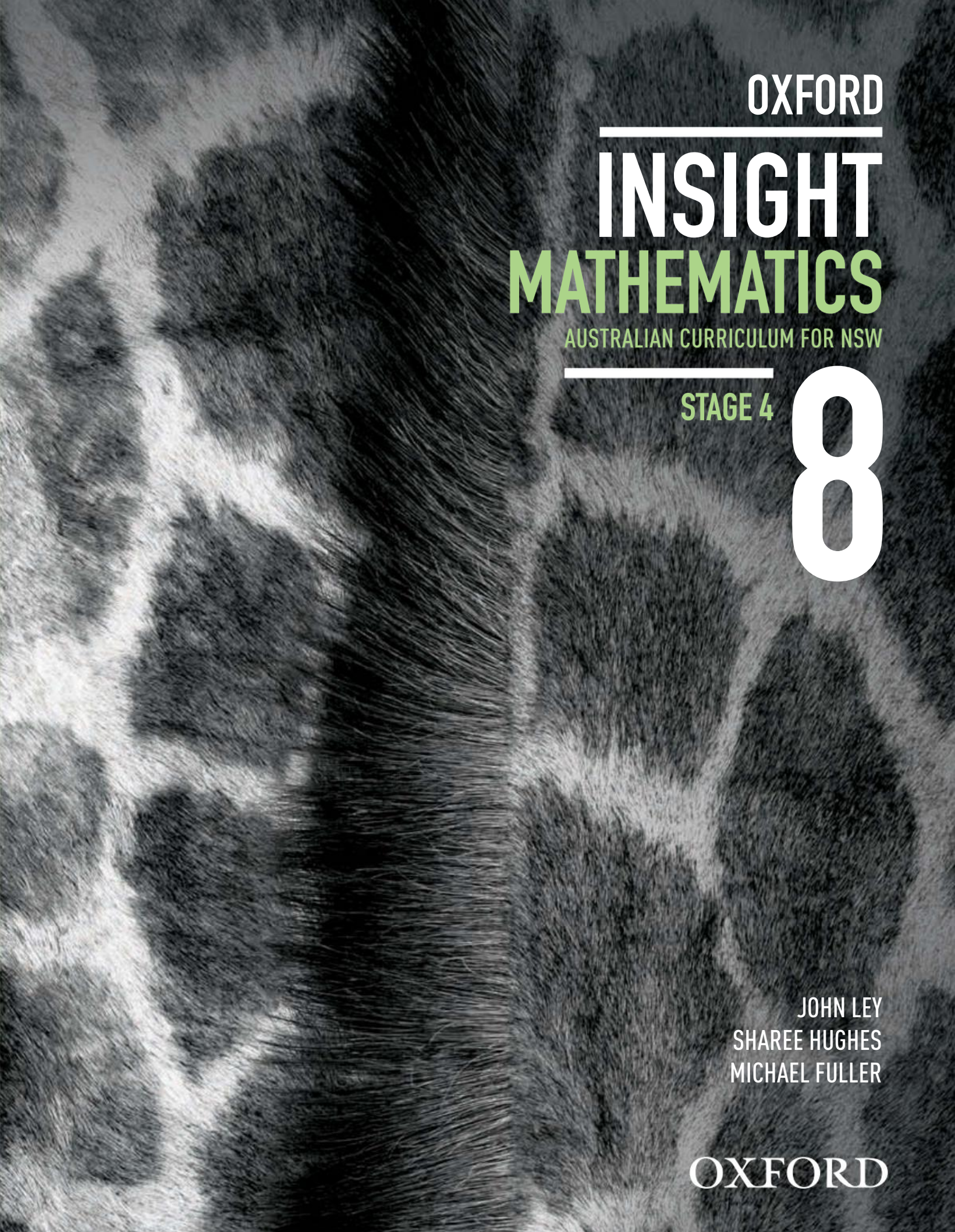

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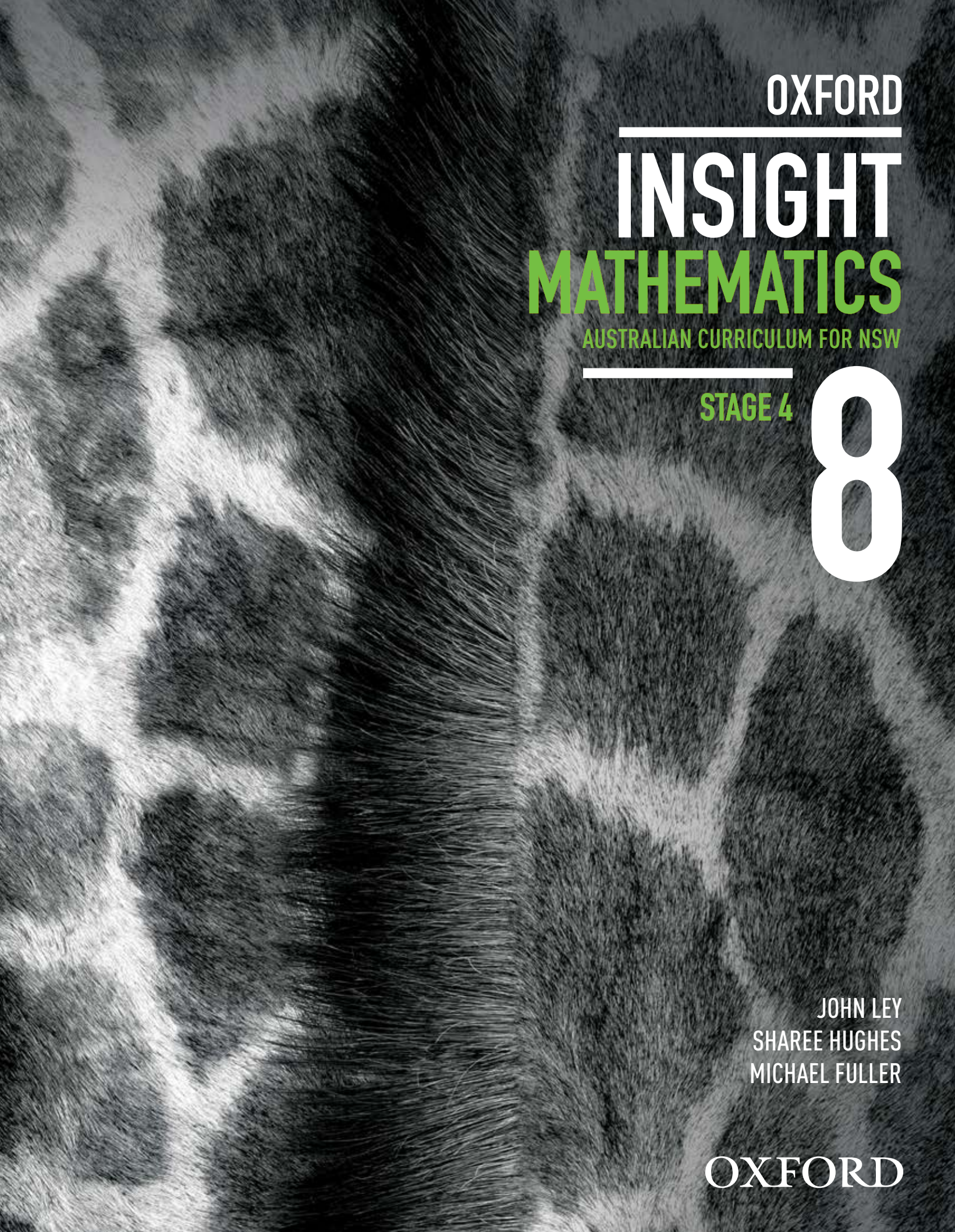
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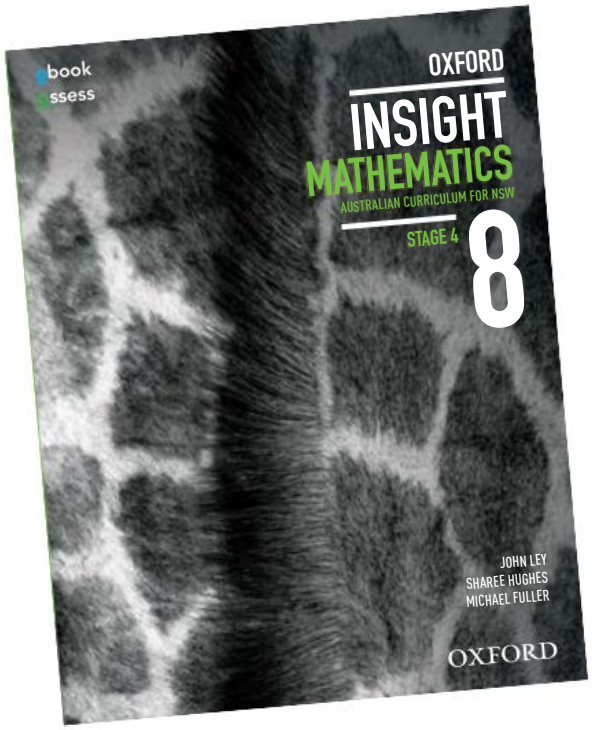

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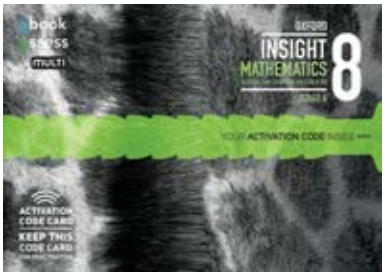

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Chapter	Name	Outcomes	NSW Syllabus references	AC references
1	Review of Year 7			
2	Ratios and rates	MA4–1WM, MA4–2WM, MA4–3WM, MA4–7NA	S4 N&A Ratio and rates	ACMNA188
3	Congruence	MA4–1WM, MA4–2WM, MA4–3WM, MA4–17MG	S4 M&G Properties of geometrical figures 1, S4 M&G Properties of geometrical figures 2	ACMMG200, ACMMG201, ACMMG202
4	Number operations and index laws	MA4–1WM, MA4–2WM, MA4–3WM, MA4–4NA, MA4–9NA	S4 N&A Indices	ACMNA149, ACMNA182, ACMNA183
5	Perimeter, area and capacity	MA4-1WM, MA4-2WM, MA3-9MG, MA3-10MG, MA3-11MG, MA4-12MG, MA4-13MG, MA4-14MG	S4 M&G Length, S4 M&G Area, S4 M&G Volume	ACMMG195, ACMMG196, ACMMG198
CR 2–5 Cumulative review chapters 2–5				
6	Time	MA4-1WM, MA4-2WM, MA3-13MG, MA4-15MG	S4 M&G Time	ACMMG199
7	Percentages	MA4-1WM, MA4-2WM, MA4-3WM, MA4-5NA, MA4-6NA	S4 N&A Fractions, decimals and percentages, S4 N&A Financial mathematics	ACMNA187, ACMNA189
8	Circles and cylinders	MA4-1WM, MA4-2WM, MA3-9MG, MA4-12MG, MA4-13MG, MA4-14MG	S4 M&G Length, S4 M&G Volume	ACMMG186, ACMMG197, ACMMG217
9	Data collection and analysis	MA4-1WM, MA4-2WM, MA4-3WM, MA3-18SP, MA4-19SP, MA4-20SP	S4 S&P Data collection and representation, S4 S&P Single variable data analysis	ACMSP206, ACMSP207, ACMSP284, ACMSP293
10	Pythagoras' theorem	MA4-1WM, MA4-2WM, MA4-16MG	S4 M&G Right-angled triangles (Pythagoras, part)	ACMMG222, ACMNA186
CR 6–10 Cumulative review chapters 6–10				
11	Algebra	MA4-1WM, MA4-2WM, MA4-3WM, MA4-8NA	S4 N&A Algebraic techniques 1, S4 N&A Algebraic techniques 2 (parts)	ACMNA190, ACMNA191, ACMNA192
12	Probability and Venn diagrams	MA4-1WM, MA4-2WM, MA4-3WM, MA4-21SP	S4 S&P Probability 2	ACMSP204, ACMSP205, ACMSP292
13	Linear equations	MA4-1WM, MA4-2WM, MA4-3WM, MA4-10NA	S4 N&A Equations	ACMNA194
14	Coordinate geometry and straight lines	MA4-1WM, MA4-3WM, MA4-11NA	S4 N&A Linear relationships	ACMNA193, ACMNA194
15	Triangles and quadrilaterals	MA4-1WM, MA4-2WM, MA4-3WM, MA4-17MG	M&G Properties of geometrical figures 1	ACMMG165, ACMMG166
CR 11–15 Cumulative review chapters 11–15				

1

Review of Year 7

This chapter deals with a review of material from the Year 7 text.

At the end of this chapter you should be able to:

- use operations with positive and negative integers including graphs
- understand the properties of angles and parallel lines
- understand basic number facts and indices
- work with fractions, decimals and percentages
- draw and build solid shapes
- work with algebraic expressions
- understand transformation and symmetry
- understand chance
- represent and interpret statistical data
- calculate area, surface area and volume
- solve linear equations
- describe the properties of geometrical figures.

A Integers

Exercise 1A

1 Plot the following points on a number line.

a 3, 6, 7, 8 **b** 5, 3, 9, 4

2 **a** Plot the numbers 5 and 10 on a number line.

b Write a statement using $<$ to describe the numbers.

c Write a statement using $>$ to describe the numbers.

d Write two whole numbers between 5 and 10.

e Write three other numbers between 5 and 10.

3 Represent the following operations on a number line and hence find the answer.

a $2 + 4$ **b** $3 + 5 - 2$ **c** $2 \times 7 - 3$

4 Write the opposites of the following statements.

a withdrawing \$5 **b** an increase of 2 cm in length

c a loss of 1 kg in weight **d** a rise of 4°C in temperature

5 State the combined effect of the following.

a a deposit of \$6 followed by a withdrawal of \$7

b a deposit of \$6 followed by a withdrawal of \$3

c a deposit of \$6 followed by a withdrawal of \$14

6 If east is the positive direction, write directed numbers for a journey:

a 7 km west **b** 6 km east **c** 8 km east then 4 km west

7 Plot the following whole numbers on the number line, then write them in ascending order.

a $-8, -5, -6, -9, -4$ **b** $-7, -2, -8, 0, 4$

8 Plot the following numbers on the number line, then write them in ascending order.

a $1\frac{1}{2}, 2\frac{1}{4}, 0, 1, -\frac{1}{2}$ **b** $-1.3, 1.2, -1.6, 0, -1.4$

9 Using a number line to help you, insert $>$ or $<$ symbols to make the following statements true.

a $-5 \quad -6$ **b** $-2 \quad 2$ **c** $0 \quad -3$

10 Simplify the following using a number line.

a $-4 - 3$ **b** $-7 - (-5)$ **c** $6 - (-5)$

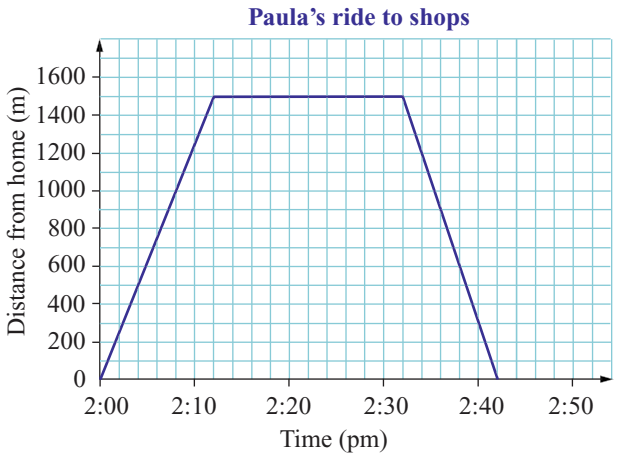
d $4 - 2 + 3$ **e** $5 - 2 - 7$ **f** $-3 - 4 - 5$

11 Plot the following points on the number plane.

$A(2, 4)$, $B(-2, -4)$, $C(3, -4)$, $D(2, -1)$, $E(3, -5)$, $F(4, 0)$

12 Draw a travel graph to describe this journey. Kyle left home at 10 am to visit a friend. He walked 2 kilometres to the bus stop taking 25 minutes, then waited 10 minutes for the bus. The bus journey took 40 minutes to travel 10 kilometres. Kyle then walked 1 kilometre in 15 minutes to his friend's house and stayed for $1\frac{1}{2}$ hours. After that he was driven home. The drive home took 20 minutes.

13 Paula rides her bike from home to the shops and back home again. The graph shows Paula's journey.



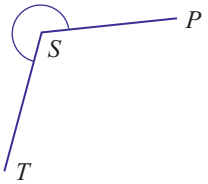
- **a** At what time did Paula leave home?
- **b** Where was Paula at 2:05 pm?
- **c** At what times was Paula 1 kilometre from home?
- **d** How far did she travel in the first 8 minutes?
- **e** What was her speed in the first 8 minutes?
- **f** What is the meaning of the horizontal section of the graph?
- **g** When was Paula stopped?
- **h** How far are the shops from Paula's home?
- **i** How long did she spend at the shops?
- **j** How long did the return journey home take?
- **k** What was her speed for the journey home?
- **l** What was the total distance travelled by Paula for the whole journey?
- **m** What was the average speed for the whole journey, not including the time she was at the shops?

B Angles and parallel lines

Exercise 1B

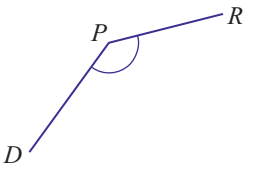
1 Name the angles shown using:

a **i** three letters



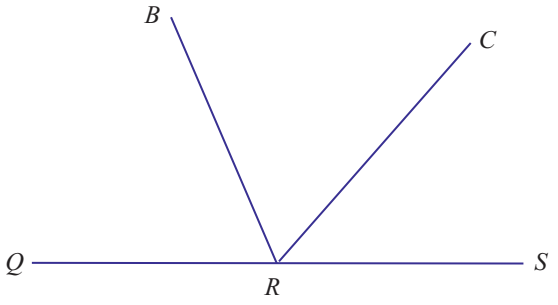
ii the vertex only.

b



2 Measure each of the angles in the diagram.

- **a** $\angle QRB$
- **b** $\angle BRS$
- **c** $\angle CRS$
- **d** $\angle BRC$
- **e** $\angle QRC$
- **f** $\angle QRS$



3 **a** Draw an interval AB approximately 5 cm long. With A as the vertex, draw angles of:

i 38° **ii** 152° **iii** 194° **iv** 253° **v** 345°

b Classify each of the angles in part **a**.

4 **a** In your own words give the meaning of complementary angles.

b Draw a diagram showing a pair of complementary angles.

c Find angles complementary to:

i 40° **ii** 18° **iii** 45° **iv** 83°

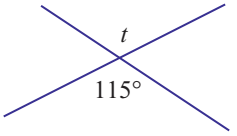
5 **a** Draw a diagram showing a pair of supplementary angles.

b Find angles supplementary to:

i 125° **ii** 80° **iii** 71° **iv** 118°

6 Find the value of the pronumerals and give a reason for your answer.

a



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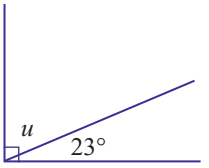

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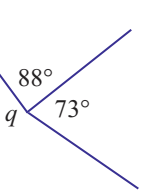

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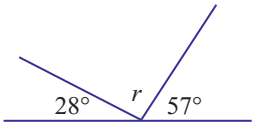

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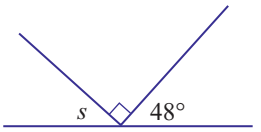

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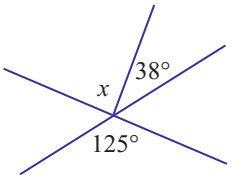
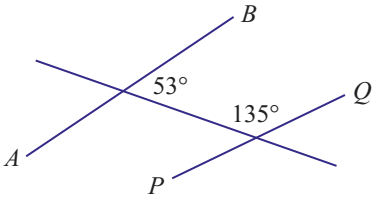


7 State whether or not lines PQ and AB are parallel. Give a reason for your answer.

a



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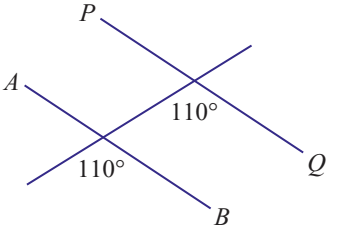
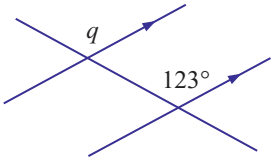


8 Find the value of the pronumerals and state a reason for your answer.

a



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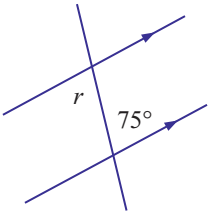

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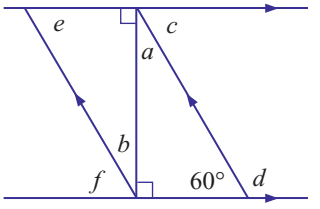


C Number and indices

Exercise 1C

1 Write the multiples of:

a 2 between 13 and 35 **b** 9 between 26 and 73 **c** 4 that are less than 65

2 **a** List the multiples of 8 less than 100.

b List the multiples of 10 less than 100.

c Write the common multiples of 8 and 10 that are less than 100.

d What is the LCM of 8 and 10?

3 **a** Write the factors of 20.

b Write the factors of 35.

c What is the HCF of 20 and 35?

4 Use a factor tree to write the following numbers as a product of prime factors.

a 50 **b** 130 **c** 520

5 Use the method of division by primes to write the following numbers as a product of prime factors.

a 140 **b** 230 **c** 540

6 **a** Write 25 and 40 as a product of prime factors.

b Find the HCF of 25 and 40.

c Find the LCM of 25 and 40.

7 **a** Write 180 and 240 as a product of prime factors.

b Find the HCF of 180 and 240.

c Find the LCM of 180 and 240.

8 Find the HCF and LCM of the following pairs of numbers. First write each number as a product of prime factors.

a 70 and 84 **b** 60 and 90 **c** 280 and 400

9 Find the following.

a $\sqrt{9}$ **b** $\sqrt{1}$ **c** $\sqrt{169}$ **d** $\sqrt[3]{125}$ **e** $\sqrt[3]{729}$ **f** $\sqrt[3]{1331}$

10 Which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 are factors of:

a 4884? **b** 23 400? **c** 161 040?

11 Between which two numbers does $\sqrt{70}$ lie?

12 Evaluate:

a $986 \div 29$ **b** $992 \div 64$

13 Evaluate:

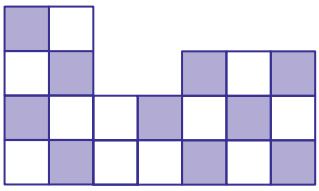
a $16 - 4 \times 2$ **b** $3^2 + 7 \times 2^3$ **c** $\sqrt{\dfrac{10^2 + 8}{3}}$

D Fractions

Exercise 1D

1 What fraction of the diagram shown is:

a shaded? **b** unshaded?



2 If $\frac{5}{13}$ of a diagram is shaded, what fraction is unshaded?

3 Convert $\frac{47}{9}$ to a mixed number.

4 Convert $8\frac{4}{7}$ to an improper fraction.

5 Complete: $\dfrac{7}{12} = \dfrac{49}{\square}$

6 Arrange in ascending order: $\frac{7}{10}$, $\frac{4}{5}$, $\frac{9}{20}$

7 State the reciprocal of $\frac{5}{8}$.

8 Simplify the following.

a $\frac{4}{5}+\frac{1}{3}$ **b** $\frac{11}{12}-\frac{1}{3}$ **c** $\frac{3}{7}\times\frac{1}{4}$ **d** $\frac{5}{6}\div\frac{1}{4}$

9 Simplify $\frac{1}{2}\times\frac{2}{3}+\frac{1}{4}$.

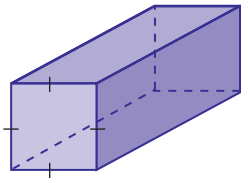
10 Katzurina donated $\frac{2}{11}$ of her weekly income to charity. If her weekly income is $495, how much did she donate?

E Drawing and building solids

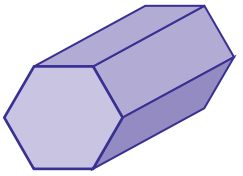
Exercise 1E

1 Name these solids.

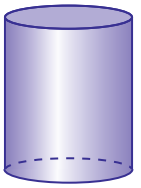
a



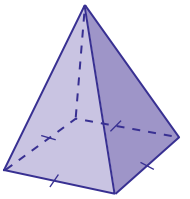
b



c



d



2 What is the geometrical name of the solid that best describes:

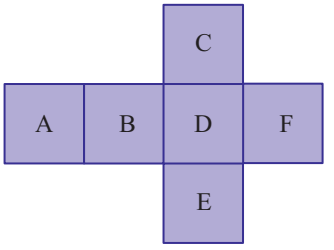
a a tennis ball?

b a cereal packet?

c can of baked beans?

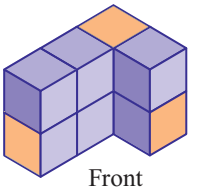
3 The diagram below shows the net of a cube with a different letter on each face. When the cube is assembled, which letter will be opposite the letter:

a A? **b** B?



4 **a** Build the solid shown using cubes.

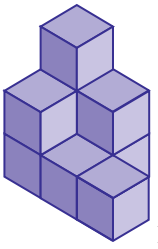
b Add a cube to each of the orange faces and sketch the resulting solid on isometric grid paper.



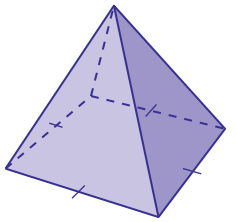
5 Build the given solid and then sketch the view from the:

a front **b** back **c** top

d left side **e** right side.

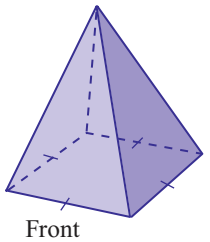


6 Is this solid oblique or right? Explain your answer.



7 Sketch the view of this prism from the:

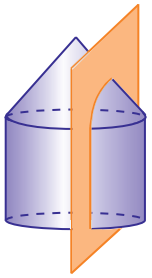
a top **b** front **c** side.



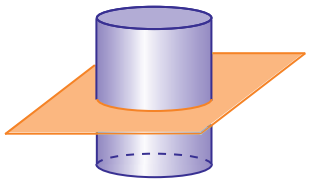
Front

8 Sketch the cross-section when these solids are cut as shown.

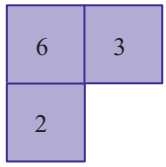
a



b



9 The numbers on the plan give the numbers of cubes in each stack of the solid. Build the solid and sketch the view of the solid from corner A on isometric grid paper.



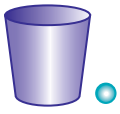
A

F Algebra and variables

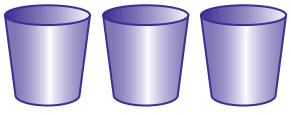
Exercise 1F

1 If there are p marbles in each cup, write algebraic expressions for the total number of marbles in each of the following diagrams.

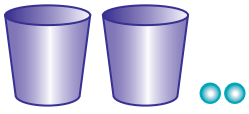
a



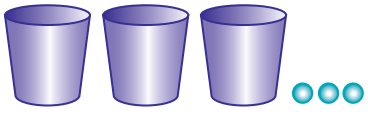
b



c



d



2 Simplify the following.

a $6 \times p$ **b** $g \times r$ **c** $m \times 5$

d $8 \times a \times b$ **e** $3 \times m \times m$ **f** $5 \times a + 3 \times q$

3 Insert multiplication signs to show the meaning of:

a $3p$ **b** ab **c** m^2 **d** $5x^2$ **e** $6pq$

4 Simplify the following.

a $p + p + p$ **b** $y + y + y + y + y$ **c** $z \times 1$

d $3pq \times 1$ **e** $0 \times 5p$

5 If $m = 3$ and $n = 4$, evaluate the following.

a mn **b** $5mn$ **c** $7m - 3n$ **d** n^2 **e** $4n^2$

6 Write the following in fraction form.

a $t \div 2$ **b** $g \div r$ **c** $r \div g$ **d** $4w \div 7$ **e** $3 \div 2x$

7 Show the meaning of the following expressions by inserting a division sign.

a $\frac{k}{3}$ **b** $\frac{4}{m}$ **c** $\frac{p}{q}$ **d** $\frac{3e}{4}$ **e** $\frac{mn}{t}$

8 If $p = 4$ and $q = 5$, evaluate the following.

a $\frac{q}{5}$ **b** $\frac{24}{p}$ **c** $\frac{5p}{q}$ **d** $\frac{4q}{p}$ **e** $\frac{5p}{2q}$

9 If $p = 7$ and $q = 3$, evaluate the following.

a $3(p + 1)$ **b** $4(q - 3)$ **c** $q(q + 1)$ **d** $5(q - 4)$ **e** $pq(p - 5)$

10 If $p = 12$ and $q = 5$, evaluate the following.

a $\frac{p + 9}{3}$ **b** $\frac{q - 3}{2}$ **c** $\frac{26}{p + 1}$ **d** $\frac{p + 6}{q + 1}$ **e** $\frac{3p + 3}{q + 8}$

G Fractions, decimals and percentages

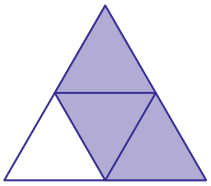
Exercise 1G

1 Complete the following table.

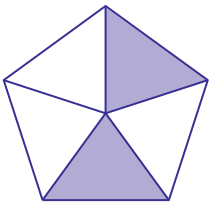
	Fraction	Decimal	Percentage
a	$\frac{1}{10}$		
b		0.2	
c			25%
d	$\frac{1}{3}$		
e		0.375	
f			50%
g	$\frac{5}{8}$		
h			$66\frac{2}{3}\%$
i		0.85	
j	$\frac{98}{100}$		

2 What percentage of these diagrams is:

a **i** shaded? **ii** unshaded?



b



3 Express each percentage as a simplified fraction.

a 25% **b** 60% **c** 72% **d** 86%

4 **a** Write 59 out of 100 as a percentage.

b Emily scored 78 out of 100 in her Science exam. Write this as a percentage.

c Write 33 out of 50 as a percentage.

d Linda scored 19 out of 25 in her Japanese exam. Write this as a percentage.

5 Express these percentages as decimals.

a 43% **b** 29% **c** 123% **d** 0.35%

6 Express the following as percentages.

a $\frac{1}{4}$ **b** 0.312 **c** 4.8 **d** $\frac{2}{3}$ **e** $\frac{5}{8}$

7 **a** Write 78 g as a percentage of 500 g.

b Write 38c as a percentage of \$2.

c Write 35 kg as a percentage of 140 kg.

8 Convert each to a percentage and arrange in ascending order: $\frac{3}{5}$, 68%, 0.48, $\frac{2}{3}$

9 **a** Increase 40 m by 12%.

b Katherine buys pens for 50c each. She sells them at an increased price of 150%. What is the selling price?

c Decrease 200 m by 30%.

10 In a class of 28 students there are 13 boys. Write the ratio of boys to girls.

11 Express each ratio in simplest form.

a 12 : 40 **b** 30 : 108

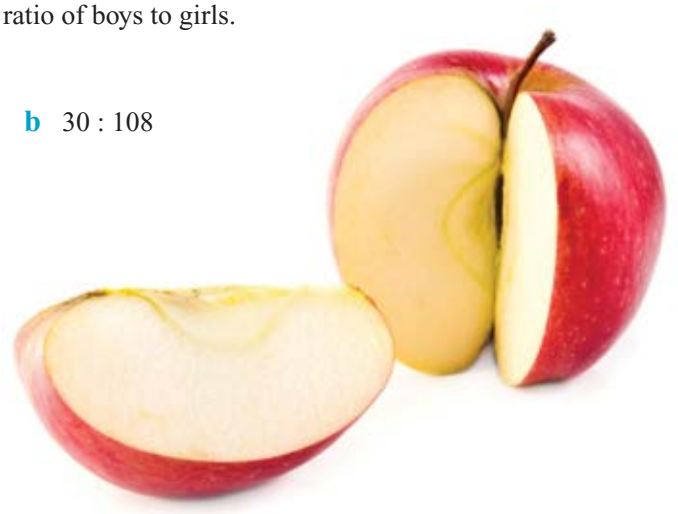
12 State the value of 8 in 0.148 507.

13 Express $\frac{7}{100} + \frac{9}{1000}$ as a decimal.

14 **a** Express 0.08 as a fraction.

b Express $\frac{3}{4}$ as a decimal.

c Express $\frac{7}{9}$ as a decimal.



15 Write 15.0775 correct to the nearest hundredth.

16 Insert one of $>$, $<$ or $=$ to make the following statements correct.

a 21.12 21.012 **b** 19.6 19.60

17 The decimal number closest to 0.47 is:

A 0.45 **B** 0.41
C 0.5 **D** 0.05

18 Simplify the following.

a $4.8 \div 0.4 \times 6$ **b** $1.2 \times 0.86 \times 3$
c $16.6 + 2.38 + 4.7$

19 Stacey purchased 15.4 m of fabric. She intends to make three shirts. Each shirt requires 4.25 m of fabric.

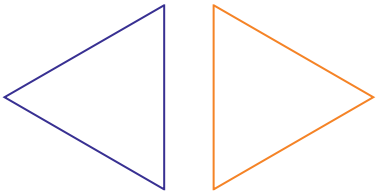
a How much fabric is used for making the shirts?
b What length of fabric remains?

H Transformations and symmetry

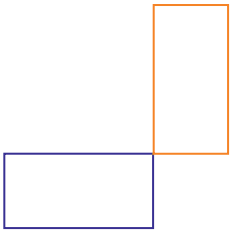
Exercise 1H

1 State whether the original (blue) figure has been translated, rotated or reflected to its new position.

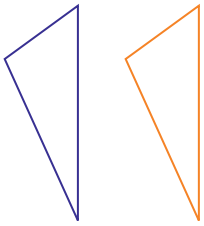
a



b

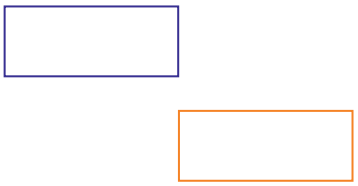


c

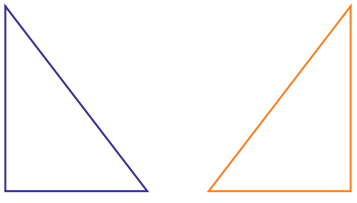


2 State whether the original (blue) figure has been translated, rotated or reflected to its new position.

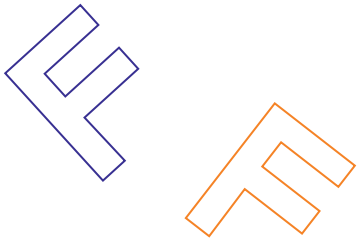
a



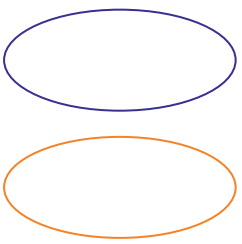
b



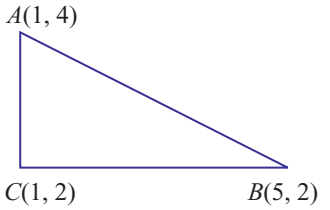
c



d



3 Plot this triangle onto grid paper. Show its new position and the coordinates of B when it is:



a translated 2 units right and 1 unit down

b rotated clockwise about C through a $\frac{1}{4}$ of a turn

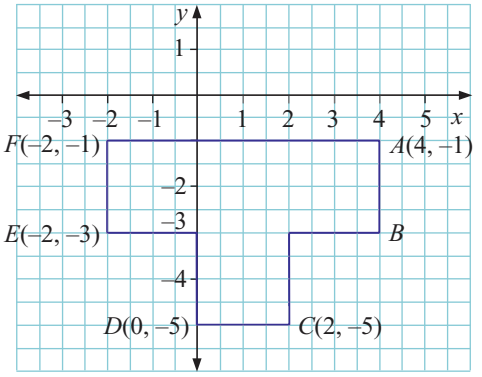
c reflected in the line CB.

4 **a** On the shape shown, what are the coordinates of B?

b Translate the shape 3 units left and 2 unit up. What are the coordinates of B'?

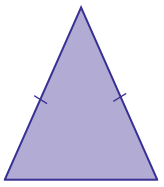
c Rotate the shape clockwise about A through a $\frac{1}{2}$ of a turn (2 right angles). What are the coordinates of B'?

d Reflect the shape in the line AB. What are the coordinates of B'?

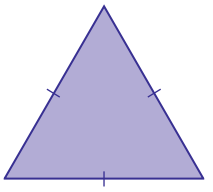


5 Draw all the axes of symmetry of the following shapes.

a



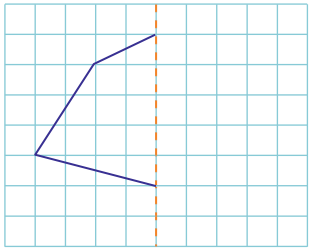
b



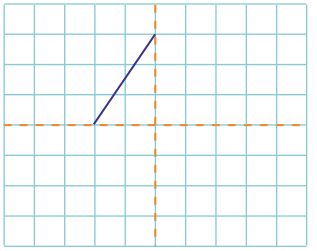
6 Which of the shapes in question **5** has rotational symmetry? State the order of symmetry.

7 Complete the following shapes given the axes of symmetry.

a

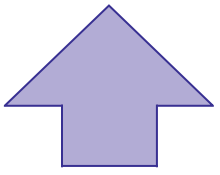


b



8 Copy the following shapes and draw all the axes of symmetry.

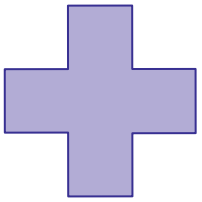
a



b

9 Do the following shapes have rotational symmetry? If so, state to order of symmetry.

a



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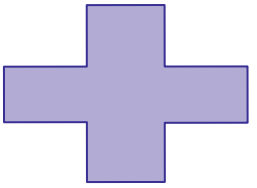
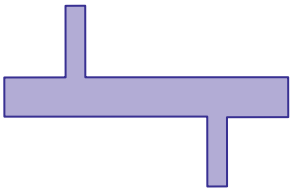


c

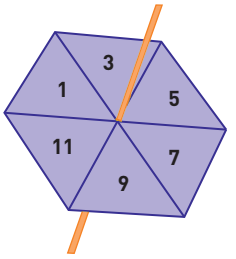


I Probability

Exercise 1I

1 A hat contains 1 pink, 1 red, 1 blue and 1 green ticket. One ticket is chosen at random.
- **a** List the sample space.
- **b** What is the probability of selecting the red ticket?

2 **a** List the sample space of this spinner.
- **b** What is the probability of spinning a 7?



3 Twenty cards with the numbers 1 to 20 written on them are shuffled and one card is chosen at random.
- **a** List the sample space.
- **b** What is the probability that the selected card has 13 written on it?

4 A box of ice-creams contains 8 chocolate, 5 strawberry and 3 vanilla. One ice-cream is chosen at random.
- **a** How many ice-creams are in the box?
- **b** How many ice-creams are strawberry?
- **c** What is the probability of selecting a strawberry ice-cream?

5 A box of chocolates contains 11 hard-centred and 9 soft-centred chocolates. One chocolate is chosen at random.
- **a** How many chocolates are in the box?
- **b** How many are soft-centred?
- **c** Kaisha likes soft-centred chocolates. What is the probability that she selects a soft-centred chocolate?

6 One card is selected at random from a normal deck of 52 cards. What is the probability that it is:
- **a** a club?
- **b** a black card?
- **c** a black ace?

7 In Year 8, 65% of students walk to school, 30% catch a bus and 5% are driven. Find the probability that a student selected at random:
- **a** walks
- **b** is driven
- **c** catches a bus
- **d** does not walk
- **e** does not catch a bus.

8 **a** Write a statement describing a probability of:
- **i** 0
- **ii** 10%
- **iii** 50%
- **iv** $\frac{85}{100}$
- **v** 1

b Estimate the probability associated with each phrase.
- **i** no chance
- **ii** very likely
- **iii** certain

9 A coin is tossed once. It has heads on one side and tails on the other. Describe an event that would be:
- **a** certain
- **b** impossible
- **c** of even chance.

10 A deck of 8 cards labelled A, B, C, D, E, F, G and H is shuffled and one card is selected at random. Describe events with a probability of:

a 0 **b** 1 **c** $\frac{1}{8}$ **d** $\frac{1}{4}$ **e** $\frac{7}{8}$

11 A spinner has four equally sized sectors coloured red, blue, green and yellow. It is spun once. What is the probability that the colour is:

a blue? **b** not blue? **c** green or yellow? **d** not green or yellow?

12 A box of chocolates has milk and dark chocolates. One chocolate is selected at random. The probability of selecting a milk chocolate is $\frac{3}{10}$. What is the probability of selecting a dark chocolate?

J Data investigation

Exercise 1J

1 What is the difference between primary and secondary data?

2 State whether a census or sample survey would be used for each of these investigations. Discuss your answers in groups.

a the weights of the members of a football team

b the number of people who prefer vanilla ice-cream

c the number of pets owned by students in a class

3 Suggest the possible bias in each of these samples.

a people at a bus stop **b** people selected from an electoral role

4 Classify these as nominal or numerical variables.

a hair colour **b** height **c** distance travelled

5 **a** Construct a frequency distribution table for this data for winning margins in a series of soccer matches.

2	2	3	0	0	2	0	4	3	1	3	5
0	3	3	3	1	1	2	2	0	0	3	1
0	1	2	1	0	4	5	3	0	1	0	2
5	1	4	3	4	2	3	0	2	3	4	0
1	2	2	0	0	2	3	2	0	4	0	2

b Draw a frequency histogram and polygon for this information.

c **i** How many soccer matches were played?

ii How many winning margins of 2 were there?

iii What does a winning margin of 0 mean?

6 **a** Draw a frequency distribution table to show the following information:

8	9	5	10	7	6	6	7	5	1	8	6
2	8	6	2	6	4	9	4	7	4	9	2
6	5	9	10	10	1	7	6	2	2	3	1

b How many numbers were 8 or more?

7 Draw a stem-and-leaf plot for the following information. Use stems of 12, 13, 14, 15, 16.

146	145	128	138	161	150	149	142	132	142
150	145	151	131	148	145	132	145	144	153

8 **a** Draw a column graph for the data in this table.

b Draw a dot plot showing this information.

Drink	Frequency
Soft drink	18
Still water	10
Juice	3
Tea/coffee	2
Other	2

9 This table shows the favourite holiday destinations of 90 Year 8 students.

Destination	Number of students	Sector angle
Snowfields	28	
Gold Coast	35	
NSW South Coast	15	
NSW North Coast	12	

a Copy and complete the table, calculating the sector angles.

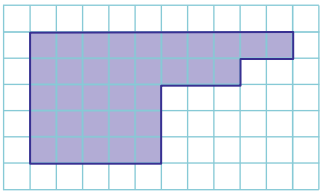
b Draw a sector graph for this information. Shade the sectors and include a title and a legend.

c Draw a divided bar graph of length 18 cm displaying the information.

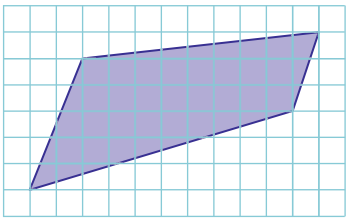
K Area, surface area and volume

Exercise 1K

1 Find the area of this shape.

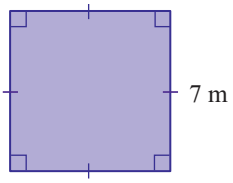


2 By counting squares, find the area of this shape.



3 Find the area of the following shapes.

a



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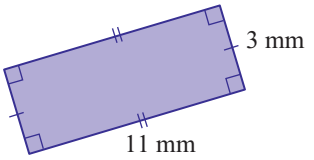

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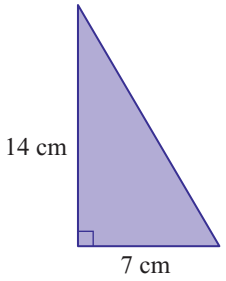
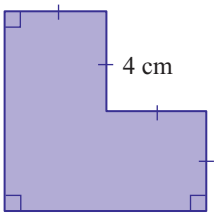


d



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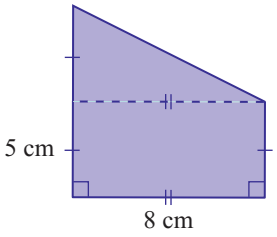
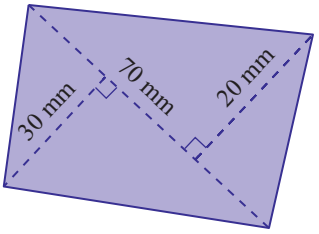
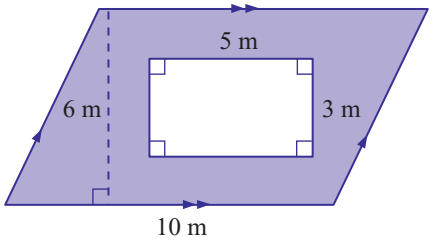


f

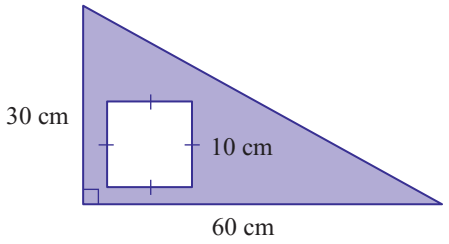


4 Find each shaded area.

a

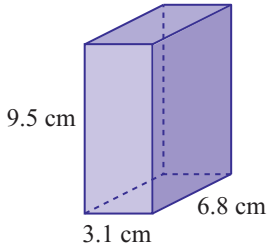


b

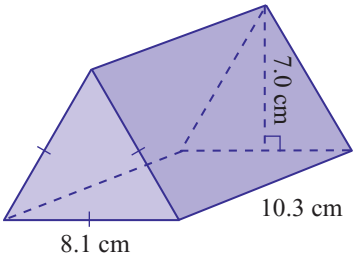


5 Calculate the surface area of each solid.

a

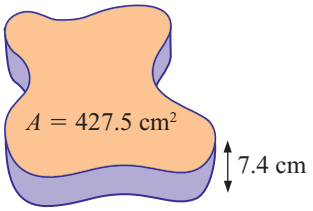


b

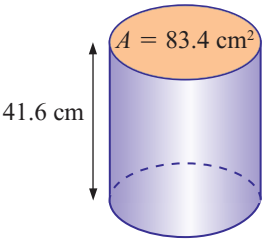


6 Calculate the volume of each solid.

a

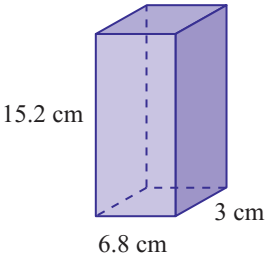


b

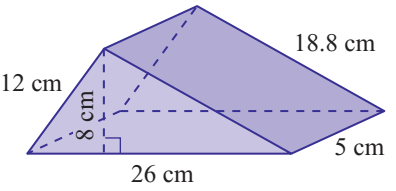


7 Calculate the volume of each solid.

a

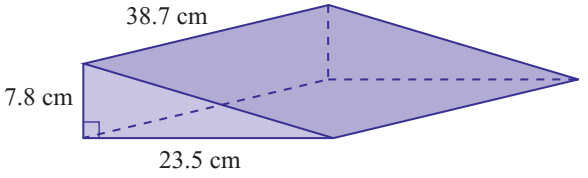


b

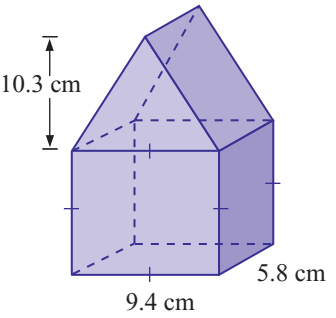


8 Calculate the volume of each solid.

a

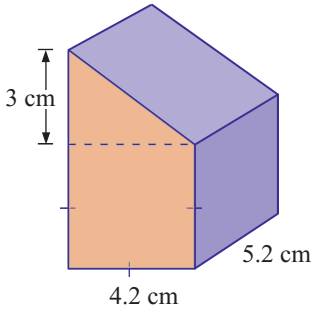


b



9 Calculate the volumes of these composite solids.

a



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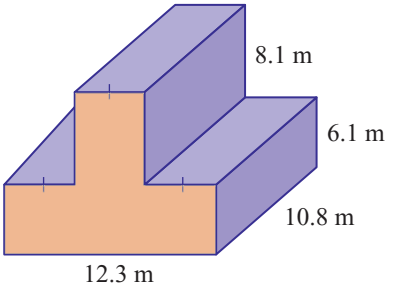


L Data measures

Exercise 1L

1 For the scores 11, 14, 15, 19, 19, 21 find the:

a mean **b** mode **c** median **d** range.

2 For the scores in this stem-and-leaf plot find the:

a mean **b** mode

c median **d** range.

Stem	Leaf
2	7 8 8
3	0 0 1 2 3 4 5 6 6
4	1 2 4 4 4 6 8
5	3 5 7 8
6	2 3

3 The back-to-back stem-and-leaf plot compares the marks gained by class A and class B in their half-yearly Mathematics exam.

a Find the mean, mode, median and range for each class.

b Which class performed better? Explain your answer.

Class B Leaf	Stem	Class A Leaf
2 1	2	8 8
6 4 2 1	3	0 3 5 6
6 5 3 1 0	4	0 2 6 6 8
1 1 0	5	3 6 9
6	6	7

4 **a** Complete this frequency distribution table.

b Calculate the mean correct to 1 decimal place.

Score (x)	Frequency (f)	$f \times x$
8	6	
9	11	
10	15	
11	12	
12	8	
13	7	
14	8	
	$\Sigma f =$	$\Sigma fx =$

5 Find the mode and range of each set of scores.

a

Score (x)	Frequency (f)
11	6
12	14
13	5
14	11
15	4

b

Score (x)	Frequency (f)
53	28
54	36
55	12
56	45
57	33

6 Complete the table to find the mean, mode, median and range for this stem-and-leaf plot.

Stem	Leaf	f	$f \times$ stem	Sum of leaves
8	4 7 9	3		
9	0 0 3 4 5 5 5 7 8 9			
10	1 1 2 3 4 7 9 9			
11	2 3 4 5 8 8 9 9 9 9			
12	0 1 3 7 9 9 9			
13	0 1 1 1 2 2 3 4 8 9			
14	1 1 1 2 8 9			

7 The marks for a class in English, Mathematics and Science tests are given.

Mark	English frequency	Mathematics frequency	Science frequency
1	3	0	5
2	3	1	4
3	3	3	3
4	3	5	2
5	3	6	1
6	3	6	1
7	3	5	2
8	3	3	3
9	3	1	4
10	3	0	5

a For each subject calculate the mean, median and mode.
b Which of the measures in part a show a difference in the results?
c In which subject did the class perform best?
d Display this data in cumulative histograms.

Linear equations

Exercise 1M

1 Show each step required to backtrack from the equation $4x + 12 = 0$ to x. Solve for x.

2 Solve the following equations.

a $x + 11 = 17$
b $x + 9 = -6$
c $4x = 36$
d $-9x = 63$
e $3y + 18 = 29$
f $5 - 4p = -47$
g $4d + 8 = 3d - 12$
h $18 + 7c = 32 - 3c$
i $3(m + 6) = 2(m - 1)$
j $8(q - 5) = -3(10 + 3q)$

3 Solve the following equations.

a $\frac{4p}{5} = 6$ **b** $\frac{3x + 12}{7} = 12$

4 Is the given value for the pronumeral a solution to the equation?

a $5d + 12 = 28;\ \ d = 3$ **b** $\frac{x}{5} + 7 = 24;\ \ x = 3\frac{2}{5}$

5 Complete this table to solve the equation $3x - 4 = 23$ using the guess, check and refine method.

x	5	11	8	9
$3x - 4$	11			

6 Starting with x, write the expression produced if you:

a add 5 **b** subtract 8 **c** multiply by 4

d divide by 7 **e** multiply by 3 then add 5 **f** divide by 8 and subtract 3

g multiply by 5 then divide by 4 **h** add 6 then multiply by 4 **i** subtract 5 then divide by 3

7 Complete the following flowcharts.

a $x \xrightarrow{\times 8} \square$ **b** $w \xrightarrow{-4} \square$ **c** $z \xrightarrow{\div 7} \square$

d $m \xrightarrow{+11} \square$ **e** $y \xrightarrow{\times 3} \square \xrightarrow{-13} \square$ **f** $a \xrightarrow{+4} \square \xrightarrow{\times 3} \square$

g $x \xrightarrow{\div 7} \square \xrightarrow{+22} \square$ **h** $w \xrightarrow{+5} \square \xrightarrow{\div 7} \square$ **i** $x \xrightarrow{\times 2} \square \xrightarrow{\div 7} \square$

8 Complete the following flowcharts.

a $x \xrightarrow{+4} \square$ **b** $y \xrightarrow{-7} \square$ **c** $q \xrightarrow{\times 9} \square$

d $m \xrightarrow{\div 6} \square$ **e** $k \xrightarrow{\times 4} \square \xrightarrow{+8} \square$ **f** $z \xrightarrow{+3} \square \xrightarrow{\times 5} \square$

g $y \xrightarrow{\div 7} \square \xrightarrow{-2} \square$ **h** $z \xrightarrow{-5} \square \xrightarrow{\div 4} \square$ **i** $x \xrightarrow{\times 9} \square \xrightarrow{+5} \square$

j $w \xrightarrow{\times 5} \square \xrightarrow{\div 8} \square$ **k** $p \xrightarrow{\div 8} \square \xrightarrow{\times 5} \square$ **l** $n \xrightarrow{-5} \square \xrightarrow{\times 8} \square$

9 Solve the following equations.

a $\frac{3a}{8} = 4$ **b** $\frac{7g}{9} = 11$ **c** $\frac{4x - 5}{7} = 3$

d $\frac{3d + 7}{2} = 5$ **e** $\frac{8e - 5}{11} = -3$ **f** $\frac{4f + 7}{5} = -7$

g $\frac{3k}{7} = \frac{2}{5}$ **h** $\frac{7x}{5} = \frac{3}{4}$ **i** $\frac{4p}{5} = \frac{2}{3}$

10 Do the following to generate equations that can be solved in two steps and have the solution $x = 10$.

a Multiply both sides by 3 then add 5 to both sides.

b Multiply both sides by 2 then subtract 7 from both sides.

c Add 3 to both sides then multiply both sides by 5.

d Subtract 1 from both sides then multiply both sides by 4.

e Divide both sides by 2 then add 9 to both sides.

f Divide both sides by 5 then subtract 4 from both sides.

g Add 8 to both sides then divide both sides by 2.

h Subtract 6 from both sides then divide both sides by 2.

11 The time needed to roast a piece of meat is 20 min plus 30 min for every 0.5 kg of meat to be cooked. This information can be expressed by the formula $t = 20 + 30w$, where t is the number of minutes needed and w is the number of 0.5 kg of meat.

a Calculate the time needed to roast a piece of meat that weighs 2.5 kg.

b A piece of meat was roasted for 110 min, using this formula. What was the weight of meat cooked?

12 To build a pattern of rectangles, the number of matches needed (N) is given by $N = 5r + 1$, where r is the number of rectangles.

a How many matches are needed to build 7 rectangles?

b How many rectangles could be built using 61 matches?

13 Sylvie is a salesperson. She is paid a weekly commission ($C). Her commission is calculated by the formula $C = 200 + \frac{S}{20}$ where S is the value, in dollars, of her sales (the goods she sells).

a How much did she earn in the week in which the value of her sales was $20 000?

b One week her commission was $640. What was the value of her sales?

N Triangles and quadrilaterals

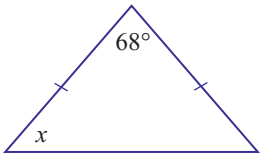
Exercise 1N

1 Construct a triangle with side lengths 13 cm, 12 cm and 5 cm.

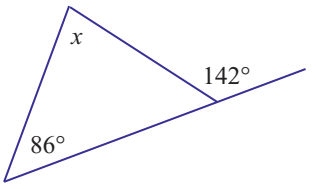
2 Draw a right-angled isosceles triangle.

3 Find x, giving a reason for your answer.

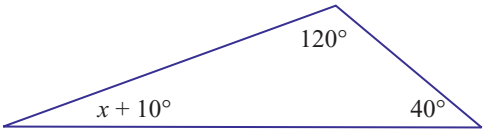
a



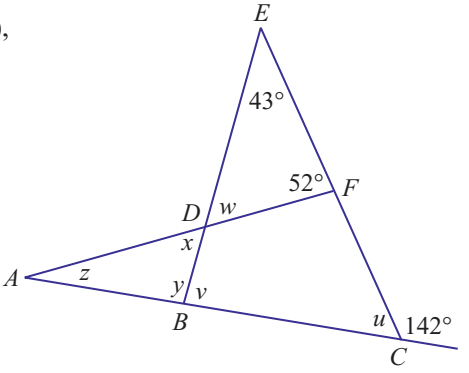
b



4 Write an equation and solve it to determine the value of x. Give a reason for your answer.



5 Find the value of the pronumerals in alphabetical order (u to z), giving reasons for your answers.



6 **a** Draw a rhombus.

b List the properties of a rhombus.

7 **a** Draw a convex quadrilateral.

b Comment on the diagonals of your diagram.

2

Ratios and rates

This chapter deals with the comparisons of quantities.

At the end of this chapter you should be able to:

- express statements as ratios
- simplify ratios, including ratios as fractions and decimals
- use the unitary method
- divide a quantity into a given ratio
- compare and calculate rates using given information
- use scale effectively.

NSW Syllabus references: S4 N&A Ratio and rates
Outcomes: MA4–1WM, MA4–2WM, MA4–3WM, MA4–7NA
NUMBER & ALGEBRA – ACMNA188

Diagnostic test

1 What is the ratio of squares : circles : triangles?

A 8 : 7 : 5 **B** 7 : 8 : 5
C 5 : 7 : 8 **D** 7 : 5 : 8

2 Express Ben's money to Aidan's money as a ratio, given that Ben has $16 and Aidan has $25.

A 16 : 25 **B** 25 : 16
C $25 : $16 **D** $16 : $25

3 In a class of 20 students $\frac{2}{5}$ are boys. Express this as a ratio of boys to girls.

A 5 : 2 **B** 3 : 2
C 5 : 3 **D** 2 : 3

4 Express 4 : 7 as a fraction.

A $\frac{4}{11}$ **B** $\frac{4}{7}$ **C** $\frac{7}{4}$ **D** $\frac{7}{11}$

5 Express $\frac{3}{8}$ as a ratio.

A 8 : 3 **B** 3 : 11
C 11 : 3 **D** 3 : 8

6 Determine the value of x when $\frac{2}{3} = \frac{x}{45}$.

A 15 **B** 30 **C** 52.6 **D** 67

7 Find the value of y when $\frac{81}{90} = \frac{y}{10}$.

A 7 **B** 9 **C** 11 **D** 13

8 Simplify 24 : 56 : 40.

A 3 : 7 : 5 **B** 5 : 3 : 7
C 3 : 5 : 7 **D** 7 : 5 : 3

9 Simplify 1.5 : 6.

A 4 : 1 **B** 3 : 12
C 1 : 4 **D** 12 : 3

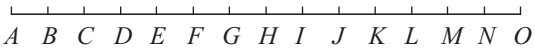
10 The ratio of Brendan's savings to expenses is 2 : 9. Calculate Brendan's savings when his expenses are $135.

A $30 **B** $67.50
C $58.20 **D** $15

11 The ratio of a girl's height to her mother's height is 4 : 9. Calculate the mother's height given that the girl's height is 80 cm.

A 35.5 cm **B** 96 cm
C 180 cm **D** 204 cm

12 Which point divides the line segment AO into the ratio 2 : 5?



A C **B** E
C K **D** M

13 Divide $800 into the ratio 3 : 17.

A $340, $120 **B** $266.67, $47.06
C $300, $1700 **D** $120, $680

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA188.

A Ratio review

A ratio is a comparison of like quantities.

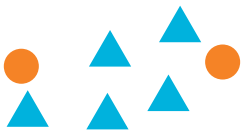
A ratio is written in the form $a : b$ (meaning 'a to b') where a and b are called the terms of the ratio and they are expressed in the same units.

- *Ratios have no units.*
  To express John's money as a ratio to Ben's money when John has \$8 and Ben has \$13, we write:
  John to Ben $= 8 : 13$
- *Order is important for ratios.*
  $4 : 1$ is not the same as $1 : 4$.
  $4 : 1 = \frac{4}{1}$ but $1 : 4 = \frac{1}{4}$
- *Each part of a ratio can be expressed as a fraction of the whole.*
  For example, in the ratio $5 : 9$, the first part is $\frac{5}{14}$ of the whole and the second part is $\frac{9}{14}$ of the whole.
  In general, for the ratio $a : b$, the first part, a, is $\frac{a}{a+b}$ of the whole and the second part, b, is $\frac{b}{a+b}$ of the whole.
- *Terms must be expressed in the same unit.*
  To express 75c as a ratio of \$2.33:
  change 75c to dollars, so $0.75 : 2.33$
  or change \$2.33 to cents, so $75 : 233$

Exercise 2A

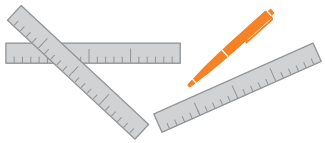
1 Express each of the following as a ratio.

a circles to triangles



b stars to moons

c pens to rulers



2 David has \$4 and Carla has \$7. Find these ratios.

a David's money to Carla's money

b Carla's money to David's money

3 'My pocket money is three times your pocket money', says Gerard to Alison.
What is the ratio of:

a **i** Gerard's pocket money to Alison's?

ii Alison's pocket money to Gerard's?

b Do we know how much pocket money each person receives?

4 Peter can cycle twice as fast as Amy. Find these ratios.

a **i** Amy's cycling speed to Peter's speed

ii Peter's cycling speed to Amy's speed

b Do we know the cycling speed of each person?

5 **a** Are the ratios 1 : 8 and 8 : 1 equal? Give a reason for your answer.

b If $a : b = b : a$, what can be said about a and b?

EXAMPLE 1

A recipe for children's playdough lists 4 cups of flour, 2 cups of water and 1 cup of salt.

a What is the ratio of flour to water?

b What is the ratio of water to the total number of cups required?

c What fraction of the dough mixture is salt?

Flour	:	water	:	salt
4 cups	:	2 cups	:	1 cup

Total is $4 + 2 + 1 = 7$ cups

a Ratio of flour to water is $4 : 2 = 2 : 1$.

b Ratio of water to total cups is $2 : 7$.

c Fraction of salt is 1 cup out of 7 cups or $\frac{1}{7}$.

It's a good idea to summarise the given information.

6 The ratio of sand to cement in a mortar mix is 5 : 2.

a What fraction of the mortar mix is sand?

b What fraction of the mortar mix is cement?

7 Feather blue is a pale blue paint made by mixing 1 part blue to 8 parts of white paint.

a What is the ratio of blue to white paint?

b What is the ratio of white to blue paint?

c What fraction of feather blue is blue paint?

d What fraction of feather blue is white paint?

e Alf mixed the blue and white paints in the ratio 8 : 1. Describe the result.

8 In a class of 30 students, $\frac{3}{5}$ of the students are boys.

a What fraction of the class is girls?

b What is the ratio of boys to girls?

c What is the ratio of girls to boys?

d Do we know how many students are boys? If so, how many?

e Do we know how many students are girls? If so, how many?

9 A box of jellybeans contains 4 blue, 5 orange, 6 red, 3 purple and 2 green jellybeans.

a What fraction of the jellybeans are blue?

b Which colour is the least common?

c Which colour is the most common?

d What fraction of the jellybeans are purple?

e What is the ratio of orange jellybeans to the total number of jellybeans in the box?

EXAMPLE 2

For each of the following diagrams find the:

i ratio of shaded area to unshaded area

ii ratio of shaded area to the total area

iii fraction of the total area that is shaded.

a

b

a **i** shaded : unshaded = 1 : 3

ii shaded : total = 1 : 4

iii Fraction of total area shaded is $\frac{1}{4}$.

b **i** shaded : unshaded = 4 : 1

ii shaded : total = 4 : 5

iii Fraction of total area shaded is $\frac{4}{5}$.

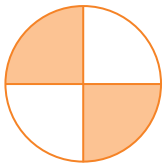
10 For each of the following diagrams find the:

i ratio of shaded area to unshaded area

ii ratio of shaded area to the total area

iii fraction of the total area that is shaded.

a



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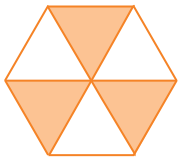

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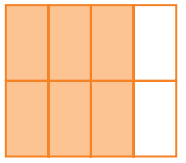

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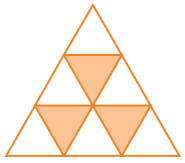
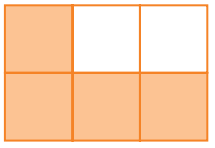
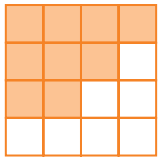


e



f



EXAMPLE 3

For each ratio, express the first part as a fraction of the whole.

a 4 : 5 **b** 1 : 9 **c** 7 : 2

a The ratio 4 : 5 has 9 parts in total. The fraction is $\frac{4}{9}$.

b The ratio 1 : 9 has 10 parts in total. The fraction is $\frac{1}{10}$.

c The ratio 7 : 2 has 9 parts in total. The fraction is $\frac{7}{9}$.

When converting a ratio to a fraction, first find the total number of parts by adding the numbers in the ratio, then put the part you want as a fraction over the total.

11 Each ratio shows the number of cats to dogs in a pet shop. For each ratio, express the number of cats as a fraction of the total number of animals.

a 2 : 3 **b** 4 : 3 **c** 5 : 4 **d** 6 : 1
e 8 : 5 **f** 5 : 7 **g** 11 : 3 **h** 2 : 5

EXAMPLE 4

Each fraction shows the first part of a ratio as a fraction of the whole. Find the ratio.

a $\frac{2}{3}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$

a For $\frac{2}{3}$, the first part of the ratio is 2.
The second part of the ratio is $3 - 2 = 1$.
The ratio is 2 : 1.

The denominator of a fraction is the total number of parts. When converting to a ratio, the numerator of the fraction is the first number and the remaining parts are the second number.

b For $\frac{1}{4}$, the first part of the ratio is 1.
This leaves 3 parts out of a total of 4 parts. The ratio is 1 : 3.

c For $\frac{3}{8}$, the first part of the ratio is 3.
This leaves 5 part out of the total of 8 parts. The ratio is 3 : 5.

12 Each fraction shows the first part of a ratio as a fraction of the whole. Find the ratio.

a $\frac{3}{4}$ **b** $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{5}{7}$
e $\frac{4}{5}$ **f** $\frac{2}{9}$ **g** $\frac{6}{7}$ **h** $\frac{5}{9}$

13 Flour, water and salt is mixed in the ratio of 4 : 2 : 1 to form playdough.

a What fraction of the playdough is water?
b What fraction of the playdough is salt?

EXAMPLE 5

Express each of the following as a ratio.

a 7 cm to 3 m **b** 73 mL to 2 L **c** 3 h to 17 min

a Convert 3 m to 300 cm. The ratio is 7 cm to 300 cm or 7 : 300.
b Convert 2 L to 2000 mL. The ratio is 73 mL to 2000 mL or 73 : 2000.
c Convert 3 h to 180 min. The ratio is 180 min to 17 min or 180 : 17.

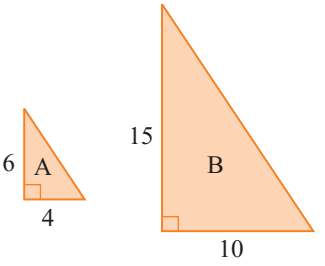
14 Express each of the following as a ratio.

a 1 m to 1 cm **b** 7 mL to 1 L **c** 11 s to 1 min
d 3 km to 173 m **e** 2 h to 13 min **f** 67 m to 1 km
g 3 L to 87 mL **h** 1 km to 27 m **i** 13 cm to 3 mm

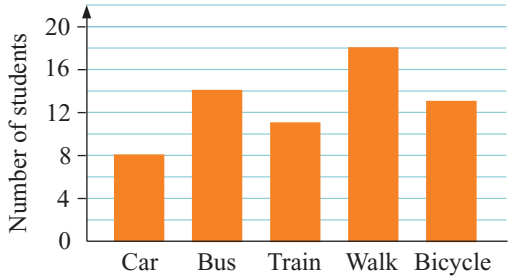
15 In this diagram, what is the ratio of the:

a base of triangle A to the base of triangle B?
b height of triangle A to the height of triangle B?
c area of triangle A to the area of triangle B?

Hint: Area of a triangle $= \frac{1}{2}b \times h$ or $\frac{b \times h}{2}$.



16 The bar graph represents the results of a survey to determine the method by which students travel to school.



a Find the total number of students surveyed.

b Write as a ratio:

i students travelling by car : students who walk

ii students travelling by bus : total number of students surveyed.

c What fraction of the students surveyed travel to school by train?

d To 1 decimal place, what percentage of these student travels by:

i train? **ii** bus? **iii** car?

Investigation 1 Gears on a bike

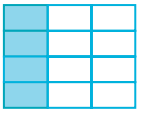
A bicycle has a chain that joins a cog on the front chainwheel to a cog on the back wheel. The gears on a bicycle can be varied by changing the position of the chain on the front chainwheel and on the rear cog.

If the front cog has 25 teeth and the rear cog has 18 teeth, then the gear ratio is 25 : 18 or $\frac{25}{18} : 1 = 1.39 : 1$.

A bicycle has two cogs on the front chainwheel with 25 teeth and 45 teeth respectively. The back wheel has three cogs with 28, 25 and 16 teeth respectively. If the chain can be used in any position on the two front cogs and on the three back cogs, find each ratio for the number of teeth on one of the front cogs to the number of teeth on one of the rear cogs in the form $n : 1$.

B Equivalent ratios review

Just as there are many ways of writing the same fraction, there are many ways of writing the same ratios. Ratios that can be simplified to the same ratio are said to be equivalent ratios. The following diagrams show this.

Diagram	Ratio of shaded : unshaded	Fraction shaded
	1 : 2	$\frac{1}{3}$
	2 : 4	$\frac{2}{6} = \frac{1}{3}$
	4 : 8	$\frac{4}{12} = \frac{1}{3}$

From the table above it can be seen that:

- in all cases the fraction shaded is the same, so
- the ratio of shaded area : unshaded area is also the same.

$\therefore$ 1 : 2 = 2 : 4 = 4 : 8, where 1 : 2 is the simplest form of the ratio.

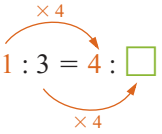
An equivalent ratio is formed by multiplying or dividing both sides of the ratio by the same non-zero number.

EXAMPLE 1

Complete these equivalent ratios by multiplying.

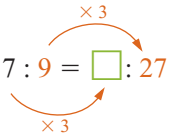
a 1 : 3 = 4 : ☐ **b** 7 : 9 = ☐ : 27

a 1 : 3 = 4 : ☐



1 : 3 = 4 : 12

b 7 : 9 = ☐ : 27



7 : 9 = 21 : 27

Exercise 2B

1 Complete these equivalent ratios by multiplying.

a 1 : 3 = 5 : ___ **b** 2 : 5 = 4 : ___ **c** 4 : 7 = 8 : ___
d 9 : 10 = 27 : ___ **e** 6 : 5 = 24 : ___ **f** 3 : 8 = 15 : ___
g 2 : 9 = ___ : 18 **h** 1 : 6 = ___ : 30 **i** 5 : 3 = ___ : 9
j 9 : 2 = ___ : 8 **k** 10 : 11 = ___ : 55 **l** 14 : 15 = ___ : 45
m 8 : 13 = 40 : ___ **n** 6 : 11 : = ___ : 99 **o** 5 : 7 = ___ : 70
p 10 : 17 = 100 : ___ **q** 3 : 2 = 33 : ___ **r** 13 : 20 = ___ : 60

EXAMPLE 2

Complete these equivalent ratios by *dividing*.

a 4 : 20 = 1 : ☐ **b** 21 : 56 = ☐ : 8

a 4 : 20 = 1 : ☐

÷ 4

4 : 20 = 1 : ☐

÷ 4

4 : 20 = 1 : 5

b 21 : 56 = ☐ : 8

÷ 7

21 : 56 = ☐ : 8

÷ 7

21 : 56 = 3 : 8

Multiplying increases and dividing decreases the numeral in the equivalent ratio.

2 Complete the following equivalent ratios by dividing.

a 2 : 10 = 1 : ___ **b** 6 : 14 = 3 : ___ **c** 9 : 3 = 3 : ___
d 12 : 15 = 4 : ___ **e** 18 : 27 = 2 : ___ **f** 100 : 120 = 5 : ___
g 20 : 32 = ___ : 8 **h** 22 : 55 = ___ : 5 **i** 15 : 40 = ___ : 8
j 42 : 60 = ___ : 10 **k** 24 : 38 = ___ : 19 **l** 4 : 36 = ___ : 9
m 35 : 14 = ___ : 2 **n** 63 : 54 = 7 : ___ **o** 24 : 18 = 4 : ___
p 75 : 120 = ___ : 8 **q** 99 : 66 = ___ : 2 **r** 62 : 93 = 2 : ___

3 Complete the following equivalent ratios.

a 4 : 11 = 24 : ___ **b** 16 : 20 = ___ : 5 **c** 15 : 21 = 5 : ___
d 28 : 42 = ___ : 3 **e** 36 : 52 = 9 : ___ **f** 7 : 5 = 28 : ___
g 16 : 80 = 1 : ___ **h** 8 : 1 = 72 : ___ **i** 1 : 3 = ___ : 39
j 88 : 55 = ___ : 5 **k** 80 : 72 = 10 : ___ **l** 9 : 13 = ___ : 52
m 15 : 2 = ___ : 12 **n** 92 : 100 = ___ : 25 **o** 38 : 40 = ___ : 20

EXAMPLE 3

Express the following ratios in simplest form.

a 42 : 48 **b** 35 : 20

a Find the HCF of 42 and 48.

42: 1, 2, 3, **6**, 7, 14, 21, 42

48: 1, 2, 3, 4, **6**, 8, 12, 16, 24, 48

HCF = 6

$42 : 48 = \frac{42}{6} : \frac{48}{6}$

$= 7 : 8$

Calculator: Use the fraction button $a\frac{b}{c}$.

Enter:

42 $a\frac{b}{c}$ 48 = 7 ⌋ 8

= 7 : 8 (ratio form)

b Find the HCF of 35 and 20.

35: 1, **5**, 7, 35

20: 1, 2, 4, **5**, 10, 20

HCF = 5

$35 : 20 = \frac{35}{5} : \frac{20}{5}$

$= 7 : 4$

Note: The fraction function on the calculator is used to simplify the ratio and the answer is a ratio not a fraction.

Enter:

35 $a\frac{b}{c}$ 20 = 1 ⌋ 3 ⌋ 4 $= 1\frac{3}{4}$

Improper fraction: SHIFT $a\frac{b}{c}$ 7 ⌋ 4

= 7 : 4 (ratio form)

4 Express the following ratios in simplest form.

a 20 : 45	**b** 16 : 40	**c** 12 : 36	**d** 30 : 50
e 42 : 66	**f** 21 : 51	**g** 25 : 85	**h** 18 : 28
i 72 : 81	**j** 54 : 84	**k** 69 : 99	**l** 240 : 360
m 100 : 150	**n** 112 : 120	**o** 108 : 232	**p** 164 : 256

5 Express the following ratios in simplest form.

a 50 : 30	**b** 36 : 12	**c** 45 : 18	**d** 48 : 27
e 88 : 24	**f** 63 : 14	**g** 84 : 22	**h** 56 : 28
i 95 : 55	**j** 72 : 12	**k** 126 : 24	**l** 180 : 16
m 174 : 21	**n** 220 : 32	**o** 230 : 160	**p** 187 : 154

EXAMPLE 4

Express the following ratios in simplest form.

a $\frac{5}{7} : \frac{3}{7}$ **b** $3 : 2\frac{2}{5}$ **c** 1.6 : 5.6

a $\frac{5}{7} : \frac{3}{7} = \frac{5}{7} \times 7 : \frac{3}{7} \times 7 = 5 : 3$

Calculator:

5 $a\frac{b}{c}$ 7 ÷ 3 $a\frac{b}{c}$ 7 = 1⌋2⌋3

SHIFT $a\frac{b}{c}$ = 5⌋3 = 5 : 3

b $3 : 2\frac{2}{5} = 3 \times 5 : \frac{12}{5} \times 5 = 15 : 12$

$= \frac{15}{3} : \frac{12}{3} = 5 : 4$

Calculator:

3 ÷ 2 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 5 = 1⌋2⌋4

SHIFT $a\frac{b}{c}$ = 5⌋4 = 5 : 4

c 1.6 : 5.6 = 1.6 × 10 : 5.6 × 10 = 16 : 56

$= \frac{16}{8} : \frac{56}{8} = 2 : 7$

Calculator:

1.6 ÷ 5.6 = SHIFT $a\frac{b}{c}$ = 2⌋7 = 2 : 7

6 Express the following as ratios in simplest form.

a $\frac{4}{9} : \frac{7}{9}$	**b** $\frac{8}{11} : \frac{5}{11}$	**c** $\frac{16}{23} : \frac{9}{23}$	**d** $\frac{12}{13} : \frac{10}{13} : \frac{14}{13}$
e $\frac{25}{40} : \frac{10}{40} : \frac{15}{40}$	**f** $2 : \frac{1}{4}$	**g** $\frac{1}{3} : 6$	**h** $\frac{4}{5} : 8$
i $2\frac{1}{3} : \frac{2}{3}$	**j** $\frac{5}{6} : 1\frac{5}{6}$	**k** $10 : 2\frac{1}{4}$	**l** $3\frac{1}{2} : 4\frac{3}{4}$
m $16\frac{1}{2} : 3\frac{1}{3}$	**n** $20\frac{1}{5} : 4\frac{1}{2}$	**o** 1.5 : 5	**p** 1.8 : 3
q 2.4 : 2.8	**r** 0.81 : 2.7	**s** 0.6 : 3.6	**t** 1.4 : 0.21

EXAMPLE 5

Express the following ratios in simplest form.

The quantities being compared must be in the same units.

a \$4 : 50c **b** 2 h : 15 min **c** 45 cm : 3 m

a \$4 : 50c
$= 400 : 50$
$= \frac{400}{50} : \frac{50}{50}$
$= 8 : 1$

b 2 h : 15 min
$= 120 \text{ min} : 15 \text{ min}$
$= \frac{120}{15} : \frac{15}{15}$
$= 8 : 1$

c 45 cm : 3 m
$= 45 \text{ cm} : 300 \text{ cm}$
$= \frac{45}{15} : \frac{300}{15}$
$= 3 : 20$

7 Express the following as ratios in simplest form.

a 25c : \$1.50 **b** \$2 : 80c **c** \$7.20 : 40c
d 1 h : 6 min **e** 40 min : $1\frac{1}{2}$ h **f** 18 h : 1 day
g 34 mm : 2 cm **h** 4 L : 750 mL **i** 3 m : 125 cm
j 540 m : 1.4 km **k** 685 g : 1.2 kg **l** $1\frac{1}{4}$ L : 540 mL
m 24 s : $3\frac{1}{4}$ min **n** 12 weeks : $2\frac{1}{2}$ years **o** 2.6 km : 800 m

8 The dimensions of a rectangular field are 1.2 km by 600 m. What is the ratio of the field's dimensions?

9 A mortar mix consists of sand, cement and gravel in the ratio 3 : 1 : 2. If two buckets of cement are used, how many buckets of sand and gravel are used?

10 A man buys goods for \$720 and sells them for \$800. Find the ratio in simplest form of:
a cost price to selling price
b selling price to profit.

11 A television set is sold with a mark up of 25% on the cost price. Find the ratio in simplest form of:
a cost price to selling price
b selling price to cost price
c profit to cost price
d selling price to profit.

12 Of the Year 8 students, 75 play soccer, 150 play netball and 125 play football. Find the ratio in simplest form of soccer players to netball players to football players.

C Problem solving: the unitary method

The unitary method is a way of finding an unknown quantity when the ratio between two quantities and the value for one quantity is known.

The unitary method involves first finding one part or one unit. This is why it is called the unitary method.

EXAMPLE 1

The ratio of tin to lead in a particular type of solder is 8 : 5.

a How much tin is needed to be mixed with 35 kg of lead?

b How much lead is needed to be mixed with 70 g of tin?

Method 1

a tin : lead

8 : 5

x : 35

where x is the amount of tin required.

$\frac{\text{tin}}{\text{lead}} = \frac{8}{5}$

$\frac{x}{35} = \frac{8}{5}$

$x = \frac{8}{5} \times 35 = \frac{280}{5} = 56$

$\therefore$ 56 kg of tin is required.

b tin : lead

8 : 5

70 : x

where x is the amount of lead required.

$\frac{\text{lead}}{\text{tin}} = \frac{5}{8}$

$\frac{x}{70} = \frac{5}{8}$

$x = \frac{5}{8} \times 70 = \frac{350}{8} = 43.75$

$\therefore$ 43.75 g of lead is required.

Method 2

a tin : lead = 8 : 5

Lead is 5 parts of the solder.

5 parts = 35 kg

1 part = $\frac{35}{5}$ kg

$\therefore$ 1 part = 7 kg

Tin is 8 parts of the solder.

8 parts = 8 × 7 kg = 56 kg

$\therefore$ 56 kg of tin is required.

b tin : lead = 8 : 5

Tin is 8 parts of the solder.

8 parts = 70 g

1 part = $\frac{70}{8}$ g

$\therefore$ 1 part = 8.75 g

Lead is 5 parts of the solder.

5 parts = 5 × 8.75 g = 43.75 g

$\therefore$ 43.75 g of lead is required.

Exercise 2C

1 Solve the following.

a $\frac{x}{15} = \frac{6}{5}$ **b** $\frac{x}{4} = \frac{9}{12}$ **c** $\frac{x}{7} = \frac{1}{4}$

d $\frac{x}{3} = 1\frac{1}{20}$ **e** $\frac{x}{2} = \frac{16}{20}$ **f** $\frac{x}{8} = \frac{12}{15}$

g $x : 5 = 6 : 10$ **h** $x : 6 = \frac{13}{15}$ **i** $x : 10 = 2 : 5$

j $6 : 15 = x : 2$ **k** $16 : 14 = 4 : x$ **l** $25 : 22 = 8 : x$

2 In a school the ratio of teachers to students is 1 : 27.

a If there are 43 teachers, how many students are at the school?

b If there are 945 students, how many teachers are at the school?

3 The ratio of Belinda's expenses to savings is 7 : 2.

a Find Belinda's expenses in a week in which she saves $80.

b Find Belinda's savings in a week in which her expenses are $126.

4 The ratio of water to cordial is 5 : 2.

a How much water is required to be mixed with 320 mL of cordial?

b How much cordial is required to be mixed with 1.3 L of water?

5 The length and breadth of a rectangle are in the ratio of 9 : 4.

a Find the length of the rectangle when the breadth is 5.2 cm.

b Find the breadth of the rectangle when the length is 128.7 cm.

6 A two-stroke petrol mix is made by mixing petrol and oil in the ratio 25 : 1.

a How many litres of petrol would be added to 500 mL of oil to make this mixture?

b How many millilitres of oil would be needed to be mixed with 8 L of petrol?

7 **a** The ratio of a boy's height to his father's is $\frac{3}{5}$. If the boy's height is 1.23 m, how tall is his father?

b The ratio of Carla's weight to Tracey's weight is $\frac{7}{9}$. If Tracey weighs 82.8 kg, how much does Carla weigh?

8 Betting odds are in the form of a ratio. Odds of 12/1 mean that $12 could be won for every $1 bet.

a How much could be won on a horse at odds 12/1 if you bet $25?

b How much did Sarah bet if she won $720 on a horse at odds 12/1?

c If Sarah decided to bet 3/5 of her winnings from part **b** and lost, how much would she lose?

d Sarah decided to bet her remaining winnings from part **b** on a race at odds of 8/1. If the horse won, how much did she win?

9 A model train is built to a scale of 1 : 150.

a If the length of the model train is 120 cm, what is the actual length of the train in metres?

b If the actual height of the train is 3 m, what is the height of the model?

10 Write out the correct list of ingredients for:

a 12 chocolate crackles **b** 36 chocolate crackles.

Chocolate crackles recipe

4 cups Rice Bubbles
$1\frac{1}{2}$ cups icing sugar
3 tablespoons cocoa
1 cup desiccated coconut
250 g copha
Makes 24

11 A bag of lollies contains a mixture of mints, toffees and caramels in the ratio 5 : 7 : 4.
If there are 25 mints in the bag, find the:

a number of caramels

b number of toffees

c total number of lollies in the bag.

EXAMPLE 2

A piece of ribbon is cut into three lengths in the ratio 3 : 5 : 7. The shortest piece is 12 cm.

a Find the length of the longest piece of ribbon.

b Find the length of the original piece of ribbon.

Method 1

a 3 : 5 : 7

12 x

$\frac{\text{long}}{\text{short}} = \frac{7}{3}$

$\frac{x}{12} = \frac{7}{3}$

$x = \frac{7}{3} \times 12$

$= \frac{84}{3}$

$= 28$ cm

Longest piece of ribbon is 28 cm.

b 3 : 5 : 7

12 : x

$\frac{\text{medium}}{\text{short}} = \frac{5}{3}$

$\frac{x}{12} = \frac{5}{3}$

$x = \frac{5}{3} \times 12$

$= \frac{60}{3}$

$= 20$ cm

Medium piece is 20 cm.

Length of original = short + medium + long

$= 12 + 20 + 28 = 60$ cm

Method 2

Ratio of lengths is 3 : 5 : 7.

Shortest length = 12 cm, so 3 parts = 12 cm.

$\therefore$ 1 part $= \frac{12}{3} = 4$ cm

a Length of longest piece = 7 parts

$= 7 \times 4$ cm

$= 28$ cm

b Length of original piece $= 3 + 5 + 7 = 15$ parts

$= 15 \times 4$ cm

$= 60$ cm

12 A triangle has side lengths in the ratio 3 : 6 : 7.

a If the longest side is 16.1 cm, find the lengths of the remaining two sides.

b Calculate the total length of the sides of the triangle.

Use the information given to find the value of one part.

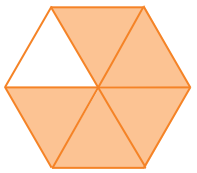
13 A piece of rope is cut into three lengths in the ratio 7 : 2 : 5. The shortest length is 1.48 m.

a Find the length of the longest piece of rope.

b Find the length of the original rope.

D Dividing a quantity in a given ratio

The hexagon shown has six equal parts, five of which are shaded.



The ratio shaded area : unshaded area = 5 : 1.

The total number of parts = 5 + 1 = 6

$\therefore$ fraction shaded = $\frac{5}{6}$

$\therefore$ fraction unshaded = $\frac{1}{6}$

When dividing a quantity into a given ratio it is often useful to represent the ratio in fraction form.

Remember the $a\frac{b}{c}$ key is your fraction key.

EXAMPLE 1

a Divide \$15 000 in the ratio 3 : 2.

b The ratio of good apples to bruised apples is 5 : 2. If 2800 apples are sorted, how many are bruised?

a Ratio is 3 : 2.

Number of parts = 3 + 2 = 5

So fractions are $\frac{3}{5}$ and $\frac{2}{5}$.

$\frac{3}{5}$ of \$15 000 = \$9000

$\frac{2}{5}$ of \$15 000 = \$6000

$\therefore$ 3 : 2 = \$9000 : \$6000

Calculator:

3 $a\frac{b}{c}$ 5 × 15 000 =

2 $a\frac{b}{c}$ 5 × 15 000 =

b Ratio is 5 : 2.

Number of parts = 5 + 2 = 7

So fractions are $\frac{5}{7}$ and $\frac{2}{7}$.

$\frac{2}{7}$ are bruised.

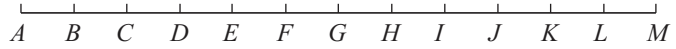
$\frac{2}{7}$ of 2800 = 800

$\therefore$ 800 apples were bruised.

2 $a\frac{b}{c}$ 7 × 2800 =

Exercise 2D

1 The line segment AM is divided into equal intervals. Which point divides AM in the following ratios?



a 5 : 7 **b** 1 : 11 **c** 1 : 3 **d** 2 : 1

2 Divide \$100 in the ratio:

a 4 : 1 **b** 3 : 2 **c** 9 : 11 **d** 5 : 3

3 Divide \$540 in the ratio:

a 2 : 1 **b** 7 : 5 **c** 4 : 5 **d** 17 : 8

4 Divide 720 kg in the ratio:

a 7 : 11 **b** 13 : 7 **c** 21 : 9 **d** 13 : 12

5 Divide 834 m in the ratio:

a 3 : 1 **b** 2 : 3 **c** 5 : 7 **d** 6 : 19

6 The town of Barten has a population of 5464. How many children are there in the town if the ratio of children to adults is 5 : 3?

7 Tropicana Drink is made by mixing orange, pineapple and passionfruit juice in the ratio 7 : 5 : 2. How much pineapple juice would be needed to make 8.4 L of Tropicana Drink?

8 The total length of the sides of a triangle is 72.6 cm and the ratio of the side lengths is 6 : 1 : 5. Calculate the length of the:

a shortest side **b** longest side.

9 A tin holds 6.24 L of two-stroke fuel. If two-stroke fuel is made from petrol and oil in the ratio 25 : 1, how much oil is in the mixture?

10 An investment fund invests in property, shares and government bonds in the ratio 5 : 4 : 3. If the total investments are $60 000, find the amounts invested in each area.

11 A brand of fertiliser is made by mixing potash, phosphates and nitrates in the ratio 3 : 2 : 3. A bag of fertiliser contains 40 kg.

a Find the number of kilograms of potash in the bag.

b Find the number of kilograms of phosphates in the bag.

12 A rectangle has a perimeter of 65 cm and the ratio of its length to its breadth is 9 : 4.

a Find the length of the rectangle.

b Find the breadth of the rectangle.

13 The ratio of the population of Tarmore to Kingsvale is 2 : 5 and the ratio of the population of Kingsvale to Westside is 2 : 3. If the total population of the three towns is 99 615, find the population of these towns.

a Tarmore **b** Kingsvale **c** Westside

Rates

Rates are comparisons of quantities of a *different* kind. So unlike ratios, rates have units.

Speed is one of the most common rates. A comparison is made between the distance travelled and the time taken.

EXAMPLE 1

Express each of the following as a rate.

a A car travels 420 km in $3\frac{1}{2}$ h. **b** Collin is paid $71.25 for working $9\frac{1}{2}$ h.

a 420 km is travelled in $3\frac{1}{2}$ h

$\therefore$ rate $= \dfrac{420 \text{ km}}{3\frac{1}{2} \text{ h}}$

$= 120$ km/h

Calculator:

420 ÷ 3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

b $71.25 pay for working $9\frac{1}{2}$ h

$\therefore$ rate $= \dfrac{\$71.25}{9\frac{1}{2}}$

$= \$7.50$ per h

71.25 ÷ 9 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

This is the average speed.

Exercise 2E

1 Complete the following.

a 22 apples bought for \$3.30 is a rate of ___c per apple.

b \$485 in rent shared between four tenants is a rate of \$___ per tenant.

c 583.2 L of water pumped into a tank in 18 minutes is a rate of ___ L/min.

d A temperature rise of 9°C in $2\frac{1}{4}$ h is a rate of ___ ___/h.

e 24 km travelled in 1 h 12 min is a rate of ___ km/h.

f 18 L of petrol consumed by a vehicle every 261 km is a rate of ___ km/L.

g 40.5 kg of seed spread over 9 m^2 is a rate of ___ kg/m^2.

h Danielle types 192 words in 3 min. This rate is ___ words/min.

i 35 oranges bought for \$4.20 is a rate of ___c per orange.

j 24 kg of peas sold for \$66.24 is a rate of ___c/kg.

k Tom runs 200 m in 50 s. This is a rate of ___ m/s.

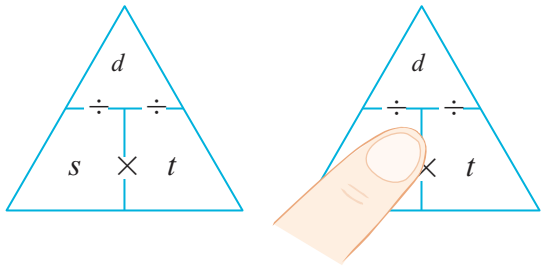
l 30 km over $\frac{1}{4}$ h is a rate of ___ km/h.

m \$173.47 for 1660 kWh of power is a rate of ___c/kWh.

This diagram is useful in distance (d), speed (s) and time (t) calculations.

Cover the quantity you want to find and read the other two quantities with the sign between them.

For example, if you want to find speed, cover s and read $s = d \div t$.



EXAMPLE 2

a A car travels at 60 km/h for 2 h and 17 min. How far does it travel?

b An aeroplane travels 2800 km in 4 h and 15 min. Calculate its speed to the nearest km.

c A train travels a distance of 770 km at a speed of 140 km/h. How long does the journey take?

a Distance = speed × time
= 60 km/h × 2 h 17 min
= 137 km

Calculator:

60 × 2 °′″ 17 °′″ =

Some calculators have a SHIFT button, others have an INV button. Check your FIX function!

b Speed = $\frac{\text{distance}}{\text{time}}$

$= \frac{2800 \text{ km}}{4 \text{ h } 15 \text{ min}}$

= 658.8

= 659 km/h

Calculator:

2800 ÷ 4 °′″ 15 °′″ =

c Time = $\frac{\text{distance}}{\text{speed}}$

$= \frac{770 \text{ km}}{140 \text{ km/h}}$

= 5.5 h or 5 h 30 min

2 **a** I walk at a rate 4.2 km/h. How far can I walk in $2\frac{1}{2}$ h?

b A train travels at a speed of 158 km/h for 4 h 12 min. Calculate the distance travelled by the train.

c A rocket travels 1021.65 km at a speed of 417 km/h. How long does it take the rocket to go this distance?

d A spacecraft travels 8500 km in 40 min. Calculate the average speed of the spacecraft in km/h.
(40 min on your calculator is 0 [°'"] 40 [°'"])

e A cyclist cycles at 18.4 km/h for 3 h 30 min. How far does the cyclist travel?

f Rory rides his motor bike at a constant speed of 92 km/h. If he travels 322 km, how long does this take?

3 A tank holds 1800 L. Currently it is leaking at a rate of 40 mL/min. How long will it take for the tank to lose:

a 500 mL? **b** 1.3 L? **c** 2.6 L? **d** 6.8 L?

4 Perfume costs $157.90 for 50 mL. How much does it cost for the following amounts?

a 20 mL **b** 80 mL **c** 130 mL **d** 170 mL

5 **a** One litre of paint covers 8 m^2. What is the area covered by:

i 3 L? **ii** 4.5 L?

b What is the amount of paint needed to cover:

i 56 m^2? **ii** 104 m^2?

6 Phillipa types at a rate of 75 words per minute.

a How long would it take her to type a 500 word essay at this rate?

b How much longer would it take Kurt to type this essay if he types at 35 words per minute?

7 The printing costs for a brochure are 1000 copies for $938 or 1500 copies for $1032.

a What is the printing cost per brochure for:

i 1000 copies? **ii** 1500 copies?

b Find the difference in cost per copy between 1500 and 1000 copies.

c If the price per copy remained fixed for 1500 or more copies, find the cost of printing 2700 brochures.

EXAMPLE 3

Which is the better buy?

A 500 g of Doggie Plus costing $3.25 **B** 750 g of Doggie Plus costing $4.65

A Rate $= \frac{325\text{c}}{500\text{ g}} = 0.65$ cents per gram

Calculator:

325 [÷] 500 [=]

B Rate $= \frac{465\text{c}}{750\text{ g}} = 0.62$ cents per gram

465 [÷] 750 [=]

The better buy is B, the 750 g costing $4.65, as each gram is cheaper!

8 Which is the better buy?

A 300 g of Weety Flakes costing $3.40

B 500 g of Weety Flakes costing $4.25

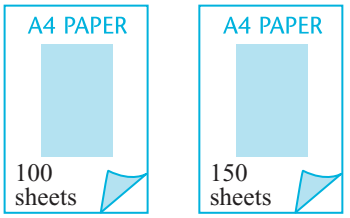
9 Which is the better buy?

A 8 m of ribbon costing $2.56

B 12 m of ribbon costing $3.60

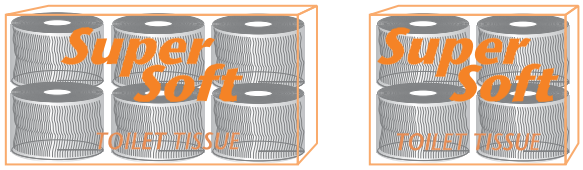
10 Which is the better buy of each product?

a



$2.40 $3.15

b



$4.32 $2.96

11 Aislenn is holidaying in Australia. She phones her parents in England and speaks for 22 minutes 5 seconds. Charges to England are $1.45 for the first 3 minutes then $0.85 for each additional minute or part thereof.

a How much did it cost her to call her parents?

b If she used a $25 phone card, how much credit remains?

12 The following table lists the rates per minute of calls. Calculate the cost of each of these calls.

a A local day call lasting 15 min 20 s

b A local night call lasting 42 min 30 s

c An STD day call lasting 33 min 12 s

d An STD economy call lasting 1 h 14 min 5 s

	Rate (cents per minute)		
	Day	Night	Economy
Local	35	47	32
STD	42	55	40

13 In cricket the strike rate is calculated using this formula.

$$\text{Strike rate} = \frac{\text{runs scored}}{\text{balls faced}} \times 100$$

$$\text{Strike rate} = \frac{R}{B} \times 100$$

Complete the following table.

	Player's name	Runs scored	Balls faced	Strike rate (1 decimal place)
a	Healey	42	30	
b	Hughes	31	22	
c	Border	18	16	
d	O'Donnell	125	98	
e	Young	71	59	

14 The success rate in netball is calculated using the following formula. Complete the following table.

$$\text{Success rate} = \frac{\text{number of goals}}{\text{number of attempts}} \times 100$$

	Player's name	Number of goals	Number of attempts	Success rate (1 decimal place)
a	Carla	4	10	
b	Susan	2	5	
c	Robert	1	4	
d	Harley	6	13	
e	Kylie	9	15	

15 On a particular day the Australian dollar was worth $US0.86.

a Calculate the value in $US of each amount.

i $A100 **ii** $A560 **iii** $A4800 **iv** $A25 000

b Calculate the value in $A of each amount.

i $US100 **ii** $US4000 **iii** $US12 000 **iv** $US45 000

16 Governments use growth rates of population to make decisions. The following table shows population and estimates of population for Africa, Asia, Oceania, America and Europe.

	Population (million)				
Year	Africa	Asia	Oceania	America	Europe
1950	221.1	1398	12.8	171.6	547.4
2000	795.7	3697	31.0	315.9	728.0
2050	1803.3	5222.1	45.8	447.9	631.9

a Calculate the average annual growth rate in population and complete the following table. Africa has already been completed. Use these values to make sure you are calculating correctly.

	Average annual rate of change in population (%)				
Year	Africa	Asia	Oceania	America	Europe
1950−2000	$\frac{795.7 - 221.1}{221.1} \times 100 = 259.88\%$ $\frac{259.88}{50} = 5.2\%$				
2000−2050	$\frac{1803.3 - 795.7}{795.7} \times 100 = 126.63\%$ $\frac{126.63}{50} = 2.5\%$				
1950−2050	$\frac{1803.3 - 221.1}{221.1} \times 100 = 715.60\%$ $\frac{715.60}{100} = 7.2\%$				

b Which region is expected to have a negative growth rate between 2000 and 2050?

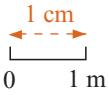
c Australia is in Oceania. Comparing Oceania with Asia, what can be said about the growth rates? Give at least two observations.

d Compare the 1950–2000 and 2000–2050 growth rates for all regions. Give a reason for the decrease in rate from one time period to the other.

e Why is it incorrect to add the growth rates for 1950–2000 and 2000–2050 to obtain the rate for 1950–2050?

F Scale drawing

The scale factor of any diagram, map or picture is the number of times that the original has been reduced or enlarged to create a scale drawing. The scale can be shown as:

- a representative fraction, for example $\frac{1}{100}$
- a ratio, for example 1 : 100 (so 1 cm ≡ 100 cm or 1 cm ≡ 1 m)
- a line or linear scale 

so 1 cm ≡ 100 cm.

≡ *means 'represents' or 'is equivalent to'.*

Enlargement

An enlargement with scale factor of 2 or 2 : 1 or $\frac{2}{1}$ means that every 2 cm on the scaled drawing represents 1 cm on the original. This can be represented as a 100% increase.


original


scaled

Reduction

A reduction with scale factor of $\frac{1}{3}$ or 1 : 3 means that every 1 cm on the scaled drawing represents 3 cm on the original.


original

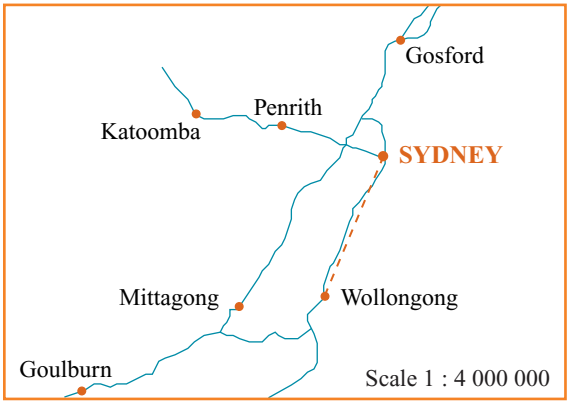

scaled

Distance maps

A scale of 1 : 4 000 000 means that 1 cm on the map represents 4 000 000 cm in real life.

So 1 cm ≡ 4 000 000 cm
≡ 40 000 m
≡ 40 km

Using a scale of 1 : 4 000 000, the distance of 2 cm from Wollongong to Sydney on the map shown on the right is a distance of 80 km in real life.
(2 cm ≡ 2 × 40 km ≡ 80 km)



Exercise 2F

1 Complete the following.

a *Scale*	*Meaning*	**b** *Scale*	*Meaning*
1 : 10	1 cm ≡ 10 cm	1 : 400 000	1 cm ≡ ____ cm
1 : 100	1 cm ≡ ____ cm	1 : 400 000	1 cm ≡ ____ m
1 : 1000	1 cm ≡ ____ cm	1 : 400 000	1 cm ≡ ____ km
1 : 1000	1 cm ≡ ____ m		

2 Express the following scales as a ratio in the form 1 : ____ if 1 cm represents:

a 400 cm **b** 16 000 cm **c** 0.05 km
d 0.8 km **e** 300 000 cm **f** 700 000 cm
g 15 000 000 cm **h** 24 000 000 cm

EXAMPLE 1

Write the scale 5 mm to 10 m as a simplified ratio.

5 mm : 10 m = 5 mm : 10 000 mm
= 5 : 10 000
= 1 : 2000 (divide both parts by 5)

The terms of the ratio must be expressed in the same units.

3 Write each of these scales as a simplified ratio.

a 4 cm : 2 m **b** 20 cm : 5.4 m **c** 50 mm : 0.8 m
d 18 m : 1.6 m **e** 84 cm : 1.2 km **f** 95 cm : 3.5 km

4 The scale on a map is 1 : 500 000. What is the actual distance if the distance on the map is:

a 0.5 cm? **b** 2.6 cm? **c** 4.8 cm? **d** 9.2 cm?

5 The scale on a map is 1 : 80 000. What is the actual distance if the distance on the map is:

a 5 cm? **b** 7 cm? **c** 10.3 cm? **d** 15.9 cm?

EXAMPLE 2

Calculate the distance on a map if the scale is 1 : 400 000 and the actual distance is 11.2 km.

Scale 1 : 400 000 or 1 cm ≡ 400 000 cm
1 cm ≡ 4000 m
1 cm ≡ 4 km

$$\text{Map distance} = \frac{\text{actual distance}}{\text{scale}}$$
$$= \frac{11.2}{4}$$
$$= 2.8 \text{ cm}$$

6 Calculate the map distance if the scale is 1 : 200 000 and the actual distance is:

a 10 km **b** 28 km **c** 37.2 km **d** 169.6 km

7 Calculate the map distance if the scale is 1 : 85 000 and the actual distance is:

a 3.4 km **b** 10.2 km **c** 21.25 km **d** 47.6 km

8 A linear scale is shown.

a What scale is being represented?

b Calculate the actual distance if the map distance is:

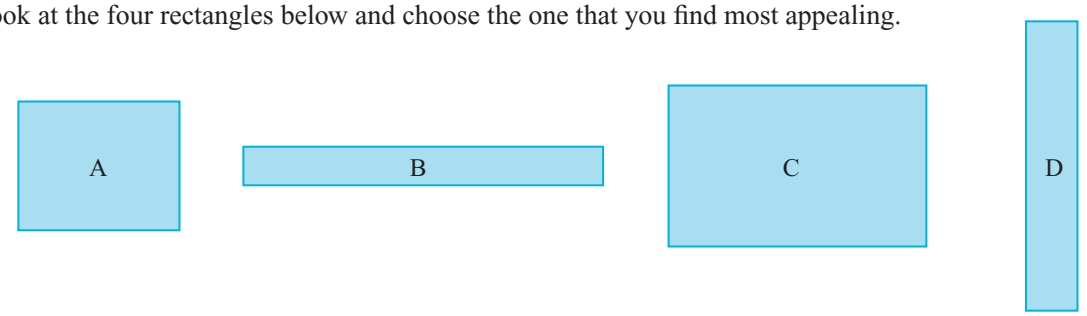
i 3 cm **ii** 5.2 cm **iii** 6.25 cm **iv** 13.6 cm

c Calculate the distance on the map if the actual distance is:

i 1200 km **ii** 1320 km **iii** 1680 km **iv** 2587.5 km

Investigation 2 The golden ratio

Look at the four rectangles below and choose the one that you find most appealing.



If you said rectangle C you would agree with many artists and architects from the past. Rectangle C is called a *golden rectangle* and is said to be one of the most visually appealing geometric shapes. The ratio of length to width in a golden rectangle is approximately 1.6 : 1; that is, the length is about 1.6 times the width. (More precisely, the ratio is 1.61803398… : 1, but 1.6 : 1 is close enough.)

The ratio 1.6 : 1 is known as the *golden ratio*. The Ancient Greeks used the golden ratio extensively in paintings, architecture, sculpture and designs on pottery.

Leonardo da Vinci suggested that the ratio of certain body measurements is close to the golden ratio. Keeping in mind that we are all different shapes and sizes, see if you can find the golden ratio using your body measurements.

Equipment needed: a tape measure and a partner

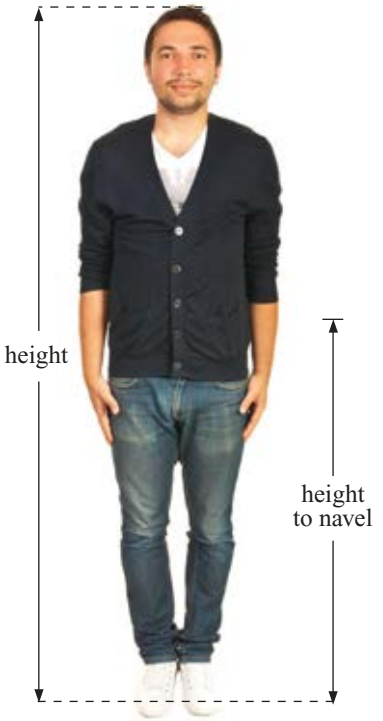
1 Ask your partner to carefully take your body measurements (in mm) as indicated in the following diagrams. Record these measurements and calculate the required ratios.

Height = ____ mm

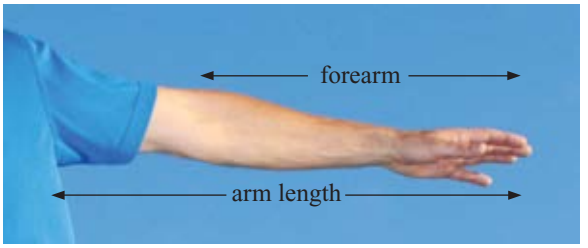
Height to navel = ____ mm

So, height : navel height = ____ : ____

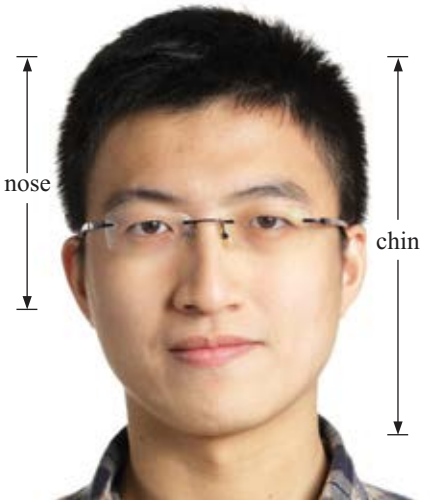
= ____ : 1



Length of arm = ____ mm (fingertip to armpit)
Length of forearm = ____ mm (fingertip to elbow)
So, arm : forearm = ____ : ____
= ____ : 1



Hairline to chin = ____ mm
Hairline to bottom of nose = ____ mm
So, hairline to chin : hairline to nose = ____ : ____
= ____ : 1



2 How do your ratios compare to the golden ratio, which is approximately 1.6 : 1?

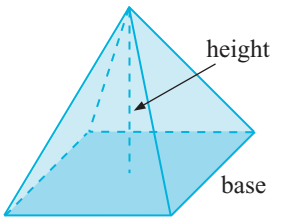
3 Compile a class list of these ratios. Are they all close to the golden ratio? (Any ratios that are a long way out may need to be recalculated.)

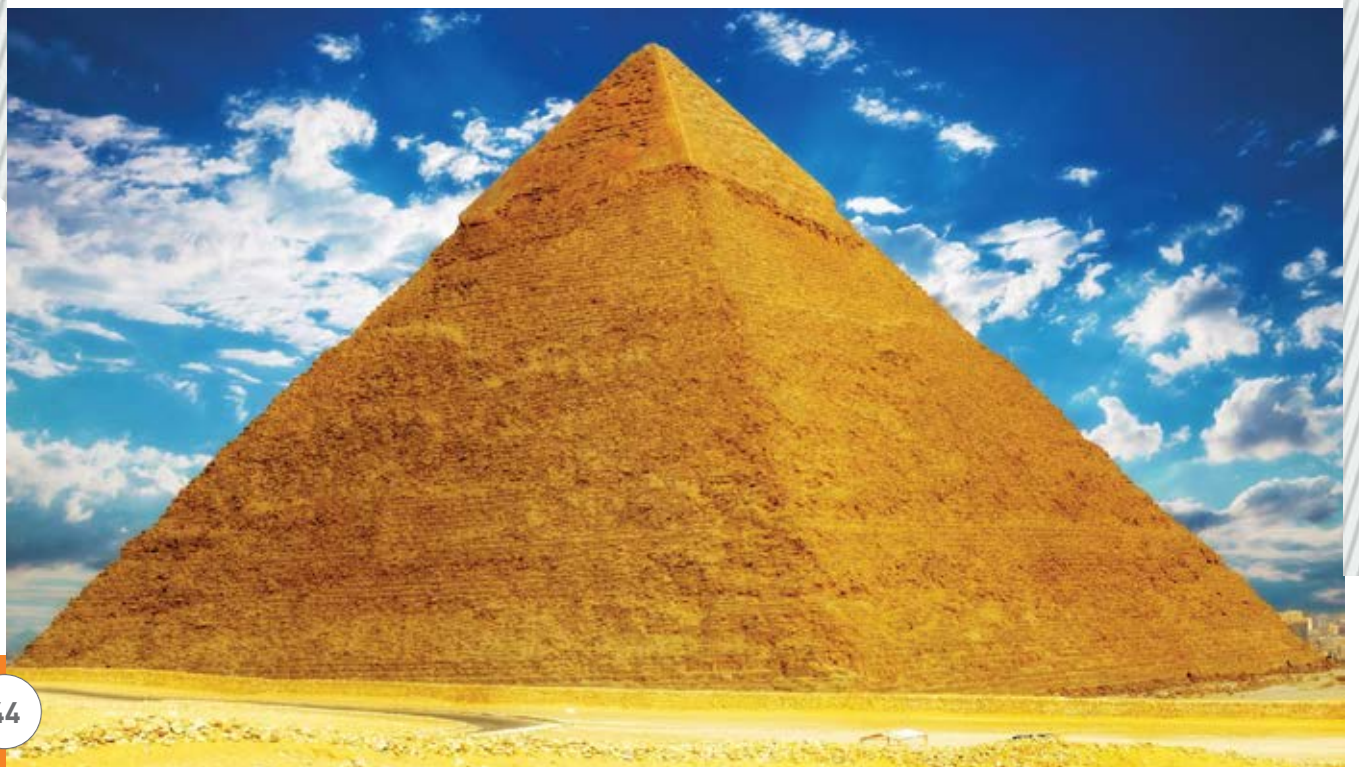
4 Can you find any other ratios of body measurements that are close to the golden ratio?

Further research

5 The Italian poet Danté stated that 'Nature is the art of God'. Find out more about the golden ratio in nature.

6 Research the dimensions of the great pyramids of ancient Egypt. Calculate the base : height ratio of several pyramids.





Language in mathematics

Creative writing

Select one of the following topics as the basis of a creative writing exercise of approximately 700–750 words. Topics could include:

- Less is best?
- I'm shrinking! Help!
- At this rate, we'll never get there!
- Comparisons can be misleading and confusing.

Terms

comparison	distance	enlarge	equivalent	express	interval
part	quantity	rate	ratio	reduce	scale factor
simplify	speed	time	unitary		

Check your skills

1 In this diagram the ratio of moons to stars is:

A 7 : 3 **B** 3 : 7
C 3 : 10 **D** 7 : 10

2 Paul weighs 62 kg, Robert weighs 94 kg and Harry weighs 84 kg. The simplified ratio of Paul's weight to Robert's weight to Harry's weight is:

A 62 : 94 : 84 **B** 31 : 47 : 42 **C** 31 : 94 : 42 **D** 62 : 47 : 42

3 In a bag of lollies containing Smarties and jubes, $\frac{4}{11}$ of the lollies are Smarties. What is the ratio of Smarties to jubes?

A 11 : 4 **B** 4 : 11 **C** 4 : 7 **D** 7 : 4

4 For the ratio 11 : 13, which fraction will express the first part as a fraction of the whole?

A $\frac{11}{24}$ **B** $\frac{24}{11}$ **C** $\frac{11}{13}$ **D** $\frac{13}{11}$

5 The fraction $\frac{22}{25}$ shows the first part of a ratio as a fraction of the whole. What is the ratio?

A 22 : 3 **B** 22 : 25 **C** 22 : 47 **D** 3 : 22

6 Determine the value of x if $\frac{4}{9} = \frac{x}{117}$.

A 263.25 **B** 52 **C** 29.25 **D** 13

7 Find the value of y if $\frac{144}{200} = \frac{18}{y}$.

A 25 **B** 13.89 **C** 12.96 **D** 0.04

8 The ratio that is equivalent to $2\frac{1}{2} : 3\frac{1}{4} : 6$ is:

A 5 : 6.5 : 12 **B** 2 : 3 : 6 **C** 10 : 13 : 24 **D** 7 : 8 : 16

9 Simplify 0.015 : 0.3 : 4.5.

A 1 : 2 : 3 **B** 1 : 20 : 300 **C** 1 : 2 : 30 **D** 1 : 20 : 30

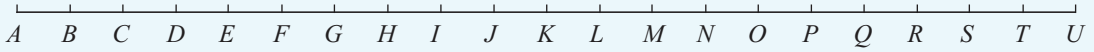
10 The ratio of cordial to water is 1 : 20. If 8.2 mL of cordial is used, the volume of water required is:

A 164 mL **B** 2.4 mL **C** 0.41 mL **D** 135 mL

11 The ratio of dogs to cats to birds is 15 : 13 : 2. How many cats are there if there are 180 dogs?

A 208 cats **B** 237 cats **C** 143 cats **D** 156 cats

12 Which point divides the line segment AU into the ratio 1 : 4?



A E **B** Q **C** J **D** P

13 Divide 2.5 L into the ratio 13 : 28 : 9.

A 37.5 mL : 70 mL : 22.5 mL **B** 6.5 mL : 14 mL : 4.5 mL

C 650 mL : 1400 mL : 450 mL **D** 65 mL : 140 mL : 45 mL

14 Calculate the cost of a phone call lasting 18 min and 20 s at a rate of 52c/min.

A $9.46 **B** $9.48 **C** $9.53 **D** $9.52

15 Two towns are 43.5 km apart. What would be the distance between the two towns on a map if the map uses a scale of 1 : 250 000?

A 0.174 cm **B** 1.74 cm **C** 17.4 cm **D** 174 cm

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–5	6–9	10, 11	12, 13	14	15
Section	A	B	C	D	E	F

2A Review set

1 What is a ratio?

2 In a class of 28 students there are 17 boys. Write the ratio of boys to girls.

3 Express each ratio in its simplest form.

a 12 : 30 **b** 84 : 108

4 Find the value of x when $3 : 7 = x : 42$.

5 Divide $1000 in the ratio 4 : 6.

6 Divide 24.6 m into the ratio 2 : 3 : 1.

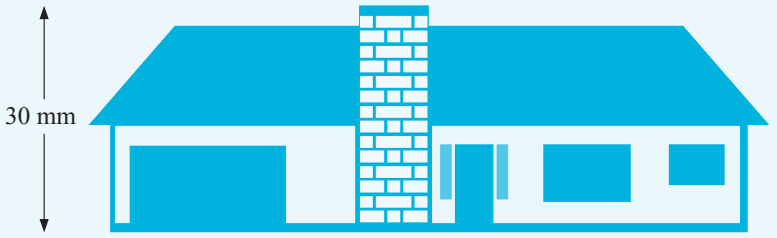
7 Susan speaks to Clara for 29 m and 42 s. Calculate the cost of the phone call if it is charged at 49c/min or part thereof.

8 A car travels at a speed of 112.6 km/h for $3\frac{1}{4}$ hours. Calculate the distance travelled.

9 A map has a scale of 1 : 250. Convert the following scale distances to real distances.

a 13 cm **b** 22.1 cm

10 The ratio of the size of a model to the size of the real building is 1 : 200. If the model has a height of 30 mm, find the actual height of the building in metres.



2B Review set

1 Why do ratios not have units?

2 A bag contains mints and fruities. If there are 18 lollies, of which 11 are mints, write the ratio of mints to fruities.

3 Express each ratio in its simplest form.

a 15 : 45 **b** 126 : 231

4 Find the value of y if $8 : 13 = 64 : y$.

5 Divide 1250 kg in the ratio 3 : 5.

6 Calculate the cost of fuel for a 381.3 km trip if petrol costs \$1.52/L and Tim's car travels 8.2 km/L.

7 Sarah bets \$130 on a horse at odds of 10/1. How much would she receive if the horse won?

8 A train travels a distance of 725.2 km at a speed of 168 km/h. How long does the journey take?

9 A map has a scale of 1 : 80. Convert the following real distances to scale distances.

a 8.8 m **b** 36 m

10 A map has a scale of 1 : 500 000. If the distance between two towns on the map is 5.3 cm, what is the actual distance between the two towns?

2C Review set

1 Express in simplest form 25 min : $1\frac{1}{4}$ hours.

2 The ratio that is equivalent to 3 : 5 is:

A 21 : 36 **B** 45 : 75

3 Find x when $\frac{4}{x} = \frac{7}{5}$.

4 Simplify the following ratios.

a 25 cm : 0.6 m **b** 360 m : 0.5 km

5 On an excursion, the ratio of teachers to students is 2 : 11. Calculate the number of students if there are 10 teachers.

6 A triangle has side lengths in the ratio 2 : 5 : 4.

a If the shortest side is 12.4 cm, find the lengths of the two remaining sides.

b Calculate the total length of the sides of the triangle.

7 Ian jogs 3.5 km in 20 minutes. Express this as a rate of km/min.

8 Which is the better buy?

A 1.2 L of Fizz Whiz Cola at $1.05 **B** 2.5 L of Fizz Whiz Cola at $2.20

9 The scale of a model aeroplane is 1 : 120. If the wingspan of the model is 17 cm, calculate the actual wingspan.

10 The actual height of a building is 825 m. If a model of the building is constructed using a scale of 1 : 1500, calculate the height of the model.

2D Review set

1 Express this ratio in simplest form: 1.2 m : 18 cm.

2 Find y when $\frac{5}{y} = \frac{3}{4}$.

3 Divide $5000 in the ratio 9 : 6 : 5.

4 Joe buys a bike for $870 and later sells it for $935. Express the following as a ratio.

a cost price : selling price **b** profit : selling price

5 The length and breadth of a rectangle are in the ratio 11 : 3.

a Find the length of the rectangle if the breadth is 7.5 cm.

b Find the breadth of the rectangle if the length is 57.2 cm.

6 If 15 oranges cost $6.30, then the cost per orange is ____ cents.

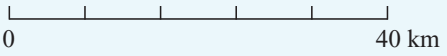
7 Which is the better buy?

A 500 g of cat food at $5.25 **B** 750 g of cat food at $7.80

8 On a particular day the $A is worth $US0.92. Calculate the value of $US1400 on that day.

9 If water costs 23c/kL, calculate the cost of 940 kL.

10 **a** Use your ruler to determine the scale of the line drawn.



b If two towns are 12.3 cm apart on a map that uses the scale above, what is the actual distance between them?

3

Congruence

This chapter deals with congruence of plane shapes.

At the end of this chapter you should be able to:

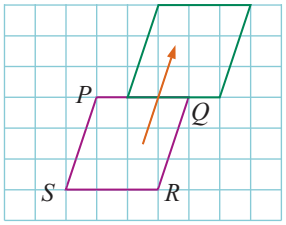
- understand the properties that determine congruence of plane shapes
- recognise which transformations create congruent figures
- establish four tests for the congruency of triangles through the construction of triangles using given conditions
- recognise which conditions do not prove congruency of triangles
- solve problems using the properties of congruent triangles
- establish the properties of triangles and quadrilaterals using congruence.

NSW Syllabus references: S4 M&G Properties of geometrical figures 1, S4 M&G Properties of geometrical figures 2
Outcomes: MA4–1WM, MA4–2WM, MA4–3WM, MA4–17MG
MEASUREMENT & GEOMETRY – ACMMG200, ACMMG201, ACMMG202

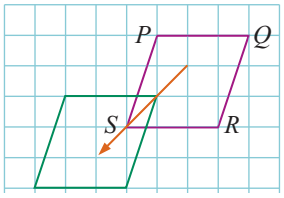
Diagnostic test

1 When the parallelogram $PQRS$ is translated 3 units left and 2 units up, the result is:

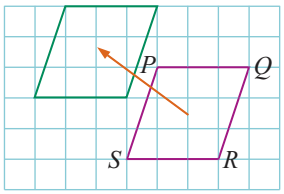
A



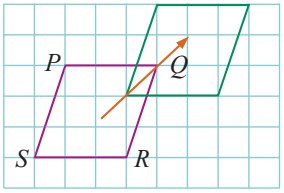
B



C

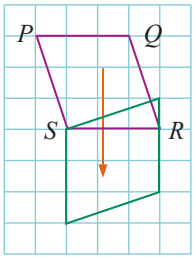


D

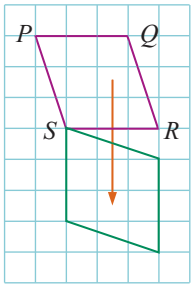


2 When the parallelogram $PQRS$ is rotated clockwise about S through 90°, the result is:

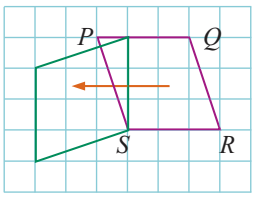
A



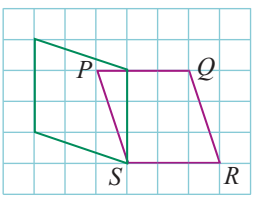
B



C

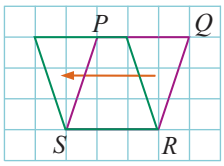


D

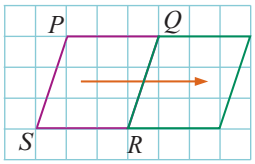


3 When the parallelogram $PQRS$ is reflected in SR, the result is:

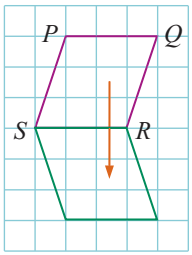
A



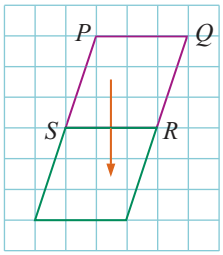
B



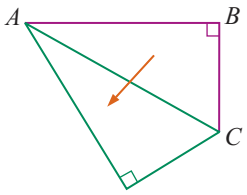
C



D



4 Triangle ABC has been:



A rotated anticlockwise about A

B reflected in AC

C translated

D rotated clockwise about A

5 When point $P(3, -2)$ is translated 2 units right and 4 units down, its image coordinates are:

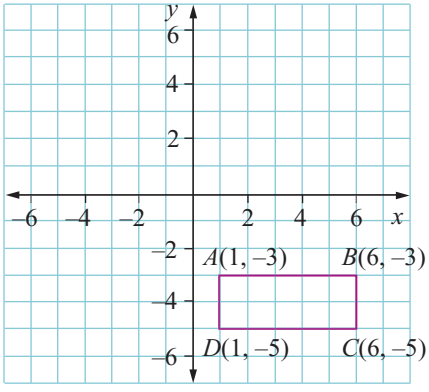
A $(1, -6)$ **B** $(5, 2)$

C $(5, -2)$ **D** $(5, -6)$

6 When rectangle $ABCD$ is reflected in the y-axis, the coordinates of the image of B are:

A $(6, 3)$ **B** $(-6, -3)$

C $(-6, 3)$ **D** $(-3, -6)$



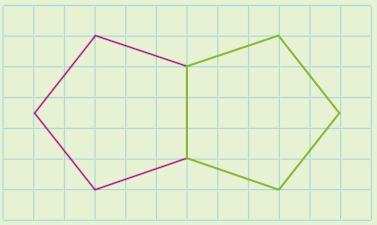
The Diagnostic test questions refer to the Year 7 outcomes from ACMMG200, ACMMG201 and ACMMG202.

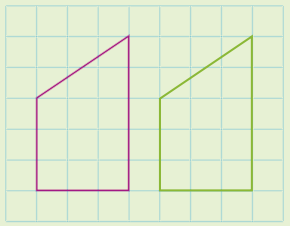
A Congruent figures

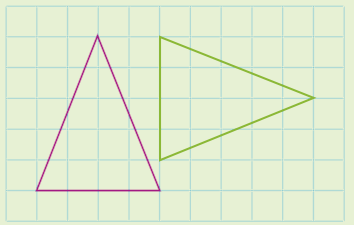
Congruent figures have exactly the same size and shape. They are identical in all respects except for position and orientation. Congruent figures are formed by performing one or more transformations, such as translation, rotation or reflection, on the original figure.

EXAMPLE 1

What transformation(s) could have been used to produce the following pairs of congruent figures?

a 

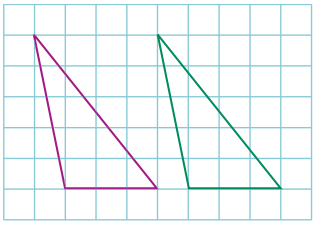
b 

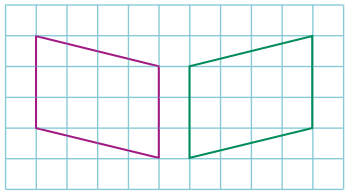
c 

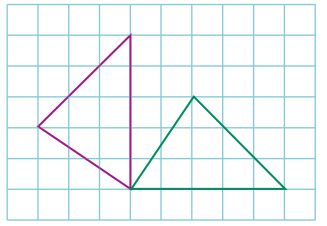
a Reflection **b** Translation **c** Rotation and translation

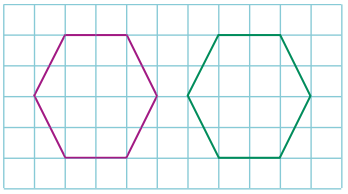
Exercise 3A

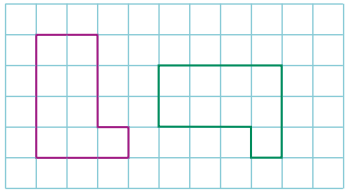
1 What transformation(s) could have been used to produce the following pairs of congruent figures?

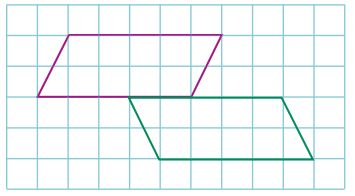
a 

b 

c 

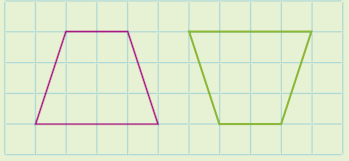
d 

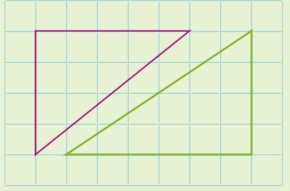
e 

f 

EXAMPLE 2

Determine whether or not the following pairs of figures are congruent.

a 

b 

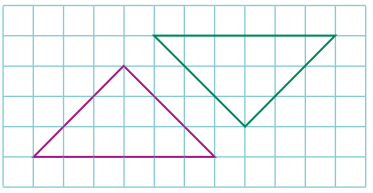
Copy the figures and cut them out. By attempting to superimpose them by a combination of rotations, reflections and translations, the figures are:

a congruent **b** not congruent

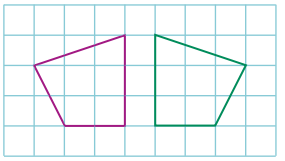
'Superimpose' means to place one figure on top of the other in such a way that one sits exactly on top of the other.

2 Determine whether or not the following pairs of figures are congruent.

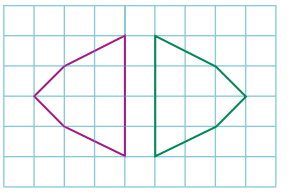
a



b



c



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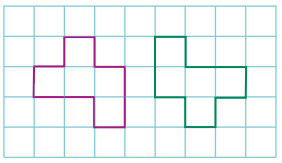

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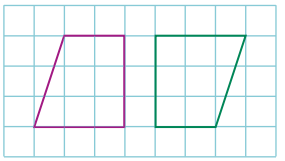

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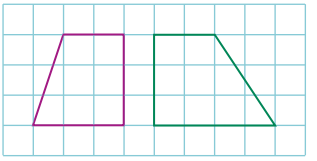


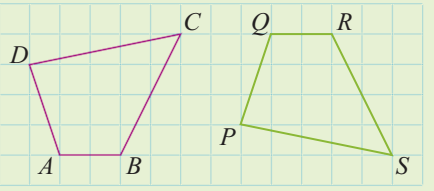
EXAMPLE 3

For the pair of congruent figures shown, copy the first figure, cut it out and superimpose it on the second figure.

a Name the matching sides and angles.

b Are the matching sides and angles equal?

c Write a congruence statement for these two figures.



a When superimposed, the matching sides are AB and QR, BC and RS, CD and SP, DA and PQ. The matching angles are $\angle A$ and $\angle Q$, $\angle B$ and $\angle R$, $\angle C$ and $\angle S$, $\angle D$ and $\angle P$.

Matching sides and angles are also referred to as corresponding sides and angles.

b Yes

c The symbol $\equiv$ means 'is congruent to'. When writing a congruence statement, the vertices are named in matching order. In this case we write quadrilateral $ABCD \equiv$ quadrilateral $QRSP$ (or $BCDA \equiv RSPQ$ or $CDAB \equiv SPQR$ or $DABC \equiv PQRS$).

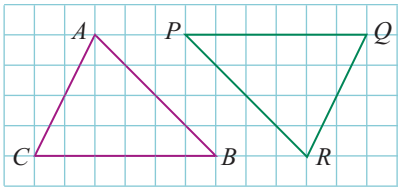
3 For each pair of congruent figures, copy the first figure, cut it out and superimpose it on the second figure.

i Name the matching sides and angles.

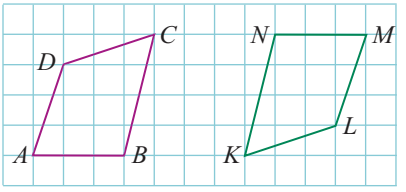
ii Are the matching sides and angles equal?

iii Write a congruence statement for the two figures.

a



b



4 **a** Check by measurement and write true or false.

i $PQ = TU$ **ii** $QR = UV$

iii $RS = VW$ **iv** $SP = WT$

b Are there four pairs of equal sides?

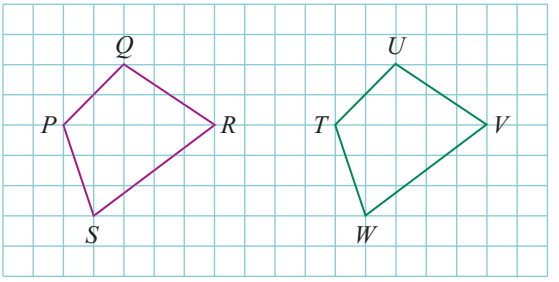
c Check by measurement and write true or false.

i $\angle P = \angle T$ **ii** $\angle Q = \angle U$

iii $\angle R = \angle V$ **iv** $\angle S = \angle W$

d Are there four pairs of equal angles?

e Are the figures congruent?



5 **a** Check by measurement and write true or false.

i $AB = KL$ **ii** $BC = LM$

iii $CD = MN$ **iv** $DA = NK$

b Are there 4 pairs of equal sides?

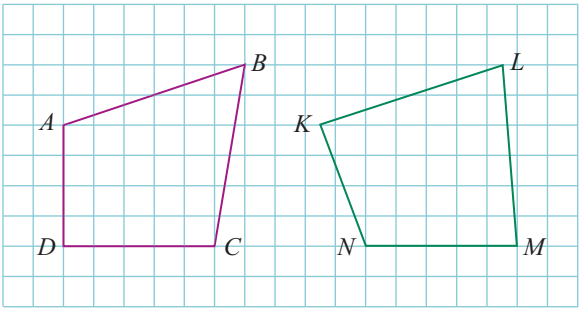
c Check by measurement and write true or false.

i $\angle A = \angle K$ **ii** $\angle B = \angle L$

iii $\angle C = \angle M$ **iv** $\angle D = \angle N$

d Are any of these pairs of angles equal?

e Are the figures congruent?



6 **a** Write true or false.

i $\angle Q = \angle J$ **ii** $\angle R = \angle K$

iii $\angle S = \angle L$ **iv** $\angle T = \angle M$

v $\angle U = \angle N$

b Are there four pairs of equal angles?

c Write true or false.

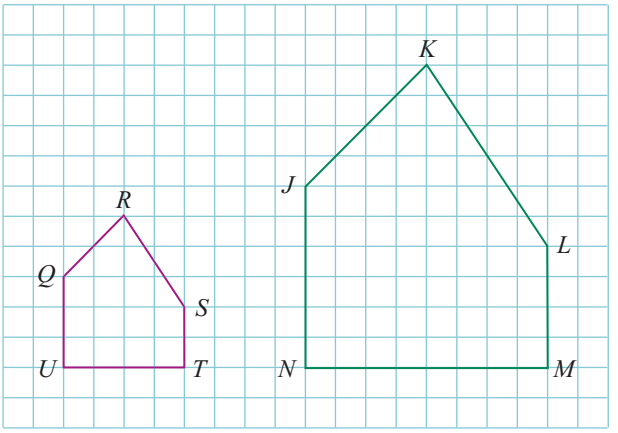
i $QR = JK$ **ii** $RS = KL$

iii $ST = LM$ **iv** $TU = MN$

v $UQ = NJ$

d Are any of these pairs of sides equal?

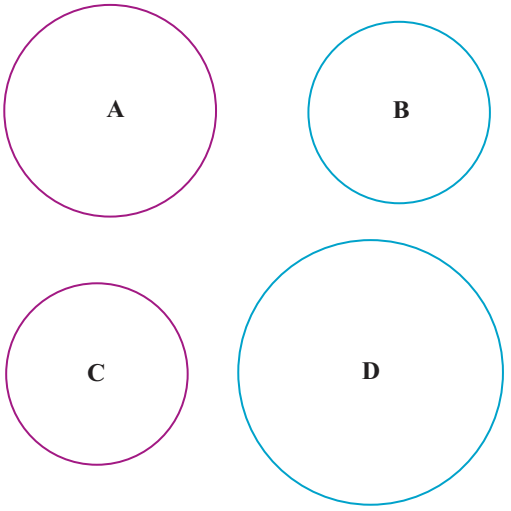
e Are the figures congruent?



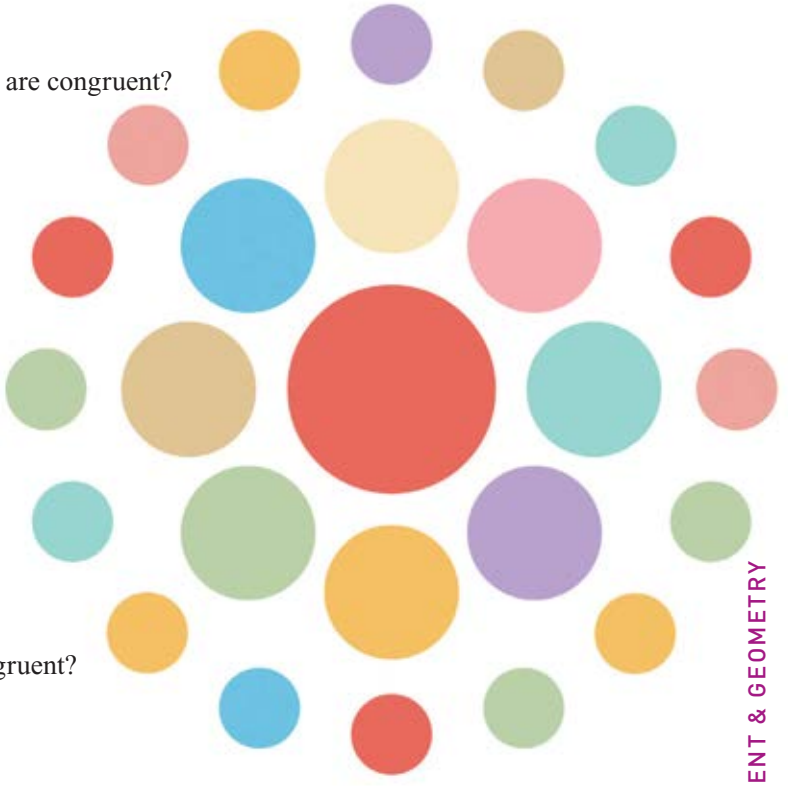
From questions **4**, **5** and **6** it can be deduced that two polygons are congruent if:

- the matching sides are equal, and
- the matching angles are equal.

7 **a** Consider the following diagram. Which circles are congruent?



b What is the condition for two circles to be congruent?

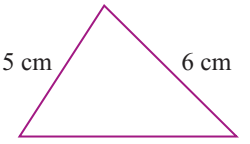


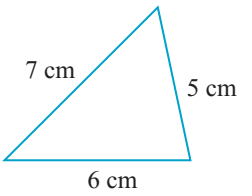
B Congruency tests

Section A shows that congruent polygons have all their matching sides and matching angles equal. This section investigates whether it is necessary to have all this information in order to show that two triangles are congruent.

Exercise 3B

1 **a** Using a pair of compasses and ruler, construct any two triangles that have three pairs of equal sides and cut them out. This is an example.



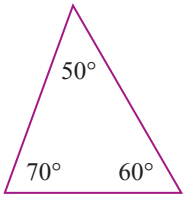


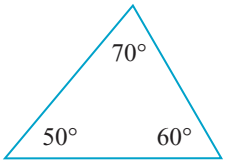
b By superimposing, determine whether or not the triangles are congruent.

c Compare your triangles with those of other students in the class. Are they congruent?

d If three sides of one triangle are equal to three sides of another triangle, are the triangles congruent?

2 **a** Using a protractor, construct any two triangles that have three pairs of equal angles. This is an example.



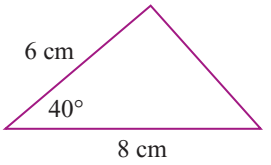


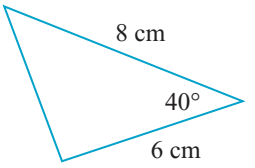
b By superimposing, determine whether the triangles are congruent.

c Compare your triangles with those of other students in the class. Are they congruent?

d If three angles of one triangle are equal to three angles of another triangle, are the triangles congruent?

3 **a** Construct any two triangles that have two pairs of equal sides and one pair of equal angles as shown.





b By superimposing the triangles, determine whether they are congruent.

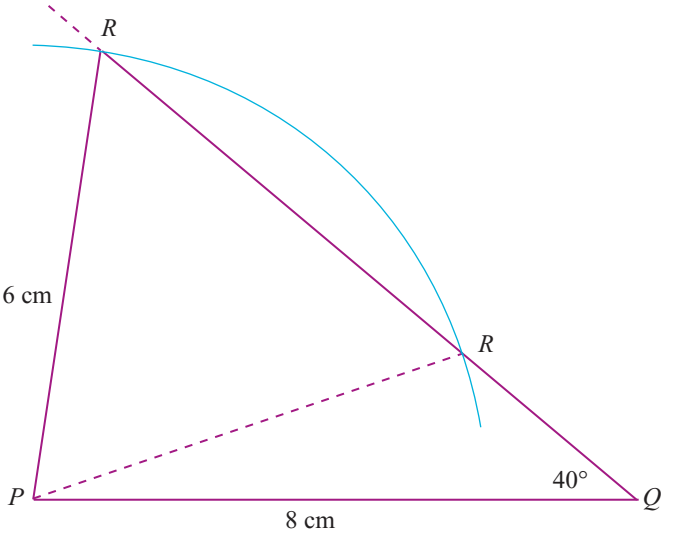
c Compare your triangles with those of other students in the class. Are they congruent?

d Notice that the given angle in each triangle is between the given sides. This is called the **included angle**. Construct the triangle below in which the angle given is not the included angle.

Step 1: Use your ruler to draw PQ to size.

Step 2: Use your protractor to draw a ray from Q so that $\angle PQR = 40°$.

Step 3: Set your compasses to a radius of 6 cm and draw the interval PR.



e **i** How many possible positions are there for R?

ii How many triangles can be drawn with these measurements?

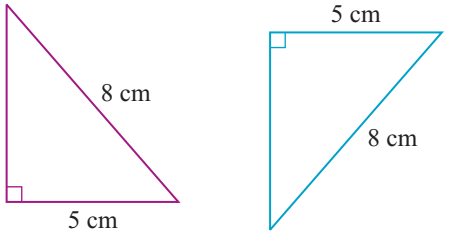
iii Are they congruent?

iv Are either of these triangles congruent to the triangles in part **a**?

f If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, are the triangles congruent?

g Discuss the results of the class and modify the statement in part **f** so that it is always true.

4 **a** Construct two right-angled triangles in which the hypotenuses and one other pair of sides are equal. This is an example.

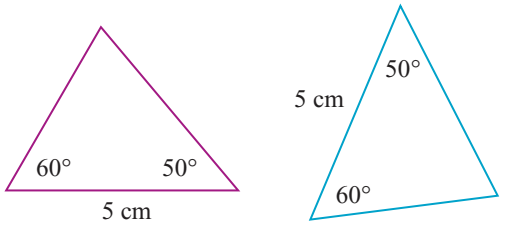


b By superimposing, determine whether the triangles are congruent.

c Compare your triangles with those of other students in the class. Are they congruent?

d If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and a side of another right-angled triangle, are the triangles congruent?

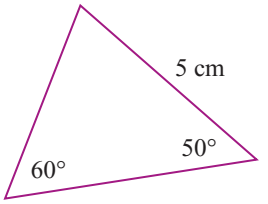
5 **a** Construct two triangles that have one pair of equal sides and two pairs of equal angles as shown.



b By superimposing, determine whether the triangles are congruent.

c Compare your triangles with those of other students in the class. Are they congruent?

d Notice that the given pair of equal sides are matching sides of the triangles. Now construct the triangle shown on the right.
Note: The side marked 5 cm in this triangle is not in the same position as in the triangles of part **a**.
Hint: Calculate the third angle of the triangle.



e By superimposing, determine whether this triangle is congruent to the triangles constructed in part **a**.

f If two angles and one side of one triangle are equal to two angles and one side of another triangle, are the triangles congruent?

g Discuss the results of the class and modify the statement in part **f** so that it is always true.

In congruent triangles, the matching sides are opposite the equal angles.

Congruency tests for triangles

From the exercise above, it can be seen that in order to prove that two triangles are congruent it is not necessary to show that all three pairs of sides and all three pairs of angles are equal.

There are four sets of minimum conditions. These are known as the congruency tests for triangles and are given below.

'Minimum conditions' means the smallest amount of information needed.

1 If three sides of one triangle are equal to three sides of another triangle, then the two triangles are congruent (SSS).

2 If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent (SAS).

3 If two angles and one side of a triangle are equal to two angles and the matching side of another triangle, then the two triangles are congruent (AAS).

4 If the hypotenuse and a second side of one right-angled triangle are equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (RHS).

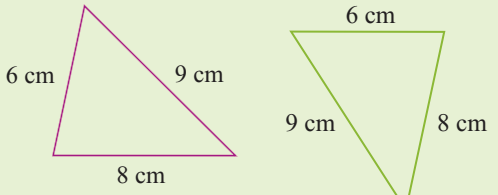
C Applying congruency tests

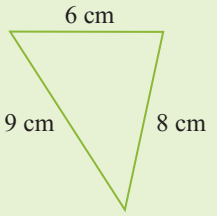
Section B listed the congruency tests for triangles. This section applies those tests.

EXAMPLE 1

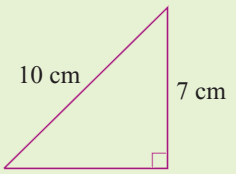
State the congruency test used to show that the following pairs of triangles are congruent.

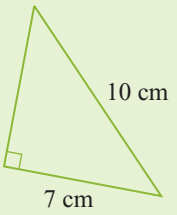
a





b





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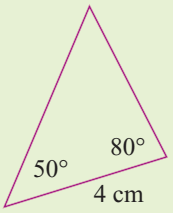

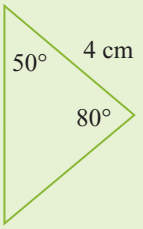

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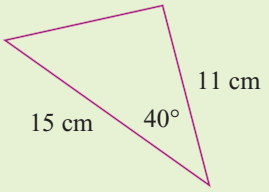
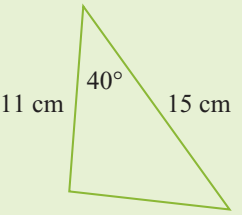




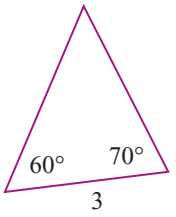
Look for pairs of equal matching sides and angles.

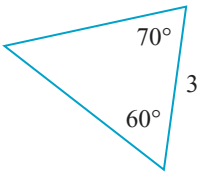
a SSS **b** RHS **c** AAS **d** SAS

Exercise 3C

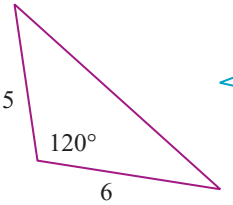
1 Find pairs of equal matching sides and angles and state the congruency test used to show that these pairs of triangles are congruent. (Diagrams are not drawn to scale. All lengths given are in centimetres.)

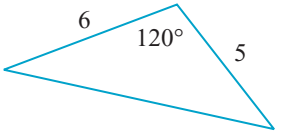
a





b





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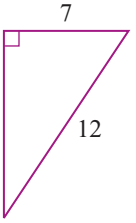

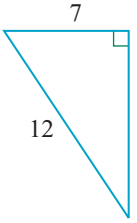

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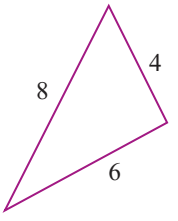

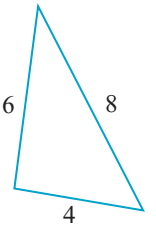

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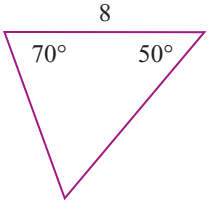

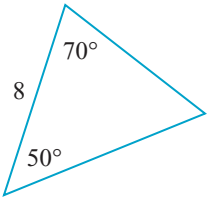

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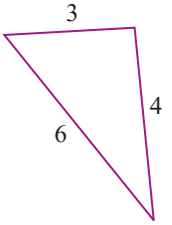

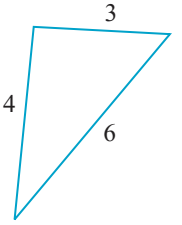

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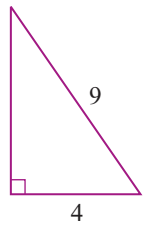

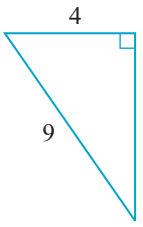

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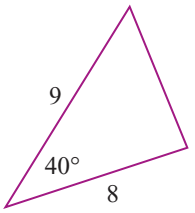

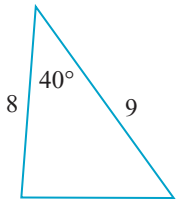

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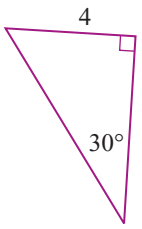

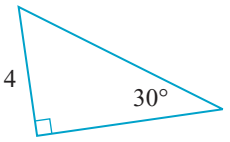

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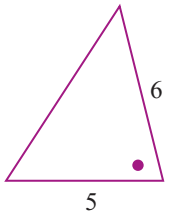

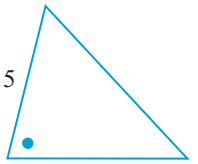

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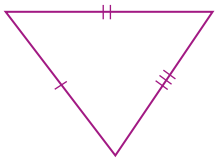

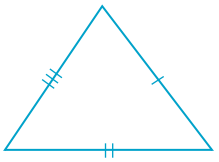

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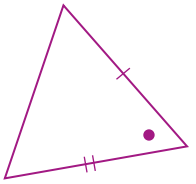

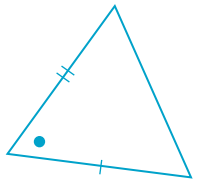

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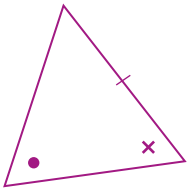

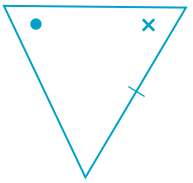

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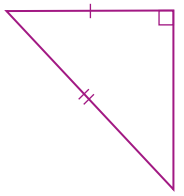
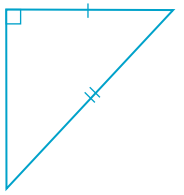
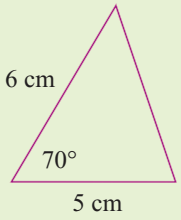


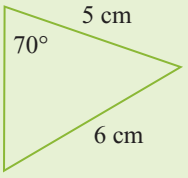


EXAMPLE 2

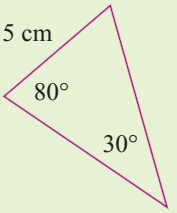
State why the following pairs of figures are *not* congruent.

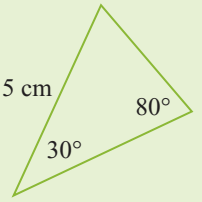
a





b





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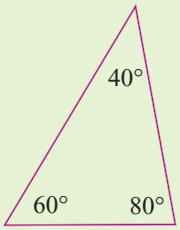

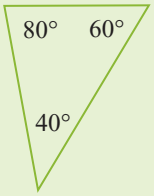

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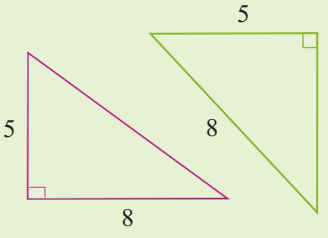


a The 70° angle in the second triangle is not the included angle.

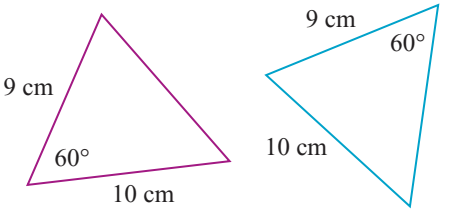
b The pair of equal sides is not a pair of matching sides in the triangles.

c Three pairs of equal angles do not necessarily make the triangles congruent. These triangles have the same shape (angles) but different size.

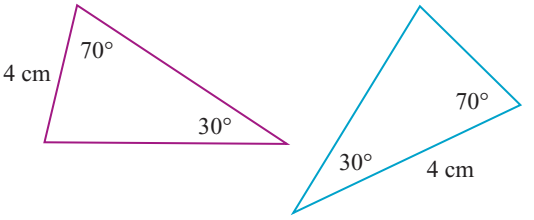
d The sides given are not matching sides.

2 Complete the following statements to explain why the figures are *not* congruent.

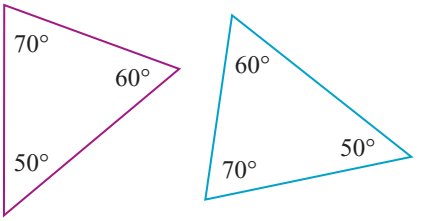
a The equal angles are not both the ____ angle.



b The equal sides are not ____ sides.

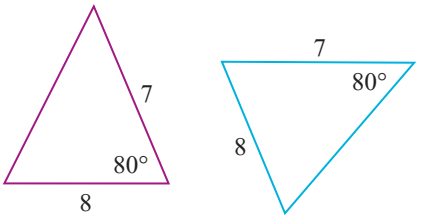


c Three pairs of equal ____ do not necessarily make the triangles congruent.

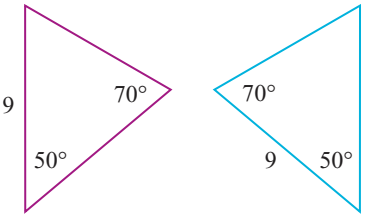


3 State why the following pairs of figures are *not* congruent. (Diagrams are not drawn to scale. All lengths given are in centimetres.)

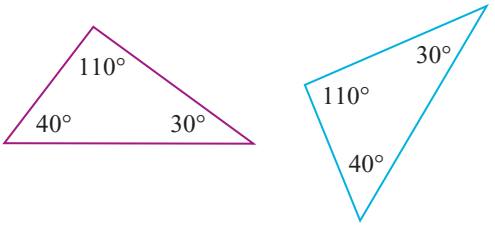
a



b



c



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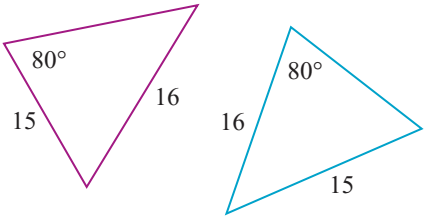

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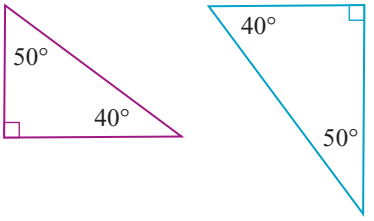

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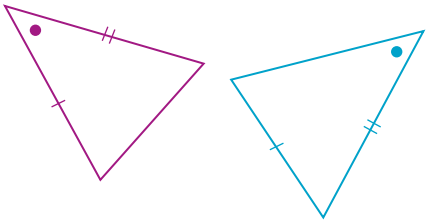

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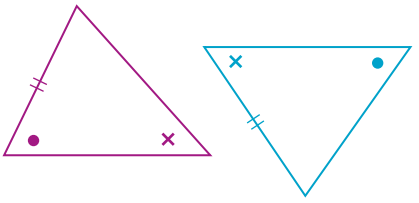

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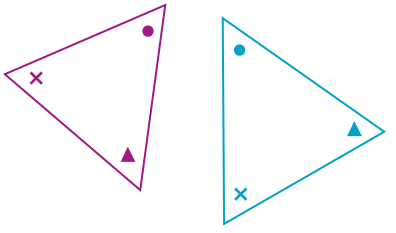
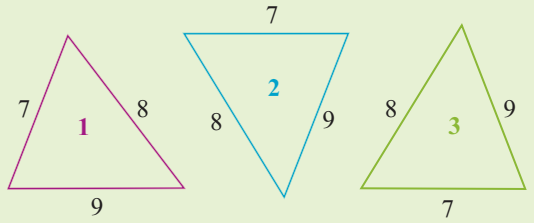


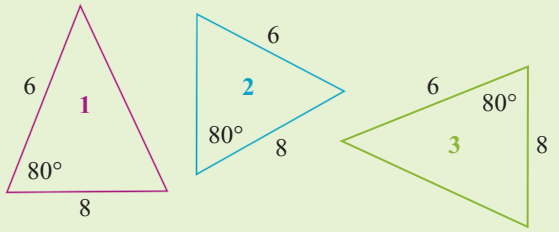
EXAMPLE 3

Which triangles are congruent? Give a reason. (All given lengths are in centimetres.)

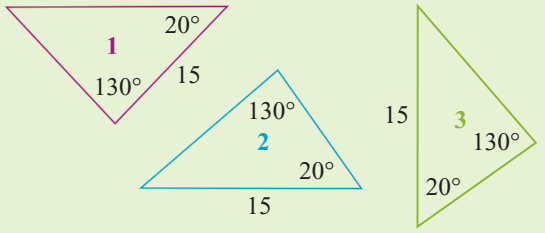
a



b



c



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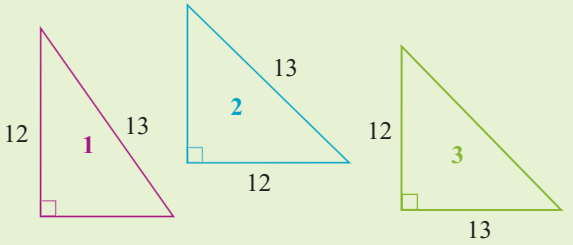


a Triangles 1, 2 and 3 (SSS)

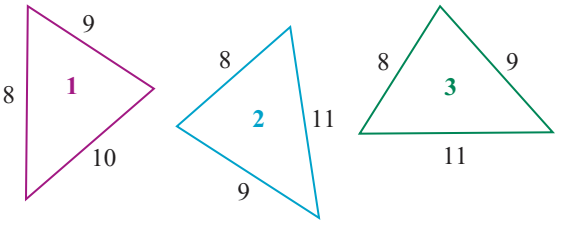
b Triangles 1 and 3 (SAS)

c Triangles 2 and 3 (AAS)

d Triangles 1 and 2 (RHS)

4 Which triangles are congruent? Give a reason. (Diagrams are not drawn to scale. All given lengths are in centimetres.)

a



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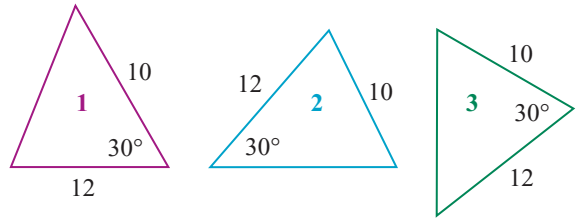

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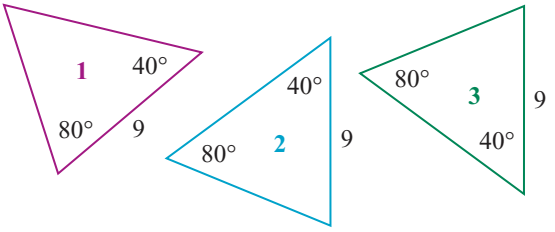

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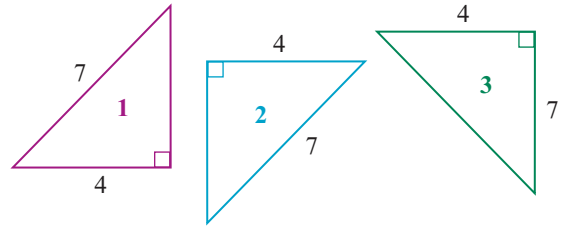

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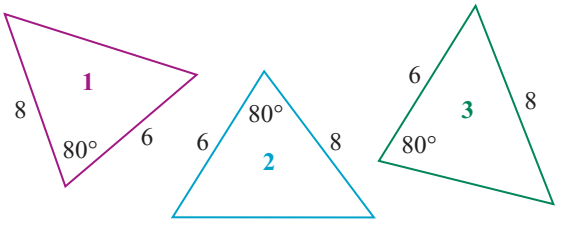

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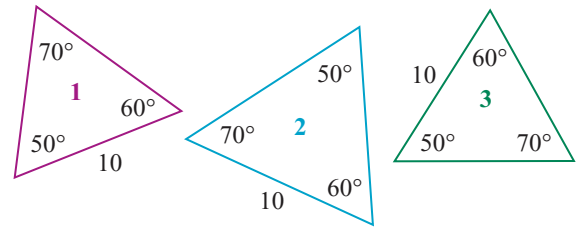

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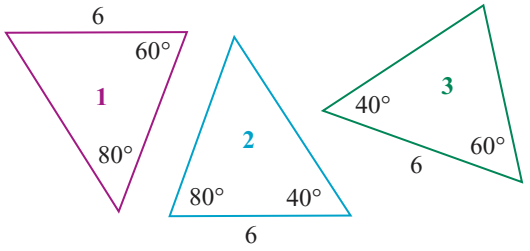

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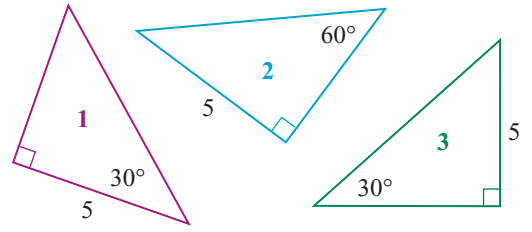
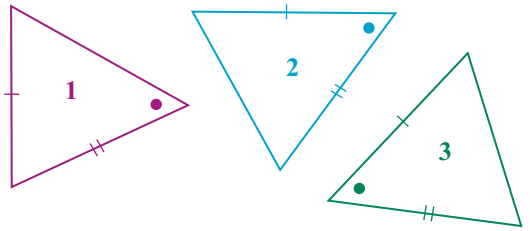


i



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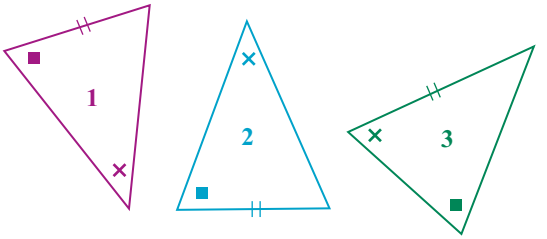
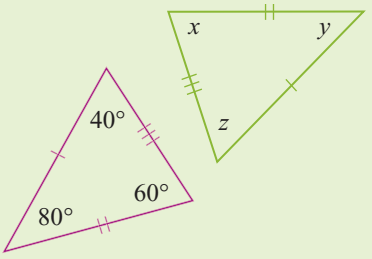


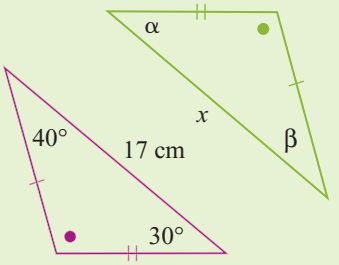
EXAMPLE 4

State why the following pairs of triangles are congruent and hence find the value of the pronumerals. (Diagrams are not drawn to scale. All lengths are in centimetres.)

a



b



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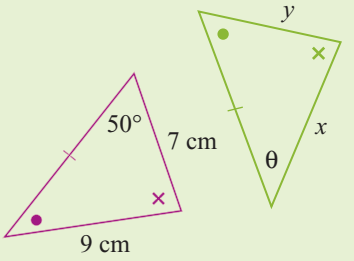


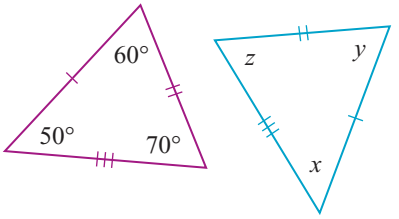
a SSS. Matching angles are opposite matching sides, hence x and 60°, y and 80°, z and 40°.

b SAS. Matching sides are opposite matching angles, hence $x = 17$ cm, $\alpha = 30°$ and $\beta = 40°$ (matching angles).

c AAS. $x = 7$ cm, $y = 9$ cm (matching sides) and $\theta = 50°$ (matching angles).

5 State why the following pairs of triangles are congruent and hence find the values of the pronumerals. (Diagrams are not drawn to scale. All lengths are in centimetres.)

a



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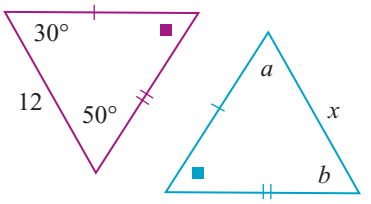

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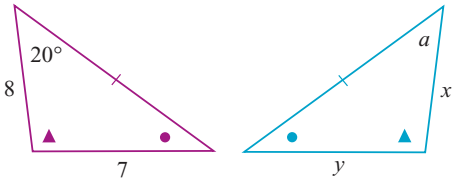

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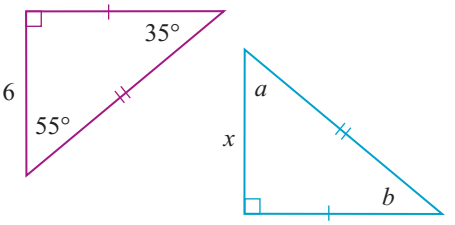

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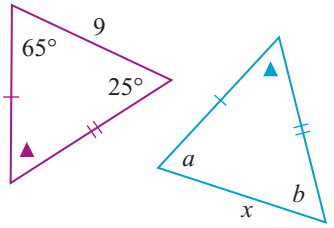

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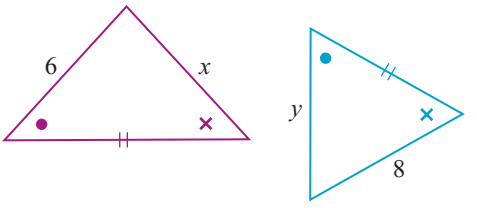

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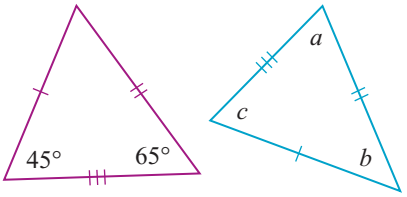

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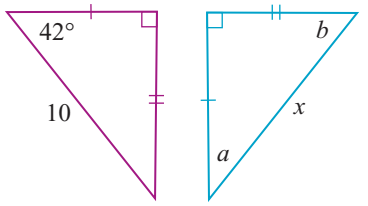

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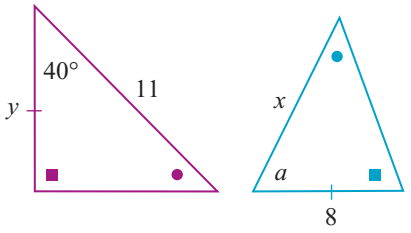
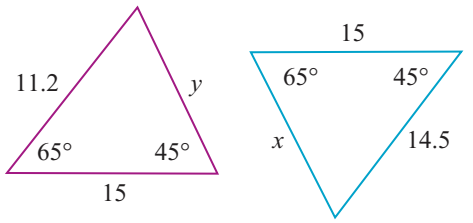


j

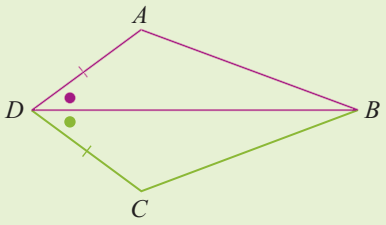


EXAMPLE 5

Prove that:

a $\triangle ABD \equiv \triangle CBD$

b $AB = CB$



a In $\triangle ABD$ and $\triangle CBD$

$AD = CD$		(given)
$DB = DB$		(common)
$\angle ADB = \angle CDB$		(given)
$\therefore \triangle ABD \equiv \triangle CBD$		(SAS)

b $AB = CB$ (matching sides of congruent triangles)

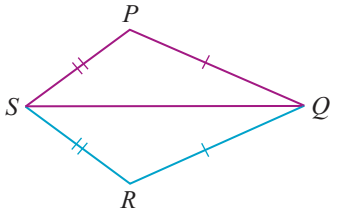
6 Complete the following proofs.

a **i** $\triangle PQS \equiv \triangle RQS$

In $\triangle PQS$ and $\triangle RQS$

$PS = RS$		(given)
$SQ = SQ$		(_____)
$PQ =$ ___		(given)
$\therefore \triangle PQS \equiv \triangle RQS$		(_____)

ii $\angle SPQ = \angle SRQ$ (_____)

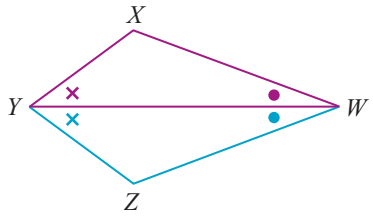


b **i** $\triangle XYW \equiv \triangle ZYW$

In $\triangle XYW$ and $\triangle ZYW$

$YW = YW$		(_____)
$\angle XYW = \angle ZYW$		(_____)
$\angle XWY =$ ___		(given)
$\therefore \triangle XYW \equiv \triangle ZYW$		(_____)

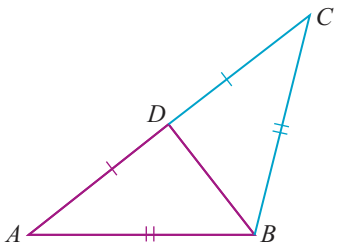
ii $XW =$ ___ (matching sides of congruent triangles)



7 Complete proofs for the following.

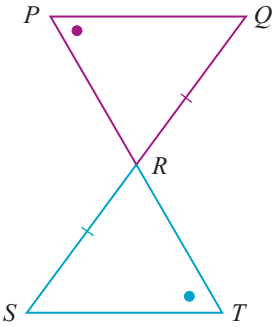
a **i** $\triangle ADB \equiv \triangle CDB$

ii $\angle ADB = \angle CDB$



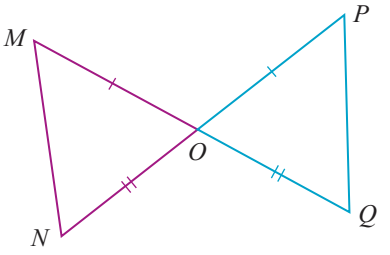
b **i** $\triangle PQR \equiv \triangle TSR$

ii $PR = RT$



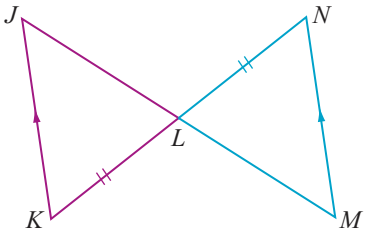
c **i** $\triangle MON \equiv \triangle POQ$

ii $MN = PQ$



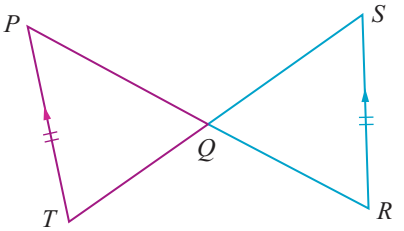
d **i** $\triangle JKL \equiv \triangle MNL$

ii $JL = ML$



e **i** $\triangle PQT \equiv \triangle RQS$

ii $TQ = SQ$



D Properties of triangles and quadrilaterals

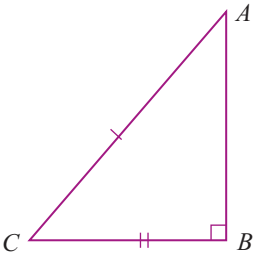
Exercise 3D

1 **a** Draw a right-angled triangle ABC as shown.

b Reflect $\triangle ABC$ in AB to form $\triangle AC'B$.

c What type of triangle is $\triangle AC'C$?

d What properties of this type of triangle ($\triangle AC'C$) are demonstrated?



2 **a** Draw any $\triangle ABC$ as shown.

b Locate the midpoint M of AB.

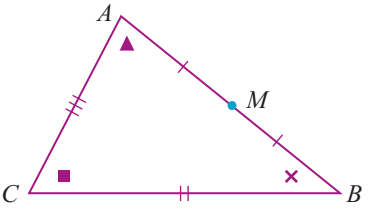
c Rotate $\triangle ABC$ clockwise about M through 180°.

d Draw the quadrilateral formed by the $\triangle ABC$ and its image.

e Mark on this quadrilateral all the equal sides and angles.

f What type of quadrilateral is formed?

g What properties of this type of quadrilateral are demonstrated?

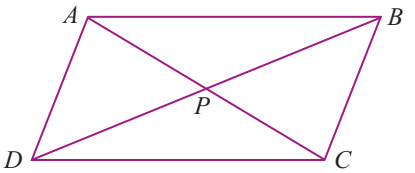


3 **a** Draw a parallelogram $ABCD$ and label the point of intersection of the diagonals P.

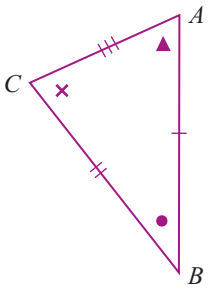
b Rotate the parallelogram clockwise about P through 180°.

c Does the parallelogram rotate onto itself?

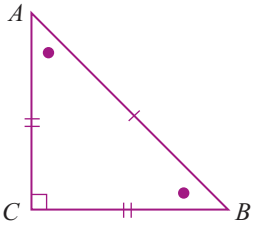
d What property of the diagonals of a parallelogram does this demonstrate?



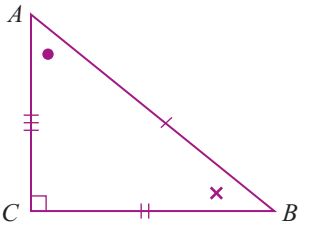
4 **a** Draw any $\triangle ABC$ as shown.
b Reflect the triangle in the line AB.
c Draw the quadrilateral formed by the $\triangle ABC$ and its image.
d Mark on this quadrilateral all the equal sides and angles.
e What type of quadrilateral is formed?
f What properties of this type of quadrilateral are demonstrated?



5 **a** Draw a right-angled isosceles $\triangle ABC$ as shown.
b Rotate the triangle clockwise about C through 90°, 180° and 270°.
c Draw the quadrilateral formed by the $\triangle ABC$ and these three images.
d Mark on this quadrilateral all the equal sides and angles.
e What type of quadrilateral is formed?
f What properties of this type of quadrilateral are demonstrated?

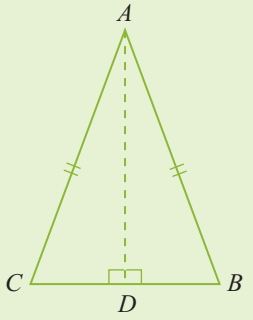


6 **a** Draw a right-angled $\triangle ABC$ as shown.
b **i** Reflect $\triangle ABC$ in BC to give $\triangle CBA'$.
ii Reflect triangle CBA' in CA' to give triangle $CA'B'$.
iii Reflect triangle $CA'B'$ in CA' to give $CB'A$.
c Draw the quadrilateral formed by the $\triangle ABC$ and these three images.
d Mark on this quadrilateral all the equal sides and angles.
e What type of quadrilateral is formed?
f What properties of this type of quadrilateral are demonstrated?



EXAMPLE 1

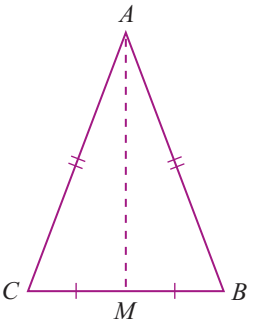
Prove that if two sides of a triangle are equal then the angles opposite the equal sides are equal.



Data: In $\triangle ABC$, $AB = AC$.
Aim: To prove $\angle ACB = \angle ABC$.
Construction: Draw AD perpendicular to CB as shown.
Proof: In $\triangle ADC$ and $\triangle ADB$

$AC = AB$	(given)
$AD = AD$	(common side)
$\angle ADC = \angle ADB = 90°$	(by construction)
$\triangle ADC \equiv \triangle ADB$	(RHS)
$\angle ACD = \angle ABD$	(matching angles of congruent triangles)
$\therefore \angle ACB = \angle ABC$	

7 Complete this alternative proof of the property proved in Example 1.

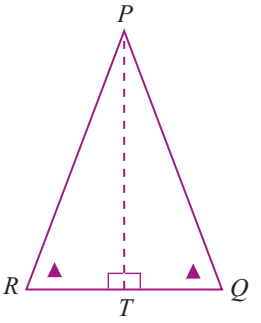


Data: In $\triangle ABC$, $AB = AC$.
Aim: To prove $\angle ACB = \angle ABC$.
Construction: Draw the line from the vertex A to the midpoint M of CB.
Proof: In $\triangle AMC$ and $\triangle AMB$

$AC =$ ___	(given)
$AM = AM$	(common side)
$MC =$ ___	(M is the midpoint of CB)
$\therefore \triangle AMC \equiv \triangle AMB$	(_____)
$\angle ACM = \angle ABM$	(_____)
$\therefore \angle ACB = \angle ABC$	

The angles opposite the equal sides of an isosceles triangle are equal.

8 Complete the following proof.

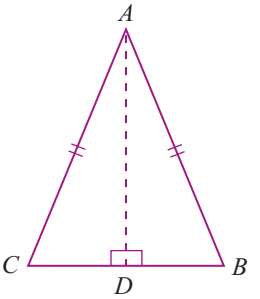


Data: In $\triangle PQR$, $\angle R = \angle Q$.
Aim: To prove that $PR = PQ$.
Construction: Draw PT perpendicular to RQ as shown.
Proof: In $\triangle PTR$ and $\triangle PTQ$:

$\angle R = \angle Q$	(_____)
$PT =$ ___	(common)
$\angle PTR = \angle$___	(construction)
$\therefore \triangle PTR \equiv \triangle PTQ$	(_____)
$\therefore PR = PQ$	(_____)

The sides opposite the equal angles of a triangle are equal.

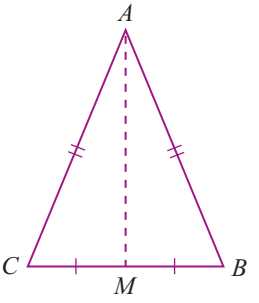
9 Prove the following.



a $\triangle ADC \equiv \triangle ADB$
b $DC = DB$

A line drawn from the vertex of an isosceles triangle perpendicular to its base bisects the base.

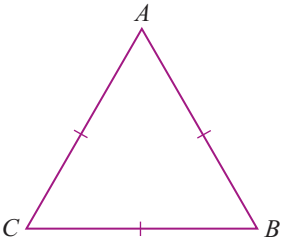
10 Prove the following.



a $\triangle AMC \equiv \triangle ABM$
b $\angle AMC = \angle AMB$
c $\angle AMC = 90°$

A line drawn from the vertex of an isosceles triangle to the midpoint of its base is perpendicular to the base.

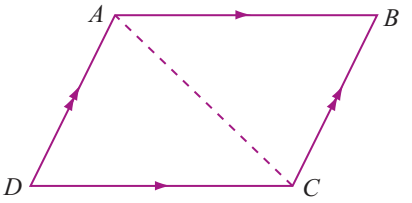
11 Complete the following proof.



Data: ABC is an equilateral triangle.
Aim: To prove that each angle of the triangle is 60°.
Proof:

$\angle B = \angle$___	($AB = AC$)
Also $\angle B = \angle$___	($CB = CA$)
$\angle A = \angle B = \angle C$	
But $\angle A + \angle B + \angle C =$ ___°	(angle sum of a triangle)
$\therefore \angle A = \angle B = \angle C =$ ___°	

12 Complete the following proof.



Data: $ABCD$ is a parallelogram. ($AB \parallel DC$ and $AD \parallel BC$)

Aim: To prove that:

a the opposite sides of a parallelogram are equal

b the opposite angles of a parallelogram are equal.

Proof: In $\triangle ABC$ and $\triangle ADC$

$AC = AC$ (_____)

$\angle BAC = \angle DCA$ (alternate angles and $AB \parallel DC$)

$\angle BCA = \angle DAC$ (_____)

$\triangle ABC \equiv \triangle ADC$ (_____)

a $\therefore AB = DC$ (matching sides of congruent triangles)

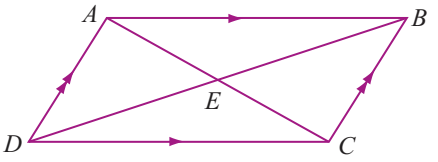
and $AD = BC$ (_____)

The opposite sides of a parallelogram are equal.

b Also $\angle ABC = \angle ADC$ (_____)

The opposite angles of a parallelogram are equal.

13 Complete the following proof.



Data: $ABCD$ is a parallelogram. ($AB \parallel DC$ and $AD \parallel BC$)

Aim: To prove that the diagonals of a parallelogram bisect each other.

Proof: In $\triangle AED$ and $\triangle CEB$

$\angle DAE = \angle$___ (alternate angles and $AD \parallel BC$)

$\angle AED = \angle BEC$ (_____)

$AD =$ ___ (opposite sides of a parallelogram are equal, proven in question **11**)

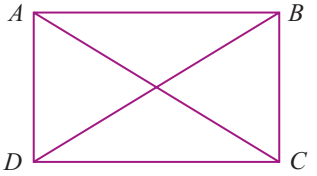
$\triangle AED \equiv \triangle CEB$ (_____)

$\therefore AE =$ ___ (matching sides of congruent triangles)

and $DE =$ ___ (_____)

The diagonals of a parallelogram bisect each other.

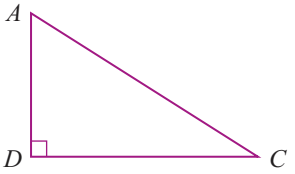
14 Complete the following proof.

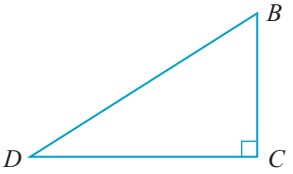


Data: $ABCD$ is a rectangle.

Aim: To prove that the diagonals of a rectangle are equal.

Proof: Drawing the triangles separately and using both figures:





In $\triangle ACD$ and $\triangle BDC$

$AD =$ ___ (opposite sides of a rectangle are equal)

$DC =$ ___ (common side)

$\angle ADC = \angle BCD$ (_____)

$\triangle ACD \equiv \triangle BDC$ (_____)

$\therefore AC =$ ___ (matching sides of congruent triangles)

The diagonals of a rectangle are equal.

Language in mathematics

1 Match each word or phrase with its meaning.

a congruent
b corresponding
c orientate
d perpendicular

A at right angles to
B point the same way
C identical
D matching

2 Write in your own words the meaning of these terms.

a superimpose
b the included angle in a triangle
c equiangular
d common side

3 Name three transformations that can be used to produce congruent figures.

4 List two properties of each triangle.

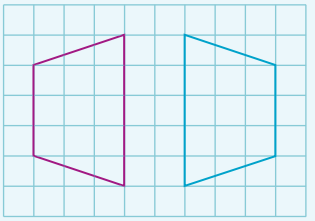
a an isosceles triangle
b an equilateral triangle

Terms

alternate angles	coincide	common side	congruence	congruency	congruent
corresponding	diagonal	equilateral	figure	identical	included angle
isosceles	kite	matching	midpoint	minimum	parallelogram
perpendicular	orientation	proof	prove	reflection	rhombus
rotation	superimpose	transformation	translation		

Check your skills

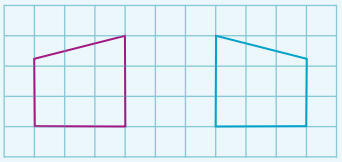
1 Which of the following transformations could not have been used to produce this pair of congruent figures?



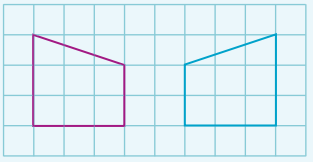
A reflection
B reflection and translation
C rotation and translation
D translation

2 Which pair of figures is not congruent?

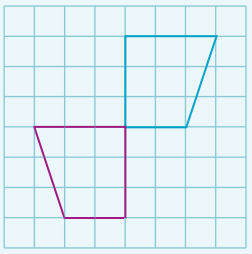
A



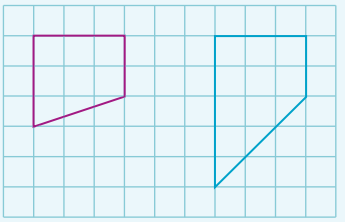
B



C



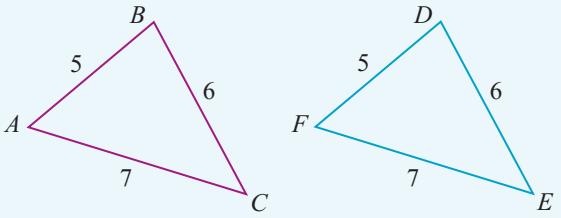
D



3 These triangles are congruent. The correct statement of congruency is:

A $\triangle ABC \equiv \triangle DEF$ **B** $\triangle ABC \equiv \triangle EFD$

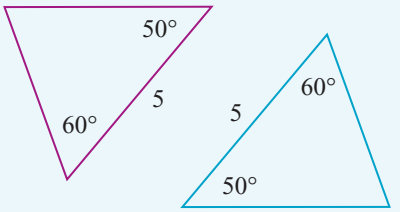
C $\triangle ABC \equiv \triangle FDE$ **D** $\triangle CBA \equiv \triangle DEF$



4 The congruency test used to show that these triangles are congruent is:

A SSS **B** SAS

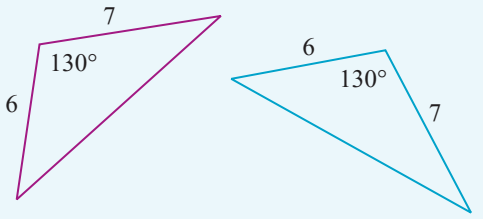
C AAS **D** RHS



5 The congruency test used to show that these triangles are congruent is:

A SSS **B** SAS

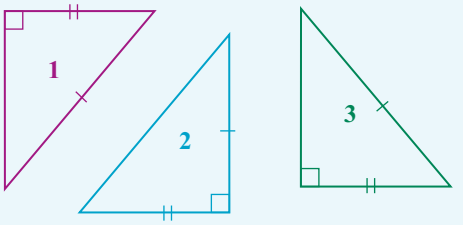
C AAS **D** RHS



6 Which triangles are congruent?

A 1 and 2 **B** 2 and 3

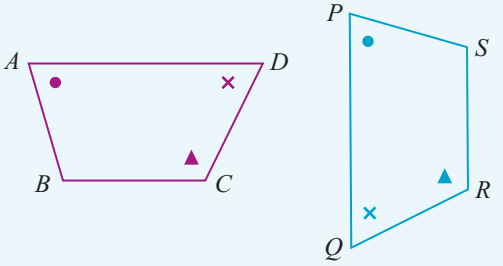
C 1 and 3 **D** 1, 2 and 3



7 The two figures shown in the diagram are congruent. The side that corresponds to AB is:

A RQ **B** PS

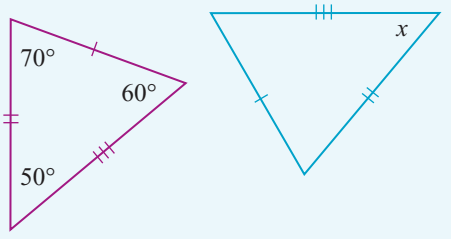
C PQ **D** QR



8 The value of the pronumeral in this pair of congruent triangles is:

A 50° **B** 60°

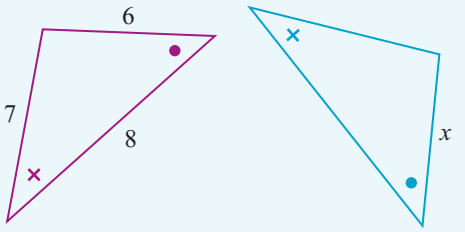
C 70° **D** 110°



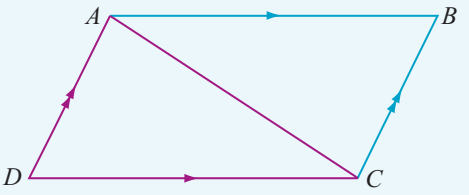
9 The value of the pronumeral in this pair of congruent triangles is:

A 5 **B** 6

C 7 **D** 18

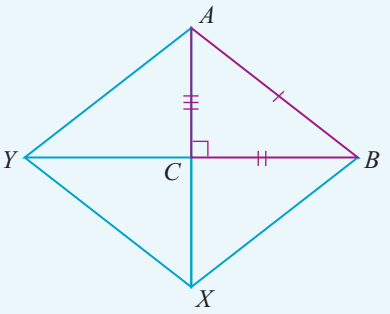


10 $ABCD$ is a parallelogram with the diagonal AC drawn. $AB \parallel DC$ and $AD \parallel BC$. In $\triangle ABC$ and $\triangle CDA$, which of the following statements is not true?



A AC is common
B $\angle BAC = \angle DCA$
C $\angle BCA = \angle DCA$
D $\angle BCA = \angle DAC$

Use the following information to answer questions **11** and **12**.
$\triangle ABC$ has been reflected in CB to give $\triangle CBX$.
$\triangle CBX$ has been reflected in CX to give $\triangle CXY$.
$\triangle CXY$ has been reflected in CY to give $\triangle CYA$.



11 The quadrilateral $ABXY$ is a:

A kite
B rhombus
C square
D rectangle

12 The property of this quadrilateral that is *not* demonstrated by these transformations is:

A the diagonals bisect the angles of the quadrilateral
B the diagonals are perpendicular
C the diagonals are equal
D the diagonals bisect each other

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

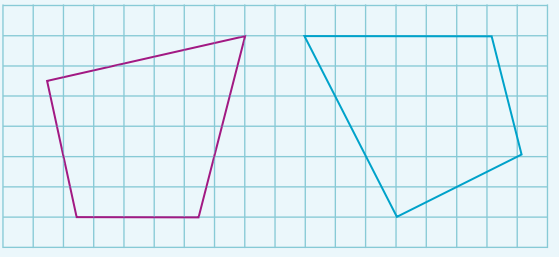
Question	1–3	4–9	10–12
Section	A	C	D

3A Review set

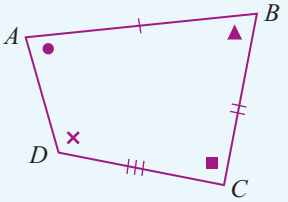
1 What are the minimum conditions necessary for:

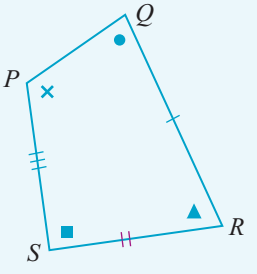
a any two shapes to be congruent?
b any two polygons to be congruent?
c two triangles to be congruent?

2 By measurement, determine whether this pair of figures is congruent.



Use this diagram to answer questions **3** and **4**.



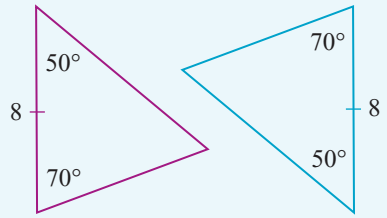


3 Write a correct statement of congruency for this pair of congruent figures.

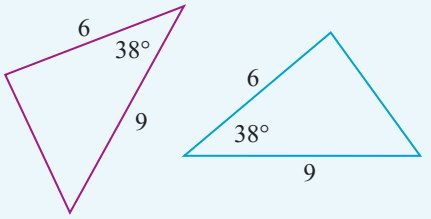
4 **a** Name the side that corresponds to AB.
b Name the angle that corresponds to $\angle C$.

5 State the congruency test used to show that each pair of triangles is congruent.

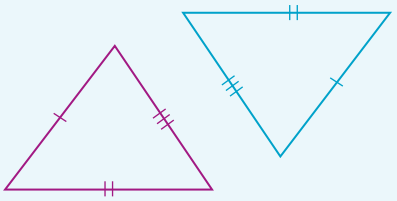
a



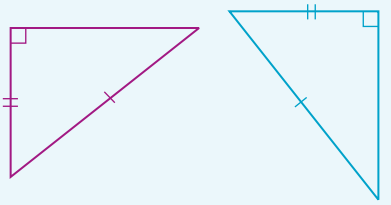
b



c

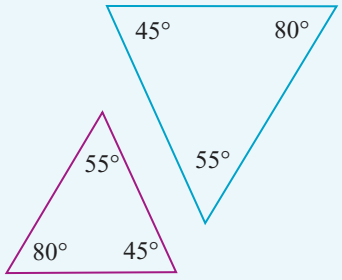


d

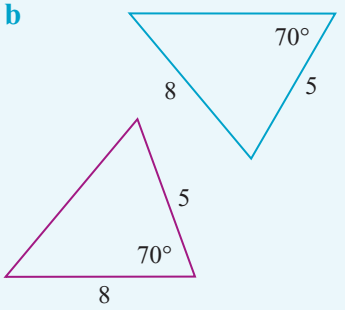


6 State why the following pairs of triangles are not congruent.

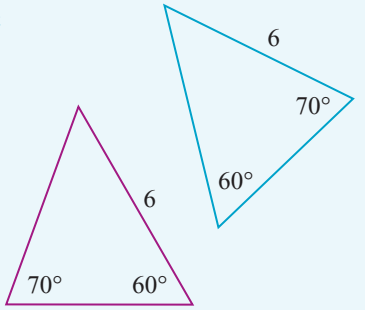
a



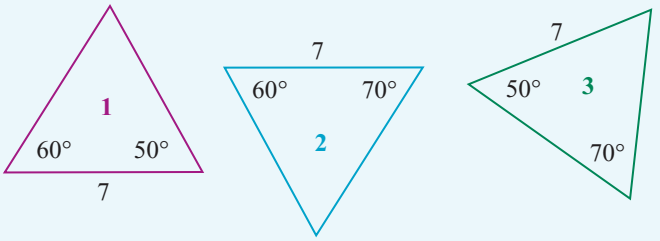
b



c

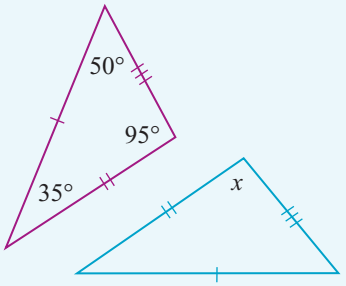


7 Which triangles are congruent?

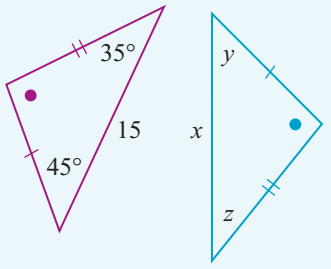


8 State why the following pairs of triangles are congruent and find the value of the pronumeral.

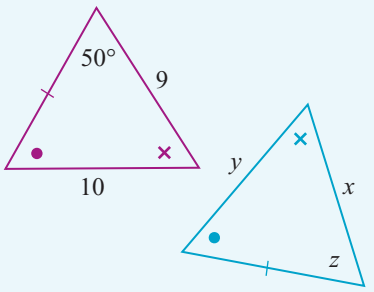
a



b



c



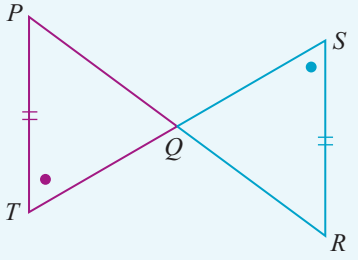
9 PR and ST are straight lines that intersect at Q. $PT = SR$ and $\angle PTQ = \angle RSQ$. Complete the proof that:

a $\triangle PQT \equiv \triangle RQS$

In $\triangle PQT$ and $\triangle RQS$

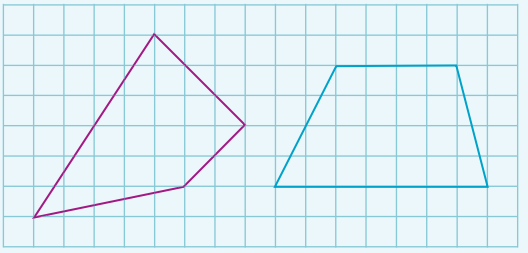
$PT =$ _____	(given)
$\angle PTQ = \angle RSQ$	(_____)
$\angle PQT = \angle RQS$	(_____)
$\therefore \triangle PQT = \triangle RQS$	(_____)

b $PQ = RQ$ (_____)

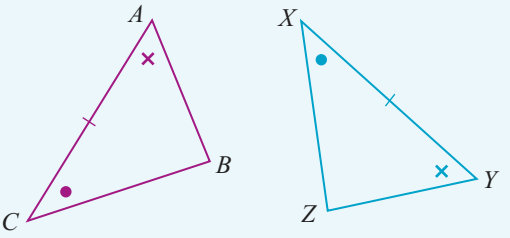


3B Review set

1 By measurement, determine whether this pair of figures is congruent.



Use this diagram to answer questions **2** and **3**.



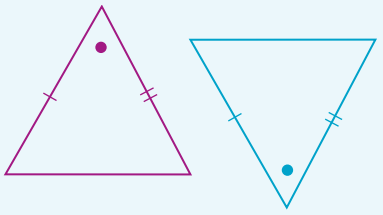
2 Write a correct statement of congruency for the pair of congruent figures.

3 **a** Name the side that corresponds to AB.

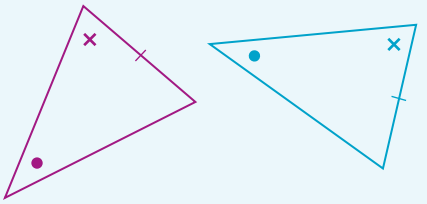
b Name the angle that corresponds to $\angle C$.

4 State the congruency test used to show that each pair of triangles is congruent.

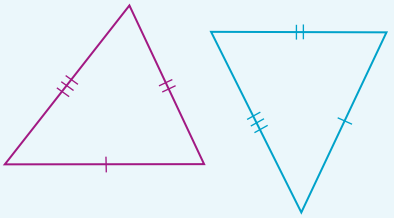
a



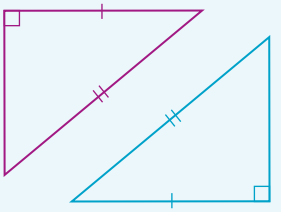
b



c

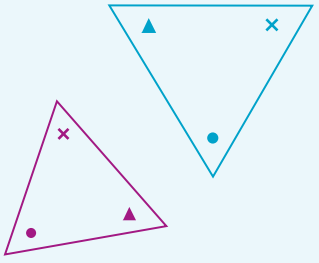


d

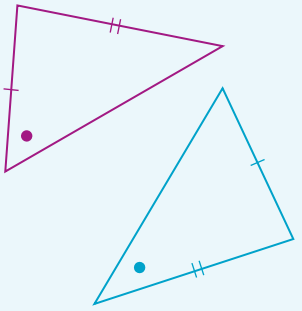


5 State why the following pairs of triangles are not congruent.

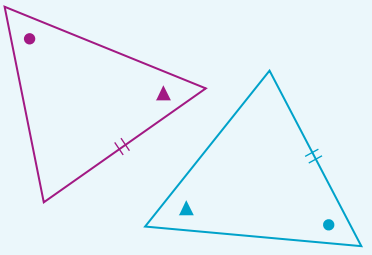
a



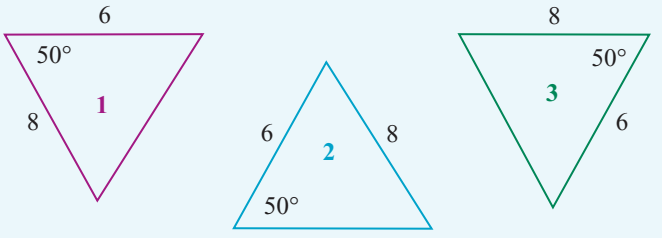
b



c

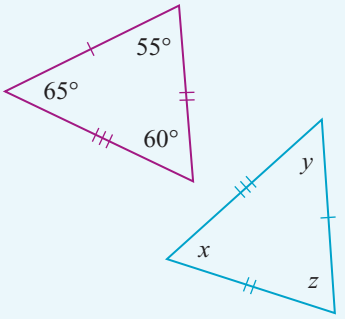


6 Which triangles are congruent?

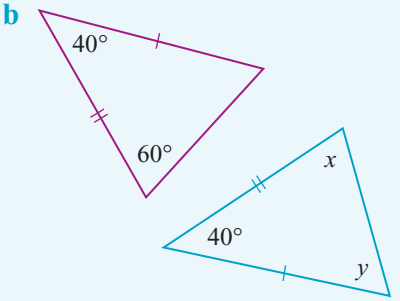


7 State why the following pairs of triangles are congruent and find the value of the pronumeral.

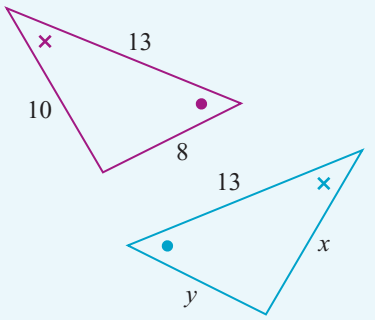
a



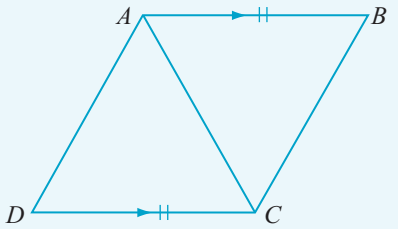
b



c



8 $ABCD$ is a quadrilateral in which $AB \parallel DC$ and $AB = DC$.
Complete the proof that:



a $\triangle ABC \equiv \triangle CDA$

In $\triangle ABC$ and $\triangle CDA$

$AB =$ _____ (_____)

$AC =$ _____ (_____)

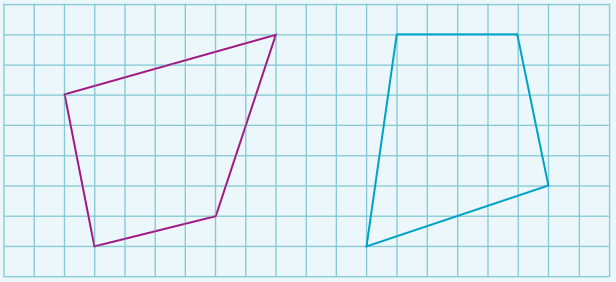
$\angle BAC = \angle DCA$ (_____)

$\therefore \triangle ABC \equiv \triangle CDA$ (_____)

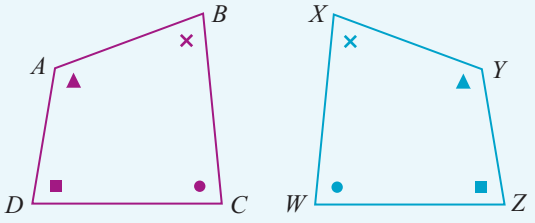
b $AD = BC$ (_____)

3C Review set

1 By measurement, determine whether this pair of figures is congruent.



Use this diagram to answer questions **2** and **3**.



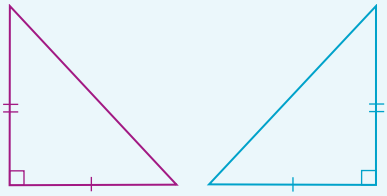
2 Write a correct statement of congruency for this pair of congruent figures.

3 **a** Name the side that corresponds to AB.

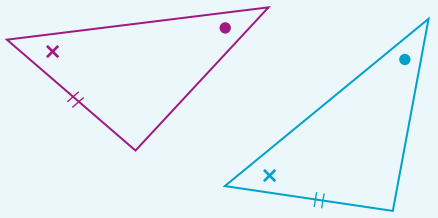
b Name the angle that corresponds to $\angle C$.

4 State the congruency test used to show that each pair of triangles is congruent.

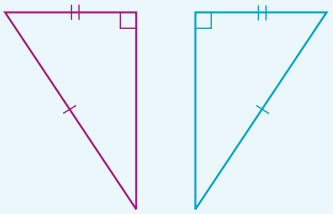
a



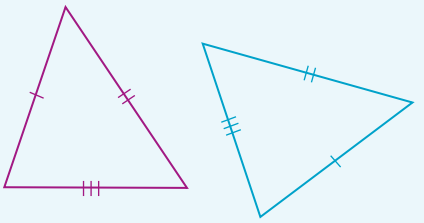
b



c

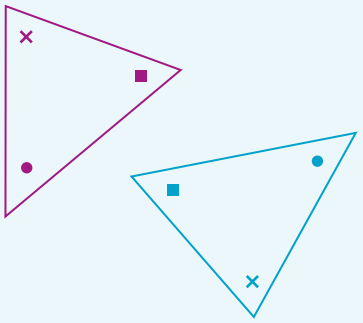


d

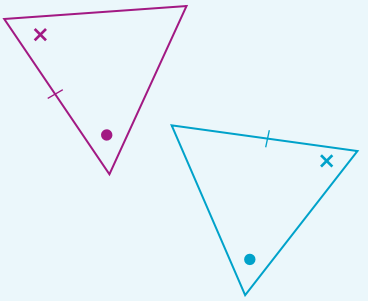


5 State why the following pairs of triangles are not congruent.

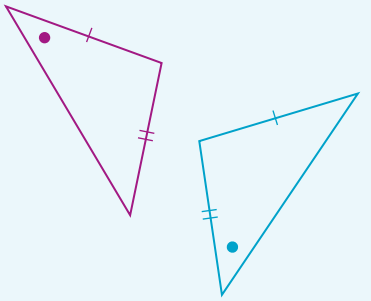
a



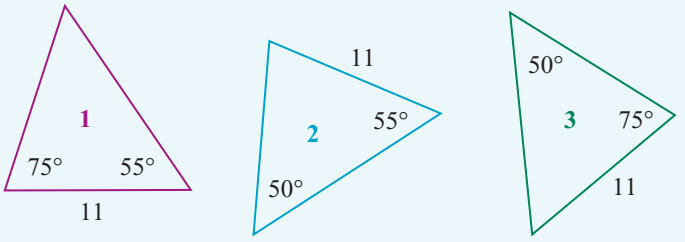
b



c

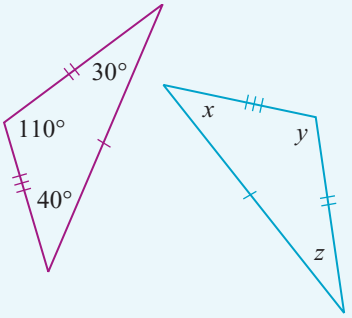


6 Which triangles are congruent?

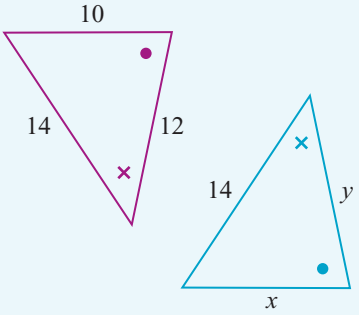


7 State why the following pairs of triangles are congruent and find the value of the pronumeral.

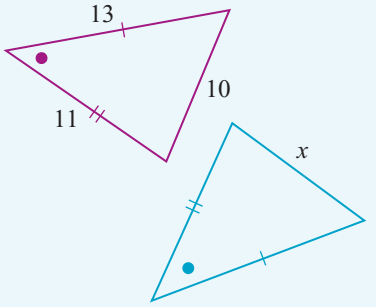
a



b



c

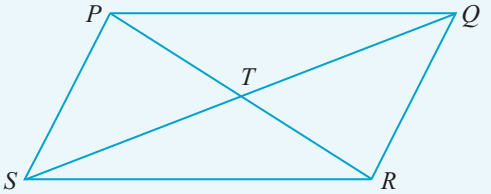


8 Complete the following proof.

Data: $PQRS$ is a parallelogram. T is the point of intersection of the diagonals PR and QS.

Aim: To prove that the diagonals bisect each other.

$PT = RT$ and $QT = ST$.



Proof: In $\triangle PQT$ and $\triangle RST$

$PQ =$ ___	(opposite sides of a parallelogram are equal)
$\angle QPT = \angle$___	(alternate angles and $PQ \parallel SR$)
$\angle PQT = \angle RST$	(_____)
$\therefore \triangle PQT \equiv \triangle RST$	(_____)
$\therefore PT =$ ___	(matching sides of congruent triangles)
and $QT = ST$	(_____)

4

Number operations and index laws

This chapter deals with number operations and indices.

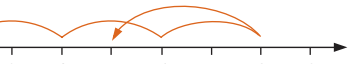
At the end of this chapter you should be able to:

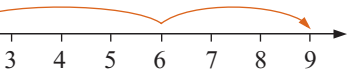
- use the terms *base*, *power*, *index* and *exponent*
- apply the concepts of index laws when multiplying and dividing
- express answers in index form and expanded form
- raise a number to a power
- explain why any base raised to the power of zero is equal to one
- confidently interchange index laws in problem solving
- multiply and divide directed numbers.

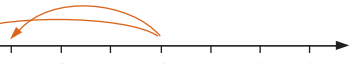
NSW Syllabus references: S4 N&A Indices
Outcomes: MA4–1WM, MA4–2WM, MA4–3WM, MA4–4NA, MA4–9NA
NUMBER & ALGEBRA – ACMNA149, ACMNA182, ACMNA183

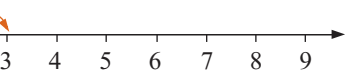
Diagnostic test

1 The operation $2 \times 4 - 3$ on a number line is:

A


B


C


D


2 A deposit of \$15 followed by a withdrawal of \$20 changes the balance by:

A \$5 less **B** \$35 more
C \$20 more **D** \$5 more

3 If up is the positive direction, write directed numbers for a lift going 4 floors up then 7 floors down.

A $+4 + 7$ **B** $+4 - 7$
C $-4 - 7$ **D** $-4 + 7$

4 The numbers $3, -5, 0, -2, -1, 1$ in ascending order are:

A $-5, 3, -2, -1, 1, 0$
B $-2, -1, 0, 1, 3, -5$
C $0, -1, 1, -2, 3, -5$
D $-5, -2, -1, 0, 1, 3$

5 $-5 + 2 =$

A 3 **B** -3 **C** 7 **D** -7

6 $-7 - 2 =$

A -5 **B** 5 **C** 9 **D** -9

7 $-2 - 5 + 6 =$

A 13 **B** -13 **C** -1 **D** 1

8 $4 + (-5) =$

A 9 **B** -9 **C** -1 **D** 1

9 $5 - (-2) =$

A 3 **B** 7 **C** -7 **D** -3

10 $3 - (-4) - 2 =$

A 5 **B** -5 **C** -3 **D** 3

11 $142 + 369 =$

A 511 **B** 510 **C** 610 **D** 227

12 $47 - 19 =$

A 66 **B** 38 **C** 32 **D** 28

13 $15 - 2 - 6 + 3 =$

A 10 **B** 14 **C** 26 **D** 2

14 $63 \times 49 =$

A 441 **B** 3087 **C** 819 **D** 2427

15 $203 \div 7 =$

A 29 **B** 28 **C** $2\frac{3}{7}$ **D** 1421

16 $683 \div 7 =$

A 4781 **B** $98\frac{2}{7}$ **C** $97\frac{4}{7}$ **D** 97

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA182 and ACMNA183.

A Index notation

Index notation is a shorthand way of writing the repeated product of numbers. As $3 \times 3 = 9$, we say that 3 squared equals 9. The 3 squared is written:

Product means '×' or 'to multiply'.

$$3^{2}$$ ← Index, power or exponent
← Base

This is called index notation. The index, power or exponent tells us how many times the base has been repeated.

EXAMPLE 1

a Write $5 \times 5 \times 5 \times 5 \times 5 \times 5$ in index form.
b Write the answer for part **a** in words.
c State which number is the base and which is the index.

a 5 is repeated 6 times. $\therefore 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$
b 5^6 means '5 to the power of 6'.
c The base is 5 and the index is 6.

Exercise 4A

1 Complete the following.

a Write $2 \times 2 \times 2 \times 2 \times 2$ in index form.
$2 \times 2 \times 2 \times 2 \times 2 = 2^{\square}$

Index, power and exponent all mean the same thing.

b Write the answer from part **a** in words.
____ means '____ to the power of _____'.

c State which number is the base and which is the index.
The base is ____ and the index is ____.

2 **a** Write $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ in index form.
b Write the answer from part **a** in words.
c State which number is the base and which is the index.

3 **a** Write the following in index form.
i $4 \times 4 \times 4$ **ii** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
b Write the answer from part **a** in words.
c State which number is the base and which is the power.

4 **a** Write the following in index form.
i $6 \times 6 \times 6 \times 6 \times 6 \times 6$ **ii** $10 \times 10 \times 10 \times 10$
b Write the answer from part **a** in words.
c State which number is the base and which is the exponent.

5 Write the following products in index form.
a $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $10 \times 10 \times 10$
c 12×12 **d** $6 \times 6 \times 6 \times 6$ **e** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

EXAMPLE 2

a Write $p \times p \times p \times p$ in index form.
b Write the answer for part **a** in words.
c State which part is the base and which is the index.

a p is repeated 4 times. $\therefore p \times p \times p \times p = p^4$
b p^4 means p to the power of 4.
c The base is p and the index is 4.

6 Complete the following.

a Write $k \times k \times k \times k \times k$ in index form. $k \times k \times k \times k \times k = k^{\square}$
b Write the answer from part **a** in words. ____ means '____ to the power of _____'.
c State which number is the base and which is the index. The base is ____ and the index is ____.

7 **a** Write the following in index form.

i $w \times w \times w \times w$ **ii** $a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

b Write the answer for part **a** in words.
c State which part is the base and which is the index.

EXAMPLE 3

Write the following in expanded form.
a 4^3 **b** h^5

Expanded form means as a repeated product.

a Base is 4 and index is 3. $\therefore$ 4 is written out 3 times.
4^3 in expanded form is $4 \times 4 \times 4$.
b Base is h and index is 5. $\therefore h$ is written out 5 times.
h^5 in expanded form is $h \times h \times h \times h \times h$.

8 Complete to write the following in expanded form.

a $5^6 =$ ___ × ___ × ___ × ___ × ___ × ___
b $12^3 =$ ___ × ___ × ___
c $j^5 =$ ___ × ____ × ___ × ___ × ___
d $v^7 =$ ___ × ___ × ___ × ___ × ___ × ___× ___

9 Write the following in expanded form.

a 1^6	**b** 7^8	**c** 6^3	**d** 2^4
e 10^2	**f** 3^2	**g** 8^5	**h** 4^5
i l^7	**j** t^2	**k** f^4	**l** a^9
m y^5	**n** d^6	**o** k^3	**p** m^8

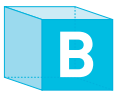
EXAMPLE 4

Use your calculator to evaluate 5^3.

5 [x^y] or [^] 3 [=] 125 *Check:* $5^3 = 5 \times 5 \times 5 = 125$

10 Use your calculator to evaluate the following.

a 3^6	**b** 5^7	**c** 4^5	**d** 8^3	**e** 9^8
f 10^3	**g** 7^4	**h** 2^8	**i** 6^4	**j** 11^3
k 1.6^4	**l** 3.8^3	**m** 4.5^4	**n** 7.4^2	**o** 6.2^3



B Multiplying numbers with the same base

EXAMPLE 1

a Write the following in expanded form.
i 3^2 **ii** 3^4 **iii** $3^2 \times 3^4$
b Write the answer for part **iii** in index form.
c Does $3^2 \times 3^4 = 3^{2+4}$?

a **i** $3^2 = 3 \times 3$
ii $3^4 = 3 \times 3 \times 3 \times 3$
iii $3^2 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
b $3^2 \times 3^4$ in index form $= 3^6$ The base, 3, is repeated 6 times.
c Yes, $3^2 \times 3^4 = 3^{2+4} = 3^6$

Exercise 4B

1 **a** Write the following in expanded form.
i $5^2 =$ ___ $\times$ ___
ii $5^7 =$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___
iii $5^2 \times 5^7 =$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___
b Write the answer to part **iii** in index form.
$5^2 \times 5^7$ in index form = ___
c Does $5^2 \times 5^7 = 5^{2+7}$? Explain.

2 **a** Write the following in expanded form.
i 7^3 **ii** 7^4 **iii** $7^3 \times 7^4$
b Write the answer to part **iii** in index form.
c Does $7^3 \times 7^4 = 7^{3+4}$?

3 **a** Write the following in expanded form.
i 6^3 **ii** 6^5 **iii** $6^3 \times 6^5$
b Write the answer to part **iii** in index form.
c Does $6^3 \times 6^5 = 6^{3+5}$?

4 **a** Write the following in expanded form.
i 10^6 **ii** 10^5 **iii** $10^6 \times 10^5$
b Write the answer in part **iii** in index form.
c Does $10^6 \times 10^5 = 10^{6+5}$?

EXAMPLE 2

Write the following in index form.

a $5^2 \times 5^4$ **b** $2^3 \times 2^7$

a $5^2 \times 5^4$

$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^{2+4} = 5^6$

Count the number of 5s.

b $2^3 \times 2^7$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{3+7} = 2^{10}$

Count the number of 2s.

5 Simplify the following by writing in index form.

a $8^4 \times 8^{10} = 8^{\square + 10} = 8^{\square}$ **b** $7^7 \times 7^2 = 7^{7+\square} = 7^{\square}$

c $9^7 \times 9^3 = 9^{\square + \square} = 9^{\square}$ **d** $5^6 \times 5^{11} = 5^{\square + \square} = 5^{\square}$

6 Simplify the following by writing in index form.

a $3^5 \times 3^4$ **b** $2^7 \times 2^5$ **c** $7^2 \times 7^8$ **d** $5^7 \times 5^2$ **e** $4^{10} \times 4^6$

f $6^9 \times 6^4$ **g** $10^5 \times 10^4$ **h** $2^{10} \times 2^{10}$ **i** $5^{20} \times 5^{10}$ **j** $3^{11} \times 3^7$

k $3^4 \times 3^6$ **l** $7^5 \times 7^8$ **m** $2^4 \times 2^4$ **n** $8^9 \times 8^{12}$ **o** $3^{14} \times 3^3$

7 Can you see a rule emerging? Complete the following statement.
Choose from these words: base, add, indices, multiplying.
When _____ numbers with the same _____, _____ the _____.
Construct your own example to explain the rule.

Look at question **6** to determine the rule.

8 **a** Write 4^3 in expanded form.
b Write $4^3 \times 4$ in expanded form.
c Write your answer for part **b** in index form.
d Is $4^3 \times 4 = 4^3 \times 4^1$? Explain.
e Hence, is 4 the same as 4^1?

9 Simplify by writing the following in index form.

a $5^4 \times 5$ **b** $3^7 \times 3$ **c** $2^9 \times 2$ **d** 5×5^8 **e** 7×7^{11}

EXAMPLE 3

a Write each in expanded form and hence find its value.
i $2^3 \times 3^4$ **ii** 6^7
b Is $2^3 \times 3^4 = 6^{3+4} = 6^7$? Use working to give a reason.

a **i** $2^3 \times 3^4 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 8 \times 81 = 648$

Bases must be the same to add indices.

ii $6^7 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 279\,936$

b No, as bases are different: $648 \neq 279\,936$

10 **a** Complete the following to write in expanded form and find the value of:
i $3^5 \times 4^2 =$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $\times 4 \times 4 =$ _____
ii $12^7 = 12 \times 12 \times$ ___ $\times$ ___ $\times$ ___ $\times 12 \times 12 = 35\,831\,808$
b Is $3^5 \times 4^2 = 12^7$? Use your answers in part **a** to explain your answer.
c Complete the statement:
You can only _____ indices if the _____ are the _____.

11 **a** Write the following in expanded form.

i $5^3 \times 2^5$ **ii** 10^8

b Is $5^3 \times 2^5 = 10^{3+5}$? Show your working and give a reason.

12 **a** Write the following in expanded form.

i $3^5 \times 3^2$ **ii** 9^7

b Is $3^5 \times 3^2 = 9^{5+2}$? Show your working and give a reason.

c What is the correct answer, in index form, for $3^5 \times 3^2$?

Remember that the base must stay the same.

13 **a** Write the following in expanded form.

i $2^3 \times 2^5$ **ii** 4^8

b Is $2^3 \times 2^5 = 4^{3+5}$? Show your working and give a reason.

c What is the correct answer in index form for $2^3 \times 2^5$?

14 **a** Write the following in expanded form.

i $5^6 \times 5^4$ **ii** 25^{10}

b Is $5^6 \times 5^4 = 25^{6+4}$? Show your working and give a reason.

c What is the correct answer in index form for $5^6 \times 5^4$?

15 Write true or false.

a $3^7 \times 3^5 = 3^{12}$ **b** $3^7 \times 2^5 = 6^{12}$ **c** $3^7 \times 3^5 = 9^{12}$

d $2^5 \times 2^4 = 2^9$ **e** $2^5 \times 5^4 = 10^9$ **f** $2^5 \times 2^4 = 4^9$

g $4^6 \times 4^7 = 4^{13}$ **h** $4^6 \times 3^7 = 12^{13}$ **i** $4^6 \times 4^7 = 16^{13}$

j $5^4 \times 3^6 = 15^{10}$ **k** $7^5 \times 7^6 = 49^{11}$ **l** $10^7 \times 10^8 = 10^{15}$

m $6^3 \times 2^5 = 12^8$ **n** $4^{10} \times 4^{10} = 4^{20}$ **o** $5^2 \times 5^9 = 25^{11}$

EXAMPLE 4

Simplify the following by writing in index form.

a $2^3 \times 2^5 \times 2^4$ **b** $3^5 \times 3^6 \times 3^3$

Remember: You can add indices if the bases are the same.

a $2^3 \times 2^5 \times 2^4 = 2^{3+5+4} = 2^{12}$

b $3^5 \times 3^6 \times 3^3 = 3^{5+6+3} = 3^{14}$

16 Complete the following to write the answer in index form.

a $4^2 \times 4 \times 4^5 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{2+1+5} = 4^{\square}$

b $6^2 \times 6^4 \times 6^3 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{\square+4+\square} = 6^{\square}$

c $10^5 \times 10^2 \times 10^8$ **d** $2^7 \times 2^{13} \times 2 \times 2^3$

e $p^3 \times p^6 \times p^2$ **f** $m^7 \times m^4 \times m^2 \times m^5$

Letters can also be a base.

17 Simplify the following by writing in index form.

a $2^4 \times 2^6 \times 2^3$ **b** $3^8 \times 3^3 \times 3^7$ **c** $5^3 \times 5^7 \times 5^4$ **d** $9^2 \times 9^5 \times 9^4$

e $4^6 \times 4^3 \times 4$ **f** $a^4 \times a^5 \times a^2$ **g** $y^7 \times y^3 \times y^2$ **h** $n^9 \times n^8 \times n^2$

i $p^6 \times p^3 \times p^{11}$ **j** $t^4 \times t^1 \times t^2$

18 Summary of findings: Complete each statement and copy it into your exercise book.
Choose from these words: 1, index, letters, bases, add.

a Only _____ indices if the _____ are the same.

b Indices is the plural of the word _____.

c Bases can be numbers or _____.

d A single digit or letter has an index value of _____.

C Raising a number to a power

EXAMPLE 1

Simplify each of the following by writing it in expanded form. Record your findings in index form.

a $(3^2)^3$ **b** $(7^3)^5$

a $(3^2)^3 = (3 \times 3)^3$
$= (3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 3^6$

b $(7^3)^5 = (7 \times 7 \times 7)^5$
$= (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7)$
$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
$= 7^{15}$

Exercise 4C

1 Simplify each of the following by writing in expanded form. Record your findings in index form.

a $(4^4)^3 = (4 \times \_\_\_ \times \_\_\_ \times 4)^3$
$= (4 \times \_\_\_ \times \_\_\_ \times 4) \times (4 \times \_\_\_ \times \_\_\_ \times 4) \times (4 \times \_\_\_ \times \_\_\_ \times 4)$
$= 4^{\square}$

b $(8^3)^2 = (8 \times \_\_\_ \times \_\_\_)^2$
$= (8 \times \_\_\_ \times \_\_\_) \times (8 \times \_\_\_ \times \_\_\_)$
$= 8^{\square}$

c $(2^5)^3 = (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_)^3$
$= (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_) \times (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_) \times (2 \times 2 \times 2 \times \_\_\_ \times \_\_\_)$
$= 2^{\square}$

d $(7^3)^5 = (7 \times 7 \times \_\_\_)^5$
$= (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_) \times (7 \times 7 \times \_\_\_)$
$= 7^{\square}$

2 Can you see a rule emerging?

a Review your answers for question **1**.
$(4^4)^3 = 4^{\square}$ $(8^3)^2 = 8^{\square}$ $(2^5)^3 = 2^{\square}$ $(7^3)^5 = 7^{\square}$
Write the rule in your own words.

b Complete the rule below based on your findings from part **a**. Copy it into your exercise book.
When raising a number to a higher power, _____ the indices.

3 Write each of the following in index form by applying the rule.

a $(3^2)^3$	**b** $(5^3)^2$	**c** $(2^3)^4$	**d** $(3^5)^3$	**e** $(7^4)^5$
f $(10^2)^5$	**g** $(4^2)^6$	**h** $(6^3)^7$	**i** $(3^8)^3$	**j** $(2^7)^{10}$
k $(3^4)^5$	**l** $(3^5)^4$	**m** $(5^2)^4$	**n** $(5^4)^2$	**o** $(9^{15})^2$

4 Write each of the following in index form by applying the rule.

a $(a^3)^2$	**b** $(b^6)^9$	**c** $(c^7)^6$	**d** $(d^5)^{11}$	**e** $(e^4)^{10}$
f $(f^2)^7$	**g** $(g^9)^4$	**h** $(h^{11})^3$	**i** $(i^4)^5$	**j** $(j^4)^2$
k $(k^7)^8$	**l** $(l^5)^8$	**m** $(m^6)^6$	**n** $(n^9)^3$	**o** $(o^9)^7$

EXAMPLE 2

Simplify the following by writing in index form.

a $(3^2)^3 \times 3^5$ **b** $7^4 \times (7^5)^6$

a $(3^2)^3 \times 3^5 = 3^6 \times 3^5$
$= 3^{11}$

b $7^4 \times (7^5)^6 = 7^4 \times 7^{30}$
$= 7^{34}$

5 Simplify the following by writing in index form.

a $(8^4)^2 \times 8^3 = 8^8 \times 8^{\square}$
$= 8^{\square}$

b $10^9 \times (10^2)^6 = 10^9 \times 10^{\square}$
$= 10^{\square}$

c $(7^4)^3 \times 7^2$ **d** $(5^5)^2 \times 5^4$ **e** $(9^2)^4 \times 9^3$ **f** $(3^6)^5 \times 3^3$

g $8^5 \times (8^3)^2$ **h** $6^2 \times (6^4)^4$ **i** $10^3 \times (10^2)^5$ **j** $4^8 \times (4^7)^{10}$

6 Simplify the following.

a $7^6 \times (7^2)^5$ **b** $(2^3)^2 \times (2^4)^3$ **c** $(5^2)^4 \times (5^3)^2$ **d** $(7^4)^2 \times (7^3)^3$

e $(3^4)^5 \times (3^2)^4$ **f** $(9^2)^5 \times (9^3)^4$ **g** $(4^6)^2 \times (4^7)^3$ **h** $(8^3)^5 \times (8^7)^7$

D Dividing numbers with the same base

EXAMPLE 1

a Write $6^8 \div 6^5$ in expanded form.

b Write your answer in index form.

a $6^8 \div 6^5 = \frac{6^8}{6^5} = \frac{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6 \times 6 \times 6}$
$= 6 \times 6 \times 6$

b Index form $= 6^3$

Exercise 4D

1 Complete the following to write each in expanded form. Express your answer in index form.

a $4^7 \div 4^3 = \frac{4 \times 4 \times 4 \times \square \times \square \times \square \times \square}{4 \times 4 \times 4}$
$= 4^{\square}$

b $9^6 \div 9^2 = \frac{9 \times 9 \times \square \times \square \times \square \times \square}{9 \times 9}$
$= 9^{\square}$

c $5^8 \div 5^5 = \frac{5 \times 5 \times 5 \times 5 \times 5 \times \square \times \square \times \square}{5 \times 5 \times 5 \times 5 \times 5}$
$= \_\_\_^{\square}$

Simplify your answers by cancelling.

d $2^{10} \div 2^6 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times \square \times \square \times \square \times \square}{2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \_\_\_^{\square}$

2 **a** Write $3^7 \div 3^2$ in expanded form. **b** Write your answer in index form.
c Is $3^7 \div 3^2 = 3^{7-2}$? Explain your answer.

3 **a** Write $7^5 \div 7^3$ in expanded form. **b** Write your answer in index form.
c Is $7^5 \div 7^3 = 7^{5-3}$? Explain your answer.

4 **a** Write $2^8 \div 2^3$ in expanded form. **b** Write your answer in index form.
c Is $2^8 \div 2^3 = 2^{8-3}$? Explain your answer.

5 **a** Write $9^8 \div 9^7$ in expanded form. **b** Write your answer in index form.
c Is $9^8 \div 9^7 = 9^{8-7}$? Explain your answer.

6 Can you see a rule emerging?

a Review your answers for questions **2** to **5**.

$3^7 \div 3^2 = 3^{\square}$ $7^5 \div 7^3 = 7^{\square}$ $2^8 \div 2^3 = 2^{\square}$ $9^8 \div 9^7 = 9^{\square}$

Write the rule in your own words.

b Complete the rule below based on your findings from part **a**. Copy it into your exercise book.

When ___ numbers with the same base ___ the indices.

7 Write in index form by applying the rule.

a $3^5 \div 3^3$	**b** $2^8 \div 2^5$	**c** $5^{10} \div 5^4$	**d** $4^9 \div 4^5$	**e** $10^{12} \div 10^7$
f $6^8 \div 6^4$	**g** $2^8 \div 2^6$	**h** $3^{11} \div 3^9$	**i** $4^7 \div 4^6$	**j** $5^4 \div 5^3$
k $3^5 \div 3^1$	**l** $5^6 \div 5^1$	**m** $2^7 \div 2$	**n** $10^4 \div 10$	**o** $7^9 \div 7$

8 Write in index form by applying the rule.

a $m^7 \div m^4$	**b** $w^{10} \div w^9$	**c** $g^7 \div g^5$	**d** $z^5 \div z^3$	**e** $p^2 \div p$
f $d^6 \div d^3$	**g** $a^8 \div a^3$	**h** $r^4 \div r^2$	**i** $b^{11} \div b^9$	**j** $h^9 \div h^4$
k $s^5 \div s^2$	**l** $t^6 \div t^2$	**m** $v^4 \div v^2$	**n** $e^8 \div e^3$	**o** $l^9 \div l^8$

EXAMPLE 2

Answer true or false. Does $6^5 \div 2^3 = 3^2$?

$$6^5 \div 2^3 = \frac{6 \times 6 \times 6 \times 6 \times 6}{2 \times 2 \times 2}$$
$$= 3 \times 3 \times 3 \times 6 \times 6$$
$$= 972$$

$3^2 = 3 \times 3 = 9$

False: $6^5 \div 2^3 \neq 3^2$ as 972 does not equal 9.

9 Complete the following to show if the answer is true or false.

a Does $9^4 \div 3^2 = 3^2$?

$$9^4 \div 3^2 = \frac{9 \times 9 \times \square \times \square}{3 \times 3}$$
$$= 729$$

$3^2 =$ ___ $\times$ ___ $= 9$

_____, as 729 ___ 9.

b Does $5^7 \div 5^3 = 5^4$?

$$5^7 \div 5^3 = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$$
$$= \_\_\_^{\square} = \_\_\_$$

$5^4 =$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $=$ ___

_____, as ___ ___ ___.

10 Can you see a rule emerging? Complete the following statement and copy it into your exercise book.
Choose from these words: subtract, dividing, same.
When _____, you can only _____ indices if the bases are the _____.

11 Write true or false.

Look at the base in the answer.

a $5^8 \div 5^5 = 5^3$	**b** $6^7 \div 2^4 = 3^3$	**c** $5^9 \div 5^3 = 1^6$	**d** $4^{10} \div 4^2 = 4^8$
e $6^5 \div 3^2 = 2^3$	**f** $2^6 \div 2^2 = 1^4$	**g** $3^9 \div 3^3 = 1^6$	**h** $10^7 \div 5^3 = 2^4$
i $8^{10} \div 2^4 = 4^6$	**j** $7^{12} \div 7^4 = 7^8$	**k** $3^9 \div 3^4 = 1^5$	**l** $10^{12} \div 5^4 = 2^8$
m $6^{15} \div 6^8 = 6^7$	**n** $2^{10} \div 2^6 = 1^4$	**o** $4^7 \div 4^6 = 4$	**p** $5^{10} \div 5^9 = 1^1$

EXAMPLE 3

Simplify the following.

a $3^5 \times 3^7 \div 3^8$ **b** $(2^8)^3 \div 2^{10}$

Working from left to right.

a $3^5 \times 3^7 \div 3^8 = 3^{5+7} \div 3^8$
$= 3^{12} \div 3^8$
$= 3^{12-8}$
$= 3^4$

b $(2^8)^3 \div 2^{10} = 2^{8\times 3} \div 2^{10}$
$= 2^{24} \div 2^{10}$
$= 2^{24-10}$
$= 2^{14}$

12 Simplify the following.

a $3^6 \times 3^4 \div 3^5$	**b** $2^5 \times 2^8 \div 2^6$	**c** $7^{10} \times 7^8 \div 7^{16}$	**d** $(5^6)^3 \div 5^{10}$
e $(4^5)^2 \div 4^7$	**f** $(3^3)^5 \div 3^7$	**g** $5^8 \div 5^3 \times 5^4$	**h** $3^6 \div 3^3 \times 3^4$
i $2^{10} \div 2^4 \times 2^3$	**j** $5^{12} \times 5^2 \div 5^4$	**k** $(2^4)^3 \div 2^9$	**l** $3^{10} \div 3^8 \times 3^5$
m $2^3 \times 2^6 \times 2^4 \div 2^7$	**n** $(5^2)^3 \times (5^4)^2 \div 5^{11}$	**o** $3^{20} \div 3^8 \div 3^7$	**p** $7^{25} \div (7^3)^5$

E The zero index

Any number raised to the power zero is equal to 1.
In general: $a^0 = 1$

EXAMPLE 1

Complete the following table to find the value of 3^0.

3^5	3^4	3^3	3^2	3^1	3^0
243	81	27			

As you read the first row of numbers, you can see that the power of the 3 is going down by one. Each missing number in the second row can be found by dividing the number before it by 3.

3^5	3^4	3^3	3^2	3^1	3^0
243	$243 \div 3 = 81$	$81 \div 3 = 27$	$27 \div 3 = 9$	$9 \div 3 = 3$	$3 \div 3 = 1$

From the table, $3^0 = 1$.

Exercise 4E

1 Complete the following table to find the value of 2^0.

2^5	2^4	2^3	$2^{\square}$	2^1	2^0
32	$32 \div 2 = 16$	$16 \div \_\_\_ = \_\_\_$	$8 \div \_\_\_ = \_\_\_$	$4 \div \_\_\_ = \_\_\_$	$2 \div \_\_\_ = \_\_\_$

2 Complete the following table to find the value of 10^0.

10^5	10^4	$10^{\square}$	$10^{\square}$	$10^{\square}$	$10^{\square}$
100 000	$\_\_\_ \div 10 = \_\_\_$	$\_\_\_ \div 10 = \_\_\_$	$\_\_\_ \div 10 = \_\_\_$	$\_\_\_ \div 10 = \_\_\_$	$\_\_\_ \div 10 = \_\_\_$

3 Complete the following table to find the value of 5^0.

5^5	5^4	5^3	5^2	5^1	5^0
3125	625				

4 Complete the following table to find the value of 4^0.

4^5	4^4	4^3	4^2	4^1	4^0
1024	256				

5 **a** Can you see a rule emerging? Review your answers from questions 1 to 4.

$2^0 = \_\_\_$ $10^0 = \_\_\_$ $5^0 = \_\_\_$ $4^0 = \_\_\_$

Write the rule in your own words.

b Complete the rule below based on your findings from part **a**. Copy it into your exercise book.

Any base raised to the power of ___ is equal to ___.

EXAMPLE 2

a Use the index laws to simplify $7^4 \div 7^4$.

b By writing in expanded form, show that $7^4 \div 7^4 = 1$.

c Hence show that $7^0 = 1$.

a Using the index laws: $7^4 \div 7^4 = 7^{4-4} = 7^0$

b $7^4 \div 7^4 = \dfrac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}} = 1$

c From parts **a** and **b**, $7^0 = 1$.

6 **a** Use the index laws to simplify $7^3 \div 7^3$.

b By writing in expanded form, show that $7^3 \div 7^3 = 1$

c Hence show that $7^0 = 1$.

7 **a** Use the index laws to simplify $9^5 \div 9^5$.

b By writing in expanded form, show that $9^5 \div 9^5 = 1$.

c Hence show that $9^0 = 1$.

8 **a** Use the index laws to simplify $6^2 \div 6^2$.
b By writing in expanded form, show that $6^2 \div 6^2 = 1$.
c Hence show that $6^0 = 1$.

9 Without using your calculator, write down the value of:

a 4^0	**b** $(23)^0$	**c** $(955)^0$	**d** $(8.67)^0$	**e** $(12.9)^0$	**f** $(\frac{3}{4})^0$
g a^0	**h** l^0	**i** m^0	**j** d^0	**k** q^0	**l** e^0

10 Use the x^y or $\wedge$ key on your calculator to find the value of:

a 7^0	**b** 13^0	**c** 29^0	**d** 5.6^0	**e** 31.7^0	**f** $(\frac{5}{8})^0$

Investigation 1 Numbers in index form

1 **a** Expand and evaluate the following.
i $2^3 = 2 \times 2 \times 2 =$ ___ **ii** $3^2 = 3 \times 3 =$ ___
b Which is the greater, 2^3 or 3^2?

2 **a** Expand and evaluate the following.
i $3^4 = 3 \times$ _____ $=$ ___ **ii** $4^3 = 4 \times$ _____ $=$ ___
b Which is the greater, 3^4 or 4^3?

3 **a** Expand and evaluate the following.
i 4^5 **ii** 5^4
b Which is the greater, 4^5 or 5^4?

4 **a** Expand and evaluate the following.
i 5^6 **ii** 6^5
b Which is the greater, 5^6 or 6^5?

5 **a** Predict which is greater: 9^{10} or 10^9?
b Check your prediction by calculation.

6 Which is larger 99^{100} or 100^{99}? Explain you answer.

7 Which is larger 99^{1000} or 100^{909}? Explain you answer.

Summary of index laws

These index laws apply when the bases are the same.

- When multiplying, add indices. $3^7 \times 3^4 = 3^{7+4} = 3^{11}$
- When dividing, subtract indices. $3^7 \div 3^4 = 3^{7-4} = 3^3$
- When raising a number to a power, multiply indices. $(3^7)^4 = 3^{7 \times 4} = 3^{28}$
- Anything to the power of zero $= 1$. $3^0 = 1$

Bases can be numbers or letters.

Exercise 4F

1 Simplify the following by applying the correct index law.

a $5^8 \times 5^2$	**b** $6^7 \times 6^9$	**c** $10^{11} \times 10^4$	**d** $12^6 \div 12^2$
e $8^7 \div 8^2$	**f** $3^6 \div 3^1$	**g** $(4^2)^3$	**h** $(5^3)^6$
i $(9^2)^{11}$	**j** 7^0	**k** 12^0	**l** 5.6^0
m $2^{18} \div 2^{15}$	**n** $(8^4)^3$	**o** $(5^2)^{14}$	**p** $\left(\frac{1}{4}\right)^0$
q $4^9 \times 4^3$	**r** $(59)^0$	**s** $14^{10} \div 14^4$	**t** $(136)^0$
u $17^8 \div 17^5$	**v** $12^7 \times 12^2$	**w** $(6^8)^3$	**x** $9^{15} \times 9^2$

2 Simplify the following by applying the correct index law.

a $(x^4)^3$	**b** $f^7 \times f^3$	**c** $(m^9)^5$	**d** $v^{11} \div v^5$
e t^0	**f** $q^9 \times q^4$	**g** $(b^{11})^5$	**h** $(y^7)^2$
i $(f^8)^2$	**j** $d^{10} \div d^6$	**k** $e^5 \times e^4$	**l** p^0
m w^0	**n** $j^6 \times j^3$	**o** $z^{20} \div z^{10}$	**p** $(a^2)^3 \div a^3$

3 Working from left to right, apply your knowledge of index laws to simplify the following.

a $2^3 \times 2^7 \times 2^6$	**b** $10^8 \div 10^4 \div 10^3$	**c** $4^7 \times 4^3 \div 4^5$	**d** $(5^2)^3 \times 5^4$
e $(2^4)^3 \times 2^2$	**f** $(6^5)^2 \times 6^3$	**g** $(8^2)^2 \times 8^4$	**h** $(9^5)^0$
i $(7^0)^2 \times 3$	**j** $(8^0)^3 \times 4$	**k** $(3^2)^0 \times (5^4)^0$	**l** $(4^5)^2 \times (4^3)^3 \div (4^6)^2$
m $(5^3)^2 \times (5^2)^0 \div (5^2)^3$	**n** $(9^2)^3 \times (9^4)^2 \times 9^0$	**o** $(7^6)^2 \div (7^3)^4 \times 7$	**p** $(6^5)^4 \div (6^3)^3 \times (6^8)^0$

4 Use the x^y or $\wedge$ key on your calculator to evaluate the following.

a 5^3	**b** 7^4	**c** 3^5	**d** 2^9
e 1.2^2	**f** 2.8^3	**g** $2^3 \times 4^2$	**h** $6^2 \times 3^3$
i $5^3 \times 1^4$	**j** $10^3 \times 2^4$	**k** $3^3 \times 10^2$	**l** $2.4^3 \times 7^2$
m $9^3 \div 3^4$	**n** $10^6 \div 5^8$	**o** $7^4 \div 2^3$	**p** $6^3 \div 5^2$

Investigation 2 Last digits in powers

1 Complete the table below. Use the pattern to find the last digits of 3^{2015}.

Power of 3	Expanded notation	Last digit
0		
1		
2	$3 \times 3 = 9$	9
3		
4		
5		
6	$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$	9
7		
8		
9		
10		

2 Complete the table below.

Power of 2	Last two digits
0	1
1	2
2	
3	
4	
5	
6	
7	
8	
9	
10	24
11	
12	
13	
14	

Power of 2	Last two digits
15	
16	
17	
18	
19	88
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	

a There is a pattern that emerges. Use the pattern to find the last two digits in the number 2^{2015}.

b Find the value of the last two digits in $2^{2^{2015}}$.

3 You are asked to evaluate 2^{3^4}.

a Calculate $(2^3)^4$.

b Calculate $2^{(3^4)}$.

c Are the answers to parts **a** and **b** equal in value? Explain if they are different.

d What is the correct answer?

4 You are asked to evaluate 3^{4^5}.

a Calculate $(3^4)^5$.

b Calculate $3^{(4^5)}$.

c Are the answers to parts **a** and **b** equal in value? Explain if they are different.

d What is the correct answer?

Investigation 3 Multiplying and dividing directed numbers

1 Complete the following number patterns.

a
$3 \times 3 =$ ___
$3 \times 2 =$ ___
$3 \times 1 =$ ___
$3 \times 0 =$ ___
$3 \times (-1) =$ ___
$3 \times (-2) =$ ___
$3 \times (-3) =$ ___

b
$-4 \times 3 =$ ___
$-4 \times 2 =$ ___
$-4 \times 1 =$ ___
$-4 \times 0 =$ ___
$-4 \times (-1) =$ ___
$-4 \times (-2) =$ ___
$-4 \times (-3) =$ ___

c
$9 \div 3 =$ ___
$6 \div 3 =$ ___
$3 \div 3 =$ ___
$0 \div 3 =$ ___
$(-3) \div 3 =$ ___
$(-6) \div 3 =$ ___
$(-9) \div 3 =$ ___

d
$-36 \div (-4) =$ ___
$-24 \div (-4) =$ ___
$-12 \div (-4) =$ ___
$0 \div (-4) =$ ___
$12 \div (-4) =$ ___
$24 \div (-4) =$ ___
$36 \div (-4) =$ ___

2 From the number patterns above, complete the following statements.

a When multiplying or dividing two numbers with the *same* sign (both positive or both negative), the answer is always _____.

b When multiplying or dividing two numbers with *different* signs, the answer is always _____.

When multiplying	When dividing
positive × positive = positive	positive ÷ positive = positive
positive × negative = negative	positive ÷ negative = negative
negative × positive = negative	negative ÷ positive = negative
negative × negative = positive	negative ÷ negative = positive

G Multiplying and dividing directed numbers

When multiplying or dividing two directed numbers:

- If the signs are the *same* (positive and positive or negative and negative), the result is *positive*.
- If the signs are *different* (positive and negative or negative and positive), the result is *negative*.

EXAMPLE 1

Simplify the following.

a -2×3 **b** $4 \times (-5)$ **c** $-6 \times (-3)$

a $-2 \times 3 = -6$ **b** $4 \times (-5) = -20$ **c** $-6 \times (-3) = +18$

Exercise 4G

1 Simplify the following.

a -7×5 **b** -3×7 **c** -4×9 **d** -6×3

e $7 \times (-4)$ **f** $8 \times (-3)$ **g** $2 \times (-3)$ **h** $8 \times (-9)$

i $-11 \times (-4)$ **j** $-6 \times (-5)$ **k** $-3 \times (-6)$ **l** $-2 \times (-7)$

m $7 \times (-8)$ **n** $-4 \times (-6)$ **o** -5×8 **p** $-8 \times (-8)$

EXAMPLE 2

Simplify the following.

a $-3 \times 2 \times (-4)$

b $(-3)^3$

c $-5 \times (-2)^2$

The square of a negative number is always positive. The cube of a negative number is always negative.

a $-3 \times 2 \times (-4)$
$= -6 \times (-4)$
$= 24$

b $(-3)^3$
$= -3 \times (-3) \times (-3)$
$= -27$

c $-5 \times (-2)^2$
$= -5 \times 4$
$= -20$

2 Simplify the following.

a $2 \times (-3) \times 5$ **b** $-3 \times (-4) \times 6$ **c** $7 \times (-3) \times 2$

d $-4 \times (-5) \times (-2)$ **e** $(-5)^2$ **f** $(-1)^2$

g $-2 \times 0 \times 4$ **h** $3 \times 3 \times (-5)$ **i** $-3 \times (-2) \times 6$

j $2 \times (-2) \times 5$ **k** $(-2) \times (-1) \times (-3)$ **l** $-1 \times 3 \times (-4)$

m $(-7)^3$ **n** $5 \times (-2) \times (-4)$ **o** $(-2)^2 \times (-6)$

EXAMPLE 3

Simplify the following.

a $36 \div (-9)$ **b** $-30 \div 6$ **c** $-50 \div (-10)$

a $36 \div (-9) = -4$ **b** $-30 \div 6 = -5$ **c** $-50 \div (-10) = 5$

3 Simplify the following.

a $-14 \div 7$ **b** $30 \div (-5)$ **c** $-16 \div (-8)$ **d** $42 \div (-7)$

e $-36 \div (-9)$ **f** $-90 \div (-10)$ **g** $-30 \div 15$ **h** $-22 \div (-11)$

i $12 \div (-3)$ **j** $44 \div (-4)$ **k** $-16 \div 4$ **l** $-30 \div (-15)$

m $-44 \div (-11)$ **n** $25 \div (-5)$ **o** $45 \div (-9)$ **p** $-48 \div (-8)$

4 Find the missing numbers.

a $15 \div \_\_\_ = -3$ **b** $18 \div \_\_\_ = -6$

c $-15 \div \_\_\_ = 5$ **d** $\_\_\_ \div 2 = -4$

e $\_\_\_ \div (-3) = 8$ **f** $\_\_\_ \div (-2) = -5$

g $\_\_\_ \div -4 = -4$ **h** $-10 \div \_\_\_ = -1$

Remember the order of operations:
1 Brackets first.
2 Next multiplication and division from left to right.
3 Then addition and subtraction.

EXAMPLE 4

Evaluate the following.

a $7 - 3 \times 5$ **b** $6 - 4 \times 5 + 2$

a $7 - 3 \times 5$ (multiply first)
$= 7 - 15$
$= -8$

b $6 - 4 \times 5 + 2$ (multiply first)
$= 6 - 20 + 2$
$= -14 + 2 = -12$

5 Simplify the following.

a $9 - 4 \times 3$ **b** $-7 + 2 \times 5$ **c** $8 + 5 - 23$

d $-60 \div 6 \times 4$ **e** $8 \times (-3) \div (-12)$ **f** $44 \div (11 - 15)$

g $(7 - 11) \times (1 - 7)$ **h** $(5 - 30) \div (-15 + 10)$ **i** $8 - 5 \times 3 - 9$

EXAMPLE 5

Evaluate the following.

a $\frac{15 + 9}{4}$ **b** $3 + \frac{15}{3}$ **c** $\frac{-5 + 8}{3} + 2$

a $\frac{15 + 9}{4}$
$= \frac{24}{4} = 6$

b $3 + \frac{15}{3}$
$= 3 + 5 = 8$

c $\frac{-5 + 8}{3} + 2$
$= \frac{3}{3} + 2 = 1 + 2 = 3$

6 Simplify the following.

a $\frac{13+7}{5}$ **b** $\frac{16-6}{2}$ **c** $\frac{15+9}{8}$ **d** $\frac{-4+18}{2}$

e $\frac{-5+11}{3}$ **f** $\frac{7-15}{4}$ **g** $7+\frac{16}{8}$ **h** $4-\frac{12}{3}$

i $9+\frac{14}{7}$ **j** $\frac{-3+12}{3}+3$ **k** $\frac{-6-8}{7}+6$ **l** $\frac{-3-5}{4}-2$

EXAMPLE 6

Use your calculator to evaluate the following.

a $-28-57$

b $-38+5\times(-19)$

c $-243+23\times(-18)$

To make a number negative on the calculator, use the ± key.

a $-28-57=-85$

28 ± − 57 =

or DAL ± 28 − 57 =

b $-38+5\times(-19)=-133$

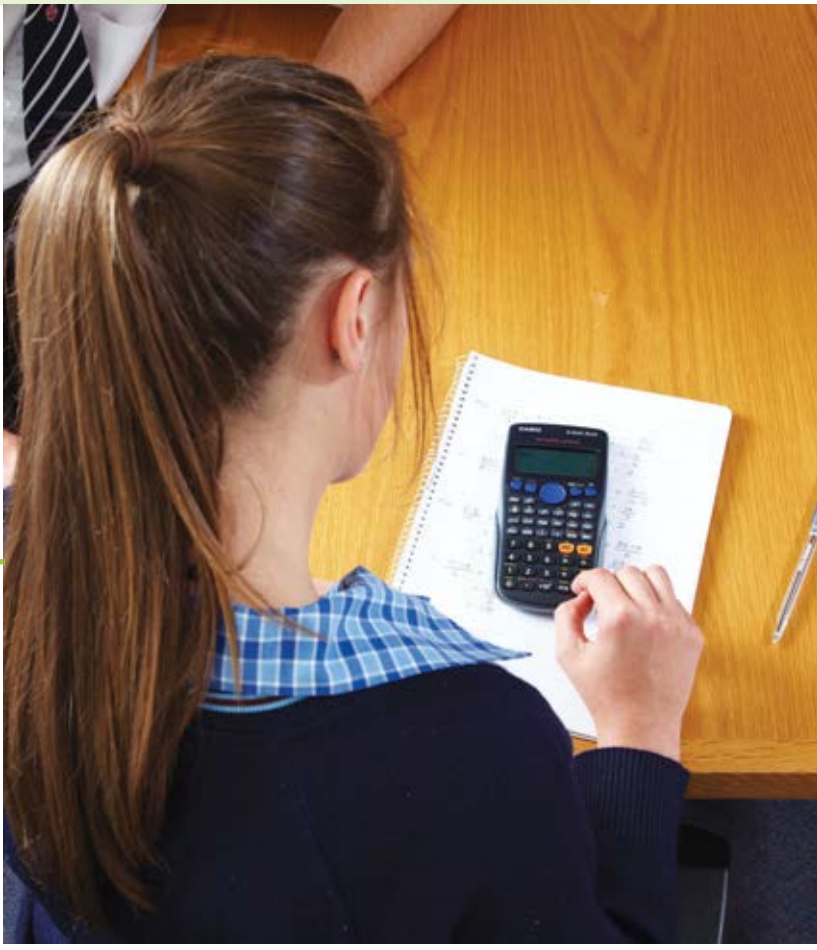
38 ± + 5 × 19 ± =

or DAL ± 38 + 5 × ± 19 =

c $-243+23\times(-18)=-657$

243 ± + 23 × 8 ± =

or DAL ± 243 + 23 × ± 8 =



7 Use your calculator to evaluate the following.

a $28-147$ **b** $-53+27$

c -28×37 **d** -41×38

e $-1484\div14$ **f** $-39-53$

g $18\times(-47)$ **h** $48-211$

i $83-9\times21$ **j** $-28+14\times32$

k $-43-21\times72$ **l** $49+63\times(-28)$

m $(27-35)\times14$ **n** $(63-85)\times(14-26)$

o $(-38-57)\times(-81)$

8 Simplify the following using your calculator.

a $-428+192-845+1291-478+293-482-311$

b $6852-4915-3211-1698-2139+4821$

c $242\times(-83)+(-93)\times45-26\times89$

Language in mathematics

1 Write the following in words.

a 3^5 **b** 8^2 **c** 2^3 **d** z^4

2 Write numerical expressions for these numbers.

a seven squared **b** four cubed

c six to the power five **d** two to the fourth

3 Replace the vowels to make words that mean the same as 'power'.

a _nd_x **b** _xp_n_nt

4 When writing numbers in index notation, what name is given to:

a the number that is being repeated? **b** the number of times it is repeated?

5 Explain why $2^3 \times 2^4 \neq 4^7$.

6 Explain why $2^6 \times 2^3 = 2^9$.

7 Explain why $10^5 \div 10^3 \neq 1^2$.

8 Explain why $4^8 \div 4^5 = 4^3$.

9 Simplify $9^5 \div 9^5$ to prove that any number to a power divided by same number to the same power is equal to 1. Explain your findings.

10 Explain why $(2^3)^3 = 2^9$.

Terms

base	index	power	exponent	explain	indices
simplify	expanded	expression	multiply	divide	raised

Check your skills

1 When written in index form $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ is:

A 5555555 **B** 5^7 **C** 7^5 **D** $(5555555)^7$

2 For the number 8^3:

A The base is 8 and the index is 3. **B** The base is 3 and the index is 8.

C The base is 3 and the index is 24. **D** The base is 8 and the index is 24.

3 6^4 means:

A 6 times 4 **B** 6^5

C 6666 **D** 6 to the power of 4

4 Which two words are also used to describe the term 'index'?

A power, product **B** raised, equivalent

C power, exponent **D** exponent, base

5 In expanded form 1^6 is:

A $1 \times 1 \times 1 \times 1 \times 1 \times 1$ **B** 16

C 6 **D** 1×6

6 When evaluated, 11^2 is:

A 22 **B** 1111 **C** 211 **D** 121

7 When simplified in index form, $3^8 \times 3^8$ is:

A 3^{16} **B** 9^8 **C** 6^{16} **D** 9^{16}

8 When simplified in index form, 5×5^4 is:

A 55^4 **B** 5^5 **C** 5^4 **D** 5^6

9 When simplified in index form, $2^6 \times 2 \times 2^3$ is:

A 6^{10} **B** 2^9 **C** 2^{10} **D** 8^9

10 When simplified in index form, $p^7 \times p^5 \times p^6$ is:

A p^{18} **B** $3p^{18}$ **C** p^6 **D** $3p^3$

11 In expanded form $(3^3)^4$ means:

A $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **B** $33 \times 33 \times 33 \times 33$

C 3^{12} **D** $9 \times 9 \times 9 \times 9$

12 When simplified in index form, $(6^2)^3$ is:

A 12^5 **B** 6^6 **C** 12^6 **D** 36^3

13 When simplified in index form, $(e^4)^7$ is:

A e^{11} **B** e^3 **C** $4e^7$ **D** e^{28}

14 When simplified in index form, $(4^2)^2 \times 4^2$ is:

A 16^8 **B** 4^6 **C** 32^6 **D** 8^6

15 When simplified in index form, $9^5 \div 9^3$ is:

A 9^2 **B** 9^{15} **C** 1^2 **D** 9^9

16 When simplified in index form, $h^{13} \div h^7$ is:

A h^5 **B** h^6 **C** h^{20} **D** h^8

17 When simplified in index form, $5^3 \times 5^2 \div 5^4$ is:

A 5^2 **B** 5^1 **C** 5^{24} **D** 5^4

18 When evaluated $7^0 + 3^0 + p^0$ is:

A p **B** $2p$ **C** 11 **D** 3

19 When evaluated using a calculator, $5^2 \times 8$ is:

A 80 **B** 40 **C** 416 **D** 200

20 When evaluated using a calculator, $9^3 \div 3^2$ is:

A 720 **B** 81 **C** 21 **D** 61

21 $7 \times (-3) =$

A 4 **B** -10 **C** 10 **D** -21

22 $8 \times (-9) =$

A 72 **B** -72 **C** 17 **D** -1

23 $-12 \div 3 =$

A -36 **B** -4 **C** 4 **D** 36

24 $-18 \div (-9) =$

A -2 **B** 2 **C** 27 **D** -27

25 $\_\_\_ \div 2 = -6 \ \therefore \_\_\_ =$

A 3 **B** -3 **C** 12 **D** -12

26 $9 - 5 \times 2 - 3 =$

A 5 **B** 14 **C** -11 **D** -4

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–6	7–10	11–14	15–17	18	19, 20	21–26
Section	A	B	C	D	E	F	G

4A Review set

1 Write the following in index form.

a $2 \times 2 \times 2 \times 2$ **b** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

2 For the following numbers, write the base and index.

a 7^9 **b** 3^{10}

3 Write the following in expanded form.

a 3^5 **b** 7^6

4 Use a calculator to evaluate the following.

a 3^8 **b** 5^6

5 Write true or false.

a $3^4 \times 2^5 = 6^9$ **b** $8^6 \div 4^2 = 2^4$

6 Simplify the following, leaving the answer in index form.

a $5^{12} \times 5^{20}$ **b** $(4^5)^3$ **c** $2^8 \div 2^6$

d $7^5 \times 7$ **e** $5^6 \times 5^7 \div 5^9$

7 Simplify the following, leaving the answer in index form.

a $p^6 \times p^2$ **b** $(m^2)^7$ **c** $f^7 \div f$

d $l^6 \times l$ **e** $w^3 \times w^5 \div w^4$

8 Explain why $5^4 \times 5^3 = 5^7$.

9 Explain why $2^8 \div 2^4 \neq 2^2$.

10 Explain why $4^0 \times 9^0 + 3^0 = 2$.

11 Simplify the following.

a $3 \times (-3) \times 5$ **b** $-5 \times (-4) \times 2$ **c** $-5 \times (-3) \times 3$

12 Simplify the following.

a $-28 \div 7$ **b** $60 \div (-5)$ **c** $-64 \div (-8)$

13 Simplify the following.

a $9 - 2 \times 3$ **b** $-7 + 4 \times 5$ **c** $9 + 15 - 23$

d $-36 \div 6 \times 4$ **e** $6 \times (-8) \div (-12)$ **f** $36 \div (11 - 15)$

4B Review set

1 Write the following in index form.

a $7 \times 7 \times 7 \times 7 \times 7$ **b** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

2 For the following numbers, write the base and index.

a 3^8 **b** 5^2

3 Write the following in expanded form.

a 6^4 **b** 7^3

4 Use a calculator to evaluate the following.

a 2^9 **b** 3^6

5 Simplify the following, leaving the answer in index form.

a $3^{10} \times 3^3$ **b** $(7^2)^6$ **c** $4^{10} \div 4^5$ **d** $6^{12} \div 6$ **e** $(2^5)^4 \times 2^{10}$

6 Write true or false.

a $4^5 \times 2^6 = 8^{11}$ **b** $5^7 \div 5^3 = 1^4$

7 Simplify the following, leaving the answer in index form.

a $c^{11} \times c^5$ **b** $(k^3)^{12}$ **c** $y^8 \div y^2$ **d** $z^9 \times z^0$ **e** $x^{10} \div x^5 \times x^3$

8 Explain why $4^7 \div 4^2 = 4^5$.

9 Explain why $6^6 \times 6^2 \neq 36^8$.

10 Explain why $(3^2)^4 = 3^8$.

11 Simplify the following.

a $4 \times (-3) \times 2$ **b** $-4 \times (-3) \times 4$ **c** $-3 \times (-4) \times 3$

12 Simplify the following.

a $-28 \div (-4)$ **b** $-30 \div (-5)$ **c** $48 \div (-8)$

13 Simplify the following.

a $9 - 2 \times 4$ **b** $-9 + 4 \times 5$ **c** $10 + 15 - 21$

d $-36 \div 4 \times (-5)$ **e** $-9 \times (-8) \div 6$ **f** $36 \div (10 - 6)$

4C Review set

1 Write the following in index form.

a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **b** $10 \times 10 \times 10$

2 For the following numbers, write the base and index.

a 8^4 **b** 12^2

3 Write the following in expanded form.

a 6^2 **b** 5^7

4 Use a calculator to evaluate the following.

a 5^4 **b** 3^7

5 Simplify the following, leaving the answer in index form.

a $9^{12} \times 9^7$ **b** $(2^3)^5$ **c** $5^{12} \div 5^4$

d $6^9 \times 6$ **e** $(2^8)^2 \times (2^{10})^3$

6 Write true or false.

a $7^4 \times 7^4 = 7^{16}$ **b** $9^2 \div 9 = 9$

7 Simplify the following, leaving the answer in index form.

a $j^7 \times j^9$ **b** $n^8 \div n^2$ **c** $(p^9)^3$

d $s^7 \times s^7 \times s$ **e** $9 \times f^0$

8 Explain why $(2^4)^3 \neq 2^7$.

9 Explain why $6^{10} \div 6^4 \times 6 = 6^7$.

10 Explain why $4^3 \div 4 < 4^7 \div 4^3$.

11 Simplify the following.

a $5 \times (-2) \times 3$ **b** $-6 \times (-3) \times 3$ **c** $-4 \times (-2) \times 5$

12 Simplify the following.

a $14 \div (-7)$ **b** $-25 \div (-5)$ **c** $-72 \div (-8)$

13 Simplify the following.

a $19 - 4 \times 3$ **b** $-17 + 4 \times 3$ **c** $19 + 5 - 8$

d $-54 \div 9 \times (-2)$ **e** $-7 \times 6 \div (-3)$ **f** $36 \div (-9) \times (-2)$

4D Review set

1 Write the following in index form.

a $6 \times 6 \times 6 \times 6 \times 6 \times 6$ **b** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

2 For the following numbers, write the base and index.

a 6^8 **b** 3^{10}

3 Write the following in expanded form.

a 4^5 **b** 2^7

4 Use a calculator to evaluate the following.

a 6^4 **b** 2^{12}

5 Simplify the following, leaving the answer in index form.

a $5^{20} \times 5^{20}$ **b** $(3^7)^{10}$ **c** $5^{16} \div 5^{12}$

d $4^{23} \div 4$ **e** $3^{30} \times (3^5)^3$

6 Write true or false.

a $2^4 \times 2^5 = 2^9$ **b** $7^6 \div 7^4 = 7^2$

7 Simplify the following, leaving the answer in index form.

a $p^0 \div k^0$ **b** $15 \times m^0$ **c** $j^0 + k^0 - m^0$

d $(f^2)^0$ **e** $(g^4)^0 + (h^0)^5$

8 Explain why $h^0 + m^0 = 2$.

9 Explain why $(j^5)^0 \times 5 = 5$.

10 Explain why $m^0 + d^0 = 2 \times f^0$.

11 Simplify the following.

a $3 \times (-4) \times 5$ **b** $-3 \times (-5) \times 2$ **c** $-2 \times (-2) \times (-2)$

12 Simplify the following.

a $15 \div (-3)$ **b** $27 \div (-9)$ **c** $(-36) \div 9$

13 Simplify the following.

a $15 - 2 \times 3$ **b** $20 - 4 \times 8$ **c** $17 + 4 \times (-3)$

d $-8 \div 2 \times (-5)$ **e** $-3 \times (-5) \times 6$ **f** $-8 \times (-6) \div 4$

5

Perimeter, area and capacity

This chapter deals with calculations of perimeter and area and conversion of units of volume to capacity.

At the end of this chapter you should be able to:

- find the perimeter of a quadrilateral
- find the area of a trapezium, rhombus and kite
- choose and convert between units of measurement for area, volume and capacity.

NSW Syllabus references: S4 M&G Length, S4 M&G Area, S4 M&G Volume
Outcomes: MA4-1WM, MA4-2WM, MA3-9MG, MA3-10MG, MA3-11MG, MA4-12MG, MA4-13MG, MA4-14MG
MEASUREMENT & GEOMETRY – ACMMG195, ACMMG196, ACMMG198

Diagnostic test

1 Measure the length of the interval AB correct to the nearest millimetre.

A 8 cm **B** 8.5 cm
C 9 cm **D** 8.4 cm

2 Estimate the length of a student's school desk.

A 60 cm **B** 80 cm
C 100 cm **D** 120 cm

3 Estimate 23.8 cm to the nearest cm.

A 23 cm **B** 24 cm
C 28 cm **D** 25 cm

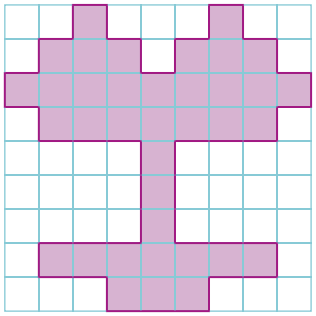
4 0.43 km = ___ cm.

A 430 **B** 4300
C 43 000 **D** 430 000

5 Which statement below is correct?

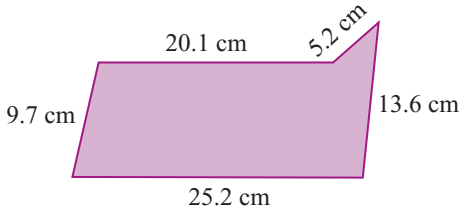
A 52 m = 520 cm
B 0.7 km = 70 000 mm
C 800 000 mm = 80 m
D 463 000 mm = 0.463 km

6 Determine the perimeter of the shape shown.



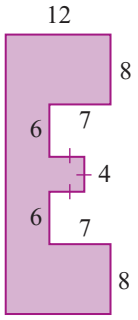
A 52 units **B** 61 units
C 56 units **D** 64 units

7 Calculate the perimeter of the shape shown.



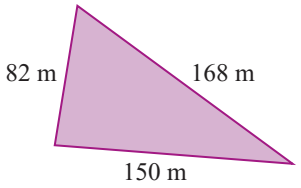
A 71.1 cm **B** 69.3 cm
C 73.8 cm **D** 64.1 cm

8 Calculate the perimeter of the shape shown. Measurements are in cm.



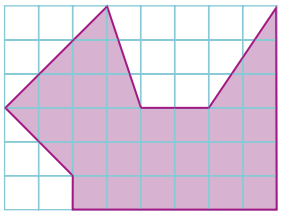
A 102 cm **B** 94 cm
C 110 cm **D** 88 cm

9 This diagram shows Joe's paddock. Calculate the cost of fencing the paddock if fencing costs $52.90 per metre.



A $20 360 **B** $22 112.20
C $21 160 **D** $2174.40

10 Estimate the area of the shaded shape to the nearest unit^2.

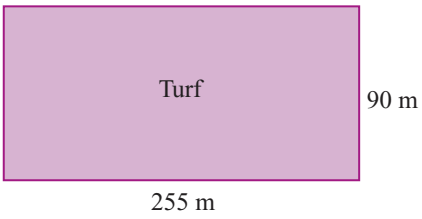


A 30 units^2 **B** 20 units^2
C 35 units^2 **D** 25 units^2

11 The area of a rectangle is 84 cm^2. Calculate its length if the breadth is 7 cm.

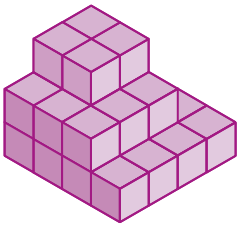
A 8 cm **B** 14 cm
C 7 cm **D** 12 cm

12 Calculate the cost of turfing the oval shown below, if each roll of turf covers an area of 0.75 m^2 and costs $1.80.



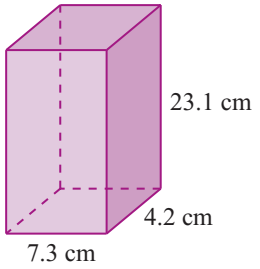
A $28 917 **B** $30 982.50
C $55 080 **D** $540

13 If the solid below is made from 1 cm^3 cubes, calculate the volume of the solid.



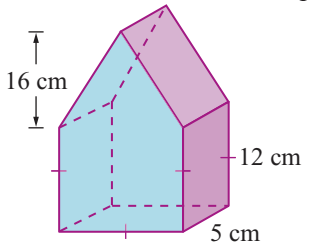
A 18 cm^3 **B** 29 cm^3
C 16 cm^3 **D** 30 cm^3

14 Calculate the volume of the solid below, correct to the nearest cm^3.



A 709 cm^3 **B** 708 cm^3
C 705 cm^3 **D** 710 cm^3

15 Calculate the volume of this composite prism.



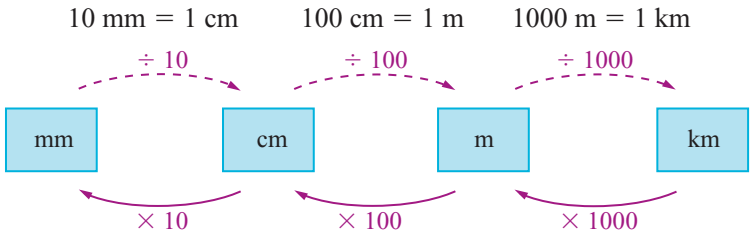
A 1680 cm^3 **B** 1350 cm^3
C 1200 cm^3 **D** 960 cm^3

The Diagnostic test questions refer to the Stage 4 outcome ACMMG159 and Stage 3 outcome ACMMG160.

A Area and perimeter review

This section reviews conversions, perimeter and area.

Length conversions



EXAMPLE 1

Convert these lengths.
a 50 mm to cm **b** 8.6 km to m

a $50 \text{ mm} = \frac{50}{10} \text{ cm}$
$= 5 \text{ cm}$

b $8.6 \text{ km} = 8.6 \times 1000 \text{ m}$
$= 8600 \text{ m}$

Exercise 5A

1 Convert these lengths.

a 21 cm to mm
b 180 mm to cm
c 3500 m to km
d 4 cm to mm
e 2.3 m to cm
f 1.8 km to m
g 200 mm to cm
h 280 cm to m
i 5.2 m to cm
j 8.3 cm to mm
k 6.3 km to m
l 0.03 m to cm
m 0.05 km to cm
n 3.2 m to mm
o 83 000 cm to km

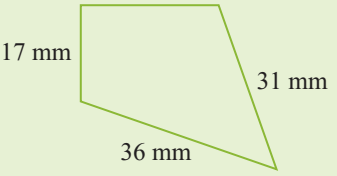
Perimeter

EXAMPLE 2

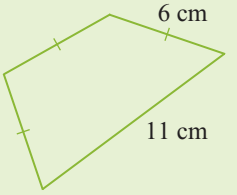
Find the perimeter of each of the following figures.

a

b



c



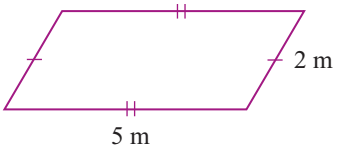
a Perimeter
$= 9 + 10 + 12$ cm
$= 31$ cm

b Perimeter
$= 17 + 24 + 31 + 36$ mm
$= 108$ mm

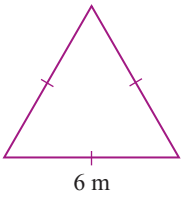
c Perimeter
$= 11 + (3 \times 6)$ cm
$= 11 + 18 = 29$
$= 29$ cm

2 Find the perimeter of each of each figure.

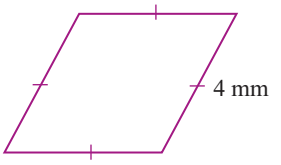
a



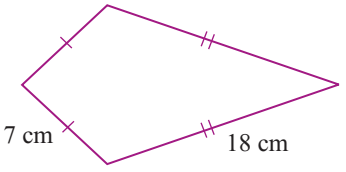
b



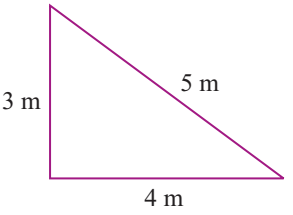
c



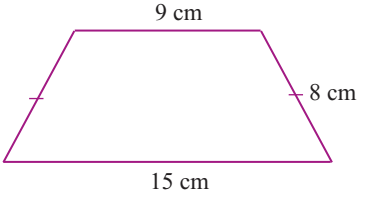
d



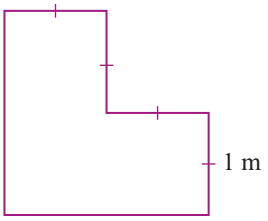
e



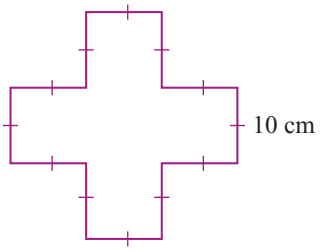
f



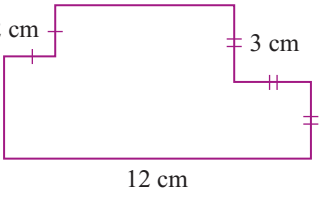
g



h



i

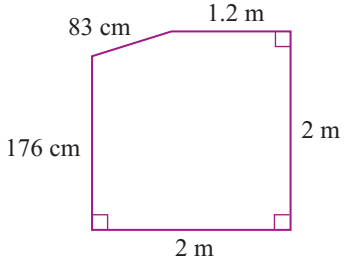


3 A rectangular paddock 220 m by 300 m is to be fenced. Find the total length of fencing required.

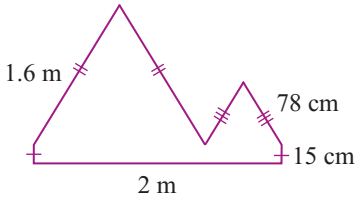
4 A rectangular swimming pool 22 m by 8 m is surrounded by a path of width 1 metre. What is the perimeter around the outside edge of the path?

5 Find the perimeter of each shape. Give your answer in metres.

a

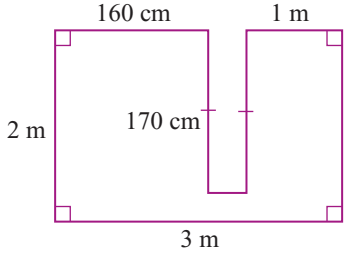


b

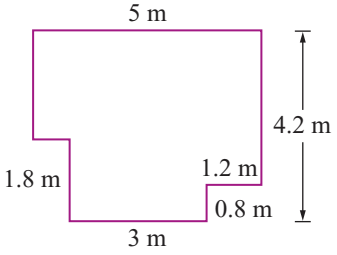


Convert all lengths to the same unit before finding the perimeter.

c



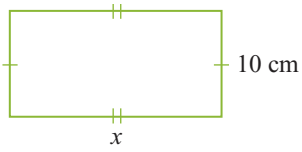
d



EXAMPLE 3

The perimeter of a rectangle is 50 cm. If the length of one side is 10 cm, calculate the length of other side.

Step 1: Draw and label a diagram.
Let any side be 10 cm.



In a rectangle, if one side is 10 cm, the corresponding side is also 10 cm.

Step 2: Solve.

$10 + 10 + x + x = 50$ cm

$20 + 2x = 50$

$2x = 50 - 20 = 30$

$x = \frac{30}{2}$

$= 15$ cm

or

$P = 50$ cm

2 known sides $= 2 \times 10 = 20$ cm

$50 - 20 = 30$ cm remaining

Remaining sides are equal

$\therefore \frac{30}{2} = 15$ cm

So, the rectangle has side lengths of 10 cm and 15 cm to have a perimeter of 50 cm.

6 Determine the value of the other side in each rectangle given the perimeter and one side length.

a Perimeter = 62 m, one side = 11 cm

b Perimeter = 216 cm, one side = 24 cm

c Perimeter = 96 cm, one side = 19 cm

d Perimeter = 78 cm, one side = 8 cm

e Perimeter = 28.82 cm, one side = 7.8 cm

f Perimeter = 41.2 cm, one side = 15.4 cm

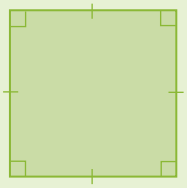
7 Determine the side length of a square given the following perimeters.

A square has four equal sides.

a 80 cm **b** 244 cm **c** 336 cm
d 261.2 cm **e** 671.6 cm **f** 1950 cm

EXAMPLE 4

Calculate the area of this square.

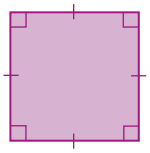


4.3 mm

Area = side × side
= 4.3 × 4.3
= 18.49 mm^2

8 Complete to find the area of each square.

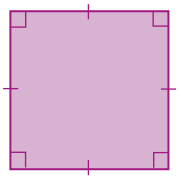
a



8 cm

$A = s \times s$
$= 8 \times$ ___
= ___ cm^2

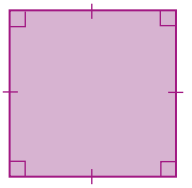
b



14 cm

$A = s \times s$
= ___ × 14
= ___ cm^2

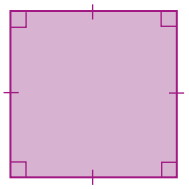
c



19 cm

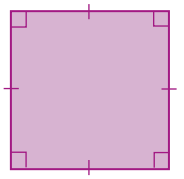
$A = s \times s$
= ___ × ___
= ___ cm^2

d



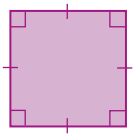
11.7 cm

e



8.9 cm

f



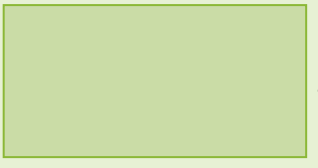
3.5 cm

9 Calculate the area of each square given the following side lengths.

a 3.9 cm **b** 2.4 mm **c** 8.7 m **d** 0.85 m

EXAMPLE 5

Calculate the area of this rectangle.



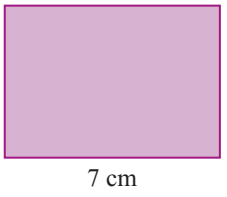
4.3 cm

8.6 cm

Area = length × breadth
= 4.3 × 8.6
= 36.98 cm^2

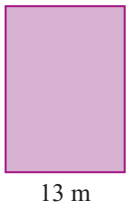
10 Complete to find the area of each rectangle.

a



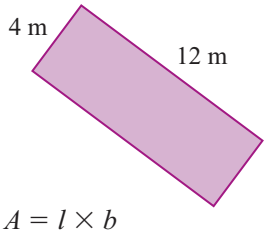
$A = l \times b$
$= 7 \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

b



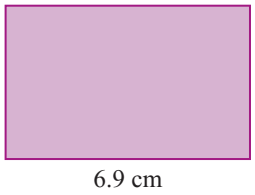
$A = l \times b$
$= \_\_\_ \times 13$
$= \_\_\_ \text{ cm}^2$

c



$A = l \times b$
$= \_\_\_ \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

d



$A = l \times b$
$= \_\_\_ \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

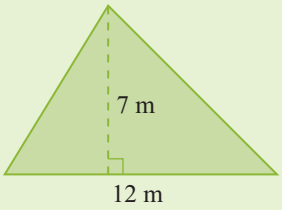
11 Find the area of each rectangle, given the length and breadth.

a length = 12 cm, breadth = 3 cm
b length = 18 cm, breadth = 5 cm
c length = 16 mm, breadth = 6 mm
d length = 4 m, breadth = 5 m
e length = 5 cm, breadth = 7 cm
f length = 6.2 cm, breadth = 3 cm

EXAMPLE 6

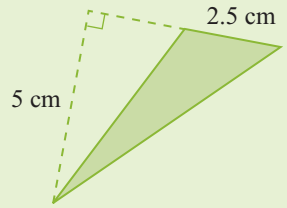
Find the area of each triangle.

a



b

c



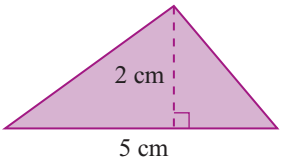
a Area $= \frac{1}{2}$base $\times$ height
$= \frac{1}{2} \times 12 \times 7$
$= 42 \text{ m}^2$

b Area $= \frac{1}{2}$base $\times$ height
$= \frac{1}{2} \times 7 \times 3$
$= 10.5 \text{ m}^2$

c Area $= \frac{1}{2}$base $\times$ height
$= \frac{1}{2} \times 2.5 \times 5$
$= 6.25 \text{ cm}^2$

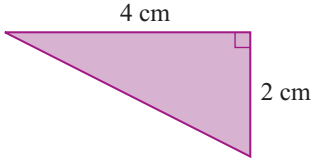
12 Complete to find the area of each triangle.

a



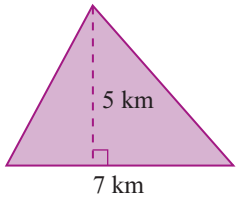
$A = \frac{1}{2}b \times h$
$= \frac{1}{2} \times 5 \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

b



$A = \frac{1}{2}b \times h$
$= \frac{1}{2} \times \_\_\_ \times 2$
$= \_\_\_ \text{ cm}^2$

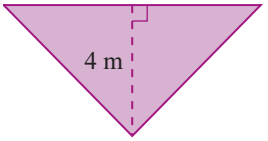
c



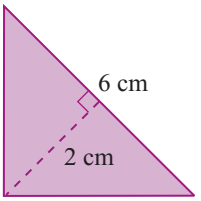
$A = \frac{1}{2}b \times h$
$= \frac{1}{2} \times \_\_\_ \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

13 Find the area of each triangle.

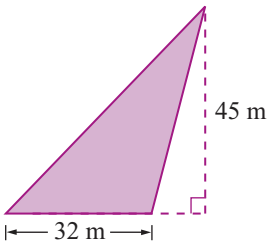
a



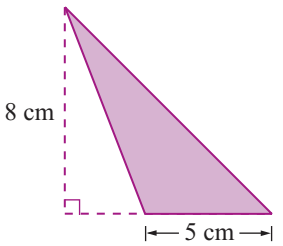
b



c

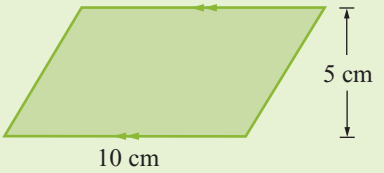


d



EXAMPLE 7

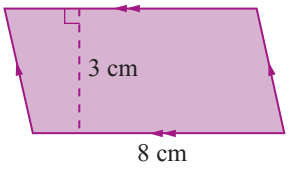
Find the area of this parallelogram.



Area = breadth × height
= 10 × 5
= 50 cm²

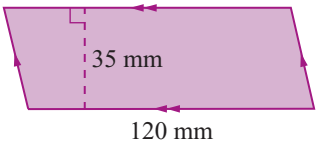
14 Complete to find the area of each parallelogram.

a



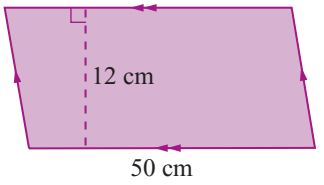
$A = b \times h$
$= \_\_\_ \times 3$
$= \_\_\_ \text{ cm}^2$

b



$A = b \times h$
$= \_\_\_ \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

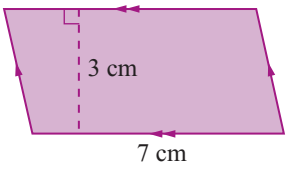
c



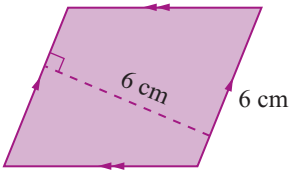
$A = b \times h$
$= \_\_\_ \times \_\_\_$
$= \_\_\_ \text{ cm}^2$

15 Find the area of each parallelogram.

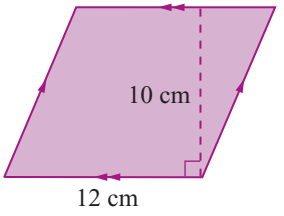
a



b



c

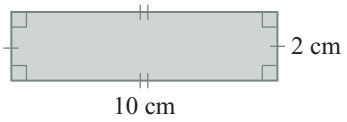


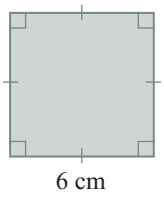
Investigation 1 Area and perimeter

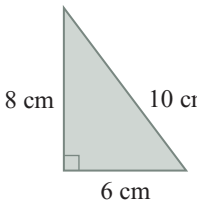
1 Calculate the perimeter and area of the shapes in each of the following sets to complete the table below.

	Square		Rectangle		Triangle	
	Perimeter	**Area**	**Perimeter**	**Area**	**Perimeter**	**Area**
Set 1						
Set 2						
Set 3						

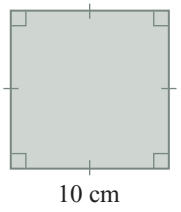
Set 1

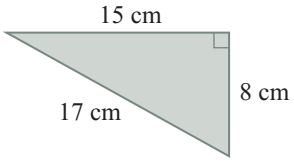


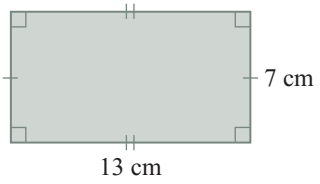




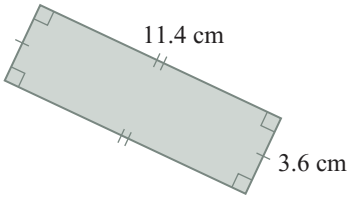
Set 2

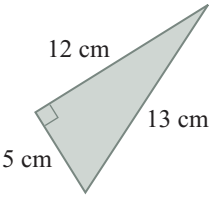


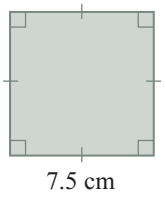




Set 3







2 If shapes have the same perimeter, do they also have the same area? Why or why not? Explain giving reasons for your answer.

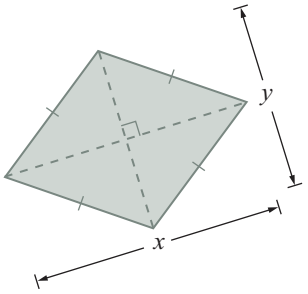
3 Construct two shapes that have the same perimeter and the same area. Give their dimensions.

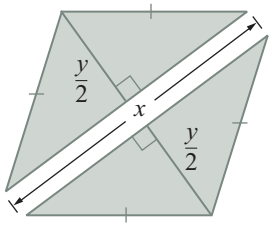
Investigation 2 Formulas for area

Use this formula to find expressions for the area of a rhombus, a kite and a trapezium.

1 Rhombus

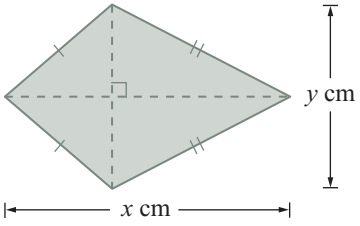
Use these diagrams to find expressions for the area of a rhombus with diagonals x and y units in length.

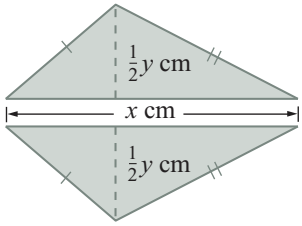




2 Kite

The formula for the area of a kite is the same as that for a rhombus. Compare this derivation with your expressions from question **1**.

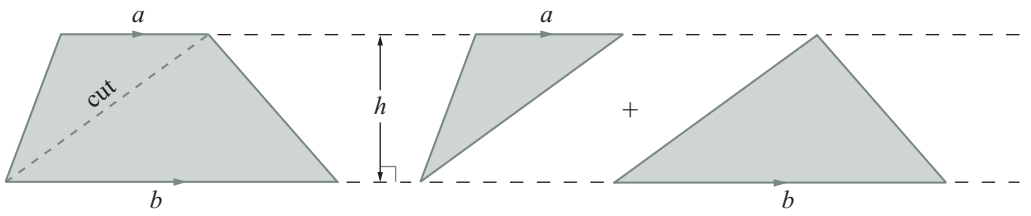




$$A = \frac{1}{2} \times x \times (\frac{1}{2}y) + \frac{1}{2} \times x \times (\frac{1}{2}y)$$
$$= \frac{1}{4}xy + \frac{1}{4}xy = \frac{1}{2}xy$$

3 Trapezium

Use these diagrams to find an expression for the area of a trapezium.



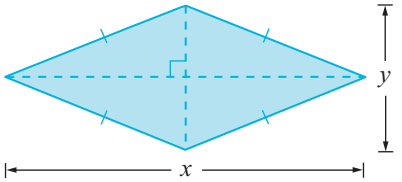
B Area of special quadrilaterals

From Investigation 2, the following formulas have been developed.

Rhombus

$A = \frac{1}{2}xy$

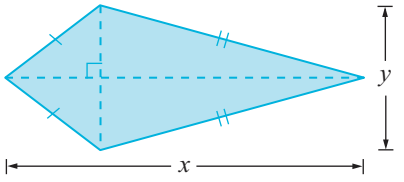
$A = \frac{1}{2} \times$ product of the lengths of the diagonals



Kite

$A = \frac{1}{2}xy$

$A = \frac{1}{2} \times$ product of the lengths of the diagonals

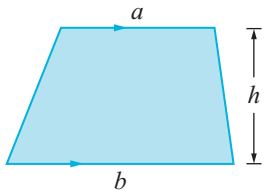


Trapezium

$A = \frac{1}{2}ah + \frac{1}{2}bh$

$= \frac{1}{2}h(a + b)$ or $A = \left(\frac{a + b}{2}\right)h$

$A =$ product of half the height and the sum of the lengths of the parallel sides
or product of the height and the average of the lengths of the parallel sides

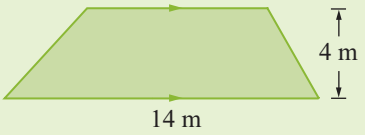


Note: The height is the perpendicular distance between the two parallel sides. Sometimes it is a side, but usually it is not.

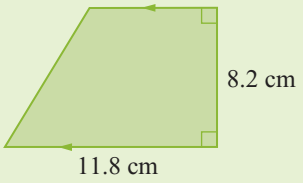
EXAMPLE 1

Find the area of each trapezium.

a



b



a Area $= \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times 4 \times (11 + 14)$

$= 2 \times 25$

$= 50 \text{ m}^2$

b Area $= \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times 8.2 \times (7.3 + 11.8)$

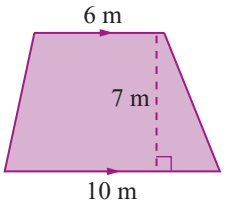
$= 4.1 \times 19.1$

$= 78.31 \text{ cm}^2$

Exercise 5B

1 Complete to find the area of each trapezium.

a

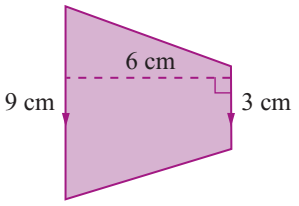


$A = \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times \_\_\_ \times (6 + \_\_\_)$

$= \_\_\_ \text{ m}^2$

b

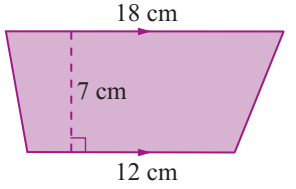


$A = \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times 6 \times (\_\_\_ + \_\_\_)$

$= \_\_\_ \text{ cm}^2$

c



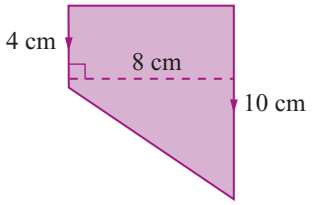
$A = \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times \_\_\_ \times (\_\_\_ + \_\_\_)$

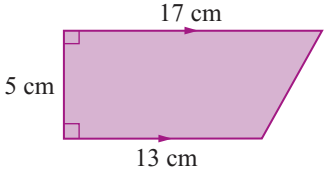
$= \_\_\_ \text{ cm}^2$

2 Find the area of each trapezium.

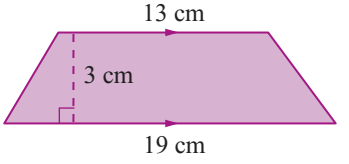
a



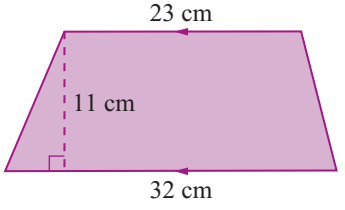
b



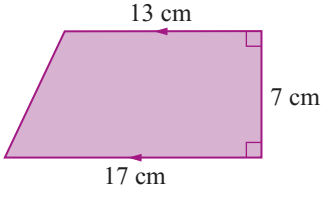
c



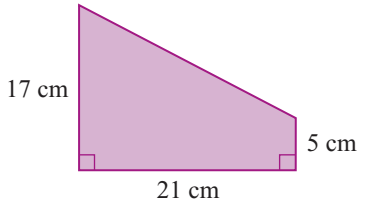
d



e



f



3 Find the area of each trapezium, given:

a height = 5 cm, a = 14 cm, b = 8 cm

b height = 4 cm, a = 5 cm, b = 9 cm

c height = 7 cm, a = 6 cm, b = 8 cm

d height = 6 mm, a = 11 mm, b = 15 mm

e height = 12 mm, a = 9 mm, b = 18 mm

f height = 6 cm, a = 5.3 cm, b = 3.2 cm

g height = 3.2 m, a = 6.9 m, b = 9 m

h height = 9.5 cm, a = 4.8 cm, b = 15.3 cm

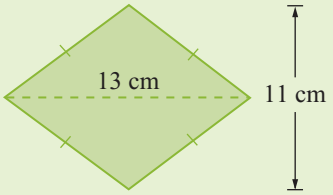
i height = 4.1 cm, a = 3.7 cm, b = 10.2 cm

j height = 2.8 m, a = 4.5 m, b = 1.7 m

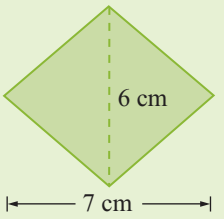
EXAMPLE 2

Find the area of each rhombus.

a



b



a Area $= \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 13 \times 11$

$= 71.5 \text{ cm}^2$

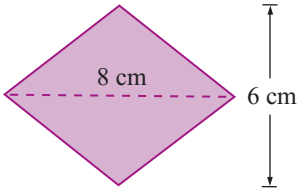
b Area $= \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 6 \times 7$

$= 21 \text{ cm}^2$

4 Complete to find the area of each rhombus.

a

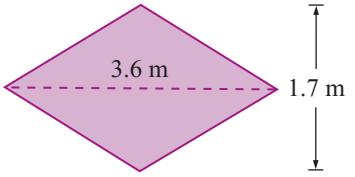


$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 8 \times \_\_\_$

$= \_\_\_ \text{ cm}^2$

b

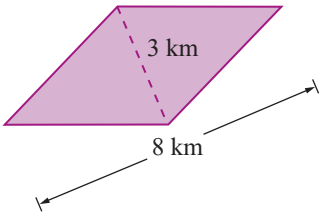


$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times \_\_\_ \times 1.7$

$= \_\_\_ \text{ m}^2$

c



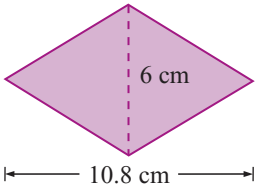
$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times \_\_\_ \times \_\_\_$

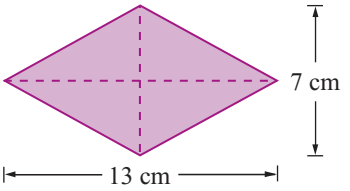
$= \_\_\_ \text{ km}^2$

5 Find the area of each rhombus.

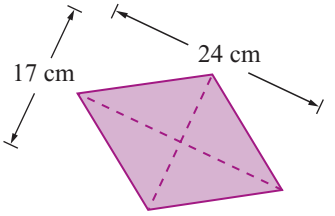
a



b



c



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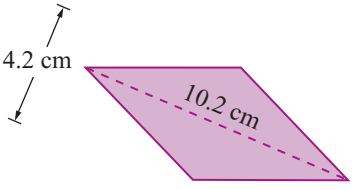

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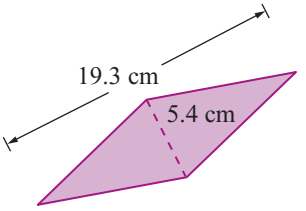

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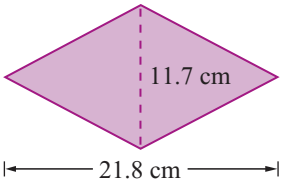
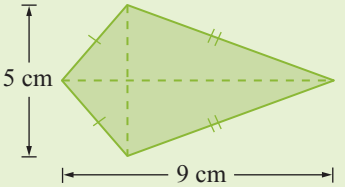


EXAMPLE 3

Find the area of this kite.

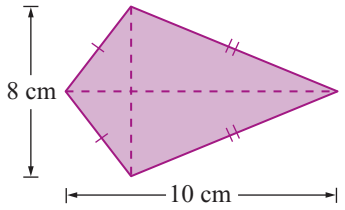


Area $= \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 9 \times 5 = 22.5 \text{ cm}^2$

6 Complete to find the area of each kite.

a

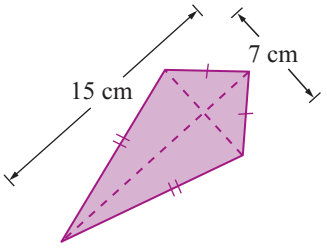


$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 10 \times$ ___

$=$ ___ cm^2

b

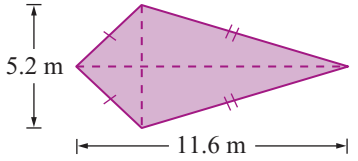


$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times$ ___ $\times 7$

$=$ ___ cm^2

c



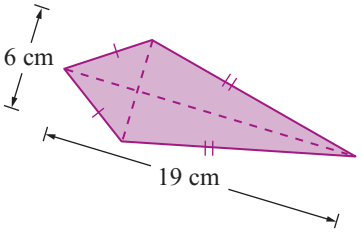
$A = \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times$ ___ $\times$ ___

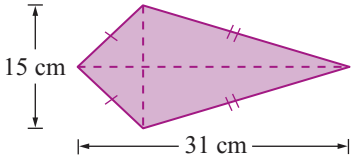
$=$ ___ m^2

7 Find the area of each kite.

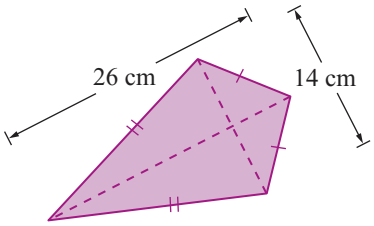
a



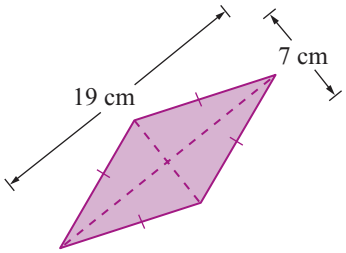
b



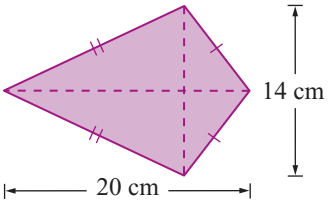
c



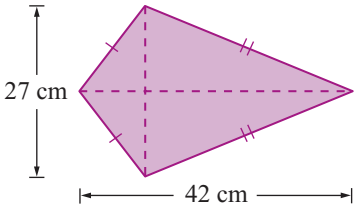
d



e



f

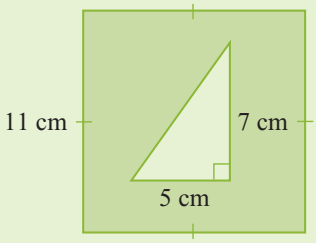


8 Calculate the area of each rhombus or kite with the diagonals x and y given.

a $x = 4$ cm, $y = 19$ cm
b $x = 3$ m, $y = 15$ m
c $x = 9$ cm, $y = 8$ cm
d $x = 12$ mm, $y = 5$ mm
e $x = 17$ mm, $y = 11$ mm
f $x = 4.6$ cm, $y = 10.8$ cm
g $x = 3.1$ cm, $y = 18.1$ cm
h $x = 11.6$ mm, $y = 6.3$ mm
i $x = 0.9$ cm, $y = 7.2$ cm

EXAMPLE 4

Calculate the area of the shaded region.



Shaded area = larger area − smaller area
= area of square − area of triangle

Area of the square $= s \times s$
$= 11 \times 11$
$= 121$ cm^2

Area of a triangle $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 5 \times 7$
$= \frac{1}{2} \times 35$
$= 17.5$ cm^2

$\therefore$ Shaded area $= 121 - 17.5 = 103.5$ cm^2

9 Complete to find the shaded area in each figure.

a Larger area $= l \times b$

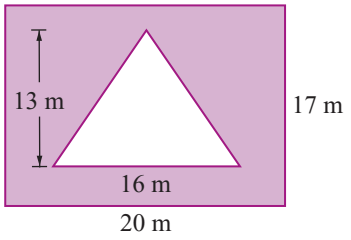
$= (20 \times \_\_\_) = \_\_\_ \text{ m}^2$

Smaller area $= \frac{1}{2}b \times h$

$= \frac{1}{2} \times 16 \times \_\_\_ = \_\_\_ \text{ m}^2$

Shaded area = larger area − smaller area

$= \_\_\_ - \_\_\_ = \_\_\_ \text{ m}^2$



b Larger area $= \frac{1}{2} \times h \times (a + b)$

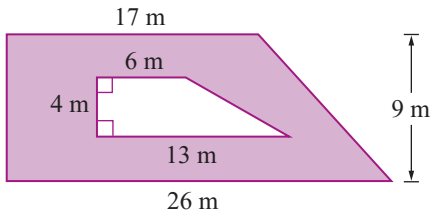
$= \frac{1}{2} \times 9 \times (17 + \_\_\_) = \_\_\_ \text{ m}^2$

Smaller area $= \frac{1}{2} \times h \times (a + b)$

$= \frac{1}{2} \times 4 \times (6 + \_\_\_) = \_\_\_ \text{ m}^2$

Shaded area = larger area − smaller area

$= \_\_\_ - \_\_\_ = \_\_\_ \text{ m}^2$



c Larger area $= s \times s$

$= (14 \times \_\_\_) = \_\_\_ \text{ cm}^2$

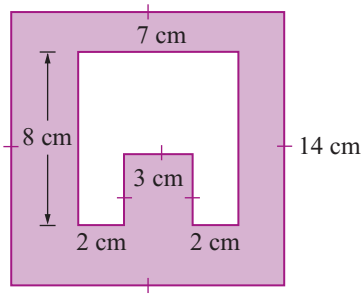
Smaller area $= l \times b - s \times s$

$= 7 \times 8 - 3 \times \_\_\_$

$= \_\_\_ - \_\_\_ = \_\_\_ \text{ cm}^2$

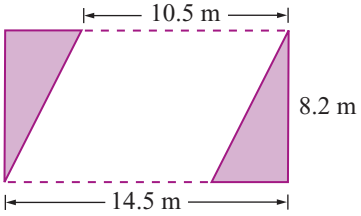
Shaded area = larger area − smaller area

$= \_\_\_ - 47 = \_\_\_ \text{ cm}^2$

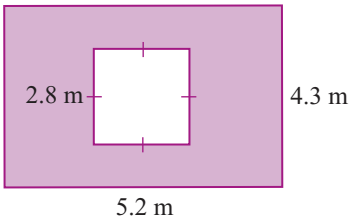


10 Calculate the area of the shaded region in each figure.

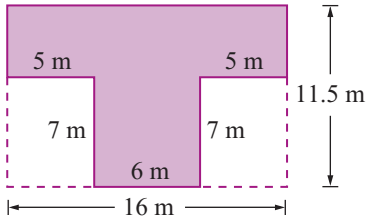
a



b



c



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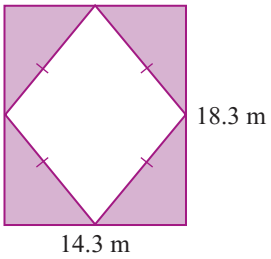
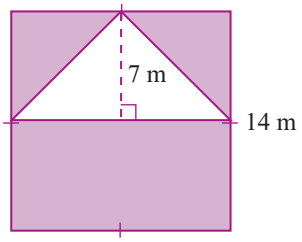
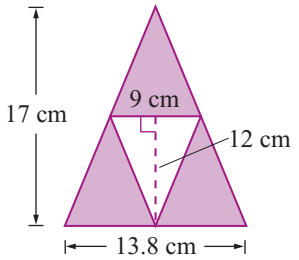


e

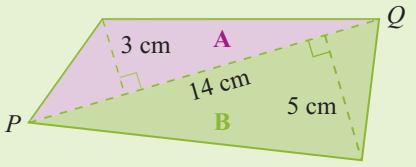


f



EXAMPLE 5

Find the area of this quadrilateral.

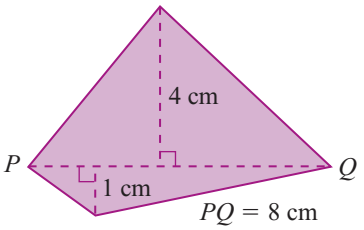


Area = area of triangle A + area of triangle B

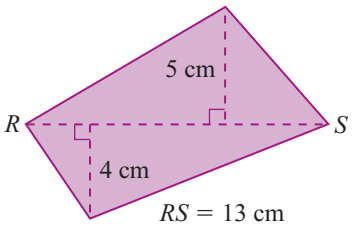
$= \frac{1}{2} \times 14 \times 3 + \frac{1}{2} \times 14 \times 5 = 56 \text{ cm}^2$

11 Find the areas of these quadrilaterals.

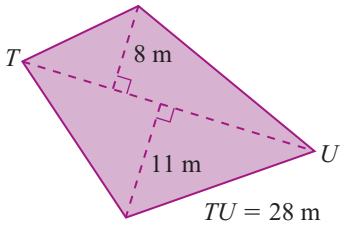
a



b

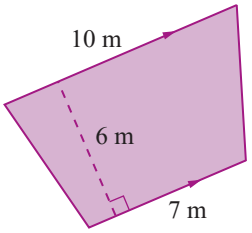


c

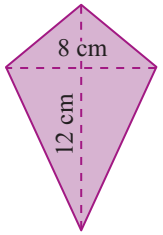


12 Use the correct formula to find the area of these quadrilaterals.

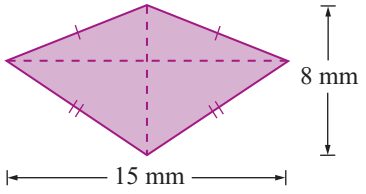
a



b



c



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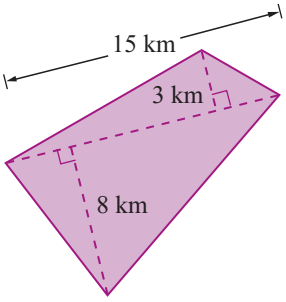
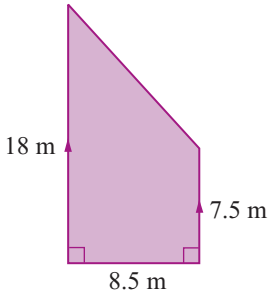


e



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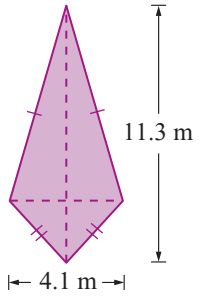
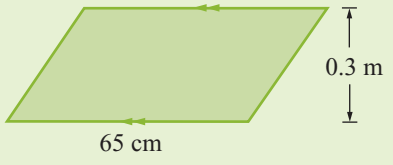


EXAMPLE 6

Calculate the area of this parallelogram in:

a cm^2 **b** m^2



a Convert 0.3 m to cm

$0.3 \text{ m} = 0.3 \times 100 \text{ cm}$

$= 30 \text{ cm}$

Area $= bh$

$= 65 \times 30$

$= 1950 \text{ cm}^2$

b Convert 65 cm to m

$65 \text{ cm} = 65 \div 100 \text{ m}$

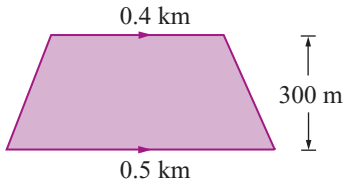
$= 0.65 \text{ m}$

Area $= bh$

$= 0.65 \times 0.3$

$= 0.195 \text{ m}^2$

13 **a** Complete to calculate the area of the trapezium in km^2.



Convert 300 m to km: 300 m = 300 ÷ ___ km = ___ km

$A = \frac{1}{2}h($___ + ___$)$

$= \frac{1}{2} \times 0.3 \times ($___ + ___$)$

$= 0.135$ km^2

b Complete to calculate the area of the trapezium in m^2.

Convert 0.4 km to m: 0.4 km = 0.4 × ___ m = 400 m

Convert 0.5 km to m: 0.5 km = 0.5 × ___ m = ___ m

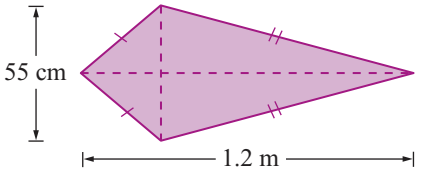
$A = \frac{1}{2}$___$(a + b)$

$= \frac{1}{2} \times$ ___ $\times ($___ $+ 500)$

$= 135\,000$ m^2

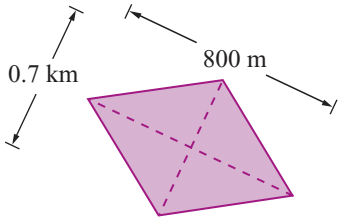
14 **a** Calculate the area of this kite in:

i cm^2 **ii** m^2



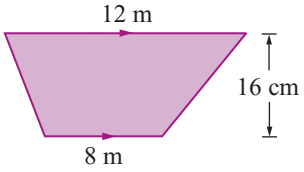
b Calculate the area of this rhombus in:

i m^2 **ii** km^2



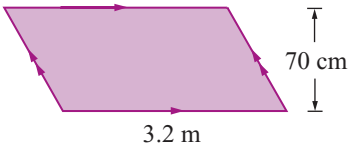
c Calculate the area of this trapezium in:

i cm^2 **ii** m^2



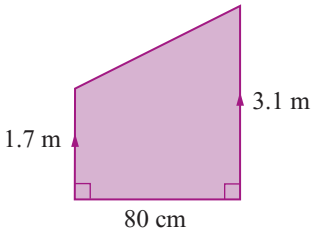
d Calculate the area of this parallelogram in:

i m^2 **ii** cm^2



e Calculate the area of this trapezium in:

i cm^2 **ii** m^2



Investigation 3 Area and perimeter

1 This is a practical method used to find the perimeter of different rectangles with the same area. You will need 1 cm grid paper to complete this part of the investigation.

a First draw as many rectangles as you can with an area of 36 cm^2. Do this by colouring in 36 of the squares on the grid paper, making sure that the shape is a rectangle. Alternatively, use 36 tiles or blocks to make the rectangles.

b What are the dimensions of the 36 cm^2 rectangle that has the largest perimeter?

c What are the dimensions of the 36 cm^2 rectangle that has the smallest perimeter?

2 This is a theoretical method for finding the perimeter of different rectangles with the same area. Again use a rectangle that has an area of 36 cm^2.

a Complete this table. The breadth can be calculated by dividing the area by the length.

Length (cm)	Breadth (cm)	Area (cm^2)	Perimeter (cm)
1		36	
2		36	
3		36	
4		36	
6		36	
9		36	
12		36	
18		36	
36		36	

b What are the dimensions of the 36 cm^2 rectangle with the largest perimeter?

c What are the dimensions of the 36 cm^2 rectangle with the smallest perimeter?

d What name is given to this rectangle?

e It is suggested that there was no need to use values of length greater than 6 to find the rectangles with the largest and smallest perimeters. Explain you answer.

3 A rectangle has area 64 cm^2.

a Complete this table.

Length (cm)	Breadth (cm)	Area (cm^2)	Perimeter (cm)
1		64	
2		64	
4		64	
8		64	

b What are the dimensions of the 64 cm^2 rectangle with the largest perimeter?

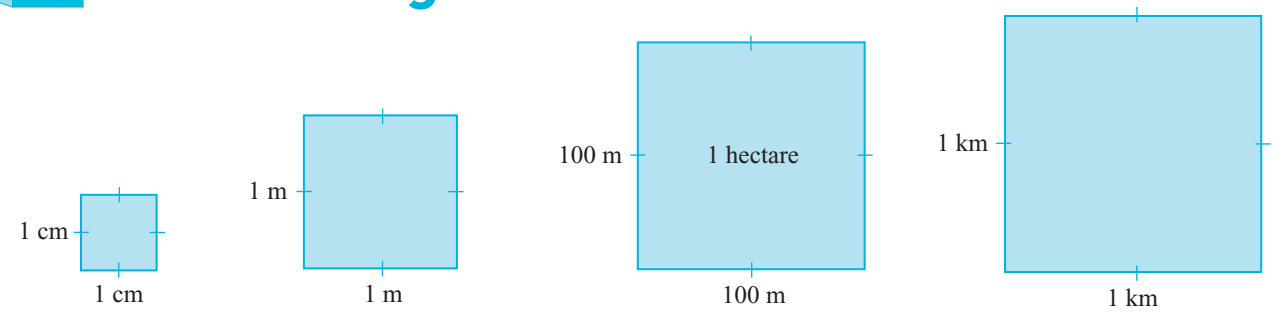
c What are the dimensions of the 64 cm^2 rectangle with the smallest perimeter?

d What name is given to this rectangle?

e Only values up to 8 were used for the length to find the rectangle with the largest and smallest perimeters. Explain why values of 16, 32 and 64 were not used.

4 Comment on the statement: 'The rectangle of fixed area that has the smallest perimeter is the square.'

C Converting area units



1 cm × 1 cm = 1 cm²

1 m × 1 m = 1 m²

100 m × 100 m = 10 000 m² = 1 ha

1 km × 1 km = 1 km² = 100 ha

Exercise 5C

1 Explain why:

a 1 cm × 1 cm = 1 cm²

b 1 m × 1 m = 1 m²

2 Explain why:

a 100 m × 100 m = 10 000 m² (1 ha)

b 1 km × 1 km = 1 km²

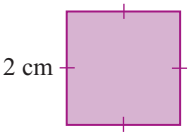
3 Is 1000 m × 1000 m = 1 km²? Explain.

1000 m × 1000 m = 1 000 000 m² = 100 ha

4 Explain why 1 km² = 100 ha.

5 **a** Find the area, in square centimetres, of this square.

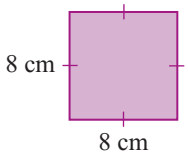
b Convert the sides to millimetres. Find the area of the square in square millimetres.



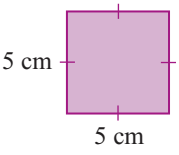
6 Find the area of the squares shown in:

i square centimetres **ii** square millimetres

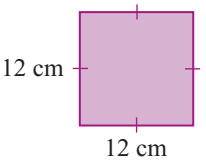
a



b



c



7 Complete the following.

a If 64 cm² = ___ mm², 6400 mm² = ___ cm².

b If 25 cm² = ___ mm², 2500 mm² = ___ cm².

c If 144 cm² = ___ mm², 14 400 mm² = ___ cm².

d To convert from cm² to mm² multiply by ___.

e To convert from mm² to cm² divide by ___.

8 Complete the table below for squares with the side lengths given.

	Areas of squares		
Side lengths	**m²**	**cm²**	**mm²**
2 m	2 × 2 = 4 m²	200 × 200 = ___ cm²	2000 × 2000 = ___ mm²
3 m			
4 m			

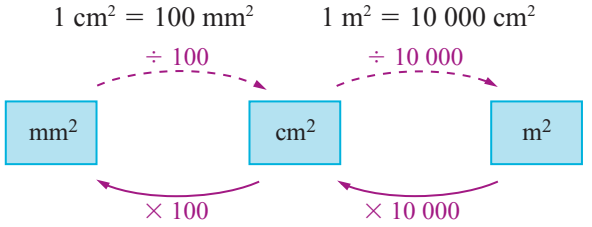
9 Complete the following statements using the table in question **8**.

a To convert from m^2 to cm^2 multiply by ___.

b To convert from cm^2 to m^2 divide by ___.

From the previous questions the following conversion chart can be drawn.

Area conversions



EXAMPLE 1

Convert:

a $8\ m^2$ to cm^2

b $3\ cm^2$ to mm^2

a $8 \times 10\,000 = 80\,000\ cm^2$

b $3 \times 100 = 300\ mm^2$

10 Complete the following.

a $11\ m^2$ to cm^2
$= 11 \times 10\,000$
$=$ ___ cm^2

b $0.6\ m^2$ to cm^2
$= 0.6 \times$ ___
$=$ ___ cm^2

c $0.04\ cm^2$ to mm^2
$=$ ___ $\times$ ___ mm^2
$=$ ___ mm^2

11 Complete the following conversions by multiplying.

a $4\ m^2 =$ ___ cm^2
b $12\ m^2 =$ ___ cm^2
c $0.3\ m^2 =$ ___ cm^2
d $0.5\ m^2 =$ ___ cm^2
e $0.09\ m^2 =$ ___ cm^2
f $0.006\ m^2 =$ ___ cm^2
g $141\ cm^2 =$ ___ mm^2
h $12\ cm^2 =$ ___ mm^2
i $0.1\ cm^2 =$ ___ mm^2
j $0.47\ cm^2 =$ ___ mm^2
k $0.06\ cm^2 =$ ___ mm^2
l $0.012\ cm^2 =$ ___ mm^2

EXAMPLE 2

Convert:

a $850\ mm^2$ to cm^2

b $250\,000\ cm^2$ to m^2

a $850 \div 100 = 8.5\ cm^2$

b $250\,000 \div 10\,000 = 25\ m^2$

12 Complete the following conversions.

a $14\,000\ mm^2$ to cm^2
$= 14\,000 \div$ ___
$=$ ___ cm^2

b $63\ mm^2$ to cm^2
$= 63 \div$ ___
$=$ ___ cm^2

c $530\ cm^2$ to m^2
$= 530 \div$ ___
$=$ ___ m^2

13 Complete the following conversions by dividing.

a $12\,000\ mm^2 =$ ___ cm^2
b $15\,600\ mm^2 =$ ___ cm^2
c $9300\ mm^2 =$ ___ cm^2
d $720\ mm^2 =$ ___ cm^2
e $58\ mm^2 =$ ___ cm^2
f $6\ mm^2 =$ ___ cm^2
g $7\,000\,000\ cm^2 =$ ___ m^2
h $3\,250\,000\ cm^2 =$ ___ m^2
i $182\,000\ cm^2 =$ ___ m^2
j $96\,000\ cm^2 =$ ___ m^2
k $3500\ cm^2 =$ ___ m^2
l $420\ cm^2 =$ ___ m^2

14 Complete the following conversions by multiplying or dividing.

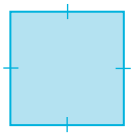
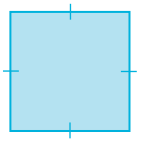
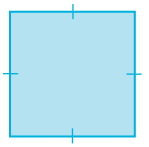
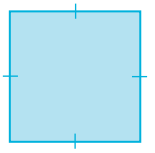
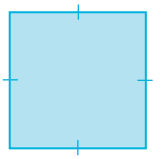
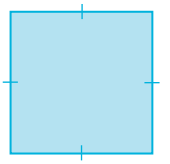
a 6 cm^2 = ___ mm^2 **b** 13 cm^2 = ___ mm^2 **c** 3.5 cm^2 = ___ mm^2

d 0.4 cm^2 = ___ mm^2 **e** 600 mm^2 = ___ cm^2 **f** 700 mm^2 = ___ cm^2

g 850 mm^2 = ___ cm^2 **h** 24 mm^2 = ___ cm^2 **i** 3 m^2 = ___ cm^2

j 15.8 m^2 = ___ cm^2 **k** 40 000 cm^2 = ___ m^2 **l** 60 500 cm^2 = ___ m^2

15 a Complete the following table.

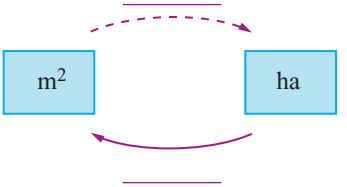
	 100 m	 500 m	 800 m	 1000 m	 5000 m	 8000 m
m^2	100 × 100 = 10 000 m^2	___ × ___ = ___ m^2	800 × 800 = ___ m^2	___ × ___ = ___ m^2	5000 × 5000 = ___ m^2	___ × ___ = ___ m^2
ha	= 1 ha	= ___ ha	= 64 ha	= ___ ha	= 2500 ha	= ___ ha

b Use the table to complete the following statements.

i To convert m^2 to hectares divide by ___.

ii To convert hectares to m^2 multiply by ___.

c Complete this conversion diagram.



16 Complete the following conversions.

a 5 ha = ___ m^2 **b** 17 ha = ___ m^2 **c** 0.9 ha = ___ m^2

d 0.23 ha = ___ m^2 **e** 1.4 ha = ___ m^2 **f** 40 000 m^2 = ___ ha

g 62 000 m^2 = ___ ha **h** 180 000 m^2 = ___ ha **i** 9 000 000 m^2 = ___ ha

j 226 000 000 m^2 = ___ ha **k** 342 680 000 m^2 = ___ ha **l** 890 000 m^2 = ___ ha

17 Complete the following conversions by multiplying or dividing.

a 20 000 m^2 = ___ ha **b** 30 000 m^2 = __ ha **c** 11 500 m^2 = ___ ha

d 48 900 m^2 = ___ ha **e** 4 ha = ___ m^2 **f** 6 ha = ___ m^2

g 3.6 ha = ___ m^2 **h** 4.93 ha = ___ m^2 **i** 0.03 ha = ___ m^2

j 1.15 ha = ___ m^2 **k** 2.1 ha = ___ m^2 **l** 0.005 ha = ___ m^2

D Volume and capacity

Volume refers to the amount of space occupied by an object.

Capacity refers to the quantity, usually of liquid, that can be contained by a solid.

1 L (1000 mL) = 1000 cm^3

and 1 mL = 1 cm^3

EXAMPLE 1

What is the capacity in mL of a container that can hold the following volumes.

a 50 cm^3 **b** 72 cm^3 **c** 6000 cm^3

The conversion is 1 cm^3 = 1 mL.

a 50 cm^3 = 50 mL **b** 72 cm^3 = 72 mL **c** 6000 cm^3 = 6000 mL

Exercise 5D

1 Find the capacity in mL of a container that has a volume of:

a 30 cm^3 **b** 60 cm^3 **c** 20 cm^3 **d** 25 cm^3
e 85 cm^3 **f** 45 cm^3 **g** 4 cm^3 **h** 34 cm^3
i 53 cm^3 **j** 500 cm^3 **k** 700 cm^3 **l** 600 cm^3
m 5000 cm^3 **n** 3000 cm^3 **o** 11 000 cm^3 **p** 7.5 cm^3

EXAMPLE 2

What is the volume in cm^3 of a container that has a capacity of:

a 20 mL? **b** 63 mL? **c** 4000 mL?

The conversion is 1 mL = 1 cm^3.

a 20 mL = 20 cm^3 **b** 63 mL = 63 cm^3 **c** 4000 mL = 4000 cm^3

2 Find the volume in cm^3 of a container that has a capacity of:

a 50 mL **b** 70 mL **c** 30 mL **d** 45 mL
e 25 mL **f** 65 mL **g** 6 mL **h** 54 mL
i 21 mL **j** 400 mL **k** 600 mL **l** 900 mL
m 7000 mL **n** 5000 mL **o** 12 000 mL **p** 7.5 mL

EXAMPLE 3

What is the capacity in litres of a container that can hold:

a 5000 cm^3? **b** 3250 cm^3? **c** 97 820 cm^3?

The conversion is 1000 cm^3 = 1 L.

a 5000 cm^3 = 5000 ÷ 1000
= 5 L

b 3250 cm^3 = 3250 ÷ 1000
= 3.25 L

c 97 820 cm^3 = 97 820 ÷ 1000
= 97.82 L

3 Complete to find the capacity in litres of a container that can hold the following volumes.

a 3000 cm^3 = 3000 ÷ 1000
= ___ L

b 6000 cm^3 = 6000 ÷ ___
= ___ L

c 2000 cm^3 = ___ ÷ ___
= ___ L

4 Find the capacity in litres of a container that can hold the following volumes.

a 75 000 cm^3 **b** 35 000 cm^3 **c** 65 000 cm^3
d 4200 cm^3 **e** 3400 cm^3 **f** 5300 cm^3
g 2535 cm^3 **h** 3773 cm^3 **i** 7688 cm^3

Dividing by 1000 moves the decimal point 3 places to the left.

EXAMPLE 4

Find the volume in cm^3 of a container that has a capacity of:

a 2 L **b** 5.2 L **c** 4.652 L

The conversion is 1 L = 1000 cm^3.

a $2\text{ L} = 2 \times 1000$
$= 2000\text{ cm}^3$

b $5.2\text{ L} = 5.2 \times 1000$
$= 5200\text{ cm}^3$

c $4.652\text{ L} = 4.652 \times 1000$
$= 4652\text{ cm}^3$

5 Complete to find the volume in cm^3 of a container that has a capacity of:

a $3\text{ L} = 3 \times 1000$
$= \_\_\text{ cm}^3$

b $8\text{ L} = 8 \times \_\_\_$
$= \_\_\text{ cm}^3$

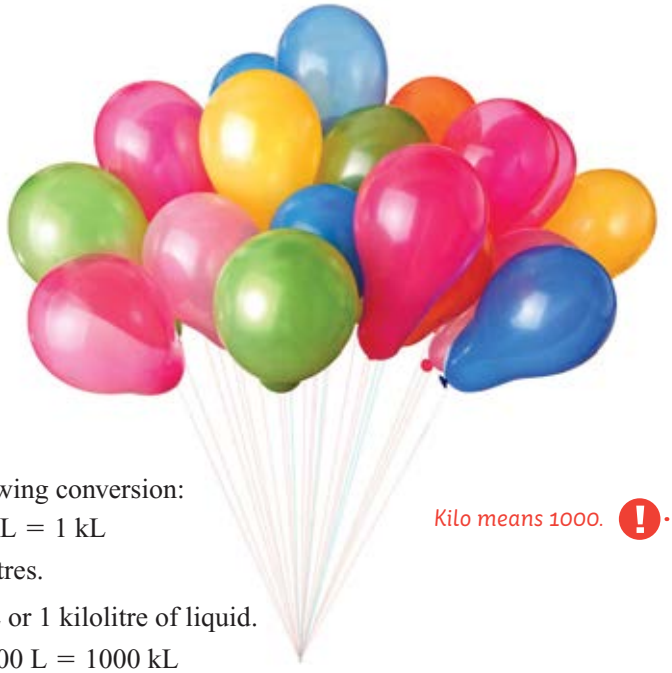
c $7\text{ L} = \_\_\_ \times \_\_\_$
$= \_\_\text{ cm}^3$

6 Find the volume in cm^3 of a container that has a capacity of:

a 4.2 L **b** 5.3 L **c** 8.9 L **d** 6.45 L
e 5.44 L **f** 3.21 L **g** 4.495 L **h** 6.293 L
i 8.443 L **j** 70 L **k** 50 L **l** 120 L

7 Which measure of capacity, mL or L, would be used for the volume held by these containers?

a teacup
b ice-cream container
c large plastic milk container
d small can of baked beans
e large bottle of soft drink
f can of drink
g small milk carton
h home fish tank
i party balloon



When dealing with larger volumes we use the following conversion:

$$1\text{ m}^3 = 1000\text{ L} = 1\text{ kL}$$

Kilo means 1000.

1 kL means 1 kilolitre. This is equivalent to 1000 litres.

A cube with sides length of 1 m would hold 1000 L or 1 kilolitre of liquid.

$$1\text{ ML} = 1\,000\,000\text{ L} = 1000\text{ kL}$$

EXAMPLE 5

Calculate the volume occupied by the following amounts of liquid.

a 8000 L **b** 2500 L **c** 7.2 kL

The conversion is 1 m^3 = 1000 L or 1 kL.

a $8000\text{ L} = 8000 \div 1000$
$= 8\text{ m}^3$

b $2500\text{ L} = 2500 \div 1000$
$= 2.5\text{ m}^3$

c $7.2\text{ kL} = 7.2 \times 1$
$= 7.2\text{ m}^3$

8 Complete to calculate the volume occupied by the following amounts of liquid.

a $4000\text{ L} = 4000 \div 1000$
$= \_\_\_\text{ m}^3$

b $330\text{ L} = 330 \div \_\_\_$
$= \_\_\_\text{ m}^3$

c $0.4\text{ kL} = 0.4 \times \_\_\_$
$= \_\_\_\text{ m}^3$

9 Calculate the space occupied by the following volumes of liquid.

a 12 000 L	**b** 9500 L	**c** 7250 L	**d** 670 L
e 136 L	**f** 25 L	**g** 12.5 kL	**h** 8.3 kL
i 5 kL	**j** 0.6 kL	**k** 0.75 kL	**l** 0.09 kL

EXAMPLE 6

Convert each to kilolitres.

a $0.007\ m^3$ **b** $8.85\ m^3$ **c** $1250\ m^3$

The conversion is $1\ m^3 = 1\ kL$.

a $0.007\ m^3 = 0.007 \div 1$
$= 0.007\ kL$

b $8.85\ m^3 = 8.85 \div 1$
$= 8.85\ kL$

c $1250\ m^3 = 1250 \div 1$
$= 1250\ kL$

10 Complete to convert each volume to kilolitres.

a $0.05\ m^3 = 0.05 \div 1$
$= \_\_\ kL$

b $580\ m^3 = 580 \div \_\_\_$
$= \_\_\ kL$

c $7000\ m^3 = 7000 \div \_\_\_$
$= \_\_\ kL$

11 Convert each volume to kilolitres.

a $0.01\ m^3$	**b** $0.9\ m^3$	**c** $480\ m^3$	**d** $295\ m^3$
e $890\ m^3$	**f** $6500\ m^3$	**g** $7200\ m^3$	**h** $9000\ m^3$
i $12\ 940\ m^3$	**j** $14\ 750\ m^3$	**k** $18\ 500\ m^3$	**l** $23\ 000\ m^3$

12 Complete to convert each volume to ML.

a $2000\ m^3 = 2000 \div 1000$
$= \_\_\_\ ML$

b $600\ m^3 = 600 \div \_\_\_$
$= \_\_\_\ ML$

c $0.8\ m^3 = 0.8 \div \_\_\_$
$= \_\_\_\ ML$

d $6000\ m^3$ **e** $420\ m^3$ **f** $75\ m^3$

13 Complete to convert to m^3.

a $5\ ML = 5 \times 1000$
$= \_\_\_\ m^3$

b $6.2\ ML = 6.2 \times \_\_\_$
$= \_\_\_\ m^3$

c $50\ ML = \_\_\_ \times \_\_\_$
$= \_\_\_\ m^3$

d 28 ML **e** 15.62 ML **f** 0.3 ML

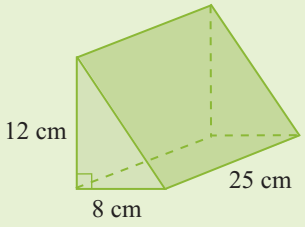
EXAMPLE 7

For this prism find its:

a volume in cm^3

b capacity in mL

c capacity in L.



Remember $1\ cm^3 = 1\ mL$
$1000\ cm^3 = 1\ L$
$1\ m^3 = 1000\ L = 1\ kL$

a V = area of base × height
$= (\frac{1}{2} \times b \times h) \times$ height
$= (\frac{1}{2} \times 8 \times 12) \times 25$
$= 1200\ cm^3$

b $1\ cm^3 = 1\ mL$
$\therefore$ Capacity $= 1200\ mL$

c $1\ L = 1000\ mL$
$\therefore$ Capacity $= \frac{1200}{1000}\ L = 1.2\ L$

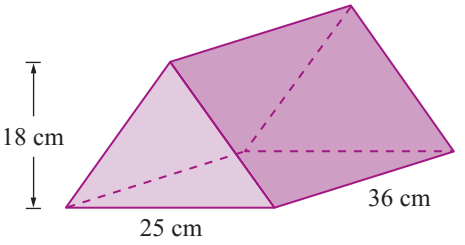
14 **a** Complete to find the volume of this prism in cm³.

V = area of base × height

$= \frac{1}{2}(b \times h) \times H$

$= \frac{1}{2} \times (\_\_\_ \times 18) \times \_\_\_$

$= \_\_\_ \times \_\_\_ = \_\_\_ \text{ cm}^3$



b Complete to find the capacity of this prism in mL.

$1 \text{ cm}^3 = 1 \text{ mL} \quad \therefore \text{Capacity} = \_\_\_ \text{ mL}$

c Complete to find the capacity of this prism in L.

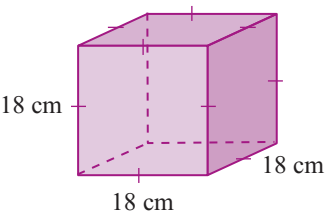
$1 \text{ L} = 1000 \text{ mL} \quad \therefore \text{Capacity} = \frac{\square}{1000} \text{ L} = \_\_ \text{ L}$

15 This cube has a side length of 18 cm.

a Find the volume in cm³.

b Find the capacity in mL.

c Find the capacity in L.



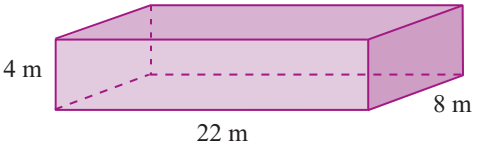
16 A rectangular prism measures 22 m by 4 m by 8 m.

a Find the volume in cm³.

b Find the capacity in mL.

c Find the capacity in L.

d Find the capacity in kL.



17 A rectangular water storage tank measures 2.9 m by 7.5 m by 2.5 m.

a Find its volume in cm³.

b Find its capacity in L.

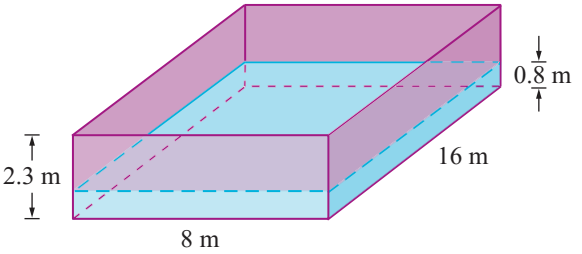
18 The internal dimensions of a refrigerator are height = 1.8 m, width = 84 cm and depth = 60 cm.

a Find the internal volume in cm³.

b Find the capacity in L.

Change all measurements to the same units.

19 The diagram shows a swimming pool 16 m long and and 8 m wide.



a How many litres of water are needed to fill the pool to a depth of 0.8 m?

b How much more water is needed to fill the pool to a depth of 2.3 m?

c Calculate the cost of filling the pool from empty if water costs 25.8 cents per kilolitre.

Language in mathematics

1 Here is a find-a-word puzzle. Find all the words in the following list:

area, base, breadth, calculate, capacity, container, conversion, convert, dimensions, estimate, height, kilolitre, length, liquid, litre, kite, measure, millilitre, object, perimeter, prefix, prism, quadrilateral, rectangle, rhombus, trapezium, volume, weight

K	I	L	O	L	I	T	R	E	A	R	H	O	M	B	U	S
T	H	D	A	E	R	B	A	S	E	F	N	T	B	J	D	F
C	D	B	H	E	I	G	H	T	C	T	O	C	K	G	I	E
C	X	E	Z	P	R	E	F	I	X	H	I	E	M	H	M	I
T	R	A	P	E	Z	I	U	M	I	G	S	J	I	U	E	L
A	M	E	A	S	U	R	E	A	Y	I	R	B	L	Y	N	E
E	B	N	W	K	V	L	L	T	H	E	E	O	L	J	S	R
L	L	I	Q	U	I	D	I	E	T	W	V	N	I	A	I	T
G	E	A	A	T	Y	C	T	N	M	Q	N	Z	G	R	O	I
N	N	T	U	X	A	P	R	I	S	M	O	M	R	E	N	L
A	G	N	W	P	T	R	E	V	N	O	C	Q	T	A	S	I
T	T	C	A	L	C	U	L	A	T	E	O	I	M	S	L	L
C	H	C	O	N	T	A	I	N	E	R	K	T	A	P	C	L
E	W	L	A	R	E	T	A	L	I	R	D	A	U	Q	U	I
R	P	E	R	I	M	E	T	E	R	Y	N	R	R	M	S	M

2 Write two common metric units of:

a perimeter **b** area **c** capacity

3 What does each of these prefixes mean?

a milli- **b** centi- **c** kilo-

4 Complete the following words, used in this chapter, by inserting the vowels.

sp__c__ pr__sm c__nv__ rs__ __n s__rf__c__

p__r__m__t__r v__l__m__ l__tr__ c__p__c__ty

5 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.

heigt lenth liquid mililitre occupy uniform

6 Use a dictionary to find two meanings of each word.

a kite **b** solid **c** capacity

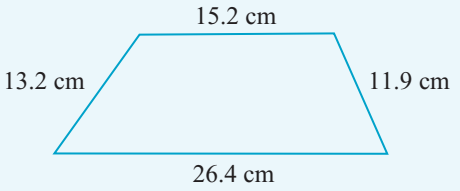
Terms

area base capacity centimetre conversion kilometre
kite parallelogram perimeter prism rectangle rhombus
square trapezium volume

Check your skills

1 The perimeter of this shape. is:

A 65.4 cm **B** 59.7 cm
C 66.7 cm **D** 64 cm



2 The area of a square is 49 cm^2. Its side length is:

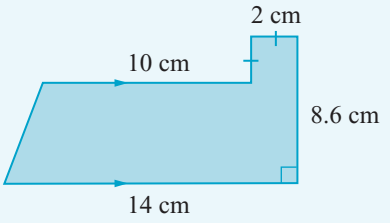
A 12.25 cm **B** 14 cm **C** 7 cm **D** 24.5 cm

3 The area of a triangle with a base of 15.2 cm and height of 6 cm is:

A 21.2 cm^2 **B** 45.6 cm^2 **C** 64.7 cm^2 **D** 91.2 cm^2

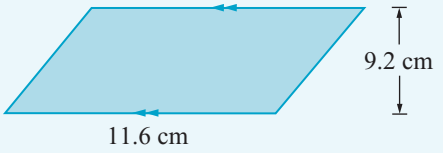
4 The area of this composite shape is:

A 89.8 cm^2 **B** 175.6 cm^2
C 120.4 cm^2 **D** 223.6 cm^2



5 The area of the parallelogram shown is:

A 41.6 cm^2 **B** 20.8 cm^2
C 106.72 cm^2 **D** 123.05 cm^2

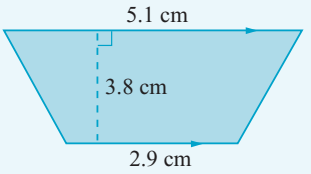


6 The formula $A = \frac{1}{2}xy$ could be used to find the area of a:

A parallelogram **B** trapezium
C rhombus **D** all of these

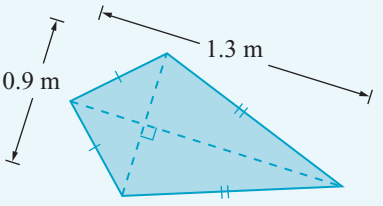
7 The area of this trapezium is closest to:

A 15.2 cm^2 **B** 30.4 cm^2
C 56.202 cm^2 **D** 28.101 cm^2



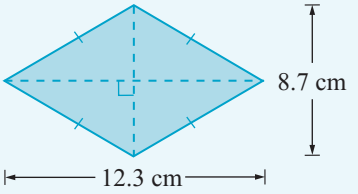
8 The area of this kite is:

A 1.17 m^2 **B** 2.34 m^2
C 0.585 m^2 **D** 2.2 m^2



9 The area of this rhombus is:

A 107.01 cm^2 **B** 13.37625 cm^2
C 26.7525 cm^2 **D** 53.505 cm^2



10 The conversion of 7 m^2 to cm^2 is:

A 70 cm^2 **B** 700 cm^2
C 7000 cm^2 **D** 70 000 cm^2

11 The conversion of 300 cm^2 to m^2 is:

A 0.03 m^2 **B** 0.3 m^2 **C** 3 m^2 **D** 30 m^2

12 The conversion of 0.08 ha to m^2 is:

A 8 m^2 **B** 80 m^2 **C** 800 m^2 **D** 8000 m^2

13 The volume occupied by 2.6 kL is:

A 2.6 m^3 **B** 0.26 m^3 **C** 0.026 m^3 **D** 26 m^3

14 The volume occupied by 0.8 kL. is:

A 0.008 m^3 **B** 0.8 m^3 **C** 8 m^3 **D** 80 m^3

15 The conversion of 1.73 m^3 to kilolitres is:

A 1.73 kL **B** 17.3 kL **C** 173 kL **D** 1730 kL

16 The conversion of 0.09 m^3 to kilolitres is:

A 0.9 kL **B** 0.009 kL **C** 0.09 kL **D** 0.0009 kL

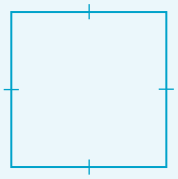
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–5	6–9	10–12	13–16
Section	A	B	C	D

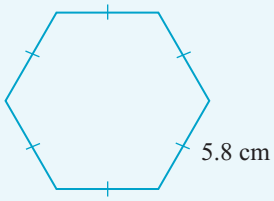
5A Review set

1 Calculate the perimeter of each shape.

a

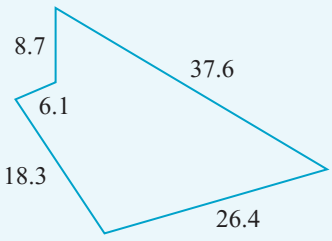


b

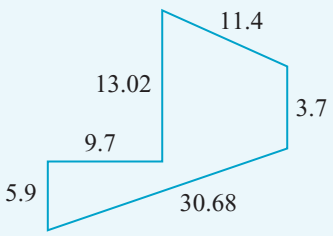


2 Calculate the perimeter of each shape. All measurements are in centimetres.

a



b

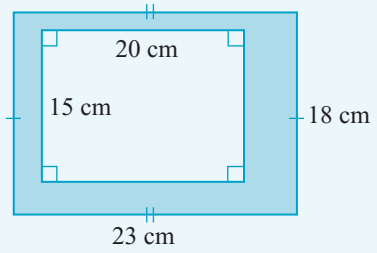


3 Calculate the area of a square with sides 9.7 cm.

4 Calculate the area of a parallelogram with a base of 12 cm and a height of 7.3 cm.

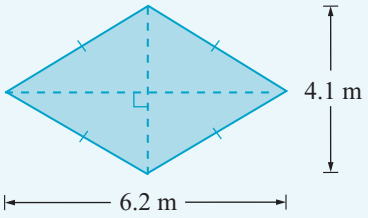
5 If shapes have the same perimeter, do they have the same area? Why or why not? Explain, giving reasons for your answer.

6 Calculate the area of the shaded region.

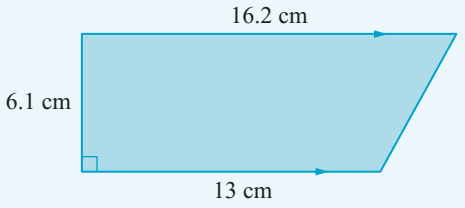


7 Find the area of these plane shapes.

a



b

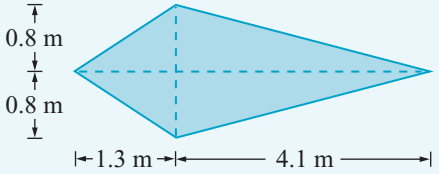


8 Write the formula for the area of these shapes.

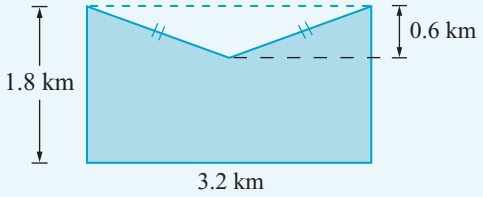
a rectangle **b** kite **c** rhombus

9 Calculate the area of each shape.

a



b



10 Complete these conversions.

a 5 cm^2 = ___ mm^2 **b** 1130 cm^2 = ___ m^2 **c** 480 mm^2 = ___ cm^2

d 2.3 m^2 = ___ cm^2 **e** 6.88 cm^2 = ___ mm^2 **f** 2.5 m^2 = ___ ha

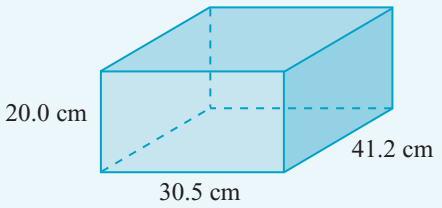
g 40 cm^3 = ___ L **h** 63 kL = ___ m^3 **i** 0.02 ML = ___ m^3

11 The capacity of a container is 800 mL. What is the total capacity, in litres, of six of these containers?

12 A small fish tank is in the shape of a rectangular prism with dimensions 20.0 cm × 30.5 cm × 41.2 cm.

a Calculate the volume of the fish tank.

b What is the capacity of the tank in litres?

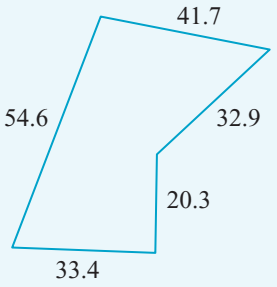


5B Review set

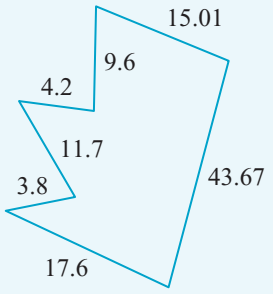
1 Calculate the perimeter of a rectangle with width 18.6 cm and length 25.9 cm.

2 Calculate the perimeter of each shape. All measurements are in centimetres.

a

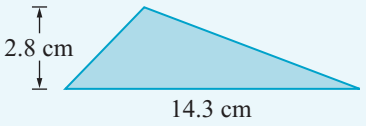


b

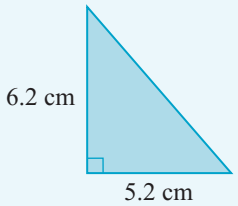


3 Calculate the area of each shape.

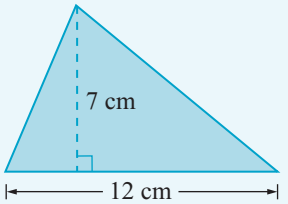
a



b



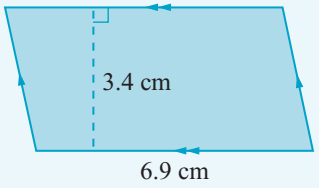
c



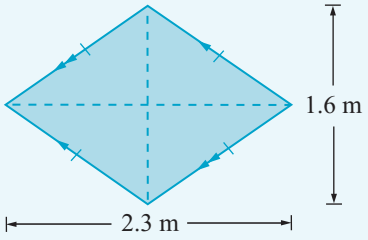
4 Define the terms *perimeter*, *area* and *volume*.

5 Find the area of these plane shapes.

a



b

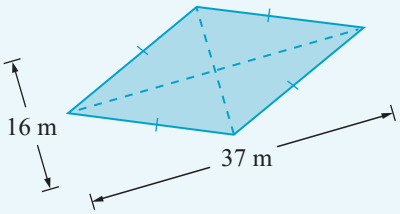


6 Write the formula for the area of each shape.

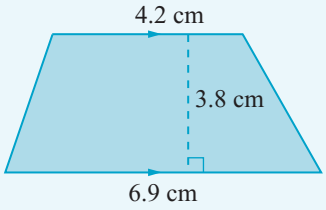
a square **b** triangle **c** parallelogram

7 Calculate the areas of each shape.

a



b



8 Complete these conversions.

a $2\text{ cm}^2 = \_\_\_\ \text{mm}^2$ **b** $300\text{ cm}^2 = \_\_\_\ \text{m}^2$ **c** $930\text{ mm}^2 = \_\_\_\ \text{cm}^2$

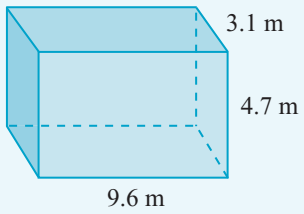
d $4.2\text{ m}^2 = \_\_\_\ \text{cm}^2$ **e** $33.9\text{ cm}^2 = \_\_\_\ \text{mm}^2$ **f** $2700\text{ m}^2 = \_\_\_\ \text{ha}$

g $600\text{ cm}^3 = \_\_\_\ \text{L}$ **h** $15.3\text{ kL} = \_\_\_\ \text{m}^3$ **i** $0.004\text{ ML} = \_\_\_\ \text{m}^3$

9 A rectangular tank is used to store water.

a Calculate its volume in m^3.

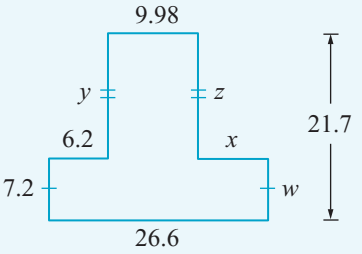
b Calculate its capacity in litres.



5C Review set

1 A regular octagon has a perimeter of 138.4 cm. Calculate the length of each side.

2 Find the length of each side marked with a pronumeral, then calculate the perimeter. All measurements are in centimetres.



3 Calculate the area of a rectangle with side lengths of 15.2 cm and 8.4 cm.

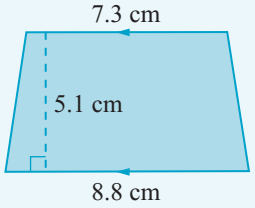
4 Which shape has the greater area?

A a rectangle 41.6×12

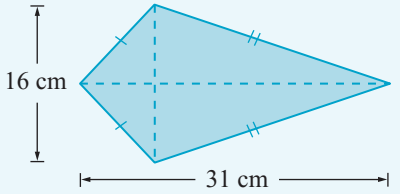
B a square 22.5×22.5

5 Find the area of these plane shapes.

a



b

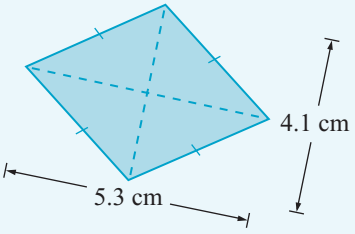


6 Write the formulas for the areas of these shapes.

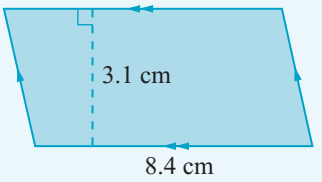
a rhombus **b** parallelogram **c** rectangle

7 Calculate the area of each shape.

a



b



8 Complete these conversions.

a $4\ cm^2 =$ ___ mm^2 **b** $700\ cm^2 =$ ___ m^2

c $530\ mm^2 =$ ___ cm^2 **d** $9.5\ m^2 =$ ___ cm^2

e $32.9\ cm^2 =$ ___ mm^2 **f** $7300\ m^2 =$ ___ ha

g $6000\ cm^3 =$ ___ L **h** $2\ kL =$ ___ m^3

i $5000\ m^3 =$ ___ ML **j** $3.25\ L =$ ___ cm^3

9 Calculate in cm^3, the space occupied by the following volumes of water.

a 2 L **b** 0.86 L

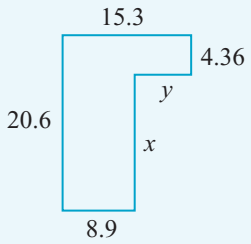
10 Calculate the capacity in litres of a container with a volume of:

a $600\ cm^3$ **b** $18\ 000\ cm^3$

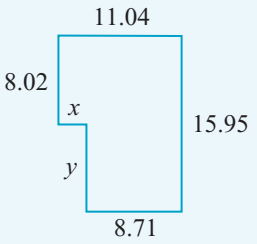
5D Review set

1 Calculate the length of each side marked with a pronumeral, then calculate the perimeter. All measurements are in centimetres.

a

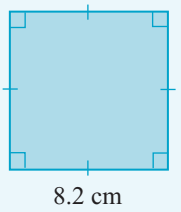


b

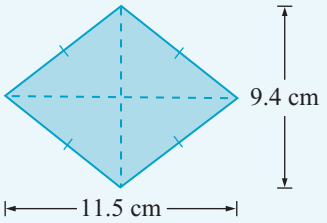


2 Calculate the area of each shape.

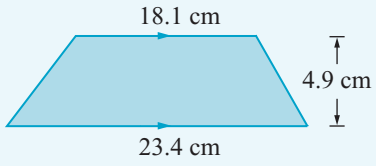
a



b



c

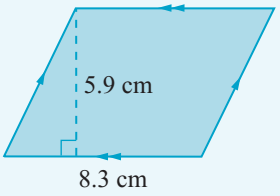


3 How many 1 cm cubes would fit into a rectangular prism with dimensions 14 cm × 8 cm × 5 cm?

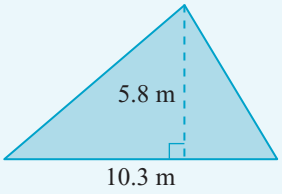
4 Explain the relationship between capacity and volume.

5 Find the area of these plane shapes.

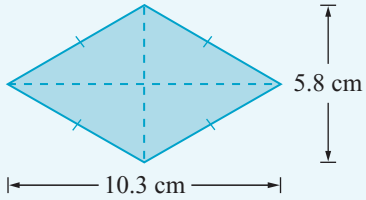
a



b



c

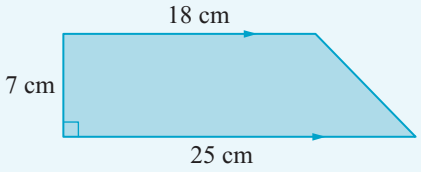


6 Write the formula for the area of the following shapes.

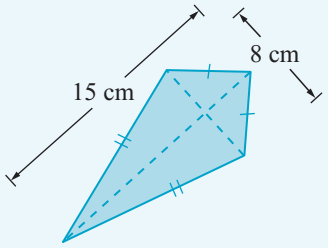
a rectangle **b** rhombus **c** trapezium

7 Calculate the area of each shape.

a



b



8 Complete these conversions.

a 5 cm^2 = ___ mm^2 **b** 400 cm^2 = ___ mm^2 **c** 7500 mm^2 = ___ cm^2

d 8.5 m^2 = ___ cm^2 **e** 5400 cm^2 = ___ m^2 **f** 60 000 m^2 = ___ ha

g 1 cm^2 = ___ mm^2 **h** 1 kL = ___ mL **i** 1 kL = ___ cm^3

j 1 m^3 = ___ L **k** 3 m^3 = ___ kL **l** 2 ML = ___ m^3

9 A rectangular swimming pool is 7 m wide, 15 m long and 1.8 m deep.

a How many litres of water are needed to fill the pool?

b Calculate the cost of filling the pool if water costs 18.3 cents per kilolitre.

Chapters 2–5 Cumulative review

1 **a** What is a ratio?

b Why don't ratios have units?

c Express this ratio in simplest form: 1.2 m : 42 cm.

d Express these ratios in simplest form.

i 25 min : $1\frac{1}{2}$ h **ii** 1.2 m : 0.9 m **iii** 30 : 45 **iv** 125 : 75

e A bag contains red and green frogs. There are 24 frogs of which 16 are red. Write the ratio of red frogs to green frogs.

f A bag contains mints and fruities. There are 18 lollies of which 5 are mints. Write the ratio of mints to fruities.

g Find x when $\frac{5}{x} = \frac{8}{3}$.

h Mick buys a trail bike for $1150 and later sells it for $1300. Write these ratios.

i cost price : selling price **ii** profit : selling price

i Find the value of x if $5 : 12 = 25 : x$.

j The length and breadth of a rectangle are in the ratio 11 : 4.

i Find the length of the rectangle when the breadth is 6.2 cm.

ii Find the breadth of the rectangle when the length is 38.5 cm.

k Which is the better buy?

A 1.2 L of Fizz-Whiz Cola at $1.15

B 2.5 L of Fizz-Whiz Cola at $2.35

l Divide 1500 kg in the ratio 7 : 8.

m Divide $2000 in the ratio 13 : 7 : 5.

n Mark cycles 8 km in 18 min. Express this as a rate in km/min correct to 1 decimal place.

o If 18 apples cost $5.40, what is the cost per apple in cents?

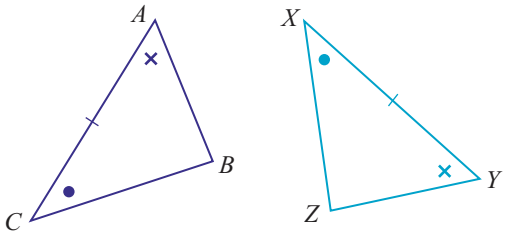
p A map has a scale 1 : 200. Convert a scale distance of 3.2 cm to an actual distance.

q The actual height of a new building is to be 625 m. If a model of the building is constructed using a scale of 1 : 1500, calculate the height of the model.

2 **a** **i** Write a correct statement of congruency for this pair of congruent figures.

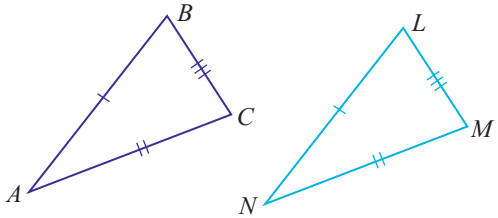
ii Name the side that corresponds to BC.

iii Name the angle that corresponds to angle B.



b Write congruence statements for these pairs of congruent triangles.

i



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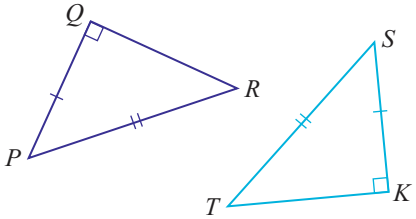

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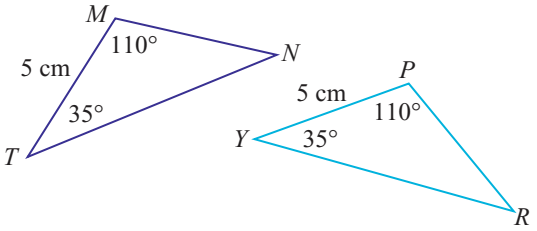

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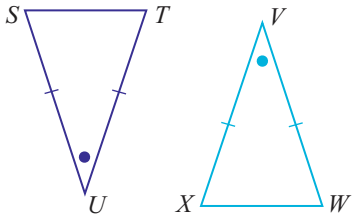
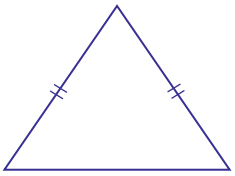


c Divide each shape into two congruent figures.

i



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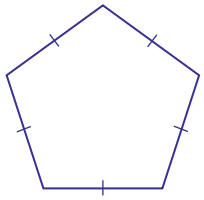
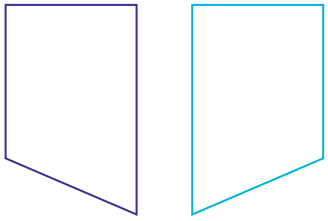


iii

d What transformation(s) could have been used to produce the following pairs of congruent figures?

i



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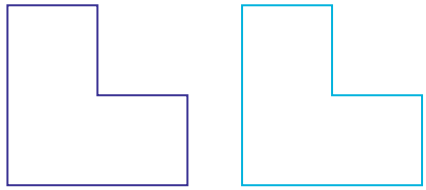
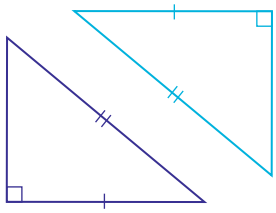
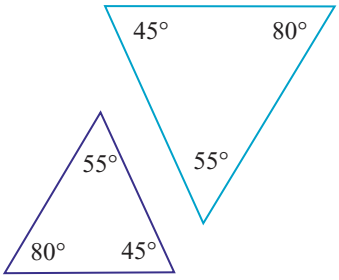


e State the congruency test used to show that these triangles are congruent.

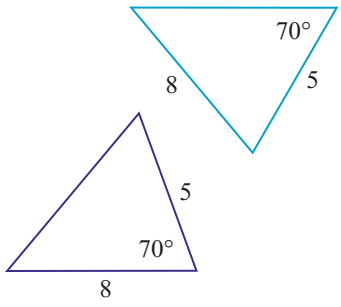


f State why the following pairs of triangles are not congruent.

i



ii



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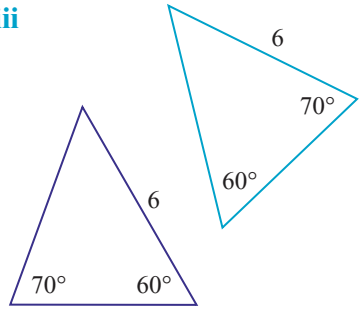
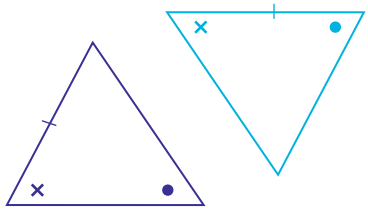
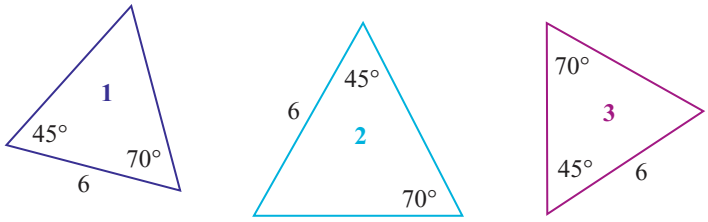


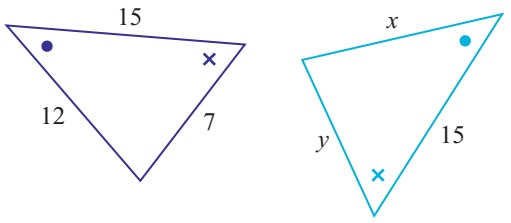
g State why this pair of triangles is not congruent.



h Which triangles are congruent? Give a reason for your answer.



i State why this pair of triangles is congruent, and hence find the values of the pronumerals.



j Complete the following proof.

Data: In $\triangle PQR$, $\angle R = \angle Q$.

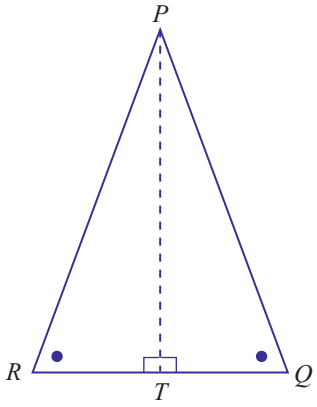
Aim: To prove $PR = PQ$.

Construction: Draw PT perpendicular to RQ as shown.

Proof: In $\triangle PTR$ and $\triangle PTQ$

$\angle R = \angle Q$	(_____)
$PT =$ ___	(common side)
$\angle PTR = \angle$___	(by construction)
$\therefore \triangle PTR \equiv \triangle PTQ$	(_____)
$\therefore PR = PQ$	(_____)

The sides opposite the equal angles of a triangle are equal.



k Complete the following proof.

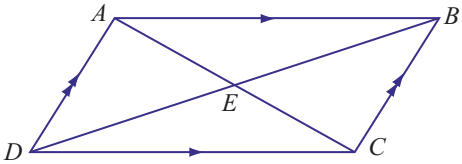
Data: ABCD is a parallelogram. (*AB* || *DC* and *AD* || *BC*)

Aim: To prove that the diagonals of a parallelogram bisect each other.

Proof: In $\triangle AED$ and $\triangle CEB$

$\angle DAE = \angle$___	(alternate angles and *AD* || *BC*)
$\angle AED = \angle BEC$	(_____)
$AD =$ ___	(opposite sides of a parallelogram are equal)
$\triangle AED \equiv \triangle CEB$	(_____)
$\therefore AE =$ ___	(matching sides of congruent triangles)
and $DE =$ ___	(_____)

The diagonals of a parallelogram bisect each other.



3 **a** Write $2 \times 2 \times 2 \times 2 \times 2 \times 2$ in index form.

b Write the base and index of 4^7.

c Write 3^5 in expanded form.

d Is $5^3 \times 2^4 = 10^7$? Explain.

e Use a calculator to evaluate the following.

i 5^4 **ii** 6^5 **iii** 2^{15}

f Simplify the following, leaving the answer in index form.

i $5^{12} \times 5^3$ **ii** $(3^5)^4$ **iii** $7 \times x^0$

g Simplify:

i $7^8 \times 7^7$ **ii** $(5^3)^4$ **iii** $3^{10} \div 3^6$

h Explain why $(3^4)^5 \neq 3^9$.

i Simplify:

i $5 \times (-2) \times 3$ **ii** $-6 \times (-3) \times 3$ **iii** $-4 \times (-2) \times 5$

j Simplify:

i $15 \div (-3)$ **ii** $27 \div (-9)$ **iii** $-36 \div 9$

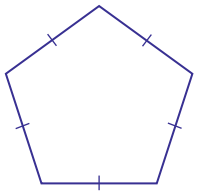
k Simplify the following.

i $9 - 2 \times 4$ **ii** $-9 + 4 \times 5$ **iii** $10 + 15 - 21$

iv $-36 \div 6 \times 4$ **v** $6 \times (-8) \div (-12)$ **vi** $36 \div (11 - 15)$

4 a Calculate the perimeter of each shape.

i



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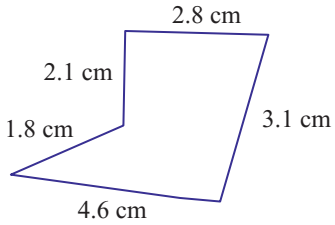
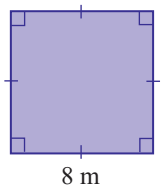


b A square has a perimeter of 16.8 cm. What is the length of each side?

c Find the area of each shape.

i



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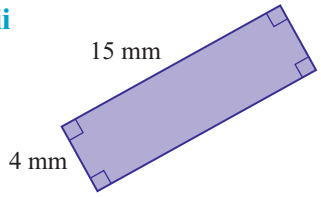

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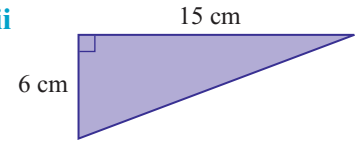

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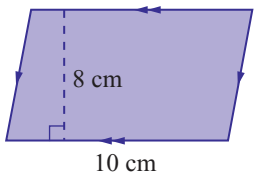

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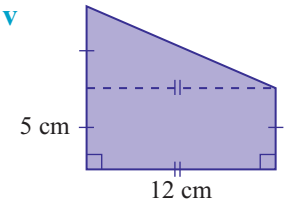

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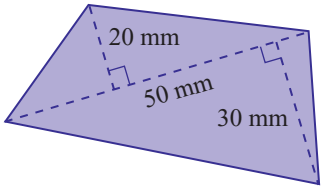

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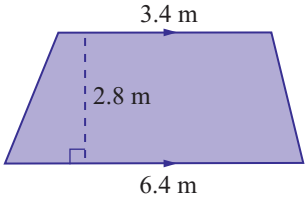

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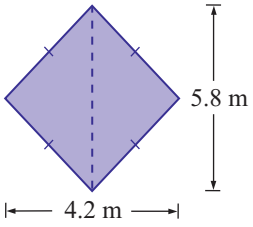

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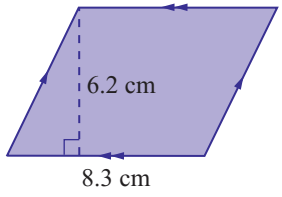
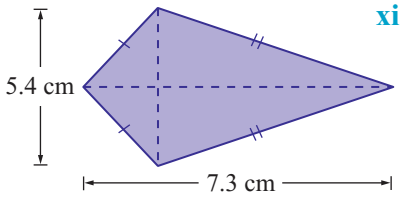


x



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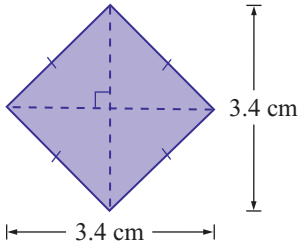

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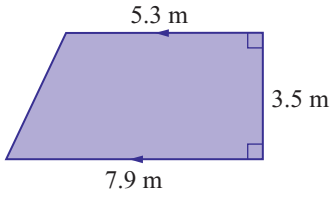
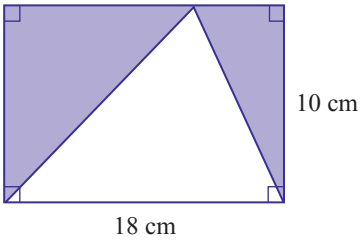


d Calculate the shaded area of each shape.

i



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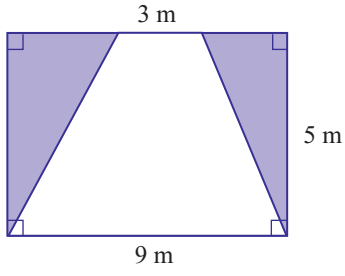

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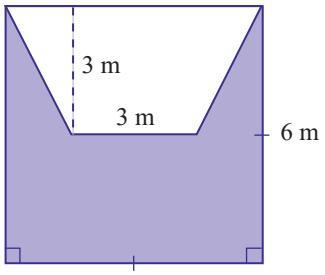

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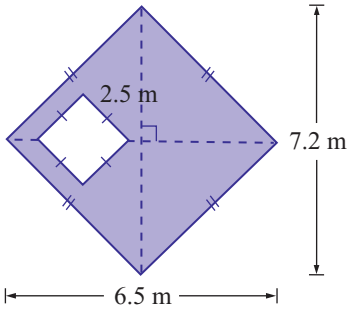


e Complete the following conversions.

i 1 cm^3 = ___ mm^3 **ii** 1 m^3 = ___ cm^3

iii kL = ___ mL **iv** 1 m^3 = ___ L

f Complete these conversions.

i 4 cm^2 = ___ mm^2 **ii** 120 cm^2 = ___ m^2

iii 340 mm^2 = ___ cm^2 **iv** 4.1 m^2 = ___ cm^2

v 6.32 cm^2 = ___ mm^2 **vi** 3.9 m^2 = ___ ha

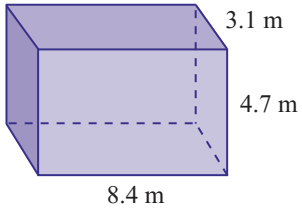
vii 3.9 ha = ___ m^2 **viii** 95 cm^3 = ___ L

ix 58 kL = ___ m^3 **x** 0.05 ML = ___ m^3

g A rectangular tank is used to store water.

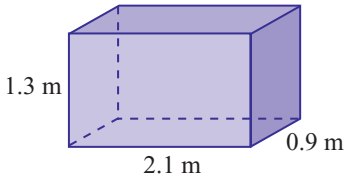
i Calculate its volume in m^3.

ii Calculate its capacity in litres.



h Calculate the volume of these rectangular prisms.

i



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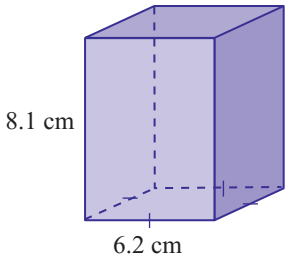


i A water tank is rectangular in shape and has dimensions 1.5 m by 0.5 m by 3.2 m.

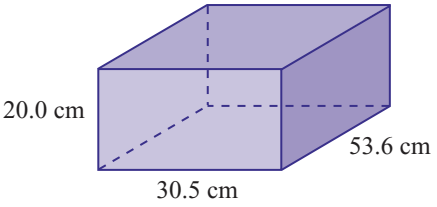
i What is its volume in m^3?

ii What is its capacity in litres?

j A small fish tank is in the shape of a rectangular prism with dimensions 20.0 cm × 30.5 cm × 53.6 cm.

i Calculate the volume of the fish tank.

ii What is the capacity of the tank in litres?



k A rectangular swimming pool is 5 m wide, 12 m long and 1.8 m deep.

i How many litres of water are needed to fill the pool?

ii Calculate the cost of filling the pool if water costs 14.3 cents per kilolitre.

6

Time

This chapter deals with time.

At the end of this chapter you should be able to:

- add and subtract time
- compare times
- calculate time differences
- perform calculations involving time
- perform calculations across different time zones
- use the $^{\circ}\,'\,''$ key on your calculator to
  - interpret calculator displays
  - add and subtract times
  - round times to the nearest hour and minute.

NSW Syllabus references: S4 M&G Time
Outcomes: MA4-1WM, MA4-2WM, MA3-13MG, MA4-15MG
MEASUREMENT & GEOMETRY – ACMMG199

Diagnostic test

1 How many hours in 2 days?

A 12 **B** 24 **C** 48 **D** 36

2 How many hours in $4\frac{1}{2}$ days?

A 135 h **B** 54 h
C 108 h **D** 99 h

3 How many minutes in $1\frac{1}{2}$ days?

A 90 **B** 84 **C** 2160 **D** 2880

4 How many seconds in $4\frac{1}{2}$ minutes?

A 270 **B** 240 **C** 135 **D** 120

5 How many minutes in 2 hours?

A 66 **B** 86 **C** 120 **D** 135

6 The time shown on this clock face is:

A 5 past 2
B 2 past 5
C 10 past 2
D 10 past 5

7 1645 h is:

A quarter to five **B** forty-five past six
C 4:45 am **D** 16:45 am

8 Five past seven in the morning in digital time is:

A 5:07 am **B** 5:07 pm
C 7:05 pm **D** 7:05 am

9 As analogue time, 1918 h is:

A 7:18 am **B** 7:18 pm
C 19:18 am **D** 19:18 pm

10 Gavin caught the 7:43 am bus from Wentworthville station to school. If the journey takes 35 min, at what time does he arrive at school?

A 8:20 am **B** 8:18 am
C 8:13 am **D** 8:15 am

Questions **11** and **12** refer to the train timetable below.

11 At what time does the 3:34 pm train from Harris Park arrive at Milsons Point?

A 4:02 pm **B** 4:12 pm
C 4:15 pm **D** 4:27 pm

12 Sarah caught the 3:21 pm train from Parramatta. How long did it take her to reach Town Hall?

A 30 min **B** 25 min
C 27 min **D** 29 min

		pm	pm	pm	pm	pm	pm	pm	pm
PARRAMATTA dep.		3:13	3:21	3:24	3:28	3:33	3:39	3:46	3:51
Harris Park		3:14				3:34		3:47	3:52
Granville		3:16	3:23	3:26		3:36	3:41	3:49	3:54
Auburn		3:20				3:39		3:53	3:58
Lidcombe	S	3:24		3:32		3:43		3:57	4:02
STRATHFIELD arr.		3:29	3:33	3:37		3:48	3:51	4:02	4:08
dep.		3:30	3:34	3:38		3:49	3:52	4:03	4:09
BURWOOD		3:31		3:39					4:10
REDFERN		3:41	3:45	3:49		3:59	4:03	4:14	4:22
CENTRAL arr.	S	3:44	3:47	3:52		4:02	4:05	4:17	4:24
dep.		3:45	3:48	3:53		4:03	4:06	4:18	4:25
TOWN HALL	S	3:47	3:50	3:55		4:05	4:08	4:20	4:27
WYNYARD	S	3:50	3:53	3:58		4:08	4:11	4:23	4:30
Milsons Point	S	3:54	3:57	4:02		4:12	4:15	4:27	4:34
NORTH SYDNEY		3:56	3:59	4:04		4:14	4:17	4:29	4:36

The Diagnostic test questions refer to the Year 7 outcomes from ACMMG110 and ACMMG139.

A Analogue clocks review

The short hand is the **hour** hand. It is pointing to 12 so it is 12 o'clock.
The long hand is the **minute** hand. It is pointing to 2 so it is 10 minutes past 12.
The other hand is the **second** hand. It is pointing to 7 so 35 seconds have passed.
Thus the time shown is: 'ten minutes and thirty-five seconds past twelve'.

12 : 10 : 35

hours : minutes : seconds

The time shown on this clock is 'ten minutes to six' or 'five-fifty' or 5:50.

The time shown on this clock is 'twenty-five minutes past three' or 'three twenty-five' or 3:25.

Exercise 6A

1 State the time shown by each clock.

a

b

c

d

2 Draw a clock face to show each of the following times.

a six fifty-five
b twenty minutes past two
c eleven forty-five
d five minutes to four

3 State the time shown by each clock, including seconds.

a

b

c

d

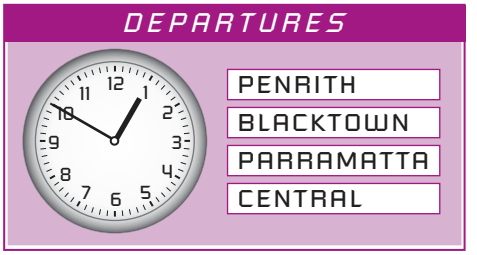
4 Draw a clock face to show each of the following times.

a fifteen minutes and ten seconds past three
b two twenty-eight and thirty-five seconds
c ten minutes and thirteen seconds to twelve
d twelve minutes and forty-nine seconds past seven
e six twenty-four and forty seconds
f one forty and thirty-three seconds.

5 Nicole wanted to catch a train from Penrith to Central. She looked at the board and decided to catch the fast train, which departed at the time shown on the board.

a How long must she wait for the train if she arrived at Penrith Station at twenty-five to one?

b If Nicole expects to arrive at Central Station at a quarter to two, how long will the journey take?



B Digital displays review

When time is shown in numerical form, the display usually indicates if it is am or pm.

This watch face shows 'eleven twenty-two and thirty-seven seconds pm'.

When reading times we usually do not need the seconds, so would say 'eleven twenty-two' or 'twenty-two minutes past eleven'.

The time displayed here is 'six zero two pm' (two minutes past six at night).

The time displayed here is 'two forty-two am' (forty-two minutes past two in the morning).

Forty-two minutes past two is also 'eighteen minutes to three'.

Exercise 6B

1 Write each of the following times as it would appear on a digital display.

Be careful! 10:50 is 'ten to eleven'.

a five minutes past seven

b twelve seventeen

c six minutes past twelve

d eleven fifty-three

e ten to seven

f a quarter to three

g half past two

h twenty-five to nine

i twelve minutes to six

j nine minutes to four

k eleven minutes to ten

l eight minutes to twelve

2 Match each digital clock to an analogue clock showing the same time.

a 07:52:04 **b** 12:09:36 **c** 04:41:52 **d** 01:13:21

i 

ii 

iii 

iv 

3 Determine the following times. Write the answer as it would appear on a digital display.

a 15 min after 10:50 am

b 17 min before 7:43 am

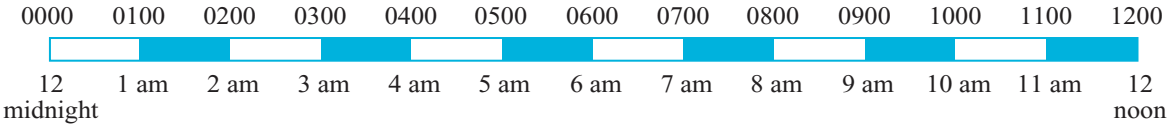
c 32 min after 3:54 pm

d 24 min before 2:08 pm

C 24-hour time review

Often 24-hour time is used, such as for airline schedules and on many electrical appliances. 24-hour time uses 4-digit numbers, without any dots of separation; so, for example, we write 1245 not 12:45.

The charts below help explain 24-hour time.

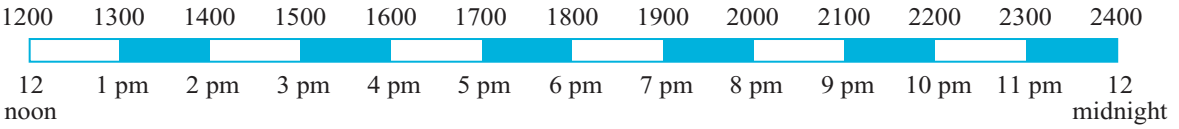


0100 reads 01 hundred hours

0800 reads 08 hundred hours

1200 reads 12 hundred hours

From midnight until noon is am.



1400 reads 14 hundred hours

2000 reads 20 hundred hours

2300 reads 23 hundred hours

For pm times, 12 hours is added to the analogue time. ∴ 1 pm = 1300 (1 + 12 = 13).

Here are some more times.

24-hour time	12-hour time
0220	2:20 am
0531	5:31 am
1204	12:04 pm
1549	3:49 pm
2117	9:17 pm

Remember after 1200 it is pm.

Exercise 6C

1 Write the following 24-hour times as analogue times (am or pm).

a 1406 **b** 0052 **c** 2002 **d** 0128

e 0815 **f** 2323 **g** 1918 **h** 1111

i 1255 **j** 1739 **k** 0634 **l** 2242

2 Convert the following to 24-hour time.

a 7:32 pm	**b** 5:20 am	**c** 1:32 am	**d** 12:52 pm
e 4:30 pm	**f** 1:47 pm	**g** 6:09 am	**h** 8:17 am
i 9:25 pm	**j** 11:01 pm	**k** 12 midnight	**l** 10:29 pm

3 Order each set of times, which occur on the same day, from earliest to latest.

a five past seven at night, 1926, 8:31 pm, 1304

b two thirty in the morning, 8 past midnight, 9:06 am, 0805, 1715

c 1204, 11:45 pm, midday, seven twenty-three at night, 1038, 12:54 pm

EXAMPLE 1

Calculate the following in 24-hour time.

a 0800 + $2\frac{1}{2}$ h

b 1718 − 1 h 43 min

Convert 24-hour time to am or pm.
Add or subtract.
Express the answer in 24-hour time.

a 8:00 am + 2 h 30 min = 10:30 am
which is 1030 in 24-hour time.

b 5:18 pm − 1 h 43 min = 3:35pm
which is 1535 in 24-hour time.

4 Calculate the following in 24-hour time.

a 0500 + $2\frac{1}{2}$ h	**b** 1030 − $1\frac{1}{2}$ h	**c** 0945 − 14 min
d 1432 + 25 min	**e** 1508 + 1 h 46 min	**f** 1223 − 2 h 21 min

5 Write the time in 24-hour notation for each of the following.

a 15 minutes after 1356 hours

b 3 hours after 1728 hours

c 48 minutes before 0904 hours

d $2\frac{1}{2}$ hours before 1016 hours

6 Brad decided to record the following TV programs. Determine the 24-hour times he would use to set up his recorder to record these programs.

	TV program	Analogue time	24-hour time
a	*Play School*	4:30 pm to 5:30 pm	___ to ___
b	*My Animal Friends*	6:00 am to 6:30 am	___ to ___
c	*The Late Show*	11:25 pm to 1:30 am	___ to ___
d	*The Pyramids*	8:30 pm to 10:00 pm	___ to ___

7 Using the data in the following table, state the time (am or pm) each flight arrives at Brisbane Airport.

	Flight number	Origin	Arrival	Gate#
a	QF650	Melbourne	0410	21
b	QF777	Adelaide	0950	20
c	QF798	Sydney	1120	8
d	QF680	Perth	1840	23
e	QF747	Townsville	2130	19

8 Sandeep woke at 0550 hours. At 0630 h he started working at his computer. At 1015 h he stopped working and decided to go and play golf. He started playing golf at 1108 and finished at 1546. Answer the following in am or pm times.

a At what time did Sandeep wake up?
b At what time did he start working?
c At what time did he stop working?
d For how long did Sandeep play golf?

D Solving time problems

EXAMPLE 1

A movie starts at 1:40 pm and finishes at 4:10 pm. What is the duration of the movie?

1:40 to 2:00 is 20 min
2:00 to 4:00 is 2 h
4:00 to 4:10 is 10 min
Duration of the movie = 2 h 30 min

Exercise 6D

1 Calculate the number of minutes between the times given below (all times are pm) and 1500 hours.

a one thirty
b twelve minutes past one
c twenty-one past one
d one forty-three
e 1:39
f twelve twenty-five

2 Calculate the number of minutes between the times given below (all times are am) and 1310 hours.

a half past eleven
b eleven fifty-three
c twenty-two past eleven
d forty-five minutes to midday

3 Determine these times.

a 15 min after 0840 hours
b 17 min before 1149 hours
c 32 min after 1749 hours
d 24 min before 1523 hours

EXAMPLE 2

Calculate the following in 24-hour time.

a $0900 + 1\frac{1}{4}$ h

b 1743 − 1 h 18 min

a 0900 + 1 h 15 min = 1015

b 1743 − 1 h = 1643
1643 − 18 min = 1625

4 Complete the following, writing the answer in 24-hour time.

a $0700 + 3\frac{1}{2}$ h = ___ + 3 h 30 min
= 10___

b 1643 − 2 h 12 min = ___ − 2___
= ___31

5 Calculate the following in 24-hour time.

a 0730 + 3 h

b $1250 - 1\frac{1}{2}$ h

c 1245 − 45 min

d 1327 + 42 min

e 2243 + 1 h 52 min

f 0025 − 3 h 20 min

EXAMPLE 3

Calculate the following times.

a 4 h 20 min + 2 h 53 min

b 7 h 13 min − 4 h 46 min

a

hours	*minutes*	
4	20	Place the hours and minutes in columns.
+ 2	53	
6	73	Add the minutes and record the answer.
= 1 h 13 min		Convert 73 min to 1 h 13 min.
1	13	Place this answer in the hours and minutes column.
7	13	Add the hours to obtain an answer.

∴ 4 h 20 min + 2 h 53 min = 7 h 13 min

b Convert 7 h 13 min to 6 h 73 min by taking 1 h (60 min) from 7 h.

hours	*minutes*	
6	73	Place the hours and minutes in columns.
− 4	46	
2	27	Subtract the hours and minutes and record the answer.

∴ 7 h 13 min − 4 h 46 min = 2 h 27 min

6 Complete to calculate the following.

a 3 h 25 min + 4 h 42 min

hours	*minutes*
3	___
+ ___	42
___	67
= 1 h ___ min	
1	___
8 h	7 min

b 9 h 14 min − 3 h 28 min
Convert 9 h 14 min to 8 h ___ min by taking 1 h (60 min) from 9 h.

hours	*minutes*
8	___
− 3	28
___ h	46 min

7 Calculate the following.

a 1 h 15 min + 3 h 20 min
b 6 h 50 min − 2 h 20 min
c 5 h 42 min + 1 h 34 min
d 3 h 25 min − 1 h 40 min
e 2 h 18 min + 55 min
f 1 h 13 min − 48 min
g 4 h 5 min + 1 h 57 min
h 2 h 37 min − 1 h 45 min
i 8 h 55 min + 2 h 55 min
j 7 h 12 min − 4 h 22 min
k 10 h 38 min + 4 h 49 min
l 8 h 52 min − 6 h 58 min

8 Calculate the following.

a 5 min 10 s + 8 min 20 s
b 12 min 32 s + 9 min 51 s
c 13 min 38 s + 15 min 42 s
d 9 min 51 s + 11 min 59 s
e 4 min 41 s − 2 min 27 s
f 7 min 35 s − 5 min 43 s
g 14 min 5 s − 8 min 39 s
h 3 min 12 s − 1 min 58 s
i 53 min 17 s − 9 min 26 s

EXAMPLE 4

Calculate the following.

a 4 days 13 h + 3 days 22 h
b 3 days 7 h − 1 day 23 h

a

	days	*hours*	
	4	13	Place the days and hours in columns.
+	3	22	
	7	35	Add the hours and record the answer.
		= 1 d 11 h	Convert 35 h to 1 day 11 h.
	1	11	Place this answer in the days and hours column.
	8	11	Add the days to obtain an answer.

∴ 4 days 13 h + 3 days 22 h = 8 days 11 h

b Convert 7 h to 31 h by taking 1 day (24 h) from 3 days giving 2 days 31 h.

	days	*hours*	
	2	31	Place the days and hours in columns.
−	1	23	
	1	8	Subtract days and hours and record the answer.

∴ 3 days 7 h − 1 day 23 h = 1 day 8 h

9 Calculate the following.

a 3 days 16 h + 4 days 18 h
b 2 days 21 h + 3 days 23 h
c 5 days 17 h − 2 days 10 h
d 4 days 2 h − 1 days 17 h
e 2 weeks 2 days + 1 weeks 6 days
f 5 weeks 5 days + 7 weeks 6 days
g 5 weeks 1 day − 2 weeks 3 days
h 11 weeks 2 days − 4 weeks 5 days

EXAMPLE 5

Calculate the elapsed time between:

a 9:38 am and 2:50 pm **b** 6:42 pm and 9:25 pm

a Convert 2:50 pm to 24-hour time: 2:50 pm to 1450

hours	*minutes*
14	50
− 9	38
5 h	12 min

∴ Time between 9:38 am and 2:50 pm is 5 h 12 min.

A period of time that has passed is known as elapsed time.

b Convert both to 24-hour time: 6:42 pm to 1842 and 9:25 pm to 2125

hours	*minutes*
20	85
− 18	42
2	43

Convert 21 h 25 min to 20 h 85 min.

∴ Time between 6:42 pm and 9:25 pm is 2 h 43 min.

10 Calculate the elapsed time between:

a 5:31 am and 10:50 am **b** 8:27 am and 11:48 am **c** 2:08 pm and 7:36 pm
d 4:23 pm and 10:58 pm **e** 8:17 am and 3:42 pm **f** 6:08 am and 5:37 pm
g 0457 and 1412 **h** 1140 and 2225 **i** 0312 and 1303

EXAMPLE 6

Calculate the elapsed time between 7:45 am Monday and 1:22 pm Wednesday of the same week.

7:45 am Monday to 7:45 am Wednesday = 2 days
7:45 am Wednesday to 1:22 pm Wednesday = 12 h 82 min − 7 h 45 min = 5 h 37 min
Total amount of time between 7:45 am Monday to 1:22 pm Wednesday = 2 days 5 h 37 min

11 Calculate the elapsed time (in days, hours, minutes) between the following.

a 4:38 pm Tuesday to 6:30 am Thursday **b** 0709 Saturday to 1216 Wednesday
c 0652 Monday to 2015 pm Friday **d** 1:46 pm Sunday to 7:25 pm Monday
e 10:17 am Wednesday to 9:38 am Saturday **f** 12:24 am Friday to 7:15 am Wednesday

E Using a calculator

Time calculations can be done on your calculator. The times are entered into the calculator in this order: hours, minutes, seconds.

Enter the hours and press the o ' " key. Next enter the minutes and press o ' " again. Then finally enter the seconds and press o ' " a third time. If there are no hours to be entered, press 0 followed by the o ' " key, as the calculator records the information in the order hours, minutes, seconds.

EXAMPLE 1

Use your calculator to enter the following times.

a 2 h **b** 4 h 10 min **c** 6 h 25 min 50 s **d** 17 min **e** 32 s

Check your calculator manual to be sure you are using the $^{\circ}\,'\,''$ *key correctly.*

		Calculator steps	Display	Actual time
a	2 h	2 $^{\circ}\,'\,''$ =	2° 0′ 0″	2 h, 0 min, 0 s
b	4 h 10 min	4 $^{\circ}\,'\,''$ 10 $^{\circ}\,'\,''$ =	4° 10′ 0″	4 h, 10 min, 0 s
c	6 h 25 min 50 s	6 $^{\circ}\,'\,''$ 25 $^{\circ}\,'\,''$ 50 $^{\circ}\,'\,''$ =	6° 25′ 50″	6 h, 25 min, 50 s
d	17 min	0 $^{\circ}\,'\,''$ 17 $^{\circ}\,'\,''$ =	0° 17′ 0″	0 h, 17 min, 0
e	32 s	0 $^{\circ}\,'\,''$ 0 $^{\circ}\,'\,''$ 32 $^{\circ}\,'\,''$ =	0° 0′ 32″	0 h, 0 min, 32 s

Exercise 6E

1 Enter the following times and record the calculator display shown.

a 5 h **b** 4 h **c** 3 h 12 min
d 7 h 23 min **e** 2 h 45 min 18 s **f** 9 h 2 min 36 s
g 36 min **h** 55 s **i** 6 h 59 min 45 s

EXAMPLE 2

Use your calculator to evaluate the following.

a 4 h 15 min + 2 h 58 min **b** 18 min + 5 h 12 min − 3 h 15 min 20 s

a 4 $^{\circ}\,'\,''$ 15 $^{\circ}\,'\,''$ + 2 $^{\circ}\,'\,''$ 58 $^{\circ}\,'\,''$ = 7° 13′ 0″

4 h 15 min + 2 h 58 min = 7 h 13 min 0 s

b 0 $^{\circ}\,'\,''$ 18 $^{\circ}\,'\,''$ + 5 $^{\circ}\,'\,''$ 12 $^{\circ}\,'\,''$ − 3 $^{\circ}\,'\,''$ 15 $^{\circ}\,'\,''$ 20 $^{\circ}\,'\,''$ = 2° 14′ 40″

18 min + 5 h 12 min − 3 h 15 min 20 s = 2 h 14 min 40 s

2 Use your calculator to evaluate the following.

a 6 h 11 min + 3 h 28 min **b** 7 h 27 min + 2 h 13 min
c 5 h 42 min + 52 min **d** 9 h − 3 h 5 min 24 s
e 2 h 19 min 8 s − 38 min 43 s **f** 8 h 18 min 41 s − 5 h 29 min 58 s
g 3 h + 2 h 14 min − 1 h 33 min 52 s **h** 7 min + 58 s + 3 h + 58 min 40 s
i 2 h 18 min − 53 min 22 s + 45 s **j** 1 h 18 s − 36 min + 5 min 26 s
k 6 h 13 min 20 s − 37 min 14 s + 52 min 25 s − 1 h 12 min 32 s
l 51 min + 3 h 24 min 17 s − 1 h 9 min 56 s + 38 min 13 s

3 Convert the following to decimals correct to 3 decimal places by entering the time using the $^\circ\,'\,''$ button then pressing SHIFT $^\circ\,'\,''$.

a 2 h
b 5 h 13 min
c 6 h 28 min
d 3 h 32 min
e 1 h 48 min 52 s
f 2 h 36 s
g 46 min
h 25 min
i 24 s
j 16 s
k 8 min 55 s
l 29 min 47 s

4 Complete the table below. Remember:

- 1 h = 60 min
- To simplify $\frac{5}{60}$ use your fraction button. 5 $a\frac{b}{c}$ 60 = 1 ⌋ 12 $\therefore \frac{1}{12}$
- Press $a\frac{b}{c}$ again to express the answer in decimal form. 1 ⌋ 12 $a\frac{b}{c}$ 0.0833.

Minutes	Fraction		Decimal (2 decimal places)
	60	Simplified	
5	$\frac{5}{60}$	$\frac{1}{12}$	0.08
6	$\frac{6}{60}$		
12	$\frac{\square}{60}$		
15	$\frac{15}{60}$	$\frac{1}{4}$	0.25
18			
20			
24			
27			
30			
36			
40		$\frac{2}{3}$	0.67
42			
45			
48			
54			
60			

EXAMPLE 3

Express each time in hours, minutes and seconds.

a $2\frac{1}{4}$ h
b $4\frac{5}{9}$ h

a 2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 4 = $^\circ\,'\,''$ 2° 15′ 0″

$2\frac{1}{4}$ h = 2 h 15 min 0 s

b 4 $a\frac{b}{c}$ 5 $a\frac{b}{c}$ 9 = $^\circ\,'\,''$ 4° 33′ 20″

$4\frac{5}{9}$ h = 4 h 33 min 20 s

5 Express each time in hours, minutes and seconds.

a $1\frac{1}{2}$ h **b** $3\frac{3}{4}$ h **c** $6\frac{4}{5}$ h **d** $4\frac{2}{3}$ h **e** $5\frac{5}{6}$ h

f $8\frac{5}{12}$ h **g** $2\frac{4}{9}$ h **h** $7\frac{7}{8}$ h **i** $3\frac{3}{5}$ h **j** $6\frac{1}{6}$ h

k $10\frac{5}{8}$ h **l** $11\frac{6}{13}$ h **m** $5\frac{4}{11}$ h **n** $8\frac{1}{7}$ h **o** $12\frac{3}{8}$ h

EXAMPLE 4

Use your calculator to express the following decimals as hours, minutes and seconds.

a 4.36 **b** 7.42 **c** 0.457

a 4.36 = ° ′ ″

= 4° 21′ 36″

4 h 21 min 36 s

b 7.42 = ° ′ ″

= 7° 25′ 12″

7 h 25 min 12 s

c 0.457 = ° ′ ″

= 0° 27′ 25.2″

0 h 27 min 25.2 s

6 Express the following decimals as hours, minutes and seconds.

a 2.68 **b** 1.05 **c** 4.82 **d** 3.6 **e** 5.4

f 8 **g** 0.5 **h** 0.2 **i** 0.4 **j** 0.86

k 0.34 **l** 0.574 **m** 10.04 **n** 0.045 **o** 3.987 64

EXAMPLE 5

Round the calculator displays as stated.

a 3° 59° 15″ to the nearest hour **b** 5° 34° 22″ to the nearest minute

a 59 min > 30 min ∴ round up

= 4 h

b 22 s < 30 s ∴ round down

= 5 h 34 min

1 h = 60 min, 59 min > 30, round up

1 min = 60 s, 22 s < 30, round down

7 Round these calculator display times as stated.

a to the nearest hour

i 2° 42′ 0″ **ii** 3° 28′ 0″ **iii** 5° 30′ 0″ **iv** 6° 18′ 25″

v 12° 53′ 42.6″ **vi** 4° 12′ 37.9″ **vii** 1° 52′ 18.2″ **viii** 7° 30′ 58.6″

b to the nearest minute

i 0° 27′ 46″ **ii** 0° 32′ 18″ **iii** 2° 13′ 25.6″ **iv** 1° 48′ 31.2″

v 8° 41′ 0.87″ **vi** 9° 16′ 52.6″ **vii** 0° 32′ 16.9″ **viii** 1° 56′ 30″

8 Each week Rob has six English lessons that each last 38 minutes.

a How much time does he spend in English lessons each week?

b If a term consists of 10 weeks, how much time is spent in English lessons each term?

9 A History teacher marked 220 essays in 26 hours 35 minutes. Find the average time spent marking one essay. (To calculate the average find: time taken ÷ number of essays.)

10 If I spend 10 minutes every night writing summaries for Science, how many hours and minutes of summaries will I have written in the month of March? (There are 31 days in March.)

EXAMPLE 6

Use your calculator to find the elapsed time between 3:40 am and 4:28 pm.

Step 1: Convert both times to 24-hour time.
3:40 am = 0340, 4:28 pm = 1628 (pm, so add 12 hours)

Step 2: Use the calculator as follows: 16 [°′″] 28 [°′″] [−] 3 [°′″] 40 [°′″] [=]

The calculator display will show 12° 48′ 0″, which is 12 h and 48 min.

11 Use your calculator to determine the elapsed time between:

a 3:20 am and 9:43 am
b 11:17 am and 6:28 pm
c 10:53 am and 3:19 pm
d 4:20 pm and 6:43 pm
e noon and twenty to seven pm
f 0728 hours and 1519 hours

F Calendars

Here are some helpful hints to remembering the number of days in each month.

Verse: *Thirty days has September, April, June and November.*
All the rest have thirty-one, except February, which has twenty-eight days clear, and twenty-nine in each leap year.

Knuckles: The knuckles have 31 days.
Between the knuckles have 30 days.
Except February, which has 28 days and 29 days in each leap year.

Leap years

Leap years occur every 4 years and have 366 days (the extra day being in February). To determine if a year is a leap year:

- Divide the year by 4. If $\frac{\text{year}}{4}$ is a whole number, the year is a leap year.
- If the year is a century year, divide by 400. If $\frac{\text{year}}{400}$ is a whole number, the year is a leap year.

For example:

Leap years		*Non-leap years*	
$\frac{1988}{4} = 497$	√	$\frac{2009}{4} = 502.25$	x
$\frac{2000}{400} = 5$	√	$\frac{1800}{400} = 4.5$	x

For leap years the answer must be whole numbers. Hint: Refer to number of days in a year.

Exercise 6F

Use the calendar below to complete this exercise.

January

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	10	21	22	23	24
25	26	27	28	29	20	31

February

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

March

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	10	21
22	23	24	25	26	27	28
29	30	31				

April

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

May

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

July

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

August

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

September

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

October

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

November

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

December

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

1 **a** Without knowing the year, how can you determine whether or not the year is a leap year?

b Is this calendar a leap year?

c Australia Day is 26 January. On what day of the week does this fall?

d Christmas Day is 25 December. On what day of the week does this fall?

e How many days between 17 April and 3 June (both dates inclusive)?

f Rod went to the doctor on 4 September. If he needs to make another appointment in five weeks time, what date is his next appointment?

g Blake booked a skiing trip to commence on 13 July. If it is now 20 April, how many weeks must be wait until he goes skiing?

h Briony plays water polo on the second Saturday of the month from August to December. List the dates on which she plays water polo.

i Rachel's credit card statement date is 19 October. She has 31 days to make the payment owing before interest is charged. By what date must she pay October's statement amount to avoid paying interest?

j Con works part-time for a butcher. He must give three weeks notice if he is going to resign. If he decides to leave on 23 August, when must he give notice to his employer?

k Determine the number of school days for your current school year during:

i Term I **ii** Term II **ii** Term III **iv** Term IV

EXAMPLE 1

Susan was born on 23/7/1999. Calculate her age as at 16/10/2014.

Years: 23/7/1999 to 23/7/2014 is 15 years.
Months: 23/7/2014 to 23/9/2014 is 2 months.
Days: September has 30 days.
$\therefore$ 23/9/2014 to the end of the month is 7 days.
Then add the 16 days of October: $7 + 16 = 23$ days
Susan's age is 15 years 2 months 23 days.

2 Calculate each person's age as at today's date given their date of birth.

a Tony: 14/10/1991 **b** Marilyn: 25/2/1996 **c** Chloe: 31/8/2004
d Honey: 4/5/1983 **e** Melissa: 11/6/1972 **f** Troy: 1/12/1998

Investigation 1 Birthdays do change!

In 2012 Wally's birthday fell on a Sunday. In 2013 it fell on a Monday and in 2014 it fell on a Tuesday. Between leap years, the day on which a particular event occurs, such as a birthday, advances by one.

1 What happens when a birthday occurs in a leap years?

2 Investigate why there is a change from each day of the week.

Investigation 2 1 month = 4 weeks

February is the only month that has 28 days, except in a leap year when February has 29 days. Investigate and show why there are not 4 weeks in every month.

Can you determine a better average for the number of weeks in each month?

Investigation 3 Leap years

Investigate why we have a leap year every 4 years. Why is it every 4 years? Where does the extra day come from?

Investigation 4 Calendar dates

1 Research the development of the current Western calendar.

2 Find out what the Gregorian calendar is.

3 Compare the calendars of Christian, Orthodox, Muslim, Jewish, Chinese and Aboriginal peoples.

4 List the days of great importance to these peoples and place these dates on your own calendar.

G Using timetables and charts

Exercise 6G

1 Here is part of the Carlingford–City train timetable.

Carlingford–City Timetable							
	pm	pm	pm	pm	pm	pm	pm
Carlingford	3:32	4:11	4:45	5:23	5:56	6:26	6:52
Telopea	3:34	4:13	4:47	5:25	5:57	6:28	6:54
Dundas	3:36	4:15	4:49	5:27	5:59	6:30	6:56
Rydalmere	3:38	4:17	4:51	5:29	6:01	6:32	6:58
Camellia	3:40	4:19	4:53	5:31	6:03	6:34	7:00
Rosehill UA	3:42	4:21	4:55	5:33	6:05	6:36	7:02
Clyde..... arr	3:45	4:24	4:58	5:36	6:08	6:39	7:05
dep	3:51	4:26	5:00	5:48	6:18	6:48	7:06
Lidcombe..... arr							
dep	3:57	4:31	5:06	5:54	6:24	6:54	7:12
Strathfield..... arr	4:02	4:36	5:11	5:59	6:29	6:59	7:18
dep	4:03	4:37	5:12	6:00	6:30	7:00	7:23
Central..... arr	4:17	4:50	5:26	6:14	6:44	7:14	7:36
dep	4:18	4:51	5:27	6:15	6:45	7:15	7:37
Town Hall	4:21	4:54	5:30	6:18	6:48	7:18	7:40
Wynyard	4:24	4:57	5:33	6:20	6:50	7:20	7:42

a i I am meeting a friend at Town Hall station. Which is the latest train from Carlingford that I can catch to arrive before 7:00 pm?

ii At what time does this train leave Lidcombe station?

b If I miss the 6:26 pm train at Carlingford, what would be the earliest time I could arrive at Wynyard?

c i How long does the 4:36 pm train from Strathfield take to get to Wynyard?

ii How long does the 4:47 pm train from Telopea take to get to Central?

iii How long does the 6:30 pm train from Dundas take to get to Town Hall?

iv How long does the 4:21 pm train from Rosehill take to get to Strathfield?

d i Calculate the time taken for the 3:32, 4:11, 4:45, 5:23 and 5:56 Carlingford trains to travel to Wynyard.

ii Why are the times different?

2 Below is part of the North Sydney–Richmond/Emu Plains train timetable.

North Sydney–Richmond/Emu Plains																		
	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am	am
NORTHSYDNEY	9:30	9:39		9:54		10:00	10:09	10:24		10:30	10:39		10:54		11:00	11:09	11:24	11:30
Milsons Point	9:32	9:41		9:56		10:02	10:11	10:26		10:32	10:41		10:56		11:02	11:11	11:26	11:32
WYNYARD	9:35	9:44		9:59		10:05	10:14	10:29		10:35	10:44		10:59		11:05	11:14	11:29	11:35
TOWN HALL	9:38	9:47		10:02		10:08	10:17	10:32		10:38	10:47		11:02		11:08	11:17	11:32	11:38
CENTRAL arr	9:41	9:50		10:05		10:11	10:20	10:35		10:41	10:50		11:05		11:11	11:20	11:35	11:41
dep	9:42	9:51	10:02	10:06		10:12	10:21	10:36		10:42	10:51	11:02	11:06		11:12	11:21	11:36	11:42
REDFERN	9:43	9:52		10:07		10:13	10:22	10:37		10:43	10:52		11:07		11:13	11:22	11:37	11:43
BURWOOD	9.53					10:23				10:53					11:23			11:53
STRATHFIELD arr	9:55	10:03	10:13	10:18		10:25	10:33	10:48		10:55	11:03	11:13	11:18		11:25	11:33	11:48	11:55
dep	9:56	10:04	10:14	10:19		10:26	10:34	10:49		10:56	11:04	11:14	11:19		11:26	11:34	11:49	11:56
Flemington	9:58					10:28				10:58					11:28			11:58
Lidcombe	10:01	10:09				10:31	10:39			11:01	11:09				11:31	11:39		12:01
Auburn	10:03					10:33				11:03					11:33			12:03
Clyde	10:06					10:36				11:06					11:36			12:06
Granville	10:08	10:15		10:29		10:38	10:45	10:59		11:08	11:15		11:29		11:38	11:45	11:59	12:08
Harris Park	10:09				10:29	10:39			10:59	11:09				11:29	11:39			12:09
PARRAMATTA arr	10:11	10:17	10:24	10:31	10:31	10:41	10:47	11:01	11:01	11:11	11:17	11:24	11:31	11:31	11:41	11:47	12:01	12:11
dep	10:12	10:18	10:25	10:32	10:32	10:42	10:48	11:02	11:02	11:12	11:18	11:25	11:32	11:32	11:42	11:48	12:02	12:12
Westmead	10:14	10:20	10:27	10:34	10:34	10:44	10:50	11:04	11:04	11:14	11:20	11:27	11:34	11:34	11:44	11:50	12:04	12:14
Wentworthville		10:22			10:36		10:52		11:06		11:22			11:36		11:52		
Pendle Hill		10:24			10:39		10:54		11:09		11:24			11:39		11:54		
Toongabbie		10:27			10:41		10:57		11:11		11:27			11:41		11:57		
Seven Hills	10:20	10:30		10:40	10:44	10:50	11:00	11:10	11:14	11:20	11:30		11:40	11:44	11:50	12:00	12:10	12:20
BLACKTOWN arr		10:34			10:48		11:04		11:18		11:34			11:48		12:04		
dep	10:24	10:35	10:36	10:44		10:54	11:05	11:14		11:24	11:35	11:36	11:44		11:54	12:05	12:14	12:24
Marayong		10:38					11:08				11:38					12:07		
Quakers Hill		10:41					11:11				11:41					12:11		
Schofields		10:48					11:18				11:48					12:18		
Riverstone		10:51					11:21				11:51					12:21		
Vineyard		10:54									11:54							
Mulgrave		10:58									11:58							
Windsor		11:02									12:02							
Clarendon		11:05									12:05							
East Richmond		11:08									12:08							
Richmond		11:10									12:10							
Doonside	10:27			10:47		10:57		11:17		11:27			11:47		11:57		12:17	12:27
Rooty Hill	10:30			10:50		11:00		11:20		11:30			11:50		12:00		12:20	12:30
MOUNT DRUITT	10:32			10:53		11:02		11:23		11:32			11:53		12:02		12:23	12:32
St Marys	10:37			10:57		11:07		11:27		11:37			11:57		12:07		12:27	12:37
Werrington	10:39			10:59		11:09		11:29		11:39			11:59		12:09		12:29	12:39
Kingswood	10:43			11:03		11:13		11:33		11:43			12:03		12:13		12:33	12:43
PENRITH	10:46		10:49	11:06		11:16		11:36		11:46		11:49	12:06		12:16		12:36	12:46
Emu Plains			10:52									11:52						
Proceeds to			LTH									KTO						

a **i** How long does the 9:30 am train from North Sydney take to get to Penrith?

ii How long does the 10:02 am train from Town Hall take to get to Seven Hills?

iii How long does the 11:26 am train from Strathfield take to get to Werrington?

iv How long does the 11:52 am train from Wentworthville take to get to Riverstone?

b **i** If I arrive at Blacktown at 10:34 am, which train did I catch from Milsons Point?
ii If I arrive at Auburn at 11:03 am, which train did I catch from Redfern?
iii If I arrive at Werrington at 12:09 pm, which train did I catch from Clyde?
iv If I arrive at Penrith at 12:46 pm, which train did I catch from Westmead?

c I am travelling from Burwood to Clarendon to meet a friend for Sunday lunch at 12:30 pm.
i Can I catch a train directly from Burwood to Clarendon?
ii What is the closest station to Burwood from which I can get a connecting train to Clarendon?
iii If I am to be on time, at what time should I depart Burwood?
iv How long has the journey taken from Burwood to Clarendon?
v If I am being picked up at the station at 12:30 pm, how long must I wait?

3 Part of a gym program is shown. Use it to answer the following questions.

a Sarah gets to the gym at 6:18 am. She decides to do yoga and aerobics.
i How long will she have to wait before the class starts?
ii How much exercise will she complete this morning?

b Carla completes the 8 am jazzercise and all the classes until the end of the 11:30 yoga class. For how long does she exercise?

c Dylan gets to the gym in time for the 3:30 pm Pump class. The class takes 50 minutes. He then takes 20 minutes to shower and change and a further 25 minutes to walk home. At what time does he get home?

6:30 am	Yoga
7:15 am	Aerobics
8:00 am	Jazzercise
8:30 am	Yoga
9:30 am	Aerobics
10:00 am	Step class
10:30 am	Boxing
11:30 am	Yoga
12:30 pm	Beginners step class
1:30 pm	Advanced aerobics
2:30 pm	Stretch class
3:00 pm	Yoga
3:30 pm	Pump

4 Use the table on the right to answer these questions.

a How many days are there between:
i the new moon and a full moon?
ii the 1st quarter and the 3rd quarter?

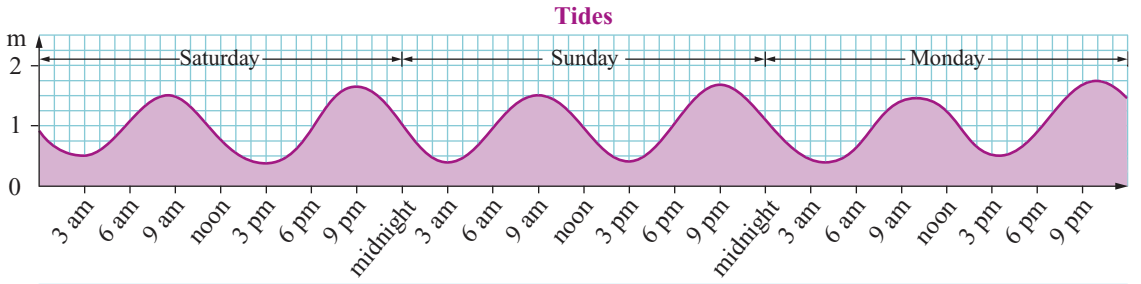
b Determine the elapsed time between:
i sunrise and sunset today
ii moonrise and moonset tomorrow.

IN THE SKY

	Sunrise	Sunset	Moonrise	Moonset
TODAY	6:55 am	4:53 pm	3:53 am	3:04 pm
TOMORROW	6:56 am	4:53 pm	4:52 am	3:38 pm

New moon	*1st quarter*	*Full moon*	*3rd quarter*
11 June	18 June	25 June	3 July

5 Use the tidal chart to answer the questions below.



2:30 am	8:34 am	2:40 pm	8:54 pm	3:06 am	9:10 am	3:09 pm	9:26 pm	3:45 am	9:46 am	3:39 pm	10:00 pm
0.4m	1.5 m	0.4 m	1.6 m	0.4 m	1.5 m	0.4 m	1.7 m	0.4 m	1.4 m	0.5 m	1.7 m

Tides information: • Port Stephens: + 5 min • Jervis Bay: + 8 min • Port Hacking: + 8 min

a How many low tides occur each day?

b How many high tides occur each day?

c On Saturday at what times is there a:

i high tide? **ii** low tide?

d On Sunday, Allan went fishing between 5 pm and 8 pm. Explain what was happening with the tides while he was fishing.

e State the times and the height of each tidal change on Monday.

f **i** How long did it take to change from low tide to high tide on Sunday morning?

ii Did it take the same length of time to go from low to high tide on Sunday afternoon? If not, what was the time difference?

g State the time of the first high tide on Monday at the following locations.

i Port Stephens **ii** Port Hacking

6 Use this bus timetable to answer these questions.

Blacktown to Seven Hills via Kings Langley																	
Weekdays																	
Showing Route Number	718	718	718	718	718	718	718	718	718	718	718	718	718	718	718	718	718
	am														pm		
Westpoint (Patrick St)						7:32		8:41	9.00	9:24	10:02	10:31	11:03	11:31	12:02	12:31	1:02
BLACKTOWN City arriv			6:07	6:33	6:48	7:35	8:06	8:44	9:05	9:31	10:03	10:33	11:04	11:33	12:03	12:33	1:03
BLACKTOWN Station			6:11	6:40	7:08	7:37	8:10	8:48	9:07	9:37	10:07	10:37	11:07	11:37	12:07	12:37	1:07
Sunnyholt Rd/Cambridge St			6:15	6:44	7:12	7:43	8:14	8:52	9:11	9:41	10:11	10:41	11:11	11:41	12:11	12:41	1:11
Vardys Rd/Green St	5:24	5:50	6:20	6:49	7:17	7:48	8:19	8:57	9:16	9:46	10:16	10:46	11:16	11:46	12:16	12:46	1:16
KINGS LANGLEY Shops	5:28	5:55	6:25	6:54	7:22	7:53	8:24	9:02	9:21	9:51	10:21	10:51	11:21	11:51	12:21	12:51	1:21
Whitby Rd /James Cook Dr	5:31	5:58	6:28	6:57	7:25	7:56	8:27	9:05	9:24	9:54	10:24	10:54	11:24	11:54	12:24	12:54	1:24
Seven Hills North School	5:35	6:03	6:33	7:02	7:31	8:02	8:33	9:10	9:29	9:59	10:29	10:59	11:29	11:59	12:29	12:59	1:29
SEVEN HILLS Station	5:40	6:09	6:39	7:09	7:39	8:10	8:40	9:16	9:35	10:05	10:35	11:05	11:35	12:05	12:35	1:05	1:35
SEVEN HILLS to City	5:46	6:16	6:46	7:16	7:45	8:16	8:45	9:27	9:45	10:15	10:45	11:15	11:45	12:15	12:45	1:15	1:45
SEVEN HILLS Shops						8:13	8:44	9:17	9:39	10:06	10:36	11:09	11:39	12:09	12:39	1:09	1:39

a On Friday, Caryn decided to catch a bus from Cambridge Street to Seven Hills station.

i If she arrives at Cambridge Street at 10:23 am, how long does she have to wait for the bus to arrive?

ii At what time would she arrive at Seven Hills station?

iii How long does she have to wait for a bus to the city?

b Greg caught the 12:02 pm bus from Westpoint (Patrick Street) to Kings Langley shops.

i How long did the journey take?

ii Greg stayed at Kings Langley shops for 45 min and realised that he would have to go to Seven Hills shops to complete his shopping. At what time did the next bus arrive?

iii How long did Greg have to wait at Kings Langley for the bus after shopping for 45 min?

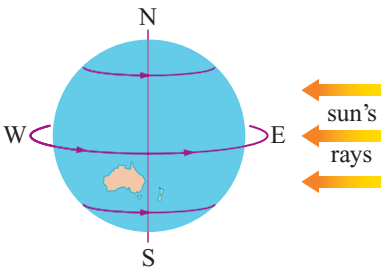
iv At what time did Greg arrive at Seven Hills?

v If he shopped for a further 1 h and 13 min, at what time did he complete his shopping?

H Time zones

Earth rotates from west to east about its axis causing day and night on Earth. Due to this rotation, when the sun rises in Sydney:

- Perth is still in darkness
- Wellington (NZ) has already had a couple of hours of sunlight.

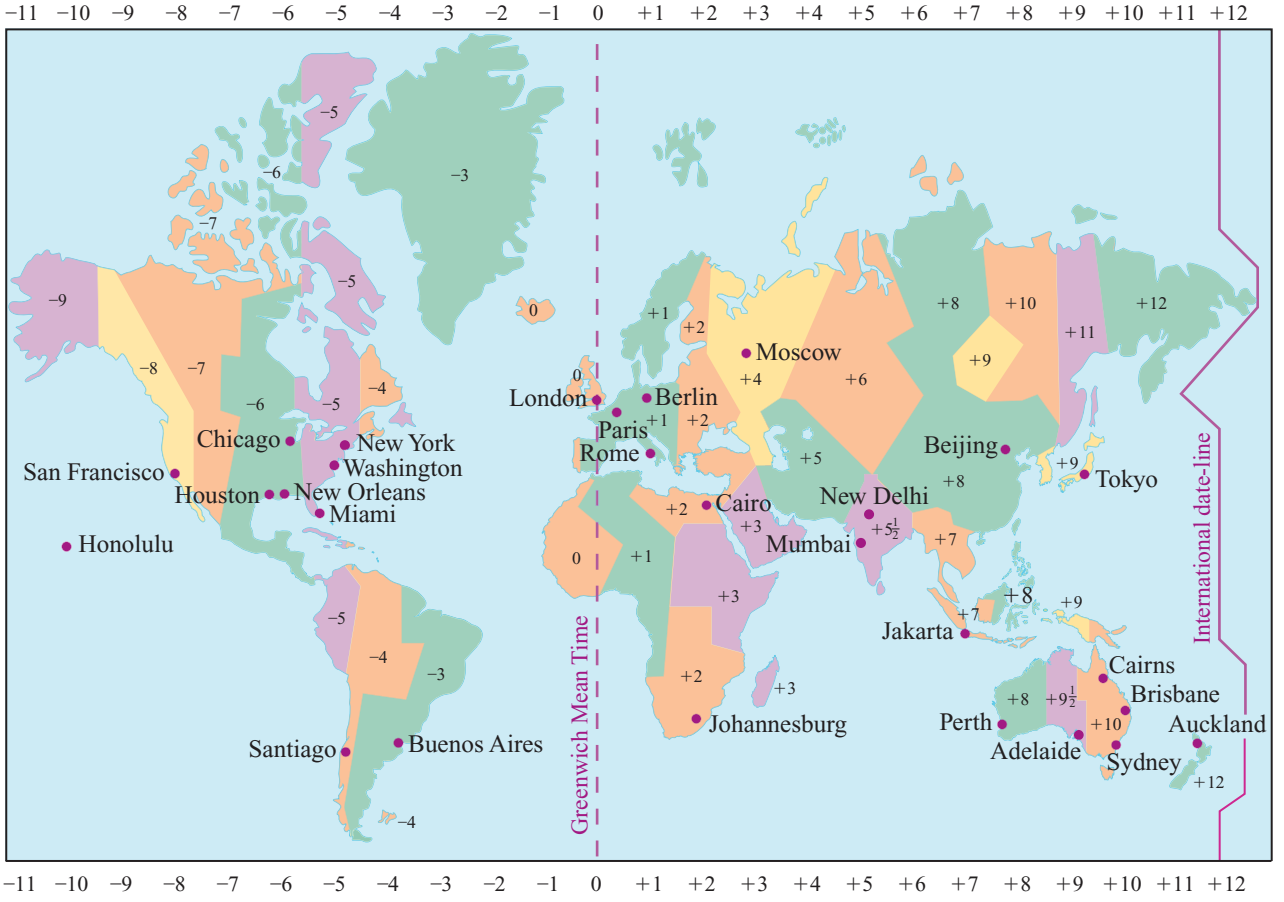


Earth rotates 360° in 1 day, so 360° = 24 hours

- 15° = 1 hour (360 ÷ 24 = 15)
- 1° = 4 minutes ($\frac{1}{15}$ of 60 min = 4 min)

Earth is divided into 24 times zones called meridians. These are generally based on the lines of longitude, but variations occur as a result of following the borders of countries, states, regions or natural boundaries (often rivers or mountains). These measurements begin at Greenwich in London (longitude 0°). The first or prime line (meridian) is the starting point for 12 time zones west of Greenwich and 12 time zones east of Greenwich. Every place within a zone uses the same time. The **standard time zones** are mostly measured in $\frac{1}{2}$ or 1 hour units. A simplified **World Time Zones** map is shown below.

World Time Zones



The International Date Line is 180° east (west) of the prime meridian. Each new day begins in the time zone to the west of the International Date Line and works its way slowly around the world over the next 24 hours.

- Time along the prime meridian is Greenwich Mean Time (GMT).
- Places to the east of the prime meridian are ahead of GMT.
- Places to the west of the prime meridian are behind GMT.
- The numbers in the zones show how many hours have to be added or subtracted from GMT to work out the standard time for that zone.

EXAMPLE 1

If it is 12 noon in Greenwich, what is the standard time in:

a Sydney? **b** New York?

a Sydney is in a zone marked +10.
Standard time is 10 hours ahead of GMT.
∴ Standard time in Sydney is 10:00 pm (12:00 midday + 10 hours).

b New York is in a zone marked −5.
Standard time in New York is 5 hours behind GMT.
∴ Standard time in New York is 7:00 am (12:00 midday − 5 hours).

EXAMPLE 2

Sydney is 10 hours ahead of London. If it is 6:00 pm in Sydney, what is the time in London?

Sydney	*London*
6:00 pm	6:00 pm (−10 h)
(+10 h)	6:00 pm → 12:00 midday → 8:00 am
	6 h + 4 h

∴ If it is 6:00 pm in Sydney, it is 8:00 am in London.

Exercise 6H

Use the World Time Zones map on the previous page to answer these questions.

1 If it is 12 noon in Greenwich, what is the standard time in:
a San Francisco? **b** Houston? **c** Perth? **d** Tokyo?

2 If it is 12 midnight in Greenwich, what is the standard time in:
a Johannesburg? **b** Paris? **c** New York? **d** Santiago?

3 If it is 4:00 am on Monday in Greenwich, what is the standard time in:
a Cairns? **b** Adelaide? **c** Mumbai? **d** Moscow?

4 If it is 11:15 pm on Saturday in Greenwich, what is the standard time in:
a Adelaide? **b** Cairo? **c** Brisbane? **d** Beijing?

5 Rod flew from Sydney to London on 15 April. The flight left Sydney at 2:50 pm and took 23 hours.
a On which day did Rod arrive in London?
b At what time did Rod arrive in London?

EXAMPLE 3

Chicago is 6 hours behind GMT and Tokyo is 9 hours ahead of GMT.

a If it is 3 pm Tuesday in Chicago, what is the time in Tokyo?

b If it is 1 pm Friday in Tokyo, what is the time in Chicago?

a Chicago is -6 and Tokyo is $+9$, so Tokyo is $+15$ hours ahead of Chicago.
3 pm + 9 hours = midnight Wednesday
midnight + 6 hours = 6 am Wednesday $\therefore$ It is 6 am Wednesday in Tokyo.

3 pm Tuesday + 15 h
= 3 pm + 9 h + 6 h

b Chicago is 15 hours behind Tokyo.
1 pm Friday − 13 hours = midnight
midnight Friday − 2 hours = 10 pm Thursday $\therefore$ It is 10 pm Thursday in Chicago.

1 pm Friday − 15 h
= 1 pm − 13 h − 2 h

6 The information below shows major cities throughout the world and the number of hours that they are ahead or behind Greenwich time.

Adelaide ($+9\frac{1}{2}$)	Auckland ($+12$)
Buenos Aires (-3)	Chicago (-6)
Honolulu ($-10\frac{1}{2}$)	Jakarta ($+7$)
Miami (-5)	New Orleans (-6)
Perth ($+8$)	Rome ($+1$)
Tokyo ($+9$)	Washington (-5)

Given the time in one city, use the information above to calculate the time in the other city.

a 7:40 am in Auckland is ____ in Miami.

b 1:35 pm in New Orleans is ____ in Honolulu.

c 5:50 am in Jakarta is ____ in Perth.

d 6:30 pm in Washington is ____ in Rome.

e 4:00 pm in Chicago is ____ in Adelaide.

f 2:27 am in Tokyo is ____ in Buenos Aires.

7 A soccer match is played in Rome at 5 pm Saturday local time. If it is being televised live, at what time will it start in:

a Perth? **b** Jakarta?

c Chicago? **d** Tokyo?

8 A live broadcast commences at 7 pm Wednesday local time in Miami. At what time will it start in:

a Perth? **b** Auckland? **c** Washington? **d** Rome?

Investigation 5 International Date Line

1 Use the Internet or library to research the following questions.

a What can you find out about the International Date Line?

b Why is it not a straight line?

c How is it possible to gain or lose a day while travelling throughout the world?

2 Write a one-page report explaining the purpose of the International Date Line.

Investigation 6 Standard time zones

1 An international company has offices in San Francisco, London and Sydney. Use the World Time Zones map in Section 6H to find the local time in San Francisco and Sydney when it is 12 noon in London.

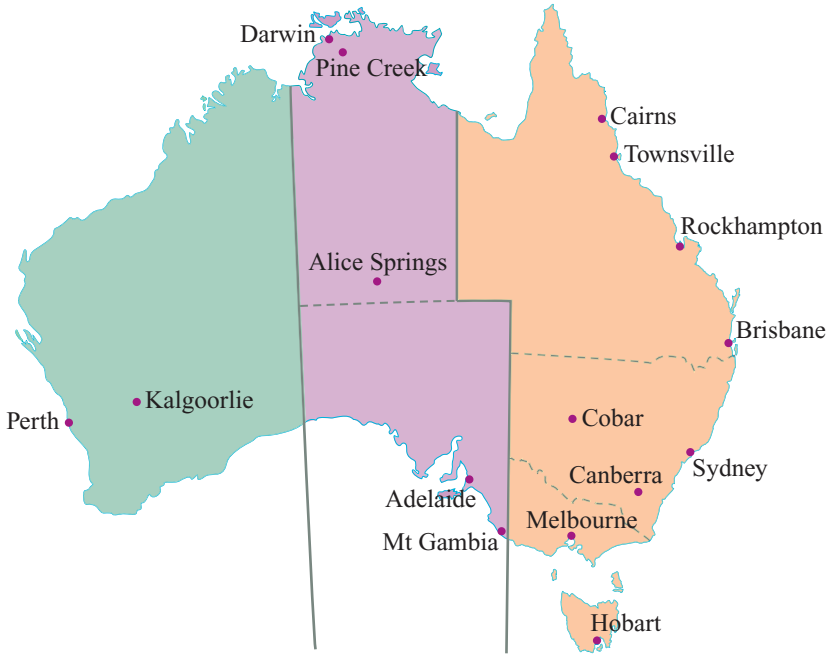
2 Complete the following table, which compares standard times in the three cities.

San Francisco	London	Sydney
	12 noon (Monday)	
	2 pm	
	4 pm	
	⋮	
	12 noon (Tuesday)	

3 The directors in the three cities wish to organise a three-way teleconference. What would be a convenient time to hold the conference?

Time zones in Australia

Australia has three different time zones that are shown on the map.



Use the Australian Time Zones map to answer questions **9** and **10**.

9 If it is 2:00 pm in Melbourne, what is the time in:

a Perth? **b** Sydney? **c** Hobart?
d Cairns? **e** Alice Springs? **f** Adelaide?

10 If it is 5:20 am in Perth, what is the time in:

a Cobar? **b** Darwin? **c** Rockhampton?
d Brisbane? **e** Kalgoorlie? **f** Adelaide?

Investigation 7 Daylight saving time

Some states of Australia use daylight saving time.

1 Explain the meaning of daylight saving.

2 Find out which states adopt daylight saving.

3 When does daylight saving start and finish in these states?

4 Complete the following table to compare times in these cities during the months when daylight saving operates.

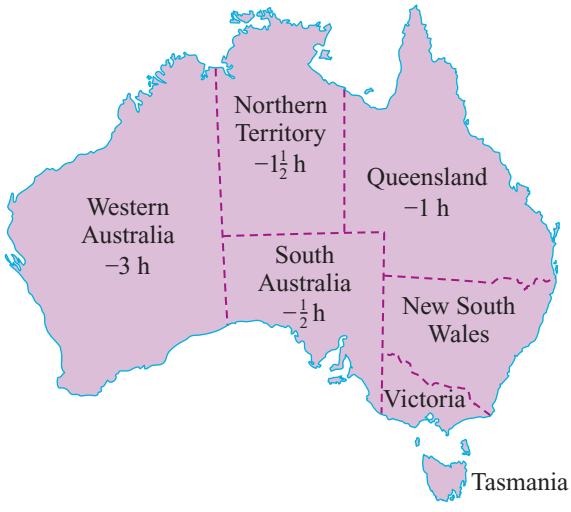
City	Time	Time	Time
Brisbane		12 noon	
Sydney	9 am		
Canberra			
Melbourne			
Hobart			2 pm
Adelaide	8:30 am		
Darwin			
Perth			

5 A plane leaves Adelaide at 8 am, Adelaide daylight saving time, and flies to Hobart. The flight time is $1\frac{1}{2}$ h. At what time will the plane arrive in Hobart?

11 During daylight saving time there are five different time zones in Australia as shown on the map. If the time in Sydney is 7:00 am, what is the time in:

a Perth? **b** Brisbane?

c Melbourne? **d** Darwin?



12 If it is 4:00 pm in Darwin during daylight saving time, what is the time in:

a Perth? **b** Hobart? **c** Adelaide? **d** Sydney?

13 Determine the arrival times for:

a a $4\frac{1}{2}$ hour flight from Melbourne to Perth taken in June and departing from Melbourne at 6:00 am

b a $10\frac{1}{2}$ hour drive from Sydney to Brisbane during daylight saving time and leaving Sydney at 7:20 am.

Language in mathematics

1 The following words are all linked with time:

equinox	century	rhythm	curfew	decade	pendulum
duration	millennium	solstice	meridian	synchronise	almanac

a Find the meaning of each word.

b Think of another five words that are linked or associated with time.

c Prepare a 2-minute talk about one of the words listed above.

Terms

24-hour time	analogue clock	calendar	convert
daylight saving time	degrees	degrees, minutes, seconds	digital display
elapsed time	Greenwich Mean Time (GMT)	International Date Line	leap year
prime meridian	time zone	timetable	World Time Zones

Check your skills

1 The time shown on this clock face is:

A 6 past 2 **B** 2 past 6

C 10 past 2 **D** 10 past 6

2 Five past eight in the morning in digital time is:

A 5:08 am **B** 5:08 pm **C** 8:05 pm **D** 8:05 am

3 Which time is equivalent to 1445 h?

A quarter to three **B** forty-five past four **C** 2:45 am **D** 16:45 am

4 Which time is 2018 in analogue time?

A 8:18 am **B** 8:18 pm **C** 20:18 am **D** 20:18 pm

5 The elapsed time between 10.17 am and 2.54 pm is:

A 7 h 23 min **B** 4 h 37 min **C** 7 h 37 min **D** 4 h 23 min

6 Calculate 4 h 18 min + 2 h 46 min.

A 6 h 54 min **B** 6 h 4 min **C** 7 h 64 min **D** 7 h 4 min

7 Calculate 2 days 21 h + 1 day 18 h.

A 4 days 15 h **B** 3 days 39 h **C** 3 days 15 h **D** 4 days 30 h

8 Calculate 10 h 24 min − 4 h 49 min.

A 5 h 35 min **B** 6 h 35 min **C** 6 h 25 min **D** 5 h 25 min

9 Calculate 5 days 18 h + 9 days 22 h in weeks, days and hours.

A 2 weeks 38 h **B** 1 week 6 days 40 h

C 2 weeks 2 days 20 h **D** 2 weeks 1 day 16 h

10 Solve 5 h 18 min + 2 h 55 min − 3 h 26 min.

A 5 h 49 min **B** 4 h 47 min **C** 8 h 52 min **D** 11 h 39 min

11 Convert $5\frac{5}{8}$ h to hours, minutes and seconds.

A 5° 5′ 8″ **B** 5° 37′ 30″ **C** 5° 8′ 5″ **D** 5° 38′ 40″

12 Round this calculator display to the nearest minute. 3° 15′ 32″

A 4 h **B** 3 h 15 min **C** 3 h 15 min 30 s **D** 3 h 16 min

13 Express 4.7 h in hours and minutes.

A 4 h 7 min **B** 4 h 70 min **C** 4 h 0.7 min **D** 4 h 42 min

14 Using your calculator, convert $2\frac{4}{9}$ h to hours, minutes and seconds.

A 2 h 4 min 9 s **B** 2 h 49 min 0 s **C** 2 h 26 min 40 s **D** 2 h 9 min 4 s

15 Round this calculator display to the nearest minute. 1° 37′ 30″

A 1 h 38 min **B** 1 h 37 min **C** 1 h 30 min **D** 1 h 36 min

16 Express 0.86 h to the nearest minute.

A 51 min **B** 52 min **C** 36 min **D** 47 min

17 How many days in a leap year?

A 365 **B** 366 **C** 52 **D** 12

Questions **18** and **19** refer to the train timetable below.

	pm	pm	pm	pm	pm	pm	pm	pm
Wentworthville		3:15			3:28		3:40	
Westmead	3:09	3:17	3:20	3:24	3:30	3:35	3:42	3:47
PARRAMATTA arr.	3:12	3:20	3:23	3:27	3:32	3:38	3:45	3:50
dep.	3:13	3:21	3:24	3:28	3:33	3:39	3:46	3:51
Harris Park	3:14				3:34		3:47	3:52
Granville	3:16	3:23	3:26		3:36	3:41	3:49	3:54
Clyde	3:18						3:51	
Auburn	3:20				3:39		3:53	3:58
Lidcombe	3:24		3:32		3:43		3:57	4:02
Flemington								
STRATHFIELD arr.	3:29	3:33	3:37		3:48	3:51	4:02	4:08
dep.	3:30	3:34	3:38		3:49	3:52	4:03	4:09
BURWOOD	3:31		3:39					4:10
REDFERN	3:41	3:45	3:49		3:59	4:03	4:14	4:22
CENTRAL arr.	3:44	3:47	3:52		4:02	4:05	4:17	4:24
dep.	3:45	3:48	3:53		4:03	4:06	4:18	4:25
TOWN HALL	3:47	3:50	3:55		4:05	4:08	4:20	4:27
WYNYARD	3:50	3:53	3:58		4:08	4:11	4:23	4:30
Milsons Point	3:54	3:57	4:02		4:12	4:15	4:27	4:34
NORTH SYDNEY	3:56	3:59	4:04		4:14	4:17	4:29	4:36
Proceeds to	BEW	GDO	CAM		GDO	BEW	GDO	

18 At what time does the 3:34 pm train from Harris Park arrive at Milsons Point?

A 4:02 pm **B** 4:12 pm **C** 4:15 pm **D** 4:27 pm

19 Sarah caught the 3:21 pm train from Parramatta. How long did it take her to reach Town Hall?

A 30 min **B** 25 min **C** 27 min **D** 29 min

20 Sydney is 10 hours ahead of London. If it is 8.45 pm on Sunday in Sydney, what time is it in London?
A 6:45 pm **B** 10:45 am **C** 10:45 pm **D** 6:45 am

21 Tokyo is 9 hours ahead of London. If it is 3:15 pm in Tokyo, what time is it in London?
A 6:15 am **B** 8:45 am **C** 10:15 pm **D** 12:15 pm

22 San Francisco is 8 hours behind London. If it is 11:09 am in London, what time is it in San Francisco?
A 7:09 pm **B** 4:51 am **C** 7:51 am **D** 3:09 am

23 It is 3:00 pm in Perth. The time in Sydney is:
A 5:00 pm **B** 1:00 pm **C** 3:00 pm **D** 3:00 am

24 Tokyo is 9 h ahead of London. If it is 4:35 am on Saturday 4 April in Tokyo, what time and day is it in London?
A 1:35 pm Saturday 4 April **B** 7:35 pm Friday 3 April
C 1:35 am Friday 3 April **D** 7:35 am Saturday 4 April

25 Perth is 3 h behind Sydney during daylight saving time. If I left Sydney at 7:23 am and arrived in Perth $4\frac{1}{2}$ h later, at what time would I arrive in Perth (local time)?
A 11:53 am **B** 2:53 am **C** 8:53 am **D** 3:53 am

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2	3–4	5–10	11–16	17	18, 19	20–25
Section	A	B	C	D	E	F	G	H

6A Review set

1 Write 'four fifteen in the afternoon' as it would appear on a digital display.

2 Write 1532 as analogue time. State whether it is am or pm.

3 Convert 1:28 pm to 24-hour time.

4 Calculate the following.
a 2 h 24 min + 3 h 56 min **b** 4 h 18 min − 2 h 36 min

5 If Sergio caught the bus at 7.52 am, at what time did he arrive at work if the bus trip took 42 min?

6 High tide is at 6:23 am and low tide is at 10:03 am. Find the time difference between high tide and low tide.

7 Convert $3\frac{1}{3}$ h to hours and minutes.

8 Round this calculator display to the nearest minute. 2° 15′ 36.2″

9 Calculate the elapsed time between 3:27 pm Monday and 7:20 am Tuesday.

10 What is a leap year?

11 Calculate the elapsed time between 0620 hours Tuesday and 1340 hours Friday.

12 Use the World Time Zones map on page 153 to find the local time in Houston when it is 4:30 pm in Greenwich.

13 Use the Australian Time Zones map on page 156 to find the local time in Perth when it is 6 pm in Melbourne.

6B Review set

1 Write 'nineteen minutes to midnight' as it would appear on a digital display.

2 State 2117 h as analogue time (am or pm).

3 Convert 'five past three in the afternoon' to 24-hour time.

4 Calculate the following.

a 3 h 16 min + 2 h 51 min

b 4 h 18 min 22 s − 1 h 55 min 49 s

5 Alex watched the late night movie from 9:45 pm until 12:08 am.

a How long was the late night movie?

b If 37 min of this time was spent on commercials, calculate the actual length of the movie.

6 The sun rises at 5:17 am and sets at 7:38 pm. Calculate the hours and minutes of daylight.

7 Convert $7\frac{3}{7}$ h to hours, minutes and seconds.

8 Round the answer for question **7** to the nearest minute.

9 Calculate the time between 0743 Saturday and 1523 Thursday.

10 Calculate the elapsed time between 2144 hours Monday and 0325 hours Saturday.

11 Use the World Time Zones map on page 153 to find the local time in Jakarta when it is 5 pm in Miami.

12 It is 7 pm daylight saving time in Sydney. What is the time in Perth?

6C Review set

1 State the time shown on each clock.

a

b

2 Write the following digital displays in 24-hour time.

a 12:15 am

b 6:46 pm

3 Calculate the elapsed time between 0505 and 0837.

4 Calculate the 'shelf life' of a product purchased on 18/6/15 by referring to these 'best-by' dates.

a 21/3/16

b 12/10/16

5 Calculate 2 h 13 min 14 s − 1 h 29 min 36 s.

6 Calculate 4 days 15 h − 3 days 8 h.

7 Calculate the time between 0341 and 1427.

8 Calculate the time elapsed between 1:42 am Monday and 6:14 pm Thursday.

9 Round these calculator displays to the nearest minute.

a 12° 15′42.3″ **b** 6° 38′17″

10 Ben sprained his ankle on 3 August. It was 24 days before he could walk without limping. On what date did he walk without limping?

11 Use the World Time Zones map on page 153 to find the local time in Greenwich when it is 3:27 pm in Perth.

12 Use the Australian Time Zones map on page 156 to find the local time in Adelaide when it is 3 am in Perth.

6D Review set

1 State the time shown on each clock.

a

b

2 Write the following digital displays in 24-hour time.

a 4:38 am **b** 11:22 pm

3 Calculate the elapsed time between 0932 and 1517.

4 Calculate 3 h 9 min 6 s + 4 h 24 min 53 s.

5 Brad caught the 11:48 am train from Penrith to Sydney. He arrived at 12:36 pm. How long did the journey take him?

6 Calculate 4 days 3 h − 1 day 16 h.

7 Calculate the time between 1518 Monday and 0429 Wednesday.

8 Round these calculator displays to the nearest minute.

a 4° 58′17″ **b** 6° 34′11″

9 Calculate the 'shelf life' of a product purchased on 25/8/15 by referring to these 'best-by' dates.

a 21/3/16 **b** 18/5/16

10 Jemima saw the doctor on 18 March. If she must make another appointment in 6 weeks time, what is the date of her next appointment?

11 Ace caught the 11:48 pm train from Penrith to Sydney. If he arrived at Central at 12:36 pm, how long did the journey take?

12 Use the World Time Zones map on page 153 to find the local time in Buenos Aires when it is 6 pm in Rome.

13 It is 6 am daylight saving time in Adelaide. What is the time in Brisbane?

7

Percentages

This chapter deals with percentages.

At the end of this chapter you should be able to:

- solve problems involving percentages
- determine a percentage of an amount
- increase and decrease by a percentage
- calculate the percentage increase and decrease
- express loss/profit as a percentage of the cost price
- solve problems involving profit and loss
- perform calculations involving GST.

NSW Syllabus references: S4 N&A Fractions, decimals and percentages, S4 N&A Financial mathematics
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-5NA, MA4-6NA
NUMBER & ALGEBRA – ACMNA187, ACMNA189

Diagnostic test

1 Convert $\frac{87}{10}$ to a decimal.

A 0.87 **B** 0.087
C 8.7 **D** 0.807

2 Convert 0.061 to a fraction.

A $\frac{61}{60\,000}$ **B** $\frac{61}{1000}$
C $\frac{61}{100}$ **D** $\frac{61}{10}$

3 Which statement below is correct?

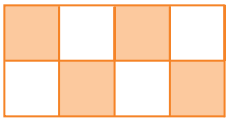
A $0.8 < 0.08$ **B** $0.8 = \frac{8}{100}$
C $0.8 > 0.88$ **D** $0.8 = \frac{8}{10}$

4 Round 0.998 71 correct to 2 decimal places.

A 1.00 **B** 0.99
C 0.10 **D** 9.9

5 What percentage of the shape is shaded?

A $\frac{1}{2}$ **B** 50%
C 0.5 **D** $\frac{4}{8}$ %



6 Express 41% as a fraction.

A $4\frac{1}{10}$ **B** 0.41 **C** 0.041 **D** $\frac{41}{100}$

7 Express 125% as a simplified fraction.

A $1\frac{1}{4}$ **B** $\frac{125}{100}$ **C** $1\frac{25}{100}$ **D** $\frac{25}{20}$

8 Express $8\frac{1}{5}$% as a decimal.

A 0.0082 **B** 0.082
C 0.82 **D** 8.2

9 Express $\frac{4}{5}$ as a percentage.

A 20% **B** 40% **C** 60% **D** 80%

10 Express 3.56 as a percentage.

A 356% **B** 35.6%
C 3.56% **D** 0.356%

11 Arrange 0.927, $\frac{24}{25}$ and $91\frac{1}{4}$% in ascending order.

A $91\frac{1}{4}$%, 0.927, $\frac{24}{25}$ **B** 0.927, $\frac{24}{25}$, $91\frac{1}{4}$%
C 0.927, $91\frac{1}{4}$%, $\frac{24}{25}$ **D** $\frac{24}{25}$, $91\frac{1}{4}$%, 0.927

12 Arrange 0.0621, 64% and $\frac{3}{5}$ in descending order.

A 0.0621, 64%, $\frac{3}{5}$ **B** 64%, $\frac{3}{5}$, 0.0621
C $\frac{3}{5}$, 0.0621, 64% **D** 64%, 0.0621, $\frac{3}{5}$

13 Express \$270 as a percentage of \$300.

A 90% **B** 111.1%
C 11.1% **D** 10%

14 Express 330 mL as a percentage of 2 L.

A 16 500% **B** 1650%
C 165% **D** 16.5%

15 Find 12% of 90 km.

A 10.8 km **B** 12.2 km
C 14.6 km **D** 18.3 km

16 Find $12\frac{1}{2}$% of 400 m.

A 0.05 m **B** 0.5 m
C 5 m **D** 50 m

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA157 and ACMNA158.

Investigation 1 Working out percentages

This investigation revises the calculation of percentages without the use of a calculator.

1 This is a method used to find 5% of $300.

a To find 5% means 5 for every 100 or $5 for every $100. This can be shown in a table.

	Amount ($)	Percentage amount ($)
	100	5
	100	5
	100	5
Total	300	15

$300 = $100 + $100 + $100 so 5% of $300 is $5 + $5 + $5 = $15

So 5% of $300 is $15.

b Use this method to find:

i 2% of $300 **ii** 5% of $200 **iii** 6% of $400 **iv** 3% of $1000

2 A similar method is used to find 4% of $350.

a To find 4% means 4 for every 100 or $4 for every $100.

If there is $4 for every $100, there must be $2 for every $50. This can be shown in a table.

	Amount ($)	Percentage amount ($)
	100	4
	100	4
	100	4
	50	2
Total	350	14

$350 = $100 + $100 + $100 + $50 so 4% of $350 is $4 + $4 + $4 + $2 = $14

So 4% of $350 is $14.

b Use this method to find:

i 3% of $350 **ii** 4% of $250 **iii** 6% of $450 **iv** 3% of $1050

3 A similar method is used to find 2.5% of $300.

a To find 2.5% means 2.5 for every 100 or $2.50 for every $100. This can be shown in a table.

	Amount ($)	Percentage amount ($)
	100	2.5
	100	2.5
	100	2.5
Total	300	7.5

$300 = $100 + $100 + $100 so 2.5% of $300 is $2.50 + $2.50 + $2.50 = $7.50

So 2.5% of $300 is $7.50.

b Use this method to find:

i 2.5% of $300 **ii** 5.5% of $200 **iii** 6.5% of $400 **iv** 3.5% of $1000

4 The methods in questions **1**, **2** and **3** are combined to find 3.5% of $450.

a To find 3.5% means 3.50 for every 100 or $3.50 for every $100.
If there is $3.50 for every $100, there must be $1.75 for every $50. This can be shown in a table.

	Amount ($)	Percentage amount ($)
	100	3.50
	100	3.50
	100	3.50
	100	3.50
	50	1.75
Total	450	15.75

$450 = $100 + $100 + $100 + $100 + $50
So 3.5% of $450 is $3.50 + $3.50 + $3.50 + $3.50 + $1.75 = $15.75

b Use this method to find:

i 3.5% of $150 **ii** 4.5% of $250
iii 6.5% of $350 **iv** 3.5% of $950

5 This method can be used to find the percentage of any amount. Find 5.2% of $340.

a To find 5.2% means 5.2 for every 100 or $5.20 for every $100.
If there is $5.20 for every $100, there must be $0.52 for every $10, so $4 \times 0.52 = $2.08 for $40.
This can be shown in a table.

	Amount ($)	Percentage amount ($)
	100	5.20
	100	5.20
	100	5.20
	40	2.08
Total	450	17.68

$340 = $100 + $100 + $100 + $40
So 5.2% of $340 is $5.20 + $5.20 + $5.20 + $2.08 = $17.68

b Use this method to find:

i 3.6% of $120
ii 4.1% of $280
iii 6.3% of $310
iv 7.8% of $980

A Percentages review

Percentages are used every day. A percentage is a way of writing a fraction with a denominator of 100.
For example $13\% = \frac{13}{100}$.

EXAMPLE 1

Express each percentage as a fraction in simplest form.

a 23% **b** 65% **c** 190% **d** $7\frac{1}{2}\%$

a $23\% = \frac{23}{100}$

b $65\% = \frac{65}{100} = \frac{13}{20}$

c $190\% = \frac{190}{100}$
$= 1\frac{90}{100} = 1\frac{9}{10}$

d $7\frac{1}{2}\% = \frac{(2 \times 7) + 1}{200}$
$= \frac{15}{200} = \frac{3}{40}$

Remember how to change to improper fractions.

Exercise 7A

1 Express each percentage as a fraction in simplest form.

a 51% **b** 89% **c** 47% **d** 61% **e** 97%
f 42% **g** 65% **h** 75% **i** 18% **j** 45%
k 50% **l** 36% **m** 54% **n** 98% **o** 66%

2 Express each percentage as a whole or mixed numeral in simplest form.

a 100% **b** 400% **c** 250% **d** 375% **e** 190%
f 620% **g** 554% **h** 236% **i** 708% **j** 1230%

3 Express each percentage as a fraction in simplest form.

a $9\frac{1}{3}\%$ **b** $15\frac{1}{4}\%$ **c** $10\frac{4}{5}\%$ **d** $5\frac{1}{6}\%$ **e** $8\frac{2}{3}\%$

EXAMPLE 2

Express each percentage as a decimal.

a 16% **b** 225%

a $16\% = \frac{16}{100} = 16 \div 100 = 0.16$

b $225\% = \frac{225}{100} = 225 \div 100 = 2.25$

When ÷ by 100, move the decimal point 2 places to the left. Fill any empty places with zeros.

4 Express each percentage as a decimal.

a 8% **b** 9% **c** 46% **d** 65% **e** 58%
f 2% **g** 26% **h** 4% **i** 77% **j** 84%

5 Express each percentage as a decimal.

a 306% **b** 154% **c** 263% **d** 856% **e** 287%
f 742% **g** 733% **h** 113% **i** 922% **j** 569%

EXAMPLE 3

Express each number as a percentage.

a $\frac{2}{5}$ **b** $7\frac{1}{4}$ **c** 2 **d** 8.5

a $\frac{2}{5} \times \frac{100}{1} = 40\%$

b $7\frac{1}{4} \times \frac{100}{1}$
$= \frac{29}{4} \times \frac{100}{1}$
$= \frac{2900}{4} = 725\%$

c $2 \times \frac{100}{1} = 200\%$

d $8.5 \times \frac{100}{1} = 850\%$

- *When × by 100, move the decimal point 2 places to the right.*
- *To change a fraction to a percentage, multiply by 100.*

6 Express the following as a percentage.

a $\frac{1}{4}$ **b** $5\frac{1}{2}$ **c** 8 **d** 6.3 **e** $\frac{3}{20}$
f $9\frac{1}{2}$ **g** 0.05 **h** $2\frac{1}{2}$ **i** 7.2 **j** $\frac{4}{50}$
k 1.6 **l** 17 **m** 0.8 **n** 0.4 **o** $\frac{9}{10}$
p 0.86 **q** $5\frac{3}{4}$ **r** 0.001 **s** 6 **t** 15

7 Express each fraction as a percentage correct to 2 decimal places. (Use your calculator.)

a $\frac{6}{11}$ **b** $\frac{12}{13}$ **c** $\frac{7}{8}$ **d** $\frac{5}{9}$ **e** $\frac{15}{16}$ **f** $\frac{17}{19}$

8 Complete the table by writing equivalent fractions, decimals and percentages on each line.

	Fraction	Decimal	Percentage
a		0.4	
b	$\frac{7}{10}$		
c			20%
d		0.05	
e	$\frac{4}{5}$		
f			37%
g		7.08	
h	$1\frac{3}{5}$		
i			0.6%
j			82%
k		11.002	
l	$\frac{3}{8}$		

9 Arrange each set in ascending order.

a 72%, $\frac{1}{4}$, 0.92 **b** $\frac{1}{5}$, 86%, 0.09 **c** $\frac{7}{8}$, 0.17, 23% **d** $\frac{3}{5}$, 0.582, $62\frac{1}{2}\%$
e $\frac{5}{6}$, 70%, 0.44 **f** 35%, 0.36, $\frac{2}{5}$ **g** 0.74, 52%, $\frac{18}{20}$ **h** $\frac{3}{8}$, 62%, 0.37

Investigation 2 Percentage symbol

Investigate the history of the percentage symbol.

How has it changed over time to become the symbol that it is now?

B Percentages of quantities

To express one quantity as a percentage of another:

- change both quantities to the same unit (if necessary)
- write $\frac{\text{first quantity}}{\text{second quantity}} \times 100\%$.

To find a percentage of a quantity:

- express each percentage as a fraction in simplest form
- replace 'of' by '$\times$' and calculate the answer.

EXAMPLE 1

Express the first quantity as a percentage of the second quantity.

a 38 cm, 40 cm **b** 42 cm, 1.2 m **c** 2 weeks, 20 days

Use $\frac{\text{first quantity}}{\text{second quantity}} \times 100\%$

a So $\frac{38}{40} \times \frac{100}{1}\% = 95\%$

$\therefore$ 38 cm is 95% of 40 cm.

b Convert to cm: that is, 42 cm, 120 cm.

So $\frac{42}{120} \times \frac{100}{1}\% = 35\%$

$\therefore$ 42 cm is 35% of 1.2 m.

c Convert to days: that is, 14 days, 20 days.

So, $\frac{14}{20} \times \frac{100}{1}\% = 70\%$

$\therefore$ 2 weeks is 70% of 20 days.

Exercise 7B

1 Express the first quantity as a percentage of the second quantity.

a \$6, \$15 **b** 10 km, 50 km **c** 4 h, 25 h

d 18 min, 50 min **e** 70 m, 125 m **f** \$88, \$440

g 60 L, 200 L **h** 27 kg, 50 kg **i** 54 min, 75 min

j 25 h, 100 h **k** 32 L, 64 L **l** 45 m, 180 m

2 What percentage is the first quantity of the second?

a 28 cm : 1.4 m **b** 72 cm : $1\frac{1}{2}$ m **c** 1.8 m : 60 cm

d 810 g : 4.05 kg **e** 156 g : 0.24 kg **f** 3.62 kg : 400 g

g \$0.60 : \$2 **h** 85c : \$5 **i** 5.4 L : 600 mL

j 18 h : 1 day **k** 12 h : 2 days **l** \$2.55 : \$1.25

m 6 months : 2 years **n** 21 months : $3\frac{1}{2}$ years **o** 24 months : 5 years

EXAMPLE 2

Calculate the following.

a 20% of 40 m

$$20\% \text{ of } 40 \text{ m} = \frac{20}{100} \times \frac{40}{1}$$
$$= \frac{800}{100}$$
$$= 8 \text{ m}$$

b $12\frac{1}{2}\%$ of \$40

$$12\tfrac{1}{2}\% = \frac{25}{200}$$
$$12\tfrac{1}{2}\% \text{ of } \$40 = \frac{25}{200} \times \frac{40}{1}$$
$$= \frac{1000}{200}$$
$$= 5 \text{ m}$$

3 Calculate:

a 45% of \$260 **b** 64% of 500 L **c** 75% of \$240
d 17% of 50 **e** 12% of 64 kg **f** 18% of 80 m
g $62\frac{1}{2}\%$ of \$320 **h** $66\frac{2}{3}\%$ of 180 m **i** 32% of 308 kg
j 72% of 210 L **k** 21% of \$544 **l** 13% of \$126

4 Convert each percentage to a decimal, then calculate:

a 4% of \$120 **b** 9% of 220 L **c** 6% of 40 m
d 15% of 600 kg **e** 13% of \$160 **f** 52% of 1600 km

C Percentage change

Two methods can be used to increase or decrease an amount by a percentage.

To increase means to add to the original amount.
To decrease means to subtract from the original amount.

Method 1

To calculate the percentage of the amount:

new amount = old amount + increase
new amount = old amount − decrease

Method 2

Find the percentage of the amount once you have increased or decreased the percentage.

EXAMPLE 1

Using method 1:

a increase \$50 by 70%

b decrease \$50 by 70%

First find 70% of \$50.

$0.70 \times 50 = \$35$ or $\frac{70}{100} \times 50 = \$35$

Remember to find the new amount first, then add or subtract it from the original amount.

a New amount = old amount + increase
= \$50 + \$35
= \$85

b New amount = old amount − decrease
= \$50 − \$35
= \$15

Exercise 7C

1 Complete to find the increased amounts using method 1.

a \$400 by 15%

$\frac{15}{100} \times \_\_\_ = \_\_\_$

New amount = \$400 + ___

= \$460

b 200 g by 40%

$\frac{\square}{\square} \times 200 = \_\_\_$

New amount = 200 g + ___

= ___ g

2 Increase the following amounts using method 1.

a \$300 by 15% **b** 100 g by 40% **c** 100 L by 8%

d \$90 by 70% **e** 68 by 20% **f** 380 by 50%

3 Decrease the following amounts using method 1.

a \$24 by 50% **b** 60 by 70% **c** 50 kg by 40%

d 90 m by 30% **e** 150 km by 8% **f** 80 L by 70%

EXAMPLE 2

a Find the percentage of an amount that is needed to increase the amount by 88%.

b Find the percentage of an amount that is needed to decrease the amount by 14%.

a 100% + 88% = 188%

b 100% − 14% = 86%

4 Find the percentage of an amount that is needed to increase the amount by:

a 11% **b** 38% **c** 55% **d** 92% **e** 68%

f 86% **g** 39% **h** 107% **i** 156% **j** 213%

5 Find the percentage of an amount that is needed to decrease the amount by:

a 9% **b** 15% **c** 18% **d** 23% **e** 32%

f 47% **g** 66% **h** 51% **i** 95% **j** 78%

EXAMPLE 3

Using method 2:

a increase \$50 by 70%

b decrease \$50 by 70%

a 100% + 70% = 170%

170% of \$50 $= \frac{170}{100} \times 50$

= \$85

b 100% − 70% = 30%

30% of \$50 = 0.3 × 50

= \$15

6 Complete to find the increased amounts using method 2.

a 70 km by 10%

100% + 10% = ___%

Find 110% of 70 km.

∴ 1.1 × 70 = ___ km

b 20 t by 80%

100% + 80% = ___%

Find 180% of 20 t.

$\therefore \frac{180}{100} \times 20 = \_\_\_$ t

7 Increase the following using method 2.

a 70 km by 10%
b 60 L by 80%
c $1200 by 60%
d 9 L by 70%
e 28 km by 50%
f 15 m by 5%

8 Decrease the following using method 2.

a 20 t by 80%
b 40 km by 70%
c 55 L by 60%
d $900 by 50%
e 60 g by 40%
f 70 m by 10%

9 Read the following problems. Decide whether it is an increase or a decrease then solve.

a A baker increased the price of chocolate mud cakes sold to restaurants by 70%. What is the new selling price of a chocolate mud cake if the original price was $5.00?
b Kayla's home in Mountain View Estate was purchased for $160 000. Its value has increased by 62%. Calculate its present value.
c A sound system priced at $980 is reduced by 15% during a sale. Calculate the sale price of the sound system.
d Al's Cars has discounted all cars by 10% for the weekend. Calculate the discounted price of a car valued at $22 000.
e Jessica purchased watches for $5.50 and marked them up by 80% before selling them in her jewellery store. Calculate the selling price of the watches to the nearest dollar.

10 In an analysis of the Rugby League grand final the following statistics were gathered.

Aspect	Winning team	Losing team
Time in possession (min)	48	32
Line breaks	15	11
Completed sets	18	12
Tackles	235	303

a Calculate the percentage time in possession for the winning team.
b Calculate the percentage of total tackles for each team.
c Compare the percentage time in possession with the percentage of tackles. What comment can be made?

Investigation 3 Comparing percentages

This table is taken from a breakfast cereal package.

Guideline daily intake for adults

	Guideline daily intake	Per serving	% DI
Energy	8700 kJ	640 kJ	
Total fat	70 g	3.5 g	5%
Saturated fat	24 g	0.7 g	
Sugars	90 g	0.4 g	<1%
Sodium	2300 mg	5 mg	<1%
Fibre	30 g	3.9 g	

1. Calculate the missing % DI (daily intake) values.
2. Compare the guidelines for your favourite breakfast cereal with the values for this cereal.
3. Another cereal has 8 g of sugar per serving. Calculate the % DI of sugar from one serve of this other cereal. How does this compare with the guidelines above?
4. Compare the % DI for sugar, sodium and fat of other packaged foods.

D Calculating percentage change

To calculate a percentage increase or decrease, the following formula is used:

$$\%\text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

$$\%\text{ decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$$

We always compare with the original amount, so this becomes the denominator.

EXAMPLE 1

Find the percentage increase.

a \$120 to \$150 **b** \$48 to \$216

a Increase $= \$150 - \$120$
$= \$30$

$$\%\text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$
$$= \frac{{}^{1}\cancel{30}}{{}^{4}\cancel{120}} \times 100\%$$
$$= \frac{100}{4}$$
$$= 25\%$$

$\therefore$ \$120 to \$150 is a 25% increase.

b Increase $= \$216 - \$48$
$= \$168$

$$\%\text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$
$$= \frac{{}^{7}\cancel{168}}{{}^{2}\cancel{48}} \times 100\%$$
$$= \frac{700}{2}$$
$$= 350\%$$

$\therefore$ \$48 to \$216 is a 350% increase.

Exercise 7D

1 Complete to find the percentage increase.

a 48 g to 60 g

Increase $= 60 - 48$

$= \_\_\_$ g

$\%\text{ increase} = \frac{12}{48} \times 100\%$

$= \_\_\_\%$

$\therefore$ 48 g to 60 g is a ___% increase.

b 30 L to 45 L

Increase $= 45 - \_\_\_$

$= \_\_\_$ L

$\%\text{ increase} = \frac{15}{\square} \times 100\%$

$= \_\_\_\%$

$\therefore$ 30 L to 45 L is a ___% increase.

2 Calculate the percentage increase.

a 50 g to 80 g
b 30 L to 75 L
c 12 min to 1 h
d \$120 to \$228
e 16 m to 20 m
f 20 g to 52 g
g 120 L to 180 L
h 150 kg to 210 kg
i 30 km to 150 km

EXAMPLE 2

Find the percentage decrease.

a 80 L to 60 L

a Decrease $= 80 - 60$

$= 20$

$\%\text{ decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$

$= \frac{{}^{1}\cancel{20}}{{}^{4}\cancel{80}} \times 100\%$

$= \frac{100}{4}$

$= 25\%$

$\therefore$ 80 L to 60 L is a 25% decrease.

b 72 kg to 36 kg

b Decrease $= 72 - 36$

$= 36$

$\%\text{ decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$

$= \frac{{}^{1}\cancel{36}}{{}^{2}\cancel{72}} \times 100\%$

$= \frac{100}{2}$

$= 50\%$

$\therefore$ 72 kg to 36 kg is a 50% decrease.

3 Complete to find the percentage decrease.

a 70 L to 42 L

Decrease $= 70 - 42$

$= \_\_\_$ L

$\%\text{ decrease} = \frac{28}{70} \times 100\%$

$= \_\_\_\%$

$\therefore$ 70 L to 42 L is a ___% decrease.

b 200 m to 50 m

Decrease $= 200 - \_\_\_$

$= \_\_\_$ m

$\%\text{ decrease} = \frac{\square}{200} \times 100\%$

$= \_\_\_\%$

$\therefore$ 200 m to 50 m is a ___% decrease.

4 Calculate the percentage decrease.

a 80 L to 32 L
b 200 m to 120 m
c 90 km to 27 km
d \$105 to \$63
e 16 t to 4 t
f \$560 to \$420
g 800 km to 200 km
h 54 m to 27 m
i 35 t to 7 t

5 This table shows the approximate cost of using various size TVs. The annual cost is based on 7 hours of viewing per day with electricity charged at 25 cents/kWh.

TV screen size	Energy/Star rating	Annual cost ($)
138 cm/54 inch	★★½	$217
	★★★★★★	$67
106 cm/42 inch	★★½	$139
	★★★★★★	$43
80 cm/32 inch	★★½	$83
	★★★★★★	$27

a For each size TV, calculate the percentage saving per year if the 6-star rating TV is used rather than the $2\frac{1}{2}$ star rating TV.

b Calculate the percentage saving from the most costly to the least costly TV.

6 The table shows the cost of various forms of lighting. The annual cost is based on 3 hours per day with electricity charged at 25 cents/kWh.

Lighting	Annual cost ($)
Compact fluorescent globe: 15 W	$3.92
Compact fluorescent globe: 20 W	$5.22
Fluorescent tube: 18 W	$6.81
Fluorescent tube: 25 W	$11.36
12-volt halogen downlight: 50 W	$14.77
Incandescent globe: 75 W	$19.31
Incandescent globe: 100 W	$26.13

Calculate the percentage saving in making a change from:

a a 100 W incandescent globe to a 75 W globe

b an 18 W fluorescent tube to a 15 W compact fluorescent globe

c a 75 W incandescent globe to a 20 W compact fluorescent globe.

E The unitary method

Here we are given some information about a quantity and its percentage of an unknown total quantity. We could then be asked to find the total quantity or another percentage of it.

Remember, the total unknown quantity is 100%. If we find the value of 1%, then we only need to multiply by 100 to find 100%.

For example, if 12 g is 10% of the weight then 10% is 12 g.

So 1% is $\frac{12}{10}$ g

$\therefore 100\% = \frac{12}{10}\text{ g} \times 100 = 120\text{ g}$

So $\frac{\text{quantity}}{\text{percentage given}} \times 100 = \text{total unknown quantity}$

It is called the unitary method because we find 1% first.

EXAMPLE 1

Find 100% of a quantity if:

a 23% is \$78.20

a 23% is \$78.20

1% is $\frac{\$78.20}{23}$

100% is $\frac{\$78.20}{23} \times 100 = \$340$

$\therefore$ 100% is \$340.

Calculator:

78.20 × 100 ÷ 23 =

b 46.5% is 186 kg

b 46.5% is 186 kg

1% is $\frac{186}{46.5}$ kg

100% is $\frac{186}{46.5} \times 100\text{ kg} = 400\text{ kg}$

$\therefore$ 100% is 400 kg.

186 × 100 ÷ 46.5 =

Exercise 7E

1 Find 100% of a quantity if:

a 10% is 82 m
b 23% is 483 g
c 78% is 733.2 km
d 49% is 110.25
e 12.9% is 29.67 g
f 5.08% is \$2.54
g $2\frac{1}{2}$% is 125 mL
h $38\frac{1}{4}$% is 19 125 kg
i 125% is \$3.40

2 34% of students at a school use public transport. If 204 students use public transport, how many students attend the school?

3 Darko scored 81% for a test. If his actual score was $121\frac{1}{2}$, what was the maximum possible mark for the test?

4 A newspaper saves \$26 860 in printing costs per year by purchasing recycled paper. If printing costs have been reduced by 31.6%, calculate the original printing costs.

5 Find the original price of items with these tags.

a

b

c

EXAMPLE 2

a The original price of a TV is increased by 30% to $323.70. Calculate the original price.

b A car is reduced by 30% to $7000. Calculate the original price.

a 130% is $323.70

1% is $\frac{323.70}{130}$

100% is $\frac{323.70}{130} \times 100 = \$249$

∴ The original price was $249.

b 30% reduction so $7000 is 70% of the original price.

1% is $\frac{7000}{70}$

100% is $\frac{7000}{70} \times 100 = \$10\,000$

∴ The original price was $10 000.

6 Find the original price for each of these items.

a A phone is increased by 25% to $640.

b A lamp is increased by 30% to $179.40.

c A suit is increased by 45% to $578.55.

d A formal dress is increased by 55% to $744.

e A computer game is increased by 60% to $136.

7 Find the original price for each of these items.

a A car is reduced by 30% to $2800.

b A game is reduced by 70% to $28.50.

c A dress is reduced by 60% to $180.

d A motor cycle is reduced by 15% to $2125.

EXAMPLE 3

a Find 35% of a quantity if 20% of the quantity is 110 L.
b Find 87% of a quantity if 32% of the quantity is 108.8 m.

a 20% = 110 L

1% is $\frac{110}{20}$

35% is $\frac{110}{20} \times 35 = 192.5$ L

∴ 35% of the quantity is 192.5 L.

Calculator:

110 ÷ 20 × 35 =

b 32% = 108.8 m

1% is $\frac{108.8}{32}$ m

87% is $\frac{108.8}{32} \times 87 = 295.8$ m

∴ 87% of the quantity is 295.8 m.

108.8 ÷ 32 × 87 =

8 **a** Find 8% of a quantity if 15% of the quantity is $34.50.
b Find 42% of a quantity if 6% of the quantity is 53.4 kg.
c Find 58% of a quantity if 13.4% of the quantity is 294.8 L.
d Find 93% of a quantity if 0.6% of the quantity is 9 km.
e Find 61.2% of a quantity if $15\frac{1}{4}$% of the quantity is 109.8 m.
f Find $18\frac{1}{4}$% of a quantity if 72% of the quantity is $3288.96.

F Profit and loss

Profit is the difference between the cost of an item and its selling price.

Profit = selling price − cost price

For example, a painting is purchased for $2000 and later sold for $3000. The profit is $3000 − $2000 = $1000.

Loss occurs if the selling price of an item is lower than the cost price.

Loss = cost price − selling price

Exercise 7F

1 Define the following terms.
a cost price
b selling price
c profit
d loss

2 Calculate the profit on each item when:
a selling price = $700 cost price = $500 profit = 700 − 500 = ____
b selling price = $520 cost price = $380 profit = ____ − 380 = ____
c selling price = $975 cost price = $482 profit = ____ − ____ = ____

3 Calculate the loss on each item when:
a cost price = $800 selling price = $530 loss = 800 − 530 = ____
b cost price = $945 selling price = $645 loss = ____ − 645 = ____
c cost price = $1050 selling price = $822 loss = ____ − ____ = ____

4 Read each statement below.

- **i** State whether a profit or a loss was made.
- **ii** Calculate the profit or loss.

a Sarah purchased a car for $8600. She later sold it for $5200.

b Kyle paid $754 for a painting. He later sold it for $2550.

c A house purchased in 1984 for $183 000 was sold in 2003 for $367 000.

d Dianna purchased shares worth $56 400 in 2007. In 2013 the value of the shares was $38 140.

EXAMPLE 1

A television purchased for $1000 was later sold for $650.

a Calculate the loss.

b Express the loss as a percentage of the cost price.

a Loss = cost price − selling price

= $1000 − $650 = $350

b Loss as a % of cost $= \frac{\text{loss}}{\text{cost price}} \times 100$

$= \frac{350}{1000} \times 100 = 35\%$

5 A computer purchased for $4000 is later sold for $2500.

a Calculate the loss.

b Express the loss as a percentage of the cost price correct to 1 decimal place.

6 A car purchased for $60 000 is later sold for $45 000.

a Calculate the loss.

b Express the loss as a percentage of the cost price correct to 1 decimal place.

7 A motorcycle was purchased for $24 000. If it was sold for $16 000, calculate the loss as a percentage of the cost price.

EXAMPLE 2

A vase purchased for $80 was later sold for $480.

a Calculate the profit.

b Express the profit as a percentage of the cost price.

a Profit = selling price − cost price

= $480 − $80 = $400

b Profit as a % of cost price $= \frac{\text{profit}}{\text{cost price}} \times 100$

$= \frac{400}{80} \times 100 = 500\%$

8 An antique set of drawers purchased for $5000 was later sold for $12 000.

a Calculate the profit.

b Express the profit as a percentage of the cost price.

9 A diamond ring valued at $8000 is later sold for $10 000. Calculate the profit as a percentage of the cost price.

10 A painting purchased for $650 is sold for $1000. Calculate the profit as a percentage of the cost price correct to 1 decimal place.

EXAMPLE 3

a A car with a cost price of $5200 is sold at a profit of 15%. Calculate the selling price.

b A computer with a cost price of $715 is sold at a loss of 27%. Calculate the selling price.

a Percentage $= 100\% + 15\% = 115\%$

Selling price $= \frac{115}{100} \times 5200 = \$5980$

b Percentage $= 100\% - 27\% = 73\%$

Selling price $= \frac{73}{100} \times 715 = \$521.95$

11 Calculate the selling price of each item.

a A computer with a cost price of $740 is sold at a profit of 15%.

b A phone with a cost price of $420 is sold at a profit of 35%.

c A car with a cost price of $6500 is sold at a loss of 30%.

d A dress with a cost price of $350 is sold at a loss of 45%.

e A computer game with a cost price of $120 is sold at a loss of 25%.

f A holiday with a cost price of $899 is sold at a profit of 40%.

EXAMPLE 4

A chair is sold for $319. This is a profit of 45%.
Calculate the cost price of the chair.

Selling percentage $= 100\% + 45\%$
$= 145\%$

145% is $319

1% is $\frac{319}{145}$

100% is $\frac{319}{145} \times 100 = \$220$

$\therefore$ The cost price was $220.

12 Calculate the cost price of these items.

a selling price is $169 for a profit of 30%
b selling price is $318.60 for a profit of 18%
c selling price is $111.54 for a profit of 43%
d selling price is $110 for a profit of 120%
e selling price is $245 for a profit of 75%
f selling price is $1320 for a profit of 340%

EXAMPLE 5

A video game console was sold for $132. This was a loss of 40%. Calculate the cost price.

Selling percentage $= 100\% - 40\% = 60\%$
60% is $132
1% is $\frac{132}{60}$
100% is $\frac{132}{60} \times 100 = \$220$
$\therefore$ The cost price was $220.

13 Calculate the cost price of these items.

a selling price is $144 for a loss of 60%
b selling price is $297.50 for a loss of 15%
c selling price is $923 for a loss of 35%
d selling price is $48.40 for a loss of 45%
e selling price is $15 for a loss of 80%
f selling price is $39.95 for a loss of 47%

14 **a** A car was purchased for $8000 and later sold for $8400.
i Calculate the profit. **ii** Express the profit as a percentage of the cost price.
b A motorcycle was purchased for $22 000 and later sold for $17 500.
i Calculate the loss. **ii** Express the loss as a percentage of the cost price.
c A boat with cost price of $3800 is sold at a profit of 12%.
i Calculate the profit. **ii** Calculate the selling price.
d A computer with a cost price of $1200 is sold at a loss of 35%.
i Calculate the loss. **ii** Calculate the selling price.
e A clock is sold for $276. This is a profit of 32%.
i Calculate the cost price of the clock. **ii** Calculate the profit.
f A smart phone was sold for $210. This was a loss of 45%.
i Calculate the cost price. **ii** Calculate the loss.

15 **a** A car with cost price of $45 800 is sold at a profit of 26%.
- **i** Calculate the profit.
- **ii** Calculate the selling price.

b A tablet computer was sold for $430. This was a loss of 23%.
- **i** Calculate the cost price.
- **ii** Calculate the loss.

c A motorcycle is sold for $6310. This is a profit of 11%.
- **i** Calculate the cost price of the motorcycle.
- **ii** Calculate the profit.

d An antique table was purchased for $16 500 and later sold for $9250.
- **i** Calculate the loss.
- **ii** Express the loss as a percentage of the cost price.

e A painting was purchased for $860 and later sold for $1100.
- **i** Calculate the profit.
- **ii** Express the profit as a percentage of the cost price.

f A gold necklace with a cost price of $3840 is sold at a loss of 47%.
- **i** Calculate the loss.
- **ii** Calculate the selling price.

16 A car is purchased for $5200 and later sold for a profit of 28%. The person who sold the car wants to buy it back and is told the price to buy it back must give the new seller a profit of 15%. How much does it cost to buy the car back?

G Goods and services tax (GST)

The GST is a federal tax applied to most goods and services in Australia. It is calculated at the rate of 10% of the purchase price of the goods or services.

The price excluding the GST (that is the price before the GST is added) is written 'price excluding GST'.

The price including the GST (that is the price after the GST is added) is written 'price including GST'.

EXAMPLE 1

a Calculate the GST and the price including GST of a camera with a listed price excluding GST of $710.

b Calculate the price including GST of a mobile phone with a listed price excluding GST of $299.

a GST $= 10\%$ of $\$710$
$= 0.1 \times \$710$
$= \$71$

Price including GST $= \$710 + \$71$
$= \$781$

b Price including GST = list price + 10% of the list price
= 110% of the list price
$= 1.10 \times \$299$
$= \$328.90$

Exercise 7G

1 Calculate the GST and the price including GST of the following items with listed prices that exclude GST.

a microwave oven $440
b computer $3690
c TV repairs $258
d DVD player $397
e plumber's bill for services $1800

2 Calculate the price including GST on the following items with listed prices that exclude GST.

a car battery $95
b ticket to Rugby Final $225
c bottle of wine $17
d printer repairs $336
e electrician's bill $457
f bicycle $399

EXAMPLE 2

A TV is advertised with a listed price of $899, price including GST.

a Calculate the GST included on the price.
b Calculate the pre-GST price.

The simple method for calculating the GST in these situations is called the 'GST rule of thumb':
GST = price including GST ÷ 11

a $$\text{GST} = \frac{\$899}{11}$$
$= \$81.73$ to the nearest cent

b Pre-GST price $= \$899 - \$81.73$
$= \$817.27$

3 For the following items with listed prices that include GST, use the GST rule of thumb to calculate:

i the GST included in the price of each item
ii the pre-GST price.

a TV $1189
b lounge suite $4970
c BBQ chicken $10.89
d perfume $148
e dress $124
f tablet computer $499

4 Find the amounts missing from the following invoices.

a *Tax invoice*
Services rendered = $850
GST = ____
Total including GST = ____

b *Tax invoice*
Services rendered = ____
GST = $48.80
Total including GST = $536.80

c *Tax invoice*
Taxable items
Shirt $69.95
Tie $29.95
Total including GST = $99.90
GST included in total = ____

d *Tax invoice*
Taxable items
5 CDs @ $32.90 including GST = ____
GST included in total = ____

5 A docking station is valued at $100 before adding the GST. After the GST is added the sale price is $110.

a Explain how the sale price of $110 was achieved.

b John concluded that the price of the docking station before the addition of GST is really equivalent to $110 × 0.90 or $110 × 90%. He calculates that the price before GST was added was $99. Is John correct? Explain your answer.

6 Complete the following to find the values of these amounts when they are increased by 10%.

a $150: $150 × 110% = $150 × 1.1 = $___

b $220: $220 × 110% = $220 × ___ = $___

c $370: $370 × 110% = $___ × ___ = $___

d $400: $400 × 110% = $___ × ___ = $___

7 Use the answers from question **6** and decrease each amount by 10%.

a $165 × 90% = $165 × 0.9 = $___

b $___ × 90% = $___ × 0.9 = $___

c $___ × 90% = $___ × 0.9 = $___

d $___ × 90% = $___ × 0.9 = $___

8 **a** What do you notice about the amounts obtained in the last column of questions **6** and **7**?

b Is the following statement true or false?
When an amount is increased by a percentage, and the new amount is then decreased by the same percentage, the result is the original amount.

c Explain the reasoning for your findings.

Investigation 4 Taxation percentages in other countries (extension)

Australia has a GST of 10%. Other countries have similar taxes, sometimes called a VAT or value added tax, of different percentage amounts. The 10% used in Australia means the calculations are simpler than for the percentages used in other countries. The rule of thumb that can be used for 10% cannot be used for any other amount.

This investigation examines the VAT of some other countries, which are listed in the table.

Country	Percentage rate
Belgium	21.0%
Luxemburg	15.0%
Germany	19.0%
France	20%
Ireland	23%

Country	Percentage rate
Italy	22.0%
Netherlands	21.0%
South Africa	14.0%
Switzerland	8%
New Zealand	13%

Complete the following calculations in Australian dollars. For example, to find the VAT on a sports bag priced at $A75 in France, calculate 20% of $75 as the VAT rate in France is 20%. The answer is $A15.

1 Calculate the VAT on a watch listed at $A650 in these countries.
a Italy **b** Luxemburg **c** South Africa **d** New Zealand

2 Calculate the VAT on a box of chocolates listed at $A15 in these countries.
a New Zealand **b** France **c** Switzerland **d** Netherlands

3 Calculate the VAT on a camera listed at $A230 in these countries.
a Belgium **b** Germany **c** Ireland **d** Italy

To calculate the amount of VAT included in the cost of a mobile phone priced at $A199 in South Africa, the unitary method must be used, as the VAT is not 10% and the rule of thumb only works for tax rates of 10%. The VAT rate in South Africa is 14% so:

114% is $A199

So 1% is $\frac{199}{114}$

Then 14% is $\frac{199}{114} \times 14$ as the VAT rate is 14%.

The VAT amount is $A24.44.

4 Use the unitary method to find the amount of VAT included in the cost of a smart phone priced at $A625 in these countries.
a Belgium **b** Germany **c** South Africa **d** New Zealand

5 Use the unitary method to find the amount of VAT included in a plane fare quoted at $A322 in these countries:
a Italy **b** Luxemburg **c** Ireland **d** Switzerland

6 Use the unitary method to find the amount of VAT included in the cost of a tablet computer priced at $A299 in these countries.
a New Zealand **b** France **c** Switzerland **d** Netherlands

7 A camera is priced at $A347 in Germany. The price includes VAT. Calculate the cost of the same camera including VAT if purchased in:
a Italy **b** Luxemburg **c** Ireland **d** New Zealand.

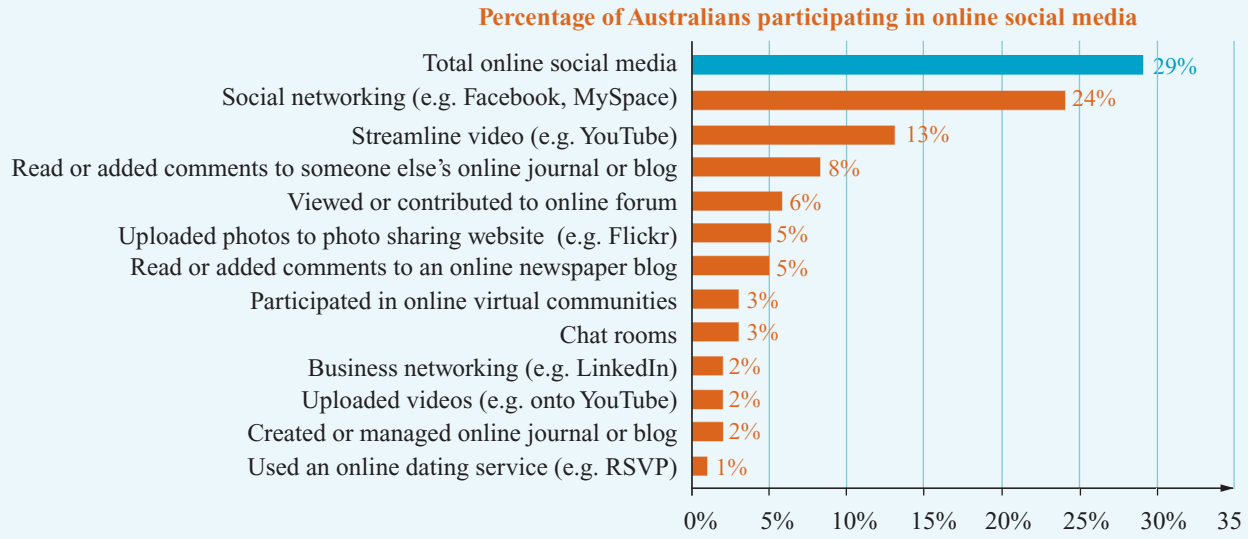
8 A carry-on airline bag is priced at $A99 in Ireland. The price includes VAT. Calculate the cost of the same bag including VAT if purchased in:
a Belgium **b** Germany **c** South Africa **d** Italy.

Language in mathematics

Online social media

An estimated 29% of Australians (5.2 million) participated in some type of online social media activity in an average 4-week period, according to Roy Morgan Research data for the year ended March 2010.

The most popular social media activity was visiting social networking sites such as Facebook and MySpace, with 24% of Australians 14+ years of age using them in an average 4-week period. Streamed videos, such as YouTube, were used by 13%; online journals and blogs by 8%; and online forums by 6%.



Source: http://www.roymorganonlinestore.com/News/1118---Over-a-quarter-of-Australians-participate-i.aspx

Jane Ianniello, Roy Morgan Research International Director of Tourism, Travel & Leisure, stated:

> 'An increasing percentage of Australians are participating in online social media. The good news for the Australian tourism industry is that people participating in online social media are more likely than the average Australian to take holidays, both domestic and overseas. They are also more likely to provide advice to their friends and family about holiday and travel, so they are potentially useful advocates for a tourism destination. To this end social media represents a real opportunity to the tourism industry.'

This information highlights the positive impacts that social media can have for the tourism industry.

1 Do you think that the statistical information is sufficient to support the claims that social media positively impacts upon the tourism industry? Explain your answer.

2 Why do you think social media has become such a powerful force in today's modern economy? It may be helpful to ask your classmates what types of social media they access, when, how often and for what purpose. How do you think the figures above would have changed since 2010?

3 Research a holiday destination of your choice online.

- **a** Is there sufficient information online to make an informed decision?
- **b** Is there evidence of social media to assist you in making your choices regarding this destination?
- **c** Do you find the use of social media helpful or a deterrent?

4 In class debate that: *Social media enables us as consumers to make informed choices*. Run a series of debates regarding the impact of social media in our lives. Tourism is only one area. Undoubtedly you will be able to look at the many areas of our everyday life that are impacted both positively and negatively by the forces of social media.

Terms

ascending	compare	conversion	convert	cost price
decrease	decimal	descending	discount	estimate
equivalent	fraction	increase	improper fraction	loss
mixed numeral	percentage	place value	profit	proper fraction
rounding	selling price	simplify		

Check your skills

1 Express 0.08% as a decimal.

A 0.0008 **B** 0.008 **C** 0.08 **D** 0.8

2 Express $5\frac{5}{6}$ as a percentage.

A 58.3 **B** 5.83 **C** 58.33 **D** 583.3

3 Express 82 L as a percentage of 90 L.

A 0.91% **B** 9.1% **C** 91.1% **D** 911.1%

4 Express 32 min as a percentage of $1\frac{1}{4}$ h.

A 0.426% **B** 4.26% **C** 42.6% **D** 426.6%

5 Find $52\frac{1}{2}$% of \$7500.

A \$3937.50 **B** \$3562.50 **C** \$3973.50 **D** \$3937.50

6 Increase 620 kg by 12%.

A 74.4 kg **B** 545.6 kg **C** 694.4 kg **D** 644.4 kg

7 Decrease \$230 by 56%.

A \$358.80 **B** \$101.20 **C** \$149.50 **D** \$105.80

8 Calculate the percentage increase from 36 kg to 65 kg to the nearest per cent.

A 55% **B** 80% **C** 45% **D** 81%

9 What is 100% of a quantity if 27% is 189?

A 1.89 **B** 7 **C** 51.03 **D** 700

10 The original price of a laptop is increased by 35% to \$1146.15. What was the original price?

A \$3274.71 **B** \$1146.15 **C** \$849 **D** \$401.15

11 If 65% of a quantity is 572 what is 42% of the quantity?

A \$371.80 **B** \$369.60 **C** \$240.24 **D** \$156.16

12 Brendan and Tiarne purchased a house in 2007 for \$430 000. They sold it in 2013 for \$572 000. Express the profit as a percentage of the purchase price.

A 33% **B** 25% **C** 30% **D** 75%

13 A car with cost price of \$6000 is sold at a loss of 45%. What is the selling price?

A \$2700 **B** \$3300 **C** \$8700 **D** \$9300

14 A ring is sold for $840. This is a profit of 40%. What was the cost price of the ring?
A $2100 **B** $504 **C** $600 **D** $336

15 An item valued at $675 needs to have 10% GST added. What is the selling price inclusive of GST?
A $607.50 **B** $668.25 **C** $742.50 **D** $67.50

16 A watch is advertised at $199 including GST. What was the pre-GST cost of the watch?
A $19.90 **B** $179.10 **C** $18.09 **D** 180.91

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1, 2	3–5	6, 7	8	9–11	12–14	15, 16
Section	A	B	C	D	E	F	G

7A Review set

1 Find 15% of 200 kg.

2 Express 26 kg as a percentage of 78 kg, correct to 2 decimal places.

3 Increase 100 by 40%.

4 Decrease 280 by 25%.

5 The amount of water in a tank increases from 80 L to 135 L. What is the percentage increase?

6 Find the percentage decrease from 85 kg to 68 kg.

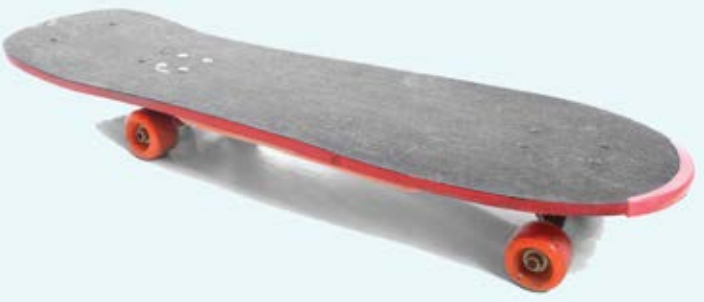
7 Find 100% of a quantity if 37% is $155.40.

8 The price of a bicycle is increased by 60% to $638.40. Calculate the original price.

9 An antique gun purchased for $1200 was later sold for $2900.
a Calculate the profit.
b Express the profit as a percentage of the cost price.

10 A necklace with a cost price of $149 is sold at a loss of 12%. Calculate the selling price.

11 A skateboard was sold for $76.50. This was a loss of 15%. Calculate the cost price.



12 A microwave oven is listed at $189 including GST.
a Calculate the GST.
b Calculate the pre-GST price.

7B Review set

1 Robyn scored 73 out of 100 in a Science exam. Express her result as a percentage.

2 Write 55 g as a percentage of 250 g.

3 Write \$0.24 as a percentage of \$2.40.

4 Find 53% of \$400.

5 Find $5\frac{1}{4}$% of 200 kg.

6 Daniel earns \$720 per week. He spends 46% of his income on rent and household expenditure, 22% on entertainment and the remainder is placed in a savings account. Calculate the amount of money Daniel allocates each week to:

a rent and household expenditure **b** entertainment **c** savings

7 Express 45 cm as a percentage of 1.2 m.

8 Decrease 14 m by 8%.

9 Joe purchases bananas for 20c each. If he sells them at an increased price of 160%, what is the selling price of a banana?

10 Find 100% of a quantity if 62% is \$264.12.

11 A tennis racquet was sold for \$135. This was a profit of 43%. Calculate the cost price.

12 Calculate the GST on a DVD player listed at \$89 excluding GST.

7C Review set

1 Express 4.2 kg as a percentage of 800 g.

2 Express 64 L as a percentage of 80 L.

3 Find 42% of 5000 km.

4 Increase 84 m by 6%.

5 Find the percentage decrease from 135 L to 61 L.

6 Find 100% of a quantity if 135% is \$75.60.

7 The price of a concert ticket is increased by 140% to \$124.60. Calculate the original price.

8 A signed West Tigers jumper purchased for \$800 was sold for \$2000.

a Calculate the profit.

b Express the profit as a percentage of the cost price.

9 An Eels jumper was sold for $35. This was a loss of 76%. Calculate the cost price.

10 Craig purchased a mountain bike for $1800. He later sold it for $1332. Express the loss as a percentage of the cost price.

11 Calculate the GST on a dinner set listed at $385 excluding GST.

12 Calculate the pre-GST price of a tracksuit listed at $99 including GST.

7D Review set

1 Express 4.5 m as a percentage of 120 cm.

2 Express 660 g as a percentage of 1.2 kg.

3 Find 72% of $80.

4 Decrease $6500 by 28%.

5 Over a period of time the value of a house increased by 15% to $564 000. Find the original value of the house, to the nearest dollar.

6 Calculate the percentage increase from 48 kg to 91 kg.

7 Calculate the percentage decrease from 112 m to 78 m.

8 Find 100% of a quantity if 38% is 29.64 kg.

9 A brochure advertises jackets for 30% off the original price. Calculate the original cost if the sale price is $455.

10 The price of a concert ticket is increased by 74% to $374.10. Calculate the original price.

11 A radio-controlled plane with a cost price of $349 is sold at a loss of 23%. Calculate the selling price.

12 A collector card was sold for $475. This was a profit of 35%. Calculate the cost price.

13 Find the GST on an item marked at $1980 including 10% GST.

14 Calculate the selling price of an item valued at $95 if 10% GST must be added.

8

Circles and cylinders

This chapter deals with circles and cylinders.

At the end of the chapter you should be able to:

- name the parts of a circle
- calculate the circumference of a circle
- find the length of an arc
- calculate the perimeter of composite figures containing circles or parts of circles
- calculate the radius of a circle given the circumference
- calculate the area of circles and sectors
- calculate the area of composite figures containing circles or parts of circles
- calculate the volume of a cylinder
- calculate the volume of composite figures containing cylinders.

NSW Syllabus references: S4 M&G Length, S4 M&G Volume
Outcomes: MA4-1WM, MA4-2WM, MA3-9MG, MA4-12MG, MA4-13MG, MA4-14MG
MEASUREMENT & GEOMETRY – ACMMG186, ACMMG197, ACMMG217

Diagnostic test

1 40 mm is equal to:
- **A** 4 cm
- **B** 400 cm
- **C** 0.4 cm
- **D** 4000 cm

2 5 cm 3 mm is equal to:
- **A** 53 cm
- **B** 5.3 cm
- **C** 0.53 cm
- **D** 5300 cm

3 43.2 cm is equal to:
- **A** 4.32 mm
- **B** 432 mm
- **C** 0.432 mm
- **D** 4320 mm

4 7 m is equal to:
- **A** 70 cm
- **B** 7000 cm
- **C** 0.7 cm
- **D** 700 cm

5 4 m 65 cm is equal to:
- **A** 46.5 cm
- **B** 465 cm
- **C** 4650 cm
- **D** 4.65 cm

6 3.25 m is equal to:
- **A** 32.5 cm
- **B** 3.25 cm
- **C** 325 cm
- **D** 3250 cm

7 157 mm is equal to:
- **A** 15.7 cm
- **B** 1.57 cm
- **C** 1570 cm
- **D** 15 700 cm

8 5.439 km is equal to:
- **A** 543.9 m
- **B** 54.39 m
- **C** 5 439 000 m
- **D** 5439 m

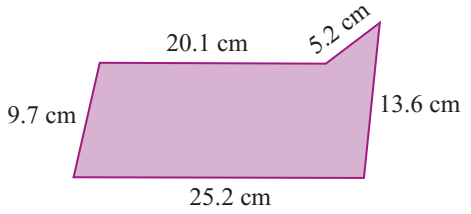
9 12 500 m is equal to:
- **A** 125 km
- **B** 12.5 km
- **C** 1.25 km
- **D** 12 500 000 km

10 Estimate 23.8 cm to the nearest cm.
- **A** 23 cm
- **B** 24 cm
- **C** 28 cm
- **D** 25 cm

11 0.43 km is equal to:
- **A** 430 cm
- **B** 4300 cm
- **C** 43 000 cm
- **D** 430 000 cm

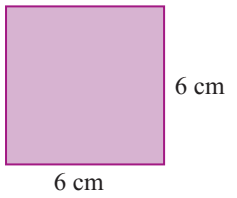
12 Which statement below is correct?
- **A** 52 m = 520 cm
- **B** 0.7 km = 70 000 mm
- **C** 800 000 mm = 80 m
- **D** 463 000 mm = 0.463 km

13 The perimeter of this shape is:



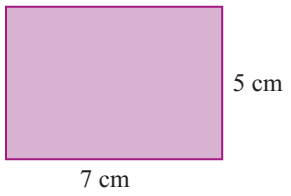
- **A** 71.1 cm
- **B** 69.3 cm
- **C** 73.8 cm
- **D** 64.1 cm

14 The area of this square is:



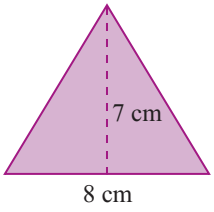
- **A** 36 cm
- **B** 36 cm^2
- **C** 24 cm
- **D** 24 cm^2

15 The area of this rectangle is:



- **A** 35 cm
- **B** 35 cm^2
- **C** 12 cm^2
- **D** 24 cm^2

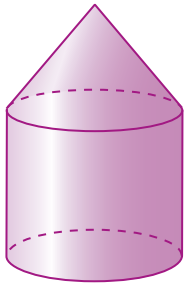
16 The area of this triangle is:



- **A** 28 cm
- **B** 28 cm^2
- **C** 15 cm
- **D** 56 cm^2

17 The geometrical name that best describes the shape of a matchbox is:
- **A** square prism
- **B** sphere
- **C** rectangular pyramid
- **D** rectangular prism

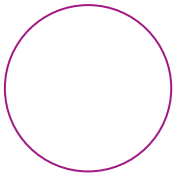
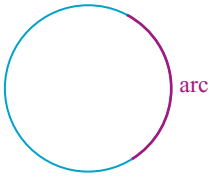
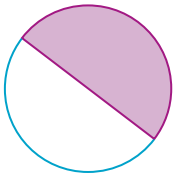
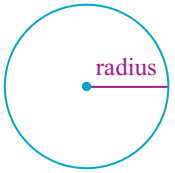
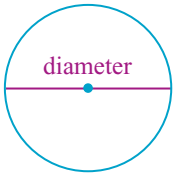
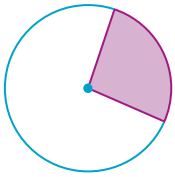
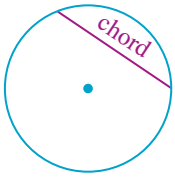
18 The solids that have been combined to make the object shown are:

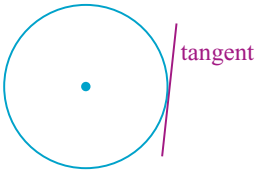
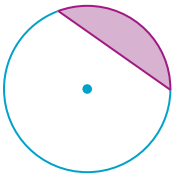
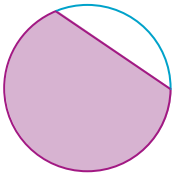


- **A** a cone and a rectangular prism
- **B** a pyramid and a hemisphere
- **C** a pyramid and a cylinder
- **D** a cone and a cylinder

The Diagnostic test questions refer to the Year 7 outcomes from ACMMG136, ACMMG159 and ACMMG161.

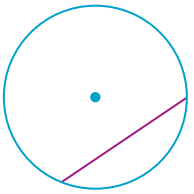
A Parts of a circle

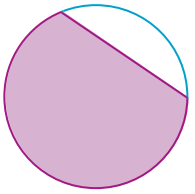
Name	Diagram	Definition
Circumference		The perimeter of a circle
Arc		Part of the circumference
Semicircle		Half of a full circle
Centre		The point inside the circle which is equidistant from all points on the circle (Equidistant means 'equal distance'.)
Radius		An interval drawn from the centre of the circle to the circumference
Diameter		An interval passing through the centre that divides the circle in half (Diameter $= 2 \times$ radius)
Sector		An area cut off by two radii
Chord		An interval joining two points on the circumference, but not passing through the centre

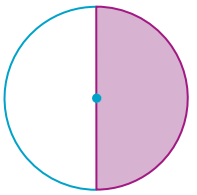
Name	Diagram	Definition
Tangent		A line on the exterior of the circle that touches the circumference at one point
Minor segment		The smaller of the two regions inside a circle bounded by the circle and a chord
Major segment		The larger of the two regions inside a circle bounded by the circle and a chord

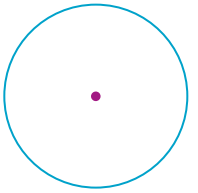
Exercise 8A

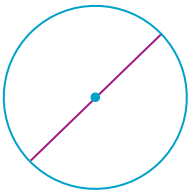
1 Name the features shown in purple for each of these circles.

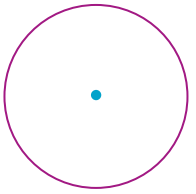
a 

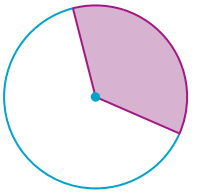
b 

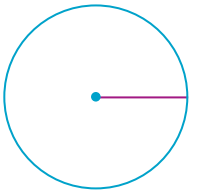
c 

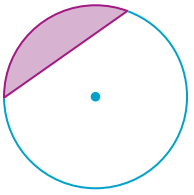
d 

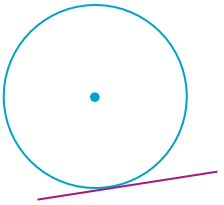
e 

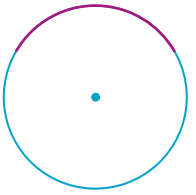
f 

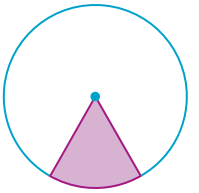
g 

h 

i 

j 

k 

l 

2 What features of circles can be seen in the examples below?

a 

b 

c

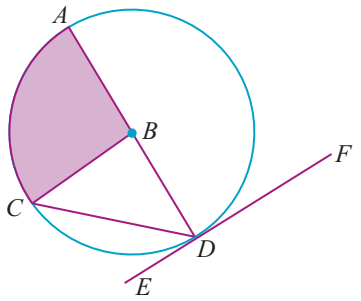
d

3 **a** Name the feature shown by these intervals.

i AB **ii** AD **iii** EF

iv AC **v** CD **vi** DB

b Which feature is represented by the shaded region?

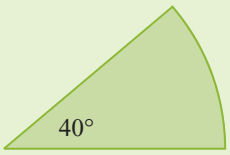


EXAMPLE 1

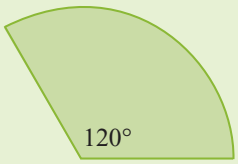
What fraction of a circle is represented by the sectors below?

A full revolution measures 360°.

a



b



a The sector represents $\frac{40°}{360°} = \frac{1}{9}$

Calculator:

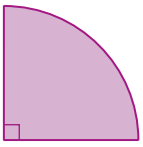
40 $a\frac{b}{c}$ 360 = shows 1 ⌋ 9 $= \frac{1}{9}$

b The sector represents $\frac{120°}{360°} = \frac{1}{3}$

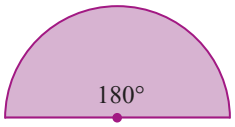
120 $a\frac{b}{c}$ 360 = shows 1 ⌋ 3 $= \frac{1}{3}$

4 What fraction of a circle is represented by the following sectors?

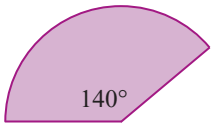
a



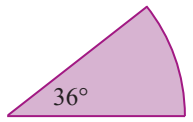
b



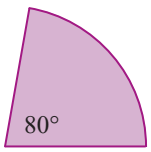
c



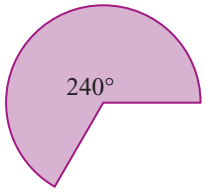
d



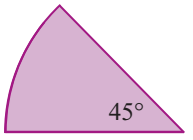
e



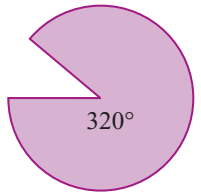
f



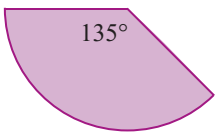
g



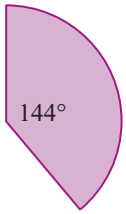
h



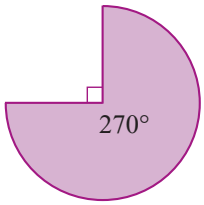
i



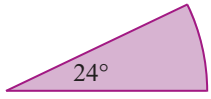
j



k



l

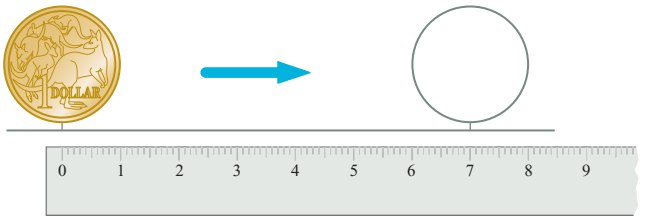


Investigation 1 Circumference of a circle

The perimeter of a circle is the distance around its boundary. In the case of the circle, the perimeter is called the circumference.

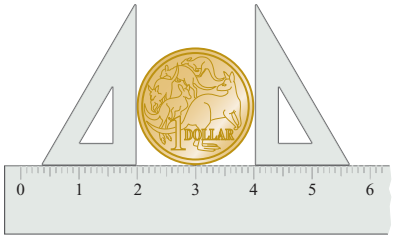
1 To measure the circumference of a coin, roll it against a ruler for one rotation and measure the length of the interval.
Measure the circumference of these coins.

a 10 cent **b** 20 cent
c \$1 **d** \$2



2 To measure the diameter of a coin, which is much easier than measuring the circumference, use a ruler and two set squares as shown. Measure the diameter of the following coins using this method.

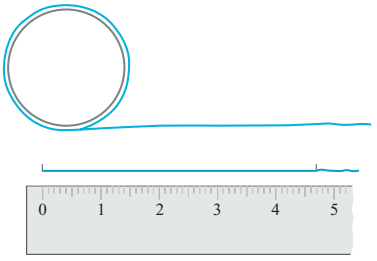
a 10 cent **b** 20 cent
c \$1 **d** \$2



3 Use the information from questions **1** and **2** to complete the following table.

Coins	Circumference, C	Diameter, d	$C \div d$
10c			
20c			
\$1			
\$2			

4 The following diagrams show how to use a piece of string to trace the perimeter of a circle to measure the circumference. A ruler can be used to measure the diameter.
Step 1: Trace around the perimeter with string.

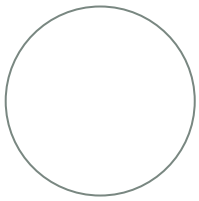


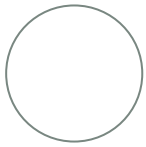
Step 2: Straighten and measure the length of string required to trace the perimeter.

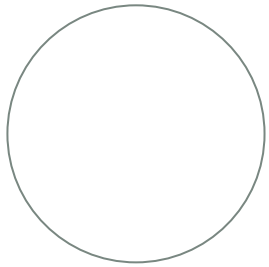
The circumference of this circle ≈ 4.7 cm.

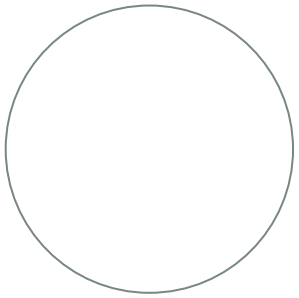
Measure the circumference and diameter of each circle drawn on the next page and record the information in the table below. Answer to the nearest mm.

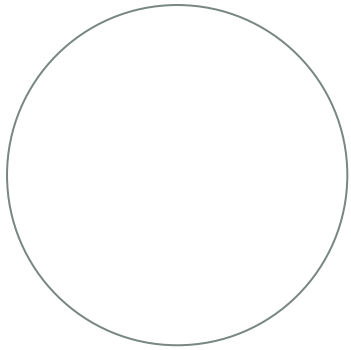
	Circumference, C	Diameter, d	$C \div d$
a			
b			
c			
d			
e			

a 

b 

c 

d 

e 

5 Complete the following table by filling in the third column. Round your answers correct to 1 decimal place.

	Circumference	Diameter	$C \div d$
a	6.3 cm	2 cm	
b	22.0 cm	7 cm	
c	28.3 cm	9 cm	
d	15.7 cm	5 cm	
e	34.6 cm	11 cm	

	Circumference	Diameter	$C \div d$
f	4.7 cm	1.5 cm	
g	7.2 cm	2.3 cm	
h	18.2 cm	5.8 cm	
i	29.5 cm	9.4 cm	
j	33.3 cm	10.6 cm	

6 From questions **3**, **4** and **5**, what have you discovered about $\frac{\text{circumference } (C)}{\text{diameter } (d)}$?

Pi (π)

You should have discovered that $\frac{C}{d} \approx 3.1$.

The actual value of this ratio is 3.141 596 265 4…, a decimal that continues forever. It is referred to as pi and is given the symbol π, which is a letter from the Greek alphabet. Pi is an irrational number, meaning that an exact value in decimal or fraction form cannot be given, so approximations are used.

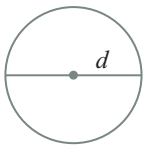
Approximations for π are listed in the table below.

Fraction	Decimal	Calculator
$3\frac{1}{7}$ or $\frac{22}{7}$	3.14 (2 decimal places)	π EXP

$\frac{22}{7}$ is 3.14 accurate to 2 decimal places.

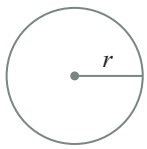
To calculate the circumference of a circle use:

$C = \pi d$



Or, as the diameter is twice the radius use:

$C = 2\pi r$

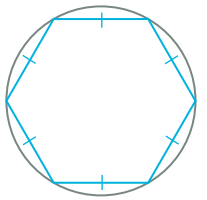


7 There have been many approximations for π throughout history. Compare those given below to the correct value of π.

19 century BCE	Babylonians	$\frac{25}{8}$	9 century BCE	Yajnavalkys	$\frac{339}{108}$
2nd century CE	Ptolemy	$\frac{377}{120}$	263 CE	Liu Hui	$\frac{3927}{1250}$

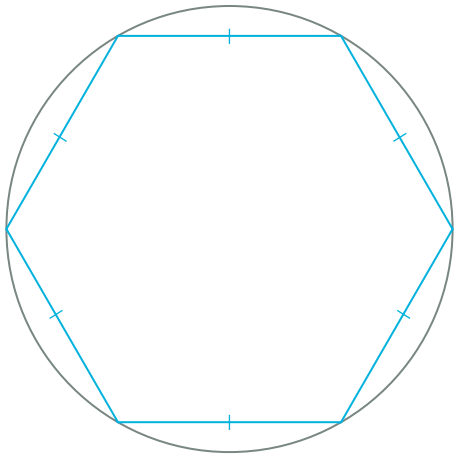
Investigation 2 The value of pi (π) using hexagons

1 The diagram shows a hexagon of side length 1.3 cm enclosed by a circle.



- **a** Calculate the perimeter of the hexagon.
- **b** The diameter of the circle is 2.5 cm. From investigation 1, you discovered that $C = \pi d$. Thus $\pi = \frac{C}{d}$. Calculate π if the circumference of the circle is 7.8 cm.

2 **a** Measure the side length of this hexagon.



- **b** Calculate the perimeter of the hexagon.
- **c** Check that the diameter of the circle is 5.9 cm.
- **d** Calculate π if the circumference $= 18.5$ cm, using $\pi = \frac{C}{d}$.
- **e** From questions **1** and **2**, what do you notice about the value of π?

3 **a** Construct a hexagon with side lengths 5 cm.

- **b** Using your compass, draw a circle around the hexagon so that the circumference touches the edges.
- **c** Calculate the perimeter of the hexagon.
- **d** Use a piece of string to trace the outside or circumference of the circle. Straighten the string and use a ruler to measure the circumference.
- **e** Measure the diameter of the circle. Be as accurate as possible.
- **f** Calculate the value of π, if $\pi = \frac{C}{d}$.
- **g** What conclusions can you draw about the value of π?

Note that a hexagon has 60° angles at its centre.

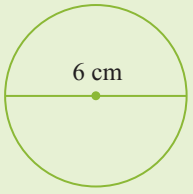
B Circumference of a circle

The perimeter of a circle is the distance around its boundary. The perimeter of a circle is called the circumference. From Investigation 2, $C = \pi d$ or $C = 2\pi r$.

EXAMPLE 1

Determine the circumference of each circle to the level of accuracy stated.

a exact value



b correct to 2 decimal places

a $C = \pi d$

$C = \pi \times 6$

$= 6\pi$ cm

b $C = 2\pi r$

$C = 2 \times \pi \times 4.5$

$= 28.27$ cm

Calculator:

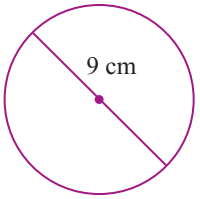
2 × π × 4.5 =

Find out how to access pi (π) on your calculator.

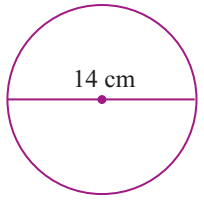
Exercise 8B

1 Calculate the circumference of each circle, giving both the exact answer and the answer correct to 2 decimal places.

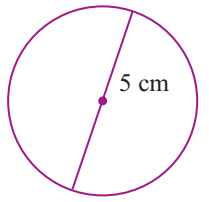
a



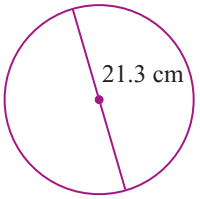
b



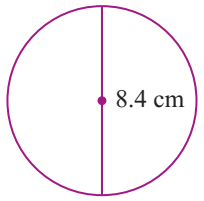
c



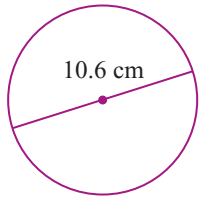
d



e

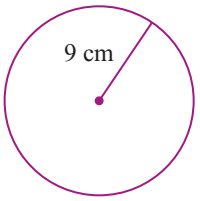


f

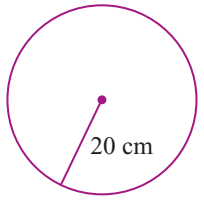


2 Calculate the circumference of each circle, giving both the exact answer and the answer correct to 2 decimal places.

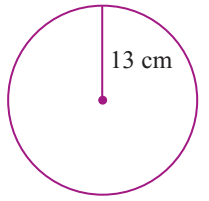
a



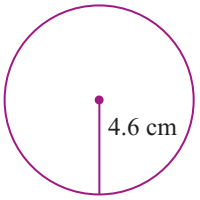
b



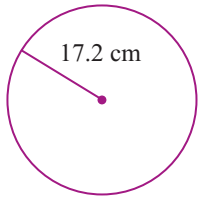
c



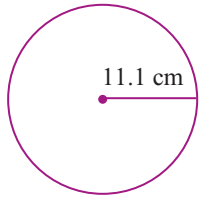
d



e

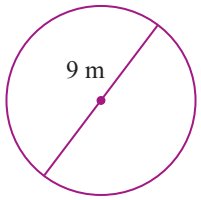


f

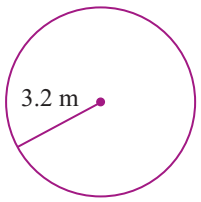


3 Calculate the circumference of each circle correct to 1 decimal place.

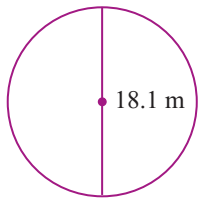
a



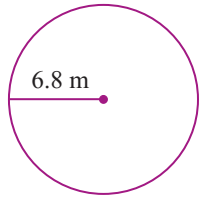
b



c



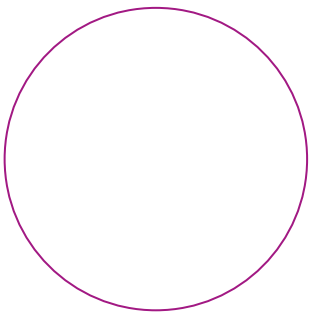
d



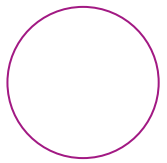
e diameter = 12.6 m **f** radius = 7.9 m **g** radius = 2.5 m **h** diameter = 8.64 m

4 Calculate the circumference of these circles correct to 1 decimal place by first measuring the diameter.

a



b



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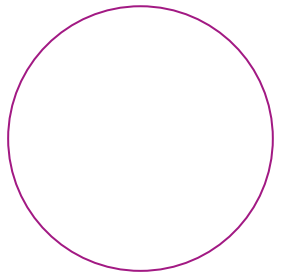
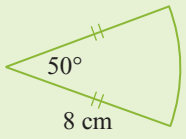


EXAMPLE 2

Calculate the length of each arc correct to 2 decimal places.

a



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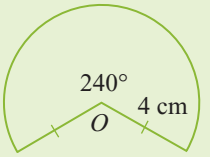


a Fraction of circumference $= \frac{50}{360} = \frac{5}{36}$

Length of arc $= \frac{5}{36} \times 2\pi r$

$= \frac{5}{36} \times 2 \times \pi \times 8$

$= \frac{20\pi}{9} = 6.98$ cm

Calculator:

5 $a\frac{b}{c}$ 36 × 2 × π × 8 =

b Fraction of circumference $= \frac{240}{360} = \frac{2}{3}$

Length of arc $= \frac{2}{3} \times 2\pi r$

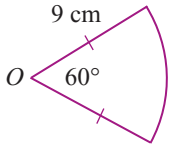
$= \frac{2}{3} \times 2 \times \pi \times 4$

$= \frac{16\pi}{3} = 16.76$ cm

2 $a\frac{b}{c}$ 3 × 2 × π × 4 =

5 Calculate the length of these arcs. Give both the exact answer and the answer correct to 2 decimal places.

a



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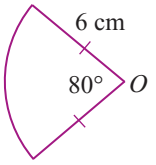

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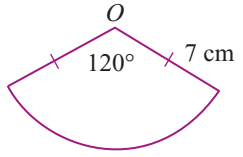

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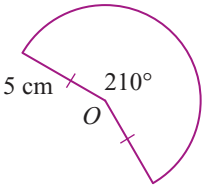

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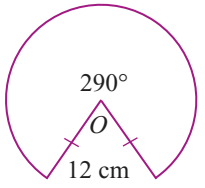

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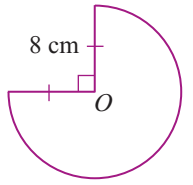
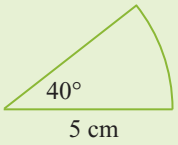


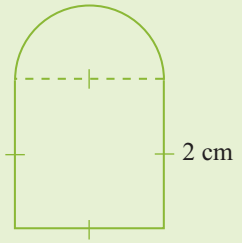
EXAMPLE 3

Calculate the perimeter of each figure correct to 1 decimal place.

a



b



Include the straight sides in the perimeter too.

a The shape is a sector.

Fraction of circumference $= \frac{40}{360} = \frac{1}{9}$

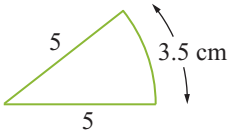
Length of arc $= \frac{1}{9}$ of $2\pi r$

$= \frac{1}{9} \times 2 \times \pi \times 5 \approx 3.5$ cm

Perimeter = sum of all sides

$= 3.5 + 5 + 5$

$= 13.5$ cm



b Shape consists of a semicircle plus a square.

Circumference of semicircle is $\frac{1}{2}$ of πd.

Length of arc $= \frac{1}{2}$ of πd

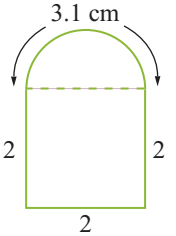
$= \frac{1}{2} \times \pi \times 2$

≈ 3.1 cm

Perimeter = sum of all sides

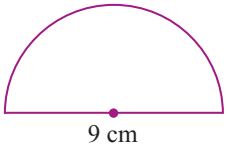
$= 3.1 + 2 + 2 + 2$

$= 9.1$ cm

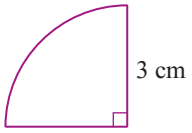


6 Calculate the perimeter of each figure correct to 1 decimal place.

a



b



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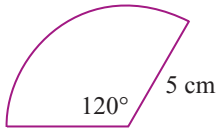
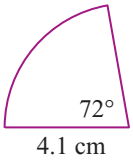
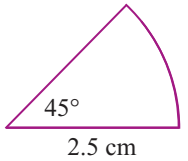


d



e



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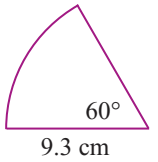

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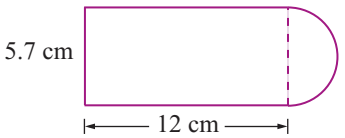
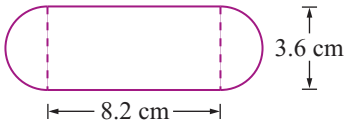
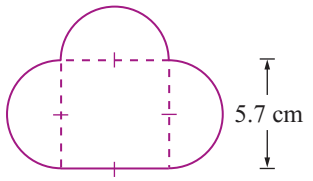


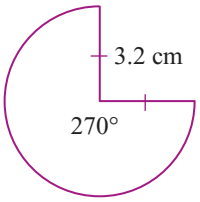
h



i



j



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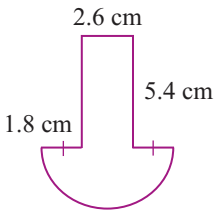

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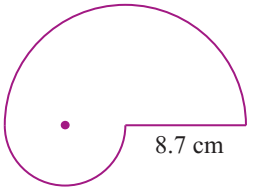


7 The diameter of a bicycle tyre is 40 cm.

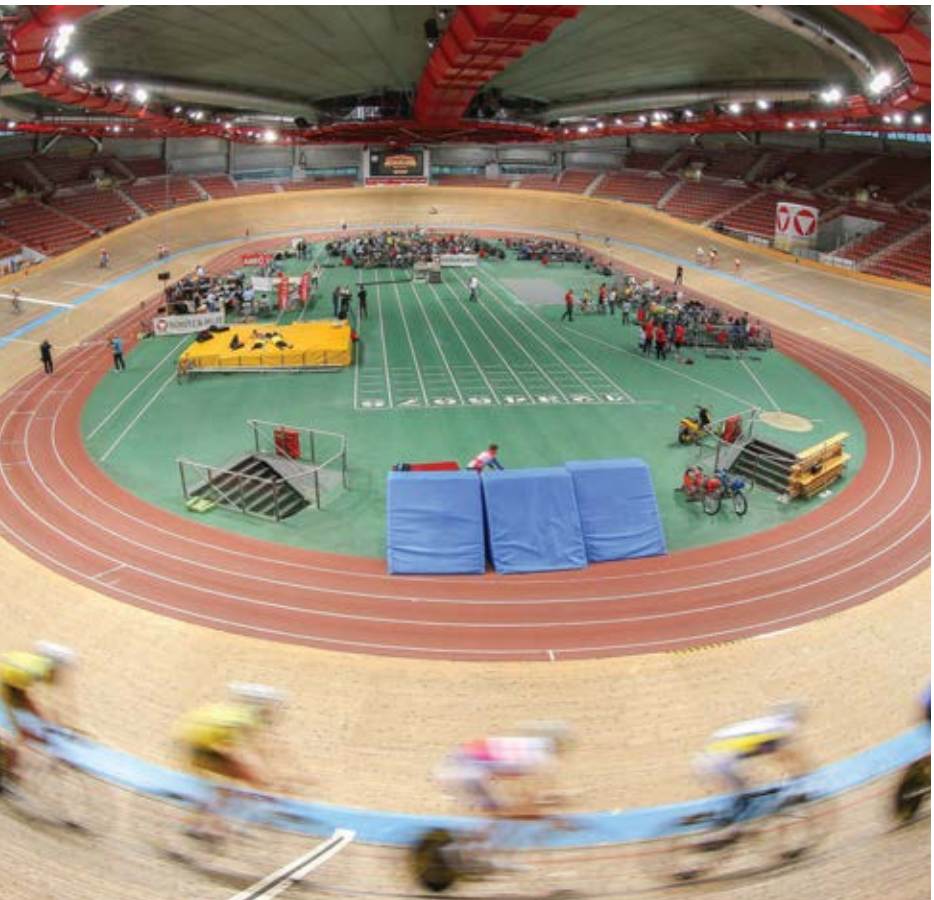
a Calculate the distance travelled by the tyre in one revolution. Give your answer to the nearest centimetre.

b How far, in metres, will the tyre travel in 500 revolutions? (Use your answer from part **a**.)

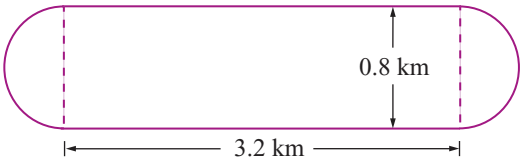
c Using your answer from part **a**, calculate the number of full revolutions required to travel:

i 1 km **ii** 7 km **iii** 13 km

Hint: Number of revolutions $= \dfrac{\text{distance to travel (m)}}{\text{distance of revolution (m)}}$



8 **a** A cyclist cycles the track shown. Calculate the distance travelled in one lap to the nearest metre.



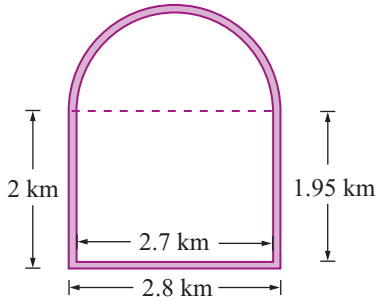
b How many complete laps would be required for the cyclist to travel:

i 100 km?

ii 175 km?

iii 234 km?

9 Danijella and Joan walk along a track every morning. Danijella walks along the outside perimeter of the track shown and Joan walks along the inside perimeter.



a How far does Danijella walk, in metres?
b How far does Joan walk, in metres?
c How much further does Danijella walk than Joan?
d If Danijella walks to a speed of 1 km/10 min, how long should it take her to complete her morning walk? (Approximate your answer to the nearest minute.)

C Finding the diameter and radius from the circumference

As $C = \pi d$, then $d = \frac{C}{\pi}$

EXAMPLE 1

The circumference of a circle is 60 cm. What is its:
a diameter to 1 decimal place? **b** radius to 1 decimal place?

a $C = \pi \times d$
$\therefore 60 = \pi \times d$
$d = \frac{60}{\pi}$
≈ 19.1 cm

Calculator:
60 ÷ π = 19.098 593 …

b $r = \frac{d}{2}$
$= \frac{19.098\ 593\ \ldots}{2}$
$= 9.549\ \ldots$
≈ 9.5 cm

Remember that the radius is half the diameter.

Exercise 8C

1 Using the circumference given for each circle, determine, correct to 1 decimal place, the:
i diameter **ii** radius.

a 15 cm **b** 37 cm **c** 85 cm **d** 100 cm
e 131 cm **f** 175 cm **g** 40 cm **h** 9.4 cm
i 12.7 cm **j** 188.5 cm **k** 282.7 cm **l** 43.9 cm

2 A machine makes circular paper plates of circumference 50 cm. Find the diameter of the plates correct to 1 decimal place.

3 A trundle wheel has a circumference of 1 m. What is the radius of the trundle wheel correct to 1 decimal place?

4 A car tyre makes 637 complete revolutions to travel 1 km.

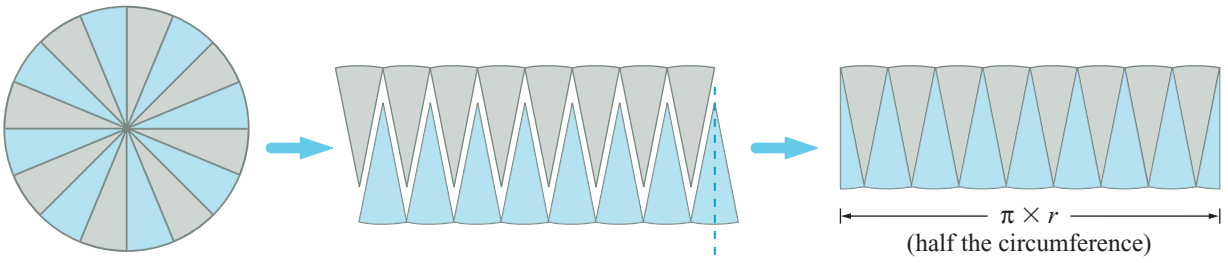
a Calculate the distance travelled by the car tyre in one revolution correct to 2 decimal places.

b Calculate the diameter of the car's wheel to the nearest centimetre.

Investigation 3 Area of a circle

Use the following diagrams to find an expression for the area of a circle.

The circle has a number of sectors drawn in it. It is then cut in half and both halves are cut along the radii. The two halves are put together and, with a little trimming, form a plane shape similar to a rectangle.



Circumference $= 2 \times \pi \times r$

The length of the rectangle is half the circumference and the width of the rectangle is approximately the radius.

Complete the following.

Area of a rectangle: A = length × breadth

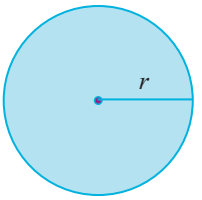
Area of the circle cut-outs: A = ____ × ____ = πr^2

∴ Area of a circle with radius r units: A = ____

Note: The more sectors the circle is cut into, the better the rectangle, but the dimensions remain the same!

D Area of a circle

The area of a circle is the number of square units inside the circle. From Investigation 3, the area of a circle is $A = \pi r^2$.

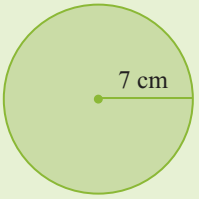


Remember: Area is measured in units².

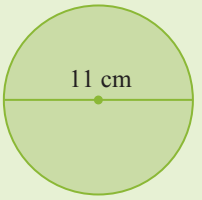
EXAMPLE 1

Calculate the area of these circles correct to 1 decimal place.

a



b



a $A = \pi r^2$
$= \pi \times 7^2$
$= 49\pi$
$\approx 153.9 \text{ cm}^2$

Calculator:

π × 7 x^2 =

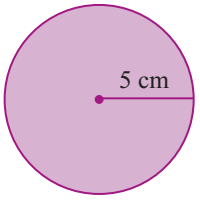
b $A = \pi r^2$
$= \pi \times 5.5^2$ (as $r = \frac{11}{2} = 5.5$)
$\approx 95.0 \text{ cm}^2$

π × 5.5 x^2 =

Exercise 8D

1 Calculate the exact area of each circle in terms of π.

a



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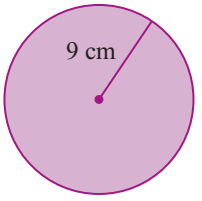
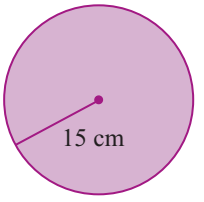
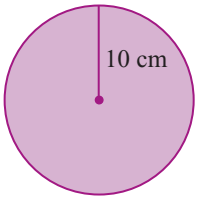


c



d



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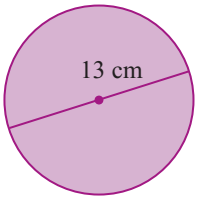
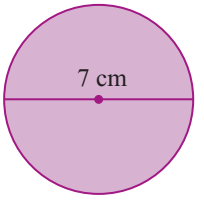
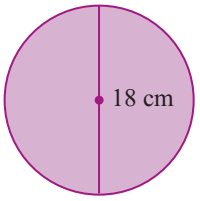


f



2 Calculate the area of each circle correct to 1 decimal place.

a



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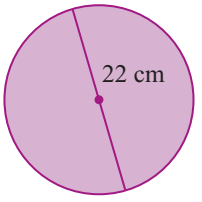

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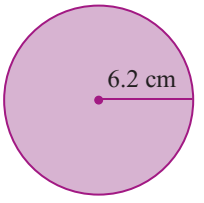
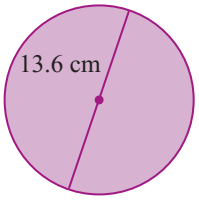


d



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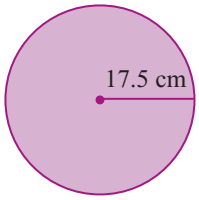
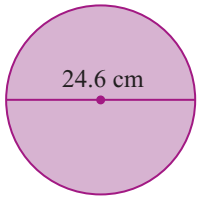


f



g radius = 4.9 cm

h diameter = 12.3 cm

i radius = 15.1 cm

j diameter = $3\frac{1}{4}$ cm

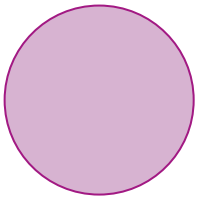
k radius = $6\frac{3}{8}$ cm

l diameter = $5\frac{1}{2}$ cm

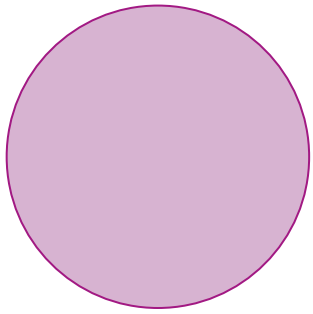
When using a calculator use brackets. ∴ $(6\frac{3}{8})^2$.

3 Measure the diameter of each circle. Calculate the area of each correct to 1 decimal place.

a



b



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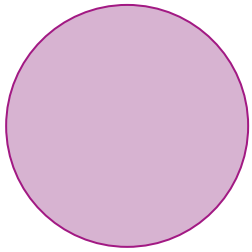
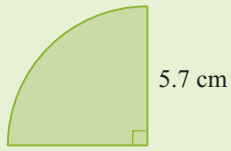


EXAMPLE 2

For this shape given, determine:

a the fraction of a circle that is drawn

b the area to the nearest whole cm².



A right angle is 90°.

a Fraction of circle $= \frac{90°}{360°}$

$= \frac{1}{4}$ of a circle

Calculator:

90 $a\frac{b}{c}$ 360 =

b $A = \frac{1}{4}$ of a whole circle

$= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 5.7^2$

$= 25.517\ldots$

≈ 26 cm²

1 $a\frac{b}{c}$ 4 × π × 5.7 x^2 =

4 Complete to calculate the area of a quarter of a circle of radius 3.7 cm.

$A = \frac{1}{4}$ of a _____ circle

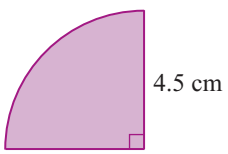
$= \frac{1}{4}\pi$ ___ ^{2}

$= \frac{1}{4} \times \pi \times$ ___ ^{2}

$=$ ___ ≈ 11 cm²

5 Determine the fraction of a circle that is drawn, then calculate the area to the nearest cm².

a



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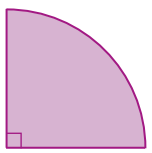

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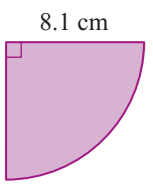
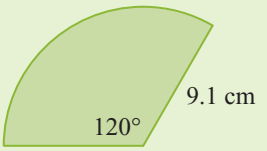


EXAMPLE 3

For the shape given, determine:

a the fraction of a circle that is drawn

b the area to the nearest cm^2.



a Fraction of circle $= \frac{120°}{360°}$

$= \frac{1}{3}$ of a circle

b $A = \frac{1}{3}$ of a whole circle

$= \frac{1}{3}\pi r^2$

$= \frac{1}{3} \times \pi \times 9.1^2$

$= 86.718 \ldots$

≈ 87 cm^2

6 Complete to calculate the area of part of a circle with radius 7.4 cm and an angle of 150°.

a Fraction of circle $= \frac{150°}{\square°}$

$= \frac{\square}{12}$ of a circle

b $A = \frac{\square}{12}$ of a whole circle

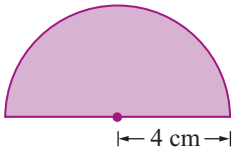
$= \frac{\square}{12} \times \pi \times r^{\square}$

$= \frac{\square}{12} \times \pi \times (\_\_)^2$

$= \_\_\_ \approx 72$ cm^2

7 Determine what fraction of a circle is drawn, then calculate the area to the nearest cm^2.

a



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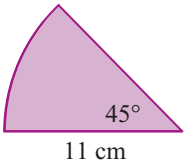

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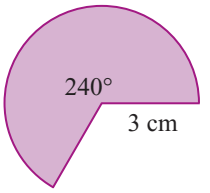

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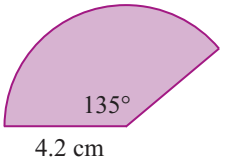

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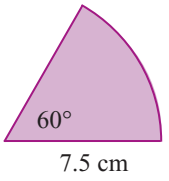

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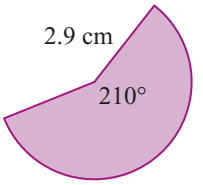
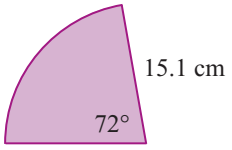
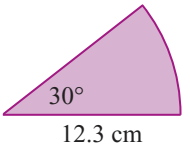


g



h



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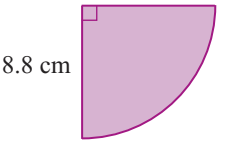
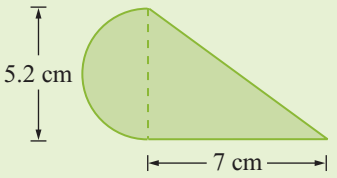


EXAMPLE 4

Calculate the area of the composite figure correct to 1 decimal place.



Only round off at the end.

The shape consists of a semicircle and a triangle.

Remember area formulas.

Area 1: semicircle

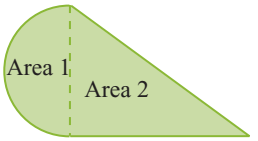
$d = 5.2$ cm

$\therefore r = \frac{5.2}{2} = 2.6$ cm

$A = \frac{\pi r^2}{2}$

$= \pi \times \frac{(2.6)^2}{2}$

≈ 10.6 cm^2



Area 2: triangle

$A = \frac{1}{2}(b \times h)$ or $\frac{b \times h}{2}$

$= \frac{7 \times 5.2}{2}$

≈ 18.2 cm^2

Calculator:

π × 2.6 x^2 ÷ 2 =

7 × 5.2 ÷ 2 =

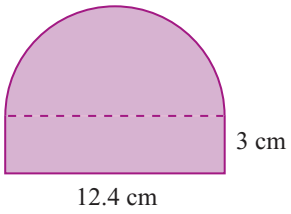
Total area $=$ area 1 $+$ area 2

$\approx 10.6 + 18.2$

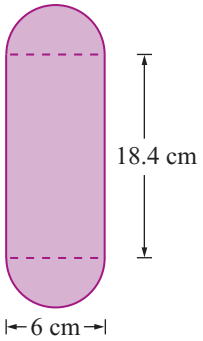
$= 28.8$ cm^2

8 Determine the area of the following composite shapes correct to 1 decimal place.

a



b



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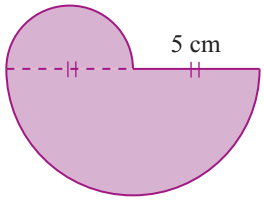
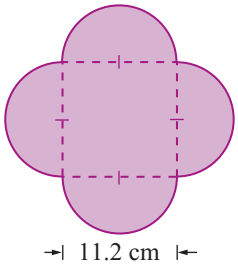


d



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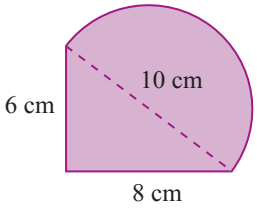

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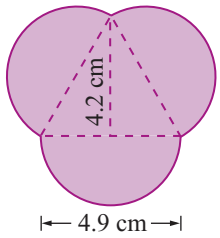

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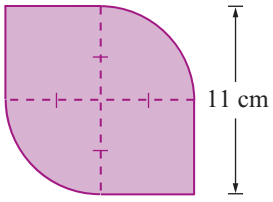

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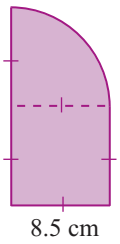
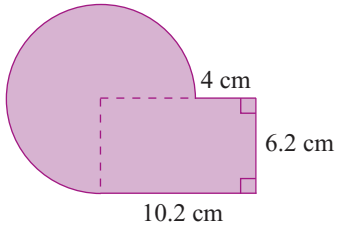


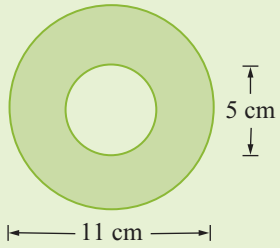
i



EXAMPLE 5

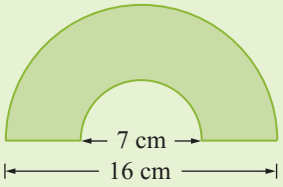
Determine the area of the shaded region correct to 1 decimal place.

a



$r = 2.5$ and $R = 5.5$

b



$r = 3.5$ and $R = 8$

a Area of shaded region = area of big circle − area of small circle

Remember to find the radius.

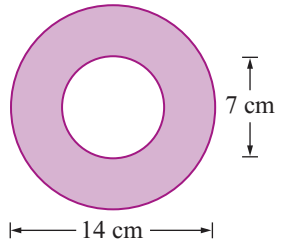
$$= \pi R^2 - \pi r^2$$
$$= \pi \times 5.5^2 - \pi \times 2.5^2$$
$$\approx 95.033 - 19.635$$
$$= 75.398$$
$$\approx 75.4 \text{ cm}^2$$

b Area of shaded region = area of big semicircle − area of small semicircle

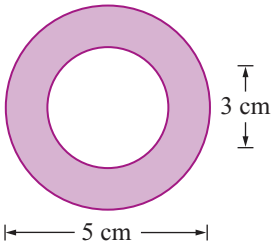
$$= \frac{\pi R^2}{2} - \frac{\pi r^2}{2}$$
$$= \pi \times \frac{8^2}{2} - \pi \times \frac{3.5^2}{2}$$
$$\approx 100.531 - 19.242$$
$$= 81.289$$
$$\approx 81.3 \text{ cm}^2$$

9 Determine the area of each shaded region correct to 1 decimal place.

a



b



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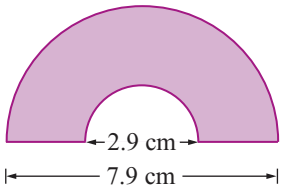

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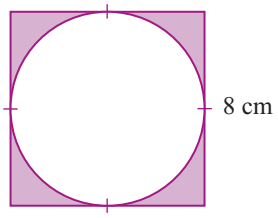

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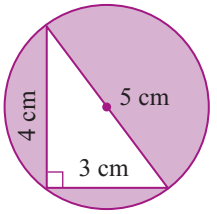

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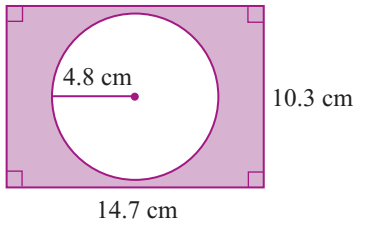
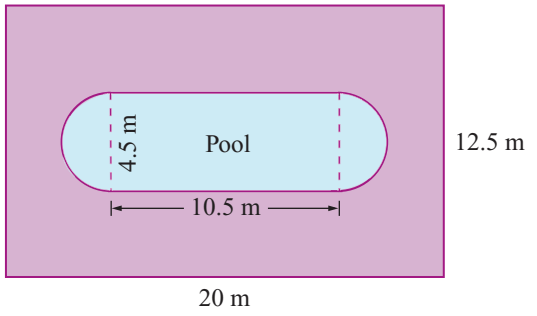


10 The diagram shows a rectangular yard with a pool surrounded by paving.

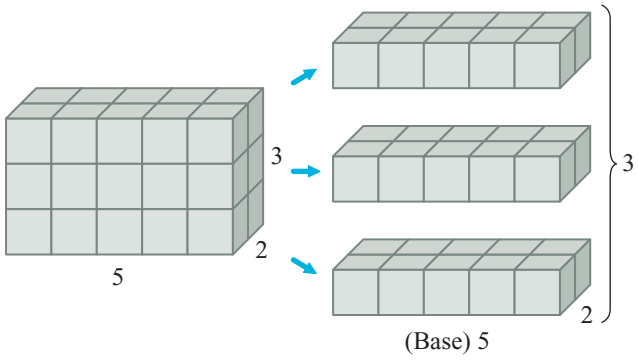
- **a** Calculate the area occupied by the swimming pool correct to 1 decimal place.
- **b** Calculate the paved area correct to 1 decimal place.
- **c** Calculate the cost of paving if paving costs $45.00 per square metre.



Investigation 4 How to calculate volume

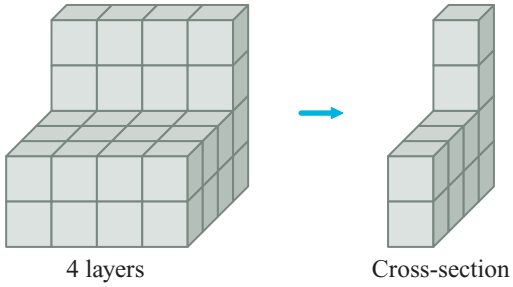
1 **a** A cross-section is formed by cutting a prism parallel to its base. To calculate the volume of the rectangular prism shown, count the number of cubic units used to build the prism.

- **b** Count the number of cubic units in the base.
- **c** How many layers are used to form the rectangular prism?
- **d** Multiply your answers from parts **b** and **c**. Is this the same answer you found in part **a** by counting the number of cubic units?



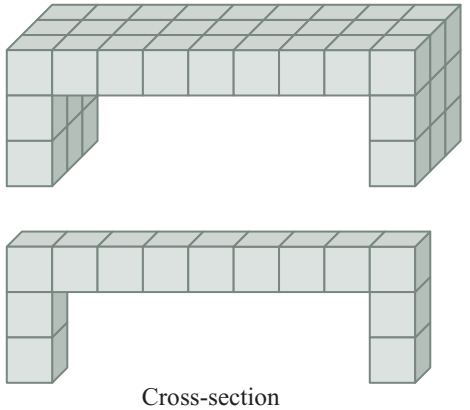
2 **a** Calculate the volume of the prism shown by counting the number of cubic units required for the construction.

- **b** Refer to the cross-section as drawn. How many cubic units are required to construct the cross-section?
- **c** How many layers of the cross-section are needed to construct the prism?
- **d** If the answers of parts **b** and **c** are multiplied, is the result the same as the answer in part **a**?



3 **a** The base is the cross-section. Count the number of cubic units in the base of this prism.

- **b** How many layers of the cross-section are used to construct the prism?
- **c** To calculate the volume of the prism:
  - **i** multiply the area of the base by the number of layers, and
  - **ii** count the number of cubic units used to construct the prism by referring to the diagram.
- **d** Are both answers from part **c** the same?



So a formula to determine volume can be derived.

Volume = area of the base (cross-section) × height of the prism

The number of cubic units in any prism = the number of cubic units in the base × the number of layers.

E Volume of a cylinder

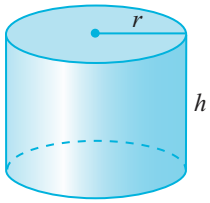
The cross-section (base) of a cylinder is a circle. The volume of any solid is found by multiplying the area of the cross-section (base) by its height, as shown in Investigation 4.

So, the volume of a cylinder is found by:

V = area of base (circle) × height

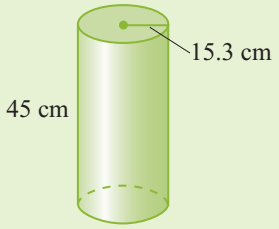
$V = Ah$

$V = \pi r^2 h$



EXAMPLE 1

Calculate the volume of the cylinder to the nearest cm^3.



$V = \pi r^2 h$

$= \pi \times 15.3^2 \times 45$

$= 33\,093.694 \approx 33\,094$ cm^3

Calculator: π × 15.3 x^2 × 45 =

Exercise 8E

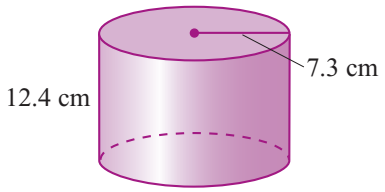
1 Complete to calculate the volume of the cylinder to the nearest cm^3.

$V = \pi$__$^2 h$

$= \pi \times$ ___$^2 \times 12.4$

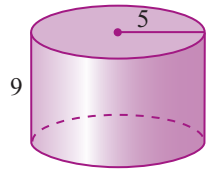
$=$ ___

≈ 2076 cm^3

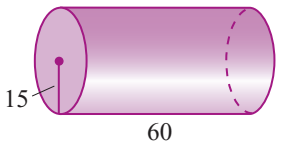


2 Calculate the volume of each cylinder to the nearest cm^3. All dimensions are in centimetres.

a



b



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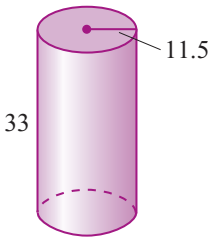
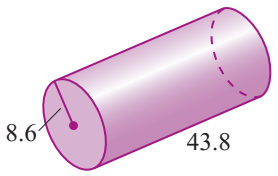
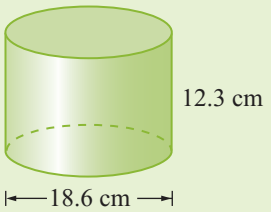


d



EXAMPLE 2

Calculate the volume of the cylinder to the nearest cm^3.



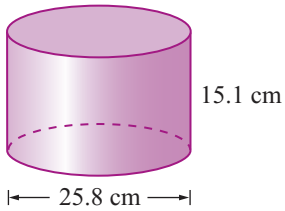
$V = \pi r^2 h$
$= \pi \times (9.3)^2 \times 12.3$
$= 3342.11\ldots$
$= 3342\ cm^3$

$d = 18.6\ cm$
$\therefore r = 18.6 \div 2$
$= 9.3\ cm$

3 Complete to calculate the volume of the cylinder correct to the nearest cm^3.

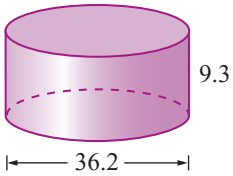
$V =$ ___ r^2 ___
$=$ ___ $\times$ (___)$^2 \times 15.1$
$=$ ___
$=$ 7894 ___

$d = 25.8\ cm$
$\therefore r = 25.8 \div$ ___
$=$ ___ cm

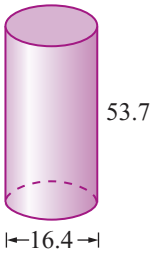


4 Calculate the volume of each cylinder to the nearest cm^3. All dimensions are in centimetres.

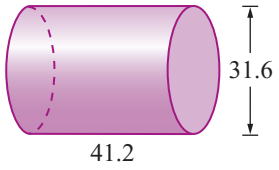
a



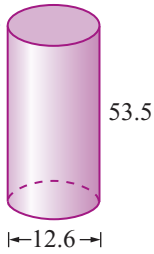
b



c

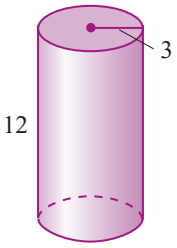


d

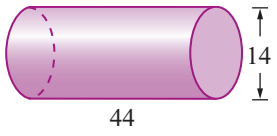


5 Calculate the volume of each cylinder to the nearest cm^3. All dimensions are in centimetres.

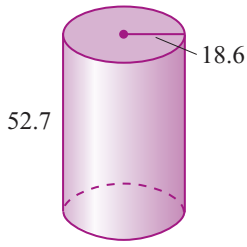
a



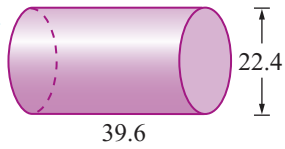
b



c



d

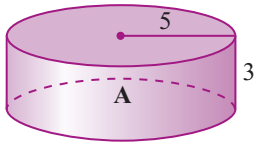


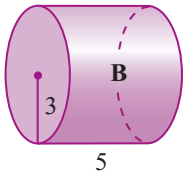
6 Use the dimensions below to calculate the volumes of these cylinders correct to 1 decimal place.

a $r = 5$ cm, $h = 12.2$ cm
b $d = 13$ cm, $h = 26$ cm
c $r = 6.4$ cm, $h = 15.9$ cm
d $d = 9$ cm, $h = 42$ cm
e $d = 37.4$ cm, $h = 150$ cm
f $r = 14.6$ cm, $h = 80$ cm

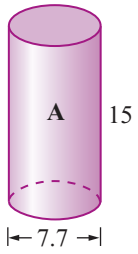
7 Which cylinder (A or B) in each pair has the greater volume? All dimensions are in centimetres.

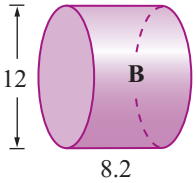
a





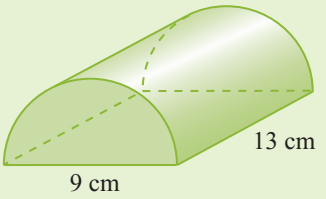
b





EXAMPLE 3

Calculate the volume to the nearest cm^3.



The diagram shows half a cylinder.

So, $V = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 4.5^2 \times 13$

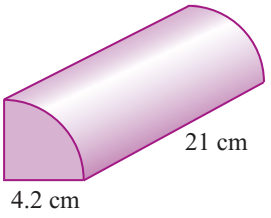
$= 413.512\,133\ldots$

$= 414\ cm^3$

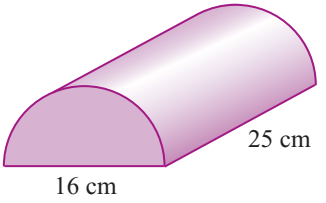
Calculator: 1 $a\frac{b}{c}$ 2 × π × 4.5 x^2 × 13 =

8 Calculate the volume of each solid to the nearest cm^3.

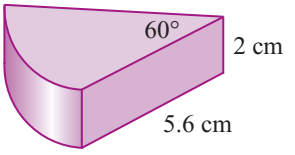
a



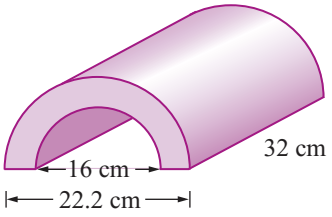
b



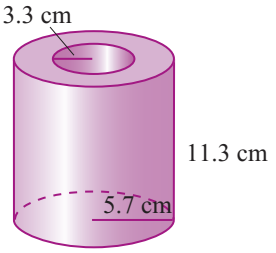
c



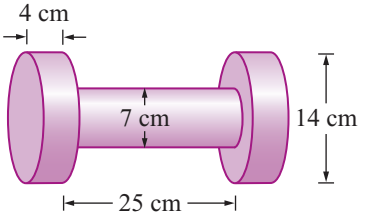
d



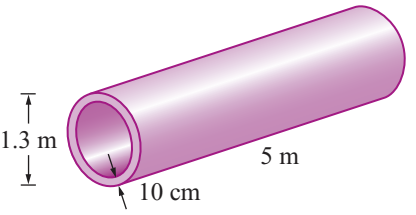
e



f



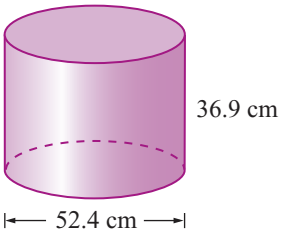
9 Giant concrete sewer pipes are 5 m long with an outer diameter of 1.3 m. The walls of the pipes are 10 cm thick. What volume of concrete is required to make each pipe? Express your answer in m^3 correct to 2 decimal places.



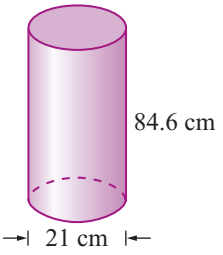
10 **i** Calculate the volume of each cylinder to the nearest cm^3.

ii Calculate the capacity of each cylinder in litres. (1 L = 1000 cm^3)

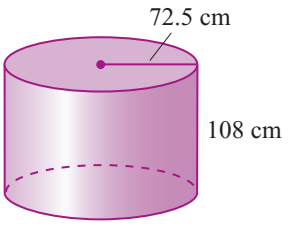
a



b



c



11 A cylindrical tank has a base radius of 2.8 m and a height of 3.1 m. Find the capacity of the tank in kilolitres. (1 m^3 = 1 kL)

Language in mathematics

The Racing Bike

Metallic silver and blue, a marvellous sight.
Powering along the highway at phenomenal speeds.
Pedal power spinning my wheels.
Racing along with the breeze at my heels.
A remarkably swift machine am I.

1 Write another three stanzas for this poem *The Racing Bike*, which has been started for you.

2 What is personification? Explain how it has been used in the first stanza of the poem.

3 What is the meaning of the following words?

a marvellous **b** phenomenal **c** metallic **d** swift

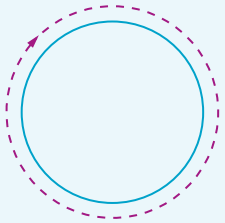
4 Write a poem of your own choice. Select a topic associated with a circular shape, for example, wheels, patterns, pie charts, circles. Incorporate into your poem two examples of any of the following:
metaphor, simile, assonance, alliteration, personification

Terms

arc	area	centre	chord	circle	circumference	cylinder
diameter	perimeter	pi (π)	radius	sector	segment	semicircle
tangent	volume					

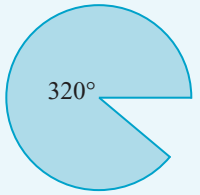
Check your skills

1 The feature shown on this circle is:



A circumference **B** diameter
C arc **D** radius

2 What fraction of a circle is represented by the sector shown?



A $\frac{1}{9}$ **B** $\frac{3}{9}$
C $\frac{8}{9}$ **D** $\frac{5}{9}$

3 Evaluate $\frac{\pi \times 41.6}{2}$ cm correct to 1 decimal place.

A 130.7 cm **B** 65.3 cm **C** 194.6 cm **D** 261.4 cm

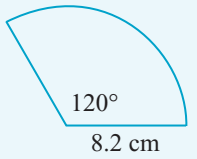
4 Calculate the circumference of a circle correct to 1 decimal place if the diameter is 4.8 cm.

A 15.1 cm **B** 30.2 cm **C** 7.6 cm **D** 21.3 cm

5 Calculate the circumference of a circle correct to 1 decimal place if the radius is 12.05 cm.

A 44.1 cm **B** 78.5 cm **C** 37.9 cm **D** 75.7 cm

6 Determine the length of this arc correct to 1 decimal place.

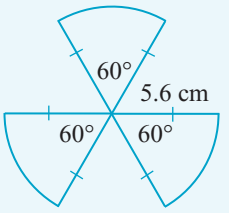


A 51.5 cm **B** 25.8 cm
C 17.2 cm **D** 33.6 cm

7 Calculate the perimeter of the shape in question **6** correct to 1 decimal place.

A 17.2 cm **B** 25.4 cm **C** 30.6 cm **D** 33.6 cm

8 Calculate the perimeter of this composite figure to the nearest cm.



A 40 cm **B** 48 cm
C 51 cm **D** 54 cm

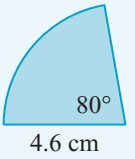
9 The circumference of a circle is 477.5 cm. Calculate its radius to the nearest whole number.

A 70 cm **B** 76 cm **C** 82 cm **D** 85 cm

10 Calculate the area of a circle to the nearest cm^2 given that the radius is 9.02 cm.

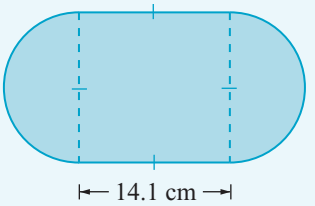
A 28 cm^2 **B** 89 cm^2 **C** 266 cm^2 **D** 256 cm^2

11 Calculate the area of this shape correct to 1 decimal place.



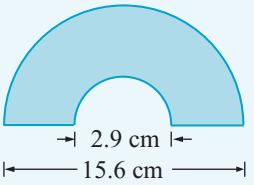
A 14.8 cm^2 **B** 22.2 cm^2
C 7.4 cm^2 **D** 66.5 cm^2

12 Calculate the area of this composite figure correct to 2 decimal places.



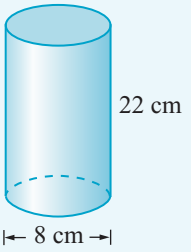
A 375.52 cm^2 **B** 354.96 cm^2
C 586.21 cm^2 **D** 823.39 cm^2

13 Calculate the area of the shaded region correct to 1 decimal place.



A 41.7 cm^2 **B** 53.9 cm^2
C 107.8 cm^2 **D** 92.3 cm^2

14 Calculate the volume of the cylindrical flask to the nearest cm^3.



A 1106 cm^3 **B** 1257 cm^3
C 4021 cm^3 **D** 4423 cm^3

15 Calculate the capacity of the flask in question **14** to the nearest litre.
($1\ cm^3 = 1$ mL)

A 1 L **B** 2 L
C 3 L **D** 4 L

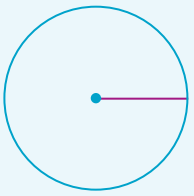
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4–8	9	10–13	14, 15
Section	A	B	C	D	E

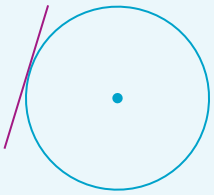
8A Review set

1 Name the feature shown in purple on each circle.

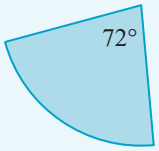
a



b



2 What fraction of a circle is represented by this sector?



3 Write the formula for the circumference of a circle when given the diameter.

4 Calculate the circumference of a circle with a diameter of 13.9 cm, correct to 1 decimal place.

5 Write the formula for the circumference of a circle when given the radius.

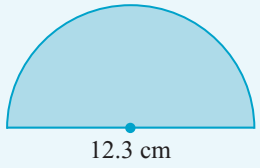
6 Calculate the circumference of a circle with a radius of 5.72 cm, correct to 2 decimal places.

7 Write down the formula for calculating the area of a circle.

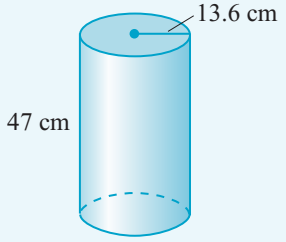
8 Calculate the area of a circle correct to 1 decimal place given:

a radius = 9 cm **b** diameter = 4.7 cm

9 Calculate the area of this shape correct to 1 decimal place.



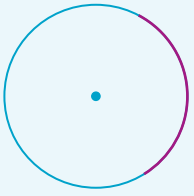
10 Calculate the volume of this cylinder to the nearest cm^3.



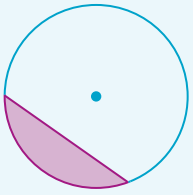
8B Review set

1 Name the feature shown in purple on each circle.

a

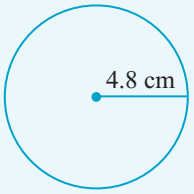


b

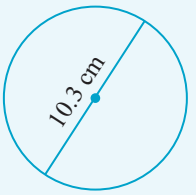


2 Calculate the circumference of each circle correct to 1 decimal place.

a

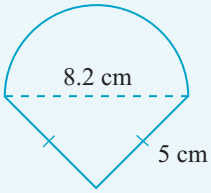


b

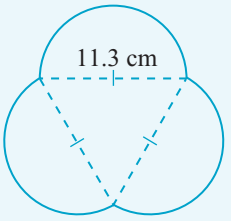


3 Calculate the perimeter of each shape correct to 2 decimal places.

a

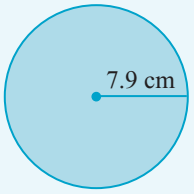


b

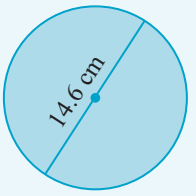


4 Calculate the area of each circle correct to 1 decimal place.

a

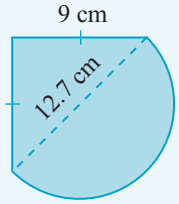


b

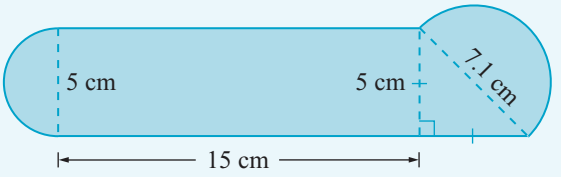


5 Calculate the area of these shapes correct to 2 decimal places.

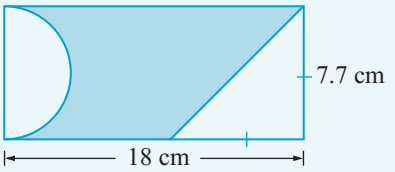
a



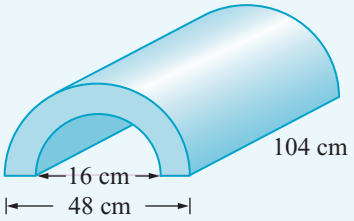
b



6 Determine the area of the shaded region to the nearest cm^2.



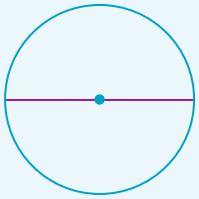
7 Calculate the volume of this solid to the nearest cm^3.



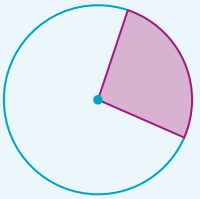
8C Review set

1 Name the feature shown in purple on each circle.

a



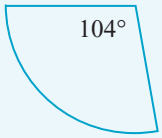
b



2 What fraction of a circle is represented by each sector?

a

b



3 Calculate the length of each arc in question **2** correct to 1 decimal place given:

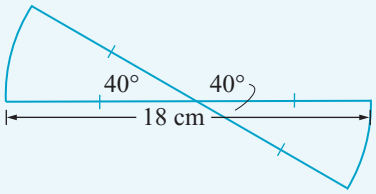
i $r = 4.2$ cm **ii** $r = 8.3$ cm

4 Calculate the area of each sector in question **2** to the nearest cm^2 given:

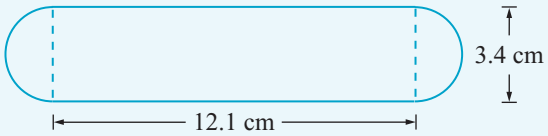
i $r = 6$ cm **ii** $r = 11.2$ cm

5 Calculate the perimeter of each shape correct to 1 decimal place.

a



b



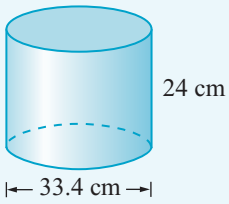
6 The minute hand of a clock is 12.3 cm long. How far, correct to 1 decimal place, will the tip of the hand travel in:

a 1 hour? **b** 1 day?

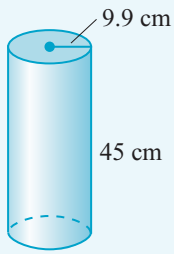
7 **i** Calculate the volume of each cylinder to the nearest cm^3.

ii Calculate the capacity of each cylinder in litres.

a



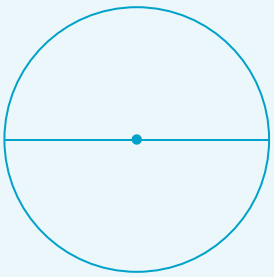
b



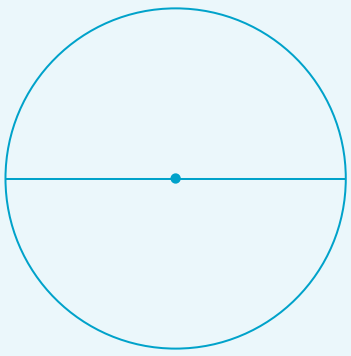
8D Review set

1 Measure the diameter of each circle then calculate the circumference to the nearest cm.

a

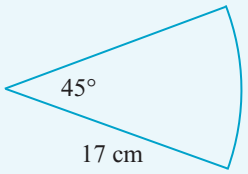


b

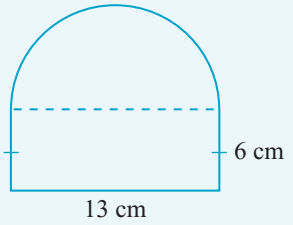


2 Calculate the total perimeter of each figure correct to 2 decimal places.

a



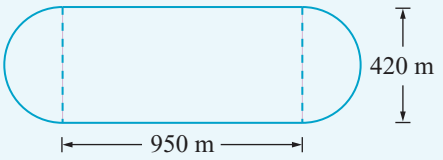
b



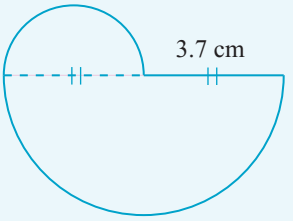
3 An athlete runs along the perimeter of this track.

a Calculate the length of one lap to the nearest metre.

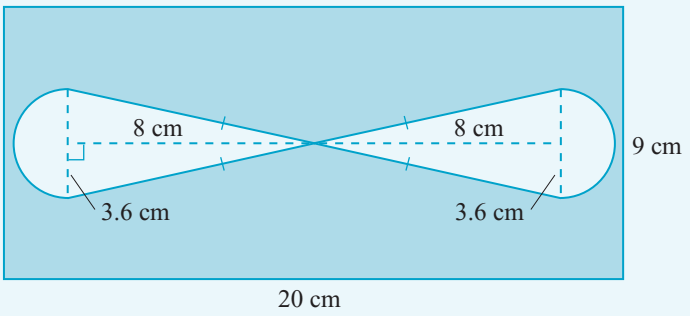
b If the athlete needs to run 32 km, how many complete laps of the track must she run?



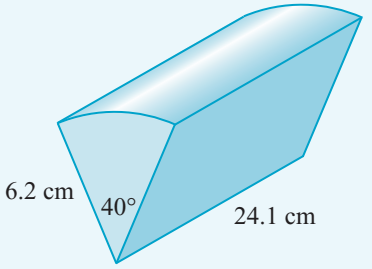
4 Calculate the area of this shape correct to 1 decimal place.



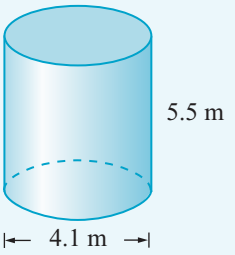
5 Calculate the area of the shaded region to the nearest cm^2.

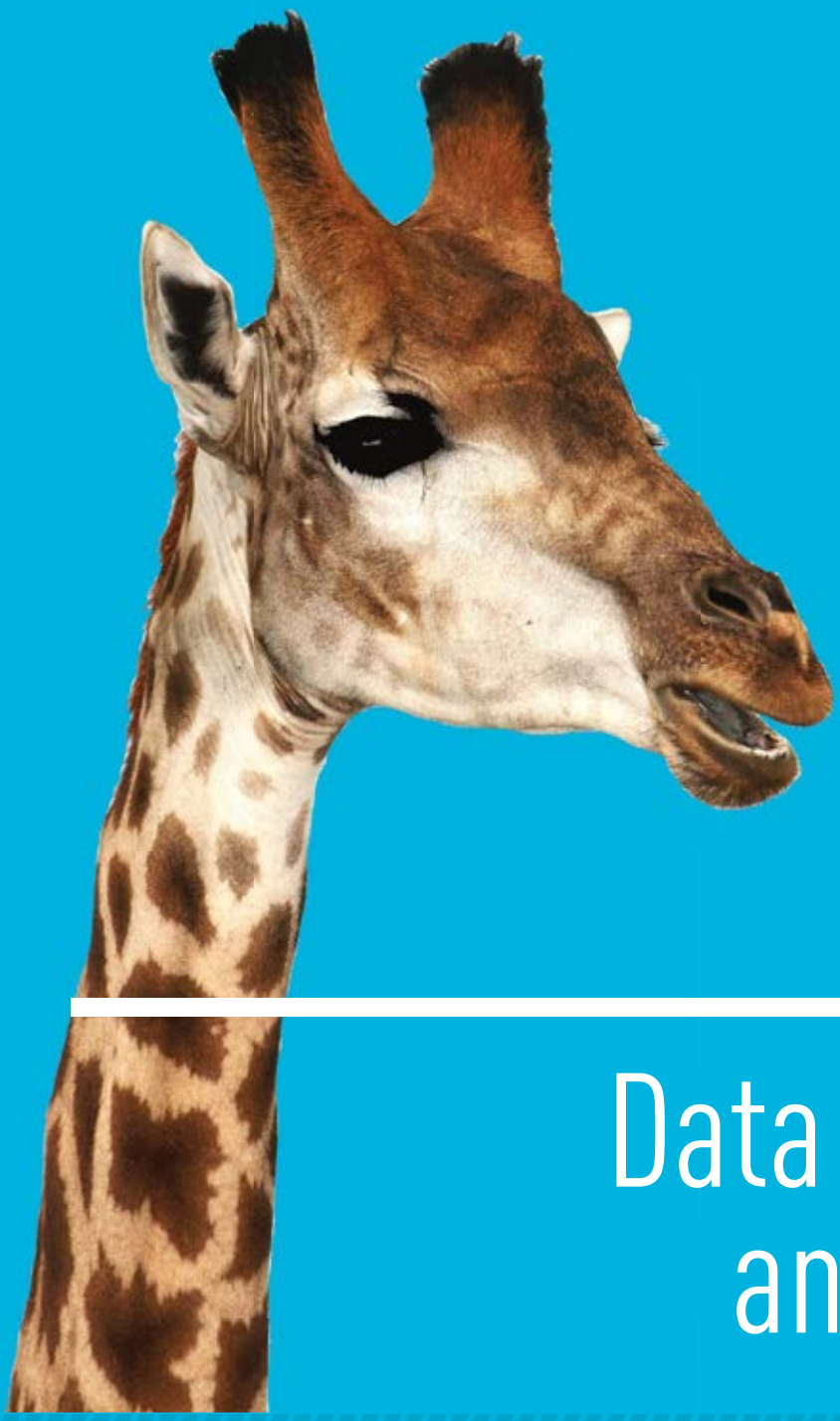


6 Calculate the volume of this solid to the nearest cm^3.



7 A water tank is 4.1 m in diameter and has a height of 5.5 m. Calculate the capacity of the water tank to the nearest litre.





9

Data collection and analysis

This chapter deals with collecting data, the use of the most appropriate measure to represent a set of data and the variation of means and proportions in samples.

At the end of this chapter you should be able to:

- recognise variables as categorical or numerical
- understand the differences between collecting data by observation, census and sample
- collect data using a random process
- identify issues that may make it difficult to obtain representative data
- question the selection of data used to support a particular viewpoint.

NSW Syllabus references: S4 S&P Data collection and representation, S4 S&P Single variable data analysis
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA3-18SP, MA4-19SP, MA4-20SP
STATISTICS & PROBABILITY – ACMSP206, ACMSP207, ACMSP284, ACMSP293

Diagnostic test

1 Which of the following is not a source of primary data?
A observations **B** questionnaires
C surveys **D** magazines

2 Which of the following is not a source of secondary data?
A the internet **B** newspapers
C interviews **D** research papers

3 If Jenny wanted to find out the most popular sport in Year 8, she would use primary data.
A true **B** false

4 If Kyle wanted to determine the mean age of Year 8 students in NSW he would use primary data.
A true **B** false

5 A survey is being designed to find out the different types of transport people use to get to work. Which of the following would not be a relevant question?
A How often do you drive to work?
B How often do you travel to work by public transport?
C Do you own a car?
D What type of public transport do you use to get to work?

6 Which type of question is likely to get the most varied responses?
A tick-box **B** open-ended
C biased **D** scaled-response

7 The mean of 8, 10, 7, 5, 12, 6 is:
A 9 **B** 8 **C** 7 **D** $45\frac{1}{3}$

8 The mode of 8, 10, 7, 5, 14, 8 is:
A 5 **B** 6 **C** 7 **D** 8

9 The range of 8, 10, 7, 5, 14, 8 is:
A 7 **B** 8 **C** 9 **D** 10

10 The median of 8, 10, 7, 5, 14, 8 is:
A 9 **B** 8 **C** 7 **D** 6

Use this frequency distribution table to answer questions **11** to **14**.

Score	19	20	21	22	23
Frequency	8	8	12	6	8

11 The mean is closest to:
A 4 **B** 20 **C** 21 **D** 22

12 The mode is:
A 4 **B** 20 **C** 21 **D** 22

13 The range is:
A 4 **B** 20 **C** 21 **D** 22

14 The median is:
A 4 **B** 20 **C** 21 **D** 22

Use this stem-and-leaf plot to answer questions **15** to **18**.

Stem	Leaf
9	7 9
10	0 0 3 4 8 8
11	4 6 6 9
12	2 2 2 5 6 8
13	4 5 6 9
14	0 3

15 The mean is:
A 2.57 **B** 16 **C** 119 **D** 122

16 The mode is:
A 2 **B** 97 **C** 122 **D** 143

17 The median is:
A 11.5 **B** 119 **C** 120.5 **D** 122

18 The range is:
A 5 **B** 46 **C** 122 **D** 143

The Diagnostic test questions refer to the Year 7 outcomes from ACMSP169 and ACMSP171.

A Variables

Statistics refers to the science of collecting facts and information. The information is organised into a form that can easily be analysed and from which conclusions and predictions can accurately be made. The facts or pieces of information collected are called data. Data may be collected by observing, counting, measuring and asking questions.

When we collect data we are looking for information about a particular property or characteristic of a group of people or objects. This property or characteristic is known in statistics as the variable. It is something measurable or observable that is expected to change over time or between individual observations. Examples of variables include eye colour, height, method of travel and the outcome when a die is rolled.

Variables are defined as categorical or numerical.

- A categorical (or nominal) variable is one that describes a characteristic or names the categories into which the variable can be sorted. For example, eye colour can be described as brown, blue, grey, green, etc. Eye colour is a categorical variable. Categorical variables are often referred to as being qualitative variables.
- A numerical (or quantitative) variable is one that has a numerical value. Numerical variables can be discrete or continuous. The values of discrete variables are exact and increase in steps. They are often the result of counting. Examples include the number of children in a family and the number of marks in a test.

  The values of continuous variables lie within a certain range. They are usually the result of measuring. For example, if we say that the weight of a person is 48 kg to the nearest kilogram, their exact weight could be anywhere between 47.5 kg and 48.5 kg. Weight is a continuous numerical variable.

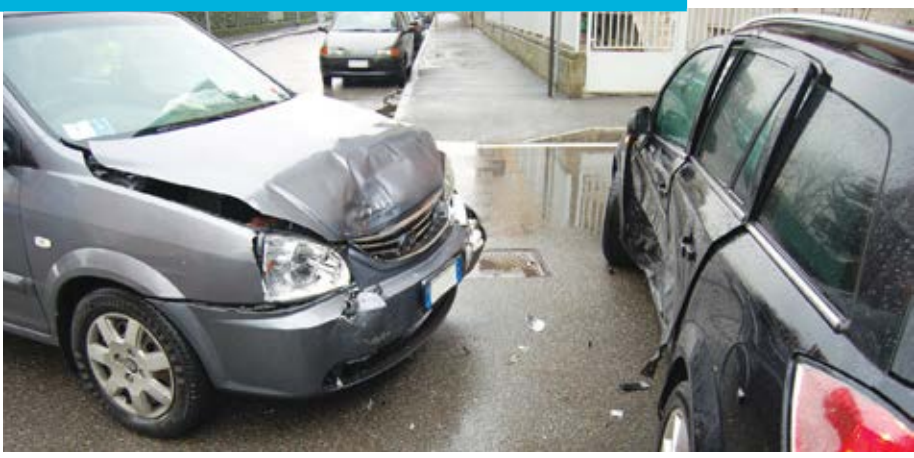
EXAMPLE 1

Classify these variables as categorical, discrete numerical or continuous numerical.

a The gender of children in a family
b The heights of Year 8 students
c The number of heads when two coins are tossed
d The rating scale for the responses to a questionnaire given as 1 = dislike, 2 = neutral, 3 = like
e The energy ratings of refrigerators given a 1-star, 2-star or 3-star ratings

a Gender can be sorted into the categories male or female. It is a categorical variable.
b Height is the result of measuring. It is a continuous numerical variable.
c The number of heads when two coins are tossed is the result of counting. It is a discrete numerical variable.
d Even though the ratings on the scale are numbers, they are the categories into which the responses can be sorted. It is a categorical variable.
e Each rating is a description of the efficiency of the refrigerator or names the categories into which the efficiency can be sorted. Energy rating is a categorical variable.

Exercise 9A



1 Classify each variable as categorical, discrete numerical or continuous numerical.

- **a** number of family pets
- **b** types of family pets
- **c** age
- **d** number of marks on a test
- **e** amount of rain in a day
- **f** number of accidents at an intersection
- **g** sizes of apples (small, medium or large)
- **h** hair colour
- **i** favourite TV show
- **j** responses to a survey question (1 strongly disagree, 2 disagree, 3 don't know, 4 agree, 5 strongly agree)
- **k** number of diners in a restaurant
- **l** fuel consumption
- **m** amount of money in a purse
- **n** health of patients after being given medicine (worse, the same, better)
- **o** temperature during the day
- **p** length of a line interval
- **q** opinions of students of the food at the canteen
- **r** time spent doing homework
- **s** quality of service at a bank (poor, average, good)

2 Give a further three examples of:

a categorical variables **b** discrete numerical variables **c** continuous numerical variables.

B Collecting data

When collecting data it is necessary to determine from whom, or what, the information is to be collected. The target population is the entire group of people or objects from which the information is to be gathered. For example, if we wanted information about the most popular colours of new cars sold in a particular year, the target population would be all new cards sold in that year. If we wanted information about the type of road accidents in a given town, the population would be all road accidents in that town.

EXAMPLE 1

What or who would be the target population if we wanted to collect information about:

a the number of siblings of the students in a Year 8 class?

b the quality ratings of new dishwashers?

a Target population would be all the students in the class.

b Target population would be all new dishwashers produced.

Techniques for collecting data can involve a census, a sample or observation.

In a census, data about every individual in the population is collected. The individuals can be people or objects. A census is detailed and accurate, but it is expensive, time consuming and often not practical.

In a sample, data is collected from a portion of the population. It is not as detailed or accurate as a census but it is quicker and cheaper. Conclusions drawn from samples always involve some degree of error.

Observation is a way of gathering data by watching events and/or noting characteristics in their natural setting. It can be expensive and time consuming but has the advantage of collecting information where and when it occurs.

EXAMPLE 2

Would a census, sample or observation be used to collect data on:

a the life of batteries?
b the most popular brand of toothpaste?
c the causes of road accidents in New South Wales?
d the direction of travel of vehicles through an intersection?

a Sample: If every battery produced was tested there would not be any left for sale.
b Sample: It would be impractical to interview everyone.
c Census: Information about every accident would need to be analysed.
d Observation.

Exercise 9B

1 What or who would be the target population if we wanted to collect information about:

a the most popular make of new cars?
b changes to the school uniform?
c the price of houses?
d the success of a new treatment for ticks and fleas on dogs?
e the suitability of a new bus timetable on a particular route?

2 Would a census, sample or observation be used to collect data on:

a the number of goals scored each week by a netball team?
b the average number of children in an Australian family?
c the number of vehicles crossing an intersection?
d the sports played by Year 8 students?
e the number of matches in a box?
f the native animals living in an area?
g the most popular TV show in Australia?
h the heights of 15-year-old males?
i the method of travel to school by all students?
j the amount of rubbish littering a street?
k the suitability of a train timetable?
l the income of Australians?
m the outcomes of rolling a die?

3 Give a further three examples of data that could be collected by using:

a a census **b** a sample **c** observation.

Investigation 1 Selecting random samples

A random sample of 5 students from the class can be selected in a number of ways.

1 **a** Write the names of all the students in your class on pieces of paper.
b Put the pieces of paper in a box, shake the box and select 5 pieces of paper.
c List the 5 names selected.

2 **a** Take the list of students in the class and assign each a number from 01 to 30 (or the number of students in the class). Write the numbers on cards.
b Shuffle and then deal 5 cards.
c List the names of the students whose numbers have been selected.

3 Use the list of numbers assigned to each student in question **2**.
a Open a phone book at any page. Start at the last 2 digits in the first phone number. If this number is between 1 and 30 (inclusive) write it down, if not skip to the next phone number down the column.
b Repeat until you have selected 5 random numbers.
c List the names of the students whose numbers have been selected. (You can start at any phone number chosen at random by, say, closing your eyes and placing the point of your pencil anywhere on the page. Start at the phone number closest to this point).

4 Most modern scientific calculators have a random number generator. On Casio calculators this function key is labelled RAN#.

By pressing SHIFT (RAN#) = the calculator produces a random number from 0 to 0.999, for example 0.397.
To obtain a random number from 1 to 30, multiply the result by 30 and round up to the next digit.
0.397 × 30 = 11.82 = 12 (rounded up).
To obtain a random number from 1 to 27, multiply by 27, etc.

a Generate 5 random numbers from 1 to 30 (inclusive) using a calculator.
b List the names of the students whose numbers have been generated.

5 **a** Divide the number of students in the class by 5 and round up to the nearest whole number, for example $28 \div 5 = 5.6$ (or 6 rounded up).
b Select every 6th name from the class list. (You may need to loop back to the start of the list to get the 5th random number.) This is called a systematic random sample.

To select a random sample of 5 boys and 5 girls from the class:

6 **a** Divide the class list into boys and girls and assign a number (say from 01 to 15) to each boy and girl.
b **i** Generate 5 random numbers from 1 to 15 using a calculator.
ii List the names of the boys who numbers have been generated.
c **i** Generate another 5 random numbers from 1 to 15 using a calculator.
ii List the names of the girls whose numbers have been generated.

C Obtaining data through sampling

In this section you will collect data using a random sample chosen from a population. A random sample is one in which every member of the population has an equal chance of being selected.

Exercise 9C

Use the data in the table below to answer the following questions.

Name	Student number	Age (years, months)	Number of children in family	Height (cm)	Hair colour
Tom	01	14 y 7 m	2	168	black
Francis	02	13 y 9 m	1	156	brown
Adelle	03	13 y 11 m	2	158	blond
Ali	04	13 y 10 m	3	163	black
Daniel	05	14 y 2 m	1	164	red
George	06	13 y 11 m	4	159	brown
Greta	07	14 y 5 m	2	162	brown
John	08	14 y 4 m	2	165	black
Harry	09	13 y 11 m	3	164	blond
Isabella	10	14 y 8 m	3	159	brown
Noah	11	13 y 11 m	1	167	blond
Kylie	12	14 y 10 m	2	155	black
Sylvia	13	14 y 5 m	4	152	brown
Abdul	14	14 y 6 m	2	164	black
Jenny	15	13 y 11 m	2	158	red
Louise	16	14 y 8 m	1	159	blond
Matt	17	13 y 11 m	3	162	black
Harley	18	13 y 9 m	2	165	brown
Ken	19	14 y 6 m	3	161	brown
Jacqueline	20	14 y 3 m	2	160	black
Maria	21	13 y 10 m	2	157	brown
Penny	22	13 y 7 m	3	158	blond
Zac	23	13 y 10 m	1	163	blond
Vera	24	14 y 1 m	2	150	black
Rima	25	14 y 6 m	5	156	black
Kenji	26	13 y 8 m	2	169	black
Bill	27	14 y 2 m	3	168	blond
An Dien	28	13 y 6 m	2	152	black
Samantha	29	13 y 11 m	3	158	brown
Najit	30	14 y 0 m	2	160	black
Class mean		**14 y 1 m**	2.3	160.4	

1 **a** **i** Use random numbers chosen from a phone book to select a sample of 5 students.
ii Calculate the mean age of these 5 students.
b **i** Select another random sample of 10 students.
ii Calculate the mean age of these 10 students.
c Which sample is better at estimating the mean age of the class?

2 **a** **i** Use random numbers generated by a calculator to select a sample of 5 students.
ii Calculate the mean number of children in each of the families of these 5 students.
b **i** Select another random sample of 10 students.
ii Calculate the mean number of children in each of the families of these 10 students.
c Which sample is better at estimating the mean number of children in the families of students in the class?

3 **a** **i** Select any student at random. (Close your eyes and place the tip of your pencil anywhere on the list.)
ii Starting at this student, select every 5th student in the list until you have randomly selected 6 students. (You may need to loop back to the start of the list.)
iii Calculate the mean height of these students.
b **i** Select any student at random.
ii Starting at this student, select every 3rd student in the list until you have randomly selected 10 students.
iii Calculate the height of these 10 students.
c Which sample is better at estimating the mean height of the class?

4 **a** **i** Choose a random sample of 5 students.
ii What proportion of students in this sample have black hair? (Write the answer as a percentage.)
b **i** Choose another random sample of 10 students.
ii What proportion of students in this sample have black hair?
c In which sample is the proportion of students with black hair closer to the proportion of students in the class with black hair?

Investigation 2 Data constraints

Investigate, discuss and identify issues that make it difficult to obtain representative data from either primary or secondary sources. What constraints limit the collection of data or result in unreliable data?

For example, lack of proximity to the area where data could be collected, lack of access to digital technologies or cultural sensitivities might influence the result.

D Bias in samples

A biased sample is one in which the data has been unfairly influenced by the collection process.
When checking the data from a sample that is being used to support a particular viewpoint, some of the things you should consider are:

1. Does every member of the target population have an equal chance of being selected?
2. Is the sample large enough?
3. Who collected the data? Does the group collecting the data have an interest in how the results turn out?
4. Is the data from a recent study? Could the findings have since changed?

EXAMPLE 1

Suggest any possible bias in each of these samples.

a a phone survey on Monday morning
b a survey of people taken at a bus terminal
c a survey at a darts competition
d five people tested with a new drug to cure cancer

a The sample would be biased towards people who are at home on Monday morning. It does not include people who are at work or carrying out some other activity such as shopping.
b The survey would be biased towards people who use the bus terminal. It does not include people who use other methods of transport.
c The survey would be biased towards people who attend darts competitions. For example, there would probably be more males than females.
d The sample is not big enough to make a valid conclusion.

Exercise 9D

1 Suggest any possible bias in each of the following samples.
a a phone survey on Friday night
b voting intentions taken at a Labor Party political rally
c responses to a survey conducted by email
d survey of people in the Star bank
e questionnaire given to people selected from the electoral roll
f opinions collected on a street corner
g opinions of five vets testing 'Fleagone' spray

2 Comment on any possible bias in the following situations.
a A survey on household spending is needed. You take a random sample from the electoral roll of a Sydney harbour-side suburb.
b A study on the hazards of smoking is carried out by a tobacco company.
c The results of a study done 15 years ago found that second-hand cigarette smoke is not hazardous.
d A company executive's report of sales found that sales had doubled in a week.
e A survey of the number of people who can touch their toes is done with students from a gymnastics class.
f To study the health of birds in an area, food is used to attract them to traps.
g You want to learn the opinions of parents in your school district regarding adult literacy education. You send out an invitation to parents to attend an information meeting.

E Review of measures of central tendency

It is often useful to represent a set of data by a single value that is considered to be central and typical of the data. This number is sometimes simply referred to as an 'average', but statisticians prefer to be more concise and refer to three numbers, known as the mean, median and mode, as measures of central tendency (or sometimes as measures of location).

EXAMPLE 1

For each set of scores, find the:

i mean **ii** median **iii** mode **iv** range.

a 21, 18, 22, 16, 19, 16

b

Score	Frequency
7	2
8	5
9	6
10	8
11	4

a i Mean $= \dfrac{\text{sum of all the scores}}{\text{number of scores}}$

$= \dfrac{21 + 18 + 22 + 16 + 19 + 16}{6} = \dfrac{112}{6}$

$= 18.7$ (1 decimal place)

ii The median is the middle score after all scores are arranged in ascending order. If there are two middle scores, average them to get the median. Half the (remaining) scores are less than the median and half the (remaining) scores are greater than the median.

Arranged in ascending order the scores are 16, 16, 18, 19, 21, 22.

Median $= \dfrac{18 + 19}{2} = 18.5$

iii Mode = score that occurs most often = 16

iv Range = highest score − lowest score

$= 22 - 16 = 6$

b i

Score (x)	Frequency (f)	$f \times x$
7	2	14
8	5	40
9	6	54
10	8	80
11	4	44
Total	**25**	**232**

Mean $= \dfrac{232}{25} = 9.28$

iii Mode = 10

ii

Score	Frequency	
7	2	1st, 2nd score
8	5	3rd to 7th scores
9	6	8th to 13th scores
10	8	
11	4	

Median = 13th score

$= 9$

iv Range $= 11 - 7 = 4$

Exercise 9E

1 Find the mean, median, mode and range of these scores.

a 17, 24, 19, 25, 19, 22

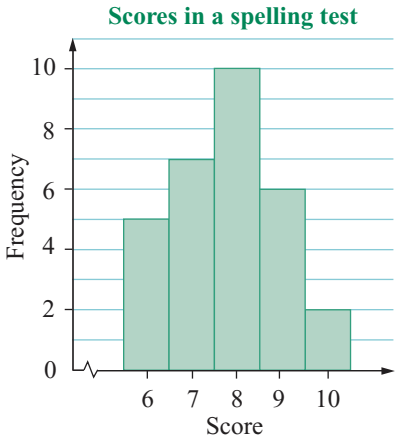
b

Score	Frequency
4	4
5	6
6	8
7	7
8	5

c

Stem	Leaves
2	0 3 9
3	2 4 5 8
4	1 7 7 8 9 9
5	4 6 6 6 7
6	4 9

d



F Clusters, gaps and outliers

A cluster occurs when the data seems to be gathered around a particular value or values.
An outlier is a data value that is unusually high or low compared with the rest of the data.

EXAMPLE 1

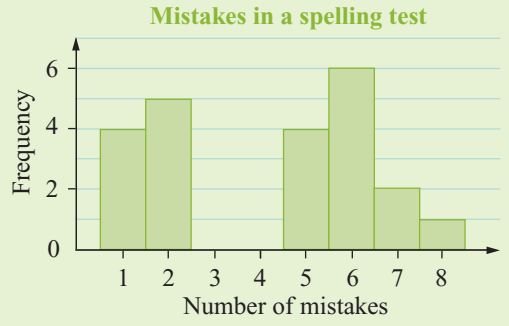
Identify any clusters, outliers or gaps in the following sets of data.

a Monthly rainfall: 25 mm, 16 mm, 6 mm, 27 mm, 28 mm, 96 mm

b

Stem	Leaf
2	2 6
3	2 7
4	0 5 9
5	0 1 1 2 3 4 4 7 7 7 8 8 9
6	1 3 3

c



a 96 mm is an outlier. It is much larger than all the other data values.
(There is a large gap between 96 and the other scores.)

b There is a cluster of scores in the 'fifties'.

c This data has a gap between 2 and 5. No students made 3 or 4 mistakes.

Exercise 9F

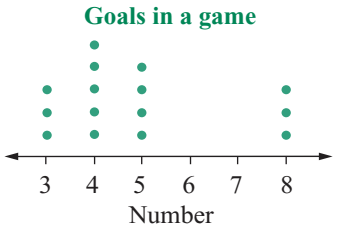
1 Identify any clusters, outliers or gaps in the following sets of data.

a 13, 14, 15, 15, 17, 104

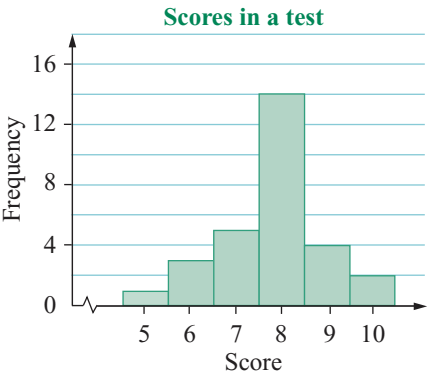
b

Stem	Leaves
4	8
5	3 6
6	0 1 1 3 4 4 4 6 8 8 8 9 9
7	5
8	1

c



d



e

Stem	Leaves
5	2 4 4 7 9
6	0 5 6 6
7	
8	3 8
9	1 1 6 8 1

f 17, 215, 216, 216, 218, 220

G Using measures of central tendency

The mean, median and mode are each called a measure of central tendency—a measure that is central and typical of all the data. However, there are situations when not all these measures are representative of the data and it is necessary to choose the most appropriate measure for that situation.

EXAMPLE 1

a Find the mean, median, mode and range of each set of scores.

i 3, 5, 5, 7, 9 **ii** 3, 5, 5, 7, 90

b Draw a dot plot for each set of data and mark the position of the mean, median and mode.

c Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

d Discuss the use of range as a measure of spread.

a i Mean $= \frac{29}{5} = 5.8$

Median $= 5$

Mode $= 5$

Range $= 6$

ii Mean $= \frac{110}{5} = 22$

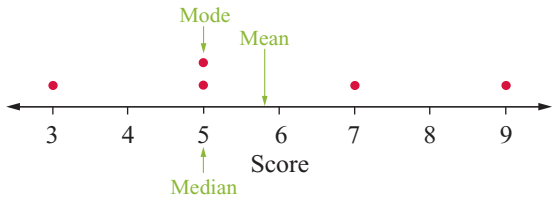
Median $= 5$

Mode $= 5$

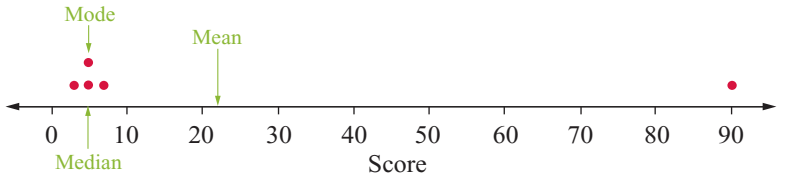
Range $= 87$

EXAMPLE 1 CONTINUED

b i



ii



c In the first data set, the mean, median and mode are all central and typical values of the scores. Any are appropriate as a measure of central tendency.
In the second data set, the mean is no longer a central value as it is larger than 4 of the 5 scores and is not typical of the value of any of the scores. It is not appropriate to use as a measure of central tendency.
The two sets of data are the same except for the last score.
As the mean is calculated using the value of every score, it is greatly affected by outliers. When the 9 in the first set is replaced by 90, much larger than the other scores, the mean changes significantly from 5.8 to 22. The mean is not an appropriate measure of central tendency if the data has an outlier.
Note: The median and mode remain unchanged despite the presence of the outlier in the second set of data and are appropriate to use as measures of central tendency.

An outlier is sometimes caused by an error in measurement or recording.

d i The range is a good measure of the spread of the scores.

ii The range is greatly affected by the outlier and is not a useful measure of the spread of this set of scores.

Exercise 9G

1 a Find the mean, median and mode of the scores in each data set.

i 7, 9, 9, 10, 12 **ii** 7, 9, 9, 10, 80

b Draw a dot plot for each set of data and mark the position of the mean, median and mode.

c Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

d Discuss the use of the range as a measure of spread.

2 Comment on the use of the mean as a measure of central tendency for these data sets.

a

Score	Frequency
12	4
13	3
14	7
15	5
16	2

b

Score	Frequency
31	4
32	6
33	8
34	7
35	5
136	1

EXAMPLE 2

a The heights of students in a school netball team were measured and recorded as 166 cm, 170 cm, 168 cm, 67 cm, 170 cm, 169 cm.

- **i** Calculate the mean, median and mode of this data.
- **ii** Identify the outlier in this set of data.
- **iii** Ignore the outlier and calculate the mean, median and mode of the remaining 5 scores.
- **iv** Should the outlier be included when reporting the mean, median and mode for this data? Why or why not?

b The scores of 5 students on a mechanical aptitude test were recorded as 18, 23, 21, 20, 52.

- **i** Calculate the mean, median and mode of this data.
- **ii** Identify the outlier in this set of data.
- **iii** Ignore the outlier and calculate the mean, median and mode of the remaining 4 scores.
- **iv** Should the outlier be included when reporting the mean, median and mode for this data? Why or why not?

a i Mean $= \frac{166 + 170 + 168 + 67 + 170 + 169}{6} = \frac{910}{6} = 151.7$ cm (1 decimal place)

Note: The mean is not central or typical of the data.

Arrange the data in order: 67, 166, 168, 169, 170, 170

Median $= \frac{168 + 169}{2} = 168.5$ cm

Mode $= 170$

ii Outlier $= 67$ cm

iii Mean $= \frac{166 + 170 + 168 + 170 + 169}{5} = \frac{843}{5} = 168.6$

Median $= 169$

Mode $= 170$

iv In this case it is reasonably obvious that the value 67 cm is a measurement or recording error, as it is not likely that any girl in the netball team would be 67 cm tall. In this case, the outlier could be ignored and the mean, median and mode would then all be central and typical of the data.

b i Mean $= \frac{18 + 23 + 21 + 20 + 52}{5} = 26.8$

Note: The mean is not central or typical of the data.

Arrange the data in order: 18, 20, 21, 23, 52

Median $= 21$

There is no mode.

ii Outlier $= 52$

iii Mean $= 20.5$

Median $= 20.5$

There is no mode.

iv In this case the outlier could be the result of one of the students having an exceptionally high mechanical aptitude compared with the others, so the outlier should be included in the reporting even though, by including it, the mean is not a central or typical value.

Note: The median is the best measure of central tendency with or without the outlier included.

Each case needs to be justified on the basis of the data collected. An outlier may be caused by instrument error or malfunction, or by a recording or transcription error and so it may be justifiable to ignore it when reporting measures of central tendency. However, an outlier could also be the result of natural deviation, in which case the data may need to be investigated more closely.

3 A metal rod was measured by 6 students and the results were recorded as 112 mm, 111 mm, 110, mm, 13 mm, 112 mm, 112 mm.

- **a** Calculate the mean, median and mode of this data.
- **b** Identify the outlier in this set of data.
- **c** Ignore the outlier and calculate the mean, median and mode of the remaining 5 scores.
- **d** Should the outlier by included in reporting the mean, median and mode for this data? Why or why not?

4 The times (in minutes) taken to travel to work in a 5 day week were recorded as 17, 15, 16, 18, 55.

- **a** Calculate the mean, median and mode of this data.
- **b** Identify the outlier in this set of data.
- **c** Ignore the outlier and calculate the mean, median and mode of the remaining 4 scores.
- **d** Should the outlier by included in reporting the mean, median and mode for this data? Why or why not?

EXAMPLE 3

a Find the mean, median and mode of these scores.

i 3, 3, 3, 4, 6, 9, 9 **ii** 3, 4, 5, 6, 9

b Draw a dot plot for each set of data and mark the position of the mean, median and mode.

c Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

a i Mean $= \frac{37}{7} = 5.3$ (1 decimal place)

Median $= 4$

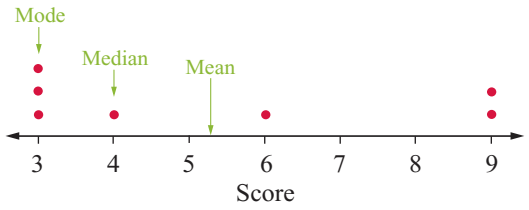
Mode $= 3$

ii Mean $= \frac{27}{5} = 5.4$

Median $= 5$

There is no mode.

b i



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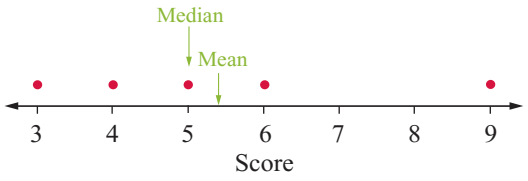


c In the first data set, the mode occurs at the end of the data set. It is not a central value. The mean or median would be the most appropriate measure of central tendency.

Note: There are situations when the mode is important even if it is not a central value. For example, the most common size of shoe would be of importance to a manufacturer of shoes.

In the second data set, there is no mode. The mean or median would be the most appropriate measure of central tendency.

5 a Find the mean, median and mode of these scores.

i 7, 7, 7, 9, 10, 11, 11, 12 **ii** 21, 24, 25, 29, 30

- **b** Draw a dot plot for each set of data and mark the position of the mean, median and mode.
- **c** Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

6 Comment on the use of the mode as a measure of central tendency for these data sets.

a

Score	Frequency
22	7
23	3
24	4
25	5
26	2

b

Score	Frequency
4	2
5	5
6	8
7	6
9	3

c

Score	Frequency
4	3
5	5
6	4
7	2
8	8

EXAMPLE 4

a Find the mean, median and mode of these scores.

i 1, 4, 5, 8, 8, 8, 8 **ii** 2, 2, 2, 2, 2, 4, 5, 7, 7

b Draw a dot plot for each set of data and mark the position of the mean, median and mode.

c Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

a i Mean $= \frac{42}{7}$

$= 6$

Median $= 8$

Mode $= 8$

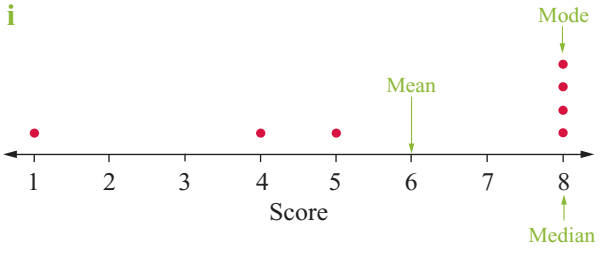
ii Mean $= \frac{33}{9}$

$= 3.7$ (1 decimal place)

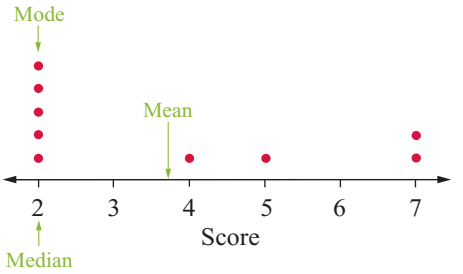
Median $= 2$

Mode $= 2$

b i



ii



c For both data sets, the median and the mode occur at the end of the data set; that is, they are not central values. The mean would be the most appropriate measure of central tendency to use for this pattern of data.

7 **a** Find the mean, median and mode of these scores.

i 5, 5, 5, 7, 8 **ii** 4, 5, 7, 8, 8, 8, 8

b Draw a dot plot for each set of data and mark the position of the mean, median and mode.

c Compare and discuss the use of the mean, median and mode as measures of central tendency for these data sets.

8 Comment on the use of the median as a measure of central tendency for these data sets.

a

Score	Frequency
13	9
14	2
15	3
16	2
17	1

b

Score	Frequency
5	5
6	9
7	12
8	10
9	4

c

Score	Frequency
46	1
47	3
48	4
49	5
50	16

9 The number of goals scored in each match by a soccer team after 10 matches is 0, 0, 2, 0, 1, 0, 2, 0, 1, 1.

a Find the mean, median and mode of these scores.

b Which of these measures would be of the greatest importance to the coach of the team?

10 The country of origin is listed for a group of 82 immigrants.

a Find if possible, the mean, median and mode of this data.

b Complete: For non-numerical data, the only relevant measure is the _____.

Country	Frequency
Britain	17
New Zealand	31
India	11
Malaysia	9
Iraq	14

11 A small company has 8 employees whose annual incomes are listed below.

4 machine operators	$42 000 each
2 tradesmen	$83 000 each
1 accountant	$126 000
1 owner/manager	$498 000

a Find the mean income for the employees.

b **i** How many employees earn less than the mean income?

ii How many employees earn more than the mean income?

c What is the median income?

d What is the modal income?

e **i** If you were a machine operator, which measure would you use to support the argument for an increase in wages for employees?

ii If you were the manager, which measure would you use to counter the argument for a wage rise?

f Which measure of central tendency is the most appropriate to represent the incomes of the employees of this company? Discuss with your class.

12 The prices of cars sold by a dealer one weekend were $6900, $18 900, $26 100, $19 990, $21 000.

a The dealer claims that the average price of cars he sells is $18 578. Which measure of central tendency has he used for his 'average'?

b Is this measure appropriate to represent the prices of the cars he sells? Give a reason.

c Which measure do you think is the most representative of the data? Give a reason.

13 The office manager at a medical practice measures the time 10 people, chosen at random, wait to see a doctor. The waiting times (to the nearest minute) were 16, 11, 8, 9, 12, 16, 16, 7, 16, 10.

a Find the mean, median and mode of these waiting times.

b Discuss the relative merits of these three measures for this data.

14 Kara scored the following marks out of 10 in six topic tests in Mathematics: 4, 5, 2, 5, 5, 3.

a The teacher reported that Kara's average mark was 4 but Kara said her average was 5. Which measures of central tendency have the teacher and Kara used?

b Which measure of central tendency do you think is the most representative of this data? Give a reason.

H Comparing sample summary statistics

Often it is not feasible to investigate the properties of a whole population so we only collect data from a small part of the population. The part of the population investigated is called a sample. We use the properties and characteristics of the sample to provide an estimate of the properties and characteristics of the population.

Exercise 9H

The number of mistakes made by 100 students in a spelling test is shown below. (No student made more than 9 mistakes.)

48047	45381	33232	35178	46971	75879	31457	22016	08694	58112
96170	91910	28868	52251	99727	32681	90071	98873	79846	50953

Consider samples of size 20 taken from this population.

1 **a** Complete the frequency distribution table shown for the first group of 20 students (start at the first number 4 and work from left to right).

Note that this is not a random sample.

Number of mistakes	Tally	Frequency
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

b Find the mean, median, mode and range of the number of mistakes made by this sample of students.

2 Repeat question **1** for the second group of 20 students.

3 Repeat question **1** for the third group of 20 students.

4 Repeat question **1** for the fourth group of 20 students.

5 Repeat question **1** for the fifth group of 20 students.

6 **a** Choose any student at random.
b Starting at this student select every 5th student until you have a systematic random sample of 20 students.
c Find the mean, median, mode and range for this sample.

7 **a** **i** Use a calculator to generate 20 random numbers from 1 to 100.
ii Let each of these random numbers be the position of a student in the list above and list the number of mistakes made by these students. (For example, if the random numbers generated are 03, 09, 13, 22, …, list the number of mistakes made by the 3rd, 9th, 13th, 22nd, … student.)
b Find the mean, median, mode and range for this sample.

8 Summarise the results from the samples of 20 students taken above, in a table like the one shown below.

	1st 20	2nd 20	3rd 20	4th 20	5th 20	6th 20	7th 20
Mean							
Median							
Mode							
Range							

a Which of these measures (mean, median and mode) varies the most between samples?

b Which of these measures varies the least?

9 **a** Determine the mean, median and mode for the whole population.

b Compare these with the sample statistics. Discuss.

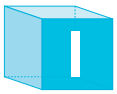
10 **a** **i** How does the range vary in the 7 samples of 20 students?

ii Compare this with the range for the population. Comment.

b Consider samples of size 5 taken from this population.

i What is the range of each of the first 10 groups of 5 students?

ii Compare and discuss sample size and variation in the value of the range.



Variation of sample mean and proportion

A key factor in the use of samples is the determination of how large the sample should be in order to give a good estimate of the properties of the whole population. Consider the following results, recorded in groups of 5, when a normal six-sided die is rolled 200 times. The results are summarised in the table below.

34664	14624	31362	51242	63611	42553	63144	45213	56443	54346
52415	33663	55244	65132	63514	62453	12646	35236	24546	13251
43356	64132	21634	46323	55651	26435	53142	25145	26513	42214
26563	21264	23245	61224	32616	11326	21621	16231	21652	31643

Score	Frequency	Proportion of total number of scores
1	31	$\frac{31}{200} = 16.5\%$
2	37	$\frac{37}{200} = 18.5\%$
3	34	$\frac{34}{200} = 17\%$
4	34	$\frac{34}{200} = 17\%$
5	28	$\frac{28}{100} = 14\%$
6	36	$\frac{36}{200} = 18\%$
Total	**200**	**100%**

Mean $= \frac{699}{200} = 3.5$ (1 decimal place)

EXAMPLE 1

Consider the proportion of 6s in samples of size 5 from the previous results for die rolls. Compare these with the population proportion.

In statistics 'the proportion of 6s' is often referred to as 'the relative frequency of the score 6'.

Proportion of 6s in sample of 5 $= \frac{\text{number of 6s}}{5}$

a Complete the following table.

Proportion of 6s in				
1st group of 5	Last group of 5	11th group of 5	20th group of 5	29th group of 5

b **i** What is the lowest proportion of 6s in these 5 samples?

ii What is the highest proportion of 6s in these 5 samples?

c In how many of these samples is the proportion in the sample approximately the same as in the population?

d Do you think that a sample of size 5 is big enough to provide a good estimate of the proportion of 6s in the population? Give a reason.

a

Proportion of 6s in				
1st group of 5	last group of 5	11th group of 5	20th group of 5	29th group of 5
$\frac{2}{5} = 40\%$	$\frac{1}{5} = 20\%$	$\frac{0}{5} = 0\%$	$\frac{0}{5} = 0\%$	$\frac{1}{5} = 20\%$

b **i** Lowest = 0%

ii Highest = 40%

Note that these are not random samples.

c There are 2 sample proportions (20% and 20%) that are approximately the same as the population proportion.

d No. Only 2 of the 5 samples have proportions close to the population proportion of 18%.

Exercise 9I

1 Consider the proportion of 6s in some samples of size 10 for the data given in the introduction to this section.

a Complete the following table.

Proportion of 6s in				
1st group of 10	Last group of 10	7th group of 10	12th group of 10	17th group of 10

b Complete the following.

i The lowest proportion of 6s in these 5 samples is ___.

ii The highest proportion of 6s in these 5 samples is ___.

c In how many of these samples is the proportion in the sample approximately the same as in the population?

d Do you think that a sample of size 10 is big enough to provide a good estimate of the proportion of 6s in the population? Give a reason for your answer.

2 Consider the proportion of 6s in some samples of size 20 for the data given in the introduction to this section.

a Complete the following table.

Proportion of 6s in				
1st group of 20	Last group of 20	4th group of 20	Random sample of 20	Random sample of 20

b **i** What is the lowest proportion of 6s in these 5 samples?

ii What is the highest proportion of 6s in these 5 samples?

c In how many of these samples is the proportion in the sample approximately the same as in the population?

d Do you think that a sample of size 20 is big enough to provide a good estimate of the proportion of 6s in the population? Give a reason for your answer.

3 Consider the data given in the introduction to this section and questions **1** and **2** above. For what sample size of this population do you seem to get the most sample proportions close to the population proportion?

EXAMPLE 2

Combine the information from the 5 samples in Example 1 into one of size 25. Is the proportion of 6s in this sample a good estimate of the population proportion?

Combining the information from our 5 samples:

$$\text{Proportion of 6s} = \frac{2+1+0+0+1}{5+5+5+5+5}$$
$$= \frac{4}{25}$$
$$= 16\%$$

Yes, this is close to the population proportion of 18%.

4 Combine the information from the 5 samples in question **1** into one of size 50. Complete the following to find if the proportion of 6s in this sample a good estimate of the population proportion.

$$\text{Proportion of 6s} = \frac{\square+\square+\square+\square+\square}{10+10+10+10+10} = \frac{\square}{50}$$
$$= \_\_\_\%$$

5 Combine the information from the 5 samples in question **2** into one of size 100. Is the proportion of 6s in this sample a good estimate of the population proportion?

6 Using the information gathered in the above questions, comment on how the size of a sample affects the estimate of the population proportion.

EXAMPLE 3

Consider the means of samples of size 5 taken from the data at the beginning of Section I and compare this with the population mean.

a Complete the table.

Sample	1st 5	Last 5	15th	27th	37th
Mean					

b **i** What is the lowest sample mean?

ii What is the highest sample mean?

c In how many of these samples is the mean of the sample approximately the same as that of the population?

d Do you think that a sample of size 5 is big enough to provide a good estimate of the mean of the population? Give a reason.

a

Sample	1st 5	Last 5	15th	27th	37th
Mean	$\frac{3+4+6+6+4}{5}$ $= 4.6$	$\frac{3+1+6+4+3}{5}$ $= 3.4$	$\frac{6+3+5+1+4}{5}$ $= 3.8$	$\frac{5+3+1+4+2}{5}$ $= 3$	$\frac{2+1+6+2+1}{5}$ $= 2.4$

b **i** Lowest sample mean $= 2.4$

ii Highest sample mean $= 4.6$

c Population mean $= 3.5$

Let's take 'within 10% of' to indicate 'approximately the same as'.

$$10\% \text{ of population mean} = \frac{10}{100} \times 3.5$$
$$= 0.35$$

Now $3.5 - 0.35 = 3.15$ and $3.5 + 0.35 = 3.85$, hence we will consider any sample means between 3.15 and 3.85 to be 'approximately the same as' the population mean.

There are two sample means (3.4 and 3.8) that are approximately the same as the population mean.

d No. Only 2 of the 5 sample means are approximately the same as the population mean.

7 Consider the means of samples of size 10 and compare this with the population mean.

a Complete the following table.

Sample	1st 10	Last 10	5th 10	14th 10	17th 10
Mean	$\frac{\square}{10} =$ ___	$\frac{\square}{10} =$ ___	$\frac{\square}{10} =$ ___	$\frac{\square}{10} =$ ___	$\frac{\square}{10} =$ ___

b Complete the following.

i The lowest sample mean is ___.

ii The highest sample mean is ___.

c In how many of these samples is the mean approximately the same as that of the population?

Complete the following:

Number of sample means that lie between 3.15 and 3.85 = ___

d Do you think that a sample of size 10 is big enough to provide a good estimate of the mean of the population? Give a reason for your answer.

8 Consider the means of samples of size 20 and compare them with the population mean.

a Complete the following table.

Sample	1st 20	Last 20	2nd 20	Random sample of 20	Random sample of 20
Mean	$\frac{\square}{20} = \_\_$	$\frac{\square}{20} = \_\_$	$\frac{\square}{20} = \_\_$		

b **i** What is the lowest sample mean?

ii What is the highest sample mean?

c In how many of these samples is the mean of the sample approximately the same as that of the population?

d Do you think that a sample of size 20 is big enough to provide a good estimate of the mean of the population? Give a reason for your answer.

9 Consider Example 3 and questions **7** and **8** above. For what sample size of this population do you seem to get the most sample means close to the population mean?

EXAMPLE 4

Use the information in Example 3 as listed in the table below.

a Find the mean of the sample means for the first:

i 3 samples **ii** 4 samples **iii** 5 samples

b Is the mean of the sample means in part **a** approximately the same as the population mean?

a

Sample	1st 5	Last 5	15th	27th	37th
Mean	4.6	3.4	3.8	3	2.4

i Mean of first 3 sample means $= \frac{\text{sum of sample means}}{3}$

$= \frac{4.6 + 3.4 + 3.8}{3}$

$= 3.9$ (1 decimal place)

ii Mean of first 4 sample means $= \frac{\text{sum of sample means}}{4}$

$= \frac{4.6 + 3.4 + 3.8 + 3}{4}$

$= 3.7$

iii Mean of first 5 sample means $= \frac{\text{sum of sample means}}{5}$

$= \frac{4.6 + 3.4 + 3.8 + 3 + 2.4}{5}$

$= 3.4$ (1 decimal place)

b As the number of samples increases, the mean of the sample means gets closer to the mean of the population.

10 Use the information in question **7**.

a Find the mean of the sample means for the first:

i 3 samples **ii** 4 samples **iii** 5 samples.

b Is the mean of the sample means approximately the same as the population mean?

11 Use the information in question **8**.

a Find the mean of the sample means for the first:

i 3 samples **ii** 4 samples **iii** 5 samples.

b Is the mean of the sample means approximately the same as the population mean?

Summary

From the previous exercises you should have discovered that the proportions and means in samples from the same population vary. In general:

- the larger the sample size the closer the sample proportion is to the population proportion
- the larger the sample size the closer the sample mean is to the population mean
- as the number of samples increases, the mean of the sample means gets closer to the mean of the population.

Investigation 3 Investigating data

1 Investigate, discuss and list the different categories that random samples could be based on if choosing from:

a a census of the population of Australia (such as gender, state, …)

b data concerning personal expenditure

c data concerning immigrants to Australia.

2 Undertake an investigation into:

a voting habits. Suggest reasons why the summary statistics might differ if samples were chosen on the basis of postcodes.

b amount of homework set by teachers. Suggest reasons why the summary statistics might differ if samples were chosen on the basis of age.

c weights of 14-year-old students. Suggest reasons why the summary statistics might differ if samples were chosen on the basis of gender.

d the number of faulty items produced by workers in a factory. Suggest reasons why the summary statistics might differ if samples were chosen on the basis of the time of the quality control check.

e the number of people who have health care cover. Suggest reasons why the summary statistics might differ if samples were chosen on the basis of private income.

Investigation 4 Practical activities

1 When a drawing pin is tossed we say that it can land either pin up or pin down.

a i In pairs, toss a drawing pin 100 times and record the results in a table like the one shown below.

Outcome	Tally	Frequency	Proportion
Pin up			
Pin down			

ii Complete the proportion column.

iii Use the information from this sample to estimate the number of times the drawing pin would land pin up, if it was tossed 1 000 000 times.

b i Put the information from your sample on the board and combine the information for the whole class in a table like the one below.

Outcome	Proportion	Combined proportion
Pin up		
Pin down		

ii Use the class information to find a better estimate of the number of times a drawing pin would land pin up if it was tossed 1 000 000 times.

iii Explain why the answer to part **ii** is a better estimate.

2 Suppose a coin is tossed 20 times and the following results occurred.

H T T T H H T H T T H T H T T H H H T H

The 'run lengths' of tails (i.e. the number of times in a row that a tail occurs) are 3, 1, 2, 1, 2, 1 and the mean run length $= \frac{3 + 1 + 2 + 1 + 2 + 1}{6} = 1.67$.

a i In pairs, toss a coin 100 times and record the results in the order in which they occur.

ii Write down the run lengths for tails.

iii Calculate the mean run length for tails in this sample of 100 tosses.

b i Put the information from your sample on the board in a table like the one below.

Sample	1	2	3			
Sample mean						

ii Use the information in this table to find a better estimate of the mean run length for tails if a coin is tossed a large number of times.

iii Explain why the answer to part **ii** is a better estimate.

c Theoretically, when a coin is tossed a large number of times, the expected mean run length of tails is 2. Compare the answers above with this theoretical value and discuss this with your class.

J Capture–recapture technique (extension)

The **capture–recapture** technique is a method used by fish and wildlife managers to collect data about the size of various animal populations.

A random sample of the animals in a population is captured and tagged or marked in some way. The tagged animals are then released back into the population. An appropriate time is allowed for them to mix completely and then another random sample of animals is captured. The number of tagged animals in this sample is used to estimate the total population, based on the assumption that the proportion of tagged animals in the total population is the same as the proportion in the second sample. This technique is used when other methods are not feasible because of budget, time or practical reasons.

EXAMPLE 1

A biologist captures a random sample of 120 fish from a lake. Each fish is tagged and released back into the lake. A week later another random sample of 80 fish is captured and it is found that 6 of them have tags.

a If we let N represent the total number of fish in the lake, write down an expression for the fraction of tagged fish in the lake.

b Write down the fraction of tagged fish in the second sample.

c Assuming that the fraction of tagged fish in the second sample is the same as the fraction of tagged fish in the lake, estimate the total population of fish in the lake at the time of tagging.

a $\frac{120}{N}$

b $\frac{6}{80}$

c $\frac{120}{N} = \frac{6}{80}$

$\frac{N}{120} = \frac{80}{6}$ Invert both sides of the equation.

To invert means to turn both sides upside down.

$\frac{N}{120} \times 120 = \frac{80}{6} \times 120$ Multiply both sides by 120.

$N = 1600$ $\therefore$ There were approximately 1600 fish in the lake at the time of tagging.

Exercise 9J

1 A biologist captures a random sample of 200 fish from a dam, tags them and releases them back into the dam. A week later another random sample of 90 fish is captured and it is found that 12 of them have tags. Complete the following to estimate the fish population at the time of tagging.

a If we let N represent the total number of fish in the lake, an expression for the fraction of tagged fish $= \frac{\square}{N}$.

b The fraction of tagged fish in the second sample $= \frac{\square}{90}$.

c Assume that the fraction of tagged fish in the second sample is the same as the fraction of tagged fish in the lake.

$\frac{\square}{N} = \frac{\square}{90}$

$\frac{N}{\square} = \frac{90}{\square}$ Inverting both sides of the equation.

$\frac{N}{\square} \times \_\_\_ = \frac{90}{\square} \times \_\_\_$ Multiplying both sides by ___.

$N = \_\_\_$

2 A park ranger captures a random sample of 50 rabbits. Each rabbit is tagged and released. A few days later he catches another 50 rabbits in the same area and finds that 4 of them are tagged.

- **a** If we let N represent the total number of rabbits, write down an expression for the fraction of tagged rabbits in this area.
- **b** Write down the fraction of tagged rabbits in the second sample.
- **c** Assuming that the fraction of tagged rabbits in the second sample is the same as the fraction of tagged rabbits in the area, write an equation and solve it to estimate the total population of rabbits.

3 A scientist captures 60 mice from a forest, puts leg bands on them and releases them back into the forest. A week later he captures a second sample of 50 mice and finds that 3 have leg bands.

- **a** If we let N represent the total number of mice, write down an expression for the fraction of tagged mice in the forest.
- **b** Write down the fraction of tagged mice in the second sample.
- **c** Assuming that the fraction of tagged mice in the second sample is the same as the fraction of tagged mice in the forest, write an equation and solve it to estimate the total population of mice.

4 A scientist at a trout farm takes 200 trout from a pond, tags them and releases them back into the pond. A week later he catches 100 trout and finds that 5 of them are tagged. Estimate the number of trout in the pond.

5 A researcher investigating the kangaroo population in a particular area catches and tags a random sample of 40 kangaroos before releasing them back into the same area. A week later he catches another sample of 40 kangaroos and finds that 8 of them are tagged. Estimate the number of kangaroos in this area.

6 To estimate the number of deer in a national park, a ranger captures 30 deer, tags them and releases them back into the park. Six months later she returns and captures 20 deer in each of three different areas of the park. The number of tagged animals in each sample is shown in the table.

Sample	Number tagged	Population estimate
1	3	
2	1	
3	2	

- **a** Estimate the total population of deer in the Park using each of the 3 samples taken. .
- **b** Using the mean of these three samples, estimate the population of deer in the Park.
- **c** If the three samples are combined to form one sample, that is, we assume that the number found tagged is 6 out of 60, what will be the population estimate?
- **d** Which of these estimates do you think is the most reliable? Discuss with your class.
- **e** Discuss any factors that might make this method unreliable.

Investigation 5 Simulating capture–recapture

Investigate, discuss and identify issues that make it difficult to obtain representative data from either primary or secondary sources. What constraints limit the collection of data or result in unreliable data; for example, lack of proximity to the data collection area, lack of access to digital technologies or cultural sensitivities?

You can simulate the capture-recapture technique as follows:
Place a large (unknown) number of toothpicks in a bowl. Take out a small number (say 10) and mark (tag) each with a marker pen. Put the marked toothpicks back in the bowl and mix them thoroughly. Then take a handful of the toothpicks from the bowl and determine what fraction of the sample is marked. Use this fraction to estimate the number of toothpicks in the bowl. Repeat the simulation by marking 20 toothpicks. Check your estimates by actually counting the number of toothpicks in the bowl.

Language in mathematics

1 **a** Explain the differences between a population, a sample taken from a population and an observation when collecting data.

b Investigate the practicalities of collecting data through a census compared to a sample.

i Discuss the limitations of using a census, such as population size.

ii What factors do you think would affect the accuracy of the results of a sample?

c List some advantages and disadvantages of using a census, a sample and observation to collect data.

2 Complete the following statements using the words from the list given below. (Some words are used more than once.)

population, proportions, larger, increases, means, larger, closer, sample, mean, vary

From the preceding exercises you should have discovered that _____ and _____ in samples vary. In general, the _____ the sample size the _____ the _____ proportion is to the _____ proportion. Also the _____ the sample size the _____ the sample _____ is to the _____ mean and as the number of samples _____, the mean of the _____ means gets closer to the _____ of the population.

3 In a school of 920 students there are six Year 8 classes with 30 students in each. Use the following questions to help you describe how you would use sampling to find the mean height of Year 8 students in this school.

a What facts do you know from the information in the question?

b What information given do you not need?

c What does the problem want you to find?

d What strategies will you use to solve the problem?

When you have finished, explain and justify your method to the class.

Terms

accuracy	ascending	average	bias	categorical
census	central tendency	characteristic	continuous	cluster
data	defective	descending	discrete	extreme
faulty	mean	median	measure	modal
mode	nominal	numerical	outlier	observation
population	prediction	primary	properties	proportion
qualitative	quantitative	random sample	range	representative
sample	secondary	systematic random sample		variable

Check your skills

1 Which of the following is a discrete numerical variable?

A time to do homework

B number of cars in a car park

C star energy ratings of clothes dryers

D hair colour

2 Which of the following is not an example of a continuous numerical variable?

A number of heads when 3 coins are tossed

B heights of netball players

C weights of hockey team members

D daily maximum temperature

3 The response scale for a survey question is given below.

1 = strongly disagree, 2 = disagree, 3 = don't know, 4 = agree, 5 = strongly agree

This type of data is classified as:

A discrete numerical **B** continuous numerical

C categorical **D** agreeable

4 'All the retailers of TV sets' would be the target population if we wanted to collect information about:

A the most popular television show **B** the most popular make of television set

C the number of people who watch television **D** the most popular sports

5 To collect data on the age of people in your local area, you would use:

A a census **B** a sample

C observation **D** an experiment

6 Which of the following is an example of data that could be collected by observation?

A the life of a battery **B** the gender of high school principals in NSW

C the number of car accidents caused by speed **D** the direction of cars passing the front of the school

7 A random sample is one for which:

A every member of the population has an equal chance of being selected

B the mean of the data is the same as the mean of the population

C all the properties of the sample are identical to the population

D all of the above

8 Which of the data sets below contains an outlier?

A 7, 11, 15, 17, 20 **B** 0, 0, 0, 0, 4, 5

C 698, 701, 712, 719, 730 **D** 48, 51, 52, 54, 256

9 In which of the data sets in question **8** is the median not a central score?

10 Six cards are drawn from a pack and the suit of each card is recorded.

spade, heart, club, spade, diamond, spade

Which of the following measures cannot be found for this data?

A mode **B** frequency of spades

C proportion of diamonds **D** mean

11 The price of home units sold by a real estate agent one month are listed.

$296 000, $297 000, $345 000, $386 000 and $622 000

Which of the following is the most appropriate measure of central tendency to represent this data?

A mean **B** mode **C** median **D** range

12 Which of the following measures varies the least between samples?

A mean **B** mode **C** median **D** enlargement

13 Which of the following statements is not true?

A The larger the sample size the closer the sample proportion is to the population proportion.

B The larger the sample size the closer the sample mean is to the population mean.

C As the number of samples increases, the sample proportion increases.

D As the number of samples increases, the mean of the sample means gets closer to the mean of the population.

14 Samples of 50, 100, 200 and 500 items produced by a machine are taken and the proportion of defective items in each sample is recorded as shown in the table below.

Sample size	50	100	200	500
Proportion of defective items	$\frac{3}{50} = 6\%$	$\frac{4}{100} = 4\%$	$\frac{7}{200} = 3.5\%$	$\frac{18}{500} = 3.6\%$

Which of the above sample sizes will give the best estimate of the proportion of defective items in the population?

A 50 **B** 100 **C** 200 **D** 500

15 A park ranger captures, tags and releases 20 wallabies in a national park. A week later 35 wallabies are captured and 4 are found to be tagged. The number of wallabies in the park is estimated to be:

A 35 **B** 700 **C** 175 **D** 350

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–3	4–6	7	8	9–11	12	13–14	15
Section	A	B	C	F	G	H	I	J

9A Review set

1 Classify the following variables as categorical, discrete numerical or continuous numerical.

a number of children in a family **b** weights of Year 8 students

c number of tails when 3 coins are tossed **d** prices of cars in a sales yard

e the rating scale for the responses to a questionnaire given as 1 = dislike, 2 = neutral, 3 = like

2 What or who would be the target population if we wanted to collect information about:

a the health problems of smokers?

b the types of injuries suffered by cyclists?

3 Would a census, sample or observation be used to collect data on these variables.

a sizes of oranges picked from an orchard

b most popular brand of peanut butter

c number of runs made by a cricketer in a season

d number of koalas in a particular area

4 Explain two simple practical methods for choosing a random sample of 5 students from a class of 28 students.

5 Explain any possible bias in the following:

a A poll about gun control is taken at a meeting of the Shooters Party.

b A salesman claimed that his sales had trebled in a week.

6 Write down the outlier in each of the following data sets.

a 0, 51, 52, 52, 54 **b** 13, 14, 15, 15, 17, 18, 79

7 For the sets of data given below, which of the mean, median or mode is not a central value of the data?

a 6, 8, 9, 9, 11, 94 **b** 3, 3, 4, 5, 7, 8 **c** 1, 4, 5, 6, 6, 6, 6

8 The sexes of a group of 5 students chosen at random from a Year 8 class are M F F M F.
For this data, find, where possible, the:

a mean **b** mode **c** median.

9 The weights (kg) of seven 1-year-old puppies of the same breed were recorded as shown below.
13, 13, 14, 15, 16, 17, 88

a For the given weights, find the mean, median and mode.

b Identify the outlier.

c Ignore the outlier and calculate the mean, median and mode.

d Should the outlier be included when reporting the mean, median and mode for this data.

9B Review set

1 Classify the following variables as categorical, discrete numerical or continuous numerical.

a income of factory workers **b** occupations of employed people

c types of coins in a purse **d** daily humidity

e the energy star ratings of air conditioners

2 What or who would be the target population if we wanted to collect information about:

a the sports played by Year 8 students? **b** the selling price of homes in a suburb?

3 Would a census, sample or observation be used to collect data on these variables?

a most popular radio station in Sydney **b** responses of patients to a new medication

c suitability of a new train timetable **d** number of cars passing the school

4 Explain how to choose a random sample of 5 students from a class of 30 students using a telephone book.

5 Explain any possible bias in the following:

a 95% of people who changed to Cheapas Insurance said they saved at least $100 on their policy.

b Greenpeace put a survey in their monthly newsletter on attitudes to climate change.

6 Which of the measures mean, median, mode and range is distorted by an outlier?

7 The eye colours of a group of 6 students are recorded below:

blue, green, grey, blue, brown, blue

Which, if any, of the measures mean, median and mode can be used with this data?

8 The times, to the nearest minute, that 10 students, chosen at random, spent waiting for a school bus are given.

19, 20, 29, 28, 29, 18, 20, 27, 19, 29

a Find the mean, median and mode of these times.

b Discuss the appropriateness of each of these measures to represent this data.

9 A small company has 9 employees. Their annual incomes are listed below.

7 warehouse workers	$46 000 each
Warehouse supervisor	$65 000
Owner/manager	$184 000

a Calculate the mean, median and mode of the annual income for this company.

b Discuss the appropriateness of each of these measures to represent this data.

9C Review set

1 Classify the following variables as categorical, discrete numerical or continuous numerical.

a area of blocks of land

b number of milk cows on farms

c country of birth

d attitude to TV show

e time to complete assignment

2 What or who would be the target population if we wanted to collect information about:

a the price of new cars?

b family income of school parents?

3 Would a census, sample or observation be used to collect data on:

a the number of loaves of bread sold each week by a supermarket?

b the most popular colour of new cars?

c the number of goals scored each week by a soccer team?

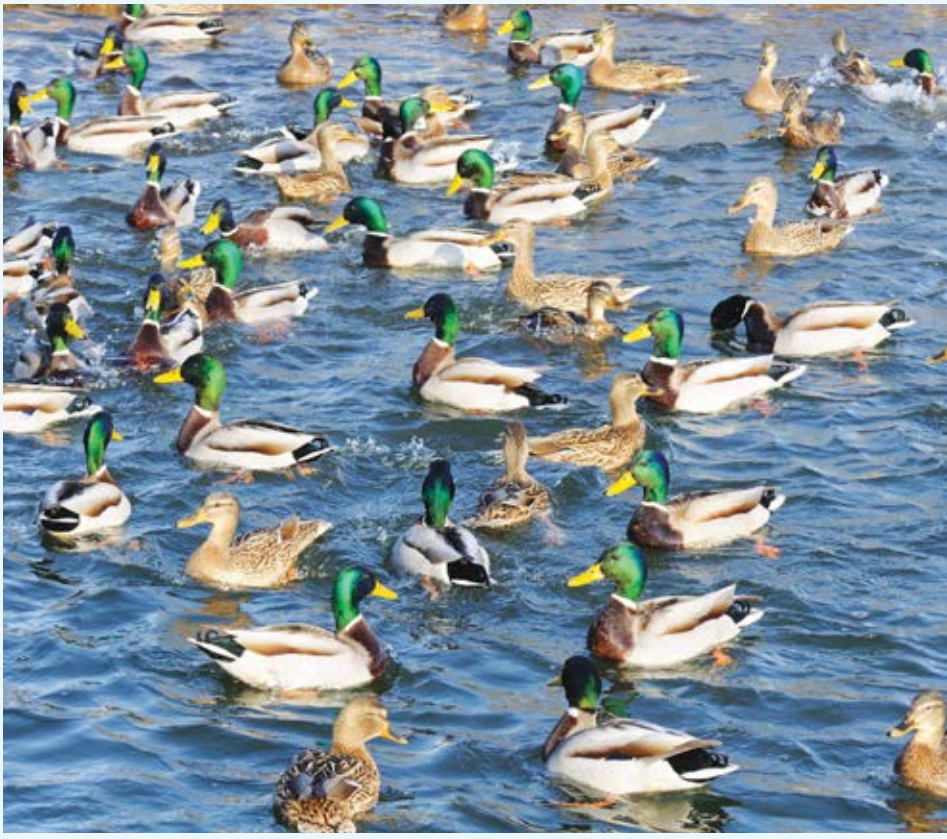
d the number of ducks living by a pond?

4 Explain how to choose a random sample of 5 students from a class of 30 students using a scientific calculator.

5 Explain any possible bias in the following claims.

a 9 out of 10 dentists recommend Brite toothpaste.

b A study commissioned by the Hotels Association showed that consuming 3 standard drinks a day was beneficial to a person's health.

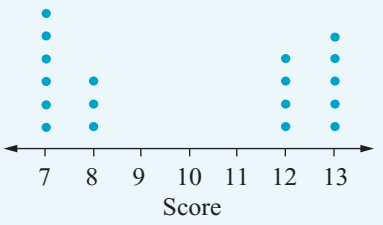


6 Identify any clusters, gaps or outliers in the following data sets.

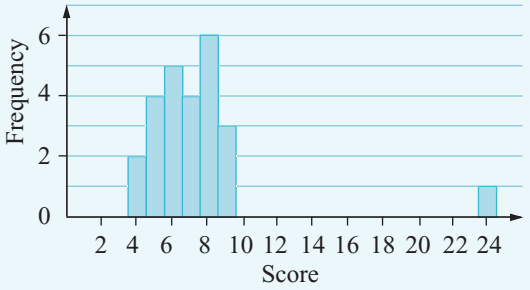
a

Stem	Leaves
3	9
4	8 1 2 2 5 5 6 8 9
5	0 0 3 4 6 7 7 9
6	4 8
7	1

b



c



7 The makes of car owned by the mothers of 10 students chosen at random were Toyota, Mazda, Toyota, Subaru, Ford, VW, Mazda, Toyota, Hyundai, Daihatsu. Find, where possible, the mean, mode and median for this data.

8 **a** Find the mean, median, mode and range of the scores 2, 2, 2, 3, 3, 4, 5, 6, 6, 7.

b If the final score (7) is replaced by 70, which of the measures in part **a** will change?

9 The number of sick days had by 20 workers in a year is listed below:

0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 10, 32

a Find the mean, median and mode of the number of sick days.

b Discuss the appropriateness of each of these measures to represent this data.

9D Review set

1 Classify the following variables as categorical, discrete numerical or continuous numerical.

a number of hours of study **b** religion of students **c** length of metal bars

d cost of cars **e** type of employment (full-time, part-time, casual)

2 What or who would be the target population if we wanted to collect information about:

a the main activity of farms in a region? **b** the quality of bicycle helmets?

3 Would a census, sample or observation be used to collect data on:

a the number of 'heads' when a coin is tossed?

b the number of senior students in a school who smoke cigarettes?

c the stopping distances of cars?

d the reasons people use taxis?

4 Explain how to choose a random sample of 5 students from a class of 30 students using a systematic random sample.

5 Explain any possible bias in the following:

a 500 surveys were posted and the 100 which were returned were analysed and conclusions drawn

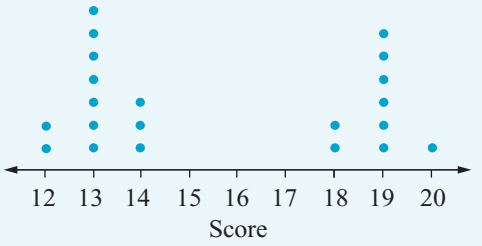
b A doctor who believed he had developed a cure for arthritis tested his patients and reported on the results.

6 Identify any clusters, gaps or outliers in the following sets of data.

a

Stem	Leaves
2	4 7
3	0 2 3 3 6 8 9 9
4	0 0 1 1 1 2 4 7 8 9
5	1 2 5
6	
7	
8	
9	4

b



7 The weights of the forwards in a Year 8 rugby team were measured and recorded, to the nearest kilogram, as:
52, 49, 51, 53, 151, 48, 50, 51

a Identify the outlier.

b Calculate the mean, median, mode and range.

i Include the outlier. **ii** Exclude the outlier.

c Should the outlier be included when reporting the mean, median, mode and range for this data? Give a reason for your answer.

8 The lengths of time (minutes) that a consultant spent on the phone for her first 10 calls of the day are given.
2, 5, 4, 1, 3, 2, 55, 3, 4, 1

a Identify the outlier.

b Calculate the mean, median, mode and range.

i Include the outlier. **ii** Exclude the outlier.

c Should the outlier by included when reporting the mean, median, mode and range for this data? Give a reason for your answer.

9 **a** Calculate the mean, median and mode of these data sets.

i 7, 7, 8, 9, 9, 9, 10, 93 **ii** 13, 14, 14, 15, 15, 16, 17 **iii**

Score	Frequency
5	7
6	2
7	1
8	2
9	3

b For which of these sets of data is the mean, median or mode not a central value of the data?

10

Pythagoras' theorem

This chapter deals with Pythagoras' theorem.

At the end of this chapter you should be able to:

- identify the hypotenuse in a right-angled triangle
- establish the relationship between the side lengths of a right-angled triangle
- write Pythagoras' theorem for right-angled triangles
- use Pythagoras' theorem to establish that a triangle has a right angle
- understand Pythagorean triads
- find the length of the hypotenuse given the other two sides
- find a side of a right-angled triangle given one side and the hypotenuse
- use Pythagoras' theorem to solve worded problems
- solve simple quadratic equations resulting in surds.

NSW Syllabus references: S4 M&G Right-angled triangles (Pythagoras, part)
Outcomes: MA4-1WM, MA4-2WM, MA4-16MG
MEASUREMENT & GEOMETRY – ACMMG222, ACMNA186

Diagnostic test

1 4 + 5 equals:
A 1 **B** 4 **C** 8 **D** 9

2 8 + 7 equals:
A 1 **B** 15 **C** 17 **D** 19

3 40 + 30 equals:
A 7 **B** 70 **C** 700 **D** 10

4 24 + 59 equals:
A 713 **B** 35 **C** 73 **D** 83

5 649 + 588 equals:
A 1237 **B** 927 **C** 827 **D** 837

6 3479 + 65 equals:
A 9970 **B** 4129 **C** 3534 **D** 3544

7 The best estimate for 1687 + 489 is:
A 2000 **B** 2200 **C** 2500 **D** 3000

8 7 − 3 equals:
A 1 **B** 4 **C** 5 **D** 10

9 15 − 7 equals:
A 22 **B** 9 **C** 8 **D** 7

10 90 − 30 equals:
A 66 **B** 60 **C** 600 **D** 10

11 59 − 25 equals:
A 24 **B** 34 **C** 83 **D** 26

12 638 − 289 equals:
A 349 **B** 359 **C** 449 **D** 459

13 2143 − 385 equals:
A 1758 **B** 1768 **C** 1752 **D** 2242

14 The best estimate for 347 × 65 is:
A 18 000 **B** 21 000
C 24 000 **D** 27 000

15 The best estimate for 1687 × 48 is:
A 75 000 **B** 80 000
C 85 000 **D** 90 000

16 Find $\sqrt{5 \times 5 \times 7 \times 7 \times 11 \times 11}$.
A $5 \times 7 \times 11$
B $5 \times 5 \times 7 \times 7 \times 11 \times 11$
C 5×7
D 3

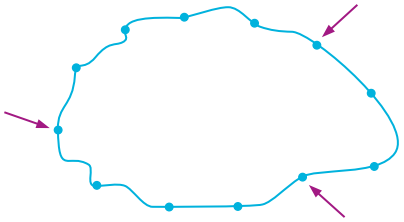
The Diagnostic test questions refer to the Year 7 outcomes from ACMNA123, ACMNA149 and ACMNA150.

A Right-angled triangles

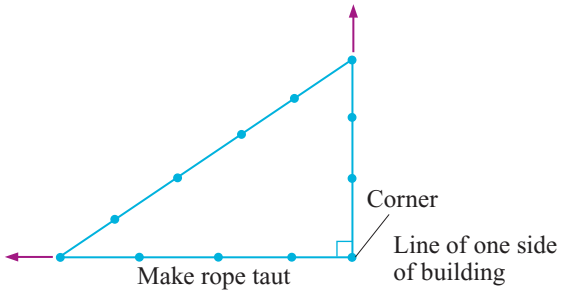
Right-angled corners

The problem of constructing right-angled corners, whether for the corners of buildings or for the division of land into rectangular fields, was overcome at least 5000 years ago.

Without the use of the sophisticated measuring devices available today, the Ancient Egyptians used a rope with 12 equally spaced knots tied along it. The rope was used to form a triangle with sides in the ratio of 3 : 4 : 5. When the rope was pulled tight to form a triangle, a right angle formed at the vertex between the sides of length 3 and 4 units.



Take hold of knots at arrows

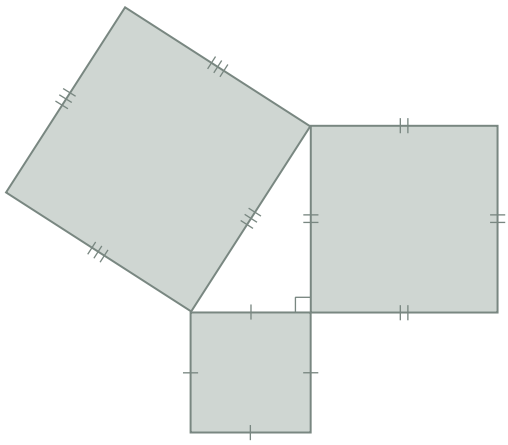


This triangle is, in fact, the simplest right-angled triangle with sides of integer length. You could make your own knotted rope to test this.

Right-angled triangles are frequently observed in real-life situations, as shown below.

Investigation 1 Pythagoras' rule or theorem

1. Draw any right-angled triangle.
2. Construct squares on each of the sides.
3. Measure and calculate the area of each square. Area = (side length)^2.
4. Is the area of the largest square equal to the sum of the areas of the other two squares?
5. Repeat this investigation twice to check your results.



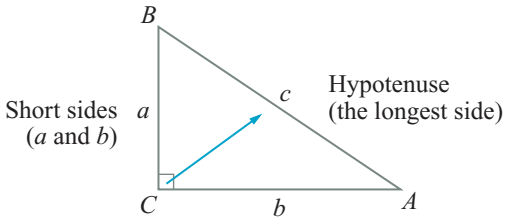
There is a special relationship between the sides of a right-angled triangle, called **Pythagoras' rule** or **theorem** (named after a famous Greek mathematician).

Pythagoras' theorem states that:

$$(\text{hypotenuse})^2 = (\text{short side 1})^2 + (\text{short side 2})^2$$

Thus in $\triangle ABC$: $c^2 = a^2 + b^2$

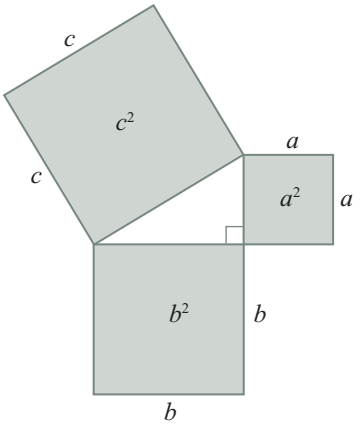
The hypotenuse is the longest side. It is opposite the right angle.



In any right-angled triangle, the square of the longest side is equal to the sum of the squares of the two smaller sides.

In geometric form, Pythagoras' theorem is:

> In any right-angled triangle, the square on the hypotenuse is equal in area to the sum of the areas of the squares on the other two sides, or $c^2 = a^2 + b^2$.

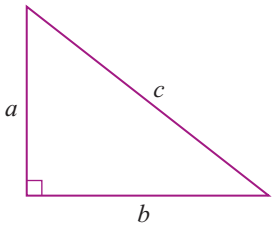


Exercise 10A explores at this relationship.

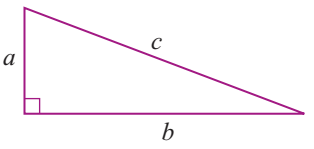
Exercise 10A

1 Measure each of the sides of these right-angled triangles and complete a table like the one below.

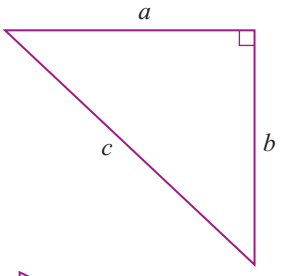
a



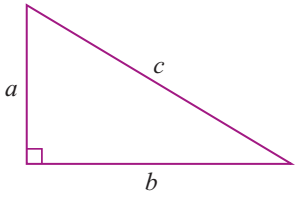
b



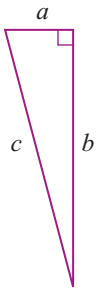
c



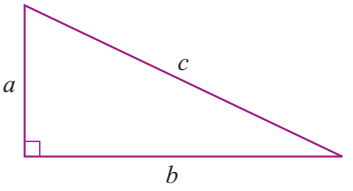
d



e



f



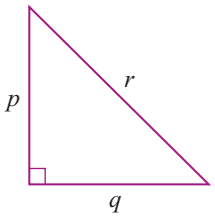
a	b	a^2	b^2	$a^2 + b^2$	$\sqrt{a^2 + b^2}$	c
⋮	⋮	⋮	⋮	⋮	⋮	⋮

What do you notice about the last two columns?

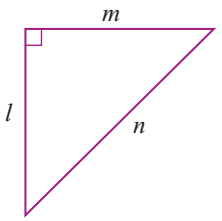
2 Draw three right-angled triangles of your own and add their side lengths to the table.

3 Identify the hypotenuse in each triangle. Remember that the hypotenuse is opposite the right angle, and it is the longest side in a right-angled triangle.

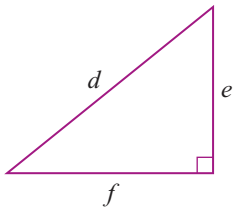
a



b



c



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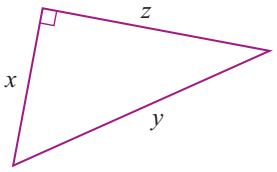
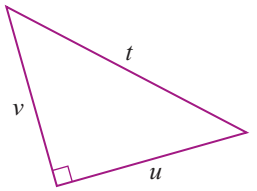


e



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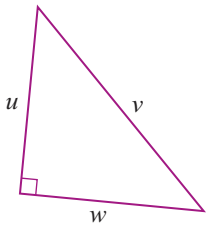
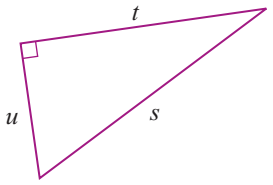


g



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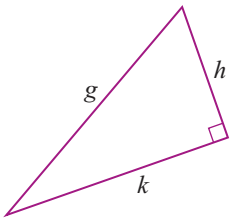
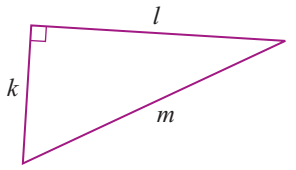


i



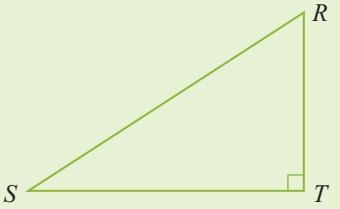
4 Using Pythagoras' theorem, write the relationship between the lengths of the sides of the right-angled triangles in question **3**.

EXAMPLE 1

Write Pythagoras' theorem for the given triangle.

a Use side names.

b Use lower case letters for sides opposite given angles.

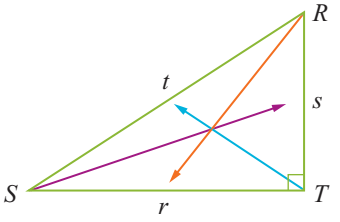


Pythagoras' theorem may be written using side names or by using lower case letters representing the sides opposite the given angles.

a The rule is $RS^2 = RT^2 + TS^2$.

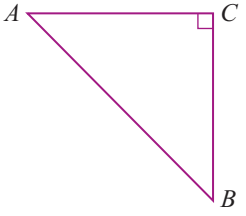
b Label the side opposite T as t, R as r, S as s.

The rule is $t^2 = s^2 + r^2$.

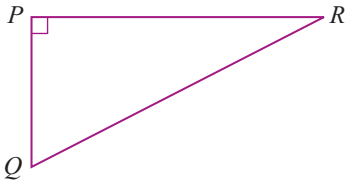


5 Use Pythagoras' theorem to express the equation for these right-angled triangles in two different ways.

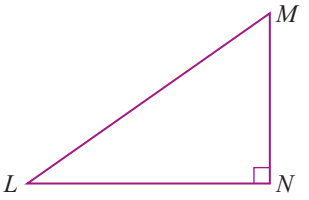
a



b



c



B Converse of Pythagoras' theorem

The converse of Pythagoras' theorem can be used to determine if a triangle is right-angled.

If, in a triangle, the sum of the squares of the two shorter sides is equal to the square of the longest side, then the triangle is right-angled.

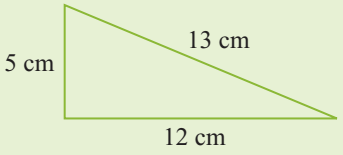
The converse of 'if A is true then B is true' is 'if B is true then A is true'.

- If $a^2 + b^2 = c^2$, then the triangle is right-angled.
- If $a^2 + b^2 \neq c^2$, then the triangle *is not* right-angled.

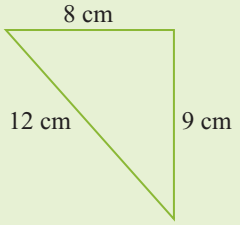
EXAMPLE 1

Determine if each triangle is right-angled by using Pythagoras' theorem.

a



b



a $a^2 + b^2 = 12^2 + 5^2$
$= 144 + 25$
$= 169$
and $c^2 = 13^2$
$= 169$
∴ The triangle is right-angled.

Calculator:

12 x^2 + 5 x^2 = 169

13 x^2 = 169

b $a^2 + b^2 = 8^2 + 9^2$
$= 64 + 81$
$= 145$
and $c^2 = 12^2$
$= 144$
∴ The triangle is not right-angled.

The longest side or measurement is used for the hypotenuse, which is side c.

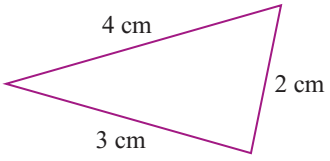
8 x^2 + 9 x^2 = 145

12 x^2 = 144

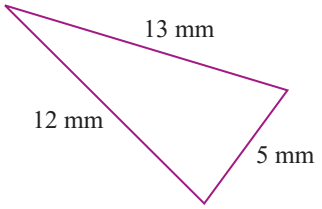
Exercise 10B

1 The following figures are not drawn accurately. Which of the triangles is a right-angled triangle? Show all your working.

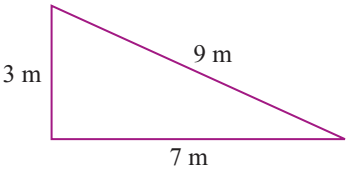
a



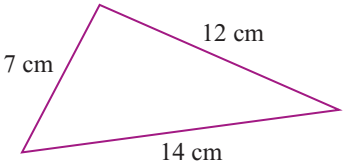
b



c



d



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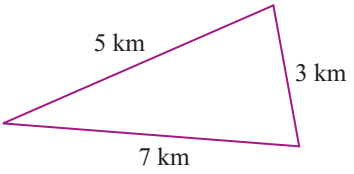
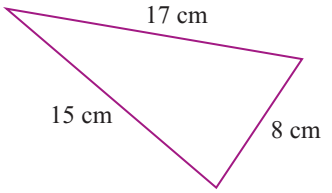
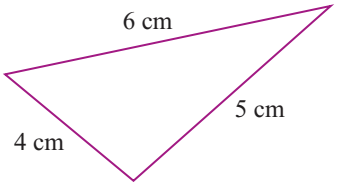


f



g



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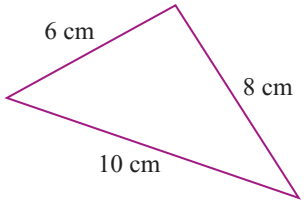
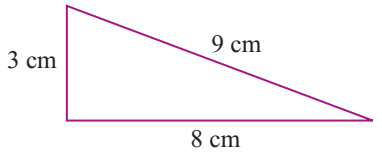


i



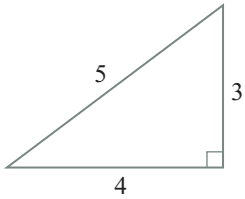
2 Use Pythagoras' theorem to determine if triangles with these side lengths are right-angled.

a 3 cm, 4 cm and 5 cm
b 7 cm, 9 cm and 12 cm
c 11 cm, 13 cm and 18 cm
d 21 cm, 28 cm and 35 cm

Investigation 2 Pythagorean triads

The simplest right-angled triangle with integer side lengths is the 3–4–5 triangle. The numbers satisfy the rule $3^2 + 4^2 = 5^2$.

As the numbers (3, 4, 5) obey the rule $a^2 + b^2 = c^2$ they are called a Pythagorean triad or triple. Other Pythagorean triads include (5, 12, 13), (7, 24, 25) and (8, 15, 17).



Any Pythagorean triad can be used to create other Pythagorean triads. One way of doing this is to multiply each of the three numbers by the same number. So (3, 4, 5) becomes (6, 8, 10) by multiplying each number by 2.

A spreadsheet program may be used to generate tables of Pythagorean triads. Here is an example.

	A	B	C	D	E	F
1	**Pythagorean triples based on 3, 4, 5 triad**					
2	***n***	**3*n***	**4*n***	**5*n***	**Sum of sides squared**	**Longest side squared**
3	1	3	4	5	25	25
4	2	6	8	10	100	100
5	3	9	12	15	225	225
6	4	12	16	20	400	400
7	5	15	20	25	625	625
8	6	18	24	30	900	900

The formulas used in the spreadsheet are shown below.

	A	B	C	D	E	F
1	**Pythagorean triples based on 3, 4, 5 triad**					
2	***n***	**3*n***	**4*n***	**5*n***	**Sum of sides squared**	**Longest side squared**
3	1	= A3*3	= A3*4	= A3*5	= B3*B3 + C3*C3	= D3*D3
4	= A3 + 1	= A4*3	= A4*4	= A4*5	= B4*B4 + C4*C4	= D4*D4
5	= A4 + 1	= A5*3	= A5*4	= A5*5	= B5*B5 + C5*C5	= D5*D5

1 Create your own tables using a spreadsheet and the Pythagorean triads (5, 12, 13), (7, 24, 25) and (8, 15, 17).

2 Use these formulas to create other tables of values.

a $2n + 1, 2n^2 + 2n, 2n^2 + 2n + 1$
b $n, \frac{n^2 - 1}{2}, \frac{n^2 + 1}{2}$, n must be an odd number.

3 Investigate some Pythagorean triads of your own. Write a report about your findings.

C Using Pythagoras' theorem to find the hypotenuse

Pythagoras' theorem is used to determine an unknown side length of a right-angled triangle when two other side lengths are known. Before doing this, practise the calculator steps required.

Exercise 10C

1 Use the x^2 key on your calculator to evaluate the following.

a 3^2	**b** 4^2	**c** 10^2	**d** 8^2
e 11^2	**f** 17^2	**g** 15^2	**h** 22^2
i 0.6^2	**j** 0.5^2	**k** 1.8^2	**l** 2.3^2
m 4.9^2	**n** 5.2^2	**o** 6.83^2	**p** 9.54^2

2 Use the $\sqrt{\ }$ key on your calculator to evaluate the following.

a $\sqrt{25}$	**b** $\sqrt{49}$	**c** $\sqrt{36}$	**d** $\sqrt{144}$
e $\sqrt{169}$	**f** $\sqrt{400}$	**g** $\sqrt{625}$	**h** $\sqrt{1089}$
i $\sqrt{0.04}$	**j** $\sqrt{0.81}$	**k** $\sqrt{1.21}$	**l** $\sqrt{4.41}$
m $\sqrt{13.69}$	**n** $\sqrt{29.16}$	**o** $\sqrt{237.16}$	**p** $\sqrt{400.8004}$

3 Use the $\sqrt{\ }$ key and FIX function on your calculator to round the following as stated.

The word 'surd' is the name for square root numbers that do not work out exactly.

a Round correct to 1 decimal place.

i $\sqrt{18}$	**ii** $\sqrt{7}$	**iii** $\sqrt{23}$	**iv** $\sqrt{82}$

b Round correct to 2 decimal places.

i $\sqrt{215}$	**ii** $\sqrt{386}$	**iii** $\sqrt{436}$	**iv** $\sqrt{721}$

c Round correct to 3 decimal places.

i $\sqrt{0.7}$	**ii** $\sqrt{1.9}$	**iii** $\sqrt{2.85}$	**iv** $\sqrt{6.04}$

d Round correct to 4 decimal places.

i $\sqrt{12.93}$	**ii** $\sqrt{8.062}$	**iii** $\sqrt{156.4}$	**iv** $\sqrt{387.69}$

4 $\sqrt{2} = 1.414\,213\,562$ and $\sqrt{3} = 1.732\,050\,808$ correct to 9 decimal places.

a Write $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{7}$ correct to 9 decimal places.

b Do any of your answers terminate or recur?

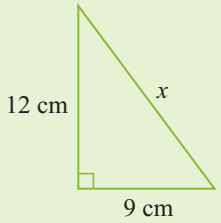
c Is $\sqrt{4}$ irrational? Explain.

Decimal numbers that are both non-terminating and non-recurring are called irrational numbers.

EXAMPLE 1

Calculate the length of the hypotenuse in each right-angled triangle.

a



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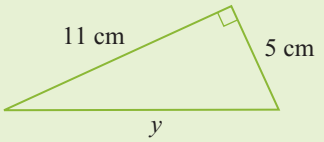

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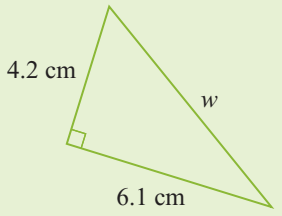


EXAMPLE 1 CONTINUED

a $a^2 + b^2 = c^2$

$12^2 + 9^2 = x^2$

$144 + 81 = x^2$

$\therefore 225 = x^2$

$\sqrt{225} = x$

$\therefore x = 15$

The hypotenuse is 15 cm.

b $a^2 + b^2 = c^2$

$11^2 + 5^2 = y^2$

$121 + 25 = y^2$

$\therefore 146 = y^2$

$\therefore y = \sqrt{146}$

The hypotenuse is $\sqrt{146}$ cm (surd form).

c $a^2 + b^2 = c^2$

$4.2^2 + 6.1^2 = w^2$

$17.64 + 37.21 = w^2$

$\therefore 54.85 = w^2$

$\sqrt{54.85} = w$

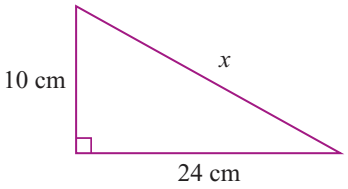
$\therefore w = 7.4$

The hypotenuse is 7.4 cm (1 decimal place).

Notice that when calculating the 'unknown' side, the answer may be a whole number, expressed as a surd or as a decimal correct to a certain number of decimal places.

5 Use Pythagoras' theorem to calculate the length of the hypotenuse. (All answers are integers.)

a



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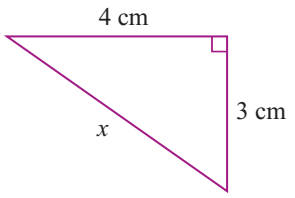

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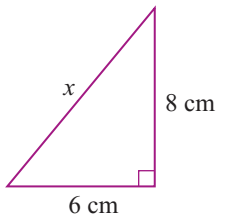

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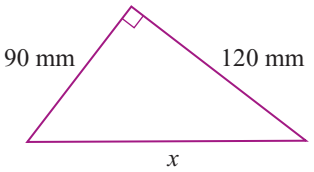

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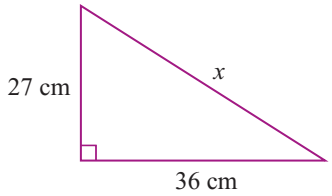

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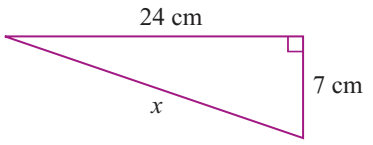

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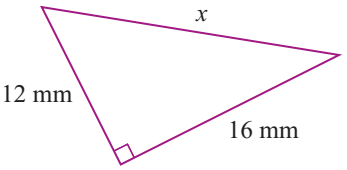

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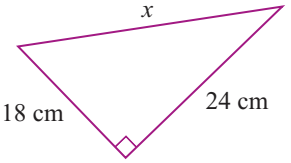
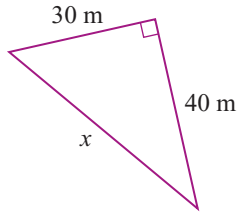
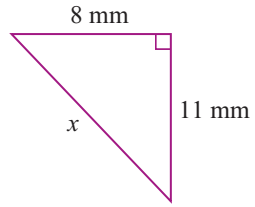


i



6 Calculate the length of the hypotenuse expressing your answer in surd ($\sqrt{\ }$) form.

a



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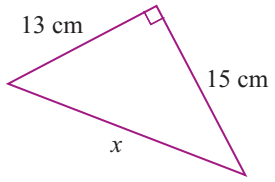

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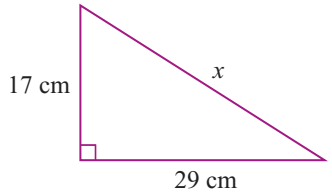

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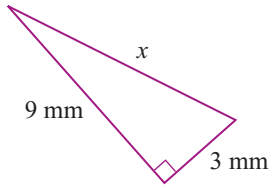

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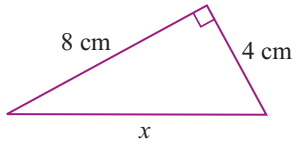

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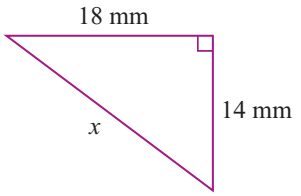

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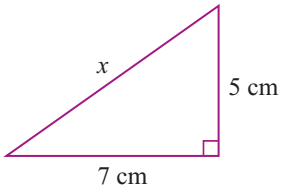

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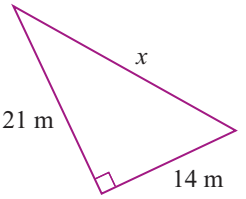

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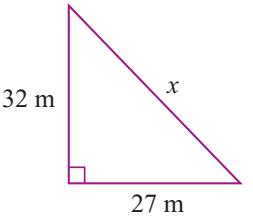
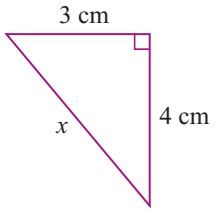


7 Find the length of the hypotenuse in each right-angled triangle correct to 1 decimal place.

a



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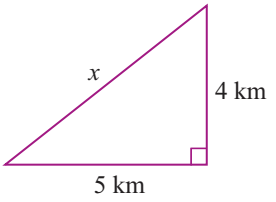

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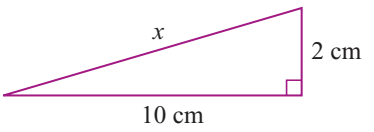

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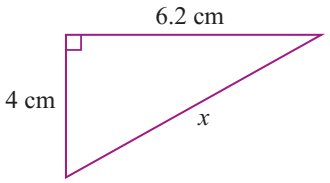

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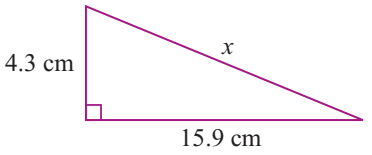

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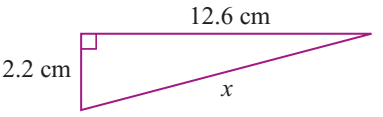

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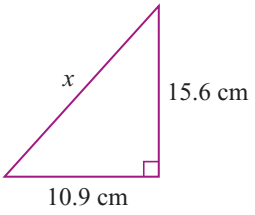

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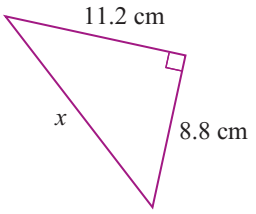

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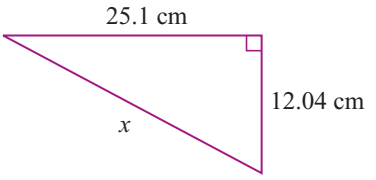
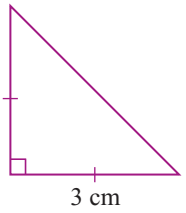


8 Find, correct to 3 decimal places, the length of the hypotenuse in each of these isosceles triangles.

a



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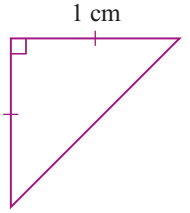

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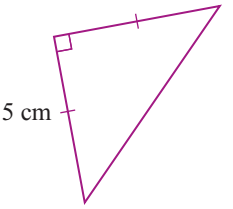
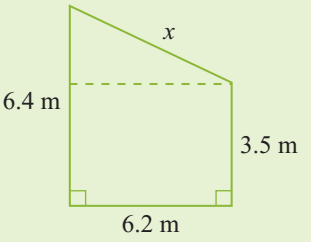


EXAMPLE 2

Calculate the value of x in the diagram correct to 1 decimal place.



Look at the shaded triangle.

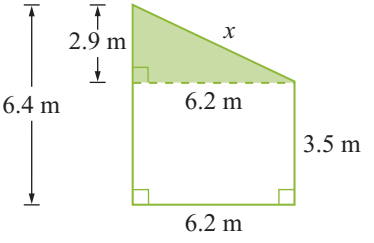
Short side of triangle $= 6.4 - 3.5 = 2.9$ m.

$c^2 = a^2 + b^2$

$x^2 = 2.9^2 + 6.2^2 = 46.85$

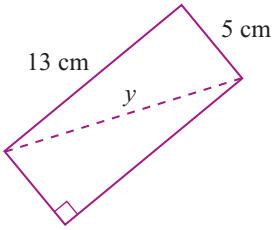
$\therefore x = \sqrt{46.85} = 6.8447\ldots$

$x = 6.8$ (1 decimal place)



9 Calculate the value of each pronumeral correct to 1 decimal place.

a



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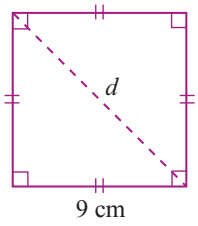

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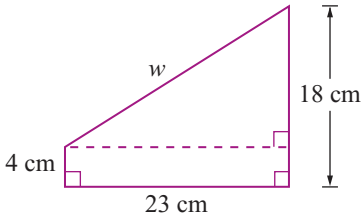

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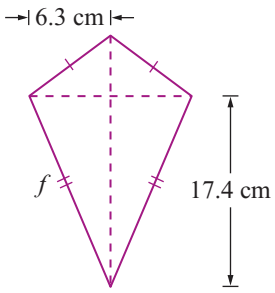

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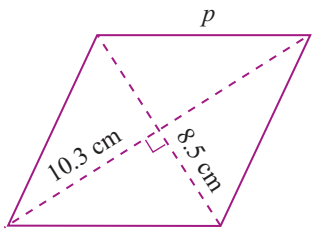

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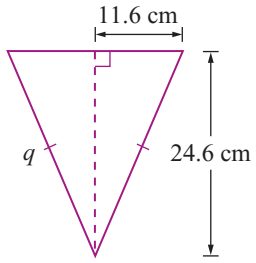
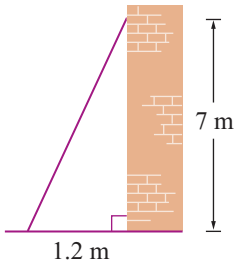
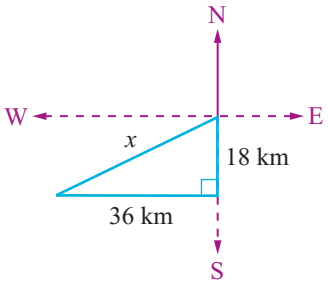


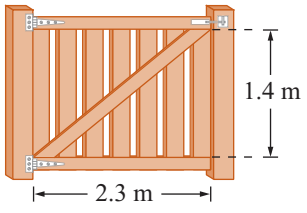
10 The foot of a ladder is placed 1.2 m from a vertical wall. If the ladder reaches 7 m up the wall, what is the length of the ladder to the nearest centimetre?



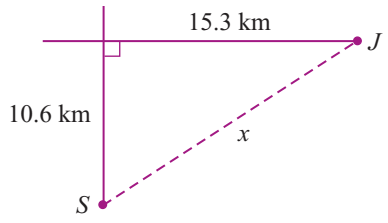
11 A ship sails 18 km due south and then 36 km due west. How far is the ship from its starting point correct to 1 decimal place?



12 A gate is 2.3 m wide with a height of 1.4 m. Calculate the length of the diagonal of the gate correct to 1 decimal place.



13 Two roads meet at right angles. From the intersection, John rides 15.3 km along one road while Sharee rides 10.6 km along the other. Find how far apart they are correct to 1 decimal place.



D Using Pythagoras' theorem to find the shorter sides

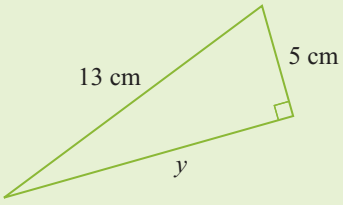
Pythagoras' theorem can also be used to calculate the length of either shorter side as shown in Example 1 below.
Rearrange the rule $c^2 = a^2 + b^2$ by subtracting b^2 from both sides to give $a^2 = c^2 - b^2$.
Use this to find the length of a short side.

Don't forget to find the square root at the end.

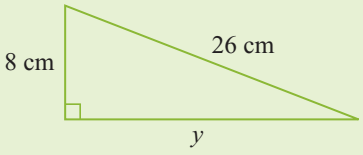
EXAMPLE 1

Calculate the value of the pronumeral y that marks one of the shorter sides to the level of accuracy stated.

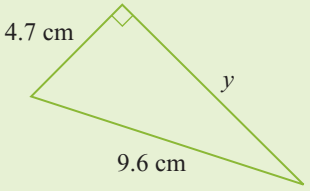
a



b surd form



c correct to 1 decimal place



a $a^2 = c^2 - b^2$
$y^2 = 13^2 - 5^2$
$= 169 - 25$
$= 144$
$\therefore y = \sqrt{144}$
$= 12$ cm

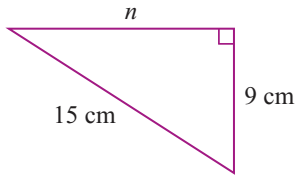
b $a^2 = c^2 - b^2$
$y^2 = 26^2 - 8^2$
$= 676 - 64$
$= 612$
$\therefore y = \sqrt{612}$ cm (surd form)

c $a^2 = c^2 - b^2$
$y^2 = 9.6^2 - 4.7^2$
$= 92.16 - 22.09$
$= 70.07$
$\therefore y = \sqrt{70.07}$
$= 8.4$ cm (1 decimal place)

Exercise 10D

1 Find the value of the pronumeral in each triangle. All answers are integers.

a



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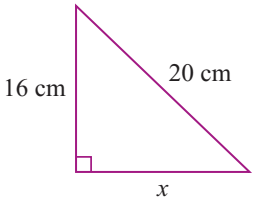

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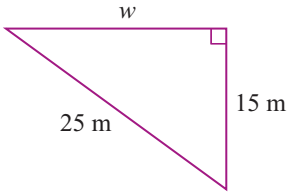

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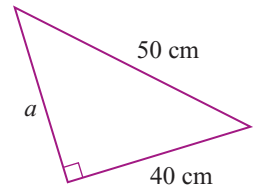

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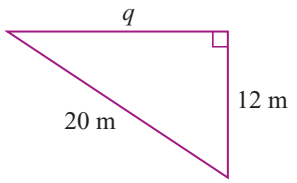

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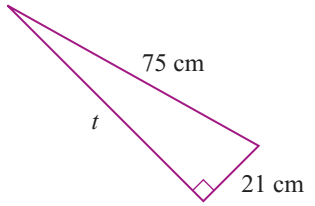

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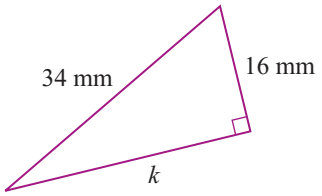

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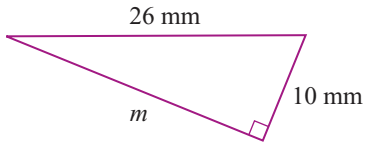

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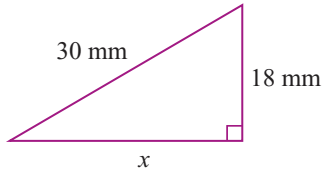
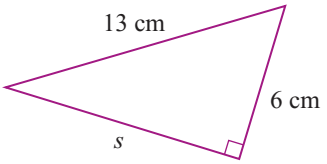


2 Find the length of the unknown short side giving your answer as a surd ($\sqrt{\ }$).

a



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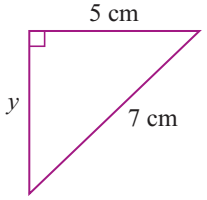

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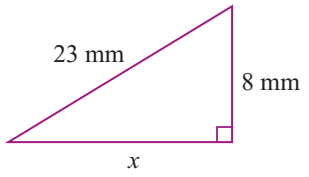

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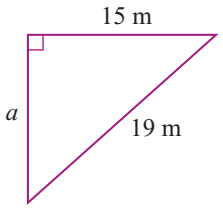

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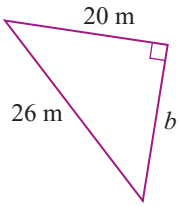

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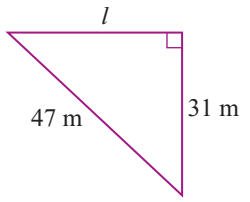

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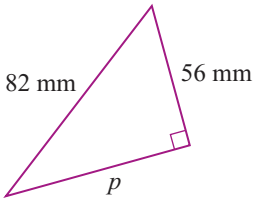

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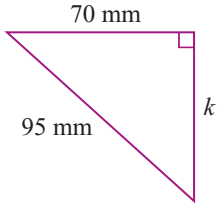

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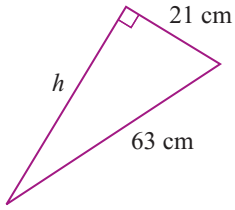
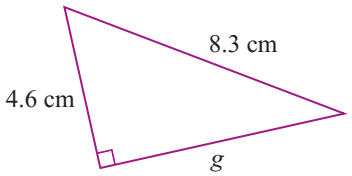


3 Find the length of the unknown short side correct to 1 decimal place.

a



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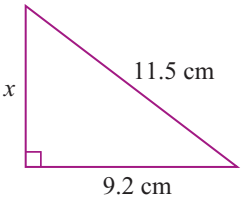

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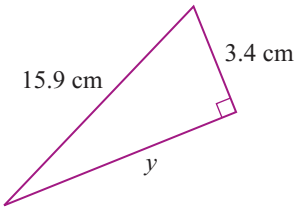

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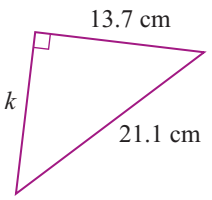

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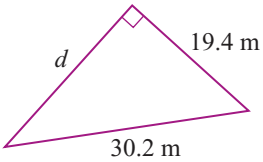

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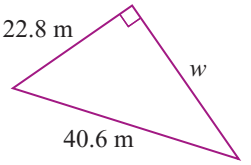
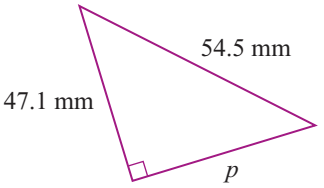
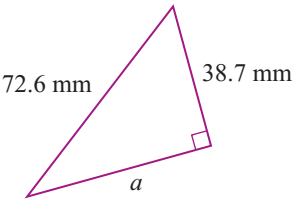


g



h



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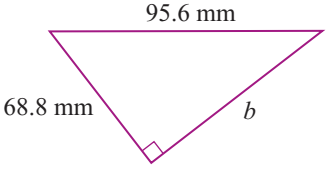
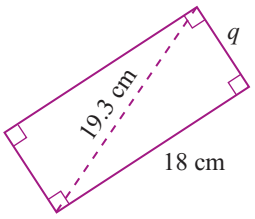


4 Find the value of each pronumeral correct to 2 decimal places.

a



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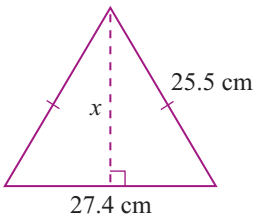

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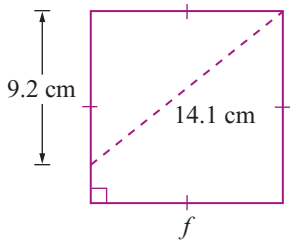

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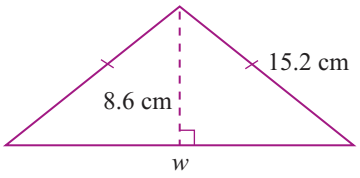

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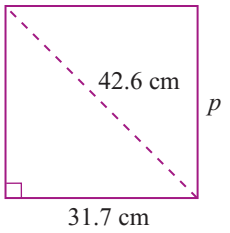

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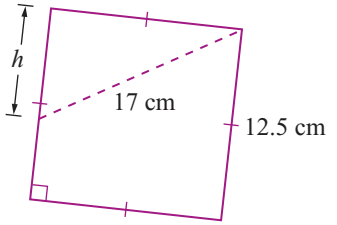

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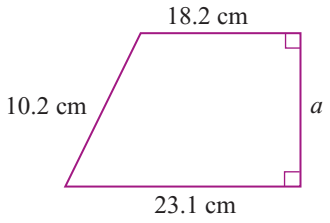

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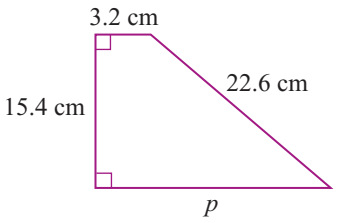

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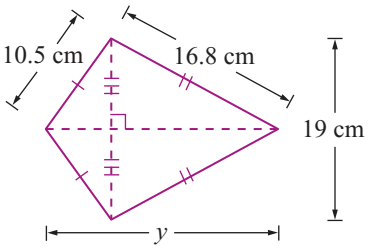
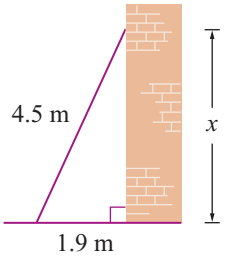
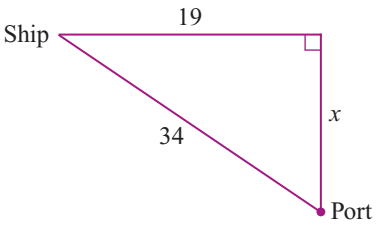


5 A ladder 4.5 m in length is placed 1.9 m from the base of a vertical wall. How far does the ladder reach up the wall to the nearest centimetre?



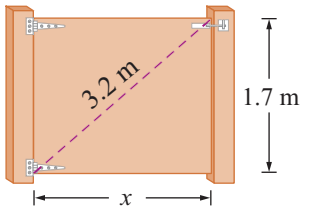
6 The diagram shows that a ship sails 34 nautical miles from its port. It then sails 19 nautical miles due east, so that it is directly north of the port. How far is the ship from the port?



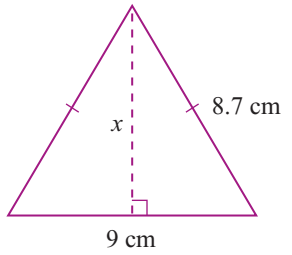
7 A gate has a height of 1.7 m and a diagonal of 3.2 m.

a Find the width of the gate to the nearest centimetre.

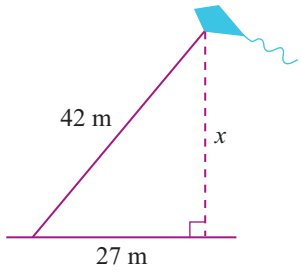
b Calculate the area of the piece of wood used to make the gate.



8 For this triangle, use the dimensions given to find the length of the altitude of the triangle from the apex to the base to the nearest millimetre.



9 A kite is flying with 42 m of string which is anchored to the ground. If the horizontal distance from where the string is anchored is 27 m, find the height of the kite above the ground to the nearest centimetre.



E Problem solving using Pythagoras' theorem

- Draw a neat, clear diagram of the situation.
- Mark on the diagram known lengths and right angles.
- Use a symbol, such as x, to represent the unknown length.
- Decide whether you are finding the hypotenuse or one of the short sides.
- Write down the Pythagorean theorem for the given situation.
- Write your answer in sentence form (where necessary).

Exercise 10E

Give your answers to 2 decimal places as necessary.

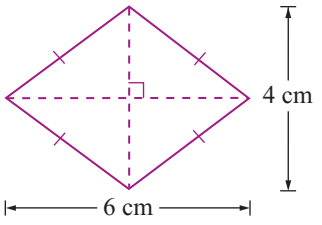
1 Find the length of a diagonal of a 12 cm by 12 cm square.

2 Find the length of a diagonal of a 7 cm by 11 cm rectangle.

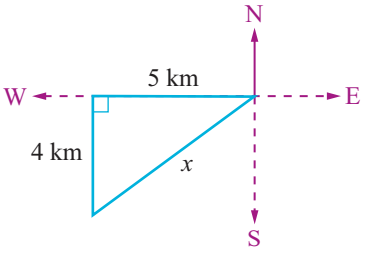
3 What is the longest length of iron rod that can be placed on the floor of a 4 m by 5 m garden shed?

4 A gate has height 1.2 m and diagonal 2.3 m. How wide is the gate?

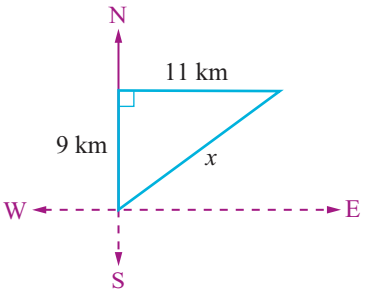
5 A rhombus has diagonals of length 4 cm and 6 cm. Find the length of its sides.



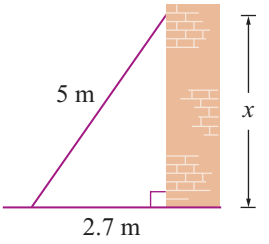
6 A ship sails 5 km due west and then 4 km due south. How far is the ship from its starting point?



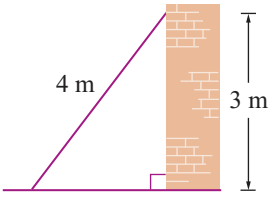
7 A ship sails 9 km north then 11 km east. How far is the ship from its starting point?



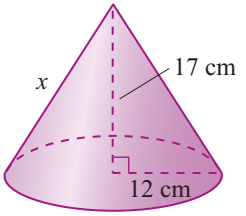
8 A ladder 5 m long is placed against a wall. It is 2.7 m from the wall. How far up the wall does the ladder reach?



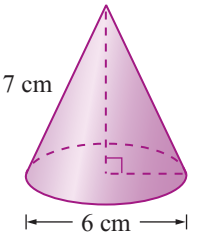
9 A ladder is 4 m long and reaches 3 m up the wall. How far is the foot of the ladder from the base of the wall?



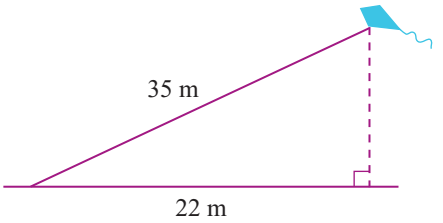
10 A cone has height 17 cm and radius 12 cm. Calculate the slant height.



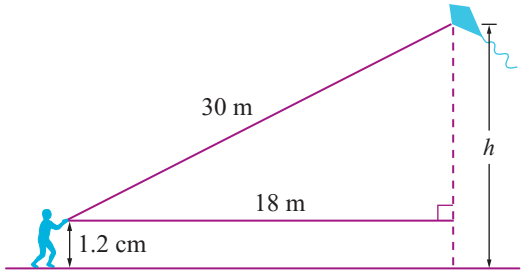
11 A cone has slant height of 7 cm and the diameter of its base is 6 cm. Find the height of the cone given that the cone is symmetrical.



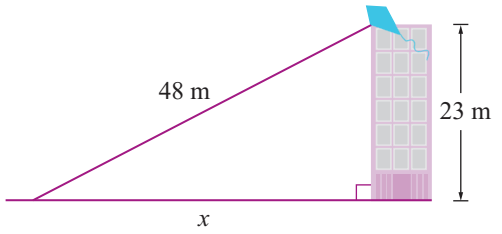
12 A kite is flying with 35 m of string. The string is anchored to the ground. If the horizontal distance from where the string is anchored is 22 m, find the height of the kite above the ground.



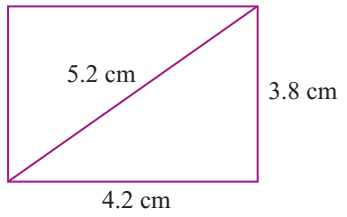
13 Oliver holds the end of the string of a kite 1.2 m above the ground. The string is 30 m long and the horizontal distance to the kite is 18 m. Find the height of the kite above the ground.



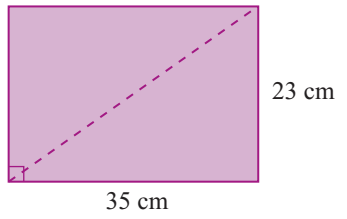
14 A kite is flying with 48 m of string let out. At this time it is level with the top of a 23 m tall building. What is the horizontal distance from the kite to the end of the string?



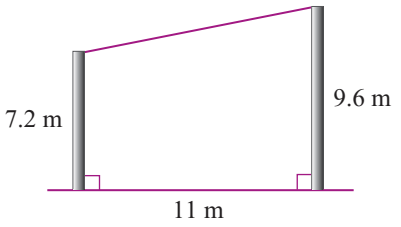
15 Korabita measures out an area of ground to be concreted as shown in the diagram. To check the floor is rectangular he measures the diagonal to be 5.2 m. Is the area measured a rectangle? Explain your answer.



16 The size of a computer screen size is the measure of the diagonal rounded to the nearest centimetre. What size would be quoted for the computer screen shown in the diagram?

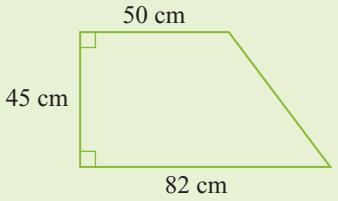


17 Two posts are 7.2 m tall and 9.6 m tall and 11 m apart on level ground. Calculate the length of string required to stretch from the top of one post to the top of the other.



EXAMPLE 1

Find the perimeter of this plane figure to the nearest centimetre.

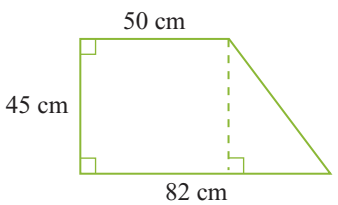


Draw a perpendicular line to make a right-angled triangle.
The triangle has height 45 cm and length $82 - 50 = 32$ cm.
Use Pythagoras' theorem to find the length of the hypotenuse.

$c^2 = 45^2 + 32^2 = 3049$

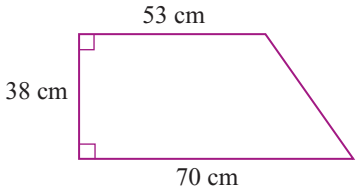
$c = \sqrt{3049} = 55.2$

Perimeter $= 82 + 45 + 50 + 55.2 = 232.2 = 232$ cm

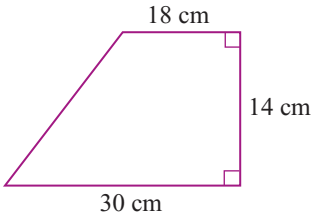


18 Find the perimeters of these figures correct to the nearest centimetre.

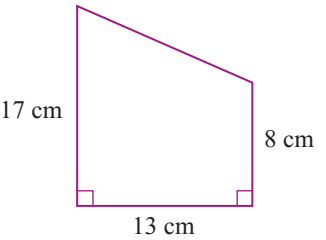
a



b



c



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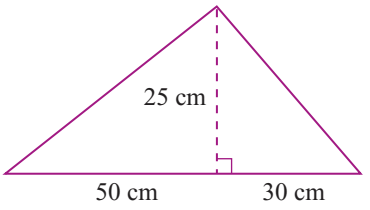

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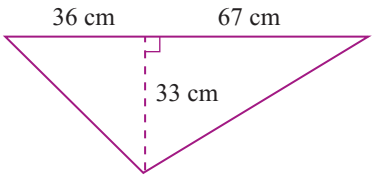

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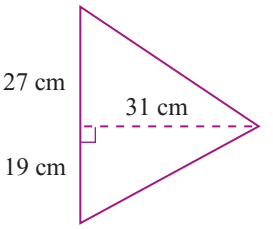

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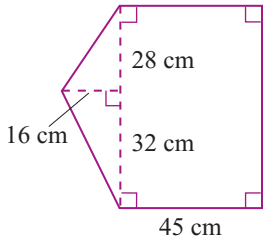

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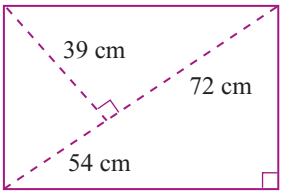

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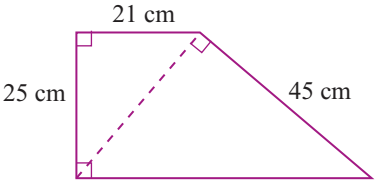
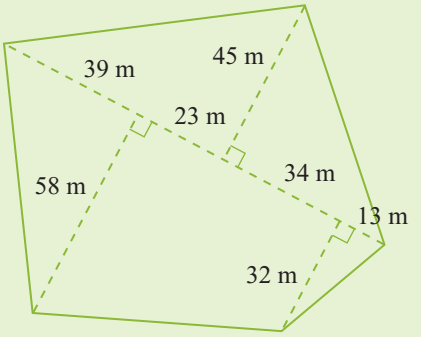


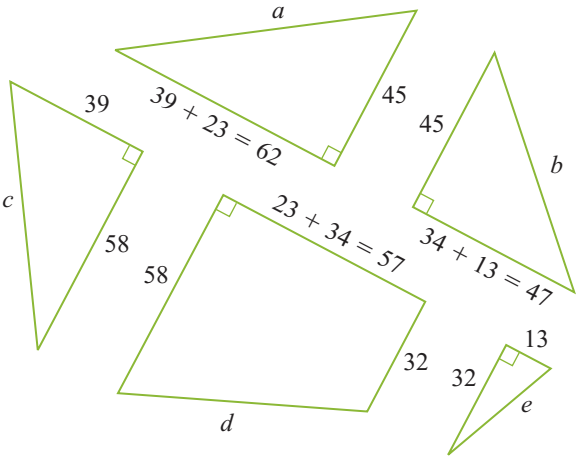
EXAMPLE 2

A surveyor makes the following measurements of a field. By using Pythagoras' theorem to find the missing side lengths, calculate the perimeter.



Split the figure into a number of triangles and a trapezium and write the dimensions on each known side. Write a letter on each side that needs to be calculated.

$a^2 = 45^2 + 62^2$
$= 5869$
$\therefore a = \sqrt{5869} = 76.6$
$b^2 = 45^2 + 47^2$
$= 4234$
$\therefore b = \sqrt{4234} = 65.1$
$c^2 = 39^2 + 58^2$
$= 4885$
$\therefore c = \sqrt{4885} = 69.9$
$e^2 = 32^2 + 13^2$
$= 1193$
$\therefore e = \sqrt{1193} = 34.5$



EXAMPLE 2 CONTINUED

Calculate d using the technique from the Example 1.

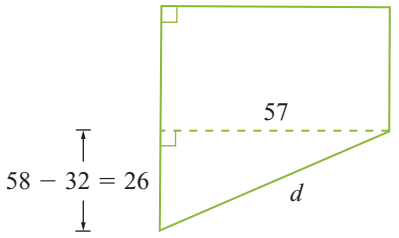
$d^2 = 57^2 + 26^2 = 3925$

$\therefore d = \sqrt{3925} = 62.6$

Perimeter $= 76.6 + 65.1 + 69.9 + 34.5 + 62.6$

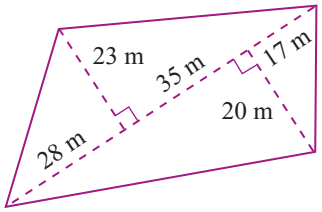
$= 308.7$ m

≈ 309 m

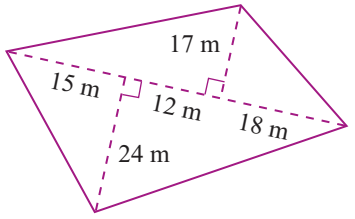


19 A surveyor makes the following measurements of a field. Use Pythagoras' theorem to find the missing side lengths and calculate the perimeter.

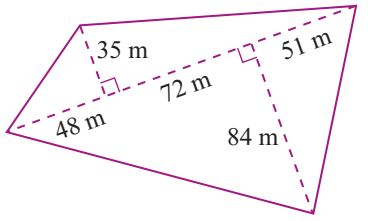
a



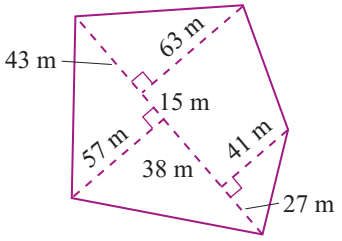
b



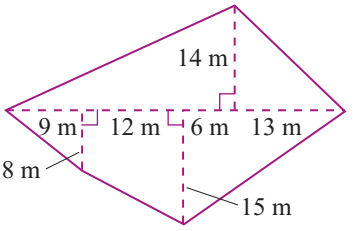
c



d



e



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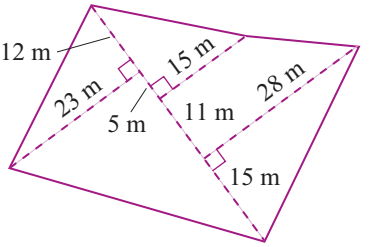
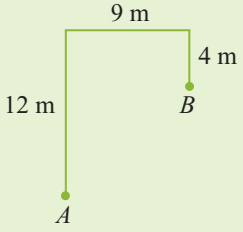


Extension

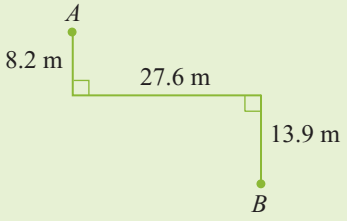
EXAMPLE 3

Find the distance AB correct to 2 decimal places.

a

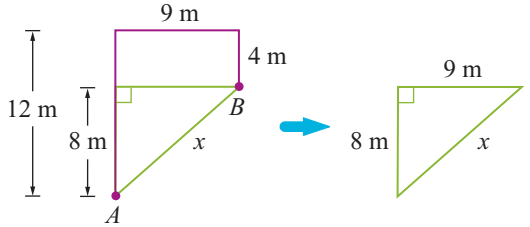


b



Joint the points A and B to form a right-angled triangle.

a



$a^2 + b^2 = c^2$

$9^2 + 8^2 = x^2$

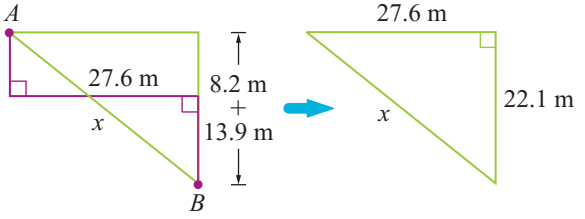
$81 + 64 = x^2$

$x^2 = 145$

$\therefore x = \sqrt{145} = 12.04$

AB is 12.04 m (2 decimal places)

b



$a^2 + b^2 = c^2$

$22.1^2 + 27.6^2 = x^2$

$488.41 + 761.76 = x^2$

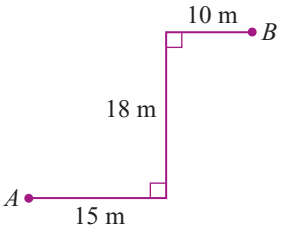
$x^2 = 1250.17$

$\therefore x = \sqrt{1250.17} = 35.36$

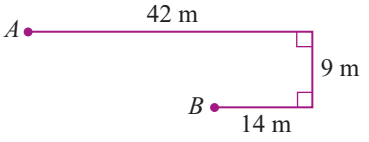
AB is 35.36 m (2 decimal places)

20 Find the distance AB correct to 2 decimal places.

a



b



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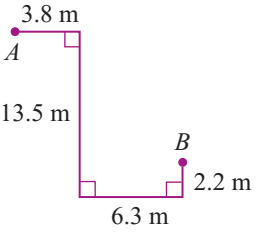
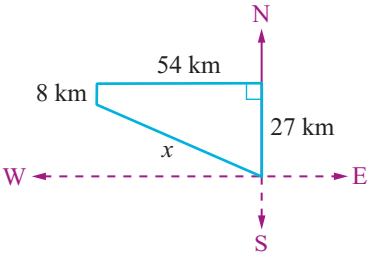
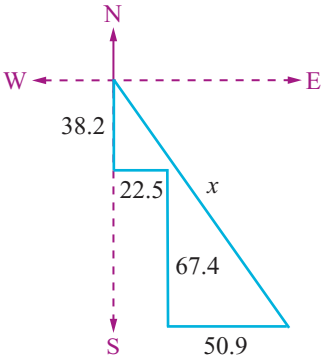


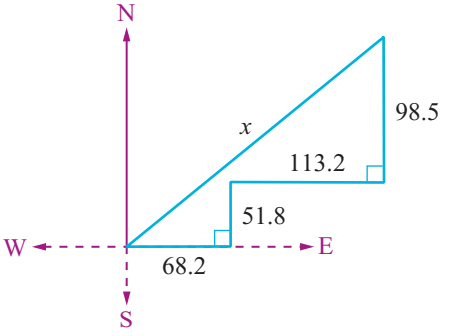
21 A cyclist rides 27 km north, then turns directly west and cycles 54 km, and finally turns south and rides a further 8 km. Calculate the cyclist's distance from his starting point, giving the answer to the nearest metre.



22 A ship sails 38.2 km due south then turns directly east and sails a further 22.5 km. From this point it is decided that the ship is to sail a further 67.4 km due south and then finally sail directly east for a further 50.9 km. Calculate the ship's distance from its original starting point correct to 1 decimal place.



23 A ship sails 68.2 km due east then turns directly north and sails a further 51.8 km. From this point the ship sails a further 113.2 km east and then turns directly north and sails 98.5 km. Calculate the ship's distance from its original starting point correct to 1 decimal place.



Investigation 3 Square numbers

1 Find 3×3 and -3×-3. Compare the answers.

2 Find these square numbers.

a 4×4 and -4×-4

b 5×5 and -5×-5

c -6×-6 and 6×6

d -10×-10 and 10×10

3 What do you notice about the answers to each part in question **2**?

4 The solution to the equation $x^2 = 49$ is a number that when multiplied by itself gives 49. What are the two answers?

5 The solution to $x^2 = 64$ is $x = 8$ or -8. Explain why there are two answers.

6 Are there always two solutions for $x^2 = c$ where c is a number? Consider when $x > 0$ and when $x < 0$.

F Simple quadratic equations

From Investigation 3 there are two solutions to the equation $x^2 = c$.

This equation is called a quadratic equation because the variable x has a power of 2.

EXAMPLE 1

Solve these equations.

a $x^2 = 25$

b $x^2 = 169$

a $x^2 = 25$

$x = \pm\sqrt{25}$ take the square root of both sides

$= \pm 5$

b $x^2 = 169$

$x = \pm\sqrt{169}$

$= \pm 13$

± is the symbol for plus or minus.

Exercise 10F

1 Solve these quadratic equations.

a $x^2 = 9$ **b** $x^2 = 16$ **c** $x^2 = 64$ **d** $x^2 = 144$

e $x^2 = 49$ **f** $x^2 = 121$ **g** $x^2 = 36$ **h** $x^2 = 81$

i $x^2 = 100$ **j** $x^2 = 4$ **k** $x^2 = 225$ **l** $x^2 = 289$

EXAMPLE 2

Solve these equations.

a $x^2 = 10$

b $x^2 = 43$

a $x^2 = 10$

$x = \pm\sqrt{10}$

$= \pm 3.16$ (2 decimal places)

b $x^2 = 43$

$x = \pm\sqrt{43}$

$= \pm 6.56$ (2 decimal places)

2 Solve these equations, giving your answers correct to 2 decimal places.

a $x^2 = 12$	**b** $x^2 = 51$	**c** $x^2 = 19$
d $x^2 = 47$	**e** $x^2 = 83$	**f** $x^2 = 28$
g $x^2 = 68$	**h** $x^2 = 91$	**i** $x^2 = 193$
j $x^2 = 200$	**k** $x^2 = 155$	**l** $x^2 = 357$

Extension

EXAMPLE 3

Solve these equations.

a $5x^2 = 80$ **b** $3x^2 = 75$

a
$$5x^2 = 80$$
$$\frac{5x^2}{5} = \frac{80}{5}$$
$$x^2 = 16$$
$$x = \pm\sqrt{16}$$
$$= \pm 4$$

b
$$3x^2 = 75$$
$$\frac{3x^2}{3} = \frac{75}{3}$$
$$x^2 = 25$$
$$x = \pm\sqrt{25}$$
$$= \pm 5$$

Quadratic equations have two answers.

3 Solve these equations.

a $2x^2 = 18$	**b** $5x^2 = 180$	**c** $8x^2 = 72$
d $3x^2 = 48$	**e** $7x^2 = 175$	**f** $10x^2 = 160$
g $7x^2 = 252$	**h** $6x^2 = 294$	**i** $5x^2 = 125$
j $3x^2 = 507$	**k** $7x^2 = 112$	**l** $6x^2 = 726$

Extension

EXAMPLE 4

Solve these equations.

a $9x^2 = 25$ **b** $81x^2 = 49$

a
$$9x^2 = 25$$
$$\frac{9x^2}{9} = \frac{25}{9}$$
$$x^2 = \frac{25}{9}$$
$$x = \pm\frac{\sqrt{25}}{\sqrt{9}}$$
$$= \pm\frac{5}{3}$$

b
$$81x^2 = 49$$
$$\frac{81x^2}{81} = \frac{49}{81}$$
$$x^2 = \frac{49}{81}$$
$$x = \pm\frac{\sqrt{49}}{\sqrt{81}}$$
$$= \pm\frac{7}{9}$$

4 Solve these equations.

a $4x^2 = 49$	**b** $100x^2 = 81$	**c** $25x^2 = 16$
d $81x^2 = 16$	**e** $49x^2 = 144$	**f** $121x^2 = 64$
g $144x^2 = 49$	**h** $81x^2 = 100$	**i** $64x^2 = 25$
j $169x^2 = 121$	**k** $162x^2 = 98$	**l** $108x^2 = 75$

Extension

EXAMPLE 5

Solve these equations.

a $3x^2 = 21$ **b** $5x^2 = 12$

a $3x^2 = 21$

$\frac{3x^2}{3} = \frac{21}{3}$

$x^2 = 7$

$x = \pm\sqrt{7}$

$= \pm 2.65$ (2 decimal places)

b $5x^2 = 12$

$\frac{5x^2}{5} = \frac{12}{5}$

$x^2 = \frac{12}{5}$

$x = \pm\sqrt{\frac{12}{5}}$

$= \pm 1.55$ (2 decimal places)

5 Solve these equations. giving answers correct to 2 decimal places if necessary.

a $7x^2 = 56$ **b** $4x^2 = 12$ **c** $11x^2 = 66$

d $13x^2 = 47$ **e** $7x^2 = 18$ **f** $5x^2 = 23$

Investigation 4 Square roots (extension)

Suppose the square root is the only key on your calculator that does not work. How can $\sqrt{5}$ be determined? Following is a method.

Step 1: Make an estimate of $\sqrt{5}$ and call it a.

Step 2: Find the average of a and $\frac{5}{a}$, and call it b.

Step 3: Find the average of b and $\frac{5}{b}$, and call it c.

Step 4: And so on.

For example, suppose $\sqrt{5} \approx 2$; that is, $a = 2$.

$\therefore b = \frac{1}{2}\left(2 + \frac{5}{2}\right) = 2.25$

$\therefore c = \frac{1}{2}\left(2.25 + \frac{5}{2.25}\right) \approx 2.2361$

$\therefore d =$ ___

$\therefore e =$ ___

$\therefore f =$ ___

1 Find d, e and f in the above.

2 Find $\sqrt{5}$ from your calculator. Does the method appear to work?

3 Start with $a = 3$ and find b, c, d, e and f. Does $\sqrt{5}$ result?

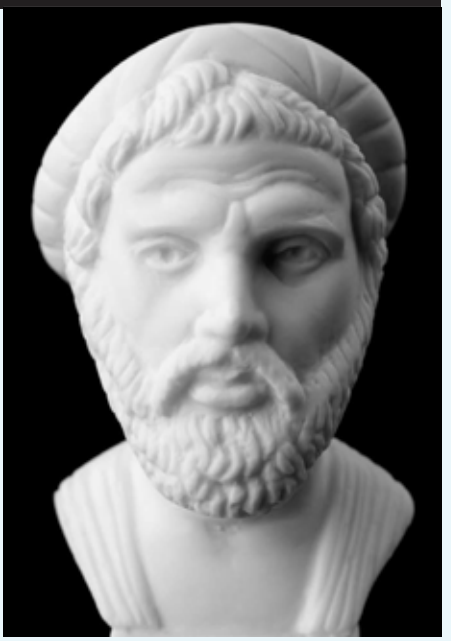
4 Can you explain why the method works?
Hint: $\sqrt{5}$ is a solution of the equation $x^2 = 5$, and notice that as the process continues $b \approx c$, $c \approx d$, $d \approx e$, …; that is, $x \approx \frac{1}{2}\left(x + \frac{5}{x}\right)$.

5 Explain how you would modify the above method to calculate:

a $\sqrt{7}$ **b** $\sqrt{k}$

Language in mathematics

Pythagoras

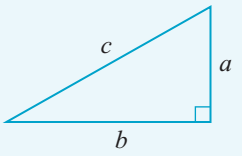


Pythagoras (approximately 580 BCE to 500 BCE) was born in Samos, Greece, and became a famous mathematician and philosopher. He founded the Pythagorean brotherhood, a religious order that influenced the development of mathematics and philosophy. The motto of this order was 'All is number'. As none of his original writings have survived, it is difficult to determine which work was done by Pythagoras and which is a result of the work of some of his followers.

The Pythagorean school of thought was applied not only to areas of mathematics but also to astronomy, music theory and acoustics among other topics. In fact, it was recognised by Copernicus that some of Pythagoras' ideas on astronomy helped him develop his theories on the rotation of the planets around the Sun.

The study of square numbers (1, 4, 9, 16, …) and rectangular numbers (2, 6, 12, 20, …) is credited to the Pythagorean school of study.

However, the best known of Pythagoras' mathematical ideas is his theory relating the lengths of the sides of a right-angled triangle. For a triangle as shown in the diagram, $a^2 + b^2 = c^2$.



Whether or not Pythagoras was himself directly responsible for some of the ideas credited to him is not particularly important. What is evident is that, as a result of his teaching and motivation, people were inspired to think creatively about mathematics, philosophy and associated areas such as music and astronomy.

A proof of Pythagoras' theorem

In the era of Pythagoras, algebra was almost non-existent and so it is likely that he discovered the relationship between the sides of a right-angled triangle by geometric means, such as the one that follows.

Consider a square that has been subdivided into four identical (congruent) triangles and a smaller square, as shown in Figure 1. We realign the four triangles as in Figure 2.

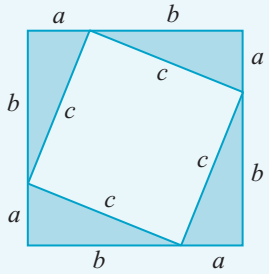


Figure 1

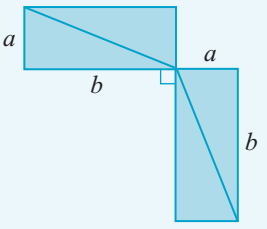


Figure 2

Now complete a square as in Figure 3. The original square and the final square are equal in size. (Both have sides of length $a + b$.)

Finally remove the four triangles from both Figure 1 and Figure 3. The remaining areas are equal.

Hence, $a^2 + b^2 = c^2$.

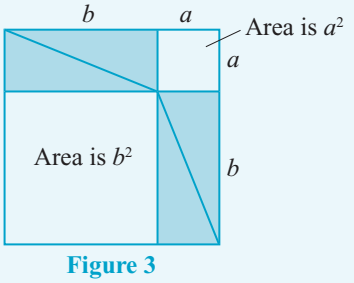


Figure 3

Answer these questions after reading the information on Pythagoras.

1 Approximately how old was Pythagoras when he died?

2 Who was Pythagoras? What was his foundation?

3 Why is it difficult to determine the precise findings of Pythagoras?

4 Into what other areas did the Pythagorean school of thought apply?

5 What benefits were derived from Pythagoras' teachings?

Recall that even before Pythagoras, the 3–4–5 triangle was used to construct right-angled corners for buildings.

6 In your own words, explain the proof of Pythagoras' theorem.

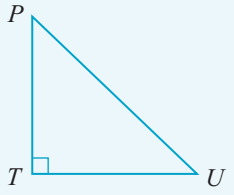
7 Research a famous mathematician. Write a two-page report on the contributions and discoveries of the mathematician of your choice.

Terms

diagonal	hypotenuse	Pythagoras' theorem	Pythagorean triad	right-angled triangle
side	square number	square root	surd	triangle

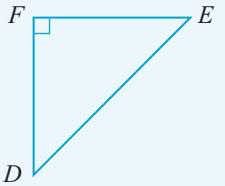
Check your skills

1 An expression for Pythagoras' theorem in this triangle is:



A $PU^2 + TU^2 = PT^2$
B $PT^2 + TU^2 = PU^2$
C $PU^2 + TP^2 = UT^2$
D $TU^2 + PU^2 = PT^2$

2 An expression for Pythagoras' theorem in this triangle is:

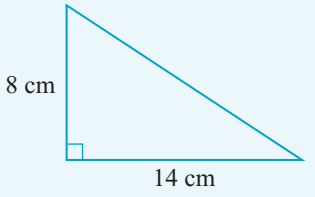


A $d^2 + e^2 = f^2$
B $e^2 + f^2 = d^2$
C $f^2 + e^2 = d^2$
D $d^2 + f^2 = e^2$

3 Which of the triangles with sides listed is not right-angled?

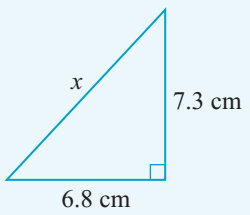
A 6 cm, 8 cm, 10 cm
B 10 cm, 24 cm, 26 cm
C 5 cm, 12 cm, 13 cm
D 9 cm, 12 cm, 18 cm

4 The length of the hypotenuse of this triangle is:



A 11.5 cm
B 16.1 cm
C 132 cm
D 260 cm

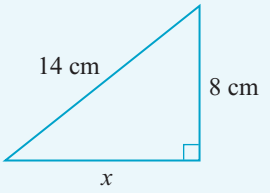
5 The length of the side marked x is closest to:



A 99.53 cm
B 9.98 cm
C 7.05 cm
D 2.7 cm

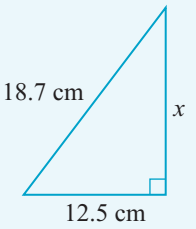
6 The length of the side marked x is closest to:

A 132 cm **B** 260 cm
C 16.1 cm **D** 11.5 cm



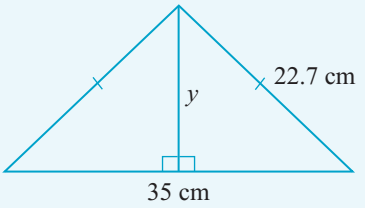
7 The length of the side marked x is closest to:

A 193.44 cm **B** 13.9 cm
C 505.94 cm **D** 22.5 cm

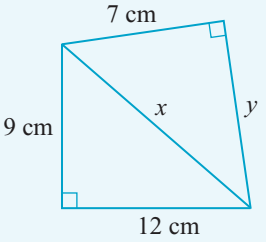


8 The value of y in the diagram is closest to:

A 40.2 **B** 26.6
C 28.66 **D** 14.5



Use this diagram to answer questions **9** and **10**.



9 The value of x is closest to:

A 15 **B** 21
C 3.9 **D** 4.58

10 The value of y is closest to:

A 19.80 **B** 16.55 **C** 13.3 **D** 14

11 The side length of a square with diagonal 15 cm is closest to:

A 21.2 cm **B** 10.6 cm **C** 3.87 cm **D** 5.48 cm

12 What is the solution to $x^2 = 36$?

A $x = 6$ **B** $x = \pm 6$ **C** $x = 1296$ **D** $x = \pm 1296$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3	4, 5	6–8	9–11	12
Section	A	B	C	D	E	F

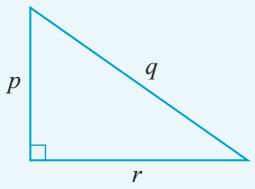
10A Review set

1 For the following triangles:

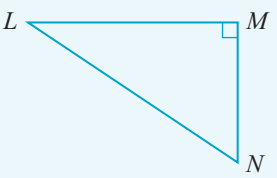
i Which side is the hypotenuse?

ii Write an expression for Pythagoras' theorem for the triangle.

a

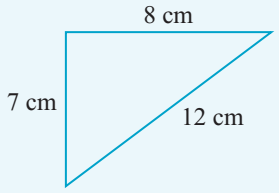


b

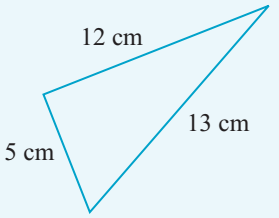


2 Show whether the following triangles are right-angled.

a



b

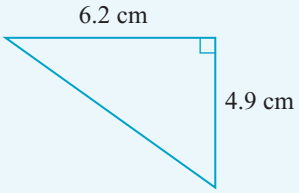


3 **a** Find the value of 7^2.

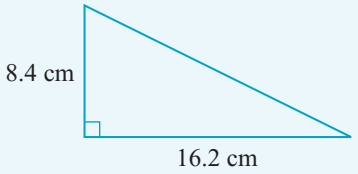
b Calculate the value of $\sqrt{85}$ correct to 1 decimal place.

4 Find the length of the hypotenuse correct to 1 decimal place.

a

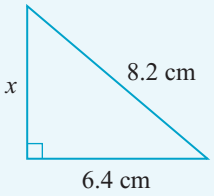


b

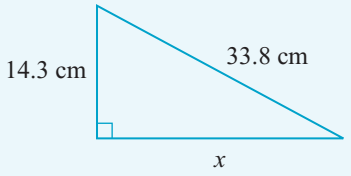


5 Find the length of the third side.

a

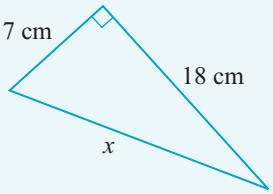


b

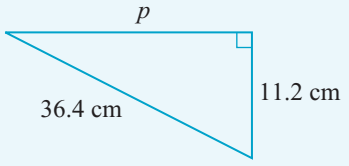


6 Find the value of the pronumeral in each of the following correct to 1 decimal place.

a



b

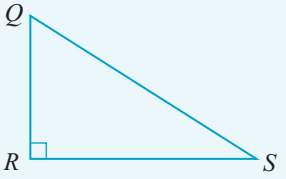


7 Calculate the length of the diagonal of a square with side length 25 cm correct to 2 decimal places.

8 Solve $x^2 = 144$.

10B Review set

1 Write an expression for Pythagoras' theorem for this triangle.



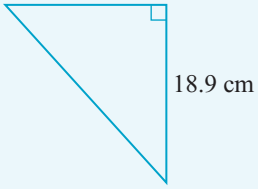
2 Is the triangle with sides 7 cm, 24 cm and 25 cm right-angled? Explain.

3 Calculate the following correct to 2 decimal places:

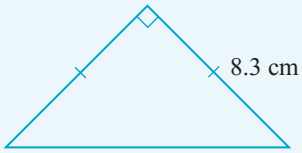
a 15.3^2 **b** $\sqrt{121}$ **c** $\sqrt{59.8}$

4 Find the length of the hypotenuse correct to 1 decimal place.

a

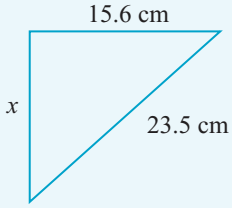


b

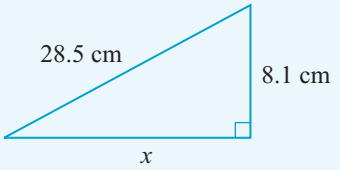


5 Find the length of the third side correct to 1 decimal place.

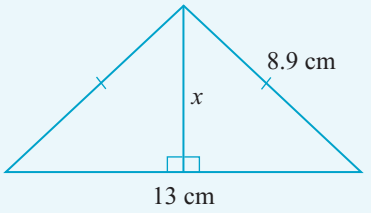
a



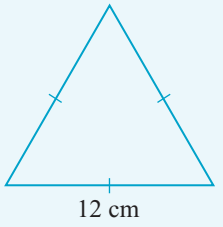
b



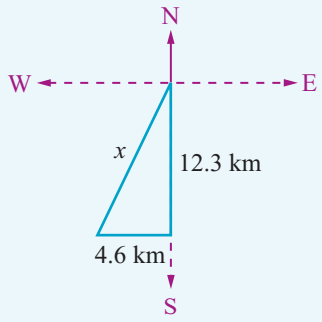
6 Find the length of the 'unknown' side correct to 1 decimal place.



7 Find the height of this equilateral triangle correct to 1 decimal place.



8 Ashya jogs 12.3 km south and then turns directly west and jogs a further 4.6 km. Calculate her distance from her original starting point to the nearest kilometre.

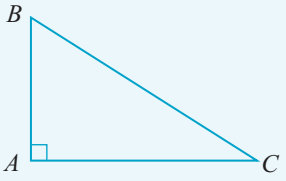


10C Review set

1 Consider this right-angled triangle.

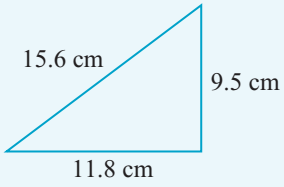
a Which side is the hypotenuse?

b Write an expression for Pythagoras' theorem for the triangle.

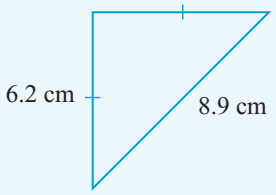


2 Show whether these triangles are right-angled.

a



b

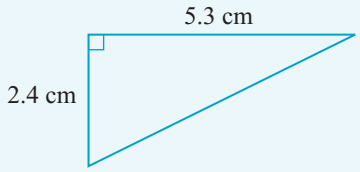


3 Evaluate:

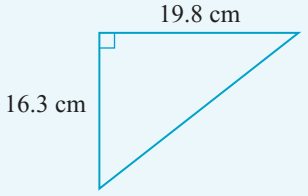
a 5^2 **b** 18^2 **c** 11.6^2

4 Find the length of the hypotenuse correct to 1 decimal place.

a

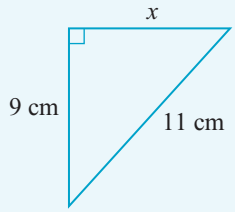


b

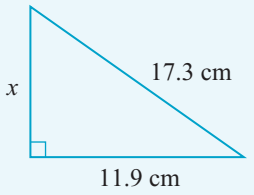


5 Find the length of the third side correct to 1 decimal place.

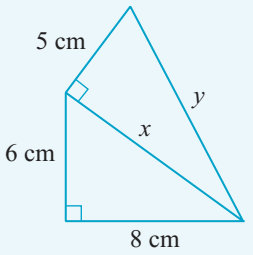
a



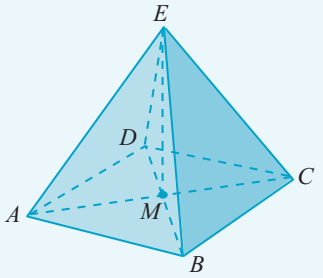
b



6 Find the values of x and y in the diagram.



7 A pyramid has a square base $ABCD$. The apex of the pyramid is vertically above the point M where the diagonals AC and BD intersect. If all edges of the pyramid have the same length of 10 cm, find the height of the pyramid to the nearest mm.

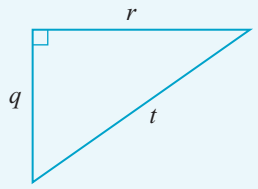


8 Solve $x^2 = 17$. Give your answer in exact form.

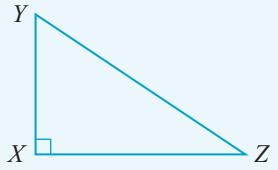
10D Review set

1 Write an expression for Pythagoras' theorem for these triangles.

a



b



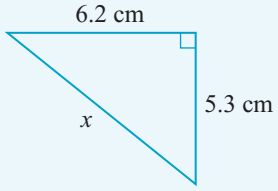
2 Is the triangle with sides 5.6 cm, 8.3 cm and 12.9 cm right-angled? Explain.

3 Evaluate:

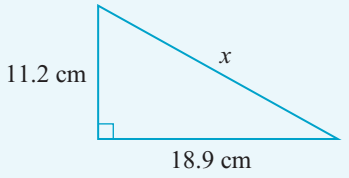
a $\sqrt{49}$ **b** $\sqrt{144}$ **c** $\sqrt{225}$

4 Find the length of the hypotenuse correct to 1 decimal place.

a

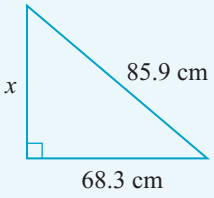


b

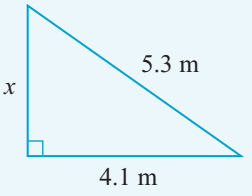


5 Find the length of the third side correct to 1 decimal place.

a

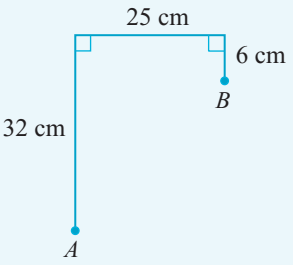


b

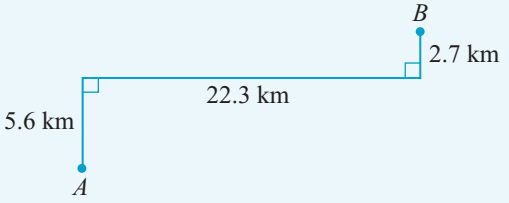


6 Find the distance AB in the following figures. (*Hint:* It is necessary to draw an additional line.)

a



b



7 A ship sails 72 nautical miles north, turns west and sails 48 nautical miles and then turns due south and sails a further 15 nautical miles. (1 nautical mile = 1 NM)

a Draw a diagram to depict the ship's journey.

b Calculate the distance of the ship from its starting point correct to 2 decimal places.

8 Find the length of a side of a square with a diagonal length of 10 cm, giving the answer correct to 1 decimal place.

Chapters 6–10 Cumulative review

1 **a** Calculate the time between 0515 and 0807.

b How many hours and minutes are there between 0823 and 1504 on the same day?

c Rod saw the doctor on 15 March. If he is to return in 5 weeks, what will be the date?

d Jessica caught the bus at 8:13 am and arrived at netball at 9:07 am. How long did the journey take?

e If Sergio caught the bus at 7.42 am, at what time did he arrive at work if the bus trip took 47 min?

f High tide is at 5:37 am and low tide is at 10:14 am Calculate the time difference between high and low tide.

g Write these calculator displays as times in hours and minutes.

i $4° 21′ 36″$ **ii** $7° 25′ 12″$

h Calculate:

i 4 h 18 min + 3 h 53 min **ii** 6 h 23 min 5 s − 2 h 47 min 23 s

i If sunrise is 5:23 am and sunset is 7:41 pm, calculate the length of the day.

j Jana watched a live comedy show from 8:45 pm until 1:12 am.

i How long was the comedy show?

ii If there were 53 minutes of commercials, calculate the actual length of the comedy show.

k Convert $5\frac{5}{7}$ h to:

i hours, minutes and seconds **ii** hours and minutes.

l Calculate the time between 0657 Monday to 1520 Wednesday.

m Calculate the time between 2:27 pm Monday and 7:21 am Tuesday.

n Use the World Time Zones map on page153 to find the local time in Houston when it is 3:25 am in Greenwich.

o Use the World Time Zones map on page 153 to find the local time in Miami when it is 6 pm in Jakarta.

p Use the Australian Time Zones map on page 156 to find the local time in Perth when it is 3 pm in Melbourne.

q It is 2 pm daylight saving time in Sydney. What is the time in Perth?

2 **a** Find 15% of 300 kg.

b Write 75 g as a percentage of 250 g.

c Find 37% of $500.

d Find $5\frac{1}{4}$ % of 400 kg.

e Express 33 kg as a percentage of 87 kg, correct to 2 decimal places.

f Increase 300 by 70%.

g Decrease 320 by 25%.

h Calculate the percentage increase from 63 kg to 75 kg.

i Calculate the percentage decrease from $275 to $180.

j The price of a concert ticket is increased by 130% to $222.30. Calculate the original price.

k Find 100% of a quantity if 53% is $112.36.

l Find 100% of a quantity if 145% is $126.15.

m The price of a concert ticket is increased by 65% to $206.25. Calculate the original price.

n A model that cost $428 is sold at a loss of 18%. Calculate the selling price.

o A tennis racquet was sold for $272.63. This was a profit of 37%. Calculate the cost price.

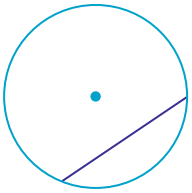
p Craig purchased a mountain bike for $3200. He later sold it for $2475. Express the loss as a percentage of the cost price.

q Calculate the GST on a jacket listed at $149 excluding GST.

r Calculate the pre-GST price of a camera listed at $132 including GST.

3 **a** Name the features shown in purple on these circles.

i



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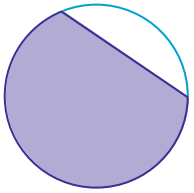

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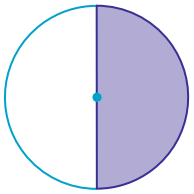

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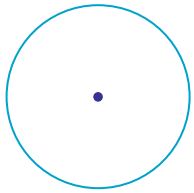

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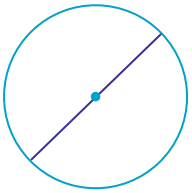

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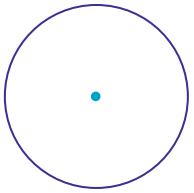

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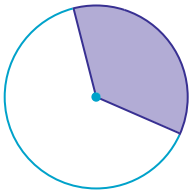

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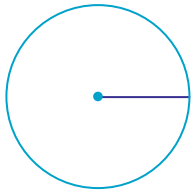

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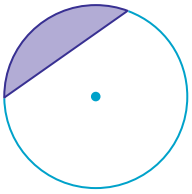

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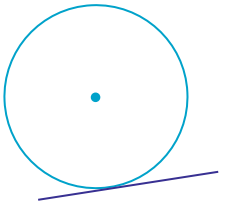

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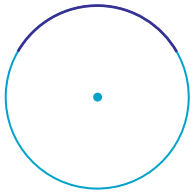
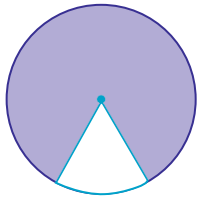


xii



b Write the formula for the circumference of a circle when given the diameter.

c Calculate the circumference of a circle with a diameter of 15.7 cm, correct to 1 decimal place.

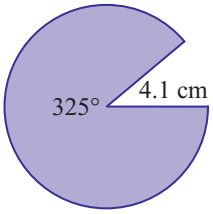
d Write the formula for the circumference of a circle when given the radius.

e Calculate the circumference of a circle with a radius of 8.63 cm, correct to 2 decimal places.

f Calculate the circumference and area of a circle of radius 3.2 m. Answer correct to 1 decimal place.

g What fraction of a circle is represented by each sector?

i



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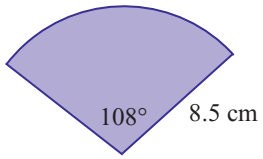
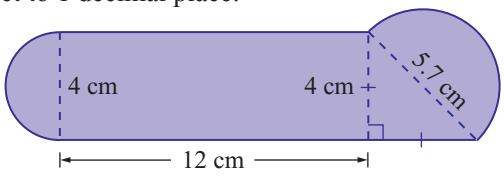
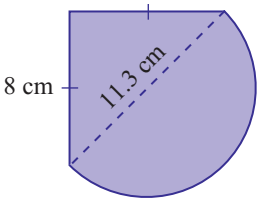


h Calculate the arc length and area of the sectors in part **g** correct to 1 decimal place.

i Calculate the area and perimeter of this shape correct to 2 decimal places.

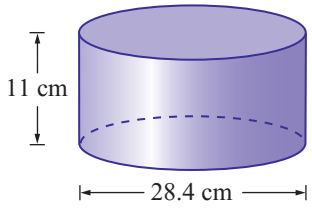


j Calculate the area and perimeter of this shape correct to 1 decimal place.



k For these cylindrical tanks, calculate the volume to the nearest cm^3, and capacity to the nearest litre.

i



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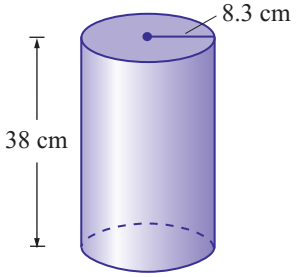
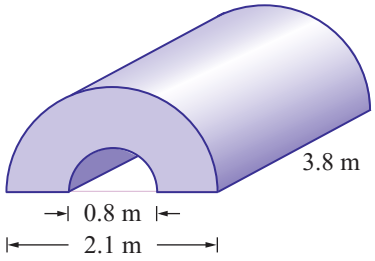


l Calculate the volume of this solid to the nearest 0.1 m^3.



4 **a** Define these statistical terms.

i census **ii** sample **iii** nominal variable

b Classify these variables as categorical (nominal), discrete numerical or continuous numerical.

i hair colour **ii** height

iii temperature **iv** number of apples on a tree

c Classify the following variables as categorical, discrete numerical or continuous numerical.

i income of factory workers **ii** occupations of employed people

iii types of coins in a purse **iv** daily humidity

iv the energy star ratings of air conditioners

d Would a census or a sample be used to investigate the number of people who use a particular brand of toothpaste? Why?

e State the advantages and disadvantages of using a census to collect information.

f Who or what would be the target population if we wanted to collect data about:

i the sports played by Year 8 students? **ii** the quality of bicycle helmets?

g **i** Draw a frequency distribution table for these scores.

9, 10, 6, 11, 8, 7, 7, 8, 6, 2, 9, 7, 3, 9, 7, 3, 7, 5, 10, 5, 8, 5, 10, 3, 7, 6, 10, 11, 11, 2

ii How many scores were 9 or more?

iii Draw a dot plot for these scores.

h Draw a stem-and-leaf plot for these scores.

35, 34, 17, 27, 50, 39, 38, 31, 21, 31, 39, 34, 40, 20, 37, 34, 21, 34, 33, 42

5 **a** For the scores 23, 25, 24, 23, 26, 29, 23, 21 find the:

i mean **ii** median **iii** mode **iv** range.

b For the scores in this frequency distribution, find the:

i mean **ii** median

iii mode **iv** range.

Score	Frequency
19	11
20	23
21	15
22	6
23	5
24	1

c For the scores in this stem-and-leaf plot, find the:

i mean **ii** median

iii mode **iv** range.

Stem	Leaves
8	2 3 4 9
9	1 1 1 6 8 8
10	1 2 5 7 7 9 9
11	1 1 2 3 7 8
12	1 4 4 4 4 5 7

d Calculate the mean, median and mode of these data sets.

i 7, 7, 8, 9, 9, 9, 10, 93

ii 13, 14, 14, 15, 15, 16, 17

iii

Score	Frequency
5	7
6	2
7	1
8	2
9	3

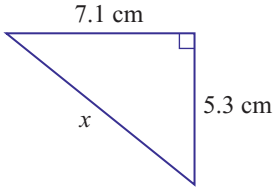
e For which of the sets of data in part **d** is the mean, median or mode not a central value of the data?

6 a Calculate to 1 decimal place:

i $\sqrt{76}$ **ii** $\sqrt{93}$ **iii** $\sqrt{423}$ **iv** $\sqrt{11}$

b Find the length of the hypotenuse correct to 1 decimal place.

i



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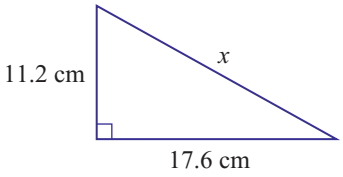
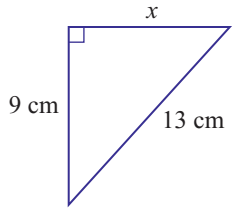


c Find the length of the third side correct to 1 decimal place.

i



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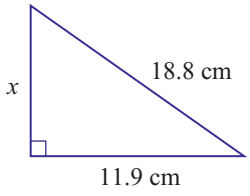
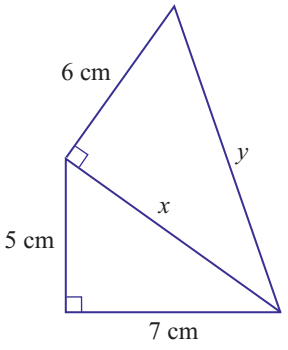


d Is the triangle with sides 8 cm, 10 cm and 13 cm right-angled? Explain why or why not.

e Calculate the length of the diagonal of a square of side length 18 cm. Give the answer correct to 1 decimal place.

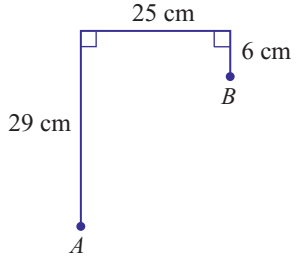
f A ship sails 84 km north, then turns west and sails 62 km. How far is the ship from its starting point?

g Find the values of x and y in this quadrilateral.

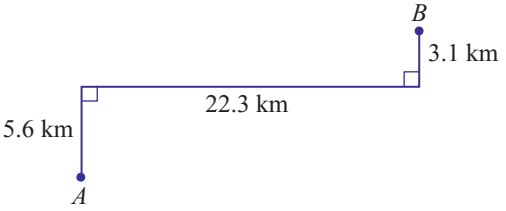


h Find the distance AB in the following figures. (*Hint:* It is necessary to draw an additional line.)

i



ii



i Solve $x^2 = 121$.

j Solve $x^2 = 19$. Give your answer in exact form.

11

Algebra

This chapter deals with the simplification of algebraic expressions and the application of the distributive law to expanding and factorising algebraic expression.

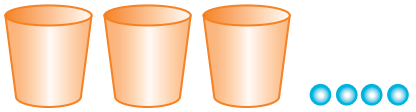
At the end of this chapter you should be able to:

- simplify algebraic expressions involving the four operations
- apply the distributive law to the expansion of algebraic expressions
- factorise algebraic expressions by identifying numerical factors.

NSW Syllabus references: S4 N&A Algebraic techniques 1, S4 N&A Algebraic techniques 2 (parts)
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-8NA
NUMBER & ALGEBRA – ACMNA190, ACMNA191, ACMNA192

Diagnostic test

1 The diagram shows 3 cups and 4 marbles.



If there are n marbles in each cup, an expression for the total number of marbles is:

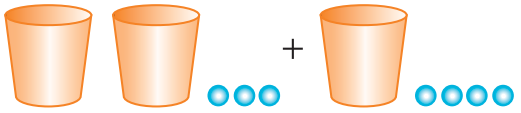
A $3 \times n + 4$ **B** $4 \times n + 3$
C $4 \times n + 3 \times n$ **D** $3 \times (n + 4)$

2 The algebraic expression $y + y + y + y$ is equivalent to:

A $4 \times y$ **B** y^4
C $4 + y$ **D** $yyyy$

3 Using the diagram below, if there are k marbles in each cup, then $(2 \times k + 3) + (k + 4) =$

A $10 \times k$ **B** $3 \times k + 7$
C $2 \times k + 7$ **D** 10



4 The value of the expression $3 \times z + 5$ if $z = 2$ is:

A 21 **B** 11 **C** 37 **D** 30

5 Which of the following expressions does *not* simplify to $24p$?

A $24 \times p$ **B** $2 \times 4 \times p$
C $6 \times 4 \times p$ **D** $3 \times 8 \times p$

6 When simplified, $1 \times 3y =$

A $13y$ **B** $3y$ **C** 1 **D** $4y$

7 When simplified, $4z \times 0 =$

A $4z$ **B** $40z$ **C** 0 **D** 1

Use the values $x = 4$ and $y = 5$ for questions **8**–**10**.

8 The value of $3xy$ is:

A 345 **B** 17 **C** 12 **D** 60

9 The value of $3x - 2y$ is:

A 2 **B** 9 **C** 50 **D** 24

10 The value of $2x^2$ is:

A 64 **B** 32 **C** 576 **D** 16

11 Which of the following expressions is *not* equivalent to $\frac{3k}{2m}$?

A $3k \div 2m$ **B** $3 \times k \div 2m$
C $3 \times k \div (2 \times m)$ **D** $2m \div 3k$

12 If $y = 6$, then $\frac{4y}{2} =$

A 23 **B** 12 **C** 26 **D** 43

13 If $k = 5$, the value of $3(k + 2)$ is:

A 21 **B** 17 **C** 37 **D** 11

14 If $k = 5$, the value of $k(6 - k)$ is:

A 25 **B** 31 **C** 5 **D** 6

15 If $k = 5$, the value of $\frac{k + 10}{5}$ is:

A 11 **B** 3 **C** 7 **D** 10

16 If $k = 5$, the value of $\frac{12}{8 - k}$ is:

A $1\frac{3}{10}$ **B** 4 **C** 36 **D** $-1\frac{1}{10}$

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA190, ACMNA191 and ACMNA192.

A Adding and subtracting algebraic terms

EXAMPLE 1

Simplify the following by adding or subtracting.

a $4k + 6k$ **b** $7w - 3w$ **c** $4k + 7w$

a If k represents the number of marbles in a cup, then $4k$ or $4 \times k$ = number of marbles in 4 cups and $6k$ or $6 \times k$ = number of marbles in 6 cups.
Thus $4k + 6k$ = number of marbles in $(4 + 6)$ cups $= 10 \times k$ or $10k$.

b If w represents the number of marbles in a box, then $7w$ or $7 \times w$ = number of marbles in 7 boxes and $3w$ or $3 \times w$ = number of marbles in 3 boxes.
Thus $7w - 3w$ = number of marbles in $(7 - 3)$ boxes $= 4 \times w$ or $4w$.

c If k represents the number of marbles in a cup and if w represents the number of marbles in a box, then $4k + 7w$ = number of marbles in 4 cups plus the number of marbles in 7 boxes.
Because the units (cups and boxes) are different, we cannot simplify this algebraic expression.

Note: Algebraic terms with the same pronumeral (same unit) are called like terms. Like algebraic terms can be added or subtracted.

Algebraic terms with different pronumerals (different units) are called unlike terms. Unlike terms cannot be added or subtracted. For example:

- $5a$, $14a$ and $23a$ are like terms (same pronumeral a)
- $5a$ and $14a^2$ are unlike terms (different pronumeral forms, a and a^2)
- $5p$, $14a^2q$ and $73d$ are unlike terms (different pronumerals p, a^2q, d)

EXAMPLE 2

Simplify where possible.

a $7x + 3x$ **b** $9w - 3w$ **c** $7x + 9w$
d $8y - y$ **e** $6k + 7$

a $7x + 3x = 7 \times x + 3 \times x$
$= (7 + 3) \times x$
$= 10 \times x = 10x$

b $9w - 3w = 9 \times w - 3 \times w$
$= (9 - 3) \times w$
$= 6 \times w = 6w$

c $7x + 9w$ cannot be simplified as $7x$ and $9w$ are unlike terms.

d $8y - y = 8 \times y - 1 \times y$
$= (8 - 1) \times y$
$= 7 \times y = 7y$

e $6k + 7$ cannot be simplified as $6k$ and 7 are unlike terms.

Exercise 11A

1 Complete the following to simplify.

a $6a + 9a = \_\_\_ \times a + \_\_\_ \times a$
$= (\_\_\_ + \_\_\_) \times a$
$= \_\_\_ \times a = \_\_\_$

b $14y - 6y = \_\_\_ \times y - \_\_\_ \times y$
$= (\_\_\_ - \_\_\_) \times y$
$= \_\_\_ \times y = \_\_\_$

2 Evaluate the expressions $3y + 6y$ and $9y$ using the following values.

a $y = 1$ **b** $y = 5$ **c** $y = -2$

Are the values of $3y + 6y$ and $9y$ always the same?

3 Evaluate the expressions $7k - 3k$ and $4k$ using the following values.

a $k = 5$ **b** $k = 8$ **c** $k = -3$

Are the values of $7k - 3k$ and $4k$ always the same?

The order of pronumerals in the like terms may be different, for example xy and yx are like terms.

4 Simplify where possible.

a $4a + 7a$	**b** $6t + 5t$	**c** $8m + m$	**d** $10p - 3p$
e $8x - 5x$	**f** $4b - b$	**g** $6a + 3b$	**h** $7y - 4z$
i $11t^2 + 6t^2$	**j** $8m^2 - 4m^2$	**k** $9k + 5m$	**l** $5ab + 6ab$
m $5ab + 2bc$	**n** $6ab^2 - 4ab^2$	**o** $3a^2b + 5ab^2$	**p** $9pq - 9qp$
q $6g^3 + g^3$	**r** $p + p$	**s** $9x + 24$	**t** $24st^2 - 15t^2s$
u $7mn^2 - 3n^2m$	**v** $32x^2y - 21yx^2$	**w** $8f - 7f$	**x** $11c - c$

EXAMPLE 3

Simplify the following.

a $8n - 11n$ **b** $-3b + 7b$ **c** $-4x^2 - 5x^2$

a $8n - 11n = (8 - 11) \times n$
$= -3n$

b $-3b + 7b = (-3 + 7) \times b$
$= 4b$

c $-4x^2 - 5x^2 = (-4 - 5) \times x^2$
$= -9x^2$

5 Collect like terms to simplify.

a $4k - 7k$
$= (\_\_\_ - \_\_\_) \times k = \_\_\_$

b $-2w + 5w$
$= (\_\_\_ + \_\_\_) \times w = \_\_\_$

c $-3t - 2t$
$= (\_\_\_ - \_\_\_) \times t = \_\_\_$

6 Simplify by collecting like terms.

a $5n - 6n$	**b** $8p - 10p$	**c** $-7c + 5c$	**d** $-4q - 2q$
e $-2d + 5d$	**f** $3ac - 12ac$	**g** $6q - 9q$	**h** $-c + 8c$
i $yz - 5yz$	**j** $-9l + 14l$	**k** $-4p^2 + 3p^2$	**l** $15x^2 - 17x^2$
m $3xy^2 - 5xy^2$	**n** $-3pq^2 - 8pq^2$	**o** $-6t^2 + 3t^2$	**p** $b - 3b$

EXAMPLE 4

Simplify the following.

a $-4f - 5f + 6f$ **b** $qr - 8rq + 3rq$

The term rq can be rearranged to qr.

a $-4f - 5f + 6f = -9f + 6f$
$= -3f$

b $qr - 8rq + 3rq = -7qr + 3qr$
$= -4qr$

7 Complete the following to simplify.

a $-3x - 5x + 4x$
$= \_\_\_ + 4x$
$= \_\_\_$

b $2pq + 4pq - 3pq$
$= \_\_\_ - 3pq$
$= \_\_\_$

c $2z - 6z + 3z$
$= \_\_\_ + 3z$
$= \_\_\_$

8 Simplify by collecting like terms.

a $7d + 3d - 4d$
b $8pq - 5pq - 6pq$
c $c + 3c - 5c$
d $9q^2 + 5q^2 - q^2$
e $-4m + m - 2m$
f $18r - 2r - 7r$
g $-a + 2a - 5a$
h $-3r + 2r - r$
i $4a^2b - 3a^2b - ba^2$
j $-17x^2y - 5x^2y + 3x^2y$
k $-d^2 + 7d^2 - 13d^2$
l $k^2m - 5k^2m + mk^2$

When collecting like terms remember to include the + or − sign in front of the like terms.

EXAMPLE 5

Simplify the following by collecting like terms.

a $6m - 7 + 2m$
b $4x^2y + 5p - 3yx^2$

a $6m - 7 + 2m = (6m + 2m) - 7$
$= 8m - 7$

b $4x^2y + 5p - 3yx^2 = (4x^2y - 3x^2y) + 5p$
$= x^2y + 5p$

9 Complete the following to simplify.

a $5y - 3 + 4y$
$= (\_\_\_ + \_\_\_) - 3$
$= \_\_\_ - 3$

b $7p + 3q + 5p$
$= (\_\_\_ + \_\_\_) + 3q$
$= \_\_\_ + 3q$

10 Simplify by collecting like terms.

a $7 + 10 - 5m$
b $3a + 6 + 5a$
c $4 + 2n - n$
d $6 - 7q + 12$
e $9 - 5k + 4$
f $3f + 12 + 8f$
g $x^2y - 5 + 2yx^2$
h $13w - 6 - 4w$
i $a^2b + 9 + 11a^2b$
j $5q^2 - 6 + q^2$
k $5x - 4x - 2$
l $p + 3p - 9$
m $6t + 4 - 5t$
n $7k - k - 6l$
o $d^2 + 5d^2 - 3d$
p $4n - n + 2n^2$
q $3c + 2c^2 + 5c$
r $6 + 8n^2 - 5n^2$
s $11mn - 2m + 4nm$
t $5a^2b + 2ab^2 - 4a^2b$

EXAMPLE 6

Simplify by collecting like terms.

a $8 + 4z - 10$
b $-2w - 9t - 5w$
c $-6x - 3y + 2x$

a $8 + 4z - 10 = 4z + 8 - 10$
$= 4z - 2$
or
$8 + 4z - 10 = -2 + 4z$
$= 4z - 2$

b $-2w - 9t - 5w = -2w - 5w - 9t$
$= -7w - 9t$
or
$-2w - 9t - 5w = -9t - 2w - 5w$
$= -9t - 7w$

c $-6x - 3y + 2x = -6x + 2x - 3y$
$= -4x - 3y$ (or $-3y - 4x$)

11 Simplify by collecting like terms.

a $6 + 5k - 9$
b $3 - 4s - 7$
c $5m + 2n - 7m$
d $2x - 8y - 5x$
e $5t - 9t + 7u$
f $2a - 4b - 7a$
g $-5d + 4e + 3d$
h $-8z - 4w + 2z$
i $-a - 3b - 2a$
j $6m - 8n - 2m$
k $-3y - 2w - y$
l $-6k + m + 2k$

EXAMPLE 7

Simplify by collecting like terms.

a $9x + 7 - 3x + 10$ **b** $5a + 7b - 3a + 2b$

a $9x + 7 - 3x + 10 = 9x - 3x + 7 + 10$
$= 6x + 17$

b $5a + 7b - 3a + 2b = 5a - 3a + 7b + 2b$
$= 2a + 9b$

12 Complete the following.

a $5x + 9 - 2x - 7$
$= 5x - 2x + 9 - 7$
$= \_\_\_ + \_\_\_$

b $7w - 4z - 3w + 2z$
$= 7w - 3w - 4z + 2z$
$= \_\_\_ - \_\_\_$

c $-3 + 4d + 7 - d$
$= -3 + 7 + 4d - d$
$= \_\_\_ + \_\_\_$

13 Simplify the following expressions by collecting like terms.

a $4a + 6 + 2a + 5$
b $11c^2 + c - 2c^2 + c$
c $x + y + x + y$
d $q^2 + 5 + q^2 - 3$
e $6t + 3v - 2v + 3t$
f $q + 4d + 3q - d$
g $12p + p^2 + 5p^2 - 8p$
h $-8l + 4 - 5l + 6$
i $-n - 2 - 5n - 1$
j $m^2 - m - m^2 - 3m$
k $-d^2 - 7d + 3d^2 - 4d$
l $25 + 8m - 9 + 2m$
m $-13 + 6n - n + 5$
n $6p - 3m + 6p - 3m$
o $7x^2 + 4xy + 8yx - 2x$

14 Simplify by collecting like terms.

a $3c - 7 + 2d - 9 + 6c$
b $n - dc + 3cd + 5n - 8 + 2$
c $-4q - 2 + r - q + 8 + 2q$
d $7e + e^2 + 3 - e^2 - 5e + 6$
e $8 - 3c + 9 - 6 - 4c + 2d - d$
f $15l + 2 - 3l + 7c - 12c + 6$
g $-7 - c + a + 10 + c - 11 - a - c - 8$
h $d - qr + r - q - qr + 6r - 3q + 4d$
i $-12r + s + 6 - t + 5 - 9s + 12r + 7t - 3$
j $ac + 2pa - 6a + 7 - 5ap + 9ac + 12$

B Multiplying algebraic terms

EXAMPLE 1

Simplify the following.

a $p \times s$ **b** $p \times p$ **c** $3p \times s$ **d** $5 \times 2k$ **e** $3p \times 2s$ **f** $4pq \times 3ps$

a $p \times s = ps$ (leave out the multiplication sign between the pronumerals)

b $p \times p = pp$ or p^2

c $3p \times s = 3 \times p \times s = 3ps$ (leave out the multiplication sign between the numbers and pronumerals)

d $5 \times 2k = 5 \times 2 \times k$ (split into numerical and pronumeral parts)
$= 10 \times k = 10k$

e $3p \times 2s = 3 \times p \times 2 \times s$ (split into numerical and pronumeral parts)
$= 3 \times 2 \times p \times s$ (group the numbers together and the pronumerals together)
$= 6 \times ps = 6ps$

f $4pq \times 3ps = 4 \times p \times q \times 3 \times p \times s$ (split into numerical and pronumeral parts)
$= 4 \times 3 \times p \times q \times p \times s$ (group the numbers and pronumerals together)
$= 4 \times 3 \times p \times p \times q \times s$
$= 12 \times p^2qs = 12p^2qs$

To multiply algebraic terms, multiply the numbers and multiply the pronumerals. We usually write the pronumerals in alphabetical order.

Exercise 11B

1 Complete the following to simplify.

a $5t \times w = \_\_\_ \times \_\_\_ \times w = \_\_\_$

b $4 \times 3m = 4 \times \_\_\_ \times \_\_\_ = \_\_\_$

c $7y \times 2 = \_\_\_ \times y \times \_\_\_$
$= \_\_\_ \times \_\_\_ \times y = \_\_\_$

d $3p^2 \times 7q = \_\_\_ \times p^2 \times \_\_\_ \times q$
$= \_\_\_ \times \_\_\_ \times p^2 \times q = \_\_\_$

e $2ab \times 3a = \_\_\_ \times a \times b \times \_\_\_ \times a$
$= \_\_\_ \times \_\_\_ \times a \times a \times b = \_\_\_$

2 Simplify the following.

a $4x \times y$ **b** $3k \times m$ **c** $x \times 5y$ **d** $4 \times 7w$

e $5 \times 4k$ **f** $6 \times 10p$ **g** $2x \times 8$ **h** $6z \times 3$

i $3m \times 4n$ **j** $6v \times 2w$ **k** $4p^2 \times 7q$ **l** $5a \times 6b^2$

m $4ab \times 5c$ **n** $3xz \times 6xy$ **o** $10pq \times 2qr$ **p** $5bc \times 7bc$

q $2 \times 3a \times 4b$ **r** $2a \times 3b \times 4c$ **s** $4p \times 5q \times 2r$ **t** $3a \times 4a \times 3c$

EXAMPLE 2

Simplify the following.

a $-5 \times 3t$

b $-2m \times -3n$

Remember: When multiplying two integers:
If the signs are the same, the answer is positive.
If the signs are different, the answer is negative.

a $-5 \times 3t = -5 \times 3 \times t$
$= -15t$

b $-2m \times -3n = -2 \times m \times -3 \times n$
$= -2 \times -3 \times m \times n$
$= 6mn$

3 Simplify the following.

a $-2 \times 5x$ **b** $-5 \times 4y$ **c** $-6 \times -2w$ **d** $-4 \times -8z$

e $4 \times -3m$ **f** $10 \times -8p$ **g** $-3m \times 2n$ **h** $4a \times -5b$

i $-6x \times -2y$ **j** $-9s \times -2t$ **k** $-4p^2 \times 6q$ **l** $-5a \times 8a$

m $-2p \times -5p$ **n** $4mn \times -2mp$ **o** $-5abc \times -6b$ **p** $-7mn \times 4kn$

C Dividing algebraic terms

When dividing algebraic terms containing pronumerals and numbers, follow these steps.

Step 1: Write the division as a fraction.
Step 2: Cancel the numbers, if possible.
Step 3: Cancel the pronumerals, if possible.
Step 4: Write your answer as a fraction.

Notice that $\frac{3}{1} = 3$, but $\frac{1}{3} \neq 3$.

Remember: Cancel means divide the numerator and denominator by the same number or pronumeral.

EXAMPLE 1

Simplify the following.

a $5x \div 10$ **b** $21x \div 7x$ **c** $\frac{5f}{15f}$ **d** $\frac{8xy}{6y}$

a $5x \div 10 = \frac{5x}{10}$
$= \frac{{}^{1}\cancel{5}x}{{}^{2}\cancel{10}}$
$= \frac{x}{2}$

b $21x \div 7x = \frac{21x}{7x}$
$= \frac{{}^{3}21{}^{1}\cancel{x}}{{}^{1}\cancel{7}{}^{1}\cancel{x}}$
$= 3$

c $\frac{5f}{15f} = \frac{{}^{1}\cancel{5}{}^{1}\cancel{f}}{{}^{3}\cancel{15}{}^{1}\cancel{f}}$
$= \frac{1}{3}$

d $\frac{8xy}{6y} = \frac{{}^{4}\cancel{8}x{}^{1}\cancel{y}}{{}^{2}\cancel{6}{}^{1}\cancel{y}}$
$= \frac{4x}{3}$

Exercise 11C

1 Complete the following to simplify.

a $10y \div 15 = \frac{10y}{\square}$
$= \frac{2y}{\square}$

b $8m \div 12m = \frac{\square}{12m}$
$= \frac{\square}{3}$

c $6x \div 8xy = \frac{\square}{8xy}$
$= \frac{\square}{4y}$

2 Simplify the following.

a $9x \div 18$ **b** $3m \div 12$ **c** $5p \div 25$ **d** $16d \div 4$ **e** $\frac{10c}{2}$

f $\frac{8a}{4}$ **g** $\frac{6a}{12a}$ **h** $\frac{44m}{22m}$ **i** $\frac{12a}{15a}$ **j** $\frac{20d}{10d}$

k $\frac{3f}{9f}$ **l** $\frac{4t}{20t}$ **m** $\frac{18p}{20d}$ **n** $\frac{6xy}{15x}$ **o** $\frac{24ab}{36bc}$

p $\frac{16r}{20qr}$ **q** $\frac{8yz}{40xyz}$ **r** $\frac{70dkl}{10klm}$ **s** $\frac{15pqr}{12q}$ **t** $\frac{14mn}{35mp}$

EXAMPLE 2

Simplify the following.

a $\frac{-xy}{-y}$ **b** $\frac{40ac}{-10ac}$ **c** $\frac{-16x^2y}{-8x}$

Remember: When dividing two integers:
If the signs are the same, the result is positive.
If the signs are different, the result is negative.

a $\frac{-xy}{-y} = \frac{\cancel{-}\cancel{xy}}{\cancel{-}\cancel{y}}$
$= x$

b $\frac{40ac}{-10ac} = \frac{{}^{4}\cancel{40ac}}{-{}^{1}\cancel{10ac}}$
$= -4$

c $\frac{-16x^2y}{-8x} = \frac{\cancel{-}{}^{2}\cancel{16}x^2y}{\cancel{-}{}^{1}\cancel{8}x}$
$= 2xy$

3 Complete the following to simplify.

a $\frac{15ab}{-20ac} = \frac{15ab}{\square}$

$= \frac{3b}{\square}$

$= -\frac{3b}{\square}$

b $\frac{-x}{xy} = \frac{\square}{xy}$

$= \frac{\square}{y}$

$= -\frac{\square}{y}$

4 Simplify the following.

a $\frac{-50d}{10d}$ **b** $\frac{-12fg}{-18g}$ **c** $\frac{-6kl}{9l}$ **d** $\frac{12fg}{-3gh}$

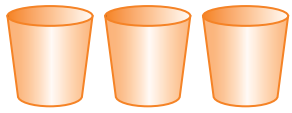
e $\frac{-36lm}{9m}$ **f** $\frac{-90y}{99x}$ **g** $\frac{-45c}{-15ac}$ **h** $\frac{-a^2bc}{b^2c}$

i $\frac{-mn}{n^2}$ **j** $\frac{x^2yz^2}{-y^2x}$ **k** $-64p \div -16q$ **l** $-6ac \div -9c$

m $-2mn \div -8mn$ **n** $-10 \div 5mn$ **o** $-3q \div q$ **p** $21pq \div -3p^2$

Investigation 1 The distributive law

1 The diagram shows 3 cups. Complete the following.



a If each cup holds 5 marbles, then
total number of marbles $= 5 + 5 + 5 =$ ___ $\times 5$.

b If each cup holds w marbles then
total number of marbles $= w + w + w =$ ___ $\times w$.

c If each cup holds $w + 5$ marbles, then
total number of marbles $= w + 5 + w + 5 + w + 5 =$ ___ $\times (w + 5)$.

2 **a** Complete the following.

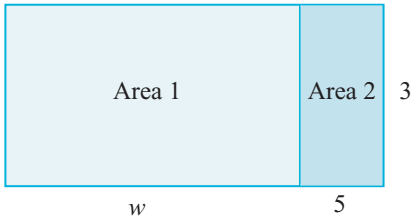
$3(w + 5) = 3 \times (w + 5)$

$= w + 5 + w + 5 + w + 5$

$= w + w + w + 5 + 5 + 5$

$=$ ___ $\times w +$ ___ $\times 5 =$ ___

b Complete the following.



Area of large rectangle = length $\times$ breadth

$=$ ___ $\times (w + 5)$

Area of rectangle 1 = length $\times$ breadth

$=$ ___ $\times w$

Area of rectangle 2 = length $\times$ breadth

$=$ ___ $\times 5$

Area of large rectangle = area of rectangle 1 + area of rectangle 2

$3 \times (w + 5) =$ ___ $\times w +$ ___ $\times 5$

c Complete the table.

	$w = 2$	$w = 3$	$w = 5$	$w = 10$
$3(w + 5)$	$= 3 \times (2 + 5)$ $= 3 \times$ ___ $=$ ___			
$3w + 15$	$= 3 \times 2 + 15$ $=$ ___ $+ 15 =$ ___			

D Removing grouping symbols

From Investigation 1 we see that $3 \times (w + 5) = 3 \times w + 3 \times 5$.

This is an example of a general result in algebra known as the distributive law. To write an algebraic expression without grouping symbols, each term inside the grouping symbols must be multiplied by the term outside.

$a \times (b + c) = a \times b + a \times c$ or $a(b + c) = ab + ac$

$a \times (b - c) = a \times b - a \times c$ or $a(b - c) = ab - ac$

In algebra, using the distributive law to remove grouping symbols is also called expanding.

EXAMPLE 1

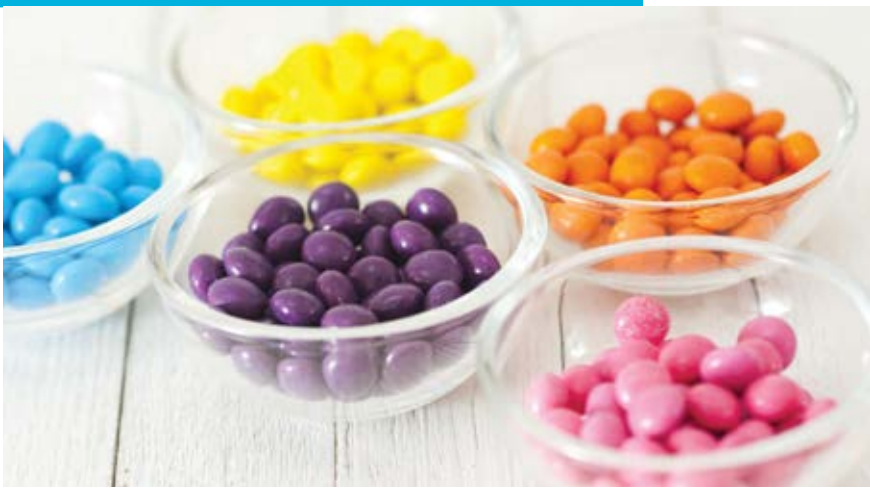
Expand these expressions.

a $4(p + 1)$ **b** $12(5 - r)$

a $4(p + 1) = 4 \times p + 4 \times 1$
$= 4p + 4$

b $12(5 - r) = 12 \times 5 - 12 \times r$
$= 60 - 12r$

Exercise 11D



1 Complete the following to expand the expressions.

a $5(d + 4) = \_\_\_ \times d + \_\_\_ \times 4$
$= \_\_\_ + \_\_\_$

b $4(y - 3) = \_\_\_ \times y - \_\_\_ \times 3$
$= \_\_\_ - \_\_\_$

c $3(6 - m) = \_\_\_ \times 6 - \_\_\_ \times m$
$= \_\_\_ - \_\_\_$

d $2(q + 7) = 2 \times \_\_\_ + 2 \times \_\_\_$
$= \_\_\_ + \_\_\_$

e $6(b - 2) = 6 \times \_\_\_ - 6 \times \_\_\_$
$= \_\_\_ - \_\_\_$

2 Expand the following expressions.

a $4(b + 3)$	**b** $12(k + 8)$	**c** $7(c - 5)$	**d** $6(d - 3)$
e $2(y - 11)$	**f** $9(a + 10)$	**g** $10(j + 9)$	**h** $8(m + 2)$
i $7(q - 2)$	**j** $5(l - 6)$	**k** $4(2 - c)$	**l** $3(r + 6)$
m $9(7 - t)$	**n** $4(v + 12)$	**o** $6(8 - n)$	**p** $6(x - 2)$

EXAMPLE 2

Expand the following expressions.

a $5(3x + 2)$ **b** $4(7 - 2x)$

a $5(3x + 2) = 5 \times 3x + 5 \times 2$
$= 15x + 10$

b $4(7 - 2x) = 4 \times 7 - 4 \times 2x$
$= 28 - 8x$

3 Complete the following to expand.

a $4(3z + 2) = \_\_\_ \times 3z + \_\_\_ \times 2$
$= \_\_\_ + \_\_\_$

b $5(2y - 3) = \_\_\_ \times 2y - \_\_\_ \times 3$
$= \_\_\_ - \_\_\_$

c $3(6 + 4k) = 3 \times \_\_\_ + 3 \times \_\_\_$
$= \_\_\_ + \_\_\_$

4 Expand the following expressions.

a $3(2m + 6)$ **b** $5(4d + 5)$ **c** $9(3p + 8)$ **d** $7(5c - 4)$
e $10(2p - 2)$ **f** $12(4c - 3)$ **g** $6(6k + 10)$ **h** $2(13n + 5)$
i $10(7a - 6)$ **j** $8(7l - 3)$ **k** $11(2h + 8)$ **l** $4(15k - 5)$
m $13(6x + 2)$ **n** $7(10w - 9)$ **o** $5(11j + 7)$ **p** $3(9q - 4)$

5 Explain the difference between each pair of expressions.

a $2x + 1$ and $2(x + 1)$ **b** $5p - 8$ and $5(p - 8)$

EXAMPLE 3

Expand the following expressions.

a $x(x + 2)$ **b** $y(z - x)$

a $x(x + 2) = x \times x + x \times 2$
$= x^2 + 2x$

b $y(z - x) = y \times z - y \times x$
$= yz - xy$

6 Complete the following to expand.

a $m(m + 3) = \_\_\_ \times m + \_\_\_ \times 3$
$= \_\_\_ + \_\_\_$

b $p(q - r) = \_\_\_ \times q - \_\_\_ \times r$
$= \_\_\_ - \_\_\_$

7 Expand the following expressions.

a $x(x + 5)$ **b** $q(q + 13)$ **c** $a(a + 8)$ **d** $z(z + 11)$
e $t(t - 6)$ **f** $m(m - 10)$ **g** $d(3 - d)$ **h** $r(r - 17)$
i $a(c - 4)$ **j** $b(d + a)$ **k** $x(y - z)$ **l** $m(n + c)$
m $j(k - h)$ **n** $d(f + g)$ **o** $e(c - d)$ **p** $r(x - y)$

EXAMPLE 4

Simplify the following expressions.

a $3x(x - 5)$ **b** $5m(2n + 6m)$

a $3x(x - 5) = 3x \times x - 3x \times 5$
$= 3x^2 - 15x$

b $5m(2n + 6m) = 5m \times 2n + 5m \times 6m$
$= 10mn + 30m^2$

8 Complete the following to expand.

a $4t(t - 3) = \_\_\_ \times t - \_\_\_ \times 3$
$= \_\_\_ - \_\_\_$

b $3x(2y + 5z) = \_\_\_ \times 2y + \_\_\_ \times 5z$
$= \_\_\_ + \_\_\_$

9 Expand the following expressions.

a $8m(m + 3)$ **b** $5c(c + 6)$ **c** $3r(11 + r)$ **d** $11q(q - 1)$
e $4x(2 - x)$ **f** $10a(7 - a)$ **g** $4a(2a + 7)$ **h** $9b(11b + 5)$
i $5f(4 - 4f)$ **j** $6d(d - f)$ **k** $3k(8 - 4k)$ **l** $12l(3 - 2l)$
m $5p(2p - 3n)$ **n** $7c(5c + 2d)$ **o** $6n(6m - 5n)$ **p** $4x(4x - 3z)$

EXAMPLE 5

Expand and simplify by collecting like terms.

a $5(p + 7) + 2p - 9$ **b** $3(x + 8) + 7(x - 9)$

a
$5(p + 7) + 2p - 9$
$= 5p + 35 + 2p - 9$
$= 7p + 26$

b
$3(x + 8) + 7(x - 9)$
$= 3x + 24 + 7x - 63$
$= 10x - 39$

10 Complete the following to simplify.

a
$3(x + 5) + 2x - 7$
$= \_\_\_ \times x + \_\_\_ \times 5 + 2x - 7$
$= \_\_\_x + \_\_\_ + 2x - 7$
$= \_\_\_ + \_\_\_$

b
$2(p + 5) + 4(p - 3)$
$= \_\_\_ \times p + \_\_\_ \times 5 + \_\_\_ \times p - \_\_\_ \times 3$
$= \_\_\_p + \_\_\_ + \_\_\_p - \_\_\_$
$= \_\_\_ - \_\_\_$

11 Expand and simplify by collecting like terms.

a $7(a + 8) + 5a$ **b** $9(p - 5) - 3$ **c** $6(c + 8) + 4c$
d $8(d - 7) - 4d$ **e** $5(q + 4) + 10q$ **f** $11(m - 7) + 15$
g $4(n + 6) + 3n - 10$ **h** $2(b - 7) + 3b + 12$ **i** $3x - 19 + 3(5 - 2x)$
j $7w - 8 + 5(w + 1)$ **k** $9(f - 3) + 8 - 6f$ **l** $6n - 10 + 2(n - 7)$
m $10y + 22 + 2(y - 10) + 3y$ **n** $7c + 3(6 - 4c) + 11 - 2c$ **o** $4(y - 6) - 3 + 5y$

12 Expand and simplify by collecting like terms.

a $2(x + 7) + 4(x + 8)$ **b** $4(d + 5) + 3(d - 2)$ **c** $8(n - 3) + 7(n - 4)$
d $3(q - 6) + 9(q - 7)$ **e** $7(f - 8) + 2(f - 9)$ **f** $10(c - 6) + 2(c - 2)$
g $x(x + 5) + 2(x - 4)$ **h** $y(y - 6) + 4(y + 2)$ **i** $w(w - 8) + w(w - 9)$
j $5(2m + 7) + 3(4m - 8)$ **k** $4(3t + 6) + 3(2t + 4)$ **l** $9(2a - 1) + 10(4a + 7)$
m $10(4a - 2) + 2a(3a - 5)$ **n** $6c(c - 7) + 2c(c + 8)$ **o** $4d(3 - 2d) + 3d(2d + 1)$

Remember: When multiplying two terms:
If the signs are the same, the result is positive.
If the signs are different, the result is negative.

EXAMPLE 6

Expand the following.

a $-5(y + 6)$ **b** $-p(p - m)$ **c** $-(3x + 2y)$

a
$-5(y + 6)$
$= (-5) \times y + (-5) \times 6$
$= -5y - 30$

b
$-p(p - m)$
$= (-p) \times p - (-p) \times (m)$
$= -p^2 - (-pm)$
$= -p^2 + pm$

c
$-(3x + 2y)$
$= -1(3x + 2y)$
$= (-1) \times 3x + (-1) \times 2y$
$= -3x - 2y$

$-(3x + 2y)$ is the same as $-1 \times (3x + 2y)$.

13 Complete the following to expand.

a
$-4(y + 3)$
$= (\_\_\_) \times y + (\_\_\_) \times 3$
$= -4y + (-12)$
$= \_\_\_ - \_\_\_$

b
$-x(x - y)$
$= (\_\_\_) \times x - (\_\_\_) \times y$
$= \_\_\_ - (\_\_\_)$
$= \_\_\_ + \_\_\_$

c
$-(4k + 3m)$
$= (\_\_\_) \times 4k + (\_\_\_) \times 3m$
$= \_\_\_ + (\_\_\_)$
$= \_\_\_ - \_\_\_$

14 Expand the following.

a $-6(a + 10)$ **b** $-4(b + 8)$ **c** $-9(k + 9)$ **d** $-3(c - 3)$
e $-5(f - 7)$ **f** $-10(d - 6)$ **g** $-7(m + 5)$ **h** $-2(n + 10)$
i $-11(h + 11)$ **j** $-10(2p - 7)$ **k** $-8(3m - 3)$ **l** $-5(7q - 8)$

15 Expand the following.

a $-p(p + 7)$ **b** $-w(w + 8)$ **c** $-d(d + 11)$ **d** $-s(s - 3)$
e $-x(x - 6)$ **f** $-f(f - 14)$ **g** $-m(n + 5)$ **h** $-a(y + 2)$
i $-k(m + 10)$ **j** $-3t(2t - p)$ **k** $-4y(5y - c)$ **l** $-8n(8n - 4m)$

16 Expand the following.

a $-(x + 2)$ **b** $-(y + 3)$ **c** $-(a + 7)$ **d** $-(n - 11)$
e $-(g - 5)$ **f** $-(b - 4)$ **g** $-(6 + g)$ **h** $-(3 + k)$
i $-(l + 13)$ **j** $-(2p - 7)$ **k** $-(5n - 8)$ **l** $-(10d - 11)$

Extension

EXAMPLE 7

Expand and simplify by collecting like terms.

a $5(x + 3) - 4(x - 7)$ **b** $9m(m + 7n) - 3m(2n + 3m)$

a
$5(x + 3) - 4(x - 7)$
$= 5x + 15 - 4x + 28$
$= 5x - 4x + 15 + 28$
$= x + 43$

b
$9m(m + 7n) - 3m(2n + 3m)$
$= 9m^2 + 63mn - 6mn - 9m^2$
$= 9m^2 - 9m^2 + 63mn - 6mn$
$= 57mn$

17 Expand and collect like terms.

a $5(p + 7) + 3p$ **b** $12(c - 8) + 29$
c $4x + 7(x - 5) + 10$ **d** $6(d - 1) + 2d$
e $3(q - 4) + 2q + 9$ **f** $15 + 2(m - 7) - 5m$
g $10(n + 8) - (6n - 3)$ **h** $9a + 14 + 2(a - 9)$
i $16s - 17 - 5(s - 4) + 6$ **j** $9(x - 8) - (x + 12)$
k $11(w + 2) - (w - 2)$ **l** $3(z + 12) - (z + 18)$
m $2(d - 7) + 5(d - 8)$ **n** $4(k + 2) - 2(k + 3)$
o $8(p - 6) - 3(p - 10)$ **p** $y(y + 8) - y(y - 9)$
q $n(n - 3) - 2(n - 6)$ **r** $w(w + 4) - 5(w - 7)$
s $5c(2c - 6) - 3c(c - 7)$ **t** $8a(2a - 1) - 2a(3a + 4)$
u $10d(d + 2) - 7d(2d - 4)$ **v** $4f - 7(f + 6) - 3(f - 10)$
w $5c(c + 7) - 8(c - 9)$ **x** $12 + 3(n - 1) - 2(n - 6)$

18 Expand and simplify each expression by collecting like terms.

a $4(x + 7) - 3(x - 5) + 2(x - 9)$ **b** $2(c + 13) - 5(c + 4) + 9(c - 6)$
c $8(n - 6) + 12 - 5n - 4(n - 14)$ **d** $12f + 30 + 4(f - 12) + 11 - 9f$
e $-5(d - 11) - 8(d + 7) - 2(d - 5)$ **f** $-(p - 7) - 3(p + 5) + 17 - 10p$
g $-7y(y - 4) - 6y(2y + 8) + 12y$ **h** $2a(a - 3) + 5a(a + 6) - 3a(9 - 2a)$
i $-9k + 15 + 3k(4 - 2k) - 6k(7 + 2k)$ **j** $4b(2c + 8b) - 2c(5b - 7c) + 2b(9c - 3b)$

E Highest common factor

The factors of a given number are the numbers that will divide into the given number exactly (that is leaving no remainder). The common factors of two given numbers are the numbers that are factors of both given numbers. The highest common factor (HCF) of two numbers is the largest of the common factors.

EXAMPLE 1

Find the HCF of 18 and 24.

The factors of 18 are 1, 2, 3, 6, 9, 18:
$18 \div 1 = 18$, $18 \div 2 = 9$, $18 \div 3 = 6$, $18 \div 9 = 2$ and $18 \div 18 = 1$
(from $18 = 1 \times 18$, $18 = 2 \times 9$, $18 = 3 \times 6$)
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24:
$24 \div 1 = 24$, $24 \div 2 = 12$, $24 \div 3 = 8$, $24 \div 4 = 6$, $24 \div 6 = 4$, $24 \div 8 = 3$, $24 \div 12 = 2$, $24 \div 24 = 1$
(from $24 = 1 \times 24$, $24 = 2 \times 12$, $24 = 3 \times 8$, $24 = 4 \times 6$)
The common factors of 18 and 24 are 1, 2, 3, 6.
The HCF of 18 and 24 = 6.

Exercise 11E

1 **a** Complete the following to find the HCF of 12 and 20.
The factors of 12 are _____.
The factors of 20 are _____.
The common factors of 12 and 20 are _____.
The HCF of 12 and 20 is ___.

b Complete the following to find the HCF of 16 and 24.
The factors of 16 are _____.
The factors of 24 are _____.
The common factors of 16 and 24 are _____.
The HCF of 16 and 24 is ___.

2 Find the HCF of these pairs of numbers.

a 12 and 15	**b** 10 and 8	**c** 27 and 33	**d** 21 and 3	**e** 10 and 20
f 16 and 24	**g** 27 and 18	**h** 30 and 40	**i** 36 and 48	**j** 48 and 60

EXAMPLE 2

Determine the HCF of these algebraic terms.
a $18x$ and 24 **b** $16k$ and $8m$

a Find the HCF of the numerical parts of each term.
HCF of 18 and 24 = 6
x is not a factor of both terms, so HCF of $18x$ and 24 = 6

b Find the HCF of the numerical parts of each term.
HCF of 16 and 8 = 8
k and m have no common factors, so HCF of $16k$ and $8m$ = 8

3 **a** Find the HCF of the numerical parts of $12m$ and $18n$.
HCF of 12 and 18 is ___.
HCF of $12m$ and $18n$ is ___.
b Find the HCF of the numerical parts of $16x$ and $20y$.
HCF of 16 and 20 is ___.
HCF of $16x$ and $20y$ is ___.

4 Find the HCF of each pair of terms.

a $8a$ and 12	**b** $18x$ and $15y$	**c** $10a$ and $15b$	**d** $6m$ and $12n$
e $12g$ and 8	**f** $9w$ and 27	**g** $15t$ and $10v$	**h** $24p$ and $12q$

F Factorising algebraic expressions

Using the distributive law to expand: $4(x + 5) = 4 \times x + 4 \times 5$
$= 4x + 20$

Reversing the process: $4x + 20 = 4 \times x + 4 \times 5$
$= 4 \times (x + 5)$ or $4(x + 5)$

The algebraic expression $4x + 20$ has been factorised because it has been written as the product of its factors 4 and $(x + 5)$.

EXAMPLE 1

Factorise the following.

a $5 \times y + 5 \times 3$ **b** $7 \times 3p - 7 \times 2q$

a $5 \times y + 5 \times 3$
$= 5 \times (y + 3)$ or $5(y + 3)$

b $7 \times 3p - 7 \times 2q$
$= 7 \times (3p - 2q)$ or $7(3p - 2q)$

Exercise 11F

1 Complete the following.
a $4 \times k + 4 \times 3 =$ ___ $\times (k + 3)$ or ___$(k + 3)$
b $5 \times a + 5 \times b =$ ___ $\times (a + b)$ or ___$(a + b)$
c $3 \times 2k - 3 \times 5m =$ ___ $\times (2k - 5m)$ or___
d $8 \times 3p - 8 \times 2q =$ ___ $\times (3p - 2q)$ or ___
e $6 \times a^2 + 6 \times d =$ ___ $\times (a^2 + d)$ or ___
f $5 \times t + 5 \times 4 = 5 \times ($___ + ___$)$ or ___
g $4 \times b + 4 \times c = 4 \times ($___ + ___$)$ or ___
h $3 \times 4m + 3 \times 5n = 3 \times ($___ + ___$)$ or ___
i $7 \times 3x - 7 \times 4y = 7 \times ($___ − ___$)$ or ___
j $9 \times 6w - 9 \times 5v = 9 \times ($___ − ___$)$ or ___

2 Factorise the following.

a $3 \times w + 3 \times 5$	**b** $5 \times b + 5 \times 7$	**c** $4 \times a - 4 \times b$	**d** $10 \times m - 10 \times n$
e $2 \times 5p + 2 \times 3q$	**f** $3 \times 4d + 3 \times 9e$	**g** $7 \times 4w - 7 \times 2v$	**h** $9 \times 5x + 9 \times 4y$
i $6 \times 7g + 6 \times 3k$	**j** $8 \times 7t - 8 \times v$	**k** $6 \times 5m - 6 \times m^2$	**l** $15 \times y^2 + 15 \times z^2$

EXAMPLE 2

Factorise these expressions.

a $15a + 10$ **b** $12a - 9b$

a *Step 1:* Find the HCF of the numerical parts of each term:
HCF of 15 and 10 = 5
Step 2: Factorise each term using the HCF as one factor:
$15a = 5 \times 3a$ and $10 = 5 \times 2$
Hence $15a + 10 = 5 \times 3a + 5 \times 2$
$= 5(3a + 2)$

b *Step 1:* Find the HCF of the numerical parts of each term:
HCF of 12 and 9 = 3
Step 2: Factorise each term using the HCF as one factor:
$12a = 3 \times 4a$ and $9b = 3 \times 3b$
Hence $12a - 9b = 3 \times 4a - 3 \times 3b$
$= 3(4a - 3b)$

3 Complete the following to factorise each expression.

a $9z + 12$
Step 1: Find the HCF of the numerical parts of each term:
HCF of 9 and 12 = ___
Step 2: Factorise each term using the HCF as one factor:
$9z =$ ___ $\times$ ___ and $12 =$ ___ $\times$ ___
Hence $9z + 12 =$ ___ $\times$ ___ + ___ $\times$ ___
= ___(___ + ___)

b $20w - 25y$
Step 1: Find the HCF of the numerical parts of each term:
HCF of 20 and 25 = ___
Step 2: Factorise each term using the HCF as one factor:
$20w =$ ___ $\times$ ___ and $25y =$ ___ $\times$ ___
Hence $20w - 25y =$ ___ $\times$ ___ − ___ $\times$ ___
= ___(___ − ___)

4 Factorise the following.

a $8a + 10$	**b** $9d + 6z$	**c** $8q + 16r$	**d** $2k - 12$
e $3l - 15m$	**f** $4 + 6d$	**g** $9h - 9k$	**h** $10 - 6c$
i $14w + 20x$	**j** $10p - 20$	**k** $22 - 11x$	**l** $12y + 15z$
m $10a - 25b$	**n** $12p + 8q$	**o** $16k - 24m$	**p** $7t + 7$
q $18x + 12y$	**r** $30v - 20w$	**s** $20 - 15z$	**t** $8x + 6y + 4z$

EXAMPLE 3

Factorise the following by finding a negative common factor.

a $-3x + 6$ **b** $-8d - 6p$

a *Step 1:* Find the highest negative common factor of the numerical parts of each term:
HCF of −3 and 6 = −3

EXAMPLE 3 CONTINUED

Step 2: Factorise each term using the negative HCF as one factor:

$-3x = -3 \times x$ and $6 = -3 \times -2$

Hence $-3x + 6 = -3 \times x + -3 \times -2$

$= -3(x + -2)$

$= -3(x - 2)$

Adding −2 is the same as subtracting 2.

b *Step 1:* Find the highest negative common factor of the numerical parts of each term:

HCF of -8 and $-6 = -2$

Step 2: Factorise each term using the negative HCF as one factor:

$-8d = -2 \times 4d$ and $-6 = -2 \times 3p$

Hence $-8d - 6p = -8d + -6p$

$= -2 \times 4d + -2 \times 3p$

$= -2(4d + 3p)$

Subtracting 6p is the same as adding −6p.

5 Complete the following to factorise by finding a negative common factor.

a $-5x + 10$

Step 1: Find the highest negative common factor of the numerical parts of each term:

HCF of -5 and $10 =$ ___

Step 2: Factorise each term using the negative HCF as one factor:

$-5x = -$___ $\times$ ___ and $10 = -$___ $\times$ ___

Hence $-5x + 10 = -$___ $\times$ ___ $+ -$___ $\times$ ___

$= -$___(___ $+$ ___)

$= -$___(___ $-$ ___)

b $-12p - 4q$

Step 1: Find the highest negative common factor of the numerical parts of each term:

HCF of -12 and $-4 = -$___

Step 2: Factorise each term using the negative HCF as one factor:

$-12p = -$___ $\times$ ___ and $-4q = -$___ $\times$ ___

Hence $-12p - 4q = -12p +$ ___

$= -$___ $\times$ ___ $+ -$___ $\times$ ___

$= -$___(___ $+$ ___)

6 Factorise the following by finding a negative common factor.

a $-10x + 5$ **b** $-6d + 4f$ **c** $-4p - 2$ **d** $-3y - 9z$ **e** $-14p - 7$ **f** $-20c + 15q$

EXAMPLE 4

Factorise the following.

a $ab + bc$ **b** $x^2 + x$

a *Step 1:* The factors of ab are 1, a, b, ab.

The factors of bc are 1, b, c, bc.

Hence the HCF of ab and bc is b.

Step 2: $ab = b \times a$

$bc = b \times c$

Hence $ab + bc = b \times a + b \times c$

$= b \times (a + c)$

$= b(a + c)$

b *Step 1:* The factors of x^2 are 1, x, x^2.

The factors of x are 1, x.

Hence the HCF of x^2 and x is x.

Step 2: $x^2 = x \times x$

$x = x \times 1$

Hence $x^2 + x = x \times x + x \times 1$

$= x \times (x + 1)$

$= x(x + 1)$

7 Complete the following to factorise the expressions.

a $pq + qt$

Step 1: The factors of pq are _____.
The factors of qt are _____.
Hence the HCF of pq and qt is ___.

Step 2: $pq =$ ___ $\times$ ___ and $qt =$ ___ $\times$ ___
Hence $pq + qt =$ ___ $\times$ ___ + ___ $\times$ ___
= ___ $\times$ (___ + ___)
= ___(___ + ___)

b $k^2 - k$

Step 1: The factors of k^2 are _____.
The factors of k are _____.
Hence the HCF of k^2 and k is ___.

Step 2: $k^2 =$ ___ $\times$ ___ and $k =$ ___ $\times$ ___
Hence $k^2 - k =$ ___ $\times$ ___ − ___ $\times$ ___
= ___ $\times$ (___ − ___)
= ___(___ − ___)

8 Factorise these expressions.

a $ab + ac$ **b** $xy - yz$ **c** $km - mn$ **d** $y^2 + y$ **e** $m^2 - m$

EXAMPLE 5

Factorise these expressions.

a $ab + 3b$ **b** $x^2 + 4x$

a *Step 1:* The factors of ab are 1, a, b, ab.
The factors of $3b$ are 1, 3, b, $3b$.
Hence the HCF of ab and $3b$ is b.

Step 2: $ab = b \times a$
$3b = b \times 3$
Hence $ab + 3b = b \times a + b \times 3$
$= b \times (a + 3)$
$= b(a + 3)$

b *Step 1:* The factors of x^2 are 1, x, x^2.
The factors of $4x$ are 1, 2, 4, x, $2x$, $4x$.
Hence the HCF of x^2 and $4x$ is x.

Step 2: $x^2 = x \times x$
$4x = x \times 4$
Hence $x^2 + 4x = x \times x + x \times 4$
$= x \times (x + 4)$
$= x(x + 4)$

9 Complete the following to factorise the expressions.

a $xy + 5x$

Step 1 The factors of xy are _____.
The factors of $5x$ are _____.
Hence the HCF of xy and $5x$ is _____.

Step 2 $xy =$ ___ $\times$ ___ and $5x =$ ___ $\times$ ___
Hence $xy + 5x =$ ___ $\times$ ___ + ___ $\times$ ___
= ___ $\times$ (___ + ___)
= ___(___ + ___)

b $k^2 - 6k$

Step 1 The factors of k^2 are _____.
The factors of $6k$ are _____.
Hence the HCF of k^2 and $6k$ is ___.

Step 2 $k^2 = \_\_\_ \times \_\_\_$ and $6k = \_\_\_ \times \_\_\_$

Hence $k^2 - 6k = \_\_\_ \times \_\_\_ - \_\_\_ \times \_\_\_$

$= \_\_\_ \times (\_\_\_ - \_\_\_)$

$= \_\_\_(\_\_\_ - \_\_\_)$

10 Factorise these expressions.

a $xy + 5y$ **b** $pq - 7p$ **c** $ab + 4a$ **d** $m^2 - 3m$ **e** $k^2 - 8k$

EXAMPLE 6

Factorise the following expressions.

a $4xy + 6xz$ **b** $8x^2 - 12x$

a *Step 1:* The factors of $4xy$ are 1, 2, 4, x, $2x$, $4x$, y, $2y$, $4y$, xy, $2xy$, $4xy$.

The factors of $6xz$ are 1, 2, 3, 6, x, $2x$, $3x$, $6x$, z, $2z$, $3z$, $6z$, xz, $2xz$, $3xz$, $6xz$.

Hence the HCF of $4xy$ and $6xz$ is $2x$

We can simplify this process by combining the HCF of 4 and 6 ($= 2$) with the HCF of xy and xz ($= x$) to give HCF of $4xy$ and $6xz = 2x$.

Step 2 $4xy = 2x \times 2y$ and $6xz = 2x \times 3z$

Hence $4xy + 6xz = 2x \times 2y + 2x \times 3z$

$= 2x \times (2y + 3z)$

$= 2x(2y + 3z)$

b *Step 1* The HCF of 8 and 12 is 4.

The HCF of x^2 and x is x.

Hence the HCF of $8x^2$ and $12x$ is $4x$

Step 2 $8x^2 = 4x \times 2x$ and $12x = 4x \times 3$

Hence $8x^2 - 12x = 4x \times 2x - 4x \times 3$

$= 4x \times (2x - 3)$

$= 4x(2x - 3)$

11 Complete the following to factorise the expressions.

a $6xy + 12xz$

Step 1 The HCF of 6 and 12 is ___.

The HCF of xy and xz is ___.

Hence the HCF of $6xy$ and $12xz$ is ___.

Step 2 $6xy = \_\_\_ \times \_\_\_$ and $12xz = \_\_\_ \times \_\_\_$

Hence $6xy + 12xz = \_\_\_ \times \_\_\_ + \_\_\_ \times \_\_\_$

$= \_\_\_ \times (\_\_\_ + \_\_\_)$

$= \_\_\_(\_\_\_ + \_\_\_)$

b $10k^2 - 15k$

Step 1 The HCF of 10 and 15 is ___.

The HCF of k^2 and k is ___.

Hence the HCF of $10k^2$ and $15k$ is ___.

Step 2 $10k^2 = \_\_\_ \times \_\_\_$ and $15k = \_\_\_ \times \_\_\_$

Hence $10k^2 - 15k = \_\_\_ \times \_\_\_ - \_\_\_ \times \_\_\_$

$= \_\_\_ \times (\_\_\_ - \_\_\_)$

$= \_\_\_(\_\_\_ - \_\_\_)$

12 Factorise the following.

a $4ab + 6ac$	**b** $12pq - 8qt$	**c** $5km + 10k$
d $9m - 12mn$	**e** $8t + 16tw$	**f** $5k^2 - 15k$
g $6m^2 - 9m$	**h** $20a^2 - 25a$	**i** $8k + 10k^2$
j $14p - 18p^2$	**k** $16pq + 20pr$	**l** $8xy - 24yz$

13 Factorise the following.

a $5xyz - 7xz$	**b** $24a^2bc + 18ab$	**c** $12a + 9b + 15c$
d $5x^2 - 10x + 20$	**e** $6ab + 10bc + 4abc$	**f** $16xy^2z - 24xyz$
g $x^2y^3 + x^3y^2$	**h** $7a^2b^3 - 4ab^2$	**i** $-ab^2 - a^2$
j $-8k^2mn^3 + 2km^2n$	**k** $-12abc - 9bcd$	**l** $-18xyz + 12x^2yz$

G Word problems

An important part of solving word problems in algebra is to be able to translate certain key words into mathematical operations. A list of some words and the corresponding mathematical operation is shown in the table below.

Key words	Mathematical operation
added to, sum, total of, more than, increased by, combined	$+$
subtracted from, difference between, less than, decreased by	$-$
multiplied by, times, product, increased or decreased by a factor of	$\times$
divided by, quotient, out of, per	$\div$

EXAMPLE 1

Write algebraic expressions for each statement. Simplify where possible.

a the sum of $5k$ and $3m$ **b** $2w$ more than $9w$ **c** $4x$ increased by $6x$

a The sum of 2 and 7 is $2 + 7 = 9$,
hence the sum of $5k$ and $3m$ is $5k + 3m$.

b 5 more than 7 is $7 + 5 = 12$,
hence $2w$ more than $9w$ is $9w + 2w = 11w$.

c 6 increased by 2 is $6 + 2 = 8$,
hence $4x$ increased by $6x$ is $4x + 6x = 10x$.

Exercise 11G

1 Write algebraic expressions for each statement. Simplify where possible.

a the sum of $2a$ and 3b	**b** the sum of $6x$ and $7x$
c k more than m	**d** $4t$ more than $8t$
e $2x$ increased by $4y$	**f** $7y$ increased by $8y$
g $7p$ is added to $5p$	**h** $6a$ and $3a$ combined

EXAMPLE 2

Write algebraic expressions for each statement. Simplify where possible.

a the difference between $9m$ and $4n$ **b** $6z$ decreased by $2z$

c 4ab less than 7ab

a The difference between 10 and 6 is $10 - 6 = 4$,
hence the difference between $9m$ and $4n$ is $9m - 4n$.

b 10 decreased by 4 is $10 - 4 = 6$,
hence $6z$ decreased by $2z$ is $6z - 2z = 4z$.

c 5 less than 7 is $7 - 5 = 2$,
hence $4ab$ less than $7ab$ is $7ab - 4ab = 3ab$.

Note: We assume here that 'the difference between' means the order 'the first quantity minus the second quantity'.

Note the order of subtraction. For example, if Peter earns \$5 per hour less than you, his hourly rate is your rate minus \$5.

2 Write algebraic expressions for each statement. Simplify where possible.

a the difference between $2w$ and $3v$ **b** the difference between $8k$ and $5k$

c $4x$ decreased by $6y$ **d** $9t$ decreased by $4t$

e $2s$ less than $8r$ **f** $5u$ less than $9u$

EXAMPLE 3

Write algebraic expressions for each statement. Simplify where possible.

a the product of $5k$ and $3m$ **b** $4a$ increased by a factor of 3

a The product of 4 and 7 is $4 \times 7 = 28$,
hence the product of $5k$ and 3m is $5k \times 3m = 15km$.

b 5 increased by a factor of 2 is $5 \times 2 = 10$,
hence $4a$ increased by a factor of 3 is $4a \times 3 = 12a$.

3 Write algebraic expressions for each statement. Simplify where possible.

a the product of $2w$ and 3 **b** the product of $6d$ and $2e$

c the product of $4a$ and $5b$ **d** $5q$ is increased by a factor of 2

e $6x$ is increased by a factor of 5 **f** $2y$ is increased by a factor of $3z$

EXAMPLE 4

Write algebraic expressions for each statement. Simplify where possible.

a the quotient of $4x$ and $3y$ **b** $7t$ out of $9t$

a The quotient of 20 and 5 is $20 \div 5 = 4$, hence the quotient of $4x$ and $3y$ is $4x \div 3y = \frac{4x}{3y}$.

b 6 out of 8 is $\frac{6}{8} = \frac{3}{4}$, hence $7t$ out of $9t$ is $\frac{7t}{9t} = \frac{7}{9}$.

4 Write algebraic expressions for each statement. Simplify where possible.

a the quotient of $4x$ and $5y$ **b** the quotient of $3a$ and $11a$

c the quotient of $5t$ and 15 **d** $2w$ out of 7

e 6 out of $8p$ **f** $4ab$ out of $12b^2$

5 Write algebraic expressions for:

i the sum of **ii** the difference between

iii product of **iv** quotient of

Simplify where possible.

a $5x$ and $3y$ **b** $5x$ and $4x$ **c** $5xy$ and $6x$

d $(x + 4)$ and 3 **e** $(2x + 5)$ and $2x$ **f** $4a$ and $(2a + 1)$

6 Write algebraic expressions for each statement. Simplify where possible.

a $7xy$ increased by $2xy$ **b** $8ab$ decreased by $3ab$

c $2k$ more than $9k$ **d** $3w$ less than $7w$

e $5xy$ increased by a factor of 3 **f** $5k + 3m$ increased by a factor of $2k$

g the quotient of $12x$ and $15xy$ **h** $7km$ out of $8k$

i 4 less than the product of 3 and k **j** 3 more than the difference between m and 7

7 **a** Yvonne is $(m + 2)$ years old.

i How old will she be in 5 years time?

ii How old was she 3 years ago?

b Yvonne's mother is double Yvonne's age at present.

i How old is Yvonne's mother now?

ii How old will Yvonne's mother be in 3 years time?

iii How old was Yvonne's mother 6 years ago?

8 Write an algebraic expression for (and simplify where possible) the perimeter and area of a rectangle given its length and breadth in the table.

Length	Breadth	Perimeter	Area
a	b		
$3x$	$2y$		
4	$z + 2$		
y	$x - 2$		
$a + 3$	7		

9 Write an algebraic expression for (and simplify where possible) the speed of a car that travels

a x kilometres in 3 hours **b** 120 km in y hours

c $10a$ kilometres in 5 hours **d** $24mn$ kilometres in $6n$ hours

10 Write algebraic expressions for the following statements.

a the cost of x kg of mince steak at \$$y$ per kg

b 10% of x

c the cost of a dress with marked price \$$x$ if a 10% discount is allowed

d four consecutive integers starting with n

e three consecutive even integers starting with n

Language in mathematics

1 Explain in words, with examples, the meaning of these terms.

a like and unlike terms
b highest common factor
c expand
d factorise

2 Illustrate three different sets of grouping symbols used in Mathematics and list their names.

3 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.
numericle, distributive, symbal, remainder, simplify, algebrake

Terms

algebra	algebraic	common	distributive law	expand
factor	factorising	grouping symbols	highest common factor	
like terms	pronumeral	numerical	remainder	unlike terms

Check your skills

1 $4a + 3a =$
A $7aa$ **B** $43a$ **C** $7a$ **D** $7aa$

2 $5k - k =$
A 4 **B** 5 **C** $4kk$ **D** $4k$

3 $3w - 5 + 4w - 1 =$
A $7w - 6$ **B** $7w - 4$ **C** $-w - 6$ **D** $-w - 4$

4 $10p + 15 + 6p - 20 =$
A $16p - 7$ **B** $16p + 5$ **C** $16p - 35$ **D** $16p - 5$

5 $-4x \times -5y =$
A $-20xy$ **B** $45xy$ **C** $20xy$ **D** $9xy$

6 $8 \div 2k =$
A $4k$ **B** $\frac{4}{k}$ **C** $\frac{1}{4k}$ **D** $\frac{k}{4}$

7 $6(3p + 5)$ when expanded is:
A $18p + 5$ **B** $18p + 30$ **C** $3p + 30$ **D** $63p + 65$

8 $3x(2y - 5z)$ when expanded is:
A $6xy - 15xz$ **B** $3xy - 15xz$ **C** $6xy - 5xz$ **D** $5xy - 8xz$

9 $-2(t + 3)$ when expanded is:
A $-2t + 6$ **B** $-2t + 1$ **C** $-2t - 3$ **D** $-2t - 6$

10 The highest common factor of 12 and 24 is:
A 12 **B** 24 **C** 4 **D** 6

11 The highest common factor of $8x$ and $20y$ is:

A $4x$ **B** $4y$ **C** 4 **D** $4xy$

12 When factorised $9p + 12$ is:

A $9(p + 12)$ **B** $3(3p + 12)$ **C** $3(3p + 4)$ **D** $3p(3p + 3)$

13 When factorised $8m - 4n$ is:

A $4m(m - n)$ **B** $4(2m - n)$ **C** $4(2m - 4n)$ **D** $8(m - 4n)$

14 When factorised $8k^2 - 12k$ is:

A $8k(k - 12)$ **B** $4k(2k - 3)$ **C** $4k^2(2 - 3k)$ **D** $4k(2 - 3k)$

15 An algebraic expression for $2y$ less than $7x$ is:

A $5xy$ **B** 5 **C** $7x - 2y$ **D** $2y - 7x$

16 An algebraic expression for 2 more than the product of a and b is:

A $ab + 2$ **B** $2ab$ **C** $a + b + 2$ **D** $2(a + b)$

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–4	5	6	7–9	10, 11	12–14	15, 16
Section	A	B	C	D	E	F	G

11A Review set

1 Simplify:

a $4s + 7s$ **b** $10k - 4k$ **c** $5w + 3 + 2w + 8$

d $5p - 2q + 3p$ **e** $7x + 1 - 4x + 7$ **f** $3a - 2b - 4a + b$

g $4(g + 5) + 3(g - 2)$

2 Simplify:

a $4x \times 5y$ **b** $-2p \times 7$ **c** $-3a \times 6b$

3 Simplify:

a $\frac{8h}{2}$ **b** $\frac{g}{3g}$ **c** $\frac{12a}{4a}$ **d** $\frac{14w}{21vw}$ **e** $\frac{6a}{9ab}$

4 Expand:

a $3(5m + 3)$ **b** $2a(3a - 4b)$ **c** $-5(k - 6)$

5 Find the highest common factor of:

a 16 and 18 **b** $12a$ and $9b$ **c** $18x$ and $12y$ **d** $9x$ and $3x^2$

6 Factorise the following.

a $6k + 9$ **b** $8t - 16$ **c** $24w + 18v$

d $-4s - 6$ **e** $5ab + 10a$ **f** $9p^2 - 12p$

7 Write an algebraic expression for each statement. Simplify where possible.

a the sum of x and 7
b the difference between $6x$ and $2x$
c the product of $2k$ and $3k$
d the quotient of $15t$ and 5
e increase $5m$ by 2
f increase $5m$ by a factor of 2
g decrease $8z$ by $3z$
h 3 less than $10p$

11B Review set

1 Simplify:

a $5q + 4q$
b $9m - 4m$
c $4a + 7 + 2a + 5$
d $7x - 4y + x$
e $6m - 5 - 3m + 9$
f $5w - 3z - 6w + 4z$
g $3(2h + 5) + 2(4h - 3)$

2 Simplify:

a $5k \times 4m$
b $-3d \times 5c$
c $-2m \times -3n$

3 Simplify:

a $\frac{9y}{3}$
b $\frac{a}{5a}$
c $\frac{10x}{5x}$
d $\frac{9ab}{12a}$
e $\frac{10xy}{10y}$

4 Expand:

a $4(5x + 7)$
b $3a(5a - 2b)$
c $-5(2m - 3)$

5 Find the highest common factor of these terms.

a 24 and 18
b $8w$ and $6v$
c $16a$ and $12b$
d $12a$ and $4ab$

6 Factorise the following.

a $8t + 12$
b $10k - 15$
c $18x + 12y$
d $-6t + 9$
e $4b^2 - 3b$
f $10pqr + 15pqt$

7 Write an algebraic expression for each statement. Simplify where possible.

a the total of $3h$ and $6h$
b $2y$ less than $6y$
c $4t$ out of $10t$
d the product of $6x$ and $5y$
e $4q$ more than $11q$
f $2ab$ increased by a factor of 3
g the quotient of $12w$ and $4w$
h the product of x and y decreased by 8

11C Review set

1 Simplify:

a $10h + h$
b $9d - d$
c $8z + 9 + 2z + 3$
d $7p - 3q - 2q$
e $4a - 3 + 3a - 4$
f $x - 3y - 4x + 2y$
g $5(2t + 3) + 4(3t - 7)$

2 Simplify:

a $5r \times 3s$
b $4g \times -3$
c $-4p \times -4q$

3 Simplify:

a $\frac{6y}{2}$
b $\frac{w}{3w}$
c $\frac{10a}{2a}$
d $\frac{12p}{15pq}$
e $\frac{8ab}{12b}$

4 Expand:

a $6(3v - 1)$ **b** $5x(2x + 3y)$ **c** $-3(4d - 5e)$

5 Find the highest common factor of:

a 24 and 36 **b** $21s$ and $24t$ **c** $28x$ and $35y$ **d** $9a^2$ and $12a$

6 Factorise the following.

a $12p - 18$ **b** $15t + 25u$ **c** $20k - 16m$

d $-5q - 10$ **e** $18km - 12kn$ **f** $6a^2b + 15ab^2$

7 Write an algebraic expression for each statement. Simplify where possible.

a the difference between $7xy$ and $2x$ **b** $5t$ more than $8t$

c increase $4z$ by a factor of $3w$ **d** $2p$ less than $9p$

e $5ab$ out of $10abc$ **f** decrease $10pq$ by $6pq$

g the sum of $2a + 3b$ and $a + 5b$ **h** the product of 3 and $x + y$

11D Review set

1 Simplify:

a $7k + k$ **b** $9m - 3m$ **c** $6b + 7e - 4b + 2e$

d $10 - 4t - 2$ **e** $8a - 4b - 7a + 7b$ **f** $12x - 5(2x - 3)$

g $5(3w - 7) - 2(4w + 1)$

2 Simplify:

a $7p \times 5$ **b** $-3a \times 9b$ **c** $-4x \times 3y \times -5z$

3 Simplify:

a $\frac{8k}{2}$ **b** $\frac{3a}{a}$ **c** $\frac{10}{4x}$ **d** $\frac{-5z}{15wz}$ **e** $\frac{-12k}{-4kmn}$

4 Expand:

a $6(2a - 3)$ **b** $-4x(5x + 7)$ **c** $-3ab(2ab - 7a)$

5 Find the highest common factor of:

a 36 and 48 **b** $18x$ and $24y$ **c** $16m$ and $12m$ **d** $24xy$ and $9x$

6 Factorise the following.

a $9k + 15$ **b** $12m - 8n$ **c** $-6t + 9$

d $-10x - 5y$ **e** $5a^2 - 7a$ **f** $12abc + 15ac$

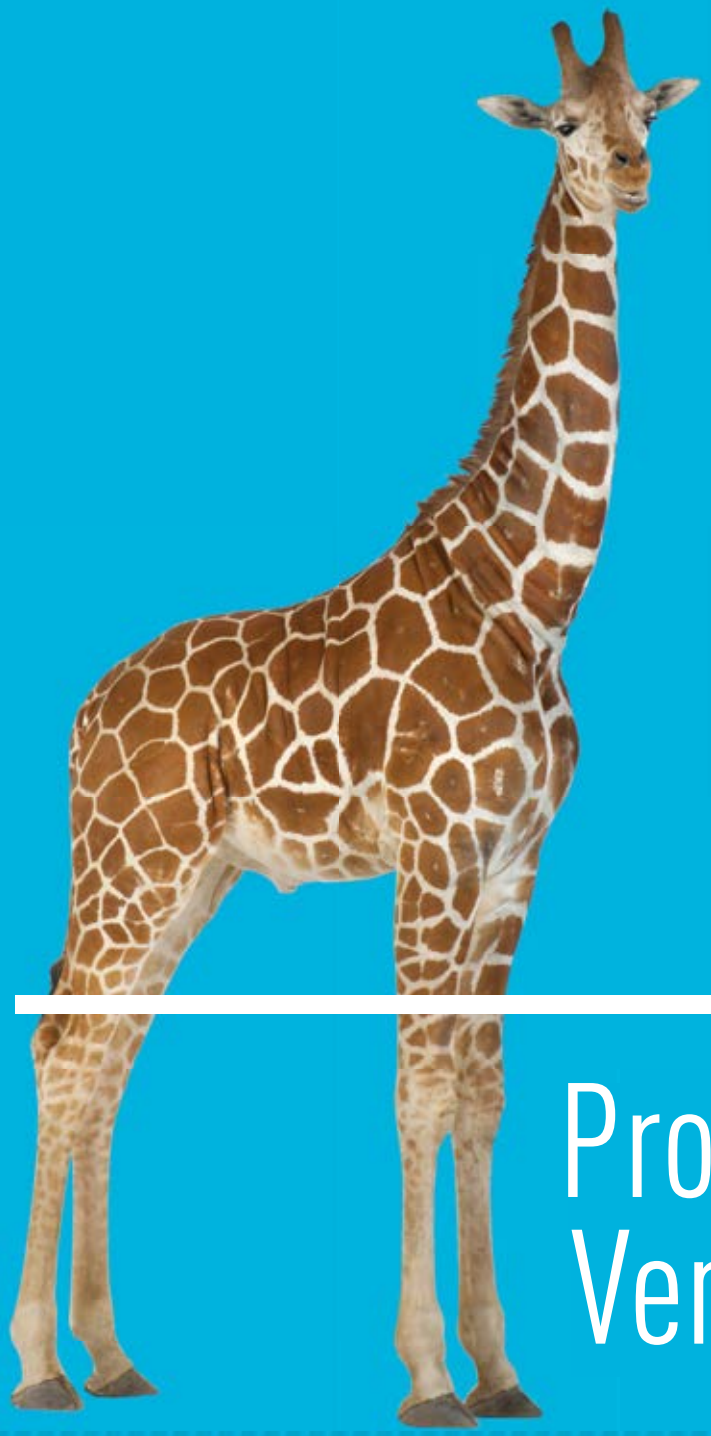
7 Write an algebraic expression for each statement. Simplify where possible.

a $3x$ more than the difference between x and 7 **b** $2k$ less than the sum of $4k$ and 3

c increase $m + n$ by a factor of 3 **d** p more than the product of $3p$ and 5

e the quotient of $8wz$ and $10w$ **f** $7n$ less than $3n$

g $9km$ out of $15k^2$ **h** 25% of y



12

Probability and Venn diagrams

This chapter deals with further ideas in chance.

At the end of this chapter you should be able to:

- identify complementary events and use the sum of probabilities to solve problems
- describe events using language of 'at least', exclusive 'or' (*A* or *B* but not both), inclusive 'or' (*A* or *B* or both) and 'and'
- represent such events in two-way tables and Venn diagrams and solve related problems.

NSW Syllabus references: S4 S&P Probability 2

Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-21SP

STATISTICS & PROBABILITY – ACMSP204, ACMSP205, ACMSP292

Diagnostic test

1 A normal six-sided die is thrown once. The number of outcomes in the sample space is:
A 1 **B** 3 **C** 5 **D** 6

2 A ticket is selected at random from a hat containing 1 blue, 2 red and 1 green ticket. The sample space is:
A {blue, red, green}
B {red}
C {blue, red, red, green}
D {not red}

3 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of randomly selecting a purple ball is:
A 1 **B** 3 **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

4 The probability of an event is $\frac{2}{5}$. As a percentage this is:
A 20% **B** 40%
C 50% **D** none of these

5 A hat contains 3 blue and 5 black tickets. If one ticket is chosen at random from the hat, the probability that it is blue is:
A $\frac{1}{2}$ **B** $\frac{3}{5}$ **C** 3 **D** $\frac{3}{8}$

6 A letter is chosen randomly from the word PROBABILITY. The probability that the letter chosen is B is:
A $\frac{1}{11}$ **B** $\frac{2}{11}$ **C** 1 **D** $\frac{2}{9}$

7 A term describing a probability of about 80% is:
A certain **B** highly probable
C evens **D** low probability

8 A coin is tossed once. An impossible event would be getting:
A heads **B** tails
C heads or tails **D** a six

9 When a normal six-sided die is rolled, the probability of getting a 7 is:
A $\frac{7}{6}$ **B** $\frac{6}{7}$ **C** $\frac{1}{6}$ **D** 0

10 If an event is certain to happen, its probability is:
A 0 **B** $\frac{1}{2}$ **C** 1 **D** 2

The Diagnostic test questions refer to the Year 7 outcomes from ACMSP167 and ACMSP168.

A Probability review

The sample space (S) of an experiment is the set of all possible outcomes. For example, if a normal six-sided die is rolled, S = {1, 2, 3, 4, 5, 6}.

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

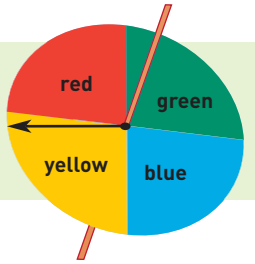
For convenience this may be written as $P(E) = \frac{n(E)}{n(\text{S})}$

where $P(E)$ = probability of the event E
$n(E)$ = number of outcomes favourable to event E
$n(\text{S})$ = number of possible outcomes.

EXAMPLE 1

A spinner is divided into 4 equal portions coloured red, blue, green and yellow. If the spinner is spun, find the probability that it lands on:

a red **b** green **c** blue or yellow



Sample space, S = {R, B, G, Y} Number of possible outcomes, $n(\text{S}) = 4$

a Favourable outcomes = {R} Number of favourable outcomes = 1
$P(\text{R}) = \frac{1}{4}$

b Favourable outcomes = {G} Number of favourable outcomes = 1
$P(\text{R}) = \frac{1}{4}$

c Favourable outcomes = {B, Y} Number of favourable outcomes = 2
$P(\text{B or Y}) = \frac{2}{4} = \frac{1}{2}$

Exercise 12A

1 A bag contains 1 red (R), 1 blue (B), 1 green (G), 1 yellow (Y) and 1 white (W) marble. One marble is chosen at random from the bag. Complete to find the following probabilities.

S = {_____} Number of possible equally likely outcomes = _____

a Probability that the marble is red:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{R})$ = _____

b Probability that the marble is green:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{G})$ = _____

c Probability that the marble is red or green:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{R or G})$ = _____

d Probability that the marble is blue or white:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{B or W})$ = _____

e Probability that the marble is blue or yellow or white:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{B or Y or W})$ = _____

2 A normal six-sided die is rolled. Find the probability of getting:

a 6 **b** 3 **c** 3 or 6
d an even number **e** an odd number **f** a number > 3

3 The numbers 0, 1, 2, 3, … 9 are written on 10 cards. The cards are shuffled and one is selected at random. Find the probability that it shows:

a 3 **b** 9 **c** 3 or 9
d an even number **e** an odd number **f** a number > 6
g a multiple of 4

4 The numbers 1, 2, 3, …7 are written on cards. The cards are shuffled and one is selected at random. Find the probability that it is:

a 4 **b** 5 **c** 4 or 5
d an even number **e** an odd number **f** a number > 4

5 The letters of the alphabet are written on cards. The cards are shuffled and one is selected at random. Find the probability that the letter shown is:

a K **b** Z **c** K or Z
d a vowel **e** a consonant **f** a letter of the word RANDOM

EXAMPLE 2

A hat contains 3 red, 4 blue and 5 green tickets. If one ticket is chosen at random, what is the probability that it is:

a red? **b** blue? **c** red or blue? **d** green? **e** blue or green?

S = {R, R, R, B, B, B, B, G, G, G, G, G}
Number of possible equally likely outcomes = 12

a Favourable outcomes = {R, R, R} Number of favourable outcomes = 3
$P(\text{R}) = \frac{3}{12} = \frac{1}{4}$

b Favourable outcomes = {B, B, B, B} Number of favourable outcomes = 4
$P(\text{B}) = \frac{4}{12} = \frac{1}{3}$

c Favourable outcomes = {R, R, R, B, B, B, B} Number of favourable outcomes = 7
$P(\text{R or B}) = \frac{7}{12}$

d Favourable outcomes = {G, G, G, G, G} Number of favourable outcomes = 5
$P(\text{G}) = \frac{5}{12}$

e Favourable outcomes = {B, B, B, B, G, G, G, G, G} Number of favourable outcomes = 9
$P(\text{B or G}) = \frac{9}{12} = \frac{3}{4}$

6 A bag contains 5 blue and 2 red marbles. One marble is drawn at random. Complete to find the following probabilities.

S = {_____}
Number of possible equally likely outcomes = ___

a Probability that the marble is blue:
Favourable outcomes = {_____}
Number of favourable outcomes = ___
$P(\text{B}) =$ ___

b Probability that the marble is red:
Favourable outcomes = {_____} Number of favourable outcomes = ___
$P(\text{R}) =$ ___

c Probability that the marble is blue or red:
Favourable outcomes = {_____} Number of favourable outcomes = ___
$P(\text{B or R}) =$ ___

7 A purse contains two 5-cent coins, three 10-cent coins, four 20-coins and one 50-cent coin. A coin is chosen at random from this purse. Find the probability that the value of the coin is:

a 10 cents **b** 20 cents **c** 10 cents or 20 cents
d 5 cents **e** 5 cents or 10 cents **f** more than 10 cents.

8 One hundred tickets are sold in a raffle. What is the probability of winning the raffle if you buy:

a 1 ticket? **b** 2 tickets? **c** 5 tickets? **d** 10 tickets?

9 A card is selected at random from a normal playing pack of 52 cards. Find the probability that it is:

a the king of clubs **b** a king **c** a spade
d black **e** a red 7 **f** a jack or a queen.

10 A letter is chosen at random from the letters of the word HIPPOPOTAMUS. What is the probability that the letter chosen is:

a H? **b** O? **c** P?

11 A set of traffic lights shows green for 45 seconds, amber for 5 seconds and red for 30 seconds. When approaching this set of lights, what is the probability that it will be showing:

a green?
b amber?
c red?

12 **a** A normal six-sided die is rolled. Find the probability of getting a:

i 7 **ii** 9

b What kind of events are those in part **a**?

c Find the probability of getting:

i a number < 9 **ii** an odd or even number.

d What kind of events are those in part **c**?

From question **12**, we can see that:

- If an event is impossible then its probability is 0.
- If an event is certain then its probability is 1.

Hence, the probability of any event, E, lies in the range 0 to 1, inclusive.
That is: $0 \leqslant P(E) \leqslant 1$

13 A spinner is divided into four equal parts coloured red, yellow, green and blue. When the spinner is spun, what is the probability that it lands on:

a purple? **b** red or yellow or green or blue?

14 The numbers 1 to 7 are written on 7 cards. The cards are shuffled and one card is selected at random. Find the probability that it shows:

a an odd or even number
b a number < 10
c 9
d 0

15 A bag contains 1 red, 1 blue and 1 green marble. A marble is selected at random from this bag. Write down an event E for which:

a $P(E) = 0$
b $P(E) = 1$

16 Jack calculated the answer to a probability question in a test to be $\frac{4}{3}$. Explain why Jack's answer was wrong.

Investigation 1 Probability certainties

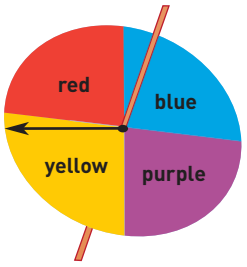
1 A coin is tossed once.

a List the sample space.
b What is the probability of heads?
c What is the probability of tails?
d Add the probability of heads and the probability of tails.
e Describe an event that results in a certainty.
f What is the probability of the event in part **e**?

2 A normal six-sided die is thrown once.

a List the sample space.
b Find these probabilities.
i $P(6)$ **ii** $P(5)$
iii $P(4)$ **iv** $P(3)$
v $P(2)$ **vi** $P(1)$
c Add the probabilities of getting 6, 5, 4, 3, 2 and 1.
d What is the probability of getting 6 or 5 or 4 or 3 or 2 or 1 when throwing a die once?
e Why is this a certainty?

3 A spinner has four equal-sized segments coloured blue, purple, red and yellow. The spinner is spun once.



a List the sample space.
b What is the probability of the outcome blue, purple, red or yellow?
c What is:
i P(blue)? **ii** P(purple)?
iii P(red)? **iv** P(yellow)?
d Add the probabilities from part **c**. What do you notice? Explain.

4 Complete:
The sum of the probabilities of all possible outcomes of a single experiment is _____.

5 A normal six-sided die is thrown once.

a Find $P(6)$.
b Find $P(1, 2, 3, 4 \text{ or } 5)$.
c Find $P(\text{not } 6)$. (*Hint:* See part **b**.)
d Add the probability of getting 6 to the probability of *not* getting 6. What do you notice? Explain your answer.

B Sum of probabilities

In Investigation 1 you found that the sum of the probabilities of all possible outcomes of an experiment is 1.

EXAMPLE 1

A bag contains red, blue and green marbles. When one marble is chosen at random from the bag, the probability that it is red is $\frac{7}{19}$ and the probability that it is blue is $\frac{4}{19}$. What is the probability that it is green?

$$\begin{aligned} P(\text{red}) + P(\text{blue}) + P(\text{green}) &= 1 \\ \frac{7}{19} + \frac{4}{19} + P(\text{green}) &= 1 \\ \frac{11}{19} + P(\text{green}) &= 1 \\ P(\text{green}) &= 1 - \frac{11}{19} \\ &= \frac{8}{19} \end{aligned}$$

The sum of the probabilities of all the possible outcomes of an experiment is 1.

Exercise 12B

1 A bag contains black, white and orange counters. If a counter is chosen at random from this bag, the probability that it is black is $\frac{6}{13}$ and the probability that it is white is $\frac{2}{13}$. Complete the following to calculate the probability that the counter is orange.

$$\begin{aligned} P(\text{black}) + P(\text{white}) + P(\text{orange}) &= \_\_\_\_ \\ \frac{6}{13} + \frac{2}{13} + P(\text{orange}) &= \_\_\_\_ \\ \_\_\_\_ + P(\text{orange}) &= \_\_\_\_ \\ P(\text{orange}) &= \_\_\_\_ - \_\_\_\_ \\ &= \_\_\_\_ \end{aligned}$$

2 A roulette wheel has red, black and green slots. When spun, the probability that the ball will stop in a red slot is $\frac{9}{19}$ and the probability that it will stop in a green slot is $\frac{1}{19}$. What is the probability that it will stop in a black slot?

3 A spinner consists of pink, blue and green sections. If the spinner is spun the probability that it will stop on blue is $\frac{1}{2}$ and the probability that it will stop on green is $\frac{1}{3}$. Find the probability that it will stop on pink.

4 When three coins are tossed, the possible outcomes are 3 heads, exactly 2 heads, exactly 1 head or no heads. If the probability of 3 heads is $\frac{1}{8}$, the probability of exactly 1 head is $\frac{3}{8}$ and the probability of getting no heads is $\frac{1}{8}$. Calculate the probability of getting exactly 2 heads.

5 A purse contains 5-cent, 10-cent, 20-cent and 50-cent coins. When a coin is chosen at random from this purse, the probability of getting a 5-cent coin is $\frac{1}{5}$, the probability of getting a 10-cent coin is $\frac{1}{3}$ and the probability of getting a 50-cent coin is $\frac{1}{10}$. Calculate the probability of getting a 20-cent coin.

C Identifying complements of events

In an experiment with sample space S, the **complement** of an event A is the set of outcomes that are in the sample space S but are not in A.

EXAMPLE 1

When a normal six-sided die is thrown, what is the complement of the event:

a throwing 2?

b throwing 1 or 2?

c throwing an even number?

a S = {1, 2, 3, 4, 5, 6}. Let A be the event 'throwing a 2'; then $A = \{2\}$.
The complement of A is the set of outcomes that are in S but are not in A.
Complement of $A = \{1, 3, 4, 5, 6\}$
The complement of A is the event 'throwing a 1, 3, 4, 5 or 6' or 'not throwing a 2'.

Write the event as a set of outcomes, even if there is only one outcome in the set.

b S = {1, 2, 3, 4, 5, 6}. Let B be the event 'throwing a 1 or 2'; then $B = \{1, 2\}$.
The complement of B is the set of outcomes that are in S but are not in B.
Complement of $B = \{3, 4, 5, 6\}$
The complement of B is the event 'throwing a 3 or 4 or 5 or 6' or 'not throwing a 1 or 2' or 'throwing a number greater than 2'.

c S = {1, 2, 3, 4, 5, 6}. Let C be the event 'throwing an even number'; then $C = \{2, 4, 6\}$.
The complement of C is the set of outcomes that are in S but are not in C.
Complement of $C = \{1, 3, 5\}$
The complement of C is the event 'throwing a 1, 3 or 5' or 'not throwing an even number' or 'throwing an odd number'.

Exercise 12C

1 A normal die is thrown. Complete the following to find the complement of each event given.

a Throwing a 6
S = {_____}. Let A be the event 'throwing a 6'; then A = {_____}.
The complement of A is the set of outcomes that are in S but are not in A.
Complement of A = {_____}
The complement of A is the event '_____' or '_____' or '_____'.

b Throwing a 5 or 6
S = {_____}. Let B be the event 'throwing a 5 or 6'; then B = {_____}.
The complement of B is the set of outcomes that are in S but are not in B.
Complement of B = {_____}
The complement of B is the event '_____' or '_____' or '_____'.

c Throwing an odd number
S = {_____}. Let C be the event 'throwing an odd number'; then C = {_____}.
The complement of C is the set of outcomes that are in S but are not in C.
Complement of C = {_____}
The complement of C is the event '_____' or '_____' or '_____'.

2 A normal die is thrown. Find the complement of each event.

a throwing 3 **b** throwing 3 or 5 **c** throwing a number greater than 2

3 The numbers 1 to 9 are written on cards and one card is chosen at random. Find the complement of these event.

a choosing 7 **b** choosing 1 or 7

c choosing an even number **d** choosing a number greater than 4

EXAMPLE 2

a A student is chosen from a class consisting of boys and girls. What is the complement of the event 'choosing a boy'?

b A marble is chosen from a bag containing black and yellow marbles .What is the complement of the event 'selecting a black marble'?

a The complement of the event 'choosing a boy' is 'not choosing a boy' or 'choosing a girl'.

b The complement of the event 'selecting a black marble' is 'not selecting a black marble' or 'selecting a yellow marble'.

4 Complete the following.

a A student is chosen from a class consisting of boys and girls. The complement of the event 'choosing a girl' is '_____ choosing a girl' or 'choosing a _____'.

b A marble is chosen from a bag containing red and blue marbles. The complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.

c A marble is chosen from a bag containing red, blue and green marbles. The complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.

d A marble is chosen from a bag containing red, blue and green marbles. The complement of the event 'choosing a red or blue marble' is '_____ choosing a red or blue marble' or 'choosing a _____ marble'.

5 A coin is tossed. What is the complement of the event 'getting a head'?

6 A spinner has four equal-sized segments coloured red, blue, green and yellow. The spinner is spun and the colour on which it stops is noted. What is the complement of the event:

a stops on red? **b** stops on red or blue?

c stops on blue or green? **d** stops on red or blue or green?

7 A card is selected from a normal playing pack. What is the complement of selecting:

a a red card?

b a spade?

c a spade or a club?

d a picture card?

8 Sydney plays Melbourne in a soccer final. What is the complement of the event:

a Sydney wins or draws?

b Sydney wins?

c a draw?

d Melbourne loses or draws?

Investigation 2 Sum of probabilities of complementary events

1 A coin is tossed once. Find these probabilities.

a $P(\text{head})$ **b** $P(\text{not head})$ **c** $P(\text{head}) + P(\text{not head})$

2 A normal six-sided die is rolled. Find:

a **i** $P(2)$ **ii** $P(\text{not } 2)$ **iii** $P(2) + P(\text{not } 2)$

b **i** $P(6)$ **ii** $P(\text{not } 6)$ **iii** $P(6) + P(\text{not } 6)$

c **i** $P(\text{even number})$ **ii** $P(\text{not even number})$ **iii** $P(\text{even number}) + P(\text{not even number})$

d **i** $P(\text{number} < 5)$ **ii** $P(\text{number not} < 5)$ **iii** $P(\text{number} < 5) + P(\text{number not} < 5)$

3 A bag contains 2 red, 5 blue and 4 white counters.
A counter is chosen at random from the bag. Find:

a **i** $P(\text{red})$ **ii** $P(\text{not red})$ **iii** $P(\text{red}) + P(\text{not red})$

b **i** $P(\text{blue})$ **ii** $P(\text{not blue})$ **iii** $P(\text{blue}) + P(\text{not blue})$

c **i** $P(\text{white})$ **ii** $P(\text{not white})$ **iii** $P(\text{white}) + P(\text{not white})$

D Probabilities of complements of events

Pairs of events in Investigation 2 are:

- getting a head, not getting a head
- getting 2, not getting 2
- getting a white counter, not getting a white counter.

These are **complementary events**. Each event is the complement of the other. The complement of the set E, called **not** E, can be written $\tilde{E}$ **or** E'.

From Investigation 2 we see that: $P(E) + P(\text{not } E) = 1$ or $P(E) + P(\tilde{E}) = 1$.

EXAMPLE 1

When two coins are tossed, the probability of getting 2 heads is $\frac{1}{4}$. What is the probability of *not* getting 2 heads when two coins are tossed?

$P(2\text{H}) + P(\text{not } 2\text{H}) = 1$

$\frac{1}{4} + P(\text{not } 2\text{H}) = 1$

$\therefore P(\text{not } 2\text{H}) = 1 - \frac{1}{4} = \frac{3}{4}$

From Example 1 we can see that $P(E) + P(\text{not } E) = 1$ is equivalent to $P(\text{not } E) = 1 - P(E)$.

Exercise 12D

1 On any weekday, the probability of a particular bus arriving at its destination on time is $\frac{9}{11}$. Complete the following to find the probability that the bus does *not* arrive on time.

$P(\text{on time}) + P(\text{not on time}) = \_\_\_$

$\frac{9}{11} + P(\text{not on time}) = \_\_\_$

$\therefore P(\text{not on time}) = \_\_\_ - \frac{\square}{\square} = \frac{\square}{\square}$

2 The probability of choosing a 20-cent coin from a particular purse is $\frac{3}{10}$. When a coin is chosen at random from this purse, what is the probability that it is not a 20-cent coin?

3 When a lolly is selected at random from a bag, the probability that it is red is $\frac{5}{13}$. What is the probability that the lolly is not red?

4 The letters of a word are written on cards and placed in a hat. When a card is chosen at random from this hat, the probability that the letter is M is $\frac{2}{7}$. What is the probability that the letter is not M?

5 On approaching a set of traffic lights, the probability that the light will be red is $\frac{30}{67}$. What is the probability that the light will not be red?

6 On a European roulette wheel, the probability that the ball will stop on a red number is $\frac{18}{37}$. What is the probability that it will *not* stop on a red number?

7 There are 1000 tickets sold in a raffle. If Angela buys 1 ticket, what is the probability that:

a she will win the raffle?

b she will not win the raffle?

8 The numbers 1 to 9 are written on cards. The cards are shuffled and one is selected at random. Find these probabilities.

a $P(9)$ **b** $P(\text{not } 9)$

c $P(\text{even number})$ **d** $P(\text{not even number})$

e $P(\text{number} < 3)$ **f** $P(\text{number not} < 3)$

9 The numbers 0 to 9 are written on cards. The cards are shuffled and one is selected at random. Find these probabilities.

a $P(6)$ **b** $P(\text{not } 6)$ **c** $P(\text{odd number})$

d $P(\text{not odd number})$ **e** $P(\text{number} > 7)$ **f** $P(\text{number not} > 7)$

10 A bag contains 6 red, 9 blue and 8 white marbles. One marble is chosen at random from the bag. Find these probabilities.

a $P(\text{red})$ **b** $P(\text{not red})$ **c** $P(\text{not blue})$ **d** $P(\text{not white})$

E Mutually exclusive events

Two events are mutually exclusive if they cannot both happen at the same time. For example, when a die is rolled once, the events 'getting a 1' and 'getting a 4' are mutually exclusive. Only one number can occur on the uppermost face and hence both events cannot happen at the same time.

The event 'getting a number bigger than 3' and the event 'getting an even number' are not mutually exclusive, as the outcomes a 4 or a 6 are bigger than 3 and even; that is, both events can happen at the same time. (These are sometimes called inclusive events.)

EXAMPLE 1

A die is rolled. State whether events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a 2 and B is the event a 3.

b A is the event a 2 and B is the event an even number.

c A is the event a 2 and B is the event a number bigger than 4.

d A is the event a number less than 5 and B is the event a number bigger than 2.

a $A = \{2\}$, $B = \{3\}$. Both A and B cannot happen at the same time. A and B are mutually exclusive.

b $A = \{2)$, $B = \{2, 4, 6\}$. Both A and B can happen at the same time (the outcome rolling a 2). A and B are non-mutually exclusive.

c $A = \{2\}$, $B = \{5, 6\}$. Both A and B cannot happen at the same time. A and B are mutually exclusive.

d $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$. Both A and B can happen at the same time (the outcomes 3 or 4). A and B are non-mutually exclusive.

Exercise 12E

1 A die is rolled. Complete the following to determine whether the events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a 5: $A = \{\_\_\_\_\}$. B is the event a 6: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

b A is the event a 5: $A = \{\_\_\_\_\}$. B is the event an odd number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

c A is the event an even number: $A = \{\_\_\_\_\}$. B is the event an odd number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

d A is the event a multiple of 3: $A = \{\_\_\_\_\}$. B is the event an even number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

2 A card is selected at random from a normal playing pack. State whether the following events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a king. B is the event an ace.

b A is the event a king. B is the event a red card.

c A is the event a 10, B is the event a queen.

d A is the event a jack, B is the event a black card.

e A is the event a 7, B is the event a diamond.

f A is the event a club, B is the event a red card.

g A is the event a black card, B is the event an ace.

3 The numbers 0 to 9 are written on 10 cards. The cards are shuffled and one card is selected at random. State whether the following events are mutually exclusive or non-mutually exclusive.

a getting a 5, getting an even number
b getting an even number, getting a number less than 5
c getting an odd number, getting a number bigger than 5
d getting a number less than 4, getting a number bigger than 5
e getting a number less than 4, getting a number less than 7

4 A pair of dice are rolled at the same time. State whether the following events are mutually exclusive or non-mutually exclusive.

a getting a total of 6, getting a double
b getting a total of 9, getting a double

F Compound events

A compound event is an event that can be expressed as a combination of simple events.

EXAMPLE 1

A normal six-sided die is thrown. Match each statement in the left-hand column with its equivalent in the right-hand column.

a throwing a 4, 5 or 6	**A** throwing at least 3
b throwing a 1, 2, 3 or 4	**B** throwing at most 5
c throwing a 3, 4, 5 or 6	**C** throwing at most 4
d throwing a 1, 2, 3, 4, or 5	**D** throwing at least 4

a throwing a 4, 5 or 6	is equivalent to	**D** throwing at least 4
b throwing a 1, 2, 3 or 4	is equivalent to	**C** throwing at most 4
c throwing a 3, 4, 5 or 6	is equivalent to	**A** throwing at least 3
d throwing a 1, 2, 3, 4, or 5	is equivalent to	**B** throwing at most 5

Exercise 12F

1 A normal six-sided die is thrown. Complete the following.

a Throwing 4, 5 or 6 is equivalent to throwing at least ___.
b Throwing a 1, 2 or 3 is equivalent to throwing at most ___.
c Throwing a 2, 3, 4, 5 or 6 is equivalent to throwing at least ___.
d Throwing a 1 or 2 is equivalent to throwing at most ___.

2 A regular eight-sided die is rolled. Complete the following.

a Throwing 1, 2, 3 or 4 is equivalent to throwing at most ___.
b Throwing a 6, 7 or 8 is equivalent to throwing at least ___.
c Throwing a 4, 5, 6, 7 or 8 is equivalent to throwing at least ___.
d Throwing a 1, 2, 3, 4 or 5 is equivalent to throwing at most ___.

3 The numbers 0 to 9 are written on cards. A card is selected at random. List the outcomes for the event that the number on the card is:

a at most 4 **b** at least 7 **c** at least 5 **d** at most 2.

4 When two children are born, the sample space for the order of birth is S = {bb (boy followed by boy), bg (boy followed by a girl), gb (girl followed by a boy), gg (girl followed by a girl)}.
List the outcomes for each event.

a at most 1 boy **b** at least 1 boy **c** at least 1 girl **d** at most 1 girl

5 When two coins are tossed, the sample space is S = {HH, HT, TH, TT}. List the outcomes for each event.

a at least 1 tail **b** at most 1 tail **c** at least 1 head **d** at most 1 head

6 A spinner has three equal parts numbered 1, 2 and 3. When this spinner is spun twice the sample space is: S = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}. List the outcomes for these events.

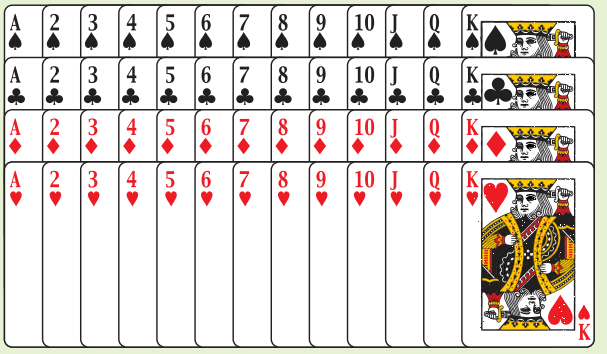
a at least one 3 **b** at most one 3

EXAMPLE 2

A card is chosen at random from a normal 52-card playing pack. List the outcomes in these events.

a choosing a jack and a red card

b choosing a club and a picture card.



a Let A be the event 'a jack' and let B be the event 'a red card'.
A = {J of hearts, J of diamonds, J of spades, J of clubs}
B = {2, 3, 4…, J, Q, K, A of hearts, 2, 3, 4…, J, Q, K, A of diamonds}
The event 'A and B' is the set of outcomes that have the attributes of both A and B; that is, the outcomes that are 'a jack and a red card'.
Hence event 'A and B' = {J of hearts, J of diamonds}

b Let X be the event 'a club' and let Y be the event 'a picture card'.
X = {2, 3, 4…, J, Q, K, A of clubs}
Y = {J, Q, K of hearts, J, Q, K of diamonds, J, Q, K of spades, J, Q, K of clubs}
The event 'X and Y' is the set of outcomes that are 'a club and a picture card'.
Hence event 'X and Y' = {J, Q, K of clubs}.

7 A card is selected at random from a normal playing pack. List the outcomes in these events.

a selecting a card that is a 10 and a heart

b selecting a card that is a 10 and a red card

c selecting a card that is black and a king

d selecting a card that is a club and even numbered

e selecting a card that is red and odd numbered

f selecting a card that is black and a picture card

EXAMPLE 3

A die is rolled. A is the event 'an odd number', B is the event 'an even number' and C is the event 'a number < 3'. List the outcomes for these events.

a A and B **b** A or B **c** B and C **d** B or C

a The event 'A and B' $= \{\ \}$ as a number cannot be both odd and even. These are mutually exclusive events.

The set { } has no outcomes and is known as an empty set.

b The event 'A or B' $= \{1, 2, 3, 4, 5, 6\}$
If the events A and B are mutually exclusive then the event 'A or B' = set of outcomes for A + set of outcomes for B.

c The event 'B and C' is the set of outcomes that belong to both B and C; that is, the outcomes that are even and less than 3. Event 'B and C' $= \{2\}$.

d The event 'B or C' is the set of outcomes that belong to B or C.
The problem raised is whether to include the outcome 2, which is in both events. If two events are not mutually exclusive, an 'or' statement needs a qualifier to determine inclusivity (the 2 is included) or exclusivity (the 2 is excluded).
The event 'B or C or both' $= \{1, 2, 4, 6\}$ is known as an 'inclusive or' compound event.
The event 'B or C but not both' $= \{1, 4, 6\}$ is known as an 'exclusive or' compound event.

EXAMPLE 4

a A card is selected at random from a pack. A is the event 'a red card', B is the event 'a black card'.
i List the outcomes for the event 'A and B'.
ii Does the event 'A or B' need a qualifier?

b A card is selected at random from a pack. A is the event 'a red card', B is the event 'an ace'.
i List the outcomes for 'A and B'.
ii Does the event 'A or B' need a qualifier?

c A person is chosen at random from a group. A is the event 'male' and B is the event 'left-handed'. Does the event 'A or B' need a qualifier?

a **i** A and $B = \{\ \}$
ii No. The events are mutually exclusive so the event 'A or B' = set of outcomes in A + set of outcomes for B.

b **i** A and $B = \{$ace of hearts, ace of diamonds$\}$
ii The events A and B are non-mutually exclusive, so the event 'A or B' needs a qualifier to indicate whether the outcomes for both events (ace of hearts, ace of diamonds) are included or excluded.

c If there are no left-handed males in the group then a qualifier is not necessary as the events are mutually exclusive. If there are left-handed males in the group then a qualifier is needed to indicate whether the left-handed males are to be included or excluded.

8 State whether a qualifier is needed for the event 'A or B'.
a A die is rolled. A is the event 'a number < 3'. B is the event ' a number > 4'.
b A die is rolled. A is the event 'an odd number'. B is the event 'a number > 4'.
c A student is chosen from a class of boys and girls. A is the event 'female'. B is the event 'wears glasses'.
d A student is chosen from a class. A is the event 'has blond hair'. B is the event 'has brown eyes'.

9 A die is rolled. A is the event 'a number > 3' and B is the event 'an odd number'. List the outcomes for these events.

a A **b** not A **c** B **d** not B

e A and B **f** A or B or both **g** A or B but not both

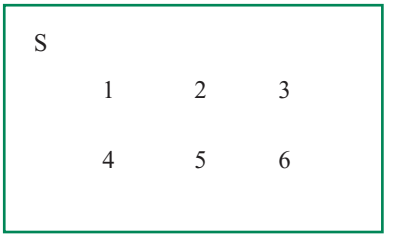
10 A card is selected at random from a normal playing pack. X is the event 'a diamond' and Y is the event 'a king'. List the outcomes for these events.

a X **b** not X **c** Y

d X and Y **e** X or Y or both **f** X or Y but not both

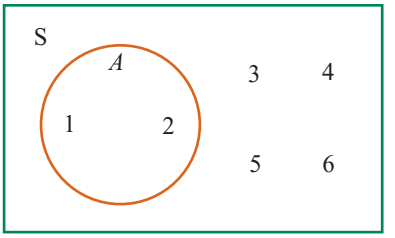
G Venn diagrams

The outcomes of an event can be represented using a Venn diagram. The sample space is represented by the region or points inside a rectangle. For example, if a die is rolled, S $= \{1, 2, 3, 4, 5, 6\}$. These outcomes are written inside a rectangle, labelled S, as shown.



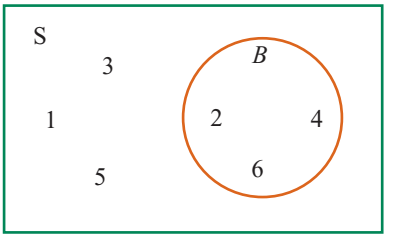
The outcomes favourable to an event are represented by points or the region inside a circle. For example, if a six-sided die is rolled and A is the event 'a number < 3', then $A = \{1, 2\}$. These outcomes are written inside a circle labelled A, as shown.

Note: All the outcomes that are not favourable to event A are outside this circle: the complement of A is the set of outcomes outside circle A. From the diagram, the event 'not A' $= \{3, 4, 5, 6\}$.



If B is the event 'an even number', then $B = \{2, 4, 6\}$. These outcomes are written inside another circle labelled B, as shown.

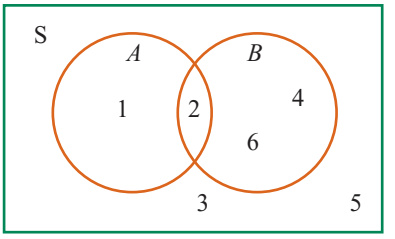
Note: All the outcomes that are not favourable to event B are outside this circle: the complement of B is the set of outcomes outside circle B. From the diagram, the event 'not B' $= \{1, 3, 5\}$.



Both events A and B can be represented in a Venn diagram.

The outcome $\{2\}$ is common (belongs) to both events and is placed in the area that is common to both circles. It is placed in the area where the two circles intersect. The outcome $\{2\}$ is 'a number < 3' and 'an even number'.

The outcomes $\{1, 2, 4, 6\}$, which are within the combined areas of both circles, including the intersection of the circles, are numbers that are 'a number < 3' or 'an even number' or both (less than 3 and even).

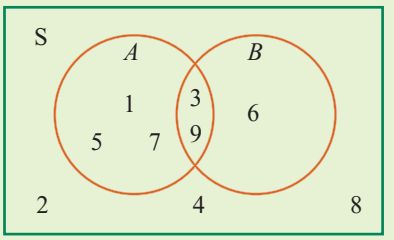


The event 'A or B or both' is an 'inclusive or' event because it includes 'both A and B'.

The outcomes $\{3, 5\}$ do not lie within either circle. These are the numbers that are neither 'less than 3' nor 'even'.

EXAMPLE 1

Numbers are written on cards, the cards shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram.

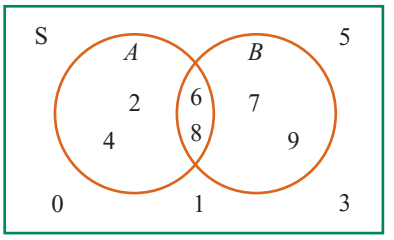


a List the sample space.
b List the set of outcomes favourable to event A.
c List the set of outcomes not favourable to the event A.
d List the set of outcomes favourable to event B.
e List the set of outcomes not favourable to the event B.
f List the set of outcomes favourable to the events 'A and B'.
g **i** List the set of outcomes favourable to the event 'A or B or both'.
ii Is this an 'inclusive or' event or an 'exclusive or' event?
h **i** List the set of outcomes favourable to the event 'A or B but not both'.
ii Is this an 'inclusive or' event or an 'exclusive or' event?
i List the outcomes that belong to neither A nor B.

a These are all the outcomes inside the rectangle. Hence S = {1, 2, 3, 4, 5, 6, 7, 8, 9}.
b These are all the outcomes inside circle A. Hence A = {1, 3, 5, 7, 9}.
c These are all the outcomes outside circle A (complement of the event A). Hence 'not A' = {2, 4, 6, 8}.
d These are all the outcomes inside circle B. Hence B = {3, 6, 9}.
e These are all the outcomes outside circle B (complement of the event B). Hence 'not B' = {1, 2, 4, 5, 7, 8}.
f These are all the outcomes in the intersection of the two circles. Hence 'A and B' = {3, 9}.
g **i** These are all the outcomes within both circles, including the intersection.
Hence 'A or B or both' = {1, 3, 5, 6, 7, 9}
ii It is an 'inclusive or' event.
h **i** These are all the outcomes within both circles, but excluding the intersection.
Hence 'A or B but not both' = {1, 5, 6, 7}.
ii It is an 'exclusive or' event.
i These are all the outcomes outside the two circles. Hence 'neither A nor B' = {2, 4, 8}.

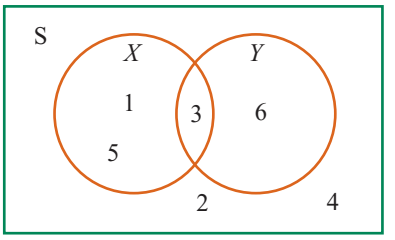
Exercise 12G

1 Numbers are written on cards, the cards are shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. Complete the following.



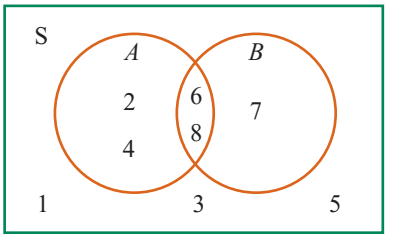
a The sample space S = {_____} (outcomes inside the rectangle)
b Event A = {_____} (outcomes inside the circle A)
c Event 'not A' = {_____} (outcomes outside the circle A)
d Event B = {_____} (outcomes inside circle B)
e Event 'not B' = {_____} (outcomes outside circle B)
f Event 'A and B' = {_____} (outcomes in the intersection of the circles)
g **i** Event 'A or B or both' = {_____} (outcomes within both circles, including the intersection)
ii Is this an 'inclusive or' event or an 'exclusive or' event?
h **i** Event 'A or B but not both' = {_____} (outcomes within both circles, excluding the intersection)
ii Is this an 'inclusive or' event or an 'exclusive or' event?
i Event 'neither A nor B' = {_____} (outcomes outside both circles)

2 A six-sided die is rolled. The outcomes for this experiment and the events X and Y are shown in the Venn diagram.



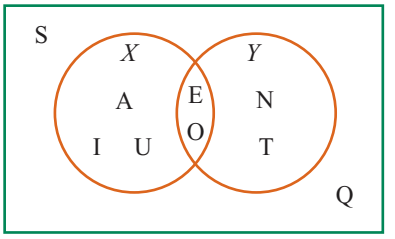
- **a** List the sample space.
- **b** List the set of outcomes for event X.
- **c** List the set of outcomes for the event 'not X'.
- **d** List the set of outcomes for event Y.
- **e** List the set of outcomes for the event 'not Y'.
- **f** List the set of outcomes for event 'X and Y'.
- **g** **i** List the set of outcomes for the event 'X or Y or both'.
  - **ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- **h** **i** List the set of outcomes for the event 'X or Y but not both'.
  - **ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- **i** List the outcomes that belong to neither X nor Y.

3 A spinner with eight equal parts numbered 1 to 8 is spun. A is the event 'the spinner lands on an even number' and B is the event 'the spinner lands on a number $>$ 5'. The outcomes for this experiment and events A and B are shown in the Venn diagram. Use the diagram to list the outcomes for the following events.



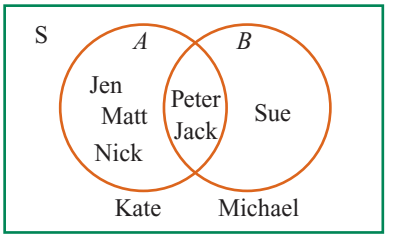
- **a** an even number
- **b** not an even number
- **c** a number $>$ 5
- **d** a number not $>$ 5
- **e** a number that is even and $>$ 5
- **f** a number that is even or $>$ 5 or both
- **g** a number that is even or $>$ 5 but not both
- **h** a number that is neither even nor $>$ 5
- **i** a number that is even but not $>$ 5
- **j** a number that is $>$ 5 but not even

4 The letters of the word EQUATION are written on cards and placed in a hat. One card is drawn at random. X is the event 'a vowel' and Y is the event 'a letter of the word NOTE'. The outcomes for this experiment and events X and Y are shown in the Venn diagram. Use the diagram to list the outcomes for the following events.



- **a** a vowel
- **b** not a vowel
- **c** a letter of the word NOTE
- **d** not a letter of the word NOTE
- **e** a vowel and a letter of the word NOTE
- **f** a vowel or a letter of the word NOTE or both
- **g** a vowel or a letter of the word NOTE but not both
- **h** neither a vowel nor a letter of the word NOTE
- **i** a vowel but not a letter of the word NOTE
- **j** a letter of the word NOTE but not a vowel

5 A person is chosen at random from a group of friends. A is the event 'has blond hair' and B is the event 'has blue eyes'. The outcomes for the experiment and events A and B are shown in the Venn diagram.

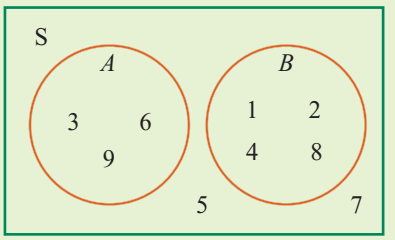


- **a** List the sample space.
- **b** List the set of outcomes for the following events.
  - **i** has blond hair
  - **ii** does not have blond hair
  - **iii** has blue eyes
  - **iv** does not have blue eyes
  - **v** has blond hair and blue eyes
  - **vi** has blond hair or blue eyes or both
  - **vii** has blond hair or blue eyes but not both
  - **viii** has neither blond hair nor blue eyes
  - **ix** has blond hair but not blue eyes
  - **x** has blue eyes but not blond hair

EXAMPLE 2

The numbers 1 to 9 are written on cards, the cards shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. List the set of outcomes for the following events.

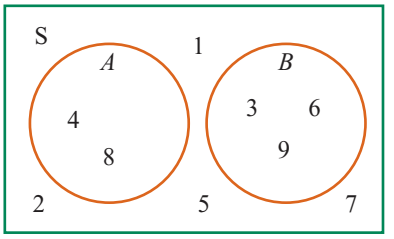
a A **b** not A **c** B **d** not B
e A and B **f** A or B **g** neither A nor B



a $A = \{3, 6, 9\}$
b not $A = \{1, 2, 4, 5, 7, 8\}$
c $B = \{1, 2, 4, 8\}$
d not $B = \{3, 5, 6, 7, 9\}$
e The events A and B have no intersection. They are mutually exclusive events: there are no outcomes of this experiment that have the properties of both A and B so 'A and B' = { }
f The event 'A or B' does not have to be qualified by 'or both' or 'but not both', as in this case, A and B are mutually exclusive events. A or $B = \{1, 2, 3, 4, 6, 8, 9\}$
g Neither A nor $B = \{5, 7\}$

6 The numbers 1 to 9 are written on cards, the cards shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. A is the event 'a multiple of 4' and B is the event 'a multiple of 3'. List the set of outcomes for the following events.

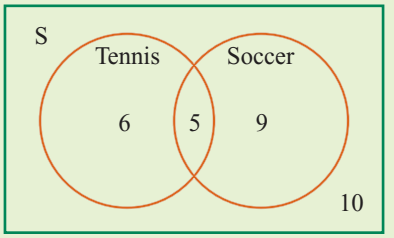
a A **b** not A **c** B **d** not B
e A and B **f** A or B **g** neither A nor B



EXAMPLE 3

This Venn diagram shows the sports played by a class of students. How many students:

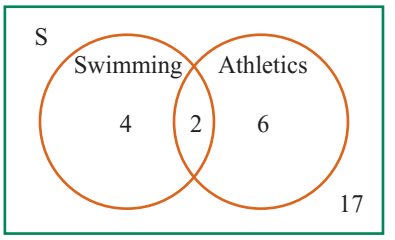
a are in the class?
b play tennis and soccer?
c play tennis?
d play soccer?
e play tennis or soccer or both?
f play tennis or soccer but not both?
g play neither tennis nor soccer?
h play tennis but not soccer?
i play soccer but not tennis?



a Number in class = 6 + 5 + 9 + 10 = 30
b Tennis and soccer = 5
c Tennis = 6 + 5 = 11
d Soccer = 9 + 5 = 14
e Tennis or soccer or both = 6 + 5 + 9 = 20
f Tennis or soccer but not both = 6 + 9 = 15
g Neither tennis nor soccer = 10
h Tennis but not soccer = 6
i Soccer but not tennis = 9

Note that the number of students, not the individual students, is shown.

7 This Venn diagram represents the number of students in a class who made the school swimming and athletics teams.



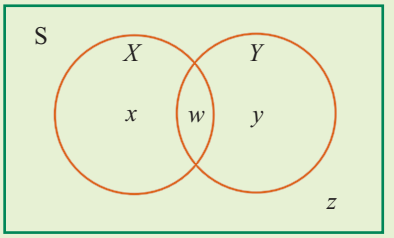
a How many students are in the class?

b How many students made:

- **i** both the swimming and athletics teams?
- **ii** the swimming team?
- **iii** the athletics team?
- **iv** the swimming team or the athletics team or both?
- **v** the swimming team or the athletics team but not both?
- **vi** neither the swimming team nor the athletics team?
- **vii** the swimming team but not the athletics team?
- **viii** the athletics team but not the swimming team?

EXAMPLE 4

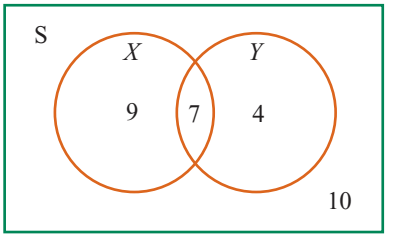
In a class of 30 students, 16 have black hair, 11 have brown eyes and 7 have both black hair and brown eyes.



a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes'. Note that the number of students, not the individual students, is shown in each region of the diagram.

b How many students:

- **i** do not have black hair?
- **ii** do not have brown eyes?
- **iii** have black hair or brown eyes or both?
- **iv** have black hair or brown eyes but not both?
- **v** have neither black hair nor brown eyes?
- **vi** have black hair but not brown eyes?
- **vii** have brown eyes but not black hair?

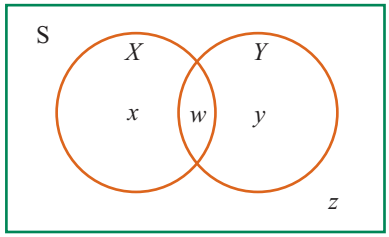


a There are 7 students with both black hair and brown eyes so $w = 7$.
This number is placed in the intersection of the circles.
There is a total of 16 students with black hair so $x + 7 = 16$.
$\therefore x = 9$
There is a total of 11 students with brown eyes so $y + 7 = 11$.
$\therefore y = 4$
There is a total of 30 students in the class so $x + w + y + z = 30$.
$9 + 7 + 4 + z = 30 \ \therefore z = 10$

b Number of students who:

- **i** do not have black hair $= 4 + 10 = 14$
- **ii** do not have brown eyes $= 9 + 10 = 19$
- **iii** have black hair or brown eyes or both $= 9 + 7 + 4 = 20$
- **iv** have black hair or brown eyes but not both $= 9 + 4 = 13$
- **v** have neither black hair nor brown eyes $= 10$
- **vi** have black hair but not brown eyes $= 9$
- **vii** have brown eyes but not black hair $= 4$

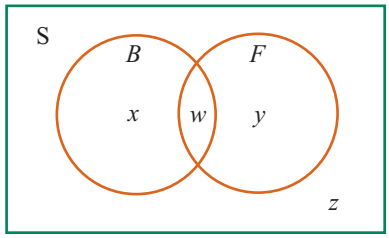
8 In a class of 29 students, 13 have black hair, 10 have brown eyes and 5 have both black hair and brown eyes.



a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes'.
Note that the number of students, not the individual students, is shown in each region of the diagram.
There are 5 students with both black hair and brown eyes so $w =$ ___. This number is placed in the intersection of the circles.
There is a total of 13 students with black hair so $x + 5 =$ ___.
$\therefore x =$ ___
There is a total of 10 students with brown eyes so $y + 5 =$ ___.
$\therefore y =$ ___
There are 29 students in the class so $x + 5 + y + z =$ ___.
___ + ___ + ___ + $z =$ ___
$\therefore z =$ ___

b The number of students who:
- **i** do not have black hair = ___
- **ii** do not have brown eyes = ___
- **iii** have black hair or brown eyes or both = ___
- **iv** have black hair or brown eyes but not both = ___
- **v** have neither black hair nor brown eyes = ___
- **vi** have black hair but not brown eyes = ___
- **vii** have brown eyes but not black hair = ___

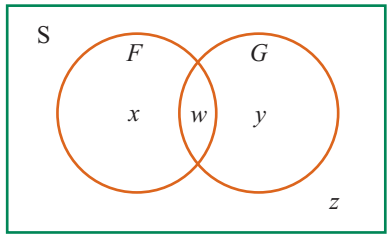
9 In a group of 25 males, 13 play basketball, 11 play football and 6 play both basketball and football.



a Complete the Venn diagram representing this information, where B is the event 'plays basketball' and F is the event 'plays football', by finding the numbers w, x, y and z.

b How many males:
- **i** do not play basketball?
- **ii** do not play football?
- **iii** play basketball or football or both?
- **iv** play basketball or football but not both?
- **v** play neither basketball nor football?
- **vi** play basketball but not football?
- **vii** play football but not basketball?

10 In a class of 30 students, 12 study French, 15 study German and 5 study both French and German.



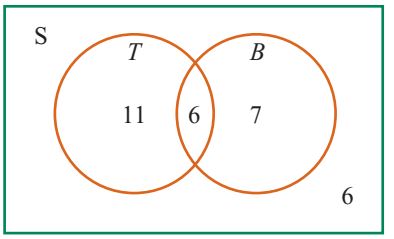
a Complete the Venn diagram representing this information, where F is the event 'studies French' and G is the event 'studies German' by finding the numbers w, x, y and z.

b How many students:
- **i** do not study French?
- **ii** do not study German?
- **iii** study French or German or both?
- **iv** study French or German but not both?
- **v** study French but not German?
- **vi** study German but not French?
- **vii** do not study a language?
- **viii** study at least one language?
- **ix** study at most one language?

EXAMPLE 5

In a survey of her class Sally found that 17 students travelled to school by train, 13 travelled by bus and 6 travelled by both train and bus. There were 30 students in Sally's class. Draw a Venn diagram to represent this information, using T is the event 'travels by train' and B is the event 'travels by bus'.

The number of students who travel by both train and bus is 6, so 6 goes in the intersection of the circles. A total of 17 students travel by train, so $17 - 6 = 11$ students travel by train only, so 11 goes in the T circle. A total of 13 students travel by bus, so $13 - 6 = 7$ students travel by bus only, so 7 goes in the B circle. There are 30 students in the class so the number who do not travel by either train or bus is $30 - (11 + 6 + 7) = 6$ so 6 goes outside the circles.



11 In a group of 19 people waiting at a bus stop one morning, 13 people had an umbrella, 4 people had a raincoat and 3 people had both an umbrella and a raincoat.

a Draw a Venn diagram to represent this information, using U for the event 'has an umbrella' and R for the event 'has a raincoat'.

b How many people in this group:

i do not have an umbrella?
ii do not have a raincoat?
iii have an umbrella or a raincoat or both?
iv have an umbrella or a raincoat but not both?
v have neither an umbrella nor a raincoat?
vi have an umbrella but not a raincoat?
vii have a raincoat but not an umbrella?

12 In a survey of a music class it was found that 9 students played the piano, 4 played the guitar and no students played both piano and guitar. There were 18 students in the class.

a Draw a Venn diagram to represent this information, using P for the event 'plays the piano' and G for the event 'plays the guitar'.

b How many students:

i do not play the piano?
ii do not play the guitar?
iii play the piano or the guitar?
iv play neither the piano nor the guitar?
v play at least one instrument?
vi play at most one instrument

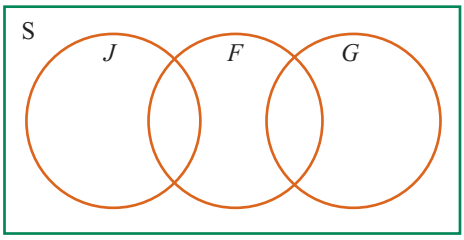
13 In a class of 30 students 25 passed the term mathematics test, 24 passed the science test and 23 students passed both tests.

a Draw a Venn diagram to represent this information using M for the event 'passed mathematics' and S for the event 'passed science'.

b How many student from this class:

i did not pass mathematics?
ii did not pass science?
iii passed neither mathematics nor science?
iv passed mathematics or science or both?
v passed mathematics or science but not both?
vi passed mathematics but not science?
vii passed science but not mathematics?
viii passed at least one subject?
ix passed at most one subject?
x failed at least one subject?
xi failed at most one subject?

14 In a class of 30 students 5 study Japanese, 13 study French, 9 study German, 2 study both Japanese and French and 5 study both French and German.

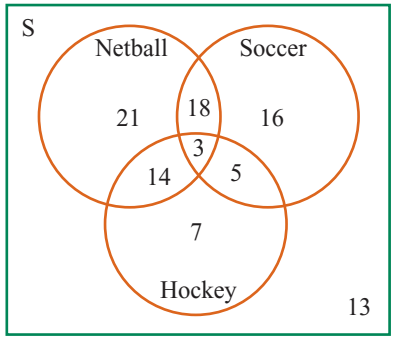


a Complete the Venn diagram where J is the event 'studies Japanese', F is the event 'studies French' and G is the event 'studies German'.

b How many students in this class:

i do not study a language?
ii do not study Japanese?
iii do not study French?
iv do not study German?
v study Japanese or French or both?
vi study Japanese or French but not both?
vii study French or German or both?
viii study French or German but not both?
ix study Japanese or German?
x study Japanese and French and German?
xi study Japanese only?
xii study French only?
xiii study German only?
xiv study exactly one language?
xv study exactly two languages?
xvi study at least one language?
xvii study at most one language

15 The Venn diagram represents the number of girls in Year 8 who play netball, soccer or hockey. How many girls:



a play netball?
b play soccer?
c play hockey?
d play netball and soccer?
e play netball and hockey?
f play soccer and hockey?
g play netball and soccer and hockey?
h netball or soccer or both?
i play netball or soccer but not both?
j play netball or hockey or both?
k play netball or hockey but not both?
l play soccer or hockey or both?
m play soccer or hockey but not both?
n do not play netball?
o do not play soccer?
p do not play hockey?
q do not play any of these sports?
r play netball but not soccer?
s play soccer but not netball?
t play netball but not hockey?
u play hockey but not netball?
v play soccer but not hockey?
w play hockey but not soccer?
x play exactly one of these sports?
y play at least one sport?
z play at most one sport?

H Two-way tables

The information collected from data and surveys can also be represented in two-way tables.

EXAMPLE 1

The table shows the data collected from a survey of a Year 8 class.

	Left-handed	Right-handed	
Male	2	14	16
Female	1	13	14
	3	27	30

Use the table to find the number of students who are:

a in the class
b male
c female
d left-handed
e right-handed
f male and left-handed
g female and right-handed
h male or right-handed or both
i male or right-handed but not both
j neither male nor right-handed
k right-handed but not male
l left-handed but not female.

a 30 students in the class
b 16 male students
c 14 female students
d 3 left-handed students
e 27 right-handed students
f 2 male and left-handed students
g 13 female and right-handed students
h 29 male or right-handed students or both
i 15 male or right-handed students but not both
j 1 neither male nor right-handed student
k 13 right-handed but not male students
l 2 left-handed but not female students

Exercise 12H

1 The information in the table was collected from a group of athletes.

	Heavy	Light	
Tall	8	9	17
Short	3	10	13
	11	19	30

Use the table to find the number of people who are:

a in the group of athletes
b tall
c short
d heavy
e light
f short and light
g short and heavy
h tall and light
i tall and heavy
j tall or light or both
k tall or light but not both
l short or heavy or both
m short or heavy but not both
n neither short nor heavy
o neither tall nor light
p heavy but not short
q light but not tall.

2 The table shows the results of a survey of a group of students.

	Born in Australia	Born overseas	
Male	87	29	116
Female	98	16	114
	185	45	230

Use the table to find how many students were:

a surveyed
b male
c female
d born in Australia
e born overseas
f male and born overseas
g female and born in Australia
h male or born overseas or both
i male or born overseas but not both
j neither male nor born overseas
k neither female nor born in Australia
l born in Australia but not male
m female but not born in Australia.

3 The table shows data collected from a group of students. How many students have:

	Black hair	Not black hair	
Brown eyes	11	6	17
Not brown eyes	5	8	13
	16	14	30

a black hair?
b brown eyes?
c black hair and brown eyes?
d black hair or brown eyes or both?
e black hair or brown eyes but not both?
f neither black hair nor brown eyes?
g black hair but not brown eyes?
h brown eyes but not black hair?

4 The table shows the results of a survey of the way a group of employees get to work each day.

	Bus	Not bus	
Train	8	v	11
Not train	y	5	z
	x	w	23

a Complete the table by finding the missing numbers.

b How many employees travel by:
i bus?
ii train?
iii bus and train?
iv bus or train or both?
v bus or train but not both?
vi neither bus nor train?
vii bus but not train?
viii train but not bus?

5 The table shows data collected from a group of students.

	Passed Mathematics	Did not pass Mathematics	
Passed English	k	3	m
Did not pass English	l	n	6
	31	p	36

a Complete the table by finding the missing numbers.
b How many students:
i passed Mathematics?
ii passed English?
iii passed Mathematics and English?
iv passed Mathematics or English or both?
v passed Mathematics or English but not both?
vi failed both subjects?
vii passed Mathematics but not English?
viii passed English but not Mathematics?

6 A group of students were surveyed about whether or not they played soccer or netball. It was found that 15 played soccer, 16 played netball, 10 played both soccer and netball and 7 played neither soccer nor netball.

	Soccer	Not soccer	
Netball			
Not netball			

a Put this information in a two-way table like the one shown and find the missing numbers.

b How many students:

i do not play soccer?

ii do not play netball?

iii play soccer or netball or both?

iv play soccer or netball but not both?

v play soccer but not netball?

vi play netball but not soccer?

7 100 people were surveyed about their smoking and drinking habits. It was found that 19 people smoked cigarettes, 13 both smoked and drank alcohol and 15 people neither smoked nor drank alcohol.

	Drink	Do not drink	
Smoke			
Do not smoke			

a Put this information in a table like the one shown and find the missing numbers.

b How many people:

i drink alcohol?

ii do not drink alcohol?

iii do not smoke?

iv smoke or drink or both?

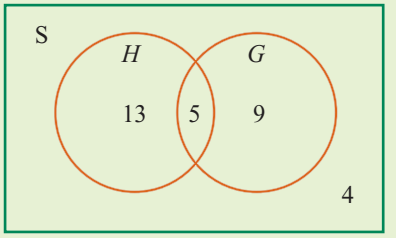
v smoke or drink but not both?

vi drink but do not smoke?

vii smoke but do not drink?

EXAMPLE 2

Information was collected from a group of students about the subjects that they studied. The number of students studying history (H) or geography (G) is shown in the Venn diagram. Represent the information in the Venn diagram in a two-way table like the one shown.



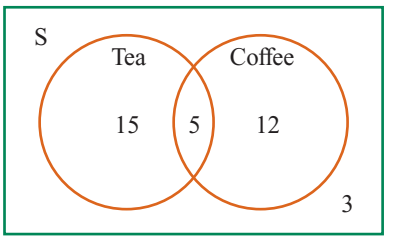
	History	Not history	
Geography			
Not geography			

There are $13 + 5 + 9 + 4 = 31$ in the group.
$13 + 5 = 18$ students study history
$9 + 5 = 14$ students study geography
5 study both history and geography
4 study neither history nor geography
9 study geography but not history
13 study history but not geography
$13 + 4 = 17$ do not study geography
$9 + 4 = 13$ students do not study history

	History	Not history	
Geography	5	9	14
Not geography	13	4	17
	18	13	31

Note: Once you have transferred some information from the Venn diagram it is sometimes possible to use your knowledge of tables to complete it.

8 Data was collected from a group of people about whether they drink tea or coffee. The information is shown in the Venn diagram.



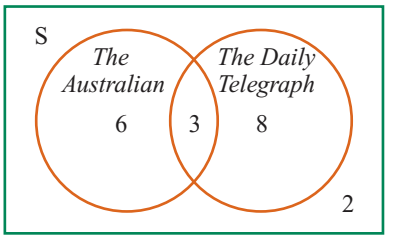
a Represent this information in a two-way table like the one below.

	Drink tea	Do not drink tea	
Drink coffee			
Do not drink coffee			

b Use the Venn diagram or the table to find how many people:

- **i** drink tea
- **ii** do not drink tea
- **iii** drink both tea and coffee
- **iv** drink tea but not coffee
- **v** drink tea or coffee but not both
- **vi** drink neither tea nor coffee
- **vii** at least one beverage
- **viii** exactly one beverage
- **ix** at most one of these beverages.

9 The people sitting in a café were surveyed about which newspaper they read. The number who read *The Australian* and *The Daily Telegraph* is shown in the Venn diagram.

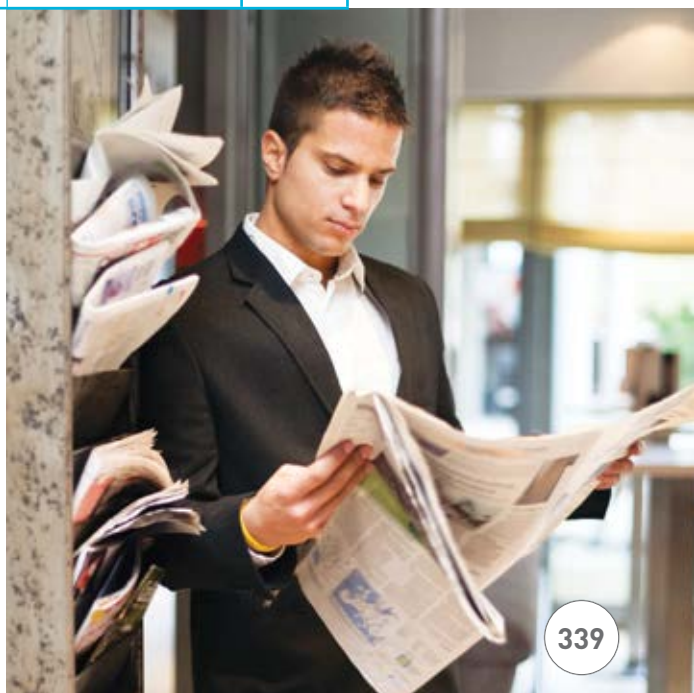


a Represent this information in a two-way table like the one shown below.

	Read *The Australian*	Do not read *The Australian*	
Read *The Daily Telegraph*			
Do not read *The Daily Telegraph*			

b How many people read:

- **i** *The Australian*?
- **ii** *The Daily Telegraph*?
- **iii** both *The Australian* and *The Daily Telegraph*?
- **iv** neither *The Australian* nor *The Daily Telegraph*?
- **v** *The Australian* but not *The Daily Telegraph*?
- **vi** *The Daily Telegraph* but not *The Australian*?
- **vii** *The Australian* or *The Daily Telegraph* or both?
- **viii** *The Australian* or *The Daily Telegraph* but not both?
- **ix** at least one newspaper?
- **x** exactly one newspaper?
- **xi** at most one newspaper?



Language in mathematics

1 Complete the following statements.

a Probability of an event $= \dfrac{\text{number of \_\_\_\_\_ outcomes}}{\text{number of \_\_\_\_\_ outcomes}}$

b If an event is impossible then its probability is _____.

c If an event is certain then its probability is _____.

d The probability of any event lies in the range _____ to _____ inclusive.

e The sum of the probabilities of all possible outcomes for an experiment is _____.

f The events 'getting a 2' and 'not getting a 2' are called _____ events.

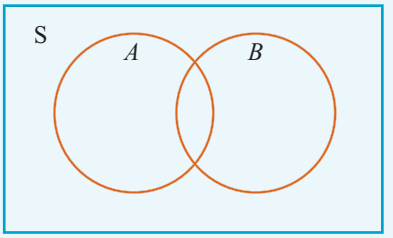
g In a Venn diagram the sample space is represented by points, or the region, inside a _____ and the outcomes favourable to an event are represented by points, or the region, inside a _____.

2 What is the complement of the event:

a selecting a blue marble?

b tossing a head?

3 The Venn diagram shows the events A and B in the sample space S. Describe in words the region where you would find the following set of outcomes.



a favourable to event A

b the complement of A

c favourable to events A and B

d favourable to events A or B or both

e favourable to events A or B but not both

Terms

certain	common	complement	complementary events		equally likely
excluding	exclusive or	divisible	favourable	impossible	including
inclusive or	intersection	outcome	possible	probability	random
sample space	shuffled	theoretical	Venn diagram		

Check your skills

1 The numbers 1 to 7 are written on cards, the cards are shuffled and one is selected at random. The probability that it is a number less than 4 is:

A $\frac{3}{7}$ **B** $\frac{4}{7}$ **C** $\frac{3}{4}$ **D** $\frac{7}{3}$

2 The letters of the word DISTRIBUTION are written on cards and one card is selected at random. The probability that the letter chosen is I is:

A $\frac{1}{10}$ **B** $\frac{1}{4}$ **C** $\frac{3}{10}$ **D** $\frac{1}{3}$

3 If the probability of an event is 1 then the event is:

A certain **B** quite likely **C** unlikely **D** impossible

4 When calculating the probability of choosing at random an apple from a bag of fruit, which of the following answers could not be correct?

A 0 **B** $\frac{3}{11}$ **C** $\frac{11}{3}$ **D** 0.3

5 A roulette wheel has red, green and black slots. When spun the probability that the ball will stop in a red slot is $\frac{18}{37}$ and the probability that it will stop in black slot is also $\frac{18}{37}$. The probability that it will stop in a green slot is:

A $\frac{18}{37}$ **B** $\frac{1}{37}$ **C** $\frac{1}{18}$ **D** $\frac{36}{37}$

6 The probability that a particular set of traffic lights is green is $\frac{43}{73}$. The probability that the light will not be green is:

A $\frac{30}{43}$ **B** $\frac{30}{73}$ **C** $\frac{43}{30}$ **D** $\frac{1}{2}$

7 If a marble is chosen from a bag containing red, blue, green and white marbles, the complement of the event 'choosing a red marble' is:

A choosing a blue marble **B** choosing a blue or green marble
C choosing a blue or white marble **D** choosing a blue or green or white marble.

8 The probability of choosing a red disc from a box is $\frac{4}{23}$. The probability of not drawing a red disc from the box is:

A $\frac{4}{23}$ **B** $-\frac{4}{23}$ **C** $\frac{19}{23}$ **D** $\frac{23}{4}$

9 A card is selected at random from a normal 52-card playing pack. Which of the following events are mutually exclusive?

A getting a queen, getting a red card **B** getting a club, getting picture card
C getting a king, getting an ace **D** getting a red 9, getting a diamond

10 The numbers 1 to 9 are written on cards and one card is selected at random. The event 'at least a 6' is equivalent to:

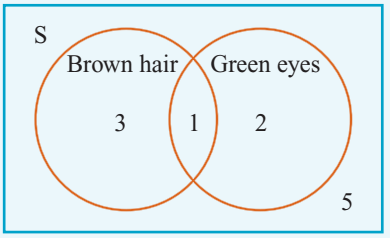
A 1, 2, 3, 4, 5 or 6 **B** 1, 2, 3, 4 or 5 **C** 6, 7, 8 or 9 **D** 7, 8 or 9

11 This table shows the possible results, including the order, when two children are born. The outcomes for the event 'at most one boy' are:

A {bg, gb} **B** {bg, gb, bb}
C {bg, gb, gg} **D** {bg, gb, bb, gg}

		1st child	
		Boy	**Girl**
2nd child	**Boy**	bb	gb
	Girl	bg	gg

The Venn diagram shows the number of people in a group of friends who have brown hair and green eyes. A person is selected at random from this group. Use the Venn diagram to answer questions **12** to **14**.



12 The number of people who have brown hair and green eyes is:

A 1 **B** 3
C 5 **D** 6

13 The number of people who do not have brown hair is:

A 2 **B** 5 **C** 7 **D** 8

14 The number of people who have brown hair or green eyes but not both is:

A 1 **B** 3 **C** 5 **D** 6

The table below shows information about a Year 8 class. Use the table to answer questions **15** to **17**.

	Passed Mathematics	Did not pass Mathematics	
Passed Science	22	2	24
Did not pass Science	x	y	5
	26	3	29

15 The missing numbers x and y are:

A 4, 1 **B** 19, 1 **C** 4, 23 **D** 19, 23

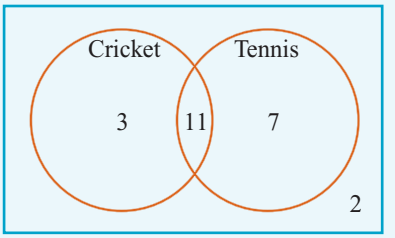
16 The number of students who passed mathematics but not science is:

A 2 **B** 4 **C** 33 **D** 26

17 The number of students who passed at least one subject is:

A 6 **B** 23 **C** 28 **D** 50

18 The information in the Venn diagram is to be represented in a two-way table like the one shown.



	Play tennis	Do not play tennis	
Play cricket		x	
Do not play cricket			
	y		

The values of x and y are:

A 3, 7 **B** 14, 7 **C** 3, 18 **D** 14, 18

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–4	5, 6	7	8	9	10, 11	12–14	15–18
Section	A	B	C	D	E	F	G	H

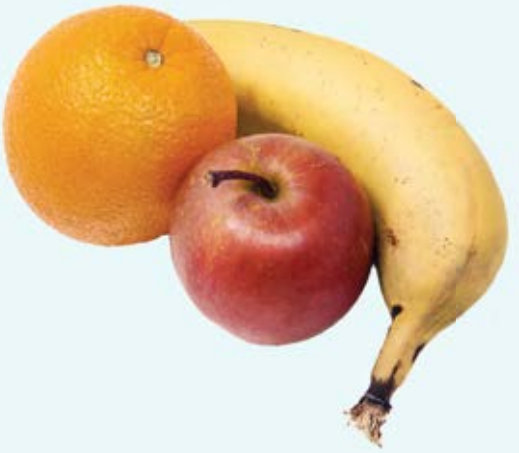
12A Review set

1 The numbers 1 to 9 are written on cards, the cards shuffled and one is selected at random. Find the probability that the number is:

a 7 **b** 7 or 8 **c** odd **d** even **e** less than 4

2 A bag contains 4 oranges, 3 apples and 2 bananas. If one piece of fruit is chosen at random from the bag, find the probability that it is:

a an apple
b an orange
c an apple or a banana
d an orange or an apple or a banana
e a peach.



3 A normal six-sided die is rolled. Write down an event E for which:

a $P(E) = 0$ **b** $P(E) = 1$

4 Red, blue and green tickets are placed in a hat. If one ticket is chosen at random from this hat, the probability that it is red is $\frac{1}{2}$ and the probability that it is blue is $\frac{2}{5}$. Calculate the probability that it is green.

5 A die is rolled. Write down the complement of these events.

a a 4 **b** an even number **c** a number less than 4

6 The letters of a word are written on cards and the cards placed in a hat. When a card is chosen at random from this hat, the probability that it is the letter E is $\frac{2}{5}$. What is the probability that it is not the letter E?

7 A coin is tossed twice. The table shows all the possible outcomes.

a S = {_____}

b Use the table to list the outcomes for the event:

i a head and a tail, in any order **ii** not two heads

iii at least one head **iv** at most one head

		1st toss	
		H	**T**
2nd toss	**H**	HH	TH
	T	HT	TT

8 The numbers 1 to 9 are written on cards, the cards are shuffled and one card is selected at random. State whether the following events are mutually exclusive or non-mutually exclusive.

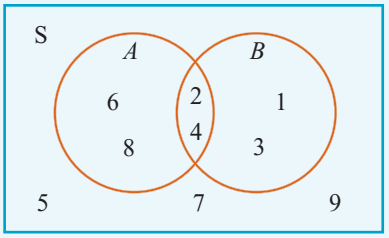
a selecting an even number, selecting a number > 4

b selecting an even number, selecting a multiple of 3

c selecting a number > 4, selecting a number < 4

d selecting a number > 3, selecting a number > 6

9 Numbers are written on cards, the cards are shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram.



a What is the number of possible outcomes for this experiment?

b List the outcomes for the following events.

i A **ii** not A

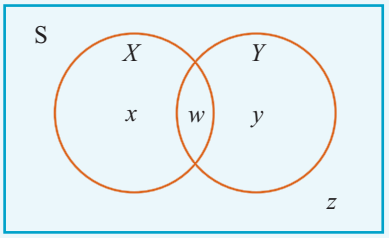
iii B **iv** not B

v A but not B **vi** B but not A

vii A and B **viii** A or B or both

ix A or B but not both **x** neither A nor B

10 In a class of 30 students, 15 have black hair, 12 have brown eyes and 7 have both black hair and brown eyes.



a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes' by finding the numbers w, x, y and z. Note that the number of students, not the individual students, is shown in each region of the diagram.

b How many students:

i have black hair? **ii** do not have black hair?

iii have brown eyes? **iv** do not have brown eyes?

v have black hair but not brown eyes? **vi** have brown eyes but not black hair?

vii have black hair and brown eyes? **viii** have black hair or brown eyes or both?

ix have black hair or brown eyes but not both? **x** have neither black hair nor brown eyes?

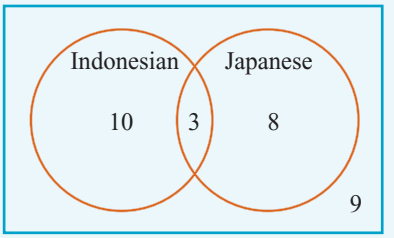
11 The table shows the data collected from a group of students about the way they get to school.

	Travel by bus	Do not travel by bus	
Walk	83	32	115
Do not walk	44	20	64
	127	52	179

How many students:

a travel by bus?
b do not walk?
c travel by bus or walk but not both?
d travel by bus and walk?
e neither walk nor travel by bus?

12 The Venn diagram shows information about the languages studied by a group of students.



a Transfer this information to a two-way table like the one below.

	Study Indonesian	Do not study Indonesian	
Study Japanese			
Do not study Japanese			

b How many students study:

i Indonesian and Japanese?
ii neither Indonesian nor Japanese?
iii Indonesian but not Japanese?
iv Indonesian or Japanese but not both?

12B Review set

1 An ordinary six-sided die is thrown. Find the probability of getting:

a a 6
b an even number
c a number > 3
d a number between 1 and 5
e a number divisible by 3

2 A letter is chosen at random from the letters of the word MISSISSIPPI. Find the probability that the letter chosen is:

a M
b P
c I
d S
e K

3 A spinner is divided into three equal parts coloured red, blue and green. Write down an event E for which:

a $P(E) = 0$
b $P(E) = 1$

4 When two coins are tossed the possible outcomes are exactly 2 heads, exactly 1 head or zero heads. Given that the probability of zero heads is $\frac{1}{4}$ and the probability of exactly 1 head is $\frac{1}{2}$, calculate the probability of getting exactly 2 heads.

5 A bag contains red, blue and green marbles. A marble is selected at random. Write down the complement of each event.

a a blue marble
b a red or blue marble
c not a blue marble

6 When a lolly is selected at random from a bag the probability that it is red is $\frac{6}{17}$. If a lolly is chosen at random from this bag, what is the probability that it is not red?

7 The numbers 1 to 9 are written on cards and one card is selected. List the outcomes for these events.

a at least 7

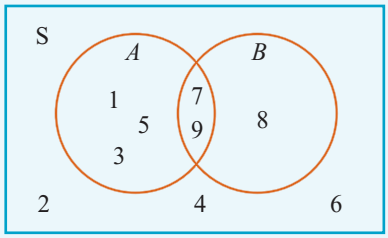
b at most 5

8 State with reasons whether the qualifiers 'or both' and 'but not both' are needed for the event 'A or B' if, when a die is rolled:

a A is the event 'an odd number', B is the event 'an even number'

b A is the event 'a number < 4', B is the event 'a number > 2'.

9 A spinner with nine equal parts numbered 1 to 9 is spun. A is the event 'the spinner lands on an odd number' and B is the event 'the spinner lands on a number > 6'. The outcomes of this experiment and events A and B are shown in the Venn diagram.



Use the diagram to list the outcomes in these events.

a an odd number

b *n*ot an odd number

c a number > 6

d a number not > 6

e a number > 6 that is not odd

f an odd number not > 6

g a number that is odd and > 6

h a number that is odd or > 6 or both

i a number that is odd or > 6 but not both

j a number that is neither odd nor > 6

10 In a group of 27 students, 12 play football, 11 play cricket and 6 play both football and cricket.

a Construct a Venn diagram to show this information, where F is the event 'plays football' and C is the event 'plays cricket'.

b How many students:

i play football but not cricket?

ii play cricket but not football?

iii play football or cricket or both?

iv play football or cricket but not both?

v neither play football nor cricket?

11 The table shows the information collected from a group of people.

	Have blond hair	**Do not have blond hair**	
Have blue eyes	x	y	13
Do not have blue eyes	4	z	15
	11	17	28

a Find the missing numbers x, y and z.

b How many students have:

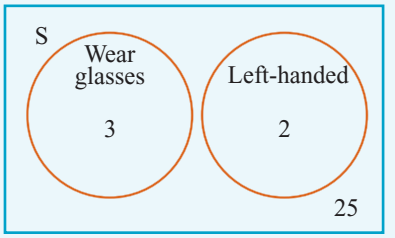
i blond hair and blue eyes?

ii neither blond hair nor blue eyes?

iii blond hair or blue eyes or both?

iv blond hair but not blue eyes?

12 The Venn diagram shows the information collected from a class of students about whether they wear glasses and whether they are left-handed.



Transfer this information to the two-way table below.

	Wear glasses	Do not wear glasses	
Left-handed			
Not left-handed			

12C Review set

1 The numbers 0 to 7 are written on cards, the cards are shuffled and one is selected at random. What is the probability that the number is:

a 7? **b** 2 or 3? **c** even?

d between 0 and 7? **e** greater than 4?

2 A card is selected at random from a normal playing pack of 52 cards. Find the probability that it is:

a the ace of spades **b** a king **c** a red 7 **d** a diamond

e black **f** red or black **g** a green 19

3 The numbers 1 to 5 are written on cards, the cards are shuffled and one is chosen at random. Write down an event E for which:

a $P(E) = 0$ **b** $P(E) = 1$

4 A purse contains 5-cent, 10-cent, 20-cent and 50-cent coins. When a coin is chosen at random from this purse, the probability of getting a 5-cent coin is $\frac{1}{8}$, the probability of getting a 10-cent coin is $\frac{2}{5}$ and the probability of getting a 50-cent coin is $\frac{1}{10}$. Calculate the probability of getting a 20-cent coin.

5 A card is selected at random from a normal playing pack. What is the complement of the event:

a a red card? **b** a spade? **c** an ace?

6 On a roulette wheel, the probability that the ball will stop in a black slot is $\frac{9}{17}$. What is the probability that it will not stop in a black slot?

7 A normal six-sided die is rolled. Write down two events A and B that are:

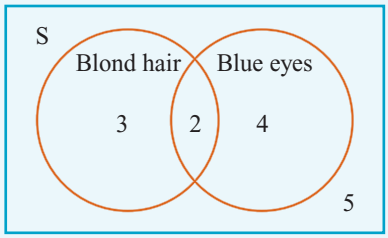
a mutually exclusive **b** non-mutually exclusive.

8 A card is chosen from a normal playing pack. List the outcomes for these events.

a a red card and a jack **b** a spade and a picture card

9 State with reasons if the qualifiers 'or both', 'but not both' are needed for the event 'A or B' given that a student is chosen from a class of boys and girls and A is the event 'a boy', B is the event 'has red hair'.

10 In a group of friends, some have blond hair and some have blue eyes. This information is shown in this Venn diagram.



a How many people are in this group?

b How many people:

i have blond hair?

ii do not have blond hair?

iii have blue eyes?

iv do not have blue eyes?

v have blond hair but not blue eyes?

vi have blue eyes but not blond hair?

vii have blond hair and blue eyes?

viii have blond hair or blue eyes or both?

xi have blond hair or blue eyes but not both?

x have neither blond hair nor blue eyes?

11 In a class of 30 students, 13 students study history, 15 study geography and 7 study both history and geography. Construct a Venn diagram to show this information. Let H be the event 'studies history' and G the event 'studies geography'.

12 **a** Represent the information in question **11** in a two-way table like the one below.

	Study History	**Do not study History**	
Study Geography			
Do not study Geography			

b How many students:

i do not study history?

ii do not study geography?

iii study history and geography?

iv study history or geography or both?

v study history or geography but not both?

vi study history but not geography?

vii study geography but not history?

viii study neither history nor geography?

ix study at least one of these subjects?

x study exactly one of these subjects?

xi study at most one of these subjects?

12D Review set

1 The numbers 1 to 21 are written on 21 cards. One card is selected at random. Find the probability that the number is:

a 10 **b** bigger than 10 **c** smaller than 10 **d** odd

e even **f** a multiple of 5 **g** contains the digit 5 **h** contains the digit 1.

2 The letters of the word PROBABILITY are written on cards and one is chosen at random. Find the probability that the letter is:

a B **b** I **c** R

d B or I **e** a vowel **f** K

3 A spinner is divided into three equal segments coloured red, blue and green. The spinner is spun. Write down an event E for which:

a $P(E) = 1$ **b** $P(E) = 0$

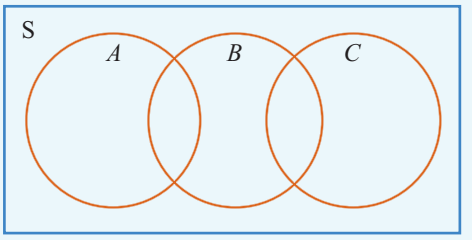
4 Black, white and purple tickets are placed in a hat and one ticket is drawn at random. Given that the probability of drawing a black ticket is $\frac{1}{3}$ and the probability of a white ticket is $\frac{1}{5}$, what is the probability of drawing a purple ticket?

5 A card is selected at random from a normal playing pack. Write down the complement of each event.

a a red card **b** a diamond

6 When a chocolate is selected at random from a box, the probability that it is a dark chocolate is 0.4. What is the probability that it is not a dark chocolate?

7 The Venn diagram shows three events A, B and C. State whether these statements are true or false.



a A and B are mutually exclusive.

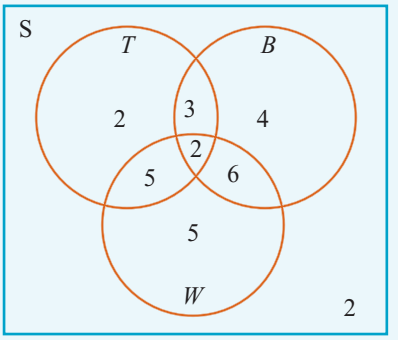
b A and C are mutually exclusive.

c B and C are non-mutually exclusive.

8 A normal six-sided die is thrown. List the outcomes for each event.

a a number > 3 **b** at least 3 **c** a number < 3 **d** at most 3

9 The Venn diagram represents the way in which the students in a class travel to school. T is the event 'travels by train', B is the event 'travels by bus' and W is the event 'walks to school'. How many students travel to school by:



a train? **b** train only?

c train and walk? **d** train or walk or both?

e train or walk but not both? **f** bus and walk?

g bus or walk but not both? **h** bus and train and walk?

i train but not by bus? **j** bus or train or both?

k none of these methods?

10 The two-way table shows the number of students in a class who study French and German.

	Study French	**Do not study French**	
Study German	3	w	11
Do not study German	x	13	19
	y	21	z

a Find the missing numbers w, x, y and z.

b Draw a Venn diagram to represent this information, using F is the event 'study French' and G is the event 'study German'.

c How many students:

i study French? **ii** do not study French?

iii study German? **iv** do not study German?

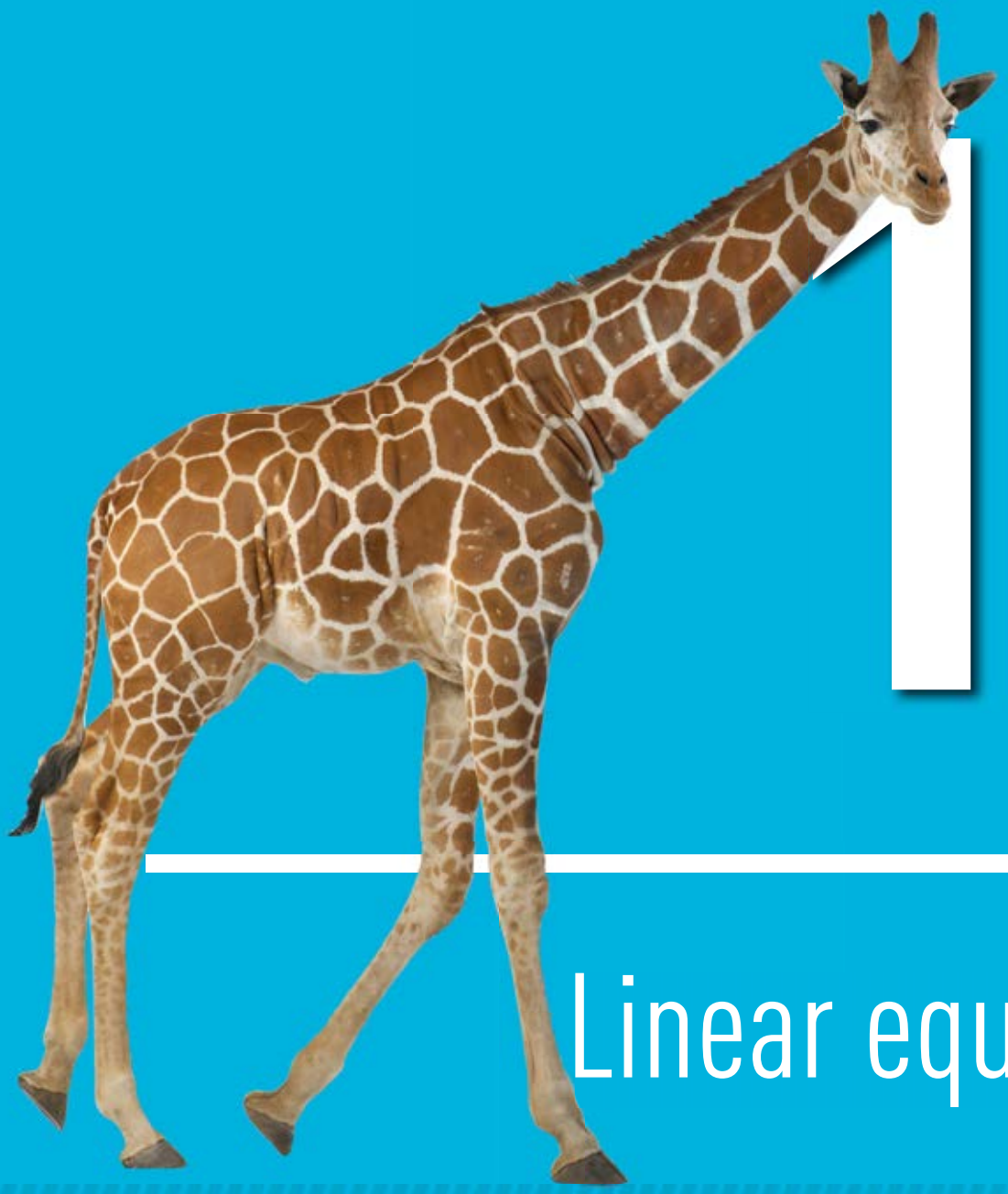
v study French and German? **vi** study French or German or both?

vii study French or German but not both? **viii** study neither French nor German?

ix study French but not German? **x** study German but not French?

xi study at least one of these languages? **xii** study at most one of these languages?

xiii study exactly one of these languages?



13

Linear equations

This chapter deals with linear equations.

At the end of this chapter you should be able to:

- solve simple linear equations using inspection and concrete materials
- solve linear equations using strategies such as backtracking
- solve equations by algebraic methods
- check solutions by substituting
- translate word problems into equations
- solve equations arising from substitution into formulas.

NSW Syllabus references: S4 N&A Equations
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-10NA
NUMBER & ALGEBRA – ACMNA194

Diagnostic test

1 Simplify $4a + 8a - 3a$.

A $6a^3$ **B** $9a$

C $12a^2 - 3a$ **D** $9a^3$

2 Simplify $11p - 3 + 4p$.

A $15p - 3$ **B** $15p^2 - 3$

C $7p - 3$ **D** $7p^2 - 3$

3 Simplify $4m - 6 - 8m + 2$.

A $-4m + 8$ **B** $4m + 4$

C $4m - 4$ **D** $-4m - 4$

4 Simplify $5 \times n \times 3 \times n \times f \times n$.

A $15nf$ **B** $5n \times 3nf$

C $15fn^3$ **D** $8n^3f$

5 Simplify $-7g \times 4h$.

A -28 **B** $-28gh$

C $4gh$ **D** $-74gh$

6 Simplify $\frac{4ab}{8b}$.

A $2a$ **B** $\frac{a^2}{2}$

C $\frac{a}{2}$ **D** $2ab$

7 Simplify $\frac{5k}{11} - \frac{3k}{11}$.

A $\frac{8k}{11}$ **B** $\frac{2k}{11}$

C $\frac{2k^2}{22}$ **D** $\frac{8k}{22}$

8 Simplify $\frac{2a}{5} + \frac{a}{3}$.

A $\frac{11a}{15}$ **B** $\frac{3a}{8}$

C $\frac{16a}{5}$ **D** $\frac{3a}{15}$

9 Expand $a(c - d)$.

A $c - ad$ **B** $ac - ad$

C $ac - d$ **D** $ac - a^2d$

10 Expand $5(m + 7)$.

A $5m + 35$ **B** $5m + 7$

C $m + 35$ **D** $5m^2 + 57$

11 Expand and simplify $4(p + 3) - 2(p - 5)$.

A $6p - 2$ **B** $2p - 2$

C $6p + 22$ **D** $2p + 22$

12 Which set below is an example of like terms?

A $\{3a, a^2\}$ **B** $\{5a^2, a^2\}$

C $\{2y, 3xy\}$ **D** $\{4a, 4\}$

13 If $m = 7$ and $n = 4$, evaluate p if $p = 2m + n$.

A 31 **B** 7 **C** 10 **D** 18

14 If $a = 6$ and $b = 8$, evaluate E if $E = \frac{ab}{2}$.

A 24 **B** 43 **C** 21 **D** 34

15 If $x = -3$ and $y = 5$, evaluate w if $w = 2x - y$.

A -11 **B** 13 **C** 11 **D** 1

16 Which set of numbers would complete this table of values?

J	1	2	3	4
$J + 3$				

A {4, 6, 9, 13} **B** {31, 32, 33, 34}

C {4, 5, 6, 7} **D** {5, 8, 11, 15}

17 State the rule that has been used for this table of values.

x	0	1	2	3	4
y	1	3	5	7	9

A $y = 2x - 2$ **B** $y = 2x + 1$

C $y = x + 1$ **D** $y = 3x - 2$

18 Write the sum of y and 7 in algebraic form.

A $y - 7$ **B** $y + 7$

C $\frac{y}{7}$ **D** $7y$

19 Write the difference between m and n multiplied by 8 in algebraic form.

A $m - 8n$ **B** $8m - n$

C $8(m - n)$ **D** $8(n + m)$

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA194.

A Equations

An equation is a mathematical sentence that indicates that two expressions have the same value.

The illustration shows the equation $x + 3 = 6$.

x is the pronumeral or the unknown.

The solution of the equation is the value of the pronumeral that makes the statement true.

By inspection, the solution to the equation $x + 3 = 6$ is that $x = 3$.

This value of x makes the statement true.

The scale balances as the LHS = RHS.

Exercise 13A

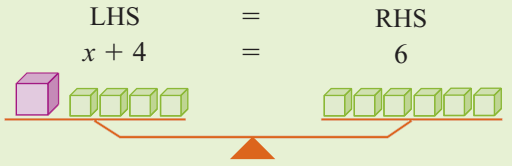
1 By inspection, find the value of the pronumeral in each equation.

a $x + 8 = 10$	**b** $x + 4 = 8$	**c** $x + 1 = 5$	**d** $y - 3 = 5$
e $y - 1 = 7$	**f** $y - 4 = 6$	**g** $m \div 5 = 2$	**h** $m \div 3 = 3$
i $m \div 4 = 2$	**j** $4p = 8$	**k** $3p = 9$	**l** $2p = 10$

Example 1 shows how concrete materials can also be used to derive a solution for an equation. From the example you can see that what you do to one side, you must also do to the other side.

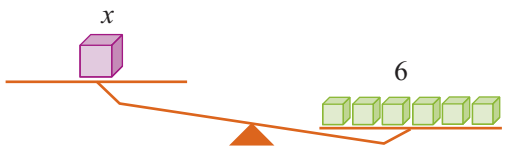
EXAMPLE 1

Find the value of x in the equation $x + 4 = 6$ using this illustration.



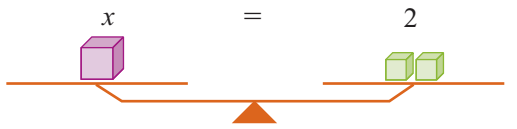
The aim is to get x ( ) by itself to find its value.

Remove 4 cubes from the LHS and x is by itself, but the scales don't balance.



To balance the scales, 4 cubes need to be taken away from the RHS.

The scales are balanced so the solution is $x = 2$.

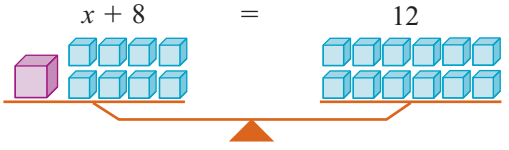


2 Consider each illustration below.

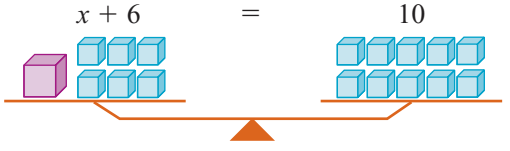
i Explain what you would need to do to both sides to find the value of the unknown.

ii State the solution of the unknown (x).

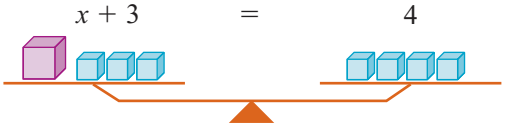
a



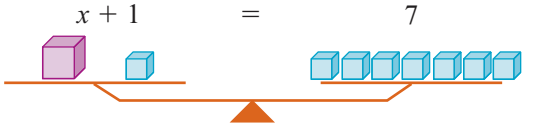
b



c



d



e $x + 2 = 5$

f $x + 7 = 12$

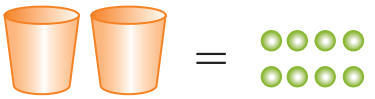
EXAMPLE 2

How many marbles are contained in each cup?

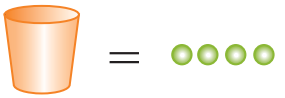
2 cups + 3 marbles = 11 marbles

Remove 3 marbles from both sides.

2 cups = 8 marbles



As there are 2 cups, divide each side by 2.
∴ 1 cup contains 4 marbles.



3 Use concrete materials, cups and counters for example, to solve the following problems.

i Explain the processes of obtaining your solution.

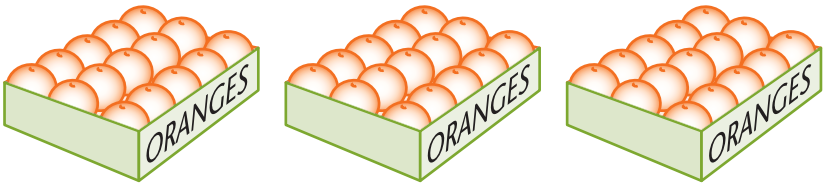
ii State your solution.

a How many pills are contained in each bottle of pills if three bottles of pills contain 36 pills.

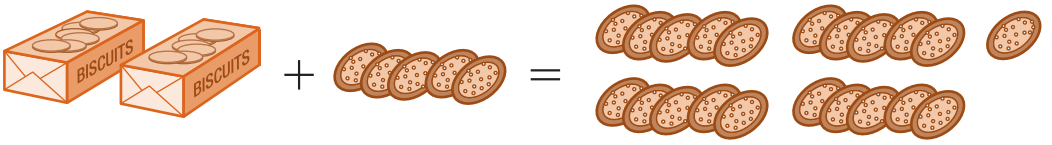
$3b = 36$

= 36 tablets

b Three boxes of oranges contain a total of 45 oranges. How many oranges are contained in each box?



c How many biscuits in each packet?



2 packets of biscuits + 5 biscuits = 21 biscuits

$2b + 5 = 21$

d How many lollies in each box?

3 boxes of lollies + 3 lollies = 27 lollies

$3l + 3 = 27$

B Backtracking

Some equations, like those outlined in Exercise 13A, can readily be solved by inspection or with the aid of concrete materials.

Backtracking is another method that involves recognising that the variable or unknown has been changed and then reversing this process. First, however, to recognise the inverse operations we have to understand how an algebraic expression has been built from a single pronumeral.

EXAMPLE 1

Use the operation shown on the arrow in the flowchart to build an algebraic expression.

a $x \xrightarrow{+5} \square$ **b** $x \xrightarrow{-5} \square$

c $x \xrightarrow{\times 5} \square$ **d** $x \xrightarrow{\div 5} \square$

a $x \xrightarrow{+5} x + 5$ **b** $x \xrightarrow{-5} x - 5$

c $x \xrightarrow{\times 5} 5x$ **d** $x \xrightarrow{\div 5} \frac{x}{5}$

Exercise 13B

1 Complete the following algebraic expressions.

a $x \xrightarrow{+7} \square$ **b** $x \xrightarrow{+2} \square$ **c** $x \xrightarrow{+9} \square$

d $x \xrightarrow{-3} \square$ **e** $x \xrightarrow{-1} \square$ **f** $x \xrightarrow{-7} \square$

g $x \xrightarrow{\times 8} \square$ **h** $x \xrightarrow{\times -4} \square$ **i** $x \xrightarrow{\times 5} \square$

j $x \xrightarrow{\div 6} \square$ **k** $x \xrightarrow{\div 2} \square$ **l** $x \xrightarrow{\div 11} \square$

EXAMPLE 2

Use the combination of operations shown to build algebraic expressions.

a $x \xrightarrow{\times 2} \square \xrightarrow{+5} \square$ **b** $x \xrightarrow{\times 4} 4x \xrightarrow{\div 7} \square$

c $x \xrightarrow{\times 5} \square \xrightarrow{\div 8} \square \xrightarrow{-2} \square$ **d** $x \xrightarrow{\times 3} \square \xrightarrow{-5} \square \xrightarrow{\div 4} \square$

a $x \xrightarrow{\times 2} 2x \xrightarrow{+5} 2x + 5$ **b** $x \xrightarrow{\times 4} 4x \xrightarrow{\div 7} \frac{4x}{7}$

c $x \xrightarrow{\times 5} 5x \xrightarrow{\div 8} \frac{5x}{8} \xrightarrow{-2} \frac{5x}{8} - 2$ **d** $x \xrightarrow{\times 3} 3x \xrightarrow{-5} 3x - 5 \xrightarrow{\div 4} \frac{3x - 5}{4}$

2 Complete the following expressions.

a $x \xrightarrow{\times 6} 6x \xrightarrow{+4} \square$

b $x \xrightarrow{\times(-9)} -9x \xrightarrow{-1} \square$

c $x \xrightarrow{\times 5} 5x \xrightarrow{+2} \square$

d $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{-8} \square$

e $x \xrightarrow{\div(-7)} \frac{x}{-7} \xrightarrow{+5} \square$

f $x \xrightarrow{\div 10} \frac{x}{10} \xrightarrow{-4} \square$

g $x \xrightarrow{\times 2} 2x \xrightarrow{+7} 2x+7 \xrightarrow{\div 6} \square$

h $x \xrightarrow{\times 5} 5x \xrightarrow{-9} 5x-9 \xrightarrow{\div 2} \square$

i $x \xrightarrow{\times(-4)} -4x \xrightarrow{-1} -4x-1 \xrightarrow{\div 7} \square$

j $x \xrightarrow{\times 6} 6x \xrightarrow{-5} 6x-5 \xrightarrow{\times 2} \square$

k $x \xrightarrow{\times 2} 2x \xrightarrow{+7} 2x+7 \xrightarrow{\times(-5)} \square$

l $x \xrightarrow{\times(-8)} -8x \xrightarrow{+1} -8x+1 \xrightarrow{\times(-3)} -3(-8x+1) \xrightarrow{\div 4} \square$

EXAMPLE 3

Use arrows to show how the following expressions have been built from the pronumeral x.

a $7x$ **b** $-3x+4$ **c** $\frac{-4(x+2)}{3}$

a $x \xrightarrow{\times 7} 7x$

b $x \xrightarrow{\times(-3)} -3x \xrightarrow{+4} -3x+4$

c $x \xrightarrow{+2} x+2 \xrightarrow{\times(-4)} -4(x+2) \xrightarrow{\div 3} \frac{-4(x+2)}{3}$

3 Use arrows to show how the following expressions were built from the pronumeral x.

a $3x$ **b** $x+5$ **c** $x-8$ **d** $-6x$ **e** $x-9$

f $\frac{x}{2}$ **g** $x+10$ **h** $\frac{x}{-4}$ **i** $5x+12$ **j** $-3x-1$

k $\frac{x}{-4}+2$ **l** $\frac{x}{5}+9$ **m** $\frac{x-8}{7}$ **n** $\frac{x+10}{13}$ **o** $\frac{5x-2}{9}$

p $\frac{3x+7}{10}$ **q** $4(x-6)$ **r** $-5(x+3)$ **s** $9(2x+7)$ **t** $-6(4x-6)$

u $\frac{3(x-6)}{5}$ **v** $\frac{-9(x+2)}{11}$ **w** $\frac{7(3x-5)}{10}$ **x** $\frac{-4(2x-8)}{13}$ **y** $\frac{5(x+1)}{-3}$

When solving equations, we need to reverse the steps or do the inverse of the operations to find the value of the single pronumeral.

So addition and subtraction are inverse operations. Multiplication and division are also inverse operations.

EXAMPLE 4

Write the inverse operation to get back to x.

a $x+5$ **b** $x-5$ **c** $5x$ **d** $\frac{x}{5}$

a $x+5 \xrightarrow{-5} x$

Inverse of $+5$ is -5.

b $x-5 \xrightarrow{+5} x$

Inverse of -5 is $+5$.

c $5x \xrightarrow{\div 5} x$

Inverse of $\times 5$ is $\div 5$.

d $\frac{x}{5} \xrightarrow{\times 5} x$

Inverse of $\div 5$ is $\times 5$.

4 Complete the following by performing the inverse operations to get back to x.

a $x + 4 \rightarrow x$

b $x + 6 \rightarrow x$

c $x + 9 \rightarrow x$

d $x - 5 \rightarrow x$

e $x - 3 \rightarrow x$

f $x - 11 \rightarrow x$

g $2x \rightarrow x$

h $-9x \rightarrow x$

i $7x \rightarrow x$

j $\frac{x}{10} \rightarrow x$

k $\frac{x}{3} \rightarrow x$

l $\frac{x}{-4} \rightarrow x$

m $6x + 2 \rightarrow 6x \rightarrow x$

n $-3x - 1 \rightarrow -3x \rightarrow x$

o $7x + 10 \rightarrow 7x \rightarrow x$

p $\frac{x}{5} - 8 \rightarrow \frac{x}{5} \rightarrow x$

q $2 + \frac{x}{3} \rightarrow \frac{x}{3} \rightarrow x$

r $\frac{x}{11} + 1 \rightarrow \frac{x}{11} \rightarrow x$

s $\frac{3x + 2}{9} \rightarrow 3x + 2 \rightarrow 3x \rightarrow x$

t $\frac{11 + 2x}{8} \rightarrow 11 + 2x \rightarrow 2x \rightarrow x$

u $\frac{-4x - 6}{7} \rightarrow -4x - 6 \rightarrow -4x \rightarrow x$

v $\frac{3(x + 7)}{4} \rightarrow 3(x + 7) \rightarrow x + 7 \rightarrow x$

w $\frac{-2(5x - 10)}{11} \rightarrow -2(5x - 10) \rightarrow 5x - 10 \rightarrow 5x \rightarrow x$

5 Show each step required to get from the following expressions back to x.

a $12x$

b $x - 8$

c $x + 11$

d $\frac{x}{5}$

e $4x - 10$

f $-9x$

g $\frac{-3x}{7}$

h $15 - 8x$

i $-x + 1$

j $3 + \frac{2x}{5}$

k $\frac{x}{4} - 1$

l $5 - \frac{x}{2}$

m $\frac{x - 9}{4}$

n $\frac{10 - x}{2}$

o $\frac{11 - 4x}{7}$

p $\frac{3x + 6}{5}$

q $2(x + 14)$

r $-3(x - 18)$

s $-2(2x + 6)$

t $9(3x - 1)$

u $\frac{7(x + 4)}{5}$

v $\frac{-4(x - 8)}{9}$

w $\frac{4(5x + 7)}{7}$

x $\frac{-3(2x - 9)}{11}$

Now that we understand how to build expressions and can do inverse operations on expressions to get back to a single pronumeral, we can solve equations using backtracking techniques.

EXAMPLE 5

Solve $4x - 3 = 17$ by backtracking.

First build the equation.
Then do the inverse operation.
The value of x is 5.

$x \xrightarrow{\times 4} 4x \xrightarrow{-3} 4x - 3$

$5 \xleftarrow{\div 4} 20 \xleftarrow{+3} 17$

6 Complete each diagram to solve the given equations by using backtracking.

a $3x + 5 = 26$

$x \xrightarrow{\times 3} 3x \xrightarrow{+ 5} 3x + 5$

$\square \xleftarrow{} 21 \xleftarrow{- 5} 26$

b $2x - 8 = 20$

$x \xrightarrow{\times 2} \square \xrightarrow{- 8} 2x - 8$

$\square \xleftarrow{} \square \xleftarrow{} 20$

c $4m - 12 = 40$

$m \xrightarrow{\times 4} 4m \longrightarrow \square$

$\square \xleftarrow{} \square \xleftarrow{} 40$

d $6y + 8 = 62$

$\square \longrightarrow 6y \longrightarrow \square$

$\square \xleftarrow{} \square \xleftarrow{} 62$

e $\frac{2p}{3} - 1 = 3$

$p \xrightarrow{\times 2} \square \xrightarrow{\div 3} \square \xrightarrow{- 1} \square$

$\square \xleftarrow{} \square \xleftarrow{} \square \xleftarrow{} 3$

f $\frac{5w}{8} + 7 = 17$

$w \longrightarrow \square \xrightarrow{\div 8} \square \longrightarrow \square$

$\square \xleftarrow{} \square \xleftarrow{} \square \xleftarrow{} 17$

g $6(d + 4) = 54$

$d \longrightarrow \square \longrightarrow \square$

$\square \xleftarrow{} \square \xleftarrow{} 54$

h $5(2a - 3) = 35$

$a \longrightarrow \square \longrightarrow \square \longrightarrow 5(2a - 3)$

$\square \xleftarrow{} \square \xleftarrow{} \square \xleftarrow{} 35$

7 Solve the following equations by backtracking.

a $4m + 8 = 52$ **b** $3d - 12 = 84$ **c** $5x + 19 = 64$

d $8p - 6 = 26$ **e** $\frac{3y}{4} + 7 = 13$ **f** $\frac{2a}{5} + 10 = 20$

g $\frac{3a + 14}{5} = 7$ **h** $\frac{4d - 8}{7} = 12$ **i** $7(2p + 3) = 126$

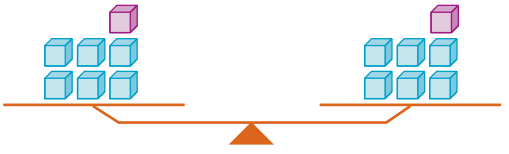
C Maintaining balance

The balance of an equation can be likened to the balance of a set of scales. Changing one side of the equation without doing the same to the other will upset the balance.

To maintain the balance, the same operation must be carried out on each side of the equation.

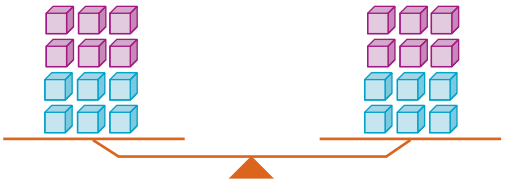
Original

The following diagrams show that balance is maintained if each side of the equation or scale is treated in the same way.

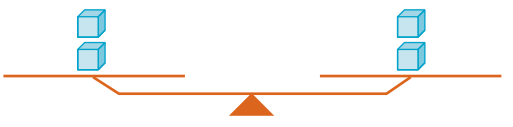


Add 1

Subtract 4



Multiply by 2



Divide by 3

Balance can be maintained if we:

- add the same amount to both sides
- subtract the same amount from both sides
- multiply both sides by the same amount
- divide both sides by the same amount.

Remember, when we are solving equations we are trying to determine the value of the pronumeral that makes the equation true. To do this we must use inverse operations to work back to the pronumeral, as you did in Exercise 13B.

EXAMPLE 1

Solve the following one-step equations by using inverse operations to balance the scales.

a $n + 7 = 13$ **b** $p - 5 = 2$ **c** $3x = 27$ **d** $\frac{R}{6} = 8$

a
$$n + 7 = 13$$
$$n + 7 - 7 = 13 - 7 \quad \text{Subtract 7 from both sides.}$$
$$n = 6$$
Check: LHS $= n + 7 = 6 + 7 = 13$
$\therefore$ LHS = RHS

Equals signs should be underneath each other when solving equations.

b
$$p - 5 = 2$$
$$p - 5 + 5 = 2 + 5 \quad \text{Add 5 to both sides.}$$
$$p = 7$$
Check: LHS $= p - 5 = 7 - 5 = 2$
$\therefore$ LHS = RHS

c
$$3x = 27$$
$$\frac{3x}{3} = \frac{27}{3} \quad \text{Divide both sides by 3.}$$
$$x = 9$$
Check: LHS $= 3x = 3 \times 9 = 27$
$\therefore$ LHS = RHS

d
$$\frac{R}{6} = 8$$
$$\frac{R}{6} \times 6 = 8 \times 6 \quad \text{Multiply both sides by 6.}$$
$$R = 48$$
Check: LHS $= \frac{R}{6} = \frac{48}{6} = 8$
$\therefore$ LHS = RHS

Exercise 13C

1 Solve each of the following one-step equations.

a $x + 11 = 17$ **b** $t + 8 = 15$ **c** $w + 9 = 21$

d $h + 5 = 13$ **e** $26 + d = 32$ **f** $31 + q = 47$

g $m - 5 = 11$ **h** $c - 9 = 17$ **i** $l - 13 = 29$

j $k - 4 = 9$ **k** $a - 16 = 30$ **l** $b - 22 = 36$

m $5y = 40$ **n** $3z = 48$ **o** $11p = 55$

p $7a = 84$ **q** $84 = 12m$ **r** $63 = 9z$

s $\frac{f}{4} = 13$ **t** $\frac{k}{10} = 9$ **u** $\frac{s}{6} = 10$

v $\frac{v}{3} = 7$ **w** $3 = \frac{y}{12}$ **x** $4 = \frac{n}{17}$

EXAMPLE 2

Solve the following one-step equations involving negative integers.

a $7 - y = -5$ **b** $-2k = 8$ **c** $x - (-3) = 11$

a
$$7 - y = -5$$
$$7 - y - 7 = -5 - 7 \quad \text{Subtract 7 from both sides.}$$
$$-y = -12$$
$$(-y) \times -1 = (-12) \times -1 \quad \text{Multiply both sides by } -1.$$
$$y = 12$$
Check: LHS $= 7 - y = 7 - 12 = -5$
$\therefore$ LHS = RHS

b
$$-2k = 8$$
$$\frac{-2k}{-2} = \frac{8}{-2} \quad \text{Divide both sides by } -2.$$
$$k = -4$$
Check: LHS $= -2k = -2 \times -4 = 8$
$\therefore$ LHS = RHS

c
$$x - (-3) = 11$$
$$x - 1(-3) = 11$$
$$x + 3 = 11$$
$$x + 3 - 3 = 11 - 3 \quad \text{Subtract 3 from both sides.}$$
$$x = 8$$
Check: LHS $= x - (-3) = 8 - (-3) = 8 + 3 = 11$
$\therefore$ LHS = RHS

Remember:
There is a 1 outside the brackets.
A negative number multiplied by a negative number results in a positive number.

2 Solve the following one-step equations involving negative integers.

a $x + 6 = 4$ **b** $d + 7 = 2$ **c** $f + 13 = -8$

d $15 + q = -5$ **e** $h + (-8) = -2$ **f** $j - (-4) = -5$

g $a - 5 = -9$ **h** $y - 7 = -11$ **i** $c - 8 = -1$

j $g - 6 = -6$ **k** $l - (-1) = 4$ **l** $-3 - k = 10$

m $10w = -3$ **n** $7h = -5$ **o** $4q = -7$

p $-5m = 40$ **q** $-3d = 18$ **r** $-2n = -20$

s $\frac{b}{4} = -2$ **t** $\frac{e}{3} = -5$ **u** $\frac{r}{6} = -6$

v $\frac{-d}{2} = 10$ **w** $\frac{-k}{8} = 4$ **x** $\frac{v}{-5} = 7$

EXAMPLE 3

Substitute to determine whether the value of x given is a solution to the equation.

a $x + 15 = 48;\ x = 33$ **b** $-4m = 28;\ x = 7$

a
$$\begin{aligned} \text{LHS} &= x + 15 \\ &= 33 + 15 \\ &= 48 \\ \text{LHS} &= \text{RHS} \end{aligned}$$
So $x = 33$ is a solution.

b
$$\begin{aligned} \text{LHS} &= -4m \\ &= -4 \times 7 \\ &= -28 \\ \text{LHS} &\neq \text{RHS} \end{aligned}$$
So $x = 7$ is not a solution.

3 Substitute to determine whether the value of the pronumeral given is a solution to the equation.

a $q + 15 = 33;\ q = 18$ **b** $w + 29 = 44;\ w = 15$

c $10 + k = 52;\ k = 32$ **d** $42 + r = 50;\ r = -6$

e $m - 18 = 7;\ m = 28$ **f** $d - 23 = 11;\ d = 34$

g $32 - n = 41;\ n = 9$ **h** $28 - k = 16;\ k = -12$

i $9b = 162;\ b = 14$ **j** $7a = 49;\ a = 7$

k $-4l = 3;\ l = -\frac{3}{4}$ **l** $-5p = 80;\ p = 16$

m $\frac{c}{5} = 12;\ c = 60$ **n** $\frac{t}{4} = -\frac{1}{2};\ t = -2$

o $\frac{y}{13} = -3;\ y = 39$ **p** $\frac{x}{21} = -8;\ x = -168$

EXAMPLE 4

Solve the following two-step equations.

a $4a + 8 = 20$ **b** $5 - 2q = 14$

a
$$\begin{aligned} 4a + 8 &= 20 \\ 4a + 8 - 8 &= 20 - 8 && \text{Subtract 8 from both sides.} \\ 4a &= 12 \\ \frac{4a}{4} &= \frac{12}{4} && \text{Divide both sides by 4.} \\ a &= 3 \end{aligned}$$
Check: $\text{LHS} = 4a + 8$
$= 4 \times 3 + 8 = 20$
$\therefore \text{LHS} = \text{RHS}$

When you check your answer you must show that the left-hand side is equal to the right-hand side.

b
$$\begin{aligned} 5 - 2q &= 14 \\ 5 - 2q - 5 &= 14 - 5 && \text{Subtract 5 from both sides.} \\ -2q &= 9 \\ \frac{-2q}{-2} &= \frac{9}{-2} && \text{Divide both sides by } -2. \\ q &= -4\tfrac{1}{2} \end{aligned}$$
Check: $\text{LHS} = 5 - 2q$
$= 5 - 2 \times -4\tfrac{1}{2} = 5 + 9 = 14$
$\therefore \text{LHS} = \text{RHS}$

4 Solve the following two-step equations showing all working.

a $7k + 9 = 44$	**b** $3d + 6 = 48$	**c** $17 + 4c = 29$
d $8 + 11b = 63$	**e** $5m - 4 = 11$	**f** $12q - 22 = 86$
g $32 - 3f = 14$	**h** $25 - 5k = 20$	**i** $58 = 26 + 8a$
j $97 = 13 + 12n$	**k** $3 = 11 - 2s$	**l** $15 = 36 - 7g$
m $12 - y = -15$	**n** $-7 - m = -24$	**o** $18 - x = -3$

5 Solve the following equations. (Answers are not whole numbers.)

a $2p + 12 = 21$	**b** $7d + 8 = 19$	**c** $4f + 25 = 52$
d $9q + 6 = 39$	**e** $11a - 10 = 14$	**f** $13k - 4 = 11$
g $5l - 17 = 6$	**h** $14w - 20 = 15$	**i** $1 + 8c = 5$
j $12 + 5n = 13$	**k** $37 + 12f = 65$	**l** $16 + 3l = 33$
m $8 - 2a = 27$	**n** $29 - 6y = 51$	**o** $13 - 4b = 36$

EXAMPLE 5

Solve the following two-step equations.

a $\frac{3x}{4} = 8$ **b** $-\frac{2}{5}y = 11$

a $\frac{3x}{4} = 8$

$4 \times \frac{3x}{4} = 8 \times 4$ Multiply both sides by 4.

$3x = 32$

$\frac{3x}{3} = \frac{32}{3}$ Divide both sides by 3.

$x = 10\frac{2}{3}$

Check: LHS $= \frac{3x}{4}$

$= 3 \times \frac{10\frac{2}{3}}{4} = \frac{3 \times \frac{32}{3}}{4} = \frac{32}{4} = 8$

$\therefore$ LHS = RHS

Multiply each side of the equation by the denominator.

b $-\frac{2}{5}y = 11$

$5 \times -\frac{2}{5}y = 11 \times 5$ Multiply both sides by 5.

$\therefore -2y = 55$

$\frac{-2y}{-2} = \frac{55}{-2}$ Divide both sides by -2.

$y = -27\frac{1}{2}$

Check: LHS $= -\frac{2}{5}y$

$= -\frac{2}{5} \times -27\frac{1}{2} = -\frac{2}{5} \times -\frac{55}{2} = 11$

$\therefore$ LHS = RHS

6 Solve the following equations.

a $\frac{2x}{3} = 2$ **b** $\frac{5d}{6} = 4$ **c** $\frac{4a}{7} = 3$ **d** $\frac{8p}{6} = 5$

e $\frac{6k}{11} = -9$ **f** $\frac{7l}{9} = -6$ **g** $\frac{10q}{13} = -2$ **h** $\frac{3m}{5} = -8$

i $\frac{5}{8}t = 7$ **j** $\frac{9}{11}x = 3$ **k** $\frac{8}{9}y = 4$ **l** $\frac{4}{7}n = 7$

m $-\frac{9}{10}a = 8$ **n** $-\frac{2}{5}w = 6$ **o** $-\frac{10}{11}f = 5$ **p** $-\frac{7}{8}c = 9$

EXAMPLE 6

Solve the following equations.

a $\frac{p-6}{4} = 5$ **b** $\frac{2d+8}{3} = 7$

a $\frac{p-6}{4} = 5$

$4 \times \frac{p-6}{4} = 5 \times 4$ Multiply both sides by 4.

$p - 6 = 20$

$p - 6 + 6 = 20 + 6$ Add 6 to both sides.

$\therefore p = 26$

Check: LHS $= \frac{p-6}{4} = \frac{26-6}{4} = \frac{20}{4} = 5$

$\therefore$ LHS = RHS

Notice that there are two parts to these equations.

b $\frac{2d+8}{3} = 7$

$3 \times \frac{2d+8}{3} = 7 \times 3$ Multiply both sides by 3.

$2d + 8 = 21$

$2d + 8 - 8 = 21 - 8$ Subtract 8 from both sides.

$2d = 13$

$\frac{2d}{2} = \frac{13}{2}$ Divide both sides by 2.

$\therefore d = 6\frac{1}{2}$

Check: LHS $= \frac{2d+8}{3} = \frac{2 \times 6\frac{1}{2} + 8}{3} = \frac{13+8}{3} = \frac{21}{3} = 7$

$\therefore$ LHS = RHS

7 Solve the following equations.

a $\frac{x+7}{5} = 6$ **b** $\frac{p+3}{7} = 4$ **c** $\frac{d+5}{2} = 9$ **d** $\frac{c-4}{3} = 11$

e $\frac{y-9}{4} = 8$ **f** $\frac{a-10}{6} = 2$ **g** $\frac{4b+2}{2} = 5$ **h** $\frac{2m+6}{4} = 7$

i $\frac{5n+8}{3} = 11$ **j** $\frac{5k-1}{9} = 6$ **k** $\frac{7c-3}{6} = 3$ **l** $\frac{9w-3}{5} = 12$

m $\frac{d+9}{5} = -3$ **n** $\frac{3m-7}{8} = -2$ **o** $\frac{5-2d}{7} = -3$ **p** $\frac{7-3d}{5} = -7$

q $\frac{3x+1}{-5} = 10$ **r** $\frac{5x-11}{-4} = -1$ **s** $\frac{8-3x}{5} = -2$ **t** $\frac{4p-5}{-3} = -5$

Investigation 1 One-to-one matching of terms

One-to-one matching of terms requires the equation on both sides of the equals sign to be broken down. Similar parts from both sides are removed and the remaining parts are the solution.

Consider the example below.

$3x + 1 = 2x + 3$

becomes $\cancel{x} + \cancel{x} + x + \cancel{1} = \cancel{x} + \cancel{x} + 2 + \cancel{1}$ Remove similar parts.

remaining part $x = 2$

1 Use one-to-one matching as the technique for solving the equations below. Show all your working.

a $5x + 4 = 4x + 7$
b $8p + 9 = 7p + 2$
c $9m - 6 = 8m - 4$
d $10d - 11 = 9d + 5$
e $6d + 4 = 5d - 6$ (*Hint:* $4 - 10 = -6$)
f $5x + 8 = 4x - 12$ (*Hint:* $8 - 20 = -12$)
g $7a + 3 = 6a - 7$
h $12y - 10 = 11y + 5$
i $3h - 6 = 2h - 5$
j $11 - 8p = 10 - 9p$
k $2g - 14 = g + 6$
l $5 - 11w = -4w - 6$

2 Does one-to-one matching work for all equations? Try to solve the following equations using the technique of one-to-one matching.

a $\frac{3x - 5}{2} = \frac{2x - 6}{3}$
b $7x - 8 = 3x + 12$

3 What are the difficulties experienced when using this strategy to solve equations?

D Equations with pronumerals on both sides

When solving equations with pronumerals on both sides of the equals sign, it is important to collect:

- the pronumerals on one side
- the numerals on the other side.

This is done by adding or subtracting pronumerals and numbers, as shown in Example 1.

EXAMPLE 1

Solve these equations.

a $4x + 7 = 6x + 3$ **b** $7y - 2 = 13 + 4y$

Remember to keep the balance.

a
$$\begin{aligned} 4x + 7 &= 6x + 3 \\ 4x + 7 - 6x &= 6x + 3 - 6x && \text{Subtract } 6x \text{ from both sides.} \\ -2x + 7 &= 3 \\ -2x + 7 - 7 &= 3 - 7 && \text{Subtract 7 from both sides.} \\ -2x &= -4 \\ \frac{-2x}{-2} &= \frac{-4}{-2} && \text{Divide both sides by } -2. \\ x &= 2 \end{aligned}$$

Check: Substitute $x = 2$ into $4x + 7 = 6x + 3$

$4 \times 2 + 7 = 6 \times 2 + 3$

$15 = 15 \quad \therefore$ LHS = RHS

b
$$\begin{aligned} 7y - 2 &= 13 + 4y \\ 7y - 2 - 4y &= 13 + 4y - 4y && \text{Subtract } 4y \text{ from both sides.} \\ 3y - 2 &= 13 \\ 3y - 2 + 2 &= 13 + 2 && \text{Add 2 to both sides.} \\ 3y &= 15 \\ \frac{3y}{3} &= \frac{15}{3} && \text{Divide both sides by 3.} \\ y &= 5 \end{aligned}$$

Check: Substitute $y = 5$ into $7y - 2 = 13 + 4y$.

$7 \times 5 - 2 = 13 + 4 \times 5$

$33 = 33 \quad \therefore$ LHS = RHS

Exercise 13D

1 Solve the following equations showing all steps.

a $5a + 8 = 3 + 6a$ **b** $3q + 11 = 3 + 4q$ **c** $6b + 10 = 16 + 4b$

d $9y + 2 = 10y - 5$ **e** $2f + 9 = 3f - 15$ **f** $h + 7 = 4h - 5$

g $3 - x = x + 7$ **h** $3d - 4 = 5d + 2$ **i** $4 - 2r = 3 - r$

j $5w + 2 = 2w + 14$ **k** $3s + 7 = 11 - s$ **l** $5 + y = 8 - 2y$

m $2x - 3 = x + 6$ **n** $5b - 9 = 1 + 6b$ **o** $3j - 5 = 7 - j$

p $5 + 2p = 11 - p$ **q** $3d - 5 = 5d + 9$ **r** $2a + 5 = 9 - 2a$

EXAMPLE 2

Substitute to determine whether the value of x given is a solution to the equation.

a $2x - 5 = 10 - 3x; \quad x = 3$ **b** $5x + 2 = 2x - 7; \quad x = 2$

a
$$\begin{aligned} \text{LHS} &= 2x - 5 \\ &= 2 \times 3 - 5 = 1 \\ \text{RHS} &= 10 - 3x \\ &= 10 - 3 \times 3 = 1 \end{aligned}$$

$\therefore$ LHS = RHS

So, $x = 3$ is a solution.

b
$$\begin{aligned} \text{LHS} &= 5x + 2 \\ &= 5 \times 2 + 2 = 12 \\ \text{RHS} &= 2x - 7 \\ &= 2 \times 2 - 7 = -3 \end{aligned}$$

$\therefore$ LHS $\neq$ RHS

So, $x = 2$ is not a solution.

2 Substitute to determine whether the value of the pronumeral given is a solution to the equation.

a $3x + 9 = 4 - 2x;\ x = -1$ **b** $7q - 5 = 3 - q;\ q = 2$

c $15 - 2y = 6 + y;\ y = 3$ **d** $2x - 3 = 7 - 4x;\ x = \frac{5}{3}$

e $5x - 7 = 3 + x;\ x = 3\frac{1}{2}$ **f** $8k + 6 = 2k - 2;\ k = -1\frac{1}{3}$

3 Solve the following equations. Check your answer by substituting the solution into the equation.

a $8x + 7 = 4x - 2$ **b** $d - 3 = 5d + 7$ **c** $3 + q = 17 + 4q$

d $15 - 3c = 2 - c$ **e** $11w - 7 = 5w + 12$ **f** $3g - 5 = 14 - 2g$

g $7k = 15 - 3k$ **h** $5k = 2k + 1$ **i** $6l = 5 - 2l$

j $x - 3 = 7x + 5$ **k** $4a - 2 = a - 6$ **l** $12 - 5b = 3b - 6$

E Equations with grouping symbols

If an equation contains grouping symbols:

- expand the grouping symbols
- then solve the equation.

EXAMPLE 1

Expand the grouping symbol, then solve the equation.

a $4(x + 7) = 38$ **b** $5(3d - 2) = 62$

a
$$4(x + 7) = 38$$
$$4x + 28 = 38$$
$$4x + 28 - 28 = 38 - 28 \quad \text{Subtract 28 from both sides.}$$
$$4x = 10$$
$$\frac{4x}{4} = \frac{10}{4} \quad \text{Divide both sides by 4.}$$
$$x = \frac{5}{2} \text{ or } 2\frac{1}{2}$$

Remember to always do the same thing to both sides of the equation.

b
$$5(3d - 2) = 62$$
$$15d - 10 = 62$$
$$15d - 10 + 10 = 62 + 10 \quad \text{Add 10 to both sides.}$$
$$15d = 72$$
$$\frac{15d}{15} = \frac{72}{15} \quad \text{Divide both sides by 15.}$$
$$d = \frac{24}{5} \text{ or } 4\frac{4}{5}$$

Exercise 13E

1 Expand the grouping symbols, then solve each equation.

a $6(x + 1) = 24$ **b** $8(t + 10) = 89$ **c** $4(l + 6) = 28$

d $2(n + 4) = -14$ **e** $12(2 + e) = -58$ **f** $9(8 + r) = 54$

g $4(m - 5) = 15$ **h** $7(p - 2) = 35$ **i** $11(c - 9) = 36$

j $6(f - 12) = 74$ **k** $5(8 - t) = -18$ **l** $13(4 - w) = -51$

2 Solve each equation.

a $6(2a + 1) = 90$	**b** $2(5m + 8) = 66$	**c** $11(3q + 4) = 77$
d $7(5b + 9) = 52$	**e** $3(6p + 7) = 147$	**f** $5(11w + 2) = 186$
g $4(3 + 3c) = -48$	**h** $10(4n - 8) = -160$	**i** $11(2 + 5d) = 132$
j $7(4 - 5x) = 13$	**k** $12(5 - 2y) = 40$	**l** $6(2 - 4w) = 96$

EXAMPLE 2

Expand the grouping symbols, then solve the equation.

a $-8(x + 9) = 8$

b $-5(3m - 5) = -20$

Remember when multiplying and dividing by positive numbers and negative numbers:
- *if the signs are the same the result is positive*
- *if the signs are different the result is negative.*

a
$$-8(x + 9) = 8$$
$$-8x - 72 = 8$$
$$-8x - 72 + 72 = 8 + 72 \quad \text{Add 72 to both sides.}$$
$$-8x = 80$$
$$\frac{-8x}{-8} = \frac{80}{-8} \quad \text{Divide both sides by } -8.$$
$$x = -10$$

b
$$-5(3m - 5) = -20$$
$$-15m + 25 = -20$$
$$-15m + 25 - 25 = -20 - 25 \quad \text{Subtract 25 from both sides.}$$
$$-15m = -45$$
$$\frac{-15m}{-15} = \frac{-45}{-15} \quad \text{Divide both sides by } -15.$$
$$m = 3$$

3 Expand the grouping symbols then solve the following equations.

a $-2(p + 7) = -4$	**b** $-3(a - 1) = -18$	**c** $-4(d + 6) = 16$
d $-8(m - 3) = 32$	**e** $-6(r - 5) = 42$	**f** $-10(x + 7) = -100$
g $-9(f + 11) = -54$	**h** $-11(q - 2) = 66$	**i** $-3(2m + 4) = -6$
j $-5(4d - 9) = 5$	**k** $-8(6 + 3a) = -24$	**l** $-7(8 - 2m) = -14$

F Substituting into a formula (extension)

Formulas contain more than one pronumeral. Like equations, the aim is to determine the value of an 'unknown' pronumeral. To do this, we need to know the values of the other pronumerals so that the values can be substituted into the given formula.

EXAMPLE 1

a The area of a rectangle is $A = lb$. Find A if $l = 6$ cm and $b = 3$ cm.

b The circumference of a circle is $C = 2\pi r$. Find C if $\pi = 3.14$ and $r = 5$ cm.

a
$$A = lb$$
$$= 6 \times 3$$
$$\therefore A = 18 \text{ cm}^2$$

b
$$C = 2\pi r$$
$$= 2 \times 3.14 \times 5$$
$$\therefore C = 31.4 \text{ cm}$$

Exercise 13F

All measurements of length for this exercise are in centimetres.

1 The area of a rectangle is $A = lb$. Find the value of A when:

a $l = 7, b = 5$ **b** $l = 9, b = 2$

c $l = 11, b = 6$ **d** $l = 3\frac{1}{2}, b = 7$

e $l = 4.5, b = 8.4$ **f** $l = 7.8, b = 6.3$

2 The area of a triangle is $A = \frac{bh}{2}$. Find the value of A when:

a $b = 5, h = 9$ **b** $b = 3, h = 11$

c $b = 10, h = 8$ **d** $b = 3.7, h = 4$

e $b = 10\frac{1}{2}, h = 2$ **f** $b = 11.3, h = 6.2$

3 The area of a trapezium is $A = \frac{1}{2}h(a + b)$. Find the value of A when:

a $h = 5, a = 2, b = 4$

b $h = 7, a = 6, b = 2$

c $h = 11, a = 5\frac{1}{2}, b = 6\frac{1}{2}$

d $h = 8.6, a = 9.2, b = 3.4$

4 The area of a circle is $A = \pi r^2$. Use the approximation to π of 3.14 to calculate the value of A given that r is:

a 3 **b** 9.6 **c** 12.4 **d** 7

5 Given the formula $y = mx + b$, find y when:

a $m = 8, x = 12, b = 9$ **b** $m = -8, x = 6, b = 4$

c $m = -7, x = \frac{1}{4}, b = 2$ **d** $m = -6.8, x = \frac{2}{5}, b = -9$

6 Given the formula $c = \sqrt{a^2 + b^2}$, find c correct to 2 decimal places when:

a $a = 4, b = 6$ **b** $a = 11, b = 8$

c $a = 2\frac{1}{2}, b = 9$ **d** $a = 7.8, b = 2.3$

7 Given the formula $C = \frac{5}{9}(F - 32)$, where C is degrees Celsius and F is degrees Fahrenheit, find C when F is:

a 10 **b** 8

c 15 **d** 18

8 Given the formula $F = 32 + \frac{9}{5}C$, where F is degrees Fahrenheit and C is degrees Celsius, find F when C is:

a 5 **b** 20

c 11 **d** 8

9 Given the formula $d = st$, where d is distance, s is speed and t is time, find the value of d when:

a $s = 9, t = 12$ **b** $s = 7.2, t = 5\frac{1}{4}$

c $s = 0.4, t = 11.6$

10 The average of three numbers is found using the formula $A = \frac{a + b + c}{3}$. Calculate the value of A when:

a $a = 1, b = 7, c = 4$
b $a = 9, b = 11, c = 13$
c $a = 4.1, b = 5, c = 3\frac{1}{2}$
d $a = 10.6, b = 9.2, c = 3.3$

EXAMPLE 2

a Use the formula $A = \frac{bh}{2}$ to find the value of h when $A = 10$ and $b = 2.5$.
b Using the formula $A = \pi r^2$ find the value of r when $A = 28.26$ and $\pi = 3.14$.

a

$A = \frac{bh}{2}$

$10 = \frac{2.5h}{2}$ Substitute the given values.

$10 \times 2 = 2.5 \times \frac{h}{2} \times 2$ Multiply both sides by 2.

$20 = 2.5 \times h$

$\frac{20}{2.5} = \frac{2.5h}{2.5}$ Divide both sides by 2.5.

$h = 8$

Check: LHS $= \frac{bh}{2} = \frac{2.5 \times 8}{2} = 10$

$\therefore$ LHS = RHS

b

$A = \pi r^2$

$28.26 = 3.14 \times r^2$ Substitute the given values.

$\frac{28.26}{3.14} = \frac{3.14 \times r^2}{3.14}$ Divide both sides by 3.14.

$9 = r^2$

$r = \sqrt{9}$ $(r > 0)$ Find the square root.

$r = 3$

Check: RHS $= \pi r^2 = 3.14 \times 9 = 28.26$

$\therefore$ LHS = RHS

11 The area of a rectangle is $A = lb$.
Find the value of b when:

a $A = 54, l = 6$
b $A = 140, l = 10$
c $A = 17.5, l = 7$
d $A = 26.4, l = 11$
e $A = 16.5, l = 5$
f $A = 114.95, l = 12.1$

12 The area of a triangle is $A = \frac{bh}{2}$.
Find the value of h when:

a $A = 60, b = 12$
b $A = 56, b = 8$
c $A = 45, b = 10$
d $A = 38.5, b = 11$
e $A = 7.5, b = 3$
f $A = 25.6, b = 6.4$

13 The area of a trapezium is $A = \frac{1}{2}h(a + b)$.

a Find the value of h when:

i $A = 22, a = 5, b = 6$ **ii** $A = 125, a = 11, b = 14$

b Find the value of b when:

i $A = 59.5, h = 7, a = 13$ **ii** $A = 46, h = 4.6, a = 9$

14 The area of a circle is $A = \pi r^2$. Use the approximation of 3.14 for π to calculate the value of r given an area of:

a 78.5 **b** 153.86 **c** 50.24 **d** 52.7834

15 The formula $y = mx + b$ is given.

a Find the value of b when:

i $y = 21, m = 2, x = 8$ **ii** $y = 47, m = 4, x = 7\frac{1}{4}$

b Find the value of m when:

i $y = 64, x = 4, b = 12$ **ii** $y = 45.7, x = 9\frac{1}{5}, b = 18.1$

16 Given the formula $c = \sqrt{a^2 + b^2}$, find b when:

a $c = 5, a = 4$ **b** $c = 10, a = 8$ **c** $c = 17, a = 8$ **d** $c = \sqrt{146}, a = 5$

17 Use the formula $C = \frac{5}{9}(F - 32)$. Find F when C has the values:

a $4\frac{4}{9}$ **b** 10 **c** $15\frac{5}{9}$ **d** $21\frac{5}{9}$

18 Given the formula $F = 32 + \frac{9}{5}C$, find C when F has values:

a $39\frac{1}{5}$ **b** $51\frac{4}{5}$ **c** $62\frac{3}{5}$ **d** 122

19 Given the formula $d = st$, find the value of s when:

a $d = 114, t = 19$ **b** $d = 748, t = 22$ **c** $d = 224.77, t = 9.1$

20 Using the formula $A = \frac{a + b + c}{3}$, calculate the value of c when:

a $A = 12, a = 7, b = 19$ **b** $A = 42, a = 5, b = 12$

G Problem solving (extension)

Equations are extremely useful for solving problems.

A problem written in words is rewritten using numbers. To represent the 'unknown' quantity or number that the problem is trying to find, a pronumeral, often x, is used.

When using equations to solve problems, follow these steps.

Method of solution

Step 1: Examine the problem. Read it carefully, perhaps two or three times.

Step 2: Select a pronumeral, usually x, to represent the 'unknown' quantity to be found.

Step 3: Set up an equation using the details of the question.

Step 4: Solve the equation.

Step 5: Write your answer to the question in sentence form.

Step 6: Check that your answer satisfies the conditions.

EXAMPLE 1

The sum of a certain number and 15 is 38. What is the number?

Let x be the number. So, $x + 15$ is the sum of the number and 15.

Remember 'sum' means to add.

$$x + 15 = 38$$
$$x + 15 - 15 = 38 - 15 \quad \text{Subtract 15 from both sides.}$$
$$x = 23$$

So the number is 23.

Check: LHS $= x + 15 = 23 + 15 = 38$

$\therefore$ LHS = RHS

EXAMPLE 2

Think of a number, treble it and subtract 11. The result is 25. What is the number?

Let x be the number. So, $3x$ is the number trebled, and $3x - 11$ is the number trebled minus 11.

$$3x - 11 = 25$$
$$3x - 11 + 11 = 25 + 11 \quad \text{Add 11 to both sides.}$$
$$3x = 36$$
$$\frac{3x}{3} = \frac{36}{3} \quad \text{Divide both sides by 3.}$$
$$x = 12$$

So the number is 12.

Check: LHS $= 3x - 11 = 3 \times 12 - 11 = 25$

$\therefore$ LHS = RHS

Exercise 13G

Let the unknown be x.

1 Write an equation for each number problem, then find a solution.

a The sum of a certain number and 18 is 46. What is the number?

b The sum of a certain number and 7 is -12. What is the number?

c A certain number minus 5 is equal to 19. What is the number?

d The difference between a certain number and 8 is -4. What is the number?

e The difference between a certain number and -2 is 6. What is the number?

f The product of a certain number and 3 is 108. What is the number?

g When a certain number is multiplied by 6 the result is 72. What is the number?

h When a certain number is divided by 9 the result is 2. What is the number?

i When a certain number is divided by -7, the result is -1.4. What is the number?

2 **a** I think of a number, double it, and subtract 3. The result is 17. What is the number?

b When the product of a certain number and 4 is added to 12, the result is 52. What is the number?

c 8 is subtracted from the product of 2 and a number. The result is -10. What is the number?

d The product of a certain number and 5 is divided by 9. The result is 10. What is the number?

e The sum of a certain number and -1 is then multiplied by 7. The result is 21. What is the number?

f Treble a certain number and subtract 12. Multiply the result by 5 and the answer is 15. What is the number?

Investigation 2 Using spreadsheets to solve equations

1 This spreadsheet can be used to solve simple linear equations.

a Solve the equation $3x - 1 = 14$. In column C an IF statement is written to display correct when the correct answer is calculated.

	A	B	C	D
1	**Solve the equation of the form ax + b = c.**			
2	a=	3		
3	b=	–1		
4	c=	14		
5				
6	LHS	RHS		
7	0	–1	incorrect	
8	1	2	incorrect	
9	2	5	incorrect	
10	3	8	incorrect	
11	4	11	incorrect	
12	5	14	correct	
13	6	17	incorrect	
14	7	20	incorrect	
15	8	23	incorrect	
16	9	26	incorrect	
17	10	29	incorrect	

The table below is the formula view of the spreadsheet showing the IF statement and the formulas for the cells. It is only necessary to type the formulas into the first row and then fill down.

	A	B	C
1	**Solve the equation of the form ax + b = c**		
…			
7	0	=$B$2*A7+$B$3	=IF(B7=$B$4,“Correct”,“incorrect”)
8	=A7+1	=$B$2*A8+$B$3	=IF(B8=$B$4,“Correct”,“incorrect”)
9	=A8+1	=$B$2*A9+$B$3	=IF(B9=$B$4,“Correct”,“incorrect”)
10	=A9+1	=$B$2*A10+$B$3	=IF(B10=$B$4,“Correct”,“incorrect”)
11	=A10+1	=$B$2*A11+$B$3	=IF(B11=$B$4,“Correct”,“incorrect”)
12	=A11+1	=$B$2*A12+$B$3	=IF(B12=$B$4,“Correct”,“incorrect”)
13	=A12+1	=$B$2*A13+$B$3	=IF(B13=$B$4,“Correct”,“incorrect”)
14	=A13+1	=$B$2*A14+$B$3	=IF(B14=$B$4,“Correct”,“incorrect”)
…	…	…	…

b Change the values in the cells B2, B3 and B4 to solve the equation $4x + 3 = 31$.

c When attempting to solve the equation $5x - 3 = 42$ the correct value of x does not appear. All the values are too small. Change the value in cell A7 to get larger values. The answer will now appear.

d When attempting to solve the equation $4x + 9 = -3$ all the values in column B are too large. Change the value in cell A7 to -10 and the answer will be displayed.

e Use the spreadsheet to solve $6x - 7 = 11$, $2x + 5 = 35$, and $7x + 16 = -5$.

2 To solve linear equations that do not have integer solutions involves a two-step process. Solve $5x - 7 = 9$.

a *Step 1:* Use the current spreadsheet to find the two whole numbers that the solution lies between by finding the last value of x for which the RHS is below the value of 9 and the first above 9. This is shown in the spreadsheet below.

	A	B	C	D
1	**Solve the equation of the form ax + b = c.**			
2	a=	5		
3	b=	–7		
4	c=	9		
5				
6	LHS	RHS		
7	0	–7	incorrect	
8	1	–2	incorrect	
9	2	3	incorrect	
10	3	8	incorrect	
11	4	13	incorrect	

So the solution lies between 3 and 4 because 9 is between 8 and 13.

b *Step 2:* Modify the spreadsheet by changing the value in cell A7 to the integer solution that is just too small, in this case $x = 3$. Edit the formula in cell A8 to add 0.1. This is shown in the spreadsheet below.

	A	B	C	D
…				
6	LHS	RHS		
7	3	8	incorrect	
8	3.1	8.5	incorrect	
9	3.2	9	correct	
10	3.3	9.5	incorrect	

c The solution is $x = 3.2$.

d Use a spreadsheet to solves these equations: $2x - 10 = 13$, $5x - 8 = 4$ and $10x + 17 = 70$.

3 **a** The spreadsheet can be modified to solve equations like $4x - 5 = -3x + 16$. Here is the formula view.

	A	B	C	D
1	**Solve the equation of the form ax + b = cx + d.**			
2	a=	4		
3	b=	–5		
4	c=	–3		
5	d=	16		
6	LHS	RHS		
7	0	=$B$2*A7+$B$3	=$B$4*A7+$B$5	=IF(B7=C7,“Correct”,“incorrect”)
8	=A7+1	=$B$2*A8+$B$3	=$B$4*A8+$B$5	=IF(B8=C8,“Correct”,“incorrect”)
9	=A8+1	=$B$2*A9+$B$3	=$B$4*A9+$B$5	=IF(B9=C9,“Correct”,“incorrect”)
…	…	…	…	…

b Use the spreadsheet to solve $5x - 5 = 2x + 7$.

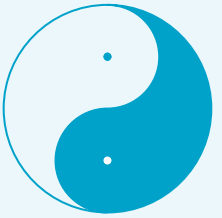
c Use the spreadsheet to solve $8x - 2 = 6x + 7$. You will need to modify the spreadsheet as the solution is not an integer.

Language in mathematics

Balance is the key to all existence

Hold a class debate on the topic 'Balance is the key to all existence'.

- Divide the class into groups of six to eight. Halve the groups to form affirmative and negative sides.
- Each team is to organise and prepare a written 3-minute speech per speaker.
- The class, acting as an audience, will evaluate each speaker's speech (content and delivery) to determine the winning team.



Terms

algebraic expression	backtracking	balance	concrete material
inspection	linear equation	number line	one-to-one matching
solution	substitution		

Check your skills

1 By inspection, state the value of the pronumeral if $5q = 40$.

A 9 **B** 5 **C** 8 **D** 6

2 Which option shows how the expression $\frac{6 - 3n}{4}$ is built?

A $6 \xrightarrow{-3} 6 - 3 \xrightarrow{\times n} 6 - 3n \xrightarrow{\div 4}$

B $n \xrightarrow{\times 3} 3n \xrightarrow{-6} 3n - 6 \xrightarrow{\div 4}$

C $3 \xrightarrow{\times n} 3n \xrightarrow{+6} 3n + 6 \xrightarrow{\div 4}$

D $n \xrightarrow{\times (-3)} -3n \xrightarrow{+6} -3n + 6 \xrightarrow{\div 4}$

3 The equation $\frac{4x - 6}{9} = 2$ has been solved using backtracking techniques. What are the values of parts **i**, **ii** and **iii** using this technique?

$x \xrightarrow{\times 4} 4x \xrightarrow{-6} 4x - 6 \xrightarrow{\div 9} \frac{4x - 6}{9}$

$\textbf{iii} \xleftarrow{\div 4} \textbf{ii} \xleftarrow{+6} \textbf{i} \xleftarrow{\times 9} 2$

A **i** 18 **ii** 26 **iii** $4\frac{1}{3}$

B **i** 18 **ii** 24 **iii** 6

C **i** 18 **ii** 22 **iii** $5\frac{1}{2}$

D **i** 18 **ii** 20 **iii** 5

4 State the value of w in the equation $\frac{2w}{7} = 8$.

A 56 **B** 24 **C** 48 **D** 28

5 State the value of g in the equation $4g - 5 = 2g - 1$.

A 28 **B** 2 **C** 17 **D** 9

6 State the value of f in the equation $6(2f - 7) = 2(4f + 8)$.

A $14\frac{1}{2}$ **B** $3\frac{3}{4}$ **C** $12\frac{1}{2}$ **D** $10\frac{1}{4}$

7 If $I = \frac{PRT}{100}$, find I when $P = \$70\,000$, $R = 8.25$ and $T = \frac{7}{12}$.

A \$336.88 **B** \$3368.75 **C** \$33 687.50 **D** \$336 875

8 Given $E = mc^2$, find m when $E = 175$ and $c = 5$.

A 7 **B** 17.5 **C** 35 **D** 40

9 Write an equation using x as the unknown number. A certain number is subtracted from eight. When it is divided by three the result is four.

A $\frac{x-8}{3} = 4$ **B** $\frac{8-x}{3} = 4$ **C** $8 - \frac{x}{3} = 4$ **D** $8 - x = \frac{4}{3}$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2, 3	4, 5	6	7, 8	9
Section	A	B	C	D	F	G

13A Review set

1 Show each step required to get from the expression $5x + 15$ back to x.

2 Solve the following equations.

a $x + 11 = 18$ **b** $x + 9 = -12$
c $4x = 22$ **d** $-9x = 58$
e $3y + 18 = 26$ **f** $5 - 4p = -72$
g $4d + 8 = 3d - 15$ **h** $18 + 7c = 32 - 4c$
i $3(m + 6) = 4(m - 1)$ **j** $8(q - 5) = -2(10 + 3q)$

3 Solve the following equations.

a $\frac{4p}{5} = 9$ **b** $\frac{3x + 12}{7} = 13$
c $\frac{m - 8}{3} = 7$ **d** $\frac{2p + 7}{5} = 3$
e $\frac{p + 5}{2} = -3$ **f** $\frac{9 - 4p}{5} = -7$

4 Solve the following equations.

a $3(x - 4) = 2$ **b** $5(3x + 2) = 9$
c $-3(p + 4) = 6$ **d** $-4(6 - 5p) = 2$
e $3x + 5 = 7x - 3$ **f** $7 + 2x = 5x - 8$

5 Is the given value for the pronumeral a solution to the equation?

a $5d + 12 = 27;\ d = 3$ **b** $\frac{x}{5} + 7 = 25;\ x = 3\frac{2}{5}$

6 If $x = 4$ and $y = 7$, find the value of $3x - 5y$.

7 The volume, V, of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Find the volume of a sphere with radius:

a 8 cm **b** 4.2 cm **c** 0.9 m

8 The formula to convert temperature measurements from degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$. Find F when:

a $C = 180°$ **b** $C = 15°$ **c** $C = 38°$

9 Solve the following equations.

a $d - 8 = 40$ **b** $x + 9 = 15$ **c** $\frac{x}{9} = 72$

d $11x = 66$ **e** $4x - 7 = 35$ **f** $3x - 12 = 70$

g $10 + 8n = 58$ **h** $12 - 6c = 78$ **i** $3(2x + 1) = 3$

10 Write an equation and solve this problem.

The sum of a certain number and 7 is 114. What is the number?

13B Review set

1 Show the steps required to get from the equation $15 - 4b = 35$ back to b.

2 Solve the following equations.

a $p - 5 = -20$ **b** $d + 8 = 3$

c $6x = -54$ **d** $-2x = -18$

e $11c + 21 = 73$ **f** $17 - 5q = -12$

g $-5(4n - 8) = 7(2n + 11)$ **h** $4(2 - 3c) = -2(5c + 3)$

3 Solve the following equations by collecting like terms.

a $7q - 7 = 18 - 2q$ **b** $-5 - 5m = 47$

4 Solve the following equations.

a $\frac{2 - 4k}{3} = -7$ **b** $13 - \frac{w}{2} = -13$

5 Solve these equations.

a $\frac{p - 7}{2} = 6$ **b** $\frac{3x - 4}{7} = 2$

c $\frac{x + 4}{3} = -1$ **d** $\frac{7 - 3p}{4} = -2$

e $5(p - 4) = 3$ **f** $6(3x - 5) = 7$

g $-4(x - 1) = 2$ **h** $-5(3 - 4p) = 1$

i $5x - 4 = 3x + 8$ **j** $4 + 5x = 12 - 3x$

6 If $a = -3$, $b = 5$ and $c = -2$, find the value of the following expressions.

a $4a + 3c - b$ **b** $\frac{5(abc)}{2}$

7 If $x = -2$ and $y = 3$ find the value of $5x - 7y$.

8 Solve the following equations.

a $d - 9 = 23$ **b** $x + 4 = 65$ **c** $\frac{x}{3} = 5$

d $11x = 121$ **e** $3x - 7 = 30$ **f** $4x - 12 = 15$

g $10 + 3n = 32$ **h** $12 - 9c = 43$ **i** $7(2x + 5) = 4$

j $4x - 2 = x + 9$ **k** $3(3 - 7x) = 4(x + 5)$ **l** $12 - 3(5 - x) = 9$

9 The formula to convert temperature measurements from degrees Fahrenheit, F, to degrees Celsius, C, is $C = \frac{5}{9}(F - 32)$. Find C when:

a $F = 248°$ **b** $F = 50°$ **c** $F = 32°$

10 Solve the following problems using equations.

a If a certain number is doubled the result is -8. What is the number?

b When four consecutive integers are added the result is 134. What are the integers?

13C Review set

1 Solve the following equations.

a $11n - 17 = -42$

b $2(q - 7) = 18$

c $-(p + 5) = 3(2p + 8)$

d $14 - \frac{3c}{5} = 11$

2 Is the given value for the pronumeral a solution to the equation?

a $-5w + 1 - 6w - 9 = -10$; $w = \frac{2}{11}$

b $2a - 1 + 3a - 5 - 8 = 4$; $a = 3\frac{3}{5}$

3 Solve these equations.

a $\frac{r + 5}{4} = 3$

b $\frac{4x - 7}{8} = 1$

c $\frac{x + 5}{-4} = 2$

d $\frac{3 - 8r}{7} = -2$

4 Given $x = -5$ and $y = 4$, find the value of $4x^2 - 6y$.

5 Solve the following equations.

a $d - 9 = 11$

b $x + 4 = 14$

c $\frac{x}{3} = 2$

d $11x = 77$

e $3x - 8 = 12$

f $7x - 12 = 19$

g $14 + 3n = 23$

h $23 - 4c = 11$

i $2(4x + 5) = 12$

6 Solve these equations.

a $4(r - 1) = 7$

b $5(2p - 7) = -5$

c $-3(x + 5) = 9$

d $-7(3x - 4) = -2$

e $4p - 7 = 5p + 9$

f $7x + 5 = -4 - 2x$

7 The surface area, A, of a cylinder is found using the formula $A = 2\pi r(r + h)$ where r is the radius and h is the height. Find the surface area of a cylinder with radius 15 cm and height 8 cm. Give the answer to the nearest whole number.

8 The time T s for a pendulum of length L m to swing back and forth once is give by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where $g \approx 10$ m/s^2. Find how long it takes for a pendulum of length 2.4 m to swing back and forth once.

9 Pythagoras' rule is $c = \sqrt{a^2 + b^2}$. Find c when:

a $a = 3, b = 4$

b $a = 5, b = 12$

c $a = 15, b = 20$

d $a = 16, b = 12$

10 Construct an equation and solve, using x as the pronumeral.

a The sum of three consecutive odd numbers is 39. What are the numbers?

b Twice a certain number is subtracted from 18. The result is the same as multiplying the number by four and adding 9. What is the number?

13D Review set

1 Solve the following equations.

a $-3 - 2p = -5$

b $7(2n + 15) = -33$

c $\frac{4c}{7} + 8 = 20$

d $3 - \frac{5p}{6} = -9$

2 Is the given value for the pronumeral a solution to the equation?

a $4(p + 7) = -3(p + 10)$; $p = 8\frac{1}{7}$

b $3m - \frac{2}{3} = 8$; $m = 2\frac{4}{9}$

3 Given $x = 12$ and $y = -7$, find the value of $6x^2 - 11y$.

4 The volume, V, of a cylinder is found using the formula $V = \pi r^2 h$ where r is the radius and h is the height. Find the volume of a cylinder with radius 12 cm and height 25 cm.

5 A formula for calculating the bend allowance, B in mm, of sheet metal is $B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$ where B is the bend allowance, T is the thickness in mm, A is the number of degrees in the angle of bend, and R is the radius of curvature in mm. Find B when $T = 1.5$, $R = 4$, $A = 116$.

6 Solve the following equations.

a $d - 5 = 11$

b $x + 7 = 14$

c $\frac{x}{6} = 2$

d $7x = 77$

e $3x - 4 = 92$

f $8x - 12 = 11$

g $11 + 7n = 53$

h $27 - 9c = 31$

i $4(3x + 5) = 13$

7 Solve these equations.

a $\frac{m - 1}{3} = 2$

b $\frac{5p + 8}{3} = 2$

c $\frac{7 + p}{2} = 3$

d $\frac{14 - 5r}{2} = -3$

8 Solve these equations.

a $5(7 + r) = 2$

b $6(3p - 1) = -4$

c $-8(x - 2) = 3$

d $-7(5x - 4) = 2$

e $7r - 1 = 5r + 9$

f $7x + 9 = -3x - 1$

9 Construct an equation and solve, using x as the pronumeral.

a 4 is subtracted from a number and the result is multiplied by -8. The answer is 50. What is the number?

b 7 is subtracted from a number and the result is multiplied by 6. The answer is the same as multiplying the number by -2 and adding 9. What is the number?

10 If $I = \frac{PRT}{100}$, find I when $P = 8500$, $R = 4.25$ and $T = \frac{5}{12}$.

11 If $T = a + (n - 1)d$, find n when $T = 80$, $a = 6$ and $d = 4$.

12 **a** Given that $V = IR$, find V when $I = 10$ and $R = 250$.

b Given that $V = IR$, find V when $I = 50$ and $R = 20$.

c Given that $W = Fs$, find W when $F = 100$ and $s = 12$.

d Given that $s = \frac{d}{t}$, find s when $d = 50$ and $t = 10$.

14

Coordinate geometry and straight lines

This chapter deals with the number plane and straight-line graphs.

At the end of this chapter you should be able to:

- plot points, including negative numbers, on the number plane
- plot points generated by rules and patterns
- find the relationship between x and y from a graph of points
- draw straight-line graphs from practical situations
- graph straight lines
- derive the equation of a straight-line graph
- understand the properties of straight-line graphs
- graph intersecting lines and find the point of intersection.

NSW Syllabus references: S4 N&A Linear relationships
Outcomes: MA4-1WM, MA4-3WM, MA4-11NA
NUMBER & ALGEBRA – ACMNA193, ACMNA194

Diagnostic test

1 The missing numbers in the pattern 2, 5, __, 11, __, 17, … are:

A 9 and 15 **B** 8 and 14
C 7 and 13 **D** 11.5 and 27.5

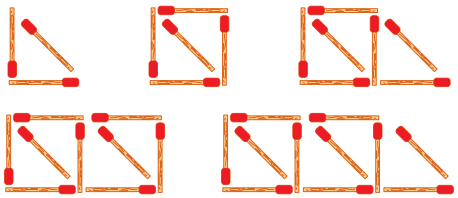
2 Using the rule 'Starting with 3, multiply the preceding term by 2 and then add 1', the first three terms of the number sequence will be:

A 3, 6, 12 **B** 3, 10, 31
C 2, 7, 22 **D** 3, 7, 15

3 If this geometric pattern of shapes is continued, the number of dots in the 10th shape will be:

A 10 **B** 100 **C** 20 **D** 50

4 This pattern of triangles was built using matches and the results were put in the table below.

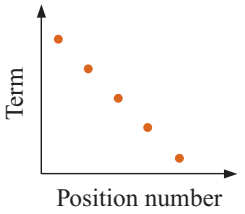


Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

Which of the following rules could have been used to form the pattern?
'Number of matches needed = ___'

A 2 × number of triangles + 1
B 3 ÷ (number of triangles − 1) × 2
C 3 × number of triangles − 1
D 2 × (number of triangles − 1) − 1

5 The values from a table representing a number sequence are graphed on the number plane shown below.



The graph shows that the number sequence is:

A increasing **B** decreasing
C constant **D** none of these

6 The number of matches needed to build a pattern of triangles is given by the rule: 'Number of matches needed = 3 × number of triangles − 1 less than the number of triangles'. If n = number of matches needed and t = number of triangles, the rule could be written algebraically as:

A $n = 3t - t - 1$ **B** $n = 3t - (t - 1)$
C $n = 3(t - t - 1)$ **D** $n = 3(t - t + 1)$

7 An algebraic rule to describe the number pattern 1, 4, 7, 10, …,
where x = the position number of the term and y = the value of the term, is:

A $y = x + 3$ **B** $x = y + 3$
C $y = 3x - 2$ **D** $x = 3y - 2$

8 The terms of a number sequence are given by the rule $u = 2n - 1$, where u represents the value of the term and n represents its position number. The first three terms of the sequence are:

A −1, 1, 3 **B** 0, 2, 4
C −1, 0, 2 **D** 1, 3, 5

The Diagnostic test questions refer to the Year 7 outcomes from ACMNA175 and ACMNA177.

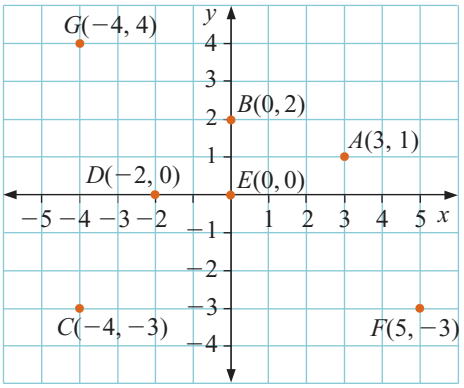
A Number plane review

The number plane was used in Year 7. The way of plotting points is across left or right, then up or down.

Remember across first; that is, the x-coordinate is first.

EXAMPLE 1

Plot the following points on a number plane.
$A(3, 1)$, $B(0, 2)$, $C(-4, -3)$, $D(-2, 0)$, $E(0, 0)$, $F(5, -3)$, $G(-4, 4)$
The point $(0, 0)$ is called the origin.

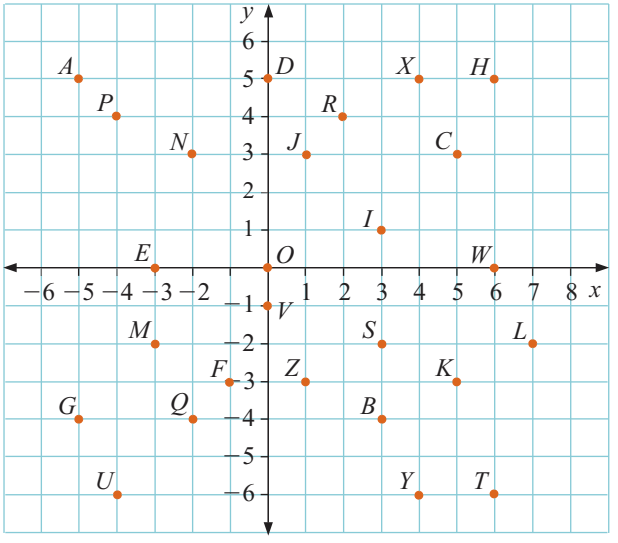


Exercise 14A

1 Plot the following points on a number plane on grid paper.

a $A(-3, -5)$	**b** $B(2, 6)$	**c** $C(5, -2)$	**d** $D(6, -1)$
e $E(1, 1)$	**f** $F(-2, -5)$	**g** $G(0, 2)$	**h** $H(3, 0)$
i $I(3, -5)$	**j** $J(-3, 0)$	**k** $K(4, 3)$	**l** $L(4, -5)$
m $M(0, -2)$	**n** $N(4, 4)$	**o** $O(0, 0)$	**p** $P(-3, -2)$
q $Q(4, -2)$	**r** $R(-5, -5)$	**s** $S(-5, 2)$	**t** $T(-1, 5)$

2 Write the coordinates of the points plotted on this number plane.



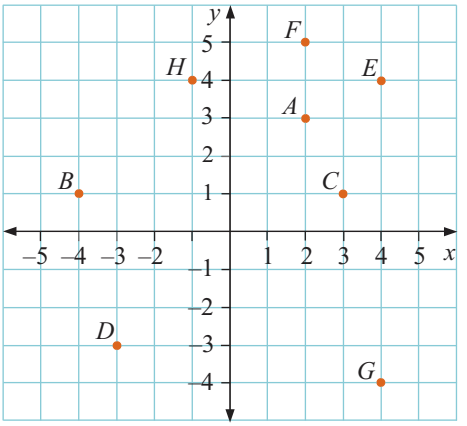
3 Eight points have been plotted on this number plane.

a Write the coordinates of the eight points.

b Name two points with the same x-coordinates. What do you notice about their positions on the number plane?

c Name two points with the same y-coordinates. What do you notice about their positions on the number plane?

d Name two points that have equal x- and y-coordinates. What do you notice about their positions on the number plane?



4 **a** Plot the points $A(-3, 3)$, $B(1, 3)$ and $C(1, -1)$ on a number plane.

b If $ABCD$ is a square, find the coordinates of the point D.

5 **a** Plot the points $P(-4, 0)$, $Q(-4, 5)$ and $R(3, 5)$ on a number plane.

b If $PQRS$ is a rectangle, find the coordinates of S.

6 **a** Plot the points $A(-3, -2)$, $B(-2, -1)$, $C(-1, 0)$, $D(0, 1)$, $E(1, 2)$ on the same number plane.

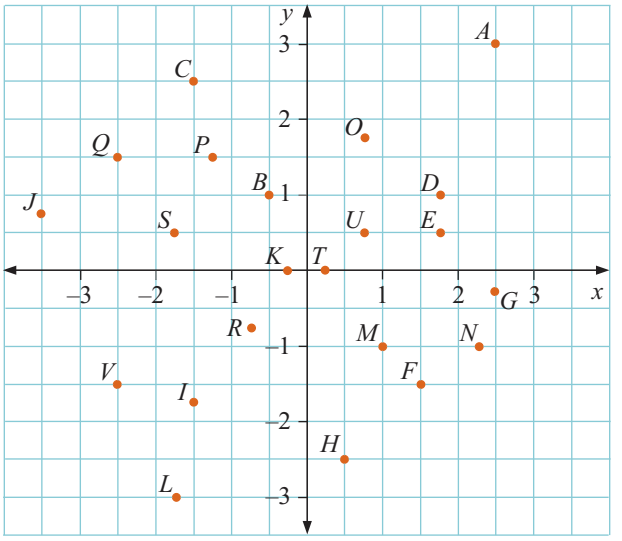
b Join the points. What do you notice?

c What are the next three points (F, G and H) if the pattern continues?

7 **a** Plot the points $A(5, 3)$, $B(4, 2)$, $C(3, 1)$, $D(2, 0)$, $E(1, -1)$ on the same number plane.

b What are the next three points (F, G and H) if the pattern continues?

8 Write the coordinates of the points on this number plane. The coordinates may not be whole numbers.



B Number patterns

Mathematics has been described as a search for patterns and relationships. An understanding of the relationships between numbers and shapes can be used to find the solutions to many problems in everyday life. In particular, algebra is one of the most important tools invented to communicate mathematical ideas.

An **arithmetic** number pattern uses only addition or subtraction.

A **geometric** number pattern uses multiplication or division. (It may use addition or subtraction also.)

EXAMPLE 1

Consider these number patterns.

a 1, 3, 5, 7, …

b 1, 3, 9, 27, …

i Describe how the pattern is being formed.

ii Write down the next three numbers in the pattern.

a **i** Starting with 1, each number is 2 more than the number before it.

Two more means to add 2.

ii So, the next three numbers in the pattern are:

$7 + 2 = 9$

$9 + 2 = 11$

$11 + 2 = 13$

b **i** Starting with 1, each number is 3 times the number before it.

Three times means to multiply by 3.

ii So, the next three numbers in the pattern are:

$27 \times 3 = 81$

$81 \times 3 = 243$

$243 \times 3 = 729$

Exercise 14B

1 For each of the following number patterns:

i Describe in words how the pattern is formed.

ii Write down the next three numbers in the pattern.

a 2, 4, 6, 8, ___

b 4, 7, 10, 13, ___

c 27, 23, 19, 15, ___

d 30, 28, 26, 24, ___

e 90, 80, 70, 60, ___

f 3, 6, 12, 24, ___

g 2, 6, 18, 54, ___

h 3, 30, 300, 3000, ___

i 1, 0.1, 0.01, 0.001, ___

j 128, 64, 32, 16, ___

2 Find the missing numbers in the following patterns.

a 3, 5, ___, 9, ___

b 2, 5, ___, 11, ___

c 4, ___, 10, ___, 16

d 30, 26, ___, 18, ___

e 51, ___, 47, ___, 43

f 2, 10, ___, 250, ___

g 2, ___, 8, ___, 32

h 625, 125, ___, 5, ___

i 800, 400, ___, ___, 50

j 3, ___, 300, ___, 30 000

C Building patterns

The patterns in questions **1** and **2** in Exercise 14B form **sequences**. Each member of the sequence is called a **term** of the sequence. For example, 1, 3, 5, 7, … is a sequence of numbers.

Many interesting sequences are formed using a **rule**.

The rule for the sequence 1, 3, 5, 7, …is: 'Start with 1 and add 2'.

1st term	2nd term	3rd term	4th term
1	$1 + 2 = 3$	$3 + 2 = 5$	$5 + 2 = 7$

EXAMPLE 1

Form a sequence of numbers by using each rule given.

a Start with 5 and add 3.

b Start with 1000 and divide by 2.

c Start with 2, multiply by 3 and then add 1.

Some number patterns need more than one operation.

a

1st term	2nd term	3rd term	4th term
5	$5 + 3 = 8$	$8 + 3 = 11$	$11 + 3 = 14$

So the sequence is 5, 8, 11, 14, …

b

1st term	2nd term	3rd term	4th term
1000	$1000 \div 2 = 500$	$500 \div 2 = 250$	$250 \div 2 = 125$

So the sequence is 1000, 500, 250, 125, …

c

1st term	2nd term	3rd term	4th term
2	$2 \times 3 + 1 = 7$	$7 \times 3 + 1 = 22$	$22 \times 3 + 1 = 67$

So the sequence is 2, 7, 22, 67, …

Exercise 14C

1 Using the rule given, find the first four terms of each number sequence.

a Start with 2 and add 3.

b Start with 5 and add 2.

c Start with 1 and add 4.

d Start with 100 and subtract 10.

e Start with 45 and subtract 3.

f Start with 50 and subtract 2.

g Start with 3 and double the preceding term.

h Start with 7 and multiply by 10.

i Start with 1 and multiply by 3.

j Start with 64 and divide by 2.

k Start with 2 and divide by 10.

l Start with 1, multiply by 4 and then add 1.

m Start with 3, multiply by 2 and then add 1.

n Start with 5, multiply by 2 and then subtract 4.

o Start with 2, multiply by 4 and then subtract 5.

p Start with 7, multiply by 2 and then subtract 9.

2 If 1 L of paint covers approximately 16 m^2, what area could be covered by:

a 1 L?

b 2 L?

c 3 L?

d 4 L?

e 5 L?

3 The height of a candle decreases by 1.5 cm every hour. If the candle is originally 10 cm high, what will be its height after:

a 1 h?

b 2 h?

c 3 h?

d 4 h?

e 5 h?

4 The cost of hiring a taxi is $5 plus $3.50 for each kilometre travelled. How much does it cost to travel:

a 1 km? **b** 2 km? **c** 3 km? **d** 4 km? **e** 5 km?

5 The time taken to roast a piece of meat is 20 min plus an extra 15 min for each $\frac{1}{2}$ kg of meat. How long will it take to roast a piece of meat that weighs:

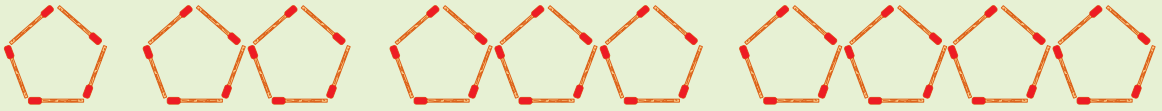
a $\frac{1}{2}$ kg? **b** 1 kg? **c** $1\frac{1}{2}$ kg? **d** 2 kg? **e** $2\frac{1}{2}$ kg?

D Graphing data

In Sections B and C number patterns were investigated. Each rule related to the previous term, such as add 3 or subtract 5. However, it would be better to relate the value of the term to its position. In this section we will use matchstick patterns to develop rules describing patterns. We will also use the number plane to show the rules in another way. The type of data used is discrete data. It is not possible to use half a match in making a shape.

EXAMPLE 1

Consider this pattern of matches.



a Complete this table.

Number of pentagons	1	2	3	4
Number of matches				

b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of pentagons and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

a

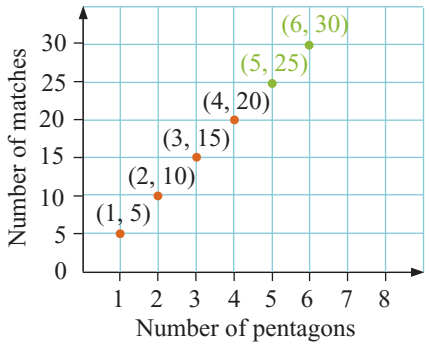
Number of pentagons	1	2	3	4
Number of matches	5	10	15	20

Do not join the dots.

b Number of matches equals five times the number of pentagons.

c (1, 5), (2, 10), (3, 15), (4, 20)

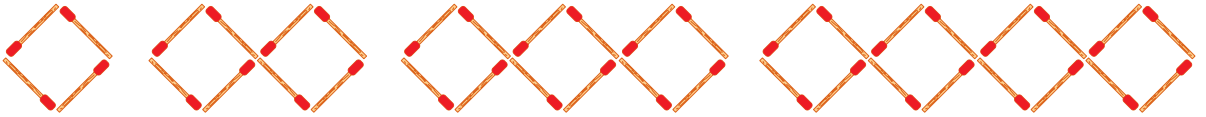
d



e (5, 25), (6, 30)

Exercise 14D

1 Consider this pattern of matches.



a Complete this table.

Number of squares	1	2	3	4
Number of matches				

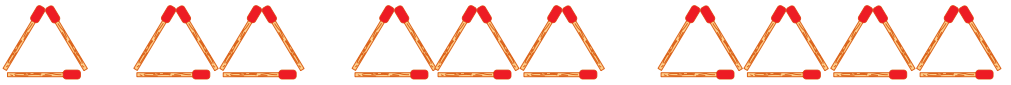
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of squares and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

2 Consider this pattern of matches.



a Complete this table.

Number of triangles	1	2	3	4
Number of matches				

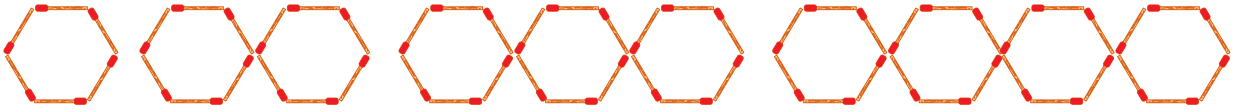
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of triangles and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

3 Consider this pattern of matches.



a Complete this table.

Number of hexagons	1	2	3	4
Number of matches				

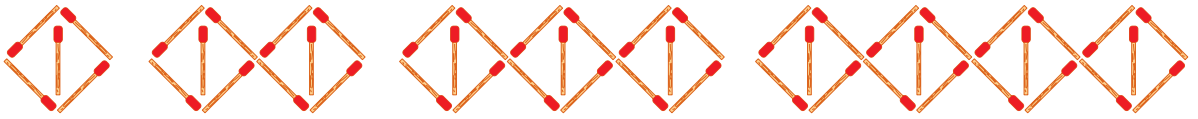
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of hexagons and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

4 Consider this pattern of matches.



a Complete this table.

Number of squares	1	2	3	4
Number of matches				

b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of squares and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

EXAMPLE 2

Consider this pattern of matches.

a Complete this table.

Number of triangles	1	2	3	4	5
Number of matches					

b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of triangles and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

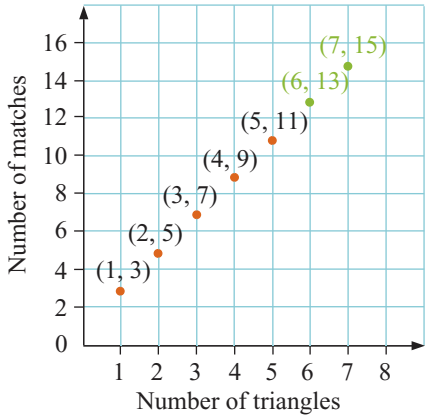
a

Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

b The number of matches goes up by 2 as the number of triangles goes up by 1, so the formula must have 2 × number of triangles. This would give 2, 4, 6, 8 and 10, but the values in the table are 3, 5, 7, 9 and 11, so a 1 must be added. So number of matches = 2 × number of triangles + 1.

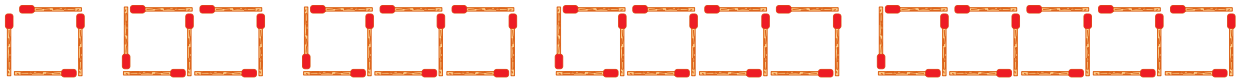
c (1, 3), (2, 5), (3, 7), (4, 9), (5, 11)

d



e (6, 13) and (7, 15)

5 Consider this pattern of matches.



a Complete this table.

Number of squares	1	2	3	4	5
Number of matches					

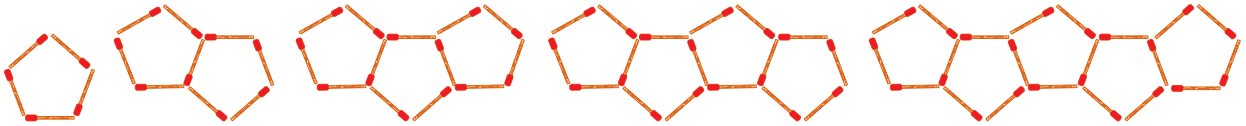
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of squares and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

6 Consider this pattern of matches.



a Complete this table.

Number of pentagons	1	2	3	4	5
Number of matches					

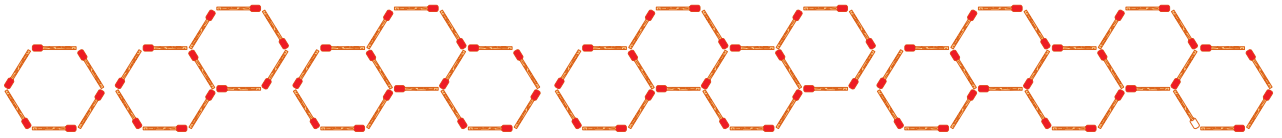
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of pentagons and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

7 **a** Consider this pattern of matches and complete the following table.



Number of hexagons	1	2	3	4	5
Number of matches					

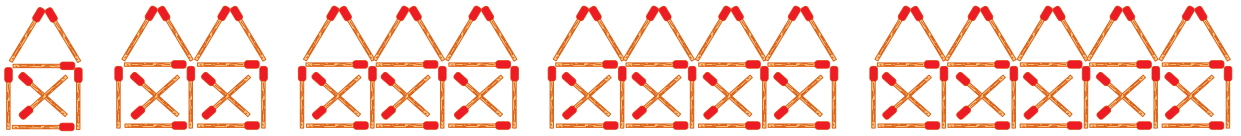
b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of hexagons and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

8 **a** Consider this pattern of matches and complete the following table.



Number of houses	1	2	3	4	5
Number of matches					

b Write a rule describing the number of matches required to make each pattern.

c Using x to represent the number of houses and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

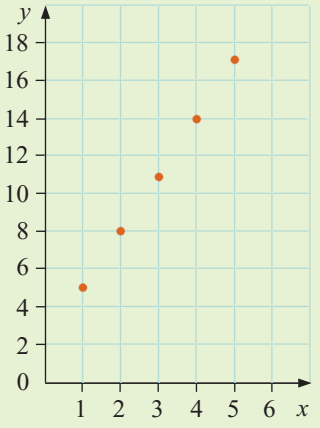
e Mark in the next two points and write their coordinates.

EXAMPLE 3

The graph shows the relationship between the number of shapes (x) in the matchstick pattern, and the number of matches for a particular matchstick pattern (y).

a Construct a table of data for this information.

b Write a rule linking x and y.



a

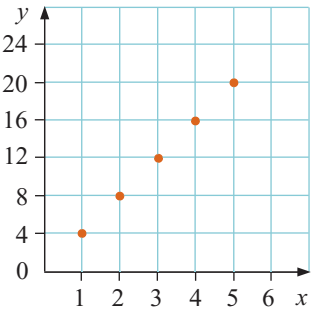
x-value	1	2	3	4	5
y-value	5	8	11	14	17

b The x-values go up by 1 and the y-values go up by 3. So $y = 3x$ must be part of the equation. The y-values for $y = 3x$ would give 3, 6, 9, 12 and 15, but the table values are 2 more, so the equation must be $y = 3x + 2$.

9 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.

a Construct a table of data for this information.

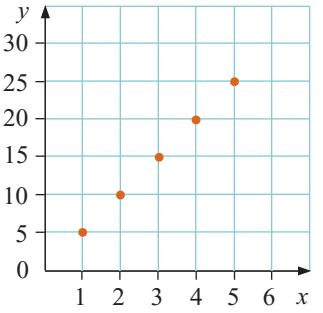
b Write a rule linking x and y.



10 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.

a Construct a table of data for this information.

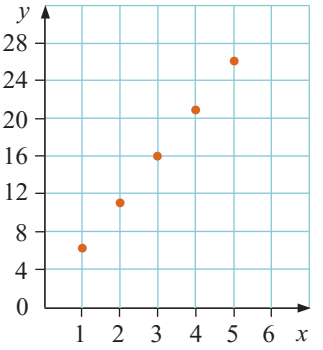
b Write a rule linking x and y.



11 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.

a Construct a table of data for this information.

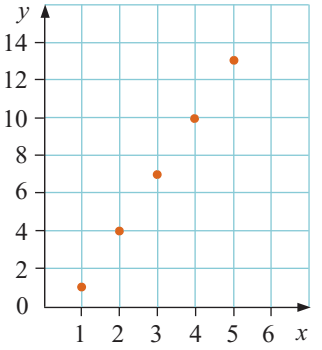
b Write a rule linking x and y.



12 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.

a Construct a table of data for this information.

b Write a rule linking x and y.



E Sketching graphs

In Section D, the values from the tables were plotted as discrete points on the number plane. As it is only possible to draw a complete pattern, it does not make sense to join the points together. It does not make sense to find the number of matches used to make $3\frac{1}{2}$ patterns.

There are other times when it does make sense to join the points together. A purchase of washing powder in bulk would show whole kilograms to be purchased as well as part kilograms. Example 1 shows this.

EXAMPLE 1

Bulk washing powder is sold for $2.00 per kilogram. The following table shows weight versus cost for various quantities of washing powder.

Weight (kg)	0	1	2	4	5	10	20
Cost ($)	0	2	4	8	10	20	40

The number of kilograms is not going up in ones!

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.

b Graph these points on the number plane. Draw a straight line through them.

Label the axis using equal divisions.

c Use the graph to find how much 3.5 kg of washing powder would cost.

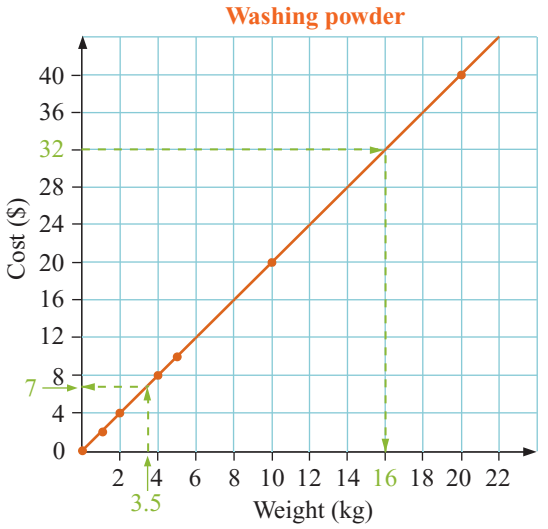
d Use the graph to find how much washing powder could be purchased for $32.

a (0, 0), (1, 2), (2, 4), (4, 8), (5, 10), (10, 20), (20, 40)

b *Note:* The line may be extended past the final point.

c Draw a line up from 3.5 on the x-axis to the graph. Draw a line across to the y-axis. From the graph, the cost is $7.00.

d Draw a line across at 32 on the y-axis to the graph. Draw a line down from the graph to the x-axis. From the graph, 16 kg can be purchased for $32.



Note: In Example 1 it makes sense to join the points, unlike the patterns in the previous exercise. It is possible to purchase fractions of a kilogram of washing powder. It does not have to be purchased in 1 kg units. Recall from Chapter 9 Section A that this type of data is called continuous.

Measurements are usually continuous variables.

Exercise 14E

1 Oranges are sold for $3.00 per kilogram. The following table shows weight versus cost for various quantities of oranges.

Weight (kg)	0	1	2	4	5	10	20
Cost ($)	0	3	6	12	15	30	60

Give the graph a heading.

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.

b Graph these points on the number plane and draw a straight line through them.

c Use the graph to find the cost of 15 kg of oranges.

d Use the graph to find how many kilogram of oranges could be purchased for $24.

2 Watermelon is sold for $2.50 per kilogram. The following table shows weight versus cost for various quantities of watermelon.

Weight (kg)	0	1	2	4	5	10	20
Cost ($)	0	2.5	5	10	12.5	25	50

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.
b Graph these points on the number plane and draw a straight line through them.
c Use the graph to find the cost of 8.5 kg of watermelon.
d Use the graph to find how much watermelon could be purchased for $37.50.

3 Cashew nuts are sold for $24.00 per kilogram. The following table shows weight versus cost for various quantities of cashew nuts.

Weight (kg)	0	1	2	5
Cost ($)	0	24	48	120

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.
b Graph these points on the number plane and draw a straight line through them.
c Use the graph to find the cost of 4.5 kg of cashews.
d Use the graph to find how many kilograms of cashews could be purchased for $84.

4 Chocolate freckles are sold for $11.00 per kilogram. The following table shows weight versus cost for various quantities of chocolate freckles.

Weight (kg)	0	1	2	5
Cost ($)	0	11	22	55

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.
b Graph these points on the number plane and draw a straight line through them.
c Use the graph to find the cost of 3.5 kg of chocolate freckles.
d Use the graph to find how many kilograms of chocolate freckles could be purchased for $30.

5 Mobile telephone calls cost $1.00 per minute.

a Complete this table of values for the cost of mobile calls.

Time (min)	0	1	2	3	4	5	10
Cost ($)	0						

b Write a set of points describing this information.

c Graph these points on the number plane and draw a straight line through them.

d Use the graph to find the cost of 7.5 minutes of calls.

e Use the graph to find how long someone could talk for $6.50.

6 Mobile telephone calls on another plan cost $1.20 per minute.

a Complete this table of values for the cost of mobile calls on this plan.

Time (min)	0	1	2	3	4	5	10
Cost ($)	0						

b Graph these points on the number plane and draw a straight line through them.

c Use the graph to find the cost of 6.5 minutes of calls.

d Use the graph to find how long someone could talk for $10.

EXAMPLE 2

The cost of hiring a car is a $10 booking fee plus $4 per kilometre.

a Complete this table of values for the car hire.

Distance (km)	0	10	20	30	40
Cost ($)					

b Sketch the graph of cost per kilometre.

c Find the cost of a journey of 25 km.

d How far can you travel for $70?

Remember to label the axes and use equal divisions on the scale.

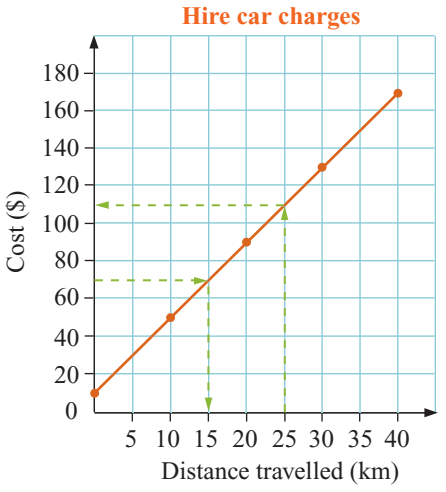
a For a journey of 10 kilometres, cost = $10 + $4 × 10 = $50
For a journey of 20 kilometres, cost = $10 + $4 × 20 = $90
For a journey of 30 kilometres, cost = $10 + $4 × 30 = $130
For a journey of 40 kilometres, cost = $10 + $4 × 40 = $170
The table of values for hiring a car is:

Distance (km)	0	10	20	30	40
Cost ($)	10	50	90	130	170

b Plot these points and draw a straight line through them.

c Draw a line up from 25 on the x-axis to the graph.
Draw a line across to the y-axis. From the graph, the cost of a 25 km journey is about $110.

d Draw a line across at 70 on the y-axis to the graph.
Draw a line down from the graph to the x-axis.
From the graph, you can travel about 15 km for $70.



7 The cost of hiring a taxi is $5.00 flagfall and $2.50 per kilometre travelled.

a Complete the table of values of taxi hire.

Distance (km)	0	10	20	30	40
Cost ($)					

b Draw a graph showing the cost of hiring the taxi.

c How much does it cost to travel 15 km?

d How far can you travel for $85?

8 The cost of hiring a taxi at night is $8.00 flagfall and $3 per kilometre travelled.

a Complete the table of costs of taxi hire.

Kilometres	0	10	20	30	40
Cost ($)					

b Draw a graph showing the cost of hiring the taxi.

c How much does it cost to travel 35 km?

d How far can you travel for $50?

9 Stephanie is paid $10 appearance money per shift plus $16 per hour worked.

a Complete the table of Stephanie's pay.

Hours	0	1	2	3	4	5
Pay ($)						

b Draw a graph showing Stephanie's pay per hour worked.

c How much is Stephanie paid for working 3.5 hours?

d How many hours must Stephanie work to earn $50?

10 Henry works as a clown. He charges $50 per party plus $50 per hour.

a Complete the table showing the cost of hiring Henry the Clown.

Number of hours	0	1	2	3	4
Cost ($)					

b Draw a graph of the cost of hiring Henry the Clown.

c How much does it cost to employ Henry the Clown for 3.5 hours?

d How long will Henry work for $175?

Investigation 1 Graphics calculator

The instructions are for a CASIO Fx 9850Gbplus.

1 Use the graphics calculator to sketch $y = 2x - 3$.

Step 1: Turn on the calculator. Select the GRAPH screen. F3 selects the type of graph.

Step 2: Choose F1, which gives $y =$. If any equations are left from previous work press F2 to delete.

Step 3: Type the equation $y = 2x - 3$ by using 2 × X,θ,T − 3 EXE.

Step 4: Press F6 to DRAW. Use F3 V-window to change the maximum and minimum values on the axes if needed. (Press EXE to return.)

Step 5: To clear the graph press F2 DEL then F1 YES.

2 Repeat from step 3 for another graph. It is possible to draw more than one graph at a time. Use Y2 then Y3 and so on without deleting.

3 **a** Sketch the graphs $y = x + 3$, $y = -3x$, $y = \frac{1}{4}x - 4$, and $y = 3x - 1$.

b Sketch some graphs of your own.

F Straight-line graphs

Section E showed how straight-line graphs could be used to model practical situations. The graphs were used to answer questions. This section examines straight-line graphs without considering the practical application. The straight lines will be extended to include negative values. Graphs are drawn from tables of values.

Note that although a straight line is defined by two points, more points are calculated to make sure there are no mistakes.

A line contains an infinite number of points.

The rules used are similar to the rules developed in Section B.

EXAMPLE 1

a Complete the table of values for $y = x + 1$.

b Draw the graph of $y = x + 1$.

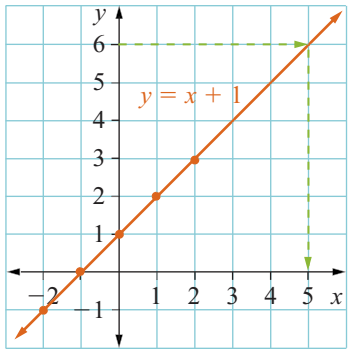
c Use the graph to solve $x + 1 = 6$.

x	−2	−1	0	1	2
y					

a When $x = -2$, $y = -2 + 1 = -1$
When $x = -1$, $y = -1 + 1 = 0$
When $x = 0$, $y = 0 + 1 = 1$
When $x = 1$, $y = 1 + 1 = 2$
When $x = 2$, $y = 2 + 1 = 3$
Use these values to complete the table.

x	−2	−1	0	1	2
y	−1	0	1	2	3

Write the equation on the graph.



b Plot these points. Draw a straight line through the points, extending the line past the points to give the graph of $y = x + 1$.

c Draw a line from $y = 6$ across to the graph then down to the x-axis. The x-value is 5; that is, $x = 5$ is the solution to $x + 1 = 6$.

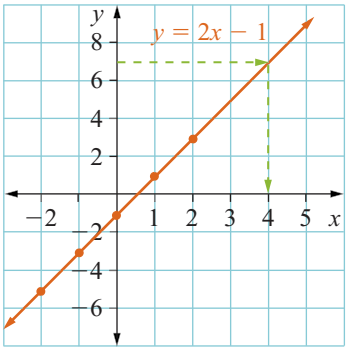
EXAMPLE 2

a Complete the table of values for $y = 2x - 1$.
b Draw the graph of $y = 2x - 1$.
c Use the graph to solve $2x - 1 = 7$.

x	-2	-1	0	1	2
y					

a When $x = -2$, $y = 2(-2) - 1 = -5$
When $x = -1$, $y = 2(-1) - 1 = -3$
When $x = 0$, $y = 2(0) - 1 = -1$
When $x = 1$, $y = 2(1) - 1 = 1$
When $x = 2$, $y = 2(2) - 1 = 3$
Use these values to complete the table.

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



b Plotting these points. Draw a straight line through the points, extending the line past the points to give the graph of $y = 2x - 1$.
c Draw a line from $y = 7$ across to the graph then down to the x-axis. The x-value is 4; that is, $x = 4$ is the solution to $2x - 1 = 7$.

Note:
- The graph must have a heading or the equation of the line.
- The x and y axes must be labelled.
- The points are plotted and the line drawn through them.
- Arrows on each end of the line show that it extends in both directions.
- Any value may be chosen for x and the corresponding y-value calculated.

Exercise 14F

1 **a** Complete the table and draw the graph $y = 2x + 1$. Some of the points are provided.
b Use the graph to solve $2x + 1 = 7$.

x	-2	-1	0	1	2
y		-1			5

2 **a** Complete the table and draw the graph $y = 3x - 2$. Some of the points are provided.
b Use the graph to solve $3x - 2 = 7$.

x	-2	-1	0	1	2
y	-8			1	

3 **a** Complete the table and draw the graph $y = 2x - 3$. Some of the points are provided.
b Use the graph to solve $2x - 3 = 4$.

x	-2	-1	0	1	2
y		-5			1

4 Use a table to draw the graphs of the following equations.
a $y = x - 2$ **b** $y = x + 4$ **c** $y = 2x + 4$
d $y = -3x + 2$ **e** $y = -x + 4$ **f** $y = 2x$
g $y = -4x + 3$ **h** $y = \frac{1}{2}x + 1$ **i** $y = 3 - x$

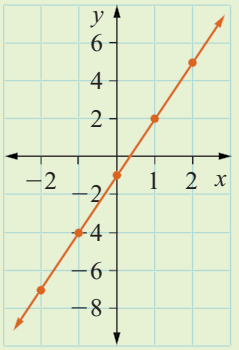
5 Use the graphs from question **4** to solve these equations.
a $x - 2 = 2$ **b** $x + 4 = 8$ **c** $2x + 4 = -4$
d $-3x + 2 = -10$ **e** $-x + 4 = 7$ **f** $2x = 7$
g $-4x + 3 = 5$ **h** $\frac{1}{2}x + 1 = -1$ **i** $3 - x = -1$

EXAMPLE 3

This graph shows a straight line.

a Use the graph to complete this table of values.

x	-2	-1	0	1	2
y					



b Write the rule describing this straight line.
The rule is of the form $y = \square x \pm \triangle$.

The symbol ± means plus or minus.

a The table of values is completed from the graph.

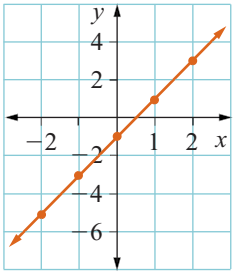
x	-2	-1	0	1	2
y	-7	-4	-1	2	5

b As x increases by 1, y increases by 3. This means that $y = 3x$ is part of the equation of the line.
When $x = 0$, $y = -1$, so the equation is $y = 3x - 1$.
To check, test another point. Test (2, 5): $5 = 3(2) - 1 = 5$ $\therefore$ The equation is correct.

6 Complete a table of values and find the equation of each of these lines.

This may be completed using a graphics calculator.

a



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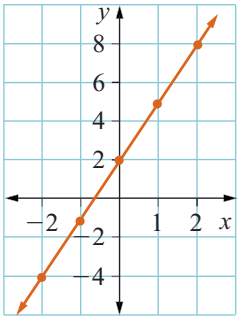

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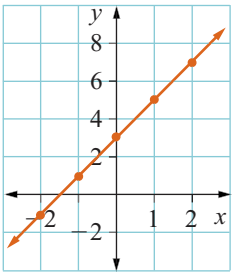

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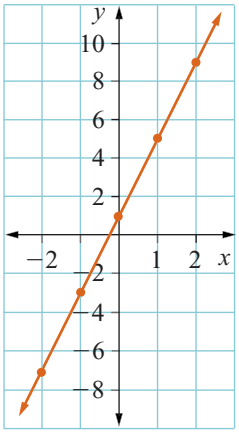

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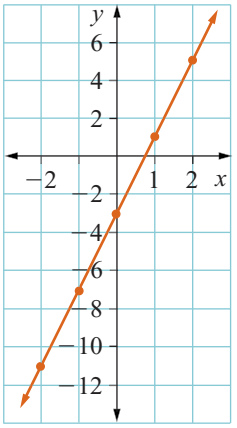

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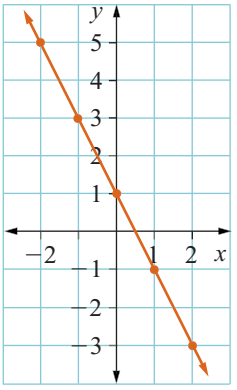

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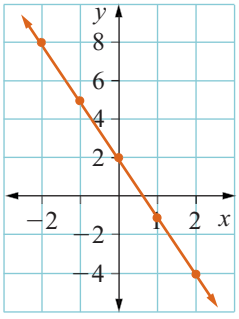
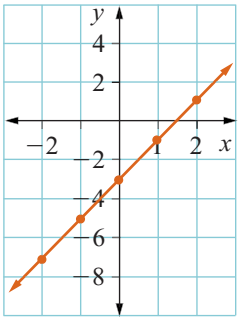
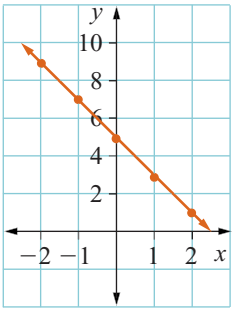


h



i



Investigation 2 Linear relationships

1 **a** Using a 0.5 cm grid, draw these graphs on the same number plane.
$y = 3x + 1, y = 3x - 1, y = 3x, y = 3x + 2$
b What do you notice about all four graphs? Explain.
c Without plotting points, add the graph of $y = 3x + 3$ to your number plane in part **a**. Explain how you knew what to draw.

2 **a** On the number plane from question **1**, draw these graphs.
$y = 2x + 1, y = 3x + 1, y = x + 1$
b What do you notice about all three graphs? Explain.
c Without plotting points, add the graph of $y = 4x + 1$ to your number plane. Explain how you knew what to draw.

3 **a** On another number plane draw the graphs of $y = x + 1, y = -x + 1$.
b What do you notice about these two graphs? Explain.
c On a second number plane draw graphs of $y = x$ and $y = -x$.
d What do you notice about these two graphs? Explain.
e On a third number plane draw graphs of $y = 2x + 1$ and $y = -2x + 1$.
f What do you notice about these two graphs? Explain.
g How can you decide if a graph is increasing or decreasing based on the equation?

G Comparing equations

In Investigation 2 you found the following properties of straight-line graphs.

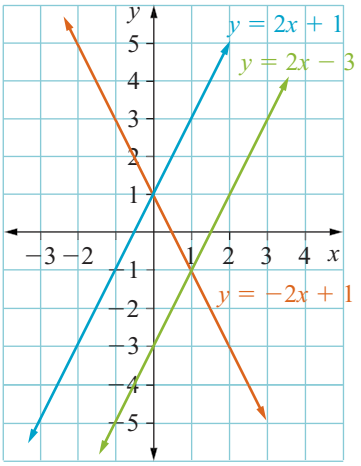
The coefficient of x is the number in front of the x.

1 If the coefficient of x is the same in each equation, the lines are parallel.
For example, $y = 2x + 1$ and $y = 2x - 3$ are parallel.

2 The constant term (the term without x) is where the line cuts the y-axis.
For example, $y = -2x + 1$ cuts the y-axis at $y = 1$. This is the y-intercept.

3 Lines with the coefficient of x equal but opposite in sign have the same slope but in opposite directions.

4 As we move from left to right, lines with a positive coefficient of x have an 'uphill' slope. Lines with a negative coefficient of x have a 'downhill' slope.



EXAMPLE 1

Explain the similarities of and differences between these lines.
$y = 4x + 1,\ y = 4x - 3,\ y = 4x$

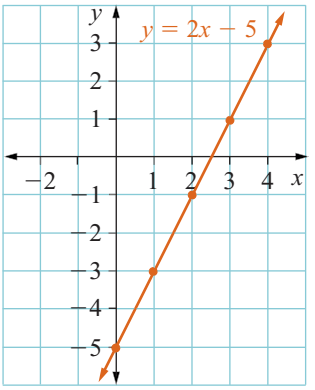
These lines are all parallel because the coefficient of x is the same in all equations.
The lines cut the y-axis at different points; that is, at 1, -3 and 0 respectively.

Exercise 14G

1 **a** Without drawing graphs, explain the similarities of and differences between the lines:
$y = 2x + 3,\ y = 2x - 1,\ y = 2x + 1$
b Draw each line to check.

2 Explain the similarities of and differences between each group of lines, giving reasons for your answers.
a $y = 2x + 2,\ y = 3x + 2,\ y = -x + 2$
b $y = 3x + 1,\ y = -3x + 1$
c $y = 5x,\ y = -5x$
d $y = -x + 2,\ y = -x + 1,\ y = -x - 3$

3 This is the graph of $y = 2x - 5$.
a The graph of the line $y = 2x - 3$ is to be drawn on the same number plane. Explain which aspects would be the same and which would be different.
b Draw the graph to check.



4 Compare and contrast the graphs below with the graph of $y = x + 2$.
a $y = x + 1$
b $y = 2x + 2$
c $y = -x + 2$
d $y = x$

5 Classify the following linear equations into groups. Describe similar features and explain why each graph is in the particular group. Graphs may be in more than one group.
$y = 2x + 3,\ y = -3x - 2,\ y = 3x + 3, y = 2x - 1,\ y = -3x,\ y = -2x,$
$y = 4x + 3,\ y = 3x,\ y = -3x - 5,\ y = 2x,\ y = 5x - 2$

EXAMPLE 2

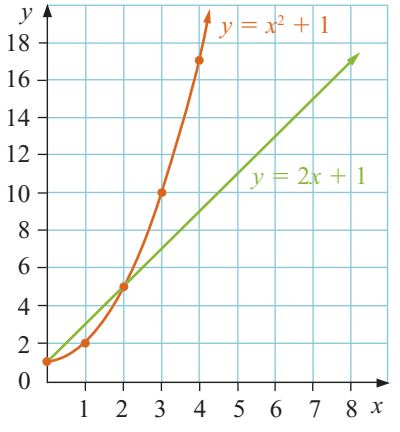
a Complete the table of values for the graph of $y = x^2 + 1$.
b Draw the graph of $y = x^2 + 1$.
c Draw the graph of $y = 2x + 1$ on the same number plane.
d Comment on the two graphs.

x	0	1	2	3	4
y					

a $y = x^2 + 1$

x	0	1	2	3	4
y	1	2	5	10	17

b Plot the values, drawing a smooth curve through the points.
c Draw the graph of $y = 2x + 1$.
d Both graphs have a y-intercept of 1, but $y = x^2 + 1$ is not a straight line. They intersect at the points (2, 5) and (0, 1).



6 **a** Complete the table of values for the graph of $y = x^2 + 3$.
b Draw the graph of $y = x^2 + 3$.
c Draw the graph of $y = 2x + 3$ on the same number plane.
d Comment on the two graphs.

x	0	1	2	3	4
y					

7 **a** Complete the table of values for the graph of $y = x^3 + 1$.
b Draw the graph of $y = x^3 + 1$.
c Draw the graph of $y = x + 1$ on the same number plane.
d Comment on the two graphs.

x	0	1	2	3
y				

8 The graphs $y = x^2 + 1$, $y = x^2 + 3$ and $y = x^3 + 1$ are all non-linear graphs. Group the equations below into linear and non-linear groups. Give reasons for your answer.

$y = 2x - 5$, $y = x^2 - 5$, $y = -x + 1$, $y = x^3 + 4$, $y = 7x + 5$, $y = x^4 + 2$

H Intersecting lines

In previous sections, sometimes lines intersected and other lines did not. If two lines intersect, the point at which they intersect is the simultaneous solution of each equation.

EXAMPLE 1

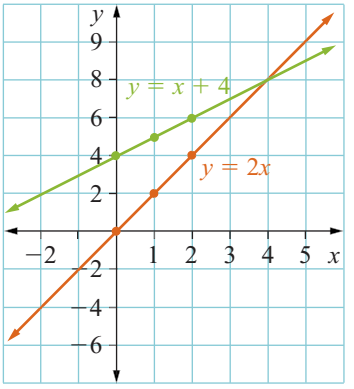
a On the same number plane, draw the graphs of $y = x + 4$ and $y = 2x$.
b Find the point of intersection.

a $y = x + 4$

x	0	1	2
y	4	5	6

$y = 2x$

x	0	1	2
y	0	2	4



b The point of intersection is (4, 8). Check by substitution.

$y = x + 4$ $\qquad$ $y = 2x$

$y = 4 + 4 = 8$ $\qquad$ $y = 2 \times 4 = 8$

$\therefore$ (4, 8) is a solution to both equations.

Exercise 14H

1 **a** On the same number plane, draw the graphs of $y = x + 3$ and $y = 2x$.
b Find the point of intersection.

2 Repeat question **1** for each of these pairs of equations.

a $y = x + 1$, $y = 2x$
b $y = x + 2$, $y = 2x + 1$
c $y = x + 3$, $y = 4x$
d $y = x + 5$, $y = 2x + 1$
e $y = x + 3$, $y = 2x - 1$
f $y = 2x - 2$, $y = 3x - 7$

3 **a** On the same number plane, draw the graphs of $y = 2x + 1$ and $y = 2x + 4$.
b Is it possible to find the point of intersection of these graphs? Explain.
c What is necessary for two straight-line graphs not to intersect?

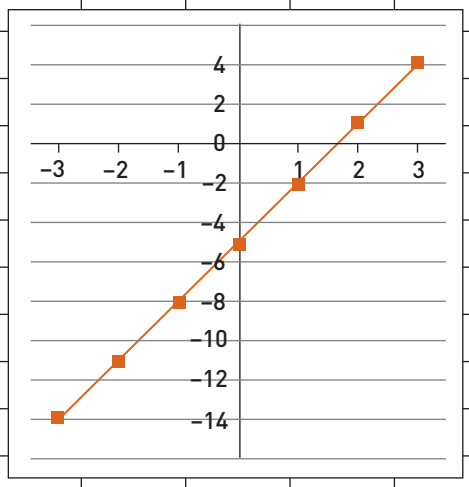
Investigation 3 Using spreadsheets to graph straight lines

1 A spreadsheet can be used to graph equations. Use a spreadsheet to graph $y = 3x - 5$.

a Copy the data from the spreadsheet shown below.

b The formula for cell B7 is =$B$3*B6+$B$4 which is copied across row 7.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Graphing straight lines**												
2													
3	m=	3											
4	b=	–5											
5													
6	x	–3	–2	–1	0	1	2	3					
7	y	–14	–11	–8	–5	–2	1	4					
8													
9													
10													
11													
12													



c Change the value of m and b in the spreadsheet to graph $y = 2x + 7$.

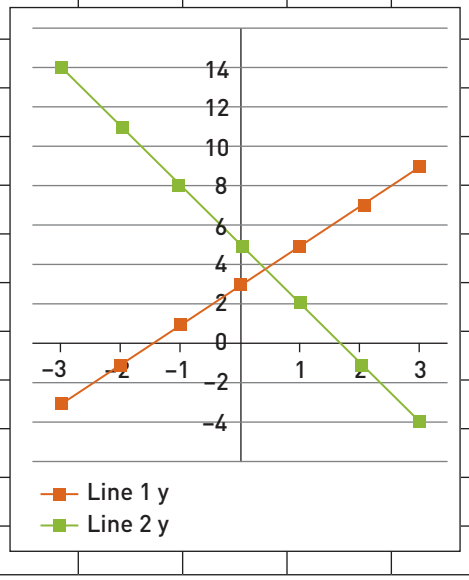
d Use the spreadsheet to draw other straight-line graphs from the exercises.

2 Spreadsheets can graph intersecting lines.

a Copy the data from the spreadsheet below.

b The formula for cell B7 is =$B$3*B6+$B$4, which is copied across row 7.
The formula for cell B8 =$E$3*B6+$E$4, which is copied across line 8.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Graphing straight lines**												
2	**Line 1**			**Line 2**									
3	m=	2		m=	–3								
4	b=	3		b=	5								
5													
6	x	–3	–2	–1	0	1	2	3					
7	Line 1 y	–3	–1	1	3	5	7	9					
8	Line 2 y	14	11	8	5	2	–1	–4					
9													
10													
11													
12													
13													



c Change the value of m and b in the spreadsheet to graph other pairs of lines.

Language in mathematics

René Descartes

René Descartes, a great mathematician and philosopher, was born in 1596. As a child he caused his father such problems that his father is supposed to have said that he was 'only fit to be bound in calf-skin'. His schooling occurred in the very strict upbringing of a Jesuit school. In fact, in the latter part of his schooling he decided to give up study and learn by 'travelling and observing'.

One of Descartes' most famous writings is called the *Discourse on Method*. This was originally written in French and is considered to have greatly influenced philosophical thought. One of his main rules was 'never to accept anything as true which I do not clearly and distinctly see to be so'.

Descartes' greatest claim to fame as a mathematician is that he is credited with the idea of representing points by using ordered number pairs on coordinate axes. Such a plane of numbers has been called the Cartesian plane, from his name.

Recently, it has been pointed out that Descartes may not have been the first to use coordinates. However, there is little doubt that he was the first to:

- use x for unknown variables
- use index notation
- prove the impossibility of trisecting angles using a compass and ruler only.

Descartes died in 1650 as a result of pneumonia. He was buried in Sweden, a Protestant country, in a graveyard usually saved for non-baptised children. His body was moved to Paris and reburied in 1667.

Read this article about the French mathematician René Descartes and then answer the questions.

1 How old was Descartes when he died?

2 Describe the way in which Descartes wanted to learn.

3 What was his greatest claim to fame?

4 **a** What was his main rule?
b Rewrite this in your own words.

5 For how many years was he buried before being relocated to Paris?

6 Write a three-sentence summary of the article.

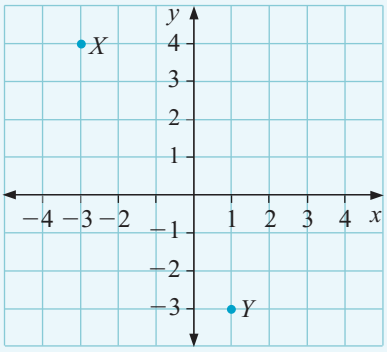
7 List three mathematical concepts introduced by Descartes.

Terms

coefficient	constant term	continuous variable	coordinates	curve
data	discrete data	equation	graph	grid reference
intercept	intersecting lines	linear equation	negative number	non-linear graph
number plane	origin	parallel	pattern	point
relationship	rule	solution	straight line	value

Check your skills

1 The coordinates of the points X and Y on this number plane are:



A (4, −3) and (−3, 1) **B** (4, −3) and (1, −3)
C (−3, 4) and (−3, 1) **D** (−3, 4) and (1, −3)

2 The missing numbers in the pattern 3, 7, ___, 15, ___, 23 are:

A 10 and 18 **B** 12 and 20
C 11 and 20 **D** 11 and 19

3 Using the rule 'starting with 2, multiply the preceding term by 3 and then subtract 1', the first three terms of the number sequence would be:

A 2, 5, 15 **B** 2, 5, 14 **C** 2, 4, 8 **D** 3, 6, 18

4 The type of data that has a straight line drawn through the points is:

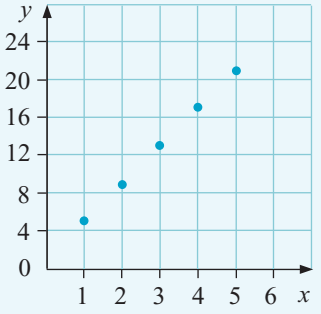
A discrete **B** integer **C** decimal **D** continuous

5 A rule describing the information in the table below is:

A $y = x + 3$ **B** $y = x + 6$ **C** $y = 3x$ **D** $y = 4x$

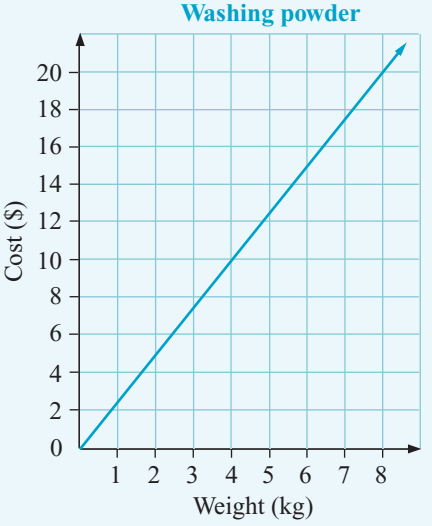
Number of squares	1	2	3	4	5
Number of matches	4	8	12	16	20

6 This graph shows the relationship between x and y.
The rule linking x and y is:



A $y = 5x$ **B** $y = 4x + 1$
C $y = 5x - 1$ **D** $y = 4x$

This graph refers to questions **7** and **8**.



7 The cost of 6 kg of washing powder is:

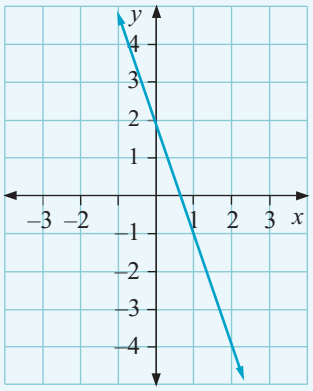
A $5 **B** $12
C $15 **D** $20

8 The amount of washing powder in kilograms that can be purchased for $18 is closest to:

A 6 **B** 6.5
C 7.2 **D** 7.8

9 The graph of $y = -3x + 2$ is closest to:

A



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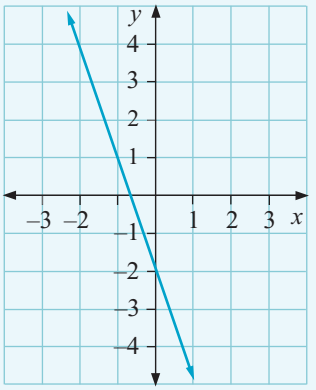

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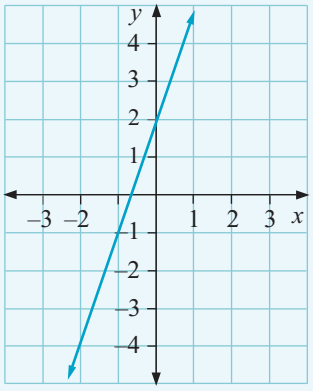

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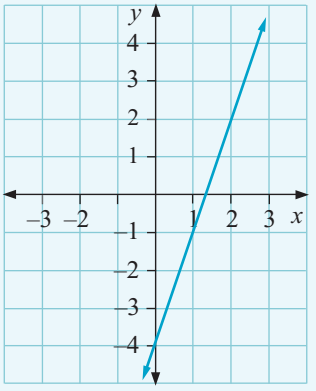


10 The correct statement describing the graphs of $y = 3x - 1$ and $y = 3x + 1$ is that the lines:

A are parallel **B** pass through (3, 0) **C** pass through (0, 1) **D** intersect

11 The correct statement describing the graphs of $y = -3x + 2$ and $y = 2x + 2$ is that the lines:

A are parallel **B** do not intersect **C** pass through (2, 0) **D** pass through (0, 2)

12 From the graphs of $y = 2x + 1$ and $y = x + 3$ the point of intersection is:

A (2, 1) **B** (2, 5) **C** (2, 3) **D** (4, 7)

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2	3	4–6	7, 8	9	10–11	12
Section	A	B	C	D	E	F	G	H

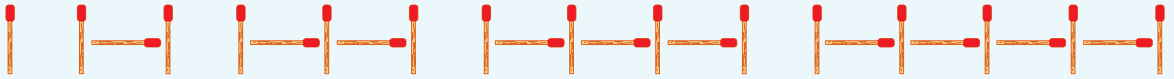
14A Review set

1 Plot these points on a number plane: $A(0, -2)$, $B(-1, -3)$, $C(2, -4)$, $D(-3, 1)$, $E(2, 3)$.

2 **a** Plot the points $A(-2, 4)$, $B(2, 4)$ and $C(2, 0)$.

b If $ABCD$ is a square, find the coordinates of the point D.

3 Consider this pattern of matches.



a Complete this table.

Number of patterns	1	2	3	4	5
Number of matches					

b Write a rule describing the number of matches required to make each pattern.

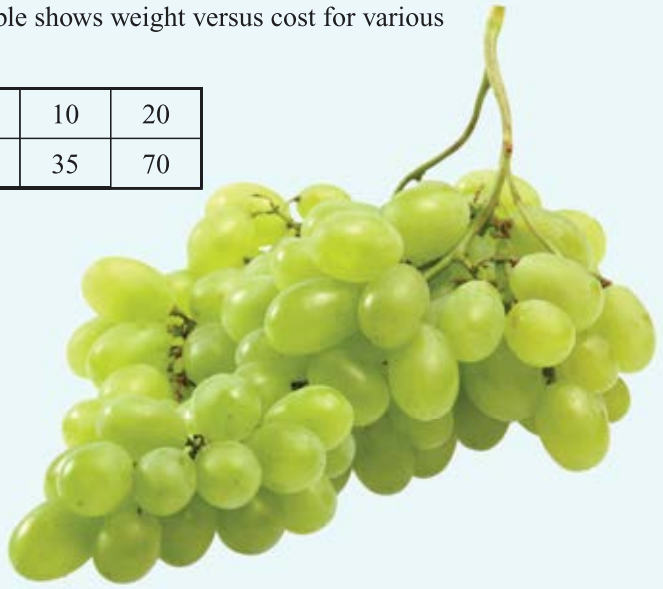
c Using x to represent the number of patterns and y to represent the number of matches, write a set of points describing this information.

d Graph these points on the number plane.

e Mark in the next two points and write their coordinates.

4 Grapes are sold for $3.50 per kilogram. The following table shows weight versus cost for various quantities of grapes.

Number of kg	0	1	2	4	5	10	20
Cost ($)	0	3.5	7	14	17.5	35	70



a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.

b Graph these points on the number plane and draw a straight line through them.

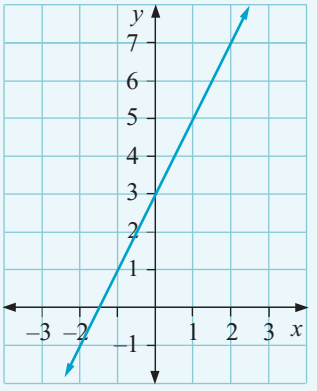
c Use the graph to find how much 8.5 kg of grapes would cost.

d Use the graph to find what weight of grapes could be purchased for $63.

5 Complete the table and draw the graph for $y = 3x - 2$. Some of the points are provided.

x	-2	-1	0	1	2
y		-5			4

6 The graph shows a straight line.



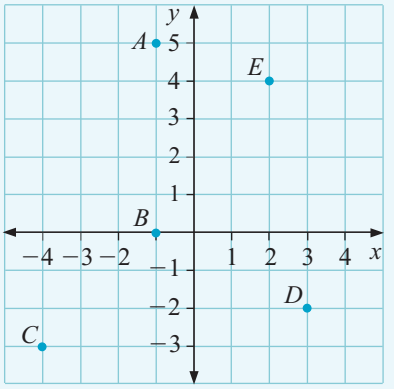
a Use the graph to complete this table of values.

x	-2	-1	0	1	2
y					

b Write the rule describing this straight line. The rule is of the form $y = \square x \pm \triangle$.

14B Review set

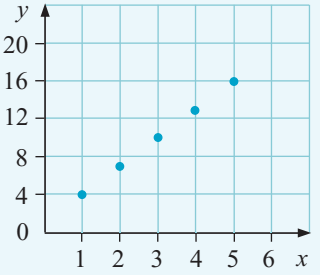
1 Write the coordinates of the points marked on this number plane.



2 **a** Plot the points $P(-3, -2)$, $Q(2, -2)$ and $R(3, 1)$ on a number plane.

b If $PQRS$ is a parallelogram, find the coordinates of the point S.

3 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.



a Construct a table of data for this information.

b Write a rule linking x and y.

4 The cost of hiring a wedding car is \$200 flagfall and \$90 per hour.

a Complete the table of costs of hire.

Number of hours	0	1	2	3	4
Cost (\$)					

b Draw a graph showing the cost of hiring the wedding car.

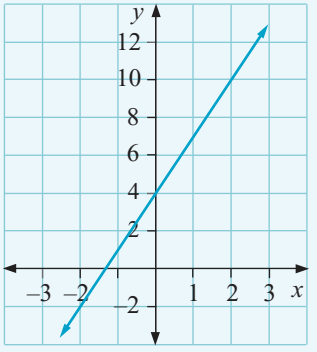
c How much does it cost to hire the car for $2\frac{1}{2}$ hours?

d For how long can you hire the car for \$520?

5 Complete the table and draw the graph for $y = 4x - 3$.

x	-2	-1	0	1	2
y					

6 This graph shows a straight line.



a Use the graph to complete this table of values.

x	-2	-1	0	1	2
y					

b Write the rule describing this straight line. The rule is of the form $y = \square x \pm \triangle$.

14C Review set

1 Plot these points on a number plane: $L(-4, -3)$, $M(5, 0)$, $N(-3, 4)$, $O(0, 0)$, $P(1, 3)$ and $Q(2, -5)$.

2 **a** Plot the points $A(-5, -3)$, $B(-4, -2)$, $C(-3, -1)$, $D(-2, 0)$ and $E(-1, 1)$ on the same number plane.
b Join the points. What do you notice?
c What are the next three points F, G and H if the pattern continues?

3 Consider this pattern of matches.

a Complete this table.

x	1	2	3	4	5
y					

b Write a rule describing the number of matches required to make each pattern.
c Using x to represent the number of squares and y to represent the number of matches, write a set of points describing this information.
d What are the next two points? Write their coordinates.

4 Pistachio nuts are sold for \$18.00 per kilogram. This table shows weight versus cost for various quantities of pistachio nuts.

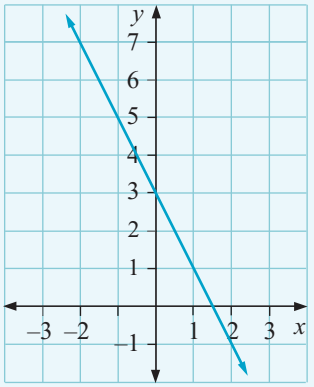
Number of kg	0	1	2	5
Cost (\$)	0	18	36	90

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.
b Graph these points on the number plane and draw a straight line through them.
c Use the graph to find how much 3.5 kg of pistachios would cost.
d Use the graph to find how many kilograms of pistachios could be purchased for \$72.

5 Complete the table and draw the graph for $y = 5x - 3$.

x	-2	-1	0	1	2
y					

6 This graph shows a straight line.



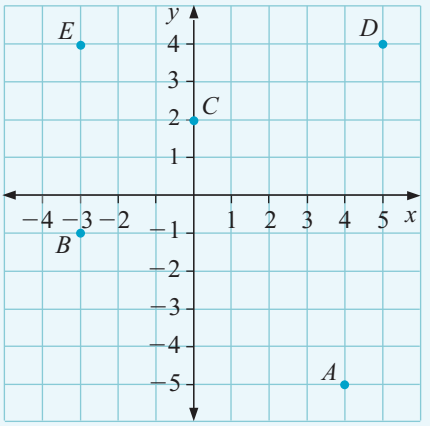
a Use the graph to complete this table of values.

x	-2	-1	0	1	2
y					

b Write the rule describing this straight line. The rule is of the form $y = \square x \pm \triangle$.

14D Review set

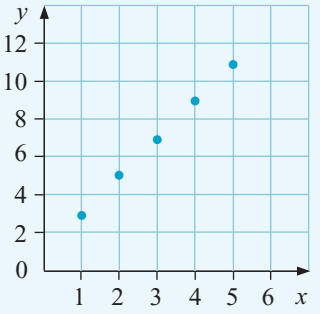
1 Write the coordinates of these points.



2 **a** Plot the points $A(6, 3)$, $B(5, 2)$, C$(4, 1)$, $D(3, 0)$ and $E(2, -1)$ on the same number plane.

b What are the next three points F, G and H if the pattern continues?

3 The graph shows the relationship between x, the number of shapes in the matchstick pattern, and y, the number of matches for a particular matchstick pattern.



a Construct a table of data for this information.

b Write a rule linking x and y.

4 The cost of hiring a jukebox is \$100 plus \$25 per hour.

a Complete the table of costs of hire.

Number of hours	0	1	2	3	4
Cost (\$)					

b Draw a graph showing the cost of hiring the jukebox.

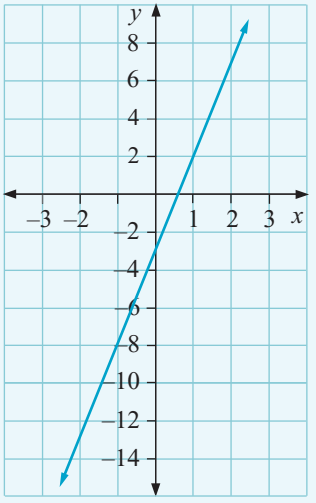
c How much does it cost to hire the jukebox for $3\frac{1}{2}$ hours?

d For how long can you hire the jukebox for \$160?

5 Complete the table and draw the graph for $y = 3x - 4$.

x	-2	-1	0	1	2
y					

6 This graph shows a straight line.



a Use the graph to complete this table of values.

x	-2	-1	0	1	2
y					

b Write the rule describing this straight line. The rule is of the form $y = \square x \pm \triangle$.

15

Triangles and quadrilaterals

This chapter deals with the properties of geometrical figures.

At the end of this chapter you should be able to:

- use correct terminology and notation
- construct triangles
- classify triangles based on their side and angle properties
- use the angle sum of a triangle property
- use the properties of types of triangles
- use the exterior angle of a triangle theorem
- classify quadrilaterals
- use the properties of quadrilaterals including angle sum.

NSW Syllabus references: M&G Properties of geometrical figures 1
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-17MG
MEASUREMENT & GEOMETRY – ACMMG165, ACMMG166

Diagnostic test

1 How many degrees in a right angle?

A 360° **B** 270° **C** 90° **D** 180°

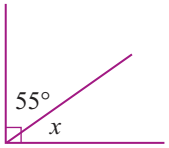
2 How many degrees in a straight angle?

A 360° **B** 270° **C** 90° **D** 180°

3 How many degrees in a revolution?

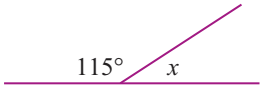
A 360° **B** 270° **C** 90° **D** 180°

4 What is the value of x?



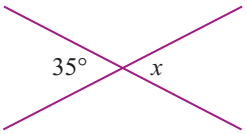
A 35° **B** 55° **C** 135° **D** 305°

5 What is the value of x?



A 35° **B** 65° **C** 145° **D** 245°

6 What is the value of x?



A 35° **B** 55° **C** 135° **D** 325°

7 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

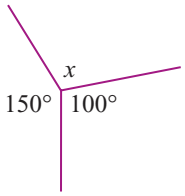
8 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

9 What is this type of angle?

A revolution **B** acute
C obtuse **D** reflex

10 What is the value of x?

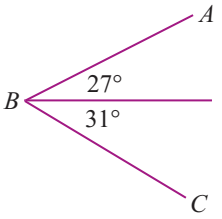


A 110° **B** 80° **C** 30° **D** 10°

11 An acute angle is:

A equal to 180° **B** $>$ 90° but $<$ 180°
C $>$ 0° but $<$ 90° **D** equal to 90°

12 Calculate the size of $\angle ABC$.

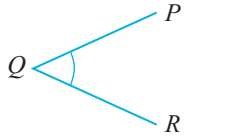
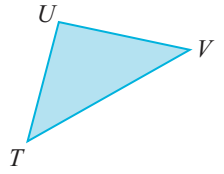
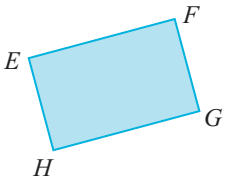
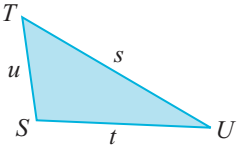
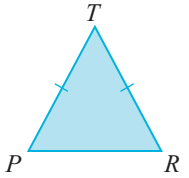
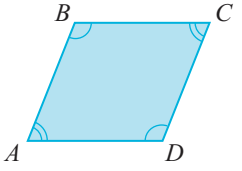
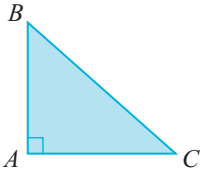


A 4° **B** 32° **C** 58° **D** 122°

The Diagnostic test questions refer to the Year 7 outcomes from ACMMG165 and ACMMG16.

A Notation review

Conventions are used to label diagrams and describe geometric figures and their properties. This section reviews these conventions and consolidates some others.

Convention	Example
A point is labelled with a capital letter. The example shows point B.	
An interval is named using its endpoints. The example shows the interval DC.	
A line is named using any two points on it. The example shows the line PR.	
An angle is named using its arms and vertex. The example could be named angle PQR, angle RQP or angle Q (written $\angle PQR$, $\angle RQP$ or $\angle Q$, or $\hat{PQR}$, $\hat{RQP}$ or $\hat{Q}$ respectively).	
Triangles are named using the three angles or vertices. The example shows $\triangle TUV$.	
Other figures are labelled using the vertices in a cyclic order. The example shows a quadrilateral $EFGH$.	
Sides of triangles are named in two ways. In this example, the longest side can be named using two points, TU or UT, or the lower case letter of the opposite vertex, s.	
Equal sides are labelled with the same markings. The example shows $TR = TP$.	
Equal angles are labelled with the same markings. The example shows $\angle B = \angle D$ and $\angle A = \angle C$.	
Right angles are shown with a square symbol. The example shows $\angle BAC = 90°$.	

Exercise 15A

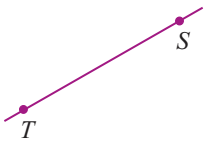
1 Draw a diagram showing each feature.

a Point Q **b** Interval XY **c** Line EF **d** Angle BAC

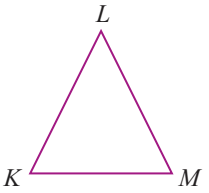
e Triangle MNO **f** Rectangle $PQRS$ **g** Hexagon $ABCDEF$ **h** Triangle with sides g, h and i

2 Describe the features shown in the following diagrams.

a

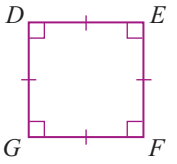


b



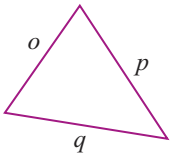
c

d

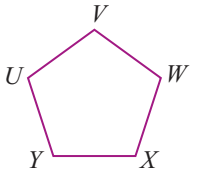


e

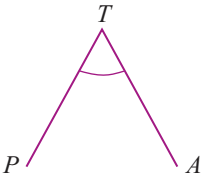
f



g

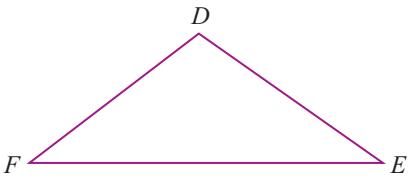


h

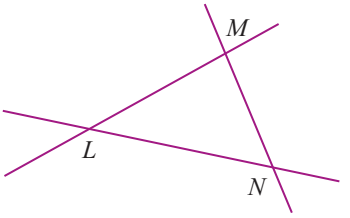


3 **a** Name this line in three different ways.

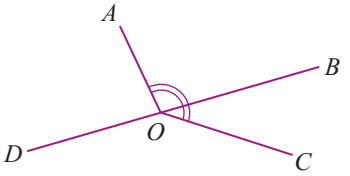
b Name the side opposite angle D in two different ways.



c Name the three lines in this diagram.

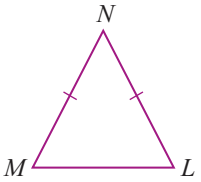


d Name the angle marked and explain why it is not $\angle O$.

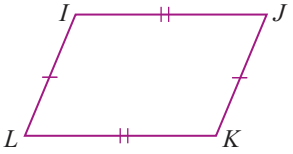


4 List the equal sides in each diagram.

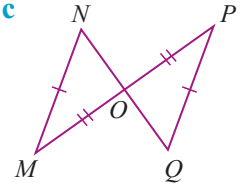
a



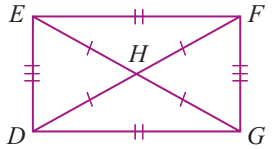
b



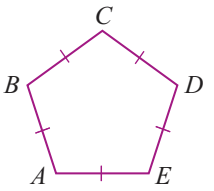
c



d



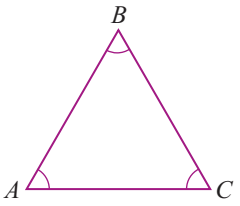
e



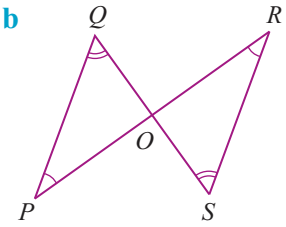
More than one marking may be used if more than one pair of sides are equal.

5 List the equal angles in each diagram.

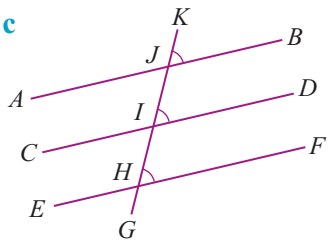
a



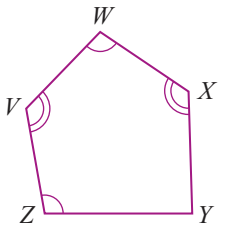
b



c



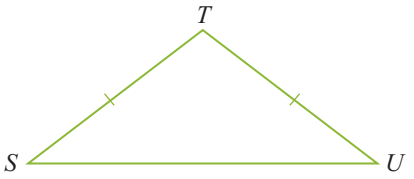
d



EXAMPLE 1

Draw a sketch of triangle STU with $\angle STU$ obtuse and $ST = TU$.

$\angle STU$ is greater than 90°.
ST and TU are labelled as equal.



6 Draw sketches showing the following information.

a Triangle DEF with $DE = DF$

b Triangle XYZ with $\angle Y = \angle Z$ and side YZ the longest side

c Triangle LMN with $\angle NLM = 90°$ and $NL = LM$

d Quadrilateral $PQRS$ with $PQ = RS$ and $\angle PSR = 90°$

e Quadrilateral $DEFG$ with $DE = GF$, $EF = DG$ and $\angle DEF = 90°$

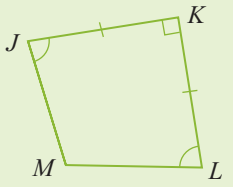
f Quadrilateral $ABCD$ with $AB = AD = 5$ cm, $\angle BAD = 80°$, $BC = 6$ cm and $CD = 7$ cm

g Quadrilateral $PQRS$ with $PQ = 5$ cm, $PS = 6$ cm, $QR = RS = 7$ cm and $\angle QPS = 110°$

h Quadrilateral $KLMN$ with $KL = 6$ cm, $LM = 5$ cm, $KN = 7$ cm, $\angle LKN = 70°$ and $\angle KLM = 130°$

EXAMPLE 2

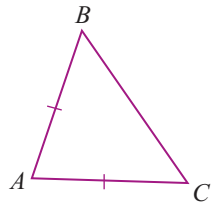
Describe the information shown in the diagram.



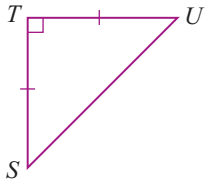
Quadrilateral $JKLM$ with $JK = KL$, $\angle MJK = \angle KLM$ and $\angle JKL = 90°$.

7 Describe the information shown in the following diagrams.

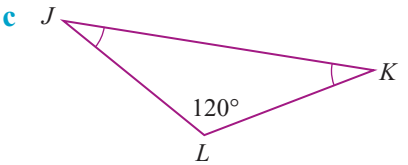
a



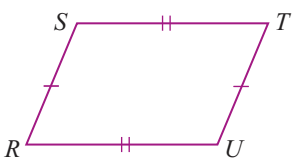
b



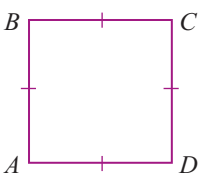
c



d



e



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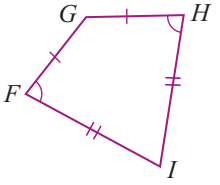


B Constructing triangles

A **triangle** is a three-sided plane shape with straight sides.

To construct a triangle, you need information about the lengths of the sides or the sizes of the angles. You also need a ruler, compasses and protractor.

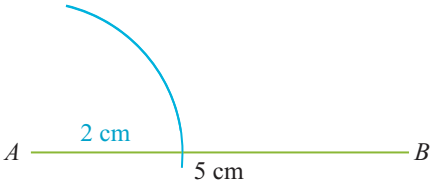
EXAMPLE 1

Constructing a triangle given three sides

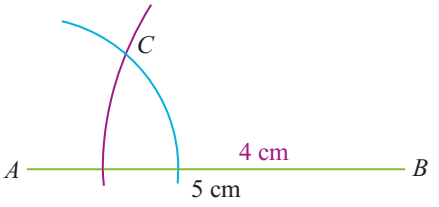
Use a ruler and compasses to construct $\triangle ABC$ with side lengths $AB = 5$ cm, $BC = 4$ cm and $AC = 2$ cm.

Step 1: Draw an interval AB 5 cm in length.

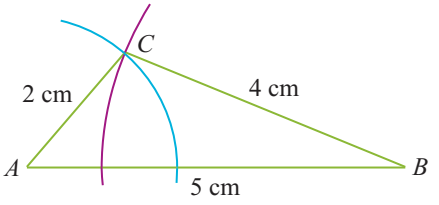
Step 2: Set the compasses to 2 cm. Draw an arc from point A.



Step 3: Set the compasses to 4 cm. Draw an arc from point B. Mark the point C where the arcs intersect.



Step 4: Draw the remaining sides (join points A and B to point C with straight lines).



Exercise 15B

1 Complete the following steps to construct $\triangle PQR$ with side lengths $PQ = 2$ cm, $QR = 3$ cm and $PR = 4$ cm.

Step 1: Draw an interval PQ 2 cm in length.

Step 2: Set the compasses to 3 cm. Draw an arc from point Q.

Step 3: Set the compasses to 4 cm. Draw an arc from point P. Mark point R where the arcs intersect.

Step 4: Join points Q and P to point R.

2 Construct the following triangles.

a $\triangle LMN$ with $LM = 6$ cm, $MN = 4$ cm, $LN = 5$ cm

b $\triangle DEF$ with $DE = 5$ cm, $EF = 3.5$ cm, $DF = 4.5$ cm

c $\triangle DKM$ with $DK = 4$ cm, $KM = 2.5$ cm, $DM = 4.2$ cm

d $\triangle OPR$ with $OP = 4$ cm, $PR = 6.5$ cm, $OR = 4$ cm

3 **a** For each triangle constructed in question **2**, measure the three angles and write them on the diagram.

b What do you notice about the angle opposite the:

i longest side? **ii** shortest side?

c Complete:

The _____ angle is opposite the longest side.

4 Use the triangles constructed in question **2** to complete this table.

	Longest side	Sum of other 2 sides	Is longest side greater than sum of the other 2 sides?
a			
b			
c			
d			

5 Try to construct triangles with the following side lengths.

a 1 cm, 7 cm and 9 cm **b** 5.2 cm, 8 cm and 6 cm

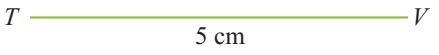
For each case explain why it is or is not possible.

EXAMPLE 2

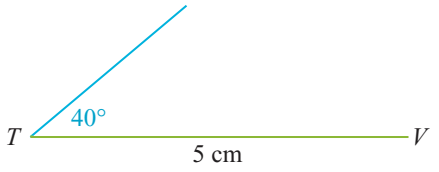
Constructing a triangle given two sides and the included angle

Use a ruler and protractor to construct $\triangle TVU$ with side lengths $TV = 5$ cm and $TU = 2$ cm and $\angle VTU = 40°$.

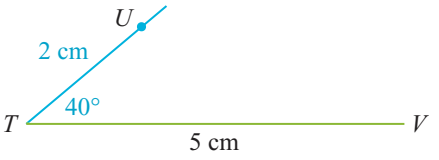
Step 1: Draw a line interval TV of length 5 cm.



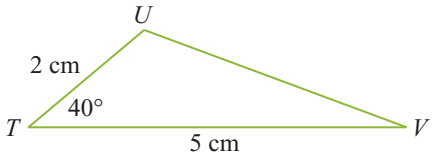
Step 2: At point T, use a protractor to construct an angle of 40° with line TV.



Step 2: Measure 2 cm along the arm of the angle. Mark this point U.



Step 4: Join points U and V to form the triangle.



6 Complete the following steps to construct $\triangle ABC$ with $AB = 6$ cm, $BC = 4$ cm and $\angle B = 70°$.

Step 1: Draw a line interval AB of length 6 cm.

Step 2: At point B, use a protractor to construct an angle of 70° with line AB.

Step 3: Measure 4 cm along the arm of the angle. Mark this point C.

Step 4: Join points C and A to form the triangle.

7 Construct the following triangles by drawing the angle first.

a $\triangle ABC$ with $AB = 3$ cm, $BC = 4$ cm, $\angle B = 110°$

b $\triangle KLM$ with $KL = 4$ cm, $LM = 2$ cm, $\angle L = 75°$

c $\triangle XTR$ with $XT = 3$ cm, $TR = 4$ cm, $\angle T = 135°$

d $\triangle LST$ with $LT = 5$ cm, $TS = 1.5$ cm, $\angle T = 65°$

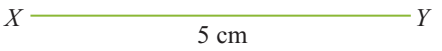
EXAMPLE 3

Constructing a triangle given two angles and a side

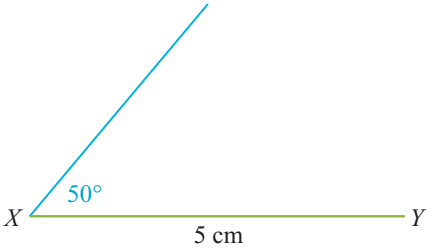
Use a ruler and protractor to construct $\triangle XYZ$ where XY is 5 cm in length, $\angle X = 50°$ and $\angle Y = 40°$.

The given side is called the **baseline**.

Step 1: Draw a line interval XY of length 5 cm.

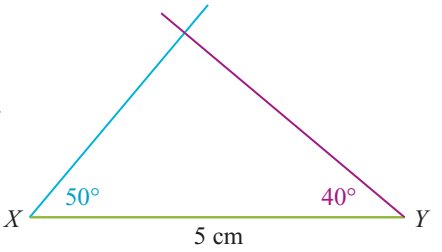


Step 2: At point X, use a protractor to construct an angle of 50° with line XY.

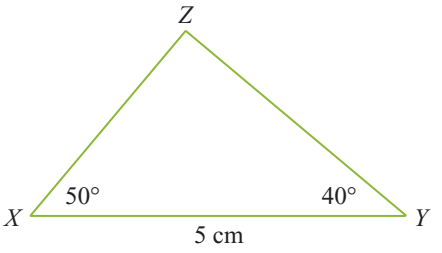


Step 3: At point Y, use a protractor to construct an angle of 40° with line XY.

Extend the arms of the angles until they cross at point Z.



Step 4: Join points X, Y and Z to form a triangle.



8 Complete the following steps to construct $\triangle PQR$ with $PQ = 4$ cm, $\angle P = 60°$ and $\angle Q = 40°$.

Step 1: Draw a line interval PQ of length 4 cm.

Step 2: At point P, use a protractor to construct an angle of 60° with line PQ.

Step 3: At point Q, use a protractor to construct an angle of 40° with line PQ.
Extend the arms of each angle until they cross at point R.

Step 4: Join points P, Q and R to form a triangle.

9 Construct the following triangles by drawing the baseline first.

a $\triangle XYZ$ with $XZ = 5$ cm, $\angle X = 50°$, $\angle Z = 30°$

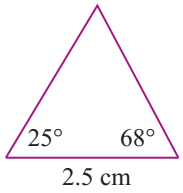
b $\triangle ABC$ with $AB = 4$ cm, $\angle B = 50°$, $\angle A = 70°$

c $\triangle MRL$ with $ML = 6$ cm, $\angle M = 90°$, $\angle L = 25°$

d $\triangle PQR$ with $PR = 5$ cm, $\angle P = 20°$, $\angle R = 110°$

10 Make accurate full-sized drawings of these triangles using the dimensions given.

a



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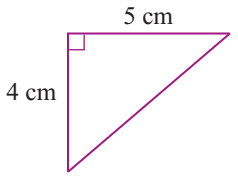

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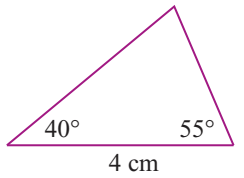

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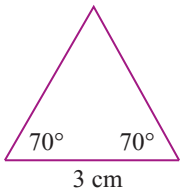
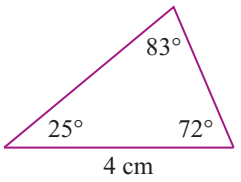


e



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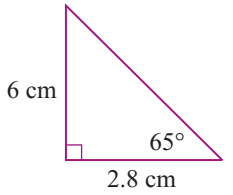
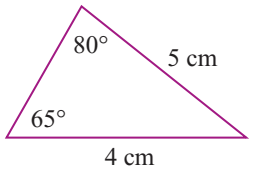


11 Is it possible to construct the triangle shown? Explain why or why not.



12 **a** Construct two different triangles with sides 5 cm, 4 cm and an angle of 50°.

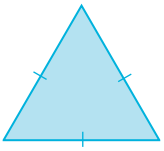
b Write a set of instructions for constructing each triangle.

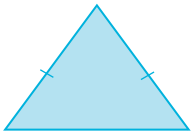
C Classifying triangles

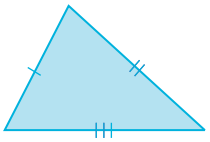
Triangles are classified according to their sides, or their angles, or their sides and angles.

Sides

An **equilateral** triangle has all sides equal in length.
An **isosceles** triangle has two sides equal in length.
A **scalene** triangle has no sides equal in length.

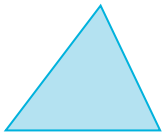

Equilateral

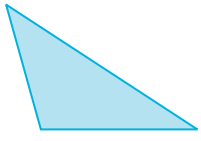

Isosceles

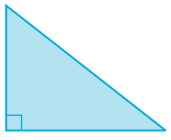

Scalene

Angles

An **acute-angled** triangle has all three angles acute.
An **obtuse-angled** triangle has one obtuse angle.
A **right-angled** triangle has one right angle.


Acute-angled

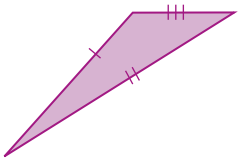

Obtuse-angled

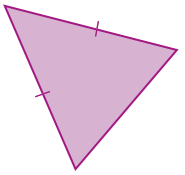

Right-angled

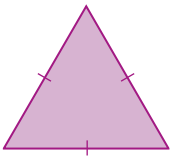
When describing triangles, give the angle name first, followed by the side name. For example, we write obtuse-angled isosceles triangle, *not* isosceles obtuse-angled triangle.

Exercise 15C

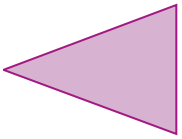
1 Classify these triangles according to their sides.

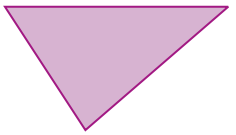
a 

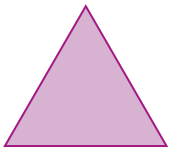
b 

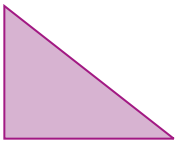
c 

2 Measure the lengths of the sides and classify the following triangles.

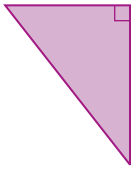
a 

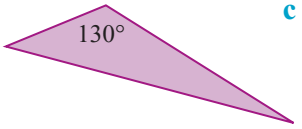
b 

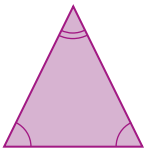
c 

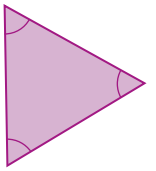
d 

3 Classify these triangles according to their angles.

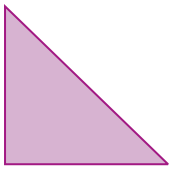
a 

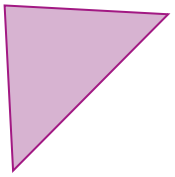
b 

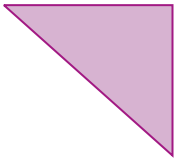
c 

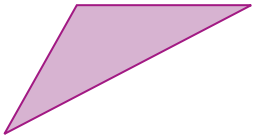
d 

4 Measure the angles and classify the following triangles.

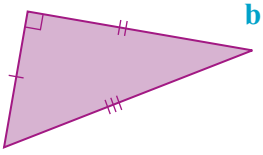
a 

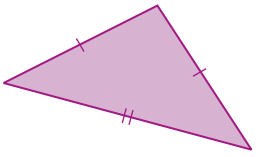
b 

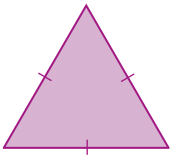
c 

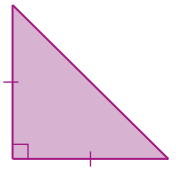
d 

5 Classify each triangle by both its angles and its sides.

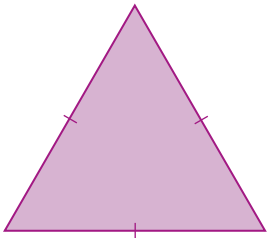
a 

b 

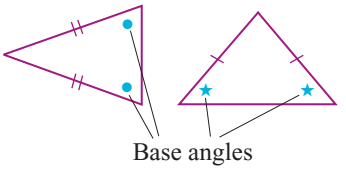
c 

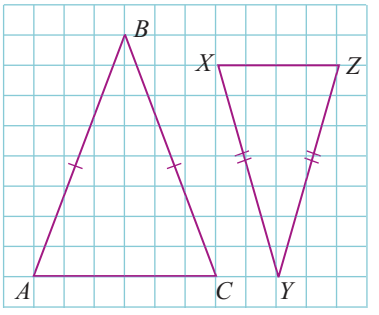
d 

6 Measure the interior angles of this equilateral triangle.
Complete the following.
The interior angles of an equilateral triangle are _____ _____ .



7 In an isosceles triangle, the angles opposite the two equal sides are called **base angles**.
Measure the base angles in triangles *ABC* and *XYZ*, then complete the following:
The base angles of an isosceles triangle are _____ .



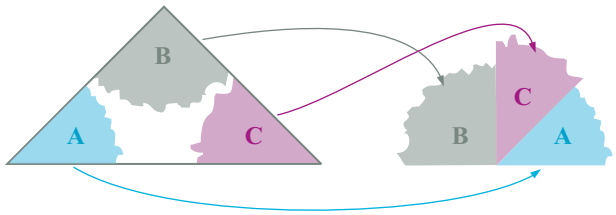


Investigation 1 Sum of the angles of a triangle

Equipment needed: ruler, paper, scissors.

1 On a piece of paper draw any triangle and label the angles A, B and C on the inside of the triangle. Cut out the triangle.

2 Tear off each of the three angles and place them adjacent to each other in any order. What do you notice?



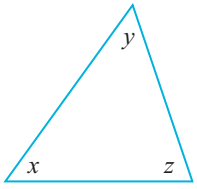
3 Repeat with triangles of different dimensions. Do not forget obtuse-angled triangles.

4 What proposal can you make about the sum of the angles of a triangle?

D Angle sum of a triangle

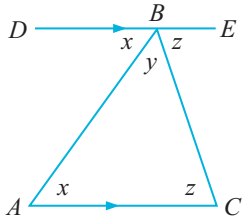
The **angle sum** of a triangle is 180°; that is, in any triangle, no matter what shape, the sum of the angles is 180°.

In the triangle shown, $x + y + z = 180°$.



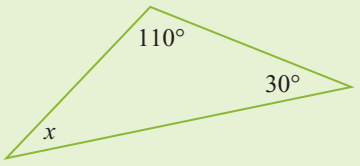
You would have discovered this in Investigation 1.

Proof: Consider any $\triangle ABC$ with interior angles x, y and z.
Draw DE parallel to AC through B.
Now $\angle ABD = x$ (equal alternate angles, $DE \parallel AC$)
Also $\angle CBE = z$ (equal alternate angles, $DE \parallel AC$)
$x + y + z = 180°$ (adjacent angles on a straight line)
$\therefore$ The angle sum of a triangle is 180°.



EXAMPLE 1

Calculate the value of the pronumeral. Show working and provide a reason for your answer.



$x + 110° + 30° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 140° = 180°$
$x = 180° - 140° = 40°$
Alternatively: $x = 180° - 110° - 30°$ (Angle sum of a triangle is 180°.)
$\therefore x = 180° - 140° = 40°$

Exercise 15D

1 Complete the following to calculate the value of the pronumeral.

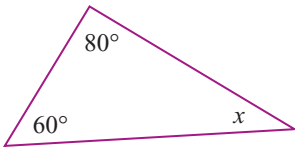
$x +$ ___ $+$ ___ $= 180°$ (Angle sum of a triangle is ___.)

$\therefore x +$ ___ $= 180°$

$x = 180° -$ ___ $=$ ___

Alternatively: $x = 180° -$ ___ $-$ ___ (___ ___ of a triangle is ___.)

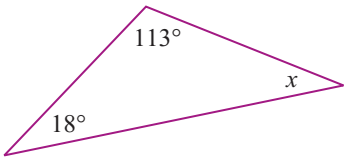
$\therefore x = 180° -$ ___ $=$ ___



Use the fact that the angles of a triangle always add up to 180°.

2 Calculate the value of the pronumeral in each of the following. Show all working.

a



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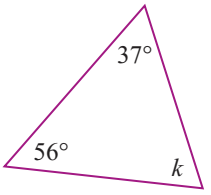

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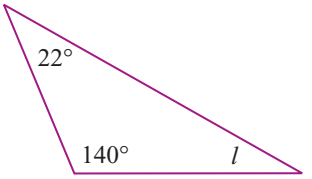

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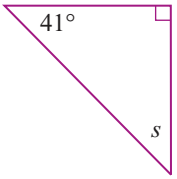

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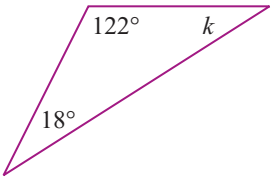

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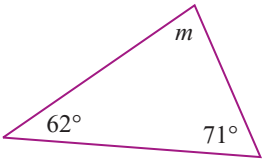
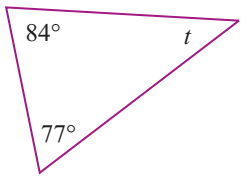


g



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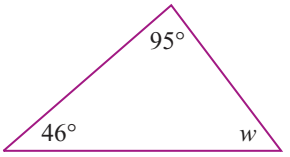

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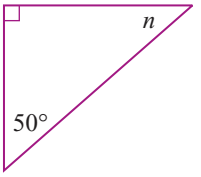

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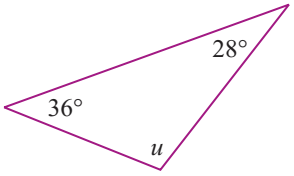

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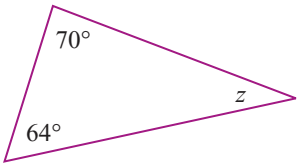

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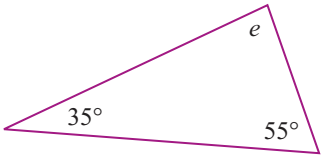

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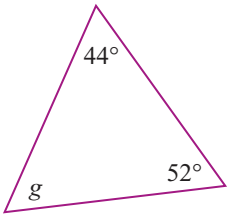

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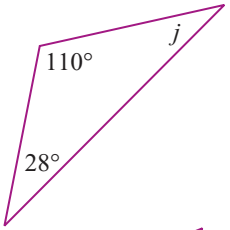
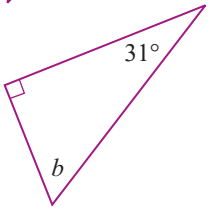
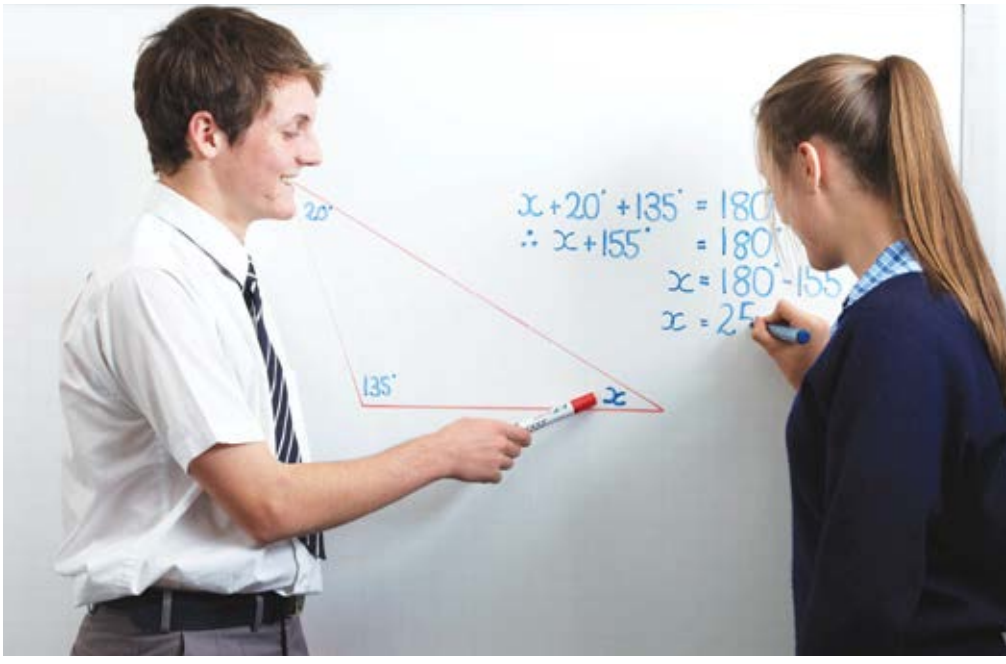


o

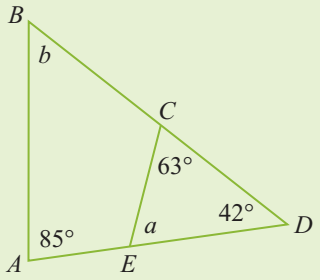




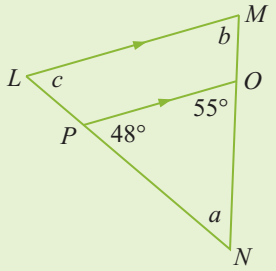
EXAMPLE 2

Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



b



a In $\triangle CDE$, $a + 63° + 42° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore a + 105° = 180°$

$a = 180° - 105° = 75°$

In $\triangle BDA$, $b + 85° + 42° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore b + 127° = 180°$

$b = 180° - 127° = 53°$

b In $\triangle PON$, $a + 48° + 55° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore a + 103° = 180°$

$a = 180° - 103° = 77°$

Also $b = 55°$ (corresponding angles, $PO \parallel LM$)

and $c = 48°$ (corresponding angles, $PO \parallel LM$)

3 Complete the following to find the values of the pronumerals.

a In $\triangle BCD$,

$a + \_\_\_ + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore a + \_\_\_ = 180°$

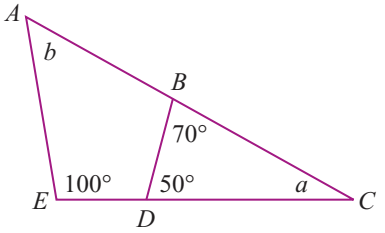
$a = 180° - \_\_\_ = \_\_\_$

In $\triangle ACE$,

$b + 100° + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore b + \_\_\_ = 180°$

$b = 180° - \_\_\_ = \_\_\_$



b In $\triangle QRS$,

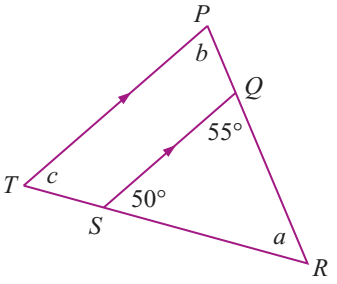
$a + \_\_\_ + \_\_\_ = 180°$ (Angle sum of a triangle is ___.)

$\therefore a + \_\_\_ = 180°$

$a = 180° - \_\_\_ = \_\_\_$

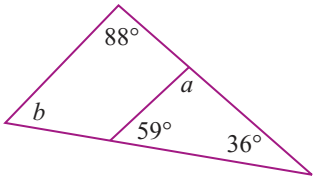
Also, $b = \_\_\_$ (corresponding angles, $TP \parallel SQ$)

and $c = \_\_\_$ (_____ angles, ___ $\parallel$ ___)



4 Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



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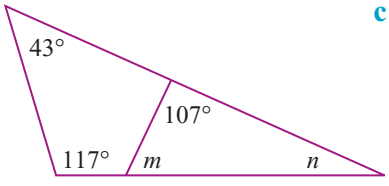
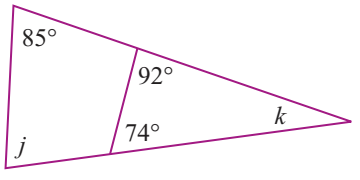
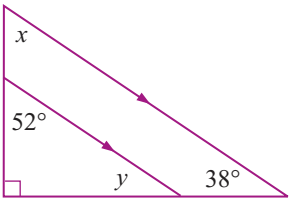


c



MEASUREMENT & GEOMETRY

d



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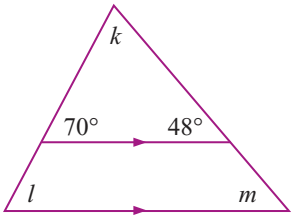

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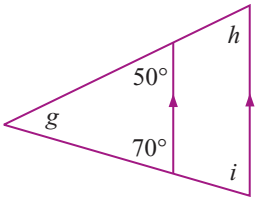
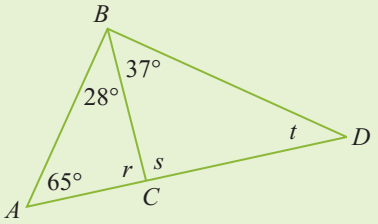


EXAMPLE 3

Find the values of the pronumerals in each of the following, giving reasons for your answers.

a



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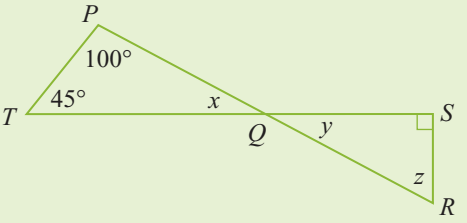


a In $\triangle ABC$, $28° + 65° + r = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 93° + r = 180°$

$r = 180° - 93° = 87°$

At C, $s = 180° - 87° = 93°$ (adjacent angles on a straight line)

In $\triangle BDC$, $37° + 93° + t = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 130° + t = 180°$

$t = 180° - 130° = 50°$

b In $\triangle PQT$, $x + 45° + 100° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore x + 145° = 180°$

$x = 180° - 145° = 35°$

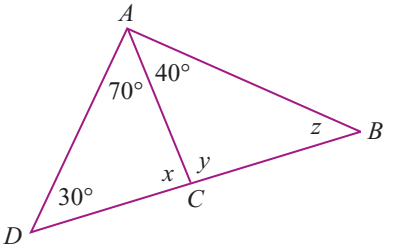
At Q, $y = 35°$ (Vertically opposite angles are equal.)

In $\triangle QSR$, $z + 90° + 35° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore z + 125° = 180°$

$z = 180° - 125° = 55°$

5 Complete the following to find the values of the pronumerals.



a In $\triangle DAC$,

$x + 70° + 30° =$ ___ (Angle sum of a triangle is ___.)

$\therefore x + 100° =$ ___

$x =$ ___ $- 100° =$ ___

At C, $y = 180° -$ ___ (adjacent angles on a straight line)

$=$ ___

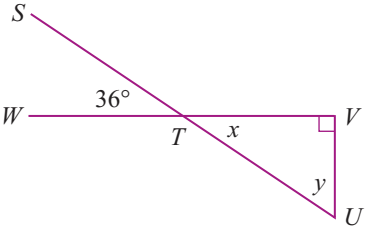
In $\triangle ABC$,

$z + 40° +$ ___ $= 180°$ (Angle sum of a triangle is ___.)

$\therefore z +$ ___ $= 180°$

$z = 180° -$ ___ $=$ ___

b At T, $x =$ ___ (____ ____ angles are equal.)



In $\triangle TVU$,

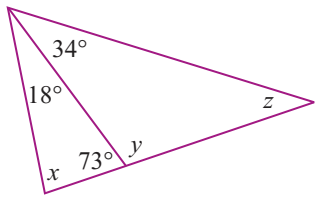
$y + 90° +$ ___ $= 180°$ (Angle sum of a triangle is ___.)

$\therefore y +$ ___ $= 180°$

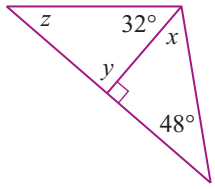
$y = 180° -$ ___ $=$ ___

6 Find the values of the pronumerals in these triangles, giving reasons for your answers.

a



b



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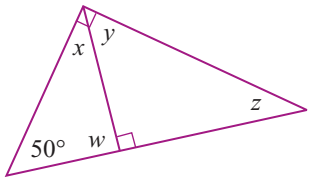

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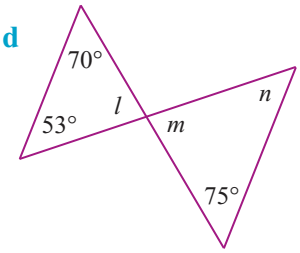

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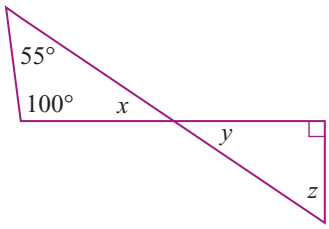

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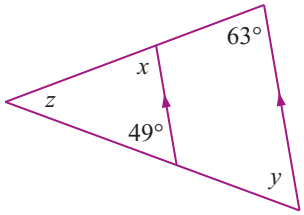

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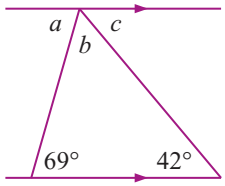

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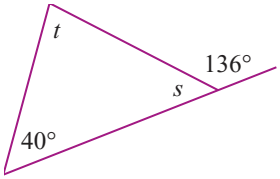

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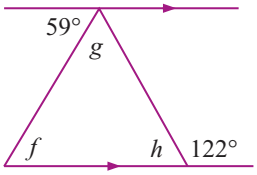
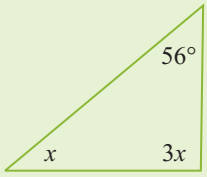


You should always give reasons to justify your working.

EXAMPLE 4

Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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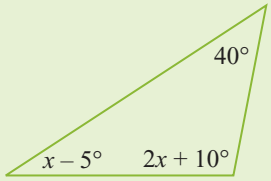


a $x + 3x + 56° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 4x + 56° = 180°$

$4x = 180° - 56° = 124°$

$x = \frac{124°}{4} = 31°$

$\therefore$ Unknown angles are: $x = 31°$

and $3x = 3 \times 31° = 93°$

b $(x - 5°) + (2x + 10°) + 40° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 3x + 45° = 180°$

$3x = 180° - 45° = 135°$

$x = \frac{135°}{3} = 45°$

$\therefore$ Unknown angles are: $x - 5° = 45° - 5° = 40°$

and $2x + 10° = 2 \times 45° + 10° = 90° + 10° = 100°$

7 Complete the following to find the value of each pronumeral. Hence find the unknown angles.

a $2x + 3x + 70° =$ ___ (Angle sum of a triangle is ___.)

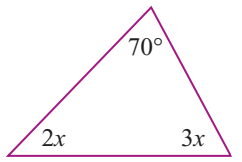
$\therefore$ ___ $x + 70° =$ ___

___ $x =$ ___ $- 70° =$ ___

$x = \frac{\square}{5} =$ ___

$\therefore$ Unknown angles are: $2x = 2 \times$ ___ $=$ ___

and $3x = 3 \times$ ___ $=$ ___



b $(2x - 5°) + (x + 10°) +$ ___ $= 180°$ (Angle sum of a triangle is 180°.)

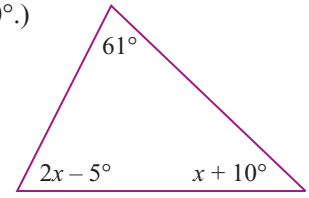
$\therefore 3x +$ ___ $= 180°$

$3x =$ ___ $-$ ___ $=$ ___

$x = \frac{\square}{3} =$ ___

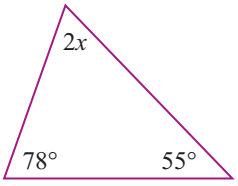
$\therefore$ Unknown angles are: $2x - 5° = 2 \times$ ___ $- 5° =$ ___

and $x + 10° =$ ___ $+$ ___ $=$ ___



8 Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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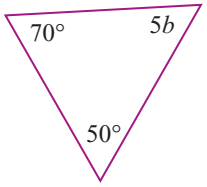

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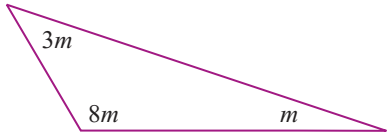

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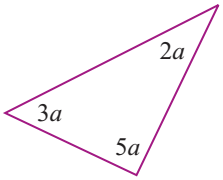

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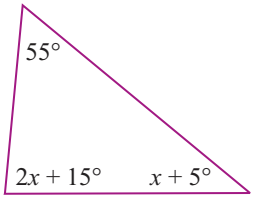

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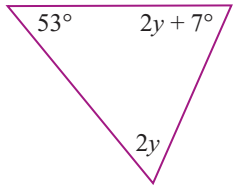
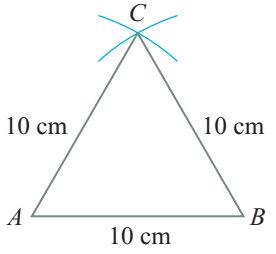


Investigation 2 Equilateral triangles

Equipment needed: compasses, ruler, paper, scissors.

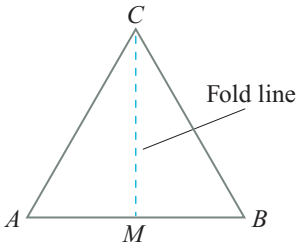
1 Construct an equilateral triangle with sides of length 10 cm, using your compasses and ruler (see section B of this chapter).

2 Accurately cut out the triangle and name the vertices A, B and C.



3 Fold the triangle in half so that vertex A matches exactly with vertex B. Unfold the triangle.

- **a** Comment on the lengths of AM and BM.
- **b** Explain why $\angle CMA = \angle CMB$. What is their degree measure? Why?
- **c** What can be said about $\angle CAM$ and $\angle CBM$?



4 Repeat the procedure in question **3**, folding so that vertices B and C match up.

5 Repeat the procedure in question **3**, folding so that vertices A and C match up.

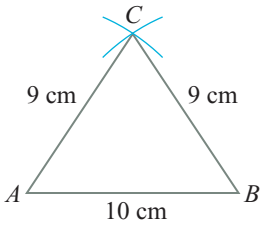
6 What conclusions can you make about:

- **a** the three angles at A, B and C?
- **b** each fold line?

Investigation 3 Isosceles triangles

Equipment needed: compasses, ruler, paper, scissors.

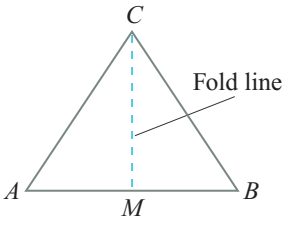
1 Construct an isosceles triangle ABC with AB of length 10 cm and $AC = BC = 9$ cm using compasses and ruler.



2 Cut out the triangle.

3 Fold the triangle in half so that vertices A and B coincide. Unfold the triangle. What can be said about:

- **a** the lengths of AM and BM?
- **b** the measures of angles CAM and CBM?
- **c** the measures of angles ACM and BCM?
- **d** the measures of angles AMC and BMC?



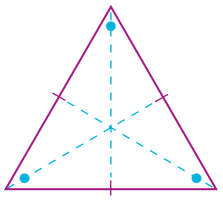
4 Repeat question **3**, folding so that vertices A and C coincide. What do you notice?

E Properties of equilateral and isosceles triangles

From Investigations 2 and 3, you have discovered the following properties of equilateral and isosceles triangles.

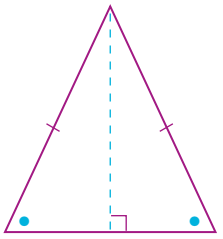
Properties of an equilateral triangle

- All sides are equal.
- All angles are 60°.
- It has three axes of symmetry.



Properties of an isosceles triangle

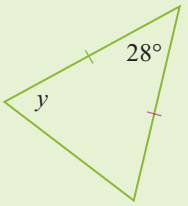
- Two sides are equal.
- The angles opposite the equal sides (the base angles) are equal.
- It has one axis of symmetry.



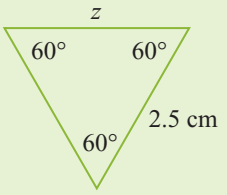
EXAMPLE 1

Find the value of the pronumeral in each triangle, giving a reason.

a



b



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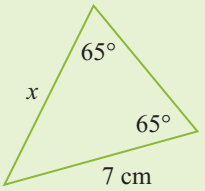

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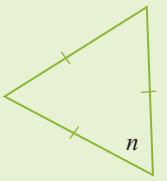


a The third angle of the triangle is also y. (Base angles of an isosceles triangle are equal.)

$\therefore 28° + y + y = 180°$ (Angle sum of a triangle is 180°.)

$2y = 180° - 28°$

$= 152°$

$y = \frac{152°}{2} = 76°$

b $z = 2.5$ cm (Sides of an equilateral triangle are equal.)

c $x = 7$ cm (Sides opposite base angles of an isosceles triangle are equal.)

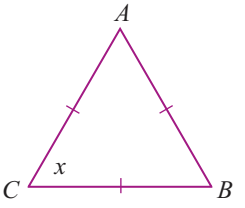
d $n = 60°$ (Angles of an equilateral triangle are 60°.)

Exercise 15E

1 Complete the following to find the value of the pronumeral in each triangle.

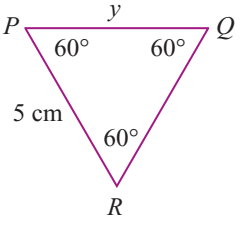
a $\triangle ABC$ is an ____ triangle. (____ sides are equal.)

$\therefore x =$ ____ (Angles of an ____ triangle are ____.)



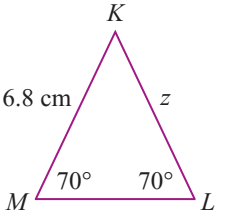
b $\triangle PQR$ is an ____ triangle. (____ angles are equal.)

$\therefore y =$ ____ cm (Sides of an ____ triangle are ____.)



c $\triangle KLM$ is an ____ triangle. (____ angles are equal.)

$\therefore z =$ ____ (Sides opposite ____ angles of an ____ triangle are ____.)



d $\triangle XYZ$ is an ____ triangle. (____ sides are equal.)

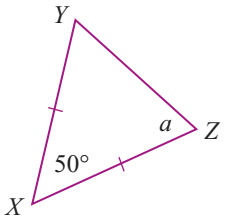
$\therefore \angle Y = a$ (____ angles of an ____ triangle are ____.)

$a + a + 50° =$ ____ (Angle sum of a triangle is ____.)

$2a + 50° =$ ____

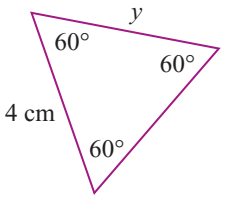
$2a =$ ____ $- 50° =$ ____

$a = \frac{\square}{2} =$ ____



2 Find the values of the pronumerals, giving reasons for your answers.

a



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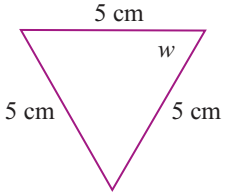

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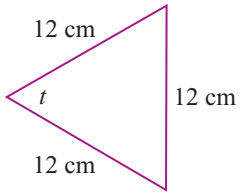

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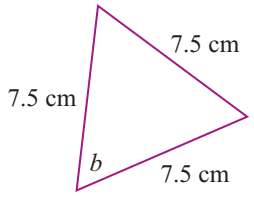

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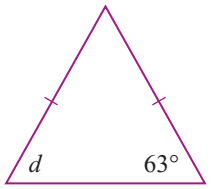

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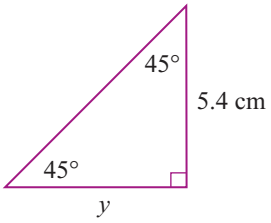

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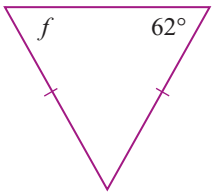

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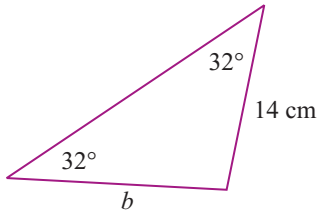

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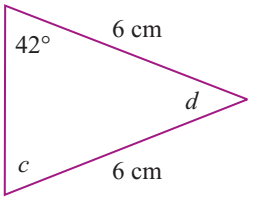

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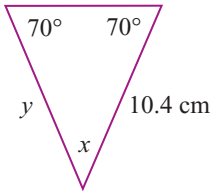

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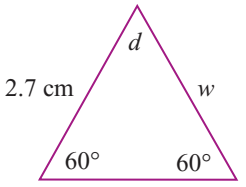

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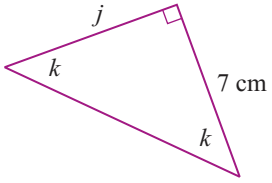

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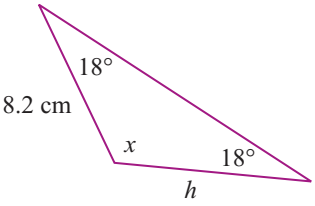

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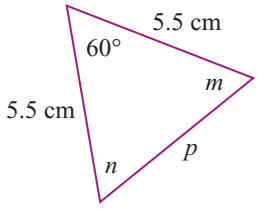
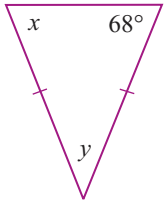


3 Find the values of x and y in each of the following. Provide reasons for your answers.

a



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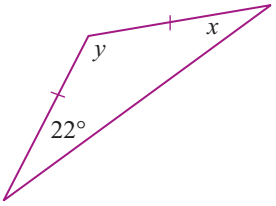

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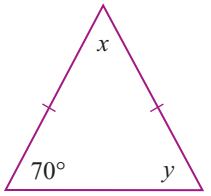

MEASUREMENT & GEOMETRY

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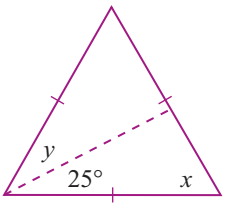

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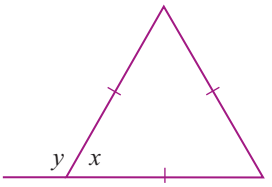

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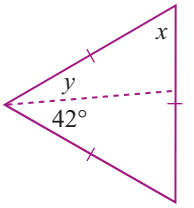

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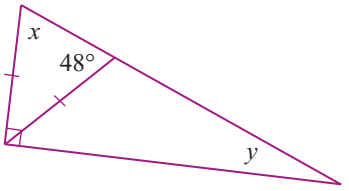

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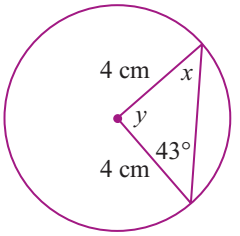

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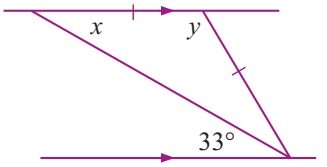
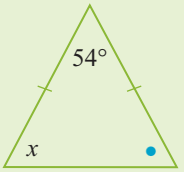


EXAMPLE 2

Write an equation and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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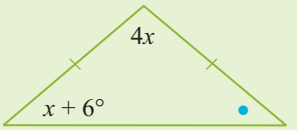


a Isosceles triangles have base angles equal.

$\therefore$ Angle marked • $= x$

$x + x + 54° = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 2x + 54° = 180°$

$2x = 180° - 54° = 126°$

$x = \frac{126°}{2} = 63°$

$\therefore$ Unknown angles are both 63° (base angles).

b Isosceles triangles have base angles equal.

$\therefore$ Angle marked • $= x + 6°$

$4x + (x + 6°) + (x + 6°) = 180°$ (Angle sum of a triangle is 180°.)

$\therefore 6x + 12° = 180°$

$6x = 180° - 12° = 168°$

$x = \frac{168°}{6} = 28°$

$\therefore$ Unknown angles are $4x = 4 \times 28° = 112°$

and $x + 6° = 28° + 6° = 34°$ (base angles)

4 Complete the following to find the values of the pronumerals. Hence find the unknown angles.

a Isosceles triangles have base angles equal.

$\therefore$ Angle marked • $=$ ____

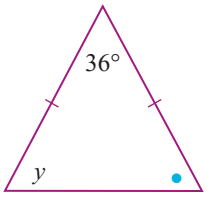
$y + y + 36° =$ ____ (____ ____ of a triangle is ____.)

$\therefore$ __$y + 36° =$ ____

__$y =$ ____ $- 36° =$ ____

$y = \frac{\square}{\square} =$ ____

$\therefore$ Unknown angles are both ____ (base angles).



b Isosceles triangles have ____ angles ____.

$\therefore$ Angle marked • = ____

$2x + (x + 16°) +$ ____ $= 180°$ (____ ____ of a triangle is ____.)

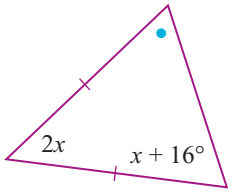
$\therefore$ ___$x + 32° = 180°$

___$x = 180° - 32° = 148°$

$x = \frac{148°}{\square} =$ ____

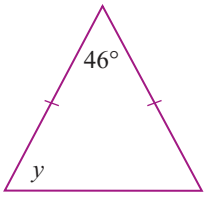
$\therefore$ Unknown angles are $2x = 2 \times$ ____ = ____

and $x + 16°$ = ____ + 16° = ____ (base angles)



5 Write an equation for each and solve it to find the value of the pronumeral. Hence find the unknown angles.

a



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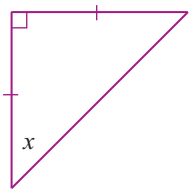

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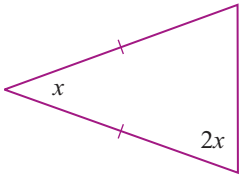

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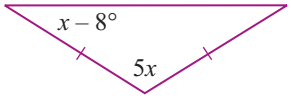

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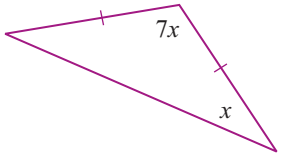

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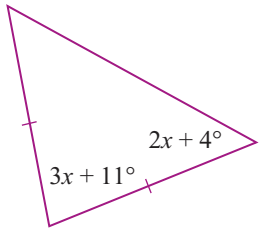
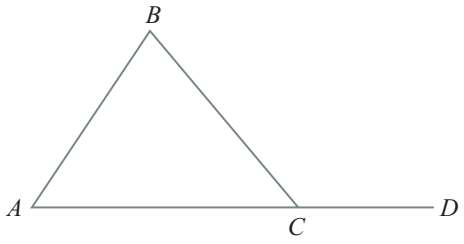


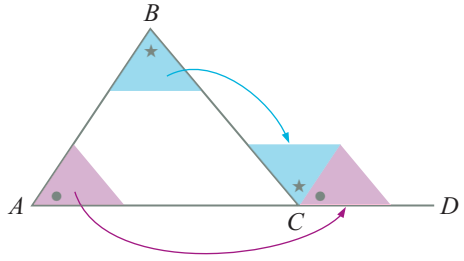
Investigation 4 Exterior angle of a triangle

An **exterior angle** of a triangle is the angle formed outside the triangle when a side is extended.
Equipment needed: ruler, paper, scissors.

1 Draw any triangle ABC and extend the side AC to D. $\angle BCD$ is an exterior angle of the triangle.



2 Cut off the angles A and B and place them next to each other on the exterior angle BCD, as shown. Is $\angle BCD = \angle A + \angle B$?



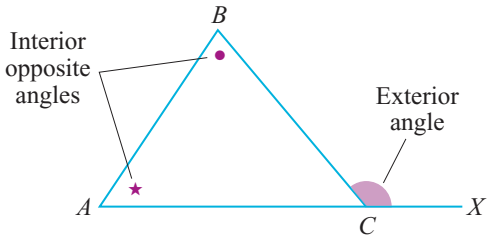
3 Repeat with triangles of different dimensions. Do not forget to try obtuse-angled triangles.

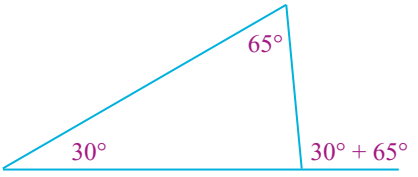
4 What proposal can you make about the exterior angle of a triangle?

F Exterior angle of a triangle

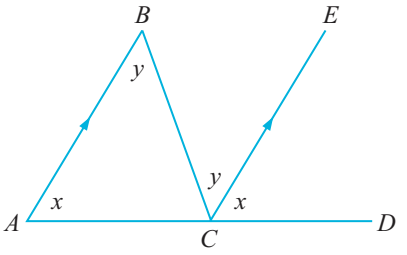
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

You would have discovered this fact in Investigation 4.





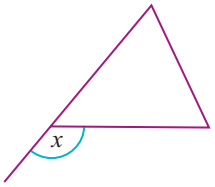
Proof: Consider any $\triangle ABC$ with $\angle BAC = x$ and $\angle ABC = y$.
Extend AC to D and draw CE parallel to AB.
Now $\angle BAC = \angle ECD = x$ (equal corresponding angles, $AB \parallel CE$)
Also $\angle ABC = \angle BCE = y$ (equal alternate angles, $AB \parallel CE$)
$\therefore \angle BCD = x + y = \angle BAC + \angle ABC$
$\therefore$ The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



Exercise 15F

1 Copy these diagrams and shade the two interior angles opposite the exterior angle labelled x.

a



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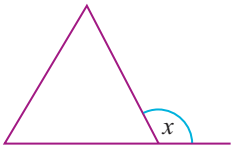

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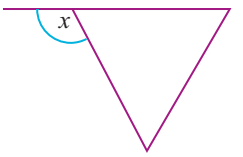

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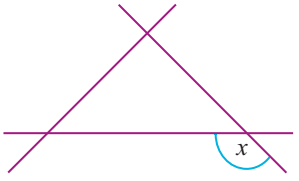

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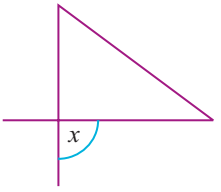

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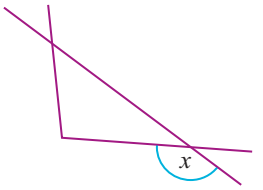
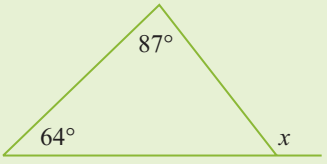


EXAMPLE 1

Find the value of the pronumeral in each triangle, giving a reason.

a



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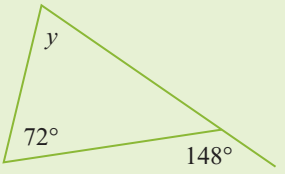
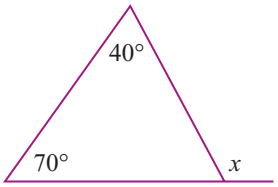


a $x = 87° + 64°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$\therefore x = 151°$

b $148° = y + 72°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
or $y + 72° = 148°$
$\therefore y = 148° - 72° = 76°$

2 Complete the following to find the value of the pronumeral in each triangle.

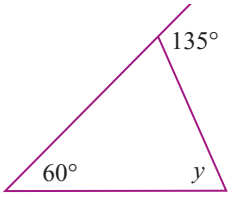
a



$x = \_\_\_ + \_\_\_$ (Exterior angle of a triangle equals sum of two ____ ____ angles.)

$\therefore x = \_\_\_$

b



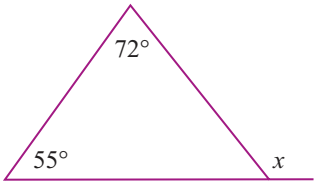
$135° = y + \_\_\_$ (____ angle of a triangle equals sum of two ____ ____ angles.)

or $y + \_\_\_ = 135°$

$\therefore y = 135° - \_\_\_ = \_\_\_$

3 Find the value of the pronumeral in each triangle.

a



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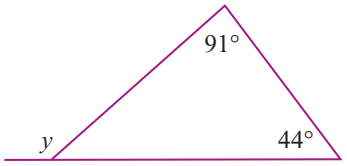

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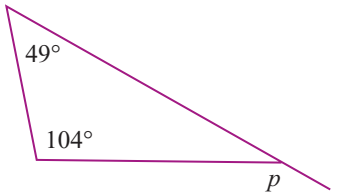

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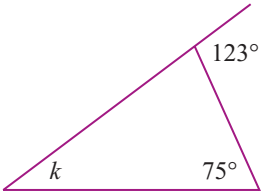

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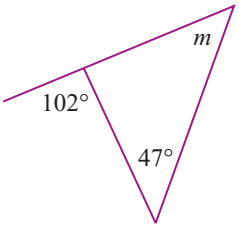

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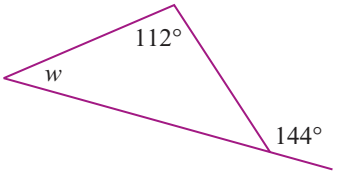
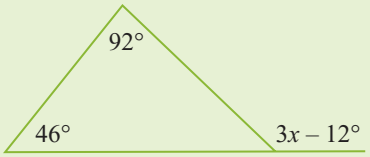


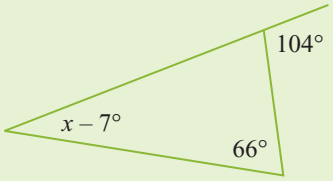
EXAMPLE 2

Write an equation for each triangle and solve it to find the value of the pronumeral.

a



b



a $3x - 12° = 92° + 46° = 138°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$\therefore 3x = 138° + 12° = 150°$

$x = \frac{150°}{3}$

$= 50°$

b $104° = (x - 7°) + 66°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

$104° = x + 59°$

or $x + 59° = 104°$

$x = 104° - 59°$

$= 45°$

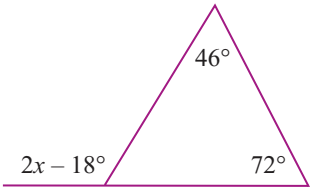
4 Complete the following to find the value of the pronumeral and the unknown angles with pronumerals.

a $2x - 18° = \_\_\_ + \_\_\_ = \_\_\_$ (Exterior angle of a triangle equals sum of ____ ____ ____ angles.)

$\therefore 2x = \_\_\_ + 18° = \_\_\_$

$x = \frac{\square}{2} = \_\_\_$

$\therefore$ Unknown angle $= 2x - 18° = \_\_\_ - \_\_\_ = \_\_\_$



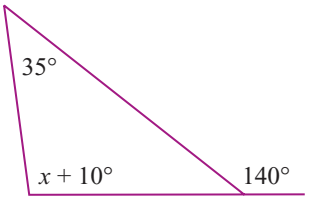
b $140° = (x + 10°) + \_\_\_$ (____ angle of a triangle equals ___ of ____ ____ ____ angles.)

$140° = x + \_\_\_$

or $x + \_\_\_ = 140°$

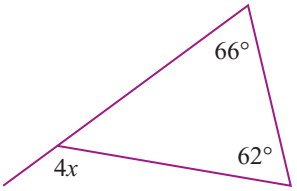
$x = 140° - \_\_\_ = \_\_\_$

$\therefore$ Unknown angle $= x + 10° = \_\_\_ + \_\_\_ = \_\_\_$



5 Write an equation for each triangle and solve it to find the value of the pronumeral and the unknown angles.

a



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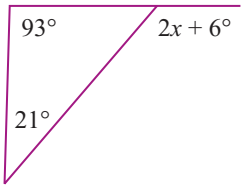

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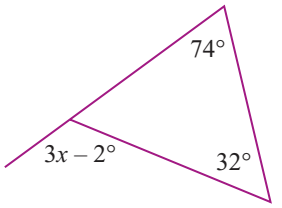

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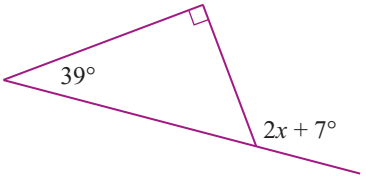

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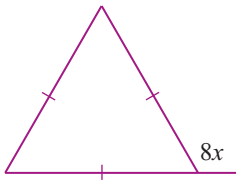

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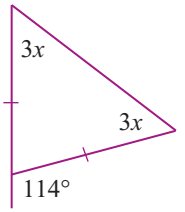
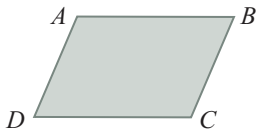


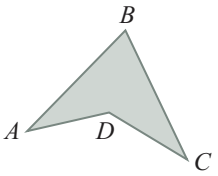
Investigation 5 Convex and concave quadrilaterals

1 A **quadrilateral** is a four-sided plane shape with straight sides. Sketch each of these quadrilaterals.

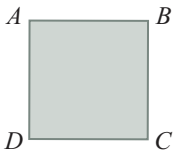
a



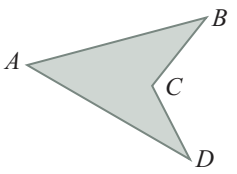
b



c



d



2 In each quadrilateral, draw both diagonals *AC* and *BD*.

3 If all diagonals are inside the quadrilateral, it is **convex**. Which quadrilaterals are convex?

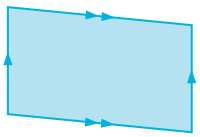
4 If one of the diagonals lies outside the quadrilateral, the quadrilateral is **concave**. Which quadrilaterals are concave?

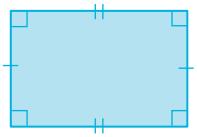
5 Examine the angles in each quadrilateral in question **1**. Make a comment about the angles in a:

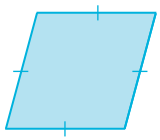
a convex quadrilateral

b concave quadrilateral.

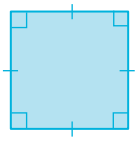
G Properties of special quadrilaterals


Parallelogram

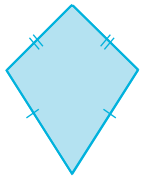

Rectangle


Rhombus

- A **parallelogram** is a quadrilateral that has opposite sides parallel.
- A **rectangle** is a quadrilateral with four equal angles of 90°.
- A **rhombus** is a quadrilateral in which all sides are equal in length (an **equilateral quadrilateral**).


Square


Trapezium

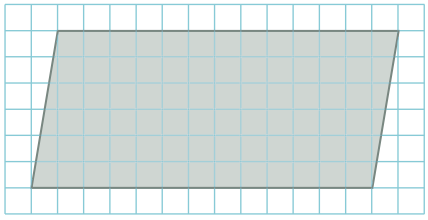

Kite

- A **square** is a rhombus with four equal angles of 90°, or a rectangle with all sides equal in length.
- A **trapezium** is a convex quadrilateral that has a pair of parallel opposite sides.
- A **kite** is a convex quadrilateral with two pairs of adjacent sides equal.

Investigation 6 Properties of quadrilaterals

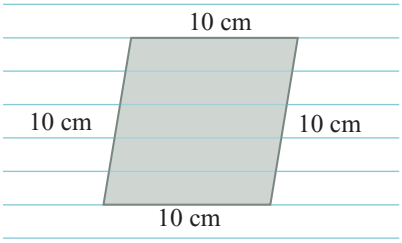
Equipment needed: compasses, ruler, square grid paper, lined paper.

1 Draw a parallelogram using a ruler and square grid paper as shown. Use it to check the following **properties of a parallelogram**:
- Opposite sides are parallel.
- Opposite sides are equal in length.
- Opposite angles are equal.
- Diagonals **bisect** each other (i.e. cut each other in half).



2 Draw a rectangle on square grid paper and use it to check the following **properties of a rectangle**:
- Opposite sides are parallel.
- Opposite sides are equal in length.
- All angles are right angles.
- Diagonals are equal in length.
- Diagonals bisect each other.

3 Construct a rhombus using a ruler, compasses and lined paper as shown. Why is the figure a rhombus? Use it to check the following **properties of a rhombus**:
- Opposite sides are parallel.
- All sides are equal in length.
- Opposite angles are equal.
- Diagonals bisect each other at right angles.
- Diagonals bisect the angles at each vertex.



4 A square is a special case of a rhombus and a rectangle and therefore has all of their properties. On squared grid paper, draw a square and use it to check the following **properties of a square**:

- Opposite sides are parallel.
- All sides are equal in length.
- All angles are right angles.
- Diagonals are equal in length.
- Diagonals bisect each other at right angles.
- Diagonals bisect the angles at each vertex.

5 Draw a trapezium on square grid paper and use it to check the following **properties of a trapezium**:

- Exactly one pair of opposite sides is parallel.

6 Draw a kite on square grid paper and use it to check the following **properties of a kite**:

- Two pairs of adjacent sides are equal.
- One pair of opposite angles are equal.
- Diagonals meet at right angles, one diagonal being bisected.
- One diagonal bisects the angles at its endpoints.

Properties of geometrical figures are important facts that need to be remembered.

Exercise 15G

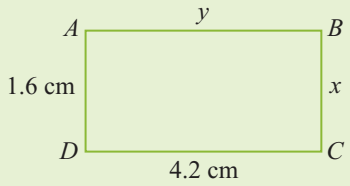
1 Complete the following table by writing yes or no.

Property	Parallelogram	Rectangle	Rhombus	Square	Trapezium	Kite
Opposite sides parallel						
Opposite sides equal						
Opposite angles equal						
All sides equal						
Adjacent sides perpendicular						
Diagonals equal						
Adjacent sides equal						
Diagonals bisect each other						
Diagonals meet at right angles						
Diagonals bisect vertices						
All angles 90°						

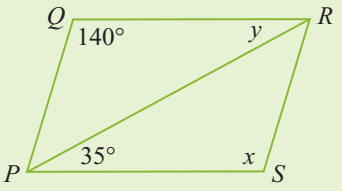
EXAMPLE 1

Use the properties of special quadrilaterals to find the values of the pronumerals, giving reasons.

a *ABCD* is a rectangle.



b *PQRS* is a parallelogram.



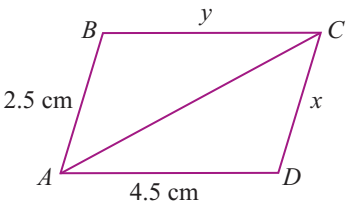
a $x = 1.6$ cm and $y = 4.2$ cm (Opposite sides of a rectangle are equal.)

b $x = 140°$ (Opposite angles of a parallelogram are equal.)

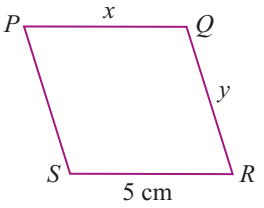
$y = 35°$ (Equal alternate angles, as $QR \parallel PS$.)

2 Use the properties of special quadrilaterals to find the values of the pronumerals, giving reasons.

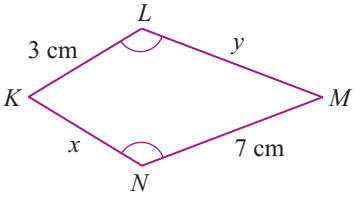
a *ABCD* is a parallelogram.



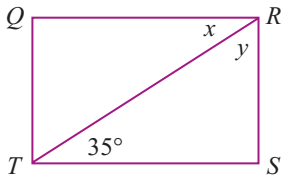
b *PQRS* is a rhombus.



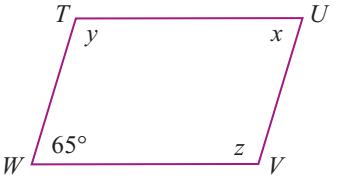
c *KLMN* is a kite.



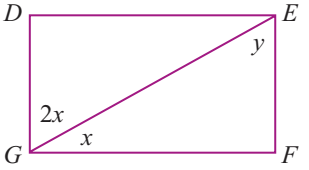
d *QRST* is a rectangle.



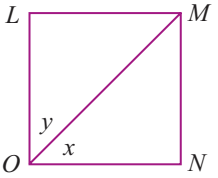
e *TUVW* is a parallelogram.



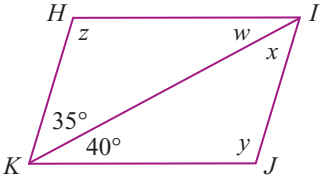
f *DEFG* is a rectangle.



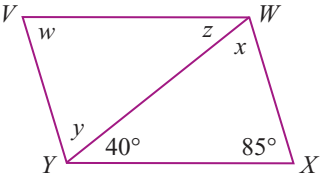
g *LMNO* is a square.



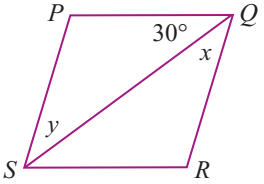
h *HIJK* is a parallelogram.



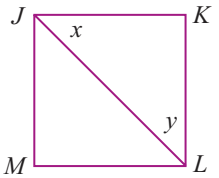
i *VWXY* is a parallelogram.



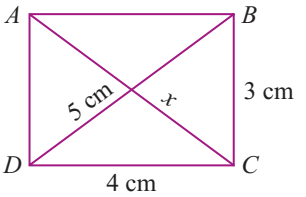
j *PQRS* is a rhombus.



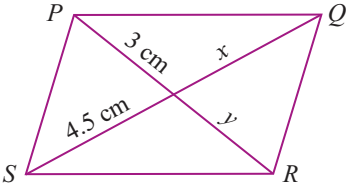
k *JKLM* is a square.



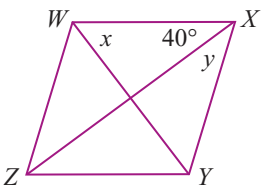
l *ABCD* is a rectangle.



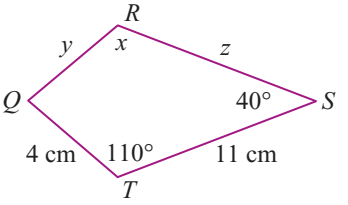
m *PQRS* is a parallelogram.



n *WXYZ* is a rhombus.



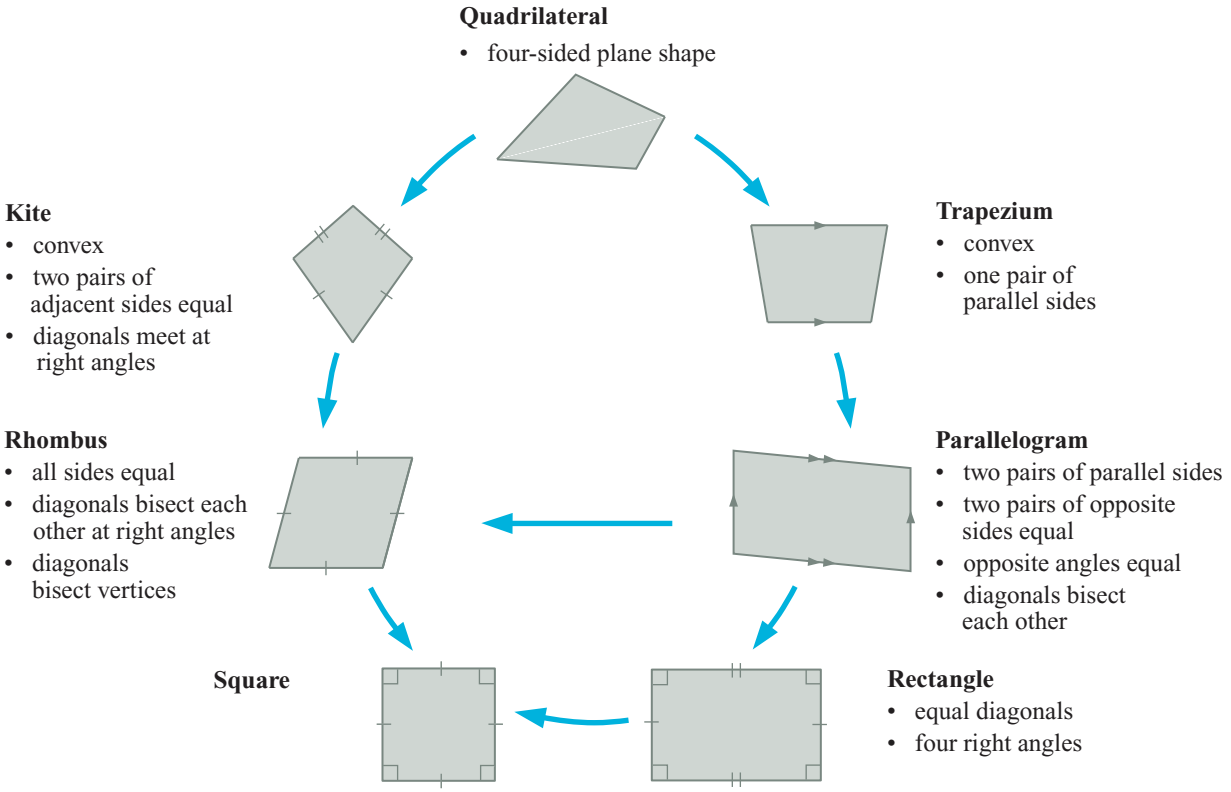
o *QRST* is a kite.



3 Identify and sketch quadrilaterals with the following properties.

- **a** One pair of sides are parallel.
- **b** Two pairs sides are of parallel.
- **c** One pair of sides are parallel and one pair of opposite sides are equal.
- **d** One pair of sides are parallel and one pair of adjacent sides are equal.
- **e** One pair of sides are parallel and one angle is a right angle.
- **f** One pair of sides are parallel and opposite angles are equal.
- **g** All angles are right angles.
- **h** Diagonals meet at right angles.
- **i** Diagonals meet at right angles and one pair of opposite angles are equal.
- **j** Diagonals bisect each other.
- **k** Diagonals bisect each other at right angles.
- **l** Diagonals are equal.
- **m** Diagonals are equal and bisect each other.
- **n** Diagonals are equal and bisect each other at right angles.

Investigation 7 Flowchart for quadrilaterals



The flowchart shows the properties of each quadrilateral. Each quadrilateral has its own properties along with the properties of the shape before it. For example:

- A rhombus has its own properties as well as those of a kite and a quadrilateral.
- A square has all the properties of a rhombus and a rectangle.

Are the following statements true or false?

1 All rectangles are parallelograms.

2 All parallelograms are rectangles.

3 All squares are rectangles.

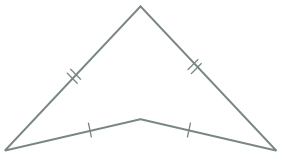
4 All rectangles are squares.

5 Every rhombus is a parallelogram.

6 Every parallelogram is a rhombus.

7 A rhombus is a special case of a kite.

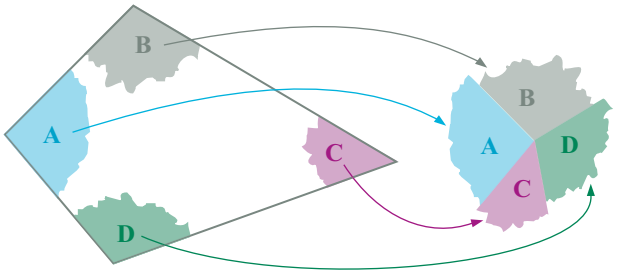
8 The quadrilateral shown on the right is a kite.



Investigation 8 Angle sum of a quadrilateral

Equipment needed: paper, scissors.

1 Draw any quadrilateral on a piece of paper and label the vertices A, B, C and D on the inside of the quadrilateral. Cut out the quadrilateral.



2 Tear off the four angles and place them adjacent to each other, in any order, with all vertices meeting but not overlapping. What do you notice?

3 Repeat this experiment with a few other quadrilaterals, including a concave quadrilateral. What do you notice?

4 Write a statement about the sum of the angles of a quadrilateral.

H Angle sum of a quadrilateral

You would have discovered this in Investigation 8.

The angle sum of a quadrilateral is 360°; that is, for any quadrilateral, no matter what shape, the sum of the angles is 360°.

As you have already learned:

- the angle sum of a triangle equals 180°
- all quadrilaterals can be divided into two triangles.

It would appear reasonable that the angle sum of a quadrilateral is equal to twice the angle sum of a triangle; that is, $2 \times 180° = 360°$.

Proof: Consider any quadrilateral $ABCD$.
Draw the diagonal BD.

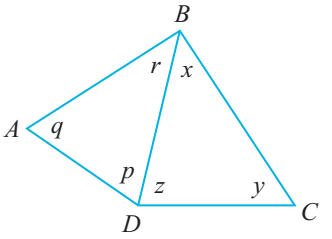
$p + q + r = 180°$ (Angle sum of $\triangle ABD = 180°$.)

$x + y + z = 180°$ (Angle sum of $\triangle BCD = 180°$.)

$\therefore q + (r + x) + y + (z + p) = 360°$

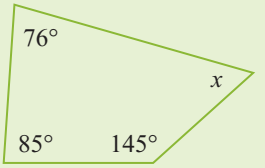
$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\therefore$ The angle sum of a quadrilateral is 360°.



EXAMPLE 1

Find the value of the pronumeral, giving a reason for your answer.



$x + 145° + 85° + 76° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore x + 306° = 360°$

$x = 360° - 306°$

$= 54°$

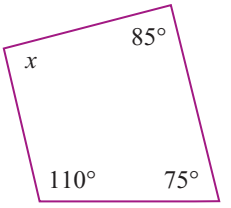
Exercise 15H

1 Complete the following to find the value of the pronumeral.

$x + 85° + 75° + 110° =$ ___ (Angle sum of a quadrilateral is ___.)

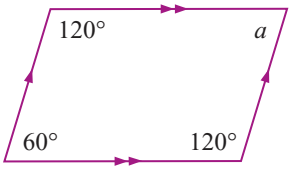
$\therefore x +$ ___ $=$ ___

$x =$ ___ $-$ ___ $=$ ___



2 Find the values of the pronumerals, giving reasons for your answers.

a



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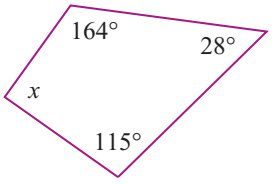
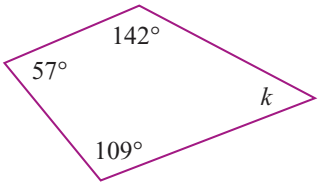
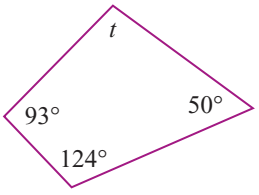


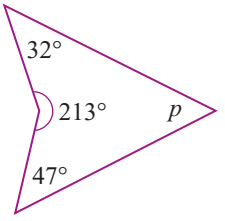
c



d



e



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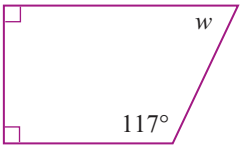
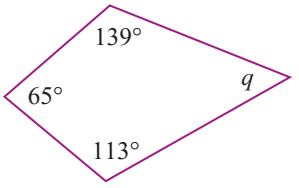
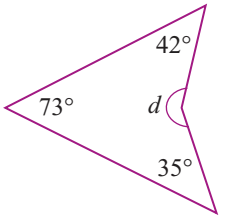


g

h

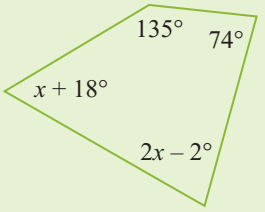


i



EXAMPLE 2

Write an equation and solve it to find the value of the pronumeral.
Hence find the unknown angles.



$(x + 18°) + (2x - 2°) + 74° + 135° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore 3x + 225° = 360°$ Collect like terms and solve the equation.

$3x = 360° - 225°$

$= 135°$

$x = \frac{135°}{3}$

$= 45°$

$\therefore$ Unknown angles are:

$x + 18° = 45° + 18° = 63°$

and $2x - 2° = 2 \times 45° - 2° = 88°$

3 Complete the following to find the value of the pronumeral. Hence find the unknown angles.

$(2x - 10°) + (x + 50°) +$ ___ + ___ = ___ (Angle sum of a quadrilateral is ___.)

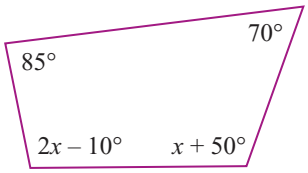
$\therefore 3x +$ ___ = ___

$3x =$ ___ − ___

$=$ ___

$x = \frac{\square}{\square}$

$=$ ___



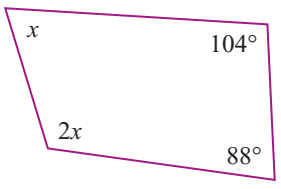
$\therefore$ Unknown angles are:

$2x - 10° = 2 \times$ ___ $- 10° =$ ___

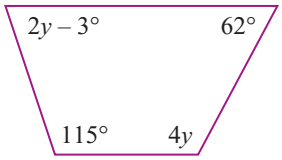
and $x + 50°$ = ___ + 50° = ___

4 Write an equation for each of the following and solve it to find the value of the pronumeral. Hence find the unknown angles.

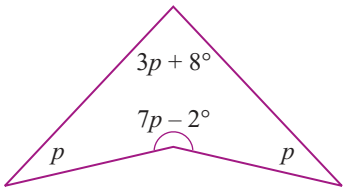
a



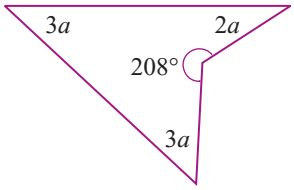
b



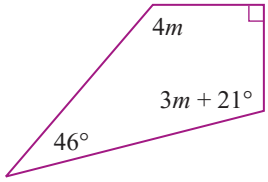
c



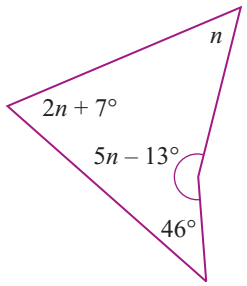
d



e



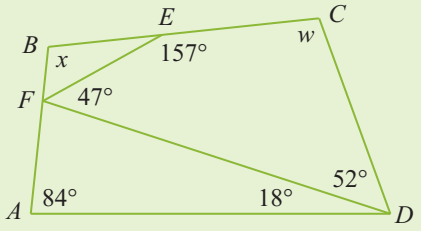
f



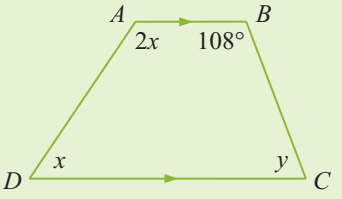
EXAMPLE 3

Find the values of the pronumerals in each quadrilateral, giving reasons for your answers.

a



b



a In quadrilateral $FECD$,

$w + 157° + 47° + 52° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore w + 256° = 360°$

$w = 360° - 256° = 104°$

In quadrilateral $ABCD$,

$84° + x + 104° + 52° + 18° = 360°$ (Angle sum of a quadrilateral is 360°.)

$\therefore x + 258° = 360°$

$x = 360° - 258° = 102°$

b $\angle B + \angle C = 180°$ (Sum of co-interior angles is 180°, $AB \parallel DC$.)

$\therefore 108° + y = 180°$

$y = 180° - 108 = 72°$

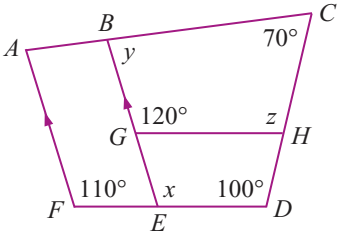
$\angle A + \angle D = 180°$ (Sum of co-interior angles is 180°, $AB \parallel DC$.)

$\therefore 2x + x = 180°$

$3x = 180°$

$x = \frac{180°}{3} = 60°$

5 Complete the following to find the values of the pronumerals.



$x =$ ___ (Corresponding angles as $AF \parallel$ ___.)

In quadrilateral $BCDE$,

$y + 70° + 100° +$ ___ $=$ ___ (Angle sum of a quadrilateral is ___.)

$\therefore y +$ ___ $=$ ___

$y =$ ___ $-$ ___ $=$ ___

In quadrilateral $BCHG$,

$z + 120° +$ ___ $+ 70° =$ ___ (Angle sum of a quadrilateral is ___.)

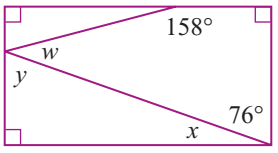
$\therefore z +$ ___ $=$ ___

$z =$ ___ $-$ ___ $=$ ___

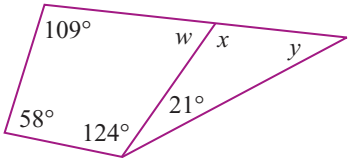
In question 6 find the values of the pronumerals in alphabetical order w, x, y.

6 Find the values of the pronumerals, giving reasons for your answers.

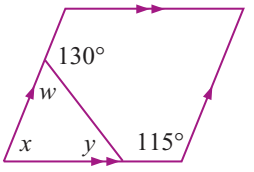
a



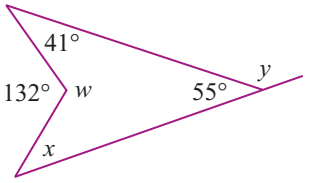
b



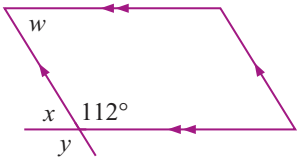
c



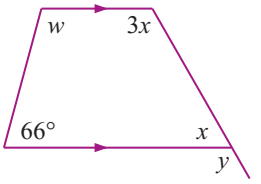
d



e



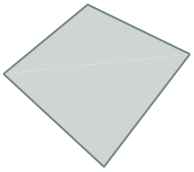
f



Investigation 9 Composite figures

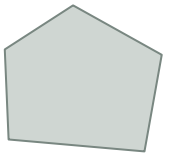
Copy each of the following figures and divide them into the plane shapes specified.

1



a 2 triangles
b 4 triangles

2

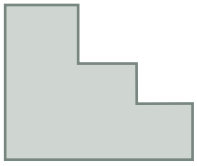


a 3 triangles
b 5 triangles
c 1 quadrilateral and 1 triangle

3

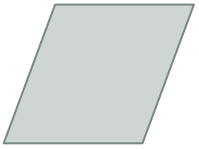
2 rectangles (2 ways)

4



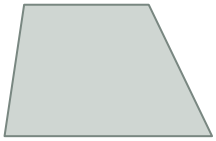
3 rectangles
In how many ways can you do this?

5



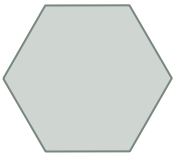
a 2 triangles
b 1 rectangle and 2 triangles
c 1 trapezium and 1 triangle
d 1 rhombus and 1 parallelogram

6



a 2 triangles
b 1 rectangle and 2 triangles
c 1 parallelogram and 1 triangle

7



a 2 quadrilaterals
b 4 triangles
c 6 triangles
d 1 quadrilateral and 2 triangles

8

a 1 square and 1 rectangle
b 1 triangles
c 1 trapezium and 1 triangle
d 1 parallelogram and 2 triangles
e 1 kite and 4 triangles

Language in mathematics

Here is a find-a-word puzzle.

P	T	I	D	F	I	S	E	L	E	C	S	O	S	I	N	S
R	R	R	N	O	I	T	A	U	Q	E	M	D	U	T	Q	E
O	A	E	E	R	I	N	A	T	U	U	N	G	M	U	X	L
N	H	A	T	E	G	S	C	S	I	D	E	S	A	T	L	E
U	C	S	X	L	A	U	N	Z	L	D	A	R	E	Q	A	L
M	W	O	E	U	R	A	E	D	A	K	E	R	D	R	U	G
E	O	N	I	T	L	P	R	A	T	T	I	E	C	R	Q	N
R	L	I	S	M	A	A	A	L	E	O	S	T	F	A	E	A
A	F	N	S	R	W	C	D	P	R	O	V	E	E	I	N	I
L	O	G	T	A	A	D	T	I	A	S	U	B	M	O	H	R
C	N	G	Q	U	A	D	R	I	L	A	T	E	R	A	L	T

1 Find all the words in the following list:

add, angle, arc, arm, construct, draw, equal, equation, equilateral, extend, exterior, flowchart, isosceles, kite, pronumeral, prove, quadrilateral, reasoning, rhombus, sides, square, sum, trapezium, triangle

2 The remaining letters reveal a message. Working from the top, left to right, place the remaining letters in the order in which they appear to reveal the message.

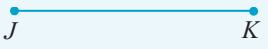
Terms

acute angle	adjacent	angle	angle sum	base angles	baseline
bisect	compasses	concave	construct	convex	diagonal
equilateral	exterior	interior	interval	isosceles	kite
line	obtuse angle	parallelogram	protractor	quadrilateral	rectangle
rhombus	right angle	scalene	square	trapezium	triangle

Check your skills

Note: Due to the practical nature of this topic, not all outcomes can be assessed here.

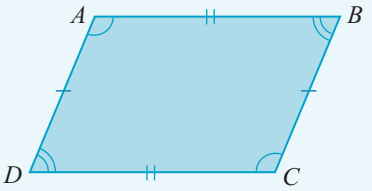
1 The diagram shows:



A line J **B** interval J

C line JK **D** interval JK

2 This diagram shows:



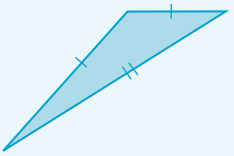
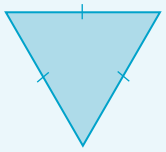
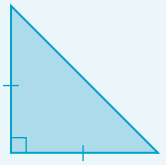
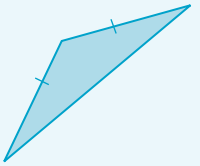
A quadrilateral $ABCD$

B a quadrilateral with four equal sides

C a quadrilateral with no equal angles

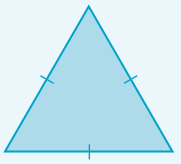
D all of the above

3 Which diagram shows a right-angled isosceles triangle?

A  **B**  **C**  **D** 

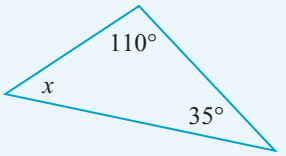
4 The side name for this triangle is:

A acute-angled **B** obtuse-angled
C right-angled **D** equilateral



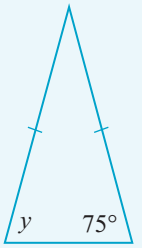
5 The value of x in the diagram is:

A 145° **B** 35°
C 215° **D** 180°



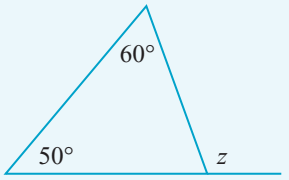
6 The value of y in the diagram is:

A 75° **B** 150°
C 105° **D** cannot find



7 The value of z in the diagram is:

A 50° **B** 60°
C 70° **D** 110°



8 A concave quadrilateral has:

A all sides equal **B** all angles equal
C all diagonals inside the figure **D** none of the above properties

9 The quadrilateral that has all its angles equal is the:

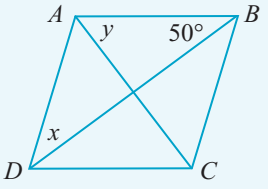
A rhombus **B** rectangle **C** parallelogram **D** kite

10 The quadrilateral that has exactly two sides parallel is the:

A rhombus **B** rectangle **C** parallelogram **D** trapezium

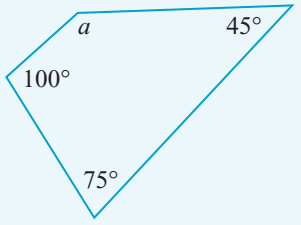
11 The values of x and y in rhombus $ABCD$ are:

A $x = 50°, y = 50°$ **B** $x = 50°, y = 40°$
C $x = 40°, y = 50°$ **D** $x = 40°, y = 40°$



12 The value of a in the diagram is:

A 140° **B** 75°
C 220° **D** 360°



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5	6	7	8–11	12
Section	A	C	D	E	F	G	H

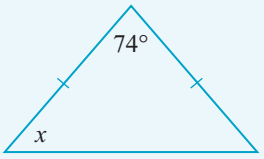
15A Review set

1 Construct a triangle with side lengths 10 cm, 8 cm and 6 cm.

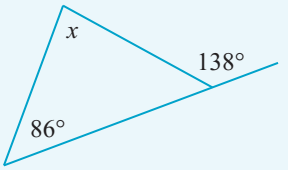
2 Draw an obtuse-angled isosceles triangle.

3 Find the value of x, giving reasons.

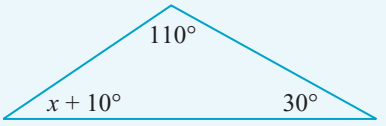
a



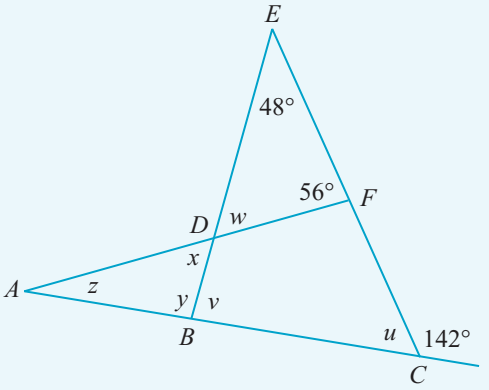
b



4 Write an equation and solve it to determine the value of the unknown angle.



5 Find the values of the pronumerals, in alphabetical order (u to z), giving reasons for your answers.

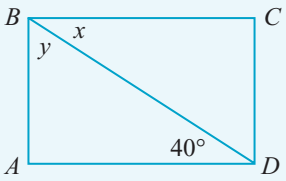


6 **a** Draw a parallelogram.
b List the properties of a parallelogram.

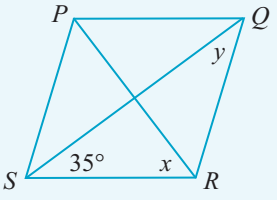
7 **a** Draw a convex quadrilateral.
b Comment on the diagonals of your diagram.

8 Find the values of the pronumerals, giving reasons.

a $ABCD$ is a rectangle.



b $PQRS$ is a rhombus.



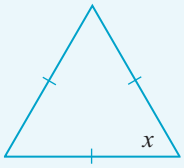
15B Review set

1 Construct a triangle with side lengths 4 cm and 7 cm that form an angle of 65°.

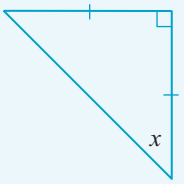
2 Draw an acute-angled scalene triangle.

3 Find the value of x, giving reasons.

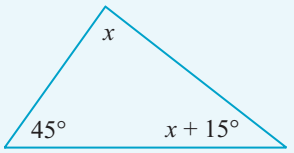
a



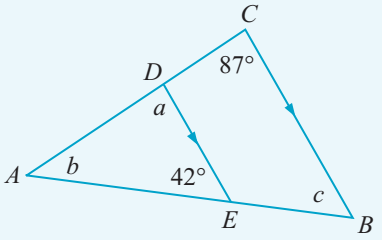
b



4 Write an equation and solve it to determine the values of the unknown angles.



5 Find the value of the pronumerals, giving reasons for your answers.



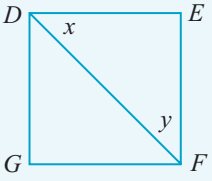
6 **a** Draw a rectangle.

b List the properties of a rectangle.

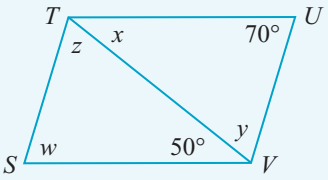
7 Give a definition for a concave quadrilateral.

8 Find the values of the pronumerals, giving reasons.

a $DEFG$ is a square.



b $STUV$ is a parallelogram.



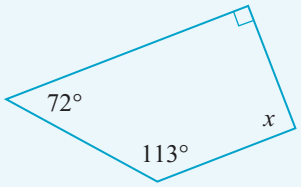
15C Review set

1 Construct a triangle with angles of 30° and 120°, joined by a side of length 6 cm.

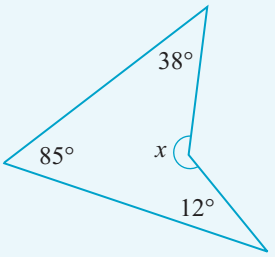
2 Draw a right-angled isosceles triangle.

3 Find the value of x, giving reasons.

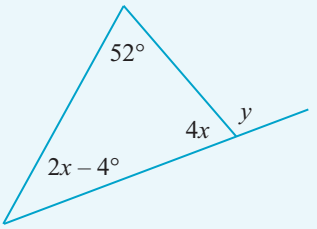
a



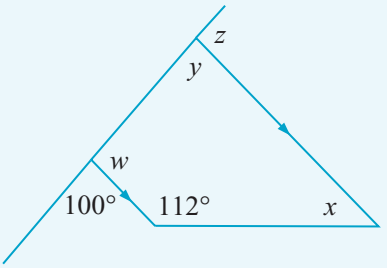
b



4 Write an equation and solve it to determine the values of the unknown angles.



5 Find the values of the pronumerals, giving reasons for your answers.



6 **a** Draw a trapezium.

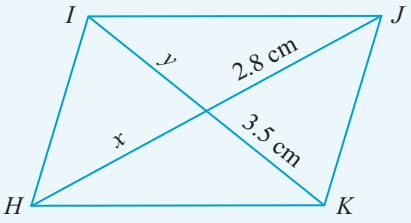
b List the properties of a trapezium.

7 **a** Sketch a quadrilateral in which the diagonals bisect each other at right angles.

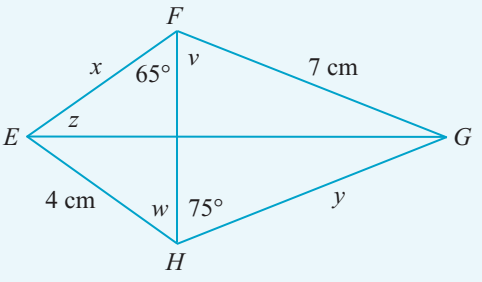
b What is the name(s) given to this quadrilateral?

8 Find the values of the pronumerals, giving reasons.

a *HIJK* is a parallelogram.



b *EFGH* is a kite.



Chapters 11–15 Cumulative review

1 a Simplify:

i $n \times n$ **ii** $5 \times p \times p \times p \times p$

b Show the meaning of:

i p^2 **ii** $3a^2$

c If $w = 3$ evaluate:

i w^2 **ii** $3w^2$

d Simplify:

i $4 \times 5t$ **ii** $6m \times 5$ **iii** $6a \times b \times 5c$

e Simplify:

i $6p + 5p$ **ii** $8m - m$ **iii** $6p - 6p$ **iv** $6x^2 - 3x^2 - x^2$

f If $a = 5$ and $b = 12$, find the value of:

i $5ab$ **ii** $\frac{3ab}{5}$ **iii** $2a - b$ **iv** $\frac{b}{a+1}$

g Simplify:

i $2a + 5a - 3b$ **ii** $2pq + 5pq - 3pq$ **iii** $8k + 5 - 3k$ **iv** $-5y + 2y$

v $3y^2 - 8y^2$ **vi** $-3x^2 + 5x^2$ **vii** $-a - a$ **viii** $9a - 9a + 9a$

h Simplify:

i $\frac{8h}{2}$ **ii** $\frac{g}{3g}$ **iii** $\frac{12a}{4a}$ **iv** $\frac{14vw}{21w}$ **v** $\frac{6a}{9ab}$

i Simplify:

i $2x + 5x$ **ii** $-5 \times 2a$ **iii** $\frac{bc}{c}$ **iv** $3x \times 4y$

j Expand:

i $3(x + 2)$ **ii** $-5(x - 3)$ **iii** $-4(x + 1)$

iv $4(5x + 7)$ **v** $3a(5a - 2b)$ **vi** $-5(2m - 3)$

k Factorise:

i $px + 3p$ **ii** $mn - mt$ **iii** $-4x - 8$

iv $12p - 18$ **v** $15t + 25u$ **vi** $20k - 16m$

vii $-5q - 10$ **viii** $18km - 12kn$ **ix** $6a^2b + 15ab^2$

2 a An eight-sided die with the numbers 1 to 8 is thrown once.

i List the sample space.

ii What is the probability of a 6?

b A box of ice-creams has 5 chocolate, 3 strawberry, 2 caramel and 2 vanilla.

i How many ice-creams are in the box?

ii How many are strawberry?

iii What is the probability of selecting a strawberry ice-cream?

c A letter is chosen from the word FLUFFY. What is the probability that it is:

i U? **ii** F?

d Write a statement describing a probability of $\frac{1}{2}$.

e Estimate the percentage probability for the statement 'very likely'.

f A six-sided die is rolled once. Describe events with a probability of:

i $\frac{1}{6}$ **ii** 0 **iii** $\frac{1}{2}$ **iv** 1

g A money box contains 10-cent coins and $1 coins. The probability of selecting a 10-cent coin is $\frac{7}{12}$. What is the probability of selecting a $1 coin?

3 **a** A die is rolled. Write the complement of these events.

i a 3 **ii** an even number **iii** a number greater than 2

b The letters of a word are written on cards and the cards are placed in a hat. When a card is chosen at random from this hat, the probability of the letter E is $\frac{3}{7}$. What is the probability that it is not the letter E?

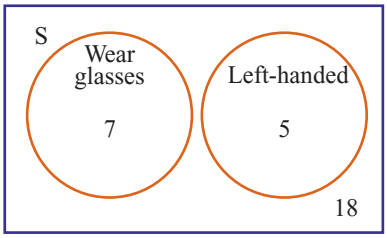
c In a group of 30 males, 13 play football, 11 play cricket and 7 play both football and cricket. Construct a Venn diagram to show this information, then use the diagram to find how many males in this group:

i play football but not cricket **ii** play cricket but not football

iii play football or cricket or both **iv** play football or cricket but not both

v play neither football nor cricket.

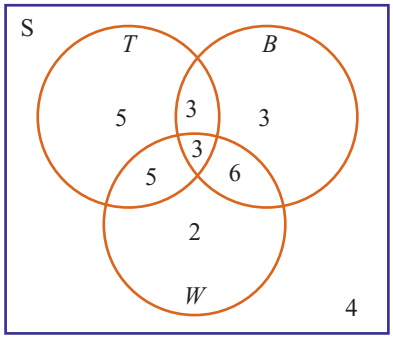
4 **a** The Venn diagram shows the information collected from a class of students about whether they wear glasses and whether they are left-handed.



Transfer this information to the two-way table below.

	Wear glasses	Do not wear glasses	
Left-handed			
Not left-handed			

b The Venn diagram represents the ways in which the students in a class travel to school. T is the event 'travels by train', B is the event 'travels by bus' and W is the event 'walks to school'. How many students travel to school by:



i train? **ii** train only?

iii train and walk? **iv** train or walk or both?

v train or walk but not both? **vi** bus and walk?

vii bus or walk but not both? **viii** bus and train and walk?

ix train but not by bus? **x**

only bus or only train or both?

xi none of these methods?

5 Solve the following equations.

a $p - 5 = -15$ **b** $d + 6 = 3$ **c** $7x = -49$

d $-3x = -24$ **e** $12c - 21 = 39$ **f** $14 - 3x = -10$

g $-3(2x - 5) = 4(3x + 8)$ **h** $3(5 - 2x) = -4(3x - 1)$ **i** $\frac{2 - 4k}{3} = -5$

j $4x - 5 + 3x = 10 - 2x + 8$ **k** $\frac{3x + 10}{7} = -4$ **l** $14 - \frac{x}{3} = 10$

6 **a** Is $x = 5$ a solution to $7x - 9 = 14$?

b The product of a certain number and 6 is 138. Write an equation and find the number.

c If $a = -3$, $b = 5$ and $c = -2$, find the value of:

i $3a - 5b + 2c$ **ii** $\frac{6abc}{60}$

d Five is subtracted from a number and the result is multiplied by 3. The answer is 21. What is the number?

7 **a** Given the formula $c = \sqrt{a^2 + b^2}$, find c when:

i $b = 5, a = 6$ **ii** $b = 15, a = 8$ **iii** $b = 17, a = 12$ **iv** $b = \sqrt{147}, a = 7$

b Use the formula $C = \frac{5}{9}(F - 32)$ to find F when C has the values:

i $10\frac{4}{9}$ **ii** 22 **iii** $12\frac{5}{9}$ **iv** $43\frac{5}{9}$

c Given the formula $F = 32 + \frac{9}{5}C$, find C when F has values:

i $39\frac{1}{5}$ **ii** $51\frac{4}{5}$ **iii** $62\frac{3}{5}$ **iv** 122

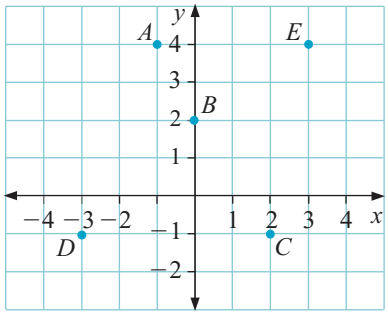
d Given the formula $d = st$, find the value of s when:

i $d = 114, t = 19$ **ii** $d = 748, t = 44$ **iii** $d = 264.67, t = 13.3$

e Using the formula $A = \frac{a + b + c}{3}$, calculate the value of c when:

i $A = 15, a = 7, b = 19$ **ii** $A = 42, a = 7, b = 12$

8 a Write the coordinates of the points shown on this number plane.



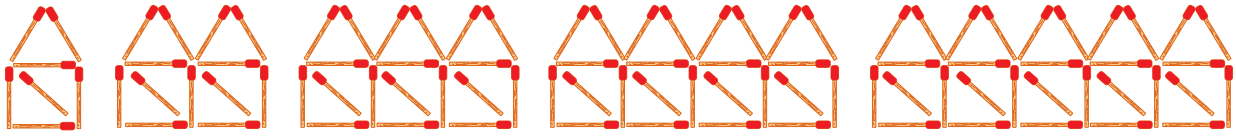
b Plot these points on a number plane.

$P(-3, -5)$, $Q(2, 4)$, $R(-2, 1)$, $S(0, -3)$, $T(4, -1)$

c **i** Plot the points $A(5, 2)$, $B(4, 1)$, $C(3, 0)$, $D(2, -1)$ on a number plane.

ii Write the coordinates of the next three points E, F and G if the pattern continues.

d Consider this pattern of matches.



i Complete the table.

Number of houses	1	2	3	4	5
Number of matches					

ii Write a rule describing the number of matches required to make each pattern.

iii Using x to represent the number of squares and y to represent the number of matches, write of set of points describing this information.

iv Write the coordinates of the next two points.

e Beans are sold for \$4.50 per kilogram. The following table shows weight versus cost for various quantities of beans.

Weight (kg)	0	1	2	4	5	10	20
Cost (\$)	0	4.50	9	18	22.50	45	90

i Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of points describing this information.

ii Graph these points on the number plane and draw a straight line through them.

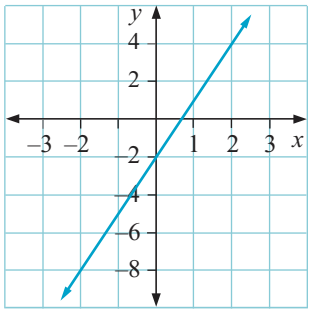
iii Use the graph to find how much 8.5 kg of beans would cost.

iv Use the graph to find what weight of beans could be purchased for \$70.

f Complete this table and draw the graph $y = 2x - 3$.

x	−2	−1	0	1	2
y					

g This graph shows a straight line.



i Use the graph to complete this table of values.

x	−2	−1	0	1	2
y					

ii Write the rule describing this straight line. The rule is of the form $y = \square x \pm \triangle$.

9 **a** Construct a triangle with side lengths 5 cm, 12 cm and 13 cm.

b Draw these triangles.

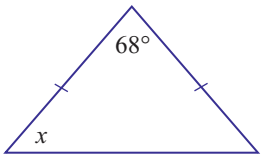
i Obtuse-angled scalene triangle

ii Right-angled isosceles triangle

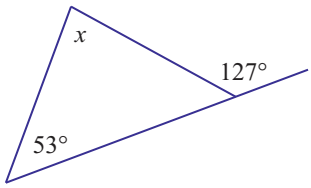
iii Acute-angled triangle

c Find the value of x, giving reason(s).

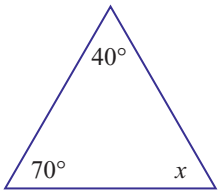
i



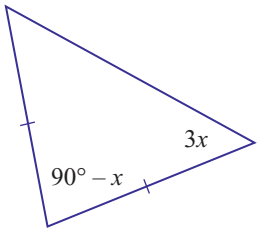
ii



iii



d Write an equation and solve it to find the value of x.
Hence find the unknown angles.

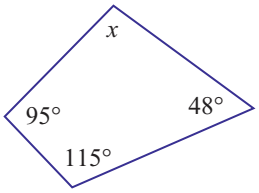


10 **a** Draw each of these plane shapes and list their properties.

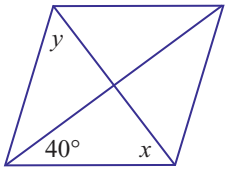
i Trapezium **ii** Rhombus **iii** Kite **iv** Rectangle

b Find the values of the pronumerals, giving reason(s).

i

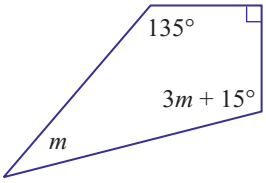


ii



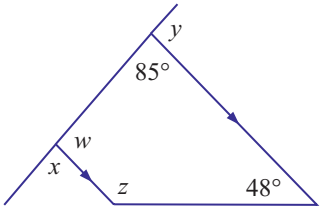
Rhombus

c Write an equation and solve it to find the value of m.
Hence find the unknown angles.

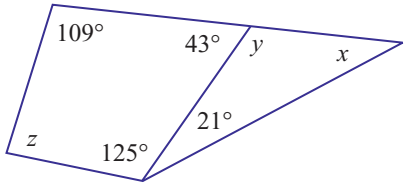


11 Find the values of the pronumerals, giving reasons.

a



b

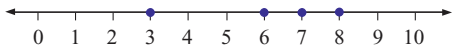


CUMULATIVE REVIEW 11–15

CHAPTER 1 REVIEW OF YEAR 7

Exercise 1A

1 a



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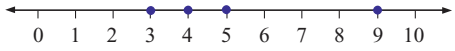
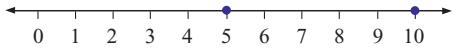


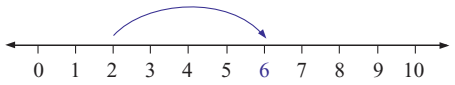
2 a



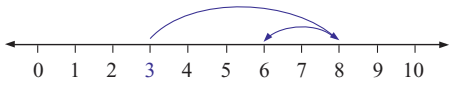
b $5 < 10$ **c** $10 > 5$

d 6, 7 and others **e** 8, 9, $7\frac{1}{2}$ and others

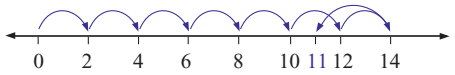
3 a



b



c



4 a Depositing $5

b Decrease of 2 cm in length

c A gain of 1 kg in weight

d A fall of 4°C in temperature

5 a Withdrawal of $1

b Deposit of $3

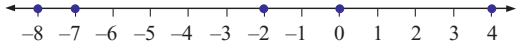
c Withdrawal of $8

6 a -7 **b** $+6$ **c** $+4$

7 a

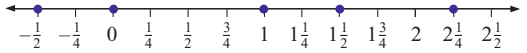
$-9, -8, -6, -5, -4$

b



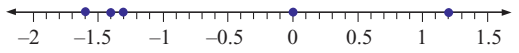
$-8, -7, -2, 0, 4$

8 a



$-\frac{1}{2}, 0, 1, 1\frac{1}{2}, 2\frac{1}{4}$

b



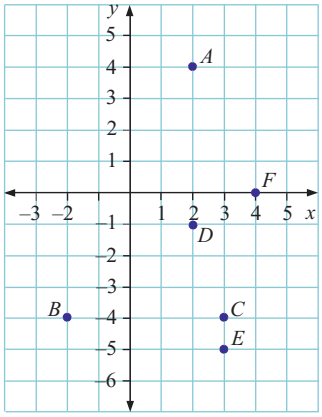
$-1.6, -1.4, -1.3, 0, 1.2$

9 a $-5 > -6$ **b** $-2 < 2$ **c** $0 > -3$

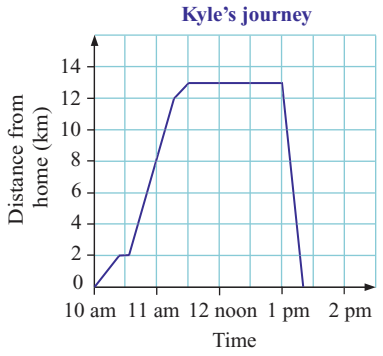
10 a -7 **b** -2 **c** 11

d 5 **e** -4 **f** -12

11



12



13 a 2 pm **b** 650 m from home

c 2:08 pm, 2:35 pm **d** 1000 m

e 125 m/min **f** She stopped.

g 2:12 pm to 2:32 pm **h** 1500 m

i 20 min **j** 10 min

k 150 m/min **l** 3000 m

m 136.4 m/min

Exercise 1B

1 a i $\angle PST$ **ii** $\angle S$

b i $\angle DPR$ **ii** $\angle P$

2 a $\angle QRB = 67°$ **b** $\angle BRS = 113°$ **c** $\angle CRS = 49°$

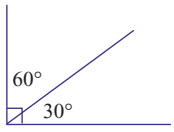
d $\angle BRC = 64°$ **e** $\angle QRC = 131°$ **f** $\angle QRS = 180°$

3 b i Acute **ii** Obtuse **iii** Reflex

iv Reflex **v** Reflex

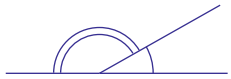
4 a Two angles that add to 90° are complementary.

b



c i 50° **ii** 72° **iii** 45° **iv** 7°

5 a



b i 55° **ii** 100° **iii** 109° **iv** 62°

6 a $t = 115°$ vertically opposite angles

b $u = 67°$ complementary angles

c $q = 199°$ angles at a point (revolution)

d $r = 95°$ angles in a straight line

e $s = 42°$ angles in a straight line

f $x = 87°$ vertically opposite

7 a No, co-interior angles are not supplementary.

b Yes, corresponding angles are equal.

8 a $q = 123°$ (corresponding angles)

b $r = 75°$ (alternate angles)

c $a = 30°$ (angle sum of $\triangle = 180°$)

$b = 30°$ (alternate angles)

$c = 60°$ (alternate angles)

$d = 120°$ (straight line $= 180°$)

$e = 60°$ (corresponding angle to c)

$f = 60°$ (corresponding angle)

Exercise 1C

1 a 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34

b 27, 36, 45, 54, 63, 72

c 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64

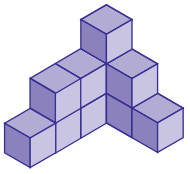
2 **a** 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
b 10, 20, 30, 40, 50, 60, 70, 80, 90
c 40, 80 **d** 40
3 **a** 1, 2, 4, 5, 10, 20 **b** 1, 5, 7, 35 **c** 5
4 **a** $50 = 2 \times 5^2$ **b** $130 = 2 \times 5 \times 13$
c $520 = 2^3 \times 5 \times 13$
5 **a** $140 = 2^2 \times 5 \times 7$ **b** $230 = 2 \times 5 \times 23$
c $540 = 2^2 \times 3^3 \times 5$
6 **a** $25 = 5 \times 5$, $40 = 2^3 \times 5$
b 5 **c** 200
7 **a** $180 = 2^2 \times 3^2 \times 5$, $240 = 2^4 \times 3 \times 5$
b 60 ($2^2 \times 3 \times 5$) **c** 720 ($2^4 \times 3^2 \times 5$)
8 **a** $70 = 2 \times 5 \times 7$, $84 = 2^2 \times 3 \times 7$, HCF = 14, LCM = 420
b $60 = 2^2 \times 3 \times 5$, $90 = 2 \times 3^2 \times 5$, HCF = 30, LCM = 180
c $280 = 2^3 \times 5 \times 7$, $400 = 2^4 \times 5^2$, HCF = 40, LCM = 2800
9 **a** ± 3 **b** ± 1 **c** ± 13 **d** 5 **e** 9 **f** 11
10 **a** 2, 3, 4, 6, 11, 12
b 2, 3, 4, 5, 6, 8, 9, 10, 12
c 2, 3, 4, 5, 6, 8, 10, 11, 12
11 8 and 9 or -8 and -9
12 **a** 34 **b** 15.5
13 **a** 8 **b** 65 **c** 6

Exercise 1D

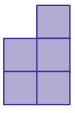
1 **a** $\frac{10}{21}$ **b** $\frac{11}{21}$
2 $\frac{8}{13}$ **3** $5\frac{2}{9}$
4 $\frac{60}{7}$ **5** $\frac{49}{84}$
6 $\frac{9}{20}, \frac{7}{10}, \frac{4}{5}$ **7** $\frac{8}{5}$
8 **a** $1\frac{2}{15}$ **b** $\frac{7}{12}$ **c** $\frac{3}{28}$ **d** $3\frac{1}{3}$
9 $\frac{7}{12}$ **10** \$90

Exercise 1E

1 **a** Square prism **b** Hexagonal prism
c Cylinder **d** Square pyramid
2 **a** Sphere **b** Rectangular prism
c Cylinder
3 **a** D **b** F
4 **b**

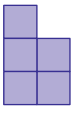


5 **a**



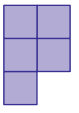
Front

b



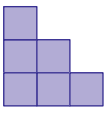
Back

c



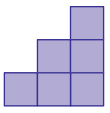
Top

d



Left side

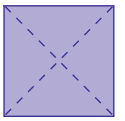
e



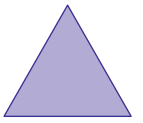
Right side

6 It is a right square pyramid because its axis is perpendicular to its base.

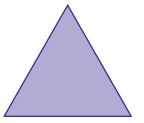
7 **a**



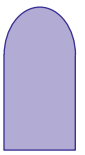
b



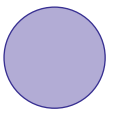
c



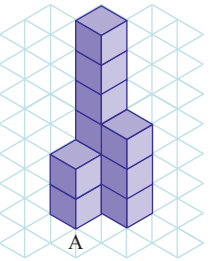
8 **a**



b



9



Exercise 1F

1 **a** $p + 1$ **b** $3p$
c $2p + 2$ **d** $3p + 3$
2 **a** $6p$ **b** gr **c** $5m$
d $8ab$ **e** $3m^2$ **f** $5a + 3q$
3 **a** $3 \times p$ **b** $a \times b$ **c** $m \times m$
d $5 \times x \times x$ **e** $6 \times p \times q$
4 **a** $3p$ **b** $5y$ **c** z **d** $3pq$ **e** 0
5 **a** 12 **b** 60 **c** 9 **d** 16 **e** 64
6 **a** $\frac{t}{2}$ **b** $\frac{g}{r}$ **c** $\frac{r}{g}$ **d** $\frac{4w}{7}$ **e** $\frac{3}{2x}$
7 **a** $k \div 3$ **b** $4 \div m$ **c** $p \div q$
d $3e \div 4$ **e** $mn \div t$
8 **a** 1 **b** 6 **c** 4 **d** 5 **e** 2
9 **a** 24 **b** 0 **c** 12 **d** -5 **e** 42
10 **a** 7 **b** 1 **c** 2 **d** 3 **e** 3

Exercise 1G

1

	Fraction	Decimal	Percentage
a	$\frac{1}{10}$	0.1	10%
b	$\frac{1}{5}$	0.2	20%
c	$\frac{1}{4}$	0.25	25%
d	$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
e	$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
f	$\frac{1}{2}$	0.5	50%
g	$\frac{5}{8}$	0.625	62.5%
h	$\frac{2}{3}$	$0.\dot{6}$	$0.66\frac{2}{3}\%$
i	$\frac{17}{20}$	0.85	85%
j	$\frac{98}{100}$	0.98	98%

2 **a** **i** 75% **ii** 25%
b **i** 40% **ii** 60%
3 **a** $\frac{1}{4}$ **b** $\frac{3}{5}$ **c** $\frac{18}{25}$ **d** $\frac{43}{50}$
4 **a** 59% **b** 78% **c** 66% **d** 76%

5 **a** 0.43 **b** 0.29 **c** 1.23 **d** 0.0035

6 **a** 25% **b** 31.2% **c** 480%
d 66.$\dot{6}$% **e** 62.5%

7 **a** 15.6% **b** 19% **c** 25%

8 0.48, $\frac{3}{5}$, $\frac{2}{3}$, 68%

9 **a** 44.8 m **b** 75 c **c** 140 m

10 13 : 15

11 **a** 3 : 10 **b** 5 : 18

12 Thousandths **13** 0.079

14 **a** $\frac{2}{25}$ **b** 0.75 **c** 0.$\dot{7}$

15 15.08

16 **a** > **b** =

17 A 0.45

18 **a** 72 **b** 3.096 **c** 23.68

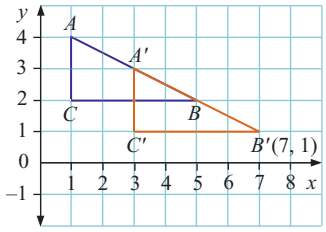
19 **a** 12.75 m **b** 2.65 m

Exercise 1H

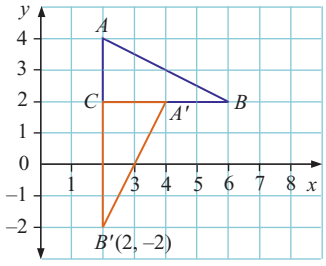
1 **a** Reflected **b** Rotated
c Translated

2 **a** Translated **b** Reflected
c Rotated **d** Translated or reflected

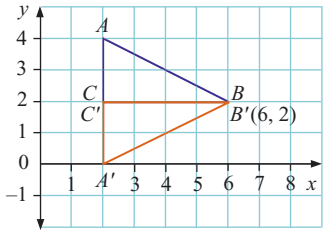
3 **a** Translated 2 right, 1 down



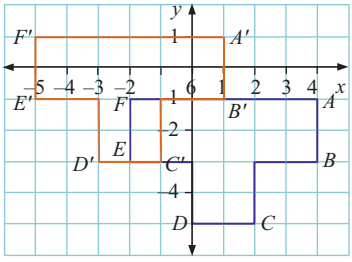
b Rotated through 90°



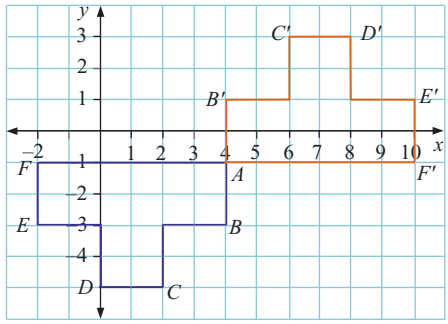
c Reflected in *CB*



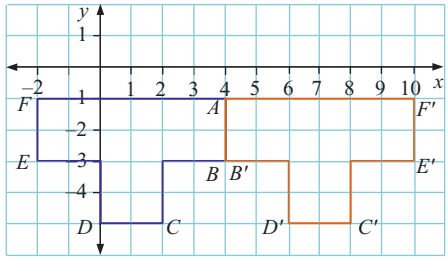
4 **a** $B(4, -3)$
b $B(1, -1)$

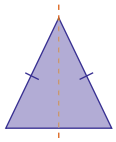
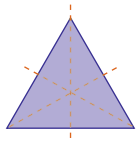


c $B(4, 1)$

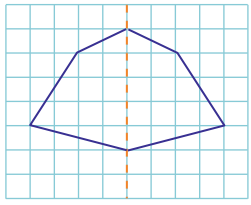
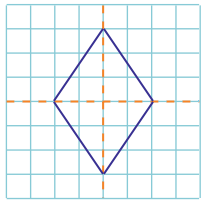


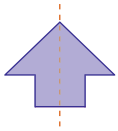
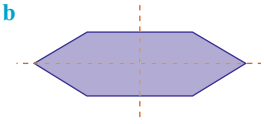
d $B(4, -3)$



5 **a**  **b** 

6 Shape **b**; order 3

7 **a**  **b** 

8 **a**  **b** 

9 **a** Yes, order 4 **b** Yes, order 2 **c** Yes, order 2

Exercise 1I

1 **a** {pink, red, blue, green} **b** $\frac{1}{4}$

2 **a** {1, 3, 5, 7, 9, 11} **b** $\frac{1}{6}$

3 **a** {the numbers 1 to 20} **b** $\frac{1}{20}$

4 **a** 16 **b** 5 **c** $\frac{5}{16}$

5 **a** 20 **b** 9 **c** $\frac{9}{20}$

6 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{26}$

7 **a** $\frac{13}{20}$ **b** $\frac{1}{20}$ **c** $\frac{3}{10}$
d $\frac{7}{20}$ **e** $\frac{7}{10}$

8 **a** **i** No chance **ii** Unlikely **iii** Even chance
iv Highly likely **v** Certain
b **i** 0 **ii** 80% **iii** 100%

9 **a** **i** Getting a head or tail **ii** Tossing a 1
iii Getting a head

10 **a** Selecting a Z
b Selecting a card from A to H
c Selecting the letter A
d Selecting an A or B
e Not selecting the letter A

11 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

12 $\frac{7}{10}$

Exercise 1J

1 Primary data is data that you collect. Secondary data is data collected from external sources such as the internet or newspapers.

2 **a** Census **b** Sample **c** Census

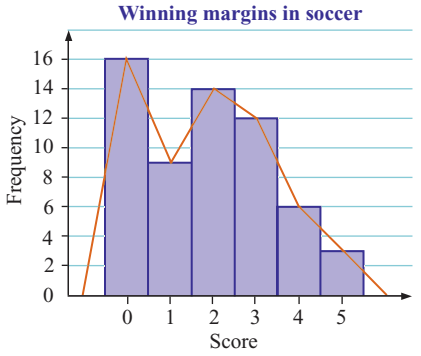
3 **a** All catch a bus so sample does not include people who use different forms of transport.
b Only people over 18 years are on the electoral role so no-one under 18 years is surveyed.

4 **a** Nominal **b** Numerical **c** Numerical

5 **a**

Score	Tally	Frequency
0	𝍸 𝍸 𝍸 |	16
1	𝍸 ||||	9
2	𝍸 𝍸 ||||	14
3	𝍸 𝍸 ||	12
4	𝍸 |	6
5	|||	3
	Total	**60**

b



c **i** 60 **ii** 14
iii The game was a draw.

6 **a**

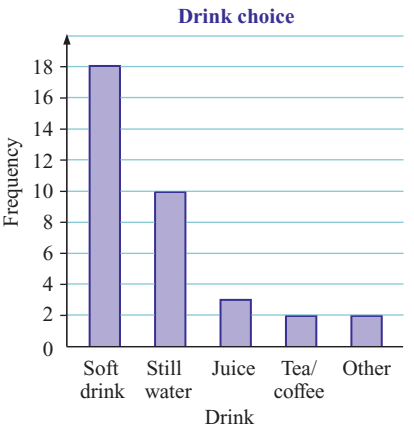
Score	Tally	Frequency
1	|||	3
2	𝍸	5
3	|	1
4	|||	3
5	|||	3
6	𝍸 ||	7
7	||||	4
8	|||	3
9	||	4
10	|||	3
	Total	**36**

b 10

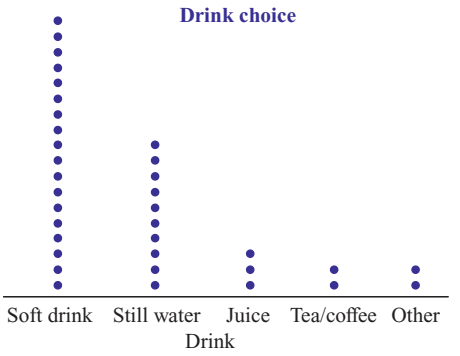
7

Stem	Leaf
12	8
13	1 2 2 8
14	2 2 4 5 5 5 5 6 8 9
15	0 0 1 3
16	1

8 **a**

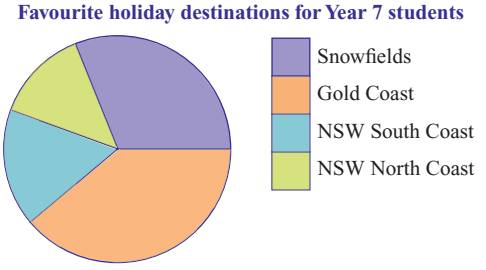


b



9 **a** Snowfields 112° Gold Coast 140°
NSW South Coast 60° NSW North Coast 48°

b



c Bar graph is a quarter size.

Exercise 1K

1 Area = 33 units^2 **2** ≈35 units^2

3 **a** 49 m^2 **b** 33 mm^2 **c** 49 cm^2
d 48 cm^2 **e** 60 cm^2 **f** 1750 mm^2

4 **a** 45 m^2 **b** 800 cm^2

5 **a** 230.26 cm^3 **b** 306.99 cm^3

6 **a** 3163.5 cm^3 **b** 3469.44 cm^3

7 **a** 310.08 cm^3 **b** 520 cm^3

8 **a** 3546.86 cm^3 **b** 793.27 cm^3

9 **a** 124.49 cm^2 **b** 1168.99 m^2

Exercise 1L

1 **a** 16.5 **b** 19 **c** 17 **d** 10

2 **a** 41.48 **b** 44 **c** 41 **d** 36

3 Class A: mean = 43.13, median = 42, mode = 28 and 46, range = 39
Class B: mean = 40.6, median = 41, mode = 51, range = 45

4 **a**

Score (x)	Frequency (f)	$f \times x$
8	6	48
9	11	99
10	15	150
11	12	132
12	8	96
13	7	91
14	8	112
	$\Sigma f = 67$	$\Sigma fx = 728$

b 10.9

5 **a** Mode = 12, range = 4
b Mode = 56, range = 4

6

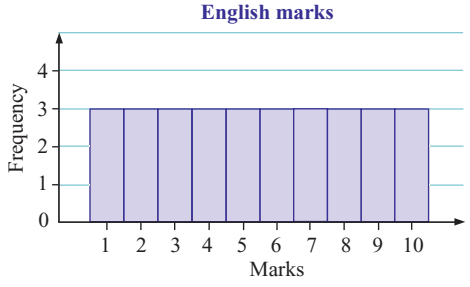
Stem	Leaf	f	$f \times$ stem	Sum of leaves
8	4 7 9	3	240	20
9	0 0 3 4 5 5 5 7 8 9	10	900	46
10	1 1 2 3 4 7 9 9	8	800	36
11	2 3 4 5 8 8 9 9 9 9	10	1100	66
12	0 1 3 7 9 9 9	7	840	38
13	0 1 1 1 2 2 3 4 8 9	10	1300	31
14	1 1 1 2 8 9	6	840	22
	Total	54	6020	259

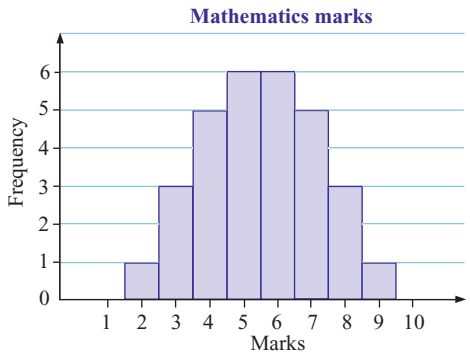
Mean = 116.3, median = 118.5, mode = 119, range = 65

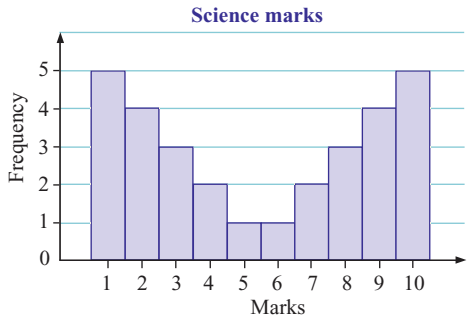
7 **a** English: mean = 5.5, median = 5.5, no mode
Mathematics: mean = 5.5, median = 5.5, mode = 5, 6
Science: mean = 5.5, median = 5.5, mode = 1, 10

b Mode **c** Science

d







Exercise 1M

1 -12, $\div 4$; $x = -3$

2 **a** $x = 6$ **b** $x = -15$ **c** $x = 9$
d $x = -7$ **e** $y = \frac{11}{3}$ **f** $p = 13$
g $d = -20$ **h** $c = \frac{14}{10} = \frac{7}{5}$ **i** $m = -20$
j $q = \frac{10}{17}$

3 **a** $p = \frac{15}{2}$ **b** $x = 24$

4 **a** No **b** No

5

x	5	11	8	9
$3x - 4$	11	29	20	23

$x = 9$

6 **a** $x + 5$ **b** $x - 8$ **c** $4x$
d $\frac{x}{7}$ **e** $3x + 5$ **f** $\frac{x}{8} - 3$
g $\frac{5x}{4}$ **h** $4(x + 6)$ **i** $\frac{x - 5}{3}$

7 **a** $8x$ **b** $w - 4$ **c** $z \div 7$
d $m + 11$ **e** $3y - 13$ **f** $3(a + 4)$
g $\frac{x}{7} + 22$ **h** $\frac{w + 5}{7}$ **i** $\frac{2x}{7}$

8 **a** $x + 4$ **b** $y - 7$ **c** $9q$
d $\frac{m}{6}$ **e** $4k + 8$ **f** $5(z + 3)$
g $\frac{y}{7} - 2$ **h** $\frac{z - 5}{4}$ **i** $9x + 5$
j $\frac{5w}{8}$ **k** $\frac{5p}{8}$ **l** $8(n - 5)$

9 **a** $a = 10\frac{2}{3}$ **b** $g = 14\frac{1}{7}$ **c** $x = 6.5$
d $d = 1$ **e** $e = -3.5$ **f** $f = -10.5$
g $k = \frac{14}{15}$ **h** $x = \frac{15}{28}$ **i** $p = \frac{5}{6}$

10 **a** $3x + 5 = 35$ **b** $2x - 7 = 13$ **c** $5x + 15 = 65$
d $4x - 4 = 36$ **e** $\frac{x}{2} + 9 = 14$ **f** $\frac{x}{5} - 4 = -2$
g $\frac{x + 8}{2} = 9$ **h** $\frac{x - 6}{2} = 2$

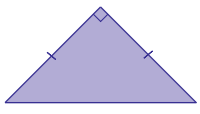
11 **a** 170 min **b** 1.5 kg

12 **a** 36 **b** 12

13 **a** \$1200 **b** \$8800

Exercise 1N

2

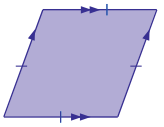


3 **a** $x = 56°$, base angles of isosceles $\triangle$
b $x = 56°$, exterior angle of $\triangle$

4 $x = 180 - 120 - 40 - 10$, $x = 10°$, angle sum of $\triangle$

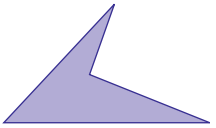
5 $u = 38°$ (supplementary angles)
$v = 99°$ (angle sum of △)
$w = 85°$ (angle sum of △)
$x = 85°$ (vertically opposite w)
$y = 81°$ (supplementary angle of v)
$z = 14°$ (angle sum of △)

6 a



b A rhombus has 2 axes of symmetry, 4 sides equal, diagonals bisect each other at right angles, opposite angles equal and opposite sides parallel.

7 a



b 1 diagonal inside
1 diagonal outside

CHAPTER 2 RATIOS AND RATES

Diagnostic test

1 B **2** A **3** D **4** B **5** D
6 B **7** B **8** A **9** C **10** A
11 C **12** B **13** D

Exercise 2A

1 a 2 : 5 **b** 3 : 2 **c** 1 : 3
2 a 4 : 7 **b** 7 : 4
3 a i 3 : 1 **ii** 1 : 3 **b** No
4 a i 1 : 2 **ii** 2 : 1 **b** No
5 a No as $\frac{1}{8} \neq \frac{8}{1}$ **b** $a = b$
6 a $\frac{5}{7}$ **b** $\frac{2}{7}$
7 a 1 : 8 **b** 8 : 1 **c** $\frac{1}{9}$ **d** $\frac{8}{9}$
e The paint would be mostly blue, $\frac{8}{9}$ blue and $\frac{1}{9}$ white, and so much darker than feather blue.
8 a $\frac{2}{5}$ **b** 3 : 2 **c** 2 : 3
d Yes, 18 **e** Yes, 12
9 a $\frac{1}{5}$ **b** Green **c** Red **d** $\frac{3}{20}$ **e** 1 : 4
10 a i 2 : 2 = 1: 1 **ii** 2 : 4 = 1 : 2 **iii** $\frac{2}{4} = \frac{1}{2}$
b i 3 : 3 = 1 : 1 **ii** 3 : 6 = 1 : 2 **iii** $\frac{3}{6} = \frac{1}{2}$
c i 6 : 2 = 3 : 1 **ii** 6 : 8 = 3 : 4 **iii** $\frac{6}{8} = \frac{3}{4}$
d i 3 : 6 = 1 : 2 **ii** 3 : 9 = 1 : 3 **iii** $\frac{3}{9} = \frac{1}{3}$
e i 4 : 2 = 2 : 1 **ii** 4 : 6 = 2 : 3 **iii** $\frac{4}{6} = \frac{2}{3}$
f i 9 : 7 **ii** 9 : 16 **iii** $\frac{9}{16}$
11 a $\frac{2}{5}$ **b** $\frac{4}{7}$ **c** $\frac{5}{9}$ **d** $\frac{6}{7}$
e $\frac{8}{13}$ **f** $\frac{5}{12}$ **g** $\frac{11}{14}$ **h** $\frac{2}{7}$
12 a 3 : 1 **b** 1 : 4 **c** 3 : 2 **d** 5 : 2
e 4 : 1 **f** 2 : 7 **g** 6 : 1 **h** 5 : 4
13 a $\frac{2}{7}$ **b** $\frac{1}{7}$
14 a 100 : 1 **b** 7 : 1000 **c** 11 : 60
d 3000 : 173 **e** 120 : 13 **f** 67 : 1000
g 3000 : 87 **h** 1000 : 27 **i** 130 : 3
15 a 4 : 10 = 2 : 5 **b** 6 : 15 = 2 : 5 **c** 4 : 25
16 a 64
b i 4 : 9 **ii** 7 : 32 **c** $\frac{11}{64}$
d i 17.2% **ii** 21.9% **iii** 12.5%

Exercise 2B

1 a 15 **b** 10 **c** 14 **d** 30
e 20 **f** 40 **g** 4 **h** 5
i 15 **j** 36 **k** 50 **l** 42
m 65 **n** 54 **o** 50 **p** 170
q 22 **r** 39
2 a 5 **b** 7 **c** 1 **d** 5
e 3 **f** 6 **g** 5 **h** 2
i 3 **j** 7 **k** 12 **l** 1
m 5 **n** 6 **o** 3 **p** 5
q 3 **r** 3
3 a 66 **b** 4 **c** 7 **d** 2
e 13 **f** 20 **g** 5 **h** 9
i 13 **j** 8 **k** 9 **l** 36
m 90 **n** 23 **o** 19
4 a 4 : 9 **b** 2 : 5 **c** 1 : 3 **d** 3 : 5
e 7 : 11 **f** 7 : 17 **g** 5 : 17 **h** 9 : 14
i 8 : 9 **j** 9 : 14 **k** 23 : 33 **l** 2 : 3
m 2 : 3 **n** 14 : 15 **o** 27 : 58 **p** 41 : 64
5 a 5 : 3 **b** 3 : 1 **c** 5 : 2 **d** 16 : 9
e 11 : 3 **f** 9 : 2 **g** 42 : 11 **h** 2 : 1
i 19 : 11 **j** 6 : 1 **k** 21 : 4 **l** 45 : 4
m 58 : 7 **n** 55 : 8 **o** 23 : 16 **p** 17 : 14
6 a 4 : 7 **b** 8 : 5 **c** 16 : 9 **d** 6 : 5 : 7
e 5 : 2 : 3 **f** 8 : 1 **g** 1 : 18 **h** 1 : 10
i 7 : 2 **j** 5 : 11 **k** 40 : 9 **l** 14 : 19
m 99 : 20 **n** 202 : 45 **o** 3 : 10 **p** 3 : 5
q 6 : 7 **r** 3 : 10 **s** 1 : 6 **t** 20 : 3
7 a 1 : 6 **b** 5 : 2 **c** 18 : 1 **d** 10 : 1
e 4 : 9 **f** 3 : 4 **g** 17 : 10 **h** 16 : 3
i 12 : 5 **j** 27 : 70 **k** 137 : 240 **l** 125 : 54
m 8 : 65 **n** 6 : 65 **o** 13 : 4
8 2 : 1 **9** Sand 6, gravel 4
10 a 9 : 10 **b** 10 : 1
11 a 4 : 5 **b** 5 : 4 **c** 1 : 4 **d** 5 : 1
12 3 : 6 : 5

Exercise 2C

1 a 18 **b** 3 **c** $\frac{7}{4}$ **d** $\frac{63}{20}$ **e** $\frac{8}{5}$ **f** $\frac{32}{5}$
g 3 **h** $\frac{26}{5}$ **i** 4 **j** $\frac{4}{5}$ **k** $\frac{7}{2}$ **l** $\frac{176}{25}$
2 a 1161 **b** 35
3 a \$280 **b** \$36
4 a 800 mL **b** 520 mL
5 a 11.7 cm **b** 57.2 cm
6 a 12.5 L **b** 320 mL
7 a 2.05 m **b** 64.4 kg
8 a \$300 **b** \$60
c \$432 **d** \$2304
9 a 180 m **b** 2 cm

10 **a** 2 cups Rice Bubbles, $\frac{3}{4}$ cup icing sugar, $1\frac{1}{2}$ tablespoons cocoa, $\frac{1}{2}$ cup desiccated coconut, 125 g copha
b 6 cups Rice Bubbles, $2\frac{1}{4}$ cups icing sugar $4\frac{1}{2}$ tablespoons cocoa, $1\frac{1}{2}$ cups desiccated coconut 375 g copha
11 **a** 20 **b** 35 **c** 80
12 **a** 6.9 cm, 13.8 cm **b** 36.8 cm
13 **a** 5.18 m **b** 10.36 m

Exercise 2D

1 **a** *F* **b** *B* **c** *D* **d** *I*
2 **a** \$80, \$20 **b** \$60, \$40
c \$45, \$55 **d** \$62.50, \$37.50
3 **a** \$360, \$180 **b** \$315, \$225
c \$240, \$300 **d** \$367.20, \$172.80
4 **a** 280 kg, 440 kg **b** 468 kg, 252 kg
c 504 kg, 216 kg **d** 374.4 kg, 345.6 kg
5 **a** 625.5 m, 208.5 m **b** 333.6 m, 500.4 m
c 347.5 m, 486.5 m **d** 200.16 m, 633.84 m
6 3415 children **7** 3 L
8 **a** 6.05 cm **b** 36.3 cm
9 0.24 L
10 Property \$25 000, shares \$20 000, bonds \$15 000
11 **a** 15 kg **b** 10 kg
12 **a** 22.5 cm **b** 10 cm
13 **a** 13 740 **b** 34 350 **c** 51 525

Exercise 2E

1 **a** 15 **b** 121.25 **c** 32.4 **d** 4°
e 20 **f** 14.5 **g** 4.5 **h** 64
i 12 **j** 276 **k** 4 **l** 120
m 10.45
2 **a** 10.5 km **b** 663.6 km **c** 2 h 27 min
d 12 750 km/h **e** 64.4 km **f** $3\frac{1}{2}$ h
3 **a** 12.5 min **b** 32.5 min
c 1 h 5 min **d** 2 h 50 min
4 **a** \$63.16 **b** \$252.64
c \$410.54 **d** \$536.86
5 **a** **i** 24 m^2 **ii** 36 m^2
b **i** 7 L **ii** 13 L
6 **a** 6 min 40 s **b** 7 min 37 s
7 **a** **i** \$0.938 or 93.8c per copy
ii 68.8c per copy
b 25c **c** \$1857.60
8 B **9** B
10 **a** 150 sheets **b** 6 rolls
11 **a** \$18.45 **b** \$6.55
12 **a** \$5.37 **b** \$19.98 **c** \$13.94 **d** \$29.63

13

	Player's name	Runs scored	Balls faced	Strike rate (1 decimal place)
a	Healey	42	30	140
b	Hughes	31	22	140.9
c	Border	18	16	112.5
d	O'Donnell	125	98	127.6
e	Young	71	59	120.3

14

	Player's name	Number of goals	Number of attempts	Success rate (1 decimal place)
a	Carla	4	10	40
b	Susan	2	5	40
c	Robert	1	4	25
d	Harley	6	13	46.2
e	Kylie	9	15	60

15 **a** **i** \$US86 **ii** \$US481.60
iii \$US4128 **iv** \$US21 500
b **i** \$A116.28 **ii** \$A4651.16
iii \$A13 953.49 **iv** \$A52 325.58

16 **a**

	Average annual rate of change in population (%)				
Year	Africa	Asia	Oceania	America	Europe
1950–2000	5.20	3.29	2.84	1.68	0.66
2000–2050	2.53	0.83	0.95	0.84	–0.26
1950–2050	7.15	2.74	2.58	1.61	0.15

b Europe
c Asia has a greater growth rate in 1950–2000, but Oceania is greater in 2000–2050. Asia has a greater growth rate in 1950–2050.
d The growth rates decrease from the 1950–2000 rate to the predicted 2000–2050 rate for all regions. This may be due to many factors, such as financial factors reducing family size or a lower birth rate.
e The growth rate for 1950–2050 must be divided by the population in 1950, while the growth rate for 2000–2050 is divided by the population in 2000.

Exercise 2F

1 **a** 1 cm ≡ 100 cm **b** 1 cm ≡ 400 000 cm
1 cm ≡ 1000 cm 1 cm ≡ 4000 m
1 cm ≡ 10 m 1 cm ≡ 4 km
2 **a** 1 : 400 **b** 1 : 16 000
c 1 : 5000 **d** 1 : 80 000
e 1 : 300 000 **f** 1 : 700 000
g 1 : 15 000 000 **h** 1 : 24 000 000
3 **a** 1 : 50 **b** 1 : 27 **c** 1 : 16
d 45 : 4 **e** 7 : 10 000 **f** 19 : 70 000
4 **a** 2.5 km **b** 13 km
c 24 km **d** 46 km
5 **a** 4 km **b** 5.6 km
c 8.24 km **d** 12.72 km
6 **a** 5 cm **b** 14 cm
c 18.6 cm **d** 84.8 cm
7 **a** 4 cm **b** 12 cm
c 25 cm **d** 56 cm
8 **a** 1 cm = 150 km, 1 cm : 150 km
b **i** 450 km **ii** 780 km
iii 937.5 km **iv** 2040 km
c **i** 8 cm **ii** 8.8 cm
iii 11.2 cm **iv** 17.25 cm

Check your skills

1 A **2** B **3** C **4** C **5** B
6 B **7** A **8** C **9** B **10** A
11 D **12** A **13** C **14** C **15** C

Review set 2A

1 It is a comparison of like quantities.
2 17 : 11
3 **a** 2 : 5 **b** 7 : 9
4 18
5 $400, $600
6 8.2 m, 12.3 m, 4.1 m
7 $14.70
8 365.95 km
9 **a** 32.5 m **b** 55.25 m
10 6 m

Review set 2B

1 The same quantities are compared.
2 11 : 7
3 **a** 1 : 3 **b** 6 : 11
4 104
5 468.75 kg, 781.25 kg
6 $70.68
7 $1300
8 4 h 19 min
9 **a** 11 cm **b** 45 cm
10 26.5 km

Review set 2C

1 1 : 3
2 B
3 2.86
4 **a** 5 : 12 **b** 18 : 25
5 55 students
6 **a** 31 cm, 24.8 cm **b** 68.2 cm
7 0.175 km/min
8 A
9 20.4 m
10 55 cm

Review set 2D

1 20 : 3
2 $\frac{20}{3}$
3 $2250, $1500, $1250
4 **a** 174 : 187 **b** 13 : 187
5 **a** 27.5 cm **b** 15.6 cm
6 42 cents
7 B
8 $1521.74
9 $216.20
10 **a** 1 cm = 8 km **b** 98.4 km

CHAPTER 3 CONGRUENCE

Diagnostic test

1 C **2** A **3** C **4** B **5** D
6 B

Exercise 3A

1 **a** Translation **b** Reflection
c Rotation
d Translation or reflection
e Rotation, reflection and translation
f Reflection and translation
2 **a** Yes **b** Yes **c** Yes
d Yes **e** Yes **f** No
3 **a** **i** AB and RP, BC and PQ, CA and QR, A and R, B and P, C and Q
ii Yes **iii** $\triangle ABC \equiv \triangle RPQ$
b **i** AB and MN, BC and NK, CD and KL, DA and LM, A and M, B and N, C and K, D and L
ii Yes **iii** $ABCD \equiv MNKL$
4 **a** **i** True **ii** True **iii** True **iv** True
b Yes
c **i** True **ii** True **iii** True **iv** True
d Yes **e** Yes
5 **a** **i** True **ii** True **iii** True **iv** True
b Yes
c **i** False **ii** False **iii** False **iv** False
d No **e** No
6 **a** **i** True **ii** True **iii** True
iv True **v** True
b Yes
c **i** False **ii** False **iii** False
iv False **v** False
d No **e** No
7 **a** B and C **b** Radii equal

Exercise 3B

1 **b** Yes **c** Yes **d** Yes
2 **b** Not necessarily **c** Not necessarily
d Not necessarily
3 **b** Yes **c** Yes
e **i** 2 **ii** 2 **iii** No **iv** No
f No
g True if angle is the included angle.
4 **b** Yes **c** Yes **d** Yes
5 **b** Yes **c** Yes **e** No **f** No
g True if the equal sides are matching sides.

Exercise 3C

1 **a** AAS **b** SAS **c** RHS **d** SSS
e AAS **f** SSS **g** RHS **h** SAS
i AAS **j** SAS **k** SSS **l** SAS
m AAS **n** RHS
2 **a** The equal angles are not both the included angle.
b The equal sides are not matching sides.
c Three pairs of equal angles do not necessarily make the triangles congruent.
3 **a** The second 80° angle is not the included angle.
b The equal sides are not matching sides.
c Three pairs of equal angles do not necessarily make triangles congruent.
d The equal angles are not included angles.
e Three pairs of equal angles do not necessarily make triangles congruent.
f The second angle is not the included angle.
g The equal sides are not matching sides.
h Three pairs of equal angles do not necessarily make congruent triangles.
4 **a** 2 and 3 (SSS) **b** 1 and 3 (SAS)
c 2 and 3 (AAS) **d** 1 and 2 (RHS)
e 1 and 2 (SAS) **f** 1 and 3 (AAS)
g 1 and 3 (AAS) **h** 1 and 2 (AAS)
i 2 and 3 (SAS) **j** 1 and 2 (AAS)
5 **a** SSS, $x = 50°$, $y = 60°$, $z = 70°$
b SAS, $x = 12$, $a = 30°$, $b = 50°$
c AAS, $a = 20°$, $x = 8$, $y = 7$
d RHS, $x = 6$, $a = 55°$, $b = 35°$
e SAS, $x = 9$, $a = 65°$, $b = 25°$
f AAS, $x = 8$, $y = 6$
g SSS, $a = 65°$, $b = 70°$, $c = 45°$
h SAS, $x = 10$, $a = 42°$, $b = 48°$

i AAS, $x = 11$, $y = 8$, $a = 40°$

j AAS, $x = 11.2$, $y = 14.5$

6 a i $PS = RS$ (given)
$SQ = SQ$ (common side)
$PQ = RQ$ (given)
$\therefore \triangle PQS \equiv \triangle RQS$ (SSS)

ii $\angle SPQ = \angle SRQ$ (matching $\angle$s of congruent $\triangle$s)

b i In $\triangle XYW$ and $\triangle ZYW$
$YW = YW$ (common side)
$\angle XYW = \angle ZYW$ (given)
$\angle XWY = \angle ZWY$ (given)
$\therefore \triangle XYW \equiv \triangle ZYW$ (AAS)

ii $XW = ZW$ (matching sides of congruent $\triangle$s)

7 a i $BD = BD$ (common side)
$AD = CD$ (given)
$AB = CB$ (given)
$\therefore \triangle ABD \equiv \triangle CDB$ (SSS)

ii $\angle ADB = \angle CDB$ (matching $\angle$s of congruent $\triangle$s)

b i $\angle RPQ = \angle RTS$ (given)
$\angle PRQ = \angle TRS$ (vertically opposite)
$RQ = RS$ (given)
$\therefore \triangle PQR \equiv \triangle TSR$ (AAS)

ii $PR = RT$ (matching sides of congruent $\triangle$s)

c i $MO = PO$ (given)
$NO = QO$ (given)
$\angle MON = \angle POQ$ (vertically opposite)
$\therefore \triangle MON \equiv \triangle POQ$ (SAS)

ii $MN = PQ$ (matching sides of congruent $\triangle$s)

d i $\angle JLK = \angle MLN$ (vertically opposite)
$\angle JKL = \angle MNL$ (alternate $\angle$s on parallel lines)
$KL = NL$ (given)
$\therefore \triangle JKL \equiv \triangle MNL$ (AAS)

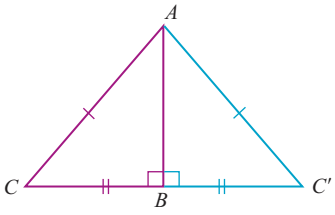
ii $JL = ML$ (matching sides of congruent $\triangle$s)

e i $\angle PQT = \angle RQS$ (vertically opposite)
$PT = RS$ (given)
$\angle PTQ = \angle RSQ$ (alternate $\angle$s on parallel lines)
$\therefore \triangle PQT \equiv \triangle RSQ$ (AAS)

ii $TQ = SQ$ (matching sides of congruent $\triangle$s)

Exercise 3D

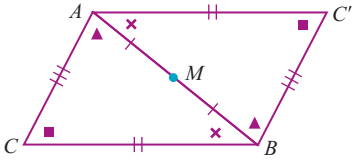
1 b



c Isosceles

d Opposite sides equal; angles opposite equal sides are equal, the line from the vertex is perpendicular to the base and bisects the base and vice versa.

2 e

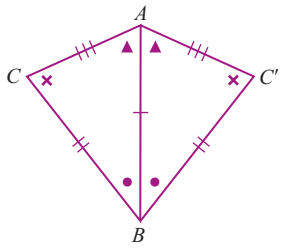


f Parallelogram

g Opposite sides equal; opposite sides parallel; opposite angles equal

3 c Yes **d** Diagonals bisect each other.

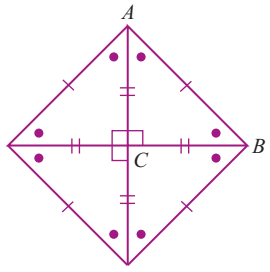
4 d



e Kite

f Two pairs of adjacent sides equal, one pair of opposite angles equal, one diagonal bisects the angles through which it passes.

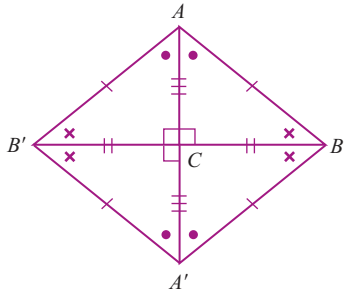
5 d



e Square

f All sides are equal, all angles are equal (and hence equal 90°), diagonals bisect each other at right angles, diagonals are equal, diagonals bisect the angles of the square. Opposite sides are parallel.

6 d



e Rhombus

f All sides equal, opposite sides are parallel, opposite angles equal, diagonals bisect each other at right angles, diagonals bisect the angles they pass through.

7 In $\triangle AMC$ and $\triangle AMB$
$AC = AB$ (given)
$AM = AM$ (common side)
$MC = MB$ (M is the midpoint of CB)
$\triangle AMC \equiv \triangle AMB$ (SSS)
$\angle ACM = \angle ABM$ (matching $\angle$s of congruent $\triangle$s)
$\therefore \angle ACB = \angle ABC$

8 $\triangle PTR$ and $\triangle PTQ$
$\angle R = \angle Q$ (given)
$PT = PT$ (common)
$\angle PTR = \angle PTQ$ (construction)
$\therefore \triangle PTR \equiv \triangle PTQ$ (AAS)
$\therefore PR = PQ$ (matching sides of congruent $\triangle$s)

11 $\angle B = \angle C$ ($AB = AC$)
Also $\angle B = \angle A$ ($CB = CA$)
$\angle A = \angle B = \angle C$
But $\angle A + \angle B + \angle C = 180°$ (angle sum of a $\triangle$)
$\therefore \angle A = \angle B = \angle C = 60°$

12 In $\triangle ABC$ and $\triangle ADC$
$AC = AC$ (common side)
$\angle BAC = \angle DCA$ (alternate $\angle$s and $AB \parallel DC$)
$\angle BCA = \angle DAC$ (alternate $\angle$s and $AD \parallel BC$)
$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)
a $\therefore AB = DC$ (matching sides of congruent $\triangle$s)
and $AD = BC$ (matching sides of congruent $\triangle$s)
The opposite sides of a parallelogram are equal.
b Also $\angle ABC = \angle ADC$
(matching angles of congruent $\triangle$s)

13 In $\triangle AED$ and $\triangle CEB$
$\angle DAE = \angle BCE$ (alternate angles and $AD \parallel BC$)
$\angle AED = \angle BEC$ (vertically opposite $\angle$s)
$AD = BC$ (opposite sides of a parallelogram are equal, proven in question **12**)
$\triangle AED \equiv \triangle CEB$ (AAS)
$\therefore AE = EC$ (matching sides of congruent $\triangle$s)
and $DE = BE$ (matching sides of congruent $\triangle$s)

14 In $\triangle ACD$ and $\triangle BDC$
$AD = BC$ (opposite sides of a rectangle are equal)
$DC = CD$ (common side)
$\angle ADC = \angle BCD$ (both 90°)
$\triangle ACD \equiv \triangle BDC$ (SAS)
$\therefore AC = BD$ (matching sides of congruent $\triangle$s)

Language in mathematics

1 **a** C **b** D **c** B **d** A
3 Translation, rotation, reflection
4 **a** Two sides equal, angles opposite equal sides are equal.
b All sides equal, all angles equal.

Check your skills

1 D **2** D **3** C **4** C **5** B
6 C **7** B **8** A **9** B **10** C
11 B **12** C

Review set 3A

1 **a** One can be superimposed on the other by any combination of translations, rotations and reflections.
b All the matching sides are equal and all the matching angles are equal.
c SSS, SAS, AAS, RHS
2 Yes
3 $ABCD \equiv QRSP$
4 **a** QR **b** $\angle S$
5 **a** AAS **b** SAS **c** SSS **d** RHS
6 **a** Three pairs of equal angles do not necessarily make the triangles congruent.
b The second 70° angle is not the included angle.
c The equal sides are not matching sides.
7 1 and 3
8 **a** SSS, $x = 95°$
b SAS, $x = 15$, $y = 45°$, $z = 35°$
c AAS, $x = 9$, $y = 10$, $z = 50°$
9 **a** In $\triangle PQT$ and $\triangle RQS$
$PT = SR$ (given)
$\angle PTQ = \angle RSQ$ (given)
$\triangle PQT = \angle RQS$ (vertically opposite $\angle$s)
$\therefore \triangle PQT = \triangle RQS$ (AAS)
b $\therefore PQ = RQ$ (matching sides of congruent $\triangle$s)

Review set 3B

1 No
2 $\triangle ABC \equiv \triangle YZX$
3 **a** YZ **b** $\angle X$
4 **a** SAS **b** AAS **c** SSS **d** RHS
5 **a** Three pairs of equal angles do not necessarily make the triangles congruent.
b The equal angles are not the included angles.
c The equal sides are not matching sides.
6 1 and 3
7 **a** SSS, $x = 60°$, $y = 65°$, $z = 55°$
b SAS, $x = 60°$, $y = 80°$
c AAS, $x = 10$, $y = 8$
8 **a** In $\triangle ABC$ and $\triangle CDA$
$AB = DC$ (given)
$AC = AC$ (common side)
$\angle BAC = \angle DCA$ (alternate angles and $AB \parallel DC$)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)
b $\therefore AD = BC$ (matching sides of congruent $\triangle$s)

Review set 3C

1 No
2 $ABCD \equiv YXWZ$
3 **a** YX **b** $\angle W$
4 **a** SAS **b** AAS **c** RHS **d** SSS
5 **a** Three pairs of equal angles do not necessarily make the triangles congruent.
b The equal sides are not matching sides.
c The equal angles are not included angles.
6 1, 2 and 3
7 **a** SSS, $x = 40°$, $y = 110°$, $z = 30°$
b AAS, $x = 10$, $y = 12$ **c** SAS, $x = 10$
8 In $\triangle PQT$ and $\triangle RST$
$PQ = RS$ (opposite sides of a parallelogram are equal)
$\angle QPT = \angle SRT$ (alternate angles and $PQ \parallel SR$)
$\angle PQT = \angle RST$ (alternate angles and $PQ \parallel SR$)
$\triangle PQT \equiv \triangle RST$ (AAS)
$\therefore PT = RT$ (matching sides of congruent $\triangle$s)
and $QT = ST$ (matching sides of congruent $\triangle$s)

CHAPTER 4 NUMBER OPERATIONS AND INDEX LAWS

Diagnostic test

1 A **2** A **3** B **4** D **5** B
6 D **7** C **8** C **9** B **10** A
11 A **12** D **13** A **14** B **15** A
16 C

Exercise 4A

1 **a** $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
b 2^5 means '2 to the power of 5'.
c The base is 2 and the index is 5.
2 **a** 7^8 **b** 7 to the power of 8
c The base is 7 and the index is 8.
3 **a** **i** 4^3 **ii** 9^8
b **i** 4 to the power of 3 **ii** 9 to the power of 8
c **i** The base is 4 and power is 3.
ii The base is 9 and power is 8.

4 **a** **i** 6^6 **ii** 10^4
b **i** 6 to the power of 6 **ii** 10 to the power of 4
c **i** The base is 6 and exponent is 6.
ii The base is 10 and exponent is 4.
5 **a** 8^9 **b** 10^3 **c** 12^2 **d** 6^4 **e** 9^7
6 **a** $k \times k \times k \times k \times k = k^5$
b k^5 means 'k to the power of 5'.
c The base is k and the index is 5.
7 **a** **i** w^4 **ii** a^{10}
b **i** w to the power of 4 **ii** a to the power of 10
c **i** The base is w and the index is 4.
ii The base is a and the index is 10.
8 **a** $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$
b $12^3 = 12 \times 12 \times 12$
c $j^5 = j \times j \times j \times j \times j$
d $v^7 = v \times v \times v \times v \times v \times v \times v$
9 **a** $1 \times 1 \times 1 \times 1 \times 1 \times 1$
b $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
c $6 \times 6 \times 6$
d $2 \times 2 \times 2 \times 2$
e 10×10
f 3×3
g $8 \times 8 \times 8 \times 8 \times 8$
h $4 \times 4 \times 4 \times 4 \times 4$
i $l \times l \times l \times l \times l \times l \times l$
j $t \times t$
k $f \times f \times f \times f$
l $a \times a \times a \times a \times a \times a \times a \times a \times a$
m $y \times y \times y \times y \times y$
n $d \times d \times d \times d \times d \times d$
o $k \times k \times k$
p $m \times m \times m \times m \times m \times m \times m \times m$
10 **a** 729 **b** 78 125 **c** 1024
d 512 **e** 43 046 721 **f** 1000
g 2401 **h** 256 **i** 1296
j 1331 **k** 6.5536 **l** 54.872
m 410.0625 **n** 54.76 **o** 238.328

Exercise 4B

1 **a** **i** $5^2 = 5 \times 5$
ii $5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
iii $5^2 \times 5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
b $5^2 \times 5^7$ in index form $= 5^9$
c Yes, 5 is being multiplied 9 (2 + 7) times.
2 **a** **i** $7 \times 7 \times 7$ **ii** $7 \times 7 \times 7 \times 7$
iii $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
b 7^7 **c** Yes
3 **a** **i** $6 \times 6 \times 6$ **ii** $6 \times 6 \times 6 \times 6 \times 6$
iii $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
b 6^8 **c** Yes
4 **a** **i** $10 \times 10 \times 10 \times 10 \times 10 \times 10$
ii $10 \times 10 \times 10 \times 10 \times 10$
iii $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
b 10^{11} **c** Yes
5 **a** $8^4 \times 8^{10} = 8^{4+10} = 8^{14}$
b $7^7 \times 7^2 = 7^{7+2} = 7^9$
c $9^7 \times 9^3 = 9^{7+3} = 9^{10}$
d $5^6 \times 5^{11} = 5^{6+11} = 5^{17}$
6 **a** 3^9 **b** 2^{12} **c** 7^{10} **d** 5^9
e 4^{16} **f** 6^{13} **g** 10^9 **h** 2^{20}
i 5^{30} **j** 3^{18} **k** 3^{10} **l** 7^{13}
m 2^8 **n** 8^{21} **o** 3^{17}

7 Rule: When multiplying numbers with the same base, add the indices.
8 **a** $4 \times 4 \times 4$
b $4 \times 4 \times 4 \times 4$
c 4^4
d Yes, 4 is being multiplied 4 times.
e Yes
9 **a** 5^5 **b** 3^8 **c** 2^{10} **d** 5^9 **e** 7^{12}
10 **a** **i** $3 \times 3 \times 3 \times 3 \times 3 \times 4 \times 4 = 3888$
ii $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 35\,831\,808$
b No, as $3888 \neq 35\,831\,808$.
c You can only add indices if the bases are the same.
11 **a** **i** $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 = 4000$
ii $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 100\,000\,000$
b No, as $4000 \neq 100\,000\,000$.
12 **a** **i** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187$
ii $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 4\,782\,969$
b No, as $2187 \neq 4\,782\,969$.
c 3^7
13 **a** **i** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$
ii $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 65\,536$
b No, as $256 \neq 65\,536$.
c 2^8
14 **a** **i** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 9\,765\,625$
ii $25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 \times 25 = 95\,367\,431\,640\,000$
b No, as they are not equal.
c 5^{10}
15 **a** True **b** False **c** False **d** True
e False **f** False **g** True **h** False
i False **j** False **k** False **l** True
m False **n** True **o** False
16 **a** $4^{2+1+5} = 4^8$ **b** $6^{2+4+3} = 6^9$
c 10^{15} **d** 2^{24} **e** p^{11} **f** m^{18}
17 **a** 2^{13} **b** 3^{18} **c** 5^{14} **d** 9^{11} **e** 4^{10}
f a^{11} **g** y^{12} **h** n^{19} **i** p^{20} **j** t^7
18 **a** Only add indices if the bases are the same.
b Indices is the plural of the word index.
c Bases can be numbers or letters.
d A single digit or letter has an index value of 1.

Exercise 4C

1 **a** $(4 \times 4 \times 4 \times 4)^3 = (4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4) = 4^{12}$
b $(8 \times 8 \times 8)^2 = (8 \times 8 \times 8) \times (8 \times 8 \times 8) = 8^6$
c $(2 \times 2 \times 2 \times 2 \times 2)^3 = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) = 2^{15}$
d $(7 \times 7 \times 7)^5 = (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7) \times (7 \times 7 \times 7) = 7^{15}$
2 **a** $(4^4)^3 = 4^{12}$ $(8^3)^2 = 8^6$ $(2^5)^3 = 2^{15}$ $(7^3)^5 = 7^{15}$
b When raising a number to a higher power, multiply the indices.
3 **a** 3^6 **b** 5^6 **c** 2^{12} **d** 3^{15}
e 7^{20} **f** 10^{10} **g** 4^{12} **h** 6^{21}
i 3^{24} **j** 2^{70} **k** 3^{20} **l** 3^{20}
m 5^8 **n** 5^8 **o** 9^{30}

4 **a** a^6 **b** b^{54} **c** c^{42} **d** d^{55}
e e^{40} **f** f^{14} **g** g^{36} **h** h^{33}
i i^{20} **j** j^8 **k** k^{56} **l** l^{40}
m m^{36} **n** n^{27} **o** o^{63}

5 **a** $(8^4)^2 \times 8^3 = 8^8 \times 8^3 = 8^{11}$
b $10^9 \times (10^2)^6 = 10^9 \times 10^{12} = 10^{21}$
c 7^{14} **d** 5^{14} **e** 9^{11} **f** 3^{33}
g 8^{11} **h** 6^{18} **i** 10^{13} **j** 4^{78}

6 **a** 7^{16} **b** 2^{18} **c** 5^{14} **d** 7^{17}
e 3^{28} **f** 9^{22} **g** 4^{33} **h** 8^{64}

Exercise 4D

1 **a** $\dfrac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4} = 4^4$

b $\dfrac{9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9} = 9^4$

c $\dfrac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = 5^3$

d $\dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2^4$

2 **a** $\dfrac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$

b 3^5

c Yes, five (= 7 − 2) 3s remain.

3 **a** $\dfrac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7}$

b 7^2

c Yes, two (= 5 − 3) 7s remain.

4 **a** $\dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$

b 2^5

c Yes, five (= 8 − 3) 2s remain.

5 **a** $\dfrac{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9}$

b 9^1

c Yes, one (= 8 − 7) 9 remains.

6 **a** $3^7 \div 3^2 = 3^5$ $7^5 \div 7^3 = 7^2$
$2^8 \div 2^3 = 2^5$ $9^8 \div 9^7 = 9^1$

b When dividing numbers with the same base, subtract the indices.

7 **a** 3^2 **b** 2^3 **c** 5^6 **d** 4^4
e 10^5 **f** 6^4 **g** 2^2 **h** 3^2
i 4^1 **j** 5^1 **k** 3^4 **l** 5^5
m 2^6 **n** 10^3 **o** 7^8

8 **a** m^3 **b** w^1 **c** g^2 **d** z^2
e p^1 **f** d^3 **g** a^5 **h** r^2
i b^2 **j** h^5 **k** s^3 **l** t^4
m v^2 **n** e^5 **o** l^1

9 **a** $9^4 \div 3^2 = \dfrac{9 \times 9 \times 9 \times 9}{3 \times 3} = 729$
$3^2 = 3 \times 3 = 9$
False, as $729 \neq 9$.

b $5^7 \div 5^3 = \dfrac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5^4 = 625$
$5^4 = 5 \times 5 \times 5 \times 5 = 625$
True, as $625 = 625$.

10 When dividing, you can only subtract indices if the bases are the same.

11 **a** True **b** False **c** False **d** True
e False **f** False **g** False **h** False
i False **j** True **k** False **l** False
m True **n** False **o** True **p** False

12 **a** 3^5 **b** 2^7 **c** 7^2 **d** 5^8
e 4^3 **f** 3^8 **g** 5^9 **h** 3^7
i 2^9 **j** 5^{10} **k** 2^3 **l** 3^7
m 2^6 **n** 5^3 **p** 3^5 **p** 7^{10}

Exercise 4E

1

2^5	2^4	2^3	2^2	2^1	2^0
32	$32 \div 2 = 16$	$16 \div 2 = 8$	$8 \div 2 = 4$	$4 \div 2 = 2$	$2 \div 2 = 1$

2

10^5	10^4	10^3	10^2	10^1	10^0
100 000	10 000	1000	100	10	1

3

5^5	5^4	5^3	5^2	5^1	5^0
3125	625	125	25	5	1

4

4^5	4^4	4^3	4^2	4^1	4^0
1024	256	64	16	4	1

5 **a** $2^0 = 1$ $10^0 = 1$ $5^0 = 1$ $4^0 = 1$

b Any base raised to the power of zero is equal to 1.

6 **a** 7^0

b $\dfrac{7 \times 7 \times 7}{7 \times 7 \times 7} = \dfrac{1}{1} = 1$

c $7^0 = 1$ (no 7s remaining)

7 **a** 9^0

b $\dfrac{9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9 \times 9 \times 9 \times 9} = \dfrac{1}{1} = 1$

c $9^0 = 1$ (no 9s remaining)

8 **a** 6^0

b $\dfrac{6 \times 6}{6 \times 6} = \dfrac{1}{1} = 1$

c $6^0 = 1$ (no 6s remaining)

9 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1
g 1 **h** 1 **i** 1 **j** 1 **k** 1 **l** 1

10 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1

Exercise 4F

1 **a** 5^{10} **b** 6^{16} **c** 10^{15} **d** 12^4
e 8^5 **f** 3^5 **g** 4^6 **h** 5^{18}
i 9^{22} **j** 1 **k** 1 **l** 1
m 2^3 **n** 8^{12} **o** 5^{28} **p** 1
q 4^{12} **r** 1 **s** 14^6 **t** 1
u 17^3 **v** 12^9 **w** 6^{24} **x** 9^{17}

2 **a** x^{12} **b** f^{10} **c** m^{45} **d** v^6
e 1 **f** q^{13} **g** b^{55} **h** y^{14}
i f^{16} **j** d^4 **k** e^9 **l** 1
m 1 **n** j^9 **o** z^{10} **p** a^3

3 **a** 2^{16} **b** 10^1 **c** 4^5 **d** 5^{10}
e 2^{14} **f** 6^{13} **g** 8^8 **h** 1
i 3 **j** 4 **k** 1 **l** 4^7
m 1 **n** 9^{14} **o** 7 **p** 6^{11}

4 **a** 125 **b** 2401 **c** 243 **d** 512
e 1.44 **f** 21.952 **g** 128 **h** 972
i 125 **j** 16 000 **k** 2700 **l** 677.376
m 9 **n** 2.56 **o** 300.125 **p** 8.64

Exercise 4G

1 **a** −35 **b** −21 **c** −36 **d** −18
e −28 **f** −24 **g** −6 **h** −72
i 44 **j** 30 **k** 18 **l** 14
m −56 **n** 24 **o** −40 **p** 64

2 **a** -30 **b** 72 **c** -42 **d** -40
e 25 **f** 1 **g** 0 **h** -45
i 36 **j** -20 **k** -6 **l** 12
m -343 **n** 40 **o** -24

3 **a** -2 **b** -6 **c** 2 **d** -6
e 4 **f** 9 **g** -2 **h** 2
i -4 **j** -11 **k** -4 **l** 2
m 4 **n** -5 **o** -5 **p** 6

4 **a** -5 **b** -3 **c** -3 **d** -8
e -24 **f** 10 **g** 16 **h** 10

5 **a** -3 **b** 3 **c** -10 **d** -40
e 2 **f** -11 **g** 24 **h** 5
i -16

6 **a** 4 **b** 5 **c** 3 **d** 7
e 2 **f** -2 **g** 9 **h** 0
i 11 **j** 6 **k** 4 **l** -4

7 **a** -119 **b** -26 **c** -1036 **d** -1558
e -106 **f** -92 **g** -846 **h** -163
i -106 **j** 420 **k** -1555 **l** -1715
m -112 **n** 264 **o** 7695

8 **a** -768 **b** -290 **c** $-26\,585$

Language in mathematics

1 **a** Three to the power of five
b Eight to the power of two
c Two to the power of three
d z to the power of four

2 **a** 7^2 **b** 4^3 **c** 6^5 **d** 2^4

3 **a** Index **b** Exponent

4 **a** Base **b** Index, power or exponent

5 $2^3 \times 2^4 = 8 \times 16 = 128$ $4^7 = 16\,384$
$128 \neq 16\,384$ so $2^3 \times 2^4 \neq 4^7$

6 $2^6 \times 2^3 = 64 \times 8 = 512$ $2^9 = 512$
$512 = 512$ so $2^6 \times 2^3 = 2^9$

7 $10^5 \div 10^3 = \dfrac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10} = 10^2$
$1^2 = 1$
$10^2 \neq 1$ so $10^5 \div 10^3 \neq 10^2$

8 $4^8 \div 4^5 = \dfrac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4} = 4^3$
so $4^8 \div 4^5 = 4^3$

9 $\dfrac{9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9 \times 9 \times 9} = \dfrac{1}{1} = 1$

10 $(2^3)^3 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 512$
$2^9 = 512$
$512 = 512$ so $(2^3)^3 = 2^9$

Check your skills

1 B	**2** A	**3** D	**4** C	**5** A
6 D	**7** A	**8** B	**9** C	**10** A
11 A	**12** B	**13** D	**14** B	**15** A
16 B	**17** B	**18** D	**19** D	**20** B
21 D	**22** B	**23** B	**24** B	**25** D
26 D				

Review set 4A

1 **a** 2^4 **b** 5^7

2 **a** Base 7, index 9 **b** Base 3, index 10

3 **a** $3^5 = 3 \times 3 \times 3 \times 3 \times 3$
b $7^6 = 7 \times 7 \times 7 \times 7 \times 7 \times 7$

4 **a** $3^8 = 6561$ **b** $5^6 = 15\,625$

5 **a** False **b** False

6 **a** 5^{32} **b** 4^{15} **c** 2^2 **d** 7^6 **e** 5^4

7 **a** p^8 **b** m^{14} **c** f^6 **d** l^7 **e** w^4

8 Add indices: $5^{4+3} = 5^7$

9 Subtract indices: $2^{8-4} = 2^4 \neq 2^2$

10 Anything to the power of zero $= 1$:
$4^0 \times 9^0 + 3^0 = (1 \times 1) + 1 = 2$

11 **a** -45 **b** 40 **c** 45

12 **a** -4 **b** -12 **c** 8

13 **a** 3 **b** 13 **c** 1
d -24 **e** 4 **f** -9

Review set 4B

1 **a** 7^5 **b** 9^7

2 **a** Base 3, index 8 **b** Base 5, index 2

3 **a** $6^4 = 6 \times 6 \times 6 \times 6$ **b** $7^3 = 7 \times 7 \times 7$

4 **a** 512 **b** 729

5 **a** 3^{13} **b** 7^{12} **c** 4^5
d 6^{11} **e** 2^{30}

6 **a** False **b** False

7 **a** c^{16} **b** k^{36} **c** y^6
d z^9 **e** x^8

8 Subtract indices: $4^{7-2} = 4^5$

9 Add indices: $6^{6+2} = 6^8 \neq 36^8$

10 Multiply indices: $3^{2 \times 4} = 3^8$

11 **a** -24 **b** 48 **c** 36

12 **a** 7 **b** 6 **c** -6

13 **a** 1 **b** 11 **c** 4
d 45 **e** 12 **f** 9

Review set 4C

1 **a** 2^8 **b** 10^3

2 **a** Base 8, index 4 **b** Base 12, index 2

3 **a** $6^2 = 6 \times 6$
b $5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

4 **a** 625 **b** 2187

5 **a** 9^{19} **b** 2^{15} **c** 5^8 **d** 6^{10} **e** 2^{46}

6 **a** False **b** True

7 **a** j^{16} **b** n^6 **c** p^{27} **d** s^{15} **e** 9

8 Multiply indices: $2^{4 \times 3} = 2^{12} \neq 2^7$

9 Subtract and add indices: $6^{10-4+1} = 6^7$

10 Subtract indices: $4^{3-1} = 4^2$
Subtract indices: $4^{7-3} = 4^4$
$4^2 < 4^4$

11 **a** -30 **b** 54 **c** 40

12 **a** -2 **b** 5 **c** 9

13 **a** 7 **b** -5 **c** 16
d 12 **e** 14 **f** 8

Review set 4D

1 **a** 6^5 **b** 5^8

2 **a** Base 6, index 8 **b** Base 3, index 10

3 **a** $4^5 = 4 \times 4 \times 4 \times 4 \times 4$
b $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

4 **a** $6^4 = 1296$ **b** $2^{12} = 4096$

5 **a** 5^{40} **b** 3^{70} **c** 5^4
d 4^{22} **e** 3^{45}

6 **a** True **b** True

7 **a** 1 **b** 15 **c** 1 **d** 1 **e** 2

8 Anything to the power of zero $= 1$: $h^0 + m^0 = 1 + 1 = 2$

9 $(j^5)^0 \times 5 = 1 \times 5 = 5$

10 $m^0 + d^0 = 2 \times f^0$
$1 + 1 = 2$ and $2 \times 1 = 2$ ∴ correct
11 **a** -60 **b** 30 **c** -8
12 **a** -5 **b** -3 **c** -4
13 **a** 9 **b** -12 **c** 5
d 20 **e** 90 **f** 12

CHAPTER 5 PERIMETER, AREA AND CAPACITY

Diagnostic test

1 B **2** C **3** B **4** C **5** D
6 A **7** C **8** C **9** C **10** A
11 D **12** C **13** B **14** B **15** C

Exercise 5A

1 **a** 210 mm **b** 18 cm **c** 3.5 km
d 40 mm **e** 230 cm **f** 1800 m
g 20 cm **h** 2.8 m **i** 520 cm
j 83 mm **k** 6300 m **l** 3 cm
m 5000 cm **n** 3200 mm **o** 0.83 km
2 **a** 14 m **b** 18 m **c** 16 mm
d 50 cm **e** 12 m **f** 40 cm
g 8 m **h** 120 cm **i** 36 cm
3 1040 m **4** 68 m
5 **a** 7.79 m **b** 7.06 m **c** 13.4 m **d** 18.4 m
6 **a** 20 cm **b** 84 cm **c** 29 cm
d 31 cm **e** 6.61 cm **f** 5.2 cm
7 **a** 20 cm **b** 61 cm **c** 84 cm
d 65.3 cm **e** 167.9 cm **f** 487.5 cm
8 **a** $8 \times 8 = 64$ cm^2 **b** $14 \times 14 = 196$ cm^2
c $19 \times 19 = 361$ cm^2 **d** 136.89 cm^2
e 79.21 cm^2 **f** 12.25 cm^2
9 **a** 15.21 cm^2 **b** 5.76 mm^2
c 75.69 m^2 **d** 0.7225 m^2
10 **a** $7 \times 5 = 35$ cm^2 **b** $18 \times 13 = 234$ cm^2
c $12 \times 4 = 48$ cm^2 **d** $4.3 \times 6.9 = 29.67$ cm^2
11 **a** 36 cm^2 **b** 90 cm^2
c 96 mm^2 **d** 20 m^2
e 35 cm^2 **f** 18.6 cm^2
12 **a** $\frac{1}{2} \times 5 \times 2 = 5$ cm^2 **b** $\frac{1}{2} \times 4 \times 2 = 4$ cm^2
c $\frac{1}{2} \times 7 \times 5 = 17.5$ km^2
13 **a** 18 m^2 **b** 6 cm^2 **c** 720 m^2 **d** 20 cm^2
14 **a** $8 \times 3 = 24$ cm^2 **b** $120 \times 35 = 4200$ mm^2
c $50 \times 12 = 600$ cm^2
15 **a** 21 cm^2 **b** 36 cm^2 **c** 120 cm^2

Exercise 5B

1 **a** $A = \frac{1}{2} \times 7 \times (6 + 10) = 56$ m^2
b $A = \frac{1}{2} \times 6 \times (3 + 9) = 36$ cm^2
c $A = \frac{1}{2} \times 7 \times (18 + 12) = 105$ cm^2
2 **a** 56 cm^2 **b** 75 cm^2 **c** 48 cm^2
d 302.5 cm^2 **e** 105 cm^2 **f** 231 cm^2
3 **a** 55 cm^2 **b** 28 cm^2 **c** 49 cm^2
d 78 mm^2 **e** 162 mm^2 **f** 25.5 cm^2
g 25.44 m^2 **h** 95.475 cm^2 **i** 28.495 cm^2
j 8.68 m^2
4 **a** $A = \frac{1}{2} \times 8 \times 6 = 24$ cm^2
b $A = \frac{1}{2} \times 3.6 \times 1.7 = 3.06$ m^2
c $A = \frac{1}{2} \times 3 \times 8 = 12$ km^2
5 **a** 32.4 cm^2 **b** 45.5 cm^2 **c** 204 cm^2
d 21.42 cm^2 **e** 52.11 cm^2 **f** 127.53 cm^2
6 **a** $A = \frac{1}{2} \times 10 \times 8 = 40$ cm^2
b $A = \frac{1}{2} \times 15 \times 7 = 52.5$ cm^2
c $A = \frac{1}{2} \times 11.6 \times 5.2 = 30.16$ m^2
7 **a** 57 cm^2 **b** 232.5 cm^2 **c** 182 cm^2
d 66.5 cm^2 **e** 140 cm^2 **f** 567 cm^2
8 **a** 38 cm^2 **b** 22.5 m^2 **c** 36 cm^2
d 30 mm^2 **e** 93.5 mm^2 **f** 24.84 cm^2
g 28.055 cm^2 **h** 36.54 mm^2 **i** 3.24 cm^2
9 **a** Larger area $= l \times b = (20 \times 17) = 340$ m^2
Smaller area $= \frac{1}{2}b \times h = \frac{1}{2} \times 16 \times 13 = 104$ m^2
Shaded area $= 340 - 104 = 236$ m^2
b Larger area $= \frac{1}{2} \times 9 \times (17 + 26) = 193.5$ m^2
Smaller area $= \frac{1}{2} \times 4 \times (6 + 13) = 38$ m^2
Shaded area $= 193.5 - 38 = 155.5$ m^2
c Larger area $= (14 \times 14) = 196$ cm^2
Smaller area $= 7 \times 8 - 3 \times 3 = 56 - 9 = 47$ cm^2
Shaded area $= 196 - 47 = 149$ cm^2
10 **a** 32.8 m^2 **b** 14.52 m^2 **c** 114 m^2
d 130.845 m^2 **e** 147 m^2 **f** 63.3 cm^2
11 **a** 20 cm^2 **b** 58.5 m^2 **c** 266 m^2
12 **a** 51 m^2 **b** 48 cm^2 **c** 60 mm^2
d 82.5 km^2 **e** 108.375 m^2 **f** 23.165 m^2
13 **a** 300 m $= 300 \div 1000$ km $= 0.3$ km
$A = \frac{1}{2}h(a + b)$
$= \frac{1}{2} \times 0.3 \times (0.4 + 0.5)$
$= 0.135$ km^2
b 0.4 km $= 0.4 \times 1000$ m $= 400$ m
0.5 km $= 0.5 \times 1000$ m $= 500$ m
$A = \frac{1}{2}h(a + b)$
$= \frac{1}{2} \times 300 \times (400 + 500)$
$= 135\,000$ m^2
14 **a** **i** 3300 cm^2 **ii** 0.33 m^2
b **i** 280 000 m^2 **ii** 0.28 km^2
c **i** 16 000 cm^2 **ii** 1.6 m^2
d **i** 2.24 m^2 **ii** 22 400 cm^2
e **i** 19 200 cm^2 **ii** 1.92 m^2

Exercise 5C

3 Yes
5 **a** 4 cm^2 **b** 400 mm^2
6 **a** **i** 64 cm^2 **ii** 6400 mm^2
b **i** 25 cm^2 **ii** 2500 mm^2
c **i** 144 cm^2 **ii** 14 400 mm^2
7 **a** If 64 cm^2 = 6400 mm^2, 6400 mm^2 = 64 cm^2.
b If 25 cm^2 = 2500 mm^2, 2500 mm^2 = 25 cm^2.
c If 144 cm^2 = 14 400 mm^2, 14 400 mm^2 = 144 cm^2.
d To convert from cm^2 to mm^2 multiply by 100.
e To convert from mm^2 to cm^2 divide by 100.

8

	Areas		
Side length	**m^2**	**cm^2**	**mm^2**
2 m	4 m^2	40 000 cm^2	4 000 000 mm^2
3 m	9 m^2	90 000 cm^2	9 000 000 mm^2
4 m	16 m^2	160 000 cm^2	16 000 000 mm^2

9 **a** To convert from m^2 to cm^2 multiply by 10 000.
b To convert from cm^2 to m^2 divide by 10 000.

10 **a** $11 \times 10\,000 = 110\,000\ cm^2$
b $0.6 \times 10\,000 = 6000\ cm^2$
c $0.04 \times 100\ mm^2 = 4\ mm^2$

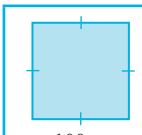
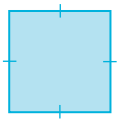
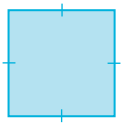
11 **a** 40 000 **b** 120 000 **c** 3000
d 5000 **e** 900 **f** 60
g 14 100 **h** 1200 **i** 10
j 47 **k** 6 **l** 1.2

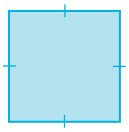
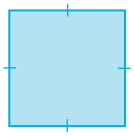
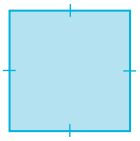
12 **a** $14\,000 \div 100 = 140\ cm^2$
b $63 \div 100 = 0.63\ cm^2$
c $530 \div 10\,000 = 0.053\ m^2$

13 **a** 120 **b** 156 **c** 93
d 7.2 **e** 0.58 **f** 0.06
g 700 **h** 325 **i** 18.2
j 9.6 **k** 0.35 **l** 0.042

14 **a** 600 **b** 1300 **c** 350
d 40 **e** 6 **f** 7
g 8.5 **h** 0.24 **i** 30 000
j 158 000 **k** 4 **l** 6.05

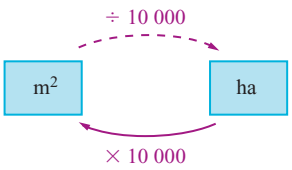
15 **a**

	 100 m	 500 m	 800 m
m^2	$100 \times 100 = 10\,000\ m^2$	$500 \times 500 = 250\,000\ m^2$	$800 \times 800 = 640\,000\ m^2$
ha	= 1 ha	= 25 ha	= 64 ha

	 1000 m	 5000 m	 8000 m
m^2	$1000 \times 1000 = 1\,000\,000\ m^2$	$5000 \times 5000 = 25\,000\,000\ m^2$	$8000 \times 8000 = 64\,000\,000\ m^2$
ha	= 100 ha	= 2500 ha	= 6400 ha

b **i** To convert m^2 to hectares divide by 10 000.
ii To convert hectares to m^2 multiply by 10 000.

c



16 **a** 50 000 **b** 170 000 **c** 9000
d 2300 **e** 14 000 **f** 4
g 6.2 **h** 18 **i** 900
j 22 600 **k** 34 268 **l** 89

17 **a** 2 **b** 3 **c** 1.15
d 4.89 **e** 40 000 **f** 60 000
g 36 000 **h** 49 300 **i** 300
j 11 500 **k** 21 000 **l** 50

Exercise 5D

1 **a** 30 mL **b** 60 mL **c** 20 mL
d 25 mL **e** 85 mL **f** 45 mL
g 4 mL **h** 34 mL **i** 53 mL
j 500 mL **k** 700 mL **l** 600 mL
m 5000 mL **n** 3000 mL **o** 11 000 mL
p 7.5 mL

2 **a** 50 cm^3 **b** 70 cm^3 **c** 30 cm^3
d 45 cm^3 **e** 25 cm^3 **f** 65 cm^3
g 6 cm^3 **h** 54 cm^3 **i** 21 cm^3
j 400 cm^3 **k** 600 cm^3 **l** 900 cm^3
m 7000 cm^3 **n** 5000 cm^3 **o** 12 000 cm^3
p 7.5 cm^3

3 **a** $3000\ cm^3 = 3000 \div 1000 = 3\ L$
b $6000\ cm^3 = 6000 \div 1000 = 6\ L$
c $2000\ cm^3 = 2000 \div 1000 = 2\ L$

4 **a** 75 L **b** 35 L **c** 65 L
d 4.2 L **e** 3.4 L **f** 5.3 L
g 2.535 L **h** 3.773 L **i** 7.688 L

5 **a** $3\ L = 3 \times 1000 = 3000\ cm^3$
b $8\ L = 8 \times 1000 = 8000\ cm^3$
c $7\ L = 7 \times 1000 = 7000\ cm^3$

6 **a** 4200 cm^3 **b** 5300 cm^3 **c** 8900 cm^3
d 6450 cm^3 **e** 5440 cm^3 **f** 3210 cm^3
g 4495 cm^3 **h** 6293 cm^3 **i** 8443 cm^3
j 70 000 cm^3 **k** 50 000 cm^3 **l** 120 000 cm^3

7 **a** mL **b** L **c** L **d** mL **e** L
f mL **g** mL **h** L **i** mL

8 **a** $4000\ L = 4000 \div 1000 = 4\ m^3$
b $330\ L = 330 \div 1000 = 0.33\ m^3$
c $0.4\ kL = 0.4 \times 1 = 0.4\ m^3$

9 **a** 12 m^3 **b** 9.5 m^3 **c** 7.25 m^3
d 0.67 m^3 **e** 0.136 m^3 **f** 0.025 m^3
g 12.5 m^3 **h** 8.3 m^3 **i** 5 m^3
j 0.6 m^3 **k** 0.75 m^3 **l** 0.09 m^3

10 **a** $0.05\ m^3 = 0.05 \div 1 = 0.05\ kL$
b $580\ m^3 = 580 \div 1 = 580\ kL$
c $7000\ m^3 = 7000 \div 1 = 7000\ kL$

11 **a** 0.01 kL **b** 0.9 kL **c** 480 kL
d 295 kL **e** 890 kL **f** 6500 kL
g 7200 kL **h** 9000 kL **i** 12 940 kL
j 14 750 kL **k** 18 500 kL **l** 23 000 kL

12 **a** $2000\ m^3 = 2000 \div 1000 = 2\ ML$
b $600\ m^3 = 600 \div 1000 = 0.6\ ML$
c $0.8\ m^3 = 0.8 \div 1000 = 0.0008\ ML$
d 6 ML **e** 0.42 ML **f** 0.075 ML

13 **a** $5\ ML = 5 \times 1000 = 5000\ m^3$
b $6.2\ ML = 6.2 \times 1000 = 6200\ m^3$
c $50\ ML = 50 \times 1000 = 50\,000\ m^3$
d 28 000 m^3 **e** 15 620 m^3 **f** 300 m^3

14 **a** $\frac{1}{2} \times (25 \times 18) \times 36$
$= 225 \times 36$
$= 8100\ cm^3$
b $1\ cm^3 = 1\ mL \therefore$ Capacity $= 8100\ mL$
c $1\ L = 1000\ mL \therefore$ Capacity $= \frac{8100}{1000}\ L = 8.1\ L$

15 **a** 5832 cm^3 **b** 5832 mL **c** 5.832 L

16 **a** 704 000 000 cm^3 **b** 704 000 000 mL
c 704 000 L **d** 704 kL

17 **a** 54 375 000 cm^3 **b** 54 375 L

18 **a** 907 200 cm^3 **b** 907.2 L

19 **a** 102 400 L **b** 192 000 L **c** $75.96

Language in mathematics

1

K	I	L	O	L	I	T	R	E	A	R	H	O	M	B	U	S
T	H	D	A	E	R	B	A	S	E	F	N	T	B	J	D	F
C	D	B	H	E	I	G	H	T	C	T	O	C	K	G	I	E
C	X	E	Z	P	R	E	F	I	X	H	I	E	M	H	M	I
T	R	A	P	E	Z	I	U	M	I	G	S	J	I	U	E	L
A	M	E	A	S	U	R	E	A	Y	I	R	B	L	Y	N	E
E	B	N	W	K	V	L	L	T	H	E	E	O	L	J	S	R
L	L	I	Q	U	I	D	I	E	T	W	V	N	I	A	I	T
G	E	A	A	T	Y	C	T	N	M	Q	N	Z	G	R	O	I
N	N	T	U	X	A	P	R	I	S	M	O	M	R	E	N	L
A	G	N	W	P	T	R	E	V	N	O	C	Q	T	A	S	I
T	T	C	A	L	C	U	L	A	T	E	O	I	M	S	L	L
C	H	C	O	N	T	A	I	N	E	R	K	T	A	P	C	L
E	W	L	A	R	E	T	A	L	I	R	D	A	U	Q	U	I
R	P	E	R	I	M	E	T	E	R	Y	N	R	R	M	S	M

2 Two each of:
- **a** mm, cm, m, km
- **b** mm^2, cm^2, m^2, km^2
- **c** mL, L, kL, ML

3 **a** one-thousandth ($\frac{1}{1000}$) **b** one-hundredth ($\frac{1}{100}$) **c** one-thousand (1000)

4 space, prism, conversion, surface, perimeter, volume, litre, capacity

5 height, length, millilitre

Check your skills

1 C **2** C **3** B **4** A **5** C
6 C **7** A **8** C **9** D **10** D
11 A **12** C **13** A **14** B **15** A
16 C

Review set 5A

1 **a** 17.2 cm **b** 34.8 cm
2 **a** 97.1 cm **b** 74.4 cm
3 94.09 cm^2 **4** 87.6 cm^2
5 No **6** 114 cm^2
7 **a** 12.71 m^2 **b** 89.06 cm^2
8 **a** $A = lb$ **b** $A = \frac{1}{2}xy$ **c** $A = \frac{1}{2}xy$
9 **a** 4.32 m^2 **b** 4.8 km^2
10 **a** 500 mm^2 **b** 0.1130 cm^2 **c** 4.8 cm^2
d 23 000 cm^2 **e** 688 mm^2 **f** 0.000 25 ha
g 0.04 L **h** 63 m^3 **i** 20 m^3
11 4.8 L
12 **a** 25 132 cm^3 **b** 25.132 L

Review set 5B

1 89 cm
2 **a** 182.9 cm **b** 105.58 cm
3 **a** 20.02 cm^2 **b** 16.12 cm^2 **c** 42 cm^2
4 Perimeter is the distance around a shape. Area is the amount of space inside a flat object. Volume is the space occupied by an object.
5 **a** 23.46 cm^2 **b** 1.84 m^2
6 **a** $A = s^2$ **b** $A = \frac{1}{2}bh$ **c** $A = bh$
7 **a** 296 m^2 **b** 21.09 cm^2
8 **a** 200 mm^2 **b** 0.03 m^2 **c** 9.3 cm^2
d 42 000 cm^2 **e** 3390 mm^2 **f** 0.27 ha
g 0.6 L **h** 15.3 m^3 **i** 4 m^3
9 **a** 139.872 m^3 **b** 139 872 L

Review set 5C

1 17.3 cm
2 $x = 10.42$ cm, $y = 14.5$ cm, $z = 14.5$ cm, $w = 7.2$ cm
perimeter = 96.6 cm
3 127.68 cm^2
4 B Square by 7.05 cm^2
5 **a** 41.055 cm^2 **b** 248 cm^2
6 **a** $A = \frac{1}{2}xy$ **b** $A = bh$ **c** $A = lb$
7 **a** 10.865 cm^2 **b** 26.04 cm^2
8 **a** 400 mm^2 **b** 0.07 m^2
c 5.3 cm^2 **d** 95 000 cm^2
e 3290 cm^2 **f** 0.73 ha
g 6 L **h** 2 m^3
i 5 ML **j** 3250 cm^3
9 **a** 2000 cm^3 **b** 860 cm^3
10 **a** 0.6 L **b** 18 L

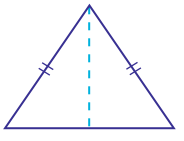
Review set 5D

1 **a** $x = 16.24$ cm, $y = 6.4$ cm; perimeter = 71.8 cm
b $x = 2.33$ cm, $y = 7.93$ cm; perimeter = 53.98 cm
2 **a** 67.24 cm^2 **b** 54.05 cm^2 **c** 101.675 cm^2
3 560 cubes
4 Volume is the amount of space in m^3 or other linear unit. Capacity is the amount of liquid that would be held, given in litres or other unit.
5 **a** 48.97 cm^2 **b** 29.87 m^2 **c** 29.87 cm^2
6 **a** $A = lb$ **b** $A = \frac{1}{2}xy$
c $A = \frac{1}{2}h(a + b)$
7 **a** 150.5 cm^2 **b** 60 cm^2
8 **a** 500 mm^2 **b** 40 000 mm^2
c 75 cm^2 **d** 85 000 cm^2
e 0.54 m^2 **f** 6 ha
g 100 mm^2 **h** 1 000 000 mL
i 1 000 000 cm^3 **j** 1000 L
k 3 kL **l** 2000 m^3
9 **a** 189 000 L **b** $34.59

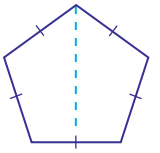
CUMULATIVE REVIEW CHAPTERS 2–5

1 **a** It is a comparison of like quantities.
b No units as they are like quantities.
c 20 : 7
d **i** 5 : 18 **ii** 4 : 3 **iii** 2 : 3 **iv** 5 : 3
e 2 : 1 **f** 5 : 13
g $x = \frac{15}{8}$
h **i** 23 : 26 **ii** 3 : 26
i $x = 60$
j **i** 17.05 cm **ii** 14 cm
k B **l** 700 kg : 800 kg
m \$1040 : \$560 : \$400 **n** 0.4 km/min
o 30 cents **p** 6.4 m
q $416\frac{2}{3}$ mm

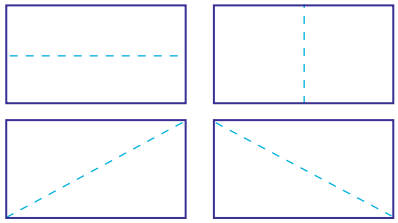
2 **a** **i** $\triangle ABC \equiv \triangle YZX$ **ii** ZX **iii** $\angle Z$
b **i** $\triangle ABC \equiv \triangle NLM$ **ii** $\triangle PQR \equiv \triangle SKT$
iii $\triangle MNT \equiv \triangle PRY$ **iv** $\triangle STU \equiv \triangle XWV$
c **i**



ii



iii



d **i** Reflection **ii** Translation

e RHS

f **i** Equal angles is not a congruence test.

ii $\angle 70°$ is opposite side 8 units in one triangle but not the other triangle.

iii Side 6 is opposite 70° in one triangle and opposite 60° in the other.

g The equal sides are not corresponding sides.

h 2 and 3

i AAS, $x = 12$, $y = 7$

j *Proof:* In $\triangle PTR$ and $\triangle PTQ$
$\angle R = \angle Q$ (given)
$PT = PT$ (common side)
$\angle PTR = \angle PTQ$ (by construction)
$\therefore \triangle PTR \equiv \triangle PTQ$ (AAS)
$\therefore PR = PQ$ (matching sides of congruent $\triangle$s)

k *Proof:* In $\triangle AED$ and $\triangle CEB$
$\angle DAE = \angle BCE$ (alternate angles and $AD \parallel BC$)
$\angle AED = \angle BEC$ (vertically opposite angles)
$AD = BC$ (opposite sides of a parallelogram equal)
$\triangle AED \equiv \triangle CEB$ (*AAS*)
$\therefore AE = EC$ (matching sides of congruent $\triangle$s)
and $DE = BE$ (matching sides of congruent $\triangle$s)

3 **a** 2^6 **b** Base = 4, index = 7

c $3 \times 3 \times 3 \times 3 \times 3$ **d** No, unequal bases

e **i** 625 **ii** 7776 **iii** 32 768

f **i** 5^{15} **ii** 3^{20} **iii** 7

g **i** 7^{15} **ii** 5^{12} **iii** 3^4

h $(3^4)^5 = 3^{20}$ not 3^9 as $3^4 \times 3^5 = 3^9$

i **i** −30 **ii** 54 **iii** 40

j **i** −5 **ii** −3 **iii** −4

k **i** 1 **ii** 11 **iii** 4

iv −24 **v** 4 **vi** −9

4 **a** **i** 17 cm **ii** 14.4 cm

b 4.2 cm

c **i** 64 m^2 **ii** 60 mm^2

iii 45 cm^2 **iv** 80 cm^2

v 90 cm^2 **vi** 1250 mm^2

vii 13.72 m^2 **viii** 12.18 m^2

ix 51.46 cm^2 **x** 19.71 cm^2

xi 5.78 mm^2 **xii** 23.1 m^2

d **i** 90 cm^2 **ii** 15 m^2

iii 22.5 m^2 **iv** 17.15 m^2

e **i** 1 cm^3 = 1000 mm^3 **ii** 1 m^3 = 1 000 000 cm^3

iii kL = 1 000 000 mL **iv** 1 m^3 = 1000 L

f **i** 4 cm^2 = 400 mm^2 **ii** 120 cm^2 = 0.012 m^2

iii 340 mm^2 = 3.4 cm^2 **iv** 4.1 m^2 = 41 000 cm^2

v 6.32 cm^2 = 632 mm^2 **vi** 3.9 m^2 = 0.00 039 ha

vii 39 000 m^2 **viii** 95 cm^3 = 0.095 L

ix 58 kL = 58 m^3 **x** 0.05 ML = 50 m^3

g **i** 122.388 m^3 **ii** 122 388 L

h **i** 2.457 m^3 **ii** 311.364 cm^3

i **i** 2.4 m^3 **ii** 2400 L

j **i** 32 696 cm^3 **ii** 32.696 L

k **i** 108 kL **ii** $15.44

CHAPTER 6 TIME

Diagnostic test

1 C **2** C **3** C **4** A **5** C
6 D **7** A **8** D **9** B **10** B
11 B **12** D

Exercise 6A

1 **a** 8 o'clock **b** 12 o'clock
c 9:20 **d** 3:35

2 **a**

b

c

d

3 **a** 1:20:45 **b** 11:05:20
c 10:33:26 **d** 9:21:55

4 **a**

b

c

d

e

f

5 **a** 15 min **b** 55 min

Exercise 6B

1 **a** 7:05 **b** 12:17 **c** 12:06 **d** 11:53
e 6:50 **f** 2:45 **g** 2:30 **h** 8:35
i 5:48 **j** 3:51 **k** 9:49 **l** 11:52

2 a and i, b and iv, c and ii, d and iii

3 **a** 11:05 am **b** 07:26 am
c 04:26 pm **d** 01:44 pm

Exercise 6C

1 **a** 2:06 pm **b** 12:52 am **c** 8:02 pm
d 1:28 am **e** 8:15 am **f** 11:23 pm
g 7:18 pm **h** 11:11 am **i** 12:55 pm
j 5:39 pm **k** 6:34 am **l** 10:42 pm

2 **a** 1932 **b** 0520 **c** 0132
d 1252 **e** 1630 **f** 1347
g 0609 **h** 0817 **i** 2125
j 2301 **k** 2400 **l** 2229

3 **a** 1304, five past seven at night, 1926 and 8:31
b 8 past midnight, two thirty in the morning, 0805, 9:06, 1715
c 1038, midday, 1204, 12:54, seven twenty-three at night, 11:45 pm

4 **a** 0730 **b** 0900 **c** 0931
d 1457 **e** 1654 **f** 1002

5 **a** 1411 **b** 2028 **c** 0816
d 0746

6 **a** 1630 to 1730 **b** 0600 to 0630
c 2325 to 0130 **d** 2030 to 2200

7 **a** 4:10 am **b** 9:50 am **c** 11:20 am
d 6:40 pm **e** 9:30 pm

8 **a** 5:50 am **b** 6.30 am
c 10.15 am **d** 4 h and 38 min

Exercise 6D

1 **a** 1 h 30 min **b** 1 h 48 min **c** 1 h 39 min
d 1 h 17 min **e** 1 h 21 min **f** 1 h 35 min

2 **a** 1 h 40 min **b** 1 h 17 min
c 1 h 48 min **d** 1 h 55 min

3 **a** 0855 h **b** 1132 h
c 1821 h **d** 1459 h

4 **a** 0700 + 0330 = 1030 h **b** 1643 − 0212 = 1431 h

5 **a** 1030 h **b** 1120 h **c** 1140 h
d 1409 h **e** 0035 h **f** 2105 h

6 **a** 3 h 25 min + 4 h 42 min

hours	*minutes*
3	25
+ 4	42
7	67

= 1 h 7 min

1	7
8 h	7 min

b Convert 9 h 14 min to 8 h 74 min by taking 1 h (60 min) from 9 h.

hours	*minutes*
8	74
− 3	28
5 h	46 min

7 **a** 4 h 35 min **b** 4 h 30 min
c 7 h 16 min **d** 1 h 45 min
e 3 h 13 min **f** 25 min
g 6 h 2 min **h** 52 min
i 11 h 50 min **j** 2 h 50 min
k 15 h 27 min **l** 1 h 54 min

8 **a** 13 min 30 s **b** 22 min 23 s
c 29 min 20 s **d** 21 min 50 s
e 2 min 14 s **f** 1 min 52 s
g 5 min 26 s **h** 1 min 14 s
i 43 min 51 s

9 **a** 8 days 10 h **b** 6 days 20 h
c 3 days 7 h **d** 2 days 9 h
e 4 wk 1 d **f** 13 wk 4 d
g 2 wk 5 d **h** 6 wk 4 d

10 **a** 5 h 19 min **b** 3 h 21 min
c 5 h 28 min **d** 6 h 35 min
e 7 h 25 min **f** 11 h 29 min
g 9 h 15 min **h** 10 h 45 min
i 9 h 51 min

11 **a** 1 days 13 h 52 min **b** 4 days 5 h 7 min
c 4 days 13 h 23 min **d** 1 day 5 h 39 min
e 2 days 23 h 21 min **f** 5 days 6 h 51 min

Exercise 6E

1 **a** 5° 0′ 0″ **b** 4° 0′ 0″ **c** 3° 12′ 0″
d 7° 23′ 0″ **e** 2° 45′ 18″ **f** 9° 02′ 36″
g 0° 36′ 0″ **h** 0° 0′ 55″ **i** 6° 59′ 45″

2 **a** 9 h 39 min **b** 9 h 40 min
c 6 h 34 min **d** 5 h 54 min 36 s
e 1 h 40 min 25 s **f** 2 h 48 min 43 s
g 3 h 40 min 8 s **h** 4 h 6 min 38 s
i 1 h 25 min 23 s **j** 29 min 44 s
k 5 h 15 min 59 s **l** 3 h 43 min 34 s

3 **a** 2.000 **b** 5.217 **c** 6.467
d 3.533 **e** 1.814 **f** 2.010
g 0.767 **h** 0.417 **i** 0.007
j 0.004 **k** 0.149 **l** 0.496

4

Minutes	Fraction		Decimal (2 decimal places)
	60	Simplified	
5	$\frac{5}{60}$	$\frac{1}{12}$	0.08
6	$\frac{6}{60}$	$\frac{1}{10}$	0.10
12	$\frac{12}{60}$	$\frac{1}{5}$	0.20
15	$\frac{15}{60}$	$\frac{1}{4}$	0.25
18	$\frac{18}{60}$	$\frac{3}{10}$	0.30
20	$\frac{20}{60}$	$\frac{1}{3}$	0.33
24	$\frac{24}{60}$	$\frac{2}{5}$	0.40
27	$\frac{27}{60}$	$\frac{9}{20}$	0.45
30	$\frac{30}{60}$	$\frac{1}{2}$	0.50
36	$\frac{36}{60}$	$\frac{3}{5}$	0.60
40	$\frac{40}{60}$	$\frac{2}{3}$	0.67
42	$\frac{42}{60}$	$\frac{7}{10}$	0.70
45	$\frac{45}{60}$	$\frac{3}{4}$	0.75
48	$\frac{48}{60}$	$\frac{4}{5}$	0.80
54	$\frac{54}{60}$	$\frac{9}{10}$	0.90
60	$\frac{60}{60}$	1	1

5 **a** 1 h 30 min 0 s **b** 3 h 45 min 0 s
c 6 h 48 min 0 s **d** 4 h 40 min 0 s
e 5 h 50 min 0 s **f** 8 h 25 min 0 s
g 2 h 26 min 40 s **h** 7 h 52 min 30 s
i 3 h 36 min 0 s **j** 6 h 10 min 0 s
k 10 h 37 min 30 s **l** 11 h 27 min 41.54 s
m 5 h 21 min 49.09 s **n** 8 h 8 min 34.29 s
o 12 h 22 min 30 s

6 **a** 2 h 40 min 48 s **b** 1 h 3 min 0 s
c 4 h 49 min 12 s **d** 3 h 36 min 0 s
e 5 h 24 min 0 s **f** 8 h 0 min 0 s
g 0 h 30 min 0 s **h** 0 h 12 min 0 s
i 0 h 24 min 0 s **j** 0 h 51 min 36 s
k 0 h 20 min 24 s **l** 0 h 34 min 26.4 s
m 10 h 2 min 24 s **n** 0 h 2 min 42 s
o 3 h 59 min 15.5 s

7 **a** **i** 3° **ii** 3° **iii** 6° **iv** 6°
v 13° **vi** 4° **vii** 2° **viii** 8°
b **i** 0° 28′ **ii** 0° 32′ **iii** 2° 13′ **iv** 1° 49′
v 8° 41′ **vi** 9° 17′ **vii** 0° 32′ **viii** 1° 57′

8 **a** 228 min or 3 h 48 min **b** 2280 min or 38 h
9 7 min 15 s **10** 5 h 10 min
11 **a** 6 h 23 min **b** 7 h 11 min
c 4 h 26 min **d** 2 h 23 min
e 6 h 40 min **f** 7 h 51 min

Exercise 6F

1 **a** If there are 29 days in February then it is a leap year.
b There are only 28 days in February, therefore it is not a leap year.
c Monday **d** Friday
e 48 days **f** 9 October
g 12 weeks
h 8 August, 12 September, 10 October, 14 November, 12 December
i 19 November **j** 2 August

Exercise 6G

1 **a** **i** 5:56 pm **ii** 6:24 pm
b 7:42 pm
c **i** 21 min **ii** 39 min
iii 48 min **iv** 15 min
d **i** 3:32 pm takes 52 min, 4:11 pm takes 46 min, 4:45 pm takes 48 min, 5:23 pm takes 57 min, 5:56 pm takes 54 min.
ii Times vary due to peak times.
2 **a** **i** 1 h 16 min **ii** 38 min
iii 43 min **iv** 29 min
b **i** 9:41 am **ii** 10:43 am
iii 11:36 am **iv** 12:14 pm
c **i** No **ii** Strathfield
iii 10:53 am **iv** 1 h 12 min
v 25 min
3 **a** **i** 12 min **ii** 1 h 30 min
b 4 h 30 min **c** 5:05 pm
4 **a** **i** 14 days **ii** 15 days
b **i** 9 h 58 min **ii** 10 h 46 min
5 **a** 2 **b** 2
c **i** 8:34 am, 8:54 pm **ii** 2.30 am, 2:40 pm
d The tide was going from low to high.
e Low tide at 3:45 am, height 0.4 m
Low tide at 3:39 pm, height 0.5 m
High tide at 9:46 am, height 1.4 m
High tide at 10:00 pm, height 1.7 m
f **i** 6 h 4 min **ii** No. 13 min difference
g **i** 9:51 am **ii** 9:54 am
6 **a** **i** 18 min **ii** 11:05 am **iii** 10 min
b **i** 19 min **ii** 1:21 pm **iii** 15 min
iv 1:39 pm **v** 2:52 pm

Exercise 6H

1 **a** 4 am **b** 6 am **c** 8 pm **d** 9 pm
2 **a** 2 am **b** 1 am **c** 7 pm **d** 8 pm
3 **a** 2 pm **b** 1:30 pm **c** 9:30 am **d** 8 am
4 **a** 8:45 am **b** 1:15 am **c** 9:15 am **d** 7:15 am
5 **a** 16 April **b** 3:50 am
6 **a** 2:40 pm **b** 9:05 am **c** 6:50 am
d 12:30 am **e** 7:30 am **f** 2:27 pm
7 **a** Midnight **b** 11 pm
c 10 am **d** 1 am Sunday
8 **a** 8 am Thursday **b** 12 noon Thursday
c 7 pm Wednesday **d** 1 am Thursday
9 **a** 12:00 pm **b** 2:00 pm **c** 2:00 pm
d 2:00 pm **e** 1:30 pm **f** 1:30 pm
10 **a** 7:20 am **b** 6:50 am **c** 7:20 am
d 7:20 am **e** 5:20 am **f** 6:50 am
11 **a** 4:00 am **b** 6:00 am
c 7:00 am **d** 5:30 am
12 **a** 2:30 pm **b** 5:30 pm
c 5:00 pm **d** 5:30 pm
13 **a** 8:30 am **b** 4:50 pm

Check your skills

1 D	**2** A	**3** A	**4** B	**5** B
6 D	**7** A	**8** A	**9** D	**10** B
11 B	**12** D	**13** D	**14** C	**15** A
16 B	**17** B	**18** B	**19** D	**20** B
21 A	**22** D	**23** A	**24** B	**25** C

Review set 6A

1 4:15 pm **2** 3:32 pm
3 1328
4 **a** 6 h 20 min **b** 1 h 42 min
5 8:34 am **6** 3 h 40 min
7 3 h 20 min **8** 2° 16°
9 15 h 53 min
10 A year in which there are 366 days
11 55 h 20 min **12** 10:30 am
13 4 pm

Review set 6B

1 11:41 pm
2 9:17 pm
3 1505
4 **a** 6 h 7 min **b** 2 h 22 min 33 s
5 **a** 2 h 23 min **b** 1 h 46 min
6 14 h 21 min **7** 7 h 25 min 43 s
8 7 h 26 min **9** 5 days 7 h 40 min
10 101 h 41 min **11** 5 am
12 4 pm

Review set 6C

1 **a** 2:40 or 20 minutes to 3 o'clock
b 10:50 or 10 minutes to 11 o'clock
2 **a** 0015 **b** 1846
3 3 h 32 min
4 **a** 247 days **b** 452 days
5 43 min 38 s **6** 1 day 7 h
7 10 h 46 min **8** 3 days 16 h 32 min
9 **a** 12 h 16 min **b** 6 h 38 min
10 27 August **11** 7:27 am
12 4:30 am

Review set 6D

1 **a** 2:35 or 25 minutes to 3 o'clock
b 12:21 or 21 minutes past 12 o'clock
2 **a** 0438 **b** 2322
3 5 h 45 min **4** 7 h 33 min 59 s
5 48 min **6** 2 days 11 h
7 1 day 13 h 11 min
8 **a** 4° 58′ **b** 6° 34′
9 **a** 210 days **b** 268 days
10 29 April **11** 48 min
12 2 pm **13** 5:30 am

CHAPTER 7 PERCENTAGES

Diagnostic test

1 C	2 B	3 D	4 A	5 B
6 D	7 A	8 B	9 D	10 A
11 A	12 B	13 A	14 D	15 A
16 D				

Exercise 7A

1 **a** $\frac{51}{100}$ **b** $\frac{89}{100}$ **c** $\frac{47}{100}$
d $\frac{61}{100}$ **e** $\frac{97}{100}$ **f** $\frac{42}{100} = \frac{21}{50}$
g $\frac{65}{100} = \frac{13}{20}$ **h** $\frac{75}{100} = \frac{3}{4}$ **i** $\frac{18}{100} = \frac{9}{50}$
j $\frac{45}{100} = \frac{9}{20}$ **k** $\frac{50}{100} = \frac{1}{2}$ **l** $\frac{36}{100} = \frac{9}{25}$
m $\frac{54}{100} = \frac{27}{50}$ **n** $\frac{98}{100} = \frac{49}{50}$ **o** $\frac{66}{100} = \frac{33}{50}$

2 **a** 1 **b** 4 **c** $2\frac{1}{2}$ **d** $3\frac{3}{4}$ **e** $1\frac{9}{10}$
f $6\frac{1}{5}$ **g** $5\frac{27}{50}$ **h** $2\frac{9}{25}$ **i** $7\frac{2}{25}$ **j** $12\frac{3}{10}$

3 **a** $\frac{7}{75}$ **b** $\frac{61}{400}$ **c** $\frac{27}{250}$ **d** $\frac{31}{600}$ **e** $\frac{13}{150}$

4 **a** 0.08 **b** 0.09 **c** 0.46 **d** 0.65
e 0.58 **f** 0.02 **g** 0.26 **h** 0.04
i 0.77 **j** 0.84

5 **a** 3.06 **b** 1.54 **c** 2.63 **d** 8.56
e 2.87 **f** 7.42 **g** 7.33 **h** 1.13
i 9.22 **j** 5.69

6 **a** 25% **b** 550% **c** 800%
d 630% **e** 15% **f** 950%
g 5% **h** 250% **i** 720%
j 8% **k** 160% **l** 1700%
m 80% **n** 40% **o** 90%
p 86% **q** 575% **r** 0.1%
s 600% **t** 1500%

7 **a** 54.55% **b** 92.31% **c** 87.50%
d 55.56% **e** 93.75% **f** 89.47%

8 **a** $\frac{2}{5}$ 0.4 40% **b** $\frac{7}{10}$ 0.7 70%
c $\frac{1}{5}$ 0.2 20% **d** $\frac{1}{20}$ 0.05 5%
e $\frac{4}{5}$ 0.8 80% **f** $\frac{37}{100}$ 0.37 37%
g $7\frac{4}{5}$ 7.08 708% **h** $1\frac{3}{5}$ 1.6 160%
i $\frac{3}{500}$ 0.006 0.6 **j** $\frac{41}{50}$ 0.82 82%
k $11\frac{1}{500}$ 11.002 1100.2% **l** $\frac{3}{8}$ 0.375 37.5%

9 **a** $\frac{1}{4}$, 72%, 0.92 **b** 0.09, $\frac{1}{5}$, 86%
c 0.17, 23%, $\frac{7}{8}$ **d** 0.582, $\frac{3}{5}$, $62\frac{1}{2}$%
e 0.44, 70%, $\frac{5}{6}$ **f** 35%, 0.36, $\frac{2}{5}$
g 52%, 0.74, $\frac{18}{20}$ **h** 0.37, $\frac{3}{8}$, 62%

Exercise 7B

1 **a** 40% **b** 20% **c** 16% **d** 36%
e 56% **f** 20% **g** 30% **h** 54%
i 72% **j** 25% **k** 50% **l** 25%

2 **a** 20% **b** 48% **c** 300% **d** 20%
e 65% **f** 905% **g** 30% **h** 17%
i 900% **j** 75% **k** 25% **l** 204%
m 25% **n** 50% **o** 40%

3 **a** \$117 **b** 320 L **c** \$180
d 8.50 **e** 7.68 kg **f** 14.4 m
g \$200 **h** 120 m **i** 98.56 kg
j 151.2 L **k** \$114.24 **l** \$16.38

4 **a** \$4.80 **b** 19.8 L **c** 2.4 m
d 90 kg **e** \$20.80 **f** 832 km

Exercise 7C

1 **a** $\frac{15}{100} \times 400 = 60$
New amount = \$400 + \$60 = \$460
b $\frac{40}{100} \times 200 = 80$
New amount = 200 g + 80 g = 280 g

2 **a** \$345 **b** 140 g **c** 108 L
d \$153 **e** 81.60 **f** 570

3 **a** \$12 **b** 18 **c** 30 kg
d 63 m **e** 138 km **f** 24 L

4 **a** 111% **b** 138% **c** 155% **d** 192%
e 168% **f** 186% **g** 139% **h** 207%
i 256% **j** 313%

5 **a** 91% **b** 85% **c** 82% **d** 77%
e 68% **f** 53% **g** 34% **h** 49%
i 5% **j** 22%

6 **a** 100% + 10% = 110%
$\therefore 1.1 \times 70 = 77$ km
b 100% + 80% = 180%
$\therefore \frac{180}{100} \times 20 = 365$ t

7 **a** 77 km **b** 108 L **c** \$1920 **d** 15.30 L
e 42 km **f** 15.75 m

8 **a** 4 t **b** 12 km **c** 22 L **d** \$450
e 36 g **f** 63 m

9 **a** Increase, \$8.50 **b** Increase, \$259 200
c Decrease, \$833 **d** Decrease, \$19 800
e Increase, \$9.90 or \$10 to nearest dollar

10 **a** 60% **b** 44% and 56%
c More time in possession, fewer tackles made.

Exercise 7D

1 **a** Increase = 60 − 48 = 12 g
% increase = $\frac{12}{48} \times 100\% = 25\%$
$\therefore$ 48 g to 60 g is a 25% increase.
b Increase = 45 − 30 = 15 L
% increase = $\frac{15}{30} \times 100\% = 50\%$
$\therefore$ 30 L to 45 L is a 50% increase.

2 **a** 60% **b** 150% **c** 400%
d 90% **e** 25% **f** 160%
g 50% **h** 40% **i** 400%

3 **a** Decrease = 70 − 42 = 28 L
% decrease = $\frac{28}{70} \times 100\% = 40\%$
$\therefore$ 70 L to 42 L is a 40% decrease.
b Decrease = 200 − 50 = 150 m
% decrease = $\frac{150}{200} \times 100\% = 75\%$
$\therefore$ 200 m to 50 m is a 75% decrease.

4 **a** 60% **b** 40% **c** 70%
d 40% **e** 75% **f** 25%
g 75% **h** 50% **i** 80%

5 **a** 69%, 69%, 67% **b** 88%

6 **a** 26% **b** 42.4% **c** 73%

Exercise 7E

1 **a** 820 m **b** 2100 g **c** 940 km
d 225 **e** 230 g **f** $50
g 5000 mL **h** 50 000 kg **i** $2.72
2 600 students
3 150
4 $85 000
5 **a** $250 **b** $1027 **c** $220
6 **a** $512 **b** $138 **c** $399
d $480 **e** $85
7 **a** $4000 **b** $95 **c** $450 **d** $2500
8 **a** $18.40 **b** 373.8 kg **c** 1276 L
d 1395 km **e** 440.64 m **f** $833.66

Exercise 7F

1 **a** The cost of an item, usually before adjustments to sell.
b The amount an item is sold for.
c Selling price minus cost price.
d Occurs if selling price is less than cost price.
2 **a** Profit = 700 − 500 = $200
b Profit = 520 − 380 = $140
c Profit = 975 − 482 = $493
3 **a** Loss = 800 − 530 = $270
b Loss = 945 − 645 = $300
c Loss = 1050 − 822 = $228
4 **a** **i** Loss **ii** $3400
b **i** Profit **ii** $1796
c **i** Profit **ii** $184 000
d **i** Loss **ii** $18 260
5 **a** $1500 **b** 37.5%
6 **a** $15 000 **b** 25%
7 33.3%
8 **a** $7000 **b** 140%
9 25% **10** 53.8%
11 **a** $851 **b** $567 **c** $4550
d $192.50 **e** $90 **f** $1258.60
12 **a** $130 **b** $270 **c** $78
d $50 **e** $140 **f** $300
13 **a** $360 **b** $350 **c** $1420
d $88 **e** $75 **f** $75.38
14 **a** **i** $400 **ii** 5%
b **i** $4500 **ii** 20.45%
c **i** $456 **ii** $4256
d **i** $420 **ii** $780
e **i** $209.09 **ii** $66.91
f **i** $381.82 **ii** $171.82
15 **a** **i** $11 908 **ii** $57 708
b **i** $558.44 **ii** $128.44
c **i** $5684.68 **ii** $625.32
d **i** $7250 **ii** 44%
e **i** $240 **ii** 28%
f **i** $1804.80 **ii** $2035.20
16 $7654.40

Exercise 7G

1 **a** $44, $484 **b** $369, $4059
c $25.80, $283.80 **d** $39.70, $436.70
e $180, $1980
2 **a** $104.50 **b** $247.50
c $18.70 **d** $369.60
e $502.70 **f** $438.90
3 **a** **i** $108.09 **ii** $1080.90
b **i** 451.82 **ii** $4518.18
c **i** $0.99 **ii** $9.90
d **i** $13.45 **ii** $134.55
e **i** $11.27 **ii** $112.73
f **i** $45.36 **ii** $453.64
4 **a** Services rendered = $850, GST = $85
Total including GST = $935
b Services rendered = $488, GST = $48.80
Total including GST = $536.80
c Total including GST = $99.90
GST included in total = $9.08
d 5 CDs @ $32.90 including GST = $164.50
GST included in total = $14.95
5 **a** 10% of $100 = $10 so 100 + 10 = $110
b Not correct. John is finding 90% of an amount that is 110% not 100% of the original amount.
6 **a** $150: $150 × 110% = 150 × 1.1 = $165
b $220: $220 × 110% = 220 × 1.1 = $242
c $370: $370 × 110% = 370 × 1.1 = $407
d $400: $400 × 110% = 400 × 1.1 = $440
7 **a** $165 × 90% = 165 × 0.9 = $148.50
b $242 × 90% = 242 × 0.9 = $217.80
c $407 × 90% = 407 × 0.9 = $366.30
d $440 × 90% = 440 × 0.9 = $396.00
8 **a** Amounts change, in fact they have decreased.
b False. 100% is increased by 10% = 110%.
c When 110% is decreased by 10% it means that 10% of a larger amount is being found and then deducted.

Check your skills

1 A **2** D **3** C **4** C **5** A
6 C **7** B **8** D **9** D **10** C
11 B **12** A **13** B **14** C **15** C
16 D

Review set 7A

1 30 kg **2** 33.33%
3 140 **4** 210
5 68.75% **6** 20%
7 $420 **8** $399
9 **a** $1700 **b** 141.7%
10 $131.12 **11** $90
12 **a** $17.18 **b** $171.82

Review set 7B

1 73% **2** 22% **3** 10%
4 $212 **5** 10.5 kg
6 **a** $331.20 **b** $158.40 **c** $230.40
7 37.5% **8** 12.88 m
9 32c **10** $426
11 $94.41 **12** $8.90

Review set 7C

1 525% **2** 80%
3 2100 km **4** 89.04 m
5 54.8% **6** $56
7 $89
8 **a** $1200 **b** 150%
9 $145.83 **10** 26%
11 $38.50 **12** $90

Review set 7D

1 375%
2 55%
3 $57.60
4 $4680
5 $490 434.78, so $490 435
6 89.6%
7 30.4%
8 78 kg
9 $650
10 $215
11 $268.73
12 $351.85
13 $180
14 $104.50

CHAPTER 8 CIRCLES AND CYLINDERS

Diagnostic test

1 A **2** B **3** B **4** D **5** B
6 C **7** A **8** D **9** B **10** B
11 C **12** D **13** C **14** B **15** B
16 B **17** D **18** D

Exercise 8A

1 **a** Chord **b** Major segment **c** Semicircle
d Centre **e** Diameter **f** Circumference
g Sector **h** Radius **i** Minor segment
j Tangent **k** Arc **l** Sector

2 **a** Sector, radius, arc
b Centre, circumference
c Arc
d Centre, circumference, radius

3 **a** **i** Radius **ii** Diameter **iii** Tangent
iv Arc **v** Chord **vi** Radius
b Sector

4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{7}{18}$ **d** $\frac{1}{10}$ **e** $\frac{2}{9}$ **f** $\frac{2}{3}$
g $\frac{1}{8}$ **h** $\frac{8}{9}$ **i** $\frac{3}{8}$ **j** $\frac{2}{5}$ **k** $\frac{3}{4}$ **l** $\frac{1}{15}$

Exercise 8B

1 **a** $9\pi = 28.27$ cm **b** $14\pi = 43.98$ cm
c $5\pi = 15.71$ cm **d** $21.3\pi = 66.92$ cm
e $8.4\pi = 26.39$ cm **f** $10.6\pi = 33.30$ cm

2 **a** $18\pi = 56.55$ cm **b** $40\pi = 125.66$ cm
c $26\pi = 81.68$ cm **d** $9.2\pi = 28.90$ cm
e $34.4\pi = 108.07$ cm **f** $22.2\pi = 69.74$ cm

3 **a** 28.3 m **b** 20.1 m **c** 56.9 m
d 42.7 m **e** 39.6 m **f** 49.6 m
g 15.7 m **h** 27.1 m

4 **a** 12.6 cm **b** 6.3 cm **c** 11.0 cm

5 **a** $\frac{1}{6} \times 2 \times \pi \times 9 = 3\pi = 9.42$ cm
b $\frac{2}{9} \times 2 \times \pi \times 6 = \frac{8\pi}{3} = 8.38$ cm
c $\frac{1}{3} \times 2 \times \pi \times 7 = \frac{14\pi}{3} = 14.66$ cm
d $\frac{7}{12} \times 2 \times \pi \times 5 = \frac{35\pi}{6} = 18.33$ cm
e $\frac{29}{36} \times 2 \times \pi \times 12 = \frac{58\pi}{3} = 60.74$ cm
f $\frac{3}{4} \times 2 \times \pi \times 8 = 12\pi = 37.70$ cm

6 **a** 23.1 cm **b** 10.7 cm **c** 20.5 cm
d 13.4 cm **e** 7.0 cm **f** 28.3 cm
g 38.7 cm **h** 27.7 cm **i** 32.6 cm
j 21.5 cm **k** 26.7 cm **l** 49.7 cm

7 **a** 126 cm **b** 630 m
c **i** 794 **ii** 5556 **iii** 10 318

8 **a** 8.913 km
b **i** 12 **ii** 20 **iii** 27

9 **a** 11 198 m **b** 10 841 m
c 357 m **d** 1 h 52 min

Exercise 8C

1 **a** **i** 4.8 cm **ii** 2.4 cm
b **i** 11.8 cm **ii** 5.9 cm
c **i** 27.1 cm **ii** 13.5 cm
d **i** 31.8 cm **ii** 15.9 cm
e **i** 41.7 cm **ii** 20.8 cm
f **i** 55.7 cm **ii** 27.9 cm
g **i** 12.7 cm **ii** 6.4 cm
h **i** 3.0 cm **ii** 1.5 cm
i **i** 4.0 cm **ii** 2.0 cm
j **i** 60.0 cm **ii** 30.0 cm
k **i** 90.0 cm **ii** 45.0 cm
l **i** 14.0 cm **ii** 7.0 cm

2 15.9 cm **3** 15.9 cm

4 **a** 1.57 m **b** 50 cm

Exercise 8D

1 **a** 25π **b** 81π **c** 225π
d 100π **e** 42.25π **f** 12.25π

2 **a** 254.5 cm^2 **b** 380.1 cm^2 **c** 120.8 cm^2
d 145.3 cm^2 **e** 962.1 cm^2 **f** 475.3 cm^2
g 75.4 cm^2 **h** 118.8 cm^2 **i** 716.3 cm^2
j 8.3 cm^2 **k** 127.7 cm^2 **l** 23.8 cm^2

3 **a** 4.9 cm^2 **b** 12.6 cm^2 **c** 8.0 cm^2

4 $A = \frac{1}{4}$ of a whole circle
$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \pi \times 3.7^2$
$= 10.752 \ldots = 11$ cm^2

5 **a** 16 cm^2 **b** 4 cm^2 **c** 52 cm^2

6 **a** Fraction of a circle $= \frac{150°}{360°}$
$= \frac{5}{12}$ of a circle
b $A = \frac{5}{12}$ of a whole circle
$= \frac{5}{12} \times \pi \times r^2 = \frac{5}{12} \times \pi \times (7.4)^2$
$= 71.680 \ldots = 72$ cm^2

7 **a** $\frac{1}{2}$, 25 cm^2 **b** $\frac{1}{8}$, 48 cm^2 **c** $\frac{2}{3}$, 19 cm^2
d $\frac{3}{8}$, 21 cm^2 **e** $\frac{1}{6}$, 29 cm^2 **f** $\frac{7}{12}$, 15 cm^2
g $\frac{1}{5}$,143 cm^2 **h** $\frac{1}{12}$, 40 cm^2 **i** $\frac{1}{4}$, 61 cm^2

8 **a** 97.6 cm^2 **b** 138.7 cm^2 **c** 49.1 cm^2
d 322.5 cm^2 **e** 63.3 cm^2 **f** 38.6 cm^2
g 108.0 cm^2 **h** 129.0 cm^2 **i** 153.8 cm^2

9 **a** 115.5 cm^2 **b** 12.6 cm^2 **c** 21.2 cm^2
d 13.7 cm^2 **e** 13.6 cm^2 **f** 79.0 cm^2

10 **a** 63.2 m^2 **b** 186.8 m^2 **c** $8406

Exercise 8E

1 $V = \pi r^2 h$
$= \pi \times 7.3^2 \times 12.4$
$= 2075.951 \ldots = 2076$ cm^3

2 **a** 707 cm^3 **b** 42 412 cm^3
c 13 711 cm^3 **d** 10 177 cm^3

3 $V = \pi r^2 h$
$= \pi \times 12.9^2 \times 15.1$
$= 7894.165 \ldots$
$= 7894 \text{ cm}^3$

$d = 25.8 \text{ cm}$
$\therefore r = 25.8 \div 2$
$= 12.9 \text{ cm}$

4 **a** 9572 cm^3 **b** 11 344 cm^3
c 32 312 cm^3 **d** 6671 cm^3

5 **a** 339 cm^3 **b** 6773 cm^3
c 57 278 cm^3 **d** 15 606 cm^3

6 **a** 958.2 cm^3 **b** 3451.0 cm^3
c 2046.0 cm^3 **d** 2671.9 cm^3
e 164 787.5 cm^3 **f** 53 573.0 cm^3

7 **a** A **b** B

8 **a** 291 cm^3 **b** 2513 cm^3 **c** 33 cm^3
d 2976 cm^3 **e** 767 cm^3 **f** 2194 cm^3

9 1.88 m^3

10 **a** **i** 79 575 cm^3 **ii** 79.575 L
b **i** 29 302 cm^3 **ii** 29.302 L
c **i** 1 783 404 cm^3 **ii** 1783.404 L

11 76.35 kL

Check your skills

1 A	**2** C	**3** B	**4** A	**5** D
6 C	**7** D	**8** C	**9** B	**10** D
11 A	**12** B	**13** D	**14** A	**15** A

Review set 8A

1 **a** Radius **b** Tangent
2 $\frac{1}{5}$ **3** $C = \pi d$
4 43.7 cm **5** $C = 2\pi r$
6 35.94 cm **7** $A = \pi r^2$
8 **a** 254.5 cm^2 **b** 17.3 cm^2
9 59.4 cm^2 **10** 27 310 cm^3

Review set 8B

1 **a** Arc **b** Minor segment
2 **a** 30.2 cm **b** 32.4 cm
3 **a** 22.88 cm **b** 53.25 cm
4 **a** 196.1 cm^2 **b** 167.4 cm^2
5 **a** 103.84 cm^2 **b** 117.11 cm^2
6 86 cm^2 **7** 83 642 cm^3

Review set 8C

1 **a** Diameter **b** Sector
2 **a** $\frac{67}{72}$ **b** $\frac{13}{45}$
3 **a** **i** 24.6 cm **ii** 48.5 cm
b **i** 7.6 cm **ii** 15.1 cm
4 **a** **i** 105 cm^2 **ii** 367 cm^2
b **i** 33 cm^2 **ii** 114 cm^2
5 **a** 48.6 cm **b** 34.9 cm
6 **a** 77.3 cm **b** 1854.8 cm
7 **a** **i** 21 028 cm^3 **ii** 21.028 L
b **i** 13 856 cm^3 **ii** 13.856 L

Review set 8D

1 **a** 3.5 cm, 11 cm **b** 4.5 cm, 14 cm
2 **a** 47.35 cm **b** 45.42 cm
3 **a** 3219 m **b** 10 laps
4 26.9 cm^2 **5** 141 cm^2
6 323 cm^3 **7** 72.6 m^3 or 72 614 L

CHAPTER 9 DATA COLLECTION AND ANALYSIS

Diagnostic test

1 D	**2** C	**3** A	**4** B	**5** C
6 B	**7** B	**8** D	**9** C	**10** B
11 C	**12** C	**13** A	**14** C	**15** C
16 C	**17** C	**18** B		

Exercise 9A

1 **a** Discrete numerical **b** Categorical
c Continuous numerical **d** Discrete numerical
e Continuous numerical **f** Discrete numerical
g Categorical **h** Categorical
i Categorical **j** Categorical
k Discrete numerical **l** Continuous numerical
m Discrete numerical **n** Categorical
o Continuous numerical **p** Continuous numerical
q Categorical **r** Continuous numerical
s Categorical

Exercise 9B

1 **a** All new cars **b** All students in the school
c All houses sold **d** All the dogs treated
e All the people who use the bus route
2 **a** Census **b** Census or sample
c Observation **d** Census
e Sample **f** Observation
g Sample **h** Sample
i Census **j** Observation
k Sample **l** Census or sample
m Observation

Exercise 9D

1 **a** Biased towards people who are home on Friday night, have a phone and choose to answer it. (Some people only have a mobile phone, some people filter calls through an answering machine.)
b The sample would be biased towards the Labor Party.
c Only selects people who use email.
d Only selects people who use the Star bank.
e Only selects people 18 years or older.
f Only selects people who pass this street corner (live in this area, are fit enough to walk, …).
g If the sample is only 5 vets then the sample is too small.

2 **a** The people who live in a harbour-side suburb are more likely to be high income earners whose household spending habits would be different from those of people living in less affluent suburbs.
b The tobacco company would have an interest in showing that there are no hazards of smoking and would likely ignore any adverse findings.
c The information is very old. It is quite possible that the results of more recent studies would be different.
d If the company sold 1 item the first week and 2 items the second week then their sales would have doubled. Neither result is very good for the company.
e The students in a gymnastics class are very likely to be more flexible than the general population.

f It is possible that more hungry or sick birds would be attracted to the traps. The sample would be biased against healthy birds.
g The parents who are illiterate won't be able to read the invitation.

Exercise 9E

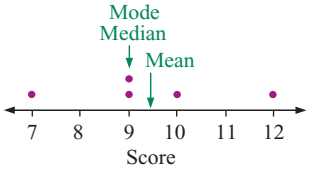
1 a Mean = 21, median = 20.5, mode = 19, range = 8
b Mean = 6.1, median = 6, mode = 6, range = 4
c Mean = 45.2, median = 47.5, mode = 56, range = 49
d Mean = 7.8, median = 8, mode = 8, range = 4

Exercise 9F

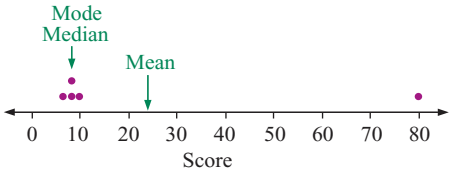
1 a Outlier = 104
b Cluster in the 60s
c Gap between 5 and 8
d Scores evenly distributed about 8
e Gap. No scores in the 70s
f Outlier = 17

Exercise 9G

1 a i Mean = 9.4, median = 9, mode = 9, range = 5
ii Mean = 23, median = 9, mode = 9, range = 73
b i



ii



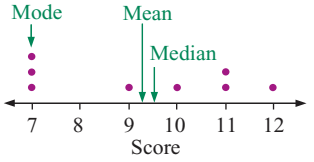
c i Mean, median and mode are all central/typical.
ii Mean is not central. It is affected by the outlier.
d i Range is good measure of spread.
ii Range is not a good measure of spread because of the outlier.

2 a Mean is appropriate.
b Mean is not central or typical as it is significantly affected by the outlier 136.

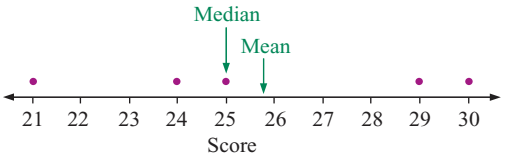
3 a Mean = 95 mm, median = 111.5 mm, mode = 112 mm
b 13 mm
c Mean = 111.4 mm, median = 112 mm, mode = 112 mm
d No. The measurement 13 mm is obviously an error.

4 a Mean = 24.2 min, median = 17 min. No mode.
b Outlier = 55 min
c Mean = 16.5 min, median = 16.5 min. No mode.
d Yes. The value 55 is not necessarily an error.

5 a i Mean = 9.25, median = 9.5, mode = 7
ii Mean = 25.8, median = 25, no mode
b i



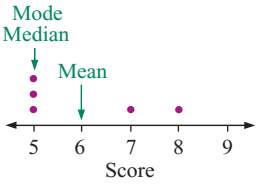
ii



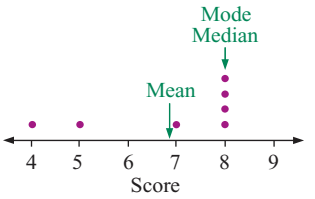
c i Mean and median are appropriate. Mode is not central.
ii Mean and median are appropriate. There is no mode.

6 a Mode occurs at one of the ends of the data set; that is, it is not central.
b Mode is appropriate.
c Mode occurs at the end of the data; that is, it is not central.

7 a i Mean = 6, median = 5, mode = 5
ii Mean = 6.9, median = 8, mode = 8
b i



ii



c i Median and mode occur at an end of the data; that is, they are not central. Mean is central.
ii Median and mode occur at an end of the data; that is, they are not central. Mean is appropriate.

8 a Median (= 13) is not central.
b Median (= 7) is central.
c Median (= 50) is not central.

9 a Mean = 0.7, median = 0.5, mode = 0
b Mode and median

10 a Mode = New Zealand. It is not possible to find mean or median.
b For non-numerical data (categorical data), the only relevant measure is the mode.

11 a $119 750
b i 6
ii 2
c $62 500
d $42 000
e i Mode
ii Mean
f Median. Half the incomes are less than it and half are greater than it.

12 a Mean
b No. Four of the five cars he sold were more expensive than his average.
c Median ($19 990). Half the prices are less than it and half are greater than it.

13 a Mean = 12.1 min, median = 11.5 min, mode = 16 min
b Mean and median are central and typical. Mode is not central but is important because more people had to wait this length of time than any other.

14 a Teacher has used mean. Kara has used mode.
b Median (4.5) because half her marks are less than it and half are higher than it.

Exercise 9H

1 a

Number of mistakes	Tally	f
0	|	1
1	||	2
2	||	2
3	𝍸	5
4	|||	3
5	||	2
6		0
7	||	2
8	|||	3
9		0

b Mean $= \frac{81}{20} = 4.05$, median $= 3.5$, mode $= 3$, range $= 8$

2 a

Number of mistakes	Tally	f
0	|	1
1	|||	3
2	||	2
3	|	1
4	||	2
5	||	2
6	||	2
7	||||	4
8	|	1
9	||	2

b Mean $= \frac{94}{20} = 4.7$, median $= 5.5$, mode $= 7$ range $= 9$

3 a

Number of mistakes	Tally	f
0	|||	3
1	𝍸	5
2	|	1
3		0
4	|	1
5	|	1
6	||	2
7	|	1
8	||	2
9	||||	4

b Mean $= \frac{87}{20} = 4.35$, median $= 4.5$, mode $= 1$, range $= 9$

4 a

Number of mistakes	Tally	f
0		0
1	||	2
2	𝍸	5
3	|	1
4		0
5	||	2
6	||	2
7	||	2
8	||||	4
9	||	2

b Mean $= \frac{101}{20} = 5.05$, median $= 5.5$, mode $= 2$, range $= 9$

5 a

Number of mistakes	Tally	f
0	|||	3
1	|	1
2		0
3	||	2
4	|	1
5	||	2
6	|	1
7	|||	3
8	|||	3
9	||||	4

b Mean $= \frac{108}{20} = 5.4$, median $= 6.5$, mode $= 9$, range $= 9$

8

	1st 20	2nd 20	3rd 20	4th 20	5th 20
Mean	4.05	4.7	4.35	5.05	5.4
Median	3.5	5.5	4.5	5.5	6.5
Mode	3	7	1	2	9
Range	8	9	9	9	9

a Mode **b** Mean

9 a Mean $= \frac{548}{120} = 4.6$, median $= 5$, mode $= 1$

10 a i Very little variation.
ii The sample ranges are very close to the population range (= 9).

b i

Sample	1	2	3	4	5	6	7	8	9	10
Range	8	7	1	7	8	4	6	6	9	7

Range varies from 1 to 9.
ii The larger the sample size the less variation in the value of the range.

Exercise 9I

1 a

Proportion of 6s in sample size 10				
1st group	Last group	7th group	12th group	17th group
$\frac{3}{10} = 30\%$	$\frac{2}{10} = 20\%$	$\frac{1}{10} = 10\%$	$\frac{2}{10} = 20\%$	$\frac{1}{10} = 10\%$

b i Lowest proportion $= \frac{1}{10}$ or 10%
ii Highest proportion $= \frac{3}{10}$ or 30%

c 2

d No. Only 2 of the 5 samples have a proportion of 6s close to the population proportion.

2 a

Proportion of 6s in		
1st group of 20	Last group of 20	4th group of 20
$\frac{4}{20} = 20\%$	$\frac{4}{20} = 20\%$	$\frac{3}{20} = 15\%$

3 Samples of size 20.

4 Proportion of 6s $= \dfrac{3+2+1+2+1}{10+10+10+10+10} = \dfrac{9}{50}$
$= 18\%$

This is the same as the population proportion.

6 The larger the sample size the better the estimate of the population proportion.

7 a

Group	1st 10	Last 10	5th 10	14th 10	17th 10
Mean	4.0	3.3	4.4	3.2	3.1

b i The lowest sample mean is 3.1.

ii The highest sample mean is 4.4.

c 2

d No. Only 2 of the 5 sample means are approximately the same as the population mean.

8 a

Group	1st 20	Last 20	2nd 20
Mean	3.45	2.9	3.45

9 Samples of size 20.

10 a i 3.9 **ii** 3.725 **iii** 3.6

b As the number of samples increases, the mean of the sample means gets closer to the population mean.

11 a i 3.3

b As the number of samples increases, the mean of the sample means gets closer to the population mean.

Exercise 9J

1 a If we let N represent the total number of fish in the lake, an expression for the fraction of tagged fish in the lake $= \frac{200}{N}$.

b The fraction of tagged fish in the second sample $= \frac{12}{90}$.

c Assuming that the fraction of tagged fish in the second sample is the same as the fraction in the lake.

$\frac{200}{N} = \frac{12}{90}$ Inverting both sides of the equation.

$\frac{N}{200} = \frac{90}{12}$ Multiplying both sides by 200.

$\frac{N}{200} \times 200 = \frac{90}{12} \times 200$

$N = 1500$

2 a $\frac{50}{N}$ **b** $\frac{4}{50}$ **c** 625

3 a $\frac{60}{N}$ **b** $\frac{3}{50}$ **c** 1000

4 4000 **5** 200

6 a

Sample	Number tagged	Population estimate
1	3	200
2	2	600
3	1	300

b ≈370 **c** 300

Language in mathematics

2 From the preceding exercises you should have discovered that proportions and means in samples vary. In general, the larger the sample size the closer the sample proportion is to the population proportion. Also the larger the sample size the closer the sample mean is to the population mean and as the number of samples increases the mean of the sample means gets closer to the mean of the population.

Check your skills

1 B **2** A **3** C **4** B **5** A
6 D **7** A **8** D **9** B **10** D
11 C **12** A **13** C **14** D **15** C

Review set 9A

1 a Discrete numerical **b** Continuous numerical
c Discrete numerical **d** Discrete numerical
e Categorical

2 a All smokers

b All cyclists who have suffered injuries

3 a Sample **b** Sample
c Census **d** Observation

4 Write the names of the 28 students on pieces of paper. Put the pieces of paper in a hat, shake and select 5 names. Using a class list, assign the numbers 1 to 28 to each of the students. Write the numbers 1 to 28 on cards, shuffle and deal 5. List the names of the students whose numbers have been selected.

5 a Members of the Shooters Party would be unlikely to be in favour of more gun control.

b If the salesman sold 1 item in the first week and 3 in the second week then his sales would have trebled. The sales for neither week are good for the company.

6 a 0 **b** 79

7 a Mean **b** Mode **c** Median

8 Mean and median are not possible. Mode = female

9 a i Mean = 25, median = 15, mode = 13

b Outlier is 88

c i Mean = 14.7, median = 14.5, mode = 13

d No. The outlier is obviously an error.

Review set 9B

1 a Discrete numerical **b** Categorical
c Categorical **d** Continuous numerical
e Categorical

2 a All Year 8 students

b The selling price of all homes recently sold in that suburb

3 a Sample **b** Census
c Sample **d** Observation

4 Take a list of the students in the class and assign each a number from 01 up to 30. Open a phone book at any page. Start at the last 2 digits in the first phone number. If this number is between 1 and 30 (inclusive) write it down, if not skip to the next phone number down the column. Repeat until you have selected 5 random numbers. List the names of the students whose numbers have been selected.

5 **a** Only people who would save money would bother to change insurance companies.
b Members of the Greenpeace organisation would all have similar views on climate change.
6 Mean and range
7 Mode
8 **a** Mean = 23.8, median = 23.5, mode = 29
b Mean and median are central; mode is not central.
9 **a** Mean = \$63 444, median = \$46 000, mode = \$46 000

Review set 9C

1 **a** Continuous numerical **b** Discrete numerical
c Categorical **d** Categorical
e Continuous numerical
2 **a** All new cars
b All parents of the school's students
3 **a** Census **b** Sample
c Census **d** Observation
4 On a CASIO calculator, press SHIFT (RAN#) =.
This produces a random number from 0 to 1. To obtain a random number from 1 to 30, multiply the result by 30 and round up to the next digit. Generate 5 random numbers from 1 to 30 (inclusive) using a calculator. List the names of the students whose numbers have been generated.
5 **a** If the sample is only 10 dentists then the sample is too small.
b The Hotels Association would have an interest in showing that drinking alcohol was beneficial to health and would likely ignore any adverse findings.
6 **a** Cluster in 40s and 50s.
b Gap between 8 and 12.
c Outlier = 24
7 Mode = Toyota
8 **a** Mean = 4, median = 3.5, mode = 2, range = 5
b Mean and range.
9 **a** Mean = 3.45, median = 1.5, mode = 0

Review set 9D

1 **a** Discrete numerical **b** Categorical
c Continuous numerical **d** Discrete numerical
e Categorical
2 **a** The managers of all farms in the region
b All makes of bicycle helmets
3 **a** Observation **b** Census
c Sample **d** Sample
4 Divide the number of students in the class by 5 and round up to the nearest whole number: $30 \div 5 = 6$. Select every 6th name from the class list.
5 **a** The sample only contains the responses of the 100 people who bothered to return the survey. These people have probably returned the survey because they have strong views on the subject. The sample should also include the views of those people who have little interest in the subject.
b The doctor already believes he has developed a cure so is more likely to report positive results. The people who report on the success of the treatment should be different from the people who administer the treatment.
6 **a** Cluster in 30s and 40s. Outlier = 94
b Clusters around 13 and 19. Gap between 14 and 18.
7 **a** 151
b **i** Mean = 63.1, median = 51, mode = 51, range = 103
ii Mean = 50.6, median = 51, mode = 51. range = 5
c No. The outlier is an obvious error.
8 **a** 55
b **i** Mean = 8, median = 3, no mode, range = 54
ii Mean = 2.8, median = 3, no mode, range = 4
c Yes, even though the mean is not a central value. There may be reasons for the outlier.
9 **a** **i** Mean = 19, median = 9, mode = 9
ii Mean = 14.9, median = 15, mode = 14, 15
iii Mean = 6.5, median = 6, mode = 5
b **i** Mean is not central because of the outlier 93.
ii All are central. **iii** Mode is not central.

CHAPTER 10 PYTHAGORAS' THEOREM

Diagnostic test

1 D **2** B **3** B **4** D **5** A
6 D **7** B **8** B **9** C **10** B
11 B **12** A **13** A **14** C **15** C
16 A

Exercise 10A

1 $\sqrt{a^2 + b^2} = c$
3 **a** r **b** n **c** d **d** y **e** t
f v **g** s **h** g **i** m
4 **a** $r^2 = p^2 + q^2$ **b** $n^2 = m^2 + l^2$ **c** $d^2 = e^2 + f^2$
d $y^2 = x^2 + z^2$ **e** $t^2 = u^2 + v^2$ **f** $v^2 = u^2 + w^2$
g $s^2 = u^2 + t^2$ **h** $g^2 = h^2 + k^2$ **i** $m^2 = k^2 + l^2$
5 **a** $AB^2 = AC^2 + BC^2$, $c^2 = b^2 + a^2$
b $QR^2 = PQ^2 + PR^2$, $p^2 = r^2 + q^2$
c $ML^2 = LN^2 + MN^2$, $n^2 = m^2 + l^2$

Exercise 10B

1 **a** No **b** Yes **c** No **d** No **e** No
f Yes **g** No **h** Yes **i** No
2 **a** Yes **b** No **c** No **d** Yes

Exercise 10C

1 **a** 9 **b** 16 **c** 100 **d** 64
e 121 **f** 289 **g** 225 **h** 484
i 0.36 **j** 0.25 **k** 3.24 **l** 5.29
m 24.01 **n** 27.04 **o** 46.6489 **p** 91.0116
2 **a** 5 **b** 7 **c** 6 **d** 12
e 13 **f** 20 **g** 25 **h** 33
i 0.2 **j** 0.9 **k** 1.1 **l** 2.1
m 3.7 **n** 5.4 **o** 15.4 **p** 20.02
3 **a** **i** 4.2 **ii** 2.6 **iii** 4.8 **iv** 9.1
b **i** 14.66 **ii** 19.65 **iii** 20.88 **iv** 26.85
c **i** 0.837 **ii** 1.378 **iii** 1.688 **iv** 2.458
d **i** 3.5958 **ii** 2.8394 **iii** 12.5060 **iv** 19.6898
4 **a** $\sqrt{5} = 2.236\,067\,977$
$\sqrt{6} = 2.449\,489\,743$
$\sqrt{7} = 2.645\,751\,311$
b No **c** No, as $\sqrt{4} = 2$.

5 **a** 26 cm **b** 5 cm **c** 10 cm
d 150 mm **e** 45 cm **f** 25 cm
g 20 mm **h** 30 cm **i** 50 m
6 **a** $\sqrt{185}$ mm **b** $\sqrt{394}$ cm **c** $\sqrt{1130}$ cm
d $\sqrt{90}$ mm **e** $\sqrt{80}$ cm **f** $\sqrt{520}$ mm
g $\sqrt{74}$ cm **h** $\sqrt{637}$ m **i** $\sqrt{1753}$ m
7 **a** 5.0 cm **b** 6.4 km **c** 10.2 cm
d 7.4 cm **e** 16.5 cm **f** 12.8 cm
g 19.0 cm **h** 14.2 cm **i** 27.8 cm
8 **a** 4.243 cm **b** 1.414 cm **c** 7.071 cm
9 **a** 13.9 cm **b** 12.7 cm **c** 26.9 cm
d 18.5 cm **e** 13.4 cm **f** 27.2 cm
10 7.10 m **11** 40.2 km
12 2.7 m **13** 18.6 km

Exercise 10D

1 **a** 12 cm **b** 12 cm **c** 20 m
d 30 cm **e** 16 m **f** 72 cm
g 30 mm **h** 24 mm **i** 24 mm
2 **a** $\sqrt{133}$ cm **b** $\sqrt{24}$ cm **c** $\sqrt{465}$ mm
d $\sqrt{136}$ m **e** $\sqrt{276}$ m **f** $\sqrt{1248}$ m
g $\sqrt{3588}$ mm **h** $\sqrt{4125}$ mm **i** $\sqrt{3528}$ mm
3 **a** 6.9 cm **b** 6.9 cm **c** 15.5 cm
d 16.0 cm **e** 23.1 m **f** 33.6 m
g 27.4 mm **h** 61.4 mm **i** 66.4 mm
4 **a** 6.96 cm **b** 21.51 cm **c** 10.69 cm
d 25.07 cm **e** 28.46 cm **f** 11.52 cm
g 8.95 cm **h** 19.74 cm **i** 18.33 cm
5 4.1 m **6** 28.2 nautical miles
7 **a** 2.71 m **b** 4.6 m^2
8 7.4 cm **9** 32.17 m

Exercise 10E

1 16.97 cm **2** 13.04 cm
3 6.40 m **4** 1.96 m
5 3.61 cm **6** 6.40 km
7 14.21 km **8** 4.21 m
9 2.65 m **10** 20.81 cm
11 6.32 cm **12** 27.22 m
13 25.2 m **14** 42.13 m
15 No, diagonal should be 5.7 m.
16 42 cm **17** 11.26 m
18 **a** 203 cm **b** 80 cm **c** 54 cm
d 175 cm **e** 227 cm **f** 123 cm
g 218 cm **h** 297 cm **i** 147 cm
19 **a** 185 m **b** 123 m **c** 432 m
d 351 m **e** 100 m **f** 136 m
20 **a** 30.81 m **b** 29.41 m **c** 15.16 m
21 57.245 km **22** 128.6 km
23 235.6 km

Exercise 10F

1 **a** $x = \pm 3$ **b** $x = \pm 4$ **c** $x = \pm 8$
d $x = \pm 12$ **e** $x = \pm 7$ **f** $x = \pm 11$
g $x = \pm 6$ **h** $x = \pm 9$ **i** $x = \pm 10$
j $x = \pm 2$ **k** $x = \pm 15$ **l** $x = \pm 17$
2 **a** $x = \pm 3.46$ **b** $x = \pm 7.14$ **c** $x = \pm 4.36$
d $x = \pm 6.86$ **e** $x = \pm 9.11$ **f** $x = \pm 5.29$
g $x = \pm 8.25$ **h** $x = \pm 9.54$ **i** $x = \pm 13.89$
j $x = \pm 14.14$ **k** $x = \pm 12.45$ **l** $x = \pm 18.89$
3 **a** $x = \pm 3$ **b** $x = \pm 6$ **c** $x = \pm 3$
d $x = \pm 4$ **e** $x = \pm 5$ **f** $x = \pm 4$
g $x = \pm 6$ **h** $x = \pm 7$ **i** $x = \pm 5$
j $x = \pm 13$ **k** $x = \pm 4$ **l** $x = \pm 11$
4 **a** $x = \pm\frac{7}{2}$ **b** $x = \pm\frac{9}{10}$ **c** $x = \pm\frac{4}{5}$
d $x = \pm\frac{4}{9}$ **e** $x = \pm\frac{12}{7}$ **f** $x = \pm\frac{8}{11}$
g $x = \pm\frac{7}{12}$ **h** $x = \pm\frac{10}{9}$ **i** $x = \pm\frac{5}{8}$
j $x = \pm\frac{11}{13}$ **k** $x = \pm\frac{7}{9}$ **l** $x = \pm\frac{5}{6}$
5 **a** $x = \pm 2.83$ **b** $x = \pm 1.73$ **c** $x = \pm 2.45$
d $x = \pm 1.90$ **e** $x = \pm 1.60$ **f** $x = \pm 2.14$

Language in mathematics

1 80 years
2 A famous mathematician and philosopher; Pythagorean brotherhood
3 No original writings survive.
4 Astronomy, music theory and acoustics
5 Square and rectangular numbers, Pythagoras' theorem relating the length of the sides of a right-angled triangle
6 Construct squares and rectangles.

Check your skills

1 B **2** A **3** D **4** B **5** B
6 D **7** B **8** D **9** A **10** C
11 B **12** B

Review set 10A

1 **a** **i** q **ii** $q^2 = p^2 + r^2$
b **i** LN **ii** $LN^2 = ML^2 + MN^2$
2 **a** No, $7^2 + 8^2 \neq 12^2$ **b** Yes, $13^2 = 12^2 + 5^2$
3 **a** 49 **b** 9.2
4 **a** 7.9 cm **b** 18.2 cm
5 **a** 5.1 cm **b** 30.6 cm
6 **a** 19.3 cm **b** 34.6 cm
7 35.36 cm **8** $x = \pm 12$

Review set 10B

1 $QS^2 = RS^2 + RQ^2$ **2** Yes, $7^2 + 24^2 = 25^2$
3 **a** 234.09 **b** 11 **c** 7.73
4 **a** 25.0 cm **b** 11.7 cm
5 **a** 17.6 cm **b** 27.3 cm
6 6.1 cm **7** 10.4 cm
8 13 km

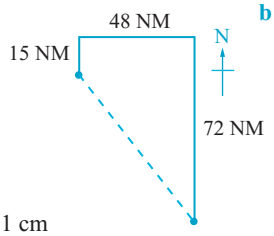
Review set 10C

1 **a** BC **b** $BC^2 = AC^2 + BA^2$
2 **a** No, $9.5^2 + 11.8^2 \neq 15.6^2$
b No, $6.2^2 + 6.2^2 \neq 8.9^2$
3 **a** 25 **b** 324 **c** 134.56
4 **a** 5.8 cm **b** 25.6 cm
5 **a** 6.3 cm **b** 12.6 cm
6 $x = 10$ cm, $y = 11.18$ cm
7 71 mm **8** $x = \pm\sqrt{17}$

Review set 10D

1 **a** $t^2 = q^2 + r^2$ **b** $YZ^2 = XZ^2 + YX^2$
2 No, $5.6^2 + 8.3^2 \neq 12.9^2$
3 **a** 7 **b** 12 **c** 15
4 **a** 8.2 cm **b** 22.0 cm

5 **a** 52.1 cm **b** 3.4 cm
6 **a** 36.1 cm **b** 23.8 km
7 **a** **b** 74.52 NM



8 7.1 cm

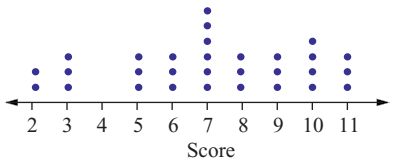
CUMULATIVE REVIEW CHAPTERS 6–10

1 **a** 2 h 52 min **b** 6 h 41 min **c** 19 April
d 54 min **e** 8:29 am **f** 4 h 37 min
g **i** 4 h 22 min **ii** 7 h 25 min
h **i** 8 h 11 min **ii** 3 h 35 min 42 s
i 14 h 18 min
j **i** 4 h 27 min **ii** 3 h 34 min
k **i** 5 h 42 min 51 s **ii** 5 h 43 min
l 2 days 8 h 23 min **m** 16 h 54 min
n 9:25 pm on the previous day
o 6 am **p** 1 pm **q** 11 am
2 **a** 45 kg **b** 30% **c** $185
d 21 kg **e** 37.9% **f** 510
g 240 **h** 19% **i** 34.5%
j $96.65 **k** $212 **l** $87
m $125 **n** $350.96 **o** $199
p 22.7% **q** $14.90 **r** $120
3 **a** **i** Chord **ii** Major segment
iii Semicircle **iv** Centre
v Diameter **vi** Circumference
vii Minor sector **viii** Radius
ix Minor segment **x** Tangent
xi Minor arc **xii** Major sector
b $C = \pi d$ **c** 49.3 cm
d $C = 2\pi r$ **e** 54.22 cm
f $C = 20.1$ m, $A = 32.2$ m^2
g **i** $\frac{325}{360} = \frac{65}{72}$ **ii** $\frac{108}{360} = \frac{3}{10}$
h **i** $C = 23.3$ cm, $A = 47.7$ cm^2
ii $C = 16.0$ cm, $A = 68.1$ cm^2
i $A = 75.04$ cm^2, $P = 43.2$ cm
j $A = 82.14$ cm^2, $P = 33.75$ cm
k **i** $V = 6968$ cm^3, capacity = 7 L
ii $V = 8224$ cm^3, capacity = 8 L
l 5.6 m^3
4 **a** **i** A census is a survey of the entire population.
ii A sample is a survey of part of a population.
iii A nominal variable describes a characteristic or names the categories into which the variable can be sorted.
b **i** Categorical (nominal) **ii** Numerical continuous
iii Numerical continuous **iv** Numerical discrete
c **i** Discrete numerical **ii** Categorical
iii Discrete numerical **iv** Continuous numerical
v Discrete numerical
d Sample: too expensive to ask everyone.
e Census: Advantages are it is accurate, detailed. Disadvantages are it is expensive, time consuming and often impractical.
f **i** Year 8 students **ii** Helmet ratings

g **i**

Mark (score)	Tally	Frequency
2	||	2
3	|||	3
4		0
5	|||	3
6	|||	3
7	𝍸 |	6
8	|||	3
9	|||	3
10	||||	4
11	|||	3
	Total	**30**

ii 10
iii



h

Stem	Leaf
1	7
2	0 1 1 7
3	1 1 3 4 4 4 4 5 7 8 9 9
4	0 2
5	0

5 **a** **i** 24.25 **ii** 23.5 **iii** 23 **iv** 8
b **i** 20.57 **ii** 20 **iii** 20 **iv** 5
c **i** 106.47 **ii** 108 **iii** 124 **iv** 45
d **i** Mean = 19, median = 9, mode = 9
ii Mean = 14.9, median = 15, mode = 14, 15
iii Mean = 6.5, median = 6, mode = 5
e **i** Mean not central **ii** All central
iii Mode not central
6 **a** **i** 8.7 **ii** 9.6 **iii** 20.6 **iv** 3.3
b **i** 8.9 cm **ii** 20.9 cm
c **i** 9.4 cm **ii** 14.6 cm
d No, $8^2 + 10^2 \neq 13^2$
e 25.5 cm **f** 104.4 km
g $x = 8.6$ cm, $y = 10.5$ cm
h **i** 34.0 cm **ii** 23.9 km
i $x = \pm 11$ **j** $x = \pm\sqrt{19}$

CHAPTER 11 ALGEBRA

Diagnostic test

1 A **2** A **3** B **4** B **5** B
6 B **7** C **8** D **9** A **10** B
11 D **12** B **13** A **14** C **15** B
16 B

Exercise 11A

1 **a** $6 \times a + 9 \times a = (6 + 9) \times a$
$= 15 \times a = 15a$
b $14 \times y - 6 \times y = (14 - 6) \times y$
$= 8 \times y = 8y$
2 **a** $3y + 6y = 3 + 6 = 9$, $9y = 9$
b $3y + 6y = 15 + 30 = 45$, $9y = 45$
c $3y + 6y = -6 + (-12) = -18$, $9y = -18$
Yes

3 **a** $7k - 3k = 35 - 15 = 20$, $4k = 20$
b $7k - 3k = 56 - 24 = 32$, $4k = 32$
c $7k - 3k = -21 - (-9) = -12$, $4k = -12$
Yes

4 **a** $11a$ **b** $11t$ **c** $9m$
d $7p$ **e** $3x$ **f** $3b$
g $6a + 3b$ **h** $7y - 4z$ **i** $17t^2$
j $4m^2$ **k** $9k + 5m$ **l** $11ab$
m $5ab + 2bc$ **n** $2ab^2$ **o** $3a^2b + 5ab^2$
p 0 **q** $7g^3$ **r** $2p$
s $9x + 24$ **t** $9st^2$ **u** $4mn^2$
v $11x^2y$ **w** f **x** $10c$

5 **a** $(4 - 7) \times k = -3k$ **b** $(-2 + 5) \times w = 3w$
c $(-3 - 2) \times t = -5t$

6 **a** $-n$ **b** $-2p$ **c** $-2c$ **d** $-6q$
e $3d$ **f** $-9ac$ **g** $-3q$ **h** $7c$
i $-4yz$ **j** $5l$ **k** $-p^2$ **l** $-2x^2$
m $-2xy^2$ **n** $-11pq^2$ **o** $-3t^2$ **p** $-2b$

7 **a** $-8x + 4x = -4x$ **b** $6pq - 3pq = 3pq$
c $-4z + 3z = -z$

8 **a** $6d$ **b** $-3pq$ **c** $-c$ **d** $13q^2$
e $-5m$ **f** $9r$ **g** $-4a$ **h** $-2r$
i 0 **j** $-19x^2y$ **k** $-7d^2$ **l** $-3k^2m$

9 **a** $(5y + 4y) - 3 = 9y - 3$
b $(7p + 5p) + 3q = 12p + 3q$

10 **a** $17 - 5m$ **b** $8a + 6$
c $4 + n$ **d** $18 - 7q$
e $13 - 5k$ **f** $11f + 12$
g $3x^2y - 5$ **h** $9w - 6$
i $12a^2b + 9$ **j** $6q^2 - 6$
k $x - 2$ **l** $4p - 9$
m $t + 4$ **n** $6k - 6l$
o $6d^2 - 3d$ **p** $3n + 2n^2$
q $2c^2 + 8c$ **r** $6 + 3n^2$
s $15mn - 2m$ **t** $a^2b + 2ab^2$

11 **a** $-3 + 5k$ or $5k - 3$ **b** $-4 - 4s$ or $-4s - 4$
c $-2m + 2n$ or $2n - 2m$ **d** $-3x - 8y$ or $-8y - 3x$
e $-4t + 7u$ or $7u - 4t$ **f** $-5a - 4b$ or $-4b - 5a$
g $-2d + 4e$ or $4e - 2d$ **h** $-6z - 4w$ or $-4w - 6z$
i $-3a - 3b$ or $-3b - 3a$ **j** $4m - 8n$
k $-4v - 2w$ or $-2w - 4v$ **l** $-4k + m$ or $m - 4k$

12 **a** $3x + 2$ **b** $4w - 2z$
c $4 + 3d$

13 **a** $6a + 11$ **b** $9c^2 + 2c$
c $2x + 2y$ **d** $2q^2 + 2$
e $9t + v$ **f** $4q + 3d$
g $4p + 6p^2$ **h** $-13l + 10$
i $-6n - 3$ **j** $-4m$
k $2d^2 - 11d$ **l** $16 + 10m$
m $5n - 8$ **n** $12p - 6m$
o $7x^2 + 12xy - 2x$

14 **a** $9c + 2d - 16$ **b** $6n + 2cd - 6$
c $-3q + r + 6$ **d** $2e + 9$
e $11 - 7c + d$ **f** $12l - 5c + 8$
g $-16 - c$ **h** $5d - 2qr + 7r - 4q$
i $-8s + 6t + 8$ **j** $10ac - 3ap - 6a + 19$

Exercise 11B

1 **a** $5 \times t \times w = 5tw$
b $4 \times 3 \times m = 12m$
c $7 \times y \times 2 = 7 \times 2 \times y = 14y$
d $3 \times p^2 \times 7 \times q = 3 \times 7 \times p^2 \times q = 21p^2q$
e $2 \times a \times b \times 3 \times a = 2 \times 3 \times a \times a \times b = 6a^2b$

2 **a** $4xy$ **b** $3km$ **c** $5xy$
d $28w$ **e** $20k$ **f** $60p$
g $16t$ **h** $18z$ **i** $12mn$
j $12vw$ **k** $28p^2q$ **l** $3ab^2$
m $20abc$ **n** $18x^2yz$ **o** $20pq^2r$
p $35b^2c^2$ **q** $24ab$ **r** $24abc$
s $40pqr$ **t** $36a^2c$

3 **a** $-10x$ **b** $-20y$ **c** $12w$
d $32z$ **e** $-12m$ **f** $-80p$
g $-6mn$ **h** $-20ab$ **i** $12xy$
j $18st$ **k** $-24p^2q$ **l** $-40a^2$
m $10p^2$ **n** $-8m^2np$ **o** $30ab^2c$
p $-28kmn^2$

Exercise 11C

1 **a** $\frac{\cancel{10}^{2}y}{\cancel{15}_{3}} = \frac{2y}{3}$ **c** $\frac{\cancel{8}^{2}\cancel{m}^{1}}{\cancel{12}_{3}\cancel{m}_{1}} = \frac{2}{3}$ **c** $\frac{\cancel{6}^{3}\cancel{x}^{1}}{\cancel{8}_{4}\cancel{xy}_{1}} = \frac{3}{4y}$

2 **a** $\frac{x}{2}$ **b** $\frac{m}{4}$ **c** $\frac{p}{5}$ **d** $4d$ **e** $5c$
f $2a$ **g** $\frac{1}{2}$ **h** 2 **i** $\frac{4}{5}$ **j** 2
k $\frac{1}{3}$ **l** $\frac{1}{5}$ **m** $\frac{9p}{10d}$ **n** $\frac{2y}{5}$ **o** $\frac{2a}{3c}$
p $\frac{4}{5q}$ **q** $\frac{1}{5x}$ **r** $\frac{7d}{m}$ **s** $\frac{5pr}{4}$ **t** $\frac{2n}{5p}$

3 **a** $\frac{\cancel{15}^{3}\cancel{ab}^{1}}{-\cancel{20}_{4}\cancel{ac}_{1}} = -\frac{3b}{4c}$ **b** $\frac{-\cancel{x}^{1}}{\cancel{xy}_{1}} = -\frac{1}{y}$

4 **a** -5 **b** $\frac{2f}{3}$ **c** $-\frac{2k}{3}$ **d** $-\frac{4f}{h}$
e $-4l$ **f** $-\frac{10y}{11x}$ **g** $\frac{3}{a}$ **h** $-\frac{a^2}{b}$
i $-\frac{m}{n}$ **j** $-\frac{xz^2}{y}$ **k** $\frac{4p}{q}$ **l** $\frac{2a}{3}$
m $\frac{1}{4}$ **n** $-\frac{2}{mn}$ **o** -3 **p** $-\frac{7q}{p}$

Exercise 11D

1 **a** $5 \times d + 5 \times 4 = 5d + 20$
b $4 \times y - 4 \times 3 = 4y - 12$
c $3 \times 6 - 3 \times m = 18 - 3m$
d $2 \times q + 2 \times 7 = 2q + 14$
e $6 \times b - 6 \times 2 = 6b - 12$

2 **a** $4b + 12$ **b** $12k + 96$ **c** $7c - 35$
d $6d - 18$ **e** $2y - 22$ **f** $9a + 90$
g $10j + 90$ **h** $8m + 16$ **i** $7q - 14$
j $5l - 30$ **k** $8 - 4c$ **l** $3r + 18$
m $63 - 9t$ **n** $4v + 48$ **o** $48 - 6n$
p $6x - 12$

3 **a** $4 \times 3z + 4 \times 2 = 12z + 8$
b $5 \times 2y - 5 \times 3 = 10y - 15$
c $3 \times 6 + 3 \times 4k = 18 + 12k$

4 **a** $6m + 18$ **b** $20d + 25$
c $27p + 72$ **d** $35c - 28$
e $20p - 20$ **f** $48c - 36$
g $36k + 60$ **h** $26n + 10$
i $70a - 60$ **j** $56l - 24$
k $22h + 88$ **l** $60k - 20$
m $78x + 26$ **n** $70w - 63$
o $55j + 35$ **p** $27q - 12$

5 **a** $2(x + 1) = 2x + 2 \neq 2x + 1$
b $5(p - 8) = 5p - 40 \neq 5p - 8$

6 **a** $m \times m + m \times 3 = m^2 + 3m$
b $p \times q - p \times r = pq - pr$

7 **a** $x^2 + 5x$ **b** $q^2 + 13q$
c $a^2 + 8a$ **d** $z^2 + 11z$
e $t^2 - 6t$ **f** $m^2 - 10m$
g $3d - d^2$ **h** $r^2 - 17r$
i $ac - 4a$ **j** $bd + ba$
k $xy - xz$ **l** $mn + mc$
m $jk - jh$ **n** $df + dg$
o $ec - ed$ **p** $rx - ry$

8 **a** $4t \times t - 4t \times 3 = 4t^2 - 12t$
b $3x \times 2y + 3x \times 5z = 6xy + 15xz$

9 **a** $8m^2 + 24m$ **b** $5c^2 + 30c$
c $33r + 3r^2$ **d** $11q^2 - 11q$
e $8x - 4x^2$ **f** $70a - 10a^2$
g $8a^2 + 28a$ **h** $99b^2 + 45b$
i $20f - 20f^2$ **j** $6d^2 - 6df$
k $24k - 12k^2$ **l** $36l - 24l^2$
m $10p^2 - 15pn$ **n** $35c^2 + 14cd$
o $36mn - 30n^2$ **p** $16x^2 - 12xz$

10 **a** $3 \times x + 3 \times 5 + 2x - 7$
$= 3x + 15 + 2x - 7 = 5x + 8$
b $2 \times p + 2 \times 5 + 4 \times p - 4 \times 3$
$= 2p + 10 + 4p - 12 = 6p - 2$

11 **a** $12a + 56$ **b** $9p - 48$
c $10c + 48$ **d** $4d - 56$
e $15q + 20$ **f** $11m - 62$
g $7n + 14$ **h** $5b - 2$
i $-3x - 4$ **j** $12w - 3$
k $3f - 19$ **l** $8n - 24$
m $15y + 2$ **n** $-7c + 29$
o $9y - 27$

12 **a** $6x + 46$ **b** $7d + 14$
c $15n - 52$ **d** $12q - 81$
e $9f - 74$ **f** $12c - 64$
g $x^2 + 7x - 8$ **h** $y^2 - 2y + 8$
i $2w^2 - 17w$ **j** $22m + 11$
k $18t + 36$ **l** $58a + 61$
m $6a^2 + 30a - 20$ **n** $8c^2 - 26c$
o $-2d^2 + 15d$

13 **a** $(-4) \times y + (-4) \times 3$
$= -4y + (-12) = -4y - 12$
b $(-x) \times x - (-x) \times y$
$= -x^2 - (-xy) = -x^2 + xy$
c $(-1) \times 4k + (-1) \times 3m$
$= -4k + (-3m) = -4k - 3m$

14 **a** $-6a - 60$ **b** $-4b - 32$
c $-9k - 81$ **d** $-3c + 9$
e $-5f + 35$ **f** $-10d + 60$
g $-7m - 35$ **h** $-2n - 20$
i $-11h - 121$ **j** $-20p + 70$
k $-24m + 24$ **l** $-35q + 40$

15 **a** $-p^2 - 7p$ **b** $-w^2 - 8w$
c $-d^2 - 11d$ **d** $-s^2 + 3s$
e $-x^2 + 6x$ **f** $-f^2 + 14f$
g $-mn - 5m$ **h** $-ay - 2a$
i $-km - 10k$ **j** $-6t^2 + 3pt$
k $-20y^2 + 4cy$ **l** $-64n^2 + 32mn$

16 **a** $-x - 2$ **b** $-y - 3$
c $-a - 7$ **d** $-n + 11$
e $-g + 5$ **f** $-b + 4$
g $-6 - g$ **h** $-3 - k$
i $-l - 13$ **j** $-2p + 7$
k $-5n + 8$ **l** $-10d + 11$

17 **a** $8p + 35$ **b** $12c - 67$
c $11x - 25$ **d** $8d - 6$
e $5q - 3$ **f** $-3m + 1$
g $4n + 83$ **h** $11a - 4$
i $11s + 9$ **j** $8x - 84$
k $10w + 24$ **l** $2z + 18$
m $7d - 54$ **n** $2k + 2$
o $5p - 18$ **p** $17y$
q $n^2 - 5n + 12$ **r** $w^2 - w + 35$
s $7c^2 - 9c$ **t** $10a^2 - 16a$
u $-4d^2 + 48d$ **v** $-6f - 12$
w $5c^2 + 27c + 72$ **x** $-n + 21$

18 **a** $3x + 25$ **b** $6c - 48$
c $n + 20$ **d** $7f - 7$
e $-15d + 9$ **f** $-14p + 9$
g $-19y^2 - 8y$ **h** $13a^2 - 3a$
i $-18k^2 - 39k + 15$ **j** $26b^2 + 14c^2 + 16bc$

Exercise 11E

1 **a** The factors of 12 are 1, 2, 3, 4, 6, 12.
The factors of 20 are 1, 2, 4, 5, 10, 20.
Common factors of 12 and 20 are 1, 2, 4.
HCF = 4
b The factors of 16 are 1, 2, 4, 8, 16.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
Common factors of 16 and 24 are 1, 2, 4, 8.
HCF = 8

2 **a** 3 **b** 2 **c** 3 **d** 3
e 10 **f** 8 **g** 9 **h** 10
i 12 **j** 12

3 **a** HCF of 12 and 18 is 6.
HCF of $12m$ and $18n$ is 6.
b HCF of 16 and 20 is 4.
HCF of $16x$ and $20y$ is 4.

4 **a** 4 **b** 3 **c** 5 **d** 6
e 4 **f** 9 **g** 5 **h** 12

Exercise 11F

1 **a** $4 \times (k + 3) = 4(k + 3)$
b $5 \times (a + b) = 5(a + b)$
c $3 \times (2k - 5m) = 3(2k - 5m)$
d $8 \times (3p - 2q) = 8(3p - 2q)$
e $6 \times (a^2 + d) = 6(a^2 + d)$
f $5 \times (t + 4) = 5(t + 4)$
g $4 \times (b + c) = 4(b + c)$
h $3 \times (4m + 5n) = 3(4m + 5n)$
i $7 \times (3x - 4y) = 7(3x - 4y)$
j $9 \times (6w - 5v) = 9(6w - 5v)$

2 **a** $3(w + 5)$ **b** $5(b + 7)$
c $4(a - b)$ **d** $10(m - n)$
e $2(5p + 3q)$ **f** $3(4d + 9e)$
g $7(4w - 2v)$ **h** $9(5x + 4y)$
i $6(7g + 3k)$ **j** $8(7t - v)$
k $6(5m - m^2)$ **l** $15(y^2 + z^2)$

3 **a** HCF of 9 and 12 = 3
$9z = 3 \times 3z$ and $12 = 3 \times 4$
Hence $9z + 12 = 3 \times 3z + 3 \times 4$
$= 3(3z + 4)$
b HCF of 20 and 25 = 5
$20w = 5 \times 4w$ and $25y = 5 \times 5y$
Hence $20w - 25y = 5 \times 4w - 5 \times 5y$
$= 5(4w - 5y)$

4 **a** $2(4a + 5)$ **b** $3(3d + 2z)$
c $8(q + 2r)$ **d** $2(k - 6)$
e $3(l - 5m)$ **f** $2(2 + 3d)$
g $9(h - k)$ **h** $2(5 - 3c)$
i $2(7w + 10x)$ **j** $10(p - 2)$
k $11(2 - x)$ **l** $3(4y + 5z)$
m $5(2a - 5b)$ **n** $4(3p + 2q)$
o $8(2k - 3m)$ **p** $7(t + 1)$
q $6(3x + 2y)$ **r** $10(3v - 2w)$
s $5(4 - 3z)$ **t** $2(4x + 3y + 2z)$

5 **a** HCF of -5 and $10 = -5$
$-5x = -5 \times x$ and $10 = -5 \times -2$
Hence $-5x + 10 = -5 \times x + -5 \times -2$
$= -5(x + -2)$
$= -5(x - 2)$

b HCF of -12 and $-4 = -4$
$-12p = -4 \times 3p$ and $-4q = -4 \times q$
Hence $-12p - 4q = -12p + (-4q)$
$= -4 \times 3p + -4 \times q$
$= -4(3p + q)$

6 **a** $-5(2x - 1)$ **b** $-2(3d - 2f)$
c $-2(2p + 1)$ **d** $-3(y + 3z)$
e $-7(2p + 1)$ **f** $-5(4c - 3q)$

7 **a** The factors of pq are 1, p, q, pq.
The factors of qt are 1, t, q, qt.
Hence the HCF of pq and qt is q.
$pq = q \times p$ and $qt = q \times t$
Hence $pq + qt = q \times p + q \times t$
$= q(p + t)$

b The factors of k^2 are 1, k, k^2.
The factors of k are 1, k.
Hence the HCF of k^2 and k is k.
$k^2 = k \times k$ and $k = k \times 1$
Hence $k^2 - k = k \times k - k \times 1$
$= k(k - 1)$

8 **a** $a(b + c)$ **b** $y(x - z)$ **c** $m(k - n)$
d $y(y + 1)$ **e** $m(m - 1)$

9 **a** The factors of xy are 1, x, y, xy.
The factors of $5x$ are 1, 5, x, $5x$.
Hence the HCF of xy and $5x$ is x.
$xy = x \times y$ and $5x = x \times 5$
Hence $xy + 5x = x \times y + x \times 5$
$= x(y + 5)$

b The factors of k^2 are 1, k, k^2.
The factors of $6k$ are 1, 2, 3, 6, k, $2k$, $3k$, $6k$.
Hence the HCF of k^2 and $6k$ is k.
$k^2 = k \times k$ and $6k = k \times 6$
Hence $k^2 - 6k = k \times k - k \times 6$
$= k(k - 6)$

10 **a** $y(x + 5)$ **b** $p(q - 7)$ **c** $a(b + 4)$
d $m(m - 3)$ **e** $k(k - 8)$

11 **a** The HCF of 6 and 12 is 6.
The HCF of xy and xz is x.
Hence the HCF of $6xy$ and $12xz$ is $6x$.
$6xy = 6x \times y$ and $12xz = 6x \times 2z$
Hence $6xy + 12xz = 6x \times y + 6x \times 2z$
$= 6x(y + 2z)$

b The HCF of 10 and 15 is 5.
The HCF of k^2 and k is k.
Hence the HCF of $10k^2$ and $15k$ is $5k$.
$10k^2 = 5k \times 2k$ and $15k = 5k \times 3$
Hence $10k^2 - 15k = 5k \times 2k - 5k \times 3$
$= 5k(2k - 3)$

12 **a** $2a(2b + 3c)$ **b** $4q(3p - 2t)$
c $5k(m + 2)$ **d** $3m(3 - 4n)$
e $8t(1 + 2w)$ **f** $5k(k - 3)$
g $3m(2m - 3)$ **h** $5a(4a - 5)$
i $2k(4 + 5k)$ **j** $2p(7 - 9p)$
k $4p(4q + 5r)$ **l** $8y(x - 3z)$

13 **a** $xz(5y - 7)$ **b** $6ab(4ac + 3)$
c $3(4a + 3b + 5c)$ **d** $5(x^2 - 2x + 4)$
e $2b(3a + 5c + 2ac)$ **f** $8xyz(2y - 3)$
g $x^2y^2(y + x)$ **h** $ab^2(7ab - 4)$
i $-a(b^2 + a)$ **j** $-2kmn(4kn^2 - m)$
k $-3bc(4a + 3d)$ **l** $-6xyz(3 - 2x)$

Exercise 11G

1 **a** $2a + 3b$ **b** $6x + 7x = 13x$
c $m + k$ **d** $8t + 4t = 12t$
e $2x + 4y$ **f** $7y + 8y = 15y$
g $5p + 7p = 12p$ **h** $6a + 3a = 9a$

2 **a** $2w - 3v$ **b** $8k - 5k = 3k$
c $4x - 6y$ **d** $9t - 4t = 5t$
e $8r - 2s$ **f** $9u - 5u = 4u$

3 **a** $2w \times 3 = 6w$ **b** $6d \times 2e = 12de$
c $4a \times 5b = 20ab$ **d** $5q \times 2 = 10q$
e $6x \times 5 = 30x$ **f** $2y \times 3z = 6yz$

4 **a** $\frac{4x}{5y}$ **b** $\frac{3a}{11a} = \frac{3}{11}$ **c** $\frac{5t}{15} = \frac{t}{3}$
d $\frac{2w}{7}$ **e** $\frac{6}{8p} = \frac{3}{4p}$ **f** $\frac{4ab}{12b^2} = \frac{a}{3b}$

5 **a** **i** $5x + 3y$ **ii** $5x - 3y$
iii $5x \times 3y = 15xy$ **iv** $\frac{5x}{3y}$

b **i** $5x + 4x = 9x$ **ii** $5x - 4x = x$
iii $5x \times 4x = 20x^2$ **iv** $\frac{5x}{4x} = \frac{5}{4}$

c **i** $5xy + 6x$ **ii** $5xy - 6x$
iii $5xy \times 6x = 30x^2y$ **iv** $\frac{5xy}{6x} = \frac{5y}{6}$

d **i** $(x + 4) + 3 = x + 7$
ii $(x + 4) - 3 = x + 1$
iii $3(x + 4) = 3x + 12$
iv $\frac{x + 4}{3}$

e **i** $(2x + 5) + 2x = 4x + 5$
ii $(2x + 5) - 2x = 5$
iii $2x(2x + 5) = 4x^2 + 10x$
iv $\frac{2x + 5}{2x}$

f **i** $4a + (2a + 1) = 6a + 1$
ii $4a - (2a + 1) = 2a - 1$
iii $4a(2a + 1) = 8a^2 + 4a$
iv $\frac{4a}{2a + 1}$

6 **a** $7xy + 2xy = 9xy$ **b** $8ab - 3ab = 5ab$
c $9k + 2k = 11k$ **d** $7w - 3w = 4w$
e $5xy \times 3 = 15xy$
f $2k(5k + 3m) = 10k^2 + 6km$
g $\frac{12x}{15xy} = \frac{4}{5y}$ **h** $\frac{7km}{8k} = \frac{7m}{8}$
i $3 \times k - 4 = 3k - 4$
j $(m - 7) + 3 = m - 4$

7 **a** **i** $(m + 2) + 5 = m + 7$
ii $(m + 2) - 3 = m - 1$

b **i** $2(m + 2) = 2m + 4$
ii $(2m + 4) + 3 = 2m + 7$
iii $(2m + 4) - 6 = 2m - 2$

8

Length	Breadth	Perimeter	Area
a	b	$2a + 2b$	ab
$3x$	$2y$	$2 \times 3x + 2 \times 2y = 6x + 4y$	$6xy$
4	$z + 2$	$2 \times 4 + 2 \times (z + 2) = 2z + 12$	$4(z + 2) = 4z + 8$
y	$x - 2$	$2 \times y + 2 \times (x - 2) = 2y + 2x - 4$	$y(x - 2) = xy - 2y$
$a + 3$	7	$2 \times (a + 3) + 2 \times 7 = 2a + 20$	$7(a + 3) = 7a + 21$

9 **a** $\frac{x}{3}$ km/h **b** $\frac{120}{y}$ km/h
c $\frac{10a}{5}$ km/h $= 2a$ km/h **d** $\frac{24mn}{6n}$ km/h $= 4m$ km/h

10 **a** $\$xy$
b $\frac{10}{100} \times x = \frac{1}{10} \times x$ or $\frac{x}{10}$
c $\$x - \frac{\$x}{10} = \frac{\$9x}{10}$
d $n, n + 1, n + 2, n + 3$ **e** $n, n + 2, n + 4$

Language in mathematics

2 ( ) parentheses
[ ] square brackets
{ } braces
3 numerical, symbol, algebraic

Check your skills

1 C **2** D **3** A **4** D **5** C
6 B **7** B **8** A **9** D **10** A
11 C **12** C **13** B **14** B **15** C
16 A

Review set 11A

1 **a** $11s$ **b** $6k$
c $7w + 11$ **d** $8p - 2q$
e $3x + 8$ **f** $-a - b$
g $7g + 14$
2 **a** $20xy$ **b** $-14p$
c $-18ab$
3 **a** $4h$ **b** $\frac{1}{3}$ **c** 3 **d** $\frac{2}{3v}$ **e** $\frac{2}{3b}$
4 **a** $15m + 9$ **b** $6a^2 - 8ab$
c $-5k + 30$
5 **a** 2 **b** 3 **c** 6 **d** $3x$
6 **a** $3(2k + 3)$ **b** $8(t - 2)$
c $6(4w + 3v)$ **d** $-2(2s + 3)$
e $5a(b + 2)$ **f** $3p(3p - 4)$
7 **a** $x + 7$ **b** $4x$ **c** $6k^2$ **d** $3t$
e $5m + 2$ **f** $10m$ **g** $5z$ **h** $10p - 3$

Review set 11B

1 **a** $9q$ **b** $5m$
c $6a + 12$ **d** $8x - 4y$
e $3m + 4$ **f** $-w + z$
g $14h + 9$
2 **a** 20 km **b** $-15cd$ **c** $6mn$
3 **a** $3y$ **b** $\frac{1}{5}$ **c** 2 **d** $\frac{3b}{4}$ **e** x
4 **a** $20x + 28$ **b** $15a^2 - 6ab$
c $-10m + 15$
5 **a** 6 **b** 2 **c** 4 **d** $4a$
6 **a** $4(2t + 3)$ **b** $5(2k - 3)$
c $6(3x + 2y)$ **d** $-3(2t - 3)$
e $b(4b - 3)$ **f** $5pq(2r + 3t)$
7 **a** $9h$ **b** $4y$ **c** $\frac{2}{5}$ **d** $30xy$
e $15q$ **f** $6ab$ **g** 3 **h** $xy - 8$

Review set 11C

1 **a** $11h$ **b** $8d$
c $10z + 12$ **d** $7p - 5q$
e $7a - 7$ **f** $-3x - y$
g $22t - 13$
2 **a** $15rs$ **b** $-12g$ **c** $16pq$
3 **a** $3y$ **b** $\frac{1}{3}$ **c** 5 **d** $\frac{4}{5q}$ **e** $\frac{2a}{3}$
4 **a** $18v - 6$ **b** $10x^2 + 15xy$
c $-12d + 15e$
5 **a** 12 **b** 3 **c** 7 **d** $3a$
6 **a** $6(2p - 3)$ **b** $5(3t + 5u)$
c $4(5k - 4m)$ **d** $-5(q + 2)$
e $6k(3m - 2n)$ **f** $3ab(2a + 5b)$
7 **a** $7xy - 2x$ **b** $13t$ **c** $12wz$ **d** $7p$
e $\frac{1}{2}c$ **f** $4pq$ **g** $3a + 8b$ **h** $3(x + y)$

Review set 11D

1 **a** $8k$ **b** $6m$
c $2b + 9e$ **d** $8 - 4t$
e $a + 3b$ **f** $2x + 15$
g $7w - 37$
2 **a** $35p$ **b** $-27ab$ **c** $60xyz$
3 **a** $4k$ **b** 3 **c** $\frac{5}{2x}$ **d** $-\frac{1}{3w}$ **e** $\frac{3}{mn}$
4 **a** $12a - 18$ **b** $-20x^2 - 28x$
c $-6a^2b^2 + 21a^2b$
5 **a** 12 **b** 6 **c** $4m$ **d** $3x$
6 **a** $3(3k + 5)$ **b** $4(3m - 2n)$
c $-3(2t - 3)$ **d** $-5(2x + y)$
e $a(5a - 7)$ **f** $3ac(4b + 5)$
7 **a** $4x - 7$ **b** $2k + 3$ **c** $3(m + n)$ **d** $16p$
e $\frac{4z}{5}$ **f** $-4n$ **g** $\frac{3m}{5k}$ **h** $\frac{y}{4}$

CHAPTER 12 PROBABILITY AND VENN DIAGRAMS

Diagnostic test

1 D **2** C **3** C **4** B **5** D
6 B **7** B **8** D **9** D **10** C

Exercise 12A

1 S = {R, B, G, Y, W}
Number of possible outcomes = 5
a Favourable outcomes = {R}
Number of favourable outcomes = 1
$P(\text{R}) = \frac{1}{5}$
b Favourable outcomes = {G}
Number of favourable outcomes = 1
$P(\text{G}) = \frac{1}{5}$
c Favourable outcomes = {R, G}
Number of favourable outcomes = 2
$P(\text{R or G}) = \frac{2}{5}$

d Favourable outcomes = {B, W}
Number of favourable outcomes = 2
P(B or W) = $\frac{2}{5}$

e Favourable outcomes = {B, Y, W}
Number of favourable outcomes = 3
P(B or Y or W)= $\frac{3}{5}$

2 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{1}{2}$
e $\frac{1}{2}$ **f** $\frac{1}{2}$

3 **a** $\frac{1}{10}$ **b** $\frac{1}{10}$ **c** $\frac{1}{5}$ **d** $\frac{2}{5}$
e $\frac{1}{2}$ **f** $\frac{3}{10}$ **g** $\frac{1}{5}$

4 **a** $\frac{1}{7}$ **b** $\frac{1}{7}$ **c** $\frac{2}{7}$ **d** $\frac{3}{7}$
e $\frac{4}{7}$ **f** $\frac{3}{7}$

5 **a** $\frac{1}{26}$ **b** $\frac{1}{26}$ **c** $\frac{1}{13}$ **d** $\frac{5}{26}$
e $\frac{21}{26}$ **f** $\frac{3}{13}$

6 S = {B, B, B, B, B, R, R}
Number of possible equally likely outcomes = 7
a Favourable outcomes = {B, B, B, B, B},
Number of favourable outcomes = 5
P(B) = $\frac{5}{7}$
b Favourable outcomes = {R, R}
Number of favourable outcomes = 2
P(R) = $\frac{2}{7}$
c Favourable outcomes = {B, B, B, B, B, R, R}
Number of favourable outcomes = 7
P(B or R) = 1

7 **a** $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{7}{10}$ **d** $\frac{1}{5}$
e $\frac{1}{2}$ **f** $\frac{1}{2}$

8 **a** $\frac{1}{100}$ **b** $\frac{1}{50}$ **c** $\frac{1}{20}$ **d** $\frac{1}{10}$

9 **a** $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$
e $\frac{1}{26}$ **f** $\frac{2}{13}$

10 **a** $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$

11 **a** $\frac{9}{16}$ **b** $\frac{1}{16}$ **c** $\frac{3}{8}$

12 **a** **i** 0 **ii** 0
b Impossible
c **i** 1 **ii** 1
d Certain

13 **a** 0 **b** 1

14 **a** 1 **b** 1 **c** 0 **d** 0

15 **a** Selecting a white marble
b Selecting a red or blue or green marble

16 The probability of an event cannot be greater than 1.

Exercise 12B

1 P(black) + P(white) + P(orange) = 1
$\frac{6}{13} + \frac{2}{13} + P$(orange) = 1
$\frac{8}{13} + P$(orange) = 1
P(orange) = $1 - \frac{8}{13} = \frac{5}{13}$

2 $\frac{9}{19}$ **3** $\frac{1}{6}$

4 $\frac{3}{8}$ **5** $\frac{11}{30}$

Exercise 12C

1 **a** S = {1, 2, 3, 4, 5, 6}
A = {6}
Complement of A = {1, 2, 3, 4, 5}
The complement of A is the event 'throwing 1, 2, 3, 4 or 5' or 'not throwing a 6' or 'throwing a number < 6'.
b S = {1, 2, 3, 4, 5, 6}
B = {5, 6}
Complement of B = {1, 2, 3, 4}
The complement of B is the event 'throwing 1, 2, 3 or 4' or 'not throwing 5 or 6' or 'throwing a number < 5'.
c S = {1, 2, 3, 4, 5, 6}
C = {1, 3, 5}
Complement of C = {2, 4, 6}
The complement of C is the event 'throwing 2, 4 or 6' or 'not throwing an odd number' or 'throwing an even number'.

2 **a** 'Throwing 1, 2, 4, 5 or 6' or 'not throwing 3'
b 'Throwing 1, 2, 4 or 6' or 'not throwing 3 or 5'
c 'Throwing a 1 or 2' or 'not throwing a number > 2' or 'throwing a number < 3'

3 **a** 'Choosing 1, 2, 3, 4, 5, 6, 8 or 9' or 'not choosing 7'
b 'Choosing 2, 3, 4, 5, 6, 8 or 9' or 'not choosing 1 or 7'
c 'Choosing 1, 3, 5, 7 or 9' or 'not choosing an even number' or 'choosing an odd number'
d Choosing 1, 2, 3 or 4' or 'not choosing a number > 4' or 'choosing a number < 5'

4 **a** If a student is chosen from a class consisting of boys and girls, the complement of the event 'choosing a girl' is 'not choosing a girl' or 'choosing a boy'.
b If a marble is chosen from a bag containing red and blue marbles, the complement of the event 'choosing a red marble' is 'not choosing a red marble' or 'choosing a blue marble'.
c If a marble is chosen from a bag containing red, blue and green marbles, the complement of the event 'choosing a red marble' is 'not choosing a red marble' or 'choosing a blue or a green marble'.
d If a marble is chosen from a bag containing red, blue and green marbles, the complement of the event 'choosing a red or blue marble' is 'not choosing a red or blue marble' or choosing a green marble'.

5 'Getting a tail' or 'not getting a head'

6 **a** Stops on blue, green or yellow
b Stops on green or yellow
c Stops on red or yellow
d Stops on yellow

7 **a** Selecting a black card
b Selecting a club, diamond or heart
c Selecting a diamond or heart
d Selecting a numbered card

8 **a** 'Sydney loses' or 'Melbourne wins'
b 'Sydney loses or draws' or 'Melbourne wins or draws'
c A win by Sydney or Melbourne
d 'Melbourne wins' or 'Sydney loses'

Exercise 12D

1 P(on time) + (not on time) = 1
$\frac{9}{11} + P$(not on time) = 1
$\therefore P$(not on time) = $1 - \frac{9}{11} = \frac{2}{11}$

2 $\frac{7}{10}$ **3** $\frac{8}{13}$ **4** $\frac{5}{7}$

5 $\frac{37}{67}$ **6** $\frac{19}{37}$

7 **a** $\frac{1}{1000}$ **b** $\frac{999}{1000}$

8 **a** $\frac{1}{9}$ **b** $\frac{8}{9}$ **c** $\frac{4}{9}$ **d** $\frac{5}{9}$
e $\frac{2}{9}$ **f** $\frac{7}{9}$

9 **a** $\frac{1}{10}$ **b** $\frac{9}{10}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{1}{5}$ **f** $\frac{4}{5}$

10 **a** $\frac{6}{23}$ **b** $\frac{17}{23}$ **c** $\frac{14}{23}$ **d** $\frac{15}{23}$

Exercise 12E

1 **a** A is the event a 5: $A = \{5\}$.
B is the event a 6: $B = \{6\}$.
Hence the events A and B are mutually exclusive.
b A is the event a 5: A $= \{5\}$.
B is the event an odd number: $B = \{1, 3, 5\}$.
Hence the events A and B are non-mutually exclusive.
c A is the event an even number: $= \{2, 4, 6\}$.
B is the event an odd number: $B = \{1, 3, 5\}$.
Hence the events A and B are mutually exclusive.
d A is the event a multiple of 3: $= \{3, 6\}$.
B is the event an even number: $B = \{2, 4, 6\}$.
Hence the events A and B are non-mutually exclusive.

2 **a** Mutually exclusive
b Non-mutually exclusive
c Mutually exclusive
d Non-mutually exclusive
e Non-mutually exclusive
f Mutually exclusive
g Non-mutually exclusive

3 **a** Mutually exclusive
b Non-mutually exclusive
c Non-mutually exclusive
d Mutually exclusive
e Non-mutually exclusive

4 **a** Non-mutually exclusive
b Mutually exclusive

Exercise 12F

1 **a** at least 4 **b** at most 3
c at least 2 **d** at most 2

2 **a** at most 4 **b** at least 6
c at least 4 **d** at most 5

3 **a** {0, 1, 2, 3, 4} **b** {7, 8, 9}
c {5, 6, 7, 8, 9} **d** {0, 1, 2}

4 **a** {gg, gb, bg} **b** {bg, gb, bb}
c {gb, bg, gg} **d** {bb, bg, gb}

5 **a** {HT, TH, TT} **b** {HH, HT, TH}
c {HT, TH, HH} **d** {TT, HT, TH}

6 **a** {(1, 3), (2, 3), (3, 1), (3, 2), (3, 3)}
b {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2)}

7 **a** {10 of hearts}
b {10 of hearts, 10 of diamonds}
c {king of clubs, king of spades}
d {2, 4, 6, 8, 10 of clubs}
e {1, 3, 5, 7, 9 of hearts, 1, 3, 5, 7, 9 of diamonds}
f {J, Q, K of clubs, J, Q, K of spades}

8 **a** No **b** Yes
c If there are no females who wear glasses, no qualifier is needed. If there are females who wear glasses, a qualifier is needed.
d If there are no students who have blond hair and brown eyes, no qualifier is needed. If there are students who have blond hair and brown eyes, a qualifier is needed.

9 **a** {4, 5, 6} **b** {1, 2, 3}
c {1, 3, 5} **d** {2, 4, 6}
e {5} **f** {1, 3, 4, 5, 6}
g {1, 3, 4, 6}

10 **a** {2, 3, 4, …,10, J, Q, K, A of diamonds}
b {2, 3, 4, …, J, Q, K, A of hearts, 2, 3, 4, …, J, Q, K, A of clubs, 2, 3, 4, …, J, Q, K, A of spades}
c {K hearts, K diamonds, K clubs, K spades}
d {K diamonds}
e {2, 3, 4, …10, J, Q, K, A of diamonds, K hearts, K clubs, K spades}
f {2, 3, 4, …10, J, Q, A of diamonds, K hearts, K clubs, K spades}

Exercise 12G

1 **a** Sample space S $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
b Event $A = \{2, 4, 6, 8\}$
c Event 'not A' $= \{0, 1, 3, 5, 7, 9\}$
d Event B $\{6, 7, 8, 9\}$
e Event 'not B' $= \{0, 1, 2, 3, 4, 5\}$
f Event 'A and B' $= \{6, 8\}$
g **i** Event 'A or B or both' $= \{2, 4, 6, 7, 8, 9\}$
ii Inclusive or
h **i** Event 'A or B but not both' $= \{2, 4, 7, 9\}$
ii Exclusive or
i Event neither A nor $B = \{0, 1, 3, 5\}$

2 **a** {1, 2, 3, 4, 5, 6} **b** {1, 3, 5}
c {2, 4, 6} **d** {3, 6}
e {1, 2, 4, 5} **f** {3}
g **i** {1, 3, 5, 6} **ii** Inclusive or
h **i** {1, 5, 6} **ii** Exclusive or
i {2, 4}

3 **a** {2, 4, 6, 8} **b** {1, 3, 5, 7}
c {6, 7, 8} **d** {1, 2, 3, 4, 5}
e {6, 8} **f** {2, 4, 6, 7, 8}
g {2, 4, 7} **h** {1, 3, 5}
i {2, 4} **j** {7}

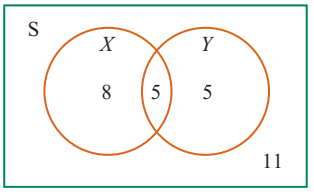
4 **a** {A, E, I, O, U} **b** {N, Q, T}
c {N, O, T, E} **d** {A, I, Q, U}
e (E, O) **f** {A, E, I, N, O, T, U}
g {A, I, N, T, U} **h** {Q}
i {A, I, U} **j** {N, T}

5 **a** {Jen, Matt, Nick, Peter, Jack, Sue, Kate, Michael}
b **i** {Jen, Matt, Nick, Peter, Jack}
ii {Sue, Kate, Michael}
iii {Peter, Jack, Sue}
iv {Jen, Matt, Nick, Kate, Michael}
v {Peter, Jack}
vi {Jen, Matt, Nick, Peter, Jack, Sue}
vii {Jen, Matt, Nick, Sue}
viii {Kate, Michael}
ix {Jen, Matt, Nick}
x {Sue}

6 **a** {4, 8} **b** {1, 2, 3, 5, 6, 7, 9}
c {3, 6, 9} **d** {1, 2, 4, 5, 7, 8}
e { } **f** {3, 4, 6, 8, 9}
g {1, 2, 5, 7}

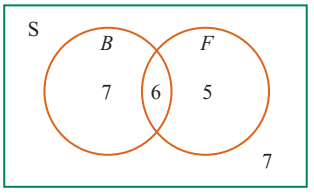
7 **a** 29
b **i** 2 **ii** 6 **iii** 8 **iv** 12
v 10 **vi** 17 **vii** 4 **viii** 6

8 **a** There are 5 students with both black hair and brown eyes so $w = 5$. This number is placed in the intersection of the circles.
There is a total of 13 students (in set X) with black hair so $x + 5 = 13$. $\therefore x = 8$
There is a total of 10 students (in set Y) with brown eyes so $y + 5 = 10$. $\therefore y = 5$
There are 29 students in the class so
$x + 5 + y + z = 29$
so $8 + 5 + 5 + z = 29$ $\therefore z = 11$



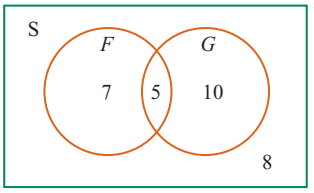
b **i** do not have black hair = 16
ii do not have brown eyes = 19
iii have black hair or brown eyes or both = 18
iv have black hair or brown eyes but not both = 13
v have neither black hair nor brown eyes = 11
vi have black hair but not brown eyes = 8
vii have brown eyes but not black hair = 5

9 **a** $w = 6, x = 7, y = 5, z = 7$



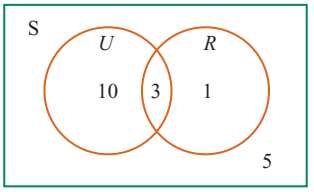
b **i** 12 **ii** 14 **iii** 18 **iv** 12
v 7 **vi** 7 **vii** 5

10 **a** $w = 5, x = 7, y = 10, z = 8$



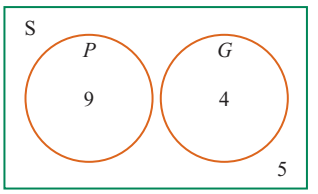
b **i** 18 **ii** 15 **iii** 22 **iv** 17
v 7 **vi** 10 **vii** 8 **viii** 22
ix 25

11 **a**



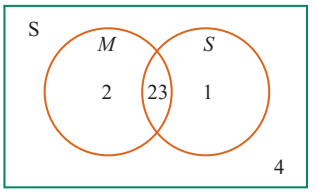
b **i** 6 **ii** 15 **iii** 14 **iv** 11
v 5 **vi** 10 **vii** 1

12 **a**



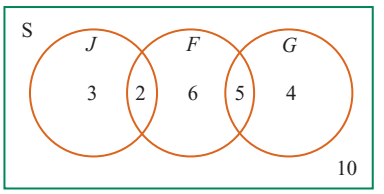
b **i** 9 **ii** 14 **iii** 13 **iv** 5
v 13 **vi** 18

13 **a**



b **i** 5 **ii** 6 **iii** 4 **iv** 26
v 3 **vi** 2 **vii** 1 **viii** 26
ix 7 **x** 7 **xi** 3

14 **a**



b **i** 10 **ii** 25 **iii** 17 **iv** 21
v 16 **vi** 14 **vii** 17 **viii** 12
ix 14 **x** 0 **xi** 3 **xii** 6
xiii 4 **xiv** 13 **xv** 7 **xvi** 20
xvii 23

15 **a** 56 **b** 42 **c** 29 **d** 21
e 17 **f** 8 **g** 3 **h** 77
i 56 **j** 68 **k** 51 **l** 63
m 55 **n** 41 **o** 55 **p** 68
q 13 **r** 35 **s** 21 **t** 39
u 12 **v** 34 **w** 21 **x** 44
y 84 **z** 57

Exercise 12H

1 **a** 30 **b** 17 **c** 13 **d** 11
e 19 **f** 10 **g** 3 **h** 9
i 8 **j** 27 **k** 18 **l** 21
m 18 **n** 9 **o** 3 **p** 8
q 10

2 **a** 230 **b** 116 **c** 114 **d** 185
e 45 **f** 29 **g** 98 **h** 132
i 103 **j** 98 **k** 29 **l** 98
m 16

3 **a** 16 **b** 17 **c** 11 **d** 22
e 11 **f** 8 **g** 5 **h** 6

4 **a** $v = 3, w = 8, x = 15, y = 7, z = 12$
b **i** 15 **ii** 11 **iii** 8 **iv** 18
v 10 **vi** 5 **vii** 7 **viii** 3

5 **a** $k = 27, l = 4, m = 30, n = 2, p = 5$
b **i** 31 **ii** 30 **iii** 27 **iv** 34
v 7 **vi** 2 **vii** 4 **viii** 3

6 **a**

	Soccer	Not soccer	
Netball	10	6	16
Not netball	5	7	12
	15	13	28

b **i** 13 **ii** 12 **iii** 21 **iv** 11
v 5 **vi** 6

7 a

	Drink	Do not drink	
Smoke	13	6	19
Do not smoke	66	15	81
	79	21	100

b **i** 79 **ii** 21 **iii** 81 **iv** 85
v 72 **vi** 66 **vii** 6

8 a

	Drink tea	Do not drink tea	
Drink coffee	5	12	17
Do not drink coffee	15	3	18
	20	15	35

b **i** 20 **ii** 15 **iii** 5 **iv** 15
v 27 **vi** 3 **vii** 32 **viii** 27
ix 30

9 a

	Read *The Australian*	Do not read *The Australian*	
Read *The Daily Telegraph*	3	8	11
Do not read *The Daily Telegraph*	6	2	8
	9	10	19

b **i** 9 **ii** 11 **iii** 3 **iv** 2
v 6 **vi** 8 **vii** 17 **viii** 14
ix 17 **x** 14 **xi** 16

Language in mathematics

1 a Probability $= \dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
b 0 **c** 1 **d** 0 to 1 **e** 1
f complementary **g** rectangle, circle
2 a Not selecting a blue marble
b Not tossing a head
3 a Inside circle A
b Outside circle A (but within the rectangle)
c In the intersection of the circles
d Inside circles A and B, including the intersection
e Inside circles A and B but not including the intersection

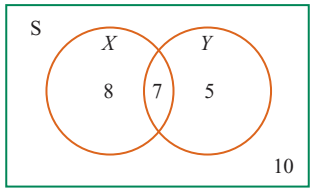
Check your skills

1 A **2** B **3** A **4** C **5** B
6 B **7** D **8** C **9** C **10** C
11 C **12** A **13** C **14** C **15** A
16 B **17** C **18** C

Review set 12A

1 a $\frac{1}{9}$ **b** $\frac{2}{9}$ **c** $\frac{5}{9}$ **d** $\frac{4}{9}$ **e** $\frac{1}{3}$
2 a $\frac{1}{3}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$ **d** 1 **e** 0
3 a Rolling a number > 6
b Rolling an odd or even number
4 $\frac{1}{10}$
5 a {1, 2, 3, 5, 6} or 'not a 4'
b {1, 3, 5}or 'not an even number or an odd number'
c {4, 5, 6}or 'a number > 3'
6 $\frac{3}{5}$

7 a {HH, HT, TH, TT}
b **i** {HT, TH} **ii** {HT, TH, TT}
iii {HT, TH, HH} **iv** {HT, TH, TT}
8 a Non-mutually exclusive **b** Non-mutually exclusive
c Mutually exclusive **d** Non-mutually exclusive
9 a 9
b **i** {2, 4, 6, 8} **ii** {1, 3, 5, 7, 9}
iii {1, 2, 3, 4} **iv** {5, 6, 7, 8, 9}
v {6, 8} **vi** {1, 3}
vii {2, 4} **viii** {1, 2, 3, 4, 6, 8}
ix {1, 3, 6, 8} **x** {5, 7, 9}
10 a



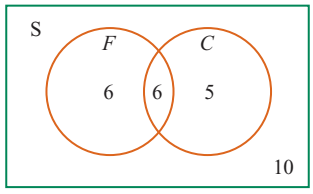
b **i** 15 **ii** 15 **iii** 12 **iv** 18
v 8 **vi** 5 **vii** 7 **viii** 20
ix 13 **x** 10
11 a 127 **b** 64 **c** 76 **d** 83 **e** 20
12 a

	Study Indonesian	Do not study Indonesian	
Study Japanese	3	8	11
Do not study Japanese	10	9	19
	13	17	30

b **i** 3 **ii** 9 **iii** 10 **iv** 18

Review set 12B

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{1}{3}$
2 a $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{4}{11}$ **d** $\frac{4}{11}$ **e** 0
3 a Spinning a colour other than red, blue or green
b Spinning red, blue or green
4 $\frac{1}{4}$
5 a A red or green marble **b** A green marble
c A blue marble
6 $\frac{11}{17}$
7 a {7, 8, 9} **b** {1, 2, 3, 4, 5}
8 a Qualifiers not needed as A and B are mutually exclusive events.
b Qualifiers are needed as A and B are non-mutually exclusive events.
9 a {1, 3, 5, 7, 9} **b** {2, 4, 6, 8}
c {7, 8, 9} **d** {1, 2, 3, 4, 5}
e {8} **f** {1, 3, 5}
g {7, 9} **h** {1, 3, 5, 7, 8, 9}
i {1, 3, 5, 8} **j** {2, 4, 6}
10 a



b **i** 6 **ii** 5 **iii** 17 **iv** 11 **v** 10
11 a $x = 7, y = 6, z = 11$
b **i** 7 **ii** 11 **iii** 17 **iv** 4

12

	Wear glasses	Do not wear glasses	
Left-handed	0	2	2
Not left-handed	3	25	28
	3	27	30

Review set 12C

1 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{4}$ **e** $\frac{3}{8}$

2 **a** $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{26}$ **d** $\frac{1}{4}$ **e** $\frac{1}{2}$
f 1 **g** 0

3 **a** Choosing a number > 5
b Choosing a number < 6

4 $\frac{3}{8}$

5 **a** A black card
b A club, diamond or heart
c Not an ace

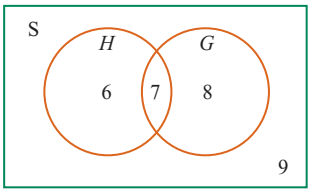
6 $\frac{8}{17}$

8 **a** {J hearts, J diamonds} **b** {J, Q, K of spades}

9 If there are no red-headed boys events A and B are mutually exclusive and no qualifiers are needed. If there are boys with red hair then the events A and B are non-mutually exclusive and the qualifiers are needed.

10 **a** 14
b **i** 5 **ii** 9 **iii** 6 **iv** 8
v 3 **vi** 4 **vii** 2 **viii** 9
ix 7 **x** 5

11



12 **a**

	History	Not History	
Geography	7	8	15
Not Geography	6	9	15
	13	17	30

b **i** 17 **ii** 15 **iii** 7 **iv** 21
v 14 **vi** 6 **vii** 8 **viii** 9
ix 21 **x** 14 **xi** 23

Review set 12D

1 **a** $\frac{1}{21}$ **b** $\frac{11}{21}$ **c** $\frac{9}{21}$ **d** $\frac{11}{21}$
e $\frac{10}{21}$ **f** $\frac{4}{21}$ **g** $\frac{2}{21}$ **h** $\frac{12}{21}$

2 **a** $\frac{2}{11}$ **b** $\frac{2}{11}$ **c** $\frac{1}{11}$ **d** $\frac{4}{11}$
e $\frac{4}{11}$ **f** 0

3 **a** Spinning a red, blue or green
b Spinning a colour other than red, blue or green

4 $\frac{7}{15}$

5 **a** 'Not a red card' or 'a black card'
b 'Not a diamond' or 'a club, spade or heart'

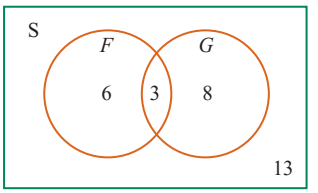
6 0.6

7 **a** False **b** True **c** True

8 **a** {4, 5, 6} **b** {3, 4, 5, 6}
c {1, 2} **d** {1, 2, 3}

9 **a** 12 **b** 2 **c** 7 **d** 23
e 16 **f** 8 **g** 17 **h** 2
i 7 **j** 22 **k** 2

10 **a** $w = 8, x = 6, y = 9, z = 30$
b



c **i** 9 **ii** 21 **iii** 11 **iv** 19
v 3 **vi** 17 **vii** 14 **viii** 13
ix 6 **x** 8 **xi** 17 **xii** 27
xiii 14

CHAPTER 13 LINEAR EQUATIONS

Diagnostic Test

1 B **2** A **3** D **4** C **5** B
6 C **7** B **8** A **9** B **10** A
11 D **12** B **13** D **14** A **15** A
16 C **17** B **18** B **19** C

Exercise 13A

1 **a** 2 **b** 4 **c** 4
d 8 **e** 8 **f** 10
g 10 **h** 9 **i** 8
j 2 **k** 3 **l** 5

2 **a** **i** Take 8 cubes from both LHS and RHS.
ii $x = 4$
b **i** Take 6 cubes from both LHS and RHS.
ii $x = 4$
c **i** Take 3 cubes from both LHS and RHS.
ii $x = 1$
d **i** Take 1 cube from both LHS and RHS.
ii $x = 6$
e **i** Take 2 cubes from both LHS and RHS.
ii $x = 3$
f **i** Take 7 cubes from both LHS and RHS.
ii $x = 5$

3 **a** **i** As there are 3 pill bottles = 36 tablets, divide 36 by 3.
ii $b = 12$
b **i** As there are 3 boxes of oranges equal to 45, divide 45 by 3.
ii $O = 15$
c **i** 2 packets + 5 biscuits = 21 biscuits, so take 5 biscuits from both LHS and RHS. Now 2 packets = 16 biscuits. Divide 16 ÷ 2 = 8.
ii $b = 8$
d **i** Take 3 lollies from each side then 3 boxes = 24 lollies. Divide both sides by 3.
1 box = 8 lollies.
ii $l = 8$

Exercise 13B

1 **a** $x + 7$ **b** $x + 2$ **c** $x + 9$
d $x - 3$ **e** $x - 1$ **f** $x - 7$
g $8x$ **h** $-4x$ **i** $5x$
j $\frac{x}{6}$ **k** $\frac{x}{2}$ **l** $\frac{x}{11}$

2 **a** $6x + 4$ **b** $-9x - 1$
c $5x + 2$ **d** $\frac{x}{3} - 8$
e $\frac{x}{-7} + 5$ **f** $\frac{x}{10} - 4$
g $\frac{2x + 7}{6}$ **h** $\frac{5x - 9}{2}$
i $\frac{-4x - 1}{7}$ **j** $2(6x - 5)$
k $-5(2x + 7)$ **l** $\frac{-3(-8x + 1)}{4}$

3 **a** $x \xrightarrow{\times 3} 3x$ **b** $x \xrightarrow{+ 5} x + 5$
c $x \xrightarrow{- 8} x - 8$ **d** $x \xrightarrow{\times (-6)} -6x$
e $x \xrightarrow{- 9} x - 9$ **f** $x \xrightarrow{\div 2} \frac{x}{2}$
g $x \xrightarrow{+ 10} x + 10$ **h** $x \xrightarrow{\div (-4)} \frac{x}{-4}$
i $x \xrightarrow{\times 5} 5x \xrightarrow{+ 12} 5x + 12$
j $x \xrightarrow{\times (-3)} -3x \xrightarrow{- 1} -3x - 1$
k $x \xrightarrow{\div (-4)} \frac{x}{-4} \xrightarrow{+ 2} \frac{x}{-4} + 2$
l $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{+ 9} \frac{x}{5} + 9$
m $x \xrightarrow{- 8} x - 8 \xrightarrow{\div 7} \frac{x - 8}{7}$
n $x \xrightarrow{+ 10} x + 10 \xrightarrow{\div 13} \frac{x + 10}{13}$
o $x \xrightarrow{\times 5} 5x \xrightarrow{- 2} 5x - 2 \xrightarrow{\div 9} \frac{5x - 2}{9}$
p $x \xrightarrow{\times 3} 3x \xrightarrow{+ 7} 3x + 7 \xrightarrow{\div 10} \frac{3x + 7}{10}$
q $x \xrightarrow{- 6} x - 6 \xrightarrow{\times 4} 4(x - 6)$
r $x \xrightarrow{+ 3} x + 3 \xrightarrow{\times (-5)} -5(x + 3)$
s $x \xrightarrow{\times 2} 2x \xrightarrow{+ 7} 2x + 7 \xrightarrow{\times 9} 9(2x + 7)$
t $x \xrightarrow{\times 4} 4x \xrightarrow{- 6} 4x - 6 \xrightarrow{\times (-6)} -6(4x - 6)$
u $x \xrightarrow{- 6} x - 6 \xrightarrow{\times 3} 3(x - 6) \xrightarrow{\div 5} \frac{3(x - 6)}{5}$
v $x \xrightarrow{+ 2} x + 2 \xrightarrow{\times (-9)} -9(x + 2) \xrightarrow{\div 11} \frac{-9(x + 2)}{11}$
w $x \xrightarrow{\times 3} 3x \xrightarrow{- 5} 3x - 5 \xrightarrow{\times 7} 7(3x - 5) \xrightarrow{\div 10} \frac{7(3x - 5)}{10}$
x $x \xrightarrow{\times 2} 2x \xrightarrow{- 8} 2x - 8 \xrightarrow{\times (-4)} -4(2x - 8) \xrightarrow{\div 13} \frac{-4(2x - 8)}{13}$
y $x \xrightarrow{+ 1} x + 1 \xrightarrow{\times 5} 5(x + 1) \xrightarrow{\div (-3)} \frac{5(x + 1)}{-3}$

4 **a** $- 4$ **b** $- 6$
c $- 9$ **d** $+ 5$
e $+ 3$ **f** $+ 11$
g $\div 2$ **h** $\div -9$
i $\div 7$ **j** $\times 10$
k $\times 3$ **l** $\times -4$
m $- 2, \div 6$ **n** $+ 1, \div -3$
o $- 10, \div 7$ **p** $+ 8, \times 5$
q $- 2, \times 3$ **r** $- 1, \times 11$
s $\times 9, - 2, \div 3$ **t** $\times 8, - 11, \div 2$
u $\times 7, + 6, \div -4$ **v** $\times 4, \div 3, - 7$
w $\times 11, \div -2, + 10, \div 5$

5 **a** $12x \xrightarrow{\div 12} x$ **b** $x - 8 \xrightarrow{+ 8} x$
c $x + 11 \xrightarrow{- 11} x$ **d** $\frac{x}{5} \xrightarrow{\times 5} x$
e $4x - 10 \xrightarrow{+ 10} 4x \xrightarrow{\div 4} x$
f $-9x \xrightarrow{\div (-9)} x$
g $\frac{-3x}{7} \xrightarrow{\times 7} -3x \xrightarrow{\div (-3)} x$
h $15 - 8x \xrightarrow{- 15} -8x \xrightarrow{\div (-8)} x$
i $-x + 1 \xrightarrow{- 1} -x \xrightarrow{\div (-1)} x$
j $3 + \frac{2x}{5} \xrightarrow{- 3} \frac{2x}{5} \xrightarrow{\times 5} 2x \xrightarrow{\div 2} x$
k $\frac{x}{4} - 1 \xrightarrow{+ 1} \frac{x}{4} \xrightarrow{\times 4} x$
l $5 - \frac{x}{2} \xrightarrow{- 5} \frac{-x}{2} \xrightarrow{\times 2} -x \xrightarrow{\div (-1)} x$
m $\frac{x - 9}{4} \xrightarrow{\times 4} x - 9 \xrightarrow{+ 9} x$
n $\frac{10 - x}{2} \xrightarrow{\times 2} 10 - x \xrightarrow{- 10} -x \xrightarrow{\div (-1)} x$
o $\frac{11 - 4x}{7} \xrightarrow{\times 7} 11 - 4x \xrightarrow{- 11} -4x \xrightarrow{\div (-4)} x$
p $\frac{3x + 6}{5} \xrightarrow{\times 5} 3x + 6 \xrightarrow{- 6} 3x \xrightarrow{\div 3} x$
q $2(x + 14) \xrightarrow{\div 2} x + 14 \xrightarrow{- 14} x$
r $-3(x - 18) \xrightarrow{\div (-3)} x - 18 \xrightarrow{+ 18} x$
s $-2(2x + 6) \xrightarrow{\div (-2)} 2x + 6 \xrightarrow{- 6} 2x \xrightarrow{\div 2} x$
t $9(3x - 1) \xrightarrow{\div 9} 3x - 1 \xrightarrow{+ 1} 3x \xrightarrow{\div 3} x$
u $\frac{7(x + 4)}{5} \xrightarrow{\times 5} 7(x + 4) \xrightarrow{\div 7} x + 4 \xrightarrow{- 4} x$
v $\frac{-4(x - 8)}{9} \xrightarrow{\times 9} -4(x - 8) \xrightarrow{\div (-4)} x - 8 \xrightarrow{+ 8} x$
w $\frac{4(5x + 7)}{7} \xrightarrow{\times 7} 4(5x + 7) \xrightarrow{\div 4} 5x + 7 \xrightarrow{- 7} 5x \xrightarrow{\div 5} x$

x $\boxed{\frac{-3(2x-9)}{11}} \xrightarrow{\times 11} \boxed{-3(2x-9)} \xrightarrow{\div(-3)} \boxed{2x-9} \xrightarrow{+9} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

6 a $3x + 5 = 26$

$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+5} \boxed{3x+5}$

$\boxed{7} \xleftarrow{\div 3} \boxed{21} \xleftarrow{-5} \boxed{26}$

b $2x - 8 = 20$

$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-8} \boxed{2x-8}$

$\boxed{14} \xleftarrow{\div 2} \boxed{28} \xleftarrow{+8} \boxed{20}$

c $4m - 12 = 40$

$\boxed{m} \xrightarrow{\times 4} \boxed{4m} \xrightarrow{-12} \boxed{4m-12}$

$\boxed{13} \xleftarrow{\div 4} \boxed{52} \xleftarrow{+12} \boxed{40}$

d $6y + 8 = 62$

$\boxed{y} \xrightarrow{\times 6} \boxed{6y} \xrightarrow{+8} \boxed{6y+8}$

$\boxed{9} \xleftarrow{\div 6} \boxed{54} \xleftarrow{-8} \boxed{62}$

e $\frac{2p}{3} - 1 = 3$

$\boxed{p} \xrightarrow{\times 2} \boxed{2p} \xrightarrow{\div 3} \boxed{\frac{2p}{3}} \xrightarrow{-1} \boxed{\frac{2p}{3}-1}$

$\boxed{6} \xleftarrow{\div 2} \boxed{12} \xleftarrow{\times 3} \boxed{4} \xleftarrow{+1} \boxed{3}$

f $\frac{5w}{8} + 7 = 17$

$\boxed{w} \xrightarrow{\times 5} \boxed{5w} \xrightarrow{\div 8} \boxed{\frac{5w}{8}} \xrightarrow{+7} \boxed{\frac{5w}{8}+7}$

$\boxed{16} \xleftarrow{\div 5} \boxed{80} \xleftarrow{\times 8} \boxed{10} \xleftarrow{-7} \boxed{17}$

g $6(d + 4) = 54$

$\boxed{d} \xrightarrow{+4} \boxed{d+4} \xrightarrow{\times 6} \boxed{6(d+4)}$

$\boxed{5} \xleftarrow{-4} \boxed{9} \xleftarrow{\div 6} \boxed{54}$

h $5(2a - 3) = 35$

$\boxed{a} \xrightarrow{\times 2} \boxed{2a} \xrightarrow{-3} \boxed{2a-3} \xrightarrow{\times 5} \boxed{5(2a-3)}$

$\boxed{5} \xleftarrow{\div 2} \boxed{10} \xleftarrow{+3} \boxed{7} \xleftarrow{\div 5} \boxed{35}$

7 a $m = 11$ **b** $d = 32$ **c** $x = 9$
d $p = 4$ **e** $y = 8$ **f** $a = 25$
g $a = 7$ **h** $d = 23$ **i** $p = 7\frac{1}{2}$

Exercise 13C

1 a $x = 6$ **b** $t = 7$ **c** $w = 12$
d $h = 8$ **e** $d = 6$ **f** $q = 16$
g $m = 16$ **h** $c = 26$ **i** $l = 42$
j $k = 13$ **k** $a = 46$ **l** $b = 58$
m $y = 8$ **n** $z = 16$ **o** $p = 5$
p $a = 12$ **q** $m = 7$ **r** $z = 7$
s $f = 52$ **t** $k = 90$ **u** $s = 60$
v $v = 21$ **w** $y = 36$ **x** $n = 68$

2 a $x = -2$ **b** $d = -5$ **c** $f = -21$
d $q = -20$ **e** $h = 6$ **f** $j = -9$
g $a = -4$ **h** $y = -4$ **i** $c = 7$
j $g = 0$ **k** $l = 3$ **l** $k = -13$
m $w = -\frac{3}{10}$ **n** $h = -\frac{5}{7}$ **o** $q = -\frac{7}{4}$
p $m = -8$ **q** $d = -6$ **r** $n = 10$
s $b = -8$ **t** $e = -15$ **u** $r = -36$
v $d = -20$ **w** $k = -32$ **x** $v = -35$

3 a Yes **b** Yes **c** No **d** No
e No **f** Yes **g** No **h** No
i No **j** Yes **k** Yes **l** No
m Yes **n** Yes **o** No **p** Yes

4 a $k = 5$ **b** $d = 14$ **c** $c = 3$
d $b = 5$ **e** $m = 3$ **f** $q = 9$
g $f = 6$ **h** $k = 1$ **i** $a = 4$
j $n = 7$ **k** $s = 4$ **l** $g = 3$
m $y = 27$ **n** $m = 17$ **o** $x = 21$

5 a $p = \frac{9}{2} = 4\frac{1}{2}$ **b** $d = \frac{11}{7} = 1\frac{4}{7}$
c $f = \frac{27}{4} = 6\frac{3}{4}$ **d** $q = \frac{33}{9} = 3\frac{2}{3}$
e $a = \frac{24}{11} = 2\frac{2}{11}$ **f** $k = \frac{15}{13} = 1\frac{2}{13}$
g $l = \frac{23}{5} = 4\frac{3}{5}$ **h** $w = \frac{35}{14} = 2\frac{1}{2}$
i $c = \frac{1}{2}$ **j** $n = \frac{1}{5}$
k $f = \frac{28}{12} = 2\frac{1}{3}$ **l** $l = \frac{17}{3} = 5\frac{2}{3}$
m $a = -\frac{19}{2} = -9\frac{1}{2}$ **n** $y = -\frac{22}{6} = 3\frac{2}{3}$
o $b = -\frac{23}{4} = -5\frac{3}{4}$

6 a $x = 3$ **b** $d = \frac{24}{5} = 4\frac{4}{5}$
c $a = \frac{21}{4} = 5\frac{1}{4}$ **d** $p = \frac{30}{8} = 3\frac{3}{4}$
e $k = -\frac{99}{6} = -16\frac{1}{2}$ **f** $l = -\frac{54}{7} = -7\frac{5}{7}$
g $q = -\frac{26}{10} = -2\frac{3}{5}$ **h** $m = -\frac{40}{3} = -13\frac{1}{3}$
i $t = \frac{56}{5} = 11\frac{1}{5}$ **j** $x = \frac{33}{9} = 3\frac{2}{3}$
k $y = \frac{36}{8} = 4\frac{1}{2}$ **l** $n = \frac{49}{4} = 12\frac{1}{4}$
m $a = -\frac{80}{9} = -8\frac{8}{9}$ **n** $w = -15$
o $f = -\frac{55}{10} = -5\frac{1}{2}$ **p** $c = -\frac{72}{7} = -10\frac{2}{7}$

7 a $x = 23$ **b** $p = 25$ **c** $d = 13$
d $c = 37$ **e** $y = 41$ **f** $a = 22$
g $b = 2$ **h** $m = 11$ **i** $n = 5$
j $k = 11$ **k** $c = 3$ **l** $w = 7$
m $d = -24$ **n** $m = -3$ **o** $d = 13$
p $d = 14$ **q** $x = -17$ **r** $x = 3$
s $x = 6$ **t** $p = 5$

Exercise 13D

1 a $a = 5$ **b** $q = 8$ **c** $b = 3$
d $y = 7$ **e** $f = 24$ **f** $h = 4$
g $x = -2$ **h** $d = -3$ **i** $r = 1$
j $w = 4$ **k** $s = 1$ **l** $y = 1$
m $x = 9$ **n** $b = -10$ **o** $j = 3$
p $p = 2$ **q** $d = -7$ **r** $a = 1$

2 a Yes **b** No **c** Yes
d Yes **e** No **f** Yes

3 a $x = -2\frac{1}{4}$ **b** $d = -2\frac{1}{2}$ **c** $q = -4\frac{2}{3}$
d $c = 6\frac{1}{2}$ **e** $w = \frac{19}{6} = 3\frac{1}{6}$ **f** $g = \frac{19}{5} = 3\frac{4}{5}$
g $k = 1\frac{1}{2}$ **h** $k = \frac{1}{3}$ **i** $l = \frac{5}{8}$
j $x = -1\frac{1}{3}$ **k** $a = -1\frac{1}{3}$ **l** $b = 2\frac{1}{4}$

Exercise 13E

1 **a** $x = 3$ **b** $t = 1\frac{1}{8}$ **c** $l = 1$
d $n = -11$ **e** $e = -6\frac{5}{6}$ **f** $r = -2$
g $m = 8\frac{3}{4}$ **h** $p = 7$ **i** $c = 12\frac{3}{11}$
j $f = 24\frac{1}{3}$ **k** $t = 11\frac{3}{5}$ **l** $w = 7\frac{12}{13}$

2 **a** $a = 7$ **b** $m = 5$ **c** $q = 1$
d $b = -\frac{11}{35}$ **e** $p = 7$ **f** $w = 3\frac{1}{5}$
g $c = -5$ **h** $n = -2$ **i** $d = 2$
j $x = \frac{3}{7}$ **k** $y = \frac{5}{6}$ **l** $w = -3\frac{1}{2}$

3 **a** $p = -5$ **b** $a = 7$ **c** $d = -10$
d $m = -1$ **e** $r = -2$ **f** $x = 3$
g $f = -5$ **h** $q = -4$ **i** $m = -1$
j $d = 2$ **k** $a = -1$ **l** $m = 3$

Exercise 13F

1 **a** 35 cm² **b** 18 cm² **c** 66 cm²
d 24.5 cm² **e** 37.8 cm² **f** 49.14 cm²

2 **a** 22.5 cm² **b** 16.5 cm² **c** 40 cm²
d 7.4 cm² **e** 10.5 cm² **f** 35.03 cm²

3 **a** 15 cm² **b** 28 cm²
c 66 cm² **d** 54.18 cm²

4 **a** 28.26 cm² **b** 289.38 cm²
c 482.81 cm² **d** 153.86 cm²

5 **a** 105 **b** −44
c 0.25 **d** −11.72

6 **a** 7.21 cm **b** 13.60 cm
c 9.34 cm **d** 8.13 cm

7 **a** $-12\frac{2}{9}°$ **b** $-13\frac{1}{3}°$ **c** $-9\frac{4}{9}°$ **d** $-7\frac{7}{9}°$

8 **a** 41° **b** 68° **c** 51° **d** 46°

9 **a** 108 cm **b** 37.8 cm **c** 4.64 cm

10 **a** 4 **b** 11 **c** 4.2 **d** 7.7

11 **a** 9 cm **b** 14 cm **c** 2.5 cm
d 2.4 cm **e** 3.3 cm **f** 9.5 cm

12 **a** 10 cm **b** 14 cm **c** 9 cm
d 7 cm **e** 5 cm **f** 8 cm

13 **a** **i** 4 cm **ii** 10 cm
b **i** 4 cm **ii** 11 cm

14 **a** 5 cm **b** 7 cm **c** 4 cm **d** 4.1 cm

15 **a** **i** 5 **ii** 18
b **i** 13 **ii** 3

16 **a** 3 cm **b** 6 cm **c** 15 cm **d** 11 cm

17 **a** 40° **b** 50° **c** 60° **d** $70\frac{4}{5}°$

18 **a** 4° **b** 11° **c** 17° **d** 50°

19 **a** 6 cm **b** 34 cm **c** 24.7 cm

20 **a** 10 **b** 109

Exercise 13G

1 **a** $x + 18 = 46; x = 28$ **b** $x + 7 = -12; x = -19$
c $x - 5 = 19; x = 24$ **d** $x - 8 = -4; x = 4$
e $x - (-2) = 6; x = 4$ **f** $3x = 108; x = 36$
g $6x = 72; x = 12$ **h** $\frac{x}{9} = 2; x = 18$
i $-\frac{x}{7} = -1.4; x = 9.8$

2 **a** $2x - 3 = 17; x = 10$ **b** $4x + 12 = 52; x = 10$
c $2x - 8 = -10; x = -1$ **d** $\frac{5x}{9} = 10; x = 18$
e $7(x - 1) = 21; x = 4$ **f** $5(3x - 12) = 15; x = 5$

Check your skills

1 C **2** D **3** B **4** D **5** B
6 A **7** B **8** A **9** B

Review set 13A

1 $5x + 15$ $\xrightarrow{-15}$ $5x$ $\xrightarrow{\div 5}$ x

2 **a** $x = 7$ **b** $x = -21$ **c** $x = 5\frac{1}{2}$
d $x = -6\frac{4}{9}$ **e** $y = 2\frac{2}{3}$ **f** $p = 19\frac{1}{4}$
g $d = -23$ **h** $c = 1\frac{3}{11}$ **i** $m = 22$
j $q = 1\frac{3}{7}$

3 **a** $p = 11\frac{1}{4}$ **b** $x = 26\frac{1}{3}$ **c** $m = 29$
d $p = 4$ **e** $p = -11$ **f** $p = 11$

4 **a** $x = 4\frac{2}{3}$ **b** $x = -\frac{1}{15}$ **c** $p = -6$
d $p = \frac{13}{10}$ **e** $x = 2$ **f** $x = 5$

5 **a** Yes **b** No

6 −23

7 **a** 2145 cm³ **b** 310 cm³ **c** 3.1 m³

8 **a** 356° **b** 59° **c** 100.4°

9 **a** $d = 48$ **b** $x = 6$ **c** $x = 648$
d $x = 6$ **e** $x = 10\frac{1}{2}$ **f** $x = 27\frac{1}{3}$
g $n = 6$ **h** $c = -11$ **i** $x = 0$

10 $x + 7 = 114; x = 107$

Review set 13B

1 b $\xrightarrow{\times(-4)}$ $-4b$ $\xrightarrow{+15}$ $15 - 4b$
-5 $\xleftarrow{\div(-4)}$ 20 $\xleftarrow{-15}$ 35

2 **a** $p = -15$ **b** $d = -5$ **c** $x = -9$
d $x = 9$ **e** $c = 4\frac{8}{11}$ **f** $q = 5\frac{4}{5}$
g $n = -1\frac{3}{34}$ **h** $c = 7$

3 **a** $q = 2\frac{7}{9}$ **b** $m = -10\frac{2}{5}$

4 **a** $k = 5\frac{3}{4}$ **b** $w = 52$

5 **a** $p = 19$ **b** $x = 6$ **c** $x = -7$
d $p = 5$ **e** $p = 4\frac{3}{5}$ **f** $x = \frac{37}{18}$
g $x = \frac{1}{2}$ **h** $p = \frac{4}{5}$ **i** $x = 6$
j $x = 1$

6 **a** −23 **b** 75

7 −31

8 **a** $d = 32$ **b** $x = 61$ **c** $x = 15$
d $x = 11$ **e** $x = 12\frac{1}{3}$ **f** $x = 6\frac{3}{4}$
g $n = 7\frac{1}{3}$ **h** $x = -3\frac{4}{9}$ **i** $x = -2\frac{3}{14}$
j $x = 3\frac{2}{3}$ **k** $x = -\frac{11}{25}$ **l** $x = 4$

9 **a** 120° **b** 10° **c** 0°

10 **a** $2x = -8; x = -4$ **b** 32, 33, 34, 35

Review set 13C

1 **a** $n = -2\frac{3}{11}$ **b** $q = 16$
c $p = -4\frac{1}{7}$ **d** $c = 5$

2 **a** Yes **b** Yes

3 **a** $r = 7$ **b** $x = 3\frac{3}{4}$
c $x = -13$ **d** $r = 2\frac{1}{8}$

4 76

5 **a** $d = 20$ **b** $x = 10$ **c** $x = 6$
d $x = 7$ **e** $x = 6\frac{2}{3}$ **f** $x = 4\frac{3}{7}$
g $n = 3$ **h** $c = 3$ **i** $x = \frac{1}{4}$

6 **a** $r = 2\frac{3}{4}$ **b** $p = 3$ **c** $x = -8$
d $x = 1\frac{3}{7}$ **e** $p = -16$ **f** $x = -1$

7 2168 cm^2 **8** 3.1 s

9 **a** 5 **b** 13 **c** 25 **d** 20

10 **a** $x + 1 + x + 3 + x + 5 = 39$; 11, 13, 15
b $18 - 2x = 4x + 9$; $x = 1\frac{1}{2}$

Review set 13D

1 **a** $p = 1$ **b** $n = -9\frac{6}{7}$
c $c = 21$ **d** $p = 14\frac{2}{5}$

2 **a** No **b** No

3 941 **4** 11 310 cm^3

5 $B = 9.6$ mm

6 **a** $d = 16$ **b** $x = 7$ **c** $x = 12$
d $x = 11$ **e** $x = 32$ **f** $x = 2\frac{7}{8}$
g $n = 6$ **h** $c = -\frac{4}{9}$ **i** $x = -\frac{7}{12}$

7 **a** $m = 7$ **b** $p = -\frac{2}{5}$
c $p = -1$ **d** $r = 4$

8 **a** $r = -6\frac{3}{5}$ **b** $p = \frac{1}{9}$ **c** $x = 1\frac{5}{8}$
d $x = \frac{26}{35}$ **e** $r = 5$ **f** $x = -1$

9 **a** $-8(x - 4) = 50$; $x = -2\frac{1}{4}$
b $6(x - 7) = -2x + 9$; $x = 6\frac{3}{8}$

10 $I = 150\frac{25}{48}$ or \$150.52

11 $T = 19\frac{1}{2}$

12 **a** 2500 **b** 1000 **c** 1200 **d** 5

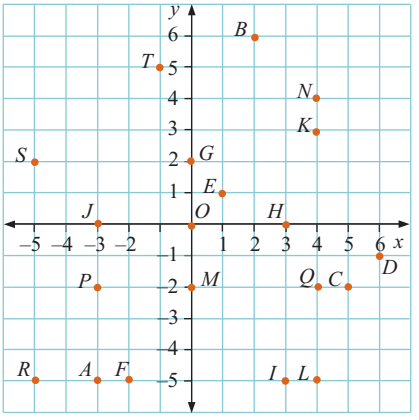
CHAPTER 14 COORDINATE GEOMETRY AND STRAIGHT LINES

Diagnostic test

1 B **2** D **3** B **4** A **5** B
6 B **7** C **8** D

Exercise 14A

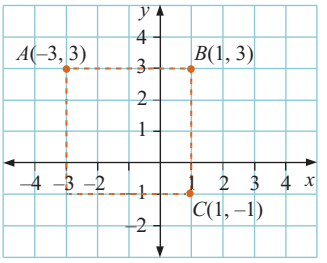
1



2 $A(-5, 5)$, $B(3, -4)$, $C(5, 3)$, $D(0, 5)$, $E(-3, 0)$, $F(-1, -3)$, $G(-5, -4)$, $H(6, 5)$, $I(3, 1)$, $J(1, 3)$, $K(5, -3)$, $L(7, -2)$, $M(-3, -2)$, $N(-2, 3)$, $O(0, 0)$, $P(-4, 4)$, $Q(-2, -4)$, $R(2, 4)$, $S(3, -2)$, $T(6, -6)$, $U(-4, -6)$, $V(0, -1)$, $W(6, 0)$, $X(4, 5)$, $Y(4, -6)$, $Z(1, -3)$

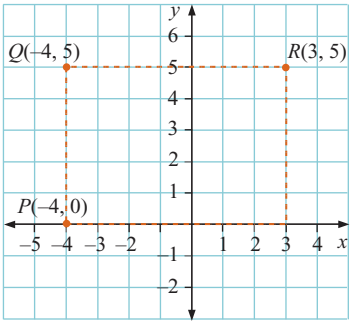
3 **a** $A(2, 3)$, $B(-4, 1)$, $C(3, 1)$, $D(-3, -3)$, $E(4, 4)$, $F(2, 5)$, $G(4, -4)$, $H(-1, 4)$
b A and F and E and G as they are on the same vertical line.
c B and C and E and H as they are on the same horizontal line.
d D and E as they are diagonally opposite each other.

4 **a**



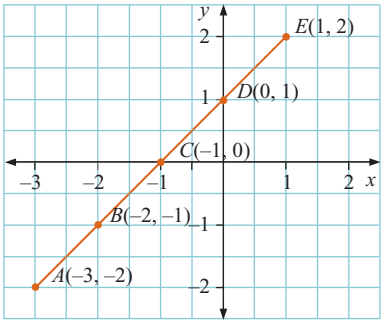
b $D(-3, -1)$

5 **a**



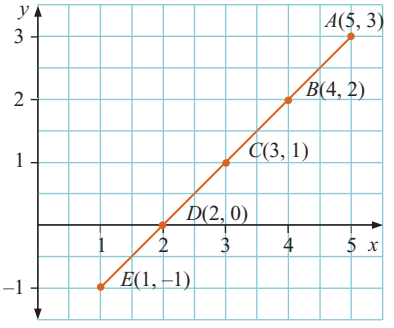
b $S(3, 0)$

6 **a**



b They are on a straight line.
c $F(2, 3)$, $G(3, 4)$, $H(4, 5)$

7 **a**



b $F(0, -2)$, $G(-1, -3)$, $H(-2, -4)$

8 $A(2\frac{1}{2}, 3)$, $B(-\frac{1}{2}, 1)$, $C(-1\frac{1}{2}, 2\frac{1}{2})$, $D(1\frac{3}{4}, 1)$, $E(1\frac{3}{4}, \frac{1}{2})$, $F(1\frac{1}{2}, -1\frac{1}{2})$, $G(2\frac{1}{2}, -\frac{1}{4})$, $H(\frac{1}{2}, -2\frac{1}{2})$, $I(-1\frac{1}{2}, -1\frac{3}{4})$, $J(-3\frac{1}{2}, \frac{3}{4})$, $K(-\frac{1}{4}, 0)$, $L(-1\frac{3}{4}, -3)$, $M(1, -1)$, $N(2\frac{1}{4}, -1)$, $O(\frac{3}{4}, 1\frac{3}{4})$, $P(-1\frac{1}{4}, 1\frac{1}{2})$, $Q(-2\frac{1}{2}, 1\frac{1}{2})$, $R(-\frac{3}{4}, -\frac{3}{4})$, $S(-1\frac{3}{4}, \frac{1}{2})$, $T(\frac{1}{4}, 0)$, $U(\frac{3}{4}, \frac{1}{2})$, $V(-2\frac{1}{2}, -1\frac{1}{2})$

Exercise 14B

1 a i Starting with 2, each number is 2 more than the number before it.
ii 10, 12, 14
b i Starting with 4, each number is 3 more than the number before it.
ii 16, 19, 22
c i Starting with 27, each number is 4 less than the number before it.
ii 11, 7, 3
d i Starting with 30, each number is 2 less than the number before it.
ii 22, 20, 18
e i Starting with 90, each number is 10 less than the number before it.
ii 50, 40, 30
f i Starting with 3, each number is 2 times the number before it.
ii 48, 96, 192
g i Starting with 2, each number is 3 times the number before it.
ii 162, 486, 1458
h i Starting with 3, each number is 10 times the number before it.
ii 30 000, 300 000, 3 000 000
i i Starting with 1, each number is $\frac{1}{10}$ of (or 0.1 times) the number before it.
ii 0.0001, 0.000 01, 0.000 001
j i Starting with 128, each number is $\frac{1}{2}$ of the number before it.
ii 8, 4, 2

2 a 7, 11 **b** 8, 14 **c** 7, 13
d 22, 14 **e** 49, 45 **f** 50, 1250
g 4, 16 **h** 25, 1 **i** 200, 100
j 30, 3000

Exercise 14C

1 a 2, 5, 8, 11, … **b** 5, 7, 9, 11, …
c 1, 5, 9, 13, … **d** 100, 90, 80, 70, …
e 45, 42, 39, 36, … **f** 50, 48, 46, 44, …
g 3, 6, 12, 24, … **h** 7, 70, 700, 7000, …
i 1, 3, 9, 27, … **j** 64, 32, 16, 8, …
k 2, 0.2, 0.02, 0.002, … **l** 1, 5, 21, 85, …
m 3, 7, 15, 31, … **n** 5, 6, 8, 12, …
o 2, 3, 7, 23, … **p** 7, 5, 1, −7, …

2 a 16 m² **b** 32 m² **c** 48 m²
d 64 m² **e** 80 m²

3 a 8.5 cm **b** 7 cm **c** 5.5 cm
d 4 cm **e** 2.5 cm

4 a \$8.50 **b** \$12 **c** \$15.50
d \$19 **e** \$22.50

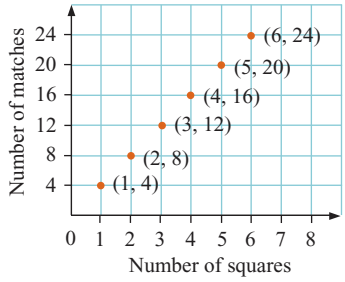
5 a 35 min **b** 50 min **c** 65 min
d 80 min **e** 95 min

Exercise 14D

1 a

Number of squares	1	2	3	4
Number of matches	4	8	12	16

b Number of matches equals four times the number of squares.
c (1, 4), (2, 8), (3, 12), (4, 16)
d

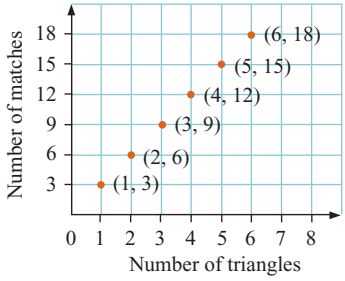


e (5, 20), (6, 24)

2 a

Number of triangles	1	2	3	4
Number of matches	3	6	9	12

b Number of matches equals three times the number of triangles.
c (1, 3), (2, 6), (3, 9), (4, 12)
d

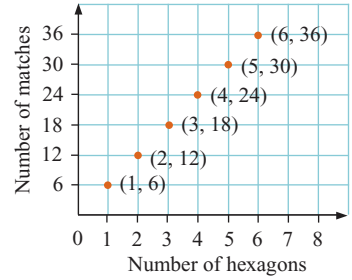


e (5, 15), (6, 18)

3 a

Number of hexagons	1	2	3	4
Number of matches	6	12	18	24

b The number of matches equals six times the number of hexagons.
c (1, 6), (2, 12), (3, 18), (4, 24)
d



e (5, 30), (6, 36)

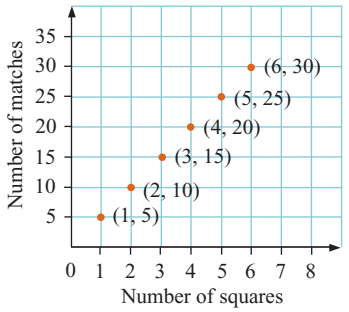
4 a

Number of squares	1	2	3	4
Number of matches	5	10	15	20

b The number of matches equals five times the number of squares.

c (1, 5), (2, 10), (3, 15), (4, 20)

d



e (5, 25), (6, 30)

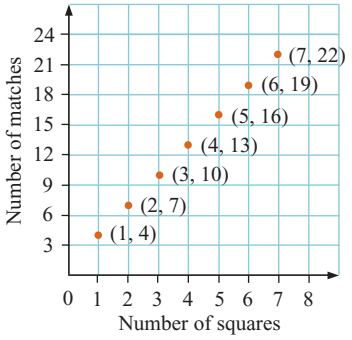
5 a

Number of squares	1	2	3	4	5
Number of matches	4	7	10	13	16

b Number of matches equals three times the number of squares plus 1.

c $y = 3x + 1$; (1, 4), (2, 7), (3, 10), (4, 13), (5, 16)

d



e (6, 19), (7, 22)

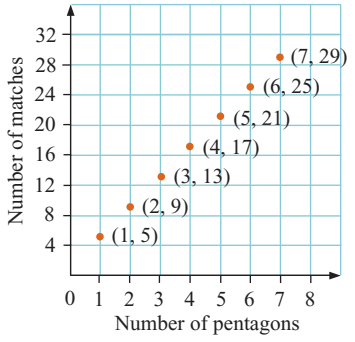
6 a

Number of pentagons	1	2	3	4	5
Number of matches	5	9	13	17	21

b Number of matches equals four times the number of pentagons plus 1.

c $y = 4x + 1$; (1, 5), (2, 9), (3, 13), (4, 17), (5, 21)

d



e (6, 25), (7, 29)

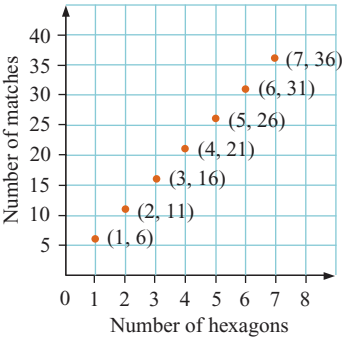
7 a

Number of hexagons	1	2	3	4	5
Number of matches	6	11	16	21	26

b Number of matches equals five times the number of hexagons plus one.

c $y = 5x + 1$; (1, 6), (2, 11), (3, 16), (4, 21), (5, 26)

d



e (6, 31), (7, 36)

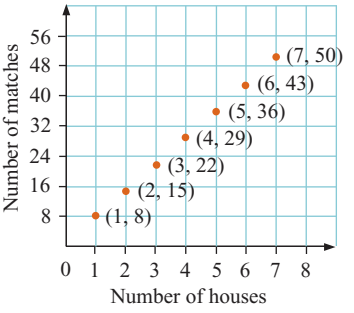
8 a

Number of houses	1	2	3	4	5
Number of matches	8	15	22	29	36

b The number of matches equals seven times the number of shapes plus 1.

c $y = 7x + 1$; (1, 8), (2, 15), (3, 22), (4, 29), (5, 26)

d



e (6, 43), (7, 50)

9 a

Number of shapes	1	2	3	4	5
Number of matches	4	8	12	16	20

b $y = 4x$

10 a

Number of shapes	1	2	3	4	5
Number of matches	5	10	15	20	25

b $y = 5 \times x = 5x$

11 a

Number of shapes	1	2	3	4	5
Number of matches	6	11	16	21	26

b $y = 5 \times x + 1 = 5x + 1$

12 a

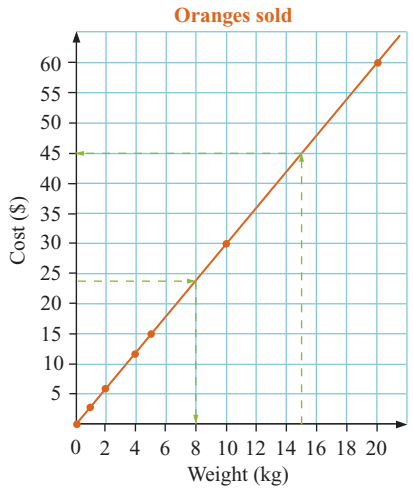
Number of shapes	1	2	3	4	5
Number of matches	1	4	7	10	13

b $y = 3 \times x - 2 = 3x - 2$

Exercise 14E

1 **a** (0, 0), (1, 3), (2, 6), (4, 12), (5, 15), (10, 30), (20, 60)

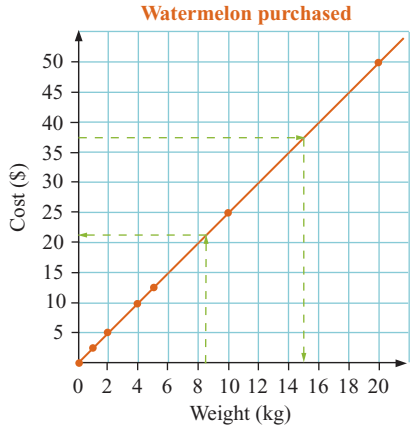
b



c $45 **d** 8 kg

2 **a** (0, 0), (1, 2.5), (2, 5), (4, 10), (5, 12.5), (10, 25), (20, 50)

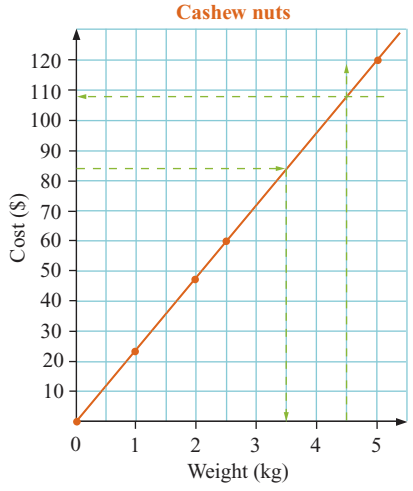
b



c $21.25 **d** 15 kg

3 **a** (0, 0), (1, 24), (2, 48), (5, 120)

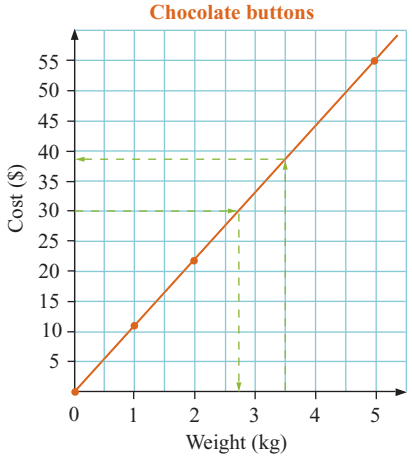
b



c $108 **d** $3\frac{1}{2}$ kg

4 **a** (0, 0), (1, 11), (2, 22), (5, 55)

b



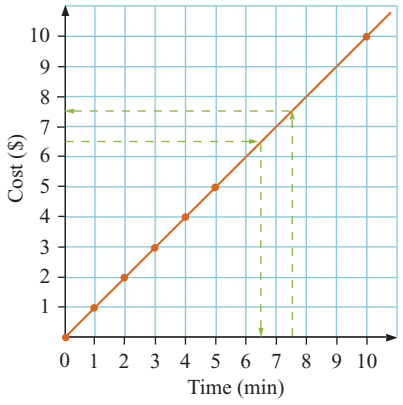
c $38.50 **d** ≈2.7 kg

5 **a**

Time (min)	0	1	2	3	4	5	10
Cost ($)	0	1	2	3	4	5	10

b (0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (10, 10)

c

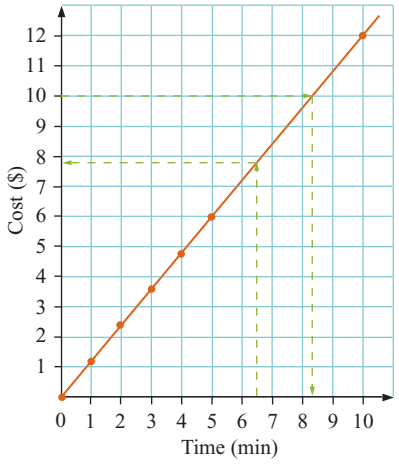


d $7.50 **e** 6.5 min

6 **a**

Time (min)	0	1	2	3	4	5	10
Cost ($)	0	1.20	2.40	3.60	4.80	6.00	12.00

b

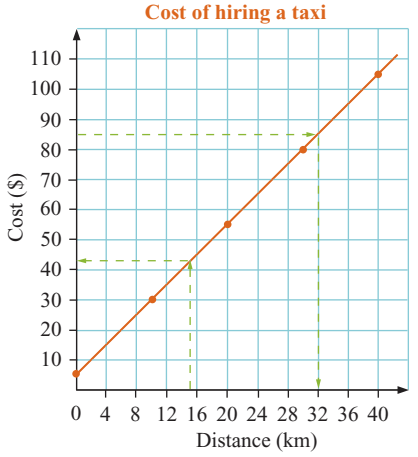


c ≈$7.80 **d** ≈8.3 min

7 a

Distance (km)	0	10	20	30	40
Cost in dollars	5	30	55	80	105

b

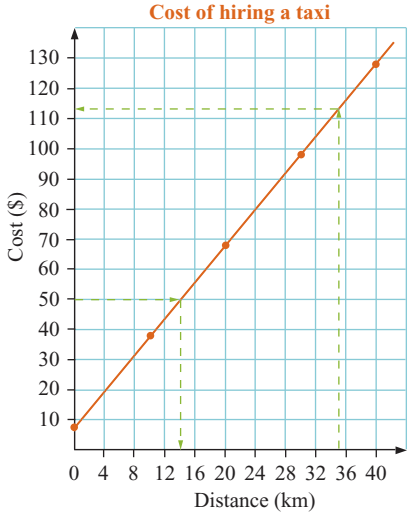


c $42.50 **d** 32 km

8 a

Kilometres	0	10	20	30	40
Cost in dollars	8	38	68	98	128

b

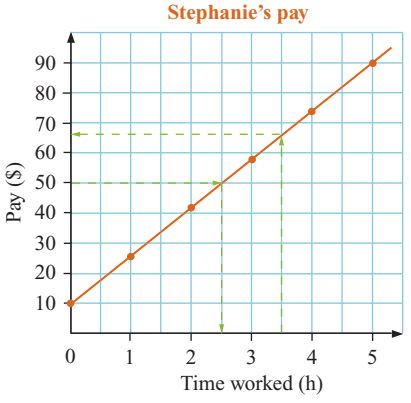


c $113 **d** 14 km

9 a

Hours	0	1	2	3	4	5
Pay ($)	10	26	42	58	74	90

b

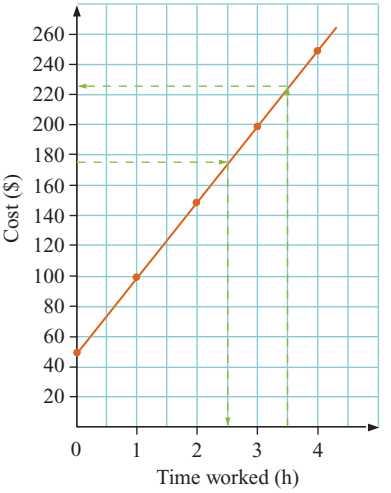


c ≈$66 **d** $2\frac{1}{2}$ hours

10 a

Number of hours	0	1	2	3	4
Cost ($)	50	100	150	200	250

b

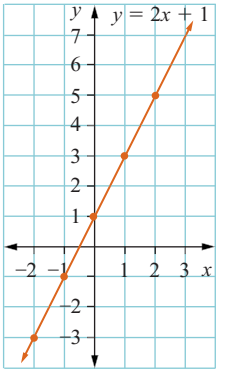


c ≈$225 **d** $2\frac{1}{2}$ hours

Exercise 14F

1 a

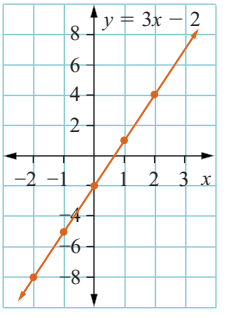
x	−2	−1	0	1	2
y	−3	−1	1	3	5



b $x = 3$

2 a

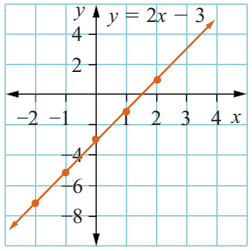
x	−2	−1	0	1	2
y	−8	−5	−2	1	4



b $x = 3$

3 a

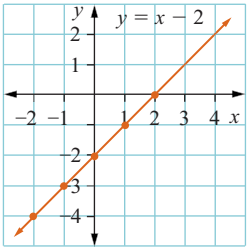
x	−2	−1	0	1	2
y	−7	−5	−3	−1	1



b $y = 3\frac{1}{2}$

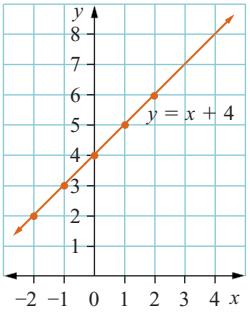
4 a

x	−2	−1	0	1	2
y	−4	−3	−2	−1	0



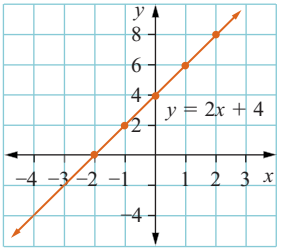
b

x	−2	−1	0	1	2
y	2	3	4	5	6



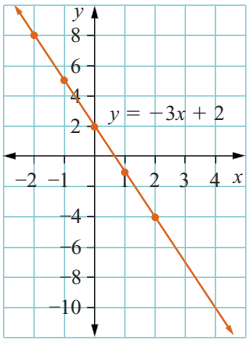
c

x	−2	−1	0	1	2
y	0	2	4	6	8



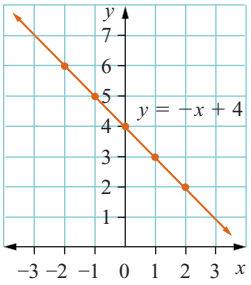
d

x	−2	−1	0	1	2
y	8	5	2	−1	−4



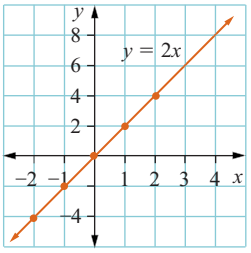
e

x	−2	−1	0	1	2
y	6	5	4	3	2



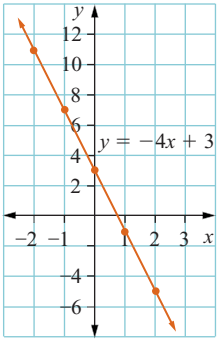
f

x	−2	−1	0	1	2
y	−4	−2	0	2	4



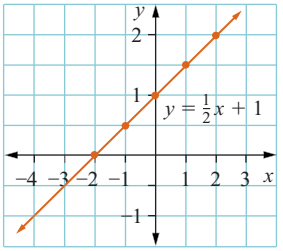
g

x	−2	−1	0	1	2
y	11	7	3	−1	−5



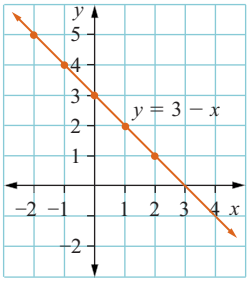
h

x	-2	-1	0	1	2
y	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2



i

x	-2	-1	0	1	2
y	5	4	3	2	1



5 **a** $x = 4$ **b** $x = 4$ **c** $x = -4$
d $x = 4$ **e** $x = -3$ **f** $x = 3\frac{1}{2}$
g $x = -\frac{1}{2}$ **h** $x = -4$ **i** $x = 4$

6 **a** $y = 2x - 1$

x	-2	-1	0	1	2
y	-5	-3	-1	1	3

b $y = 3x + 2$

x	-2	-1	0	1	2
y	-4	-1	2	5	8

c $y = 2x + 3$

x	-2	-1	0	1	2
y	-1	1	3	5	7

d $y = 4x + 1$

x	-2	-1	0	1	2
y	-7	-3	1	5	9

e $y = 4x - 3$

x	-2	-1	0	1	2
y	-11	-7	-3	1	5

f $y = -2x + 1$

x	-2	-1	0	1	2
y	5	3	1	-1	-3

g $y = -3x + 2$

x	-2	-1	0	1	2
y	8	5	2	-1	-4

h $y = 2x - 3$

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

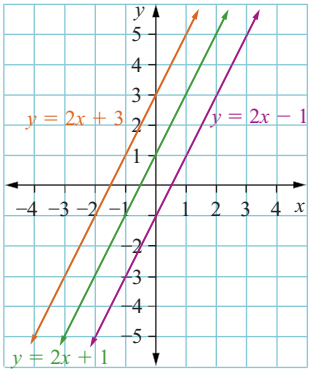
i $y = -2x + 5$

x	-2	-1	0	1	2
y	9	7	5	3	1

Exercise 14G

1 **a** The lines are parallel and cut the y-axis at 3, -1 and 1 respectively.

b



2 **a** They all intersect at (0, 2), but have different gradients.
b They intersect at (0, 1) and have the same gradient but in opposite directions.
c They intersect at (0, 0) and have the same gradient but in opposite directions.
d These lines are parallel. They have y-intercepts of 2, 1, and -3 respectively.

3 **a** The gradient or slope is the same, but the new graph intersects the y-axis at -3.

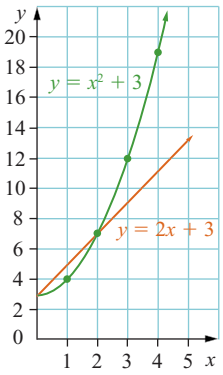
4 **a** The lines are parallel and cut the y-axis at different points.
b $y = 2x + 2$ is steeper than $y = x + 2$; they both cut the y-axis at 2.
c Both graphs intersect the y-axis at (0, 2) and have the same slope but in opposite directions.
d The lines are parallel but they cut the y-axis at different points.

5 Parallel: $y = 2x + 3$, $y = 2x - 1$, $y = 2x$
Parallel: $y = -3x - 2$, $y = -3x$, $y = -3x - 5$
Parallel: $y = 3x + 3$, $y = 3x$
Pass through same point: $y = 2x + 3$, $y = 3x + 3$, $y = 4x + 3$
Pass through same point: $y = -3x - 2$, $y = 5x - 2$

6 a

x	0	1	2	3	4
y	3	4	7	12	19

b

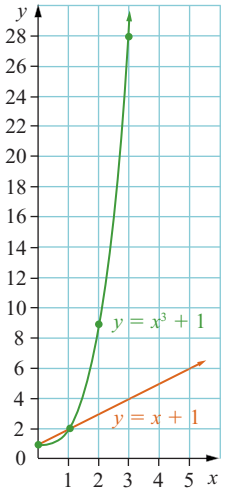


d The graphs intersect at (0, 3) and (2, 7).
$y = x^2 + 3$ is not a straight line.

7 a

x	0	1	2	3
y	1	2	9	28

b, c

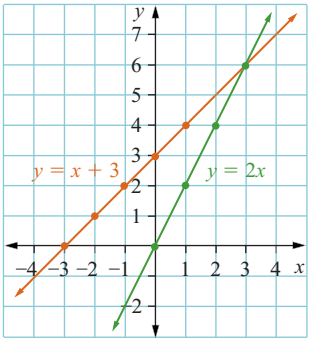


d $y = x^3 + 1$ is not a straight line. The graphs intersect at (0, 1) and (1, 2).

8 Linear: $y = 2x - 5$, $y = -x + 1$, $y = 7x + 5$
Non-linear: $y = x^2 - 5$, $y = x^3 + 4$, $y = x^4 + 2$

Exercise 14H

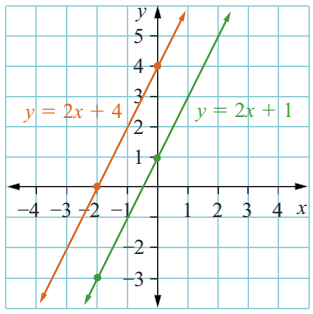
1 a



b (3, 6)

2 a (1, 2) **b** (1, 3) **c** (1, 4)
d (4, 9) **e** (4, 7) **f** (5, 8)

3 a



b No, as these graphs are parallel and will not intersect.
c They must be parallel, ∴ the coefficient of x must be the same.

Language in mathematics

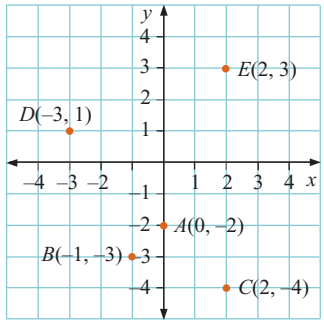
1 54 years
2 By travelling and observing
3 His idea of representing points by using ordered pairs on coordinate axes.
4 a His main rule was 'never to accept anything as true which I do not clearly and distinctly see to be so'.
b Believe only what you understand.
5 17 years
7 Use of index notation and the Cartesian plane; impossibility of trisecting angles using a compass and ruler only, and use of x for unknown variables.

Check your skills

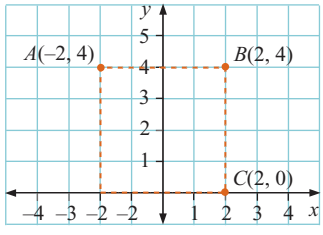
1 D **2** D **3** B **4** D **5** D
6 B **7** C **8** C **9** A **10** A
11 D **12** B

Review set 14A

1



2 a



b $D(-2, 0)$

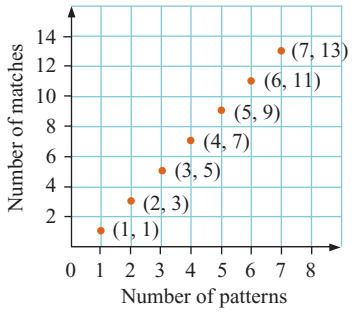
3 a

Number of patterns	1	2	3	4	5
Number of matches	1	3	5	7	9

b Number of matches is equal to two times the number of patterns minus 1.

c $y = 2x - 1$; (1, 1), (2, 3), (3, 5), (4, 7), (5, 9)

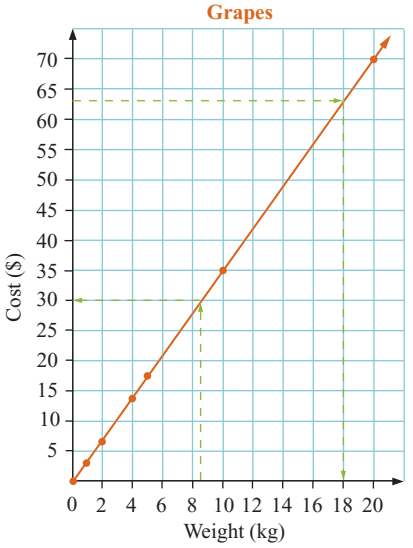
d



e (6, 11), (7, 13)

4 a (0, 0), (1, 3.5), (2, 7), (4, 14), (5, 17.5), (10, 35), (20, 70)

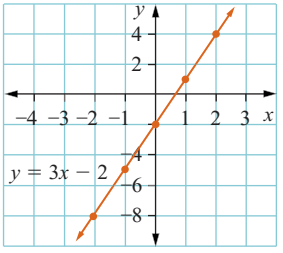
b



c ≈$30 **d** 18 kg

5

x	−2	−1	0	1	2
y	−8	−5	−2	1	4



6 a

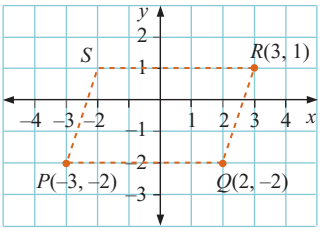
x	−2	−1	0	1	2
y	−1	1	3	5	7

b $y = 2x + 3$

Review set 14B

1 $A(-1, 5)$, $B(-1, 0)$, $C(-4, -3)$, $D(3, -2)$, $E(2, 4)$

2 a



b $S(-2, 1)$

3 a

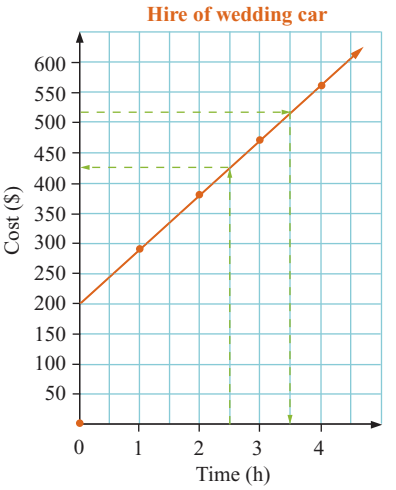
x	1	2	3	4	5
y	4	7	10	13	16

b $y = 3x + 1$

4 a

Number of hours	0	1	2	3	4
Cost ($)	200	290	380	470	560

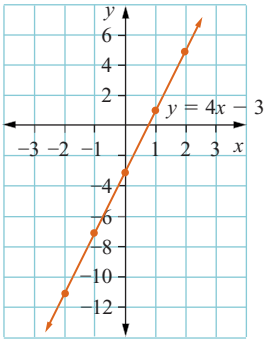
b



c $425 **d** $\approx 3\frac{1}{2}$ h

5

x	−2	−1	0	1	2
y	−11	−7	−3	1	5



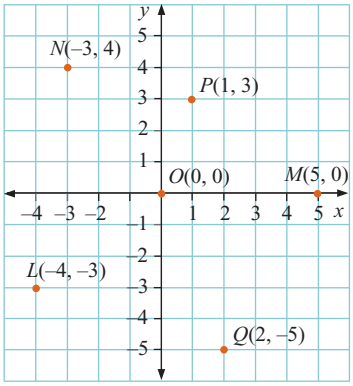
6 a

x	−2	−1	0	1	2
y	−2	1	4	7	10

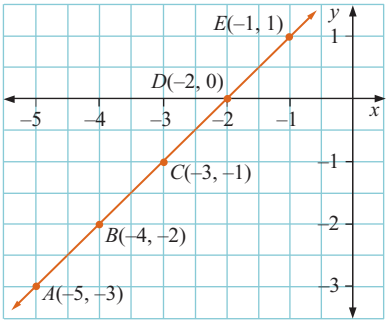
b $y = 3x + 4$

Review set 14C

1



2 a



b It is a straight line.

c $F(0, 2)$, $G(1, 3)$, $H(2, 4)$

3 a

x	1	2	3	4	5
y	5	9	13	17	21

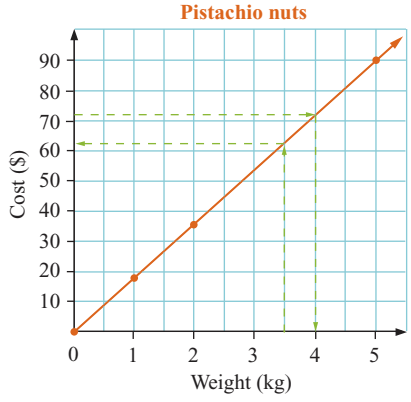
b Number of matches equals four times the number of squares + 1.

c $y = 4x + 1$; (1, 5), (2, 9), (3, 13), (4, 17), (5, 21)

d (6, 25), (7, 29)

4 a (0, 0), (1, 18), (2, 36), (5, 90)

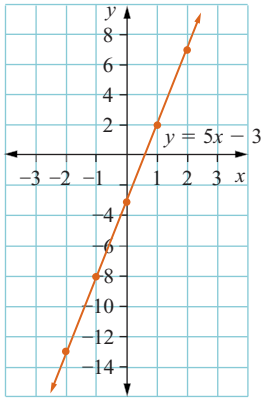
b



c $63 **d** 4 kg

5

x	−2	−1	0	1	2
y	−13	−8	−3	2	7



6 a

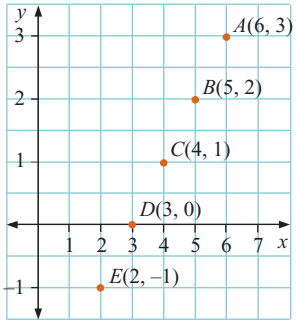
x	−2	−1	0	1	2
y	7	5	3	1	−1

b $y = -2x + 3$

Review set 14D

1 $A(4, -5)$, $B(-3, -1)$, $C(0, 2)$, $D(5, 4)$, $E(-3, 4)$

2 a



b $F(1, -2)$, $G(0, -3)$, $H(-1, -4)$

3 a

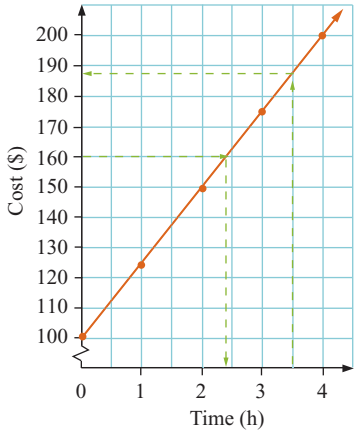
x	1	2	3	4	5
y	3	5	7	9	11

b $y = 2x + 1$

4 a

Number of hours	0	1	2	3	4
Cost ($)	100	125	150	175	200

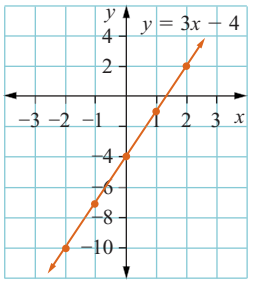
b



c ≈$188 **d** 2.4 h or 2 h 24 min

5

x	−2	−1	0	1	2
y	−10	−7	−4	−1	2



6 a

x	−2	−1	0	1	2
y	−13	−8	−3	2	7

b $y = 5x - 3$

CHAPTER 15 TRIANGLES AND QUADRILATERALS

Diagnostic test

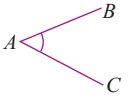
1 C **2** D **3** A **4** A **5** B
6 A **7** D **8** C **9** B **10** A
11 C **12** C

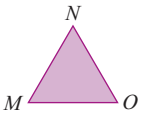
Exercise 15A

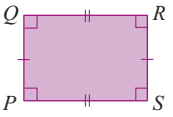
1 a 

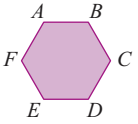
b 

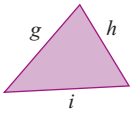
c 

d 

e 

f 

g 

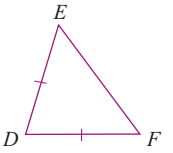
h 

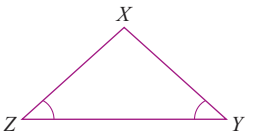
2 a Line TS **b** Triangle KLM
c Point R **d** Square $DEFG$
e Interval OP
f Triangle with sides o, p, q or triangle OPQ
g Pentagon $UVWXY$ **h** Angle PTA or $\angle T$

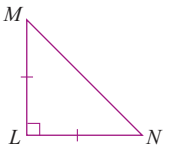
3 a TP, PT, TR, RP, RT, PR **b** FE, d, EF
c LM, MN, LN
d $\angle AOC$ as more than one angle has its vertex at O.

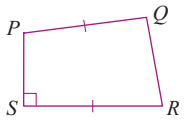
4 a $MN = NL$
b $IJ = LK$, $IL = JK$
c $MN = QP$, $MO = OP$
d $EF = DG$, $FG = ED$, $HF = HG = HE = HD$
e $BC = CD = DE = EA = AB$

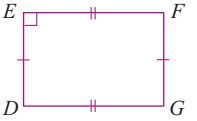
5 a $\angle BAC = \angle CBA = \angle ACB$, or $\angle A = \angle B = \angle C$
b $\angle PQO = \angle OSR$, $\angle ORS = \angle OPQ$, or $\angle Q = \angle S$, $\angle R = \angle P$
c $\angle KJB = \angle JID = \angle IHF$
d $\angle WVZ = \angle WXY$, $\angle VZY = \angle VWX$, or $\angle V = \angle X$, $\angle Z = \angle W$

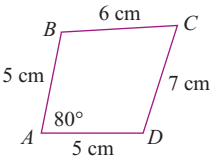
6 a 

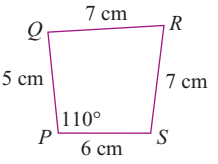
b 

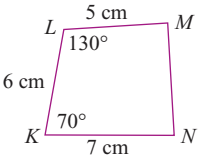
c 

d 

e 

f 

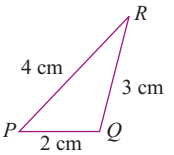
g 

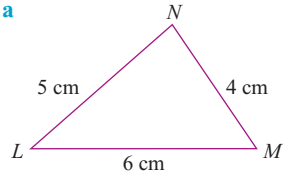
h 

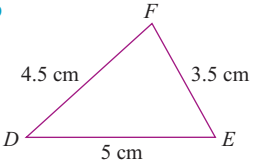
7 a Triangle ABC, $AB = AC$
b Triangle STU, $ST = UT$, $\angle T = 90°$
c Triangle JKL, $\angle L = 120°$, $\angle J = \angle K$
d Quadrilateral $RSTU$, $ST = UR$, $SR = TU$
e Quadrilateral $ABCD$, $AD = DC = CB = AB$
f Quadrilateral $FGHI$, $FG = GH$, $\angle F = \angle H$, $FI = IH$

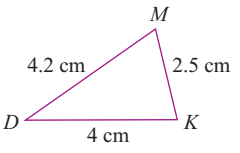
Exercise 15B

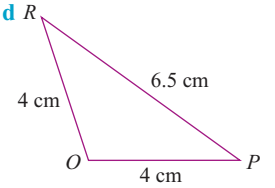
Constructions are shown at half size.

1 

2 a 

b 

c 

d 

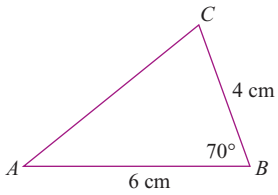
3 b i Largest angle **ii** Smallest angle
c The largest angle is opposite the longest side.

4

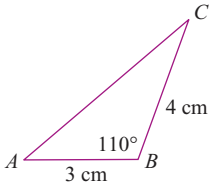
	Longest side	Sum of other 2 sides	Is longest side greater than sum of other 2 sides?
a	$LM = 6$ cm	9 cm	No
b	$DE = 5$ cm	8 cm	No
c	$DM = 4.2$ cm	6.5 cm	No
d	$PR = 6.5$ cm	8 cm	No

5 **a** No, $1 + 7 < 9$ **b** Yes, $5.2 + 6 > 8$

6



7 **a**



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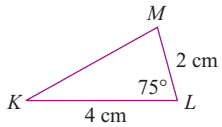

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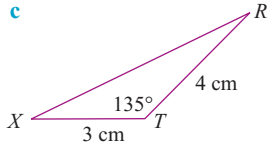

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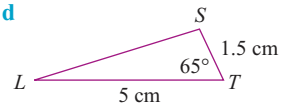

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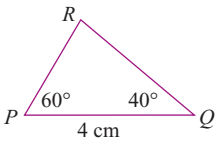
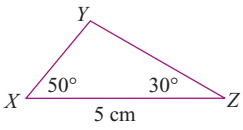


9 **a**



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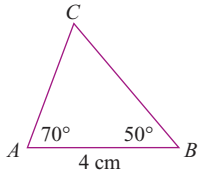

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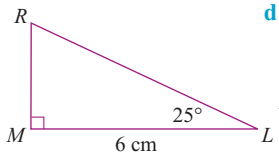

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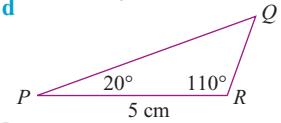


11 No, the largest angle (80°) should be opposite the longest side.

Exercise 15C

1 **a** Scalene **b** Isosceles
c Equilateral

2 **a** Isosceles **b** Scalene
c Equilateral **d** Scalene

3 **a** Right-angled **b** Obtuse-angled
c Acute-angled **d** Acute-angled

4 **a** Right-angled **b** Acute-angled
c Right-angled **d** Obtuse-angled

5 **a** Right-angled scalene
b Obtuse-angled isosceles
c Acute-angled equilateral
d Right-angled isosceles

6 The interior angles of an equilateral triangle are all 60°.

7 The base angles of an isosceles triangle are equal.

Exercise 15D

1 $x + 60° + 80° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 140° = 180°$
$x = 180° - 140° = 40°$
or $x = 180° - 60° - 80°$
(Angle sum of a triangle is 180°.)
$\therefore x = 180° - 140° = 40°$

2 **a** $x = 49°$ **b** $k = 87°$
c $l = 18°$ **d** $s = 49°$
e $k = 40°$ **f** $m = 47°$
g $t = 19°$ **h** $w = 39°$
i $n = 40°$ **j** $u = 116°$
k $z = 46°$ **l** $e = 90°$
m $g = 84°$ **n** $j = 42°$
o $b = 59°$

3 **a** $a + 50° + 70° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore a + 120° = 180°$
$a = 180° - 120° = 60°$
$b + 100° + 60° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore b + 160° = 180°$
$b = 180° - 160° = 20°$

b $a + 50° + 55° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore a + 105° = 180°$
$a = 180° - 105° = 75°$
$b = 55°$ (Corresponding angles as $TP \parallel SQ$.)
$c = 50°$ (Corresponding angles as $TP \parallel SQ$.)

4 **a** $a = 85°$ (Angle sum of a triangle is 180°.)
$b = 56°$ (Angle sum of a triangle is 180°.)

b $n = 20°$ (Angle sum of a triangle is 180°.)
$m = 53°$ (Angle sum of a triangle is 180°.)

c $k = 14°$ (Angle sum of a triangle is 180°.)
$j = 81°$ (Angle sum of a triangle is 180°.)

d $x = 52°$ (Angle sum of a triangle is 180°, or corresponding angles on parallel lines are equal.)
$y = 38°$(Angle sum of a triangle is 180°, or corresponding angles on parallel lines are equal.)

e $k = 62°$ (Angle sum of a triangle is 180°.)
$l = 70°$ (Corresponding angles on parallel lines are equal.)
$m = 48°$ (Corresponding angles on parallel lines are equal.)

f $g = 60°$ (Angle sum of a triangle is 180°.)
$h = 50°$ (Corresponding angles on parallel lines are equal.)
$i = 70°$ (Corresponding angles on parallel lines are equal.)

5 **a** $x + 70° + 30° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore x + 100° = 180°$
$x = 180° - 100° = 80°$
$y = 180° - 80° = 100°$
(Adjacent angles on a straight line are supplementary.)
$z + 40° + 100° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore z + 140° = 180°$
$z = 180° - 140° = 40°$

b $x = 36°$ (Vertically opposite angles equal.)
$y + 90° + 36° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore y + 126° = 180°$
$y = 180° - 126° = 54°$

6 **a** $x = 89°$ (Angle sum of a triangle is 180°.)
$y = 107°$ (Adjacent angles on a straight line are supplementary.)
$z = 39°$ (Angle sum of a triangle is 180°.)

b $x = 42°$ (Angle sum of a triangle is 180°.)
$y = 90°$ (Adjacent angles on a straight line are supplementary.)
$z = 58°$ (Angle sum of a triangle is 180°.)

c $w = 90°$ (Adjacent angles on a straight line are supplementary.)
$x = 40°$ (Angle sum of a triangle is 180°.)
$y = 50°$ (Sum of complementary adjacent angles is 90°.)
$z = 40°$ (Angle sum of a triangle is 180°.)

d $l = 57°$ (Angle sum of a triangle is 180°.)
$m = 57°$ (Vertically opposite angles are equal.)
$n = 48°$ (Angle sum of a triangle is 180°.)

e $x = 25°$ (Angle sum of a triangle is 180°.)
$y = 25°$ (Vertically opposite angles are equal.)
$z = 65°$ (Angle sum of a triangle is 180°.)

f $x = 63°$ (Corresponding angles on parallel lines are equal.)
$y = 49°$ (Corresponding angles on parallel lines are equal.)
$z = 68°$ (Angle sum of a triangle is 180°.)

g $a = 69°$ (Alternate angles are equal.)
$b = 69°$ (Angle sum of a triangle is 180°.)
$c = 42°$ (Alternate angles are equal, or adjacent angles on a straight line are supplementary.)

h $s = 44°$ (Adjacent angles on a straight line are supplementary.)
$t = 96°$ (Angle sum of a triangle is 180°.)

i $f = 59°$ (Alternate angles on parallel lines are equal.)
$h = 58°$ (Adjacent angles on a straight line are supplementary.)
$g = 63°$ (Angle sum of a triangle is 180°.)

7 a $2x + 3x + 70° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore 5x + 70° = 180°$
$5x = 180° - 70° = 110°$
$x = \frac{110°}{5} = 22°$
$\therefore$ Unknown angles are:
$2x = 2 \times 22° = 44°$
and $3x = 3 \times 22° = 66°$

b $(2x - 5) + (x + 10°) + 61° = 180°$ (Angle sum of a triangle is 180°.)
$\therefore 3x + 66° = 180°$
$3x = 180° - 66° = 114°$
$x = \frac{114°}{3} = 38°$
$\therefore$ Unknown angles are:
$2x - 5° = 2 \times 38° - 5° = 71°$
and $x + 10° = 38° + 10° = 48°$

8 a $2x + 55° + 78° = 180°$, $x = 23\frac{1}{2}°$; 47°

b $5b + 70° + 50° = 180°$, $b = 12°$; 60°

c $3m + 8m + m = 180°$, $m = 15°$; 15°, 45°, 120°

d $3a + 5a + 2a = 180°$, $a = 18°$; 36°, 54°, 90°

e $(x + 5°) + (2x + 15°) + 55° = 180°$, $x = 35°$; 40°, 85°

f $2y + (2y + 7°) + 53° = 180°$, $y = 30°$; 60°, 67°

Exercise 15E

1 a $\triangle ABC$ is an equilateral triangle. (All sides are equal.)
$\therefore x = 60°$ (Angles of an equilateral triangle are 60°.)

b $\triangle PQR$ is an equilateral triangle. (All angles are equal.)
$\therefore y = 5$ cm (Sides of an equilateral triangle are equal.)

c $\triangle KLM$ is an isosceles triangle. (Base angles are equal.)
$\therefore z = 6.8$ cm (Sides opposite base angles of an isosceles triangle are equal.)

d $\triangle XYZ$ is an isosceles triangle. (Two sides are equal.)
$\therefore \angle Y = a$ (Base angles of an isosceles triangle are equal.)
$a + a + 50° = 180°$ (Angle sum of a triangle is 180°.)
$2a + 50° = 180°$
$2a = 180° - 50° = 130°$
$a = \frac{130°}{2} = 65°$

2 a $y = 4$ cm (Sides of equilateral triangle are equal.)

b $w = 60°$ (Angles of an equilateral triangle are 60°.)

c $t = 60°$ (Angles of an equilateral triangle are 60°.)

d $b = 60°$ (Angles of an equilateral triangle are 60°.)

e $d = 63°$ (Base angles of an isosceles triangle are equal.)

f $p = 8$ cm (Sides opposite base angles of an isosceles triangle are equal.)

g $y = 5.4$ cm (Sides opposite base angles of an isosceles triangle are equal.)

h $f = 62°$ (Base angles of an isosceles triangle are equal.)

i $b = 14$ cm (Sides opposite base angles of an isosceles triangle are equal.)

j $c = 42°$ (Base angles of an isosceles triangle are equal.)
$d = 96°$ (Angle sum of a triangle is 180°.)

k $x = 40°$ (Angle sum of a triangle is 180°.)
$y = 10.4$ cm (Sides opposite base angles of an isosceles triangle are equal.)

l $d = 60°$ (Angle sum of a triangle is 180°.)
$w = 2.7$ cm (Sides of an equilateral triangle are equal.)

m $j = 7$ cm (Sides opposite base angles of an isosceles triangle are equal.)
$k = 45°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

n $h = 8.2$ cm (Sides opposite base angles of an isosceles triangle are equal.)
$x = 144°$ (Angle sum of a triangle is 180°.)

o $m = n$ (Base angles of an isosceles triangle are equal.)
$m = n = 60°$ (Angle sum of a triangle is 180°.)
$p = 5.5$ cm (Triangle is equilateral, and sides of an equilateral triangle are equal.)

3 a $x = 68°$ (Base angles of an isosceles triangle are equal.)
$y = 44°$ (Angle sum of a triangle is 180°.)

b $x = 22°$ (Base angles of an isosceles triangle are equal.)
$y = 136°$ (Angle sum of a triangle is 180°.)

c $y = 70°$ (Base angles of an isosceles triangle are equal.)
$x = 40°$ (Angle sum of a triangle is 180°.)

d $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 35°$ (Adjacent angles adding to 60°, and angles of an equilateral triangle are 60°.)

e $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 120°$ (adjacent angles on a straight line)

f $x = 60°$ (Angles of an equilateral triangle are 60°.)
$y = 18°$ (Adjacent angles adding to 60°, and angles of an equilateral triangle are 60°.)

g $x = 48°$ (Base angles of an isosceles triangle are equal.)
$y = 42°$ (Angle sum of a triangle is 180°.)
h $x = 43°$ (Base angles of an isosceles triangle are equal.)
$y = 94°$ (Angle sum of a triangle is 180°.)
i $x = 33°$ (Alternate angles on parallel lines are equal.)
$y = 114°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

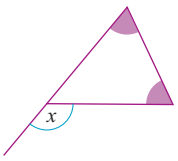
4 a Isosceles triangles have base angles equal.
$\therefore$ Angle marked • $= y$
$y + y + 36° = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 2y + 36° = 180°$
$2y = 180° - 36° = 144°$
$y = \frac{144°}{2} = 72°$
$\therefore$ Unknown angles are both 72° (base angles).

b Isosceles triangles have base angles equal.
$\therefore$ Angle marked • $= x + 16°$
$2x + (x + 16°) + (x + 16°) = 180°$
(Angle sum of a triangle is 180°.)
$\therefore 4x + 32° = 180°$
$4x = 180° - 32° = 148°$
$x = \frac{148°}{4} = 37°$
$\therefore$ Unknown angles are:
$2x = 2 \times 37° = 74°$
and $x + 16° = 37° + 16° = 53°$ (base angles)

5 a $2y + 46° = 180°$, $y = 67°$; 67°, 67°
b $2x + 90° = 180°$, $x = 45°$; 45°, 45°
c $x + 2x + 2x = 180°$, $x = 36°$; 36°, 72°, 72°
d $(x - 8°) + (x - 8°) + 5x = 180°$,
$x = 28°$; 20°, 20°, 140°
e $7x + x + x = 180°$, $x = 20°$; 140°, 20°, 20°
f $(3x + 11°) + (2x + 4°) + (2x + 4°) = 180°$,
$x = 23°$; 80°, 50°, 50°

Exercise 15F

1 a



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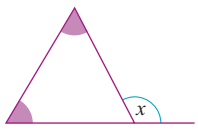

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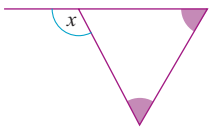

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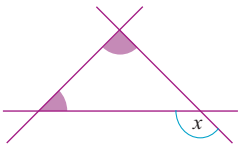

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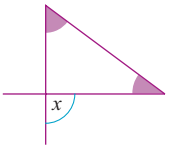

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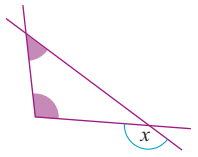


2 a $x = 70° + 40° = 110°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
b $135° = y + 60°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
or $y + 60° = 135°$
$\therefore y = 135° - 60° = 75°$

3 a $x = 127°$ **b** $y = 135°$ **c** $p = 153°$
d $k = 48°$ **e** $m = 55°$ **f** $w = 32°$

4 a $2x - 18° = 46° + 72° = 118°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$\therefore 2x = 118° + 18° = 136°$
$x = \frac{136°}{2} = 68°$
$\therefore$ Unknown angle $= 2x - 18° = 136° - 18° = 118°$

b $140° = (x + 10°) + 35°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$140° = x + 45°$
or $x + 45° = 140°$
$\therefore x = 140° - 45° = 95°$
$\therefore$ Unknown angle $= x + 10° = 95° + 10° = 105°$

5 a $4x = 128°$, $x = 32°$; 128°
b $2x + 6° = 114°$, $x = 54°$; 114°
c $3x - 2° = 106°$, $x = 36°$; 106°
d $2x + 7° = 129°$, $x = 61°$; 129°
e $8x = 120°$, $x = 15°$; 120°
f $3x + 3x = 114°$, $x = 19°$; 57°, 57°

Exercise 15G

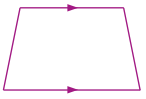
1

Property	Parallelogram	Rectangle	Rhombus
Opposite sides parallel	Y	Y	Y
Opposite sides equal	Y	Y	Y
Opposite angles equal	Y	Y	Y
All sides equal	N	N	Y
Adjacent sides perpendicular	N	Y	N
Diagonals equal	N	Y	N
Adjacent sides equal	N	N	Y
Diagonals bisect each other	Y	Y	Y
Diagonals meet at right angles	N	N	Y
Diagonals bisect vertices	N	N	Y
All angles 90°	N	Y	N

Property	Square	Trapezium	Kite
Opposite sides parallel	Y	Y, one pair	N
Opposite sides equal	Y	N	N
Opposite angles equal	Y	N	Y, one pair
All sides equal	Y	N	N
Adjacent sides perpendicular	Y	N	N
Diagonals equal	Y	N	N
Adjacent sides equal	Y	N	Y, two pairs
Diagonals bisect each other	Y	N	Y, one diagonal
Diagonals meet at right angles	Y	N	Y
Diagonals bisect vertices	Y	N	Y, one diagonal
All angles 90°	Y	N	N

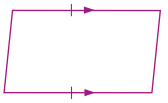
2 a $x = 2.5$ cm, $y = 4.5$ cm (Opposite sides of a parallelogram are equal.)
b $x = y = 5$ cm (All sides of a rhombus are equal.)
c $x = 3$ cm, $y = 7$ cm (Adjacent sides of a kite are equal.)
d $x = 35°$ (Alternate angles, $QR \parallel TS$.)
$y = 55°$ (All angles of a rectangle are 90°, and adjacent complementary angles.)
e $x = 65°$ (Opposite angles of a parallelogram are equal.)
$y = 115°$ (Co-interior angles add to 180°, $TU \parallel WV$.)
$z = 115°$ (Co-interior angles add to 180°, $TW \parallel UV$, or opposite angles of a parallelogram are equal.)
f $x = 30°$ ($2x + x = 90°$; all angles of a rectangle are 90°.)
$y = 60°$ ($y = 2x$; alternate angles, $DQ \parallel EF$, or angle sum of a triangle, $\angle F = 90°$.)
g $x = y = 45°$ (All angles of a square are 90°, and the diagonals of a square bisect its vertices.)
h $w = 40°$ (*A*lternate angles, $HI \parallel KJ$.)
$x = 35°$ (*A*lternate angles, $HK \parallel IJ$.)
$y = 105°$ (Angle sum of a triangle is 180°.)
$z = 105°$ (Opposite angles of a parallelogram are equal; or co-interior angles add to 180°, $HI \parallel KJ$.)
i $w = 85°$ (Opposite angles of a parallelogram are equal.)
$x = 55°$ (Angle sum of a triangle is 180°.)
$y = 55°$ (*A*lternate angles, $VY \parallel WX$.)
$z = 40°$ (*A*lternate angles, $VW \parallel YX$.)
j $x = 30°$ (Diagonals of a rhombus bisect its vertices.)
$y = 30°$ (*A*lternate angles, $PS \parallel QR$, or base angles of an isosceles triangle are equal.)
k $x = y = 45°$ (All angles of a square are 90°, and diagonals of a square bisect its vertices.)
l $x = 5$ cm (Diagonals of a rectangle are equal and bisect each other.)
m $x = 4.5$ cm, $y = 3$ cm (Diagonals of a parallelogram bisect each other.)
n $x = 50°$ (Diagonals of a rhombus bisect each other at right angles, and angle sum of a triangle is 180°.)
$y = 40°$ (Diagonals of a rhombus bisect its vertices.)
o $x = 110°$ (One pair of opposite angles are equal.)
$y = 4$ cm (Adjacent sides are equal.)
$z = 11$ cm (Adjacent sides are equal.)

3 Examples:

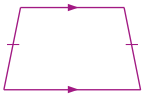
a

Trapezium

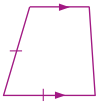
b

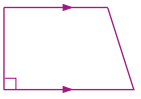
Parallelogram

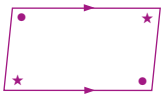
c

Parallelogram

or


Trapezium (isosceles)

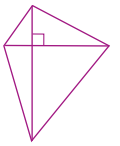
d

Trapezium

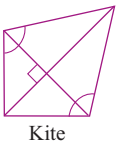
e

Trapezium

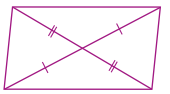
f

Parallelogram

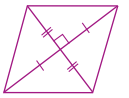
g

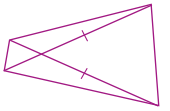
Rectangle

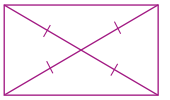
h

Quadrilateral

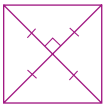
i

Kite

j

Parallelogram

k

Rhombus

l

Quadrilateral

m

Rectangle

n

Square

Exercise 15H

1 $x + 85° + 75° + 110° = 360°$ (Angle sum of a quadrilateral is 360°.)
$\therefore x + 270° = 360°$
$x = 360° - 270°$
$= 90°$

2 a $a = 60°$ (Opposite angles in a parallelogram are equal.)
b $x = 53°$ (Angle sum of a quadrilateral is 360°.)
c $k = 52°$ (Angle sum of a quadrilateral is 360°.)
d $t = 93°$ (Angle sum of a quadrilateral is 360°.)
e $p = 68°$ (Angle sum of a quadrilateral is 360°.)
f $w = 63°$ (Angle sum of a quadrilateral is 360°.)
g $m = 120°$ (Angle sum of a quadrilateral is 360°.)
h $q = 43°$ (Angle sum of a quadrilateral is 360°.)
i $d = 210°$ (Angle sum of a quadrilateral is 360°.)

3 $(2x - 10°) + (x + 50°) + 70° + 85° = 360°$
(Angle sum of a quadrilateral is 360°.)
$3x + 195° = 360°$
$\therefore 3x = 360° - 195°$
$= 165°$
$x = \frac{165°}{3}$
$= 55°$
$\therefore$ Unknown angles are:
$2x - 10° = 2 \times 55° - 10° = 100°$
and $x + 50° = 55° + 50° = 105°$

4 a $x + 2x + 88° + 104° = 360°$, $x = 56°$;
unknown angles are 56°, 112°
b $(2y - 3°) + 62° + 4y + 115° = 360°$, $y = 31°$;
unknown angles are 59°, 124°
c $(3p + 8°) + p + (7p - 2°) + p = 360°$, $p = 29.5°$;
unknown angles are 96.5°, 29.5°, 204.5°, 29.5°
d $3a + 3a + 2a + 208° = 360°$, $a = 19°$;
unknown angles are 57°, 57°, 38°
e $4m + 90° + (3m + 21°) + 46° = 360°$, $m = 29°$;
unknown angles are 116°, 108°
f $(2n + 7°) + n + (5n - 13°) + 46° = 360°$, $n = 40°$;
unknown angles are 87°, 40°, 187°

5 $x = 110°$ (Corresponding angles as $AF \parallel BE$.)
$y + 70° + 100° + 110° = 360°$ (Angle sum of a quadrilateral is 360°.)
$\therefore y + 280° = 360°$
$y = 360° - 280° = 80°$
$z + 120° + 80° + 70° = 360°$ (Angle sum of a quadrilateral is 360°.)
$\therefore z + 270° = 360°$
$z = 360° - 270° = 90°$

6 a $w = 36°$ (Angle sum of a quadrilateral is 360°.)
$x = 14°$ (Adjacent complementary angles)
$y = 76°$ (Angle sum of a triangle is 180°.)

b $w = 69°$ (Angle sum of a quadrilateral is 360°.)
$x = 111°$ (Adjacent angles on a straight line add to 180°.)
$y = 48°$ (Angle sum of a triangle is 180°.)

c $w = 50°$ (Adjacent angles on a straight line add to 180°.)
$x = 65°$ (Sum of co-interior angles is 180°.)
$y = 65°$ (Angle sum of a triangle is 180°.)

d $w = 228°$ (Angle sum at a point is 360°.)
$x = 36°$ (Angle sum of a quadrilateral is 360°.)
$y = 125°$ (Adjacent angles on a straight line add to 180°.)

e $w = 68°$ (Sum of co-interior angles is 180°.)
$x = 68°$ (Alternate angles are equal, or adjacent angles on a straight line.)
$y = 112°$ (Vertically opposite angles are equal, or adjacent angles on a straight line.)

f $w = 114°$ (Sum of co-interior angles is 180°.)
$x = 45°$ ($3x + x = 180°$, sum of co-interior angles is 180°; or $3x + x + 114° + 66° = 360°$, angle sum of a quadrilateral is 360°.)
$y = 135°$ (Corresponding angles are equal, or adjacent angles on a straight line.)

Language in mathematics

1

P	T	I	D	F	I	S	E	L	E	C	S	O	S	I	N	S
R	R	R	N	O	I	T	A	U	Q	E	M	D	U	T	Q	E
O	A	E	E	R	I	N	A	T	U	U	N	G	M	U	X	L
N	H	A	T	E	G	S	C	S	I	D	E	S	A	T	L	E
U	C	S	X	L	A	U	N	Z	L	D	A	R	E	Q	A	L
M	W	O	E	U	R	A	E	D	A	K	E	R	D	R	U	G
E	O	N	I	T	L	P	R	A	T	T	I	E	C	R	Q	N
R	L	I	S	M	A	A	A	L	E	O	S	T	F	A	E	A
A	F	N	S	R	W	C	D	P	R	O	V	E	E	I	N	I
L	O	G	T	A	A	D	T	I	A	S	U	B	M	O	H	R
C	N	G	Q	U	A	D	R	I	L	A	T	E	R	A	L	T

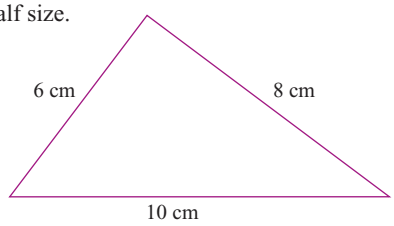
2 I find triangles and quadrilaterals fascinating.

Check your skills

1 D **2** A **3** C **4** D **5** B
6 A **7** D **8** D **9** B **10** D
11 B **12** A

Review set 15A

1 Shown at half size.



2 Example:

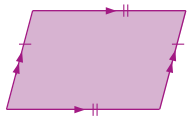
3 a $x = 53°$ (Base angles of isosceles triangles are equal.)

b $x = 52°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)

4 $(x + 10°) + 110° + 30° = 180°$
$x = 30°$
$\therefore$ Unknown angle is 40°.

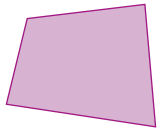
5 $u = 38°$ (Adjacent angles on a straight line add to 180°.)
$v = 94°$ (Angle sum of a triangle is 180°.)
$w = 76°$ (Angle sum of a triangle is 180°.)
$x = 76°$ (Vertically opposite angles are equal.)
$y = 86°$ (Adjacent angles on a straight line; or exterior angle of $\triangle EBC$ equals sum of two interior angles.)
$z = 18°$ (Angle sum of a triangle is 180°.)

6 a Example:



b Two pairs of equal and parallel sides; opposite angles are equal; diagonals bisect each other

7 a Example:



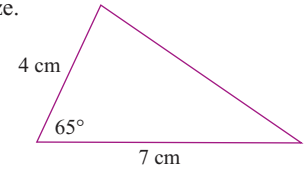
b Diagonals are all inside the quadrilateral.

8 a $x = 40°$ (Alternate angles are equal, $BC \parallel AD$.)
$y = 50°$ (All angles of a rectangle equal 90°, and adjacent complementary angles.)

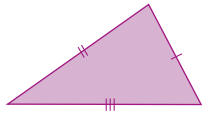
b $x = 55°$ (Diagonals of a rhombus bisect each other at right angles, and angle sum of a triangle is 180°.)
$y = 35°$ (Sides of a rhombus are equal, $SR = RQ$, and base angles in an isosceles triangle are equal.)

Review set 15B

1 Shown at half size.



2 Example:

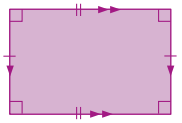


3 **a** $x = 60°$ (Angles of an equilateral triangle are 60°.)
b $x = 45°$ (Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

4 $x + 45° + (x + 15°) = 180°$, $x = 60°$
$\therefore$ Unknown angles are 60° and 75°.

5 $a = 87°$ (Corresponding angles are equal, $DE \parallel CB$.)
$b = 51°$ (Angle sum of a triangle is 180°.)
$c = 42°$ (Corresponding angles are equal, $DE \parallel CB$.)

6 **a** Example:



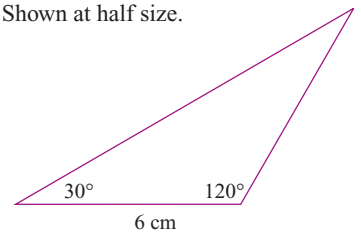
b Opposite sides are equal and parallel; diagonals are equal and bisect each other; all angles are 90°.

7 A concave quadrilateral has one diagonal outside the quadrilateral, or one interior angle greater than 180°.

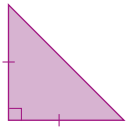
8 **a** $x = y = 45°$ (All angles in a square equal 90°, and diagonals bisect the angles at each vertex.)
b $w = 70°$ (Opposite angles of a parallelogram are equal.)
$x = 50°$ (Alternate angles are equal, $TU \parallel SV$.)
$y = 60°$ (Angle sum of a triangle is 180°; or sum of co-interior angles is 180°, $TU \parallel SV$.)
$z = 60°$ (Alternate angles, $TS \parallel UV$; or angle sum of a triangle is 180°.)

Review set 15C

1 Shown at half size.



2



3 **a** $x = 85°$ (Angle sum of a quadrilateral is 360°.)
b $x = 225°$ (Angle sum of a quadrilateral is 360°.)

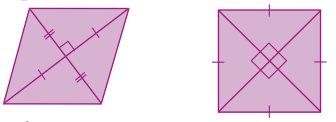
4 $(2x - 4°) + 52° + 4x = 180°$
(Angle sum of a triangle is 180°.) $\therefore x = 22°$
Unknown angles are 40° and 88°.
$y = 92°$ (Exterior angle of a triangle equals sum of two interior opposite angles, or adjacent angles on a straight line.)

5 $w = 80°$ (Adjacent angles on a straight line add to 180°.)
$x = 68°$ (Sum of co-interior angles on parallel lines is 180°.)
$y = 100°$ (Corresponding angles on parallel lines are equal, or angle sum of a quadrilateral is 360°.)
$z = 80°$ (Corresponding angles on parallel lines are equal, or adjacent angles on a straight line add to 180°.)

6 **a** Example:

b One pair of opposite sides parallel

7 **a** Examples:



b Rhombus or square

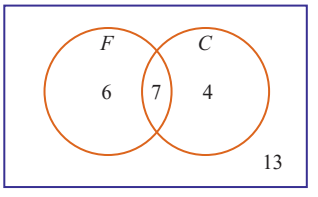
8 **a** $x = 2.8$ cm, $y = 3.5$ cm
(Diagonals of a parallelogram bisect each other.)
b $x = 4$ cm, $y = 7$ cm
(Two pairs of adjacent sides of a kite are equal.)
$v = 75°$ (Base angles of an isosceles triangle are equal, $GF = GH$.)
$w = 65°$ (Base angles of an isosceles triangle are equal, $EF = EH$.)
$z = 25°$ (Diagonals of a kite meet at right angles, and angle sum of a triangle is 180°.)

CUMULATIVE REVIEW CHAPTERS 11–15

1 **a** **i** n^2 **ii** $5p^4$
b **i** $p \times p$ **ii** $3 \times a \times a$
c **i** 9 **ii** 27
d **i** $20t$ **ii** $30m$ **iii** $30abc$
e **i** $11p$ **ii** $7m$ **iii** 0 **iv** $2x^2$
f **i** 300 **ii** 36 **iii** -2 **iv** 2
g **i** $7a - 3b$ **ii** $4pq$
iii $5k + 5$ **iv** $-3y$
v $-5y^2$ **vi** $2x^2$
vii $-2a$ **viii** $9a$
h **i** $4h$ **ii** $\frac{1}{3}$ **iii** 3 **iv** $\frac{2v}{3}$ **v** $\frac{2}{3b}$
i **i** $7x$ **ii** $-10a$ **iii** b **iv** $12xy$
j **i** $3x + 6$ **ii** $-5x + 15$
iii $-4x - 4$ **iv** $20x + 28$
v $15a^2 - 6ab$ **vi** $-10m + 15$
k **i** $p(x + 3)$ **ii** $m(n - t)$
iii $-4(x + 2)$ **iv** $6(2p - 3)$
v $5(3t + 5u)$ **vi** $4(5k - 4m)$
vii $-5(q + 2)$ **viii** $6k(3m - 2n)$
ix $3ab(2a + 5b)$

2 **a** **i** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **ii** $\frac{1}{8}$
b **i** 12 **ii** 3 **iii** $\frac{3}{12}$ or $\frac{1}{4}$
c **i** $\frac{1}{6}$ **ii** $\frac{3}{6} = \frac{1}{2}$
d Even chance **e** 90%
f **i** Rolling a die and getting a 3
ii Rolling a 7 on a normal die
iii Getting an even number
iv Rolling a die and getting a number from 1 to 6
g $\frac{5}{12}$

3 **a** **i** Not a 3 **ii** An odd number
iii A number less than or equal to 2
b $\frac{4}{7}$
c



i 6 **ii** 4 **iii** 17
iv 10 **v** 13

4 a

	Wear glasses	Do not wear glasses	
Left-handed	0	5	5
Not left-handed	7	18	25
	7	23	30

b **i** 16 **ii** 5 **iii** 5 **iv** 12
v 7 **vi** 6 **vii** 5 **viii** 3
ix 10 **x** 11 **xi** 4

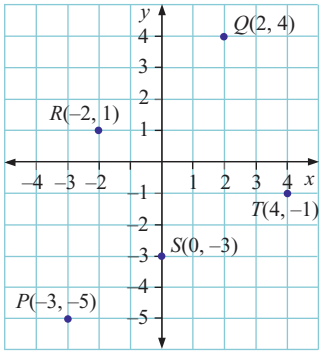
5 a $p = -10$ **b** $d = -3$ **c** $x = -7$
d $x = 8$ **e** $c = 5$ **f** $x = 8$
g $x = -\frac{17}{18}$ **h** $x = -\frac{11}{6}$ **i** $k = \frac{17}{4}$
j $x = \frac{23}{9}$ **k** $x = -\frac{38}{3}$ **l** $x = 12$

6 a No **b** $6x = 138$; $x = 23$
c **i** -38 **ii** 3
d The number is 12.

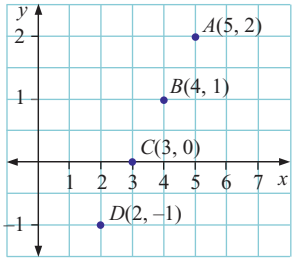
7 a **i** 7.8 **ii** 17 **iii** 20.8 **iv** 14
b **i** 50.8 **ii** 71.6 **iii** 54.6 **iv** 110.4
c **i** 4 **ii** 11 **iii** 17 **iv** 50
d **i** 6 **ii** 17 **iii** 19.9
e **i** 19 **ii** 107

8 a $A(-1, 4)$, $B(0, 2)$, $C(2, -1)$, $D(-3, -1)$, $E(3, 4)$

b



c **i**



ii $E(1, -2)$, $F(0, -3)$, $G(-1, -4)$

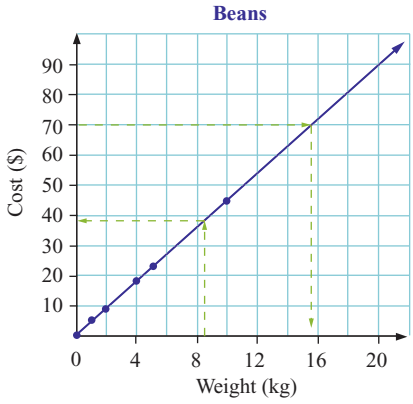
d **i**

Number of matches	1	2	3	4	5
Number of matches	7	13	19	25	31

ii Number of matches equals six times the number of shapes plus one; $y = 6x + 1$
iii (1, 7), (2, 13), (3, 19), (4, 25), (5, 31)
iv (6, 37), (7, 43)

e **i** (0, 0), (1, 4.5), (2, 9), (4, 18), (5, 22.5), (10, 45), (20, 90)

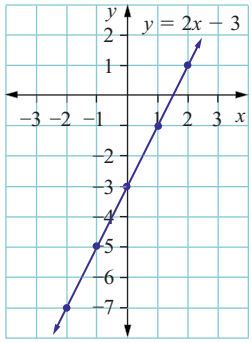
ii



iii ≈\$38 **iv** ≈15.5 kg

f

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

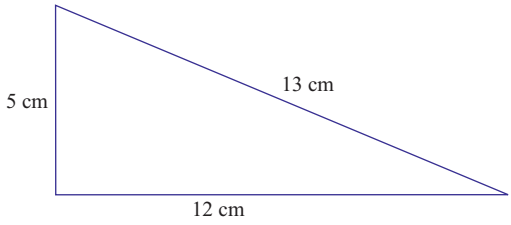


g **i**

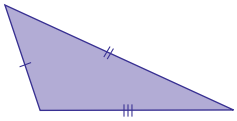
x	-2	-1	0	1	2
y	-8	-5	-2	1	4

ii $y = 3x - 2$

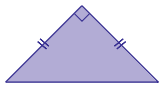
9 a Shown at half size.



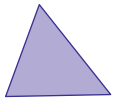
b **i** Example:



ii Example:



iii Example:



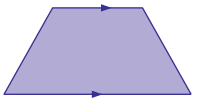
c **i** $2x = 180° - 68° = 112°$
$\therefore x = 56°$
(Base angles of an isosceles triangle are equal, and angle sum of a triangle is 180°.)

ii $127° = x + 53°$
$\therefore x = 74°$
(Exterior angle of a triangle equals sum of the two interior opposite angles.)

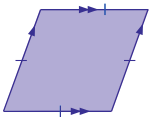
iii $x = 70°$ (Angle sum of a triangle is 180°.)

d $(90° - x) + 3x + 3x = 180°$
$\therefore x = 18°$
$\therefore$ Unknown angles are 72° and 54°.

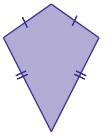
10 **a** **i** One pair of parallel sides



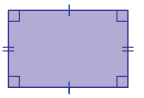
ii Two pairs of parallel sides.
All sides equal.
Opposite angles equal.
Diagonals bisect at right angles.
Diagonals bisect vertices.



iii Two pairs of adjacent sides equal.
One pair of opposite angles equal.
Diagonals intersect at right angles.
One diagonal bisects vertices and bisects other diagonal.



iv Two pairs of equal parallel sides.
All angles 90°.
Equal diagonals that bisect each other.



b **i** $x + 48° + 115° + 95° = 360°$
$\therefore x = 102°$
(Angle sum of a quadrilateral is 360°.)

ii $x + 40° = 90°$
$\therefore x = 50°$
(Diagonals of a rhombus intersect at 90°, and angle sum of a triangle is 180°.)
$y = x = 50°$ (Sides of a rhombus are equal, and base angles of an isosceles triangle are equal.)

c $m + 135° + 90° + (3m + 15°) = 360°$
$\therefore m = 30°$
$\therefore$ Unknown angles are 30° and 105°.

11 **a** $w = 95°$ (Sum of co-interior angles is 180°.)
$x = 85°$ (Corresponding angles are equal, or adjacent angles on a straight line add to 180°.)
$y = 95°$ (Adjacent angles on a straight line add to 180°, or corresponding angles are equal.)
$z = 132°$ (Sum of co-interior angles is 180°, or angle sum of a quadrilateral is 360°.)

b $x = 22°$ (Exterior angle of a triangle equals sum of two interior opposite angles.)
$y = 137°$ (Adjacent angles on a straight line add to 180°, or angle sum of a triangle is 180°.)
$z = 83°$ (Angle sum of a quadrilateral is 360°.)

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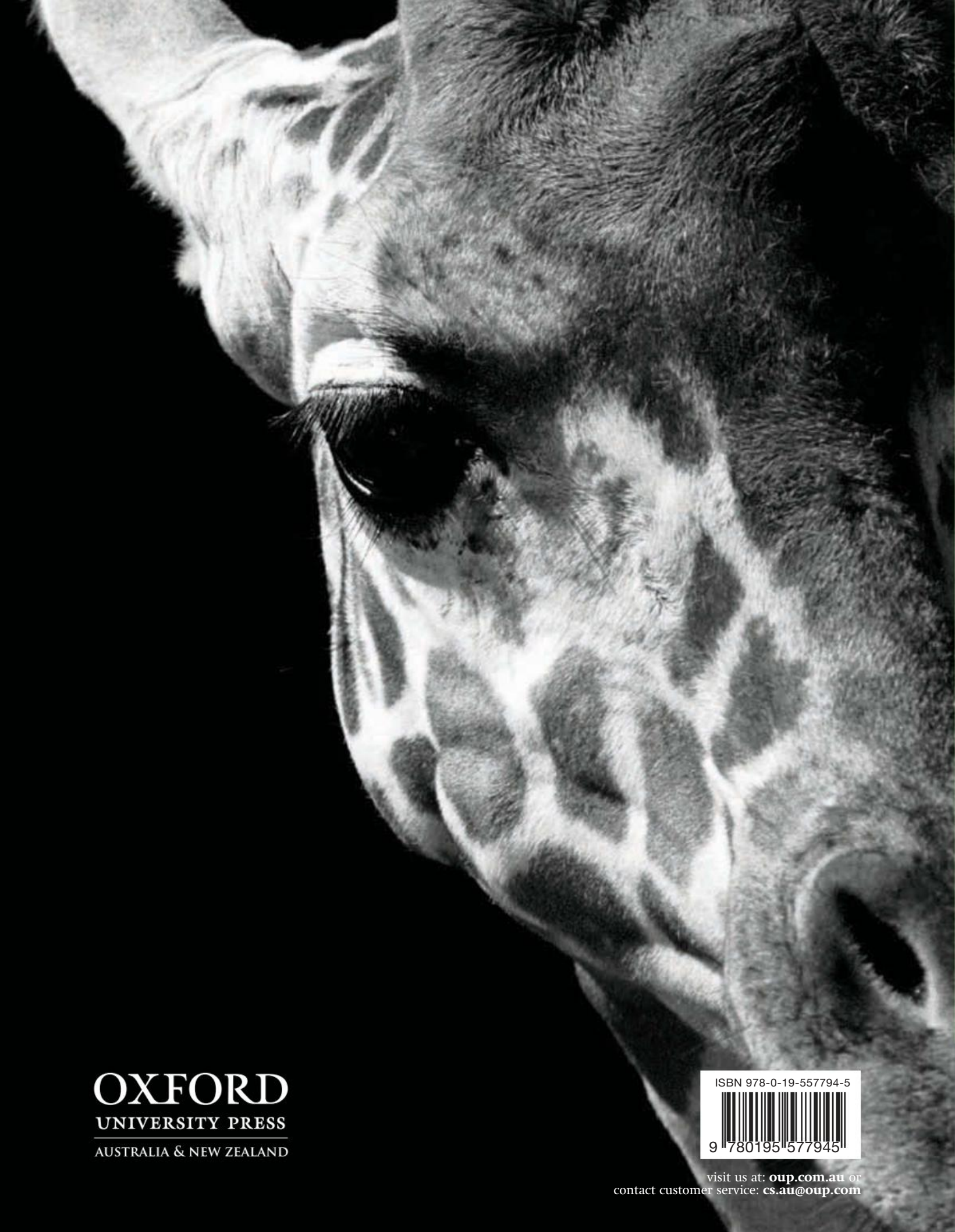

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