Mathematics

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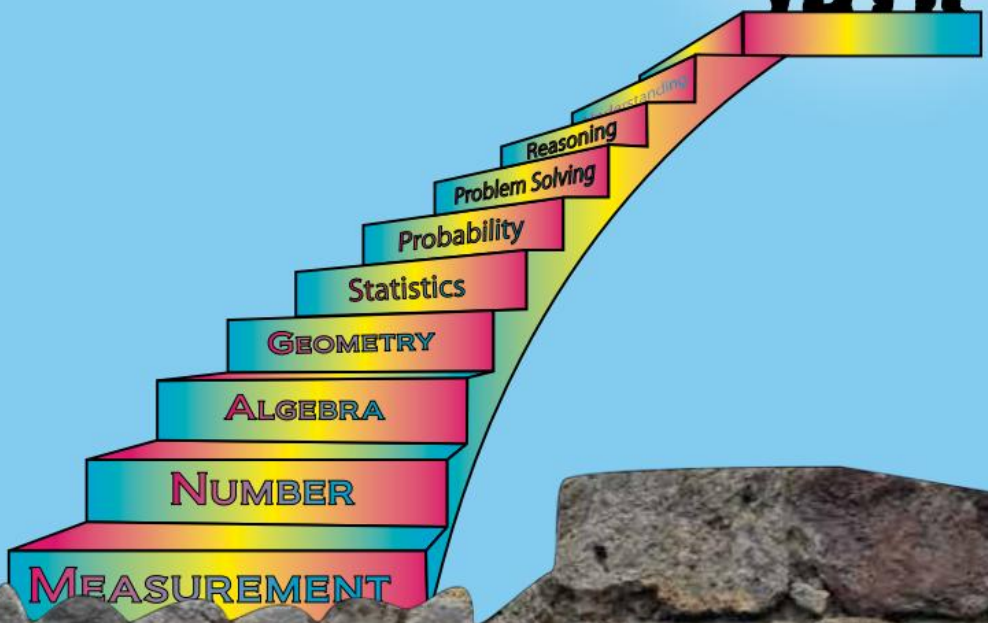
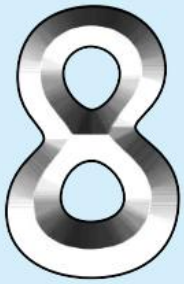


National Curriculum

Dr Terry Dwyer

Mathematics

National Curriculum



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The cover: Maths is the stairway to success in our technological economy and society.

"In today's world, economic access and full citizenship depend crucially on math and science literacy." - Robert P. Moses.

"Mathematics makes a significant contribution to our modern society; its basic skills are vital for the life opportunities of the youth. Let us all embrace and encourage the study of this indispensable subject."
- Alfred L. Teye.

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Preface

This text has been written for Year 8 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

Each chapter, apart from Review, contains:

- ★ Numerous worked examples
- ★ Numerous sets of graded exercises
- ★ An open-ended rich task
- ★ Mental computation
- ★ Technology in mathematics
- ★ Investigations
- ★ Puzzles
- ★ NAPLAN questions
- ★ Maths competition preparation
- ★ A mathematics game
- ★ A mathematics trick
- ★ A bit of mathematics history
- ★ Careers using mathematics
- ★ Chapter review

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Resources

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- ★ **Workprogram**
- ★ **Study guides** for each term.
- ★ Detailed **lesson plans** for each term.
- ★ Sample **assessment items**.
- ★ **Ebook**/Interactive **PDF** of this textbook.

Index Laws

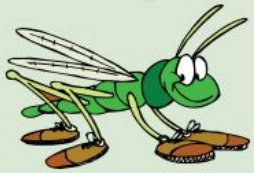
1

- ★ Use index notation with numbers to establish the index laws with positive integral indices and the zero index.
- ★ Evaluate numbers expressed as powers of positive integers.

A TASK

Napier's bones are multiplication tables written on strips of bones (or paper).

Logarithms are indices with base 10.



- Make a set of Napier's bones (There is plenty of information on the Internet).
- Learn how to use Napier's bones to multiply numbers such as 463×8.
- Choose an audience and demonstrate your Napier's bones.

A LITTLE BIT OF HISTORY

Logarithms are useful because they are a convenient way of handling large numbers. Lorgarithms are used in finance, physics, chemistry, agriculture, geology and many other fields.

1614 Napier invented a form of logarithm, based on algebra, to simplify mathematical calculations.

1620 Burgi used a form of logarithms, based on geometry, to simplify mathematical calculations.

1624 Napier and Biggs invent the modern form of logarithms.

Logarithms and Multiplication
Before calculators, large multiplication was very difficult and logarithms were used.

Example:

$$\begin{aligned} 6\times 2 &= 10^{0.778}\times 10^{0.301} \\ &= 10^{0.778+0.301} \\ &= 10^{1.079} \\ &= 11.995 \end{aligned}$$

Can you show that $6 = 10^{0.778}$?

Warmup

Exercise 1.1

Calculate each of the following:

1	1×1	**2**	2×2	**3**	3×3	**4**	4×4
5	5×5	**6**	1×1×1	**7**	2×2×2	**8**	3×3×3
9	4×4×4	**10**	5×5×5	**11**	1×1×1×1	**12**	2×2×2×2
13	3×3×3×3	**14**	4×4×4×4	**15**	5×5×5×5		

16 Copy and complete the following table:

Squares are often used. (Area square = side^2)

Cubes happen now and then. (Volume cube = side^3)

	Square	Cube	Fourth	Fifth
1				
2				
3				
4				
5				

17 10×10

18 10×10×10

19 10×10×10×10

20 10×10×10×10×10

21 10×10×10×10×10×10

22 10×10×10×10×10×10×10

4×4×4×4×4×4×4

On a calculator: 4 ^ 7 = 16384

The larger the power, the more useful a calculator will become.

23 2×2×2×2×2×2

24 5×5×5×5×5×5×5

25 1×1×1×1×1×1×1×1

26 2×2×2×2×2×2×2×2

27 How many 10s are multiplied together to equal 1 million?

28 Calculate 10^9 (10^9 is generally known as 1 billion).

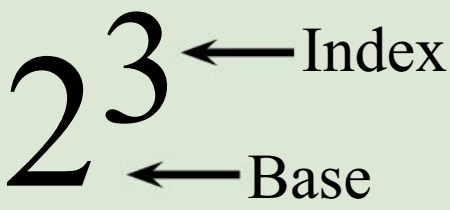
29 1 googol is known as 10^{100}. Can your calculator handle a number this large?

30 Write a terabyte in full (A terabyte drive (HDD) will hold 10^{12} bytes.).

31 Write 500 gigabytes in full (A 500 gigabyte drive will hold 500×10^9 bytes).

Indices

A convenient way of writing $2\times2\times2$ is



Exercise 1.2

Write the following in index form:

$2\times2\times2\times2\times2$ $= 2^5$	$a\times a\times a\times a$ $= a^4$

1	$4\times4\times4$	**2**	$2\times2\times2\times2$	**3**	$a\times a\times a$
4	$10\times10\times10$	**5**	$b\times b\times b\times b\times b$	**6**	$h\times h\times h$
7	$m\times m\times m\times m\times m$	**8**	$9\times9\times9\times9$	**9**	$3\times3\times3\times3\times3\times3$

Exercise 1.3

Write the following in factor form:

3^4 $= 3\times3\times3\times3$	b^3 $= b\times b\times b$

1	4^3	**2**	b^4	**3**	5^2
4	2^7	**5**	6^2	**6**	m^5
7	x^4	**8**	p^5	**9**	1^4

Exercise 1.4

Write the following in index form:

$2\times2\times2\times4\times4$ $= 2^3\times4^2$	$abbaaab$ $= a^4\times b^3$

1	$aabbbaa$	**2**	$3\times3\times3\times2\times2$	**3**	$abaaababb$
4	$2\times3\times2\times3\times2\times3\times2$	**5**	$bggggbbbg$	**6**	$zzzzzzzzzzz$
7	$ppqrppqqrrrp$	**8**	$2\times2\times2\times3\times4\times4\times4\times3$	**9**	$4gg4g4g4gg$

Accountants examine and advise on the finances, records and compliance of clients. Accountancy normally involves a lot of public contact.

- Relevant school subjects are English, Mathematics, IT.
- Courses normally involve a Degree with a major in accounting.

Index Law 1

Multiplying Indices:

$2^4 \times 2^2 = 2\times2\times2\times2 \times 2\times2$
$= 2^6$

Is the same as

Multiplying Indices:

$2^4 \times 2^2 = 2^{4+2}$
$= 2^6$

Multiplying Indices:

$a^3 \times a^2 = a\times a\times a \times a\times a$
$= a^5$

Is the same as

Multiplying Indices:

$a^3 \times a^2 = a^{3+2}$
$= a^5$

Index Law 1

$a^m \times a^n = a^{m+n}$

Exercise 1.5

Simplify and write the following in index form:

This only works if the bases are the same.

$2^3 \times 2^2$

$= 2\times2\times2 \times 2\times2$
$= 2^5$

or $2^3 \times 2^2 = 2^{3+2} = 2^5$

$a^2 \times a^5$

$= a\times a \times a\times a\times a\times a\times a$
$= a^7$

or $a^2 \times a^5 = a^{2+5} = a^7$

1	$2^2\times2^3$	**2**	$3^3\times3^2$	**3**	$2^4\times2^2$	**4**	$4^4\times4^3$
5	$a^2\times a^2$	**6**	$b^3\times b^2$	**7**	$z^5\times z^3$	**8**	$w^4\times w^3$
9	$2^3\times2^1$	**10**	$t^2\times t^5$	**11**	$5^2\times5^3$	**12**	$10^4\times10^5$
13	$3^2\times3^4$	**14**	$7^3\times7^2$	**15**	$d^2\times d^3$	**16**	$m^3\times m^3$
17	$c^4\times c^3$	**18**	$10^2\times10^4$	**19**	$a^2\times a^3$	**20**	$b^5\times b$
21	$h^5\times h^2$	**22**	$z^3\times z^2$	**23**	$n^5\times n^4$	**24**	$10^5\times10^2$

This only works if the bases are the same.

$3^2\times3^3\times3^2 = 3^{2+3+2}$
$= 3^7$

$a^2\times a^5\times a = a^{2+5+1}$
$= a^8$

25	$2^2\times2^3\times2^2$	**26**	$3^4\times3^3\times3^2$	**27**	$a^2\times a^3\times a^3$	**28**	$u^2\times u^2\times u^3$
29	$2\times2^3\times2^3$	**30**	$p^3\times p^3\times p$	**31**	$d^2\times d^3\times d^4$	**32**	$m^3\times m^3\times m^3$
33	$c^4\times c^3\times c$	**34**	$y^2\times y^4\times y^2$	**35**	$a^2\times a^3\times a^2$	**36**	$b^5\times b\times b^2$
37	$h^5\times h^2$	**38**	$z^3\times z^2\times z^4$	**39**	$b^5\times b^2\times b$	**40**	$z^3\times z^2\times z^4$

Index Law 2

Dividing Indices:

$2^4 \div 2^2 = \frac{2\times2\times2\times2}{2\times2} = 2\times2$

$= 2^2$

Is the same as

Dividing Indices:

$2^4 \div 2^2 = 2^{4-2}$

$= 2^2$

Dividing Indices:

$a^3 \div a^2 = \frac{a\times a\times a}{a\times a}$

$= a$

Is the same as

Dividing Indices:

$a^3 \div a^2 = a^{3-2}$

$= a$

Index Law 2

$a^m \div a^n = a^{m-n}$

Exercise 1.6

Simplify and write the following in index form:

This only works if the bases are the same.

$2^3 \div 2^2 = \frac{2\times2\times2}{2\times2}$

$= 2$

or $2^3 \div 2^2 = 2^{3-2} = 2$

$a^6 \div a^2 = \frac{a\times a\times a\times a\times a\times a}{a\times a}$

$= a\times a\times a\times a$

$= a^4$

or $a^6 \div a^2 = a^{6-2} = a^4$

1	$2^5 \div 2^3$	**2**	$3^3 \div 3^2$	**3**	$2^4 \div 2^2$	**4**	$4^4 \div 4^3$
5	$a^6 \div a^2$	**6**	$b^7 \div b^2$	**7**	$10^5 \div 10^3$	**8**	$w^4 \div w^3$
9	$2^3 \div 2^1$	**10**	$t^7 \div t^4$	**11**	$5^6 \div 5^4$	**12**	$10^5 \div 10^2$
13	$3^6 \div 3^4$	**14**	$7^3 \div 7^2$	**15**	$10^8 \div 10^5$	**16**	$3^4 \div 3^3$
17	$a^4 \div a^3$	**18**	$u^4 \div u^2$	**19**	$2^9 \div 2^3$	**20**	$p^3 \div p^3$

21 $\frac{m^4}{m^2}$ **22** $\frac{e^5}{e^2}$

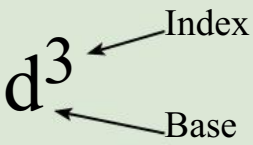
23 $\frac{c^7}{c^4}$ **24** $\frac{10^4}{10^2}$

25 $\frac{a^6}{a^5}$ **26** $\frac{3^7}{3^4}$

$m^4 \div m^2 = \frac{m^4}{m^2}$

They are the same thing.

Summary



Exercise 1.7
Write each of the following in index form:

$3\times3\times3\times3 = 3^4$	$d\times d\times d = d^3$

1	$1\times1\times1$	**2**	$2\times2\times2\times2$	**3**	$3\times3\times3\times3$
4	$4\times4\times4\times4\times4$	**5**	5×5	**6**	$1\times1\times1\times1\times1$
7	$2\times2\times2$	**8**	$3\times3\times3$	**9**	10×10
10	$10\times10\times10$	**11**	$10\times10\times10\times10$	**12**	$b\times b\times b\times b\times b\times b$
13	$n\times n\times n\times n\times n\times n\times n$	**14**	$p\times p\times p\times p\times p$	**15**	$a\times a\times a\times a\times a\times a$

Exercise 1.8
Write each of the following in factor form:

$4^3 = 4\times4\times4$	$w^4 = w\times w\times w\times w$

1	2^3	**2**	d^4	**3**	6^2
4	2^5	**5**	g^4	**6**	1^3
7	2^8	**8**	h^7	**9**	1^4

Exercise 1.9
Simplify and write the following in index form:

$2^2\times2^4 = 2^{2+4} = 2^6$	$y^2\times y^3 = y^{2+3} = y^5$

Index Law 1

$a^m\times a^n = a^{m+n}$

1	$2^2\times2^3$	**2**	$3^2\times3^2$	**3**	$5^4\times5^2$	**4**	$2^4\times2^3$
5	$a^3\times a^2$	**6**	$b^3\times b^1$	**7**	$k^5\times k^3$	**8**	$p^4\times p^2$
9	$10^3\times10^1$	**10**	$10^4\times10$	**11**	$1^2\times1^3$	**12**	$h^4\times h^5$

Exercise 1.10
Simplify and write the following in index form:

$3^4\div3^2 = 3^{4-2} = 3^2$	$m^6\div m^3 = m^{6-3} = m^3$

Index Law 2

$a^m\div a^n = a^{m-n}$

1	$2^4\div2^3$	**2**	$3^5\div3^2$	**3**	$5^4\div5^2$	**4**	$4^5\div4^3$
5	$c^6\div c^2$	**6**	$b^8\div b^2$	**7**	$10^5\div10^3$	**8**	$w^5\div w^3$
9	$\frac{2^5}{2^3}$	**10**	$\frac{10^9}{10^3}$	**11**	$\frac{f^6}{f^4}$	**12**	$\frac{z^6}{z^5}$

Index Law 3

Power Indices:

$(2^3)^2 = (2\times2\times2)^2$
$= (2\times2\times2)\times(2\times2\times2)$
$= 2^6$

Is the same as

Power Indices:

$(2^3)^2 = 2^{3\times2}$
$= 2^6$

Power Indices:

$(a^2)^3 = (a\times a)^3$
$= (a\times a)\times(a\times a)\times(a\times a)$
$= a^6$

Is the same as

Power Indices:

$(a^2)^3 = a^{2\times3}$
$= a^6$

Index Law 3

$(a^m)^n = a^{m\times n}$

Exercise 1.11

Simplify and write the following in index form:

$(3^2)^3 = (3\times3)^3$ $= (3\times3)\times(3\times3)\times(3\times3)$ $= 3^6$ or $(3^2)^3 = 3^{2\times3} = 3^6$	$(b^4)^2 = (b\times b\times b\times b)^2$ $= (b\times b\times b\times b)\times(b\times b\times b\times b)$ $= b^8$ or $(b^4)^2 = b^{4\times2} = b^8$

1	$(2^2)^3$	**2**	$(2^3)^3$	**3**	$(3^2)^3$	**4**	$(4^2)^3$
5	$(d^2)^2$	**6**	$(b^3)^4$	**7**	$(t^2)^4$	**8**	$(n^2)^5$
9	$(5^2)^2$	**10**	$(s^2)^5$	**11**	$(m^2)^3$	**12**	$(3^4)^2$
13	$(5^3)^3$	**14**	$(10^2)^3$	**15**	$(10)^3$	**16**	$(g^2)^5$
17	$(10^3)^2$	**18**	$(h^4)^3$	**19**	$(d^2)^4$	**20**	$(2^5)^3$

$3^4\times(3^2)^3 = 3^4\times3^6 = 3^{10}$	$(b^4)^2b^3 = b^8\times b^3 = b^{11}$

21	$2^2\times(2^2)^3$	**22**	$2^2\times(2^3)^3$	**23**	$(3^2)^3\times3^3$	**24**	$4^4\times(4^2)^3$
25	$d^3(d^2)^2$	**26**	$b^4(b^3)^4$	**27**	$(t^2)^4t^5$	**28**	$n^3(n^2)^5$
29	$(5^2)^2\times5$	**30**	$s^2(s^2)^5$	**31**	$m(m^2)^3$	**32**	$3\times(3^4)^2$

Zero Index

Zero Index:
$2^4 \div 2^4 = 1$

or $2^4 \div 2^4 = 2^{4-4}$
$= 2^0$
Which must be = 1

Is the same as

Zero Index:
$2^0 = 1$

Zero Index:
$p^3 \div p^3 = 1$

or $p^3 \div p^3 = p^{3-3}$
$= p^0$
Which must be = 1

Is the same as

Zero Index:
$p^0 = 1$

Zero Index

$a^0 = 1$

Try 2^0 on your calculator. Does it work?

Exercise 1.12

Simplify each of the following:

		Anything to the power of 0 is 1.
$3^0 = 1$	$h^0 = 1$	

1 2^0	**2** 5^0	**3** b^0	**4** k^0
5 a^0	**6** 3^0	**7** 10^0	**8** d^0
9 1^0	**10** w^0	11 x^0	**12** a^0

		Anything to the power of 0 is 1.
$3\times5^0 = 3\times1 = 3$	$5b^0 = 5\times1 = 5$	

13 5×2^0	**14** $3a^0$	**15** 6×4^0	**16** 2×1^0
17 $7w^0$	**18** $3e^0$	**19** 9×3^0	**20** 8×2^0
21 $4g^0$	**22** $5n^0$	**23** 2×2^0	**24** 5×5^0
25 $d^0(d^2)^2$	**26** $b^0(b^3)^2$	**27** $(t^2)^4t^0$	**28** 5×5^0
29 $(5^0)^2\times5$	**30** $s^2(s^0)^5$	**31** $m(m^2)^0$	**32** $3\times(3^0)^2$

Mental Computation

Exercise 1.13

1 Spell index.
2 Write in index form: $3\times3\times3\times3\times3$
3 Write in factor form: 2^3
4 Simplify: $m^3\times m^2$
5 Simplify: $2^8\div2^5$
6 Simplify: 5^0
7 Change $\frac{1}{2}$ to a decimal.
8 Change 0.25 to a fraction.
9 Change $2\frac{3}{5}$ to a vulgar fraction.
10 I buy a \$6.40 magazine with \$10. How much change?

Employers expect employees to be good at mental computation.

Exercise 1.14

1 Spell indices (Plural of index).
2 Write in index form: $2\times2\times2\times2$
3 Write in factor form: 4^3
4 Simplify: $d^3\times d^2$
5 Simplify: $3^5\div3^2$
6 Simplify: a^0
7 Change $\frac{1}{4}$ to a decimal.
8 Change 0.75 to a fraction.
9 Change $3\frac{2}{3}$ to a vulgar fraction.
10 I buy a \$7.20 magazine with \$10. How much change?

Which day is the strongest day of the week?
Sunday. The rest of the days are weekdays.

Exercise 1.15

1 Spell multiplication.
2 Write in index form: $5\times5\times5\times5\times5$
3 Write in factor form: 3^4
4 Simplify: $a^2\times a^4$
5 Simplify: $5^4\div5^2$
6 Simplify: 4^0
7 Change $\frac{3}{4}$ to a decimal.
8 Change 0.5 to a fraction.
9 Change $4\frac{1}{4}$ to a vulgar fraction.
10 I buy a \$5.60 magazine with \$10. How much change?

The average distance to the Sun is 1.5×10^{11} m.

NAPLAN Questions

Exercise 1.16

1 Calculate each of the following:

a) 4.27×10^3 **b)** 4.27×10^6 **c)** 4.27×10^9

2 $5.16^3+2.73^4$ is closest to:

a) 3 000 **b)** 30 000 **c)** 300 000

3 Which card below has the same value as: 2^5

a) 3^3 **b)** 2×4^2 **c)** 4×3^2

4 Solve for b in:

a) $2^b = 16$ **b)** $2^b=64$ **c)** $3^b = 81$

The Earth has a diameter of 1.3×10^7 m.

5 What is the value of: $3\times10^4+6\times10^3+2\times10^2+4\times10^1$?

6 15^2 is between:

a) 150 and 200 **b)** 200 and 250 **c)** 250 and 300

7 Calculate each of the following:

a) 2^3-2^2 **b)** 3^3-3^2 **c)** 2^3+2^4

8 Which is the same as $3^2\times3^2$

a) $3\times3\times3$ **b)** $3\times3\times3\times3$ **c)** $3\times3\times3\times3\times3$

9 The area of a square is 64 cm^2. What is the length of each side?

10 The area of a cube is 64 cm^3. What is the length of each side?

11 Which is the same as $2^4\times4^2$

a) 2^6 **b)** 2^8 **c)** 2^{10}

12 Calculate each of the following:

a) $\frac{2^5}{2^3}$ **b)** $\frac{3^4}{3^3}$ **c)** $\frac{10^4}{10^2}$

d) $(\sqrt{4})^3$ **e)** $(\sqrt{16})^3$ **f)** $(\sqrt[3]{8})^3$

13 35 000 000 is the same as:

a) 3.5×10^6 **b)** 3.5×10^7 **c)** 3.5×10^8

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 1.17

1 Find the value of each of the following:

a) $(0.2)^2$ **b)** $(0.03)^2$ **c)** $(0.2)^3$

d) $10 + 10^2 + 10^3$ **e)** $2 + 2^2 + 2^3 + 2^4$ **f)** $10 \times 10^2 \times 10^3$

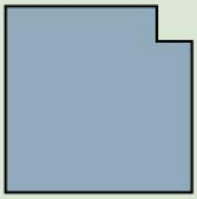
g) $2^2 - 1^2$ **h)** $3^2 - 2^2$ **i)** $4^2 - 3^2$

2 Aila went up 5 floors, down 6 floors, down 2 floors, up 7 floors. Aila was then on the 16th floor. On which floor did Aila start?

3 Find the value of x in each of the following:

a) $\frac{1}{4} + \frac{2}{x} = \frac{11}{12}$ **b)** $\frac{2}{x} - \frac{1}{3} = \frac{1}{15}$ **c)** $\frac{1}{4} + \frac{1}{x} = \frac{1}{3}$

4 A square of area 25 cm^2 is cut from the corner of a square of area 625 cm^2. What is now the perimeter of the shape?



5 Calculate each of the following:

a) $\frac{2^6 \times 2^3}{2^2 \times 2^7}$ **b)** $\frac{3^4 \times 3^3}{3^2 \times 3^4}$ **c)** $\frac{4^5 \times 4^3}{4^3 \times 4^4}$

What is the last digit in 4^{15}?

$4^1 = 4$
$4^2 = 16$ Last digit pattern is: 4, 6, 4, 6, 4, ...
$4^3 = 64$
$4^4 = 256$ The index, 15, is odd
$4^5 = 1024$ The last digit in 4^{15} is 4

6 What is the last digit in 4^{32}?

7 What is the last digit in 4^{27}?

8 What is the last digit in 54^{15}?

9 What is the last digit in 364^{73}?

How many beans can you put into an empty bag?

One. After that the bag isn't empty.

What is the highest power of 2 that divides exactly into 100?

$100 \div 2 = 50$, $50 \div 2 = 25$. 2 will divide into 100 twice.
The highest power of 2 that divides exactly into 100 is 2^2

10 What is the highest power of 2 that divides exactly into 1000?

Investigations

Investigation 1.1 Powers

What is the last digit in 3^{23}, 3^{68}?

1 Write down the powers of 3 from 31 to 310 (or use a spreadsheet).
2 Look for a pattern in the last digit of each number.
3 Can you now see the answer (The example in the Competition questions may be useful)?

Investigation 1.2 A million steps?

How far will a million steps take you?

More than 10 kilometres?

How long will it take you to walk a million steps?

Investigation 1.3 A shortcut for squaring numbers?

Complete the pattern:

$2^2 = 3\times1 + 1$ $\quad$ $6^2 =$
$3^2 = 4\times2 + 1$ $\quad$ $7^2 =$
$4^2 = 5\times3 + 1$ $\quad$ $8^2 =$
$5^2 = 6\times4 + 1$ $\quad$ $9^2 =$

What larger numbers would be easy to square using this method?

Investigation 1.4 Computer numbers?

A byte in computer memory holds one character. This introduces special names for large units of memory. For example, a gigabyte is 1 000 000 000 bytes in decimal and 2^{30} = 1 073 741 824 bytes in binary.

Research and complete the following:

Term	Decimal	Binary
Kilobyte	1 000	1 024
Megabyte		
Gigabyte		
Terabyte		
Petabyte		
Exabyte		
Zettabyte		
Yottabyte		

A Couple of Puzzles

Exercise 1.18

Complete each of the following magic squares so that the sum of any row, column, or diagonal is 15 (Also use each of the numbers from 1 to 9).

a)

	1	
	5	
4	9	

b)

	5	
8	1	6

c)

4		2
3		
8		

d)

	7	
8		

e)

	3	
6		

f)

	3	
2		

A Game

Twenty. Make twenty strokes on a piece of paper.

- ➔ Each of two players take turns to mark off one, two, or three of the strokes.
- ➔ The winner is the person who crosses off the last stroke.

Play a couple of games and try to determine a winning strategy.

A Sweet Trick

1	Ask your audience to choose a number from 1 to 9.		6
2	Add 29.	6 + 29	= 34
3	Cross out the last digit.		= 3
4	Multiply by 10.	3 x 10	= 30
5	Add 4.	30 + 4	= 34
6	Multiply by 3.	34 x 3	= 102
7	Subtract 2.	102 – 2	= 100

The answer is always 100.

You could embellish this trick by having the audience turning to the page of the answer etc.

Technology

Technology 1.1

Use a spreadsheet to calculate powers of the numbers 1 to 10.
Also create a graph of each of the powers.

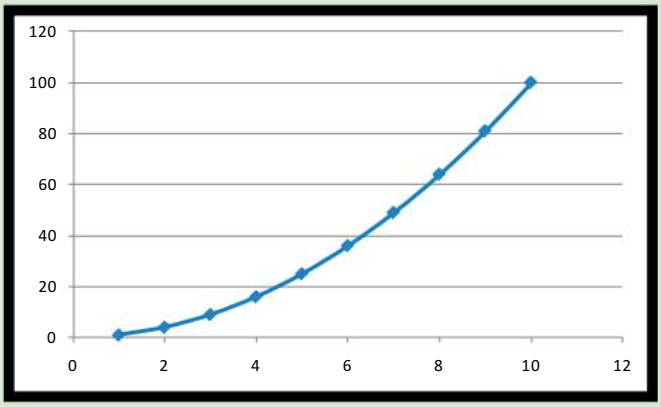
	Square	Cube	Fourth	Fifth
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	
5	25	125		
6				
7				
8				
9				
10				

= a2^5

The ^ is the power

A graph of 2^x

{x=1,2,3,4,5,6,7,8,9,10}

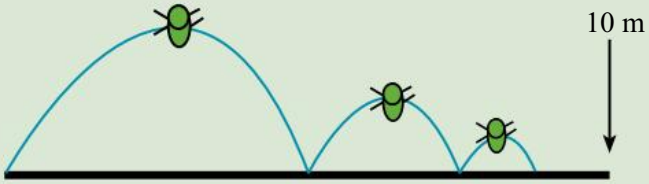


Technology 1.2

Florence the flea is planning a 10 m journey. Florence's first jump will be 5 m, then 2.5 m, then 1.25 m. Florence is jumping half the remaining distance each time.
How many jumps will it take for Florence to cover the 10 m?

Jump	Jump Distance	Distance left
1	5	5
2	2.5	2.5
3	1.25	1.25
4		
5		

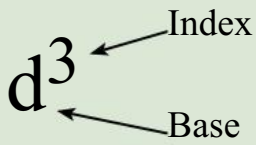
1 Set up a spreadsheet similar to this one.
2 Extend the spreadsheet until the distance left is 0.



Chapter Review 1

Exercise 1.19

1 Write each of the following in index form:



$3\times3\times3\times3 = 3^4$	$d\times d\times d = d^3$

a) $1\times1\times1\times1$ **b)** $4\times4\times4\times4$ **c)** $3\times3\times3\times3\times3\times3$
d) $g\times g\times g\times g\times g\times g\times g$ **e)** $a\times a\times a\times a\times a$ **f)** $10\times10\times10\times10\times10\times10$

2 Simplify and write the following in index form:

$2^2\times2^4 = 2^{2+4} = 2^6$	$y^2\times y^3 = y^{2+3} = y^5$

Index Law 1

$a^m\times a^n = a^{m+n}$

a) $4^2\times4^3$ **b)** $2^2\times2^2$ **c)** $10^4\times10^2$ **d)** $5^4\times5^3$
e) $a^5\times a^2$ **f)** $b^3\times b^1$ **g)** $p^7\times p^3$ **h)** $w^4\times w^2$
i) $10^5\times10^1$ **j)** 10×10^5 **k)** $1^5\times1^3$ **l)** $h^5\times h^5$

3 Simplify and write the following in index form:

$3^4\div3^2 = 3^{4-2} = 3^2$	$m^6\div m^3 = m^{6-3} = m^3$

Index Law 2

$a^m\div a^n = a^{m-n}$

a) $10^4\div10^3$ **b)** $4^5\div4^2$ **c)** $3^7\div3^2$ **d)** $2^5\div2^3$
e) $a^6\div a^2$ **f)** $n^7\div n^2$ **g)** $10^7\div10^3$ **h)** $1^5\div1^3$
i) $\frac{4^5}{4^3}$ **j)** $\frac{t^7}{t^5}$ **k)** $\frac{10^3}{10^1}$ **l)** $\frac{u^7}{u^2}$

4 Simplify and write the following in index form:

$(3^2)^3 = 3^{2\times3} = 3^6$	$(b^4)^2 = b^{4\times2} = b^8$

Index Law 3

$(a^m)^n = a^{m\times n}$

a) $(3^2)^3$ **b)** $(2^3)^3$ **c)** $(5^2)^3$ **d)** $(10^2)^3$
e) $(10^3)^2$ **f)** $(b^4)^3$ **g)** $(u^2)^5$ **h)** $(1^5)^3$
i) $(7^2)^3$ **j)** $(3^3)^3$ **k)** $(2^2)^3\times2^3$ **l)** $4^5\times(4^2)^3$

5 Simplify each of the following:

$3^0 = 1$	$h^0 = 1$

Zero Index

$a^0 = 1$

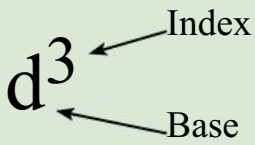
a) 3^0 **b)** 10^0 **c)** c^0 **d)** $5n^0$

The Earth's atmosphere extends out for 1×10^7 m.

Chapter Review 2

Exercise 1.20

1 Write each of the following in index form:



$3\times3\times3\times3 = 3^4$	$d\times d\times d = d^3$

a) $2\times2\times2$ **b)** $3\times3\times3\times3\times3$ **c)** $d\times d\times d\times d\times d\times d$
d) $10\times10\times10\times10\times10$ **e)** $z\times z\times z\times z\times z\times z$ **f)** $1\times1\times1\times1\times1\times1\times1$

2 Simplify and write the following in index form:

$2^2\times2^4 = 2^{2+4} = 2^6$	$y^2\times y^3 = y^{2+3} = y^5$

Index Law 1

$a^m\times a^n = a^{m+n}$

a) $3^2\times3^4$ **b)** $1^2\times1^2$ **c)** $10^3\times10^5$ **d)** $t^2\times t^3$
e) $m^5\times m^3$ **f)** $3^3\times3^1$ **g)** $p^6\times p^3$ **h)** $5^4\times5^5$
i) $10^1\times10^3$ **j)** 10×10^6 **k)** $1^4\times1^3$ **l)** $j^2\times j^5$

3 Simplify and write the following in index form:

$3^4\div3^2 = 3^{4-2} = 3^2$	$m^6\div m^3 = m^{6-3} = m^3$

Index Law 2

$a^m\div a^n = a^{m-n}$

a) $3^4\div3^3$ **b)** $e^4\div e^2$ **c)** $2^7\div2^2$ **d)** $1^5\div1^3$
e) $g^5\div g^2$ **f)** $m^6\div m^3$ **g)** $10^9\div10^3$ **h)** $q^5\div q^3$
i) $\frac{h^3}{h^2}$ **j)** $\frac{4^6}{4^2}$ **k)** $\frac{10^6}{10^3}$ **l)** $\frac{v^8}{v^5}$

4 Simplify and write the following in index form:

$(3^2)^3 = 3^{2\times3} = 3^6$	$(b^4)^2 = b^{4\times2} = b^8$

Index Law 3

$(a^m)^n = a^{m\times n}$

a) $(4^2)^3$ **b)** $(3^4)^3$ **c)** $(7^3)^2$ **d)** $(b^2)^3$
e) $(10^2)^2$ **f)** $(e^4)^2$ **g)** $(h^2)^4$ **h)** $(1^2)^3$
i) $(5^2)^3$ **j)** $(8^3)^2$ **k)** $(2^2)^3\times2^4$ **l)** $3^5\times(3^2)^3$

5 Simplify each of the following:

$3^0 = 1$	$h^0 = 1$

Zero Index

$a^0 = 1$

a) 2^0 **b)** y^0 **c)** 9^0 **d)** $3s^0$

There were 7×10^9 people on Earth in 2010.

Integers 2

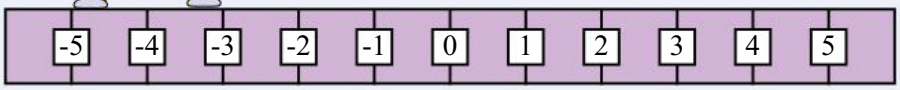
- ★ Carry out the four operations with integers.
- ★ Use efficient mental and written strategies.
- ★ Use appropriate digital technologies.

A TASK

Mark out −7 to 0 to 7 on the playground.

- What is −3 − 2?
- Have someone stand at −3.
- Step backward 2 steps.
- What is the answer?

By pasting a series of digital photos, make a slide show showing how to add and subtract integers.



A LITTLE BIT OF HISTORY

Zero = nothing, empty.
0 = the symbol for empty, nothing.
Zero = 0 = nought = naught = nil = null.

'How can nothing be something?' - Ancient Greeks.

100 AD Subtract a negative number from zero to make it positive.
525 AD Nulla, a Roman word meaning nothing.
628 AD Rules for the use of zero:
Example: 0 + 0 = 0

$a \times 0 = 0$
$0 \times a = 0$
$0 \div a = 0$
$a \div 0 = ??$ No answer.

Integers

Integers are:		
	Positive whole numbers:	1, 2, 3, 4, 5, 6, ...
and	Zero:	0
and	Negative whole numbers:	¯1, ¯2, ¯3, ¯4, ¯5, ¯6, ...

8 and +8 are the same thing.

Exercise 2.1

Change the following words to an integer:

A loss of 3kg. ¯3	A rate drop of 25 points. ¯25	8 floors up. 8

1 A drop of 3°C.
2 A increase of 12%.
3 I lost 12 marbles.
4 A reduction of 15%
5 A profit of $45
6 A gain of 30 cm.
7 A loss of $2 million.
8 No movement.
9 15 m under the sea.
10 A rise of 4 m.
11 A profit of $450.
12 $3000 decrease.
13 $80 over the value.
14 $31 under value.
15 Drop of 12 floors.
16 Zero change.

Assume that up is positive and down is negative:

17 50 m down.
18 20 units down.
19 Move 4 places down.
20 Up 3 levels.
21 Down 2 amps.
22 Up 50 basis points.
23 Neither up nor down.
24 48 steps down.

Assume that to the right is positive and that to the left is negative:

25 Move 8 places to the right.
26 Don't move.
27 6 clicks to the left.
28 9 clicks to the right.
29 Move the biro 5 to the left.
30 Move 3 places to the left.

Z is the set of Integers. **Z** = {... −4, −3, −2, −1, 0, 1, 2, 3, 4, ... }

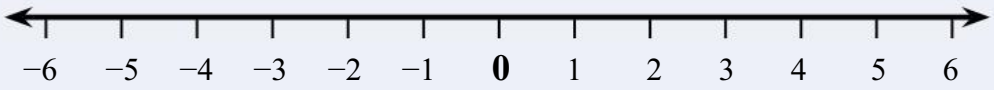
Numbers such as 3.5, −721.2, 2½ are not integers.

Z comes from the German Zahl, meaning number.

Ordering Integers

Integers can be placed on a number line.

Integers on the right are larger than integers on the left.



'less than'
$^{-}1 < 2$
$^{-}1$ is to the left of 2

'greater than'
$5 > {}^{-}2$
5 is to the right of $^{-}2$

Exercise 2.2

Copy each pair of numbers and place the correct < or > between them.

$3 \quad {}^{-}1$	$^{-}4 \quad 2$	$^{-}3 \quad {}^{-}6$
3 is to the right of $^{-}1$	$^{-}4$ is to the left of 2	$^{-}3$ is to the right of $^{-}6$
3 is greater than $^{-}1$	$^{-}4$ is less than 2	$^{-}3$ is greater than $^{-}6$
$3 > {}^{-}1$	$^{-}4 < 2$	$^{-}3 > {}^{-}6$

1	5 $^{-}2$	**2**	$^{-}3$ 1	**3**	5 $^{-}2$
4	$^{-}4$ 0	**5**	0 $^{-}1$	**6**	$^{-}3$ $^{-}2$
7	2 6	**8**	4 $^{-}2$	**9**	$^{-}3$ 6
10	$^{-}5$ $^{-}6$	**11**	2 0	**12**	$^{-}4$ $^{-}1$
13	6 $^{-}1$	**14**	$^{-}4$ $^{-}3$	**15**	5 $^{-}2$
16	13 5	**17**	$^{-}11$ $^{-}1$	**18**	$^{-}15$ $^{-}11$

Exercise 2.3

Arrange the integers in ascending order (smallest to largest):

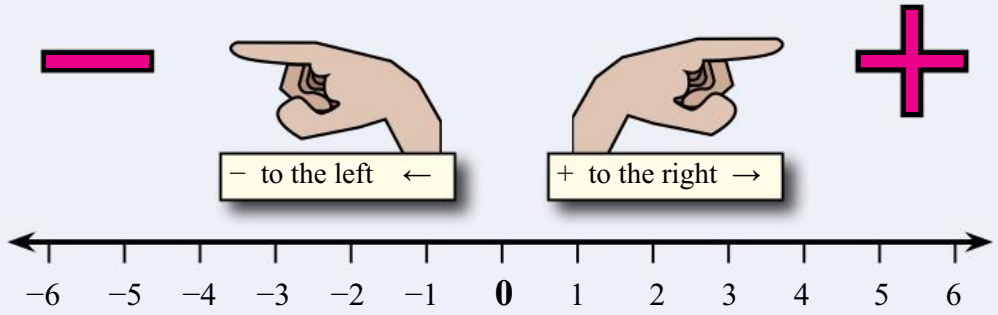
6, $^{-}1$, 2, $^{-}5$, $^{-}4$

$^{-}5, {}^{-}4, {}^{-}1, 2, 6$

From left to right on the number line.

1	3, 0, 1, 5	**2**	1, $^{-}1$, 0, $^{-}2$	**3**	6, $^{-}3$, 1, $^{-}2$
4	$^{-}5$, 3, 2, $^{-}3$, 0	**5**	$^{-}6$, 2, $^{-}1$, $^{-}4$	**6**	$^{-}5$, 5, 1, $^{-}3$, 2
7	1, $^{-}2$, $^{-}1$, $^{-}3$, 2	**8**	3, 1, $^{-}4$, 6, $^{-}6$	**9**	$^{-}4$, 2, 4, $^{-}3$, 6
10	2, 6, $^{-}6$, 0, 4	**11**	5, $^{-}3$, 6, $^{-}4$, 1	**12**	$^{-}7$, 8, 3, $^{-}3$, 4
13	11, $^{-}3$, $^{-}4$, $^{-}9$, 2	**14**	$^{-}2$, $^{-}3$, $^{-}1$, $^{-}12$, 9	**15**	$^{-}13$, 15, 13, $^{-}12$, $^{-}7$

Adding Integers



Exercise 2.4

Add the following integers:

$3 + \bar{4}$ Put your biro on 0. Move biro 3 to right. Move biro 4 to the left. Your biro is at $\bar{1}$. $3 + \bar{4} = \bar{1}$	$\bar{5} + 3$ Put your biro on 0. Move biro 5 to left. Move biro 3 to the right. Your biro is at $\bar{2}$. $\bar{5} + 3 = \bar{2}$

1	$4 + 3$	**2**	$5 + \bar{3}$	**3**	$\bar{2} + 4$
4	$\bar{1} + \bar{4}$	**5**	$7 + 6$	**6**	$6 + \bar{2}$
7	$\bar{7} + 1$	**8**	$\bar{4} + \bar{2}$	**9**	$12 + 13$
10	$4 + \bar{6}$	**11**	$\bar{3} + 5$	**12**	$\bar{2} + \bar{3}$
13	$7 + 8$	**14**	$8 + \bar{3}$	**15**	$\bar{5} + 8$
16	$13 + 8$	**17**	$12 + \bar{5}$	**18**	$\bar{9} + 6$
19	$\bar{2} + \bar{7}$	**20**	$13 + 17$	**21**	$\bar{12} + 8$

Exercise 2.5

Copy and complete the following addition tables:

1

+	$\bar{2}$	$\bar{1}$	0	1
$\bar{2}$				
$\bar{1}$				
0				
1				

$1 + \bar{2} = \bar{\bar{1}}$

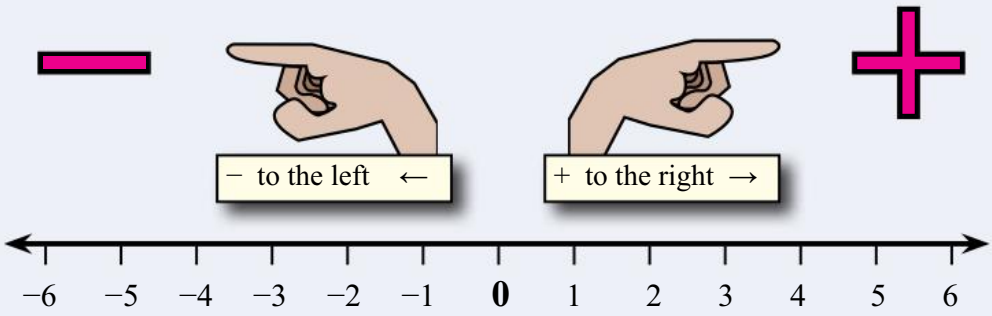
$0 + 1 = 1$

2

+	$\bar{10}$	$\bar{5}$	0	5
$\bar{10}$				
$\bar{5}$				
0				
5				

$\bar{5} + \bar{5} = \bar{10}$

Subtracting Integers



Exercise 2.6

Calculate the following:

$2 - 4$ Put your biro on 0. Move biro 2 to right. Move biro 4 to the left. Your biro is at $^{-}2$. $2 - 4 = {}^{-}2$	$^{-}3 - 2$ Put your biro on 0. Move biro 3 to left. Move biro 2 to the left. Your biro is at $^{-}5$. $^{-}3 - 2 = {}^{-}5$

1	$5 - 3$	**2**	$3 - 4$	**3**	$^{-}3 + 1$	**4**	$^{-}4 - 2$
5	$6 - 5$	**6**	$2 - 5$	**7**	$^{-}6 + 3$	**8**	$^{-}3 - 4$
9	$4 - 1$	**10**	$2 - 6$	**11**	$^{-}3 + 5$	**12**	$^{-}1 - 3$
13	$7 - 2$	**14**	$6 - 9$	**15**	$^{-}8 + 7$	**16**	$^{-}5 - 1$
17	$9 - 15$	**18**	$^{-}12 + 10$	**19**	$10 - 9$	**20**	$^{-}15 + 10$

− is the opposite direction of +
− (−) is opposite of − thus +

$- {}^{-}3 = 3$ $\quad - {}^{-}1 = 1$
$- {}^{-}8 = 8$ $\quad - {}^{-}5 = 5$

$3 - {}^{-}2$ Put your biro on 0. Move biro 3 to right. $- {}^{-}2 = 2$ Move biro 2 to the right. Your biro is at 5. $3 - {}^{-}2 = 5$	$^{-}5 - {}^{-}1$ Put your biro on 0. Move biro 5 to left. $- {}^{-}1 = 1$ Move biro 1 to the right. Your biro is at $^{-}4$. $^{-}5 - {}^{-}1 = {}^{-}4$

21	$1 - {}^{-}4$	**22**	$5 - {}^{-}1$	**23**	$^{-}3 - {}^{-}2$	**24**	$^{-}2 - {}^{-}4$
25	$5 - {}^{-}3$	**26**	$4 - {}^{-}3$	**27**	$^{-}1 - {}^{-}7$	**28**	$^{-}6 - {}^{-}2$
29	$3 - {}^{-}1$	**30**	$5 - {}^{-}2$	**31**	$^{-}3 - {}^{-}1$	**32**	$^{-}4 - {}^{-}2$
33	$2 - {}^{-}6$	**34**	$8 - {}^{-}4$	**35**	$^{-}8 - {}^{-}3$	**36**	$^{-}4 - {}^{-}6$
37	$9 - {}^{-}12$	**38**	$2 - {}^{-}7$	**39**	$^{-}5 - {}^{-}4$	**40**	$^{-}3 - {}^{-}12$

Multiplying Integers

Integers are used in millions of everyday situations.

Exercise 2.7

Calculate each of the following:

$3 \times 4 = 3$ lots of 4 $= 4 + 4 + 4$ $= 12$	$2 \times \bar{3} = 2$ lots of $\bar{3}$ $= \bar{3} + \bar{3}$ $= \bar{6}$
$5 \times \bar{2} = 5$ lots of $\bar{2}$ $= \bar{2} + \bar{2} + \bar{2} + \bar{2} + \bar{2}$ $= \bar{10}$	$\bar{4} \times 2 = 2 \times \bar{4}$ $= 2$ lots of $\bar{4}$ $= \bar{4} + \bar{4}$ $= \bar{8}$

1	5×2	**2**	$5 \times \bar{2}$	**3**	$2 \times \bar{5}$	**4**	$\bar{5} \times 2$
5	3×4	**6**	$3 \times \bar{4}$	**7**	$4 \times \bar{3}$	**8**	$\bar{3} \times 4$
9	4×2	**10**	$4 \times \bar{2}$	**11**	$2 \times \bar{4}$	**12**	$\bar{4} \times 2$
13	7×0	**14**	0×7	**15**	$0 \times \bar{7}$	**16**	$\bar{7} \times 0$
17	1×15	**18**	$15 \times \bar{1}$	**19**	$1 \times \bar{15}$	**20**	$\bar{15} \times 1$

21

×	$\bar{2}$	$\bar{1}$	0	1
1				
2				
3				
4				

$3 \times \bar{2} = \bar{6}$

$4 \times \bar{1} = \bar{4}$

22

$1 \times 0 = 0$

×	$\bar{2}$	$\bar{1}$	0	1
2				
1				
0				
$\bar{1}$				

$\bar{1} \times \bar{2} = 2$ What else can it be?

$\bar{3} \times \bar{4} = \bar{}$ (3 lots of $\bar{4}$) $= \bar{}$ ($\bar{4} + \bar{4} + \bar{4}$) $= \bar{}\,\bar{}12$ $= 12$	$\bar{2} \times \bar{5} = \bar{}$ (2 lots of $\bar{5}$) $= \bar{}$ ($\bar{5} + \bar{5}$) $= \bar{}\,\bar{}10$ $= 10$

23	$\bar{4} \times \bar{2}$	**24**	$\bar{3} \times \bar{2}$	**25**	$\bar{3} \times \bar{3}$
26	$\bar{4} \times \bar{5}$	**27**	$\bar{1} \times \bar{2}$	**28**	$\bar{3} \times \bar{5}$
29	3×6	**30**	$4 \times \bar{5}$	**31**	$\bar{4} \times 2$
32	$\bar{5} \times \bar{2}$	**33**	7×2	**34**	$6 \times \bar{5}$
35	$\bar{4} \times 5$	**36**	$\bar{4} \times \bar{4}$	**37**	5×7
38	$8 \times \bar{2}$	**39**	$\bar{3} \times 7$	**40**	$\bar{8} \times \bar{3}$

Menu

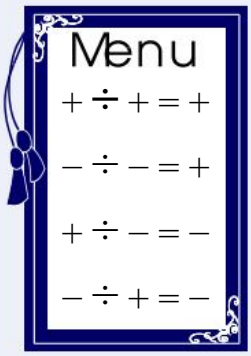
$+ \times + = +$

$- \times - = +$

$+ \times - = -$

$- \times + = -$

Dividing Integers



Exercise 2.8

Calculate each of the following:

$8 \div 4 = 2$	$9 \div {}^{-}3 = {}^{-}3$
${}^{-}10 \div 2 = {}^{-}5$	${}^{-}12 \div {}^{-}2 = 6$

1 $6 \div 2$ **2** $4 \div {}^{-}2$ **3** ${}^{-}20 \div 5$
4 ${}^{-}5 \div {}^{-}5$ **5** $8 \div 4$ **6** $12 \div {}^{-}4$
7 ${}^{-}4 \div 2$ **8** ${}^{-}30 \div {}^{-}5$ **9** $12 \div 2$ **10** $14 \div {}^{-}2$
11 ${}^{-}22 \div 11$ **12** ${}^{-}4 \div {}^{-}2$ **13** $7 \div 1$ **14** $20 \div {}^{-}4$
15 ${}^{-}33 \div 11$ **16** ${}^{-}27 \div {}^{-}3$ **17** $15 \div 3$ **18** $15 \div {}^{-}1$
19 ${}^{-}18 \div 6$ **20** ${}^{-}15 \div {}^{-}5$ **21** $16 \div 4$ **22** $16 \div {}^{-}4$

23

÷	${}^{-}4$	${}^{-}2$	2	4
${}^{-}8$				
${}^{-}4$				
0				
4				

$0 \div {}^{-}4 = 0$

${}^{-}4 \div 2 = {}^{-}2$

24

÷	${}^{-}10$	${}^{-}5$	5	10
${}^{-}40$				
${}^{-}20$				
0				
20				

${}^{-}20 \div {}^{-}5 = 4$

$20 \div {}^{-}10 = {}^{-}2$

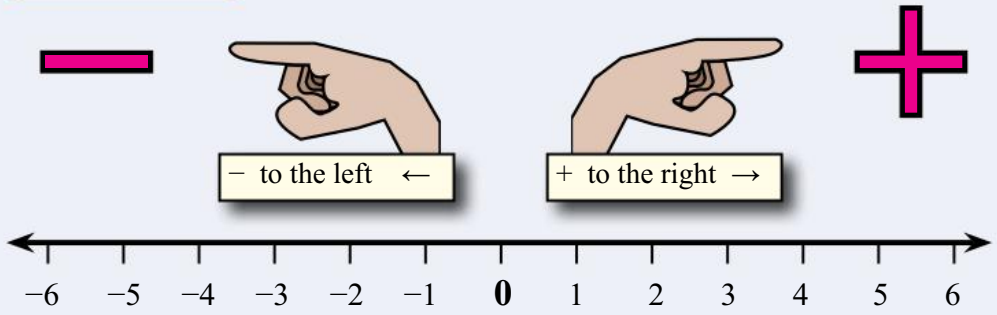
Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

$4 + ({}^{-}5 + 3) \times {}^{-}3$
$= 4 + {}^{-}2 \times {}^{-}3$
$= 4 + 6$
$= 10$

${}^{-}15 - 5 \times (2 + {}^{-}1)$
$= {}^{-}15 - 5 \times 1$
$= {}^{-}15 - 5$
$= {}^{-}20$

25 $4 + 2 \times {}^{-}5$ **26** $6 \div {}^{-}2 - 4$
27 ${}^{-}6 + (5 \times 2)$ **28** $({}^{-}18 \div 6) - 2$
29 $(20 - 10) \div {}^{-}2$ **30** $5 - (6 + {}^{-}2)$
31 ${}^{-}8 \div 4 - 4$ **32** $8 - 4 \times {}^{-}1$
33 ${}^{-}9 \div {}^{-}3 + 2$ **34** $12 \div ({}^{-}4 + 2)$

All Together



Exercise 2.9

Calculate the following:

$^{-}5 + 2$ Put your biro on 0. Move biro 5 to the left. Move biro 2 to the right. Your biro is at $^{-}3$. $^{-}5 + 2 = {}^{-}3$	$^{-}3 - {}^{-}2$ Put your biro on 0. Move biro 3 to the left. $- {}^{-}2 = 2$ Move biro 2 to the right. Your biro is at $^{-}1$. $^{-}3 - {}^{-}2 = {}^{-}1$

1	$5 + 3$	**2**	$6 + {}^{-}3$	**3**	$^{-}3 + 2$	**4**	$^{-}5 + {}^{-}1$
5	$8 - 5$	**6**	$^{-}4 - 4$	**7**	$7 - {}^{-}1$	**8**	$^{-}5 - {}^{-}2$
9	$10 + 15$	**10**	$5 + {}^{-}7$	**11**	$^{-}6 + 5$	**12**	$^{-}2 + {}^{-}4$
13	$19 - 13$	**14**	$^{-}3 - 5$	**15**	$3 - {}^{-}8$	**16**	$^{-}9 - {}^{-}2$
17	$5 - {}^{-}4$	**18**	$^{-}8 - 2$	**19**	$^{-}15 - {}^{-}10$	**20**	$4 - {}^{-}4$

$2 \times {}^{-}3$ = 2 lots of $^{-}3$ $= {}^{-}3 + {}^{-}3$ $= {}^{-}6$	$^{-}10 \div 2 = {}^{-}5$ $^{-}12 \div {}^{-}2 = 6$	× and ÷	+	−
		+	+	−
		−	−	+

21	5×6	**22**	$3 \times {}^{-}6$	**23**	$^{-}5 \times 2$	**24**	$^{-}6 \times {}^{-}4$
25	$8 \div 2$	**26**	$12 \div {}^{-}6$	**27**	$^{-}18 \div 3$	**28**	$^{-}15 \div {}^{-}3$
29	$12 \div 4$	**30**	$12 \div {}^{-}3$	**31**	$^{-}12 \div 6$	**32**	$^{-}45 \div {}^{-}9$

33 Herman owes each of five friends $30. What is Herman's balance.

34 Mia owes each of six friends $45. What is Mia's balance?

35 Lauren borrowed $35, repaid $17, and borrowed another $18. What is Lauren's balance?

36 Jack is driving at 100 km/h. Jack slows down by 20 km/h and then a further 30 km/h. What is Jack's speed?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 2.10

1 Spell integer.
2 $^{-}4 + 5$
3 $3 \times {}^{-}2$
4 $^{-}4 \div {}^{-}2$
5 24×5
6 Write in index form: $3\times3\times3\times3\times3$
7 Simplify: $m^3 \times m^2$
8 Simplify: 5^0
9 10% of 36
10 If I paid \$10 deposit and 10 payments of \$10. How much did I pay?

10% of 36 = 3.6

Exercise 2.11

1 Spell multiplication.
2 $^{-}3 + 2$
3 $^{-}3 \times {}^{-}2$
4 $4 \div {}^{-}2$
5 48×5
6 Write in index form: $2\times2\times2\times2\times2\times2$
7 Simplify: $d^4 \div d^2$
8 Simplify: 8^0
9 10% of 43
10 If I paid \$20 deposit and 10 payments of \$5. How much did I pay?

To multiply by 5:
Multiply by 10 then divide by 2.

$48\times5 = 48\times10\div2 = 480\div2 = 240.$

Exercise 2.12

1 Spell division.
2 $2 + {}^{-}5$
3 $2 \times {}^{-}4$
4 $^{-}6 \div 2$
5 68×5
6 Write in index form: $5\times5\times5\times5$
7 Simplify: $x^3 \times x^4$
8 Simplify: g^0
9 10% of 41
10 If I paid \$50 deposit and 10 payments of \$5. How much did I pay?

I wondered why the cricketball kept getting bigger.

Then it hit me.

Sports Psychologist assist sportspeople in maximising performance through managing their mental wellbeing.

- School subjects normally involve English and Mathematics.
- Courses usually involve a University degree with a major in psychology.

NAPLAN Questions

Exercise 2.13

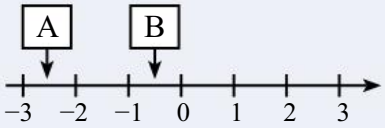
1 Calculate:

a) $^{-}5 + 2$ **b)** $^{-}3 + 2$ **c)** $4 - {}^{-}2$

d) $^{-}3 \times 2$ **e)** $4 \div {}^{-}2$ **f)** $^{-}3 \times {}^{-}4$

g) $2^2 - 3^2$ h) $^{-}9 \div {}^{-}3 + 2$ **i)** $12 \div ({}^{-}4 + 2)$

2 Estimate the value of A and B:



3 Given: $y = 8 - 2x$ find the value of y when $x = 1.5$

4 Find the value of: $x^2 + 2x + 4$ when $x = {}^{-}1$

Mathematics is useful in almost every career that one can think of.

5 Arrange the following integers in ascending order:

a) $0, {}^{-}3, 1, {}^{-}2, 2$ **b)** $6.7, {}^{-}6.7, {}^{-}7.6, 7.6$

6 What are the coordinates of:

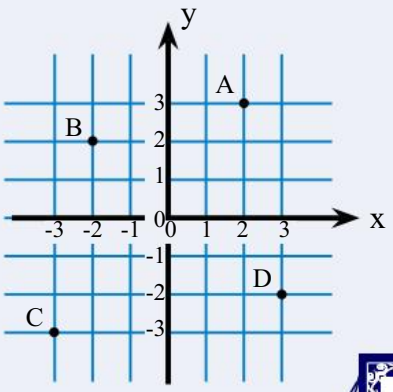
a) A?

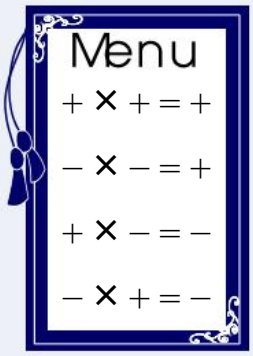
b) B?

c) C?

d) D?

x coordinate first,
y coordinate second.





7 What is the value of $a^3 - b^3$ when $a = 1$ and $b = -1$?

8 Find the value of $5b^2$ when $b = -2$

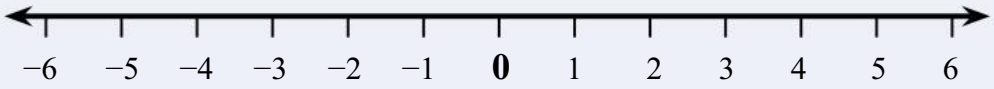
9 Solve for x: $3x + 5 = {}^{-}1$

10 $x + y = 1$
$xy = -20$
What are the values of x and y that satisfy both equations?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 2.14



1 Kade is standing on $^{-}3$ and facing towards the zero when she is given the following instructions:

- Move 3 units forward.
- Move 4 units backward.
- Move 7 units forward.

Where is Kade now standing?

Our mobile phone would not have been possible without mathematics.

2 Calculate:

a) $5 - 4 + 3 - 2 + 1$

b) $1 - 2 + 3 - 4 + 5$

c) $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$

d) $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10$

e) $100 - 99 + 98 - 97 + \ldots\ldots\ldots + 4 - 3 + 2 - 1$

f) $1 - 2 + 3 - 4 + \ldots\ldots\ldots + 97 - 98 + 99 - 100$

3 $m = x + 5,\ n = 9 - y$

If $x = {}^{-}2$ and $y = {}^{-}1$, What is the value of:

a) m? **b)** n? **c)** m − n?

4 Solve each of the following equations:

$3x + 4 = 1$ $3x = 1 - 4$ $3x = {}^{-}3$ $x = {}^{-}3 \div 3$ $x = {}^{-}1$	$3x - 4 = {}^{-}10$ $3x = {}^{-}10 + 4$ $3x = {}^{-}6$ $x = {}^{-}6 \div 3$ $x = {}^{-}2$

a) $2x + 4 = 2$ **b)** $3x + 7 = 4$ **c)** $6x + 5 = {}^{-}1$

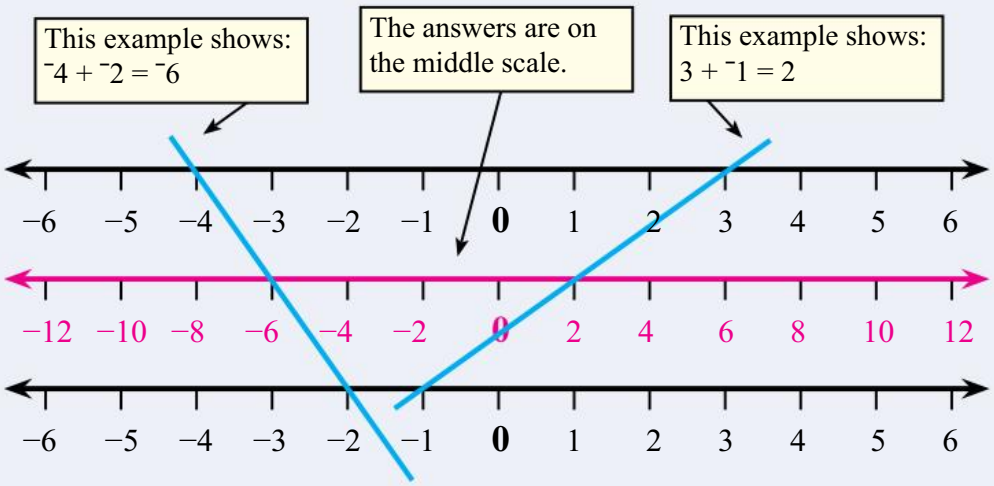
d) $2x - 5 = 9$ **e)** $3x - 6 = {}^{-}12$ **f)** $6x - 5 = {}^{-}17$

Investigations

Investigation 2.1 Integer Nomogram

A nomogram is a diagram used for calculations.
A nomogram normally has three scales.

1 Make the following Nomogram for adding integers.



2 Use the Nomogram to answer the following:

a) ⁻5 + 2 **b)** ⁻3 + 2 **c)** 4 + ⁻2

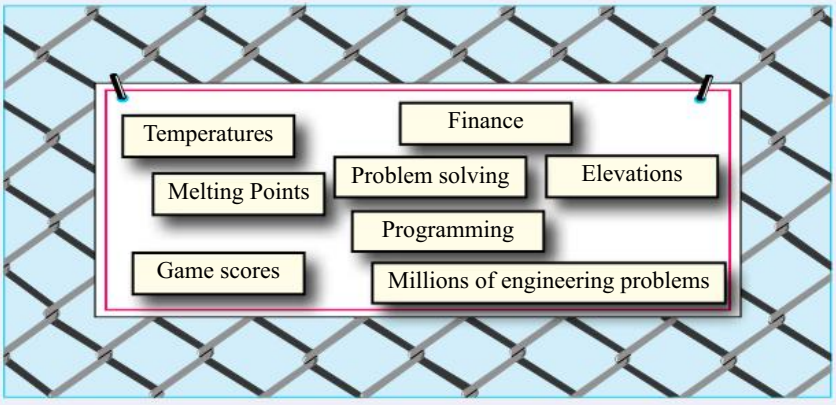
3 Can you make a Nomogram for multiplying integers?
Use your Nomogram to answer the following:

a) ⁻5 × 2 **b)** 3 × ⁻2 **c)** ⁻4 × ⁻1

Investigation 2.2 Integer Collage

A collage is an art form that may include cuttings from newspapers, brochures, magazines, coloured papers, photographs and other objects, glued to a board of paper or other material.

Make a collage of practical integers.



A Couple of Puzzles

Exercise 2.15

1 Complete each grid based on the following rules:

- The numbers 1, 2, 3, and 4 each can only be used once per row.
- The numbers 1, 2, 3, and 4 each can only be used once per column.
- The numbers 1, 2, 3, and 4 each can only be used once per 2×2 box.

a)

1	2		
		3	4

b)

			1
			2
			3
			4

A Game

Two players take it in turns to play nine holes of **Integer Golf**.

1 A player tosses a die and subtracts 4 from the number showing.

2 The player records the result on their card.

3 The player with the lowest score after nine holes wins.

Die	Score	
6	2	2 over par
5	1	bogey
4	0	par
3	⁻1	birdie
2	⁻2	eagle
1	⁻3	albatross

	1	2	3	4	5	6	7	8	9
Joel	0/0	⁻2/⁻2	1/⁻1	1/0	2/2	1/3	0/3	⁻2/1	⁻3/⁻2
Ella	⁻3/⁻3	⁻2/⁻5	1/⁻4	⁻3/⁻7	2/⁻5	2/⁻3	1/⁻2	⁻3/⁻5	0/⁻5

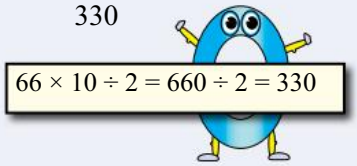
Ella wins with a score of 5 under.

A Sweet Trick

1 Ask your audience to write down any five consecutive numbers between ten and one hundred. 64, 65, 66, 67, 68, 69

2 Ask them for their middle number. 66

3 You tell them the sum of their five numbers. 330



Technology

Technology 2.1 Calculators and Integers

Practice the following on your calculator:

a) $5 - 3$ Enter: [5] [−] [3] [=] to give the answer [2]

b) $3 - 4$ Enter: [3] [−] [4] [=] to give the answer [$^{-}1$]

[+/−] or [(−)] is normally on the bottom row of your calculator.

c) $^{-}3 + 1$ Enter: [+/−] [3] [+] [2] [=] to give the answer [$^{-}2$]

d) $^{-}4 - 2$ Enter: [+/−] [4] [−] [2] [=] to give the answer [$^{-}6$]

e) $1 - {}^{-}4$ Enter: [+/−] [3] [+] [2] [=] to give the answer [$^{-}2$]

g) $^{-}3 - {}^{-}2$ Enter: [+/−] [3] [−] [+/−] [2] [=] to give the answer [$^{-}1$]

Now use your calculator on Exercise 2.16 Question 5.

Technology 2.2 Integer WebQuests

WebQuests are learning activities based on information on the Web.
A WebQuest can provide an opportunity to experience aspects of Integers that would not normally be available in a classroom.

1 Search the Internet for 'Integer WebQuest'.

2 Select and complete one of the WebQuests.

Can you complete a WebQuest from start to finish?

Technology 2.3 Integer Games and Applets

Integer games provide some fun while building and strengthening your Integer skills.

Search the internet for some of the many interactive Integer games and applets.

What is the difference between a Java applet and a Flash applet?

Chapter Review 1

Exercise 2.16

1 Change the following words to an integer:

a) A drop of 2°C. **b)** A increase of 15%.

c) A profit of $25. **d)** Shortened by 12 cm.

2 Copy each pair of numbers and place the correct < or > between them.

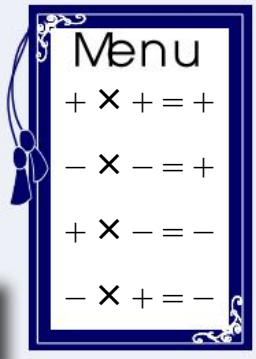
a) 3 $\bar{}2$ **b)** $\bar{}4$ 1 **c)** $\bar{}2$ $\bar{}5$

3 Arrange the integers in ascending order (smallest to largest):

a) 3, 0, $\bar{}1$, $\bar{}5$ **b)** 2, $\bar{}1$, 0, $\bar{}2$ **c)** $\bar{}4$, $\bar{}3$, 1, $\bar{}2$

4 Calculate the following:

a) $6 - 4$	**b)** $\bar{}2 + 1$	**c)** $\bar{}5 - 4$
d) $7 - \bar{}2$	**e)** $\bar{}5 - \bar{}2$	**f)** 2×2
g) $4 \times \bar{}3$	**h)** $\bar{}2 \times 3$	**i)** $\bar{}5 \times \bar{}2$
j) $9 \div 3$	**k)** $6 \div \bar{}2$	**l)** $\bar{}15 \div 5$
m) $\bar{}8 \div \bar{}4$	**n)** $6 + 3 \times \bar{}5$	**o)** $8 \div \bar{}2 - 1$
p) $(\bar{}6 + 5) \times 2$	**q)** $\bar{}16 \div (6 - 2)$	**r)** $10 \div (\bar{}4 + 2)$

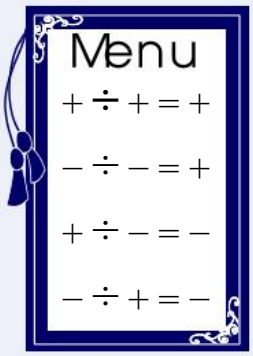


Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

$-\bar{}3 = 3$ $-\bar{}1 = 1$
$-\bar{}8 = 8$ $-\bar{}5 = 5$

5 Calculate the following:

a) $7 - 5$	**b)** $\bar{}4 + 2$	**c)** $\bar{}6 - 1$
d) $2 - \bar{}3$	**e)** $\bar{}5 - \bar{}1$	**f)** 1×2
g) $3 \times \bar{}5$	**h)** $\bar{}5 \times 2$	**i)** $\bar{}4 \times \bar{}2$
j) $6 \div 2$	**k)** $10 \div \bar{}2$	**l)** $\bar{}20 \div 5$
m) $\bar{}5 \div \bar{}5$	**n)** $6 + 2 \times \bar{}3$	**o)** $6 \div \bar{}2 - 4$
p) $(\bar{}6 + 5) \times 2$	**q)** $\bar{}18 \div (9 - 7)$	**r)** $12 \div (\bar{}5 + 2)$



6 Joseph owes each of six friends $40.
What is Joseph's balance.

7 The temperature at midnight was 4 °C. If the temperature dropped a further 9 °C, what is then the temperature?

8 Ashley borrowed $150, repaid $70, and borrowed another $20.
What is Ashley's balance?

9 Abhaya is driving at 110 km/h. Abhaya slows down by 30 km/h and then a further 30 km/h. What is Abhaya's speed?

10 The Roman civilisation began around 500 BC and finished around 475 AD.
How long did the Roman civilisation last?

Chapter Review 2

Exercise 2.17

1 Change the following words to an integer:

a) A loss of \$250 million. **b)** A rise of 15 m.

c) \$500 under value. **d)** No change.

2 Copy each pair of numbers and place the correct < or > between them.

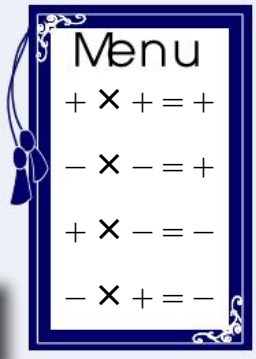
a) 5 ⁻1 **b)** ⁻3 3 **c)** ⁻4 ⁻3

3 Arrange the integers in ascending order (smallest to largest):

a) 2, ⁻4, ⁻2, 3 **b)** 4, ⁻2, ⁻4, 0 **c)** ⁻3, 1, ⁻2, ⁻1

4 Calculate the following:

a) $2 - 3$	**b)** $^{-}7 + 2$	**c)** $^{-}1 - 3$
d) $4 - {}^{-}5$	**e)** $^{-}2 - {}^{-}5$	**f)** 7×3
g) $2 \times {}^{-}6$	**h)** $^{-}8 \times 1$	**i)** $^{-}4 \times {}^{-}1$
j) $12 \div 6$	**k)** $14 \div {}^{-}2$	**l)** $^{-}10 \div 2$
m) $^{-}6 \div {}^{-}2$	**n)** $4 + 2 \times {}^{-}3$	**o)** $6 \div {}^{-}3 - 5$
p) $({}^{-}2 + 3) \times 4$	**q)** $^{-}14 \div (9 - 2)$	**r)** $15 \div ({}^{-}4 + 1)$



Order of Operations:

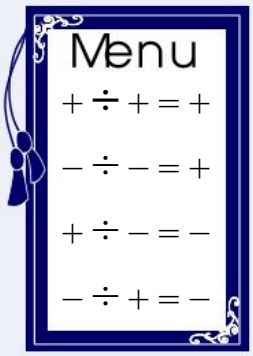
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

$-{}^{-}3 = 3$ $-{}^{-}1 = 1$

$-{}^{-}8 = 8$ $-{}^{-}5 = 5$

5 Calculate the following:

a) $4 - 1$	**b)** $^{-}2 + 2$	**c)** $^{-}9 - 2$
d) $1 - {}^{-}4$	**e)** $^{-}4 - {}^{-}2$	**f)** 4×3
g) $2 \times {}^{-}3$	**h)** $^{-}3 \times 5$	**i)** $^{-}3 \times {}^{-}3$
j) $3 \div 3$	**k)** $9 \div {}^{-}3$	**l)** $^{-}16 \div 8$
m) $^{-}18 \div {}^{-}3$	**n)** $10 + 3 \times {}^{-}2$	**o)** $9 \div {}^{-}3 - 5$
p) $({}^{-}8 + 2) \times 4$	**q)** $^{-}24 \div (4 - 8)$	**r)** $12 \div ({}^{-}6 + 3)$



6 Andre owes each of four friends \$70.
What is Andre's balance.

7 The temperature at midnight was −4°C. If the temperature dropped a further 3°C, what is then the temperature?

8 Ming borrowed \$350, repaid \$220, and borrowed another \$60.
What is Ming's balance?

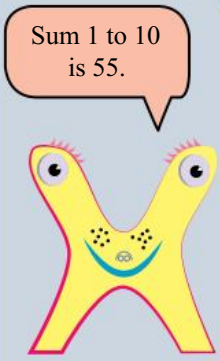
9 A submarine at 120 m below sea level rose 50 m. What is the position of the submarine relative to sea level?

10 Mercury has a melting point of −40°C and a boiling point of 357°C. What is Mercury's temperature range from melting point to boiling point?

Algebra 3

- ★ Extend and apply the distributive law to the expansion of algebraic expressions.
- ★ Factorise algebraic expressions by identifying (highest common factor) of numeric and algebraic expressions.
- ★ Simplify algebraic expressions involving the four operations.

1+2+3+ ..... +98+99+100 = ???



A TASK

Supposedly, Gauss's teacher asked the class to sum the integers from 1 to 100 while the teacher talked to a parent. As the teacher walked out of the room to see the parent, Gauss called out the answer.

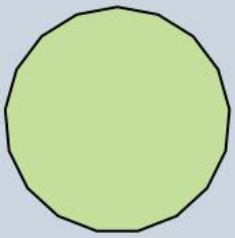
- What is the shortcut that Gauss probably used?
- Can the shortcut be used for other problems?
- Brainstorm a number of real life applications.
- Report your findings (poster, oral report, etc.).

A LITTLE BIT OF HISTORY

Karl Friederick Gauss (1777-1855) was one of the greatest mathematicians of all time.

Before Gauss entered the Göttingen University in 1795, he had independently discovered Bode's law, the binomial theorem, the arithmetic and geometric mean, the law of quadratic reciprocity, and the prime number theorem.

In his early twenties, Gauss constructed a heptakaidecagon, a regular polygon with 17 sides, using only a straight edge and a compass. A construction that had puzzled mathematicians for hundreds of years.



Gauss was so proud that he wanted a heptakaidecagon on his tombstone. The stonemason said it would be too difficult and would look like a circe.

Warm Up

2x and 2×x are the same thing.

x÷3 and $\frac{x}{3}$ are the same thing.

Exercise 3.1

Let x represent the total number of students in a maths class. Write an algebraic expression for the number of students in the classroom if:

a)	4 students were absent.	$x - 4$
b)	2 new students entered the room.	$x + 2$
c)	y students from another class entered the room.	$x + y$
d)	the number of students in the room doubled.	$2x$
e)	one-third of the students left the room.	$x - \frac{x}{3}$

1 Let x represent the total number of students in a maths class. Write an algebraic expression for the number of students in the classroom if:

a) 3 students were absent.
b) 5 new students entered the room.
c) b students from another class entered the room.
d) the number of students in the room tripled.
e) half of the students left the room.

2 Let a represent the amount of money ($) in the cookie jar. Write an algebraic expression for the amount of money in the cookie jar for each of the following:

a) $1.50 is added to the jar.
b) $5.25 is taken out of the jar.
c) $b is added to the jar.
d) $t is taken out of the jar.
e) the amount of money in the jar is doubled.

Algebra is an efficient way of solving millions and millions of problems.

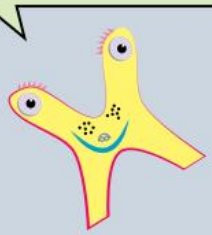
3 The cupboard contains x dinner plates, y soup plates, and z cups. Write algebraic expressions for each following situation:

a) the total number of dinner plates, soup plates, and cups.
b) the total number of plates (no cups).
c) the number of cups in the cupboard after 3 cups are taken.
d) the number of cups in the cupboard after 5 cups are added.

4 Write algebraic expressions for each of the following:

a) the sum of x and y.
b) add a and 7.
c) twice b plus 10.
d) the difference between m and n.
e) the product of a and g.
f) b cubed.
g) triple x and add five times the value of p.
h) square x and subtract double x then add 7.
i) the product of m and the square of c.

"I was x years old in the year x^2." Augustus De Morgan (when asked about his age).



Substitution

When using substitution in algebra, a variable such as x or y is replaced with its value.

Exercise 3.2

Find the value of $2x - 1$ if $x = 3$	Find the value of $4a - b^2$ if $a = 2$, $b = 3$
$2x - 1 = 2\times3 - 1$ $= 6 - 1$ $= 5$	$4a - b^2 = 4\times2 - 3^2$ $= 8 - 9$ $= -1$

1 Find the value of each of the following algebraic expressions given that $x = 4$ and $y = 2$.

a) $3x$ **b)** $2y$ **c)** $x + 5$

d) $y - 5$ **e)** $x \div 6$ **f)** $\frac{y}{3}$

g) $x + y$ **h)** $x - y$ **i)** $y - x$

j) $3x^2 - 5$ **k)** $5x - 2y$ **l)** $5y - 2y^2 + 10$

2 If $x = 4$, what is the value of $\frac{3x}{2x-2}$?

3 $y = 8 - 3x$, what is the value of y when $x = 1.25$?

4 $B = 3h^2$ What is the value of B when $h = 25$?

5 What is the value of $2x^2 - 5x + 7$ when $x = -2$?

The area A, of a rectangle of length l, and width w, is given by the algebraic formula: $A = lw$.

Find the area of the rectangle if length = 12.2 m and width = 4.6 m

$A = lw$
$= 12.2 \text{ m} \times 4.6 \text{ m}$
$= 56.12 \text{ m}^2$

6 The area, A, of a rectangle of length, l, and width, w, is given by the algebraic formula: $A = lw$. Find the area of each of the following rectangles:

a) length = 6 cm and width = 5 cm.

b) length = 12.2 m and width = 4.6 m.

7 The perimeter, P, of a rectangle of length, l, and breadth, b, can be calculated using either the algebraic formula: $P = 2(l + b)$ or the formula: $P = 2l + 2b$. Calculate the perimeter of each of the following rectangles using **both** formulas:

a) length 5 cm and breadth 2 cm.

b) length 16.5 cm and breadth 9.7 cm.

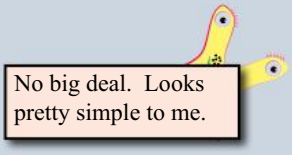
8 The area of a triangle, A, with base b, and height h, is given by the algebraic formula: $A = 0.5bh$. Calculate the area of each of the following triangles:

a) base of 6 mm and height of 2 mm.

b) base of 8.2 cm and height of 5.8 cm.

Algebraic + and −

Algebra is fundamental to higher level mathematics. Algebraic addition and subtraction are very essential parts of algebra.



Exercise 3.3

Simplify the following expressions:

$4x + 3x = 7x$	$5y - 7y = {}^{-}2y$	$9b + 5b - b = 13b$

1	$2x + 6x$	**2**	$5a - 3a$	**3**	$3c + 6c$
4	$7e - 2e$	**5**	$12c + 7c$	**6**	$3z - 8z$
7	$9h + 6h$	**8**	$8k - 5k$	**9**	$5d + 7d$
10	$6f - f$	**11**	$16o + 2o$	**12**	$4x - 5x$
13	$17y + 4y$	**14**	$9w - 10w$	**15**	$3a + 7a$
16	$6s + 3s + 5s$	**17**	$8d - 7d + 4d$	**18**	$17a - 7a - 6a$
19	$7x + 2x + x$	**20**	$4u - 3u - 5u$	**21**	$-6d + 4d + 5d$
22	$8x + 5x - 3x - x$	**23**	$8a + 3a - 4a - 5a$		
24	$-4g + 3g - 5g + g$	**25**	$6y + y + 2y + 7y$		

The key is to only join together the terms that are alike.

$7a + 5e - 4a = 3a + 5e$	$5y^2 + 4y + y - 2y^2 = 3y^2 + 5y$

26	$8x + 5x + 2a$	**27**	$7a + 4b - 4a + 2b$
28	$6b + 3g - 5b + g$	**29**	$9y + y + 2p - 7y$
30	$5t + 6 - 7t + 4$	**31**	$m + 5n - 3n + 5m - n$
32	$3b^2 - 8 + 5b^2 - 3$	**33**	$7a^3 + 7b - 5a^3 + 2b$
34	$4xy + 7 + 2xy - 5$	**35**	$3as^2 + 5d^5 - 2as^2 + 2d^5$
36	$10x + 3x - 8x + 7y$	**37**	$a + 5x^2y^3 - 6a - 2x^2y^3 - 5a$
38	$12x + 7 + 2x - 3$	**39**	$^{-}14x + 6y + 9x - 8y + y$

Exercise 3.4

Use substitution to check if the following expressions are equivalent:

$5a + 2a - 8 + 6a + 4 = 12a - 4$	Using $a = 2$
LHS $= 5a + 2a - 8 + 6a + 4$ $= 5\times2 + 2\times2 - 8 + 6\times2 + 4$ $= 10 + 4 - 8 + 12 + 4$ $= 22$	RHS $= 12a - 4$ $= 12\times2 - 4$ $= 24 - 4$ $= 20$ Not equivalent

Use any value you wish for the variables other than 0.

1	$2x + 3x = 8x$	**2**	$9x - 7x + 3x = 5x$
3	$7a - 4 + 8 - 3a = 5a + 4$	**4**	$6 + 2b + 6b - 7 - 7b = b - 1$
5	$10x + 6x - 8x + 7y = 8x + 7y$	**6**	$2a + 5x + 6a - 2x - 5a = 3a + 4x$
7	$14x + 2 + 2x - 8 = 16x - 6$	**8**	$^{-}14x + 6y + 9x - 8y + y = {}^{-}23x - 2y$

Algebraic Multiplication

Exercise 3.5
Simplify the following expressions:

Multiple the numbers.
Multiply the letters.

$4 \times 3a = 4 \times 3 \times a$ $= 12a$	$3d \times 5e = 3 \times d \times 5 \times e$ $= 15de$	$10b \times \frac{1}{5} = 10 \times \frac{1}{5} \times b$ $= 2b$

1 $2 \times 5a$ **2** $3 \times 7a$ **3** $3 \times 6c$
4 $7 \times 2e$ **5** $4 \times 7n$ **6** $3 \times 8m$
7 $8h \times 4$ **8** $5a \times 3$ **9** $6 \times 2s$
10 $4 \times 3d$ **11** $4 \times 9b$ **12** $e \times 4$
13 $4w \times 2$ **14** $3 \times p$ **15** $6 \times 2v$
16 $9d \times \frac{1}{3}$ **17** $\frac{p}{2} \times 6$ **18** $12r \times \frac{1}{4}$
19 $3a \times 4b$ **20** $2f \times 2w$ **21** $7s \times 2y$
22 $5c \times 2b$ **23** $8t \times 3d$ **24** $g \times 7k$
25 $9n \times 3p$ **26** $6h \times 5e$ **27** $4g \times 5f$
28 $\frac{m}{2} \times 4n$ **29** $10r \times \frac{p}{2}$ **30** $8m \times \frac{k}{4}$

Multiply the numbers.
Multiply the letters.

+ times − = −
− times + = −
− times − = +

$4a \times 3a = 4 \times 3 \times a \times a$ $= 12a^2$	$5mn \times {}^-2m^2n = 5 \times {}^-2 \times m \times m^2 \times n \times n$ $= {}^-10m^3n^2$

$m \times m^2 = m \times m \times m = m^3$

31 $4g \times 3g$ **32** $3t \times 4t$
33 $9h \times 3h$ **34** $7d \times 5d$
35 $8s \times 2s^2$ **36** $4g^2 \times 5f \times 2g$ **37** $3e \times e^2 \times 2d$
38 $5m \times {}^-2n$ **39** $2a \times {}^-3a$ **40** $3b \times {}^-4b$
41 $4ab \times 3a$ **42** $6p \times {}^-2p^2$ **43** $9w \times {}^-3w^2$
44 $5mn \times {}^-3m^2n$ **45** $7pn \times {}^-4p^2n$ **46** $4ab \times {}^-6a^2b$
47 ${}^-3p^2d \times {}^-2pd$ **48** ${}^-4h^2 \times {}^-4h$ **49** ${}^-4a^2b^2c \times {}^-5a^2bc$

Electronic Engineers design, develop, test and maintain electronic parts and systems used in computers, communications, navigation, industry and entertainment. They may specialise as a biomedical engineer, computer engineer or mechatronic engineer.

- Relevant school subjects are Mathematics, Physics
- Courses range from Universtity Bachelor degrees to Graduate diplomas.

Algebraic Division

$6d \div 4d$ and $\frac{6d}{4d}$ and $\frac{3d}{2d}$ and $\frac{3}{2}$ are the same thing.

Exercise 3.6

Simplify the following expressions:

$8b \div 2 = \frac{8b}{2} = 4b$	$6d \div 4d = \frac{6d}{4d} = \frac{3}{2}$	$18ay \div 4a = \frac{18ay}{4a} = \frac{9y}{2}$

1	$8a \div 4$	**2**	$9y \div 3$	**3**	$6c \div 2$
4	$12e \div 6$	**5**	$14n \div 7$	**6**	$8m \div 2$
7	$20f \div 4$	**8**	$18n \div 9$	**9**	$20h \div 5$
10	$5g \div 10$	**11**	$4a \div 12$	**12**	$3s \div 9$
13	$7m \div 14$	**14**	$2d \div 10$	**15**	$6t \div 18$
16	$20p \div 6$	**17**	$12b \div 8$	**18**	$18d \div 4$
19	$16k \div 6$	**20**	$12d \div 10$	**21**	$15s \div 10$
22	$20y \div 15$	**23**	$14f \div 4$	**24**	$6g \div 4$
25	$6g \div 4g$	**26**	$24d \div 10d$	**27**	$24p \div 9p$
28	$4g \div 6g$	**29**	$12a \div 8a$	**30**	$8v \div 14v$
31	$12ay \div 4a$	**32**	$14de \div 4e$	**33**	$18dg \div 4d$
34	$6tw \div 24t$	**35**	$5st \div 15st$	**36**	$12a \div 18ab$

$-6m \div 4mn = \frac{-6m}{4mn} = \frac{-3}{2n}$

Divide the numbers.
Divide the letters

+ divided by − = −
− divided by + = −
− divided by − = +

37	$-8d \div 4$	**38**	$-6q \div 3$	**39**	$-12h \div 2h$
40	$-4g \div -2g$	**41**	$-12a \div 4a$	**42**	$-8v \div -4v$
43	$-12ay \div -4a$	**44**	$-14de \div -7e$	**45**	$-18dg \div 4d$
46	$-6tw \div 24t$	**47**	$15st \div -5st$	**48**	$-12a \div -18ab$
49	$-9mg \div -6mn$	**50**	$18mpq \div -6mp$	**51**	$24st \div -9st$
52	$-16bc \div 24b$	**53**	$-28ad \div -12d$	**54**	$-24dwz \div 36dw$

Calculators are very good at handling fractions:
See Technology 3.1

Distributive Law

The distributive law:
a(b + c) = ab + ac

Each term inside the brackets: **b** and **c** is multiplied by the term outside the brackets: **a** to give: **ab + ac**

ie., **a** is distributed through the brackets.

Exercise 3.7

Expand each of the following:

4(a + 3) = 4a + 12 | 3(2b − 5) = 6b − 15

Multiply each inside term by the outside term.

1	4(b + 3)	**2**	5(c + 2)	**3**	2(a + 7)
4	3(g + 1)	**5**	6(h + 5)	**6**	7(n + 4)
7	5(m + 3)	**8**	3(y + 2)	**9**	9(w + 6)
10	3(2c + 4)	**11**	4(3e + 4)	**12**	5(4v + 3)
13	7(3y + 1)	**14**	6(3x + 5)	**15**	2(12g + 5)
16	2(5a + d)	**17**	5(w + h)	**18**	3(9q + 4d)
19	2(2z − 4)	**20**	5(2s − 4)	**21**	3(4d − 3)
22	4(3f − 7)	**23**	8(2a − 5)	**24**	6(5h − 6)
25	3(2u − 4)	**26**	4(7p − 6)	**27**	5(7d − 9)
28	8(s − 3t)	**29**	7(5n − 8)	**30**	2(9u − 15)

$5w(3w - 2m) = 15w^2 - 10wm$

$5w \times 3w = 15ww = 15w^2$

31	t(2t + 3)	**32**	3z(4z + 5)	**33**	6g(4g + 5)
34	2d(3d − 4)	**35**	3n(2n − 4)	**36**	m(7m − 2)
37	y(3y + 2r)	**38**	5b(3b − 2c)	**39**	6u(3b − 4u)

Exercise 3.8

Use substitution to check if the following expressions are equivalent:

8(2x − 4) = 16x − 32

Using x = 3

Use any value you wish for the variables other than 0.

LHS = 8(2x − 4)
= 8(2×3 − 4)
= 16

RHS = 16x − 32
= 16×3 − 32
= 16 thus equivalent

1	2(x + 3) = 2x + 6	**2**	9(x − 3) = 9x − 27
3	7(a − 4) = 7a − 24	**4**	6(2b + 6) = 12b − 36
5	10(x − 8) = 10x − 80	**6**	$2a(5a - 2) = 5a^2 + 4a$
7	14(y − 8) = 14y − 102	**8**	$6y(8y + 1) = 48y^2 + 6y$
9	$t(2t + 3) = 2t^2 + 3t$	**10**	$3z(4z + 5) = 12z^2 + 5z$
11	$6g(4g + 5) = 24g^2 + 30$	**12**	$2d(3d - 4) = 6d^2 - 8d$
13	3n(2n − 4) = 6n − 8n	**14**	$m(7m - 2) = 7m^2 - 2m$
15	$y(3y + 2r) = 3y^2 + 2r$	**16**	5b(3b − 2c) = 15b − 10bc
17	6u(3b − 4u) = 18bu − 24u	**18**	5(6b + 3u) = 30b + 15u

Distributive Law

Exercise 3.9
Expand each of the following:

+ times + = +
+ times − = −
− times + = −
− times − = +

$^{-}4(a + 3) = {}^{-}4\times a + {}^{-}4\times 3$ $= {}^{-}4a - 12$	$^{-}3(2b - 5) = {}^{-}3\times 2b - {}^{-}3\times 5$ $= {}^{-}6b + 15$

1	$^{-}2(a + 3)$	**2**	$^{-}5(r + 2)$	**3**	$^{-}2(c + 4)$
4	$^{-}4(2d + 5)$	**5**	$^{-}6(3h + 1)$	**6**	$^{-}7(4x + 7)$
7	$^{-}5(2m - 4)$	**8**	$^{-}3(y + 2)$	**9**	$^{-}9(w + 6)$
10	$^{-}3(2c - 4)$	**11**	$^{-}4(3e - 4)$	**12**	$^{-}5(4v + 3)$
13	$^{-}t(2t + 3)$	**14**	$^{-}3z(4z - 2)$	**15**	$^{-}6g(4g + 5)$
16	$^{-}2d(3d - 4)$	**17**	$^{-}8n(2n - 4)$	**18**	$^{-}m(3m - 5)$
19	$^{-}p(3 + 2p)$	**20**	$^{-}4e(3e - 2c)$	**21**	$^{-}4u(2b - 4u)$
22	$^{-}2y(3y - 4)$	**23**	$^{-}3s(5s - 4)$	**24**	$^{-}m(7m - 2)$
25	$^{-}z(3z + 2r)$	**26**	$^{-}5q(9q - 2d)$	**27**	$^{-}6t(3t - 5u)$

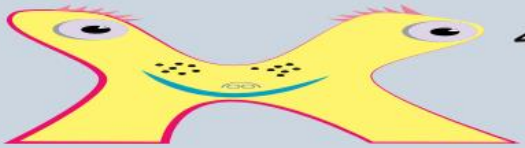
Mixed Problems

The Distributive Law: $a(b + c) = ab + ac$

Exercise 3.10
Expand and simplify if necessary:

$a(a - b) = a\times a - a\times b$ $= a^2 - ab$	$^{-}2(3b - 4) = {}^{-}2\times 3b - {}^{-}2\times 4$ $= {}^{-}6b + 8$	$4 \times 37 = 4(30 + 7)$ $= 120 + 28$ $= 148$

1	$a(b + c)$	**2**	$5(a + 3)$	**3**	$26\times 101 = 26(100 + 1)$
4	$x(b + c)$	**5**	$^{-}3(b - 4)$	**6**	$4\times 53 = 4(50 + 3)$
7	$a(a + d)$	**8**	$^{-}2(x + 2)$	**9**	$5\times 42 = 5(40 + 2)$
10	$x(2x + 3)$	**11**	$6(3x + 5)$	**12**	$3\times 63 = 3(60 + 3)$
13	$a(5a - d)$	**14**	$^{-}5(w + h)$	**15**	$7\times 45 = 7(40 + 5)$
16	$d(2s - 4)$	**17**	$^{-}5(2s - 4)$	**18**	$3\times 19 = 3(20 - 1)$
19	$n(3n - 7)$	**20**	$8(2a - 5)$	**21**	$6\times 48 = 6(50 - 2)$
22	$3(2u - 4)$	**23**	$^{-}4(7p - 6)$	**24**	$5\times 67 = 5(70 - 3)$
25	$8(s - 3t)$	**26**	$7(5n - 8)$	**27**	$15\times 102 = 15(100 + 2)$



Distribute - to spread out, to cover everything.

Factorising

Factorising is the opposite of distributing.

Factorising: $ab + ac = a(b + c)$

Exercise 3.11

Factorise the following algebraic expressions:

$2x + 4 = 2(x + 2)$	$3b - 6 = 3(b - 2)$	$4p^2 - 8p = 4p(p - 2)$

1	$2x + 6$	**2**	$3a + 6$	**3**	$2c + 4$
4	$2x - 6$	**5**	$3a - 6$	**6**	$5g + 10$
7	$5a + 15b$	**8**	$2m + 8n$	**9**	$2a^2 + 3a$
10	$5x^2 + 3x$	**11**	$7t^2 - 5t$	**12**	$3s^2 - 4s$
13	$2x^2 + 6x$	**14**	$3y^2 + 6y$	**15**	$2r^2 - 4r$
16	$4a + 8d$	**17**	$3w - 9$	**18**	$5b + 10$

Exercise 3.12

Find the highest common factor of each of the following pairs of terms:

3a and 6a The factors of 3a are: 3, a The factors of 6a are: 2, 3, 6, a The highest common factor = 3a	4ef and 8fg The factors of 4ef are: 2, 4, e, f The factors of 6a are: 2, 4, 8, f, g The highest common factor = 4f

1	3x and 6x	**2**	4a and 8ab	**3**	6ab and 10a
4	4b and 6ab	**5**	3xy and 9y	**6**	5s + 10d
7	3g and 12	**8**	12e and 4	**9**	8ab and 12abc
10	6p and 14pq	**11**	4p and 16	**12**	8a and 2b
13	6 and 12y	**14**	5xy and 15yz	**15**	15ef and 27fg
16	16g and 8	**17**	16h and 64hij	**18**	14rt and 35t

Exercise 3.13

Factorise the following algebraic expressions:

$2 + 4x = 2\times1 + 2\times2x$ $= 2(1 + 2x)$ Check: $2(1 + 2x) = 2 + 4x$	$4m + 12mn = 4m\times1 + 4m\times3n$ $= 4m(1 + 3n)$ Check: $4m(1 + 3n) = 4m + 12mn$	$6b - 10ab = 2b\times3 - 2b\times5a$ $= 2b(3 - 5a)$ Check: $2b(3 - 5a) = 6b - 10ab$

1	$3 + 6x$	**2**	$4a + 8ab$	**3**	$6ab - 10a$
4	$4b + 6ab$	**5**	$3xy - 9y$	**6**	$5s + 10d$
7	$3g - 12$	**8**	$12e + 4$	**9**	$8ab + 12abc$
10	$6p + 14pq$	**11**	$4p - 16$	**12**	$8a + 2b$
13	$6 + 12y$	**14**	$5xy - 15yz$	**15**	$15ef + 27fg$
16	$16g - 8$	**17**	$16h + 64hij$	**18**	$14rt + 35t$

Factorising

Exercise 3.14

Find the highest common factor of each of the following pairs of terms:

4v and $^{-}$6v The factors of 4v are: 2, 4, v The factors of 6a are: 2, 3, 6, v The highest common factor = 2v	$^{-}$4ef and $^{-}$8fg The factors of 4ef are: 2, 4, e, f The factors of 8fg are: 2, 4, 8, f, g The negative sign is common The highest common factor = $^{-}$4f

1	4x and $^{-}$6x	**2**	4a and 10ab	**3**	$^{-}$2xy and $^{-}$10y
4	$^{-}$4b and 8ab	**5**	$^{-}$3ay and $^{-}$9y	**6**	5s + 10d
7	6g and $^{-}$10	**8**	15e and 3	**9**	$^{-}$8df and 12adf
10	6p and $^{-}$14pq	**11**	$^{-}$3m and 18	**12**	4a and $^{-}$2b
13	$^{-}$8 and 12u	**14**	5mn and 15no	**15**	$^{-}$12ef and $^{-}$27fg
16	14g and $^{-}$8	**17**	$^{-}$24j and $^{-}$64hij	**18**	10bn and $^{-}$35n

Factorising:

$$ab + ac = a(b + c)$$
$$ab - ac = a(b - c)$$
$$^{-}ab + ac = a(^{-}b + c)$$
$$^{-}ab - ac = {}^{-}a(b + c)$$

Exercise 3.15

Factorise each of the following algebraic expressions:

$2x - 4 = 2\times x - 2\times 2$ $= 2(x - 2)$ Check: $2(x - 2) = 2x - 4$	$4v - 6v = 2v\times 1 - 2v\times 3$ $= 2v(1 - 3)$ Check: $2v(1 - 3) = 2v - 6v$	$^{-}4ef - 8fg = {}^{-}4f\times e + {}^{-}4f\times 2g$ $= {}^{-}4f(e + 2g)$ Check: $^{-}4f(e + 2g) = {}^{-}4ef - 8fg$

1	4x − 6	**2**	4a + 10ab	**3**	$^{-}$2xy − 10y
4	$^{-}$4b + 8ab	**5**	$^{-}$3ay − 9y	**6**	5s + 10d
7	6g − 10	**8**	15e + 3e	**9**	$^{-}$8df + 12adf
10	6p − 14pq	**11**	$^{-}$3m + 18	**12**	4a − 2b
13	$^{-}$8 + 12u	**14**	5mn + 15no	**15**	$^{-}$12ef − 27fg
16	14g − 8	**17**	$^{-}$24j − 64hij	**18**	10bn − 35n

See Exercise 3.8 for an example.

Exercise 3.16

Use substitution to check if the following expressions are equivalent:

1	$2x + 6 = 2(x + 3)$	**2**	$9x - 18 = 9(x - 2)$
3	$^{-}7a - 28 = {}^{-}7(a + 4)$	**4**	$12b - 36 = 6(2b + 6)$
5	$10x - 80 = 10(x - 8)$	**6**	$^{-}5a - 20a = {}^{-}5a(1 + 4)$
7	$14y - 102 = 14(y - 8)$	**8**	$48y^2 + 6y = 6y(8y + 1)$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 3.17

1 Spell distributive
2 What is the value of $2x - 1$, if $x = 2$?
3 $^{-}4 + 2$
4 $2 \times {}^{-}3$
5 Simplify: $6a - 2a$
6 Simplify: $2g \times 3g$
7 Distribute: $5(2a + 3)$
8 Factorise: $3x - 6$
9 Write in index form: $3\times3\times3\times3\times3\times3\times3$
10 10% of 72

10% of 72 = 7.2

They that would perfect their work must first sharpen their tools - Confucius.

Exercise 3.18

1 Spell substitution
2 What is the value of $2x + 1$, if $x = 2$?
3 $^{-}3 - 2$
4 $^{-}5 \times {}^{-}3$
5 Simplify: $4m + 2m$
6 Simplify: $8b \div 4$
7 Distribute: $3(2x - 1)$
8 Factorise: $4x - 6$
9 Write in index form: $5\times5\times5\times5$
10 10% of 38

Exercise 3.19

1 Spell factorise
2 What is the value of $3x - 2$, if $x = 3$?
3 $5 + {}^{-}4$
4 $6 \div {}^{-}3$
5 Simplify: $5b - 2b$
6 Simplify: $5 \times {}^{-}3c$
7 Distribute: $4(2c - 1)$
8 Factorise: $5x + 15$
9 Write in index form: $2\times2\times2\times2\times2\times2$
10 10% of 47

How do you expand 3(x +2)?

3 (x + 2)

NAPLAN Questions

Exercise 3.20

1 If a = 3, what is the value of 6a?

2 If x = 4, what is the value of $\frac{3x}{2x-2}$?

3 y = 8 − 3x, what is the value of y when x = 2.25?

4 What is the value of $6x - 2x^2 + 75$ when x = ⁻1?

5 $B = 3h^2$ What is the value of B when h = 25?

6 Given that T = 6n + 7500, what is n when T = 7650?

7 $P = \frac{4q}{15r}$, what is the value of r when P = 0.2 and q = 9?

One method is to substitute values for x until both expressions are the same.

8 25 x Δ = 30 What is the value of Δ?

9 y = 2x + 5
y = 5x − 7 What value of x satisfies both of these equations?

10 y = 4x − 2
y = 2x + 8 What value of x satisfies both of these equations?

11 A rule for a pattern is multiply by three and then add 2. The first three numbers of this pattern are: 5, 17, 53, ... What is the fifth number in this pattern?

12 A rule for a pattern is to add two and then multiply by 3. The first three numbers of this pattern are: 9, 33, 105, ... What is the fifth number in this pattern?

13 What is the value of $a^2 + b^2$ when a = 2 and b = ⁻3?

14 What is the value of $4t^2$ when t = ⁻3?

15 Expand: 3(4a + 1)

3(2x −1) + 3 + ? = 8x +5 What term makes this equation true for all values of x?

Left Hand Side = 3(2x – 1) + 3 + ?
= 6x – 3 + 3 + ?
= 6x + ? Thus need 2x + 5 to be equal to 8x + 5

16 2(2x − 1) + 6 + ? = 5x + 7 What term makes this equation true for all values of x?

17 A number is multiplied by itself and then 5 is added. The answer is 14. What is the number?

18 Two numbers added together equal 7. The two numbers multiplied together equal 12. What are the two numbers?

19 Two numbers added together equal ⁻3. The two numbers multiplied together equal ⁻10. What are the two numbers?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

−− = +
−+ = −
+ − = −
++ = +

Exercise 3.21

1 Simplify each of the following:

4a − 2 − (3a − 3)

= 4a – 2 – 3a + 3 {Distribute the – and – ¯3 = 3 }
= a + 1 {4a – 3a = a and –2+3 = 1 }

a) 3b − 4 − (2b − 6)
b) 6n + 2 − (4n − 4)
c) 5x − 2 − (2x − 4)
d) 6a − 4b − (4a − 5b)
e) 5w − 3(w − 2)
f) 7u − 2(u − 4)
g) 4x − (3x − 4)
h) 5y − (3y + 3)
i) 3d − 2(2d + 4)
j) 6h − 4(2h − 2)
k) (3a + 2b) − (2a − 4b)
l) (6g + 4h) − (5g − 3h)

If x = 4, find the value of $\frac{\sqrt{20+x^2}}{\sqrt{20-x^2}}$

$= \frac{\sqrt{20+4^2}}{\sqrt{20-4^2}}$ {Substitute 4 for x }

$= \frac{\sqrt{36}}{\sqrt{4}} = \frac{6}{2}$ $= 3$

2 If x = 5, find the value of $\sqrt{x^2-4^2}$

3 If x = 8, find the value of $\sqrt{x^2-2x+1}$

4 If a = 8 and b = ¯8, find the value of $\frac{b}{a-b}$

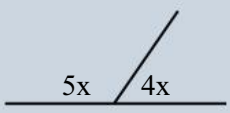
5 If a = 0.4, b = 0.2, c = 1.4, find the value of $\frac{bc}{a}$

x − 25 + x + 25 = 180

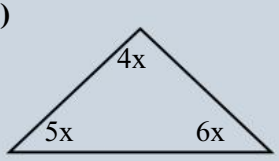
6 When one-sixth of a number is subtracted from itself, the result is 45. What is the number?

7 For each of the following, write an equation and then solve the equation:

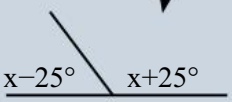
a)



b)



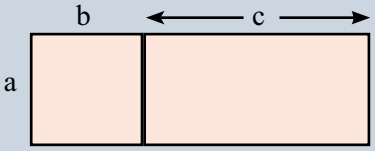
c)



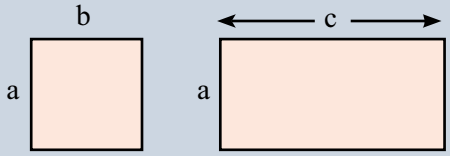
Investigations

Investigation 3.1 **The Distributive law: a(b + c) = ab + ac**

1 Write an algebraic expression for the area of the rectangle shown on the right.



2 Write an algebraic expression for each of the two rectangles shown on the right and sum them together.



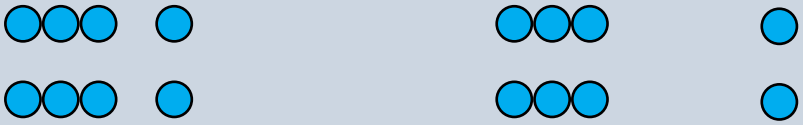
3 What do you notice?

Investigation 3.2 **The Distributive law: a(b + c) = ab + ac**

Use centicubes or counters to prove the distributive law for each of the following instances:

1 2(3 + 1) = 2x3 + 2x1

2 lots of (3 + 1) is the same as 2 lots of 3 and 2 lots of 1



2 3(2 + 1)= 3x2 + 3x1

3 2(4 + 2) = 2x4 + 2x2

4 4(3 + 2) = 4x3 + 4x2

Investigation 3.3

The Distributive Law helps with mental math.

For example 4×53
$= 4 \times 50 + 4 \times 3$
$= 200 + 12$
$= 212$

Brainstorm a number of mental math problems that would be made easier with the use of the distributive law.

Technology

Technology 3.1 Simplifying fractions

Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ [15] [$a\frac{b}{c}$] [35] [=] [3r7] meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ [18] [$a\frac{b}{c}$] [4] [=] [4r1r2] meaning $4\frac{1}{2}$

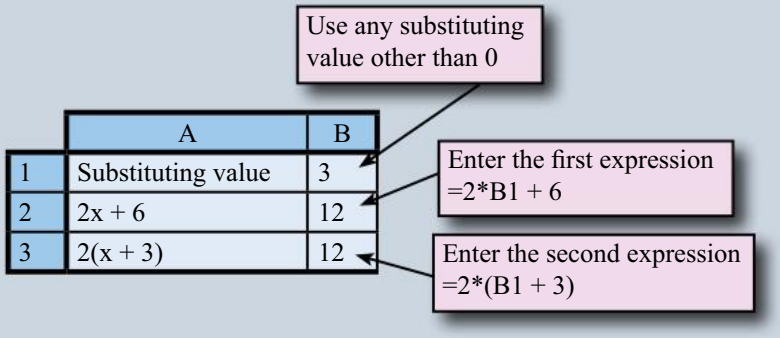
To change to a vulgar fraction: [2ndF] [$a\frac{b}{c}$] to give [9r2] ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:

a) 3 : 9 **b)** 9 : 12 **c)** 16 : 24
d) 2.1 : 3.5 **e)** 14.4 : 12.6 **f)** 256 : 1024

Technology 3.2 Substitution

Use a spreadsheet to check that the following expressions are equivalent:



Thus the expressions 2x + 6 and 2(x + 3) are equivalent.

Are the following statements correct?

1 $2x + 6 = 2(x + 3)$ **2** $9x - 18 = 9(x - 2)$
3 $^{-}7a - 28 = {}^{-}7(a + 4)$ **4** $12b - 36 = 6(2b + 6)$
5 $10x - 80 = 10(x - 8)$ **6** $^{-}5a^2 - 4a = {}^{-}2a(5a + 2)$
7 $14y - 102 = 14(y - 8)$ **8** $48y^2 + 6y = 6y(8y + 1)$

Technology 3.3 Expanding and factorising

Graphics calculators are capable of expanding and factorising:

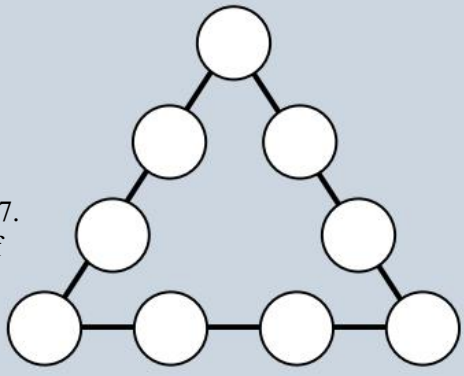
1 Choose **expand** from the algebra menu.
2 Enter the algebraic expression: $3(4x - 5)$ to produce $12x - 15$

1 Choose **factor** from the algebra menu.
2 Enter the algebraic expression: $2x - 15$ to produce $3(4x - 5)$

A Couple of Puzzles

Exercise 3.22

1 Place each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 in each circle so that the sum along each side is 17. Then see if you can arrange each of the numbers so that the sum is 20



2 Maddie makes Easter cards at a cost of $0.90 each and sells them for $2.50 each. How many cards does Maddie need to sell to make a profit of $120?

A Game

Go is an ancient oriental game, similar to naughts and crosses, played on a square grid by two players.

1 Each player takes turns to place X and O on the grid.

2 The winner is the player with five in a row, column, or diagonal.

	X								
O	O	O						O	
		X	O		X				
	O	O	X	O	X				
	X	X	O	X	O				
		O			X		X		X
						O			

A Sweet Trick

1 Ask your audience to roll a pair of dice while you are not looking.

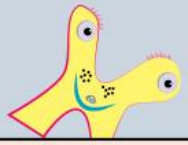
2 Ask them to multiply the number on the first die by 5 — $2\times5 = 10$

3 Add 3 — $10+3 = 13$

4 Double the answer — $13\times2 = 26$

5 Add the number on the second die — $26+4 = 30$

6 Add 5 — $30+5 = 35$



If you subtract 11 from their answer (35 - 11 =24), you have the number on the first die (2), and the number on the second die (4).

Chapter Review 1

Exercise 3.23

1 Let x represent the amount of money ($) in the cookie jar. Write an algebraic expression for the amount of money in the cookie jar for each of the following:

a) $5.50 is added to the jar.
b) $3.25 is taken out of the jar.
c) $b is added to the jar.
d) $t is taken out of the jar.
e) the amount of money in the jar is doubled.

2 Simplify the following expressions:

a)	$2x + 5x$	**b)**	$7h - 3h$	**c)**	$8c \div 4$
d)	$2 \times 3a$	**e)**	$3z - 8z$	**f)**	$15s \div 10$
g)	$4u - 2u - 3u$	**h)**	$9n \times 3p$	**i)**	$6t \div 18$
j)	$^{-}6d + 4d + 5d$	**k)**	$2a \times {}^{-}3a$	**l)**	$18dg \div 4d$
m)	$3m^3 \times 2m$	**n)**	$4a^4 \times 3a$	**o)**	$2d^3 \times 2d^2$
p)	$3b^2 - 8 + 6b^2 - 3$	**q)**	$5mn \times {}^{-}3m^2n$	**r)**	$^{-}8a \div 6ab$
s)	$6d^3 \div 2d$	**t)**	$10u^2 \div 5u^2$	**u)**	$18p^3 \div 6p^2$

3 Expand each of the following expressions:

a)	$2(t + 5)$	**b)**	$6(2w - 5)$	**c)**	$3(2m - 4)$
d)	$g(2g + 3)$	**e)**	$5b(3b - 2c)$	**f)**	$^{-}5(w + h)$
g)	$^{-}3(7h - 4)$	**h)**	$3(30a - 1)$	**i)**	$14(10x + 2)$
j)	$8(4c - 5)$	**k)**	$^{-}4(8b - 9)$	**l)**	$^{-}6(2p - 6)$

4 Factorise each of the following expressions

a)	$3x + 12$	**b)**	$4a - 6$	**c)**	$6c + 8$
d)	$5t + 15$	**e)**	$2f - 6$	**f)**	$2w - 10$
g)	$3 + 6h$	**h)**	$4de + 8e$	**i)**	$6a - 10ab$
j)	$4p + 10pq$	**k)**	$^{-}4b + 12ab$	**l)**	$^{-}3st - 6$
m)	$^{-}3k + 18$	**n)**	$^{-}15dg - 27fg^2$	**o)**	$^{-}24cd - 48bcd$

5 Use substitution to check that the following expressions are equivalent:

a)	$2x + 4 = 2(x + 2)$	**b)**	$9(x - 3) = 9x - 27$
c)	$^{-}7a - 14 = {}^{-}7(a - 2)$	**d)**	$10(c - 3) = 10c + 30$
e)	$5(2x - 15) = 10x - 75$	**f)**	$^{-}6c^2 - 4c = {}^{-}2c(3c + 2)$
g)	$14y(y^2 - 12) = 14y^3 - 148$	**h)**	$48y^2 + 6y = 6y(8y + 1)$

6 If b = 5, what is the value of $\frac{3x}{2x-2}$?

7 What is the value of $7x - x^2 + 10$ when $x = {}^{-}1$?

8 $A = 5c^3$ What is the value of A when c = 5?

Q Why is April 1st so tired?

A You'd be tired too, after 31 days of March.

9 Simplify each of the following expressions:

a)	$4(x + 3) - 4$	**b)**	$5(y - 3) + 2y + 1$
c)	$2(2p + 4) - 10$	**d)**	$4(2h - 2) + 3(h + 1)$
e)	$9a - 4 + 3(2a - 6)$	**f)**	$6d + 2 + 2(4d - 3)$

Chapter Review 2

Exercise 3.24

1 Write an algebraic expression for each of the following:

a) the sum of x and y
b) 5 plus double b
c) triple d minus 45
d) add 9 to h
e) twice the sum of double h and 3

2 Simplify the following expressions:

a) $3d + 4d$
b) $9m - 4m$
c) $8f \div 2$
d) $3 \times 5v$
e) $2p - 5p$
f) $16n \div 12$
g) $6g - 2g - 3g$
h) $6b \times 2b$
i) $8t \div 18$
j) $^{-}6a + 3a + a$
k) $4w \times {}^{-}3w$
l) $18tu \div 10u$
m) $2c^3 \times 3c$
n) $5a^4 \times 3a$
o) $6s^3 \times 2s^2$
p) $4b^2 - 7 + 6b^2 - 2$
q) $8bc \times {}^{-}3b^2c$
r) $^{-}8a \div 12ab$
s) $24d^3 \div 14d$
t) $4h^2 \div 12h^2$
u) $18p^3 \div 21p^2$

3 Expand each of the following expressions:

a) $3(r + 5)$
b) $5(4w - 2)$
c) $4(3m - 3)$
d) $z(2z + 7)$
e) $5b(b - 3c)$
f) $^{-}3(u + v)$
g) $^{-}8(2h - 1)$
h) $4(60x - 1)$
i) $15(100y + 3)$
j) $9(2k - 4)$
k) $^{-}4(8c - 3)$
l) $^{-}6(5v - 2)$

4 Factorise each of the following expressions

a) $3x + 9$
b) $4y - 8$
c) $6z + 8$
d) $5a + 20$
e) $2f - 6$
f) $2g - 16$
g) $3 + 15h$
h) $4ab + 16b$
i) $6c - 22bc$
j) $4d + 10cd$
k) $^{-}5j + 30jk$
l) $^{-}3mn - 9$
m) $^{-}3p + 18$
n) $^{-}15qr - 21qr^2$
o) $^{-}24st - 36tuv$

5 Use substitution to check that the following expressions are equivalent:

a) $3d + 12 = 3(d + 4)$
b) $7(h - 4) = 7h + 28$
c) $^{-}24b - 48 = {}^{-}12(2b + 4)$
d) $8(e - 2) = 8e - 16$
e) $5(3n - 8) = 15n - 40$
f) $^{-}2c^2 - 6c = {}^{-}2c(c + 3)$
g) $15s(s^2 - 6) = 15y^3 - 90$
h) $32y^2 + 12y = 4y(8y + 2)$

6 If b = 6, what is the value of $\frac{3x}{2x-2}$?

7 What is the value of $x^2 + 3x - 7$ when $x = {}^{-}2$?

8 $B = 6d^4$ What is the value of A when c = 2?

Why didn't the Romans find algebra challenging?
Because X was always equal to 10.

9 Simplify each of the following expressions:

a) $3(x + 5) - 7$
b) $2(y - 5) + 4y + 16$
c) $5(p + 6) - 30$
d) $3(2d - 1) + 2(d + 3)$
e) $6v - 4 + 2(2v - 8)$
f) $5f + 9 + 3(2f - 2)$

Probability

4

- ★ Understand that probabilities range between 0 to 1.
- ★ Identify complementary events.
- ★ Identify the complement of familiar events.
- ★ Use the sum of probabilities to solve problems.

A TASK

Given the following on course odds, it is possible to place a bet on each entrant and show a profit no matter which entrant wins.

And how often does this happen?

Askance	**3 : 1**
Belittle	**3 : 1**
Candid	**8 : 1**
Devolve	**10 : 1**

If $10 is placed on Belittle and Belittle wins, then $30 is returned plus the $10 bet, making a total of $40.

a) Given a purse of $100, how much should be placed on each entrant to return a profit no matter who wins?

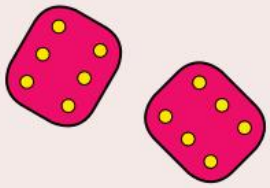
b) How can you identify situations where it is possible to bet on each entrant and make a profit?

A LITTLE BIT OF HISTORY

In 1654, a french nobleman thought that gambling even money on at least one double six appearing in 24 throws of two dice might be good value.

The nobleman asked two famous French mathematicians, Blaise Pascal and Pierre de Fermat, to decide if betting even money on double six was as good as it appeared. In solving the problem, Pascal and Fermat began the development of the mathematical theory of probability.

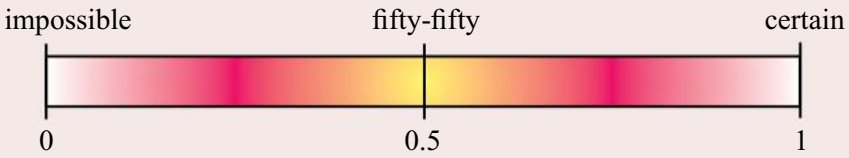
It isn't good value. Why not?



Probability

Probability is the chance of something happening.

The chance can be expressed as a decimal, fraction, pecentage, ratio, or as words.



Exercise 4.1

1 Copy the above probability scale and add each of the following to the scale.

a) The day after Thursday will be Friday.
b) You will watch TV tonight.
c) You will eat bread today.
d) It will rain tomorrow.
e) You will have your driver's licence before you are 20.
f) Throw a coin and it will land tails up.
g) You will send a text message today.
h) A snowball will keep cool in hell.
i) The new baby will be a girl.

Next time you get a chance, find out about Buckley's chance.

2 Brainstorm each of the following:

a) We have Buckley's chance of winning the match.
b) The probability of tossing a coin and getting a head is fifty-fifty, or 0.5 or ½.
If a coin is tossed 100 times, will the number of heads be 50?
c) Almost every gambling game involving money is unfair.
d) The probability of rolling a die and getting a 4 is one-sixth, or 0.17, or $^1/_6$.
If a die is rolled 60 times, will the number of 4s be 10?
e) The probability of winning first prize in a lottery is $\frac{1}{1000000}$ or 0.000001.
This is so close to 0 that we have Buckley's chance of winning the lottery.
f) The probability of winning first prize in a lottery is $\frac{1}{1000000}$ or 0.000001.
How many tickets would we have to buy to have a fifty-fifty chance of winning the lottery?

3 Investigate the experimental probability of tossing a coin.
Toss a coin 50 times. Copy and complete the following summary:

No of Heads =		No of Tails =	
Probability of heads	$= \frac{\quad}{50}$	Probability of tails	$= \frac{\quad}{50}$
	= 0.		= 0.

4 When tossing a coin the theoretical probabilities are:
Probability of Head = ½ =0.5 Probability of Tail = ½ = 0.5

a) If a coin is tossed 40 times, how many heads would be expected?

b) If a coin is tossed 40 times, how many tails would be expected?

c) If a coin is tossed 400 times, how many heads would be expected?

5 Investigate the experimental probability of tossing a die.
Toss a die 60 times. Copy and complete the following summary:

Number of 1s	=	Probability of 1 = $\frac{\quad}{60}$ = 0.
Number of 2s	=	Probability of 2 = $\frac{\quad}{60}$ = 0.
Number of 3s	=	Probability of 3 = $\frac{\quad}{60}$ = 0.
Number of 4s	=	Probability of 4 = $\frac{\quad}{60}$ = 0.
Number of 5s	=	Probability of 5= $\frac{\quad}{60}$ 0.
Number of 6s	=	Probability of 6= $\frac{\quad}{60}$ = 0.

6 When tossing a die the theoretical probabilities are:

Probability of 1= $\frac{1}{6}$ =0.167 Probability of 2= $\frac{1}{6}$ =0.167

Probability of 3= $\frac{1}{6}$ =0.167 Probability of 4= $\frac{1}{6}$ =0.167

Probability of 5= $\frac{1}{6}$ =0.167 Probability of 6= $\frac{1}{6}$ =0.167

a) If a die is tossed 30 times, how many 1s would be expected?

b) If a die is tossed 30 times, how many 2s would be expected?

c) If a die is tossed 30 times, how many 6s would be expected?

d) If a die is tossed 300 times, how many 6s would be expected?

Theoretical Probability

Theoretical probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

The sample space is a list of all possible outcomes.

A standard 6-sided die is thrown. What is the probability of each of the following happening:

a) 3? **b)** odd? **c)** >4?

Sample space = {1,2,3,4,5,6}

a) P(3) = $\frac{1}{6}$ {there is one 3}

b) P(odd) = $\frac{3}{6} = \frac{1}{2}$ {1,3,5 are odd}

c) P(<4) = $\frac{3}{6} = \frac{1}{2}$ {1,2,3 are <4}

Eight balls are numbered 1 to 8. What is the probability of selecting a ball that shows:

a) 2? **b)** even? **c)** >6?

Sample space = {1,2,3,4,5,6,7,8}

a) P(2) = $\frac{1}{6}$ {there is one 2}

b) P(even) = $\frac{4}{8} = \frac{1}{2}$ {2,4,6,8 are even}

c) P(>6) = $\frac{2}{8} = \frac{1}{4}$ {7,8 are >6}

Exercise 4.2

1 A six-sided die is thrown. What is the probability of each of the following:

a) 3? **b)** even? **c)** odd?
d) 5? **e)** <3? **f)** >4?
g) >6? **h)** a number divisible by 3?

2 An eight-sided die is thrown. What is the probability of each of the following:

a) 5? **b)** even? **c)** odd?
d) 7? **e)** <5? **f)** >7?
g) >5? **h)** a number divisible by 3?

3 A bag contains five blue marbles and three red marbles. What is the probability of taking a marble from the bag that is:

a) red? **b)** blue? **c)** not red?
d) not blue? **e)** blue or red? **f)** white?

4 A bag contains six white marbles and four yellow marbles. What is the probability of taking a marble from the bag that is:

a) white? **b)** yellow? **c)** not white?
d) not yellow? **e)** white or yellow? **f)** red?

Statisticians put mathematics to scientific use to design, collect, and interpret data experiments in many fields such as economics, medicine, psychology, marketing, public health, biology, sports, and others.

- Relevant school subjects are mathematics.
- Courses range from Bachelor degrees to Masters degrees and PhD.

The Complement

If the probability of an event is P(A), then the probability that the event would **not** occur is 1 – P(A).

Complementary:
Two parts that make a whole, one being the negation of the other.

Write the complement of each of the following: **a)** The ball is red. **b)** An even will show. **c)** William will fall ill. **a)** The ball is not red **b)** An odd will show. **c)** William will not fall ill	Ten balls are numbered 1 to 10. What is the probability of selecting a ball that shows: **a)** a multiple of 3? **b)** not a multiple of 3? **a)** $= \frac{3}{10}$ **b)** $= 1 - \frac{3}{10} = \frac{7}{10}$

Exercise 4.3

1 Write the complement of each of the following:

a) The ball is white.
b) An odd number will show.
c) Mia will win the prize.
d) The coin will show a tail.
e) It will rain today.

Complement:

P(notA) = 1 − P(A)

2 Six balls are numbered 1 to 6. What is the probability of selecting a ball that shows:

a) 1? **b)** not 1? **c)** odd?
d) even? **e)** <3? **f)** not <3?

3 Sixteen balls are numbered 1 to 16. What is the probability of selecting a ball that shows:

a) 10? **b)** not 10? **c)** odd?
d) even? **e)** >14? **f)** not >14?
g) a multiple of 3? **h)** not a multiple of 3?
i) a number divisible by 4?

4 The tickets in a staff raffle are numbered 1 to 50. What is the probability that the winning ticket is:

a) 13? **b)** not 13? **c)** odd?
d) even? **e)** <11? **f)** not < 11?
g) a multiple of 13? **h)** not a multiple of 13?

5 There are 1 000 tickets in a raffle raising money for the Junior Soccer club. If you bought five tickets, what are your chances of:

a) winning the raffle?
b) not winning the raffle?

6 If a letter is chosen at random from the word EXPERIMENTAL, what is the probability that the letter is (vowels are a,e,i,o,u):

a) a vowel? **b)** the letter A? **c)** a consonant?
d) the letter E? **e)** the letter R? **f)** the letter A or E?
g) the letter O? **h)** not the letter M? **i)** not the letter E?

7 If a letter is chosen at random from the word THEORETICAL, what is the probability that the letter is:

a) a vowel? **b)** the letter A? **c)** a consonant?
d) the letter E? **e)** the letter T? **f)** the letter T or E?
g) the letter M? **h)** not the letter T? **i)** not the letter E?

8 A bag contains ten red marbles, six blue marbles, four green, and four orange marbles. What is the probability that the first marble taken from the bag is:

a) red? **b)** blue? **c)** green?
d) orange? **e)** not red? **f)** not green?
g) not orange? **h)** blue or green? **i)** yellow?
j) not blue? **k)** not green? **l)** not yellow?

9 Using the occupation table and the average weekly earnings graph below, what is the probability that a person chosen at random will:

Managers	1 500 000
Professionals	2 300 000
Trades	1 600 000
Personal service	1 000 000
Clerical	1 600 000
Sales	1 000 000
Machinery	700 000
Labourers	1 200 000
TOTAL	**10 900 000**

a) be a manager?
b) be a professional?
c) be in sales?
d) earn more than $2000 per week on average?
e) earn less than $1000 per week on average?

A Year 8 student is flipping a coin while answering a true-false test. At the end of the test, the student is again flipping the coin.

"What are you doing?" asks the teacher. "Checking my answers." replied the student.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 4.4

1 Spell probability.
2 What is the value of $3x - 2$, if $x = 3$?
3 Simplify: $m^3 \times m^2$
4 $^{-}4 + 2$
5 Simplify: $5b - 2b$.
6 A 6-sided die is tossed, what is P(>4)?
7 Distribute: $5(2a + 3)$.
8 Factorise: $5x + 15$.
9 P(winning) = 0.05, what is P(not winning)?
10 $^{-}3 \times {}^{-}2$

The 50-50-90 rule: Anytime you have a 50-50 chance of getting something right, there's a 90% chance you'll get it wrong.

Exercise 4.5

1 Spell complement.
2 $^{-}2 - 3$
3 Simplify: $n^2 \times n^5$
4 Simplify: $2a + 5a$.
5 A coin is tossed 30 times, how many tails expected?
6 An 8-sided die is tossed, what is P(divisible by 3)?
7 Distribute: $3(2x + 4)$.
8 Factorise: $4c + 12$.
9 P(winning) = 0.15, what is P(not winning)?
10 $^{-}3 \times 4$

The latest survey shows that 3 out of 4 people make up 75% of the World's population.

Exercise 4.6

1 Spell theoretical.
2 $^{-}3 + 5$
3 Simplify: $x^3 \times x^5$
4 Simplify: $2b + 3b$.
5 A coin is tossed 40 times, how many heads expected?
6 A 10-sided die is tossed, what is P(divisible by 5)?
7 Distribute: $5(2t - 3)$.
8 Factorise: $3h + 9$.
9 P(losing) = 0.75, what is P(not losing)?
10 $^{-}6 + 3$

NAPLAN Questions

Exercise 4.7

1 What is the probability of rolling a 4 on a six-sided die?

2 A jar of jelly beans contains jelly beans with the following colours: 30 black, 25 white, 25 red and 20 green.
What is the chance of picking a white jelly bean?

3 A bowl contains 25 red apples and 15 green apples. What is the chance of choosing a green apple?

4 A bag contains 50 marbles of which 20 are red, the others are yellow. What is the chance of picking a red marble from the bag?

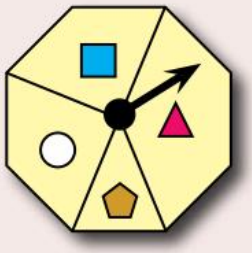
5 A traffic light is amber and red for 30 seconds. The traffic light is green for 25 seconds. What is the probability that the traffic light is green?

6 There are 30 Australian stamps and 20 New Zealand stamps. What is the probability of choosing an Australian stamp?

7 The following numbers are to be put into the stem-and-leaf plot below
45, 55, 57, 58, 59, 62, 67, 69, 70, 73, 73, 75. Which score is missing?

4	5
5	7 8 9
6	2 7 9
7	0 3 3 5

8 Sienna spins the arrow 100 times. Which is the most likely table of results?



a)

Shape	Number
■	25
▲	30
⬟	25
●	20

b)

Shape	Number
■	25
▲	35
⬟	15
●	25

9 A coin is tossed twice. There are 4 possible outcomes. What is the probability of getting 1 head and 1 tail in any order?

10 A coin is tossed 3 times. There are 8 possible outcomes. What is the probability of getting 2 heads and 1 tail in any order?

11 Two dice are thrown. There are 36 possible outcomes. What is the probability of getting two sixes?

Competition Questions

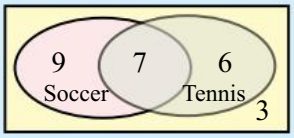
Exercise 4.8

In a class of 25 students, 16 students play soccer, 13 students play tennis, and 7 students play both soccer and tennis. Find the probability that

a) a student plays tennis only.

b) a student plays neither tennis nor soccer.

Step 1: Draw a Venn diagram



Step 2: Calculate the probabilities

P(tennis only) = $\frac{6}{25}$

P(neither tennis nor soccer) = $\frac{3}{25}$

1 In a class of 27 students, 11 students play an instrument, 14 students have a job, and 8 students play an instrument and have a job. Find the probability that:

a) a student has a job but doesn't play an instrument.

b) a student doesn't have a job and doesn't play an instrument.

2 In a sports club of 150 members, 88 play golf, 63 play bowls, and 45 play golf and bowls. Find the probability that:

a) a member plays golf only.

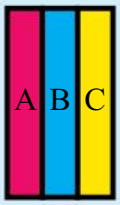
b) a member doesn't play golf or bowls.

Three books, A, B, and C, are randomly placed on a shelf. What is the probability that A and B will be next to each other?

List all possible arrangements of the three books:

ABC ACB BAC BCA CAB CBA

P(AB next to each other)= $\frac{4}{6} = \frac{2}{3}$



3 Three books, A, B, and C are randomly placed on a shelf. What is the probability that B and C will be next to each other?

4 Three digits, 1, 2, and 3, are randomly arranged to form a three-digit number. What is the probability that number is greater than 200?

5 Three digits, 1, 2, and 7 are randomly arranged to form a three-digit number. Find the probability that:

a) the number is even.

b) the number is odd.

c) the first and the last digit is odd.

d) the number is greater than 100.

6 Two girls and one boy are randomly arranged in a row. Find the probability that:

a) the two girls are together.

b) the boy is on one end.

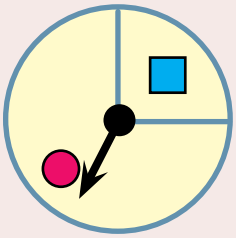
c) a girl is on one end.

Investigations

Investigation 4.1
A weekend street festival, including a parade with marching bands, is being planned.

Such street festivals are devastated by wet weather. Undertake some research to recommend a weekend in the year that has the least chance of rain.

Investigation 4.2
Make a spinner similar to the one shown.
The ■ covers 90° and the ● covers 270°.



a) What is the probability of a ■?

b) What is the probability of a ●?

c) Spin the spinner 80 times and record the results.
Does the experimental probability agree with the theoretical probability?

d) Can you simulate the spinner on a spreadsheet (See Technology 4.3)?

Investigation 4.3
Play the following game with a die:
You score 2 points if you throw a number greater than 4.
Your partner scores 1 point if they throw an even number.
The person with the highest score, after 30 throws, wins.

Comment on the fairness of the game.
Use theoretical probability to support your point of view.

Investigation 4.4
A staff social club raises money by conducting raffles. A typical raffle sells 50 tickets at $1 each with a $25 voucher as the one and only prize. A staff member buys 5 tickets in the raffle because the member believes that this gives better value. Simulate the raffle and comment on the staff member's strategy.

Investigation 4.5 The value of pi?

a) Accurately measure the length of a match. Rule long parallel lines on a large sheet of paper so that the distance between the lines is exactly double the length of a match.

b) From a height of about a metre, drop all the matches in a box onto the sheet of paper.

c) $\pi = \dfrac{Total\ number\ of\ matches}{Number\ of\ matches\ touching\ a\ line}$

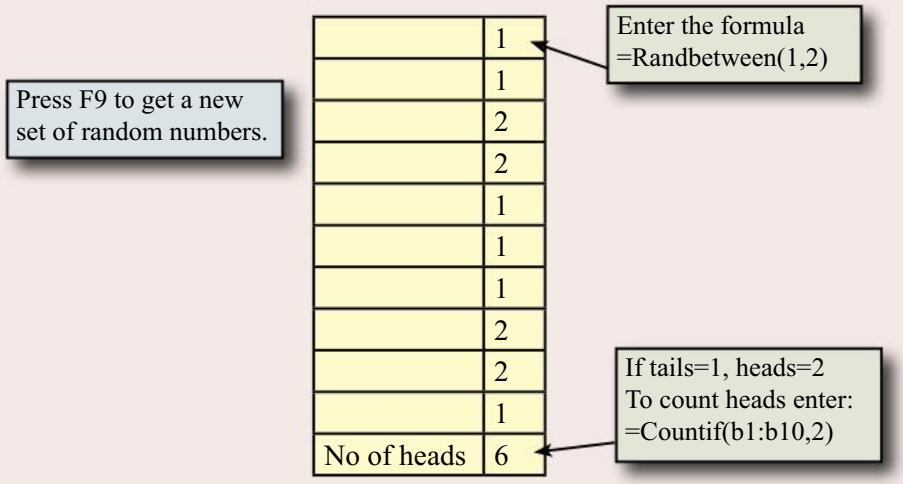
Technology

Technology 4.1
Use the Rand (random) on a calculator to simulate the throwing of a die.

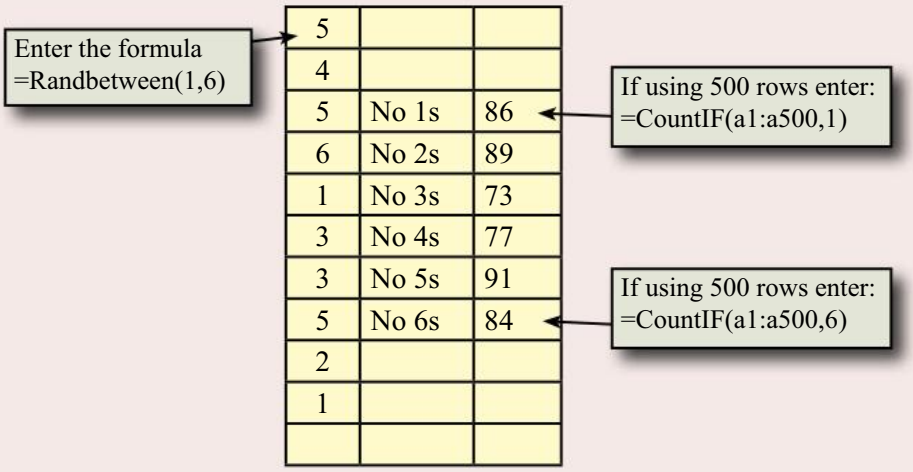
A throw = [2ndF] [Rand] [x] [6] [+] [0.5] [=] {Round the answer}

The random function gives a number between 0 and 1.
This is scaled to give a number from 1 to 6.

Technology 4.2
Use a spreadsheet to simulate the tossing of a coin. This is a great way to estimate the experimental probability of a head or tail. Use the Help to find out about the Round function and the Rand function.



Technology 4.3
Use a spreadsheet to simulate the throwing of a die. This is a great way to estimate the experimental probability of the numbers 1, 2, 3, 4, 5, 6.

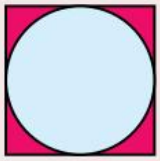


A Couple of Puzzles

Exercise 4.9

1 When a number is doubled and the result is added to five, the answer is seventeen. What is the number?

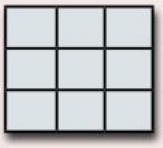
2 The circle fits exactly inside the square. If the radius of the circle is 12 cm, what is the perimeter of the square?



3 Heron took three hours and twenty minutes to make the trip to meet Kara. If Heron left at 7:35 am, when did he meet Kara?

4 In the previous year, 45 of the 60 lambs were male. If 100 lambs are born this year, how many are expected to be male?

5 Place each of the numbers: –4, –3, –2, –1, 0, 1, 2, 3, 4 in the square so that the sum of each row, column, and diagonal is 0.



A Game

Two-up is considered a traditional true-blue Australian gambling game. The game is illegal unless played on ANZAC day or in a casino. The game is played with just two coins and a ring of smooth area of ground.

Kip.	A piece of wood on which the coins are placed for spinning.
Ringer.	The person in charge of the game.
Spinner.	Player spinning the coins.

The Ringer yells 'Come in Spinner' and a Spinner enters the ring. Bets are made between a pair of people. One person may bet \$5 on heads and the other person will bet \$5 on tails. The winner takes the \$10.

The Spinner uses the kip to throw the two pennies into the air. If two heads show then the Ringer yells 'heads'. If two tails show then the Ringer yells 'tails'. If a head and a tail show then the Ringer yells 'odds' and the pennies are thrown until either heads or tails is yelled. The Spinner remains in the ring while heads are being thrown and is replaced when tails is thrown.

Play the game with centicubes?

Is the game fair?

Chapter Review 1

Exercise 4.10

1 Write a definition of probability.

2 Determine the probability of each of the following events and mark the result on the above probability scale.

a) Roll a 2 with a six-sided die.

b) I will watch TV today.

c) Win first prize in a lottery with one ticket (Assume 1 000 000 tickets in the lottery).

3 When tossing a coin the theoretical probabilities are:
Probability of Head = ½ =0.5 Probability of Tail = ½ = 0.5

a) If a coin is tossed 50 times, how many heads would be expected?

b) If a coin is tossed 50 times, how many tails would be expected?

c) If a coin is tossed 500 times, how many heads would be expected?

4 A six-sided die is thrown. What is the probability of each of the following:

a) 5? **b)** even? **c)** odd?

d) 2? **e)** <2? **f)** >2?

g) >6? **h)** a number divisible by 2?

5 Write the complement of each of the following:

a) The ball is red.

b) An odd number will show.

c) Riley will win the lottery.

d) The coin will show a tail.

e) It will be fine today.

6 There are 1 000 tickets in a raffle raising money for the Junior Cricket club. If you bought five tickets, what are your chances of:

a) winning the raffle?

b) not winning the raffle?

7 A bag contains six red marbles, ten blue marbles, and four green marbles. What is the probability that the first marble taken from the bag is:

a) red? **b)** blue? **c)** green?

d) orange? **e)** not red? **f)** not green?

g) not blue? **h)** blue or green?

8 The letters of the word DUD are randomly arranged in a row.
Find the probability that:

a) the two Ds are together.

b) the letter D is either at the beginning or at the end.

c) U occupies the first place.

Chapter Review 2

Exercise 4.11

1 Write a definition of probability.

2 Determine the probability of each of the following events and mark the result on the above probability scale.

a) The letter A will be randomly selected from the letters of the word PROBABILITY

b) I will send a text message today.

c) I will spend more than $10 today.

d) I will lose a biro today.

3 When tossing a coin the theoretical probabilities are:
Probability of Head = ½ =0.5 Probability of Tail = ½ = 0.5

a) If a coin is tossed 60 times, how many heads would be expected?

b) If a coin is tossed 60 times, how many tails would be expected?

c) If a coin is tossed 600 times, how many heads would be expected?

4 A six-sided die is thrown. What is the probability of each of the following:

a) 6? **b)** even? **c)** odd?

d) 4? **e)** <4? **f)** >4?

g) <1? **h)** a number divisible by 3?

5 Write the complement of each of the following:

a) The ball is blue.

b) An even number will show.

c) Ella will win the lottery.

d) The coin will show a head.

e) It will rain today.

6 There are 1 000 tickets in a raffle raising money for the Junior netball club. If you bought ten tickets, what are your chances of:

a) winning the raffle?

b) not winning the raffle?

7 A bag contains five red marbles, five blue marbles, and ten green marbles. What is the probability that the first marble taken from the bag is:

a) red? **b)** blue? **c)** green:

d) orange? **e)** not red? **f)** not green:

g) not blue? **h)** blue or green?

8 Four books, A, B, and C are randomly placed on a shelf.
Find the probability that

a) A and B are together.

b) A and B are at each end.

Review 1

Chapter 11 Index Laws

A convenient way of writing $2\times2\times2$ is 2^3 (3 ← Index, 2 ← Base)

Index Law 1
$a^m \times a^n = a^{m+n}$

Index Law 2
$a^m \div a^n = a^{m-n}$

Index Law 3
$(a^m)^n = a^{m\times n}$

Zero Index
$a^0 = 1$

Chapter 2 Integers

Integers are:

	Positive whole numbers:	1, 2, 3, 4, 5, 6, ...
and	Zero:	0
and	Negative whole numbers:	$^-1$, $^-2$, $^-3$, $^-4$, $^-5$, $^-6$, ...

Integers can be placed on a number line.

Integers on the right are larger than integers on the left.

Chapter 3 Algebra

★ $2a + 3a = 5a$, $4b - 6b = {}^-2b$, $3c \times {}^-2c^2 = {}^-6c^3$, $18ad \div 3a = 6d$

★ Distributive law: $a(b+c) = ab + ac$

★ Factorising: $ab + ac = a(b + c)$

Chapter 4 Probability

★ Probability is the chance of something happening. The chance can be expressed as a decimal, fraction, pecentage, ratio, or as words.

★ Theoretical probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

★ The complement of A is **not** A. P(not A) = 1 – P(A)

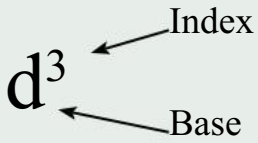
Review 1

Exercise 5.1 Mental computation

1 Spell factorise.
2 What is the value of $4x - 3$, if $x = 2$?
3 $^{-}7 + 2$
4 $^{-}4 \div {}^{-}2$
5 Simplify: $7c - 3c$.
6 A 6-sided die is tossed, what is P(>4)?
7 Distribute: $3(2b + 4)$.
8 Factorise: $5a + 20$.
9 P(winning) = 0.25, what is P(not winning)?
10 Simplify: $x^3 \times x^2$

Exercise 5.2

1 Write each of the following in index form:



$3\times3\times3\times3 = 3^4$	$d\times d\times d = d^3$

a) $2\times2\times2$ **b)** $3\times3\times3\times3\times3$ **c)** $d\times d\times d\times d\times d\times d$
d) $10\times10\times10\times10\times10$ **e)** $z\times z\times z\times z\times z\times z$ **f)** $1\times1\times1\times1\times1\times1\times1$

2 Simplify and write the following in index form:

$2^2\times2^4 = 2^{2+4} = 2^6$	$y^3\times y^3 = y^{2+3} = y^5$

Index Law 1
$a^m \times a^n = a^{m+n}$

a) $3^2\times3^4$ **b)** $1^2\times1^2$ **c)** $10^3\times10^5$ **d)** $t^2\times t^3$
e) $m^5\times m^3$ **f)** $3^3\times3^1$ **g)** $p^6\times p^3$ **h)** $5^4\times5^5$
i) $10^1\times10^3$ **j)** 10×10^6 **k)** $1^4\times1^3$ **l)** $j^2\times j^5$

3 Simplify and write the following in index form:

$3^4\div3^2 = 3^{4-2} = 3^2$	$m^6\div m^3 = m^{6-3} = m^3$

Index Law 2
$a^m \div a^n = a^{m-n}$

a) $3^4\div3^3$ **b)** $e^4\div e^2$ **c)** $2^7\div2^2$ **d)** $1^5\div1^3$
e) $g^5\div g^2$ **f)** $m^6\div m^3$ **g)** $10^9\div10^3$ **h)** $q^5\div q^3$
i) $\frac{h^3}{h^2}$ **j)** $\frac{4^6}{4^2}$ **k)** $\frac{10^6}{10^3}$ **l)** $\frac{v^8}{v^5}$

4 Simplify and write the following in index form:

$(3^2)^3 = 3^{2\times3} = 3^6$	$(b^4)^2 = b^{4\times2} = b^8$

Index Law 3
$(a^m)^n = a^{m\times n}$

a) $(4^2)^3$ **b)** $(3^4)^3$ **c)** $(7^3)^2$ **d)** $(b^2)^3$
e) $(5^2)^3$ **f)** $(8^3)^2$ **g)** $(2^2)^3\times2^4$ **h)** $3^5\times(3^2)^3$

5 Simplify each of the following:
a) 2^0 **b)** y^0 **c)** 9^0 **d)** $3s^0$

6 Change the following words to an integer:

a) A loss of $250 million. **b)** A rise of 15 m.

c) $500 under value. **d)** No change.

7 Copy each pair of numbers and place the correct < or > between them.

a) 5 ⁻1 **b)** ⁻3 3 **c)** ⁻4 ⁻3

8 Arrange the integers in ascending order (smallest to largest):

a) 2, ⁻4, ⁻2, 3 **b)** 4, ⁻2, ⁻4, 0 **c)** ⁻3, 1, ⁻2, ⁻1

9 Calculate the following:

a) $2 - 3$	**b)** $^{-}7 + 2$	**c)** $^{-}1 - 3$
d) $4 - {}^{-}5$	**e)** $^{-}2 - {}^{-}5$	**f)** 7×3
g) $2 \times {}^{-}6$	**h)** $^{-}8 \times 1$	**i)** $^{-}4 \times {}^{-}1$
j) $12 \div 6$	**k)** $14 \div {}^{-}2$	**l)** $^{-}10 \div 2$
m) $^{-}6 \div {}^{-}2$	**n)** $4 + 2 \times {}^{-}3$	**o)** $6 \div {}^{-}3 - 5$
p) $(^{-}2 + 3) \times 4$	**q)** $^{-}14 \div (9 - 2)$	**r)** $15 \div (^{-}4 + 1)$

$-{}^{-}3 = 3$ $\quad -{}^{-}1 = 1$

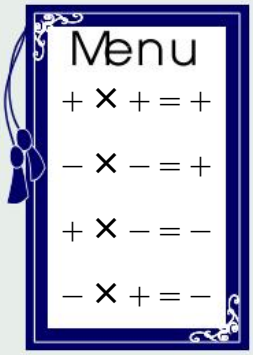
$-{}^{-}8 = 8$ $\quad -{}^{-}5 = 5$

Order of Operations:

1 ( ) brackets first.

2 × and ÷ from left to right.

3 + and − from left to right.



10 Andre owes each of four friends $70.
What is Andre's balance?

11 The temperature at midnight was −4°C. If the temperature dropped a further 3°C, what is then the temperature?

12 Ming borrowed $350, repaid $220, and borrowed another $60.
What is Ming's balance?

13 A submarine at 120 m below sea level rose 50 m. What is the position of the submarine relative to sea level?

14 Mercury has a melting point of −40°C and a boiling point of 357°C. What is Mercury's temperature range from melting point to boiling point?

15 Write an algebraic expression for each of the following:

a) the sum of a and b. **b)** the product of x and y.

c) c is increased by 5. **d)** h is decreased by 2x.

e) the product of x and triple y.

16 Simplify the following expressions:

a) $2a + 7a$	**b)** $5b - 3b$	**c)** $6c \div 3$
d) $3 \times 3d$	**e)** $15f \div 10$	**f)** $3h - 2h - 5h$
g) $4e \times 3g$	**h)** $^{-}3i + 5i + 5i$	**i)** $3j \times {}^{-}3j$
j) $12mn \div 8n$	**k)** $4g^3 \times 2g$	**l)** $4a^2 - 7 + 6a^2 - 1$
m) $3xy \times {}^{-}3x^2y$	**n)** $12v^2 \div 3v^2$	**o)** $12p^3 \div 10p^2$

17 Expand each of the following expressions:

a) $3(b + 5)$ **b)** $3(2a - 6)$ **c)** $5(3d - 1)$

d) $7(4x - 1)$ **e)** $^{-}2(4y - 7)$ **f)** $^{-}6(3z - 2)$

18 Factorise each of the following expressions:

a) $3d + 6$ **b)** $5e - 15$ **c)** $6f + 9$

d) $4d + 18de$ **e)** $^{-}2b + 10ab$ **f)** $^{-}3xy - 21$

g) $^{-}6t + 14$ **h)** $^{-}15g - 21g^2$ **i)** $^{-}24mn - 28mno$

19 Use substitution to check that the following expressions are equivalent:

a) $3x + 9 = 3(x + 3)$ **b)** $^{-}7(s + 3) = {}^{-}7s - 21$

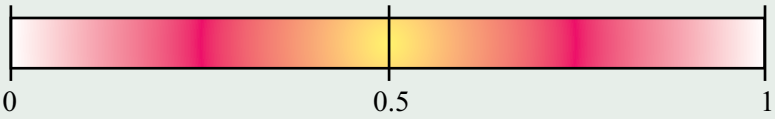
c) $5(c - 3) = 5c + 15$ **d)** $45v^2 + 15v = 15v(3v + 1)$

20 Simplify each of the following expressions:

a) $2(a + 3) - 5$ **b)** $4(b - 2) + 3b + 1$

c) $5(2k + 2) - 10$ **d)** $3(6n - 1) + 5(n + 2)$

21 Write a definition of probability.



22 Determine the probability of each of the following events and mark the result on a copy of the above probability scale:

a) The letter A will be randomly selected from the letters of the word ALGEBRA.

b) I will watch more than 2 hours of TV today.

23 When tossing a coin the theoretical probabilities are:
Probability of Head = ½ =0.5 Probability of Tail = ½ = 0.5

a) If a coin is tossed 50 times, how many heads would be expected?

b) If a coin is tossed 50 times, how many tails would be expected?

c) If a coin is tossed 500 times, how many heads would be expected?

24 A six-sided die is thrown. What is the probability of each of the following?

a) 2? **b)** even? **c)** odd?

d) 3? **e)** <3? **f)** a number divisible by 2?

25 Write the complement of each of the following:

a) The number is 3. **b)** An odd number will show.

26 There are 500 tickets in a raffle raising money for the Junior basketball club. If you bought ten tickets, what are your chances of:

a) winning the raffle? **b)** not winning the raffle?

27 A bag contains ten red marbles, ten blue marbles, and five green marbles. What is the probability that the first marble taken from the bag is:

a) red? **b)** not red? **c)** not green?

28 Three books, A, B, and C are randomly placed on a shelf.
Find the probability that:

a) A and B are together. **b)** A and B are at each end.

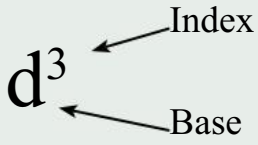
Review 2

Exercise 5.3 Mental computation

1 Spell probability.
2 What is the value of $3x - 5$, if $x = 3$?
3 $7 - {}^{-}3$
4 $7 \times {}^{-}3$
5 Simplify: $8m - 2m$.
6 A 6-sided die is tossed, what is P(<4)?
7 Distribute: $3(3d + 2)$.
8 Factorise: $6h + 10$.
9 P(winning) = 0.35, what is P(not winning)?
10 Simplify: $x^3 \times x^4$

Exercise 5.4

1 Write each of the following in index form:



$3\times3\times3\times3 = 3^4$	$d\times d\times d = d^3$

a) $5\times5\times5\times5$ **b)** $2\times2\times2\times2\times2$ **c)** $w\times w\times w\times w\times w\times w$
d) $10\times10\times10$ **e)** $y\times y\times y\times y\times y\times y$ **f)** $1\times1\times1\times1\times1\times1$

2 Simplify and write the following in index form:

Index Law 1
$a^m \times a^n = a^{m+n}$

$2^2\times2^4 = 2^{2+4} = 2^6$	$y^3\times y^3 = y^{2+3} = y^5$

a) $4^2\times4^5$ **b)** $1^2\times1^5$ **c)** $10^2\times10^5$ **d)** $t^6\times t^3$
e) $n^6\times n^3$ **f)** $5^3\times5^1$ **g)** $h^6\times h^4$ **h)** $7^4\times7^7$
i) $10^1\times10^5$ **j)** 10×10^4 **k)** $1^2\times1^3$ **l)** $z^3\times z^5$

3 Simplify and write the following in index form:

Index Law 2
$a^m \div a^n = a^{m-n}$

$3^4\div3^2 = 3^{4-2} = 3^2$	$m^6\div m^3 = m^{6-3} = m^3$

a) $3^7\div3^3$ **b)** $d^6\div d^2$ **c)** $3^8\div3^3$ **d)** $1^7\div1^2$
e) $m^6\div m^3$ **f)** $d^7\div d^5$ **g)** $10^8\div10^2$ **h)** $s^5\div s$
i) $\frac{a^5}{a^2}$ **j)** $\frac{2^6}{2^3}$ **k)** $\frac{10^6}{10^4}$ **l)** $\frac{x^7}{x^4}$

4 Simplify and write the following in index form:

Index Law 3
$(a^m)^n = a^{m\times n}$

$(3^2)^3 = 3^{2x3} = 3^6$	$(b^4)^2 = b^{4x2} = b^8$

a) $(2^3)^3$ **b)** $(3^4)^2$ **c)** $(4^3)^2$ **d)** $(x^2)^4$
e) $(y^3)^3$ **f)** $(5^3)^4$ **g)** $(2^3)^2\times2^5$ **h)** $3^4\times(3^2)^4$

5 Simplify each of the following:

a) 5^0 **b)** 1^0 **c)** 3^0 **d)** $2y^0$

6 Change the following words to an integer:

a) A drop of 5°C. **b)** A increase of 25%.

c) A profit of $150. **d)** Shortened by 5 cm.

7 Copy each pair of numbers and place the correct < or > between them.

a) 2 $^{-}2$ **b)** $^{-}1$ 2 **c)** $^{-}2$ $^{-}3$

8 Arrange the integers in ascending order (smallest to largest):

a) $^{-}1, ^{-}3, ^{-}2, 3$ **b)** $^{-}1, 1, ^{-}2, 0$ **c)** $2, ^{-}3, ^{-}4, ^{-}1$

9 Calculate the following:

a) $5 - 3$	**b)** $^{-}4 + 1$	**c)** $^{-}4 - 1$
d) $2 - ^{-}3$	**e)** $^{-}1 - ^{-}6$	**f)** 6×2
g) $5 \times ^{-}2$	**h)** $^{-}2 \times 4$	**i)** $^{-}3 \times ^{-}2$
j) $10 \div 5$	**k)** $12 \div ^{-}2$	**l)** $^{-}14 \div 7$
m) $^{-}4 \div ^{-}4$	**n)** $4 + 3 \times ^{-}1$	**o)** $8 \div ^{-}4 - 2$
p) $5 \times (^{-}3 + 1)$	**q)** $^{-}10 \div (7 - 2)$	**r)** $16 \div (^{-}3 + 1)$

$-^{-}3 = 3$ $-^{-}1 = 1$

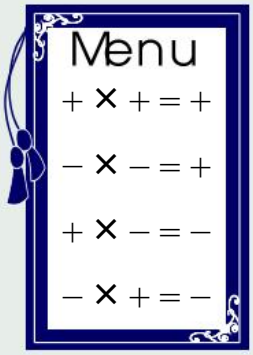
$-^{-}8 = 8$ $-^{-}5 = 5$

Order of Operations:

1 ( ) brackets first.

2 × and ÷ from left to right.

3 + and − from left to right.



10 Ethan owes each of five friends $60.
What is Ethan's balance.

11 The temperature at midnight was −4°C. If the temperature dropped a further 3°C, what is then the temperature?

12 Yin borrowed $230, repaid $90, and borrowed another $40.
What is Yin's balance?

13 A submarine at 60 m below sea level rose 20 m. What is the position of the submarine relative to sea level?

14 The Egyptian civilisation began around 3100 BC and finished around 30 BC.
How long did the Egyptian civilisation last?

16 Write an algebraic expression for each of the following:

a) the sum of c and d. **b)** the product of x and y.

c) b minus 5. **d)** r divided by a.

e) double x plus x cubed.

17 Simplify the following expressions:

a) $3y + 2y$	**b)** $6d - 3d$	**c)** $12c \div 8$
d) $6 \times 5a$	**e)** $15a \div 5$	**f)** $5u - 3u - 3u$
g) $3n \times 4p$	**h)** $^{-}9e + 2e + 5e$	**i)** $4g \times ^{-}3g$
j) $21dg \div 14d$	**k)** $4h^3 \times 2h$	**l)** $2a^2 - 7 + 6a^2 - 3$
m) $2ab \times ^{-}3a^2b$	**n)** $15d^3 \div 3d^2$	**o)** $18p^3 \div 12p^2$

18 Expand each of the following expressions:

a) $2(n + 3)$ **b)** $6(3w - 5)$ **c)** $7(2p - 1)$

d) $6(4m - 2)$ **e)** $^{-}3(8k - 9)$ **f)** $^{-}6(2t - 3)$

19 Factorise each of the following expressions

a) $6x + 12$ **b)** $4r - 6$ **c)** $10c + 8$

d) $4j + 14jk$ **e)** $^{-}4e + 12ef$ **f)** $^{-}3h - 6$

g) $^{-}3v + 18$ **h)** $^{-}15wx - 21wx^2$ **i)** $^{-}21ab - 48abc$

20 Use substitution to check that the following expressions are equivalent:

a) $2x + 6 = 2(x + 3)$ **b)** $^{-}8d - 14 = {}^{-}2(4d - 7)$

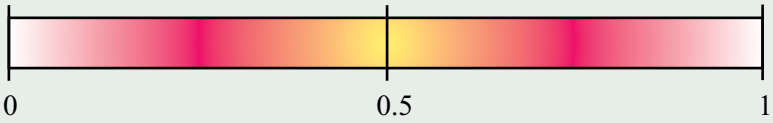
c) $5(r - 3) = 5r + 15$ **d)** $48s^2 + 6s = 6s(8s + 1)$

21 Simplify each of the following expressions:

a) $4(a + 3) - 4$ **b)** $2(z - 3) + 5z + 1$

c) $2(2b + 4) - 10$ **d)** $5(2h - 1) + 3(h + 1)$

22 Write a definition of probability.



23 Determine the probability of each of the following events and mark the result on the above probability scale.

a) The letter I will be randomly selected from the letters of the word DISTRIBUTIVE

b) I will be in bed before 9:30 tonight.

24 When tossing a coin the theoretical probabilities are:
Probability of Head = ½ =0.5 Probability of Tail = ½ = 0.5

a) If a coin is tossed 70 times, how many heads would be expected?

b) If a coin is tossed 70 times, how many tails would be expected?

c) If a coin is tossed 700 times, how many heads would be expected?

25 A six-sided die is thrown. What is the probability of each of the following?

a) 1 **b)** even **c)** odd

d) 5 **e)** <5 **f)** a number divisible by 3

26 Write the complement of each of the following:

a) I will pass the test. **b)** An even number will show.

27 There are 200 tickets in a raffle raising money for the Junior softball club. If you bought ten tickets, what are your chances of:

a) winning the raffle? **b)** not winning the raffle?

28 A bag contains five red marbles, ten blue marbles, and ten green marbles. What is the probability that the first marble taken from the bag is

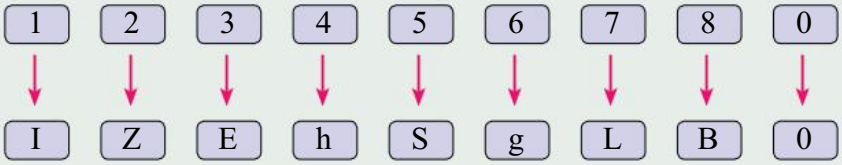
a) red **b)** not red **c)** not green

29 Three books, A, B, and C are randomly placed on a shelf.
Find the probability that

a) A and C are together. **b)** A and C are not together.

Make the Calculator Talk

When the following numbers are entered into a calculator and the calculator is turned upside down, the numbers become the letters:



Who can get:
- the most words?
- the longest word?

1 Use a brainstorming method to form words with these letters:
Examples: hello, booze, eels, globe.

2 Choose a word and work out the associated number.
Check by entering the number and turn the calculator upside.
Example: hello = 0.7734 use the decimal point to get the 0 in front).

3 Invent a calculation that gives the number as the answer.
Example: (6.5468 − 5) ÷ 2 (The answer is 0.7734 which is hello)

4 Make up a story that has the word as the answer.

Q: How do you keep a blonde in suspense?

A: Present her with a mirror and tell her to wait for the other person to say "**(6.5468 − 5) ÷ 2**"

What should you say when you meet a monster with two heads?

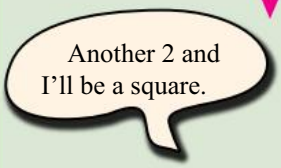
6×(0.1289÷100000+0.1289)

What occurs in two days of the week, in six months of the year, and once in a year?

(20530.587÷12.81−6.7) ÷ 532

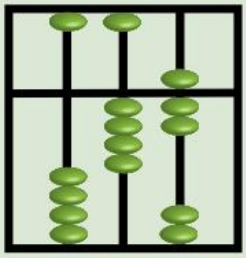
Real Numbers 6

- ★ Recognise terminating, recurring and non-terminating decimals and choose their appropriate representations.
- ★ Investigate the concept of irrational numbers, including π.
- ★ Understand that the real number system includes irrational numbers and that certain subsets of the real number system have particular properties.



A TASK

The Abacus was probably the first and most enduring of calculating devices. The Abacus can add, subtract, multiply, divide, square root, and cube root with blinding speed and accuracy.



- Experiment with an Internet 'Abacus applet'.
- Learn to do either +, −, x, or ÷ on the Abacus.
- Make an Abacus.
- Choose an audience and demonstrate your Abacus.

A LITTLE BIT OF HISTORY

Our modern society owes much to our Hindu-Arabic number system.

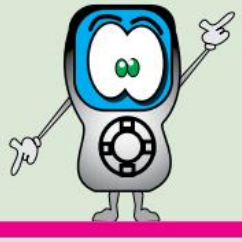
The Hindu-Arabic number system is simpler and more efficient than other number systems such as the Roman number system and the Egyptian number system because it:

- uses the digit 0.
- uses only 10 digits to represent all numbers.
- has a place value system.

The Hindu-Arabic system eventually dominated because it was better at multiplication.

The modern controversy:

Calculators and computers are a more efficient way of dealing with numbers. Why are we still using pen and paper?



Our Number System

A number system is a set of numbers used to count, compare, calculate, etc.

- Babylonian Number System
- Roman Number System
- Mayan Number System
- Egyptian Number System
- Many other Number System

Our Hindu-Arabic number system is simpler and more efficient than other number systems.

Real Numbers

Rational Numbers

Can be expressed as a ratio a/b where a and b are integers.

- **Integers.** ⁻3, 5, ⁻18, 0, 3, 9
- **Fractions.** 1/2, 3/4, 5/3, 6/7
- **Decimals.** (Either terminate or recurr.)

Irrational Numbers

Cannot be expressed as a ratio a/b (a and b are integers).

- **Surds.** $\sqrt{2} = 1.4142135...$, $\sqrt{3} = 1.7320508...$
- **Special numbers.** $\pi = 3.1415926...$, $e = 2.7182818...$

Complex Numbers

a + bi
a and b are real numbers.
i is the imaginary part.

Rational Numbers: Integers

A **ratio**nal number is a number that can be written as a ratio a/b where a and b are integers.

Integers are **ratio**nal numbers because they can be written as a ratio a/b where a and b are integers.

$$\frac{a}{b}$$

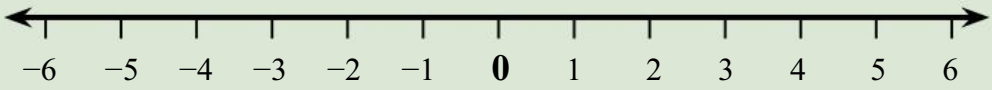
The denominator cannot be 0

Integers are:

	Positive whole numbers:	1, 2, 3, 4, 5, 6, ...
and	Zero:	0
and	Negative whole numbers:	¯1, ¯2, ¯3, ¯4, ¯5, ¯6, ...

Integers can be placed on a number line.

Integers on the right are larger than integers on the left.



Exercise 6.1

Show that each of the following integers are Rational numbers.

$$\text{¯}5 = \frac{-5}{1} \qquad 8 = \frac{8}{1}$$

Because the integer can be written as a ratio of two integers, it is a **Ratio**nal number.

1 5 **2** ¯4 **3** ¯2

4 ¯9 **5** 15 **6** 0

Exercise 6.2

Arrange the integers in ascending order (smallest to largest):

6, ¯1, 2, ¯5, ¯4

¯5, ¯4, ¯1, 2, 6

From left to right on the number line.

1 2, 0, 3, 1 **2** ¯1, 3, 0, ¯3 **3** 6, ¯2, ¯1, 2

4 5, ¯2, 1, ¯4, 0 **5** ¯8, ¯3, ¯1, ¯2 **6** 4, 6, ¯2, ¯3, 1

7 ¯1, ¯2, ¯4, 2, ¯3 **8** 2, 3, ¯1, ¯2, ¯5 **9** ¯7, 1, 3, ¯3, ¯1

Rational Numbers: Fractions

$\frac{a}{b}$ ← Numerator; Denominator

A rational number is a number that can be written as a ratio a/b where a and b are integers.

Proper fraction $a < b$ Examples: $\frac{3}{5}, \frac{15}{17}, \frac{165}{219}$

Improper fraction $a > b$ Examples: $\frac{3}{2}, \frac{9}{2}, \frac{518}{37}$

Mixed number whole number and fraction. Examples: $2\frac{3}{4}, 3\frac{2}{5}, 1\frac{19}{24}$

Exercise 6.3

1 Change each of the following mixed numbers to an improper fraction:

$$2\frac{3}{5} = 2 + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

This might be easier:

$2 \times 5 + 3 = 13$

Thus: $\frac{13}{5}$

a) $2\frac{1}{2}$ **b)** $1\frac{3}{5}$ **c)** $1\frac{2}{5}$ **d)** $2\frac{2}{3}$

e) $1\frac{1}{8}$ **f)** $3\frac{3}{4}$ **g)** $2\frac{1}{4}$ **h)** $3\frac{7}{10}$

2 Change each of the following improper fractions to a mixed number:

$\frac{13}{5}$ $13 \div 5 = 2$ remainder 3

$= 2\frac{3}{5}$

Q Who invented fractions?
A Henry the 1/8th.

a) $\frac{5}{2}$ **b)** $\frac{7}{3}$ **c)** $\frac{9}{4}$ **d)** $\frac{9}{5}$

e) $\frac{8}{5}$ **f)** $\frac{10}{3}$ **g)** $\frac{19}{7}$ **h)** $\frac{37}{11}$

Library Technicians assist librarians and other information managers to organise and operate systems for handling recorded material and files.

- Relevant school subjects are English, Mathematics, IT.
- Courses normally involve a Diploma of library and information services.

3 Change the following decimals to mixed numbers:

2.4 $= 2\frac{4}{10}$	3.92 $= 3\frac{92}{100}$	15.718 $= 15\frac{718}{1000}$

a) 5.7 **b)** 4.9 **c)** 8.58 **d)** 9.27

e) 3.333 **f)** 15.417 **g)** 4.01 **h)** 0.0004

4 Change the following mixed numbers to a decimal:

$5\frac{8}{10} = 5.8$	$2\frac{7}{100} = 2.07$	$40\frac{35}{1000} = 40.035$

a) $3\frac{4}{10}$ **b)** $4\frac{13}{100}$ **c)** $12\frac{7}{10}$ **d)** $5\frac{97}{100}$

e) $6\frac{1}{10}$ **f)** $72\frac{56}{100}$ **g)** $5\frac{3}{100}$ **h)** $7\frac{196}{1000}$

$\frac{a}{b}$

If b is not a prime number (other than 2 or 5) then a **Terminating decimal**

If b has a prime factor (other than 2 or 5) then a **Recurring decimal**

Exercise 6.4

Change the following fractions to a decimal:

$\frac{3}{5} = 0.6$ ← Terminating decimal	
$\frac{2}{9} = 0.22222222.. = 0.\dot{2}$ →	☺ **Recurring decimals** repeat the same sequence without stopping. ☺ It is sometimes useful to write a line over the repeated sequence. ☺ Some decimals take a long time to recur: ☺ $\frac{5}{7} = 0.714285714285714285714285...$
$\frac{9}{11} = 0.8181818181... = 0.\overline{81}$ →	

1 $\frac{1}{10}$ **2** $\frac{1}{3}$ **3** $\frac{3}{4}$ **4** $\frac{2}{3}$

5 $\frac{4}{5}$ **6** $\frac{2}{9}$ **7** $\frac{1}{2}$ **8** $\frac{4}{11}$

Decimals, Percentages, Fractions

Fractions, decimals and percentages are used hundreds of thousands of times every day for comparison and analysis. Fractions, decimals and percentages are different representations of the same number.

Exercise 6.5

1 Write each of the following decimals as a percentage and as a fraction:

$0.3 = 0.3 \times 100$
$= 30\%$

$= \frac{30}{100}$

$= \frac{3}{10}$

× 100
To change a decimal to a percent.

÷ 100
To change a percent to a fraction.

a) 0.1	**b)** 0.2	**c)** 0.3	**d)** 0.4
e) 0.5	**f)** 0.6	**g)** 0.7	**h)** 0.8
i) 0.9	**j)** 1	**k)** 0.25	**l)** 0.75

$0.0675 = 0.0675 \times 100$
$= 6.75\%$

$= \frac{6.75}{100}$

$= \frac{27}{400}$

Use a calculator to simplify complicated fractions.
See Technology 6.2

m) 0.05	**n)** 3	**o)** 1.1	**p)** 1.5
q) 2	**r)** 4	**s)** 1.4	**t)** 6.25
u) 0.15	**v)** 0.08	**w)** 0.0625	**x)** 3.75

2 Write each of the following percentages as a fraction and as a decimal:

$30\% = \frac{30}{100} = \frac{3}{10}$ $= 0.3$	$112\% = \frac{112}{100} = \frac{28}{25}$ $= 1.12$	$4.25\% = \frac{4.25}{100} = \frac{17}{400}$ $= 0.0425$

a) 10%	**b)** 20%	**c)** 30%	**d)** 40%
e) 50%	**f)** 60%	**g)** 70%	**h)** 80%
i) 90%	**j)** 100%	**k)** 1%	**l)** 5%
m) 7%	**n)** 12%	**o)** 120%	**p)** 150%
q) 200%	**r)** 500%	**s)** 156%	**t)** 217%
u) 6.2%	**v)** 8.7%	**w)** 12.25%	**x)** 5.75%

Rounding to two decimal places, first look at the third decimal place:

56.231694 ↑ less than 5 thus **56.23**	27.01769 ↑ 5 or more thus **27.02**	1.07276 ↑ less than 5 thus **1.07**	4.79634216 ↑ 5 or more thus **4.80**

3 Write each of the following fractions as a decimal and as a percentage (if necessary round to 2 decimal places):

$\frac{3}{5} = 0.6$

$= 60\%$

Learn these fractions and their decimals by heart - these fractions are very common.

a) $\frac{1}{10}$ **b)** $\frac{2}{10}$ **c)** $\frac{3}{10}$ **d)** $\frac{4}{10}$

e) $\frac{5}{10}$ **f)** $\frac{6}{10}$ **g)** $\frac{7}{10}$ **h)** $\frac{8}{10}$

i) $\frac{9}{10}$ **j)** $\frac{10}{10}$ **k)** $\frac{1}{2}$ **l)** $\frac{1}{3}$

m) $\frac{2}{3}$ **n)** $\frac{1}{4}$ **o)** $\frac{2}{4}$ **p)** $\frac{3}{4}$

q) $\frac{1}{5}$ **r)** $\frac{2}{5}$ **s)** $\frac{3}{5}$ **t)** $\frac{4}{5}$

u) $\frac{5}{5}$ **v)** $\frac{1}{8}$ **w)** $\frac{3}{8}$ **x)** $\frac{5}{8}$

4 Write each of the following as a fraction, as a decimal, and as a percentage:

a) One-quarter of the students were absent on the last day of school.
b) One-fifth of the tomatoes had to be thrown out.
c) The trust fund reported a 15% return on investment.
d) Water makes up approximately 75% of a person's body.
e) Peron pays a marginal tax of 37%.
f) The dodgy loan rate was set at 26%.
g) GST is a tax of 10c in the dollar.
h) The poll showed that around 7 in 10 politicians favoured a policy change.
i) There are 581 boys in the school of 1 000 students.
j) Lara scored 9 out of 10 for the test.
k) Sunny scored 21 out of 50 for the test.
l) Viroy scored 46 out of 50 for the test.
m) Wal scored 64 out of 100 for the test.
n) Gani scored 63 out of 100 for the test.
o) 99.95% of the students passed the Mathematics test.

Irrational Numbers

An Irrational number is a number that cannot be written as a ratio of two integers.

The test:
Does the decimal terminate or recurr?

Rational number - the decimal either terminates (ends), or recurrs (repeats).

Irrational number - the decimal doesn't terminate (end) or recurr (repeat).

Exercise 6.6
Are the following numbers rational or irrational?

$\frac{5}{7}$ = 0.714285714285714285714285714285714285
The numbers 714285 recurr (repeat) thus the number is a rational number.

√3 = 1.732050807568877293527446341505872
There appears to be no recurring numbers, thus √3 is an irrational number.

1 Is 0.818181818181818181818181818181818181 (= $\frac{9}{11}$) a rational number or an irrational number?

2 Is √2 a rational number or an irrational number?
√2 = 1.414213562373095048801688724209698

3 Is π (pi) a rational number or an irrational number?
π = 3.141592653589793238462643383279502
(π is the ratio of the circumference of a circle to the diameter of the circle.)

4 Is √4 a rational number or an irrational number?
√4 = 2.0

5 Is 0.615384615384615384615384615384615384 (= $\frac{8}{13}$) a rational number or an irrational number?

6 Is e (Euler's Number) a rational number or an irrational number?
e = 2.718281828459045235360287471352662

7 Is φ (The Golden Ratio) a rational number or an irrational number?
φ = 1.618033988749894848204586834365638

8 Is 0.578947368421052631578947368421052631578947 (= $\frac{11}{19}$)
a rational number or an irrational number?

Mental Computation

Exercise 6.7

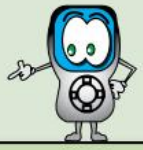
1 Spell terminating.
2 Change 0.3 to a fraction.
3 Change $\frac{3}{5}$ to a decimal.
4 What is the value of $3x - 2$, if $x = 2$?
5 Simplify: $7a - 2a$.
6 Change $2\frac{3}{4}$ to an improper fraction.
7 $5 - {}^{-}4$
8 $5 \times {}^{-}4$
9 Round 5.2641 to 2 decimal places.
10 Simplify: $x^2 \times x^4$

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 6.8

1 Spell recurring.
2 Change 30% to a fraction.
3 Change $\frac{2}{5}$ to a decimal.
4 What is the value of $4x - 1$, if $x = 2$?
5 Simplify: $6b - 4b$.
6 Change $2\frac{2}{5}$ to an improper fraction.
7 $7 - {}^{-}2$
8 ${}^{-}7 \times {}^{-}2$
9 Round 2.3782 to 2 decimal places.
10 Simplify: $x^3 \times x^2$

1 What is 3Q + 7Q?
2 10Q.
1 You're welcome.



Why are the following fractions not Australian?

$$\frac{a}{c}, \frac{b}{c}, \frac{c}{c}$$

Their numerators are all over Cs.

Exercise 6.9

1 Spell improper.
2 Change 0.4 to a fraction.
3 Change $\frac{3}{4}$ to a percentage.
4 What is the value of $2x - 3$, if $x = 4$?
5 Simplify: $8d - 6d$.
6 Change $3\frac{1}{3}$ to an improper fraction.
7 $8 - {}^{-}3$
8 ${}^{-}8 \times 3$
9 Round 4.2164 to 2 decimal places.
10 Simplify: $x^4 \times x^3$

NAPLAN Questions

Exercise 6.10

1 What is exactly halfway between $1\frac{1}{2}$ and $2\frac{1}{2}$?

2 What is exactly halfway between $1\frac{1}{2}$ and $3\frac{1}{2}$?

3 What is exactly halfway between $1\frac{3}{4}$ and $3\frac{1}{4}$?

4 How much time between 5:30 am and 2:15 pm?

5 How much time between 6:12 am and 3:05 pm?

6 The bucket of potting mix contained three-quarters of a bucket of compost. How much compost is needed to make 3 buckets of potting mix?

7 Find the value of:

a) $4.8 \div 0.4$ **b)** $4 \times (200 - 6)$ **c)** $45 - 24 \div 6$

8 Write $3\frac{2}{3}$ as a vulgar fraction.

9 Write $\frac{15}{4}$ as a mixed numeral.

10 Write $5\frac{3}{4}$ as a decimal fraction.

11 Write $\frac{3}{4}$ as a percentage.

12 Estimate: $27 \times 12 + 39 \times 23$

13 Estimate: $\sqrt{500}$

14 A bag has 15 red marbles and 20 blue marbles. What fraction of the marbles are blue?

15 The pizza is cut into 12 equal slices. How many slices are left after two-thirds of the pizza is eaten?

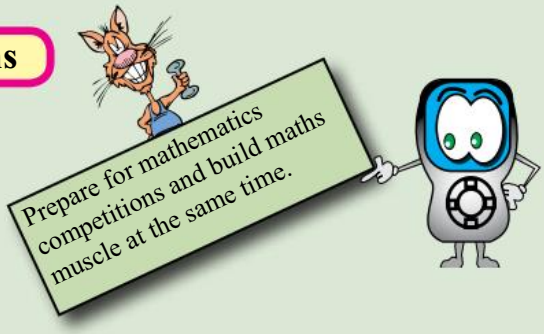
16 The painting contract returned $20 000. Labour and materials cost 50% and 30% of the remainder was paid in tax. How much was left?

17 Chloe completed 40% of the task on the first day and 40% of the remaining task on the second day. What percentage of the whole task did Chloe complete on the second day?

18 Which number has the largest value?

a) 1/5 **b)** 0.21 **c)** $(0.4)^2$ **d)** $\sqrt{0.04}$

Competition Questions



Exercise 6.11

1 What is the sum of the greatest number and the least number of:

0.67, 0.7, 0.76, 0.77, 0.6, 0.66 ?

2 Which of the following fractions is the largest?

a) $\frac{111}{222}$ **b)** $\frac{222}{333}$ **c)** $\frac{333}{444}$ **d)** $\frac{444}{555}$

3 Find the value of $\frac{0.25}{5}$

4 Find the value of ½(3.5 + 2.5)

5 What is the value of $2 + \frac{3}{100} + \frac{4}{1000}$?

6 Find the value of each of the following:

a) $\frac{1}{2}+\frac{1}{3}$ **b)** $\frac{1}{2}-\frac{1}{4}$ **c)** $\frac{2}{3}\text{x}\frac{3}{2}$

7 Find the value of x in each of the following:

a) $\frac{1}{4}=\frac{2}{x}$ **b)** $\frac{2}{x}=\frac{1}{3}$ **c)** $\frac{1}{4}+\frac{1}{x}=\frac{1}{3}$

8 How many minutes in three-quarters of an hour?

9 A bus journey takes two and one-quarter hours. If the bus leaves at 9:10 am, when would the bus be expected to arrive?

10 Two-thirds of a number is twelve. What is the number?

11 The recipe for potting mix suggested that 8 buckets of compost be mixed with 3 buckets of soil. Sienna decided to use 3/4 of the amount of compost and the same amount of soil. What fraction of the potting mix will be compost?

12 Beryl received ½ of the inheritance. Zirl received ⅓ of the inheritance. If there was $300 000 left, what was the value of the inheritance?

13 Riley is about to travel on a bus and knows that the fare is greater than $1.00 and less than $2.00. What is the minimum number of coins he must carry to be sure of being able to give the exact fare (Assume that the available coins are 5c, 10c, 20c, 50c, $1, and $2)?

Investigations

Investigation 6.1 Pen/Paper or Calculator/Computer

- The reason the Hindu-Arabic number system has dominated other number systems is that it is concise and efficient.
- The calculator/computer is more efficient than pen/paper so why continue with the pen/paper?

Investigate why the pen/paper method still has a place in our modern society.
Can your group brainstorm 5 advantages/disadvantages of each method?

Pen/Paper	Calculator/Computer
1. An efficient way of thinking 2. Less efficient in crunching numbers. 3. 4. 5.	1. Can't think. 2. More efficient in dealing with number crunching. 3. 4. 5.

Investigation 6.2 Changing recurring decimals to fractions

There are some great tricks for changing a recurring decimal into a fraction.

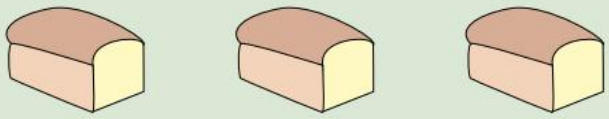
Use search phrases such as "recurring decimals" to find a neat method.

> Can you change 0.45454545.. into a fraction (the answer is $\frac{5}{11}$)?
>
> Can you change: 3.2323232323.. into a fraction?

Investigation 6.3 Practical fractions

The Egyptians used unit fractions (the numerator = 1). For example:

Divide 3 loaves of bread equally among five people.



Egyptian method: Each person gets 1/3 and 1/5 and 1/15 of a loaf of bread.

Our method: Each person gets 3/5 of a loaf of bread.

a) Use the Egyptian method to divide 3 pizzas among 5 people.
b) Use our method to divide 3 pizzas among 5 people.
c) Which method is easiest?
d) Which method is fairest?
e) What is the Egyptian method of dividing 3 pizzas among 4 people?

A Couple of Puzzles

Exercise 6.12

Find the missing figures in each of the following problems:

1 $\frac{*}{5}+\frac{1}{5}=\frac{3}{5}$ **2** $\frac{5}{7}-\frac{*}{7}=\frac{3}{7}$ **3** $\frac{4}{3}\times\frac{2}{*}=\frac{8}{15}$ **4** $\frac{1}{2}\div\frac{3}{*}=\frac{5}{6}$

5
$$\begin{array}{r} *2.*7 \\ +1*.08 \\ \hline 35.7* \end{array}$$

6
$$\begin{array}{r} *4.9* \\ -3*.*7 \\ \hline 54.59 \end{array}$$

7
$$\begin{array}{r} *.7 \\ \times 4 \\ \hline 22.8 \end{array}$$

8 $*.4\div 2 = 3.*$

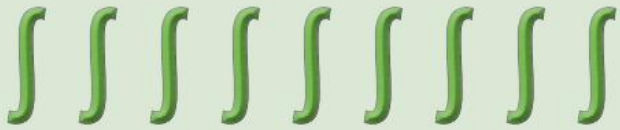
9 What is $\frac{1}{3}$ of $\frac{1}{3}$ of $\frac{1}{3}$ of a pie?

10 It is suggested that one cigarette reduces a person's life by $5\frac{1}{2}$ minutes.

If a person smokes a packet of 25 cigarettes every day for 15 years, how many days have been lost?

A Game

Nines. Make nine marks on a piece of paper. Each of two players take turns to mark off one or two of the marks. The loser is the person who crosses off the last mark.



A Sweet Trick

1	Ask your audience to write a 4 digit number.	8372
2	They rearrange the numbers in any order.	3278
3	They find the difference.	5094
4	They circle any number except zero.	5 0 9 (4)
5	They read out the numbers not circled, in any order.	9 0 5
6	You tell them which number they circled.	4

Sum the digits repeatedly to a single digit and take from 9.

9+0+5 = 14, 1+4 = 5, 9−5 = **4**.

Technology

Technology 6.1 Calculators and fractions

Calculators, with $a\frac{b}{c}$, are very good at handling fractions.

Experiment with the fraction button to learn how to do each of the following:

1 Show that $\frac{7}{12}\times\frac{6}{5} = \frac{7}{10} = 0.7$

2 Show that $\frac{3}{4}\div\frac{5}{12} = 1\frac{4}{5} = \frac{9}{5} = 1.8$

3 Show that $1\frac{3}{5}\times 2\frac{1}{4} = 3\frac{3}{5} = \frac{18}{5} = 3.6$

Use the fraction button on your calculator to check your answers to the previous exercises.

Technology 6.2 Calculators and fractions

Use a calculator to simplify $\frac{6.75}{100}$

An optimist says the glass is half empty.
A pessimist says the glass is half full.
An engineer says the glass is twice as big as it needs to be.

Change $\frac{6.75}{100}$ to $\frac{6\frac{3}{4}}{100}$ and enter this in your calculator.

Enter will then give $\frac{27}{400}$

Or try 675/10000 on your calculator.

Use your calculator to simplify the following fractions:

a) $\frac{8.75}{100}$ **b)** $\frac{4.2}{100}$ **c)** $\frac{5.6}{10}$ **d)** $\frac{2.25}{10}$

e) $\frac{6.4}{25}$ **f)** $\frac{9.8}{8}$ **g)** $\frac{12.75}{24}$ **h)** $\frac{2.25}{5}$

Technology 6.3 Fraction games and applets

Search the internet for some of the many fraction games and applets.

Give special attention to games and applets that demonstrate mixed numbers, multiplying fractions and dividing fractions.

What is the difference between a Java applet and a Flash applet?

Chapter Review 1

Exercise 6.13

1 Show that each of the following integers are Rational numbers.

a) 3 **b)** $^{-}7$ **c)** 6

d) $^{-}4$ **e)** 1 **f)** 0

2 Arrange the integers in ascending order (smallest to largest):

a) 5, 2, 0, 1 **b)** $^{-}2, ^{-}3, 0, ^{-}1$ **c)** $^{-}4, 3, ^{-}1, 1$

d) $^{-}3, ^{-}1, 2, ^{-}2, 0$ **e)** $^{-}4, ^{-}2, ^{-}3, ^{-}5$ **f)** $2, 4, ^{-}3, 3, 0$

g) $^{-}2, ^{-}1, ^{-}4, 5, 3$ **h)** $^{-}2, 1, ^{-}1, 2, 3$ **i)** $^{-}1, 2, 3, ^{-}4, ^{-}5$

3 Write each of the following decimals as a percentage and as a fraction:

a) 0.1 **b)** 0.5 **c)** 1.3 **d)** 6.25

e) 0.15 **f)** 0.06 **g)** 0.75 **h)** 0.8

4 Write each of the following percentages as a decimal and as a fraction:

a) 10% **b)** 70% **c)** 25% **d)** 1%

e) 200% **f)** 12% **g)** 12.75% **h)** 6.25%

5 Write each of the following fractions as a decimal and as a percentage (if necessary round to 2 decimal places):

a) $\frac{1}{10}$ **b)** $\frac{1}{4}$ **c)** $\frac{3}{10}$ **d)** $\frac{2}{5}$

e) $\frac{3}{4}$ **f)** $\frac{1}{3}$ **g)** $\frac{2}{3}$ **h)** $\frac{8}{10}$

6 Write each of the following as a fraction, as a decimal, and as a percentage:

a) Two-fifths of the oranges had to be thrown out.

b) Water makes up approximately 75% of a person's body.

c) The dodgy loan rate was set at 24%.

d) The loan rate was set at 6.8%.

7 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{2}$ **b)** $1\frac{3}{5}$ **c)** $1\frac{2}{5}$ **d)** $2\frac{2}{3}$

e) $1\frac{1}{8}$ **f)** $3\frac{3}{4}$ **g)** $2\frac{1}{4}$ **h)** $3\frac{7}{10}$

8 Change each of the following improper fractions to a mixed number:

a) $\frac{5}{2}$ **b)** $\frac{7}{3}$ **c)** $\frac{9}{4}$ **d)** $\frac{9}{5}$

e) $\frac{8}{5}$ **f)** $\frac{10}{3}$ **g)** $\frac{19}{7}$ **h)** $\frac{37}{11}$

9 Is 0.307692307692307692307692307692307692307 ($= \frac{4}{13}$) a rational number or an irrational number?

10 Is $\sqrt{5}$ a rational number or an irrational number?
$\sqrt{5}$ = 2.236067977499789696409173668731 3

Chapter Review 2

Exercise 6.14

1 Show that each of the following integers are Rational numbers.

a) 2 **b)** $^{-}9$ **c)** $^{-}3$

d) $^{-}8$ **e)** 10 **f)** 0

2 Arrange the integers in ascending order (smallest to largest):

a) 5, 0, 3, 2 **b)** $^{-}1$, 0, $^{-}3$, $^{-}2$ **c)** 3, $^{-}2$, $^{-}1$, $^{-}3$

d) $^{-}2$, $^{-}1$, 1, 2, 0 **e)** $^{-}4$, $^{-}1$, $^{-}3$, $^{-}2$ **f)** 4, 5, $^{-}3$, $^{-}1$, 1

g) $^{-}4$, $^{-}2$, 1, 2, $^{-}3$ **h)** 1, 5, $^{-}1$, $^{-}2$, $^{-}5$ **i)** $^{-}8$, 9, 8, $^{-}9$, $^{-}7$

3 Write each of the following decimals as a percentage and as a fraction:

a) 0.2 **b)** 0.6 **c)** 2.4 **d)** 1.75

e) 0.35 **f)** 0.05 **g)** 0.25 **h)** 0.4

4 Write each of the following percentages as a decimal and as a fraction:

a) 10% **b)** 50% **c)** 75% **d)** 2%

e) 400% **f)** 15% **g)** 10.6% **h)** 7.25%

5 Write each of the following fractions as a decimal and as a percentage (if necessary round to 2 decimal places):

a) $\frac{1}{10}$ **b)** $\frac{4}{10}$ **c)** $\frac{1}{2}$ **d)** $\frac{3}{5}$

e) $\frac{5}{10}$ **f)** $\frac{1}{4}$ **g)** $\frac{3}{4}$ **h)** $\frac{8}{10}$

6 Write each of the following as a fraction, as a decimal, and as a percentage:

a) Three-quarters of the pumpkins were diseased.

b) Jody achieved eighteen out of twenty for the test.

c) The dodgy loan rate was set at 21%.

d) The loan rate was set at 6.25%.

7 Change each of the following mixed numbers to an improper fraction:

a) $2\frac{1}{4}$ **b)** $3\frac{3}{4}$ **c)** $3\frac{7}{10}$ **d)** $5\frac{4}{5}$

e) $3\frac{3}{8}$ **f)** $1\frac{3}{5}$ **g)** $2\frac{1}{2}$ **h)** $1\frac{2}{5}$

8 Change each of the following improper fractions to a mixed number:

a) $\frac{3}{2}$ **b)** $\frac{9}{2}$ **c)** $\frac{9}{5}$ **d)** $\frac{6}{5}$

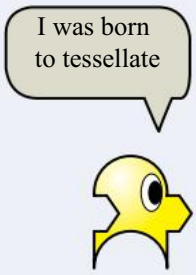
e) $\frac{7}{4}$ **f)** $\frac{17}{4}$ **g)** $\frac{33}{8}$ **h)** $\frac{37}{13}$

9 Is 0.6363636363636363636363636363636363636363636 ($= \frac{7}{11}$) a rational number or an irrational number?

10 Is $\sqrt{11}$ a rational number or an irrational number?
$\sqrt{11}$ = 3.316624790355399849114932736 6707

Congruence 7

- ★ Two figures are congruent if one shape lies exactly on top of the other after one or more transformations (translation, reflection, rotation).
- ★ Solve problems using properties of congruent figures, justifying reasoning and making generalisations.
- ★ The minimal conditions for congruence (SSS, SAS, ASA and RHS) and the conditions that do not prescribe congruence (ASS, AAA).
- ★ Plot the vertices of two-dimensional shapes on the Cartesian plane, translating, rotating or reflecting the shape and using coordinates to describe the transformation.



A TASK

The entrance to the libray is to be tiled. Design a tessellation that could be used to cover the floor of the entrance.

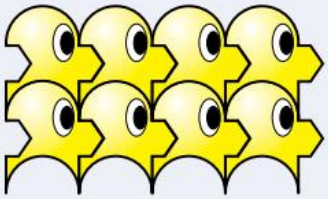
- Research tessellations using basic shapes such as squares, equilateral triangles and even circles.
- Research how MC Escher made tessellations.
- Create your own design.
- Present your design.

A LITTLE BIT OF HISTORY

4000 BC	Sumerian tesselations.
3000 BC	Tessellations in the form of mosaics are used in Mesopotamia.
1619	Kepler wrote about regular and semiregular tessellation.
1891	Fedorov proved that tiling of the plane is based on 17 different groups of isometries.
1936	MC Escher begins to create hundreds of amazing tessellations.
2008	Tessellation software allows for realistic visualisation in movies and games (eg. large armies etc).

Tessellations (to pave) are everywhere in our society from ancient mosaics and architecture to modern art, floors, and quiltmaking. Tessellations also occur in nature from beehives to rock structures.

See Investigation 7.1 to make your own tessellation.

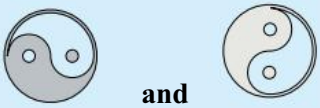


Congruence

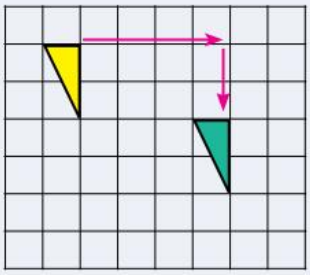
Congruent shapes have the same shape and size.
They fit exactly on top of each other.

The following pairs of shapes are congruent:

1  **and**
The second shape has been reflected

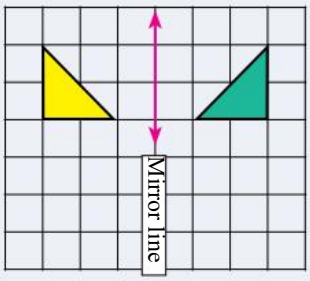
2  **and**
The second shape has been translated and rotated

A **translation** slides a shape across a page.
A translation produces a congruent shape.



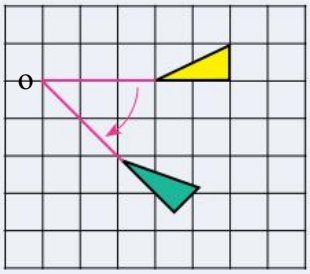
The triangle has been translated
4 units to the right
and 2 units down

A **reflection** is a mirror image.
A reflection produces a congruent shape.



The triangle has been reflected across the mirror line

A **rotation** turns a shape about a point.
A rotation produces a congruent shape.



The triangle has been rotated 45º around the point O

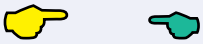
Exercise 7.1

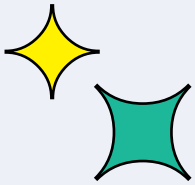
Which of the following pairs of shapes are congruent (If cut out, they should fit exactly on top of each other)?

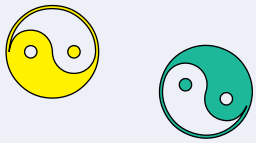
1 

2 

3 

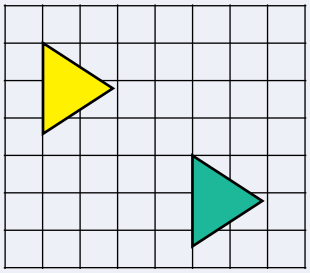
4 

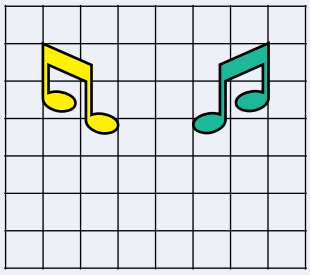
5 

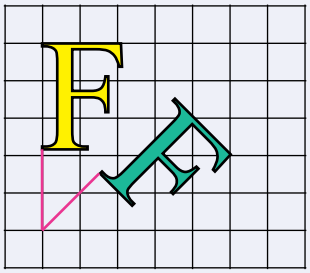
6 

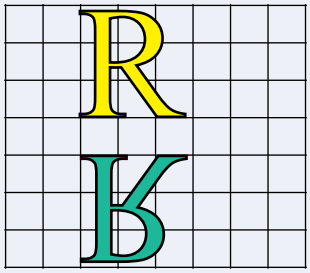
Exercise 7.2

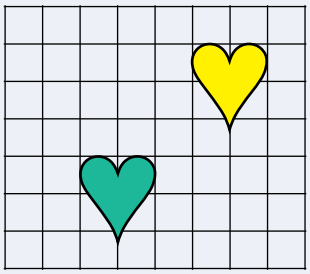
Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

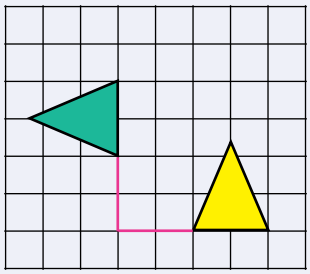
1 

2 

3 

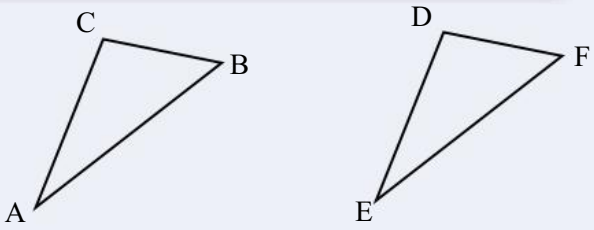
4 

5 

6 

Congruent Triangles

Congruent triangles have exactly the same shape and size.
The symbol for congruence is ≡ or ≅

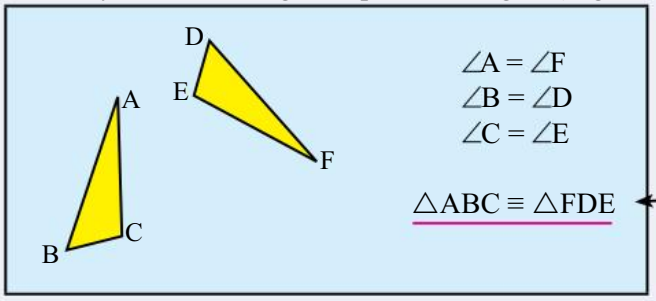


Triangle ABC is congruent to triangle EFD or **△ABC ≡ △EFD**

The **angles must be named** in matching order:
∠A = ∠E
∠B = ∠F
∠C = ∠D

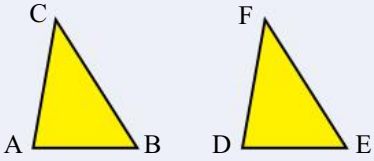
The **sides must be named** in matching order:
side AB = side EF
side BC = side FD
side CA = side DE

Exercise 7.3
Correctly name the congruent pair of triangles (angles and sides must match):



If the angles are in matching order then the sides will be in matching order.
And vice versa.

1



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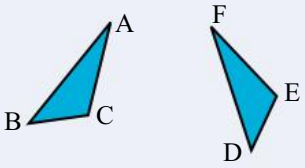

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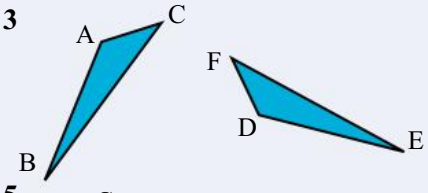

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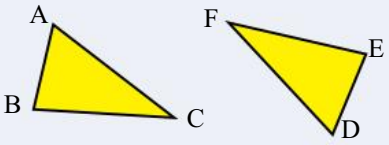

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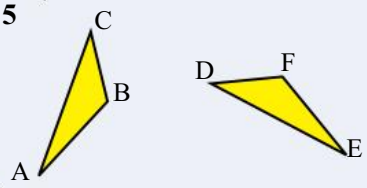

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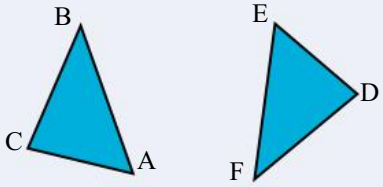
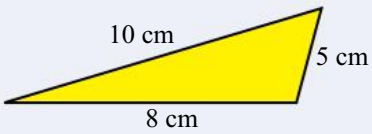


Tests for Congruent Triangles

Activity: The SSS test - If the three sides match are they congruent?

1. Draw a triangle with sides of 5 cm, 8 cm, and 10 cm.
2. Match your triangle with other people in the classroom who have also drawn a triangle with sides of 5 cm, 8 cm, and 10 cm.
3. Are the triangles congruent?

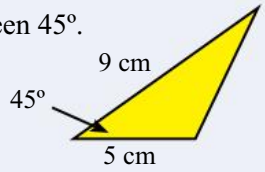


Activity: The AAA test - If the three angles match are they congruent?

1. Draw a triangle with angles of 60º, 20º, and 100º.
2. Match your triangle with other people in the classroom who have also drawn a triangle with angles of 60º, 20º, and 100º.
3. Are the triangles congruent?

Activity: The SAS test - If two sides and the angle in between the two sides match are they congruent?

1. Draw a triangle with sides of 5 cm and 9 cm with the angle in between 45º.
2. Match your triangle with other people in the classroom who have also drawn a triangle with sides of 5 cm and 9 cm with the angle in between 45º.
3. Are the triangles congruent?

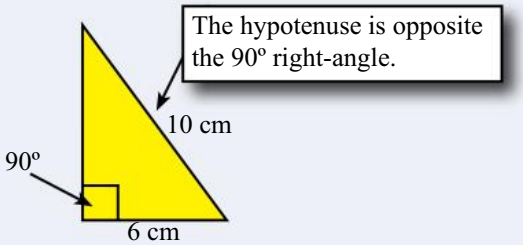


Activity: The ASA test - If two angles and a side match are they congruent?

1. Draw a triangle with angles of 30º and 45º and a side of 8 cm.
2. Match your triangle with other people in the classroom who have also drawn a triangle with angles of 30º and 45º and a side of 8 cm.
3. Are the triangles congruent?

Activity: The RHS test - If a right-angle, the hypotenuse and another side match are they congruent?

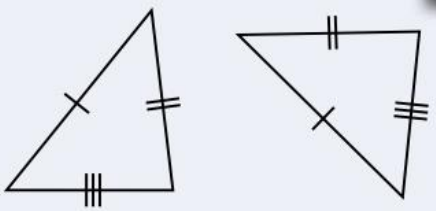
1. Draw a triangle with a right-angle (90º), a hypotenuse of 10 cm, and another side of 6 cm.
2. Match your triangle with others who have also drawn a triangle with a right-angle (90º) a hypotenuse of 10 cm, and another side of 6 cm.
3. Are the triangles congruent?



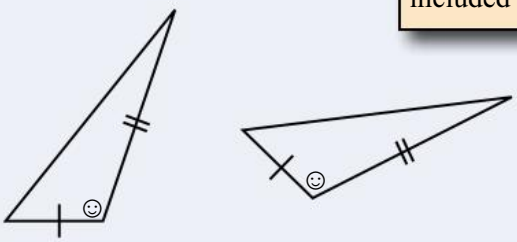
Tests for Congruent Triangles

The activities on the previous page lead to four conditions that each provide a test of whether two triangles are congruent:

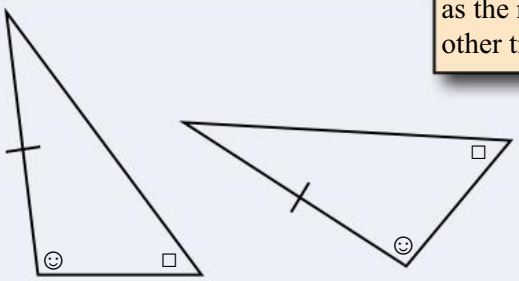
SSS (side, side, side). Two triangles are congruent if the three sides in one triangle are the same length as the matching sides on the other triangle.



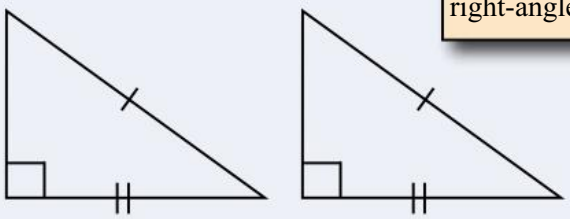
SAS (side, angle, side). Two triangles are congruent if two sides and the included angle in one triangle are the same size as the matching sides and included angle on the other triangle.



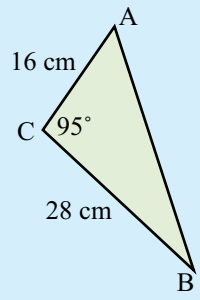
ASA (angle, side, angle). Two triangles are congruent if two angles and a side in one triangle are the same size as the matching two angles and side on the other triangle.

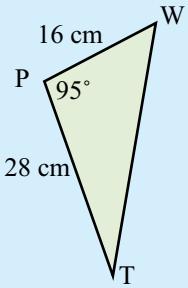


RHS (right-angle, hypotenuse, side) Two triangles are congruent if the hypotenuse and another side in one right-angled triangle are the same size as the matching hypotenuse and another side on the other right-angled triangle.



Exercise 7.4 Use the tests for congruence to test whether the following pairs of triangles are congruent:

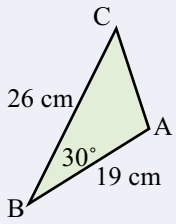


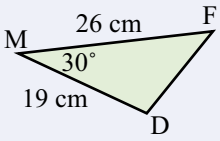


side AC = side WP
$\angle C = \angle P$ {angle inbetween}
side CB = side PT

$\triangle ACB \equiv \triangle WPT$ {**SAS**}

1





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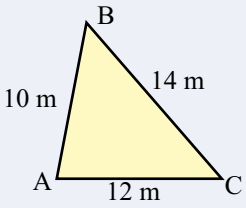

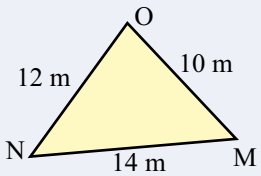

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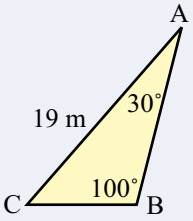

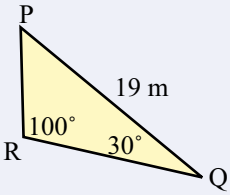

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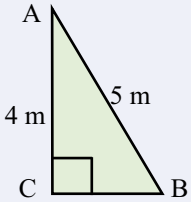

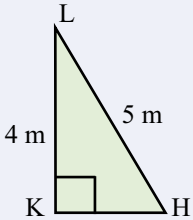

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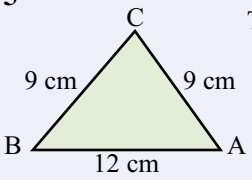

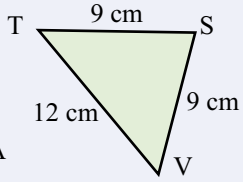

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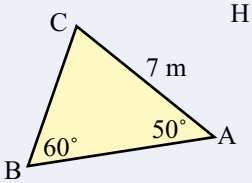

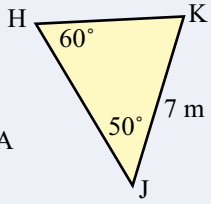

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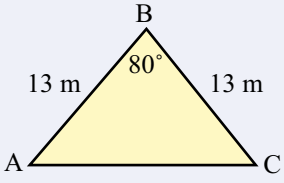

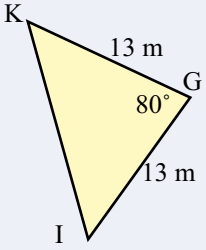

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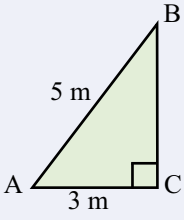

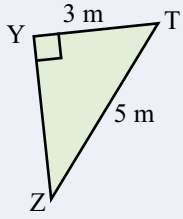

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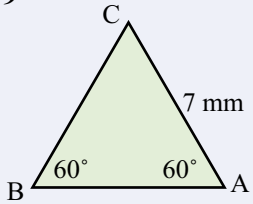

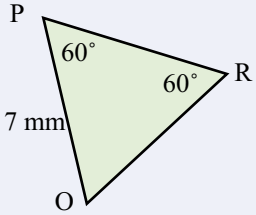

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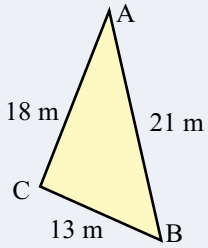
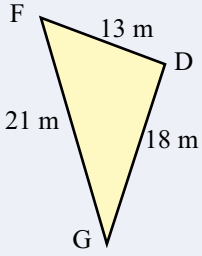
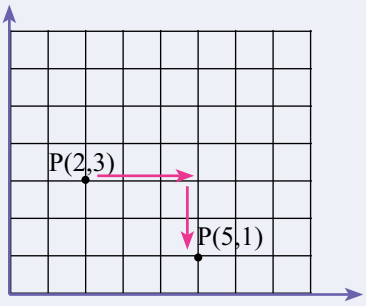




Exercise 7.5

What are the new coordinates of P(2,3) after P is translated 3 units right and 2 units down?

= P(2+3,3–2)
= P(5,1)



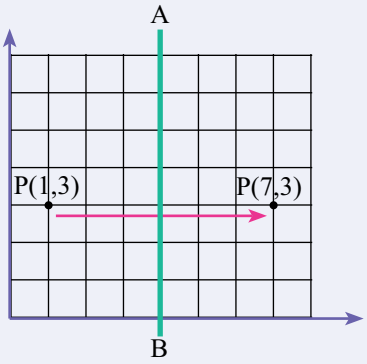
1 What are the new coordinates of P(1,4) after P is translated 2 units right and 2 units down?

2 What are the new coordinates of P(2,3) after P is translated 4 units right and 1 unit down?

3 What are the new coordinates of P(5,4) after P is translated 3 units left and 2 units up?

What are the new coordinates of P(1,3) after P is reflected in the line AB?

= P(1+2x3,3)
= P(7,3)



4 What are the new coordinates of P(1,4) after P is reflected in the line AB?

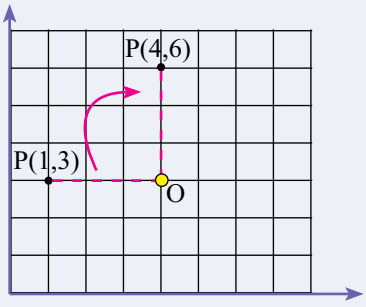
5 What are the new coordinates of P(1,2) after P is reflected in the line AB?

6 What are the new coordinates of P(2,4) after P is reflected in the line AB?

What are the new coordinates of P(1,3) after P is rotated 90° clockwise about the point O?

= P(4,6)

{Plot the points to work out what the new coordinates will be.}



7 What are the new coordinates of P(2,3) after P is rotated 90° clockwise about the point O?

8 What are the new coordinates of P(0,3) after P is rotated 90° clockwise about the point O?

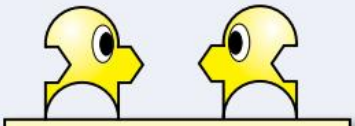
9 What are the new coordinates of P(1,3) after P is rotated 90° anti-clockwise about the point O?

Mental Computation

Exercise 7.6

You need to be a good mental athlete because many everyday problems are solved mentally.

1 Spell transformation.
2 Change 0.3 to a fraction.
3 Change $\frac{2}{5}$ to a decimal.
4 Translate P(2,3) 2 units to the right and 4 units up.
5 $^{-}5 + 3$
6 Change $\frac{8}{5}$ to a mixed numeral.
7 Simplify: $6b - 4b$.
8 What are the four tests for congruent triangles?
9 Round 5.2641 to 2 decimal places.
10 Simplify: $x^5 \times x^2$



In just two days from now, tomorrow will be yesterday.

Exercise 7.7

1 Spell congruent.
2 Change 0.5 to a fraction.
3 Change $\frac{1}{4}$ to a decimal.
4 Translate P(1,3) 3 units to the right and 2 units down.
5 $^{-}2 + 3$
6 Change $1\frac{2}{3}$ to an improper fraction.
7 Simplify: $3x + 2x$.
8 What are the four tests for congruent triangles?
9 Round 6.3475 to 2 decimal places.
10 Simplify: $x^3 \times x^4$

Exercise 7.8

Tomorrow is often the busiest day of the week - Spanish Proverb.

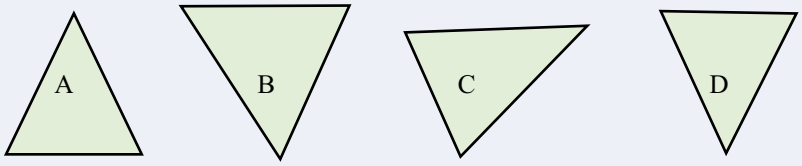
1 Spell reflection.
2 Change 0.25 to a fraction.
3 Change $\frac{3}{4}$ to a decimal.
4 Translate P(5,3) 3 units to the left and 1 unit down.
5 $^{-}4 + 6$
6 Change $\frac{7}{3}$ to an improper fraction.
7 Simplify: $5c - 2c$.
8 What are the four tests for congruent triangles?
9 Round 9.2173 to 2 decimal places.
10 Simplify: $x^2 \times x^4$

NAPLAN Questions

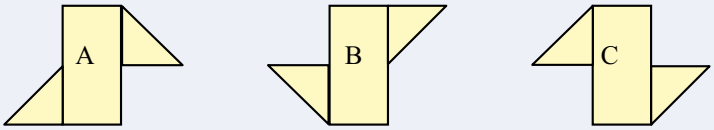
Exercise 7.9

1 Which pair of the following shapes are congruent?

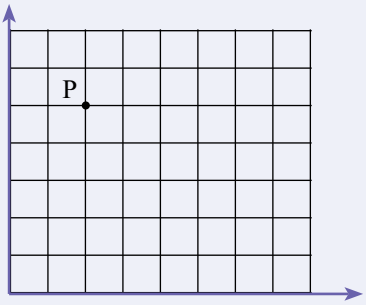
a)



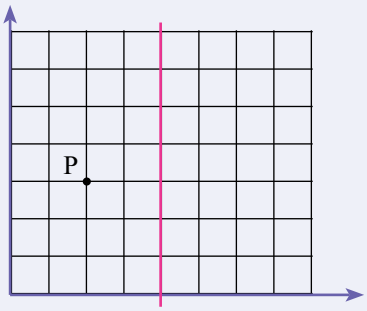
b)



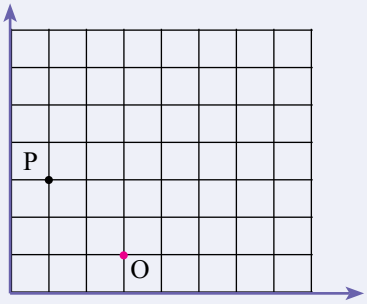
2 The point P(2,5) is translated 3 units down and 4 units to the right. What are the new coordinates of point P?



3 The point P(2,3) is reflected in the mirror (the red line). What are the new coordinates of point P?



4 The point P(1,3) is rotated 90 clockwise about O. What are the new coordinates of point P?



5 SSS, SAS, AAS, and RHS are tests for congruent triangles. What would be the test for congruent circles?

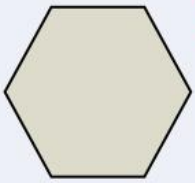
Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 7.10

1 Divide the regular hexagon, all sides equal, into:

2 congruent shapes
3 congruent shapes
4 congruent shapes
6 congruent shapes



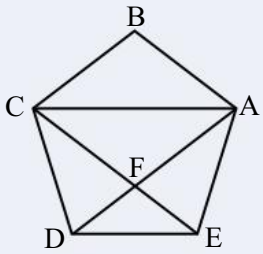
2 Can you use a single straight line to divide each of the following letters into two congruent shapes?

A B C D E F

3 Does the diagonal divide the rectangle, the parallelogram, the rhombus, and the kite into two congruent shapes?

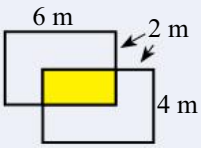
4 What does the letter F look like after being reflected in a mirror?

5 Name three pairs of congruent triangles in the regular pentagon below:

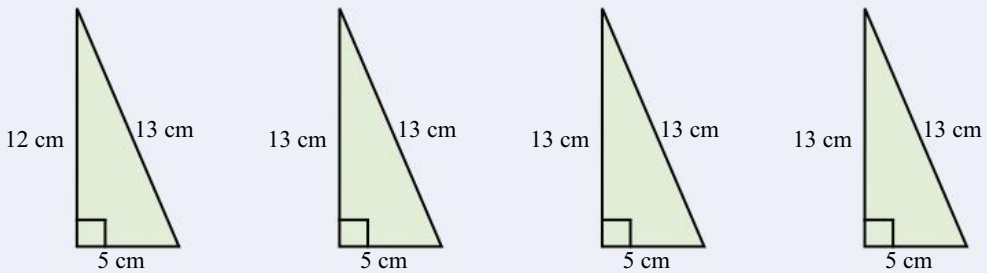


The statement below is true.
The statement above is false.

6 What is the area of the intersection of the two congruent rectangles?



7 Rearrange the four congruent triangles to form a square with a hollow square in the middle. What is the area of the middle square?



Technology

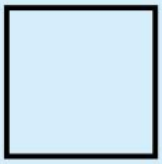
Technology 7.1 Tessellations
There are numerous tessellation activities and games on the Internet. Experiment with some of these activities.

Technology 7.2 Tessellations with LOGO
LOGO is a computer programming language created in 1967 for educational use.

- LOGO is essentially a turtle with a pen on the monitor.
- The turtle will draw geometric shapes when given movement commands.
- The LOGO programming language is a powerful programming language.
- LOGO is considered an ideal introduction to computer programming.

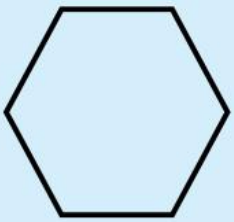
To draw a square:

```
PENDOWN
FORWARD 100
LEFT 90
FORWARD 100
LEFT 90
FORWARD 100
LEFT 90
FORWARD 100
LEFT 90
PENUP
```



To draw a regular hexagon

```
REPEAT 6 [FD 100 LEFT 60]
```



There are a number of free LOGO applications on the Internet and many tutorials, even including tutorials on LOGO and tessellations.

After just a short time learning the language you will be able to start making tessellations with LOGO.

Technology 7.3 Tessellation with other software

a) 'Kali' allows you to draw some wonderful tessellations.

b) 'Tess' also allows you to draw some wonderful tessellations as demonstrated on their website.

c) 'Tessellation Exploration' all kinds of transformations of basic shapes.

d) 'Tessellate' an online Javascript tessellation activity.

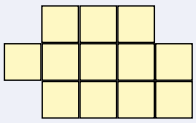
Graphic Designers or Commercial Artists design and produce art and layouts for advertising, magazines, newspapers, books, websites etc.

- Relevant school subjects are English and Mathematics.
- Courses range from Diploma, to Advanced Diplomas, and University Degrees.

Investigations

Investigation 7.1 Make your own tessellation

Tessellations are about designing congruent shapes that fill a space without gaps or overlapping. Tessellations may be formed by joining translated, reflected, and rotated congruent shapes.



The square is a basic shape that will tessellate:

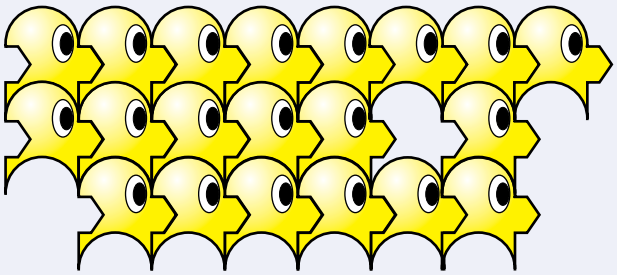
1 Select a shape that will tessellate.
A square is a good shape to start with.

2 Cut a piece from the bottom and add it to the top.

3 Cut a piece from the left and add it to the right.

4 This shape, based on a square, should now tessellate.
Decorate the piece.

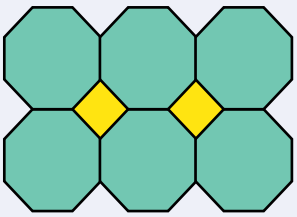
5 Make multiple copies and tessellate - use a photocopier or tessellation software.



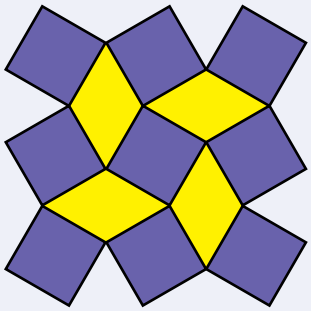
Investigation 7.2 Regular polygon tessellation

Experiment with tessellations of combinations of polygons. Try combinations of shapes such as triangles, squares, parallelograms, pentagons, hexagons etc:

Squares and Octagons

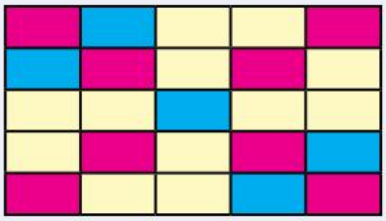


Squares and Parallelograms



A Couple of Puzzles

Exercise 7.11



1 What percentage of the rug is blue?
What percentage of the rug is yellow?

2 Hume exercises 30 mins each day.
How many hours does Hume exercise in a 30 day month?

3 I have four daughters.
Every daughter has a brother.
How many children?

I am a nobody.
Nobody is perfect.
Therefore I am perfect.

4 A is three times the age of B. In ten years time, A will be twice the age of B.
How old are A and B?

5

This is a well known puzzle.
Join all 9 dots using only 4 straight lines.
Do not lift your pencil from the page.

A Game

Odds and Evens is naughts and crosses played with the numbers.

2, 4, 6, 8, 10 1, 3, 5, 7, 9

1 One player uses the odd numbers,
the other player uses the even numbers.

2 When you use a number cross it off
because it can't be used more than once.

3 The winner is the first to get a sum of 15
either horizontally, vertically, or diagonally.

6	4	7
	10	5
		3

A Sweet Trick

All the flowers of tomorrow
are in the seeds of yesterday.

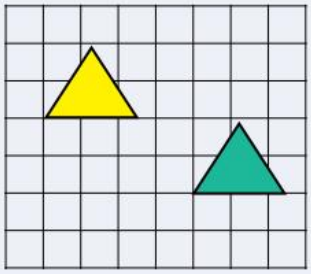
1	Ask your audience to choose their favourite number from 1 to 9.	7
2	Write the number three times.	777
3	Sum the digits of the number.	7+7+7 = 21
4	Multiply the answer by 37	21 x 37 = 777

Chapter Review 1

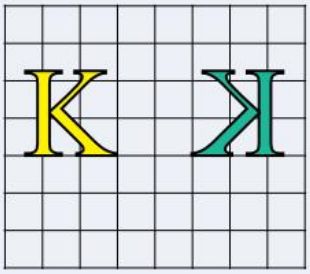
Exercise 7.12

1 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

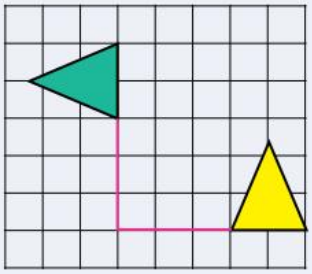
a)



b)

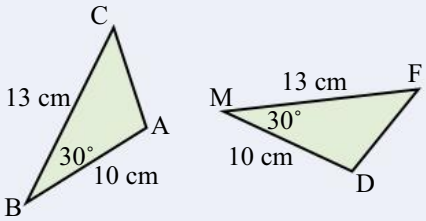


c)

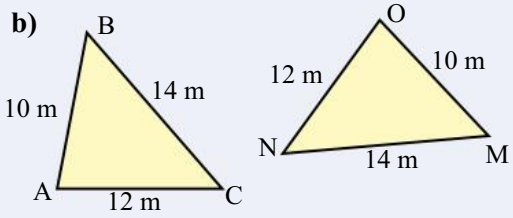


2 Use the tests for congruence to test whether the following pairs of triangles are congruent:

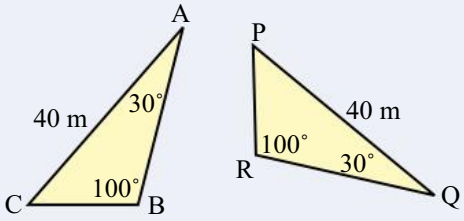
a)



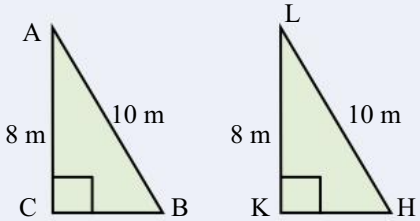
b)



c)

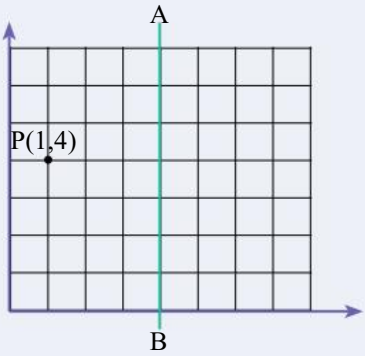


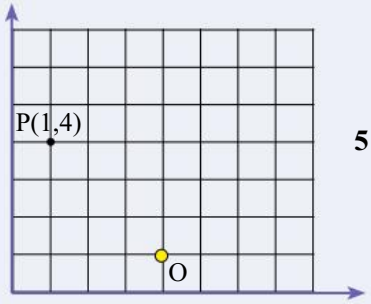
d)



3 What are the new coordinates of P(3,5) after P is translated 3 units left and 3 units up?

4 What are the new coordinates of P(1,4) after P is reflected in the line AB?





5 What are the new coordinates of P(1,4) after P is rotated 90° clockwise about the point O?

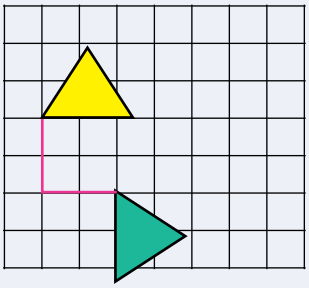
I told you to be home by a quarter of twelve.
A quarter of twelve is 3.

Chapter Review 2

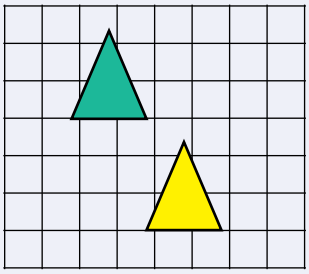
Exercise 7.13

1 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

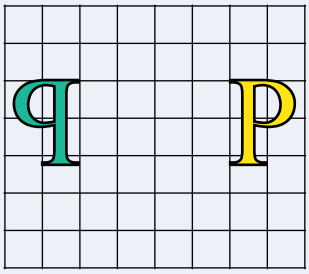
a)



b)

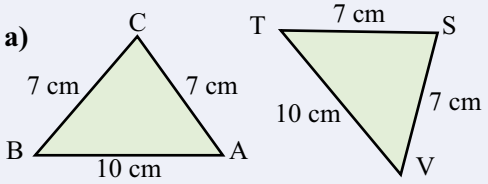


c)

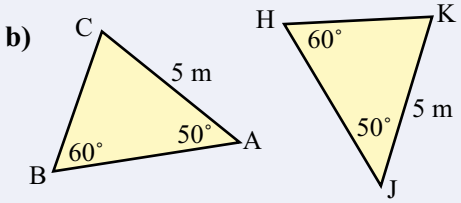


2 Use the tests for congruence to test whether the following pairs of triangles are congruent:

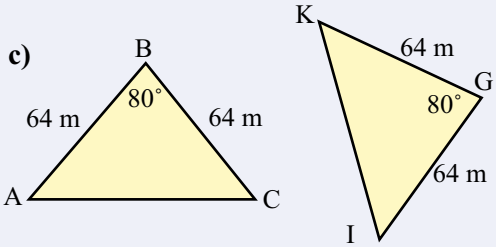
a)



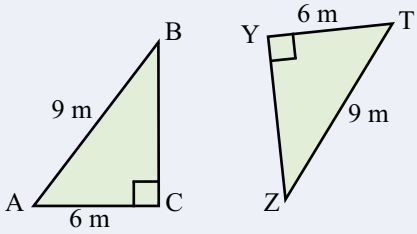
b)



c)

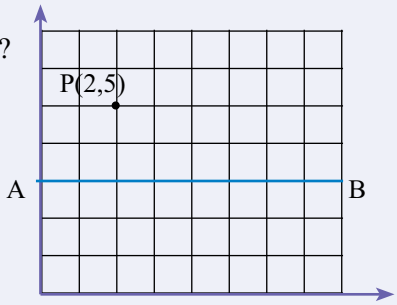


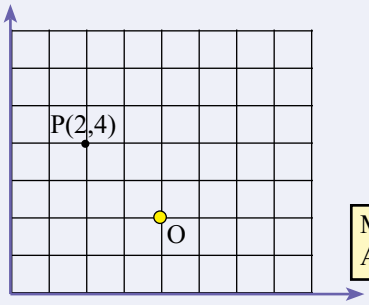
d)



3 What are the new coordinates of P(2,6) after P is translated 4 units right and 5 units down?

4 What are the new coordinates of P(2,5) after P is reflected in the line AB?





5 What are the new coordinates of P(2,4) after P is rotated 45° clockwise about the point O?

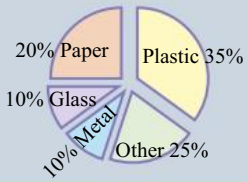
My short-term memory is not as sharp as it used to be.
Also, my short-term memory is not as sharp as it used to be.

Data

8

- ★ Use sample properties to predict characteristics of the population.
- ★ Use displays of data to explore and investigate effects.
- ★ Explore the practicalities and implications of obtaining representative data.
- ★ Understand that making decisions and drawing conclusions based on data may differ from those based on preferences and beliefs.
- ★ Investigate the effect of individual data values, including outliers, on the mean and median.

See Investigation 8.1 for the steps in conducting research.



A TASK

Conduct research on the playground rubbish.

- Plan the research (Collect data how many times per day, where, what, why rubbish dropped?)
- Collect data (Volume of rubbish in bin, volume of rubbish around bin, attitudes to rubbish).
- Data representation.
- Analyse the data and make a conclusion.
- Present your research.

A LITTLE BIT OF HISTORY

In Roman times a census involved the registration of citizens and their property for taxation purposes.

1828	The first census of New South Wales.
1881	The first Australian census.
1901	Section 51 (xi) of the Constitution contains powers to make laws about the "census and statistics".
1905	Commonwealth Bureau of Census and Statistics formed: The Census shall be taken in the year 1911, and in every tenth year thereafter.
1961	The census is held every five years from 1961.

Census, an official survey of people, and their age, gender, health, income, etc.

Collecting Data

Census
A collection of data from the **whole** population.

Sample
A collection of data from **part** of the population.

Example
What percentage of people with lung cancer are smokers? Assume 9000 new cases of lung cancer each year.

A census would collect data from each of the 9000 people.

A sample would collect data from a part of the 9000 people.

A **Census** reflects the opinions of the whole population

Advantages of samples	**Disadvantages of samples**
☺ A sample is more economical.	☻ Tricky to get a good sample.
☺ Information can be gathered more quickly.	☻ The sample must be small but large enough to reflect the opinion of the population.
☺ A sample uses less resources.	

Exercise 8.1
What is wrong with each of the following:

100 randomly selected Year 8 students were asked if they had a part time job. 47 students replied yes. The conclusion was that 47% of the school's population had a part time job.

While the sample size seems large enought, the sample isn't representative. The other Year levels weren't sampled.
A stratified sample, with samples taken from each Year level, would be better.

The Internet has sample size calculators.

The smaller the sample, the larger the error.

1 5 friends were asked if they liked maths. 4 said yes. The conclusion was that 80% of the school's population like maths.

2 150 randomly selected Year 12 students were asked about their exercise habits. The conclusion was that 87% of the school's population exercised less than 30 minutes per day.

3 5 Year 7s, 5 Year 8s, 5 Year 9s, 5 Year 10s, 5 Year 11s and 5 Year 12s were asked about their study habits. The conclusion was that between 30% and 80% of students study for less that 2 hours per night.

4 Survey forms, asking what music should be played at the upcoming school dance, were placed in the Administration block along with the response box.

Exercise 8.2

Complete a frequency table for each of the following data:

Eye colour	Tally	Frequency
Amber	|	1
Blue	𝍸 | |	7
Brown	𝍸 𝍸 |	11
Gray	| |	2
Green	|	1
Hazel	| |	2
	Total =	24

The 1st column lists **all possible** outcomes.

The **tally** is used **during** the survey.

The **frequency** is the number of times each outcome occurs..

1 A classroom survey of eye colour:

Brown	Blue	Blue	Brown
Brown	Blue	Hazel	Brown
Green	Brown	Blue	Amber
Blue	Brown	Green	Hazel
Brown	Blue	Green	

There is a lot of information about "eye colours" on the Internet.

2 How people in the class travelled to school today:

Walk	Bus	Bus	Walk	Bus	Bus
Bicycle	Car	Car	Train	Car	Walk
Train	Walk	Car	Train	Train	Car
Car	Car	Bus	Bus	Train	Walk
Bus	Car	Car	Bus		

3 The number of TVs at each class person's home:

1	2	2	2	3	1
2	2	4	1	2	2
1	2	2	3	2	3
1	2	2	2	2	2
2	2				

4 A survey on whether there is too much violence on TV:

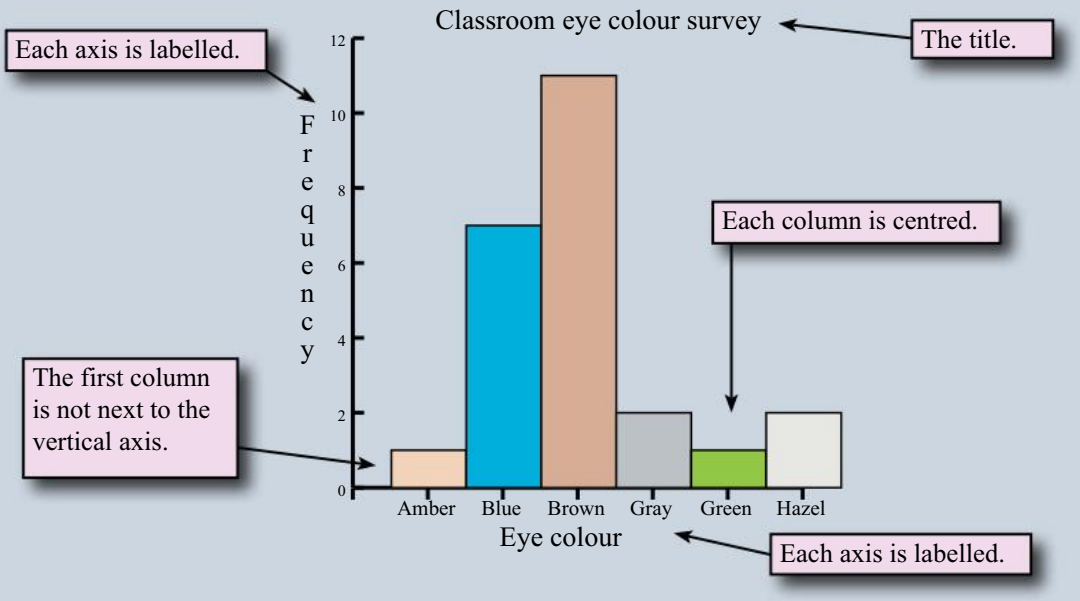
Agree	Disagree	Disagree	Disagree	Agree
Strongly disagree	Agree	Agree	Disagree	Disagree
Disagree	Strongly agree	Agree	Disagree	Disagree
Disagree	Agree	Disagree	Disagree	Disagree
Agree	Disagree	Disagree	Strongle Agree	

5 A dice was rolled 65 times:

6	2	4	6	4	2	4	2	6	1	1	3	5
5	1	3	5	5	3	3	5	1	4	4	5	2
2	5	6	4	6	6	5	1	5	3	2	5	3
2	1	3	3	1	4	1	1	3	5	2	6	5
6	2	5	3	2	1	3	4	2	4	2	1	2

Frequency column graph

A frequency column graph or histogram allows a visual assesment of the data



Exercise 8.3

Draw a frequency column graph for each of the following surveys after first constructing a frequency table:

1 Is there a need to change the Australian flag?:

No	No	No	Yes	No	No
No	No	Yes	No	No	Yes
No	No	Yes	No	No	No
No	Yes	Yes	No	No	No
No	Yes	Yes			

2 The Netball team's season results:

Loss	Win	Loss	Loss	Loss	Loss	Win
Loss	Win	Win	Win	Win	Win	Loss
Loss	Win	Loss	Win	Win	Win	Loss

3 A survey on the need to wear hats in the schoolgrounds:

Agree	Disagree	Agree	Agree	Agree
Disagree	Agree	Agree	Disagree	Disagree
Agree	Strongly agree	Agree	Strongly agree	Disagree
Disagree	Disagree	Disagree	Agree	Disagree
Disagree	Agree	Strongly agree	Strongly Agree	Agree

4 The age of each person in the classroom:

14	14	15	14	14	15
14	13	15	14	14	13
14	14	15	16	14	14
14	13	13	14	14	15
13	14	15	14	14	

Descriptive Statistics

The first step in analysing data is to describe the data using descriptive statistics.

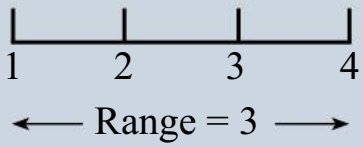
The Range
describes the **spread** of the data.

The range is the difference between the smallest and the largest data values. The range is the simplest description of the spread of the data.

Exercise 8.4
Find the range of each of the following set of scores:

1, 2, 3, 4, 4

Range = largest smallest
Range = 4 – 1
Range = 3



1 4, 7, 15, 23, 31, 43
2 4, 15, 32, 83, 112, 145, 231
3 2, 7, 2, 6, 3, 2, 6, 7
4 1, 3, 3, 5, 1, 2, 7, 4
5 232, 235, 236, 232, 231
6 425, 424, 421, 423, 424, 427
7 25.2, 26.1, 25.8, 23.2, 24.6
8 -2, -3, -5, -4, -1, -2, -6

The Mode
describes the **middle** of the data.

The mode is the score that occurs most often. There are several problems with the mode as a central measure.

Exercise 8.5
Find the mode of each of the following set of scores:

5, 3, 3, 4, 3, 5, 3 Mode = 3 {Occurs 4 times}	2, 5, 8, 5, 4, 2, 1, 3 Mode = 2 and 5 {Twice each} This set of scores is bimodal

1 4, 6, 6, 3, 2, 3, 6
2 4, 5, 2, 3, 4, 2, 4, 5
3 2, 5, 2, 1, 6, 1, 1, 5
4 1, 4, 3, 3, 4, 5, 6, 2
5 5, 5, 5, 3, 4, 3, 3, 5
6 1, 2, 3, 2, 1, 2, 4, 1
7 32, 35, 36, 32, 31, 33
8 7.1, 7.3, 7.1, 7.2, 7.1
9 -1, -3, -4, -2, -1, -2, -1
10 a, b, a, c, b, a, d, b, a

Descriptive Statistics

The Median
describes the **middle** of the data.

The median is the middle of a set of scores. The median is a central measure that ignores extreme high scores and extreme low scores.

Exercise 8.6
Find the median of each of the following set of scores:

5, 7, 3, 4, 3, 5, 3 Put the scores in ascending order 3, 3, 3, 4, 5, 5, 7 Median = 4 {4 is in the middle }	2, 5, 7, 5, 4, 2, 1, 3 Put the scores in ascending order 1, 2, 2, 3, 4, 5, 5, 7 Median = 3.5 {Average of 3 & 4}

1 4, 6, 6, 3, 2, 3, 6
2 4, 5, 2, 3, 4, 2, 4, 5
3 2, 5, 2, 1, 6, 1, 1, 5
4 1, 2, 3, 4, 1, 2, 4, 3
5 32, 98, 36, 32, 31, 33
6 21, 24, 23, 23, 24, 56
7 5.2, 6.1, 5.8, 3.2, 4.6
8 -1, -3, -4, -2, -1, -2, -1

The Mean
describes the **middle** of the data.

The mean is the sum of the scores divided by the number of scores. The mean is also called the **average**. The mean is heavily affected by extreme scores.

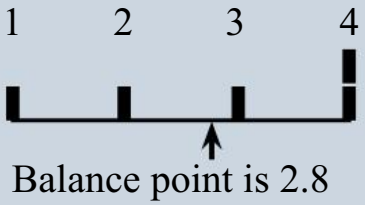
Exercise 8.7
Find the mean of each of the following set of scores:

1, 2, 3, 4, 4

$$mean = \frac{Sum\ of\ scores}{No\ of\ scores}$$

$$mean = \frac{1+2+3+4+4}{5}$$

mean = 2.8



1 1, 2, 3, 4, 4, 4
2 1, 2, 3, 4, 4, 80 ← Notice how the 80 affects the mean.
3 1, 1, 2, 2, 3, 4, 4, 4, 4, 4
4 1, 6, 6, 2, 2, 3
5 31, 36, 36, 32, 32, 33
6 81, 86, 86, 82, 82, 83
7 101,106,106,102,102,103
8 721,726,726,722,722,723
9 5.1, 5.6, 5.6, 5.2, 5.2, 5.3
10 −1, −6, −6, −2, −2, −3

Analysing Data

8A believe they are a better mathematics class than 8B. Analyse the following data, taken from a recent mathematics competition, and make a comment about 8As assumed mathematical superiority (Coding: High Distinction = 4, Distinction = 3, Credit = 2, Participation = 1).

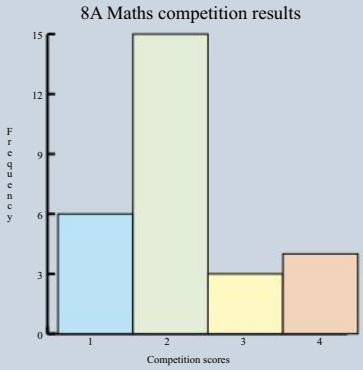
8A							8B						
4	3	2	2	3	4	1	1	2	2	2	2	2	2
2	2	4	1	2	2	2	2	2	2	2	2	3	3
1	2	1	2	2	1	2	2	2	2	2	3	2	2
2	1	2	4	2	2	3	3	2	2	3	2	3	

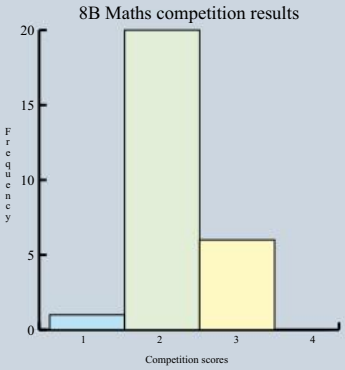
First, complete a frequency table for each class:

8A	Tally	Frequency
1	𝍸 /	6
2	𝍸 𝍸 𝍸	15
3	///	3
4	////	4
		28

8B	Tally	Frequency
1	/	1
2	𝍸 𝍸 𝍸 𝍸	20
3	𝍸 /	6
4		0
		27

Second, complete a frequency column for each class:





Third, calculate the descriptive statistics for each class:

	8A	8B
Mode	2	2
Median	2	2
Mean	2.18	2.19
Range	3	2

Technology such as graphic calculators, spreadsheets, computer software are very good at this.
See Technology 8.1,8.2,8.2

Fourth, make a comment (refer to the original question).
The frequency table shows 8A has 4 High Distinction students and 6 Participation students compared to 8B with no High Distinction students and 1 Participation student. The 8A histogram shows a skew to the left while 8B has a slight skew to the right. Because there are no extreme scores, the mean is probably the best measure and shows a slight advantage to 8B despite 8A having a larger number of High Distinctions.

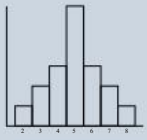
Exercise 8.8

1 Did the Year 8s appreciate the concert more than the Year 9s (The concert was rated on a scale from 1 to 5 with 1=bad, 2=poor, 3=fair, 4=good, 5=excellent)?

Year 8						Year 9					
4	2	5	4	3	3	4	3	3	2	4	3
5	4	3	4	3	2	4	1	5	2	4	3
1	5	3	3	4	2	2	4	1	3	2	4
4	3	3	4	2	5	3	2	1	5	2	4
3	5	4	3	4	1	2	4	3	5	5	1
2	4	3	3	4	3	3	2	3	2	4	2
4	2	4	3	5	4	3	1	5			

2 Are the above concert data sets symmetrical?

A frequency column graph is symmetrical if a vertical line can divide the graph into roughly two equal halves.
In a perfectly symmetrical data set,
the mean, the median, and the mode are the same.



A symmetrical data set.

3 In deciding whether to continue with an advertising campaign, data about the number of customers per day just before the campaign started and the number of customers per day after the campaign started was collected. Analyse the data and suggest whether the advertising campaign is making a difference.

Before						After					
18	19	17	22	20	21	18	19	19	20	19	23
20	19	19	22	19	21	20	21	20	22	22	21
18	19	20	34	18	22	22	19	19	22	20	21
20	20	18	22	20	19	21	21	21	20	19	21
21	18	21	20	19		20					

Outliers are unusual values.
- Are they a mistake?
- Should you delete them?

4 Referring to the data in Question 3 above:

a) Are the above data sets symmetrical or skew?

b) Are there any outliers?

c) Would you prefer to use mean or median as a central measure?

The mean is normally used.
The median is used when there are extreme values.

Well.
Don't take my word for it. Find the mean and median of these house prices.

The median is used for house prices:
$590 000, $750 000, $600 000, $945 000, $4 500 000

The $4 500 000 doesn't affect the median as much as the mean.

Mental Computation

Exercise 8.9

1 Spell median.

Given the data set: 2,2,3,4,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Change $\frac{3}{5}$ to a decimal.

7 Translate P(2,3) 2 units to the right and 4 units up.

8 Change $\frac{9}{5}$ to a mixed numeral.

9 What are the four tests for congruent triangles?

10 Round 17.3271 to 2 decimal places.

Mental computation helps you learn how to think.

Exercise 8.10

1 Spell range.

Given the data set: 2,3,3,3,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Change $\frac{1}{4}$ to a decimal.

7 Translate P(1,3) 3 units to the right and 2 units down.

8 Change $\frac{7}{4}$ to a mixed numeral.

9 What are the four tests for congruent triangles?

10 Round 66.6666 to 2 decimal places.

You do not find what you do not seek - English Proverb.

Exercise 8.11

1 Spell average.

Given the data set: 1,2,2,2,5

2 What is the range?

3 What is the mode?

4 What is the median?

5 What is the mean?

6 Change $\frac{4}{5}$ to a decimal.

7 Translate P(5,4) 2 units to the left and 4 units down.

8 Change $\frac{8}{3}$ to a mixed numeral.

9 What are the four tests for congruent triangles?

10 Round 43.2149 to 2 decimal places.

Two statisticians are at a firing range.
One shoots 10 cm to the left of the bulls eye.
The other shoots 10 cm to the right.
Both are happy because on average they got a bulls eye.

NAPLAN Questions

Exercise 8.12

1 Find the mean (average) exercise time:

Wednesday	Thursday	Friday	Saturday	Sunday
30 minutes	45 minutes	20 minutes	35 minutes	1 hour 10 minutes

2 Given the scores: 3, 4, 5, 5, 5, 5, 8, 10, 90

a) What is the mean, median, and mode?

b) What is the mean, median, and mode after 90 is removed?

c) Which is affected the most by removing 90, the mean, median, or mode?

3 **a)** Find the mean, median, and mode of the following: 1, 2, 2, 2, 3, 3, 4

b) Is Mean > Median = Mode true?

4 Jack is spending five days work experience in the city. A return train ticket from his relative's place to the city will cost $7.50 each day and a weekly ticket will cost $25. What is Jack's daily saving if he uses a weekly ticket?

After 10 innings, the average number of runs per innings was 13.
After 15 innings the average number of runs per innings was 16.
How many runs were scored in the last 5 innings?

After 10 innings:

$$average = \frac{total}{no\ innings}$$

$$13 = \frac{total}{10}$$

total for 10 innings $= 13 \times 10$
$= 130$

After 15 innings:

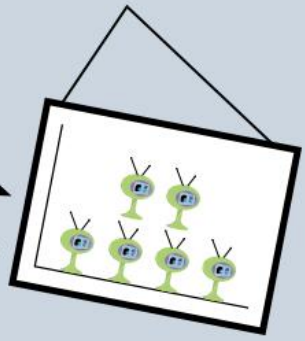
$$average = \frac{total}{no\ innings}$$

$$16 = \frac{total}{15}$$

total for 15 innings $= 16 \times 15$
$= 240$

No runs last 5 innings $= 240 - 130$
$= 110$

5 After 10 innings, the average number of runs was 16.
After 15 innings the average number of runs was 13.
How many runs were scored in the last 5 innings?



6 This frequency column graph was constructed from 1, 2, 2, 2, 3, 3, 4. Where is the mistake in the frequency column graph?

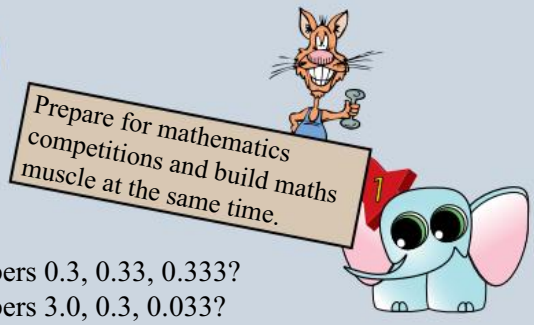
7 Consider the faulty frequency column graph:

a) find the mean, median, and mode.

b) compare this with the original data set: 1, 2, 2, 2, 3, 3, 4. What has happened to the mean, the median, and the mode?

8 Given the scores: 6, 6, 6, 7, 7, 7, 50. Which central measure is affected the most by removing 50: the mean, median, or mode?

Competition Questions



Exercise 8.13

1 What is the average of the numbers 0.3, 0.33, 0.333?

2 What is the average of the numbers 3.0, 0.3, 0.033?

3 What is the average of the numbers 33, 3.3, 0.33?

4 If 50 million people have total savings of $10 billion, what is the average savings per person?

5 If 5 million people have total savings of $4 billion, what is the average savings per person?

6 If 45 million people inhabit 45 000 km^2, what is the average number of people per km^2?

A person has an average of 48 after three tests. What mark must the person get on the fourth test so that the average of the four tests is 50?

$48 = \frac{total}{3}$ Total after 3 tests = 144	$50 = \frac{total}{4}$ Total after 4 tests = 200 Thus need 200– 144 = 56 on the 4th test.

7 A person has an average of 81 after three tests. What mark must the person get on the fourth test so that the average of the four tests is 85?

8 A person has an average of 65 after four tests. What mark must the person get on the fifth test so that the average of the five tests is 70?

9 The average of five numbers is 4. When the sixth number is added, the average is 5. What is the sixth number?

10 The scores for the first three innings were 25, 3, and 12. What was the score for the fourth innings if the average for all four innings was 12?

11 The average for the first three innings was 12. The fourth and fifth innings produced scores of 24 and 6. If after the sixth innings the average was 14, what was the score for the sixth innings?

A cricketer scored 84 and this raised the cricketer's average from 36 to 39. What must the cricketer score in the next innings to raise the average to 40?

Assuming n innings: $\frac{36n + 84}{n + 1} = 39$ $36n + 84 = 39n + 39$ $n = 15$ (15 innings)	Assume next score is x: $\frac{39 \times 16 + x}{17} = 40$ $624 + x = 680$ $x = 56$

12 A cricketer scored 90 and this raised the cricketer's average from 26 to 28. What must the cricketer score in the next innings to raise the average to 30?

13 A cricketer scored 80 and this raised the cricketer's average from 30 to 32. What must the cricketer score in the next innings to raise the average to 35?

14 A cricketer scored 80 and this raised the cricketer's average from 30 to 32. What did the cricketer score in the next innings to raise the average to 40?

Investigations

Investigation 8.1 Undertake real-life research.

1 Form a team and brainstorm an appropriate problem or issue.

2 Plan the research
- Define the overall research question.
- Define subset research questions.
- Decide how to obtain data to answer the research questions.
- Consider ways of ensuring that the data collection is unbiased.
- Consider the equipment needed for the research.

3 Conduct the research
- Rehearse the data collection method.
- Collect the data.

If it's zero degrees outside today and it will be twice as cold tomorrow, how cold will it be?

4 Analyse the data
- Look for errors/outliers and decide what to do with the errors/outliers.
- Calculate the appropriate descriptive statistics.
- Choose an appropriate method of presentation (Histograms etc).

5 Report the conclusions
- Match the analysis with the research questions.
- Do the answers to the research questions indicate further research questions?

Investigation 8.2

Collect the heights of a Year 12 or Year 10 form class or maths class and the heights of your own form class or maths class. Construct frequency column graphs.

a) While it is expected that Year 12s are taller than Year 8s, does the Year 8 frequency column graph have a similar shape to the Year 12 frequency column graph?

b) Is the data symmetrical?

Does the mean of your class have a value close to the mean of the Year 12 class?

Does the median of your class have a value close to the median of the Year 12 class?

Investigation 8.3 Online statistical activities

There are a large number of national and international online statistical activities for Year 8 students. These activities generally involve the collection of data from your class to form a large national data set or international data set. This then forms the basis of infomed research on many relevant topics.

Become involved in one of these activities.

Statisticians design, collect, and analyse data using a variety of statistical techniques to provide information for a large range of clients.
- Relevant school subjects are Mathematics.
- Courses range from Bachelor degrees to Master degrees.

Technology

Technology 8.1 The Calculator

Scientific calculators will calculate descriptive statistics such as mean and sum.

1 Change the calculator mode to Stat or SD

2 Clear the calculator.

2 Enter a number then press M+

3 Repeat entering a number and then pressing M+

4 Find the $\bar{x}$ button, this is the mean.

5 Find the $\sum x$ button, this is the sum of the numbers.

Check your calculator's manual for Statistical calculations

Technology 8.2 The Spreadsheet

Most spreadsheets will calculate a massive number of descriptive statistics **and** draw a frequency polygon:

1 Enter a set of data into the spreadsheet.

2 Enter formulas for the mean, median, and mode.

	A	B	C
1	2	mean =	2.6
2	2	median =	2
3	2	mode =	2
4	3		
5	4		

=mean(A1:A5)

=median(A1:A5)

=mode(A1:A5)

Technology 8.3 The Graphics calculator

A graphics calculator will automatically calculate a large number of descriptive statistics **and** draw a frequency column graph:

1 Select the STAT menu, EDIT, and enter data into one of the lists.

2 Return to the main screen.

3 Select the STAT menu, Calc, 1_Stats and enter L1.

L1	L2	L3
2		
2		
2		
3		
4		

→

1_Stats

$\bar{x}$ = 2.6

median = 2

n = 5

$\sum x$ = 13

+ heaps more.

To draw a frequency column graph:

1 Enter the numbers in L1 and the frequency in L2.

2 Use STATPLOT to set up the graph Xlist= L1 and Frequency = L2.

3 Use Zoom and ZooMSTAT to fit the graph if necessary.

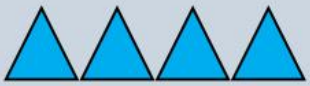
4 TRACE.

L1	L2	L3
2	3	
3	1	
4	1	

→

A Couple of Puzzles

Exercise 8.14



1 Join these four triangles together to make one large triangle.

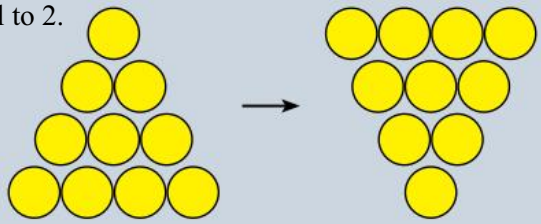
2 How many integers between 1 and 100 have the sum of their digits equal to 10 (82 is one such integer)?

3 The numbers from 1 to 20 are put in a box. What is the probability of selecting a prime number from the box?

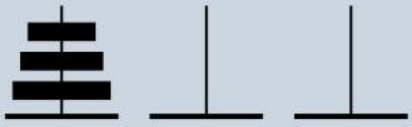
4 What are the next three numbers in the sequence: 1, 3, 6, 10, …

5 Five positive integers have their mode, median, and mean all exactly equal to 2. What are the five numbers?

6 By moving just three coins, turn the triangle of coins upside down.



A Game



Can you do it in 7 moves? Do you want to try 4 disks? You will hear more about this well-known puzzle.

The Tower of Hanoi is a game played by one person. The aim of the game is to move the three disks from the left peg to the right peg. The game can be played with different sized coins.

1 Only one disk can be moved at a time.

2 A larger disk must never be above a smaller disk

There are many Tower of Hanoi Applets on the Internet. Try one of them.

A Sweet Trick

Ask for a number

1 Double the number

2 Add 6

3 Divide by 2

4 Subtract the original number

The answer is 3

$= (2x + 6) \div 2 - x$
$= x + 3 - x$
$= 3$
No matter what number you give me, the answer will be 3.

What about this trick?

1 Double the number

2 Add 10

3 Divide by 2

4 Subtract the original number

Chapter Review 1

Exercise 8.15

1 What is wrong with each of the following:

a) 15 classmates were asked if they intended going to the social. 9 said yes. The conclusion was that 60% of the school's population would go to the social.

b) Survey forms, asking about what music should be played at the upcoming school dance, were placed in the library along with the response box.

2 Find the range, mean, median, and mode of each of the following data sets:

a) 5, 2, 3, 4, 5, 4, 3, 4 **b)** 6.5, 6.5, 6.5, 6.6, 6.8, 6.7, 6.5

3 Which central measure, mean, mode, or median would be most useful in each of the following cases?

a) The average height of Year 8 students

b) The average number of animals in each family

c) The average wage

4 Draw a frequency column graph for each of the following surveys after first constructing a frequency table:

a) A coin was thrown 38 times:

H	H	T	T	H	T	H	T	T	T	H
H	T	H	T	H	T	H	H	T	T	T
T	H	H	T	T	H	H	T	T	H	H
T	T	H	T	H						

b) A survey on whether mobile phones are a classroom distraction:

Disagree	Strongly agree	Agree	Disagree	Disagree
Disagree	Agree	Agree	Disagree	Disagree
Disagree	Agree	Agree	Agree	Disagree
Agree	Disagree	Disagree	Disagree	Agree
Agree	Disagree	Disagree	Disagree	Strongly disagree

5 Suspecting that there were different distributions of students in form groups, year level data was taken from two form groups. Analyse the data and make a comment.

Red form						Blue form					
7	8	8	12	9	9	8	7	10	8	10	10
10	12	11	7	8	10	9	11	10	10	12	11
11	8	12	11	12	7	11	9	7	7	8	7
7	7	8	7	12	8	11	12	10	12	11	12
9	8	10	11			7	10	10	12	8	10

6 A person has an average of 80 after four tests. What mark must the person get on the fifth test so that the average of the five tests is 85?

Chapter Review 2

Exercise 8.15

1 What is wrong with each of the following:

a) 5 Year 7s, 5 Year 8s, 5 Year 9s, 5 Year 10s, were asked about their study habits. The conclusion was that between 30% and 80% of students study for less that 2 hours per night.

b) Survey forms, asking about what music should be played at the upcoming school dance, were placed in the library along with the response box.

2 Find the range, mean, median, and mode of each of the following data sets:

a) 3, 4, 3, 2, 5, 2, 3, 5 **b)** 1.2, 1.4, 1.1, 1.3, 1.3, 1.2, 1.2

3 Which central measure, mean, mode, or median would be most useful in each of the following cases?

a) The average weight of Year 8 students.

b) The most popular shoe size.

c) The average house price.

4 Draw a frequency column graph for each of the following surveys after first constructing a frequency table:

a) A die was thrown 36 times:

5	2	1	5	6	5	1	3	4	1
6	2	6	6	5	3	2	6	5	4
4	1	6	1	3	3	3	1	6	3
2	4	3	2	1	4				

b) A survey on whether rubbish in the schoolgrounds is a problem:

Disagree	Strongly agree	Agree	Disagree	Disagree
Strongly disagree	Agree	Disagree	Disagree	Disagree
Agree	Disagree	Disagree	Agree	Agree
Disagree	Strongly agree	Agree	Disagree	Disagree
Agree	Disagree	Strongly agree		

5 Suspecting that drivers were more careless with their car speed on public holidays, radar gun readings rounded to the nearest 5 km/h, were taken in a 50 km/h zone were taken on a normal workday and the public holiday the next day. Analyse the data and make a comment.

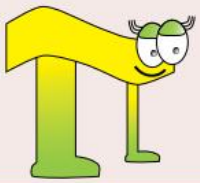
Workday						Public holiday					
50	40	50	45	45	50	50	55	50	50	50	65
50	40	40	35	45	40	50	55	60	50	40	50
50	50	55	60	70	55	55	50	40	65	70	50
50	40	40	55	45	45	55	50	60	50	55	50
50	60	70	55	50	50	50	55	45	50	55	50

6 A person has an average of 45 after four tests. What mark must the person get on the fifth test so that the average of the five tests is 50?

Circles

- ★ Investigate the relationship between features of circles such as circumference, area, radius and diameter.
- ★ Use formulas to solve problems involving circumference and area.
- ★ Investigate the circumference and area of circles with materials or by measuring, to establish an understanding of formulas.
- ★ Investigate the area of circles using a square grid or by rearranging a circle divided into sectors.

Hi I'm Pi.
My name rhymes with hi and bye.



A TASK

π is a very important mathematical and physical constant and is often used in engineering, science and mathematics.

- Research the history of π.
- π is irrational. What does this mean?
- Design an experiment to calculate π.
- Report your findings (poster, oral report, etc.).

A LITTLE BIT OF HISTORY

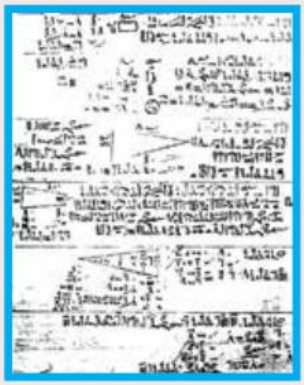
The Ahmes Papyrus is about 6 m long and 35 cm wide. It was a copy, made around 1650 BC, of a 200 year old version.

The Papyrus contains 87 mathematical problems.

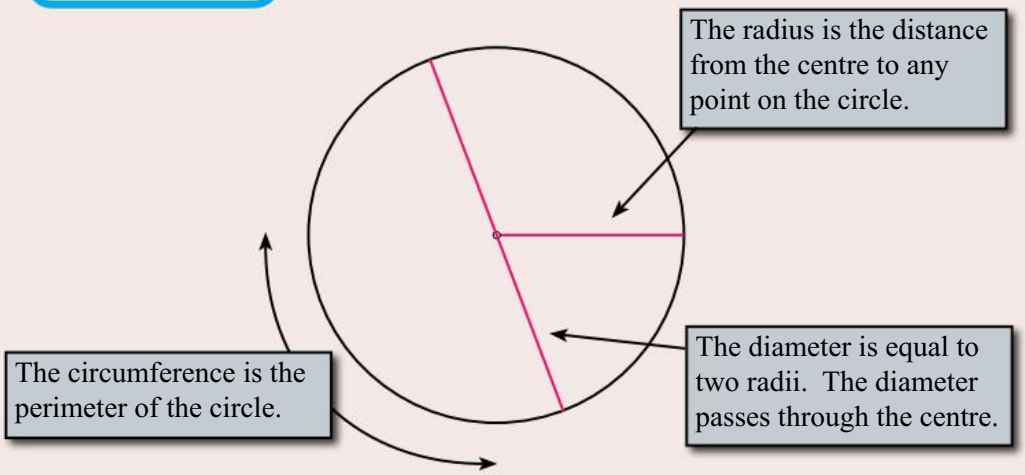
The Papyrus has a formula for the area of a circle: $Area = \frac{64d^2}{81}$

This is the same as: $Area = \frac{256r^2}{81}$

Thus their $\pi = 3.16$



The Circle



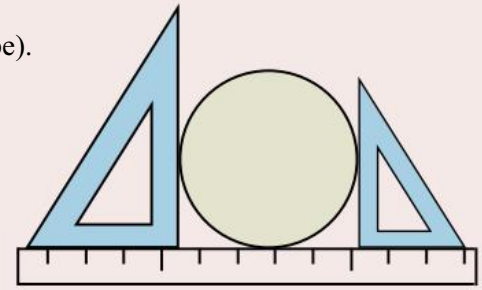
Circumference

What is the relationship between the circumference and the diameter?

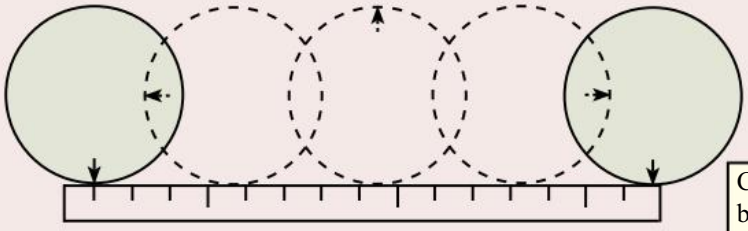
Exercise 9.1

1 Get some circles
(e.g., drink cans, coins, toilet rolls, pipe).

2 Measure the diameter.



3 Measure the circumference.



Can you think of a better way to measure the circumference?

4 Calculate the ratio

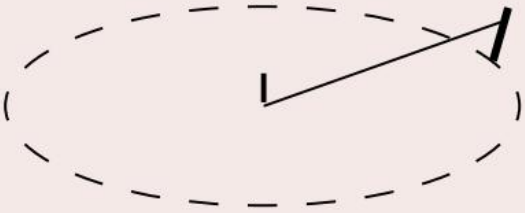
Object	Diameter (d)	Circumference (C)	C ÷ d
Jar of curry paste Drink can	6.5 cm	21.2 cm	21.2 ÷ 6.5 = 3.26

It has been known for hundreds of years

That $\mathbf{C \div d}$

is always between 3.14 and 3.15

Will using larger circles give a better result?



Exercise 9.2

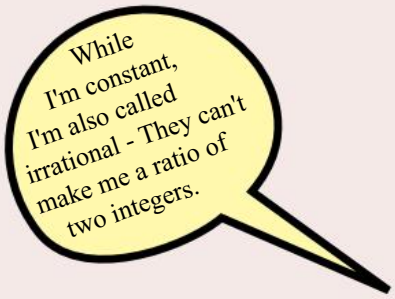
1 Use a piece of chalk and a measuring tape to draw a circle with a radius of 2 m.

2 Carefully use the tape measure to measure the circumference.

3 Draw larger and larger circles.

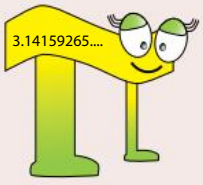
4 Calculate the ratio of the circumference to the diameter for each circle:

Circle	Diameter (d)	Circumference (C)	C ÷ d
Radius = 2 m	4 m		
Radius = 3 m	6 m		



Q. Divide the Sun's circumference by its diameter and what do you get?

A. Pi in the sky.

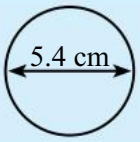
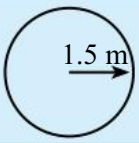


Circumference

Thus, for a circle: Circumference ÷ diameter= **π**
or Circumference = **π**×diameter {multiply both sides by diameter}
or Circumference = 2**π**radius {because diameter = 2 × radius}

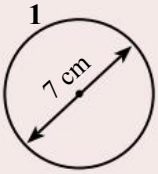
C = πd **or** **C = 2πr**

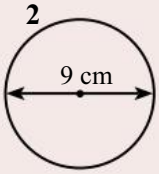
Exercise 9.3
Calculate the circumference of each of the following circles
(Answer correct to 2 decimal places):

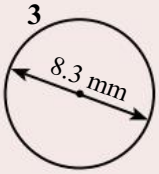
C = πd = π × 5.4 = 16.96 cm 	C = 2πr = 2 × π × 1.5 = 9.42 m 

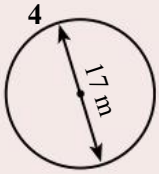
Rounding to two decimal places:

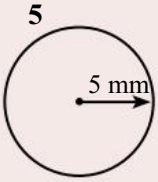
56.231694 less than 5 thus **56.23**	27.01769 5 or more thus **27.02**	1.07276 less than 5 thus **1.07**	4.79634216 5 or more thus **4.80**

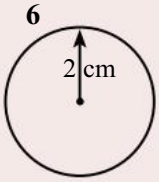
1 

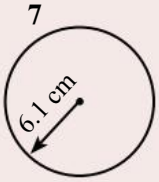
2 

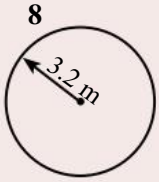
3 

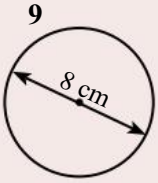
4 

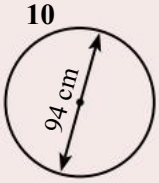
5 

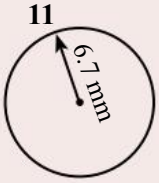
6 

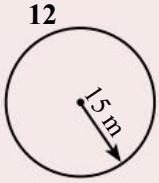
7 

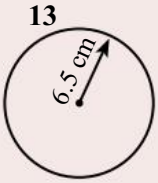
8 

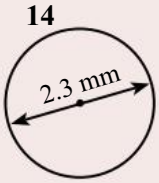
9 

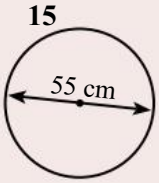
10 

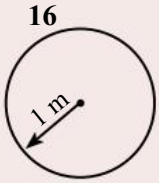
11 

12 

13 

14 

15 

16 

Perimeter

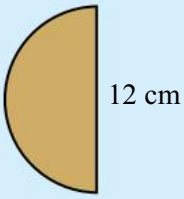
The perimeter is the distance around the outside of the shape.

Exercise 9.4

Find the perimeter of each of the following shapes:

Circumference halfcircle $= \pi D \div 2$
$= \pi \times 12 \div 2$
$= 18.85$ cm

Perimeter $= 12 + 18.85$
$= 30.85$ cm



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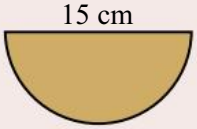

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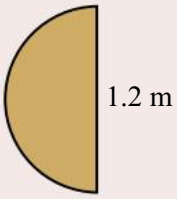

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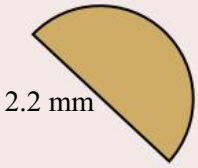

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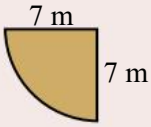

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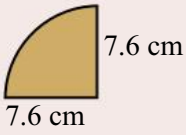

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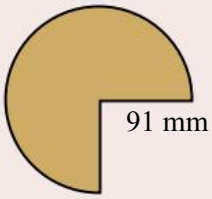
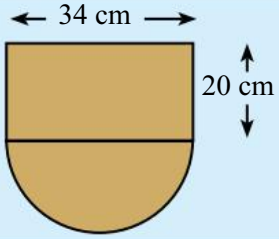


Circumference halfcircle $= \pi D \div 2$
$= \pi \times 34 \div 2$
$= 53.41$ cm

Perimeter $= 53.41 + 20 + 20 + 34$
$= 127.41$ cm



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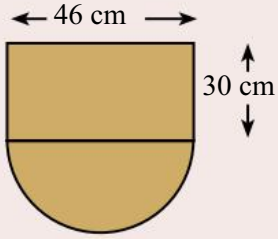

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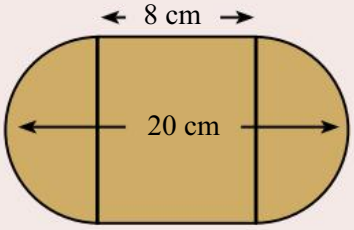


9 A circular flower bed has a diameter of 2.3 m. What is the length of the edge of the bed?

10 If the moon is assumed to be spherical with a radius of 1720 km, what is the circumference of the moon?

11 To make a trundle wheel with a circumference of 1 m, what would be the radius of the wheel?

Area of a Circle

What is the relationship between the area and the square of the radius?

Exercise 9.5

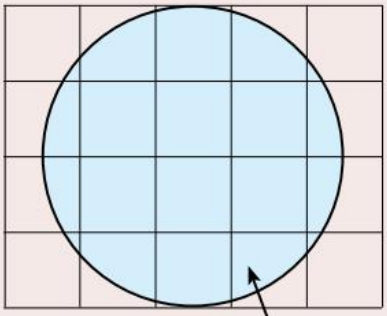
1 Get some circles
(e.g., drink cans, coins, toilet rolls, pipe).

2 Outline the circle on paper with 1 cm grid.

3 Measure the radius (halve the diameter?).

4 Estimate the area by counting the squares

5 Calculate the ratio of the area and the square of the radius.



Is this about 12 squares?

Object	Radius (r)	Area (A)	$A \div r^2$
Perfume bottle Drink can	$4 \div 2 = 2$ cm	12 cm^2	$12 \div 2^2 = 3.0$

It has been known for hundreds of years

That $A \div r^2$

is always between 3.14 and 3.15

Thus $A \div r^2 = \pi$ **or** $A = \pi r^2$

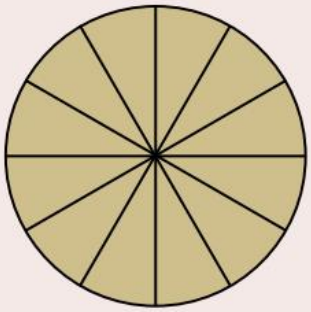
Exercise 9.6

1 Draw a circle with a radius of 10 cm.

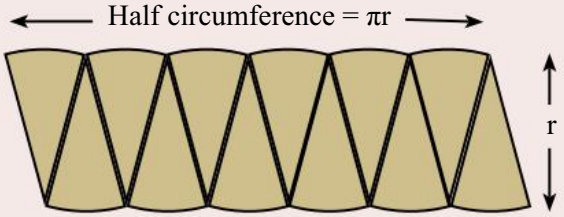
2 Divide the circle into 12 sectors.
(each sector $360^\circ \div 12 = 30^\circ$)

3 Cut out seach sector.

4 Arrange the sectors into a rectangle.



5 What is the area of the rectangle (length × breadth)?

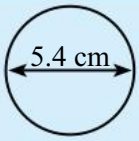
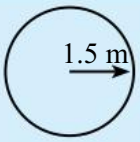


$A = \pi r \times r$

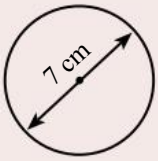
$A = \pi r^2$

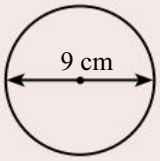
Exercise 9.7

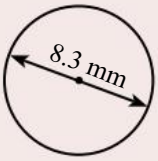
Calculate the area of each of the following circles (Correct to 2 decimal places):

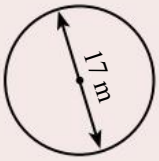
A $= \pi r^2$ $= \pi \times (5.4 \div 2)^2$ $= 22.90\ cm^2$ 	A $= \pi r^2$ $= \pi \times 1.5^2$ $= 7.07\ m^2$ 

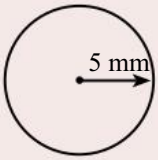
Using the calculator: $A = \pi \times (5.4 \div 2)^2$

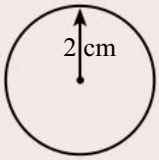
1 

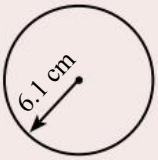
2 

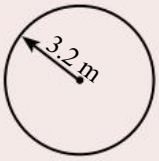
3 

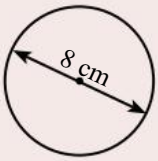
4 

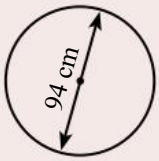
5 

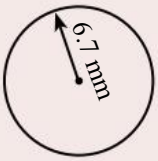
6 

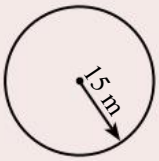
7 

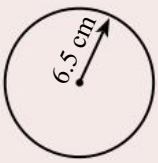
8 

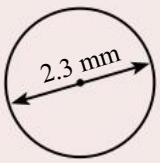
9 

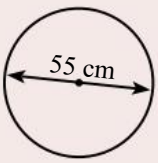
10 

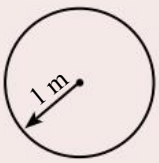
11 

12 

13 

14 

15 

16 

Broadcasting Technicians install, start up, maintain and repair equipment used for the transmission and reception of television and radio broadcast signals. They may specialise as a wireless communications technician.

- Relevant school subjects are Mathematics, Science.
- Courses usually involve an apprenticeship/certificate.

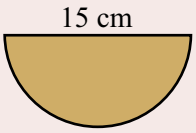
Exercise 9.8

Find the area of each of the following shapes:

Area of halfcircle

$= \pi r^2 \div 2$
$= \pi \times 6^2 \div 2$
$= 56.55 \text{ cm}^2$

1



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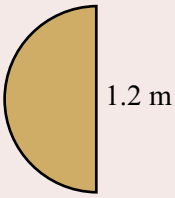

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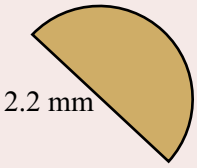

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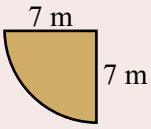

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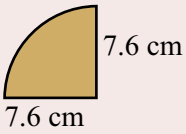

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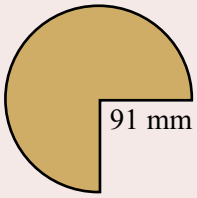
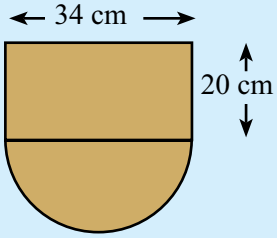


Area = Area halfcircle + Area rectangle
$= \pi r^2 \div 2$ + lxb
$= \pi \times 17^2 \div 2$ + 34 × 20
= 453.96 + 680
$= 1133.96 \text{ cm}^2$



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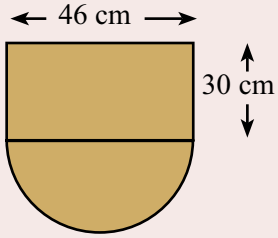

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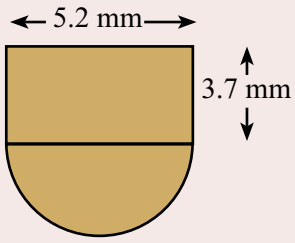

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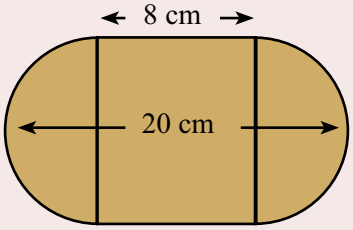

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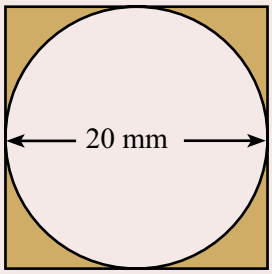


Mental Computation

Exercise 9.9

1 Spell perimeter.
2 What is the formula for the perimeter of a circle?
3 What is the formula for the area of a circle?
4 What is the symbol for pi?
5 Why is pi irrational?

Given the data set: 2,2,3,4,5

6 What is the median?
7 What is the mean?

8 Translate P(2,3) 3 units to the right and 4 units up.

9 Change $2\frac{1}{3}$ to a vulgar fraction.

10 Round 64.8219 to 2 decimal places.

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 9.10

1 Spell circumference.
2 What is the formula for the circumference of a circle?
3 What is the formula for the area of a circle?
4 What is the symbol for pi?
5 Why is pi irrational?

Given the data set: 1,2,3,3,6

6 What is the median?
7 What is the mean?

8 Translate P(5,2) 3 units to the left and 3 units down.

9 Change $3\frac{1}{2}$ to a vulgar fraction.

10 Round 9.8563 to 2 decimal places.

The blast radius of a grenade is always greater than the distance you can jump - Murphy's laws of combat.

Exercise 9.11

1 Spell diameter.
2 What is the formula for the circumference of a circle?
3 What is the formula for the area of a circle?
4 What is the symbol for pi?
5 Why is pi irrational?

Given the data set: 1,1,2,3,3

6 What is the median?
7 What is the mean?

8 Translate P(2,3) 4 units to the right and 4 units up.

9 Change $2\frac{2}{3}$ to a vulgar fraction.

10 Round 88.776 to 2 decimal places.

The real secret of success is enthusiasm - Walter Chrysler, entrepreneur.

NAPLAN Questions

Exercise 9.12

1 The diameter of a circular table top is 1.7 metres. What is the circumference to the nearest metre?

2 The circumference of a wheel is 2.3 metres. What is the radius of the wheel to the nearest centimetre?

3 The area, A, of a circle is given by the formula $A = \pi r^2$, where r is the radius. When r = 4.6 cm, what is the area of the circle?

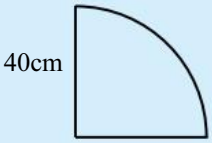
4 The area, A, of a circle is given by the formula $A = \pi r^2$, where r is the radius. When A = 90 cm^2, what is the radius to the nearest centimetre?

5 The volume, V, of a cylinder is given by the formula: $V = \pi r^2 h$, where r is the radius of the base of the cone and h is the height of the cone. What is the volume of a cylinder with base radius of 5 cm and height of 7 cm?

The quarter circle shown has a radius of 40 cm.

a) What is the curved distance?

b) What is the perimeter?



$C_{quartercircle} = \pi D \div 4$

$= \pi \times 80 \div 4$

$= 62.83$ cm

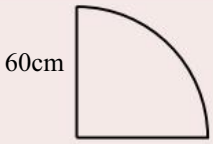
Perimeter $= 40 + 40 + 62.83$

$= 142.83$ cm

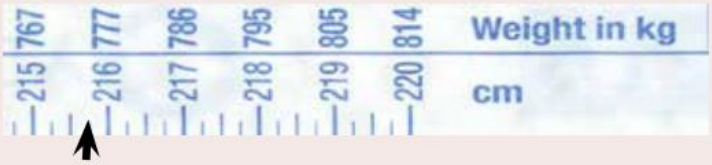
6 The quarter circle shown has a radius of 60 cm.

a) What is the curved distance?

b) What is the perimeter?



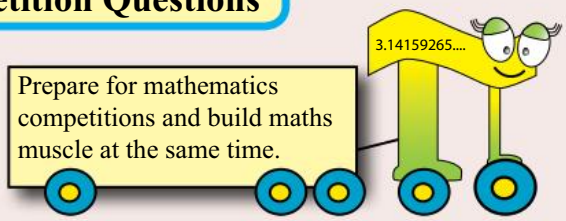
7 The tape was placed around the horse, exactly behind the front legs as required. What is the weight of the horse?



8 The diameter of Jupiter is approximately 3 times the diameter of Neptune and 12 times the diameter of Venus. How many times larger is the diameter of Neptune than the diameter of Venus?

9 The diameter of Jupiter is approximately 12 times the diameter of Venus and 21 times the diameter of Mars. How many times larger is the diameter of Venus than the diameter of Mars?

Competition Questions



Make sure the units are the same.

Exercise 9.13

1 How many times will a 700 mm diameter bicycle wheel turn in covering 5 km?

2 The radius of a circle has been doubled. What has happened to the circumference of the circle?

3 The radius of a circle has been tripled. What has happened to the circumference of the circle?

The radius of a circle has been doubled. What has happened to the area of the circle?

Circle of radius r: $A = \pi r^2$

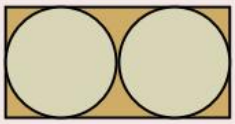
Circle of radius 2r: $A = \pi(2r)^2 = 4\pi r^2$

The area is 4 times the size

4 The radius of a circle has been tripled. What has happened to the area of the circle?

5 The radius of a circle has quadrupled. What has happened to the area of the circle?

6 A rectangle is drawn around two circles. Each circle has a radius of 3 cm. What is the area of the rectangle?



7 Assuming that the Earth's radius is 6400 km, at what average speed would you have to travel to get around the world in 80 days.

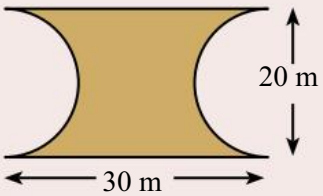
8 A belt is placed around the Earth's equator.

a) What would be the length of the belt assuming a diameter of 12800 km?

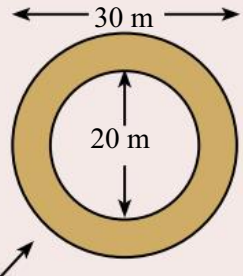
b) If 10 m was added to the length of the belt, what would be the distance between the belt and the Earth's surface?

9 Find the shaded area of each of the following shapes:

a)



b)



This is an annulus - the area between two circles

Investigations

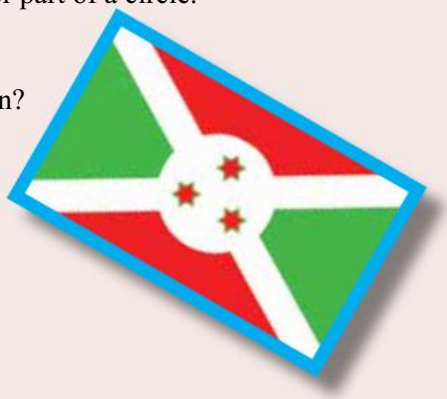
Investigation 9.1 Flags and circles

The flags of many countries contain a circle or part of a circle.

1 The Olympic flag.
- **a)** Draw and colour the flag.
- **b)** What do the circles and colours mean?

2 The United Nations flag.
- **a)** Draw and colour the flag.
- **b)** What do the circles mean?

3 Find another flag with circles.
- **a)** Draw and colour the flag.
- **b)** What is the meaning?



Investigation 9.2 Calculating Pi

One method is to calculate as many terms of the following infinite sum as you need to get the accuracy that you want:

$$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 + 4/13 - 4/15 + ...$$

Complete the following until you are satisfied with your answer:

2 terms	$\pi = 4 - 4/3$	$= 2.666666$
3 terms	$\pi = 4 - 4/3 + 4/5 - 4/7$	$= 3.466666$
4 terms	$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9$	$= 3.339683$
5 terms	$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11$	$= 2.976046$
6 terms	$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 + 4/13$	$= 3.283738$
7 terms	$\pi =$	
8 terms	$\pi =$	
9 terms	$\pi =$	
10 terms	$\pi =$	

There is probably an easier way - Can you think of one?

Calculators use infinite series for many of their functions - they are much faster than we are.

Investigation 9.3 Pi in Pizza

1 Calculate the area of each of the following Pizzas and use your answer to decide which pizza offers the best value:

Size	Diameter	Price
Small	18 cm	$12
Medium	25 cm	$15
Large	30 cm	$18
Monster	45 cm	$40

Q What did the farmer use to make crop circles?
A A protractor.

2 Investigate the pricing at your local Pizza. Are there large differences in value for various sizes?

Technology

Technology 9.1 Circumference of planets

Use a spreadsheet to calculate the circumference of the planets:

Enter the formula: =pi()*B1

Enter the formula: =Round(C1,0)

Planet	Diameter	Circumference	Rounded 0 places
Mercury	4842 km	15211.59163	15212
Venus	12232 km		
Earth	13462 km		
Mars	6754 km		
Jupiter	137614 km		
Saturn	114678 km		
Uranus	52242 km		
Neptune	49056 km		

Technology 9.2 Calculating Area

Use a graphics calculator to find the area of circles:

You will need to find the variable button to enter x.

Press **Y=** and enter the formula for area: πx^2

Press **Calc** and enter the radius. The area will then be given.

Technology 9.3 Circle Applets

There are many Internet applets showing all kinds of activities involving circles. Use a search phrase such as "circle applet" and investigate the many applets.

Technology 9.4 Calculating Pi

Use a spreadsheet to sum an infinite series:

$$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 + 4/13 - 4/15 + ...$$

Term	
1	4
2	-1.333333333
3	0.8
Sum =	

Enter the formula =(-1)^(A2-1)*4/(2*A2-1)

Do you know how this works?

A Couple of Puzzles

Exercise 9.14

1 This a well known puzzle.
Move just 2 matches so that the coin is no longer inside the wine glass.

2 Fred is 30 years older than Ted.
Their total age is 31 years.
How old is Ted? (The answer is not 1).

3 The chance of tossing a coin and getting a tail is 0.5.
What is the chance of two people tossing a coin and both getting a tail?

A Game

Bingo.

1 25 problems are made so that each answer is equal to each of the numbers from 1 to 25. Eg. 3x4+1=13, 26-12=14, half of 30 = 15, etc.

2 Each member of the class makes a 3x3 square and writes a number from 1 to 25 in each of the nine squares

3 Each of the 25 problems are randomly called out and each time the answer occurs on a class member's 3x3 square, the square is marked.

4 The winner calls BINGO when any three in a row, column, diagonal is marked on their square.

5 Many extensions to this game are possible.

A Sweet Trick

1	Ask your audience to write down any set of ascending numbers without showing you	14689	
2	Ask them to multiply by 10	14689x10	= 146890
3	Subtract the first number from the second number	146890 - 14689	= 132201
4	Add 9	132201+9	= 132210
5	Ask them to cross out any number except 0, and ask them to tell you the remaining numbers.	remaining are 1,3,2,1,0	
6	You tell them they crossed out a 2.		

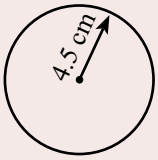
1 + 3 + 2 + 1 = 7
Subtract 7 from 9 to give the crossed out number.

Chapter Review 1

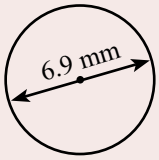
Exercise 9.15

1 Calculate the circumference of each of the following circles (2 decimal places):

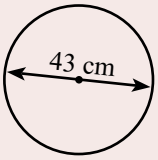
a)



b)



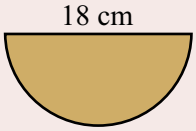
c)



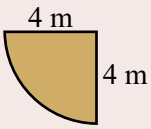
d)

2 Find the perimeter of each of the following shapes (2 decimal places):

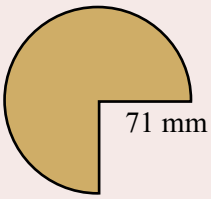
a)



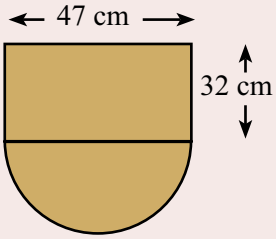
b)



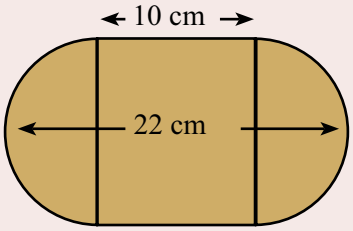
c)



d)

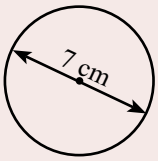


e)

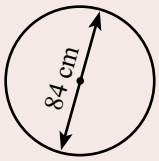


3 Calculate the area of each of the following circles (2 decimal places):

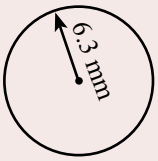
a)



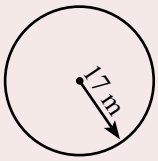
b)



c)

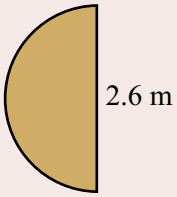


d)

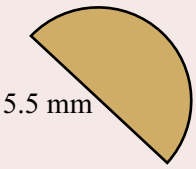


4 Find the area of each of the following shapes (2 decimal places):

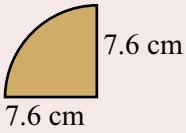
a)



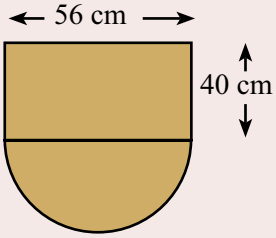
b)



c)



d)



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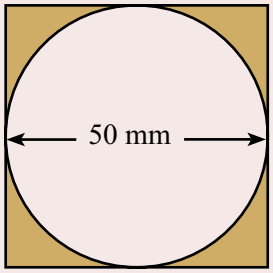
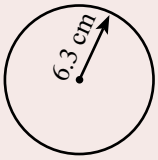


Chapter Review 2

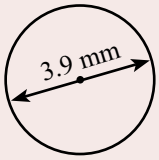
Exercise 9.16

1 Calculate the circumference of each of the following circles (2 decimal places):

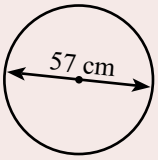
a)



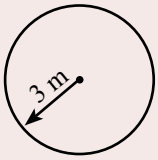
b)



c)

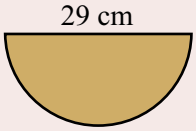


d)

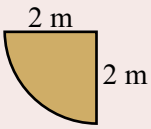


2 Find the perimeter of each of the following shapes (2 decimal places):

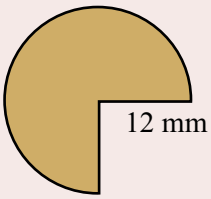
a)



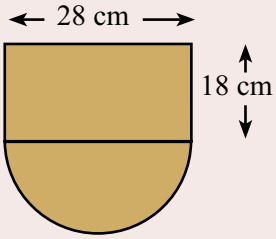
b)



c)



d)



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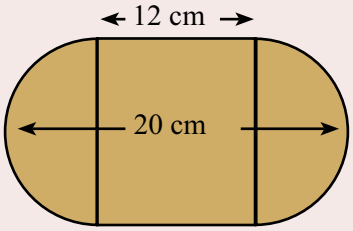
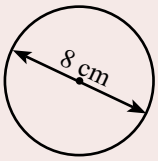
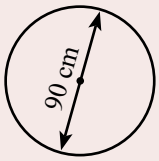


3 Calculate the area of each of the following circles (2 decimal places):

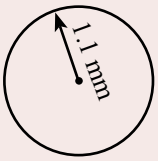
a)



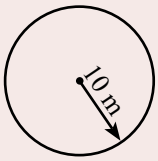
b)



c)

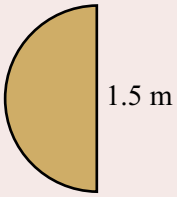


d)

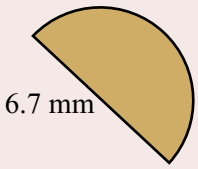


4 Find the area of each of the following shapes (2 decimal places):

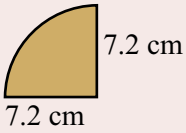
a)



b)



c)



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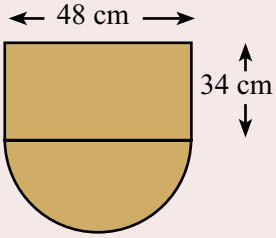

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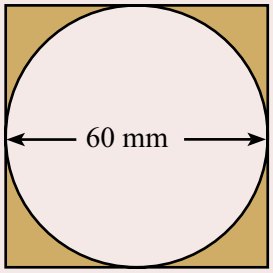


Review 2

10

Chapter 6 Real Numbers

Rational Numbers

Can be expressed as a ratio a/b where a and b are integers.

Integers.
$^{-}3, 5, ^{-}18, 0, 3, 9$

Fractions.
1/2, 3/4, 5/3, 6/7

Decimals.
(Either terminate or recurr.)

Irrational Numbers

Cannot be expressed as a ratio a/b where a and b are integers.

Surds.
$\sqrt{2} = 1.4142135...$
$\sqrt{3} = 1.7320508...$

Special numbers.
$\pi = 3.1415926...$
$e = 2.7182818...$

Chapter 7 Congruence

Tests of congruent triangles:
SSS (side, side, side).
SAS (side, angle, side).
ASA (angle, side, angle).
RHS (right-angle, hypotenuse, side).

Congruent shapes have the same shape and size.
A **translation** slides a shape across a page.
A **reflection** is a mirror image.
A **rotation** turns a shape about a point.

Chapter 8 Data Investigation

Census
A collection of data from the **whole** population.

Sample
A collection of data from **part** of the population.

The range is the difference between the smallest and largest value.
The mode is the score that occurs most often.
The median is the middle of a set of scores (Not affected by extreme values).
The mean is the sum divided by the number of values (affected by extreme values.

Chapter 9 Circles

Circumference of a circle

$$C = \pi d$$

The perimeter is the distance around the outside of the shape.

Area of a circle

$$A = \pi r^2$$

π
is irrational because it cannot be expressed as a ratio

Review 1

Exercise 10.1 Mental computation

1 Spell circumference.
2 What is the formula for the perimeter of a circle?
3 What is the formula for the area of a circle?
4 What is the symbol for pi?
5 Why is pi irrational?

Given the data set: 2,2,3,4,5

6 What is the median?
7 What is the mean?
8 Translate P(2,3) 2 units to the right and 4 units up.
9 Change $3\frac{1}{2}$ to a vulgar fraction.
10 Round 6.7284 to two decimal places.

Exercise 10.2

1 Show that each of the following integers are Rational numbers.
a) 5 **b)** $^{-}4$ **c)** 13

2 Arrange the integers in ascending order (smallest to largest):
a) 4, $^{-}1$, 0, $^{-}2$ **b)** 1, $^{-}2$, 0, $^{-}1$ **c)** $^{-}3$, 2, $^{-}1$, 1

3 Write each of the following decimals as a percentage and as a fraction:
a) 0.1 **b)** 0.15 **c)** 2.25 **d)** 0.05

4 Write each of the following percentages as a decimal and as a fraction:
a) 10% **b)** 70% **c)** 25% **d)** 2.25%

5 Write each of the following fractions as a decimal and as a percentage (if necessary round to 2 decimal places):
a) $\frac{1}{10}$ **b)** $\frac{2}{5}$ **c)** $\frac{7}{10}$ **d)** $\frac{3}{4}$

6 Write each of the following as a fraction, as a decimal, and as a percentage:
a) Two-fifths of the oranges had to be thrown out.
b) Water makes up approximately 75% of a person's body.
c) The dodgy loan rate was set at 21%.

7 Change each of the following mixed numbers to an improper fraction:
a) $2\frac{1}{4}$ **b)** $1\frac{3}{5}$ **c)** $3\frac{3}{4}$ **d)** $5\frac{1}{3}$

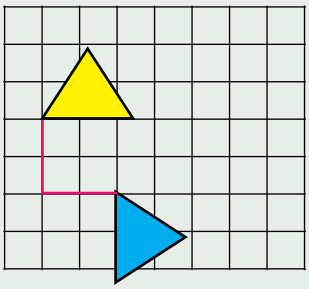
8 Change each of the following improper fractions to a mixed number:
a) $\frac{5}{3}$ **b)** $\frac{10}{3}$ **c)** $\frac{11}{4}$ **d)** $\frac{17}{5}$

9 Is 0.6363636363636363636363636363636363636363636 ($=\frac{7}{11}$) a rational number or an irrational number?

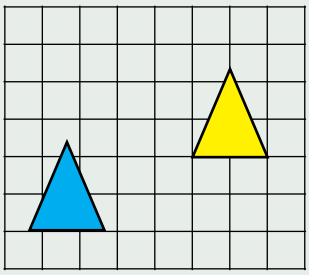
10 Is $\sqrt{6}$ a rational number or an irrational number?
$\sqrt{6}$ = 2.449489742783178098197284074705 9

11 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

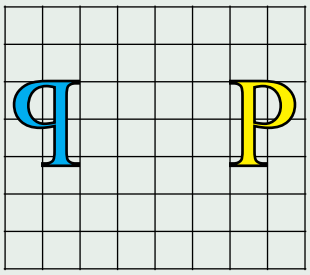
a)



b)

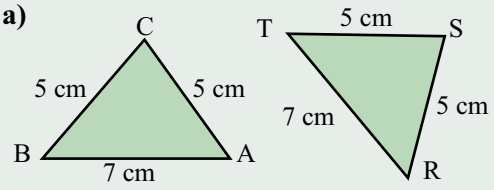


c)

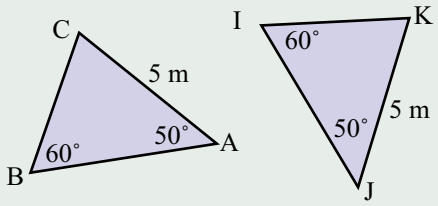


12 Use the tests for congruence to test whether the following pairs of triangles are congruent:

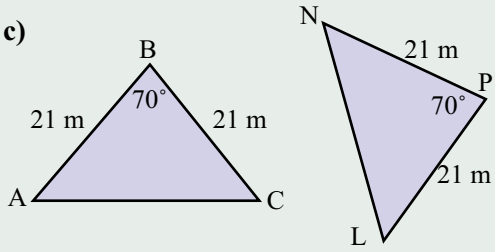
a)



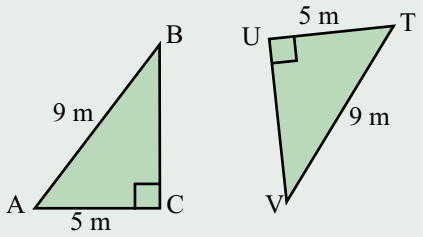
b)



c)



d)



13 What are the new coordinates of P(3,2) after P is translated 4 units right and 6 units down?

14 What is wrong with the following:
3 Year 7s, 3 Year 8s, 3 Year 9s, 3 Year 10s, were asked about their study habits. The conclusion was that between 50% and 80% of students study for less that 2 hours per night.

15 Find the range, mean, median, and mode of each of the following data sets:

a) 3, 4, 3, 2, 3, 2, 3, 5

b) 2.2, 2.4, 2.1, 2.3, 2.3, 2.2, 2.2

16 Which central measure, mean, mode, or median would be most useful in each of the following cases?

a) The average weight of Year 8 students.

b) The most popular dress size.

c) The average house price.

17 Draw a frequency column graph for the survey on the appropriateness of the tuckshop food, after first constructing a frequency table:

Agree	Strongly agree	Agree	Disagree	Disagree
Strongly Agree	Agree	Disagree	Agree	Disagree
Agree	Disagree	Disagree	Agree	Agree
Agree	Strongly agree	Agree	Disagree	Disagree
Agree	Disagree	Strongly agree		

18 Surveys of the playground rubbish were conducted both before and after a campaign raising awareness of littering (4-excellent, 3-good, 2-fair, 1-poor). Analyse the data and make a comment.

Before						After					
3	2	3	2	2	3	3	4	3	3	3	4
3	2	2	1	2	2	3	4	4	3	2	3
3	3	4	4	4	4	4	3	2	4	4	3
3	2	2	4	2	2	4	3	4	3	4	3
3	4	4	4	3	3	3	4	2	3	4	3

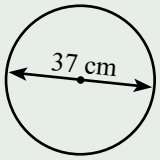
19 A person has an average of 48 after five tests. What mark must the person get on the sixth test so that the average of the six tests is 50?

20 Calculate the circumference and the area of each of the following circles (2 decimal places):

a)

b)

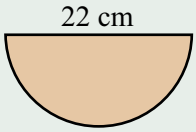
c)



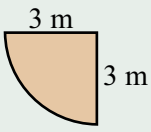
d)

21 Find the perimeter and the area of each of the following shapes (2 decimal places):

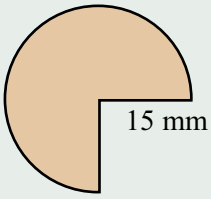
a)



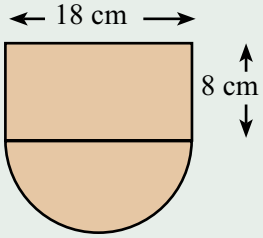
b)



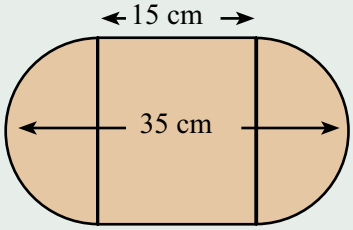
c)



d)



e)



Review 2

Exercise 10.3 Mental computation

1 Spell congruence.
2 What is the formula for the perimeter of a circle?
3 What is the formula for the area of a circle?
4 What is the symbol for pi?
5 Why is pi irrational?

Given the data set: 1,1,1,2,5

6 What is the median?
7 What is the mean?
8 Translate P(3,4) 3 units to the right and 3 units down.
9 Change $2\frac{2}{3}$ to a vulgar fraction.
10 Round 43.918362 to two decimal places.

Exercise 10.4

1 Show that each of the following integers are Rational numbers.
a) 2 **b)** $^{-}2$ **c)** $^{-}65$

2 Arrange the integers in ascending order (smallest to largest):
a) 2, $^{-}3$, 0, $^{-}1$ **b)** 4, $^{-}3$, 1, $^{-}2$ **c)** $^{-}1$, $^{-}2$, 0, 1

3 Write each of the following decimals as a percentage and as a fraction:
a) 0.2 **b)** 0.25 **c)** 3.5 **d)** 0.05

4 Write each of the following percentages as a decimal and as a fraction:
a) 10% **b)** 40% **c)** 25% **d)** 275%

5 Write each of the following fractions as a decimal and as a percentage (if necessary round to 2 decimal places):
a) $\frac{1}{10}$ **b)** $\frac{1}{5}$ **c)** $\frac{3}{10}$ **d)** $\frac{1}{4}$

6 Write each of the following as a fraction, as a decimal, and as a percentage:
a) About 30% of the World's surface is land..
b) The dodgy loan rate was set at 18%.

7 Change each of the following mixed numbers to an improper fraction:
a) $1\frac{3}{4}$ **b)** $2\frac{1}{5}$ **c)** $4\frac{2}{3}$ **d)** $5\frac{3}{5}$

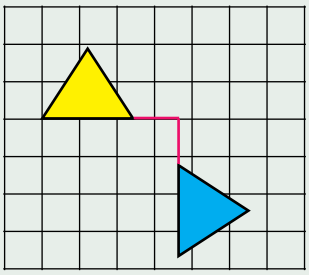
8 Change each of the following improper fractions to a mixed number:
a) $\frac{5}{2}$ **b)** $\frac{8}{3}$ **c)** $\frac{15}{4}$ **d)** $\frac{23}{6}$

9 Is 3.1176470588235294 1176470588235294 1176470588 ($=\frac{53}{17}$) a rational number or an irrational number?

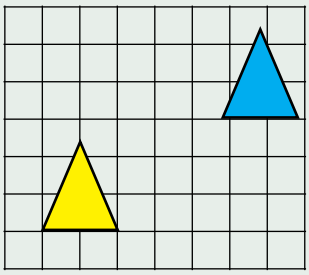
10 Is π a rational number or an irrational number?
π = 3.14159265358979323846264338327950288419716939937510528097494

11 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

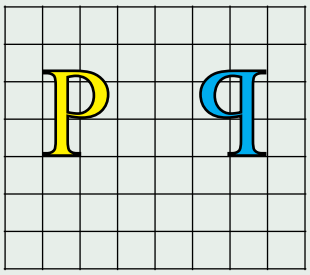
a)



b)

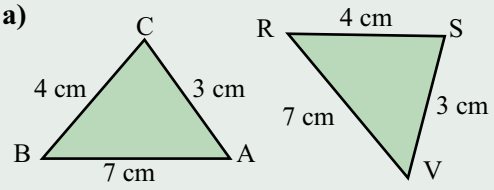


c)

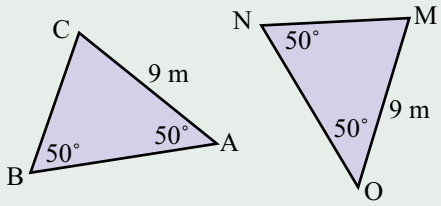


12 Use the tests for congruence to test whether the following pairs of triangles are congruent:

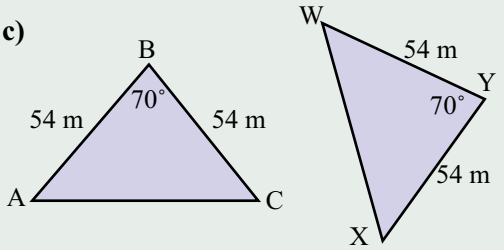
a)



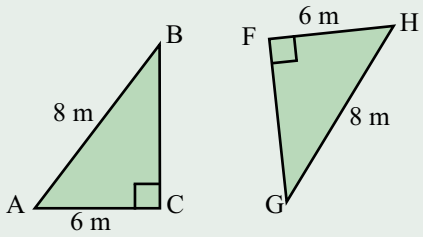
b)



c)



d)



13 What are the new coordinates of P(5,7) after P is translated 4 units right and 9 units down?

14 What is wrong with the following:
Survey forms, asking about what music should be played at the upcoming school dance, were placed in the library along with the response box.

15 Find the range, mean, median, and mode of each of the following data sets:

a) 2, 1, 1, 2, 3, 2, 4, 5

b) 6.2, 6.4, 6.1, 6.3, 6.4, 6.2, 6.4

16 Which central measure, mean, mode, or median would be most useful in each of the following cases?

a) The average weight of Year 8 students.

b) The most popular shoe size.

c) The average wage.

15 Draw a frequency column graph for 40 throws of a die after first constructing a frequency table:

1	2	1	5	6	5	6	3	4	1
6	2	6	6	5	3	2	6	5	4
5	1	6	1	3	3	3	2	6	3
2	4	3	2	6	4	1	6	4	4

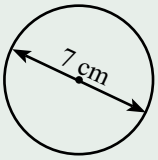
16 Curious about the reaction times of younger students, the following samples of reaction 'distance' were obtained (The distance a ruler falls between fingers before it is caught: 5-very quick, 4-quick, 3-average, 2-slow, 1-very slow). Analyse the data and make a comment.

Year 6						Year 8					
3	1	3	4	3	5	4	3	4	5	3	3
3	5	4	3	4	1	2	5	3	5	3	5
3	4	1	3	4	3	4	4	3	3	2	3
3	2	3	3	3	4	4	3	4	5	2	3

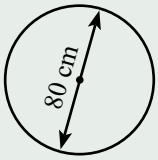
17 A person has an average of 76 after four tests. What mark must the person get on the fifth test so that the average of the five tests is 80?

18 Calculate the circumference and the area of each of the following circles (2 decimal places):

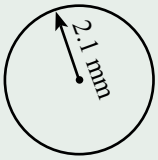
a)



b)



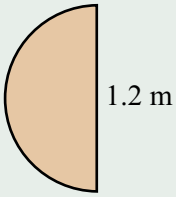
c)



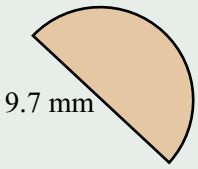
d)

19 Find the perimeter and the area of each of the shaded shapes (2 decimal places):

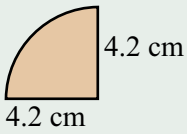
a)



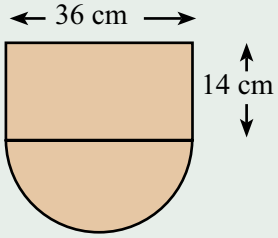
b)



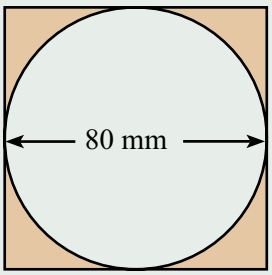
c)



d)



e)

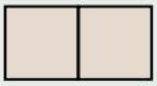


Isometric Drawing

Isometric grids can be found on the Internet.

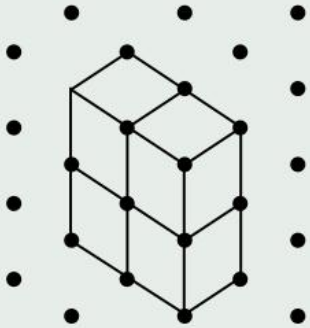
Example:
Make an isometric drawing of the following:

Top

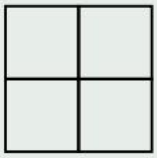
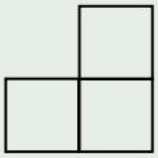
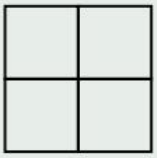
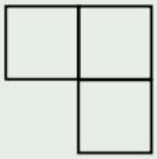
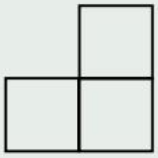
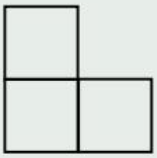
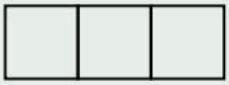
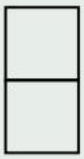
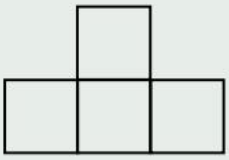
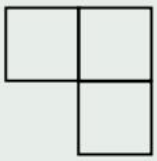
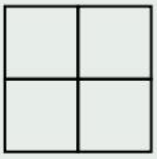
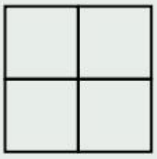


Side

Front



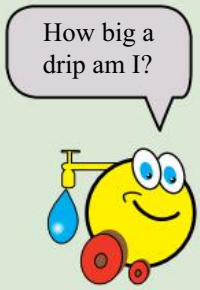
Make an isometric drawing of each of the following:

	Top	Side	Front
a)			
b)			
c)			
d)			

Ratio & Rate 11

- ★ Use percentages to solve problems, including those involving mark-ups, discounts, profit and loss and GST.
- ★ Express profit and loss as a percentage of cost or selling price, compare the difference.
- ★ Investigate the methods used in retail stores to express discounts.
- ★ Solve a range of problems involving rates and ratios, with and without digital technologies
- ★ Understand that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem.
- ★ Calculate population rates in Australia and Asia and explain their difference.

A TASK



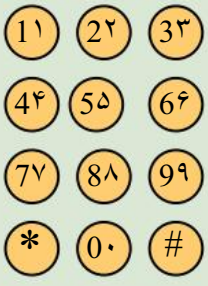
"A dripping tap can waste 5000 litres of water a year".

- Is this possible?
- Estimate the litres of water lost by a dripping tap in a year.
- Estimate the number of dripping taps in the local council area.
- How much water is wasted each year by dripping taps in the local council area?
- How do you fix a dripping tap?
- Write a report about dripping taps.
- Can you get it published in the local paper?

A LITTLE BIT OF HISTORY

Our number system is based on a very old number system known as the Hindu-Arabic system (around 500 AD).

While our numbers and the Arabic numbers have evolved to now look quite different, base 10 was very important in developing our mathematical knowledge.



Ratio Warm Up

A Ratio is a comparison of two quantities of the same type in the same units.

A ratio can be written as 3: 10, a fraction $\frac{3}{10}$, a decimal 0.3, a percentage 30%

Exercise 11.1

Write the following comparisons as ratios:

The mathematical symbol for ratio is

18 people passed the test and 5 failed.	
a) What is the ratio of pass to fail?	18 : 5
b) What is the ratio of fail to pass?	5 : 18
c) What is the ratio of pass to the total?	18 : 23
d) What is the ratio of fail to the total	5 : 23

1 The Maths class has 9 girls and 14 boys.

a) What is the ratio of girls to boys?

b) What is the ratio of boys to girls?

c) What is the ratio of girls to the total number in the class?

d) What is the ratio of boys to the total number in the class?

2 Last month there were 24 sunny days and 7 cloudy days.

a) What is the ratio of sunny to cloudy days?

b) What is the ratio of cloudy to sunny days?

c) What is the ratio of sunny to the total number of days in the month?

d) What is the ratio of cloudy to the total number of days in the month?

3 Write each of the following ratios as a fraction, a decimal and a percentage:

	fraction	decimal	percentage
1 : 4	$\frac{1}{4}$	0.25	25%
5 : 2	$\frac{5}{2}$	2.5	250%

Make a percentage by multiplying by 100.

a) 1 : 2 **b)** 1 : 10 **c)** 1 : 4 **d)** 3 : 4

e) 1 : 5 **f)** 2 : 5 **g)** 2 : 10 **h)** 4 : 5

i) 3 : 10 **j)** 4 : 10 **k)** 6 : 10 **l)** 7 : 10

m) 8 : 10 **n)** 3 : 5 **o)** 9 : 10

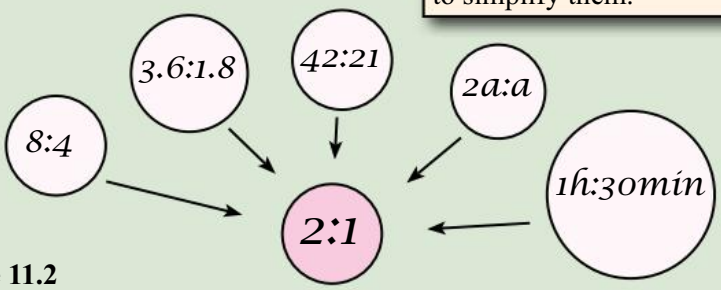
4 Write each of the following fractions as a ratio, a decimal and a percentage:

a) $\frac{1}{2}$ **b)** $\frac{3}{10}$ **c)** $\frac{4}{5}$ **d)** $\frac{1}{4}$

e) $\frac{3}{4}$ **f)** $2\frac{1}{2}$ **g)** $3\frac{1}{10}$ **h)** $6\frac{3}{5}$

Ratio

When ratios are large or awkward it sometimes helps to simplify them.



Exercise 11.2

Simplify the following ratios:

15 : 9 $= \frac{15}{9}$ $= \frac{3 \times 5}{3 \times 3}$ $= \frac{5}{3}$ = 5 : 3	4.5 : 5.5 $= \frac{4.5 \times 10}{5.5 \times 10}$ $= \frac{45}{55}$ $= \frac{5 \times 9}{5 \times 11}$ $= \frac{9}{11}$ = 9 : 11	8b : 4b $= \frac{8b}{4b}$ $= \frac{8}{4}$ $= \frac{2}{1}$ = 2 : 1

A calculator is good at simplifying ratios (see Technology 11.1).

1	8 : 4	**2**	3 : 12	**3**	12 : 16
4	3 : 9	**5**	6 : 9	**6**	15 : 5
7	2.5 : 1.5	**8**	2.4 : 1.8	**9**	5 : 1.5
10	2.8 : 2.1	**11**	4.0 : 1.2	**12**	8a : 12a
13	2c : 8c	**14**	9x : 15x	**15**	1.44y : 7.2y

Exercise 11.3

The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a total mixture of 4 L?	Rose food is 1/5 of the mixture = 1/5×4 L = 0.8 L or 800 mL

1 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 2 L?

2 The fruit juice is made by mixing juice concentrate and water in the ratio of 1 : 3. How much juice concentrate is needed to make 2 L of fruit juice?

3 The lunch bill for the 7 guests came to $154. If Amelia paid for 3 of the guests and Jack paid for the other 4 guests, how much did each pay?

4 Sophie paid $7 towards the Lottery ticket and Noah paid $3. If the Lottery ticket returned $30, how much should each receive?

5 The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 2 : 3. If the mixer takes 30 shovelfulls, how many shovelfulls of cement is needed?

6 A line 20 cm long is to be divided in the ratio of 1.2 : 1.8, where should the line be divided?

7 The chainsaw petrol is made by mixing two-stroke oil and unleaded petrol in the ratio of 1 : 25. How much two-stroke oil is needed to make 2 L of chainsaw petrol?

Percentages

Percentages are used hundreds of thousands of times every day for comparison and analysis.

Calculations often involve finding a percentage of an amount.

Exercise 11.4

1 Write each of the following percentages as a fraction and as a decimal:

$30\% = \frac{30}{100} = \frac{3}{10}$ $= 0.3$	$112\% = \frac{112}{100}$ $= 1.12$	$4.25\% = \frac{4.25}{100}$ $= 0.0425$

a) 10% **b)** 20% **c)** 30% **d)** 40%
e) 50% **f)** 60% **g)** 70% **h)** 80%
i) 90% **j)** 100% **k)** 3% **l)** 5%
m) 8% **n)** 1% **o)** 110% **p)** 150%
q) 200% **r)** 500% **s)** 143% **t)** 417%
u) 6.2% **v)** 8.7% **w)** 12.25% **x)** 5.75%

2 Calculate each of the following:

Find 20% of 80 $= 0.2 \times 80$ $= 16$	Find 37% of 176 $= 0.37 \times 176$ $= 65.12$	Find $13\frac{1}{5}\%$ of 65 $= 0.132 \times 65$ $= 8.58$

a) 25% of 60 **b)** 10% of 65 **c)** 75% of 24 **d)** 30% of 70
e) 50% of 80 **f)** 40% of 30 **g)** 60% of 250 **h)** 80% of 25
i) $12\frac{1}{2}\%$ of 80 **j)** $37\frac{1}{2}\%$ of 80 **k)** $62\frac{1}{2}\%$ of 80 **l)** $21\frac{3}{5}\%$ of 80

m) About 70% of a 60 kg person's weight is water. How much of the 60 kg is water ?

n) 40% of a \$560 wage is spent on rent. How much is the rent?

o) If 27% of a sale is profit, how much profit in a sale of \$4650?

p) Lenny pays 37% tax on an income of \$68 500 pa. How much tax does Lenny pay?

q) The management fund returned 11.5% on an investment of \$74 500. How much was returned?

r) Approximately 3.5% of the World's population of 7 billion people are Indonesians. Roughly how many people are Indonesian?

s) How many people in Oceania/Australia given that they make up approximately 0.5% of the World's population?

I was given a 70% rise. 5% pay rise and 65% work increase.

Exercise 11.5

Add GST (10%) to an item priced at $37	A discount of 15% is offered on a sub-woofer priced at $230. What is the new price?
= 37 + 10% of 37 = 37 + 0.1 × 37 = $40.70	= 230 − 15% of 230 = 230 − 0.15 × 230 = $195.50

1 Add GST (10%) to the price of each of the following items:

a) A sandwich @ $2.40
b) Cable subscription @ $55
c) A watch @ $48.50
d) A filing cabinet @ $240
e) A hotel room @ $145
f) Haircut @ $33
g) A telephone bill @ $125.60
h) Electricity account @ $171.60

The GST is a tax of 10% on most supplies of goods and services in Australia from 1 July 2000.

2 Discount each of the following prices by 25%

a) A DVD player @ $224
b) A coffee table @ $119
c) A running shoe @ $105
d) An MP3 player @ $275
e) A bedside clock @ $23.50
f) A ball gown @ $859
g) A calculator @ $18.50
h) A DVD @ $27.99

3 An insurance premium of $455.50 is given a 10% discount. What is then the new cost of the premium?

4 A health insurance fee of $164.10 per month is supported by a Federal Government rebate of 30%. What is the fee after the 30% rebate?

5 A car insurance discount of 7.5% is given to drivers 55 and over. What would a 56 year-old actually pay for a car insurance premium of $368.50?

6 A wage increase of 3.5% has just been granted. Adjust the following wages:

a) $975 **b)** $534.50 **c)** $308.20 **d)** $2560.35

7 A fortnightly wage of $970 is increased by 3.8%. What is the new wage?

8 A fixed deposit of $5430 is increased by an interest payment of 4.2%. How much money is now in the fixed deposit?

9 If Indonesia's growth rate is 1.16% per year, and their population is 245 million, what will be their population next year?

10 If Australia's growth rate is 1.2% per year, and their population is 23 million, what will be their population next year?

11 A house insurance premium of $340 attracts a stamp duty of 5.5%, and a GST of 10% of the premium. What is the total insurance premium?

Express profit and loss as a percentage of cost price or selling price.

Exercise 11.6

Mia bought a block of land for $120 000 and because of financial difficulty, quickly sold the block of land for $100 000.		
Find the loss = 120 000 - 100 000 = $20 000	Find the loss as a percentage of the buying price. $= \frac{20000 \times 100}{120000}$ = 16.67%	Find the loss as a percentage of the selling price. $= \frac{20000 \times 100}{100000}$ = 20%

1 Ethan bought a Year 7 maths textbook for $65 and a year later sold it on ebay for $23 (after deducting ebay commission and postage costs).
- **a)** Find the loss.
- **b)** Find the loss as a percentage of the buying price.
- **c)** Find the loss as a percentage of the selling price.
- **d)** Which percentage loss sounds better as far as Ethan is concerned?

2 Ella bought a 2 bedroom unit 12 years ago for $95 000 and recently sold it for $280 000 (after deducting selling costs).
- **a)** Find the profit.
- **b)** Find the profit as a percentage of the buying price.
- **c)** Find the profit as a percentage of the selling price.
- **d)** Which percentage profit sounds better as far as Ella is concerned?

3 A cattle buyer bought a pen of steers from a farmer for $12 500 and sold them to a feedlot for $15 000.
- **a)** Find the profit.
- **b)** Find the profit as a percentage of the buying price.
- **c)** Find the profit as a percentage of the selling price.
- **d)** Which percentage profit would the buyer mention to the farmer?
- **e)** Which percentage profit would the buyer mention to their employer?

4 A phone has a retail price of $85 and a recommended retail price of $105.
- **a)** What is the new price after a discount of 20% off $85?
- **b)** What is the new price after a discount of 30% off $105?
- **c)** Which sounds more impressive? Which is the better deal?

5 Find the discounted price of a CD:
- **a)** A discount of 20% on the marked price of $26.
- **b)** A discount of 30% on the recommended retail price of $32.
- **c)** Which sounds more impressive? Which is the better deal?

6 Find the discounted price of a phone:
- **a)** A discount of 15% on the marked price of $89.
- **b)** A discount of 30% on the recommended retail price of $112.
- **c)** Which sounds more impressive? Which is the better deal?

Rates

A rate is a comparison of quantities of different kinds.

Example: Rent of $380 per week compares the quantity $ with the quantity week.

Exercise 11.7

A house is let for $245 per week. What is the rent for 4 weeks? $= \frac{245 \times 4}{1}$ $= \$980$	If the price of diesel is $1.65 per litre, what is the cost of 32.6 L of diesel? $= \frac{1.65 \times 32.6}{1}$ $= \$53.79$

1 A two bedroom unit is being let for $465 pw (per week). What is the rent for:
a) 4 weeks? **b)** 6 months (26 weeks)? **c)** 1 year (52 weeks)?

2 Aamina earns a salary of $2254.30 per fortnight, how much is earned in:
a) a month (4 weeks)?
b) 6 months (26 weeks)?
c) 1 year (52 weeks)?
d) 1 decade (10 years)?

The first text message "**Merry Christmas**" was sent from a computer to a phone in 1992.

3 A prepaid mobile phone deal offers 28c per text message, what would be the charge for 80 text messages?

4 The prepaid company offers one rate of 49 cents per 30 secs to any mobile or landline within the country. What would it cost for a 8 min 30 sec call?

5 It has been reported that the average mobile phone subscriber in the Phillipines sends 27 text messages per day. How many text messages would be sent in a 31 day month?

6 It was necessary to buy timber at $9.85 per metre. How much did it cost for:
a) 2 m? **b)** 4 m? **c)** 3.2 m?
d) 850 mm? **e)** 1 630 mm? **f)** 27 900 m?

7 The Internet connection costs $37.50 per month, how much for a year?

8 The price of the material is $17.40 per metre. What would be the cost of 7 m?

9 If the price of unleaded petrol is $1.86 per litre, what is the cost of 40 L of petrol?

10 If I travel at an average speed of 90 km/h, how far will I travel in 5 hours?

11 If the plane is descending at a rate of 500 m per minute, how far will the plane descend in 5 mins?

12 If I travel at an average speed of 80 km/h, how far will I travel in 2 hours?

13 If I ride my pushbike at an average speed of 25 km/h, how far will I travel in 12 mins?

14 The instructions on the lawn fertiliser bag suggests that a rate of 2.5 kg per 100 m^2 be used for the couch lawn. How much fertiliser is needed for a 800 m^2 lawn?

Unitary Method

The unitary method is a useful method for comparing one rate with another rate.

The unitary method first finds the rate for one quantity (usually 100 if the second quantity is larger than 100).

Exercise 11.8

1 Find the rate per one or 100 in each of the following:

247 runs in 50 overs $= \frac{247}{50}$ runs in $\frac{50}{50}$ overs = 4.94 runs in 1 over	\$4.15 for 220 g $= \$\frac{4.15}{2.2}$ for $\frac{220}{2.2}$ g = \$1.89 for 100 g

a) 220 runs in 50 overs
b) 196 runs in 45 overs
c) Walk 5000 m in 40 mins
d) Walk 15 km in 2 hours
e) \$5 for 2 kg
f) \$3 for 1.5 kg
g) \$3.99 for 220 g
h) \$4.25 for 250 g
i) 42 L used to go 600 km
j) 54 L used to go 650 km
k) Walk 42 m in 60 secs
l) Run 520 m in 80 secs
m) \$8.16 for 850 g
n) \$6.90 for 750 g

2 Which is the best buy?

\$4.15 for 750 mL of vegetable oil	or	\$10.15 for 2 L?
$= \frac{4.15}{7.5}$ for $\frac{750}{7.5}$		$= \frac{10.15}{20}$ for $\frac{2000}{20}$
= \$0.55 for 100 mL		= \$0.50 for 100 mL
The 2 L is the best buy.		

a) \$3.49 for 750 mL of vegetable oil or \$10.35 for 2 L?
b) \$5.07 for 500 g of breakfast cereal or \$4.05 for 420 g?
c) \$3.27 for 200 g of white chocolate biscuits or \$4.37 for 300 g?
d) \$3.99 for 205 g macaroni cheese or \$8.15 for 500 g?
e) \$5.74 for 500 mL of body wash or \$2.69 for 240 mL of body wash?
f) \$8.33 for 90 mL of roll on deodorant or \$4.99 for 70 mL?

Albert Einstein proposed the theory of relativity in 1905. As your speed comes close to 299 792 km/s, the speed of light, time slows down.

Suppose you spend 10 Earth years on a machine travelling at 270 000 km/s and time slowed by 50%. You would be 18 while your friends would be 23?

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally

Exercise 11.9

1 Spell percentage.
2 Divide \$50 in the ratio of 7 : 3
3 Rent is \$350 per week. What is the rent for 4 weeks?
4 What is 10% of 23?
5 What is 20% of 23?
6 An \$80 item has a discount of 10%. What is the new price?
7 Add GST (10%) to \$60.
8 Simplify the ratio 12 : 8
9 Which is the best buy: 200 g for \$5 or 500 g for \$10?
10 If the price of unleaded petrol was \$1.50 per litre, what would it cost to fill a 50 L tank?

Why is 0.4047 hectares like a bad tooth? Because it is an acre.

Exercise 11.10

1 Spell unitary.
2 Divide \$60 in the ratio of 4 : 2
3 Rent is \$450 per week. What is the rent for 4 weeks?
4 What is 10% of 34?
5 What is 20% of 34?
6 An \$70 item has a discount of 10%. What is the new price?
7 Add GST (10%) to \$50.
8 Simplify the ratio 6 : 16
9 Which is the best buy: 100 g for \$10 or 300 g for \$25:
10 If the price of unleaded petrol was \$1.60 per litre, what would it cost to fill a 50 L tank?

If you're not lighting any candles, don't complain about being in the dark.

Exercise 11.11

1 Spell discount.
2 Divide \$70 in the ratio of 4 : 3
3 Rent is \$550 per week. What is the rent for 4 weeks?
4 What is 10% of 41?
5 What is 20% of 41?
6 An \$60 item has a discount of 10%. What is the new price?
7 Add GST (10%) to \$90.
8 Simplify the ratio 10 : 14
9 Which is the best buy: 200 g for \$3 or 50 g for \$1?
10 If the price of unleaded petrol was \$1.70 per litre, what would it cost to fill a 50 L tank?

NAPLAN Questions

Exercise 11.12

1 William worked for 6 hours and was paid $22 per hour. How much did William earn?

2 Isabella bought a box of 100 stamps for $65. What is the cost of 1 stamp?

3 The ratio of cattle to sheep is 1 : 4. If there are 20 sheep, how many cattle?

4 There is 45 kg of mortar. The ratio of cement to sand in the mortar mix is 1 to 4. How many kg of cement is in the mortar?

5 Cooper needs to travel by bus for 21 days in June. A daily ticket will cost him $5.80 and a monthly ticket will cost him $109.20. What is his average daily saving if Cooper buys a monthly ticket?

6 A bike priced at $220 was discounted by 20%. What is the discounted price?

7 In 2010 there were 120 Year 12 Students. 48 of them were offered a place at a University. What percentage of students weren't offered a University place?

8 What percentage of the marbles are white?

Marbles	Number
Red	4
Orange	2
Green	3
Blue	8
White	?
TOTAL	**20**

9 Change $3\frac{2}{3}$ to a vulgar fraction.

10 Change $\frac{111}{5}$ to a mixed fraction.

Mixed fraction	Vulgar fraction
$2\frac{2}{3}$	$\frac{8}{3}$

11 A cake has 3 blue candles and 9 white candles. What fraction of the candles are blue?

12 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 20%? Use the shortcut to increase $4 by 20%

13 On Wednesday Chloe completed 70% of the project. On Thursday she completed 50% of the remainder of the project. What percentage of the whole project did she complete on Thursday (Assume 100 at the start of the project)?

14 The paddock grew 800 pumpkins. One-quarter of them were unsuitable for market. 20% of the marketable pumpkins were exported. How many pumpkins were exported?

15 If 1 Australian dollar buys 0.90 US dollars, how many US dollars could be bought with 20 Australian dollars using this exchange rate?

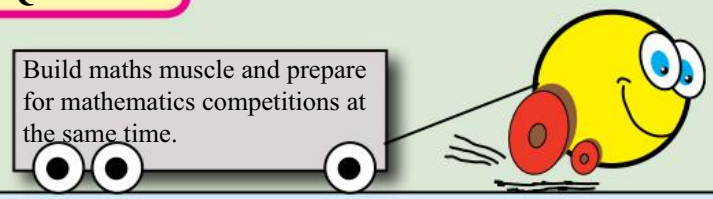
16 The car uses 5 L of petrol per 100 km. How far would the car travel on $30 of petrol at $1.50 per litre?

17 The distance from A to B is four times the distance from B to C. The distance from A to C is 150 metres. What is the distance from A to B?

Competition Questions

Exercise 11.13

Build maths muscle and prepare for mathematics competitions at the same time.



On a farm with 50 horses and 70 cattle, 30% of the horses are lame and 10% of the cattle are lame. What percentage of the animals are lame?

Lame horses = 0.3 x 50 = 15

Lame cattle = 0.1 x 70 = 7

$$\% \text{ lame} = \frac{(15+7)x\,100}{(50+70)}$$

% lame = 18.33

1 On a farm with 10 horses and 40 cattle, 10% of the horses are lame and 20% of the cattle are lame. What percentage of the animals are lame?

2 The junior orchestra consists of 20 Year 9 students and 30 Year 8 students. If 15% of the Year 9s play the trumpet and 20% of the Year 8s play the trumpet, what percentage of the orchestra play the trumpet?

A shortcut for finding the price before GST was added: divide by 1.1 Why?

When GST (10%) is added to an item, the price is \$66. What was the price of the item before GST was added? Let p be initial price.

66 = p + 0.1p

66 = p(1 + 0.1)

66 = p × 1.1

66 ÷ 1.1 = p

\$60 = Price before GST

When an item is discounted by 15%, the price is \$34. What was the price of the item before it was discounted?

34 = p − 0.1p

34 = p(1 − 0.1)

34 = p × 0.85

34 ÷ 0.85 = p

\$40 = Price before discount

3 When GST (10%) is added to an item, the price is \$88. What was the price of the item before GST was added?

4 When GST (10%) is added to an item, the price is \$77. What was the price of the item before GST was added?

5 When an item is discounted by 15%, the price is \$68. What was the price of the item before it was discounted?

6 When an item is discounted by 25%, the price is \$45. What was the price of the item before it was discounted?

7 Simplify the ratio 24 : 36 : 15 : 27

8 A, B, C, D share \$500 in the ratio 1 : 2 : 3 : 4. How much does B get?

9 A, B, C, D share in the ratio 1 : 2 : 3 : 4. If B gets \$500, what was the original total?

10 Neba bought some cushions. If GST (10%) hadn't been added to the cost, Neba would have been able to buy an extra cushion. How many cushions did Neba buy?

Investigations

Investigation 11.1 Gear Ratios

Bring a bike into the classroom, and when in lowest gear, turn it upside down.

1 Turn the pedals through 10 revolutions.

2 Count the number of times the rear wheel turns.

3 Determine the gear ratio (the ratio of pedal turns to rear wheel turns).

4 Repeat for the top gear.

Why use gears?
Why the difference in gear ratios between the lowest gear and the highest gear?
Can you name 10 other machines that use gears?
Can you find the gear ratio by counting the teeth?

Investigation 11.2 Population Comparison

Use the Internet to investigate Australia's population change:

- current population.
- current birth rate.
- current death rate.
- current immigration rate.

Can you predict Australia's population in ten years time?
Compare this with an investigation of Indonesia's population change.
Compare this with an investigation of other Asian countries.

Investigation 11.3 Real Life

Applications of percentages and rates are everywhere.

1 Can you make a list of 20 applications of percentages?

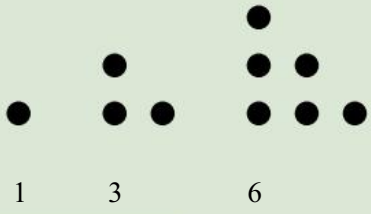
2 Can you make a list of 20 applications of rates?

Investigation 11.4 Triangular Numbers

The numbers 1, 3, 6, 10, …. are Triangular numbers because they form triangles.

1 What are the next four Triangular numbers?

2 What do you notice about the sum of any two consecutive Triangular numbers?



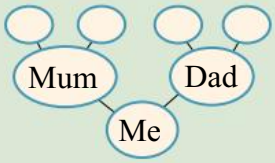
Astronomers study the movement, origins, and properties of the stars, planets, and galaxies.

- Relevant school subjects are Mathematics and Physics.
- Courses range from Universtity Bachelor degrees to Master degrees.

A Couple of Puzzles

Exercise 11.14

1 The digits of 122 are 1, 2, and 2, and 1 + 2 + 2 = 5. Find ten other numbers whose digits add to give 5.



2 Write 15 as the sum of three consecutive numbers.

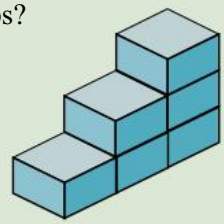
3 Write 15 as the sum of five consecutive numbers.

4 How many great-great-great-grandparents did you have?

5 How long would it take to count to one million if you counted once every second?

6 Score a century. Use +, –, x, ÷, and brackets between the numbers 1 2 3 4 5 6 7 8 9 to make exactly100.

7 Blocko is building a set of steps out of blocks and the first 3 steps are shown. How many blocks will be needed to build 10 steps?



A Game

Elevens Make eleven marks on a piece of paper.
Each of two players take turns to mark off one or two of the marks.
The loser is the person who crosses off the last mark.

A Sweet Trick

1 Put 20 objects in a container (eg. centicubes, coins).

2 Turn your back and ask someone to take from 1 to 9 objects and put them in their pocket. **They take 6**

3 Ask them to count the number left. **14 are left**

4 Ask them to total the digits of the number left. **1 + 4 = 5**

5 Ask them to take that many from the container and to add them to their pocket. **They take 5**

6 Ask them to take as many as they like from those left in the container and to keep these in their hand. **They take 3**

7 You tell them how many they have in their hand.

Count the number left in the container, ie 6, and subtract from 9.
9 - 6 = 3. They have 3 in their hand.

Practice the trick a couple of times before you use it.

Technology

Technology 11.1 Simplifying ratios

Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ [15] [$a\frac{b}{c}$] [35] [=] [3r7] meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ [18] [$a\frac{b}{c}$] [4] [=] [4r1r2] meaning $4\frac{1}{2}$

To change to a vulgar fraction: [2ndF] [$a\frac{b}{c}$] to give [9r2] ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:

a) 3 : 9 **b)** 9 : 12 **c)** 16 : 24

d) 2.1 : 3.5 **e)** 14.4 : 12.6 **f)** 256 : 1024

5 out of 4 people can't do fractions.

Technology 11.2 Paying off a loan

Jade has taken out a $5 500 loan to pay for a home entertainment system. Jade is paying off the loan at $280 a month. Jade would really appreciate it if you could produce a graph showing what is going on. The interest rate is 10.5% pa paid monthly (ie. the interest rate is 10.5÷12= 0.875% per month).

1 Set up a spreadsheet similar to the one below.

Enter: =b2-c2+d2

*Enter: =0.00875*b2*

	a	b	c	d
1	Month	Balance	Repayment	Interest
2	0	5500	280	48
3	1	5268	280	46
4	2	5034	280	44
5	3	4798	280	42
6	4	4560	280	40
7	5	4320	280	38

2 Use Chart Wizard to produce the graph (Month and balance only).

3 If Jade were to increase repayments to $300 a month, what difference would it make?

4 If the interest rate were to increase to 15.5%, what difference would it make?

Chapter Review 1

Exercise 11.15

1 Write each of the following ratios as a fraction, a decimal and a percentage:

a) 1 : 2 **b)** 2 : 5 **c)** 3 : 4

d) 3 : 10 **e)** 2 : 3 **f)** 7 : 10

2 Simplify the following ratios:

a) 8 : 4 **b)** 9 : 12 **c)** 12 : 16

d) 2.5 : 1.5 **e)** 2.4 : 1.8 **f)** 3.5c : 2.1c

3 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 4 L?

4 The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 3 : 2. If the mixer takes 30 shovelfulls, how many shovelfulls of sand is needed?

5 Calculate each of the following:

a) 25% of 60 **b)** 30% of 70 **c)** $12\frac{1}{2}$% of 80

d) About 70% of a 60 kg person's weight is water. How much of the 60 kg is water ?

6 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 20%? Use the shortcut to increase $40 by 20%

7 Find the discounted price of a phone:?

a) A discount of 15% on the marked price of $95.

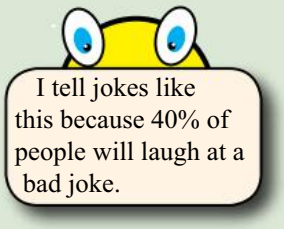
b) A discount of 30% on the recommended retail price of $115.

c) Which sounds more impressive? Which is the better deal?

8 A car insurance discount of 7.5% is given to drivers 55 and over. What would a 56 year-old actually pay for a car insurance premium of $475.50?

9 When GST (10%) is added to an item, the price is $88. What was the price of the item before GST was added?

In what month do people eat the least?
February – it's the shortest month.



10 A prepaid mobile phone deal offers 28c per text message, what would be the charge for 80 text messages?

11 The instructions on the lawn fertiliser bag suggests that a rate of 3.5 kg per 100 m^2 be used for the couch lawn.
How much fertiliser is needed for a 600 m^2 lawn?

12 Find the rate per one or 100 in each of the following:

a) 210 runs in 50 overs. **b)** 48 L used to go 600 km.

13 Which is the best buy?

a) $5.27 for 500 g of breakfast cereal or $4.95 for 450 g?

b) $8.53 for 90 mL of roll on deodorant or $6.99 for 70 mL?

c) $3.37 for 150 g of white chocolate biscuits or $4.05 for 200 g?

Chapter Review 2

Exercise 11.16

1 Write each of the following ratios as a fraction, a decimal and a percentage:

a) 1 : 2 **b)** 3 : 5 **c)** 1 : 4

d) 5 : 10 **e)** 1 : 3 **f)** 2 : 5

2 Simplify the following ratios:

a) 5 : 10 **b)** 6 : 15 **c)** 15 : 25

d) 1.5 : 2.5 **e)** 1.8 : 2.4 **f)** 3.6a : 2.4a

3 The rose food is mixed with water in the ratio of 1 : 3. How much rose food is needed to make a mixture of 2 L?

4 The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 3 : 2. If the mixer takes 30 shovelfulls, how many shovelfulls of gravel is needed?

5 Calculate each of the following:

a) 15% of 50 **b)** 30% of 80 **c)** $37\frac{1}{2}$% of 40

d) About 70% of a person's weight is water. How much of a 60 kg person is water?

6 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 30%? Use the shortcut to increase $50 by 30%

7 A monthly rent return of $1650 is reduced by a 7% management fee. How much is paid to the landlord?

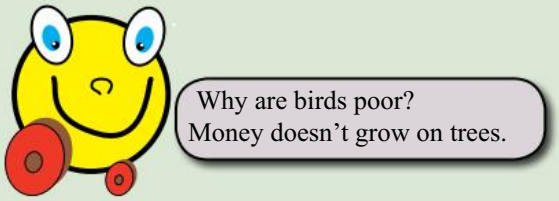
8 Find the discounted price of a phone:?

a) A discount of 15% on the marked price of $105.

b) A discount of 25% on the recommended retail price of $125.

c) Which sounds more impressive? Which is the better deal?

9 When GST (10%) is added to an item, the price is $99. What was the price of the item before GST was added?



10 A prepaid mobile phone deal offers 29c per text message, what would be the charge for 45 text messages?

11 The prepaid company offers one rate of 68 cents per 30 secs to any mobile or landline within the country. What would it cost for a 3 min 30 sec call?

12 Find the rate per one or 100 in each of the following:

a) 240 runs in 50 overs. **b)** 54 L used to go 600 km.

13 Which is the best buy?

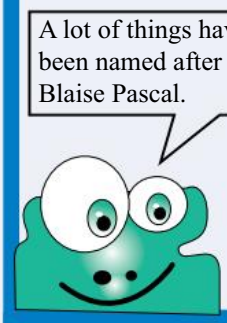
a) $6.36 for 500 g of chips or $5.85 for 440 g?

b) $1.49 for 750 mL of softdrink or $3.15 for 2 L?

c) $3.74 for 70 mL of insect spray or $4.19 for 105 mL of insect spray?

Linear Equations 12

- ★ Solve linear equations using algebraic and graphical techniques.
- ★ Use variables to symbolise simple linear equations and use a variety of strategies to solve them.
- ★ Solve equations using concrete materials, such as the balance model, and explain the need to do the same thing to each side of the equation.



A TASK

Pascal's Triangle:

- Research Pascal's Triangle.
- Find out about the Triangular Numbers and where they may be found in Pascal's Triangle.
- Find another famous number pattern in Pascal's Triangle.
- Describe another pattern that you have noticed.

A LITTLE BIT OF HISTORY

Pascal's Triangle was named after the French mathematician Blaise Pascal (1623−1662).

Pascal's triangle contains many patterns. The following is just one of thousands of patterns:

The sum of each row is a power of 2

1	= 1	$= 2^0$
1+1	= 2	$= 2^1$
1+2+1	= 4	$= 2^2$
1+3+3+1	= 8	$= 2^3$
1+4+6+4+1	= 16	$= 2^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Can you work out the next couple of lines in Pascal's Triangle?

Linear Patterns

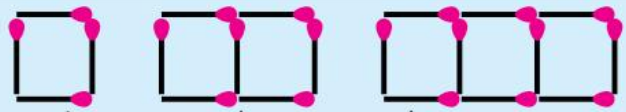
Exercise 12.1

For each of the following patterns:

a) Count the number of matches needed for each step, extend the pattern and summarise the pattern in a **table**.

b) Write a **function** for the pattern.

c) **Check** that the function is correct:



Table

Step	1	2	3	4	5
Matches	4	7	10	13	16

Function = 3×step + 1

Check. When step = 5, Function = 3×5+1 = 16

Extra 3 matches for each step
3×step

First step is 3+1 (= 4 matches).

Function = 3×step+1

1

2

3

4

5

Linear Patterns

Exercise 12.2

Write a function for the pattern in each of the following tables:

A linear pattern increases by the same amount each step.

x	1	2	3	4	5	10	20
y	3	5	7	9	11	21	41

Extra 2 for each x → **2x**

First step is 2+1. → **y = 2x + 1**

Check your answer:
y = 2x + 1
y = 2×5 +1 {Checking for x=5}
y = 11 {correct}

1

x	1	2	3	4	5	10	20
y	4	6	8	10	12	22	42

2

x	1	2	3	4	5	10	20
y	4	7	10	13	16	31	61

3

x	1	2	3	4	5	10	20
y	1	3	5	7	9	19	39

4

x	1	2	3	4	5	10	20
y	6	8	10	12	14	24	44

Exercise 12.3

For each of the following:

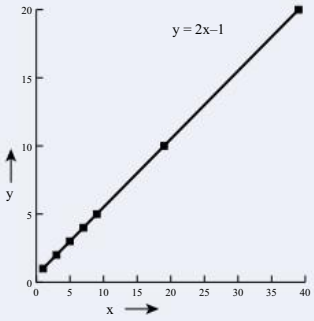
a) Use the function to complete the table.

b) Graph the function:

This is a linear pattern because ....

x	1	2	3	4	5	10	20
y = 2x − 1	1	3	5	7	9	19	

x=1, y=2×1−1=1 (1,1)
x=2, y=2×2−1=3 (2,3)
x=3, y=2×3−1=5 (3,5)
x=4, y=2×4−1=7 etc. (4,7)



1

x	1	2	3	4	5	10	20
y = 3x + 1	4	7					

2

x	1	2	3	4	5	10	20
y = 2x + 2	4	6					

Solving Linear Equations

To solve an equation is to find the value of the unknown number (the variable) in the equation.

The formal way to solve an equation is to choose to do the same thing to each side of the equation until only the unknown number remains on one side of the equation.

Exercise 12.4

Solve each of the following equations:

$x + 5 = 9$ $x + 5 - 5 = 9 - 5$ $x = 4$	$5a = 15$ $5a \div 5 = 15 \div 5$ $a = 3$
$c - 4 = 7$ $c - 4 + 4 = 7 + 4$ $c = 11$	$d \div 3 = 5$ $d \div 3 \times 3 = 5 \times 3$ $d = 15$

The inverse of + is −
The inverse of − is +
The inverse of × is ÷
The inverse of ÷ is ×

1 $x + 5 = 9$ **2** $b - 7 = 11$ **3** $5y = 25$
4 $t \div 4 = 3$ **5** $7x = 28$ **6** $x + 7 = 16$
7 $x - 8 = 17$ **8** $6x = 36$ **9** $x \div 5 = 7$
10 $j - 8 = 3$ **11** $a + 11 = 43$ **12** $21m = 105$
13 $z \div 7 = 11$ **14** $2w = 48$ **15** $x - 29 = 41$

$4(d + 3) = 28$
$4d + 12 = 28$ {Distributive law}
$4d = 28 - 12$ {inverse of + is − }
$4d = 16$
$d = 16 \div 4$ {inverse of × is ÷ }
$d = 4$

16 $3(x + 4) = 18$ **17** $2(h - 3) = 4$ **18** $2(a + 3) = 14$
19 $3(t - 5) = 12$ **20** $5(b + 4) = 15$ **21** $4(n - 7) = 12$
22 $6(m - 2) = 18$ **23** $4(x + 2) = 16$ **24** $2(f + 5) = 10$
25 $4(p - 3) = 20$ **26** $5(a + 6) = 25$ **27** $7(x - 3) = 28$

$5(2x + 3) = 25$
$10x + 15 = 25$ {Distributive law}
$10x = 25 - 15$ {inverse of + is − }
$10x = 10$
$x = 10 \div 10$ {inverse of × is ÷ }
$x = 1$

28 $3(x - 4) = 12$ **29** $2(x - 3) = 14$ **30** $3(x + 5) = 18$
31 $4(2x - 2) = 16$ **32** $5(2d + 7) = 65$ **33** $5(2x - 3) = 15$
34 $2(2x - 5) = 10$ **35** $7(3x + 2) = 35$ **36** $4(5x - 4) = 24$

Solving Linear Equations

Exercise 12.5

Solve each of the following equations:

$$
\begin{aligned}
5x + 3 - 3x &= 11 \\
2x + 3 &= 11 && \{5x - 3x = 2x\} \\
2x &= 11 - 3 && \{\text{inverse of } + \text{ is } - \} \\
2x &= 8 \\
x &= 8 \div 2 && \{\text{inverse of } \times \text{ is } \div \} \\
x &= 4
\end{aligned}
$$

If the unknown occurs more than once, put the unknowns together (5x−3x=2x).

1 $3a + 2a = 15$
2 $4x + 5 + 2x = 23$
3 $7d + 6 - 3d = 18$
4 $9s - 4 - 2s = 17$
5 $6u + 3u - 5 + u = 25$
6 $2w + 4w + 7 - 3w = 16$

$$
\begin{aligned}
3(2x - 3) - 4x &= 17 \\
6x - 9 - 4x &= 17 && \{\text{Distributive law}\} \\
2x - 9 &= 17 && \{6x - 4x = 2x\} \\
2x &= 17 + 9 && \{\text{inverse of } - \text{ is } + \} \\
2x &= 26 \\
x &= 26 \div 2 && \{\text{inverse of } \times \text{ is } \div \} \\
x &= 13
\end{aligned}
$$

7 $2(x - 3) + 3x = 9$
8 $5(x - 3) - 2x = 9$
9 $3(x + 2) + 2x = 11$
10 $3(2x - 2) + 4x = 14$
11 $3x + 2(x + 2) = 14$
12 $x + 3(x - 1) = 5$
13 $2x + 3(x - 1) + 3x = 13$
14 $5x + 2(2x + 3) - x + 5 = 35$

$$
\begin{aligned}
5x - 3 &= 2x + 9 \\
5x - 3 - 2x &= 9 && \{\text{Get the 2x to the other side}\} \\
3x - 3 &= 9 && \{5x - 2 = 3x\} \\
3x &= 9 + 3 && \{\text{inverse of } - \text{ is } + \} \\
3x &= 12 \\
x &= 12 \div 3 && \{\text{inverse of } \times \text{ is } \div \} \\
x &= 4
\end{aligned}
$$

If the unknown is on both sides of the =, put the unknowns together .

15 $5x - 7 = 2x - 1$
16 $5x - 8 = 2x + 4$
17 $6d + 3 = 2d + 27$
18 $5g + 15 = 3g + 19$
19 $2x + 2 = 9 - 5x$
20 $5h + 4 = 4h + 7$
21 $5h + 15 = 3h + 19$
22 $3n + 22 = 2n + 34$
23 $8p - 10 = 5 + 3p$
24 $6y + 3 = y + 28$
25 $7a + 12 = 30 - 2a$
26 $5w - 4 = 5 - 4w$
27 $5y - 3 = 9 - y$
28 $4(t - 2) = 2t + 4$
29 $3(m + 3) + 2 = 15 - m$
30 $4x + 5 - x = 2(3 - x) + 4$

Exercise 12.6

For each of the following functions:

a) Evaluate the function for values of 1, 2, 3, 4, 5.

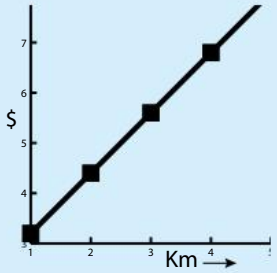
b) Draw a graph of the function.

The function is linear because it increases by the same amount of \$1.20 each step.

The taxi charge, in \$, is given by the function
C = 1.2k + 2, where k is kilometres travelled.

When k=1, C = 1.2×**1**+2 = \$3.20
k=2, C = 1.2×**2**+2 = \$4.40
k=3, C = 1.2×**3**+2 = \$5.60
k=4, C = 1.2×**4**+2 = \$6.80
k=5, C = 1.2×**5**+2 = \$8.00

k	1	2	3	4	5
C = 1.2x + 2	3.20	4.40	5.60	6.80	8.00



1 The taxi charge, in \$, is given by the function C = 1.3k + 2, where k is the number of kilometres travelled.

2 A taxi charges \$4 for the flag fall and then \$2.50 per kilometre. Thus the taxi charge function is C = 2.5k + 4.

3 The assistant real estate agent's weekly pay, in \$, is calculated by the function P = 50h + 200, where h is the number of houses sold.

4 The assistant real estate agent is paid \$250 per week and then \$100 for every house sold. Thus the assistant real estate agent's weekly pay function is given by: P = 100h + 250.

5 The plumber's charge, in \$, is given by the function: C = 40h + 60, where h is the number of hours of work.

6 The plumber charges \$60 for the call-out fee and then \$80 per hour. Thus the plumber's charge function is: C = 80t + 60.

7 When travelling at an average speed of 80 km per hour, the distance travelled is given by the function D = 80t, where t is the number of hours of travel.

8 When travelling at an average speed of 90 km per hour, the distance travelled is given by the function D = 90t, where t is the number of hours of travel.

9 The profit from the social is given by the function: P = 15n − 120, where n is the number of people attending.

10 People attending the social will pay \$25 each and it will cost \$250 to hire the room. Thus the profit function is P = 25n − 250.

11 The weekly pay, in \$, is given by W = 30n + 100, where n is the number of wicker chairs made.

12 Jon is paid a retainer of \$150 per week and also receives \$2 for each bag he makes. Thus the function for his weekly pay is given by: W = 2b + 150.

Solving Linear Functions

Exercise 12.7

The taxi charge, in $, is given by the function $C = 1.3k + 2$, where k is the number of kilometres travelled.

a) What is the charge to travel 4 km?

b) If the charge was $11.10, how far did the taxi travel?

a) $k = 4$, $C = 1.3\times\mathbf{4}+2 = \$7.20$

b) $1.3k + 2 = 11.1$

$1.3k = 11.1 - 2$ {The inverse of + is −}

$1.3k = 9.1$

$k = 9.1 \div 1.3$ {The inverse of × is ÷}

$k = 7$

The taxi travelled 7 km for a charge of $11.10

Solve equations means changing things until the unknown is on its own.

1 The taxi charge, in $, is given by the function $C = 2.2k + 3$, where k is the number of kilometres travelled.

a) What is the charge to travel 8 km?

b) Find the distance travelled if the taxi charge was $27.20.

2 The assistant real estate agent's weekly pay, in $, is calculated by the function $P = 50h + 200$, where h is the number of houses sold.

a) What is the weekly pay if 15 houses were sold?

b) If the pay for the week was $1 100, how many houses were sold?

3 The plumber's charge, in $, is given by the function $C = 70h + 80$, where h is the number of hours of work.

a) What is the charge for 5 hours work?

b) How many hours did the plumber work if the charge was $570?

4 When travelling at an average speed of 80 km per hour, the distance travelled is given by the function $D = 80t$, where t is the number of hours of travel.

a) What distance will be covered in 5 hours?

b) How many hours of travel did it take to cover a distance of 260 km?

5 The time, in seconds, for reheating donuts in a microwave is given by $T = 15n + 10$, where n is the number of donuts.

a) How long will it take to reheat 6 donuts.

b) How many donuts will be reheated in a time of 145 seconds?

6 The total mass of a truck, in tonnes, is given by the function $M = 0.08c + 2$, where c is the number of cartons.

a) What is the mass of the truck if there are 50 cartons on the truck?

b) If the total mass is 6.88 tonnes, how many cartons are on the truck?

7 The time, in minutes, for medium roasting of beef in a conventional oven at 200˚C is given by the function $T = 50m + 25$, where m is the mass of the roast in kilograms.

a) How long will it take to cook a 2.5 kg roast?

b) If the timer is set for 187.5 minutes, what is the mass of the roast?

Solving Word Problems

Exercise 12.8

A taxi charges \$3 for the flag fall and then \$2.50 per kilometre.

a) What is the fare to travel 8 km?

b) If the fare was \$19.25, how far did the taxi travel?

\$2.50 per km ▶ $2.5k$

\$3 flagfall ▶ $2.5k + 3$

First find a function.

a) $k = 8$, $C = 2.5\times\mathbf{8}+3 = \$23.00$

These equations can also be solved with technology. See Technology 12.1 and 12.2.

b) $2.5k + 3 = 19.25$

$2.5k = 19.25 - 3$ {The inverse of + is −}

$2.5k = 16.25$

$k = 16.25 \div 2.5$ {The inverse of × is ÷}

$k = 6.5$

The taxi travelled 6.5 km for a charge of \$19.25

1 A taxi charges \$3 for the flag fall and then \$2.75 per kilometre.
- **a)** What is the fare to travel 12 km?
- **b)** If the fare was \$33.25, how far did the taxi travel?

2 The plumber charges \$50 to be called out and then \$75 per hour?
- **a)** What will the plumber charge for a 3 hour job?
- **b)** If the plumber charged \$237.50, how long did the plumber work?

3 For the social it will cost \$325 to hire the room and each ticket will cost \$35.
- **a)** What will be the profit if 45 people attend the social?
- **b)** If the profit is \$515, how many people attended the social?

4 The truck has a weight of 4.5 tonnes and each pallet weighs 0.75 tonnes.
- **a)** What is the total weight if the truck carries 3 pallets?
- **b)** If the total weight is 8.25 tonnes, how many pallets were put on the truck?

5 To print the flyers, the printer charges an upfront \$35 and then \$0.15 per flyer.
- **a)** What will the printer charge for 1000 flyers?
- **b)** How many flyers were printed if the cost was \$410?

6 Ella has \$135 in the bank and plans to add \$25 every week.
- **a)** How much will Ella have after 14 weeks?
- **b)** How many weeks will Ella need to save to have \$810 in the bank?

7 The Internet phone company offers a 2.75 cents flagfall and 4.72 cents/min.
- **a)** How much for an Internet call that lasts 4 minutes?
- **b)** If an Internet call costs 33.43 cents, how long was the call?

8 Riley works in a design studio and is paid a weekly retainer of \$350 together with a piece rate of \$55 per completed image.
- **a)** What is Riley's weekly pay in which he completes 11 images?
- **b)** If Riley's weekly pay is \$1615, how many images did he complete?

9 The cyclist travels at an average speed of 25 km/h.
- **a)** How far will the cyclist travel in 5 hours?
- **b)** How long will it take the cyclist to travel 300 km?

Mental Computation

Mental computation can make problems easier and quicker.

Exercise 12.9

1 Spell linear.
2 Solve: $5x = 15$
3 Divide \$50 in the ratio of 3 : 2
4 Solve: $3x + 2 = 11$
5 Simplify: $m^2 \times m^3$
6 Add GST (10%) to \$50.
7 Simplify: 5^0
8 A taxi charges \$3 flagfall and \$2.50 per km. What is the function?
9 Change $3\frac{1}{2}$ to a vulgar fraction.
10 Simplify: $2^8 \div 2^6$

What are the warmest months of the year?

September, November, and December, because they have embers in them

Exercise 12.10

1 Spell equation.
2 Solve: $x \div 4 = 3$
3 Divide \$60 in the ratio of 2 : 1
4 Solve: $2x - 5 = 3$
5 Simplify: $b^2 \times b^4$
6 Add GST (10%) to \$60.
7 Simplify: 4^0
8 A call costs 15 cents flagfall and 98 cents per minute. What is the function?
9 Change $2\frac{2}{3}$ to a vulgar fraction.
10 Simplify: $5^5 \div 5^3$

The secret of getting ahead is getting started.

Exercise 12.11

1 Spell function.
2 Solve: $x - 3 = 6$
3 Divide \$70 in the ratio of 5 : 2
4 Solve: $4x - 2 = 6$
5 Simplify: $d^2 \times d^3$
6 Add GST (10%) to \$70.
7 Simplify: 7^0
8 The plumber charges \$70 callout and \$80 per hour. What is the function?
9 Change $2\frac{3}{4}$ to a vulgar fraction.
10 Simplify: $3^6 \div 3^3$

Laboratory Officers help scientists by preparing samples, carrying out experiments, measuring and recording and presenting results.

- Relevant school subjects are English and Mathematics.
- Courses range from Traineeships to Certificate and Diploma courses.

NAPLAN Questions

Exercise 12.12

1 If x = 4, what is the value of 5x − 3?

2 What is the function for each of the following?

a)

x	1	2	3	4	5
y	3	5	7	9	11

b)

x	1	2	3	4	5
y	0.5	2.5	4.5	6.5	8.5

3 y = 15 − 3x. What is the value of y when x = 4.74?

4 What is the next number in the following pattern?

5 The time, in minutes, for medium roasting of beef in a conventional oven at 200˚C is given by the function T = 50m + 25, where m is the mass of the roast in kilograms. What is the mass of a roast that needs 187.5 minutes to cook?

Q If you were running a race, and you passed the person in 2nd place, what place would you be in now?
A You would be in 2nd place.

6 What is the next number in each of the following patterns?

a)

x	1	2	3	4	5
y	1	4	9	16	?

b)

x	1	2	3	4	5
y	2	5	10	17	?

7 Expand each of the following:

a) 4(2g − 3). **b)** 3(2x + 1) **c)** 5(3y − 2)

8 Solve each of the following:

a) When a number is doubled and then 13 is added the answer is 21.

b) When a number is tripled and 7 subtracted, the answer is 17.

9 Solve each of the following:

a) 4a − 3 = 2a + 7 **b)** 5n + 5 = 2n + 26

c) 3(4x − 2) + 5x = 3x + 22 **d)** 2(3x +2) − 5x = 3

10 Write a function for each of the following patterns:

a)

b)

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 12.13

What would be the third number from the left of the 40th row of the pattern?	1	2	3	4	5
	6	7	8	9	10
	11	12	13	14	15

In the third column, the pattern is 3, 8, 13, …..
Each row increases by 5 thus: **Row x 5**
The first row is 3 = 5 **– 2**.
The function = Row x 5 − 2
= 40 x 5 − 2
= 198

1 **a)** What would be the second number from the left of the 10th row of the pattern?

1	2	3	4
5	6	7	8

b) What would be the third number from the left of the 20th row of the pattern?

9	10	11	12

2 **a)** What would be the first number from the left of the 10th row of the pattern?

2	4	6	8
10	12	14	16

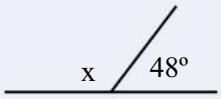
b) What would be the second number from the left of the 30th row of the pattern?

18	20	22	24

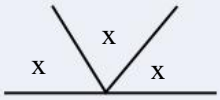
3 For each of the following, write an equation and then solve the equation:

The perimeter of a triangle is 50 m. If each side is 2x, 3x, and 5x, what is the value of x?
2x + 3x + 5x = 50
10x = 50
x = 50 ÷ 10
x = 5 m

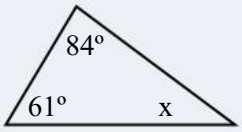
a)



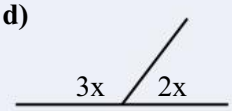
b)



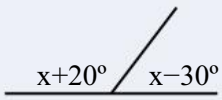
c)



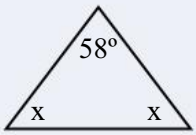
d)



e)



f)



4 The perimeter of a triangle is 30 m. If each side is 2x, 3x, and 5x, what is the value of x?

5 The three sides of an equilateral triangle are x + 3, 2x − 2, and 4y. What is the value of x and y

6 A number minus three-fifths of the number is twelve. What is the number?

Technology

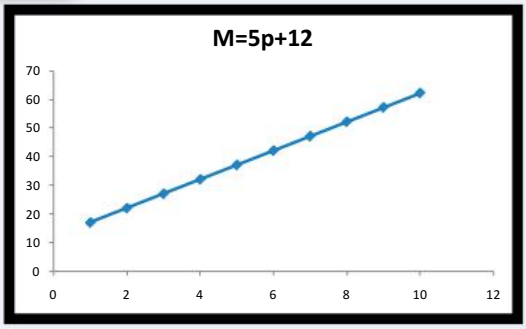
Technology 12.1 Linear equations and the spreadsheet

The mass of an envelope is 12 g and the mass of a sheet of paper is 5 g.

$$M = 5p + 12$$

Pages	Mass
1	17
2	22
3	27
4	52
5	57
6	42
7	47
8	52
9	57
10	62

Enter the formula:
=5*a2+12



a) What is the mass of an envelope and 5 sheets of paper?
b) If the mass is 52 g, how many sheets of paper?

Technology 12.2 Linear equations and the Graphics Calculator

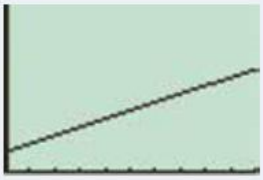
The mass of an envelope is 12 g and the mass of a sheet of paper is 5 g.

$$M = 5p + 12$$

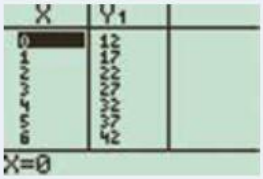
Press **Y=** and enter the function 5x + 12.

Set the **window** to:
Xmin = 0
Xmax = 10
Ymin = 0
Ymax = 100

Press **GRAPH** to see a plot of the function.



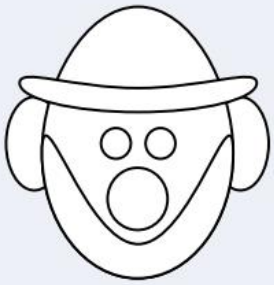
Press **TABLE** to see a table of the values.



Use **CALC** to answer the questions.

A Couple of Puzzles

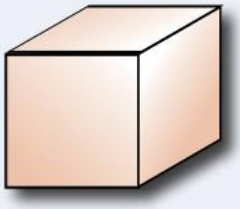
Exercise 12.14



1 This clown must be coloured:
with only three colours and,
no two adjacent areas can be coloured the same.

2 Who am I?
If you multiply me by 5 and then add 105,
the answer is 225.

3 What day of the week is 185 days after Friday?



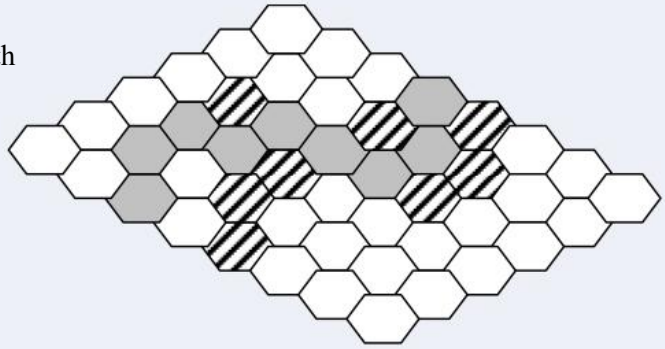
4 What is the sum of the number of:
faces + edges + vertices of a cube?

5 x is the units digit in the number 573 79x.
If the number is divisible by 11, what is x?

A Game

Hex.
Two players take turns in marking a hex cell.
The winner is the player with a continuous line from one side to the other.

Hex grids can be found on the Internet. You can also play the game on the Internet.



A Sweet Trick

The answer is always 7.

1	Ask your audience to choose any number (other than 0)	143	
2	Double the number	143 × 2	= 286
3	Add 12	286 + 12	= 287
4	Multiply by 5	287 × 5	= 1435
5	Keep only the last digit		5
6	Square the number	5 × 5	= 25
7	Total the digits	2 + 5	= 7

Investigations

Investigations 12.1 Step fitness formula

The step fitness test is a simple way of assessing aerobic fitness.

1 Step with one foot onto a 40 cm chair, bench, or box and then again step with the other foot onto the chair, bench, or box.

2 Repeat this for 3 minutes with about 25 steps every minute.

3 Wait for 20 seconds and then count your pulse, p, for 15 seconds.

4 For females: **Fitness = 110 – 1.68p**
For males: **Fitness = 66 – 0.74p**

Excellent	>55
Good	50-55
Average	45-50
Poor	<45

p is the number of heartbeats in 15s.

Investigations 12.2 Step fitness formula

Use either Technology 12.1 or Technology 12.2 to sketch graphs of the fitness formulas.

a) Are the formulas linear?
b) Are the formulas realistic models of fitness?
c) How would you improve the formulas?

Investigation 12.3 Walking fitness formula

The walk test is another way of assessing aerobic fitness. You will need your weight, w, in kilograms and your age, a, in years.

1 Walk, without running, as fast as you can for 1600 m. Measure the time, t, in decimal minutes (eg 13 mins 42 s = 13 mins 42/60 s = 13.7).

2 Count your pulse, p, for 15 seconds as soon as you finish.

3 For males: **Fitness = 140 - 0.2w - 0.4a -3.3t - 0.6p**
For females: **Fitness = 130 - 0.2w - 0.4a - 3.3t - 0.6p**

'It does not matter how slowly you go so long as you do not stop.' Confucius

Chapter Review 1

"Who laughs last, thinks slowest."
- a sort of Murphy's law.

Exercise 12.15

1 For each of the following tables, write a function for the pattern in the table.

a)

x	1	2	3	4	5	10	20
y	4	7	10	13	16	31	61

b)

x	1	2	3	4	5	10	20
y	2	7	15	20	25	47	97

2 For each of the following, use the function to complete the table and, graph the function.

a)

x	1	2	3	4	5	10	20
y = 2x + 1	3	5					

b)

x	1	2	3	4	5	10	20
y = 2x − 1	1	3					

3 Solve each of the following equations:

a) $x + 4 = 9$ **b)** $a - 7 = 12$ **c)** $6y = 24$

d) $3(x - 4) = 18$ **e)** $2(h + 3) = 12$ **f)** $2(a + 3) = 14$

g) $3a + 4a = 35$ **h)** $6x + 5 + 2x = 29$ **i)** $2(x - 2) + 3x = 6$

j) $5(x - 2) - 2x = 2$ **k)** $5y - 7 = 2y - 1$ **l)** $5x - 8 = 2x + 4$

4 A taxi charges \$2 for the flag fall and then \$3.25 per kilometre.

a) Write a function for the taxi charge.

b) What is the fare to travel 10 km?

c) If the fare was \$57.25, how far did the taxi travel?

5 For the social it will cost \$250 to hire the room and each ticket will cost \$30.

a) Write a function for profit for the social.

b) What will be the profit if 45 people attend the social?

c) If the profit is \$1370, how many people attended the social?

6 **a)** What would be the second number from the left of the 20th row of the pattern?

b) What would be the third number from the left of the 25th row of the pattern?

1	3	6	9
12	15	18	21
24	27	30	33

Every day, innumeracy affects 8 out of 5 people.

Chapter Review 2

Exercise 12.16

1 For each of the following tables, write a function for the pattern in the table:

a)

x	1	2	3	4	5	10	20
y	3	5	7	9	11	21	41

b)

x	1	2	3	4	5	10	20
y	5	7	9	11	13	23	43

The latest survey shows that 3 out of 4 people make up 75% of the world's population.

2 For each of the following use the function to complete the table and, graph the function:

a)

x	1	2	3	4	5	10	20
y = 3x + 1	4	7					

b)

x	1	2	3	4	5	10	20
y = 3x − 1	2	5					

3 Solve each of the following equations:

a) $b + 7 = 8$ **b)** $d - 7 = 8$ **c)** $4x = 24$

d) $2(x - 3) = 14$ **e)** $3(h + 5) = 27$ **f)** $2(n + 1) = 16$

g) $2g + 6g = 32$ **h)** $2x + 1 + 3x = 31$ **i)** $2(x - 4) + 3x = 7$

j) $5(c - 3) - 2c = 6$ **k)** $6r - 9 = 2r - 1$ **l)** $5p - 8 = 2p + 10$

4 The truck has a weight of 7.5 tonnes and each pallet weighs 0.55 tonnes.

a) Write a function for the total weight of the truck.

b) What is the total weight if the truck carries 7 pallets?

c) If the total weight is 12.45 tonnes, how many pallets were put on the truck?

5 Ella has $165 in the bank and plans to add $35 every week.

a) Write a function for Ella's total savings.

b) How much will Ella have after 15 weeks?

c) How many weeks will Ella need to save to have $970 in the bank?

6 **a)** What would be the third number from the left of the 10th row of the pattern?

b) What would be the fourth number from the left of the 30th row of the pattern?

1	5	9	13
17	21	25	29
33	37	41	45

Why are waiters good at sums?

Because they know their tables.

Data

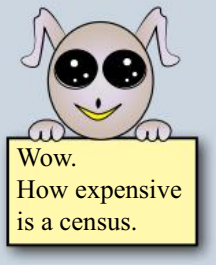
13

- ★ Explore the practicalities and implications of obtaining representative data.
- ★ Understand that making decisions and drawing conclusions based on data may differ from those based on preferences and beliefs.
- ★ Explore the variation of means and proportions in representative data.
- ★ Investigate an international issue where media reporting and the use of data reflects different cultural or social emphases.
- ★ Use sample properties to predict characteristics of the population.

A TASK

Brainstorm an opinion poll question (canteen food, induction day, suggestion box, global warming, etc).

- Design your sample (size?, how to avoid bias).
- Collect data.
- Plot data.
- Estimate the population opinion.
- Present your opinion poll .



A LITTLE BIT OF HISTORY

500 BC	First recorded census in Rome. Penalty for not participating was loss of property and enslavement.
1086 AD	William the Conquerer took a census of anything that could be taxed: families, servants, land, and livestock.
1790	US began a census every 10 years.
1881	The first Australian census. The census is now held every five years from 1961.

Census, an official survey of all of the people, and their age, gender, health, income, etc.

Population data, such as a census, is expensive. The 2006 Australian census cost $300 million. The first US census in 1790 cost $44 000. The 2010 US census cost $15 billion.

Sample Data

Collecting data is the foundation of an intelligent world. The better the data the better our decisions.

Sample
A collection of data from **part** of the population.

Sample Bias:
The sample doesn't represent the population.

Sample Size:
Must be small enough to be economical but large enough to represent the population.

Data may be collected from existing records, surveys, observations, and interviews. The objective is to collect quality data with a minimum of effort.

Exercise 13.1

Comment on each of the following sampling techniques?

The school canteen asked 50 Year 10 students if they would buy soup during winter. 20 said yes. The conclusion was that 40% ($\frac{20}{50}\times 100$)of the school's population would buy soup in winter.

The size seems large enough.
There appears to be bias because the other Year levels weren't sampled, and it doesn't appear that the students were randomly sampled.
Because of the bias there is likely to be a large error.

The Internet has sample size calculators.

The smaller the sample, the larger the error.

1 10 friends were asked if they had a part time job. 6 said yes. The conclusion was that 60% of the school's population had a part time job.

2 150 randomly selected Year 12 students were asked about their exercise habits. The conclusion was that 87% of the school's population exercised less than 30 minutes per day.

3 20 randomly selected students were asked about their TV viewing. 13 indicated that they watched more than 2 hours of TV each night. The conclusion was that 65% ($\frac{13}{20}\times 100$)of the school's population watches more than 2 hours of TV each night.

4 Survey forms, asking about what music should be played at the upcoming school dance, were placed in the Administration block along with the response box.

5 A count of cars passing the school during lunch hour showed that one-half of the cars were white. The conclusion was that 50% of the cars in the town are white.

6 5 Year 7s, 5 Year 8s, 5 Year 9s, 5 Year 10s, 5 Year 11s and 5 Year 12s were asked about their opinion of school bus travel. Only 15% of respondents were unhappy with travelling on the school bus.

Describing Sample Data

Mean, median and spread are usually used to summarise the sample data.

Exercise 13.2

The Mean is the sum of the scores divided by the number of scores. The mean is also called the **average**. The mean is heavily affected by extreme scores

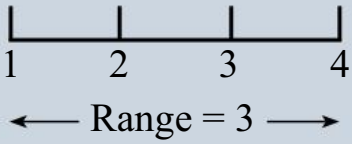
1 Find the mean of each of the following set of scores:

1, 2, 3, 4, 4

$$mean = \frac{Sum\ of\ scores}{No\ of\ scores}$$

$$mean = \frac{1+2+3+4+4}{5}$$

$mean = 2.8$



a) 1, 2, 2, 3, 4, 4
b) 35, 32, 31, 36, 36, 39
c) 21, 76, 63, 73, 88
d) 3.1, 7.6, 9.6, 5.2, 4.2, 7.3, 5.8

The Median is the middle of a set of scores. The median is a central measure that ignores extreme high scores and extreme low scores.

2 Find the median of each of the following set of scores:

5, 7, 3, 4, 3, 5, 3

Put the scores in ascending order
3, 3, 3, 4, 5, 5, 7

Median = 4 {4 is in the middle }

2, 5, 7, 5, 4, 2, 1, 3

Put the scores in ascending order
1, 2, 2, 3, 4, 5, 5, 7

Median = 3.5 {Average of 3 & 4}

a) 4, 7, 8, 3, 1, 3, 5
b) 42, 77, 35, 31, 55
c) 31, 34, 33, 33, 34, 36
d) 2.2, 5.1, 5.7, 4.2, 6.6

The Range is the simplest description of the spread of the data. The range is the difference between the smallest and the largest data values.

3 Find the range of each of the following set of scores:

1, 2, 3, 4, 4

Range = largest – smallest
= 4 – 1
= 3

a) 2, 5, 13, 27, 42, 45
b) 46, 54, 61, 69, 73, 85
c) 620, 638, 659, 683, 712, 737
c) 3.2, 4.1, 4.8, 5.2, 5.6

Sample Size

The larger the sample size the better the population estimation.

The larger the sample size the more resources used (in time, money etc).

1 100 random numbers from 1 to 10 are written on pieces of paper and put in a bucket.
2 5 numbers are randomly selected and their mean, median, and range found.
3 The numbers are put back in the bucket and then 10 numbers are randomly selected and their mean, median, and range found.
4 The following table was completed:

Sample Size	Mean	Median	Range
5	4.6	4	7
10	5.7	6	8
15	5.5	6	9
20	4.9	4.5	9
25	4.92	5	9
100	5.04	5	9

The population

A sample size of 20 has mean, median, range close to the population.

Is a sample size of 20 from 100 a good balance of error and resources?

Exercise 13.3

1 Write 100 random numbers from 1 to 10 on pieces of paper and put them in a bucket.
2 Randomly select 5 numbers and find their mean, median, and range.
3 Randomly select 10 numbers and find their mean, median, and range.
4 Randomly select 15 numbers and find their mean, median, and range.
5 Complete the following table:

Sample Size	Mean	Median	Range
5			
10			
15			
20			
25			
100			

From a population of 100 what is the best sample size?

6 Which sample size gives the best balance between population estimation and least effort?

Sample Bias

Bias: influence in an unfair way.

A sample with bias is a waste of time.

Exercise 13.4

1 Bias, unfair influence, appears in many different forms.
Research and make a list of 20 biases.

Ethnic	Bias in the form of racism or regionalism.
Gender	Bias, either for or against, a gender.
Media	Bias of journalists in covering and reporting events.
Personal	Bias for personal gain.
Political	Bias, either for or against, a political party or philosophy.
Religious	Bias, either for or against, religion, faith or beliefs.

2 A random sample helps avoids bias because each member of the population has an equal chance of being selected in the sample.
Select a random sample of 5 students from your class

a) Number each member of your class from 1 to 25 (or 28 if there are 28 people in your class).

b) Generate 5 random numbers (See Technology 13.1).

1	Oliver
2	Amelia
3	Charlotte
4	Noah
5	Chloe
6	Jack
7	Isabella
8	Ethan
9	.....
10	.....

5 Random numbers:

6	Thus Jack
21	
13	
3	Thus Charlotte
17	

3 A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.
How many students from each Year level of the following school should be randomly selected in a sample size of 60 students?

Year	Number
7	200
8	200
9	150
10	50

Total = 600

Year 8 fraction = 200/600=0.33

Number to select = 0.33 × 60 = 20

Exercise 13.5

35 students in a random sample of 50 students have brown eyes. Estimate the number of students in a school of 400 with brown eyes.

% brown eyes $= \frac{35}{50} \times 100$
$= 70\%$

No students $= 70\%$ of 400
$= \frac{70}{100} \times 400$
$= 280$

Conclusion: 280 students have brown eyes.

A really rough error?
$= \sqrt{\frac{p(1-p)}{n}}$
$= \sqrt{\frac{70(1-70)}{50}}$
$= 7\%$ error
$= 7\%$ of 280

No students $= 280 \pm 20$

Number of brown eyed students between 300 and 240.

1 30 students in a random sample of 50 students have brown eyes. Estimate the number of students in the school of 600 with brown eyes.

2 15 students in a random sample of 25 students work part time. Estimate the number of students in the school of 700 students who work part time.

3 12 students in a random sample of 60 students indicated that they would buy soup from the canteen in Winter. Estimate the number of students in the school of 240 students who would buy soup from the canteen in Winter.

4 A randomly selected sample of 500 households suggests that 150 of the households regularly watch a TV news segment. Estimate the number of households, from the population of 12 million households, that regularly watch a TV news segment.

5 34 rose cuttings were treated with root hormone and planted in early March. 18 of the rose cuttings were successful.. Estimate the number of successful rose plants that would take from 4000 rose cuttings.

6 A randomly selected sample of 80 respondents suggests that 22 of the respondents favour the policies of the Prime Minister. Estimate the number of people, from a population of 23 million people, who favour the policies of the Prime Minister.

7 75% of the people who had answered the online question "Do you favour an increase in the GST rate?" had responded with a no.

a) Estimate the number of people from a population of 245 million people that do not favour an increase in the GST rate.

b) Indicate two possible sources of bias in the online survey.

Mental Computation

Mental computation helps you learn how to think.

Exercise 13.6

1 Spell random.
2 Give an example of sampling bias.
3 Write in index form: $2\times2\times2\times2$
4 Solve: $2x + 5 = 11$
5 Simplify: $n^2\times n^3$
6 What is the mean of: 1,3,3,3?
7 Add GST (10%) to $40.
8 A taxi charges $3 flagfall and $2 per km. What is the function?
9 Change $2\frac{2}{3}$ to a vulgar fraction.
10 Simplify: $3^5\div3^2$

Statisticians put their head in the oven and their feet in the refrigerator: "On average they feel fine."

Exercise 13.7

1 Spell bias.
2 Give an example of sampling bias.
3 Write in index form: $5\times5\times5\times5\times5\times5$
4 Solve: $5x + 2 = 17$
5 Simplify: $a^2\times a^5$
6 What is the median of: 1,2,4,5,6?
7 Add GST (10%) to $90.
8 A plumber charges $60 callout and $50 per hour. What is the function?
9 Change $3\frac{1}{2}$ to a vulgar fraction.
10 Simplify: $6^4\div6^2$

"Fall seven times, stand up eight."
- Japanese proverb

Exercise 13.8

1 Spell population.
2 Give an example of sampling bias.
3 Write in index form: $3\times3\times3\times3\times3$
4 Solve: $3x + 7 = 13$
5 Simplify: $b^2\times b^4$
6 What is the mean of: 1,2,3,4?
7 Add GST (10%) to $30.
8 A call costs 15 cents flagfall and 98 cents per minute. What is the function?
9 Change $2\frac{3}{4}$ to a vulgar fraction.
10 Simplify: $6^3\div6^2$

NAPLAN Questions

Exercise 13.9

1 Find the mean (average) travelling time:

Monday	Tuesday	Wednesday	Thursday	Friday
20 mins	25 minutes	20 minutes	15 minutes	1 hour 10 minutes

2 Given the scores: 2, 2, 3, 3, 4, 4, 4, 6, 50

a) What is the mean, median, and mode?

b) What is the mean, median, and mode after 50 is removed?

c) Which is affected the most by removing 50, the mean, median, or mode?

3 **a)** Find the mean, median, and mode of the following: 1, 1, 2, 2, 2, 3, 4

b) Is Mean > Median = Mode true?

4 Sienna is spending five days with her sister in the city. What is Sienna's daily saving if she uses a weekly ticket instead of a daily return ticket?

5 Mia scored 16, 13, 8, 18 and 17 points in the first 5 games of the season. Mia then scored 12 points in the sixth game. After including the sixth game, what has happened to the:

a) mean? **b)** median? **c)** range?

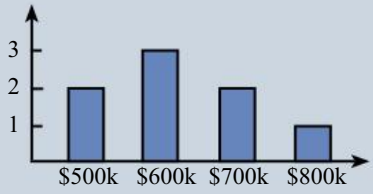
Attack the question:
- Skim the question to get an overview.
- Reread carefully, underline key words and numbers.

6 Given the numbers: 1 3 3 4 5 6 8 find the

a) mean **b)** mode **c)** median **d)** range

7 Given the numbers: 21 23 23 24 25 26 28, find the

a) mean **b)** mode **c)** median **d)** range



8 The graph shows a real estate agent's home sales for April.

a) Find the mean and the median.

b) The $800 000 sale fell through. Find the new mean and median.

Competition Questions

Prepare for Mathematics Competitions and build maths muscle at the same time.

Exercise 13.10

1 What is the average of the numbers: 0.2, 0.22, 0.222?

2 What is the average of the numbers: 2.0, 2.2, 2.22?

3 What is the average of the numbers: 20, 22, 22.2?

The average of four numbers is 10. If 2 is subtracted from each number, what is then the average of the four numbers?

If 2 is subtracted from each number, then 2 is subtracted from the average.
The new average = 10 - 2 = 8

4 The average of four numbers is 20. If 5 is subtracted from each number, what is then the average of the four numbers?

5 The average of four numbers is 36. If 6 is subtracted from each number, what is then the average of the four numbers?

6 The average of eight numbers is 48. If 4 is added to each number, what is then the average of the eight numbers?

7 If 5 million people have total savings of $4 billion, what is the average savings per person?

8 If 50 million people have total savings of $10 billion, what is the average savings per person?

9 If 45 million people inhabit 45 000 km^2, what is the average number of people per km^2?

In a set of five numbers, the average of the first two numbers is 8 and the average of the last three numbers is 6. What is the average of the five numbers?

$$8 = \frac{total}{2}$$

Total of first two numbers = 16

$$6 = \frac{total}{3}$$

Total of last two numbers = 18

Total of the five numbers = 16+18 = 34

$$mean = \frac{34}{5}$$

average = 6.8

10 In a set of five numbers, the average of the first two numbers is 7 and the average of the last three numbers is 6. What is the average of the five numbers?

11 In a set of ten numbers, the average of the first seven numbers is 8 and the average of the last three numbers is 9. What is the average of the ten numbers?

"A journey of a thousand miles begins with a single step." - Confucius

Investigations

Investigation 13.1 How many centicubes in the bag?

1 Have someone put a large number of centicubes in a bucket and ask them to cover the bucket with a cloth so that you can't see into the bucket and you don't know how many centicubes are in the bucket.

2 Reach into the covered bucket and get 30 centicubes from the bucket. Put a small mark on the centicubes and put them back into the bucket. Shake the bucket.

3 Remove a sample of 40 centicubes from the bucket and count the number of marked centicubes.

4 The number of centicubes in the bucket $= \dfrac{40}{\text{No of marked centicubes}} \times 30$

5 Repeat the sampling with what you think may be a better number of marked centicubes and a better sample size.

How can tagging a few fish in a dam help work out how many fish are in the dam?

How would I use this method to estimate how many people live in a town?

Investigation 13.2 TV Ratings

- ☺ How are TV ratings made?
- ☺ Comment on the sample size.
- ☺ How is sampling bias avoided?
- ☺ Conduct your own TV Rating.

Investigation 13.3 Opinion Polls

- ☺ How are Opinion polls made?
- ☺ Comment on the sample size.
- ☺ How is sampling bias avoided?
- ☺ Find examples of Opinion polls that have been very wrong.
- ☺ Conduct your own Opinion poll.

Investigation 13.4 Media Bias

- ☺ Internet search "media bias examples".
- ☺ Find media articles that involve sampling.
- ☺ Is the sample size sufficient?
- ☺ Is the sample biased?

If you want three opinions, just ask two statisticians.

Technology

Technology 13.1 Methods of randomly selecting 5 class members

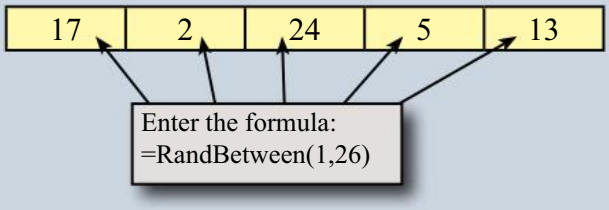
Assuming 26 people in the class.

a) Pen and Paper Method:
Give each member of your class a number from 1 to 26.
Write each number from 1 to 26 on a piece of paper (same sized pieces?).
Put the numbers in a container and randomly select 5 pieces of paper.

b) Calculator Method:
Give each member of your class a number from 1 to 26).
Find the random (Rand) button.
Repeat 5 times:

Rand × **26** + **1** =

c) Spreadsheet Method:
Give each member of your class a number from 1 to 26.
Use the RandBetween to give random numbers from 1 to 26.



d) Graphics Calculator Method
Give each member of your class a number from 1 to 26.
Choose the random from the calculator (Math → Prob → randINT).

Enter: **randInt(1,26,5)**

This will give 5 random numbers from 1 to 26.

d) Internet Method
Give each member of your class a number from 1 to 26.
Find a "random number generator" on the Internet.
Follow the instructions.

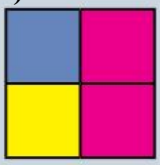
Biostatisticians work in hospitals and with health authorities to study the link between diseases and their causes.
- Relevant school subjects are English and Mathematics.
- Courses usually involve a university degree.

A Couple of Puzzles

Exercise 13.11

1 Which is the missing square?

a)



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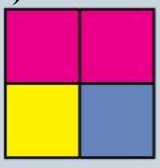

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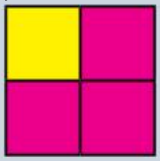
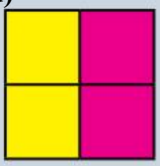
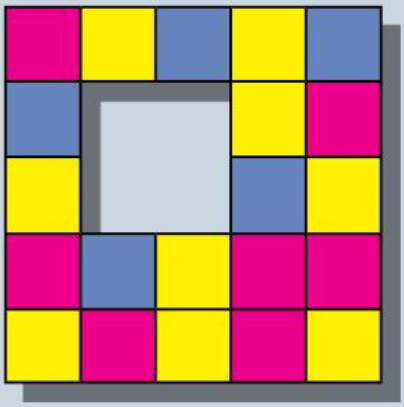


d)





A Game

Technology 13.1 shows how to use the calculator random button.

Calculator Cricket

1 **Repeat**
Player 1 generates a random number;
Sum the numbers;
Until the number is zero.

2 **Repeat**
Player 2 generates a random number;
Sum the numbers;
Until the number is zero.

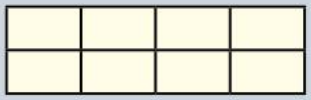
3 The winner is the player with the highest score.

Use the tenths digit?

If the random number is 0.2834 then the score is 2

If the random number is 0.0218 then the player is out.

A Sweet Trick



Make sure that the pieces of paper with 2 torn edges go to the even people.

Make sure that the pieces of paper with 3 torn edges go to the odd people.

1 Tear a sheet of paper into eight roughly equal sized pieces of paper.

2 Give each piece to a person and ask four people to write any even number on their piece and ask the other four people to write any odd number on their piece of paper.

3 Ask them to put the eight pieces face down on a table and to mix them up.

4 You, with appropriate theatre, indicate which pieces have an odd number and which have an even number (jewelry on a string swinging in a circle over odd and in a straight line over even?).

Chapter Review 1

Exercise 13.12

1 Comment on the following sampling technique:
10 friends were asked if they had a school locker. 7 said yes. The conclusion was that 70% of the school's population had a school locker.

2 An opinion poll concludes that: "48% favour the Government and 52% favour the opposition." Why should the opinion poll also report the sample size and the method of sampling?

3 Find the mean, median, and range of each of the following sets of data:

a) 2, 3, 5, 5, 5, 7, 9 **b)** 22, 23, 25, 25, 25, 27, 29

c) 540, 542, 553, 568 **d)** 2.4, 2.5, 2.8, 2.8, 2.9

35 students in a random sample of 50 students have brown eyes. Estimate the number of students in a school of 400 with brown eyes.

% brown eyes $= \frac{35}{50} \times 100$
$= 70\%$

No students $= 70\%$ of 400
$= \frac{70}{100} \times 400$
$= 280$

Conclusion: 280 students have brown eyes.

A really rough error?
$= \sqrt{\frac{p(1-p)}{n}}$
$= \sqrt{\frac{70(1-70)}{50}}$
$= 7\%$ error
$= 7\%$ of 280

No students $= 280 \pm 20$

4 22 students in a random sample of 50 students have blue eyes. Estimate the number of students in the school of 450 with blue eyes.

5 A randomly selected sample of 800 households suggests that 124 of the households regularly watch a TV local news segment. Estimate the number of households, from the population of 10 million households, that regularly watch a TV local news segment.

6 A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.
How many students from each Year level of the following school should be randomly selected in a sample size of 70 students?

Year	Number
7	100
8	100
9	100
10	50

Did you know that the great majority of people have more than the average number of toes?

Chapter Review 2

Exercise 13.13

1 Comment on the following sampling technique:
15 friends were asked if there were enough rubbish bins in the school grounds. 12 said yes. The conclusion was that 80% of the school's population were satisfied with the number of rubbish bins in the school grounds.

2 An opinion poll concludes that: "74% favour an increase in taxes to pay for better educational facilities and 26% oppose the increase in taxes." Why should the opinion poll also report the sample size and the method of sampling?

3 Find the mean, median, and range of each of the following sets of data:

a) 1, 4, 4, 4, 6, 7, 8

b) 51, 54, 54, 54, 56, 57, 58

c) 1540, 1542, 1553, 1568

d) 6.4, 6.5, 6.8, 6.8, 6.9

35 students in a random sample of 50 students have brown eyes. Estimate the number of students in a school of 400 with brown eyes.

% brown eyes $= \frac{35}{50} \times 100$
$= 70\%$

No students $= 70\%$ of 400
$= \frac{70}{100} \times 400$
$= 280$

Conclusion: 280 students have brown eyes.

A really rough error?
$= \sqrt{\frac{p(1-p)}{n}}$
$= \sqrt{\frac{70(1-70)}{50}}$
$= 7\%$ error

$= 7\%$ of 280

No students $= 280 \pm 20$

4 35 students in a random sample of 50 students have dark hair. Estimate the number of students in the school of 1450 with dark hair.

5 A randomly selected sample of 600 households suggests that 179 of the households regularly watch a TV sports segment. Estimate the number of households, from the population of 8 million households, that regularly watch a TV sports segment.

6 A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.
How many students from each Year level of the following school should be randomly selected in a sample size of 100 students?

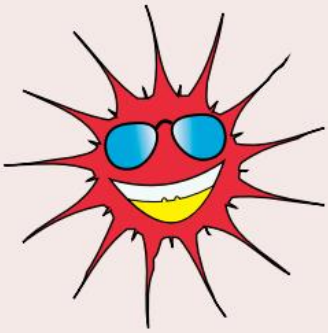
Year	Number
7	250
8	250
9	300
10	250

Q: How many statisticians does it take to change a light bulb?
A: One - plus or minus three.

Time

14

- ★ Solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.
- ★ Calculate travel times given the start and finish time in 12-hour and 24-hour time including where the start and end time are in different time zones.
- ★ Identify regions in Australia and countries in Asia that are in the same time zone.



A TASK

Calculate the circumference of the Earth by using Eratosthenes' method.

- Research Eratosthenes' method.
- Conduct your experiment.
- Calculate Earth's circumference.
- Publish your findings in the local newspaper.

The sun circles the Earth in 24 hours. The Sun thus moves 360° in 24 hours or 15° in one hour.

A LITTLE BIT OF HISTORY

Eratosthenes taught at the University of Alexandria around 240 BC.

Eratosthenes estimated the circumference of the Earth to be about 38 500 km.

His estimation was made by noting that when a vertical stick had no shadow at Cyrene, now Libya, at the same time a vertical stick at Alexandria 770 km away casts a shadow of 1/50 th of a circle.

12-hour time

1 day = 24 hours
1 hour = 60 mins
am = morning ✓
pm = afternoon ✓

Exercise 14.1

What is the time 3 hours after 6:15 am?

= 6 hours 15 mins + 3 hours
= 9 hours 15 mins
= 9:15 am

What is the time 45 mins before 9:20 pm?

= 9 hours + 20 mins − 45 mins
= 8 hours + 80 mins − 45 mins
= 8 hours + 35 mins
= 8:35 pm

Change 1 hour to 60 mins.

1 What is the time:
- **a)** 4 hours after 7:30 am?
- **b)** 5 hours before 10:20 am
- **c)** 2 hours 15 minutes after 9:15 am?
- **d)** 7 hours 20 minutes before 3:10 pm?
- **e)** 6 hours 33 minutes after 3:37 pm?

How much time between 9:15 pm and 3:20 am?

From 9:15 pm to 10 pm	= 0 hours	45 mins
From 10 pm to midnight	= 2 hours	
From midnight to 3:20 am	= 3 hours	20 mins
Total	= 5 hours + 65 mins	
	= 6 hours 5 mins	

65 mins = 1 hr 5 mins.

Use a calculator? See Technology 14.1

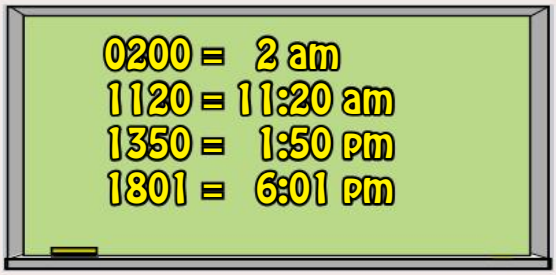
2 How much time between:
- **a)** 3:10 pm and 8:30 pm
- **b)** 6:45 am and 8:30 am
- **c)** 1:53 am and 10:19 am
- **d)** 11:51 pm and 4:37 am
- **e)** 11:15 am and 2:20 pm
- **f)** 8:55 am and 3:10 pm

3 John leaves home at 7:15 am and takes 10 mins to walk to the bus station. He waits 5 mins for the bus and the bus takes 25 mins to travel to his workplace. At what time does John get to work?

4 The plane leaves Sydney at 8:45 am and arrives in Melbourne at 11:10 am. How long is the flight?

5 The train leaves Brisbane at 10:25 pm on Tuesday and arrives in Melbourne at 8:15 am Wednesday. How long is the train trip?

24-hour time



Exercise 14.2

1 Convert the following 12-hour times to 24-hour time:

a)	10:15 am	**b)**	7:12 am	**c)**	9:05 am
d)	3:10 pm	**e)**	6:30 pm	**f)**	11:45 pm
g)	1:20 am	**h)**	8:42 pm	**i)**	10:00 am

6:20 am = 0620
5:20 pm = 1200 + 0520
= 1720

2 Convert the following 24-hour times to 12-hour time:

a)	0230	**b)**	0645	**c)**	0817
d)	1156	**e)**	1320	**f)**	1515
g)	1810	**h)**	2040	**i)**	2325

0715 = 7:15 am
1630 = 1200 + 0430
= 4:30 pm

How much time between 0530 and 1240?

$$\begin{array}{rr} 12 & 40 \\ 05 & 30 \; - \\ \hline 07 & 10 \end{array} = 7 \text{ hours } 10 \text{ minutes}$$

How much time between 1350 and 1810?

$$\begin{array}{rrcrr} 18 & 10 & = & 17 & 70 \\ 13 & 50 & & 13 & 50 \; - \\ \hline & & & 04 & 20 \end{array}$$

= 4 hours 20 minutes

3 How much time between (24-hour times):

a)	1115 and 1420	**b)**	0805 and 1510
c)	1010 and 1540	**d)**	0645 and 0830
e)	0153 and 1019	**f)**	0437 and 2351

4 The plane leaves Hobart at 0640 and arrives in Sydney at 0950
How long is the flight?

5 The plane leaves Hobart at 0905 and arrives in Canberra at 1235
How long is the flight?

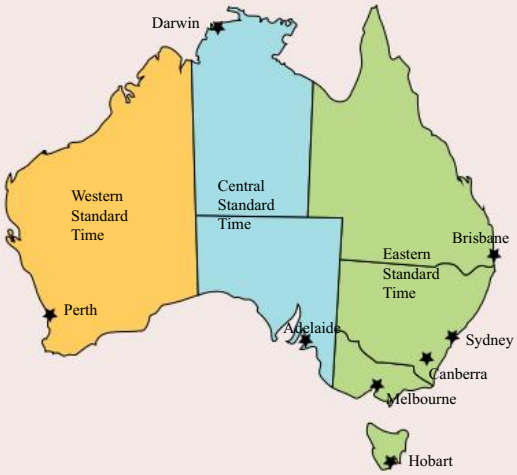
6 The ferry leaves Melbourne at 0625 and arrives in Hobart at 1315
How long is the trip?

7 The train leaves Melbourne at 0725 pm on Saturday and arrives in Brisbane at 0910 am Sunday. How long is the train trip?

Time n. 1. a method of measuring the passage of time.

Time Zones

Australia has 3 time zones.



Western	Central	Eastern
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

What is always behind the times?
The back of the clock.

Exercise 14.3

Use the above information to answer the following questions:

If the time in Sydney is 10 am, what is the time in Perth? 8 am {Perth is 2 hours earlier}	If the time in Darwin is 4 pm, what is the time in Hobart. 4:30 pm {Hobart is 30 mins later}

1 If the time in Adelaide is 8:30 pm, what is the time in

a) Brisbane **b)** Canberra **c)** Darwin **d)** Hobart
e) Melbourne **f)** Perth **g)** Sydney?

2 If the time in Brisbane is 10 am, what is the time in

a) Adelaide **b)** Canberra **c)** Darwin **d)** Hobart
e) Melbourne **f)** Perth **g)** Sydney?

3 If the time in Canberra is 3 pm, what is the time in

a) Adelaide **b)** Brisbane **c)** Darwin **d)** Hobart
e) Melbourne **f)** Perth **g)** Sydney?

4 If the time in Darwin is 11 pm, what is the time in

a) Adelaide **b)** Brisbane **c)** Canberra **d)** Hobart
e) Melbourne **f)** Perth **g)** Sydney?

5 If the time in Hobart is 6 am, what is the time in

a) Adelaide **b)** Brisbane **c)** Canberra **d)** Darwin
e) Melbourne **f)** Perth **g)** Sydney?

6 A person in Perth wishes to phone a business in Sydney before the business in Sydney closes at 5pm. Before what time in Perth should the person phone?

Time Zones

Exercise 14.4

The plane leaves Brisbane at 7:55 pm and arrives in Adelaide at 9:50 pm.
How long is the flight (Brisbane and Adelaide are in different time zones)?

Brisbane time 7:55 pm = Adelaide time 7:25 pm {30 min difference}

Now find time between 7:25 pm and 9:50 pm

	9 hours	50 mins
	7 hours	25 mins −
Trip time	2 hours	25 mins

We are not considering daylight saving times at this stage.

1 The plane leaves Brisbane at 8:25 pm and arrives in Adelaide at 11:35 pm.
How long is the flight (Brisbane and Adelaide are in different time zones)?

2 Given the following Flight Reviews, how long is each flight?

a)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Brisbane **3:55 PM**	Adelaide **7:05 PM**	KP 416 Economy	20 kg

b)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Sydney **1920**	Adelaide **2140**	RT 551 Economy	23 kg

c)

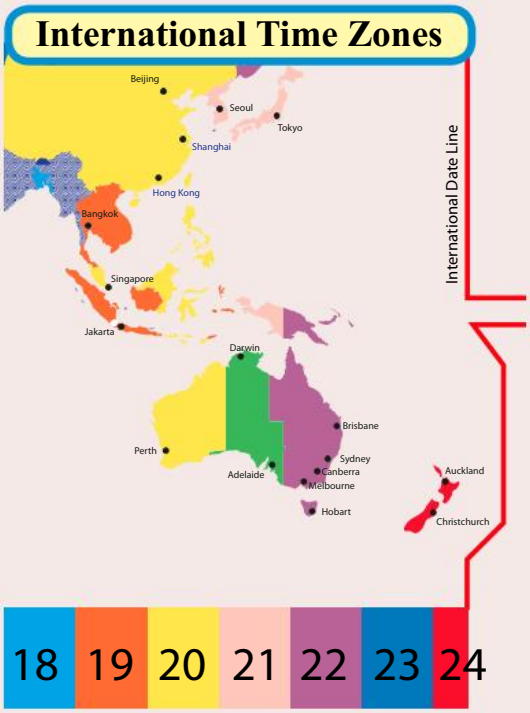
DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Melbourne **1405**	Darwin **1650**	DM 105 Economy	20 kg

d)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Perth **10:40 PM**	Hobart **5:10 am**	DM 675 Economy	20 kg

3 Given the following train connections, find the length of time for each journey.

	Connections	Day Depart	Time Depart	Day Arrive	Time Arrive
a)	Sydney-Perth	Saturday	2:55 pm	Tuesday	9:10 am
b)	Melbourne-Perth	Saturday	7:35 am	Tuesday	9:10 am
c)	Adelaide-Perth	Sunday	6:40 pm	Tuesday	9:10 am
d)	Perth-Adelaide	Sunday	11:55 am	Tuesday	7:20 am
e)	Perth-Melbourne	Sunday	11:55 am	Wednesday	6:40 pm
f)	Perth-Sydney	Sunday	11:55 am	Wednesday	10:15 am



The sun circles the Earth in 24 hours. The Sun thus moves 360° in 24 hours or 15° in one hour.

The Earth is divided into 24 time zones (lines of meridian). Each zone is 1 hour apart (15°).

We are not considering daylight saving times at this stage.

Sydney is in 22 hr time zone.
Bangkok is in 19 hr time zone.
Sydney is 3 hrs ahead of Bangkok.

Exercise 14.5

The plane leaves Sydney at 1805 and arrives in Bangkok at 2310.
How long is the flight?

Sydney time 1805 pm = Bangkok time 1505 {3 hrs difference}
Now find time between 2310 and 1505

	23hours	10 mins
	15 hours	05 mins −
Trip time	8 hours	5 mins

1 Given the following Flight Reviews, how long is each flight?

a)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Sydney **1155**	Hong Kong **1750**	ST 46 Economy	18 kg

b)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Melbourne **1805**	Auckland **2340**	BZ 1 Economy	20 kg

c)

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Christchurch **5:40 am**	Tokyo **4:25 pm**	BZ 51 Economy	20 kg

Mental Computation

Employees are expected to be competent at mental computation.

Exercise 14.6

1 Spell time zone.
2 How many hours in 2 days?
3 Write in index form: $5\times5\times5\times5\times5\times5$
4 Solve: $5x - 4 = 11$
5 Convert 24-hour time 1935 to 12-hour time.
6 What is the mean of: 1,2,3,4?
7 Add GST (10%) to $80.
8 A taxi charges $3 flagfall and $2.90 per km. What is the function?
9 If the Sun covers 360° in 24 hours, how many degrees in 1 hour?
10 Simplify: $4^5 \div 4^3$

Stretch your arm out and spread your hand. Your spread hand covers about 1 hour of the sky. ie., 15°.

Exercise 14.7

1 Spell twenty-four.
2 How many hours in 3 days?
3 Write in index form: $2\times2\times2\times2\times2$
4 Solve: $2x + 9 = 3$
5 Convert 12-hour time 3:20pm to 24-hour time.
6 What is the mean of: 1,2,2,3?
7 Add GST (10%) to $23.
8 A plumber charges $50 callout and $70 per hour. What is the function?
9 If the Sun covers 360° in 24 hours, how many degrees in 2 hours?
10 Simplify: $2^5 \times 2^3$

It is not enough to stare up the steps, we must step up the stairs - Vaclav Havel.

Exercise 14.8

1 Spell International.
2 How many hours in 4 days?
3 Write in index form: $a\times a\times a\times a\times a\times a$
4 Solve: $4x + 15 = 7$
5 Convert 24-hour time 1023 to 12-hour time.
6 What is the mean of: 2,2,3,3?
7 Add GST (10%) to $34.
8 The wage involves a retainer of $450 and $50 per sale. What is the function?
9 If the Sun covers 360° in 24 hours, how many degrees in 3 hours?
10 Simplify: $x^5 \div x^2$

NAPLAN Questions

Exercise 14.9

1 Eun walked at a speed of 8 km per hour for 30 minutes. How far did she walk?

2 The table shows the time that Cooper spent on data recording last week.
What is the average of these times?

Data Recording	
Monday	4 hours
Tuesday	2 hours
Wednesday	5 hours
Thursday	2 hours
Friday	6 hours

3 How much time between:
- **a)** 6:25am and 9:40 am?
- **b)** 9:05am and 3:15 pm?
- **c)** 4:52am and 6:15pm?
- **d)** 1410 and 1855 (24-hour time)?
- **e)** 0850 and 1510 (24-hour time)?

4 John leaves Sydney at 1320 and arrives in Singapore at 1840 (Singapore time).
If Sydney time is 2 hours ahead of Singapore time, how long did the journey take?

5 The distance on the map from Kuala Lumpur to Singapore is 1.6 cm. How long will it take to travel from Kuala Lumpur to Singapore at a speed of 100 km per hour?

Scale: 1cm = 200km

6 Jack worked for 6 hours at $25 per hour. How much did Jack earn?

7 Riley drove for 6 hours at 80 km per hour. What distance did Riley travel?

8 Li drove 450 km in 6 hours. What was Li's average speed?

9 Sonoko drove 500 km in 6 hours. If her average speed for the first 300 km was 90 km per hour, how long did it take to travel the last 200 km?

10 Charlotte drove 330 km in 6 hours. If her average speed for the first 150 km was 60 km per hour, how long did it take to travel the rest of the journey?

11 Sophie is 5 years older than her brother Liam and one-quarter her mother's age. If Sophie's mother is 52 years old, how old is Liam?

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 14.10

If a 1¾ hour training session starts at 11:20 am, when should it finish?

11 hrs	20 mins	
1 hr	45 mins	+
12 hrs	65 mins	
13 hrs	5 mins = 1:05pm	

1 If a 1¼ hour training session starts at 6:20 am, when will it finish?

2 A movie lasts 1 hour 35minutes. If the movie starts at 8:45 pm, when will it finish?

3 Jay catches the train at 6:10 am and knows that the train will take 1 hour 20 minutes minutes to get to Dutton Park. When will Jay arrive at Dutton Park?

A train left Alpha at 11:43 am and arrived in Emerald at 3:30 pm on the same day. How long did the journey take?

3:30 pm	= 15 hours 30 minutes	= 14 hours	90 minutes	
11:43 am	= 11 hours 43 minutes	= 11 hours	43 minutes	–
		3 hours	47 minutes	

4 A train left Ingham at 10:51 am and arrived in Innisfail at 1:59 pm. How long did the journey take?

5 A train left Benalla at 9:25 am and arrived in Wagga Wagga at 2:03 pm. How long did the journey take?

6 A train left Port Agusta at 10:25 am and arrived in Ceduna at 2:10 pm. How long did the journey take?

7 How many hours from 8 am Monday until 5 pm Friday of the same week?

8 At our school the first class starts each morning at 9:00 am and the last class finishes at 3:10 pm. We have a morning break of 30 minutes and a lunch break of 50 minutes. How much time is spent in class?

9 To the nearest year, how long is 1 million minutes?

10 Kayla completed her Masters degree in March 2011. If the Masters took Kayla 3 years and 5 months, when did Kayla start?

11 Jay can run three times as fast as she can walk. If Jay runs half the distance and walks the rest of the way to work, it takes her 32 mins. How long would it take Jay to walk to work?

12 How many times a day do the hands of a clock form a right angle (90°)?

Technology

Technology 14.1
Use the degree, minute, second button on your calculator to add and subtract times.
The button looks something like: [D°M'S] or [° ' '']

1 Class starts at 10:23am and finishes 11:13am, how long is the class?

[**11**] [D°M'S] [**13**] [−] [**10**] [D°M'S] [**23**] [=] 50 minutes

2 Use a calculator to redo the time problems in Exercise 14.1 and 14.2.

Technology 14.2
Use the degree, minute, second button on your calculator to change normal time to decimal time, and change decimal time to normal time.

1 Change 2 hours 24 minutes to decimal time.

[**2**] [D°M'S] [**24**] [=] [**2nd F**] [D°M'S] to give 2.4 hours.

2 Change 2.15 hours to normal time.

[**2.15**] [**2nd F**] [D°M'S] [=] to give 2 hours 9 minutes

Technology 14.3
Use a spreadsheet to calculate standard times in each Australian city relative to Eastern Standard Time (EST).

Handling time in a spreadsheet is a bit tricky at first.

To enter a time based on the 12-hour clock, type a space and then **a** or **p** after the time; for example, **3:00 p**.

To do calculations, enter the time in quotes eg A2 – "1:30".

EST	Perth	Adelaide	Darwin	Brisbane	Canberra	Hobart	Melbourne	Sydney
3:00pm	1:00pm	2:30pm	2:30pm	3:00pm	3:00pm	3:00pm	3:00pm	3:00pm

Enter 3:00p

Enter the formula: =A2 – "0:30"

Format the cells by: <format>,<cells>,<time>

Technology 14.4
Design a spreadsheet to show times in each Australian city during daylight saving time.

Investigations

Investigation 14.1 The Moon

1 Lunar month?

a) What are the phases of the moon?

b) How long does it take the moon to complete a full cycle of phases?

c) On which phase of the moon is the best fishing? Why?

d) Some gardeners use the phases of the moon for planting. Why?

e) How does the phase of the moon affect Easter?

2 Blue Moon

a) What is a blue moon?

b) How often does a Blue Moon occur?

3 The moon and the tides.
Make diagrams to explain each of the following:

a) How does the moon affect the tides?

b) What are lunar tides?

c) What are spring tides?

d) What are neap tides?

4 Find out about some full moon influences.

Investigation 14.2 Sundials

1 Use the Internet to find out about sundials.

2 Make a sundial.

The shadow stick was the earliest form of sundial. People judged the time of day by the length and position of the stick's shadow.

Gnomon is the name for the shadow stick (Greek for 'the one that knows'.)

Investigation 14.3 Australia and Asia

1 Find a time zone map of Asia on the Internet.

2 Identify regions in Australia and countries in Asia that are in the same time zone.

3 Name a city in Asia that has the same time as Perth.

4 Name a city in Asia that has the same time as Melbourne and Sydney.

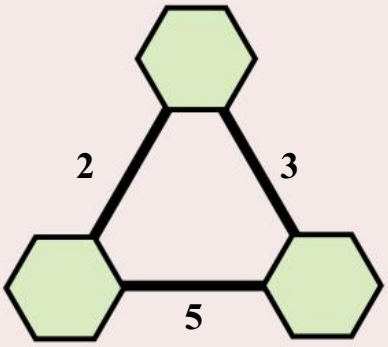
Petroleum Engineers use an understanding of of the behaviour of oil, water and gas under high pressure in rock to recover hydrocarbons (oil and gas).

- Relevant school subjects are English and Mathematics.
- Courses normally involve a University Degree with a lot of mathematics.

A Couple of Puzzles

Exercise 14.11

1 Put a number in each corner so that the differences are as shown.



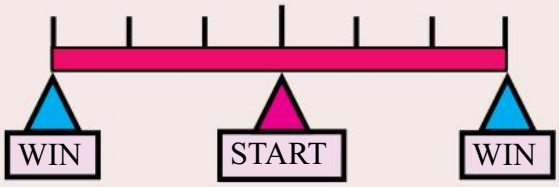
2 Two mothers and two daughters each ate a whole boiled egg for breakfast. How is this possible if there were only three eggs?

3 Do you know the answer to the sum without adding each number?

```
    10000
   111000
  1111100
 11111110
+ 111111111
```

A Game

Tug of War



Player 1 Player 2

1 Each of two players starts with 50.
2 Each player writes down, in secret, how many points they are going to use in this move.
3 The player with the highest points moves their marker one place from the START to the WIN.
4 Each player subtracts their points from the total.

The winner is the first to reach their WIN.

A Sweet Trick

Multiply up to 20×20 in your head.

This is an impressive trick that doesn't take too much practice.

Example: 16 × 17

1	Add the first number to the units of the second.	16+7 = 23
2	Multiply by 10.	23×10 = 230
3	Multiply the units digits of each number.	6×7 = 42
4	Sum the two answers.	230 + 42 = 272

Practice this first on paper. Then practice mentally.

Chapter Review 1

Exercise 14.12

1 What is the time:

a) 5 hours after 6:10 am?

b) 3 hours 15 minutes after 8:15 am?

c) 4 hours 30 minutes before 3:10 pm?

2 How much time between:

a) 3:20 pm and 9:30 pm?

b) 7:50 am and 9:30 am?

c) 11:15 am and 2:20 pm?

d) 8:55 am and 3:10 pm?

3 Kon leaves home at 7:05 am and takes 15 mins to walk to the bus station. He waits 5 mins for the bus and the bus takes 35 mins to travel to his workplace. At what time does Kon get to work?

4 Convert the following 12-hour times to 24-hour time:

a) 9:05 am **b)** 7:42 am **c)** 3:10 pm

5 Convert the following 24-hour times to 12-hour time:

a) 0150 **b)** 0915 **c)** 1835

6 How much time between (24-hour times):

a) 1015 and 1435?

b) 0645 and 1310 ?

7 The plane leaves Sydney at 1640 and arrives in Hobart at 1950. How long is the flight?

Q Why is a clock like a river?
A Because it won't run for long without winding.

Western	Central	Eastern
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

8 If the time in Brisbane is 9 am, what is the time in

a) Adelaide? **b)** Canberra? **c)** Darwin? **d)** Hobart?

e) Melbourne? **f)** Perth? **g)** Sydney?

9 The plane leaves Adelaide at 7:20 am and arrives in Melbourne at 9:15 am. How long is the flight (Melbourne and Adelaide are in different time zones)?

10 Given the following Flight Review, how long is the flight?

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Brisbane **5:35 PM**	Adelaide **8:45 PM**	KP 416 Economy	20 kg

Chapter Review 2

> Whether you think you can or whether you think you can't, you're right! - Henry Ford

Exercise 14.13

1 What is the time:

a) 3 hours after 4:50 pm?

b) 5 hours 25 minutes after 10:15 am?

c) 4 hours 20 minutes before 2:10 pm?

2 How much time between:

a) 5:20 am and 8:50 am?

b) 2:50 pm and 6:10 pm?

c) 10:35 am and 3:10 pm?

d) 9:50 am and 4:05 pm?

3 Kerry leaves home at 8:15 am and takes 5 mins to walk to the bus station. She waits 5 mins for the bus and the bus takes 25 mins to travel to her workplace. At what time does Kerry get to work?

4 Convert the following 12-hour times to 24-hour time:

a) 8:45 am **b)** 11:17 am **c)** 6:18 pm

5 Convert the following 24-hour times to 12-hour time:

a) 0305 **b)** 1055 **c)** 2105

6 How much time between (24-hour times):

a) 0615 and 1035?

b) 0935 and 1520?

7 The plane leaves Canberra at 1010 and arrives in Melbourne at 1150 How long is the flight?

> There is an ancient invention still used in some parts of the world today that allows people to see through walls. What is it?
>
> A Window

Western	Central	Eastern
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

8 If the time in Sydney is 5 pm, what is the time in

a) Adelaide? **b)** Canberra? **c)** Darwin? **d)** Hobart?

e) Melbourne? **f)** Perth? **g)** Brisbane?

9 The plane leaves Perth at 5:55 am and arrives in Canberra at 11:05 am. How long is the flight (Perth and Canberra are in different time zones)?

10 Given the following Flight Review, how long is the flight?

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Darwin **10:05 AM**	Brisbane **2:55 PM**	KP 411 Economy	20 kg

Review 3

15

Chapter 11 Ratio and Rate

★ A ratio is a comparison of two quantities of the same type in the same units.

★ A ratio can be written as 3: 10, a fraction $\frac{3}{10}$, a decimal 0.3, a percentage 30%

★ A rate is a comparison of quantities of different kinds (Rent of $380 per week).

★ The unitary method is a useful method for comparing one rate with another rate.

Chapter 12 Linear Equations

Identify patterns: 

Extra 3 matches for each step 3×step
First step is 3+1 matches.
Function = 3×step+1

Solve equations: $7a + 12 = 30 - 2a$

Evaluate, graph and solve linear functions

Chapter 13 Statistical Measures

Sample
A collection of data from **part** of the population.

Sample Bias:
The sample doesn't represent the population.

Sample Size:
Must be small enough to be economical but large enough to represent the population.

Given: 1, 2, 2, 3, 4 Mean = $\frac{12}{5}$ = 2.4, Median = 2, Range = 4 – 1 =3

Chapter 14 Time

12-hour time
1 day = 24 hours 1 hour = 60 mins am = morning pm = afternoon
24-hour time
0200 = 2 am 1120 = 11:20 am 1350 = 1:50 pm 2001 = 10:01 pm

Australia has 3 time zones.

Western	**Central**	**Eastern**
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

Review 1

Exercise 15.1 Mental Computation

1 Spell percentage.
2 How many hours in 2 days?
3 Write in index form: $2\times2\times2\times2\times2$
4 Solve: $3x - 5 = 7$
5 Convert 24-hour time 1530 to 12-hour time.
6 What is the mean of: 1,2,2,3?
7 Add GST (10%) to $30.
8 A taxi charges $3 flagfall and $3.20 per km. What is the function?
9 If the Sun covers 360° in 24 hours, how many degrees in 1 hour?
10 Simplify: $3^5 \div 3^3$

Exercise 15.2

1 Write each of the following ratios as a fraction, a decimal and a percentage:
a) 1 : 2 **b)** 2 : 5 **c)** 1 : 4
d) 2 : 4 **e)** 3 : 10 **f)** 3 : 5

2 Simplify the following ratios:
a) 2 : 6 **b)** 3 : 15 **c)** 12 : 20
d) 1.6 : 2.4 **e)** 1.5 : 2.5 **f)** 1.2b : 2.4b

3 The rose food is mixed with water in the ratio of 1 : 4. How much rose food is needed to make a mixture of 2 L?

4 The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 2 : 3. If the mixer takes 30 shovelfulls, how many shovelfulls of sand is needed?

5 Calculate each of the following:
a) 20% of 40. **b)** 25% of 80. **c)** $37\frac{1}{2}$% of 64.

6 Add GST (10%) to the price of each of the following items:
a) A haircut @ $105. **b)** A chocolate @ $5.65.

7 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 20%? Use the shortcut to increase $50 by 20%

8 Find the discounted price of a phone:?
a) A discount of 15% on the marked price of $125.
b) A discount of 25% on the recommended retail price of $145.
c) Which sounds more impressive? Which is the better deal?

9 A prepaid mobile phone deal offers 28c per text message, what would be the charge for 56 text messages?

10 The prepaid company offers one rate of 49 cents per 30 secs to any mobile or landline within the country. What would it cost for a 4 min 30 sec call?

11 Find the rate per one or 100 in each of the following:
a) 220 runs in 50 overs. **b)** 54 L used to go 600 km.

12 Which is the best buy?
a) $5.36 for 400 g of candy or $5.95 for 440 g?
b) $2.45 for 750 mL of fruit juice or $4.85 for 2 L?
c) $4.74 for 750 mL of olive oil or $6.14 for 1 L of olive oil?

13 Write a function for the pattern in each of the following tables:

a)

x	1	2	3	4	5
y	5	8	11	14	17

b)

x	1	2	3	4	5
y	2	7	12	17	22

14 For each of the following, use the function to complete the table, and then graph the function.

a)

x	1	2	3	4	5	10	20
y = 2x + 1	3	5					

b)

x	1	2	3	4	5	10	20
y = 3x − 2	1	4					

15 Solve each of the following equations:

a) $a + 3 = 8$ **b)** $b - 4 = 9$ **c)** $5c = 15$

d) $2(x - 6) = 10$ **e)** $4(x + 5) = 28$ **f)** $2(x + 6) = 8$

g) $2d + 5d = 28$ **h)** $4x + 1 + 2x = 25$ **i)** $2(x - 3) + 4x = 30$

j) $4(c - 1) - 2c = 6$ **k)** $6x - 5 = 2x - 1$ **l)** $7m - 8 = 2m + 12$

16 The truck has a weight of 3.5 tonnes and each pallet weighs 0.45 tonnes.

a) Write a function for the total weight of the truck.

b) What is the total weight if the truck carries 8 pallets?

c) If the total weight is 5.75 tonnes, how many pallets were put on the truck?

17 Ethan has $145 in the bank and plans to add $45 every week.

a) Write a function for Ethan's total savings.

b) How much will Ethan have after 12 weeks?

c) How many weeks will Ethan need to save to have $640 in the bank?

18 Comment on the following sampling technique:
10 friends were asked if there was enough seating in the school grounds.
7 said yes. The conclusion was that 70% of the school's population were satisfied with the seating in the school grounds.

19 An opinion poll concludes that: "54% supported the Government's approach to education and 46% opposed the Government's approach." Why should the opinion poll also report the sample size and the method of sampling?

20 Find the mean, median, and range of each of the following sets of data:

a) 1, 1, 2, 2, 2, 3, 4 **b)** 71, 71, 72, 72, 72, 73, 74

c) 640, 652, 663, 668 **d)** 2.4, 2.5, 2.8, 2.8, 2.9

35 students in a random sample of 50 students have brown eyes.
Estimate the number of students in a school of 400 with brown eyes.

$$\% \text{ brown eyes} = \frac{35}{50} \times 100$$
$$= 70\%$$
$$\text{No students} = 70\% \text{ of } 400$$
$$= \frac{70}{100} \times 400$$
$$= 280$$

Conclusion: 280 students have brown eyes.

If you were in a dark room with a candle, a woodstove, a match and a gas lamp which do you light first?

The match.

21 32 students in a random sample of 50 students have brown eyes. Estimate the number of students in the school of 1030 with brown eyes.

22 A randomly selected sample of 600 households suggests that 279 of the households regularly watch a cooking segment. Estimate the number of households, from the population of 6 million households, that regularly watch the cooking segment.

23 A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample. How many students from each Year level of the following school should be randomly selected in a sample size of 60 students?

Year	Number
7	150
8	150
9	100
10	100

24 What is the time 2 hours 30 minutes after 8:15 am?

25 How much time between 9:50 am and 1:20 pm?

26 Konrad leaves home at 6:45 am and takes 15 mins to walk to the bus station. He waits 5 mins for the bus and the bus takes 45 mins to travel to his workplace. At what time does Konrad get to work?

27 Convert the following 12-hour times to 24-hour time:
a) 4:35 am **b)** 11:46 am **c)** 9:13 pm

28 Convert the following 24-hour times to 12-hour time:
a) 0530 **b)** 1010 **c)** 2332

Australia has 3 time zones.

Western	Central	Eastern
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

29 If the time in Hobart is 5 pm, what is the time in

a) Adelaide? **b)** Canberra? **c)** Darwin? **d)** Brisbane?
e) Melbourne? **f)** Perth? **g)** Sydney?

30 Given the following Flight Review, how long is the flight?

DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Perth **5:05 PM**	Canberra **11:55 PM**	DF 16 Economy	20 kg

Review 2

Exercise 15.3 Mental Computation

1 Spell equation.
2 How many hours in 3 days?
3 Write in index form: $3\times3\times3\times3$
4 Solve: $3x - 4 = 5$
5 Convert 24-hour time 1845 to 12-hour time.
6 What is the mean of: 1,3,5?
7 Add GST (10%) to $40.
8 A phone call costs 25c flagfall and 92c per minute. What is the function?
9 If the Sun covers 360° in 24 hours, how many degrees in 2 hours?
10 Simplify: $x^5 \div x^2$

Exercise 15.4

1 Write each of the following ratios as a fraction, a decimal and a percentage:

a) 1 : 2 **b)** 3 : 10 **c)** 1 : 4
d) 3 : 6 **e)** 2 : 3 **f)** 2 : 5

2 Simplify the following ratios:

a) 5 : 15 **b)** 6 : 12 **c)** 8 : 6
d) 1.2 : 1.8 **e)** 1.4 : 2.1 **f)** 2.4s : 3.2s

3 The rose food is mixed with water in the ratio of 1 : 3. How much rose food is needed to make a mixture of 2 L?

4 The concrete is to be made of cement, sand, and gravel in the ratio of 1 : 3 : 2. If the mixer takes 30 shovelfulls, how many shovelfulls of cement is needed?

5 Calculate each of the following:

a) 15% of 40 **b)** 40% of 60 **c)** $37\frac{1}{2}$% of 80

6 Add GST (10%) to the price of each of the following items:

a) A phone @ $125 **b)** A magazine @ $8.45

7 A shortcut for adding GST (10%) is to multiply by 1.1. What would be the shortcut for adding 40%? Use the shortcut to increase $50 by 40%

8 Find the discounted price of a phone:?

a) A discount of 20% on the marked price of $185.

b) A discount of 30% on the recommended retail price of $205.

c) Which sounds more impressive? Which is the better deal?

9 A prepaid mobile phone deal offers 29c per text message, what would be the charge for 83 text messages?

10 The prepaid company offers one rate of 43 cents per 30 secs to any mobile or landline within the country. What would it cost for a 2 min 30 sec call?

11 Find the rate per one or 100 in each of the following:

a) 265 runs in 50 overs **b)** 66 L used to go 600 km

12 Which is the best buy:

a) $12.45 for 500 g of chocolate or $15.05 for 750 g?

b) $4.52 for 2 L of fruit juice or $5.19 for 3 L?

13 Write a function for the pattern in each of the following tables:

a)

x	1	2	3	4	5	10	20
y	4	7	10	13	16	31	61

b)

x	1	2	3	4	5	10	20
y	2	8	14	20	26	56	116

14 For each of the following, use the function to complete the table, and then graph the function.

a)

x	1	2	3	4	5	10	20
$y = 2x + 3$	5	7					

b)

x	1	2	3	4	5	10	20
$y = 3x - 2$	1	4					

15 Solve each of the following equations:

a) $a + 5 = 7$ **b)** $b - 3 = 9$ **c)** $5x = 15$

d) $3(x - 2) = 12$ **e)** $3(y + 3) = 30$ **f)** $2(u + 4) = 18$

g) $3k + 4k = 21$ **h)** $2x + 3 + 5x = 24$ **i)** $2(x - 3) + 4x = 18$

j) $4(d - 3) - 2d = 4$ **k)** $9p - 7 = 3p - 1$ **l)** $4t - 3 = 2t + 7$

16 A taxi charges $3 for the flag fall and then $3.50 per kilometre.

a) Write a function for the taxi charge.

b) What is the fare to travel 10 km?

c) If the fare was $45, how far did the taxi travel?

17 For the social it will cost $150 to hire the room and each ticket will cost $25.

a) Write a function for profit for the social.

b) What will be the profit if 32 people attend the social?

c) If the profit is $525, how many people attended the social?

18 Comment on the following sampling technique:
5 friends were asked about the music at the school social 2 said they were satisfied. The conclusion was that 40% of the school's population were satisfied with the music at the school social.

19 An opinion poll concludes that: "52% supported the Government's position on environmental protection and 48% opposed the Government's position." Why should the opinion poll also report the sample size and the method of sampling?

20 Find the mean, median, and range of each of the following sets of data:

a) 3, 3, 3, 4, 5, 7, 9

b) 33, 33, 33, 34, 35, 37, 39

c) 70, 72, 73, 75

d) 1.4, 1.5, 1.5, 1.6, 1.9

35 students in a random sample of 50 students have brown eyes.
Estimate the number of students in a school of 400 with brown eyes.

$$\text{\% brown eyes} = \frac{35}{50} \times 100$$
$$= 70\%$$
$$\text{No students} = 70\% \text{ of } 400$$
$$= \frac{70}{100} \times 400$$
$$= 280$$

Conclusion: 280 students have brown eyes.

I start with the letter e.
I end with the letter e.
I contain only one letter.
Yet I am not the letter e!
What am I?

An Envelope.

21 46 students in a random sample of 75 students have a part-time job. Estimate the number of students in the school of 1730 with a part-time job.

22 A randomly selected sample of 900 households suggests that 146 of the households regularly watch a sporting segment. Estimate the number of households, from the population of 6 million households, that regularly watch the sporting segment.

23 A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample. How many students from each Year level of the following school should be randomly selected in a sample size of 90 students?

Year	Number
7	120
8	120
9	110
10	100

24 What is the time 1 hours 35 minutes after 9:20 am?

25 How much time between 8:05 am and 4:30 pm?

26 Kale leaves home at 7:15 am and takes 10 mins to walk to the bus station. He waits 5 mins for the bus and the bus takes 50 mins to travel to his workplace. At what time does Kale get to work?

27 Convert the following 12-hour times to 24-hour time:
a) 3:19 am **b)** 10:23 am **c)** 11:18 pm

28 Convert the following 24-hour times to 12-hour time:
a) 0812 **b)** 1146 **c)** 2238

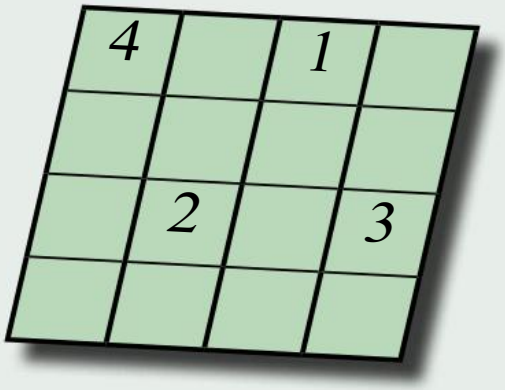
Australia has 3 time zones.

Western	Central	Eastern
EST−2hrs	EST−30mins	EST
Perth	Darwin Adelaide	Brisbane Sydney Camberra Melbourne Hobart
7:00 am	8:30 am	9:00 am

29 If the time in Darwin is 9 pm, what is the time in
a) Adelaide? **b)** Canberra? **c)** Hobart? **d)** Brisbane?
e) Melbourne? **f)** Perth? **g)** Sydney?

30 Given the following Flight Review, how long is the flight?

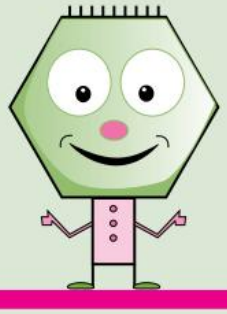
DEPARTING	ARRIVING	FLIGHT	BAGGAGE
Brisbane **8:35 AM**	Adelaide **1:05 PM**	AL 4 Economy	20 kg



☺ Fill in the blanks
☺ Use only 1, 2, 3, or 4.
☺ Each number once only per row.
☺ Each number once only per column.

Linear Relationships 16

- ★ Plot linear relationships on the Cartesian plane.
- ★ Plot points for tables of values from non-rule-based data.
- ★ Solve linear equations using algebraic and graphical techniques.
- ★ Use variables to symbolise simple linear equations.



A TASK

How far does a stone fall in one second?

- Brainstorm ideas about how to answer the question.
- Experiment with falling stones.
- Find the formula (Physics students study the formula).
- Brainstorm practical uses for the formula.
- Present your results.

A LITTLE BIT OF HISTORY

Sir Isaac Newton (1642-1727) was one of the greatest mathematicians of all time.

In 1687, Newton published *Philosophiae Naturalis Principia Mathematic*. The book introduced Newton's three laws of motion (inertia, action/reaction, and acceleration proportional to force), and the application of his laws of motion to the tides, the moon, and the planets.

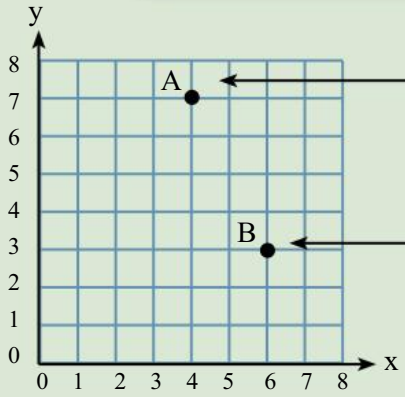
Newton also invented calculus, formulated classical mechanics, significantly contributed to the study of optics (Newton's rings, the colours of white light) and chemistry.

The story is that Newton developed algebraic formulas for falling objects after an apple fell on his head.

Ordered Pairs

In order to fix a point in two-dimensional space, we use ordered pairs (x,y).
The position of the point is referenced by x on the x-axis and y on the y-axis.

To plot points
- Put your biro on the Origin (0,0).
- Move your biro horizontally, the x value.
- Move your biro vertically, the y value.

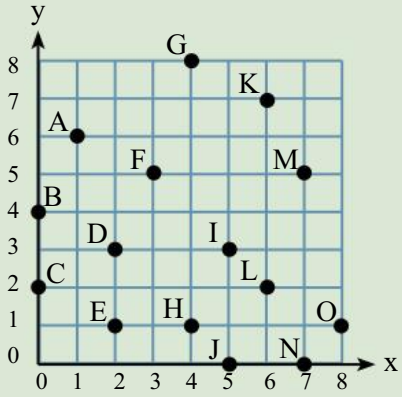


A (4,7) 4 along the x-axis.
7 along the y-axis.

B(6,3) 6 along the x-axis.
3 along the y-axis.

Exercise 16.1

1 Write the coordinates of each of the points in the graph below.



The x first, then the y.

2 Plot the following points on a set of Cartesian axes:

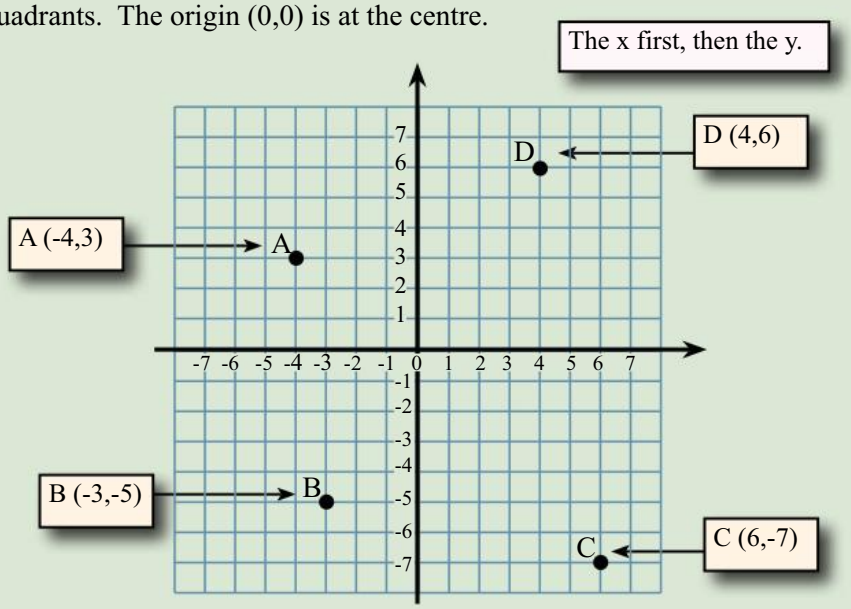
a) A(1,5)	**b)** B(5,5)	**c)** C(3,0)
d) D(1,6)	**e)** E(0,7)	**f)** F(5,3)
g) G(0,0)	**h)** H(7,1)	**i)** I(1,1)

3 What is it? Plot the following points on a sheet of grid paper, joining each point to the previous point with a continuous line.
(3,2) (4,0) (5,2) (7,3) (5,4) (4,6) (3,4) (1,3) (3,2)

4 Make your own "What is it?"

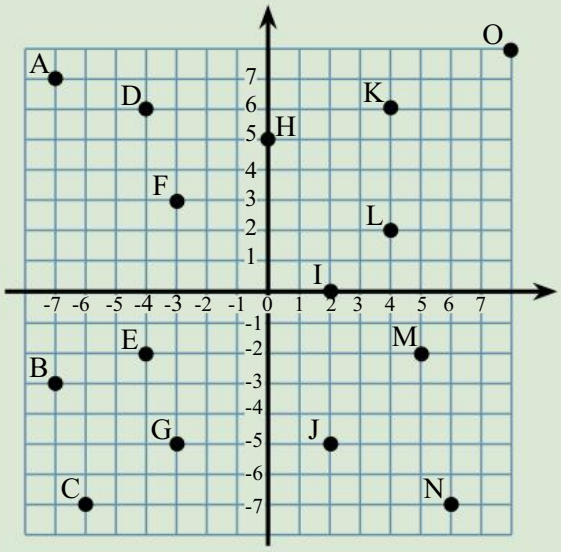
The Four Quadrants

The use of ordered pairs to fix a point in two-dimensional space can be extended to negative numbers as well as positive numbers. The cartesian space is then divided into four quadrants. The origin (0,0) is at the centre.



Exercise 16.2

1 Write the coordinates of each of the points in the graph below:



2 Plot the following points on a set of Cartesian axes:

a)	A(1,−5)	**b)**	B(−5,5)	**c)**	C(−3,0)	**d)**	D(1, −6)
e)	E(0,−7)	**f)**	F(−5,−3)	**g)**	G(0,0)	**h)**	H(−7,−1)
i)	I(−1,1)	**j)**	J(3,−1)	**k)**	K(0,−4)	**l)**	L(−1,5)

Linear Models

A hint to the enormous value of mathematics begins to appear when we are able to translate a problem into a mathematical model and use the model to solve problems.

Exercise 16.3

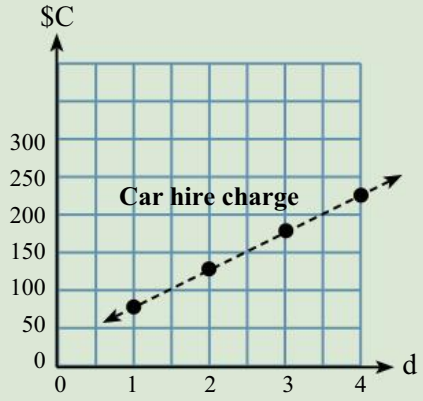
For each of the following data, plot the points and write a mathematical model:

Car hire rate

No of days (d):	1	2	3	4
Charge ($C):	70	120	170	220

Each step increases by 50: Thus $C = 50d

The first step is 50 + 20: Thus $C = 50d + 20



A linear model increases by the same amount each step.

1 The car hire rate:

No of days (d)	1	2	3	4
Hire charge ($C)	70	110	150	190

2 The car hire rate:

No of days (d)	1	2	3	4
Hire charge ($C)	100	160	220	280

3 The car hire rate:

No of days (d)	1	2	3	4
Hire charge ($C)	100	155	210	265

4 The car hire rate:

No of days (d)	1	2	3	4
Hire charge ($C)	80	140	200	260

5 The taxi charge:

No of km (k)	1	2	3	4
Taxi charge ($C)	4	6	8	10

6 The taxi charge:

No of km (k)	1	2	3	4
Taxi charge ($C)	4.00	5.50	7.00	8.50

7 The beaker temperature:

No of minutes (t)	1	2	3	4
Temperature°C (T)	25	32	39	46

Use of mathematical models to solve problems is the basis of many many careers: Finance, biology, geology, meteorology, engineering, economics, research, statistics, etc, etc, etc, etc.

Model Your World

Age of car in years (a)	1	2	3	4
Yearly repair cost ($C)	320	600	880	1160

a) Plot the points and write a mathematical model.

Each step increases by 280: $C = 280a

The first step is 280+40 $C = 280a + 40

b) What would be the expected yearly repair cost of a 5 year-old car.?**a = 5,**

$C = 280×5 + 40

$C = 1440

c) If the yearly repair bill is $1000, what is the expected age of the car?

$C = 1000, 1000 = 280a + 40

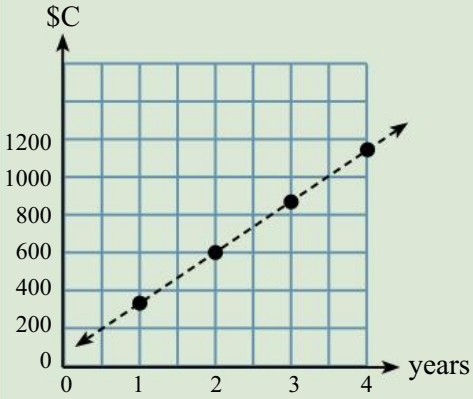
1000 − 40 = 280a

960 = 280a

960÷280 = a

3.4 = a

The car is about 3.5 years old



Can you use the above graph to solve these problems?

A massive number of real-world problems are solved by creating mathematical models.

Exercise 16.4

1

Age of car in years (a)	1	2	3	4
Yearly repair cost ($C)	180	320	460	600

a) Plot the points and write a mathematical model.

b) What would be the expected yearly repair cost of a 6 year-old car?

c) If the yearly repair bill is $390, what is the expected age of the car?

2

Weeks (w)	1	2	3	4
Bank balance ($S)	250	275	300	325

a) Plot the points and write a mathematical model.

b) How much in the bank after 5 weeks?

c) How long will it take to have a balance of $750?

3

Share dividend ($d)	1	2	3	4
Share price ($S)	22	29	36	43

a) Plot the points and write a mathematical model.

b) What is the expected share price for a dividend of $2.50

c) If the share price is $26.20, what is the expected dividend?

Exercise 16.5

A linear function increases by the same amount each step.

Given the function, $y = 2x + 1$

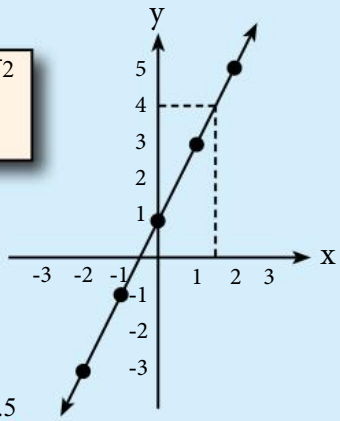
a) Copy and complete the following table.

x	-2	-1	0	1	2
y = 2x + 1					

When $x = ^-2$, $y = 2\times^-2 + 1 = ^-3$
When $x = ^-1$, $y = 2\times^-1 + 1 = ^-1$
When $x = 0$, $y = 2\times0 + 1 = 1$
When $x = 1$, $y = 2\times1 + 1 = 3$
When $x = 2$, $y = 2\times2 + 1 = 5$

$2\times^-2$ = two lots of $^-2$
$= ^-2\ ^-2$
$= ^-4$

x	-2	-1	0	1	2
y = 2x + 1	-3	-1	1	3	5



b) Draw a graph of the function.

c) From the graph, when y = 4, what is x? x = 1.5

For each of the following:

a) Copy and complete the following table.

b) Draw a graph of the function.

c) From the graph, when y = 4, what is x?

1

x	-2	-1	0	1	2
y = 2x + 2					

2

x	-2	-1	0	1	2
y = 2x + 3					

3

x	-2	-1	0	1	2
y = 2x + 5					

4

x	-2	-1	0	1	2
y = 2x − 1					

5

x	-2	-1	0	1	2
y = 2x − 4					

6

x	-2	-1	0	1	2
y = 3x + 1					

7

x	-2	-1	0	1	2
y = 3x + 2					

8

x	-2	-1	0	1	2
y = 3x + 4					

9

x	-2	-1	0	1	2
y = 3x − 1					

10

x	-2	-1	0	1	2
y = 3x − 2					

Non-Linear Functions

A non-linear function **does not** increase by the same amount each step.

Exercise 16.6

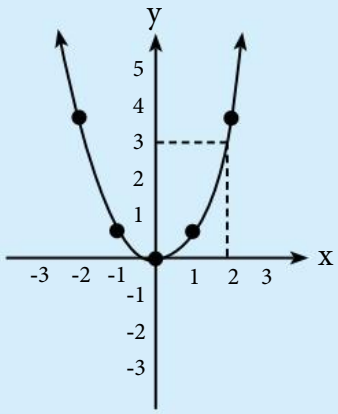
Given the function, $y = x^2$

a) Copy and complete the following table.

x	$^-2$	$^-1$	0	1	2
$y = x^2$					

When $x = ^-2$, $y = ^-2 \times ^-2 = 4$
When $x = ^-1$, $y = ^-1 \times ^-1 = 1$
When $x = 0$, $y = 0 \times 0 = 0$
When $x = 1$, $y = 1 \times 1 = 1$
When $x = 2$, $y = 2 \times 2 = 4$

x	$^-2$	$^-1$	0	1	2
$y = x^2$	4	1	0	1	4



b) Draw a graph of the function.

c) From the graph, when $y = 3$, what is x? $x = 1.7$

For each of the following:

a) Copy and complete the following table.

b) Draw a graph of the function.

c) From the graph, when $y = 3$, what is x?

1

x	-2	-1	0	1	2
$y = x^2$					

2

x	-2	-1	0	1	2
$y = x^2 + 1$					

3

x	-2	-1	0	1	2
$y = x^2 + 2$					

4

x	-2	-1	0	1	2
$y = x^2 + 3$					

5

x	-2	-1	0	1	2
$y = x^2 - 1$					

6

x	-2	-1	0	1	2
$y = x^2 - 2$					

7

x	-2	-1	0	1	2
$y = x(x + 1)$					

8

x	-2	-1	0	1	2
$y = x(x - 1)$					

9

x	-2	-1	0	1	2
$y = x^3$					

10

x	-2	-1	0	1	2
$y = 1/x$					

Non-Linear Functions

A non-linear function **does not** increase by the same amount each step.

Exercise 16.7

The following table shows 10 consecutive days of the Australian All Ordinaries, All Ords, share price index.

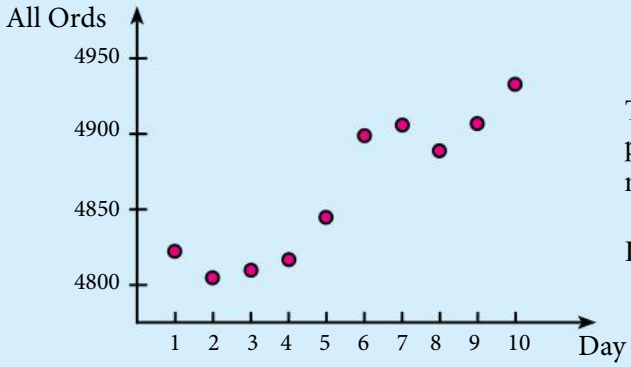
There are enormous sums of money invested in share markets.

a) Plot the points.

b) Can you detect a pattern?

c) Can you guess what the share price index for the next day?

Day	1	2	3	4	5	6	7	8	9	10
All Ords	4824	4810	4816	4822	4843	4896	4910	4872	4914	4933



There doesn't appear to be a pattern apart from an upward movement.

It was 4886 the next day.

1 The following table shows 10 consecutive days of China's Shanghai Composite, SSEC, share price index.

Australia's economic health is said to be tied to China's economic health.

a) Plot the points.

b) Can you detect a pattern?

c) Can you guess what the next All Ords share price index will be?

Day	1	2	3	4	5	6	7	8	9	10
SSEC	2821	2833	2783	2812	2831	2817	2772	2696	2708	2753

2 The following table shows 10 consecutive days of the American David Jones, DJ, share price index.

It has been said that if America sneezed then Australia would catch a cold.

a) Plot the points.

b) Can you detect a pattern?

c) Can you guess what the next All Ords share price index will be?

Day	1	2	3	4	5	6	7	8	9	10
DJ	11717	11697	11672	11639	11674	11754	11732	11784	11834	11824

3 The following table shows 10 consecutive days of the Indonesian Jakarta, JKSE, share price index.

Indonesia, 250 million people, is one of Australia's neighbours in Southeast Asia.

a) Plot the points.

b) Can you detect a pattern?

c) Can you guess what the share price index for the next day?

Day	1	2	3	4	5	6	7	8	9	10
JKSE	3734	3631	3478	3455	3555	3565	3569	3536	3549	3534

Mental Computation

$4 = 2 \times 2$
To multiply by 4, double twice.

$16 \times 4 = 32 \times 2 = 64$

Exercise 16.8

1 Spell Equation.
2 16×4
3 $y = 2x + 3$. Find y when $x = -1$
4 $y = 2x + 3$. Find x when $y = 7$
5 What is the function for the sequence: 3, 5, 7, 9, ...?
6 Convert 24-hour time 1830 to 12-hour time.
7 Solve: $2x - 3 = 4$
8 What is the median of: 1,2,2,3,4?
9 Write in index form: $2 \times 2 \times 2 \times 2$
10 Simplify: $b^4 \times b^2$

Many everyday problems are solved mentally.

Exercise 16.9

1 Spell Linear.
2 17×4
3 $y = 3x + 2$. Find y when $x = -1$
4 $y = 3x + 3$. Find x when $y = 12$
5 What is the function for the sequence: 2, 5, 8, 11, ...?
6 Convert 12-hour time 4:50pm to 12-hour time.
7 Solve: $2x + 5 = 2$
8 What is the median of: 1,2,3,3,4?
9 Write in index form: $3 \times 3 \times 3 \times 3 \times 3 \times 3$
10 Simplify: $b^5 \div b^2$

Nothing is all wrong. Even a busted clock is right twice a day.

Exercise 16.10

1 Spell Coordinates.
2 25×4
3 $y = 2x + 5$. Find y when $x = -1$
4 $y = 2x + 5$. Find x when $y = 15$
5 What is the function for the sequence: 5, 9, 13, 17, ...?
6 Convert 24-hour time 1510 to 12-hour time.
7 Solve: $5x - 5 = 4$
8 What is the median of: 1,2,3,4,5?
9 Write in index form: $a \times a \times a \times a$
10 Simplify: $(x^5)^2$

Get a degree majoring in maths and biology and become a **Data Analyst** for the Department of Primary Industries. You could, among many things, use satellite technology to model the population of pests and their eradication.

NAPLAN Questions

Exercise 16.11

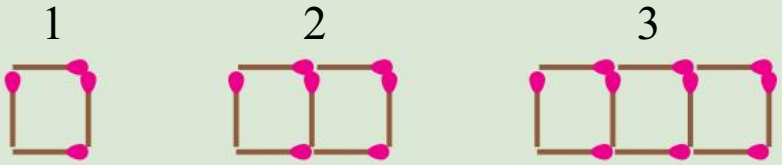
1 What is the missing number?

$$4 \times ? + 12 = 72$$

2 What is the next number in the following pattern?

× 5 − 2	→	3, 8, 13, ?

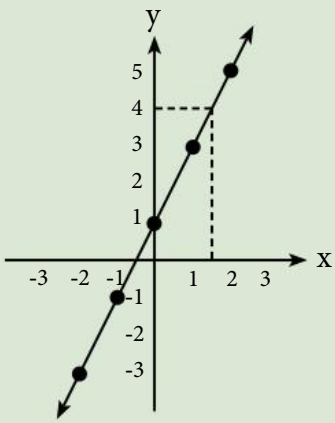
3 How many matches in the 12th shape?



4 Write a linear model of the following plumber's fee:

Hours (h)	1	2	3	4
Fee ($F)	90	140	190	240

5 Write the linear model represented by this graph.



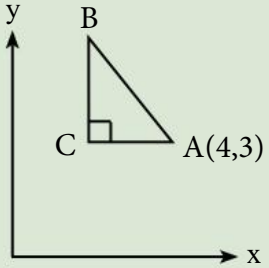
6 Solve each of the following:

a) When a number is doubled and then 15 is added the answer is 23.

b) When a number is tripled and 9 subtracted, the answer is 24.

c) A number is squared and then added to 11 to give the answer 20. What is the number?

7 What are the coordinates of B (AB = 5, BC = 4, CA = 3)?



Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 16.12

1 An approximate formula for converting kilometres, k, to miles, m, is $m = \frac{5k}{8}$. How many miles in 32 kilometres?

2 Does the point (2,2) lie on the line: $y = 2x + 5$?

3 The point (100, 8) lies on the line $20x + y = 2008$.
Find another point that lies on this line?

4 What is the equation of each of the following lines?
a) Each step increases by 2 and it cuts the y-axis at (0,1).
b) Each step decreases by $^{-}2$ and it cuts the y-axis at (0,$^{-}1$).

Write a mathematical model of the following printing costs:

No. books	Cost
2 000	\$7 000
3 000	\$8 500
5 000	\$11 500
10 000	\$19 000

Let b=no. of thousands
Each step increases by \$1500: Thus C = 1500b
When b = 1 (ie 1000), cost=\$5 500
The first step is 1500+4000

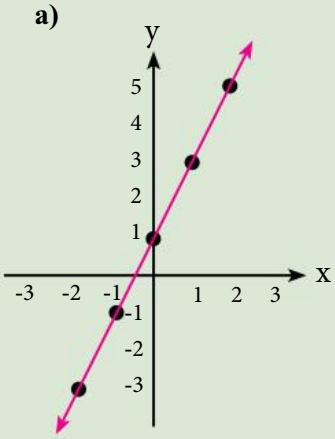
Thus **C = 1500b + 4000**

5 Write a mathematical model of the following printing costs:

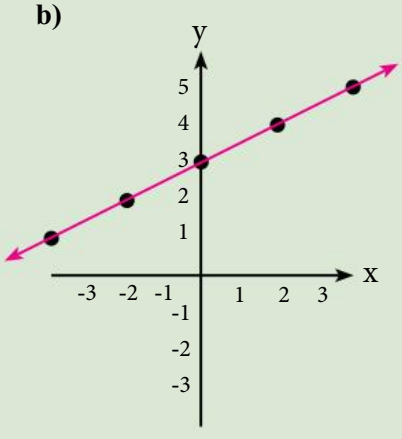
No. books	Cost
3 000	\$12 000
5 000	\$15 000
7 000	\$18 000
10 000	\$22 500

6 What is the equation of each of the following lines?

a)



b)



Investigations

Investigation 16.1 A linear relationship?

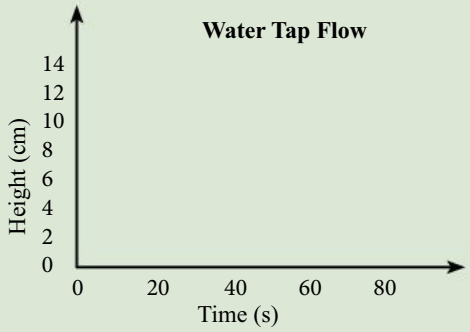
You need:
- a bucket
- a ruler
- a stopwatch

1 Turn on a tap and let the water run at a constant rate.

2 Ready the bucket by putting a ruler inside the bucket.

3 Put the bucket under the tap and record the height of water every 10 seconds (You may wish to record every 30 s or so dependent upon how much water is flowing from the tap).

4 Draw a graph.



5 **a)** How will the graph change if the water runs faster?
b) How will the graph change if the water runs slower?
c) Is the graph linear if the sides of the bucket are vertical?
d) Is the graph linear if the sides of the bucket are not vertical?

Investigation 16.2 What is it?

- ✦ Plot the following points on a sheet of grid paper.
- ✦ Choose a scale that will include all points in a set.
- ✦ Join the points in a set together (Don't join the first to the last).
- ✦ Do the same for each set. Eventualy a shape will appear.

1 **a)** (−15,3) (−10,8) (−10,3) (−8,3) (−5,7)
b) (−8,0) (−6,−6) (−8,−8) (−5,−8)
c) (4,−6) (3,−8) (6,−8) (10,10)
d) (−8,0) (−12,−2) (−16,2) (−15,3) (−15,1)
e) (−5,−8) (−3,−4) (4,−6)
f) (10,10) (8,4) (−10,8)
g) Draw a circle with radius 0.5 at centre (−12,4)

2 Make your own What is it?
a) Draw or copy an image on a Cartesian plane.
b) Write the ordered pairs of key points.
c) Test your "What is it?" on a classmate.

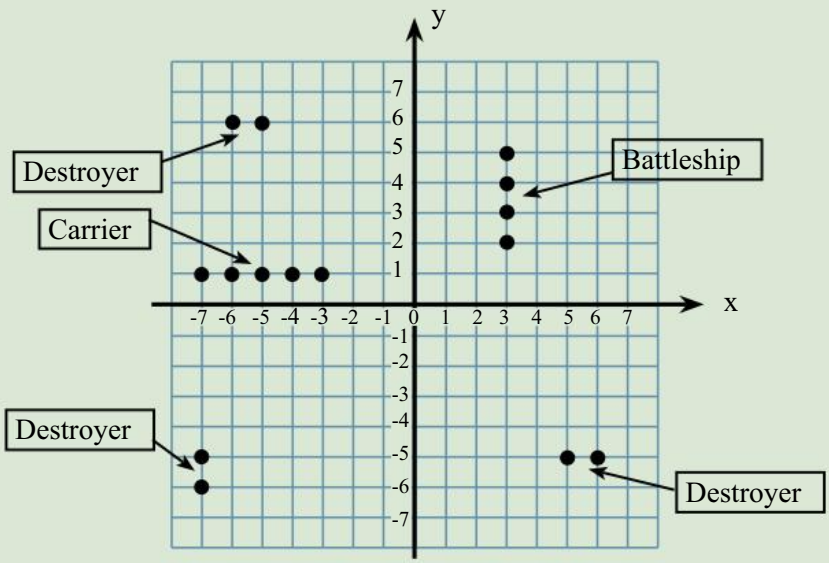
A Game

Battleships.

Each of two players set their navy on a Cartesian plane (Aircraft carrier - 5 spots, Battleship - 4 spots, 3 Destroyers -2 spots each).

The aim of the game is to destroy the opponent's navy.

- Each person takes it in turn to fire a shot by calling out an ordered pair e.g., (−3, 2).
- When a shot is fired, the opponent calls out either hit or miss.



A Sweet Trick

For this trick you will need five cards. Write the numbers 1, 2, 3, 4, 5 in black on one side of each of the five cards and the numbers 6, 7, 8, 9, 10 in red on the other side. The 6 on the back of the 1, the 7 on the back of the 2, the 8 on the back of the 3 etc.

1 Without looking, ask your audience to drop the cards onto the floor.
2 Ask them to tell you how many red cards are facing up.
3 Ask them to total the numbers on the five cards.
4 You call out the answer before they can add them up.

For every red card add 5 to 15.
If there are 3 red cards, add 3x5 to 15.
The total facing is 30.

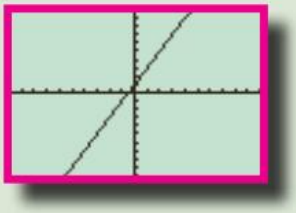
Technology

Technology 16.1
Use a Graphics Calculator to plot the equations in Exercise 16.5

x	-2	-1	0	1	2
y = 2x + 1					

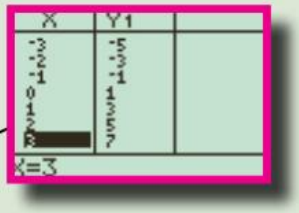
Press [Y =] and enter the equation eg., 2x + 1

Press [Graph] to see a plot of the equation



Press [Table] to see a table of the values

x	-2	-1	0	1	2
y = 2x + 1	-3	-1	1	3	5



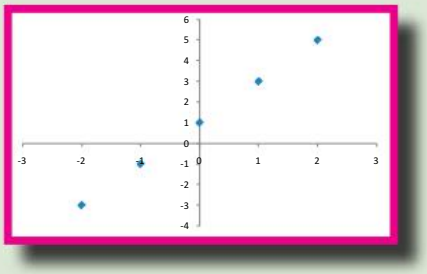
Technology 16.2
Use a Graphics Calculator, as above, to experiment with non-linear tables of data (See Exercise 16.7).

Technology 16.3
Use a Spreadsheet to plot the equations in Exercise 16.5 and 16.6.

x	-2	-1	0	1	2
y = 2x + 1	-3	-1	1	3	5

Enter the formula:
= 2*B1+1

Use the Chart (Scatter) to plot the equation:



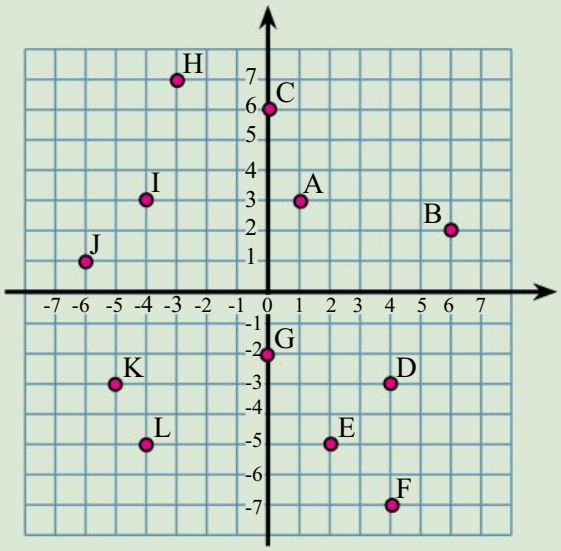
Technology 16.4 Applets
There are a very large number of "graph, function, plotter" applets on the Internet.

There are also applets that let you experiment with "parallel lines".

Chapter Review 1

Exercise 16.13

1 Write the coordinates of each of the points in the graph below:



The x distance first.
The y distance second.

How many times can you subtract 7 from 83, and what is left afterwards?
I can subtract it as many times as I want, and it leaves 76 every time.

2 Plot the following points on a set of Cartesian axes:

a) A(2,4) **b)** B(5,1) **c)** C(¯2,6) **d)** D(1,¯2)
e) E(¯2,¯4) **f)** F(0,0) **g)** G(0,3) **h)** H(¯7,0)

3

No of days (d)	1	2	3	4
Hire charge ($C)	100	145	190	235

a) Plot the points.
b) Write a mathematical model for the car hire rate.

4

Weeks (w)	1	2	3	4
Bank balance ($S)	250	350	450	550

a) Plot the points and write a mathematical model.
b) How much in the bank after 6 weeks?
c) How long will it take to have a balance of $950?

5

x	-2	-1	0	1	2
y = 2x + 2					

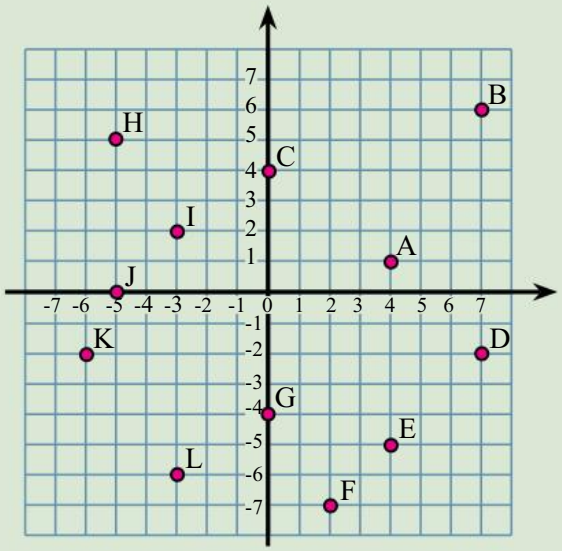
a) Copy and complete the following table.
b) Draw a graph of the function.
c) From the graph, when y = 4, what is x?

Money is an article which may be used as a universal passport to everywhere except heaven, and as a universal provider for everything except happiness. -- **Wall Street Journal**

Chapter Review 2

Exercise 16.13

1 Write the coordinates of each of the points in the graph below:



The x distance first.
The y distance second.

2 Plot the following points on a set of Cartesian axes:

a) A(4,6) **b)** B(2,7) **c)** C(3,¯2) **d)** D(4, ¯1)
e) E(¯2,5) **f)** F(0,0) **g)** G(¯4,0) **h)** H(0,¯6)

3

No of days (d)	1	2	3	4
Hire charge ($C)	80	120	160	200

A circle is a round straight line with a hole in the middle.

a) Plot the points.
b) Write a mathematical model for the car hire rate.

4

Weeks (w)	1	2	3	4
Bank balance ($S)	120	170	220	270

a) Plot the points and write a mathematical model.
b) How much in the bank after 7 weeks?
c) How long will it take to have a balance of $920?

5

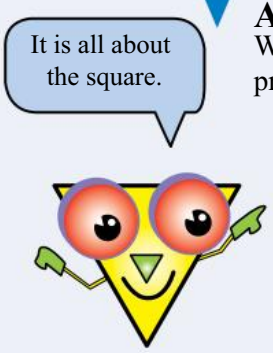
x	-2	-1	0	1	2
y = 2x + 3					

a) Copy and complete the following table.
b) Draw a graph of the function.
c) From the graph, when y = 6, what is x?

Don't worry about moving slowly, worry about standing still.
-- Chinese proverb

Measurement 17

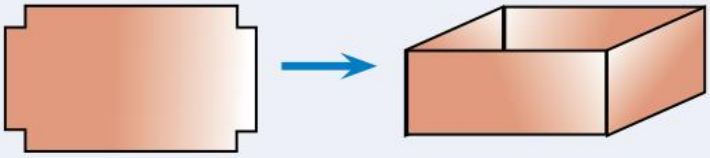
- ★ Find perimeters and areas of parallelograms, rhombuses and kites.
- ★ Explore the use of parallelograms, rhombuses and kites in art and architecture.
- ★ Develop the formulas for volumes of rectangular and triangular prisms and prisms in general.
- ★ Use formulas to solve problems involving volume.
- ★ Investigate the relationship between volumes of rectangular and triangular prisms.



A TASK

Who can make the largest possible container, rectangular prism, from an A4 sheet of paper?

- Get an A4 sheet of paper.
- How big a square will you cut from each corner?
- Cut the square from each of the four corners.
- Fold up the sides of the container.
- Measure the length, width, height.
- Calculate volume = length×breadth×height
- Who has the largest volume?



A LITTLE BIT OF HISTORY

King Henry I ruled that a standard yard was the distance from the tip of his nose to the tip of his outstretched arm. Henry was the King of England from 1100 - 1135 AD and his yard is still in use today (1 yard ≈ 91.4 cm).

How do you compare in size to King Henry I?
How far is it from the tip of your nose to the tip of your outstretched arm?

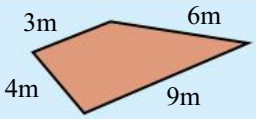
Perimeter

Perimeter: the distance around the outside of a shape.

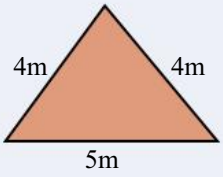
Exercise 17.1

Find the perimeter of each of the following shapes:

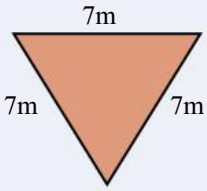
Perimeter = 3m + 6m + 9m + 4m
Perimeter = 22 m



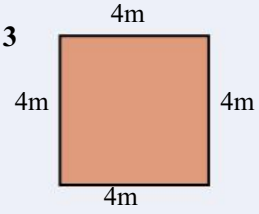
1



2



3



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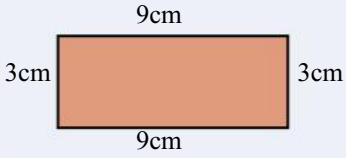
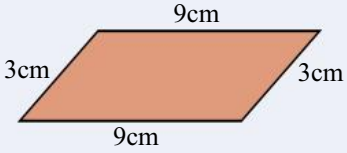
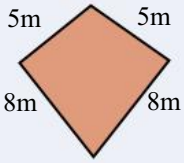


5



6



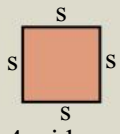
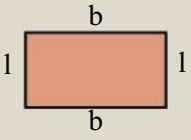
7 The Sydney Cricket ground can contain a football field of approximately 147 m by 136 m. What distance would be covered by walking around the field?

8 A paddock in the shape of a rectangle 145 m by 226 m is to have a fence with 4 strands of barbed wire.

a) How much barbed wire is needed?

b) If a 400 m roll of barbed wire costs $95, what would be the cost of the wire?

Perimeter formulas

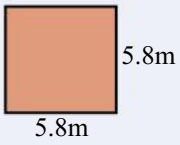
Square	**Rectangle**
	
Perimeter = 4×side = 4s	Perimeter = 2×length + 2×breadth **or** = 2(l + b)

Exercise 17.2

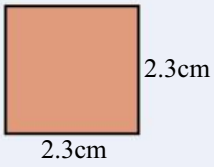
Calculate the perimeter of each of the following shapes:

Perimeter:
around the rim
peri = around
meter = a measure

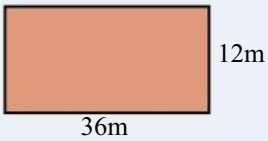
1



2

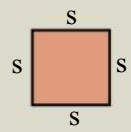
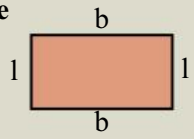
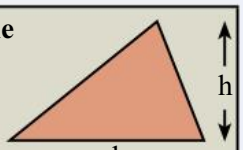


3



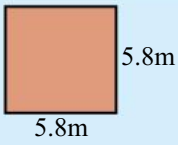
4

Area

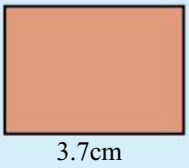
Square	Rectangle	Triangle
 Area = s^2	 Area = l×b	 Area = ½bh

Exercise 17.3

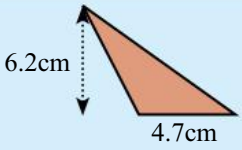
Calculate the area of each of the following shapes:



Area $= s^2$
$= (5.8m)^2$
$= 33.64\ m^2$

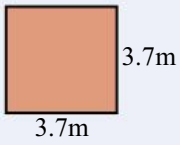


Area $= l \times b$
$= 3.7cm \times 2.3cm$
$= 8.51\ cm^2$

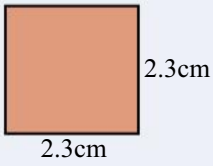


Area $= \frac{1}{2}bh$
$= 0.5 \times 4.7cm \times 6.2cm$
$= 14.57\ cm^2$

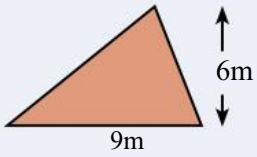
1



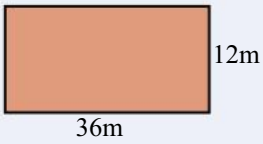
2



3



4



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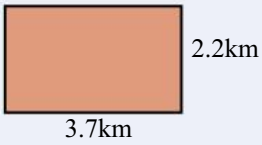
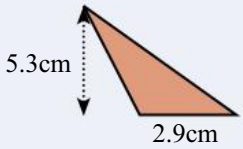


6



7 A kitchen bench top is 1.4 m by 2.9 m. How many square metres of laminate is needed to cover the top of the bench?

8 A rectangular paddock is 124 m by 111 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000m^2)?

A hectare is the area of a square 100 m by 100 m.

9 A triangular road sign has a base of 20 cm and a perpendicular height of 32 cm. What is the area of the road sign?

10 A bedroom is 3.3 m by 2.8 m. How many square metres of carpet is needed to cover the floor of the room?

11 A paddock, in the shape of a triangle, has a base of 648 m and a perpendicular height of 457 m. What is the area of the paddock in square metres and hectares?

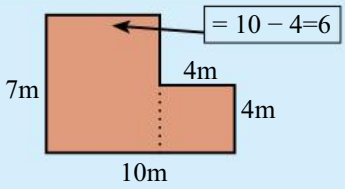
12 The builder wants to put a 2 m wide concrete path around the outside of a 12 m square building. What is the area of the path?

Composite Shapes

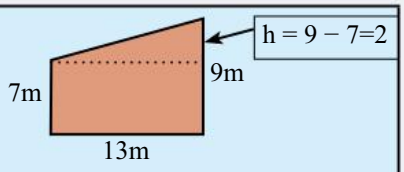
Composite shapes can be squares, rectangles, triangles etc composed together.

Exercise 17.4

Calculate the area of each of the following composite shapes:

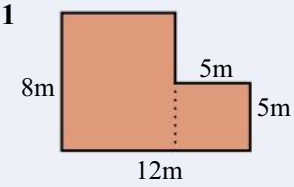


Area = rectangle+square
= lb + s^2
= 7m×6m + 4m×4m
= $42m^2$ + $16m^2$
= 58 m^2

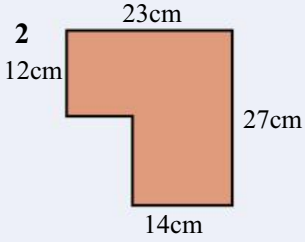


Area = rectangle+triangle
= ½bh + lb
= 0.5×13m×2m + 13m×7m
= $13m^2$ + 91 m^2
= 104 m^2

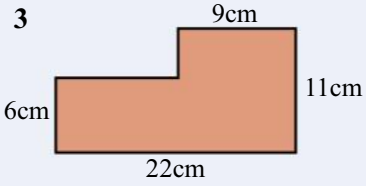
1



2



3



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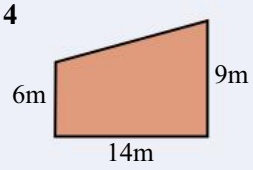

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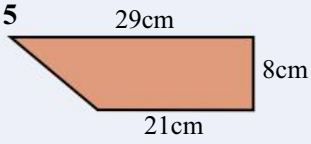

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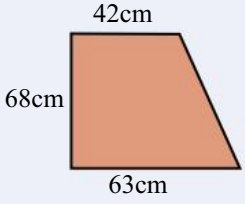
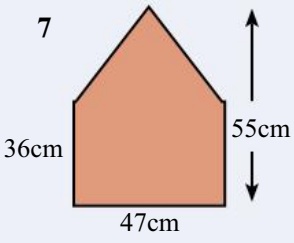


7



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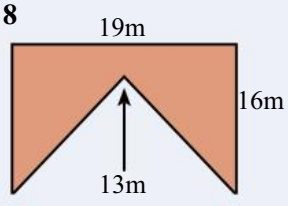
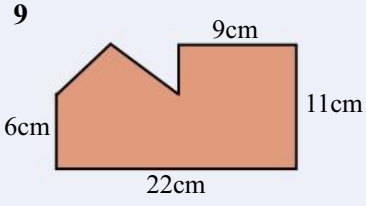


9

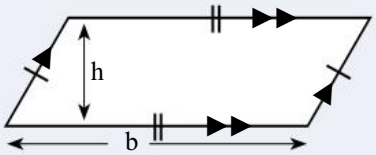


Optometrists use a variety of instruments and tests to detect eye conditions. They manage eye problems and refer severe cases to opthalmologists.

- Relevant school subjects are Mathematics, Physics, Biology, Chemistry.
- Courses range from Universtity Bachelor degrees to Graduate diplomas.

Parallelograms

A parallelogram is a quadrilateral with opposite sides parallel.



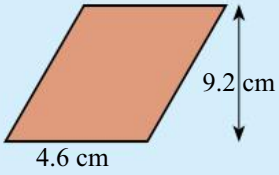
Area of a parallelogram = bh

Why is the area = bh?
See Investigation 17.1

Exercise 17.5

Calculate the area of each of the following shapes:

Area = bh
= 4.6cm × 9.2cm
= 42.32 cm^2



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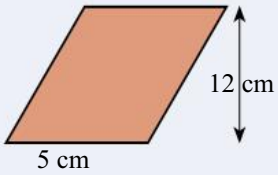

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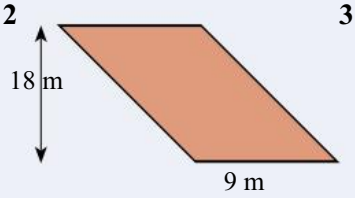

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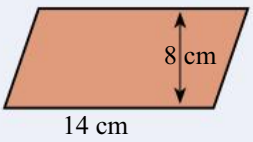

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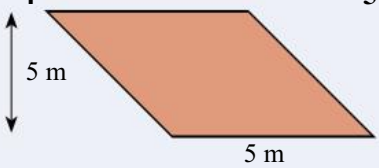

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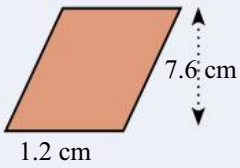

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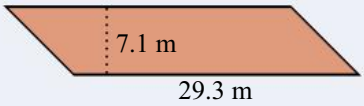

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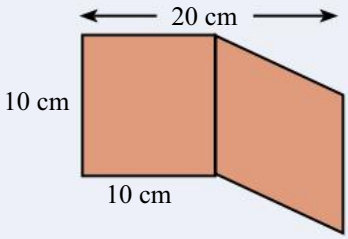

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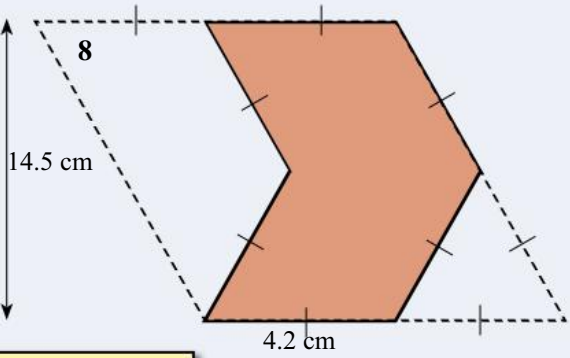
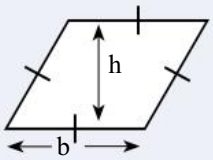


Parallelograms get a bit beyond themselves - they are just two triangles really.

Special Quadrilaterals

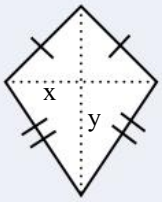
A **rhombus** is a quadrilateral with all four sides equal.



Why is the area = bh?
See Investigation 17.2

Area of a Rhombus = bh

A **kite** is a quadrilateral with two pairs of adjacent sides equal.

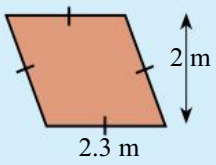


x and y are the lengths of the diagonals.
Why is the area = ½xy?
See Investigation 17.3

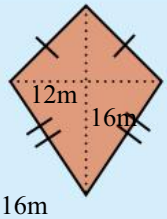
Area of a kite = ½xy

Exercise 17.6

Calculate the area of each of the following shapes:

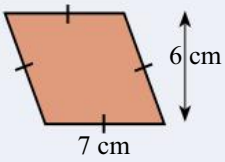


Area = bh
= 2.3m × 2m
= 4.6 m²

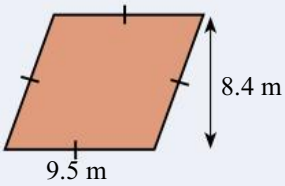


Area = ½xy
= 0.5 × 12m × 16m
= 96 m²

1



2



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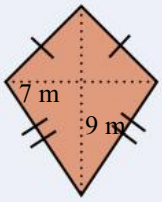

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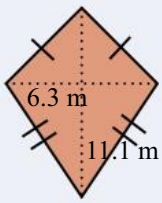

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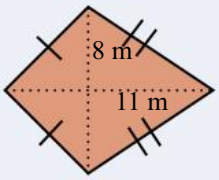

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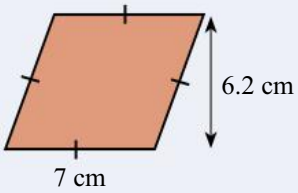
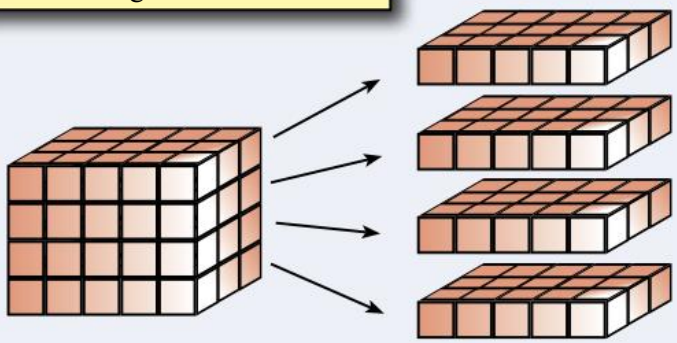


Rectangular Prisms

A **rectangular prism** is a column with a rectangular base.

The volume is the space occupied by the prism.



Volume = 60? Count them.

Height = 4

No units in base =5x3

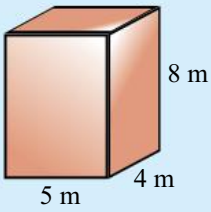
Volume of prism = Area of base x Height

=15 × 4 = 60

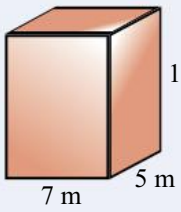
Exercise 17.7

Find the volume of each of the following prisms:

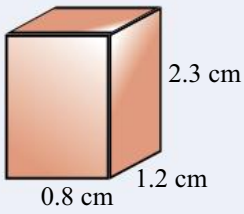
Volume = Area of base x Height
= 5cm × 4cm × 8cm
= 160 cm³



1



2



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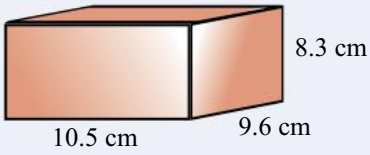
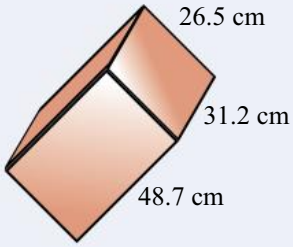


4



5 A milk carton has a base of 10 cm by 10 cm and a height of 20 cm. How many litres of milk will the carton hold (1000 cm³ = 1 L)?

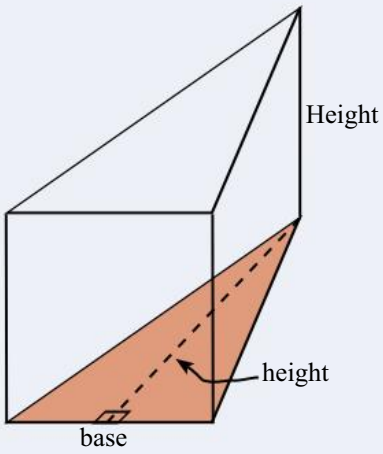
6 A bin has a base of 22 cm by 22 cm and height 64 cm. How many litres of water would the bin hold (1000 cm³ = 1 L)?

7 How many litres of water is needed to fill a swimming pool 50 m long, 10 m wide, and 1.6 m deep (1 m³ = 1000 L)?

Triangular Prism

A **triangular prism** is a column with a triangular base.

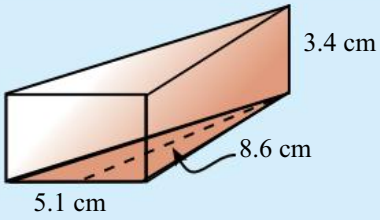
The volume is the space occupied by the prism.



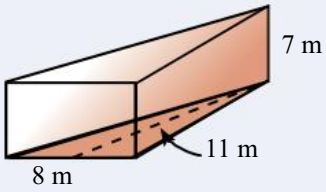
Volume of prism = **Area of base** × **Height**
= Area of triangle × Height
= ½base × height × Height

Exercise 17.8
Find the volume of each of the following prisms:

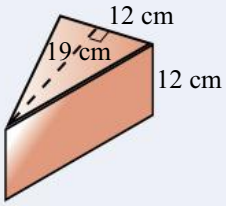
Volume = Area of base x Height
= ½ ×5 .1cm × 8.6cm × 3.4cm
= 74.56 cm^3



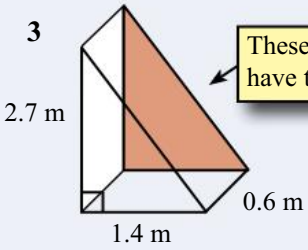
1



2

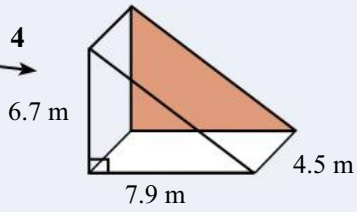


3



These triangular prisms have their base at the side

4



5 A triangular prism of length 30 cm has a triangular base with a perpendicular height of 5 cm and a base of 6 cm. How many grams of chocolate will the prism hold (1 cm^3 of chocolate = 1.5 g)?

This growth appeared when I was in prism.

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 17.9

1 Spell parallelogram.
2 Divide $50 in the ratio of 7 : 3
3 What is 20% of 23?
4 What is the formula for the area of a triangle?
5 What is the formula for the area of a parallelogram?
6 What is the formula for the volume of a rectangular prism?
7 What is the perimeter of a rectangle 4 m by 8 m?
8 What is the area of a rectangle 4 m by 8 m?
9 What is the volume of a rectangular prism with height 6 cm and base area 15 cm^2?
10 How many m^2 in a hectare?

Why did Chloe put a clock under her desk?
She wanted to work overtime.

Exercise 17.10

1 Spell rhombus.
2 Divide $60 in the ratio of 2 : 4
3 What is 20% of 42?
4 What is the formula for the area of a rectangle?
5 What is the formula for the area of a rhombus?
6 What is the formula for the volume of a triangular prism?
7 What is the perimeter of a rectangle 5 m by 9 m?
8 What is the area of a triangle base 3 cm and perpendicular height 8 cm?
9 Volume of a rectangular prism: height 6 cm, base 4 cm x 5 cm?
10 How many m^2 in a hectare?

It's always too soon to quit.
- Norman Vincent Peale

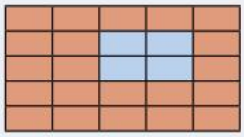
Exercise 17.11

1 Spell triangular prism.
2 Divide $70 in the ratio of 3 : 4
3 What is 20% of 34?
4 What is the formula for the area of a triangle?
5 What is the formula for the area of a kite
6 What is the formula for the volume of a rectangular prism?
7 What is the perimeter of a rectangle 6 m by 4 m?
8 What is the area of a rectangle 6 m by 4 m?
9 Volume of a triangular prism: height 8 m, base area 25 m^2?
10 How hectares in a paddock 100 m by 200 m?

NAPLAN Questions

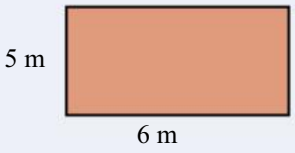
Exercise 17.12

1 The total area of the blue rectangles in the middle is 24 m^2. What is the total area of the rust rectangles?



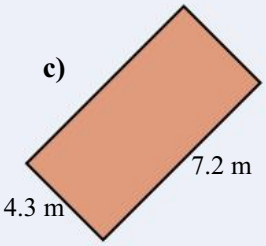
2 Which of the following has the largest area?

a)



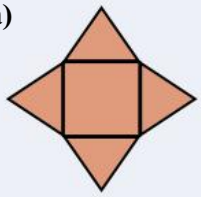
b)

c)

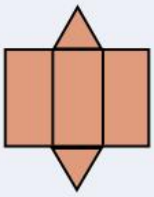


3 Either make or draw a diagram of what each of the following nets will make.

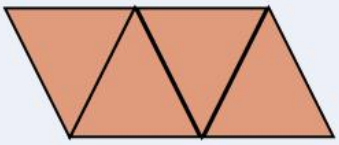
a)



b)



c)



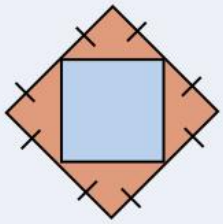
4 A photocopier is used to enlarge a 4 cm by 5 cm photo. The photocopy is twice as long and twice as wide. How many times larger is the area of the photocopy compared to the area of the original photo?

5 Find the volume of a cone with radius 5 cm and height 7 cm.

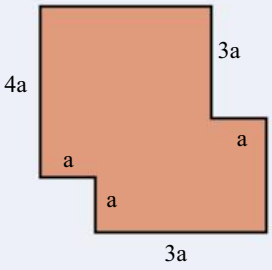
The volume of a cone is: $V = \dfrac{\pi r^2 h}{3}$

6 The area of a rectangle is 96 cm^2 and the width is 8 cm. What is the length of the rectangle?

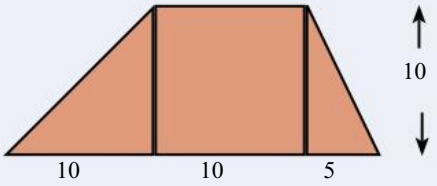
7 What is the ratio of the area of the inner blue square to the area of the larger rust square?



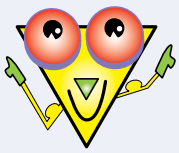
8 What is the area of the floor plan?



9 What is the area of the trapezium?



Competition Questions

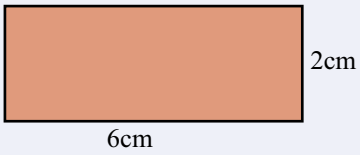


Build maths muscle and prepare for mathematics competitions at the same time.

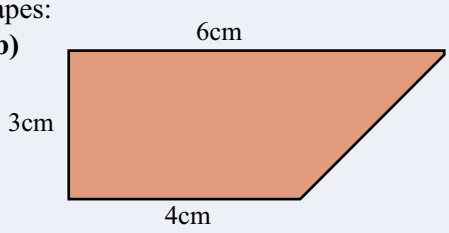
Exercise 17.13

1 Find the area of each of the following shapes:

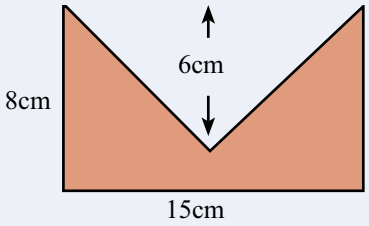
a)



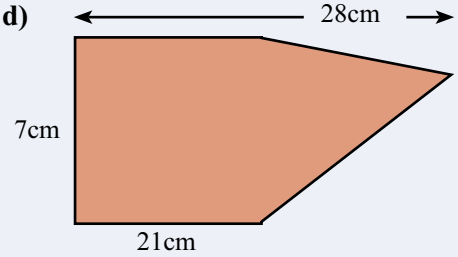
b)



c)



d)



The area of a square is 64 cm^2.
What is the perimeter of the square?

Area of square = side^2
64 = side^2
$\sqrt{64}$ = side
Perimeter = 4 x side
Perimeter =4 x 8 = 32 cm

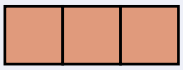
25 mm of rain on a flat roof puts 2500 L of water into the tank. If all of the rain on the roof goes into the tank, what is the area of the roof?

Volume = 2500 L
Area of roof x depth = 2500x1000 cm^2
Area of roof x 2.5 cm = 2500000 cm^2
Area of roof = 2500000 ÷ 2.5
= 1000000 cm^2
Area of roof = 100 m^2

2 A box measures 20 cm by 30 cm by 40 cm. What is the volume, in litres, of the box? 1000 cm^3 = 1 L

3 A box of baking soda measures 3 cm by 5 cm by 7 cm. How many teaspoons of baking soda are expected to be in the box (1 teaspoon = 5 cm^3)?

4 A shape is made by putting these three squares together. If the perimeter is 32 cm, what is the area of the shape?



5 The area of a square is 36 cm^2. What is the perimeter of the square?

6 60 centicubes are glued together to form a rectangular based prism. If the area of the base is 20 cm^2, what is the height of the prism?

7 20 mm of rain on a flat roof puts 2000 L of water into the tank. If all of the rain on the roof goes into the tank, what is the area of the roof?

8 50 mm of rain on a flat roof puts 3000 L of water into the tank. If all of the rain on the roof goes into the tank, what is the area of the roof?

Investigations

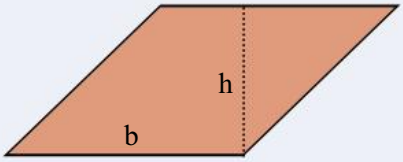
Investigation 17.1 The area of a parallelogram

A parallelogram is a quadrilateral with opposite sides parallel (and with equal length).

1. Make a copy of the above parallelogram - include the letters.
2. Cut along the dotted line and rearrange the pieces to make a rectangle?
3. What is the area of the rectangle? Check that this is the formula for the area of a parallelogram.
4. Paste your results into your maths pad.

Investigation 17.2 The area of a rhombus

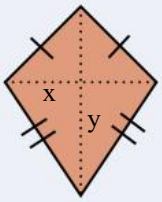
A rhombus is a quadrilateral with all four sides equal in length.
A rhombus is a special parallelogram.



1. Make a copy of the above rhombus - include the letters.
2. Cut along the dotted line and rearrange the pieces to make a rectangle?
3. What is the area of the rectangle? Check that this is the formula for the area of a rhombus.
4. Paste your results into your maths pad.

Investigation 17.3 The area of a kite

A **kite** is a quadrilateral with two pairs of adjacent sides equal



1. Make a copy of the above trapezium - include the lettering.
2. Cut along the dotted lines and rearrange the pieces to make a familiar figure..
3. What is the area of the familiar figure? Check that this is the formula for the area of a kite.
4. Paste your results into your maths pad.

Investigations

Investigation 17.4 Parallelogram, rhombus, and kite in structures?

a) Find five images of structures using a parallelograms, rhombuses, or kites.

b) As an architect you are to design a dome:

The dome is to be interesting and attractive.
The dome is to be strong and safe.
The dome is to be based on the use of parallelograms, rhombuses, or kites.
Make a model of your dome using straws.

Investigation 17.5 How big is 1 cubic centimetre (cm^3)?

To become familiar with the volume of a cubic centimetre, make a 1 cm by 1 cm by 1 cm box or use centicubes.

Use the cubic centimetre to estimate volumes in your classroom:

- The volume of a calculator.
- the volume of a pencil case, etc.

$1000\ cm^3 = 1\ L$

How close were your estimates to the actual areas?

Investigation 17.6 How big is 1 cubic metre (m^3)?

To become familiar with the space of a cubic metre, make a 1 m by 1 m by 1 m frame

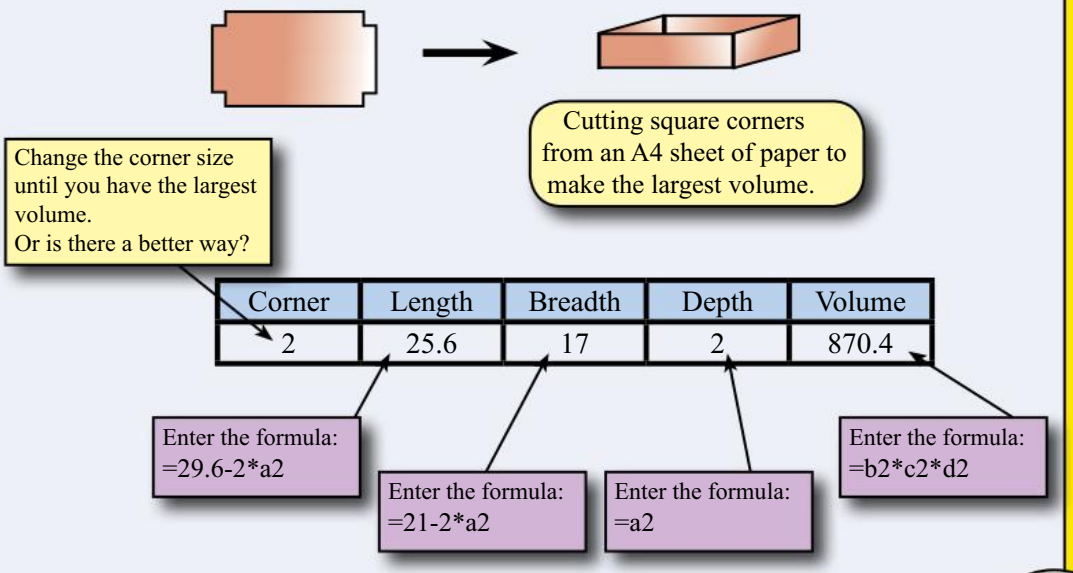
Use the cubic metre to estimate volumes in your classroom:

- The volume of a desk.
- The volume of the classroom.

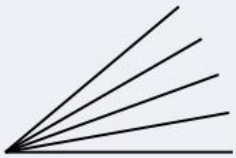
How close were your estimates to the actual volumes?

Technology

Use a spreadsheet to help with the task at the beginning of the chapter.



A Couple of Puzzles



Exercise 17.14

1 How many acute angles?
An acute angle is an angle less than 90°

2 Who am I? If you multiply me by 16 and then add 45, the answer is 285.

3 A train is 1 km long. If the train is travelling 1 km every 5 minutes, how long does the train take to pass through a 2 km tunnel (the answer is not 10 mins)?

4 A 1 metre square sheet of cardboard is cut into 1 millimetre squares. If these squares are placed side by side, how long would they be?

A Game

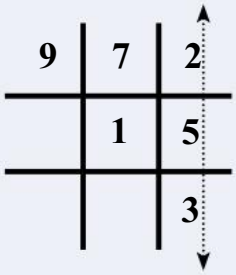
Tens is naughts and crosses played with numbers.
The winner is the person who gets a total of **10**

1, 2, 3, 4, 5, 6, 7, 8, 9

1 Take it in turns to use one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9

2 When you use a number, cross it off, because it can't be used more than once.

3 The winner is the first person to get a sum of 10 either horizontally, vertically, or diagonally



A Sweet Trick

1 You write down the numbers: 1 2 3 4 5 6 7 9 {No 8}

2 Ask your audience to circle their favourite number. 1 2 3 (4) 5 6 7 9

3 Ask your audience to multiply by 36. 12345679 x 36

4 Hey Presto! The answer is nine of their favourite number. **4 4 4 4 4 4 4 4 4**

Multiply the circled number by 9 (4 x 9 = 36).

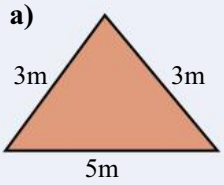
Remember - there is no 8

Chapter Review 1

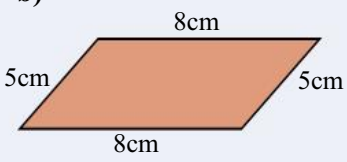
Exercise 17.15

1 Calculate the perimeter of each of the following shapes:

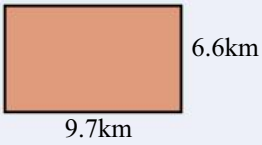
a)



b)

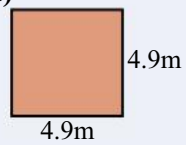


c)



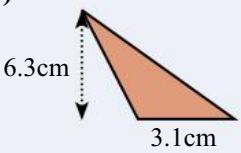
2 Calculate the area of each of the following shapes:

a)

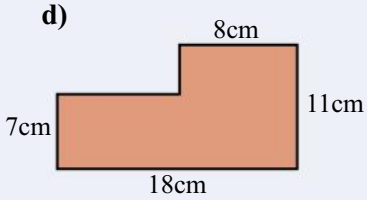


b)

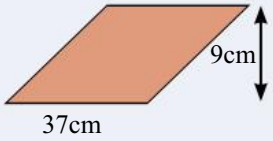
c)



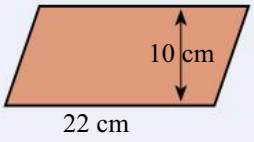
d)



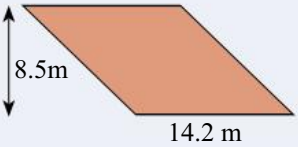
e)



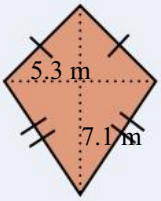
f)



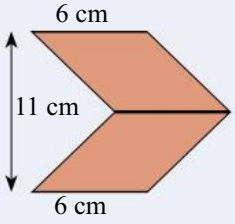
g)



h)

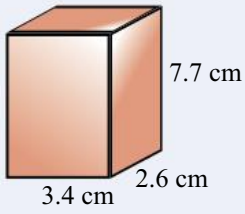


i)

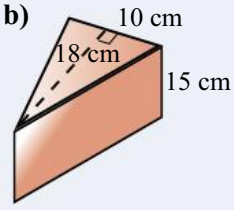


3 Find the volume of each of the following prisms:

a)



b)



What grows larger the more you take away?
A hole.

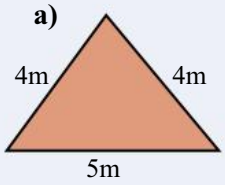
When is it correct to say "I is"?
I is the letter before J.

Chapter Review 2

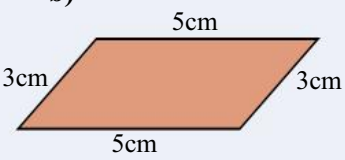
Exercise 17.16

1 Calculate the perimeter of each of the following shapes:

a)



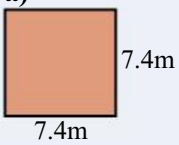
b)



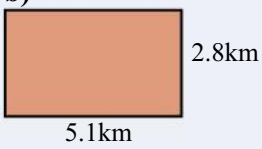
c)

2 Calculate the area of each of the following shapes:

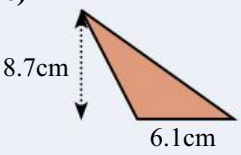
a)



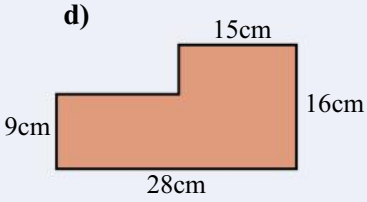
b)



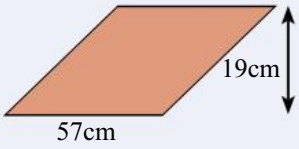
c)



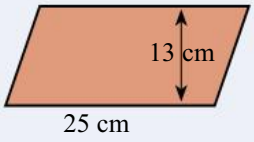
d)



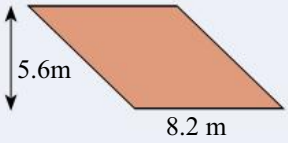
e)



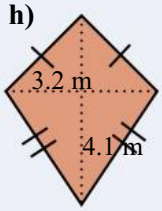
f)



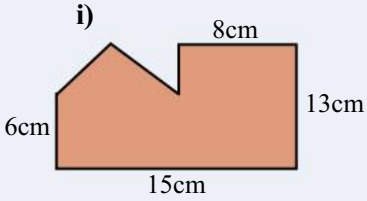
g)



h)

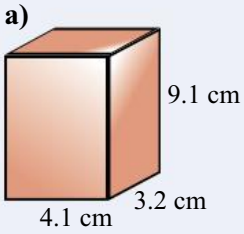


i)

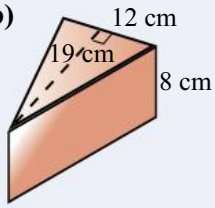


3 Find the volume of each of the following prisms:

a)



b)



Our greatest glory is not in never falling, but in rising every time we fall - Confucius.

Probability

18

- ★ Understand that representing data in Venn diagrams or two-way tables facilitates the calculation of probabilities.
- ★ Use Venn diagrams and two-way tables to calculate probabilities for events satisfying 'and', 'or', 'given' and 'not' conditions.
- ★ Pose 'and', 'or', 'not' and 'given' probability questions about objects or people.
- ★ Collect data to answer the questions using Venn diagrams or two-way tables.

A TASK

Conduct research on a gambling game.

- Choose a 2 dice game.
- Simulate the game a large number of times.
- Investigate the theoretical probabilities.
- Is the game fair?
- Present your findings.

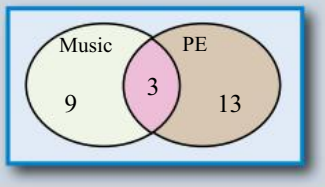
A LITTLE BIT OF HISTORY

Leonhard Euler (1707 - 1783), a Swiss mathematician, used "Eulerian Circles" to represent problems.

John Venn (1834 - 1923) expanded the use of Eulerian Circles and they are now known as "Venn Diagrams."

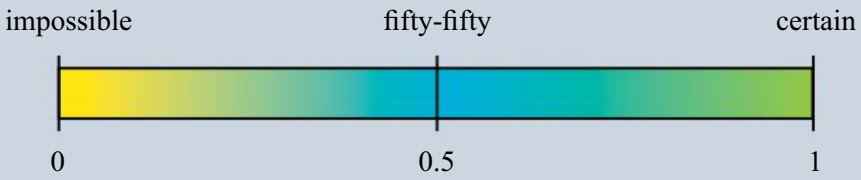
Venn Diagrams are used to represent and solve problems in probability, logic, statistics, computer science, and linguistics.

So many things involve probability. Insurance is about probability. Weather prediction is about probability. Atomic theory is about probability. Etc, etc, etc.



Warm Up

Probability is the chance of something happening.



Exercise 18.1

1 Copy the above probability scale and add each of the following to the scale.

a) The day after Monday will be Tuesday.

b) We have Buckley's chance of winning the match.

c) Throw a coin and it will show a head.

d) There is a 25% chance that it will rain today.

e) We have a 3 in 4 chance of picking the winner.

f) The probability of winning first prize in a lottery is $\frac{1}{1000000}$ or 0.000001.

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

A standard 6-sided die is thrown. What is the probability of each of the following happening?

a) 3 **b)** odd **c)** <4

Sample space = {1,2,3,4,5,6}

a) $P(3) = \frac{1}{6}$ {there is one 3}

b) $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$ {1,3,5 are odd}

c) $P(<4) = \frac{3}{6} = \frac{1}{2}$ {1,2,3 are <4}

Eight balls are numbered 1 to 8. What is the probability of selecting a ball that shows:

a) 2? **b)** even? **c)** >6?

Sample space = {1,2,3,4,5,6,7,8}

a) $P(2) = \frac{1}{8}$ {there is one 2}

b) $P(\text{even}) = \frac{4}{8} = \frac{1}{2}$ {2,4,6,8 are even}

c) $P(>6) = \frac{2}{8} = \frac{1}{4}$ {7,8 are >6}

2 A six-sided die is thrown. What is the probability of each of the following?

a) 3 **b)** even **c)** odd

d) 4 **e)** <3 **f)** >3

g) >6 **h)** a number divisible by 3

3 A bag contains five blue marbles and three red marbles. What is the probability of taking a marble from the bag that is:

a) red? **b)** blue? **c)** not red?

d) not blue? **e)** blue or red? **f)** white?

Venn Diagrams

Venn diagrams are a good way of representing probability.

Exercise 18.2

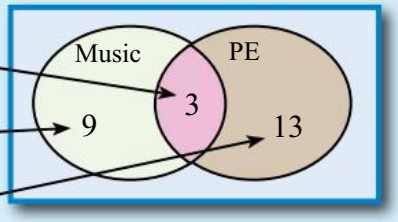
In a class of 25 students, 12 students study music, and 16 students study PE.

a) Draw a Venn Diagram

12 + 16 = 28 which is 3 more than 25
Thus 3 study music and PE

12 - 3 = 9.
Thus 9 study music only

16 - 3 = 13
Thus 13 study PE only

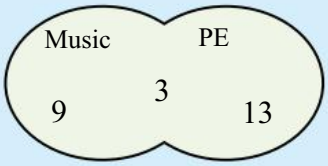


b) Find the probability that a student studies:

Music **and** PE

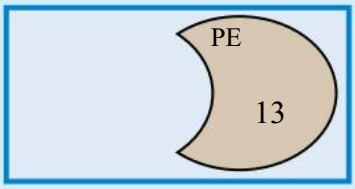
$= \frac{3}{25} = 0.12$

Music **or** PE



$= \frac{25}{25} = 1$

not music



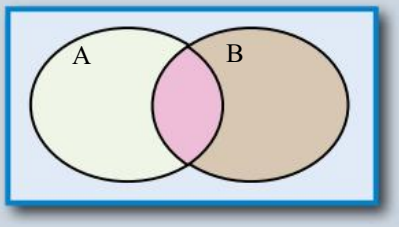
$= \frac{13}{25} = 0.52$

1 In a class of 25 students, 13 students study music, and 17 students study PE. Draw a Venn Diagram and find the probability that a student:
- **a)** studies music and PE.
- **b)** studies music or PE.
- **c)** does not study music.
- **d)** does not study PE.

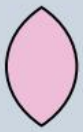
2 In a class of 21 students, 12 students play cricket, and 16 students play netball. Draw a Venn Diagram and find the probability that a student:
- **a)** plays cricket and netball.
- **b)** plays cricket or netball.
- **c)** does not play cricket.
- **d)** does not play netball.

3 In a class of 28 students, 27 students passed Maths, and 26 students passed English. Draw a Venn Diagram and find the probability that a student:
- **a)** passed maths and English.
- **b)** passed maths or English.
- **c)** did not pass Maths.
- **d)** did not pass English.

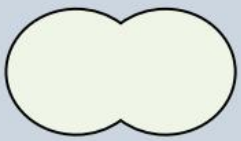
Venn Diagrams



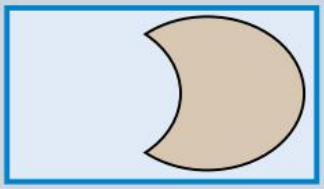
A **and** B



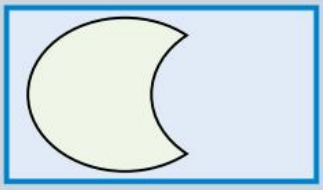
A **or** B



not A



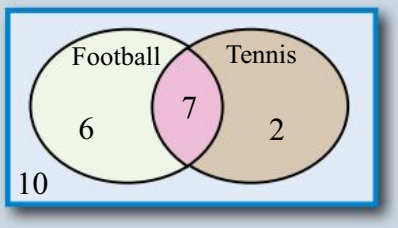
not B



Exercise 18.3

1 In a class of 25 students, 13 students play football, 9 students play tennis, and 7 students play both football and tennis. Draw a Venn Diagram and find the probability that a student:

a) plays football **and** tennis.
b) plays football **or** tennis.
c) does **not** play football.
d) does **not** play tennis.
e) does **not** play football or tennis.



2 In a class of 27 students, 11 students play an instrument, 14 students have a job, and 8 students play an instrument and have a job. Draw a Venn Diagram and find the probability that a student:

a) plays an instrument **and** has a job.
b) plays an instrument **or** has a job.
c) does **not** play an instrument.
d) does **not** have a job.
e) does **not** play an instrument or has a job.

3 In a sports club of 150 members, 88 play golf, 63 play bowls, and 45 play golf and bowls. Draw a Venn Diagram and find the probability that a member:

a) plays golf **and** plays bowls.
b) plays golf **or** plays bowls.
c) does **not** play golf.
d) does **not** play bowls.
e) does **not** play golf or bowls.

Two-Way Tables

Two-way tables are useful when calculating the theoretical probability of two events.

Exercise 18.4

1 Investigate the experimental probability when tossing two coins.
Toss two coins 50 times. Use a tally similar to below

2 Combine your results with the results of other members in the class.

HH	H and T	TT
14	23	13

→

	50 throws	400 throws
P(HH)	$\frac{14}{50} = 0.28$	
P(HandT)	$\frac{23}{50} = 0.46$	
P(TT)	$\frac{13}{50} = 0.26$	

3 Calculate the theoretical probability by using a **Two-Way Table.**

Coin 2 \ Coin 1	H	T
H	HH	HT
T	TH	TT

→

P(HH) $= \frac{1}{4} = 0.25$

P(HandT) $= \frac{2}{4} = 0.5$

P(TT) $= \frac{1}{4} = 0.25$

4 Did your experimental probabilities approach the theoretical probabilities?

5 Assuming that the chances of a girl or boy being born is equal, use a **Two-Way Table** to determine the theoretical probabilities for a family of two children:

a) P(2 girls).
b) P(1 girl and1 boy).
c) P(2 boys).

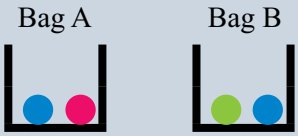
In a Two-Way Table.
One event on one side.
Another event on the other side.

Child 2 \ Child 1	G	B
G		
B		

Exercise 18.5

1 Bag A contains 1 blue and 1 red marble. Bag B contains 1 blue and 1 green marble. One marble is randomly chosen from Bag A and one marble is chosen from Bag B. Use a **Two-Way Table** to calculate the theoretical probabilities of getting:

a) a blue and a green marble.
b) two blue marbles.
c) a red and a green marble.



2 A coin is tossed and a die is rolled. Use a Two-Way Table to calculate the probability of obtaining:

a) a tail and a 6.
b) a head and a 1.
c) a head and an even number.

Die \ Coin	H	T
1		
2		
3		
4		
5		
6		

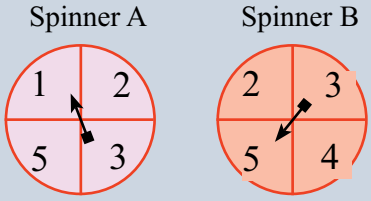
3 Two dice are thrown and the two numbers are added together. Use a Two-Way Table to calculate the probability of obtaining:

a) a total of 12.
b) a total of 2.
c) a total of 10.
d) a total of 3.
e) a total of 7.

First Die \ Second Die	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

4 Spinner A and Spinner B are spun to produce a two digit number. The spinners are showing the two digit number 15. Use a Two-Way Table to calculate the probabilities of obtaining:

a) a two-digit number less than 20.
b) a two-digit number greater than 40.
c) a two-digit number divisible by 5.
d) a two-digit number divisible by 3.



Spinner A \ Spinner B	2	3	4	5
1				15
2				
3				
5				

Mental Computation

Mental computation helps people prepare for problems in everyday life.

Exercise 18.6

1 Spell Venn Diagram.
2 Convert 3000 cm to m
3 Solve: $2x - 1 = 3$
4 A die is thrown. What is P(even)?
5 $y = 3x + 2$. Find y when $x = ^{-}1$
6 What is the function for the sequence: 4, 7, 10, 13, ...?
7 Two coins are tossed. What is P(HH)?
8 In a class of 25 students. 15 play tennis and 13 play football. How many play tennis and football?
9 What is the formula for the area of a triangle?
10 Simplify: $a^6 \times a^2$

Anything you do can get you shot, including nothing
- Murphy's Laws of Combat.

Exercise 18.7

1 Spell Probability.
2 Convert 5000 m to km
3 Solve: $3x - 2 = 4$
4 A die is thrown. What is P(odd)?
5 $y = 4x + 5$. Find y when $x = ^{-}2$
6 What is the function for the sequence: 2, 7, 12, 17, ...?
7 Two coins are tossed. What is P(H and T)?
8 In a class of 23 students. 13 play netball and 17 play football. How many play netball and football?
9 What is the formula for the area of a parallelogram?
10 Simplify: $a^3 \times a^2$

Exercise 18.8

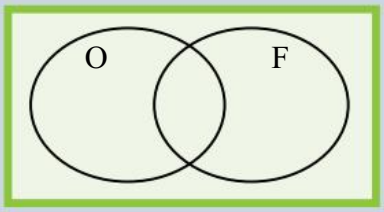
Its not whether you tried and failed but whether you failed to try.

1 Spell Theoretical.
2 Convert 6000 cm to m
3 Solve: $2x - 3 = 4$
4 A die is thrown. What is P(<3)?
5 $y = 2x + 2$. Find y when $x = ^{-}1$
6 What is the function for the sequence: 3, 7, 11, 15, ...?
7 Two coins are tossed. What is P(TT)?
8 In a class of 27 students. 12 play softball and 19 play football. How many play softball and football?
9 What is the perimeter of a rectangle 4 m by 8 m?
10 Simplify: $c^6 \div c^2$

NAPLAN Questions

Exercise 18.9

1 What is the probability of rolling a 5 on a six-sided die?



2 O is the set of odd numbers less than 15.
F is the set of multiples of 5 less than 15.
What is the largest number that belongs to O **and** F.

3 A jar of jelly beans contains 3 black, 2 white, and 4 green jelly beans.
What is the chance of picking a black jelly bean?

4 A traffic light is amber and red for 30 seconds and green for 25 seconds.
What is the chance that the traffic light is green?

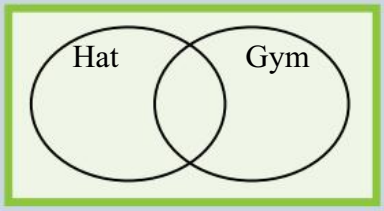
5 A coin is tossed twice. There are 4 possible outcomes. What is the probability of getting 1 head and 1 tail in any order?

		Second Coin: H	Second Coin: T
First Coin	H	H,H	H,T
	T	T,H	T,T

6 Two dice are thrown.
What is the probability of getting two sixes?

First Die \ Second Die	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

7 When the class of 27 students went to PE, 18 students wore a hat and 15 students wore gym shoes. How many students wore a hat **and** gym shoes?



8 A survey of the class produced the following table.
What is the probability that a classmate with black hair has blue eyes?

		Eye Colour: Brown	Eye Colour: Blue	Eye Colour: Other
Hair Colour	Black	7	4	3
	Blond	2	3	3
	Other	2	3	1

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 18.10

1 A bag contains two blue marbles, one red marble, and three green marbles. If a marble is randomly selected from the bag, what is the most likely colour of the marble?

2 A survey of the class produced the following table. What is the probability that a classmate has neither brown hair nor blue eyes?

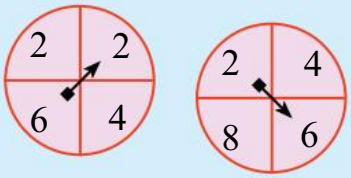
		Eye Colour		
		Brown	Blue	Other
Hair Colour	Black	7	4	3
	Blond	2	3	3
	Other	2	3	1

3 How many numbers between 101 and 200 are divisible by both 3 and 5?

The two spinners show a total of 8.

a) How many different ways can a total of 6 be obtained?

b) If the spinners are spun again, what is the probability of a total of 10?



First draw up a two-way table

Spinner 1 \ Spinner 2	2	4	6	8
2	4	6	8	10
2	4	6	8	10
4	6	8	10	12
6	8	10	12	14

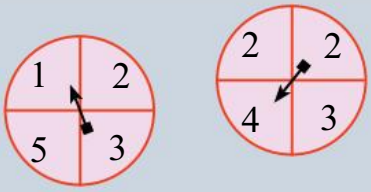
a) A total of 6 in 3 ways.

b) P(total=10) = 4/16 = 0.25

4 The two spinners show a total of 5.

a) How many different ways can a total of 5 be obtained?

b) If the spinners are spun again, what is the probability of a total of 4?



5 Using the cards below:

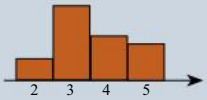
a) How many two digit numbers, divisible by 5, can be made?

b) How many three digit numbers, divisible by 2, can be made?

Hint: Draw up a sample space.

Investigations

Investigation 18.1 Throwing two dice



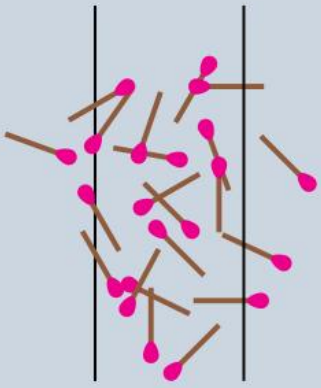
1 Obtain 2 dice.
2 Draw up a chart similar to below.
3 Throw the two dice and cross a square above the total.

2	3	4	5	6	7	8	9	10	11	12

Can you predict what the bar chart will look like after about 100 throws of the two dice?

Investigation 18.2 The value of pi?

1 Accurately measure the length of a match.
Rule two long parallel lines on a large sheet of paper so that the distance between the lines is exactly double the length of a match.
2 From a height of about a metre, drop all the matches in a box onto the sheet of paper.



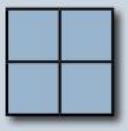
3 $\pi = \dfrac{\text{Total number of matches}}{\text{Number of matches touching a line}}$

Enironmental Engineers are concerned with the assessment of environmental impact, pollution control, and management of natural resources.
- Relevant school subjects are Mathematics, Chemistry, and Biology.
- Environmental engineer courses range from Bachelor degrees, Graduate Diplomas to Masters degrees.

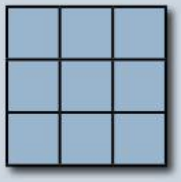
A Couple of Puzzles

Exercise 18.11

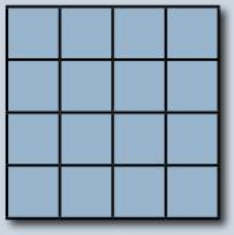
1 For each of the following, count the number of squares in each diagram and complete the calculation:



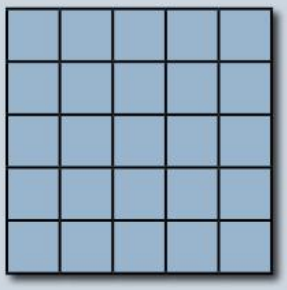
a) $1^2 + 2^2 =$



b) $1^2 + 2^2 + 3^2 =$



c) $1^2 + 2^2 + 3^2 + 4^2 =$



d) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 =$

And the connection between the number of squares and the calculation is?

2 Use any of the symbols +, −, x, ÷, or ( ) to make the following statements true:

a) 4 4 4 4 = 16

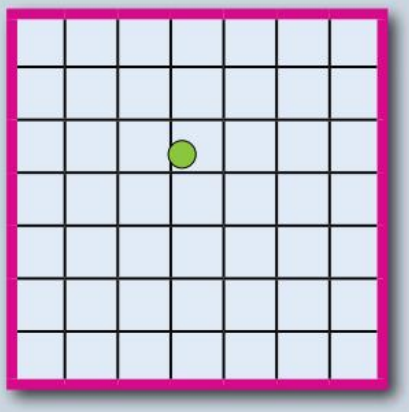
b) 4 4 4 4 = 6

c) 4 4 4 4 = 7

A Game

Coin in the square is a popular fairground game and is played by throwing a coin, say 20c, onto a table marked with squares. If the coin lands inside a square without touching a line then a prize is given, say double your money.

1 Make a large grid of 5cm squares.
2 Place the grid on a table.
3 Make a foot line to keep an outstretched hand about 50cm from the nearest edge of the grid.
4 Throw a 20c coin onto the table. Did the coin fall within a square?



5 What do you think your chances are of throwing a 20c coin onto the table and doubling your money? What would be fairer odds?

Estimate the odds of **coin in the square:**

Experimental probability (100 throws)	Theoretical probability?
P(coin in square) = $= \frac{\text{No. in square}}{100}$	P(coin in square) = ????

A Sweet Trick

1 Place 30 items in front of your audience.
2 Turn around and ask your audience to roughly select half of them and to count the number of items in their half.
3 Ask them to sum the digits.
4 Subtract the answer from their half.
5 Ask them to take some of the items and tell you how many are left.
6 You tell them how many they have.

There will always be 9 left.

14
1 + 4 = 5
14 − 5 = 9

They take 5 leaving 4

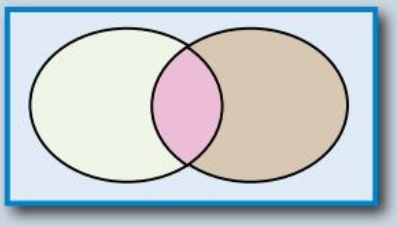
9 − 4 = 5
They have 5.

Chapter Review 1

Exercise 18.12

1 In a class of 23 students, 15 students study music, 11 students study PE, and 3 students study music and PE.
Draw a Venn Diagram and find the probability that a student:

a) studies music and PE.
b) studies music or PE.
c) does not study music.
d) does not study PE.



2 In a class of 26 students, 11 students play an instrument, 13 students have a job, and 4 students play an instrument and have a job. Draw a Venn Diagram and find the probability that a student:

a) plays an instrument and has a job.
b) plays an instrument or has a job.
c) does not play an instrument.
d) does not have a job.
e) does not play an instrument or has a job.

3 Assuming that the chances of a girl or boy being born is equal, use a Two-Way Table to determine the theoretical probabilities for a family of two children:

a) P(2 girls).
b) P(1 girl and1 boy).
c) P(2 boys).

4 A coin is tossed and a die is rolled.
Use a Two-Way Table to calculate the probability of obtaining:

a) a tail and a 3.
b) a head and a 1.
c) a head and an even number.

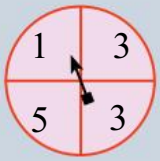
Coin

Die	H	T
1		
2		
3		
4		
5		
6		

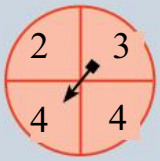
5 Spinner A and Spinner B are spun to produce a two digit number.
The spinners are showing the two digit number 14.
Use a Two-Way Table to calculate the probabilities of obtaining:

a) a two-digit number less than 20.
b) a two-digit number greater than 40.
c) a two-digit number divisible by 3.
d) a two-digit number divisible by 3.

Spinner A



Spinner B



Spinner B

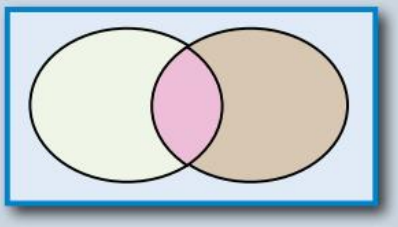
Spinner A	2	3	4	4
1				14
3				
3				
5				

Chapter Review 2

Exercise 18.13

1 In a class of 23 students, 13 students study music, 17 students study PE, and 7 students study music and PE.
Draw a Venn Diagram and find the probability that a student:

a) studies music and PE.
b) studies music or PE.
c) does not study music.
d) does not study PE.



2 In a class of 22 students, 9 students play an instrument, 8 students have a job, and 3 students play an instrument and have a job. Draw a Venn Diagram and find the probability that a student:

a) plays an instrument and has a job.
b) plays an instrument or has a job.
c) does not play an instrument.
d) does not have a job.
e) does not play an instrument or has a job.

3 Assuming that the chances of a girl or boy being born is equal, use a Two-Way Table to determine the theoretical probabilities for a family of two children:

a) P(2 girls).
b) P(1 girl and1 boy).
c) P(2 boys).

4 A coin is tossed and a die is rolled.
Use a Two-Way Table to calculate the probability of obtaining:

a) a tail and a 5.
b) a head and a 6.
c) a tail and an odd number.

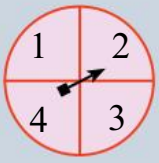
Coin

Die	H	T
1		
2		
3		
4		
5		
6		

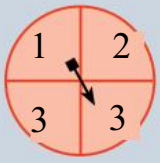
5 Spinner A and Spinner B are spun to produce a two digit number.
The spinners are showing the two digit number 23.
Use a Two-Way Table to calculate the probabilities of obtaining:

a) a two-digit number less than 20.
b) a two-digit number greater than 40.
c) a two-digit number divisible by 11.
d) a two-digit number divisible by 3.

Spinner A



Spinner B



Spinner B

Spinner A	1	2	3	3
1				
2			23	
3				
4				

Congruence 19

- ★ Develop the conditions for congruence of triangles.
- ★ Construct triangles using the conditions for congruence.
- ★ Solve problems using the properties of congruent figures.
- ★ Establish of the conditions for congruence (SSS, SAS, ASA and RHS) to solve problems.
- ★ Establish the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites.

A division by zero fallacy:

$0 \times 1 = 0$

$0 \times 2 = 0$

Thus: $0 \times 1 = 0 \times 2$

$\frac{0 \times 1}{0} = \frac{0 \times 2}{0}$

$\therefore 1 = 2$

The fallacy here is the assumption that it is possible to divide by 0.

A TASK

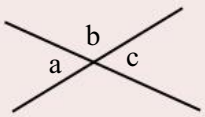
False proof:

- Investigate division by zero.
- Find a division by zero fallacy.
- Present the fallacy, or your own version, to your audience.

A LITTLE BIT OF HISTORY

Thales (624 BC - 547 BC) appears to have been the first known Greek mathematician and also appears to have been the first to use proof by logical reasoning.

Vertically opposite angles are equal $a = c$



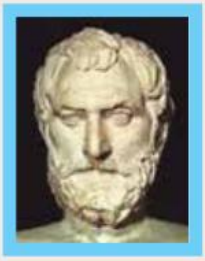
Thales' general proof:

$b + a = 180°$

$b + c = 180°$

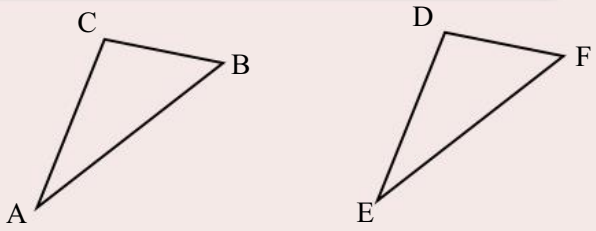
$\therefore a = c$

It is claimed that Thales measured the height of the Pyramids by noting the length of the shadow when the length of his own shadow was the same as his height.



Constructing Triangles

Congruent triangles have exactly the same shape and size.
The symbol for congruence is ≡ or ≅

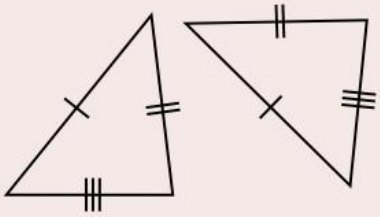


Triangle ABC is congruent to triangle EFD $\triangle ABC \equiv \triangle EFD$

The **angles must be named** in matching order:
$\angle A = \angle E$
$\angle B = \angle F$
$\angle C = \angle D$

The **sides must be named** in matching order:
side AB = side EF
side BC = side FD
side CA = side DE

SSS (side, side, side). Two triangles are congruent if the three sides in one triangle are the same length as the matching sides on the other triangle.

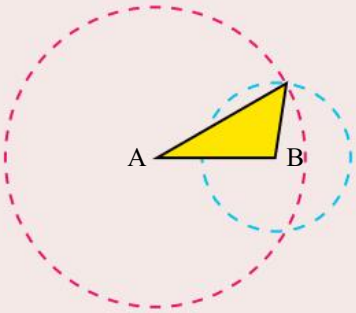


Exercise 19.1 SSS Given 3 sides, construct a triangle
Draw a triangle with sides of 8 cm, 10 cm, and 5 cm.

a) Use a ruler and draw an 8 cm line.

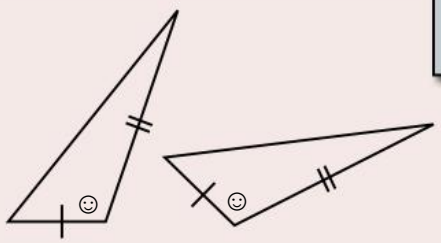
b) Set your compass to 10 cm and draw a circle with centre A.
c) Set your compass to 5 cm and draw a circle with centre B.
d) Use a ruler to complete the triangle.

Your triangle should exactly match the other triangles in the classroom?
They should be congruent.



SAS (side, angle, side).

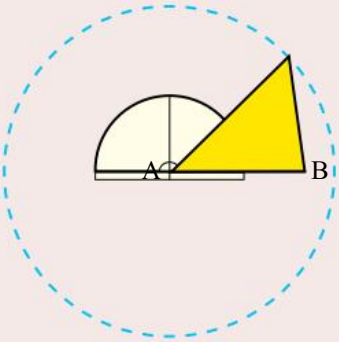
Two triangles are congruent if two sides and the included angle in one triangle are the same size as the matching sides and included angle on the other triangle.



Exercise 19.2 SAS **Given 2 sides and the angle in between the two sides, construct a triangle.**

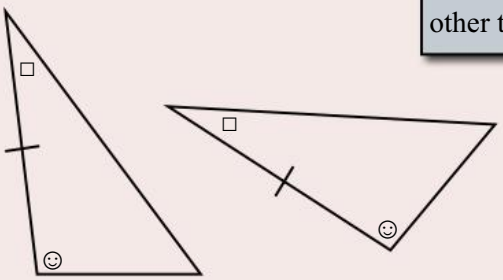
Draw a triangle with sides of 9 cm and 11 cm and the angle in between 45º

a) Use a ruler and draw an 9 cm line.
b) Set your protractor on AB with the centre on A and mark 45º
c) Set your compass to 11 cm and draw a circle with centre A.
d) Use a ruler to complete the triangle.



ASA (angle, side, angle).

Two triangles are congruent if two angles and a side in one triangle are the same size as the matching two angles and side on the other triangle.

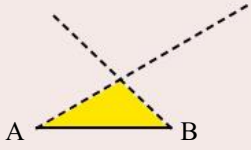


Your triangle should exactly match the other triangles in the classroom?
They should be congruent.

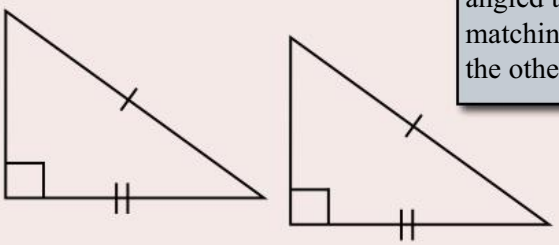
Exercise 19.3 ASA **Given two angles and a side construct a triangle**

Draw a triangle with angles of 30º and 45º and a side of 8 cm.

a) Use a ruler and draw an 8 cm line.
b) Set your protractor on AB with the centre on A and mark 30º
c) Set your protractor on AB with the centre on B and mark 45º
d) Use a ruler to complete the triangle.



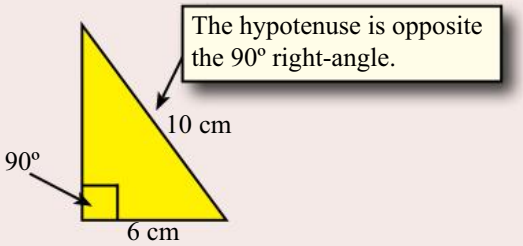
RHS (right-angle, hypotenuse, side) Two triangles are congruent if the hypotenuse and another side in one right-angled triangle are the same size as the matching hypotenuse and another side on the other right-angled triangle.



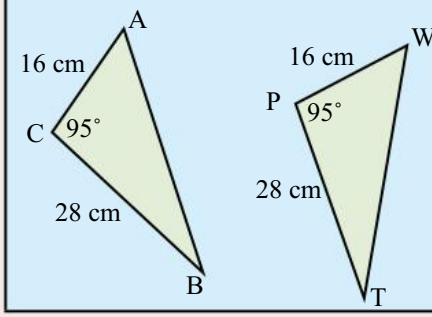
Exercise 19.4 RHS **Given the hypotenuse, a right-angle and another side, construct a triangle.**

Draw a triangle with a right-angle (90º), a hypotenuse of 10 cm, and another side of 6 cm.

a) Use a ruler and draw an 6 cm line.
b) Set your protractor on AB with the centre on A and mark 90º
c) Set your compass to 10 cm and draw a circle with centre B.
d) Use a ruler to complete the triangle.



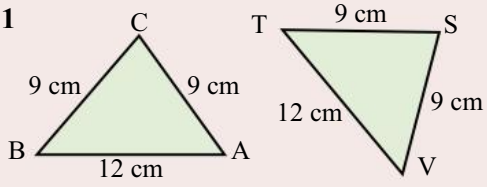
Exercise 19.5 Use the tests for congruence to test whether the following pairs of triangles are congruent:



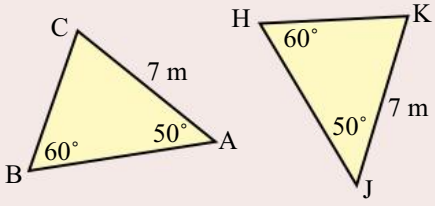
side CB = side PT
$\angle C = \angle P$ {angle inbetween}
side AC = side WP

$\triangle ACB \equiv \triangle WPT$ {**SAS**}

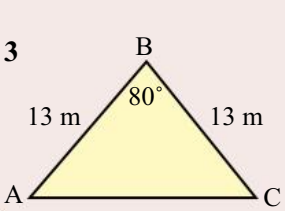
1

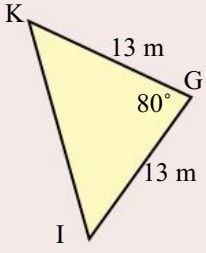


2



3





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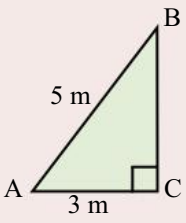
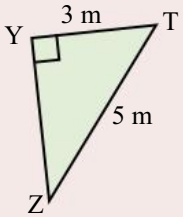




Exercise 19.6

Find the unknowns (each pair of triangles are congruent):

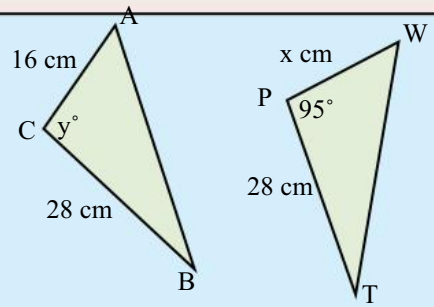
$\triangle ACB \equiv \triangle WPT$

$AC = WP$ {matching sides}

$\therefore 16 \text{ cm} = x$

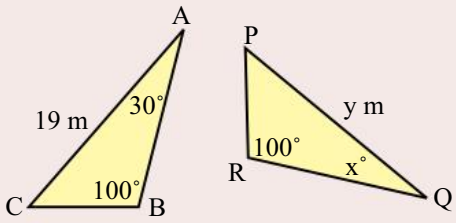
$\angle ACB = \angle WPT$ {matching angles}

$\therefore y = 95^\circ$



This proof shows that y= 95º.
This proof is based on congruent triangle reasoning.

1



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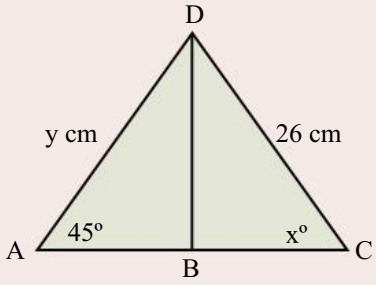

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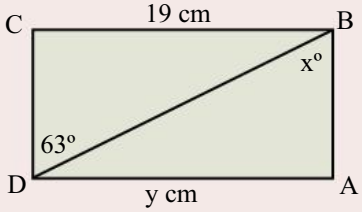

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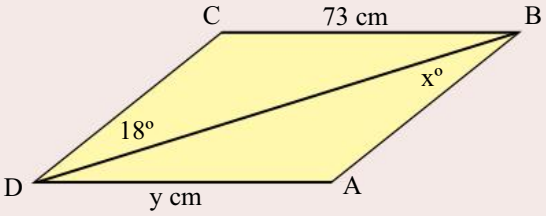

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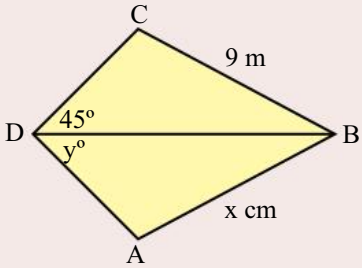

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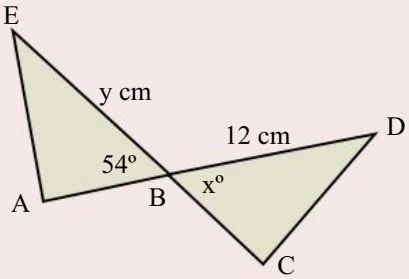
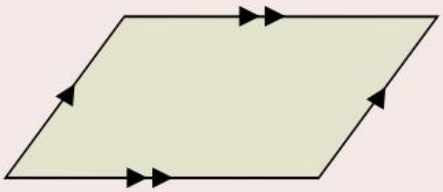


Parallelogram

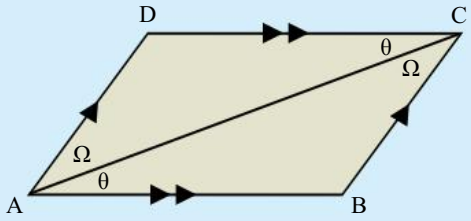
Parallelogram
A quadrilateral with opposite sides parallel.

Opposite sides are equal.
Opposite angles are equal.
The diagonals bisect each other.



Exercise 19.7

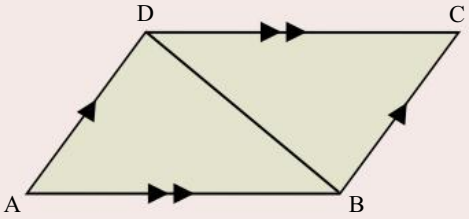
Prove that the diagonal AC cuts the parallelogram into two congruent triangles.



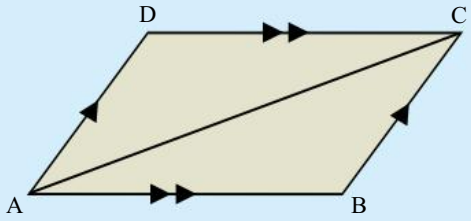
$\angle DCA = \angle BAC$ {DC // AB}
$CA = AC$ {same side}
$\angle CAD = \angle ACB$ {AD // BC}

$\therefore \triangle ADC \equiv \triangle CBA$ {ASA}

1 Prove that the diagonal DB cuts the parallelogram into two congruent triangles.



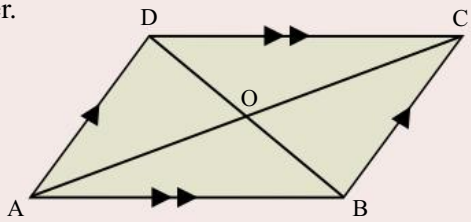
Prove that the opposite angles, $\angle ADC$ and $\angle CBA$, are equal.



$\triangle ADC \equiv \triangle CBA$ {ASA}
$\therefore \angle ADC = \angle CBA$ {matching angles}

2 Prove that the opposite sides, AD and CB are equal.
3 Prove that the opposite sides, DC and BA are equal.
4 Prove that the diagonals bisect each other.

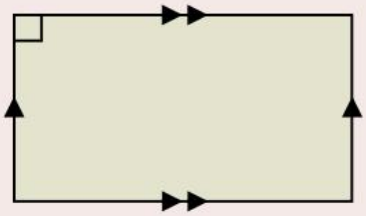
Hint:
First prove AO = OC.
Then prove DO = OB.
The diagonals must then bisect each other.



Rectangle

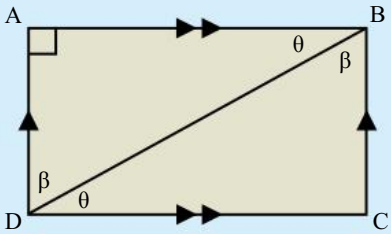
Rectangle
A parallelogram with a right-angle.

Opposite sides are equal.
All angles are right-angles.
The diagonals are equal.
The diagonals bisect each other.



Exercise 19.8

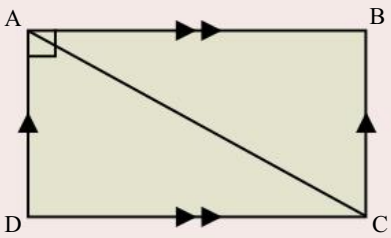
Prove that the diagonal BD cuts the rectangle into two congruent triangles.



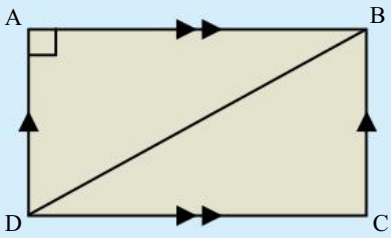
$\angle ABD = \angle CDB$ {AB // CD}
$BD = DB$ {same side}
$\angle BDA = \angle DBC$ {AD // CB}

$\therefore \triangle ABD \equiv \triangle CDB$ {ASA}

1 Prove that the diagonal AC cuts the rectangle into two congruent triangles.

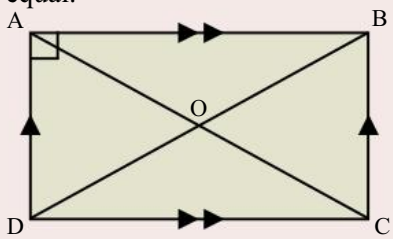


Prove that $\angle BCD$ is a right-angle.



$\triangle DAB \equiv \triangle BCD$ {ASA}
$\therefore 90° = \angle BCD$ {matching angles}

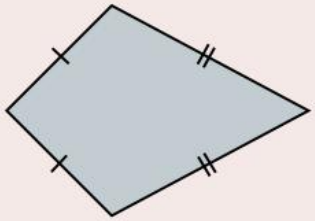
2 Prove that the opposite sides, DA and BC are equal.
3 Prove that the opposite sides, AB and CD are equal.
4 Prove that the diagonals bisect each other.



Kite

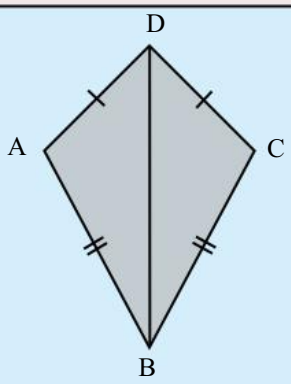
Kite
A quadrilateral with two pairs of adjacent sides equal.

Opposite angles are equal.
The longer diagonal bisects the shorter diagonal.
Diagonals intersect at right-angles.



Exercise 19.9

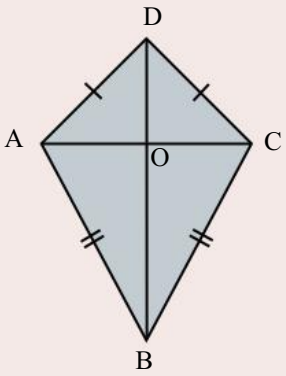
Prove that the diagonal DB cuts the kite into two congruent triangles.



AD = CD {property of kite}
BA = BC {property of kite}
DB = DB {same side}

$\therefore$ $\triangle BAD \equiv \triangle BCD$ {SSS}

1 Prove that $\angle BAD = \angle BCD$.



2 Prove that the longer diagonal bisects the shorter diagonal.
a) Prove that $\triangle AOD \equiv \triangle COD$.
b) Prove that AO = CO.

3 Prove that the diagonals intersect at right angles.
a) Prove that $\triangle AOD \equiv \triangle COD$.
b) Prove that $\angle AOD = \angle COD$.
c) $\angle AOD + \angle COD = 180°$ $\therefore$?

Building Surveyors assess building plans and interpret building laws to enforce building construction and safety regulations.
- Relevant school subjects are Mathematics and English.
- Courses range from Diplomas to University Degrees.

Mental Computation

Exercise 19.10

1 Spell Congruent.
2 What does SSS mean?
3 What is the symbol for congruent?
4 Add 10% to $40.
5 Convert 4 000 cm to m
6 $y = 3x + 1$. Find y when $x = ^{-}2$
7 Two coins are tossed. What is P(HH)?
8 In a class of 20 students. 15 play tennis and 13 play football. How many play tennis and football?
9 Solve: $4x - 5 = 1$
10 Simplify: $a^5 \times a^2$

You need to be a good mental athlete because many everyday problems are solved mentally

Exercise 19.11

1 Spell Parallelogram.
2 What does ASA mean?
3 What is the symbol for congruent?
4 Add 10% to $50.
5 Convert 5 000 m to km
6 $y = 3x + 1$. Find y when $x = ^{-}1$
7 Two coins are tossed. What is P(TT)?
8 In a class of 25 students. 14 play tennis and 13 play football. How many play tennis and football?
9 Solve: $2x + 5 = 8$
10 Simplify: $b^5 \div b^2$

A proof is evidence for the truth of a statement.

Exercise 19.12

1 Spell Rectangle.
2 What does SAS mean?
3 What is the symbol for congruent?
4 Add 10% to $60.
5 Convert 5 000 cm to m
6 $y = 2x + 5$. Find y when $x = ^{-}3$
7 Two coins are tossed. What is P(TandH)?
8 In a class of 20 students. 12 play tennis and 11 play football. How many play tennis and football?
9 Solve: $3x - 2 = 4$
10 Simplify: $(a^4)^2$

All humans are mortal.
Socrates is a human.
Socrates is mortal.

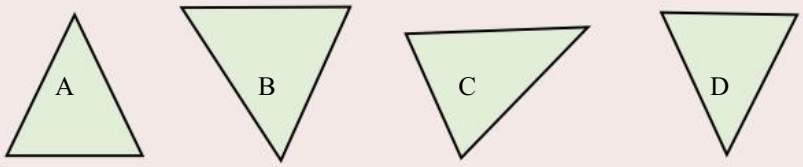
Thus deductive reasoning has advanced mathematics and consequently our human society.

NAPLAN Questions

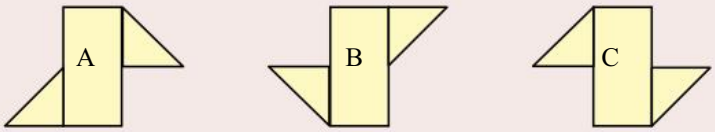
Exercise 19.13

1 Which pair of the following shapes are congruent?

a)



b)



2 Which statements are true?

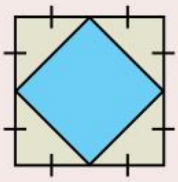
a) The diagonals of a rectangle are equal.

b) The diagonals of a parallelogram bisect each other.

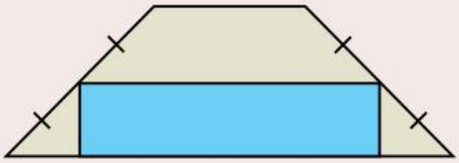
c) A kite has two pairs of equal sides.

3 A quadrilateral has one pair of parallel sides.
What is the name of the quadrilateral?

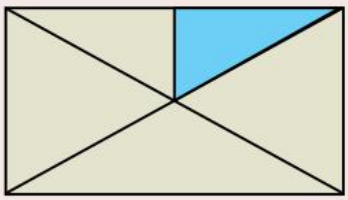
4 What fraction of the square is shaded blue?
The blue shape is a square.



5 What fraction of the trapezium has been shaded blue?
The blue shape is a rectangle.



6 What fraction of the following rectangle has been shaded blue?
The blue triangle is a right-angled triangle.



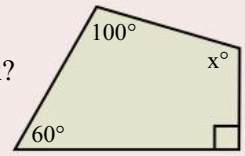
7 SSS, SAS, AAS, and RHS are tests for congruent triangles.
What would be the test for congruent circles?

Competition Questions

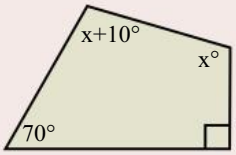
Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 19.14

1 What is the value of x?



2 What is the value of x?

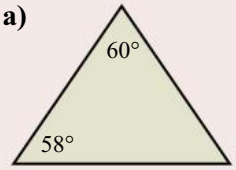


Quadrilateral
Sum of internal angles = 360°

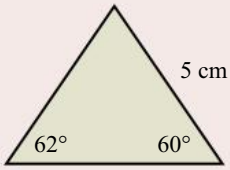
3 Does the diagonal divide the rectangle, the parallelogram, the rhombus, and the kite into two congruent shapes?

4 Which of the following triangles are congruent?

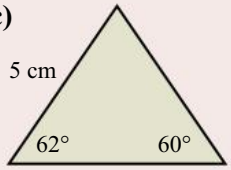
a)



b)



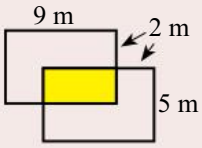
c)



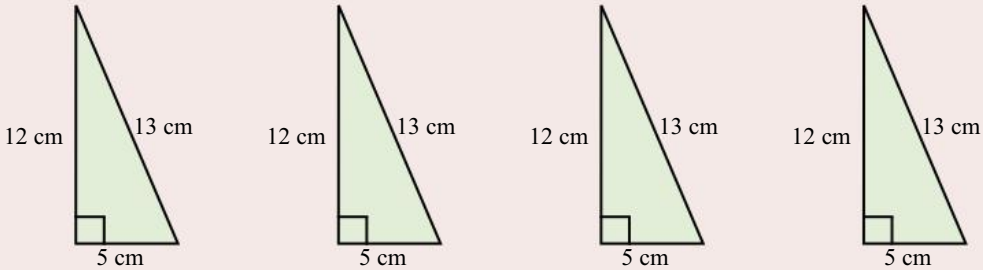
5 Three vertices of a rectangle are (−4,3), (2,3), and (−4,−2). What are the coordinates of the fourth vertex?

6 Three vertices of a parallelogram are (−4,3), (2,3), and (−4,−2). Name three possible coordinates of the fourth vertex.

7 What is the area of the intersection of the two congruent rectangles?



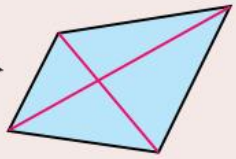
8 Rearrange the four congruent triangles to form a square with a hollow square in the middle. What is the area of the middle square?



Investigations

Investigation 19.1 How many diagonals?

A quadrilateral has two diagonals.



How many diagonals in a pentagon?

How many diagonals in a hexagon?

No of sides	4	5	6	7	8	9	10		100
No of diagonals	2								

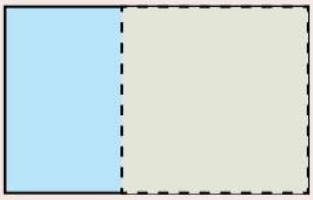
Investigation 19.2 Golden Rectangle

It is believed that the Golden Ratio produces the most pleasing shape.

This rectangle is a Golden Rectangle. Does it look good?

Any rectangle which has the ratio 1 : 1.618 is called a **Golden Rectangle**.

Have a go.

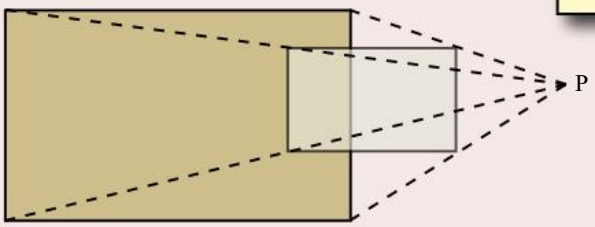


Start with a Golden Rectangle.
Repeat
Take a square and you still have a Golden Rectangle
Until nothing left?

Investigation 19.3 Rectangle Reduction

a) Draw a rectangle.
b) Place a point, P, outside the rectangle.
c) Draw lines from the vertices to P.
d) Join the midpoints of the lines.

Investigate:
- The size of the new rectangle?
- P in different places?
- 0.25 length of lines?
- Start with a parallelogram?



Technology

Technology 19.1 Geometry Software

There are a considerable number of Geometry Software applications available through the Internet.

1 Conduct a search for Geometry Software

2 Write a brief review of a selected three applications:

- **a)** Price.
- **b)** Suitability for the work in this chapter.
- **c)** Ease of use.

3 Which appears to be the better Geometry application.

4 Test it.

5 Present the software to an audience and lead them through some exercises.

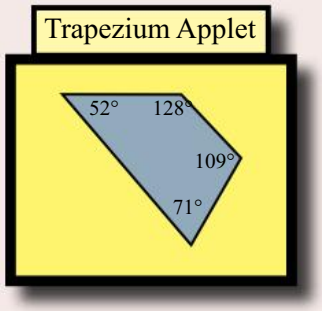
Technology 19.2 Shape Applets

1 Search the Internet for "drawing math shapes".

2 Experiment with the variety of applets that let you explore triangles, parallelograms, rectangles, trapeziums, kites etc.

3 Note if they also allow you to experiment with congruent shapes.

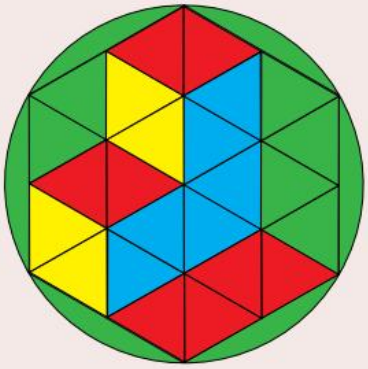
4 If you are impressed with one of the sites, present the site to an audience and lead them through some exercises.



Technology 19.3 Geometric Patterns

Use Geometry Software to draw a spectacular geometric pattern.

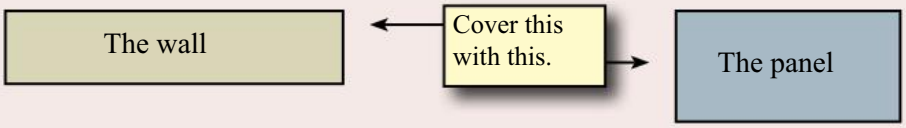
- Search the Internet for ideas.
- Don't overlook the use of standard drawing software.



A Couple of Puzzles

Exercise 19.15

1 A builder has an exterior wall 9 m by 2 m that needs to be covered with a 6 m by 3 m wall cladding panel. What is the best way to cut the panel?



2 Fred is 30 years older than Ted. Their total age is 31 years. How old is Ted (The answer is not 1)?

A Game

Combination.

The winner is the person with the largest total (two to five players).

1 Copy the table.

2 When it is your turn, throw a dice four times.

3 Then try to use (), +, –, x, ÷, or ^ to equal one of the uncrossed numbers in the table.

4 If successful, cross out the number and add it to your total.

5 It is then the next person's turn.

4, 1, 5, 1
(4x5)^(1+1) = 400

Which numbers will be the hardest to get?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
60	70	80	90	100	150	200	250	300	400

A Sweet Trick

Show your audience copies of the following sums and ask them to decide which sum is the bigger?

987654321	123456789
087654321	123456780
007654321	123456700
000654321	123456000
000054321	123450000
000004321	123400000
000000321	123000000
000000021	120000000
+ 000000001	+ 100000000

Incredibly, the two totals are the same.

Sum = 1 083 676 269

Chapter Review 1

Exercise 19.16

1 Using a ruler, a protractor, and a pair of compasses, construct the following triangles (leave all construction marks):

a) three sides SSS; 8 cm, 6 cm, and 6 cm.

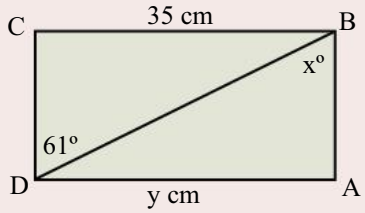
b) two sides and included angle SAS; 7 cm, 45°, and 9 cm.

c) angle, side, angle ASA; 60°, 8 cm, 30°.

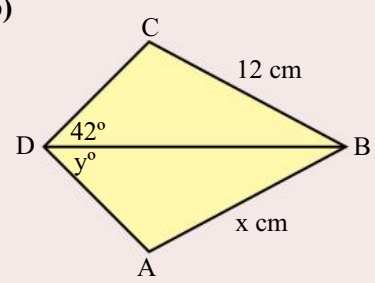
d) right-angle, hypotenuse, side RHS; 90°, 10 cm, 6 cm.

2 Find the unknowns (each pair of triangles are congruent):

a)



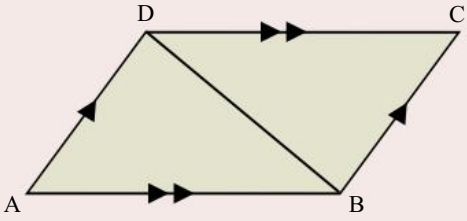
b)



3 For the given parallelogram ABCD:

a) Prove that the diagonal DB cuts the parallelogram into two congruent triangles.

b) Prove that the opposite sides, AD and CB are equal.

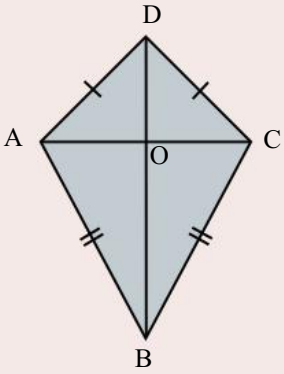


4 For the given kite ABCD:

a) Prove that the diagonal DB cuts the kite into two congruent triangles.

b) Prove that the longer diagonal bisects the shorter diagonal.

c) Prove that the diagonals intersect at right angles.



Chapter Review 2

Exercise 19.17

1 Using a ruler, a protractor, and a pair of compasses, construct the following triangles (leave all construction marks):

a) three sides SSS; 5 cm, 8 cm, and 6 cm.

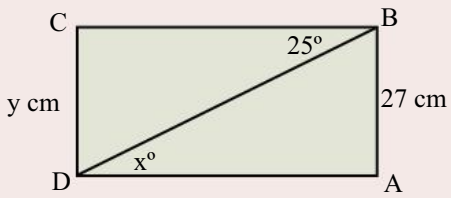
b) two sides and included angle SAS; 7 cm, 60°, and 7 cm.

c) angle, side, angle ASA; 45°, 6 cm, 90°.

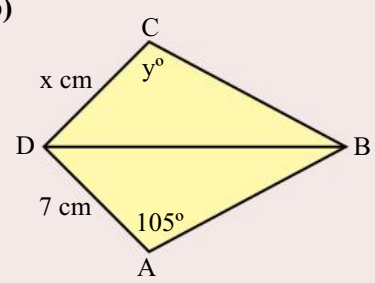
d) right-angle, hypotenuse, side RHS; 90°, 10 cm, 8 cm.

2 Find the unknowns (each pair of triangles are congruent):

a)



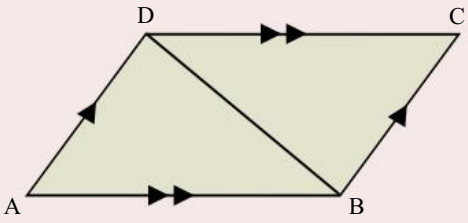
b)



3 For the given parallelogram ABCD:

a) Prove that the diagonal DB cuts the parallelogram into two congruent triangles.

b) Prove that the opposite sides, AD and BC are equal.

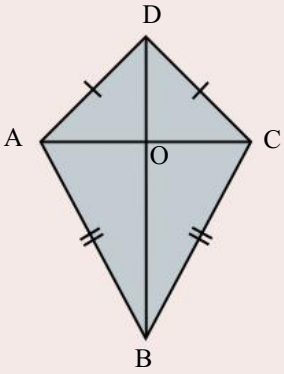


4 For the given kite ABCD:

a) Prove that the diagonal DB cuts the kite into two congruent triangles.

b) Prove that the longer diagonal bisects the shorter diagonal.

c) Prove that the diagonals intersect at right angles.



Confucius:
I hear, I know.
I see, I remember.
I do, I understand.

Review 4

20

Chapter 16 Linear Relationships

Age of car in years (x)	1	2	3	4
Yearly repair cost ($Y)	320	600	880	1160

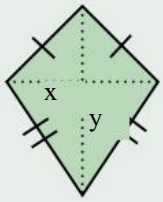
Each step increases by 280: $Y = 280x
The first step is 280+40 $Y = 280x + 40

A linear function increases by the same amount each step.

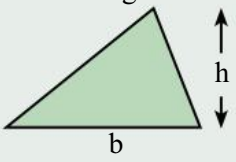
A non-linear function **does not** increase by the same amount each step.

Chapter 17 Measurement

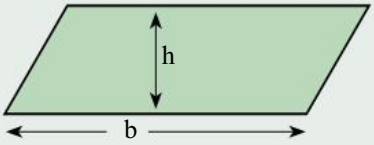
Perimeter is the distance around the outside of a shape.
Volume of prism = Area of base × Height.

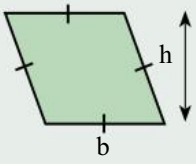


Area of square = s^2
Area of rectangle = lxb
Area of triangle = ½bh



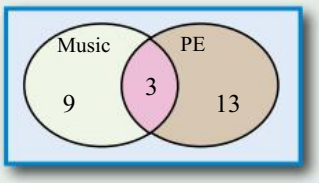
Area of parallelogram = bh
Area of rhombus = bh
Area of kite = ½xy





Chapter 18 Probability

In a class of 25 students, 12 students study music, and 16 students study PE.
12 + 16 = 28 which is 3 more than 25
Thus 3 study music and PE



In a Two-Way Table.
One event on one side.
Another event on the other side.

Chapter 19 Congruence

Parallelogram
A quadrilateral with opposite sides parallel.
- Opposite sides are equal.
- Opposite angles are equal.
- The diagonals bisect each other.

Rectangle
A parallelogram with a right-angle.
- Opposite sides are equal.
- All angles are right-angles.
- The diagonals are equal.
- The diagonals bisect each other.

Kite
A quadrilateral with two pairs of adjacent sides equal.
- Opposite angles are equal.
- The longer diagonal bisects the shorter diagonal.
- Diagonals intersect at right-angles.

Review 1

Exercise 20.1 Mental Computation

4 = 2×2
To multiply by 4, double twice.
15×4 = 30×2 = 60

1 Spell Congruent.
2 What does SSS mean?
3 What is the symbol for congruent?
4 15x4.
5 Convert 70 000 cm to m
6 y = 5x + 1. Find y when x = ¯2
7 Two coins are tossed. What is P(HH)?
8 In a class of 24 students. 15 play tennis and 13 play football.
How many play tennis and football?
9 Solve: 2x − 7 = 1
10 Simplify: $a^6 \times a^2$

Exercise 20.2

1 Plot the following points on a set of Cartesian axes:
a) A(1,−3) **b)** B(−2,3) **c)** C(−2, −4) **d)** D(0, 0)

2

Weeks (w)	1	2	3	4
Bank balance ($S)	200	250	300	350

a) Plot the points and write a mathematical model.
b) How much in the bank after 8 weeks?
c) How long will it take to have a balance of $750?

3

x	-2	-1	0	1	2
y = 2x + 1					

a) Copy and complete the above table.
b) Draw a graph of the function.
c) From the graph, when y = 4, what is x?

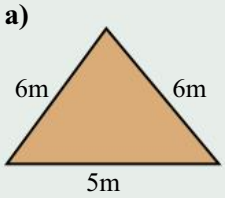
What do all ships weigh regardless of size?

Anchor.

4 Convert:
a) 7.2 metres to centimetres. **b)** 14.2 kilometres to metres.
c) 650 metres to kilometres. **d)** 981 centimetres to metres.
e) 7 000 000 centimetres to kilometres.
f) 20 000 centimetres to kilometres.

5 Calculate the perimeter of each of the following shapes:

a)



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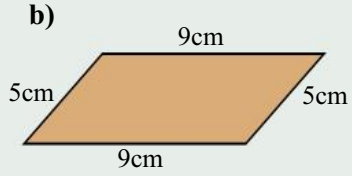
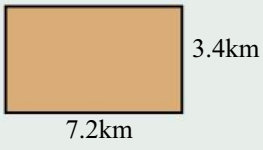
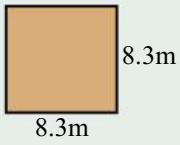


c)

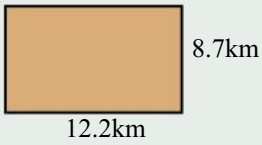


6 Calculate the area of each of the following shapes:

a)



b)



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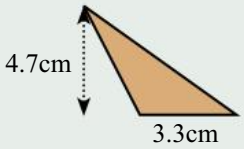
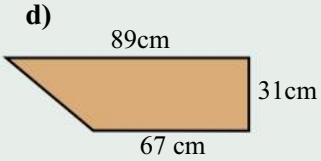
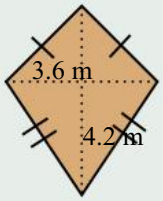


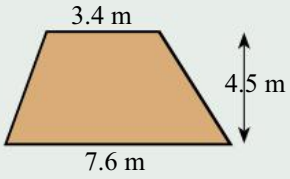
d)



e)

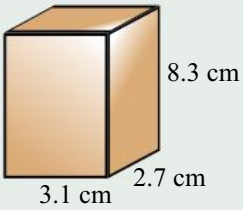


f)

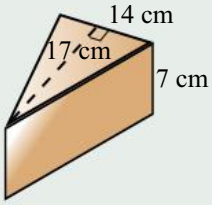


7 Find the volume of each of the following prisms:

a)

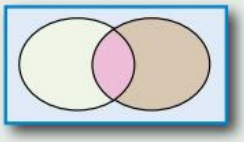


b)



8 In a class of 26 students, 11 students play an instrument, 8 students have a job, and 5 students play an instrument and have a job. Draw a Venn Diagram and find the probability that a student:

a) plays an instrument and has a job.
b) plays an instrument or has a job.
c) does not play an instrument.
d) does not have a job.



9 Assuming that the chances of a girl or boy being born is equal, use a Two-Way Table to determine the theoretical probabilities for a family of two children:

a) P(2 girls).
b) P(1 girl and1 boy).
c) P(2 boys).

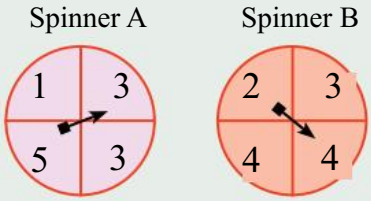
10 A coin is tossed and a die is rolled. Use a Two-Way Table to calculate the probability of obtaining:

a) a tail and a 4.
b) a head and a 5.
c) a head and an even number.

Die \ Coin	H	T
1		
2		
3		
4		
5		
6		

11 Spinner A and Spinner B are spun to produce a two digit number.
The spinners are showing the two digit number 34.
Use a Two-Way Table to calculate the probabilities of obtaining:

a) a two-digit number less than 20.
b) a two-digit number greater than 30.
c) a two-digit number divisible by 5.
d) a two-digit number divisible by 3.



Spinner B

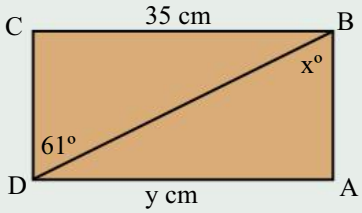
Spinner A	2	3	4	4
1				
3			34	
3				
5				

12 Using a ruler, a protractor, and a pair of compasses, construct the following triangles (leave all construction marks):

a) three sides SSS; 8 cm, 6 cm, and 6 cm.
b) two sides and included angle SAS; 7 cm, 45°, and 9 cm.
c) angle, side, angle ASA; 60°, 8 cm, 30°.
d) right-angle, hypotenuse, side RHS; 90°, 10 cm, 6 cm.

13 Find the unknowns (each pair of triangles are congruent):

a)



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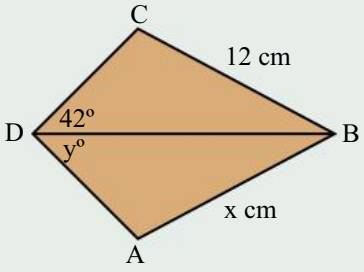
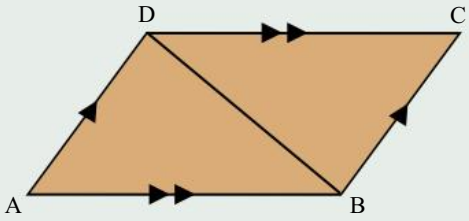


14 For the given parallelogram ABCD:

a) Prove that the diagonal DB cuts the parallelogram into two congruent triangles.
b) Prove that the opposite sides, AD and BC are equal.
c) Prove that the opposite angles are equal.
d) Prove that the diagonals bisect each other.



Review 2

Exercise 20.3 Mental Computation

4 = 2×2
To multiply by 4, double twice.
16×4 = 32×2 = 64

1 Spell Probability.
2 What does SAS mean?
3 What is the symbol for congruent?
4 16x4.
5 Convert 7000 cm to m
6 y = 4x + 1. Find y when x = ¯2
7 Two coins are tossed. What is P(TT)
8 In a class of 25 students. 15 play tennis and 13 play football.
How many play tennis and football?
9 Solve: 2x + 5 = 1
10 Simplify: $a^6 \times a^2$

Exercise 20.4

1 Plot the following points on a set of Cartesian axes:
a) A(2,−1) **b)** B(−2,1) **c)** C(−2, −1) **d)** D(0, 0)

2

Weeks (w)	1	2	3	4
Bank balance ($S)	150	250	350	450

a) Plot the points and write a mathematical model.
b) How much in the bank after 6 weeks?
c) How long will it take to have a balance of $850?

3

x	-2	-1	0	1	2
y = 3x + 1					

a) Copy and complete the following table.
b) Draw a graph of the function.
c) From the graph, when y = 7, what is x?

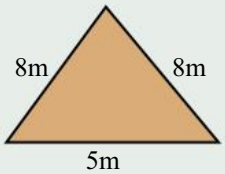
What part of a clock is always old?

The second hand.

4 Convert:
a) 4.5 metres to centimetres. **b)** 24.23 kilometres to metres.
c) 7650 metres to kilometres. **d)** 2340 centimetres to metres.
e) 3 000 000 centimetres to kilometres.
f) 50 000 centimetres to kilometres.

5 Calculate the perimeter of each of the following shapes:

a)



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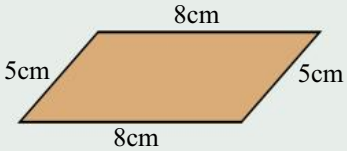
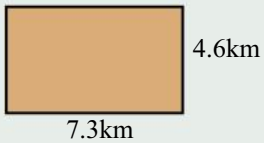
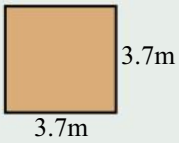


c)



6 Calculate the area of each of the following shapes:

a)



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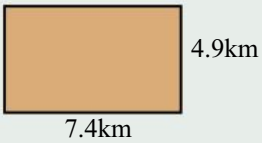
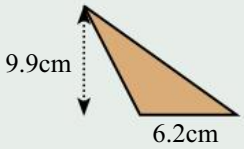
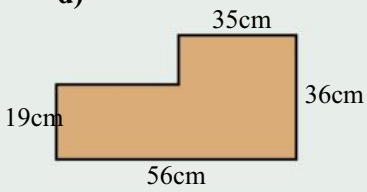


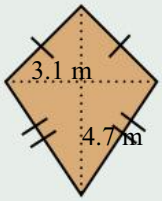
c)



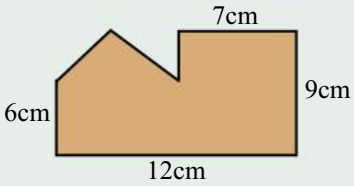
d)



e)

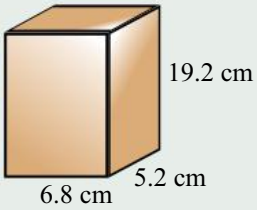


f)

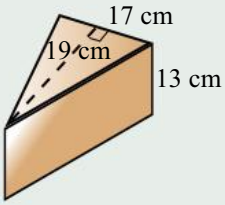


7 Find the volume of each of the following prisms:

a)

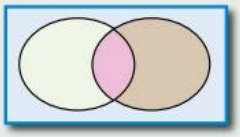


b)



8 In a class of 25 students, 10 students study music, 12 students study PE, and 6 students study music and study PE. Draw a Venn Diagram and find the probability that a student:

a) studies music and PE.
b) studies music or PE.
c) does not study music.
d) does not study PE.



9 Assuming that the chances of a girl or boy being born is equal, use a Two-Way Table to determine the theoretical probabilities for a family of two children:

a) P(2 girls).
b) P(1 girl and 1 boy).
c) P(2 boys).

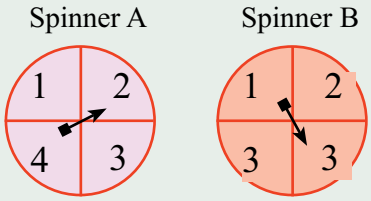
10 A coin is tossed and a die is rolled. Use a Two-Way Table to calculate the probability of obtaining:

a) a tail and a 4.
b) a head and a 1.
c) a tail and an odd number.

Die \ Coin	H	T
1		
2		
3		
4		
5		
6		

11 Spinner A and Spinner B are spun to produce a two digit number.
The spinners are showing the two digit number 23.
Use a Two-Way Table to calculate the probabilities of obtaining:

a) a two-digit number less than 30.
b) a two-digit number greater than 30.
c) a two-digit number divisible by 5.
d) a two-digit number divisible by 3.



Spinner B

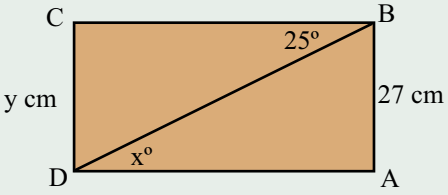
Spinner A	1	2	3	3
1				
2			23	
3				
4				

12 Using a ruler, a protractor, and a pair of compasses, construct the following triangles (leave all construction marks):

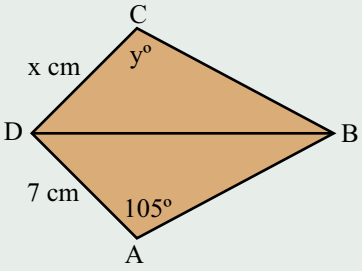
a) three sides SSS; 8 cm, 6 cm, and 6 cm.
b) two sides and included angle SAS; 7 cm, 45°, and 9 cm.
c) angle, side, angle ASA; 60°, 8 cm, 30°.
d) right-angle, hypotenuse, side RHS; 90°, 10 cm, 6 cm.

13 Find the unknowns (each pair of triangles are congruent):

a)



b)



14 For the given parallelogram ABCD:

a) Prove that the diagonal DB cuts the parallelogram into two congruent triangles.
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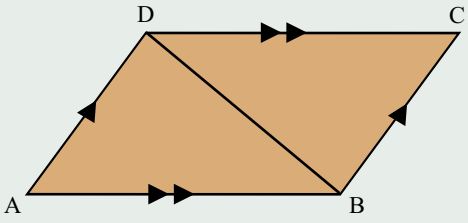
c) Prove that the diagonals bisect each other.



THE GOLDEN RECTANGLE

The Golden Rectangle is said to have the proportions,
neither too long nor too square, that are the most pleasing to the eye.

When the length is divided by the width the ratio is about 1.6.

Actually the Golden Ratio is defined as:

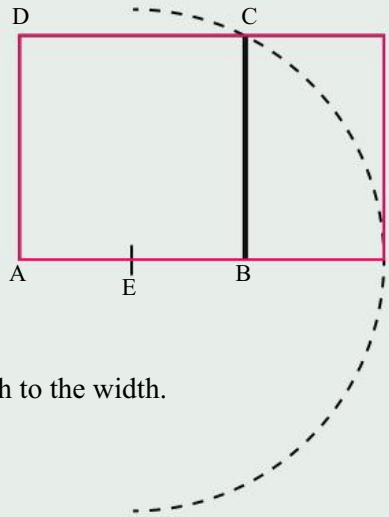
$$\frac{\text{width}}{\text{length}} = \frac{\text{length}}{(\text{length} + \text{width})}$$

$$\text{length} = \frac{\text{width}(1+\sqrt{5})}{2}$$

The Golden Ratio is 1.618034.....

This rectangle is a Golden Rectangle.

How to construct a Golden Rectangle:



1 Make a square: ABCD

2 Find the midpoint E of AB

3 Set the radius of a compass to EC

4 Draw an arc with centre E as shown

5 Complete the Golden Rectangle.
Check by calculating the ratio of the length to the width.

Golden Rectangle Experiment
Make a variety of rectangles and ask people to choose which rectangle has the most pleasing shape.

Rectangle	Length	Width	Length ÷ Width
An A4 sheet of paper			
Photograph frame			
Picture frame			
Other rectangles			

Glossary

Acute − An acute angle is a sharp angle between 0° and 90°.

Angle − An angle is the measure of turn between two lines. Angles are measured in degrees from 0° to 360°, eg. 147°. In later studies other measures such as radians will be introduced.

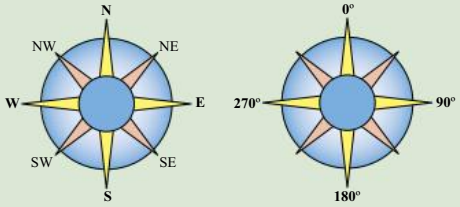
Angle sum of a polygon −
The sum of the interior angles of a triangle is 180°.
The sum of the interior angles of a quadrilateral is 360°.
The sum of the interior angles of a pentagon is 540°.
The general rule: Sum interior angles = (no sides − 2)x 180°

Ascending order of numbers is an order from smallest to largest.
Example: 2, 3, 5, 10 is in ascending order.

Average − An average is a central measure. Average and mean are the same.
(The mode, and median, although different, are also central measures).

Area − The area is the amount of surface.
Area of rectangle = length x breadth
Area of triangle = 1/2 x base x height
Area of circle = π x radius^2 = πr^2
Area of parallelogram = base x height

Bearing − The bearing is the angle measured clockwise from North.
The bearing of North is 0°.
The bearing of East is 90°.
The bearing of South is 180°.
The bearing of South West is 225°.
The bearing of West is 270°.



Bias − Unfair sampling. The sample does not represent the population.

Census − Collection of data from the entire population.

Centimetre − A centimetre is one-hundredth of a metre. 100 cm = 1 m.

Circumference − The circumference is the distance around the outside of a circle.
C = 2πr or C = πd

Complementary angles are angles that sum to 90°.
Example: 40° and 50° are complementary angles.

Complementary events - The complement of any event (A) is the event (not A). The probabilities of complementary events add to 1.

Composite number has more than two factors.
Example: 8 has factors of 1, 2, 4, 8. 8 is a composite number.

Compound stem-and-leaf plot − A compound stem-and-leaf plot has two stem-and-leaf plots joined together.

Congruent objects have the same shape and the same size.

The symbol for congruence is ≡ or ≅
The tests of congruent triangles are:
SSS (side, side, side).
SAS (side, angle, side).
ASA (angle, side, angle).
RHS (right-angle, hypotenuse, side).

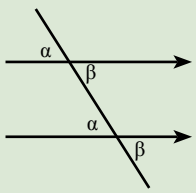
Consecutive numbers are numbers that follow one another.
Example: 3, 4, 5 are consecutive numbers.

Continuous numbers are numbers that can have any value. Weight is continuous because the weight of an object can be any number on the number line. Discrete numbers can have only certain values - the number of people in the class must be a whole number. There can't be 4.62 people in the class.

Compound Interest - Compound interest arises when interest is added to the principal. The interest that has been added also earns interest. This addition of interest to the principal is called compounding. Eg A bank account may have its interest compounded every year: in this case, an account with $1000 initial principal and 20% interest per year would have a balance of $1200 at the end of the first year, $1440 at the end of the second year, and so on.

Coordinates − An ordered pair on numbers that fix a point in the cartesian plane.
Example: P(2,5). The point P is 2 units to the right and 5 units up from the origin (0,0).

Corresponding angles − matching angles when a line cuts a pair of lines.
If the lines are parallel, the corresponding angles are equal.



α is one pair of corresponding angles.
β is another pair of corresponding angles.

Cube − A cube is a three-dimensional object with all six faces congruent and each face having the shape of a square. A cube is one of the five platonic solids.

Cubed − A cubed number is the number multiplied by itself three times.
Example: Two cubed = $2^3 = 8$.

Cubic centimetre is the amount of space occupied by a cube with each side of length 1 cm. The unit is 1 cm^3 (1 L = 1 000 cm^3).

Cubic metre is the amount of space occupied by a cube with each side of length 1 m. The unit is 1 m^3 (1 m^3 = 1 000 L).

Data - Information collected for analysis or reference.

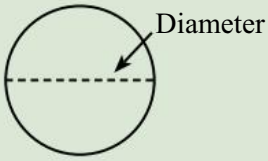
Decagon is a polygon with 10 sides and 10 angles.

Decimal place − The number of places after the decimal point.
Example: 5.281 has three decimal places.

Denominator in a fraction is the number at the bottom.

Descending order of numbers is an order from largest to smallest.
Example: 10, 5, 3, 2 is in descending order.

Diameter − The diameter of a circle is the length of the line joining two points on the circle and that passes through the centre of the circle.



Die − A die is a cube with each of the numbers 1, 2, 3, 4, 5, 6 on each of the six faces.
The opposite sides of a die sum to seven.
Die is singular, dice is plural.

Digit − A digit is a single number.
Example: The number 435 has the digits 4, 3, and 5.

Discrete numbers are numbers that can only have certain values, normally whole numbers.
Example: The number of people in the class is discrete (Can't be 4.62 people).

Distributive law − Each term in the brackets is multiplied by the term outside the brackets. Example: $3(a + 5) = 3a + 15$

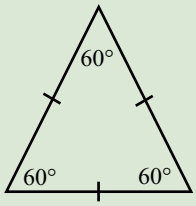
Dividend − The dividend is the number being divided. In 45 ÷7, 45 is the dividend.

Divisor − The divisor is the number dividing. In 45 ÷7, 7 is the divisor.

Dodecahedron − A dodecahedron is a three-dimensional object with all twelve faces congruent and each face having the shape of a regular pentagon (5 sides of equal length). A dodecahedron is one of the five platonic solids.

Equation − An equation is a mathematical sentence with an equals sign.
Example: 2x + 5 = 9 is an equation.

An **equilateral triangle** is a triangle with three equal sides.
Each of the three angles in an equilateral triangle are 60°.



Estimate − To make an approximate guess of the answer.
Example: An estimate of 43 x 26 is 40 x 30 = 1200

Evaluate − To evaluate an expression is to find the value of the expression.
Example: Evaluate 2x(3−1)
2x(3−1) = 2x2 = 4

Even numbers are numbers that are exactly divisible by 2.
Example: 2, 4, 6, 8, 10 are even numbers.

Expand − Each term in the brackets is multiplied by the term outside the brackets.
Example: 3(a + 5) = 3a + 15

Factors − The factors of a number are the numbers which divide exactly into the number.
Example: The factors of 6 are 1, 2, 3, 6.

Factorise − To make into a product.
Example: 3a + 15 = 3(a + 3)

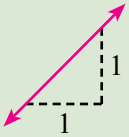
Finite − A definite number.
Example: {2, 3, 1, 6 } has a finite number of elements. It has 4 elements.
The opposite of finite is infinite.

Formula − A formula is an equation.
The formula for the perimeter of a circle is: $C = 2\pi r$

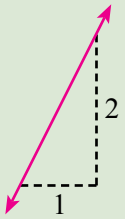
Frequency − The number of times a number occurs.
Example: 2, 3, 2, 3, 2, 2 The frequency of 2 is 4 {2 occurs 4 times}

Gram − A gram is a measure of mass and is one thousandth of a kilogram.
1 000 g = 1 kg

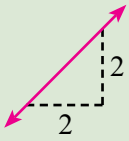
Gradient − A measure of the slope.
Example: The gradient of the line $y = 2x - 1$ is 2. $m = 2$



m=1



m=2



m=1

Greater than − The symbol for greater than is >
Example: 6 is greater than 4, 6 > 4

Heptagon − A polygon with seven sides and seven angles.

Hexagon − A hexagon is a polygon with six sides and six angles.

Highest common factor − The largest factor that is common.
Example: The highest common factor of 12 {1,2,3,4,6,12} and 8 {1,2,4,8} is 4.

Hypotenuse - The longest side in a right-angled triangle.
The hypotenuse is opposite the right-angle.

Icosahedron − An icosahedron is a three-dimensional object with all twenty faces congruent and each face having the shape of an equilateral triangle.
A icosahedron is one of the five platonic solids.

Improper fraction − An improper fraction or vulgar fraction is a fraction with the numerator larger than the denominator.
Example: $\frac{5}{3}$ is an improper fraction.

Index − The power when a number is written in index form.
Example: $9 = 3^2$. The index is 2.

Infinite − Too large/many to be counted. Not finite.

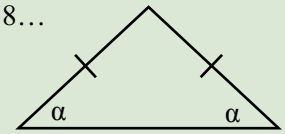
Integers are whole positive numbers and whole negative numbers.
Example: −2, 3, 4, −7 are integers. 3.4 is not an integer.

Intersection − The point where two lines cross each other. The common numbers in a set of two numbers.

Irrational number − An irrational number is a number that cannot be written as a common fraction or as a decimal fraction that terminates or recurs.

Example: π is irrational because it cannot be written as a decimal that terminates or recurs. $\pi = 3.14159265358\ldots$

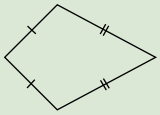
$\sqrt{2}$ is irrational = 1.41421356….



Isosceles − A triangle with two sides of equal lengths.

The angles opposite the equal sides are equal.

Kite − A quadrilateral with two pairs of adjacent sides equal.



Kilogram − A kilogram is a measure of mass. 1 kg = 1 000 g. 1 tonne = 1 000 kg.

Kilometre − A kilometre is a measure of length. 1 km = 1 000 m.

Latitude − The latitude of a position on Earth is the angle North or South of the Equator.

Adelaide's latitude is 34.55°S, Adelaide's longitude is 138.35°E.
Brisbane's latitude is 27.28°S, Brisbane's longitude is 153.01°E.
Canberra's latitude is 35.27°S, Canberra's longitude is 149.12°E.
Darwin's latitude is 12.28°S, Darwin's longitude is 130.50°E.
Hobart's latitude is 42.53°S, Hobart's longitude is 147.19°E.
Melbourne's latitude is 37.82°S, Melbourne's longitude is 144.95°E.
Perth's latitude is 31.95°S, Brisbane's longitude is 115.83°E.
Sydney's latitude is 33.52°S, Sydney's longitude is 151.13°E.

Litre − A litre is a measure of volume. 1 kg = 1 000 mL.

Longitude − The longitude of a position on Earth is the angle East or West of the line of meridian through Greenwich. See Latitude for examples.

Mean − The mean of a set of numbers is the sum of the numbers divided by the number of numbers. The mean of 2, 4, 5, 7 =

Median − The median of a set of numbers is the middle number when the numbers have been put in order.

Example: Find the median of: 4, 5, 2, 3, 6, 7, 2
In order: 2, 2, 3, 4, 5, 6, 7
The median is 4
Find the median of: 1, 3, 1, 0, 4, 3
In order: 0, 1, 1, 3, 3, 4
The median is the mean of 1 and 3 = 2.

Metre − The metre, m, is the standard measure of length.

Millimetre − A millimetre is one thousandth of a metre. 1 m = 1 000 mm.

Mixed number − A mixed number consists of a whole number and a fraction.

Example: $2\frac{3}{5}$

Mode − The mode of a set of numbers is the number that occurs the most.
Example: 2, 4, 3, 3, 5, 3. The mode is 3 (3 occurs three times).
1, 5, 4, 1, 5, 3. The mode is 1 and 5 (bimodal).

Net − The net of a solid is the shape that can be folded to make the solid.

Numerator − The numerator is the top number in a fraction.

Obtuse angle − An obtuse angle is an angle between 90° and 180°.

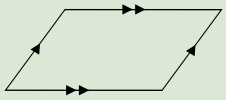
Octagon − An octagon is a polygon with eight sides and eight angles.

An **octahedron** is a three-dimensional object with all eight faces congruent and each face having the shape of an equilateral triangle. A octahedron is one of the five platonic solids.

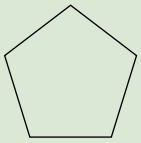
Odd numbers are numbers that are not exactly divisible by 2.
Example: 1, 3, 5, 7, 9 are odd numbers.

Obtuse angle − An angle greater than 90° and less than 180°.

Parallelogram − A parallelogram is a quadrilateral, four sided figure, in which the opposite sides are parallel.



Pentagon − A pentagon is a polygon with five sides and five angles.



Per annum − Per year.

Percentage − A percentage is a fraction of 100. $43\% = \frac{43}{100}$

Perimeter − The perimeter is the distance around the outside edge of a figure.

Perpendicular lines are lines that are at 90° to each other.

Pi, π, is the ratio of the circumference of a circle to the diameter.
$\pi = 3.14159265358\ldots$

Polygons are shapes made up of straight lines. Triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), hexagons (6 sides) etc are polygons.

Polyhedron − A solid shape with flat sides. Cube, dodecahedron, icosahedron, etc

Power − The power of a number is the number of times the number is multiplied by itself.
Example: $3x3x3x3x3 = 3^5$. {3 to the fifth power

Probability − The chance of an event happening.
Probability ranges from a low of 0 (no chance) to a high of 1 (certain).

$$P(event) = \frac{No\ of\ \ favourable\ outcomes}{Total\ no\ of\ \ outcomes}$$

Prime number − A prime number is a number with just two factors, 1 and itself.
2, 3, 5, 7, 11, 13 are prime numbers. 1 is not a prime number.

Prism − A prism is a three-dimensional shape in which the base shape is repeated from the bottom to the top.

Cylinder.
A circular based prism.

Probability is the chance of an event happening.
If a die is thrown, the chance of a 3 showing is

Quadratic − An equation in which the highest power of x is 2
Example: $y = 2x^2 - 5x + 3$

Quadrilateral − A quadrilateral is a figure with four straight lines.

Quartile − A value that divides the data in quarters.
Upper quartile, median, lower quartile.

Quotient − The quotient is the result of a division.
Example: The quotient of $10 \div 5$ is 2.

Radius − The radius of a circle is the distance from the centre of a circle to a point on the circle.

Range − The difference between the highest and the lowest value.

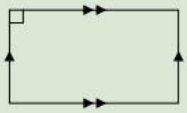
Ratio − A ratio is a comparison of two quantities. A certain two stroke petrol is made by mixing one part of two stroke oil to 32 parts of unleaded petrol (1: 32).

Rational number − A rational number is a number that can be written as a common fraction or as a decimal fraction that either terminates or recurs.

Example: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{-2}{3}$

Example: π is irrational because it cannot be written as a decimal fraction that terminates or recurs. $\pi = 3.14159265358\ldots$

Rectangle − A rectangle is a four sided figure in which the opposite sides are parallel and the internal angles are 90° (right-angles).



Rectangular prism − A rectangular prism is a prism in which the base is a rectangle.

Right-angle − A right-angle is 90°.

Rounding - Giving an approximation of a number using the nearest more convenient number is called rounding. When rounding to the nearest ten, 11, 12, 13 and 14, round to 10, whereas 15, 16, 17, 18 and 19 round to 20.

Similar figures have the same shape. Congruent figures have a the same shape and the same size.

Square − A square is figure with four equal sides and each internal angle of size 90°.

Square centimetre, cm^2, is the area occupied by a square with each side of length 1 cm.

Square metre, m^2, is the area occupied by a square with each side of length 1 m ($1\ m^2 = 10\ 000\ cm^2$).

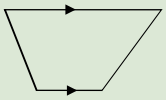
Symmetry - Property of regularity in shape by, for example, reflection or rotation. The letter T is symmetrical by reflection, the letter Z is symmetrical by rotation, the letter H is symmetrical by both reflection and rotation, the letter R is not symmetrical.

Surface area - The surface area of an object is the sum of the area of the various faces that make up the object.

Tonne − A tonne, t, is a measure of mass (1 t = 1 000 kg).

Transformation - A movement of figures and objects. The transformations translation (slide), rotation (turn) and reflection (flip) do not change the size or shape of the figure or object.

Trapezium − A trapezium is a four-sided figure with one pair of opposite sides parallel.



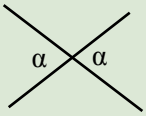
Triangle − A triangle is a figure with three sides.

Two-way table − A table that shows the sample space of a two-stage experiment.

Variables are letters used in equations, formulas, and expressions.
Example: x is a variable in the equation: $3x + 4 = 12$.

Vertex − The corner point of an angle.

Vertically opposite angles − A pair of non-adjacent angles formed when two lines intersect. Vertically opposite angles are equal.



Vinculum − The horizontal line separating the numerator from the denominator.

Volume of a figure is a measure of the amount of space occupied by the figure.
Example: The volume of a prism = area of base x height

Whole numbers are the positive counting numbers.
Example: 0, 1, 2, 3, 4, 5, etc.

x-intercept − The point where the graph cuts the x-axis.

y-intercept − The point where the graph cuts the y-axis.

Answers

	Square	Cube	Fourth	Fifth
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125

Exercise 1.1 **1** 1 **2** 4 **3** 9 **4** 16 **5** 25 **6** 1 **7** 8 **8** 27 **9** 64 **10** 125 **11** 1 **12** 16 **13** 81 **14** 256 **15** 625 **17** 100 **18** 1000 **19** 10 000 **20** 100 000 **21** 1 000 000 **22** 10 000 000 **23** 64 **24** 78125 **25** 1 **26** 256 **27** 6 **28** 1 000 000 000 **29** normally no **30** 1 000 000 000 000 **31** 500 000 000 000

Exercise 1.2 **1** 4^3 **2** 2^4 **3** a^3 **4** 10^3 **5** b^5 **6** h^3 **7** m^5 **8** 9^4 **9** 3^6

Exercise 1.3 **1** 4×4×4 **2** b×b×b×b **3** 5×5 **4** 2×2×2×2×2×2×2 **5** 6×6 **6** m×m×m×m×m **7** x×x×x×x **8** p×p×p×p×p **9** 1×1×1×1

Exercise 1.4 **1** $a^4 \times b^3$ **2** $2^2 \times 3^3$ **3** $a^5 \times b^4$ **4** $2^4 \times 3^3$ **5** $b^4 \times g^5$ **6** z^{11} **7** $p^5 \times q^3 \times r^4$ **8** $2^3 \times 3^2 \times 4^3$ **9** $4^4 \times g^6$

Exercise 1.5 **1** 2^5 **2** 3^5 **3** 2^6 **4** 4^7 **5** a^4 **6** b^5 **7** z^8 **8** w^7 **9** 2^4 **10** t^7 **11** 5^5 **12** 10^9 **13** 3^6 **14** 7^5 **15** d^5 **16** m^6 **17** c^7 **18** 10^6 **19** a^5 **20** b^6 **21** h^7 **22** z^5 **23** n^9 **24** 10^7 **25** 2^7 **26** 3^9 **27** a^8 **28** u^7 **29** 2^7 **30** p^7 **31** d^9 **32** m^9 **33** c^8 **34** y^8 **35** a^7 **36** b^8 **37** h^7 **38** z^9 **39** b^8 **40** z^9

Exercise 1.6 **1** 2^2 **2** 3 **3** 2^2 **4** 4 **5** a^4 **6** b^5 **7** 10^2 **8** w **9** 2^2 **10** t^3 **11** 5^2 **12** 10^3 **13** 3^2 **14** 7 **15** 10^3 **16** 3 **17** a **18** u^2 **19** 2^6 **20** 1 **21** m^2 **22** e^3 **23** c^3 **24** 10^2 **25** a **26** 3^3

Exercise 1.7 **1** 1^3 **2** 2^4 **3** 3^4 **4** 4^5 **5** 5^2 **6** 1^5 **7** 2^3 **8** 3^3 **9** 10^2 **10** 10^3 **11** 10^4 **12** b^6 **13** n^7 **14** p^5 **15** a^6

Exercise 1.8 **1** 2×2×2 **2** d×d×d×d **3** 6×6 **4** 2×2×2×2×2 **5** g×g×g×g **6** 1×1×1 **7** 2×2×2×2×2×2×2×2 **8** h×h×h×h×h×h×h×h **9** 1×1×1×1

Exercise 1.9 **1** 2^5 **2** 3^5 **3** 5^6 **4** 2^7 **5** a^5 **6** b^4 **7** k^8 **8** p^6 **9** 10^4 **10** 10^5 **11** 1^5 **12** h^9

Exercise 1.10 **1** 2 **2** 3^3 **3** 5^2 **4** 4^2 **5** c^4 **6** b^6 **7** 10^2 **8** w^2 **9** 2^2 **10** 10^6 **11** f^2 **12** z

Exercise 1.11 **1** 2^6 **2** 2^9 **3** 3^6 **4** 4^6 **5** d^4 **6** b^{12} **7** t^8 **8** n^{10} **9** 5^4 **10** s^{10} **11** m^6 **12** 3^8 **13** 5^9 **14** 10^6 **15** 10^3 **16** g^{10} **17** 10^6 **18** h^{12} **19** d^8 **20** 2^{15} **21** 2^8 **22** 2^{11} **23** 3^9 **24** 4^{10} **25** d^7 **26** b^{16} **27** t^{13} **28** n^{13} **29** 5^5 **30** s^{12} **31** m^7 **32** 3^9

Exercise 1.12 **1** 1 **2** 1 **3** 1 **4** 1 **5** 1 **6** 1 **7** 1 **8** 1 **9** 1 **10** 1 **11** 1 **12** 1 **13** 5 **14** 3 **15** 6 **16** 2 **17** 7 **18** 3 **19** 9 **20** 8 **21** 4 **22** 5 **23** 2 **24** 5 **25** d^4 **26** b^6 **27** t^8 **28** 5 **29** 5 **30** s^2 **31** m **32** 3

Exercise 1.13 **2** 3^5 **3** 2×2×2 **4** m^5 **5** 2^3 **6** 1 **7** 0.5 **8** 1/4 **9** 13/5 **10** $3.60

Exercise 1.14 **2** 2^4 **3** 4×4×4 **4** d^5 **5** 3^3 **6** 1 **7** 0.25 **8** 3/4 **9** 11/3 **10** $2.80

Exercise 1.15 **2** 5^5 **3** 3×3×3×3 **4** a^6 **5** 5^2 **6** 1 **7** 0.75 **8** 1/2 **9** 17/4 **10** $4.40

Exercise 1.16 **1a** 4270 **1b** 4 270 000 **1c** 4 270 000 000 **2** 2a **3** b **4a** b=4 **4b** b=6 **4c** b=4 **5** 36 240 **6** b **7a** 4 **7b** 18 **7c** 24 **8** b **9** 8cm **10** 4cm **11** b **12a** 4 **12b** 3 **12c** 100 **12d** 8 **12e** 64 **12f** 8 **13** b

Exercise 1.17 **1a** 0.04 **1b** 0.0009 **1c** 0.008 **1d** 1110 **1e** 30 **1f** 1 000 000 **1g** 3 **1h** 5 **1i** 7 **2** 12 **3a** x=3 **3b** x=5 **3c** x=12 **4** 100cm **5a** 1 **5b** 3 **5c** 4 **6** 6 **7** 4 **8** 4 **9** 4 **10** 2^3

Exercise 1 **18** Puzzles

8	1	6	4	9	2	4	9	2	6	7	2	8	3	4	4	3	8
3	5	7	3	5	7	3	5	7	1	5	9	1	5	9	9	5	1
4	9	2	8	1	6	8	1	6	8	3	4	6	7	2	2	7	6

Exercise 1.19 **1a** 1^4 **1b** 4^4 **1c** 3^6 **1d** g^7 **1e** a^5 **1f** 10^6 **2a** 4^5 **2b** 2^4 **2c** 10^6 **2d** 5^7 **2e** a^7 **2f** b^4 **2g** p^{10} **2h** w^6 **2i** 10^6 **2j** 10^6 **2k** 1^8 **2l** h^{10} **3a** 10^1 **3b** 4^3 **3c** 3^5 **3d** 2^2 **3e** a^4 **3f** n^5 **3g** 10^4 **3h** 1^2 **3i** 4^2 **3j** t^2 **3k** 10^2 **3l** u^5 **4a** 3^6 **4b** 2^9 **4c** 5^6 **4d** 10^6 **4e** 10^6 **4f** b^{12} **4g** u^{10} **4h** 1^{15} **4i** 7^6 **4j** 3^9 **4k** 2^9 **4l** 4^{11} **5a** 1 **5b** 1 **5c** 1 **5d** 8

Exercise 1.20 **1a** 2^3 **1b** 3^5 **1c** d^6 **1d** 10^5 **1e** z^6 **1f** 1^7 **2a** 3^6 **2b** 1^4 **2c** 10^8 **2d** t^5 **2e** m^8 **2f** 3^4 **2g** p^9 **2h** 5^9 **2i** 10^4 **2j** 10^7 **2k** 1^7 **2l** j^7 **3a** 3 **3b** e^2 **3c** 2^5 **3d** 1^2 **3e** g^3 **3f** m^3 **3g** 10^6 **3h** q^2 **3i** h **3j** 4^4 **3k** 10^3 **2l** v^3 **4a** 4^6 **4b** 3^{12} **4c** 7^6 **4d** b^6 **4e** 10^4 **4f** e^8 **4g** h^8 **4h** 1^6 **4i** 5^6 **4j** 8^6 **4k** 2^{10} **4l** 3^{11} **5a** 1 **5b** 1 **5c** 1 **5d** 3

Exercise 2.1 **1** ¯3 **2** 12 **3** ¯12 **4** ¯15 **5** 45 **6** 30 **7** ¯2 **8** 0 **9** ¯15 **10** 4 **11** 450 **12** ¯3000 **13** 80 **14** ¯31 **15** ¯12 **16** 0 **17** ¯50 **18** ¯20 **19** ¯4 **20** 3 **21** ¯2 **22** 50 **23** 0 **24** ¯48 **25** 8 **26** 0 **27** ¯6 **28** 9 **29** ¯5 **30** ¯3

Exercise 2.2 **1** 5 > ¯2 **2** ¯3 < 1 **3** 5 > ¯2 **4** ¯4 < 0 **5** 0 > ¯1 **6** ¯3 < ¯2 **7** 2 < 6 **8** 4 > ¯2 **9** ¯3 < 6 **10** ¯5 > ¯6 **11** 2 > 0 **12** ¯4 < ¯1 **13** 6 > ¯1 **14** ¯4 < ¯3 **15** 5 > ¯2 **16** 13 > 5 **17** ¯11 < ¯1 **18** ¯15 < ¯11

Exercise 2.3 **1** 0,1,3,5 **2** ¯2,¯1,0,1 **3** ¯3,¯2,1,6 **4** ¯5,¯3,0,2,3 **5** ¯6,¯4,¯1,2 **6** ¯5,¯3,1,2,5 **7** ¯3,¯2,¯1,1,2 **8** ¯6,¯4,1,3,6 **9** ¯4,¯3,2,4,6 **10** ¯6,0,2,4,6 **11** ¯4,¯3,1,5,6 **12** ¯7,¯3,3,4,8 **13** ¯9,¯4,¯3,2,11 **14** ¯12,¯3,¯2,¯1,9 **15** ¯13,¯12,¯7,13,15

Exercise 2.4 **1** 7 **2** 2 **3** 2 **4** ¯5 **5** 13 **6** 4 **7** ¯6 **8** ¯6 **9** 25 **10** ¯2 **11** 2 **12** ¯5 **13** 15 **14** 5 **15** 3 **16** 21 **17** 7 **18** ¯3 **19** ¯9 **20** 30 **21** ¯4

Exercise 2.5 **1**

+	⁻2	⁻1	0	1
⁻2	⁻4	⁻3	⁻2	⁻1
⁻1	⁻3	⁻2	⁻1	0
0	⁻2	⁻1	0	1
1	⁻1	0	1	2

2

+	⁻10	⁻5	0	5
⁻10	⁻20	⁻15	⁻10	⁻5
⁻5	⁻15	⁻10	⁻5	0
0	⁻10	⁻5	0	5
5	⁻5	0	5	10

Exercise 2.6 **1** 2 **2** ⁻1 **3** ⁻2 **4** ⁻6 **5** 1 **6** ⁻3 **7** ⁻3 **8** ⁻7 **9** 3 **10** ⁻4 **11** 2 **12** ⁻4 **13** 5 **14** ⁻3 **15** ⁻1 **16** ⁻6 **17** ⁻6 **18** ⁻2 **19** 1 **20** ⁻5 **21** 5 **22** 6 **23** ⁻1 **24** 2 **25** 8 **26** 7 **27** 6 **28** ⁻4 **29** 4 **30** 7 **31** ⁻2 **32** ⁻2 **33** 8 **34** 12 **35** ⁻5 **36** 2 **37** 21 **38** 9 **39** ⁻1 **40** 9

Exercise 2.7 **1** 10 **2** ⁻10 **3** ⁻10 **4** ⁻10 **5** 12 **6** ⁻12 **7** ⁻12 **8** ⁻12 **9** 8 **10** ⁻8 **11** ⁻8 **12** ⁻8 **13** 0 **14** 0 **15** 0 **16** 0 **17** 15

×	⁻2	⁻1	0	1
1	⁻2	⁻1	0	1
2	⁻4	⁻2	0	2
3	⁻6	⁻3	0	3
4	⁻8	⁻4	0	4

22

×	⁻2	⁻1	0	1
2	⁻4	⁻2	0	2
1	⁻2	⁻1	0	1
0	0	0	0	3
⁻1	2	1	0	⁻1

18 ⁻15 **19** ⁻15 **20** ⁻15 **21** **23** 8 **24** 6 **25** 9 **26** 20 **27** 2 **28** 15 **29** 18 **30** ⁻20 **31** ⁻8 **32** 10 **33** 14 **34** ⁻30 **35** ⁻20 **36** 16 **37** 35 **38** ⁻16 **39** ⁻21 **40** 24

Exercise 2.8 **1** 3 **2** ⁻2 **3** ⁻4 **4** 1 **5** 2 **6** ⁻3 **7** ⁻2 **8** 6 **9** 6 **10** ⁻7 **11** ⁻2 **12** 2 **13** 7 **14** ⁻5 **15** ⁻3 **16** 9 **17** 5 **18** ⁻15

÷	⁻4	⁻2	2	4
⁻8	2	4	⁻4	⁻2
⁻4	1	2	⁻2	⁻1
0	0	0	0	0
4	⁻1	⁻2	2	1

24

÷	⁻10	⁻5	5	10
⁻40	4	8	⁻8	⁻4
⁻20	2	4	⁻4	⁻2
0	0	0	0	0
20	⁻2	⁻4	4	2

19 ⁻3 **20** 3 **21** 4 **22** ⁻4 **23** **25** ⁻6 **26** ⁻7 **27** 4 **28** ⁻5 **29** ⁻5 **30** 1 **31** ⁻6 **32** 12 **33** 5 **34** ⁻6

Exercise 2.9 **1** 8 **2** 3 **3** ⁻1 **4** ⁻6 **5** 3 **6** ⁻8 **7** 8 **8** ⁻3 **9** 25 **10** ⁻2 **11** ⁻1 **12** ⁻6 **13** 6 **14** ⁻8 **15** 11 **16** ⁻7 **17** 9 **18** ⁻10 **19** ⁻5 **20** 8 **21** 30 **22** ⁻18 **23** ⁻10 **24** ⁻24 **25** 4 **26** ⁻2 **27** ⁻6 **28** 5 **29** 3 **30** ⁻4 **31** ⁻2 **32** 5 **33** ⁻150 **34** ⁻270 **35** ⁻36 **36** 50

Exercise 2.10 **2** 1 **3** ⁻6 **4** 2 **5** 120 **6** 3^5 **7** m^5 **8** 1 **9** 3.6 **10** $110

Exercise 2.11 **2** ⁻1 **3** 6 **4** ⁻2 **5** 240 **6** 2^6 **7** d^2 **8** 1 **9** 4.3 **10** $70

Exercise 2.12 **2** ⁻3 **3** ⁻8 **4** ⁻3 **5** 340 **6** 5^4 **7** x^7 **8** 1 **9** 4.1 **10** $100

Exercise 2.13 **1a** ⁻3 **1b** ⁻1 **1c** 6 **1d** ⁻6 **1e** ⁻2 **1f** 12 **1g** ⁻5 **1h** 5 **1i** ⁻6 **2** A=⁻2.5, B= ⁻0.5 **3** 5 **4** 3 **5a** ⁻3,⁻2,0,1,2 **5b** ⁻7.6,⁻6.7,6.7,7.6 **6** A(2,3), B(⁻2,2), C(⁻3,⁻3), D(3,⁻2) **7** 2 **8** 20 **9** x=⁻2 **10** x=5,y=⁻4

Exercise 2.14 **1** 3 **2a** 3 **2b** 3 **2c** 5 **2d** ⁻5 **2e** 50 **2f** ⁻50 **3a** 3 **3b** 10 **3c** ⁻7 **4a** x=⁻1 **4b** x=⁻1 **4c** x=⁻1 **4d** x=7 **4e** x=⁻2 **4f** x=⁻2

Exercise 2.15 **1a)**

1	2	4	3
3	4	2	1
4	3	1	2
2	1	3	4

1b)

2	4	3	1
1	3	4	2
3	1	2	3
4	2	1	4

Exercise 2.16 **1a** ⁻2 **1b** 15 **1c** 25 **1d** ⁻12 **2a** 3>⁻2 **2b** ⁻4<1 **2c** ⁻2>⁻5 **3a** ⁻5, ⁻1, 0 3 **3b** ⁻2, ⁻1, 0, 2 **3c** ⁻4, ⁻3, ⁻2, 1 **4a** 2 **4b** ⁻1 **4c** ⁻9 **4d** 9 **4e** ⁻3 **4f** 4 **4g** ⁻12 **4h** ⁻6 **4i** 10 **4j** 3 **4k** ⁻3 **4l** ⁻3 **4m** 2 **4n** ⁻9 **4o** ⁻5 **4p** ⁻2 **4q** ⁻4 **4r** ⁻5 **5a** 2 **5b** ⁻2 **5c** ⁻7 **5d** 5 **5e** ⁻4 **5f** 2 **5g** ⁻15 **5h** ⁻10 **5i** 8 **5j** 3 **5k** ⁻5 **5l** ⁻4 **5m** 1 **5n** 0 **5o** ⁻7 **5p** ⁻2 **5q** 9 **5r** ⁻4 **6** ⁻$240 **7** ⁻5°C **8** ⁻$100 **9** 50km/h **10** 975 years

Exercise 2.17 **1a** ⁻250 **1b** 15 **1c** ⁻500 **1d** 0 **2a** 5>⁻1 **2b** ⁻3<3 **2c** ⁻4>⁻3 **3a** ⁻4, ⁻2, 2 3 **3b** ⁻4, ⁻2, 0, 4 **3c** ⁻3, ⁻2, ⁻1, 1 **4a** ⁻1 **4b** ⁻5 **4c** ⁻4 **4d** 9 **4e** 3 **4f** 21 **4g** ⁻12 **4h** ⁻8 **4i** 4 **4j** 2 **4k** ⁻7 **4l** ⁻5 **4m** 3 **4n** ⁻2 **4o** ⁻7 **4p** 4 **4q** ⁻2 **4r** ⁻5 **5a** 3 **5b** 0 **5c** ⁻11 **5d** 5 **5e** ⁻2 **5f** 12 **5g** ⁻6 **5h** ⁻15 **5i** 9 **5j** 1 **5k** ⁻3 **5l** ⁻2 **5m** 6 **5n** 4 **5o** ⁻8 **5p** ⁻24 **5q** 6 **5r** ⁻4 **6** ⁻$280 **7** ⁻7°C **8** ⁻$190 **9** ⁻70m **10** 397°C

Exercise 3.1 **1a** x−3 **1b** x+5 **1c** x+b **1d** 3x **1e** x−x/2 or x/2 or 0.5x **2a** a+1.5 **2b** a−5.25 **2c** a+b **2d** a−t **2e** 2a **3a** x+y+z **3b** x+y **3c** z−3 **3d** z+5 **4a** x+y **4b** a+7 **4c** 2b+10 **4d** m−n **4e** ag **4f** b^3 **4g** 3x+5p **4h** x^2-2x+7 **4i** mc^2

Exercise 3.2 **1a** 12 **1b** 4 **1c** 9 **1d** ¯3 **1e** 2/3 **1f** 2/3 **1g** 6 **1h** 2 **1i** ¯2 **1j** 43 **1k** 16 **1l** 12 **2** 2 **3** 4.25 **4** 1875 **5** 25 **6a** $30cm^2$ **6b** $56.12m^2$ **7a** 14cm **7b** 52.4cm **8a** $6mm^2$ **8b** $23.78cm^2$

Exercise 3.3 **1** 8x **2** 2a **3** 9c **4** 5e **5** 19c **6** ¯5z **7** 15h **8** 3k **9** 12d **10** 5f **11** 18o **12** ¯x **13** 21y **14** ¯w **15** 10a **16** 14s **17** 5d **18** 4a **19** 10x **20** ¯4u **21** 3d **22** 9x **23** 2a **24** ¯5g **25** 16y **26** 13x+2a **27** 3a+6b **28** b+4g **29** 3y+2p **30** ¯2t+10 **31** 6m+n **32** $8b^2-11$ **33** $2a^3+9b$ **34** 6xy+2 **35** as^2+7d^5 **36** 5x+7y **37** $-10a+3x^2y^3$ **38** 14x+4 **39** −5x−y

Exercise 3.4 **1** not equivalent **2** equivalent **3** not equivalent **4** equivalent **5** equivalent **6** not equivalent **7** equivalent **8** not equivalent

Exercise 3.5 **1** 10a **2** 21a **3** 18c **4** 14e **5** 28n **6** 24m **7** 32h **8** 15a **9** 12s **10** 12d **11** 36b **12** 4e **13** 8w **14** 3p **15** 12v **16** 3d **17** 3p **18** 3r **19** 12ab **20** 4fw **21** 14sy **22** 10bc **23** 24dt **24** 7gk **25** 27np **26** 30eh **27** 20fg **28** 2mn **29** 5pr **30** 2km **31** $12g^2$ **32** $12t^2$ **33** $27h^2$ **34** $35d^2$ **35** $16s^3$ **36** $40fg^3$ **37** $6de^3$ **38** ¯10mn **39** ¯$6a^2$ **40** ¯$12b^2$ **41** $12a^2b$ **42** ¯$12p^3$ **43** ¯$27w^3$ **44** ¯$15m^3n^2$ **45** ¯$28n^2p^3$ **46** ¯$24a^3b^2$ **47** $6d^2p^3$ **48** $16h^3$ **49** $20a^4b^3c^2$

Exercise 3.6 **1** 2a **2** 3y **3** 3c **4** 2e **5** 2n **6** 4m **7** 5f **8** 2n **9** 4h **10** g/2 **11** a/3 **12** s/3 **13** m/2 **14** d/5 **15** t/3 **16** 10p/3 **17** 3b/2 **18** 9d/2 **19** 8k/3 **20** 6d/5 **21** 3s/2 **22** 4y/3 **23** 7f/2 **24** 3g/2 **25** 3/2 **26** 12/5 **27** 8/3 **28** 2/3 **29** 3/2 **30** 4/7 **31** 3y **32** 7d/2 **33** 9g/2 **34** w/4 **35** 1/3 **36** 2/(3b) **37** ¯2d **38** ¯2q **39** ¯6 **40** 2 **41** ¯3 **42** 2 **43** 3y **44** 2d **45** ¯9g/2 **46** ¯w/4 **47** ¯3 **48** 2/(3b) **49** 3g/(2n) **50** ¯3q **51** ¯8/3 **52** ¯2c/3 **53** 7a/3 **54** ¯2z/3

Exercise 3.7 **1** 4b+12 **2** 5c+10 **3** 2a+14 **4** 3g+3 **5** 6h+30 **6** 7n+28 **7** 5m+15 **8** 3y+6 **9** 9w+54 **10** 6c+12 **11** 12e+16 **12** 20v+15 **13** 21y+7 **14** 18x+30 **15** 24g+10 **16** 10a+2d **17** 5w+5h **18** 27q+12d **19** 4z−8 **20** 10s−20 **21** 12d−9 **22** 12f−28 **23** 16a−40 **24** 30h−36 **25** 6u−12 **26** 28p−24 **27** 35d−45 **28** 8s−24t **29** 35n−56 **30** 18u−30 **31** $2t^2+3t$ **32** $12z^2+15z$ **33** $24g^2+30g$ **34** $6d^2-8d$ **35** $6n^2-12n$ **36** $7m^2-2m$ **37** $3y^2+2ry$ **38** $15b^2-10bc$ **39** $18bu-24u^2$

Exercise 3.8 **1** equivalent **2** equivalent **3** not equivalent **4** not equivalent **5** equivalent **6** not equivalent **7** not equivalent **8** equivalent **9** equivalent **10** not equivalent **11** not equivalent **12** equivalent **13** not equivalent **14** equivalent **15** not equivalent **16** not equivalent **17** not equivalent **18** equivalent

Exercise 3.9 **1** ¯2a−6 **2** ¯5r −10 **3** ¯2c−8 **4** ¯8d−20 **5** ¯18h−6 **6** ¯28x−49 **7** ¯10m+20 **8** ¯3y−6 **9** ¯9w−54 **10** ¯6c+12 **11** ¯12e+16 **12** ¯20v−15 **13** $¯2t^2-3t$ **14** $¯12z^2+6z$ **15** $¯24g^2-30g$ **16** $¯6d^2+8d$ **17** $¯16n^2+32n$ **18** $¯3m^2+5m$ **19** ¯3p−$2p^2$ **20** ¯$12e^2+8ce$ **21** ¯$8bu+16u^2$ **22** ¯$6y^2+8y$ **23** ¯$15s^2+12s$ **24** ¯$7m^2+2m$ **25** ¯$3z^2-2rz$ **26** ¯$45q^2+10dq$ **27** ¯$18t^2+30tu$

Exercise 3.10 **1** ab+ac **2** 5a+15 **3** 2626 **4** bx+cx **5** ¯3b+12 **6** 212 **7** a^2+ad **8** ¯2x−4 **9** 210 **10** $2x^2+3x$ **11** 18x+30 **12** 189 **13** $5a^2-ad$ **14** ¯5w−5h **15** 315 **16** 2ds−4d **17** ¯10s+20 **18** 57 **19** $3n^2-7n$ **20** 16a−40 **21** 288 **22** 6u−12 **23** ¯28p+24 **24** 335 **25** 8s−24t **26** 35n−56 **27** 1530

Exercise 3.11 **1** 2(x+3) **2** 3(a+2) **3** 2(c+2) **4** 2(x−3) **5** 3(a−2) **6** 5(g+2) **7** 5(a+3b) **8** 2(m+4n) **9** a(2a+3) **10** x(5x+3) **11** t(7t−5) **12** s(3s−4) **13** 2x(x+3) **14** 3y(y+2) **15** 2r(r−2) **16** 4(a+2d) **17** 3(w−3) **18** 5(b+2)

Exercise 3.12 **1** 3x **2** 4a **3** 2a **4** 2b **5** 3y **6** 5 **7** 3 **8** 4 **9** 4ab **10** 2p **11** 4 **12** 2 **13** 6 **14** 5y **15** 3f **16** 8 **17** 16h **18** 7t

Exercise 3.13 **1** 3(1+2x) **2** 4a(1+2b) **3** 2a(3b−5) **4** 2b(2+3a) **5** 3y(x−3) **6** 5(s+2d) **7** 3(g−4) **8** 4(3e+1) **9** 4ab(2+3c) **10** 2p(3+7q) **11** 4(p−4) **12** 2(4a+b) **13** 6(1+2y) **14** 5y(x−3z) **15** 3f(5e+9g) **16** 8(2g−1) **17** 16h(1+4ij) **18** 7t(2r+5)

Exercise 3.14 **1** 2x **2** 2a **3** ¯2y **4** 4b **5** ¯3y **6** 5 **7** 2 **8** 3 **9** 4df **10** 2p **11** 3 **12** 2 **13** 4 **14** 5n **15** ¯3f **16** 2 **17** ¯8j **18** 5n

Exercise 3.15 **1** 2(x−3) **2** 2a(2+5b) **3** ¯2y(x+5) **4** 4b(−1+2a) **5** ¯3y(a+3) **6** 5(s+2d) **7** 2(3g−5) **8** 3e(5e+1) **9** 4df(¯2+3a) **10** 2p(3−7q) **11** 3(−m+6) **12** 2(2a−b) **13** 4(¯2+3u) **14** 5n(m+3o) **15** ¯3f(4e+9g) **16** 2(7g−4) **17** ¯8j(3+8hi) **18** 5n(2b−7)

Exercise 3.16 **1** equivalent **2** equivalent **3** equivalent **4** not equivalent **5** equivalent **6** equivalent **7** not equivalent **8** equivalent

Exercise 3.17 **2** 3 **3** ¯2 **4** ¯6 **5** 4a **6** $6g^2$ **7** 10a+15 **8** 3(x−2) **9** 3^7 **10** 7.2

Exercise 3.18 **2** 5 **3** ¯5 **4** 15 **5** 6m **6** 2b **7** 6x−3 **8** 2(2x−3) **9** 5^4 **10** 3.8

Exercise 3.19 **2** 7 **3** 1 **4** ¯2 **5** 3b **6** ¯15c **7** 8c−4 **8** 5(x+3) **9** 2^6 **10** 4.7

Exercise 3.20 **1** 18 **2** 2 **3** 1.25 **4** 67 **5** 1875 **6** 25 **7** 12 **8** 1.2 **9** x=4 **10** x=5 **11** 485 **12** 969 **13** 13 **14** 36 **15** 12a+3 **16** x +3 **17** 3 (or ¯3) **18** 4 and 3 **19** ¯5 and 2

Exercise 3.21 **1a** b+2 **1b** 2n+6 **1c** 3x+2 **1d** 2a+b **1e** 2w+6 **1f** 5u +8 **1g** x +4 **1h** 2y−3 **1i** ¯d−8 **1j** ¯2h+8 **1k** a+6b **1l** g+7h **2** 3 **3** 7 **4** ¯0.5 **5** 0.7 **6** 54 **7a** x=20° **7b** x=12° **7c** x=90°

Exercise 3.22 **1** 3482, 3761, 2591 then 5429, 5861, 9371 **2** 75

Exercise 3.23 **1a** x+5.5 **1b** x−3.25 **1c** x+b **1d** x−t **1e** 2x **2a** 7x **2b** 4h **2c** 2c **2d** 6a **2e** ¯5z **2f** 3s/2 **2g** −u **2h** 27np **2i** t/3 **2j** 3d **2k** ¯6a^2 **2l** 9g/2 **2m** 6m^4 **2n** 12a^5 **2o** 4d^5 **2p** 9b^2−11 **2q** ¯15m^3n^2 **2r** ¯4/(3b) **2s** 3d^2 **2t** 2 **2u** 3p **3a** 2t+10 **3b** 12w−30 **3c** 6m−12 **3d** 2g^2+3g **3e** 15b^2−10bc **3f** ¯5w−5h **3g** ¯21h+12 **3h** 90a−3 **3i** 140x+28 **3j** 32c−40 **3k** ¯32b+36 **3l** ¯12p+36 **4a** 3(x+4) **4b** 2(2a−3) **4c** 2(3c+4) **4d** 5(t+3) **4e** 2(f−3) **4f** 2(w−5) **4g** 3(1+2h) **4h** 4e(d+2) **4i** 2a(3−5b) **4j** 2p(2+5q) **4k** 4b(¯1+3a) **4l** ¯3(st+2) **4m** 3(¯k+6) **4n** ¯3g(5d+9fg) **4o** ¯24cd(1+2b) **5a** equivalent **5b** equivalent **5c** not equivalent **5d** not equivalent **5e** equivalent **5f** equivalent **5g** not equivalent **5h** equivalent **6** 15/8 **7** 2 **8** 625 **9a** 4x+8 **9b** 7y−14 **9c** 4p−2 **9d** 11h −5 **9e** 15a−22 **9f** 14d−4

Exercise 3.24 **1a** x+y **1b** 2b+5 **1c** 3d−45 **1d** h+9 **1e** 2(2h+3) **2a** 7d **2b** 5m **2c** 4f **2d** 15v **2e** ¯3p **2f** 4n/3 **2g** g **2h** 12b^2 **2i** 4t/9 **2j** ¯2a **2k** ¯12w^2 **2l** 9t/5 **2m** 6c^4 **2n** 15a^5 **2o** 12s^5 **2p** 10b^2−9 **2q** ¯24b^3c^2 **2r** ¯2/(3b) **2s** 12d^2/7 **2t** 1/3 **2u** 6p/7 **3a** 3r+15 **3b** 20w−10 **3c** 12m−12 **3d** 2z^2+7z **3e** 5b^2−15bc **3f** ¯3u−3v **3g** ¯16h+8 **3h** 240x−4 **3i** 1500y+45 **3j** 18k−36 **3k** ¯32c+12 **3l** ¯30v+12 **4a** 3(x+3) **4b** 4(y−2) **4c** 2(3z+4) **4d** 5(a+4) **4e** 2(f−3) **4f** 2(g−8) **4g** 3(1+5h) **4h** 4b(a+4) **4i** 2c(3−11b) **4j** 2d(2+5c) **4k** 5j(¯1+6k) **4l** ¯3(mn+3) **4m** 3(−p+6) **4n** ¯3qr(5+7r) **4o** ¯12t(2s+3uv) **5a** equivalent **5b** not equivalent **5c** equivalent **5d** equivalent **5e** equivalent **5f** equivalent **5g** not equivalent **5h** not equivalent **6** 9/5 **7** ¯9 **8** 96 **9a** 3x+8 **9b** 6y+6 **9c** 5p **9d** 8d+3 **9e** 10v−20 **9f** 11f+3

Exercise 4.1 **4a** 20 **4b** 20 **4c** 200 **6a** 5 **6b** 5 **6c** 5 **6d** 50

Exercise 4.2 **1a** 1/6 **1b** 1/2 **1c** 1/2 **1d** 1/6 **1e** 1/3 **1f** 1/3 **1g** 0 **1h** 1/3 **2a** 1/8 **2b** 1/2 **2c** 1/2 **2d** 1/8 **2e** 1/2 **2f** 1/8 **2g** 3/8 **2h** 1/4 **3a** 3/8 **3b** 5/8 **3c** 5/8 **3d** 3/8 **3e** 1 **3f** 0 **4a** 3/5 **4b** 2/5 **4c** 2/5 **4d** 3/5 **4e** 1 **4f** 0

Exercise 4.3 **1a** The ball is not white **1b** An even number will show **1c** Mia will not win the prize **1d** The coin will show a head **1e** It will not rain today **2a** 1/6 **2b** 5/6 **2c** 1/2 **2d** 1/2 **2e** 1/3 **2f** 2/3 **3a** 1/16 **3b** 15/16 **3c** 1/2 **3d** 1/2 **3e** 1/8 **3f** 7/8 **3g** 5/16 **3h** 11/16 **3i** 1/4 **4a** 1/50 **4b** 49/50 **4c** 1/2 **4d** 1/2 **4e** 1/5 **4f** 4/5 **4g** 3/50 **4h** 47/50 **5a** 1/200 **5b** 199/200 **6a** 5/12 **6b** 1/12 **6c** 7/12 **6d** 1/4 **6e** 1/12 **6f** 1/3 **6g** 0 **6h** 11/12 **6i** 3/4 **7a** 5/11 **7b** 1/11 **7c** 6/11 **7d** 2/11 **7e** 2/11 **7f** 4/11 **7g** 0 **7h** 9/11 **7i** 9/11 **8a** 5/12 **8b** 1/4 **8c** 1/6 **8d** 1/6 **8e** 7/12 **8f** 5/6 **8g** 5/6 **8h** 5/12 **8i** 0 **8j** 3/4 **8k** 5/6 **8l** 1 **9a** 15/109 **9b** 23/109 **9c** 10/109 **9d** 38/109 **9e** 22/109

Exercise 4.4 **2** 7 **3** m^5 **4** ¯2 **5** 3b **6** 1/3 **7** 10a+15 **8** 5(x+3) **9** 0.95 **10** 6

Exercise 4.5 **2** ¯5 **3** n^7 **4** 7a **5** 15 **6** 1/4 **7** 6x+12 **8** 4(c+3) **9** 0.85 **10** ¯12

Exercise 4.6 **2** 2 **3** x^8 **4** 5b **5** 20 **6** 1/5 **7** 10t−15 **8** 3(h+3) **9** 0.25 **10** ¯3

Exercise 4.7 **1** 1/6 **2** 1/4 **3** 15/40=3/8 **4** 2/5 **5** 25/55=5/11 **6** 3/5 **7** 55 **8** b) **9** 1/2 **10** 3/8 **11** 1/36

Exercise 4.8 **1a** 6/27=2/9 **1b** 10/27 **2a** 43/150 **2b** 44/150=22/75 **3** 4/6=2/3 **4** 4/6=2/3 **5a** 2/6=1/3 **5b** 4/6=2/3 **5c** 2/6=1/3 **5d** 1 **6a** 2/3 **6b** 2/3 **6c** 1

Exercise 4.9 **1** 6 **2** 96cm **3** 10:55am **4** 75 **5** one answer 1st row −1,4,−3 next row −2,0,2 then 3,−4,1

Exercise 4.10 **1** Probability is the chance of something happening **2a** 1/6=0.17 **2c** 1/1000000=0.0000001 **3a** 25heads **3b** 25tails **3c** 250heads **4a** 1/6 **4b** 1/2 **4c** 1/2 **4d** 1/6 **4e** 1/6 **4f** 2/3 **4g** 0 **4h** 1/2 **5a** The ball is not red **5b** An even number will show **5c** Riley will not win the lottery **5d** The coin will show a head **5e** It will not be fine today **6a** 5/1000=1/200 **6b** 199/200 **7a** 3/10 **7b** 1/2 **7c** 1/5 **7d** 0 **7e** 7/10 **7f** 4/5 **7g** 1/2 **7h** 7/10 **8a** 2/3 **8b** 1 **8c** 1/3

Exercise 4.11 **1** Probability is the chance of something happening **2a** 1/11=0.09 **3a** 30heads **3b** 30tails **3c** 300heads **4a** 1/6 **4b** 1/2 **4c** 1/2 **4d** 1/6 **4e** 1/2 **4f** 1/3 **4g** 0 **4h** 1/3 **5a** The ball is not blue **5b** An odd number will show **5c** Ella will not win the lottery **5d** The coin will show a tail **5e** It will not rain today **6a** 1/100 **6b** 99/100 **7a** 1/4 **7b** 1/4 **7c** 1/2 **7d** 0 **7e** 3/4 **7f** 1/2 **7g** 3/4 **7h** 3/4 **8a** 2/3 **8b** 1/3

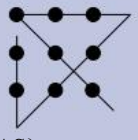
Exercise 5.1 **2** 5 **3** ¯5 **4** 2 **5** 4c **6** 1/3 **7** 6b+12 **8** 5(a+4) **9** 0.75 **10** x^5
Exercise 5.2 **1a** 2^3 **1b** 3^5 **1c** d^6 **1d** 10^5 **1e** z^6 **1f** 1^7=1 **2a** 3^6 **2b** 1^4=1 **2c** 10^8 **2d** t^5 **2e** m^8 **2f** 3^4 **2g** p^9 **2h** 5^9 **2i** 10^4 **2j** 10^7 **2k** 1^7=1 **2l** j^7 **3a** 3 **3b** e^2 **3c** 2^5 **3d** 1^2=1 **3e** g^3 **3f** m^3 **3g** 10^6 **3h** q^2 **3i** h **3j** 4^4 **3k** 10^3 **3l** v^3 **4a** 4^6 **4b** 3^{12} **4c** 7^6 **4d** b^6 **4e** 5^6 **4f** 8^6 **4g** 2^{10} **4h** 3^{11} **5a** 1 **5b** 1 **5c** 1 **5d** 3 **6a** ¯$250million **6b** 15m **6c** ¯$500 **6d** 0 **7a** 5>¯1 **7b** ¯3<3 **7c** ¯4<¯3 **8a** ¯4,¯2,2,3 **8b** ¯4,¯2,0,4 **8c** ¯3,¯2,¯1,1 **9a** ¯1 **9b** ¯5 **9c** ¯4 **9d** 9 **9e** 3 **9f** 21 **9g** ¯12 **9h** ¯8 **9i** 4 **9j** 2 **9k** ¯7 **9l** ¯5 **9m** 3 **9n** ¯2 **9o** ¯7 **9p** 4 **9q** ¯2 **9r** ¯5 **10** ¯$280 **11** ¯7°C **12** ¯$190 **13** 70m **14** 397°C
15a a+b **15b** xy **15c** c+5 **15d** h−2x **15e** x3y or 3xy **16a** 9a **16b** 2b **16c** 2c **16d** 9d **16e** 3f/2 or 1.5f **16f** ¯4h **16g** 12eg **16h** 7i **16i** −$9j^2$ **16j** 3m/2 or 1.5m **16k** $8g^4$ **16l** $10a^2$−8 **16m** $^-9x^3y^2$ **16n** 4 **16o** 6p/5 or 1.2p **17a** 3b+15 **17b** 6a−18 **17c** 15d−5 **17d** 28x−7 **17e** ¯8y+14 **17f** ¯18z+12 **18a** 3(d+2) **18b** 5(e−3) **18c** 3(2f+3) **18d** 2d(2+9e) **18e** 2b(¯1+5a) **18f** ¯3(xy+7) **18g** 2(¯3t+7) **18h** ¯3g(5+7g) **18i** ¯4mn(6+7o) **19a** equivalent **19b** equivalent **19c** not equivalent **19d** equivalent **20a** 2a+1 **20b** 7b−7 **20c** 10k **20d** 23n+7
21 Probability is the chance of something happening **22a** 2/7=0.29 **23a** 25 **23b** 25 **23c** 250 **24a** 1/6 **24b** 1/2 **24c** 1/2 **24d** 1/6 **24e** 1/3 **24f** 1/2 **25a** The number is not 3 **25b** An even number will show **26a** 1/50 **26b** 49/50 **27a** 2/5 or 0.4 **27b** 3/5 or 0.6 **27c** 4/5 or 0.8 **28a** 2/3 **28b** 1/3
Exercise 5.3 **2** 4 **3** 10 **4** ¯21 **5** 6m **6** 1/2 **7** 9d+6 **8** 2(3h+5) **9** 0.65 **10** x^7
Exercise 5.4 **1a** 5^4 **1b** 2^5 **1c** w^6 **1d** 10^3 **1e** y^6 **1f** 1^6 =1 **2a** 4^7 **2b** 1^7 =1 **2c** 10^7 **2d** t^9 **2e** n^9 **2f** 5^4 **2g** h^{10} **2h** 7^{11} **2i** 10^6 **2j** 10^5 **2k** 1^5 =1 **2l** z^8 **3a** 3^4 **3b** d^4 **3c** 3^5 **3d** 1^5 =1 **3e** m^3 **3f** d^2 **3g** 10^6 **3h** s^4 **3i** a^3 **3j** 2^3 **3k** 10^2 **3l** x^3 **4a** 2^9 **4b** 3^8 **4c** 4^6 **4d** x^8 **4e** y^9 **4f** 5^{12} **4g** 2^{11} **4h** 3^{12} **5a** 1 **5b** 1 **5c** 1 **5d** 2 **6a** ¯5°C **6b** 25% **6c** $150 **6d** ¯5 **7a** 2>¯2 **7b** ¯1<2 **7c** ¯2>¯3 **8a** ¯3,¯2,¯1,3 **8b** ¯2,¯1,0,1 **8c** ¯4,¯3,¯1,2 **9a** 2 **9b** ¯3 **9c** ¯5 **9d** 5 **9e** 5 **9f** 12 **9g** ¯10 **9h** ¯8 **9i** 6 **9j** 2 **9k** ¯6 **9l** ¯2 **9m** 1 **9n** 1 **9o** ¯4 **9p** ¯10 **9q** ¯2 **9r** ¯8 **10** $300 **11** ¯7°C **12** ¯$180 **13** ¯40m **14** 3070years

Exercise 5.4 **1a** 1/2, 0.5, 50% **1b** 3/10, 0.3, 30% **1c** 1/4, 0.25, 25% **1d** 1/2, 0.5, 50% **1e** 2/3, 0.67, 67% **1f** 2/5, 0.4, 40% **2a** 1:3 **2b** 1:2 **2c** 4:3 **2d** 2:3 **2e** 2:3 **2f** 3:4 **3** 500mL **4** 5 shovelfulls **5a** 6 **5b** 24 **5c** 30 **6a** $137.50 **6b** $9.30 **7** x1.4, $70 **8a** $148 **8b** $143.50 **8c** The 30% sounds better, 30% is the better deal **9** $24.07 **10** $2.15 **11a** 5.3 runs per over **11b** 11L per 100km **12a** $15.05 per 750g **12b** $5.19 for 3L **13a** 21m **13b** 26cm **13c** 23.8km **14a** $13.69m^2$ **14b** $36.26km^2$ **14c** $30.69cm^2$ **14d** $1659cm^2$ **14e** $1235cm^2$ **4f** $56.94m^2$ **14g** $14.57m^2$ **14h** $100.5cm^2$ **15a** $678.912cm^2$ **15b** $2099.5cm^3$ **16a** c+d **16b** xy **16c** b−5 **16d** r/a **16e** $2x+x^3$ **17a** 5y **17b** 3d **17c** 3c/2 **17d** 30a **17e** 3a **17f** −u **17g** 12np **17h** ¯2e **17i** $^-12g^2$ **17j** 3g/2 **17k** $8h^4$ **17l** $8a^2$−10 **17m** $^-6a^3b^2$ **17n** 5d **17o** 3p/2 **18a** 2n+6 **18b** 18w−30 **18c** 14p−7 **18d** 24m−12 **18e** ¯24k+27 **18f** ¯12t+18 **19a** 6(x+2) **19b** 2(2r−3) **19c** 2(5c+4) **19d** 2j(2+7k) **19e** 4e(¯1+3f) **19f** ¯3(h+2) **19g** 3(−v+6) **19h** ¯3wx(5+7x) **19i** ¯3ab(7+16c) **20a** equivalent **20b** not equivalent **20c** not equivalent **20d** equivalent **21a** 4a+8 **21b** 7z−5 **21c** 4b−2 **21d** 13h−2 **22** Probability is the chance of something happening **23a** 1/4 **24a** 35 **24b** 35 **24c** 350 **25a** 1/6 **25b** 1/2 **25c** 1/2 **25d** 1/6 **25e** 2/3 **25f** 1/3 **26a** I will not pass the test **26b** An odd number will show **27a** 1/20 **27b** 19/20 **28a** 1/5 **28b** 4/5 **28c** 3/5 **29a** 2/3 **29b** 1/3

Exercise 6.1 **1** 5/2 **2** ¯4/1 **3** ¯2/1 **4** ¯9/1 **5** 15/1 **6** 0/1
Exercise 6.2 **1** 0,1,2,3 **2** ¯3,¯1,0,3 **3** ¯2,¯1,2,6 **4** ¯4,¯2,0,1,5 **5** ¯8,¯3,¯2,¯1 **6** ¯3,¯2,1,4,6 **7** ¯4,¯3,¯2,¯1,2 **8** ¯5,¯2,¯1,2,3 **9** ¯7,¯3,¯1,1,3
Exercise 6.3 **1a** 5/2 **1b** 8/5 **1c** 7/5 **1d** 8/3 **1e** 9/8 **1f** 15/4 **1g** 9/4 **1h** 37/10 **2a** 2/1/2 **2b** 2/1/3 **2c** 2/1/4 **2d** 1/4/5 **2e** 1/3/5 **2f** 3/1/3 **2g** 2/5/7 **2h** 3/4/11 **3a** 5/7/10 **3b** 4/9/10 **3c** 8/58/100 **3d** 9/27/100 **3e** 3/333/1000 **3f** 15/417/1000 **3g** 4/1/100 **3h** 4/10000 **4a** 3.4 **4b** 4.13 **4c** 12.7 **4d** 5.97 **4e** 6.1 **4f** 72.56 **4g** 5.03 **4h** 7.196
Exercise 6.4 **1** 0.1 **2** $0.\overline{3}$ **3** 0.75 **4** $0.\overline{6}$ **5** 0.8 **6** $0.\overline{2}$ **7** 0.5 **8** $0.\overline{36}$
Exercise 6.5 **1a** 10%, 10/100=1/10 **1b** 20%, 20/100=1/5 **1c** 30%, 30/100=3/10 **1d** 40%, 40/100=2/5 **1e** 50%, 50/100=1/2 **1f** 60%, 60/100=3/5 **1g** 70%, 70/100=7/10 **1h** 80%, 80/100=4/5 **1i** 90%, 90/100=9/10 **1j** 100%, 100/100=1 **1k** 25%, 25/100=1/4 **1l** 75%, 75/100=3/4 **1m** 5%, 5/100=1/20 **1n** 300%, 300/100=3 **1o** 110%, 110/100=11/10 **1p** 150%, 150/100=3/2 **1q** 200%, 200/100=2 **1r** 400%, 400/100=4 **1s** 140%, 140/100=7/5 **1t** 625%, 625/100=25/4 **1u** 15%, 15/100=3/20 **1v** 8%, 8/100=2/25 **1w** 6.25%, 6.25/100=1/16 1x 375%, 375/100=15/4 **2a** 10/100=1/10=0.1 **2b** 20/100=1/5=0.2 **2c** 30/100=3/10=0.3 **2d** 40/100=2/5=0.4 **2e** 50/100=1/2=0.5 **2f** 60/100=3/5=0.6 **2g** 70/100=7/10=0.7 **2h** 80/100=4/5=0.8 **2i** 90/100=9/10=0.9 **2j** 100/100=1=1 **2k** 1/100=0.01 **2l** 5/100=1/20=0.05 **2m** 7/100=0.07 **2n** 12/100=3/25=0.12 **2o** 120/100=6/5=1.2 **2p** 150/100=3/2=1.5 **2q** 200/100=2=2 **2r** 500/100=5=5 **2s** 156/100=39/25=1.56 **2t** 217/100=2.17 **2u** 6.2/100=31/500=0.062 **2v** 8.7/100=87/1000=0.087 **2w** 12.25/100=49/400=0.1225 **2x** 5.75/100=23/400=0.0575 **3a** 0.1, 10% **3b** 0.2, 20% **3c** 0.3, 30% **3d** 0.4, 40% **3e** 0.5, 50% **3f** 0.6, 60% **3g** 0.7, 70% **3h** 0.8, 80% **3i** 0.9, 90% **3j** 1, 100% **3k** 0.5, 50% **3l** 0.33, 33% **3m** 0.67, 67% **3o** 0.5, 50% **3p** 0.75, 75% **3q** 0.2, 20% **3r** 0.4, 40% **3s** 0.6, 60% **3t** 0.8, 80%

3u 1, 100% **3v** 0.125, 12.5% **3w** 0.375, 37.5% **3x** 0.625, 62.5% **4a** 1/4, 0.25, 25% **4b** 1/5, 0.25, 25%
4c 15/100=3/20, 0.15, 15% **4d** 75/100=3/4, 0.75, 75% **4e** 37/100, 0.37, 37% **4f** 26/100=13/50, 0.26, 26%
4g 10/100=1/10, 0.1, 10% **4h** 7/10, 0.7, 70% **4i** 581/1000, 0.581, 58.1% **4j** 9/10, 0.9, 90%
4k 21/50, 0.42, 42% **4l** 46/50=23/25, 0.92, 92% **4m** 64/100=16/25, 0.64, 64% **4n** 63/100, 0.63, 63%
4o 99.95/100=9995/10000=1999/2000, 0.9995, 99.95%
Exercise 6.6 **1** rational **2** irrational **3** irrational **4** rational **5** rational **6** irrational **7** irrational **8** rational
Exercise 6.7 **2** 3/10 **3** 0.6 **4** 4 **5** 5a **6** 11/4 **7** 9 **8** $^{-}$20 **9** 5.26 **10** x^6
Exercise 6.8 **2** 3/10 **3** 0.4 **4** 7 **5** 2b **6** 12/5 **7** 9 **8** 14 **9** 2.38 **10** x^5
Exercise 6.9 **2** 2/5 **3** 75% **4** 5 **5** 2d **6** 10/3 **7** 11 **8** $^{-}$24 **9** 4.22 **10** x^7
Exercise 6.10 **1** 2 **2** 2/1/2 **3** 2/1/2 **4** 8hrs 45min **5** 8hrs 53min **6** 2/1/4 **7a** 12 **7b** 776 **7c** 41 **8** 11/3 **9** 3/3/4
10 5.75 **11** 75% **12** 1100 **13** 22 **14** 20/35=4/7 **15** 4 **16** $7000 **17** 24% **18** 0.21
Exercise 6.11 **1** 1.37 **2** 444/555 **3** 0.05 **4** 3 **5** 2.034 **6a** 5/6 **6b** 1/4 **6c** 1 **7a** x=8 **7b** x=6 **7c** x=12 **8** 45mins
9 11:25am **10** 18 **11** 2/3 **12** $1 800 000 **13** 7
Exercise 6.12 **1** 2 **2** 2 **3** 5 **4** 5 **5** 22.67+13.08=35.75 **6** 84.96−30.37=54.59 **7** 5.7×4=22.8
8 6.4÷2=3.2 or 7.4÷2=3.7 **9** 1/27 **10** 523days
Exercise 6.13 **1a** 3/1 **1b** $^{-}$7/1 **1c** 6/1 **1d** $^{-}$4/1 **1e** 1/1 **1f** 0/1 **2a** 0,1,2,5 **2b** $^{-}$3,$^{-}$2,$^{-}$1,0 **2c** $^{-}$4,$^{-}$1,1,3 **2d** $^{-}$3,$^{-}$2,$^{-}$1,0,2
2e $^{-}$5,$^{-}$4,$^{-}$3,$^{-}$2 **2f** $^{-}$3,0,2,3,4 **2g** $^{-}$4,$^{-}$2,$^{-}$1,3,5 **2h** $^{-}$2,$^{-}$1,1,2,3 **2i** $^{-}$5,$^{-}$4,$^{-}$1,2,3 **3a** 10%, 1/10 **3b** 50%, 1/2
3c 130%, 13/10 **3d** 625%, 625/100=25/4 **3e** 15%, 15/100=3/20 **3f** 6%, 6/100=3/50 **3g** 75%, 3/4
3h 80%, 80/100=4/5 **4a** 0.1, 10/100=1/10 **4b** 0.7, 70/100=7/10 **4c** 0.25, 25/100=1/4 **4d** 0.01, 1/100
4e 2, 200/100=2 **4f** 0.12, 12/100=3/25 **4g** 0.1275, 12.75/100=1275/10000=51/400
4h 0.0625, 6.25/100=625/10000=1/16 **5a** 0.1, 10% **5b** 0.25, 25% **5c** 0.3, 30% **5d** 0.4, 40% **5e** 0.75, 75%
5f 0.33, 33% **5g** 0.67, 67% **5h** 0.8, 80% **6a** 2/5, 0.4, 40% **6b** 75/100=3/4, 0.75, 75%
6c 24/100=6/25, 0.24, 24% **6d** 6.8/100=68/1000=17/250, 0.068, 6.8% **7a** 5/2 **7b** 8/5 **7c** 7/5 **7d** 8/3 **7e** 9/8
7f 15/4 **7g** 9/4 **7h** 37/10 **8a** 2/1/2 **8b** 2/1/3 **8c** 2/1/4 **8d** 1/4/5 **8e** 1/3/5 **8f** 3/1/3 **8g** 2/5/7 **8h** 3/4/11 **9** rational
10 irrational
Exercise 6.14 **1a** 2/1 **1b** $^{-}$9/1 **1c** $^{-}$3/1 **1d** $^{-}$8/1 **1e** 10/1 **1f** 0/1 **2a** 0,2,3,5 **2b** $^{-}$3,$^{-}$2,$^{-}$1,0 **2c** $^{-}$3,$^{-}$2,$^{-}$1,3 **2d**
$^{-}$2,$^{-}$1,0,1,2 **2e** $^{-}$4,$^{-}$3,$^{-}$2,$^{-}$1 **2f** $^{-}$3,$^{-}$1,1,4,5 **2g** $^{-}$4,$^{-}$3,$^{-}$2,1,2 **2h** $^{-}$5,$^{-}$2,$^{-}$1,1,5 **2i** $^{-}$9,$^{-}$8,$^{-}$7,8,9 **3a** 20%, 2/10=1/5
3b 60%, 6/10=3/5 **3c** 240%, 24/10=12/5 **3d** 175%, 175/100=7/4 **3e** 35%, 35/100=7/20 **3f** 5%, 5/100=1/20
3g 25%, 25/100=1/4 **3h** 40%, 4/10=2/5 **4a** 0.1, 1/10 **4b** 0.5, 1/2 **4c** 0.75, 3/4 **4d** 0.02, 2/100=1/50
4e 4, 400/100=4 **4f** 0.15, 15/100=3/20 **4g** 0.11, 510.6/100=106/1000=53/50
4h 0.0725, 7.25/100=725/10000=29/400 **5a** 0.1, 10% **5b** 0.4, 40% **5c** 0.5, 50% **5d** 0.6, 60% **5e** 0.5, 50%
5f 0.25, 25% **5g** 0.75, 75% **5h** 0.8, 80% **6a** 3/4, 0.75, 75% **6b** 18/20=9/10, 0.9, 90% **6c** 21/100, 0.21, 21%
6d 6.25/100=625/10000=1/16, 0.0625, 6.25% **7a** 9/4 **7b** 15/4 **7c** 37/10 **7d** 29/5 **7e** 27/8 **7f** 8/5 **7g** 5/2 **7h** 7/5
8a 1/1/2 **8b** 4/1/2 **8c** 1/4/5 **8d** 1/1/5 **8e** 1/3/4 **8f** 4/1/4 **8g** 4/1/8 **8h** 2/11/13 **9** rational **10** irrational

Exercise 7.1 1, 2, 3, and 6 are congruent.
Exercise 7.2 **1** translation **2** reflection **3** rotation **4** reflection **5** translation **6** rotation
Exercise 7.3 **1** △ABC≡△DEF **2** △ABC≡△FDE **3** △ABC≡△DEF **4** △ABC≡△DEF **5** △ABC≡△EFD
6 △ABC≡△EDF
Exercise 7.4 **1** △ACB≡△DFM{SAS} **2** △ACB≡△ONM{SSS} **3** △ACB≡△QPR{ASA}
4 △ACB≡△LKH{RHS} **5** △ACB≡△VST{SSS} **6** △ACB≡△JKH{ASA} **7** △ACB≡△KIG{SAS}
8 △ACB≡△TYZ{RHS} **9** △ACB≡△POR{ASA} **10** △ACB≡△GDF{SSS}
Exercise 7.5 **1** P(3,2) **2** P(6,2) **3** P(2,6) **4** P(7,4) **5** P(7,2) **6** P(6,4) **7** P(4,5) **8** P(4,7) **9** P(4,0)
Exercise 7.6 **2** 3/10 **3** 0.4 **4** P(4,7) **5** $^{-}$2 **6** 1/3/5 **7** 2b **8** SSS, SAS, ASA, RHS **9** 5.26 **10** x^7
Exercise 7.7 **2** 1/2 **3** 0.25 **4** P(4,1) **5** 1 **6** 5/3 **7** 5x **8** SSS, SAS, ASA, RHS **9** 6.35 **10** x^7
Exercise 7.8 **2** 1/4 **3** 0.75 **4** P(2,2) **5** 2 **6** 2/1/3 **7** 3c **8** SSS, SAS, ASA, RHS **9** 9.22 **10** x^6
Exercise 7.9 **1a** A and D **1b** Aand B **2** P(6,2) **3** P(6,3) **4** P(5,3) **5** R {circles with the same radius}
Exercise 7.10 **1**  **2** A B C D E **3** yes except for one diagonal of the kite **4** 
5 △CDE≡△AED △CAD≡△ACE, △CFD≡△AFE **6** 8m^2 **7** 49cm^2
Exercise 7.11 **1** Blue=20%, Yellow=48% **2** 15hrs **3** 5 **4** A=30, B=10 **5**



Exercise 7.12 **1a** translation **1b** reflection **1c** rotation **2a** △ACB≡△DFM{SAS}
2b △ACB≡△ONM{SSS} **2c** △ACB≡△QPR{ASA} **2d** △ACB≡△LKH{RHS} **3** P(0,8) **4** P(7,4) **5** P(7,4)

Exercise 7.13 **1a** rotation **1b** translation **1c** reflection **2a** △ACB≡△VST{SSS} **2b** △ACB≡△JHK{ASA} **2c** △ACB≡△KIG{SAS} **2d** △ACB≡△TYZ{RHS} **3** P(6,1) **4** P(2,1) **5** P(6,4)

Exercise 8.1 **1** The sample is too small and not representative of the school population **2** The sample size appears large enough however only Year 12s were sampled **3** The error is so large as to make the conclusion of little use **4** The sample is restricted to students aware of the survey and willing to put in the effort

Exercise 8.2

1

Eye colour	Freq
Amber	1
Blue	6
Brown	7
Green	3
Hazel	2
	19

2

Travel mode	Freq
Bicycle	1
Bus	8
Car	9
Train	5
Walk	5
	28

3

No TV	Freq
1	5
2	17
3	3
4	1
	26

4

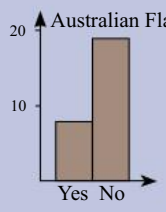
TV violence	Freq
S Agree	2
Agree	7
Disagree	14
S Disagree	1
	24

5

Die	Freq
1	11
2	13
3	11
4	9
5	13
6	8
	65

Exercise 8.3

1



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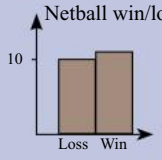
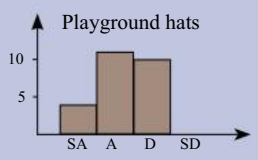
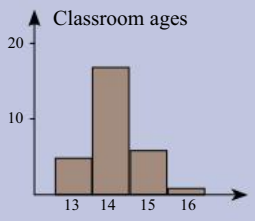


3



4



Exercise 8.4 **1** 39 **2** 227 **3** 5 **4** 6 **5** 5 **6** 6 **7** 2.9 **8** 5

Exercise 8.5 **1** 6 **2** 4 **3** 1 **4** 3 and 4 **5** 5 **6** 1 and 2 **7** 32 **8** 7.1 **9** −1 **10** a

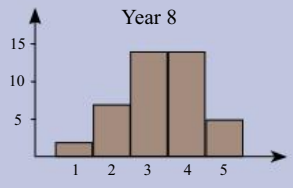
Exercise 8.6 **1** 4 **2** 4 **3** 2 **4** 2.5 **5** 32.5 **6** 23.5 **7** 5.2 **8** −2

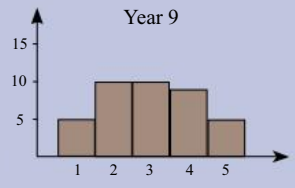
Exercise 8.7 **1** 3 **2** 15.67 **3** 2.9 **4** 3.33 **5** 33.33 **6** 83.33 **7** 103.33 **8** 723.33 **9** 5.33 **10** −3.33

Exercise 8.8 **1** The frequency table shows Year 8 has more 3, 4, and 5 ratings than Year 9. The Year 8 histogram is slightly skewed to the right while the Year 9 histogram is skewed to the left. The are no extreme values thus the mean is probably the best measure and shows a higher rating by the Year 8s. The Year 8s appreciated the concert more than the Year 9s.

Year 8	
Mode	3and4
Median	3
Mean	3.31
Range	4

Year 9	
Mode	2and3
Median	3
Mean	2.97
Range	4



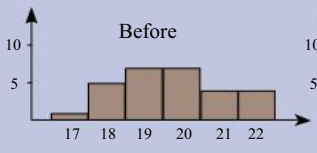


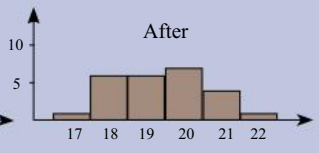
2 Both sets are not symmetrical although close.

3 34 is an outlier and thus deleted as probably a mistake and it would have a large influence on the overall statistics. Without outliers the mean is the best measure and shows after to be larger. The advertising campaign has increased the number of customers

Before	
Mode	19&20
Median	20
Mean	19.71
Range	5

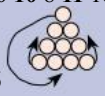
After	
Mode	21
Median	20
Mean	20.4
Range	5





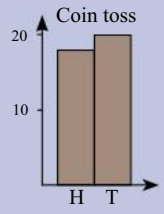
4a the data sets are skew **4b** 34 in the before is an outlier **4c** after deleting the one outlier, the mean is probably the best central measure

Exercise 8.9 **2** 3 **3** 2 **4** 3 **5** 3.2 **6** 0.6 **7** P(4,7) **8** 1/4/5 **9** SSS, ASA, SAS, RHS **10** 17.33
Exercise 8.10 **2** 3 **3** 3 **4** 3 **5** 3.2 **6** 0.25 **7** P(4,1) **8** 1/3/4 **9** SSS, ASA, SAS, RHS **10** 66.67
Exercise 8.11 **2** 4 **3** 2 **4** 2 **5** 2.4 **6** 0.8 **7** P(3,0) **8** 2/2/3 **9** SSS, ASA, SAS, RHS **10** 43.21
Exercise 8.12 **1** 40mins **2a** mean=15, median=5, mode=5 **2b** mean=5.625, median=5, mode=5 **2c** mean **3a** mean=2.43, median=2, mode=2 **3b** Mean>Median=Mode is true **4** $2.50 **5** 35 **6** the 2 column is wrong **7a** mean=2.5, median=2.5, mode=2and3 **7b** the mean, median, and mode have increased **8** the mean
Exercise 8.13 **1** 0.321 **2** 1.111 **3** 12.21 **4** $200 **5** $800 **6** 1000 **7** 97 **8** 90 **9** 10 **10** 8 **11** 18 **12** 94 **13** 110 **14** 240

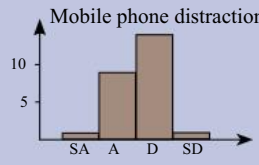
Exercise 8.14 **1**  **2** 8 **3** 8/20=0.4 **4** 15,21,28 **5** 2,2,2,2,2 is one answer **6** 

Exercise 8.15 **1a** The sample is small and not representative **1b** The survey is highly selective in that only those who know and are willing may complete the survey **2a** range=3, mean=3.75, median=4, mode=4 **2b** range=0.3, mean=6.59, median=6.5, mode=6.5 **3a** mean - if no outlier **3b** mode **3c** median

4a)



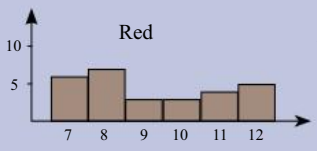
4b)

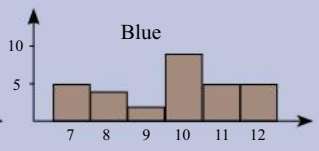


5 The frequency graphs and the statistics show that the two distributions are different **6** 105

Red	
Mode	8
Median	9
Mean	9.25
Range	5

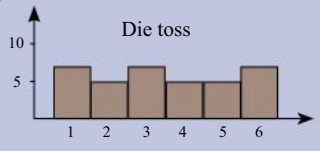
Blue	
Mode	10
Median	10
Mean	9.67
Range	5



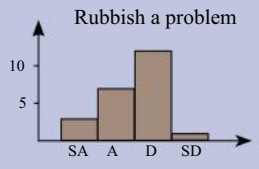


Exercise 8.16 **1a** The error is too large for the data to be useful **1b** The survey is highly selective in that only those who know of the survey and are willing may complete the survey
2a range=3, mean=3.375, median=3, mode=3 **2b** range=0.3, mean=1.24, median=1.2, mode=1.2
3a mean - if no outlier **3b** mode **3c** median

4a)

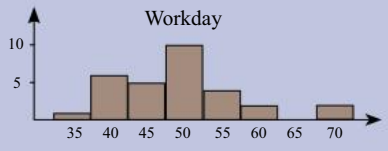


4b)

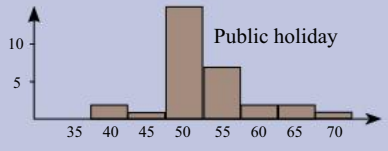


5 The public holiday frequency graph shows a skew to the right compared to the workday graph. It may be concluded that drivers drive faster on public holidays. **6** 70

Workday	
Mode	50
Median	50
Mean	49.33
Range	35



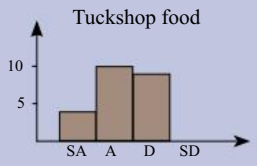
Public holiday	
Mode	50
Median	50
Mean	52.67
Range	30



Exercise 9.3 **1** 21.99cm **2** 28.27cm **3** 26.08mm **4** 53.41m **5** 31.42mm **6** 12.57cm **7** 38.33cm **8** 20.11m **9** 25.13cm **10** 295.31cm **11** 42.10mm **12** 94.25m **13** 40.84cm **14** 7.23mm **15** 172.79cm **16** 6.28m
Exercise 9.4 **1** 38.56cm **2** 3.08m **3** 5.66mm **4** 25.00m **5** 27.14cm **6** 610.83mm **7** 178.26cm **8** 53.70cm **9** 7.23m **10** 10807km **11** 0.16m
Exercise 9.7 **1** 38.48cm^2 **2** 63.62cm^2 **3** 54.11mm^2 **4** 226.98m^2 **5** 78.54mm^2 **6** 12.57cm^2 **7** 116.90cm^2 **8** 32.17m^2 **9** 50.27cm^2 **10** 6939.78cm^2 **11** 141.03mm^2 **12** 706.86m^2 **13** 132.73cm^2 **14** 4.15mm^2 **15** 2375.83cm^2 **16** 3.14m^2
Exercise 9.8 **1** 88.36cm^2 **2** 0.57m^2 **3** 1.90mm^2 **4** 38.48m^2 **5** 45.36cm^2 **6** 19511.65mm^2 **7** 2210.95cm^2 **8** 29.86mm^2 **9** 209.10cm^2 **10** 85.84mm^2
Exercise 9.9 **2** C=πD **3** A=πr^2 **4** π **5** π cannot be expressed as a ratio of two integers **6** 3 **7** 3.2 **8** P(5,7) **9** 7/3 **10** 64.28
Exercise 9.10 **2** C=πD **3** A=πr^2 **4** π **5** π cannot be expressed as a ratio of two integers **6** 3 **7** 3 **8** P(2,−1) **9** 7/2 **10** 9.86
Exercise 9.11 **2** C=πD **3** A=πr^2 **4** π **5** π cannot be expressed as a ratio of two integers **6** 2 **7** 2 **8** P(6,7) **9** 8/3 **10** 88.78
Exercise 9.12 **1** 5m **2** 37cm **3** 66.48cm^2 **4** 5cm **5** 549.78cm^3 **6a** 94.25cm **6b** 214.25cm **7** 775kg **8** 4times **9** 1.75times or 7/4larger
Exercise 9.13 **1** 2274 **2** doubled **3** tripled **4** 9 times **5** 16times **6** 72cm^2 **7** 20.94 km/hr **8a** 40212.39km **8b** 1.59m **9a** 285.84m^2 **9b** 392.70m^2
Exercise 9.14 **1**  **2** 0.5 **3** 1/4 or 0.25
Exercise 9.15 **1a** 28.27cm **1b** 21.68mm **1c** 135.09cm **1d** 12.57m **2a** 46.27cm **2b** 14.28m **2c** 476.58mm **2d** 184.83cm **2e** 57.70cm **3a** 38.48cm^2 **3b** 5541.77cm^2 **3c** 124.69mm^2 **3d** 907.92m^2 **4a** 2.65m^2 **4b** 11.88mm^2 **4c** 45.36cm^2 **4d** 3471.50cm^2 **4e** 536.50mm^2
Exercise 9.16 **1a** 39.58cm **1b** 12.25mm **1c** 179.07cm **1d** 18.85m **2a** 74.55cm **2b** 7.14m **2c** 80.55mm **2d** 107.98cm **2e** 49.13cm **3a** 50.27cm^2 **3b** 6361.73cm^2 **3c** 3.80mm^2 **3d** 314.16m^2 **4a** 0.88m^2 **4b** 17.63mm^2 **4c** 40.72cm^2 **4d** 2536.78cm^2 **4e** 772.57mm^2

Exercise 10.1 **2** C=πD **3** A=πr^2 **4** π **5** cannot be expressed as a ratio of two integers **6** 3 **7** 3.2 **8** P(4,7) **9** 7/2 **10** 6.73
Exercise 10.2 **1a** 5/1 **1b** ¯4/1 **1c** 13/1 **2a** ¯2,¯1,0,4 **2b** ¯2,¯1,0,1 **2c** ¯3,¯1,1,2 **3a** 10%, 1/10 **3b** 15%, 15/100=3/20 **3c** 225%, 225/100=9/4 **3d** 5%, 5/100=1/20 **4a** 0.1, 1/10 **4b** 0.7, 7/10 **4c** 0.25, 25/100=1/4 **4d** 0.0225, 2.25/100=225/10000=9/400 **5a** 0.1, 10% **5b** 0.4, 40% **5c** 0.7, 70% **5d** 0.75, 75% **6a** 2/5, 0.4, 40% **6b** 3/4, 0.75, 75% **6c** 21/100, 0.21, 21% **7a** 9/4, **7b** 8/5, **7c** 15/4 **7d** 16/3 **8a** 1/2/3 **8b** 3/1/13 **8c** 2/3/4 **8d** 3/2/5 **9** rational **10** irrational **11a** rotation **11b** translation **11c** reflection **12a** △ACB≡△RST{SSS} **12b** △ACB≡△JKI{ASA} **12c** △ACB≡△NLP{SAS} **12d** △ACB≡△TUV{RHS} **13** P(7,−4) **14** Small sample size and the error is too large to be useful **15a** range=3, mean=3.125, median=3, mode=3 **15b** range=0.3, mean=2.24, median=2.2, mode=2.2 **16a** mean **16b** mode **16c** median
17

Tuckshop food	
Strongly agree	4
Agree	10
Disagree	9
Strongly disagree	0



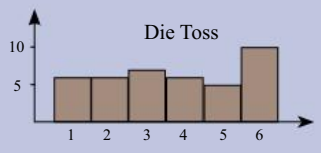
18 The after frequency graph shows a significant skew to the right and with the increased mean indicates that the campaign was a success.

Before	
Mode	2
Median	3
Mean	2.83
Range	3

After	
Mode	3
Median	3
Mean	3.30
Range	2

Exercise 10.2 **19** 60 **20a** C=27.02cm A=58.09cm^2 **20b** C=8.48mm, A=5.73mm^2
20c C=116.24cm, A=1075.21cm^2 **20d** C=12.57m, A=12.57m^2 **21a** P=56.56cm, A=190.07cm^2
21b P=10.71m, A=7.07m^2 **21c** P=100.69mm, A=530.14mm^2 **21d** P=62.27cm, A=271.23cm^2
21e P=92.83cm, A=614.16cm^2
Exercise 10.3 **2** C=πD **3** A=πr^2 **4** π **5** cannot be expressed as a ratio of two integers **6** 1 **7** 2 **8** P(6,1) **9** 8/3
10 43.92
Exercise 10.4 **1a** 2/1 **1b** ¯2/1 **1c** ¯65/1 **2a** ¯3,¯1,0,2 **2b** ¯3,¯2,1,4 **2c** ¯2,¯1,0,1 **3a** 20%,20/100=1/5
3b 25%, 25/100=1/4 **3c** 350%, 350/100=3/1/2 **3d** 5%,5/100=1/20 **4a** 0.1, 1/10 **4b** 0.4,4/10=2/5
4c 0.25,25/100=1/4 **4d** 2.75, 275/100=11/4 **5a** 0.1, 10% **5b** 0.2, 20% **5c** 0.3, 30% **5d** 0.25, 25%
6a 30/100=3/10, 0.3, 30% **6b** 18/100=9/50, 0.18, 18% **7a** 7/4 **7b** 11/5 **7c** 14/3 **7d** 28/5 **8a** 2/1/2 **8b** 2/2/3
8c 3/3/4 **8d** 3/5/6 **9** rational **10** irrational **11a** rotation **11b** translation **11c** reflection
12a △ACB≡△VSR{SSS} **12b** △ASB≡△OMN{ASA} **12c** △ACB≡△WXY{SAS}
12d △ACB≡△HFG{RHS} **13** P(9,−2) **14** Bias in that some may respond more than once, some may not be bothered to respond, some may not know about the survey **15a** range=4, mean=2.5, median=2, mode=2
15b range=0.3, mean=6.29, median=6.3, mode=6.4 **16a** mean **16b** mode **16c** median
17

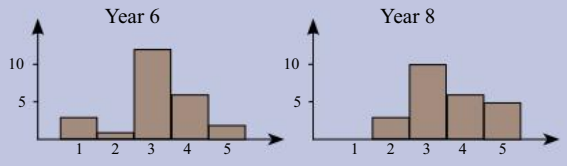
Die Toss	
1	6
2	6
3	7
4	6
5	5
6	10



18 Both frequency graphs have a similar shape with the Year 8 shifted further to the right. Together with the mean this suggests that the Year 8s have a shorter reaction distance than the Year 6s.

Year 6	
Mode	3
Median	3
Mean	3.125
Range	4

Year 8	
Mode	3
Median	3
Mean	3.54
Range	4



19 96 **20a** C=21.99cm A=38.48cm^2 **20b** C=251.33mm, A=5026.55mm^2 **20c** C=13.19cm, A=13.85cm^2
20d C=125.66m, A=1256.64m^2 **21a** P=3.08m, A=0.57m^2 **21b** P=24.94mm, A=36.95m^2
21c P=15.00cm, A=13.85cm^2 **21d** P=120.55cm, A=1012.94cm^2 **21e** P=320mm, A=1373.45mm^2

Exercise 11.1 **1a** 9:14 **1b** 14:9 **1c** 9:23 **1d** 14:23 **2a** 24:7 **2b** 7:24 **2c** 24:31 **2d** 7:31 **3a** 1/2, 0.5, 50%
3b 1/10, 0.1, 10% **3c** 1/4, 0.25, 25% **3d** 3/4, 0.75, 75% **3e** 1/5, 0.2, 20% **3f** 2/5, 0.4, 40% **3g** 2/10, 0.2, 20%
3h 4/5, 0.8, 80% **3i** 3/10, 0.3, 30% **3j** 4/10, 0.4, 40% **3k** 6/10, 0.6, 60% **3l** 7/10, 0.7, 70% **3m** 8/10, 0.8, 80%
3n 3/5, 0.6, 60% **3o** 9/10, 0.9, 90% **4a** 1:2, 0.5, 50% **4b** 3:10, 0.3, 30% **4c** 4:5, 0.8, 80% **4d** 1:4, 0.25, 25%
4e 3:4, 0.75, 75% **4f** 5:2, 2.5, 250% **4g** 31:10, 3.1, 310% **4h** 33:5, 6.6, 660%
Exercise 11.2 **1** 2:1 **2** 1:4 **3** 3:4 **4** 1:3 **5** 2:3 **6** 3:1 **7** 5:3 **8** 4:3 **9** 10:3 **10** 4:3 **11** 10:3 **12** 2:3 **13** 1:4 **14** 3:5
15 2:1
Exercise 11.3 **1** 0.4L or 400mL **2** 0.5L or 500mL **3** $66 and $88 **4** $21 and $9 **5** 5 shovelfulls **6** 8cm
7 77mL or approx 80mL
Exercise 11.4 **1a** 1/10, 0.1 **1b** 2/10=1/5, 0.2 **1c** 3/10, 0.3 **1d** 4/10=2/5. 0.4 **1e** 1/2, 0.5 **1f** 3/5, 0.6
1g 7/10, 0.7 **1h** 4/5, 0.8 **1i** 9/10, 0.9 **1j** 1/1, 1 **1k** 3/100, 0.03 **1l** 15/100=1/20, 0.05 **1m** 8/100=2/25, 0.08
1n 1/100, 0.01 **1o** 110/100=11/10, 1.1 **1p** 150/100=3/2, 1.5 **1q** 200/100=2/1, 2 **1r** 500/100=5/1, 5
1s 143/100, 1.43 **1t** 417/100,4.17 **1u** 6.2/100=62/1000=3.1/50, 0.062 **1v** 8.7/100=87/1000, 0.087
1w 12.25/100=1225/10000=49/400, 0.1225 **1x** 5.75/100=575/10000=23/400, 0.0575 **2a** 15 **2b** 6.5 **2c** 18
2d 21 **2e** 40 **2f** 12 **2g** 150 **2h** 20 **2i** 10 **2j** 30 **2k** 50 **2l** 17.28 **2m** 42kg **2n** $224 **2o** $1255.50 **2p** $25 3454
2q $8567.50 **2r** 245million **2s** 35million
Exercise 11.5 **1a** $2.64 **1b** $60.50 **1c** $53.35 **1d** $264 **1e** $159.50 **1f** $36.30 **1g** $138.16 **1h** $188.76 **2a** $168
2b $89.25 **2c** 78.75 **2d** $206.25 **2e** $17.63 **2f** $644.25 **2g** $13.88 **2h** $20.99 **3** $409.95 **4** $114.87 **5** $340.86
6a $1009.13 **6b** $553.21 **6c** $318.99 **6d** $2649.96 **7** $1006.86 **8** $5658.06 **9** 247.84million **10** 23.28million
11 $392.70

Exercise 11.6 **1a** $42 **1b** 65% **1c** 183% **1d** % of buying price 65% **2a** $185 000 **2b** 195% **2c** 66% **2d** % of buying price 195% **3a** $2500 **3b** 20% **3c** 17% **3d** % of selling price 17% **3e** % of buying price 17% **4a** $68 **4b** $73.50 **4c** 30% sounds better **4d** 20% is the better deal **5a** $20.80 **5b** $22.40 **5c** 30% sounds better, 20% is the better deal **6a** $75.65 **6b** $78.40 30% sounds better, 15% is the better deal.
Exercise 11.7 **1a** $1860 **1b** $12 090 **1c** $24 180 **2a** $4508.60 **2b** $29 305.90 **2c** $58 611.80 **2d** $586 118 **3** $22.40 **4** $8.33 **5** 837 **6a** $19.70 **6b** $39.40 **6c** $31.52 **6d** $8.37 **6e** $16.06 **6f** $274 815 **7** $450 **8** $121.80 **9** $74.40 **10** 450km **11** 2.5km **12** 160km **13** 5km **14** 20kg
Exercise 11.8 **1a** 4.4 runs per over **1b** 4.36 runs per over **1c** 125m per min **1d** 7.5km per hr **1e** $2.50 per kg **1f** $2 per kg **1g** $1.81 per 100g **1h** $1.70 per 100g **1i** 7L per 100km **1j** 8.31L per 100km **1k** 0.7m per sec **1l** 6.5m per sec **1m** $0.96 per 100g **1n** $0.92 per 100g **2a** $3.49 for 750mL **2b** $4.05 for 420g **2c** $4.37 for 300g **2d** $8.15 for 500g **2e** $2.69 for 240mL **2f** $4.99 for 70mL
Exercise 11.9 **2** $35and $15 **3** $1400 **4** 2.3 **5** 4.6 **6** $72 **7** $66 **8** 3:2 **9** 500g for $10 **10** $75
Exercise 11.10 **2** $40 and $20 **3** $1800 **4** 3.4 **5** 6.8 **6** $63 **7** $55 **8** 3:8 **9** 300g for $25 **10** $80
Exercise 11.11 **2** $40 and $30 **3** $2200 **4** 4.1 **5** 8.2 **6** $54 **7** $99 **8** 5:7 **9** 200g for $3 **10** $85
Exercise 11.12 **1** $132 **2** $0.65 **3** 5 **4** 9kg **5** $0.60 **6** $176 **7** 60% **8** 15% **9** 11/3 **10** 22/1/5 **11** 1/4 **12** multiply by 1.2 $4.80 **13** 15% **14** 120 **15** 18 **16** 400km **17** 120m
Exercise 11.13 **1** 18 **2** 18 **3** $80 **4** $70 **5** $80 **6** $60 **7** 8:12:5:9 **8** $100 **9** $2500 **10** 10
Exercise 11.14 **1** 14, 23, 32, 41, 50, 104, 131, 140, 203, 221 **2** 4+5+6 **3** 1+2+3+4+5 **4** 32 **5** 11days 13.8hours **6** 1+2+3+4+5+6+7+8x9 **7** 55
Exercise 11.15 **1a** 1/2, 0.5, 50% **1b** 2/5, 0.4, 40% **1c** 3/4, 0.75, 75% **1d** 3/10, 0.3, 30% **1e** 2/3, 0.67, 67% **1f** 7/10, 0.7, 70% **2a** 2:1 **2b** 3:4 **2c** 3:4 **2d** 5:3 **2e** 4:3 **2f** 5:3 **3** 0.8L or 800mL **4** 15 shovelfulls **5a** 15 **5b** 21 **5c** 10 **5d** 42kg **6** x1.2, $48 **7a** $80.75 **7b** $80.50 **7c** 30% sounds better, both about the same **8** $439.84 **9** $80 **10** $22.40 **11** 21kg **12a** 4.2 runs per over **12b** 8L per 100km **13a** $5.27 for 500g **13b** $8.53 for 90mL **13c** $4.05 for 200g
Exercise 11.16 **1a** 1/2, 0.5, 50% **1b** 3/5, 0.6, 60% **1c** 1/4, 0.25, 25% **1d** 1/2, 0.5, 50% **1e** 1/3, 0.33, 33% **1f** 2/5, 0.4, 40% **2a** 1:2 **2b** 2:5 **2c** 3:5 **2d** 3:5 **2e** 3:4 **2f** 3:2 **3** 0.5L or 500mL **4** 10 shovelfulls **5a** 7.5 **5b** 24 **5c** 15 **5d** 42kg **6** x1.3, $65 **7** $1534.50 **8a** $89.25 **8b** $93.75 **8c** 25% sounds better, 15% is the better deal **9** $90 **10** $13.05 **11** $4.76 **12a** 4.8 runs per over **12b** 9L per 100km **13a** $6.36 for 500g **13b** $3.15 for 2L **13c** $4.19 for 105mL

Exercise 12.1 **1** Function=2×step+1 **2** Function=2×step+2 **3** Function=3×step **4** Function=3×step−1 **5** Function=2×step−1
Exercise 12.2 **1** y=2x+2 **2** y=3x+1 **3** y=2x−1 **4** y=2x+4
Exercise 12.3 **1** Plot the points (1,4), (2,7), (3,10), (4,13), (5,16), (10,31), (20,61)
2 Plot the points (1,4), (2,6), (3,8), (4,10), (5,12), (10,22), (20,42)
Exercise 12.4 **1** x=4 **2** b=18 **3** y=5 **4** t=12 **5** x=4 **6** x=9 **7** x=25 **8** x=6 **9** x=35 **10** j=11 **11** a=32 **12** m=5 **13** z=77 **14** w=24 **15** x=70 **16** x=2 **17** h=5 **18** a=4 **19** t=9 **20** b=¯1 **21** n=10 **22** m=5 **23** x=2 **24** f=0 **25** p=8 **26** a=¯1 **27** x=7 **28** x=8 **29** x=10 **30** x=1 **31** x=3 **32** d=3 **33** x=3 **34** x=5 **35** x=1 **36** x=2
Exercise 12.5 **1** a=3 **2** x=3 **3** d=3 **4** s=3 **5** u=3 **6** w=3 **7** x=3 **8** x=8 **9** x=1 **10** x=2 **11** x=2 **12** x=2 **13** x=2 **14** x=3 **15** x=2 **16** x=4 **17** d=6 **18** g=2 **19** x=1 **20** h=3 **21** h=2 **22** n=12 **23** p=3 **24** y=5 **25** a=2 **26** w=1 **27** y=2 **28** t=6 **29** m=1 **30** x=1
Exercise 12.6 **1** Plot the points (1,3.3), (2,4.6), (3,5.9), (4,7.2), (5,8.5) **2** Plot the points (1,6.5), (2,9), (3,11.5), (4,14), (5,16.5) **3** Plot the points (1,250), (2,300), (3,350), (4,400), (5,450) **4** Plot the points (1,350), (2,450), (3,550), (4,650), (5,750) **5** Plot the points (1,100), (2,140), (3,180), (4,220), (5,260) **6** Plot the points (1,140), (2,220), (3,300), (4,380), (5,460) **7** Plot the points (1,80), (2,160), (3,240), (4,320), (5,400) **8** Plot the points (1,90), (2,180), (3,270), (4,360), (5,450) **9** Plot the points (1,−105), (2,−90), (3,−75), (4,−60), (5,−45) **10** Plot the points (1, -225), (2,-200), (3,-175), (4,-150), (5,-125) **11** Plot the points (1,130), (2,160), (3,190), (4,220), (5,250) **12** Plot the points (1,152), (2,154), (3,156), (4,158), (5,160)
Exercise 12.7 **1a** $20.60 **1b** 11km **2a** $950 **2b** 18 **3a** $430 **3b** 7hrs **4a** 400km **4b** 3hrs 15mins **5a** 100secs **5b** 9donuts **6a** 6tonnes **6b** 61cartons **7a** 150mins **7b** 3.25kg

Exercise 12.8 **1a** C=2.75k+3, $36 **1b** 11km **2a** C=75t+50, $275 **2b** 2.5hrs **3a** P=35t−325, $1250 **3b** 24people **4a** W=0.75t+4.5, 6.75tonnes **4b** 5pallets **5a** C=0.15t+35, $185 **5b** 2500flyers **6a** S=25w+135, $485 **6b** 27weeks **7a** C=4.72t+2.75, 21.63cents **7b** 6.5mins **8a** P=55i+350, $955 **8b** 23images **9a** D=25t, 125km **9b** 12hrs
Exercise 12.9 **2** x=3 **3** $30 and $20 **4** x=3 **5** m^5 **6** $55 **7** 1 **8** Charge=2.50k+3 **9** 7/2 **10** 2^2
Exercise 12.10 **2** x=12 **3** $40 and $20 **4** x=4 **5** b^6 **6** $66 **7** 1 **8** Charge=98t+15 **9** 8/3 **10** 5^2
Exercise 12.11 **2** x=9 **3** $50 and $20 **4** x=2 **5** d^5 **6** $77 **7** 1 **8** Charge=80h+70 **9** 11/4 **10** 3^3
Exercise 12.12 **1** 17 **2a** y=2x+1 **2b** y=2x−1.5 **3** 0.78 **4** 20 **5** 3.25kg **6a** 25 **6b** 26 **7a** 8g−12 **7b** 6x+3 **7c** 15y−10 **8a** x=4 **8b** x=8 **9a** a=5 **9b** n=7 **9c** x=2 **9d** x=¯1 **10a** Function=3×step+1 **10b** Function=4×step−1
Exercise 13.13 **1a** 38 **1b** 79 **2a** 74 **2b** 236 **3a** x=132° **3b** x=60° **3c** x=35° **3d** x=36° **3e** x=95° **3f** x=61° **4** x=3m **5** x=5, y=2 **6** x=30

Exercise 12.14 **1** many solutions **2** 24 **3** Monday **4** 26 **5** x=3
Exercise 12.15 **1a** y=3x+1 **1b** y=5x−3 **2a** plot the points (1,3), (2,5), (3,7), (4,9), (5,11), (10,21), (20,41) **2b** plot the points (1,1), (2,3), (3,5), (4,7), (5,9), (10,19), (20,39) **3a** x=5 **3b** a=19 **3c** y=4 **3d** x=10 **3e** h=3 **3f** a=4 **3g** a=5 **3h** x=3 **3i** x=2 **3j** x=4 **3k** y=2 **3l** x=4 **4a** Charge=3.25d+2 **4b** $34.50 **4c** 17km **5a** Profit=30p−250 **5b** $1100 **5c** 54people **6a** 231 **6b** 294
Exercise 12.16 **1a** y=2x+1 **1b** y=2x+3 **2a** plot the points (1,4), (2,7), (3,10), (4,13), (5,16), (10,31), (20,61) **2b** plot the points (1,2), (2,5), (3,8), (4,11), (5,14), (10,29), (20,59) **3a** b=1 **3b** d=15 **3c** x=6 **3d** x=10 **3e** h=4 **3f** n=7 **3g** g=4 **3h** x=6 **3i** x=3 **3j** c=7 **3k** r=2 **3l** p=6 **4a** Weight=0.55p+7.5 **4b** 11.35tonnes **4c** 9pallets **5a** Saving=35w+165 **5b** $690 **5c** 23weeks **6a** 153 **6b** 477

Exercise 13.1 **1** The sample size appears small and the sample is biased because it doesn't represent the population **2** The sample size appears large enough but has bias because only the Year 12s were sampled **3** The sample size may be small **4** The sample will be biased for many reasons - a person may respond more than once, some may be apathetic and not respond, some may not know about the survey **5** no indication of the size of the sample, bias because the sample is limited to lunchhour and cars moving past the school **6** small sample and bias because few of the sampled students may travel on the bus
Exercise 13.2 **1a** 2.67 **1b** 34.83 **1c** 64.2 **1d** 6.11 **2a** 4 **2b** 42 **2c** 33.5 **2d** 5.1 **3a** 43 **2b** 39 **2c** 117 **3c** 2.4
Exercise 13.4 **1**

Cultural	interpreting and judging phenomena in terms particular to one's own culture.
Ethnic or racial	racism, regionalism and tribalism.
Geographical	describing a dispute as it is conducted in one country, when the dispute is framed differently elsewhere.
Media	real or perceived bias of journalists and news producers within the mass media, in the selection of which events will be reported and how they are covered
Gender	including sexism, homophilia and heteronormativity.
Linguistic:	bias, favoring certain languages
Political	bias in favor of or against a particular political party, philosophy, policy or candidate.
Corporate	bias in favor of a business.
Advertising	bias with observations motivated for selling an opinion rather than using objectivity.
Sociological	bias in favor of a society's ideals. bias for groups needs/wants.
Entertainment	bias in favor of entertaining an audience
Personal	bias for personal gain.
Religious	bias for or against religion, faith or beliefs;
Sensationalist	favoring the exceptional over the ordinary. This includes emphasizing, distorting, or fabricating exceptional news to boost commercial ratings.

3 Total=600 Year7=200/600*60=20, Year8=200/600*60=20, Year9=150/600*60=15, Year10=50/600*60=5
Exercise 13.5 **1** 360±25 **2** 420±41 **3** 48±6 **4** 3 600 000±75 000 **5** 2118±340 **6** 6.3million **7a** 184million **7b** some people may reply more than once, some people may not bother to reply, some people may not know about the survey, some people may not have access and many other reasons.
Exercise 13.6 **2** Personal bias **3** 2^4 **4** x=3 **5** n^5 **6** 2.5 **7** $44 **8** Charge=2k+3 **9** 8/3 **10** 3^3

Exercise 13.7 **2** Personal bias **3** 5^6 **4** x=3 **5** a^7 **6** 4 **7** $99 **8** Charge=50t+60 **9** 7/2 **10** 6^2
Exercise 13.8 **2** Personal bias **3** 3^5 **4** x=2 **5** b^6 **6** 2.5 **7** $33 **8** Charge=98t+15 **9** 11/4 **10** 6
Exercise 13.9 **1** 30mins **2a** mean=8.67, median=4, mode=4 **2b** mean=3.5, median=3.5, mode=4
2c the mean is affected the most **3a** mean=2.14, median=2, mode=2 **3b** true **4** $2.80
5a the mean has decreased **5b** the median has decreased **5c** the range has stayed the same **6a** mean=4.29
6b mode=3 **6c** median=4 **6d** range=7 **7a** mean=24.29 **7b** mode=23 **7c** median=24 **7d** range=7
8a mean=$625k, median=$600k **8b** mean=$600k, median=$600k
Exercise 13.10 **1** 0.214 **2** 2.14 **3** 21.4 **4** 15 **5** 30 **6** 52 **7** $800 **8** $200 **9** 1000 people per km^2 **10** 6.4 **11** 8.3
Exercise 13.11 **1** 
Exercise 13.12 **1** The sample size appears small and is biased because only friends were sampled
2 Sample size would give an indication of the error and sampling method would indicate the level of bias
3a mean=5.14, median=5, range=7 **3b** mean=25.14, median=25, range=7 **3c** mean=550.75, median=547.5, range=28 **3d** mean=2.68, median=2.8, range=0.5 **4** 198 **5** 1.55million **6** Total=350, Year7=100/350*70=20, Year8=100/350*70=20, Year9=100/350*70=20, Year10=50/350*70=10
Exercise 13.13 **1** The sample size appears small and is biased because only friends were sampled
2 Sample size would give an indication of the error and sampling method would indicate the level of bias
3a mean=4.86, median=4, range=7 **3b** mean=54.68, median=54, range=7 **3c** mean=1550.75, median=1547.5, range=28 **3d** mean=6.68, median=6.8, range=0.5 **4** 1015 **5** 2.39million **6** Total=1050, Year7=250/1050*100=24, Year8=250/1050*100=24, Year9=300/1050*100=28, Year10=250/1050*100=24

Exercise 14.1 **1a** 11:30am **1b** 5:20am **1c** 11.30am **1d** 7:50am **1e** 10:10pm **2a** 5h 20min **2b** 1h 45min **2c** 8h 26min **2d** 4h 46min **2e** 3h 5min **2f** 6h 15min **3** 7:55am **4** 2h 25min **5** 9h 50min
Exercise 14.2 **1a** 1015 **1b** 0712 **1c** 0905 **1d** 1510 **1e** 1830 **1f** 2345 **1g** 0120 **1h** 1042 **1i** 1000 **2a** 2:30am **2b** 6:45am **2c** 8:17am **2d** 11:56am **2e** 1:20pm **2f** 3:15pm **2g** 6:10pm **2h** 8:40pm **2i** 11:25pm **3a** 3h 5min **3b** 7h 5min **3c** 5h 30min **3d** 1h 45min **3e** 8h 26min **3f** 19h 14min **4** 3h 10min **5** 3h 30min **6** 6h 50min **7** 13h 45min
Exercise 14.3 **1a** 9pm **1b** 9pm **1c** 8:30pm **1d** 9pm **1e** 9pm **1f** 7pm **1g** 9pm **2a** 9:30am **2b** 10am **2c** 9:30am **2d** 10am **2e** 10am **2f** 8am **2g** 10am **3a** 2:30pm **2b** 3pm **3c** 2:30pm **3d** 3pm **3e** 3pm **3f** 1pm **3g** 3pm **4a** 11pm **4b** 11:30pm **4c** 11:30pm **4d** 11:30pm **4e** 11:30pm **4f** 9:30pm **4g** 11:30pm **5a** 5:30am **5b** 6am **5c** 6am **5d** 5:30am **5e** 6am **5f** 4am **5g** 6am **6** 3pm
Exercise 14.4 **1** 3h 40min **2a** 3h 40min **2b** 2h 50min **2c** 3h 15min **2d** 4h 30min **3a** 68h 15min **3b** 74h 35min **3c** 40h 0min **3d** 41h 25min **3e** 76h 45min **3f** 68h 20min
Exercise 14.5 **1a** 7h 55min **1c** 3h 35min **1c** 13h 45min
Exercise 14.6 **2** 48h **3** 5^6 **4** x=3 **5** 7:35pm **6** 2.5 **7** $88 **8** Charge=2.90k+3 **9** 15° **10** 4^2
Exercise 14.7 **2** 72h **3** 2^5 **4** x=¯3 **5** 1520 **6** 2 **7** $25.30 **8** Charge=70t+50 **9** 30° **10** 2^8
Exercise 14.8 **2** 96h **3** a^6 **4** x=¯2 **5** 10:23am **6** 2.5 **7** $37.40 **8** Wage=50s+450 **9** 45° **10** x^3
Exercise 14.9 **1** 4km **2** 3.8h or 3h 48mins **3a** 3h 15min **3b** 6h 10min **3c** 13h 23min **3d** 4h 45min **3e** 6h 20min **4** 7h 20min **5** 3h 12min **6** $150 **7** 480km **8** 75km/h **9** 2h 40min **10** 3h 30min **11** 8 years old
Exercise 14.10 **1** 7:35am **2** 10:20pm **3** 7:30am **4** 3h 8min **5** 4h 38min **6** 3h 45min **7** 105h **8** 4h 50min **9** 2 years **10** October 2008 **11** 48mins **12** 44
Exercise 14.11 **1** Many answers eg 7,5,2 **2** Grandmother, mother, daughter **3** 123 454 321
Exercise 14.12 **1a** 11:10am **1b** 11:30am **1c** 10:40am **2a** 6h 10min **2b** 1h 40min **2c** 3h 5min **2d** 6h 15min **3** 8am **4a** 0905 **4b** 0742 **4c** 1510 **5a** 1:50am **5b** 9:15am **5c** 6:35pm **6a** 4h 20min **6b** 6h 25min **7** 3h 10min **8a** 8:30am **8b** 9am **8c** 8:30am **8d** 9am **8e** 9am **8f** 7am **8g** 9am **9** 1h 25min **10** 3h 40min
Exercise 14.13 **1a** 7:50pm **1b** 3:40pm **1c** 9:50am **2a** 3h 30min **2b** 3h 20min **2c** 4h 35min **2d** 6h 15min **3** 8:50am **4a** 0845 **4b** 1117 **4c** 1818 **5a** 3:05am **5b** 10:55am **5c** 9:05pm **6a** 4h 20min **6b** 5h 45min **7** 1h 40min **8a** 4:30pm **8b** 5pm **8c** 4:30pm **8d** 5pm **8e** 5pm **8f** 3pm **8g** 5pm **9** 3h 10min **10** 4h 20min

Exercise 15.1 **2** 48h **3** 2^5 **4** x=4 **5** 3:30pm **6** 2 **7** $33 **8** Charge=3.20d+3 **9** 15° **10** 3^2
Exercise 5.2 **1a** 1/2, 0.5, 50% **1b** 2/5, 0.4, 40% **1c** 1/4, 0.25, 25% **1d** 1/2, 0.5, 50% **1e** 3/10, 0.3, 30% **1f** 3/5, 0.6, 60% **2a** 1:3 **2b** 1:5 **2c** 3:5 **2d** 2:3 **2e** 3:5 **2f** 1:2 **3** 0.4L=400mL **4** 10 shovelfulls **5a** 8 **5b** 20 **5c** 24 **6a** $115.50 **6b** $6.22 **7** x1.2, $60 **8a** $106.25 **8b** $108.75 **8c** The 25% sounds better, 15% is the better deal **9** $15.68 **10** $4.41 **11a** 4.4 runs per over **11b** 9L per 100km **12a** $5.36 per 400g **12b** $4.85 for 2L **12c** $6.14 for 1L **13a** y=3x+2 **13b** y=5x−3
14a Plot the points (1,3), (2,5), (3,7), (4,9), (5,11), (10,21), (20,41)
14b Plot the points (1,1), (2,4), (3,7), (4,10), (5,13), (10,28), (20,58) **15a** a=5 **15b** b=13 **15c** c=3 **15d** x=11 **15e** x=2 **15f** x=¯2 **15g** d=4 **15h** x=4 **15i** x=4 **15j** c=5 **15k** x=1 **15l** m=4 **16a** Weight=0.45p+3.5 **16b** 7.1tonnes **16c** 5pallets **17a** Savings=45w+145 **17b** $685 **17c** 11weeks
18 the sample size appears to be small and biased in that only friends were sampled
19 The sample size indicates the error and the sampling method indicates the bias
20a mean=2.14, median=2,range=3 **20b** mean=72.14, median=72, range=3
20c mean=655.75, median=657.5, range=28 **20d** mean=2.68, median=2.8, range=0.5 **21** 659 **22** 2.8million **23** Total=500, Year7=150/500*60=18, Year8= 150/500*60=18, Year9=100/500*60=12, Year10=100/500*60=12 **24** 10:45am **25** 3h 30min **26** 7:50am **27a** 0435 **27b** 1146 **27c** 1113 **28a** 5:30am **28b** 10:10am **28c** 11:32pm **29a** 4:30pm **29b** 5pm **29c** 4:30pm **29d** 5pm **29e** 5pm **29f** 3pm **29g** 5pm **30** 4h 50min
Exercise 15.1 **2** 72h **3** 3^4 **4** x=3 **5** 6:45pm **6** 3 **7** $44 **8** Charge=92m+25 **9** 30° **10** x^3
Exercise 5.3 **1a** 1/2, 0.5, 50% **1b** 3/10, 0.3, 30% **1c** 1/4, 0.25, 25% **1d** 1/2, 0.5, 50% **1e** 2/3, 0.67, 67% **1f** 2/5, 0.4, 40% **2a** 1:3 **2b** 1:2 **2c** 4:3 **2d** 2:3 **2e** 2:3 **2f** 3:4 **3** 0.5L=500mL **4** 5 shovelfulls **5a** 6 **5b** 24 **5c** 30 **6a** $137.50 **6b** $9.30 **7** x1.4, $70 **8a** $148 **8b** $143.50 **8c** The 30% sounds better, 30% is the better deal **9** $24.07 **10** $2.15 **11a** 5.3 runs per over **11b** 11L per 100km **12a** $15.05 per 750g **12b** $5.19 for 3L **13a** y=3x+1 **13b** y=6x−4 **14a** Plot the points (1,5), (2,7), (3,9), (4,11), (5,13), (10,23), (20,43)
14b Plot the points (1,1), (2,4), (3,7), (4,10), (5,13), (10,28), (20,58) **15a** a=2 **15b** b=12 **15c** x=3 **15d** x=6 **15e** y=7 **15f** u=5 **15g** k=3 **15h** x=3 **15i** x=4 **15j** d=8 **15k** p=1 **15l** t=5 **16a** Charge=3.5d+3 **16b** $38 **16c** 12km **17a** Profit=25t−150 **17b** $650 **17c** 27people **18** the sample size appears to be small and biased in that only friends were sampled **19** The sample size indicates the error and the sampling method indicates the bias
20a mean=4.86, median=4,range=6
20b mean=34.86, median=34, range=6 **20c** mean=72.5, median=72.5, range=5
20d mean=1.58, median=1.5, range=0.5 **21** 1061 **22** 970 000 **23** Total=450, Year7=120/450*90=24, Year8=120/450*90=24, Year9=110/450*90=22, Year10=100/450*90=20 **24** 10:55am **25** 8h 25min **26** 8:20am **27a** 0319 **27b** 1023 **27c** 2318 **28a** 8:12am **28b** 11:46am **28c** 10:38pm **29a** 9pm **29b** 9:30pm **29c** 9:30pm **29d** 9:30pm **29e** 9:30pm **29f** 7:30pm **29g** 9:30pm **30** 5h

Exercise 16.1 **1** A(1,6), B(0,4), C(0,2), D(2,3), E(2,1), F(3,5), G(4,8), H(4,1), I(5,3), J(5,0), K(6,7), L(6,2), M(7,5), N(7,0), O(8,1) **Exercise 16.2** **1** A(−7,7), B(−7,−3), C(−6,−7), D(−4,6), E(−4,−2), F(−3,3),G(−3,−5), H(0,5), I(2,0), J(2,−5), K(4,6), L(4,2), M(5,−2), N(6,−7), O(8,8) **Exercise 16.3** **1** $C=40d+30 **2** $C=60d+40 **3** $C=55d+45 **4** $C=60d+20 **5** $C=2k+2 **6** $C=1.5k+2.5 **7** T=7t+18 **Exercise 16.4** **1a** $C=4aa+140 **1b** $380 **1c** 6.25years **2a** $S=25w+225 **2b** $350 **2c** 21weeks **3a** $S=7d+15 **3b** $32.50 **3c** $1.60

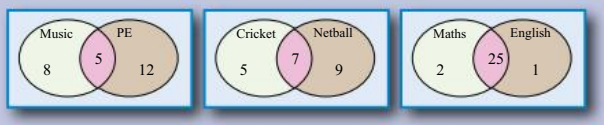
Exercise 16.5
1 Plot the points (−2,−2), (−1,0), (0,2), (1,4), (2,6) y=4,x=1
2 Plot the points (−2,−1), (−1,1), (0,3), (1,5), (2,7) y=4,x=0.5
3 Plot the points (−2,1), (−1,3), (0,5), (1,7), (2,9) y=4, x=−0.5
4 Plot the points (−2,−5), (−1,−3), (0,−1), (1,1), (2,3) y=4,x=2.5
5 Plot the points (−2,−8), (−1,−6), (0,−4), (1,−2), (2,0) y=4,x=4
6 Plot the points (−2,−5), (−1,−2), (0,1), (1,4), (2,7) y=4,x=1
7 Plot the points (−2,−4), (−1,−1), (0,2), (1,5), (2,8) y=4,x=0.67
8 Plot the points (−2,−2), (−1,1), (0,4), (1,7), (2,10) y=4,x=0)
9 Plot the points (−2,−7), (−1,−4), (0,−1), (1,2), (2,5) y=4,x=1.67
10 Plot the points (−2,−8), (−1,−5), (0,−2), (1,1), (2,4) y=4,x=2

Exercise 16.6
1 Plot the points (−2,4), (−1,1), (0,0), (1,1), (2,4) y=3,x=1.7 or x=¯1.7
2 Plot the points (−2,5), (−1,2), (0,1), (1,2), (2,5) y=3,x=1.4 or x=¯1.4
3 Plot the points (−2,6), (−1,3), (0,2), (1,3), (2,6) y=3, x=1 or x=¯1
4 Plot the points (−2,7), (−1,4), (0,3), (1,4), (2,7) y=3,x=0
5 Plot the points (−2,3), (−1,0), (0,−1), (1,0), (2,3) y=3,x=2 or x=¯2
6 Plot the points (−2,2), (−1,−1), (0,−2), (1,−1), (2,2) y=3,x=2.4 or x=¯2.4
7 Plot the points (−2,2), (−1,0), (0,0), (1,2), (2,6) y=3,x=1.3
8 Plot the points (−2,6), (−1,2), (0,0), (1,0), (2,2) y=3,x=2.3)
9 Plot the points (−2,−8), (−1,−1), (0,0), (1,1), (2,8) y=3,x=1.4
10 Plot the points (−2,−0.5), (−1,−1), (0,unknown - cannot divide by 0), (1,1), (2,0.5) y=3,x=0.33
Exercise 16.7 **1** It was 2669 the next day **2** It was 11825 the next day **3** It was 3453 the next day
Exercise 16.8 **2** 64 **3** y=1 **4** x=2 **5** function=2×step+1 **6** 6:30pm **7** 3.5 **8** 2 **9** 2^4 **10** b^6
Exercise 16.9 **2** 68 **3** y=¯1 **4** x=3 **5** function=3×step−1 **6** 1650 **7** √10 **8** ¯1.5 **9** 3^6 **10** b^3
Exercise 16.10 **2** 100 **3** y=3 **4** x=5 **5** function=4×step+1 **6** 3:10pm **7** 1.8 **8** 3 **9** a^4 **10** x^{10}
Exercise 16.11 **1** 15 **2** 18 **3** 37 **4** $F=50h+40 **5** y=2x+1 **6a** 4 **6b** 11 **6c** 3 (or ¯3) **7** B(1,7)
Exercise 16.12 **1** 20miles **2** no **3** (10,1808) many other points **4a** y=2x+1 **4b** y=¯2x−1 **5** Cost=3000b+3000 **6a** y=2x+1 **6b** y=0.5x+3
Exercise 16.13 **1** A(1,3), B(6,2), C(0,6), D(4,¯3), E(2,¯5), F(4,¯7), G(0,¯2), H(¯3,7), I(¯4,3), J(¯6,1), K(¯5,¯3), L(¯4,¯5) **3b** $C=45d+55 **4a** $S=100w+150 **3b** $750 **3c** 8weeks **4** Plot the points (−2,−2), (−1,0), (0,2), (1,4), (2,6) y=4,x=1
Exercise 16.14 **1** A(4,1), B(7,6), C(0,4), D(7,¯2), E(4,¯5), F(2,¯7), G(0,¯4), H(¯5,5), I(¯3,2), J(¯5,0), K(¯6,¯2), L(¯3,¯6) **3b** $C=40d+40 **4a** $S=50w+70 **4b** $420 **4c** 17weeks **5** Plot the points (−2,−1), (−1,1), (0,3), (1,5), (2,7) y=6,x=1.5

Exercise 17.1 **1** 13m **2** 21m **3** 16m **4** 24cm **5** 24cm **6** 26m **7** 566m **8a** 2968m **8b** $760
Exercise 17.2 **1** 23.2m **2** 9.2cm **3** 96m **4** 11.8km
Exercise 17.3 **1** 13.69m^2 **2** 5.29cm^2 **3** 27m^2 **4** 432m^2 **5** 8.14km^2 **6** 7.685cm^2 **7** 4.06m^2 **8** 13 764m^2, 1.38ha **9** 320cm^2 **10** 9.24m^2 **11** 148 068m^2, 14.81ha **12** 112m^2
Exercise 17.4 **1** 81m^2 **2** 486cm^2 **3** 177cm^2 **4** 105m^2 **5** 200cm^2 **6** 3570cm^2 **7** 2138.5cm^2 **8** 180.5m^2 **9** 209.5cm^2
Exercise 17.5 **1** 60cm^2 **2** 162m^2 **3** 112cm^2 **4** 25m^2 **5** 9.12cm^2 **6** 208.03m^2 **7** 200cm^2 **8** 60.9cm^2
Exercise 17.6 **1** 42cm^2 **2** 79.8m^2 **3** 31.5m^2 **4** 34.965m^2 **5** 44m^2 **6** 43.4cm^2
Exercise 17.7 **1** 490m^3 **2** 2.208cm^3 **3** 836.64cm^3 **4** 40 265.16cm^3 **5** 2 000cm^3=2L **6** 30 976cm^3=30.98L **7** 800 000 000cm^3=800 000L
Exercise 17.8 **1** 308m^3 **2** 1368cm^3 **3** 1.134m^3 **4** 119.09m^3 **5** 450cm^3=675g
Exercise 17.9 **2** $35:$15 **3** 4.6 **4** Area = ½bh **5** Area=bh **6** Volume=Areabasexh **7** 24m **8** 32m^2 **9** 90cm^3 **10** 10 000m^2
Exercise 17.10 **2** $20:$40 **3** 8.4 **4** Area = lb **5** Area=bh **6** Volume=½Areabasexh **7** 28m **8** 12m^2 **9** 120cm^3 **10** 10000m^2
Exercise 17.11 **2** $30:$40 **3** 6.8 **4** Area = ½bh **5** Area=½xy **6** Volume=Areabasexh **7** 20m **8** 24m^2 **9** 100m^3 **10** 2ha
Exercise 17.12 **1** 126m^2 **2** c) 4.3mx7.2m **3a** square based pyramid **3b** triangular prism **3c** triangular based pyramid **4** 4 times larger **5** 183.26 cm^3 **6** 12cm **7** 1:2 **8** 16a^2 **9** 175units^2
Exercise 17.13 **1a** 12cm^2 **1b** 15cm^2 **1c** 75cm^2 **1d** 171.5cm^2 **2** 24L **3** 21teaspoons **4** 48cm^2 **5** 24cm **6** 3cm **7** 100m^2 **8** 60m^2
Exercise 17.14 **1** 10 **2** 15 **3** 15mins **4** 1km
Exercise 17.15 **1a** 11m **1b** 26cm **1c** 32.6km **2a** 24.01m^2 **2b** 28.5km^2 **2c** 9.765cm^2 **2d** 158cm^2 **2e** 333cm^2 **2f** 220cm^2 **2g** 120.7m^2 **2h** 18.815m^2 **2i** 66cm^2 **3a** 68.068cm^3 **3b** 1350cm^3
Exercise 17.16 **1a** 13m **1b** 16cm **1c** 26.6km **2a** 54.76m^2 **2b** 14.28km^2 **2c** 26.535cm^2 **2d** 357cm^2 **2e** 1083cm^2 **2f** 325cm^2 **2g** 45.92m^2 **2h** 6.56m^2 **2i** 170.5cm^2 **3a** 119.392cm^3 **3b** 912cm^3

Exercise 18.1 **1a** 1 **1b** 0 **1c** 0.5 **1d** .25 **1e** 0.75 **1f** 0.000001 **2a** 1/6 **2b** 1/2 **2c** 1/2 **2d** 1/6 **2e** 1/3 **2f** 1/2 **2g** 0 **2h** 1/3 **3a** 3/8 **3b** 5/8 **3c** 5/8 **3d** 3/8 **3e** 1 **3f** 0

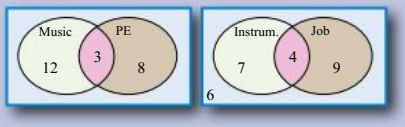
Exercise 18.2 **1a** 1/5 **1b** 1 **1c** 12/25 **1d** 8/25
2b 1/3 **2b** 1 **2c** 3/7 **2d** 5/21
3b 25/28 **3b** 1 **3c** 1/28 **3d** 1/14



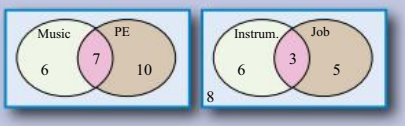
Exercise 18.3 **1a** 7/25 **1b** 3/5 **1c** 12/25 **1d** 16/25 **1e** 2/5
2a 8/27 **2b** 17/27 **2c** 16/27 **2d** 13/27 **2e** 10/27
3a 45/150=3/10 **3b** 106/150=53/75 **3c** 62/150=31/75
3d 87/150=29/50 **3e** 44/150=22/75

Exercise 18.4 **5a** 0.25 **5b** 0.5 **5c** 0.25
Exercise 18.5 **1a** 0.25 **1b** 0.25 **1c** 0.25 **2a** 1/12 **2b** 1/12 **2c** 1/4 **3a** 1/36 **3b** 1/36 **3c** 3/36=1/12 **3d** 2/36=1/18 **3e** 6/36=1/6 **4a** 4/16=1/4 **4b** 4/16=1/4 **4c** 4/16=1/4 **4d** 5/16
Exercise 18.6 **2** 30m **3** x=2 **4** 1/2 **5** $^-1$ **6** y=3x+1 **7** 1/4 **8** 3 **9** ½bh **10** a^8
Exercise 18.7 **2** 5km **3** x=2 **4** 1/2 **5** $^-3$ **6** y=5x−3 **7** 1/2 **8** 7 **9** bh **10** a^5
Exercise 18.8 **2** 60m **3** x=3.5 **4** 1/3 **5** 0 **6** y=4x−1 **7** 1/4 **8** 4 **9** 24m **10** c^4
Exercise 18.9 **1** 5 **2** 5 **3** 3/9=1/3 **4** 25/55=5/11 **5** 1/2=0.5 **6** 1/36 **7** 6 **8** 4/14=2/7
Exercise 18.10 **1** green **2** 7/28=1/4 **3** 7 **4a** 4 **4b** 3/16 **5a** 4 **5b** 24
Exercise 18.11 **1a** 5 **1b** 14 **1c** 30 **1d** 55 The number of squares and the answer to the calculation is the same **2a** 4+4+4+4=16 **2b** 4+(4+4)÷4=6 **2c** 4+4−4÷4=7 or 44÷4−4=7

Exercise 18.12 **1a** 3/23 **1b** 23/23=1 **1c** 8/23 **1d** 15/23
2a 4/26=2/13 **2b** 20/26=10/13 **2c** 15/26
2d 13/26=1/2 **2e** 6/26=3/13
3a 1/4 **3b** 1/2 **3c** 1/4 **4a** 1/12 **4b** 1/12 **4c** 3/12=1/4
5a 4/16=1/4 **5b** 4/16=1/4 **5c** 5/16



Exercise 18.13 **1a** 7/23 **1b** 23/23=1 **1c** 10/23 **1d** 6/23
2a 3/22 **2b** 14/22=7/11 **2c** 13/22
2d 14/22=7/11 **2e** 8/22=4/11
3a 1/4 **3b** 1/2 **3c** 1/4 **4a** 1/12 **4b** 1/12 **4c** 3/12=1/4
5a 4/16=1/4 **5b** 4/16=1/4 **5c** 4/16=1/4 **5d** 5/16



Exercise 19.5 **1** sideBA=sideTV, side CB=sideST, side AC=sideVS, △ACB ≡ △VST {**SSS**}
2 ∠A = ∠J, ∠B = ∠H, side AC=side JK, △ACB ≡ △JKH {**ASA**}
3 side BA=side GK, ∠B = ∠G, side CB=side IG, △ACB ≡ △KIG {**SAS**}
4 ∠C = ∠Y {right-angle}, side BA=side ZT {hypotenuse}, side AC=sideTY, △ACB ≡ △TYZ {**RHS**}

Exercise 19.6 **1** △ACB ≡ △QPR. AC=QP {matching sides} ∴19m=y, ∠BAC=∠RQP {matching angles} ∴30°=x
2 △ABD ≡ △CBD. AD=CD {matching sides} ∴y=26cm, ∠DAB=∠DCB {matching angles} ∴45°=x
3 △ABD ≡ △CDB. AD=CB {matching sides} ∴y=19cm, ∠ABD=∠CDB {matching angles} ∴x=63°
4 △ABD ≡ △CDB. DA=BC {matching sides} ∴y=73cm, ∠ABD=∠CDB {matching angles} ∴x=18°
5 △ABD ≡ △CBD. AB=CB {matching sides} ∴x=9m, ∠BDA=∠BDC {matching angles} ∴y=45°
6 △ABE ≡ △CBD. BE=BD {matching sides} ∴y=12cm, ∠ABE=∠CBD {matching angles} ∴54°=x

Exercise 19.7
1 ∠ADB= ∠CBD {AD // CB}, DB= BD {same side}, ∠DBA= ∠BDC {AB // CD}, ∴△ADB ≡ △CBD {**ASA**}
2 △ADC ≡ △CBA {**ASA**}, ∴AD = CB {matching sides}
3 △ADC ≡ △CBA {**ASA**}, ∴DC = BA {matching sides}

Exercise 19.7

4 sideAB=sideCD, ∠ABO=∠CDO {AB // CD}, ∠OAB=∠OCD {AB // CD}, ∴ △ABO ≡ △CDO {**ASA**}
△ABO ≡ △CDO {**ASA**}, BO=DO {matching sides}, OA=OC {matching sides} ∴ diagonals bisect each other.

Exercise 19.8

1 ∠CAB= ∠ACD {BC // DA}, CA= AC {same side}, ∠BCA= ∠DAC {BC // DA},
∴ △CAB ≡ △ACD {**ASA**}

2 △DAB ≡ △BCD {**ASA**}, ∴ DA = BC {matching sides}

3 △DAB ≡ △BCD **ASA**}, ∴ AB = CD {matching sides}

4 sideAB=sideCD, ∠ABO=∠CDO {AB // CD}, ∠OAB=∠OCD {AB // CD}, ∴ △ABO ≡ △CDO {**ASA**}
∴ BO=DO {matching sides}, ∴ OA=OC {matching sides} ∴ diagonals bisect each other.

Exercise 19.9

1 △BAD ≡ △BCD {**ASA**}, ∴ ∠BAD = ∠BCD {matching angles}

2 DA=DC {property of kite}, OD=OD {common side}, ∠ODA=∠ODC {matching angle},
∴ △AOD ≡ △COD {**ASA**} AO=CO {matching sides} The longer diagonal bisects the shorter diagonal.

3 △AOD ≡ △COD {**ASA**} ∠AOD=∠COD {matching angles}, ∠AOD+∠COD = 180°
∴ ∠AOD=90°, ∠COD=90°

Exercise 19.10 **2** side, side, side for congruency **3** ≡ or ≅ **4** $44 **5** 40m **6** ¯5 **7** 1/4 **8** 8 **9** 1.5 **10** a^7

Exercise 19.11 **2** angle, side, angle for congruency **3** ≡ or ≅ **4** $55 **5** 5km **6** ¯2 **7** 1/4 **8** 2 **9** 1.5 **10** b^3

Exercise 19.12 **2** side, angle, side for congruency **3** ≡ or ≅ **4** $66 **5** 50m **6** ¯1 **7** 1/2 **8** 3 **9** 2 **10** a^8

Exercise 19.13 **1a** A and D **1b** A and B **2** all three are true **3** trapezium **4** 1/2 **5** 1/2 **6** 1/8 **7** R {same radius}

Exercise 19.14 **1** 110° **2** 95° **3** yes **4** 4a and 4c **5** (2,−2) **6** (2,8), (−10,−2), and (2,−2) **7** 21m^2

8  49cm^2

Exercise 19.15 **2** 0.5years

Exercise 19.16 **2a** x=61°, y=35cm **2b** x=12cm, y=42°

3a ∠ABD= ∠CDB {AB // CD}, BD= DB {same side}, ∠BDA= ∠DBC {DA // BC},
∴ △ABD ≡ △CDB {**ASA**}

3b △ABD ≡ △CDB {**ASA**} AD=CB {matching sides}

4a AD=CD {property of kite}, BA=BC {property of kite}, DB=DB {same side}
∴ △ADB ≡ △CDB {**SSS**}

4b DA=DC {property of kite}, OD=OD {common side}, ∠ODA=∠ODC {matching angle},
∴ △AOD ≡ △COD {**ASA**} AO=CO {matching sides}
∴ The longer diagonal bisects the shorter diagonal.

4c △AOD ≡ △COD {**ASA**} ∠AOD=∠COD {matching angles}, ∠AOD+∠COD = 180°
∴ ∠AOD=90°, ∠COD=90°

Exercise 19.17 **2a** x=25°, y=27cm **2b** x=7cm, y=105°

3a ∠ABD= ∠CDB {AB // CD}, BD= DB {same side}, ∠BDA= ∠DBC {DA // BC},
∴ △ABD ≡ △CDB {**ASA**}

3b △ABD ≡ △CDB {**ASA**} AD=CB {matching sides}

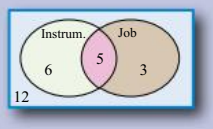
4a AD=CD {property of kite}, BA=BC {property of kite}, DB=DB {same side}
∴ △ADB ≡ △CDB {**SSS**}

4b DA=DC {property of kite}, OD=OD {common side}, ∠ODA=∠ODC {matching angle},
∴ △AOD ≡ △COD {**ASA**} AO=CO {matching sides}
∴ The longer diagonal bisects the shorter diagonal.

4c △AOD ≡ △COD {**ASA**} ∠AOD=∠COD {matching angles}, ∠AOD+∠COD = 180°
∴ ∠AOD=90°, ∠COD=90°

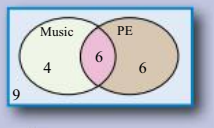
Exercise 20.1 **2** side, side, side for congruency **3** ≡ or ≅ **4** 60 **5** 700m **6** ¯9 **7** 1/4 **8** 4 **9** 4 **10** a^8

Exercise 20.2 **2a** $S=50w+150 **2b** $550 **2c** 12weeks **3a** Plot the points (−2,−3), (−1,−1), (0,1), (1,3), (2,5)
3b x=1.5 **4a** 720cm **4b** 14 200m **4c** 0.65km **4d** 9.81m **4e** 70km **4f** 0.2km **5a** 17m **5b** 28cm **5c** 10.6km
6a 68.89m^2 **6b** 106.14km^2 **6c** 7.76cm^2 **6d** 2418cm^2 **6e** 7.56m^2 **6f** 24.75m^2
7a 69.47cm^3 **7b** 833cm^3 **8a** 5/26 **8b** 14/26=7/13 **8c** 15/26 **8d** 18/26=9/13
9a 1/4 **9b** 1/2 **9c** 1/4 **10a** 1/12 **10b** 1/12 **10c** 3/12=1/4
11a 4/16=1/4 **11b** 12/16=3/4 **11c** 0 **11d** 5/16
13a x=61°, y=35cm **13b** x=12cm, y=42°
14a ∠ABD= ∠CDB {AB // CD}, BD= DB {same side}, ∠BDA= ∠DBC {DA // BC},
∴ △ABD ≡ △CDB {**ASA**}
14b △ABD ≡ △CDB {**ASA**} AD=CB {matching sides}
14c △ABD ≡ △CDB {**ASA**} ∠BAD=∠DCB {matching angles}
14d Assume diagonals intersect at O. sideAB=sideCD, ∠ABO=∠CDO {AB//CD},
∠OAB=∠OCD {AB//CD},
∴ △ABO ≡ △CDO {**ASA**}
BO=DO {matching sides}, OA=OC {matching sides}
∴ diagonals bisect each other.



Exercise 20.3 **2** side, angle, side for congruency **3** ≡ or ≅ **4** 64 **5** 70m **6** ¯7 **7** 1/4 **8** 3 **9** ¯2 **10** a^8

Exercise 20.4 **2a** $S=100w+50 **2b** $650 **2c** 8weeks **3a** Plot the points (−2,−5), (−1,−2), (0,1), (1,4), (2,7)
3b x=2 **4a** 450cm **4b** 24 230m **4c** 7.65km **4d** 23.4m **4e** 30km **4f** 0.5km **5a** 21m **5b** 26cm **5c** 23.8km
6a 13.69m^2 **6b** 36.26km^2 **6c** 30.69cm^2 **6d** 1625cm^2 **6e** 7.29m^2 **6f** 100.5cm^2
7a 678.91cm^3 **7b** 2099.5cm^3 **8a** 6/25 **8b** 16/25 **8c** 15/25=3/5 **8d** 13/25
9a 1/4 **9b** 1/2 **9c** 1/4 **10a** 1/12 **10b** 1/12 **10c** 1/4
11a 8/16=1/2 **11b** 8/16=1/2 **11c** 0 **11d** 5/16
13a x=25°, y=27cm **13b** x=7cm, y=105°
14a ∠ABD= ∠CDB {AB // CD}, BD= DB {same side}, ∠BDA= ∠DBC {DA // BC},
∴ △ABD ≡ △CDB {**ASA**}
14b △ABD ≡ △CDB {**ASA**} AD=CB {matching sides}
14c △ABD ≡ △CDB {**ASA**} ∠BAD=∠DCB {matching angles}
14d Assume diagonals intersect at O. sideAB=sideCD, ∠ABO=∠CDO {AB//CD},
∠OAB=∠OCD {AB//CD},
∴ △ABO ≡ △CDO {**ASA**}
BO=DO {matching sides}, OA=OC {matching sides}
∴ diagonals bisect each other.



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