

WE15

The wall at the local church needs a support to keep it in place (as shown). The support is positioned on the ground 10 m from the base of the wall. There is a 5-m vertical prop for this support placed 6 m from the wall. How far up the wall does the support reach?



Thinking	Working
1 Draw the two triangles separately. Label each of the points on each triangle. (Here, the point *A* is common to both triangles.) Label the required length *x*.	
2 Show that the triangles are similar.	$\angle BAE = \angle CAD$ (common angle) $\angle BEA = \angle CDA = 90°$ (given in diagram) $\angle ABE = \angle ACD$ (angles in triangles $= 180°$) $\therefore \Delta ABE \sim \Delta ACD$ (AAA)
3 Use the fact that the ratio of the matching sides is equal to form an equation.	$\frac{CD}{BE} = \frac{AD}{AE}$

4 Solve the equation.

$$\frac{x}{5} = \frac{10}{4}$$
$$x = \frac{10}{4} \times 5$$
$$= \frac{5}{2} \times \frac{5}{1}$$
$$= \frac{25}{2}$$
$$= 12.5 \text{ m}$$

5 Answer the question (including units). $\therefore$ The support reaches 12.5 m up the wall.

6.6 Solving problems using similar triangles

Navigator

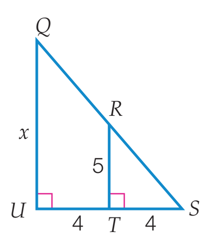
Q1 (a), (c), Q2, Q3, Q4, Q5 , Q6, Q7, Q9, Q10	Q1 (b), (c), Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1 (b), (d), Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10

Answers page 641

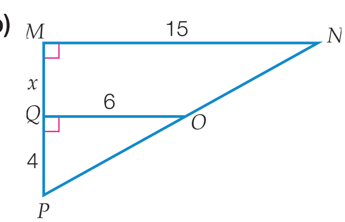
Fluency

1 In each of the following diagrams, show that the two triangles are similar and then find the value of x. WE14

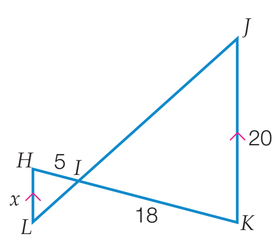
(a)



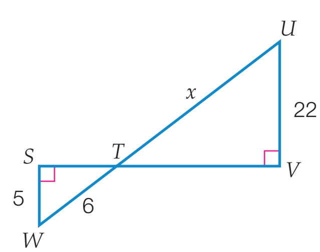
(b)



(c)



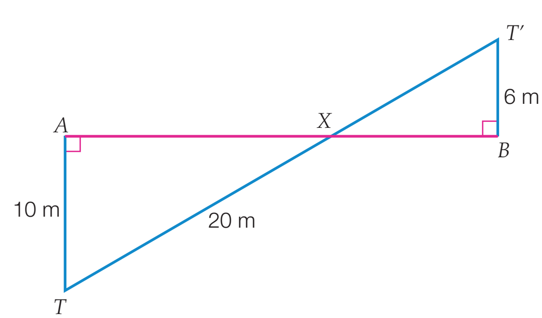
(d)



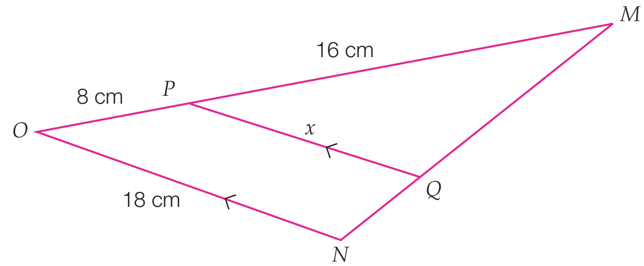
2 A tap T is placed on one end of a 10 m wall. A 20 m water pipe runs from the tap to a Point X as shown on the diagram. It is to be extended to another tap at T' on another wall BT' 6 m long. WE15

(a) Show that the triangles ATX and $BT'X$ in the diagram are similar.

(b) Calculate the total length of new pipe XT' required.



3 The diagram below shows two similar triangles.



(a) Which of the following equations could be used to determine the length of PQ?

A $\frac{8}{18} = \frac{x}{16}$ **B** $\frac{x}{18} = \frac{8}{16}$ **C** $\frac{x}{18} = \frac{16}{24}$ **D** $\frac{x}{16} = \frac{8}{24}$

(b) The scale factor that produces the larger triangle from the smaller is:

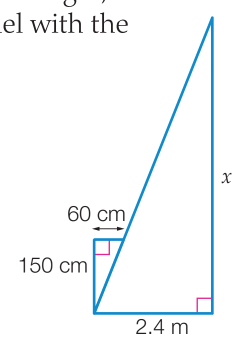
A 0.5 **B** $0.\dot{6}$ **C** 1.5 **D** 2

Understanding

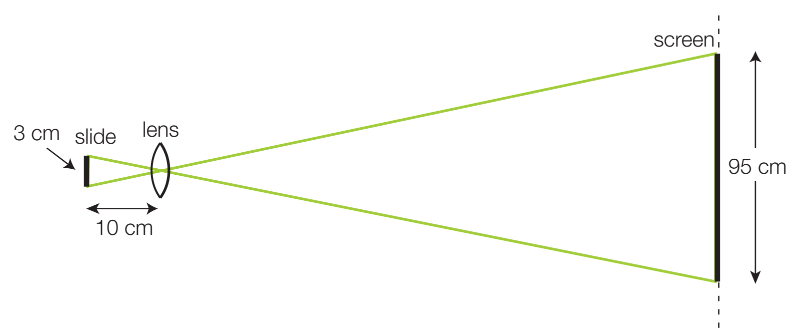
4 Noah is standing on a long ladder leaning against a wall. The base of the ladder is 2.4 m from the base of the wall. His friend Sean is holding the ladder to prevent it wobbling. Sean is standing with his shoulders 150 cm directly above the base of the ladder. His arms, 60 cm in length, are stretched horizontally, parallel with the ground, to hold the ladder.

A diagram of the information is shown.

How far up the wall does the ladder reach?



5 A light source projects the image of a photographic slide onto a screen, as shown in the diagram.

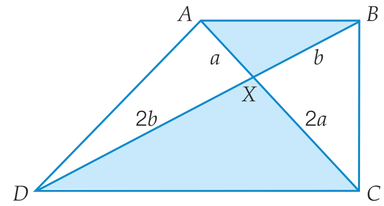


The slide is 3 cm high and positioned 10 cm behind a lens. Calculate the distance between the lens and the screen if the image on the screen is 95 cm high.

6 Martin plays tennis at night under floodlights. When he stands 4.5 m from the base of a floodlight, he casts a shadow 1.4 m long. If Martin is 1.7 m tall, how high is the floodlight?

Reasoning

7 A quadrilateral $ABCD$ is drawn as shown with diagonals intersecting at Point X.



(a) Show that the shaded triangles are similar.

(b) Hence show that $ABCD$ is a trapezium.

(c) Will the non-shaded triangles also be similar? Explain why or why not.

8 The leader of the local Scout troop was explaining to the Scouts how to use a stick to estimate the height of a tree.

Step 1: Have someone hold the stick vertically on the ground.

Step 2: Lie on the ground in a position so that, when looking at the tree on a line of sight that goes through the stick, the top of the tree and the top of the stick are in line.

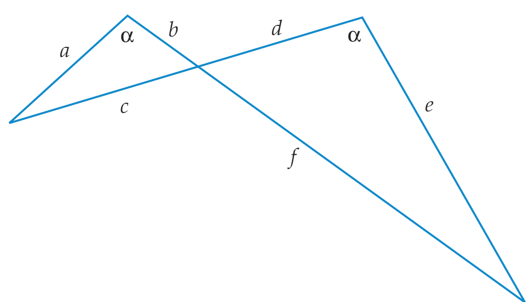
Step 3: Unfortunately the leader had forgotten Step 3.

(a) Draw a diagram of the situation and use your knowledge of similar triangles to complete the instructions.

(b) List any practical problems you can see with this method.

Open-ended

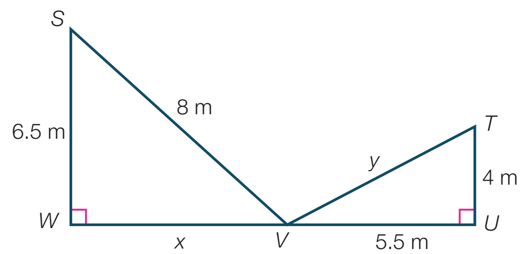
9 State a value for each of the variables in the diagram so that the two triangles are similar.



Hint

Be sure that $a + b > c$.

10 The teacher drew the following diagram on the board and asked the students to calculate the lengths of x and y. Santino's answers are shown.



Santino's answers:

To calculate x:

$$\frac{x}{5.5} = \frac{6.5}{4}$$

$$x = \frac{6.5}{4} \times 5.5$$

$$x = 8.94 \text{ m}$$

To calculate y:

$$\frac{y}{8} = \frac{4}{6.5}$$

$$y = \frac{4}{6.5} \times 8$$

$$y = 4.92 \text{ m}$$

(a) Explain why the values Santino calculated for x and y are not correct.

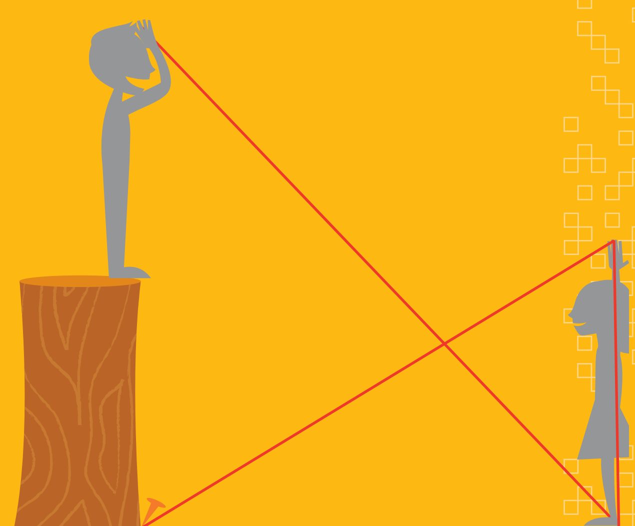
(b) What method could Santino use to calculate the values of x and y?

Outside the Square

Problem solving

Curiously crossed

Gina and Gerard are twins who are both exactly 1.5 metres tall. Gina has a very long rope. Gerard attaches one end of it to the bottom of a 1.5-metre tree stump with a tent peg. Gina picks up the rope and holds it above her head as high and as tightly as she can so it is 2 m above the ground. She lets the rest of the rope fall down behind her back and stands on it. Gerard then picks up the other end of the rope and stands on the tree stump so the rope crosses itself. This end of the rope just reaches the top of his head. How high off the ground do the ropes cross?



Strategy options

- Draw a diagram.
- Test all possible combinations.
- Break problem into manageable parts.
- Look for a pattern.

Measuring shadows to find heights

Equipment required: measuring tape (or metre ruler), chalk

Thales was a mathematician who lived from 625 to 547 BC. With his knowledge of similar triangles, he was able to determine the heights of the Egyptian Pyramids. He realised that when the length of his shadow was the same as his height, the same was true for all objects nearby.

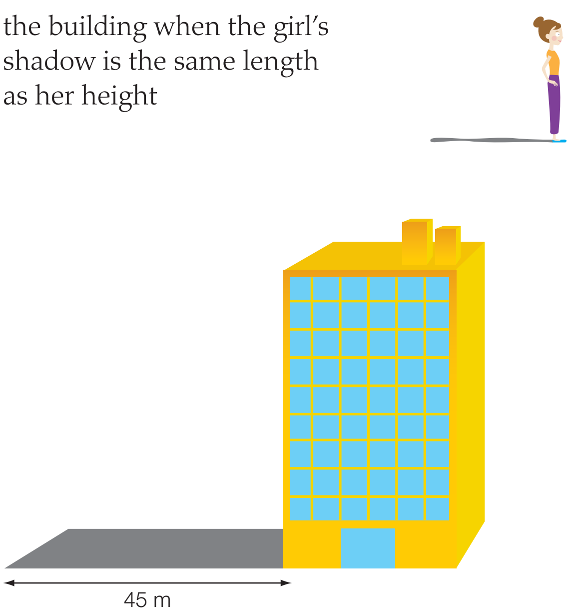
The Big Question

How can knowledge of similar triangle properties be used to measure tall objects?

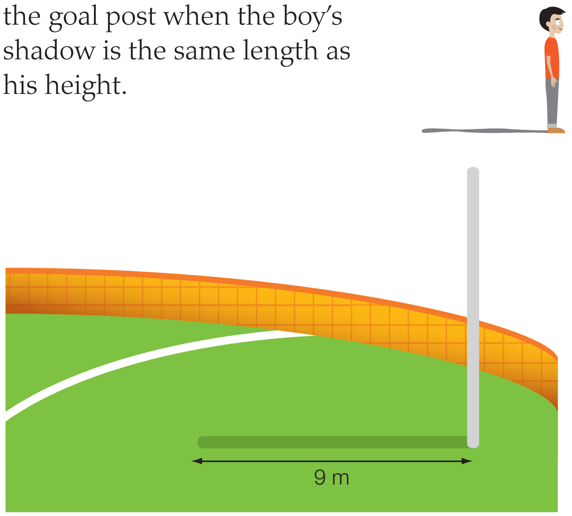
Engage

1 Based on Thales' observation, determine the height of:

(a) the building when the girl's shadow is the same length as her height



(b) the goal post when the boy's shadow is the same length as his height.



(c) A zoo keeper was feeding a giraffe when he noticed that the giraffe's shadow was three times as long as his own shadow. This was observed when the zoo keeper's shadow equalled his height. If the zoo keeper's height is 1.8 m, what is the height of the giraffe?

Thales' observation is useful, but what if we want to know the height of an object when our shadow does not equal our height?

Explore

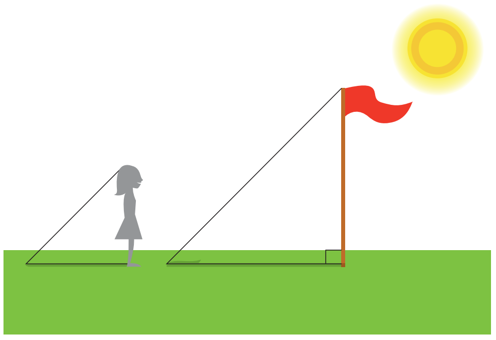
2 Choose an object that is too tall to measure with measuring tape, such as a flagpole, goal post or building.

3 Follow the steps below to calculate its height.

Step 1: Record your height, in metres, to the nearest cm (e.g. 1.54 m).

Step 2: Estimate the height of the tall object, so you can compare it with its height calculated later.

Step 3: Measure the length of your shadow and the length of the shadow of the object of unknown height.



Step 4: Draw a diagram showing the two similar triangles, stating all measurements. Label the unknown height x.

Step 5: Using the knowledge that corresponding sides of similar triangles have the same ratio, calculate the height of the tall object. Record the calculated height.

Strategy options

- Draw a diagram.
- Make a table.

Explain

4 How do you know that the triangle formed by the object and its shadow and the triangle formed by you and your shadow are similar triangles?

Elaborate

5 What was the height of your chosen object? Choose another tall object and find its height using the same method.

Evaluate

6 What problems did you encounter while performing this investigation? How did/could you overcome these?

7 Did you find any shortcuts for calculating the heights of the tall objects you investigated?

8 How accurate do you believe this method is for determining the heights of tall objects? Explain.

9 What changes, if any, would you make to this investigation to make the measurements and findings more accurate?

10 List three things that could not be measured in this way.

Extend

11 Are two people with shadows the same length at the same time of day the same height? Explain.

12 Record your shadow length at hourly intervals.

(a) At what time is the length of your shadow equal to your height?

(b) At what time is the length of your shadow zero?

(c) Is there a relationship between time of day and shadow length?

13 Thales of Miletus is the earliest known Greek mathematician. Find out more about him and his discoveries.

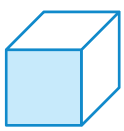
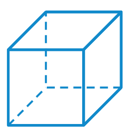
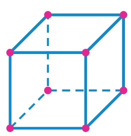
Solids and nets

6.7

A **solid** is a three-dimensional (3D) shape or object. Solids are made up of flat, two-dimensional 2D shapes called **faces**. Two faces meet at a line called an **edge**. Three or more faces meet at a **vertex**. The plural of vertex is vertices.

A solid where all faces are polygons is known as a **polyhedron**. The plural of polyhedron is polyhedra. All edges are straight lines.

A polyhedron whose faces are all identical regular polygons is called a **regular polyhedron**.

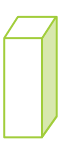
Face	Edge	Vertex
The flat part of a solid.  A cube has 6 faces.	The line segment where two faces meet.  A cube has 12 edges.	A corner where several edges and faces meet.  A cube has 8 vertices.

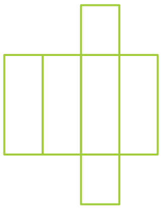
Nets

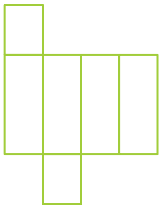
Boxes and containers come in many different shapes and sizes. Boxes and containers are made by drawing an outline on a flat surface and then folding the outline into the three-dimensional solid.

A **net** is the 2D outline of a solid that shows all the faces. A net can be folded to make the 3D solid.

Solids can have more than one net.







Special types of polyhedra

- **Prisms** have a polygon base and a regular cross-section (any cut parallel to the base gives an identical shape to the base).
- Pyramids have a polygon base but all other faces are triangles, so the shape tapers to a point.
- Platonic solids have identical polygon faces. There are only five platonic solids.
- Archimedean solids consist of two or more regular polygons but are not prisms. There are 13 Archimedean solids.

6.7 Solids and nets

Navigator

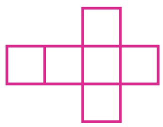
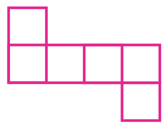
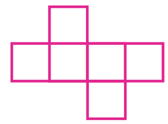
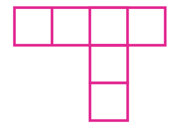
Answers page 641

Q1, Q2 (a), Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q14	Q1, Q2 (a), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14	Q1, Q2 (b), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14

Equipment required: scissors, nets, thread, glue and a ruler for Questions **7** and **8**

Fluency

1 Which of the following could not be a net for a cube?

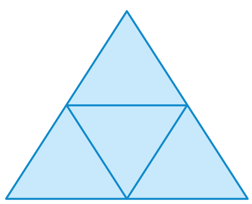
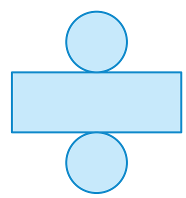
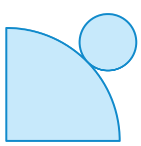
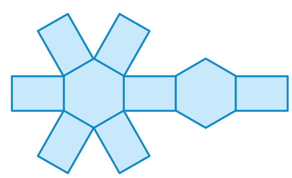
A 
B 
C 
D 

2 Draw a possible net for each of the following solids.

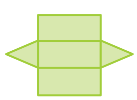
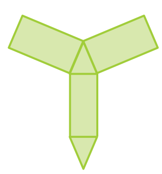
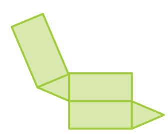
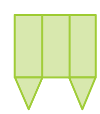
(a) a rectangular prism with dimensions of length 5 cm, width 2 cm and height 3 cm

(b) a pentagonal prism of side length 2 cm and height 5 cm

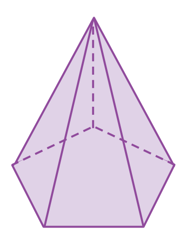
3 Name the solids that the nets below would fold into.

(a) 
(b) 
(c) 
(d) 

4 Which of the following nets *cannot* be folded to make a triangular prism?

A 
B 
C 
D 

5 Which description matches the shape shown?



A 5 faces, 10 edges, 5 vertices

B 6 faces, 10 edges, 6 vertices

C 10 faces, 10 edges, 6 vertices

D 6 faces, 9 edges, 6 vertices

Understanding

6 Draw the net of the external packaging for the following objects (e.g. no folding flaps).

(a)

(b)

(c)

7 Draw a net of a cube with side lengths of 3 cm on cardboard, cut it out and construct a solid cube.

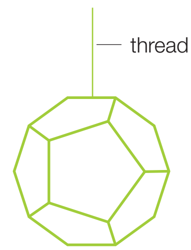
8 Follow these steps to construct solids from the nets shown. (Nets are available on Pearson Reader.)

PP Blackline Master

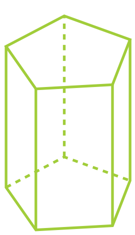
(i) Copy and cut out each net, including the tabs, and then crease along the fold lines so that a sharp line is formed.

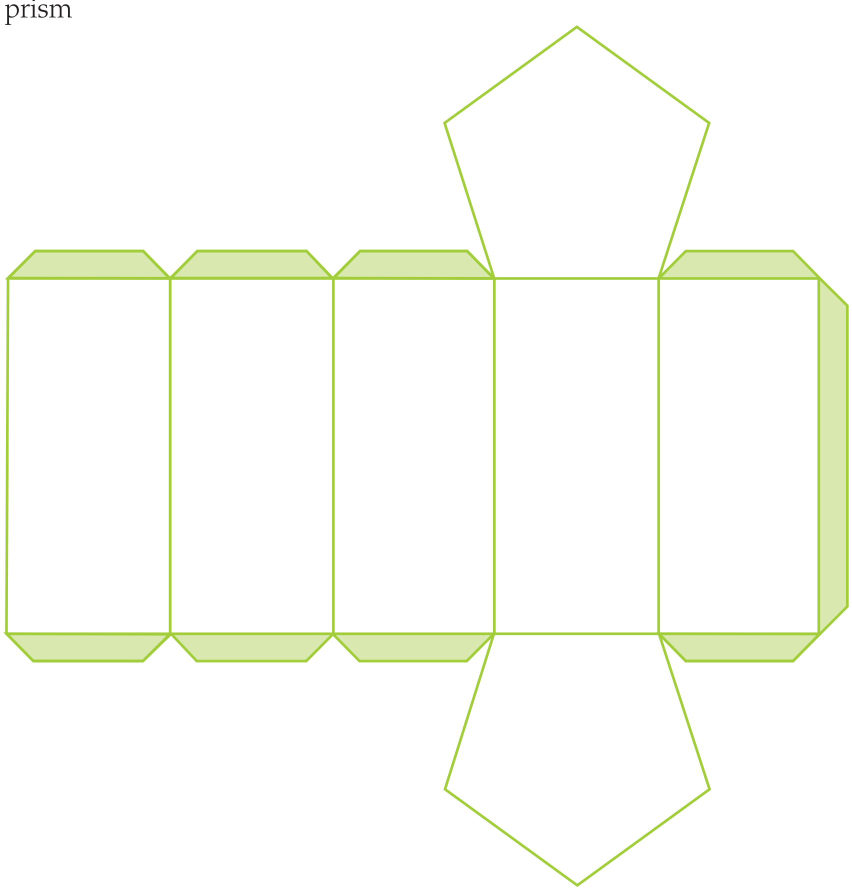
(ii) Fold the nets so the marked fold lines are on the inside of the model.

(iii) Insert a length of thread in your models before glueing the last tab. The thread may be used for hanging the models in a display later on.

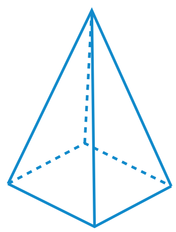


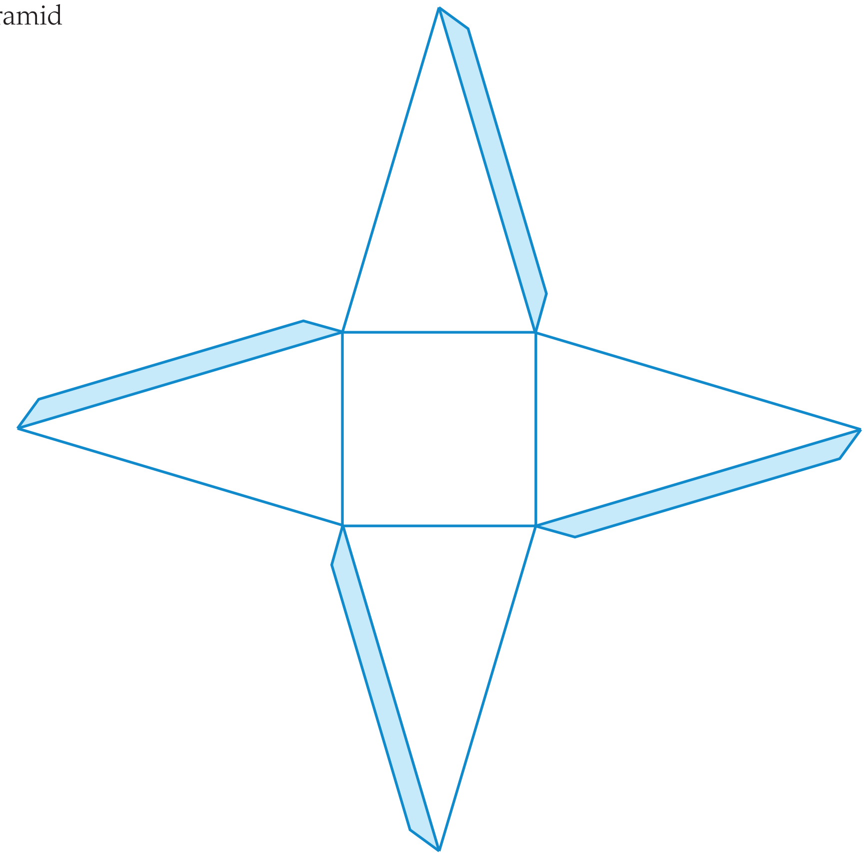
(a) a pentagonal prism





(b) a square pyramid



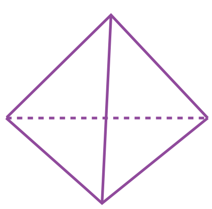


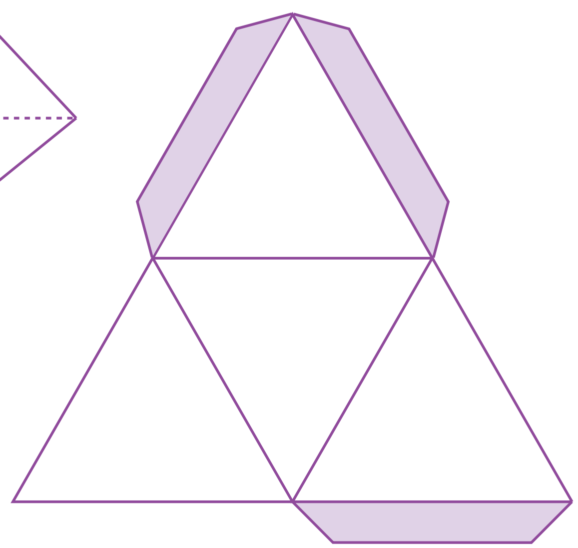
(c) a tetrahedron:

- has four identical triangular faces
- is a rectangular polyhedron, as each face is an equilateral triangle
- is a triangular pyramid.

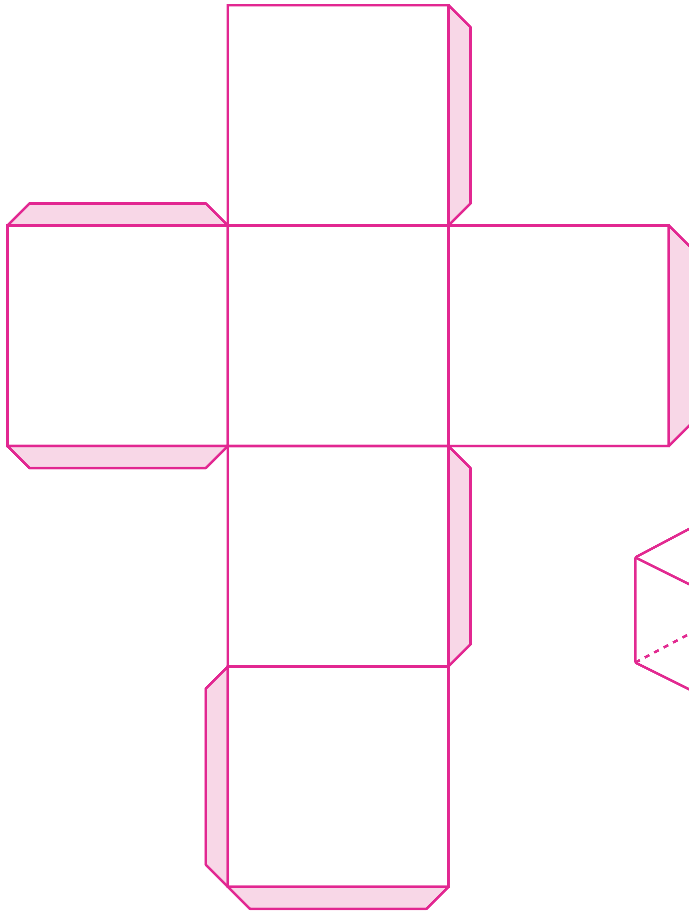
Hint

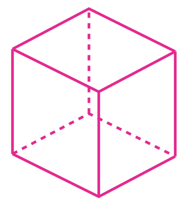
A tetrahedron is very strong. The atoms in diamonds are arranged in a lattice of tetrahedrons. Diamond is one of the hardest substances known.





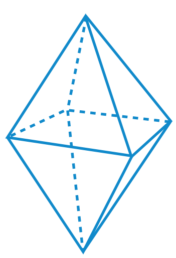
(d) a cube

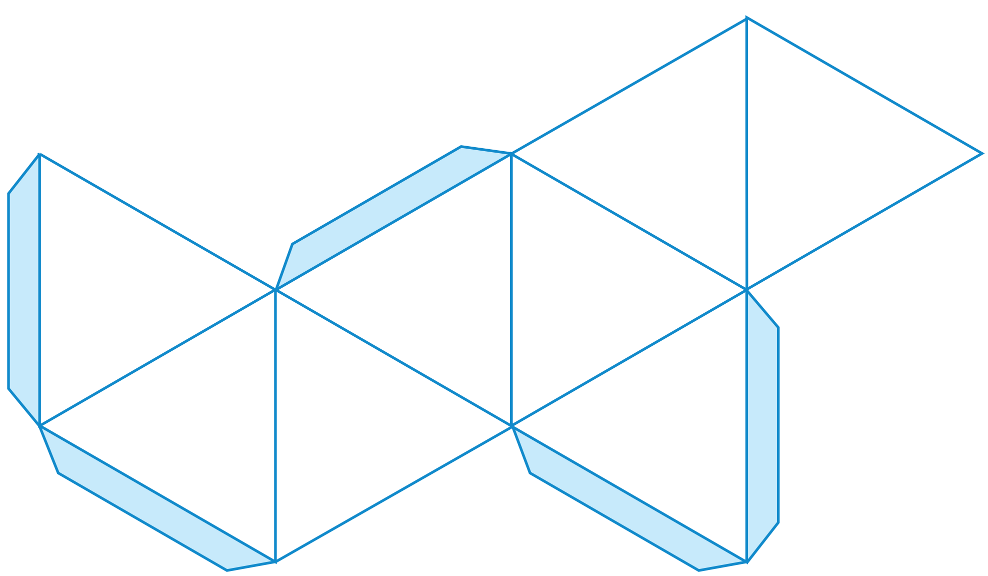




(e) an octahedron:

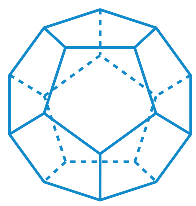
- has eight identical triangular faces
- is a regular polyhedron, as each face is a equilateral triangle
- can be thought of as two square-based pyramids joined together at the bases.

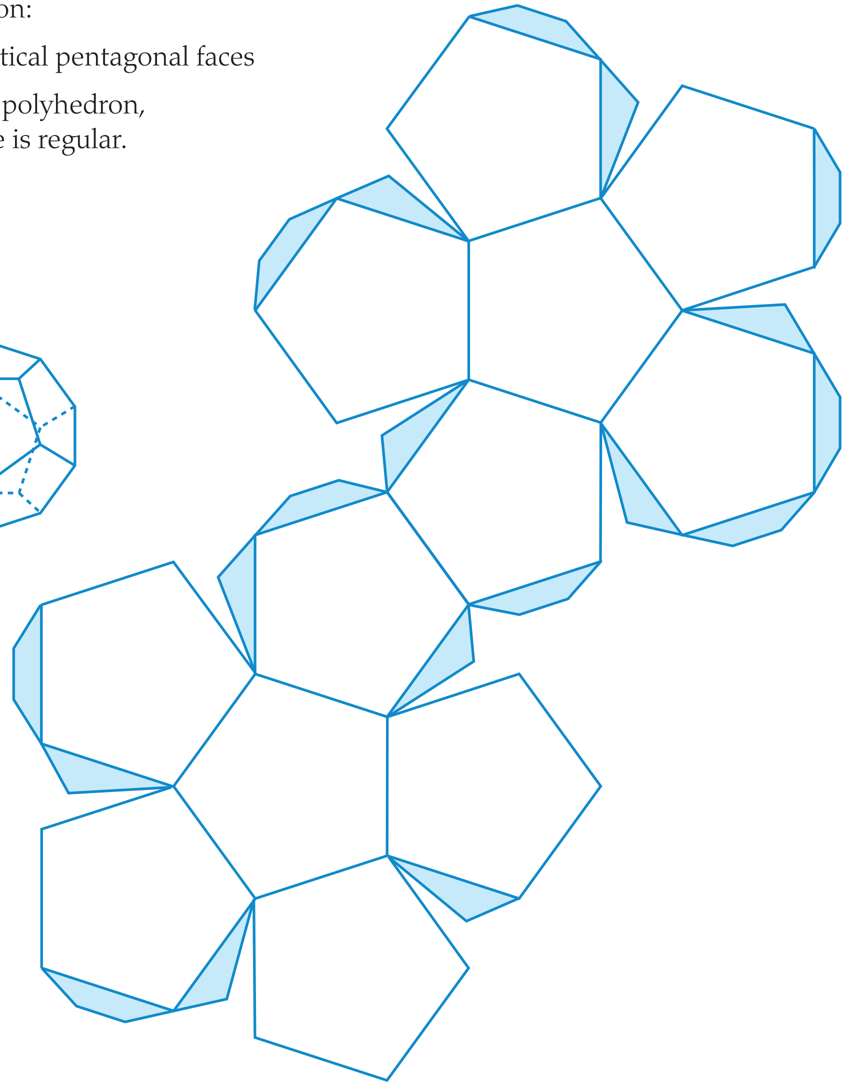




(f) a dodecahedron:

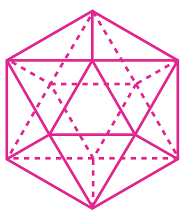
- has 12 identical pentagonal faces
- is a regular polyhedron, as each face is regular.





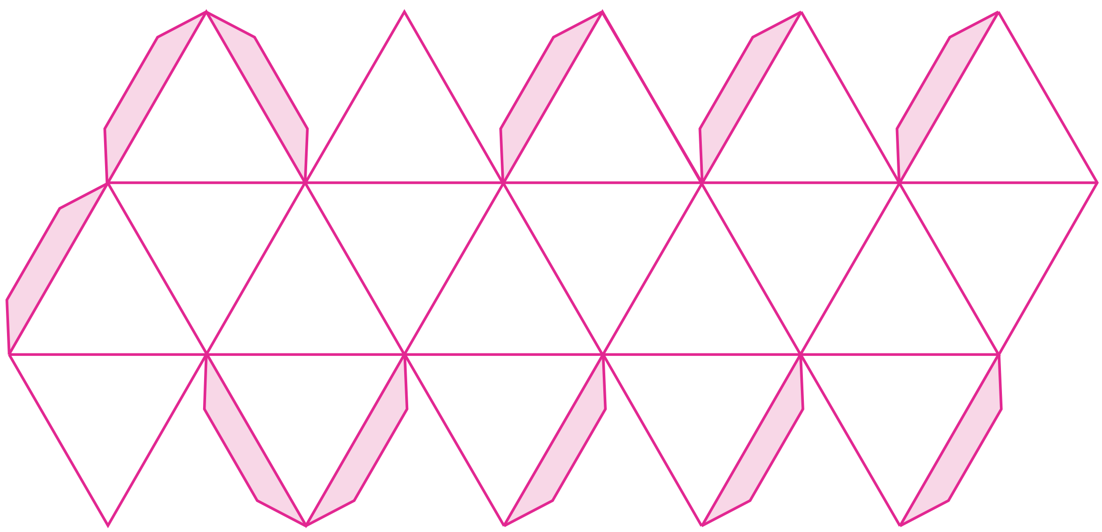
(g) an icosahedron:

- has 20 equilateral triangle faces
- is a regular polyhedron.

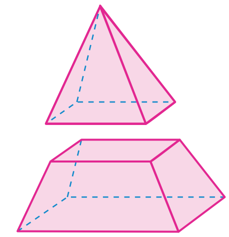


Hint

The chicken pox virus has the shape of an icosahedron, a solid with 20 triangular faces.

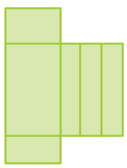


Reasoning

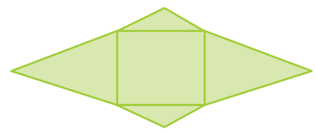


9 A square pyramid has been cut into two, as shown. How many faces, vertices and edges does each part have?

10 Roland drew nets of two solids shown below.



Net of a rectangular prism



Net of a rectangular pyramid

(a) Explain why the two nets above will not work.

(b) Correct Roland's drawings.

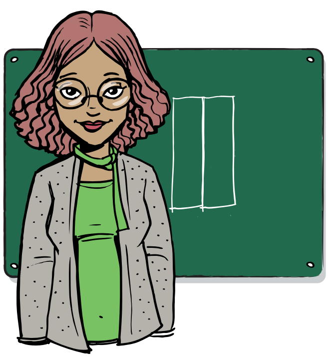
Open-ended

11 Draw a net of a solid that has nine faces.

12 State three solids whose nets have no rectangles.

13 Can a solid have only three faces? Can a polyhedron have only three faces? Explain your answers.

14 Saxon can only see part of the net the teacher is talking about. He can see two quadrilaterals. Draw a possible net and the solid it can fold into.



Outside the Square Puzzle

Matching cubes

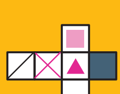
Match the cubes with their net.

Cube 1

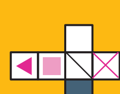
Cube 2

Cube 3

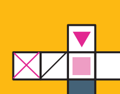
Cube 4



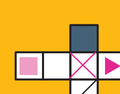
Net 1



Net 2



Net 3



Net 4

Challenge 6

Note: In this section the diagrams are not drawn to scale.

1 In ΔABC, sides AC and BC are equal and side AB is shorter than AC.

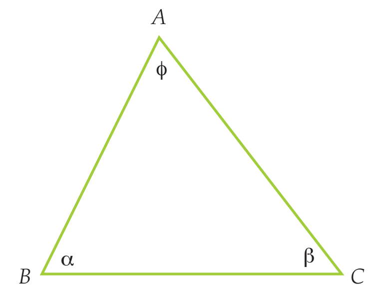
Which statement is true?

A α, β and ϕ are all equal

B $\alpha = \beta$

C $\beta = \phi$

D $\alpha = \phi$



2 How can you use six matches to form four equilateral triangles, each with a side length of one match?

3 In the diagram, JLM and KLN are straight lines. $JK = KL = LJ$.

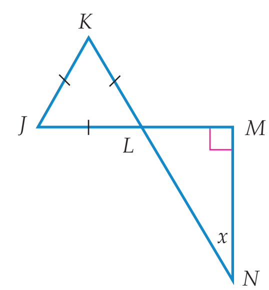
$\angle LMN = 90°$. What is the size of x?

A 15°

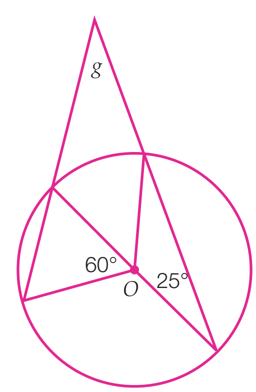
B 30°

C 45°

D 60°



4 Find the value of g in this diagram.



5 In the quadrilateral $PQRS$, $PS = PR = PQ$.

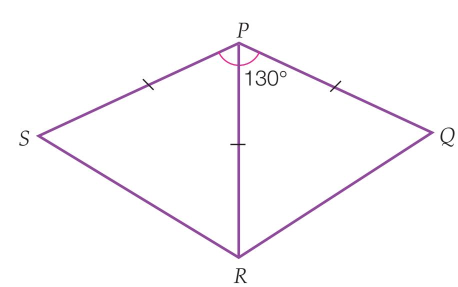
If $\angle SPQ = 130°$, then the size of $\angle SRQ$ is:

A 25°

B 115°

C 125°

D 135°



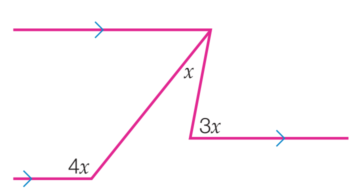
6 $\angle ABC$, $\angle CBD$ and $\angle DBA$ together make up an angle of revolution. If $\angle ABC$ is acute and $\angle CBD$ is obtuse, which of the following are possible for $\angle DBA$? There may be more than one answer.

A acute **B** obtuse **C** reflex **D** straight

7 What is the value of x?

A 22.5 **B** 30

C 36 **D** 45



Chapter review 6

D.I.Y. Summary

Key Words

alternate angles	edge	polygon	similar
centre of enlargement	enlargement	polyhedron	similar triangles
co-interior angles	exterior angle	prism	solid
congruent	faces	reduction	vertex
congruent triangles	interior angle	regular polyhedron	
corresponding angles	net	scale factor	

Copy and complete the following, using the words and phrases from the list where appropriate, to write a summary for the chapter. A word or phrase may be used more than once.

1 When a transversal cuts a pair of parallel lines, the pairs of __________ are equal, the pairs of __________ are equal, and pairs of __________ are supplementary.

2 __________ triangles are exactly the same shape and size.

3 The __________ of a triangle is equal to the sum of the interior opposite angles.

4 Multiplying the sides of a shape by a __________ greater than one will produce an __________ of the shape.

5 Which of the tests for similar triangles states that pairs of matching sides must be in the same ratio?

6 Which of the tests for similar triangles states that the matching hypotenuses and one other pair of sides of right-angled triangles must be in the same ratio?

7 Which of the tests for similar triangles states that one pair of matching angles must be equal and the pairs of matching sides on either side of the angle must be in the same ratio?

8 __________ have the same shape, but not necessarily the same size.

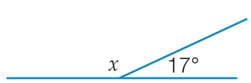
9 A __________ can be folded to make a three-dimensional solid.

Fluency

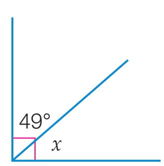
1 Find the value of x in the following diagrams:

Ex. 6.1

(a)



(b)



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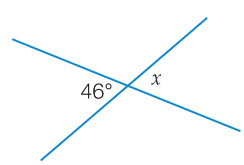

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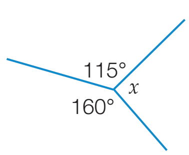
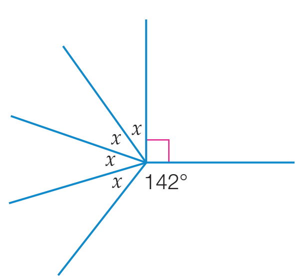


(e)



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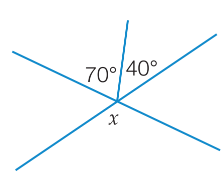
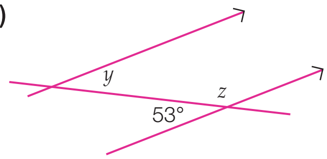


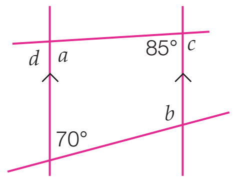
2 Find the value of the variables in the following diagrams:

Ex 6.1

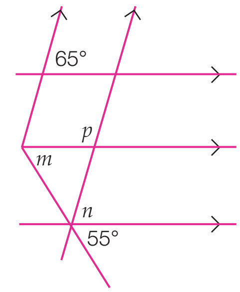
(a)



(b)



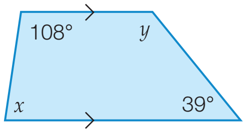
(c)



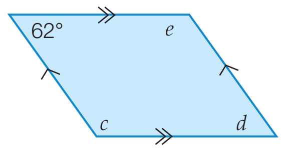
3 Find the value of the variables in the following diagrams:

Ex 6.3

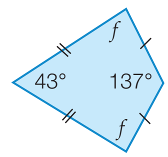
(a)



(b)



(c)



4 Match the name of the shape with its correct description.

Ex 6.3

(a) Adjacent sides of equal length, no parallel sides

(b) One pair of opposite sides parallel

(c) Four right angles, opposite sides equal in length

(d) Two sides equal in length

A Rectangle

B Kite

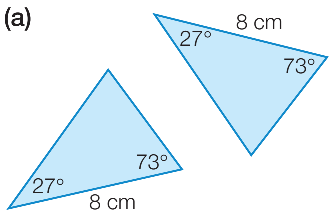
C Isosceles triangle

D Trapezium

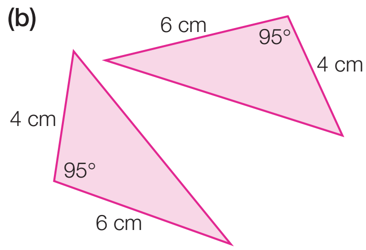
5 Explain why each of the following pairs of triangles are congruent.

Ex 6.2

(a)



(b)



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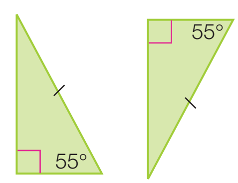
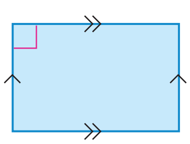


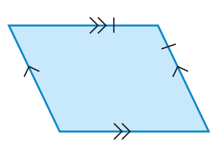
6 Identify each of the following shapes using the name that has all the properties shown.

Ex 6.3

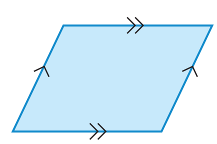
(a)



(b)



(c)

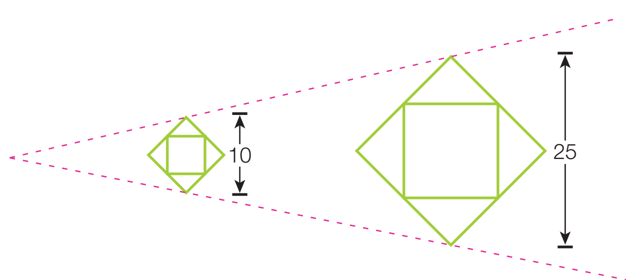


7 In the figure below, what is the scale factor for:

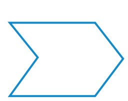
Ex 6.4

(a) the reduction

(b) the enlargement?



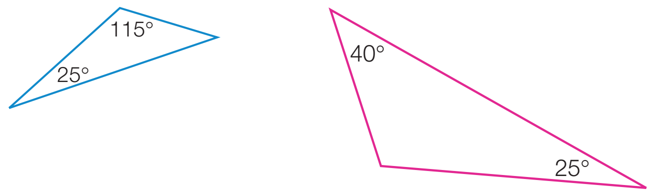
8 Enlarge the following shape by a factor of 2.4.



Ex 6.4

9 Show that the following pair of triangles are similar.

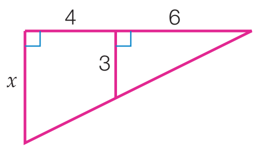
Ex.6.5



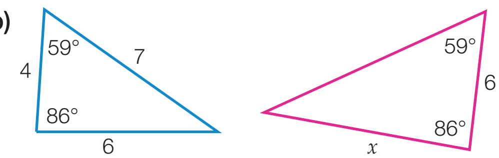
10 Find the value of x in the following diagrams.

Ex.6.5

(a)

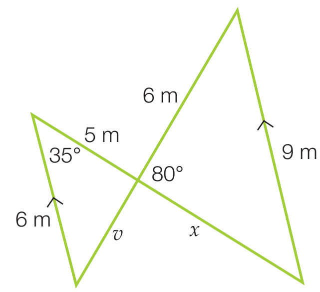


(b)



11 Show that the following pair of triangles are similar, then find the lengths of the unknown sides.

Ex.6.6



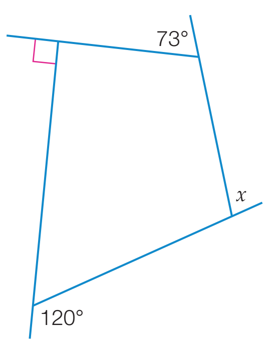
12 Draw the net for a regular tetrahedron.

Ex.6.7

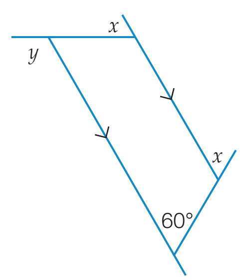
Understanding

13 Find the value of the variables in each of these quadrilaterals.

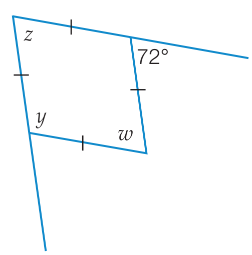
(a)



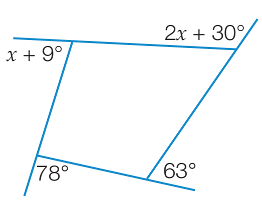
(b)



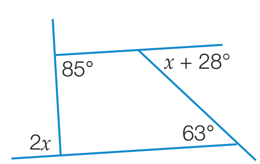
(c)



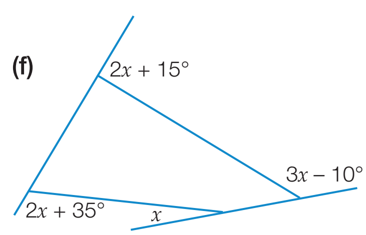
(d)



(e)

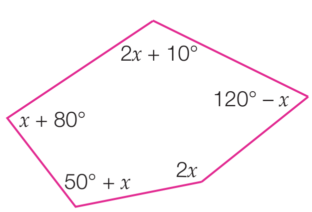


(f)

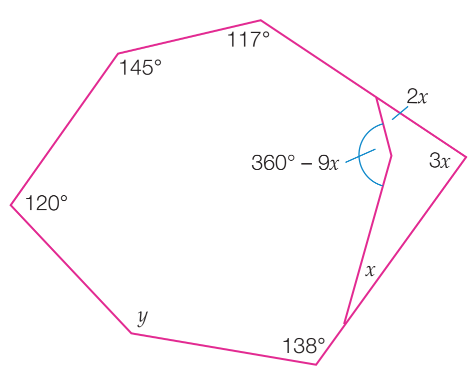


14 Find the values of the variables in each of these shapes.

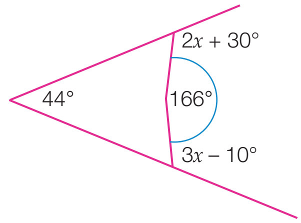
(a)



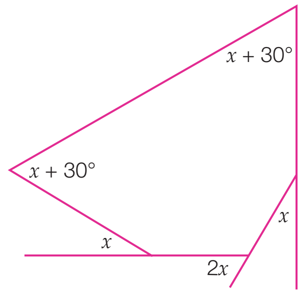
(b)



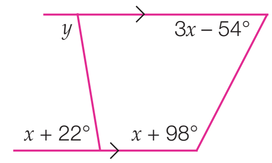
(c)



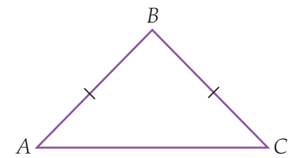
(d)



(e)



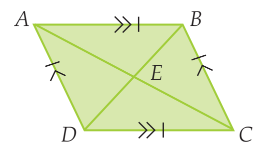
15 In ΔABC, $BA = BC$



(a) Prove that $\angle BAC = \angle BCA$ by drawing the line BD perpendicular to AC, meeting it at D.

(b) What property of isosceles triangles have you just proved?

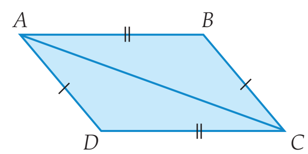
16 **(a)** For the diagram shown, prove AC ⊥ BD.



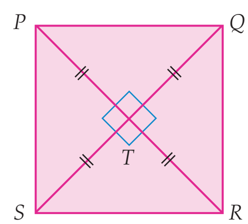
(b) What special property of a rhombus have you just proved?

17 Show the following.

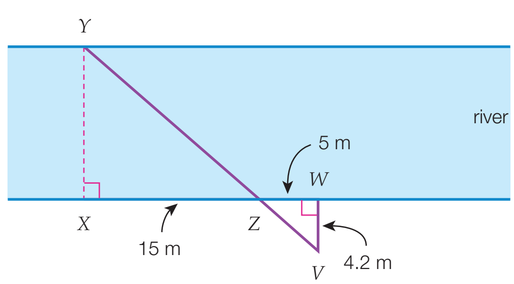
(a) If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.



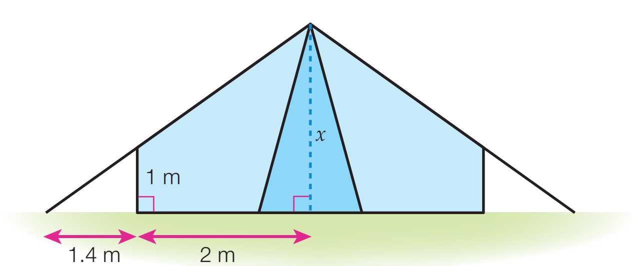
(b) If the diagonals of a quadrilateral are equal and bisect each other at right angles, then the quadrilateral is a square.



18 Angela is on a walk through the forest when she comes to a river. She wants to know the distance across the river. She measures the distances shown and draws a diagram to help her. Find the distance across the river.

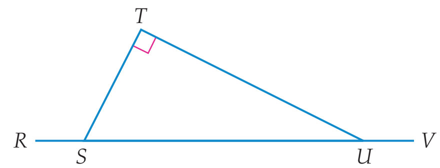


19 Use your knowledge of similar triangles to calculate the height of the tent shown.

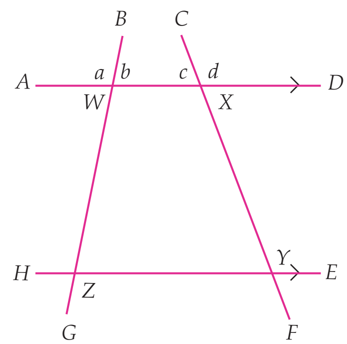


Reasoning

20 Prove that $\angle TSR + \angle TUV = 270°$



21 Use the diagram to prove that the sum of the interior angles of a trapezium is 360°.



22 A postcard measuring 11 cm × 8.5 cm is enlarged so that its new length and width are 120% the size of the original dimensions.

(a) What is the scale factor of the enlargement?

(b) What are the dimensions of the enlarged postcard?

(c) Find the area of **(i)** the original postcard and **(ii)** the enlarged image.

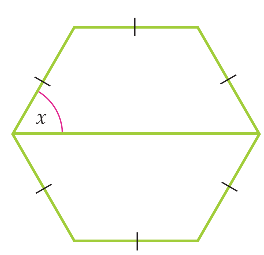
(d) Find the ratio $\frac{\text{area of photocopied image}}{\text{area of original postcard}}$. Explain why this ratio is not the same as the scale factor you found in **(a)**.

(e) Photocopiers enlarge images according to area, not length. What percentage enlargement would you need to enter into the photocopier to achieve the same enlarged postcard?

NAPLAN practice 6

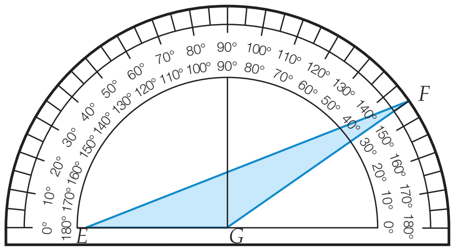
Numeracy: Non-calculator

1 Two isosceles trapeziums fit together to make the regular hexagon shown.



What is the value of x?

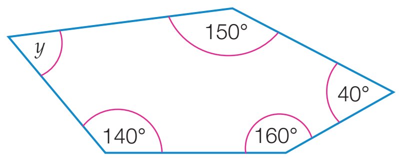
2 ΔEFG is shaded below. What is the value of $\angle FGE$?



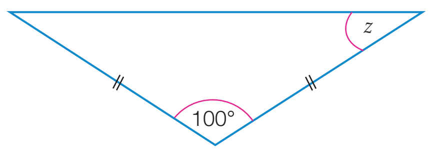
A 35° **B** 45° **C** 145° **D** 155°

Numeracy: Calculator allowed

3 What is the value of y?

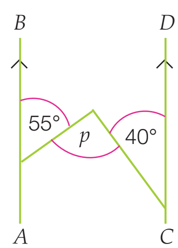


4 What is the value of z?



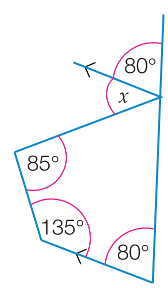
5 In the diagram, AB and CD are parallel.

What is the value of p?



6 In the diagram, a pair of parallel lines are shown.

What is the value of x?



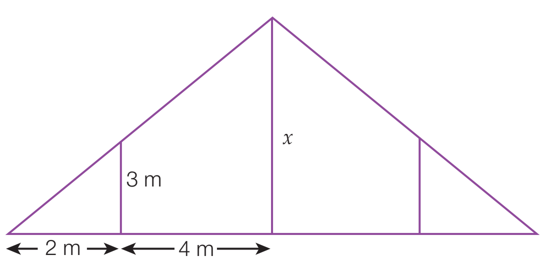
7 Liam is 1.5 m tall. His shadow is 2 m long. A tree nearby has an 8 m shadow. How tall is the tree?

A 7.5 m **B** 7 m **C** 6 m **D** 4 m

8 The support beams for a triangular barn are being put in place. How long will the support beam in the centre of the barn need to be?

A 6 m **B** 9 m

C 12 m **D** 15 m



9 A poster measuring 1.5 m × 0.9 m is enlarged by a scale factor of 3. What is the area of the enlarged poster?

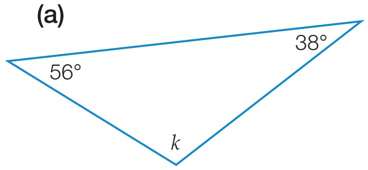
Mixed review C

Fluency

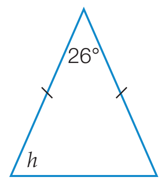
1 Find the perimeter of a semicircle of radius 12 m. Write your answer correct to two decimal places. Ex.4.1

2 Find the value of the variables in each of the following. Ex.6.1

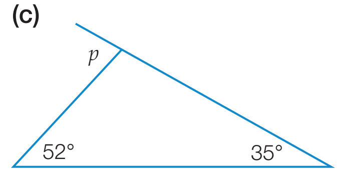
(a)



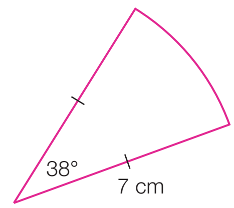
(b)



(c)

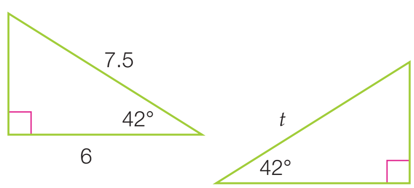


3 Find the area of the sector shown in the diagram. Give your answer correct to two decimal places. Ex.4.2

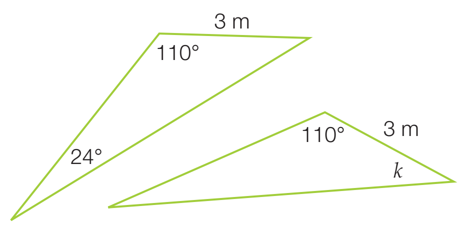


4 Find the values of the variables in the following congruent triangles. Ex.6.2

(a)



(b)



5 Find the surface area of a cylinder of height 10 cm and a base radius of 4 cm. Give your answer correct to two decimal places. Ex.4.3

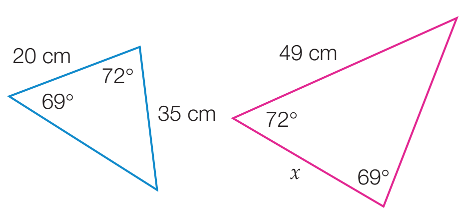
6 Calculate the simple interest in the following cases. Ex.1.5

(a) $P = \$5000$, $R = 4.7\%$ p.a., $T = 3$ years

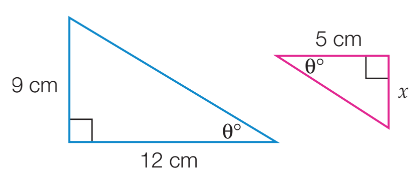
(b) $P = \$690$, $R = 8.7\%$ p.a., $T = 1.5$ years

7 Given that the following triangles are similar, state the scale factor and find the value of the variable. (The blue triangles are the original.) Ex.6.5

(a)



(b)



8 Write out the rules for the area of a: Ex.4.2

(a) kite (b) rhombus (c) trapezium

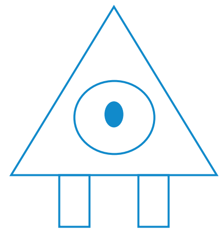
9 Factorise the following expressions: Ex 3.8

(a) $y^2 + 7y + 3y + 21$ (b) $cd + 3c + 4d + 12$ (c) $t^2 - 4t - 5t + 20$

10 For the illustration shown: Ex 6.4

(a) Enlarge by a scale factor of 3

(b) Reduce by a scale factor of $\frac{1}{2}$.



11 Find the gradient of the line joining each of the following pairs of points. Ex 5.5

(a) (0, 4) and (5, 14) (b) (12, 8) and (5, 15) (c) (-8, -11) and (6, -4)

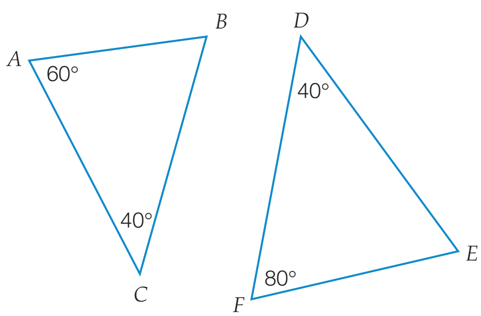
12 Solve the following equations: Ex 5.1

(a) $\frac{3x}{5} - 7 = 4$ (b) $\frac{4x + 7}{4} = 5$ (c) $\frac{2(3x + 8)}{5} = -2$

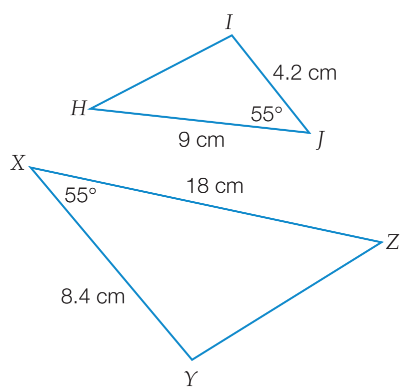
Understanding

13 Show that the following pairs of triangles are similar.

(a)

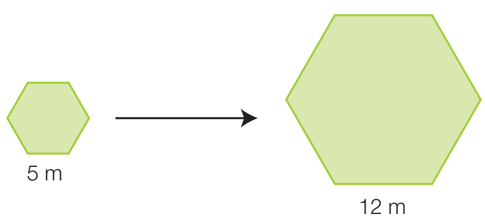


(b)

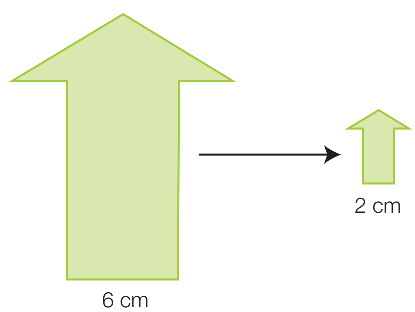


14 What scale factors have been used in the following?

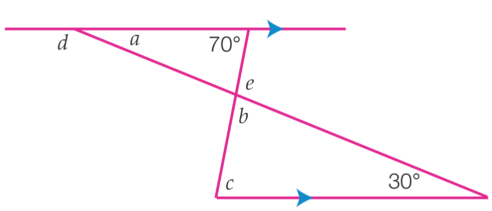
(a)



(b)

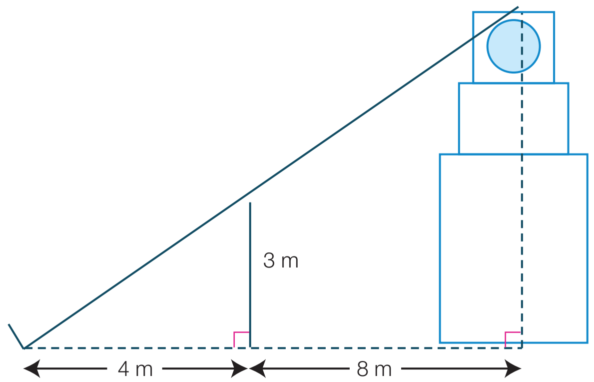


15 Find the values of the variables in the diagram below.



Reasoning

16 The clock on the Town Hall is being repaired. A safety wire has been attached to the top of the Town Hall, as shown. How high is the Town Hall?



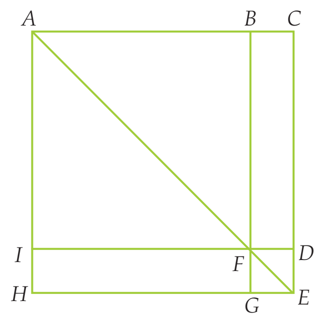
17 **(a)** Expand the brackets in the expression $x(x + 4) - 21$.

(b) Factorise this new expression as the product of two brackets.

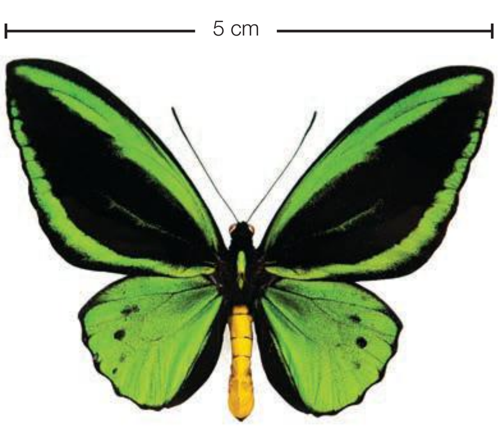
(c) What value of x in this new expression gives 110×100?

(d) Use the last answer to show that $103 \times 107 - 21 = 110 \times 100$.

18 The diagram below has three different sized triangles. Name one example of each triangle and show that all three are similar.



19 What scale factor is needed to enlarge this image of a butterfly to a width of 27 cm?



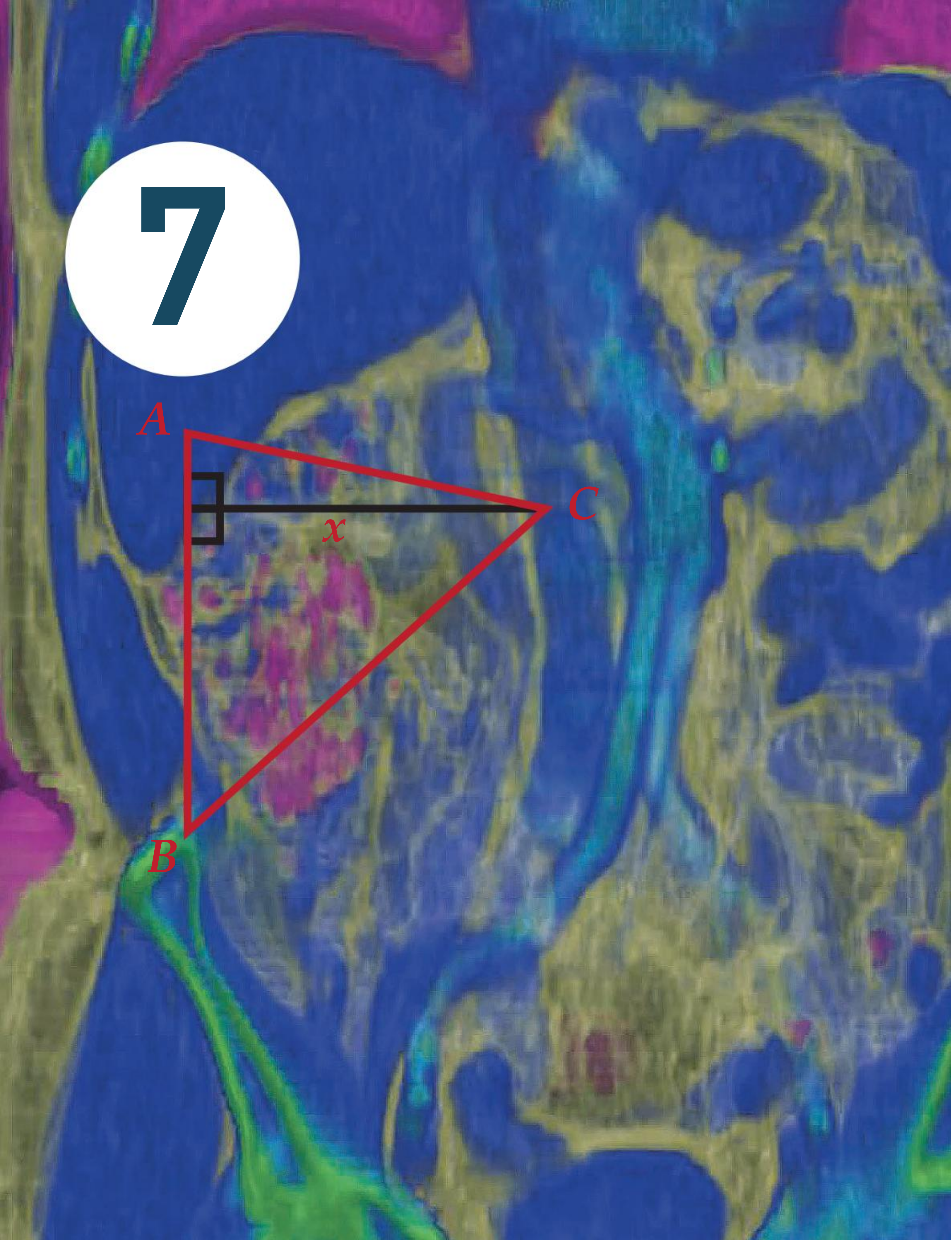
20 A standard size photo is 10 cm × 15 cm.

(a) If the photo is enlarged to 30 cm × 45 cm, what is the scale factor of the enlargement?

(b) Calculate the area of **(i)** the standard size photo and **(ii)** the enlargement.

(c) Calculate the ratio $\dfrac{\text{area of enlargement}}{\text{area of standard size}}$.

(d) Compare your answers to **(a)** and **(c)**. What do you notice?


7
A
C
x
B

Trigonometry

7

Operating with triangles

Advances in digital imagery use mathematics to help save patient's lives.

A CAT or CT (Computed Tomography) scan uses a combination of X-rays and computer technology to produce an internal 'slice' of the body, showing bones, muscles, blood vessels and all internal organs. Trigonometry can be used on a scanned image to locate the exact position of tumours, infections or other abnormalities. So, doctors now not only know what is wrong before operating on a patient, but they can also calculate the exact position where they must operate. Five hundred years ago, Leonardo da Vinci was stealing the bodies of dead prisoners and dissecting them to view inside the human body. Now you need only push a button.

Forum

Explain why CAT scans are said to have 'revolutionised the medical world'. Why is accuracy important when calculating distances inside the human body? A CAT scan can be used to find the location of an abnormality inside the body. What other information should a doctor know about an abnormality before operating?

Why learn this?

Trigonometry is the study of the relationships between the angles and sides of triangles and is essential in many fields. Trigonometry enables people to use the stars to navigate through open seas. It allows engineers and architects to calculate lengths and angles in their designs for buildings and structures. Surveyors and cartographers use trigonometry to calculate distances between landmarks and create accurate maps; oceanographers use trigonometry to study the movement of waves and tides; and optometrists use trigonometry to help them determine which type of glasses will help correct someone's vision.

After completing this chapter you will be able to:

- identify and find values for the three trigonomeric ratios, given a reference angle
- use trigonometry to find side lengths in right-angled triangles
- use trigonometry to find angles in right-angled triangles
- use angles of elevation and depression to solve problems using trigonometry
- use trigonometry to solve problems involving bearings.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R7.1

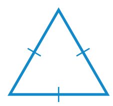
1 Solve the following equations. Where necessary, express your answer as a decimal correct to four decimal places.

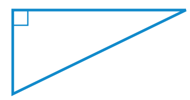
(a) $2x + 16 = 41$ **(b)** $38 - x = 25$ **(c)** $x^2 + 10 = 74$

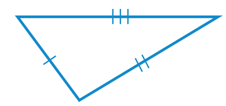
(d) $2x^2 = 98$ **(e)** $\frac{x}{15} = 0.356$ **(f)** $\frac{30}{x} = 0.4783$

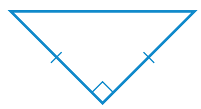
PP Worksheet R7.2

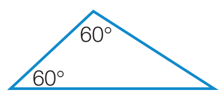
2 State whether the following triangles are: **(i)** equilateral, scalene or isosceles and **(ii)** acute angled, obtuse angled or right angled.

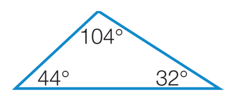
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

PP Worksheet R7.3

3 **(a)** For the formula $F = ma$, find F if $m = 6$ and $a = 7$.

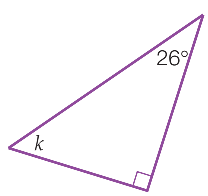
(b) Find S if $a = 21$, $b = 24$ and $c = 35$ in the formula $S = \frac{a + b + c}{2}$.

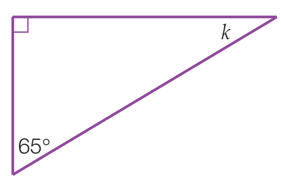
(c) For the formula $E = \frac{1}{2}mv^2$, find E when $m = 3$ and $v = 6$.

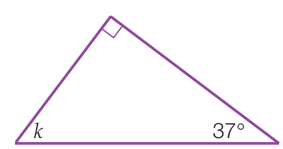
(d) For the formula $E = mx^2$, find m when $E = 100$ and $x = 5$.

PP Worksheet R7.4

4 Find the value of k in each of these triangles.

(a) 

(b) 

(c) 

PP Worksheet R7.5

5 Simplify the following ratios and then express the answer as a fraction.

(a) $8 : 10$ **(b)** $36 : 24$ **(c)** $\frac{5}{6} : \frac{2}{3}$

(d) $3\frac{1}{3} : 2\frac{1}{2}$ **(e)** 54 cm : 72 cm **(f)** 3.2 m : 400 cm

Key Words

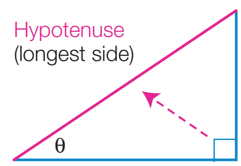
adjacent side	compass bearing	opposite side	trigonometry
angle of depression	cosine	reference angle	trigonometric ratio
angle of elevation	degree	sine	true bearing
bearing	hypotenuse	tangent	

Introduction to trigonometry

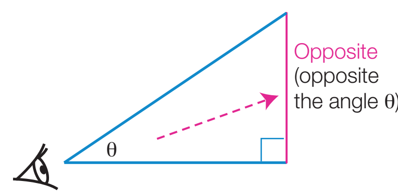
7.1

Trigonometry is the study of the relationship between the side lengths and angles in right-angled triangles. There are three ratios that compare the different lengths of the sides of a right-angled triangle. These ratios are called **trigonometric ratios**. To define these trigonometric ratios we need to name the sides of the right-angled triangle. When naming the sides, we need to specify which of the other two angles we are referring to, because it can affect the name of the side. In the diagrams below, the Greek letter theta, with the symbol θ, is used to identify the **reference angle**. Other commonly used symbols are α, β and ϕ (the letters alpha, beta and phi). At this level, angles are measured in the unit of **degrees**.

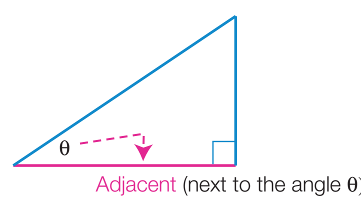
- The **hypotenuse** is always the longest side. It is always opposite the right angle.



- The **opposite side** is the side opposite the angle θ. To help you identify it, imagine that you are standing looking at the angle θ. From this view, the opposite side is behind θ.



- The **adjacent side** is the side next to the angle θ. It is between the right angle and θ.

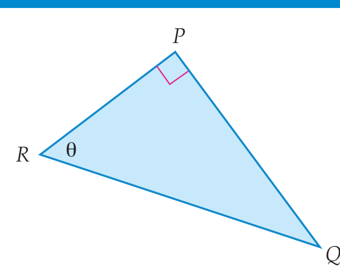


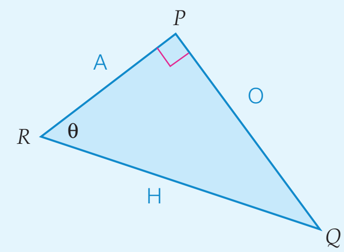
The side names can be abbreviated to H, O and A. The opposite and adjacent sides of a triangle are dependent on which angle is the reference angle.

Each of the three trigonometric ratios, which we will learn more about in the next section, uses two of the three sides of the triangle.

Worked Example 1

In the following triangle, label the hypotenuse (H), the opposite (O) and the adjacent (A) sides.



Thinking	Working
1 The hypotenuse is opposite the right angle. (*RQ* is the hypotenuse.) **2** The adjacent side is next to the reference angle θ. (*PR* is the adjacent side.) **3** Find the side opposite to the reference angle. (*PQ* is the opposite side.) Label the triangle accordingly.	

Worked Example 2

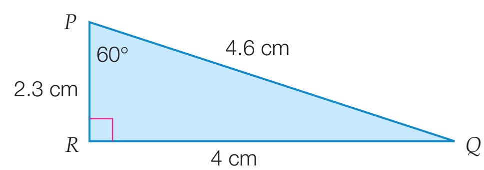
WE2

For the triangle pictured, with a reference angle of 60°, what is the length of:

(a) the hypotenuse

(b) the opposite side

(c) the adjacent side?



Thinking	Working
(a) Identify the longest side. This is the hypotenuse.	*(a) Length of hypotenuse = 4.6 cm.*
(b) Identify the side opposite the reference angle. This is the opposite side.	*(b) Length of opposite side = 4 cm.*
(c) Identify the side next to the reference angle. This is the adjacent side.	*(c) Length of adjacent side = 2.3 cm.*

7.1 Introduction to trigonometry

Navigator

Answers page 644

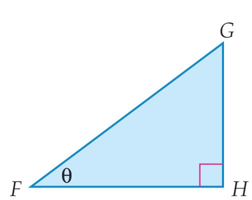
Q1 (a)–(c), Q2 (a)–(c), Q3 (a), (b), Q4, Q5, Q6, Q7, Q8, Q9, Q10	Q1 (a), (c), (e), Q2 (a), (c), (e), Q3 (a), (b), Q4, Q6, Q7, Q8, Q9, Q10	Q1 (d)–(f), Q2 (d)–(f), Q3 (b), (c), Q4, Q6, Q7, Q8, Q9, Q10

Fluency

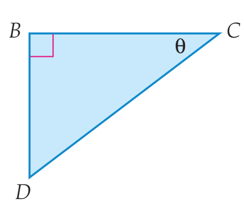
WE1

1 Copy each of the following triangles and use the reference angle to identify the hypotenuse, the opposite side and the adjacent side. Label these sides with H, O and A, respectively.

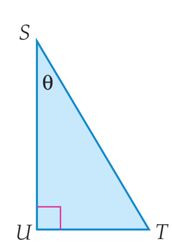
(a)

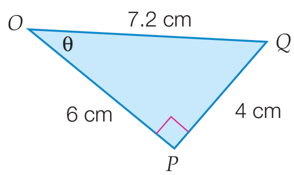


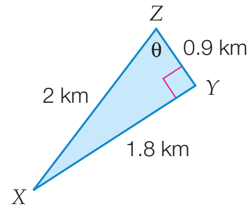
(b)

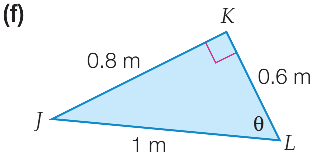


(c)



(d)


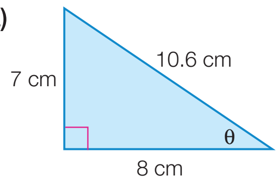
(e)


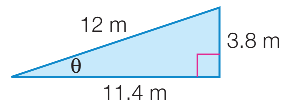
(f)


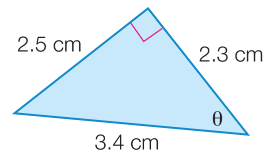
2 For each of the following triangles, state the length of the:

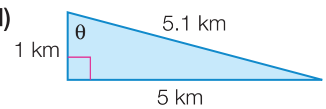
WE2

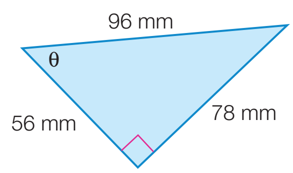
(i) hypotenuse
(ii) opposite side
(iii) adjacent side.

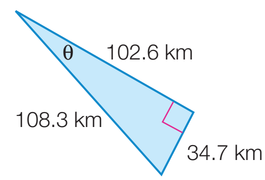
(a)


(b)


(c)


(d)


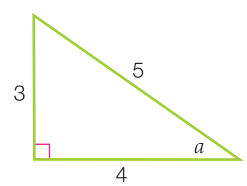
(e)


(f)


3 (a) Which of the following statements is incorrect?

A The hypotenuse is the longest side in a right-angled triangle.
B The hypotenuse is opposite the right angle.
C The shorter side next to the reference angle is called the adjacent side.
D A right-angled triangle always has two sides equal.

(b) From this triangle determine which of the following represents the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for the angle a:



A $\frac{3}{5}$ B $\frac{3}{4}$

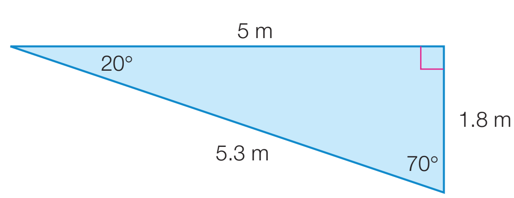
C $\frac{4}{5}$ D $\frac{4}{3}$

(c) Using the diagram from part **(b)**, what would be the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for the other acute angle in the triangle?

A $\frac{3}{5}$ B $\frac{4}{5}$ C $\frac{4}{3}$ D $\frac{3}{4}$

Understanding

4 Consider the following triangle:



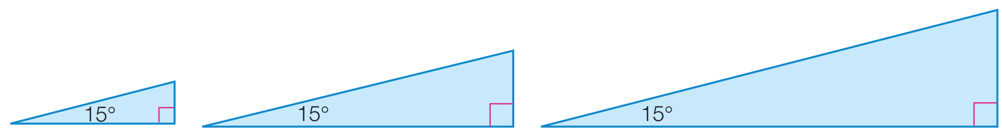
(a) State the length of the hypotenuse.
(b) If 20° is the reference angle, state the length of the opposite side.
(c) If the adjacent side is 1.8 m long, what is the size of the reference angle?

5 A right-angled triangle has three sides of 5, 12 and 13.

(a) Which of these sides would be the hypotenuse?

(b) Draw a diagram of the triangle with the vertices labelled *A*, *B* and *C*, with the right angle at *B*. Mark the lengths on the sides.

6 **(a)** In the following right-angled triangles, measure the length of each side.

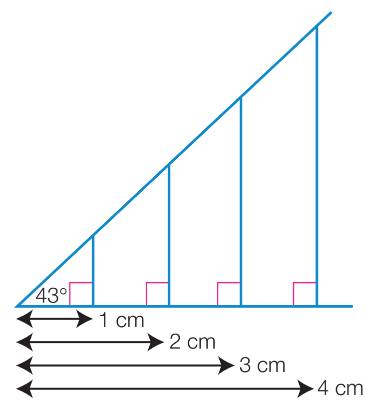


(b) For each triangle in **(a)**, use 15° as the reference angle to find the value of $\frac{\text{opposite}}{\text{hypotenuse}}$.

(c) For each triangle in **(a)**, use 15° as the reference angle to find the value of $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Reasoning

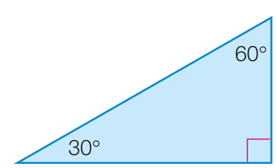
7 **(a)** Use a protractor to draw an angle of 43°. Mark off distances of 1 cm, 2 cm, 3 cm and 4 cm and rule a perpendicular line at each point. A diagram is shown here.



(b) Measure the heights of the four triangles created and, using 43° as the reference angle, calculate the ratio $\frac{\text{opposite}}{\text{adjacent}}$ in each triangle. Write your ratios in decimal form, rounded to one decimal place.

(c) The four triangles that you have drawn are similar triangles because they all have equal angles (the AAA similarity test). How does this help you explain your answer to **(b)**?

8 **(a)** Use a ruler and protractor to accurately draw three differently sized triangles with angles of 30°, 60° and 90°. One triangle is shown here.



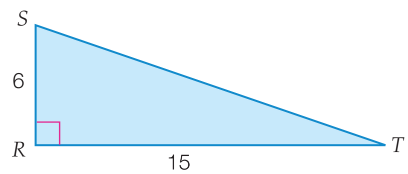
(b) Use your triangles from **(a)** to copy and complete the table below. Write measurements to the nearest mm.

Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°						
2	30°						
3	30°						
1	60°						
2	60°						
3	60°						

(c) What do you notice about the values calculated in the last three columns?

(d) Because the three triangles drawn in **(a)** are of different sizes but have the same angles, they are similar triangles. What do you know about the side lengths in similar triangles?

9 The teacher drew the diagram on the right on the board. He asked the students to complete the triangle and label the hypotenuse, opposite and adjacent sides.



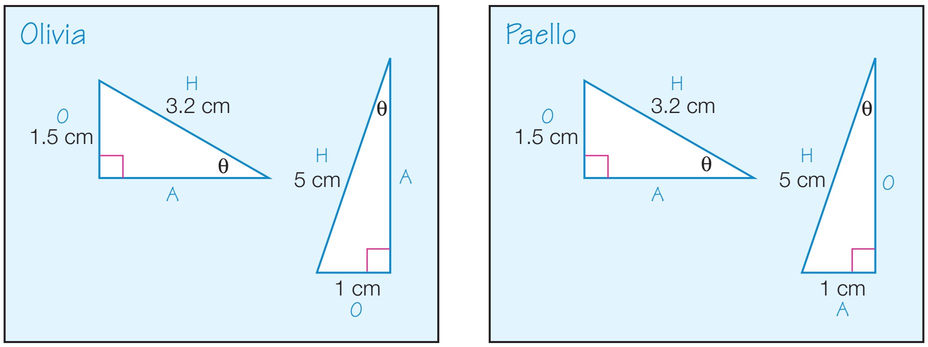
Nash said, 'The opposite side is 15'. Aisha said, 'The opposite side is 6'. They are both right.

(a) What piece of information did the teacher forgot to add to the triangle that makes it possible for both students to be correct?

(b) Draw the completed triangles of Nash and Aisha.

Open-ended

10 A class was given two right-angled triangles and asked to label the hypotenuse, opposite and adjacent sides, relative to the reference angle θ. The answers of two students, Olivia and Paello, are shown below:



Paello says to Olivia: 'Your answer is different to mine. I think it's wrong, because the adjacent side is always the one on the bottom of a right-angled triangle'.

Write what you would say to Paello to explain and correct his misunderstanding.

Outside the Square Puzzle

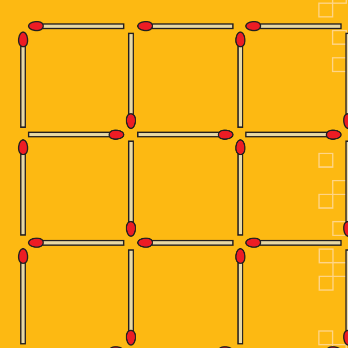
Matchstick madness

Starting with the arrangement of matchsticks shown, how could you:

(a) Remove four matches to leave five identical squares?

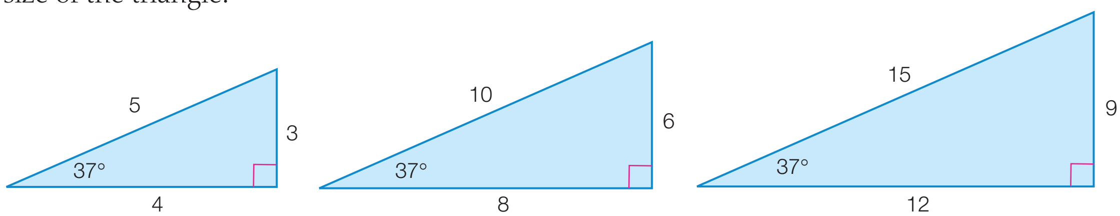
(b) Remove eight matches to leave four identical squares?

(c) Remove six matches to leave three different squares?



7.2 Trigonometric ratios

All triangles with three angles the same are similar. One of the properties of similar triangles is that the ratios between matching side lengths are constant, regardless of the size of the triangle.



The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is the same for each of the above triangles: $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$.

The same can be said for the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$. These three trigonometric ratios are called **sine** (sin), **cosine** (cos) and **tangent** (tan). Each ratio uses two of the three sides of a right-angled triangle. For a specific reference angle, the value of a particular ratio is always the same, regardless of the size of the triangle. For example:

The sine of 37° (sin 37°) = 0.60 (correct to two decimal places)

The cosine of 37° (cos 37°) = 0.80 (correct to two decimal places)

The tangent of 37° (tan 37°) = 0.75 (correct to two decimal places)

The sine of θ is given by:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

The cosine of θ is given by:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

The tangent of θ is given by:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

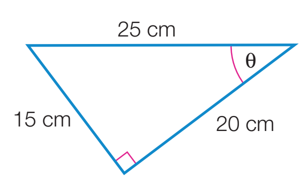
A common acronym to help with remembering the trigonometric ratios is:

SOH CAH TOA

Worked Example 3

WE3

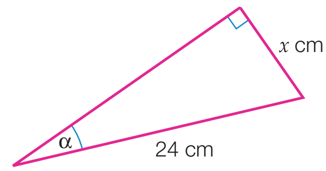
Find the exact value of tan θ.



Thinking	Working
1 Write the rule for the tangent ratio.	$\tan\theta = \frac{O}{A}$
2 Identify the opposite and adjacent sides and substitute the lengths.	$\tan\theta = \frac{15}{20}$
3 Simplify if possible.	$= \frac{3}{4}$

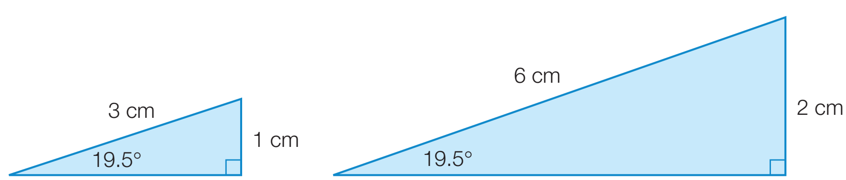
Worked Example 4 WE4

If $\sin \alpha = \frac{1}{3}$, use the sine ratio to find the value of x in the following triangle.



Thinking	Working
1 Write the rule for the sine ratio.	$\sin \alpha = \frac{O}{H}$
2 Identify the hypotenuse and the opposite side and substitute the known values.	$\frac{1}{3} = \frac{x}{24}$
3 Solve the equation for x.	$\frac{1}{3} \times 24 = x$ $x = 8$ cm

The trigonometric ratio for a particular reference angle can be found by drawing the triangle and measuring the required side lengths. The right-angled triangle below shows that the sine ratio for 19.5° is approximately $\frac{1}{3}$.



This means that for right-angled triangles with a reference angle of 19.5°, the opposite side length will always be very close to one-third the length of the hypotenuse.

> **Warning**
> This measuring technique is provided to demonstrate how the trigonometric ratios are found and requires an accurately drawn scale diagram. In the next section you will learn how to find these values accurately by using your calculator. You should never assume that a diagram in a textbook or on a test is drawn to scale.

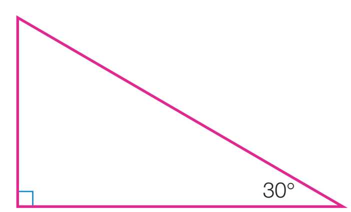
Worked Example 5 WE5

Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

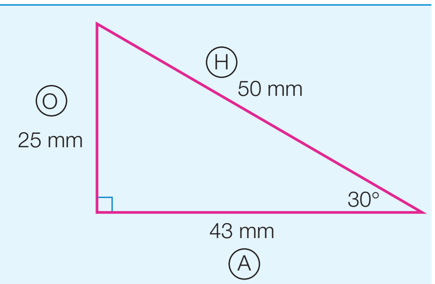
(a) sin 30°

(b) cos 30°

(c) tan 30°



Thinking	Working
1 Measure the sides, putting measurements on the triangle	
2 Label the hypotenuse (H), opposite (O) and adjacent (A).	



(a) Use SOH to calculate sin 30°.	(a) $\sin 30° = \frac{O}{H} = \frac{25}{50} = \frac{1}{2} = 0.50$
(b) Use CAH to calculate cos 30°.	(b) $\cos 30° = \frac{A}{H} = \frac{43}{50} = 0.86$
(c) Use TOA to calculate tan 30°.	(c) $\tan 30° = \frac{O}{A} = \frac{25}{43} = 0.58$

7.2 Trigonometric ratios

Navigator

Answers page 645

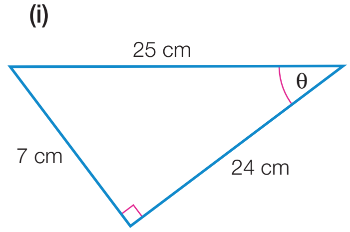
Q1 (a) (i), (b) (i), (c) (i), Q2 (a) (i), (b) (i), (c) (i), Q3, Q4, Q6 (a), (b), Q7, Q8, Q10, Q11, Q12, Q15	Q1 (a) (ii), (b) (ii), (c) (ii), Q2 (a) (ii), (b) (ii), (c) (ii), Q3, Q4, Q6 (b), (c), Q7, Q9, Q10, Q11, Q12, Q14, Q15	Q1 (a) (iii), (b) (iii), (c) (iii), Q2 (a) (iii), (b) (iii), (c) (iii), Q3, Q4, Q5, Q6 (c), (d), Q8, Q9, Q11, Q12, Q13, Q15

Fluency

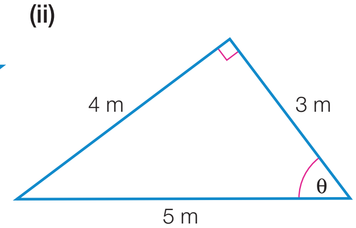
WE3

1 **(a)** Find the exact value of tan θ in each triangle.

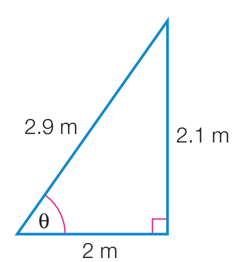
(i)



(ii)

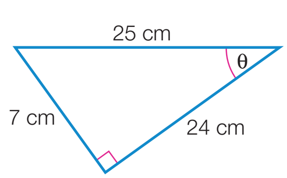


(iii)

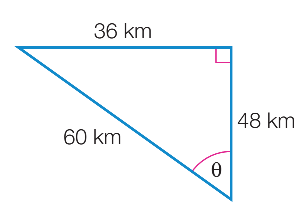


(b) Find the exact value of sin θ in each triangle.

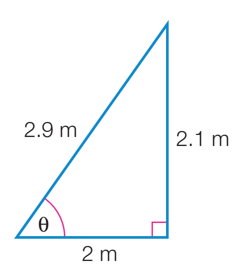
(i)



(ii)

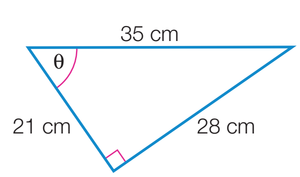


(iii)

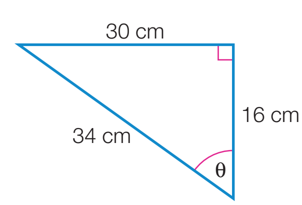


(c) Find the exact value of cos θ in each triangle.

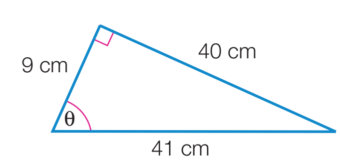
(i)



(ii)



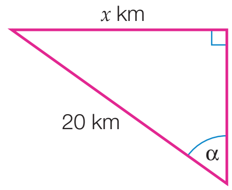
(iii)



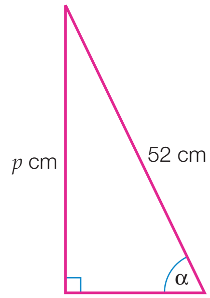
2 **(a)** Find the value of the variable, correct to two decimal places, in the following triangles, where sin α =

WE4

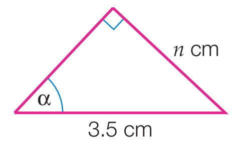
(i) $\frac{1}{3}$



(ii) $\frac{3}{4}$

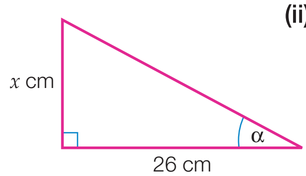


(iii) $\frac{1}{5}$

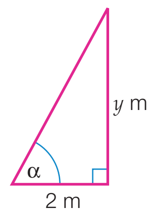


(b) Find the value of the variable, correct to two decimal places, in the following triangles, where tan α =

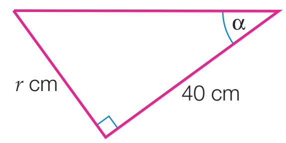
(i) $\frac{1}{2}$



(ii) 3

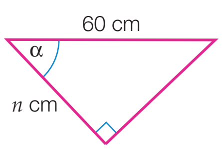


(iii) 0.52

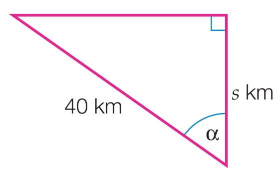


(c) Find the value of the variable, correct to two decimal places, in the following triangles, where cos α =

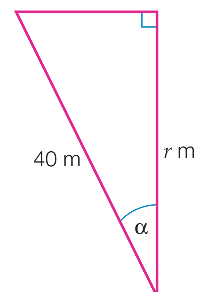
(i) $\frac{4}{5}$



(ii) 0.3



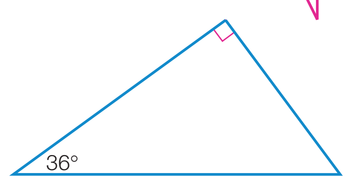
(iii) 0.56



3 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

WE5

(a) sin 36° **(b)** cos 36° **(c)** tan 36°



4 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

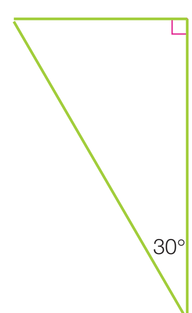
(i) tan 30° **(ii)** cos 30°

(iii) sin 30° **(iv)** sin 60°

(v) tan 60° **(vi)** cos 60°

(b) What do you notice about the values of cos 30° and sin 60°?

(c) What do you notice about the values of cos 60° and sin 30°?



Hint
What is the value of the unlabelled angle in the diagram?

5 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

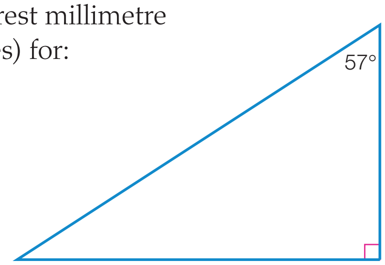
(i) cos 57° **(ii)** tan 57°

(iii) sin 57° **(iv)** sin 33°

(v) cos 33° **(vi)** tan 33°

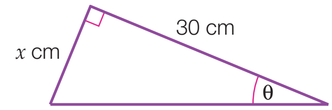
(b) What do you notice about the values for sin 57° and cos 33°?

(c) What do you notice about the values for sin 33° and cos 57°?



Hint
What is the value of the third angle in this triangle?

Understanding

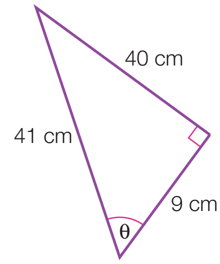


6 (a) If $\tan\theta = \frac{2}{3}$, then the value of x in this triangle is:

A 15 B 20

C 40 D 45

(b) In this diagram, $\cos\theta$ is:



A $\frac{9}{41}$ B $\frac{40}{41}$

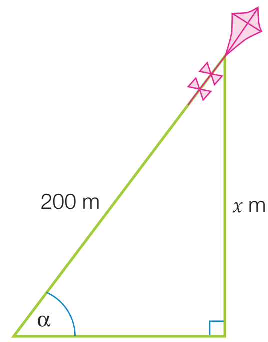
C $\frac{40}{9}$ D $\frac{41}{9}$

(c) The value for sin 71° would be the same as:

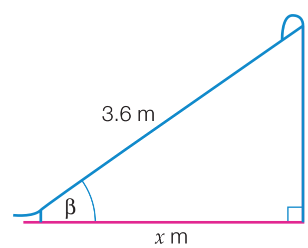
A cos 71° B cos 19°

C sin 19° D cos 29°

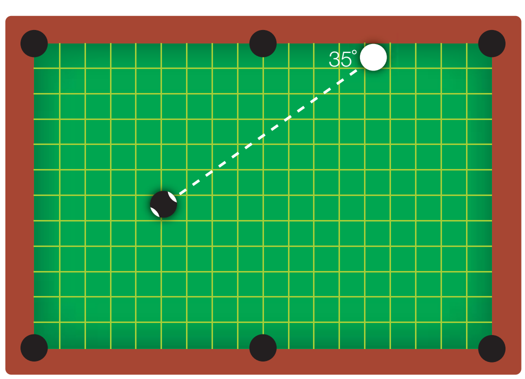
7 One windy day, Nam was flying a kite. Nam's kite string was 200 m long. If $\sin\alpha = \frac{1}{2}$, how high above the ground was the kite?



8 A slippery dip is 3.6 m long. If $\cos\beta = \frac{7}{9}$, find the distance between the foot of the slippery dip and the bottom of the supporting tower.



9 In a game of pool, the cue ball is at the edge of the table and is hit 152 cm, at an angle of 35° with the side of the table. It hits the black ball, which is 125 cm further down the table. Calculate the value of cos 35°, correct to two decimal places.



Reasoning

10 A 7.5 m ladder is leaning against a wall at an angle of 45°. The bottom of the ladder is 5.3 m from the base of the wall.

(a) Draw a diagram to scale (1 cm = 1 m) showing the information provided.

(b) What type of triangle is formed by the ladder, the wall and the ground?

(c) How far up the wall does the ladder reach?

(d) Explain how you can find the answer to **(c)** without doing any calculations.

11 A slide that is 5 m long is to be built in a playground. The recommended angle of the slide with the ground is 37°. Parents are concerned that the slide may be too high for the children.

(a) If $\sin 37° = \frac{3}{5}$, what is the height of the slide with this recommended slide angle?

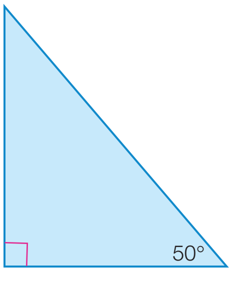
(b) The parents are only happy if the height at the top of the slide is no greater than 2.5 m. What changes could be made so that this can happen?

12 The sine of an angle is $\frac{12}{13}$.

(a) Draw the triangle and label the sides. **(b)** Calculate the length of the other side.

(c) Determine the cosine ratio.

13 A right-angled triangle with a reference angle of 50° is shown.



(a) Measure the side lengths (in mm) and then calculate the values of sin 50°, cos 50° and tan 50°, correct to three decimal places.

(b) In what way is the value of tan 50° different to sin 50° and cos 50°? Explain why this happens.

(c) Draw a right-angled triangle that has:

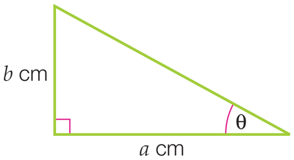
(i) a tan θ value of less than 1

(ii) a tan θ value of more than 1.

(d) Can the values of sin θ and cos θ be greater than 1? Why/why not?

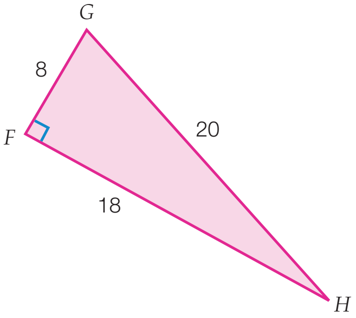
Open-ended

14 In the following triangle, $\tan \theta = \frac{7}{10}$. Write down at least three possible pairs of values for the lengths of sides a and b.



15 The teacher of a Year 9 class drew the following triangle on the board and asked the students to find the sin and cos ratios for the angles at G and H.

After completing the task, Rashira looked at her answers and noticed that she had written $\sin H = \frac{8}{20}$ and $\cos G = \frac{8}{20}$. She said to her friend, 'They're the same!' That can't be right, can it? How would you explain to Rashira that she has the correct answers?



7.3 Using trigonometry to find side lengths

Recall that…

The sine of θ is given by:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

SOH

The cosine of θ is given by:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

CAH

The tangent of θ is given by:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

TOA

To use trigonometry to find side lengths in right-angled triangles, you first need to identify which trigonometric ratio to use. To do this, identify which side lengths are involved in the problem. For example, if the adjacent side and the hypotenuse are the lengths of interest, then the cosine ratio will be used.

To calculate an unknown side length of a right-angled triangle using trigonometry:

1 Draw the right-angled triangle, showing all known information.

2 Label the sides with O, A and H in relation to the given angle.

3 Use SOH–CAH–TOA to decide which trigonometric ratio to use.

4 Form an equation and solve for the unknown.

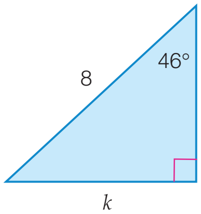
Using a calculator to find a trigonometric ratio

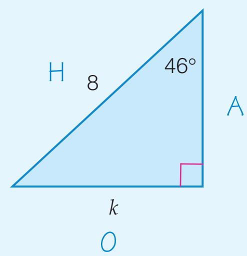
As stated previously, the value of a particular trigonometric ratio is always the same for a particular angle. We can use our calculator to find the value of a ratio by pressing one of the following keys: **sin** **cos** **tan**. For example, to find the sine of 67°, you would press **sin** **6** **7** **=**. Many trigonometric ratio values are long decimals. It is more accurate to use the full value in your calculations and round your final answer than to use a rounded ratio in your calculations. Make sure your calculator is set to degrees mode.

Worked Example 6

WE6

Find the value of k, correct to two decimal places.

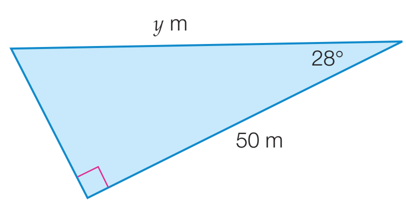


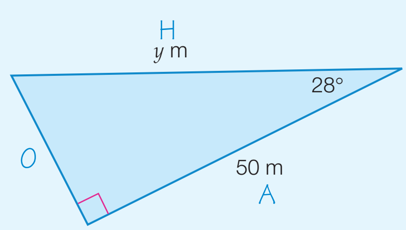
Thinking	Working
1 Use the reference angle to label the sides of the triangle H, O and A.	
2 List the information given.	$\theta = 46°$ $H = 8$ $O = k$
3 Identify the sides involved in the problem (H and O) and hence the trigonometric ratio that will need to be used (sine). Write out the rule for the required ratio.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
4 Substitute the values.	$\sin 46° = \dfrac{k}{8}$
5 Rearrange to make the unknown the subject of the equation (here we multiply both sides by 8).	$k = 8 \times \sin 46°$
6 Use a calculator to determine k, correct to two decimal places.	≈ 5.75 (2 d.p.)

Worked Example 7

WE7

Find the value of y, correct to two decimal places.



Thinking	Working
1 Use the reference angle to label the sides of the triangle H, O and A.	
2 List the information given.	$\theta = 28°$ $A = 50$ m $H = y$ m
3 Identify the sides involved in the problem (H and A) and hence the trigonometric ratio that will need to be used (cosine). Write the rule for the required ratio.	$\cos\theta = \dfrac{A}{H}$

4 Substitute the values. $\cos 28° = \dfrac{50}{y}$

5 Rearrange to make the unknown the subject of the equation (here we multiply both sides by y, then divide by cos 28°).

$$y \times \cos 28° = 50$$
$$y = \frac{50}{\cos 28°}$$

6 Use a calculator to determine y, correct to two decimal places. ≈ 56.63 m (2 d.p.)

7.3 Using trigonometry to find side lengths

Navigator

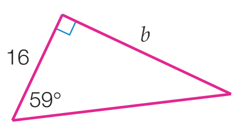
Answers page 645

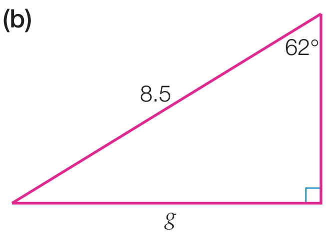
Q1, (a)–(c), Q2 (a)–(c), Q3, Q5, Q6, Q7, Q8, Q9, Q11, Q12	Q1 (a), (c), (e), Q2 (a), (c), (e), Q3, Q5, Q7, Q9, Q10, Q11, Q12, Q13	Q1 (d)–(f), Q2 (d)–(f), Q4, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14

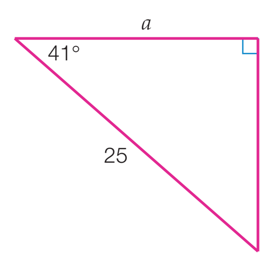
Fluency

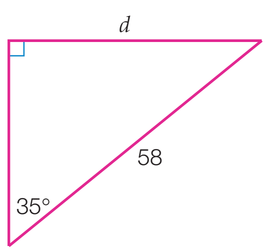
WE6

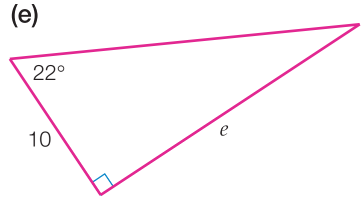
1 Find the value of each variable correct to two decimal places.

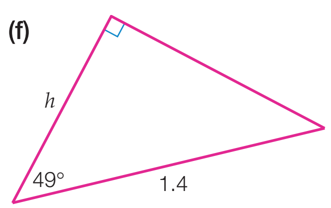
(a)


(b)


(c)


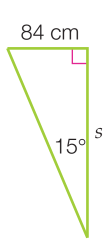
(d)


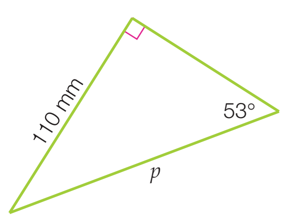
(e)


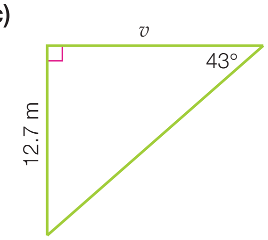
(f)


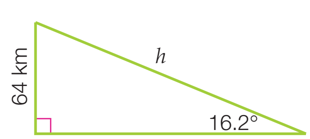
WE7

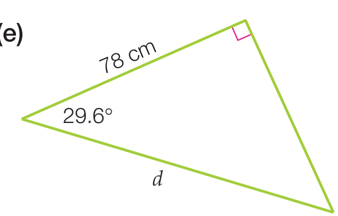
2 Find the value of the variable (correct to two decimal places) in each of the following triangles.

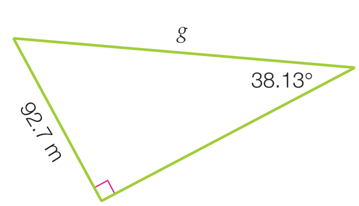
(a)


(b)


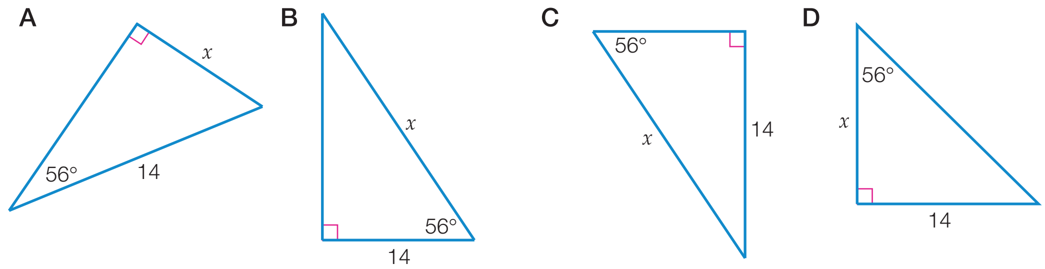
(c)


(d)


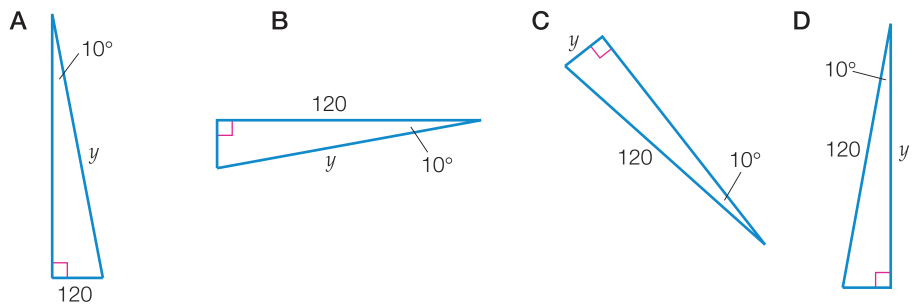
(e)


(f)


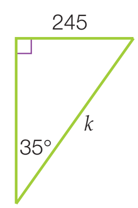
3 (a) For which of the following triangles can $\sin 56° = \dfrac{14}{x}$ be solved to give x?



(b) For which of the following triangles can $\cos 10° = \dfrac{y}{120}$ be solved to give y?



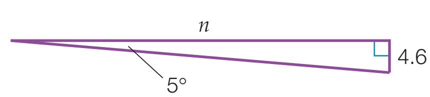
4 (a) The value of k could be found by using the equation:



A $\sin 35° = \dfrac{k}{245}$ B $\tan 35° = \dfrac{245}{k}$

C $\sin 35° = \dfrac{245}{k}$ D $\cos 35° = \dfrac{245}{k}$

(b) The value of n could be found by using the equation:



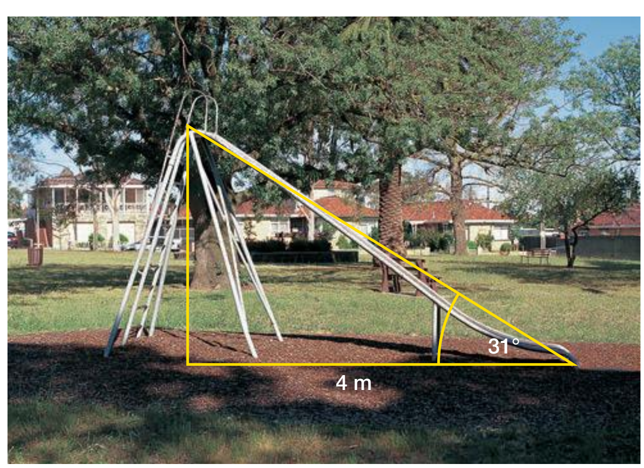
A $\sin 5° = \dfrac{4.6}{n}$ B $\cos 5° = \dfrac{4.6}{n}$

C $\tan 5° = \dfrac{n}{4.6}$ D $\tan 5° = \dfrac{4.6}{n}$

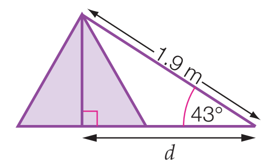
Understanding

For the following questions, express all answers correct to two decimal places.

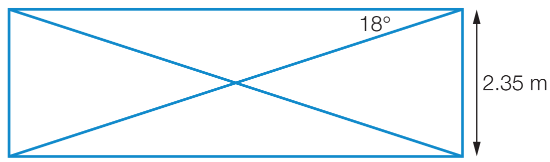
5 Use the information given in the photograph to find the height of the slide.



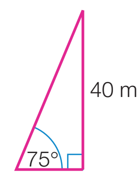
6 Calculate how far away a tent peg is from a tent pole if the 1.9 m support wire is pegged so that it makes an angle of 43° with the ground.



7 A rectangular gate has diagonal wooden supports that make an angle of 18° to the horizontal sections of the frame. If the gate is 2.35 m high, how long would one of the diagonal supports be?



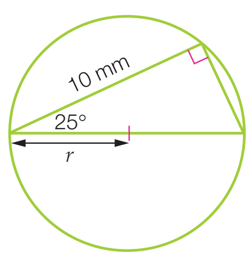
8 Members of a stunt team wish to be catapulted into the air in a capsule at an angle of 75° to the horizontal. Assuming they travel along a straight path, what horizontal distance will they have covered when they have reached a height of 40 m?



Reasoning

9 A road sign is in the shape of an equilateral triangle with a vertical measurement of 41.2 cm. How long is each side of the road sign?

10 Find the radius of this circle.



Open-ended

11 An angle in a right-angled triangle measures 63°. Give two possible pairs of lengths of the adjacent side and the hypotenuse.

12 Shown is Caillan's working for a trigonometry question.

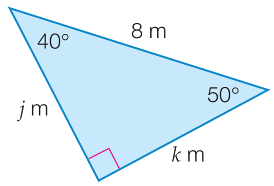
(a) What might the question have been? (Draw the triangle).

(b) Calculate the length of the third side of the triangle.

$$\cos 15° = \frac{20}{y}$$
$$y \times \cos 15° = 20$$
$$y = \frac{20}{\cos 15°}$$
$$y = 20.7 \text{ cm}$$

13 The following answers were given by a student on a trigonometry test.

(i) Find the value of k.



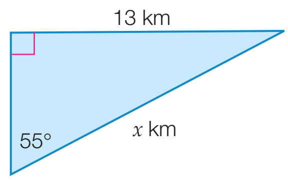
$\cos\theta = \frac{A}{H}$

$\cos 40° = \frac{k}{8}$

$8 \times \cos 40° = k$

$k = 6.13$ m

(ii) Find the value of x.



$\sin\theta = \frac{O}{H}$

$\sin 55° = \frac{13}{x}$

$13 \times \sin 55° = x$

$x = 10.65$ km

(a) Explain the mistake the student has made in each question.

(b) Show the correct calculations and answers.

14 Explain the differences between using Pythagoras' Theorem to find an unknown side of a right-angled triangle and using trigonometric ratios. For each method, state the minimum information that must be known in order to apply the method successfully.

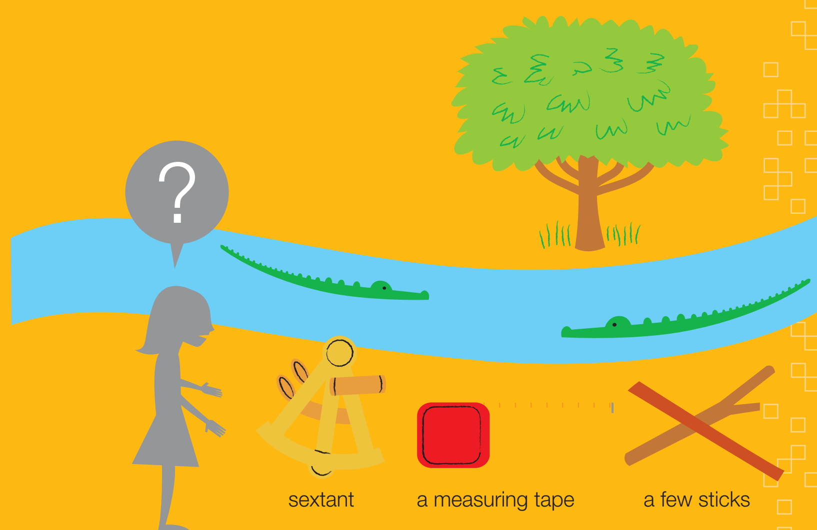
Outside the Square — Problem solving

Finding the height of a tree

Your task is to find the height of a tree that is on the other side of a river that cannot be crossed. You have been given a 100 m tape measure and a sextant.

A sextant is an instrument for measuring the angle from your horizontal line of sight to the top of another object. This object may be higher or lower than your line of sight. You do not have to reach the object to be able to find the angle, but it must be within sight.

How can you find the height of the tree?



Strategy options

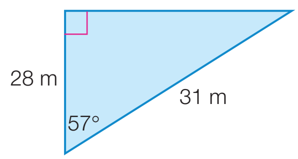
- Draw a picture.
- Break the problem into manageable parts.

Half-time 7

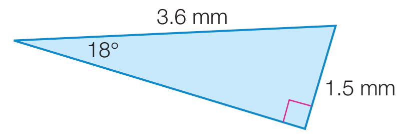
Ex 7.2

1 Write the letter names (O, A or H) of the two sides that would be used in any calculations.

(a)



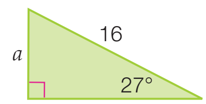
(b)



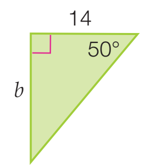
Ex 7.3

2 Find the value of each variable, correct to two decimal places.

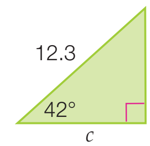
(a)



(b)

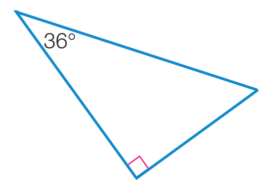


(c)



Ex 7.2

3 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to two decimal places) for:

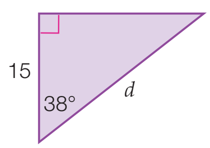


(a) sin 36° (b) cos 36° (c) tan 36°

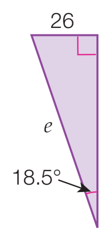
Ex 7.3

4 Find the value of each variable, correct to two decimal places.

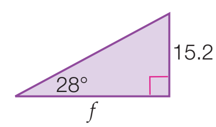
(a)



(b)



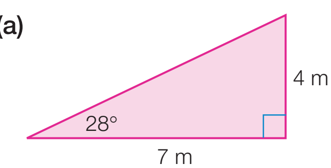
(c)



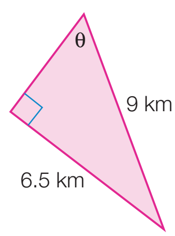
Ex 7.1

5 Label the sides of the following right-angled triangles as hypotenuse (H), opposite (O) and adjacent (A).

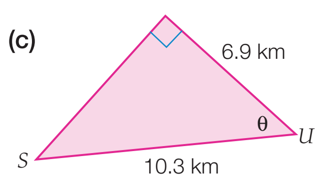
(a)



(b)

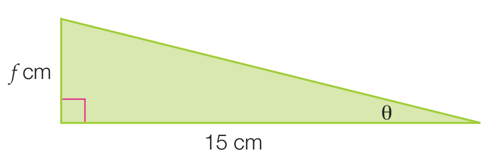


(c)



Ex 7.2

6 If $\tan \theta = \frac{2}{5}$, what is the value of f?



Using trigonometry to find angles

7.4

If we are asked to solve an equation such as $3x - 5 = 22$, we would use the 'inverse operations' of $+ 5$ and $\div 3$. These inverse operations reverse the effect of the initial operations ($\times 3$, -5) used to form the equation. We will now look at a similar idea involving the trigonometric ratios. In the previous section, we used a specific angle to find a trigonometric ratio. The inverse of this is to use a specific ratio to find the corresponding angles, for example, finding the value of θ given $\sin \theta = 0.45$.

To use the trigonometric ratios in right-angled triangles to find an angle, two of the side lengths must be known. We have seen that the trigonometric ratios sine, cosine and tangent are the ratios of two specific side lengths of a right-angled triangle. If we know two side lengths, they will form one of the trigonometric ratios. We can then find which angle corresponds to this ratio by rearranging the trigonometric equation to make the angle the subject. This requires the use of the trigonometric inverse.

Trigonometric ratio	Inverse
sine of θ $\sin \theta = \frac{O}{H}$	$\theta = \sin^{-1}\left(\frac{O}{H}\right)$
cosine of θ $\cos \theta = \frac{A}{H}$	$\theta = \cos^{-1}\left(\frac{A}{H}\right)$
tangent of θ $\tan \theta = \frac{O}{A}$	$\theta = \tan^{-1}\left(\frac{O}{A}\right)$

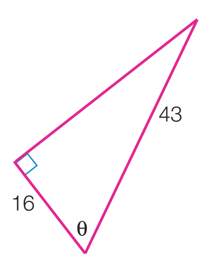
Note that the notation used here can appear to be misleading. The $^{-1}$ does not literally mean 'to the power of -1'. When talking about $\sin^{-1}$ you should always say 'inverse sine', not 'sine to the power of negative one'.

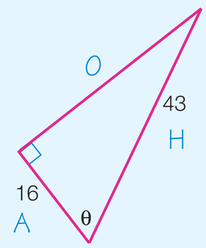
On most calculators, the inverse trigonometric functions are found by pressing **2nd F**, **SHIFT** or **INV** before pressing **sin**, **cos** or **tan**.

Worked Example 8

WE8

Find the value of θ correct to the nearest degree.

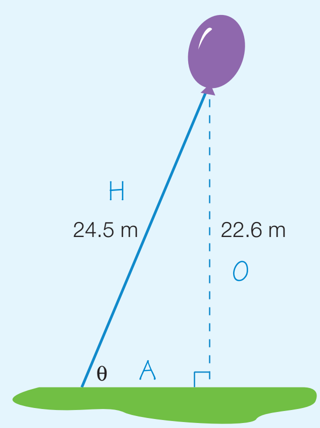


Thinking	Working
1 Label the sides of the triangle O, A and H.	
2 Identify the sides involved in the problem (adjacent and hypotenuse) and hence the trigonometric ratio that will need to be used (cosine). Write the rule for the required ratio.	$\cos\theta = \frac{A}{H}$
3 Substitute the values.	$\cos\theta = \frac{16}{43}$
4 Rearrange to make θ the subject.	$\theta = \cos^{-1}\left(\frac{16}{43}\right)$
5 Use a calculator to determine θ, correct to the nearest degree.	$\approx 68°$

Worked Example 9

WE9

A helium-filled balloon is attached to the ground by a 24.5 m string. A light breeze is blowing so that the base of the balloon is at a height of 22.6 m above the ground. What is the smaller of the two angles, to the nearest degree, that the string makes with the ground?

Thinking	Working
1 Draw a diagram that shows a right-angled triangle with all the given information. Label the sides O, A and H.	
2 Identify the sides involved in the problem (opposite and hypotenuse) and hence the trigonometric ratio that will need to be used (sine). Write the rule for the required ratio.	$\sin\theta = \frac{O}{H}$

3	Substitute the values.	$\sin\theta = \dfrac{22.6}{24.5}$
4	Rearrange to make θ the subject.	$\theta = \sin^{-1}\left(\dfrac{22.6}{24.5}\right)$
5	Use a calculator to determine θ.	$\approx 67.287°$
6	Answer the question, giving your answer correct to the nearest degree.	The string makes an angle of 67° with the ground.

7.4 Using trigonometry to find angles

Navigator

Q1, (a)–(c), Q2, Q3, Q4, Q5, Q6, Q8, Q9	Q1 (c)–(e), Q2, Q3, Q4, Q5, Q7, Q8, Q9	Q1 (f)–(h), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

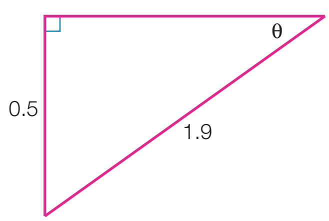
Answers page 646

Fluency

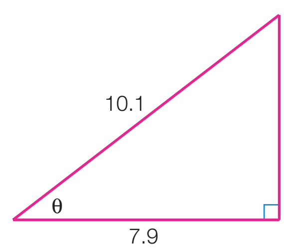
1 Find the value of θ, correct to the nearest degree, in each of the following.

WE8

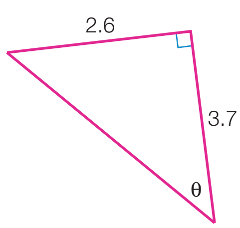
(a)



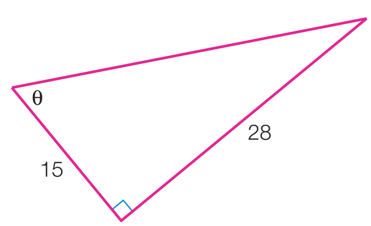
(b)



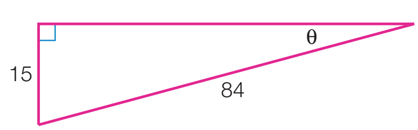
(c)



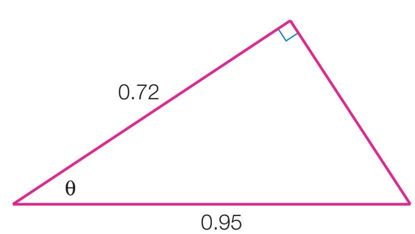
(d)



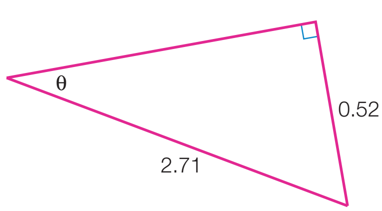
(e)



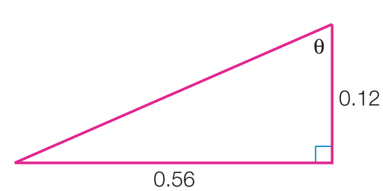
(f)



(g)

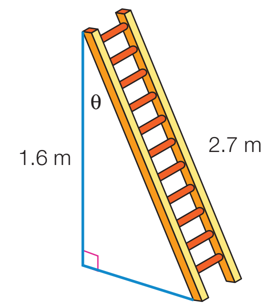


(h)



WE9

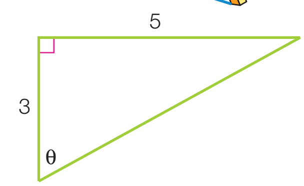
2 The top of a 2.7-m ladder, leaning against a wall, reaches a height of 1.6 m. What angle, to the nearest degree, does the ladder make with the wall?



3 **(a)** The value of θ can be found by using the equation:

A $\cos\theta = \frac{3}{5}$ **B** $\tan\theta = \frac{5}{3}$

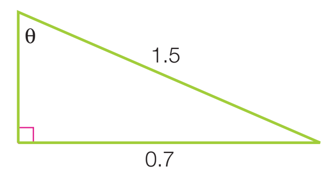
C $\sin\theta = \frac{3}{5}$ **D** $\tan\theta = \frac{3}{5}$



(b) The value of θ in this triangle would be equal to:

A $\tan^{-1}\left(\frac{0.7}{1.5}\right)$ **B** $\cos^{-1}\left(\frac{1.5}{0.7}\right)$

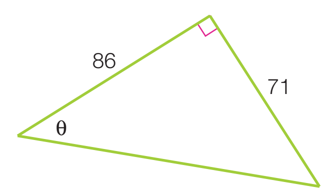
C $\sin^{-1}\left(\frac{0.7}{1.5}\right)$ **D** $\cos^{-1}\left(\frac{0.7}{1.5}\right)$



(c) The value of θ in this triangle would be equal to:

A $\cos^{-1}\left(\frac{71}{86}\right)$ **B** $\tan^{-1}\left(\frac{86}{71}\right)$

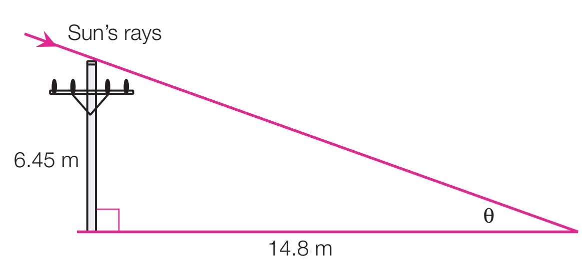
C $\sin^{-1}\left(\frac{71}{86}\right)$ **D** $\tan^{-1}\left(\frac{71}{86}\right)$



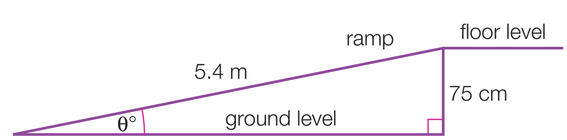
Understanding

4 A children's seesaw is made from a plank of wood 3.4 m long supported in the middle, as shown in the photograph. If the greatest height reached by one end of the seesaw is 0.9 m, what angle does the seesaw make with the ground? Round your answer to the nearest degree.

5 Find (to the nearest degree) the angle of the Sun's rays, θ, when a 6.45 m high power pole casts a shadow 14.8 m long.



6 A builder uses a ramp to bring in sand in a wheelbarrow from ground level to floor level inside a building under construction. Floor level is 75 cm above ground level. If the ramp is 5.4 m long, what angle, to the nearest degree, does the ramp make with the ground?



Reasoning

7 (a) When the opposite side and hypotenuse are similar in length, sin θ is large. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of sin θ.

(b) When the opposite side is small and the hypotenuse is large, sin θ is very small. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of sin θ.

(c) Complete the following sentences:

(i) When the adjacent side and hypotenuse are similar in length, cos θ is ________.

(ii) When the adjacent side is small and the hypotenuse is large, cos θ is ________.

8 One angle between 0° and 90° gives a trigonometric ratio of 1.

(a) What does a ratio of 1 tell you about the lengths of the two sides involved?

(b) Which ratio is it? Explain how you know.

(c) What type of right-angled triangle gives this ratio a value of 1?

(d) What is the value of the angle involved?

Open-ended

9 Two of the side lengths of a right-angled triangle are 9 and 15. What could the reference angle be?

10 An access ramp for a motorised wheelchair must make an angle with the horizontal that is between 6° and 9°. Find three different ramp lengths and the angles they must be placed at to give smooth access to a platform that is 65 cm high.

Outside the Square

Puzzle

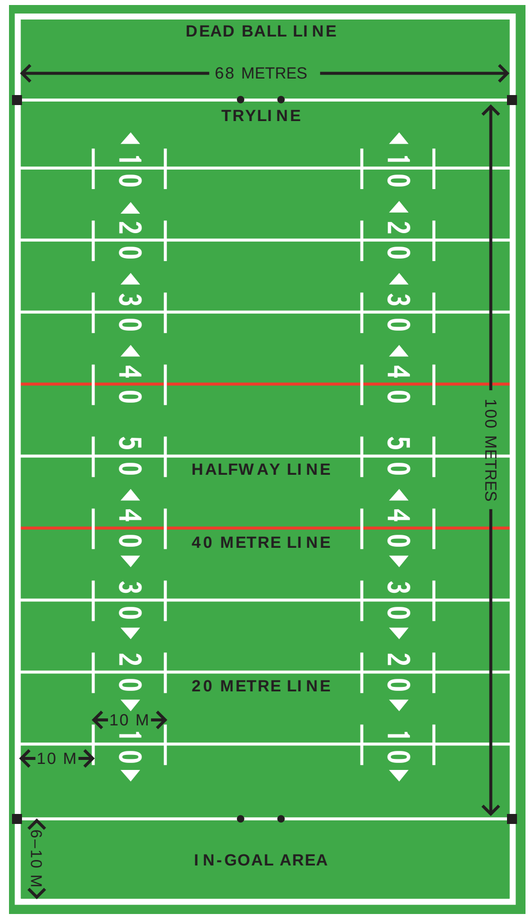
Sudoku

Copy the following Sudoku puzzle and complete the blank squares such that each row, column and smaller 3 × 3 square contains each of the digits 1–9 only once.

9					7	3	1	
7		2		9			4	8
	8		6	5	4			
		8			9	5	6	1
	1		4		6		7	
3	2	6	1			9		
			7	4	2			
6	4			1		8		9
	3	1						7

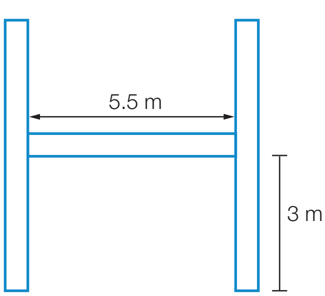
Investigation

Kicking goals in rugby



Equipment required: Calculator

To score in the game of Rugby League, teams need to move the ball down the field and place it over the try line. This is called a 'try' and is worth 4 points. After a try has been scored, the team can gain an extra 2 points by kicking a goal, known as a 'conversion'. A goal kicker must attempt to kick the goal from a position on the field anywhere on a line perpendicular to the tryline at the point where the try was scored. The goal consists of two posts 5.5 m apart, joined by a horizontal bar 3 m off the ground to make an 'H' shape. To score a goal, the ball must be kicked between the vertical posts and over the horizontal bar.



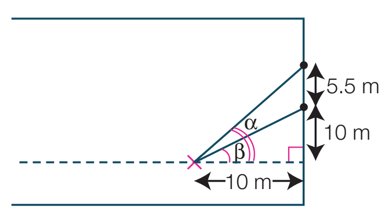
The Big Question

Where is the best position to kick from in order to maximise the chances of kicking a goal?

Engage

1 Imagine that a try is scored 10 m from the near goal post. Draw a diagram of the rugby field showing the goal posts, goal line and a line perpendicular to the goal line, 10 m from the near goal post.

Mark the goal kicker's position with an X, 10 m from the try line. Rule two lines from this point, one to the near goal post and one to the far post. Mark the angles formed by these lines with the line perpendicular to the goal line as α and β.



2 Separately draw the two right-angled triangles formed in the diagram that would be used to calculate the angles α and β. Calculate the kicking angle when standing 10 m from the try line.

3 Describe how the kicking angle can be calculated using the two angles.

Explore

4 Calculate the kicking angle for distances 0 and 20 m from the goal line after a try has been scored 10 m from the closer goal post. Record your results in a table.

Strategy options

- Draw a picture.
- Draw a table.
- Break the problem into manageable parts.

Explain

5 Do your results show a significant difference in the kicking angle at different positions? Would it make a significant difference to a player's ability to kick a goal?

Elaborate

6 Where is the best position for the goal kicker to kick from in order to maximise the angle between the goals (when a try has been scored 10 m from the closer goal post)?

7 (a) How far can a good rugby player kick? How would kicking distance affect where the kicker positions the ball?

(b) What other factors would the goal kicker have to take into account when choosing the position to kick from?

8 How might a rugby player use the information you have found in this Investigation? What other sports or situations might this type of investigation be useful for?

Evaluate

9 Explain the method of calculating the optimal kicking angle.

10 Did you develop any shortcuts when calculating each of the goal-kicking positions? If so, what were they?

11 What other methods could be used to find the optimal kicking angle, without using trigonometry?

12 Do you believe you have found the optimal goal-kicking position? How could you attain a more accurate result?

Extend

13 Investigate other try positions along the try line, such as closer to or further away from a goal post.

14 Is there a relationship between the optimal kicking angle and the placement of the try? Find out by drawing a rugby field and marking the best position to kick from for various try placements.

Mathspace

Adventure on Sohcahtoa Island

Your class has been taken on an excursion to Sohcahtoa Island. You land at Hypotenuse Harbour and are given a map of the island with places of interest clearly marked. Your map has vertical lines separating horizontal distances of 300 m. Your excursion is a competition.

Equipment required: 2–4 brains, 1 die, a counter for each player, a calculator for each player

How to win:

Travel from Hypotenuse Harbour to Port Pythagoras in the shortest possible distance, under the conditions of the competition.

How to play:

All players begin at Hypotenuse Harbour. You must first go to Cosine High School to collect supplies. You will need to throw a 2 or more to do this. After leaving the high school, you must visit at least three landmarks on the island along the way before arriving at the port. To get to Port Pythagoras, you may only travel along the marked roads, take the ferry or row across the lake.

On your turn:

1. Roll the die. Multiply your die roll by 300 m (the distance between the lines on the map). This is the maximum horizontal distance you can cover.
2. Determine which landmarks lie within this distance to the east or west of where you are. You may choose to move east or west, depending on which landmark you wish to visit. For example, if you roll a 3, you can move 300, 600 or 900 m east or west from your location.
3. Travel to your chosen landmark along a marked route. Calculate the distance you have travelled using the horizontal distance, the given angle and the correct trigonometric ratio. Record the distance travelled (to the nearest whole number).
4. Keep a running total of the distance you have travelled and that of your competition, adding the distance after each move. (Remember, your aim is to reach Port Pythagoras with the smallest possible total.) If there are more than two players, ensure that every player has their distance calculated by at least one other person.
5. If there is no place of interest you can reach from your throw, you cannot move and must add 100 m onto your distance total.
6. You may choose to stay where you are rather than take a route you don't want to follow. The penalty is an extra 500 m added to your total distance.
7. The game ends when all players reach Port Pythagoras. The winner is the player with the smallest total.

Sample move:

A player is at Isosceles Boat Hire. They roll a 2 and could choose to travel 2 × 300 = 600 m east to go to Sine Cinema or Vertex Museum, 600 m west to get to Perpendicular Cliffs or 300 m west to get to Tangent Beach Lookout. To get to Picnic Paradise, they would need to throw a 3 or more.

The distance from Isosceles Boat Hire along the road to Sine Cinema is calculated by

$$H = \frac{A}{\cos(40° + 15°)}$$

$$= \frac{600}{\cos 55°}$$

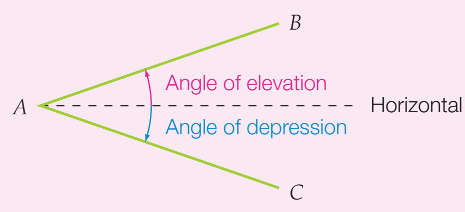
$$= 1046 \text{ m}$$


N
W
E
S
300m
300 m
Trigonometry Oval
60°
12°
Tangent Beach Lookout
14°
64°
Vertex Museum
41°
70°
Port Pythagoras
Equilateral Swimming Pool
20°
Triangular Point
42°
Isosceles Boat Hire
60°
40°
15°
LAKE ADJACENT
Cosine High School
20°
70°
20°
20°
75°
Picnic Paradise
40°
25°
20°
Government House
Hypotenuse Harbour
45°
Sine Cinema
25°
12°
Perpendicular Cliffs
48°
15°

7.5 Applications of trigonometry

The **angle of elevation** from A to B is the angle the upward-sloping line AB makes with the horizontal.

The **angle of depression** from A to C is the angle the downward-sloping line AC makes with the horizontal.

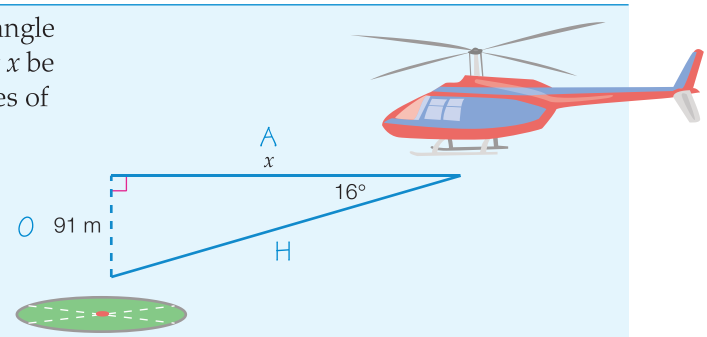


If you were positioned at A, you would need to *raise* your eyes from a horizontal line of sight to view an object at B.

You would need to *lower* your eyes from a horizontal line of sight to view an object at C.

Worked Example 10

A helicopter, flying at a constant altitude of 91 m, sights the middle of a landing pad at an angle of depression of 16°. How far, in metres, must the helicopter travel to be directly over the landing spot at an altitude of 91 m?

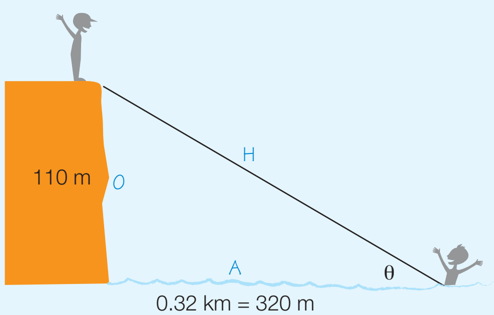
Thinking	Working
1 Draw a diagram that shows a right angle with all the known information. Let x be the required distance. Label the sides of the triangle O, A and H.	
2 Write down all the given information.	$\theta = 16°$ $O = 91$ m $A = x$ m
3 Identify the sides involved in the problem (opposite and adjacent) and hence the trigonometric ratio that will need to be used (tangent). Write out the rule for the required ratio.	$\tan\theta = \frac{O}{A}$
4 Substitute the values.	$\tan 16° = \frac{91}{x}$

5	Rearrange to make x the subject.	$x \tan 16° = 91$ $x = \dfrac{91}{\tan 16°}$
6	Use a calculator to determine x.	≈ 317.35 (2 d.p.)
7	Answer the question, giving your answer correct to the nearest metre.	The helicopter must travel 317 m before it is directly over the landing pad.

Worked Example 11

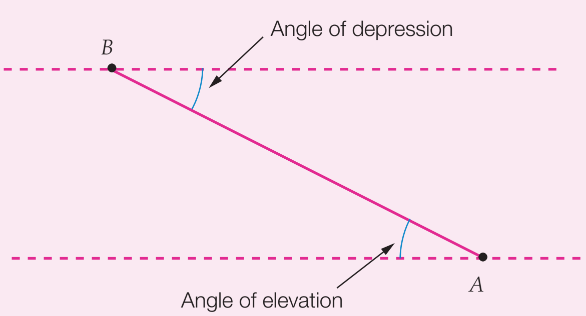
WE11

A swimmer, 0.32 km from the base of a cliff, looks up and waves to her friend who is at the top of the 110 m cliff. Find the angle of elevation from the swimmer to the top of the cliff. Give your answer to the nearest degree.

Thinking		Working
1	Draw a diagram. Let θ be the unknown angle. Convert measurements to the same units. Label your diagram with O, A and H.	
2	Identify the sides involved in the problem (opposite and adjacent) and hence the trigonometric ratio that will need to be used (tangent). Write out the rule for the required ratio.	$\tan\theta = \dfrac{O}{A}$
3	Substitute the values.	$\tan\theta = \dfrac{110}{320}$
4	Rearrange to make θ the subject.	$\theta = \tan^{-1}\left(\dfrac{110}{320}\right)$
5	Use a calculator to determine θ.	≈ 18.97 (2 d.p.)
6	Answer the question, giving your answer correct to the nearest degree.	The angle of elevation from the swimmer to the top of the cliff is 19° (to the nearest degree).

Angles of elevation and depression are always measured from the horizontal.

The angle of elevation from Object *A* to Object *B* is the same as the angle of depression from Object *B* to Object *A*. (Since the line *AB* crosses between two parallel lines, the angle of elevation and the angle of depression are alternate angles. Hence they are equal.)

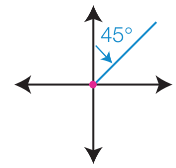


Bearings

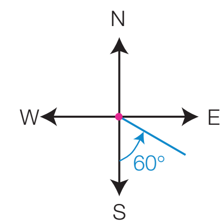
Bearings are used to indicate the direction of one object from another object or point and are used extensively in navigation, orienteering and map reading.

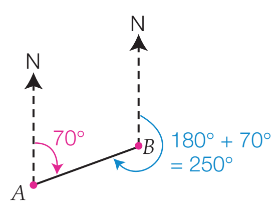
There are two types of bearings:

True bearings: A true bearing describes a direction measured as an angle clockwise from north. True bearings are always written using three figures. For example, an angle of 45° measured from north would be written as 045°T.



Compass bearings: A compass bearing describes a direction measured as an angle either side of north or south. For example, an angle of S60°E is found by facing south and then turning 60° towards the east.





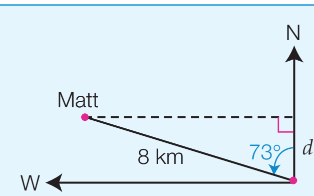
Note that the bearing from one point A to another point B is not the same as the bearing from B to A.

Bearing of B from A = 070°T or N70°E
Bearing of A from B = 250°T or S70°W

Worked Example 12

WE12

Matt hikes for 8 km from his base camp on a bearing of N73°W. How far north is Matt from his base camp? Give your answer correct to one decimal place.

Thinking	Working
1 Draw a diagram showing all the known information. Let d be the required distance.	
2 Identify the sides involved in the problem (adjacent and hypotenuse) and hence the trigonometric ratio that will need to be used (cosine). Write out the rule for the required ratio.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
3 Substitute the values.	$\cos 73° = \dfrac{d}{8}$
4 Rearrange to make d the subject.	$d = 8 \times \cos 73°$
5 Use a calculator to determine d.	≈ 2.339 km
6 Answer the question, giving your answer correct to one decimal place.	Matt is 2.3 km north of the base camp.

7.5 Applications of trigonometry

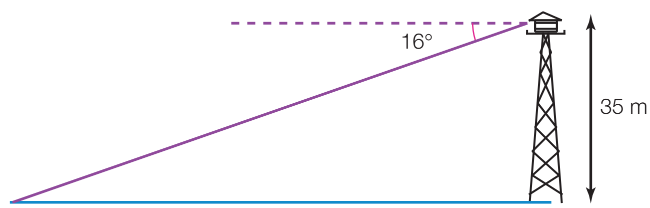
Navigator

Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q14, Q15	Q2, Q3, Q4, Q6, Q8, Q9, Q11, Q12, Q14, Q16, Q17	Answers page 646

Fluency

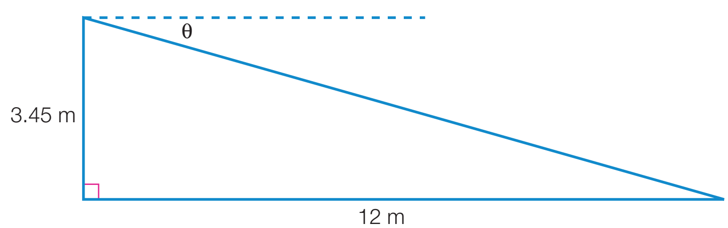
Find all angles to the nearest degree and all lengths to two decimal places, unless told otherwise.

1 Michelle, a forestry officer on watch from a lookout 35 m above the ground, notices a small brush fire at an angle of depression of 16°. How far, in metres, would a fire-fighting aircraft, currently directly over the lookout, need to fly to drop its load on the fire?



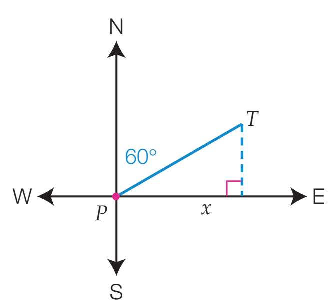
WE10

2 The crew on a fishing boat spot a shark near the surface of the water nearby. They estimate that the horizontal distance between themselves and the shark is 12 m. If the crew are 3.45 m above sea level, find the angle of depression to the shark.



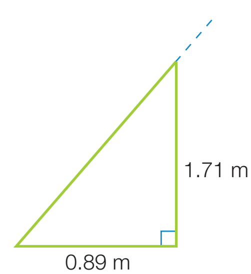
WE11

3 A train leaves Paddistone station and travels 40 km on a bearing of 060°T to Tarrigal station. How far east of Paddistone station is Tarrigal station?



WE12

4 Find the angle of elevation of the Sun (to two decimal places) if Phillip is 1.71 m tall and he casts a 0.89 m shadow.



5 A 5.8-m tree casts a shadow across the ground. If the angle of elevation of the Sun is 55°, the length of the shadow is:

A 4.1 m **B** 4.8 m

C 7.1 m **D** 10.1 m

Understanding

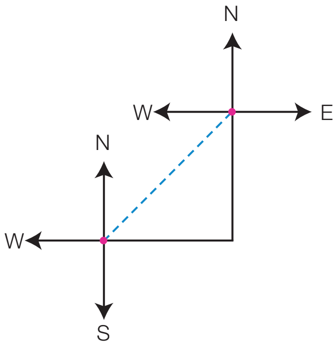
6 A man on a horse rode 4 km due south before turning and travelling 6.5 km due west. The bearing the horse and man must travel on to return to their starting point is:

A N31.6°E

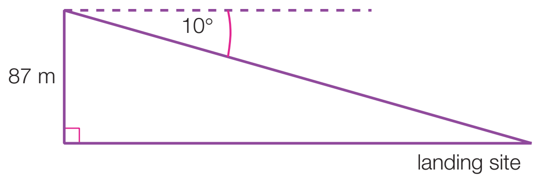
B N58.4°E

C S31.6°W

D S58.4°W



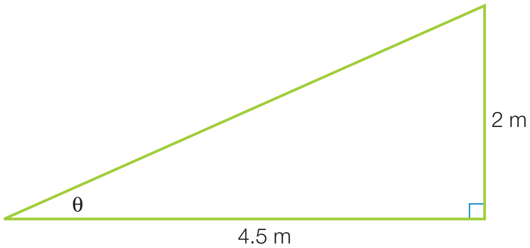
7 From a hot air balloon, the angle of depression to a marked landing site is 10°. If the observation deck in the balloon is 87 m above the ground, the horizontal distance the balloon needs to travel to reach the landing site is:



A 86 m **B** 88 m **C** 493 m **D** 501 m

8 A lifeguard spots a swimmer in distress from a lifeguard station 2.8 m high. If the angle of depression from the lifeguard to the swimmer is 17°, find the distance to the swimmer along the lifeguard's line of sight.

9 Martin has stopped his car at an intersection. He is 4.5 m from the base of a traffic signal, which is red, and judges the position of the green light to be 2 m higher than his horizontal line of sight. Through what angle of elevation must he raise his eyes to view the green light signal?



Reasoning

10 A ship travels 45 km on a bearing of N42°W.

(a) How far north of its starting point is the ship?

(b) How far west of its starting point is the ship?

11 A hiker walks 2.4 km east, then walks S75°W until she is due south of her starting position.

(a) Draw a diagram of the scenario.

(b) How far is she from her starting point?

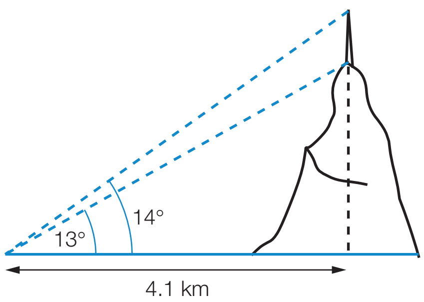
(c) How far did she walk on the bearing of S75°W before she was due south of her starting position?

12 Sarah is standing 100 m due south of a tower. Dougal is standing 140 m due west of the same tower. Using both compass bearings and true bearings, find the bearing of:

(a) Dougal from Sarah

(b) Sarah from Dougal.

13 A communications tower is on top of a nearby mountain. From a position 4.1 km away from the base of the mountain, the angle of elevation to the bottom of the tower is 13°, whereas the angle of elevation to the top of the tower is 14°. Find the height of the tower to the nearest metre.



14 From her vantage point on a cliff, Maria sights two swimmers in a direct line in front of her at angles of depression of 38.6° and 53.9°. If Maria is 50 m above the water level, find the distance between the two swimmers.

Open-ended

15 The angle of elevation from Ned's car to the top of a nearby tree is 27°. Give two pairs of possible values for the height of the tree and the horizontal distance of the tree from the car.

16 Draw a diagram that shows the angle of elevation and the angle of depression between two objects. Explain why these two angles are equal.

17 Germaine's maths test contained the following question:

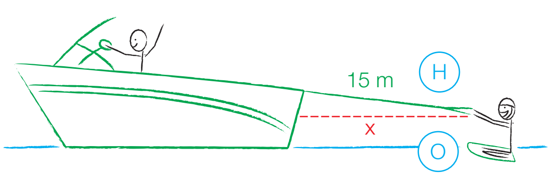
'Bianca was kneeboarding behind a boat on a 15 m rope. If the rope has an angle of depression of 4°, how far is Bianca from the back of the boat?'

Here is Germaine's answer. The teacher had marked it as incorrect.

(a) Identify the mistake and calculate the correct answer.

(b) How would you explain to Germaine the mistake she made?

Germaine's answer:



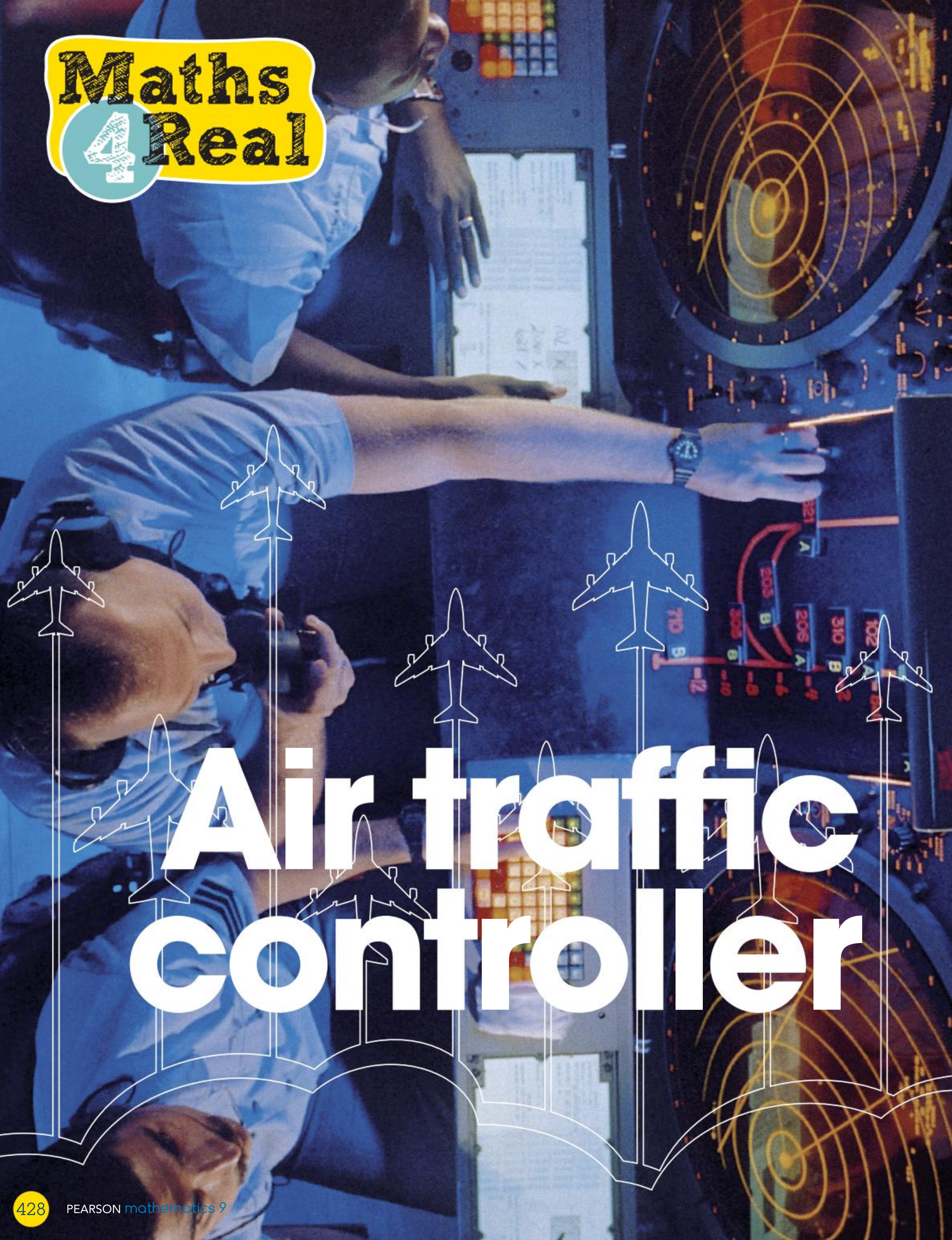
$\sin 4° = \dfrac{x}{15 \text{ m}}$

$x = \sin 4° \times 15$

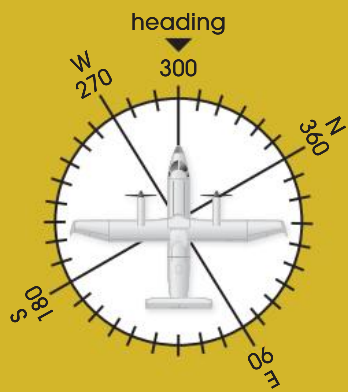
$x = 1.05$ m

Maths 4 Real

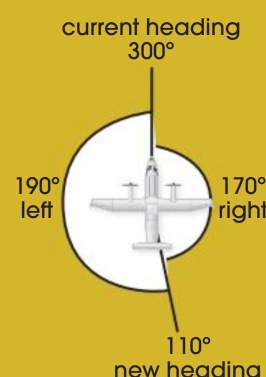
Air traffic controller



The airspace above Australia and all the aircraft that fly in it are controlled by air traffic controllers on the ground. The responsibility of the air traffic controllers is to keep all aircraft within their air space from colliding, particularly when they are coming in to land. They do this by placing the arriving aircraft into two main groups (jets and turboprops) by slowing them down, speeding them up or putting them in holding patterns. When a plane is approaching an airport or landing strip, an air traffic controller will give the pilot instructions as to the direction they need to approach from. All directions to the pilot are given in degrees measured by a heading indicator. In the heading indicator, 360° is north, 90° is east and so on, in a clockwise direction around from north.



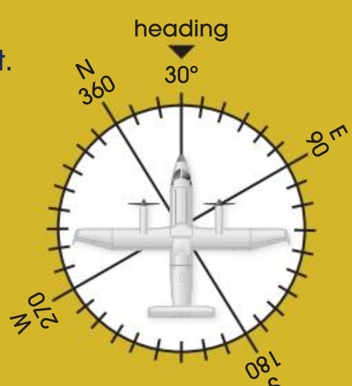
When giving instructions to a plane to change to a new heading, the plane is instructed to turn left or right, usually through the shorter angle. For example, if a plane is currently on a heading of 300° and needs to change its heading to 110°, it can turn 170° to the right or 190° to the left. Because 170° is the shorter angle of the two, the plane is instructed to turn right by 170°.

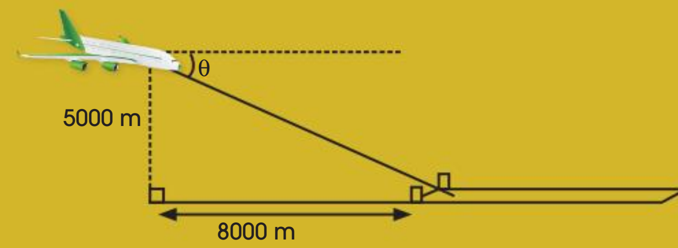


1 Refer to the heading indicator on the right.

(a) In which direction is the plane currently heading?

(b) The plane needs to change its heading to 200°. Find the turning required for left and right. Would you instruct the pilot to turn to the left or to the right? Explain your answer.





2 A plane is about to make its approach to land. It is 8000 m from the end of the runway and 5000 m high. At what angle would you instruct the pilot to make the approach to land?

The air traffic controller's problem

A plane bound for Canberra is flying on a current heading of 325° at an altitude of 30 000 feet (for altitude, the imperial unit of feet is used instead of metres). You have been notified that Canberra airport is closed due to fog, so the plane will have to be diverted to another airfield. The plane must first turn to a heading of 150° and then descend 30 000 feet at an angle of 30°.

3 Should you tell the plane to turn to the left or the right, and by how much? Draw a diagram to help you.

4 **(a)** What distance along the ground (in feet) has the plane travelled from the beginning to the end of its descent of 30 000 feet? Draw a diagram to help you.

(b) Using the conversion factor that 1 foot = 0.3048 m, calculate how many metres the plane has travelled (to the nearest metre).

Research

Investigate a career as an air traffic controller. What is their role? Why is this job important? What equipment do they use? What qualifications are required? What other mathematical skills are required to perform the job?

Challenge 7

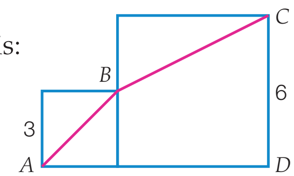
1 If m, n are integers, where $m > n$ and $n \geq 1$, then $m^2 - n^2$, $2mn$, $m^2 + n^2$ are Pythagorean triples (i.e. a set of three numbers that obey Pythagoras' Theorem).

(a) Show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$.

(b) Find the Pythagorean triples with no common factors, whose smallest number is:

(i) 3 (ii) 5 (iii) 7 (iv) 8 (v) 9

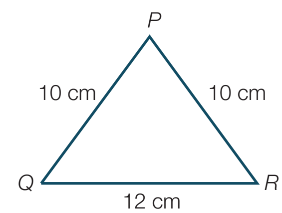
2 A square of side length 3 cm is placed next to a square of side length 6 cm as shown. The perimeter of the quadrilateral $ABCD$ is:



A $3\sqrt{2}$ cm B $3\sqrt{5}$ cm

C $6\sqrt{2}$ cm D $3(5 + \sqrt{2} + \sqrt{5})$ cm

3 In ΔPQR, $PQ = PR = 10$ cm, $QR = 12$ cm.



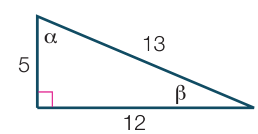
(a) What type of triangle is PQR?

(b) The perpendicular from P to QR meets QR at S. Calculate the length of PS.

(c) The value of sin R is:

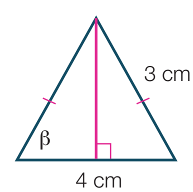
A $\frac{4}{5}$ B $\frac{2}{3}$ C $\frac{3}{5}$ D $\frac{3}{10}$

4 Which of the following is equivalent to $\cos\alpha°$?



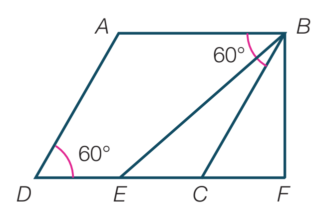
A $\sin\alpha°$ B $\cos\beta°$ C $\tan\beta°$ D $\sin\beta°$

5 For the triangle shown, which statement is correct?



A $\sin\beta = \frac{3}{4}$ B $\cos\beta = \frac{4}{3}$ C $\cos\beta = \frac{2}{3}$ D $\tan\beta = \frac{5}{2}$

6 $ABCD$ is a rhombus in which the opposite angles B and D are each 60°. CD is bisected at E and BE is drawn. BF is perpendicular to DC produced. Let the sides of the rhombus be $2x$ units.



(a) Write down the length of EC.

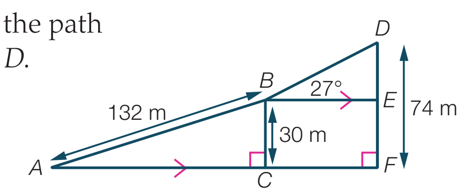
(b) Use trigonometry to find the length of CF.

(c) Calculate the length of BF as a surd.

(d) Calculate the size of $\angle EBF$.

(e) Hence write down the size of $\angle EBC$, correct to the nearest minute.

7 A mountain bike rider rides up a hill. The slope of the path he follows is shown in the diagram from A to B to D.



(a) The first stage of the track from A to B is 132 m long. The track rises 30 m over this distance (i.e. $BC = 30$ m).

Find the angle of elevation (θ), to the nearest degree, of the track AB.

(b) If the second stage of the track is inclined at 27° to the horizontal, find the length of the track BD to the nearest metre.

(c) The winning time for a rider to climb the hill from A to D was 1 min and 30 s. Calculate the average speed of the bike in metres per second (m/s). Give your answer correct to one decimal place.

Chapter review 7

D.I.Y. Summary

Key Words

adjacent side	compass bearing	opposite side	trigonometry
angle of depression	cosine	reference angle	trigonometric ratio
angle of elevation	degree	sine	true bearing
bearing	hypotenuse	tangent	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 When using the trigonometric ratios we must know which angle is to be used. We call this angle the ___________.

2 The ___________ of an angle is found by dividing the opposite side length by the length of the adjacent side.

3 The only trigonometric ratio that can have a value greater than 1 is ___________.

4 A ___________ is measured from the north or from the south.

5 The ___________ is measured from the horizontal up to the line of sight, whereas the ___________ is measured from the horizontal down to the line of sight.

6 A ___________ is measured clockwise from north.

7 (a) Draw a right-angled triangle, with one of the angles labelled θ.

(b) Label the hypotenuse with an H.

(c) Label the side adjacent to θ with an A.

(d) Label the opposite side with an O.

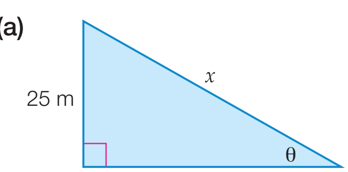
8 Write down the formulas for the three trigonometric ratios.

9 Draw diagrams showing angles of elevation and angles of depression.

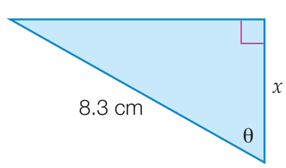
Fluency

1 Name the two labelled sides of the following triangles as opposite, adjacent or hypotenuse and give the trigonometric ratio that would be used to calculate the unknown side. Ex.7.1

(a)



(b)



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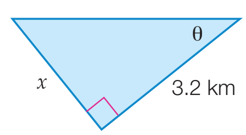
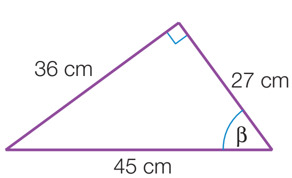
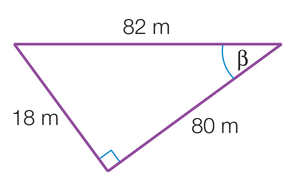


2 Find the value of $\tan \beta°$ for each triangle, correct to four decimal places where appropriate. Ex.7.2

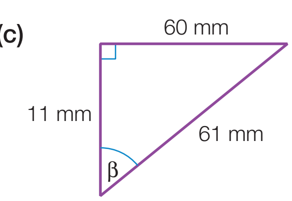
(a)



(b)

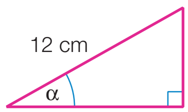


(c)

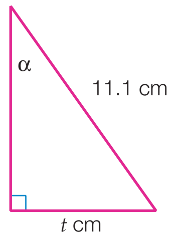


3 If $\sin \alpha = \frac{1}{3}$, find the value of the variable for each triangle, correct to two decimal places. **Ex 7.2**

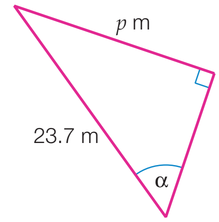
(a)



(b)

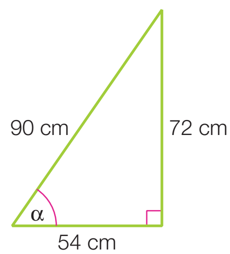


(c)

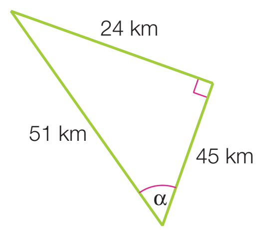


4 Find the exact value of $\cos \alpha°$ for each triangle. **Ex 7.2**

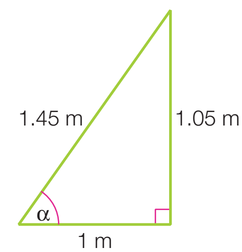
(a)



(b)

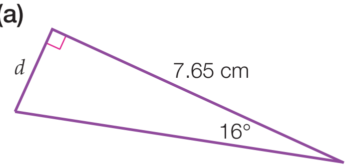


(c)

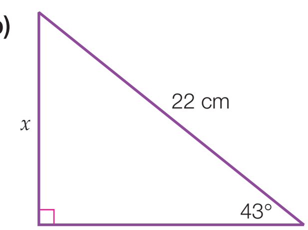


5 Find the value of the unknowns in the following triangles. Give your answers correct to two decimal places for side lengths and the nearest degree for angles. **Ex 7.3, 7.4**

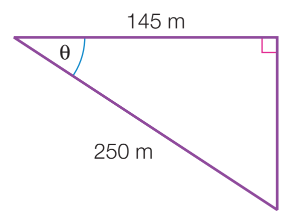
(a)



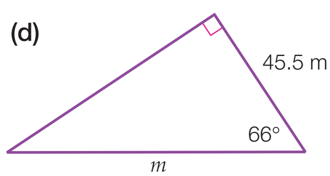
(b)



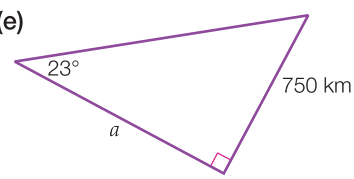
(c)



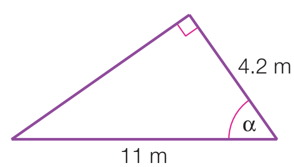
(d)



(e)



(f)



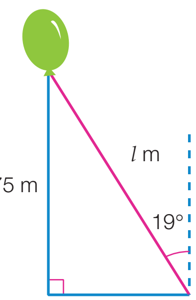
6 A kite is attached to a string that is 15 m long. If the string makes an angle of 26° to the horizontal, the height of the kite is: **Ex 7.5**

A 6.6 m **B** 7.4 m **C** 13.5 m **D** 34.2 m

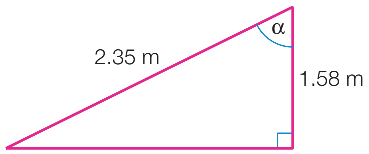
7 A hiker travels 13.2 km on a bearing of 125°T. How far south and east is the hiker from the starting position? Express your answer correct to two decimal places. **Ex 7.5**

Understanding

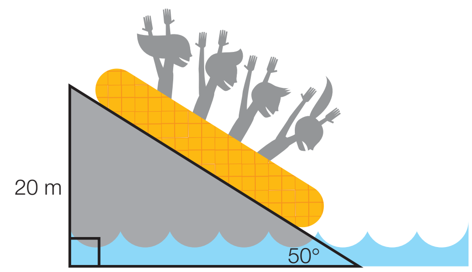
8 A balloon on the end of a string is being blown by the wind. If the balloon is 31.75 m above the ground and the string makes an angle of 19° to the vertical, find the length of the string.



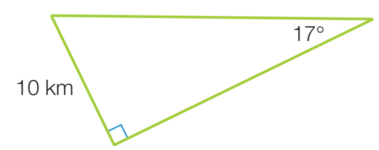
9 A children's slide has a vertical ladder 1.58 m high. Find the angle the slide makes with this ladder if it is 2.35 m long.



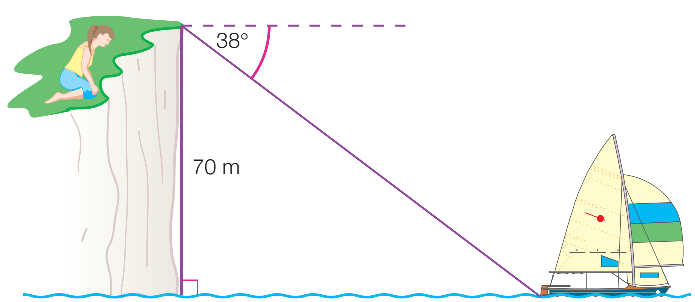
10 At an amusement park, a scary water raft ride descends a slide at an angle of 50° to the horizontal. If the slide starts 20 m above the ground, how long is the slide?



11 An orienteering trail follows a triangular course as shown in the diagram. Find the total length of the course, correct to one decimal place.



12 The angle of depression from the top of a 70 m cliff to a boat out at sea is 38°. How far away is the boat from the base of the cliff?



13 A submarine travelled 56 km on a bearing of 025°T from Point A to Point B. How far to the north has the submarine travelled from its starting point?

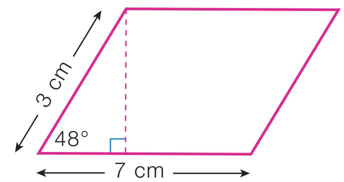
14 A four-wheel drive solar car was being tested in the Simpson Desert. It travelled north for 25 km, then turned and travelled 14 km due west. The driver would like to travel straight back to the starting point. What bearing must the car travel to return to its starting point, to the nearest degree?

Reasoning

15 A 365 cm ladder leaning against a wall makes an angle of 68.2° with the ground. If the ladder slips 76 cm down the wall, what angle (correct to one decimal place) does it now make with the ground?

16 **(a)** Find the height of this parallelogram. Give your answer correct to two decimal places.

(b) Hence find the area of this parallelogram.

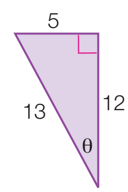


17 When Colin is 85 m away from a building he notices Spiderman at an angle of elevation of 62° climbing up the side of the building. What will be the angle of elevation when Spiderman has climbed up a further 10 m? Give your answer correct to one decimal place.

NAPLAN practice 7

Numeracy: Non-calculator

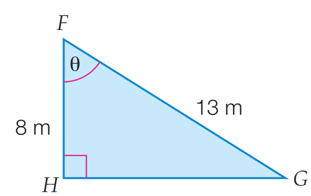
1 In the diagram on the right, the exact value of $\sin\theta$ is:



A $\frac{5}{12}$ **B** $\frac{5}{13}$

C $\frac{12}{13}$ **D** $\frac{13}{12}$

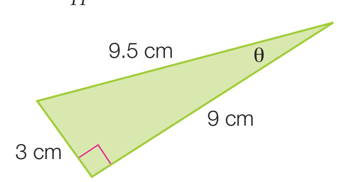
2 Which trigonometric ratio would be used to calculate angle θ?



A sin **B** cos

C tan **D** Pythagoras' Theorem

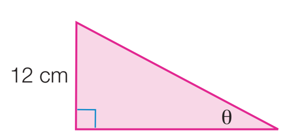
3 What is the tan ratio for θ in the triangle shown?



A $\frac{1}{3}$ **B** $\frac{9}{9.5}$

C $\frac{3}{9.5}$ **D** $\frac{9.5}{9}$

4 If $\tan\theta = \frac{3}{4}$, then the value of the variable in the diagram on the right is:

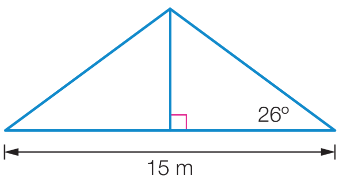


A 3 cm **B** 4 cm

C 9 cm **D** 16 cm

Numeracy: Calculator allowed

5 Part of a roof frame is shown. The frame has two symmetrical sides. What is the minimum length of timber needed for the diagonal parts of the frame?



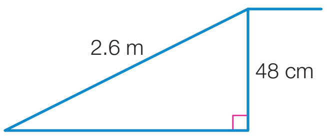
A 8 m B 9 m

C 16 m D 17 m

6 An aeroplane is a horizontal distance of 4000 m from the airport, flying at 2500 m, when it is instructed to descend for landing. To the nearest degree, at what angle will the aeroplane need to descend?

A 32° B 39° C 51° D 58°

7 A wheelchair ramp stretches from the ground to the top of a step 48 cm high. The ramp is 2.6 m long. To the nearest degree, what angle does the ramp make with the ground?



A 10° B 11°

C 32° D 33°

8 A rescue helicopter hovering 95 m above the ocean spots a swimmer in distress at an angle of depression of 32°. What horizontal distance will the helicopter need to travel to be directly above the swimmer?

A 95 m B 112 m C 152 m D 179 m

8

Statistics and probability

8

Predicting risk

What do tsunamis and the global financial crisis have in common? The answer is probability and statistics!

Risk analysis is a process used by businesses and government agencies to assess the likely effects of various events. A government must assess the probability and likely impact of natural disasters, such as a tsunami, and allocate resources accordingly. Similarly, businesses must assess the probability of unknown events happening, such as the global financial crisis of 2009, and allocate resources to limit their risk.

Forum

What issues may insurers consider when assessing your likelihood to claim on car insurance? Why do they bother doing this anyway? In a business context, a company may be thinking about introducing a new product. How do you think probability would play a part in their decision-making processes?

Why learn this?

The most common application of probability you have seen is probably associated with gambling and, for statistics, it is probably sports related. However, probability and statistics reach much further into our lives than these examples. Investing money, insurance and saving are areas where probability plays a significant role. Probability and statistics also play a part in the decision making by big business and in preparations for natural disasters.

After completing this chapter you will be able to:

- list all outcomes for two-step chance experiments using tree diagrams and arrays
- understand the difference between experiments that involve 'with replacement' and those 'without replacement'
- calculate probabilities for events, including the use of 'and' and 'or'
- calculate relative frequency from given or collected data
- investigate how surveys estimate population means and medians
- collect data from secondary sources
- construct back-to-back stem-and-leaf plots and histograms
- describe data using words such as 'skewed' and 'symmetric'
- describe and interpret data sets in terms of location and spread
- investigate techniques for collecting data.

Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R8.1

1 To one decimal place, calculate the mean, median and range of the following data:

20, 17, 24, 28, 32, 14, 21, 18, 14, 22, 16, 25

PP Worksheet R8.2

2 A normal die is rolled once. Calculate the following probabilities, expressing your answers in fraction form:

(a) Pr(rolling a 2) **(b)** Pr(not rolling a multiple of 3) **(c)** Pr(rolling 4 or 5)

PP Worksheet R8.3

3 Find the mean, correct to two decimal places, and median of the following data set:

Number of brothers and sisters	0	1	2	3	4	5	6
Frequency	12	17	23	20	18	4	2

PP Worksheet R8.4

4 List the sample space for drawing a particular suit from a normal pack of playing cards.

Key Words

back-to-back histogram
back-to-back stem plot
categorical data
certain
chance
complementary outcomes
continuous data
discrete data
impossible
measure of location
median class interval
mutually exclusive
negative skew
observational data
ordinal data
positive skew
primary data
probability
qualitative data
quantitative data
random sample
relative frequency
sample space
sampling with replacement
sampling without replacement
secondary data
skewed graph
stem plot
symmetrical graph
tree diagram
two-way table

Investigating data

8.1

Data types

Data can be divided into two broad groups:

- **quantitative data** sets, where the data consist of numerical values
- **qualitative data** sets, where the data consist of categories or descriptive terms.

Quantitative data

When we count the number of students in a class the answer is always a whole number. For example, it is not possible for there to be 23.6 students in a class. When we ask our friends their shoe size we get answers like 5 or $6\frac{1}{2}$. The sizes are either a whole number or a half size. It is not possible to buy shoes of size $5\frac{2}{3}$. In cases like these, where there is a set of clearly distinct values that can be obtained, we say we have a **discrete data** set.

In theory, when we measure our height any decimal value is possible. In a practical sense, our measurements are limited to the accuracy of our measuring devices; however, if you are 156 cm tall, then at some stage you must have been 155.9 cm or 155.926 cm or 155.9264 cm. Data sets of this type are described as **continuous data** sets and are usually associated with items that are measured using a measuring device such as a tape measure or scales.

One of the easiest ways to distinguish between the two quantitative data types is to ask the question 'Is the data set measured or counted?' If it is counted, it will be discrete; if it is measured, it will be continuous.

Worked Example 1

WE1

Decide whether the following data sets are discrete or continuous.

(a) The number of goals scored by each of the teams in each of their games in the 2010 FIFA World Cup.

(b) The crowd size at each of the games in the 2010 FIFA World Cup.

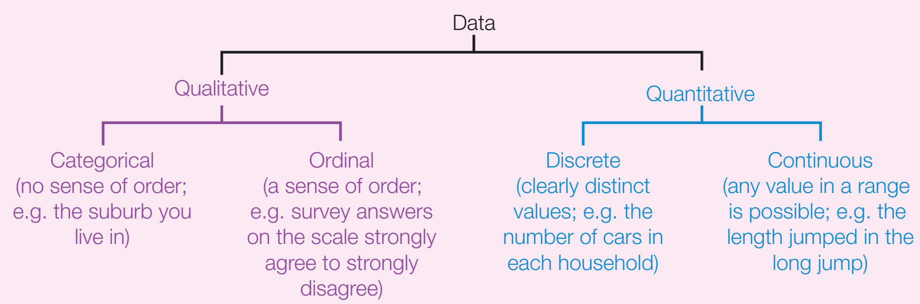
(c) The height of each of the players at the 2010 FIFA World Cup.

Thinking	Working
(a) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a counted value.	(a) Discrete data
(b) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a counted value.	(b) Discrete data
(c) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a measured value.	(c) Continuous data

Qualitative data

Qualitative data are data that are not numerical. Just as quantitative data can be split into two types, so can qualitative data. When we record information such as the colour of the cars in the car park, we are dealing with **categorical data**. When there is some order associated with the categories, then we have **ordinal data**. This order may involve words such as strongly agree or disagree, but may also be coded with numbers, for example, when a survey asks you to rate your opinion on a scale of 1–5, where 1 may mean strongly agree and 5 may mean strongly disagree. Even though there are numbers involved, it does not mean the data type is quantitative—the numbers are really just codes for the categories.

The different types of data can be summarised in the following diagram:



Investigating data-related questions

Some data-related questions can be answered by looking for existing data and conducting an analysis of those data. The internet provides you with ready access to mountains of data, but it is sometimes hard to find exactly what you want. These sorts of data are called **secondary data**; they are data we have not collected ourselves. Other questions can only be answered if we collect data ourselves. A simple example would be conducting a survey to find the level of support for a change to a school's uniform. Data of this type are called **primary data**.

Imagine we were asked, 'Which Australian state or territory has the worst road death data?' This is an example where we can analyse secondary data. An internet search may identify the Australian Government site that provides many data related to road safety.

For the purposes of this example, data will be used from this site, in particular data current to the end of 2008.

The number of road deaths for each state and territory for 2008 is summarised in the following table.

State/territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14

This seems to indicate that NSW is the most dangerous state. But is this a fair conclusion? Surely there are more people in NSW, so shouldn't we look at some other comparison?

This next table includes the death rate per 100 000 people in each of the states and territories.

State/territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14
Road deaths per 100 000 people	5.70	5.72	7.64	6.18	9.66	8.03	34.10	4.07

Well, that tells a different story! Things don't look too safe in the Northern Territory and things look much better for NSW and Victoria.

Is there any other comparison we could look at? What about considering the number of registered vehicles?

This table includes the deaths per 10 000 registered vehicles.

State/Territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14
Road deaths per 100 000 people	5.70	5.72	7.64	6.18	9.66	8.03	34.10	4.07
Road deaths per 10 000 vehicles	0.88	0.77	1.03	0.84	1.20	1.02	6.10	0.58

Again, we see the Northern Territory producing the worst result and the ACT and Victoria coming out best.

So, what comment can we make? The data seem to support the view that in 2008 the Northern Territory was the most dangerous place to be on the roads and that the ACT was the safest.

It may be relevant to mention that in 2008 there was no speed limit on the open road in the Northern Territory, but there is one now.

Conducting a survey

Surveys are conducted all the time in today's modern society. We are surveyed about a wide variety of things, from our voting preference to the TV programs we watch or the radio station we listen to. As part of a consultation process, we are surveyed about our opinions before major changes are made to things that will affect our lives. Surveys can be conducted by filling in a printed copy, over the phone, by personal interview or over the internet. However, before you can conduct a survey, there are many things to be decided to ensure your survey gives you usable and accurate information.

Designing a survey

First, you need to design the survey instrument and decide how you will conduct the survey. Let us suppose that your school is thinking about changing the design of the school jumper. It would be sensible for a committee to come up with a small range of alternatives that can then be put before the school community. The design of your survey instrument may be to simply have pictures of each design, with an opportunity to rank in order of preference. The current jumper should be included as one of the options. You need to keep in mind that any questions you ask must have definite answers that can be tallied easily. The next decision is then to administer the survey. This includes making a decision about the type of survey administered, such as an interview or printed questionnaire. The next question to be asked is who should be surveyed. In this case there are three different interest groups to consider: students, parents, teachers.

Running a survey

Then, you need to decide how many of each group should be surveyed. To survey the entire school population (conduct a census) may be feasible in small schools (<500 students), but in schools where there are over 1500 students this would be very time consuming and difficult. A representative sample of the school population would normally be used.

Analysing survey results

Once you obtain your results, you need to analyse them. How do you organise and display your data? What do you do if there is no clear preference? Do you use a preferential voting system to allocate second preferences from those whose response was not for one of the favourites? Preferential voting is used in our state and federal elections. Because voting is compulsory in Australia for all eligible voters, an election here is a census. However, in the USA voting is optional, so the voters constitute a (self-selected) sample.

Observational data

Another way of collecting primary data is to simply observe it using our eyes or our ears. For example, you could go outside and count the number of vehicles that pass the school in each of ten 3-minute periods. Data collected in this way is called **observational data**. However, care does need to be exercised when drawing up the observation charts. As an example, think about observing and recording the colour of the vehicles passing your school. How many colours should you be looking for? (For example, is dark green different to green?) You will almost certainly need to have a category called 'other' to catch all those difficult colours.

8.1 Investigating data

Navigator

Answers page 647

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11

Fluency

WE1

1 For each of these data sets, state whether it is discrete or continuous.
  - (a) The number of apples in each 2-kg bag at the supermarket.
  - (b) The weight of each bag of apples individually selected by customers in a fruit shop.
  - (c) The head circumference of each player in a basketball team.
  - (d) The number of matches in each of 30 boxes of matches checked during quality control.
  - (e) The time taken by each competitor to complete the 100 m freestyle at the most recent Olympic Games.
  - (f) The greatest recorded height cleared by each competitor in the pole vault at the most recent Commonwealth Games.
  - (g) The number of children in each family on your street.
  - (h) The average time it takes each student in your class to get to school each day.

2 For each of these data sets, state whether it is quantitative or qualitative.

  - (a) The colour of each car in the car park at the supermarket.
  - (b) The eye colour of each student in your class.
  - (c) The amount of pocket money received each week by each member of your class.
  - (d) The number of grains of sand in each of 10 separate matchboxes filled at the beach.
  - (e) The mass of each car in the carpark at your school.
  - (f) The mass of each student in your class, recorded correct to the nearest kg.

(g) The response of each person surveyed to a question with possible answers: agree, unsure, disagree.

(h) The party of choice for each person voting in a state election.

3 (i) For each of the data sets in Question **2** that you said was quantitative, state whether it is discrete or continuous.

(ii) For each of the data sets in Question **2** that you said was qualitative, state whether it is categorical or ordinal.

4 Some road death data related to motorcyclists are included in the following table.

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Number of deaths	55	43	72	17	36	8	10	4
Deaths per 10 000 registered motorcyclists	3.71	3.16	5.17	4.30	4.67	6.51	20.28	3.94

Make some comments about the data, with reference to safety and dangerous places for motorcyclists.

Understanding

5 Choose the most precise data type for each of the following.

(a) Height of players in the Australian netball team

A discrete **B** continuous **C** categorical **D** qualitative

(b) Shoe size of players in the National Basketball League

A discrete **B** continuous **C** categorical **D** ordinal

(c) Eyesight test results of umpires on a cricket umpires panel

A discrete **B** categorical **C** qualitative **D** ordinal

6 Chad is conducting a survey related to the school canteen. The first few questions from Chad's survey are shown below.

Question 1: tHow often do you buy something from the school canteen?

Question 2: What is the best thing about the school canteen?

Question 3: What items should be available at the school canteen that are not presently available?

His friend Narelle tells Chad that his questions are too open and he would be better off including some suggested answers to his questions that can be chosen by the people surveyed.

Chad agrees with this suggestion. Give some options that Chad may include for each of these questions.

7 The local football club is investigating new jumper designs. They surveyed members, and other interested parties, offering four choices: the current design (Option 1) and three new designs (Options 2, 3 and 4). The results of the survey were as follows.

Option 1	Option 2	Option 3	Option 4
25%	30%	24%	21%

(a) Do you think the survey supports a change of jumper? If so, which design should be chosen?

(b) What advice would you give another club about conducting a similar survey?

Reasoning

8 Sophie records the number of days each week the maximum temperature is greater than 22°C. Explain why Sophie's set of data is discrete and not continuous.

9 One of the questions on a statistics test was 'Is height continuous data or discrete data?'

Joe's answer was as follows:

Height is discrete because when I was measured in Physical Education class I was told by Mr Keeley that I was 172.2 cm. This is a discrete value.

Ngarla's answer was as follows:

Height is continuous because every value between two numbers is possible. At some stage I was 145 cm tall and sometime later I was 146 cm tall. There must have been a time when I was 145.6 cm tall, or 145.68 cm tall and even 145.687 cm tall. It is all to do with how accurately we measure the height.

Make some comments about these two answers. Are either of them correct? How would you answer the question?

Open-ended

10 **(a)** Write two data sets that are discrete.
(b) Write two data sets that are continuous.
(c) Write two data sets that are categorical.
(d) Write two data sets that are ordinal.

11 Your teacher has directed you to construct a recording sheet related to the type of vehicles that pass your school in a certain period of time.

(a) How many different types of vehicle do you think should be included on your sheet?

(b) If left to design the recording sheet individually, do you think other students will come up with the same list as you?

(c) Would it be a problem if different students had different sheets and were recording data on different streets that bordered your school?

Outside the Square — Puzzle

Equation squares

Copy and complete the following puzzle by filling in all the blank spaces so that all the equations are true. A number may be repeated in a row or column, but all numbers must be positive whole numbers.

Now that you have completed that task, see if you can construct another puzzle of your own. Resist the temptation to put zero in the bottom right-hand corner! Why would that make the puzzle easier to complete?

2	×		+		=	5
×		×		×		×
	×	4	−		=	
+		+		−		−
4	−		+	1	=	
=		=		=		=
8	+		+		=	23

Interpreting data

8.2

We have just seen that a census is a data set for an entire population and a sample is when we have information about only a portion of the population. When dealing with a sample, we usually then want to make predictions about the whole population based on the results for the sample. For this to work we need to make sure that our sample is representative of the whole population. We need to make sure our sample is not biased. This means making sure that the views of one part of a population are not over-emphasised in the results. For example, if we were trying to find out the average number of children per family in our local area and we stood outside the local child care centre and surveyed people there, we would probably get a value that was too high because we are unlikely to include any people in our survey who do not have children.

At this stage it is also important to say that a population does not have to be 'people'. For example, if we were trying to establish the rate of occurrence of various species in a marine park we would have to ensure that we included some shore areas as well as some at sea because not every species will live permanently in sea water.

This year we will concentrate on simple random samples of the population. A **random sample** is one in which every member of the population has the same chance of being selected. We could achieve this, in a class setting for example, by writing the name of each person on a separate sheet of paper and then selecting names from a hat. For bigger populations, we may assign a number to each person and then use random numbers to identify those people selected.

Random numbers

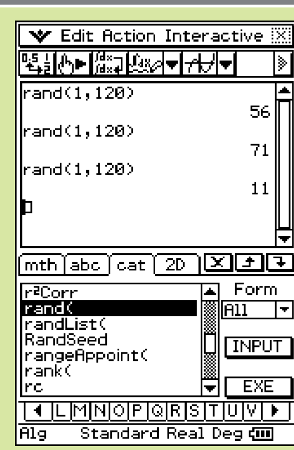
A random number generator will produce sequences of digits that can be used to match identification numbers for the population. A scientific calculator may produce random numbers like 0.695, 0.419, 0.14, 0.616 … because these are three-digit decimals, we would interpret them to represent numbers 695, 419, 140, 616 … in the population. Note how we interpreted the two-digit decimal. It was really 0.140, but the calculator doesn't show the trailing zero for the decimal. If our population only went to 500, we would ignore any numbers higher than this and keep producing random numbers until we had enough. Any repeated numbers would be used only once. Many scientific calculators, and all CAS, will produce random integers that are much more convenient for our purposes.

Using the TI-Nspire CAS

The screen shot shows what to do at the calculator screen. You can just type the letters, and don't worry about the capital letter. Each time you press enter (or enter) you get a random number in the range 1–120.

Using the ClassPad

Go to the Main screen and press Keyboard. Then press cat > R and highlight rand(. Then tap INPUT and type 1,120). Each time you press EXE you get a new random number in the stated range.



Interpreting survey data

Having done the work of collecting the data and calculating statistics related to them, the next step is the interpretation of the data. This is an important skill that is also useful when we are trying to interpret media reports related to statistics. One of the ways we describe data sets is by using a single value to represent the whole data set. This value is called a **measure of location**. You should be familiar with the three measures of location: mean, median and mode. In everyday language the word 'average' is often used to represent the mean. However, you do need to be careful because not everybody follows this convention. You will need to look at the information provided, or the context of the question, to decide which measure of location is being used.

Both the mean and median are often used in statistical reports. The mean can be affected by extreme values, so in cases like this it may be better to use the median. If the data set is relatively symmetrical in its distribution, then either the mean or median does a good job. This issue will be taken up later in this chapter.

In practical terms, especially when we are dealing with continuous data sets, the mode is not used very often. This is because there may very well be no data value that is present even twice.

Worked Example 2

WE2

The following data were taken from the Australian Bureau of Statistics Average Weekly Earnings bulletin (ABS: 6302.0 August 2010) released on 18 November 2010.

Males	Full-time adult ordinary time earnings	Full-time adult total earnings	All employees total earnings
2009			
May	1268.70	1335.70	1120.50
August	1289.30	1355.80	1138.70
November	1310.50	1377.00	1160.10
2010			
February	1326.30	1394.30	1175.30
May	1336.90	1407.40	1183.60
August	1343.90	1417.20	1188.10

According to the Explanatory Notes, estimates of average weekly earnings are calculated by dividing estimates of weekly total earnings by estimates of the number of employees.

(a) Which measure of location is being used here?

(b) Can you see any potential problems with using this as the measure of location?

(c) By what percentage did the full-time ordinary male earnings increase from May 2009 to August 2010? Express your answer correct to two decimal places.

(d) Why are the figures in the third column lower than the equivalent figures in the other two columns?

Thinking	Working
(a) Use the explanation to decide which measure it matches.	(a) This is the mean.

(b) Give a brief explanation based on your existing knowledge.	(b) The mean can be affected by extreme values, whether they are very low or very high.
(c) Find the difference between the two figures and convert to a percentage.	(c) $1343.90 - 1268.70 = 75.20$ Convert to percentage change, based on original value: $\frac{75.20}{1268.70} \times 100\% = 5.93\%$
(d) Give a brief explanation based on your understanding of the figures.	(d) This column includes males who are only employed part-time. These males would naturally earn less per week than full-time employees.

8.2 Interpreting data

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q9, Q12, Q15	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14

Answers page 648

Equipment required: scientific calculator or CAS for Questions **2**, **6** and **15**

Fluency

1 The following data were taken from the Australian Bureau of Statistics Average Weekly Earnings bulletin (ABS: 6302.0 August 2010) released on 18 November 2010.

WE2

Females	Full-time adult ordinary time earnings	Full-time adult total earnings	All employees total earnings
2009			
May	1054.60	1069.10	729.90
August	1066.20	1080.20	737.50
November	1079.40	1093.80	747.30
2010			
February	1093.10	1108.10	757.40
May	1105.70	1121.20	766.40
August	1116.70	1132.40	774.20

According to the Explanatory Notes, estimates of average weekly earnings are calculated by dividing estimates of weekly total earnings by estimates of the number of employees.

(a) By what percentage did the full-time ordinary female earnings increase from May 2009 to August 2010? Express your answer correct to two decimal places.

(b) In August 2010, for females, what percentage of *Full-time adult ordinary time earnings* was the *All employees total earnings*? Express your answer correct to two decimal places.

(c) Calculate this same percentage for males. You will need to look back to the figures in Worked Example 2.

(d) Make some comment about the difference between the two percentages obtained in **(b)** and **(c)**. Include at least one reason why this is the case.

2 (a) Select a random sample of 10 out of a population of whole numbers from 1 to 100 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

(b) Select a random sample of 5 out of a population of 25 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

3 When choosing the random samples in Question **2**, if you get two identical numbers you should:

A ignore one of them and select another

B keep the two identical numbers

C discard both

D choose another selection process

4 (a) Which one of these data sets could not be obtained by observation?

A the number of traffic lights along a road

B the proportion of people wearing coats

C the number of Holdens in a carpark

D the proportion of secondary students in Queensland born in Brisbane

(b) Which one of these numbers could be obtained using a survey?

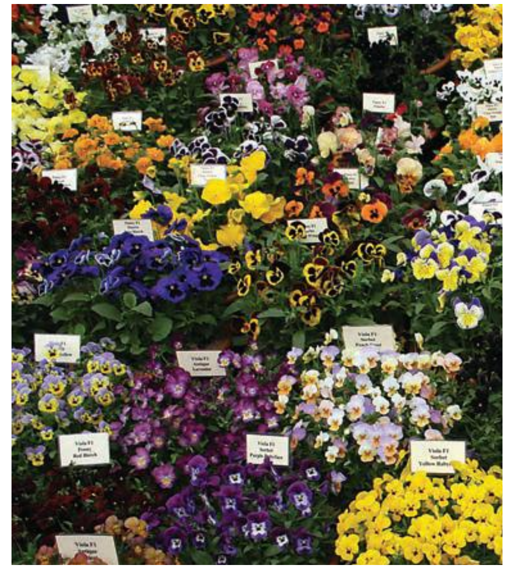
A the proportion of blue cars parked in a street

B the number of farms in the Mallee

C the proportion of students in a school who use a bus to get to school

D the number of National Trust houses in Australia

Understanding



5 In a sample of 50 seedlings, nine had white flowers, 12 had mauve flowers, 18 had crimson flowers, two had pink flowers, eight had magenta flowers, and one had purple flowers. If this sample was representative of the general population, what mean number of seedlings would be needed if a garden designer required:

(a) 32 magenta **(b)** 10 pink **(c)** 120 mauve?

6 A random number generator can be used to simulate the rolling of a die. The following are the results for a group of 30 rolls:

5, 5, 5, 3, 5, 2, 5, 6, 3, 1, 3, 3, 2, 2, 5, 6, 1, 1, 6, 1, 4, 6, 3, 3, 5, 5, 4, 1, 1, 5

(a) Which number appeared the most often?

(b) Do you think every group of 30 rolls would produce the same results?

(c) Use a random number generator to produce three lots of 30 rolls and comment on your results.

7 Yvette has a large packet of jelly beans. She knows the bag contains 160 jelly beans, of which 20 are black, 40 are pink, 60 are blue and the rest are orange. Yvette makes a random selection of 20 jelly beans from the packet. Her selection contains more pink jelly beans than blue ones, which surprises her. Why do you think this may have surprised Yvette?

8 The Bureau of Meteorology records the following data for the North Adelaide weather station with regards to rainfall, measured in mm. These data are based on records kept since 1883.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	21.5	20.2	25.0	44.1	64.6	73.4	68.3	64.4	55.1	45.7	31.0	27.6
Median	16.1	9.9	18.4	38.0	57.8	67.6	63.9	63.5	49.7	44.6	25.8	22.3

(a) Why are the mean figures consistently higher than the median figures?

(b) Based on this, for any given month, is the actual rainfall recorded likely to be less than the mean value or the median value?

(c) If you were lobbying for the building a new dam, which figures would you quote in your media releases?

(d) The biggest difference between the two figures is for February. Why do think this is the case?

Reasoning

9 The SRC at your school has been informed by the people who work at the school canteen that the manners displayed by students have been falling away. The SRC is concerned by this and decides to do some research.

(a) The SRC decides to conduct a survey with 50 students. The school population is 740. How would you recommend the SRC should go about selecting the 50 students to survey?

Concern is then raised that the students may not give absolutely truthful answers on the survey, so it is decided to observe interactions in the canteen.

(b) Can you see any problems with this approach?

(c) How would you suggest the SRC investigate this issue?

10 Each year the US company JD Power and Associates releases the results of a survey related to the reliability of cars and other vehicles over a 3-year period. The graph following shows the results for 43 700 owners of cars built in 2008. The survey is designed to predict the reliability of vehicles, not to make any statements about the latest vehicles, from any company.

(a) Which brand did the best in this survey?

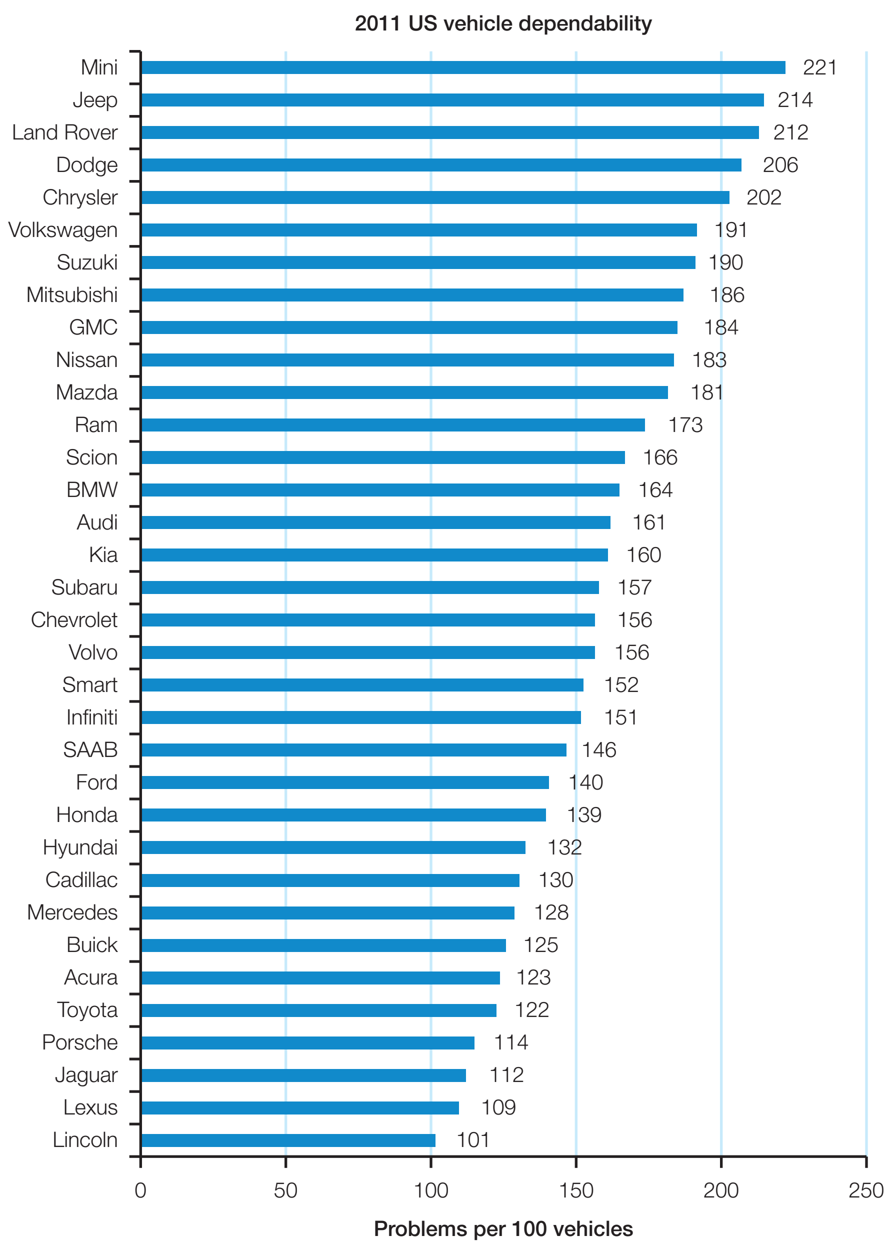
(b) What is the median number of problems per 100 vehicles?

(c) Take a random sample of five brands and find the median value.

(d) Take three more random samples of five brands and find the median value.

(e) Take five random samples of 10 brands and find the median value for each.

(f) Compare the median values you found for the samples of five and 10 brands with the population median value.



11 A firm has the following personnel:

Executives	11	Managers	26
Salespeople	34	Office workers	89

The owners of the firm wish to find out to what degree the firm's employees would support the setting up of a gymnasium for employees' use. They want to know what facilities employees would like the gym to have and how often it may be used. They decide to survey the employees.

(a) What is the population in this case?

(b) Would it be fair to sample four employees from each category? Whose opinion would this favour?

(c) Given the relatively small total population, what other option should be examined?

12 A large jar contains 25 black marbles, 34 blue marbles and 11 white marbles.

(a) A sample of 10 marbles is drawn from the jar. What would you expect the sample to contain if it truly represented the contents of the jar?

(b) If you drew five such samples from the jar, would you expect them all to be the same?

13 Some newspapers pose a daily question and ask readers to respond either by telephone or by logging onto a website.

(a) Does this represent a random sample?

Recently, the *Herald Sun*, published in Melbourne, posed the question 'Do you approve of a bus tour cashing in on Victoria's tragic crimes?' The results were 35.5% Yes and 64.5% No. In smaller type, the actual numbers were displayed: 103 Yes and 187 No.

(b) Do you think this sample size is big enough to draw any conclusions about the broader population?

(c) What would that population actually be: the readers of the *Herald Sun* or the residents of Melbourne, or Victoria or Australia? Write a sentence or two giving your thoughts about this.

Open-ended

14 The Office for National Statistics is the United Kingdom's equivalent of our Bureau of Statistics. In November 2009 they released a statistical bulletin titled *2009 Annual Survey of Hours and Earnings*. Some of the statistics published are reproduced below.

Median gross weekly earnings

		Employee jobs paid at adult rates and not affected by absence		
		Full-time	**Part-time**	**All**
£ per week April 2009	Men	531.1	143.6	491.0
	Women	426.4	155.6	309.8
	All	488.7	152.9	397.3

(a) What percentage of the male income for full-time employees does the female income for full-time employees represent?

(b) What do you notice about the part-time figure for women?

(c) What additional information would you like before making a definitive statement about the reason for the result found in **(b)**?

The following additional information was also published.

Median gross hourly earnings, excluding overtime

		Employee on adult rates and not affected by absence		
		Full-time	Part-time	All
£ per hour April 2009	Men	12.97	7.71	12.42
	Women	11.39	7.86	9.68
	All	12.34	7.83	10.99

(d) How does this additional information affect your answer to (c)? Is this now sufficient information to state why females working part-time are paid more, on average, than men?

Further to the information already provided, the Bulletin also tells us that, in April 2009, the mean hourly earnings for men employed full-time was £16.07 compared with £13.43 for women, whereas for part-time employees the figures were £11.98 for men with £10.40 for women.

(e) What is different about these mean figures compared with the median figures? Which figures would you be more inclined to use if you were writing a newspaper article about them?

15 (a) Use a random number generator of some kind to produce 20 numbers from 1 to 20. Record your results for each number individually in a frequency table. (Repeated numbers are counted.)

(b) Use the same method to produce 60 numbers from 1 to 20. Again, record your answers in a frequency table. (Repeated numbers are counted.)

(c) Collate the results from the rest of the class, recording your results in a frequency table.

(d) What conclusion can you draw from this work?

Outside the Square

Puzzle

Code breaking

To solve these puzzles you need to use the clues to find the number that matches the letter. In each case the number is a whole number from 1 to 6. The same whole number may be repeated in a particular code.

Code 1

A	B	C	D	E	F

$A + B = 8 \quad D \times E = 15 \quad B + F = 8 \quad C \times E = 25 \quad D - A = 1$

Code 2

A	B	C	D	E	F

$D \times E = 2 \quad A \times F = 9 \quad C - E = 0 \quad B \div E = 2 \quad B + E = 6$

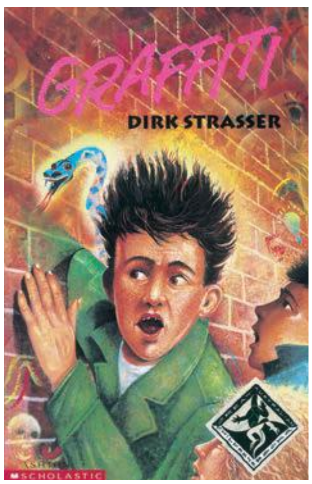
Investigation

Reading matters!!

Equipment required: 1 brain, 3–4 brains for the Extend section, a couple of novels, some newspapers

Book reviewers often consider the 'readability' level of a novel. The number of long words can give an impression of a novel, but a more precise indication can be found by applying the test developed by Edward Fry.

The Fry readability test



The following steps make up the Fry readability test. Just read through them now, you don't need to apply them until later.

Step 1 Select a passage at random from the novel and count 100 words.

Step 2 Count the number of sentences (to the nearest sentence) in the 100 words.

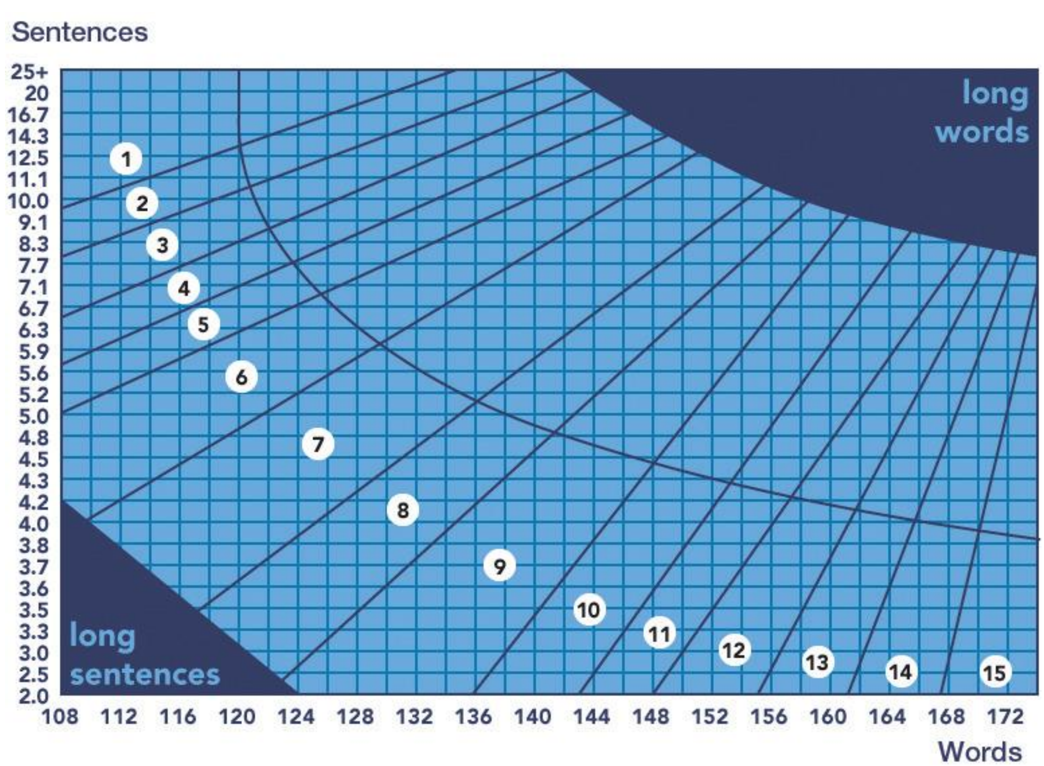
Step 3 Sound the number of syllables in the 100 words (e.g. the word 'graffiti' has three syllables: gra-ffi-ti).

Step 4 Repeat steps 1–3 for two other passages.

Step 5 Find the mean number of sentences and syllables.

Passage number	Number of sentences in 100 words	Number of syllables in 100 words
1		
2		
3		
Mean values		

Step 6 Use the mean values and the chart shown to determine the readability level.



The Big Question

Are the novels you read written to a suitable readability level?

Engage

1 A 100-word passage from the novel *Graffiti* appears below.

> I moved to the side and checked my shirt pocket. People behind me streamed past. Where did I put my ticket? I know I put it somewhere where I wouldn't lose it. Was that a policeman over there? Why was he staring at me? Stay calm, I told myself. Is it in my backpack? In my jacket pocket? No. I started to sweat like I do during a football match. I could see myself in gaol, eating bread and water. Aunt Vivien would keep promising to visit me, but she wouldn't turn up. I'd never get the chance to have my tooth fixed. Serving a life sentence with a gap in my mouth—I couldn't think of anything worse. Help!
>
> D. Strasser, *Graffiti*, Ashton Scholastic, p. 11

(a) How many sentences does it contain?

(b) How many syllables are in the passage?

(c) Use the chart and your answers to **(a)** and **(b)** above to determine a readability level based on just one passage. Why is it desirable to average results for three passages?

Explore

2 Check the readability level of the novel you are reading at present. Randomly choose a page, identify a passage that is 100 words long and work out the number of sentences and syllables. Repeat this for two other randomly selected passages and then find the mean number of sentences and the mean number of syllables.

(To randomly choose a page, just open the book wherever you like, pick the start of any paragraph you like and count 100 words from there. Then, open the book to another page and choose the start of any paragraph, which will be the starting point for your second lot of 100 words. Then, open the book to another page and follow the same process.)

3 What is the readability level of the novel you chose?

Strategy options

- Break problem into manageable parts.
- Make a table.

Explain

4 Do you think if you chose another three passages from the same novel that you would get a similar result? Check your theory by randomly choosing another three passages and performing the calculations.

(Just repeat the process you used above to randomly choose your three passages of 100 words.)

5 Summarise your findings and make a statement about the suitability of the novel for a person of your age, based on readability level.

For the purposes of this investigation, how close would you consider close enough? For example, if you are 15 and the calculated reading age was 14, would you think this suitable? What about if it were 10?

Elaborate

6 Choose another novel you have read recently and check its readability level.

Evaluate

7 Are the novels you are reading of a suitable readability level?

8 Do you think you would ever go through this process before reading a novel and then decide on the basis of the readability level whether to read the book?

9 Does understanding about readability levels help you understand why you may find some books too difficult for you to read, whereas other books are too simple?

10 Do you think the readability level of a novel is the only thing that should be considered when choosing a novel to read?

Extend

11 Obtain a newspaper and perform readability tests on different sections within it (e.g. main news section, sports, leisure etc.). Is there a difference in readability levels within a paper? Why?

12 Compare readability levels within a different newspaper and comment on your findings. Work in small teams so you can investigate more aspects of each newspaper.

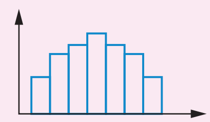
8.3 Statistics from grouped data

Symmetry and skew

When we look at statistical graphs, such as histograms, we need to describe what we can see. A **symmetrical graph** has frequencies that are evenly distributed about the median or middle value, whereas a **skewed graph** has higher frequencies on one side of the median.

- If a graph is symmetrical, or close to symmetrical, then the mean, median and mode will be very close to each other.

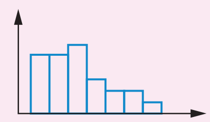
Symmetrical



- If most of the results fall on the left of the graph, we say it has **positive skew**. The mode will be on the left of the median and the mean will be on the right.

  mode < median < mean

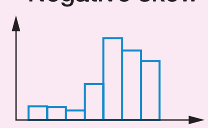
Positive skew



- If most results fall on the right of the graph, we say it has **negative skew**. The mode will be on the right of the median and the mean will be on the left.

  mean < median < mode

Negative skew



Worked Example 3

Decide, using the following statistics for each data set, whether the data sets are symmetrical or skewed. If the data is skewed, state whether the skew is positive or negative.

(a) mode = 87
mean = 87.5
median = 87.2

(b) mode = 16
mean = 22.5
median = 19.3

(c) mode = 43
mean = 28
median = 35.6

Thinking	Working
(a) 1 Consider whether the mean, median and mode are close together.	(a) The values of the mean, median and mode are close together.
2 If so, the data set is symmetrical.	The data set is symmetrical.
(b) 1 Consider whether the mean, median and mode are close together.	(b) The values of the mean, median and mode are not close together. The mean is on the right of the median and the mode is on the left of the median.
2 If not, look at which side of the median the mean and the mode are located.	Therefore, the data are positively skewed.

(c) 1 Consider whether the mean, median and mode are close together.	*(c) The values of the mean, median and mode are not close together. The mean is on the left of the median and the mode is on the right of the median.*
2 If not, look at which side of the median the mean and the mode are located.	*Therefore, the data is negatively skewed.*

Grouped data

When we are dealing with a large data set, we have two choices: (i) we can deal with the raw data; or (ii) we can group the data. If we group the data, we lose some accuracy in the calculation of the statistics, but this is normally balanced against the reduced time and effort required to get those statistics. When choosing the group, or bin, size, we aim to have between 5 and 10 groups, or bins. Further to this, we like to have group sizes of 2, 5, 10, 20, 50 etc. It would be quite unusual to settle on a bin size of 7, for example.

When we have grouped data we need a value to represent each group of values. This value is called the class centre or class midpoint and is found by adding the endpoints together and dividing by 2.

Finding the mean

To find the mean of grouped data, we carry out the following steps.

Step 1 Find the class centre or midpoint by adding the endpoints of the class interval and dividing by 2. We enter these values in a column labelled x.

Step 2 Multiply the midpoint value, x, by the frequency, f, and enter this product in a new column labelled fx.

Step 3 Total all the values in the fx column and write the sum as Σfx (sigma fx).

Step 4 We then divide Σfx by the sum of all the frequencies Σf. This will give us the mean.

$$\text{mean} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\Sigma fx}{\Sigma f}$$

Finding the median class

Because we have grouped the data, we have lost some detail regarding the individual values; so, instead of finding the median as a second measure of location, we find the **median class interval**. This is simply the group, or bin, that contains the median value.

Worked Example 4

WE4

A manufacturer has orders from 50 different retailers for a particular product each week. One week the number of orders were summarised as follows.

Number of orders	Frequency (f)
100–149	5
150–199	9
200–249	7
250–299	8
300–349	11
350–399	6
400–449	0
450–499	4

Use the grouped data to:

(a) calculate the mean number of orders. Give your answer correct to two decimal places.

(b) calculate the median class interval for the number of orders

(c) draw a bar chart of the orders

(d) describe the spread of the data.

Thinking	Working

(a) 1 To calculate the mean you need to find the class centre and multiply this value by the frequency. You will need to add two columns to the frequency table.

(a)

Number of orders	x	Frequency	fx
100–149	124.5	5	622.5
150–199	174.5	9	1570.5
200–249	224.5	7	1571.5
250–299	274.5	8	2196.0
300–349	324.5	11	3569.5
350–399	374.5	6	2247.0
400–449	424.5	0	0.0
450–499	474.5	4	189.0
		$\sum f = 50$	$\sum fx = \$75$

2 The mean is then found by calculating: $\frac{\Sigma fx}{\Sigma f}$. It does not need to be a whole number.

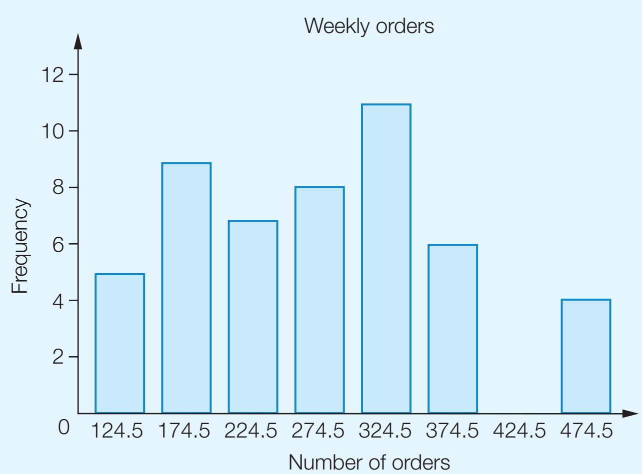
$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{13\,675}{50}$$
$$= 273.50$$

(b) Find the median class by counting down the frequency column until the middle is found. (Here, there are 50 values, so the median is between the 25th and 26th values.)

(b) Both the 25th and 26th values are in the 250–299 class, so the median class interval is somewhere between 250 and 299 orders.

(c) Draw a bar chart in the usual way. Use the midpoint values as the labels for the horizontal axis.

(c)



(d) Describe the shape of the bar chart.

(d) The graph is not symmetrical. It is slightly negatively skewed.There do not appear to be any outliers.

Worked Example 5

WE5

A wool producer wants to keep some of his lambs for breeding. To do this, the lambs must reach a certain breeding weight. He decides to use statistics to help him plan a breeding program. The data in the frequency table gives the weight, in kg, of the lambs in his flock.

Weight (kg)	Frequency
25–<30	5
30–<35	12
35–<40	14
40–<45	16
45–<50	13
50–<55	10
55–<60	2

(a) Calculate the mean weight of the lambs. Give your answer correct to two decimal places.

(b) Calculate the median class interval for the weights of the lambs.

(c) Draw a histogram of the data.

(d) Describe the spread of the data in terms of its skewness.

Thinking

Working

(a) 1 To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Weight (kg)	x	Frequency	fx
25–<30	27.5	5	137.5
30–<35	32.5	12	390
35–<40	37.5	14	525
40–<45	42.5	16	680
45–<50	47.5	13	617.5
50–<55	52.5	10	525
55–<60	57.5	2	115
		$\sum f = 72$	$\sum fx = 2990$

2 The mean is then found by calculating: $\dfrac{\sum fx}{\sum f}$

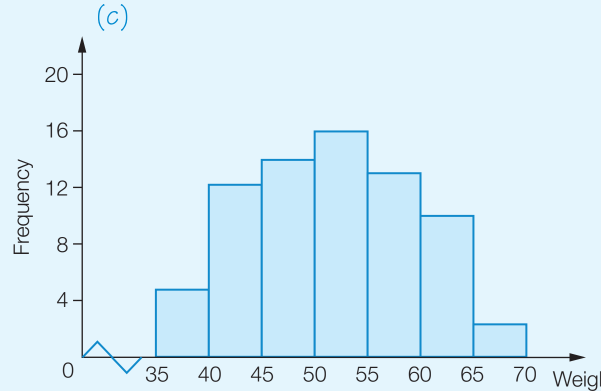
$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{2990}{72} = 41.53 \text{ kg (correct to 2 d.p.)}$$

(b) Find the median class by counting down the frequency column until the middle is found. (Here, there are 72 values, so the median is between the 36th and 37th values.)

(b) Both the 36th and 37th values are in the 40–<45 class, so the median class interval is between 40 and 45 kg.

(c) Draw a histogram in the usual way, remembering to leave half a column width before starting the first column.

(c)



(d) Describe the shape of the histogram.

(d) The graph has a symmetrical shape.

Skewness and stem plots

Stem plots are useful to determine whether our data is symmetrical or skewed because they look like frequency graphs turned on their side.

Positive skew

STEM	LEAF
0	1 3 4 4 5
1	2 5 8 3 5 6
2	3 3 4 5 6 8
3	0 2 4 4
4	4 5
5	2
6	8

Symmetrical

STEM	LEAF
0	1
1	2 5 8
2	3 3 4 5 6 8
3	0 2 4 4 6 8 9
4	4 5 7 7 8 9
5	2 3 6
6	8

Negative skew

STEM	LEAF
0	1 2
1	4 8 9
2	1 1 4 6
3	0 2 6 7 7
4	4 4 5 6 7 8 9
5	0 1 3 4 5
6	4 5 5 5 6 7 8

The advantage of the stem plot compared to the frequency graph is that we have not lost any of the raw data, so we can still work out accurate values for the mean, median and mode. However, a stem plot that looks perfectly symmetrical may not be. As an example, although the symmetrical stem plot shown above looks to be perfectly symmetrical, based on shape, if we calculate the mean and the median we will find they are not exactly the same. The median is 34, whereas the mean is 35.07—you can check these for yourself.

Using technology

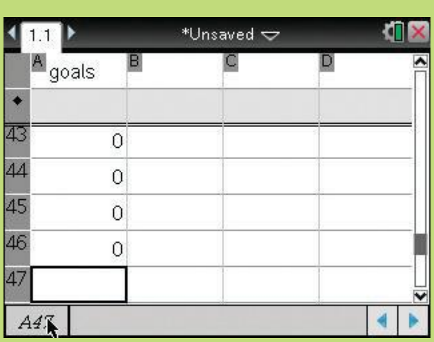
If we have a large list of raw data spread out over a large range, it can take some time to sort it into class intervals and then draw a frequency graph in order to comment about its symmetry or skewness. However, technology, such as a CAS, makes this much easier. We will look at a relatively small data set to illustrate the process, but the size of the data set does not change the way we would go about the task.

We will use the number of goals scored prior to the beginning of the 2010 AFL season by each player on the list of the Brisbane Lions.

25	4	1	134	575	198	27	2	106	14	73	0	0
84	49	423	66	0	55	153	17	1	3	19	0	0
11	1	6	15	17	57	0	40	19	0	0	26	0
2	0	0	0	0	0	0						

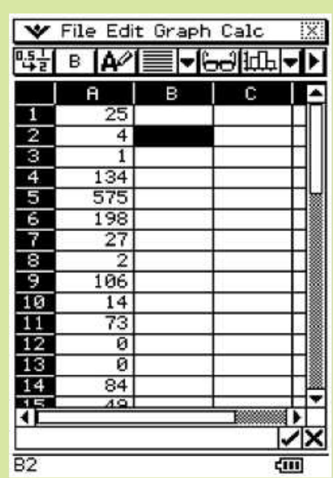
Using the TI-Nspire CAS

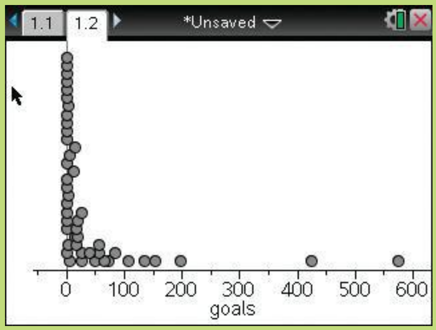
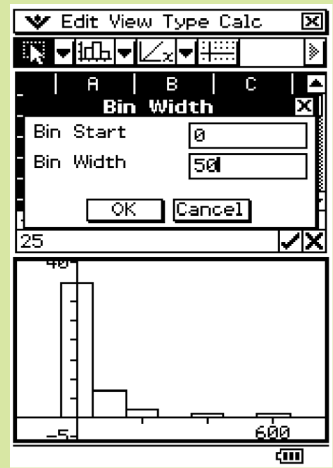
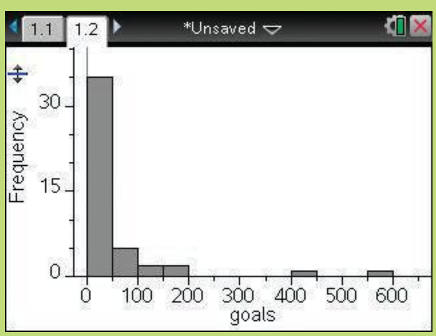
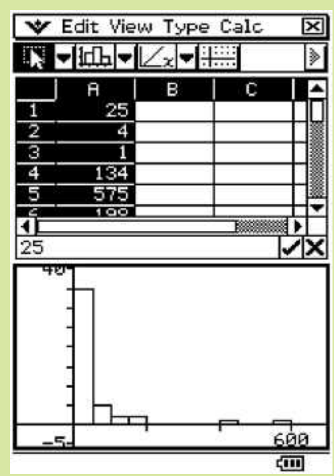
1 Press on and choose a new Lists & Spreadsheets page. Scroll up to beside the A and enter ‘goals’ as the name of the column. You need this to be able to draw a graph. Now scroll down and enter the data, starting in the row labelled ‘1’.



Using the ClassPad

1 Tap Menu and choose Spreadsheet. Enter the data in column A.

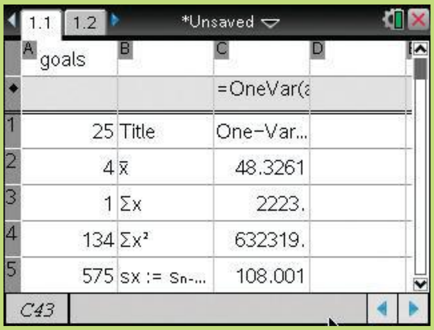


Using the TI-Nspire CAS	Using the ClassPad
2 Now, press ctrl I (or ctrl I) to insert a new page and choose Add Data & Statistics. This will show as a scattering of points. Move the cursor to below the horizontal axis and press and choose 'goals'. This will then create the equivalent of a dot plot. 	2 Highlight column A and select Graph Histogram. A histogram is drawn immediately. Select Calc > Bin Width. Make Bin Start 0 and Bin Width 50 and tap OK. 
3 Now, press menu > Plot Type > Histogram to create the histogram. 	3 The histogram is now in the form you want. 
4 This has chosen the class interval for you, in this case, 50. If you want to change it, you can, by pressing menu > Plot Properties > Histogram Properties > Bin Settings and choose the width you want.	4 If you had wanted a different interval size you could have made that choice in the dialog box.
5 The histogram shows that the data are positively skewed and that there are probably two outliers, 423 and 575. Before you started you probably suspected these would be outliers!	5 The histogram shows that the data are positively skewed and that there are probably two outliers, 423 and 575. Before you started you probably suspected these would be outliers!

Using the TI-Nspire CAS	Using the ClassPad

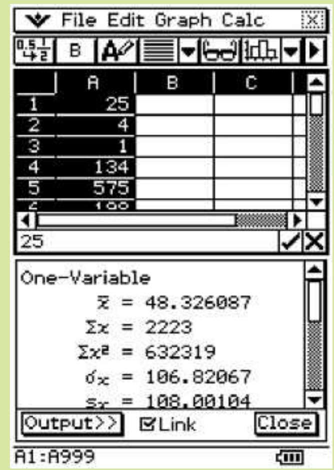
Using the TI-Nspire CAS

6 Press ctrl ◂ to go back to the data screen and press menu > Statistics > Stat Calculations > One-Variable Statistics and say OK to Num of Lists being 1 and OK to the X1 List being in a[ ]. You will have to scroll up to see the results.



Using the ClassPad

6 Now, with column A highlighted, tap Calc > One-Variable. All the summary statistics are listed. Tap on Output. You can choose where to paste your statistics. If you are happy with the displayed columns for Labels and Results, tap on Paste.



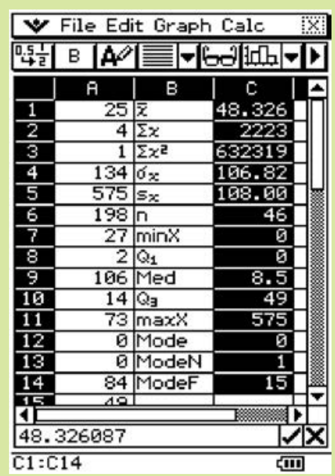
Using the TI-Nspire CAS

7 Because the screen is not big enough to see the statistics all at the same time, we will summarise:

Mean = 48.3261
Median = 8.5
IQR = 49
Range = 575

Using the ClassPad

7



In summary:
Mean = 48.3261
Median = 8.5
Mode = 0
IQR = 49 ($Q_3 - Q_1$)
Range = 575 (max – min)

If we are dealing with grouped data, then we need to use the midpoint of the class interval to represent the score and enter this in the first column. We then enter the frequencies in the second column and proceed in much the same way.

8.3 Statistics from grouped data

Navigator

Answers page 648

Q1, Q2 (a), Q3 (a), Q4, Q5, Q6, Q7, Q8, Q13, Q14, Q15	Q1, Q2 (a), Q3 (a), Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q14, Q15	Q1, Q2 (b), Q3 (b), Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q14, Q15

Equipment required: CAS calculator or Excel spreadsheet may be used for Question **5**

Fluency

WE3

1 Decide, using the following statistics for each data set, whether the data set is symmetrical or skewed. If the data are skewed, state whether the skew is positive or negative.

(a) mode = 68
mean = 60
median = 63.5

(b) mode = 132
mean = 158
median = 144

(c) mode = 235
mean = 233
median = 234

WE4

2 **(a)** A bakery records the number of sales of a new type of specialty bread over a 60-day period. The number of sales of the bread are summarised in the table.

Number of loaves	Frequency (f)
60–69	2
70–79	2
80–89	3
90–99	6
100–109	12
110–119	13
120–129	15
130–139	7

Use the grouped data to:

(i) calculate the mean number of sales (give your answer correct to two decimal places).

(ii) calculate the median class interval for the number of sales

(iii) draw a bar chart of the sales

(iv) describe the spread of the data.

(b) The number of cars in a council car park was recorded every hour over a 60-hour period starting from midnight. The data are summarised below.

Number of cars	Frequency (f)
0–49	9
50–99	8
100–149	17
150–199	5
200–249	13
250–299	8

Use the grouped data to:

(i) calculate the mean number of cars (give your answer correct to two decimal places).

(ii) calculate the median class interval for the number of cars

(iii) draw a bar chart of the cars

(iv) describe the spread of the data.

WE5

3 **(a)** A horse trainer has a large stable of horses. He needs to keep records of each horse's weight. The weights are entered into a frequency table so that the trainer can gather some statistics on the horses in his stable. The data in the frequency table give the weights, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	3
300–<350	13
350–<400	17
400–<450	20
450–<500	16
500–<550	14
550–<600	4

(i) Calculate the mean weight of the horses. Give your answer correct to the nearest kilogram.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Draw a histogram of the data.

(iv) Describe the spread of the data in terms of its skewness.

(b) Another horse trainer has a stable of horses. He also keeps records of each horse's weight. The weights are entered into a frequency table so the trainer can gather some statistics on the horses in his stable. The data in the frequency table give the weights, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	5
300–<350	11
350–<400	21
400–<450	27
450–<500	18
500–<550	11
550–<600	7

(i) Calculate the mean weight of the horses. Give your answer correct to the nearest kilogram.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Draw a histogram of the data.

(iv) Describe the spread of the data in terms of its skewness.

4 For each of the following stem plots, calculate the **(i)** mean, **(ii)** median and **(iii)** comment on the skewness of the data. Give your answer correct to two decimal places.

(a)

STEM	LEAF
4	8
5	2 3 4 5 6 7 7
6	4 5 5 6 7
7	3 4 5
8	2
9	1
10	9

(b)

STEM	LEAF
11	5 7
12	4 5
13	2 3 4 9
14	0 0 2 3 4
15	6 7 8 9 9 9
16	1 3 3 4 5 6
17	0 5 6 7 8

(c)

STEM	LEAF
21	0 2
22	1 8 8
23	0 1 2 3
24	1 2 5 6 7 9
25	1 2 3
26	1 1 3
27	1 2

5 If you have access to a CAS calculator or an Excel spreadsheet, use it to help you with this question; otherwise complete it using the normal methods.

The numbers that follow are the career points scored by the players on the list of the Brisbane Broncos at the start of the 2010 NRL season.

0	0	0	32	4	0	238	8	28	0
16	302	4	0	130	8	1163	114	16	8
88	8	0	0	582	0	16	44	0	0
60	62	82	0	0	56	36			

(a) Using grouped data, draw a histogram and use it to comment on the symmetry or skewness of the data.

(b) Do you think any of the values might be a possible outlier? If so, which ones?

(c) Using grouped data, calculate the mean and median class of the data.

Hint
To find the class centre, add the endpoints and divide by 2.

6 (a) The class centre for a class interval of 0–19 is:

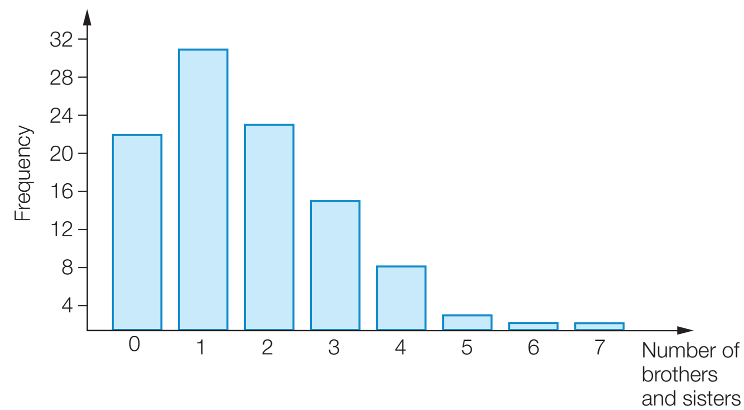
A 9 B 9.5 C 10 D 10.5

(b) The class centre for a class interval of 10–20 is:

A 14.5 B 15 C 15.25 D 15.5

Understanding

7 The following graph shows the number of brothers and sisters for each of the students in Year 8.



(a) Describe the data in terms of its skewness.

(b) Estimate the mean number of brothers and sisters.

(c) Now, calculate the mean number of brothers and sisters. How good was your estimate?

(d) Do you think this is a realistic spread of brothers and sisters? What could you do to check?

8 Find the class intervals for each of the following lists of class centres for discrete data.

(a) 5.5, 15.5, 25.5, 35.5, 45.5

(b) 2, 7, 12, 17, 22

(c) 24.5, 74.5, 124.5, 174.5, 224.5

(d) 25.5, 75.5, 125.5, 175.5, 225.5

(e) 11, 14, 17, 20, 23

(f) 27, 32, 37, 42, 47

9 The following frequency table shows the time taken, in minutes, by the students at a particular school to complete a cross-country race.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

(a) Calculate the mean time taken by the students to complete the cross-country race.

(b) Draw a histogram and use it to describe the data set.

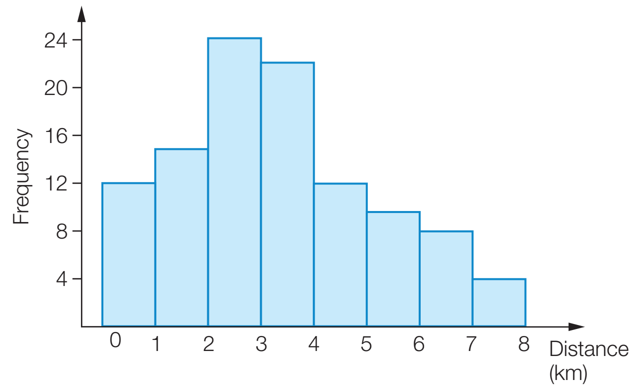
(c) Five students did not complete the race within the time limit. How would their results affect the mean time taken?

(d) How might you record their times in order to include them in the histogram?

10 The following histogram shows the distance travelled to school, measured in km, by each of the students in Year 9 at Westbow High School.

(a) Describe the graph in terms of symmetry or skewness.

(b) Make an estimate of the mean distance travelled to school.



Reasoning

11 The following table shows the number of AFL games played by each of the 45 players on the list of the Sydney Swans at the beginning of the 2010 season.

Number of games	Frequency
0–39	22
40–79	7
80–119	7
120–159	3
160–199	2
200–239	3
240–279	1

(a) Calculate the mean number of games played.

(b) State the game interval that represents the median class.

(c) Which of these measures of location do you think does the better job of describing the number of games played?

(d) In fact, 13 of the players have yet to play a game. Recalculate the two measures of location, leaving out these players.

(e) Does this give a better representation of the measures of location for the number of games? Explain your reasoning.

12 (a) For discrete data, what can you tell about a class interval if the class centre is a whole number?

(b) For discrete data, what can you tell about a class interval if the class centre is a decimal number with a decimal part of .5?

(c) For discrete data, can a class centre ever be other than a whole number or a .5 number? Explain your reasoning.

Open-ended

13 (a) Write two different lists of class intervals that have class centres 5 apart.

(b) Write two different lists of class intervals that have class centres 10 apart.

14 (a) Construct a stem plot with at least 20 values such that the median is bigger than the mean.

(b) Construct a stem plot with at least 20 values such that the median is smaller than the mean.

15 Draw a symmetrical frequency graph that has at least 20 values, using class intervals of 0–4, 5–9 etc. and then use it to construct a frequency table.

(a) Estimate the mean from the frequency table.

(b) Now make up the actual data values and calculate the mean.

(c) Comment on what you find.

Outside the Square Puzzle

Complete the patterns

In each case you need to find the next number in the pattern.

1

4	8	10	11	11.5	

2

3	4	9	24	81	144	

3

4	2	3	1	2	0	

4

4	5	9	14	23	

5 Now, see if you can make up another four of your own and get one of your classmates to try to solve them.

Comparing data sets

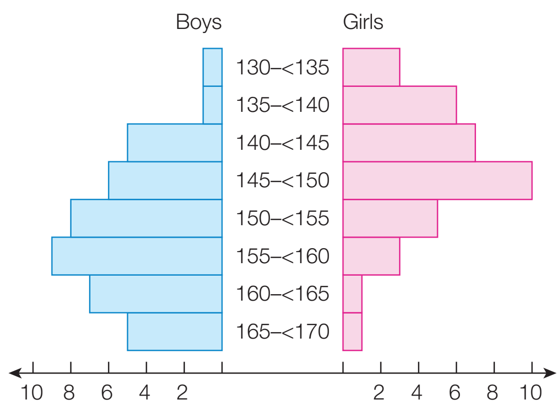
8.4

We often want to compare data sets that are similar. For instance, we may want to see whether boys and girls of the same age are the same height. If we were just looking at the height of boys, we may draw a stem plot if there weren't too many data values or a histogram if the data set was bigger. To compare the two we draw a back-to-back stem-and-leaf plot or a back-to-back histogram.

back-to-back stem-and-leaf plot

Boys	Stem	Girls
4	13_L	4 4
9 8	13_H	6 7 9
3 3 2	14_L	0 1 1 3 4
9 8 6 6 5 5	14_H	5 5 5 7
3 2 2 0 0	15_L	0 1 4
8	15_H	5 6
4	16_L	3
6	16_H	

back-to-back histogram



For the back-to-back stem plot, the right-hand side stem plot is displayed normally. The left-hand side stem plot is displayed similarly but is a mirror image of a normal stem plot. In the case of the back-to-back histogram, we draw the bars horizontally rather than vertically as we usually do.

Worked Example 6

WE6

The weights (in kg) of the players on the 2010 playing lists for each of the two AFL teams based in South Australia are as follows.

Adelaide:	99	85	100	85	99	84	82	81	80	80	93	97	75
	82	90	86	84	102	84	87	82	83	99	76	87	74
	98	92	79	89	79	82	98	86	89	75	88	82	77
	92	86	87	80	87	84	81	80					
Port Adelaide:	84	82	85	90	87	80	80	80	83	83	80	98	79
	91	85	99	84	83	96	100	88	85	98	93	86	98
	95	78	99	80	89	87	98	92	90	84	95	82	94
	85	100	80	80	70	85	101	75					

(a) Find the range for the complete data set.

(b) Use the range to decide on a suitable class interval for a stem plot.

(c) Draw a back-to-back stem plot to display the data and make a statement about which team seems to be the heavier

Thinking	Working
(a) Find the range by subtracting the smallest value from the largest value.	(a) $102 - 70 = 32$
(b) We prefer somewhere between 5 and 10 intervals.	(b) We will have a class interval of 5, with the first interval being 7_L, that is 70–74.
(c) 1 Construct the back-to-back stem plot.	(c)

Adelaide	Stem	Port Adelaide
4	7_L	0
9 9 7 6 5 5	7_H	5 8 9
4 4 4 4 3 2 2 2 2 2 1 1 0 0 0 0	8_L	0 0 0 0 0 0 0 2 2 3 3 3 4 4 4
9 9 8 7 7 7 7 6 6 6 5 5	8_H	5 5 5 5 5 6 7 7 8 9
3 2 2 0	9_L	0 0 1 2 3 4
9 9 9 8 8 7	9_H	5 5 6 8 8 8 8 9
2 0	10_L	0 0 1

2 Make a statement comparing the data sets.	It would seem that Port Adelaide is the heavier team because it has more players weighing 90 kg and over. It also has fewer players weighing less than 80 kg.

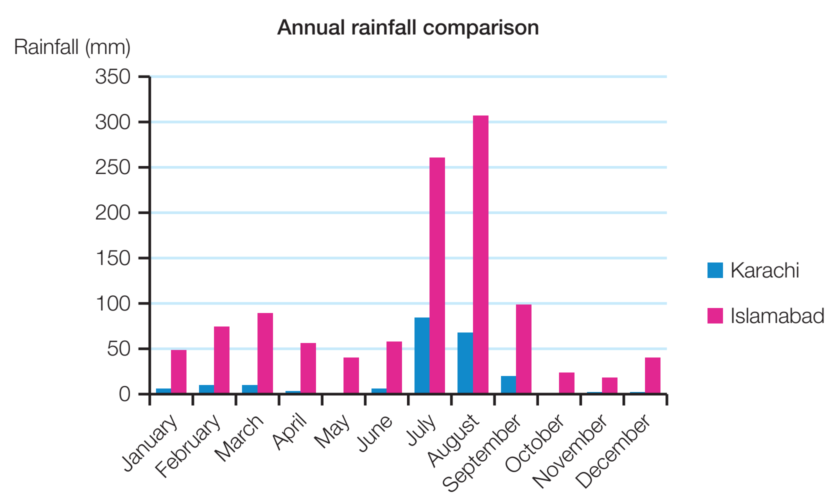
Comparative graphs

We can also compare data sets by looking at side-by-side column or bar graphs. This usually involves some interpretation of what is shown in the graph.

Worked Example 7

WE7

Pakistan is quite diverse climatically. The average monthly rainfall, in mm, for two of its largest cities is shown below.



(a) Describe the average monthly rainfall for the two cities using just what you can see in the graph.

(b) What makes this a difficult task?

(c) What additional information would you like to have to make a more detailed analysis of the average monthly rainfall?

Thinking	Working
(a) Look at the graph and make sure you know which colour represents which city. Describe the graph as best you can.	(a) Islamabad is much wetter than Karachi, but both follow the same general pattern of having the most rain in July, August and September.
(b) Think about potential sources of difficulty. Are any data missing? Is it easy to estimate the relevant values?	(b) The rainfall figures for Karachi are so low that it is difficult to see them sometimes.
(c) Think about what would make it easier to produce some statistical values.	(c) If we had the actual numbers, rather than just the graphical view, we would have a better chance of conducting a sensible statistical analysis.

8.4 Comparing data sets

Navigator

Q1, Q2, Q3, Q4, Q6, Q9	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Answers page 650

Fluency

1 The heights (in cm) of the players on the 2010 playing lists for each of the two AFL teams based in West Australia are as follows.

WE6

Fremantle:	174	191	176	182	188	185	189	180	190	182	186
	181	186	189	190	192	194	193	186	193	194	188
	191	191	199	184	193	192	203	211	184	188	181
	180	194	177	187	182	184	191	191	192	189	194
	184	186									
West Coast Eagles:	195	183	178	189	188	179	188	201	186	185	186
	183	187	189	196	194	184	189	203	192	169	192
	191	186	187	185	186	198	195	196	189	189	195
	183	189	190	194	182	191	167	202	183	175	186
	189	181									

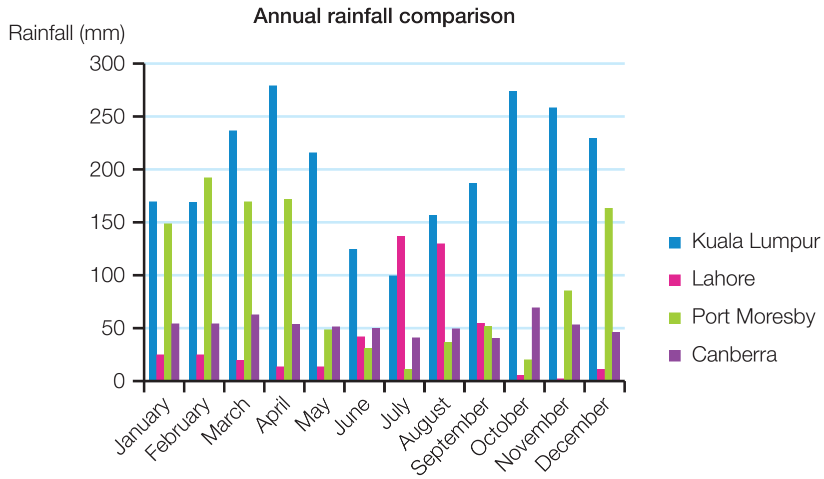
(a) Find the range for each team.

(b) Use the range to decide on a suitable class interval for a stem plot.

(c) Draw a back-to-back stem plot to display the data and make a statement about which team seems to be the taller.

WE7

2 The following graph shows the annual monthly rainfall, in mm, for Kuala Lumpur (Malaysia), Lahore (Pakistan), Port Moresby (Papua New Guinea) and Canberra (Australia).



Although it is interesting to compare data such as these for different places around the world, this graph does not quite work.

(a) What would you consider to be the biggest problem associated with this graph?

(b) How could you overcome this problem?

(c) Estimate the monthly rainfall figures for Kuala Lumpur and Port Moresby and use these figures to draw a new comparative graph.

(d) Use your estimated figures to calculate the mean, median and range rainfall for each of these two cities. Then use these figures, and your graph, to compare the rainfall in each of these two cities.

3 The US PGA is one of the major events in the world of golf. The table below shows a summary of the scoring in the first two rounds of the 2010 event.

Score	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
Frequency Day 1	0	1	4	6	10	22	17	21	25	13	10	6	8	4	3	1	2	1
Frequency Day 2	4	2	11	8	14	23	13	13	18	19	6	10	3	2	4	2	1	1

(a) Group the data in 2s, with the first group being 66–67, and draw a back-to-back histogram of the data.

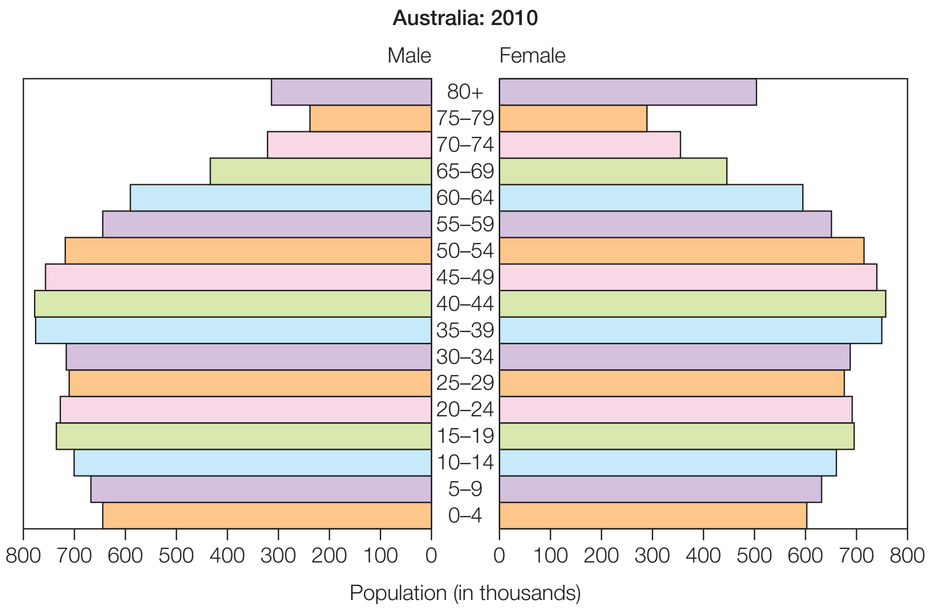
(b) Why was it appropriate to group the data in the way done in **(a)**?

(c) Use the histogram to help you decide on which day scoring was easier. In golf, the lower the score the better it is.

(d) Use the raw data in the frequency table to calculate the mean and median scores for each day. Do these values confirm what the histogram shows?

Understanding

4



(a) For which age groups does it appear there are more males than females?

(b) Why are the 80+ bars longer than the 75–79 bars?

(c) How useful do you think this graph is in helping you compare male to female age distribution in Australia?

5 Are Year 9 girls taller or shorter than Year 9 boys? One way to investigate this is to sample some Year 9 students. However, the Australian Bureau of Statistics has organised this for you already through the Census at Schools program. One of the features of the program is that you can sample Australian students based on many different criteria.

From that website, the following data about a sample of 50 Year 9 Victorian students in 2010 were obtained.

Male heights (cm):	176	181	154	177	191	190	180	175	164	166
	147	188	175	159	157	183	173	174	177	185
Female heights (cm):	145	162	160	167	153	155	162	178	163	167
	164	161	174	165	171	164	160	170	169	162
	174	167	154	153	168	144	165	160	168	172

(a) Draw a back-to-back stem plot of the data. Use a class interval of 5 (i.e. 16_L and 16_H).

(b) What conclusion would you draw from the stem plot drawn?

(c) Is it a concern that there are more girls than boys in the sample? Explain your reasoning here.

(d) Do you think another sample would produce the same results? Give some detail in your answer.

(e) Do you think a sample from another state or territory would give different results? What do you think would be reasonable ranges in which you would find the heights of Year 9 Australian students for males and females?

6 The average monthly rainfall, in mm, for four places in Australia is shown below.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	276.5	307.6	179.0	66.3	32.5	20.5	13.8	16.4	10.9	24.9	60.2	131.7
B	17.5	11.6	21.6	18.3	41.7	61.4	56.1	49.6	40.6	27.4	18.4	18.4
C	24.5	24.5	20.3	17.4	22.5	22.0	18.9	18.5	20.4	24.5	20.7	21.7
D	87.3	86.6	111.4	154.6	197.1	211.2	225.6	234.9	200.1	173.7	139.3	113.9

(a) Calculate the mean, median and range for each of these places.

(b) Describe the rainfall pattern for each place, including some mention of the seasons of the year.

(c) The four places, in no particular order, are Broken Hill (in outback NSW), Port Lincoln (a coastal town in SA), Weeaproinah (in southern Vic) and Townsville (in tropical north Qld). You are now to decide which place is which. Include some justification for your choices.

Reasoning

7 In Question **3** you looked at some data from the 2010 US PGA event. In golf events such as this, which are played over four rounds, approximately half the field is eliminated after the first two rounds and does not compete in the last two rounds. The following table shows the total score for each player for the first two rounds and the third and fourth rounds, if they were still competing.

Rounds 1 and 2:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	1	1	2	10	5	7	13	9	9	14	15	11	10	10
Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	7	5	5	4	2	4	5	0	1	2	0	1	1	

Rounds 3 and 4:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	0	2	3	4	6	2	5	5	4	9	8	9	2	4
Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	2	3	2	1	0	0	0	0	0	0	0	0	0	

(a) Given we prefer between 5 and 10 groups, explain why grouping in 5s, with a first group of, say, 136–140, would be the best way to group the data if we were to draw a back-to-back histogram of the data.

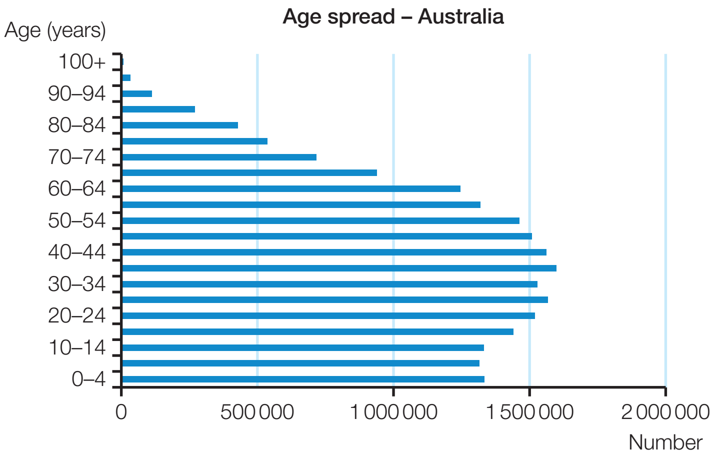
(b) What is the concern with this grouping for the Rounds 3 and 4 data?

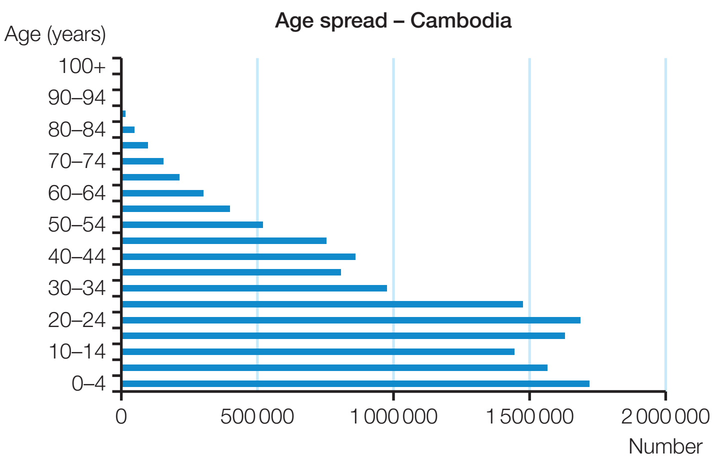
(c) Go ahead and draw the histogram using this grouping.

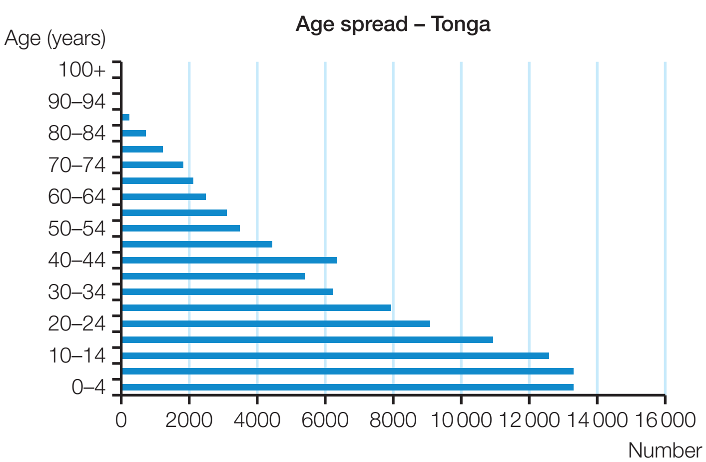
(d) Why do you think the two sides of the histogram seem so different?

(e) Find the mean and median for each data set and use these to add some further detail to your comments.

8 The following graphs show the age spread for people in three countries.







Compare the ages of people in the three countries using as much detail as you can.

Open-ended

9 Construct a back-to-back stem plot where the mean of the right-hand side is greater than the median and, on the left-hand side, the mean is less than the median. State what the data may represent.

10 The website of the Australian Bureau of Statistics has some great animated population pyramids of the Australian population that can be broken down into states or viewed in various ways to provide more information. Go to this website and investigate what you can do. Write a short report about this.

Mathspace

Rating The Ratings

Who decides which shows remain on TV and which are 'canned' after only two episodes?

Why is your favourite show suddenly moved from 8:30 p.m. Friday to 7 p.m. Tuesday—just when you have to go to basketball training? Does it matter when a certain show is on?

TV networks pour millions of dollars into buying or making shows they hope will be successful. A lot of this money is gained from selling advertising space during shows (those 2-minute sections of commercials, or 'ads', that break up programs) to companies looking to sell a product. To get the best value from their advertising, these companies need to ensure that their ad is shown in front of as large an audience as possible. Not just a large audience, but preferably their 'target audience', the people most likely to buy their product.

How do we know who's watching what?

In order to collect data on who is watching which show, a small electronic device, known as a 'peoplemeter', is installed on each of the TVs in selected households around Australia (with the permission of those living in the house). The peoplemeter records minute-by-minute information on whether the TV is being viewed at the time, and what is being viewed. This information is transmitted overnight from the peoplemeter via a phone line to the central database of a company called OzTAM (Australian Television Audience Management). OzTAM collates and organises all the information received and makes it available for people such as network schedulers and advertising executives to use. This information is known as 'the ratings' because it can be used to tell which programs are the most and least popular.

Sampling the population

In 2011, 3035 households across the five major Australian cities (Sydney, Melbourne, Brisbane, Perth and Adelaide) had a peoplemeter installed. Another 2015 peoplemeters were installed across regional areas of New South Wales, Victoria, Queensland and Tasmania, and a further 100 in regional Western Australia. The households with the peoplemeters installed make up what is called the 'Ratings Panel'.

1 (a) How many households in total make up the Ratings Panel?

(b) Given that the decisions made using the information collected from the Ratings Panel will affect all TV-watching Australians, do you think the number of households with a peoplemeter is a big enough sample ?

How is the sample chosen?

To decide in which households to place the peoplemeters, a large face-to-face survey is conducted. Although not a census, the number of households surveyed is five to ten times the size of the final sample that makes up the Ratings Panel. The survey collects some general information that is used to define the characteristics of the overall population, such as the gender, ages, occupations and education of the people in the household, the number and type of TVs in the household and who has the responsibility of buying the groceries for the household. The households surveyed are then classified into groups, based on household size, location and TV ownership. Households are then invited from each group to form the Ratings Panel by having a peoplemeter installed.

2 (a) Use your answer to Question 1(a) to determine the number of households that would have been surveyed originally. Give your answer as a range.

(b) Why do you think one of the questions is about who is responsible for buying the groceries for the household?

(c) Why do you think the households are classified into the groups described?

Producing the ratings

Imagine that the grand final of Channel 8's talent show 'The Y Factor' has just been shown. Data from the Ratings Panel peoplemeters showed that 1986 of the Panel's households tuned in to some or all of the show.

Use the following information to help you answer the questions below:

- There are approximately 8.5 million households in Australia.
- The average number of people in a household is 2.5.

3 (a) What percentage of Ratings Panel households tuned in?

(b) Assuming that the Panel's viewing is representative of the rest of Australia, how many Australian households tuned in?

(c) How many people does this represent?

4 Are there any programs that you think would produce quite different results in different states, or areas within states? You do not necessarily have to confine your thinking to programs that are high ratings winners.

Skewing the data?

As well as detecting and sending program data from the TV itself, the peoplemeter has a special remote control that can be used by members of the household to register the number and ages of people watching a particular show.

5 How could this capability be used to distort, or 'skew', the data generated about the household? What would be the reasons for doing so?

Using the data

The viewing data collected from the Ratings Panel can be grouped into categories, such as the gender, age group and occupation of the viewer. This means that advertisers can determine what their 'target audience' is most likely to be watching (i.e. the type of person most likely to purchase the type of product being advertised.) OzTAM's data can be split into the following age groups for both men and women:

0–15	13–24	18+	25+	30–49	40–64
5–12	16+	18–29	25–44	35–54	55+
10–15	16–24	18–49	25–54	40+	55–64
13–17	16–54	18–54	25–54*	40–54	65+

*with children

6 Imagine that you were advertising each of the products listed below. Select the viewer data groups you believe would represent your target market(s).

(a) a new model of ute or 4WD car

(b) a new type of hair colour that covers grey hair

(c) a new computer program that helps children learn to read

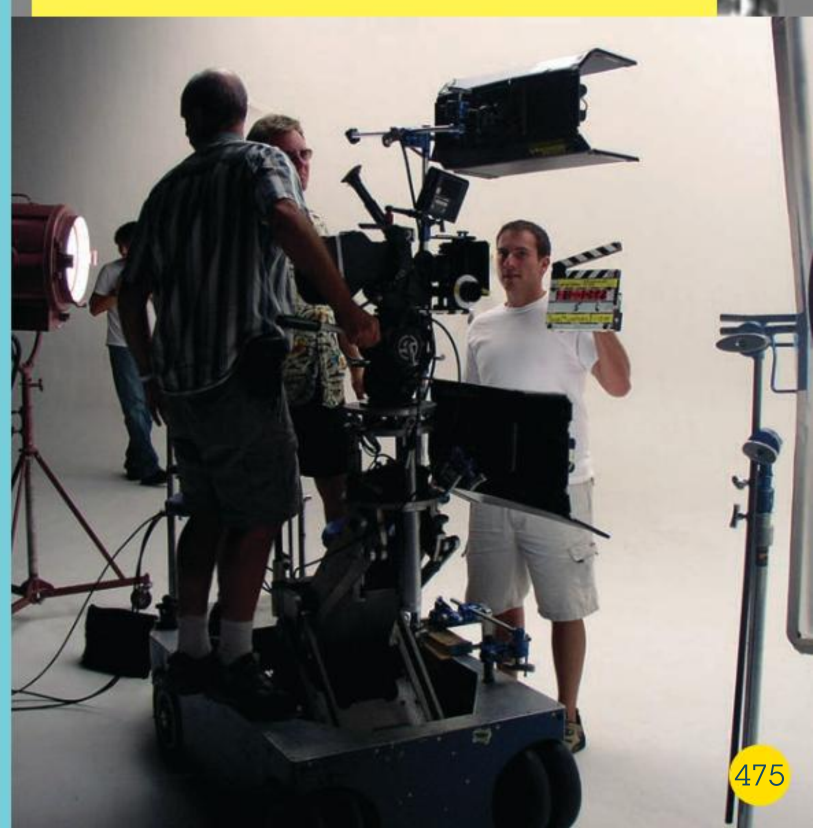
(d) a new 'energy' drink

Research

7 In the music industry, the Australian Recording Industry Association (ARIA) produces weekly charts of music, such as the Top 50 Singles Chart.

(a) How are charts such as this compiled?

(b) How do you think the information these sorts of charts provides compares with the OzTam information?



Maths 4 Real

COUNTING THE COUNTRY

The Australian Census is completed every 5 years. A genuine attempt is made to collect data about every person in Australia at the time of the census. The next census is due in 2016.

1 Why do you think the Government goes to the expense and effort of conducting a census every 5 years? Wouldn't it be a lot easier to just collect some sample data?

The Census is only one of the services provided by the Australian Bureau of Statistics (ABS). The ABS collects data and distributes information about many aspects of Australian life.

On 4 May 2011, the ABS estimated the Australian population to be 22 599 195 with a population growth rate of one new person every 1 min 37 s.

2 At this rate, what was the Australian population on 1 July 2011?

The following table shows some recent data related to the Australian population.

Year	Population ($\times 10^3$)	Median age (years)	% Born in Europe	% Born in Asia
1994	17 855	33.4	13.5	4.7
1995	18 072	33.7	13.3	4.9
1996	18 311	34.0	13.2	5.1
1997	18 518	34.4	13.0	5.3
1998	18 711	34.8	12.8	5.3
1999	18 926	35.1	12.5	5.3
2000	19 153	35.4	12.3	5.4
2002	19 413	35.7	12.0	5.5
2002	19 641	36.0	11.9	5.7
2003	19 873	36.2	11.7	5.7
2004	20 111	36.4	11.7	5.7

Note: These questions can be answered using technology such as Excel or a CAS.

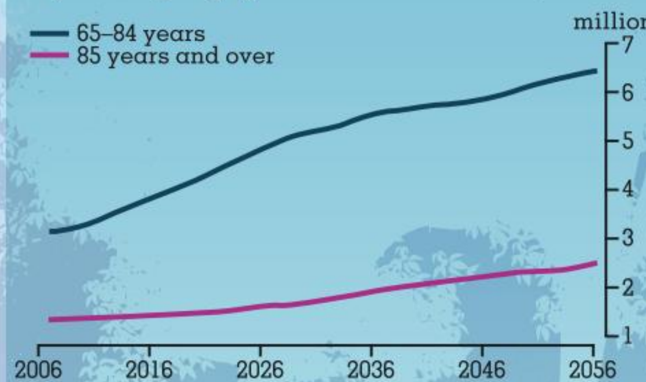
3 Use the data in the table to draw a line graph of the population over the decade. Extend your graph to predict the population in 2011. How does this estimate compare with the figure given earlier in this task?

4 Draw a line graph showing both the percentage of the population born in Europe and the percentage born in Asia. Do you think the percentage born in Asia will ever be greater than the percentage born in Europe? If so, when do you think this will occur?

5 The median age increased by 3 years in the period shown in the table. What does this tell you about Australia's population?

The following graph shows projected figures related to older people in the Australian population.

Projected aged populations: 2007[(a)] – 2056, Series B



(a) Data for 2007 is estimated, all other years are projections

Source: *Population Projections, Australia. 2006 to 2101* (ABS cat no. 3222.0)

6 How might a graph like this be used by local, state and federal members of government when making policy decisions?

Half-time 8

Ex 8.2

1 For each of the following stem plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to two decimal places.

(a)

STEM	LEAF
3	0 1 9 9
4	4 5 6 7 9
5	3 8 9 9
6	4 5 6
7	7
8	0 1

(b)

STEM	LEAF
12	1 4 7
13	2 5 6 9
14	3 4 7 8 9
15	0 1 2 3 4
16	4 6 7 7
17	1 3 4

Ex 8.1

2 Choose the most precise data type for each of the following.

(a) The rating, on a scale of 1–5, given by members of the audience at a movie premiere.

A discrete **B** ordinal **C** categorical **D** qualitative

(b) The time taken by each of the competitors to complete the marathon at the 2012 Olympic Games.

A discrete **B** continuous **C** categorical **D** qualitative

(c) The number of buckets of apples picked by each of the pickers at an orchard during one day.

A discrete **B** continuous **C** categorical **D** ordinal

Ex 8.1

3 A local company thinks it needs to change its corporate image and so has hired a graphic artist to produce a new company logo. The graphic artist has come up with three possible logos. You are responsible for finding out which logo should be adopted by the company. Explain how you would go about reaching a decision.

Ex 8.4

4 The following table shows the heights of 40 students sampled from the Year 9 students at a particular school. The measurement has been taken to the nearest cm. The teacher knows that one group is boys and the other is girls, but has forgotten which is which. Draw a back-to-back stem plot and use this to help you decide which is which.

Group 1:	164	153	149	167	181	175	154	167	177	189
	155	166	180	170	173	155	156	183	177	160
Group 2:	172	166	155	161	172	155	151	158	167	171
	166	171	173	155	160	160	179	166	154	153

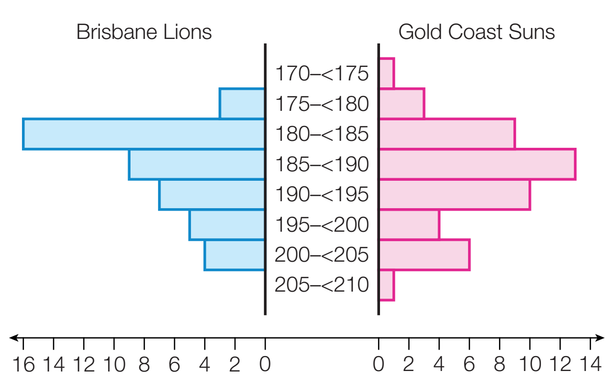
Ex 8.4

5 The following back-to-back histogram shows the height of the players on the 2011 playing lists of the two Queensland-based AFL teams.

(a) Which team appears to be taller?

(b) What other information would you like to more fully answer this question?

(c) The mean and median heights for the Brisbane Lions are 188 cm and 186 cm, respectively, whereas for the Gold Coast Suns they are 189 cm and 188 cm, respectively. Does this information make it easier for you to compare the sides?



Understanding probability

8.5

The **probability** or **chance** of any event must lie within the range 0 to 1. If the probability is 0, we say the event is **impossible**. For example, the probability of rolling a number greater than 6 with a normal die is 0. If the probability is 1, we say the event is **certain**. For example, the probability that the Prime Minister of Australia is an Australian citizen is 1.

We assign probabilities based on assumptions, or estimate the value based on data collected, or combine the two in some way. For example, when we are flipping a coin we almost certainly assume that the coin is fair; that is, that it has an equal chance of landing with either heads or tails showing. So we say $\Pr(H) = \frac{1}{2}$. If we have been observing the colour of cars that pass the school, we may have collected data such as the following.

Colour	White	Blue	Green	Grey	Brown	Silver	Other
Frequency	58	13	7	15	10	21	46

From the data collected, we can estimate that $\Pr(\text{white}) = \frac{58}{170} = \frac{29}{85} \approx 0.34$. If we assume these results are an accurate reflection of cars in general, we would conclude that approximately one-third of cars are white.

In cases where we can reasonably assume that all the outcomes are equally likely, we can use the following rule.

$$\Pr(\text{successful event}) = \frac{\text{number of successful equally likely outcomes}}{\text{total number of possible equally likely outcomes}}$$

When dealing with probability we must remember that the values lie in the range 0–1 if we are dealing with fractions or decimals, or 0–100 if we are dealing with percentages.

$0 \le \Pr(\text{event}) \le 1$

$0\% \le \Pr(\text{event}) \le 100\%$

Worked Example 8

WE8

Narelle has been having some trouble with her car. Over the course of the past 14 days she has failed to get it to start on 8 days. What is the estimated probability that Narelle will be unable to start her car on the 15th day?

Thinking	Working
1 State the relevant information we have.	Narelle has failed to start the car on 8 of 14 days.
2 Estimate the probability based on this information.	$\Pr(\text{Narelle is unable to start the car}) = \frac{8}{14}$

3 Cancel to express the fraction in simplest form (if necessary).	$= \frac{\cancel{8}^{4}}{\cancel{14}_{7}}$ $= \frac{4}{7}$

Complementary events

When dealing with probability, we often use just two outcomes: success and failure. These are **complementary outcomes** because the Pr(success) + Pr(failure) = 1. Consider a normal pack of cards that has four suits: hearts, diamonds, clubs and spades. When drawing a card from the pack, we may be interested in Pr(getting a heart) or Pr(not getting a heart). The outcome 'not getting a heart' is the complement of the outcome 'getting a heart'. The sum of the probability of an outcome and its complement is always 1.

To make things easier we often assign letters to represent the outcomes. For example we may let H stand for the outcome 'drawing a heart from the pack'. We could then write $\Pr(H) = \frac{1}{4}$. Using this style of notation we use not H to represent the complement of event H. So, $\Pr(\text{not } H) = 1 - \frac{1}{4} = \frac{3}{4}$.

In general, $\Pr(A) + \Pr(\text{not } A) = 1$, where A and not A are complementary events.

This can also be expressed as:

$$\Pr(\text{not } A) = 1 - \Pr(A)$$

By stating the rule in this way we ensure that the two events are mutually exclusive; this is a necessary condition for complementary events.

Worked Example 9

WE9

In a box containing 24 chocolates, there are 10 cream-centred and 8 hard-centred, and the rest are filled with nuts. One chocolate is chosen at random. Find the probability of choosing:

(a) a cream-centred chocolate

(b) a hard-centred chocolate

(c) a chocolate filled with nuts

(d) a chocolate that is not cream-centred

(e) a chocolate that is not filled with nuts

Thinking	Working
Assign letters to stand for each of the basic events. By saying we choose at random, we are saying each chocolate has the same chance of being selected, so we are dealing with equally likely outcomes.	Let C = choosing a cream-centred chocolate H = choosing a hard-centred chocolate N = choosing a chocolate filled with nuts
(a) $\Pr(C) = \frac{\text{no. of cream-centred chocolates}}{\text{total no. of chocolates}}$	(a) $\Pr(C) = \frac{10}{24}$ $= \frac{5}{12}$

(b) $\Pr(H) = \dfrac{\text{no. of hard-centred chocolates}}{\text{total no. of chocolates}}$

(b) $\Pr(H) = \dfrac{8}{24}$

$= \dfrac{1}{3}$

(c) The number of chocolates that are filled with nuts $= 24 - 10 - 8$
$= 6$

(c) $\Pr(N) = \dfrac{6}{24}$

$= \dfrac{1}{4}$

(d) not C = choosing a chocolate that is *not* cream-centred

(d) $\Pr(\text{not } C) = 1 - \Pr(C)$

$= 1 - \dfrac{5}{12}$

$= \dfrac{7}{12}$

or $\Pr(\text{not C}) = \dfrac{\text{no. of chocolates not cream-centred}}{\text{total no. of chocolates}}$

$\Pr(\text{not } C) = \dfrac{8 + 6}{24}$

$= \dfrac{14}{24}$

$= \dfrac{7}{12}$

(e) not N = choosing a chocolate that is *not* filled with nuts

$\Pr(\text{not } N) = \dfrac{\text{no. of chocolates not filled with nuts}}{\text{total no. of chocolates}}$

(e) $\Pr(\text{not } N) = 1 - \Pr(N)$

$= 1 - \dfrac{1}{4}$

$= \dfrac{3}{4}$

Relative frequency

When conducting probability experiments we often have different numbers of trials. To compare the outcomes for such different experiments we use the **relative frequency** of the results. This is usually expressed as a fraction, decimal or percentage and is found by dividing the number of outcomes for a particular result by the total number of outcomes.

Worked Example 10

WE10

Five coins were flipped a different number of times each and the results recorded in the table below.

	Coin 1	Coin 2	Coin 3	Coin 4	Coin 5
Heads	21	14	16	54	109
Tails	17	19	20	49	117

(a) What was the relative frequency of heads for Coin 3?

(b) For which coin was the relative frequency of tails greatest?

Thinking	Working
(a) Add the two values for the coin of interest and divide this into the number of heads for that coin. Then express in simplest fraction form and decimal form.	(a) Relative frequency of heads $= \frac{16}{36}$ $= \frac{4}{9}$ $= 0.\overline{4}$
(b) 1 Calculate the relative frequency of tails for each coin. Recording the results as decimals, to three decimal places, will make the comparison easier. Redraw the table using the relative frequencies.	(b)
2 Answer the question by finding the highest relative frequency of tails.	Coin 2 has the highest relative frequency of tails.

	Coin 1	Coin 2	Coin 3	Coin 4	Coin 5
Tails	0.447	0.576	0.556	0.476	0.518

$$\text{Relative frequency} = \frac{\text{number of sucessful trials}}{\text{number of trials}}$$

This last Worked Example raises the question of what a biased set of results would look like. For example, do you think any of the five coins in Worked Example **10** are actually biased or are the results simply within the bounds of variability we would accept from a small set of results?

8.5 Understanding probability

Navigator

Answers page 652

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q11, Q12, Q15, Q16, Q18, Q22	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q20, Q22, Q23	Q1, Q2, Q3, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q23

Fluency

WE8

1 Madelene and Pieter enjoy playing against each other in ten-pin bowling. Over the past few years Madelene has won 26 times and Pieter has won 28 times. Estimate the probability that Madelene will win their next game.

WE9

2 To decide which lane he would swim in for the butterfly swimming event, Rohan drew a numbered token by random selection from a bag. The bag contained eight different tokens numbered from 1 to 8 to represent the eight lanes of the pool. Find the probability that Rohan:

(a) swims in lane 1

(b) swims in lane 8

(c) does not swim in lane 8

(d) swims in an even-numbered lane

(e) swims in one of lanes 2, 3 or 4.

3 A fruit bowl contains one each of the following fruits: apple, banana, orange, apricot, pear, nectarine and peach. The following table gives the results for five separate experiments in which fruits were chosen by random selection from the bowl (and replaced after selection). WE10

Fruit	Experiment 1	Experiment 2	Experiment 3	Experiment 4	Experiment 5
Apple	5	4	3	5	4
Banana	2	3	3	6	2
Orange	6	6	3	2	7
Apricot	7	8	4	7	8
Pear	3	9	4	9	4
Nectarine	2	10	4	2	6
Peach	4	2	5	5	2

Answer the following, expressing all relative frequencies to two decimal places.

(a) For which experiment was the relative frequency of bananas greatest, and what was it?

(b) For which experiment was the relative frequency of bananas least, and what was it?

(c) For which experiment was the relative frequency of apples greatest, and what was it?

(d) For which experiment was the relative frequency of apples least, and what was it?

(e) Which fruit, in which experiment, had the lowest relative frequency, and what was it?

(f) Which fruit, in which experiment, had the highest relative frequency?

4 Your class has collected the following data from observations of the cars passing your school.

Make	Holden	Ford	Toyota	Mazda	Honda	Hyundai	Other
Frequency	35	27	31	15	7	10	42

Assuming that these figures are a reliable estimate of all cars on the road, find the probability, written as a decimal correct to two decimal places, that the next car observed is:

(a) a Ford (b) a Holden or a Mazda (c) not a Toyota.

5 Elliot, Kris and Austin are UNO fanatics. So far this year Elliot has won 45 games, Kris has won 38 games and Austin has won 67 games. Estimate an approximate value for the probability for each to win the next game.

6 (a) The probability of choosing by random selection a black jelly bean from the selection of black and pink jelly beans shown is:

A $\frac{1}{4}$ B $\frac{2}{5}$ C $\frac{3}{5}$ D $\frac{2}{3}$

(b) The probability of choosing by random selection a blue ball from a bag containing only six blue balls is:

A 0 B $\frac{1}{5}$ C $\frac{1}{6}$ D 1

(c) Aaron has passed eight out of the last 10 maths tests. Based on these data, the probability that he will *not* pass his next maths test is:

A 0 B $\frac{1}{5}$ C $\frac{4}{5}$ D 1

7 Five dice were rolled a different number of times each and the results recorded in the table below.

	Die 1	Die 2	Die 3	Die 4	Die 5
1	10	14	21	31	18
2	12	19	28	28	22
3	13	21	23	30	21
4	11	13	25	26	17
5	14	17	22	19	16
6	19	15	26	24	19

(a) For which die was the relative frequency of 1 the highest?

A die 1 B die 2 C die 4 D die 5

(b) For which die was the relative frequency of 6 the lowest?

A die 1 B die 2 C die 4 D die 5

(c) Put in order, from smallest to largest, the relative frequency for a 2 appearing by random selection.

(d) Put in order, from smallest to largest, the relative frequency for a 5 appearing by random selection.

8 Bill has beaten Dario in 15 of their last 25 badminton matches. The estimated probability that Dario will win the next match is:

A $\frac{1}{4}$ B $\frac{3}{8}$

C $\frac{2}{5}$ D $\frac{3}{5}$

9 A basketballer has scored the following points in the last 20 games:

22 15 19 27 16 31 40 29 19 26 14 22 33 41 44 32 31 20 24 30

Using these past results as a guide, estimate the following probabilities.

(a) The probability that the basketballer scores less than 20 points in the next game is:

A $\frac{1}{5}$ B $\frac{1}{4}$ C $\frac{3}{10}$ D $\frac{1}{3}$

(b) The probability that the basketballer scores more than 35 points in the next game is:

A $\frac{1}{10}$ B $\frac{3}{20}$ C $\frac{1}{5}$ D $\frac{1}{4}$

(c) The probability that the basketballer scores between 21 and 34 points in the next game is:

A $\frac{9}{20}$ B $\frac{11}{20}$ C $\frac{2}{3}$ D $\frac{9}{11}$

10 The maximum daily temperatures were recorded for the previous fortnight. They were:

23°C, 25°C, 19°C, 18°C, 20°C, 21°C, 21°C, 24°C, 29°C, 28°C, 24°C, 25°C, 20°C, 22°C

Using these as a guide, estimate the probability that the next maximum daily temperature is:

(a) 20°C **(b)** below 20°C **(c)** above 20°C

(d) below 30°C **(e)** above 30°C **(f)** above 25°C

(g) Find the sum of your answers to **(a)**, **(b)** and **(c)**. Do you obtain the result you expect?

11 A die numbered 1–6 is biased so that Pr(1) = Pr(2) = Pr(3) = $\frac{1}{4}$ and Pr(4) = Pr(5) = Pr(6).

(a) What is the fractional value for Pr(4)?

(b) Find Pr(even number).

(c) Find Pr(not a multiple of 3).

12 If the probability of choosing by random selection a red jelly bean from a packet is $\frac{4}{9}$, what is the probability of not selecting a red jelly bean?

Understanding

13 A teacher compiled this table of absences from his class for first term.

No. of students absent	No. of days
0	10
1	15
2	12
3	9
4	3
5	1

(a) How many school days were there in Term 1? Assume for the remainder of this question that the figures in the table are an indication of what could be expected in Term 2.

(b) Based on the data collected, what is the estimated probability that on any particular day in Term 2 the number of absentees will be:

(i) zero students **(ii)** two students **(iii)** five students

(iv) less than four students **(v)** more than one student **(vi)** at least one student?

(c) Express each of the fractional answers from **(b)** as a decimal, correct to two decimal places if necessary.

(d) Express each of the fractional answers from **(b)** as a percentage, correct to two decimal places if necessary.

14 The table below gives the life expectancy for men at various ages.

Current age (years)	20	25	30	35	40	45	50
Further life expectancy (years)	52.1	47.4	42.8	38.2	33.8	29.6	25.5

Current age (years)	55	60	65	70	75	80	85
Further life expectancy (years)	21.7	18.2	15.0	12.1	9.6	7.5	5.7

(a) What is the expected age of death for a man currently:

(i) 20 years of age **(ii)** 30 years of age **(iii)** 40 years of age

(iv) 55 years of age **(v)** 65 years of age **(vi)** 75 years of age?

(b) What do you notice about the figures? Is the figure constant?

Half of all men of a particular age are expected to live to at least their current life expectancy figure. So, half of all men currently 25 years of age are expected to be alive at the age of 72.4. The further life expectancy for a 72-year-old is 11.0 years.

(c) What fraction of men currently 25 years of age would you expect to live to the age of 83 years?

(d) Who would be interested in these figures, and why?

15 Find:

(a) Pr(being born on 17 June). Ignore leap years.

(b) Pr(being born in June).

(c) Pr(being born on the 17th of any month).

(d) Pr(being born in a month containing exactly 30 days).

(e) Pr(not being born in February).

16 A fair six-sided die is rolled. Find the probability of obtaining:

(a) a 2

(b) a 7

(c) either a 1 or a 2

(d) an odd number

(e) a number larger than 2

(f) a number between 0 and 7.

17 For each of the following experiments, complete the following steps.

(i) Predict the number of times each of the outcomes will be obtained.

(ii) Conduct the experiment and record the outcomes.

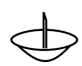
(iii) Draw up a table of results.

(iv) Use the table to calculate the probabilities for each of the outcomes achieved in the experiment.

(v) Compare your predictions with the actual results.

(a) Toss two coins together 60 times. Record how many times the outcomes are two heads, or two tails, or one head one tail.

(b) Drop a drawing pin onto your page 100 times.

The two outcomes are point up:  and point down: 

(c) Use the random number key on your calculator. Push the button 100 times and note how many times each digit appears in the first place after the decimal point.

(d) Open to any page in a paperback novel. Take the first paragraph and record how often each letter occurs.

18 Chandra opens the batting for his local cricket club in the Under 14s. Over the season so far he has scored the following runs.

34, 21, 1, 14, 45, 23, 35, 0, 10, 56, 36, 12

Use this as a guide to estimate the following for his next score.

(a) Pr(score from 0 to 9)

(b) Pr(score from 10 to 19)

(c) Pr(score from 20 to 29)

(d) Pr(score from 30 to 39)

(e) Pr(score from 40 to 49)

(f) Pr(score from 50 to 59)

(g) Pr(score greater than or equal to 60)

(h) Add the probabilities in **(a)** to **(g)** and explain the meaning of your answer.

(i) Chandra is asked to play in the seniors, instead of the U14s, in his next match. Could we use these same figures to predict the likelihood of his score? Explain your answer.

Reasoning

19 Over the whole basketball season Eloise scored with two-thirds of the free throws she had. In the Grand Final she is lining up for her third free throw. The opposition coach believes Eloise is sure to miss because she has scored with her first two. Explain why you think the coach is right or wrong about this.

20 To help choose his lotto numbers Sylvester records the frequency with which numbered balls are drawn out over a 2-year period.

(a) Offer two reasons why Sylvester might think this information is useful.

(b) In your opinion, is this information useful? Explain your opinion in a sentence or two.

21 A biased die, marked 1 to 6, is such that:

$\Pr(6) = \Pr(2) = \Pr(1) = \frac{1}{12}$ $\Pr(\text{not } 6) = 5 \times \Pr(5) + \Pr(1)$ $\Pr(3) = \Pr(4) + \Pr(2)$

Find the probability of rolling each number.

Open-ended

22 You have an unbiased eight-sided die, with faces numbered 1 to 8. Write an event for each of the following probabilities.

(a) $\Pr(A) = \frac{1}{4}$ **(b)** $\Pr(B) = \frac{1}{2}$ **(c)** $\Pr(C) = \frac{5}{8}$

23 In some board games, such as *Monopoly*, you need to throw a double 6 on a normal six-sided die before you can begin.

(a) Is there something special about double 6? Is it harder, or easier, to throw than, say, double 3?

(b) **(i)** Roll two dice and record the number of trials needed to get to each of the double results. Complete the table below to help you.

Event	Trial no.
1, 1	
2, 2	
3, 3	
4, 4	
5, 5	
6, 6	

You do not have to get 1, 1 before 2, 2. Just keep counting. For example, 2, 2 may occur as trial 12 and 5, 5 as trial 30 etc.

(ii) Repeat this experiment 10 times and find the mean number of trials for each of the six doubles to occur.

(iii) Answer part **(a)** of this question again.

Investigation

Game simulator

Equipment required: 2 or more brains, 1 normal six-sided die, calculator or CAS (optional)

The Big Question

Can we simulate results for a variety of sports using a die?

Have you ever wondered why your favourite computer game is not exactly the same every time you play it? Inside the program is a random number generator that is used to make decisions about the path a game will follow each time. Owing to the nature of random numbers, we would not expect the same decision to be made each time. To illustrate this we will look at some popular games that can be simulated using random numbers. The instructions say to use dice to get the numbers, but you could use your calculator or CAS to get the random numbers if that is more convenient for you.

Engage

1 To simulate a game of Aussie Rules football, play a game of Dice AFL.

Rules

Each team rolls the die twice. The first roll represents the number of goals that are kicked in a quarter and the second roll represents the number of behinds kicked.

Repeat this another three times and record your results in the table below.

Total the points for each team using 6 points for a goal and 1 point for a behind.

	Team A		Team B	
	Goals	Behinds	Goals	Behinds
1st quarter				
2nd quarter				
3rd quarter				
4th quarter				
Total				

Explore

2 Now play a game of Dice Rugby League.

Scoring in the NRL is 4 points for a try, which then leads to a conversion kick at goal in an attempt to gain a further 2 points. Teams may kick a field goal, which is worth a single point at any time. For the convenience of this simulation, we will assume the conversion rate in the real game is approximately 50%.

Rules

Each team has 20 rolls of a single die, taken in turns.

A roll of 1 represents a field goal: score 1 point.

A roll of 4 represents a try: score 4 points. In addition, you have a bonus roll, where an even number represents the conversion and scores 2 additional points.

Any other number scores zero points.

Strategy options

- Make a table.

Explain

3 For the AFL game you played:

(a) What is the lowest possible score for a quarter?

(b) What is the lowest possible score for the game?

(c) What is the highest possible score for a quarter?

(d) What is the highest possible score for the game?

4 For the Rugby League game you played:

(a) What is the lowest possible score for the game?

(b) What is the highest possible score for the game?

5 Using your answers to Questions **3** and **4**, how realistic do you think the scores were?

6 Are there limitations obvious to you with this simulation?

7 What assumptions have been made about the game to make it relatively simple to play?

8 Overall, comment on how well the die simulates the real game.

Elaborate

9 Now try Dice Cricket.

Rules

Each player has 10 cricketers.

The players score runs by rolling the die and scoring runs until they roll a 5, at which point the cricketer is out and the next cricketer begins his/her innings. When all 10 cricketers have been dismissed, their runs are added to give the total score for the team. The second team now has their turn to bat. The winner is the team with the higher number of runs.

10 What is the highest score recorded for a single cricketer in either team?

11 How many cricketers scored a duck (no score)?

12 Do you think Dice Cricket simulates cricket better than the games of football played earlier? Why or why not?

Evaluate

13 Could you use something other that a die to improve your game and make it more realistic?

14 Are all the situations that occur in the game covered by the simulation?

15 Should the number allocations be changed?

16 How is this number allocation related to probability?

Extend

17 Drawing on your experience of the games played in this Investigation:

(a) Create your own simulation of a well-known sport.

(b) Write out the rules for your simulation so that another person could play it with no need to ask any clarifying questions.

(c) Explain how you decided upon the numbers you allocated to simulate various events in the game.

(d) How did probability help with this?

(e) Do you think it is important to play the game before you finalise the design?

Technology Exploration Spreadsheet

How many heads?

Versions of this Exploration for other technologies are available in Pearson Reader.

PP Technology

We can use a spreadsheet not only to model tossing coins and rolling dice, but also to organise the results.

1 If we let 0 represent tails and 1 represent heads, then the spreadsheet can 'simulate' tossing a coin for us. In Excel, type the following formula into cell A1: =RANDBETWEEN(0,1). This formula asks Excel to RANDomly (randomly) select an integer in the range represented by the two values entered.

2 All that work for one toss of a coin! Click back on A1 and move the mouse to the bottom right-hand corner. The cursor should become a thin black cross—this is the 'fill handle'. Drag this cross to E1. You will now have five coins being tossed. Drag the highlighted cells (A1 to E1) in the same manner down 10 rows and you get 50 coins being tossed.

3 Excel can organise the results, that is, count the number of heads and tails tossed. In A12 enter the formula =SUM(A1:E10) and press ENTER. What does the result represent? In a blank cell continually press the DELETE key. What happens? Record the number of heads for 30 separate sets of 50 coin tosses. What was the smallest number of heads observed? What was the highest number of heads observed? Find the mean number of heads observed.

4 Rolling a die can be simulated in a similar way. Double-click the Sheet2 tab at the bottom of the workbook and rename it Dice Rolling. In A1 on this sheet enter the formula =RANDBETWEEN(1,6). Create 50 rolls of a die using the fill handle.

5 Getting Excel to collate the results this time requires a bit more thought. In A12 enter Face and in B12 enter Frequency. In A13–A18 enter the numbers 1–6. In B13 enter the formula =COUNTIF($A$1:$E$10,A13). This formula looks in cells A1 to E10 and COUNTs them IF they are the same as the value in A13 (i.e. it counts how many 1s there are). The $ signs make the cell reference absolute, which means it does not change when we copy it into a new cell. The A13 does not have the dollar sign because we want it to change each time. Now fill the formula down into cells B14–B18.

	A	B	C	D	E	F
1	1	6	6	3	4	
2	2	5	6	3	3	
3	6	6	4	2	1	
4	3	2	4	6	2	
5	6	6	2	3	3	
6	4	4	1	4	6	
7	4	4	4	6	5	
8	5	3	5	3	2	
9	4	4	5	6	6	
10	6	4	1	5	1	
11						
12	Face	Frequency				
13	1	5				
14	2	6				
15	3	8				
16	4	12				
17	5	6				
18	6	13				
19						
20						

We can then press the Delete key in a blank cell to get a new set of 50 results that are counted in a frequency table. In this way we can rapidly produce many hundreds of results and check the long-term probability of any particular result.

Record in your exercise books the number of 4s rolled in 30 separate sets of 50 rolls. A sample set of results is given below:

9	5	3	8	8	8	8	9	9	10
6	8	11	13	9	6	6	14	14	15
12	3	8	5	5	10	7	12	9	9

Theoretical probability predicts $\frac{1}{6}$ of the rolls should be a 4. How did our trial fare? The 30 results supplied give a total of 259 fours from 1500 results. This represents 0.1727 of the results. The decimal equivalent of $\frac{1}{6}$ is 0.1667, so the result is quite close.

How did your own set of 30 trials fare?

6 Collate all your class results and check how close this comes to what theoretical probability predicts. Write a sentence or two about this.

Taking it further

7 How can we organise rolling two dice simultaneously, noting the total and keeping a record of the results?

On a new sheet use column A to roll a die 50 times (using A1–A50) and likewise in Column B. Get the total of A1 and B1 by writing an appropriate formula in C1 and copy this down the column. Then set up the frequency table in a similar manner to your previous work, using Sum and Frequency as the headings.

8 What additional consideration would need to be given if we were interested in the difference between the two results rather than the sum?

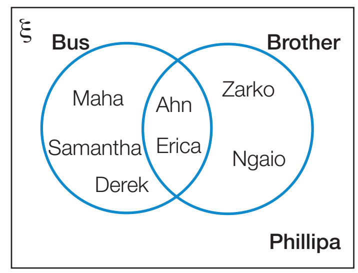
Probability events

8.6

Venn diagrams

In Year 8 you were introduced to Venn diagrams. There are three situations for which Venn diagrams are really useful. These are when we are dealing with the words 'and', 'or' and 'not'.

Let us imagine we have asked our friends some questions. The results have been summarised in a Venn diagram.



Who travelled to school today by bus?

Maha, Samantha, Derek, Ahn and Erica

Who has a brother?

Ahn, Erica, Zarko and Ngaio

Who travelled to school today by bus and has a brother?

Ahn and Erica

Who travelled to school today by bus or has a brother?

Maha, Samantha, Derek, Ahn, Erica, Zarko and Ngaio

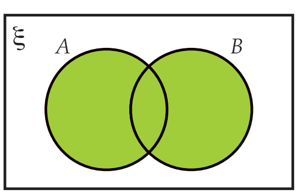
Who did not travel to school by bus today?

Zarko, Ngaio and Phillipa

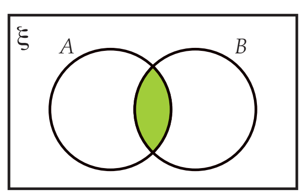
How would we most precisely describe Phillipa? *She did not travel to school by bus today and she does not have a brother.*

Shading can be used on Venn diagrams to represent the three key situations.

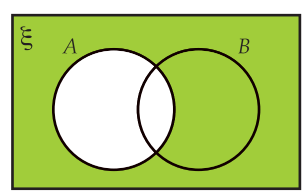
A or B



A and B



A' (not A)



Further applications of the basic probability rule

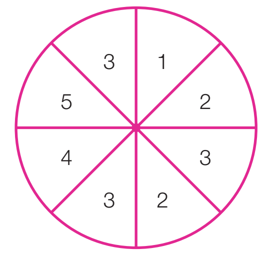
We can also encounter the words and, 'or' and 'not' in situations where we can just count the number of successful outcomes and apply the basic rule of probability that applies when we are dealing with equally likely outcomes:

$$\text{Pr(event)} = \frac{\text{number of favourable equally likely outsomes}}{\text{total number of equally likely outcomes}}$$

Worked Example 11

WE11

The spinner shown in the diagram on the right is spun once.
Find the following probabilities.



(a) Pr(2)

(b) Pr(4)

(c) Pr(2 or 4)

(d) Pr(even number)

(e) Pr(not 2)

Thinking	Working
(a) Find the probability using the basic probability rule.	(a) $Pr(2) = \frac{2}{8} = \frac{1}{4}$
(b) Find the probability using the rule.	(b) $Pr(4) = \frac{1}{8}$
(c) Find the probability using the rule.	(c) $Pr(2 \text{ or } 4) = \frac{3}{8}$
(d) Find the probability using the rule.	(d) $Pr(\text{even number}) = \frac{3}{8}$
(e) Find the probablility using the rule.	(e) $Pr(\text{not } 2) = \frac{6}{8} = \frac{3}{4}$

Mutually exclusive events

When we roll a normal six-sided die there are many different events that can occur. Let us look at some of them, and their associated probability.

Pr(rolling a 1) = $\frac{1}{6}$

Pr(rolling a 3) = $\frac{1}{6}$

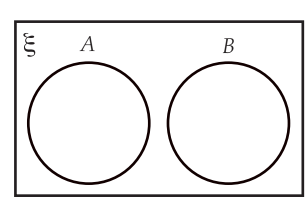
Pr(rolling an even number) = $\frac{1}{2}$

Pr(rolling a multiple of 3) = $\frac{1}{3}$

It is not possible to roll a 1 and a 3 at the same time. We say these events are **mutually exclusive**. Conversely, the events 'rolling a 3' and 'rolling a multiple of 3' are not mutually exclusive because they can occur at the same time. This would be if we actually rolled a 3.

Mutually exclusive events can be represented by the Venn diagram shown. Note that there is no overlap.

If an element belongs to set A it cannot belong to set B also.



8.6 Probability events

Navigator

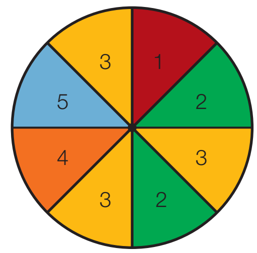
Q1, Q2, Q3, Q4, Q5, Q8 (a)–(c), Q9, Q12	Q1, Q2, Q5, Q6, Q7, Q8, Q9, Q12	Q1, Q2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Answers page 653

Fluency

WE11

1 The spinner shown at right is spun once. Find the following probabilities.



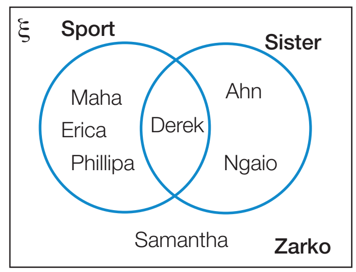
(a) Pr(3) **(b)** Pr(5)

(c) Pr(3 or 5) **(d)** Pr(odd number)

(e) Pr(5 or even number) **(f)** Pr(5 or odd number)

(g) Pr(not 3 or 4) **(h)** Pr(not even)

2 You have asked your friends some questions and summarised their answers in the Venn diagram shown. The questions were:



- Who has a sister?
- Who regularly plays sport at the weekend?

Use the Venn diagram to answer the following questions.

(a) Who has a sister?

(b) Who does not regularly play sport at the weekend?

(c) Who has a sister and regularly plays sport at the weekend?

(d) Who has a sister or regularly plays sport at the weekend?

3 A basket contains three red, five green and two blue scarves. Find the probability that a scarf drawn at random will be:

(a) red or green

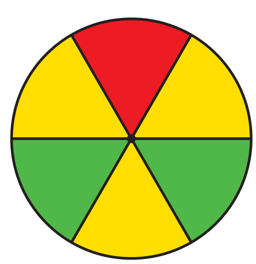
(b) red or blue

(c) green or blue.

4 A drawer contains six black, eight brown and four white socks. Find the probability that a sock drawn at random from the drawer will be:

(a) black or brown **(b)** black or white **(c)** brown or white.

5 A spinner is coloured as shown in the diagram. Use it to find:



(a) Pr(green)

(b) Pr(yellow)

(c) Pr(yellow or green)

(d) Pr(yellow or red).

6 A bowl contains 30 balls, numbered from 1 to 30. A ball is drawn from the bowl. Find:

(a) Pr(2 or 10) **(b)** Pr(even or 7)

(c) Pr(odd or multiple of 8) **(d)** Pr(multiple of 7 or multiple of 8)

(e) Pr(multiple of 4 or multiple of 9) **(f)** Pr(4 or multiple of 5).

7 Which of the following pairs of events is not mutually exclusive?

A obtaining a head on the toss of a coin—obtaining a tail on the toss of a coin

B choosing an odd-numbered horse in a race—choosing horse number 7 in a race

C winning first prize in a raffle with a blue ticket—winning first prize in a raffle with a yellow ticket

D playing for England in the World Cup—playing for Brazil in the World Cup

8 A bowl contains 20 identical counters with a different one of the numbers from 1 to 20 written on it. The following sets have been defined.

A = 'multiples of 4' B = 'multiples of 3' C = '1, 3, 5, 7, 9' D = '1, 2, 3, 5, 8, 13'

(a) State whether or not each of the following pairs of sets are mutually exclusive.

(i) A and B **(ii)** A and C **(iii)** A and D

(iv) B and C **(v)** B and D **(vi)** C and D

(b) Find the probability of each of A, B, C and D if a single counter is drawn from the bowl.

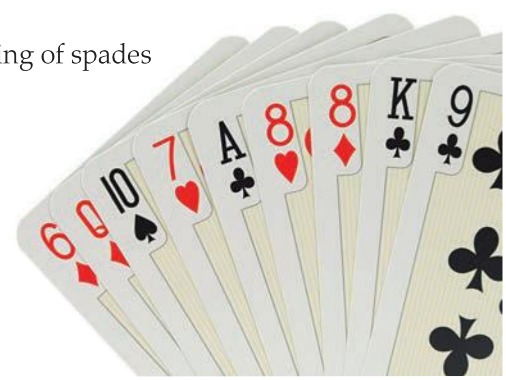
Understanding

9 An ordinary pack of playing cards consists of 52 cards, half red and half black. The red cards are divided into two suits: hearts and diamonds. The black cards are divided into two suits: clubs and spades. Each suit is made up of 13 cards: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.

If a card is drawn at random from the pack, what is the probability that the card is:

(a) black **(b)** a diamond

(c) black or a diamond **(d)** a diamond or the king of spades



(e) a king or an ace or a 6

(f) the king of spades or the queen of diamonds or the jack of hearts

(g) the king of diamonds or a spade or the ace of hearts

(h) not a king **(i)** not a diamond

(j) neither a king nor a queen **(k)** a spade or a king

(l) a king or a queen or a red card?

10 Consider the letters of the word *left* and the letters of the word *right*. A bag contains 26 pieces of card, each with a different one of the letters of the alphabet written on it. A letter is drawn at random from the bag. Find the following probabilities.

(a) The letter is in the word *left*. **(b)** The letter is in the word *right*.

(c) The letter is in either word.

(d) Explain in two different ways how you can get the answer to part **(c)**.

(e) The letter is in both words. **(f)** The letter is not in *left*.

(g) The letter is not in *right*. **(h)** The letter is in neither word.

Reasoning

11 We know that if two events, A and B, are mutually exclusive then
$\Pr(A) + \Pr(B) = \Pr(A \text{ or } B)$. The opposite of this is also true:
If $\Pr(A) + \Pr(B) \neq \Pr(A \text{ or } B)$ then the events are not mutually exclusive.

Consider the following situation where a red die and a blue die are rolled at the same time.

Let A = the red die shows 3
B = the blue die shows 4
C = the sum of the two numbers showing is 6
D = the difference between the two numbers showing is 1

(a) Copy and complete the following diagram to show the sample space.

		Red die					
		1	2	3	4	5	6
Blue die	1	1, 1	2, 1				
	2	1, 2					
	3						
	4						
	5						
	6						

(b) Calculate:

(i) Pr(A) **(ii)** Pr(B) **(iii)** Pr(C) **(iv)** Pr(D)

(c) Calculate:

(i) Pr(A or B) **(ii)** Pr(A or C) **(iii)** Pr(A or D)
(iv) Pr(B or C) **(v)** Pr(B or D) **(vi)** Pr(C or D)

(d) Are any of the pairs of events mutually exclusive?

Open-ended

12 **(a)** Write a pair of events that are mutually exclusive.

(b) Write a pair of events that are not mutually exclusive.

13 Draw a Venn diagram with three overlapping circles and write some questions like those in Question **2**.

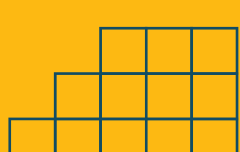
Outside the Square Game

Bigger is better

Equipment required: 1 or more brains, 10 small pieces of paper or small counters of the same size.

Preparation: Write the digits 0 1 2 3 4 5 6 7 8 9 on separate pieces of paper or counters, turn them over and shuffle them around.

Each player should copy the following grid.



How to win: The aim of the game is to form the largest total possible when the three numbers formed by completing the grid are added.

How to play: Randomly select one of the small pieces of paper to reveal a digit. Each player must then write this digit in one of the squares in their grid.

The piece of paper is then returned to the pile where they are all shuffled before another one is drawn. Each player then writes this digit in a square on their grid.

The process is repeated until all the squares in the grid are filled, so 12 digits will need to be drawn in order to fill the 12 squares of each player's grid.

Each player then adds the three-, four- and five-digit numbers they have formed.

If playing alone, you should play a number of times, each time aiming to improve on your highest score.

8.7 Representing probability

Multiple events

There are many situations that involve more than one outcome. Finding probabilities in these situations is made easier if you draw up a table showing all the possible outcomes. The full list of outcomes is called the **sample space.**

For example, when we roll two dice we may be interested in the individual results so we can find probabilities such as Pr(result includes an even number). In a case like this, we would represent the outcomes as (1, 1), (1, 2) etc. Conversely, if we were interested in the total of the two numbers we would just write that total. You need to read the questions carefully and then decide which format is most appropriate for your display. We can show all the outcomes in the sample space using a **two-way table** as shown below.

Worked Example 12

WE12

Ann-Marie has rolled two dice and is interested in the sum of the two numbers showing uppermost.

(a) List the sample space showing all possible outcomes.

(b) Using the sample space, find:

(i) Pr(total of 7) **(ii)** Pr(total is less than 6)

Thinking	Working
(a) Use a table as a way of showing the sample space.	(a) See table below. There are 36 possible outcomes, giving the sample space: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
(b) (i) Count the number of results from the table that give the totals required, and put this over 36.	(b) (i) For a total of 7, there are 6 outcomes, therefore: Pr(total of 7) = $\frac{6}{36} = \frac{1}{6}$
(ii) Do not include totals of 6, because the question says *less* than 6.	(ii) For a total less than 6, there are 10 outcomes, therefore: Pr(less than 6) = $\frac{10}{36} = \frac{5}{18}$

Die No. 1 (columns), Die No. 2 (rows)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

For multiple events:

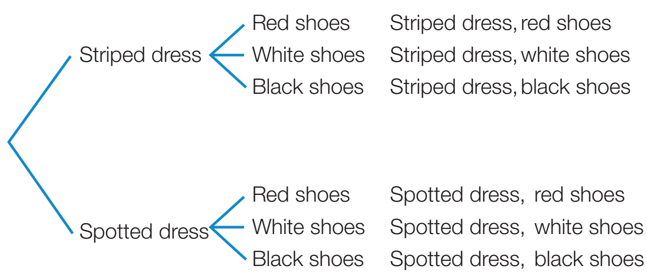
1 Draw up a table to show the sample space.

2 Count the number of successful outcomes.

3 Use $\text{Pr(success)} = \dfrac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely possible outcomes}}$

Tree diagrams

Another method of representing the sample space is to use a **tree diagram**. Consider the situation where we wish to find the probability of Bianca wearing red shoes and a spotted dress if she can choose to wear either her striped or spotted dress and match this with either red, white or black shoes. (Assume that she has no preference for either dress or any pair of shoes.)



The tree diagram clearly shows the six equally likely outcomes, so the probability of wearing the spotted dress with red shoes is $\frac{1}{6}$ because each event is equally likely.

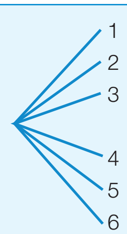
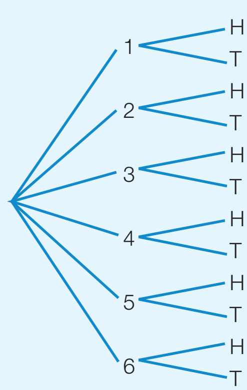
Worked Example 13

WE13

A die is thrown and then a coin is flipped.

(a) Use a tree diagram to list the sample space.

(b) Find Pr(an even number followed by a head).

Thinking	Working
(a) 1 Draw the first part of the tree with the number of branches to match the number of outcomes for the first event. In this case six, 1, 2, 3, 4, 5 or 6.	(a) 
2 To each of the branches add the number of branches to match the number of outcomes for the second event. In this case there are two outcomes, head (H) or tail (T).	

3 List the sample space.	(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)
(b) 1 Count the number of equally likely successful outcomes and the total number of outcomes.	(b) 3 and 12
2 Find the probability using the formula: Pr(success) $= \frac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely outcomes}}$ Simplify if possible.	Pr(even number and head) $= \frac{3}{12}$ $= \frac{1}{4}$

Sampling with and without replacement

If, for example, we were drawing cards from a pack and we replaced the card before the next draw, this is called **sampling with replacement**. Situations like this are relatively easy to deal with because the probability stays the same for each draw. However, if we did not replace the card, which is more likely if we are playing a card game, then the probability changes at each draw. This is called **sampling without replacement**. It is best to write out the sample space and work out the probabilities from there.

Worked Example 14

WE14

A bowl contains three apples and two pears. Joshua wants two pieces of fruit and doesn't care what he has, so he just randomly chooses two pieces of fruit from the bowl.

(a) Write out the sample space, making sure you distinguish between different pieces of the same fruit.

(b) Find the probability that Joshua chooses:

(i) two apples **(ii)** two pears **(iii)** two different pieces of fruit.

Thinking	Working
(a) 1 Decide on the symbols to use for the sample space.	(a) We will use A1, A2, A3, P1 and P2 to represent the fruit.
2 Write out the sample space in a systematic way.	A1A2, A1A3, A1P1, A1P2, A2A1, A2A3, A2P1, A2P2, A3A1, A3A2, A3P1, A3P2, P1A1, P1A2, P1A3, P1P2, P2A1, P2A2, P2A3, P2P1
(b) (i) Count the number of successful outcomes and divide by the total number of outcomes.	(b) $\frac{6}{20} = \frac{3}{10}$
(ii) Count the number of successful outcomes and divide by the total number of outcomes.	$\frac{2}{20} = \frac{1}{10}$
(iii) Count the number of successful outcomes and divide by the total number of outcomes.	$\frac{12}{20} = \frac{6}{10} = \frac{3}{5}$

8.7 Representing probability

Navigator

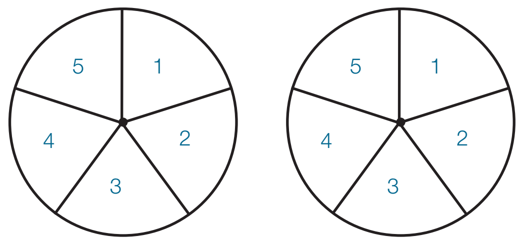
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q11, Q12	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12

Answers page 653

Fluency

For each of the following questions, it may be a good idea to first draw up a sample space using one of the techniques covered.

1 Con has two spinners as shown here. Draw a sample space to help you answer the questions. If Con spins them both, find the probability of getting: WE12



(a) A total of 6

(b) A total of 10

(c) A total of less than 7

(d) A total not equal to 5.

2 A coin is flipped and then a six-sided die is rolled. Find the probability of: WE13

(a) A head followed by a 6 **(b)** A head followed by an even number

(c) A head followed by a number less than 5 **(d)** A tail followed by a multiple of 3

(e) Not getting a head or a 5.

3 A bowl contains four apples and two bananas. Joshua wants two pieces of fruit and doesn't care what he has so he just randomly chooses two pieces of fruit from the bowl. WE14

(a) Write out the sample space, making sure you distinguish between different pieces of the same fruit.

(b) Find the probability that Joshua chooses:

(i) two apples **(ii)** two bananas **(iii)** two different pieces of fruit.

4 The menu at Shirley's local restaurant has four entrées: prawn cocktail, oysters, pâté and satay; and five main courses: lamb, fish, beef, pork and poultry.

(a) List all the possible different meals.

(b) Find, assuming random selection:

(i) Pr(pâté and pork)

(ii) Pr(oysters or fish)

(c) Shirley does not like satay or lamb. If she chooses a dish at random, what are the chances she has a meal she will enjoy?

5 Heath has four suits (black, blue, grey and brown) and five shirts (white, blue, yellow, pink and beige). Heath randomly chooses his clothes each day.

(a) The probability that Heath wears the grey suit is:

A $\frac{1}{4}$ **B** $\frac{1}{9}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(b) The probability that Heath wears the blue suit and the blue shirt is:

A $\frac{1}{20}$ B $\frac{7}{24}$ C $\frac{1}{2}$ D $\frac{2}{3}$

(c) The probability that Heath wears the black suit or the yellow shirt is:

A $\frac{1}{60}$ B $\frac{1}{9}$ C $\frac{2}{9}$ D $\frac{2}{5}$

Understanding

6 Aldo is making his choice for elective classes for Year 10. He must choose one subject from each of these subject blocks.

Block 1	Block 2
French	German
Art	Metalwork
ICT	Bonus Maths
	Drama

(a) List all the possible subject combinations that Aldo can choose.

(b) Find the following, assuming the subjects are chosen randomly:

(i) Pr(Art and Drama) (ii) Pr(chooses exactly one language)

(iii) Pr(not ICT) (iv) Pr(at least one language)

(v) Pr(neither Art nor German)

(c) If he *must* do Bonus Maths, what is the probability that Aldo also does French?

7 Angela is on holiday at the Solaris Beach Resort. She needs to choose her activities for the next day. Her choices are shown in the following table. Assume she makes one choice from each session by random selection.

Session 1	Session 2
Wind surfing	Water skiing
Scuba diving	Wave boarding
Body boarding	Wave pool
	Surfing

(a) How many different combinations of two activities can Angela select?

(b) Find the following:

(i) Pr(wind surfing and wave boarding)

(ii) Pr(scuba diving and any of the Session 2 activities)

(iii) Pr(body boarding and not surfing)

(c) Angela decides that she will finish the day in the wave pool. In how many ways can she now plan her day?

8 A bowl contains two blue marbles, two green marbles and two yellow marbles. Two marbles are drawn from the bowl.

(a) If the marbles are not replaced after being drawn, find the probability that:

(i) each marble is blue (ii) one marble is blue and the other yellow

(iii) each marble is a different colour (iv) neither marble is green.

(b) Recalculate the same probabilities if the marbles are now replaced after being drawn.

Reasoning

9 (a) A coin is tossed. List the elements in the sample space.

(b) How many outcomes are possible?

(c) A 10c coin and a 20c coin are tossed. List the elements in the sample space.

(d) How many outcomes are possible?

(e) A 10c coin, 20c coin and a 50c coin are tossed. Write out the sample space.

(f) How many events are possible?

(g) Is there a connection between the number of outcomes possible for each coin and the total number of outcomes possible for the combined events? Explain your answer.

10 A particular cube has two green faces, two blue faces and two red faces. It is rolled eight times and the face uppermost is recorded.

(a) How many elements are there in the sample space? (*Hint*: Don't try to write them out—apply the rule from Question **9**.)

(b) What is the probability that eight green faces in a row will occur?

(c) Why is **(b)** are of one of the easiest questions that could be asked about this event?

(d) What are the other equivalent questions with the same answer?

(e) Consider the question *'What is the probability that none of the faces will be green?'* This seems like an application of complementary probabilities to the event stated in **(b)**, but it is not quite as easy as that. Why not?

Open-ended

11 Construct a situation that has a sample space of at least 15 elements. Make up at least four questions to go with your sample space. Your teacher may get you to share your questions with other members of the class.

12 Make up some questions associated with a bowl of marbles. Make sure your questions include some with 'and', 'or' and 'not'.

Outside the Square — Problem solving

Birthday cards

Two of your friends happen to have their birthdays close together. You decide to post them each a birthday card. You purchase the cards and write a nice message inside each card. You then address the two envelopes but get distracted when you are putting the cards in the envelopes. What is the probability that you put the cards in the correct envelopes?

Now consider the same question if there were three friends, so three cards and three envelopes. What is the probability that you get all the cards correct? What about getting just one of them correct? What about getting none of them correct? Can you get exactly two of them correct? Explain your answer.

Strategy options

- Make a table.
- Test all possible combinations.

Challenge 8

1 This is a set of scores: 18, 20, 20, 25, 37.

A whole number score between 18 and 37 is added to the set of scores.

For the new set of scores, indicate whether each of the following statements is correct or incorrect.

(a) The mean of the scores could stay the same.

(b) The mean of the scores could decrease.

(c) The median of the scores could stay the same.

(d) The median of the scores could decrease.

2 (a) A set of five numbers has a range of 5 and a mean of 5. None of the numbers is 5. Write a set of possible numbers.

(b) Add a sixth number to your set so as to increase the mean to 6. (The range will change.)

3 A factor of the number 48 is chosen at random. What is the probability that the chosen factor does not itself have a factor of 3?

4 A prime number die has the numbers 2, 3, 5, 7, 11 and 13 on it. Two such dice are thrown. What is the probability that:

(a) the sum shown is even

(b) the number on one die is larger than the number on the other?

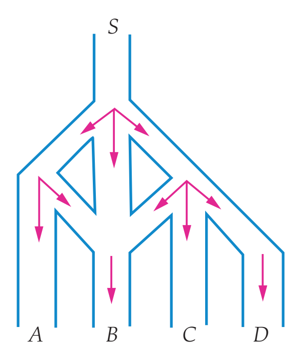
5 A barrel contains 18 marbles that are coloured white, black or green. If you draw one marble at random, then you are most likely to draw a black, and you are twice as likely to draw a white compared with a green. What could the contents of the barrel be? (There are three possible answers.)

6 It is estimated that 85% of students in Australia have internet access at home.

(a) Two students are selected at random. What is the probability that neither of them has internet access at home?

(b) Based on a recent survey, 60% of students who have internet access at home have also joined Facebook. A student is selected at random. What is the probability that the student has internet access at home but has not joined Facebook?

7 Rémy the mouse is put in a maze. He starts at Point S and can only move in the direction of the arrows; he cannot double back. At any junction he is equally likely to choose any of the forward paths.



(a) Show that the probability that Rémy ends up at Point A is $\frac{1}{6}$.

(b) Find the probability that Rémy ends up at Point B.

(c) Find the probability that Rémy ends up at Point C.

(d) Find the probability that Rémy ends up at Point D.

Chapter review 8

D.I.Y. Summary

Key Words

back-to-back histogram	mutually exclusive	sample space
back-to-back stem plot	negative skew	sampling with replacement
categorical data	observational data	sampling without replacement
certain	ordinal data	secondary data
chance	positive skew	skewed graph
complementary outcomes	primary data	stem plot
continuous data	probability	symmetrical graph
discrete data	qualitative data	tree diagram
impossible	quantitative data	two-way table
measure of location	random sample	
median class interval	relative frequency	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1. The ___________ or ___________ of an event lies between 0 and 1 inclusive.
2. If we collect data, such as recording the types of vehicles that pass the school, we are dealing with ___________.
3. Data collected by another person or agency that you have accessed are called ___________.
4. Non-numerical data that are simply divided into categories are called ___________ .
5. The general name for any type of numerical data is ___________.
6. Exact data values that are countable are called ___________.
7. The mean and the median are the two most commonly used ___________ .
8. If a data set has its values bunched to the right of the histogram, it is said to display ___________.
9. An event with a probability of 1 is said to be ___________.
10. We can use a ___________ or a ___________ to assist in finding the sample space.
11. If the mean, median and mode are close together, a histogram of the data is likely to be a ___________.
12. Any data value that arises from measurement is going to be ___________.
13. Events that cannot occur at the same time are said to be ___________.

Fluency

1 State whether the following data sets are discrete or continuous. **Ex 8.1**

(a) the times taken by competitors to complete the Sydney–Hobart yacht race in a particular year

(b) the number of occupants in cars that go through a toll booth

2 A survey was conducted at the local football club concerning a possible change to the club jumper. The following table summarises the percentage support for each of the three new designs as well as the current design. **Ex 8.1**

	Existing design	New design #1	New design #2	New design #3
Junior players	36	25	19	20
Junior parents	78	12	8	2
Senior players	46	31	18	5
Supporters	42	25	12	21
Sponsors	19	34	21	26

As the member of the club committee in charge of the survey, what recommendation would you make to the club? Give as much detail as you think appropriate.

3 **(a)** Which one of the following could not be carried out by observation? **Ex 8.2**

A the number of pot holes in a particular stretch of road

B the percentage of students in NSW who were born outside NSW

C the number of vehicles in the car park at the MCG that are not cars

D the percentage of students in your school who have red hair

(b) Which one of these numbers could be obtained using a survey?

A the percentage of people in the crowd at the SCG who travelled by public transport

B the number of historic sites along the Hume Highway between Melbourne and Sydney

C the number of paintings in the MONA outside Hobart

D the number of red cars in the long-term car park at Tullamarine airport

4 For each of the following stem plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to two decimal places. **Ex 8.3**

(a)

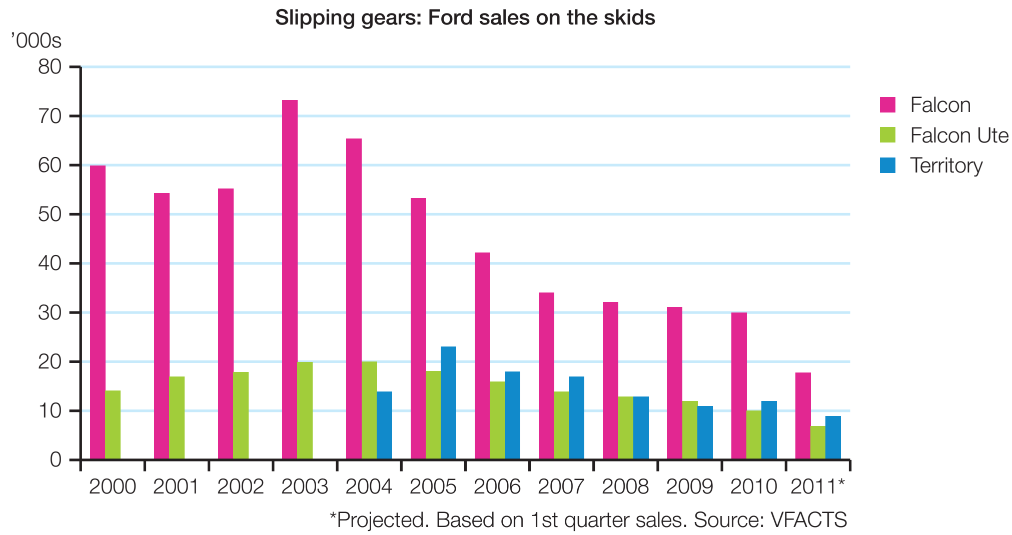
STEM	LEAF
4	0 1
5	4 5
6	2 4 5
7	4 5 6 7 8
8	1 3 5 5 6 6
9	0 0 2 4 5

(b)

STEM	LEAF
23	0 3 7 8 8
24	2 5 6 9
25	3 4 7 8 9
26	3
27	5 6
28	0

5 The following graph appeared in *The Age* (15 April 2011, p. 2).

Ex. 8.4



(a) What has happened to the annual sales of the Falcon since the introduction of the Territory?

(b) Compare the trend in the sales of the Falcon with the sales of the Falcon Ute across the years shown in the graph.

(c) For 2004, estimate the sales of the three vehicle types shown in the graph.

(d) The number of Falcon Utes sold in 2004 was how many percent of the number of Falcons sold in 2004?

(e) The number of Territorys sold in 2004 was how many percent of Falcons sold in 2004?

(f) Repeat **(c)**–**(e)** for 2010.

(g) Make a comment about the percentages you have just calculated.

(h) What could you say about the total sales of these three Ford vehicles?

6 Last year I bought a packet of a particular brand of biscuits every week. Of these, 40 packets contained broken biscuits. Based on these previous results, what is the estimated probability that the next packet I buy will contain broken biscuits?

Ex. 8.5

7 A jar contains five coloured blocks, one of each of the colours red, white, blue, green and gold. In an experiment in which 20 blocks were drawn from the jar, one at a time with replacement, the following results were obtained.

Ex. 8.5

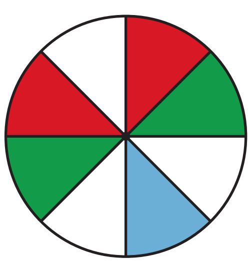
Colour	Red	White	Blue	Green	Gold
Frequency	4	3	1	7	5

(a) What is the relative frequency of drawing out a white block?

(b) What is the relative frequency of drawing out a gold block?

8 A spinner, coloured as shown on the right, is spun once. Find:

Ex. 8.6



(a) Pr(white)

(b) Pr(green or red)

(c) Pr(not white)

(d) Pr(neither green nor red)

(e) Pr(white or green or red)

9 Two normal dice are rolled at the same time.

(a) How many elements are there in the sample space?

(b) Find the probability of the following events:

(i) Pr(total of 5) **(ii)** Pr(total less than 11)

(iii) Pr(at least one even number in the pair) **(iv)** Pr(no prime numbers in the pair)

Ex. 8.7

Understanding

10 An area 20 m × 20 m is pegged out in a nature reserve and the number of various species within the area is counted. The count includes three Coast Wattle, 12 Bidgee Widgee, seven Karkalla, four White Correa, six Running Postman and two Moonah. The nature reserve has a total area of 7.5 hectares. (Remember, 1 ha = 10 000 m^2.)

(a) If the control area is representative of the entire reserve, state the number of each species you would expect to find in the reserve.

(b) Do you think it is reasonable to make this assumption about the relationship between the control area and the entire reserve? What would you do to improve the reliability of your estimate?

11 Do Year 9 girls have longer or shorter arm spans than Year 9 boys?

The following data about a sample of 50 Year 9 Victorian students in 2010 were obtained from the Census at Schools website.

Male arm span (cm):	166	174	163	178	198	145	180	175	176	171
	133	178	165	152	163	191	169	173	178	182
Female arm span (cm):	158	165	166	170	153	157	151	180	168	160
	170	165	155	153	171	164	159	170	169	164
	173	163	156	149	171	129	154	150	166	172

(a) Draw a back-to-back stem plot of the data. Use a class interval of 10.

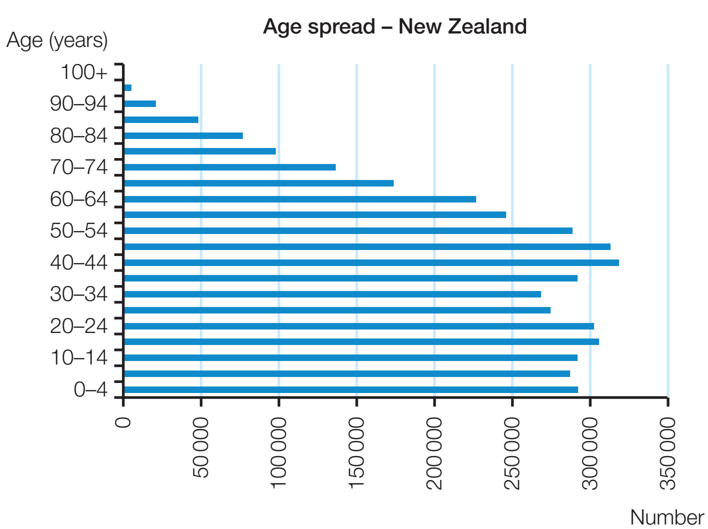
(b) What conclusion would you draw from the stem plot drawn?

(c) Is it a concern that there are more girls than boys in the sample?

(d) Does this mean there are more Year 9 girls than boys in Victorian schools? Explain your reasoning here.

(e) How different would the results be if they were for Year 7 students or Year 11 students? Give some detail in your answer.

12 The following graph shows the age spread of the population of New Zealand.



Describe the age spread of the population of New Zealand using as much detail as you can.

13 The coach of the local junior soccer team has recorded the number of goals his team scores each week in the competition over a number of years.

Assume that the figures in the table are an indication of what can be expected from his team in the future.

Number of goals	Number of games
0	5
1	9
2	7
3	4
4	3
5	1

(a) What is the estimated probability that in the next game the number of goals scored will be:

(i) Five goals **(ii)** Less than two goals

(iii) No more than four goals **(iv)** Either two or four goals?

(Give your answers in fractional form.)

(b) State each of the probabilities from **(a)** as a decimal, correct to two decimal places, and as a whole number percentage.

14 A die is tossed twice. What is the probability of tossing:

(a) A pair of sixes?

(b) Any pair of identical numbers?

(c) Two even numbers?

Reasoning

15 Some newspapers pose a daily question and ask readers to respond either by telephone or by logging onto a website. Recently, the *Herald Sun*, published in Melbourne, posed the question 'Should youth detainees be barred from watching real life stories of crime?' The results were 87.8% Yes and 12.2% No. In smaller type, the actual numbers were displayed: 922 Yes and 128 No.

(a) Do you think you could have predicted the result for this question?

(b) Rephrase the question so it is not so pointed. Do you think the results would be the same if you asked this question?

(c) Do you think this sample size is big enough to draw any conclusions about the broader population?

(d) What would that population actually be—the readers of the *Herald Sun* or the residents of Melbourne, or Victoria, or Australia? Write a sentence or two giving your thoughts about this.

(e) What would you need to do if you wanted to improve the reliability of these results?

16 Two five-sided dice, one green and one blue, are rolled at the same time.

(a) Draw up a table to show the sample space possible.

(b) Find the following probabilities:

(i) Pr(total of 9) **(ii)** Pr(total less than 8)

(iii) Pr(total no more than 5) **(iv)** Pr(at least one prime number)

(v) Pr(difference between numbers less than 3) **(vi)** Pr(at least one odd number)

(vii) Pr(both numbers even) **(viii)** Pr(not a double)

(c) Are the events 'getting a 5 on the green die' and 'getting a 3 on the blue die' mutually exclusive? Explain your answer.

17 Every year *The Economist* publishes what it calls the Big Mac Index. This compares the cost of a Big Mac in various countries around the world using $US as the comparative currency. This index can give the reader a simple view of the relative cost of (some) goods in these various countries. For 2010 the following figures were published.

Country	Big Mac price ($US)	Country	Big Mac price ($US)	Country	Big Mac price ($US)
United States	3.73	Hong Kong	1.90	Russia	2.33
Argentina	3.56	Hungary	3.33	Saudi Arabia	2.67
Australia	3.84	Indonesia	2.51	Singapore	3.08
Brazil	4.91	Israel	3.86	South Africa	2.45
Britain	3.48	Japan	3.67	Sri Lanka	2.82
Canada	4.00	Latvia	2.80	Sweden	1.86
Chile	3.34	Lithuania	2.71	Switzerland	6.56
China	1.95	Malaysia	2.19	South Korea	6.19
Colombia	4.39	Mexico	2.50	Taiwan	2.34
Costa Rica	3.83	New Zealand	3.59	Thailand	2.17
Czech Republic	3.43	Norway	7.20	Turkey	3.89
Denmark	4.90	Pakistan	2.46	UAE	2.99
Eqypt	2.28	Peru	3.54	Ukraine	1.84
Estonia	2.62	Philippines	2.19	Uruguay	1.74
Euro area	4.33	Poland	2.60		

(a) Which country has the lowest price and what is that price?

(b) Which country has the highest price and what is it?

(c) What is the range of prices?

(d) You want to draw a histogram showing this data. Why is a price range of 50 cents appropriate in this case?

(e) Draw the histogram, using $1.50–< $2.00 as the first interval.

The mean of the data is 3.336 and the median is 3.205.

(f) Take a random sample of five prices and find the mean and median of those five prices.

(g) Take another random sample of five prices and find the mean and median of those five prices.

(h) Take two different random samples of 10 prices and find the mean and median for each group of 10.

(i) Which size sample would you expect to do a better job of representing the whole list? Did your sampling process confirm this?

NAPLAN practice 8

Numeracy: Non-calculator

1 The probability of rolling a 3 on a standard six-sided die is:

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{6}$ D $\frac{3}{4}$

2 For each of the following data sets, state whether it is discrete or continuous.

(a) the number of potatoes in each 5-kg bag at the supermarket

(b) the wrist circumference of each player on the list of the AFL side Greater Western Sydney

(c) the time taken by each competitor to complete a particular stage of the *Tour de France*

(d) the number of brothers and sisters for each of the students in your class

Numeracy: Calculator allowed

Questions **3** and **4** refer to the following frequency table.

Value	2	3	4	5	6	7	8	9
Frequency	4	5	2	1	8	5	7	2

3 The median of the data is:

A 4 B 5.5 C 6 D 8

4 The interquartile range of the data is:

A 3 B 5 C 6 D 8

5 The spellcheck on Bob's computer goes crazy and rearranges the letters in a word in a random order. If the letters in the word FRIEND are rearranged randomly, what is the probability that the new arrangement ends with either an i or an e?

A $\frac{1}{3}$ B $\frac{2}{5}$ C $\frac{2}{4}$ D $\frac{2}{3}$

6 Ros has a 70% chance of passing her English exam and an 80% chance of passing mathematics. The probability that Ros will pass both exams is:

A 1.5 B 0.56 C 0.44 D 0.06

7 This list shows the number of films that nine members of a film club watched in April.

Number of films watched	0, 1, 2, 2, 3, 4, 5, 5, 5

Which of the following is true for this data?

A Mean > median = mode B Mean < median < mode

C Mean = median = mode D Mean = median < mode

9

Non-linear relationships and proportion 9

'The mathematics of fear.' Roller coasters are designed to give thrills with no spills, so it is very important to get the maths right!

The very first roller coasters were probably slides built in the 16th century in Russia. By 1817 there were two roller coasters in France that had cars locked to the track similar to those in operation today.

Cyclone, a high-speed gravity roller coaster, opened in Dreamworld in December 2001, and it is still the tallest high-speed gravity roller coaster in the Southern Hemisphere. It reaches speeds of up to 85 km/h. Roller coasters are mathematically marvellous machines. One of the curves used in the design of roller coasters is the parabola.

A parabola is the graph of a type of equation known as a quadratic!

Forum

How do you think the speed of Cyclone is generated? What effect do you think there would be on the speed or fear factor if the drop was made steeper? Is there a limit to how steep the drop could be? Why? Would more people in the carriage increase or decrease the speed?

Why learn this?

Linear equations in the form of $y = mx + b$ are only one way variables can be related. Non-linear equations have variables that are related in other ways that do not graph as straight lines. Recognising the shape of the graphs of these different types of relationships, knowing how to transform them and solving non-linear equations mean we can now use mathematics to solve a greater range of problems. The shape of a quadratic is a symmetric curve called a parabola and its shape is the path of a ball thrown over a distance or any other object (projectile) such as a bullet, missile or arrow when it is fired. Circles appear everywhere in everyday life: in cylinders, in the design of kitchenware (pots, pans, cups, bowls), and in buildings and infrastructure. Growth and decay are modelled with exponential relationships. Hyperbolas are used to graph the relationship between many variables, such as pressure and volume, that are related inversely. As one increases, the other decreases. Non-linear relationships are used extensively in physics to explore and interpret the world and how things work.

After completing this chapter you will be able to:

- plot and sketch graphs showing non-linear relationships between two variables
- recognise the equations for parabolas, circles, exponentials and hyperbolas
- use algebra to solve some types of quadratics algebraically
- understand how the solution to non-linear equations is related to the graph of the relationship
- solve real-life applications of non-linear relationships, taking constraints into account
- solve problems involving direct and inverse proportion.

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

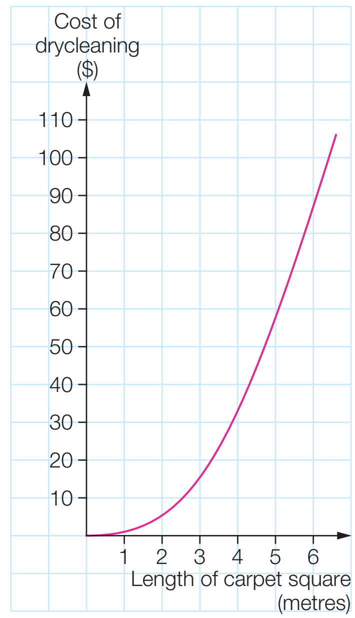
1 **(a)** If $p = 2m^2 - 3m + 5$, find the value of p when:

(i) $m = 1$ **(ii)** $m = 0$ **(iii)** $m = -1$

(b) Find the value of $x(x - 3)$ when:

(i) $x = 5$ **(ii)** $x = -3$ **(iii)** $x = 0$ **(iv)** $x = 3$

2 Consider this graph showing the cost charged to dryclean carpet squares of different lengths.



(a) Find the approximate cost to dryclean a carpet square with a length of:

(i) 3 m **(ii)** 5.5 m

(b) Find the approximate length of the carpet square if the drycleaning cost is:

(i) \$48.00 **(ii)** \$18.75

3 Expand the following.

(a) $3x(x + 4)$ **(b)** $(x + 2)(x + 5)$

(c) $(x - 7)(x + 3)$ **(d)** $(x - 5)(x + 5)$

4 Factorise the following.

(a) $x^2 + 2x$ **(b)** $x^2 - 16$ **(c)** $x^2 + 12x + 36$

5 Factorise the following.

(a) $x^2 + 2x + 5x + 10$ **(b)** $x^2 - 2x - 3x + 6$

(c) $x^2 - 14x + x - 14$ **(d)** $x^2 - 7x - 4x + 28$

6 Solve for x.

(a) $3x + 5 = 7$ **(b)** $5x - 2 = 11$ **(c)** $6x - 5 = -11$

Key Words

asymptote	dilation factor	parabola	translation
axis of symmetry	direct proportion	quadratic equation	turning point
circle	exponential	quadratic relationship	vertex
coefficient	hyperbola	rectangular hyperbola	
constant of proportionality	inverse proportion	reflection	
dilation	Null Factor Law	transformation	

Quadratic relationships

9.1

Up until now, the only relationship between two variables that you have studied has been linear. Linear relationships graph as straight lines and a linear equation is written in the form $y = mx + b$, where m is the gradient and b is the y-intercept of the graph.

$y = 3x - 4$ and $y = -2x + 7$ are examples of linear equations.

However, not all relationships between two variables are linear. In this chapter we are going to explore some of these non-linear relationships.

A **quadratic relationship** is a very common type of non-linear relationship. The graph of a quadratic relationship is a special shape called a **parabola** and a **quadratic equation** is written in the form of $y = ax^2 + bx + c$.

$y = x^2 + 3x - 4$ and $y = -2x^2 + 9$ are examples of quadratic equations where x is the independent and y is the dependent variable.

All quadratic equations have a term where the highest power of the independent variable is 2, the dependent variable has a highest power of 1, and all powers are integers.

> The general equation of a quadration equation is given by
> $y = ax^2 + bx + c$, where $a \neq 0$

Worked Example 1

State which of the following are quadratic relationships.

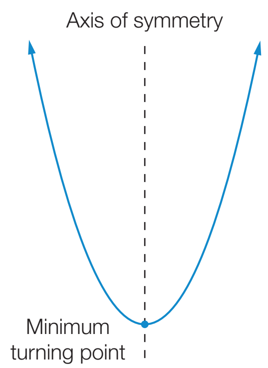
(a) $g = 8f + 6f^2$ **(b)** $p^2 = r - 3r^2$ **(c)** $d = 4b^3 - b^2 + 5$

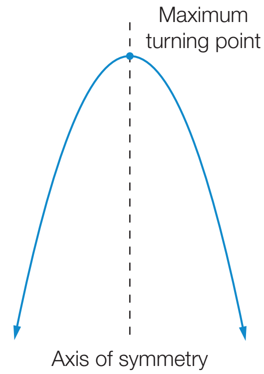
Thinking	Working
(a) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic	(a) The highest power of f is 2 and the highest power of g is 1. This is a quadratic relationship.
(b) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic.	(b) The highest power of r is 2 and the highest power of p is 2. This is not a quadratic relationship.
(c) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic.	(c) The highest power of b is 3 and the highest power of d is 1. This is not a quadratic relationship.

Key features of a parabola

Axis of symmetry and turning point

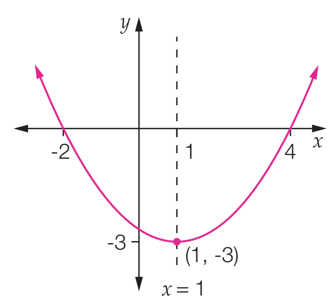
A parabola can be drawn as  or . Each parabola has an axis of symmetry and a turning point.





An **axis of symmetry** is an imaginary line that divides the parabola into two mirror-image or symmetrical halves. The **turning point** or **vertex** is where the graph changes direction. A minimum turning point occurs where the graph changes direction at the lowest point (the lowest y-value) of the graph. A maximum turning point occurs where the graph changes direction at the highest point (the highest y-value) of the graph.

The parabola shown on the right has a minimum turning point at (1, -3) and an axis of symmetry at $x = 1$.



The graph of a quadratic can be plotted using the following steps:

1. Complete a table of values.
2. Draw a set of appropriately sized and scaled axes.
3. Plot the points from the table.
4. Join the points with a smooth curve.
5. Label the graph, the turning point and the axis of symmetry.

Although the graph of a quadratic is always a parabola, graphs that have the same axis of symmetry and turning point as $y = x^2$ can be thinner or wider than $y = x^2$ and can have a maximum turning point instead of the minimum turning point of $y = x^2$.

Worked Example 2

WE2

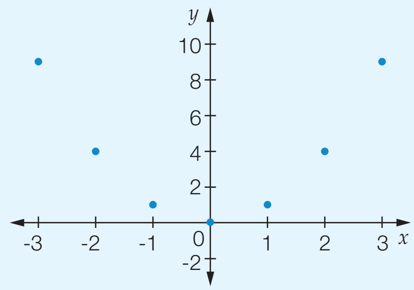
Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = x^2$ **(b)** $y = 2x^2$ **(c)** $y = -x^2$

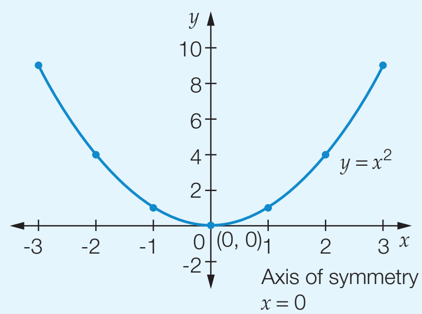
Thinking	Working
(a) 1 Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.	(a) $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

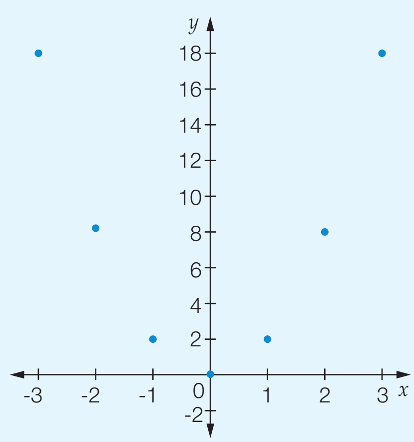


(b) 1 Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

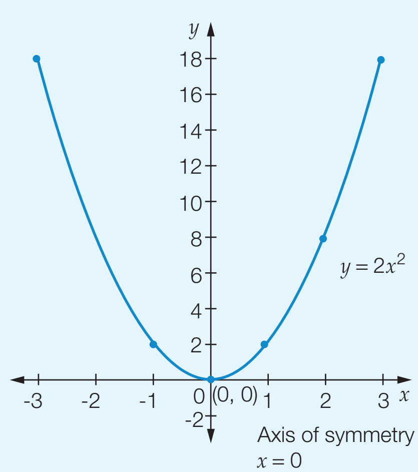
(b) $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

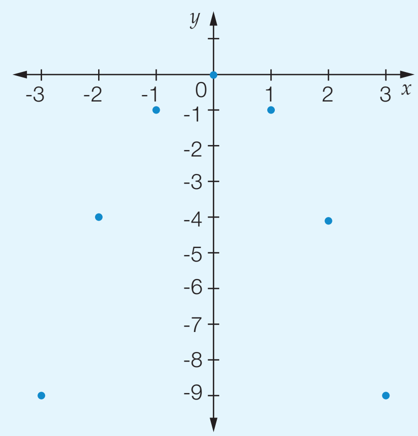


(c) **1** Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

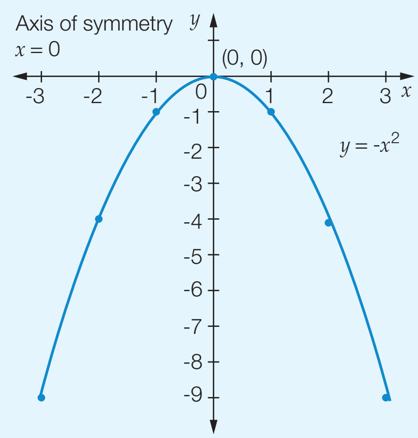
(c) $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.



Not all parabolas have a turning point at the origin, and not all parabolas are symmetrical around the y-axis.

Worked Example 3

WE3

Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = 2x^2 - 3$ **b)** $y = 2 - x^2$ **c)** $y = x^2 - 2x - 3$

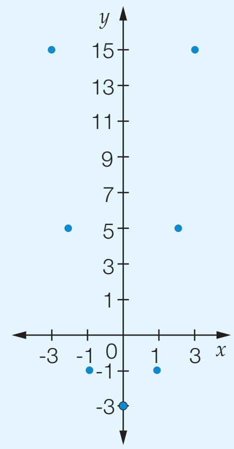
Thinking **Working**

(a) **1** Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

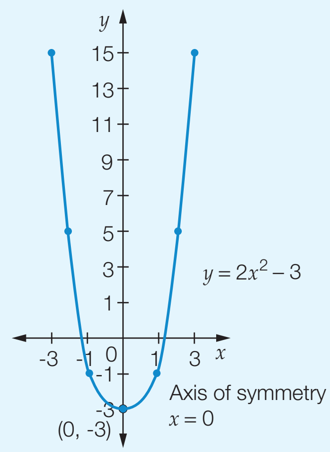
(a) $y = 2x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	15	5	-1	-3	-1	5	15

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

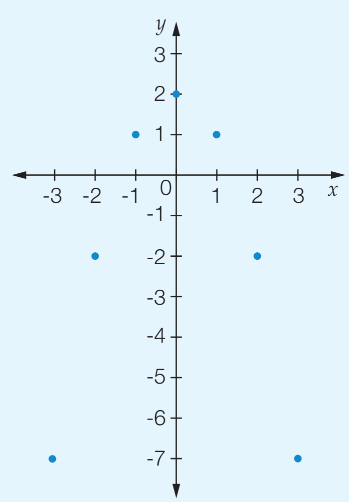


(b) 1 Draw a table of values using the x-values in the question, (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

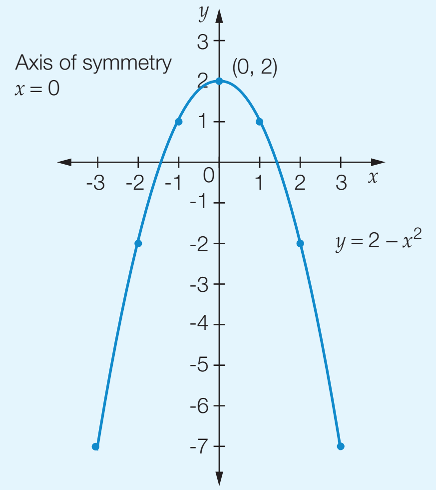
(b) $y = 2 - x^2$

x	-3	-2	-1	0	1	2	3
y	-7	-2	1	2	1	-2	-7

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the asix of symmetry.

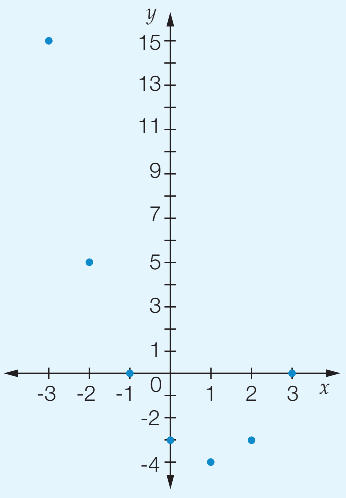


(c) 1 Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

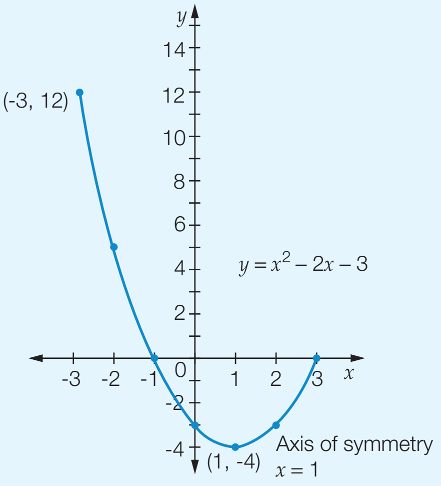
(c) $y = x^2 - 2x - 3$

x	-3	-2	-1	0	1	2	3
y	12	5	0	-3	-4	-3	0

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



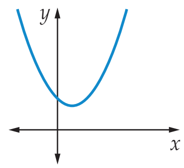
3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.



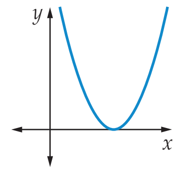
x- and y-intercepts of parabolas

All quadratic graphs will intersect the y-axis. This point is known as the y-intercept.

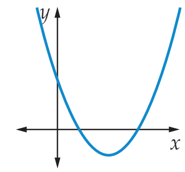
Many quadratic graphs never intersect the x-axis. Some quadratic graphs only touch the x-axis at one point. All other quadratics intersect the x-axis at two points.



No x-intercepts



1 x-intercept



2 x-intercepts

All quadratics have a y-intercept.

A quadratic graph may have one, two or no x-intercepts.

Worked Example 4

WE4

For the graph shown, write down:

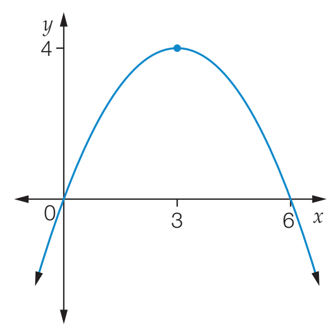
(a) the coordinates of the turning point

(b) the type of turning point

(c) the equation of the axis of symmetry

(d) the y-intercept

(e) the x-intercepts.



Thinking	Working
(a) Identify the point where the graph changes direction. This is the turning point.	(a) There is a turning point at (3, 4).
(b) Determine whether the turning point is at the maximum (or greatest) value of y or the minimum (or least) value of y.	(b) Because $y = 4$ is the maximum value of y, then this is a maximum turning point.
(c) Find the line that divides the graphs into symmetrical halves. Notice that the axis of symmetry goes through the turning point.	(c) The axis of symmetry is the line $x = 3$.
(d) Find the point where the graph cuts the y-axis (y-intercept).	(d) The y-intercept is (0, 0).
(e) Find the points where the graph cuts the x-axis (x-intercepts).	(e) The x-intercepts are (0, 0) and (6, 0).

Worked Example 5

WE 5

For the equation:

(a) $y = x^2 - 4x + 5$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = x^2 - 4x + 3$

(i) Complete the table of values.

x	-2	-1	0	1	2	3	4	5
y								
(x, y)								

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write down the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write down the equation of the axis of symmetry of the parabola.

(vi) Write down the coordinates of the y-intercept.

(vii) Write down the coordinates of the x-intercepts.

(viii) From the graph, find the values of x when $y = 8$. Hence, state the values of x when the given equation is equal to 8. (Answer correct to one decimal place.)

Thinking / Working

(a) (i) Complete the table of values by substituting values. For example, when $x = -2$:

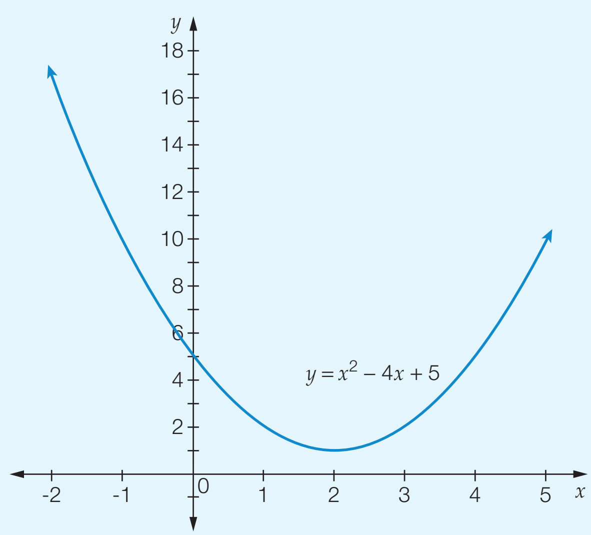
$y = (-2)^2 - 4 \times (-2) + 5$
$= 4 + 8 + 5$
$= 17$

(a) (i) $y = x^2 - 4x + 5$

x	-2	-1	0	1	2	3	4	5
y	17	10	5	2	1	2	5	10
(x, y)	(-2, 17)	(-1, 10)	(0, 5)	(1, 2)	(2, 1)	(3, 2)	(4, 5)	(5, 10)

(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



(iii) Identify the point where the graph changes direction.

(iii) The turning point is (2, 1).

(iv) Determine whether the turning point is where the maximum or the minimum value of y occurs.

(iv) $y = 1$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is a vertical line passing through the turning point.

(v) The axis of symmetry is $x = 2$.

(vi) Identify the point where the graph cuts the y-axis.

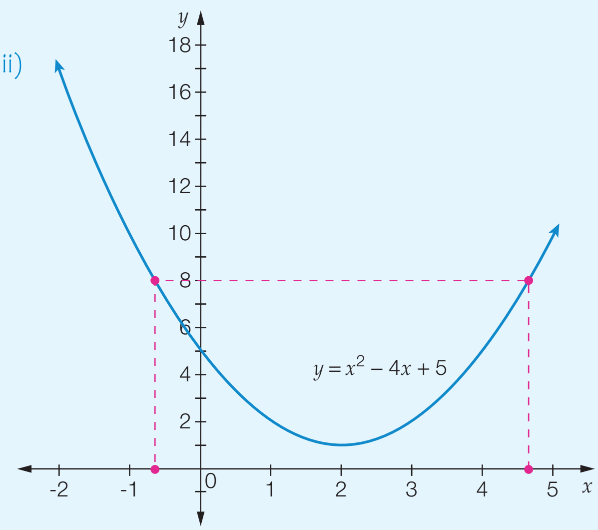
(vi) The y-intercept is (0, 5).

(vii) Identify the points, if any, where the graph cuts the x-axis.

(vii) There are no x-intercepts.

(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. Read off the corresponding values of x.

(viii)



The graph shows $x \approx 4.6$ and -0.6 when $y = 8$.

So, $x^2 - 4x + 5 = 8$ when $x \approx 4.6$ and -0.6.

(b) **(i)** Complete the table of values by substituting values. For example, when $x = -2$:

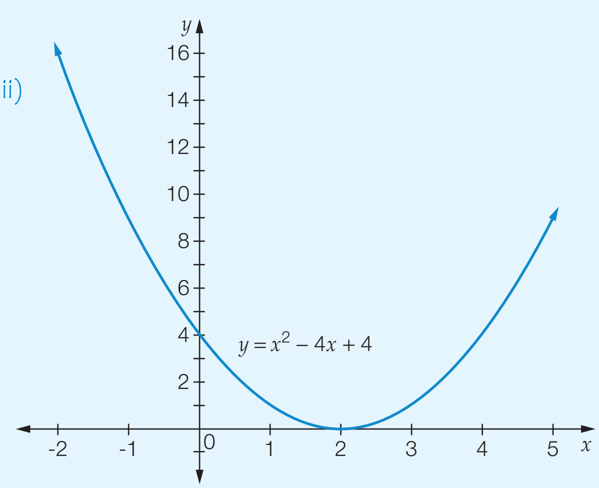
$$y = (-2)^2 - 4 \times (-2) + 4$$
$$= 4 + 8 + 4$$
$$= 16$$

(b) (i) $y = x^2 - 4x + 4$

x	-2	-1	0	1	2	3	4	5
y	16	9	4	2	01	1	4	9
(x, y)	(-2, 16)	(-1, 9)	(0, 4)	(1, 1)	(2, 0)	(3, 1)	(4, 4)	(5, 9)

(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



(iii) Identify the point where the graph changes direction.

(iii) The turning point is (2, 0).

(iv) Determine whether the turning point is where the maximum or minimum value y occurs.

(iv) $y = 0$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is a vertical line passing through the turning point.

(v) The axis of symmetry is $x = 2$.

(vi) Identify the point where the graph cuts the y-axis.

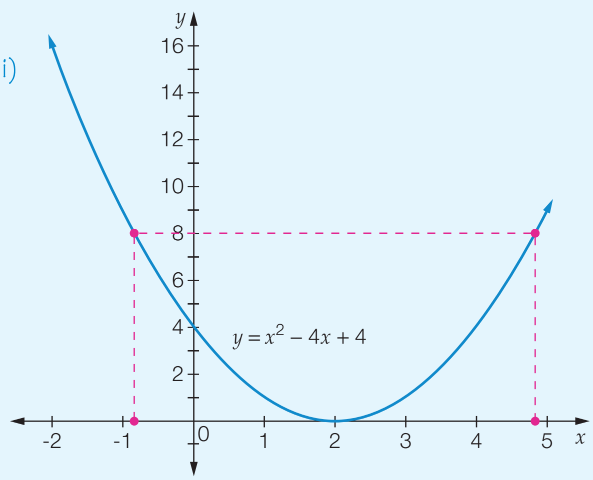
(vi) The y-intercept is (0, 4).

(vii) Identify the point, if any, where the graph cuts the x-axis.

(vii) There is one x-intercept at (2, 0).

(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. Read off the corresponding values of x.

(viii)



The graph shows $x \approx 4.8$ and -0.8 when $y = 8$.

So, $x^2 - 4x + 4 = 8$ when $x \approx 4.8$ and -0.8.

(c) (i) Complete the table of values by substituting values. For example, when $x = -2$:

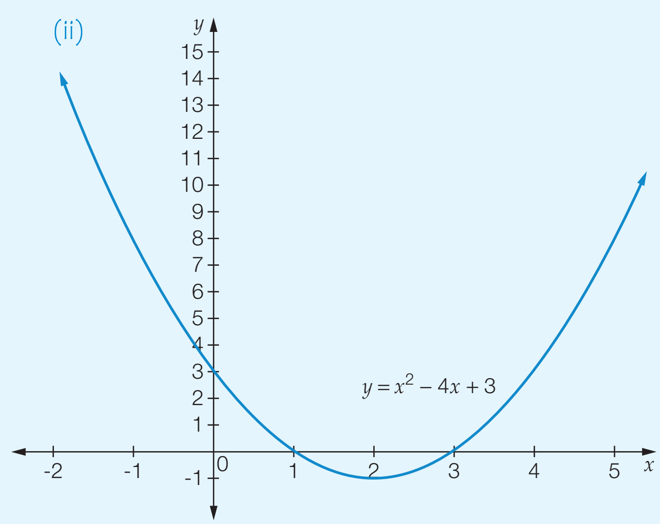
$$y = (-2)^2 - 4 \times (-2) + 3$$
$$= 4 + 8 + 3$$
$$= 15$$

(c) (i) $y = x^2 - 4x + 3$

x	-2	-1	0	1	2	3	4	5
y	15	8	3	0	-1	0	3	8
(x, y)	(-2, 15)	(-1, 8)	(0, 3)	(1, 0)	(2, -1)	(3, 0)	(4, 3)	(5, 8)

(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



(iii) Identify the point where the graph changes direction.

(iii) The turning point is (2, -1).

(iv) Determine whether the turning point is where the maximum value of y occurs, or where its minimum value occurs.

(iv) $y = -1$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is a vertical line passing through the turning point.

(v) The axis of symmetry is $x = 2$.

(vi) Identify the point where the graph cuts the y-axis.

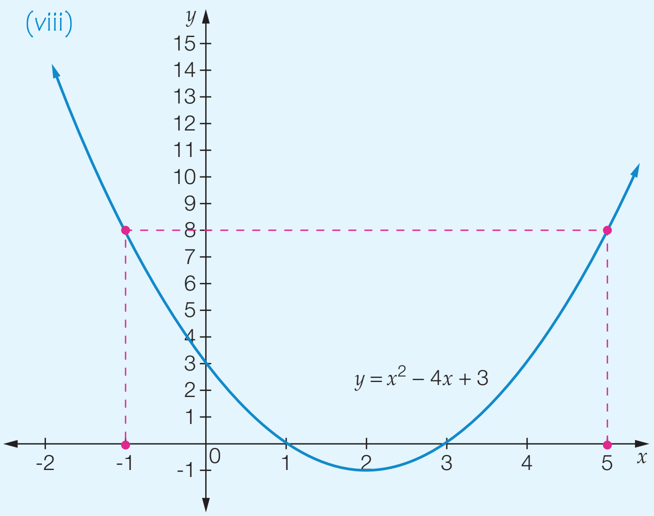
(vi) The y-intercept is (0, 3).

(vii) Identify the points where the graph cuts the x-axis.

(vii) There are two x-intercepts, one at (1, 0) and one at (3, 0).

(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. Read off the corresponding values of x.

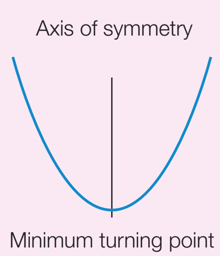
(viii)

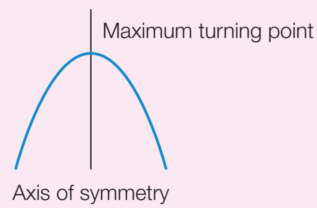


The graph shows $x = -1$ and 5 when $y = 8$.
So, $x^2 - 4x + 3 = 8$ when $x = -1$ and 5.

Features of the parabola:

- Every parabola is symmetrical about the axis of symmetry, which is a vertical line with an equation of the form $x = a$ where a is a constant.
- Every parabola has a vertex or turning point, which is where the graph changes direction.
- A parabola can have a minimum turning point or a maximum turning point.
- All parabolas have a y-intercept.
- A parabola can have zero, one or two x-intercepts.





9.1 Quadratic relationships

Navigator

Answers page 656

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q21, Q22, Q24	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q19, Q20, Q22, Q23	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q17, Q18, Q19, Q20, Q21, Q22, Q23

Equipment required: graph paper for most questions

Fluency

WE1

1 State which of the following are quadratic relationships.

(a) $m = 4k^2 - 6$ (b) $a = -b^2$ (c) $s = 4t^4 - 5t^2 + 7$

(d) $r = 4w - 3$ (e) $l = 2 - 3n^2$ (f) $a^2 = 9 - x^2$

WE2

2 Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = 3x^2$ (b) $y = -2x^2$ (c) $y = \frac{x^2}{2}$

WE3

3 Plot the graph of the following quadratic equations for integer values of x between -4 and 4 inclusive.

(a) $y = 3x^2 - 3$ (b) $y = -2x^2 + 3$ (c) $y = \frac{x^2}{2} - 2$

(d) $y = 3x^2 - 6x$ (e) $y = -2x^2 + 5x + 3$ (f) $y = \frac{x^2}{2} - \frac{3x}{2} - 2$

WE4

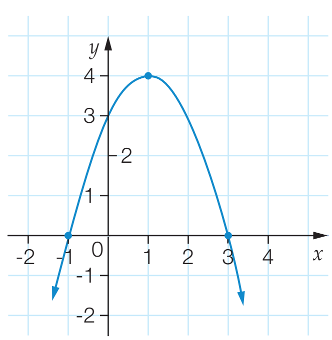
4 For each of the following graphs, write down:

(i) the coordinates of the turning point (ii) the type of turning point

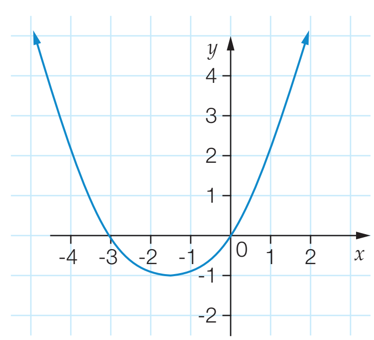
(iii) the equation of the axis of symmetry (iv) the y-intercept

(v) the x-intercepts.

(a)



(b)



WE5

5 For each of the following equations:

(a) $y = x^2 - 2x + 2$ (b) $y = x^2 - 2x - 3$ (c) $y = x^2 - 2x + 1$

x	-2	-1	0	1	2	3	4	5
y								

(i) Copy and complete the table of values.

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write down the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write down the equation of the axis of symmetry of the parabola.

(vi) Write down the coordinates of the y-intercept.

(vii) Write down the coordinates of the x-intercepts.

(viii) From the graphs, find the values of x when $y = 5$. Hence, state the values of x when the given equation is equal to 5.

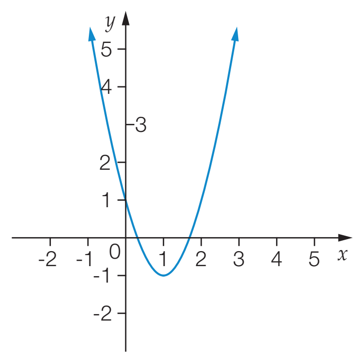
6 (a) Which of the following does not show a quadratic relationship?

A $v = w - 4w^2$ B $f^2 = 3d^2 - 1$ C $p = 6r^2 - 7$ D $k^2 = t$

(b) If $p = 3m^2 + 4$, then if m equals 5, p will be equal to:

A 19 B 34 C 75 D 79

7 The graph of $y = 2x^2 - 4x + 1$ is shown.



(a) For this graph the turning point would be:

A (1, 0) B (1, 1)

C (1, -1) D (-1, 1)

(b) The value(s) of x for $2x^2 - 4x + 1 = 0$ would be approximately:

A -0.8 and 1.8 B 1 and 0

C 0.3 and 1.7 D 0 and 2

(c) The value(s) of x for $2x^2 - 4x + 1 = 1$ would be approximately:

A 0 and 2 B 0.3 and 1.7 C 0 and 1 D -0.8 and 1.8

Understanding

8 Compare your graphs in Question **2 (a)–(c)** to your graphs in Questions **3 (a)–(c)**.

Comment on any similarities and differences.

9 Compare your graphs in Question **2 (a)–(c)** to your graphs in Questions **3 (d)–(f)**.

Comment on any similarities and differences.

10 (a) Complete the following table of values for $y = x^2 - 4$. (The first column has been completed for you.)

x	-4	-3	-2	-1	0	1	2	3	4
y	12								
(x, y)	(-4, 12)								

(b) Plot the points onto a number plane. Join them with a smooth curve.

(c) What are the coordinates of the turning point?

(d) Does the parabola have a maximum or a minimum turning point?

(e) Write down the equation of the axis of symmetry of the parabola.

(f) Write down the coordinates of the y-intercept.

(g) Write down the coordinates of the x-intercepts.

(h) From the graph, find the value of y when $x = 3.5$ (correct to one decimal place).

(i) From the graph, find the values of x when $y = -2$ (correct to one decimal place).

(j) State the values of x when $x^2 - 4 = 5$. Hence, find the values of x when $x^2 - 9 = 0$.

11 **(a)** Complete the following table of values for $y = -2x^2 - 6x$.

x	-5	-4	-3	-2	-1	0	1	2
x^2								

(b) Plot the graph of $y = -2x^2 - 6x$ using graph paper.

(c) Write down the coordinates of the turning point. State whether the parabola has a maximum or a minimum turning point.

(d) Write down the coordinates of any x- and y-intercepts.

(e) What are the x-values when $y = 4$?

(f) If $y = 4$, what does $-2x^2 - 6x$ equal? Hence, state the x-values when $-2x^2 - 6x = 4$. These x-values are the solution to the equation $-2x^2 - 6x = 4$.

(g) Use the graph to find the x-values that solve the equation $-2x^2 - 6x = -8$.

(h) Find the x-values that solve $-2x^2 - 6x = 0$.

12 **(a)** Complete a table of values for $y = x^2 - x - 2$ for integer values of x between -3 and 5.

(b) Plot the graph of $y = x^2 - x - 2$ for the points in your table.

(c) Write down the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(d) Write down the coordinates of any x- and y-intercepts.

(e) Use the parabola to solve x in each of the following equations.

(i) $x^2 - x - 2 = 0$ **(ii)** $x^2 - x - 2 = 4$ **(iii)** $x^2 - x - 2 = 2$

(iv) $x^2 - x - 2 = -2.25$ **(v)** $x^2 - x - 2 = -4$ **(vi)** $x^2 - x - 2 = 1\frac{1}{2}$

13 **(a)** Plot the graph of $y = x^2 + 6x + 10$. Use integer values of x between -7 and 1.

(b) What are the coordinates of the turning point?

(c) Write down the coordinates of any x- and y-intercepts.

(d) Use the graph to solve each of the following equations.

(i) $x^2 + 6x + 10 = 1$ **(ii)** $x^2 + 6x + 10 = 10$

(iii) $x^2 + 6x + 10 = 0$ **(iv)** $x^2 + 6x + 10 = 15$

14 **(a)** Plot the graphs of the following parabolas on separate axes on graph paper. (Remember to complete a table of values first.)

(i) $y = -x^2 + 4x$ for $-1 \le x \le 5$

(ii) $y = x^2 + 3x - 10$ for $-6 \le x \le 3$

(iii) $y = -x^2 + 2x + 3$ for $-2 \le x \le 4$

(b) Use your graphs to solve the following equations to find x.

(i) $-x^2 + 4x = 0$ **(ii)** $x^2 + 3x - 10 = 0$ **(iii)** $-x^2 + 2x + 3 = 4$

(iv) $-x^2 + 4x = 4$ **(v)** $x^2 + 3x - 10 = 8$ **(vi)** $-x^2 + 2x + 3 = 0$

(vii) $-x^2 + 4x = 2$ **(viii)** $x^2 + 3x - 10 = 4$ **(ix)** $-x^2 + 2x + 3 = -2$

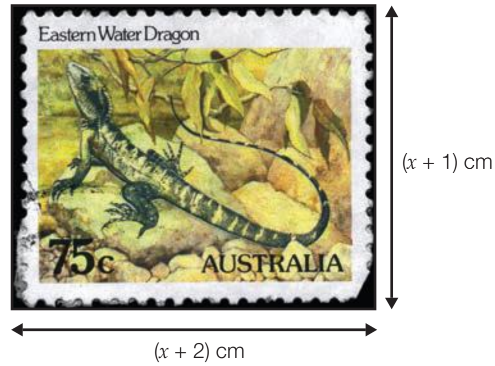
Reasoning

15 A postage stamp has a length of $(x + 2)$ cm and a width of $(x + 1)$ cm.

(a) Write an equation for the area, h, of the postage stamp in expanded form.

(b) Complete a table of values for x and A.

(c) Plot a graph of your equation showing x on the horizontal axis (use 1 cm = 1 cm) and A on the vertical axis (use 1 cm = 1 cm^2).



Hint
Use only realistic values of x for your table of values.

(d) Use the graph to find the value x represents if the area of the stamp is 8 cm^2. Did you obtain one or two values for x in this problem? Explain.

(e) What is the length and width of the stamp if its area is 8 cm^2? (Give your answers correct to one decimal place.)

16 Karrie, a keen golfer, hits a golf ball along the fairway. The height of the golf ball, h, in metres, at any time, t, in seconds, during its flight is given by the equation $h = -5t^2 + 20t$.

(a) Complete a table of values for t and h.

(b) Plot the graph of $h = -5t^2 + 20t$. Show the values of t on the horizontal axis (use 1 cm = 0.25 s) and values for h on the vertical axis (use 1 cm = 2 m).

Use the graph to answer the following questions.

(c) How high does the ball get above the ground?

(d) How long does it take to reach this maximum height?

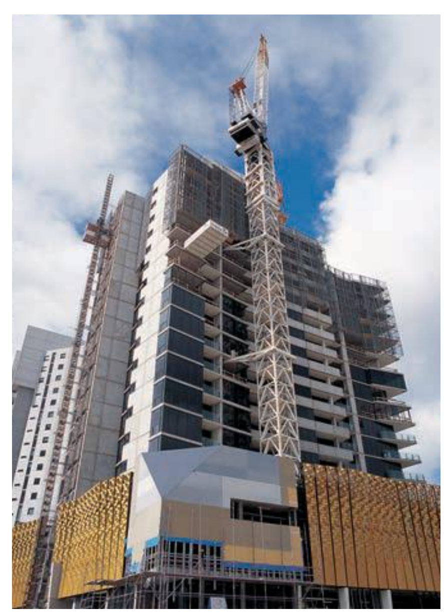
(e) When does the ball first reach a height of 12 m?

(f) How long does it take before the ball hits the ground?

(g) For how long was the ball at least 8 m above the ground?

17 A worker's vacuum flask was accidentally dropped from the top of a multistorey building. The relationship between the height, h, above the ground of the flask (in metres) and the time, t, for it to fall to that height (in seconds) was found to be:

$h = 125 - 5t^2$



(a) Complete a table of values for t and h.

(b) Plot the graph of $h = 125 - 5t^2$. Show values of t on the horizontal axis (use 1 cm = 1 s) and values for the height h on the vertical axis (use 1 cm = 20 m).

(c) What is the height of the building?

(d) How long did it take for the flask to hit the ground?

(e) How high above the ground was the flask after 2.5 s? (Answer correct to the nearest metre.)

(f) What distance, to the nearest metre, had the flask fallen after 2.5 s?

(g) How long did it take for the flask to fall 50 m? (Answer correct to one decimal place.)

(h) How long did it take for the flask to fall 100 m? (Answer correct to one decimal place.)

(i) What distance did the flask fall during the fourth second?

(j) During which second did the flask fall the greatest distance?

Hint
The first second occurs between $t = 0$ and $t = 1$.

18 The speed of a car was tracked by radar in a police helicopter over a period of 10 minutes. The police found that the car's speed could be represented by the equation, $s = 3t^2 - 30t + 135$, where s is the speed of the car in km/h after t minutes have passed.

(a) Plot a graph representing this information, showing values of t on the horizontal axis. (Use 1 cm = 2 minutes) and values of s on the vertical axis (use 1 cm = 20 km/h).

Use the graph to answer the following questions.

(b) What was the maximum speed recorded for the car?

(c) What was the minimum speed recorded for the car?

(d) What was the speed of the car after 3 minutes?

(e) What was the speed of the car after 7 minutes?

(f) When was the car travelling at 80 km/h?

(g) When was the car travelling at 120 km/h?

(h) The speed limit along this section of road is 100 km/h. In which time intervals was the car driven above the speed limit?

19 The entrance to a one-way railway tunnel forms an arch. At any particular distance, d, across the base of the tunnel, starting from the left, the height, h, of the arch can be found using the equation $h = -d^2 + 6d$. Both h and d are in metres.

(a) Plot the graph of $h = -d^2 + 6d$. (Show d on the horizontal axis.)

Use the graph to answer the following questions.

(b) How wide is the base of the tunnel?

(c) What is the maximum height of the tunnel?

(d) How high is the tunnel 1 m along the base from either side of the tunnel?

For a train to pass safely through the tunnel there must be a minimum vertical clearance of 1 m between the train and the arch. Assume the train has a rectangular cross-section. Use this information to answer the following questions.

(e) If a particular train happened to be 4 m wide, what would be its maximum allowable height.

(f) What is the maximum allowable height of a train that is 3 m wide?

(g) Find the maximum width of a train if it just fits through the tunnel and is 4.8 m high.

(h) Decide whether trains of the following dimensions would fit through the tunnel.

(i) width 2.6 m and height 6 m **(ii)** width 3.8 m and height 4.5 m.

20 The volume of liquid in a container can be found using the equation $V = 3t^2 + 4t$, where V is in cm^3 and t is the number of seconds after pouring commences.

(a) Plot the graph of $V = 3t^2 + 4t$. (Use values of t from 0 to 7 and show t on the horizontal axis.)

Use your graph to answer the following questions.

(b) Find the amount of liquid in the container at the start.

(c) Find the amount of liquid in the container after 4 minutes.

(d) The container can hold only 132 cm^3. How long, in seconds, does it take to fill the container?

(e) How much liquid is added in the first 2 seconds?

(f) What are the feasible values for time and for volume?

Open-ended

21 (a) Write down at least three expressions that are quadratic expressions.

(b) Write down at least three expressions that are not quadratic expressions.

22 Make up at least two quadratic equations. For each equation:

(a) Draw up a table of values.

(b) Plot the graph.

(c) Find the turning point.

(d) Find the value of any x- and y-intercepts.

23 (a) If the length of a rectangle is 3 cm longer than its width, write down the area of the rectangle.

(b) Draw a graph of this relationship.

(c) Choose a value for the area of the rectangle and use your graph to find the dimensions of your rectangle correct to two decimal places

24 Josh is having some trouble filling in a table of values for a quadratic equation. Explain what he is doing wrong.

(a) $y = -x^2 + 2x$

x	-3	-2	-1	0
y	3	0	-1	0

(b) $y = 2x^2 + 1$

x	-2	-1	0	1	2
y	17	5	1	5	17

Outside the Square Puzzle

Hitori

Copy and complete the following grid by colouring in cells according to the rules given below.

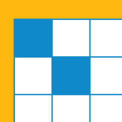
5	2	2	3	4
5	6	1	2	2
5	5	4	5	1
6	3	1	1	5
5	1	3	4	2

Rules:

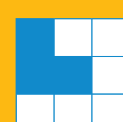
1 Each number must not be used more than once in each row and column. When there is more than one number of a particular value (e.g. there are two 4s), all but one must be coloured in.

2 No cell that has been coloured in can have another coloured-in cell adjacent to it except diagonally (i.e. the coloured cells cannot be touching horizontally or vertically, but diagonally is OK).

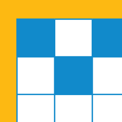
3 The white cells must all be joined or continuous.



Correct



Incorrect (adjacent cells coloured)

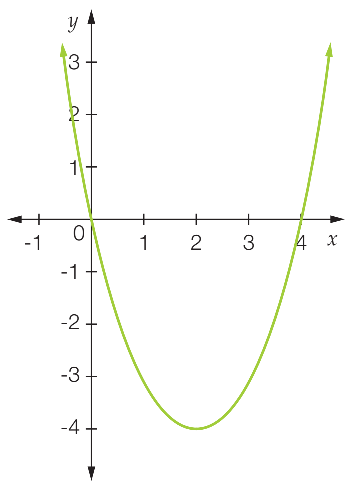


Incorrect (uncoloured cells don't form a single area)

It may be helpful to circle numbers that are to be included (e.g. numbers directly next to coloured cells).

9.2 Solving quadratic equations

To solve an equation means to find the values that make the equation true. We can find solutions to quadratic equations such as $x^2 - 4x = 0$ by locating the points where the graph of $y = x^2 - 4x$ intersects the graph of $y = 0$ (the x-axis).



The x-intercepts of $y = x^2 - 4x$ are 0 and 4.
So, the solution to $x^2 - 4x = 0$ is $x = 0$ or 4.

Graphical methods of solution are time-consuming and can be inaccurate if the solutions involve fractions or surds. We need to be able to solve quadratic equations algebraically.

We use the Null Factor Law to solve these equations.

The Null Factor Law

If the product of two or more numbers is zero, then at least one of the numbers must be zero.

If $ab = 0$, then $a = 0$, $b = 0$ or both a and $b = 0$.

To use the Null Factor Law to solve quadratic equations, two requirements need to be met.

1 One side of the equation must equal zero.
2 The other side must be the product of factors. (Factorisation may be necessary to achieve this.)

When quadratic equations were plotted in the previous section, all of the parabolas had a y-intercept. Some parabolas had no x-intercepts, some had one x-intercept, and the rest had two x-intercepts. The x-intercepts occur when the graph crosses the x-axis, when $y = 0$. We can read off these values from a graph, but we need to be able to find these values algebraically. We can do this by solving the quadratic equation when it equals zero.

We use the Null Factor Law to obtain solutions without having to draw graphs.

For linear equations there is only one x-intercept, so there is only one solution when $y = 0$.

$2x = 0$	or	$2(x - 4) = 0$
$x = 0$		$x - 4 = 0$
		$x = 4$

Because there can be zero, one or two x-intercepts, there can be zero, one or two solutions when $y = 0$.

$x^2 + 4 = 0$	$x^2 = 0$	$x^2 - 4x = 0$
no solutions	$x = 0$	$x(x - 4) = 0$
	one solution	$x = 0$ or $(x - 4) = 0$
		$x = 0$ or $x = 4$
		two solutions

Worked Example 6

WE6

Solve the following equations using the Null Factor Law.

(a) $x(x + 3) = 0$ **(b)** $(x + 3)(x - 2) = 0$ **(c)** $(x - 1)^2 = 0$

Thinking	Working
(a) 1 Write down the equation.	(a) $x(x + 3) = 0$
2 Use the Null Factor Law.	$x = 0$ or $x + 3 = 0$
3 Solve each individual equation.	$x = 0$ or $x = -3$
4 State your solution.	The solution to $x(x + 3) = 0$ is $x = 0$ or $x = -3$.
(b) 1 Write down the equation.	(b) $(x + 3)(x - 2) = 0$
2 Use the Null Factor Law.	$x + 3 = 0$ or $x - 2 = 0$
3 Solve each individual equation.	$x = -3$ or $x = 2$
4 State your solution.	The solution to $(x + 3)(x - 2) = 0$ is $x = -3$ or $x = 2$
(c) 1 Write down the equation.	(c) $(x - 1)^2 = 0$
2 Rewrite as a product.	$(x - 1)(x - 1) = 0$
3 Use the Null Factor Law. Because both factors are identical (it is a repeated factor), there is only one equation to solve.	$x - 1 = 0$
4 Solve each individual equation.	$x = 1$
5 State your solution.	The solution to $(x - 1)^2 = 0$ is $x = 1$.

If the quadratic expression is not in a factorised form, we must try to factorise it first. Not all quadratics will factorise to give real number solutions.

Solving non-factorised quadratics

For equations in the form of $ax^2 + bx = 0$:

- make sure the right-hand side (RHS) is equal to zero
- identify the highest common factor (HCF) and factorise
- use the Null Factor Law to find solutions.

Worked Example 7

WE7

Solve the following equations using the Null Factor Law.

(a) $2x^2 + 4x = 0$ **(b)** $7x^2 = x$

Thinking		Working
(a)	**1** Write the equation.	(a) $2x^2 + 4x = 0$
	2 Factorise the left-hand side (LHS) by finding the HCF.	$2x(x + 2) = 0$
	3 Apply the Null Factor Law.	$2x = 0$ or $x + 2 = 0$
	4 Solve each equation.	$x = 0 \quad x = -2$
	5 State your answer.	$x = 0$ or -2
(b)	**1** Write the equation.	(b) $7x^2 = x$
	2 Rearrange to make one side equal to zero.	$7x^2 - x = 0$
	3 Factorise the LHS of the equation by finding the HCF.	$x(7x - 1) = 0$
	4 Apply the Null Factor Law.	$x = 0$ or $7x - 1 = 0$
	5 Solve each equation.	$x = 0$ or $7x = 1$ $x = \frac{1}{7}$
	6 State your answer.	$x = 0$ or $\frac{1}{7}$

For equations in the form of $x^2 - a^2 = 0$:

- Make sure the RHS is equal to zero.
- Recognise the difference of two squares.
- Factorise using $a^2 - b^2 = (a + b)(a - b)$.
- Use the Null Factor Law to find solutions.

Worked Example 8

WE8

Solve the following equations using the Null Factor Law.

(a) $x^2 - 49 = 0$ **(b)** $4x^2 = 81$ **(c)** $x^2 - 7 = 0$

Thinking		Working
(a)	**1** Write the equation.	(a) $x^2 - 49 = 0$
	2 Factorise the (LHS) using $a^2 - b^2 = (a + b)(a - b)$	$(x - 7)(x + 7) = 0$
	3 Apply the Null Factor Law.	$x - 7 = 0$ or $x + 7 = 0$
	4 Solve each equation and state your answer.	$x = 7$ or $x = -7$

Thinking	Working
(b) 1 Write the equation.	(b) $4x^2 = 81$
2 Rearrange the equation so that the RHS = 0.	$4x^2 - 81 = 0$
3 Write the LHS of the equations as the difference of two squares.	$(2x)^2 - (9)^2 = 0$
4 Factorise the LHS using $a^2 - b^2 = (a + b)(a - b)$.	$(2x - 9)(2x + 9) = 0$
5 Apply the Null Factor Law.	$2x - 9 = 0$ or $2x + 9 = 0$
6 Solve each equation and state your answer.	$x = \frac{9}{2}$ or $x = -\frac{9}{2}$ $x = 4\frac{1}{2}$ or $x = -4\frac{1}{2}$
(c) 1 Write the equation.	(c) $x^2 - 7 = 0$
2 Write the LHS of equation as the difference of two squares.	$x^2 - (\sqrt{7})^2 = 0$
3 Factorise the LHS using $a^2 - b^2 = (a + b)(a - b)$.	$(x - \sqrt{7})(x + \sqrt{7}) = 0$
4 Apply the Null Factor Law.	$(x - \sqrt{7}) = 0$ or $(x + \sqrt{7}) = 0$
5 Solve each equation and state your answer.	$x = \sqrt{7}$ or $x = -\sqrt{7}$

For equations in the form of $x^2 + 2ax + a^2 = 0$ or $x^2 - 2ax + a^2 = 0$

- Make sure the RHS is equal to zero.
- Recognise a perfect square.
- Factorise using $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.
- Use the Null Factor Law to find solutions.

Worked Example 9

WE9

Solve the following equations using the Null Factor Law.

(a) $x^2 - 6x + 9 = 0$ **(b)** $2x^2 + 48x = -288$

Thinking	Working
(a) 1 Use the perfect square rule to factorise the LHS.	(a) $x^2 - 6x + 9 = 0$ $(x - 3)^2 = 0$
2 Write as the product of factors.	$(x - 3)(x - 3) = 0$
3 Solve the equation and state your answer. Because there is a repeated factor, there is only one equation to solve.	$x - 3 = 0$ $x = 3$

(b) **1**	Rearrange to make one side equal to zero.	(b) $2x^2 + 48x = -288$
2	Factorise the LHS by: • taking out the HCF • using the perfect square rule to factorise the LHS.	$2x^2 + 48x + 288 = 0$ $2(x^2 + 24x + 144) = 0$ $2(x + 12)^2 = 0$
3	Divide both sides by 2.	$(x + 12)^2 = 0$
4	Write as the product of factors.	$(x + 12)(x + 12) = 0$
5	Solve the equation and state your answer. Because there is a repeated factor, there is only one equation to solve.	$x + 12 = 0$ $x = -12$

Notice that there is only one solution for these quadratics. Remember that some parabolas only touch the x-axis at one point. The equation to find the x-intercept of these parabolas will have only one solution.

9.2 Solving quadratic equations

Navigator

Answers page 659

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q10, Q13, Q14	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Fluency

WE6

1 Solve the following equations using the Null Factor Law.

(a) $x(x - 1) = 0$ **(b)** $x(x + 8) = 0$ **(c)** $x(x + 5) = 0$

(d) $(x - 4)(x - 1) = 0$ **(e)** $(x - 2)(x - 6) = 0$ **(f)** $(x - 3)(x - 9) = 0$

(g) $(x - 4)(x + 1) = 0$ **(h)** $(x - 3)(x + 7) = 0$ **(i)** $(x - 5)(x + 2) = 0$

(j) $(x + 4)(x + 6) = 0$ **(k)** $(x + 7)(x + 2) = 0$ **(l)** $(x + 3)(x + 10) = 0$

(m) $(x - 4)^2 = 0$ **(n)** $(x + 7)^2 = 0$ **(o)** $(x - 9)^2 = 0$

WE7

2 Solve the following equations using the Null Factor Law.

(a) $x^2 - 6x = 0$ **(b)** $x^2 - x = 0$ **(c)** $x^2 - 8x = 0$

(d) $x^2 + 7x = 0$ **(e)** $x^2 + 5x = 0$ **(f)** $x^2 + x = 0$

(g) $2x^2 - 8x = 0$ **(h)** $3x^2 - 9x = 0$ **(i)** $7x^2 - 28x = 0$

(j) $3x^2 + 27x = 0$ **(k)** $5x^2 + 45x = 0$ **(l)** $12x^2 + 4x = 0$

WE8

3 Solve the following equations using the Null Factor Law.

(a) $x^2 - 9 = 0$ **(b)** $x^2 - 1 = 0$ **(c)** $x^2 - 16 = 0$

(d) $9x^2 - 25 = 0$ **(e)** $25x^2 - 36 = 0$ **(f)** $49x^2 - 81 = 0$

(g) $2x^2 - 98 = 0$ **(h)** $3x^2 - 27 = 0$ **(i)** $7x^2 - 28 = 0$

(j) $5x^2 = 125$ (k) $4x^2 = 49$ (l) $9x^2 = 100$
(m) $16 - x^2 = 0$ (n) $36 - x^2 = 0$ (o) $9 - x^2 = 0$
(p) $x^2 - 11 = 0$ (q) $x^2 - 29 = 0$ (r) $x^2 - 47 = 0$

4 Solve the following equations using the Null Factor Law. WE9

(a) $x^2 - 6x + 9 = 0$ (b) $x^2 - 2x + 1 = 0$ (c) $x^2 - 8x + 16 = 0$
(d) $x^2 + 10x + 25 = 0$ (e) $x^2 + 12x + 36 = 0$ (f) $x^2 + 18x + 81 = 0$
(g) $2x^2 - 28x + 98 = 0$ (h) $3x^2 + 18x + 27 = 0$ (i) $7x^2 - 28x + 28 = 0$
(j) $x^2 - 4x = -4$ (k) $x^2 - 12x = -36$ (l) $x^2 - 10x = -25$
(m) $x^2 + 9 = 6x$ (n) $x^2 + 1 = 2x$ (o) $x^2 + 49 = 14x$

5 (a) The solution(s) to $x(x + 1) = 0$ is/are:

A $x = -1$ B $x = 1$ C $x = 0, x = -1$ D $x = 0, x = 1$

(b) The solution(s) to $(x + 2)(x - 3) = 0$ is/are:

A $x = 2, x = 3$ B $x = -2, x = 3$ C $x = 2, x = -3$ D $x = -2, x = -3$

(c) The solution(s) to $x^2 + 10x + 25 = 0$ is/are:

A $x = 2, x = 5$ B $x = -5$ C $x = 10, x = 25$ D $x = 5$

(d) The solution/s to $x^2 - 121 = 0$ is/are:

A $x = 11, x = -11$ B $x = 11$ C $x = 0, x = -11$ D $x = 0, x = 11$

Understanding

6 When three times a number is added to the square of that number, the result is seven times the number. Use an equation to find the number if it is greater than zero.

7 The product of $(2x + 1)$ and $(2x - 3)$ is equal to the product of $(x + 2)$ and $(5x - 6)$. Find x.

8 The height of a golf ball can be represented by the equation $h = 4t - t^2$, where t represents the time in seconds.

The golf ball will be 4 m above the ground when t equals:

A 0 seconds B 4 seconds C 2 seconds D Never

Reasoning

9 A square vegetable garden has one side extended by 4 m and the other side reduced by 4 m. The area is now 65 m^2. What was the side length of the original vegetable garden?

Hint
Write an equation and solve it to find the original length of the vegetable garden.

10 A graph has equation $y = (x + 1)(x - 4)$. Find the x-intercepts of this graph.

11 Determine the x-intercepts of the following quadratic relationships.

(a) $y = x^2 - 144$ (b) $y = 6x^2 + 24x$ (c) $y = x^2 - 12x + 36$

12 **(a)** Expand $(x-6)(x+2)$.

(b) Use your answer in part **(a)** to find the solution to $x^2-4x-12=0$.

(c) How are your solutions in part **(b)** related to the constant term in $y=x^2-4x-12$?

(d) Expand $(x-5)(x+1)$.

(e) Use your answer in part **(d)** to find the solution to $x^2-4x-5=0$.

(f) How are your solutions for part **(e)** related to the constant term in $y=x^2-4x-5$?

(g) Solve the quadratic equation $x^2-4x=0$.

(h) What is the value of the constant term in x^2-4x?

(i) How are your solutions for part **(g)** related to the constant term in $y=x^2-4x$?

(j) Expand $(x-1)(x-3)$.

(k) Use your answer in part **(j)** to find the solution to $x^2-4x+3=0$.

(l) How are your solutions for part **(k)** related to the constant term in $y=x^2-4x+3$?

(m) Solve the quadratic equation $x^2-4x+4=0$.

(n) How are your solutions in part **(m)** related to the constant term in $y=x^2-4x+4$?

(o) What does the constant term in the equations in parts **(c)**, **(f)**, **(i)** and **(n)** represent graphically?

(p) What do the equations in parts **(b)**, **(e)**, **(g)**, **(k)** and **(m)** all have in common?

(q) What information does the solution to these five equations give you about the graphs of each of these equations?

(r) What happens to the x-intercepts of the graphs of these equations as the constant term increases?

(s) Write a statement about the solutions to $x^2-4x+c=0$ where $c>4$. Hence, make a statement about the x-intercepts of graphs in the form of $y=x^2-4x+c$ where $c>4$.

Open-ended

13 Decide on two values of x that will be the x-intercepts of a quadratic graph. Write down at least two possible equations for the graph.

14 Louise is solving the equation $8x-x^2=16$. She writes

$8x-x^2=16$
$x(8-x)=16$
$x=16$ or $8-x=16$
$x=16$ or $x=-8$

Explain to Louise what she has done incorrectly and write the correct solution.

Outside the Square — Problem solving

Stopping distances

The distance it takes for a car to stop could be described by the equation: $y=0.005x^2+0.12x$, where y (m) is the distance it takes to stop and x is the speed (km/h) of the car.

(a) What extra distance will it take a car to stop if it is travelling at 60 km/h instead of 40 km/h through a school zone?

(b) What is the extra distance required for a car to stop if it is exceeding a 100 km/h speed limit by 10 km/h?

(c) Show that the speed of the car must be less than 12 km/h to ensure it does not hit a stationary car 2 m in front of it.

(d) What is the effect on the stopping distance of wiping 5 km/h off a 100 km/h speed?

Strategy options

- Have I seen a similar problem.
- Break problem into manageable parts.

Sketching parabolas using transformations

9.3

We have learnt previously to plot parabolas and to solve some types of quadratic equations. To sketch a graph we use key features of a graph instead of plotting points. In this section, we will explore the key features of a parabola so we can sketch parabolas without plotting points by using transformations.

A **transformation** is any change that can be made to a graph or shape using a **dilation**, **reflection**, **translation** or rotation. We will look at dilations, reflections and transformations of parabolas.

Dilation of parabolas

We know the quadratic equation in the form of $y = x^2$ graphs as a parabola when plotted.

Here we will investigate what happens to the graph when x^2 is multiplied by a constant, a, to form graphs in the form of $y = ax^2$. This is called a dilation and the **coefficient** of x^2, a, is referred to as the **dilation factor**.

Worked Example 10

WE10

(a) Plot the graph of $y = x^2$ for $-2 \le x \le 2$.

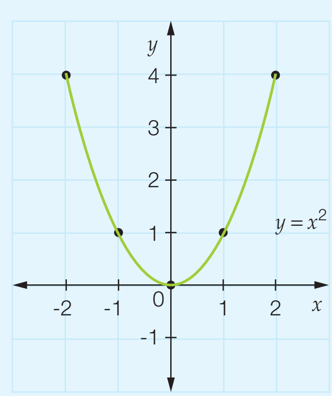
(b) Plot the graphs of **(i)** $y = -x^2$, **(ii)** $y = 2x^2$ and **(iii)** $y = \frac{1}{2}x^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$ using words narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

Thinking	Working
(a) Plot the graph of $y = x^2$.	(a)

x	-2	-1	0	1	2
y	4	-1	0	1	4



(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = -x^2$

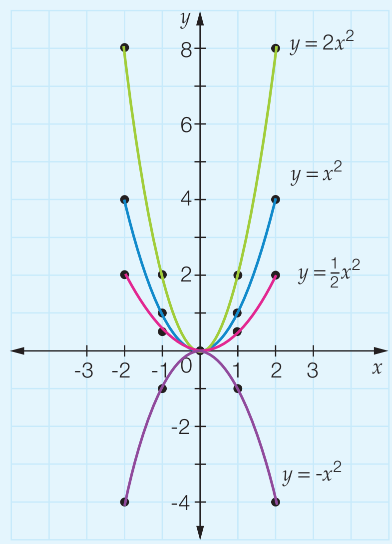
x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

(ii) $y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

(iii) $y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	2	0.5	0	0.5	2



(c) Use your graph to decide whether each graph is narrower, wider or inverted.

(c) (i) $y = -x^2$ is inverted but the same width.

(ii) $y = 2x^2$ is narrower than $y = x^2$.

(iii) $y = \frac{1}{2}x^2$ is wider than $y = x^2$.

(d) Identify the coefficient of x^2 in each equation. This is the value of a, the dilation factor.

(d) (i) The dilation factor, a, is -1.

(ii) The dilation factor, a, is 2.

(iii) The dilation factor, a, is $\frac{1}{2}$.

For equations of the form $y = ax^2$:

- When the coefficient of x^2 is negative ($a < 0$) the parabola is inverted. These graphs are reflected in the x-axis.
- When the coefficient of x^2 is greater than 1 ($a > 1$), the parabola is narrower than $y = x^2$.
- When the coefficient of x^2 is greater than zero but less than 1 ($0 < a < 1$), the parabola is wider than $y = x^2$.
- When the coefficient of x^2 is greater than -1 but less than zero ($-1 < a < 0$) the parabola is also wider than $y = x^2$.

Horizontal translations of parabolas

Now we will investigate equations that translate the parabola $y = x^2$ left and right. These are horizontal translations.

Worked Example 11

WE11

(a) Plot the graph of $y = x^2$ for $-4 \le x \le 4$

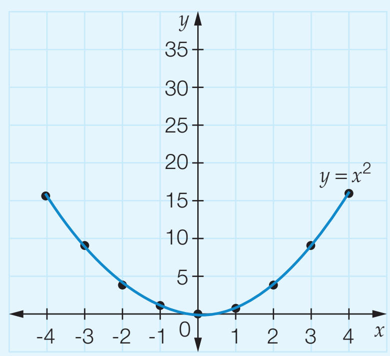
(b) Plot the graphs of **(i)** $y = (x - 1)^2$ and **(ii)** $y = (x + 2)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

Thinking	Working
(a) Plot the graph of $y = x^2$.	(a) $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16



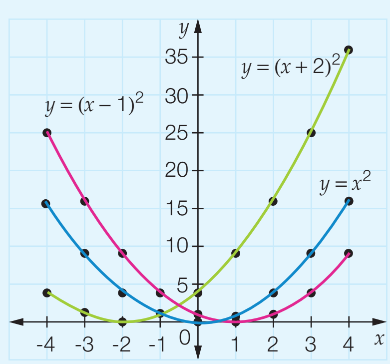
(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = (x - 1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	25	16	9	4	1	0	1	4	9

(ii) $y = (x + 2)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	4	1	0	1	4	9	16	25	36



(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.	(c) (i) The graph of $y = (x - 1)^2$ has been translated 1 units to the right. (ii) The graph of $y = (x + 2)^2$ has been translated 2 units to the left.
(d) State the x-intercept from the graph and identify the x-value of the intercept from the equation.	(d) (i) The x-intercept of $y = (x - 1)^2$ is the point (1, 0). The x-value of the intercept is $x = 1$. This is the number subtracted from x before squaring in the equation. (ii) The x-intercept of $y = (x + 2)^2$ is the point (-2, 0). The x-value of the intercept is $x = -2$. This is the number subtracted from x before squaring in the equation.

The graph of $y = (x - h)^2$ is the graph of $y = x^2$ translated h units to the right.
If the value of h is negative, this will mean a translation to the left.

Vertical translations of parabolas

Here we will investigate equations that translate the parabola $y = x^2$ up or down. These are vertical translations.

Worked Example 12

WE12

(a) Plot the graph of $y = x^2$ for $-4 \leq x \leq 4$.

(b) Plot the graphs of **(i)** $y = x^2 + 3$ and **(ii)** $y = x^2 - 4$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

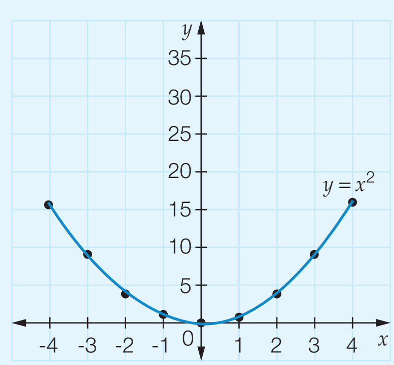
(d) State the y-intercept for each graph in part **(b)** and identify the y-intercept from the equation.

Thinking | **Working**

(a) Plot the graph of $y = x^2$.

(a)

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16



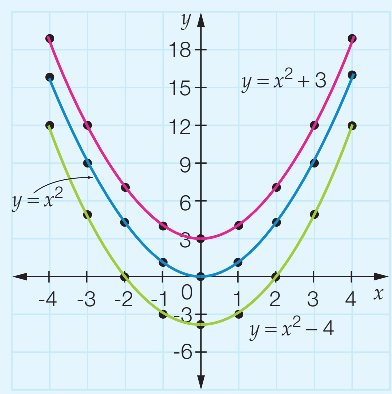
(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = x^2 + 3$

x	-4	-3	-2	-1	0	1	2	3	4
y	19	12	7	4	3	4	7	12	19

(ii) $y = x^2 - 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12



(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.

(c) (i) The graph of $y = x^2$ has been translated 3 units upwards.

(ii) The graph of $y = x^2$ has been translated 4 units downwards.

(d) State the y-intercept from the graph and identify the y-value of the intercept from the equation.

(d) (i) The y-intercept of $y = x^2 + 3$ is the point (0, 3). The y-value of the intercept is $y = 3$. This is the constant in the equation.

(ii) The y-intercept of $y = x^2 - 4$ is the point (0, -4). The y-value of the intercept is $y = -4$. This is the constant in the equation.

The graph of $y = x^2 + k$ is the graph of $y = x^2$ translated k units upwards. If the value of k is negative, this will mean a translation downwards.

Combined horizontal and vertical translations

We can combine horizontal and vertical translations with equations in the form of $y = (x - h)^2 + k$, where h is a translation to the right and k is a translation upwards.

Worked Example 13

WE13

(a) Plot the graph of $y = x^2$ for $-5 \le x \le 5$.

(b) Plot the graphs of **(i)** $y = (x - 3)^2 + 1$ and **(ii)** $y = (x + 1)^2 - 5$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

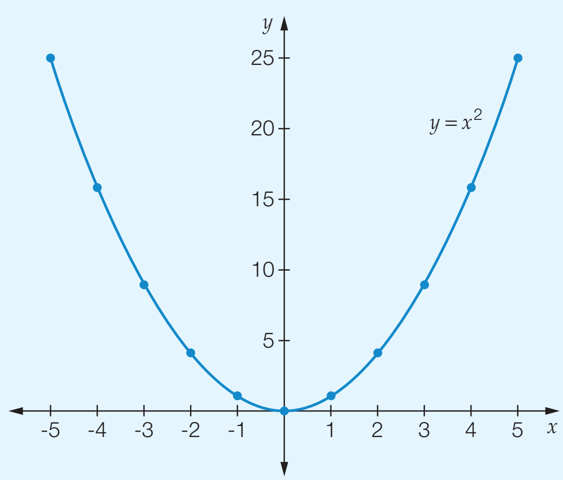
Thinking	Working

(a) 1 Construct a table of values for the given values of x.

(a) $y = x^2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

2 Plot the graph of $y = x^2$.



(b) 1 Constuct a table of values for each of the equations for the given values of x.

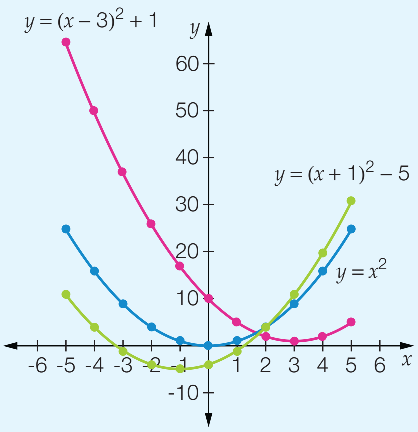
(b) (i) $y = (x - 3)^2 + 1$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	65	50	37	26	17	10	5	2	1	2	5

(ii) $y = (x + 1)^2 - 5$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	11	4	-1	-4	-5	-4	-1	4	11	20	31

2 Plot each of the graphs on the same set of axes.



(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.

(c) (i) The graph of $y = x^2$ has been translated 3 units to the right and 1 unit upwards.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left and 5 units down.

(d) Identify the y-value of the intercept from the table of values for the equation (when $x = 0$).

(d) (i) The y-intercept is (0, 10).

(ii) The y-intercept is (0, -4).

(e) Use your graph to determine how many x-intercepts there are.

(e) (i) The graph has no x-intercepts.

(ii) The graph has two x-intercepts.

The graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated h units to the right and k units upwards. If the value of h is negative, this will mean a translation to the left. If the value of k is negative, this will mean a translation downwards.

Locating x- and y-intercepts

We have seen that the y-intercept occurs where the parabola intersects the y-axis (i.e. when $x = 0$). The y-value of the y-intercept can therefore be found by substituting $x = 0$ into the quadratic equation.

If $y = x^2 - 5x + 6$
$y = 0 - 5 \times 0 + 6$ (substituting $x = 0$)
$= 6$

The coordinates of the y-intercept would be (0, 6).

We have also seen that the x-intercept occurs where the parabola intersects the x-axis when $y = 0$. The x-value of the x-intercept can therefore be found by substituting $y = 0$ into the quadratic equation.

If $y = (x - 2)(x - 3)$ (this quadratic, when expanded, gives $y = x^2 - 5x + 6$)
$0 = (x - 2)(x - 3)$ (substituting $y = 0$)
$(x - 2) = 0$ or $(x - 3) = 0$ (using the Null Factor Law)
$x = 2$ or $x = 3$

The coordinates of the x-intercepts would be (2, 0) or (3, 0).

The turning point

All quadratic graphs have a vertex or turning point. This can be a minimum value or a maximum value.

When a graph has two x-intercepts, the x-coordinate of the turning point is halfway between the two x-intercepts.

For the quadratic equation $y = (x - 3)(x + 5)$, the x-intercepts are (3, 0) and (-5, 0) and $x = -1$ is the x-coordinate of the turning point.

The corresponding y-coordinate can be found by substituting the x-value into the quadratic equation.

$$y = (-1 - 3)(-1 + 5)$$
$$y = -4 \times 4$$
$$= -16$$

When a graph has only one x-intercept, the turning point is the x-intercept.

For the quadratic equation $y = (x - 4)^2$, the x-intercept is (4, 0).

When there are no x-intercepts, the turning point is more difficult to find. We will explore ways to find the turning point in the following Worked Example.

Worked Example 14

WE 14

(a) Plot the graph of $y = (x - 1)^2 - 9$ for $-5 \leq x \leq 5$.

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x - h)^2 + k$, write down the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x - 4)(x + 2)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

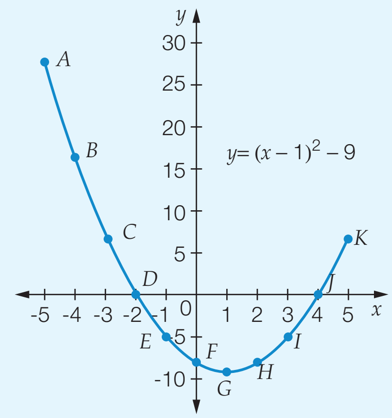
(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write down the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

Thinking	Working

(a) Draw up a table of values and plot the points on a Cartesian plane. Label the axes and the graph.

(a) $y = (x - 1)^2 - 9$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	27	16	7	0	-5	-8	-9	-8	-5	0	7



(b) Look at the graph to see where it changes direction.	(b) The turning point is (1, -9).
(c) Write down the values of h and k. Write an ordered pair (h, k). Compare this point to the point found in part **(b)**.	(c) $h = 1$ and $k = -9$ $(h, k) = (1, -9)$ Yes, it is the same point.
(d) **1** Expand the brackets using the perfect square rule and simplify.	(d) $y = (x - 1)^2 - 9$ $= x^2 - 2x + 1 - 9$ $= x^2 - 2x - 8$
2 Expand the second expression and simplify.	$(x - 4)(x + 2)$ $= x^2 - 4x + 2x - 8$ $= x^2 - 2x - 8$
3 Compare the two expressions.	$(x - 1)^2 - 9 = (x - 4)(x + 2)$
4 Equate the factorised expression to 0.	$(x - 4)(x + 2) = 0$
5 Use the Null Factor Law to find the x-intercepts.	$x = 4$ and $x = -2$
6 Find the average of the two x-values.	Average of 4 and -2 $= \frac{4 + -2}{2} = 1$
7 Compare this value with the value of x found in part **(b)**.	Yes, this is the same value.
(e) Substitute the value of x found in part **(d)** into the equation.	(e) $y = (x - 1)^2 - 9 = (1 - 1)^2 - 9 = -9$ Yes, this is the same value.
(f) **1** Write down the values of a, b and c.	(f) $a = 1$, $b = -2$, $c = -8$
2 Find $-\frac{b}{2a}$.	$-\frac{b}{2a} = -\frac{-2}{2} = 1$
3 Compare this value with the value of x found in part **(b)**.	Yes, this is the same value.

When the quadratic equation is written as $y = a(x - h)^2 + k$, we can identify the turning point as the point with coordinates (h, k).

For the quadratic equation $y = (x - 2)^2 + 7$ the coordinates of the turning point are (2, 7).

In a quadratic equation in the form of $y = ax^2 + bx + c$, the x-value of the turning point can be found using $x = -\frac{b}{2a}$.

9.3 Sketching parabolas using transformations

Navigator

Answers page 660

Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 1 & 2, Q8 Columns 1 & 2, Q9, Q10, Q11, Q12, Q13 (a)–(h), Q14, Q17, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 Columns 2 & 3, Q9, Q10, Q11, Q12, Q13 (a)–(h), Q14, Q17, Q18, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7 Columns 3 & 4, Q8 Columns 3 & 4, Q9, Q10, Q11, Q12, Q13 (i)–(p), Q14, Q15, Q16, Q17, Q18, Q20

Fluency

WE10

1 **(a)** Plot the graph of $y = x^2$ for $-3 \le x \le 3$.

(b) Plot the graphs of **(i)** $y = -2x^2$, **(ii)** $y = 3x^2$ and **(iii)** $y = \frac{x^2}{4}$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$ using the words narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

WE11

2 **(a)** Plot the graph of $y = x^2$ for $-3 \le x \le 3$.

(b) Plot the graphs of **(i)** $y = (x - 2)^2$ and **(ii)** $y = (x + 1)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

WE12

3 **(a)** Plot the graph of $y = x^2$ for $-3 \le x \le 3$.

(b) Plot the graphs of **(i)** $y = x^2 + 4$ and **(ii)** $y = x^2 - 2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the y-intercept for each graph in part **(b)** and identify the y-intercept from the equation.

WE13

4 **(a)** Plot the graph of $y = x^2$ for $-3 \le x \le 3$.

(b) Plot the graphs of **(i)** $y = (x - 1)^2 - 2$ and **(ii)** $y = (x + 2)^2 + 1$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

WE14

5 **(a)** Plot the graph of $y = (x - 2)^2 - 1$ for $-5 \le x \le 5$.

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x - h)^2 + k$, write down the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x - 1)(x - 3)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write down the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

6 (a) State whether the graph of each of these equations will be narrower or wider than the parabola $y = x^2$.

(i) $y = 5x^2$ (ii) $y = \frac{7}{8}x^2$ (iii) $y = -\frac{1}{6}x^2$ (iv) $y = -14x^2$

(b) Which of the graphs of the equations in part **(a)** would be upside down (or inverted) compared with $y = x^2$?

(c) State the dilation factor of each of the following.

(i) $y = 4x^2$ (ii) $y = \frac{1}{5}x^2$ (iii) $y = 7x^2 + 2x$ (iv) $y = \frac{2}{3}x^2 - 3x + 1$

(d) Will a parabola be narrower or wider than $y = x^2$ if it has a dilation factor of:

(i) 3 (ii) $\frac{4}{5}$ (iii) 16 (iv) $\frac{1}{7}$?

7 How is the graph of $y = x^2$ translated (or moved) to obtain each of the following parabolas?

(a) $y = x^2 + 4$ (b) $y = x^2 - 7$ (c) $y = x^2 - 1$ (d) $y = x^2 + 2$

(e) $y = x^2 - 8$ (f) $y = x^2 + 5$ (g) $y = x^2 + \frac{1}{2}$ (h) $y = x^2 - \frac{3}{4}$

8 How is the graph of $y = x^2$ translated (moved) to obtain each of the following parabolas?

(a) $y = (x - 5)^2$ (b) $y = (x + 4)^2$ (c) $y = (x + 1)^2$ (d) $y = (x - 7)^2$

(e) $y = (x + 6)^2$ (f) $y = (x - 3)^2$ (g) $y = (x - \frac{2}{3})^2$ (h) $y = (x + \frac{3}{2})$

9 For each of the following quadratic relationships:

(i) find the transformations of the graph $y = x^2$ that have taken place; and

(ii) find the turning point.

(a) $y = 2(x - 1)^2 + 3$ (b) $y = -3(x + 2)^2 - 1$ (c) $y = 4(x + 3)^2 - 2$

(d) $y = \frac{1}{3}(x + 5)^2 + 3$ (e) $y = (x + 4)^2 - 3$ (f) $y = 2(x + 3)^2 + \frac{1}{2}$

10 (a) To obtain the graph of $y = x^2 + 6$ we need to move the graph of $y = x^2$:

A 6 units to the right B 6 units to the left

C 6 units up D 6 units down

(b) To obtain the graph of $y = (x + 6)^2$ we need to move the graph of $y = x^2$:

A 6 units to the right B 6 units to the left

C 6 units up D 6 units down

(c) To obtain the graph of $y = (x - 2)^2$ we need to move the graph of $y = x^2$:

A 2 units to the right B 2 units to the left

C 2 units up D 2 units down

(d) To obtain the graph of $y = x^2 - 4$ we need to move the graph of $y = x^2$:

A 4 units to the right B 4 units to the left

C 4 units up D 4 units down

11 (a) To obtain the graph of $y = (x - 1)^2 - 3$, we need to move the graph of $y = x^2$:

A 1 unit to the right and 3 units down B 3 units to the right and 1 unit down

C 1 unit to the left and 3 units up D 3 units to the right and 1 unit down

(b) To obtain the graph of $y = 2(x - 1)^2 - 3$ we need to change the graph of $y = x^2$ by:

A dilating by a factor of 2 (makes it narrower) and moving it 1 unit to the right and 3 units down

B dilating by a factor of 3 (makes it narrower) and moving it 1 unit to the left and 3 units down

C dilating by a factor of $\frac{1}{2}$ (makes it wider) and moving it 1 unit to the right and 3 units down

D dilating by a factor of 2 (makes it narrower) and moving it 1 unit to the right and 3 units up

(c) To obtain the graph of $y = -\frac{1}{2}(x - 4)^2 + 1$, we would need to change the graph of $y = x^2$ by:

A dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the right and 1 unit up

B inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the left and 1 unit up

C inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the left and 1 unit down

D inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the right and 1 unit up

Understanding

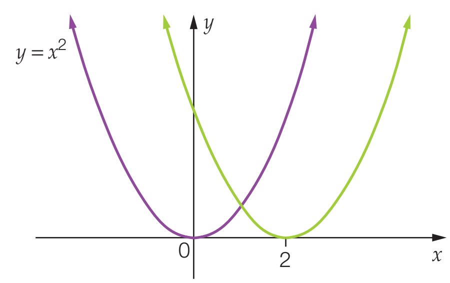
12 **(a)** The equation of the unlabelled parabola shown could be:

A $y = x^2 + 2$

B $y = x^2 - 2$

C $y = (x - 2)^2$

D $y = (x + 2)^2$



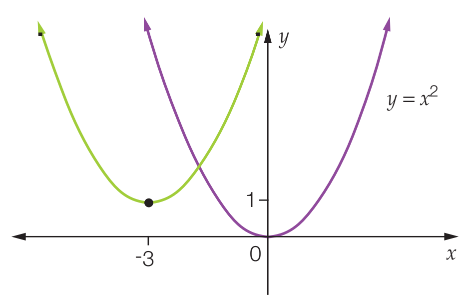
(b) The equation of the unlabelled parabola shown could be:

A $y = x^2 + 1$

B $y = (x + 3)^2 + 1$

C $y = 3x^2 + 1$

D $y = (x - 3)^2 + 1$



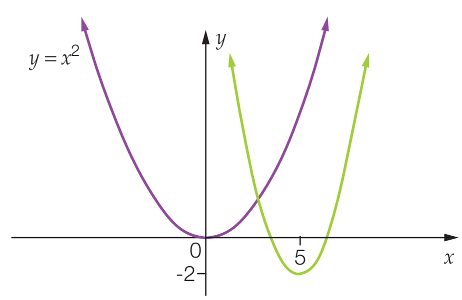
(c) The equation of the unlabelled parabola shown could be:

A $y = 2(x - 5)^2 - 2$

B $y = \frac{1}{2}(x + 5)^2 - 2$

C $y = 2(x - 5)^2 + 2$

D $y = \frac{1}{2}(x - 5)^2 - 2$



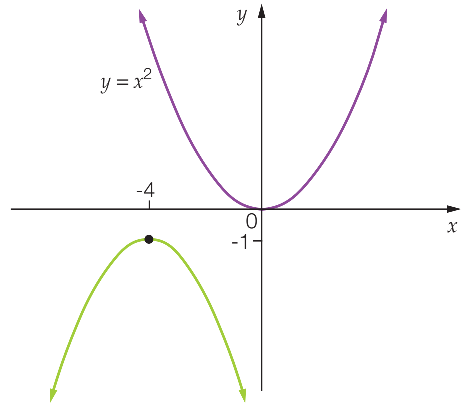
(d) The equation of the unlabelled parabola shown could be:

A $y = (x + 4)^2 - 1$

B $y = -(x + 4)^2 - 1$

C $y = (x - 4)^2 - 1$

D $y = 4(x - 4)^2 - 1$



13 State the changes that should be made to the graph of $y = x^2$ to obtain the graph of each of the following.

		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x + 1)^2 + 2$				
(b)	$y = (x - 3)^2 + 1$				
(c)	$y = 5x^2 + 4$				
(d)	$y = -x^2 - 6$				
(e)	$y = 2(x + 5)^2 - 3$				
(f)	$y = -(x - 2)^2 + 3$				
(g)	$y = -3(x + 6)^2 - 4$				
(h)	$y = \frac{1}{2}(x - 3)^2 + 1$				
(i)	$y = (x - 4)^2 - 5$				
(j)	$y = (x + 2)^2 - 7$				
(k)	$y = 3x^2 - 5$				
(l)	$y = -x^2 - 2$				
(m)	$y = 5(x - 6)^2 - 4$				
(n)	$y = -(x - 5)^2 + 2$				
(o)	$y = -4(x + 1)^2 - 3$				
(p)	$y = \frac{3}{4}(x + 4)^2 + 6$				

14 When graphed, how many of the following equations would have a shape like that of the parabola shown here?

Identify them.

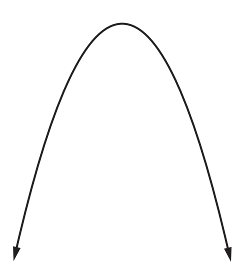
(a) $y = x^2 + 2x - 5$

(b) $y = -x^2 - 2x + 4$

(c) $y = 1 - x^2$

(d) $y = 3 - 4x + x^2$

(e) $y = 6 - 2x - x^2$



15 If the x-intercepts of a parabola are -1 and 5, the turning point could be:

A (0, 3) B (-2, 2) C (2, 4) D (1, -6)

Reasoning

16 (a) A quadratic graph has a turning point at (-2, 5) and a dilation factor of 1. Find the equation of the graph.

(b) Find the y-intercept (where $x = 0$).

(c) Draw the graph, clearly labelling the turning point and the y-intercept.

17 Find the new equation when the following transformations are completed on the graph $y = x^2$.

(a) The graph is dilated by a factor of 5, translated 3 units to the right and 2 units up.

(b) The graph is dilated by a factor of $\frac{1}{3}$, translated down 2 units and to the right 4 units.

18 (a) The sum of two numbers, x and y, is 20. Express y in terms of x.

(b) Write the product, P, in terms of x and sketch a graph of P.

(c) Use the graph to find the values of x and y that will make the product as large as possible.

Open-ended

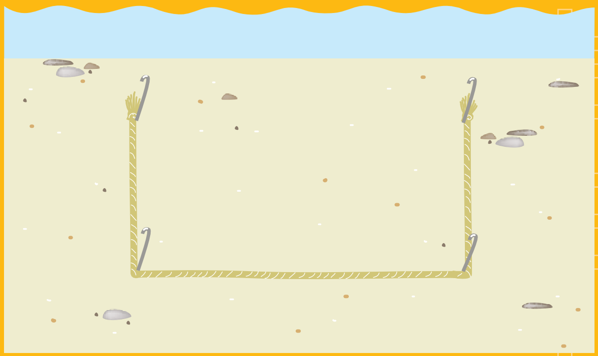
19 Write down at least two equations that show a quadratic relationship in the form $y = a(x - b)^2 + c$. For each one, write down the transformations that the graph $y = x^2$ must undergo to be the graph of the relationship you have written.

20 Write equations for three parabolas that have no x-intercepts.

Outside the Square — Problem solving

Let's go camping

You decide to go camping at a camping ground by a lake. The manager of the camping ground has designed a novel way of determining individual campsites. She gives you a 60 m long piece of rope and four pegs. The only condition is that your campsite must be rectangular. You decide to pitch your tent by the lake and only need to use the rope on three sides. Find the largest campsite that you could make.



Strategy options

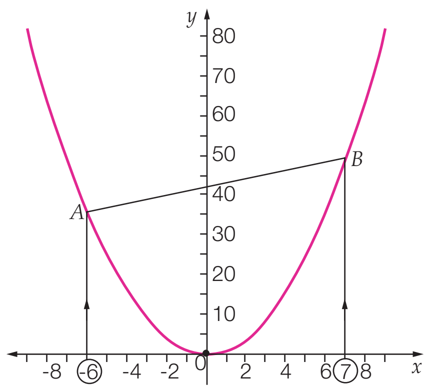
- Test all possible combinations.
- Make a table.
- Make a model.
- Draw a diagram.

Investigation

A parabolic nomogram

Equipment required: graph paper (1-cm grid with smaller divisions), one brain

A nomogram is a group of scaled lines that are usually parallel and are used to perform a calculation. The nomogram in this investigation uses a parabolic graph to calculate multiplications.



The Big Question

What makes a parabolic nomogram work?

Engage

1 **(a)** Construct an accurate version of the parabola $y = x^2$ on graph paper. Circle the values of -6 and 7 on the x-axis, then draw two vertical lines from these points, going upwards to meet the parabola. Label the points where these lines meet the parabola A and B. Draw a line joining the points A and B. Your graph should look identical to the graph shown above.

(b) Write down the y-value of the y-intercept.

2 **(a)** Write down the y-value as the y-intercept of the line segment AB.

(b) What do you notice about the numbers circled on the graph shown above?

3 Use your parabolic nomogram to find the approximate answers to the following:

(a) 3.5×8.5 **(b)** 6.2×7.4

(c) $60 \div 4.6$ **(d)** $59 \div 5.4$

Explore

4 The equation of a line can be found using $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$. This equation equates the gradient of the line joining the point (x, y) to the point (x_1, y_1) and the gradient of the line joining the point (x_2, y_2) to the point (x_1, y_1).

Use this formula to find the equation of the line joining the Points A and B on the given parabola.

Compare the y-value of the y-intercept in your equation to your answer in Question **2**.

5 Find the equation of the line joining the points on the parabola where:

(a) $x = -3.5$ and $x = 8.5$. Compare the y-value of the y-intercept in your equation to your answer in Question **3(a)**.

(b) $x = -6.2$ and $x = 7.4$. Compare the y-value of the y-intercept in your equation to your answer in Question **3(b)**.

Strategy options

- Solve a simpler problem.
- Test all possible combinations.
- Look for a pattern.

Explain

6 Can you see a relationship between the values that are multiplied and:

(a) the gradient of your equation?

(b) the y-value of the y-intercept of your equation?

Write down the possible relationships.

7 What was different about the x-intercept used in the graph from the values multiplied in Question **3**?

Elaborate

8 **(a)** Find the general equation of the line joining the point (a, a^2) and the point (b, b^2). Remember $a^2 - b^2 = (a + b)(a - b)$.

(b) Write down the gradient and the y-value of the y-intercept.

(c) Does this confirm what you found in Question **6**?

(d) Use this information to answer the Big Question.

9 What happens when you multiply a number by itself? Why does the nomogram still work?

Evaluate

10 Did you draw your graph accurately enough for your nomogram to work properly?

11 Did this activity give you a better understanding of gradients and y-intercepts ? If so, explain how.

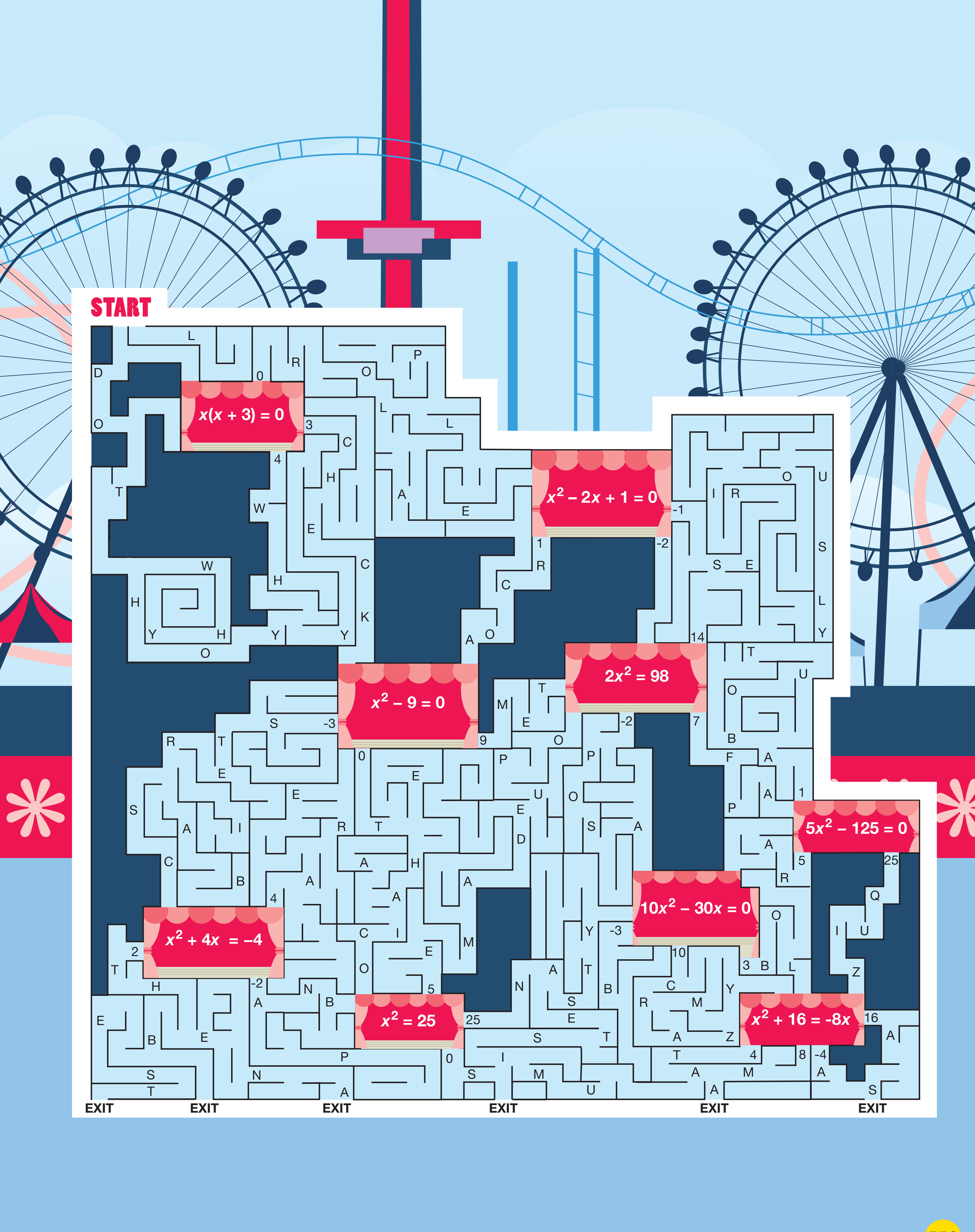
Extend

12 Describe how this nomogram could be used to find the square root of a number.

Mathspace

Quadratic maze

Welcome to 'Parabolas', a local amusement park. You can win free entry if you can find your way correctly through this quadratic maze. As you move through the maze you will need to collect the letters that lie in your path. There are nine side-shows in the maze. You must pass through each one of them. When you enter a side-show, you need to solve the equation shown, and move out through the exit that shows a correct solution. You must enter a side-show through the openings that do not have numbers across them. Once your have exited the maze, the collected letters will spell out a secret code. Crack the code and entry is free! Good luck!


START
$x(x + 3) = 0$
$x^2 - 2x + 1 = 0$
$x^2 - 9 = 0$
$2x^2 = 98$
$5x^2 - 125 = 0$
$10x^2 - 30x = 0$
$x^2 + 4x = -4$
$x^2 = 25$
$x^2 + 16 = -8x$
EXIT
EXIT
EXIT
EXIT
EXIT
EXIT

Half-time 9

Ex. 9.3

1 Consider the equation $y = -x^2 + 1$.

(a) Set up a table of values using integer values of x between -3 and 3 inclusive.

(b) Plot the points on a number plane and join them with a smooth curve.

(c) Find the coordinates of the turning point.

(d) Find the coordinates of the y-intercept.

(e) Find the coordinates of the x-intercepts.

Ex. 9.1

2 Which of the following is not a quadratic expression?

A $2x^2 + 3x + 5$ B $-x + 3x^2$ C $3x + 2$ D $4x^2 - 3x - 1$

Ex. 9.2

3 Solve each of the following.

(a) $x(x + 4) = 0$ (b) $2x(x - 3) = 0$ (c) $(x + 2)(x - 3) = 0$

Ex. 9.1

4 Lisa throws a ball upwards. The equation of the flight of the ball is $h = -5t^2 + 8t + 1.3$, where h is the height of the ball in metres and t is the time in seconds.

(a) Draw a graph showing this relationship.

(b) From the graph, find when the ball first reaches a height of 3 m.

(c) How high is the ball after 1.2 seconds?

Ex. 9.2

5 Solve each of the following.

(a) $x^2 + 4x + 4 = 0$ (b) $x^2 + 12x + 36 = 0$ (c) $x^2 - 18x + 81 = 0$

Ex. 9.3

6 (a) Complete a table of values for $y = 2x^2 + 4x$ from $x = -3$ to $x = 3$ inclusive.

(b) Plot the points on a number plane and join them using a smooth curve.

(c) Write down the coordinates of the turning point.

(d) State whether the parabola has a maximum or a minimum turning point.

(e) Write down the coordinates of the x- and y-intercepts.

Ex. 9.2

7 Solve each of the following.

(a) $x^2 - 6x = 0$ (b) $2x^2 + 20x + 50 = 0$ (c) $3x^2 = 27x$

Ex. 9.2

8 Solve each of the following.

(a) $x^2 - 64 = 0$ (b) $(x + 1)^2 = 0$ (c) $(x - 2)^2 = 4$

Ex. 9.1

9 (a) Plot the graph of $y = x^2 - 2x + 3$. Use integer values of x between -3 and 5 inclusive.

(b) What are the coordinates of the turning point?

(c) Write down the coordinates of any x- and y-intercepts.

(d) Use the graph to solve the following equations.

(i) $x^2 - 2x + 3 = 3$ (ii) $x^2 - 2x + 3 = 2$

(iii) $x^2 - 2x + 3 = 6$ (iv) $x^2 - 2x + 3 = 0$

Equipment required: 1 brain, 1 computer with Geogebra, a workbook, a pen

Versions of this Exploration for other technologies are available in Pearson Reader.

Investigating non-linear equations

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right hand corner of the screen.

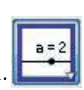
1 Click on the View menu and select 'Grid' then click on the Options menu and select 'Point Capturing' and 'On (Grid)'. Select 'Labelling' and 'No New Objects'. Select Drawing Pad and set the x-axis values from -10 to 10. Select x from the Label menu. Set the y-axis values from -8 to 8. Select y from the Label menu.

2 If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided. On both axes, select 'Distance' and set to 1.

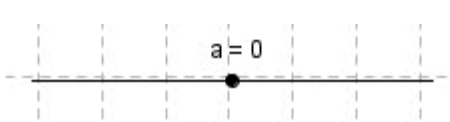
Investigating parabolas

All parabolas can be represented by the equation $y = ax^2 + bx + c$, where a represents the dilation and c represents the y-coordinate of the y-intercept (where the line cuts the y-axis).

We will now create parabolas using different values of a, b and c and investigate how the graphs change when we change these values.

3 Select the 'Slider' tool. 

Click on the right-hand side of the Graphics View, the space where the drawings appear. A pop-up box will appear. Under 'Name', type a. Change 'Increment' to 0.2. Click on the 'Slider' tab and change 'Width' to 200. Click 'Apply'. A line like this will appear on the screen.



4 Construct a second slider by repeating Step **3**, naming it b.

5 Construct a third slider by repeating Step **3**, naming it c.

6 Set the Slider values for a, b and c to 0.

7 In the Input bar at the bottom of the screen type y= x^2 (the ^ is needed to tell the program to square x). A parabola should appear on the screen.

8 In the 'Input' bar at the bottom of the screen type y=a*x^2+b*x+c (the * is needed to tell the program to multiply values). Increase the slider value for a from -5 to 5. A parabola should appear on the screen, and it should change as the slider changes.

9 Write down what you see as the slider values for a increase from -5 to 5. What happens when $a = 0$? Explain. What happens when $a = 1$? Explain. What happens when $a = -1$? Explain.

10 Set the value of a to 1 and increase the value of c from -5 to 5. Describe what happens to the parabola as the c value changes. What do you notice about the shape of each graph? What do you notice about the y-intercept for each graph? What do you notice about the x-intercepts? (Hint: $c = -4$ and -1 should help you.)

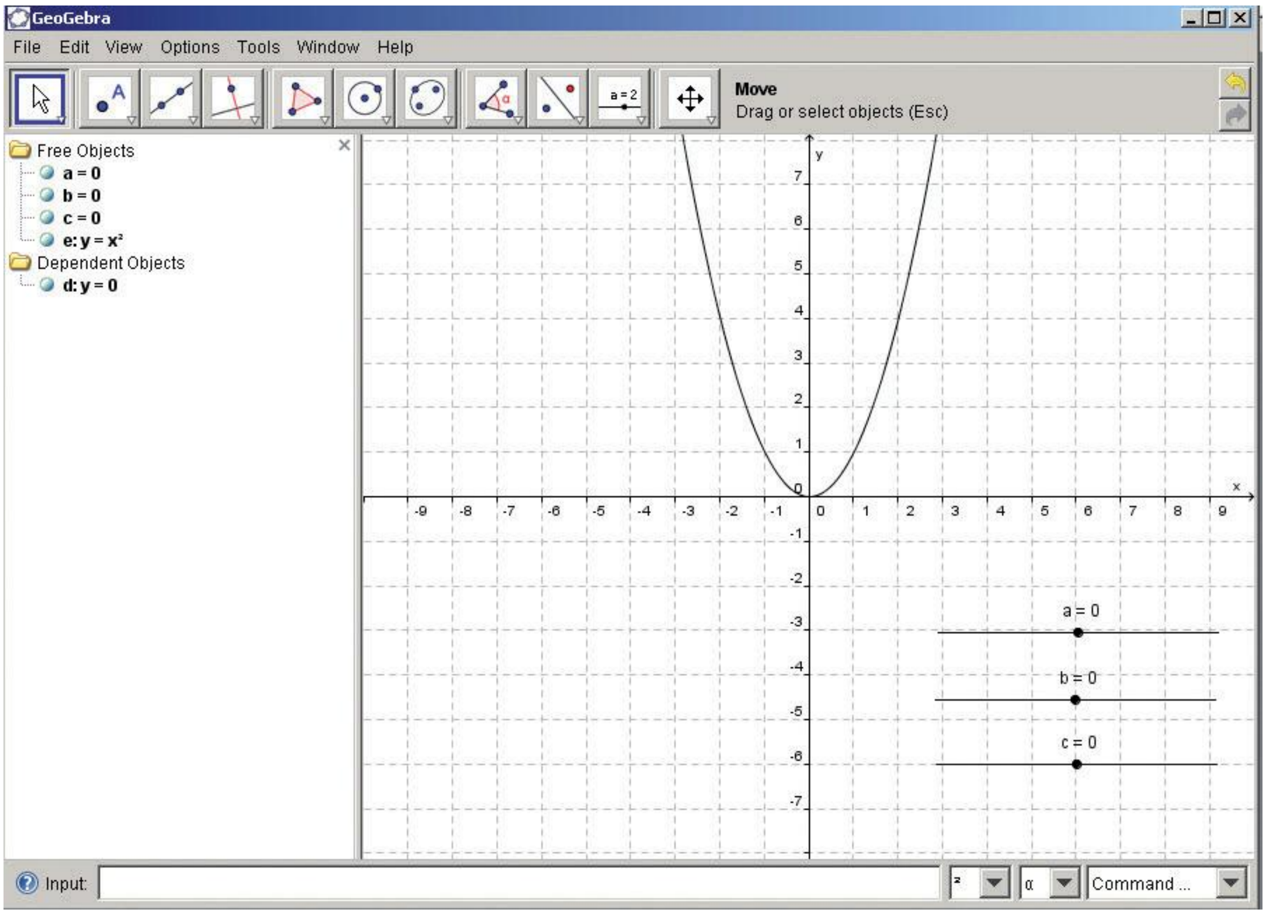
11 Leave the a value at 1 and set the c value to 0. Change the b value from -5 to 5. Write down any connection between the graphs and the b value. Use algebra to explain this connection.

You will also notice that an equation will appear in the 'Algebra View' (on the left-hand side of the screen).

12 Find a value for each of b and c so that the parabola moves to the right. Explain why you chose these values.

13 Find a value for each of b and c so that the parabola moves to the left. Explain why you chose these values.

Technology Exploration GeoGebra



Investigating circles

14 Open a new GeoGebra screen. Click on the circle tool, the sixth tool from the left, and select the circle with centre and radius.  Use this tool to draw a circle with radius 4 and centre at the origin. Use the point tool to insert a point on the circle in each quadrant. Use the coordinates of each point given in 'Algebra View' to show that $x^2 + y^2 = 16$. If you do not get 16 exactly, explain why.

15 Use the input box to enter the equation $(x–h)$^2 + $(y–k)$^2 =r^2. Set up three new sliders as you did in Step **3** and call them h, k and r (set the r values from 0 to 5). Set both h and k values to zero and r to 4. What do you see? Explain.

16 Leave the k value on 0 and the r value on 4. Change the slider value of h from -5 to 5. Write down what you see.

17 Now set the value of h to 0 and change the slider value of k from -5 to 5. Write down what you see.

18 What happens when neither h nor k is zero? Choose integer values for each of h and k. Write these values down and write down the centre of the circle that is drawn. What do you see?

19 Use the point tool to insert a point on this circle in each quadrant. Use the coordinates of each point given in 'Algebra View' to show that $(x - h)^2 + (y - k)^2 = 16$. If you do not get 16 exactly, explain why.

20 Set the values of h and k to zero and change the value of r from 0 to 5. Describe what happens.

Investigating exponentials

21 Open a new GeoGebra screen. Use the input box to enter the equation $y = a$^$(x - h) + k$. Set up three new sliders as you did in Step **3** and call them h, k and a (set the a values from 0 to 5). Set both h and k values to zero and a to 1. What do you see?

22 Leave the k value on 0 and the a value on 1. Change the slider value of h from -5 to 5. Write down what you see.

23 Now set the value of h to 0 and change the slider value of k from -5 to 5. Write down what you see.

24 What happens when neither h nor k is zero? Choose integer values for each of h and k. Write these values down and write down the equation of the asymptote that is formed. What do you see?

25 Set the values of h and k to zero and change the value of a from 0 to 5. Describe what happens.

Investigating hyperbolas

26 Open a new GeoGebra screen. Use the input box to enter the equation $y=a/(x–h)+k$. Set up three new sliders as you did in Step **21** and call them h, k and a (set the a values from -5 to 5). Set both h and k values to zero and a to 1. Copy down what you see.

27 Now set the a value to -1. Compare this graph with the graph in the previous step.

28 Leave the k value on 0 and reset the a value to 1. Change the slider value of h from -5 to 5. Write down what you see.

29 Now set the value of h to 0. Change the slider value of k from -5 to 5. Write down what you see.

30 What happens when neither h nor k is zero? Choose integer values for each of h and k. Write these values down, and write down the equation of the asymptotes that are formed. What do you see?

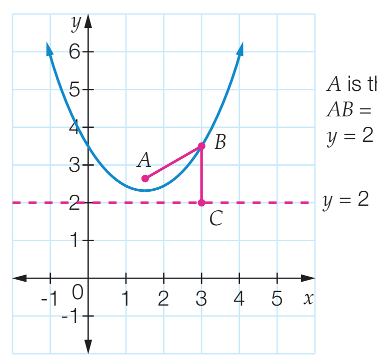
Taking it further

31 Use the ellipse tool to draw an ellipse with the foci on either a horizontal or vertical line. What is different about the equation of an ellipse from that of a circle?

32 Use the hyperbola tool to draw a hyberbola with the foci on either a horizontal or vertical line. What is different about this hyperbola from the hyperbolas you drew in Steps **26**–**30**?

33 What is different about the equation of this hyperbola from the equation of the ellipse in Step **31**?

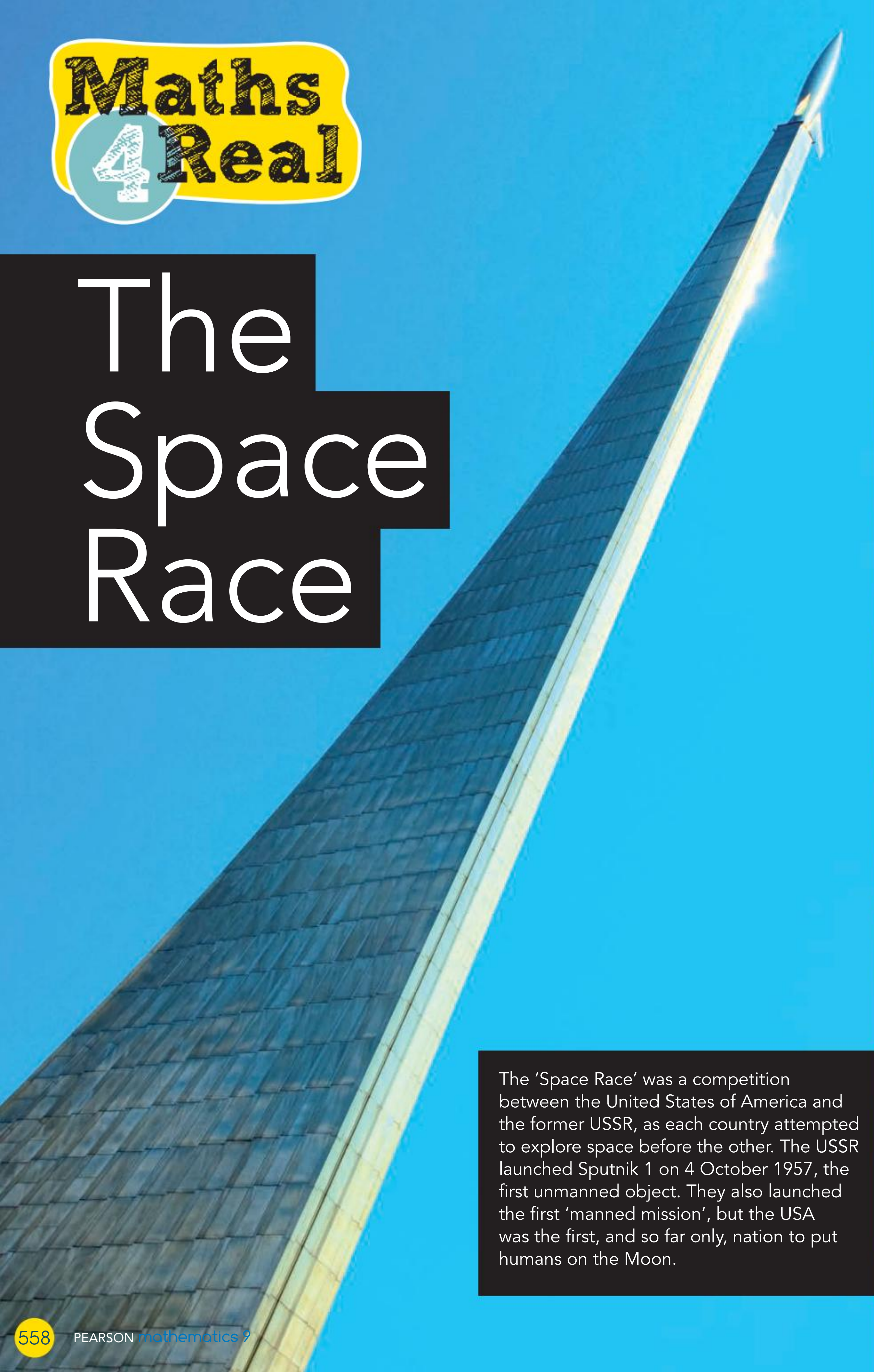
34 Use the parabola tool to draw a parabola. A directrix is a line such that the line from the focus to any point on the parabola is equal to the perpendicular distance from that point to the directrix. For example, in the diagram shown, A is the focus and the directrix is $y = 2$. You can use an axis as a directrix or enter a linear equation in the input box.



A is the focus
$AB = BC$
$y = 2$ is the directrix

Maths 4 Real

The Space Race



The 'Space Race' was a competition between the United States of America and the former USSR, as each country attempted to explore space before the other. The USSR launched Sputnik 1 on 4 October 1957, the first unmanned object. They also launched the first 'manned mission', but the USA was the first, and so far only, nation to put humans on the Moon.

Rocket science

When an object is projected vertically upwards with an initial velocity of *v* metres per second, its approximate height, *h* metres above the ground after *t* seconds is given by the equation $h = vt - 5t^2$.

1 A toy rocket is fired into the air, with the initial velocities given in the table below. Calculate the height above the ground after the times given.

	Initial velocity (m/s)	Time (s)	Height (m)
(a)	10	1	
(b)	50	7	
(c)	25	4	
(d)	40	2	
(e)	1200	120	
(f)	300	60	

2 A toy rocket is fired into the air with an initial velocity of 20 m/s.

(a) Copy and complete this table of values of *h* for the values of *t* shown.

t	0	1	2	3	4	5
h						

(b) Use your completed table of values to plot a graph of *h* and *t*.

(c) Use your graph to determine:

(i) the maximum height of the rocket

(ii) the time the rocket spent in the air.

3 Given that we are working with height above ground, is it sensible to calculate *h* for values of *t* greater than 4?

(a) Substitute the values $h = 0$ and $v = 20$ m/s and factorise the right-hand side of the equation.

(i) Find the solutions to the equation.

(ii) Which two stages of the rocket's journey do the solutions correspond to?

4 Find the time, *t*, when the rocket hits the ground, if it has been fired with an initial velocity of:

(a) 10 m/s **(b)** 100 m/s **(c)** 500 m/s

(d) 1300 m/s **(e)** 1800 m/s **(f)** 2100 m/s

Research

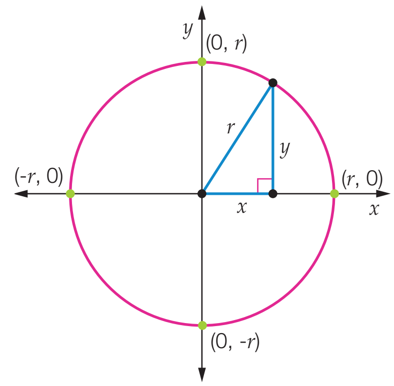
Research one of these landmark events in space exploration. How was mathematics involved in these developments?

Timeline of some major events in space exploration	
4 October 1957	USSR Sputnik 1: First satellite
17 March 1958	USA Vanguard 1: First solar-powered satellite
2 January 1959	USSR Luna 1: First rocket reaching Earth escape velocity
7 August 1959	USA Explorer 6: First photograph of Earth from space
12 April 1961	USSR Vostok 1: First manned spaceship
3 February 1966	USSR Luna 9: First soft landing on the Moon
21 July 1969	USA Apollo 11: Neil Armstrong is the first person on the Moon
23 April 1971	USSR Salyut 1: First space station
15 July 1975	USSR and USA combine in the Apollo–Soyuz Test Project

9.4 Circle relationships

Circles are important non-linear relationships. The important features of a **circle** are:

- the centre with coordinates (h, k)
- the radius, r

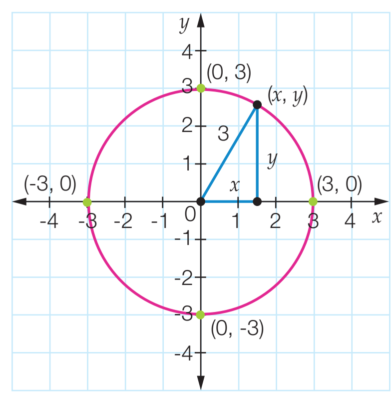


To derive the equation of a circle with the centre at the origin, we use Pythagoras' Theorem. Using the graph of a circle, we can write an equation $x^2 + y^2 = r^2$.

In circle equations, both the x and y variables are squared.

We can find four points on the circle circumference to give maximum and minimum values by adding and subtracting the radius from the centre coordinates.

For a circle with centre (0, 0) and radius r, the four points would be $(r, 0)$, $(0, r)$ $(-r, 0)$ and $(0, -r)$.



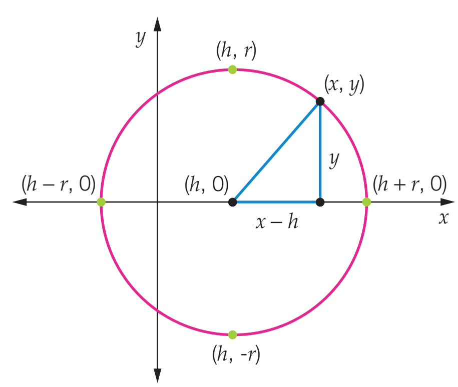
In the diagram shown, the radius is 3 units, so:

$x^2 + y^2 = z^2$
$x^2 + y^2 = 9$

In this case, the four points on the circumference would be (3, 0), (0, 3) (-3, 0) and (0, -3).

Translated circles

A circle equation can be translated horizontally.



If the circle centre is moved h units to the right, it will have the coordinates $(h, 0)$, and all points on the circle will move h units to the right:

$(x - h)^2 + y^2 = r^2$

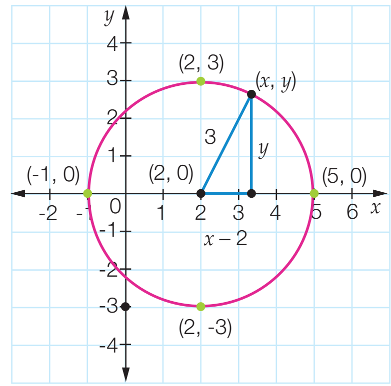
The four points on the circumference would be $(h + r, 0)$, (h, r), $(h - r, 0)$ and $(h, -r)$.

For a radius of 3 units and a centre (2, 0) the equation

$(x-2)^2+y^2=3^2$

$(x-2)^2+y^2=9$

The four points on the circumference would be (5, 0), (2, 3), (-1, 0) and (2, -3).



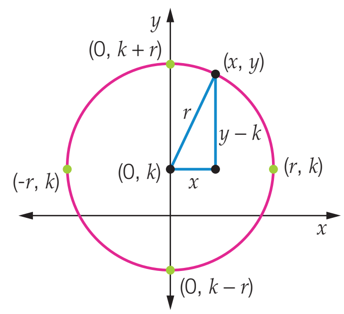
A circle equation can be translated vertically.

If the circle centre is moved k units up, it will have the coordinates (0, k) and all points on the circle will move k units up.

The equation becomes:

$x^2+(y-k)^2=r^2$

The four points on the circumference would be (r, k), (0, $k+r$), ($-r$, k) and (0, $k-r$).

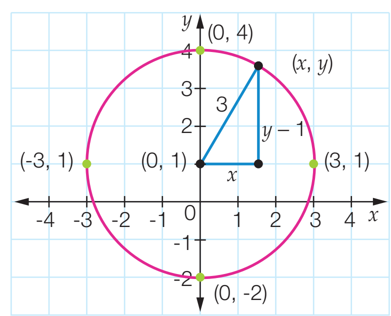


For a radius of 3 units and a centre of (0, 1), the equation of the circle is:

$x^2+(y-1)^2=3^2$

$x^2+(y-1)^2=9$

The four points on the circumference would be (3, 1), (0, 4), (-3, 1) and (0, -2).



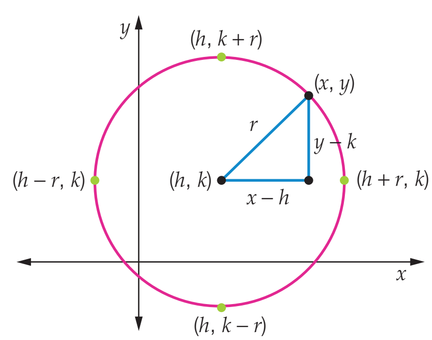
A circle can be translated both horizontally and vertically.

If the circle centre is moved h units to the right and k units up, it will have the coordinates (h, k) and all points on the circle will move h units to the right and k units up.

The equation becomes:

$(x-h)^2+(y-k)^2=r^2$

The four points on the circumference would be ($h+r$, k), (h, $k+r$), ($h-r$, k) and (h, k - r).

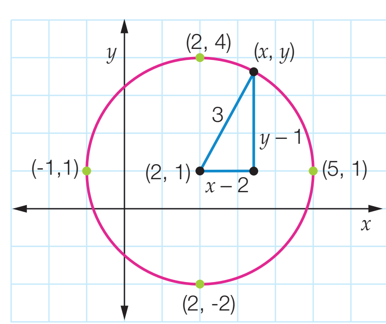


For a radius of 3 units and a centre of (2, 1), the equation will be:

$(x-2)^2+(y-1)^2=z^2$

$(x-2)^2+(y-1)^2=9$

The four points on the circumference would be (5, 1), (2, 4), (-1, 1) and (2, -2).



The equation of a circle with centre (0, 0) and radius r is:

$x^2 + y^2 = r^2$

The general equation of a circle with centre (h, k) and radius r is:

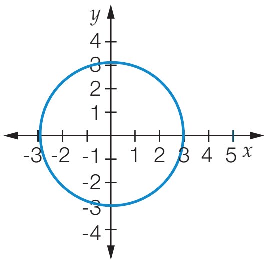
$(x - h)^2 + (y - k)^2 = r^2$

Worked Example 15

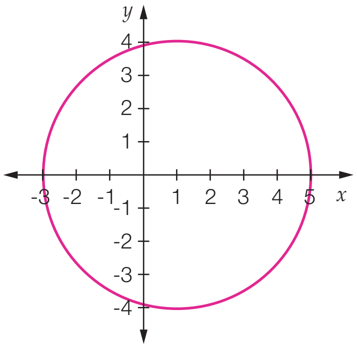
WE15

Determine the centre and radius of each of the following, and hence the equation of the circle.

(a)



(b)



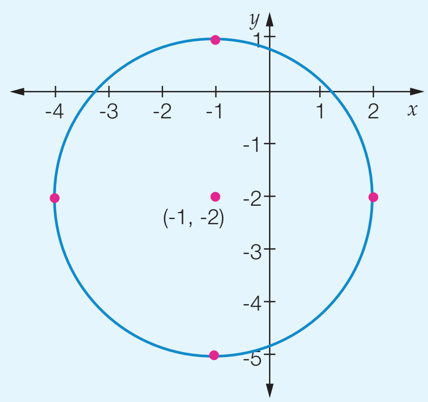
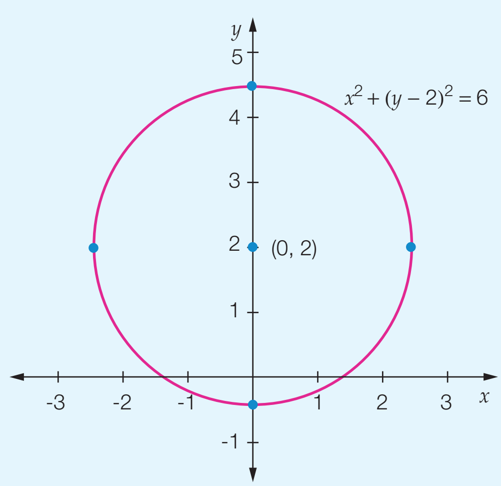
Thinking	Working
(a) 1 Determine the centre by looking at the x- and y-values that are the maximum and minimum; add them together and divide by 2.	(a) $x = \frac{3 + -3}{2} = 0$ $y = \frac{3 + -3}{2} = 0$ Centre: (0, 0)
2 Determine the radius by subtracting the minimum from the maximum for the x- or y-values and then dividing by 2.	Radius: $\frac{3 - -3}{2} = 3$
3 State the equation by substituting for h, k and r in the general form of the equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$.	$(x - 0)^2 + (y - 0)^2 = 3^2$ $x^2 + y^2 = 9$
(b) 1 Determine the centre by looking at the x- and y-values that are the maximum and minimum; add them together and divide by 2.	(b) $x = \frac{5 + -3}{2} = 1$ $y = \frac{4 + -4}{2} = 0$ Centre: (1, 0)
2 Determine the radius by subtracting the minimum from the maximum for the x-and y-values and then dividing by 2.	Radius: $\frac{5 - -3}{2} = 4$
3 State the equation by substituting for h, k and r in the general form of the equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$.	$(x - 1)^2 + (y - 0)^2 = 4^2$ $(x - 1)^2 + y^2 = 16$

Worked Example 16

WE16

Sketch each of the following graphs of circles.

(a) $(x+1)^2 + (y+2)^2 = 9$ **(b)** $x^2 + (y-2)^2 = 6$

Thinking	Working
(a) 1 Determine the centre of the circle and the radius from the equation.	(a) $(x+1)^2 + (y+2)^2 = 9$ Centre: (-1, -2) Radius: $\sqrt{9} = 3$ Points (2, -2) (-1 ,1) (-4, -2) (-1, -5)
2 Set up appropriate axes to fit the graph. **3** Using the radius and the centre, find four points of the circle and then sketch between them.	
(b) 1 Determine the centre of the circle and the radius from the equation.	(b) $x^2 + (y-2)^2 = 6$ Centre: (0, 2) Radius = $\sqrt{6}$ Points: $(\sqrt{6}, 2)$ $(0, \sqrt{6}+2)$ $(-\sqrt{6}, 2)$, $(0, \sqrt{6}-2)$
2 Set up appropriate axes to fit the graph.	≈ (2.45, 2), (0, 4.45), (-2.45, 2), (0, -0.45)
3 Using the radius and the centre, find four points of the circle and then sketch between them. If the radius is not rational, use an approximation.	

A circle can be sketched by plotting the four points where the radius can be added vertically or horizontally to the centre point. The other points of a circle are difficult to plot by hand.

Worked Example 17

WE17

A path around an area on a coordinate map is described by the equation $(x-3)^2 + (y-2)^2 = 9$.

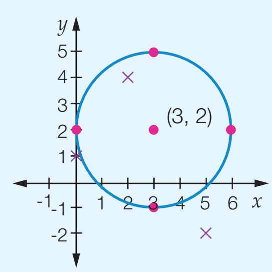
(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

(i) (0, 1) **(ii)** (2, 4) **(iii)** (5, -2)

Thinking	Working
(a) Compare the given equation to the general equation of a circle $(x-h)^2 + (y-k)^2 = r^2$ to determine h, k and r.	(a) Centre (3, 2) and radius = 3
(b) Add and subtract the radius onto the h value to find the two points on the circle in line with the centre horizontally. Add and subtract the radius to the k value to find the two points on the circle in line with the centre vertically.	(b) (0, 2), (6, 2) (3, -1) (3, 5)
(c) Plot the points on the circumference on Cartesian coordinates.	(c) 
(d) Use your graph to decide whether the given points are inside or outside the circle.	(d) (i) (0, 1) is outside the circle. (ii) (2, 4) is inside the circle. (iii) (5, -2) is outside the circle.

9.4 Circle relationships

Navigator

Answers page 663

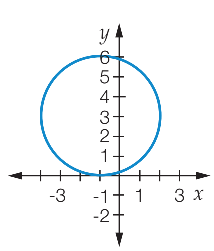
Q1, Q2, Q3, Q4, Q5, Q6, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11

Fluency

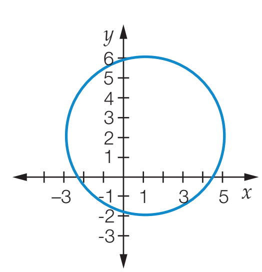
WE15

1 Determine the centre and radius of each of the following, and hence the equation of the circle.

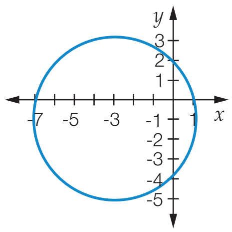
(a)



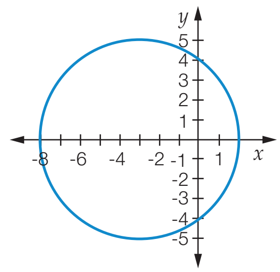
(b)



(c)



(d)



2 Sketch each of the following graphs of circles. WE16

(a) $(x+2)^2+(y+3)^2=25$

(b) $(x-2)^2+(y-1)^2=16$

(c) $(x+1)^2+(y-1)^2=4$

(d) $x^2+(y-1)^2=9$

(e) $(x+2)^2+y^2=7$

(f) $x^2+y^2=18$

3 A path around an area on a coordinate map is described by the equation $(x+1)^2+(y-3)^2=16$. WE17

(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

(i) (1, 2) **(ii)** (3, 6) **(iii)** (-2, 0)

Understanding

4 Which of the following equations would graph as a circle?

A $x^2+y=4$ **B** $x^2+(y-3)^2=5$ **C** $y^2=x^2+9$ **D** $y^2=x-3$

5 Write down the equation if the circle with equation $x^2+y^2=36$ undergoes the following transformations.

(a) a move of 4 units to the right and 3 units up

(b) a move of 6 units to the left and 4 units up

(c) a move of 7 units to the right

(d) a move of 5 units down

(e) a dilation to a radius of 7

6 What are the centre coordinates and radius of a circle with the equation $(x-5)^2+(y+3)^2=81$?

A (5, 3), 9 **B** (-5, 3), 9 **C** (5, -3), 81 **D** (5, -3), 9

Reasoning

7 A circle has an equation $(x+4)^2+(y+2)^2=25$. What would be the equation of this circle if it is translated 2 units left and 6 units up and the radius is decreased by 1 unit?

8 **(a)** Show, by substitution, that the equations $(x-8)^2+(y+8)^2=169$, $(x+9)^2+(y-9)^2=169$ and $x^2+y^2=25$ are three circles that intersect at (3, 4) and (-4, -3).

(b) Draw a graph of each of these three circles on the same set of axes.

9 A circular path with a radius of 5 metres has its centre 2 metres east and 3 metres north of a sculpture. It has been decided to move the path 9 metres further east and 1 metre further north and that its radius is to be increased by 8 metres.

(a) Write an equation to represent the original circle if the sculpture is positioned at the origin.

(b) Write an equation to represent the circle after it has been moved.

(c) Show that a point that is 1 metre west and 1 metre south of the sculpture lies on both circles.

Open-ended

10 (a) Write down two equations that would graph as circles with a centre at the origin.

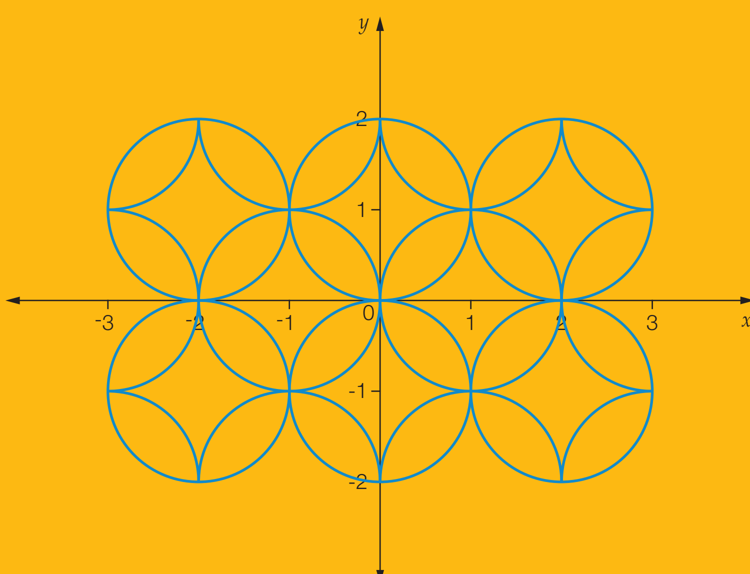
(b) Write down two equations that would graph as circles with the same radius but with centres in different quadrants.

11 Make a design with three overlapping circles and write down the equations for the circles you used in your design.

Outside the Square | Problem solving

Circles and circles

Find the equations for the eight full circles in the pattern shown.



Strategy options

- Draw a diagram.
- Solve a simpler problem.
- Look for a pattern.
- Guess and check.

Exponentials and hyperbolas EXTENSION

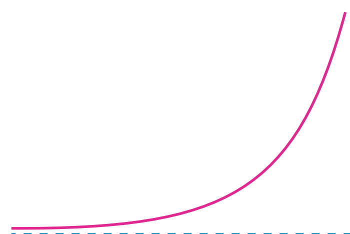
9.5

Exponential relationships

Consider cell division: one cell divides into two, and each of these cells then divides into two. Now there are four cells. If this process continues, four cells become eight, eight become 16 and so on. This process is called **exponential** growth and, in the case of cell division, is represented by the equation $y = 2^x$.

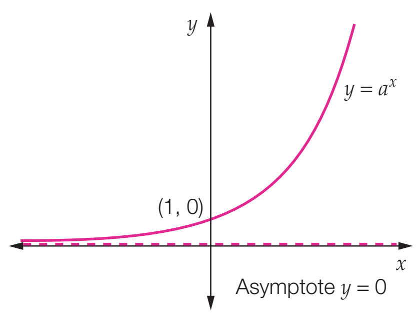
An exponential relationship is a relationship between two quantities that involves an exponent (or index), where the exponent is a variable. Many naturally occurring events involve exponential growth or decay. Exponential graphs always go in the same direction, either continually getting bigger or continually getting smaller, but, unlike straight lines, there is a limiting feature to this. Exponential graphs will always have a value that they will get closer and closer to but never reach. This value is called the **asymptote** of the graph.

An exponential curve looks like this:



The simplest exponential equations are in the form $y = a^x$ where a is the base and is any non-zero number and x is the index or exponent.

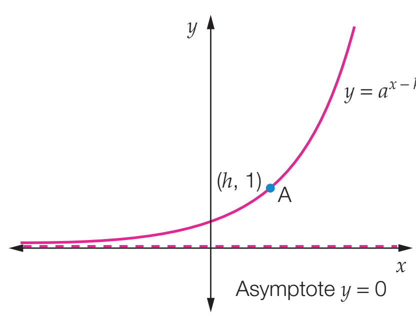
Because $a^0 = 1$, for all values of a except 0, all these exponential graphs will have a y-intercept of (0, 1). As x gets smaller (i.e. negatively large), the value of y gets closer and closer to 0, but it never reaches it. The line with the equation $y = 0$ (the x-axis) is the asymptote.



Translations of exponential graphs

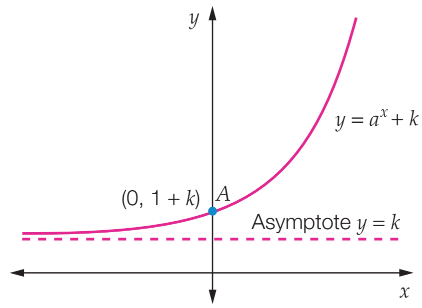
An exponential graph with equation $y = a^x$ can be translated k units to the right to give the equation $y = a^{x-h}$.

The asymptote remains $y = 0$.

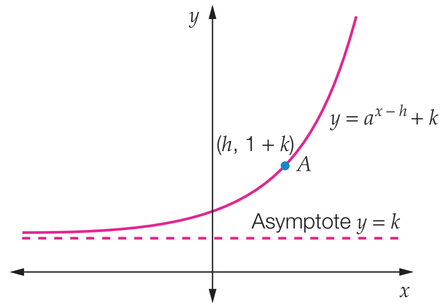


An exponential graph with an equation $y = a^x$ can be translated k units up to give the equation $y = a^x + k$.

Here the asymptote becomes $y = k$.



An exponential graph with the equation $y = a^x$ can be translated h units to the right and k units up to give the equation $y = a^{x-h} + k$.



The general equation of an exponential relationship is $y = a^{x-h} + k$, where

- a is the base
- x is the exponent
- h indicates the translation to the right
- k indicates the translation upwards
- $y = k$ is the asymptote

Worked Example 18

WE18

Plot the following graphs by setting up tables of values from -3 to 3 inclusive.

(a) $y = 2^x$ **(b)** $y = 2^{x+1}$ **(c)** $y = 2^x + 3$

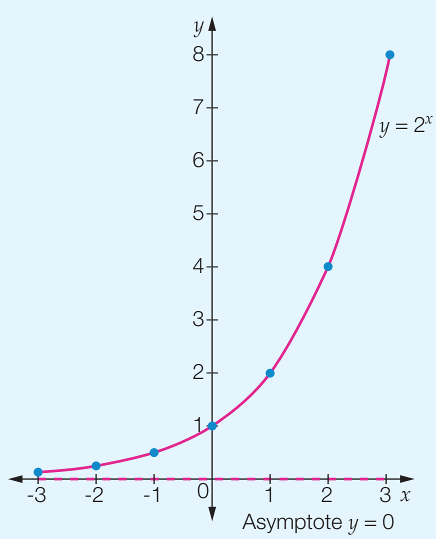
Thinking / Working

(a) 1 Set up the table and replace x with each value in the equation, taking care with negative indices.

(a)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

2 Plot the points and join them with a smooth curve.

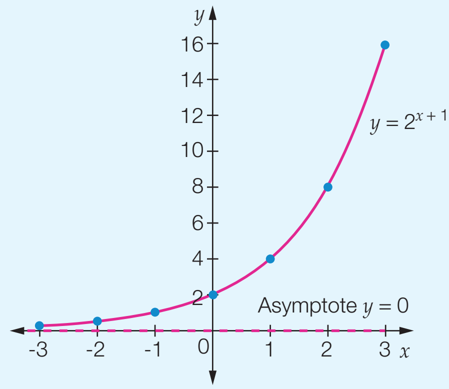


(b) **1** Set up the table and replace x with each value in the equation, taking care with negative indices.

(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

2 Plot the points and join them with a smooth curve.

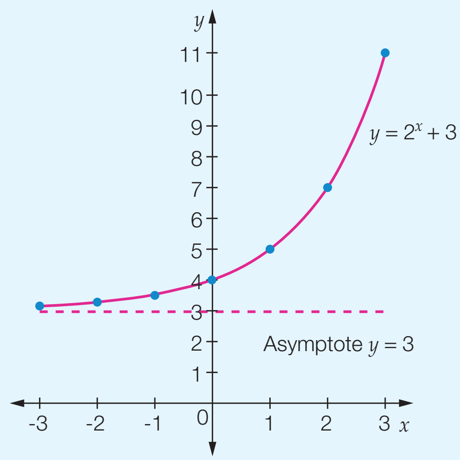


(c) **1** Set up the table and replace x with each value in the equation, taking care with negative indices.

(c)

x	-3	-2	-1	0	1	2	3
y	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	5	7	11

2 Plot the points and join them with a smooth curve.



Worked Example 19

WE19

Bacteria in a culture are multiplying rapidly. Each bacterial cell divides into two, then each of these cells divides into two and so on. This can be expressed in the form $N = 2^t$ where N is the number of bacteria and t is the time in hours.

(a) For the relationship $N = 2^t$, copy and complete the table of values below for t and N.

t	0	1	2	3	4	5	6
N							

(b) Use this table of values to plot a graph of $N = 2^t$. Join the points with a smooth curve.

(c) Use your graph to determine how long it takes for the number of bacteria to exceed 40.

(d) Evaluate N for $t = 5.3$ (give your answer correct to the nearest integer) and explain why this value is meaningless in this situation.

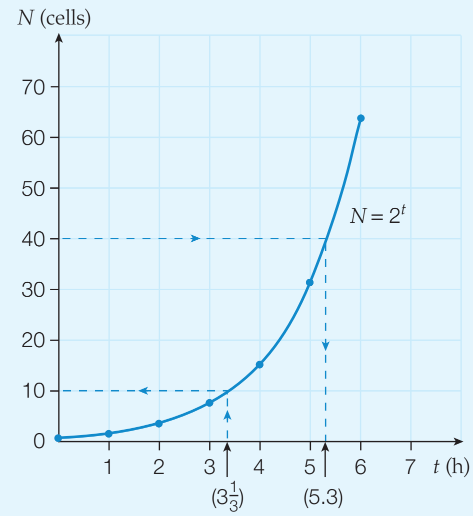
(e) Determine how many bacteria there are after 3 hours 20 minutes.

Thinking	Working
(a) Substitute for t in $N = 2^t$ to fill in the table.	(a)

t	0	1	2	3	4	5	6
N	1	2	4	8	16	32	64

(b) Plot points, using t for the horizontal axis and N for the vertical axis.

(b)



(c) Using the graph, draw a horizontal line from $N = 40$ to the curve, then drop a vertical line.

(c) From the graph, when $N = 40$, $t = 5.3$ hours. However, it takes a full hour for the 32 cells present at $t = 5$ to become 64 cells, so $t = 6$.

(d) Calculate $2^{5.3}$ using your calculator.

(d) The answer using a calculator is ≈ 39.4, but cell division only takes place at the end of each hour, so this value is meaningless in this situation.

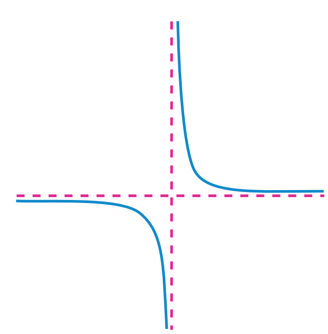
(e) Change the time into fractional units, then move up to the curve and across to the vertical axis.

(e) At 3 hours 20 minutes there will still only be 8 cells, as the next cell division does not occur until $t = 4$.

Rectangular hyperbolas

The simplest rectangular hyperbola is represented by the equation $y = \frac{1}{x}$.

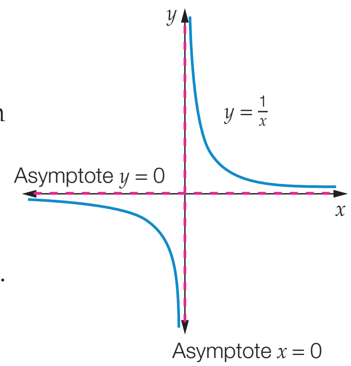
A hyperbola takes the following shape, with both horizontal and vertical asymptotes.



A table of values for the equation $y = \frac{1}{x}$ is shown below.

x	-100	-10	-5	-1	-0.1	-0.01	0	0.01	0.1	1	5	10	100
y	-0.01	-0.1	-0.2	-1	-10	-100	cannot be evaluated	100	10	1	0.2	0.1	0.001

Plotting points, we can see there are two separate parts to the graph. When $x = 0$, the y-value cannot be evaluated. When x is just greater than 0, y is extremely large. When x is just a little smaller than 0, y is extremely small (negatively large). The graph is discontinuous at $x = 0$ and there is an asymptote at $x = 0$, the y-axis.

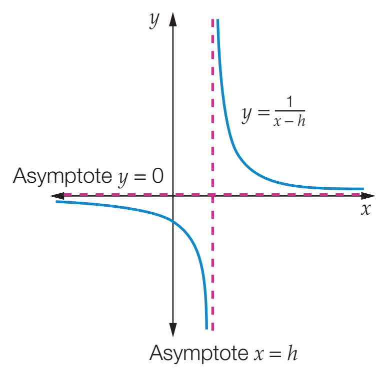


There is also no solution to the equation $0 = \frac{1}{x}$. When x is extremely large, y becomes extremely small but never reaches 0. When x is extremely small (i.e. negatively large) y becomes extremely small but negative. Once again, it can never reach 0. The line $y = 0$, or the x-axis, is also an asymptote.

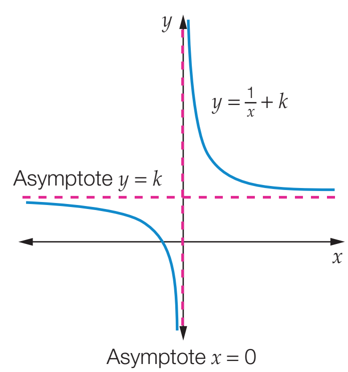
A graph of this relationship is called a **rectangular hyperbola**.

Translations of rectangular hyperbolas

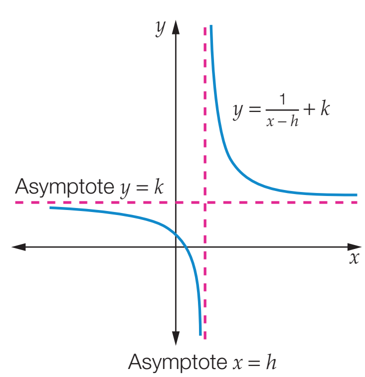
$y = \frac{1}{x-h}$ is the graph of $y = \frac{1}{x}$ translated h units to the right.



$y = \frac{1}{x} + k$ is the graph of $y = \frac{1}{x}$ translated k units upwards.



$y = \frac{1}{x-h} + k$ is the graph of $y = \frac{1}{x}$ translated h units to the right and k units upwards.



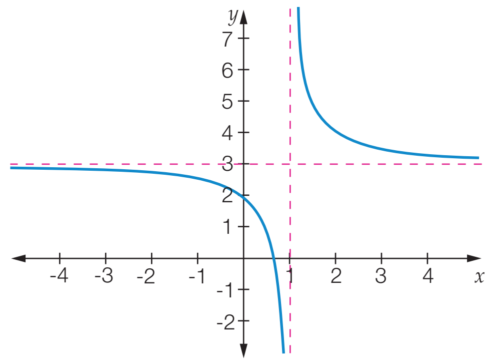
The general equation of a rectangular hyperbola is $y = \frac{a}{x-h} + k$, where a is the dilation factor, h is the translation to the right and k is the translation upwards.

Worked Example 20

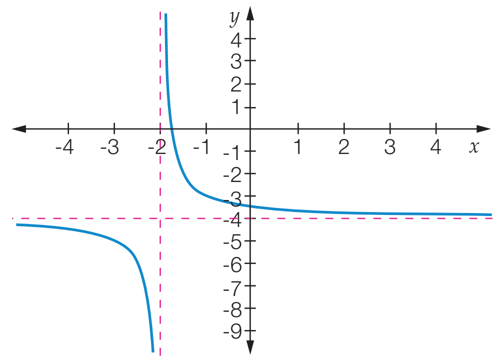
WE20

Determine the equation of each of the following hyperbolas from their graphs. Use $a = 1$ in the general equation $y = \dfrac{a}{x - h} + k$.

(a)



(b)



Thinking	Working
(a) 1 Determine the equations of the asymptotes.	(a) Asymptotes: $x = 1$, $y = 3$ $a = 1$
2 Use the equations of the asymptote to identify h and k. (Here $h = 1$ and $k = 3$.)	$h = 1$ and $k = 3$
3 Use the values of h and k in the general equation $y = \dfrac{a}{x - h} + k$.	Equation: $y = \dfrac{1}{x - 1} + 3$
(b) 1 Determine the equations of the asymptotes.	(b) Asymptotes: $x = -2$, $y = -4$ $a = 1$
2 Use the equations of the asymptote to identify h and k. (Here $h = -2$ and $k = -4$.)	$h = -2$ and $k = -4$
3 Use the values of h and k in the general equation $y = \dfrac{a}{x - h} + k$.	Equation: $y = \dfrac{1}{x - -2} + -4$ $y = \dfrac{1}{x + 2} - 4$

9.5 Exponentials and hyperbolas

EXTENSION

Navigator

Answers page 664

Q1, Q2, Q3, Q4, Q5, Q6, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9

Fluency

WE18

1 Plot the following graphs by setting up tables of values from -3 to 3 inclusive.

(a) $y = 3^x$ **(b)** $y = 3^{x+1}$ **(c)** $y = 3^x - 2$

2 A gardener decided to propagate his orchid (make new plants). He did this by dividing the bulb on his plant after it finished flowering into three. He then planted these smaller bulbs giving him three orchids in total. He continued to do this at the same time every year with every orchid he had grown from the original orchid. This situation can be modelled by the equation $P = 3^t$ where P is the number of orchids he has and t is the time in years. WE19

(Assume that a healthy orchid grows from every divided bulb and all orchids survive for at least 6 years.)

(a) For the relationship of orchids to the time in years where $P = 3^t$, copy and complete the table of values below.

t (years)	0	1	2	3	4	5	6
P (orchids)							

(b) Use this table of values to plot a graph of $P = 3^t$. Join the points with a smooth curve.

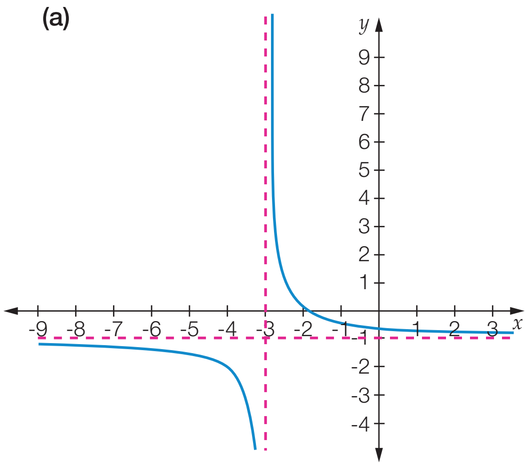
(c) Use your graph to determine how many years it takes for the number of orchids to exceed 125.

(d) Evaluate P for $t = 3.8$ (give your answer correct to the nearest integer) and explain why this value is meaningless in this situation.

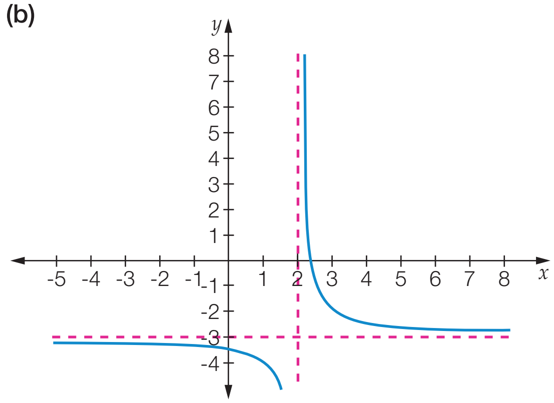
(c) Determine how many orchids there are after $5\frac{1}{2}$ years.

3 Determine the equation of each of the following hyperbolas from their graphs. Use $a = 1$ in the general equation $y = \dfrac{a}{x-h} + k$. WE20

(a)



(b)



Understanding

4 **(a)** Plot the graphs of $y = 2^x$ and $y = 3^x$ on the same set of axes.

(b) By comparing asymptotes, y-intercepts and shape, describe the similarities and differences between the two graphs.

5 Which of the following equations will not graph as a hyperbola?

A $y = \dfrac{2}{x+5} - 6$ **B** $y = 3 + \dfrac{2}{x}$ **C** $x(y-3) = 1$ **D** $y = \dfrac{2}{x^2} - 1$

6 An exponential graph of $y = 5^x$ is shifted 4 units left and 3 units down. Which of the following is the equation of the new graph?

A $y = 5^{(x-4)} + 3$ **B** $y = 5^{(x-4)} - 3$ **C** $y = 5^{(x+4)} + 3$ **D** $y = 5^{(x+4)} - 3$

Reasoning

7 The table below gives values for the temperature of the metal surface of an electric iron as it cools. The temperature, T, is in degrees Celsius and the time, m, is in minutes.

m	0	2	4	6	8	10
T	160	108	76	55	42	34

(a) Use graph paper to plot a graph of T versus m. (Put m along the horizontal axis.) Join the points with a smooth curve.

(b) What is the initial temperature of the metal surface of the iron?

(c) What is the temperature of the iron after 3 minutes?

(d) When is the temperature of the iron 50°C?

(e) Extend the curve. Predict the temperature of the iron after 12 minutes.

8 The population of a town has been increasing at a rate of 10% each year since the beginning of 2005. This can be modelled by the equation $P = 2000 \times 1.1^t$ where P is the number of people in the town t years after the beginning of 2005. Initially, when $t = 0$, the population numbered 2000.

(a) Set up a table to show the population of the town for the first 10 years after 2005.

(b) Draw a graph showing this information.

(c) From the graph, approximately how long did it take for the town to double in size?

(d) Jason said it would take 10 years for the population to double. Explain why this isn't the case.

Open-ended

9 **(a)** Substitute your own values for h and k into the equation of an exponential in the form of $y = 2^{(x-h)} + k$ and draw a graph of this relationship.

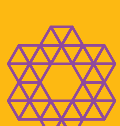
(b) Substitute your own values for h and k into the equation of a hyperbola in the form of $y = \dfrac{1}{x-h} + k$ and draw a graph of this relationship.

Outside the Square — Problem solving

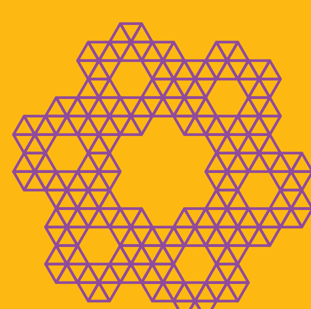
How many triangles?

Six equilateral triangles are joined to make a hexagon.

Six hexagons are joined to make the hexagon pattern as shown.



Six of these hexagon patterns are joined to make a larger pattern.



Six of these larger hexagon patterns are joined in the same way to make an even larger pattern. How many triangles would there be in this pattern?

Strategy options

- Look for a pattern.
- Make a table.

Direct proportion

9.6

Many types of relationships between quantities occur frequently in real life. The simplest type of relationship between two quantities is one in which any change in one quantity results in an identical change to the other quantity. For example, if we double one quantity, the other will double also; halve one quantity and the other will halve also. In this case we say that one quantity is in direct proportion to the other quantity.

The relationship between two quantities, say a and b, is one of **direct proportion** when a increases as b increases and the graph of the relationship passes through the origin.

A simple example of direct proportion is the relationship between distance and speed. For example, if we double the speed of travel, the distance covered in a specified time interval will double also; if we halve the speed, the distance travelled will halve also. In this case the graph showing direct variation is linear and passes through the origin. An object that has no speed is stationary and covers no distance.

$a \propto b$ means 'a is directly proportional to b'.

$a \propto b$ is not the same as $a = b$.

Consider this example. Rhonda and Michael sail their cruiser from Queenscliff at a speed of 30 km/h. After 1 hour they have travelled 30 km, after 2 hours they have travelled 60 km, and after 3 hours they have travelled 90 km. This can be shown in a table.

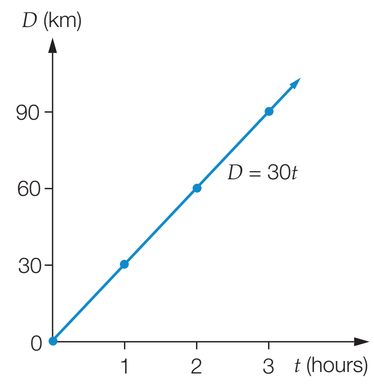
Time (hours)	0	1	2	3
Distance (km)	0	30	60	90

By considering the ratio $\frac{D}{t}$ for each pair of values we find that a constant value is obtained. This value, k, is called the **constant of proportionality**. (Note, we are assuming k is positive.)

In this case, $\frac{D}{t} = k = 30$, so $D = 30t$.

A graph of distance, D, versus time, t, can be drawn.

Notice that the graph of D versus t is a straight line that passes through the origin, (0, 0). As t increases, D increases at a proportional rate. We say that the distance is in direct proportion to time, i.e. $D \propto t$. In other words, the relationship between D and t is one of direct proportion.



In all examples of direct proportion, as one variable increases so does the other. Not all increasing trends are necessarily examples of direct proportion, but if one variable *decreases* as the other increases, it is *not* direct proportion.

Worked Example 21

WE 21

Which of the following could be examples of direct proportion?

(a)

x	1	3	6	8
y	2	6	12	18

(b)

s	1	2	3	5
t	2	8	18	50

(c)

v	1	2	3	4
w	4	2	1	0.5

(d)

x	1	2	3	4
y	5	7	7	3

Thinking	Working
(a) y is increasing as x is increasing.	(a) Could be direct proportion
(b) t is increasing as s is increasing.	(b) Could be direct proportion
(c) w is decreasing as v is increasing.	(c) Not direct proportion
(d) q is fluctuating as p is increasing.	(d) Not direct proportion

It may be that the direct proportion occurs between one quantity and another quantity raised to a positive power. The relationship is of the form $y \propto x^n$ or, as an equation, $y = kx^n$.

Consider this table of values:

x	0	1	2	3
y	0	3	12	27

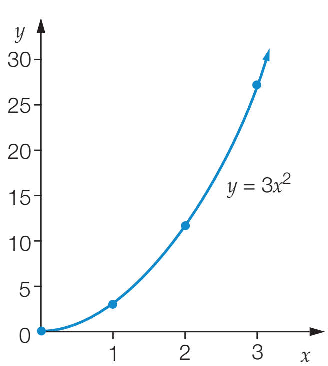
By calculating the ratio of each pair of values we find that k is not constant, even though the relationship is one of direct proportion since y increases as x increases and it passes through the origin. This relationship may be of the type $y \propto x^n$.

Set up a new table using x^2 instead of x.

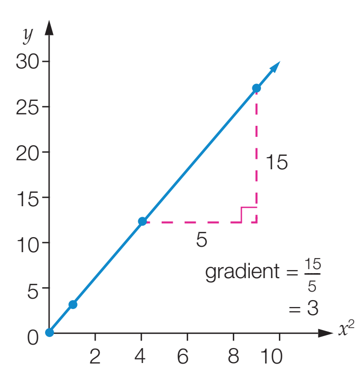
x^2	0	1	4	9
y	0	3	12	27

By calculating k for each pair of values we find that $\frac{y}{x^2} = k = 3$, so $y = 3x^2$.

By plotting the graph y against x we get a curve, indicating that y is not directly proportional to x.



However, by plotting y against x^2 we get a straight line passing through the origin with a gradient of 3. That is, $y \propto x^2$, so y is directly proportional to x^2.



In general, there is direct proportion between two variables, x and y, when

$y \propto x^n$ where n is any positive number;

or, as an equation, when

$y = kx^n$ where k is the constant of proportionality

Two variables are in direct proportion if the graph of y plotted against x^n is a linear graph passing through the origin, (0, 0).

As x increases, y increases.

We can use this knowledge to find the equation that represents the relationship between two variables. A *direct relationship* exists if a graph of one variable against another to some power results in a straight line through the origin. A *linear relationship* exists if the graph is a straight line.

Worked Example 22

WE22

Find the equation for each of the following tables of values.

(a)

x	1	3	6	8
y	2	6	12	16

(b)

s	0	1	2	3	5
t	0	2	8	18	50

(c)

a	0	1	4	9
b	0	3	6	9

Thinking	Working
(a) 1 Determine the ratio $\frac{y}{x}$ for each pair of values.	(a) $\frac{y}{x} = \frac{2}{1} = \frac{6}{3} = \frac{12}{6} = \frac{16}{8} = 2$ $\therefore k = 2$
2 The ratio is constant, so y is directly proportional to x.	$y \propto x$
3 Write the equation.	$y = kx$
4 Substitute $k = 2$ into the equation.	$y = 2x$
(b) 1 Determine the ratio $\frac{t}{s}$ for each pair of values.	(b) $\frac{t}{s} = \frac{2}{1} = 2$, $\frac{t}{s} = \frac{8}{2} = 4$, $\frac{t}{s} = \frac{18}{3} = 6$, $\frac{t}{s} = \frac{50}{5} = 10$
2 The ratio is not constant, so t is not directly proportional to s. As there is a pair of values (0, 0), the relationship may be of the form $t \propto s^n$. Notice that the ratio is increasing, so $n > 1$. Set up a table of values of t and s^2.	see table below

s^2	0	1	4	9	25
t	0	2	8	18	50

	3 Determine the ratio $\frac{t}{s^2}$ for each pair of values.	$\frac{t}{s^2} = \frac{2}{1} = \frac{8}{4} = \frac{18}{9} = \frac{50}{25} = 2$ $\therefore k = 2$
	4 The ratio is constant, so t is directly proportional to s^2.	$t \propto s^2$
	5 Write the equation.	$t = ks^2$
	6 Substitute $k = 2$ into the equation.	$t = 2s^2$
(c)	**1** Determine the ratio $\frac{b}{a}$ for each pair of values.	(c) $\frac{b}{a} = \frac{3}{1} = 3$, $\frac{b}{a} = \frac{6}{4} = \frac{3}{2}$, $\frac{b}{a} = \frac{9}{9} = 1$
	2 The ratio is not constant, so b is not directly proportional to a. As there is a pair of values (0, 0), the relationship may be of the form $b \propto a^n$. Notice that the ratio is decreasing, so $0 < n < 1$. Set up a table of values of b and $\sqrt{a}$.	see table below
	3 Determine the ratio $\frac{b}{\sqrt{a}}$ for each pair of values.	$\frac{b}{\sqrt{a}} = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = 3$ $\therefore k = 3$
	4 The ratio is constant, so b is directly proportional to $\sqrt{a}$.	$b \propto \sqrt{a}$
	5 Write the equation.	$b = k\sqrt{a}$
	6 Substitute $k = 3$ into the equation.	$b = 3\sqrt{a}$

$\sqrt{a}$	0	1	2	3
b	0	3	6	9

9.6 Direct proportion

Navigator

Answers page 666

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Fluency

WE21

1 Which of the following could be examples of direct proportion?

(a)

v	1	2	4	5
w	7	3	2	1

(b)

p	1	2	3	4
q	3	4	5	0

(c)

x	1	2	3	4
w	0	1	2	4

(d)

s	1	2	6	5
t	5	10	30	25

2 Find the equation for each of the following tables of values.

WE22

(a)

a	0	1	2	3
b	0	4	8	12

(b)

w	0	1	2	3
v	0	5	10	15

(c)

c	0	1	4	6
d	0	0.5	2	3

(d)

g	0	4	6	8
h	0	1	1.5	2

(e)

e	0	1	2	3
h	0	3	6	9

(f)

p	0	2	4	6
q	0	3	6	9

(g)

i	1	2	3	0
j	2	8	18	0

(h)

a	0	1	2	3
d	0	3	12	27

(i)

m	0	1	2	3
n	0	4	32	108

(j)

g	1	2	3	0
h	2	16	54	0

(k)

p	0	1	2	3
q	0	0.5	2	4.5

(l)

b	0	1	2	3
f	0	0.2	0.8	1.8

(m)

r	0	1	2	3
s	0	3	24	81

(n)

p	0	1	2	3
m	0	0.25	2	6.75

(o)

t	0	1	2	3
u	0	0.5	4	13.5

(p)

v	0	1	4	9
w	0	2	4	6

(q)

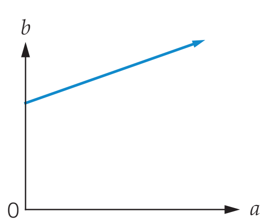
u	0	1	4	9
t	6	0.1	0.2	0.3

(r)

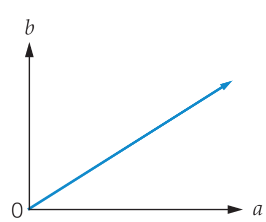
d	0	1	4	9
l	0	$\frac{1}{3}$	$\frac{2}{3}$	1

3 State whether or not each graph could be an example of direct proportion.

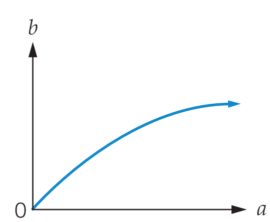
(a)



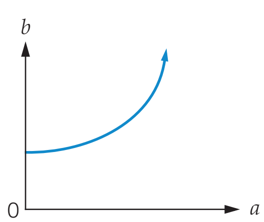
(b)



(c)



(d)



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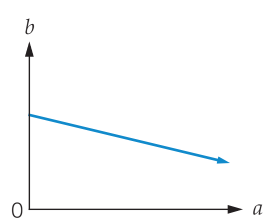
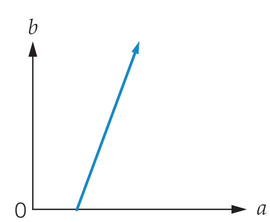


(f)



4 Express each of these relationships involving direct proportion as an equation, using k as the constant.

(a) $m \propto n$

(b) $a \propto b^2$

(c) $g \propto h^3$

(d) t is proportional to w.

(e) N is proportional to p^2.

(f) y is proportional to the cube of x.

(g) V is proportional to the square of r.

(h) L is proportional to the square root of d.

5 (a) The volume of a sphere (V) is directly proportional to the cube of its radius (r). This may be written as:

A $V \propto r^2$ B $V \propto r^3$ C $V \propto r$ D $V \propto \sqrt{r}$

(b) The radius of a cylinder (r) is directly proportional to the square root of the volume (V). This may be written as:

A $r \propto \sqrt{V}$ B $r \propto V^2$ C $r \propto V^3$ D $r \propto V$

6 State the value of the constant of proportionality in each of the following.

(a) $p = 3q$ (b) $a = 0.2b^2$ (c) $M = \frac{1}{4}n$ (d) $y = 200\sqrt{x}$

(e) $c = n^5$ (f) $F = \frac{a}{6}$ (g) $x = \frac{w^3}{20}$ (h) $P = \frac{r^4}{35}$

7 In each of the following table of values find the constant of proportionality and hence find the missing values in the table.

(a) $p \propto q$

q	5	7	9	
p	15	21		33

(b) $m \propto n^2$

n	1	3	5	
m	7		175	252

(c) $d \propto c^3$

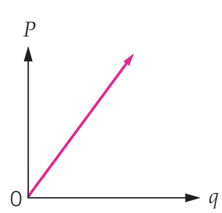
c	1		4	5
d	1	8		125

(d) $w \propto \sqrt{x}$

x	1		16	49
w	3	6	12	

8 Which graph shows direct proportion of the form $p \propto q^2$?

A



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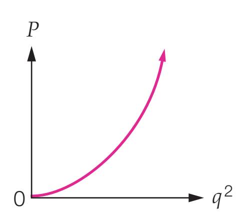
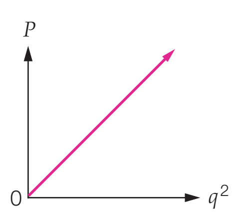
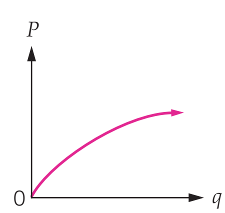


C



D



Understanding

9 A student measures the current and voltage of a simple electrical circuit to determine the value of a resistor within the circuit. The results obtained are shown in the table.

Current, *I* (amps)	0.001	0.002	0.003	0.004
Voltage, *V* (volts)	3	6	9	12

(a) Graph the relationship between I amps and V volts. Show I on the horizontal axis and V on the vertical axis.

(b) Calculate the constant of proportionality, in this case called the resistance R (ohms).

(c) Calculate the voltage if 0.01 amps of current is flowing through the circuit.

(d) What is the current flowing through the circuit when a voltage of 45 volts is measured in the circuit?

10 The following table shows the relationship between the surface area of a sphere and its radius.

Radius, *r* (cm)	2	7	15	20	50
Surface area, *A* (cm^2)	50.24	615.44	2 826	5 024	31 400

(a) Determine the constant of variation for this relationship.

(b) Express this relationship as an equation using $\pi = 3.14$.

Reasoning

11 The distance to the horizon is proportional to the square root of the height above sea level. A man standing on a cliff 32 m above sea level can see a distance of 18 km.

(a) How far could he see if he was standing on the viewing platform of a lighthouse 18 m above the cliff top?

(b) What height above sea level would he need to be to see a distance of 100 km?

12 The following table shows the circumference, surface area and volume of spheres of various radii, in similar units.

Radius, r	Circumference, C	Surface area, A	Volume, V
2	12.56	50.24	33.49
4	25.12	200.96	267.95
6	37.68	452.16	904.32
8	50.24	803.84	2143.57
10	62.80	1256.00	4186.67
12	75.36	1808.64	7234.56

(a) Find the ratio $\frac{C}{r}$, $\frac{A}{r}$ and $\frac{V}{r}$ for each pair of values.

(b) Which of the ratios is constant? Describe your results.

(c) Plot graphs of C against r, A against r and V against r.

(d) Plot a graph of A against r^2. Describe the graph.

(e) Plot a graph of V against r^3. Describe the graph.

(f) Which of the graphs show direct proportion between the quantities? Write a relationship for each of these.

(g) Calculate the constant of proportionality, k, for each of these relationships. Express each as an equation in terms of π (where $\pi = 3.14$).

Open-ended

13 **(a)** Write two different relationships of the type $y = kx^n$, where $n > 0$ and $0 < k < 1$. Use the same value of n for both of your relationships.

(b) Sketch the graph of $y = x^n$ for your chosen value of n and, on the same set of axes, sketch the two relationships you have written. Describe what happens as k decreases from 1 to 0.

Outside the Square Problem solving

Relationships between variables

Determine the relationship between x and y for each of these tables.

1

x	0	1	2	3	4
y	0	3	12	27	48

2

x	0	1	2	3	4
y	1	5	25	125	625

Strategy options

- Look for a pattern.
- Guess and check.

Inverse proportion EXTENSION

9.7

It is possible that an increase in one quantity will result in the *decrease* of the other quantity, or vice versa.

When a gas is confined to a certain volume, it exerts a pressure on the sides of the container. If the volume of the container increases, the pressure exerted by the gas decreases. If the volume of the container decreases, the pressure exerted by the gas increases. The product of the pressure and volume stays constant: $PV = K$ where P is the pressure, V is the volume and K is a constant. This can written as $P = \frac{K}{V}$.

Consider the relationship between average speed and time of travel. If we double the average speed of travel, we will halve the time taken to travel a certain distance. Similarly, if the average speed is divided by, say, a factor of 4, it will take four times as long to travel a certain distance. This type of proportion is called **inverse proportion** and the graph of P plotted against V will be a 1st-quadrant **hyperbola**, which is a non-linear graph.

For example, Rhonda and Michael decide to take a trip in their cruiser from Queenscliff to Sorrento, following the coast of Port Phillip Bay. The trip is 200 km long. The speed they will sail at depends on the time they wish the trip to take.

For example, if the trip is to take 5 hours, the average speed of the cruiser must be $\frac{200}{5}$ or 40 km/h.

For 10 hours, the average speed must be $\frac{200}{10}$ or 20 km/h.

For 20 hours, the average speed must be $\frac{200}{20}$ or 10 km/h.

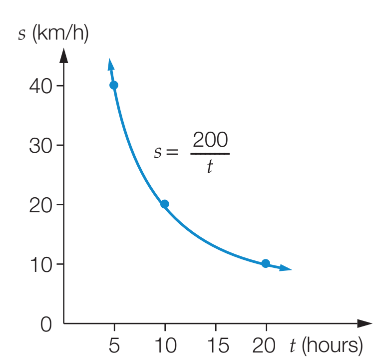
This can be shown in a table:

Time (hours)	5	10	20
Average speed (km/h)	40	20	10

For a trip of t hours, the average speed of the cruiser must be $\frac{200}{t}$ km/h.

For average speed s we can write the formula $s = \frac{200}{t}$.

A graph of s plotted against t can be drawn.



The shape of the graph is called a hyperbola.

As t increases, s decreases.

We say that the average speed *is inversely* proportional to the time, i.e. $s \propto \frac{1}{t}$.

In other words, the relationship between s and t is one of inverse proportion.

Note that only the 1st quadrant of a rectangular hyperbola is graphed.

The formula $s = \frac{200}{t}$ or $s = 200 \times \frac{1}{t}$ has a constant of proportionality of 200.

This constant of proportionality, k, can be found using the product of the two variables, i.e. $st = k = 200$.

In all examples of inverse proportion, as one variable increases the other decreases. Not all decreasing trends are necessarily examples of inverse proportion, but if one variable *increases* as the other increases, it is *not* an inverse proportion.

Worked Example 23

WE 23

Which of the following could be examples of inverse proportion?

(a)

a	1	3	6	8
b	2	6	12	16

(b)

c	1	2	3	5
d	10	5	3	1

(c)

e	1	2	3	4
f	4	2	1	0.5

Thinking	Working
(a) b is increasing as a is increasing.	(a) Not inverse proportion
(b) d is decreasing as c is increasing.	(b) Could be inverse proportion
(c) f is decreasing as e is increasing.	(c) Could be inverse proportion

In inverse proportionality, we say that one variable varies inversely as another variable that is raised to a power. The simplest case is the one in which this power is 1. If y is inversely proportional to x,

$$y \propto \frac{1}{x} \text{ where '}\propto\text{' means 'is proportional to',}$$

or, as an equation:

$$y = \frac{k}{x} \quad \text{or} \quad xy = k, \text{ where } k \text{ is the constant of proportionality}$$

This means that when x is doubled, y is halved. If x is tripled, y is divided by 3.

In general, there is inverse proportion between two variables, x and y, when

$$y \propto \frac{1}{x^n} \text{ where } n \text{ is any positive number;}$$

or, as an equation, when

$$y = \frac{k}{x^n} \quad \text{or} \quad x^n y = k, \text{ where } k \text{ is the constant of proportionality}$$

As x increases, y decreases. The graph of an inversely proportional relationship is a hyperbola.

Worked Example 24

WE 24

Find the equation of the relationship between the variables in each of the following tables of values.

(a)

x	5	10	15	20	30
y	60	30	20	15	10

(b)

F	1	2	5	10
d	200	50	8	2

Thinking		Working
(a) 1	As x increases, y decreases, so the relationship could be inverse proportion.	(a) $y \propto \frac{1}{x^n}$
2	Calculate xy to see whether there is a constant of proportionality.	$5 \times 60 = 300$ $10 \times 30 = 300$ $15 \times 20 = 300$ $20 \times 15 = 300$ $30 \times 10 = 300$
3	If the product gives a constant, the variables are inversely proportional and the constant value calculated is the constant of proportionality.	$y \propto \frac{1}{x}$ and $k = 300$
4	Express as an equation.	$y = \frac{k}{x}$ or $xy = k$
5	Substitute the value of k into the equation. Here $k = 300$.	$y = \frac{300}{x}$ or $xy = 300$
(b) 1	As F increases, d decreases, so the relationship could be inverse proportion.	(b) $d \propto \frac{1}{F^n}$
2	Calculate the product of each pair of values to see whether there is a constant of proprotionality.	$1 \times 200 = 200$ $2 \times 50 = 100$ $5 \times 8 = 40$ $10 \times 2 = 20$
3	If the product does not give a constant, set up a new table using F^2 and d.	(see table below)
4	Repeat Step 2 using F^2 and d.	$1 \times 200 = 200$ $4 \times 50 = 200$ $25 \times 8 = 200$ $100 \times 2 = 200$
5	If the product gives a constant, the variables are inversely proportional and the constant value calculated is the constant of proportionality.	$d \propto \frac{1}{F^2}$ and $k = 200$
6	Express as an equation.	$d = \frac{k}{F^2}$ or $dF^2 = k$
7	Substitute the value of k into the equation. Here $k = 200$.	$d = \frac{200}{F^2}$ or $dF^2 = 200$

F^2	1	4	25	100
d	200	50	8	2

9.7 Inverse proportion EXTENSION

Navigator

Answers page 667

Q1, Q2, Q3, Q4, Q5 Column 1, Q6, Q7, Q11	Q1, Q2, Q3, Q5 Column 2, Q6, Q7, Q8, Q10, Q11	Q1, Q2, Q3, Q5 Column 3, Q6, Q8, Q9, Q10, Q11

Fluency

WE23

1 Considering general trends, which of the following could be examples of inverse proportion?

(a)

v	1	2	4	5
w	7	3	2	1

(b)

p	1	2	3	4
q	3	4	5	0

(c)

x	1	2	3	4
y	24	12	8	6

(d)

s	1	2	3
t	9	2.25	1

WE24

2 Find the equation of the relationship between the variables in each of the following tables of values.

(a)

a	1	2	4	5
b	40	20	10	8

(b)

c	3	4	5	6
d	20	15	12	10

(c)

e	3	6	9	12
f	72	36	24	18

(d)

g	4	8	16	24
h	36	18	9	6

(e)

k	1	2	4	10
l	240	60	15	2.4

(f)

m	1	2	4	6
n	144	36	9	4

3 Express each of these relationships involving inverse proportion as an equation, using k as the constant.

(a) $p \propto \frac{1}{q}$ (b) $y \propto \frac{1}{x^2}$ (c) $a \propto \frac{1}{\sqrt{b}}$

(d) t is inversely proportional to w.

(e) V is inversely proportional to h^2.

(f) f is inversely proportional to the cube of g.

(g) L is inversely proportional to the square root of m.

(h) a is inversely proportional to the square root of d.

4 (a) The cost per passenger on a bus tour (c) is inversely proportional to the number of passengers on the tour (n). This may be written as:

A $c \propto n$ B $c \propto n^2$ C $c \propto \frac{1}{n}$ D $c \propto \frac{1}{n^2}$

(b) The thickness (t) of a pizza is inversely proportional to the square of its radius (r). This may be written as:

A $t \propto r$ B $t \propto r^2$ C $t \propto \frac{1}{r}$ D $t \propto \frac{1}{r^2}$

(c) The radius, r, of a cylinder is inversely proportional to the square root of its height, h. This may be written as:

A $r \propto h$ B $r \propto \frac{1}{h}$ C $r \propto \sqrt{h}$ D $r \propto \frac{1}{\sqrt{h}}$

5 State the value of the constant of proportionality in each of the following.

(a) $y = \frac{7}{x}$ (b) $a = \frac{6.3}{b^2}$ (c) $d = \frac{12}{g}$

(d) $p = \frac{0.97}{q^2}$ (e) $w = 5 \times \frac{1}{\sqrt{n}}$ (f) $B = 3 \times \frac{1}{\sqrt{c}}$

(g) $M = \frac{1}{2p^3}$ (h) $L = \frac{1}{5a^3}$ (i) $g = \frac{1}{0.4h}$

(j) $w = \frac{1}{1.6x}$ (k) $V = \frac{1}{0.08c^2}$ (l) $n = \frac{1}{1.25q^3}$

Understanding

6 y is inversely proportional to the cube of x. If $y = 16$ when $x = 2$, find:

(a) y when $x = 5$ (b) x when $y = 2$.

7 Average speed is inversely proportional to the time taken to travel a given distance. At an average speed of 100 km/h, a certain journey takes 2 hours to complete. However, due to a roadworks speed limit the average speed was reduced to 40 km/h. How long did it now take to travel this distance?

8 The frequency of waves of electromagnetic radiation is inversely proportional to their wavelength. If gamma rays have a frequency of 3×10^{22} Hz with a wavelength of 1×10^{-14} m, what is the wavelength of ultraviolet waves with a frequency of 1.5×10^{16} Hz?

9 The gravitational force of attraction between two bodies is inversely proportional to the square of their distance apart. The force between two objects 40 km apart is 85 units. What is the distance between the two objects when the force is 340 units?

Reasoning

10 The following results were obtained during an experiment measuring light intensity, I, from a light source on an object a distance d metres away from the light.

Distance, *d*	1	2	3	4
Intensity, *I*	720	180	80	45

(a) Plot the graph of light intensity (I) against distance (d).

(b) Plot the graph of light intensity (I) against $\frac{1}{d}$.

(c) Is I inversely proportional to d? Justify your answer.

(d) Copy and complete the following table.

d	1	2	3	4	6	8	12
d^2	1						
I	720	180	80	45			

(e) Calculate the value d^2I for each pair of values. Is this value constant?

(f) Write an equation showing the relationship between I and d.

(g) Plot the graph of I against $\frac{1}{d^2}$. Comment on the result.

(h) What is the effect on the value for I when d is doubled?

(i) What is the effect on the value for I when d is trebled?

(j) What is the effect of halving the value of d?

Open-ended

11 Write three pairs of coordinate points for three different equations of the type $y = \frac{k}{x^n}$. One pair of coordinate points must be common to all three equations.

Outside the Square — Problem solving

The mass of Earth

In 1611, Johannes Kepler (1571–1630) proposed a law of variation relating the radius (R) of orbit and time (T) in years for one orbit (called the period) of a planet or satellite orbiting the same central body (e.g. the Sun). R can be measured in gigametres, 1×10^9 m.

- The Earth orbits the Sun every year and its radius of orbit is 228 gigametres.
- The Moon orbits the Earth every 27.3 days and its radius of orbit is 380 000 km.

Kepler's law of planetary motion states that for planets or satellites orbiting the same central body, $\frac{R^3}{T^2} =$ a constant. The constant is written in physics texts as: $\frac{GM}{4\pi^2}$ where $G = 6.67 \times 10^{-11}$ and M is the mass of the central body (in kg). Use Kepler's law to calculate the mass of the Earth.

Strategy options

- Break problem into manageable parts.
- Guess and check.

Challenge 9

1 The solutions of the equation $x^2 - 13x - 48 = 0$ are:

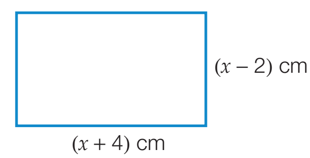
A -3, 16 B 3, -16 C 4, -12 D -6, 8

2 How many positive integers less than or equal to 3000 can be written down without using the digits 4, 5 or 6?

3 If $(x - 2)(2x + 3) = 0$, then possible values for $2x + 3$ are:

A 0 only B 0 and 3 C 0 and 7 D $-\frac{3}{2}$ and $-\frac{7}{2}$

4 The perimeter of the rectangle in the diagram is 112 cm. Its total area is:



A 44 cm^2 B 187 cm^2

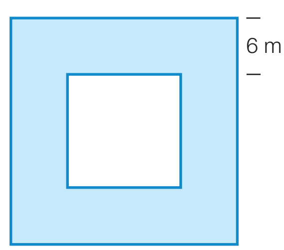
C 775 cm^2 D 3127 cm^2

5 If $u \# v$ means $2u - v$, what is the value of $6 \# (2 \# 5)$?

6 The mean of the solutions of the quadratic equation $x^2 + 10x - 11 = 0$ is:

A -10 B -5 C 5 D 10

7 A 6-m wide strip is mown around the inside edges of a square lawn. If eight-ninths of the lawn has been mowed (shaded area), calculate the area, in square metres, that still has to be mown (unshaded area).



8 Two three-digit numbers, *abc* and *def*, have the following property:

$$\begin{array}{cccc} & a & b & c \\ + & d & e & f \\ \hline 1 & 0 & 0 & 0 \end{array}$$

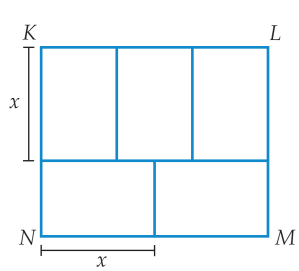
None of a, b, c, d, e and f is zero. Therefore, the value of $a + b + c + d + e + f$ is:

A 10 B 19 C 21 D 28

9 If $x + \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ equals:

A 25 B 23 C 27 D 3

10 Five identical rectangles are arranged to form a larger rectangle *KLMN*, as shown. The area of *KLMN* is 4000 cm^2. The length, x cm, of each of the identical rectangles is closest to:



A 33 cm B 35 cm

C 37 cm D 39 cm

Chapter review 9

D.I.Y. Summary

Key Words

asymptote	dilation factor	parabola	translation
axis of symmetry	direct proportion	quadratic equation	turning point
circle	exponential	quadratic relationship	vertex
coefficient	hyperbola	rectangular hyperbola	
constant of proportionality	inverse proportion	reflection	
dilation	Null Factor Law	transformation	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 A ___________ is the type of graph produced by a quadratic relationship.

2 A ___________ is an expression where the highest power of x is 2 and the highest power of y is 1.

3 The ___________ passes through the ___________ of a quadratic graph.

4 Another name for the turning point is the ___________.

5 The graph $y = x^2$ can be changed into $y = 2(x - 1)^2 + 3$ by undergoing a number of ___________s.

6 The graph $y = x^2$ is a ___________ of the graph $y = -x^2$.

7 The equation $(x - 3)^2 + (y + 4)^2 = 9$ graphs as a ___________ with a ___________ of 3.

8 The main features that a sketch curve of a quadratic relationship has are: y-intercept, x-intercepts and the ___________.

9 Using the ___________ it is possible to find the solutions for x when an equation equals 0.

10 The graph of $y = \frac{1}{x}$ is called a ___________. EXTENSION

Fluency

1 Plot the graph of $y = x^2 - 2x - 8$. Ex. 9.1

(a) Write down the coordinates of the x- and y-intercepts.

(b) Write down the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(c) Write down the equation of the axis of symmetry.

(d) From the graph, find the value of y when $x = 3$.

(e) What are the values of x when $y = 7$?

(f) Hence state the values of x when $x^2 - 2x - 8 = 7$.

(g) Write down the values of x when $x^2 - 2x - 8 = 0$.

2 Solve the following using the Null Factor Law. Ex. 9.2

(a) $2x(x + 4) = 0$ (b) $(x - 7)(x + 1) = 0$ (c) $x^2 - 16 = 0$

(d) $x^2 + 6x + 9 = 0$ (e) $x^2 = 8x$ (f) $x^2 + 36 = 12x$

3 Solve these quadratic equations. **Ex.9.2**

(a) $x^2 - 4x = 0$ **(b)** $x^2 - 6 = 0$ **(c)** $x^2 + 12 = 0$

(d) $3x^2 = 27$ **(e)** $(x - 3)(x + 7) = 11$ **(f)** $(x - 3)(x + 2) = 6$

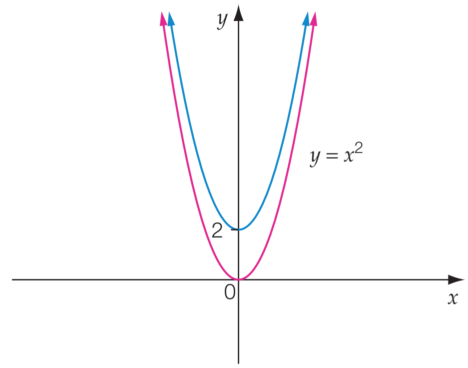
4 **(a)** The equation of the unlabelled parabola shown could be: **Ex.9.3**

A $y = x^2 + 2$

B $y = x^2 - 2$

C $y = (x - 2)^2$

D $y = (x + 2)^2$



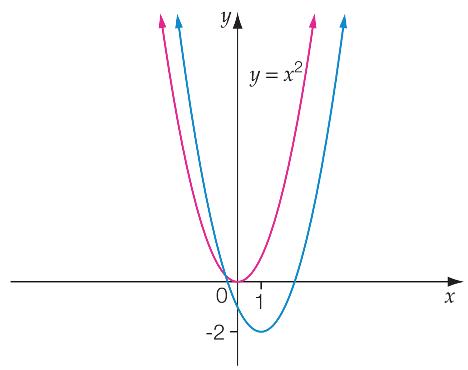
(b) The equation of the unlabelled parabola shown could be:

A $y = (x - 1)^2 - 2$

B $y = (x - 1)^2 + 2$

C $y = (x + 1)^2 + 2$

D $y = (x + 1)^2 - 2$



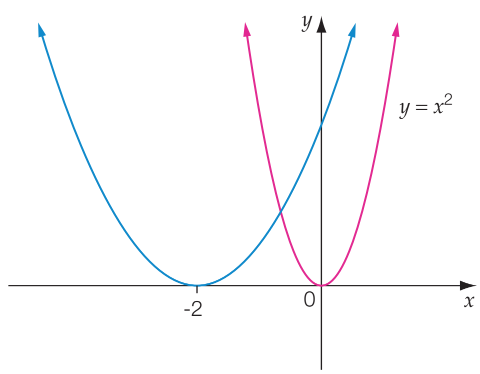
(c) The equation of the unlabelled parabola shown could be:

A $y = 2(x - 2)^2$

B $y = \frac{1}{2}(x + 2)^2$

C $y = \frac{1}{2}(x - 2)^2$

D $y = 2(x + 2)^2$



5 State the changes that should be made to the graph of $y = x^2$ to obtain the graph of each of the following. **Ex.9.3**

		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x - 1)^2 + 3$				
(b)	$y = -x^2 + 4$				
(c)	$y = -(x + 2)^2 - 1$				
(d)	$y = 3(x - 4)^2 + 2$				
(e)	$y = \frac{1}{2}(x - 1)^2 + 1$				
(f)	$y = -2(x + 2)^2 - 2$				

6 Sketch the graphs of: **Ex.9.3**

(a) $y = x^2 + 8x + 16$ **(b)** $y = -x^2 + 2x - 1$

(c) $y = 3x^2 + 9x$ **(d)** $y = x^2 - 4x + 4$

7 (a) The graph of the quadratic relationship $y = x^2 - 64$ crosses the x-axis at: **Ex 9.3**

A $x = -8$ and $x = 8$ B $x = 8$ C $x = -8$ D $x = 64$

(b) The graph of the quadratic relationship $y = x^2 - 8x + 16$ crosses the x-axis at:

A $x = 2$ and $x = 8$ B $x = -4$ and $x = 4$ C $x = 4$ D $x = -4$

8 Sketch graphs for each of the following circle equations. **Ex 9.4**

(a) $x^2 + (y - 2)^2 = 25$ (b) $(x + 3)^2 + (y - 1)^2 = 16$

9 Sketch graphs for each of the following equations showing the asymptotes. EXTENSION **Ex 9.5**

(a) $y = \dfrac{1}{x+2} - 1$ (b) $y = 2^{x-1} + 3$

10 Find the equation for each of the following tables. **Ex 9.6**

(a)

a	1	3	4
b	3	9	12

(b)

c	0	4	6
d	0	2	3

(c)

g	0	4	9
h	0	6	9

11 (a) The volume of a cube is directly proportional to the cube of its height. This may be written as: **Ex 9.6**

A $V \propto H^3$ B $V = H^3 + 1$ C $V \propto H^{1/3}$ D $V = H^{1/3}$

(b) The surface area of a cylinder is directly proportional to the square of its radius. This may be written as:

A $A = r^2 + 3r$ B $A \propto r^2$ C $A \propto \sqrt{r}$ D $A \propto \dfrac{1}{r^2}$

(c) The profit of a company is directly proportional to the square root of its number of employees. This may be written as:

A $p \propto n^2$ B $p = n^2$ C $p = \sqrt{n^2}$ D $p \propto \sqrt{n}$

12 Find the equation for each table of values. EXTENSION **Ex 9.7**

(a)

j	1	2	4
k	12	6	3

(b)

p	1	4	9
q	6	3	2

13 (a) The flexibility of a ball is inversely proportional to the cube root of its volume. This may be written as: EXTENSION **Ex 9.7**

A $f \propto V^3$ B $f \propto \dfrac{1}{V^3}$ C $f \propto \sqrt{V}$ D $f \propto \dfrac{1}{\sqrt[3]{V}}$

(b) The intensity of candlelight is inversely proportional to the square of the distance from the candle. This may be written as:

A $I \propto d^2$ B $I = d^2$ C $I \propto \dfrac{1}{d^2}$ D $I = \dfrac{1}{\sqrt{d}}$

Understanding

14 A softball thrown by an outfielder to the pitcher followed a parabolic path where its height, h metres, at any time, t seconds, is given by the relationship:

$$h = -5t^2 + 10t + 1$$

(a) Complete a table of values for this relationship.

(b) Plot the graph of this relationship on graph paper.

(c) What was the height of the softball after 0.5 seconds?

(d) When was the ball first at a height of 5 m above the ground?

(e) During which time interval was the ball above a height of 2 m?

(f) What was the maximum height of the softball during its flight?

(g) How long was the ball in flight if it was caught at a height of 1.6 m above the ground on its downward path?

15 A jet of water from an outlet level with the surface of a pond follows a parabolic path. The height of the water at any point can be described by the equation:

$$h = -d^2 + 6d$$

where h is the height of the water a horizontal distance d from the water outlet. Both h and d are in metres.

(a) Sketch the graph of $h = -d^2 + 6d$.

(b) From your sketch, state the maximum height of the water jet above the surface of the pond.

(c) What horizontal distance does the jet of water cover from its outlet back to the surface of the pond?

16 Describe the transformation that has been performed when $(x + 2)^2 + (y - 1)^2 = 25$ has been changed to $(x - 4)^2 + (y - 3)^2 = 16$.

17 **(a)** Show that the graphs of $y = 2^{x+1} + 5$ and $y = 3^{x+2} - 2$ have the same y-intercept. EXTENSION

(b) Write down the equation of the asymptotes for each graph.

Reasoning

18 Di and Sam found the following table of values, but some values were missing. Di could not see a pattern, but Sam thought she could see one.

x	1	2	3	4	5	
y	0.1		2.7		12.5	21.6

(a) Is there a relationship between the two values? If so, write the relationship.

(b) Use the relationship to fill in the missing values.

19 Leonard and Faye want to develop a vegetable garden in their backyard. They have 20 m of chicken wire and enough posts to erect a fence around the four sides of the rectangular garden.

(a) If the length of the rectangular garden is x m, find an expression (in terms of x) for the width of the garden.

(b) Show that the area of the garden, A m^2, is given by the quadratic expression $A = -x^2 + 10x$.

(c) Sketch the graph of the area of the vegetable garden, A m^2, against the length of the garden, x m. (Show length on the horizontal axis.)

(d) What should the dimensions of the garden be if the area is to be as large as possible?

(e) What is the area of the largest vegetable garden that Leonard and Faye can make?

20 A book cover has an area of 64 cm^2. If the length is x cm and the length and width add to 16 cm, find the dimensions of the book cover.

21 The sum of two numbers is 50. What are the numbers if their product is to be as large as possible?

22 A parabola has a minimum turning point at (3, 5) and a dilation factor of 1. What is the equation of the curve?

23 A circle has an equation $(x + 1)^2 + (y - 3)^2 = 121$. What would be the equation of this circle if it is translated 4 units right and 3 units down and the radius is decreased by 2 units?

24 $v^2 = u^2 + 2as$ is a formula used in motion. If $u = 0$, the formula becomes $v^2 = 2as$. If a is a constant:

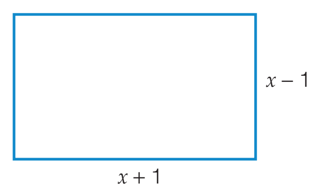
(a) what happens to v when s is doubled?

(b) what happens to s when v is reduced to $\frac{3}{4}$ of its original value?

NAPLAN practice 9

Numeracy: Non-calculator

1 Find the value of x in the following rectangle given that the area is 8 m^2.



2 Here is a table of values for x and y.

x	-3	-2	-1	0	1
y	18	8	2	0	2

Which of the following is a correct rule for y in terms of x?

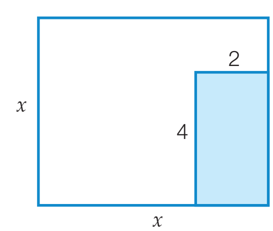
A $y = 6x$ **B** $y = 4x$ **C** $y = x^2 + 3$ **D** $y = 2x^2$

3 The solution/s to $x^2 - 4 = 0$ is/are:

A $x = -2$ **B** x = 0 **C** $x = 2$ and -2 **D** $x = 2$

Numeracy: Calculator allowed

4 Jane drew this plan for her square block of land. (All measurements are in metres.)



The shaded area shows the garage. Apart from the garage, what is the area of the land?

A $x^2 + 4x$ **B** $(x + 4)(x - 2)$

C $(x - 4)(x - 2)$ **D** $x^2 - 8$

5 Complete the table of values below that would be used to plot points for the relationship $y = x^2 - 3x + 2$.

x	-3	-2	-1	0	1	2	3
y							

6 If $x = 2$, then which of these expressions will be equal to 6?

A $x^2 + 5x - 3$ **B** $(x + 1)(x - 4)$ **C** $4x^2 - 2x + 3$ **D** $-x^2 + 4x + 2$

Mixed review D

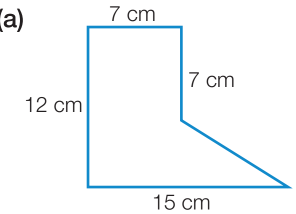
Fluency

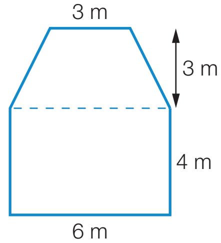
1 An investment of \$5000 is made at 3.5% p.a. for 6 years. Find the difference in interest earned if interest is compounded monthly compared with annually. **Ex.1.5**

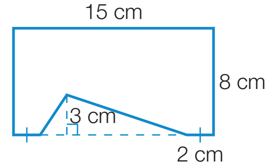
2 Factorise the following quadratic expressions. **Ex.3.8**

(a) $6a^2 - 24$ (b) $3a^2 + 6a + 3$ (c) $x^2 + 5x - 7x - 35$

3 Find the area of each of the following shapes. **Ex.4.2**

(a)


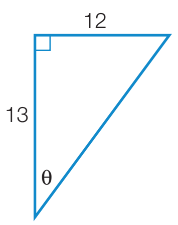
(b)


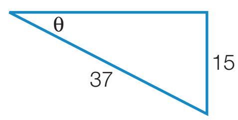
(c)


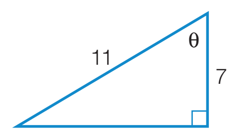
4 Solve the following equations. **Ex.5.1**

(a) $\dfrac{3a+4}{7} = -2$ (b) $\dfrac{4a+3}{2} = \dfrac{2a}{5}$ (c) $\dfrac{3a-2}{3} = \dfrac{4a-5}{2}$

5 Find the value of the pronumeral in each of the following, correct to one decimal place. **Ex.7.4**

(a)


(b)


(c)


6 Find the mean for each of the following grouped data frequency tables. Give your answers correct to two decimal places. **Ex.8.3**

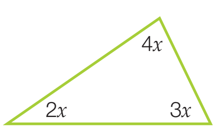
(a)

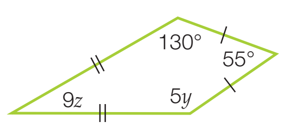
Class interval	Frequency
20–<25	8
25–<30	6
30–<35	3
35–<40	2
40–<45	5
45–<50	5

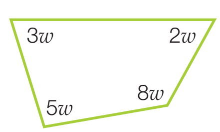
(b)

Class interval	Frequency
111–<115	12
116–<120	15
121–<125	17
126–<130	19
131–<135	13
136–<140	11

7 Find the value of the pronumeral in each figure. **Ex.5.2**

(a)


(b)


(c)


8 Solve each of the following for x. **Ex.9.2**

(a) $2x^2 - 50 = 0$ (b) $4x^2 + 2x = 0$ (c) $x^2 - 6x + 9 = 0$

9 Sketch the graphs of each of the following, clearly showing the turning point.

(a) $y = (x + 3)(x - 2)$ **(b)** $y = x^2 + 2x + 5$ **(c)** $y = 2x^2 - 4x$

Ex 9.1

10 Sketch the graph of each of the following quadratics, showing all important features.

(a) $y = x^2 + 6x - 4$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = -2x^2 + 3x$

Ex 9.1

11 Consider the equation $y = 2x^2 + x$.

Ex 9.1

(a) Draw up and fill in a table of values with x-values from -3 to 3.

(b) Use the table of values to draw the graph of the equation.

(c) Find the turning point of the graph.

(d) Determine the x- and y-intercepts for the graph.

Understanding

12 Jodie is standing on the top of a sand dune and spots a dolphin in the water. If the distance from Jodie to the dolphin along her line of sight is 100 m, find the angle of depression from Jodie to the dolphin if she is 4.3 m above sea level. Answer to one decimal place.

13 The 26 letters of the alphabet are written on 26 identical cards and placed in a bag. A card is chosen at random. What is the probability that the card chosen is:

(a) a vowel

(b) one of the letters in the word 'card'

(c) not a letter in the word 'letter'?

Reasoning

14 A parabola has x-intercepts at $x = 4$ and $x = 6$.

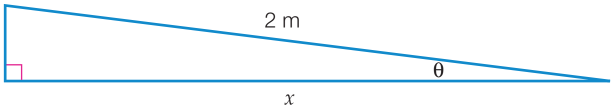
(a) Determine the x-coordinate of the turning point.

(b) If the coefficient of x^2 is 1, determine the equation of the graph.

15 A book cover has an area of 60 cm^2. If the width is 7 cm less than the length, find the dimensions of the book cover.

16 To meet Australian standards, ramps for wheelchair access that are longer than 152 cm must only rise 1 cm vertically for every 14 cm horizontal distance.

(a) What is the largest angle, θ, between the ramp and the horizontal that can be used to build a wheelchair ramp 2 m long? Answer to the nearest degree.



(b) Use your answer to **(a)** to find:

(i) the horizontal length, x

(ii) the height of the top of the ramp. Answer to the nearest mm.

Answers

Worked solutions and answers to *all* activities appear in the Teacher Companion.

Chapter 1

Recall 1 (p. 4)

1 (a) 0.17 (b) 0.057 (c) 0.00321

2 (a) 0.64 (b) 1.125 (c) 0.575
(d) $0.1\dot{6}$ (e) $0.\overline{90}$ (f) $0.\overline{714285}$

3 (a) $\frac{79}{100}$ (b) $\frac{3}{1000}$ (c) $\frac{7}{500}$
(d) $\frac{213}{200}$ (e) $\frac{507}{50}$ (f) $\frac{451}{5000}$

4 (a) 8.7 (b) 9.1 (c) 7.8 (d) $206\frac{2}{3}$

5 (a) 70% (b) 91% (c) 3.2% (d) 82.14%

6 (a) $\frac{17}{20}$ (b) $\frac{11}{50}$ (c) $\frac{67}{1000}$ (d) $\frac{1}{200}$

7 (a) 0.49 (b) 0.3 (c) 0.025 (d) 3.7

8 (a) 75 m (b) 78.75 kg (c) $28 160

9 (a) 1 m (b) 6.35 km (c) 3.982 cm

Exercise 1.1 (p. 9)

1 (a) $10 (b) $22.50 (c) $45
(d) 6.6 L (e) 264 m (f) 26.95 kg
(g) 1.8 kg (h) 43.66 kg (i) 52.8 L
(j) $17.50 (k) 187.5 m (l) 1.08 L
(m) $977.50 (n) $5.58 (o) $1333.33

2 (a) (i) 360 (ii) 324 (iii) 259.2 (iv) 576
(b) (i) 4560 (ii) 1425 (iii) 3078 (iv) 114
(c) (i) $70.57 (ii) $230.94 (iii) $68.96 (iv) $68.48
(d) (i) 4.9 s (ii) 8.53 s (iii) 9.18 s (iv) 9.47 s

3 (a) 69.2% (b) 16.8% (c) 63.6% (d) 18.9%
(e) 66.7% (f) 21.5% (g) 75.9% (h) 26.0%

4 (a) (i) 60% (ii) 8.5% (iii) 16%
(b) (i) 17% (ii) 28.3%

5 Pineapple: 1.95 L
Orange: 0.388 L
Passionfruit: 0.113 L
Guava: 0.05 L

6 37.7 kg

7 (a) $1059.71 (b) $35.84

8 (a) 8.56 s (b) 3.22%

9 (a) $\frac{7}{9}$ (b) 22.22% (c) $\frac{8}{10}$ or $\frac{4}{5}$, 20%

10 (a) 0.5, 50% (b) $\frac{9}{20}$, 45% (c) $\frac{18}{25}$, 0.72
(d) 1.25, 125% (e) $\frac{14}{25}$, 56% (f) $\frac{1}{8}$, 0.125

11 (a) 131 250 t (b) 87.5%

12 (a) $4.00 (b) $3.64 (c) 7.06%

13 79.6% 14 16 884

15 (a) (i) 120 (ii) 96
(b) (i) 390 (ii) 273
(c) When you are increasing and decreasing by the same percentage, you are not multiplying and dividing by the same number.

Open-ended – Sample answers

16 45, 59 and 80

17 (a) No, 25% of $299 is $74.75, a greater saving than $31.
(b) 25% is equivalent to $\frac{1}{4}$, which Trent could have found by dividing $300 (rounding up slightly) by 4 to get $75. Because we rounded up the actual answer will be slightly less, but still greater than $31.

Exercise 1.2 (p. 16)

1 (a) $20.40 (b) $71.25 (c) $38.39 (d) $262.47

2 (a) (i) 40% (ii) 28.6%
(b) (i) 3.6% (ii) 3.7%

3 (a) $75.45 (b) $16.80

4 (a) $2 (b) $27.57

5 (a) $175 (b) $2699

6 A 7 D 8 C

9 (a) $69.60 (b) $96

10 $735 11 $88.80

12 (a) $1434 (b) 149.4%

13 $15.71

14 (a) $19.99 (b) $1.82

15 (a) $27.18 (b) $317.27

16 54.4%

17 (a) $227.29 (b) $11 591.79 (c) $756.21 (d) 6.52%

18 She would save $4.99 by going to her favourite shoe shop.

19 (a) $1088 (b) $598
(c) 45% is applied to two different prices. First, 45% of the cost price of $750 is found and added on. Second, 45% of the selling price of $1088 is found and subtracted. This is a larger amount than the 45% mark up.
(d) 31%

20 14.9% 21 $7857.14

Open-ended – Sample answers

22 (a) 24.2%: $999 × (1 – 0.242) = $999 × 0.758 = $757.24
23%: $999 × (1 – 0.23) = $999 × 0.77 = $769.23
20.8%: $999 × (1 – 0.208) = $999 × 0.792 = $791.21

(b) Example:
Discount 20%, marked price $3062.50;
$3062.50 × 0.8 = $2450
Discount 21.5%, marked price $3121.02;
$3121.02 × 0.785 = $2450
Discount 23%, marked price $3181.82;
$3181.82 × 0.77 = $2450

23 Example: A $100 pair of shoes.

A 30% discount gives a sale price of $70. An additional 20% discount means that 20% of $70 (not the original $100) is subtracted. After a 20% discount, sale price = $56.

One 50% discount off the original $100 gives a sale price of $50.

Mathspace (p. 20)

1 Greatest gain: Option A for guitar, shoes, antique teapot; Option B for iPod, yoga mat, blue wig

2 True or false?

(a) T (b) F (c) T (d) F (e) F
(f) F (g) T (h) T (i) F

3 (b) 'To find 100% you multiply by 1 or do nothing.'

(d) 'To find 50% of something you multiply by a half (or divide by 2).'

(e) 'To find 150% you could multiply by 1.5 (or halve and multiply by 3).'

(f) 'To find 1%, find 10% and divide by 10.'

(i) 'To find 30% of something multiply by 0.3 OR find 10% and multiply by 3.'

Payment maze:

Minimum $150 + 30% -$35 – 30% + 10% – 20% = $98.56

Exercise 1.3 (p. 25)

1 $690.90 **2** $795 **3** $1530 **4** A

5 B **6** C

7 Adam – $623.20 Britta – $639.60
Con – $623.20 Deng – $442.80

8 (a) $4800 (b) $5400 (c) $6950 (d) $8000

9 115 sheep

10 (a) $656 ÷ 38 × 0.8 = $13.81 per hour (b) $339.73

11 $661.30

12 (a) $675 (b) 31.2%

13 $150 000

14 (a) 41 hours (b) 45 hours (c) 50 hours

(d) 20.5 hours overtime is exactly the same

15 (a) (i) $78.75 (ii) $1057.50

(b) $2937.50

16 (a) Option A: $40 000; Option B: $35714

(b) $25 000

(c) If Nabil expects sales less than $25 000 per week, he should choose Option A

17 (a) $1873.08 (b) $23.41 (c) $18.73

Open-ended – Sample answers

18 (a) Currently Malik is earning $16.78 × 0.4 × 32 + 17 = $231.78 a week.

Working the same number of hours in a fast food outlet, he would earn $600 ÷ 38 × 0.7 × 32 = $353.68.

Next year (assuming the same number of hours), his wages would be:

Apprentice: $16.78 × 0.52 × 32 + 17 = $296.22 a week

Fast food: $600 ÷ 38 × 0.8 × 32 = $404.21 a week

In the third year, his wages would be:

Apprentice: $16.78 × 0.7 × 32 + 17 = $392.87 a week

Fast food: $600 ÷ 38 × 0.9 × 32 = $454.74 a week

In the fourth year, his wages would be:

Apprentice: $16.78 × 0.82 × 32 +17 = $457.31 a week

Fast food: $600÷38 × 32 = $505.26

(b) Malik does not earn more than a casual fast food employee during his apprenticeship. However, he is training to become a fully qualified tradesperson capable of earning far more than a casual fast food worker. This calculation assumes 32 hours a week as a casual worker, but for casuals there is no guaranteed number of hours, and no leave. Malik's current position is more stable and secure.

19 16 hours during the week: $15.80 × 1.25 × 16 = $316

14 hours on a weekend = ($15.80 × 1.25 × 1.1 × 14) = $304.15

10 weekday hours + 5 weekend hours:
(10 × $15.80 × 1.25) + (5 × $15.80 × 1.25 × 1.1) = $306.13

Exercise 1.4 (p. 34)

1 (a) (i) $7252.20 (ii) $38 421.80
(b) (i) $16 759.50 (ii) $60 605.50
(c) (i) $34 200 (ii) $90 800
(d) (i) $3536.40 (ii) $26 039.60
(e) (i) $1204.95 (ii) $12 828.05
(f) (i) $63 550 (ii) $136 450
(g) (i) $11 538 (ii) $48 422
(h) (i) $0 (ii) $5976

2 (a) (i) $74.13 (ii) $650.87
(b) (i) $851.50 (ii) $3778.50
(c) (i) $154.74 (ii) $1107.65
(d) (i) $9.09 (ii) $166.91
(e) (i) $2190.87 (ii) $6444.37
(f) (i) $63.25 (ii) $473.79

3 (a) $57 765 (b) $10 879.50

4 B

5 (a) $48 256 (b) $308.72 (c) $1547.28 (d) $167.04

6 (a) $504 (b) $7008 (c) $4377.50

7 (a) $24 518.52 (b) $24 296.52 (c) $2744.48

8 (a) $9900 (b) $1048.08 (c) $190.38

(d) $4445.25 (e) $2027.75

9 (a) $17 550 (b) 22% (c) $11 100

(d) 14% (e) $6450

(f) If the income is split then each individual gets the benefit of the tax-free threshold and a lower percentage of income tax.

10 (a) $0.30 (b) $0.37 (c) $0.45

(d) These figures are the amount of tax paid up to the beginning of that tax bracket.

Open-ended – Sample answers

11 The tax table is designed so that those who earn more, pay more tax. This could be argued as being unfair because those who work hard and earn more are penalised, or fair because those who can afford more, can contribute more.

12 Compulsory superannuation means that Australian workers should all have an income after they retire, and are therefore less likely to fall into poverty after retirement.

Half-time 1 (p. 37)

1 (a) 73.3% (b) 14.5%

2 $65.45

3 (a) $79.20 (b) $275.80 (c) $55.28 (d) $199.33

4 Lucas should buy the $165 skateboard with a 25% discount in shop B (it costs $123.75 compared with the other one costing $130).

5 $1851.29

6 (a) 28.4% (b) 22.1%

7 (a) $7809 (b) $1527.73

8 Jeans – $72.22
Jacket – $72.31
Scarf – $13.75
In total Naomi saved $35.28

9 $758

Exercise 1.5 (p. 43)

1 (a) $16 200 (b) $980 (c) $4694.50 (d) $233.75

2 (a) $937.50 (b) $21 562.50

3 (a) (i) $1.91 (ii) $1.05

(b) (i) $1.36 (ii) $0.23

4 8.9%

5 (a) $176 (b) $378 (c) $11.07 (d) $34.98

6 B 7 $13 427.13

8 (a) $R = 0.090$, 9.0% (b) $R = 0.140$, 14.0%

9 (a) 7.9 years (b) 3.8 years

10 (a) $200 (b) $1350

11 4.6 years 12 $1455.23 13 $120 14 $8036

15 (a) (i) $0.75 (ii) $3.29

(b) $2.54

16 207 days 17 $24 000 18 $7000

19 (a)

Date		Transaction		Balance
Jul	1			$975.00
Jul	17	Deposit	$197	$1172.00
Jul	28	Withdrawal	$641	$531.00
Aug	4	Deposit	$299	$830.00
Aug	19	Deposit	$862	$1692.00
Aug	29	Withdrawal	$176	$1516.00
Sep	6	Deposit	$194	$1710.00
Sep	28	Withdrawal	$800	$910.00
Oct	16	Deposit	$426	$1336.00
Oct	27	Withdrawal	$500	$836.00
Oct	31	Balance		$836.00

(b) July: $531; August: $531; September: $910; October: $836

(c) $6.49

(d) July: $975; August: $531; September: $1516; October: $910

(e) $9.07 (f) 40%

(g) Try to make withdrawals at the start of each month.

20 (a) $2250 (b) $17 250 (c) 479.17

(d) 13% (e) $1500 (f) $1000

(g) Part (e) assumes the $15 000 is owed for the whole 3 years, whereas part (f) doesn't.

Open-ended – Sample answers

21 4% for 2 years, 8% for 1 year, 16% for 6 months

22 Michael has overcalculated the amount of days the balance was in his account. The opening balance was in his account for 8 days instead of 9, and the second balance was in his account for 12 days instead of 13.

23 As a borrower you are in debt and must repay the debt out of your own income, which gives you less money to spend. If you pay interest you will end up paying back far more than you borrowed. As a lender, you are relying on people to pay back the money you loaned. If they don't or can't repay, you have lost money.

Exercise 1.6 (p. 52)

1 (a) $11.60 (b) $14.10 (c) $10 (d) $12

2 $44.76

3 (a) $41 (b) $989.70 (c) $45.55

4 (a) 7 weeks (b) $8.95

5 (a) 31 days (b) $251.33 (c) $3.70

6 (a) $95.80 (b) $106.03 (c) $179.17/month

7 (a) $2.10

(b) Using an ATM then depositing into their landlord's account, because this saves $3.90 each month

8 (a) $48 600 (b) $8605 (c) 4.3%

9 (a) $124.48 (b) $8825.05 (c) $12 623.05
(d) $179.32 (e) $321.80
(f) People often use a credit card because they want the goods but do not have the cash to pay for them at the time. If they cannot pay the bill at the end of the month, interest and late fees are added to the balance. If several payments are missed interest is charged on the interest, and paying off the debt becomes very difficult.

Open-ended – Sample answers

10 (a) 12 months (b) $61
(c) Students' own answers.
(d) (i) Goods are able to be taken home straight away.
(ii) People do not want to pay fees, and are not sure they will have the full sum of money to avoid paying high interest at the end of the period.

11 (a) Four ATM withdrawals, two other bank ATM withdrawals, three EFTPOS transactions and five online internet transfers = $10.75
(b) Kezia could try to stop using other banks ATMs altogether, and withdraw more money from her own bank's ATM each time, so she is only being charged an ATM fee a couple of times each month.

12 Debit card: advantage—use money that you actually have
disadvantage—if you do not have the money at the time, you cannot purchase
Credit card: advantage—can purchase even if you do not actually have the cash
disadvantage—interest charged if bill not paid on time, tempted to overspend.

Exercise 1.7 (p. 60)

1 (a) (i) 1.35 kWh (ii) $0.25
(b) (i) 0.76 kWh (ii) $0.14
(c) (i) 0.422 5 kWh (ii) $0.08
(d) (i) 0.5 kWh (ii) $0.09
(e) (i) 19.2 kWh (ii) $3.55
(f) (i) 38.4 kWh (ii) $7.10
(g) (i) 1.275 kWh (ii) $0.24
(h) (i) 20.25 kWh (ii) $3.75

2 (a) 60 days (b) 84 m^3 (c) 3297 MJ
(d) $43.66 (e) $75.75

3 (a) The computer uses more energy (0.06 kWh compared with 0.053 kWh)
(b) The dishwasher uses more energy (0.615 kWh compared with 0.55 kWh)

4 (a) Sewage Disposal, Service Charges, Waterways and Drainage Charge.
(b) $91.19 (c) 92 days
(d) 68 kL (e) 185 L/day

5 (a) 18.75 kWh (b) $3.71
6 (a) 1.08 kWh (b) $74.90
7 (a) 49 L (b) 18 kL
8 (a) 0.354 kWh (b) $0.07 (c) $24.55
9 (a) $0.217 per kL
(b) A higher charge for extra water could discourage people from using it, and encourage them to save water.

10 (a) 9.5 kWh (b) 1.2 tonnes
(c) 13.3 kg of CO_2/day
(d) Consumption is higher during the winter months.
(e) Higher heating costs during winter (house heating, hot water). Alternatively, a greater number of people in the house between April and December.

Open-ended – Sample answers

11 Games console—1.5 hours—Saving of $1.30
Television (LCD)—2 hours—Saving of $18.72

12 For a 4-minute shower:
(a) 60 L (b) 21.9 kL (c) $27.59
(d) 10.95 kL of water and $13.80 saved

13 (a) heating, hot water, cooking
(b) The house has gas heating, which is in use during winter.

Exercise 1.8 (p. 65)

1 (a) $588 (b) $1920 (c) $2352
(d) $456 (e) $480 (f) $4752
2 (a) $668 (b) $611 (c) $228 (d) $114
3 (a) $2.99 (b) $6.36
4 $16.20
5 (a) Samsung GalaxyS = An extra $30 per month
(b) $1371
6 (a) (i) $696 (ii) $180 (iii) 200 MB
(b) $10.10
(c) (i) $20
(ii) If you do not use all of the monthly data allowance —it can not be used in the next month. It is lost.
(d) $10.16
7 (a) $8.49 (b) $0.79
8 (a) $4.29 (b) $19.36 (c) $580.80
(d) (i) $501.80 (ii) $6021.60
(e) $19.90

Open-ended – Sample answers

9 Three calls of 2 minutes, two calls of 5 minutes, 17 SMSs, ($19.21)
Two calls of 4 minutes, one call of 6 minutes, one call of 7 minutes, four SMSs, one MMS ($20.00)
Five calls of 90 seconds, two calls of 3 minutes, 16 SMSs, five MMSs ($19.99)

10 **(a)** No—6 cents difference

(b) Pre-paid is a good way of monitoring and limiting the amount you are spending on the phone.

11 Students' own answers.

12 Students' own answers.

Challenge 1 (p. 70)

1 Numbers are $x, x+1, x+2, x+3, x+4, x+x+4=1708$, $2x=1704$, $x=852$

Numbers are 852, 853, 854, 855, 856

2 $7^1=7, 7^2=49, 7^3=343, 7^4=2401, 7^5=16\,807, 7^6=117\,649$, $7^7=823\,543, 7^8=5\,764\,801, \dots$

The powers of 7 end in 07, 49, 43 or 01 in that order and then the pattern is repeated.

2048 is divisible by 4, so the last two digits of 7^{2048} will be the same as the last two digits of 7^4, that is 01.

Hence, $7^{2048} \div 100$ has a remainder of 1.

3 The numbers will end in 5 because numbers ending in 0 have 2 as their smallest prime factor. Will be ≥ 25.

$5=1\times 5$ (1 is not prime), $25=5\times 5$, $35=5\times 7$, $55=5\times 11$, $65=5\times 13$, $85=5\times 17$, $95=5\times 19$.

Numbers are 5, 25, 35, 55, 65, 85, 95

4 If $m=n=1$ then $m^3=n^4$. But m and n are different.
$(2^4)^3=(2^3)^4$, $m=16$, $n=8$.

$(3^4)^3=(3^3)^4$, $m=81$, $n=27$ etc. Smallest value is $m=16$.

5 **(a)** 21/02/2012, 22/02/2022, 23/02/2032, 11/02/2011, 10/02/2001 etc.

(b) 10/02/2001, 2 October, 2001; 11/02/2011, 2 November, 2011; 09/02/2090, 2 September, 2090; 01/02/2010, 2 January, 2010

6 The pattern is that the numbers increase by the next multiple of 3. The missing terms are 45, 63 and 84.

7 This is really dividing the interval between $\frac{1}{12}$ and $\frac{1}{7}$ into three equal parts.

$$\left(\frac{1}{7}-\frac{1}{12}\right)\div 3=\frac{5}{84}\times\frac{1}{3}=\frac{5}{252};$$

the numbers are $\frac{1}{12}+\frac{5}{252}=\frac{26}{252}=\frac{13}{126}$

and $\frac{26}{252}+\frac{5}{252}=\frac{31}{252}$

8 12 students, 30 min, 100 dishes; four students, 90 min, 100 dishes; eight students, 45 min, 100 dishes; eight students, 4.5 min, 10 dishes; 8 students, 40.5 min, 90 dishes. It will take 40.5 min.

9 **D**

10 **D** 1000^1 has four digits $=3\times 1+1$; $1000^2=1\,000\,000$ has seven digits $=3\times 2+1$;
so, 1000^{2010} has $3\times 2010+1=6031$ digits

11 **C** Let cost be $\$x$ so $D_1=0.1\times x$, $C_1=x-0.1x=0.9x$

$D_2=0.2\times 0.9x=0.18x$, $C_2=0.9x-0.18x=0.72x$

$D_3=\frac{1}{3}\times 0.72x=0.24x$, $C_3=0.72x-0.24x=0.48x$

So total discount $=0.52x$, i.e. 52%.

12 A **13** A, C

Chapter review 1 (p. 71)

1 **(a)** \$62.21 **(b)** \$260

2 **(a)** \$81.25% **(b)** 25%

3 **(a)** 52.50 **(b)** 26.24

4 C

5 **(a)** **(i)** \$6.16 **(ii)** \$41.09

(b) **(i)** \$117.27 **(ii)** \$35.41

6 \$415.80

7 \$831.25

8 **(a)** \$0 **(b)** \$7917 **(c)** \$12 384

9 **(a)** \$2380 **(b)** \$9380 **(c)** 195.42/month

10 \$64.13

11 **(a)** **(i)** 1.935 kWh **(ii)** \$0.38

(b) **(i)** 42 kWh **(ii)** \$8.32

(c) **(i)** 0.225 kWh **(ii)** \$0.04

12 \$24.95

13 \$989.20

14 8.4%

15 **(a)** \$138.75 **(b)** \$124.88 **(c)** 66.5%

16 \$1004.25

17 **(a)** \$44 720 **(b)** \$43 280 **(c)** \$6534

18 **(a)** \$16 **(b)** 5 weeks

19 **(a)** **(i)** \$1.11 **(ii)** \$0.49

(b) \$0.53

20 **(a)** **(i)** \$1440 **(ii)** \$5560 **(iii)** \$109

(b) \$291.67/month **(c)** \$132.53

21 **(a)** Tanya earned more money (\$516.91 compared with Freya's \$493.76).

(b) Casual workers receive more because they don't receive work benefits, including sick pay and holiday leave, and their job isn't guaranteed to be ongoing.

22 \$19.76 profit **23** \$136.89

24 **(a)** \$563.70 **(b)** \$12 112.80

NAPLAN practice 1

1 B **2** A **3** B **4** C **5** B

6 C **7** B **8** \$776.80 **9** \$10 740 **10** \$43.22

Chapter 2

Recall 2 (p. 78)

1 (a) 45.79 (b) 12.23 (c) 4.55

2 (a) 144 (b) 3025 (c) 1406.25
(d) 25 (e) 250 (f) 56.66

3 (a) 9 (b) 13 (c) 16

4 (a) 8.06 (b) 25.65 (c) 17.93

5 (a) $x = 6$ (b) $x = 81$ (c) $x = 33$

6 (a) $x = 6$ (b) $x = 6.24$ (c) $x = 18.03$
(d) $x = 8$ (e) $x = 11$ (f) $x = 36.06$

Exercise 2.1 (p. 82)

1 (a) Right-angled, $\angle ACB$ (b) Not right-angled
(c) Not right-angled (d) Not right-angled
(e) Right-angled, $\angle DFE$ (f) Right-angled, $\angle DFE$
(g) Right-angled, $\angle PRQ$ (h) Right-angled, $\angle PRQ$
(i) Not right-angled

2 (a) A (b) D (c) D

3 (a) AB, $\angle ACB$ (b) AB, $\angle ACB$
(c) AC, $\angle ABC$ (d) BC, $\angle BAC$

4 C

5 (a) Yes (b) No (c) No (d) Yes (e) Yes (f) Yes

6 (a) $a^2 + b^2 = 3^2 + 5^2$
$= 9 + 25$
$= 34$ m
$c^2 = 6^2$
$= 36$ m

(b) Jack should decrease the length of the diagonal to $\sqrt{34}$ m (5.83 m).
The frame is not square.

7 (a)

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3 cm	4 cm	5 cm	9	16	25	25
12 cm	16 cm	20 cm	144	256	400	400
60 mm	80 mm	100 mm	3600	6400	10 000	10 000
5 cm	12 cm	13 cm	25	144	169	169

(b) The last two columns.
(c) The values in these columns should be the same (allowing for slight measurement errors).

8 (a) Yes. $15^2 + 36^2 = 39^2$.
(b) Yes. $2.5^2 + 6^2 = 6.5^2$.
(c) Pythagoras' Theorem holds when the side lengths are multiplied or divided by the same value.

9 A and C are both right-angled triangles. Their side lengths are double the side lengths of the given right-angled triangles.

Open-ended – Sample answers

10 An example is 3 cm, 3 cm, 4.2 cm, $3^2 + 3^2 = 4.2^2$.

11 Examples are triangles with side lengths:
3 m, 4 m and 5 m
5 m, 12 m and 13 m
8 m, 15 m and 17 m.
Multiples of these side lengths (e.g. 6 m, 8 m and 10 m) also give right-angled triangles.

12 Students' own answers.

Exercise 2.2 (p. 91)

1 (a) 13 m (b) 10 cm (c) 15 mm
(d) 39 m (e) 26 cm (f) 65 mm

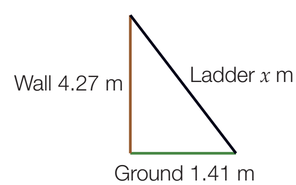
2 (a) $\sqrt{185}$ m (b) $\sqrt{180}$ m (c) $\sqrt{37.48}$ m
(d) $\sqrt{280.09}$ cm (e) $\sqrt{73}$ cm (f) $\sqrt{452}$ mm

3 (a) 17 cm (b) 34 m (c) 10 cm
(d) 5 m (e) 35 m (f) 10 cm
(g) 9.90 cm (h) 27.46 mm (i) 19.21 cm
(j) 10.20 m (k) 5.52 km (l) 13.89 m

4 (a) $x = 17$ m, $y = \sqrt{338}$ m
(b) $x = 13$ cm, $y = \sqrt{194}$ cm

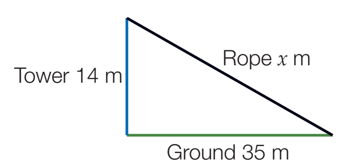
5 B 6 B 7 A

8 (a)



(b) 4.50 m

9 (a)



(b) 38 m

10 (a) $a = \sqrt{89} = 9.43$, $b = \sqrt{80} = 8.94$
(b) $d = \sqrt{337} = 18.36$, $e = \sqrt{562} = 23.71$, $f = \sqrt{4162} = 64.51$
(c) $a = \sqrt{34} = 5.83$, $b = \sqrt{68} = 8.25$
(d) $a = \sqrt{65} = 8.06$, $b = 9$, $c = \sqrt{97} = 9.85$

11 C

12 (a) $\sqrt{200} = 14.14$ cm; $1.414 = \sqrt{2}$
(b) $\sqrt{2}$; it is always $\sqrt{2}$

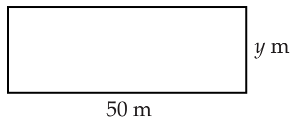
13 (a) $A = 1.41$ m ($\sqrt{2}$), $B = 1.73$ m ($\sqrt{3}$), $C = 2$ m, $D = 2.24$ m ($\sqrt{5}$), $E = 2.45$ m ($\sqrt{6}$), $F = 2.65$ m ($\sqrt{7}$)
(b) Of the 26 side lengths, 22 cannot be written as an exact decimal value. The exact values are 2 (side C), 3 (side H), 4 (side O) and 5 (side X).

14 (a) A to E to H to K (b) $3 + 3\sqrt{2}$

15 106 cm

Open-ended – Sample answers

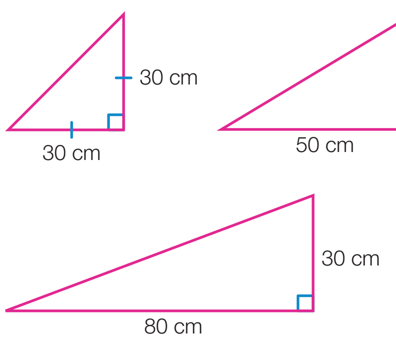
16 (a)



Distance from corner, y m	Total distance swum, x m
1	50.01 m
2	50.04 m
5	50.25 m
10	50.99 m
15	52.20 m
20	53.85 m
25	55.90 m

17 (a) horizontal length of 30 cm, ramp length of 42 cm; horizontal length of 50 cm, ramp length of 58 cm; horizontal length of 80 cm, ramp length of 85 cm.

(b)



(c) The ramp with the longest length will be the easiest for getting wheelchairs up the ramp.

Exercise 2.3 (p. 97)

1 (a) 35 **(b)** 24 **(c)** 9
(d) 40 m **(e)** 48 cm **(f)** 77
(g) $\sqrt{645}$ **(h)** $\sqrt{389.69}$ **(i)** $\sqrt{988}$
(j) $\sqrt{14\,104}$ **(k)** $\sqrt{112.5}$ m **(l)** $\sqrt{290.72}$ m

2 (a) $x = 9$ **(b)** $x = 9.90$ **(c)** $x = 3$ **(d)** $x = 8$
(e) $x = 7.48, y = 8.31$ **(f)** $x = 6.63, y = 17.32$

3 (a) B **(b)** C

4 2.42 m **5** 2.46 m

6 (a) 14.14 cm **(b)** 8.49 m **(c)** 70.71 cm

7 (a) Sides are each 4 m long **(b)** 3.46 m

Open-ended – Sample answers

8 Students' own answers; should range between 2.77 m and 3.1 m

9 (a) 12 cm and 16 cm; 10 cm and 17.32 cm
(b) 11.25 cm and 33.14 cm; 21.58 cm and 27.56 cm

Half-time 2 (p. 100)

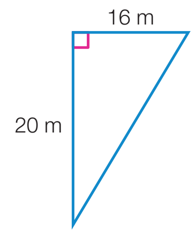
1 (a) 106.15 m **(b)** 45 m **(c)** 98.49 m

2 (a) 39.69 m **(b)** 12 m **(c)** 4.33 m

3 $a^2 + b^2 = 10^2 + 24^2 = 676$ $\quad c^2 = 26^2 = 676$

$a^2 + b^2 = c^2$, so triangle is right-angled.

4 (a)

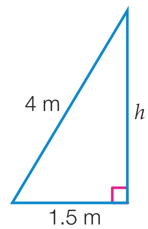


(b) 25.61 m

5 (a) Yes **(b)** Yes **(c)** No

6 $a = 65$; $b = 72$ **7** 2.29 cm

8 (a)



(b) 3.71 m

Exercise 2.4 (p. 104)

1 (a) 5.83 m **(b)** 15.80 m

2 9.95

3 (a) 1.70 m **(b)** 2.33 m

4 D **5** 3.47 m

6 (a) 44.82 m **(b)** 18.18 m

7 0.51 m **8** 1.88 km **9** D **10** D

11 11.31 cm **12** 3.14 m **13** \$1005.33

14 through mountains \$157 429; around mountains \$132 500; therefore, it is cheaper to build around the mountains.

15 \$1840 (for 8 girders)

16 (a) 9 m **(b)** 3.31 m **(c)** 15.62 m

17 (a) 1.73 m **(b)** 3.87 m **(c)** 29.21 m

Open-ended – Sample answers

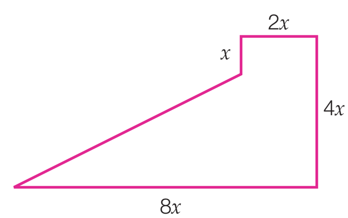
18 After 40 minutes one candle is of length 25 cm and the other candle is of length 26 cm.

Distance $= \sqrt{1^2 + 20^2} = 20.02$ cm

After 3 hours one candle is of length 7.5 cm and the other candle is of length 12 cm.

Distance $= \sqrt{4.5^2 + 20^2} = 20.5$ cm

19 (a) $x = 10$
(b) 217.08 m



Exercise 2.5 (p. 111)

1 (a) 15 m (b) 50 mm (c) 13 cm (d) 25 cm
(e) 2.5 m (f) 8.5 mm (g) 12 mm (h) 24 m
(i) 24 cm (j) 16 cm (k) 12 m (l) 6 cm

2 (a) 26 (b) 39 (c) 15
(d) 45 (e) 82 (f) 130
(g) 65 (h) 1300 (i) 116

3 B 4 C

5 (a) Yes (b) Yes (c) Yes (d) No
(e) No (f) Yes (g) No (h) No

6 192 cm

7 (a)

7	24	25
9	40	41
11	60	61

(b) The three numbers are a Pythagorean Triple.

(c) Take an odd number and square it. Subtract one from this number and divide by 2. These are the first two numbers in the triple. The third is now more than the second.

8 (a)

12	35	37
16	63	65
20	99	101

(b) The three numbers form a Pythagorean Triple.

(c) Take any two consecutive odd numbers and add them together. This is the first number in the triple. Now multiply the two odd numbers together to give the second number in the triple. The third number is two more than the second number.

9 $(5, 12, 13) \times 32 = (160, 384, 416)$

10 (a) All of the triples except for (16, 63, 65) and (36, 77, 85) satisfy Derek's rule.

(b) Neither (16, 63, 65) nor (36, 77, 85) includes a prime number.

(c) You don't need to mention the condition of having at least one prime number.

(d) In column 1 the pattern is 'Take the next odd number as the first number, square it and then halve it and use the two whole numbers either side of this.' The next triple is (15, 112, 113).

In column 2 the pattern is 'Get the next multiple of 4 as the first number, times by which multiple of 4 it is and subtract 1. Use this and the number 2 more than the second number.' The next triple is (32, 255, 257).

In column 3 the pattern is 'From the previous triple the first number is 8 more than the previous first number.'

The third number is found by adding the square of an odd number to the first number (the odd number is the next consecutive odd in the sequence 3, 5, 7, 9, …). The second number is 8 less than the third number. The next triple is (60, 221, 229).

Open-ended – Sample answers

11 Students' own answers. For example, (11, 60, 61) gives (22, 120, 122), (33, 180, 183), (44, 240, 244).

12 (a) Example: (8, 15, 17) (b) (16, 30, 34), (24, 45, 51)

Challenge 2 (p. 116)

1 (a) $b = 3 - a, a + b = 3$

(b) $KL = a\sqrt{2}$

(c) $LM = b\sqrt{2}$

(d) $KL + LM = (a + b)\sqrt{2} = 3\sqrt{2}$

2 (a) $AC = 5, AD = 13, DE^2 = 85^2 - 13^2 = 84$

(b) Area $= 0.5 \times (4 \times 3 + 5 \times 12 + 13 \times 84) = 582$ cm^2.

3 $7^2 = 49, 8^2 = 64$. Any whole number between 49 and 64.

4 $2016 = 4 \times 504 = 4 \times 4 \times 126 = 4 \times 4 \times 9 \times 14$. The smallest number is 14.

5 (a) $(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4 = (m^2 + n^2)^2$

(b) (i) $m = 2, n = 1$: 3, 4, 5

(ii) $m = 3, n = 1$: 8, 6, 10 $\rightarrow$ 3, 4, 5

$m = 3, n = 2$: 5, 12, 13

(iv) $m = 4, n = 1$: 15, 8, 17 $\rightarrow$ 8, 15, 17

(iii) $m = 4, n = 2$: 12, 16, 20 $\rightarrow$ 3, 4, 5

$m = 4, n = 3$: 7, 24, 25

(v) $m = 5, n = 1$: 24, 10, 26 $\rightarrow$ 5, 12, 13

$m = 5, n = 2$: 21, 20, 29 $\rightarrow$ 20, 21, 29

$m = 5, n = 3$; 16, 30, 34 $\rightarrow$ 8, 15, 17

$m = 5, n = 4$: 9, 40, 41

Note that when m and n are both even or odd then the triple formed has a common factor. Thus, to generate triples with no common factor, it appears that one value must be even and the other odd. This hypothesis could be investigated further.

6 $AB = 3\sqrt{2}, BC = 3\sqrt{5}$,

Perimeter $= 3\sqrt{2} + 3\sqrt{5} + 6 + 9 = 3(5 + \sqrt{2} + \sqrt{5})$ cm.

Answer D

Chapter review 2 (p. 117)

1 (a) No (b) Yes (c) No

2 (a) 75 cm (b) $\sqrt{605}$ mm (c) $\sqrt{4050}$ m

3 (a) $x = 27.33$ (b) $x = 78.33$ (c) $x = 15.23$

4 (a) $a = 40.82$ (b) $x = 9.49$ (c) $x = 9.64$ (d) $x = 13$

5 A

6 (a) 20 (b) 205 (c) 407

7 (a) It is not square (b) 72 cm or 73 cm

8 (a) $x = \sqrt{115}; y = \sqrt{556}$

(b) $x = \sqrt{40}\,; y = \sqrt{15}$

9 Ming wins by 1.58 s 10 7.03 m

11 (a) 2 m

(b) Possibly—many races are won by less than 2 m.

12 (a) Yes (b) No (c) Yes

13 (a) 19.71 m, 10 sleepers (b) $350

14 (a) 4 cm (b) 5.66 cm (c) 21.66 cm

NAPLAN practice 2

1 D 2 B 3 A 4 A

5 D 6 A 7 C 8 D

Mixed review A (p. 121)

1 9 months

2 (a) 56.25 m (b) 79.04 kg (c) $192.50

3 Not right-angled

4 (a) 72.3% (b) 17.4% (c) 6.4%

5 (a) $1401.65 (b) $777

6 (a) $0 (b) $1200 (c) $9750

7 $853.62

8 (a) (i) 3.15 kWh (ii) 60.48 cents

(b) (i) 12.6 kWh (ii) 241.92 cents ($2.4192)

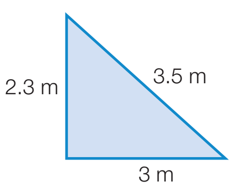
9 $11 025 10 $b = 48$, $c = 50$

11 (a) $x = 52$ (b) $x = 80$

12 Mustafa: 87.5%; Adam: 81.8%

Mustafa is the more accurate shooter

13 (a)



(b) No; $3^2 + 2.3^2 \neq 3.5^2$

14 65 cm

15 10.41% p.a.

16 685 km

17 (a) $16.45 (b) $12.67 (c) 170%

18 No, there is a 4.69 cm drop from the ramp to the platform.

19 28.93 cm

Chapter 3

Recall 3 (p. 126)

1 (a) $14x$ (b) $8y$ (c) k^2

2 (a) $4h + 2y$ (b) $8y^3 + 6x^2 - 2x$

3 (a) $12xy$ (b) $20k^2$ (c) $-60g^3$

4 (a) 2^{13} (b) 6^6 (c) 5^4 (d) 1

5 (a) $\frac{4^6}{5^9}$ (b) $2^{16} \times 5^8$ (c) $\frac{7^4 \times 3^8}{4^{10}}$

6 (a) (i) 20 cm (ii) 25 cm^2

(b) (i) 22 m (ii) 18 m^2

(c) (i) 16 mm (ii) 12 mm^2

7 (a) $4x + 8$ (b) $6x - 18$ (c) $-2x - 8$

8 (a) $6(x - 2)$ (b) $5(2x - 5)$ (c) $-3(x - 5)$

Exercise 3.1 (p. 132)

1 (a) 2^9 (b) 3^{11} (c) 5^{18} (d) a^7 (e) b^{11}

(f) $5c^5$ (g) $6d^5$ (h) $40b^8$ (i) $28b^4$ (j) $12a^6$

(k) $14a^4$ (l) $200a^7$ (m) $24a^9b^2$ (n) $27a^4b^5$ (o) $30b^7c^7$

(p) $-35a^4b^8$ (q) $36e^6f^6$ (r) $-24a^6b^{10}$

2 (a) 3^3 (b) 4^5 (c) 9^2 (d) m^3 (e) $4b^2$

(f) $5g^4$ (g) n^2 (h) $2b^2$ (i) $2c^4$ (j) $2a$

(k) $\frac{3}{2}a^4$ (l) $\frac{7}{5}e^3f^3$ (m) $2a^3b^4$ (n) $-\frac{8}{7}a^3b^2$ (o) $-12ab$

3 (a) 2^{12} (b) 3^{10} (c) 5^8 (d) 3^{18} (e) 4^{21}

(f) 7^{18} (g) 5^{10} (h) 8^{34} (i) 11^5 (j) b^{21}

(k) k^{32} (l) a^{12} (m) y^{17} (n) k^{27} (o) m^{20}

(p) g^{18} (q) y^{23} (r) t^{21} (s) p^2 (t) n^6

(u) b^3

4 (a) C (b) A

5 (a) $\frac{x^7 \times x^{12}}{x^{12}}$ (b) $\frac{6y^4 \times y^6}{y^2 \times y^5}$

$= \frac{x^{16}}{x^{12}}$ $= \frac{6y^{10}}{y^7}$

$= x^4$ $= 6y^3$

6 (a) x^9 (b) $\frac{t}{3}$ (c) c^7 (d) j^5 (e) $12a^8$ (f) $2q^3$

7 (a) A (b) C (c) C (d) C

8 (a) $5a^2$ (b) $3a^2 \times 5a^2$

9 (a) C (b) B

10 (a) m^3 (b) p^{14} (c) k^3 (d) n^{10}

11 (a) A (b) C

12 (a) 3 (b) 0 (c) -2 (d) 1

13 (a), (b)

Week	Amount saved each week (c)	Amount saved each week in index form (c)
1	5	5×2^0
2	10	5×2^1
3	20	5×2^2
4	40	5×2^3
5	80	5×2^4

(c) $a = 5 \times 2^{n-1}$ (d) $51.20

(e) $3 335 443.15

Open-ended – Sample answers

14 $-3a$ and ab^3, a^2 and $-3b^3$, $3b$ and $-a^2b^2$

15 $10a^5b^4 \div 2a^3b$, $5a^2b^5 \div b^2$, $15a^4b^3 \div 3a^2$

16 $24a^2b^3 \div 22a^2b^2 = \frac{12b}{11}$

$2 \times 3^2a \div 3ab = \frac{6}{b}$

$48a^5b^6 \div 24a^2b^3 = 2a^3b^3$

17 $m = 3, n = 8$; $m = 4, n = 6$; $m = 12, n = 2$

18 (a) The indices have been multiplied instead of added. The correct answer is x^6.

(b) The indices have been divided instead of subtracted. The correct answer is x^6.

(c) The index inside the brackets has been raised to the power of the index outside the brackets, when instead they should have been multiplied together. The correct answer is x^8.

Exercise 3.2 (p. 144)

1 (a) $6^2 \times 5^2$ (b) $11^4 \times 3^4$ (c) $12^5 \times 9^5$ (d) $16x^2$
(e) p^6q^2 (f) $a^{12}f^3$ (g) $32p^5$ (h) $p^{21}q^{35}$
(i) c^8d^{28} (j) b^3c^3 (k) $216a^{12}$ (l) $81p^{10}q^8$
(m) $49m^6n^{12}$ (n) $8a^3b^6$ (o) $81e^4f^{16}$

2 (a) $\frac{12^2}{5^2}$ (b) $\frac{7^4}{3^4}$ (c) $\frac{9^8}{2^8}$ (d) $\frac{y^3}{x^3}$
(e) $\frac{d^5}{e^5}$ (f) $\frac{m^{12}}{n^4}$ (g) $\frac{w^8}{z^2}$ (h) $\frac{r^{20}}{s^4}$
(i) $\frac{d^6}{g^2}$ (j) $\frac{81}{w^{24}}$ (k) $\frac{512}{m^6}$ (l) $\frac{8}{r^{12}}$
(m) $\frac{h^{15}}{g^{10}}$ (n) $\frac{a^{12}}{c^{20}}$ (o) $\frac{t^{12}}{v^9}$ (p) $\frac{125a^3}{b^3}$
(q) $\frac{c^{24}}{64x^6}$ (r) $\frac{f^{18}}{216g^3}$

3 (a) 12^2 (b) 20^6 (c) 56^4 (d) $(4h)^8$ (e) $(3p)^4$
(f) $(7r)^9$ (g) $(am)^5$ (h) $(ht)^7$ (i) $(pn)^8$

4 (a) 5^4 (b) 4^3 (c) 12^5 (d) $\left(\frac{r}{3}\right)^5$ (e) $\left(\frac{n}{6}\right)^7$
(f) $\left(\frac{g}{4}\right)^9$ (g) $\left(\frac{x}{y}\right)^6$ (h) $\left(\frac{k}{n}\right)^{11}$ (i) $\left(\frac{g}{h}\right)^{10}$

5 (a) 1 (b) 7 (c) 8 (d) b^2
(e) 3 (f) c^3 (g) $-3a^4$ (h) $-8g^{11}$
(i) -1 (j) 1 (k) 3 (l) 3
(m) $3x^4$ (n) k^8 (o) $\frac{n^4}{2}$ (p) a^3
(q) 3 (r) $\frac{3}{2}$ (s) $\frac{3f^2}{4}$ (t) $\frac{5s^9}{9}$
(u) $\frac{5p^2}{3}$

6 (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{625}$ (d) $\frac{1}{27}$
(e) $\frac{1}{25}$ (f) $\frac{1}{64}$ (g) $\frac{1}{x^5}$ (h) $\frac{1}{y^3}$
(i) $\frac{1}{b^7}$ (j) $\frac{3}{e^3}$ (k) $\frac{7}{x^5}$ (l) $\frac{3}{g^4}$
(m) $\frac{8}{p^5q^2}$ (n) $\frac{2}{x^3y^5}$ (o) $\frac{12}{a^3b^9}$ (p) $\frac{3p^3}{q}$
(q) $\frac{5x^2}{y^7}$ (r) $\frac{7b^2}{a^5}$

7 (a) C (b) A (c) B (d) B

8 (a) x^6y^6 (b) g^9h^9 (c) $432x^5$ (d) $400x^2y^4$
(e) $c^{12}d^9$ (f) x^6y^8

9 (a) $\frac{f^5}{g^5}$ (b) $\frac{p^{10}}{q^{10}}$ (c) $\frac{d^{27}}{q^{14}}$ (d) a^2b^2
(e) $\frac{4a^5}{b^5}$ (f) $\frac{108x^8}{y^5}$

10 (a) w^2z^5 (b) c^2e^4 (c) p^6q^3 (d) v^3w^5

11 (a) 1 (b) 1 (c) 1 (d) 1
(e) m^{12} (f) f^{20} (g) t^4 (h) 1
(i) 1 (j) $\frac{3}{5}$ (k) $\frac{4}{7}$ (l) 1
(m) $\frac{5b^6}{2}$ (n) $\frac{d^6}{6}$ (o) $\frac{d}{10}$ (p) $\frac{q^5}{15}$

12 (a) $\frac{1}{16}$ (b) $\frac{1}{27}$ (c) $\frac{1}{625}$ (d) 2 (e) 64 (f) 9
(g) $\frac{4}{9}$ (h) $\frac{3}{64}$ (i) $\frac{2}{25}$ (j) $-\frac{7}{27}$ (k) -3 (l) -27

13 (a) d^2 (b) k^5 (c) $3a$
(d) x^3y^6 (e) p^7q^2 (f) $\frac{n^4}{m^4}$
(g) $\frac{b^{12}}{a^8}$ (h) $\frac{n^{10}}{m^8}$ (i) $x^{15}y^6$

14 (a) C (b) A (c) B (d) B

15 Eva's method is faster because simplifying the expression in the brackets first before raising it to the index makes the mathematics simpler.

16 (a) 3^{2n} (b) 2^{3n+2} (c) 5^{n+1}

17 (a) $5^3, 5^2, 5^1, 5^0$ (b) $5^{-1}, 5^{-2}, 5^{-3}, 5^{-4}, 5^{-5}$
(c) $125, 25, 5, 1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \frac{1}{3125}$

18 (a) 1 divided by the original number
(b) $\frac{7}{a}$ m

19 (a) $\dfrac{n^4T^6p^2}{b^2}$

(b) (i) 10^{12}

(ii) We will obtain a positive answer, because the number inside the brackets will be squared, and a negative or a positive number squared gives a positive answer.

20 (a)

Source	Sound level (loudness, dbs)	Sound intensity, watts/m^2
Jet aircraft, 50 m away	140	100
Threshold of pain	130	10
Chainsaw, 1 m distance	110	0.1
Disco, 1 m from speaker	100	0.01
Busy traffic	80	0.0001
Vacuum cleaner, 1 m distance	70	0.000 01
Average home	50	0.000 000 1
Rustling of leaves in the distance	10	0.000 000 000 001

(b) 3

(c) The intensity of a rock concert is at the same level as the threshold of discomfort.

(d) Thunder at a sound level of 110 db.

Open-ended – Sample answers

21 (a) $(6m^2)^2 \times (n^2)^4$, $(mn^2)^4 \times 6^2$

(b) $(2m^2)^2 \div 5(n^3)^2$, $\dfrac{(4m^2n)^2}{20n^8}$

22 $(2x)^3$, x^3y^2, $(3xy)^3$

23 Students' own answers.

24 $3^2 = 9$ not 6. $(y^4)^2 = y^8$ not y^6. $6 \div 2 = 3$ not 4. $x^4 \div x^{-2} = x^6$ not x^2.

$$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{9x^4y^8}{2x^{-2}y^5} = \frac{9x^6y^3}{2}$$

Exercise 3.3 (p. 154)

1 (a) 9800 (b) 65 000 000 (c) 3 250 000
(d) 420 710 (e) 0.002 (f) 0.000 05
(g) 0.000 000 07 (h) 0.050 32 (i) 0.008 26
(j) 0.000 041 8 (k) 0.000 714 (l) 0.000 003 55

2 (a) 2.56×10^3 (b) 9.002×10^4 (c) 6.14×10^5
(d) 1.9×10^5 (e) 8.05×10^6 (f) 5.6×10^7
(g) 3.05×10^{-1} (h) 3×10^{-2} (i) 4×10^{-4}
(j) 2.01×10^{-3} (k) 7.2466×10^{-5} (l) 5.3218×10^{-7}

3 (a) 390 000 (b) 0.054 0 (c) 670
(d) 0.003 00 (e) 1700 000 (f) 0.900

4 (a) 3 (b) 4 (c) 3 (d) 2 (e) 4 (f) 2

5 (a) 3×10^{-3} seconds (b) 1.34×10^{-1} seconds
(c) $3\text{-}4 \times 10^{-1}$ seconds

6 (a) 4.0075×10^4 km (b) 1×10^{-7} m
(c) 1.5×10^{-3} m (d) 5.9×10^9 km
(e) 3×10^{-8} m (f) 1.063×10^3 °C

7 (a) 300 000 000 m/s
(b) 0.000 000 000 000 001 5 m
(c) 0.000 12 m
(d) 4 000 000 000 000 000 000 000 000 atoms
(e) 778 000 000 km
(f) 15 000 000 °C

8 A

9 (a) 1.416×10^{11} (b) 2.1286×10^{-2}
(c) 3.0×10^{11} (d) 2.0×10^{12}
(e) 6.066×10^{-4} (f) 4.299×10^{-10}
(g) $-1.379\,914 \times 10^8$ (h) $1.349\,545 \times 10^{10}$
(i) $2.689\,98 \times 10$ (j) $1.580\,64 \times 10^{12}$
(k) $2.376\,923\,077 \times 10^{11}$ (l) $3.395\,358\,605 \times 10^{14}$

10 (a) 1×10^{-14} m (b) 4×10^3 m (c) 2×10^{-5} m
(d) 2.73×10^{3}°C (e) 1.0651×10^4 athletes
(f) 1×10^{14} atoms (g) 1.5×10^8 km, 6×10^3°C
(h) 1.4×10^5 cases

11 (a) 6×10^{-3} seconds (b) 2.5×10^{-3} seconds
(c) 2×10^{-3} seconds (d) 2×10^{-3} seconds

12 (a) $9.99\text{-}9.997 \times 10^{-4}$ seconds (b) $1.9\text{-}1.95 \times 10^{-4}$ seconds

13 (a) 4.57×10^9 years (b) 6.4×10^8 years
(c) 2.4×10^9 years ago (d) 2.2×10^8 years
(e) 5×10^8 years (f) 5.3×10^8 years
(g) 1.2×10^8 years (h) 1.65×10^8 years
(i) 1.44×10^8 years
(j) 6.5×10^7 years ago and 2.51×10^8 years ago
(k) 4.5698×10^9 years

14 (a) (i) 3.6×10^{10} m (ii) $1.088\,64 \times 10^{15}$ m
(iii) $9.467\,28 \times 10^{15}$ m (b) 8.3 minutes

15 1.28 seconds

16 (a) paper (b) 1.8×10^{-3} m

17 (a) 6.235×10^{11} cells (b) 3.9904×10^{13} cells
(c) 39 904 000 000 000 cells

18 (a) 1.08×10^8 km (b) 1.08×10^{11} m
(c) 6.79×10^8 km or 6.79×10^{11} m
(d) 4.2×10^7 km or 4.2×10^{10} m

Open-ended – Sample answers

19 5×10^{-3} cm

20 2×10^{13}

Exercise 3.4 (p. 164)

1 (a) (i) $k = \frac{P}{m}$ (ii) -8

(b) (i) $b = y - mx$ (ii) -1

(c) (i) $h = \frac{C + g}{a}$ (ii) $-\frac{1}{2}$

(d) (i) $x = \frac{a - c}{m}$ (ii) -6

(e) (i) $p = \frac{kn + 2}{m}$ (ii) -53

(f) (i) $w = v(u + xy)$ (ii) 30

2 (a) $r = \sqrt{\frac{V}{\pi h}}$ (b) $v = \sqrt{\frac{2E}{m}}$ (c) $h = \sqrt{\frac{Fk}{d}}$

(d) $m = \frac{k^2 n}{49}$ (e) $b = \frac{d^2 c}{4}$ (f) $y = \frac{m^2 z}{x^2}$

3 (a) $x = z - y$ (b) $x = 3a + 2b$ (c) $x = \frac{m + n}{k}$

(d) $x = \frac{f - c}{d}$ (e) $x = rt - p$ (f) $x = vy + w$

(g) $x = \frac{kp - y}{2}$ (h) $x = \frac{mw - t}{r}$ (i) $x = \frac{mn - mp}{p}$

(j) $x = \frac{gh - e}{cd}$ (k) $x = \frac{mn + k}{ap}$ (l) $s = \frac{x - vw}{tu}$

(m) $x = \frac{y(f - b)}{d}$ (n) $x = \frac{n(t - kp)}{m}$ (o) $x = \frac{c(ab - f)}{d}$

4 B

5 (a) $t = \frac{v - u}{a}$

(b) (i) $t = 4$ s (ii) $t = 3$ s

6 (a) $r = \frac{C}{2\pi}$

(b) (i) $r = 99.95$ (ii) $r = 2.61$

7 $h = \frac{V}{\pi r^2}$; $h = 2.00$

8 $w = \frac{P}{2} - l$; $w = 19$ m

9 (a) 1.44 (b) 4 (c) ±700

(d) When calculating a square root of a number there is a negative and positive solution. The negative solution means that the object is travelling in the opposite direction to the positive solution.

10 (a) 21 g (b) Yes (the overall density is 1.02 g/cm^3)

11 (a) $\frac{2A}{h} - a$ (b) $b = 13$

(c) If we had to recalculate b for different values of A, a and h, it would be easier to have b as the subject.

12 $s = an + \frac{dn^2}{2} - \frac{dn}{2}$

It's difficult to make n the subject as there are terms with n and n^2 in this expression.

Open-ended – Sample answers

13 She has made a mistake in the final line. She should have taken b away from $\frac{2A}{h}$. $a = \frac{2A}{h} - b$; $b = \frac{2A}{h} - a$

Half-time 3 (p. 167)

1 (a) $12x^7y^2$ (b) $4x^2y^3$ (c) $\frac{3m^6n}{4}$

2 $x = \frac{z + 3y}{2y}$, $x = 2.2$

3 (a) $9x^8y^{10}$ (b) $\frac{a^6}{36b^2}$ (c) $\frac{125}{3}$

4 (a) 3.84×10^5 km (b) 5×10^2 mL

5 (a) 4 (b) 3 (c) 3

6 (a) $2^7 \times 5^4$ (b) 3×5^6 (c) 2^5

7 (a) 110 000 000 000 000 bacteria

(b) 0.000 004 4 m

8 (a) $\frac{1}{q^7}$ (b) $\frac{1}{c^9b^5}$ (c) 2 (d) $\frac{8n^4}{m^4}$

9 (a) $\frac{1}{x^5}$ (b) $\frac{3}{p^4}$ (c) $\frac{5n}{m^3}$ (d) $\frac{6r^2}{p^4q}$

10 (a) 0.300 (b) 80.0 (c) 62 100

Exercise 3.5 (p. 172)

1 (a) $2x + 10$ (b) $4a + 36$ (c) $8b + 24$

(d) $8x - 16$ (e) $9x - 18$ (f) $3k - 12$

(g) $-7x - 7$ (h) $-4d - 32$ (i) $-6x + 18$

(j) $-9c + 54$ (k) $-2x + 10$ (l) $-5x + 30$

(m) $x^2 + 3x$ (n) $m^2 + 5m$ (o) $x^2 + 9x$

(p) $-x^2 + 8x$ (q) $-x^2 + 4x$ (r) $-n^2 + 9n$

(s) $3x^2 + 6x$ (t) $2u^2 - 12u$ (u) $5x^2 - 5x$

(v) $-2x^2 - 14x$ (w) $-7q^2 - 7q$ (x) $-9b^2 + 54b$

2 (a) $5x + 2$ (b) $5y - 4$ (c) $2x - 11$

(d) $5a + 13$ (e) $2a^2 - 3a + 10$ (f) $3x^2 + 2x + 4$

(g) $7y^2 - 26y - 10$ (h) $9k^2 + 27k$

3 (a) $xy + 5x + 6y + 30$ (b) $xy + 7x + 4y + 28$

(c) $ce + 3c + de + 3d$ (d) $rt - 8r + 5t - 40$

(e) $ab - a + 2b - 2$ (f) $pq - 2p + 3q - 6$

(g) $ay - 4a - 3y + 12$ (h) $mn - 6m - 2n + 12$

(i) $km - 4k - 5m + 20$ (j) $4mp + 4np + mq + nq$

(k) $7ax + 7kx + ay + ky$ (l) $12ps + 3pt + 8rs + 2rt$

(m) $2ab - 6a + b - 3$ (n) $3x^2 + 5x - 2$

(o) $3mn - 3m + 2n - 2$ (p) $10am - 6an - 15bm + 9bn$
(q) $15cx - 20cy - 6dx + 8dy$ (r) $14pr - 21pt - 4qr + 6qt$

4 (a) $a^2 + 5a + 6$ (b) $c^2 + 8c + 15$
(c) $y^2 + 13y + 42$ (d) $x^2 - 2x - 3$
(e) $y^2 + 7y - 18$ (f) $a^2 + 5a - 84$
(g) $b^2 - 3b + 2$ (h) $x^2 - 7x + 12$
(i) $p^2 - 15p + 56$ (j) $6m^2 + 11m + 3$
(k) $8d^2 + 22d + 15$ (l) $21k^2 + 47k + 20$
(m) $8y^2 - 2y - 3$ (n) $10k^2 - 29k - 21$
(o) $8a^2 + 14a - 15$ (p) $12a^2 - 16a + 5$
(q) $12p^2 - 29p + 15$ (r) $24r^2 - 37r + 14$

5 (a) $x^2 + 2xy + y^2$ (b) $m^2 + 2mn + n^2$
(c) $a^2 + 2ab + b^2$ (d) $p^2 - 2pr + r^2$
(e) $c^2 - 2cd + d^2$ (f) $g^2 - 2gh + h^2$
(g) $4c^2 + 12cd + 9d^2$ (h) $9m^2 + 12mn + 4n^2$
(i) $25a^2 + 30ab + 9b^2$ (j) $25x^2 - 40xy + 16y^2$
(k) $4j^2 - 12jk + 9k^2$ (l) $49c^2 - 70cd + 25d^2$

6 (a) $m^2 - n^2$ (b) $p^2 - q^2$
(c) $a^2 - b^2$ (d) $9a^2 - 4b^2$
(e) $16x^2 - 9y^2$ (f) $49m^2 - 9n^2$

7 A 8 D

9 (a) $(x + 7)(x + 2)\text{cm}^2$ (b) $204\ \text{cm}^2$
(c) 6 (d) $x^2 + 9x + 14$
(e) $10^2 + 9 \times 10 + 14 = 204$

10 (a) (i) $(15 + x)$m (ii) $(3 + x)$m
(iii) $(15 + x)(3 + x)\text{m}^2$ (iv) $(45 + 18x + x^2)\text{m}^2$
(b) New garden bed has $(18x + x^2)\text{m}^2$ more area
(c) 40 m^2 extra area

11 (a) $x(x + 3)$ (b) $10x - 12$ (c) 3 cm, 4 cm

12 Two possible methods are to substitute values for x and to use the areas of rectangles.

13 (a) (i) $x^2\ \text{m}^2$ (ii) $(x - 4)$ m
(iii) $(x - 2)$ m (iv) $(x - 2)(x - 4)\ \text{m}^2$
(b) $(6x - 8)\ \text{m}^2$

Open-ended – Sample answers

14 $-15x + 6$

15 $x^2 + x - 12$

16 length = l, width = w; increase length by 5 units, increase width by 7 units; area of new rectangle = $(l + 5)(w + 7)$

Exercise 3.6 (p. 178)

1 (a) $d^2 - 2d + 1$ (b) $m^2 + 10m + 25$
(c) $k^2 + 4k + 4$ (d) $p^2 - 6p + 9$
(e) $x^2 - 16x + 64$ (f) $y^2 - 24y + 144$
(g) $m^2 + 2mn + n^2$ (h) $w^2 - 2wk + k^2$
(i) $c^2 - 12c + 36$ (j) $y^2 + 12y + 36$
(k) $x^2 + 18x + 81$ (l) $k^2 - 14k + 49$
(m) $16y^2 + 24y + 9$ (n) $4w^2 + 36w + 81$
(o) $9x^2 + 30x + 25$ (p) $36k^2 - 12k + 1$
(q) $49 - 28a + 4a^2$ (r) $9 - 48d + 64d^2$
(s) $1 - 8c + 16c^2$ (t) $36 + 84y + 49y^2$
(u) $16g^2 - 8gh + h^2$ (v) $9a^2 + 6ad + d^2$
(w) $4a^2 - 12ab + 9b^2$ (x) $25m^2 + 80mn + 64n^2$

2 (a) $c^2 - 25$ (b) $a^2 - 16$ (c) $k^2 - 64$
(d) $81 - x^2$ (e) $9 - y^2$ (f) $1 - k^2$
(g) $m^2 - n^2$ (h) $x^2 - y^2$ (i) $c^2 - d^2$
(j) $v^2 - w^2$ (k) $p^2 - q^2$ (l) $h^2 - g^2$
(m) $4m^2 - 1$ (n) $9x^2 - 16$ (o) $36b^2 - 49$
(p) $1 - 25y^2$ (q) $64 - 25a^2$ (r) $9 - 4d^2$
(s) $16a^2 - 25c^2$ (t) $9m^2 - 4n^2$ (u) $4x^2 - 49y^2$
(v) $16 - a^2b^2$ (w) $9 - c^2d^2$ (x) $25 - j^2k^2$

3 B 4 D 5 A 6 C

7 (a) (i) $x^2 + 14x + 49$ (ii) $4x^2 - 12x + 9$
(b) (i) 121 cm^2 (ii) 25 cm^2

8 (a) $x^2\ \text{m}^2$ (b) $(x^2 - 4)\text{m}^2$
(c) Square playing field is larger by 4 m^2.

9 (a) 529 (b) 2401 (c) 12 544
(d) 14.44 (e) 36.120 1 (f) 143.041 6
(g) 9996 (h) 3.91 (i) 35.998 4

10 (a) π (b) The area remains constant (π).

11 (a) $\frac{1}{2}(L + l)(L - l)$ (b) $\frac{1}{2}L^2 - \frac{1}{2}l^2$
(c) $(L + l)(L - l) = L^2 - l^2$
(d) (i) $12l^2\ \text{cm}^2$ (ii) $12l^2\ \text{cm}^2$
(e) Area of large triangle = $125l^2$% of area shaded
$= \frac{12l^2}{12.5l^2} \times 100\% = 96\%$

Open-ended – Sample answers

12 Sample answers: $a = 5$ $b = 4$; $a = 13$ $b = 5$; $a = 10$ $b = 6$; any Pythagorean triple where a = length of hypotenuse and b = either of the other side lengths.

Exercise 3.7 (p. 182)

1 (a) $3(x + 3)$ (b) $12(y + 2)$ (c) $6(1 + 7a)$
(d) $2(k - 3)$ (e) $4(m - 5)$ (f) $4(2b - 1)$
(g) $x(7 - y)$ (h) $s(4 - 9t)$ (i) $a(3 - b)$
(j) $5(3y + w + 2x)$ (k) $2(a + 2b - 4c)$ (l) $3(x - 5y + 3)$
(m) $ac(3 + 2b)$ (n) $xy(5y + 2x)$ (o) $3fg(3fh - 4g)$
(p) $6gh(h + 3g)$ (q) $8xy(2x - 5y)$ (r) $6ab(3 - 7a)$
(s) $-4(m + 5)$ (t) $-7(d + 7)$ (u) $-12(r + 5)$
(v) $(a + 3)(x + 5)$ (w) $(n - 2)(m + 9)$ (x) $(x + 5)(3 + y)$

2 (a) $5(2x + 5)$ (b) $4(4y + 9)$ (c) $7(2a - 5)$

(d) $6(2d-3)$ (e) $9(2-3m)$ (f) $3(12-5k)$

(g) $b(24ac-10+7c)$ (h) $k(5m-3n+2gh)$ (i) $c(1+4b+2a)$

(j) $x(x+3)$ (k) $y(y+6)$ (l) $k(5+k)$

(m) $m(8-3m)$ (n) $p(4p-7)$ (o) $a(9a-5)$

(p) $3d(2-3d)$ (q) $2g(11-7g)$ (r) $8a(3-4a)$

3 (a) $-2(m+6)$ (b) $-4(k+6)$ (c) $-5(h+5)$

(d) $-x(x+1)$ (e) $-y(y+3)$ (f) $-p(p+4)$

(g) $-4(4a-1)$ (h) $-7(3b-1)$ (i) $-6(5r-1)$

(j) $-8(3p-5)$ (k) $-2(9w-8)$ (l) $-9(5k+7)$

(m) $-3x(5x+12)$ (n) $-7y(2y+7)$ (o) $-4r(5r+11)$

(p) $-2(4d+2f+3g)$ (q) $-5(3a+2b+c)$ (r) $-7(2c+4d+f)$

(s) $-4x(3x+4-5y-z)$

(t) $-2k(m-2m^2+3+4k)$

(u) $-2b(5b-2c+4c^2-8a)$

4 (a) $(y-1)(4+w)$ (b) $(m-3)(5+n)$

(c) $(p-2)(5+r)$ (d) $(q+5)(p-2)$

(e) $(2x+3)(y-1)$ (f) $(3p+5)(a-1)$

(g) $(f-1)(d-6)$ (h) $(d-2)(5d-4)$

(i) $(m-6)(3m-7)$

5 C 6 A

7 (a) $8ab+12a$ (b) $4a(2b+3)$

8 (a) $(c+2)(d+5)$ (b) $c=d+3$

9 (a) $(5-x)$ m (b) $(5-x)(y+3)$ m^2

10 (a) $4x^2+xy$ cm^2 (b) $x(4x+y)$ cm^2

(c) $\pi(x-1)^2$ cm^2 (d) $\pi x^2-2\pi x+\pi$ cm^2

(e) $4x^2+xy-(\pi x^2-2\pi x+\pi)$
$=(4-\pi)x^2+(y+2\pi)x-\pi$ cm^2

(f) $\pi x^2+xy-2\pi x+\pi$ cm^2

11 (a) $(3p+2)(2p-1)$ cm^2 (b) $(3p+2)(r+2)$ cm^2

(c) $(3p+2)(2p-r-3)$

(d) If you calculate the length of the lasagne after removing the slice, it's easier to then calculate the area after the removing the slice.

New length $=2p-1-(r+2)=2p-r-3$

New area $=(3p+2)(2p-r-3)$

Open-ended – Sample answers

12 Sample answers: $3x+12xy$; $6x+9xb$; $12abx+15cx$

13 The correct answer is $-5(x-2)$. The first answer had the wrong sign in front of the constant in the brackets, and the second answer had the wrong sign and value of the constant in the brackets.

Exercise 3.8 (p. 190)

1 (a) $(x+4)(y+3)$ (b) $(p+1)(q+5)$

(c) $(m+2)(n+9)$ (d) $(a+f)(b+d)$

(e) $(p+t)(q+r)$ (f) $(k+6)(m+3n)$

(g) $(p-7)(n+1)$ (h) $(a-5)(b+1)$

(i) $(m-2)(n+7)$ (j) $(a-6)(b-4)$

(k) $(x+2)(x-8)$ (l) $(y+4)(y-6)$

2 (a) $(k+2)(p+3)$ (b) $(m+p)(n+q)$

(c) $(d+6)(c+2)$ (d) $(a-k)(d+h)$

(e) $(x-4)(y+2)$ (f) $(e-1)(g+3)$

(g) $(b+1)(c-1)$ (h) $(y+3)(x-1)$

(i) $(k+3)(m-n)$

3 (a) (i) $(x+2)(x+7)$

(ii) $x^2+9x+14$

(iii) $x^2+9x+14=(x+2)(x+7)$

(iv) $2\times 7=14$

(b) (i) $(j+5)(j+3)$

(ii) $j^2+8j+15$

(iii) $j^2+8j+15=(j+5)(j+3)$

(iv) $5\times 3=15$

(c) (i) $(z+8)(z+1)$

(ii) z^2+9z+8

(iii) $z^2+9z+8=(z+8)(z+1)$

(iv) $8\times 1=8$

(d) (i) $(x-8)(x+2)$

(ii) $x^2-6x-16$

(iii) $x^2-6x-16=(x-8)(x+2)$

(iv) $-8\times 2=-16$

(e) (i) $(y-6)(y+4)$

(ii) $y^2-2x-24$

(iii) $y^2-2x-24=(y-6)(y+4)$

(iv) $-6\times 4=-24$

(f) (i) $(x-4)(x+3)$

(ii) x^2-x-12

(iii) $x^2-x-12=(x-4)(x+3)$

(iv) $-4\times 3=-12$

(g) (i) $(a+5)(a-4)$

(ii) a^2+a-20

(iii) $a^2+a-20=(a+5)(a-4)$

(iv) $5\times -4=-20$

(h) (i) $(k+5)(k-2)$

(ii) $k^2+3k-10$

(iii) $k^2+3k-10=(k+5)(k-2)$

(iv) $5\times -2=-10$

(i) (i) $(m+4)(m-7)$

(ii) $m^2-3m-28$

(iii) $m^2-3m-28=(m+4)(m-7)$

(iv) $4\times -7=-28$

(j) (i) $(y-7)(y-3)$

(ii) $y^2-10y+21$

(iii) $y^2 - 10y + 21 = (y - 7)(y - 3)$

(iv) $-7 \times -3 = 21$

(k) (i) $(c - 5)(c - 9)$

(ii) $c^2 - 14c + 45$

(iii) $c^2 - 14c + 45 = (c - 5)(c - 9)$

(iv) $-5 \times -9 = 45$

(l) (i) $(d - 12)(d - 8)$

(ii) $d^2 - 20d + 96$

(iii) $d^2 - 20d + 96 = (d - 12)(d - 8)$

(iv) $-12 \times -8 = 96$

(m) (i) $(m + 3)^2$

(ii) $m^2 + 6m + 9$

(iii) $m^2 + 6m + 9 = (m + 3)^2$

(iv) $3 \times 3 = 9$

(n) (i) $(r + 6)^2$

(ii) $r^2 + 12r + 36$

(iii) $r^2 + 12r + 36 = (r + 6)^2$

(iv) $6 \times 6 = 36$

(o) (i) $(k + 11)^2$

(ii) $k^2 + 22k + 121$

(iii) $k^2 + 22k + 121 = (k + 11)^2$

(iv) $11 \times 11 = 121$

(p) (i) $(p - 5)^2$

(ii) $p^2 - 10p + 25$

(iii) $p^2 - 10p + 25 = (p - 5)^2$

(iv) $-5 \times -5 = 25$

(q) (i) $(d - 7)^2$

(ii) $d^2 - 14d + 49$

(iii) $d^2 - 14d + 49 = (d - 7)^2$

(iv) $-7 \times -7 = 49$

(r) (i) $(n - 8)^2$

(ii) $n^2 - 16n + 64$

(iii) $n^2 - 16n + 64 = (n - 8)^2$

(iv) $-8 \times -8 = 64$

(s) (i) $(c - 3)(c + 3)$

(ii) $c^2 - 9$

(iii) $c^2 - 9 = (c - 3)(c + 3)$

(iv) $-3 \times 3 = -9$

(t) (i) $(t - 2)(t + 2)$

(ii) $t^2 - 4$

(iii) $t^2 - 4 = (t - 2)(t + 2)$

(iv) $-2 \times 2 = -4$

(u) (i) $(b - 9)(b + 9)$

(ii) $b^2 - 81$

(iii) $b^2 - 81 = (b - 9)(b + 9)$

(iv) $-9 \times 9 = -81$

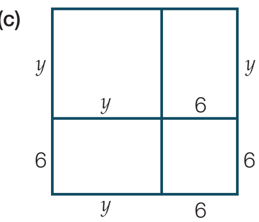
4 A

5 (a) $x^2 - 4x + 12 - 3x$ (b) $x^2 - 7x + 12$

(c) $(x - 4)(x - 3)$

6 (a) $y^2 + 12y + 36$ (b) $(y + 6)^2$

(c)



7 (a) $(a + 7)(b + c + d)$

(b) The expression would be equal to $(a + 7)(a + 7)$, which is a perfect square: $(a + 7)^2$.

8 (a) $2x$, $5x$ (b) $(x + 2)(x + 5)$

9 (a) $7y(5y - 3)$ (b) $7y$ cm by $(5y - 3)$ cm

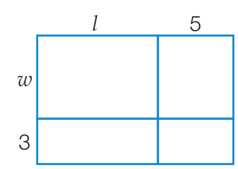
(c) $(7y + 3)(5y - 3)$ (d) 3 cm by $(5y - 3)$ cm

10 (a) $\frac{1}{2}(3x + 2y)(x - 2)$ (b) $3x + 2y$

Open-ended – Sample answers

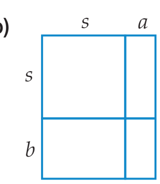
11 (a) xy and ay (b) $(x + a)(x + y)$

12 (a)



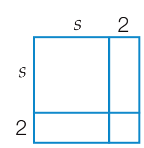
Area $= lw + 3l + 5w + 15$
or
Area $= (l + 5)(w + 3)$

(b)



Area $= s^2 + as + bs + ab$
or
Area $= (s + a)(s + b)$

(c)



Area $= s^2 + 2s + 2s + 4$
or
Area $= (s + 2)^2$

Challenge 3 (p. 192)

1 $a + b = 27$, $a \times b = 72 = 6 \times 12 = 8 \times 9 = 4 \times 18 = 3 \times 24$. Numbers are 3 and 24.

2 Let today be x. The day before yesterday is $x - 2$. These two days need to be in different years.
If today is 1 January, year y, then on 30 December, year $(y - 1)$, she is 15. Her birthday is on 31 December.
So on 31 December, year $(y - 1)$, she turns 16. On 31 December, year y, she is 17 and on 31 December, year $(y + 1)$, next year, she is 18. She said this on 1 January in the year of her 17th birthday.

3 $12^3 = 1728$, $13^3 = 2197$, $20^3 = 8000$, $21^3 = 9261$, $22^3 = 10\,648$. There are nine perfect cubes between 2000 and 10 000.

4 D: Shaded area $= (2x + 1)(x - 2) - 10 = 2x^2 - 3x - 12$.

5 $(3 \times 174)^2 = 272\,484$, so ♦ represents 4.

6 If a, b and c are positive whole numbers then at least two of them are either both odd or both even. The difference between two odd numbers will be even and the difference between two even numbers will be even; hence, at least one of $(a - b)$, $(b - c)$ or $(c - a)$ will be even and so the product will have a factor of 2.

7 Powers of 3 end in 3, 9, 7 or 1, and the pattern is repeated. $404 \div 4 = 101$, so 3^{404} ends in 1.

Powers of 4 end in 4 or 6, and the pattern is repeated. Hence, odd powers of 4 end in 4.

$3^{404} + 4^{495}$ ends in $(1 + 4 = 5)$, so it is divisible by 5.

8 $x + y = 6$; $1 + 5$, $2 + 4$
$x + z = 7$; $1 + 6$, $2 + 5$, $3 + 4$
$x + w = 9$; $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$
$y + z = 9$; $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$
$y + w = 11$; $1 + 10$, $2 + 9$, $3 + 8$, $4 + 7$, $5 + 6$
$z + w = 12$; $1 + 11$, $2 + 10$, $3 + 9$, $4 + 8$, $5 + 7$
By trial and error: if $x = 1$, $y = 5$, $z = 6$, $w = 8$, $z = 4$, so not the answer. Same result if $x = 5$. If $s = 2$, $y = 4$, $z = 5$, $w = 7$, and these results satisfy the other equations.
Numbers are 2, 4, 5 and 7.
Or, using pairs of equations you can get: $z - y = 1$, $w - y = 3$, $y - x = 2$, $w - x = 5$, So $y + x = 9$ and $z - y = 1$, so $2z = 10$, $z = 5$
$y + w = 11$ and $w - y = 3$, so $2w = 14$, $w = 7$
$x + y = 6$ and $y - x = 2$, so $2y = 8$, $y = 4$
$x + w = 9$ and $w - x = 5$, so $2x = 4$, $x = 2$

9 (a) $f = \frac{1}{u} + \frac{1}{v} = \frac{1}{f}, \frac{v+u}{uv} = \frac{1}{f}, f = \frac{uv}{u+v}$

(b) $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}, = \frac{1}{v} = \frac{u-f}{fu}, v = \frac{uf}{u-f}$

10 (a) Let the two-digit number be $10a + b$. The number formed by reversing the digits is $10b + a$. The difference $= 10a + b - (10b + a) = 9a - 9b = 9(a - b)$, which is divisible by 9.

(b) The three digit number is $100a + 10b + c$. The number formed by reversing the digits is $100c + 10b + a$. The difference $= 100a + 10b + c - (100c + 10b + a) = 99a - 99c = 99(a - c)$, which is divisible by 99 (and hence by the factors of 99).

(c) The four digit number is $1000a + 100b + 10c + d$. The number formed by reversing the digits is $1000d + 100c + 10b + a$. The difference $= 1000a + 100b + 10c + d - (1000d + 100c + 10b + a) = 999a + 99b - 99c - 999d = 9(111a + 11b - 11c - 111d)$, which is divisible by 9 (and hence by 3).

Chapter review 3 (p. 193)

1 (a) a^5b^7 (b) m^7n^8 (c) $\frac{p}{q^3}$
(d) x^2 (e) y^5 (f) c^2d

2 D

3 (a) $\frac{-5}{m}$ (b) $\frac{a^2}{b^6}$ (c) $\frac{1}{x^2y^3}$ (d) $\frac{1}{a^4b^9}$

4 (a) 2×10^{-15} mm (b) 1.5×10^{-4} kg
(c) 3.2×10^8 L (d) 4.8×10^{14}

5 (a) 54 000 000 (b) 0.000 827 3
(c) 0.000 014 (d) 7162

6 (a) $x = \frac{3 - 2z}{y}$ (b) $x = \frac{2a}{3}$
(c) $x = \frac{s - 8t^2}{t}$ (d) $x = \frac{1 + 2z}{15}$

7 A

8 (a) $4x + 28$ (b) $x^2 - 3x$ (c) $-10x^2 + 5x$

9 (a) (i) $13x - 7$ (ii) $18x^2 + 19x - 6$
(b) (i) 45 (ii) 358

10 (a) $x^2 + 10x + 16$ (b) $6a^2 + 7a - 20$

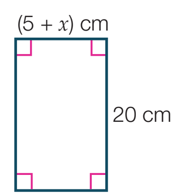
11 (a) $x^2 + 24x + 144$ (b) $25c^2 - 20cd + 4d^2$
(c) $16x^2 - 9$

12 (a) $4(2a + 3)$ (b) $-5k(3 + 4k)$
(c) $pq(5p - 3 + 2q - r)$ (d) $-6(4a + ab + 12)$

13 (a) $(a + 4)(b + 3)$ (b) $(b + 1)(a - 2)$
(c) $(b - 6)(a - 5)$ (d) $(a + 7)(b - 1)$

14 (a) 1.5×10^{-3} (b) 5.9×10^9 km

15 (a) 778 000 000 km (b) 15 000 000°C

16 (a)


(b) $(50 + 2x)$ cm (c) $2(25 + x)$
(d) $20(5 + x)\text{cm}^2$ (e) $100 + 20x$
(f) Sample answer: $x = 10$; $20 \times 15 = 100 + 20 \times 10$

17 $2x + 6$

18 $(m - 2)(k + 8)$; $m = 5$, $k = 8$

19 $5y - 3$

20 (a) 7×10^3 L
(b) (i) 4.9×10^4 L (i) 0.1 L (iii) 7.665×10^6 L
(c) The amount of blood each person has will vary between people, as well as at different times for the same person. However, this provides a suitable approximation.

21 (a) $10 + 2x$ (b) $x^2 + 5x$
(c) (i) $x^2 + 5x - 14$
(ii) $x^2 + 7x - 2x - 14$, they are the same expression
(iii) There are no sheep

NAPLAN practice 3

1 A 2 42 3 $7(2x + 3)$

4 B 5 $x = \frac{12 - y}{3}$ 6 D 7 A

Chapter 4

Recall 4 (p. 198)

1 (a) 72.1 (b) 18 210 (c) 15.7123 (d) 0.000074

2 (a) 42 mm (b) 390 cm (c) 7100 m
(d) 0.45 m (e) 0.186 m (f) 5.06 km
(g) 0.011 km (h) 0.003 72 km

3 (a) 5 m (b) 1.6 mm (c) 15.1 cm

4 (a) D (b) A (c) C (d) B

5 (a) 44 cm, 112 cm^2 (b) 48 cm, 96 cm^2
(c) 44 cm, 154 cm^2

Exercise 4.1 (p. 203)

1 (a) 34 cm (b) 24 cm (c) 60 cm
(d) 45 cm (c) 130 mm (f) 140 cm

2 (a) (i) 4.8 m (ii) 10.2 m (iii) 45.0 mm (iv) 60.0 mm
(b) (i) 14.8 km (ii) 11.3 km (iii) 128.0 cm (iv) 0.2 cm

3 (a) (i) 15 cm (ii) 43 cm
(b) (i) 7 m (ii) 13 m
(c) (i) 1470 mm (ii) 2212 mm
(d) (i) 139 m (ii) 191 m
(e) (i) 68 cm (ii) 121 cm
(f) (i) 152 cm (ii) 480 cm

4 B

5 (a) 9.1 km (b) 111.4 cm (c) 42.8 cm
(d) 112.8 cm (e) 314.2 cm (f) 127.1 cm
(g) 94.2 cm (h) 268.5 cm (i) 88.8 cm

6 12.0 cm 7 40 200 km

8 (a) 113 m (b) 22.6 seconds (c) 31.83 m

9 2.342 km

10 35.5 km

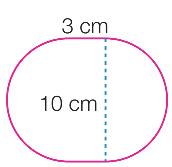
11 Just under 16 corrugations (15.9 on the calculator)

Open-ended – Sample answers

12 Students' own answers.

13 Any whole number value from 16 to 28 cm (e.g. a diameter of 20 cm gives a circumference of ≈ 62.83 cm; a diameter of 25 cm gives a circumference of ≈ 78.54 cm; a diameter of 28 cm gives a circumference of ≈ 87.96 cm).

14 Possible shape with $P \approx 37.4$ cm:



Exercise 4.2 (p. 212)

1 (a) 30 000 cm^2 (b) 1720 cm^2 (c) 7250 mm^2
(d) 60 mm^2 (e) 90 000 m^2 (f) 40 700 m^2
(g) 0.0035 ha (h) 6.95 cm^2 (i) 0.0127 m^2
(j) 0.0835 m^2 (k) 438 000 m^2 (l) 0.685 ha
(m) 0.029 m^2 (n) 3960 mm^2 (o) 15 km^2
(p) 100.8 km^2

2 (a) 8196.9 cm^2 (b) 16.6 m^2 (c) 506.3 cm^2
(d) 5.3 m^2 (e) 4.3 m^2 (f) 1.3 km^2

3 (a) 1853 cm^2 (b) 143 cm^2 (c) 1750 mm^2
(d) 240 m^2 (e) 293 cm^2 (f) 506 cm^2
(g) 6514 cm^2 (h) 628 cm^2 (i) 184 cm^2
(j) 468 cm^2 (k) 1087 cm^2 (l) 1973 cm^2

4 C

5 (a) 1408 m^2 (b) 176 kg

6 (a) (i) 112 mm^2 (ii) 1.12 cm^2
(b) (i) 225 mm^2 (ii) 2.25 cm^2
(c) (i) 127.23 mm^2 (ii) 1.27 cm^2
(d) (i) 4500 mm^2 (ii) 45 cm^2

7 8.95 m^2

8 (a) 12.44 m^2
(b) At least 623 bricks would be required to tile the path, although taking into account wastage, say 630 bricks.

9 167.6 cm^2

10 (a) 451 cm (b) 1546 cm^2 (c) 62%

11 (a) 311.1 m^2 (b) 69.1 m^2 (c) 82%

12 5.53 m^2 13 289.6 m^2 14 807 m^2 15 375 blocks

16 (a) 200 cm^2 (b) 14.14 cm

17 127° 18 11.5 m^2

19 (a) (i) 3 cm (ii) 2 cm
(b) They are the same.

20 C

Open-ended – Sample answers

21 The area of the top of the pizza is 616 cm^2, so any rectangle that gives this area with integer values: approximately 25 × 25; 22 × 28; 31 × 20 etc.

22 Area = 200 cm^2 (length 20 cm, width 10 cm); Area = 216 cm^2 (length 18 cm, width 12 cm).

Half-time 4 (p. 216)

1 (a) 540 cm (b) 29 mm (c) 30 m
(d) 111 mm (e) 57 cm (f) 107 m

2 (a) 1344 cm^2 (b) 273 cm^2 (c) 57 cm^2

3 10 100 km

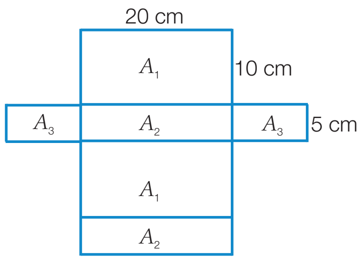
4 30 cm

5 (a) 400 mm^2 (b) 75 000 cm^2 (c) 260 000 m^2
(d) 198 000 m^2 (e) 305 ha (f) 0.0274 m^2

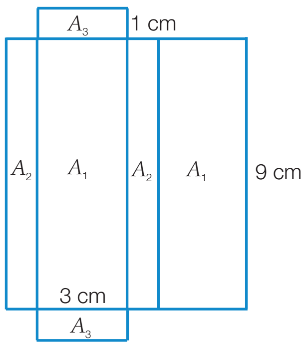
6 270 cm^2

Exercise 4.3 (p. 220)

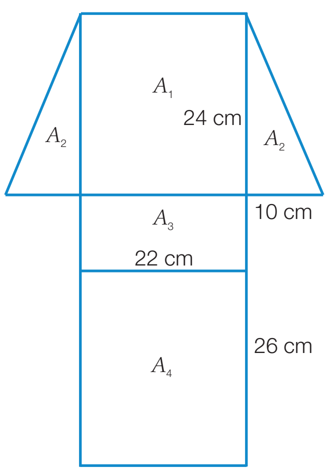
1 **(a)** 700 cm^2



(b) 78 cm^2

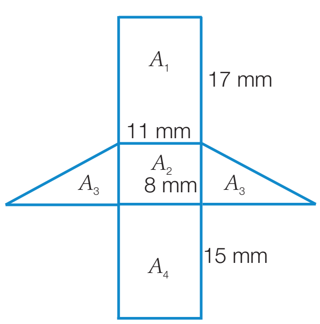


(c) 1560 cm^2

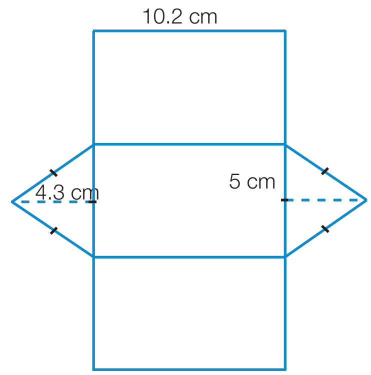


(d) 560 mm^2

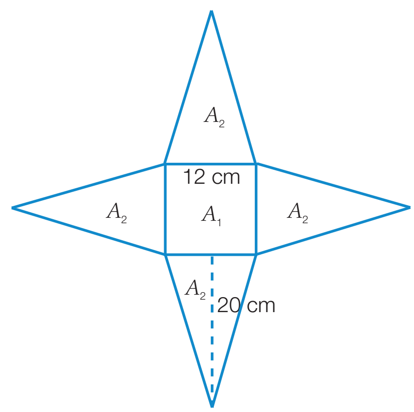
(d)



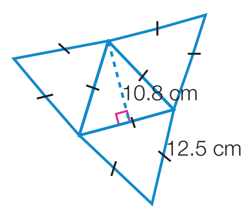
(e) 174.5 cm^2



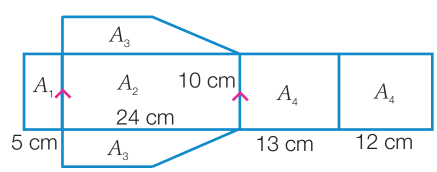
(f) 624 cm^2



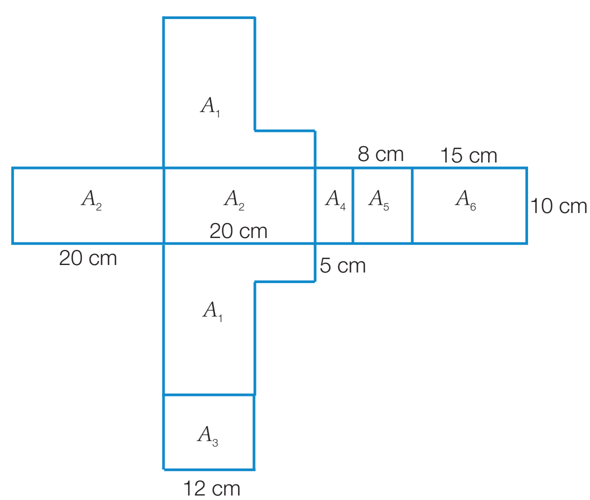
(g) 270 cm^2



(h) 720 cm^2



(i) 1360 cm^2



2 (a) 31.7 m^2 (b) 6622.5 cm^2

(c) 18.8 m^2 (d) 6432.4 mm^2

3 (a) C (b) D

4 Type 1 uses 352 cm^2; Type 2 uses 372 cm^2, so Type 1 uses less

5 109.93 m^2

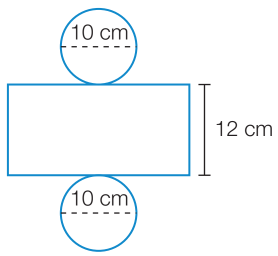
6 (a) 250 cm^2 (b) 85 cm^2

7 34 m^2

8 (a) 60 cm long, 50 cm wide (b) 30%

9 (a) 54 cm^2 (b) 162 cm^2 (c) 3.06 times

10 (a)



(b) B (c) C (d) C

Open-ended – Sample answers

11 Four times larger; possible choice, l = 10 cm, 20 cm, SA = 600 cm^2, 2400 cm^2

12 Any of these combinations (length, width, height) in centimetres: (2, 2, 119), (4, 4, 58), (6, 6, 37), (8, 8, 26), (10, 10, 19), (12, 12, 14), (16, 16, 7), (20, 20, 2)

Exercise 4.4 (p. 226)

1 (a) 600 cm^3 (b) 512 m^3 (c) 1260 mm^3

(d) 210 cm^3 (e) 63 m^3 (f) 28 274 cm^3

2 (a) 827 mL (b) 3.2 L (c) 195 kL (d) 18.2 ML

3 3040 m^3

4 (a) 20 cm (b) 12.4 m

(c) 12 cm (d) 30 mm

5 (a) D (b) C (c) B (d) A

6 100 cups

7 (a) 113 cm (b) 160 days

8 6048 cm^3

9 (a) 15 944 cm^3 (b) 10 cm

10 (a) 76 027 mL (b) 14 451 cm^3 (c) 273.7 kL

Open-ended – Sample answers

11 (a) Examples are: Medicine, cans of drink, bottles of juice.

(b) The amount of petrol in the tank of a car, daily water usage in a household.

(c) Capacity of a swimming pool, water tank capacity.

12 Possible dimensions: (a) 2 cm, 5 mm, 1 cm (b) 20 cm, 5 cm, 10 cm (c) 2 m, 50 cm, 1 m (d) 20 m, 5 m, 10 m

13 (a) (i) 56 700 cm^3 = 0.0567 m^3. Need to move two decimal places for each length conversion, hence six places overall.

(ii) 144 765 $mm^3 \approx 145$ cm^3. Need to move one decimal place for each length conversion, hence three places overall.

(b) Work in the changed units from the start; that is:

(i) 0.45 m, 0.18 m, 0.7 m (ii) 4.8 cm, 2 cm.

Mathspace (p. 230)

Origami questions

1 (a) Answers will vary; approximately 36–39 cm.

(b) Triangles, rectangle, trapezium.

(c) Triangle = $\frac{1}{2}bh$, rectangle = lw, trapezium = $\frac{1}{2}(a+b)h$.

(d) Answers will vary; approximately 79–81cm^2.

(e) Approximately 26%.

Dehydration

2 1.38 L/h 3 5 L 4 6.5 L 5 1.86 L/h

6 31 mL/min 7 1 L, no

Challenge 4 (p. 235)

1 $A = 6s^2$. $v = s^3 = 19\,683$, $s = 27$. $A = 6 \times 27^2 = 4374$ cm^2.

2 3 years of 366 days and 12 years of 365 days = 5478 days. Number of seconds = 5478 × 24 × 60 × 60 = 473 299 200.

3 Time off field = $\frac{2}{13} \times 40$ min = 6.15 min = 6 min 9 s.

4 Time: 1 + 2 + 4 + 8 + … = 24 × 60 = 1440 min.

Number of clicks = 1 more than number of terms in above time sum.

$1 + 2 = 3$, $1 + 2 + 4 = 7$, $1 + 2 + 4 + 8 = 15 = 2^4 - 1$

Find which power of 2 is ≤ 1440. $2^{10} = 1024$. He clicks his fingers 10 + 1 = 11 times.

5 Require 360 mL in each of the two containers.

Fill the 330-mL jug and use it to fill the 150-mL jug. Pour the remaining 180 mL into one container. Pour the 150 mL back into the 720-mL jug.

Repeat, giving 360 mL in one container. Pour the remaining 360 mL from the 720-mL jug into the second container.

6 Volume increases by a factor of 8, the surface area by a factor of 4.

7 Perimeter of first figure = $12\pi + 12\pi = 24\pi$.
Perimeter of second figure = $3 \times 4\pi + 12\pi = 24\pi$.

Ratio = 1 : 1.

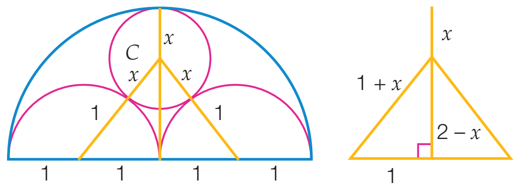
8 Each small square has an area of 4 cm^2.
Length of side = 2 cm.

Perimeter = 14 × 2 + 8 × 2 = 44 cm

9 A Let $OB = x$ cm, $OA = 2x$ cm, $AD = 4x$ cm.

Area of square = $AD^2 = 16x^2$ cm^2.
Area of annulus = $\pi(2x + x)(2x - x) = 3\pi x^2$ cm^2.

Fraction covered $\frac{3\pi x^2}{16x^2} = \frac{3\pi}{16}$.

10 

Let the radius of the circle, centre C, be x cm.

In the right-angled triangle,

$$(1+x)^2 = 1^2 + (2-x)^2$$
$$1 + 2x + x^2 = 1 + 4 - 4x + x^2$$
$$6x = 4$$
$$x = \frac{2}{3} \text{ cm.}$$

The radius of a circle centre C is $\frac{2}{3}$ cm.

Chapter review 4 (p. 236)

1 (a) 27 cm (b) 60 cm (c) 57 cm

2 (a) (i) 134 mm (ii) 304 mm
(b) (i) 66 cm (ii) 108 cm
(c) (i) 1.3 cm (ii) 3.7 cm
(d) (i) 204 mm (ii) 356 mm
(e) (i) 98.2 cm (ii) 151.8 cm
(f) (i) 6.95 m (ii) 9.65 m

3 (a) 420 mm^2 (b) 2 ha (c) 68.25 m^2
(d) 5.28 km^2 (e) 807 000 m^2 (f) 90 cm^2

4 (a) 5675 mm^2 (b) 693 cm^2 (c) 0.75 cm^2
(d) 7762 mm^2 (e) 1316.2 cm^2 (f) 4.69 m^2

5 (a) 164 cm^2 (b) 3792 cm^2 (c) 356 cm^2
(d) 603 cm^2 (e) 7.51 m^2

6 (a) 1550 cm^2 (b) 7.49 m^2 (c) 190.5 cm^2
(d) 179 cm^2

7 (a) 360 cm^3 (b) 2011 cm^3 (c) 1380 cm^3
(d) 3000 cm^3 (e) 110 mm^3 (f) 450 m^3

8 (a) 0.017 6 ha (b) 1.96 cm^2
(c) 1800 mm^2 (d) 0.002 4 m^2

9 (a) 61.34 m^2 (b) 12.30 m^2

10 147 575 m^2 11 308.86 mL 12 384 cm^2

13 (a) 7527 m^2 (b) 361 m

14 148 cm^2 15 598 cm^2

16 (a) 143 m^2 (b) 14 900 (c) 145 200 L

17 (a) 12 160 cm^3 (b) 32 cm (c) 16.75 cm

18 22 cm

19 (a) 117 810 (b) 16 h 21 min 45 sec

20 (a) 8.48 m
(b) hour hand: 0.04 cm/min,
minute hand: 0.63 cm/min

NAPLAN practice 4

1 C 2 62.4 cm^2 3 B 4 7 cm
5 D 6 A 7 62 cm 8 C

Mixed review B (p. 241)

1 (a) 212.5 (b) \$0.08 (c) 72

2 (a) $80g^2h^3$ (b) $-36k^4$ (c) $108e^3fg$

3 (a) $-8x - 16$ (b) $30y - 55y^2$ (c) $12p + 58$

4 (a) $4(m - 7)$ (b) $7d(ed + 5)$ (c) $2y(4xy - 1)$

5 (a) 14.29% (b) 75% (c) 12.5% (d) 250%

6 132 cm^2

7 (a) 20 mm^2 = 0.2 cm^2
(b) 23.67 km^2 = 23 670 000 m^2
(c) 0.007 m^2 = 70 cm^2 (d) 14.5 ha = 145 000 m^2

8 (a) x^8 (b) x^4 (c) x^4

9 (a) \$48.50 (b) \$77.60 (c) \$92.15 (d) \$145.50

10 (a) 1 (b) 5 (c) 1 (d) 5

11 (a) 178.5 cm^2 (b) 92 m^2 (c) 198 cm^2
(d) 2045.2 cm^2 (e) 17.4 cm^2 (f) 47.1 cm^2

12 (a) \$45 (b) \$1890 (c) \$7845 (d) \$18 549

13 (a) $x^2 + 4x - 21$ (b) $x^2 + 12x + 36$
(c) $4x^2 - 20x + 25$ (d) $x^2 - 64$

14 (a) 4.5×10^7 (b) 5.27×10^9
(c) 3×10^{-4} (d) 2.98×10^{-7}

15 (a) (i) 153.68 m^2 (ii) 110.98 m^3
(b) (i) 20.36 m^2 (ii) 6.87 m^3

16 (a) 7238.2 cm^3 (b) 7238.2 mL

17 (a) $12g^4h^4$ (b) $120a^7b^{11}$ (c) $\frac{xy^5}{3}$

18 216 days

19 (a) \$560 (b) \$481

20 (a) 2 (b) $\frac{3p^5q^7}{2}$ (c) $5x^5y^7$

21 5.1 m^2

22 (a) $m = \frac{2E}{v^2}$ (b) 5.3 kg

23 (a) \$364 (b) \$487.50 (c) \$606.25 (d) \$649.95

24 (a) 0.8 m (approx.) (b) 12 days

25 Surface area is multiplied by four.

26 (a) Option A: at least \$16 000; Option B: at least \$12 667.
(b) \$6000
(c) Option A is better if Bettina expects to sell less than \$6000 worth of goods per week.

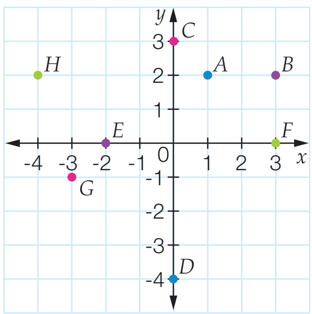
27 (a) 21.46%
(b) 21.46%
(c) They are equally efficient.

Chapter 5

Recall 5 (p. 246)

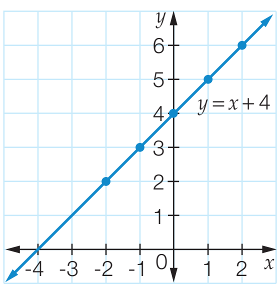
1 (a) $P = 5$ (b) $P = 11$ (c) $P = 3$ (d) $P = -13$ (e) $C = 5$
(f) $C = 17$ (g) $C = 9$ (h) $C = -7$

2



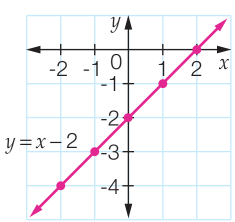
3 (a)

x	-2	-1	0	1	2
y	2	3	4	5	6



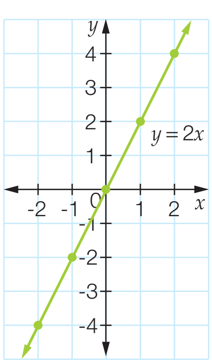
(b)

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



(c)

x	-2	-1	0	1	2
y	-4	-2	0	2	4



4 (a) $x = 6$ (b) $y = 15$ (c) $a = -10$
(d) $b = 21$ (e) $c = 12$ (f) $d = -14$

5 (a) $g = 19$ (b) $x = 42$ (c) $y = 6$
(d) $x = 35$ (e) $b = -12$ (f) $k = 36$

6 (a) $-15x - 6$ (b) $20 - 6x$ (c) $15x + 43$

7 $y = 6 - 2x$

8 (a) $h = 13$ (b) $x = 7$

Exercise 5.1 (p. 251)

1 (a) $h = -2$ (b) $b = 5$ (c) $x = -1$ (d) $r = \frac{5}{6}$ (e) $f = -4$
(f) $k = \frac{2}{5}$ (g) $x = \frac{6}{7}$ (h) $x = -9$ (i) $y = 0$ (j) $b = 6$
(k) $a = -2$ (l) $b = -3$ (m) $p = -15$ (n) $e = 4$ (o) $g = 8$
(p) $y = 2\frac{3}{5}$ (q) $p = 2\frac{3}{4}$ (r) $y = 2$

2 (a) $x = 6$ (b) $x = 7$ (c) $x = 9$ (d) $x = -9$ (e) $x = 12$
(f) $x = 6$ (g) $x = 4$ (h) $x = -2$ (i) $x = 2$ (j) $x = 3$
(k) $x = -1$ (l) $x = 6$ (m) $x = -2$ (n) $x = -8$ (o) $x = -5$

3 (a) $c = -7$ (b) $b = -8$ (c) $f = -2$ (d) $d = 7$ (e) $e = 4$
(f) $a = 6$ (g) $g = -7$ (h) $h = 20$ (i) $x = 3$ (j) $k = \frac{3}{2}$
(k) $j = 2$ (l) $y = 3$ (m) $m = 4$ (n) $n = 2$ (o) $x = 1$
(p) $y = 1$ (q) $p = -3$ (r) $x = 3$ (s) $a = 2$ (t) $x = 9$
(u) $p = 4$

4 (a) $x = 8$ (b) $x = -10$ (c) $x = -5$ (d) $x = -6$
(e) $x = -3\frac{1}{4}$ (f) $x = 5\frac{5}{6}$ (g) $x = \frac{1}{11}$ (h) $x = -\frac{4}{27}$
(i) $x = 7$

5 (a) $b = -22$ (b) $a = 21$ (c) $c = 3$ (d) $d = -8$
(e) $x = 16\frac{1}{2}$ (f) $y = 10$ (g) $g = 1$ (h) $k = 1$
(i) $m = -10$ (j) $j = \frac{3}{2}$ (k) $x = -5\frac{1}{2}$ (l) $p = -19$
(m) $x = -2\frac{1}{3}$ (n) $y = \frac{26}{41}$

6 (a) $x = 6$ (b) $x = 8$ (c) $x = 0$ (d) $x = -36$ (e) $x = -42$
(f) $x = 12$ (g) $x = 3\frac{3}{4}$ (h) $x = 0$ (i) $x = -14$ (j) $x = 15$
(k) $x = 8$ (l) $x = 22$ (m) $x = 22$ (n) $x = 57$ (o) $x = 55$

7 (a) B (b) B (c) C (d) C

8 (a) B (b) C (c) D (d) A (e) D (f) C

9 (a) D (b) B

10 (a) B (b) D (c) D (d) A

11 (a) $x + 7 = 19, x = 12$ (b) $y - 5 = 3, y = 8$
(c) $3x = 14, x = 4\frac{2}{3}$ (d) $\frac{x}{4} = -5, x = -20$
(e) $2x + 3 = 13, x = 5$ (f) $4x - 7 = -12, x = -1\frac{1}{4}$

12 (a) $x = 5$ (b) $x = -2$ (c) $x = 2$ (d) $x = -9$ (e) $x = -6$
(f) $x = 4$ (g) $x = 5$ (h) $x = 1$ (i) $x = -2$ (j) $x = 0$
(k) $x = 6$ (l) $x = -2$

13 (a) $x = -\frac{5}{8}$ (b) $x = -\frac{23}{12}$ (c) $x = -\frac{8}{15}$ (d) $x = 8\frac{1}{2}$
(e) $x = \frac{1}{4}$ (f) $x = -\frac{11}{14}$ (g) $x = -\frac{1}{6}$ (h) $x = \frac{2}{3}$
(i) $x = 2\frac{5}{6}$

14 $4x - 11 = 7 - 2x, x = 3$

15 $x + 36 = 2(x + 6), x = 24$

16 $\frac{3x + 1}{2} = \frac{x - 4}{5}, x = -1$

17 **(a)** $n + n + 2 + n + 4 = 195$, $n = 63$. The three numbers are 63, 65 and 67.

(b) $n + n + 2 + n + 4 + n + 6 = 204$, $n = 48$. The four numbers are 48, 50, 52 and 54.

(c) $n + n + 7 + n + 14 + n + 21 + n + 28 + n + 35 = 609$, $n = 84$. The six numbers are 84, 91, 98, 105, 112 and 119.

18 **(a)** $x + \frac{x}{6} = 28, x = 24$ **(b)** $x + \frac{x}{4} = 45, x = 36$

(c) $x + \frac{x}{8} = 99, x = 88$

Open-ended – Sample answers

19 If $x = -1$, $2x + 7 = 5$. If $x = -2$, $2x + 7 = 3$. If $x = -10$, $2x + 7 = -13$.

20 $\frac{x}{2} + 1 = x - 1$, $\frac{3x + 4}{2} = 8$, $5(x - 1) = 3(x + 1)$

21 $x + 2 = 2x + 1$, $2x - 7 = x - 3$, $20 - x = x + 18$

Exercise 5.2 (p. 259)

1 Kate is 15 years old and her father is 45 years old.

2 Andrew has 11 coins and Lori has six coins.

3 B **4** B

5 length = 13 m, width = 10 m

6 Annette: 300 m; Mark: 600 m; Sue: 1.2 km

7 \$3.65 **8** Peter is 16 years old. **9** 15

10 21 **11** 49°, 55°, 76° **12** 11 cm

13 Joe is 12, Kate is 15, Brett is 48 and Gabby is 30.

14 $\frac{3(3a - 8)}{4} = \frac{7(a + 3)}{5}$, $a = \$12$

Open-ended – Sample answers

15 23, 26, 30

Exercise 5.3 (p. 263)

1 **(a)** 5 **(b)** 13 **(c)** 17 **(d)** 26

(e) 10 **(f)** 25 **(g)** $\sqrt{17}$ **(h)** $\sqrt{37}$

(i) $\sqrt{61}$ **(j)** $2\sqrt{2}$ **(k)** $2\sqrt{13}$ **(l)** $4\sqrt{5}$

2 **(a)** (-1.5, 2) **(b)** (-1, 4) **(c)** (-1, -7) **(d)** (0.5, 2)

(e) (-1, 7.5) **(f)** (1.5, -0.5) **(g)** (3.5, 4.5) **(h)** (2.5, -2.5)

(i) (4.5, 5.5)

3 C **4** (1, 5.5)

5 **(a)** (9, 2) **(b)** (5, -10) **(c)** (9, 0)

6 $AB = \sqrt{(1 - -3)^2 + (1 - -2)^2} = 5$.

$BC = \sqrt{(-2 - 1)^2 + (5 - 1)^2} = 5$.

$AC = \sqrt{(-2 - -3)^2 + (5 - -2)^2} = \sqrt{50}$.

Two sides are of equal length, therefore ΔABC is isosceles.

$\sqrt{AB^2 + BC^2} = \sqrt{5^2 + 5^2} = \sqrt{50}$. Pythagoras' Theorem holds and therefore ΔABC is a right-angled triangle.

7 **(a)** Side lengths are 3.61, 3.61 and 5.10 (2 d.p.).

(b) Isosceles

8 **(a)** 5 **(b)** (3, 2) **(c)** (7, 1)

(d) From B to C, 11.18 (2 d.p.)

(e) Parallelogram

9 **(a)** Midpoint of AC = (-3, -5), midpoint of BD = (-3, -5)

(b) All sides are 4.47 (2 d.p.) **(c)** $AC = 8.49$, $BD = 2.83$

(d) Rhombus

10 $AB = \sqrt{20}$, $BC = \sqrt{45}$, $CD = \sqrt{20}$, $DA = \sqrt{45}$. Therefore opposite sides are of equal length. $AC = \sqrt{65}$, $BD = \sqrt{65}$. Therefore diagonals are also of equal length and hence $ABCD$ is a rectangle.

11 (4, 1)

12 Coordinates of $PQRS$ are $P(-2, 4)$, $Q(1, 2)$, $R(-1, 0)$, $S(-4, 2)$. PQ has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, QR has length $\sqrt{8}$ and gradient 1, RS has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, SP has length $\sqrt{8}$ and gradient 1. Therefore opposite sides have equal length and gradient and hence $PQRS$ is a parallelogram.

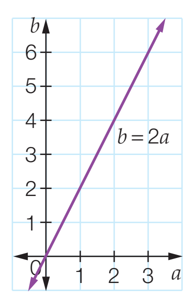
Open-ended – Sample answers

13 $A(-1, 0)$, $B(1, 0)$ and $C(0, \sqrt{3})$. $AB = 2$, $BC = 2$ and $CA = 2$. Therefore ΔABC is an equilateral triangle.

14 $A(0, 8)$, $B(3, 12)$, $C(6, 8)$ and $D(3, 0)$. $AB = 5$, $BC = 5$, $CD = \sqrt{73}$ and $DA = \sqrt{73}$. $ABCD$ has two pairs of adjacent, equal sides and is therefore a kite.

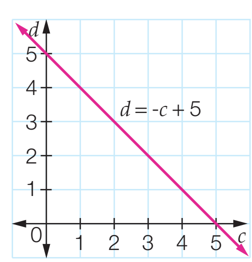
Exercise 5.4 (p. 270)

1 **(a)** **(i)**



(ii) $b = 2a$

(b) **(i)**

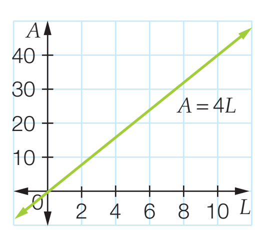


(ii) $d = -c + 5$

2 **(a)** **(i)**

L	0	10	20	30
A	0	40	80	120

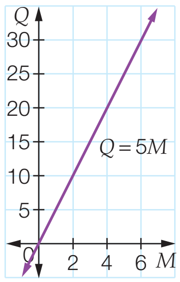
(ii)



(b) **(i)**

M	0	2	4	6
Q	0	10	20	30

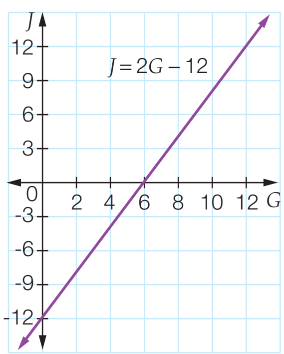
(ii)



(c) **(i)**

G	0	4	8	12
J	-12	-4	4	12

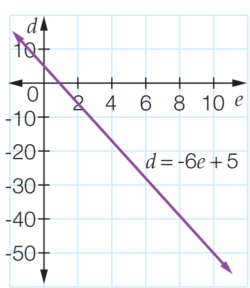
(ii)



(d) **(i)**

e	0	3	6	9
d	5	-13	-31	-49

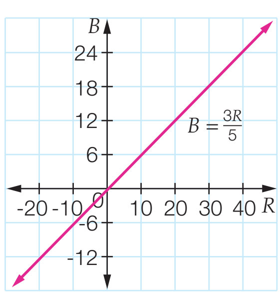
(ii)



(e) **(i)**

R	-20	0	20	40
B	-12	0	12	24

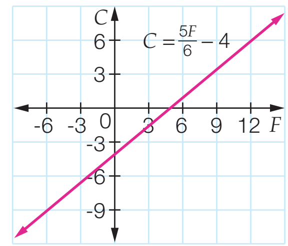
(ii)



(f) **(i)**

F	-6	0	6	12
C	-9	-4	1	6

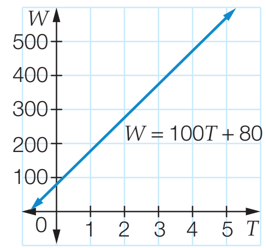
(ii)



3 **(a)**

No. televisions sold	0	1	2	3	4	5
Weekly wage ($)	80	180	280	380	480	580

(b)



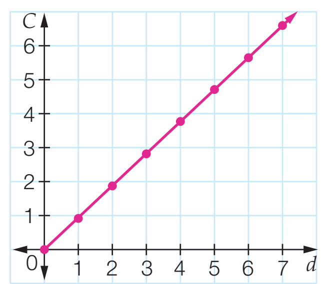
(c) Yes **(d)** $W = 100T + 80$ **(e)** $880

4 **(a)** D **(b)** B **(c)** C

5 **(a)**

No. litres	0	1	2	3	4	5	6	7
Cost ($)	0	0.95	1.90	2.85	3.80	4.75	5.70	6.65

(b)



(c) Yes, the graph is in a straight line.

(d) **(i)** $2.85 **(ii)** $2.28 **(iii)** $5.42

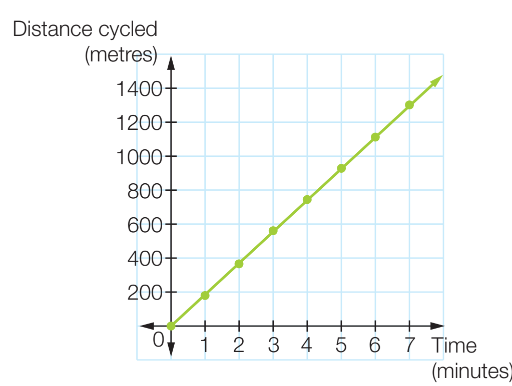
(e) **(i)** 4 L **(ii)** 2.6 L **(iii)** 5.5 L

(f) $C = 0.95d$

6 **(a)**

Time (min)	0	1	2	3	4	5	6	7
Distance cycled (m)	0	185	370	555	740	925	1110	1295

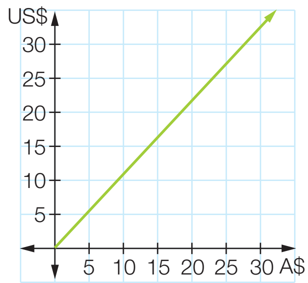
(b)



(c) 499.5 m **(d)** 3.8 min **(e)** 10.8 min

(f) $D = 185t$ **(g)** 7400 m (7.4 km)

7 (a)

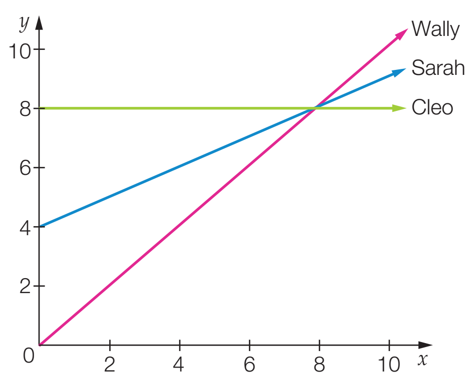


(b) US$7.35 (c) US$18.50 (d) A$5.20 (e) A$9.10

8 (a)

D	Wally	Sarah	Cleo
0	0	4	8
1	1	4.5	8
2	2	5	8
3	3	5.5	8
4	4	6	8
5	5	6.5	8
6	6	7	8
7	7	7.5	8
8	8	8	8
9	9	8.5	8
10	10	9	8

(b)



(c) 8 km

(d) (i) Wally collects the most money above 8 km.

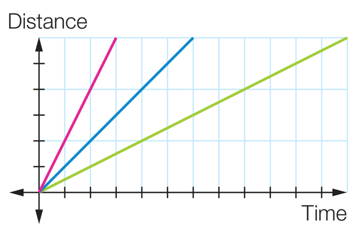
(ii) Sarah never collects the highest amount of money for any distance.

(iii) Cleo collects the most money between 0 and 8 km.

9 In the first 2 years Frank earns more than Bruno and, after that, Bruno earns more.

Open-ended – Sample answers

10



As the time taken to travel the same distance increases, the line becomes less steep (has a smaller gradient).

Half-time 5 (p. 277)

1 (a) Yes (b) No (c) No (d) Yes

2 (4.5, 5)

3 (a) $x = 9$ (b) $y = 5$ (c) $x = 16$ (d) $x = -9$ (e) $x = 3$

(f) $x = 3$ (g) $p = \frac{5}{2}$ (h) $a = -5$ (i) $m = 13$

4 13

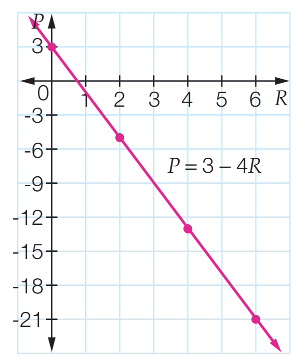
5 Kelvin weighs 54 kg, Rob weighs 60 kg

6 C 7 $x = 2$ m

8 (a)

R	0	2	4	6
P	3	-5	-13	-21

(b)



Exercise 5.5 (p. 282)

1 a: 1; b: $\frac{2}{5}$; c: $-\frac{1}{3}$; d: -2

2 a: 0; b: 0; c: undefined; d: undefined

3 (a) 2 (b) 3 (c) -4 (d) -1 (e) 2

(f) 1 (g) $-\frac{7}{12}$ (h) -1 (i) $-\frac{1}{3}$ (j) 0

(k) 0 (l) 0 (m) undefined

(n) undefined (o) undefined

4 (a) positive (b) negative

(c) zero (d) undefined

5 a: B; b: A; c: A; d: C; e: B

6 (a) a: -1; b: 3; c: $\frac{1}{3}$; d: 0; e: $-\frac{1}{3}$ (b) b (c) a

7 (a) C (b) A (c) A (d) C

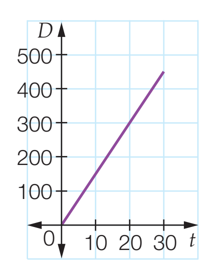
8 (a) 1200 m (b) 20 min (c) 60 m (d) 60

(e) 60 m/min

(f) (i) 360 m (ii) 720 m (iii) 1080 m (iv) $60t$ m

(g) D = $60t$

9 (a)



(b) 15 m/s or 54 km/h (c) 15

(d) $D = 15t$ (e) 795 m

10 The ramp should be 555 cm from the first step: $4 \times 13 = 52$ cm in vertical rise. Because the gradient is 1:12, the horizontal run is $12 \times 52 = 624 - 3 \times 23$ (tread of three stairs) $= 555$ cm.

11 24 gradients; there are 22 (11 positive and 11 negative due to a mirror image) that are either whole numbers or fractions. The 23rd is $m = 0$ and the 24th is undefined.

12 (a) 320 L (b) 8 min (c) -40

(d) 40 L/min (e) The volume of water is decreasing.

(f) (i) 160 L (ii) 80 L (iii) 240 L (iv) $(320 - 40t)$ L

(g) $V = 320 - 40t$

(h) Neither time nor the volume of water can be a negative value.

13 (a) 48 km/h (b) 96 km/h (c) 7 h

Open-ended – Sample answers

14 (-2, 8), (4, 4), (7, 2)

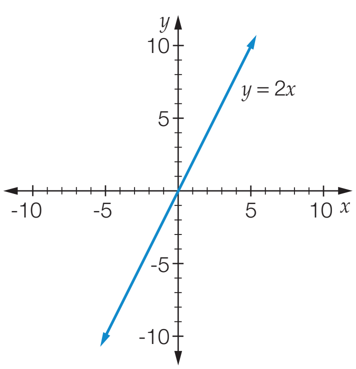
15 Sally has made a mistake by adding the negative values of x and y to the positive values instead of subtracting them from the positive values.

Exercise 5.6 (p. 297)

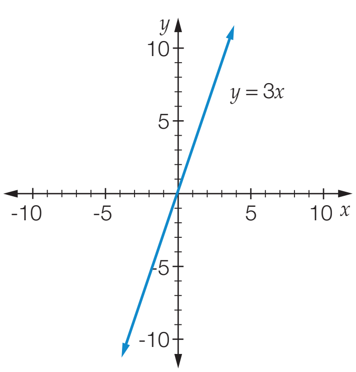
1

	Gradient	y-intercept
(a)	1	(0, 3)
(b)	2	(0, 1)
(c)	7	(0, 3)
(d)	3	(0, -2)
(e)	1	(0, -4)
(f)	-4	(0, 3)
(g)	-1	(0, 4)
(h)	-2	(0, 6)
(i)	2	(0, -5)

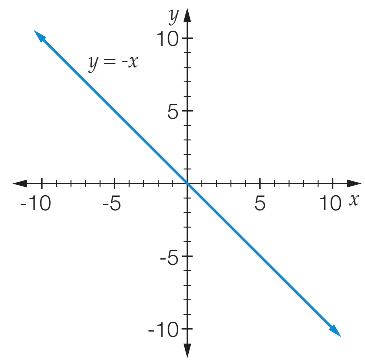
2 (a)



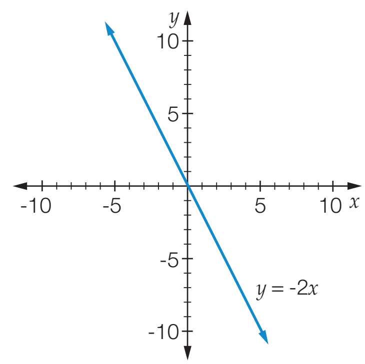
(b)



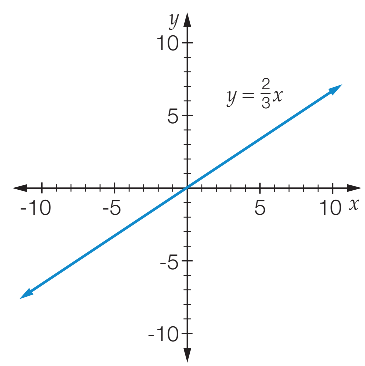
(c)



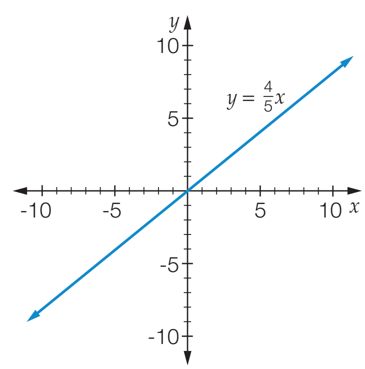
(d)



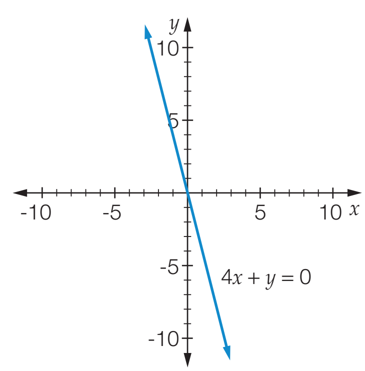
(e)



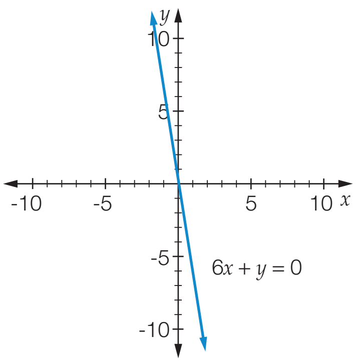
(f)



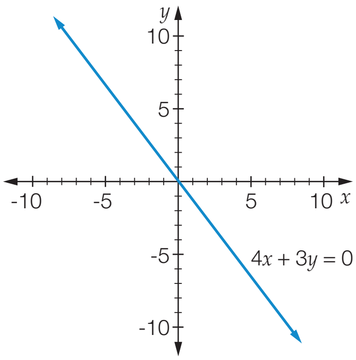
(g)



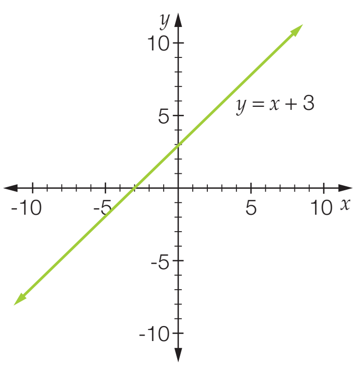
(h)



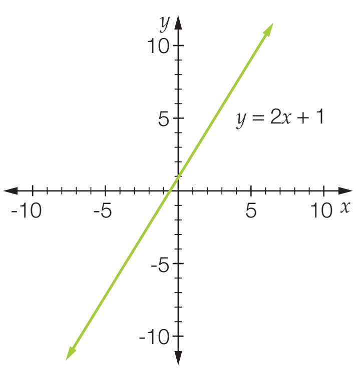
(i)



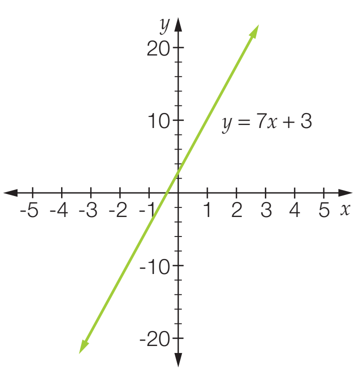
3 (a)



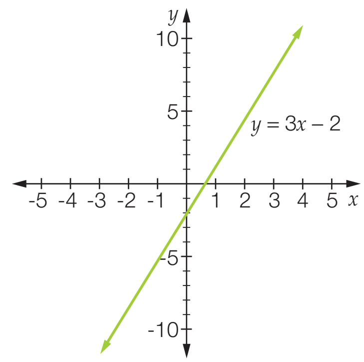
(b)



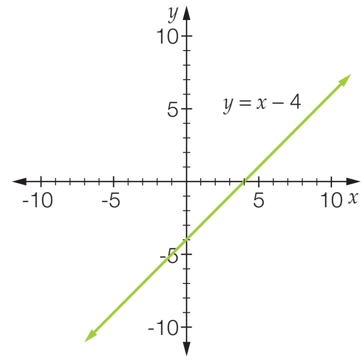
(c)



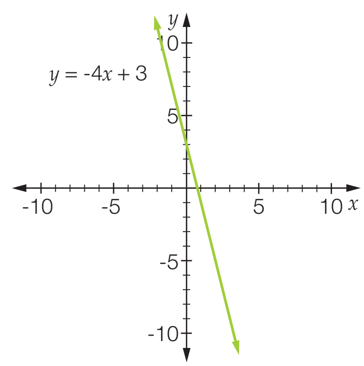
(d)



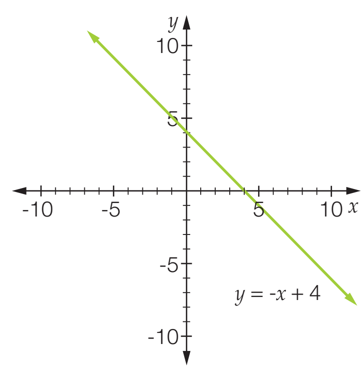
(e)



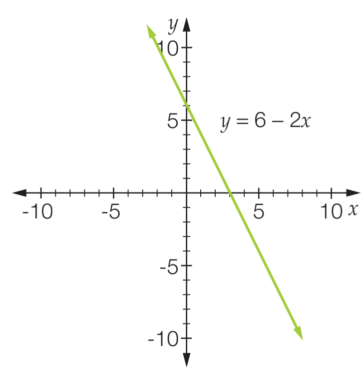
(f)



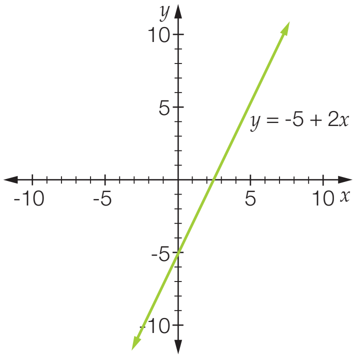
(g)



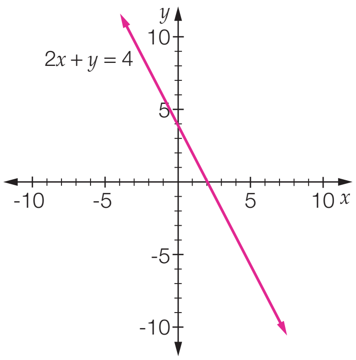
(h)



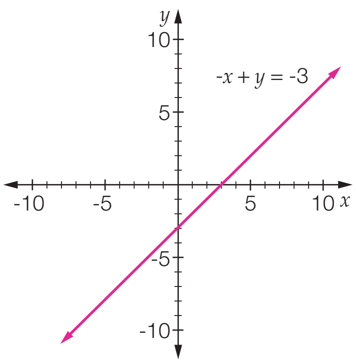
(i)



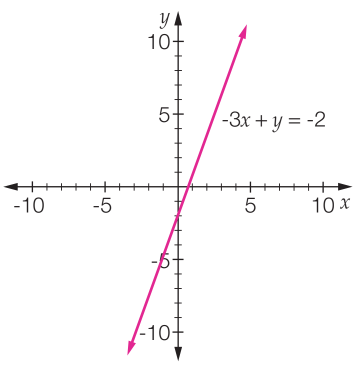
4 (a)



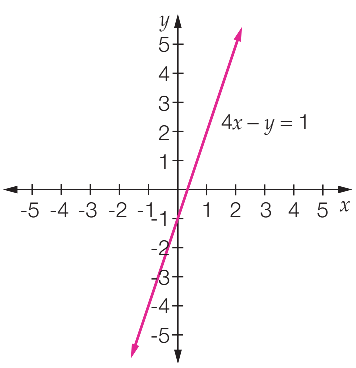
(b)



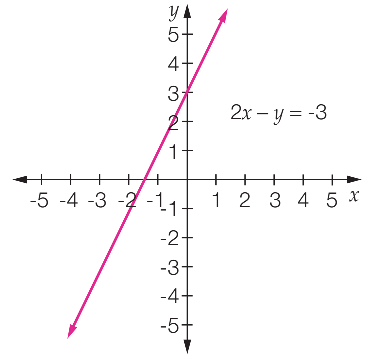
(c)



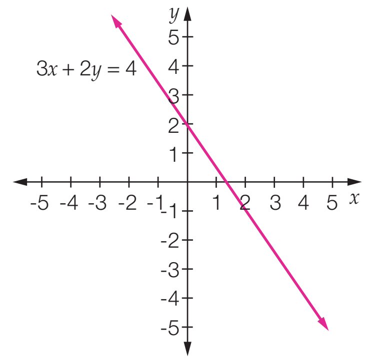
(d)



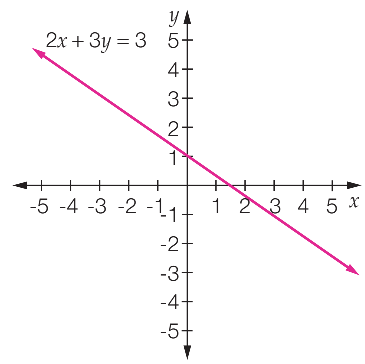
(e)



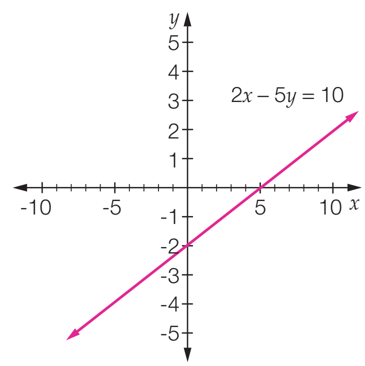
(f)



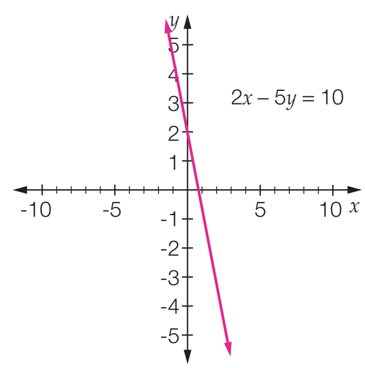
(g)



(h)



(i)



5 (a) $m = 3$ (b) $m = -4$ (c) $m = 3$

(d) $m = 3$ (e) $m = -\frac{2}{5}$ (f) $m = \frac{4}{3}$

6 (a) $-\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{3}$

(d) $-\frac{1}{3}$ (e) $\frac{5}{2}$ (f) $-\frac{3}{4}$

7 (a) $y = 3x + 4$ (b) $y = x + 5$ (c) $y = 2x$

(d) $y = 4x$ (e) $y = -5x + 1$ (f) $y = 3x - 4$

8 (a) D (b) C

9 (a) B (b) B (c) A (d) B

10 (a) D (b) A (c) C

11 (a) $-\frac{1}{2}$ (b) undefined (c) $-\frac{1}{7}$ (d) $-\frac{2}{5}$

12 (a) B (b) D (c) A (d) F

(e) G (f) C (g) E

13 (a) The lines are horizontal. (b) The lines are vertical.

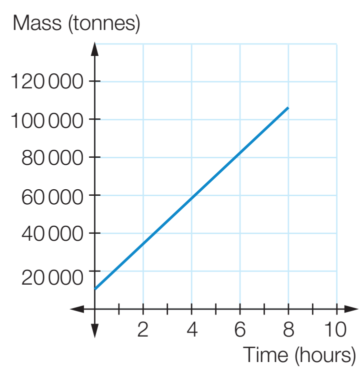
14 (a) (0, 50) (b) 25 (c) $d = 25t + 50$

(d) 50 m/min (e) 550 m

15 (a) 30 (b) 30 km/h (c) 20 km

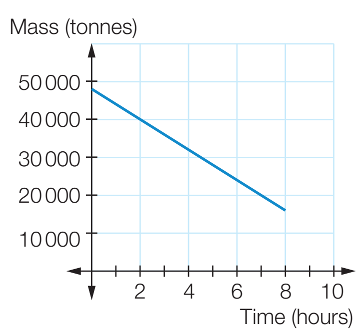
(d) $d = 100t + 90$

16 (a)



(b) 12 000 (c) $m = 12\,000t + 10\,000$

(d)



(e) $m = 4000t$ (f) 16 000 tonnes (g) 12 hours

17 Cameron is 110 km away from town A, Yvonne is 40 km away from town A.

Open-ended – Sample answers

18 (a) (0, -4), (10, 6), (15, 11) (b) (0, 2), (-5, 0), (10, 6)

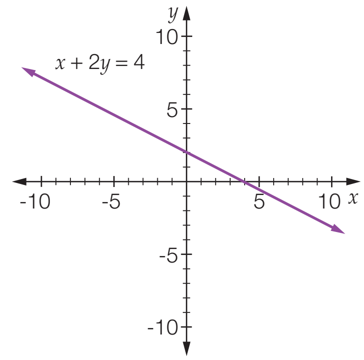
(c) (0, -0.5), (4, 0.5), (2, 0)

19 $y = x, y = x + 1, y = -x + 2, y = -x + 3$

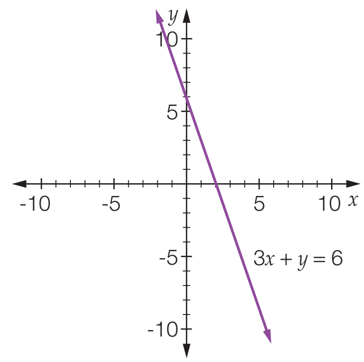
20 (a) $y = 4x - 9$ (b) $y = -\frac{1}{4}x + 2$

Exercise 5.7 (p. 306)

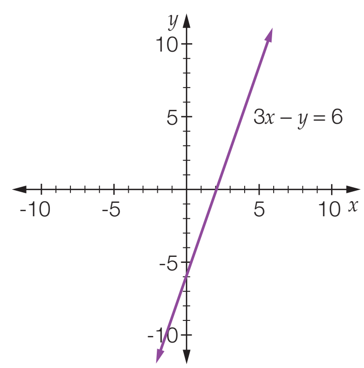
1 (a)



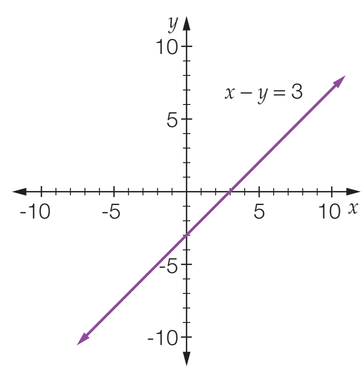
(b)



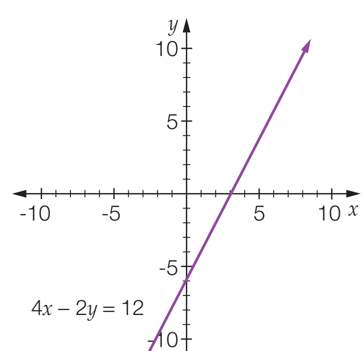
(c)



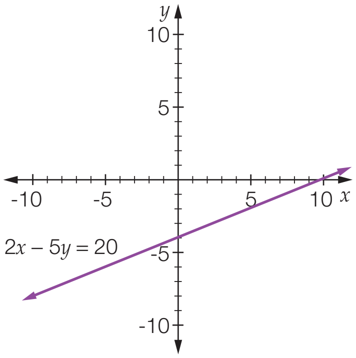
(d)



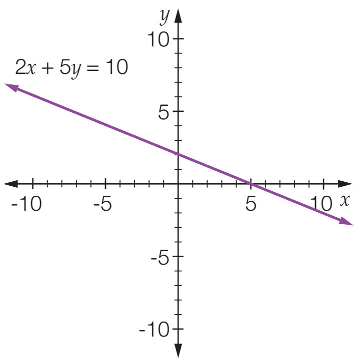
(e)



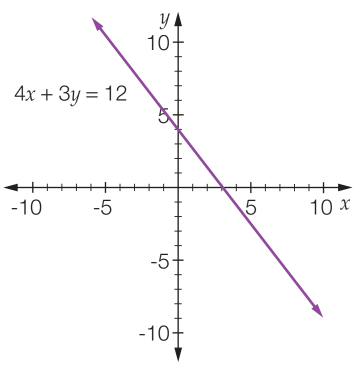
(f)



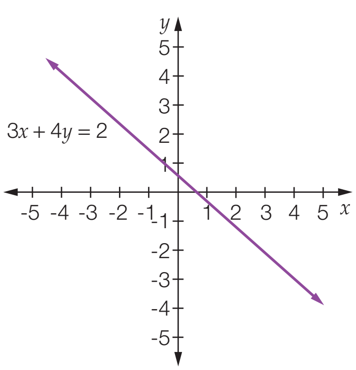
(g)



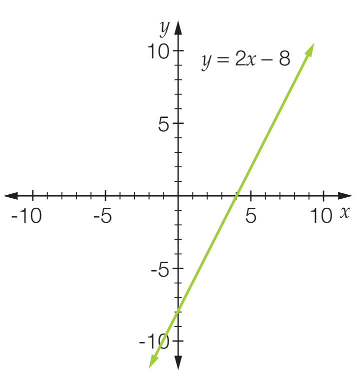
(h)



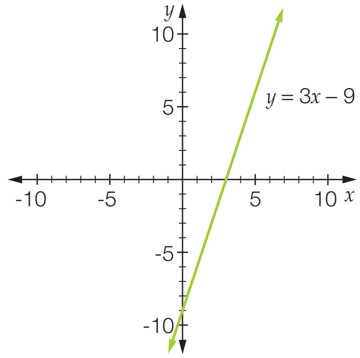
(i)



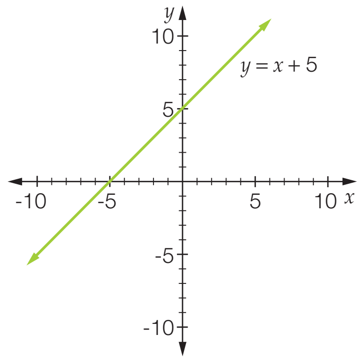
2 (a)



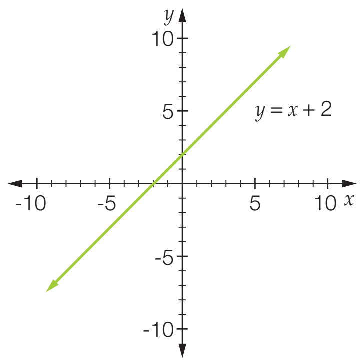
(b)



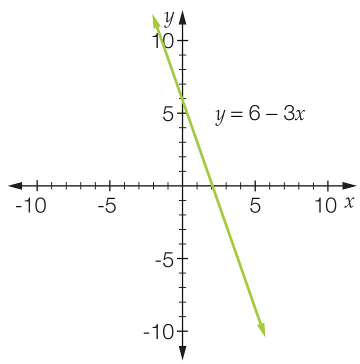
(c)



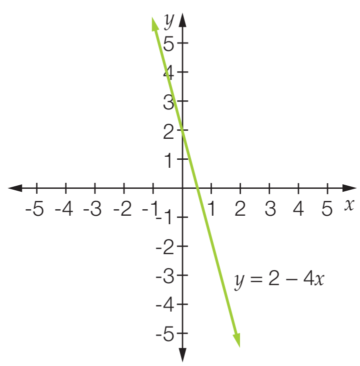
(d)



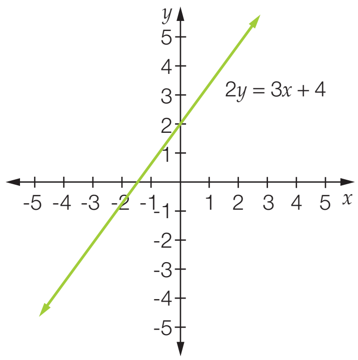
(e)



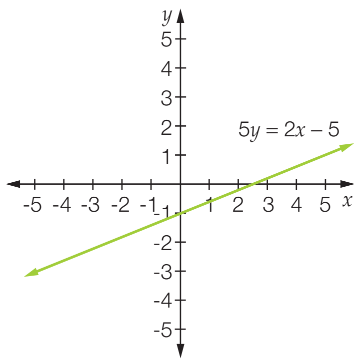
(f)



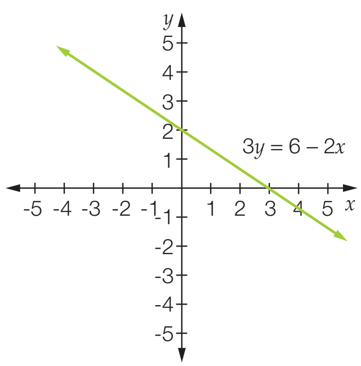
(g)



(h)



(i)



3 **(a)** C **(b)** B

4 **(a)** **(i)** \$150 **(ii)** \$100 **(iii)** $175 - 25n$

(b) $A = 175 - 25n$

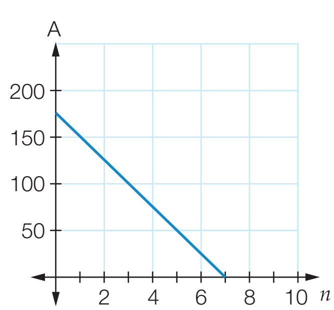
(c) **(i)** $A = 175$

(ii) This means that Todd owes \$175 at the time of purchase.

(d) **(i)** $n = 7$

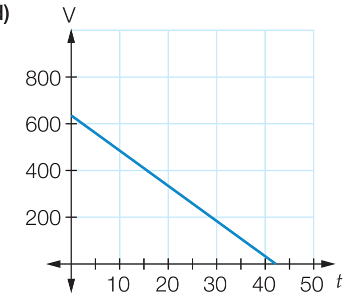
(ii) This means that Todd will have paid off the full amount in 7 weeks.

(e)



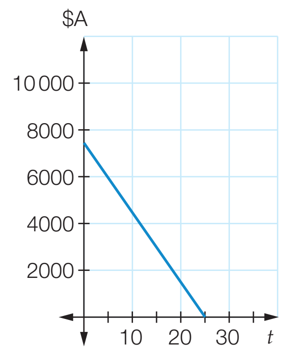
5 **(a)** 555 L **(b)** 630 L **(c)** 42 min

(d)



6 **(a)** \$7500 **(b)** 2 years and 1 month

(c)



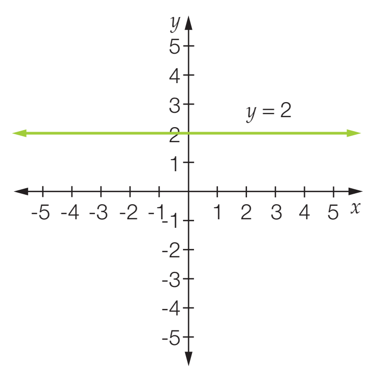
(d) $\$A = 7500 - 300t$ **(e)** \$5700 **(f)** \$4200

Open-ended – Sample answers

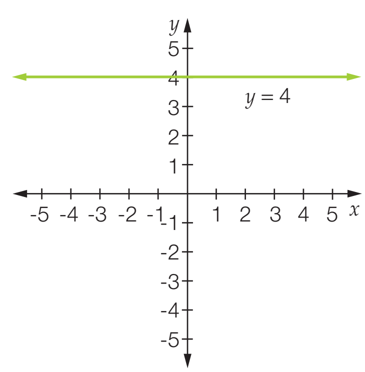
7 (-1, 0) and (0, 3); (1, 0) and (0, -3); (2, 0) and (0, -6)

Exercise 5.8 (p. 309)

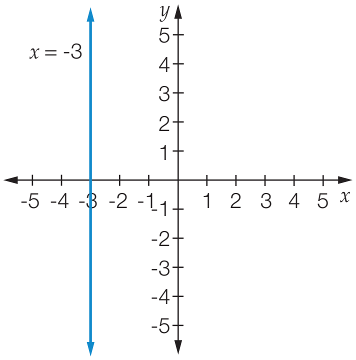
1 **(a)**



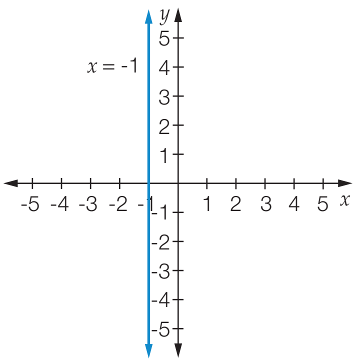
(b)



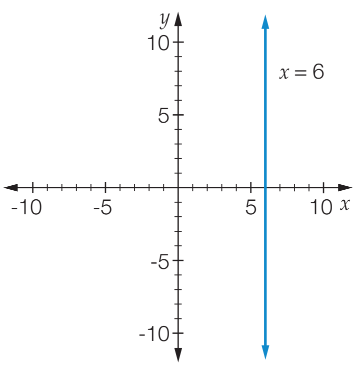
(c)



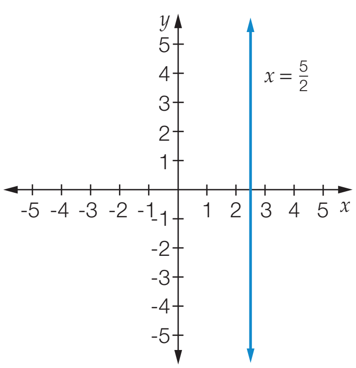
(d)



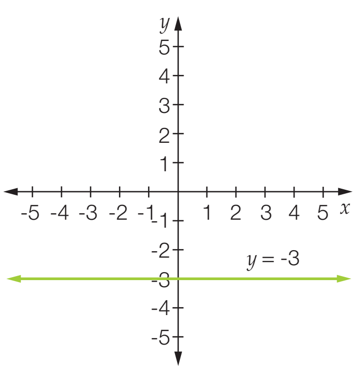
(e)



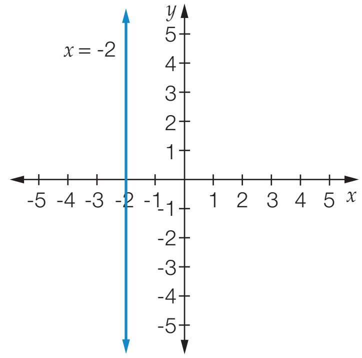
(f)



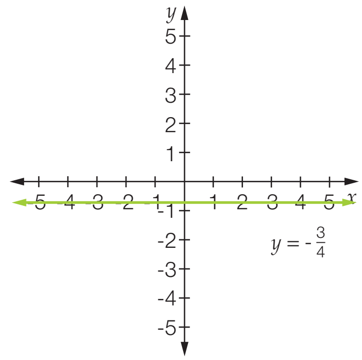
(g)



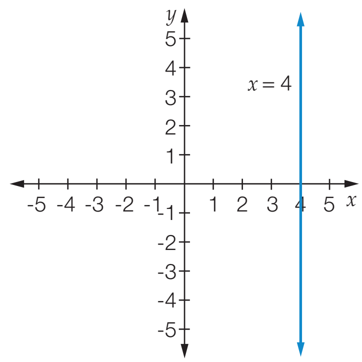
(h)



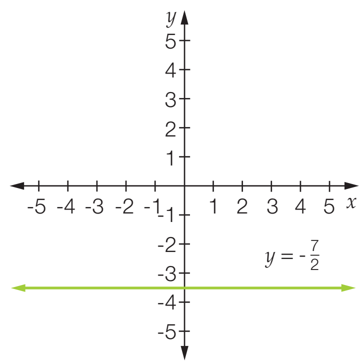
(i)



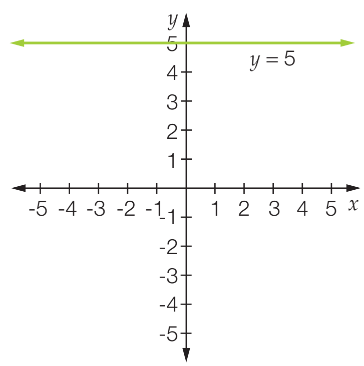
(j)



(k)



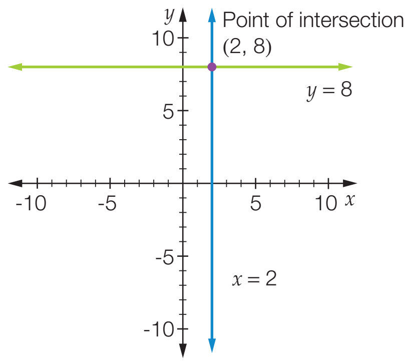
(l)



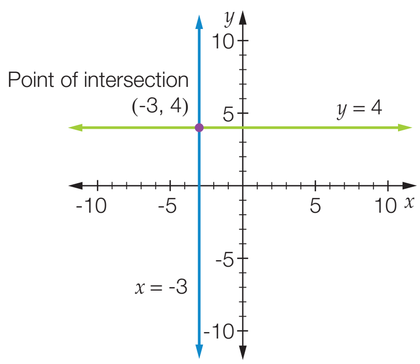
2 (a) $y = -6$ (b) $y = 2$ (c) $y = \frac{2}{3}$ (d) $y = \frac{3}{4}$

3 (a) B (b) B (c) C

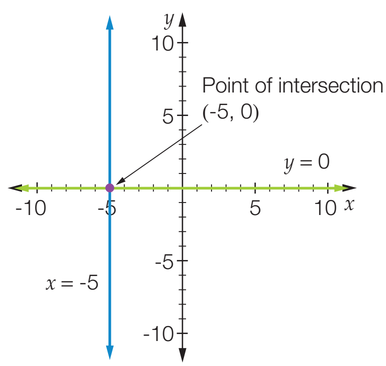
4 (a)



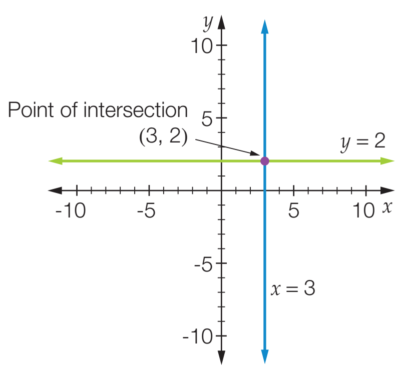
(b)



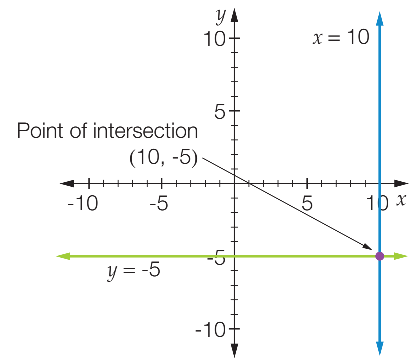
(c)



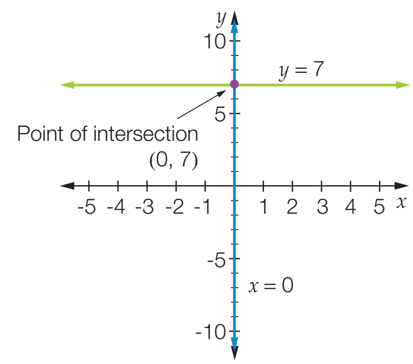
(d)



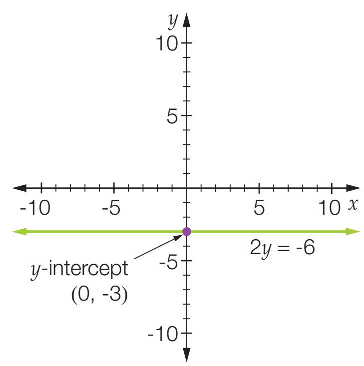
(e)



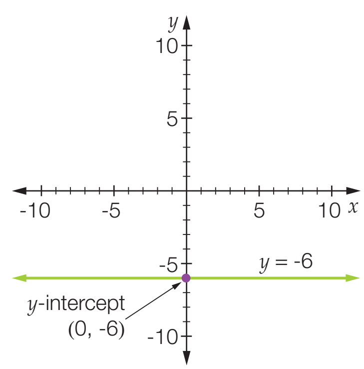
(f)



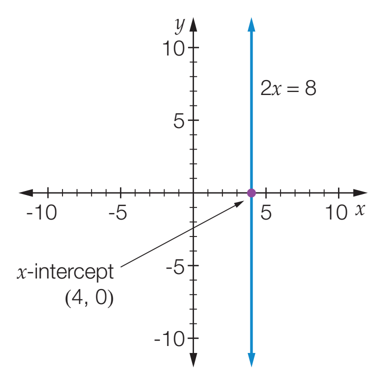
5 (a)



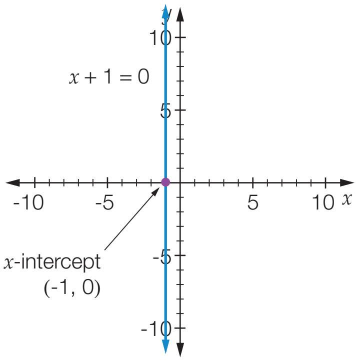
(b)



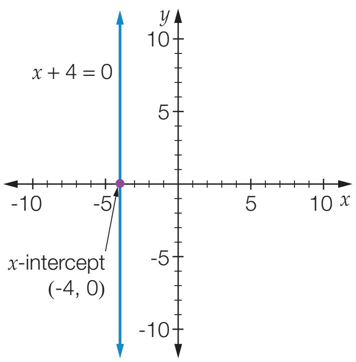
(c)



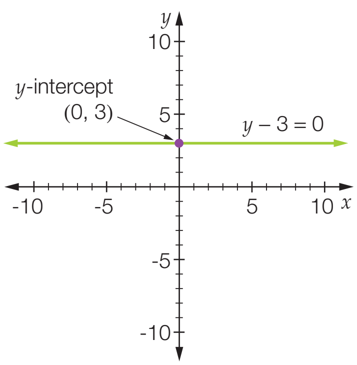
(d)


$x + 1 = 0$
x-intercept
(-1, 0)

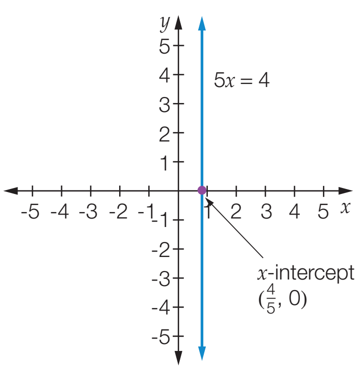
(e)


$x + 4 = 0$
x-intercept
(-4, 0)

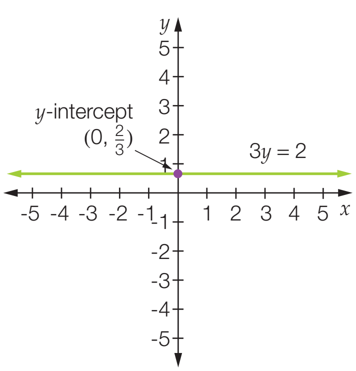
(f)


y-intercept
(0, 3)
$y - 3 = 0$

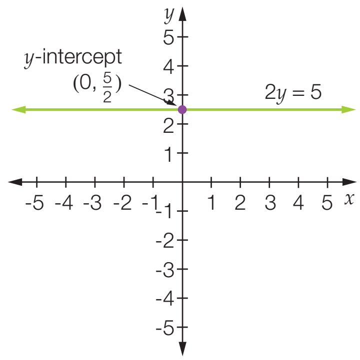
(g)


$5x = 4$
x-intercept
$(\frac{4}{5}, 0)$

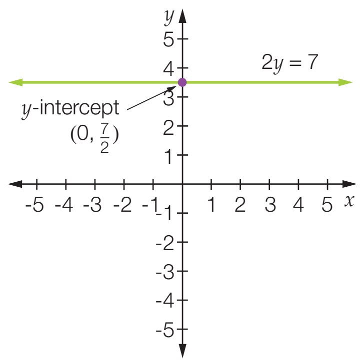
(h)


y-intercept
$(0, \frac{2}{3})$
$3y = 2$

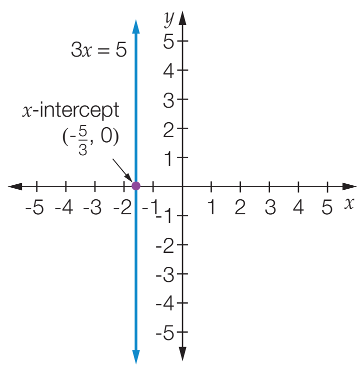
(i)


y-intercept
$(0, \frac{5}{2})$
$2y = 5$

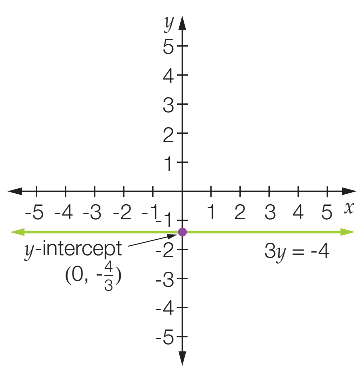
(j)


$2y = 7$
y-intercept
$(0, \frac{7}{2})$

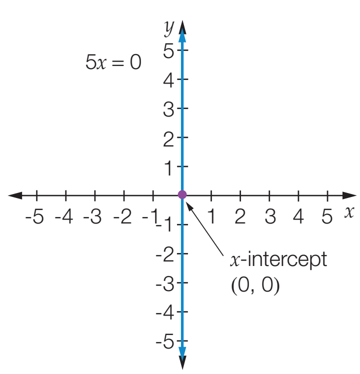
(k)


$3x = 5$
x-intercept
$(-\frac{5}{3}, 0)$

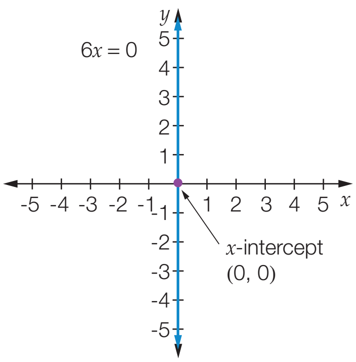
(l)


y-intercept
$(0, -\frac{4}{3})$
$3y = -4$

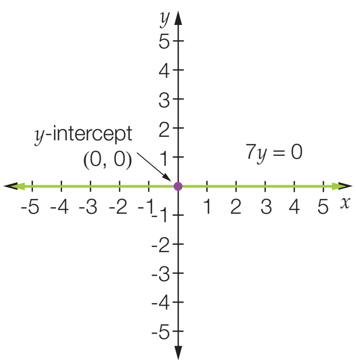
(m)


$5x = 0$
x-intercept
(0, 0)

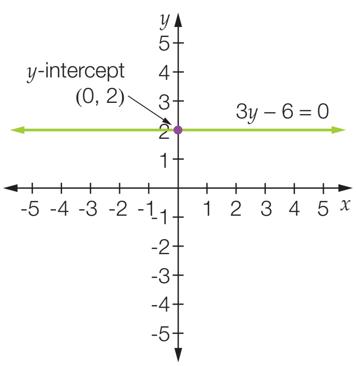
(n)



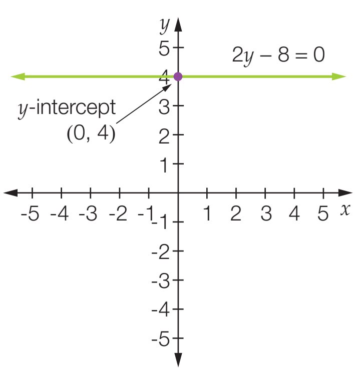
(o)



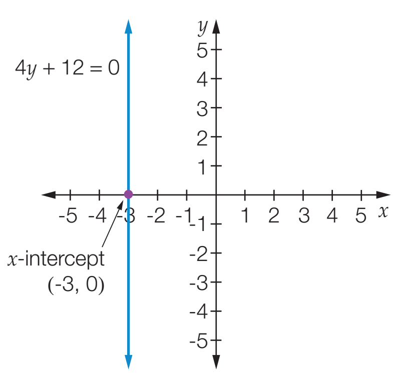
(p)



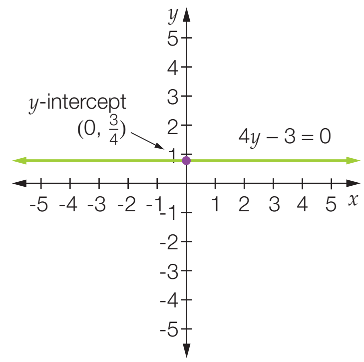
(q)



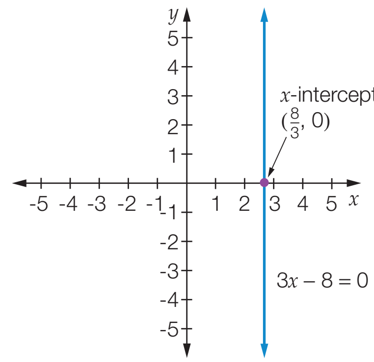
(r)



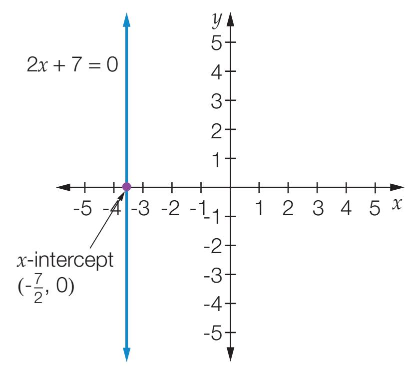
(s)



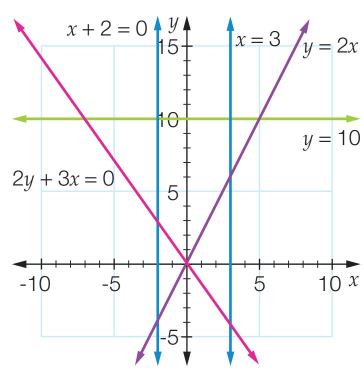
(t)



(u)

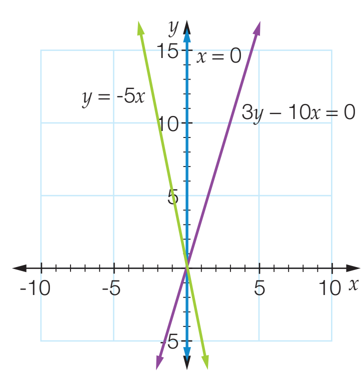


6 (a)



(b) (0, 0), (3, 6), (3, 10), (-2, 3), (-2, 10), $(-6\frac{2}{3}, 10)$, (5, 10)

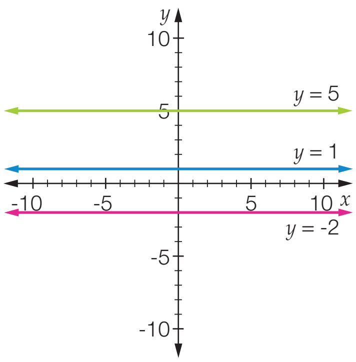
(c)



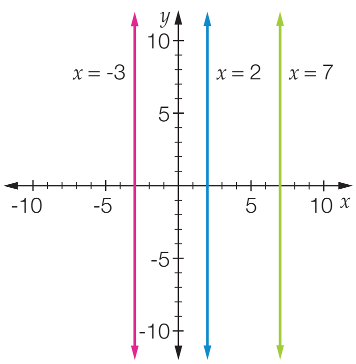
(d) Yes. It is located in the path of the road.

Open ended–Sample answers

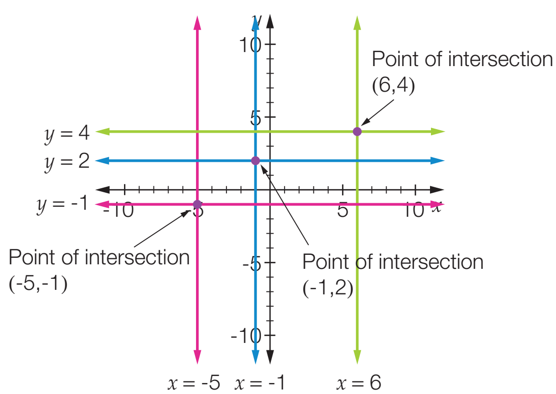
7 $y = -2, y = 1, y = 5$



8 $x = -3, x = 2, x = 7$



9



Challenge 5 (p. 313)

1 Jesse travels twice as fast as Vinny. He leaves 13 minutes after Vinny, so in the next 13 minutes he will travel twice as far as Vinny has travelled in the 26 minutes since he started. Thus, Jesse will catch up with Vinny at this time: 5.48 p.m. + 13 min = 6.01 p.m.

2 Let the bicycle cost \$$x$ per day and the car cost \$$y$ per day.

$2x + 3y = 150$ and $4x + y = 90$
$12x + 3y = 270$
$10x = 120$
$x = 12$ and $y = 42$

The bike costs \$12 per day and the car costs \$42 per day.

3 The equation of the line is $y = 2x + 3$; $w = 17$.

4 D

5 **(a)** From A to B is across 2 and up 1. From B to A it is back 2 and down 1. Hence, from C to D is back 2 and down 1, so D is (0, 3).

(b) The gradient of $AB = \frac{2-1}{3-1} = \frac{1}{2}$; the gradient of $BC = \frac{4-2}{2-3} = \frac{2}{-1} = -2$. The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

(c) The gradient of $AD = -2$, the gradient of $DC = \frac{1}{2}$. The gradients have a product of -1 and are the same as the gradients of their opposite sides.

(d) The parallel sides of the square have the same gradient.

(e) The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

6 Either plot the points, or by analysis see that from K to L is 3 across and 4 up. From L, go back 4 and up 3 to give M (-3, 6). From K, go back 4 and up 3 to give N (-6, 2). For the second set of values, from L go across 4 and down 3 to give M (5, 0). From K, go across 4 and down 3 to give N (2, -4). This process is based on the fact that the gradient of the perpendicular sides is the negative reciprocal of the gradient of KL. The hypotenuse of our triangle is still 5, the length of the side.

Answers: M (-3, 6) and N (-6, 2) or M (5, 0) and N (2, -4).

7 (1, -1)

8 **(a)** $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}$, $\frac{x-a}{x-b} = \frac{x-b-x}{x-b}$, $x - a = -b$, $x = a - b$

(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2$, $\frac{a^2-b^2}{abx} = a^2 - b^2$, $abx = 1$, $x = \frac{1}{ab}$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}$, $(a-b)(x+c) = (a+b)(x-c)$, $ax + ac - bx - bc$ $= ax - ac + bx - bc$, $2ac = 2bx$, $x = \frac{ac}{b}$.

Chapter review 5 (p. 314)

1 **(a)** $x = -5$ **(b)** $x = -1$ **(c)** $x = -45$
(d) $x = 72$ **(e)** $x = 0$ **(f)** $x = \frac{5}{2}$

2 **(a)** $x = 9$ **(b)** $x = 19$ **(c)** $x = \frac{11}{9}$ **(d)** $x = 3$

3 **(a)** $x = \frac{13}{6}$ **(b)** $x = -19$ **(c)** $x = -9$ **(d)** $x = 1$
(e) $x = \frac{3}{2}$ **(f)** $x = \frac{41}{5}$

4 **(a)** B **(b)** C

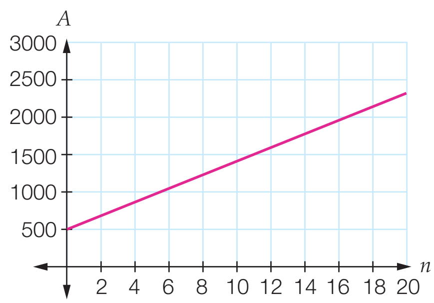
5 **(a)** $p + 4$ **(b)** $p - 11$
(c) $10 = p - 11$, $p = 21$. Pete had 21 mice originally.

6 **(a)** 10.82 (2 d.p.) **(b)** (1.5, 8)

7 **(a)**

No.weeks	0	1	2	3	4	5	6
Amount paid (\$)	500	590	680	770	860	950	1040

(b)



(c) \$1490

(d) 15 weeks

(e) $A = 500 + 90n$

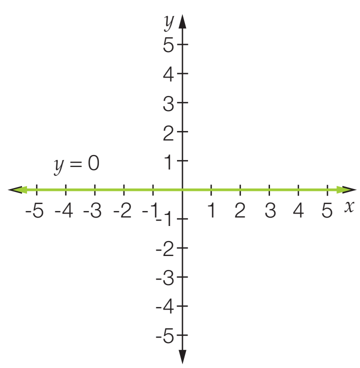
(f) \$1310

8 5

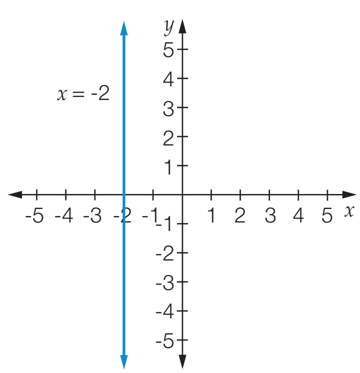
9 (a) A (b) A

10 gradient = 2, x-intercept = (-2, 0), y-intercept = (0, 4)

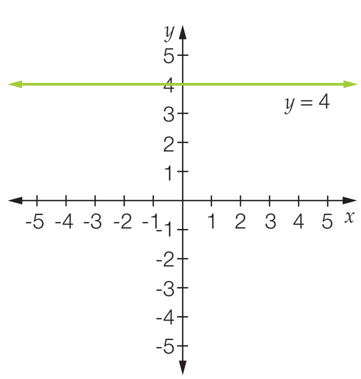
11 (a) (-2, 0), (0, 0), (1, 0), (3, 0) or any points that have a y value of zero.



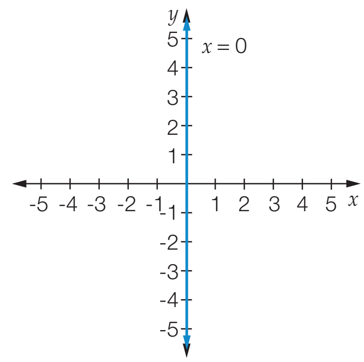
(b) (-2, -5), (-2, -1), (-2, 1), (-2, 4) or any points that have an x value of -2.



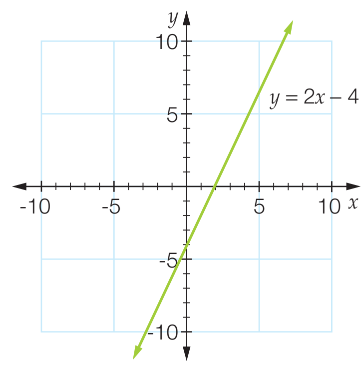
(c) (-2, 4), (0, 4), (1, 4), (3, 4) or any points that have a y value of 4.



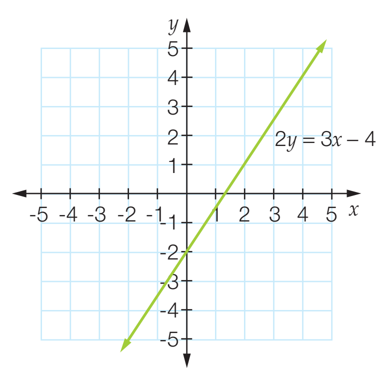
(d) (0, -5), (0, -1), (0, 1), (0, 4) or any points that have an x value of -2.



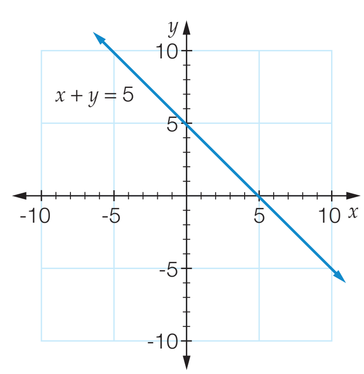
12 (a) y-intercept: (0, -4), gradient = 2



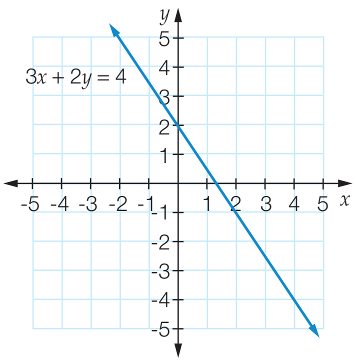
(b) y-intercept: (0, -2), gradient = $\frac{3}{2}$



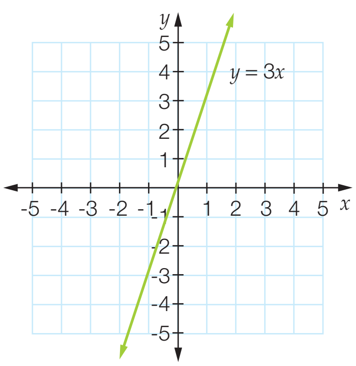
(c) y-intercept: (0, 5), gradient = -1



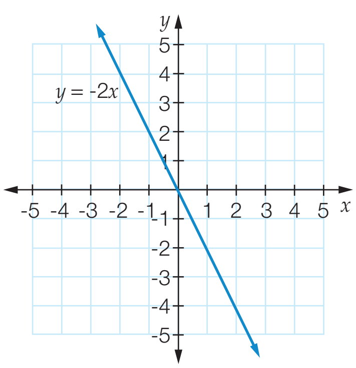
(d) y-intercept: (0, 2), gradient = $-\frac{3}{2}$



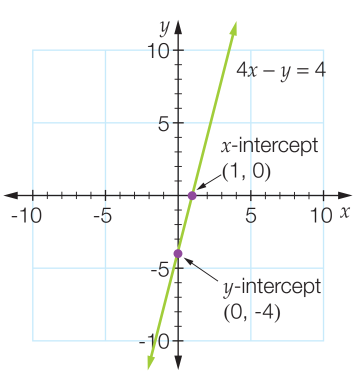
(e) y-intercept: (0, 0), gradient = 3



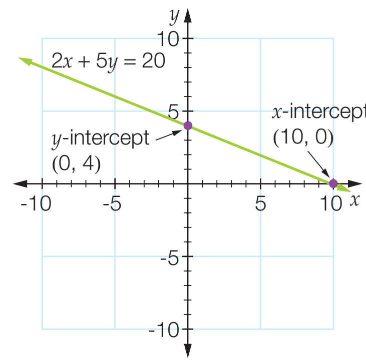
(f) y-intercept: (0, 0), gradient = -2



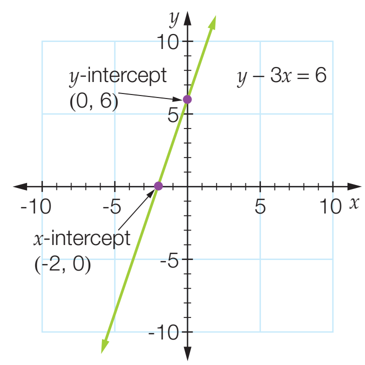
13 (a)



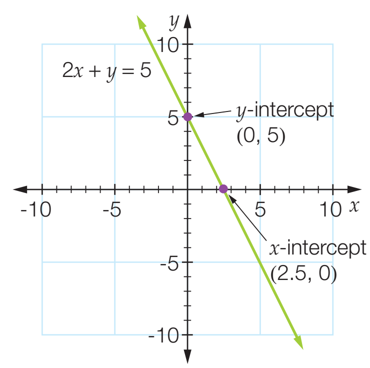
(b)



(c)

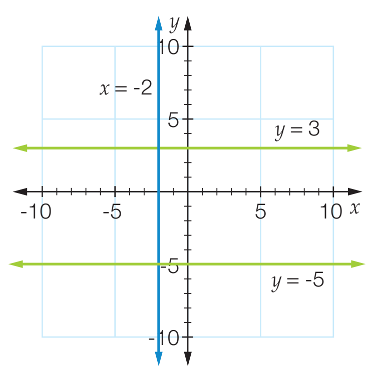


(d)



14 (a) D (b) C (c) D

15



16 (a) (0, 0) and (5, 350) (b) 70
(c) 70 m/min (d) $d = 70t$ (e) 560 m

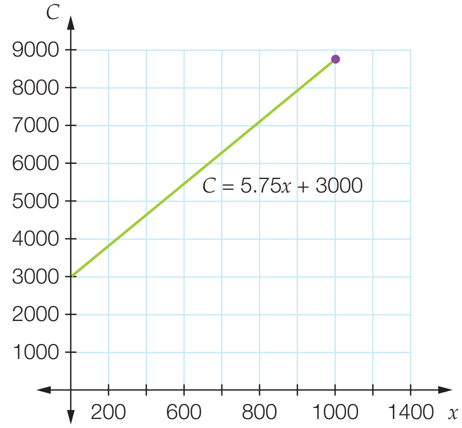
17 (a) 21, 22, 23 (b) 22, 24, 26, 28

18 $58 = 2(2x - 7) + 2x$, $x = 12$. Therefore the length is 17 m and the width is 12 m.

19 $k = 9$

20 (a) $C = 5.75x + 3000$ (b) \$5875 (c) 1043

(d)



The cost of 1000 components is \$8750.

21 (a) $C = 70 + 105t$ (b) \$910

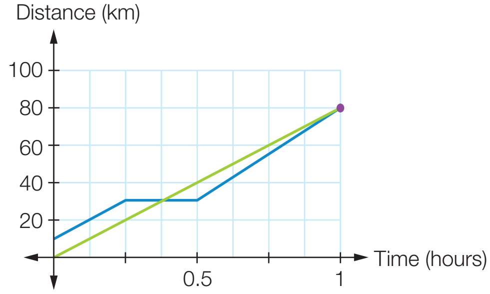
(c) 2 hours 15 minutes

22 (a) Students check own graphs carefully.

(b) 80 km (c) 80 km/h (d) 15 min

(e) 100 km/h (f) 70 km

(g)



(h) 80 km/h (i) $d = 80t$

(j) 22.5 min after she left (k) 50 km from home

NAPLAN practice 5

1 C	2 D	3 $b = 7$	4 B
5 C	6 125	7 A	8 5
9 C	10 38 cm		

Chapter 6

Recall 6 (p. 320)

1 Students to draw own angles.

2 (a) acute (b) obtuse (c) reflex (d) reflex

3 (a) 35° (b) 245°

4 D

5 $a = 46°$, $b = 46°$, $c = 46°$

6 (a) 173 (b) 64 (c) 70

7 (a) 4.8 m (b) 1.5 m

8 (a) 6 : 1 (b) 3.5 : 1 (c) 21.88 : 1 (d) 686.17 : 1

9 (a) 1 175 000 (b) 4 cups

Exercise 6.1 (p. 323)

1 (a) $a = 38°$, alternate angles on parallel lines; $b = 82°$, corresponding angles on parallel lines

(b) $f = 125°$, straight angle; $g = 125°$, corresponding angles on parallel lines; $h = 70°$, alternate angles on parallel lines

(c) $w = 25°$, straight angle; $x = 85°$, vertically opposite angle; $y = 25°$, alternate angles on parallel lines; $z = 110°$, exterior angle equals sum of two interior opposite angles

2 (a) $x = 50°$, supplementary angles

(b) $m = 30°$, angles in a revolution

(c) $a = 90°$, corresponding angles on parallel lines; $b = 117°$, corresponding angles on parallel lines, $c = 63°$, straight angle; $d = 153°$, straight angle

3 B

4 (a) $a = 50°$, cointerior angles on parallel lines; $2b + 40 = 130$, $b = 45°$, two opposite interior angles equal to exterior angle of triangle

(b) $x = 36°$, straight angle; $y = 144°$, corresponding angles on parallel lines

(c) $m = 40°$, two opposite interior angles equal to exterior angle of triangle; $n = 40°$, alternate angles on parallel lines; $r = 140°$, straight angle.

5 (a) False (b) False (c) True (d) False

6 $p = 75°$; equal angles of an isosceles triangle, $r = 75°$; corresponding angles on parallel lines

7 $y = 5x + 45$, alternate angles on parallel lines, hence $9x + 45 = 180°$, $x = 15°$, sum of angles in a triangle; $y = 5 \times 15 + 45 = 120°$; $z = 60°$, sum of angles in a triangle.

8 (a) $3x = 40 + (x + 30)$, vertically opposite angles; $2x = 70$, $x = 35°$

(b) $(2y - 40) + 40 = 180$, co-interior angles supplementary on parallel lines; $2y = 180$, $y = 90°$

(c) $65 + (90 + z - 35) = 180$, co-interior angles supplementary on parallel lines; $120 + z = 180$. $z = 60°$

Open-ended – Sample answers

9 y creates a straight angle with 2 angles. These both have the same value as x; one because it is an equal angle to x in an isosceles triangle, the other because it is an alternate angle on parallel lines to x. Therefore, $2x + y = 180$. If $x = 40°$, $y = 100°$. If $x = 50°$, $y = 80°$. If $x = 70°$, $y = 40°$.

10 (a) x and y must sum to 180°. For example, $x = 100°$ and $y = 80°$.

(b) m and n must be equal. For example, $m = n = 120°$.

(c) $t = 180 - y - (180 - n)$, $t = -y + n$

(d) $t = -(180 - x) + m$ (e) $t = -80 + 120 = 40°$

(f) $t = -(180 - 100) + 120 = 40°$ (g) Yes

Exercise 6.2 (p. 331)

1 (a) SSS (b) SSS (c) RHS
(d) SAS (e) SAS (f) SSS
(g) RHS (h) AAS (i) AAS

2 $\angle STO = \angle PRO$ (alternate angles, $ST \parallel RP$)
$\angle SOT = \angle POR$ (vertically opposite angles)
$ST = PR = 10$ cm (given)
$\Delta STO \equiv \Delta PRO$ (AAS)
$TO = RO$ (matching sides in congruent triangles)
$x = 6$ cm

3 (a) In Δs EFH and EGH,
$FH = GH$ (given)
$\angle FHE = \angle GHE$ (both 90°)
EH is a common side
$\therefore \Delta EFH \equiv \Delta EGH$ (SAS)
$\therefore \angle EFH = \angle EGH$ (matching angles in congruent triangles)

(b) ΔEFG is isosceles because it has two equal angles; i.e. $\angle EFH = \angle EGH$.

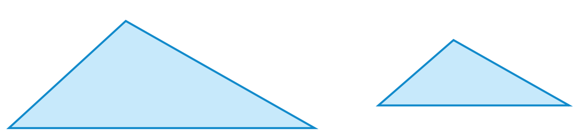
(c) If the perpendicular bisector of a side of a triangle passes through the opposite vertex, then the triangle is isosceles with the other two sides being equal in length.

(d) $EF = EG$ because they are matching sides in the congruent triangles EFH and EGH.

4 $\Delta MNO \equiv \Delta DFE$ (SSS), $\Delta ABC \equiv \Delta JKL$ (SAS), $\Delta GHI \equiv \Delta RQP$ (AAS)

5 (a) (i) SAS (ii) SAS (iii) AAS
(iv) SAS (v) AAS (vi) RHS
(b) (i) $c = 35°$ (ii) $p = 38°$ (iii) $x = 15$ cm
(iv) $g = 5$ cm (v) $x = 50$ mm (vi) $m = 75°$

6 Two triangles with three equal angles are equiangular and so are similar. To be congruent they must have at least one pair of matching sides equal. The following triangles are equiangular but not congruent.



7 Proof: In Δs ABE and DCE,
$\angle ABE = \angle DCE$ (alternate angles, $AB \parallel CD$)
$BE = CE$ (given)
$\angle AEB = \angle DEC$ (vertically opposite angles)
$\therefore \Delta ABE \equiv \Delta DCE$ (AAS congruency)
$\therefore AB = CD$ (matching sides of congruent triangles)

8 (a) In Δs XYZ and SRT, $XY = SR$ (both 17 m)
$\angle XYZ = \angle SRT$ (both 90°)
$YZ = RT$ (both 12 m)
$\therefore \Delta XYZ \equiv \Delta SRT$ (SAS)

(b) In Δs ABC and RQS, $\angle ABC = \angle RQS$ (both 90°)
$AB = RQ$ (both 11 cm)
$AC = RS$ (both 15 cm)
$\therefore \Delta ABC \equiv \Delta RQS$ (RHS)

(c) In Δs DEF and XZY, $DE = XZ$ (both 6 cm)
$EF = ZY$ (both 6 cm)
$DF = XY$ (both 8 cm)
$\therefore \Delta DEF \equiv \Delta XZY$ (SSS)

(d) In Δs FGH and PQR,
$\angle FGH = \angle PQR$ (both 80°)
$\angle FHG = \angle PRQ$ (both 70°)
$GF = QP$ (both 5 cm)
$\therefore \Delta FGH \equiv \Delta PQR$ (AAS)

(e) In Δs DEF and XVW, $\angle DFE = \angle XWV$ (both 90°)
$DE = XV$ (both 16 cm)
$FE = WV$ (both 7 cm)
$\therefore \Delta DEF \equiv \Delta XVW$ (RHS)

(f) In Δs KLM and QSR,
$\angle LKM = \angle SQR$ (both 50°)
$\angle LMK = \angle SRQ$ (both 30°)
$KM = QR$ (both 32 cm)
$\therefore \Delta KLM \equiv \Delta QSR$ (AAS)

9 In Δs TUR and TSR, $\angle TUR = \angle TSR$ (both 90°)
TR is a common side
$TU = TS$ (given)
$\therefore \Delta TUR \equiv \Delta TSR$ (RHS)

10 In Δs SQP and RPQ,
$SP = RQ$ (given)
$\angle SPQ = \angle RQP$ (given)
PQ is a common side
$\therefore \Delta SQP \equiv \Delta RPQ$ (SAS)

11 (a) In Δs ABC and CDA, $AB = CD$ (given)
$BC = DA$ (given)
AC is a common side
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)
$x = 32°$ (matching angles in congruent triangles)

(b) In Δs ADC and ABC, $\angle ADC = \angle ABC = 35°$ (given)
$\angle ACD = \angle ACB = 120°$ (given)
AC is a common side
$\therefore \Delta ADC \equiv \Delta ABC$ (AAS)
$b = 23$ cm (matching sides in congruent triangles)

(c) In Δs ABE and CDE, $\angle ABE = \angle CDE = 56°$ (given)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = CD$ (given)

$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)

$x = 23$ cm (matching sides in congruent triangles)

(d) In Δs OAB and OCB, $\angle OBA = \angle OBC = 90°$

$OA = OC$ (radii)

OB is a common side

$\therefore \Delta OAB \equiv \Delta OCB$ (RHS)

$y = 26°$ (matching angles in congruent triangles)

(e) In Δs ABO and CDO, $AB = CD$ (given)

$AO = CO$ (radii)

$BO = DO$ (radii)

$\therefore \Delta ABO \equiv \Delta CDO$ (SSS)

$y = 9$ m (matching sides in congruent triangles)

(f) In Δs ABE and DCE,

$\angle BAE = \angle CDE = 75°$(given)

$AB = DC$ (given)

$\angle AEB = \angle DEC$ (vertically opposite angles)

$\therefore \Delta ABE \equiv \Delta DCE$ (AAS)

$k = 8$ cm (matching sides in congruent triangles)

12 (a) In Δs DEG and DFG,

$DE = DF$ (given)

$EG = FG$ (G is the midpoint of EF)

GD is a common side

$\therefore \Delta DEG \equiv \Delta DFG$ (SSS)

$\therefore \angle DGE = \angle DGF$ (matching angles of congruent triangles)

But $\angle DGE + \angle DGF = 180°$, therefore $\angle DGE = 90°$ and hence $DG \perp EF$.

(b) If ΔDEF is equilateral, then all interior angles are 60°.

Because $\Delta DEG \equiv \Delta DFG$ (SSS)

then $\angle EDG = \angle FDG = 60 \div 2 = 30°$

13 In Δs KLN and KMN,

$KL = KM$ (given)

$\angle LKN = \angle MKN$ (given)

KN is a common side

$\therefore \Delta KLN \equiv \Delta KMN$ (SAS)

$\therefore LN = NM$ (matching sides of congruent triangles)

$\angle KNL = \angle KNM$ (matching angles of congruent triangles)

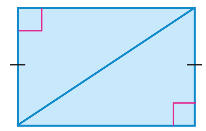
But $\angle KNL + \angle KNM = 180°$, hence $KN \perp LM$.

Therefore KN is the perpendicular bisector of LM.

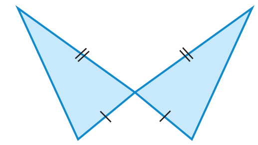
14 In Δs QPS and RPS we only have two pairs of equal sides. Either another pair of sides needs to be equal, the angles between the equal sides need to be equal, or PS perpendicular to QR, to be able to prove that the triangles are congruent. The equal angles that are given are not included by the sides.

Open-ended – Sample answers

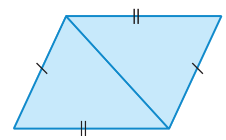
15



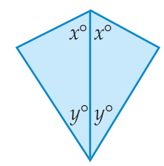
RHS



SAS

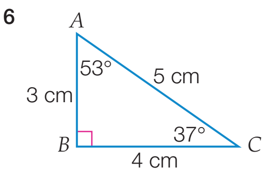


SSS

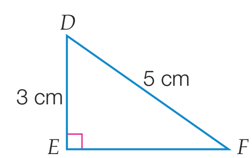


AAS

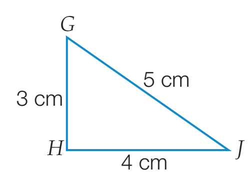
16



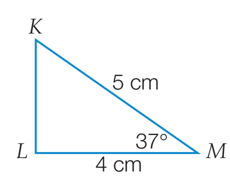
Original ΔABC



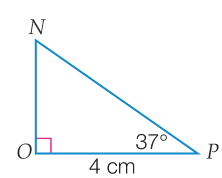
RHS



SSS



SAS



AAS

Exercise 6.3 (p. 342)

1 (a) $x = 36°$ (b) $y = 15°, z = 110°$ (c) $w = 27°$

2 (a) Join DB. In Δs DAB and ΔBCD,

$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel DC$)

DB is a common side

$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel BC$)

$\Delta DAB \equiv \Delta DCB$ (AAS)

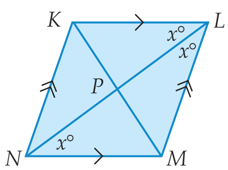
$\angle DAB = \angle BCD$ (matching angles in congruent triangles)

(b) $AB = DC$ (matching sides in congruent triangles); $AD = BC$ (matching sides in congruent triangles)

(c) If the diagonal AC is drawn, then the same steps as in (a) can be used to prove that $\Delta ADC \equiv \Delta CBD$. $\angle ADC = \angle CBA$ (matching angles in congruent triangles).

(d) The opposite angles of a parallelogram are equal. The opposite sides of a parallelogram are equal.

3 (a) Let $\angle KLN = x°$ and mark other angles that are equal to $x°$ on the diagram.



$\angle LNM = \angle KLN = x°$ (alternate angles, $KL \parallel NM$)

$LM = MN$ (adjacent sides of a rhombus)

ΔLMN is isosceles

$\angle MLN = \angle LNM = x°$ (equal angles of an isosceles triangle)

$\angle KLN = \angle MLN$ (both equal $x°$)

(b) In Δs KLP and MLP

$KL = LM$ (adjacent sides of a rhombus)

$\angle KLP = \angle MLP$ (proved in (a))

LP is a common side

$\Delta KLP \equiv \Delta MLP$ (SAS)

(c) $\angle KPL = \angle MPL$ (matching angles in congruent triangles)

$\angle KPL + \angle MPL = 180°$ ($\angle KPM = 180°$)

$\angle KPL = \angle MPL = 90°$

(d) $KP = MP$ (matching sides in congruent triangles)

(e) The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

4 $PT = TR$, $QT = TS$, $\angle PTQ = \angle QTR = \angle RTS = \angle STP = 90°$

In Δs PTQ and QTR, $PT = TR$ (given)

$\angle PTQ = \angle QTR = 90°$ (given)

QT is a common side

$\Delta PTQ \equiv \Delta QTR$ (SAS)

$PQ = QR$ (matching sides in congruent triangles)

In Δs PTQ and RTS, $PT = TR$ (given)

$\angle PTQ = \angle RTS = 90°$ (vertically opposite angles)

$QT = TS$ (given)

$\Delta PTQ \equiv \Delta RTS$ (SAS)

$\angle PQT = \angle RST$ (matching angles in congruent triangles)

$\angle PQS$ and $\angle RSQ$ are a pair of equal alternate angles contained between PQ and SR.

$PQ \parallel SR$

Repeating these steps in ΔPTS and ΔRTQ allows you to prove that $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so $PQRS$ is a rhombus.

5 C

6 (a) $x = 21°$ (b) $w = 124°$, $y = 33°$, $z = 40°$ (c) $u = 115°$

7 (a) False (b) True (c) False (d) True

8 Hexagon 9 A 10 B

11 (a) 1080° (b) 135°

12 (a) Proof: In Δs ADC and CBA,
$\angle ACD = \angle CAB$ (alternate angles, $AB \parallel DC$)
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
AC is a common side

$\therefore \Delta ADC \equiv \Delta CBA$ (AAS)

$\therefore \angle ADC = \angle CBA$ (matching angles in congruent triangles)

(b) Repeat (a) in Δs ABD and CDB to prove that they are congruent, Hence, $\angle DAB = \angle BCD$ (matching angles in congruent triangles).

13 (a) $\angle ADC = \angle CBA$ and $\angle DAB = \angle BCD$, hence the opposite angles of a parallelogram are equal.

(b) Because $\Delta ADC \equiv \Delta CBA$, then $AB = DC$ (matching sides) and $AD = BC$ (matching sides). Hence the opposite sides of a parallelogram are equal.

(c) In Δs AEB and CED,

$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel DC$)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = DC$ (opposite sides of a parallelogram are equal, proved above)

$\Delta AEB \equiv \Delta CED$ (AAS)

(d) $AE = EC$ (matching sides of congruent triangles); $BE = ED$ (matching sides of congruent triangles). Hence, AC and BD bisect each other at E.

14 (a) Proof: In Δs PRS and QSR:
$\angle PSR = \angle QRS = 90°$ (given)

$\angle SPR = \angle RQS$ (alternate angles, $PS \parallel QR$)

$PS = QR$ (given)

$\therefore \Delta PRS \equiv \Delta QSR$ (AAS)

(b) $PR = QS$ (matching sides of congruent triangles)

(c) The diagonals of a rectangle are equal.

15 (a) In Δs KLM and KNM, $KL = KN$ (given)

$LM = NM$ (given)

KM is a common side

$\Delta KLM \equiv \Delta KNM$ (SSS)

Hence, $\angle KLM = \angle KNM$ (matching angles of congruent triangles)

(b) In Δs LKX and NKX, $KL = KN$ (given)

$\angle LKX = \angle NKX$ (matching angles of congruent triangles KLM and KNM)

KX is a common side

$\Delta LKX \equiv \Delta NKX$ (SAS)

(c) $\angle LXK = \angle NXK$ (matching angles of congruent triangles), but, $\angle LXK + \angle NXK = 180°$ (LXN is a straight angle). Hence, $\angle LXK = \angle NXK = 90°$.

(d) One pair of opposite angles of a kite are equal; these are the angles between the unequal sides. The diagonals of a kite intersect at right angles.

16 (a) In Δs WYX and WYZ, $WX = WZ$ (given)

$XY = ZY$ (given)

WY is a common side

$\Delta WYX \equiv \Delta WYZ$ (SSS)

$\angle WYX = \angle WYZ$ (matching angles of congruent triangles)

(b) Repeat (a) for Δs WZX and YXZ, then $\angle WZX = \angle YZX$ (matching angles of congruent triangles).

(c) The diagonals of a rhombus bisect the angles of the rhombus.

17 (a) In Δs ACD and ACB, $AD = BC$ (given)

$DC = AB$ (given)

AC is a common side

$\Delta ACD \equiv \Delta ACB$ (SSS)

$\angle ACD = \angle ACB$ (matching angles of congruent triangles)

(b) Because $\angle DCB = 90°$, $\angle ACD = \angle ACB = 45°$.

(c) The diagonals of a square bisect the angles of the square.

18 (a) In Δs KLM and LKN, $LM = KN$ (given)

$\angle KLM = \angle LKN = 90°$ (given)

KL is a common side

$\Delta KLM \equiv \Delta LKN$ (SAS)

$KM = NL$ (matching sides of congruent triangles)

(b) ΔLKN is a right-angled isosceles triangle so $\angle KLN = \angle KNL = 45°$

ΔLMN is a right-angled isosceles triangle so $\angle MLN = \angle MNL = 45°$

Hence $\angle KLO = \angle LKO = 45°$

ΔKLO is isosceles so $KO = OL$.

Similarly you can show that ΔMLO is isosceles and $OL = OM$.

Hence $KO = OM$ (both equal OL)

Since ΔKNO is isosceles, $KO = NO$ so $LO = ON$ (both equal KO)

(c) In ΔKLO since $\angle KLO = \angle LKO = 45°$ then $\angle KOL = 90°$ (angle sum of ΔKLO is 180°)

(d) The diagonals of a square bisect each other at right angles and bisect the angles of the square.

19 In Δs UXW and WVU,

$UX = WV$ (given)

$\angle XUW = \angle VWU$ (alternate angles, $UX \parallel VW$)

UW is a common side

$\Delta UXW \equiv \Delta WVU$ (SAS)

Hence $\angle XWU = \angle VUW$ (matching angles of congruent triangles)

But these are a pair of equal alternate angles between XW and UV, so $XW \parallel UV$.

Hence $UX \parallel VW$ and $XW \parallel UV$, so $UVWX$ is a parallelogram.

20 In Δs ABC and ADC, $AB = AD$ (given)

$BC = DC$ (given)

AC is a common side

$\Delta ABC \equiv \Delta ADC$ (SSS)

Hence, $\angle ADC = \angle ABC = 90°$ (matching angles of congruent triangles). Because ΔABC is right angled and isosceles, then $\angle BAC = \angle BCA = 45°$. Similarly, $\angle DAC = \angle DCA = 45°$. Hence, $\angle DAB = \angle DCB = 90°$. Now $\angle DAB$ and $\angle ADC$ are a pair of supplementary co-interior angles between AB and DC so $AB \parallel DC$, and $\angle DAB$ and $\angle ABC$ are a pair of supplementary co-interior angles between AD and BC. Thus, $AD \parallel BC$. Hence, $ABCD$ is a parallelogram with a pair of adjacent sides equal, so $ABCD$ is a rhombus. But $ABCD$ is a rhombus with one angle a right angle, so $ABCD$ is a square.

21 In Δs EFH and GFH, $EF = HG$ (given)

$EH = GF$ (given)

FH is a common side

$\Delta EFH \equiv \Delta GFH$ (SSS)

Hence $\angle EHF = \angle GFH$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EH and FG, so $EH \parallel FG$. Similarly, $\angle DFH = \angle GHF$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EF and HG, so $EF \parallel HG$. Hence, $EFGH$ is a parallelogram. But $EF = FG$, so EFGH is a parallelogram with a pair of adjacent sides equal. Thus, $EFGH$ is a rhombus.

22 (a) $2x + 2y = 360°$ (angle sum of a quadrilateral); $x + y = 180°$. Hence, $\angle MJK + \angle JML = 180°$.

(b) $\angle MJK$ and $\angle JML$ are a pair of supplementary co-interior angles between JK and ML. Hence $JK \parallel ML$. But $\angle MJK + \angle JKL = 180°$, so $\angle MJK$ and $\angle JKL$ are a pair of supplementary co-interior angles between JM and KL. Hence, $JM \parallel KL$. $JKLM$ has both pairs of opposite sides parallel, so it is a parallelogram.

23 In Δs SPT and QRT, $ST = QT$ (given)

$\angle STP = \angle QTR$ (vertically opposite angles)

$TP = TR$ (given)

$\Delta SPT \equiv \Delta QRT$ (SAS)

Hence $\angle PST = \angle TQR$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SP and RQ, so $SP \parallel RQ$. Similarly, you can prove that $\Delta SRT \equiv \Delta QPT$ (SAS) so that $\angle RST = \angle TQP$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SR and PQ, so $SR \parallel PQ$. Hence, $SP \parallel RQ$ and $SR \parallel PQ$, so $SPQR$ is a parallelogram.

Open-ended – Sample answers

24 (a) Not true. A rhombus has four equal sides but no right angle, so it is not a square.

(b) All rectangles have their opposite sides parallel. The converse is: 'All quadrilaterals with their opposite sides parallel are rectangles'. This is false, as parallelograms have opposite sides parallel but are not always rectangles.

25 A square is a rectangle because it has two pairs of parallel sides all meeting at 90° angles, the defining features of a rectangle. However, for a rectangle to be a square its four sides must also be of equal length, which is not necessarily the case.

Half-time 6 (p. 347)

1 (a) $d = 130°$, $e = 125°$ (b) $n = 60°$, $m = 140°$, $p = 100°$

(c) $n = 47°$, $p = 133°$

2 (a) $a = 54°$, $b = 36°$ (RHS) (b) $x = 11$ cm (AAS)

(c) $y = 5$ cm (SAS) (d) $y = 108°$ (SSS)

3 (a) $a = 77°$, $b = 138°$ (b) $u = 110°$

(c) $w = 114°$, $y = 37°$, $z = 35°$

4 Proof: In Δs ABE and CDE,
$\angle ABE = \angle CDE = 40°$ (given)
$\angle AEB = \angle DEC$ (vertically opposite angles)
$BE = DE$ (given)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
$\therefore AB = CD$ (matching sides of congruent triangles)

5 *D*

6 (a) False: a rectangle does not have to have four equal sides.

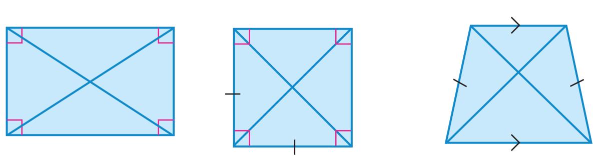
(b) True: all parallelograms have four sides.

(c) True: the diagonals of a square cut the square into four congruent isosceles triangles.

(d) False: the diagonals of a parallelogram only bisect each other at right angles when it is a rhombus.

7 Proof: In Δs KLM and KNM, $KL = KN$ (given)
$LM = NM$ (given)
MK is a common side
$\therefore \Delta KLM \equiv \Delta KNM$ (SSS)
$\angle KLM = \angle KNM$, $\angle LMK = \angle NMK$, $\angle MKL = \angle MKN$.

8 Rectangle Square Isosceles trapezium



9 $(2y - 20) + 140 = 180$ (allied angles on parallel lines), $y = 30°$

$3x = x + 10 + 40$ (vertically opposite angles), $x = 25°$

$30 + z - 25 + 35 = 180$ (allied angles on parallel lines), $z = 140°$

10 Only two sides are given equal. For congruence you need the third sides equal or the angles between the equal sides equal (i.e. need $PQ = PR$ or $\angle PSQ = \angle PSR$).

11 (a) In Δs ABC and ADC,
$AB = AD$ (given)
$BC = DC$(given)
AC is a common side
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
$\therefore \angle ABC = \angle ADC$ (matching angles in congruent triangles)

(b) A kite has one pair of opposite angles that are equal.

Exercise 6.4 (p. 352)

1 (a) 1.5 (b) 0.5 (c) 2 (d) 1

2 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

3 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

4 (a) (i) 1 cm : 2 cm (ii) 1.5 cm : 3 cm

(iii) 1.75 cm : 3.5 cm

(b) 0.5

5 (a) (i) 3 cm : 2 cm (ii) 3.3 cm : 2.2 cm

(iii) 3 cm : 2 cm (iv) 1.5 cm : 1 cm

(b) 1.5

6 C

7 Students' own answers—the dimensions of $A'B'C'D'$ should be 6 cm × 9 cm.

8 Students' own answers—side lengths of $J'K'L'M'N$ should be 10 cm.

9 (a) 3.5 (b) $\frac{3}{7}$

(c) Enlarged by a scale factor of 1.5

(d) It is not a true enlargement because only the length has been stretched, altering the actual shape of the original image.

10 4 cm^2 11 1.75

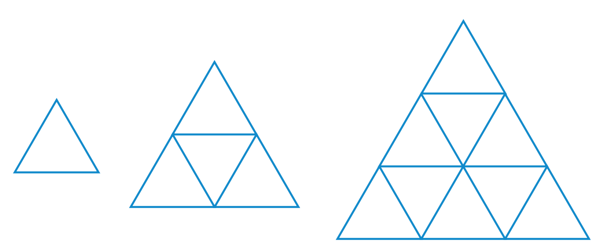
Open-ended – Sample answers

12 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

13 The area of an enlarged rectangle is the area of the original rectangle multiplied by the square of the scale factor. Area of original rectangle $= 72 \div 3^2 = 8$. The dimensions of the original rectangle are either 1×8 or 2×4.

14 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

15 (a)



(b) Scale factor of 4 has a side length of 4 cm (or area of 16 small triangles); scale factor of 9 has a side length of 9 cm (or an area of 81 small triangles).

(c) Because the side length of the medium triangle has length of two small triangles, the scale factor is 2; the small triangle has been enlarged by a scale factor of 3 to obtain the large triangle because it has a side length of three triangles.

Exercise 6.5 (p. 363)

a (a) SAS (b) AAA (c) RHS (d) SSS

2 (a) $x = 14$ cm (b) $x = 8$ m (c) $x = 7.5$ cm (d) $x = 20$ mm

3 (a) B (b) C

4 (a) C (b) C

5 (a) 2.5 (b) 0.5 (c) 0.8 (d) 1.5

6 (a) $\Delta ABC \sim \Delta EDF$ (SSS) (b) $\Delta GHK \sim \Delta GIJ$ (AAA)

(c) $\Delta LMN \sim \Delta OPN$ (AAA)

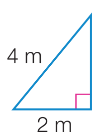
7 (a) (i) $\frac{2}{3}$ (ii) 7.5 m

(b) (i) 2.5 (ii) 8.75 cm

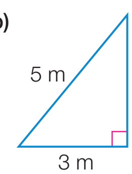
(c) (i) $\frac{3}{4}$ (ii) 5.625 m

(d) (i) 1.25 (ii) 1.28 km

8 (a)



(b)



(c) Not similar; matching side lengths do not have the same ratio.

9 (a) 1.5

(b) Students' own answers—check carefully that $QR = 5.4$ cm and $QPR = 35°$.

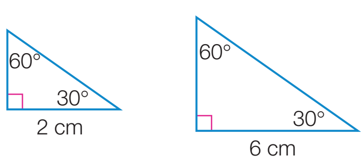
10 All angles remain the same (AAA).

11 (a) Matching angles are the same and matching side lengths are of the same ratio.

(b) $b = 20$ cm, $d = 100°$, $r = 55°$, $s = 12$ cm

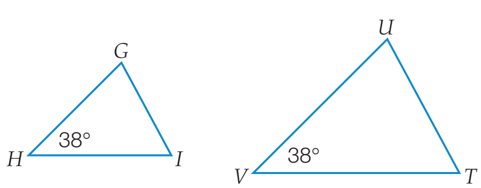
Open-ended – Sample answers

12



Scale factor = 3.

13 (a)



(b) Two side lengths (which enclose the given angle) or one more angle must be stated to justify similarity.

Exercise 6.6 (p. 369)

1 (a) AAA; $x = 10$ (b) AAA; $x = 6$

(c) AAA; $x = 5.56$ (d) AAA; $x = 26.4$

2 (a) AAA means that the triangles are similar

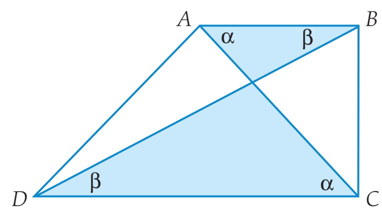
(b) 12 m

3 (a) C (b) C

4 6 m 5 316.67 cm 6 7.2 m

7 (a) Equally opposite angles at x mean that SAS shows the shaded triangles are similar.

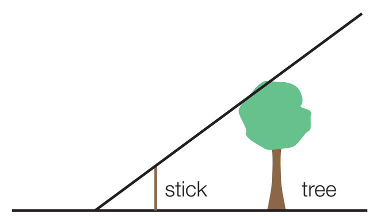
(b)



As the triangles are similar and the alternate angles are equal, AB and CD must be parallel. Hence $ABCD$ is a trapezium.

(c) No, the remaining two triangles only have one pair of vertically opposite angles, which is not enough to justify similarity.

8 (a)



Measure these three distances: from the stick to the tree; from the stick to the place on the ground you look from; and the length of the stick. Use similar triangles to find the height of the tree.

(b) Students' own answers.

Open-ended – Sample answers

9 Ratio of corresponding sides must be equal $\frac{a}{e} = \frac{b}{d} = \frac{c}{f}$.

One possible solution is: $a = 3$, $b = 2$, $c = 4$, $e = 6$, $d = 4$, $f = 8$

$\therefore \frac{3}{6} = \frac{2}{4} = \frac{4}{8}$

10 (a) The triangles might not be similar, thus cannot be calculated in this way.

(b) Pythagoras' Theorem.

Exercise 6.7 (p. 376)

1 D

2 Several possibilities for each—teacher to check.

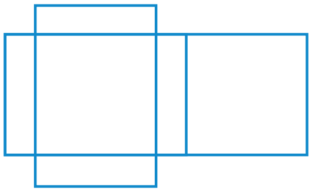
3 (a) tetrahedron (b) cylinder

(c) cone (d) hexagonal prism

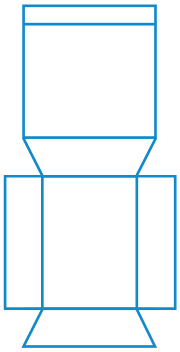
4 D 5 B

6 Sample answers—teacher to check

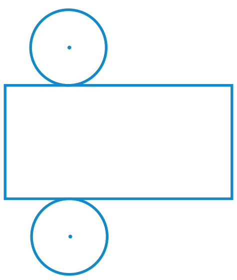
(a)



(b)



(c)



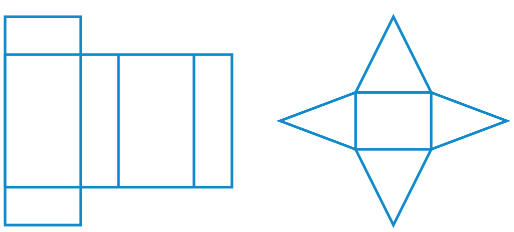
7 Students' own answers—check construction measurements carefully.

8 Students' own constructions.

9 Top part has five faces, five vertices and eight edges. Bottom part has six faces, eight vertices and twelve edges.

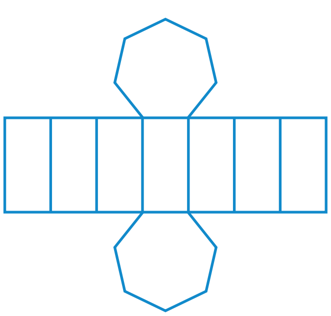
10 (a) Rectangular prism: edges that are meant to meet are of different lengths, no top to match the base. Rectangular pyramid: the triangles all need to have the same sloping side lengths to meet at the apex of the pyramid.

(b)



Open-ended – Sample answers

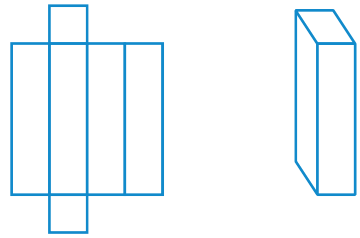
11



12 tetrahedron, octahedron, dodecahedron

13 No, a face is a flat surface; therefore, a cylinder is not suitable. A polyhedron always has more than three faces. The simplest polygon used to construct a polyhedron is a triangle and it requires a minimum of four faces to create a three-dimensional shape.

14 a rectangular prism



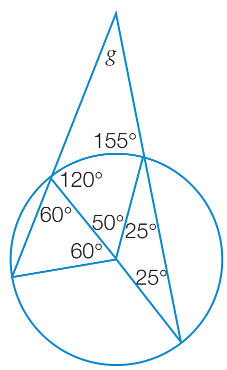
Challenge 6 (p. 382)

1 D

2 Arrange the matches as a three-dimensional tetrahedron. The four faces are equilateral triangles.

3 B

4 $g = 360 - (120 + 50 + 155) = 35°$



5 B

6 B, C, D

7 B

Chapter review 6 (p. 383)

1 (a) $x = 163°$ (b) $x = 41°$ (c) $x = 46°$

(d) $x = 85°$ (e) $x = 32°$ (f) $x = 110°$

2 (a) $y = 53°, z = 127°$ (b) $a = 95°, b = 110°, c = 95°, d = 85°$

(c) $m = 55°, n = 65°, p = 115°$

3 (a) $x = 72°, y = 141°$ (b) $c = 118°, d = 62°, e = 118°$

(c) $f = 90°$

4 (a) B (b) D (c) A (d) C

5 (a) AAS (b) SAS (c) AAS

6 (a) rectangle (b) rhombus (c) parallelogram

7 (a) 0.4 (b) 2.5

8 Students' own diagrams.

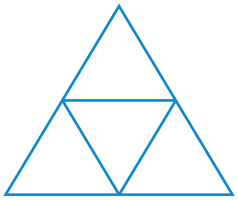
9 The missing angles in the triangles are 40° and 115°, so the triangles are similar (AAA).

10 (a) 5 (b) 9

11 A vertically opposite angle and an alternate angle on parallel lines means the triangles are similar by AAA.

$v = 4$ m, $x = 7.5$ m

12



13 (a) $x = 77$ (b) $x = 60, y = 120$
(c) $w = 72$ $y = 108, z = 72$ (d) $x = 60$
(e) $x = 40$ (f) $x = 40$

14 (a) $x = 56$ (b) $x = 24, y = 128$ (c) $x = 43.6$
(d) $x = 30$ (e) $x = 34, y = 124$

15 (a) $BD \perp AC$

In Δs ABD and CBD, $\angle BDA = \angle BDC = 90°$ ($BD \perp AC$)

$AB = CB$ (given)

BD is a common side

$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)

$\therefore \angle BAD = \angle BCD$ (matching angles of congruent triangles)

$\therefore \angle BAC = \angle BCA$

(b) The angles opposite equal sides in an isosceles triangle are equal.

16 (a) $AE = CE$

$DE = BE$

$\Delta ABE \equiv \Delta BEC \equiv \Delta CED \equiv \Delta DEA$ (SSS)

$\therefore \angle AEB = \angle BEC = \angle CED = \angle DEA = 90°$

$\therefore AC \perp BD$

(b) The diagonals of a rhombus bisect each other at right angles.

17 (a) In Δs ABC and ADC, $AB = CD$ (given)

$BC = DA$ (given)

AC is a common side

$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)

$\therefore \angle BAC = \angle DCA$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between AB and DC, so $AB \parallel DC$.

Similarly, $\angle BCA = \angle DAC$ and hence $BC \parallel AD$.

Hence $ABCD$ is a parallelogram because it is a quadrilateral with both pairs of opposite sides parallel.

(b) In Δs PQT and RQT, $PT = TR$ (given)

$\angle PTQ = \angle RTQ$ (both 90°, given)

QT is a common side

$\therefore \Delta PQT \equiv RQT$ (SAS)

$\therefore PQ = QR$ (matching sides of congruent triangles)

Similarly, you can prove that $\Delta PQT \equiv RST$ (SAS) so that $PQ = SR$.

$\angle QPT = \angle SRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PQ and SR, so $PQ \parallel SR$.

Similarly, you can prove that $\Delta PST \cong \Delta RQT$ (SAS).

$\angle SPT = \angle QRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PS and QR, so $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so it is a rhombus.

But Δs PQT and RQT are right-angled isosceles triangles, so $\angle PQT = \angle RQT = 45°$.

Hence $\angle PQR = 90°$.

$\therefore$ $PQRS$ is a square.

18 12.6 m

19 2.43 m

20 Let $\angle TSU = x°$.

$\angle TSR = (180 - x)°$ ($RSUV$ a straight line)

$\angle TUS = (90 - x)°$ (angle sum of ΔTSU)

$\angle TUV = (90 + x)°$ ($RSUV$ a straight line)

$\therefore \angle TSR + \angle TUV = (180 - x)° + (90 + x)° = 270°$

21 $a + b = 180°$ (AD a straight line)

Similarly $c + d = 180°$

Now $\angle ZWX = a$ (vertically opposite angles)

and $\angle YXW = d$ (vertically opposite angles)

$\angle YZW = b$ (corresponding angles, $AD \parallel HE$)

$\angle ZYX = c$ (corresponding angles, $AD \parallel HE$)

$\angle ZWX + \angle YXW + \angle ZYX + \angle YZW = a + d + c + b$
$= 180° + 180° = 360°$

22 (a) 1.2

(b) 13.2 cm by 10.2 cm

(c) (i) 93.5 cm^2 (ii) 134.64 cm^2

(d) 1.44—this ratio is different from the scale factor as each side has been increased by the scale factor, so the area has increased by the square of the scale factor.

(e) 144%

NAPLAN practice 6

1 60° 2 C 3 50° 4 40°
5 95° 6 40° 7 C 8 B
9 12.15 m^2

Mixed review C (p. 389)

1 61.70m

2 (a) 86° (b) 77° (c) 87°

3 16.25 cm^2

4 (a) 7.5 (b) $k = 46°$

5 351.86 cm^2

6 (a) \$705 (b) \$90.05

7 (a) Scale factor = 1.4, $x = 28$ cm

(b) Scale factor = $\frac{5}{12}$, $x = 3.75$ cm

8 (a) $A = \frac{xy}{2}$ (b) $A = \frac{xy}{2}$ (c) $A = \frac{1}{2}h(a+b)$

9 (a) $(y+7)(y+3)$ (b) $(d+3)(c+4)$ (c) $(t-4)(t-5)$

10 Students' own answers.

11 (a) 2 (b) -1 (c) $\frac{1}{2}$

12 (a) $x = \frac{55}{3}$ (b) $x = \frac{13}{4}$ (c) $x = -\frac{13}{3}$

13 (a) AAA (b) SAS, scale factor 2

14 (a) 2.4 (b) $\frac{1}{3}$

15 $a = 30°$, $b = 80°$, $c = 70°$, $d = 150°$, $e = 100°$

16 9 m

17 (a) $x^2 + 4x - 21$ (b) $(x+7)(x-3)$ (c) $x = 103$

(d) Students to show the equality of the expressions.

18 Any of ΔAEH, ΔAFI, ΔFEG, ΔACE, ΔABF, ΔFDE. All triangles have one right angle and two 45° angles
$\therefore$ they are similar triangles (AAA).

19 5.4

20 (a) 3

(b) (i) 150 cm^2 (ii) 1350 cm^2

(c) 9

(d) $9 = 3^2$, the scale factor of the area enlargement is the square of the scale factor of the enlarged lengths.

Chapter 7

Recall 7 (p. 394)

1 (a) 12.5 (b) 13 (c) ± 8 (d) ± 7

(e) 5.34 (f) 62.7221

2 (a) equilateral and acute angled

(b) scalene and right angled

(c) scalene and acute angled

(d) isosceles and right angled

(e) equilateral and acute angled

(f) scalene and obtuse angled

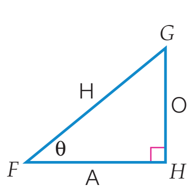
3 (a) 42 (b) 40 (c) 54 (d) 4

4 (a) 64° (b) 25° (c) 53°

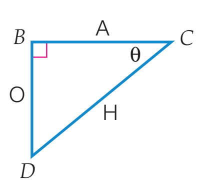
5 (a) $\frac{4}{5}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{4}{3}$ (e) $\frac{3}{4}$ (f) $\frac{4}{5}$

Exercise 7.1 (p. 396)

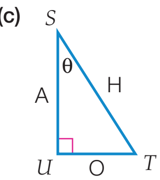
1 (a)



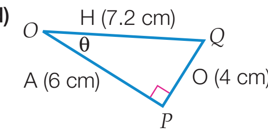
(b)



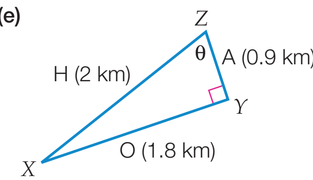
(c)



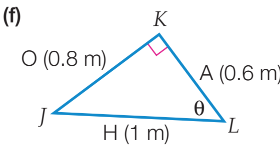
(d)



(e)



(f)



2 (a) (i) 10.6 cm (ii) 7 cm (iii) 8 cm

(b) (i) 12 m (ii) 3.8 m (iii) 11.4 m

(c) (i) 3.4 cm (ii) 2.5 cm (iii) 2.3 cm

(d) (i) 5.1 km (ii) 5 km (iii) 1 km

(e) (i) 96 mm (ii) 78 mm (iii) 56 mm

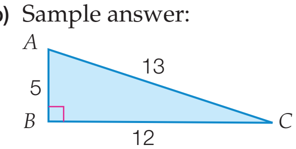
(f) (i) 108.3 km (ii) 34.7 km (iii) 102.6 km

3 (a) D (b) B (c) C

4 (a) 5.3 m (b) 1.8 m (c) 70°

5 (a) 13

(b) Sample answer:



6 (a) Triangle 1: H: 2.25 cm; O: 0.55 cm; A: 2.2 cm

Triangle 2: H: 4.25 cm; O: 1 cm; A: 4.1 cm

Triangle 3: H: 6.25 cm; O: 1.55 cm; A: 6.15 cm

(b) $\frac{0.55}{2.55}; \frac{1}{4.25}; \frac{1.55}{6.25}$

(c) $\frac{2.2}{2.25}; \frac{4.1}{4.25}; \frac{6.15}{6.25}$

7 (a) Students' own diagram.

(b) $\frac{0.9}{1} = 0.9$; $\frac{1.8}{2} = 0.9$; $\frac{2.7}{3} = 0.9$; $\frac{3.6}{4} = 0.9$

(c) Corresponding sides have an equal ratio in similar triangles.

8 (a) Students' own diagrams.

(b) Length measurements will depend on students' diagrams.

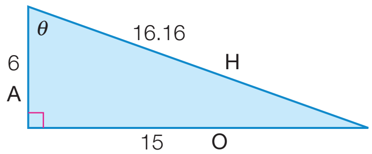
Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°				≈0.577	≈0.5	≈0.866
2	30°				≈0.577	≈0.5	≈0.866
3	30°				≈0.577	≈0.5	≈0.866
1	60°				≈1.732	≈0.866	≈0.5
2	60°				≈1.732	≈0.866	≈0.5
3	60°				≈1.732	≈0.866	≈0.5

(c) The ratios of corresponding sides are the same for each triangle.

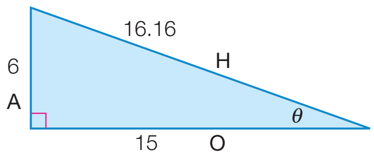
(d) Students' own response.

9 (a) The teacher has not marked in the angle.

(b) Nash



Aisha



Open-ended – Sample answers

10 The adjacent side is between the right angle and the reference angle. It is not always on the bottom of a triangle.

Exercise 7.2 (p. 402)

1 (a) (i) $\frac{7}{24}$ (ii) $\frac{4}{3}$ or $1\frac{1}{3}$ (iii) $\frac{21}{20}$ or $1\frac{1}{20}$ or 1.05

(b) (i) $\frac{7}{25}$ (ii) $\frac{3}{5}$ (iii) $\frac{21}{29}$

(c) (i) $\frac{3}{5}$ or 0.6 (ii) $\frac{8}{17}$ (iii) $\frac{9}{41}$

2 (a) (i) $x = 6.\dot{6}$ (ii) $p = 39$ (iii) $n = 0.7$

(b) (i) $x = 13$ (ii) $y = 6$ (iii) $r = 20.8$

(c) (i) $n = 48$ (ii) $s = 12$ (iii) $r = 22.4$

3 (a) 0.59 (b) 0.81 (c) 0.73

4 (a) (i) 0.58 (ii) 0.87 (iii) 0.50

(iv) 0.87 (v) 1.73 (vi) 0.50

(b) same (c) same

5 (a) (i) 0.54 (ii) 1.54 (iii) 0.84

(iv) 0.54 (v) 0.84 (vi) 0.65

(b) same (c) same

6 (a) B (b) A (c) B

7 100 m

8 2.8 m

9 0.82

10 (a) Students to draw a scale diagram.

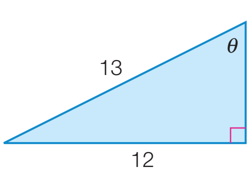
(b) right-angled isosceles triangle

(c) 5.3 m

(d) The two shorter sides of a right-angled isosceles triangle are equal.

11 (a) 3 m (b) Angle from the ground could be lowered to 30° or less.

12 (a)



(b) 5 (c) $\frac{5}{13}$

13 (a) sin 50° = 0.766; cos 50° = 0.643; tan 50 = 1.192

(b) tan 50° has a value greater than 1. The opposite side is longer than the adjacent side.

(c) (i) Any right-angled trianlge with a reference angle less than 45°.

(ii) Any right-angled triangle with a reference angle greater than 45°.

(d) sin θ and cos θ can not be greater than 1 because the hypotenuse (the bottom number on the ratio) will always be longer than the other two sides.

Open-ended – Sample answers

14 Any, as long as the ratio between b and a is 7:10. Sample answers: $a = 10, b = 7; a = 20, b = 14; a = 100, b = 70$.

15 The opposite and adjacent side change position with the two reference angles, so sin H is the same as cos G.

Exercise 7.3 (p. 408)

1 (a) 26.63 (b) 7.51 (c) 18.87 (d) 33.27

(e) 4.04 (f) 0.92

2 (a) 313.49 cm (b) 137.73 mm (c) 13.62 m

(d) 229.40km (e) 89.71 cm (f) 150.13 m

3 (a) C (b) D

4 (a) C (b) D

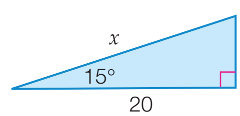
5 2.40 m 6 1.39 m 7 7.60 m

8 10.72 m 9 47.57 cm 10 5.52 mm

Open-ended – Sample answers

11 The adjacent side divided by the hypotenuse must be approximately equal to 0.45.

12 (a) A question might have been to find the hypotenuse, as shown in the triangle below.



(b) 5.36

13 (a) (i) Use of the wrong reference angle in the equation. Student has calculated the length of j.

(ii) Incorrect method of rearranging the equation. The unknown is the denominator in the equation, so the known side should be divided by the trigonometric ratio.

(b) (i) $\cos\theta = \frac{A}{H}$ (ii) $\sin\theta = \frac{O}{H}$

$\cos 50° = \frac{k}{8}$ $\sin 55° = \frac{13}{x}$

$\cos 50° \times 8 = k$ $x = \frac{13}{\sin 55°}$

$k = 5.14$ m $x = 15.87$ km

14 To find a side length with Pythagoras' Theorem, the side lengths of two sides must be known. Trigonometry can calculate side lengths with one side length and one angle known.

Half-time 7 (p. 412)

1 (a) A, H (b) O, H

2 (a) 7.26 (b) 16.68 (c) 9.14

3 (a) 0.59 (b) 0.81 (c) 0.73

4 (a) 19.04 (b) 81.94 (c) 28.59

5 (a) 4 m = opposite, 7 m = adjacent, unknown = hypotenuse.

(b) 9 km = hypotenuse, 6.5 km = opposite, unknown = adjacent

(c) 10.3 km = hypotenuse, 6.9 km = adjacent, unknown = opposite

6 $f = 6$

Exercise 7.4 (p. 415)

1 (a) 15° (b) 39° (c) 35° (d) 62°

(e) 10° (f) 41° (g) 11° (h) 78°

2 54°

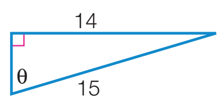
3 (a) B (b) C (c) D

4 15°

5 24°

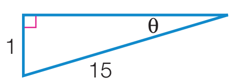
6 8°

7 (a)



$\sin\theta = 0.933$

(b)



$\sin\theta = 0.067$

(c) (i) close to 1 (ii) close to 0

8 (a) A ratio of 1 means that the two sides are equal in length.

(b) Tangent ratio—the other two ratios involve the hypotenuse, which is always longer than the other two sides.

(c) isosceles triangle

(d) 45°

Open-ended – Sample answers

9 There are four possible triangles with reference angles:

$\sin^{-1}\left(\frac{9}{15}\right) = 36.87°$, $\cos^{-1}\left(\frac{9}{15}\right) = 53.13°$, $\tan^{-1}\left(\frac{9}{15}\right) = 30.96°$,

$\tan^{-1}\left(\frac{15}{9}\right) = 59.04°$.

10 ramp length = 450 cm, $\theta = 8.3°$

ramp length = 500 cm, $\theta = 7.5°$

ramp length = 550 cm, $\theta = 6.8°$

Exercise 7.5 (p. 425)

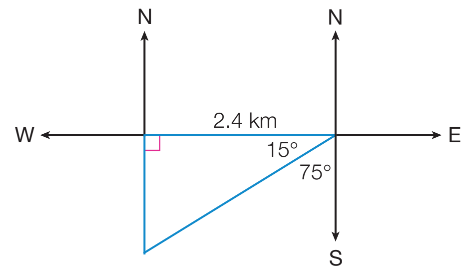
1 122.06 m 2 16° 3 34.64 km

4 63° 5 A 6 B

7 C 8 9.58 m 9 24°

10 (a) 33.44 km (b) 30.11 km

11 (a)



(b) 0.64 km (c) 2.48 km

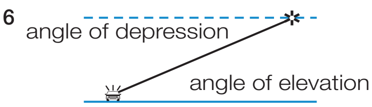
12 (a) N54°W or 306°T (b) S54°E or 126°T

13 76 m 14 26.17 m

Open-ended – Sample answers

15 Height of tree ÷ distance from car ≈ 0.51

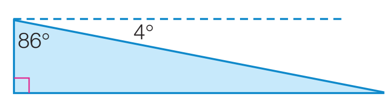
16



The two horizontal lines are parallel. The angle of elevation and the angle of depression are alternate angles, which we know are equal.

17 (a) Germaine has based her calculations on the reference angle of 4° between the rope and the boat; however, 4° is the angle of depression. The correct answer is 14.96 m.

(b) The angle of depression must go down from the horizontal, which means in this case it is not inside the triangle. The reference angle inside the triangle (to use the sine ratio) is 86°.



Challenge 7 (p. 430)

1 (a) $(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2$
$= m^4 + 2m^2n^2 + n^4$
$= (m^2 + n^2)^2$

(b) (i) $m = 2, n = 1 : 3, 4, 5$

(ii) $m = 3, n = 1 : 8, 6, 10 \rightarrow 3, 4, 5$
$m = 3, n = 2 : 5, 12, 13$

(iii) $m = 4, n = 2 : 12, 16, 20 \rightarrow 3, 4, 5$
$m = 4, n = 3 : 7, 24, 25$

(iv) $m = 4, n = 1 : 15, 8, 17 \rightarrow 8, 15, 17$

(v) $m = 5, n = 1 : 24, 10, 26 \rightarrow 5, 12, 13$
$m = 5, n = 2 : 21, 20, 29 \rightarrow 20, 21, 29$
$m = 5, n = 3 : 16, 30, 34 \rightarrow 8, 15, 17$
$m = 5, n = 4 : 9, 40, 41$

2 $AB = 3\sqrt{2}$, $BC = 3\sqrt{5}$,

Perimeter $= 3\sqrt{2} + 3\sqrt{5} + 6 + 9 = 3(5 + 2\sqrt{5})$ cm

Answer D

3 (a) Isosceles (b) $PS^2 = 100 - 36$; $PS = 8$ cm (c) A

4 D

5 C

6 (a) $EC = DE = x$

(b) $\frac{CF}{CB} = \cos 60°$, $CF = 2x \times 0.5 = x$

(c) $BF^2 = CB^2 - CF^2 = 4x^2 - x^2 = 3x^2$. $BF = \sqrt{3}x$

(d) $\tan\angle EBF = \frac{EF}{BF} = \frac{2x}{\sqrt{3}x} = \frac{2}{\sqrt{3}}$. $\angle EBF = 49°6'$. (49.1°)

(e) $\angle CBF = 30°$. $\angle EBC = 49°6' - 30° = 19°6'$

7 (a) $\sin\theta = \frac{30}{132}$, $\theta = 13.13° \approx 13°$

(b) $\sin 27 = \frac{44}{BD} = BD = \frac{44}{\sin 27} = 96.9 \approx 97$ m.

(c) Total length of track = 132 + 97 = 229 m.

Average speed = $\frac{229}{90} = 2.54 \approx 2.5$ m/s.

Chapter review 7 (p. 431)

1 (a) 25m = opposite, x = hypotenuse, sin

(b) 8.3 cm = hypotenuse, x = adjacent, cos

(c) 3.2 km = adjacent, x = opposite, tan

2 (a) 1.3333 (b) 0.225 (c) 5.4545

3 (a) $x = 4$ cm (b) $t = 3.7$ cm (c) $p = 7.9$ m

4 (a) $\frac{3}{5}$ (b) $\frac{15}{17}$ (c) $\frac{20}{29}$

5 (a) 2.19 cm (b) 15.00 cm (c) 55° (d) 111.87 m

(e) 1766.89 m (f) 68°

6 A

7 7.57 km south and 10.81 km east

8 33.58 m

9 48°

10 26.11 m

11 76.9 km

12 89.6 m

13 50.75 km

14 151°T or S29°E

15 46.1°

16 (a) 2.23 cm (b) 15.61 cm^2

17 63.4°

NAPLAN practice 7

1 B 2 B 3 A 4 D

5 D 6 A 7 B 8 C

Chapter 8

Recall 8 (p. 438)

1 mean = 20.9, median = 20.5, range = 18

2 (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$

3 mean 2.36; median 2

4 spade, club, diamond, heart

Exercise 8.1 (p. 442)

1 (a) discrete (b) continuous (c) continuous

(d) discrete (e) continuous (f) continuous

(g) discrete (h) continuous

2 (a) qualitative (b) qualitative (c) quantitative

(d) quantitative (e) quantitative (f) quantitative

(g) qualitative (h) qualitative

3 (i) (c) discrete (d) discrete

(e) continuous (f) continuous

(ii) (a) categorical (b) categorical

(g) ordinal (h) categorical

4 The safest state, with the lowest registered deaths per 10 000 registered motorcyclists, is Victoria. Although the most motorcyclists are killed in Queensland, the rate is much worse in the Northern Territory. The deaths per 10 000 motorcyclists gives a more accurate picture than the raw figures.

5 (a) B (b) A (c) D

6 Sample answers:
Question 1: Every day; once a week; at least once a month etc.
Question 2: Range of products: prices; quality of food; service etc.
Question 3: Students to provide own answer based on their own school canteen.

7 (a) 75% of those surveyed want to change but they cannot agree on what that should be. These results suggest the jumper should not be changed.

(b) Limit the choices to two. Perhaps you could have several parts to the process, narrowing the options each time.

8 The results are discrete because there is a fixed, countable number of values possible.

9 Joe's answer is incorrect because measured data is always continuous; it doesn't matter how much you round the data off. Ngarla's answer is correct, although she provides more information than necessary. It would be sufficient to say that height is a measured variable and so continuous.

Open-ended – Sample answers

10 (a) number of pets per household, number of students in each year

(b) time take to swim 50 m, height of seedlings in a science experiment

(c) favourite football team, type of pet owned

(d) genre of DVD owned: drama, comedy, romance, animation; gender: male, female

11 (a) Students will come up with their own list of vehicle types.

(b) It is unlikely every student would arrive at exactly the same list.

(c) If the intent was to collate class results, then it would be a problem if the lists were not the same.

Exercise 8.2 (p. 447)

1 (a) 5.89% (b) 69.33% (c) 88.41%

(d) The full-time figures are not far apart, but these figures are quite different. This would seem to indicate that women work fewer hours than men when part-time and that more women work part-time.

2 Students will produce their own answers.

3 A

4 (a) D (b) C

5 (a) 200 seedlings (b) 250 seedlings 500 seedlings

6 (a) 5

(b) No, every sample will have its own unique composition.

(c) Students will produce their own answers and comment on the similarities and differences in their samples.

7 There are 50% more blue jelly beans than pink in the packet so Yvette may have thought the sample should reflect the proportion of colours in the packet. This is not correct mathematical thinking.

8 (a) The mean will be affected by the years when there is quite a lot of rainfall, increasing the mean value. The median is consistently lower, as Adelaide is normally relatively dry apart from in these wet years.

(b) It is more likely to have less rain than the mean value.

(c) You would use the median figures because it is more likely that these figures will make it seem that Adelaide is drier than it may actually be.

(d) February is the driest month on whichever measure you use. So, the exceptionally wet Februarys will have more effect on the mean value.

9 (a) Assign each student a number and then use random numbers to select the sample.

(b) Student behaviour may be different if they see they are being observed.

(c) Students to provide their own answer, but something like the following may be appropriate. A combination of the two strategies given may be useful. It would also be useful to speak with the canteen workers to get more detail regarding their concerns.

10 (a) Lincoln

(b) 156.5

(c) Students will provide their own answer.

(d) Students will provide their own answer.

(e) Students will provide their own answer.

(f) Students will provide their own answer.

11 (a) 160

(b) No; this would favour executives and managers.

(c) Conduct a census instead.

12 (a) Expect 4 black, 5 blue and 2 white marbles. Note that due to rounding this adds to 11.

(b) You should not expect every sample to be the same. There are variations in every experimental situation.

13 (a) No, it is actually a self-selecting sample that is notoriously unreliable.

(b) 290 people is a very small sample size for the broader population, and it is therefore difficult to draw any conclusions. More details will be required, such as readership numbers, to assist in making judgement statements and drawing conclusions about the broader population.

(c) Although the *Herald Sun* is published in Melbourne, there is also an online version. It is hard to ascertain the readers of this newspaper. It does not take into account the difference between numbers of published papers and actual readers. One paper can be read by more than one person (e.g. if a paper is left in a café or in a public place).

Open-ended – Sample answers

14 (a) 80.29%

(b) It is higher than the equivalent figure for men.

(c) You might like to see the number of hours worked part-time and/or the pay per hour for part-time work.

(d) It shows that women seem to be paid more for part-time work than men. It helps, but is probably not all you would like to see.

(e) The means indicate that men earn more than women in part-time work, which is the opposite of what the medians tell us. If you were trying to promote the notion that women are paid less than men, then you would probably use the mean values.

15 Students will provide their own answers.

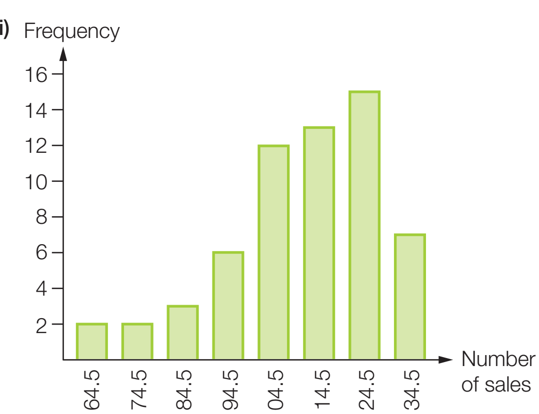
Exercise 8.3 (p. 462)

1 (a) negatively skewed (b) positively skewed

(c) symmetrical

2 (a) (i) 110.83 (ii) 110–119

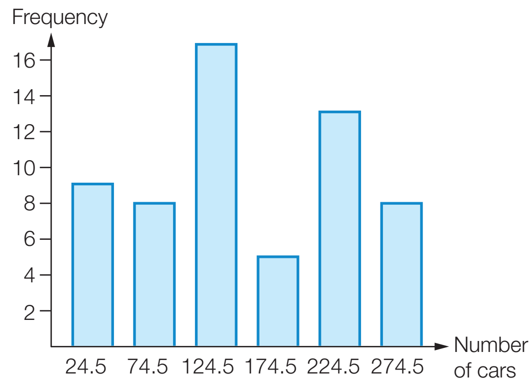
(iii)



(iv) The graph is not symmetrical. It is negatively skewed and there are no outliers.

(b) (i) 148.67 (ii) 100–149

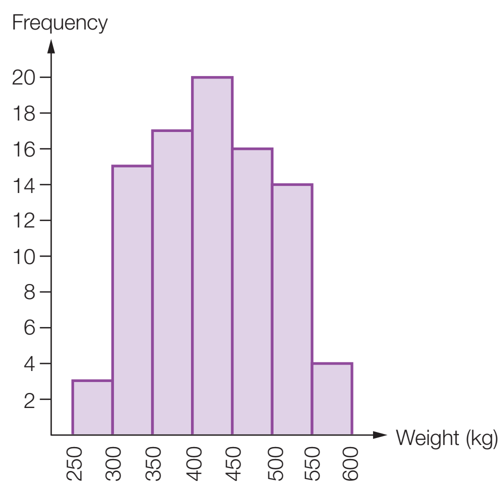
(iii)



(iv) The graph is relatively symmetrical, although the 150–199 bin is a little low for true symmetry.

3 (a) (i) 427 kg (ii) 400 – < 450 kg

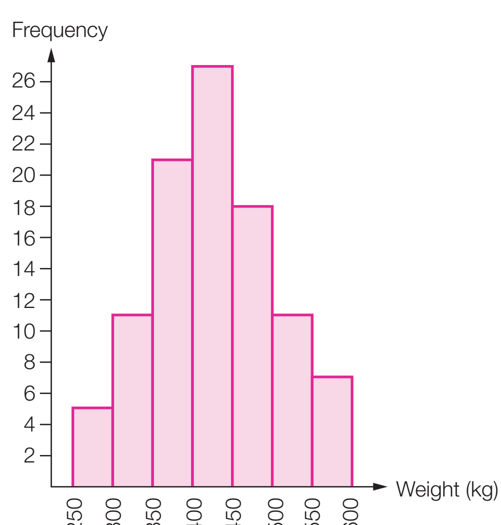
(iii)



(iv) The data look symmetrical.

(b) (i) 427 kg (ii) 400 – < 450 kg

(iii)



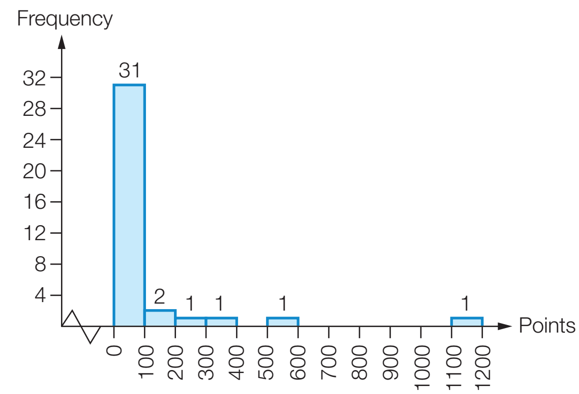
(iv) The data look symmetrical.

4 (a) (i) 66.47 (ii) 65 (iii) positively skewed

(b) (i) 151.13 (ii) 157.5 (iii) negatively skewed

(c) (i) 242.57 (ii) 245 (iii) symmetrical

5 (a)



The graph is positively skewed and it is not symmetrical.

(b) Certainly 1163 is an outlier, and an argument could be made for 582 as well.

(c) mean is 112.16, median class is 0–99

6 (a) B (b) B

7 (a) The data set is positively skewed.

(b) Students will provide their own answer, but a value about 2 would seem reasonable.

(c) 1.72; students will make their own comment about the reasonableness of their estimate.

(d) Students will make their own comment, but may suggest conducting a survey within their own school.

8 (a) 1–10; 11–20; 21–30; 31–40; 41–50

(b) 0–4; 5–9; 10–14; 15–19; 20–24

(c) 0–49; 50–99; 100–149; 150–199; 200–249

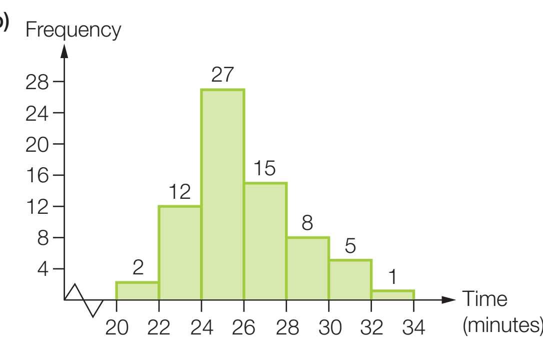
(d) 1–50; 51–100; 101–150; 151–200; 201–250

(e) 10–12; 13–15; 16–18; 19–21; 22–24

(f) 25–29; 30–34; 35–39; 40–44; 45–49

9 (a) 25.97 minutes

(b)



The data set is moderately positively skewed.

(c) Their times would increase the mean.

(d) You might just assign them a time, say 34 minutes.

10 (a) The data set is slightly positively skewed.

(b) Somewhere around 3 km would seem reasonable.

11 (a) 71.94 (b) 40–79 games

(c) In this case the mean probably does the better job

(d) Mean: 93.25; median: between 40–79 and 80–119 games, so let's say about 80 games.

(e) Both these values are probably better measures of location because they represent the players who have actually played. There is no guarantee any of those on 0 will ever play AFL.

12 (a) The two end numbers are both odd or both even.

(b) One of the end numbers is odd and the other is even.

(c) No, because there are no possibilities left after those described in (a) and (b).

Open-ended – Sample answers

13 (a) 1–5, 6–10, 11–15, etc. and 100–104, 105–109, 110–114, etc.

(b) 0–9, 10–19, 20–29 etc. and 1–10, 11–20, 21–30 etc.

14 (a) Students are to provide their own answers.

(b) Students are to provide their own answers.

15 (a) Students are to provide their own answers.

(b) Students are to provide their own answers.

(c) Students are to provide their own answers.

Exercise 8.4 (p. 469)

1 (a) 37 for Fremantle and 36 for West Coast Eagles

(b) We will have a class interval of 5, with the first being 16_H (165 – 169).

(c)

Femantle	Stem	West Coast Eagles
	16_H	7 9
4	17_L	
7 6	17_H	5 8 9
4 4 4 4 2 2 2 1 1 0 0	18_L	1 2 3 3 3 3 4
9 9 9 8 8 8 7 6 6 6 6 5	18_H	5 5 6 6 6 6 6 7 7 8 8 9 9 9 9 9 9 9
4 4 4 4 3 3 3 2 2 2 1 1 1 1 1 0 0	19_L	0 1 1 2 2 4 4
9	19_H	5 5 5 6 8
3	20_L	1 2 3
	20_H	
1	21_L	

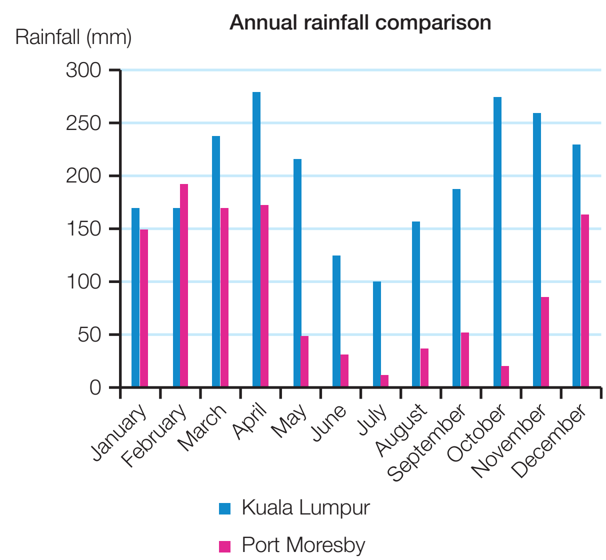
The two teams seem to be very close as far as height is concerned. Some more work would need to be done to make a more definitive statement. This may include calculating the mean and median heights for the two teams.

2 (a) The figures for Lahore, and to a lesser extent Canberra, are lost on the graph owing to the much higher rainfall in the other two places.

(b) Restrict graphs such as this to comparing things that are closer in value.

(c) The actual figures are as shown here. (Students have estimated these figures so there is no expectation that they will have exactly the same answers.)

	Kuala Lumpur	Port Moresby
January	171	150
February	169	194
March	237	170
April	279	173
May	216	49
June	126	31
July	102	12
August	157	37
September	188	53
October	275	20
November	259	87
December	230	164

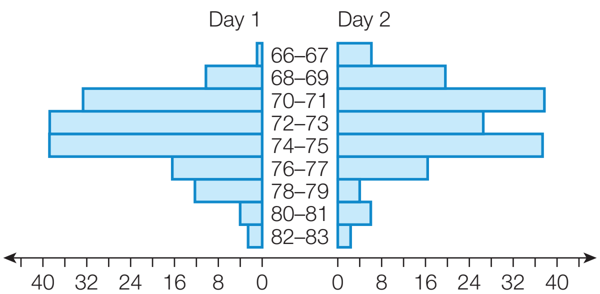


(d) Kuala Lumpur: mean: 200.75; median: 202; range: 177
Port Moresby: mean: 95; median: 70; range: 182

(These figures are again from the estimated values, so students will almost certainly have different answers.) Although both places have some months with high rainfall, Kuala Lumpur is much more consistently wet, as indicated by its much higher mean and median values. The larger range for Port Moresby is brought about by its very dry season in the middle of the year. Kuala Lumpur is wetter in every month except February and even then they are quite close.

3 (a)

Score range	Frequency Day 1	Frequency Day 2
66–67	1	6
68–69	10	19
70–71	32	37
72–73	38	26
74–75	38	37
76–77	16	16
78–79	12	5
80–81	4	6
82–83	3	2



(b) Grouping this way gave nine columns for each part of the histogram. This fits well into our preferred number of 5 to 10.

(c) Day 2 appears to have had marginally lower scores.

(d) Day 1: Mean = 73.55 and Median = 73
Day 2: Mean = 72.84 and Median = 73
This confirms the answer given in (c).

4 (a) From birth to about 49 years of age.

(b) These bars include everybody older than 79. Each of the other bars covers a 5-year range, but these cover up to 20 or 30, depending on how long people live.

(c) It gives a good picture of the differences between the ages but is not as useful comparing males to females because it is difficult to see the values exactly.

5 (a)

Male	Stem	Female
	14_L	4
7	14_H	5
4	15_L	3 3 4
9 7	15_H	5
4	16_L	0 0 0 1 2 2 2 3 4 4
6	16_H	5 5 7 7 7 8 8 9
4 3	17_L	0 1 2 4 4
7 7 6 5 5	17_H	8
3 1 0	18_L	
8 5	18_H	
1 0	19_L	

(b) Year 9 Victorian boys are clearly taller than the equivalent girls.

(c) This would normally be a concern, but even with fewer boys the seven tallest students are boys and the two shortest are girls.

(d) Another sample may, but is unlikely to, give the same results. We would, however, expect that all samples would show that the boys are taller than the girls.

(e) The same basic structure of results would probably apply for the whole of Australia. It would seem fair to say that boys would be in the range 145–195 cm and girls 140–185 cm.

6 (a) The following table shows the results.

	Mean	Median	Range
A	95.03	46.35	296.7
B	31.88	24.5	49.8
C	21.33	21.2	7.1
D	161.31	164.15	148.3

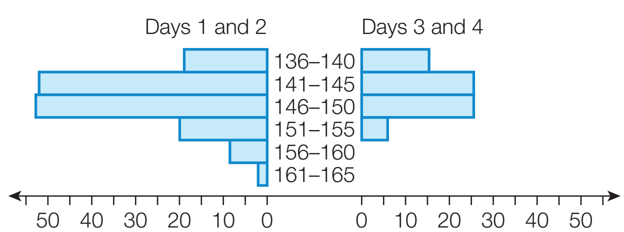
(b) A has a wet season (basically summer) and a dry season (winter and spring). This is best reflected in its very high range. It is likely to be in the tropics.
B has low annual rainfall with most rain occuring in the winter.
C is quite a dry place with a consistent rainfall pattern. This is reflected in the closeness of the mean and median values and the very low range.
D is the wettest of the four places with most of its rain coming in the winter months. However, its high range value indicated it has some relatively dry months. These occur in the summer.

(c) Tropical places have high rainfall in the summer months, so **A** must be Townsville. Broken Hill and Port Lincoln are both dry places but, as Port Lincoln is on the coast it will likely have more rain. So **B** is Port Lincoln and **C** is Broken Hill. This leaves **D** to be Weeaproinah, which is the wettest place in Victoria.

7 (a) This grouping would give six groups for the first pair of days, so it fits neatly into our preferred range of 5 to 10.

(b) There are only four groups for the school pair of days because no one scored over 153.

(c)



(d) There are only half as many results on the right-hand side and none at all in the highest two categories. However, it appears that the scores for the second pair of days are lower because there are almost as many scores in the lowest score category as on the first pair of days. As approximately half the players are eliminated after the first two rounds, it is safe to say that the better performing players will proceed to Rounds 3 and 4.

(e) For Days 1 and 2: Mean = 146.4 and Median = 146;
For Days 3 and 4: Mean = 144.6 and Median = 145.
This confirms that scoring was lower on Days 3 and 4.

8 It is important to note the different scale used in the Tonga graph compared with Australia and Cambodia. Australia has a bigger spread of population (range) compared with both Cambodia and Tonga. Cambodia has a higher birth weight than Australia. Overall, Tonga has a smaller population compared with the other two countries.

Open-ended – Sample answers

9 Students will provide their own answers.

10 Students will provide their own answers.

Mathspace (p. 474)

1 (a) 5150 (b) Students' own answer

2 (a) 25 750 to 51 500 households would have been originally surveyed.

(b) This gives information for key advertisers. They want to target the people who make the decisions about spending.

(c) This is so a fair spread of community members can be achieved. It is an attempt to remove bias from the results.

3 (a) 38.6% (b) 3 277 864 (c) 8 194 660

4 Some sporting events, such as NRL State of Origin, would be more popular in certain states (NSW and Queensland) than others. There would also be some shows directed to small community groups that would have widely divergent popularity.

5 This could be used to enter larger numbers of people and different ages than those actually present, in the hope of seeing a favourite show continue.

6 (a) male 18–54 (b) 40–64 year olds

(c) females 18–54 (d) 13–24 year olds

7 (a) These are based on sales (as reported by music shops) and electronic downloads.

(b) Possibly not as reliable, but there is not as much riding on these results.

Half-time 8 (p. 478)

1 (a) (i) 54.32 (ii) 53 (iii) positive skew

(b) (i) 149.46 (ii) 149.5 (iii) symmetrical

2 (a) B (b) B (c) A

3 Students will provide their own answer, but it should contain at least some of the following elements:

- Construct a survey instrument that shows the three new logos as well as the current one.
- Decide on important stakeholders to be surveyed—employees, customers, general public etc.
- How many of each will be surveyed?
- Do their opinions count equally?
- What will represent success for your survey?

4

Group 1	Stem	Group 2
9	14_H	
4 3	15_L	1 3 4
6 5 5	15_H	5 5 5 8
4 0	16_L	0 0 1
7 7 6	16_H	6 6 6 7
3 0	17_L	1 1 2 2 3
7 7 5	17_H	9
3 1 0	18_L	
9	18_H	

It is likely that Group 1 is the boys.

5 (a) Gold Coast Suns appear to be slightly taller. They have the tallest player and more players in the second tallest band. The Brisbane Lions have a lot of players in the 180–<185 band, which will bring down their average height.

(b) The mean and median would be the least you would require. Having the actual heights would be most useful.

(c) This extra information confirms that the Gold Coast Suns are slightly taller than the Brisbane Lions. In fact the statistics are very close.

Exercise 8.5 (p. 482)

1 $\frac{13}{27}$

2 (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$

3 (a) experiment 4; 0.17 (b) experiment 5; 0.06

(c) experiment 1; 0.17 (d) experiment 2; 0.10

(e) peach; experiment 2; 0.05 (f) pear; experiment 4; 0.25

4 (a) 0.16 (b) 0.3 (c) 0.81

5 Elliot: $\frac{3}{10}$; Kris: $\frac{19}{75}$; Austin: $\frac{67}{150}$

6 (a) B (b) D (c) B

7 (a) C (b) B

(c) Die 1, Die 4, Die 2, Die 3, Die 5

(d) Die 4, Die 5, Die 3, Die 2, Die 1

8 C

9 (a) B (b) B (c) B

10 (a) $\frac{1}{7}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) 1 (e) 0 (f) $\frac{1}{7}$

(g) 1, as expected, as these three outcomes cover all possibilities.

11 (a) $\frac{1}{12}$ (b) $\frac{5}{12}$ (c) $\frac{2}{3}$

12 $\frac{5}{9}$

13 (a) 50

(b) (i) $\frac{1}{5}$ (ii) $\frac{6}{25}$ (iii) $\frac{1}{50}$

(iv) $\frac{23}{25}$ (v) $\frac{1}{2}$ (vi) $\frac{4}{5}$

(c) (i) 0.20 (ii) 0.24 (iii) 0.02

(iv) 0.92 (v) 0.50 (vi) 0.80

(d) (i) 20% (ii) 24% (iii) 2%

(iv) 92% (v) 50% (vi) 80%

14 (a) (i) 72.1 years (ii) 72.8 years (iii) 73.8 years

(iv) 76.7 years (v) 80 years (vi) 84.6 years

(b) As you get older your expected age at death increases.

(c) $\frac{1}{4}$ (half of a half)

(d) Insurance companies; it helps them set the premiums (prices) for policies.

15 (a) $\frac{1}{365}$ (b) $\frac{6}{73}$ (c) $\frac{12}{365}$ (d) $\frac{24}{73}$ (e) $\frac{337}{365}$

16 (a) $\frac{1}{6}$ (b) 0 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$ (f) 1

17 Students' own answers

18 (a) $\frac{2}{12}=\frac{1}{6}$ (b) $\frac{3}{12}=\frac{1}{4}$ (c) $\frac{2}{12}=\frac{1}{6}$

(d) $\frac{3}{12}=\frac{1}{4}$ (e) $\frac{1}{12}$ (f) $\frac{1}{12}$ (g) 0

(h) Add to 1, and this is because all possibilities shown so far have been covered.

(i) The seniors are likely to be a different standard and these figures may not apply.

19 The coach is wrong. Pr(success) = $\frac{2}{3}$ for every throw

20 (a) Sylvester may believe that balls that have appeared frequently will continue to appear frequently. Alternatively, he may think that the balls that haven't been drawn often will be 'due' their turn.

(b) No it is not useful. The balls do not have memories and will come out randomly next time, regardless of what has happened in the past.

21 Pr(1) = Pr(2) = Pr(6) = $\frac{1}{12}$, Pr(3) = $\frac{1}{3}$, Pr(4) = $\frac{1}{4}$, Pr(5) = $\frac{1}{6}$

Open-ended – Sample answers

22 Examples of solutions:

(a) A = getting a multiple of 4

(b) B = getting an odd number

(c) C = not getting a composite number

23 (a) No, the probability of rolling any double number, whether it is a double 6, double 3 or double 1, is the same, $\frac{1}{36}$.

(b) Students' own answers

Exercise 8.6 (p. 493)

1 (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$

(e) $\frac{1}{2}$ (f) $\frac{5}{8}$ (g) $\frac{1}{2}$ (h) $\frac{5}{8}$

2 (a) Derek, Ahn and Ngaio

(b) Ahn, Ngaio, Samantha and Zarko

(c) Derek

(d) Maha, Erica, Phillipa, Derek, Ahn and Ngaio

3 (a) $\frac{4}{5}$ (b) $\frac{1}{2}$ (c) $\frac{7}{10}$

4 (a) $\frac{7}{9}$ (b) $\frac{5}{9}$ (c) $\frac{2}{3}$

5 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

6 (a) $\frac{1}{15}$ (b) $\frac{8}{15}$ (c) $\frac{3}{5}$ (d) $\frac{7}{30}$ (e) $\frac{1}{3}$ (f) $\frac{7}{30}$

7 B

8 (a) (i) No (ii) Yes (iii) No

(iv) No (v) No (vi) No

(b) $Pr(A) = \frac{1}{4}$ $Pr(B) = \frac{3}{10}$ $Pr(C) = \frac{1}{4}$ $Pr(D) = \frac{3}{10}$

9 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{7}{26}$ (e) $\frac{3}{13}$ (f) $\frac{3}{52}$

(g) $\frac{15}{52}$ (h) $\frac{12}{13}$ (i) $\frac{3}{4}$ (j) $\frac{11}{13}$ (k) $\frac{4}{13}$ (l) $\frac{15}{26}$

10 (a) $\frac{2}{13}$ (b) $\frac{5}{26}$ (c) $\frac{4}{13}$

(d) You can write out the sample space or use Pr(either) = Pr(left) + Pr(right) – Pr(both)

(e) $\frac{1}{26}$ (f) $\frac{11}{13}$ (g) $\frac{21}{26}$ (h) $\frac{9}{13}$

11 (a)

		red die					
		1	2	3	4	5	6
blue die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

(b) (i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{5}{36}$ (iv) $\frac{5}{18}$

(c) (i) $\frac{11}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{7}{18}$ (iv) $\frac{5}{18}$

(v) $\frac{7}{18}$ (vi) $\frac{5}{12}$

(d) Yes, but C and D are the only mutually exclusive pair

Open-ended – Sample answers

12 Students' own answers

13 Students will provide their own answers

Exercise 8.7 (p. 499)

1 (a) $\frac{1}{5}$ (b) $\frac{1}{25}$ (c) $\frac{3}{5}$ (d) $\frac{21}{25}$

2 (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{5}{12}$

3 (a)

A1A2	A1A3	A1A4	A1B1	A1B2
A2A1	A2A3	A2A4	A2B1	A2B2
A3A1	A3A2	A3A4	A3B1	A3B2
A4A1	A4A2	A4A3	A4B1	A4B2
B1A1	B1A2	B1A3	B1A4	B1B2
B2A1	B2A2	B2A3	B2A4	B2B1

(b) (i) $\frac{4}{9}$ (ii) $\frac{1}{9}$ (iii) $\frac{4}{9}$

4 (a) Prawn, Lamb; Prawn, Fish; Prawn, Beef; Prawn, Pork; Prawn, Poultry; Oysters, Lamb; Oysters, Fish; Oysters, Beef; Oysters, Pork; Oysters, Poultry; Pâté, Lamb; Pâté, Fish; Pâté, Beef; Pâté, Pork, Pâté, Poultry; Satay, Lamb; Satay, Fish; Satay, Beef; Satay, Pork; Satay, Poultry

(b) (i) $\frac{1}{20}$ (ii) $\frac{2}{5}$ (c) $\frac{3}{5}$

5 (a) A (b) A (c) D

6 (a) FG, FM, FB, FD, AG, AM, AB, AD, IG, IM, IB, ID

(b) (i) $\frac{1}{12}$ (ii) $\frac{5}{12}$ (iii) $\frac{2}{3}$ (iv) $\frac{1}{2}$ (v) $\frac{1}{2}$

(c) $\frac{1}{3}$

7 (a) 12

(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$

(c) 3

8 (a) (i) $\frac{1}{15}$ (ii) $\frac{4}{15}$ (iii) $\frac{4}{5}$ (iv) $\frac{2}{5}$

(b) (i) $\frac{1}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{2}{3}$ (iv) $\frac{4}{9}$

9 (a) H, T (b) 2 (c) HH, HT, TH, TT (d) 4

(e) HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(f) 8

(g) Yes. To find the number of events for a combination you multiply together the number possible for each event within the combination.

10 (a) $3^8 = 6561$ (b) $\frac{1}{6561}$

(c) There is only one way it can happen: GGGGGGGG.

(d) 'What is the probability that all the faces will be blue?' and 'What is the probability that all the faces will be red?'

(e) The complementary question to the statement in (b) would actually be: 'What is the probability that not all of the faces are green?'

(b) $\frac{1}{6}$ (c) $\frac{5}{12}$

Open-ended – Sample answers

11 Students' own answers.

12 Students will provide their own answers.

Challenge 8 (p. 502)

1 correct; correct; correct; incorrect.

2 (a) 2, 4, 6, 6, 7 (other answers may be possible)

(b) 11

3 The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

3, 6, 12, 24, 48 all have factors of 3; the other five factors do not have a factor of 3. Therefore, the probability of not having factor of 3 is $\frac{1}{2}$.

4

+	2	3	5	7	11	13
2	4	5	7	9	13	15
3	5	6	8	10	14	16
5	7	8	10	12	16	18
7	9	10	12	14	18	20
11	13	14	16	18	22	24
13	15	16	18	20	24	26

(a) $\frac{13}{18}$

(b) $\frac{5}{6}$, except when the numbers are the same, one number is always larger.

5 9 black, 6 white, 3 green; or 12 black, 4 white, 2 green; or 15 black, 2 white, 1 green

6 (a) $\frac{3}{20} \times \frac{3}{20} = \frac{9}{400} = 0.022\,5$

(b) $\frac{17}{20} \times \frac{2}{5} = \frac{17}{50} = 0.34$

7 (a) At first junction he has three choices, one of which leads to A. At the second junction he has two choices, one of which leads to A. The probability that he ends up at $A = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

(b) Probability that he ends up at $B = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{11}{18}$.

(c) Probability that he ends up at $C = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

(d) Probability that he ends up at $D = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Chapter review 8 (p. 503)

1 (a) continuous (b) discrete

2 For every group except sponsors the existing jumper has the greatest level of support. If the sponsors are more important than the other groups, then a decision to go with new design #1 may be justified, but the sponsors are not really overwhelming in their support for this. In all likelihood the current design should be retained.

3 (a) B (b) A

4 (a) (i) 75.13 (ii) 78 (iii) negative skew

(b) (i) 251.83 (ii) 251 (iii) positive skew

5 (a) The sales of the Falcon have fallen considerably. In 2004 there were approximately 64 000 sold, and by 2010 this had fallen to approximately 30 000. So, the number sold had more than halved. In this same time the sales of the Ford Territory have fluctuated but have usually been in the 10 000–15 000 range.

(b) The sales of the Falcon fell initially before seeing a resurgence in 2003, but have then fallen steadily, although up to the end of 2010 the number seeemed to have become relatively stable. The estimate for 2011 sees another fall. In the same time the sales of the Falcon Ute rose steadily to a peak in 2004 and then fell at about the same slight rate as they rose.

(c) Falcon: 65 000; Falcon Ute: 20 000; Territory: 13 000

(d) 31% (will depend on estimates)

(e) 20% (will depend on estimates)

(f) Falcon: 30 000; Falcon Ute: 10 000; Territory: 11 000; 33%; 37%

(g) The percentage comparisions for both the Falcon Ute and the Territory have increased. The Territory has almost doubled its comparative value.

(h) The total number has fallen enormously. The peak year was 2004 with approximately 98 000 vehicles and the worst year was either 2009 or 2010—approximately 51 000 vehicles each. The predictions for 2011 are worse. The total number of vehicles sold in these categories has roughly halved.

6 $\frac{10}{13}$

7 (a) $\frac{3}{20}$ (b) $\frac{1}{4}$

8 (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$ (e) $\frac{7}{8}$

9 (a) 36

(b) (i) $\frac{1}{9}$ (ii) $\frac{11}{12}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{4}$

10 **(a)** Coast Wattle: 562; Bidgee Widgee: 2250; Karkalla: 1312; White Correa: 750; Running Postman: 1125; Moonah: 375

(b) It is probably not reasonable to apply this across the entire reserve because different areas could support different vegetation types. To improve reliability you should take a few more samples from widely spaced areas of the reserve.

11

(a)

Male	Stem	Female
	12	9
3	13	
5	14	9
2	15	0 1 3 3 4 5 6 7 8 9
9 6 5 3 3	16	0 3 4 4 5 5 6 6 8 9
8 8 8 6 5 4 3 1	17	0 0 0 1 1 2 3
2 0	18	0
8 1	19	

(b) The arm span for boys is longer than for girls.

(c) Not in this case, because the longest three arm spans are all boys.

(d) No, this does not mean there are more girls than boys; it is just the way this sample was constructed.

(e) You would expect the values to be lower for Year 7 students and higher for Year 11 students. For Year 11 we would probably expect that the boys would have larger arm spans than the girls. Things are not so clear for year 7 students because it will depend on when the students undergo their growth spurt.

12 The spread of the population of New Zealand shows a bump around the ages 35–54. This is where the so-called 'baby boomers' can be found in the graph. The rest of the graph shows that the population is relatively stable until about the age of 65, when things start to decline fairly rapidly. This is the age when the number of deaths starts to increase. By the time we get to the late 70s or 80s there are only a few people left.

13 **(a)** **(i)** $\frac{1}{29}$ **(ii)** $\frac{14}{29}$ **(iii)** $\frac{28}{29}$ **(iv)** $\frac{10}{29}$

(b) **(i)** 0.03, 3% **(ii)** 0.48, 48%

(iii) 0.97, 97% **(iv)** 0.34, 34%

14 **(a)** $\frac{1}{36}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$

15 **(a)** Yes, the question is fairly transparent in what is sought for the answer.

(b) What sort of TV shows should youth detainees be allowed to watch? The problem with this question is that it is not just a Yes/No answer. The results would probably not be the same.

(c) The sample size is probably OK, but the method by which it has been obtained is not.

(d) The broader population could only be the readers of the *Herald–Sun*.

(e) The sample would need to be more representative of the population, whatever that was determined to be.

16 **(a)**

		green die				
		1	2	3	4	5
blue die	1	1, 1	2, 1	3, 1	4, 1	5, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5

(b) **(i)** $\frac{2}{25}$ **(ii)** $\frac{19}{25}$ **(iii)** $\frac{2}{5}$ **(iv)** $\frac{21}{25}$

(v) $\frac{19}{25}$ **(vi)** $\frac{21}{25}$ **(vii)** $\frac{4}{25}$ **(viii)** $\frac{4}{5}$

(c) No—Pr(5 on green) + Pr(3 on blue) ≠ Pr(5 on green or 3 on blue).

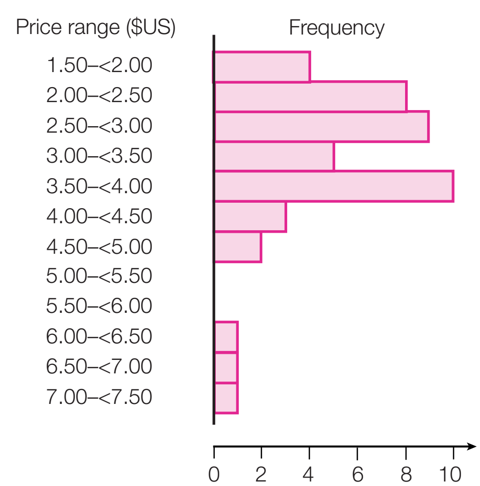
17 **(a)** Uruguay, \$1.74

(b) Norway, \$7.20

(c) \$5.46

(d) Only 50 cents or \$1 would be sensible intervals, and 50 cents gives 12 intervals, which is just a bit bigger than we normally like. However, there are lots of prices in the \$2–\$4 range, so this helps separate those prices.

(e)



(f) Students to provide their own answers.

(g) Students to provide their own answers.

(h) Students to provide their own answers.

(i) A sample of 10 would be expected to do a better job, but this may not always be the case with individuals results.

NAPLAN practice 8

1 C

2 **(a)** discrete **(b)** continuous

(c) continuous **(d)** discrete

3 C **4** B **5** A **6** B

7 D

Chapter 9

Recall 9 (p. 512)

1 (a) (i) 4 (ii) 5 (iii) 10

(b) (i) 10 (ii) 18 (iii) 0 (iv) 0

2 (a) (i) $16 (ii) $75

(b) (i) 4.6 m (ii) 3.2 m

3 (a) $3x^2 + 12x$ (b) $x^2 + 7x + 10$

(c) $x^2 - 4x - 21$ (d) $x^2 - 25$

4 (a) $x(x + 2)$ (b) $(x - 4)(x + 4)$

(c) $(x + 6)^2$

5 (a) $(x + 2)(x + 5)$ (b) $(x - 2)(x - 3)$

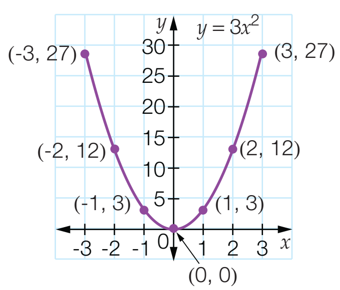
(c) $(x - 14)(x + 1)$ (d) $(x - 7)(x - 4)$

6 (a) $x = \frac{2}{3}$ (b) $x = \frac{13}{5}$ (c) $x = -1$

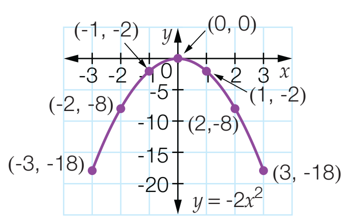
Exercise 9.1 (p. 524)

1 (a), (b) and (e) are quadratic relationships.

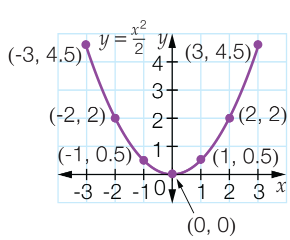
2 (a)



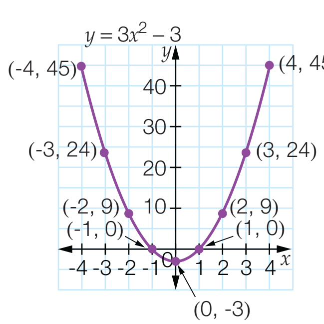
(b)



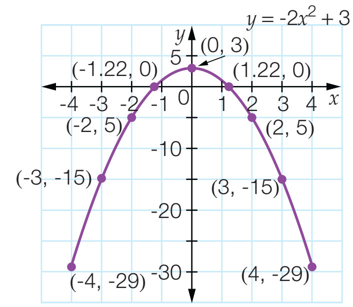
(c)



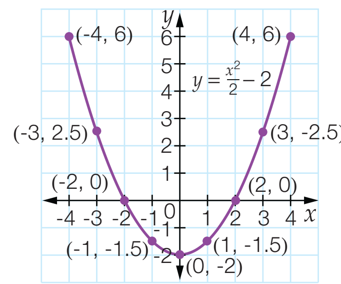
3 (a)



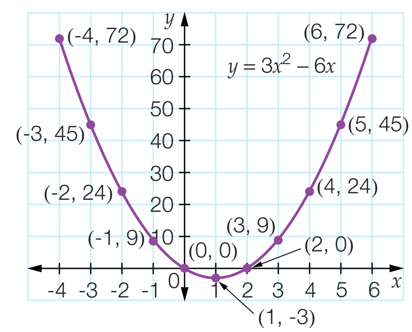
(b)



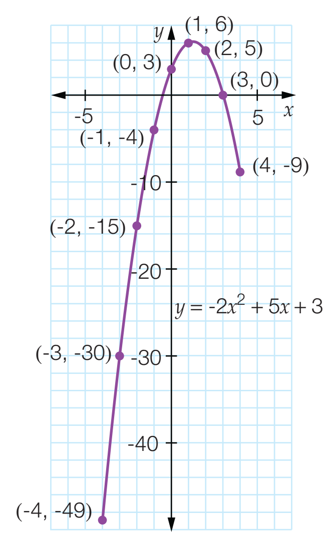
(c)



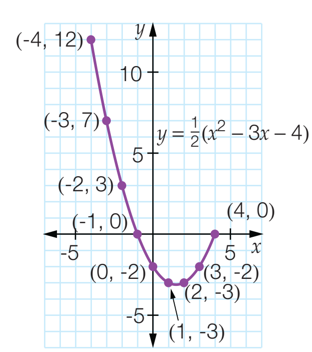
(d)



(e)



(f)



4 (a) (i) (1, 4) (ii) maximum (iii) $x = 1$

(iv) (0, 3) (v) (-1, 0) and (3, 0)

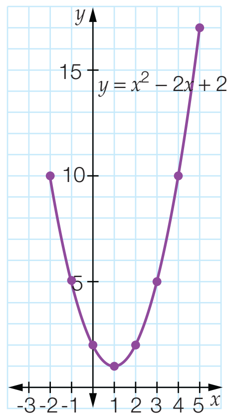
(b) **(i)** $(-\frac{3}{2}, -1)$ **(ii)** minimum **(iii)** $x = -\frac{3}{2}$

(iv) (0, 0) **(v)** (-3, 0) and (0, 0)

5 **(a)** **(i)**

x	-2	-1	0	1	2	3	4	5
y	10	5	2	1	2	5	10	17

(ii)



(iii) (1, 1) **(iv)** minimum turning point

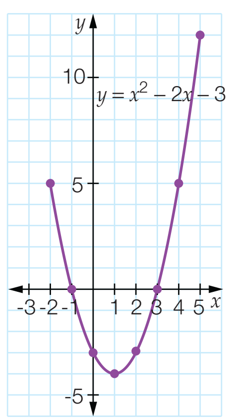
(v) $x = 1$ **(vi)** (0, 2)

(vii) no x-intercepts **(viii)** $x = -1$ or $x = 3$

(b) **(i)**

x	-2	-1	0	1	2	3	4	5
y	5	0	-3	-4	-3	0	5	12

(ii)



(iii) (1,-4) **(iv)** minimum turning point

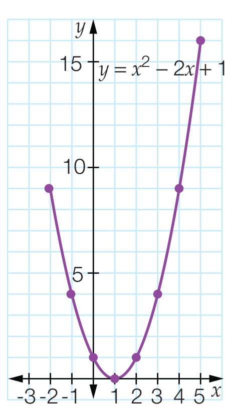
(v) x = 1 **(vi)** (0, -3)

(vii) (-1, 0) and (3, 0) **(viii)** $x = -2$ or $x = 4$

(c) **(i)**

x	-2	-1	0	1	2	3	4	5
y	9	4	1	0	1	4	9	16

(ii)



(iii) (1, 0) **(iv)** minimum turning point

(v) $x = 1$ **(vi)** (0, 1)

(vii) (1, 0) **(viii)** $x = -1.2$ or $x = 3.2$

6 **(a)** B **(b)** D

7 **(a)** C **(b)** C **(c)** A

8 **(a)** Both graphs concave up. If you move the graph of $y = 3x^2$ down 3 units, you get the graph of $y = 3x^2 - 3$.

(b) Both graphs concave down. If you move the graph of $y = -2x^2$ up 3 units, you get the graph of $y = -2x^2 + 3$.

(c) Both graphs concave up. If you move the graph of $y = \frac{x^2}{2}$ down 2 units you get the graph of $y = \frac{x^2}{2} - 2$.

9 **(a)** Both graphs concave up. The graph of $y = 3x^2 - 6x$ has the same dilation as $y = 3x^2$, but is moved to the right and down compared to $y = 3x^2$.

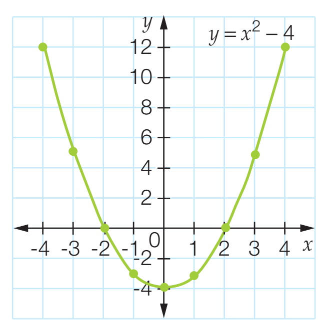
(b) Both graphs concave down. The graph of $y = -2x^2 + 5x + 3$ has the same dilation as $y = -2x^2$, but is moved to the right and upwards compared to $y = -2x^2$.

(c) Both graphs concave up. The graph of $y = \frac{x^2}{2} - \frac{3x}{2} - 2$ has the same dilation as $y = \frac{x^2}{2}$, but is moved to the right and down compared to $y = \frac{x^2}{2}$.

10 **(a)**

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12
(x, y)	(-4, 12)	(-3, 5)	(2, 0)	(-1, -3)	(0, -4)	(1, -3)	(2, 0)	(3, 5)	(4, 12)

(b)



(c) (0, -4) **(d)** minimum **(e)** $x = 0$

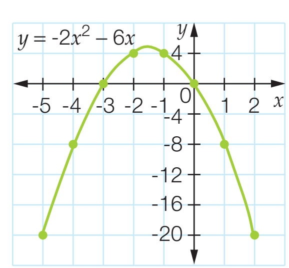
(f) (0, -4) **(g)** (-2, 0) and (2, 0) **(h)** 8.3

(i) -1.4; 1.4 **(j)** $x = -3$ or 3

11 **(a)**

x	-5	-4	-3	-2	-1	0	1	2
y	-20	-8	0	4	4	0	-8	-20

(b)



(c) $(-1\frac{1}{2}, 4\frac{1}{2})$ maximum turning point

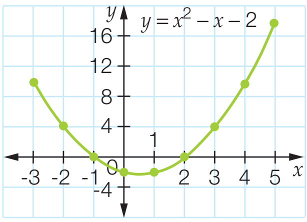
(d) (-3, 0); (0, 0) **(e)** -2; -1 **(f)** 4; $x = -2, -1$

(g) $x = -4, 1$ **(h)** $x = -3, 0$

12 (a)

x	-3	-2	-1	0	1	2	3	4	5
y	10	4	0	-2	-2	0	4	10	18

(b)



(c) $(\frac{1}{2}, -2\frac{1}{4})$; minimum turning point

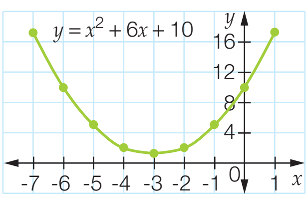
(d) (-1, 0); (2, 0); (0, -2)

(e) **(i)** $x = -1, 2$ **(ii)** $x = -2, 3$ **(iii)** $x \approx -1.6, 2.6$

(iv) $x = 0.5$ **(v)** no real x-values

(vi) $x \approx -1.4, 2.4$

13 (a)

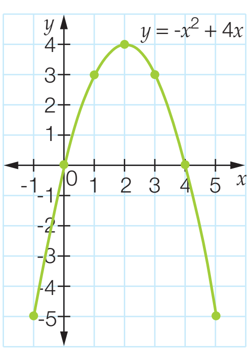


(b) (-3, 1) **(c)** no x-intercepts, (0, 10)

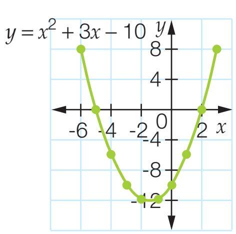
(d) **(i)** $x = -3$ **(ii)** $x = -6, 0$

(iii) no real x-values **(iv)** $x \approx -6.7, 0.7$

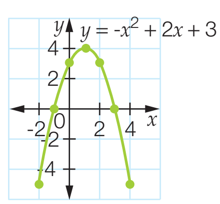
14 (a) **(i)**



(ii)



(iii)



(b) **(i)** $x = 0, 4$ **(ii)** $x = -5, 2$ **(iii)** $x = 1$

(iv) $x = 2$ **(v)** $x = -6, 3$ **(vi)** $x = -1, 3$

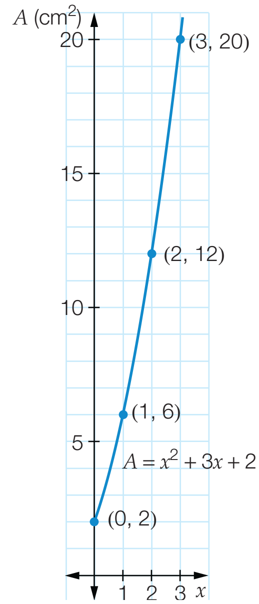
(vii) $x \approx 0.6, 3.4$ **(viii)** $x \approx -5.5, 2.5$ **(ix)** $x \approx -1.4, 3.4$

15 (a) $A = x^2 + 3x + 2$

(b)

x	0	1	2	3
A	2	6	12	20

(c)



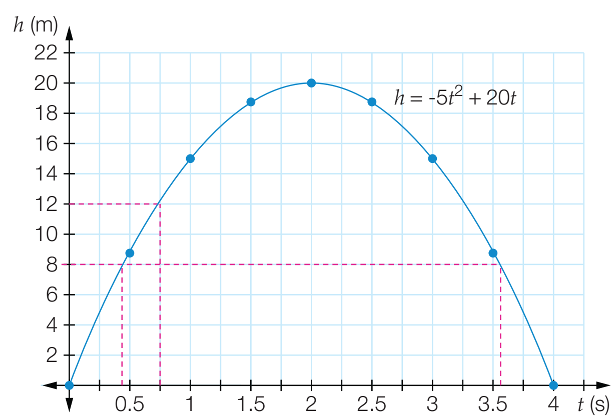
(d) $x = 1.4$. This should be the only value you obtain, because it is unrealistic for x to be negative.

(e) length = 3.4 cm, width = 2.4 cm

16 (a)

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	0	8.75	15	18.75	20	18.75	15	8.75	0

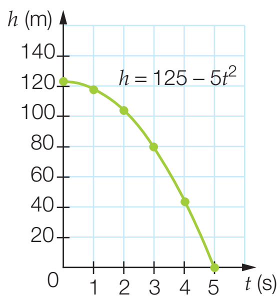
(b)



(c) 20 m **(d)** 2 s **(e)** ≈ 0.75 s **(f)** 4 s **(g)** ≈ 3.2 s

17 (a)

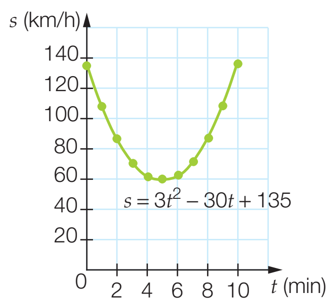
t	0	1	2	3	4	5
h	125	120	105	80	45	0

(b) 

(c) 125 m (d) 5 s (e) 94 m (f) 31 m

(g) 3.2 s (h) 4.5 s (i) 35 m

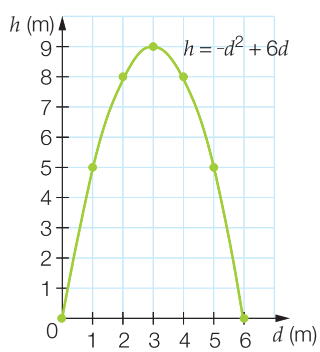
(j) fifth second i.e. between $t = 4$ and $t = 5$ s

18 (a) 

(b) 135 km/h (c) 60 km/h (d) 72 km/h

(e) 72 km/h (f) 2.4 min; 7.6 min

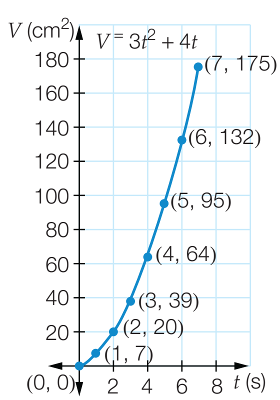
(g) 0.5 min; 9.5 min (h) $0 \le t < 1.35$ and $8.65 < t \le 10$

19 (a) 

(b) 6 m (c) 9 m (d) 5 m

(e) 4 m (f) 5.75 m (g) 3.6 m

(h) (i) Yes (ii) No

20 (a) 

(b) 0 (c) 64 cm^3 (d) 6 s (e) 20 cm^3

(f) $0 \le t \le 6, 0 \le V \le 132$

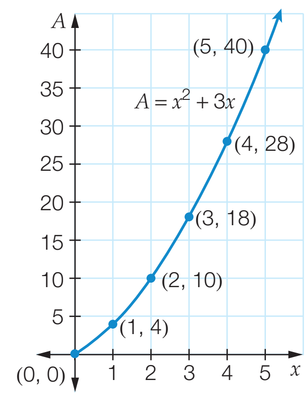
Open-ended – Sample answers

21 (a) Student answers will vary; any expressions such as $x^2 + 2x$, $3x^2 - 6x$, $\frac{1}{2}(x^2 - 3x - 4)$.

(b) Student answers will vary; any expressions such as $y + 4$, $x^2 + y^2$, $4x^3 + 3x^2 + 2x$.

22 Student answers will vary.

23 (a) $A = x^2 + 3x$, where x is the width of the rectangle.

(b) 

(c) If $A = 10$, width = 2 and length = 5.

24 (a) Josh thought that $-x^2 = (-x)^2$.

(b) Josh thought that $2x^2 = (2x)^2$.

Exercise 9.2 (p. 534)

1 (a) $x = 0$ or $x = 1$ (b) $x = 0$ or $x = -8$ (c) $x = 0$ or $x = -5$

(d) $x = 1$ or $x = 4$ (e) $x = 2$ or $x = 6$ (f) $x = 3$ or $x = 9$

(g) $x = -1$ or $x = 4$ (h) $x = -7$ or $x = 3$ (i) $x = -2$ or $x = 5$

(j) $x = -6$ or $x = -4$ (k) $x = -7$ or $x = -2$

(l) $x = -10$ or $x = -3$ (m) $x = 4$

(n) $x = -7$ (o) $x = 9$

2 (a) $x = 0$ or $x = 6$ (b) $x = 0$ or $x = 1$ (c) $x = 0$ or $x = 8$

(d) $x = -7$ or $x = 0$ (e) $x = -5$ or $x = 0$ (f) $x = -1$ or $x = 0$

(g) $x = 0$ or $x = 4$ (h) $x = 0$ or $x = 3$ (i) $x = 0$ or $x = 4$

(j) $x = -9$ or $x = 0$ (k) $x = -9$ or $x = 0$ (l) $x = -\frac{1}{3}$ or $x = 0$

3 (a) $x = -3$ or $x = 3$ (b) $x = -1$ or $x = 1$ (c) $x = -4$ or $x = 4$

(d) $x = -\frac{5}{3}$ or $x = \frac{5}{3}$ (e) $x = -\frac{6}{5}$ or $x = \frac{6}{5}$

(f) $x = -\frac{9}{7}$ or $x = \frac{9}{7}$ (g) $x = -7$ or $x = 7$ (h) $x = -3$ or $x = 3$

(i) $x = -2$ or $x = 2$ (j) $x = -5$ or $x = 5$ (k) $= -\frac{7}{2}$ or $x = \frac{7}{2}$

(l) $x = -\frac{10}{3}$ or $x = \frac{10}{3}$ (m) $x = -4$ or $= 4$

(n) $x = -6$ or $x = 6$ (o) $x = -3$ or $x = 3$

(p) $x = -\sqrt{11}$ or $x = \sqrt{11}$ (q) $x = -\sqrt{29}$ or $x = \sqrt{29}$

(r) $x = -\sqrt{47}$ or $x = \sqrt{47}$

4 (a) $x = 3$ (b) $x = 1$ (c) $x = 4$

(d) $x = -5$ (e) $x = -6$ (f) $x = -9$

(g) $x = 7$ (h) $x = -3$ (i) $x = 2$

(j) $x = 2$ (k) $x = 6$ (l) $x = 5$

(m) $x = 3$ (n) $x = 1$ (o) $x = 7$

5 (a) C (b) B (c) B (d) A

6 Let n be the number, $n^2 + 3n = 7n$; $n^2 - 4n = 0$; $n = 4$

7 $x = -9$ or $x = 1$

8 C 9 9 m 10 (-1, 0), (4, 0)

11 (a) (12, 0), (-12, 0) (b) (0, 0), (-4, 0) (c) (6, 0)

12 (a) $x^2 - 4x - 12$ (b) $x = -2$ or $x = 6$

(c) The product of the solutions equals the constant term.

(d) $x^2 - 4x - 5$ (e) $x = -1$ or $x = 5$

(f) The product of the solutions equals the constant term.

(g) $x = 0$ or $x = 4$ (h) 0

(i) The product of the solutions equals the constant term.

(j) $x^2 - 4x + 3$ (k) $x = 1$ or $x = 3$

(l) The product of the solutions equals the constant term.

(m) $x = 2$

(n) The product of the repeated solutions equals the constant term.

(o) The constant term represents the y-intercept.

(p) All equations have the first two terms, x^2 and $-4x$, in common.

(q) The solution tells you where the graphs intersect the x-axis.

(r) As the constant term increases, the distance between the x-intercepts decreases until the case where there is only one x-intercept.

(s) When $c > 4$, the equation $x^2 - 4x + c = 0$ has no solution. Hence, a graph of $y = x^2 - 4x + c$, $c > 4$ has no x-intercepts.

Open-ended – Sample answers

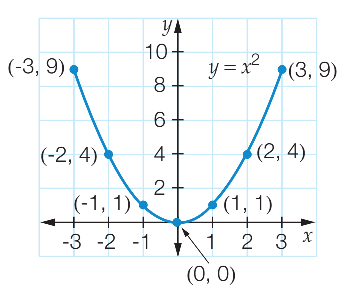
13 Let $x = 0$ or $x = 2$; two possible equations are $y = x^2 - 2x$ and $y = 2x^2 - 4x$.

14 Louise needed to use the Null Factor Law (i.e. she had to first set the equation to equal 0).

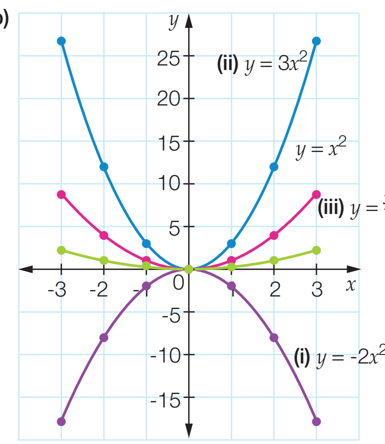
$0 = x^2 - 8x + 16$
$\therefore 0 = (x - 4)^2$
$\therefore x - 4 = 0$
$\therefore x = 4$

Exercise 9.3 (p. 546)

1 (a)



(b)



from left to right

(-3, 27)	(-3, 9)	(-3, 2.25)	(-3, -18)
(-2, 12)	(-2, 4)	(-2, 1)	(-2, -8)
(-1, 3)	(-1, 1)	(-1, 0.25)	(-1, -2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)
(1, 3)	(1, 1)	(1, 0.25)	(1, -2)
(2, 12)	(2, 4)	(2, 1)	(2, -8)
(3, 27)	(3, 9)	(3, 2.25)	(3, -18)

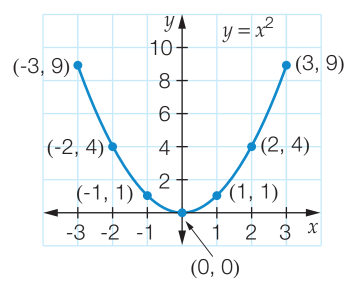
(c) (i) $y = -2x^2$ is reflected in the x-axis and narrower than $y = x^2$

(ii) $y = 3x^2$ is narrower than $y = x^2$

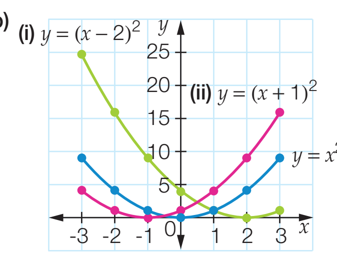
(iii) $y = \frac{1}{4}x^2$ is wider than $y = x^2$

(d) (i) -2 (ii) 3 (iii) $\frac{1}{4}$

2 (a)



(b)



from left to right

(-3, 4)	(-3, 9)	(-3, 25)
(-2, 1)	(-2, 4)	(-2, 16)
(-1, 0)	(-1, 1)	(-1, 9)
(0, 1)	(0, 0)	(0, 4)
(1, 4)	(1, 1)	(1, 1)
(2, 9)	(2, 4)	(2, 0)
(3, 16)	(3, 9)	(3, 1)

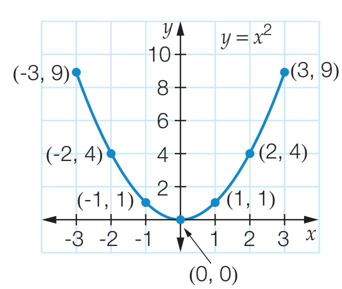
(c) (i) The graph of $y = x^2$ has been translated 2 units to the right to give the graph of $y = (x - 2)^2$.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left to give the graph of $y = (x + 1)^2$.

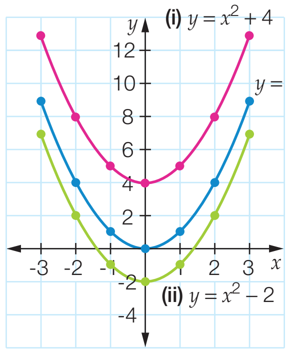
(d) (i) (2, 0). The x-value of the intercept, 2, is the number subtracted from x before squaring in the equation.

(ii) (-1, 0). The x-value of the intercept, -1, is the number subtracted from x before squaring in the equation.

3 (a)



(b)



from left to right

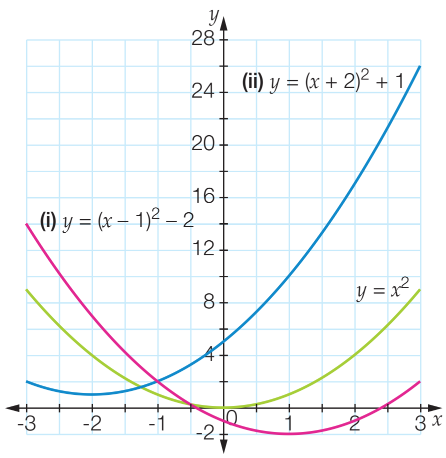
(-3, 13)	(-3, 9)	(-3, 7)
(-2, 8)	(-2, 4)	(-2, 2)
(-1, 5)	(-1, 1)	(-1, -1)
(0, 4)	(0, 0)	(0, -2)
(1, 5)	(1, 1)	(1, -1)
(2, 8)	(2, 4)	(2, 2)
(3, 13)	(3, 9)	(3, 7)

(c) (i) The graph of $y = x^2$ has been translated 4 units upwards to give the graph of $y = x^2 + 4$.

(ii) The graph of $y = x^2$ has been translated 2 units downwards to give the graph of $y = x^2 - 2$.

(d) (i) (0, 4) (ii) (0, -2)

4 (a), (b)



(c) (i) The graph of $y = (x - 1)^2 - 2$ is the graph of $y = x^2$ translated 1 unit right and 2 units down.

(ii) The graph of $y = (x + 2)^2 + 1$ is the graph of $y = x^2$ translated 2 units left and 1 unit up.

(d) (i) $y = (x - 1)^2 - 2$ has a y-intercept of -1.

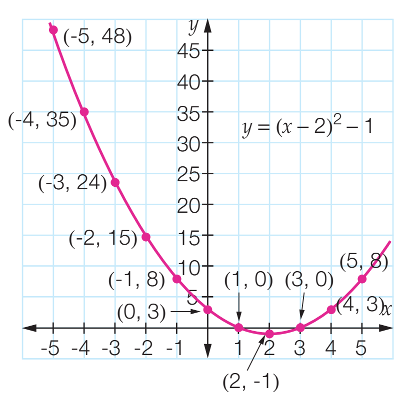
(ii) $y = (x + 2)^2 + 1$ has a y-intercept of 5.

(e) $y = x^2$ has one x-intercept.

(i) $y = (x - 1)^2 - 2$ has two x-intercepts.

(ii) $y = (x + 2)^2 + 1$ has zero x-intercepts.

5 (a)



(b) (2,-1) (c) $h = 2, k = -1$. Yes

(d) Both equations expand to $x^2 - 4x + 3$. The x-coordinates of the x-intercepts are 1 and 3, so the value of the x-coordinate of the turning point is the same as the average of these two values.

(e) $y = -1$, which is the same as the y-coordinate of the turning point.

(f) $a = 1, b = -4, c = 3$. The value of $-\frac{b}{2a}$ is 2, which is the same as the value of the x-coordinate of the turning point.

6 (a) (i) narrower (ii) wider (iii) wider (iv) narrower

(b) (iii) and (iv)

(c) (i) 4 (ii) $\frac{1}{5}$ (iii) 7 (iv) $\frac{2}{3}$

(d) (i) narrower (ii) wider (iii) narrower (iv) wider

7 (a) up 4 units (b) down 7 units

(c) down one unit (d) up 2 units

(e) down 8 units (f) up 5 units

(g) up $\frac{1}{2}$ unit (h) down $\frac{3}{4}$ unit

8 (a) 5 units to the right (b) 4 units to the left

(c) 1 unit to the left (d) 7 units to the right

(e) 6 units to the left (f) 3 units to the right

(g) $\frac{2}{3}$ unit to the right (h) $\frac{3}{2}$ units to the left

9 (a) (i) dilation factor 2, moved right 1 up 3 (ii) (1, 3)

(b) (i) reflected in x-axis, dilation factor 3, moved left 2 down 1 (ii) (-2, -1)

(c) (i) dilation factor 4, moved left 3 down 2 (ii) (-3, -2)

(d) (i) dilation factor $\frac{1}{3}$, moved left 5 up 3 (ii) (-5, 3)

(e) (i) moved left 4, down 3 (ii) (-4, -3)

(f) (i) dilation factor 2, moved left 3 up $\frac{1}{2}$ (ii) $(-3, \frac{1}{2})$

10 (a) C (b) B (c) A (d) D

11 (a) A (b) A (c) D

12 (a) C (b) B (c) A (d) B

13

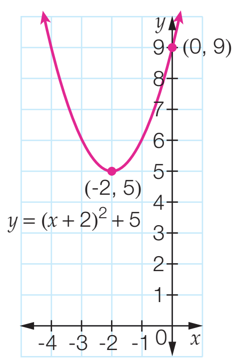
		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x + 1)^2 + 2$	–	–	1 unit left	2 units up
(b)	$y = (x - 3)^2 + 1$	–	–	3 units right	1 unit up
(c)	$y = 5x^2 + 4$	by factor of 5 (narrower)	–	–	4 units up
(d)	$y = -x^2 - 6$	–	invert	–	6 units down
(e)	$y = 2(x + 5)^2 - 3$	by factor of 2 (narrower)	–	5 units left	3 units down
(f)	$y = -(x - 2)^2 + 3$	–	invert	2 units right	3 units up
(g)	$y = -3(x + 6)^2 - 4$	by factor of 3 (narrower)	invert	6 units left	4 units down
(h)	$y = \frac{1}{2}(x - 3)^2 + 1$	by factor of $\frac{1}{2}$ (wider)	–	3 units right	1 unit up
(i)	$y = (x - 4)^2 - 5$	–	–	4 units right	5 units down
(j)	$y = (x + 2)^2 - 7$	–	–	2 units left	7 units down
(k)	$y = 3x^2 - 5$	by factor of 3 (narrower)	–	–	5 units down
(l)	$y = -x^2 - 2$	–	invert	–	2 units down
(m)	$y = 5(x - 6)^2 - 4$	by factor of 5 (narrower)	–	6 units right	4 units down
(n)	$y = -(x - 5)^2 + 2$	–	invert	5 units right	2 units up
(o)	$y = -4(x + 1)^2 - 3$	by factor of 4 (narrower)	invert	1 unit left	3 units down
(p)	$y = \frac{3}{4}(x + 4)^2 + 6$	by factor of $\frac{3}{4}$ (wider)	–	4 units left	6 units up

14 **(b)**, **(c)** and **(e)**

15 C

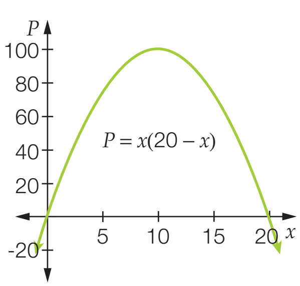
16 **(a)** $y = (x + 2)^2 + 5$ **(b)** $y = 9$

(c)



17 **(a)** $y = 5(x - 3)^2 + 2$ **(b)** $y = \frac{1}{3}(x - 4)^2 - 2$

18 **(a)** $y = 20 - x$ **(b)** $P = x(20 - x)$



(c) $x = 10$ and $y = 10$, $P = 100$

Open-ended – Sample answers

19 $y = 4(x - 2)^2 + 3$. Dilation factor of 4, translation 2 units to the right and 3 units up.

$y = -\frac{1}{2}(x + 3)^2 - 4$. Reflected in the x-axis and a dilation factor of $\frac{1}{2}$, translated 3 units to the left and 4 units down.

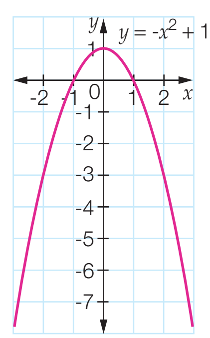
20 $y = (x - 5)^2 + 2$; $y = (x + 4)^2 + 7$; $y = -2(x - 5)^2 - 3$

Half-time 9 (p. 554)

1 **(a)** $y = -x^2 + 1$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

(b)

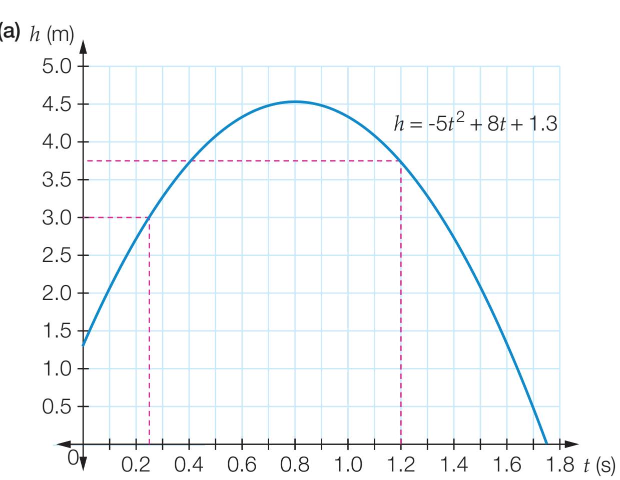


(c) (0, 1) **(d)** (0, 1) **(e)** (-1, 0) and (1, 0)

2 C

3 **(a)** $x = -4$ or $x = 0$ **(b)** $x = 0$ or $x = 3$ **(c)** $x = -2$ or $x = 3$

4 **(a)**



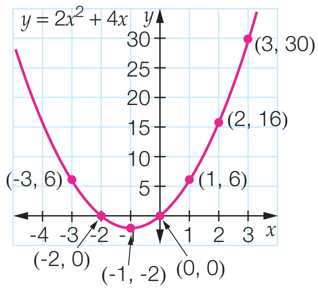
(b) 0.25 s **(c)** 3.7 m

5 (a) $x = -2$ (b) $x = -6$ (c) $x = 9$

6 (a) $y = 2x^2 + 4x$

x	-3	-2	-1	0	1	2	3
y	6	0	-2	0	6	16	30

(b)



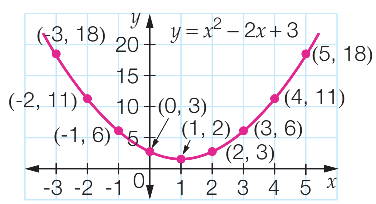
(c) (-1, -2) (d) minimum turning point

(e) x-intercepts (-2, 0) and (0, 0); y-intercept (0, 0)

7 (a) $x = 0$ or $x = 6$ (b) $x = -5$ (c) $x = 0$ or $x = 9$

8 (a) $x = -8$ or $x = 8$ (b) $x = -1$ (c) $x = 0$ or $x = 4$

9 (a)



(b) (1, 2)

(c) y-intercept is (0, 3); there are no x-intercepts

(d) (i) $x = 0$ or $x = 2$ (ii) $x = 1$

(iii) $x = -1$ or $x = 3$ (iv) No solutions

Exercise 9.4 (p. 564)

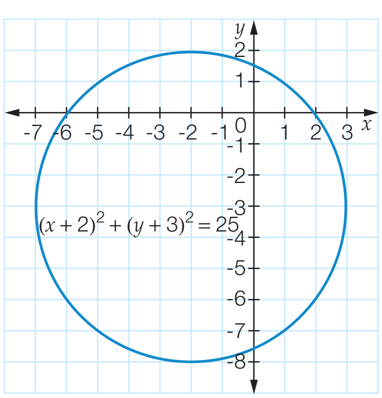
1 (a) centre: (-1, 3); radius = 3; $(x + 1)^2 + (y - 3)^2 = 9$

(b) centre: (1, 2); radius = 4; $(x - 1)^2 + (y - 2)^2 = 16$

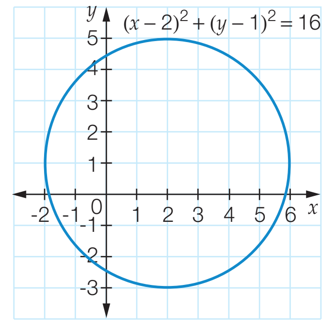
(c) centre (-3, -1); radius = 4; $(x + 3)^2 + (y + 1)^2 = 16$

(d) centre (-3, 0); radius = 5; $(x + 3)^2 + y^2 = 25$

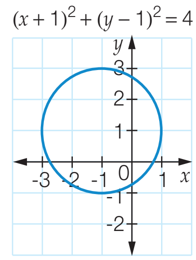
2 (a)



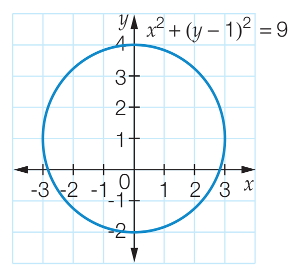
(b)



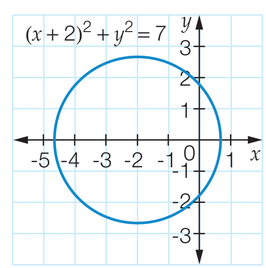
(c)



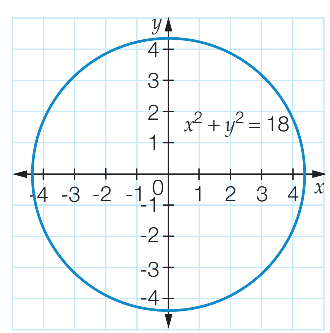
(d)



(e)



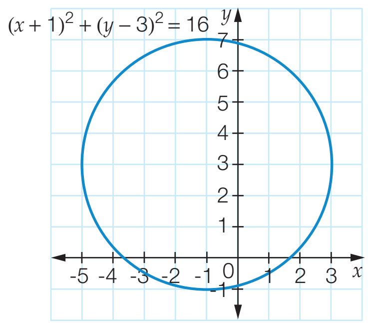
(f)



3 (a) centre = (-1, 3), radius = 4

(b) Points on circle are (3, 3), (-1, 7), (-5, 3), (-1, -1).

(c)



(d) **(i)** inside **(ii)** outside **(iii)** inside

4 B

5 **(a)** $(x-4)^2+(y-3)^2=36$

(b) $(x+6)^2+(y-4)^2=36$ **(c)** $(x-7)^2+y^2=36$

(d) $x^2+(y+5)^2=36$ **(e)** $x^2+y^2=49$

6 D

7 $(x+6)^2+(y-4)^2=16$

8 **(a)** point (3, 4)

$$(x-8)^2+(y+8)^2=169$$
$$(3-8)^2+(4+8)^2=169$$
$$(-5)^2+12^2=169$$
$$25+144=169\text{—true}$$

$$(x+9)^2+(y-9)^2=169$$
$$(3+9)^2+(4-9)^2=169$$
$$12^2+(-5)^2=169$$
$$25+144=169\text{—true}$$

$$x^2+y^2=25$$
$$3^2+4^2=25$$
$$9+16=25\text{—true}$$

Point (-4, -3)

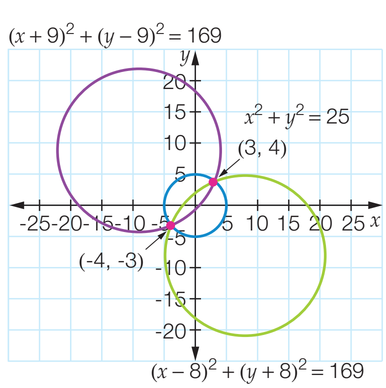
$$(x-8)^2+(y+8)^2=169$$
$$(-4-8)^2+(-3+8)^2=169$$
$$(-12)^2+5^2=169$$
$$144+25=169\text{—true}$$

$$(x+9)^2+(y-9)^2=169$$
$$(-4+9)^2+(-3-9)^2=169$$
$$5^2+(-12)^2=169$$
$$25+144=169\text{—true}$$

$$x^2+y^2=25$$
$$(-4)^2+(-3)^2=25$$
$$16+9=25\text{—true}$$

The three circles intersect at the points (3, 4) and (-4, -3).

(b)



9 **(a)** $(x-2)^2+(y-3)^2=25$ **(b)** $(x-11)^2+(y-4)^2=169$

(c) Testing point (-1, -1) in part **(a)** equation:

$(-1-2)^2+(-1-3)^2=25$—true

Testing point (-1, -1) in part **(b)** equation:

$(-1-11)^2+(-1-4)^2=169$—true

Open-ended – Sample answers

10 **(a)** Student answers will vary. Examples include $x^2+y^2=9$ and $x^2+y^2=100$.

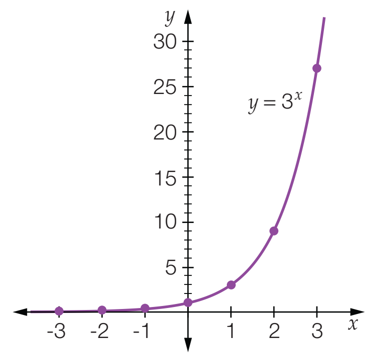
(b) Student answers will vary. Examples include $(x-5)^2+(y-6)^2=9$ and $(x+14)^2+(y+8)^2=9$.

11 Student answers will vary.

Exercise 9.5 (p. 572)

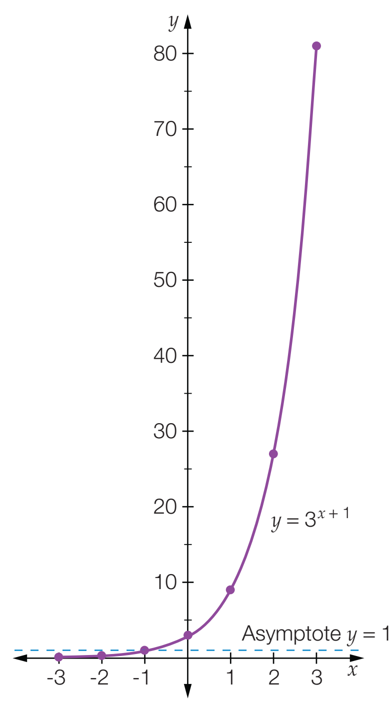
1 **(a)**

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



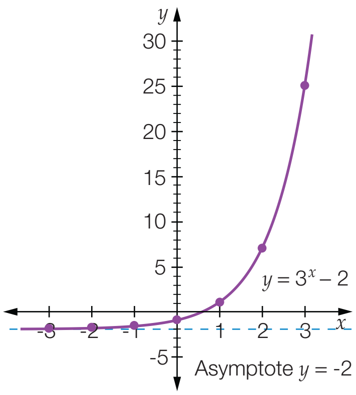
(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81



(c)

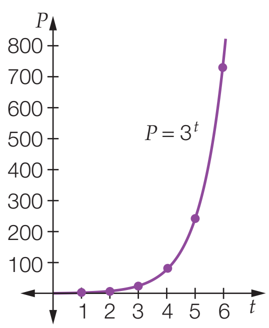
x	-3	-2	-1	0	1	2	3
y	$-\frac{53}{27}$	$-\frac{17}{9}$	$-\frac{5}{3}$	-1	1	7	25



2 (a)

t	0	1	2	3	4	5	6
P	1	3	9	27	81	243	729

(b)



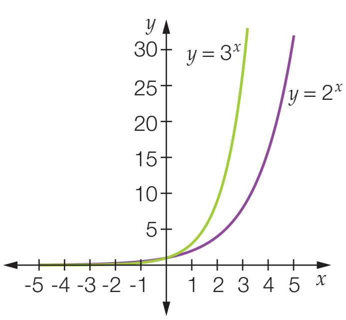
(c) 5 years

(d) 65. This value is meaningless as he never has 65 orchids. The values are not continous in this situation.

(e) 243

3 (a) $y = \dfrac{1}{x+3} - 1$ (b) $y = \dfrac{1}{x-2} - 3$

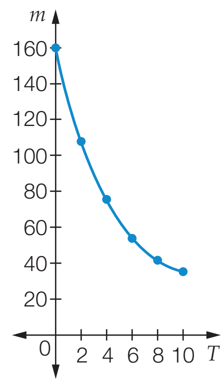
4 (a)



(b) Both graphs have a y-intercept of 1 and $y = 0$ is the equation of the asymptote for both graphs. $y = 3^x$ is steeper than $y = 2^x$.

5 D 6 D

7 (a)

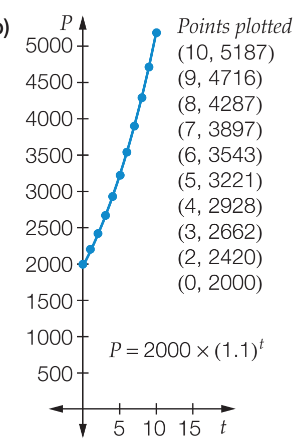


(b) 160° (c) ≈ 91° (d) ≈ 6 min 45 s (e) ≈ 28°

8 (a)

	Beginning of year					
t	2005	2006	2007	2008	2009	2010
Population, P	2000	2200	2420	2662	2928	3221
	Beginning of year					
t	2011	2012	2013	2014	2015	
Population, P	3543	3897	4287	4716	5187	

(b)

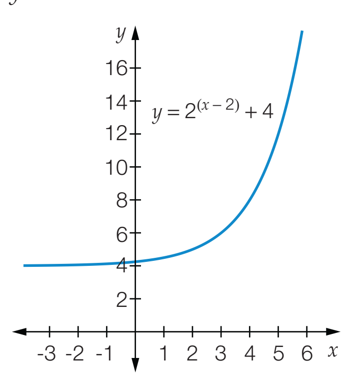


(c) 7.3 years

(d) The increase is happening on the previous year's population; the 10% is compounded annually and it would therefore take less than 10 years for the population to double.

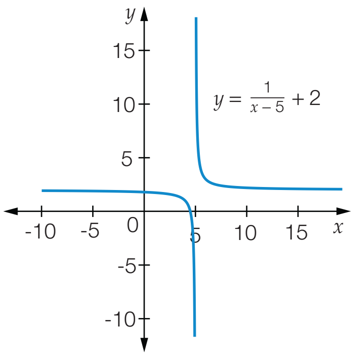
Open-ended – Sample answers

9 (a) $h = 2, k = 4$
$y = 2^{(x-2)} + 4$



(b) $h = 5, k = 2$

$y = \dfrac{1}{x-5} + 2$



Exercise 9.6 (p. 578)

1 (a) not direct proportion (decreasing)

(b) not direct proportion (fluctuating)

(c) not direct proportion (not through origin)

(d) could be direct proportion

2 (a) $b = 4a$ (b) $v = 5w$ (c) $d = \frac{1}{2}c$

(d) $h = \frac{1}{4}g$ (e) $h = 3e$ (f) $q = 1.5p$

(g) $j = 2i^2$ (h) $d = 3a^2$ (i) $n = 4m^3$

(j) $h = 2g^3$ (k) $q = \frac{1}{2}p^2$ (l) $f = \frac{1}{5}b^2$

(m) $s = 3r^3$ (n) $m = \frac{1}{4}p^3$ (o) $u = \frac{1}{2}t^3$

(p) $w = 2\sqrt{v}$ (q) $t = \frac{1}{10}\sqrt{u}$ (r) $l = \frac{1}{3}\sqrt{d}$

3 (b) is the only graph that could be an example of direct proportion

4 (a) $m = kn$ (b) $a = kb^2$ (c) $g = kh^3$

(d) $t = kw$ (e) $N = kp^2$ (f) $y = kx^3$

(g) $V = kr^2$ (h) $L = k\sqrt{d}$

5 (a) B (b) A

6 (a) 3 (b) 0.2 (c) $\frac{1}{4}$ (d) 200

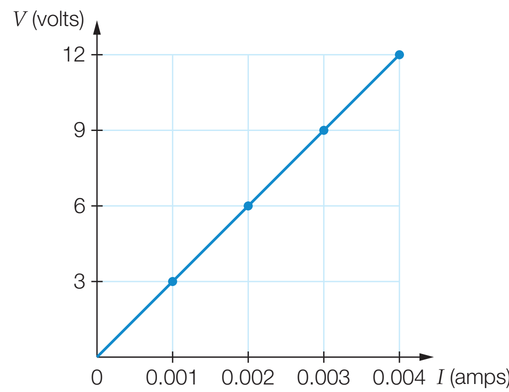
(e) 1 (f) $\frac{1}{6}$ (g) $\frac{1}{20}$ (h) $\frac{1}{35}$

7 (a) $k = 3, q = 11, p = 27$ (b) $k = 7, m = 63, n = 6$

(c) $k = 1, c = 2, d = 64$ (d) $k = 3, x = 4, w = 21$

8 C

9 (a)



(b) 3000 ohms (c) 30 volts (d) 0.015 amps

10 (a) $k = 12.56$ (b) $A = 4\pi r^2$

11 (a) 22.5 km (b) 988 m

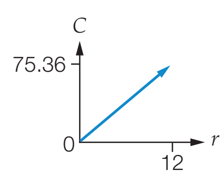
12 (a) $\dfrac{C}{r} = 6.28$

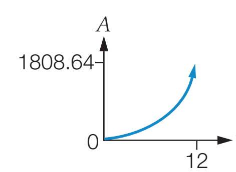
$\dfrac{A}{r} = 25.12, 50.24, 75.36, 100.48, 125.60, 150.72$

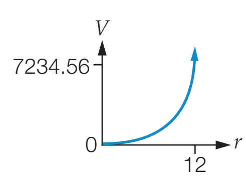
$\dfrac{V}{r} = 16.745, 66.9875, 150.72, 267.95, 418.67, 602.88$

(b) $\dfrac{C}{r} = 6.28$; $\dfrac{A}{r}$ is increasing by a constant value with each pair of values; $\dfrac{V}{r}$ seems to be no relationship

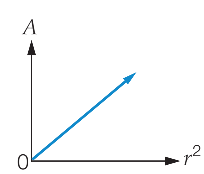
(c)



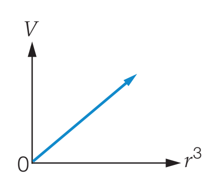




(d) linear relationship



(e) linear relationship



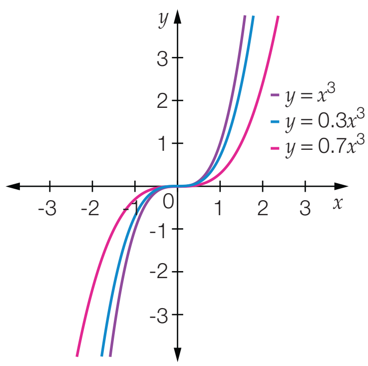
(f) $C \propto r$; $A \propto r^2$; $V \propto r^3$

(g) $k = 6.28$, $C = 2\pi r$; $k = 12.56$, $A = 4\pi r^2$; $k = 4.19$, $V = \frac{4}{3}\pi r^3$

Open-ended – Sample answers

13 (a) $y = 0.3x^3$, $y = 0.7x^3$

(b)



As k decreases from 1 to 0, the graph is dilated horizontally.

Exercise 9.7 (p. 586)

1 (a) could be inverse proportion

(b) not inverse proportion (fluctuates)

(c) could be inverse proportion

(d) could be inverse proportion

2 (a) $b = \frac{40}{a}$ (b) $d = \frac{60}{c}$ (c) $f = \frac{216}{e}$

(d) $h = \frac{144}{g}$ (e) $I = \frac{240}{k^2}$ (f) $n = \frac{144}{m^2}$

3 (a) $p = \frac{k}{q}$ (b) $y = \frac{k}{x^2}$ (c) $a = \frac{k}{\sqrt{b}}$ (d) $t = \frac{k}{w}$

(e) $V = \frac{k}{h^2}$ (f) $f = \frac{k}{g^3}$ (g) $L = \frac{k}{\sqrt{m}}$ (h) $a = \frac{k}{\sqrt{d}}$

4 (a) C (b) D (c) D

5 (a) 7 (b) 6.3 (c) 12 (d) 0.97

(e) 5 (f) 3 (g) $\frac{1}{2}$ (h) $\frac{1}{5}$

(i) $\frac{1}{0.4} = 2.5$ (j) $\frac{1}{1.6} = \frac{5}{8}$

(k) $\frac{1}{0.08} = 12.5$ (l) $\frac{1}{1.25} = 0.8$

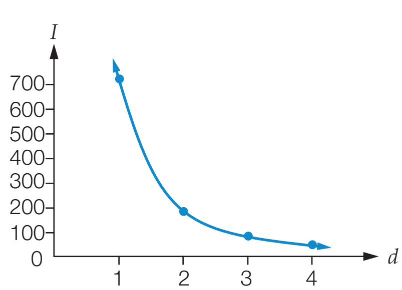
6 (a) $\frac{128}{125}$ (b) 4

7 5 hours

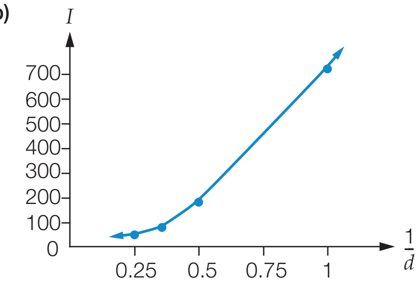
8 2×10^{-8} m

9 20 km

10 (a)



(b)



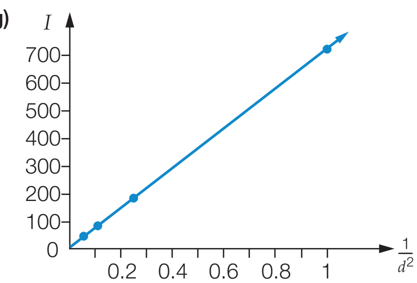
(c) No, because it is not a linear relationship.

(d)

d	1	2	3	4	6	8	12
d^2	1	4	9	16	36	64	144
I	720	180	80	45	20	11.25	5

(e) 720; yes (f) $I = \frac{720}{d^2}$

(g)



It is a linear relationship: $I \propto \frac{1}{d^2}$.

(h) I is reduced by 4 times.

(i) I is reduced by 9 times.

(j) I is increased by 4 times.

Open-ended – Sample answers

11 possible solution $y = \frac{16}{x}$, (1, 16) (2, 8) (4, 4);

$y = \frac{16}{x^2}$, (1, 16) (2, 4) (4, 1); $y = \frac{16}{\sqrt{x}}$, (1, 16), (4, 8) (16, 4)

Challenge 9 (p. 589)

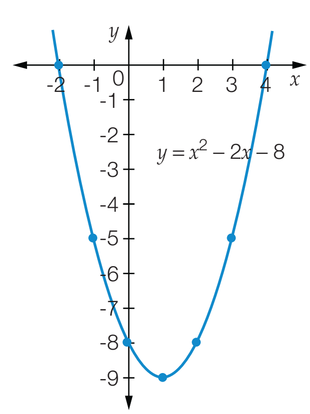
1 A **2** 1028 **3** C **4** C

5 13 **6** B **7** 36 m^2 **8** D

9 B **10** B

Chapter review 9 (p. 590)

1



(a) x-intercepts (-2, 0) and (4, 0); y-intercept (0, -8)

(b) (1, -9) minimum turning point

(c) $x = 1$ (d) $y = -5$ (e) -3 and 5

(f) $x = -3$ or $x = 5$ (g) $x = -2$ or $x = 4$

2 (a) $x = -4$ or $x = 0$ (b) $x = -1$ or $x = 7$ (c) $x = -4$ or $x = 4$

(d) $x = -3$ (e) $x = 0$ or $x = 8$ (f) $x = 6$

3 (a) $x = 0$ or $x = 4$ (b) $x = -\sqrt{6}$ or $x = \sqrt{6}$

(c) No solution (d) $x = -3$ or $x = 3$ (e) $x = -8$ or $x = 4$

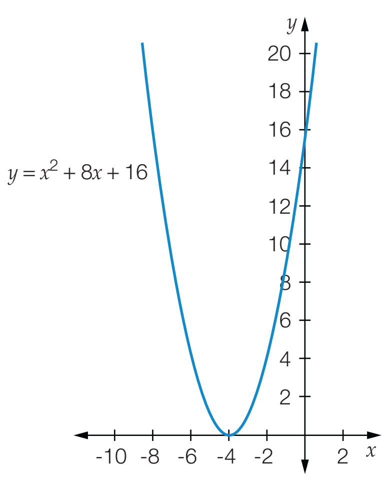
(f) $x = -3$ or $x = 4$

4 (a) A (b) A (c) B

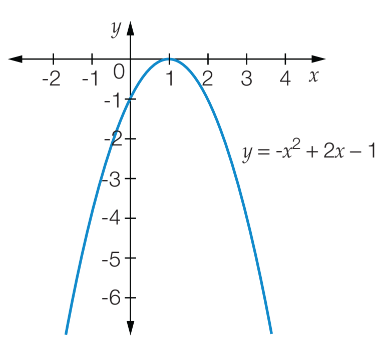
5

		Dilation	Reflection in x-axis	Translation in x direction	Translation in y direction
(a)	$y = (x - 1)^2 + 3$	No dilation	No reflection	1 unit right	3 units up
(b)	$y = -x^2 + 4$	No dilation	Invert	No translation	4 unit up
(c)	$y = -(x + 2)^2 - 1$	No dilation	Invert	2 units left	1 unit down
(d)	$y = 3(x - 4)^2 + 2$	By factor 3 (narrower)	No reflection	4 units right	2 units up
(e)	$y = \frac{1}{2}(x - 1)^2 + 1$	By factor $\frac{1}{2}$ (wider)	No reflection	1 unit right	1 unit up
(f)	$y = -2(x + 2)^2 - 2$	By factor 2 (narrower)	Invert	2 units left	2 units down

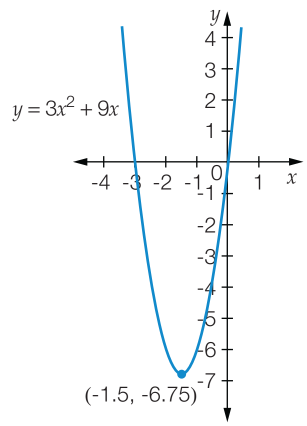
6 (a)



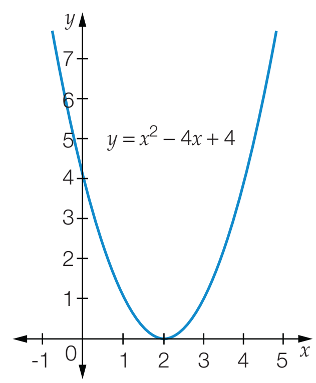
(b)



(c)

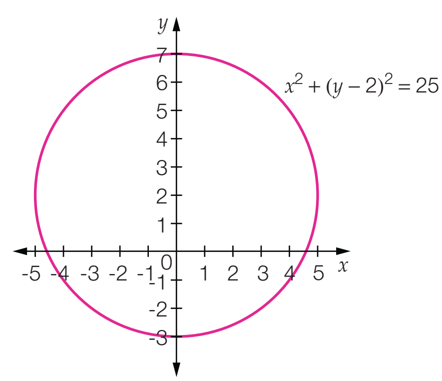


(d)

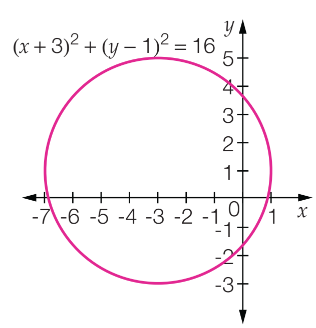


7 (a) A (b) C

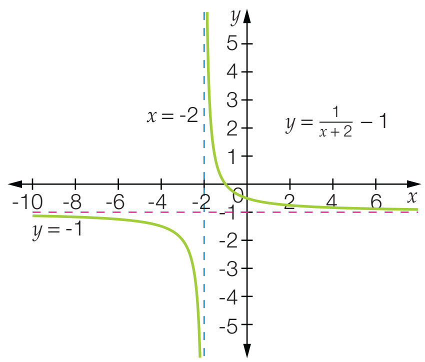
8 (a)



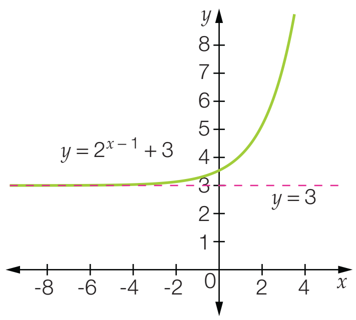
(b)



9 **(a)**



(b)



10 **(a)** $b = 3a$ **(b)** $d = \frac{c}{2}$ **(c)** $h = 3\sqrt{g}$

11 **(a)** A **(b)** B **(c)** D

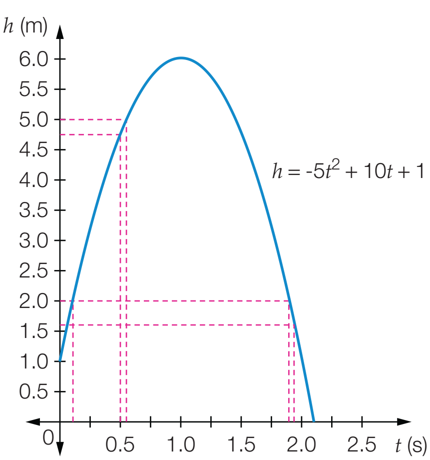
12 **(a)** $k = \frac{12}{j}$ **(b)** $q = \frac{6}{\sqrt{p}}$

13 **(a)** D **(b)** C

14 **(a)**

t	0	0.5	1	1.5	2
h	1	4.75	6	4.75	1

(b)

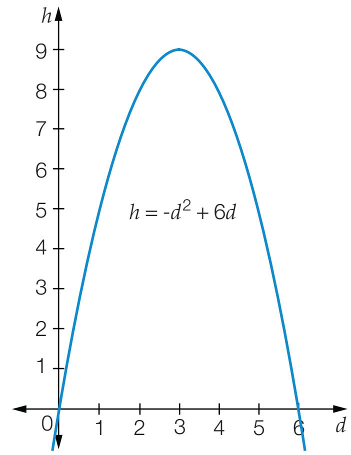


(c) 4.75 m **(d)** ≈ 0.55 s

(e) between 0.1 and 1.9 s

(f) 6 m **(g)** ≈ 1.95 s

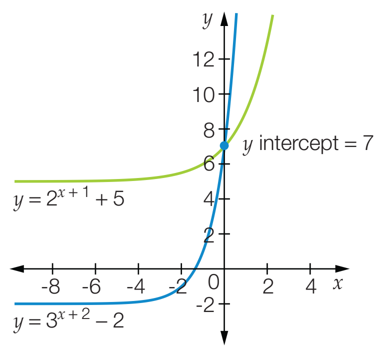
15 **(a)**



(b) 9 m **(c)** 6 m

16 The circle $(x - 4)^2 + (y - 3)^2 = 16$ has been translated 6 units to the right, 2 units upwards and the radius decreased by 1 unit compared with the circle $(x + 2)^2 + (y - 1)^2 = 25$.

17 **(a)**



(b) $y = 2^{x+1} + 5$ horizontal asymptote $y = 5$; $y = 3^{x+2} - 2$ horizontal asymptote $y = -2$

18 **(a)** Yes, $y = \frac{1}{10}x^3$

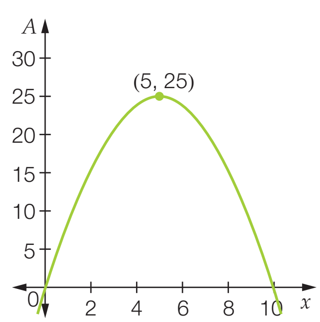
(b)

x	1	2	3	4	5	6
y	0.1	0.8	2.7	6.4	12.5	21.6

19 **(a)** Width = $10 - x$

(b) $A = \text{length} \times \text{width} = x(10 - x) = 10x - x^2 = -x^2 + 10x$

(c)



(d) 5 m × 5 m **(e)** 25 m^2

20 length = 8 cm, width = 8 cm

21 25 and 25

22 $y = (x - 3)^2 + 5$

23 $(x - 3)^2 + y^2 = 81$

24 (a) When s is doubled, v increases by $\sqrt{2}$.

(b) When v is reduced by $\frac{1}{4}$, s is reduced by $\frac{7}{16}$.

NAPLAN practice 9

1 $x = 3$ **2** D **3** C **4** D

5 $y = x^2 - 3x + 2$

x	-3	-2	-1	0	1	2	3
y	20	12	6	2	0	0	2

6 D

Mixed review D (p. 595)

1 $6166.51 - 6146.28 = \$20.23$

2 (a) $6(a+2)(a-2)$ **(b)** $3(a+1)^2$ **(c)** $(x+5)(x-7)$

3 (a) $104\ \text{cm}^2$ **(b)** $37.5\ \text{m}^2$ **(c)** 103.5cm^2

4 (a) $a = -6$ **(b)** $a = -\frac{15}{16}$ **(c)** $a = \frac{11}{6}$

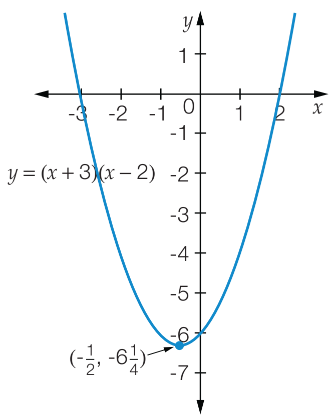
5 (a) 42.7° **(b)** 23.9° **(c)** 50.5°

6 (a) 33.36 **(b)** 125.24

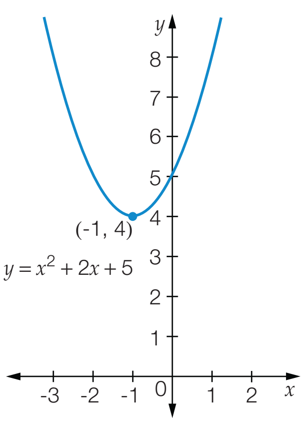
7 (a) $x = 20°$ **(b)** $y = 26°, z = 5°$ **(c)** $w = 20°$

8 (a) $x = -5$ or $x = 5$ **(b)** $x = -\frac{1}{2}$ or $x = 0$ **(c)** $x = 3$

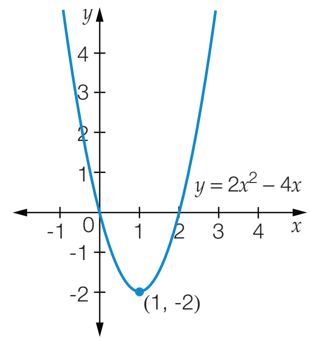
9 (a)



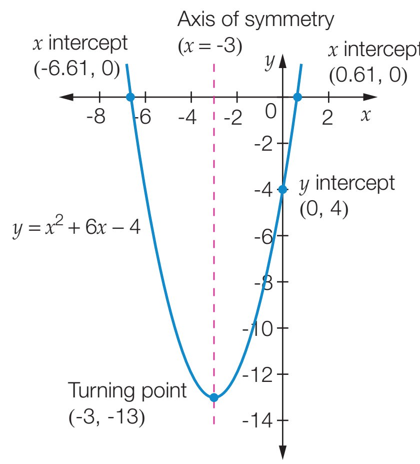
(b)



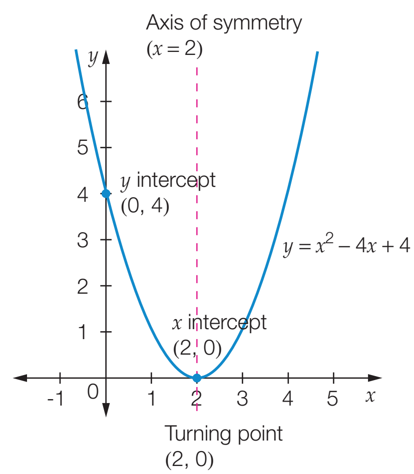
(c)



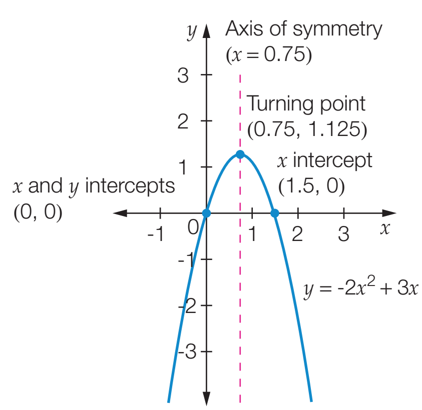
10 (a)



(b)



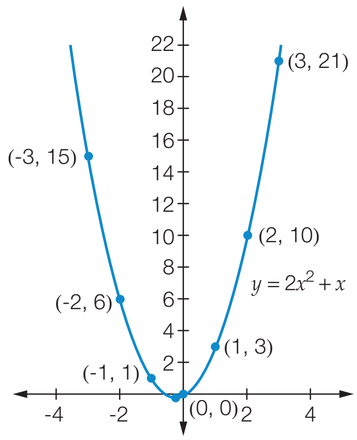
(c)



11 (a) $y = 2x^2 + x$

x	-3	-2	-1	0	1	2	3
y	15	6	1	0	3	10	21

(b)



(c) $(-\frac{1}{4}, -\frac{1}{8})$

(d) x-intercepts $(-\frac{1}{2}, 0)$ and $(0, 0)$; y-intercept $(0, 0)$

12 2.5°

13 (a) $\frac{5}{26}$ (b) $\frac{2}{13}$ (c) $\frac{11}{13}$

14 (a) $x = 5$ (b) $y = (x - 4)(x - 6)$ or $y = x^2 - 10x + 24$

15 length = 12 cm, width = 5 cm

16 (a) 4°

(b) (i) 1.995 m (ii) 140 mm

Glossary and index

adjacent side the side of a right-angled triangle beside a given or required angle (p. 395)

alternate angles angles found between two lines but on opposite sides of a transversal. Alternate angles between parallel lines are equal. (p. 322)

angle of depression the angle between the horizontal and the line of sight to a lower object (p. 422)

angle of elevation the angle between the horizontal and the line of sight to a higher (elevated) object (p. 422)

arc a section of the circumference of a circle (p. 201)

area the amount of surface within a boundary (p. 208)

asymptote a line that a graph approaches but does not cross or touch for particular values of the independent variable (p. 567)

award a document that sets out the minimum working conditions of employment for employees in a certain occupation or industry (p. 22)

axis of symmetry an imaginary line along which a mirror could be placed and the reflection would be indistinguishable from the original (p. 514)

back-to-back histogram two horizontal histograms of similar data, with the same y-axis but plotted in opposite directions on the x-axis (p. 467)

back-to-back stem plot two stem plots, one with its leaves going to the left from the stem and the other with the leaves going to the right, used to compare two sets of (related) data (p. 467)

base the number that is raised to an index or power (p. 127)

bearing indicates the direction of one object from another object or point (p. 424)

binomial an expression with two terms (p. 169)

buying on terms a method of purchase where goods can be taken home before payment, under certain conditions; also known as hire purchase (p. 49)

capacity the amount that a container can hold; usually refers to gases and liquids. Measured in units based on the litre (p. 225)

categorical data non-numerical data that is simply divided into categories (p. 440)

centre of enlargement a point of reference, (called O) from which the distance to each of the points on the original shape is measured and multiplied by the scale factor to determine the distance from O to the matching corners of the enlarged or reduced shape (p. 350)

certain associated with a probability of 1 (p. 479)

chance the likelihood of an event (p. 479)

cheque a form on which an order is written to a bank to pay the stated sum from the drawer's account (p. 49)

circle a plane curve with the equation $x^2 + y^2 = r^2$ (p. 560)

circumference the perimeter of a circle (p. 200)

coefficient the number factor in a term (p. 537)

co-interior angles angles found between two lines and on the same side of a transversal. Co-interior angles between parallel lines are supplementary. (p. 322)

commission payment for work done based on a percentage of sales made or business gained (p. 23)

compass bearing a direction measured as an angle either side of north or south (p. 424)

complementary outcomes two outcomes of an event (often success or failure) where the sum of the probabilities of each outcome add to 1 (p. 480)

congruent exactly the same in size and shape (p. 328)

congruent triangles triangles that are exactly the same shape and size. There are four tests that can be used to determine if two triangles are congruent: AAA (Angle, Angle, Angle), SAS (Side, Angle, Side), AAS (Angle, Angle, Side) and RHS (Right angle, Hypotenuse, Side). (p. 328)

constant of proportionality the constant value in a relationship that is a direct proportion (p. 575)

continuous data numerical data for which the possible values are all of the real numbers within a particular range; hence, there are an infinite number of possible outcomes that cannot be counted (p. 439)

converse a statement that reverses a statement that has been made (p. 81)

coordinates numbers used to describe a position on a Cartesian plane (p. 261)

corresponding angles angles found either both above or both below two lines and on the same side of the transversal. Corresponding angles on parallel lines are equal. (p. 322)

cosine the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for a particular angle in a right-angled triangle (p. 400)

credit card a card issued by a bank or financial institution that obtains the user credit from the institution in order to pay a business (p. 49)

cross-section a slice through a solid so that the shape is similar to that of the base. In a prism, the cross-section is uniform (identical to the base). (p. 217)

debit card similar to a credit card, but the money is deducted from the account within a few days and no interest is charged (p. 49)

deferred payment a payment option for the HECS-HELP, where payment can be put off until the borrower is earning over a certain amount (p. 49)

degree a unit used to measure the size of angles; a full turn is equal to 360° (p. 395)

dependent variable a variable whose value is dependent on the value of another variable to which it is connected by a rule (p. 247)

difference of two squares a technique for factorising quadratics: $a^2 - b^2 = (a - b)(a + b)$ (p. 178)

dilation a change in the size of a graph or shape (p. 537)

dilation factor the number that can be used to compare an original figure to its dilated size (p. 537)

direct proportion a relationship between two variables such that one variable increases as the other increases, and when one variable is 0, the other variable is also 0 (p. 575)

discount a reduction in the marked price of goods for sale (p. 13)

discrete data exact values that are also countable (e.g. number of students present) (p. 439)

distributive law the way to multiply out the brackets, e.g. $a(b + c) = ab + ac$ (p. 168)

earnings income received for performing work (p. 31)

edge a line that joins two faces of a solid (p. 375)

EFTPOS Electronic Funds Transfer at Point Of Sale, where a card is used at the time of purchase and the funds are withdrawn directly from the purchaser's account (p. 49)

enlargement a figure that has matching sides in exactly the same ratio as another smaller figure. The figures are said to be similar. (p. 349)

equation two expressions, one on either side of an equals sign (p. 162)

equivalence the left-hand side of the equation is equal to the right-hand side of the equation (p. 247)

expand multiply; used usually when multiplying brackets (p. 138)

expanded form the form of an expression in which all factors have been multiplied out fully (p. 127)

exponent *see* index

exponential describes a relationship between two quantities that involves an exponent (or index), where the exponent is the variable (p. 567)

expression a collection of terms added or subtracted together (p. 140)

exterior angle an angle formed when the side of a polygon is extended outside the shape; the angle is between the extended side and the adjacent side (p. 322)

faces flat surfaces of a solid (p. 375)

factorise to write an expression as a product of its factors (p. 181)

formula a rule that is expressed using pronumerals and an equals sign (p. 162)

general form a form in which no actual values are used and represents any possible version of that form (p. 291)

gradient the measure of a line's steepness, also called slope (p. 278)

gradient–intercept form an equation in the form $y = mx + b$, where the gradient is m and the y-value of the intercept is b (p. 291)

gross income earnings before deductions have been made (p. 31)

grouping in pairs a technique for factorising a four-term polynomial where terms are put in pairs, a common factor is taken out and the remaining factor in each pair is common (p. 187)

highest common factor the largest number that occurs in each of the sets of factors of a given group of numbers (p. 181)

hire purchase *see* buying on terms

hyperbola the shape of the graph of an inverse proportionality relationship (p. 583)

hypotenuse the longest side in a right-angled triangle (pp. 79, 395)

impossible associated with a probability of 0 (p. 479)

income money received for work and from business activities and investments, such as shares, property or savings (p. 22)

income tax a direct tax levied on earnings (income) (p. 31)

independent variable a variable in a rule that determines the value of another variable (the dependent variable) in the rule (p. 247)

index (*plural* **indices**) a number, written smaller, to the right and above another number (the base), which shows the number of times the base is multiplied by itself; also called a power or exponent (p. 127)

index form short way of writing a number multiplied by itself using a base raised to an index (p. 127)

intercept the value at which a line cuts the x- or y-axis (p. 291)

interest when money is borrowed or invested, interest is paid (either by the individual or the bank) for the use of that money (p. 40)

interior angle an angle inside a shape (p. 322)

interval the line segment formed when two points are joined (p. 261)

inverse operation an 'opposite' operation; for example, + and – or × and ÷ (p. 247)

inverse proportion a relationship between two variables in which one increases by a factor when the other decreases by the same factor (p. 583)

irrational number a number that cannot be expressed in the form $\frac{a}{b}$, where $b \neq 0$ and a and b are whole numbers (p. 88)

lay-by a system of payment where an item is paid for at intervals before being taken home after payment in full (p. 49)

linear equation an equation in which the power of the pronumeral(s) is 1 (p. 247)

linear graph a straight-line graph (p. 247)

linear relationship a relationship between two variables that, when graphed, gives a straight line (p. 247)

loss when goods are sold for less than what they were bought or made for a loss is made (p. 13)

measure of location measures of data that are representative of the data set. The median and mean are measures of location. (p. 446)

median class interval a class interval in grouped data that contains the median (middle) value (p. 456)

midpoint halfway between two given coordinates that define an interval (p. 261)

mutually exclusive not able to occur at the same time; if one event occurs, then the other cannot (p. 492)

negative skew a statistical graph that is not symmetrical, with the mean of the data on the right side of the median. It has a long tail on the left of the median, so is also called a skew to the left. (p. 455)

net a diagram consisting of each face of a solid, used to help construct these shapes (pp. 217, 375)

net income earnings after deductions have been made, also known as take-home pay (p. 31)

Null Factor Law states that if the product of factors is zero, then one or more of the factors must equal zero (p. 530)

observational data primary data obtained by direct observation (p. 442)

opposite side the side of a right-angled triangle opposite a given or required angle (p. 395)

ordinal data non-numerical data that has some sort of order associated with it (p. 440)

origin the point of intersection of the vertical and horizontal axes (p. 261)

overtime hours worked in excess of minimum hours set for a time period (p. 22)

parabola a curve that describes a quadratic relationship (p. 513)

parallel a reference to lines that lie in the same plane that are always the same distance apart and will never intersect (p. 295)

PAYG Pay As You Go, which describes the system by which income tax is paid to the government (p. 31)

perfect square a quadratic trinomial expression that can be factorised to give $(a \pm b)^2$ (p. 177)

perimeter the distance around the outside of a shape (p. 199)

perpendicular a description of lines that meet/intersect at right angles (p. 295)

piece work work for a payment per item produced (p. 24)

polygon a many-sided (or many-angled) plane shape (p. 335)

polyhedron (*plural* **polyhedra**) a solid where all faces are polygons, shapes with straight edges (pp. 217, 375)

positive skew a statistical graph that is not symmetrical, with the mean of the data on the left side of the median. It has a long tail on the right of the median, so is also called a skew to the right. (p. 455)

power *see* index

primary data data collected directly by an observer (p. 440)

principal the initial amount of money borrowed or invested (p. 40)

prism a three-dimensional shape with a uniform cross-section where the cross-section is a polygon and the sides are straight (pp. 217, 375)

probability the study of chance in which the likelihood of the occurrence of a specific event is expressed as a number between, or equal to, 0 and 1 (p. 479)

profit when goods are sold for more than what they were bought or made for a profit is made (p. 13)

proof a step-by-step logical connection of statements known to be true to verify that another statement is always true (p. 79)

Pythagoras' Theorem a famous theorem in mathematics that states that, in right-angled triangles, 'the square of the hypotenuse is equal to the sum of the squares on the other two sides'. Often expressed as $c^2 = a^2 + b^2$ (p. 79)

Pythagorean triple any three whole numbers that can be the sides of a right-angled triangle. The simplest of these is 3, 4, 5 (p. 110)

quadratic equation an equation in which the highest power of the pronumeral(s) is 2 (p. 513)

quadratic relationship a relationship between two variables where the rule for connection is a polynomial containing terms where the highest power of the pronumeral is 2 (p. 513)

quadratic trinomial a polynomial consisting of three terms where the highest order term is raised to the power of two (p. 171)

qualitative data data that are not numerical. Qualitative data can be categorical or ordinal. (p. 439)

quantitative data any type of numerical data (p. 439)

random sample sampling in which there is no pattern involved in the sampling process (p. 442)

rate of interest the percentage per year (per annum) applied to the amount borrowed or invested to calculate interest (p. 40)

rational approximation an approximate value for an irrational number (p. 88)

reciprocal the result of dividing an expression, term or number into 1. The product of an expression, term or number and its reciprocal is 1. (p. 142)

rectangular hyperbola a non-linear graph of the equation $y = \dfrac{a}{x-h} + k$ with both horizontal and vertical asymptotes (p. 571)

reduction a figure resulting from the application of a scale factor that is less than 1 (p. 349)

reference angle the angle in a right-angled triangle that determines the opposite and adjacent sides used in trigonometric ratios (p. 395)

reflection the transformation of a shape that would result from looking at the reflection of the shape in a mirror held along the line of reflection (p. 537)

regular polyhedron a polyhedron whose faces are all identical regular polygons (p. 375)

relative frequency a proportion found by dividing the number of outcomes of a particular result by the total number of trials (p. 481)

retainer a fixed amount paid to an employee (usually each week) as a base payment to which commission earned is added (p. 23)

right-angled triangle a triangle that contains a right angle (p. 79)

rise the vertical distance between two points on the Cartesian plane, used in the equation

$\text{gradient} = \dfrac{\text{rise}}{\text{run}}$ (p. 278)

run the horizontal distance between two points on the Cartesian plane, used in the equation

$\text{gradient} = \dfrac{\text{rise}}{\text{run}}$ (p. 278)

salary a fixed annual income paid at regular intervals (p. 23)

sample space a listing of all the outcomes possible for a particular experiment (p. 496)

sampling with replacement random sampling where the sample taken from any population is returned, without damage, to the population before a subsequent sample is taken. The size of the population remains constant. (p. 498)

sampling without replacement random sampling where the sample taken from any population is not returned to the population before a subsequent sample is taken. The size of the population decreases with each sample taken. (p. 498)

scale factor the number that relates the size of a map or drawing to the original it represents (p. 349)

scientific notation (standard form) a number is in scientific notation if it is written as $a \times 10^b$ where $1 \le a \le 10$ and b is an integer (p. 152)

secondary data data that have come from another source, such as the internet, and have not been collected directly (p. 440)

sector the area contained within two radii of a circle and the arc that joins them (p. 201)

significant figures the number of digits in a number that are counted or written after the number has been approximated (p. 153)

similar having the same shape but not necessarily the same size (p. 360)

similar triangles triangles where matching sides are in the same proportion and where matching angles are equal (p. 361)

simple interest the interest calculated on the initial principal for the entire term of a loan/investment (p. 40)

sine the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ for a particular angle in a right-angled triangle (p. 400)

skewed graph a statistical graph that is not symmetrical in shape (p. 445)

slope *see* gradient

solid a three-dimensional figure that has length, width and depth (p. 375)

solve find the solution (p. 247)

stem plot (stem-and-leaf plot) a statistical graph in which the leaf value represents the units digit for each value and the stem represents the other digit(s). The leaf values are ordered from smallest to largest, usually on the right-hand side of the corresponding stem values. (p. 459)

straight line the shortest distance between two points. When referring to graphs, an equation of the form $y = mx + b$. (p. 247)

subtended refers to the angle formed by an interval at a point that does not lie on that line (p. 201)

superannuation regular payments made to a fund to be accessed upon retirement (p. 31)

surd a number (irrational) that can only be expressed exactly using the radical ($\sqrt{\ }$) sign (p. 88)

surface area the total area of all the surfaces of a solid (p. 217)

symmetrical graph a graph that has an axis of symmetry that divides the graph into mirror image halves (p. 455)

tangent the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for a particular angle in a right-angled triangle (p. 400)

tax money collected by the government from Australian residents to pay for public services such as hospitals, schools, roads and security (p. 31)

tax deduction a work-related expense incurred by the taxpayer in the course of earning an income (p. 33)

tax return a document completed at the end of the financial year to determine whether a refund is due or more tax needs to be paid (p. 33)

taxable income gross income minus allowable tax deductions (p. 33)

theorem a rule that has been proved to be always true (p. 79)

transformation any change that can be made to a graph or shape using rotation, dilation, reflection or translation (p. 537)

translation the transformation of a graph that keeps the shape, size and orientation of the graph exactly the same, but changes its location; usually involving a vertical or horizontal movement (p. 537)

tree diagram a diagram consisting of lines branching out from other lines used to represent a sample space. Probabilities can be assigned to each branch and can then be used to calculate the probability of a particular outcome. (p. 497)

trigonometric ratio the ratio of sides of a right-angled triangle for particular sized angles. There are three trigonometric ratios: sine θ, cosine θ and tangent θ. (p. 395)

trigonometry the part of mathematics that studies the relationship between the measures of sides and angles in right-angled triangles (p. 395)

true bearing a bearing that has been measured in a clockwise direction from north (p. 424)

turning point the point at which the gradient of a curve changes from positive to negative or negative to positive (p. 514)

two-way table information presented in rows and columns (p. 496)

undefined a value that cannot be calculated (p. 281)

variable in algebra, an unknown amount that can vary (p. 128)

vertex the corner of a shape (pp. 375, 514); *see also* turning point

volume the amount of space inside a three-dimensional shape (p. 224)

wages income usually based on an hourly rate of pay. The rate may be higher for extra hours worked above the standard number or for work done on weekends or public holidays. (p. 22)