ACADEMIC ASSOCIATES

Study Guide

Introductory

Mathematics Methods

O.T. Lee

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Introductory Mathematics Methods

Core skills and concepts required for the
ATAR Mathematics Methods courses

Dr O. T. Lee

The Introductory Mathematics Methods Study Guide provides a checklist of core skills and concepts required for the ATAR Mathematics Methods courses.

Introductory Mathematics Methods Study Guide

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Preface

The Introductory Mathematics Methods Study Guide is a vital tool to enable students to develop the core skills and the understanding of concepts required for the year 11 and 12 ATAR Mathematics Methods Units 1, 2, 3 & 4 courses. The core skills and concepts are in line with Level 10 Advanced of the Australian Mathematics Curriculum.

It is aimed at students in year 10 as preparation for the year 11 & 12 ATAR Mathematics Methods courses. It may also be helpful for students already in year 11 studying Mathematics Methods, who may have missed out on the required essential skills and concepts.

Each chapter consists of introductory notes, worked examples and practice tasks. Some chapters include Automaticity Tasks which are critical for success in the Mathematics Methods Course. These are skills which should be automated.

Review questions are provided along with fully worked solutions for each Automaticity Task, Practice Task and Review Task.

The use of calculators is _not recommended_ for this Study Guide.

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Number Skills

1.1 POWERS & ROOTS

Instant and accurate recall of commonly used powers and roots are essential.

Automaticity Task 1.1

Complete the following table as quickly and accurately as you can.

2^1	2^2	2^3	2^4	2^5	2^6
3^1	3^2	3^3	3^4		
4^1	4^2	4^3	4^4		
5^1	5^2	5^3	5^4		
6^2	7^2	8^2	9^2	11^2	12^2
13^2	15^2	16^2	25^2	30^2	50^2

Automaticity Task 1.2

Complete the following table as quickly and accurately as you can.

$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$
$\sqrt{64}$	$\sqrt{81}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{225}$
$\sqrt[3]{8}$	$\sqrt[3]{27}$	$\sqrt[3]{64}$	$\sqrt[3]{125}$		

1.2 WORKING WITH FRACTIONS

Automaticity Task 1.3

Perform the following computations as quickly and as accurately as you can.

	Answer				
$\dfrac{1}{2}+\dfrac{1}{4}$		$\dfrac{1}{2}+\dfrac{1}{3}$		$\dfrac{1}{3}+\dfrac{1}{4}$	
$\dfrac{3}{4}+\dfrac{1}{3}$		$\dfrac{1}{6}+\dfrac{1}{3}$		$\dfrac{2}{3}+\dfrac{1}{4}$	
$\dfrac{1}{6}+\dfrac{2}{3}$		$\dfrac{3}{4}+\dfrac{1}{6}$		$\dfrac{1}{6}+\dfrac{1}{4}$	
$\dfrac{2}{3}-\dfrac{1}{4}$		$\dfrac{1}{6}-\dfrac{1}{3}$		$\dfrac{1}{4}-\dfrac{1}{3}$	
$\dfrac{1}{2}-\dfrac{1}{3}$		$\dfrac{5}{6}+\dfrac{1}{2}$		$\dfrac{1}{6}-\dfrac{3}{4}$	
$1+\dfrac{1}{a}$		$\dfrac{a}{b}-\dfrac{c}{b}$		$\dfrac{a}{b}+\dfrac{c}{d}$	

Automaticity Task 1.4

Perform the following computations as quickly and as accurately as you can.

	Answer		Answer		Answer
$\dfrac{\left(\dfrac{1}{2}\right)}{3}$		$\dfrac{\left(\dfrac{2}{3}\right)}{2}$		$\dfrac{\left(\dfrac{3}{4}\right)}{6}$	
$\dfrac{3}{\left(\dfrac{1}{4}\right)}$		$\dfrac{6}{\left(\dfrac{2}{3}\right)}$		$\dfrac{5}{\left(\dfrac{3}{5}\right)}$	
$\dfrac{\left(\dfrac{1}{10}\right)}{\left(\dfrac{3}{10}\right)}$		$\dfrac{\left(\dfrac{3}{5}\right)}{\left(\dfrac{2}{3}\right)}$		$\dfrac{\left(\dfrac{5}{6}\right)}{\left(\dfrac{1}{6}\right)}$	
$\dfrac{\left(\dfrac{4}{7}\right)}{\left(\dfrac{4}{5}\right)}$		$\dfrac{\left(\dfrac{3}{2}\right)}{\left(\dfrac{1}{4}\right)}$		$\dfrac{\left(\dfrac{3}{2}\right)}{\left(\dfrac{6}{5}\right)}$	
$\dfrac{\left(\dfrac{a}{b}\right)}{\left(\dfrac{c}{b}\right)}$		$\dfrac{\left(\dfrac{a}{b}\right)}{\left(\dfrac{a}{d}\right)}$		$\dfrac{\left(\dfrac{a}{b}\right)}{\left(\dfrac{c}{d}\right)}$	

1.3 WORKING WITH SURDS

- A rational number can be expressed as a fraction involving two integers.
  - 1.2 is a rational number as $1.2 \equiv \dfrac{6}{5}$. $0.\overline{3}$ is a rational number as $0.\overline{3} \equiv \dfrac{1}{3}$.

    $\sqrt{2}$ is not a rational number as it cannot be expressed as a fraction involving two integers.

- A surd is a number expressed using the radical sign $\sqrt{}$ that cannot be simplified into a rational number.
  - $\sqrt{2}$ is a surd but $\sqrt{4}$ is not a surd as $\sqrt{4} \equiv 2$.

    $\sqrt{8}$ is a surd as $\sqrt{8} \equiv 2\sqrt{2}$ which is a surd.

- The surd conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$.

  When a surd is multiplied with its conjugate, the result is a rational number.
  - $(a + \sqrt{b})(a - \sqrt{b}) \equiv a^2 - (\sqrt{b})^2 \equiv a^2 - b$.

Worked Example 1.1

Simplify each of the following: (a) $\sqrt{12}$ (b) $\sqrt{18} + \sqrt{32}$

(a) $\sqrt{12} = \sqrt{4 \times 3}$

        $= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

(b) $\sqrt{18} + \sqrt{32} = \sqrt{9 \times 2} + \sqrt{16 \times 2}$

                $= 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$

Worked Example 1.2

Expand and simplify each of the following: (a) $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right)$ (b) $\left(1 + \sqrt{2}\right)^2$

(a) $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right) = \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2$

                         $= 3 - 2 = 1$

(b) $\left(1 + \sqrt{2}\right)^2 = 1 + 2\sqrt{2} + \left(\sqrt{2}\right)^2$

                 $= 1 + 2 + 2\sqrt{2} = 3 + 2\sqrt{2}$

Worked Example 1.3

Rationalise and simplify each of the following: (a) $\dfrac{1}{\sqrt{3} + \sqrt{2}}$ (b) $\dfrac{2 - \sqrt{5}}{1 + \sqrt{2}}$

> *In each case, multiply both the numerator and denominator with the surd conjugate of the denominator.*

(a) $\dfrac{1}{\sqrt{3} + \sqrt{2}} = \dfrac{1}{\sqrt{3} + \sqrt{2}} \times \dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

           $= \dfrac{\sqrt{3} - \sqrt{2}}{1}$

           $= \sqrt{3} - \sqrt{2}$

(b) $\dfrac{2 - \sqrt{5}}{1 + \sqrt{2}} = \dfrac{2 - \sqrt{5}}{1 + \sqrt{2}} \times \dfrac{1 - \sqrt{2}}{1 - \sqrt{2}}$

           $= \dfrac{2 - 2\sqrt{2} - \sqrt{5} + \sqrt{2}\sqrt{5}}{1 - 2}$

           $= -2 + 2\sqrt{2} + \sqrt{5} - \sqrt{10}$

Automaticity Task 1.5

Rationalise each of the following as quickly and as accurately as you can.

Surd	Rationalised Surd	Surd	Rationalised Surd	Surd	Rationalised Surd
$\dfrac{1}{\sqrt{2}}$		$\dfrac{1}{\sqrt{3}}$		$\dfrac{2}{\sqrt{5}}$	
$\dfrac{1}{\sqrt{8}}$		$\dfrac{1}{\sqrt{12}}$		$\dfrac{1}{5\sqrt{5}}$	

Practice Task 1.1

Simplify each of the following: (a) $\sqrt{48}$ (b) $\sqrt{45}+\sqrt{125}$

Practice Task 1.2

Expand and simplify each of the following: (a) $\left(\sqrt{5}-3\right)\left(\sqrt{5}+3\right)$ (b) $\left(\sqrt{8}+\sqrt{2}\right)^{2}$

Practice Task 1.3

Rationalise and simplify each of the following: (a) $\dfrac{2}{\sqrt{20}-\sqrt{5}}$ (b) $\dfrac{1-\sqrt{2}}{3+2\sqrt{2}}$

Algebraic Expansions

The following expansions should have been committed to memory.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a - b)(a + b) = a^2 - b^2$

Worked Example 2.1

Expand and simplify: (a) $(2 + 3x)^2 - (x - 1)^2$ (b) $-2(1 - 2x)^2 + (3 - x)^2$

(a) $(2 + 3x)^2 - (x - 1)^2 = (4 + 12x + 9x^2) - (x^2 - 2x + 1)$
$\qquad\qquad\qquad\qquad\quad = 8x^2 + 14x + 3$

(b) $-2(1 - 2x)^2 + (3 - x)^2 = -2(1 - 4x + 4x^2) + (9 - 6x + x^2)$
$\qquad\qquad\qquad\qquad\qquad = -7x^2 + 2x + 7$

Worked Example 2.2

Expand and simplify: (a) $(x - 2)(x + 2) + (x - 2)^2$ (b) $(x + 3)(2 - x) - (2x - 1)(2x + 1)$

(a) $(x - 2)(x + 2) + (x - 2)^2$ (b) $(x + 3)(2 - x) - (2x - 1)(2x + 1)$
$\quad = x^2 - 4 + (x^2 - 4x + 4)$ $= (-x^2 - x + 6) - (4x^2 - 1)$
$\quad = 2x^2 - 4x$ $= -5x^2 - x + 7$

Worked Example 2.3

Expand and simplify: (a) $(2x - 1)(x^2 + 3x + 2)$ (b) $(3 - x)(x^2 - x + 1)$

(a) $(2x - 1)(x^2 + 3x + 2)$ (b) $(3 - x)(x^2 - x + 1)$
$\quad = 2x^3 + 6x^2 + 4x - x^2 - 3x - 2$ $= 3x^2 - 3x + 3 - x^3 + x^2 - x$
$\quad = 2x^3 + 5x^2 + x - 2$ $= -x^3 + 4x^2 - 4x + 3$

Worked Example 2.4

Expand and simplify: (a) $(x + 1)^3$ (b) $(1 - 2x)^3$

(a) $(x + 1)^3 = (x + 1)(x + 1)^2$ (b) $(1 - 2x)^3 = (1 - 2x)(1 - 2x)^2$
$\quad = (x + 1)(x^2 + 2x + 1)$ $= (1 - 2x)(1 - 4x + 4x^2)$
$\quad = x^3 + 2x^2 + x + x^2 + 2x + 1$ $= 1 - 4x + 4x^2 - 2x + 8x^2 - 8x^3$
$\quad = x^3 + 3x^2 + 3x + 1$ $= 1 - 6x + 12x^2 - 8x^3$

Automaticity Task 2.1

Expand and simplify as quickly and as accurately as you can.

	Simplified Expansion		Simplified Expansion
$(x + 3)^2$		$(4 - x)^2$	
$(2x - 3)^2$		$(3 - 2x)^2$	
$(x + 5)(x - 5)$		$(3x + 4)(3x - 4)$	
$(7 - x)(7 + x)$		$\left(6 - \dfrac{x}{2}\right)\left(6 + \dfrac{x}{2}\right)$	

Practice Task 2.1

Expand and simplify: (a) $(1 - 3x)^2 - (2x - 1)^2$ (b) $(x - 5)(2 - x) - (4x - 3)(4x + 3)$

Practice Task 2.2

Expand and simplify: (a) $(x - 2)(2x^2 - x + 3)$ (b) $(3 - 2x)(2x^2 + x + 1)$

Practice Task 2.3

Expand and simplify: (a) $(x - 2)^3$ (b) $(3 + 2x)^3$

Algebraic Factorization

Simple Factorization

The following results should have been committed to memory.

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a - b)(a + b)$

Worked Example 3.1

Factorise completely: (a) $12x^2 + 27x$ (b) $12x^2 - 27$

(a) $12x^2 + 27x = 3x(4x + 9)$

(b) $12x^2 - 27 = 3(4x^2 - 9)$
$$= 3(2x - 3)(2x + 3)$$

Worked Example 3.2

Factorise completely: (a) $x^2 + x - 42$ (b) $x^2 + 10x + 25$

(a) $x^2 + x - 42 = (x - 6)(x + 7)$

(b) $x^2 + 10x + 25 = (x + 5)^2$

Worked Example 3.3

Factorise completely: (a) $2x^2 - 12x + 18$ (b) $2x^2 - 9x + 9$

(a) $2x^2 - 12x + 18 = 2(x^2 - 6x + 9)$
$$= 2(x - 3)^2$$

(b) $2x^2 - 9x + 9 = (2x - 3)(x - 3)$

Worked Example 3.4

Factorise completely: (a) $-x^2 - 2x + 15$ (b) $-2x^2 - 7x + 4$

(a) $-x^2 - 2x + 15 = -(x^2 + 2x - 15)$
$$= -(x + 5)(x - 3)$$

(b) $-2x^2 - 7x + 4 = -(2x^2 + 7x - 4)$
$$= -(2x - 1)(x + 4)$$

Automaticity Task 3.1

Factorise each of the following expressions as quickly and as accurately as you can.

	Factorised expression		Factorised expression
$4x^2 - 49$		$9x^2 - 100$	
$x^2 + 8x + 16$		$x^2 - 12x + 36$	

Practice Task 3.1

Factorise completely: (a) $5x^2 - 45$ (b) $36x^2 - 64y^2$

Practice Task 3.2

Factorise completely: (a) $x(x + 2) + 3(x + 2)$ (b) $2(1 - x) - x(x - 1)$

Practice Task 3.3

Factorise completely: (a) $x^2 + 4x - 32$ (b) $2x^2 + x - 6$

Practice Task 3.4

Factorise completely: (a) $-x^2 + x + 6$ (b) $-3x^2 + 4x + 4$

Practice Task 3.5

Factorise completely: (a) $4x^2 - 4x - 3$ (b) $4x^2 - x - 3$

Chapter 4

Remainder & Factor Theorem

4.1 POLYNOMIAL DIVISION

Polynomial division is the "algebraic version" of the long division of numbers.

Worked Example 4.1

Use polynomial division to determine the quotient and remainder when $x^2 + 3x - 11$ is divided by $x - 1$.

$$
\begin{array}{r}
x + 4 \\
x - 1 \overline{\smash{)}\ x^2 + 3x - 11} \\
\underline{x^2 - x} \\
4x - 11 \\
\underline{4x - 4} \\
-7
\end{array}
$$

Hence, quotient is $x + 4$ and remainder is -7.

Worked Example 4.2

Use polynomial division to determine the quotient and remainder when $2x^3 - 5x^2 + x + 2$ is divided by $x + 1$.

$$
\begin{array}{r}
2x^2 - 7x + 8 \\
x + 1 \overline{\smash{)}\ 2x^3 - 5x^2 + x + 2} \\
\underline{2x^3 + 2x^2} \\
-7x^2 + x \\
\underline{-7x^2 - 7x} \\
8x + 2 \\
\underline{8x + 8} \\
-6
\end{array}
$$

Hence, quotient is $2x^2 - 7x + 8$ and remainder is -6.

Practice Task 4.1

Use polynomial division to determine the quotient and remainder when $x^2 - 5x + 7$ is divided by $x - 2$.

Practice Task 4.2

Use polynomial division to determine the quotient and remainder when $-x^3 + 3x^2 - x - 1$ is divided by $x + 2$.

4.2 THE REMAINDER AND FACTOR THEOREM

The Remainder Theorem

When the polynomial $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$.

The Factor Theorem

If $(x - a)$ is a factor of the polynomial $f(x)$, then $f(a) = 0$.

Conversely, if $f(a) = 0$, then $(x - a)$ is a factor of the polynomial $f(x)$.

Worked Example 4.3

Determine the remainder when $2x^3 + 5x^2 - 8x + 10$ is divided by $(x - 1)$.

Let $f(x) = 2x^3 + 5x^2 - 8x + 10$.
Using the Remainder Theorem,

$$\text{Remainder} = f(1)$$
$$= 2 + 5 - 8 + 10$$
$$= 9$$

> *Note that the Remainder provides a quick way to determine the remainder but does not provide the quotient.*

Worked Example 4.4

Show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$. Hence, factorise completely $x^3 + x^2 - 4x - 4$.

Let $f(x) = x^3 + x^2 - 4x - 4$
$$\text{Remainder} = f(2)$$
$$= 8 + 4 - 8 - 4 = 0$$
Hence, $(x - 2)$ is a factor.

Using polynomial division:

$$
\begin{array}{r}
x^2 + 3x + 2 \\
x - 2 \overline{)\,x^3 + x^2 - 4x - 4} \\
\underline{x^3 - 2x^2} \\
3x^2 - 4x \\
\underline{3x^2 - 6x} \\
2x - 4 \\
\underline{2x - 4} \\
0
\end{array}
$$

$\Rightarrow x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$
$\qquad\qquad\qquad\qquad = (x - 2)(x + 1)(x + 2)$

> *The Remainder Theorem provides the remainder. Clearly if the remainder is zero, then the divisor must be a factor.*
>
> *Hence, the Factor Theorem is a special case of the Remainder Theorem.*

> *Alternative Method*
> $x^3 + x^2 - 4x - 4 = (x - 2)(ax^2 + bx + c)$
> By inspection:
> $$a = 1$$
> $$c = 2$$
> $$b = 3$$
> $\Rightarrow x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$
> $\qquad\qquad\qquad = (x - 2)(x + 1)(x + 2)$

Practice Task 4.3

Determine the remainder when $x^3 - 2x^2 - 3x + 4$ is divided by $(x + 1)$.

Practice Task 4.4

Show that $(x - 1)$ is a factor of $f(x) = x^3 + 5x^2 + 3x - 9$. Hence, factorise completely $f(x)$.

Practice Task 4.5

Factorise completely $f(x) = x^3 - 7x - 6$.

Completion of Squares

When the quadratic expression $x^2 + bx + c$ is expressed as a completed square:

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Worked Example 5.1

Factorise completely $x^2 - 14x + 49$. Hence, complete the squares for $x^2 - 14x + 50$.

$$x^2 - 14x + 49 = (x - 7)^2$$

Clearly
$$\begin{aligned}
x^2 - 14x + 50 &= x^2 - 14x + 49 + 1 \\
&= (x - 7)^2 + 1
\end{aligned}$$

Worked Example 5.2

Express as a completed square $x^2 + 8x - 10$.

$$\begin{aligned}
x^2 + 8x - 10 &= (x^2 + 8x + 16) - 16 - 10 \\
&= (x + 4)^2 - 26
\end{aligned}$$

Worked Example 5.3

Express as a completed square $-x^2 + 6x + 3$.

$$\begin{aligned}
-x^2 + 6x + 3 &= -[x^2 - 6x - 3] \\
&= -[(x^2 - 6x + 9) - 9 - 3] \\
&= -(x - 3)^2 + 12
\end{aligned}$$

Worked Example 5.4

Express as a completed square $2x^2 + x - 4$.

$$\begin{aligned}
2x^2 + x - 4 &= 2\left[x^2 + \frac{x}{2} - 2\right] \\
&= 2\left[\left(x^2 + \frac{x}{2} + \frac{1}{16}\right) - \frac{1}{16} - 2\right] \\
&= 2\left(x + \frac{1}{4}\right)^2 - \frac{33}{8}
\end{aligned}$$

Practice Task 5.1

Factorise completely $x^2 + 16x + 64$. Hence, complete the squares for $x^2 + 16x + 60$.

Practice Task 5.2

Express as a completed square $x^2 - 12x + 20$.

Practice Task 5.3

Express as a completed square $-x^2 + 3x - 3$.

Practice Task 5.4

Express as a completed square $2x^2 + 4x - 5$.

Chapter 6

Solving Equations

06

6.1 QUADRATIC EQUATIONS

$$\text{For } ax^2 + bx + c = 0, \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Worked Example 6.1

Solve for x: (a) $x^2 + 2x - 35 = 0$ (b) $(2x - 3)^2 - 4 = 0$

(a) $x^2 + 2x - 35 = 0 \implies (x + 7)(x - 5) = 0$

$x = -7, 5$

(b) $(2x - 3)^2 - 4 = 0 \implies (2x - 3)^2 = 4$

$2x - 3 = \pm 2$

$x = \dfrac{1}{2}, \dfrac{5}{2}$

Worked Example 6.2

Solve for x: (a) $(x + 2)^2 - 5 = 0$ (b) $x^2 - 3x - 1 = 0$

(a) $(x + 2)^2 - 5 = 0 \implies (x + 2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

(b) $x^2 - 3x - 1 = 0 \implies x = \dfrac{3 \pm \sqrt{9 - 4(1)(-1)}}{2}$

$x = \dfrac{3 \pm \sqrt{13}}{2}$

6.2 CUBIC EQUATIONS

Worked Example 6.3

Solve for x in $x^3 + x^2 - 4x - 4 = 0$.

Let $f(x) = x^3 + x^2 - 4x - 4$

$f(-1) = -1 + 1 + 4 - 4 = 0$

Hence, $(x + 1)$ is a factor.

Using polynomial division:

$$
\begin{array}{r}
x^2 - 4 \\
x+1 \overline{\smash{)}\, x^3 + x^2 - 4x - 4} \\
\underline{x^3 + x^2 } \\
-4x - 4 \\
\underline{-4x - 4} \\
0
\end{array}
$$

$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$

$= (x + 1)(x + 2)(x - 2)$

Hence, $f(x) = 0 \implies x = -2, -1, 2$

Automaticity Task 6.1

Solve as quickly and as accurately as you can.

	Solution		Solution
$x^2 = 25$		$x^2 = 3$	
$(x+3)(x-8)=0$		$(3x+1)(x-4)=0$	

Practice Task 6.1

Solve for x: (a) $x^2 + 2x - 24 = 0$ (b) $(1-x)^2 - 5 = 0$

Practice Task 6.2

Solve for x: (a) $2x^2 - x = 6$ (b) $2x^2 + x - 2 = 0$

Practice Task 6.3

Solve for x in $x^3 - 6x^2 + 11x - 6 = 0$.

Indices

For $a \neq 0$:
- $a^x \times a^y = a^{x+y}$
- $\dfrac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$

- $a^o = 1$
- $a^{-x} = \dfrac{1}{a^x}$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$

Worked Example 7.1

Simplify, leaving answers with positive indices:

(a) $\dfrac{20a^3b^2}{12ab^4}$

(b) $\dfrac{15b^2}{8a^3} \times \dfrac{10a^5}{45b^3}$

(a) $\dfrac{20a^3b^2}{12ab^4} = \dfrac{5a^{3-1}}{3b^{4-2}}$

$\quad = \dfrac{5a^2}{3b^2}$

(b) $\dfrac{15b^2}{8a^3} \times \dfrac{10a^5}{45b^3} = \dfrac{5a^{5-3}}{12b^{3-2}}$

$\quad = \dfrac{5a^2}{12b}$

Worked Example 7.2

Simplify, leaving answers with positive indices:

(a) $\left(\dfrac{x^2}{5y}\right)^2$

(b) $\left(\dfrac{2a^2}{3b}\right)^{-2}$

(a) $\left(\dfrac{x^2}{5y}\right)^2 = \dfrac{(x^2)^2}{(5y)^2}$

$\quad = \dfrac{x^4}{25y^2}$

(b) $\left(\dfrac{2a^2}{3b}\right)^{-2} = \left(\dfrac{3b}{2a^2}\right)^2$

$\quad = \dfrac{9b^2}{4a^4}$

Worked Example 7.3

Simplify, leaving answers with positive indices:

(a) $\sqrt[3]{\dfrac{8a^6}{27b^3}}$

(b) $\left(\dfrac{36x^{10}}{49y^4}\right)^{-\frac{1}{2}}$

(a) $\sqrt[3]{\dfrac{8a^6}{27b^3}} = \left(\dfrac{8a^6}{27b^3}\right)^{\frac{1}{3}}$

$\quad = \dfrac{8^{\frac{1}{3}}(a^6)^{\frac{1}{3}}}{27^{\frac{1}{3}}(b^3)^{\frac{1}{3}}} = \dfrac{2a^2}{3b}$

(b) $\left(\dfrac{36x^{10}}{49y^4}\right)^{-\frac{1}{2}} = \left(\dfrac{49y^4}{36x^{10}}\right)^{\frac{1}{2}}$

$\quad = \dfrac{7y^2}{6x^5}$

Automaticity Task 7.1

Simplify as quickly and as accurately as you can, leaving answers with positive indices.

	Answer		Answer		Answer		Answer
$(x^2)^3$		$(x^4)^{\frac{1}{2}}$		$(-2x)^{-3}$		$\left(\dfrac{x}{3}\right)^{-2}$	

Practice Task 7.1

Simplify, leaving answers with positive indices: (a) $\dfrac{48a^2b^{-2}}{15a^5b^2}$ (b) $\dfrac{21b^2}{14a^{-4}} \times \dfrac{36a^{-5}}{15b}$

Practice Task 7.2

Simplify, leaving answers with positive indices: (a) $\left(\dfrac{3a}{b^{-2}}\right)^2$ (b) $\left(\dfrac{a^{\frac{1}{2}}}{4b}\right)^{-2}$

Practice Task 7.3

Simplify, leaving answers with positive indices: (a) $\left(\dfrac{64a^6}{125b^{-3}}\right)^{\frac{1}{3}}$ (b) $\sqrt{\dfrac{25a^{-8}}{144b^4}}$

Exponential Equations I

- In an indicial or exponential equation, the unknown appears as an exponent (power).

Worked Example 8.1

Solve for x: (a) $2^{3x} = 64$ (b) $3^{1+x} = \dfrac{1}{81}$

> *Express the number on the RHS as a power of the base number of the LHS.*

(a) $2^{3x} = 64 \implies 2^{3x} = 2^6$
$$3x = 6$$
$$x = 2$$

(b) $3^{1+x} = \dfrac{1}{81} \implies 3^{1+x} = 3^{-4}$
$$1 + x = -4$$
$$x = -5$$

Worked Example 8.2

Solve for x: (a) $4^{-x} = 32$ (b) $9^{2x+1} = \dfrac{1}{27^x}$

> *Express the base numbers of both sides as powers of a common base.*

(a) $4^{-x} = 32 \implies (2^2)^{-x} = 2^5$
$$2^{-2x} = 2^5$$
$$-2x = 5$$
$$x = -\frac{5}{2}$$

(b) $9^{2x+1} = \dfrac{1}{27^x} \implies (3^2)^{2x+1} = \dfrac{1}{(3^3)^x}$
$$3^{4x+2} = 3^{-3x}$$
$$4x + 2 = -3x$$
$$x = -\frac{2}{7}$$

Worked Example 8.3

Solve for x: (a) $4 \times 5^x = 100$ (b) $200 \times 2^x = 50 \times 2^{2x}$

> *Use cross-multiplication to collect all x terms to one side.*

(a) $4 \times 5^x = 100 \implies 5^x = \dfrac{100}{4}$
$$5^x = 25$$
$$x = 2$$

(b) $200 \times 2^x = 50 \times 2^{2x}$
$$\frac{200}{50} = \frac{2^{2x}}{2^x}$$
$$4 = 2^x$$
$$x = 2$$

Automaticity Task 8.1

Solve for x as quickly and as accurately as you can.

	Answer		Answer		Answer
$2^x = 64$		$3^x = 81$		$5^x = \dfrac{1}{25}$	

Practice Task 8.1

Solve for x: (a) $5^{3-x} = 625$ (b) $2^{-\frac{x}{2}} = 128$

Practice Task 8.2

Solve for x: (a) $9^{-x} = 27^{1+x}$ (b) $4^{2x-1} = 8^x$

Practice Task 8.3

Solve for x: (a) $3 \times 4^{2x} = 96$ (b) $20 \times 2^{0.5x} = 160 \times 2^{-0.1x}$

Logarithms

- For $a > 0$, if $M = a^x$, then $\log_a M = x$.

  The logarithm of M to the base a is the power a has to be raised to match M.

- For $a > 0$:   • $\log_a a = 1$ • $\log_a 1 = 0$
  - $\log_a M + \log_a N = \log_a (M \times N)$ • $\log_a M - \log_a N = \log_a \left(\dfrac{M}{N} \right)$
  - $\log_a (M^b) = b \times \log_a M$

Worked Example 9.1

Rewrite in logarithmic form: (a) $81 = 3^4$ (b) $y = 2^x$

(a) $81 = 3^4 \implies \log_3 81 = 4$ (b) $y = 2^x \implies \log_2 y = x$

Worked Example 9.2

Rewrite in exponential form: (a) $\log_5 \dfrac{1}{125} = -3$ (b) $\log_{10} 2 = x$

(a) $\log_5 \dfrac{1}{125} = -3 \implies \dfrac{1}{125} = 5^{-3}$ (b) $\log_{10} 2 = x \implies 2 = 10^x$

Worked Example 9.3

Evaluate each of the following: (a) $\log_{10} 1\,000\,000$ (b) $\log_4 256$

(a) $\log_{10} 1\,000\,000 = \log_{10} 10^6$ (b) $\log_4 256 = \log_4 4^4$
$$= 6 \times \log_{10} 10 \qquad\qquad\qquad = 4 \times \log_4 4$$
$$= 6 \times 1 = 6 \qquad\qquad\qquad\qquad = 4 \times 1 = 4$$

Worked Example 9.4

Simplify each of the following: (a) $\log_5 (x + 2) + 2\log_5 (x + 1)$ (b) $\dfrac{1}{2}\log_7 (x - 2) - \log_7 (x - 1)$

(a) $\log_5 (x + 2) + 2\log_5 (x + 1)$ (b) $\dfrac{1}{2}\log_7 (x - 2) - \log_7 (x - 1)$
$$= \log_5 (x + 2) + \log_5 (x + 1)^2 \qquad\qquad = \log_7 (x - 2)^{\frac{1}{2}} - \log_7 (x - 1)$$
$$= \log_5 [(x + 2)(x + 1)^2] \qquad\qquad\qquad = \log_7 \left(\dfrac{\sqrt{x - 2}}{x - 1} \right)$$

Automaticity Task 9.1

Evaluate as quickly and as accurately as you can.

	Answer		Answer		Answer
$\log_{10} 100$		$\log_{10} 0.1$		$\log_2 16$	

Practice Task 9.1

Express in exponential form to solve for *x*:　　(a) $\log_{10} x = 3$　　　　(b) $\log_x 81 = 2$

Practice Task 9.2

Express in logarithmic form to solve for *x*:　　(a) $2^x = 5$　　　　(b) $3^{2x+1} = 17$

Practice Task 9.3

Simplify each of the following:　　(a) $\log_6 (x^2 - 1) - 2\log_6 (x + 1)$　(b) $\log_9 (x + 3) + \dfrac{1}{2}$

Practice Task 9.4

Expand each of the following:　　(a) $\log_2 x(x + 5)$　　　(b) $\log_3 \left(\dfrac{x - 2}{x + 4} \right)$

Exponential Equations II

- For $a^x = b$ where b cannot be expressed as a power of a,
  to determine x in terms of the common logarithm ($\log_{10}$):
    - Take $\log_{10}$ on both sides. $\qquad \log_{10} a^x = \log_{10} b$
    - Apply logarithmic rule. $\qquad x \log_{10} a = \log_{10} b$
    - Isolate x. $\qquad\qquad\qquad x = \dfrac{\log_{10} b}{\log_{10} a}$

Worked Example 10.1

Solve for x in terms of $\log_{10}$ where appropriate $\qquad$ (a) $3^x = 14$ $\qquad$ (b) $4^{x+1} = 20$

(a) $3^x = 14 \quad \Rightarrow \quad \log_{10} 3^x = \log_{10} 14$

$$x \log_{10} 3 = \log_{10} 14$$

$$x = \frac{\log_{10} 14}{\log_{10} 3}$$

(b) $4^{x+1} = 20$

$$\Rightarrow \quad \log_{10} 4^{x+1} = \log_{10} 20$$

$$(x+1)\log_{10} 4 = \log_{10} 20$$

$$(x+1) = \frac{\log_{10} 20}{\log_{10} 4}$$

$$x = \frac{\log_{10} 20}{\log_{10} 4} - 1$$

Worked Example 10.2

Solve for x in terms of $\log_{10}$ where appropriate:
(a) $100(7^x) = 200$ $\qquad$ (b) $50(5^{2x}) = 400(5^{x+1})$

(a) $100(7^x) = 200$

$$7^x = \frac{200}{100} = 2$$

$$\log_{10} 7^x = \log_{10} 2$$

$$x \log_{10} 7 = \log_{10} 2$$

$$x = \frac{\log_{10} 2}{\log_{10} 7}$$

(b) $\qquad 50(5^{2x}) = 400(5^{x+1})$

$$\frac{5^{2x}}{5^{x+1}} = \frac{400}{50}$$

$$5^{2x-(x+1)} = 8$$

$$5^{x-1} = 8$$

$$(x-1)\log_{10} 5 = \log_{10} 8$$

$$(x-1) = \frac{\log_{10} 8}{\log_{10} 5}$$

$$x = \frac{\log_{10} 8}{\log_{10} 5} + 1$$

Practice Task 10.1

Solve for x in terms of $\log_{10}$ where appropriate   (a) $10^x = 50$ (b) $6^{1-x} = 100$

Practice Task 10.2

Solve for x in terms of $\log_{10}$ where appropriate:
(a) $40(8^x) = 100$ (b) $100(6^{-x}) = 300(6^{-2x-1})$

Simultaneous Equations

Worked Example 11.1

Solve for x and y: (a) $x + y = 10$ $x - y = 6$ (b) $x + 2y = 13$ $2x - y = 6$

(a)

$$x + y = 10 \qquad \text{I}$$
$$x - y = 6 \qquad \text{II}$$

$\text{I} + \text{II}$ $2x = 16 \implies x = 8, y = 2$

(b)

$$x + 2y = 13 \qquad \text{I}$$
$$2x - y = 6 \qquad \text{II}$$

$\text{I} \times 2$ $2x + 4y = 26$ Ib

$\text{Ib} - \text{II}$ $5y = 20 \implies y = 4, x = 5$

Worked Example 11.2

Solve for x and y. $3x + 2y = 6$ $2x - 5y = 23$

$$3x + 2y = 6 \qquad \text{I}$$
$$2x - 5y = 23 \qquad \text{II}$$

$\text{I} \times 2$ $6x + 4y = 12$ III
$\text{II} \times 3$ $6x - 15y = 69$ IV

$\text{III} - \text{IV}$ $19y = -57 \implies y = -3, x = 4$

Worked Example 11.3

Solve for x and y. $\qquad$ $x + y = 1$ $\qquad\qquad$ $y = x^2 + 2x + 3$

$$x + y = 1 \quad\Rightarrow\quad y = 1 - x$$

Substitute into $y = x^2 + 2x + 3$:

$$1 - x = x^2 + 2x + 3$$
$$x^2 + 3x + 2 = 0$$
$$(x + 1)(x + 2) = 0$$
$$x = -2, -1$$

Hence: $\qquad\qquad\qquad\qquad$ $x = -2, y = 3; x = -1, y = 2$

Practice Task 11.1

Solve for x and y. $x + 2y = 5$ $\qquad\qquad$ $x - 3y = 4$

Practice Task 11.2

Solve for x and y. $5x + 3y = 5$ $\qquad\qquad$ $3x + 2y = 4$

Practice Task 11.3

Solve for x and y. $x + y = 4$ $\qquad\qquad$ $y = x^2 - 3x + 1$

Lines

- Gradient of line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_1 - y_2}{x_1 - x_2}$.
- Equation of line with gradient m passing through (h, k) is $y - k = m(x - h)$.
- Parallel lines have the same gradient.
- If the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$ are perpendicular, then $m_1 \times m_2 = -1$.

Worked Example 12.1

State the equation of the line with gradient 4, passing through the point $(1, 3)$

Equation of line: $y - 3 = 4(x - 1) \;\Rightarrow\; y = 4x - 1$

Worked Example 12.2

Determine the equation of the line passing through the points $(3, 5)$ and $(5, 1)$.

Gradient of line: $m = \dfrac{5 - 1}{3 - 5} = -2$

Equation of line: $y - 5 = -2(x - 3) \;\Rightarrow\; y = -2x + 11$

Worked Example 12.3

Determine the equation of the line passing through $(-2, -3)$ and parallel to the line with equation $4x + y = 10$.

Rewrite equation of given line: $4x + y = 10 \;\Rightarrow\; y = -4x + 10$

Hence, gradient of required line: $m = -4$

Equation of required line: $y + 3 = -4(x + 2) \;\Rightarrow\; y = -4x - 11$

Worked Example 12.4

Determine the equation of the line passing through $(6, -1)$ and perpendicular to the line with equation $3x + 2y = 6$.

Rewrite equation of given line: $3x + 2y = 6 \;\Rightarrow\; 2y = -3x + 6 \;\Rightarrow\; y = -\dfrac{3}{2}x + 3$

Hence, gradient of required line: $m = \dfrac{-1}{\left(-\dfrac{3}{2}\right)} = \dfrac{2}{3}$

Equation of required line: $y + 1 = \dfrac{2}{3}(x - 6) \;\Rightarrow\; y = \dfrac{2}{3}x - 5$

Automaticity Task 12.1

In the axis provided below, sketch as quickly and accurately as you can the lines with equations $y = x + 2$, $x + y = 5$ and $x = -3$.
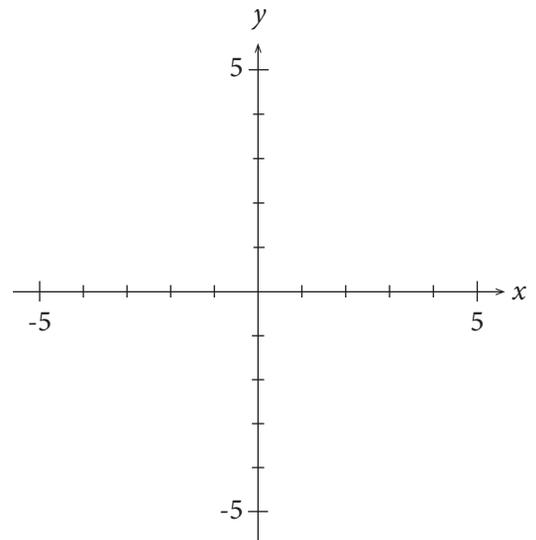
Practice Task 12.1

Determine the equation of the line passing through (2, 12) and:
(a) the point (−4, 6).
(b) parallel to the line with equation $3x + 4y = 12$.
(c) perpendicular to the line with equation $x - 3y = 9$.

Practice Task 12.2

Find the coordinates of the point of intersection between the lines $x + 2y = 11$ and $y = 2x + 3$.

Quadratics

	Factored Form $y = k(x - m)(x - n)$	Completed Square Form $y = k(x - p)^2 + q$	General Form $y = ax^2 + bx + c$
Roots	$x = m, n$	*Solve $y = 0$* *(may not have solutions)*	*Solve $y = 0$* *(may not have solutions)*
Vertical Intercept	$(0, kmn)$	$(0, kp^2 + q)$	$(0, c)$
Line of Symmetry	$x = \dfrac{m+n}{2}$	$x = p$	$x = -\dfrac{b}{2a}$
Turning Point	*Substitute $x = \dfrac{m+n}{2}$ into y.*	(p, q)	*Substitute $x = -\dfrac{b}{2a}$ into y.*

Worked Example 13.1

For each of the following quadratic curves state the coordinates of all the intercepts and turning point. Hence, sketch these curves.

(a) $y = (x - 2)(x + 4)$ (b) $y = 1 - (x + 2)^2$

(c) $y = x^2 + 4x + 5$

(a) *x*-intercepts: $(2, 0), (-4, 0)$

    y-intercept: $(0, -8)$

    LOS: $x = \dfrac{2 + (-4)}{2} = -1$

    Turning Point: $(-1, -9)$
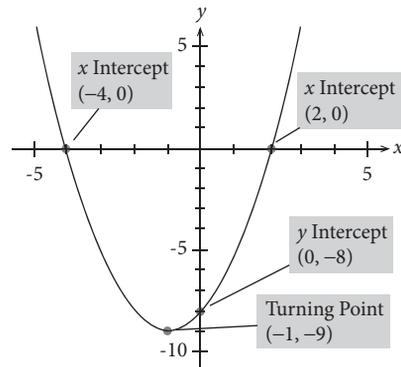
(b) For *x*-intercepts: $1 - (x + 2)^2 = 0$

                    $(x + 2)^2 = 1$

                    $(x + 2) = \pm 1 \implies x = -3, -1$

    Hence, $(-3, 0)$ & $(-1, 0)$.

    y-intercept: $(0, -3)$

    Turning Point: $(-2, 1)$
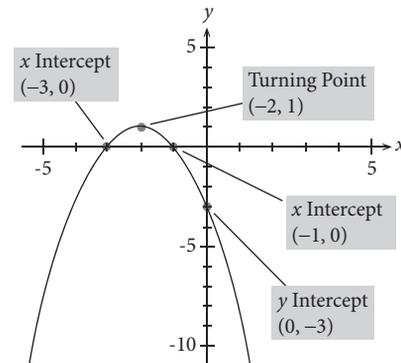
(c) For x-intercepts: $x^2 + 4x + 5 = 0$

Discriminant $\Delta = 16 - 4(1)(5) < 0$.

Hence, no x-intercepts.

y-intercept: $(0, 5)$

LOS: $x = \dfrac{-4}{2(1)} = -2$

Hence, turning point is at $(-2, 1)$.
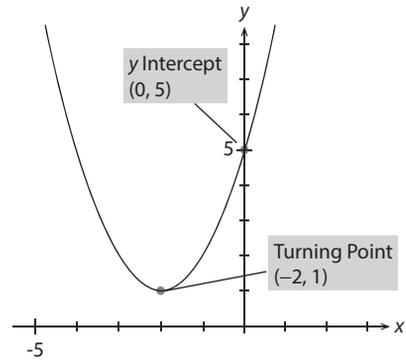
Practice Task 13.1

Sketch the curve with equation $y = 2(x - 1)^2 + 3$. State the coordinates of the intercept(s) and turning point.
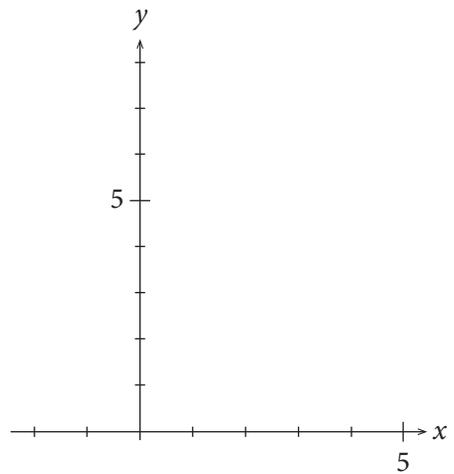
Practice Task 13.2

Sketch the curve with equation $y = -x^2 + 2x + 15$. State the coordinates of the intercept(s) and turning point.

Chapter 14

Cubics

14
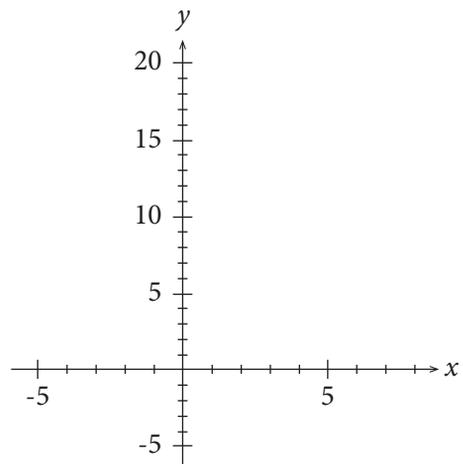
Cubics with three roots	Cubics with two roots	Cubics with one root
Cubic equation can be factorised into three distinct linear factors.	Cubic equation can be factorised into three linear factors with two identical linear factors.	Cubic equation has one linear factor and one non-reducible quadratic factor or three identical linear factors.
$y = (x+1)(2-x)(2x+1)$	$y = (x+2)(x-3)^2$	$y = (x+2)(x^2+1), y = (1-x)^3$
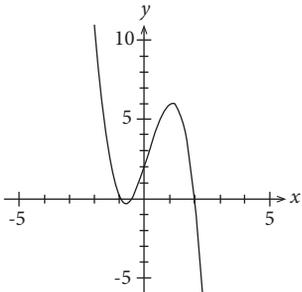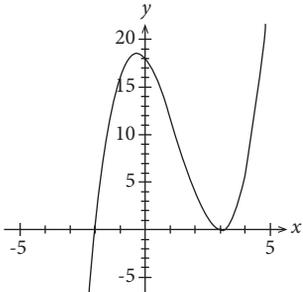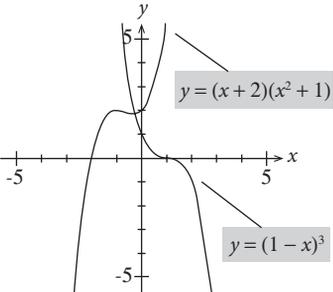
Worked Example 14.1

Sketch the curve with equation $y = x^3 + x^2 - x - 1$. Indicate all intercepts.

For x-intercepts: $x^3 + x^2 - x - 1 = 0$
Let $f(x) = x^3 + x^2 - x - 1$.
    $f(1) = 1 + 1 - 1 - 1 = 0$
Hence, $(x-1)$ is a factor.

$$\begin{array}{r} x^2 + 2x + 1 \\ x-1 \overline{) x^3 + x^2 - x - 1} \\ \underline{x^3 - x^2} \\ 2x^2 - x \\ \underline{2x^2 - 2x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

Using polynomial division:
$x^3 + x^2 - x - 1 = (x-1)(x^2 + 2x + 1)$
            $= (x-1)(x+1)^2$
For x-intercepts: $(x-1)(x+1)^2 = 0$
                $x = -1, 1$

y-intercept: $(0, -1)$
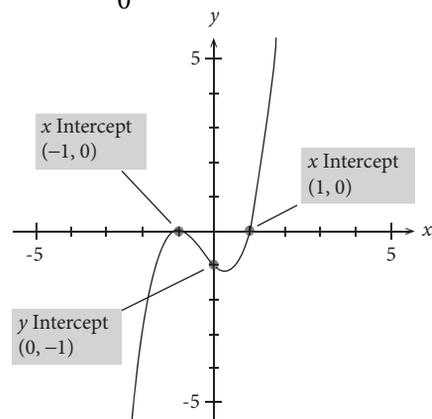
> *Notice that (−1, 0) corresponds to the repeated linear factor and is both a root and a turning point.*

x Intercept (−1, 0)
x Intercept (1, 0)
y Intercept (0, −1)

Practice Task 14.1

The accompanying diagram shows a cubic curve.
Determine the equation of this curve.

Practice Task 14.2

Sketch the curve with equation $y = 2x^3 - 5x^2 - 2x + 5$. Indicate all intercepts.

Chapter 15

Rectangular Hyperbolae

- The accompanying diagram shows the graph of the rectangular hyperbola $y = \dfrac{1}{x-2} + 1$.
  - The curve has no turning points.
  - The curve has a horizontal asymptote with equation $y = 1$.
  - The curve has a vertical asymptote with equation $x = 2$.
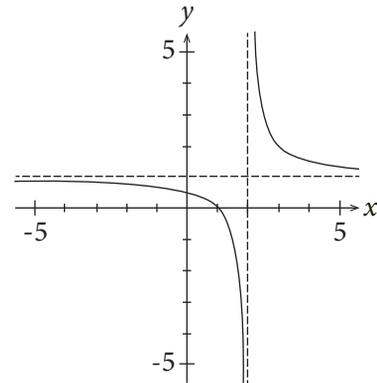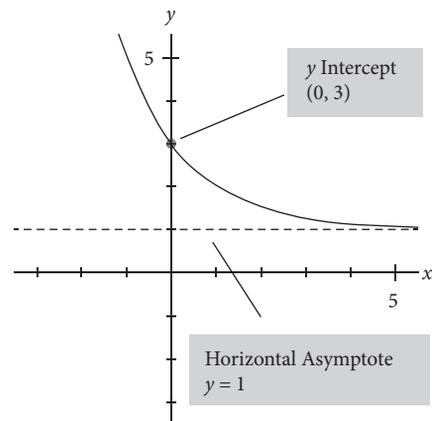
Worked Example 15.1

Determine the equation of the rectangular hyperbola shown in the accompanying diagram.

Equation is of the form $y = \dfrac{k}{x+1} - 3$

When $x = 0$, $y = -5$:

$$-5 = \frac{k}{1} - 3$$
$$k = -2.$$

Hence: $\qquad y = \dfrac{-2}{x+1} - 3$

x Intercept $\left(\dfrac{-5}{3}, 0\right)$

Vertical Asymptote $x = -1$

Horizontal Asymptote $y = -3$
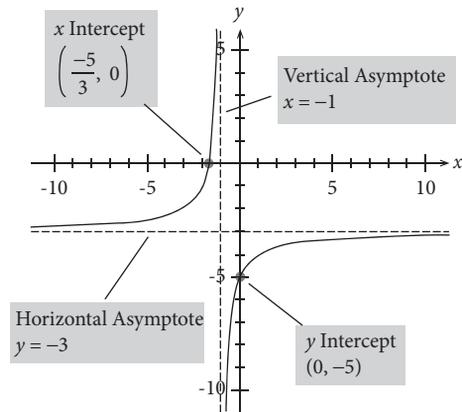
y Intercept $(0, -5)$

Worked Example 15.2

Sketch the curve with equation $y = \dfrac{1}{1-x} + 1$. State all intercepts and asymptotes.

When $x = 0$, $\qquad y = \dfrac{1}{1-0} + 1 = 2.$

Hence, vertical-intercept is $(0, 2)$.

When $y = 0$ $\qquad \dfrac{1}{1-x} + 1 = 0$
$$x = 2$$

Hence, horizontal-intercept is $(2, 0)$

Horizontal Asymptote: $\qquad y = 1$

Vertical Asymptote: $\qquad x = 1$

y Intercept $(0, 2)$

Vertical Asymptote $x = 1$

Horizontal Asymptote $y = 1$

x Intercept $(2, 0)$

Practice Task 15.1

The accompanying diagram shows a rectangular hyperbola. Determine the equation of this curve.

Vertical Asymptote
$x = 1$

x Intercept
$\left(\dfrac{5}{4}, 0 \right)$

y Intercept
$\left(0, \dfrac{-5}{2} \right)$

Horizontal Asymptote
$y = -2$
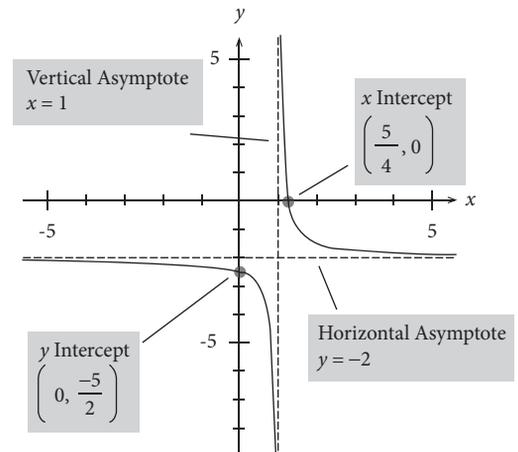
Practice Task 15.2

Sketch the curve with equation $y = 2 - \dfrac{2}{x - 3}$. State all intercepts and asymptotes.

Exponential Curves

- The accompanying diagram shows the graph of the exponential curve $y = k(a^x) + b$.

  - The curve has no turning points.
  - The curve has a horizontal asymptote with equation $y = b$.
  - The curve has a vertical intercept at $(0, k + b)$.

y Intercept
$(0, k + b)$

Horizontal Asymptote
$y = b$
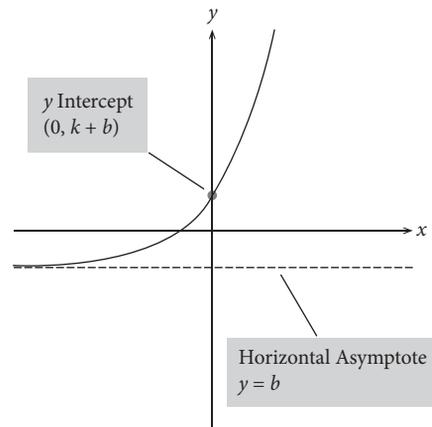
Worked Example 16.1

Sketch the curve with equation $y = 2(2^{-x}) + 1$. State the equation of the horizontal intercept and the coordinates of the vertical intercept.

> For the y-intercept: $x = 0$
> $$\Rightarrow \quad y = 2(1) + 1 = 3$$
> Hence, $(0, 3)$.
>
>
> Horizontal asymptote has equation $y = 1$.

y Intercept
$(0, 3)$

Horizontal Asymptote
$y = 1$

Worked Example 16.2

The accompanying diagram shows the graph of $y = a(5^{-x}) + b$. Determine the values of a and b.

> From sketch, horizontal asymptote is $y = -2$.
> Hence, $b = -2$.
>
> For vertical intercept: $\qquad x = 0$
> $$\Rightarrow \quad y = a + (-2)$$
> But vertical intercept is $(0, 0)$.
> Hence, $\qquad\qquad\qquad 0 = a + (-2)$
> $$\Rightarrow \quad a = 2$$
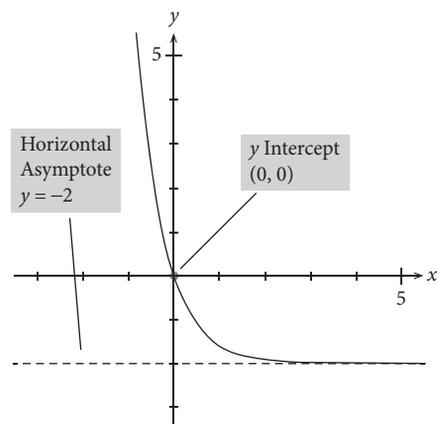
Horizontal
Asymptote
$y = -2$

y Intercept
$(0, 0)$

Practice Task 16.1

The accompanying diagram shows the graph of $y = a(3^x) + b$. Determine the values of a and b.

y

5

Horizontal
Asymptote
$y = 3$

-5

x

y Intercept
(0, 1)

Practice Task 16.2

(a) On the axes below, sketch the curve with equation $y = 3(2^x) - 4$. State the equation of the horizontal asymptote and the coordinates of the vertical intercept.

y

5

-5

x

-5

(b) On the same axes, sketch the line $x + y = 3$. Use your sketch to estimate the solution to the equation $3(2^x) = 7 - x$. Explain how you obtained your answer.

Chapter 17
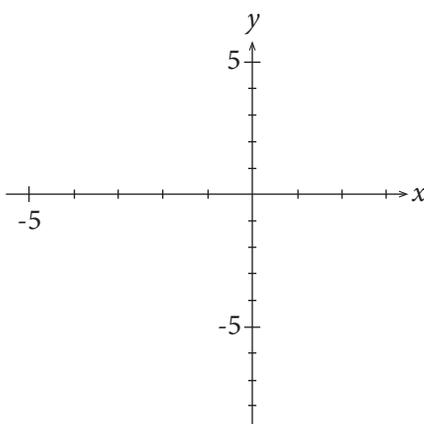
Circles

- The Cartesian equation of a circle of radius r with centre at (a, b) is given by:

$$(x - a)^2 + (y - b)^2 = r^2.$$

Worked Example 17.1

State the equation of each of the circles drawn.

(a)

(b)
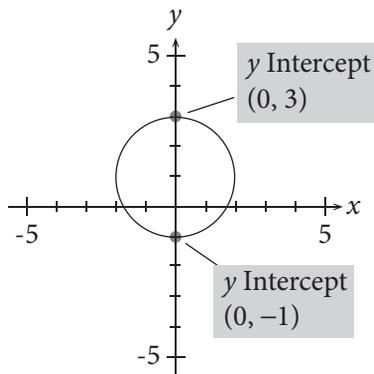
(a) Centre of circle is $(0, 1)$.

   Radius $= 2$

   Hence, equation is $x^2 + (y - 1)^2 = 4$
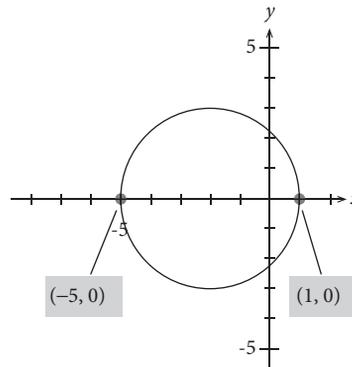
(b) Centre of circle is $(-2, 0)$.

   Radius $= 3$

   Hence, equation is $(x + 2)^2 + y^2 = 9$

Worked Example 17.2

(a) Express $x^2 - 4x$ as a completed square.

(b) Express $y^2 + 10y$ as a completed square.

(c) Hence, or otherwise, determine the coordinates of the centre and the radius of the circle with equation $x^2 - 4x + y^2 + 10y = 71$.

   (a) $x^2 - 4x \equiv (x - 2)^2 - 4$

   (b) $y^2 + 10y \equiv (y + 5)^2 - 25$

   (c) Using results from (a) and (b), $x^2 - 4x + y^2 + 10y \equiv (x - 2)^2 - 4 + (y + 5)^2 - 25$

   Hence, $x^2 - 4x + y^2 + 10y = 71$ can be rewritten as:

$$(x - 2)^2 - 4 + (y + 5)^2 - 25 = 71$$

$$\Rightarrow (x - 2)^2 + (y + 5)^2 = 100$$

   Hence, centre of circle has coordinates $(2, -5)$ and radius of circle $= 10$.

Automaticity Task 17.1

Write as quickly and as accurately as you can the equation of a circle with:

Centre	Radius	Equation	Centre	Radius	Equation
(0, 0)	2		(0, 0)	6	
(1, 0)	4		(3, 4)	5	

Practice Task 17.1
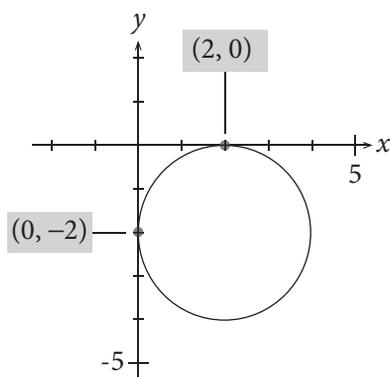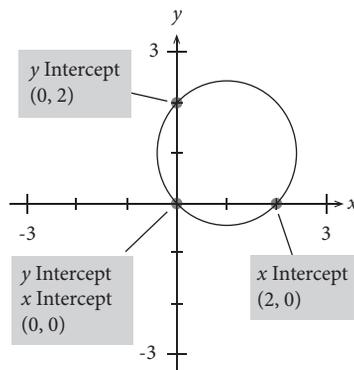
State the equation of each of the circles drawn.

(a)

(b)

Practice Task 17.2

Determine the coordinates of the centre and the radius of the circle with equation $x^2 + 6x + y^2 - 2y - 15 = 0$.

Right Triangle Trigonometry

18

SOHCAHTOA

- For Right Triangles, the label "Opposite"
  refers to the side opposite the angle of interest.
  - $\sin(x) = \dfrac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}}$ • $\cos(x) = \dfrac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}}$

  - $\tan(x) = \dfrac{\textbf{O}\text{pposite}}{\textbf{A}\text{djacent}}$
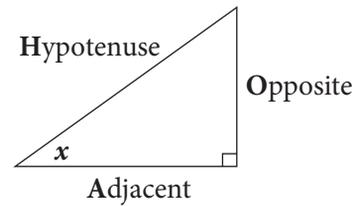
Exact Values

x	0°	30°	45°	60°	90°
$\sin(x)$	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
$\tan(x)$	0	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

Worked Example 18.1

In $\triangle ABC$, $\angle BCA = 90^{\text{o}}$, $BC = 10$ cm and $\tan \angle ABC = \dfrac{1}{2}$.
Determine the length of AC.

$$\tan \angle ABC = \frac{AC}{10} \Rightarrow AC = 10 \times \tan \angle ABC$$
$$= 10 \times \frac{1}{2} = 5 \, \text{cm}$$
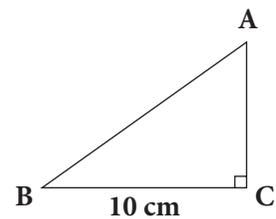
Worked Example 18.2

In $\triangle ABC$, $\angle ACB = 90^{\text{o}}$, $\angle BAC = x^{\text{o}}$, $BC = 8$ cm and $AC = 5$ cm.
Determine: (a) $\sin(x)$ (b) $\cos(x)$.

(a) Using Pythagoras Theorem: $AB = \sqrt{5^2 + 8^2} = \sqrt{89}$

Hence: $\sin(x) = \dfrac{8}{\sqrt{89}} = \dfrac{8\sqrt{89}}{89}$

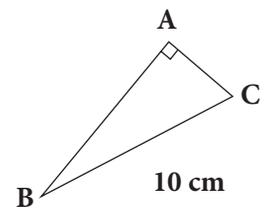(b) $\cos(x) = \dfrac{5}{\sqrt{89}} = \dfrac{5\sqrt{89}}{89}$
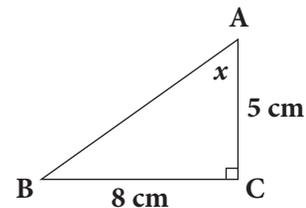
Introductory Mathematics Methods Study Guide

Automaticity Task 18.1

Complete the following table as quickly and as accurately as you can.

x	0°	30°	45°	60°	90°
sin (x)					
cos (x)					
tan (x)					

Practice Task 18.1

In $\triangle ABC$, $\angle BAC = 90º$, $BC = 10$ cm and $\cos \angle ABC = \dfrac{1}{5}$.
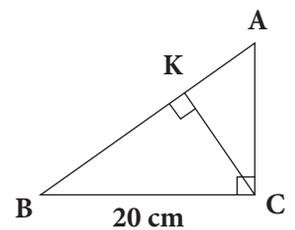
Determine the exact length of AC.

Practice Task 18.2

In $\triangle ABC$, $\angle ACB = 90º$, $BC = 20$ cm and $\cos \angle ABC = \dfrac{4}{5}$.

The point K on AB is foot of the perpendicular from C to AB.

Determine:　　　(a) BK　　　　(b) AK.

Non-Right Triangle Trigonometry

19

THE UNIT CIRCLE

- The SOHCAHTOA definitions for the trigonometric ratios do not apply for angles greater than 90°.
- The unit circle definitions for the trigonometric ratios apply to angles of any size.
- Let OR be the ray that defines angle θ.
  R is on the circumference of the unit circle.
  - The Sine function is defined as:
    sin (θ) = *y-coordinate of R*
  - The Cosine function is defined as:
    cos (θ) = *x-coordinate of R*
  - The Tangent function is defined as:
    $$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
- Using the unit circle definition of trigonometric ratios:
  - the sine ratio is positive in quadrants 1 & 2
  - the cosine ratio is positive in quadrants 1 & 4
  - the tangent ratio is positive in quadrants 1 & 3.

SINE is positive — ALL are positive

Quadrant 2 — Quadrant 1

Quadrant 3 — Quadrant 4

TANGENT is positive — COSINE is positive
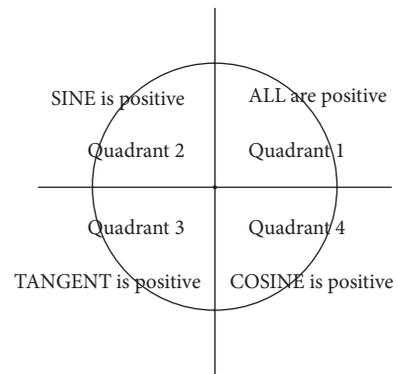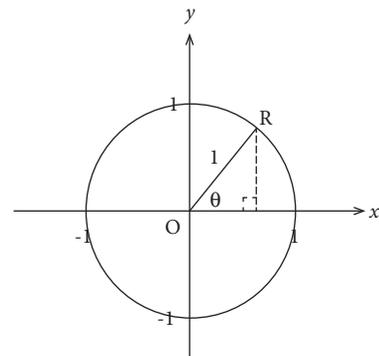
Worked Example 19.1

R is a point on a unit circle with coordinates (0.6, 0.8).
The ray OR forms an angle of size θ° with the positive *x*-axis as shown in the accompanying diagram.
Determine the value of: (a) sin θ°   (b) cos (180° − θ°)

(a) sin θ° = 0.8

(b) cos (180° − θ°) = − cos θ° = − 0.6
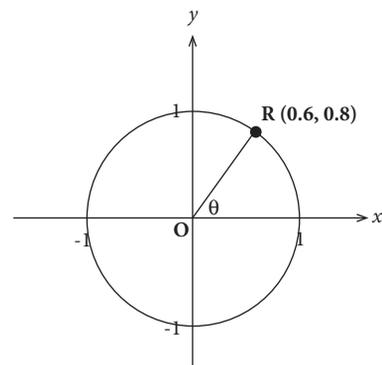
Worked Example 19.2

Determine: (a) sin 120°   (b) cos 150°   (c) tan 150°

(a)   sin 120° = sin 60°

$$= \frac{\sqrt{3}}{2}$$

(b)   cos 135° = − cos 45°

$$= -\frac{\sqrt{2}}{2}$$

(c)   tan 150° = − tan 30°

$$= -\frac{\sqrt{3}}{3}$$

Automaticity Task 19.1

Complete the following table as quickly
and as accurately as you can.

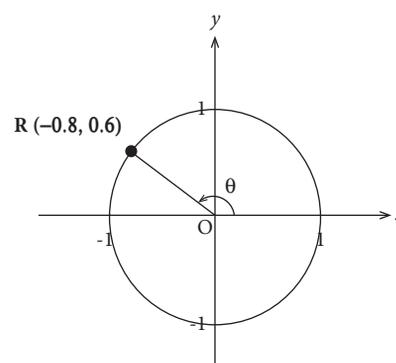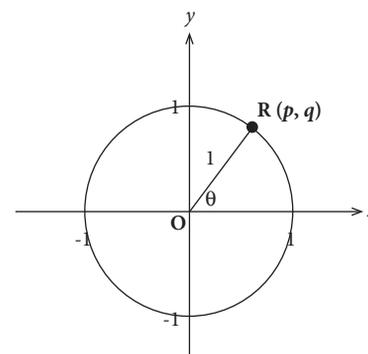| x	120°	135°	150°	180°
sin (x)				
cos (x)				
tan (x)				

Practice Task 19.1

R is a point on a unit circle with coordinates (−0.8, 0.6).
The ray OR forms an angle of size θ° with the positive
x-axis as shown in the accompanying diagram.
Determine the value of: (a) cos θ°   (b) tan (180° − θ°).

Practice Task 19.2

R is a point on a unit circle with coordinates (p, q).
The ray OR forms an angle of size θ° with the positive
x-axis as shown in the accompanying diagram. Given that
sin θ° $= \dfrac{2}{3}$, determine the value of p and q.

Chapter 20

Trigonometric Equations

- The reference angle associated with a given angle θ, is the acute angle, which the defining line (ray) makes with either the positive or the negative x-axis.
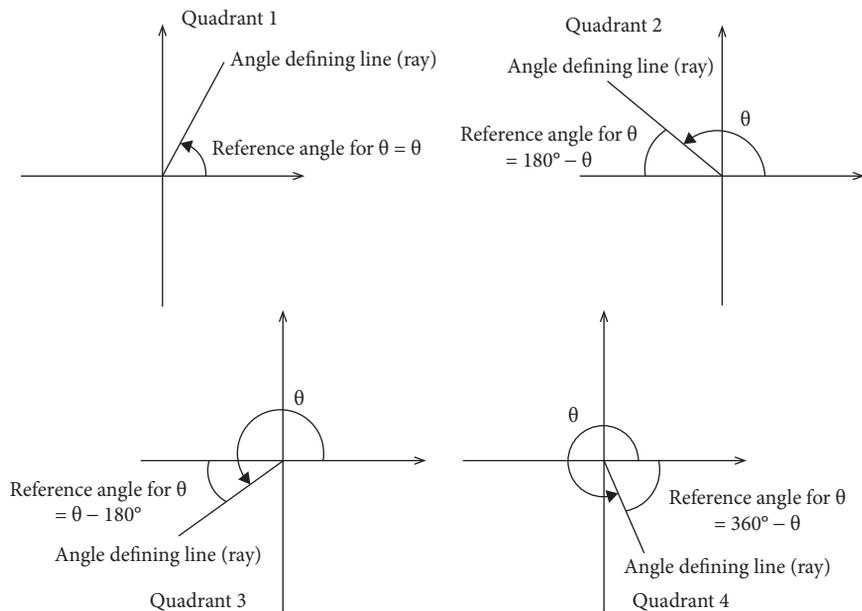
Worked Example 20.1

Solve for $x°$, where $0° \leq x \leq 360°$, if $\sin x° = \dfrac{1}{2}$.

$\sin x° = \dfrac{1}{2} \implies$ Reference angle for $x° = 30°$

The sine function is positive in quadrants 1 and 2.

Hence:
$$x° = 30°, 180° - 30°$$
$$= 30°, 150°$$

Worked Example 20.2

Solve for $x°$, where $0° \leq x \leq 360°$, if $\cos x° = -\dfrac{\sqrt{2}}{2}$.

$\cos x° = -\dfrac{\sqrt{2}}{2} \implies$ Reference angle for $x° = 45°$

The cosine function is negative in quadrants 2 and 3.

Hence:
$$x° = 180° - 45°, 180° + 45°$$
$$= 135°, 225°$$

Automaticity Task 20.1

Complete the following table as quickly and as accurately as you can.

x	210°	225°	300°	330°
sin (x)				
cos (x)				
tan (x)				

Practice Task 20.1

Solve for $x°$, where $0° \leq x \leq 360°$, if $\sin x° = -\dfrac{\sqrt{3}}{2}$.

Practice Task 20.2

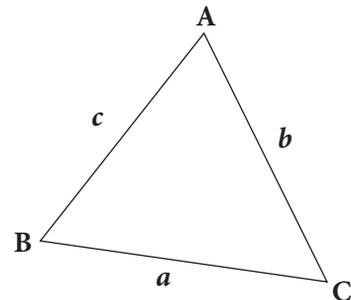Given that $\cos^{-1}\left(\dfrac{3}{4}\right) = 41.4°$, solve for $x°$, where $0° \leq x \leq 360°$, if $\cos x° = \dfrac{3}{4}$.

Practice Task 20.3

Given that $\tan^{-1}\left(\dfrac{5}{2}\right) = 68.2°$, solve for $x°$, where $0° \leq x \leq 360°$, if $\tan x° = -\dfrac{5}{2}$.

Chapter 21

The Sine Rule

- In any triangle ABC (acute or obtuse):

  - $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

  - $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

- The Sine Rule establishes a relationship between two pairs of Angle-Opposite Side in the form:

$$\frac{\text{length of side1}}{\sin (\text{angle opposite side1})} = \frac{\text{length of side2}}{\sin (\text{angle opposite side2})}$$
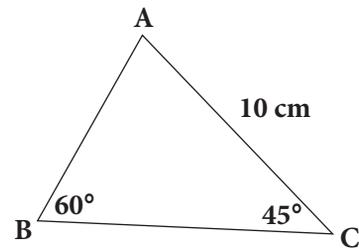
Worked Example 21.1

In $\triangle ABC$, $\angle ABC = 60°$, $\angle ACB = 45°$ and $AC = 10$ cm.
Calculate the length of AB.

Using the Sine Rule:

$$\frac{AB}{\sin 45°} = \frac{10}{\sin 60°}$$

$$AB = \frac{10}{\sin 60°} \times \sin 45°$$

$$= \frac{10}{\left(\dfrac{\sqrt{3}}{2}\right)} \times \frac{\sqrt{2}}{2} = \frac{10\sqrt{6}}{3} \text{ cm}$$

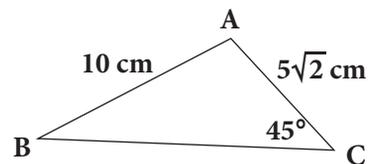## Worked Example 21.2

In $\triangle ABC$, $\angle ACB = 45°$, $AB = 10$ cm and $AC = 5\sqrt{2}$ cm.
Calculate the size of $\angle ABC$.

Using the Sine Rule:

$$\frac{\sin A\hat{B}C}{5\sqrt{2}} = \frac{\sin 45°}{10}$$

$$\sin A\hat{B}C = \frac{\sin 45°}{10} \times 5\sqrt{2}$$

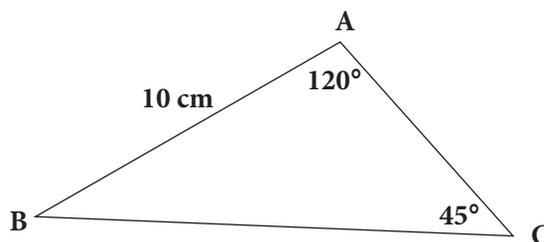$$= \frac{\left(\dfrac{\sqrt{2}}{2}\right)}{10} \times 5\sqrt{2} = \frac{1}{2}$$

> *Remember to check if an obtuse solution is possible.*

Hence: $\angle ABC = 30°$ or $180° - 30°$

$= 30°$ (Reject $150°$ as $150° + 45° > 180°$)

Practice Task 21.1

In ΔABC, ∠BAC = 120°, ∠ACB = 45° and
AB = 10 cm. Calculate the length of BC.

Practice Task 21.2

In ΔABC, ∠ABC is an acute angle such that $\sin \angle ABC = \dfrac{1}{4}$,
AB = 10 cm and AC = 8 cm. Use the calculator
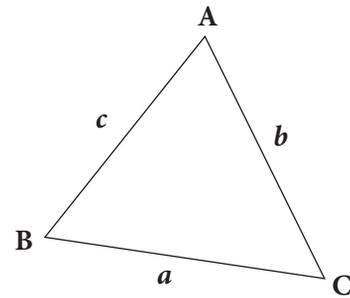screen-dump to calculate the size of ∠ACB.
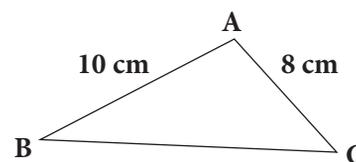
The Cosine Rule

- In any triangle ABC (acute or obtuse):
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- The Cosine Rule establishes a relationship between two sides and the included angle in the form:
  (side opposite given angle)²
  = (included side1)² + (included side2)² − 2 × (included side1) × (included side2) × cos (given angle)

Worked Example 22.1

In $\triangle ABC$, $\angle ABC = 60°$, BA = 3 cm and BC = 5 cm.
Calculate the length of AC.

Using the Cosine Rule:
$$AC^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 60°$$
$$= 9 + 25 - 2 \times 3 \times 5 \times \frac{1}{2}$$
$$= 19$$
$$AC = \sqrt{19} \text{ cm.}$$
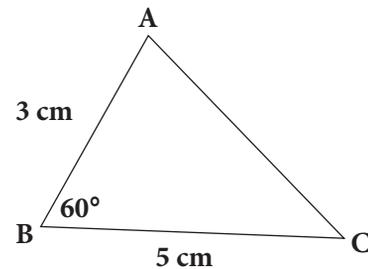
Worked Example 22.2

In $\triangle ABC$, AB = $2\sqrt{2}$ cm, AC = $\sqrt{5}$ cm and BC = 3 cm.
Calculate the size of $\angle ABC$.

Using the Cosine Rule:
$$\cos A\hat{B}C = \frac{(2\sqrt{2})^2 + 3^2 - \sqrt{5}^2}{2 \times 3 \times 2\sqrt{2}}$$
$$= \frac{8 + 9 - 5}{12\sqrt{2}}$$
$$= \frac{12}{12\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}$$
$$\Rightarrow \quad \angle ABC = 45°$$

Practice Task 22.1

In ∆ABC, ∠BAC = 120°, AB = 10 cm
and AC = 6 cm. Calculate the length of BC.

A

120°

10 cm

6 cm

B

C
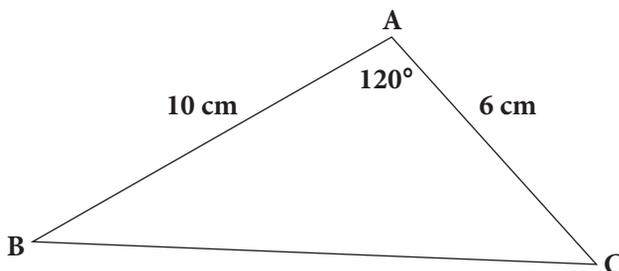
Practice Task 22.2

In ∆ABC, AB = 4 cm, AC = 5 cm and BC = 8 cm.
Use the calculator screen-dump to calculate the size of ∠BAC.

A

4 cm

5 cm

B

C

8 cm

$$\cos^{-1}\left(\frac{73}{80}\right)$$

24.1

$$\cos^{-1}\left(\frac{55}{64}\right)$$

30.8

$$\cos^{-1}\left(\frac{-23}{40}\right)$$

125.1

Chapter 23
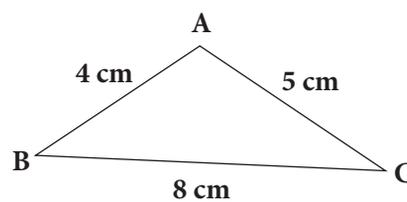
Trigonometric Graphs

- The graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ are shown below.

Properties	Properties
• Equation of mean line is $y = 0$.	• Equation of mean line is $y = 0$.
• Period = 360° • Amplitude = 1 unit	• Period = 180° • Amplitude: not applicable
• Minimum value -1 • Maximum value = 1	• Point of interest: (45°, 1)
	• Asymptotes: $x = \pm\, 90°$

- The following table shows the properties of transformed trigonometric graphs.

	Mean Line	Amplitude	Min. y-value	Max. y-value	Period	Asymptotes
$y = a \sin (bx) + d$ $y = a \cos (bx) + d$	$y = d$	$\lvert a \rvert$	$d - \lvert a \rvert$	$d + \lvert a \rvert$	$\dfrac{360°}{b}$	
$y = a \tan (bx) + d$	$y = d$				$\dfrac{180°}{b}$	$x = \pm \dfrac{90°}{b}$

Worked Example 23.1

The graph of $y = a \sin (bx) + c$ is shown in the accompanying diagram. Find the values of a, b and c.

Mean line is $y = -1$ $\Rightarrow$ $c = -1$

Period = 180° $\Rightarrow$ $d = 2$

Hence: $y = a \sin (2x) - 1$.

When $x = 45°$, $y = 0$: $0 = a \sin 90° - 1$

$\Rightarrow$ $a = 1$
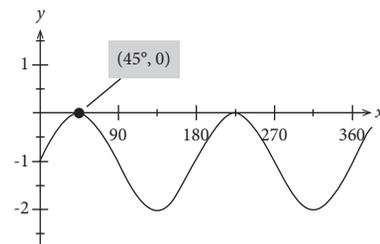
Worked Example 23.2

Sketch in the axes provided, the graph of $y = 2 \cos x + 1$.

Equation of mean line is $y = 1$.

Period = 360°   Amplitude = 2

When $x = 0$, $y = 2 \cos 0° + 1 = 3$.
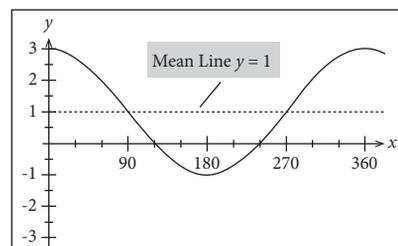
Practice Task 23.1

The graph of $y = a \cos (bx) + c$ is shown in the accompanying diagram. Find the values of a, b and c.
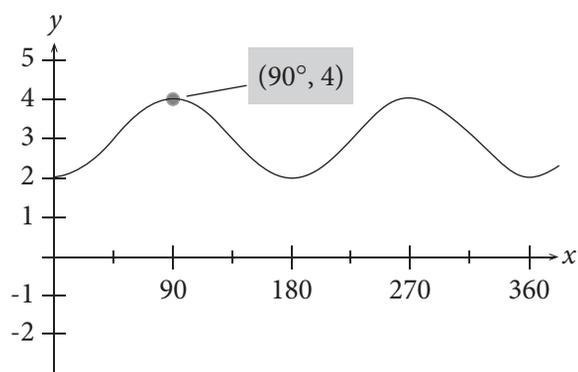
Practice Task 23.2
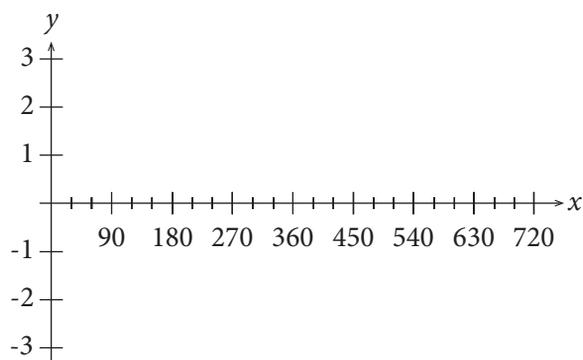
Sketch in the axes provided, the graph of $y = -2\sin\left(\dfrac{x}{2}\right)$.

Practice Task 23.3

The graph of $y = a \tan (bx) + c$ is shown in the accompanying diagram. Find the values of a, b and c.
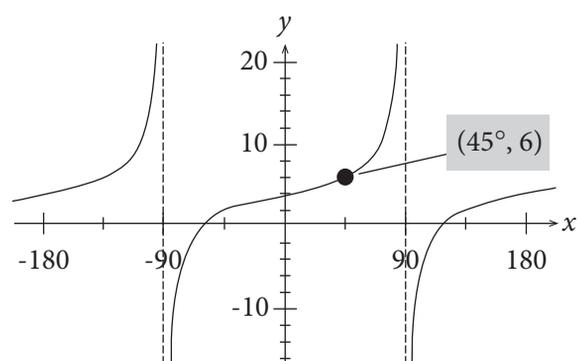
Sets & Venn Diagrams

- A set, e.g. **A** = { 1, 3, 5, 7, 9}, is a collection of elements that share a common property.
  - $n(\textbf{A})$ or $|\textbf{A}|$ denotes the number of elements in a set.
  - The accompanying table lists the symbols used to represent various set operations.
- The relationship between sets and the operations performed on them can be represented graphically using **Venn diagrams.**

Symbol	Meaning
$\in$	is an element of
$\subset$	is a subset of
$\cap$	intersection
$\cup$	union
$\textbf{A}'$ or $\overline{\textbf{A}}$	Complement of A

$$\textbf{A} \cap \textbf{B} \qquad \overline{\textbf{A}} \cap \textbf{B} \qquad \overline{\textbf{A} \cup \textbf{B}}$$
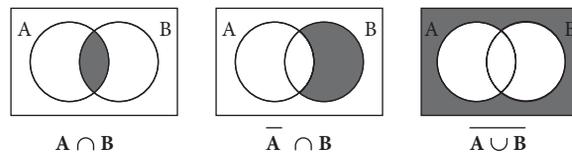
Worked Example 24.1

Let the universal set **U** = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }. Let **A** = { 2, 3, 5, 7 } & **B** = { 0, 2, 4, 6, 8 }.

Determine: (a) $\textbf{A} \cap \textbf{B}$ (b) $\textbf{A} \cup \textbf{B}$ (c) $n(\overline{\textbf{A} \cup \textbf{B}})$

(a) $\textbf{A} \cap \textbf{B} = \{\,2, 3, 5, 7\,\} \cap \{\,0, 2, 4, 6, 8\,\} = \{\,2\,\}$

(b) $\textbf{A} \cup \textbf{B} = \{\,2, 3, 5, 7\,\} \cup \{\,0, 2, 4, 6, 8\,\} = \{\,0, 2, 3, 4, 5, 6, 7, 8\,\}$

(c) $\overline{\textbf{A} \cup \textbf{B}} = \{\,1, 9\,\} \;\Rightarrow\; n(\overline{\textbf{A} \cup \textbf{B}}) = 2$

Worked Example 24.2

Given that $n(\textbf{U}) = 20$, $n(\textbf{A}) = 13$, $n(\textbf{B}) = 11$ and $n(\overline{\textbf{A} \cup \textbf{B}}) = 4$, use a Venn diagram to determine $n(\textbf{A} \cap \textbf{B})$.
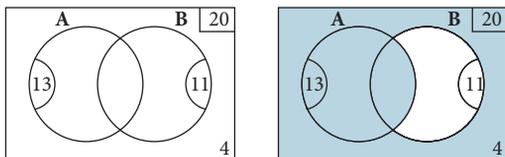
$$n(\text{shaded region}) = 13 + 4 = 17$$
$$\Rightarrow \quad n(\text{shaded unshaded region}) = 20 - 17 = 3$$
$$\Rightarrow \quad n(\textbf{A} \cap \textbf{B}) = 11 - 3 = 8$$

Automaticity Task 24.1

In the Venn diagrams provided, shade each of the following regions as quickly and accurately as you can.

$$A \cup B \qquad\qquad A \cap \overline{B} \qquad\qquad \overline{A} \cap \overline{B}$$
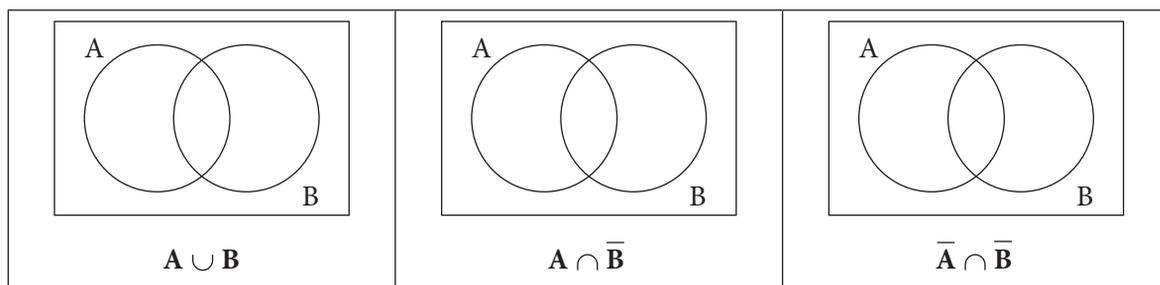
Practice Task 24.1

Let the universal set $U = \{ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 \}$. Let $A = \{ 11, 13, 17, 19 \}$ **and** $B = \{ 11, 13, 15, 17, 19 \}$.

Determine: (a) $A \cap B$ (b) $A \cup B$ (c) $n(\overline{A \cup B})$

Practice Task 24.2

In a group of 50 year 10 students, 25 played netball (**N**), 15 played basketball (**B**) and 15 played none of these two sports. Use a Venn diagram to determine the number of students that: (a) played netball only (b) played exactly one of these two sports.

- A sample space is the set of all possible outcomes in a statistical experiment.
  An event is a subset of the sample space.
  - For example, when a fair die is rolled,
    - the sample space is { 1, 2, 3, 4, 5, 6 }
    - the event that an even number is obtained
      is represented by the subset { 2, 4, 6 }.
- Where all outcomes are equally likely:
  - $P(A) = \dfrac{\text{No. of outcomes in A}}{\text{Total No. of outcomes}}$

  - $P(A \text{ given } B) = P(A|B) = \dfrac{\text{No. of outcomes in B that satisfy A}}{\text{Total No. of outcomes in B}}$

Worked Example 25.1

The accompanying table shows the number of students enrolled in the subjects listed.

Find the probability that a randomly chosen student: (a) was enrolled in Music
(b) was enrolled in Economics and Music
(c) who was enrolled in History was also enrolled in Music.

	Chemistry	Music	Total
Economics	12	4	16
History	6	8	14
Total	18	12	30

(a) $P(\text{Music}) = \dfrac{12}{30}$ (b) $P(\text{Econs} \cap \text{Music}) = \dfrac{4}{30}$ (c) $P(\text{Music} \mid \text{History}) = \dfrac{8}{14}$

Worked Example 25.2

In a group of adults, 4 had visited only London, 6 had visited Rome, 3 had visited both London and Rome and 14 had not visited either London or Rome. Calculate the probability that a randomly chosen adult:
(a) had visited London or Rome   (b) who had not visited London had visited Rome.

(a)   Using a Venn diagram:
$$n(\text{London} \cup \text{Rome}) = 6 + 4 = 10$$
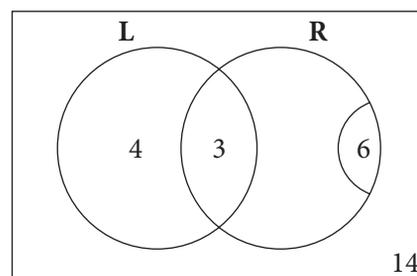$$n(\text{Total}) = 6 + 4 + 14 = 24$$
$$\text{Hence, } P(\text{London} \cup \text{Rome}) = \frac{10}{24}$$

(b)   From the Venn diagram:
$$n(\text{Not London}) = 24 - 7 = 17$$
$$n(\text{Not London} \cap \text{Rome}) = 6 - 3 = 3$$
$$\text{Hence, } P(\text{Rome} \mid \text{Not London}) = \frac{3}{17}.$$

Practice Task 25.1

In a group of 50 students, there were 10 girls enrolled in Mathematics Methods. There were as many girls enrolled in Mathematics Methods as girls not enrolled in Mathematics Methods. 32 of these students were not enrolled in Mathematics Methods. A student was randomly chosen from this group. Calculate the probability that this student:

(a) is a boy enrolled in Mathematics Methods

(b) is a boy given that this student is either a boy or enrolled in Mathematics Methods.

Practice Task 25.2

A sample of 400 people were asked to respond YES or NO to the question, "Should uranium mining be permitted in Western Australia (WA)?". The results are tabulated in the first table. The second table shows the gender distribution of those in the sample who responded YES.

	YES	NO
WA born	10%	55%
Not WA born	30%	5%

(a) Find the probability that a person chosen at random from those surveyed, voted YES and was Male.

(b) Given that a randomly selected respondent voted YES and was female, find the probability that the respondent was WA born.

	WA born	Not WA born
Female	5%	35%
Male	15%	45%

Probability II

- Tree diagrams can be used to display multi-staged outcomes.

- In the tree diagram shown, if A and B are the only outcomes of any stage, then $P(A) + P(B) = 1$.

- The probabilities associated with subsequent stages are all conditional probabilities.

- The probability of a multi-stage outcome is the product of the probabilities that trace the path to the outcome. For example, $P(A \cap C) = P(A) \times P(C \mid A)$.
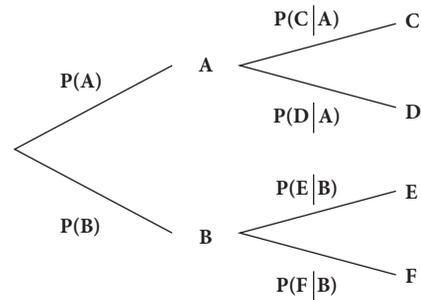
Worked Example 26.1

A box contains 6 red balls and 4 green balls. Two balls are randomly drawn from the box *with replacement*. The accompanying tree diagram displays the outcomes for the draw.

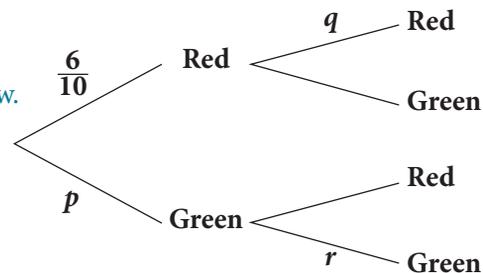(a) Determine the values of *p*, *q* and *r*.

(b) Calculate the probability of drawing:
  (i) two red balls
  (ii) two balls of the same colour.

(a) $p + \dfrac{6}{10} = 1 \Rightarrow p = \dfrac{4}{10}$ $\qquad q = \dfrac{6}{10}$ $\qquad r = \dfrac{4}{10}$

(b) (i) $P(\text{Red} \cap \text{Red}) = \dfrac{6}{10} \times \dfrac{6}{10} = \dfrac{36}{100}$

(ii) $P(\text{same colour balls}) = P(\text{Red} \cap \text{Red}) + P(\text{Green} \cap \text{Green})$

$$= \dfrac{36}{100} + \dfrac{4}{10} \times \dfrac{4}{10} = \dfrac{52}{100}$$

Worked Example 26.2

Repeat Worked Example 26.1 for draws *without replacement*.

(a) $p + \dfrac{6}{10} = 1 \Rightarrow p = \dfrac{4}{10}$ $\qquad q = \dfrac{5}{9}$ $\qquad r = \dfrac{3}{9}$

(b) (i) $P(\text{Red} \cap \text{Red}) = \dfrac{6}{10} \times \dfrac{5}{9} = \dfrac{30}{90}$

(ii) $P(\text{same colour balls}) = P(\text{Red} \cap \text{Red}) + P(\text{Green} \cap \text{Green})$

$$= \dfrac{30}{90} + \dfrac{4}{10} \times \dfrac{3}{9} = \dfrac{42}{90}$$

Practice Task 26.1

A box contains 3 red balls and 5 green balls. Two balls are randomly drawn from the box *without replacement*. The accompanying tree diagram displays the outcomes for the draw.

(a) Determine the values of p, q and r.

(b) Calculate the probability of drawing two balls of different colours.

Practice Task 26.2

60% of year 11 students in a college are girls and of these 75% are enrolled in an ATAR Mathematics course while 50% of year 11 boys in this college are enrolled in an ATAR Mathematics course.

(a) Draw a tree diagram to display the given information.

(b) Calculate the probability that a randomly chosen year 11 student from this college:
   (i) is *not* enrolled in an ATAR Mathematics course given that the student is a girl
   (ii) is a girl enrolled in an ATAR Mathematics course
   (iii) is a student enrolled in an ATAR Mathematics course
   (iv) is a girl given that this student is enrolled in an ATAR Mathematics course.

Mean & Standard Deviation

- Measures of Central Tendency (Location)
  - Median: the "middle score" when the scores have been ranked.
  - Mode: the most frequently occurring score
  - Mean: $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$
- Measures of Dispersion (Spread)
  - Range = Max Score − Min Score
  - Standard Deviation σ : an indicator of the "average distance" of the scores from the mean

$$\sigma = \frac{\sqrt{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}}{n}$$

$$= \sqrt{\left(\frac{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}{n}\right) - (\bar{x})^2}$$

Worked Example 27.1

Consider the scores: 1, 2, 3, 4, 5. Use the table provided to calculate the mean and standard deviation for these scores.

x	1	2	3	4	5
$(x - \bar{x})$					
$(x - \bar{x})^2$					

x	1	2	3	4	5
$(x - \bar{x})$	−2	−1	0	1	2
$(x - \bar{x})^2$	4	1	0	1	4

$$\bar{x} = \frac{1 + 2 + 3 + 4 + 5}{5} = 3 \qquad \sigma = \sqrt{\frac{4 + 1 + 0 + 1 + 4}{5}} = \sqrt{2}$$

Worked Example 27.2

Consider the scores: −2, −1, 0, 1, 2. Use the table provided to calculate the mean and standard deviation for these scores.

x	−2	−1	0	1	2
x^2					

x	−2	−1	0	1	2
x^2	4	1	0	1	4

$$\bar{x} = \frac{(-2) + (-1) + 0 + 1 + 2}{5} = 0 \qquad \sigma = \sqrt{\left(\frac{4 + 1 + 0 + 1 + 4}{5}\right) - 0^2} = \sqrt{2}$$

Practice Task 27.1

Consider the scores: 2, 4, 6, 8, 10. Use the table provided to calculate the mean and standard deviation for these scores.

x	2	4	6	8	10
$(x - \bar{x})$					
$(x - \bar{x})^2$					

Practice Task 27.2

Consider the scores: 1, 1, 4, 4, 5. Use the table provided to calculate the mean and standard deviation for these scores.

x	1	1	4	4	5
x^2					

Practice Task 27.3

The accompanying table show the means and standard deviations for datasets A, B and C.

Set	A	B	C
Mean	8	5	6
Standard deviation	2	2	2

(a) Determine which of these datasets are more dispersed.

(b) Determine the new mean and standard deviation if each of the scores in set:

(i) A is increased by one unit   (ii) B is doubled   (iii) C is increased by 10%

Review 1

Review Question 1.1

Expand and simplify each of the following:　　(a) $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)$　　　(b) $\left(\sqrt{27}-3\right)^2$

Review Question 1.2

Rationalise and simplify each of the following:　(a) $1+\dfrac{\sqrt{2}}{\sqrt{2}-1}$　　　　　(b) $\dfrac{1+\sqrt{3}}{5-2\sqrt{3}}$

Review Question 1.3

Expand and simplify:　　(a) $(1-3x)^2-(2x-1)^2$　　　　(b) $(x-1)(1-3x)-(1-2x)(1+2x)$

Review Question 1.4

Expand and simplify:　　(a) $(1-2x)(x^2+2x-1)$　　　　(b) $(2x-1)^3$

Review Question 1.5

Factorise completely: (a) $3x^2 - 12$ (b) $(1 - 2x) + 3x(2x - 1)$

Review Question 1.6

Factorise completely: (a) $x^2 + x - 20$ (b) $-5x^2 + 4x + 1$

Review Question 1.7

Factorise completely: (a) $8x^2 + 8x + 2$ (b) $6x^2 + x - 1$

Review Question 1.8

Use polynomial division to determine the quotient and remainder when $2x^3 - 5x^2 + 4x - 2$ is divided by $x - 1$.

Review Question 1.9

Determine the remainder when $-x^3 - 3x^2 + 5x + 10$ is divided by $(x + 2)$.

Review Question 1.10

Factorise completely $f(x) = -x^3 + 3x - 2$.

Review Question 1.11

Express as a completed square: (a) $x^2 - 8x + 10$ (b) $-2x^2 - 8x + 3$

Review Question 1.12

Solve for x: (a) $x^2 - 4x - 5 = 0$ (b) $(2 + x)^2 - 7 = 0$

Review Question 1.13

Solve for x: (a) $3x^2 - 7x + 2 = 0$ (b) $6x^2 - x - 2 = 0$

Review Question 1.14

Solve for x in $x^3 - 2x + 1 = 0$.

Review Question 1.15

Simplify, leaving answers with positive indices: (a) $\dfrac{30a^3b^{-1}}{12a^{-5}b^3}$ (b) $\dfrac{25b^{-2}}{10a^3} \times \left(\dfrac{2a^2}{b^{-1}}\right)^2$

Review Question 1.16

Simplify, leaving answers with positive indices: (a) $\sqrt{\dfrac{81a^4}{64b^{-8}}}$ (b) $\left(\dfrac{-a^3}{6\sqrt{b}}\right)^{-2}$

Review Question 1.17

Solve for x: (a) $4^{1-2x} = 64$ (b) $8^{\frac{x}{3}} = 32^{x+1}$

Review Question 1.18

Solve for x: (a) $4 \times 5^{2x-1} = 500$ (b) $10 \times 3^{0.3x} = 90 \times 3^{-0.2x}$

Review Question 1.19

Express in exponential form to solve for x: (a) $\log_{10} x = -2$ (b) $\log_x 27 = 3$

Review Question 1.20

Express in logarithmic form to solve for x: (a) $6^x = 10$ (b) $5x + 1 = 20$

Review Question 1.21

Solve for x in terms of $\log_{10}$: (a) $8^x = 20$ (b) $25(3^x) = 100(9^{x+1})$

Review Question 1.22

Expand each of the following: (a) $\log_{10} 10\,(x + 1)$ (b) $\log_5\left(\dfrac{x^2 - 1}{x + 2}\right)$

Review Question 1.23

Simplify each of the following: (a) $-2 + 3\log_{10}(x - 1)$ (b) $\log_2(x + 3) + \log_2(x - 3) + \log_2(x)$

Review Question 1.24

Solve for x and y. $2x + 3y = 13$ $3x - 4y = 11$

Review Question 1.25

Solve for x and y. $x - y = -2$ $y = x^2 - 3x - 3$

Review 2

Review Question 2.1

Determine the equation of the line passing through $(-2, 5)$ and:
(a) the point $(3, -10)$.
(b) parallel to the line with equation $y = 1 - 4x$.
(c) perpendicular to the line with equation $2x + 3y = 10$.

Review Question 2.2

Find the coordinates of the point of intersection between the lines $y = 2x - 3$ and $y = 2 - 3x$.
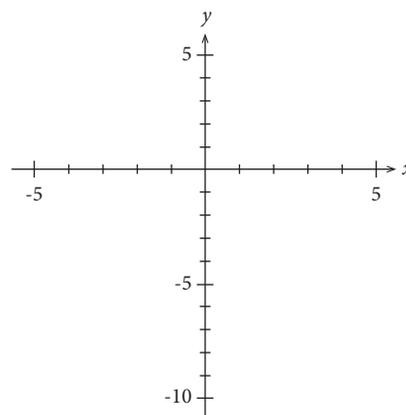
Review Question 2.3

Sketch the curve with equation $y = (x - 2)(x + 4)$. State the coordinates of all intercepts and the turning point.
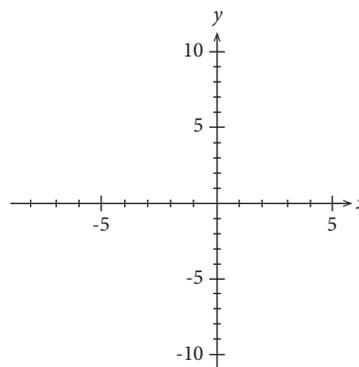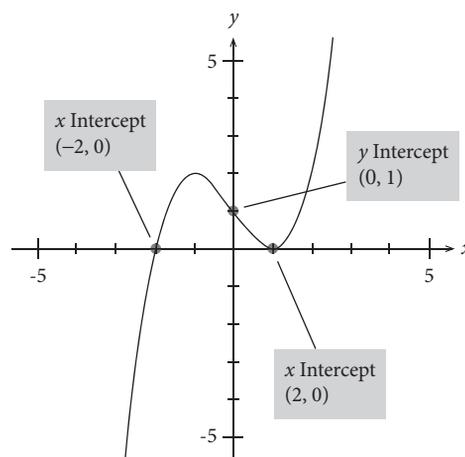
Review Question 2.4

Sketch the curve with equation $y = -x^2 + 2x - 2$. State the coordinates of the intercept(s) and turning point.

Review Question 2.5

The accompanying diagram shows a cubic curve. Determine the equation of this curve.
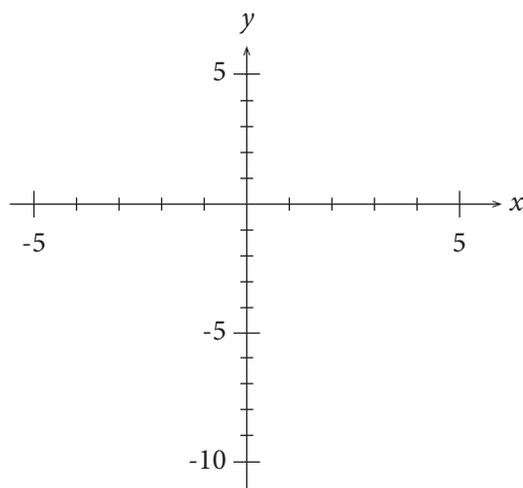

Review Question 2.6

Determine the equation of the rectangular hyperbola shown in the accompanying diagram.
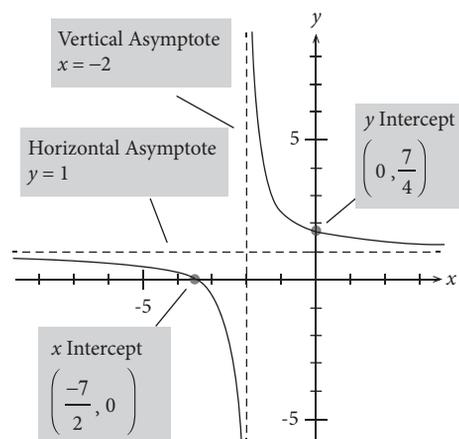
Review Question 2.7

Sketch the curve with equation $y = -x^3 + 2x^2 + 5x - 6$. Indicate all intercepts.

Review Question 2.8

Sketch the curve with equation $y = 1 + \dfrac{2}{2-x}$. State all intercepts and asymptotes.

Review Question 2.9

The accompanying diagram shows the graph of
$y = 0.5\,(2^{x+a}) + b$. Determine the values of a and b.

y

5

y Intercept
$\left(0\,,\dfrac{-7}{4}\right)$

x Intercept
$(3, 0)$

-5

5

x

Horizontal
Asymptote
$y = -2$

Review Question 2.10

(a) On the axes below, sketch the curve with equation $y = -(2^{-x}) + 2$. State the equation of the horizontal intercept and the coordinates of the intercepts.

y

5

-5

5

x

-5
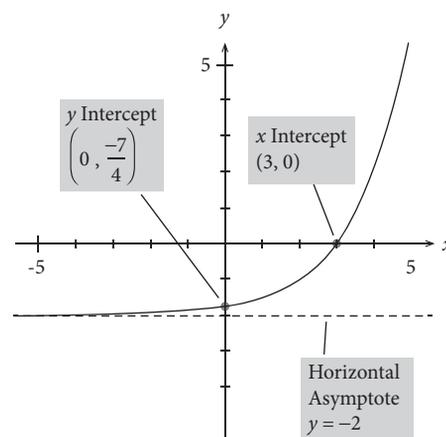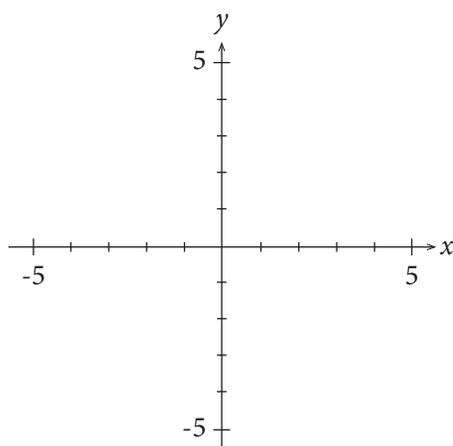
(b) On the same axes, sketch the line $y = x$. Use your sketch to estimate the solution to the equation $2^{-x} = 2 - x$. Explain how you obtained your answer.

Review Question 2.11
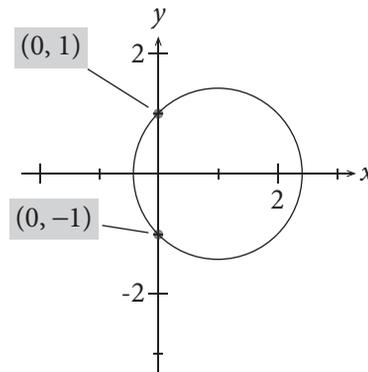
State the equation of each of the circles drawn.

(a)

(b)

Review Question 2.12

Determine the coordinates of the centre and the radius of the circle with equation $x^2 - 4x + y^2 + 8y - 16 = 0$.

Review Question 3.1

In $\triangle ABC$, $\angle ABC = 90°$, BC = 1 cm and $\sin \angle BAC = \dfrac{1}{4}$.

Determine the exact length of: (a) AC (b) AB.
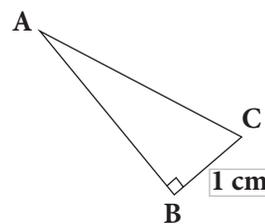
Review Question 3.2

In $\triangle ABC$, $\angle BAC = 90°$ and $\tan \angle ABC = \dfrac{4}{3}$. The point K on BC is foot of the perpendicular from A to BC and AK = 8 cm.

Determine: (a) BK   (b) AC.
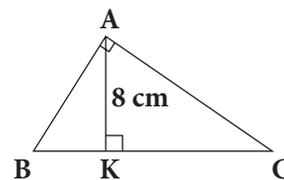
Review Question 3.3

R is a point on a unit circle with coordinates (0.6, 0.8).
The ray OR forms an angle of size $\theta°$ with the positive *x*-axis as shown in the accompanying diagram. Determine the value of:
(a) $\sin \theta°$ (b) $\cos (180° - \theta°)$ (c) $\tan (180° + \theta°)$
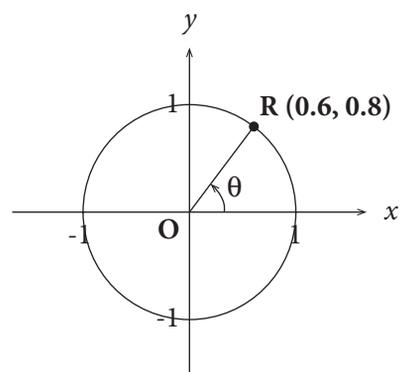
Review Question 3.4

Solve for $x°$, where $0° \leq x \leq 360°$, if $\cos x° = \dfrac{\sqrt{2}}{2}$.

Review Question 3.5

Solve for $x°$, where $0° \leq x \leq 360°$, if $\tan x° = \dfrac{\sqrt{3}}{3}$.

Review Question 3.6

Given that $\sin^{-1}\left(\dfrac{5}{8}\right) = 38.7°$, solve for $x°$, where $0° \leq x \leq 360°$, if $\sin x° = \dfrac{5}{8}$.

Review Question 3.7

Given that $\cos^{-1}\left(\dfrac{2}{7}\right) = 73.4°$, solve for $x°$, where $0° \leq x \leq 360°$, if $cos\, x° = -\dfrac{2}{7}$.

Review Question 3.8

In $\triangle ABC$, $\angle BAC = 120°$, $\angle ABC = 15°$ and $AB = \sqrt{6}$ cm. Calculate the length of BC.
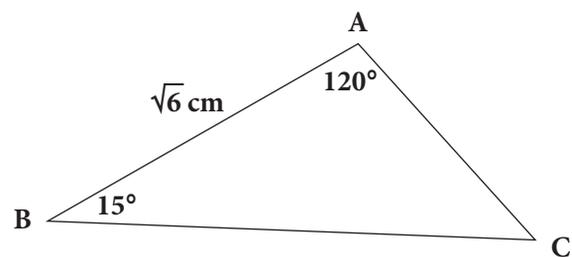
Review Question 3.9

In $\triangle ABC$, $\angle BAC$ is an obtuse angle such that $\sin \angle BAC = \dfrac{4}{5}$, AB = 8 cm and BC = 8 cm. Use the calculator screen-dump to calculate the size of:

(a) $\angle BAC$ (b) $\angle ACB$ (c) $\angle ABC$.
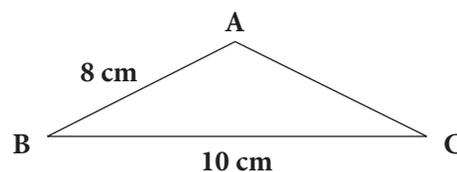
$$\sin^{-1}\left(\frac{16}{25}\right)$$
$$39.8$$

$$\sin^{-1}\left(\frac{4}{5}\right)$$
$$53.1$$

$$\sin^{-1}\left(\frac{5}{16}\right)$$
$$18.2$$

Review Question 3.10

In $\triangle ABC$, $\angle BAC = 150°$, AB = 3 cm and $AC = \sqrt{3}$ cm. Calculate the length of BC.
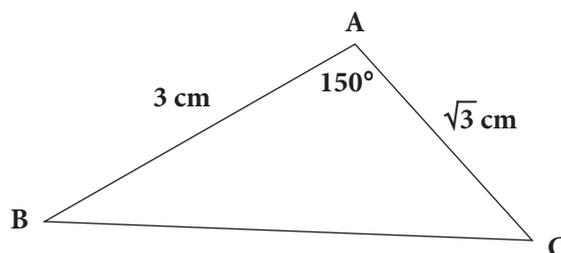
Review Question 3.11

In $\triangle ABC$, AB $= 2$ cm, AC $= 3$ cm and BC $= 4$ cm.
Use the calculator screen-dump to calculate the size of $\angle ABC$.

$$\cos^{-1}\left(\frac{-1}{4}\right)$$
104.5

$$\cos^{-1}\left(\frac{11}{16}\right)$$
46.6

$$\cos^{-1}\left(\frac{21}{24}\right)$$
29.0

Review Question 3.12

The graph of $y = a \sin(x + b) + c$ is shown in the accompanying diagram. Find the values of a, b and c.

x Intercept
(30, 0)
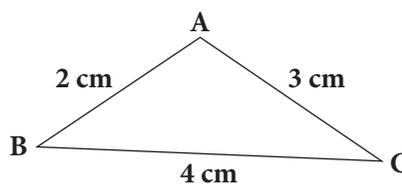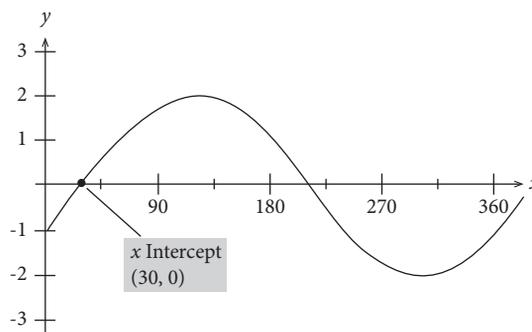
Review Question 3.13

Sketch in the axes provided, the graph of $y = \cos(2x) + 1$.
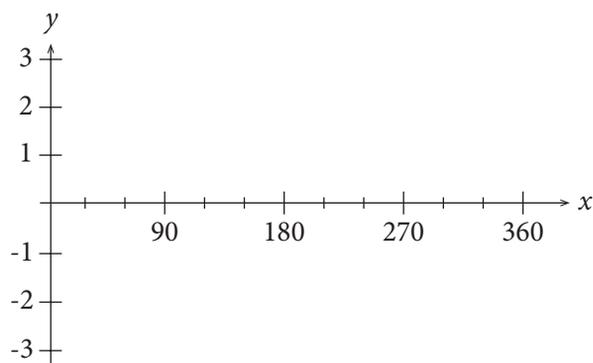
Review 4

Review Question 4.1

Let the universal set $\mathbf{U} = \{\, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \,\}$. Let $\mathbf{A} = \{\, 3, 6, 9 \,\}$
and B $= \{\, 1, 3, 5, 7, 9 \,\}$.

Determine: (a) $\mathbf{A} \cap \mathbf{B}$ (b) $\mathbf{A} \cap \overline{\mathbf{B}}$ (c) $n(\overline{\mathbf{A} \cup \mathbf{B}})$

Review Question 4.2

In a group of 40 year 10 students, 23 played hockey (**H**), 21 played cricket (**C**) and 4 played none of these two sports. Use a Venn diagram to determine the number of students that:
(a) played cricket only (b) played exactly one of these two sports.

Review Question 4.3

In a group of 50 students, there were 15 girls enrolled in Physics. There are as many girls not enrolled in Physics as boys enrolled in Physics. 25 of these students were enrolled in Physics. A student was randomly chosen from this group. Calculate the probability that this student:
(a) is a boy enrolled in Physics
(b) is a girl given that this student is not enrolled in Physics.

Review Question 4.4

A box contains 2 red balls and 3 green balls. Two balls are randomly drawn from the box *without replacement*. The accompanying tree diagram displays the outcomes for the draw.

(a) Determine the values of p, q and r.

(b) Calculate the probability of drawing two balls of the same colours.

Review Question 4.5

25% of students in a college wear glasses and of these 80% have dark hair while 10% of students who do not wear glasses have dark hair.

(a) Draw a tree diagram to display the given information.

(b) Calculate the probability that a randomly chosen student from this college:
   (i) does not have dark hair given that this student does not wear glasses
   (ii) is a student that does not have dark hair and does not wear glasses
   (iii) is a student with dark hair
   (iv) is a student that wear glasses given that this student had dark hair.

Review Question 4.6

Consider the scores: 1, 3, 5, 7, 9. Use the table provided to calculate the mean and standard deviation for these scores.

x	1	3	5	7	9
$(x-\bar{x})$					
$(x-\bar{x})^2$					

Review Question 4.7

Consider the scores: 0, 2, 3, 5, 5. Use the table provided to calculate the mean and standard deviation for these scores.

x	0	2	3	5	5
x^2					

Solutions to Automaticity Tasks

Automaticity Task 1.1

Complete the following table as quickly and accurately as you can.

2^1	2^2	2^3	2^4	2^5	2^6
2	4	8	16	32	64
3^1	3^2	3^3	3^4		
3	9	27	81		
4^1	4^2	4^3	4^4		
4	16	64	256		
5^1	5^2	5^3	5^4		
5	25	125	625		
6^2	7^2	8^2	9^2	11^2	12^2
36	49	64	81	121	144
13^2	15^2	16^2	25^2	30^2	50^2
169	225	256	625	900	2 500

Automaticity Task 1.2

Complete the following table as quickly and accurately as you can.

$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$
2	3	4	5	6	7
$\sqrt{64}$	$\sqrt{81}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{225}$
8	9	11	12	13	15
$\sqrt[3]{8}$	$\sqrt[3]{27}$	$\sqrt[3]{64}$	$\sqrt[3]{125}$		
2	3	4	5		

Automaticity Task 1.3

Perform the following computations as quickly and as accurately as you can.

	Answer				
$\frac{1}{2}+\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}+\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}+\frac{1}{4}$	$\frac{7}{12}$
$\frac{3}{4}+\frac{1}{3}$	$\frac{13}{12}$	$\frac{1}{6}+\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}+\frac{1}{4}$	$\frac{11}{12}$
$\frac{1}{6}+\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{4}+\frac{1}{6}$	$\frac{11}{12}$	$\frac{1}{6}+\frac{1}{4}$	$\frac{5}{12}$
$\frac{2}{3}-\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{6}-\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{4}-\frac{1}{3}$	$-\frac{1}{12}$
$\frac{1}{2}-\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{6}+\frac{1}{2}$	$\frac{4}{3}$	$\frac{1}{6}-\frac{3}{4}$	$-\frac{7}{12}$
$1+\frac{1}{a}$	$\frac{a+1}{a}$	$\frac{a}{b}-\frac{c}{b}$	$\frac{a-c}{b}$	$\frac{a}{b}+\frac{c}{d}$	$\frac{ad+bc}{bd}$

Automaticity Task 1.4

Perform the following computations as quickly and as accurately as you can.

	Answer		Answer		Answer
$\dfrac{\left(\frac{1}{2}\right)}{3}$	$\frac{1}{6}$	$\dfrac{\left(\frac{2}{3}\right)}{2}$	$\frac{1}{3}$	$\dfrac{\left(\frac{3}{4}\right)}{6}$	$\frac{1}{8}$
$\dfrac{3}{\left(\frac{1}{4}\right)}$	12	$\dfrac{6}{\left(\frac{2}{3}\right)}$	9	$\dfrac{5}{\left(\frac{3}{5}\right)}$	$\frac{25}{3}$
$\dfrac{\left(\frac{1}{10}\right)}{\left(\frac{3}{10}\right)}$	$\frac{1}{3}$	$\dfrac{\left(\frac{3}{5}\right)}{\left(\frac{2}{3}\right)}$	$\frac{9}{10}$	$\dfrac{\left(\frac{5}{6}\right)}{\left(\frac{1}{6}\right)}$	5
$\dfrac{\left(\frac{4}{7}\right)}{\left(\frac{4}{5}\right)}$	$\frac{5}{7}$	$\dfrac{\left(\frac{3}{2}\right)}{\left(\frac{1}{4}\right)}$	6	$\dfrac{\left(\frac{3}{2}\right)}{\left(\frac{6}{5}\right)}$	$\frac{5}{4}$
$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{b}\right)}$	$\frac{a}{c}$	$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{a}{d}\right)}$	$\frac{d}{b}$	$\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}$	$\frac{ad}{bc}$

Automaticity Task 1.5

Rationalise each of the following as quickly and as accurately as you can.

Surd	Rationalised Surd	Surd	Rationalised Surd	Surd	Rationalised Surd
$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{\sqrt{3}}$	$\dfrac{\sqrt{3}}{3}$	$\dfrac{2}{\sqrt{5}}$	$\dfrac{2\sqrt{5}}{5}$
$\dfrac{1}{\sqrt{8}}$	$\dfrac{\sqrt{2}}{4}$	$\dfrac{1}{\sqrt{12}}$	$\dfrac{\sqrt{3}}{6}$	$\dfrac{1}{5\sqrt{5}}$	$\dfrac{\sqrt{5}}{25}$

Automaticity Task 2.1

Expand and simplify as quickly and as accurately as you can.

	Simplified Expansion		Simplified Expansion
$(x+3)^2$	x^2+6x+9	$(4-x)^2$	$16-8x+x^2$
$(2x-3)^2$	$4x^2-12x+9$	$(3-2x)^2$	$9-12x+4x^2$
$(x+5)(x-5)$	x^2-25	$(3x+4)(3x-4)$	$9x^2-16$
$(7-x)(7+x)$	$49-x^2$	$\left(6-\dfrac{x}{2}\right)\left(6+\dfrac{x}{2}\right)$	$36-\dfrac{x^2}{4}$

Automaticity Task 3.1

Factorise each of the following expressions as quickly and as accurately as you can.

	Factorised expression		Factorised expression
$4x^2-49$	$(2x-7)(2x+7)$	$9x^2-100$	$(3x-10)(3x+10)$
$x^2+8x+16$	$(x+4)^2$	$x^2-12x+36$	$(x-6)^2$

Automaticity Task 6.1

Solve as quickly and as accurately as you can.

	Solution		Solution
$x^2=25$	±5	$x^2=3$	$\pm\sqrt{3}$
$(x+3)(x-8)=0$	$-3,8$	$(3x+1)(x-4)=0$	$-\dfrac{1}{3},4$

Automaticity Task 7.1

Simplify as quickly and as accurately as you can, leaving answers with positive indices.

	Answer		Answer		Answer		Answer
$(x^2)^3$	x^6	$(x^4)^{\frac{1}{2}}$	x^2	$(-2x)^{-3}$	$\dfrac{-1}{8x^3}$	$\left(\dfrac{x}{3}\right)^{-2}$	$\dfrac{9}{x^2}$

Automaticity Task 8.1

Solve for x as quickly and as accurately as you can.

	Answer		Answer		Answer
$2^x = 64$	$x = 6$	$3^x = 81$	$x = 4$	$5^x = \dfrac{1}{25}$	$x = -2$

Automaticity Task 9.1

Evaluate as quickly and as accurately as you can.

	Answer		Answer		Answer
$\log_{10} 100$	2	$\log_{10} 0.1$	-1	$\log_2 16$	4

Automaticity Task 12.1

In the axis provided below, sketch as quickly and accurately as you can the lines with equations $y = x + 2$, $x + y = 5$ and $x = -3$.
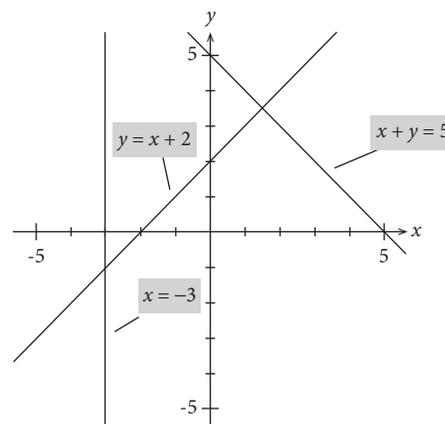
Automaticity Task 17.1

Write as quickly and as accurately as you can the equation of a circle with:

Centre	Radius	Equation	Centre	Radius	Equation
$(0, 0)$	2	$x^2 + y^2 = 4$	$(0, 0)$	6	$x^2 + y^2 = 36$
$(1, 0)$	4	$(x - 1)^2 + y^2 = 16$	$(3, 4)$	5	$(x - 3)^2 + (y - 4)^2 = 25$

Automaticity Task 18.1

Complete the following table as quickly and as accurately as you can.

x	0º	30º	45º	60º	90º
$\sin(x)$	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
$\tan(x)$	0	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

Automaticity Task 19.1

Complete the following table as quickly and as accurately as you can.

x	120º	135º	150º	180º
$\sin(x)$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
$\cos(x)$	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{\sqrt{3}}{2}$	-1
$\tan(x)$	$-\sqrt{3}$	-1	$-\dfrac{\sqrt{3}}{3}$	0

Automaticity Task 20.1

Complete the following table as quickly and as accurately as you can.

x	210º	225º	300º	330º
$\sin(x)$	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{1}{2}$
$\cos(x)$	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$
$\tan(x)$	$\dfrac{\sqrt{3}}{3}$	1	$-\sqrt{3}$	$-\dfrac{\sqrt{3}}{3}$

Automaticity Task 24.1

In the Venn diagrams provided, shade each of the following regions as quickly and accurately as you can.

$\mathbf{A} \cup \mathbf{B}$

$\mathbf{A} \cap \mathbf{\overline{B}}$

$\mathbf{\overline{A}} \cap \mathbf{\overline{B}}$
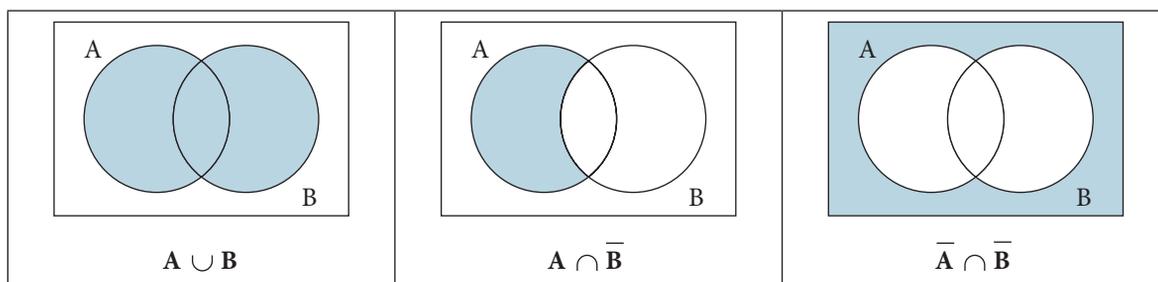
Solutions to Practice Tasks

Practice Task 1.1

Simplify each of the following: (a) $\sqrt{48}$ (b) $\sqrt{45} + \sqrt{125}$

(a) $\sqrt{48} = \sqrt{16 \times 3}$
$= \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

(b) $\sqrt{45} + \sqrt{125} = \sqrt{9 \times 5} + \sqrt{25 \times 5}$
$= 3\sqrt{5} + 5\sqrt{5} = 8\sqrt{5}$

Practice Task 1.2

Expand and simplify each of the following: (a) $\left(\sqrt{5} - 3\right)\left(\sqrt{5} + 3\right)$ (b) $\left(\sqrt{8} + \sqrt{2}\right)^2$

(a) $\left(\sqrt{5} - 3\right)\left(\sqrt{5} + 3\right) = \left(\sqrt{5}\right)^2 - (3)^2$
$= 5 - 9 = -4$

(b) $\left(\sqrt{8} + \sqrt{2}\right)^2 = \left(\sqrt{8}\right)^2 + 2\sqrt{8}\sqrt{2} + \left(\sqrt{2}\right)^2$
$= 8 + 8 + 2 = 18$

Practice Task 1.3

Rationalise and simplify each of the following: (a) $\dfrac{2}{\sqrt{20} - \sqrt{5}}$ (b) $\dfrac{1 - \sqrt{2}}{3 + 2\sqrt{2}}$

(a) $\dfrac{2}{\sqrt{20} - \sqrt{5}} = \dfrac{2}{\sqrt{20} - \sqrt{5}} \times \dfrac{\sqrt{20} + \sqrt{5}}{\sqrt{20} + \sqrt{5}}$
$= \dfrac{2\sqrt{20} + 2\sqrt{5}}{20 - 5}$
$= \dfrac{6\sqrt{5}}{15}$
$= \dfrac{2\sqrt{5}}{5}$

(b) $\dfrac{1 - \sqrt{2}}{3 + 2\sqrt{2}} = \dfrac{1 - \sqrt{2}}{3 + 2\sqrt{2}} \times \dfrac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$
$= \dfrac{3 - 2\sqrt{2} - 3\sqrt{2} + 4}{9 - 8}$
$= 7 - 5\sqrt{2}$

Practice Task 2.1

Expand and simplify: (a) $(1 - 3x)^2 - (2x - 1)^2$ (b) $(x - 5)(2 - x) - (4x - 3)(4x + 3)$

(a) $(1 - 3x)^2 - (2x - 1)^2$
$= 1 - 6x + 9x^2 - (4x^2 - 4x + 1)$
$= 5x^2 - 2x$

(b) $(x - 5)(2 - x) - (4x - 3)(4x + 3)$
$= (-x^2 + 7x - 10) - (16x^2 - 9)$
$= -17x^2 + 7x - 1$

Practice Task 2.2

Expand and simplify: (a) $(x-2)(2x^2-x+3)$ (b) $(3-2x)(2x^2+x+1)$

(a) $(x-2)(2x^2-x+3)$	(b) $(3-2x)(2x^2+x+1)$
$\quad = 2x^3-x^2+3x-4x^2+2x-6$	$\quad = 6x^2+3x+3-4x^3-2x^2-2x$
$\quad = 2x^3-5x^2+5x-6$	$\quad = -4x^3+4x^2+x+3$

Practice Task 2.3

Expand and simplify: (a) $(x-2)^3$ (b) $(3+2x)^3$

(a) $(x-2)^3 = (x-2)(x-2)^2$	(b) $(3+2x)^3 = (3+2x)(3+2x)^2$
$\quad = (x-2)\,(x^2-4x+4)$	$\quad = (3+2x)\,(9+12x+4x^2)$
$\quad = x^3-4x^2+4x-2x^2+8x-8$	$\quad = 27+36x+12x^2+18x+24x^2+8x^3$
$\quad = x^3-6x^2+12x-8$	$\quad = 27+54x+36x^2+8x^3$

Practice Task 3.1

Factorise completely: (a) $5x^2-45$ (b) $36x^2-64y^2$

(a) $5x^2-45 = 5(x^2-9)$	(b) $36x^2-64y^2 = 4(9x^2-16y^2)$
$\quad = 5(x-3)(x+3)$	$\quad = 4(3x-4y)(3x+4y)$

Practice Task 3.2

Factorise completely: (a) $x(x+2)+3(x+2)$ (b) $2(1-x)-x(x-1)$

(a) $x(x+2)+3(x+2) = (x+3)(x+2)$	(b) $2(1-x)-x(x-1) = 2(1-x)+x(1-x)$
	$\quad = (2+x)(1-x)$

Practice Task 3.3

Factorise completely: (a) $x^2+4x-32$ (b) $2x^2+x-6$

(a) $x^2+4x-32 = (x-4)(x+8)$	(b) $2x^2+x-6 = (2x-3)(x+2)$

Practice Task 3.4

Factorise completely: (a) $-x^2+x+6$ (b) $-3x^2+4x+4$

(a) $-x^2+x+6 = -(x^2-x-6)$	(b) $-3x^2+4x+4 = -(3x^2-4x-4)$
$\quad = -(x-3)(x+2)$	$\quad = -(3x+2)(x-2)$

Practice Task 3.5

Factorise completely:　(a) $4x^2 - 4x - 3$　　　(b) $4x^2 - x - 3$

(a) $4x^2 - 4x - 3 = (2x - 3)(2x + 1)$	(b) $4x^2 - x - 3 = (4x + 3)(x - 1)$

Practice Task 4.1

Use polynomial division to determine the quotient and remainder when $x^2 - 5x + 7$ is divided by $x - 2$.

$$
\begin{array}{r}
x - 3 \\
x - 2 \overline{\smash{\big)}\ x^2 - 5x + 7} \\
\underline{x^2 - 2x} \\
-3x + 7 \\
\underline{-3x + 6} \\
1
\end{array}
$$

Hence, quotient is $x - 3$ and remainder is 1.

Practice Task 4.2

Use polynomial division to determine the quotient and remainder when $-x^3 + 3x^2 - x - 1$ is divided by $x + 2$.

$$
\begin{array}{r}
-x^2 + 5x - 11 \\
x + 2 \overline{\smash{\big)}\ -x^3 + 3x^2 - x - 1} \\
\underline{-x^3 - 2x^2} \\
5x^2 - x \\
\underline{5x^2 + 10x} \\
-11x - 1 \\
\underline{-11x - 22} \\
21
\end{array}
$$

Hence, quotient is $-x^2 + 5x - 11$ and remainder is 21.

Practice Task 4.3

Determine the remainder when $x^3 - 2x^2 - 3x + 4$ is divided by $(x + 1)$.

> Let $f(x) = x^3 - 2x^2 - 3x + 4$.
> Using the Remainder Theorem,
> $$\text{Remainder} = f(-1)$$
> $$= -1 - 2 + 3 + 4$$
> $$= 4$$

Practice Task 4.4

Show that $(x - 1)$ is a factor of $f(x) = x^3 + 5x^2 + 3x - 9$. Hence, factorise completely $f(x)$.

> Remainder $= f(1)$
> $$= 1 + 5 + 3 - 9 = 0.$$
> Hence, $(x - 1)$ is a factor.
>
> Using polynomial division:
>
> $x^3 + 5x^2 + 3x - 9 = (x - 1)(x^2 + 6x + 9)$
> $$= (x - 1)(x + 3)^2$$
>
> $$\begin{array}{r} x^2 + 6x + 9 \\ x - 1 \overline{)\, x^3 + 5x^2 + 3x - 9} \\ x^3 - x^2 \\ \hline 6x^2 + 3x \\ 6x^2 - 6x \\ \hline 9x - 9 \\ 9x - 9 \\ \hline 0 \end{array}$$

Practice Task 4.5

Factorise completely $f(x) = x^3 - 7x - 6$.

> $f(1) = 1 - 7 - 6 \neq 0.$
> $f(-1) = -1 + 7 - 6 = 0$
> Hence, $(x + 1)$ is a factor.
>
> Using polynomial division:
>
> $x^3 - 7x - 6 = (x + 1)(x^2 - x - 6)$
> $$= (x + 1)(x + 2)(x - 3)$$
>
> $$\begin{array}{r} x^2 - x - 6 \\ x + 1 \overline{)\, x^3 + 0x^2 - 7x - 6} \\ x^3 + x^2 \\ \hline -x^2 - 7x \\ -x^2 - x \\ \hline -6x - 6 \\ -6x - 6 \\ \hline 0 \end{array}$$
>
> | *Note that the first factor is obtained by trial and error using the Remainder Theorem.* |
> |---|

Practice Task 5.1

Factorise completely $x^2 + 16x + 64$. Hence, complete the squares for $x^2 + 16x + 60$.

$$x^2 + 16x + 64 = (x + 8)^2$$

Clearly
$$x^2 + 16x + 60 = x^2 + 16x + 64 - 4$$
$$= (x + 8)^2 - 4$$

Practice Task 5.2

Express as a completed square $x^2 - 12x + 20$.

$$x^2 - 12x + 20 = (x^2 - 12x + 36) - 36 + 20$$
$$= (x - 6)^2 - 16$$

Practice Task 5.3

Express as a completed square $-x^2 + 3x - 3$.

$$-x^2 + 3x - 3 = -[x^2 - 3x + 3]$$
$$= -\left[\left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} + 3\right]$$
$$= -\left(x - \frac{3}{2}\right)^2 - \frac{3}{4}$$

Practice Task 5.4

Express as a completed square $2x^2 + 4x - 5$.

$$2x^2 + 4x - 5 = 2\left[x^2 + 2x - \frac{5}{2}\right]$$
$$= 2\left[(x^2 + 2x + 1) - 1 - \frac{5}{2}\right]$$
$$= 2(x + 1)^2 - 7$$

Practice Task 6.1

Solve for x: (a) $x^2 + 2x - 24 = 0$ (b) $(1 - x)^2 - 5 = 0$

(a) $x^2 + 2x - 24 = 0 \implies (x + 6)(x - 4) = 0$ (b) $(1 - x)^2 - 5 = 0 \implies (1 - x)^2 = 5$

$$x = -6,\ 4$$

$$1 - x = \pm\sqrt{5}$$
$$x = 1 \pm \sqrt{5}$$

Practice Task 6.2

Solve for x: (a) $2x^2 - x = 6$ (b) $2x^2 + x - 2 = 0$

(a) $2x^2 - x - 6 = 0$ (b) $2x^2 + x - 2 = 0$

$$\implies (2x + 3)(x - 2) = 0$$

$$x = -\frac{3}{2},\ 2$$

$$\implies x = \frac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$$

$$x = \frac{-1 \pm \sqrt{17}}{4}$$

Practice Task 6.3

Solve for x in $x^3 - 6x^2 + 11x - 6 = 0$.

Let $f(x) = x^3 - 6x^2 + 11x - 6$

$f(1) = 1 - 6 + 11 - 6 = 0$

Hence, $(x - 1)$ is a factor.

Using polynomial division:

$x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6)$

$\qquad\qquad\qquad\quad = (x - 1)(x - 2)(x - 3)$

Hence, $f(x) = 0 \implies x = 1,\ 2,\ 3$

$$
\begin{array}{r}
x^2 - 5x + 6 \\
x - 1 \overline{)\, x^3 - 6x^2 + 11x - 6} \\
\underline{x^3 - x^2} \\
-5x^2 + 11x \\
\underline{-5x^2 + 5x} \\
6x - 6 \\
6x - 6 \\
\underline{} \\
0
\end{array}
$$

Practice Task 7.1

Simplify, leaving answers with positive indices: (a) $\dfrac{48a^2 b^{-2}}{15a^5 b^2}$ (b) $\dfrac{21b^2}{14a^{-4}} \times \dfrac{36a^{-5}}{15b}$

(a) $\dfrac{48a^2 b^{-2}}{15a^5 b^2} = \dfrac{16}{5a^{5-2}b^{2-(-2)}}$

$\qquad = \dfrac{16}{5a^3 b^4}$

(b) $\dfrac{21b^2}{14a^{-4}} \times \dfrac{36a^{-5}}{15b} = \dfrac{18b^{2-1}}{5a^{-4-(-5)}}$

$\qquad = \dfrac{18b}{5a}$

Practice Task 7.2

Simplify, leaving answers with positive indices: (a) $\left(\dfrac{3a}{b^{-2}}\right)^2$ (b) $\left(\dfrac{a^{\frac{1}{2}}}{4b}\right)^{-2}$

(a) $\left(\dfrac{3a}{b^{-2}}\right)^2 = (3ab^2)^2$

   $= 9a^2b^4$

(b) $\left(\dfrac{a^{\frac{1}{2}}}{4b}\right)^{-2} = \left(\dfrac{4b}{a^{\frac{1}{2}}}\right)^2$

   $= \dfrac{16b^2}{a}$

Practice Task 7.3

Simplify, leaving answers with positive indices: (a) $\left(\dfrac{64a^6}{125b^{-3}}\right)^{\frac{1}{3}}$ (b) $\sqrt{\dfrac{25a^{-8}}{144b^4}}$

(a) $\left(\dfrac{64a^6}{125b^{-3}}\right)^{\frac{1}{3}} = \left(\dfrac{64a^6b^3}{125}\right)^{\frac{1}{3}}$

   $= \dfrac{4a^2b}{5}$

(b) $\sqrt{\dfrac{25a^{-8}}{144b^4}} = \sqrt{\dfrac{25}{144a^8b^4}}$

   $= \dfrac{5}{12a^4b^2}$

Practice Task 8.1

Solve for x: (a) $5^{3-x} = 625$ (b) $2^{-\frac{x}{2}} = 128$

(a) $5^{3-x} = 625 \;\Rightarrow\; 5^{3-x} = 5^4$

   $3 - x = 4$

   $x = -1$

(b) $2^{-\frac{x}{2}} = 128 \Rightarrow 2^{-\frac{x}{2}} = 2^7$

   $-\dfrac{x}{2} = 7$

   $x = -14$

Practice Task 8.2

Solve for x: (a) $9^{-x} = 27^{1+x}$ (b) $4^{2x-1} = 8^x$

(a) $9^{-x} = 27^{1+x} \;\Rightarrow\; (3^2)^{-x} = (3^3)^{1+x}$

   $3^{-2x} = 3^{3+3x}$

   $-2x = 3 + 3x$

   $x = -\dfrac{3}{5}$

(b) $4^{2x-1} = 8^x \;\Rightarrow\; (2^2)^{2x-1} = (2^3)^x$

   $2^{4x-2} = 2^{3x}$

   $4x - 2 = 3x$

   $x = 2$

Practice Task 8.3

Solve for x: (a) $3 \times 4^{2x} = 96$ (b) $20 \times 2^{0.5x} = 160 \times 2^{-0.1x}$

(a) $3 \times 4^{2x} = 96 \Rightarrow 4^{2x} = \dfrac{96}{3}$

$\qquad 4^{2x} = 32$

$\qquad (2^2)^{2x} = 2^5$

$\qquad 4x = 5$

$\qquad x = \dfrac{5}{4}$

(b) $\qquad 20 \times 2^{0.5x} = 160 \times 2^{-0.1x}$

$\qquad \dfrac{2^{0.5x}}{2^{-0.1x}} = \dfrac{160}{20}$

$\qquad 2^{0.5x - (-0.1x)} = 8$

$\qquad 2^{0.6x} = 2^3$

$\qquad 0.6x = 3$

$\qquad x = 5$

Practice Task 9.1

Express in exponential form to solve for x: (a) $\log_{10} x = 3$ (b) $\log_x 81 = 2$

(a) $\log_{10} x = 3 \;\Rightarrow\; x = 10^3$

$\qquad x = 1\,000$

(b) $\log_x 81 = 2 \;\Rightarrow\; 81 = x^2$

$\qquad x = 9$

Practice Task 9.2

Express in logarithmic form to solve for x: (a) $2^x = 5$ (b) $3^{2x+1} = 17$

(a) $2^x = 5 \;\Rightarrow\; x = \log_2 5$

(b) $3^{2x+1} = 17 \;\Rightarrow\; 2x + 1 = \log_3 17$

$\qquad x = \dfrac{1}{2}\left(\log_3 17 - 1\right)$

Practice Task 9.3

Simplify each of the following: (a) $\log_6 (x^2 - 1) - 2\log_6 (x + 1)$ (b) $\log_9 (x + 3) + \dfrac{1}{2}$

(a) $\log_6 (x^2 - 1) - 2\log_6 (x + 1)$

$= \log_6 (x^2 - 1) - \log_6 (x + 1)^2$

$= \log_6 \dfrac{(x^2 - 1)}{(x+1)^2} = \log_6 \dfrac{(x-1)(x+1)}{(x+1)^2}$

$= \log_6 \left(\dfrac{x-1}{x+1}\right)$

(b) $\log_9 (x + 3) + \dfrac{1}{2}$

$= \log_9 (x+3) + \dfrac{1}{2}\log_9 9$

$= \log_9 (x+3) + \log_9 9^{\frac{1}{2}}$

$= \log_9 [3(x+3)]$

Practice Task 9.4

Expand each of the following: (a) $\log_2 x(x+5)$ (b) $\log_3\left(\dfrac{x-2}{x+4}\right)$

(a) $\log_2 x(x+5) = \log_2 x + \log_2 (x+5)$ (b) $\log_3\left(\dfrac{x-2}{x+4}\right) = \log_3 (x-2) - \log_3 (x+4)$

Practice Task 10.1

Solve for x in terms of $\log_{10}$ where appropriate   (a) $10^x = 50$ (b) $6^{1-x} = 100$

(a) $10^x = 50 \Rightarrow \log_{10} 10^x = \log_{10} 50$

$\qquad\qquad x \log_{10} 10 = \log_{10} 50$

$\qquad\qquad\qquad x = \log_{10} 50$

(b) $6^{1-x} = 100$

$\qquad \Rightarrow \log_{10} 6^{1-x} = \log_{10} 100$

$\qquad (1-x)\log_{10} 6 = \log_{10} 100$

$\qquad\qquad (1-x) = \dfrac{2}{\log_{10} 6}$

$\qquad\qquad\quad x = 1 - \dfrac{2}{\log_{10} 6}$

Practice Task 10.2

Solve for x in terms of $\log_{10}$ where appropriate:

(a) $40(8^x) = 100$ (b) $100(6^{-x}) = 300(6^{-2x-1})$

(a) $\quad 40(8^x) = 100$

$\qquad 8^x = \dfrac{100}{40}$

$\qquad \log_{10} 8^x = \log_{10}\left(\dfrac{5}{2}\right)$

$\qquad x \log_{10} 8 = \log_{10}\left(\dfrac{5}{2}\right)$

$\qquad x = \dfrac{\log_{10}\left(\dfrac{5}{2}\right)}{\log_{10} 8}$

(b) $\qquad 100(6^{-x}) = 300(6^{-2x-1})$

$\qquad \dfrac{6^{-x}}{6^{-2x-1}} = \dfrac{300}{100}$

$\qquad 6^{-x-(-2x-1)} = 3$

$\qquad 6^{x+1} = 3$

$\qquad (x+1)\log_{10} 6 = \log_{10} 3$

$\qquad \left(x+1\right) = \dfrac{\log_{10} 3}{\log_{10} 6}$

$\qquad x = \dfrac{\log_{10} 3}{\log_{10} 6} - 1$

Practice Task 11.1

Solve for x and y. $x + 2y = 5$ $x - 3y = 4$

$$x + 2y = 5 \qquad \text{I}$$
$$x - 3y = 4 \qquad \text{II}$$

$$\text{I} - \text{II} \qquad\qquad 5y = 1$$
$$y = \frac{1}{5}$$

$$\text{Substitute into I:} \qquad x = 5 - \frac{2}{5}$$
$$= \frac{23}{5}$$

Practice Task 11.2

Solve for x and y. $5x + 3y = 5$ $3x + 2y = 4$

$$5x + 3y = 5 \qquad \text{I}$$
$$3x + 2y = 4 \qquad \text{II}$$

$$\text{I} \times 2 \qquad\qquad 10x + 6y = 10 \qquad \text{III}$$
$$\text{II} \times 3 \qquad\qquad 9x + 6y = 12 \qquad \text{IV}$$

$$\text{III} - \text{IV} \qquad\qquad x = -2$$
$$y = 5$$

Practice Task 11.3

Solve for x and y. $x + y = 4$ $y = x^2 - 3x + 1$

$$x + y = 4 \implies y = 4 - x$$
$$\text{Substitute into } y = x^2 - 3x + 1:$$
$$4 - x = x^2 - 3x + 1$$
$$x^2 - 2x - 3 = 0$$
$$(x + 1)(x - 3) = 0$$
$$x = -1, 3$$
$$\text{Hence:} \qquad x = -1, y = 5; x = 3, y = 1$$

Practice Task 12.1

Determine the equation of the line passing through (2, 12) and:

(a) the point (−4, 6).

(b) parallel to the line with equation $3x + 4y = 12$.

(c) perpendicular to the line with equation $x − 3y = 9$.

(a)	Gradient of line:	$m = \dfrac{12 - 6}{2 - (-4)} = 1$
	Equation of line:	$y - 12 = x - 2 \quad \Rightarrow \quad y = x + 10$
(b)	Rewrite equation of given line:	$3x + 4y = 12 \quad \Rightarrow \quad 4y = -3x + 12$
	Hence, gradient of required line:	$m = -\dfrac{3}{4}$
	Equation of required line:	$y - 12 = -\dfrac{3}{4}(x - 2) \Rightarrow y = -\dfrac{3}{4}x + \dfrac{27}{2}$
(c)	Rewrite equation of given line:	$x - 3y = 6 \quad \Rightarrow \quad 3y = x - 6$
	Hence, gradient of required line:	$m = \dfrac{-1}{\left(\dfrac{1}{3}\right)} = -3$
	Equation of required line:	$y - 12 = -3(x - 2) \quad \Rightarrow \quad y = -3x + 18$

Practice Task 12.2

Find the coordinates of the point of intersection between the lines $x + 2y = 11$ and $y = 2x + 3$.

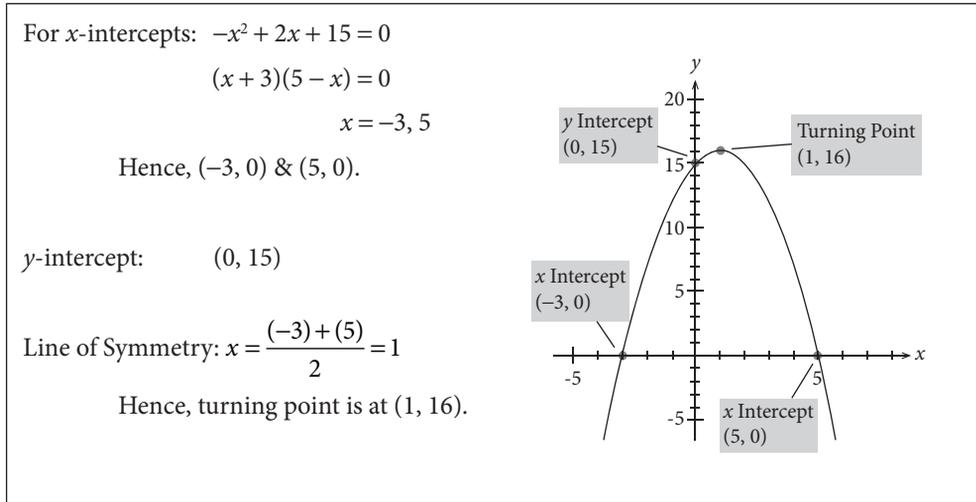|	
Substitute $y = 2x + 3$ into $x + 2y = 11$. $\quad x + 2(2x + 3) = 11$ $\qquad\qquad 5x = 5 \quad \Rightarrow \quad x = 1$ $\qquad\qquad\quad y = 5$ Hence, point of intersection is (1, 5).	OR $\qquad x + 2y = 11 \qquad$ I $\qquad\qquad 2x - y = -3 \qquad$ II I × 2 $\qquad 2x + 4y = 22 \qquad$ III III − II $\qquad 5y = 25 \quad \Rightarrow \quad y = 5, x = 1$ Hence, point of intersection is (1, 5).

Practice Task 13.1

Sketch the curve with equation $y = 2(x − 1)^2 + 3$. State the coordinates of the intercept(s) and turning point.

For x-intercepts: $\quad 2(x - 1)^2 + 3 = 0$

$\qquad\qquad\qquad 2(x - 1)^2 = -3$

Equation has no real solutions,

hence, no x-intercepts.

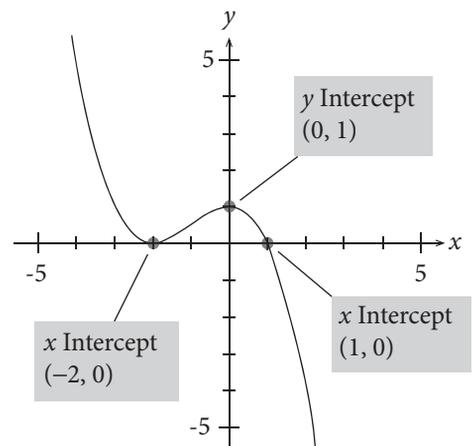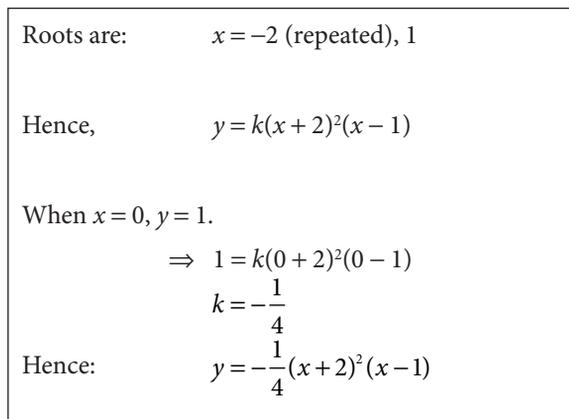y-intercept: $\qquad (0, 5)$

Turning Point: $\qquad (1, 3)$

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Practice Task 13.2

Sketch the curve with equation $y = -x^2 + 2x + 15$. State the coordinates of the intercept(s) and turning point.

For x-intercepts: $-x^2 + 2x + 15 = 0$

$$(x+3)(5-x) = 0$$

$$x = -3, 5$$

Hence, $(-3, 0)$ & $(5, 0)$.

y-intercept: $(0, 15)$

Line of Symmetry: $x = \dfrac{(-3)+(5)}{2} = 1$

Hence, turning point is at $(1, 16)$.

y Intercept
(0, 15)

Turning Point
(1, 16)

x Intercept
(−3, 0)

x Intercept
(5, 0)

Practice Task 14.1

The accompanying diagram shows a cubic curve. Determine the equation of this curve.

Roots are: $x = -2$ (repeated), 1

Hence, $y = k(x+2)^2(x-1)$

When $x = 0$, $y = 1$.

$$\Rightarrow \quad 1 = k(0+2)^2(0-1)$$

$$k = -\frac{1}{4}$$

Hence: $y = -\dfrac{1}{4}(x+2)^2(x-1)$

y Intercept
(0, 1)

x Intercept
(−2, 0)

x Intercept
(1, 0)

Practice Task 14.2

Sketch the curve with equation $y = 2x^3 - 5x^2 - 2x + 5$. Indicate all intercepts.

For x-intercepts: $2x^3 - 5x^2 - 2x + 5 = 0$

Let $f(x) = 2x^3 - 5x^2 - 2x + 5$.

$f(1) = 2 - 5 - 2 + 5 = 0$

Hence, $(x - 1)$ is a factor.

Using polynomial division:

$2x^3 - 5x^2 - 2x + 5 = (x - 1)(2x^2 - 3x - 5)$

$= (x - 1)(2x - 5)(x + 1)$

For x-intercepts: $(x - 1)(2x - 5)(x + 1) = 0$

$$x = -1, 1, \frac{5}{2}$$

y-intercept: $(0, 5)$

$$
\begin{array}{r}
2x^2 - 3x - 5 \\
x - 1 \overline{)2x^3 - 5x^2 - 2x + 5} \\
\underline{2x^3 - 2x^2} \\
-3x^2 - 2x \\
-3x^2 + 3x \\
\underline{} \\
-5x + 5 \\
-5x + 5 \\
\underline{} \\
0
\end{array}
$$

Practice Task 15.1

The accompanying diagram shows a rectangular hyperbola. Determine the equation of this curve.

Equation is of the form $y = \dfrac{k}{x - 1} - 2$

When $x = 0$, $y = \dfrac{-5}{2}$:

$$\frac{-5}{2} = \frac{k}{0 - 1} - 2$$

$$\frac{-5}{2} = -k - 2.$$

$$k = \frac{1}{2}$$

Hence: $y = \dfrac{1}{2(x - 1)} - 2$
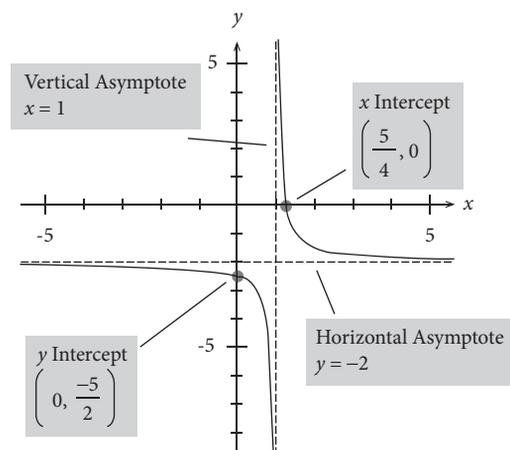
Practice Task 15.2

Sketch the curve with equation $y = 2 - \dfrac{2}{x-3}$. State all intercepts and asymptotes.

When $x = 0$, $\qquad y = 2 - \dfrac{2}{0-3} = \dfrac{8}{3}$.

Hence, vertical-intercept is $\left(0, \dfrac{8}{3}\right)$.

When $y = 0$ $\qquad 2 - \dfrac{2}{x-3} = 0$

$\qquad\qquad\qquad \dfrac{2}{x-3} = 2$

$\qquad\qquad\qquad 2(x-3) = 2$

$\qquad\qquad\qquad x = 4$

Hence, horizontal-intercepts is $(4, 0)$

Horizontal Asymptote: $\qquad y = 2$

Vertical Asymptote: $\qquad x = 3$
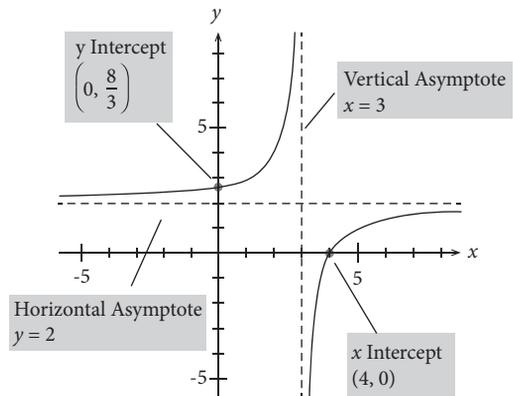
y Intercept $\left(0, \dfrac{8}{3}\right)$

Vertical Asymptote $x = 3$

Horizontal Asymptote $y = 2$

x Intercept $(4, 0)$

Practice Task 16.1

The accompanying diagram shows the graph of $y = a(3^x) + b$. Determine the values of a and b.

From sketch,

horizontal asymptote is $y = 3$.

$\qquad\qquad \Rightarrow \quad b = 3$.

For vertical intercept: $\quad x = 0$

$\qquad\qquad \Rightarrow \quad y = a + (3)$

But vertical intercept is $(0, 1)$.

Hence, $\qquad\qquad 1 = a + (3)$

$\qquad\qquad \Rightarrow \quad a = -2$
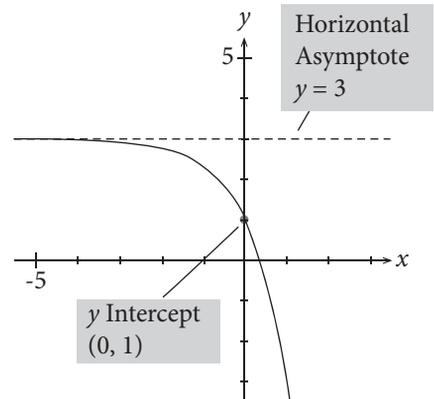
Horizontal Asymptote $y = 3$

y Intercept $(0, 1)$

Practice Task 16.2

(a) On the axes below, sketch the curve with equation $y = 3(2^x) - 4$. State the equation of the horizontal asymptote and the coordinates of the vertical intercept.
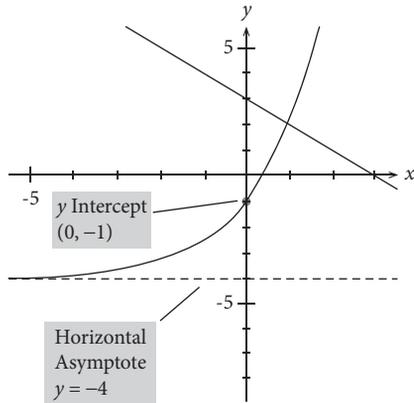
For the y-intercept:

$x = 0 \implies y = 3(1) - 4 = -1$

Hence, $(0, -1)$.

Horizontal asymptote has equation $y = -4$.

(b) On the same axes, sketch the line $x + y = 3$. Use your sketch to estimate the solution to the equation $3(2^x) = 7 - x$. Explain how you obtained your answer.
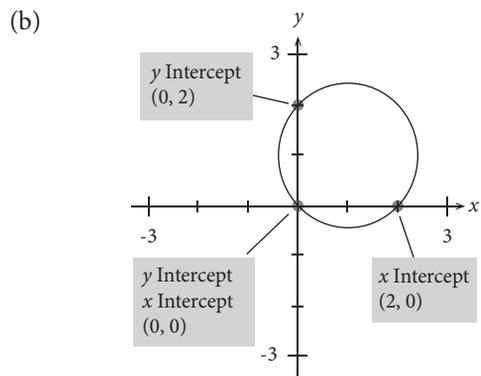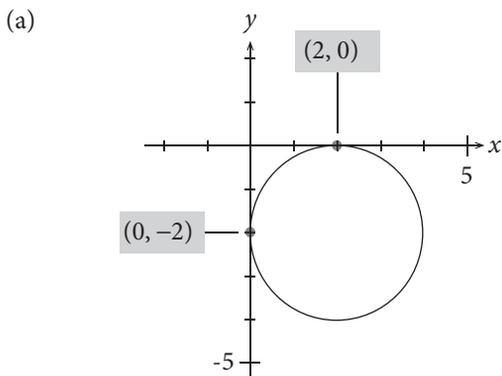
Rewrite $3(2^x) = 7 - x$ as $3(2^x) = 4 + (3 - x)$

$$3(2^x) - 4 = 3 - x$$

Hence, solution is the x-coordinate of the point of intersection between $y = 3(2^x) - 4$ and $x + y = 3$.

From sketch: $x \approx 1$.

Practice Task 17.1

State the equation of each of the circles drawn.

(a)

(b)

(a) Centre of circle is $(2, -2)$.

Radius $= 2$

Hence, equation is $(x - 2)^2 + (y + 2)^2 = 4$

(b) Centre of circle is $(1, 1)$.

Using Pythagoras Theorem:

Radius $= \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, equation is $(x - 1)^2 + (y - 1)^2 = 2$

Practice Task 17.2

Determine the coordinates of the centre and the radius of the circle with equation $x^2 + 6x + y^2 - 2y - 15 = 0$.

Rewrite as completed squares:
$$x^2 + 6x + y^2 - 2y \equiv (x + 3)^2 - 9 + (y - 1)^2 - 1$$

Hence,
$$x^2 + 6x + y^2 - 2y = 14$$

becomes:
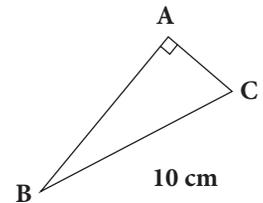$$(x + 3)^2 - 9 + (y - 1)^2 - 1 = 15$$
$$(x + 3)^2 + (y - 1)^2 = 25$$

Hence, centre of circle has coordinates $(-3, 1)$ and radius of circle = 5.

Practice Task 18.1

In $\triangle ABC$, $\angle BAC = 90°$, $BC = 10$ cm and $\cos \angle ABC = \dfrac{1}{5}$.

Determine the exact length of AC.

$$\cos \angle ABC = \frac{AB}{BC} \implies AB = 10 \times \cos \angle ABC$$
$$= 10 \times \frac{1}{5} = 2 \text{ cm}$$

Using Pythagoras Theorem:
$$AC = \sqrt{10^2 - 2^2}$$
$$= \sqrt{96}$$
$$= 4\sqrt{6} \text{ cm}$$

Practice Task 18.2

In $\triangle ABC$, $\angle ACB = 90°$, $BC = 20$ cm and $\cos \angle ABC = \dfrac{4}{5}$. The point K on AB is foot of the perpendicular from C to AB.

Determine: (a) BK   (b) AK.
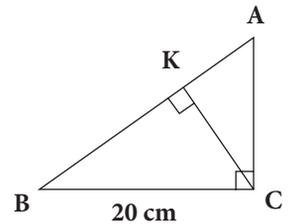
(a) In $\triangle KBC$: $\angle KBC = \angle ABC$
$$\cos KBC = \frac{BK}{BC}$$
$$\implies BK = 20 \times \frac{4}{5} = 16 \text{ cm}.$$

(b) In $\triangle ABC$: $\cos ABC = \dfrac{BC}{BA}$
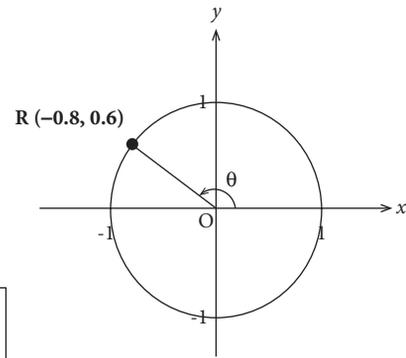$$\implies BA = \frac{20}{\left(\dfrac{4}{5}\right)} = 20 \text{ cm}.$$

Hence: $AK = 25 - 16 = 9$ cm.

Practice Task 19.1

R is a point on a unit circle with coordinates $(-0.8, 0.6)$.
The ray OR forms an angle of size $\theta°$ with the positive
x-axis as shown in the accompanying diagram.
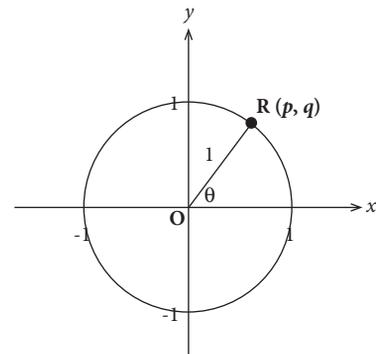Determine the value of: (a) $\cos \theta°$ (b) $\tan(180° - \theta°)$.

$$(a) \quad \cos \theta° = -0.8$$
$$(b) \quad \tan(180° - \theta°) = -\tan(\theta°)$$
$$= -\frac{\sin(\theta°)}{\cos(\theta°)}$$
$$= -\frac{0.6}{(-0.8)} = \frac{3}{4}$$

Practice Task 19.2

R is a point on a unit circle with coordinates (p, q).
The ray OR forms an angle of size $\theta°$ with the positive
x-axis as shown in the accompanying diagram. Given that
$\sin \theta° = \dfrac{2}{3}$, determine the value of p and q.

Since $\sin \theta° = \dfrac{2}{3}$, $q = \dfrac{2}{3}$
R lies on a unit circle.

Hence, using Pythagoras Theorem:
$$p^2 + q^2 = 1$$
$$p^2 = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$$
$$p = \frac{\sqrt{5}}{3} \quad \left(\text{reject } -\frac{\sqrt{5}}{3}\right)$$

Practice Task 20.1

Solve for $x°$, where $0° \le x \le 360°$, if $\sin x° = -\dfrac{\sqrt{3}}{2}$.

$\sin x° = -\dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad$ Reference angle for $x° = 60°$
The sine function is negative in quadrants 3 and 4.

Hence: $\quad x° = 180° + 60°, 360° - 60°$
$$= 240°, 300°$$

Practice Task 20.2

Given that $\cos^{-1}\left(\dfrac{3}{4}\right) = 41.4°$, solve for $x°$, where $0° \le x \le 360°$, if $\cos x° = \dfrac{3}{4}$.

$\cos x° = \dfrac{3}{4} \implies$ Reference angle for $x° = \cos^{-1}\left(\dfrac{3}{4}\right) = 41.4°$

The cosine function is positive in quadrants 1 and 4.
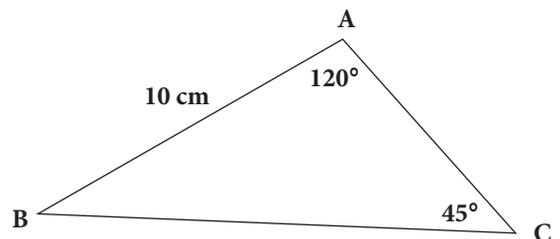
Hence: $x° = 41.4°,\ 360° - 41.4°$

$= 41.4°,\ 318.6°$

Practice Task 20.3

Given that $\tan^{-1}\left(\dfrac{5}{2}\right) = 68.2°$, solve for $x°$, where $0° \le x \le 360°$, if $\tan x° = -\dfrac{5}{2}$.

$\tan x° = -\dfrac{5}{2} \implies$ Reference angle for $x° = \tan^{-1}\left(\dfrac{5}{2}\right) = 68.2°$

The tangent function is negative in quadrants 2 and 4.

Hence: $x° = 180° - 68.2°,\ 360° - 68.2°$

$= 111.8°,\ 291.8°$

Practice Task 21.1

In $\triangle ABC$, $\angle BAC = 120°$, $\angle ACB = 45°$ and
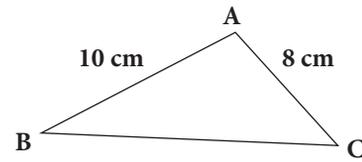$AB = 10$ cm. Calculate the length of BC.

Using the Sine Rule:

$$\frac{BC}{\sin 120°} = \frac{10}{\sin 45°}$$

$$BC = \frac{10}{\sin 45°} \times \sin 120°$$

$$= \frac{10}{\left(\dfrac{\sqrt{2}}{2}\right)} \times \frac{\sqrt{3}}{2} = 5\sqrt{6} \text{ cm}$$

Practice Task 21.2

In $\triangle ABC$, $\angle ABC$ is an acute angle such that $\sin \angle ABC = \dfrac{1}{4}$, $AB = 10$ cm and $AC = 8$ cm. Use the calculator screen-dump to calculate the size of $\angle ACB$.

Using the Sine Rule:

$$\frac{\sin A\hat{C}B}{10} = \frac{\sin A\hat{B}C}{8}$$
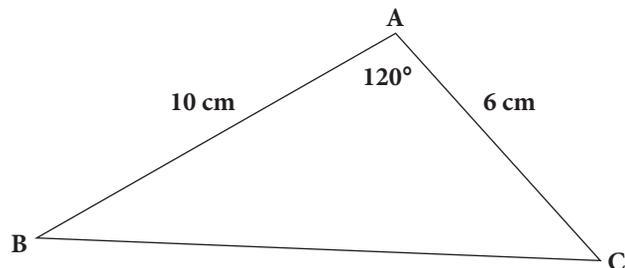
$$\sin A\hat{C}B = \frac{\sin A\hat{B}C}{8} \times 10$$

$$= \frac{\left(\dfrac{1}{4}\right)}{8} \times 10 = \frac{5}{16}$$

Hence: $\angle ACB = 18.2°$ or $180° - 18.2°$

$= 18.2°$

or $161.8°$ (Accept $161.8°$ as $161.8° + 14.5° < 180°$)

Practice Task 22.1

In $\triangle ABC$, $\angle BAC = 120°$, $AB = 10$ cm and $AC = 6$ cm.
Calculate the length of BC.

Using the Cosine Rule:

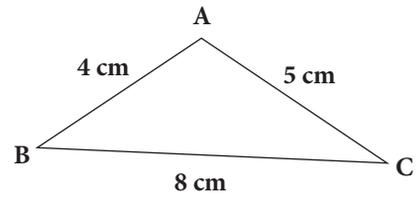$$BC^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos 120°$$

$$= 100 + 36 - 2 \times 10 \times 6 \times \left(-\frac{1}{2}\right)$$

$$= 196$$

$$BC = 14 \text{ cm.}$$

Practice Task 22.2

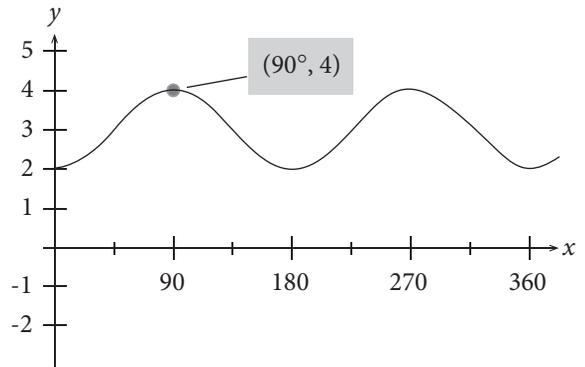In ΔABC, AB = 4 cm, AC = 5 cm and BC = 8 cm. Use the calculator screen-dump to calculate the size of ∠BAC.

A

4 cm

5 cm

B

8 cm

C

Using the Cosine Rule:
$$\cos \breve{B}AC = \frac{4^2 + 5^2 - 8^2}{2 \times 4 \times 5}$$
$$= \frac{16 + 25 - 64}{40}$$
$$= \frac{-23}{40}$$
$$\Rightarrow \angle BAC = 125.1°$$

$\cos^{-1}(\frac{73}{80})$ 24.1

$\cos^{-1}(\frac{55}{64})$ 30.8

$\cos^{-1}(\frac{-23}{40})$ 125.1

Practice Task 23.1

The graph of $y = a \cos (bx) + c$ is shown in the accompanying diagram. Find the values of a, b and c.

(90°, 4)

Mean line is $y = 3$ $\Rightarrow$ $c = 3$

Period = 180° $\Rightarrow$ $b = 2$

Hence: $y = a \cos (2x) + 3$.

When $x = 90°$, $y = 4$:
$$4 = a \cos 180° + 3$$
$$\Rightarrow a = -1$$

Practice Task 23.2

Sketch in the axes provided, the graph of $y = -2 \sin\left(\frac{x}{2}\right)$.

Mean line is $y = 0$

Period = $2 \times 360° = 720°$

When $x = 180°$, $y = -2$:
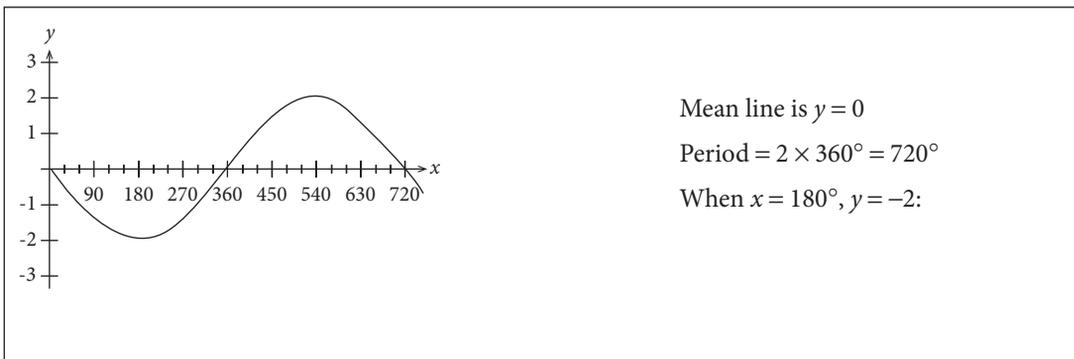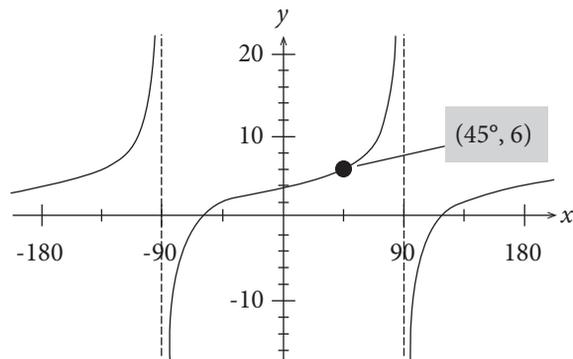
Practice Task 23.3

The graph of $y = a \tan (bx) + c$ is shown in the accompanying diagram. Find the values of a, b and c.

Mean line is $y = 4$ $\Rightarrow$ $c = 4$

Period $= 180°$ $\Rightarrow$ $b = 1$

Hence: $y = a \tan (x) + 4$.

When $x = 45°$, $y = 6$:

$6 = a \tan 45° + 4$

$\Rightarrow$ $a = 2$

Practice Task 24.1

Let the universal set $\mathbf{U} = \{ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 \}$. Let $\mathbf{A} = \{ 11, 13, 17, 19 \}$ **and $\mathbf{B} = \{ 11, 13, 15, 17, 19 \}$.**

Determine: (a) $\mathbf{A} \cap \mathbf{B}$ (b) $\mathbf{A} \cup \mathbf{B}$ (c) $n\overline{(\mathbf{A} \cup \mathbf{B})}$

(a) $A \cap B = \{ 11, 13, 17, 19 \} \cap \{ 11, 13, 15, 17, 19 \} = \{ 11, 13, 17, 19 \}$

(b) $A \cup B = \{ 11, 13, 17, 19 \} \cup \{ 11, 13, 15, 17, 19 \} = \{ 11, 13, 15, 17, 19 \}$

(c) $\overline{A \cup B} = \{ 10, 12, 14, 16, 18 \}$ $\Rightarrow$ $n\overline{(\mathbf{A} \cup \mathbf{B})} = 5$

Practice Task 24.2

In a group of 50 year 10 students, 25 played netball (**N**), 15 played basketball (**B**) and 15 played none of these two sports. Use a Venn diagram to determine the number of students that:
(a) played netball only   (b) played exactly one of these two sports.

(a) $n(\text{netball only}) = 50 - 15 - 15 = 20$

(b) $n(\text{basketball only}) = 50 - 25 - 15 = 10$

Hence,

$n(\text{played exactly one sport}) = 20 + 10$

$= 30$

Practice Task 25.1

In a group of 50 students, there were 10 girls enrolled in Mathematics Methods. There were as many girls enrolled in Mathematics Methods as girls not enrolled in Mathematics Methods. 32 of these students were not enrolled in Mathematics Methods. A student was randomly chosen from this group. Calculate the probability that this student:

(a) was a boy enrolled in Mathematics Methods

(b) was a boy given that this student was either a boy or enrolled in Mathematics Methods.

(a) Using a Venn diagram:

$$n(\text{Boy} \cap \text{MM}) = 50 - 10 - 10 - 22$$

$$= 8$$

$$P(\text{Boy} \cap \text{MM}) = \frac{8}{50}$$

(b) $n(\text{Boy} \cup \text{MM}) = 50 - 10 = 40$

$$P(\text{Boy} \mid \text{Boy} \cup \text{MM}) = \frac{30}{40}$$

Practice Task 25.2

A sample of 400 people were asked to respond YES or NO to the question, "Should uranium mining be permitted in Western Australia (WA)?". The results are tabulated in the first table. The second table shows the gender distribution of those in the sample who responded YES.

(a) Find the probability that a person chosen at random from those surveyed, voted YES and was Male.

(b) Given that a randomly selected respondent voted YES and was female, find the probability that the respondent was WA born.

	YES	NO
WA born	10%	55%
Not WA born	30%	5%

	WA born	Not WA born
Female	5%	35%
Male	15%	45%

(a) $n(\text{Yes}) = 400 \times 40\% = 160$

$$n(\text{YES} \cap \text{Male}) = 160 \times 60\% = 96$$

$$P(\text{YES} \cap \text{Male}) = \frac{96}{400}$$

(b) $n(\text{YES} \cap \text{Female}) = 160 - 96 = 64$

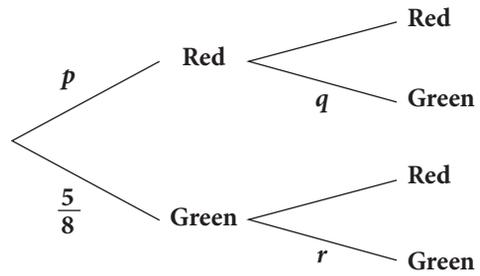$n(\text{WA born} \cap \text{Female who voted YES})$

$$= 160 \times 5\% = 8$$

$$P(\text{WA born} \mid \text{Female who voted YES}) = \frac{8}{64}$$

Practice Task 26.1

A box contains 3 red balls and 5 green balls. Two balls are randomly drawn from the box *without replacement*. The accompanying tree diagram displays the outcomes for the draw.

(a) Determine the values of p, q and r.

(b) Calculate the probability of drawing two balls of different colours.

(a) $p + \dfrac{5}{8} = 1 \Rightarrow p = \dfrac{3}{8}$ $\qquad q = \dfrac{5}{7} \qquad r = \dfrac{4}{7}$

(b) P(different coloured balls) = P(Red $\cap$ Green) + P(Green $\cap$ Red)

$$= \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{30}{56}$$

Practice Task 26.2

60% of year 11 students in a college are girls and of these 75% are enrolled in an ATAR Mathematics course while 50% of year 11 boys in this college are enrolled in an ATAR Mathematics course. (a) Draw a tree diagram to display the given information.

(b) Calculate the probability that a randomly chosen year 11 student from this college:

   (i) is *not* enrolled in an ATAR Mathematics course given that the student is a girl

  (ii) is a girl enrolled in an ATAR Mathematics course

 (iii) is a student enrolled in an ATAR Mathematics course

 (iv) is a girl given that this student is enrolled in an ATAR Mathematics course.

(a)

(b) (i) P(Not ATAR Maths $\mid$ Girl) $= 1 - \dfrac{3}{4} = \dfrac{1}{4}$

  (ii) P(Girl $\cap$ ATAR Maths) $= \dfrac{6}{10} \times \dfrac{3}{4} = \dfrac{18}{40}$

 (iii) $\qquad$ P(ATAR Maths) = P(Girl $\cap$ ATAR Maths) + P(Boy $\cap$ ATAR Maths)

$$= \frac{18}{40} + \frac{4}{10} \times \frac{5}{10} = \frac{13}{20}$$

 (iv) $\quad$ P(Girl $\mid$ ATAR Maths) $= \dfrac{\left(\dfrac{18}{40}\right)}{\left(\dfrac{13}{20}\right)} = \dfrac{9}{13}$

Practice Task 27.1

Consider the scores: 2, 4, 6, 8, 10. Use the table provided to calculate the mean and standard deviation for these scores.

x	2	4	6	8	10
$(x - \bar{x})$					
$(x - \bar{x})^2$					

x	2	4	6	8	10
$(x - \bar{x})$	−4	−2	0	2	4
$(x - \bar{x})^2$	16	4	0	4	16

$$\bar{x} = \frac{2+4+6+8+10}{5} = 6 \qquad \sigma = \sqrt{\frac{16+4+0+4+16}{5}} = 2\sqrt{2}$$

Practice Task 27.2

Consider the scores: 1, 1, 4, 4, 5. Use the table provided to calculate the mean and standard deviation for these scores.

x	1	1	4	4	5
x^2					

x	1	1	4	4	5
x^2	1	1	16	16	25

$$\bar{x} = \frac{1+1+4+4+5}{5} = 3 \qquad \sigma = \sqrt{\left(\frac{1+1+16+16+25}{5}\right) - 3^2}$$

$$= \sqrt{\left(\frac{59}{5}\right) - 9} = \sqrt{\frac{14}{5}}$$

Practice Task 27.3

The accompanying table show the means and standard deviations for datasets A, B and C.

Set	A	B	C
Mean	8	5	6
Standard deviation	2	2	2

(a) Determine which of these datasets are more dispersed.

(b) Determine the new mean and standard deviation if each of the scores in set:

    (i) A is increased by one unit   (ii) B is doubled   (iii) C is increased by 10%

(a) The dispersion in each set is the same as the standard deviations are the same.

(b) (i) The mean would increase by one unit to 9.

      The standard deviation would remain at 2 as the separation of the scores to each other remains unchanged.

  (ii) The mean would be doubled to 10.

      The standard deviation would be doubled to 4 as the separation of the scores to each other would be doubled.

  (iii) The mean would be increased by 10% to 6.6.

      The standard deviation would be increased by 10% to 2.2 as the separation of the scores to each other would be increased by 10%.

Solutions to Review 1

Review Question 1.1

Expand and simplify each of the following: (a) $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)$ (b) $\left(\sqrt{27}-3\right)^2$

(a) $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)=\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2$

$$= 7-5 = 2$$

(b) $\left(\sqrt{27}-3\right)^2 = \left(\sqrt{27}\right)^2 - 2\times3\sqrt{27} + (3)^2$

$$= 27 - 18\sqrt{3} + 9 = 36 - 18\sqrt{3}$$

Review Question 1.2

Rationalise and simplify each of the following: (a) $1+\dfrac{\sqrt{2}}{\sqrt{2}-1}$ (b) $\dfrac{1+\sqrt{3}}{5-2\sqrt{3}}$

(a) $1+\dfrac{\sqrt{2}}{\sqrt{2}-1}=1+\dfrac{\sqrt{2}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$\qquad = 1+\dfrac{\left(\sqrt{2}\left(\sqrt{2}+1\right)\right)}{2-1}$

$\qquad = 1+\left(2+\sqrt{2}\right)$

$\qquad = 3+\sqrt{2}$

(b) $\dfrac{1+\sqrt{3}}{5-2\sqrt{3}}=\dfrac{1+\sqrt{3}}{5-2\sqrt{3}}\times\dfrac{5+2\sqrt{3}}{5+2\sqrt{3}}$

$\qquad = \dfrac{5+2\sqrt{3}+5\sqrt{3}+6}{25-12}$

$\qquad = \dfrac{11}{13}+\dfrac{7\sqrt{3}}{13}$

Review Question 1.3

Expand and simplify: (a) $(1-3x)^2-(2x-1)^2$ (b) $(x-1)(1-3x)-(1-2x)(1+2x)$

(a) $(1-3x)^2-(2x-1)^2$

$\qquad = 1-6x+9x^2-(4x^2-4x+1)$

$\qquad = 5x^2-2x$

(b) $(x-1)(1-3x)-(1-2x)(1+2x)$

$\qquad = (-3x^2+4x-1)-(1-4x^2)$

$\qquad = x^2+4x-2$

Review Question 1.4

Expand and simplify: (a) $(1-2x)(x^2+2x-1)$ (b) $(2x-1)^3$

(a) $(1-2x)(x^2+2x-1)$

$\qquad = x^2+2x-1-2x^3-4x^2+2x$

$\qquad = -2x^3-3x^2+4x-1$

(b) $(2x-1)^3 = (2x-1)(2x-1)^2$

$\qquad = (2x-1)(4x^2-4x+1)$

$\qquad = 8x^3-8x^2+2x-4x^2+4x-1$

$\qquad = 8x^3-12x^2+6x-1$

Review Question 1.5

Factorise completely: (a) $3x^2 - 12$ (b) $(1 - 2x) + 3x(2x - 1)$

$$
\begin{aligned}
\text{(a)} \quad 3x^2 - 12 &= 3(x^2 - 4) \\
&= 3(x - 2)(x + 2)
\end{aligned}
\qquad
\begin{aligned}
\text{(b)} \quad (1 - 2x) + 3x(2x - 1) \\
= (1 - 2x) - 3x(1 - 2x) \\
= (1 - 2x)(1 - 3x)
\end{aligned}
$$

Review Question 1.6

Factorise completely: (a) $x^2 + x - 20$ (b) $-5x^2 + 4x + 1$

$$
\text{(a)} \quad x^2 + x - 20 = (x - 4)(x + 5)
\qquad
\begin{aligned}
\text{(b)} \quad -5x^2 + 4x + 1 &= -(5x^2 - 4x - 1) \\
&= -(5x + 1)(x - 1)
\end{aligned}
$$

Review Question 1.7

Factorise completely: (a) $8x^2 + 8x + 2$ (b) $6x^2 + x - 1$

$$
\begin{aligned}
\text{(a)} \quad 8x^2 + 8x + 2 &= 2(4x^2 + 4x + 1) \\
&= 2(2x + 1)^2
\end{aligned}
\qquad
\text{(b)} \quad 6x^2 + x - 1 = (3x - 1)(2x + 1)
$$

Review Question 1.8

Use polynomial division to determine the quotient and remainder when $2x^3 - 5x^2 + 4x - 2$ is divided by $x - 1$.

$$
\begin{array}{r}
2x^2 - 3x + 1 \\
x - 1 \overline{\smash{\big)}\, 2x^3 - 5x^2 + 4x - 2} \\
\underline{2x^3 - 2x^2} \\
-3x^2 + 4x \\
\underline{-3x^2 + 3x} \\
x - 2 \\
\underline{x - 1} \\
-1
\end{array}
$$

Hence, quotient is $2x^2 - 3x + 1$ and remainder is -1.

Review Question 1.9

Determine the remainder when $-x^3 - 3x^2 + 5x + 10$ is divided by $(x + 2)$.

Let $f(x) = -x^3 - 3x^2 + 5x + 10$.
Using the Remainder Theorem,
$$\text{Remainder} = f(-2)$$
$$= 8 - 12 - 10 + 10$$
$$= -4$$

Review Question 1.10

Factorise completely $f(x) = -x^3 + 3x - 2$.

$f(1) = -1 + 3 - 2 = 0.$
Hence, $(x - 1)$ is a factor.

Using polynomial division:

$$x^3 - 7x - 6 = (x - 1)(-x^2 - x + 2)$$
$$= -(x - 1)(x^2 + x - 2)$$
$$= -(x - 1)(x + 2)(x - 1)$$
$$= -(x - 1)^2(x + 2)$$

$$\begin{array}{r} -x^2 - x + 2 \\ x-1 \overline{)-x^3 + 0x^2 + 3x - 2} \\ \underline{-x^3 + x^2} \\ -x^2 + 3x \\ \underline{-x^2 + x} \\ 2x - 2 \\ 2x - 2 \\ \underline{} \\ 0 \end{array}$$

Review Question 1.11

Express as a completed square: (a) $x^2 - 8x + 10$ (b) $-2x^2 - 8x + 3$

(a) $x^2 - 8x + 10 = (x^2 - 8x + 16) - 16 + 10$
$$= (x - 4)^2 - 6$$

(b) $-2x^2 - 8x + 3 = -2\left[x^2 + 4x - \dfrac{3}{2}\right]$
$$= -2\left[(x^2 + 4x + 4) - 4 - \dfrac{3}{2}\right]$$
$$= -2(x + 2)^2 + 11$$

Review Question 1.12

Solve for x: (a) $x^2 - 4x - 5 = 0$ (b) $(2 + x)^2 - 7 = 0$

(a) $x^2 - 4x - 5 = 0 \implies (x + 1)(x - 5) = 0$

$\qquad\qquad\qquad\qquad x = -1, 5$

(b) $(2 + x)^2 - 7 = 0 \implies (2 + x)^2 = 7$

$\qquad\qquad\qquad\qquad 2 + x = \pm \sqrt{7}$

$\qquad\qquad\qquad\qquad x = -2 \pm \sqrt{7}$

Review Question 1.13

Solve for x: (a) $3x^2 - 7x + 2 = 0$ (b) $6x^2 - x - 2 = 0$

(a) $3x^2 - 7x + 2 = 0$

$\qquad \implies (3x - 1)(x - 2) = 0$

$\qquad\qquad\qquad x = \dfrac{1}{3}, 2$

(b) $6x^2 - x - 2 = 0$

$\qquad \implies (3x - 2)(2x + 1) = 0$

$\qquad\qquad\qquad x = -\dfrac{1}{2}, \dfrac{2}{3}$

Review Question 1.14

Solve for x in $x^3 - 2x + 1 = 0$.

Let $f(x) = x^3 - 2x + 1$

$f(1) = 1 - 2 + 1 = 0$

Hence, $(x - 1)$ is a factor.

Using polynomial division:

$x^3 - 2x + 1 = (x - 1)(x^2 + x - 1)$

Hence, $f(x) = 0 \implies x = 1, \ x^2 + x - 1 = 0$

$\qquad\qquad\qquad = 1, \dfrac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$\qquad\qquad\qquad = 1, \dfrac{-1 \pm \sqrt{5}}{2}$

$$\begin{array}{r}
x^2 + x - 1 \\
x - 1 \overline{\smash{)}\ x^3 - 0x^2 - 2x + 1} \\
\underline{x^3 - x^2} \\
x^2 - 2x \\
\underline{x^2 - x} \\
-x + 1 \\
\underline{-x + 1} \\
0
\end{array}$$

Review Question 1.15

Simplify, leaving answers with positive indices:

(a) $\dfrac{30a^3b^{-1}}{12a^{-5}b^3}$

(b) $\dfrac{25b^{-2}}{10a^3} \times \left(\dfrac{2a^2}{b^{-1}}\right)^2$

(a) $\dfrac{30a^3b^{-1}}{12a^{-5}b^3} = \dfrac{5a^{3-(-5)}}{2b^{3-(-1)}}$

$\qquad = \dfrac{5a^8}{2b^4}$

(b) $\dfrac{25b^{-2}}{10a^3} \times \left(\dfrac{2a^2}{b^{-1}}\right)^2 = \dfrac{5b^{-2}}{2a^3} \times \dfrac{4a^4}{b^{-2}}$

$\qquad = 10a$

Review Question 1.16

Simplify, leaving answers with positive indices:

(a) $\sqrt{\dfrac{81a^4}{64b^{-8}}}$

(b) $\left(\dfrac{-a^3}{6\sqrt{b}}\right)^{-2}$

(a) $\sqrt{\dfrac{81a^4}{64b^{-8}}} = \dfrac{9a^2}{8b^{-4}}$

$\qquad = \dfrac{9a^2b^4}{8}$

(b) $\left(\dfrac{-a^3}{6\sqrt{b}}\right)^{-2} = \left(\dfrac{-6\sqrt{b}}{a^3}\right)^2$

$\qquad = \dfrac{36b}{a^6}$

Review Question 1.17

Solve for x:

(a) $4^{1-2x} = 64$

(b) $8^{\frac{x}{3}} = 32^{x+1}$

(a) $4^{1-2x} = 64 \;\Rightarrow\; 4^{1-2x} = 4^3$

$\qquad 1 - 2x = 3$

$\qquad x = -1$

(b) $8^{\frac{x}{3}} = 32^{x+1} \;\Rightarrow\; (2^3)^{\frac{x}{3}} = (2^5)^{x+1}$

$\qquad x = 5x + 5$

$\qquad x = -\dfrac{5}{4}$

Review Question 1.18

Solve for x:

(a) $4 \times 5^{2x-1} = 500$

(b) $10 \times 3^{0.3x} = 90 \times 3^{-0.2x}$

(a) $4 \times 5^{2x-1} = 500 \;\Rightarrow\; 5^{2x-1} = 125$

$\qquad 5^{2x-1} = 5^3$

$\qquad 2x - 1 = 3$

$\qquad x = 2$

(b) $10 \times 3^{0.3x} = 90 \times 3^{-0.2x}$

$\qquad \dfrac{3^{0.3x}}{3^{-0.2x}} = \dfrac{90}{10}$

$\qquad 3^{0.5x} = 9$

$\qquad 0.5x = 2$

$\qquad x = 4$

Review Question 1.19

Express in exponential form to solve for x: (a) $\log_{10} x = -2$ (b) $\log_x 27 = 3$

(a) $\log_{10} x = -2 \implies x = 10^{-2}$

$\qquad\qquad\qquad\qquad x = \dfrac{1}{100}$

(b) $\log_x 27 = 3 \implies 27 = x^3$

$\qquad\qquad\qquad\qquad x = 3$

Review Question 1.20

Express in logarithmic form to solve for x: (a) $6^x = 10$ (b) $5^{x+1} = 20$

(a) $6^x = 10 \implies x = \log_6 10$

(b) $5^{x+1} = 20 \implies x + 1 = \log_5 20$

$\qquad\qquad\qquad\qquad x = \log_5 20 - 1$

Review Question 1.21

Solve for x in terms of $\log_{10}$:   (a) $8^x = 20$ (b) $25(3^x) = 100(9^{x+1})$

(a) $\log_{10} 8^x = \log_{10} 20$

$\qquad x \log_{10} 8 = \log_{10} 20$

$\qquad\qquad x = \dfrac{\log_{10} 20}{\log_{10} 8}$

(b) $\qquad 25(3^x) = 100(9^{x+1})$

$\qquad\qquad \dfrac{3^x}{9^{x+1}} = \dfrac{100}{25}$

$\qquad\qquad \dfrac{3^x}{(3^2)^{x+1}} = 4$

$\qquad\qquad 3^{x-(2x+2)} = 4$

$\qquad\qquad 3^{-x-2} = 4$

$\qquad (-x-2)\log_{10} 3 = \log_{10} 4$

$\qquad\qquad (-x-2) = \dfrac{\log_{10} 4}{\log_{10} 3}$

$\qquad\qquad x = -2 - \dfrac{\log_{10} 4}{\log_{10} 3}$

Review Question 1.22

Expand each of the following: (a) $\log_{10} 10\,(x+1)$ (b) $\log_5\left(\dfrac{x^2-1}{x+2}\right)$

(a) $\log_{10} 10\,(x+1) = \log_{10} 10 + \log_{10}(x+1)$

$\qquad\qquad\qquad\quad = 1 + \log_{10}(x+1)$

(b) $\log_5\left(\dfrac{x^2-1}{x+2}\right) = \log_5(x^2-1) - \log_5(x+2)$

$\qquad\qquad\qquad\quad = \log_5[(x-1)(x+1)] - \log_5(x+2)$

$\qquad\qquad\qquad\quad = \log_5(x-1) + \log_5(x+1) - \log_5(x+2)$

Review Question 1.23

Simplify each of the following: (a) $-2 + 3 \log_{10}(x-1)$ (b) $\log_2(x+3) + \log_2(x-3) + \log_2(x)$

(a) $-2 + 3 \log_{10}(x-1)$

$\quad = -2\log_{10} 10 + \log_{10}(x-1)^3$

$\quad = \log_{10} 10^{-2} + \log_{10}(x-1)^3$

$\quad = \log_{10}\left[\dfrac{(x-1)^3}{100}\right]$

(b) $\log_2(x+3) + \log_2(x-3) + \log_2(x)$

$\quad = \log_2[x\,(x+3)\,(x-3)]$

$\quad = \log_2[x\,(x^2-9)]$

Review Question 1.24

Solve for x and y. $2x + 3y = 13$ $3x - 4y = 11$

	$2x + 3y = 13$	I
	$3x - 4y = 11$	II
I × 3	$6x + 9y = 39$	III
II × 2	$6x - 8y = 22$	IV
III − IV	$17y = 17 \;\Rightarrow\; y = 1$	
	$x = 5$	

Review Question 1.25

Solve for x and y. $x - y = -2$ $y = x^2 - 3x - 3$

$x - y = -2 \;\Rightarrow\; y = x + 2$

$\qquad$ Substitute into $y = x^2 - 3x - 3$:

$\qquad\qquad x + 2 = x^2 - 3x - 3$

$\qquad x^2 - 4x - 5 = 0$

$\qquad (x+1)(x-5) = 0$

$\qquad\qquad x = -1, 5$

Hence:$\qquad\qquad x = -1, y = 1; x = 5, y = 7$

Solutions to Review 2

Review Question 2.1

Determine the equation of the line passing through $(-2, 5)$ and:
(a) the point $(3, -10)$.
(b) parallel to the line with equation $y = 1 - 4x$.

(c) perpendicular to the line with equation $2x + 3y = 10$.

(a) Gradient of line: $\qquad m = \dfrac{-10 - 5}{3 - (-2)} = -3$

Equation of line: $\qquad y - 5 = -3(x - (-2)) \implies y = -3x - 1$

(b) Gradient of required line: $\qquad m = -4$

Equation of required line: $\qquad y - 5 = -4\,(x - (-2)) \implies y = -4x - 3$

(c) Rewrite equation of given line: $\qquad 2x + 3y = 10 \implies 3y = -2x + 10$

Hence, gradient of required line: $\qquad m = \dfrac{-1}{\left(-\dfrac{2}{3}\right)} = \dfrac{3}{2}$

Equation of required line: $\qquad y - 5 = \dfrac{3}{2}(x - (-2)) \implies y = \dfrac{3x}{2} + 8$

Review Question 2.2

Find the coordinates of the point of intersection between the lines $y = 2x - 3$ and $y = 2 - 3x$.

$$2x - 3 = 2 - 3x$$
$$5x = 5 \implies x = 1$$
$$y = -1.$$

Hence, point of intersection is $(1, -1)$.

Review Question 2.3

Sketch the curve with equation $y = (x - 2)(x + 4)$. State the coordinates of all intercepts and the turning point.

y-intercept: $\qquad (0, -8)$.

For x-intercepts: $\quad (x - 2)(x + 4) = 0$
$$x = -4, 2$$

Equation of LOS: $\quad x = \dfrac{-4 + 2}{2} = -1$

Turning Point: $\qquad (-1, -9)$

x Intercept $(-4, 0)$

x Intercept $(2, 0)$

Turning Point $(-1, -9)$
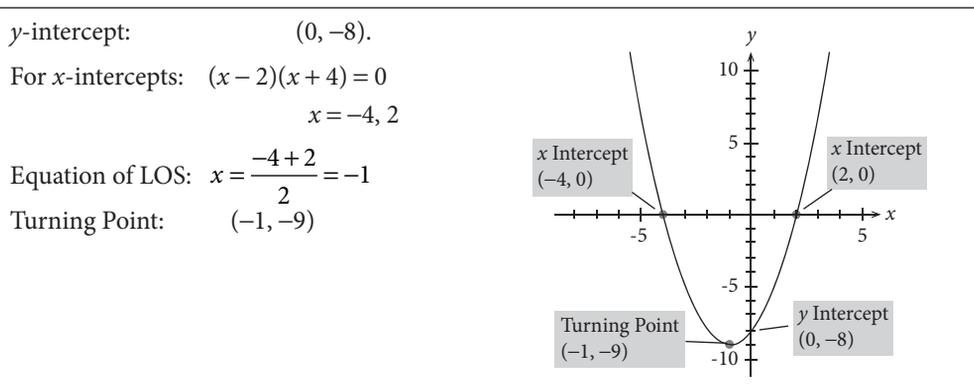
y Intercept $(0, -8)$

Review Question 2.4

Sketch the curve with equation $y = -x^2 + 2x - 2$. State the coordinates of the intercept(s) and turning point.

For x-intercepts: $-x^2 + 2x - 2 = 0$

Discriminant $\Delta = (2)^2 - 4(-1)(-2) = -4$
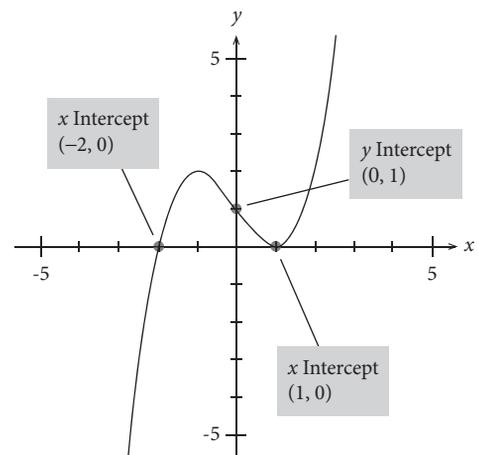
Hence, curve has no x-intercepts.

y-intercept: $(0, -2)$

Line of Symmetry: $x = \dfrac{-2}{2(-1)} = 1$

      Hence, turning point is at $(1, -1)$.

Turning Point
$(1, -1)$

y Intercept
$(0, -2)$

Review Question 2.5

The accompanying diagram shows a cubic curve. Determine the equation of this curve.

Roots are: $x = -2, 1$ (repeated)

Hence, $y = k(x + 2)(x - 1)^2$

When $x = 0$, $y = 1$.

        $\Rightarrow$ $1 = k(0 + 2)(0 - 1)^2$

        $k = \dfrac{1}{2}$

Hence: $y = \dfrac{1}{2}(x + 2)(x - 1)^2$

x Intercept
$(-2, 0)$

y Intercept
$(0, 1)$

x Intercept
$(1, 0)$

Review Question 2.6

Determine the equation of the rectangular hyperbola shown in the accompanying diagram.

Equation is of the form $y = \dfrac{k}{x + 2} + 1$

When $x = 0$, $y = \dfrac{7}{4}$: $\dfrac{7}{4} = \dfrac{k}{0 + 2} + 1$

        $\dfrac{7}{4} = \dfrac{k}{2} + 1$

        $k = \dfrac{3}{2}$

Hence: $y = \dfrac{3}{2(x + 2)} + 1$

Vertical Asymptote
$x = -2$

Horizontal Asymptote
$y = 1$

y Intercept
$\left(0, \dfrac{7}{4}\right)$
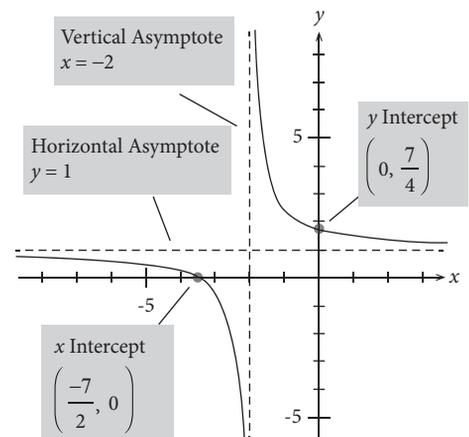
x Intercept
$\left(\dfrac{-7}{2}, 0\right)$

Review Question 2.7

Sketch the curve with equation $y = -x^3 + 2x^2 + 5x - 6$. Indicate all intercepts.

Let $f(x) = -x^3 + 2x^2 + 5x - 6$.
$\quad f(1) = -1 + 2 + 5 - 6 = 0$
Hence, $(x - 1)$ is a factor.

Using polynomial division:

$$
\begin{array}{r}
-x^2 + x + 6 \\
x - 1 \overline{)\, -x^3 + 2x^2 + 5x - 6\,} \\
\underline{-x^3 + x^2} \\
x^2 + 5x \\
\underline{x^2 - x} \\
6x - 6 \\
\underline{6x - 6} \\
0
\end{array}
$$

$-x^3 + 2x^2 + 5x - 6 = (x - 1)(-x^2 + x + 6)$
$ = -(x - 1)(x^2 - x - 6)$

For x-intercepts: $\quad -(x - 1)(x + 2)(x - 3) = 0 \implies x = -2, 1, 3$

y-intercept: $\quad (0, -6)$
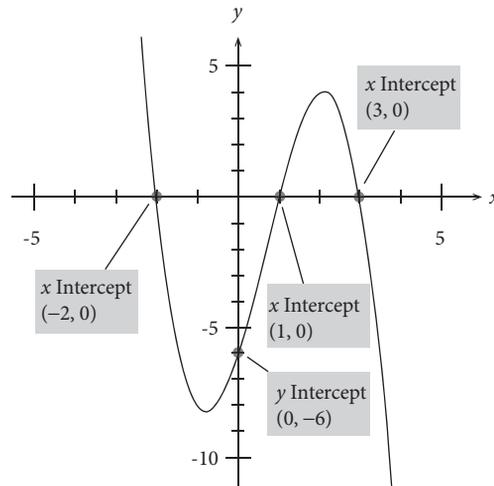
Review Question 2.8

Sketch the curve with equation $y = 1 + \dfrac{2}{2 - x}$. State all intercepts and asymptotes.

When $x = 0$, $\quad y = 1 + \dfrac{2}{2 - 0} = 2.$

Hence, vertical-intercept is $(0, 2)$.

When $y = 0$ $\quad 1 + \dfrac{2}{2 - x} = 0$
$ \dfrac{2}{2 - x} = -1$
$ -(2 - x) = 2$
$\phantom{When y = 0 \quad 1 \dfrac{2}{2}} x = 4$

Hence, horizontal-intercept is $(4, 0)$

Horizontal Asymptote: $\quad y = 1$
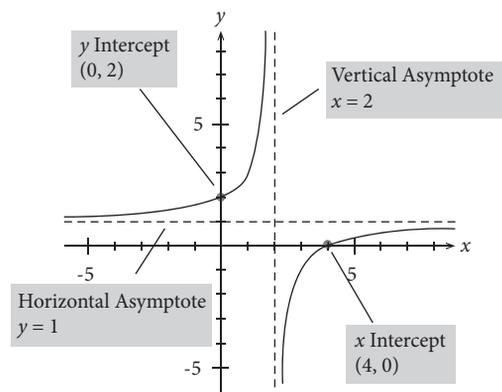
Vertical Asymptote: $\quad x = 2$

Review Question 2.9

The accompanying diagram shows the graph of $y = 0.5\,(2^{x+a}) + b$. Determine the values of a and b.
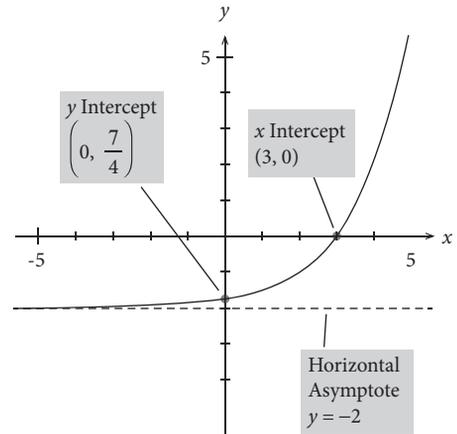
Horizontal asymptote is $y = -2$.

$$\Rightarrow\quad b = -2.$$

For vertical intercept: $x = 0$

$$\Rightarrow\quad y = 0.5\,(2^a) - 2$$

But vertical intercept is $\left(0, \dfrac{-7}{4}\right)$.
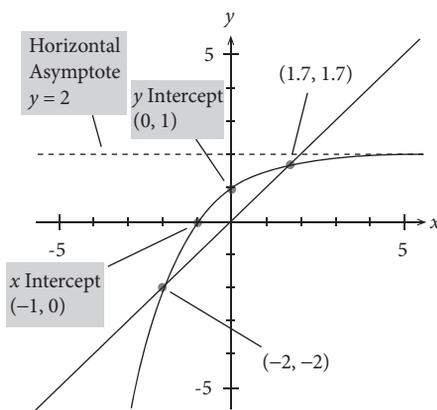
Hence, $\qquad \dfrac{-7}{4} = 0.5\,(2^a) - 2$

$$\Rightarrow\quad 0.5\,(2^a) = 2 - \frac{7}{4} = \frac{1}{4}$$

$$2^a = \frac{1}{2}$$

$$a = -1$$

Review Question 2.10

(a) On the axes below, sketch the curve with equation $y = -(2^{-x}) + 2$. State the equation of the horizontal intercept and the coordinates of the intercepts.

Horizontal asymptote is $y = 2$.

For the y-intercept:
$x = 0 \implies y = -(1) + 2 = 1$
Hence, $(0, 1)$.

For the x-intercept:
$y = 0 \implies -(2^{-x}) + 2 = 0$
$\qquad\qquad 2^{-x} = 2 \implies x = -1$
Hence, $(-1, 0)$.

(b) On the same axes, sketch the line $y = x$. Use your sketch to estimate the solution to the equation $2^{-x} = 2 - x$. Explain how you obtained your answer.

Rewrite $2^{-x} = 2 - x$ as $-(2^{-x}) + 2 = x$

Hence, solution is the x-coordinate of the point of intersection between $y = -(2^{-x}) + 2$ and $y = x$.

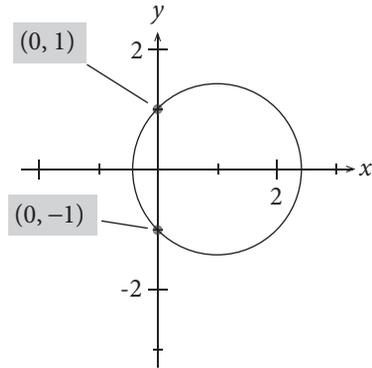From sketch: $\qquad x \approx -2$ and 1.7.

Review Question 2.11

State the equation of each of the circles drawn.

(a)

(b)
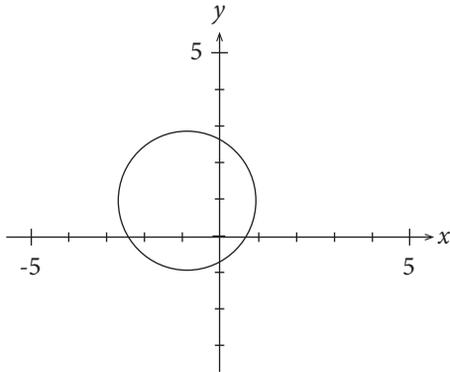
(a) Centre of circle is $(-1, 1)$. Radius $= 2$ Hence, equation is $(x + 1)^2 + (y - 1)^2 = 4$	(b) Centre of circle is $(1, 0)$. Using Pythagoras Theorem: Radius $= \sqrt{1^2 + 1^2} = \sqrt{2}$ Hence, equation is $(x - 1)^2 + y^2 = 2$

Review Question 2.12

Determine the coordinates of the centre and the radius of the circle with equation $x^2 - 4x + y^2 + 8y - 16 = 0$.

Rewrite as completed squares:
$$x^2 - 4x + y^2 + 8y \equiv (x - 2)^2 - 4 + (y + 4)^2 - 16$$

Hence, $\qquad\qquad\qquad x^2 - 4x + y^2 + 8y = 16$

becomes: $\qquad (x - 2)^2 - 4 + (y + 4)^2 - 16 = 16$

$$(x - 2)^2 + (y + 4)^2 = 36$$

Hence, centre of circle has coordinates $(2, -4)$ and radius of circle $= 6$.

Solutions to Review 3

Review Question 3.1

In $\triangle ABC$, $\angle ABC = 90°$, $BC = 1$ cm and $\sin \angle BAC = \dfrac{1}{4}$.

Determine the exact length of: (a) AC (b) AB.
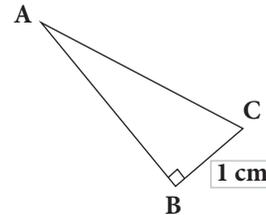
(a) $\sin \angle BAC = \dfrac{BC}{AC} \Rightarrow AC = \dfrac{BC}{\sin B\hat{A}C}$

$$= \dfrac{1}{\left(\dfrac{1}{4}\right)} = 4 \text{ cm}$$

(b) Using Pythagoras Theorem:

$$AB = \sqrt{4^2 - 1^2}$$

$$= \sqrt{15} \text{ cm}$$

Review Question 3.2

In $\triangle ABC$, $\angle BAC = 90°$ and $\tan \angle ABC = \dfrac{4}{3}$. The point K on BC is foot of the perpendicular from A to BC and $AK = 8$ cm.

Determine: (a) BK (b) AC.
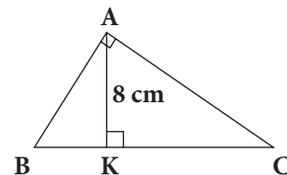
(a) In $\triangle ABK$: $\tan ABK = \dfrac{AK}{BK}$

$$\Rightarrow BK = \dfrac{8}{\left(\dfrac{4}{3}\right)} = 6 \text{ cm.}$$

(b) In $\triangle ABK$: $AB^2 = AK^2 + BK^2$

$$= 8^2 + 6^2 = 100$$

$$\Rightarrow AB = 10 \text{ cm}$$

In $\triangle ABC$: $\tan ABC = \dfrac{AC}{AB}$

$$\Rightarrow AC = 10 \times \dfrac{4}{3}$$

$$= \dfrac{40}{3} \text{ cm.}$$

Review Question 3.3

R is a point on a unit circle with coordinates (0.6, 0.8).
The ray OR forms an angle of size $\theta°$ with the positive
x-axis as shown in the accompanying diagram.
Determine the value of:
(a) $\sin \theta°$ (b) $\cos (180° - \theta°)$ (c) $\tan (180° + \theta°)$
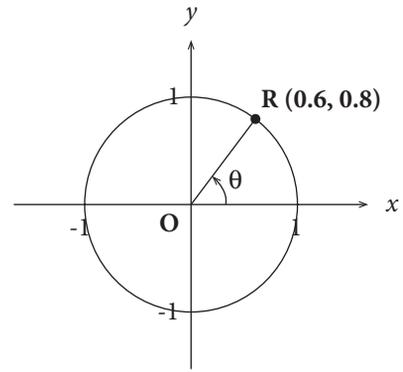
> (a) $\sin \theta° = 0.8$
>
> (b) $\cos (180° - \theta°) = - \cos (\theta°)$
>
> $= -0.6$
>
> (c) $\tan (180° + \theta°) = \tan (\theta°)$
>
> $= \dfrac{0.8}{0.6} = \dfrac{4}{3}$

Review Question 3.4

Solve for $x°$, where $0° \leq x \leq 360°$, if $\cos x° = \dfrac{\sqrt{2}}{2}$.

> $\cos x° = \dfrac{\sqrt{2}}{2}$ $\Rightarrow$ Reference angle for $x° = 45°$
>
> The cosine function is positive in quadrants 1 and 4.
>
> Hence: $x° = 45°, 360° - 45°$
>
> $= 45°, 315°$

Review Question 3.5

Solve for $x°$, where $0° \leq x \leq 360°$, if $\tan x° = -\dfrac{\sqrt{3}}{3}$.

> $\tan x° = -\dfrac{\sqrt{3}}{3}$ $\Rightarrow$ Reference angle for $x° = 30°$
>
> The tangent function is negative in quadrants 2 and 4.
>
> Hence: $x° = 180° - 30°, 360° - 30°$
>
> $= 150°, 330°$

Review Question 3.6

Given that $\sin^{-1}\left(\dfrac{5}{8}\right) = 38.7°$, solve for $x°$, where $0° \le x \le 360°$, if $\sin x° = \dfrac{5}{8}$.

$\sin x° = \dfrac{5}{8} \implies$ Reference angle for $x° = \sin^{-1}\left(\dfrac{5}{8}\right) = 38.7°$

The sine function is positive in quadrants 1 and 2.

Hence: $x° = 38.7°,\ 180° - 38.7°$

              $= 38.7°,\ 141.3°$

Review Question 3.7

Given that $\cos^{-1}\left(\dfrac{2}{7}\right) = 73.4°$, solve for $x°$, where $0° \le x \le 360°$, if $\cos x° = -\dfrac{2}{7}$.

$\cos x° = -\dfrac{2}{7} \implies$ Reference angle for $x° = \cos^{-1}\left(\dfrac{2}{7}\right) = 73.4°$

The cosine function is negative in quadrants 2 and 3.

Hence: $x° = 180° - 72.4°,\ 180° + 72.4°$

              $= 106.6°,\ 253.4°$

Review Question 3.8

In $\triangle ABC$, $\angle BAC = 120°$, $\angle ABC = 15°$ and
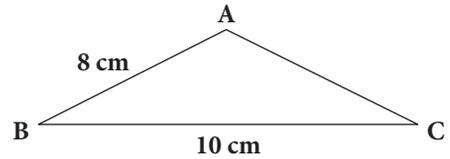$AB = \sqrt{6}$ cm. Calculate the length of BC.
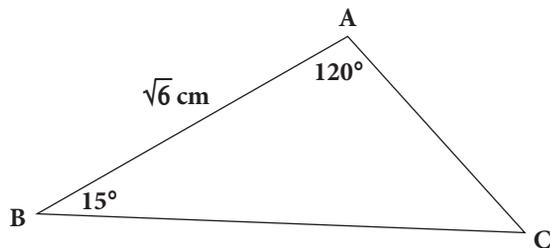
$\angle ACB = 180° - 120° - 15° = 45°$

Using the Sine Rule:

$$\dfrac{BC}{\sin 120°} = \dfrac{\sqrt{6}}{\sin 45°}$$

$$BC = \dfrac{\sqrt{6}}{\sin 45°} \times \sin 120°$$

$$= \dfrac{\sqrt{6}}{\left(\dfrac{\sqrt{2}}{2}\right)} \times \dfrac{\sqrt{3}}{2} = 3 \text{ cm}$$

Review Question 3.9

In ΔABC, ∠BAC is an obtuse angle such that
$\sin \angle BAC = \frac{4}{5}$, AB = 8 cm and BC = 8 cm. Use the calculator
screen-dump to calculate the size of:
(a) ∠BAC   (b) ∠ACB   (c) ∠ABC.

(a) ∠BAC = 180° − 53.1° = 126.9°

(b) Using the Sine Rule:

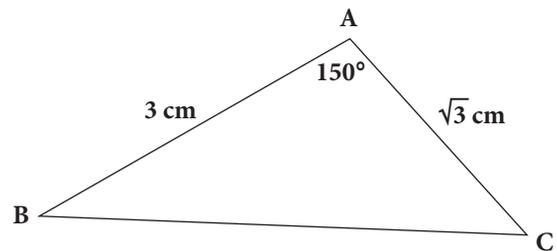$$\frac{\sin B\hat{A}C}{10} = \frac{\sin A\hat{C}B}{8}$$

$$\sin A\hat{C}B = \frac{\sin B\hat{A}C}{10} \times 8$$

$$= \frac{\left(\frac{4}{5}\right)}{10} \times 8 = \frac{16}{25}$$

Hence: ∠ACB = 39.8°

(c) ∠ABC = 180° − 39.8° − 126.9° = 13.3°

Review Question 3.10

In ΔABC, ∠BAC = 150°, AB = 3 cm and AC = $\sqrt{3}$ cm.

Calculate the length of BC.

Using the Cosine Rule:

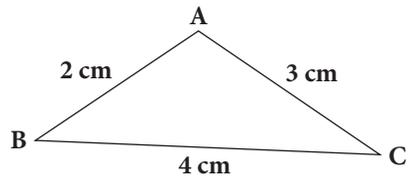$$BC^2 = 3^2 + \left(\sqrt{3}\right)^2 - 2 \times 3 \times \sqrt{3} \times \cos 150°$$

$$= 9 + 3 - 2 \times 3 \times \sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right)$$

$$= 21$$

$$BC = \sqrt{21} \text{ cm.}$$

Review Question 3.11

In $\triangle$ABC, AB = 2 cm, AC = 3 cm and BC = 4 cm. Use the calculator screen-dump to calculate the size of $\angle$ABC.
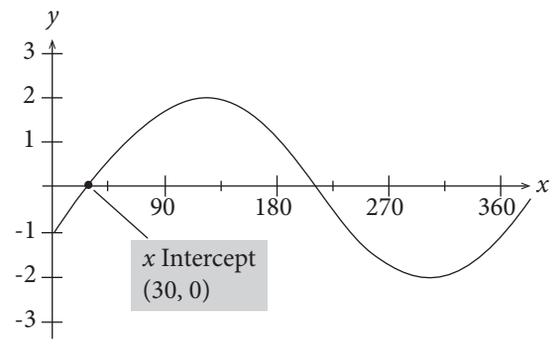
Using the Cosine Rule:
$$\cos A\hat{B}C = \frac{4^2 + 2^2 - 3^2}{2 \times 4 \times 2}$$
$$= \frac{16 + 4 - 9}{16}$$
$$= \frac{11}{16}$$
$$\Rightarrow \angle ABC = 46.6°$$

$\cos^{-1}(\frac{-1}{4})$
104.5

$\cos^{-1}(\frac{11}{16})$
46.6

$\cos^{-1}(\frac{21}{24})$
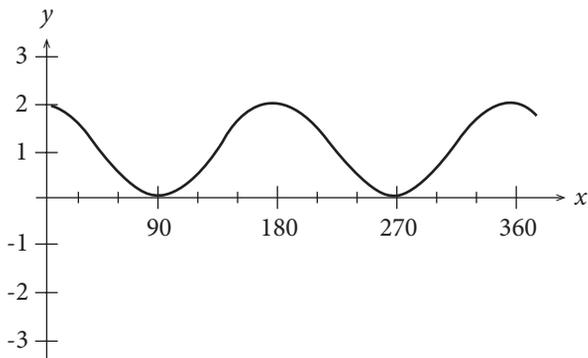29.0

Review Question 3.12

The graph of $y = a \sin(x + b) + c$ is shown in the accompanying diagram. Find the values of a, b and c.

Mean line is $y = 0$ $\Rightarrow$ $c = 0$

Amplitude = 2 $\Rightarrow$ $a = 2$

Hence: $y = 2 \sin(x + b) + 3$.

When $x = 30°$, $y = 0$:

$$0 = 2 \sin(30° + b)$$
$$0 = 30° + b$$
$$\Rightarrow b = -30°$$

x Intercept
(30, 0)

Review Question 3.13

Sketch in the axes provided, the graph of $y = \cos(2x) + 1$.

Mean line is $y = 1$

Period = $\dfrac{360°}{2} = 180°$

When $x = 0°$, $y = 2$:

Solutions to Review 4

Review Question 4.1

Let the universal set $\mathbf{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let $\mathbf{A} = \{3, 6, 9\}$
and $\mathbf{B} = \{1, 3, 5, 7, 9\}$.

Determine: (a) $\mathbf{A} \cap \mathbf{B}$ (b) $\mathbf{A} \cap \bar{\mathbf{B}}$ (c) $n(\overline{\mathbf{A} \cup \mathbf{B}})$

> (a) $\mathbf{A} \cap \mathbf{B} = \{3, 6, 9\} \cap \{1, 3, 5, 7, 9\} = \{3, 9\}$
>
> (b) $\mathbf{A} \cap \bar{\mathbf{B}} = \{3, 6, 9\} \cap \{0, 2, 4, 6, 8\} = \{6\}$
>
> (c) $\overline{\mathbf{A} \cup \mathbf{B}} = \{0, 2, 4, 8\}$ $\Rightarrow$ $n(\overline{\mathbf{A} \cup \mathbf{B}}) = 4$

Review Question 4.2

In a group of 40 year 10 students, 23 played hockey (**H**), 21 played cricket (**C**) and 4 played none of these two sports. Use a Venn diagram to determine the number of students that:
(a) played cricket only   (b) played exactly one of these two sports.

> (a) $n(\text{cricket only}) = 40 - 23 - 4 = 13$
>
> (b) $n(\text{hockey only}) = 40 - 21 - 4 = 15$
>
>   Hence,
>
> $n(\text{played exactly one sport}) = 13 + 15$
>
> $= 28$

Review Question 4.3

In a group of 50 students, there were 15 girls enrolled in Physics. There are as many girls not enrolled in Physics as boys enrolled in Physics. 25 of these students were enrolled in Physics. A student was randomly chosen from this group. Calculate the probability that this student:
(a) is a boy enrolled in Physics
(b) is a girl given that this student is not enrolled in Physics.

> (a) Using a Venn diagram:
>
> $n(\text{Boy} \cap \text{Physics}) = 25 - 15 = 10$
>
> $P(\text{Boy} \cap \text{Physics}) = \dfrac{10}{50}$
>
> (b) $n(\text{Girl} \cap \text{Not Physics}) = 10$
>
> $n(\text{Not Physics}) = 50 - 25 = 25$
>
> $P(\text{Girl} \mid \text{Not Physics}) = \dfrac{10}{25}$

Review Question 4.4

A box contains 2 red balls and 3 green balls. Two
balls are randomly drawn from the box *without replacement*.
The accompanying tree diagram displays the outcomes for the draw.
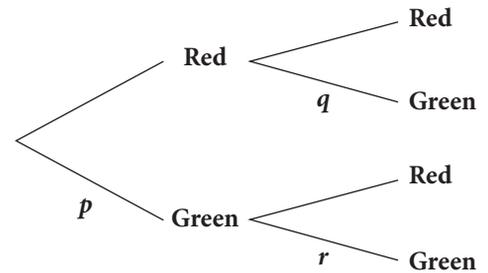
(a) Determine the values of p, q and r.

(b) Calculate the probability of drawing two balls of
the same colours.

(a) $p = \dfrac{3}{5}$ $q = \dfrac{3}{4}$ $r = \dfrac{2}{4}$

(b) P(same coloured balls) = P(Red ∩ Red) + P(Green ∩ Green)

$$= \frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{2}{4} = \frac{8}{20}$$

Review Question 4.5

25% of students in a college wear glasses and of these 80% have dark hair while 10% of students who do not wear glasses have dark hair.

(a) Draw a tree diagram to display the given information.

(b) Calculate the probability that a randomly chosen student from this college:

   (i) does not have dark hair given that this student does not wear glasses

  (ii) is a student that does not have dark hair and does not wear glasses

 (iii) is a student with dark hair

 (iv) is a student that wear glasses given that this student had dark hair.

(a)

(b)   (i) P(Not dark hair | No glasses) $= 1 - \dfrac{1}{10} = \dfrac{9}{10}$

  (ii) P(Not dark hair ∩ No glasses) $= \dfrac{3}{4} \times \dfrac{9}{10} = \dfrac{27}{40}$

 (iii) P(Dark hair) = P(Glasses ∩ Dark hair) + P(No glasses ∩ Dark hair)

$$= \left(\frac{1}{4} \times \frac{4}{5} \right) + \left(\frac{3}{4} \times \frac{1}{10} \right) = \frac{11}{40}$$

 (iv) P(Glasses | Dark hair) $= \dfrac{\left(\dfrac{4}{20} \right)}{\left(\dfrac{11}{40} \right)} = \dfrac{8}{11}$

Review Question 4.6

Consider the scores: 1, 3, 5, 7, 9. Use the table provided to calculate the mean and standard deviation for these scores.

x	1	3	5	7	9
$(x - \bar{x})$					
$(x - \bar{x})^2$					

x	1	3	5	7	9
$(x - \bar{x})$	-4	-2	0	2	4
$(x - \bar{x})^2$	16	4	0	4	16

$$\bar{x} = \frac{1+3+5+7+9}{5} = 5 \qquad \sigma = \sqrt{\frac{16+4+0+4+16}{5}} = 2\sqrt{2}$$

Review Question 4.7

Consider the scores: 0, 2, 3, 5, 5. Use the table provided to calculate the mean and standard deviation for these scores.

x	0	2	3	5	5
x^2					

x	0	2	3	5	5
x^2	0	4	9	25	25

$$\bar{x} = \frac{0+2+3+5+5}{5} = 3 \qquad \sigma = \sqrt{\left(\frac{0+4+9+25+25}{5}\right) - 3^2}$$

$$= \sqrt{\left(\frac{63}{5}\right) - 9} = \sqrt{\frac{18}{5}}$$

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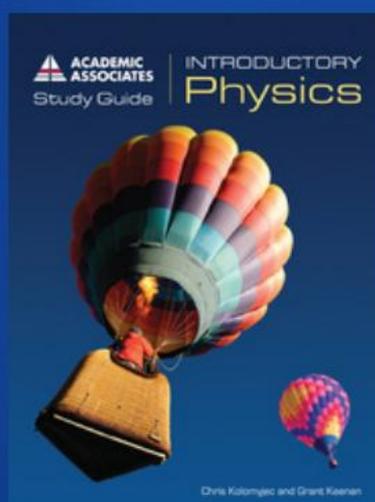
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