PEARSON

PHYSICS

WESTERN AUSTRALIA

12

2ND EDITION

CHAPTER 02

Equilibrium of forces

A well-designed building should be stable when exposed to a wide variety of forces. Engineers and architects must use their understanding of physics to predict the forces acting on a structure and thus ensure the design is safe.

When all the forces acting on an object add up to a zero net force, the body is said to be in translational equilibrium. There will be no translational acceleration if the forces acting on the object are balanced. For a structure to be completely at rest and stable, however, it is not enough that it is simply in translational equilibrium. The structure or system must also be in rotational equilibrium, which means that the sum of all the torques acting about a point are zero. When an object is in translational and rotational equilibrium, it is said to be in static equilibrium.

This chapter focuses on the concept of equilibrium, which describes the situation in which forces and torques are balanced. If there is equilibrium of translational forces, then there will be no net translational force. If there is equilibrium of torque or moments, then the object will not be rotating.

Science Understanding

- the stability of an object depends on the location of its centre of mass
- when an object experiences a net force at a distance from a pivot and at an angle to the lever arm, it will experience a torque or moment about that point, which includes applying the relationship:
  $\tau = rF\sin\theta$
- for a rigid body to be in equilibrium, the sum of the forces and the sum of the moments must be zero, which includes applying the relationships:
  $\Sigma F = 0$
  $\tau = rF\sin\theta$
  $\Sigma\tau = 0$
- static equilibrium contexts may include:
- objects leaning against a frictionless wall, e.g. a ladder
- objects pivoting about one point, e.g. a seesaw
- bridges and cantilevers
- suspension of objects by cables, e.g. signs

2.1 Torque

Many real-life situations involve objects that rotate about a pivot point, such as closing a door, using a spanner or turning a steering wheel (Figure 2.1.1). In these situations, a force acts to provide a turning effect, known as a **torque** (τ). Newton's laws use the concept of force to help explain the motion of a body in a straight line. The concept of torque is used in exactly the same way to explain changes in the rotational motion of an object.

FIGURE 2.1.1 Applying a torque to the rim or arms of a steering wheel will cause it to turn.

UNDERSTANDING TORQUE

When a turning effect is applied to an object, a number of factors must work together to cause that object to turn. First, there must be a **pivot point** around which the object will rotate; this is referred to as the **axis of rotation**. There must also be a force (F) applied to the object in such a way as to cause the object to rotate. To allow rotation, the force must not be applied through the pivot point, but rather at some perpendicular distance from it. The force applied will have a **line of action**, which must not pass through the pivot point either.

EXTENSION

Torque and moment of a force

The terms 'torque' (τ) and 'moment of a force' (M), which is usually shortened to 'moment', are used interchangeably in physics at the high school level.

The difference between these terms is in the conventions of their use. Typically, torque is used for dynamic problems in which there is an angular acceleration, and so the speed at which an object rotates is changing. Moments are typically used in static problems, often as reaction forces or internal forces in an object, and are balanced so that there is no angular acceleration. An example of how a torque is used would be in the force applied by an axle to the wheel of a car to increase its speed of rotation. A moment might be used when calculating the reaction force for a beam that is attached to a wall at one end and has a force applied on the other.

Both torque and moment are calculated using the equation $\tau = rF \sin\theta$.

Force and the pivot point

When analysing a rotating system, the position of the pivot point or axis of rotation must be considered (Figure 2.1.2). A wheel, for example, moves in a circular path around its axle. The imaginary line along the length of the axle is the axis of rotation.

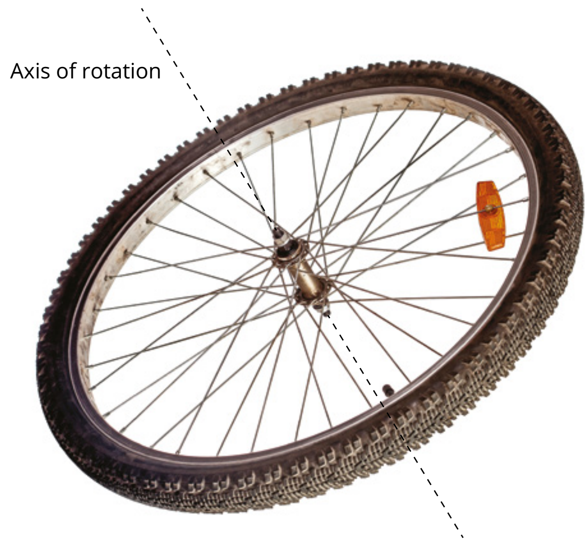


FIGURE 2.1.2 The axis of rotation.

A force applied directly towards or away from the axis of rotation will not create a turning effect on the wheel. This is because the line of action of the force passes through the pivot point (Figure 2.1.3).

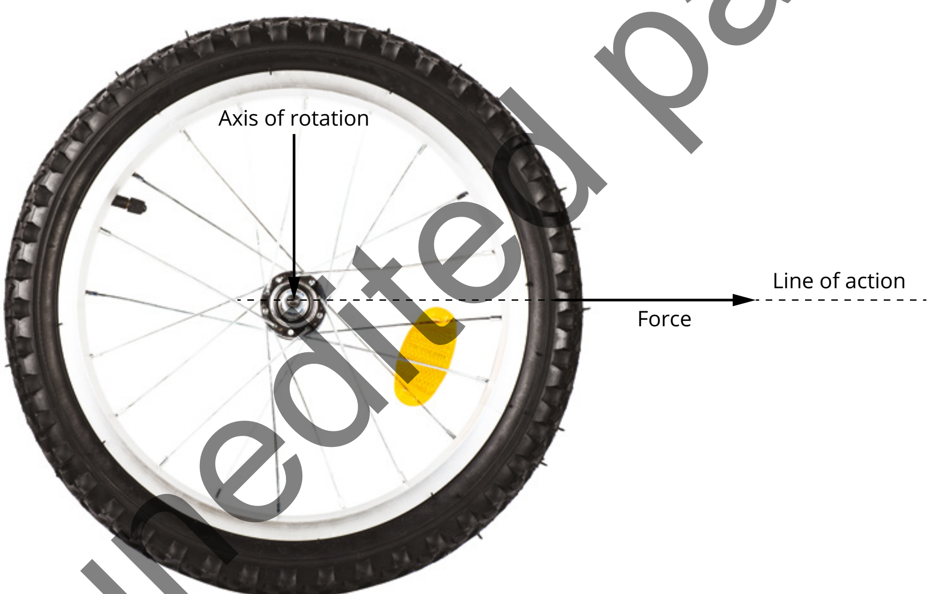


FIGURE 2.1.3 When the line of action of the force passes through the pivot point, the wheel will not experience a torque.

Torque is achieved by applying a force on the wheel at a point where the line of action of the force does not pass through the axis of rotation or pivot point. The maximum effect is achieved when the force applied is at 90° to a line drawn from the axis of rotation to the point at which the force is applied (Figure 2.1.4).

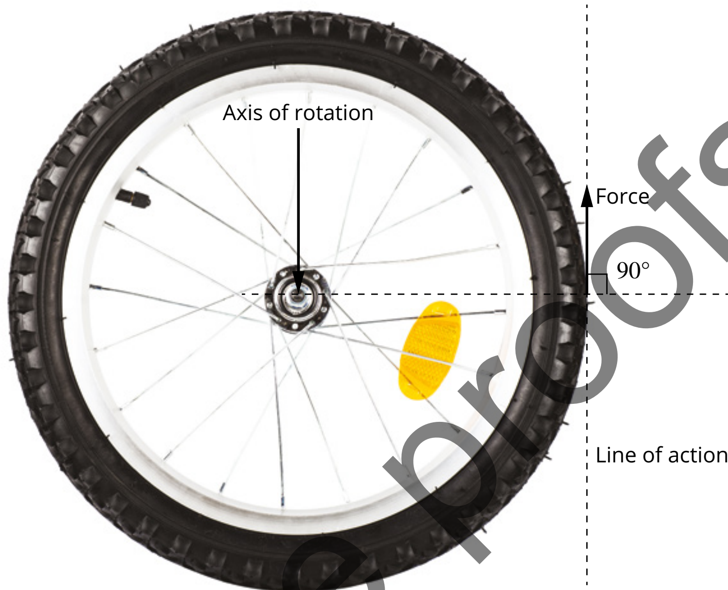


FIGURE 2.1.4 Maximum torque occurs when the force applied is perpendicular (90°) to the axis of rotation.

Magnitude of the force and the torque

The torque (τ) on an object is directly proportional to the magnitude of the force (F). If all other things are equal, a larger force will result in a larger torque (Figure 2.1.5).

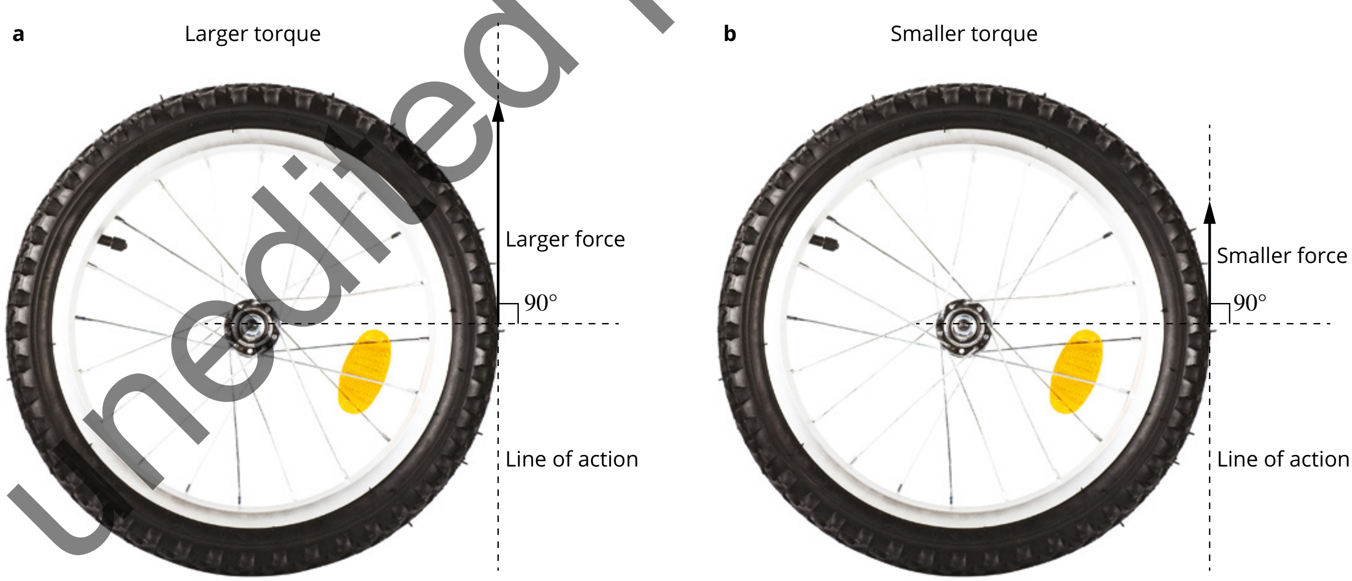


FIGURE 2.1.5 The magnitude of a force affects the torque on an object. The wheel in (a) will experience a larger torque than the wheel in (b).

Distance from the pivot point and torque

In addition to the size of the force and the angle at which it is acting, the amount of torque created is also directly proportional to the perpendicular distance between the axis of rotation and the line of action of the force. This perpendicular distance is called the **force arm** and is given the symbol $r_{\perp}$. Given that everything else is constant, the larger the force arm or perpendicular distance ($r_{\perp}$), the larger the torque (τ) (Figure 2.1.6).

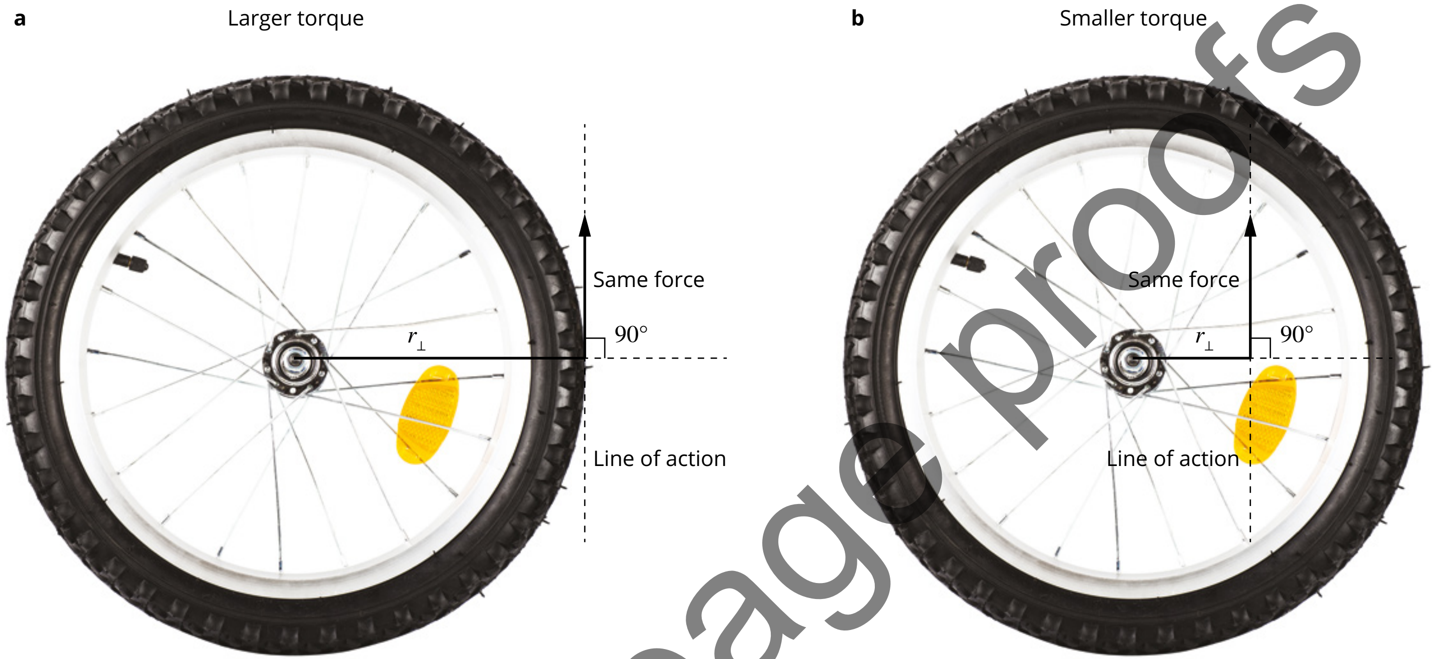


FIGURE 2.1.6 The perpendicular distance from the pivot point to the line of action of the force affects the torque on an object. The wheel in (a) will experience a larger torque than the wheel in (b).

The torque equation

The magnitude of the torque (τ) increases or decreases as the force (F) increases or decreases. The torque also increases or decreases as the force arm or the perpendicular distance from the pivot to the line of action of the force ($r_{\perp}$) increases or decreases.

Hence, the formula for calculating the magnitude of the torque is:

$\tau = r_{\perp}F$

where τ is torque (N m)

$r_{\perp}$ is the perpendicular distance from pivot to the force's line of action (m)

F is force (N).

A rotating body moves either clockwise or anticlockwise. You can determine the direction of the rotation by considering the context of the question. When two or more torques are acting, a sign convention can be applied to each torque where clockwise is considered to be positive and anticlockwise is considered to be negative. In this way, the torques can be added to determine the resultant torque.

Worked example 2.1.1

CALCULATING TORQUE

A bus driver applies a force of 45.0 N on the steering wheel of a bus as it turns a right-hand corner. The radius of the steering wheel is 30.0 cm. If the force is applied at 90.0° to the radius, calculate the torque on the steering wheel.	
Thinking	**Working**
Identify the variables involved and state them in their standard form.	$\tau = ?$ $r_{\perp} = 0.300\,\text{m}$ $F = 45.0\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_{\perp}F$ $\tau = (0.300)(45.0)$ $\tau = 13.500$
State the answer to the correct number of significant figures and include the appropriate direction (clockwise or anticlockwise).	$\tau = 13.5\,\text{N m}$ clockwise

Worked example: Try yourself 2.1.1

CALCULATING TORQUE

A force of 255 N is required to apply a torque on the steering wheel of a sports car as it turns left. The force is applied at 90.0° to the 15.5 cm radius of the steering wheel. Calculate the torque on the steering wheel.

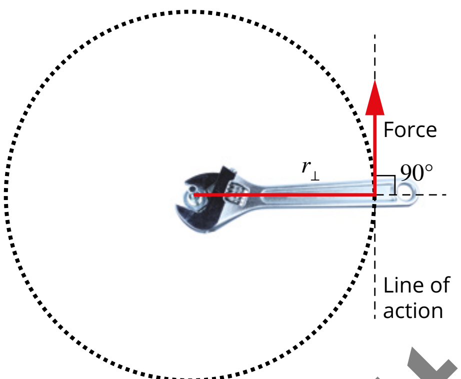


FIGURE 2.1.7 An adjustable spanner applies a torque to the nut of a nut-and-bolt system.

Torque on different objects

A torque does not need to be acting on a circular object. Any object can rotate about a point if a force is applied where the line of action of the force is not acting through the pivot point. For example, spanners can apply a torque to a nut or bolt when the pivot point is the bolt and a force is applied at right angles to the spanner (Figure 2.1.7).

Longer spanners can apply a greater torque on a nut than shorter spanners. This can be useful when trying to remove tight nuts or bolts, such as the wheel nuts on a car (Figure 2.1.8). Some wheel-nut spanners have extendable handles that increase the torque for loosening very tight nuts or tightening the nuts sufficiently so that they won't come loose.

FIGURE 2.1.8 Loosening the wheel nuts with an extended handle spanner will increase the torque on the nut.

Worked example 2.1.2

CALCULATING PERPENDICULAR DISTANCE

A car driver can apply a maximum force of 845 N on a wheel-nut spanner that is adjustable up to 30.0 cm in length. The force is applied at 90.0° to the radius. If the wheel nuts need a torque of 224 N m to remove them, what is the minimum length of the adjustable spanner at which the nuts can be loosened? State whether the spanner is long enough.

Thinking	Working
Identify the variables involved and state them in their standard form.	$\tau = 224\,\text{N m}$ $r_\perp = ?$ $F = 845.0\,\text{N}$
Apply the equation for torque. Rearrange as necessary.	$\tau = r_\perp F$ $r_\perp = \frac{\tau}{F}$ $r_\perp = \frac{(224)}{(845)}$ $r_\perp = 0.265\,09$
State the answer to the appropriate number of significant figures and with comparable units to the question.	$r_\perp = 26.5\,\text{cm}$
Compare the answer with the length of the spanner and state whether it is appropriate for this task.	As the spanner can be extended to 30.0 cm, it can be made long enough to provide the minimum perpendicular distance of 26.5 cm.

Worked example: Try yourself 2.1.2

CALCULATING PERPENDICULAR DISTANCE

A truck driver can apply a maximum force of 1020 N on a large truck wheel-nut spanner that has a length of 80.0 cm. The force is applied at 90.0° to the radius. If the truck's wheel nuts need a torque of 635 N m to make them secure, determine if the length of this spanner is sufficient for the job.

Doors are also good examples of torque in action when the hinges form the axis of rotation and the force is applied to the door at a perpendicular distance (Figure 2.1.9).

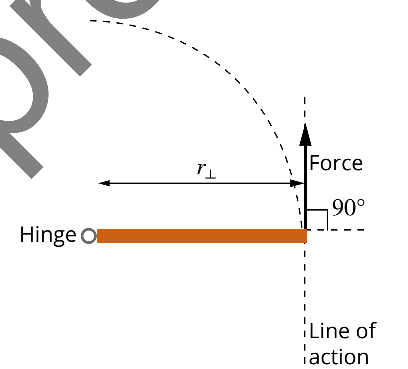


FIGURE 2.1.9 A door can have a torque applied to it, as long as the line of action of the force is not through the axis of rotation.

NON-PERPENDICULAR CALCULATIONS OF TORQUE

When the force causing a torque acts along a line that is at an angle other than 90.0° to an object, such as a door, the torque is reduced (Figure 2.1.10). In these circumstances, you can calculate the torque by using one of two approaches: either by finding the component of the force acting perpendicular to the door, or by finding the perpendicular distance from the pivot point to the line of action of the force.

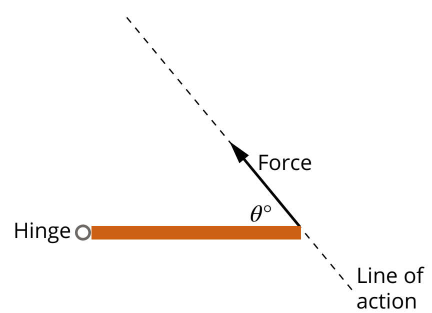


FIGURE 2.1.10 When the force causing a torque is not perpendicular to the door, the torque is reduced.

Recall that the formula for torque (τ) on an object is:

$$\tau = r_\perp F$$

This equation calculates the torque (τ) when the force (F) and the distance from the pivot point to the line of action of the force (r) are perpendicular to each other. It doesn't matter whether the radius is perpendicular to the line of action of the force, or if the force is perpendicular to the radius. You will explore both methods and see that they are equivalent.

Calculating torque using perpendicular force

The component of any force can be calculated using trigonometry. To find the component of the force that is perpendicular to a door, for example, you can use the magnitude of the force and the angle between the door and the line of action of the force (Figure 2.1.11).

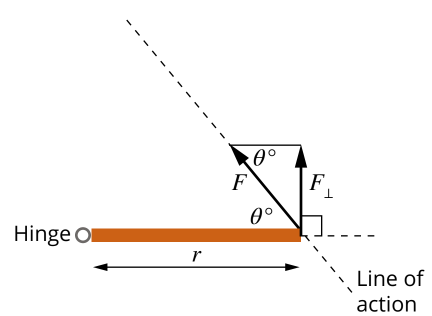


FIGURE 2.1.11 Determining the components of a force.

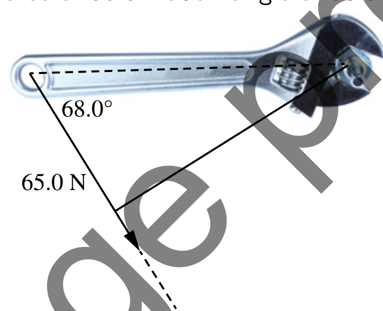
In this case, $\tau = rF_{\perp}$ and $F_{\perp} = F\sin\theta$.
Combining the two equations gives:
$\tau = rF\sin\theta$

When solving this problem, you can either calculate the perpendicular component of the force and then apply the torque equation, or simply use the combined equation. It is recommended that you begin by calculating the perpendicular force, and then use the torque equation. Once you are comfortable with that strategy, you can try using the combined equation.

Worked example 2.1.3

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A student uses a 42.0 cm long adjustable spanner to loosen a nut on their bike. The student applies a force of 65.0 N at an angle of 68.0° to the spanner.



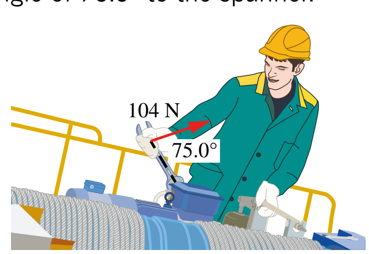
Calculate the anticlockwise torque that the student applies to the nut.

Thinking	Working
Use the trigonometric relationship $F_{\perp} = F\sin\theta$ to determine the force perpendicular to the spanner.	$F_{\perp} = F\sin\theta$ $F_{\perp} = (65.0)\sin(68.0°)$ $F_{\perp} = 60.267$ N
Convert variables to their standard units.	$r = 42.0$ cm $r = 0.420$ m
Apply the equation for torque: $\tau = rF_{\perp}$	$\tau = rF_{\perp}$ $\tau = (0.420)(60.267)$ $\tau = 25.312$
State the answer to the appropriate number of significant figures and with the appropriate units, including the direction since torque is a vector.	$\tau = 25.3$ N m anticlockwise

Worked example: Try yourself 2.1.3

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.



Calculate the clockwise torque that the mechanic applies to the nut.

Calculating torque using perpendicular radius

The perpendicular component of any distance can be calculated using Pythagoras' theory or trigonometry. To find the component of a length that is perpendicular to the line of action of the force acting on a door, construct a line from the pivot point to the line of action of the force so that it intersects the line of action at right angles (Figure 2.1.12).

In this case

$\tau = r_{\perp}F$ and $r_{\perp} = r\sin\theta$

combine to give

$\tau = r\sin\theta F$

The equation $\tau = rF\sin\theta$ is identical to $\tau = r\sin\theta F$, so either method would be appropriate for calculating the torque when the force is not acting at right angles to the object. In both cases, the component of the distance or the component of the force is always going to be less than the distance or the force itself. This decrease in value will result in a smaller torque being applied to the object. The maximum torque will always be achieved when the line of action of the force is perpendicular to the maximum distance from the pivot point to the line of action.

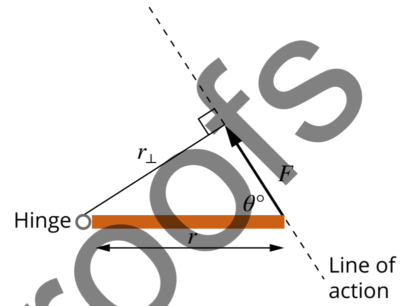
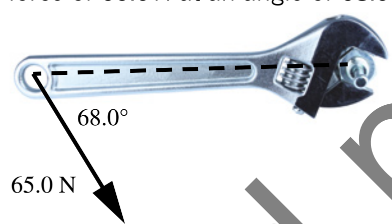


FIGURE 2.1.12 Determining the components of a distance.

Worked example 2.1.4

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF DISTANCE

A student uses a 42.0 cm long adjustable spanner to loosen a nut on their bike. The student applies a force of 65.0 N at an angle of 68.0° to the spanner.



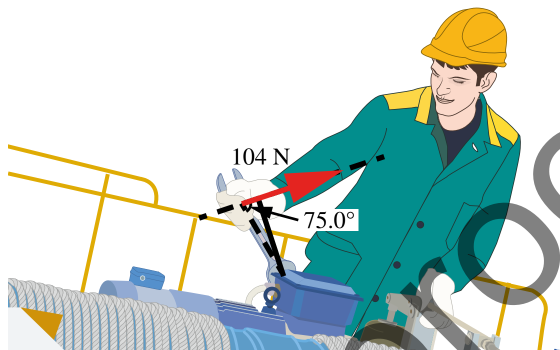
Using the perpendicular distance, calculate the anticlockwise torque that the student applies to the nut.

Thinking	Working
Convert variables to their standard units.	$r = 42.0$ cm $r = 0.420$ m
Use the trigonometric relationship $r_{\perp} = r\sin\theta$ to determine the perpendicular distance from the pivot point to the line of action of the force.	$r_{\perp} = r\sin\theta$ $r_{\perp} = (0.420)\sin(68.0°)$ $r_{\perp} = 0.38942$ m
Apply the equation for torque: $\tau = r_{\perp}F$.	$\tau = r_{\perp}F$ $\tau = (0.38942)(65.0)$ $\tau = 25.312$
State the answer with the appropriate unit, direction and significant figures. Note that this is the same as the answer to Worked example 2.1.3.	$\tau = 25.3$ N m anticlockwise

Worked example: Try yourself 2.1.4

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF DISTANCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.



Using the perpendicular distance, calculate the clockwise torque that the mechanic applies to the nut. Give your answer to three significant figures.

PHYSICS IN ACTION

The torque wrench

The extent to which a nut or bolt is tightened can be critical to the safe operation of machinery or motors, a good example being the wheel nuts on a car. If a nut or bolt is too loose then it could fall out, but if it is too tight then it could either distort the part or the bolt could break off. Both of these situations could result in expensive repairs. To avoid nuts and bolts being too loose or too tight, manufacturers use different tools and methods to estimate the amount of torque required to tighten a nut or bolt to the correct tightness.

Figure 2.1.13 illustrates different types of torque wrenches. The beam wrench is the simplest type of torque wrench; it has a flexible lever arm with a bar and scale that separates the wrench head and handle. When a torque is applied, a pointer on the scale moves to indicate the amount of torque in newton metres (N m). The click-type torque wrench can be set to apply a fixed amount of torque. When the required torque has been achieved, the wrench 'clicks' and releases itself, preventing any further tightening.

More recently, electronic torque wrenches have been developed. A force beam within the wrench converts the applied compressive or tensile force to an electrical signal that is calibrated to show the force in newton metres on a digital readout. Measurements can also be stored within the instrument's memory and transferred to a computer. The force sensors you will most likely use in physics operate on this same principle.

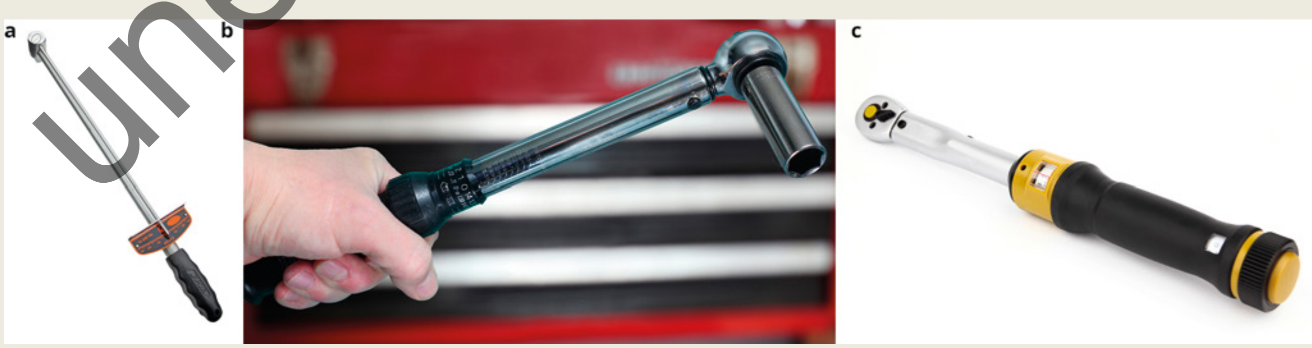


FIGURE 2.1.13 Three types of wrenches commonly used to measure the torque applied to a nut or bolt: (a) beam torque wrench, (b) click-type torque wrench, (c) digital torque wrench.

2.1 Review

SUMMARY

- Torque is a measurement of the amount of force applied to an object to make it rotate around an axis.
- The formula for calculating the magnitude of the torque is $\tau = r_{\perp}F$.
- Torque occurs when the acting force is not applied directly through the pivot point of the object.
- Maximum torque occurs when the acting force is applied at a perpendicular angle of 90° to the force arm.
- The larger the force acting on the object, the larger the torque will be.
- The longer the force arm, the greater the torque will be.
- If torque is generated by a force that is not perpendicular to the lever arm of the object, one of the two methods can be used:
  - Use the component of the force perpendicular to the length of the object to calculate torque $\tau = rF_{\perp}$
  - Use the distance from the pivot point perpendicular to the line of action of the force to calculate torque $\tau = r_{\perp}F$.
- Both methods for determining torque from non-perpendicular situations equate to $\tau = rF\sin\theta$.

KEY QUESTIONS

1 Select the correct option below to complete the following statement about torque.
When a force acts such that the line of action of the force is not directed through the pivot point of the object then:
A a torque will result
B no torque will result

2 Which of the options listed below best explains the factors affecting the torque acting on an object?
A Torque is only increased by increasing the force applied to the object.
B Torque increases when either the force or the force arm increases, or both increase.
C Torque only increases when the force arm decreases.
D Torque only increases when the force arm increases.

3 Calculate the torque applied to a bolt by a 20.0 cm long spanner that has a force of 25.0 N acting at 90.0° to its length and at the end of the spanner.

4 Calculate the radius of the wheel on a pressure valve that supplies a torque on the valve of magnitude 3.47 N m when a force of 12.0 N is applied.

5 When opening a 1.00 m wide door, the torque of the greatest magnitude is provided by:
A pushing with a force of 33.0 N at right angles to the door, in the middle of the door
B pushing with a force of 25.0 N at right angles to the door, 25.0 cm from the handle edge of the door
C pushing with a force of 50.0 N at right angles 25.0 cm from the hinges of the door
D pulling with a force of 25.0 N at right angles to the door, at the handle edge of the door

6 Demolishers wish to knock over a concrete wall. They plan to use a wrecking ball that exerts 5250 N as it hits the wall. If it hits at a point that is 3.05 m above the ground, calculate the magnitude of the torque that is developed on the wall if the wall pivots at its base.

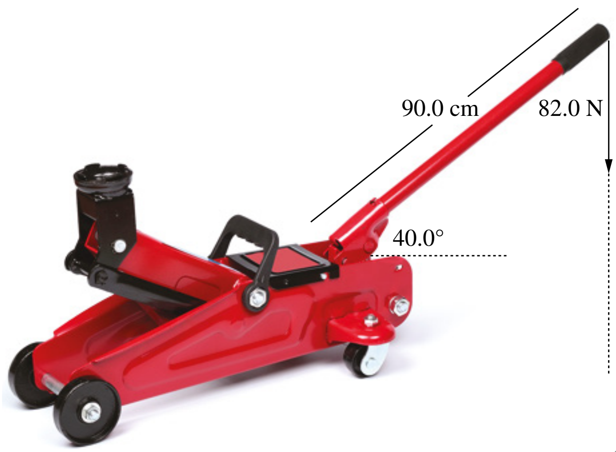
7 Calculate the length of a spanner that is used to tighten a nut to a torque of magnitude 32.1 N m when a force of 24.0 N is applied at right angles to the spanner, at the end of the spanner.

8 A camper ties a rope from the top of a 2.00 m tent pole to a peg on the ground. The rope is tightened so that the rope applies a 30.0 N force at an angle of 40.0° to the pole. Calculate the magnitude of the torque that is developed on the tent pole due to the rope if it pivots at its base.

2.1 Review *continued*

9 Penny is assembling some flat-pack furniture with an Allen key. Calculate the torque supplied on a screw by a 7.00 cm long Allen key that has a force of 8.50 N acting at 90.0° to its length, and at the end of the Allen key.

10 A mechanic uses a trolley jack to jack up a car by pushing vertically down on the end of a 90.0 cm lever with a force of 82.0 N, as shown in the diagram below. The lever is shown at an angle of 40.0° up from horizontal. Calculate the magnitude of the torque acting on the pivot.



The following information applies to questions 11 and 12.

A rope is attached at 30.0° to a freshly planted tree. The line of action of the force is along the same line as the rope, and the rope is attached 1.50 m from the base of the tree. Assume the base of the tree is the pivot point.

11 Calculate the length of the perpendicular force arm of the rope.

12 Calculate the torque on the tree if the force applied to the tree by the rope is 12.5 N and it is tensioned clockwise about the base of the tree.

2.2 Equilibrium of forces

Newton's first law states that an object will continue with its motion unless acted upon by an external unbalanced force. The situation in which the object continues with its motion, or doesn't change its velocity, occurs when the forces on the object are balanced. Balanced forces, like those in a tug of war (Figure 2.2.1), are said to be in translational equilibrium. When a tug of war starts, for example, there is an equilibrium of forces as both teams take the strain. Winning a tug of war match involves one team applying a greater force so that there is a net force on the rope, causing the rope and teams to accelerate in the winning team's direction. Once the rope and the teams are moving at a constant velocity, an equilibrium of forces exists once again.

FIGURE 2.2.1 The game 'tug of war' is an example of translational equilibrium.

CENTRE OF MASS AND STABILITY

Think about an athlete running in a 100 m sprint. In simple terms, the athlete runs in a straight line along the track, and the displacement and velocity at any time can be calculated using the principles discussed in Chapter 3 'Linear motion' in Year 11. However, the motion of the various parts of the athlete's body will differ significantly during the run. The athlete's arms and legs move in a complex manner that requires a significantly more complex analysis.

The analysis of the motion of complicated systems, such as a sprinter or high-jumper, can be simplified to the motion of a single point. In other words, the mass of the sprinter can be considered to be 'concentrated' into a single point that has travelled in a straight line. This single point is called the **centre of mass**. There is as much mass above the centre of mass as there is below it, as much mass to the left as there is to the right, and as much mass in front as there is behind it.

If an object is uniform in one dimension only (e.g. a straight piece of wire), its centre of mass will be exactly halfway along its length. The centre of mass of an object in two dimensions (e.g. a flat piece of card), will be the point that is halfway along and halfway across the object (Figure 2.2.2). It is even possible for the centre of mass to lie outside the body; for example, the centre of mass of a doughnut is located at the centre of the hole. A person's centre of mass is typically in the lower part of the back, but it will vary with the positions of the arms and legs.

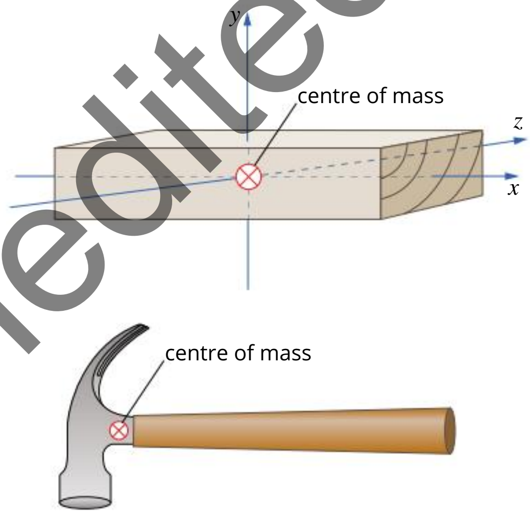


FIGURE 2.2.2 The centre of mass of a three-dimensional block of wood is shown with arrows drawn in the *x*, *y* and *z* dimensions illustrating the lines where there is equal mass on either side of the lines. The centre of mass of a non-uniform hammer shows that the centre of mass will be closer to the end with a higher density than the end with the lower density.

A concept that is closely related to centre of mass is **centre of gravity**. Instead of being a point particle whose motion equates to the whole extended body or system, the centre of gravity is the position from which the entire weight of the body or system is considered to act. As a consequence of this, the centre of gravity is the position at which the body will balance. For all practical purposes, the centre of gravity is exactly at the centre of mass. It is only when a body is so large that it is in a non-uniform gravitational field that the centre of gravity no longer coincides with the centre of mass.

There are three types of equilibrium related to the stability of an object:

- **stable equilibrium**—The object will return to its equilibrium position even when a force is applied to it, as long as the centre of mass is not moved outside the **base** of support.
- **unstable equilibrium—**The object will accelerate and will not return to its equilibrium position when a force is applied to it, since the centre of mass is readily moved outside the base of support.
- **neutral equilibrium—**The object will remain stationary no matter where it is placed, as any force applied has no effect on the relationship between the centre of mass and the base or point of support.

Figure 2.2.3 illustrates how this applies to real-life objects.

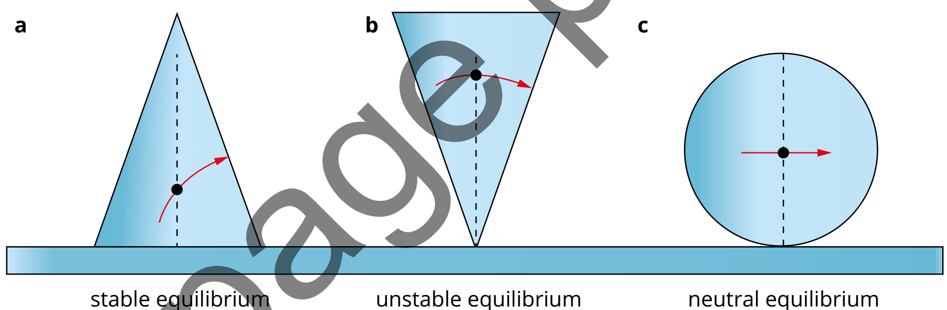


FIGURE 2.2.3 Three objects illustrating the difference between (a) stable, (b) unstable and (c) neutral equilibrium. As (a) has a broad base of support, it will return to its original position even when a force is applied to it, as long as the centre of mass is not moved outside the base of support.

When designing structures, engineers and architects want to ensure that balance and stability are maintained. Whether this occurs depends on the relative positions of the centre of gravity and the base. When a vertical line downwards from the centre of gravity passes through the base of support, the object is stable. The vertical line from the centre of gravity represents the direction of the force of gravity on the object.

In Figure 2.2.4(a), the weight of the car passes through the car's support base, between the wheels. The torque acting on the car is clockwise in this case and therefore does not cause the car to tip over. In the case of the truck, however, the weight is directed outside the truck's base of support, so an anticlockwise torque acts to tip the truck over.

The stability of an object or structure can be increased in a number of ways:

- the centre of gravity is lowered
- the width of the support base is increased
- the angle from the centre of gravity to the edge of the base is increased.

As a result of any of these, the object has to be tipped further to make the force of gravity act outside the support base. Racing cars have a very low centre of gravity to increase their stability when cornering at high speed. In a similar way, training wheels on a child's bicycle widen the support base, making it harder to tip the bicycle sideways.

These conditions also apply to objects that are supported from above. A child on a playground swing will be in stable equilibrium when the swing is hanging straight down. A gentle push will cause the swing to move but it will return to its original position once the outside force is removed.

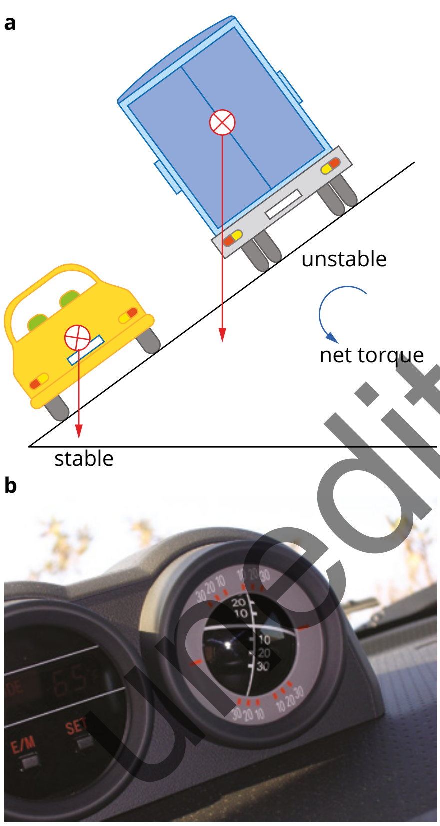


FIGURE 2.2.4 (a) The car on the incline is in stable equilibrium, while the heavily laden truck on the same incline could topple. The weight vector is outside the lower point of support for the truck, so there is no reaction force from the road to the higher wheel. (b) Modern four-wheel drives and tractors have inclinometers to warn the driver if the vehicle is in danger of tipping.

If, on the other hand, the swing is raised to a significant height and held there, then it would be in unstable equilibrium. The swing is in equilibrium only because the forces on it are balanced while it is being held. If the holding force (say a parent's hands) is removed, then the swing will start to move, swinging across the point of stable equilibrium before eventually coming to rest in a position of stable equilibrium.

TRANSLATIONAL EQUILIBRIUM

A **translational equilibrium** of forces occurs when the sum of the forces acting through the centre of mass of an object add to give a zero resultant or zero net translational force. As a net translational force causes acceleration in one direction, a zero net translational force causes no acceleration of the object. This condition is the defining aspect of a translational equilibrium of forces.

When analysing a situation involving more than one force acting on an object, translational equilibrium will exist if the sum of the forces is equal to zero: $\Sigma F = 0$. This can also be written as $F_{net} = 0$.

Vector diagrams of an equilibrium of forces in one dimension

Vector diagrams can be drawn of the forces applied to an object when they are acting in one dimension. For example, if three people are pulling to the right and three people are pulling to the left in a game of tug of war, as shown in Figure 2.2.5, then the forces are all in one dimension: left and right. You can add these forces using a vector diagram by drawing all the forces from each person head to tail, as described in Chapter 2 'Scalars and vectors' in the Year 11 student book. If the tug of war is in translational equilibrium, then all the forces should add to give a zero net force.

FIGURE 2.2.5 Tug of war with a vector addition diagram showing a net force of zero, indicating an equilibrium of forces exists.

Calculating an equilibrium of forces in one dimension

Whether a situation is in translational equilibrium or not can be determined using a consistent sign convention to represent the direction of the force vectors in one dimension. Typically, in the *x*-dimension, left is treated as negative and right as positive; similarly in the *y*-dimension, up is treated as positive and down as negative. In the *z*-dimension, forwards is positive and backwards is negative. By applying a consistent sign convention to the forces acting on an object, the addition of those forces, with their signs, will give a vector sum of zero if the situation is in translational equilibrium.

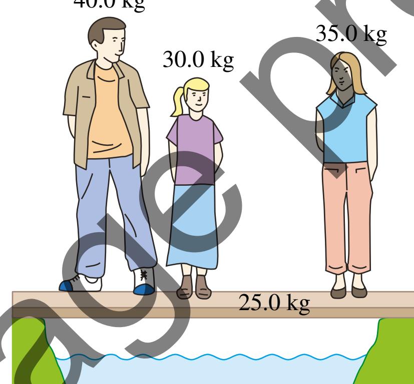
That is:

$\Sigma F_{\text{left–right}} = 0$
$\Sigma F_{\text{up–down}} = 0$
$\Sigma F_{\text{forwards–backwards}} = 0$

Worked example 2.2.1

CALCULATING TRANSLATIONAL EQUILIBRIUM IN ONE DIMENSION

Three students are standing on a plank that is bridging a small stream. The plank is supported at each end by the ground. The plank has a mass of 25.0 kg, and the students have masses of 40.0 kg, 30.0 kg and 35.0 kg. There is an upward force from the left bank on the plank of 725 N. If the plank is in translational equilibrium, calculate the force of the right bank on the plank. Use $g = 9.80\ \text{N kg}^{-1}$ when answering this question.

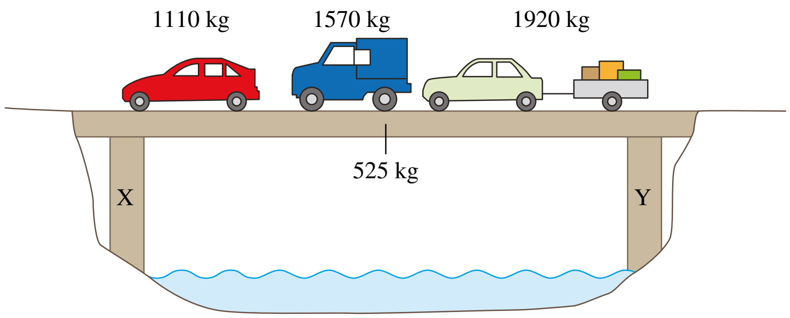


Thinking	Working
Identify the variables involved and state them with directions in their standard form.	$m_1 = 40.0$ kg $m_2 = 30.0$ kg $m_3 = 35.0$ kg $m_P = 25.0$ kg $F_{LB} = 725$ N up $g = 9.80\ \text{N kg}^{-1}$ down
Apply a sign convention to the vector data.	$F_{LB} = +725$ N $g = -9.80\ \text{N kg}^{-1}$
The object experiencing translational equilibrium is the plank.	$\Sigma F_{\text{up–down}} = 0$
Expand the equation to include each of the forces acting on the plank.	$F_1 + F_2 + F_3 + F_{WP} + F_{LB} + F_{RB} = 0$ $m_1g + m_2g + m_3g + m_Pg + F_{LB} + F_{RB} = 0$
Substitute the data into the equation and solve for the unknown.	$(40.0)(-9.80) + (30.0)(-9.80) + (35.0)(-9.80) + (25.0)(-9.80) + (725) + F_{RB} = 0$ $(-392) + (-294) + (-343) + (-245) + (725) + F_{RB} = 0$ $-549 + F_{RB} = 0$ $F_{RB} = +549$
State the answer to the appropriate number of significant figures and with the appropriate direction.	$F_{RB} = 549$ N up

Worked example: Try yourself 2.2.1

CALCULATING TRANSLATIONAL EQUILIBRIUM IN ONE DIMENSION

Three cars are parked on a beam bridge that has a mass of 525 kg. Pillar X applies a force of 2.00×10^4 N upwards. If the situation is in translational equilibrium, calculate the force provided by pillar Y. Use $g = 9.80\ \text{N kg}^{-1}$ when answering this question. Car 1 (C_1) is red; car 2 (C_2) is blue; car 3 (C_3) is light green.



SOLVING EQUILIBRIUM IN TWO DIMENSIONS

If the forces involved in the equilibrium situation are in two dimensions, there are two strategies for determining if the sum of the forces is zero: using vector diagrams or vector components.

Vector diagrams

You can draw **vector diagrams** in which vectors are added in two dimensions. This was covered in Chapter 2 'Scalars and vectors' in the Year 11 student book. In any vector addition diagram, the individual vectors are added head to tail with the net vector drawn from the tail of the first vector to the head of the last vector. In a situation in which the forces are in equilibrium, the vector addition diagram should end up with the head of the last vector finishing at the tail of the first vector. This means that the vector addition diagram forms a closed loop and therefore there is no net force (Figure 2.2.6).

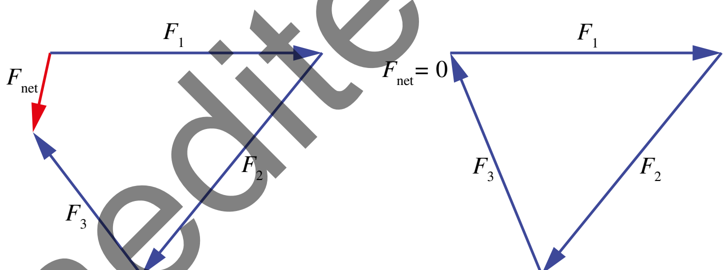


FIGURE 2.2.6 The vector diagram on the left shows three vectors added head to tail and the resultant net force vector F_{net} in red. The closed loop vector addition diagram on the right shows an equilibrium of forces, where $F_{net} = 0$.

Vector components

A vector diagram that results in a closed loop fulfils the conditions of a translational equilibrium of forces. If the three forces form a right triangle, you can use Pythagoras' theorem and trigonometry to determine the magnitude and direction of a third force that will be in equilibrium with two other forces. If the three forces are in any other triangle, then the unknown force can be found using the sine law or the cosine law, which is a more complex method. However, there is an easier approach that involves determining the **components** of the forces in two perpendicular dimensions. These force components are then added in each of their dimensions, which results in two perpendicular resultant vectors that can be added using Pythagoras' theorem and trigonometry (Figure 2.2.7).

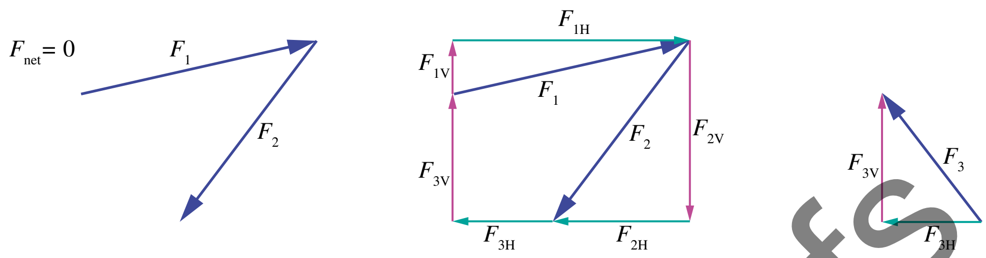


FIGURE 2.2.7 A method for finding F_3 that acts to make an equilibrium of forces with F_1 and F_2.

The guiding principle behind this method is that for the forces to be in equilibrium, the sum of the *x*-axis (horizontal or left–right) forces must equal zero, and the sum of the *y*-axis (vertical or up–down) forces must also equal zero.

$$\Sigma Fx_{\text{-axis}} = 0 \quad \text{or} \quad \Sigma F_{\text{vertical}} = 0 \quad \text{or} \quad \Sigma F_{\text{left–right}} = 0$$
$$\Sigma Fy_{\text{-axis}} = 0 \quad\quad\quad \Sigma F_{\text{horizontal}} = 0 \quad\quad\quad \Sigma F_{\text{up–down}} = 0$$

To draw a vector diagram (Figure 2.2.8):

- identify all of the forces that apply (e.g. tension, gravitational force)
- draw the vectors for each force separately and mark all known angles
- reposition the vectors so they are head to tail, adding all of the forces together.

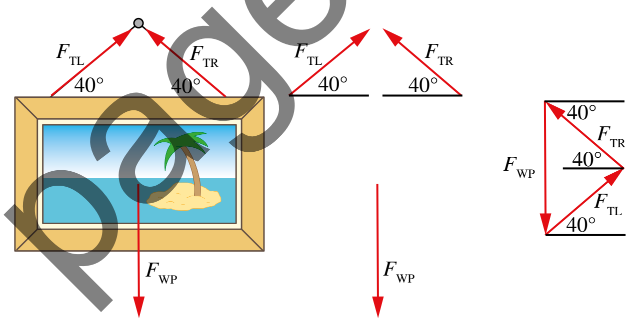
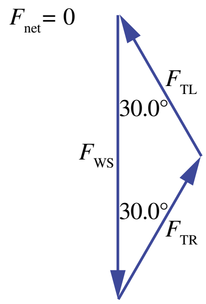
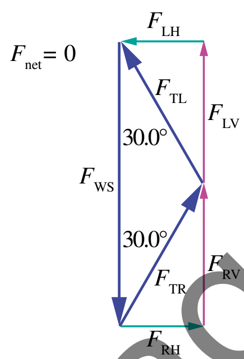
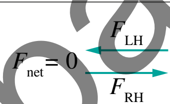
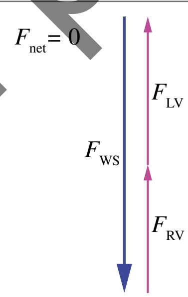


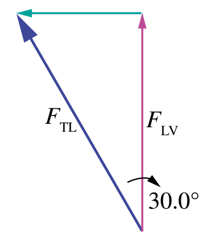
FIGURE 2.2.8 A vector diagram is drawn by identifying the vectors and then adding the vectors together, head to tail.

Worked example 2.2.2

CALCULATING TRANSLATIONAL EQUILIBRIUM IN TWO DIMENSIONS

An advertising sign is hung by two cables to the ceiling of a shop. The sign has a mass of 45.0 kg and the cables are at an angle of 30.0° to the vertical as shown in the image below. If the mass of the cables is ignored, calculate the tension in each cable when the sign is suspended. Use $g = 9.80\,\text{N kg}^{-1}$ when answering this question.

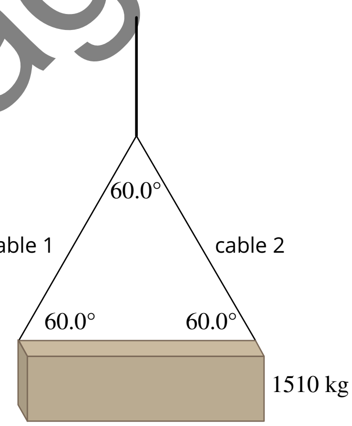
Thinking	Working
Construct a vector diagram adding all of the forces together.	 F_{WS} is the weight of the sign, F_{TL} is the tension of the left cable and F_{TR} is the tension of the right cable.
Apply left and right components and up and down components.	
In the horizontal dimension, F_{LH} is in equilibrium with F_{RH}.	
In the vertical dimension, F_{WS} is in equilibrium with F_{LV} and F_{RV}.	
Apply the equation for translational equilibrium in one dimension. F_{RV} and F_{LV} are equal in magnitude and therefore each is half of F_{WS}.	$\Sigma F_{up\text{–}down} = 0$ $F_{RV} = F_{LV}$
Expand the equation to include each of the vertical forces acting on the sign.	$F_{WS} + F_{RV} + F_{LV} = 0$ $m_S g + F_{RV} + F_{RV} = 0$
Substitute the data into the equation and solve for the unknown.	$(45.0)(-9.80) + 2F_{RV} = 0$ $-441 + 2F_{RV} = 0$ $F_{RV} = \frac{441}{2}$ $F_{RV} = F_{LV} = 220.5\,N$

Draw the right triangle with one of the vertical components of the tension and the angle.	
Use trigonometry to solve for the tension in one of the cables, which will equal the tension in the other cable. Present your answer to the appropriate number of significant figures.	$\cos\theta = \frac{F_{LV}}{F_{TL}}$ $F_{TL} = \frac{F_{LV}}{\cos\theta}$ $F_{TL} = \frac{(220.5)}{\cos(30.0°)}$ $F_{TL} = 254.61$ $F_{TL} = F_{TR} = 255\,N$

Worked example: Try yourself 2.2.2

CALCULATING TRANSLATIONAL EQUILIBRIUM IN TWO DIMENSIONS

A concrete beam of mass 1510 kg is being lifted by cables labelled 1 and 2, as shown in the diagram. The beam is moving upwards with a constant velocity of 2.00 m s^{-1}. Calculate the tension in cable 1 and cable 2. Ignore the mass of the cables and use $g = 9.80\,N\,kg^{-1}$ when answering this question.



2.2 Review

SUMMARY

- Translational equilibrium in one dimension can be represented mathematically as $\Sigma F = 0$.
- Translational equilibrium in two dimensions can be represented mathematically as $\Sigma Fx = 0$ and $\Sigma Fy = 0$.
- In two dimensions, an equilibrium of forces can be represented in a closed vector diagram.
- By calculating x and y components of the forces in equilibrium, the forces in equilibrium in each dimension can be found.

KEY QUESTIONS

1 Select the correct statement explaining when translational equilibrium occurs.

A A net force acts through the centre of mass and causes no acceleration.
B A net force acts through the centre of mass and causes an acceleration.
C No net force acts through the centre of mass resulting in an acceleration.
D No net force acts through the centre of mass and so there is no acceleration.

2 An object is in translational equilibrium. Which of the following statements applies?

A The object must be stationary.
B The object must be moving at constant velocity.
C The object must be experiencing translational acceleration.
D The object must not be experiencing translational acceleration.

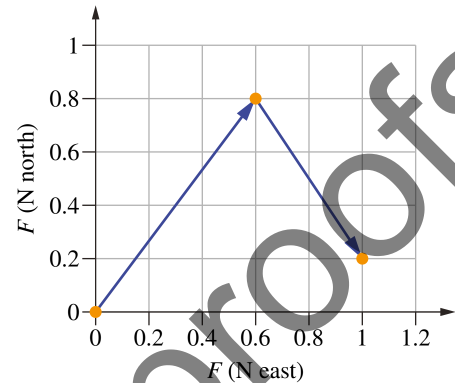
3 A string supports a birdfeeder that has a mass of 355 g. If the birdfeeder is in translational equilibrium, calculate the tension in the string supporting the birdfeeder.

4 Calculate the mass of a pendulum bob that hangs stationary from a chain when the tension in the chain is 7.50 N.

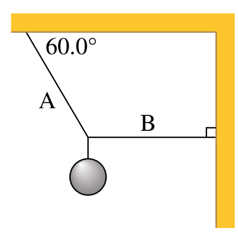
5 A chef wants to make sure that the wire cables attached to each end of a kitchen rail will be able to hold the weight of the rail, two tools and a frying pan that must hang from it. The mass of the rail is 3.05 kg, the mass of each tool is 345 g and the mass of the frying pan is 2.25 kg. Calculate the tension force in each wire cable if the load is spread evenly along the rail.

6 Two window cleaners working on the windows of a high-rise building work on a platform suspended by four cables. Tom has a mass of 79.0 kg, Jack has a mass of 68.0 kg and the mass of the platform is 225 kg. The platform is moving down the side of the building at a constant speed and all cables provide the same tension. Calculate the tension in one of the cables.

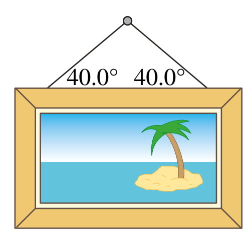
7 A shopping trolley is pushed in different directions by three children. The force vectors of two of the children are shown in the diagram. Draw the force vector from the third child that would cause the shopping trolley to be in translational equilibrium.



8 A bowling alley wants to promote its business by suspending a 112 kg fibreglass bowling ball on a frame outside its alley. The bowling ball is supported by two steel cables capable of withstanding up to 1550 N of tension before breaking. Cable A is at an angle of 60.0° to the horizontal frame member and cable B is perpendicular to the vertical frame member. Assuming the mass of the cables is negligible, calculate the tension in cable A and in cable B.



9 A picture is hung on a wall as shown in the diagram below. If the hanging wire is capable of supporting a maximum force of 40.0 N, what is the maximum mass of the picture that can be supported before the wire snaps?



10 A street performer stands on a rope tied between two posts. When the performer is standing at the centre, the rope makes an angle of 10.0° to the horizontal. Assuming the mass of the rope is negligible and that the mass of the performer is 75.0 kg, calculate the tension in the rope. Give your answer to three significant figures.

2.3 Static equilibrium

FIGURE 2.3.1 When a cyclist doesn't need to pedal, they can stand on both pedals with equal force. This causes an equal torque in the clockwise and anticlockwise directions. The pedals are in rotational equilibrium.

Objects are in translational equilibrium when the sum of the forces acting through their centre of mass equals zero. However, not all forces act through the centre of mass of an object; some forces may act in ways that generate torque or moment on an object. In order for an object to be in rotational equilibrium, the sum of the moments in a clockwise direction must balance the sum of the moments in an anticlockwise direction (Figure 2.3.1). This relationship is called the **principle of moments**.

By combining the conditions for translational equilibrium and rotational equilibrium, an object can be made to be in a state of static equilibrium. Static equilibrium will be explored in more detail in this section.

ROTATIONAL EQUILIBRIUM

A **rotational equilibrium** of torque or moments occurs about a **reference point** when the sum of all the torques acting on an object in the clockwise direction is equal to the sum of all the torques acting in the anticlockwise direction.

When analysing a situation involving more than one torque acting on an object, the principle of moments can be said to apply when rotational equilibrium exists, or the sum of the torques about a reference point is equal to zero. That is:

$\Sigma\tau = 0$

This is also represented as:

$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$

Figure 2.3.2 illustrates the masts of a boat that are in rotational equilibrium around their base of support.

FIGURE 2.3.2 To keep each mast in rotational equilibrium relative to its base, the stainless-steel cables (stays) attached to it must provide opposing torques on the mast.

When the sum of the torques is not balanced, however, the object will experience an unbalanced torque and will rotate about the reference point.

CONDITIONS FOR STATIC EQUILIBRIUM

When a body or a system is not accelerating or rotating, it must be in both translational and rotational equilibrium. This situation satisfies the conditions for **static equilibrium**, which can be represented by:

$\Sigma F = 0$ and $\Sigma\tau = 0$

This can also be shown as:

$\Sigma Fx = 0$

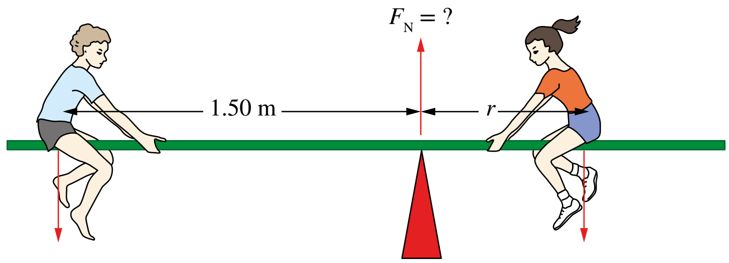
$\Sigma Fy = 0$

$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$

Worked example 2.3.1

CALCULATING STATIC EQUILIBRIUM

Two young children make a seesaw with a long plank. The boy sits on the seesaw 1.50 m from the pivot. The masses of the boy and the girl are 20.0 kg and 30.0 kg respectively. Assume that the mass of the plank is negligible. Use $g = 9.80\,\text{N kg}^{-1}$ in your calculations where required.



a Calculate the force applied to the plank by the pivot point when the children are sitting on the seesaw.

Thinking	Working
Identify the variables involved and state them with directions in their standard form.	$m_g = 30.0\,\text{kg}$ $m_b = 20.0\,\text{kg}$ $g = 9.80\,\text{N kg}^{-1}$ down
Apply a sign convention to the vector force data.	$g = -9.80\,\text{N kg}^{-1}$
Identify the object that is in translational equilibrium. This is the object on which all the forces are acting.	The object experiencing translational equilibrium is the plank.
Apply the equation for translational equilibrium in one dimension.	$\Sigma F_y = 0$
Expand the equation to include each of the forces acting on the plank.	$F_{Wg} + F_{Wb} + F_P = 0$ $m_g g + m_b g + F_P = 0$
Substitute the data into the equation and solve for the unknown.	$(30.0)(-9.80) + (20.0)(-9.80) + F_P = 0$ $(-294) + (-196) + F_P = 0$ $(-490) + F_P = 0$ $F_P = +490$
State the answer to the appropriate number of significant figures and with the appropriate direction.	$F_P = 4.90 \times 10^2\,\text{N}$ up

b Calculate where the girl has to sit in order to balance the boy.

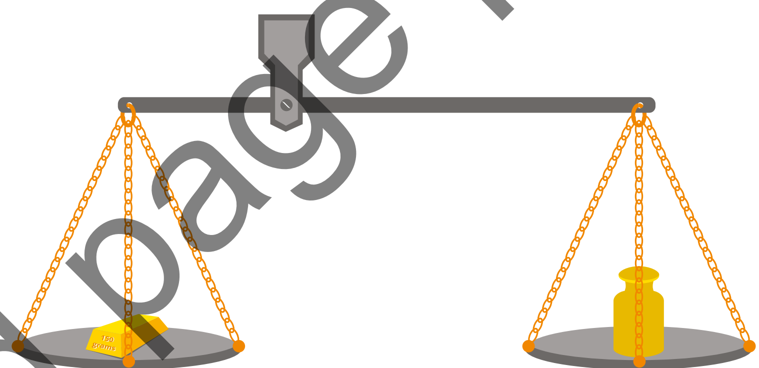
Thinking	Working
Identify the variables involved and state them in their standard form.	$m_g = 30.0\,\text{kg}$ $m_b = 20.0\,\text{kg}$ $r_{\perp b} = 1.50\,\text{m}$ $g = 9.80\,\text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the seesaw.
Identify the reference point about which the torques will be calculated.	The reference point is the pivot of the seesaw.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the girl on the seesaw provides the clockwise torque. The force of the boy on the seesaw provides the anticlockwise torque.

Apply the equation for rotational equilibrium.	$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$
Expand the equation to include each of the torques acting on the seesaw.	$F_{Wg}r_{\perp g} = F_{Wb}r_{\perp b}$
Substitute the data into the equation and solve for the unknown. State the answer to the appropriate number of significant figures.	$F_W r_{\perp g} = F_{Wb}r_{\perp b}$ $m_g g r_{\perp g} = m_b g r_{\perp b}$ $(30.0)(9.80)r_{\perp g} = (20.0)(9.80)(1.50)$ $r_{\perp g} = \frac{(20.0)(9.80)(1.50)}{(30.0)(9.80)}$ $r_{\perp g} = 1.00$ m

Worked example: Try yourself 2.3.1

CALCULATING STATIC EQUILIBRIUM

A set of scales (with one longer arm) is used to measure the mass of gold when it is larger than the standard set of masses. A lump of gold weighing 185 g is placed on the short arm, which is 10.0 cm long, and the standard masses are placed on the long arm. Use $g = 9.80\ \text{N kg}^{-1}$ in your calculations where required. Give your answers to three significant figures.



a Calculate the force applied to the scale's arm due to the pivot point if a standard mass of 50.0 g exactly balances the gold.

b Calculate the length the long arm should have in order to balance the gold.

In Worked example 2.3.1, the seesaw is in equilibrium because all the forces and torques are balanced. In solving the problem, it seemed obvious to choose the pivot as the reference point around which the torques are determined. But because the seesaw plank is in equilibrium, any point could have been chosen as the reference point. For example, you could take the reference point to be where the girl is sitting (Figure 2.3.3), meaning that the torques acting on the plank would be due to the boy and to the seesaw pivot point. The torque due to the girl becomes zero, as the lever arm distance for her will be zero. The solutions to the questions in the Worked example will still be the same for this reference point.

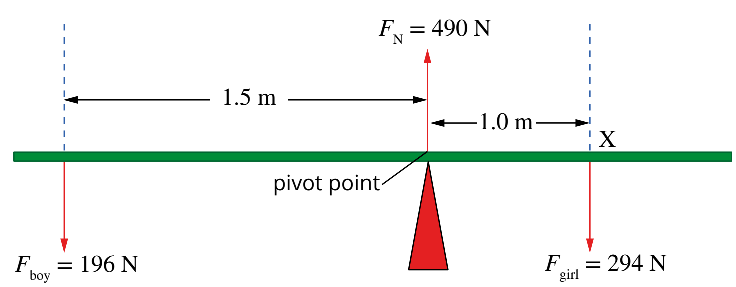


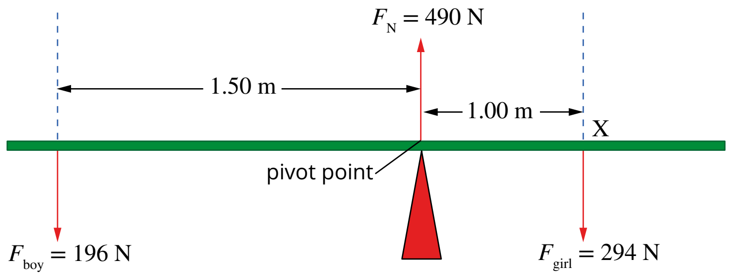
FIGURE 2.3.3 A force diagram for the seesaw problem from Worked example 2.3.1, showing that the point at which the girl sits (labelled X) has been chosen as the reference point.

Thus, the boy will create an anticlockwise torque around the girl, while the normal force at the pivot for the seesaw creates an equal clockwise torque around her. In the following Worked example, it can be verified that the seesaw is in rotational equilibrium by calculating the torques around the position of the girl. The plank will be in rotational equilibrium if the clockwise torque equals the anticlockwise torque.

Worked example 2.3.2

CALCULATING STATIC EQUILIBRIUM USING A DIFFERENT REFERENCE POINT

Verify that the seesaw plank in the image below is in rotational equilibrium about the reference point X, where the girl is sitting. The weights of the boy and girl are 196 N and 294 N respectively, and the force of the pivot on the plank is 490 N upwards. Assume that the mass of the plank is negligible.



Here, the point at which the girl is sitting (labelled X) has been chosen as the reference point. The force due to the boy acts downwards on the plank and the force due to the pivot point acts upwards on the plank.

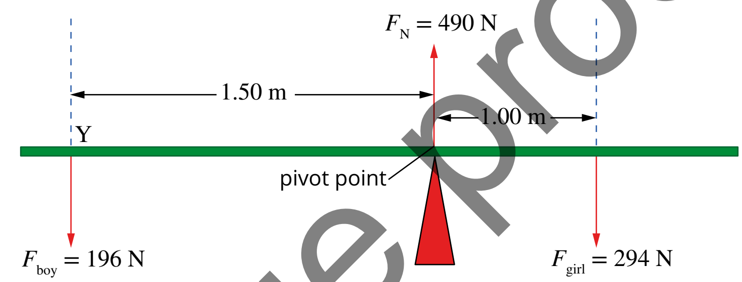
Thinking	Working
Identify the variables involved and state them in their standard form.	$F_P = 490$ N $F_g = 294$ N $F_b = 196$ N $r_{\perp b} = 1.00 + 1.50 = 2.50$ m $r_{\perp P} = 1.00$ m
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the seesaw.
Identify the reference point about which the torques will be calculated.	The reference point is the position of the girl at X.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the pivot on the plank provides the clockwise torque. The force of the boy on the plank provides the anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the seesaw. Note that the girl's torque is not included here as, being the reference point, her torque is zero.	$F_P r_{\perp P} = F_b r_{\perp b}$
Substitute the data into the equation and solve.	$F_P r_{\perp P} = F_b r_{\perp b}$ $(490)(1.00) = (196)(2.50)$ $490 = 490$

Identify the magnitude of the clockwise torque compared to the magnitude of the anticlockwise torque.	Around reference point X (the position of the girl), the clockwise torque due to the pivot point on the plank is equal to the anticlockwise torque due to the boy on the plank. So, the plank is in rotational equilibrium.

Worked example: Try yourself 2.3.2

CALCULATING STATIC EQUILIBRIUM USING A DIFFERENT REFERENCE POINT

Verify that the seesaw plank in the figure below is also in rotational equilibrium about the reference point Y, where the boy is sitting. The weights of the boy and girl are 196 N and 294 N respectively, and the force of the pivot on the plank is 490 N upwards. Assume that the plank's mass is negligible.



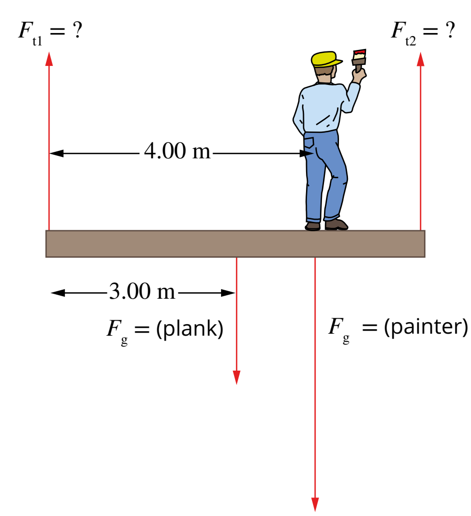
STATIC EQUILIBRIUM WITH TWO UNKNOWN FORCES

The seesaw problem is relatively straightforward since in each situation there is only one unknown force. If there are two unknown forces, the reference point can be chosen to coincide with one of the forces. Using this strategy means that the force acting at the reference point contributes no torque (since $r = 0$). The remaining unknown force can then be found using the relationship $\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$, as shown in Worked example 2.3.3.

Worked example 2.3.3

CALCULATING STATIC EQUILIBRIUM WITH TWO UNKNOWNS

A 70.0 kg painter stands 4.00 m from the end of a 6.00 m long plank that is supported by a rope at each end. The plank has a mass of 20.0 kg. Determine the tension on the left-hand rope (F_{t1}). Use $g = 9.80\ \text{N kg}^{-1}$ in your calculations where required.



Thinking	Working
Identify the variables involved and state them in their standard form.	$m_{pl} = 20.0\,kg$ $m_{pa} = 70.0\,kg$ $r_{\perp F_{t1}-F_{t2}} = 6.00\,m$ $r_{\perp c-F_{t2}} = 3.00\,m$ $r_{\perp pa-F_{t2}} = 2.00\,m$ $g = 9.80\,N\,kg^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the plank.
Identify the reference point about which the torques will be calculated. Always choose a point at which one of the unknown forces acts in order to form an equation with just one unknown variable. This avoids the need to solve simultaneous equations.	The reference point is the point at which the rope providing the tension force F_{t2} is attached.
Decide which force causes the clockwise torques and which forces cause the anticlockwise torques around the chosen reference point.	The tension force of the left-hand rope on the plank provides the clockwise torque. The force of the painter on the plank provides an anticlockwise torque. The force of gravity on the plank provides another anticlockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$
Expand the equation to include each of the torques acting on the plank. The torque of the right-hand rope is not included as it acts through the reference point.	$F_{t1}r_{\perp F_{t1}-F_{t2}} = F_{pl}r_{\perp c-F_{t2}} + F_{pa}r_{\perp pa-F_{t2}}$ $F_{t1}r_{\perp F_{t1}-F_{t2}} = m_{pl}gr_{\perp c-F_{t2}} + m_{pa}gr_{\perp pa-F_{t2}}$
Substitute the data into the equation and solve for the unknown force.	$F_{t1}(6.00) = (20.0)(9.80)(3.00) + (70)(9.80)(2.00)$ $F_{t1} = \frac{(196)(3.00) + (686)(2.00)}{(6.00)}$ $F_{t1} = \frac{(588) + (1372)}{(6.00)}$ $F_{t1} = 326.67$ $F_{t1} = 327\,N$

Work example: Try yourself 2.3.3

CALCULATING STATIC EQUILIBRIUM WITH TWO UNKNOWNS

For the painter on the plank scenario in Worked example 2.3.3, determine the tension on the right-hand rope (F_{t2}).

Another way to determine the second unknown force is to apply the conditions for translational equilibrium. You can check these values:

$\Sigma F = 0$: the sum of the two upward forces (tensions) 555 N + 327 N = 882 N.

This balances the sum of the two downward forces: 196 N + 686 N = 882 N.

OTHER STATIC EQUILIBRIUM SCENARIOS

The balanced seesaw and the supported plank are just two of many situations in which static equilibrium can occur. Other conditions in which static equilibrium exists include the following scenarios.

Cantilevers

A beam that extends beyond its support structure is called a **cantilever**. Cantilevers are common structural elements. For example, the diving board at the local pool is a cantilever.

A cantilever bridge might be used to span a river or valley (Figure 2.3.4). Pillars are built at regular intervals across a gap in order to support a number of beams. The cantilever beams are then joined at the centre of each span. The forces on the pillars are not affected by joining the beams; they remain the same as they would be if the beams were not connected. All the support for the cantilever is supplied by pillars.

Other structures involving cantilevers include shelving, awnings over the footpath outside shops and the wings of a plane.

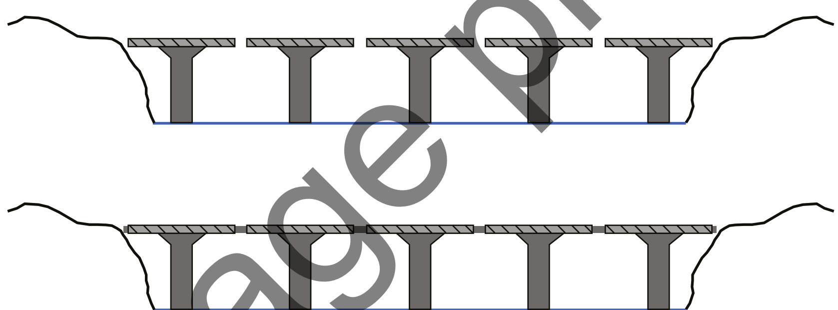


FIGURE 2.3.4 Each beam in the cantilever bridge is fully supported by the pillar below the beam. No added support is provided by connecting the beams.

PHYSICS IN ACTION

Cantilever ruler

- Is it possible to support a ruler with two fingers when one finger is not in the middle of the ruler? Try this experiment using a 1-metre ruler. A smaller ruler will do if that is all that is available.
- Place your index fingers underneath each end of the ruler to support it, as shown in Figure 2.3.5.
- Move your fingers towards each other to find the centre of mass of the ruler.
- Place your right hand at the 30 cm mark of the ruler. Now determine where you need to place your left hand if it must be placed at a point that is less than the 50 cm mark.
- You will find that, regardless of where you start and provided that the ruler is balanced, your fingers will come together at the centre of mass of the ruler. The downward force from the ruler on each finger will vary depending upon its position, and the friction between finger and ruler will vary as a result of the change in that downward force. The result is that both fingers will end up on either side of the centre of mass.

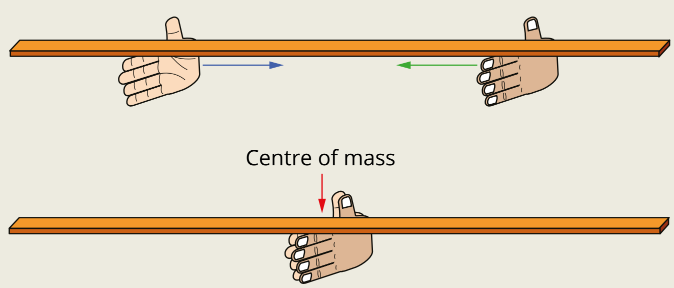


FIGURE 3.3.5 Finding the centre of mass of a ruler with your fingers.

When the centre of mass of a beam is not directly above a support, the force due to gravity acting on the centre of mass will create a torque that must be factored into the conditions of rotational equilibrium. In this case, two supports are usually required: one providing an upward force on the beam and the other providing a downward force on the beam.

Figure 2.3.6 shows a swimming pool diving board with one support applying an upward force on the board and the other support, the metal strap, applying a downward force on the board. A diving board must have a downward force on the fixed end of the board to create an opposing torque to the torque provided by the force due to gravity acting on the board.

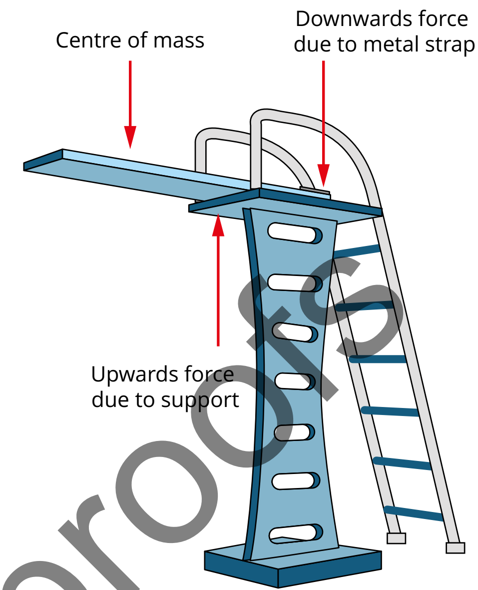
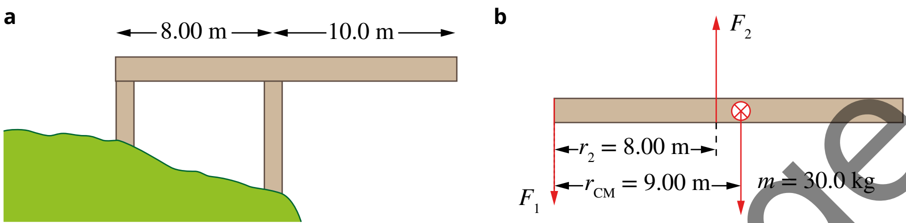


FIGURE 2.3.6 A diving board showing that a metal strap is needed to provide the downward force on the fixed end of the board.

Worked example 2.3.4

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER

A uniform cantilever beam 18.0 m long is used as a viewing platform. It extends 10.0 m beyond two supports that are 8.00 m apart. The beam has a mass of 30.0 kg. Determine the magnitude and direction of the force that the right-hand support must supply so that the beam is in static equilibrium.

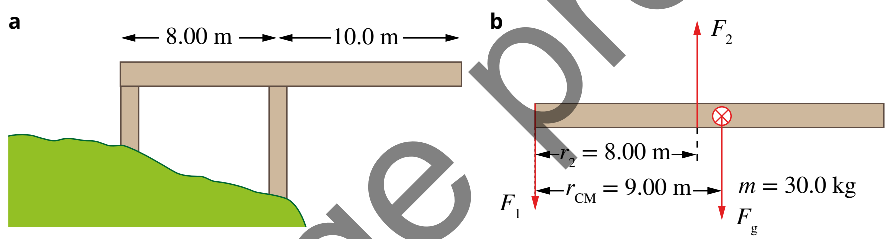


Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 30.0\ \text{kg}$ $r_{\perp F_2 - F_1} = 8.00\ \text{m}$ $r_{\perp c - F_1} = 9.00\ \text{m}$ $g = 9.80\ \text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object upon which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Identify the reference point about which the torques will be calculated. Choose the point on which the other unknown force acts to eliminate it as an unknown in the equation.	The reference point is the point at which the support providing the force F_1 is attached.
Decide which force causes the clockwise torque and which force causes the anticlockwise torque around the chosen reference point.	The force of the right-hand support on the beam provides the anticlockwise torque. The force of gravity on the beam provides the clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam. The torque of the left-hand support is not included, as it acts through the reference point.	$F_b r_{\perp c - F_1} = F_2 r_{\perp F_2 - F_1}$

Substitute the data into the equation and solve for the unknown.	$(30.0)(9.80)(9.00) = F_2(8.00)$ $F_2 = \frac{(30.0)(9.80)(9.00)}{(8.00)}$ $F_2 = \frac{(2646)}{(8.00)}$ $F_2 = 330.75$
State the direction of the force acting on the object in equilibrium and present to the appropriate number of significant figures with the appropriate units.	The force of 331 N is upwards on the beam.

Worked example: Try yourself 2.3.4

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER



Determine the magnitude and direction of the force that the left-hand support must supply so that the beam is in static equilibrium (F_1).

You can check these values using $\Sigma F = 0$.

The sum of the upward forces (F_2) is 331 N.

This balances the sum of the two downward forces, $(F_g + F_1) = 294 + 36.8 \approx 331$ N.

Struts and ties

In addition to balancing the forces on their main beams and pillars, many structures are strengthened in a number of ways. For example, a cantilever may be supported by struts or ties, as shown in Figure 2.3.7. A strut is placed below the beam and is rigid, so it will be under compression. A tie, attached from above a beam, may be either rigid or flexible and will be under tension.

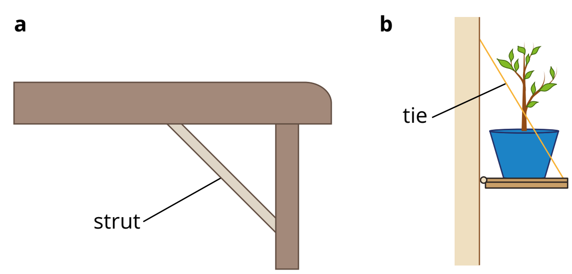
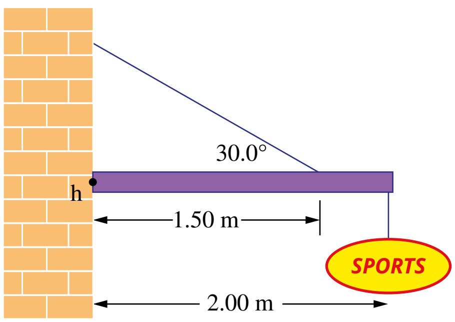


FIGURE 2.3.7 (a) A strut helps to support a cantilevered beam and is under compression. (b) A tie helps to support a fold-out shelf and is under tension.

Worked example 2.3.5

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER SUPPORTED BY A TIE

A sign of mass 10.0 kg is suspended from the end of a uniform 2.00 m long cantilevered beam. The other end of the beam is attached to the wall by a hinge labelled h. The beam has a mass of 25.0 kg and is further supported by a wire tie that makes an angle of 30.0° to the beam. The wire is attached to the beam at a point 1.50 m from the wall. Use $g = 9.80\,\text{N kg}^{-1}$ and ignore the mass of the wire for these calculations.



Calculate the tension (F_t) in the wire that is supporting the beam.

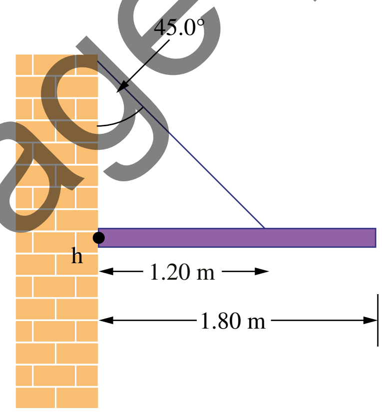
Thinking	Working
Identify the variables involved and state them in their standard form.	$m_b = 25.0\,\text{kg}$ $m_s = 10.0\,\text{kg}$ $r_{\perp b\text{–}h} = 1.00\,\text{m}$ $r_{\perp s\text{–}h} = 2.00\,\text{m}$ $r_{\perp w\text{–}h} = 1.50\,\text{m}$ $g = 9.80\,\text{N kg}^{-1}$
Identify the object that is in rotational equilibrium. This is the object on which all the torques are acting.	The object experiencing rotational equilibrium is the beam.
Identify the reference point about which the torques will be calculated. Choose the point at which the other unknown force acts to eliminate it from the equation.	The reference point is the hinge (h) at which the beam is connected to the wall. As the unknown reactionary force at the hinge passes through this reference point, it does not need to be considered.
Decide which force causes the anticlockwise torque and which forces cause the clockwise torques around the chosen reference point.	The force of the wire tie on the beam provides the anticlockwise torque. The force of gravity on the beam provides one clockwise torque. The force of gravity on the sign provides another clockwise torque.
Apply the equation for rotational equilibrium.	$\Sigma\tau_{\text{clockwise}} = \Sigma\tau_{\text{anticlockwise}}$
Expand the equation to include each of the torques acting on the beam.	$F_b r_{\perp b\text{–}h} + F_s r_{\perp s\text{–}h} = F_w r_{\perp w\text{–}h}$
Substitute the data into the equation to solve for the perpendicular distances from the force arm to the line of action of the force.	$r_{\perp w\text{–}h} = r_{\perp w\text{–}h} \sin(30°)$ $r_{\perp w\text{–}h} = 1.50 \times \sin(30°)$ $r_{\perp w\text{–}h} = 0.750\,\text{m}$

Substitute the data into the equation and solve for the unknown force. Present your answer with the correct units and to the correct number of significant figures.	$(25.0)(9.80)(1.00) + (10.0)(9.80)(2.00) = F_t\,(0.750)$ $F_t = \frac{(245) + (196)}{(0.750)}$ $F_t = \frac{(441)}{(0.750)}$ $F_t = 588.00$ $F_t = 588\,N$
By finding the perpendicular distance using the angle at which the tension force is acting, the tension force can be calculated directly by the equation. If the vertical component of the tension force was found, then the tension force could be calculated separately using the angle at which the force is acting.	

Worked example: Try yourself 2.3.5

CALCULATING THE STATIC EQUILIBRIUM OF A CANTILEVER SUPPORTED BY A TIE

A uniform 5.00 kg beam, 1.80 m long, extends from the side of a building and is supported by a wire tie that is attached to the beam 1.20 m from a hinge (h) at an angle of 45.0°.



Calculate the tension (F_t) in the wire that is supporting the beam.

2.3 Review

SUMMARY

- Static equilibrium occurs when an object experiences translational equilibrium and rotational equilibrium.
- Static equilibrium can be represented mathematically as $\Sigma F = 0$ and $\Sigma \tau = 0$.
- Rotational equilibrium is best represented mathematically as $\Sigma\tau_{clockwise} = \Sigma\tau_{anticlockwise}$.
- In calculations of static equilibrium, the reference point is the point about which the torques act.
- In calculations of static equilibrium with two unknown forces, the reference point can be placed at the point at which one of the unknown forces acts. This eliminates any torque due to this force, as the distance from the force to the reference point is zero.

KEY QUESTIONS

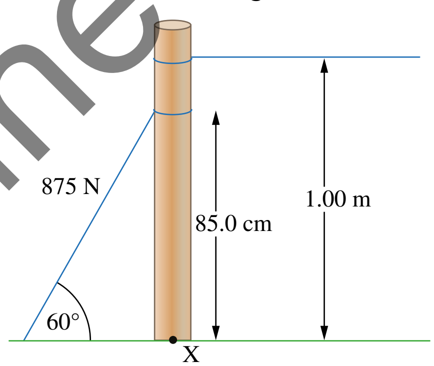
1 Select the correct statement describing the requirements for an object to be in rotational equilibrium.
  A A net torque acts about the reference point and rotation does not occur.
  B A net torque acts about the reference point and rotation occurs.
  C No net torque acts about the reference point and the rate of rotation does not increase.
  D No net torque acts about the reference point and the rate of rotation increases.

2 An adult of mass 75.0 kg sits on a seesaw at a playground with a 25.0 kg child sitting 2.25 m from the pivot point on the other side from the adult. Calculate how far from the pivot the adult must sit for the seesaw to remain balanced and horizontal.

3 Select the statement that correctly describes an object in static equilibrium.
  A The object experiences rotational equilibrium, but not translational equilibrium.
  B The object experiences rotational equilibrium and translational equilibrium.
  C The object experiences neither rotational equilibrium nor translational equilibrium.
  D The object experiences translational equilibrium, but not rotational equilibrium.

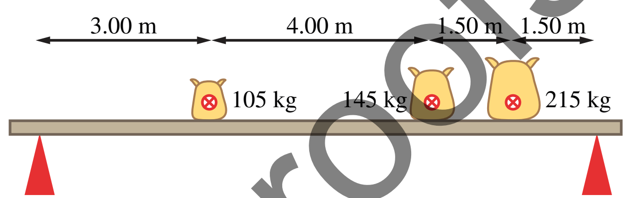
4 An adult of mass 90.0 kg sits on a seesaw at a playground with two 20.0 kg children. One child is sitting 1.50 m from the pivot point and the other is sitting 2.50 m from the pivot point, both on the other side from the adult. Calculate how far from the pivot the adult must sit for the seesaw to remain balanced and horizontal.

5 The end post of a vineyard trellis is held in position by a backstay that is under a tension of 875 N at an angle of 60.0° to the horizontal. The geometry of the situation is shown in the diagram below.



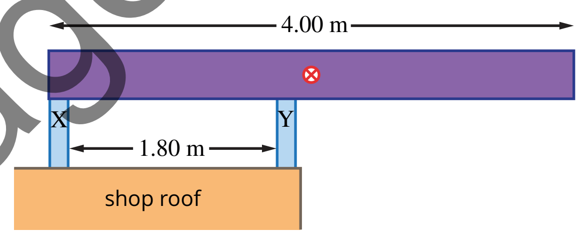
Using the base of the post, X, as the pivot point, determine the size of the tension in the vineyard trellis wire, F_t.

6 A makeshift shelf is used in a bakery to store sacks of flour. The shelf is constructed using a 10.0 m beam with a mass of 50.0 kg, with one support positioned at each end. The shelf holds sacks of mass 105 kg, 145 kg and 215 kg at the positions shown in the figure below. Calculate the forces on the beam due to the left and right supports.



The following information applies to questions 7–9.

A 4.00 m cantilever-type awning is constructed on the roof of a shop so that it shades the front. The awning has a mass of 925 kg and is supported by two supporting columns, X and Y, which produce forces on the awning of F_X and F_Y respectively.



7 Determine the force supplied by column Y on the awning (F_Y).

8 Determine the direction in which F_X acts on the awning.

9 Determine the force supplied by column X on the awning (F_X).

Chapter review

02

KEY TERMS

axis of rotation
base
cantilever
centre of gravity
centre of mass
component
force arm
line of action
neutral equilibrium
pivot point
principle of moments
reference point
rotational equilibrium
stable equilibrium
static equilibrium
torque
translational equilibrium
unstable equilibrium
vector diagram

Use $g = 9.80\,\text{m}\,\text{s}^{-2}$ to answer the following questions.

1 Which of the options best completes the following statement about torque?
'When a force acts such that the line of action of the force is directed through the pivot point of the object, …'
A a torque will result
B a torque will result only if the applied force is greater than the weight of the pivot point
C a torque will result only if the weight of the pivot point is greater than the applied force
D no torque will result

2 Tom is riding his skateboard and is in a state of translational equilibrium. Select the correct statement regarding Tom's motion.
A He must be decelerating.
B He must be maintaining a constant velocity.
C He must be experiencing a translational acceleration.
D He must be experiencing a rotational acceleration.

3 A cyclist is coasting down a road at a decreasing velocity while standing on the pedals. Which one or more of the following objects is in static equilibrium?
A the rear wheel
B the front wheel
C the front cog connected to the pedals
D the cyclist standing on the pedals

4 A child pulls down on a lever-type door handle with a force of 30.0 N. Given that the length of the handle is 12.0 cm, calculate the maximum possible torque acting on the handle's pivot.

5 A railway signalman is responsible for pushing levers that move train tracks, which switches a train from one line to another. Calculate the torque on the axle at the bottom of a 1.35 m lever if a 74.0 N force acts at an angle of 60.0° to the lever.

6 Define 'stability'. Describe the different types of stability and the factors that affect stability.

The following information applies to questions 7 and 8.
A woman whose car has a flat tyre has two wheel spanners in the boot of her car. One wheel spanner is 15.0 cm long, the other is 75.0 cm long.

7 In order to undo the wheel nuts with a minimum amount of effort, which wheel spanner should the woman select? Explain.

8 If the maximum force that the woman can apply is 45.0 N, determine the maximum torque that can be delivered to a wheel nut.

9 In the following situations, a torque is acting. In each case, identify the axis of rotation or pivot point about which the torque acts and estimate the length of the lever arm.
a A garden tap is turned on.
b A wheelbarrow is lifted by the handles.
c An object is picked up with a pair of tweezers.
d A screwdriver is used to lever open a tin of paint.

The following information applies to questions 10 and 11.
A crane with a horizontal lever arm is lifting a concrete wall of mass 2.50 tonnes. The load is 20.0 m from the axis of rotation.

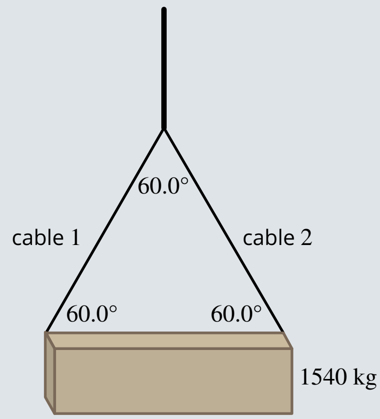
10 Calculate the torque created by this load.

11 What stops the crane from toppling over as a result of this torque?

12 Which of the following are in translational equilibrium? More than one correct answer is possible.
A a stationary elevator
B an elevator going up with constant velocity
C an aeroplane during take-off
D a container ship sailing with constant velocity
E a car plummeting off a cliff

The following information applies to questions 13 and 14.

A concrete beam of mass 1540 kg is being lifted by steel cables, as shown in the diagram.

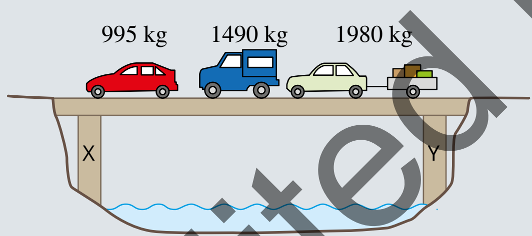


The beam is moving upwards with a constant velocity of 2.00 m s^{-1}. Ignore the mass of the cable.

13 Draw a force diagram showing all the forces acting on the beam.

14 Calculate the tension in cable 1 and cable 2. Show your answer in kN.

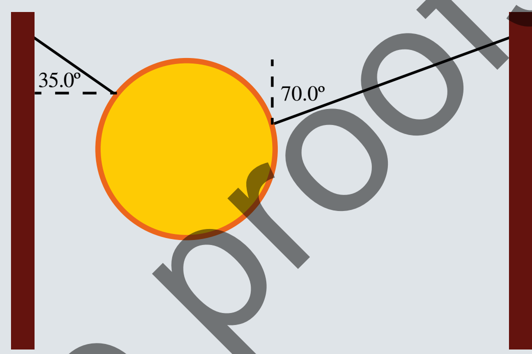
15 Three cars are crossing a beam bridge of mass 525 kg in a single line. At one instant, the left pillar (X) is providing a force of 2.00×10^4 N upwards. What is the size and direction of the force exerted by pillar Y at this instant?



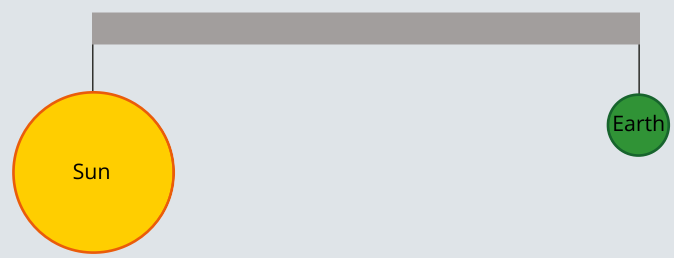
16 A dining tabletop, of mass 40.0 kg, is supported by four legs. On the table there is 4.50 kg of food. If the table is stationary, and the legs all provide the same support, calculate the force each leg would be exerting on the table.

17 A climber of mass of 86.5 kg is hanging from a rope half-way up a rock wall. Calculate the tension force provided by the rope in order for the climber to be in translational equilibrium.

18 A 12.5 kg sign is hung between two poles by two cables of negligible mass, as shown in the diagram below. The sign is in translational equilibrium. Calculate the tension in the shorter left-hand cable and the longer right-hand cable.



19 A child's toy is to be suspended above her bed from a string attached to the ceiling. The toy is made from a 1.00 m long aluminium rod of mass 10.0 g, with a 145 g model of the Sun hanging at one end of the rod and a 22.5 g model of the Earth hanging at the other end. Calculate how far from the Sun, in centimetres, the ceiling attachment string needs to be tied to the bar, so that the whole toy is in static equilibrium.



CHAPTER

01 Practical investigation

This chapter covers most of the skills needed to successfully plan and conduct a practical investigation.

Section 1.1 is a guide to designing and planning an investigation, including how to write a hypothesis, and how to identify the variables. It explains validity, reliability, precision and accuracy, to assist in planning an investigation appropriately.

Section 1.2 is a guide to conducting investigations. It describes methods for accurately collecting and recording data to uncertainty errors. It explores presenting data using tables and graphs, to aid in selecting the most appropriate format for presenting the results.

Section 1.3 explains how to discuss an investigation and draw evidence-based conclusions that relate to the hypothesis and research question.

Practical Investigation Steps

The size and scope of a practical investigation can be initially quite daunting, but establishing a task list and timeline will help break it down into manageable steps. The entire task is expected to take between 7 and 10 hours.

Here are some steps that will need to be considered in a timeline:

- Determine the topic and type of investigation.
- Research and write down the theory on which the investigation is based.
- Determine an appropriate question to answer, and formulate a hypothesis.
- Identify the independent, dependent and controlled variables.
- Select equipment and resources needed for the investigation.
- Determine an appropriate procedure (methodology), considering validity, reliability and accuracy.
- Assess the risks and ethical issues, and identify measures to address these.
- Conduct the investigation and record all data obtained.
- Analyse and evaluate the data.
- Evaluate your methods. Suggest ways of improving or extending the investigation.
- Write an evidence-based conclusion. Describe the limitations of the study.
- Write the final report. (This should not be the focus of the investigation but rather an opportunity to communicate the investigation process and your conclusions.)

Some of these tasks are larger and will require more time than others. Many will overlap. Plan a realistic approach, consult with teachers to establish school-based time constraints and fix dates for the completion of each task. Allow time for reflection and to review your earlier work.

Science Inquiry Skills

- identify, research and construct questions for investigation; propose hypotheses; and predict possible outcomes
- design investigations, including the procedure to be followed, the materials required, and the type and amount of primary and/or secondary data to be collected; conduct risk assessments; and consider research ethics
- conduct practical work, including the manipulation of devices, safely, competently and methodically for the collection of valid and reliable data
- Represent data in meaningful and useful ways, including using appropriate *Système Internationale* (SI) units and symbols, and significant figures
- organise and analyse data to identify trends, patterns and relationships
- identify sources of random and systematic uncertainty and estimate their effect on measurement results
- state absolute uncertainties in values and calculate percentage uncertainty where appropriate
- combine uncertainties in calculations to determine the overall uncertainty in a measurement (addition, subtraction, multiplication and division)
- identify anomalous data and calculate the percentage difference between the experimental results and a currently accepted value
- select, synthesise and use evidence to make and justify conclusions
- interpret a range of scientific texts and evaluate processes and conclusions by considering the available evidence, and use reasoning to construct scientific arguments
- select, construct and use appropriate representations, including text and graphical representations of empirical and theoretical relationships, to communicate conceptual understanding, solve problems and make predictions
- select, use and interpret appropriate mathematical representations, including linear and non-linear graphs and algebraic relationships representing physical systems, to solve problems and make predictions
- relate gradients and axis intercepts of linear graphs to physical quantities
- apply dimensional analysis to determine the appropriate units for calculated quantities, e.g. a gradient in a graph
- use uncertainty bars to represent the uncertainty in a value on a graph and take into account when sketching a line of best fit
- communicate to specific audiences and for specific purposes using appropriate language and nomenclature

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 12 Syllabus for teaching from 2025*.

1.1 Designing and planning the investigation

Taking the time to carefully plan and design a practical investigation before you begin will help you to maintain a clear and concise focus throughout. Preparation is essential. Ensure you understand the theory behind the investigation and prepare a detailed plan for the practical components of the investigation. This section is a guide to some of the key steps that should be taken when planning and designing a practical investigation (Figure 1.1.1).

FIGURE 1.1.1 There are many elements to a practical investigation, which may appear overwhelming to begin with. Taking a step-by-step approach will help the process and assist you in completing a worthwhile investigation.

DETERMINING THE TYPE OF INVESTIGATION

In your course, you will be expected to prepare a portfolio of the practical work you have undertaken across the range of physics areas studied. This portfolio will include evidence of a variety of science inquiry skills, including experimental design and the communication and evaluation of scientific data.

Several types of research methods are used in science. Physics investigations typically fall into two methodologies:

- analysing the slope of a linear graph
- determining the relationship between two continuous **variables**.

From your research on the topic you are investigating you may find that there is a mathematical relationship between the variables that you are exploring. It may be possible to use your algebra skills to write the mathematical relationship in the form of a linear equation, $y = mx + c$, where y is the dependent variable and x is the independent variable. A graph of the y and x variables will result in a linear graph with a slope equal to a constant, m, or collection of constants. By equating the value of the slope to these constants some meaningful analysis can be made, such as finding a value for the acceleration due to gravity or the specific heat capacity of a metal.

If, in your research, you find that there is no equation that links your variables, then you can investigate to determine what the relationship might be. By graphing your variables, you may find that the relationship is linear, or it may be inverse, exponential or even inverse squared. An investigation of this kind results in a calibration curve, which can be used to predict values of the dependent variable given particular values of the independent variable.

DEVELOPING RESEARCH QUESTIONS AND AIMS, FORMULATING HYPOTHESES AND IDENTIFYING VARIABLES

The research question, aim and hypothesis are interlinked. It is important to note that each of these can be refined as the planning of the investigation continues.

Formulating a question

A research question is a question that comes from an inquiring mind. When you are actively involved in developing your scientific understanding you will want to know what factors affect a variable, or what is the relationship between two variables. The research question poses the question that the investigation seeks to answer. For example: What is the relationship between voltage and current?

Before formulating a question, it is good practice to conduct a literature review of the topic to be investigated. You should become familiar with the relevant scientific concepts and key terms.

During this review, write down questions, correlations or equations as they arise.

Compile a list of possible ideas. Do not reject ideas that initially might seem impossible. Use these ideas to generate questions that are answerable.

Before constructing a hypothesis, formulate a question that needs an answer. This question will lead to a hypothesis when:

- the question is reduced to measurable variables
- a prediction is made based on knowledge and experience.

Evaluating your question

Once a question has been chosen, stop to evaluate the question before progressing. The question may need further refinement or even further investigation before it is suitable as a basis for an achievable and worthwhile investigation. A major planning point is to attempt something that is possible to complete in the time available or with the resources on hand. It might be a little difficult to create a particularly complicated device with the facilities available in the school laboratory.

To evaluate the question, consider the following:

- Relevance: Is the question related to the appropriate area of study?
- Clarity and measurability: Can the question lead to a clear hypothesis? If the question cannot lead to a specific hypothesis, it will be very difficult to complete the research.
- Time frame: Can the question be answered within a reasonable period of time? Is the question too broad?
- Knowledge and skills: Do you have a level of knowledge and a level of laboratory skills that will allow the question to be explored? Keep the question simple and achievable.
- Practicality: Are resources, such as laboratory equipment and materials, likely to be readily available? Keep things simple. Avoid investigations that require sophisticated or rare equipment. Equipment that is more-readily available includes timing devices, objects that could be used as projectiles, a tape measure and other common laboratory equipment.
- Safety and ethics: Consider the safety and ethical issues associated with the question you will be investigating. If there are issues, can these be addressed?
- Advice: Seek advice from your teacher about the question. Their input may prove very useful. Their experience may lead them to consider aspects of the question that you have not thought about.

Defining the aim of the investigation

An aim is a statement describing in detail what will be investigated to answer the research question. For example: The aim of the experiment is to investigate the relationship between the voltage and current in a circuit with constant resistance. Each aim should directly relate to the variables that will be referred to in the hypothesis. The aims do not need to include the details of the method.

Example

- Aim: The aim of the experiment is to investigate the relationship between mass and acceleration, when a constant force is applied.

Hypothesis

A hypothesis is a definite statement, based on previous knowledge and evidence or observations, that attempts to answer the research question. The hypothesis must relate the independent and dependent variables and describe the relationship between them. For example: Increasing the voltage supplied to a circuit with constant resistance increases the current proportionally.

Here are some further examples of hypotheses:

- For a constant force, if the mass is increased, the acceleration is decreased as an inverse relationship.
- If the value of the resistance of a circuit increases, the current flowing in the circuit will decrease as an inverse relationship.

- Assuming no heat loss to the surroundings, the temperature rise of a fixed mass of water is proportional to the time it is heated by a constant power source.
- As the height from which an object is dropped increases, the final velocity of the object will increase as a squared relationship.

There are no wrong or right hypotheses. You might formulate a hypothesis that a more experienced person will disagree with; however, the purpose of an investigation is to find the answer to a research question. If the answer to the question supports your hypothesis, then that is a positive result, as it will confirm your understanding of the concept. On the other hand, if your investigation does not support your hypothesis, then that is a useful result as well, as you can now say that your original understanding was not correct and you can change your understanding to a more scientific one. Some of you might notice that the following hypothesis will not be supported by the investigation:

- The greater the mass of a marble, the faster it will hit the ground, when dropped from the same height.

This doesn't mean that the hypothesis is wrong, but it may indicate that there was some misconception that you had that was not exposed in your literature review.

Formulating a hypothesis

A good hypothesis should:

- be a definite statement of the relationship
- include an independent and a dependent variable that is continuous and measurable
- be worded so that it can be tested in the experiment.

The hypothesis should also be falsifiable. This means that a negative outcome would disprove it. For example, the hypothesis that all apples are round cannot be proved beyond doubt, but it can be disproved, in other words, it is falsifiable. In fact, only one oval-shaped apple is needed to disprove this hypothesis. Unfalsifiable hypotheses cannot be tested by science. These include hypotheses on ethical, moral and other subjective judgements.

Variables

A good scientific hypothesis can be tested, that is, it can be supported or refuted through investigation. To be a testable hypothesis, it should be possible to measure both the change or treatment and the effect, or what will happen. The factors that can be changed, or are changed as a result of the experiment or investigation, are called the variables. An experiment or investigation determines the relationship between variables.

There are three categories of variables:

- The **independent variable** is the variable that is changed by the researcher. You must test only one independent variable in any investigation, otherwise it cannot be stated that the changes in the dependent variable are the result of changes in the independent variable.
- The **dependent variable** is the variable that may change in response to a change in the independent variable. This is the variable that will be measured or observed. You should measure only one dependent variable in any investigation. If you want to measure another dependent variable then you will need to do another investigation with another hypothesis.
- **Controlled variables** are all the variables that must be kept constant during the investigation otherwise the test cannot be fair.

Read the following example of a hypothesis.

If the cross-sectional area of a resistor is constant, the longer the wire, the greater the resistance as a linear relationship.

Identify the different variables.

- independent variable: length of wire
- dependent variable: resistance of the wire
- controlled variables: potential difference, material of the resistor, temperature of the resistor.

Completing a table like Table 1.1.1 will assist in evaluating the question or questions.

TABLE 1.1.1 Break the question down to determine the variables.

Research question	How does the power of a kettle affect the time taken to boil water?
Independent variable	the power of the kettle
Dependent variable	the time the kettle takes to boil water
Controlled variables	mass of the water, purity of the water, starting temperature of the water and kettle
Potential hypothesis	The greater the power of a kettle, the less time it will take to boil water, as an inverse relationship

Qualitative and quantitative variables

Variables are either qualitative or quantitative, with further subsets in each category.

- **Qualitative variables** can be observed but not measured; for example, describing a light globe as bright or dim. They can only be sorted into groups or categories such as brightness, type of construction material or type of device.
  - Nominal variables are categorical variables in which the order is not important; for example, the type of material or type of device.
  - Ordinal variables are categorical variables in which order is important and groups have an obvious ranking or level; for example, brightness (Figure 1.1.2).
- **Quantitative variables** can be measured. Length, area, weight, temperature and cost are all examples of quantitative data.
  - Discrete variables consist of only integer numerical values, not fractions; for example, the number of pins in a packet, the number of springs connected together, or the energy levels in atoms.
  - Continuous variables allow for any numerical value within a given range; for example, the measurement of temperature, length, mass and frequency.

In physics, you should choose continuous quantitative variables for both the independent and dependent variables. This will allow you to construct a line graph, and therefore determine the slope of the line, or the relationship between the variables.

FIGURE 1.1.2 When recording qualitative data, describe in detail how each variable will be defined. For example, if recording the brightness of light globes, light meters are a quantitative way to gather data.

WRITING THE METHODOLOGY

The methodology, or method, of your investigation is a step-by-step procedure. When detailing the method, ensure it enables you to conduct a valid, reliable and accurate investigation.

Validity

Validity refers to whether an experiment is in fact testing the hypothesis. Is the investigation obtaining data that is relevant to the question, or is it flawed?

To ensure an investigation is valid, it should be designed so that only one variable is changed at a time. The other variables must remain constant, so that meaningful conclusions can be drawn about the effect of the independent variable alone.

To ensure validity, you must carefully determine:

- the independent variable—the variable that will be changed, and how it will change
- the dependent variable—the variable that will be measured
- the controlled variables—the variables that must remain constant.

Reliability

Reliability refers to the idea that the experiment can be repeated many times and will obtain consistent results. You can maintain the investigation's reliability by:

- listing and defining the control variables and how they will be kept constant
- listing the detailed steps that you will take to conduct the experiment, describing what you will do and how you will measure and record data
- ensuring that there are enough changes of the independent variable. Typically, five changes over a wide range of the independent variable are considered sufficient.
- ensuring there are enough trials conducted for each value of the independent variable. Typically, you should conduct at least three trials repeating the experiment, then average the three measurements. This reduces random errors and allows systematic errors to be identified. If a reading differs too much from the rest (known as an outlier), discard it before averaging (Figure 1.1.3).

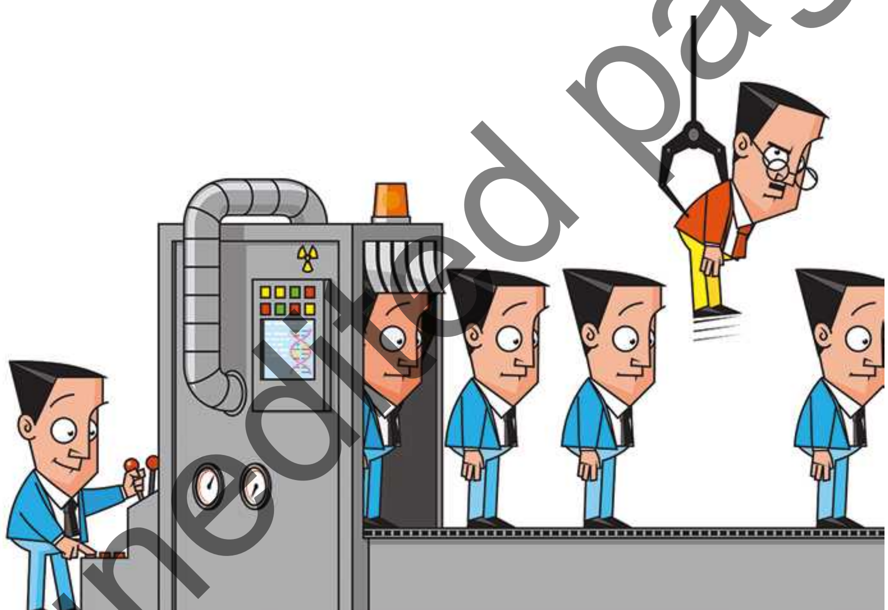


FIGURE 1.1.3 Replication increases the reliability of your investigation. It ensures that if anyone repeats the investigation they will obtain similar data.

Accuracy and precision

Precision refers to the extent to which the instrument can make repeated measures of the dependent variable that are the same for the same value of the independent variable. For example, if each measurement of the current in an electrical circuit is within 0.1 A of the others, then the device is more precise than a device for which there is a difference of 0.5 A. **Accuracy** refers to how close a measurement is to the true or accepted value.

You will need to consider if the instruments to be used are sensitive enough. Build some testing into your investigation to confirm the accuracy and reliability of the equipment and your ability to read the information obtained.

Reasonable steps to ensure the accuracy of the investigation include considering:

- the type of instrument that will be used to measure the independent and dependent variables.
- calibrating the measuring equipment by testing a standard.

Describe the materials and method in appropriate detail in your scientific reports. This should ensure that every measurement can be repeated and the same result obtained within reasonable margins of experimental uncertainty. (A margin of less than 5% is reasonable.)

Data analysis

How you will be analysing the data produced from your investigation must be considered when writing your method. A wide range of analysis tools are available. For example, tables can be used to organise data so that patterns can be established, and graphs can show relationships and comparisons. In fact, preparing an empty table showing the data that needs to be obtained will help in the planning of the investigation. See page 13 for more information on organising a data table.

Sourcing appropriate materials and technology

When designing your investigation, you will need to decide on the materials, technology and instrumentation that will be used to carry out your research. It is important to find the right balance between items that are easily accessible and those that will give you accurate results. As you move on to conducting your investigation, it will be important to take note of the quality of your chosen instruments and how this affects the accuracy and validity of your results.

Modifying the methodology

The methodology may need modifying as the investigation is carried out. The following actions will help to determine any issues in the methodology and how to modify it.

- Record everything.
- Be prepared to make changes to the approach.
- Note any difficulties encountered and the ways they were overcome. What were the failures and successes? Every test carried out can contribute to the understanding of the investigation as a whole, no matter how much of a disaster it may first appear.
- Do not panic. Go over the theory again, and talk to the teacher and other students. A different perspective can lead to a solution.

If the expected data is not obtained, don't worry. As long as it can be critically and objectively evaluated, with the limitations of the investigation identified and further investigations proposed, the work is worthwhile.

COMPLYING WITH ETHICAL AND SAFETY GUIDELINES

Ethical considerations

Some investigations require an ethics approval—consult with the teacher. In fact, when deciding on an investigation, identify all possible ethical considerations and evaluate whether those parts of the investigation are necessary or if there are ways you can reduce or mitigate them.

Occupational health and safety

While planning for an investigation, it is important to consider the potential risks to ensure the safety and yourself and others.

Everything we do has some risk involved. Risk assessments are performed to identify, assess and control hazards. A risk assessment should be performed for any situation, in the laboratory or outside in the field. Always identify the risks and control them to keep everyone safe. For example, carry out voltage–current experiments with low voltages (less than 6.0 V DC or 4 × 1.5 V batteries) coupled to resistors so that the currents in the circuits are of the order of milliamps. *At all times* avoid direct exposure to 240 V AC household voltages (Figure 1.1.4).

To identify risks, think about:

- the activity that will be carried out
- the equipment or chemicals that will be used.

The following hierarchy of risk controls is organised from most to least effective:

1. *Elimination:* Eliminate dangerous equipment, procedures or substances.
2. *Substitution:* Find different equipment, procedures or substances to use that will achieve the same result, but have less risk associated with them.
3. *Isolation:* Ensure there is a barrier between the person and the hazard. Examples include physical barriers such as guards in machines, or fume hoods to work with volatile substances.
4. *Engineering controls:* Modify equipment to reduce risks.
5. *Administrative controls:* Provide guidelines, special procedures, warning signs and information about safe behaviours for any participants.
6. *Personal protective equipment (PPE):* Wear safety glasses, lab coats, gloves and respirators etc. where appropriate. Provide these to other participants as needed.

FIGURE 1.1.4 When planning an investigation, you need to identify, assess and control hazards.

Science outdoors

Sometimes investigations and experiments will be carried out outdoors. Working outdoors has its own set of potential risks and it is equally important to consider ways to eliminate or reduce these risks.

Table 1.1.2 contains examples of risks associated with fieldwork outdoors.

TABLE 1.1.2 Risks associated with fieldwork outdoors.

Risks	Control measures
sunburn	wear sunscreen, a hat and sunglasses
hot or cold weather	wear clothing to protect against heat or cold
projectile launch	create barriers so that people know not to enter the area
trip hazards	minimise the use of cables (electrical, computer) and cover them up with matting be aware of tree roots, rocks etc.

First aid measures

Minimising the risk of injury reduces the chance of requiring first aid assistance. However, it is still important to have someone with first aid training present during practical investigations. Always tell the teacher or laboratory technician if an injury or accident happens.

Personal protective equipment

Everyone who works in a laboratory wears items that help keep them safe. This is called **personal protective equipment (PPE)** and includes:

- safety glasses
- shoes with covered tops
- disposable gloves when handling chemicals
- a disposable apron or a lab coat if there is risk of damage to clothing
- ear protection if there is risk to hearing.

1.1 Review

SUMMARY

- An aim is a statement describing in detail what will be investigated. For example: The aim of the experiment is to investigate the relationship between force, mass and acceleration.
- A hypothesis is a definite statement of the relationship between the independent and dependent variables based on previous knowledge and evidence or observations that attempts to answer the research question. For example: With the force kept constant, the acceleration decreases with increasing mass as an inverse relationship.
- Once a question has been chosen, stop to evaluate the question before progressing. The question may need further refinement or even further investigation before it is suitable as a basis for an achievable and worthwhile investigation. Make sure that it is possible to complete the activity in the time available and with the resources on hand. It might be a little difficult to create a particularly complicated device with the facilities available in the school laboratory.
- There are three categories of variables:
  - The independent variable is the variable that is changed by the researcher.
  - The dependent variable is the variable that may change in response to a change in the independent variable. This is the variable that will be measured or observed.
  - Controlled variables are all the variables that must be kept constant during the investigation so that it is a fair test.
- The methodology of your investigation is a step-by-step procedure. When detailing the methodology, ensure it complies as a valid, reliable and accurate investigation.
- It is also important to determine how many times the independent variable needs to be changed and how many trials need to be run for each change in the independent variable.
- Data analysis should be a consideration in the method. Determine how the data will be presented and analysed. A wide range of analysis tools could be used. For example, tables organise data so that patterns can be established and graphs can show relationships and comparisons.
- In every investigation you need to consider the risks and potentially hazardous situations, and act to minimise those risks.

KEY QUESTIONS

1 In a practical investigation a student changes the potential difference across a circuit by adding or subtracting batteries in series in the circuit.
  a How could the potential difference be a discrete value?
  b How could it be continuous?

2 In another experiment the student uses the following range of values to describe the brightness of a light: dazzling, bright, glowing, dim, off. What type of measurement is the variable 'brightness'?

3 Select the best hypothesis from the three options below. Give reasons for your choice.
  A Hypothesis 1: If you increase the mass of the marble that you drop, the final velocity of the marble will increase linearly.
  B Hypothesis 2: The greater the potential difference across a resistor, the greater the current through it.
  C Hypothesis 3: Different metals will have different resistances.

4 Give the correct term that describes an experiment with each of the following conditions.
  a The experiment addresses the hypothesis and aims.
  b The experiment is repeated and consistent results are obtained.
  c Appropriate, high-quality equipment is chosen and calibrated for the desired measurements.

5 A student wanted to find out how the tension in an elastic band affects the band's initial velocity when launched from their finger. State:
  a the independent variable
  b the dependent variable
  c three controlled variables.

1.2 Conducting investigations, and recording and presenting data

Once the planning and design of a practical investigation is complete, the next step is to undertake the investigation and record the results. As with the planning stages, there are key steps and skills to keep in mind to maintain high standards and minimise potential error throughout the investigation (Figure 1.2.1).

This section will focus on the best methods for conducting a practical investigation, systematically generating, recording and processing data, and then presenting it in a concise and clear manner.

FIGURE 1.2.1 When carrying out your investigation try to maintain high standards to minimise potential errors.

CONDUCTING INVESTIGATIONS TO COLLECT AND RECORD DATA

For an investigation to be scientific, it must be objective and systematic. Ensuring familiarity with the methodology and protocols before beginning will help you to achieve this.

While working, keep asking questions: Is the work biased in any way? If changes are made, how will they affect the study? Will the investigation still be valid for the aim and hypothesis?

It is essential that during the investigation the following are recorded:

- all quantitative data collected
- the methods used to collect the data
- any incident, feature or unexpected event that may have affected the quality or validity of the data.

The data recorded is the **raw data**. Usually this data needs to be processed in some manner before it can be presented. If an error occurs in the processing of the data or you decide to present the data in an alternative format, the recorded raw data will always be available for you to refer back to.

IDENTIFYING ERRORS

All practical investigations have errors associated with them. Errors can occur for a variety of reasons. It is important that potential errors are considered when planning an investigation and that measures are taken to reduce them. This ensures the investigation is as accurate as possible. Any additional errors that occur during the collection of results should also be recorded.

There are two types of errors:

- systematic errors
- random errors.

Systematic errors

A **systematic error** is an error that is consistent and will occur again if the investigation is repeated in the same way.

Systematic errors are usually a result of instruments that are not calibrated correctly or methods that are flawed.

An example of a systematic error would be if a ruler mark indicating 5 cm from 0 cm was actually only 4.9 cm from 0 cm due to a manufacturing error or shrinkage of the wood. Another example would be if the researcher repeatedly used a piece of equipment incorrectly throughout the entire investigation. Figure 1.2.2 shows how traffic police reduce systematic errors in their data collection.

FIGURE 1.2.2 To avoid a systematic error, make sure that you are using measuring equipment correctly. Laser speed guns, for example, need to be placed on a stationary support so the aim point is held on a single target point for the duration of the read.

Random errors

Random errors occur in an unpredictable manner and are generally small. Random errors are typically caused by minor, unpredictable changes in experimental conditions that lead to fluctuations around the true value. An example of a random error could be electronic noise in the circuit of an electrical instrument.

Techniques for reducing error

Designing the method carefully, including selection and use of equipment, will help reduce errors.

Appropriate equipment

Use the equipment that is best suited to the type of data being collected to validate the hypothesis. Determining the appropriate units and scale for the data will help to select the correct equipment. Using the right unit and scale will ensure that measurements are more accurate and precise, thereby minimising systematic errors.

Significant figures represent precision and reliability of a measurement. The number of significant figures used depends on the scale of the instrument. It is important to record data to the number of significant figures available from the equipment or observation. Using more or fewer significant figures can be misleading.

Review the following examples to learn more about significant figures:

- 15 has two significant figures
- 3.5 has two significant figures
- 3.50 has three significant figures
- 0.037 has two significant figures
- 1401 has four significant figures.

To calculate gravitational potential energy (E_g), the formula is $E_g = mg\Delta h$.
If $g = 9.80\,\text{m}\,\text{s}^{-2}$, mass (m) = 7.50 kg, height (h) = 0.64 m (64 cm):

$$E_g = 9.80 \times 7.50 \times 0.64 = 47.09\,\text{J}$$

When reporting data, quote the result to the least number of significant figures found in the measured values. In this example, height is accurate to two significant figures while g and mass have three significant figures, so report the result to two significant figures, for example $E_g = 47\,\text{J}$.

Although digital scales can measure to many more than two figures and calculators can give 12 figures, be sensible and follow the significant figure rules.

Calibrated equipment

Some equipment, such as some motion sensors, needs to be calibrated before use to account for the temperature at the time. Before carrying out the investigation, make sure the instruments or measuring devices are properly calibrated and functioning correctly. For example, measure the temperature and apply a correction to the speed of sound to calibrate a motion sensor if necessary.

Correct use of equipment

Use the equipment properly. Ensure training has been completed and that you have practised using the equipment before beginning the investigation. Improper use of equipment can result in inaccurate data with large errors, and the validity of the data can be compromised.

Incorrect reading of measurements is a common misuse of equipment. Make sure all the equipment needed in the investigation can be used correctly and record the instructions in detail so they can be checked if the data doesn't appear correct.

RECORDING AND PRESENTING QUANTITATIVE DATA

Raw data is unlikely to be used directly to validate the hypothesis. However, raw data is essential to the investigation and plans for collecting the raw data should be made carefully. Consider the formulas or graphs that will be used to analyse the data at the end of the investigation. This will help to determine the type of raw data that needs to be collected in order to validate the hypothesis.

For example, to calculate take-off velocity for a vertical jump, three sets of raw data will need to be collected using a force platform: the athlete's standing body weight, the ground reaction force and the time during the vertical jump. The data can then be processed to obtain the take-off impulse.

Once you have determined the data that needs to be collected, prepare a table to record the data.

ANALYSING AND PRESENTING DATA

The raw data that has been obtained needs to be presented in a way that is clear, concise and accurate.

There are several ways to present data, including tables, graphs, flow charts and diagrams. The best way of visualising the data depends on its nature. Try multiple formats before making a final decision to create the best possible presentation.

Presenting raw and processed data in tables

Tables organise data into rows and columns, and can vary in complexity according to the nature of the data. Tables can be used to organise raw and processed data or to summarise results.

Data in the table should be ordered in columns. The first column should contain the independent variable (the one being changed). Subsequent columns should include the dependent variable results from all trials, with one column for each trial. The final column should then show the average of all of the trials for the dependent variable.

Tables should have the following features:

- a descriptive title that contains both the independent and depended variables
- column headings (including the unit and the uncertainty of measurement)
- aligned figures (align the decimal points)
- the independent variable placed in the left column
- the trials of the dependent variable placed in the right columns with the average column on the end
- an overarching heading for all columns showing the dependent variable results (including the average column).

You may sometimes need to process data to assist with producing a linear graph. This processed data can be added as an additional column to the right of the average column. An example of processed data would be if I^2 needed to be plotted on a graph instead of I. Any processed data should have appropriate units and uncertainty in the heading if the units and uncertainty remain the same, or alongside the data if they vary.

Look at the table in Figure 1.2.3, which has been used to organise raw and processed data about the effect of current on voltage.

The current through a resistor at different potential differences ← clear title

	Current I (±0.01 A)				
Potential difference ΔV (±0.01 V)	Trial 1	Trial 2	Trial 3	Average	I^2 (A^2)
1.50	0.31	0.34	0.32	0.32 ± 0.02	0.10 ± 0.01
2.00	0.42	0.45	0.41	0.43 ± 0.02	0.18 ± 0.02
2.50	0.50	0.51	0.52	0.51 ± 0.01	0.26 ± 0.01
3.00	0.62	0.65	0.58	0.62 ± 0.04	0.38 ± 0.05
3.50	0.70	0.71	0.72	0.70 ± 0.01	0.49 ± 0.01

headings for each column with units and uncertainties of measurement

consistent number of significant figures

independent variable

dependent variable trials with consistent number of significant figures

averages calculated with uncertainty of averages displayed

processed data with processed uncertainties, correct significant figures and heading with appropriate units

FIGURE 1.2.3 A simple table listing the raw data obtained in the second, third and fourth columns with the common uncertainty in the overarching column heading, the average of each trial calculated in the fifth column with individual uncertainties, and the processed data, which is the average values squared along with their processed individual uncertainties, in the sixth column.

FIGURE 1.2.4 An analogue pressure scale.

A table of processed data usually presents the average values of trials, the **mean**. However, the mean on its own does not provide an accurate picture of the results. To report processed data more accurately, the uncertainty should be presented as well.

UNCERTAINTY

Measuring from an analogue scale

An analogue scale is a fixed scale from which measurements are taken. Examples of an analogue scale include rulers or pressure gauges, such as the one shown in Figure 1.2.4. When measuring with an analogue scale and the value falls between increments, an estimate should be made to one decimal place beyond the nearest increment. For example, if you measure the length of a block of wood using a ruler marked in centimetres and it aligns precisely with the 10 cm mark, the length is 10.0 cm. However, if the length is approximately halfway between the 10 and 11 cm marks, it should be recorded as 10.5 cm.

Types of uncertainties

When presenting a range of measurements for a particular value, it is essential to include both the mean and the associated **uncertainty** to accurately convey the results. In other words, the mean must be accompanied by an indication of the range of data obtained, reflecting variability in the measurements.

uncertainty = ± (half the range of the values)

For example, if the velocity, in km h^{-1}, of cars travelling down a certain road was:

46.0, 50.0, 55.0, 48.0, 50.0, 58.0 and 45.0

the average velocity would be:

$$\frac{46.0 + 50.0 + 55.0 + 48.0 + 50.0 + 58.0 + 45.0}{7} = 50.2857 = 50.3\,\text{km h}^{-1}$$

The uncertainty would therefore be half the maximum velocity minus the minimum velocity. In this case, 58.0 is the maximum velocity and 45.0 is the minimum velocity, meaning that the range is 13.0 and half the range is 6.50, so the uncertainty is ± 6.50 km h^{-1}.

This data should be presented as:

Average speed is 50.3 ± 6.50 km h^{-1}.

This is called the **uncertainty of averages** as it is taken from the calculation of the mean. Make sure that the final value is expressed to the appropriate number of significant figures, and remember that the uncertainty should never exceed the number of decimal places of the measured value itself.

There is also uncertainty associated with using measuring devices. This is called the **uncertainty of measurement**. This type of uncertainty arises from the precision of the measuring device's scale. The uncertainty of measurement varies depending on the type of measuring device you are using.

- Some devices will have a given uncertainty; for instance, a measuring cylinder might have an uncertainty of ± 0.1 mL printed on it.
- If the uncertainty is not provided, it will depend on whether the measuring device is digital or analogue.
  - If you are measuring on a digital scale, the uncertainty is the smallest possible increment. For instance, a set of scales that gives a result of 100.01 g will have an uncertainty of measurement of ± 0.01 g.
  - If the measuring device is analogue (i.e. those with a fixed scale), the uncertainty of measurement is typically half of the smallest scale division. For instance, if you use a ruler with a millimetre scale and measure the length of an object as 25.0 mm, the uncertainty would be ± 0.5 mm. It is important to note that the uncertainty should have the same number of decimal places as the measurement, but not necessarily the same number of significant figures.

When stating a final value with its uncertainty, you may have two choices to make:

1 If the uncertainty of the measuring device is different from the uncertainty of the averages, you should use the larger of the two values for the average uncertainty. This ensures your reported data reflects the limitations of the instrument, not potentially overstating the precision of the average.
2 You should consider the nature of the investigation, the measuring devices used and the method of data collection in relation to the calculated uncertainty of the average. If it was difficult to take the measurements or you experienced issues with the measuring devices, then you may choose to use a reasonable percentage uncertainty to replace the uncertainty of the averages. For example, if the percentage uncertainty of the averages is around ± 1.5%, but you believe this does not adequately represent the precision of your data collection, you might opt for a higher percentage uncertainty, such as ± 5.0%. If you make this adjustment, make sure you include a note under your data table explaining your rationale.

Calculating percentage uncertainties

Uncertainties that are displayed in the same units as the measurement are known as **absolute uncertainties**. You can use these absolute uncertainties to calculate a percentage uncertainty using the equation below.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measurement}} \times 100\%$$

Percentage uncertainties are helpful because they allow for the comparison and combination of uncertainties across different units.

Combining uncertainties

It is common to process data in a physics investigation. This may include altering a single set of data, like the column titled 'I^2' in Figure 1.2.3; or perhaps you may use two data sets to calculate a new value, for instance, combining a measured current and potential difference to calculate the power. As these are values calculated using measurements with their own uncertainties, it is crucial to combine the uncertainties to get an overall uncertainty for the calculated value.

The method for combining uncertainties depends on the type of mathematical operation used in the calculation: whether measurements are raised to a power, multiplied or divided, or added or subtracted.

Measurements raised to a power

If a measurement is raised to a power, such as squared or cubed, the percentage uncertainty associated with the measurement will be multiplied by that power. This is shown in step three of Worked example 1.2.1.

Multiplying or dividing measurements

When multiplying or dividing measurements, the total uncertainty is found by adding together the percentage uncertainties for each value. Since the measurements may not have the same units, it is important to use the percentage uncertainties rather than absolute uncertainties, which cannot be added directly. This is outlined in Worked example 1.2.1.

Worked example 1.2.1

COMBINING UNCERTAINTIES WHEN MULTIPLIED OR DIVIDED

A researcher wants to determine the density of a sample of an unknown substance that is in the shape of a cube. The following measurements are taken.

- Length of each side of the cube: ℓ = 25.5 ± 0.5 mm
- Mass of the cube: m = 32.2 ± 0.1 g

Calculate the density with the appropriate absolute uncertainty.

Thinking	Working
Convert all absolute uncertainties to percentage uncertainties.	$\%\text{uncertainty} = \frac{\text{uncertainty}}{\text{measurement}} \times 100$ $\%\text{uncertainty}_{length} = \frac{(0.5)}{(25.5)} \times 100$ $\%\text{uncertainty}_{length} = \pm 1.96078\%$ $\%\text{uncertainty}_{mass} = \frac{(0.1)}{(32.2)} \times 100$ $\%\text{uncertainty}_{mass} = \pm 0.31056\%$
Calculate the density of the cube with the data converted to the SI units of kg and m^3.	$\rho = \frac{m}{V}$ $\rho = \frac{m}{\ell^3}$ $\rho = \frac{(32.2 \times 10^{-3})}{(25.5 \times 10^{-3})^3}$ $\rho = 1.26275$ $\rho = 1.26\,\text{kg}\,\text{m}^{-3}$
As the volume is found by cubing the length value, the percentage uncertainty for the volume is three times that of the length.	$\%\ \text{uncertainty}_{volume} = 3 \times (\pm 1.96078)$ $\%\ \text{uncertainty}_{volume} = \pm 5.88235\%$
Calculate the total percentage uncertainty. As the density is found by dividing the mass by the volume, you must add the percentage uncertainties of the mass and volume.	total % uncertainty = (0.31056) + (5.88235) total % uncertainty = ± 6.19291%
Convert the total percentage uncertainty to the absolute uncertainty of the answer.	absolute uncertainty = $\pm(1.26275) \times \frac{(6.19291)}{100}$ absolute uncertainty = ± 0.078201 $\text{kg}\,\text{m}^{-3}$
State the answer to the correct number of significant figures with the absolute uncertainty to the correct decimal place and the correct units.	Therefore, the density is 1.26 ± 0.08 $\text{kg}\,\text{m}^{-3}$.

Worked example: Try yourself 1.2.1

COMBINING UNCERTAINTIES WHEN MULTIPLIED OR DIVIDED

A student wanted to investigate the electrical power drawn by a kettle. The kettle had a measured resistance of 50.0 ± 0.1 Ω and was drawing a measured current of 4.21 ± 0.01 A.

Using the formula:

$P = I^2R$

where

P = power (W)

I = current (A)

R = resistance (Ω)

calculate the power drawn by the kettle and include the uncertainty with the value.

Adding or subtracting measurements

When adding or subtracting measurements, make sure that all measurements are in the same units. To find the total uncertainty of an addition or a subtraction, add together the absolute uncertainties of the individual measurements.

Evaluating investigations using uncertainties

Uncertainties provide a useful way of evaluating the accuracy and precision of your investigation's results. They indicate the potential variability or how scattered your values are in the measurement; a low total uncertainty suggests higher precision. Additionally, you can use the uncertainty to calculate possible minimum and maximum values, which could be useful for comparing your experimental results to a known values. If the known value falls within the range of uncertainty of the experimental results, this can be an indication of accuracy.

Other descriptive statistics measures

The mean and the uncertainty are statistical measures that help describe data accurately. Other statistical measures that can be used, depending on the data obtained, are:

- **mode:** the mode is the value that appears most often in a data set. This measure is especially useful to describe qualitative or discrete data. For example, the mode of the values 0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.04 is 0.02.
- **median:** the median is the 'middle' value of an ordered list of values. For example, the median of the values 5, 5, 8, 8, 9, 10, 20 is 8. The median is particularly useful when the data range is wide or includes outliers (unusual results), which can make the mean less reliable.

Graphs

In general, tables provide more detailed data than graphs, but it is easier to observe trends and patterns in data in graphical form than in tabular form.

Graphs are used when two variables are being considered and one variable is dependent on the other. The graph shows the relationship between the variables.

Several types of graphs can be used, including line graphs, bar graphs and pie charts. The best one to use will depend on the nature of the data.

General rules to follow when making a graph (Figure 1.2.5) include the following:

- Keep the graph simple and uncluttered.
- Use a descriptive title that contains the independent and dependent variables.
- Represent the independent variable on the x-axis and the dependent variable on the y-axis.
- Make axes proportionate to the data.
- Clearly label axes with both the variable and the unit in which it is measured.

- Include error bars showing the uncertainty of each point. The error bar should extend above and below the plotted point by the uncertainty in the dependent variable and to the left and right of the plotted point by the uncertainty of the independent variable.
- If the y-axis intercept is important in your investigation, then the x-axis must not be broken (e.g. interrupted or truncated).
- Do not force the line of best fit through the origin of the graph (0, 0). If the line of best fit does not pass through the origin, it may indicate the presence of systematic errors in your investigation.

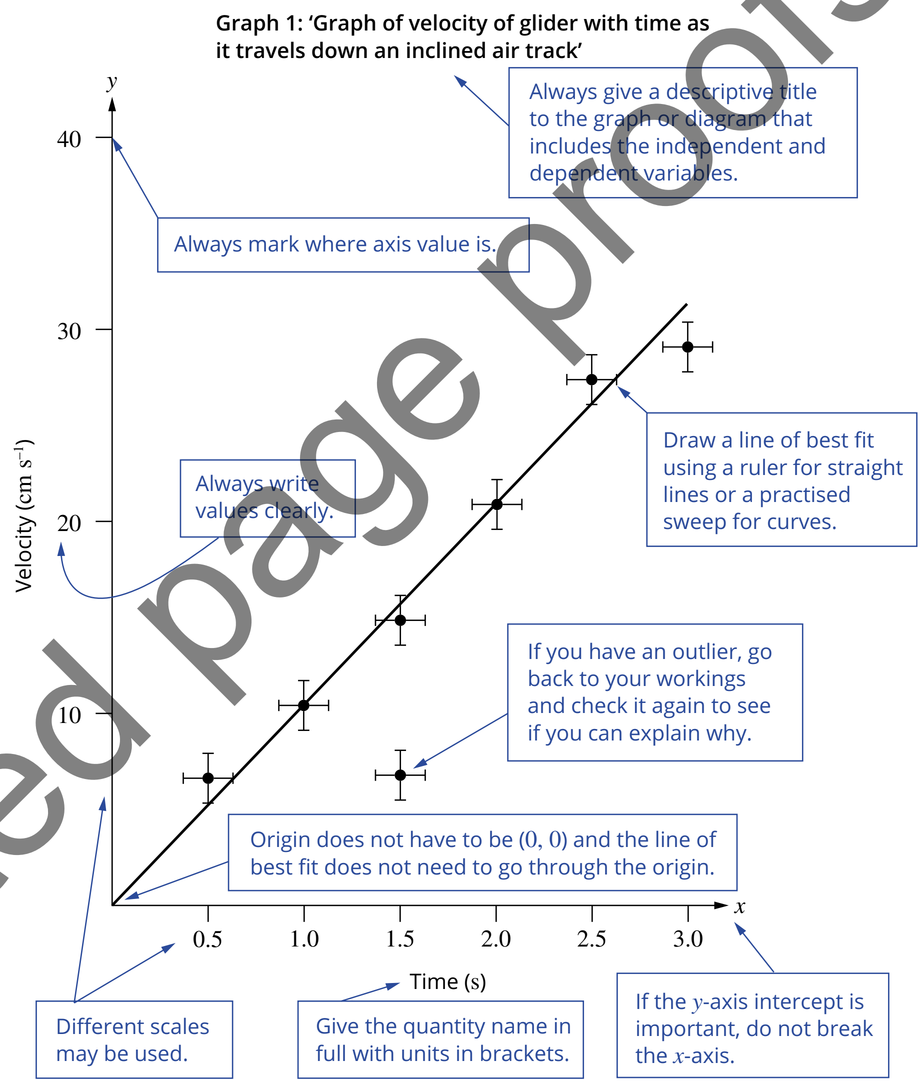


FIGURE 1.2.5 A graph shows the relationship between two variables.

Line graphs

Line graphs are a good way of representing continuous quantitative data. In a line graph, the values are plotted as a series of points with error bars on the graph. There are two ways of joining these points:

- A line can be ruled from each point to the next (Figure 1.2.6a). It shows the overall trend; it is not meant to predict the value of the points between the plotted data. These graphs are rarely used in physics.

- The points can be joined with a single smooth straight or curved line (Figure 1.2.6b). This creates a trend line, also known as a line of best fit or a curve of best fit. The line of best fit does not have to pass through every point but should go through as many error bars as possible. It is used when there is an obvious trend between the variables. These graphs are most commonly used in physics.

Outliers

Sometimes when the data is collected, there may be one point that does not fit with the trend and is clearly an error. This is called an **outlier**. An outlier is often caused by a mistake made in measuring or recording data, or from a random error in the measuring equipment. If there is an outlier, include it on the graph, but ignore it when adding a line of best fit (as in Figure 1.2.5, where the point (1.50, 6) is an outlier).

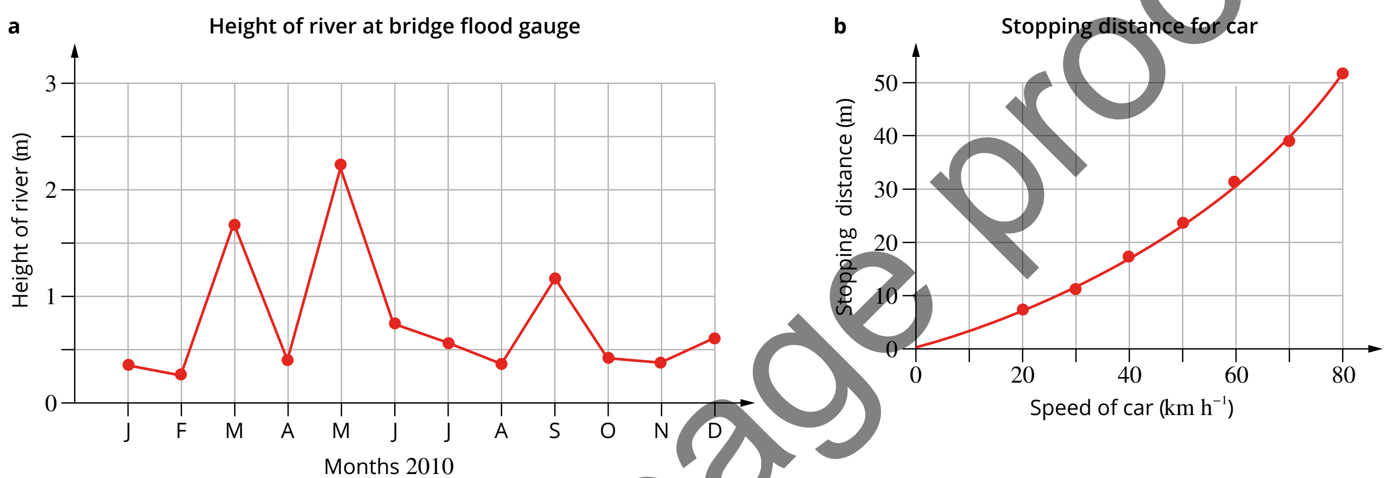


FIGURE 1.2.6 (a) The data in the graph is joined from point to point. (b) The data in graph is joined with a line of best fit, which shows the general trend.

1.2 Review

SUMMARY

- It is essential that during the investigation, the following are recorded:
  - all quantitative data collected
  - the methods used to collect the data
  - any incident, feature or unexpected event that may have affected the quality or validity of the data.
- A systematic error is an error that is consistent and will occur again if the investigation is repeated in the same way. Systematic errors are usually a result of instruments that are not calibrated correctly or methods that are flawed.
- Random errors occur in an unpredictable manner and are generally small. A random error could be, for example, the small fluctuations in the ambient temperature affecting a heat transfer investigation.
- The number of significant figures used depends on the scale of the instrument used. It is important to record data to the number of significant figures available from the equipment or observation.
- The simplest form of a table is a five-column format in which the first column contains the independent variable (the one being changed), the second to fourth columns contain the trials of the dependent variable (the one that may change in response to a change in the independent variable) and the final column contains the average of the trials.
- When there is a range of measurements of a particular value, the average must be accompanied by the uncertainty of averages. These uncertainties can be represented as absolute values or as a percentage of the measurement (percentage uncertainty).

1.2 Review *continued*

- When processing data, you must combine uncertainties:
  - If a measurement is raised to a power, the percentage uncertainty must be multiplied by the power value, i.e. if a measurement is squared, the percentage uncertainty is multiplied by two.
  - If measurements are multiplied or divided, the percentage uncertainties must be added together.
  - If measurements are added or subtracted, the absolute uncertainties must be added together.
- General rules to follow when making a graph include the following:
  - Keep the graph simple and uncluttered.
  - Use a descriptive title that contains the independent and dependent variables.
  - Represent the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
  - Make axes proportionate to the data.
  - Clearly label axes with both the variable and the unit in which it is measured.
  - Include error bars showing the uncertainty of each point. The error bar should extend above and below the plotted point by the uncertainty in the dependent variable and to the left and right of the plotted point by the uncertainty of the independent variable.
  - If the *y*-axis intercept is important in your investigation, the *x*-axis must not be broken.
  - Do not force the line of best fit through the origin of the graph (0, 0). If the line of best fit does not pass through the origin, it can indicate that there may be systematic errors in your investigation.

KEY QUESTIONS

1 The masses of 1.00 cm^3 cubes of potato were recorded and the cubes placed in distilled water. After 60 minutes, the cubes were weighed again and the difference in mass was calculated. What type of error is involved:

a if the electronic scales were not tared properly?

b if the electronic scales were affected briefly by a power surge?

2 If using the quantities mass = 7.505 kg and speed = 1.40 $m\,s^{-1}$ in a calculation, what would be the appropriate number of significant figures in the answer?

3 For the data set 21.0, 28.0, 19.0, 19.0, 25.0, 24.0, determine:

a the mean

b the mode

c the median

d the uncertainty of averages.

4 Plot the following data set with error bars, assigning each variable to the appropriate axis on the graph.

Potential difference ± 5.00% (V)	Current ± 0.01 (A)
2.07	0.06
1.56	0.05
1.24	0.04
0.93	0.03
0.63	0.02

5 How can the general pattern (trend) of the data set in Question 4 be represented once the points are plotted?

6 Compare the error bars for the current in Question 4 to the error bars for the potential difference.

1.3 Discussing investigations and drawing evidence-based conclusions

Now that the chosen topic has been thoroughly researched and the investigation has been conducted and data collected, it is time to draw it all together. The final part of the investigation involves summarising the findings in an objective, clear and concise manner.

FIGURE 1.3.1 To discuss and conclude your investigation, use the raw and processed data.

EXPLAINING RESULTS IN THE DISCUSSION

The discussion is the part of the investigation where the evaluation and explanation of the investigation methods and results takes place. It is the interpretation of what the results mean.

The key sections of the discussion are:

- analysing and evaluating data
- evaluating the investigative method
- explaining the link between investigation findings and relevant physics concepts.

When writing the discussion, consider the message you want to convey to the audience. Statements should be clear and concise. By the time the discussion concludes, the audience must have a clear idea of the context, results and implications of the investigation.

ANALYSING AND EVALUATING DATA

In the discussion, the findings of the investigation need to be analysed and interpreted.

- State whether a pattern, trend or relationship was observed between the independent and dependent variables. Describe what kind of pattern it was and specify the conditions under which it was observed.
- Were there discrepancies, deviations or anomalies in the data? If so, these should be acknowledged and explained.
- Identify any limitations in the data you have collected. Consider whether a larger sample size or further variations in the independent variable would lead to a stronger conclusion.

Trends in line graphs

Graphs are drawn to show the relationship, or trend, between two variables, as shown in Figure 1.3.2.

- Variables that change in linear or direct proportion to each other produce a straight sloping trend line.
- Variables that change exponentially in proportion to each other produce a curved trend line.
- With an inverse relationship, one variable increases as the other variable decreases.
- When there is no relationship between two variables, one variable will not change even if the other changes.

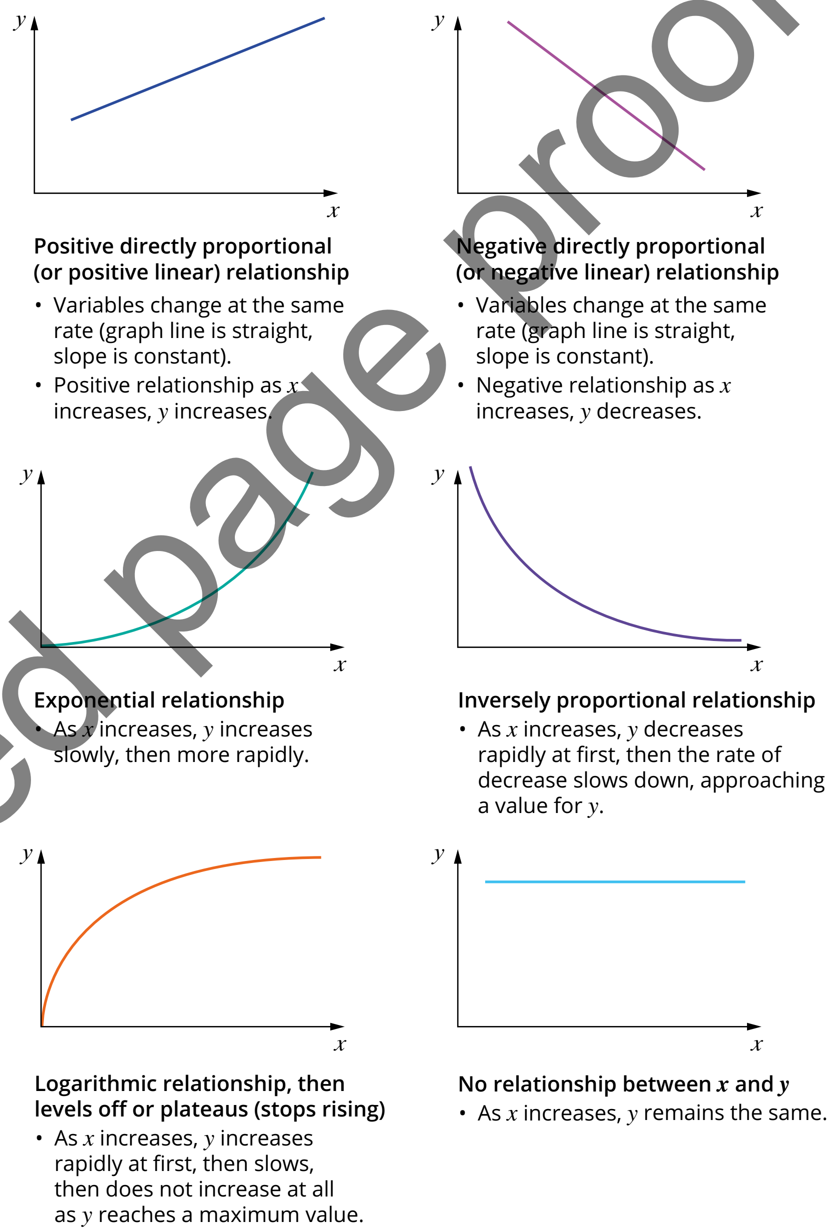


FIGURE 1.3.2 Various relationships can exist between two variables.

Remember that the results may be unexpected. This does not make the investigation a failure. However, the findings must be related to the hypothesis, aims and method.

EVALUATING THE METHOD

It is important to discuss the limitations of the investigation method. Evaluate the method and identify any issues that could have affected the validity, accuracy, precision or reliability of the data. Sources of errors and uncertainty must also be evaluated in the discussion.

Once any limitations or problems in the methodology have been identified, recommend improvements on how the investigation could be conducted if repeated; for example, suggest how bias could be minimised or eliminated.

Bias

Bias may occur in any part of the investigation method, including sampling and measurements.

Bias is a form of systematic error resulting from the researcher's personal preferences or motivations. There are many types of bias, including:

- poor definitions of both concepts and variables (e.g. classifying cricket pitch surfaces as slow or fast without defining 'slow' and 'fast')
- incorrect assumptions (e.g. assuming that footwear type, model and manufacturer do not affect ground reaction forces, and as a result failing to control this variable during an investigation on slip risk on different indoor and outdoor surfaces)
- errors in the investigation design and methodology (e.g. taking a sample of a particular group of athletes that includes one gender more than the other in the group).

Some biases cannot be eliminated, but should still be addressed in the discussion.

Accuracy and precision

In the discussion, evaluate the degree of accuracy and precision of the measurements for each variable of the hypothesis. Address the uncertainties associated with these measurements and discuss their impact on the results.

When applicable compare the chosen method to alternative methods that could have been used, evaluating the advantages and disadvantages of the selected method and the effect on the results.

If your investigation involves comparing your results to a known accepted value, you should calculate a percentage difference. The percentage difference can be calculated by using the following formula:

$$\text{percentage difference} = \frac{\text{accepted value} - \text{experimental value}}{\text{accepted value}} \times 100$$

A smaller percentage difference indicates a more accurate result. For example, a percentage difference of less than 5% suggests that the difference between the accepted value and your experimental value is not significant.

Reliability

When discussing the results, indicate the range of the data obtained from replicates. Explain how the sample size was selected. Larger samples are usually more reliable, but time and resources might have been scarce. Discuss whether the results of the investigation have been limited by the sample size.

The control group is important to the reliability of the investigation. A control group helps identify whether an uncontrolled variable has been overlooked and may explain any unexpected results.

Error

Discuss any source of systematic or random error and suggest ways of improving the investigation.

FIGURE 1.3.3 Honest evaluation and reflection play important roles in analysing methodology.

DISCUSSING RELEVANT PHYSICS CONCEPTS

To make the investigation more meaningful, it should be explained within the right context; i.e. using related physics ideas, concepts, theories and models. Within this context, explain the basis for the hypothesis.

For example, if studying the impact of temperature on linear strain of a material (e.g. a rubber band), some of the contextual information to include in the discussion could be:

- the definition of linear strain
- the functions of linear strain
- the relationship between linear strain and temperature
- definitions of material behaviour (such as plastic and elastic)
- factors known to affect linear strain
- existing knowledge on the role of temperature on linear strain
- ranges of temperatures investigated and the reason they were chosen
- materials studied and the reasons for this choice
- methods of measuring the linear strain of a material.

Relating your findings to a physics concept

Once a context is established, you can use this as a framework to discuss whether the data supported or refuted the hypothesis. Ask questions such as:

- Was the hypothesis supported?
- Has the hypothesis been fully addressed? (If not, give an explanation of why this is so and suggest what could be done to either improve or complement the investigation.)
- Do the results contradict the hypothesis? If so, why? (The explanation must be plausible and must be based on the results and previous evidence.)

Providing a theoretical context also enables comparison of the results with existing relevant research and knowledge. After identifying the major findings of the investigation, ask questions such as:

- How does the data fit with the literature?
- Does the data contradict the literature?
- Do the findings fill a gap in the literature?
- Do the findings lead to further questions?
- Can the findings be extended to another situation?

Be sure to discuss the broader implications of the findings. Implications are the bigger picture. Outlining them for the audience is an important part of the investigation. Ask questions such as:

- Do the findings contribute to or impact the existing literature and knowledge of the topic?
- Are there any practical applications for the findings?

DRAWING EVIDENCE-BASED CONCLUSIONS

A conclusion is usually a paragraph that links the collected evidence to the hypothesis and provides a justified response to the research question.

Indicate whether the hypothesis was supported or refuted, citing the evidence that led to this conclusion. Do not provide irrelevant information. Only refer to the specifics of the hypothesis and the research question, and do not make generalisations.

Read the examples given for the following hypothesis and research question.

Hypothesis: An increase in temperature will cause an increase in linear deformation (change in length) before failure.

- Poor response to the hypothesis: Linear deformation has value y_1 at temperature 1 and value y_2 at temperature 2.
- Better response to the hypothesis: An increase in temperature from 1 to 2 produces an increase in linear deformation of z in the rubber band.

Research question: Does temperature affect the maximum linear deformation the material can withstand?

- Poor response to the research question: The results show that temperature does affect the maximum deformation of a material.
- Better response to the research question: Analysis of the results indicates that increasing the temperature from 1 to 2 in the rubber band correlates with an increase in the maximum linear deformation, which is consistent with existing knowledge on how temperature influences material deformation.

REFERENCES AND ACKNOWLEDGEMENTS

All the quotations, documents, publications and ideas used in the investigation need to be acknowledged in the 'references and acknowledgements' section to avoid plagiarism and to ensure authors are credited for their work. References and acknowledgements also give credibility to the study and allow the audience to locate information sources should they wish to study the topic further.

When referencing a book, include, in this order:

- author's surname and initials
- date of publication
- title
- publisher's name
- place of publication.

For example: Moran G. et al. (2025), *Pearson Physics 12*, Pearson Education, Melbourne, Victoria.

When referencing a website, include, in this order:

- author's surname and initials, or name of organisation, or title
- year website was written or last revised
- title of webpage
- date website was accessed
- website address.

For example: Wheeling Jesuit University/Center for Educational Technologies (2013), *NASA Physics Online Course: Forces and Motion*, accessed 16 June 2015, from http://nasaphysics.cet.edu/forces-and-motion.html.

1.3 Review

SUMMARY

- The discussion is the part of the investigation where the evaluation and explanation of the investigation methods and results takes place. It is the interpretation of what the results mean.
- In the discussion, the findings of the investigation need to be analysed and interpreted.
  - State whether a pattern, trend or relationship was observed between the independent and dependent variables. Describe what kind of pattern it was and specify under what conditions it was observed.
  - Were there discrepancies, deviations or anomalies in the data? If so, these should be acknowledged and explained.
  - Identify any limitations in the data collected. Perhaps a larger sample or further variations in the independent variable would lead to a stronger conclusion.
- It is important to discuss the limitations of the investigation method. Evaluate the method and identify any issues that could have affected the validity, accuracy, precision or reliability of the data. Sources of errors and uncertainty must also be stated in the discussion, and suggestions could be given as to how to reduce these errors.

1.3 Review *continued*

- When discussing the results, indicate the range of the data obtained from replicates. Explain how the sample size was selected. Larger samples are usually more reliable, but time and resources are likely to have been scarce. Discuss whether the results of the investigation have been limited by the sample size.
- To make the investigation more meaningful, it should be explained within the right context, including the related physics ideas, concepts, theories and models. Within this context, explain the basis for the hypothesis.
- Indicate whether the hypothesis was supported or refuted and the evidence that led to this conclusion. Do not provide irrelevant information or make generalisations.

KEY QUESTIONS

1 What relationship between the variables is indicated by a sloping linear graph?

2 What relationship exists if one variable decreases as the other increases?

3 What relationship exists if both variables increase or both decrease at the same rate?

4 What might cause a sample size to be limited in an investigation?

5 Consider this investigation hypothesis: An increase in the potential difference across a single resistor in an electric circuit will cause an increase in the current through the resistor.
Improve this hypothesis given the data below:
When the current was 0.03 A, the voltage was 0.93 V and when the current was 0.04 A, the voltage was 1.86 V.

Chapter review

01

KEY TERMS

absolute uncertainty
accuracy
controlled variable
dependent variable
independent variable
mean
median
mode
outlier
personal protective equipment (PPE)
precision
qualitative variable
quantitative variable
random error
raw data
reliability
significant figures
systematic error
uncertainty
uncertainty of averages
uncertainty of measurement
validity
variable

1 What is a hypothesis and what form does it take?

2 Consider the hypothesis provided below. What are the dependent, independent and controlled variables?
Hypothesis: Releasing an arrow in archery at an angle greater or smaller than 45° will result in a shorter flight displacement (range).

3 What is the dependent variable in each of the following hypotheses?
- **a** If you push an object with a fixed mass (e.g. shot-put) with a larger force, then the acceleration of that object will be greater.
- **b** As the vertical displacement of a falling object increases, the vertical acceleration of the falling object is constant.
- **c** A springboard diver rotates faster when in a tucked position than when in a stretched (layout) position.

4 List the following types of hazard controls from the most effective to the least effective.
substitution, personal protective equipment, engineering controls, administrative controls, elimination, isolation

5 The speed of a toy car rolling down an inclined plane was measured six times. The measurements obtained (in $cm\,s^{-1}$) were 7.0, 6.5, 6.8, 7.2, 6.5, 6.5.
What is the uncertainty of the average of these values?

6 Which of the statistical measurements of mean, mode and median is most affected by an outlier?

7 What relationship between variables is indicated by a curved trend line?

8 If you hypothesise that impact force is positive directly proportional to drop height, what would you expect a graph of the data to look like?

9 What is meant by the 'limitations' of the investigation method?

10 What is 'bias' in an investigation?