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foundation mathematics 12

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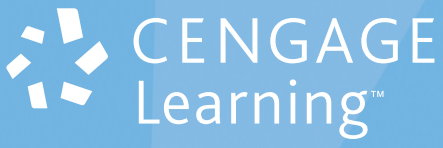
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foundation mathematics 12

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Preface

Nelson VICmaths Foundation Mathematics Units 3 and 4 has been written and updated for the 2022 Study Design. In this book, teachers will find familiar features that strongly contribute to effective mathematics teaching and learning: clear worked examples, graded exercises, investigations, technology integration, practical activities, language activities, and a combined glossary and index.

This book caters for students heading towards the workforce or further training after school, and is therefore focused on teaching useful mathematical skills and applications for life. The author team are experts in teaching and writing for senior practical mathematics courses, and have published for both state and Australian curriculums.

The authors and publishers have endeavoured to produce a practical text that captures the spirit of the course, providing relevant, current and interesting examples of mathematics being used in society and industry. The Years 11 and 12 student books cover the relevant study design with 16 short chapters written in plain English. There is an emphasis on numeracy, literacy and real-life applications, engaging students by using themes such as art and design, media bias, travel, personal and business finance, building and measurement. We wish all teachers and students using this book every success in embracing the Foundation Mathematics course.

About the authors

Sue Thomson is an experienced teacher and educational leader. She was an examination writer, assessor, marker and curriculum writer. Sue is a prolific and successful author with an interest in language development, financial literacy and making mathematics accessible to all. She has written many successful series for NSW, Queensland and WA, including the *Hands-On Maths in Practice* and *Access to General Mathematics* series.

Judy Binns is a mathematics coordinator and experienced author who has taught in urban and rural schools. She has an interest in motivating students with learning difficulties, and wide experience in teaching senior practical mathematics courses. Judy co-wrote a successful mathematics series for Years 7–8 and with Sue Thomson has written for senior practical mathematics in NSW, Queensland and WA.

Contributing authors

John Drake, **Katie Jackson** and **Joanne Magner** created the video tutorials.

Study Design grid

Area of Study	*Nelson VICmaths Foundation Mathematics 12* chapter	
1 Algebra, number and structure	**2**	Crunching numbers
	4	Measurement and accuracy
	9	Equations and formulas
	10	Ratio and variation
2 Data analysis, probability and statistics	**3**	That's biased
	7	Analysing data
	13	Fitting the data
	16	Australia and the world
3 Discrete mathematics: Financial and consumer mathematics	**1**	Investing money
	6	Borrowing money
	12	Running a business
	14	Paying tax and super
	16	Australia and the world
4 Space and measurement	**4**	Measurement and accuracy
	5	Measuring at home
	8	Creating designs
	11	From paper to reality
	15	Surface area and volume

About this book

In each chapter

Each chapter begins on a double-page spread.

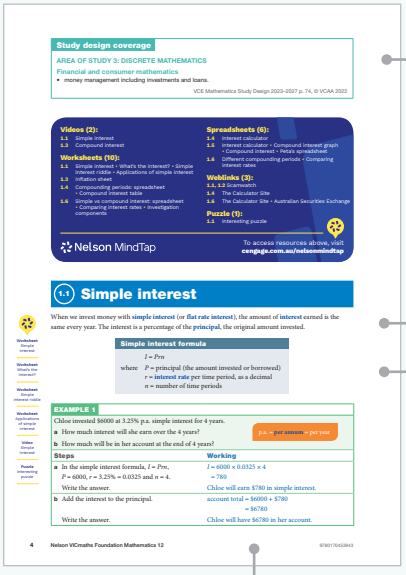


Each chapter has a **Table of contents** for the chapter.

Each chapter has a **What will we do in this chapter?** list of outcomes.

Each chapter has a **How are we ever going to use this?** list of applications.

Each chapter has a **Chapter problem** to be solved.

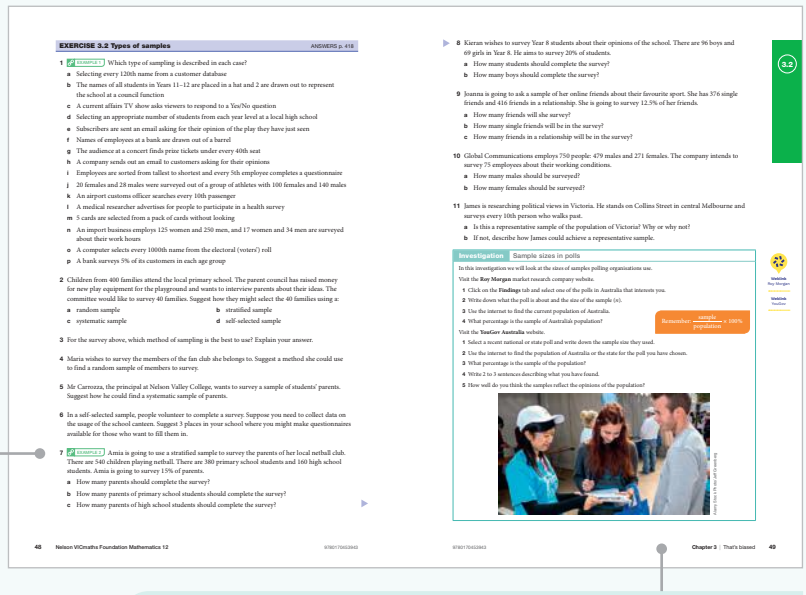


Study design coverage and extracts are shown at the start of the chapter, along with a listing of **Nelson MindTap chapter resources**.

Important words and phrases are printed in blue and listed in the **Glossary and index** at the back of the book.

Important facts and formulas are highlighted in a shaded box.

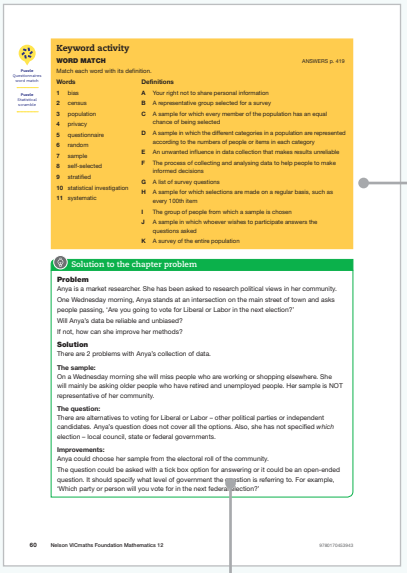
Worked examples are explained clearly step-by-step, with the mathematical working shown on the right-hand side.



Graded exercises are linked to the worked examples and include problems and realistic applications.

Investigations and **Practical activities** explore the syllabus in more detail, through group work, discovery and modelling activities. **Extended investigations** can be adapted into assessment tasks and projects.

At the end of each chapter



Keyword activity

WORD MATCH

Match each word with its definition.

Words	Definitions
1 bias	A Your right not to share personal information
2 census	B A representative group selected for a survey
3 population	C A sample for which every member of the population has an equal chance of being selected
4 privacy	
5 questionnaire	D A sample in which the different categories in a population are represented according to the numbers of people or items in each category
6 random	E An unwanted influence in data collection that makes results unreliable
7 sample	F The process of collecting and analysing data to help people to make informed decisions
8 self-selected	
9 stratified	G A list of survey questions
10 statistical investigation	H A sample for which selections are made on a regular basis, such as every 100th item
11 systematic	
	I The group of people from which a sample is chosen
	J A sample in which whoever wishes to participate answers the questions asked
	K A survey of the entire population

Solution to the chapter problem

Problem

Anya is a market researcher. She has been asked to research political views in her community. One Wednesday morning, Anya stands at an intersection on the main street of town and asks people passing, 'Are you going to vote for Liberal or Labor in the next election?'

Will Anya's data be reliable and unbiased?

If not, how can she improve her methods?

Solution

There are 2 problems with Anya's collection of data.

The sample:

On a Wednesday morning she will miss people who are working or shopping elsewhere. She will mainly be asking older people who have retired and unemployed people. Her sample is NOT representative of her community.

The question:

There are alternatives to voting for Liberal or Labor – other political parties or independent candidates. Anya's question does not cover all the options. Also, she has not specified *which* election – local council, state or federal governments.

Improvements:

Anya could choose her sample from the electoral roll of the community.

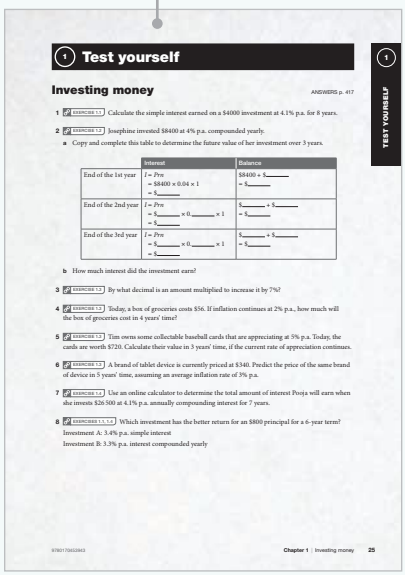
The question could be asked with a tick box option for answering or it could be an open-ended question. It should specify what level of government the question is referring to. For example, 'Which party or person will you vote for in the next federal election?'

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Keyword activity focuses on the mathematical language and terminology learned in the chapter.

Solution to the chapter problem revisits the problem introduced at the start of the chapter and presents a solution to the problem.

Test yourself contains revision linked to the relevant exercise set.



Test yourself

Investing money

ANSWERS p. 417

1 EXERCISE 1.2 Calculate the simple interest earned on a $4000 investment at 4.1% p.a. for 8 years.

2 EXERCISE 1.2 Josephine invested $8400 at 4% p.a. compounded yearly.

a Copy and complete this table to determine the future value of her investment over 3 years.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8400 \times 0.04 \times 1$ $= \$____$	$\$8400 + \$____$ $= \$____$
End of the 2nd year	$I = Prn$ $= \$____ \times 0.____ \times 1$ $= \$____$	$\$____ + \$____$ $= \$____$
End of the 3rd year	$I = Prn$ $= \$____ \times 0.____ \times 1$ $= \$____$	$\$____ + \$____$ $= \$____$

b How much interest did the investment earn?

3 EXERCISE 1.2 By what decimal is an amount multiplied to increase it by 7%?

4 EXERCISE 1.2 Today, a box of groceries costs $56. If inflation continues at 2% p.a., how much will the box of groceries cost in 4 years' time?

5 EXERCISE 1.2 Tim owns some collectable baseball cards that are appreciating at 5% p.a. Today, the cards are worth $720. Calculate their value in 3 years' time, if the current rate of appreciation continues.

6 EXERCISE 1.2 A brand of tablet device is currently priced at $340. Predict the price of the same brand of device in 5 years' time, assuming an average inflation rate of 3% p.a.

7 EXERCISE 1.3 Use an online calculator to determine the total amount of interest Pooja will earn when she invests $26 500 at 4.1% p.a. annually compounding interest for 7 years.

8 EXERCISES 1.1, 1.2 Which investment has the better return for an $800 principal for a 6-year term?

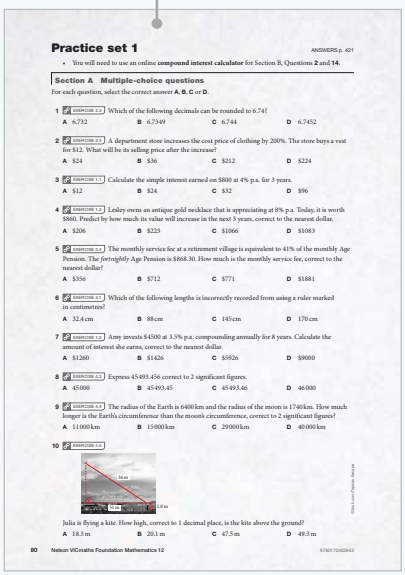
Investment A: 3.4% p.a. simple interest

Investment B: 3.3% p.a. interest compounded yearly

TEST YOURSELF 1

Chapter 1 | Investing money 25

Practice sets revise the skills and knowledge of previous chapters.



Practice set 1

ANSWERS p. 421

- You will need to use an online **compound interest calculator** for Section B, Questions **2** and **14**.

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 2.1 Which of the following decimals can be rounded to 6.74?
A 6.732 B 6.7349 C 6.744 D 6.7452

2 EXERCISE 2.1 A department store increases the cost price of clothing by 200%. The store buys a vest for $12. What will be its selling price after the increase?
A $24 B $36 C $212 D $224

3 EXERCISE 3.1 Calculate the simple interest earned on $800 at 4% p.a. for 3 years.
A $12 B $24 C $32 D $96

4 EXERCISE 1.2 Lesley owns an antique gold necklace that is appreciating at 8% p.a. Today, it is worth $860. Predict by how much its value will increase in the next 3 years, correct to the nearest dollar.
A $206 B $223 C $1066 D $1083

5 EXERCISE 2.4 The monthly service fee at a retirement village is equivalent to 41% of the monthly Age Pension. The *fortnightly* Age Pension is $868.30. How much is the monthly service fee, correct to the nearest dollar?
A $356 B $712 C $771 D $1881

6 EXERCISE 4.1 Which of the following lengths is incorrectly recorded from using a ruler marked in centimetres?
A 32.4 cm B 88 cm C 145 cm D 170 cm

7 EXERCISE 1.2 Amy invests $4500 at 3.5% p.a. compounding annually for 8 years. Calculate the amount of interest she earns, correct to the nearest dollar.
A $1260 B $1426 C $5926 D $9000

8 EXERCISE 2.3 Express 45 493.456 correct to 2 significant figures.
A 45 000 B 45 493.45 C 45 493.46 D 46 000

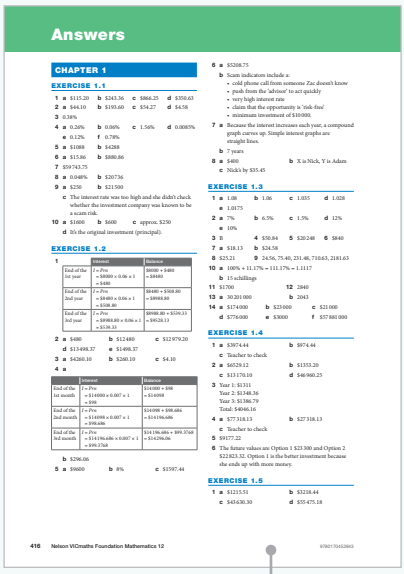
9 EXERCISE 4.4 The radius of the Earth is 6400 km and the radius of the moon is 1740 km. How much longer is the Earth's circumference than the moon's circumference, correct to 2 significant figures?
A 11 000 km B 15 000 km C 29 000 km D 40 000 km

10 EXERCISE 4.5

Julia is flying a kite. How high, correct to 1 decimal place, is the kite above the ground?
A 18.3 m B 20.1 m C 47.5 m D 49.3 m

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At the end of the book



Answers

CHAPTER 1

EXERCISE 1.1

1 a $115.20 b $243.36 c $866.25 d $350.63
2 a $44.10 b $193.60 c $54.27 d $4.58
3 0.38%
4 a 0.26% b 0.06% c 1.56% d 0.0085%
e 0.12% f 0.78%
5 a $1088 b $4288
6 a $15.86 b $880.86
7 $59 743.75
8 a 0.048% b $20 736
9 a $250 b $21 500
c The interest rate was too high and she didn't check whether the investment company was known to be a scam risk.
10 a $1600 b $600 c approx. $250
d It's the original investment (principal).

EXERCISE 1.2

1

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$480$	$\$8000 + \$480$ $= \$8480$
End of the 2nd year	$I = Prn$ $= \$8480 \times 0.06 \times 1$ $= \$508.80$	$\$8480 + \$508.80$ $= \$8988.80$
End of the 3rd year	$I = Prn$ $= \$8988.80 \times 0.06 \times 1$ $= \$539.33$	$\$8988.80 + \$539.33$ $= \$9528.13$

2 a $480 b $12 480 c $12 979.20
d $13 498.37 e $1498.37
3 a $4260.10 b $260.10 c $4.10
4 a

	Interest	Balance
End of the 1st month	$I = Prn$ $= \$14000 \times 0.007 \times 1$ $= \$98$	$\$14000 + \$98$ $= \$14098$
End of the 2nd month	$I = Prn$ $= \$14098 \times 0.007 \times 1$ $= \$98.686$	$\$14098 + \$98.686$ $= \$14196.686$
End of the 3rd month	$I = Prn$ $= \$14196.686 \times 0.007 \times 1$ $= \$99.3768$	$\$14196.686 + \$99.3768$ $= \$14296.06$

b $296.06
5 a $9600 b 8% c $1597.44
6 a $5208.75
b Scam indicators include a:
- cold phone call from someone Zac doesn't know
- push from the 'advisor' to act quickly
- very high interest rate
- claim that the opportunity is 'risk-free'
- minimum investment of $10 000.

7 a Because the interest increases each year, a compound graph curves up. Simple interest graphs are straight lines.
b 7 years
8 a $400 b X is Nick, Y is Adam
c Nick's by $35.45

EXERCISE 1.3

1 a 1.08 b 1.06 c 1.035 d 1.028
e 1.0175
2 a 7% b 6.5% c 1.5% d 12%
e 10%
3 5 4 $50.84 5 $20 248 6 $840
7 a $18.13 b $24.58
8 $25.21 9 24.56, 75.40, 231.48, 710.63, 2181.63
10 a 100% + 11.17% = 111.17% = 1.1117
b 15 schillings
11 $1700 12 2840
13 a 30 201 000 b 2043
14 a $174 000 b $23 000 c $21 000
d $776 000 e $3000 f $57 881 000

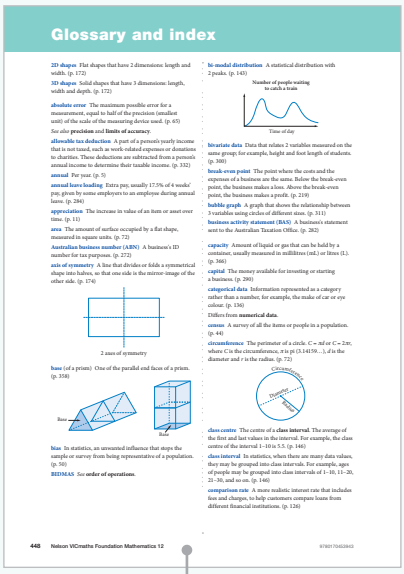
EXERCISE 1.4

1 a $397.44 b $874.44
c Teacher to check
2 a $6529.12 b $1353.20
c $13 170.10 d $46 960.25
3 Year 1: $1311
Year 2: $1 548.36
Year 3: $1 386.79
Total: $4 046.16
4 a $77 318.13 b $27 318.13
c Teacher to check
5 $9377.22
6 The future values are Option 1 $23 300 and Option 2 $22 823.32. Option 1 is the better investment because she ends up with more money.

EXERCISE 1.5

1 a $1215.91 b $3218.44
c $43 630.30 d $55 475.18

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Glossary and index

2D shapes Flat shapes that have 2 dimensions: length and width. (p. 172)

3D shapes Solid shapes that have 3 dimensions: length, width and depth. (p. 172)

absolute error The maximum possible error for a measurement, equal to half of the precision (smallest unit) of the scale of the measuring device used. (p. 65)
See also **precision** and **limits of accuracy**.

allowable tax deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. These deductions are subtracted from a person's annual income to determine their taxable income. (p. 332)

annual Per year. (p. 5)

annual leave loading Extra pay, usually 17.5% of 4 weeks' pay, given by some employers to an employee during annual leave. (p. 284)

appreciation The increase in value of an item or asset over time. (p. 11)

area The amount of surface occupied by a flat shape, measured in square units. (p. 72)

Australian business number (ABN) A business's ID number for tax purposes. (p. 272)

axis of symmetry A line that divides or folds a symmetrical shape into halves, so that one side is the mirror-image of the other side. (p. 174)

2 axes of symmetry

base (of a prism) One of the parallel end faces of a prism. (p. 358)

Base

Side

bias In statistics, an unwanted influence that stops the sample or survey from being representative of a population. (p. 56)

BEDMAS *See* **order of operations**.

bi-modal distribution A statistical distribution with 2 peaks. (p. 143)

Number of people waiting to catch a train

Time of day

bivariate data Data that relates 2 variables measured on the same group; for example, height and foot length of students. (p. 300)

break-even point The point where the costs and the expenses of a business are the same. Below the break-even point, the business makes a loss. Above the break-even point, the business makes a profit. (p. 219)

bubble graph A graph that shows the relationship between 3 variables using circles of different sizes. (p. 311)

business activity statement (BAS) A business's statement sent to the Australian Taxation Office. (p. 282)

capacity Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 366)

capital The money available for investing or starting a business. (p. 280)

categorical data Information represented as a category rather than a number, for example, the make of car or eye colour. (p. 138)
Differs from **numerical data**.

census A survey of all the items or people in a population. (p. 44)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is $\frac{22}{7}$ or 3.14159..., d is the diameter and r is the radius. (p. 72)

Circumference

Diameter

Radius

class centre The centre of a **class interval**. The average of the first and last values in the interval. For example, the class centre of the interval 1–10 is 5.5. (p. 146)

class interval In statistics, when there are many data values, they may be grouped into class intervals. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30, and so on. (p. 146)

comparison rate A more realistic interest rate that includes fees and charges, to help customers compare loans from different financial institutions. (p. 126)

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Answers (with worked solutions provided on Nelson MindTap for teachers to allocate to students.

A combined **Glossary and index**.

Nelson MindTap

Nelson MindTap is an online learning space that provides students with tailored learning experiences. Access tools and content that make learning simpler yet smarter to help you achieve VCE maths mastery.

Nelson MindTap includes an eText with integrated interactives and online assessment.

Margin links in the student book signpost multimedia student resources found on MindTap.

Nelson MindTap for students:

- **Watch** video tutorials featuring expert teacher advice to unpack new concepts and develop your understanding.
- **Revise** using quizzes, worksheets and skillsheets to practise your skills and build your confidence.
- **Navigate** your own path, accessing the content, weblinks, analytics and support as you need it.

Nelson MindTap for teachers:

- Tailor content to different learning needs – assign directly to the student, or the whole class.
- Monitor progress using assessment tools like Gradebook and Reports.
- Integrate content and assessment directly within your school's LMS for ease of access.
- Access topic tests, teaching plans and worked solutions to each exercise set.

Nelson VICmaths 11–12 series





Chapter problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested $25 000 for 3 years, earning simple interest at 4.2% per year.

Whose investment will earn more interest?

CHAPTER 1

DISCRETE MATHEMATICS:
FINANCIAL AND CONSUMER MATHEMATICS

INVESTING MONEY

1.1 Simple interest
1.2 Compound interest
1.3 Inflation and appreciation
1.4 Interest calculators
1.5 Compound interest spreadsheets
1.6 Different compounding periods
Extended investigation: My investment portfolio
Extended investigation: The sports club investment
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Calculate and compare simple and compound interest
- Use technology to calculate the future value of a compound interest investment
- Use technology to compare the progress of loans and investments, and the effects of changing interest rates and compounding periods
- Solve problems involving inflation and appreciation: situations that increase in a similar way to compound interest

How are we ever going to use this?

- Determine the chance that an investment is safe
- Choose a safe investment that will give a reasonable return
- Make calculations to check that we have received the correct investment returns
- Identify and avoid falling for financial scams

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Study design coverage

AREA OF STUDY 3: DISCRETE MATHEMATICS

Financial and consumer mathematics

- money management including investments and loans.

VCE Mathematics Study Design 2023–2027 p. 74, © VCAA 2022

Videos (2):

1.1 Simple interest
1.3 Compound interest

Worksheets (10):

1.1 Simple interest • What's the interest? • Simple interest riddle • Applications of simple interest
1.3 Inflation sheet
1.4 Compounding periods: spreadsheet • Compound interest table
1.6 Simple vs compound interest: spreadsheet • Comparing interest rates • Investigation components

Spreadsheets (6):

1.4 Interest calculator
1.5 Interest calculator • Compound interest graph • Compound interest • Peta's spreadsheet
1.6 Different compounding periods • Comparing interest rates

Weblinks (3):

1.1, 1.2 Scamwatch
1.4 The Calculator Site
1.6 The Calculator Site • Australian Securities Exchange

Puzzles (1):

1.1 Interesting puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

1.1 Simple interest

When we invest money with **simple interest** (or **flat rate interest**), the amount of **interest** earned is the same every year. The interest is a percentage of the **principal**, the original amount invested.

Worksheet Simple interest

Worksheet What's the interest?

Worksheet Simple interest riddle

Worksheet Applications of simple interest

Video Simple interest

Puzzle Interesting puzzle

Simple interest formula

$$I = Prn$$

where P = principal (the amount invested or borrowed)
r = **interest rate** per time period, as a decimal
n = number of time periods

EXAMPLE 1

Chloe invested $6000 at 3.25% p.a. simple interest for 4 years.

a How much interest will she earn over the 4 years?

b How much will be in her account at the end of 4 years?

p.a. = **per annum** = per year

Steps	Working
a In the simple interest formula, $I = Prn$, $P = 6000$, $r = 3.25\% = 0.0325$ and $n = 4$.	$I = 6000 \times 0.0325 \times 4$ $= 780$
Write the answer.	Chloe will earn $780 in simple interest.
b Add the interest to the principal.	account total = $6000 + $780 = $6780
Write the answer.	Chloe will have $6780 in her account.

Sometimes money is not invested or borrowed for a whole number of years. We need to make sure the interest rate and the time period match.



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EXAMPLE 2

Rosie invested $9500 at 4.2% p.a. simple interest for 30 months. How much interest did she earn?

Steps	Working
The interest rate is per year but the time is in months. We need to change 30 months to years by dividing by 12.	time $= \frac{30}{12}$ years
$P = 9500$, $r = 0.042$, $n = \frac{30}{12}$	$I = 9500 \times 0.042 \times \frac{30}{12}$ $= 997.5$
Write the answer.	Rosie earned $997.50 in interest.

Converting months to years

- When the interest rate is per annum (p.a.), the time must be in years.
- Divide months by 12 to change the time into years.

EXAMPLE 3

What percentage interest rate per month is equivalent to 5.52% p.a.?

Steps	Working
Divide an **annual** interest rate by 12 to change it to a monthly interest rate.	5.52% ÷ 12 = 0.46%
Write the answer.	5.52% p.a. is equivalent to 0.46% per month.

EXERCISE 1.1 Simple interest

ANSWERS p. 416

1 EXAMPLE 1 Calculate the simple interest on each principal.

a $900 at 3.2% p.a. for 4 years

b $1560 at 3.9% p.a. for 4 years

c $4500 at 5.5% p.a. for $3\frac{1}{2}$ years

d $2750 at 5.1% p.a. for $2\frac{1}{2}$ years

2 EXAMPLE 2 What is the simple interest on each investment?

a $840 at 3.5% p.a. for 18 months

b $12 800 at 6.05% p.a. for 3 months

c $2960 at $5\frac{1}{2}$% p.a. for 4 months

d $880 at $6\frac{1}{4}$% p.a. for 1 month

3 EXAMPLE 3 What percentage interest per month is equivalent to 4.56% p.a.?

4 NVM Credit Union offers investors 3.12% p.a. simple interest. Using 4 decimal places when required, express this rate of interest as a:

a monthly rate
b weekly rate
c 6-monthly rate
d daily rate
e fortnightly rate
f quarterly rate (3-monthly rate).

Divide by 52 for the weekly rate, divide by 2 for the 6-monthly rate and divide by 4 for the quarterly rate.

'Quarterly' means 4 times per year – that is, every 3 months.

5 Nina borrowed $3200 from a finance company for 2 years at 17% p.a. simple interest to buy some furniture for her home.

a How much interest did she have to pay?
b How much, including interest, did Nina have to repay the finance company?

6 Mark owed $865 on his credit card. The credit card company charged him 1 month's interest at 22% p.a.

a How much interest was he charged?
b Calculate the total amount including interest on his credit card.

7 Eddie borrowed $55 000 from a finance company to set up an online business. He borrowed the money for 6 months and was charged 17.25% p.a. interest. How much did he have to repay the finance company, including interest?

8 Kelly signed a contract to buy a new house before she sold her old house. Unfortunately, her old house did not sell in time and when it was time to pay for the new property, Kelly had to borrow $960 000 in bridging finance (high-interest, short-term loan) at 1.44% monthly.

a What daily interest rate is equivalent to 1.44% per month? Assume a month has 30 days.
b Kelly borrowed the money for 45 days. Calculate the amount of interest she had to pay.



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9 Last January, Jude received a text message from Darren, who she knew briefly years ago. He told her that he was now working for an investment company and knew of an investment opportunity that was paying 15% p.a. interest, paid every 6 months. Jude decided to invest $20 000 for 6 months in this business.

a How much interest could Jude expect her investment to earn at 15% p.a. for 1 month?
b How much, including interest, would Jude expect to receive when she withdraws her investment after 6 months?
c In July, Jude couldn't contact Darren. He had disappeared with her money. Why should Jude have been suspicious of the investment opportunity?

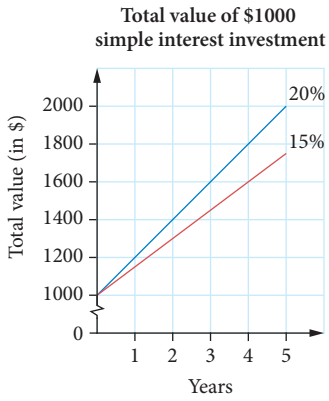
10 This graph shows the total value of a $1000 simple interest investment at 15% p.a. and 20% p.a. for 5 years.

a How much is the investment worth after 4 years at 15% p.a.?

b How much simple interest is earned in 3 years at 20% p.a.?

c How much more interest is earned in 5 years at 20% p.a. than the same period at 15% p.a.?

d Both graphs start at 1000 on the vertical axis. What is the meaning of the 1000?



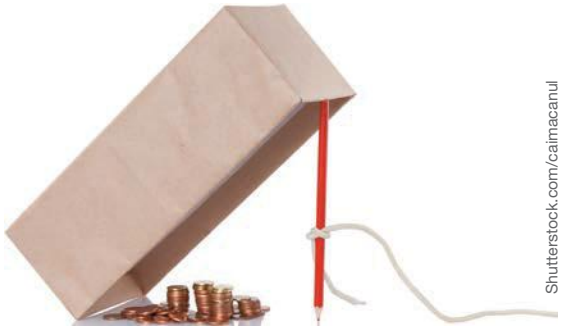
Weblink
Scamwatch

Investigation Investment scams

Visit the Australian Government's **Scamwatch** website scamwatch.gov.au and research investment scams under **Types of scams**.

Design a poster that includes the following information:

- a description of the common types of investment scams
- a list of the warning signs that an investment could be a scam
- a description of how you can protect yourself against investment scams
- advice about what you should do if you think you've been scammed.

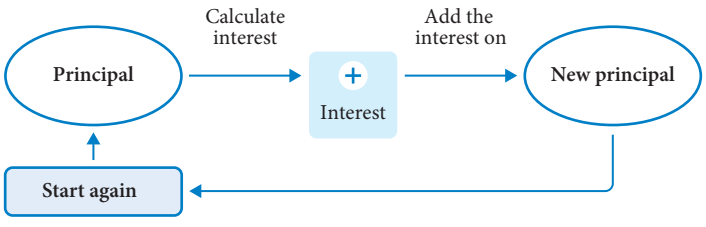

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1.2 Compound interest

There's a well-known claim often quoted by financial investors:

'Compound interest is the eighth wonder of the world.
Those who understand it, earn it. Those who don't, pay it.'

With **compound interest**, we receive 'interest on our interest'. After interest is added to the investment, the next time interest is calculated, it will be based on this larger amount. The amount of interest grows or is *compounded* as the size of the investment grows.



The size of an investment at a particular time in the future is called its **future value**.

EXAMPLE 4

Sarina invested $2000 at 5% p.a. interest compounded annually.

a Calculate the future value of Sarina's investment at the end of 3 years.

b Calculate the total interest she earned.

Steps

a We need to calculate the interest for the first year and add it to the principal before we calculate the following year's interest.

$I = Prn$, where $r = 5\% = 0.05$, $n = 1$, and P changes each year but its initial value is $2000.

Alternatively, calculate the value of 5% interest added to the principal by multiplying by 1.05 (for example, $2000 × 1.05 = $2100), giving us the answers in the Balance column straight away.

Working

	Interest	Balance
End of the 1st year	$I = \$2000 \times 0.05 \times 1$ $= \$100$	$\$2000 + \$100$ $= \$2100$
End of the 2nd year	$I = \$2100 \times 0.05 \times 1$ $= \$105$	$\$2100 + \$105$ $= \$2205$
End of the 3rd year	$I = \$2205 \times 0.05 \times 1$ $= \$110.25$	$\$2205 + \$110.25$ $= \$2315.25$

The future value of Sarina's investment is $2315.25.

b Total interest = final balance – original principal

$\text{total interest} = \$2315.25 - \$2000$
$= \$315.25$

or

add the 3 interest amounts from the Interest column of the table above.

$\text{total interest} = \$100 + \$105 + \$110.25$
$= \$315.25$

Write the answer.

The total interest earned over the 3 years was $315.25.

EXERCISE 1.2 Compound interest

ANSWERS p. 416

1 EXAMPLE 4 Yumi invested $8000 at 6% p.a. interest compounded annually for 3 years. Copy and complete this table to calculate the future value of her investment at the end of 3 years.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$_____$	$\$8000 + \$_____$ $= \$_____$
End of the 2nd year	$I = Prn$ $= \$_____ \times 0.___ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$
End of the 3rd year	$I = Prn$ $= \$_____ \times 0.___ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$

2 Jayden invested $12 000 for 3 years at 4% p.a. compounding yearly.

a How much interest did he earn in the first year?

b How much was in his account at the end of the first year?

c How much was in his account at the end of the second year?

d Calculate the future value of his investment at the end of 3 years.

e How much interest will Jayden earn during his 3-year investment?

3 Nhi invested $4000 at 3.2% p.a. for 2 years compounding annually.

a Calculate the future value of her investment at the end of the 2 years.

b How much interest will she earn during the 2 years?

c How much less interest would Nhi earn had it been simple interest rather than compound interest?

4 Suresh has saved $14 000 from his after-school job that he plans to spend on a car. He is going to invest the money for 3 months by which time he hopes to have his P-plates. Suresh's investment is going to pay 0.7% per month, interest compounded monthly.

a Copy and complete the table.

	Interest	Balance
End of the 1st month	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$_____$	$\$14\,000 + \$_____$ $= \$_____$
End of the 2nd month	$I = Prn$ $= \$_____ \times 0.___ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$
End of the 3rd month	$I = Prn$ $= \$_____ \times 0.___ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$

b How much interest will Suresh make during the 3-month investment?

5 This table shows the interest and balance calculations for Keira's investment.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$9600 \times 0.08 \times 1$ $= \$768.00$	$\$9600 + \$768.00$ $= \$10\,368$
End of the 2nd year	$I = Prn$ $= \$10\,368 \times 0.08 \times 1$ $= \$829.44$	$\$10\,368 + \$829.44$ $= \$11\,197.44$

a How much money did Keira invest?

b What was the annual rate of compound interest?

c How much interest did Keira's investment earn?

6 Last night, Zac received a phone call from an investment advisor he doesn't know telling him about an investment opportunity that is virtually risk-free. The advisor says that the offer is likely to be fully subscribed quickly, so he has to agree straight away. The minimum investment is $10 000 and the interest rate is 15% p.a. compounding annually for 3 years.

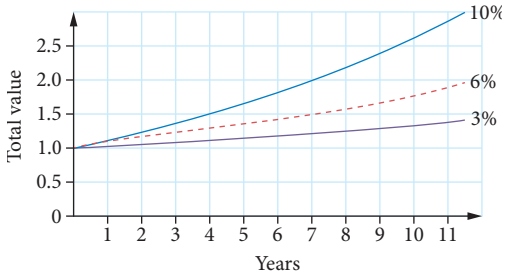
a Calculate the amount of interest the advisor claims $10 000 will receive in the next 3 years.

b What signs show that this opportunity is most probably a scam?

7 This graph shows the future value of a $1 compound investment at 3% p.a., 6% p.a. and 10% p.a.

a Describe how the shape of a compound interest graph is different from the shape of a simple interest graph.

b Approximately how long does it take for a $1 investment to double its value at 10% p.a. compound interest?

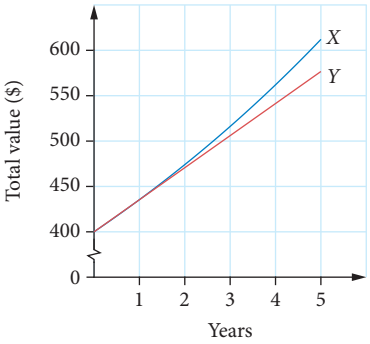


8 Nick and Adam invested the same amount of money at 9% p.a., but Nick's investment was 9% p.a. interest compounded annually, while Adam's was 9% p.a. simple interest. The graphs show the future value of their investments after 5 years.

a How much was Nick and Adam's original investment?

b Match the graphs *X* and *Y* to Adam's and Nick's investment.

c After 5 years, state whose investment is higher and by how much.



Investigation Scams involving 'free money'

Technology and the internet have changed the way we do things and have made life easier for us. But technology has also made it easier for criminals to use scams and financial fraud to rob people of their money.

1 Visit the **Scamwatch** website scamwatch.gov.au.

2 Investigate **Types of scams: Unexpected money** scams. List 4 different reasons people fall for scams.

3 Prepare a list of answers you could give to possible scammers when they are trying to convince you to do something you don't want to do.

4 Investigate **Get help**. Describe what you should do if you have sent money to someone you now think is a scammer.

Weblink
Scamwatch

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

1.3 Inflation and appreciation

We can use the same process for calculating the future value of a compound interest investment to solve problems involving inflation, appreciation, population growth and other types of growth.

Calculating compound interest can be tedious and time consuming. There is a calculator shortcut for increasing an amount by a percentage.

To **increase** a quantity by 7.5%, multiply by (100% + 7.5%) = 107.5% = 1.075.

Video
Compound interest

EXAMPLE 5

Calculate the future value of a principal of $1000 in 3 years at a compound interest rate of 4.5% p.a.

Steps	Working
First, calculate the future value after the 1st year by increasing $1000 by 4.5%. The shortcut is to multiply $1000 by 100% + 4.5% = 104.5% = 1.045.	amount at the end of the 1st year = $1000 × 1.045 = $1045
Repeat: increase $1045 by 4.5%.	amount at the end of the 2nd year = $1045 × 1.045 = $1092.025
Repeat: increase $1092.025 by 4.5%.	amount at the end of the 3rd year = $1092.025 × 1.045 = $1141.166 125
Round to the nearest cent and write the answer.	The future value of the investment is $1141.17.

Inflation is an increase in the prices of goods and services.

Appreciation is an increase in value over time of an item such as artwork, gold, a prestige car, land or a house.



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EXAMPLE 6

Emma's weekly groceries cost $180. If the current rate of inflation of 2.5% p.a. continues for the next 3 years, how much more will Emma have to pay for the same groceries in 3 years' time?

Steps	Working
Increase $180 by 2.5% for 3 years. To increase by 2.5% three times, multiply by (100% + 2.5%) = 102.5% = 1.025 three times.	future cost of the groceries = $180 × 1.025 × 1.025 × 1.025 = $193.840 312 5 = $193.84
To calculate how much more Emma will have to pay, subtract the original cost from the future cost.	extra amount = $193.84 – $180 = $13.84
Write the answer.	Emma's groceries will cost $13.84 more in 3 years.

Instead of multiplying by 1.025 3 times, try multiplying by 1.025^3 instead.

EXERCISE 1.3 Inflation and appreciation

ANSWERS p. 416

1 EXAMPLE 5 Write the decimal beginning with '1' (such as 1.09) that we would multiply an amount by to increase the amount by:

a 8% **b** 6% **c** 3.5% **d** 2.8% **e** 1.75%

2 Write the percentage that an amount increases by when the amount is multiplied by:

a 1.07 **b** 1.065 **c** 1.015 **d** 1.12 **e** 1.1

3 Dianna invested $1000 at 3.6% p.a. compound interest. Which expression is the calculation for the future value of Dianna's investment in 2 years' time?

A $1000 × 1.036 **B** $1000 × 1.036 × 1.036 **C** $1000 × 1.036 × 2

4 EXAMPLE 6 If inflation is 3.1% p.a., calculate the cost of a new pair of jeans in 4 years' time if they cost $45 today.

5 Annabel is saving to buy a new car. Prices are increasing at a constant rate of 4% p.a. The car is priced at $18 000 today. How much will a similar new car cost in 3 years' time? Answer to the nearest dollar.

6 An antique table is valued at $560 and its value is increasing at 7% p.a. If the table is kept in the same condition, how much could it be worth in 6 years' time, to the nearest dollar?

7 The average rate of Australian wage increases was 3.38% p.a. between 2010 and 2020.

a In 2010, Joe earned $13 per hour. 10 years later, Joe was in the same job. How much might he have earned per hour in 2020?

b In 2014, Maggie earned $23 per hour. How much might she earn per hour in 2016?

8 From 1951 to 2016, Australia's average rate of inflation was 5.09% per year. Imagine a soccer ball cost $1 in 1951. How much would a similar soccer ball cost in 2016, 65 years later?

Instead of multiplying by 1.0509 65 times, multiply by 1.0509^{65}.

9 Wars are a common cause of hyperinflation. Hyperinflation is when prices rise by 50% or more per month. At the end of World War II, Hungary experienced hyperinflation. The currency in Hungary at the time was called the pengo, and prices rose by 207% each day!

Increase by 207% = multiplying by 307% or 3.07.

Complete this table to show how the cost of a basket of essential food supplies changed during the Hungarian time of hyperinflation. Write values correct to 2 decimal places.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Cost of basket of food	8 pengo					

10 In 1921, after World War I, Austria's inflation was 134% p.a. or 11.17% per month.

a Use a calculation to show that we can increase a price by 11.17% by multiplying by 1.1117.

b At the beginning of 1921, a meal in Austria cost 9 Austrian schillings. What did the same meal cost 5 months later? Answer to the nearest schilling.



Getty Images/Universal History Archive

11 Jono collects mint-condition *Donald Duck* comics. One of his comics is signed by the creator, Carl Barks. Currently it is valued at $1200 and is appreciating (increasing in value) by 9% p.a. If it continues to appreciate at the current rate, how much will this comic be worth in 4 years' time? Answer correct to the nearest $100.

12 The population of a small rural town is increasing by 3% p.a. Today the population is 2450. If the annual growth continues at 3% per year, what will be the population of the town in 5 years?

13 In 2021, Australia's population was 25 800 000. If the population increases at approximately 3.2% p.a.:

a estimate Australia's population, correct to the nearest thousand, in 2026

b find when Australia's population is expected to pass 50 million.

Use a guess-and-check method.

14 This table shows the value of some prize collectables in 2021.

	Item	Value
a	Certified American baseball cards	$150 000
b	Mint-condition *Spiderman* comic book	$20 000
c	Vintage *Star Wars* costume in original packet	$18 000
d	Original Apple computer in unopened box	$670 000
e	Limited edition *Frozen* doll in unopened box	$2 500
f	Original copy of one of Da Vinci's notebooks	$50 000 000

No one knows what future rates of appreciation will be nor what particular items will become valuable collectables. If you own any collectables, their value will be higher if they are kept in their unopened, original box.

Assuming an annual rate of appreciation of 5%, calculate a predicted 2024 value of each collectable listed in the table. Answer to the nearest $1000.

Investigation Collectables

Do an online search for collectables.

- Write down a list of items that are currently valuable.
- Which collectables increase in value more quickly than others?



Shutterstock.com/Stefano Ember

Practical activity Calculating inflation

This activity is in 2 parts. Complete the first part now, then keep your results to use again in Chapter 16. It is very important that you do this part now. The values you will use later must be taken 6 to 12 months apart.

The inflation rate measures changes in prices over time. Typically, countries use the price of a 'basket of goods' in their calculations. This basket includes items like essential food, rent, electricity, transport, clothing and entertainment. It does not include investment expenses like shares and owning **real estate**.

In this investigation, you will determine the cost of some typical items in the 'Australian basket of goods' today and again when you are completing Chapter 16. Then you will be guided through the steps to calculate the rate of inflation based on your data.

- Copy the record sheet, download it or type it up in a document or spreadsheet.
- Complete the price column for each item. Make careful notes about the brand and size you are pricing. When you determine the prices again in Chapter 16, it is essential that you price identical items. When you price a take-away meal, write down the exact food items so you won't have to remember it! There are 2 spaces at the end of the table for you to include 2 essential items of your choice.

Worksheet
Inflation sheet

Inflation basket of goods record sheet

	Type or brand	Size	Price at [insert current date]	Price at [insert later date]
Box of breakfast cereal				
1 L milk				
Loaf of bread				
Jar of coffee				
Packet of 6 sausages				
1 hot BBQ chicken				
1 kg tomatoes				
4 kg potatoes				
Cup of take-away coffee				
Take-away meal				
Rent for a 2-bedroom unit for 1 week				
Family's electricity bill				
Local bus trip				
Train trip to the city				
1 L of petrol				
Pair of jeans				
Pair of shoes				
Friday evening movie ticket				
		Total		

Remember: keep this record sheet in a safe place!

1.4 Interest calculators

Worksheet
Compounding periods: spreadsheet

Worksheet
Compound interest table

Spreadsheet
Interest calculator

Weblink
The Calculator Site

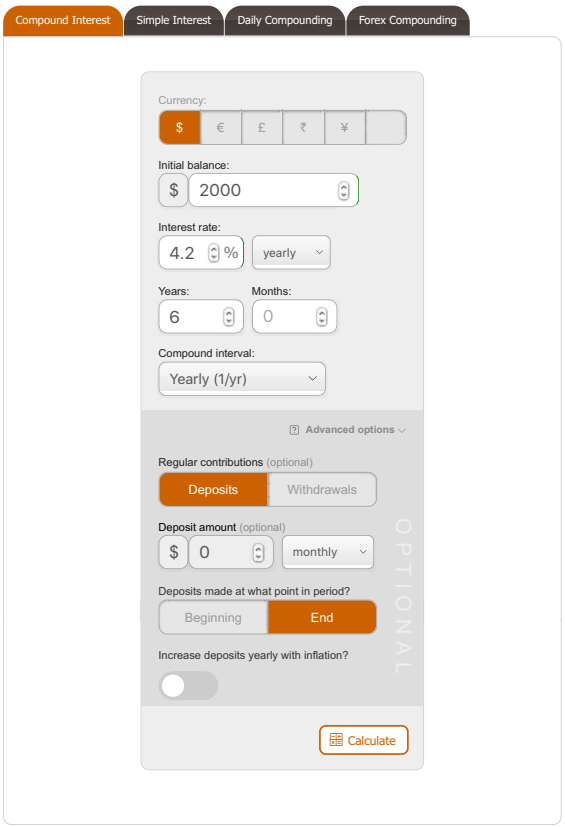
We can use an online interest calculator to calculate compound interest, such as the one on **The Calculator Site** or **MoneySmart**.

EXAMPLE 7

Jana invested $2000 in an account that earned 4.2% p.a. interest, compounded yearly for 6 years. Calculate the final value of her investment and the amount of interest she earned.

Steps

Enter the principal (initial balance), interest rate, period (6 years) and compound interval (yearly) into the **Compound Interest calculator** on thecalculatorsite.com and click **Calculate**.

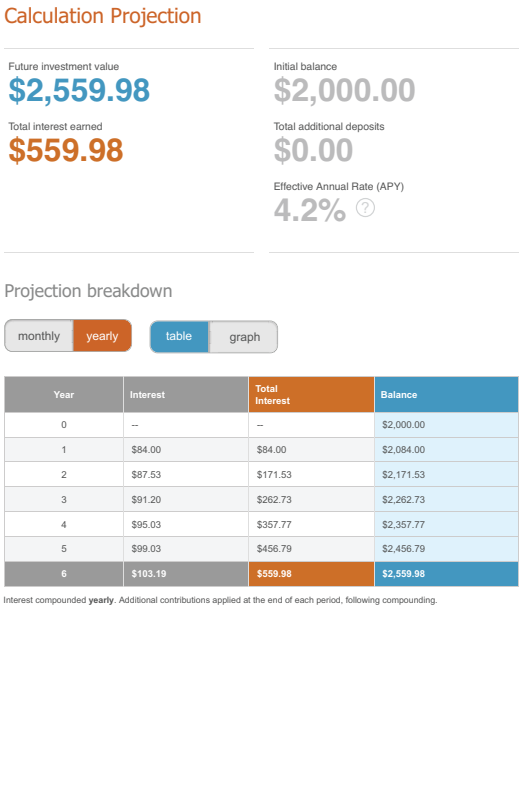


Click on the **graph** tab for a graph of the future value over the 6 years.

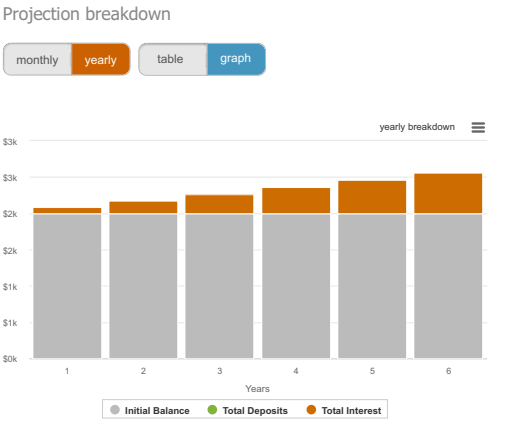
Working

final value = $2559.98

total interest = $559.98



thecalculatorsite.com



thecalculatorsite.com

EXERCISE 1.4 Interest calculators

ANSWERS p. 416

*Use an online **compound interest calculator** for each question.*

1 EXAMPLE 7 Jayne invested $3000 in an account that earned 4.1% p.a. interest, compounded annually for 7 years.

a Calculate the future value of her investment at the end of 7 years.

b How much interest will the investment earn?

c Use an online calculator to construct a graph showing the value of her investment during the 7 years.

2 Calculate the future value of each of the following investments.

	Principal	Interest rate p.a.	Term in years
a	$5500	2.9%	6
b	$1260	1.8%	4
c	$12 000	3.15%	3
d	$36 500	3.2%	8

3 Taj invested $46 000 at 2.85% p.a. annually compounding interest for 3 years. Determine the interest his investment earned each year and the total amount of interest.

4 When we lend someone money at compound interest, the calculations involved are exactly the same as when we invest in compound interest. Mr Kahill lent the soccer club $50 000 to use in upgrading club facilities. The loan was for 8 years at 5.6% p.a. annually compounding interest. The club agreed to repay the loan plus interest in one lump sum at the end of 8 years.

a How much will the club have to repay at the end of the loan?

b How much interest in total is the club being charged?

c Construct a graph to display the amount the club owes Mr Kahill for the term of the loan.

5 Determine the future value of a $6400 loan annually compounding at 3.67% p.a. for 10 years, assuming no repayments are made.

6 Sandra has $20 000 to invest for 3 years. She is considering the 2 different investments shown on the right.

Option 1	Simple interest at 5.5% p.a.
Option 2	Compound interest at 4.5% p.a.

Which option do you recommend she take? Use figures to justify your answer.

1.5 Compound interest spreadsheets

Spreadsheets are powerful tools to use for financial calculations. In this section, we will use a prepared spreadsheet and will then make one of our own.

Spreadsheet Interest calculator

Spreadsheet Compound interest graph

Spreadsheet Compound interest

EXAMPLE 8

Download the **Compound interest** spreadsheet and use it to determine the amount of interest that a $1200 investment will earn at 12% p.a. annually compounding interest for 8 years.

Steps

Insert $1200 for the principal and 12% for the interest.

Working

	A	B	C	D
1	**Compound interest spreadsheet**			
2	Only enter data in cells shaded blue.			
3				
4	Principal	$1,200.00	Annual rate of interest as a percentage	12%
5				
6				
7		**Account balance at the beginning of the year**	**Interest earned during the year**	**Account balance at the end of the year**
8	**Year 1**	$1,200.00	$144.00	$1,344.00
9	**Year 2**	$1,344.00	$161.28	$1,505.28
10	**Year 3**	$1,505.28	$180.63	$1685.91
11	**Year 4**	$1,685.91	$202.31	$1,888.22
12	**Year 5**	$1,888.22	$226.59	$2,114.81
13	**Year 6**	$2,114.81	$253.78	$2,368.59
14	**Year 7**	$2,368.59	$284.23	$2,652.82
15	**Year 8**	$2,652.82	$318.34	$2,971.16
16				

future value after 8 years = $2971.16

interest = future value – principal
= $2971.16 – $1200
= $1771.16



Shutterstock.com/Koldunova Anna

EXAMPLE 9

David often lends money to members of his family, and he wants the principal and the interest all repaid at the end of the loan. Create a spreadsheet that he can use to calculate the total amount to be repaid at the end of the loan.

Steps

The spreadsheet needs to have places to enter the principal (C3) and the interest rate (C4).

We'll enter the interest rate as a decimal to simplify our formulas.

Enter 5000 in C3 and 0.06 in C4 for a principal of $5000 and an interest rate of 6% p.a., and use 'currency' format for all the cells to display money values. The final step is to highlight the last cell containing a formula in each column and fill it down.

Now the spreadsheet is ready for David to use.

Working

In B7 we want the amount loaned.
Formula for B7: =C3

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the cells shaded green			
3		Amount loaned		
4		Interest rate as a decimal		
5				
6	Year	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7				
8				

We need 1 here as the starting year. The next year, formula for A8: =A7+1.

The amount owing at the beginning of the second year is the same as the amount owing at the end of the first year.
Formula for B8: =D7

Use *Prn* for the interest. P = B7, r = C4, n = 1, so we can leave n out. But we don't want the interest rate to change when we 'copy down', so we put $ signs in the formula for C7: =B7*$C$4

This is the amount at the beginning of the year, plus interest.
Formula for D7: =B7+C7

Here is the spreadsheet with the formulas included.

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the cells shaded green			
3		Amount loaned		
4		Interest rate as a decimal		
5				
6	Year	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	=C3	=B7*$C$4	=B7+C7
8	=A7+1	=D7		

You'll get something like this:

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the cells shaded green			
3		Amount loaned	$5,000.00	
4		Interest rate as a decimal	0.06	
5				
6	Year	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	$5,000.00	$300.00	$5,300.00
8	2	$5,300.00	$318.00	$5,618.00
9	3	$5,618.00	$337.00	$5,955.08
10	4	$5,955.08	$357.30	$6,312.38

EXERCISE 1.5 Compound interest spreadsheets

ANSWERS p. 416

Spreadsheet
Compound interest

*Use the **Compound interest** spreadsheet for questions **1** to **3**.*

1 EXAMPLE 8 Use the spreadsheet to determine the future value of each investment.

	Principal	Interest rate p.a.	Term in years
a	$1 000	5%	4
b	$2 500	4.3%	6
c	$32 500	3.75%	8
d	$50 000	2.1%	5

2 Calculate the interest earned on each investment.

	Principal	Interest rate p.a.	Term in years
a	$3 000	4%	5
b	$5 250	3.25%	4
c	$9 600	2.9%	8
d	$12 000	1.3%	6

3 Goran is saving to buy a car. He now has $5460 saved from his part-time job, which he plans to invest at 2.9% p.a. for 3 years.

a Calculate the future value of this investment.

b Goran hopes to have saved another $4000 when the investment matures. Calculate the total amount he may have available to spend on the car.

Spreadsheet
Peta's spreadsheet

4 EXAMPLE 9 Peta is creating a spreadsheet to show the future value of a compound interest investment. Download **Peta's spreadsheet**.

	A	B	C	D
1	Peta's spreadsheet			
2	Only enter data in cells shaded yellow.			
3				
4	Present value	$6,000.00		
5	Annual interest rate as a decimal	0.45		
6				
7	Year	Value at the beginning of the year	Interest	Value at the end of the year
8	1			
9				

a Write the formulas for cells B8, C8, D8 and A9, enter the formulas into the spreadsheet, then copy the formulas down to row 25.

b Peta meant to enter the decimal for 4.5% in B5 but she made a mistake. Correct her mistake.

c Use Peta's spreadsheet to determine the future value of a $24 000 investment at 4.5% annually compounding interest for 16 years.

d How much interest will this investment earn?

5 **a** Design your own spreadsheet to show the future value of a compound interest investment. Can you arrange it differently from the examples in this chapter? Add colour to make your spreadsheet easier to use.

b Use your spreadsheet to determine the amount of time it takes for a $3000 investment to double in size (that is, become $6000) when the interest rate is 4% p.a.

1.6 Different compounding periods

So far, we've only considered investments with interest compounding yearly but interest can compound monthly, quarterly, weekly or daily. The calculations, however, can be very repetitive and time consuming. Fortunately, we can use technology to do the long calculations for us.

In this section, you will need to use an online **compound interest calculator** or to download the **Different compounding periods** spreadsheet.

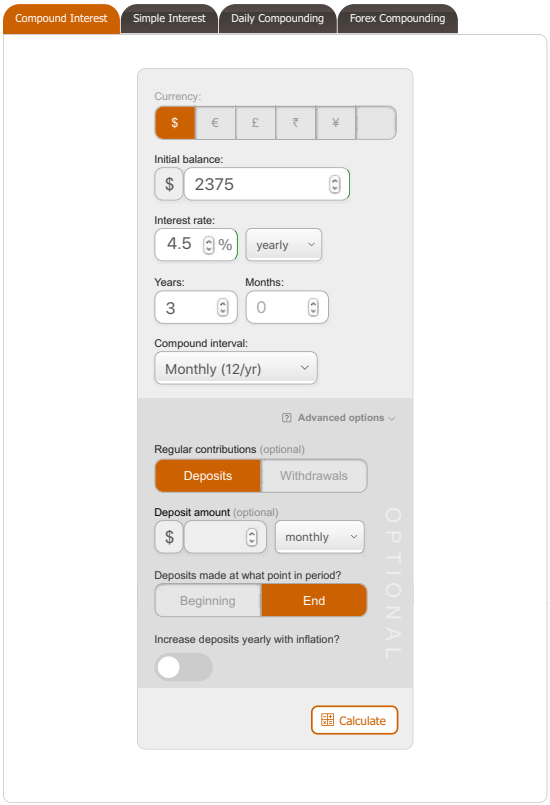
Spreadsheet
Different compounding periods

Weblink
The Calculator Site

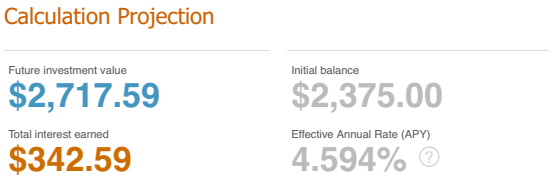
EXAMPLE 10

Zack invested $2375 for 3 years in an account that earned 4.5% p.a. interest, compounded monthly. Use technology to calculate the future value of his investment and the amount of interest he will earn.

Steps	Working
Put the following information from the question into an online **compound interest calculator**: principal: $2375 interest rate: 4.5% p.a. period: 3 years compounded monthly	



thecalculatorsite.com



thecalculatorsite.com

future value = $2717.59

total interest = $342.59

Put the information from the question into the **Different compounding periods** spreadsheet.

As interest is compounded monthly, number of periods per year = 12.

	A	B	C	D	E
1	Different compounding periods				
2					
3	How much compound interest will I earn?				
4	Only enter data in cells shaded blue.				
5					
6	Principal	$2,375.00			
7	Annual rate of interest as a percentage	4.50%		Interest rate per compounding period, as a decimal	0.00375
8	Number of compounding periods per year	12		Number of compounding periods	36
9	Length of the investment in years	3		Final value of the investment	$2,717.59
10				Interest earned during the investment	$342.59

future value = $2717.59

total interest = $342.59

EXERCISE 1.6 Different compounding periods

ANSWERS p. 417

Use the technology of your choice to answer these questions.

1 EXAMPLE 10 Determine the amount of interest each investment will earn.

	Principal	Interest rate p.a.	Term	Compounding period
a	$10 000	6%	3 years	monthly
b	$2 500	4.2%	4.5 years	quarterly
c	$1 840	3.6%	5 years	half yearly
d	$3 800	4.6%	2 years	daily

2 Marissa won $5800 in the lotto and invested it at 7.2% p.a. monthly compounding interest for 3 years.

a How much will Marissa's win grow to in 3 years?

b How much interest will Marissa's investment earn?

3 Brody saved $1660 towards a holiday for after he finishes Year 12. He invested it at 5% p.a. interest compounded monthly for 1 year. How much was in his holiday account at the end of the year?

4 Jackie received a $2500 bonus as a result of increased company profits. She is going to invest it for 1 year at 6% p.a. How much *more* interest will she earn in monthly compounding interest compared to simple interest?

5 When Kristy was made redundant at work, she was given a lump sum payment of $40 000, which she invested for 5 years. How much more interest will she earn from *daily* compounding interest than *annually* compounding interest, at 6% p.a.?

6 On his retirement, Yo-han received a lump sum superannuation payment of $450 000. He is going to invest it for 2 years. He can invest it at 6.4% p.a. monthly compounding interest or 6.55% p.a. annually compounding interest. Which investment will give him the better return? Justify your answer.



iStock.com/XiXinXing

7 Lucy has $200 000 to invest to provide for her retirement. She considers 2 different investment options.

	Investment term	Fees
Finance company	5% p.a. compounding daily	$10 per month
Managed fund	6% p.a. compounding monthly	6.5% of the interest earned

All fees are deducted at the end of the year.

Which option will give Lucy the higher return in 12 months? Explain your answer.

Investigation Different compounding periods

In this investigation, you are going to determine how changing the compounding period (how often interest is compounded) affects the amount of interest earned.

What you have to do

1 Use either an online calculator or a spreadsheet to complete the missing values in the table.

Invest $10 000 for 4 years at 8% p.a.	
Compounding period	Interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

2 Write a sentence to describe the observations you've made.

3 Check the correctness of your observation by completing a table for another investment.

Invest $50 000 for 7 years at 12% p.a. compounding interest	
Compounding period	Interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

4 Chris is going to invest $2000 for 3 years at 6% p.a. He can select either monthly compounding interest or fortnightly compounding interest. Which compounding period will give him the better return? Give a reason for your answer.

Worksheet
Simple vs compound interest: spreadsheet

Worksheet
Comparing interest rates

Spreadsheet
Comparing interest rates

Investigation | Is compound interest always better than simple interest?

In this investigation, you are going to make some calculations to help you choose the investment with the better return.

What you have to do

1. Copy the tables below.
2. Use the technology of your choice to complete the tables. To make it easier, make the principal $1000 in every calculation.
3. Complete the class discussion questions after you have finished the calculations.

Which is better: Simple or compound interest?

Part A: The interest rates and the terms are the same

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 3% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Part B: The terms are the same but the compound interest rate is higher than the simple interest rate

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5.1% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3.75% p.a.	Term: 6 years Interest rate: 4% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 8.1% p.a.	Simple interest = Compound interest = Which investment is better?

Part C: The terms are the same but the simple interest rate is higher than the compound interest rate

Simple interest	Interest compounded annually	Simple interest
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 4.8% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 2.75% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 12% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 3 years Interest rate: 12% p.a.	Term: 3 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Class discussion questions

- When the interest rates and the terms are the same, does simple or compound interest give the better return?
- When the compound interest rate is higher than the simple interest rate and the terms are the same, which type of investment produces the better return?
- When the simple interest rate is larger than the compound rate and the terms are the same, will one type of interest always give the better return?

Investigations

You are required to complete some investigations in the Foundation Mathematics course.

This diagram shows the 3 components of an investigation. You can also download a copy.

Components of an investigation

1. Formulation
- What is the investigation about?
- What do you know?
- What tools can you use?
- What extra information do you need?

→

2. Exploration
- Make calculations.
- Summarise results systematically, for example in a table, graph or spreadsheet.
- Consider your results:
  - Do they make sense?
  - Is there a pattern?
- Make a conjecture.
- Test your conjecture.

→

3. Communication
- Describe what you did.
- Summarise your results.
- Explain your conclusions.

Worksheet
Investigation components

There are many different ways to approach an investigation. You and your friends may take very different approaches and they can all be correct!

The extended investigations in this book can be adapted into assessment tasks and projects.

Extended investigation — My investment portfolio

Suppose you have $10 000 to invest, some in a term deposit and the remainder in shares. You will be using the data about the performance of your investments in Chapter 16, *Australia and the world*, at the end of the year.

- Decide the proportion of the money you want to put into each type of investment.
- Select a term deposit account and some shares to purchase.
  The **Australian Securities Exchange** website (www2.asx.com.au) can assist you with the price of shares.
- Monitor the value of your shares every fortnight. Record their value and note any dividends paid.

Weblink
Australian Securities Exchange

Extended investigation — The sports club investment

You are the treasurer for a local sports club. The club aims to build new facilities and improve the grounds when it has sufficient funds. The club currently has $150 000, which needs to be invested for the next 12 months. Club members have made the following investment suggestions.

Option	Interest	Rate
1 Hamish suggests the investment be with his local bank.	Annually compounding interest paid at the end of the year	3.2% p.a.
2 Ashleigh thinks the local credit union has a good deal.	Monthly compounding interest added to the account each month	3.19% p.a.
3 Krishna believes her bank's term deposits are the way to go.	Flat rate interest, with interest paid at maturity	3.21% p.a.
4 Stefan recommends buying bitcoins online. He thinks the club could make big money if the value of the bitcoin gets back to its previous high value.	Currently, 1 bitcoin is worth A$71 500. Its value has been as high as almost A$85 000.	

Your job is to consider the safety of each investment together with the investment return, then write a letter of recommendation to the board. Remember to include calculations or website screenshots in your letter.

You can download and use the investigation components guide.

Worksheet
Investigation components

Keyword activity

WORD MATCH

ANSWERS p. 417

Match the terms in the left column with their correct meanings in the right column.

Terms	Meaning
1 future value	**A** An illegal method used to trick people out of their money
2 compound interest	**B** Occurring every year
3 compounding period	**C** The rate of return (money earned) on an investment, usually expressed as a percentage per year
4 scam	**D** Interest that is calculated only on the original principal
5 annual	**E** How often compound interest is calculated for an investment
6 appreciate	**F** The original amount invested or borrowed
7 inflation	**G** To increase in value, such as the value of an antique
8 interest rate	**H** The value of an investment over time
9 principal	**I** An increase in the price of an item, such as the cost of bread
10 superannuation	**J** Interest that is calculated on the current value of an investment, including interest previously added to the principal
11 simple interest	**K** A regular saving fund to provide an income when you are retired

Solution to the chapter problem

Problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested $25 000 for 3 years, earning simple interest at 4.2% per year.

Whose investment will earn more interest?

Solution

Use an online calculator or spreadsheet to calculate the future value of Ryan's investment.

The principal is $25 000, the interest rate is 4% p.a. and the time is 3 years.

An online calculator shows that Ryan's investment will be worth $28 121.60 at the end of 3 years.

Ryan's interest = $28 121.60 – $25 000
= $3121.60

Use the simple interest formula to calculate Melanie's interest.

Interest = Prn

P = $25 000, r = 0.042 and n = 3

Melanie's interest = $25 000 × 0.042 × 3
= $3150

Compare the interest:

Melanie's investment will earn $3150 – $3121.60 = $28.40 more interest than Ryan's.

1 Test yourself

Investing money

ANSWERS p. 417

1 EXERCISE 1.1 Calculate the simple interest earned on a $4000 investment at 4.1% p.a. for 8 years.

2 EXERCISE 1.2 Josephine invested $8400 at 4% p.a. compounded yearly.

a Copy and complete this table to determine the future value of her investment over 3 years.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8400 \times 0.04 \times 1$ $= \$_____$	$\$8400 + \$_____$ $= \$_____$
End of the 2nd year	$I = Prn$ $= \$_____ \times 0._____ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$
End of the 3rd year	$I = Prn$ $= \$_____ \times 0._____ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$

b How much interest did the investment earn?

3 EXERCISE 1.3 By what decimal is an amount multiplied to increase it by 7%?

4 EXERCISE 1.3 Today, a box of groceries costs $56. If inflation continues at 2% p.a., how much will the box of groceries cost in 4 years' time?

5 EXERCISE 1.3 Tim owns some collectable baseball cards that are appreciating at 5% p.a. Today, the cards are worth $720. Calculate their value in 3 years' time, if the current rate of appreciation continues.

6 EXERCISE 1.3 A brand of tablet device is currently priced at $340. Predict the price of the same brand of device in 5 years' time, assuming an average inflation rate of 3% p.a.

7 EXERCISE 1.4 Use an online calculator to determine the total amount of interest Pooja will earn when she invests $26 500 at 4.1% p.a. annually compounding interest for 7 years.

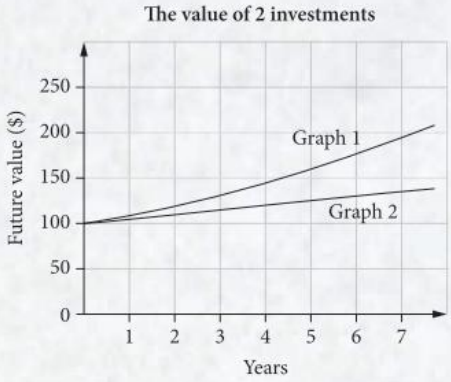
8 EXERCISES 1.1, 1.4 Which investment has the better return for an $800 principal for a 6-year term?

Investment A: 3.4% p.a. simple interest

Investment B: 3.3% p.a. interest compounded yearly

9 EXERCISE 1.5 Minh drew a graph of the future values of 2 investments. One investment is paying simple interest and the other is paying compound interest.

Which graph is which?



10 EXERCISES 1.5, 1.6 Jill used this spreadsheet to calculate the future value and interest when she invested $60 000 at 8% p.a. with monthly compounding interest.

	A	B	C	D
1	Jill's spreadsheet			
2				
3				
4		Initial investment C4	$60,000.00	
5		Annual interest rate as a decimal C5	0.08	
6		Monthly interest rate C6	0.006666667	
7				
8				
9	Number of months	Value of the investment at the beginning of the month	Monthly interest	Value of the investment at the end of the month
10	1	$60,000.00	$400.00	$60,400.00
11	2	$60,400.00	$402.67	$60,802.67
12	3	$60,802.67	$405.35	$61,208.02
13	4	$61,208.02	$408.05	$61,616.07
14	5	$61,616.07	$410.77	$62,026.85
15	6	$62,026.85	$413.51	$62,440.36
16	7	$62,440.36	$416.27	$62,856.63
17	8	$62,856.63	$419.04	$63,275.67
18	9	$63,275.67	$421.84	$63,697.51
19	10	$63,697.51	$424.65	$64,122.16
20	11	$64,122.16	$427.48	$64,549.64
21	12	$64,549.64	$430.33	$64,979.97
22	13	$64,979.97	$433.20	$65,413.17
23	14	$65,413.17	$436.09	$65,849.26
24	15	$65,849.26	$439.00	$66,288.25
25	16	$66,288.25	$441.92	$66,730.17
26	17	$66,730.17	$444.87	$67,175.04
27	18	$67,175.04	$447.83	$67,622.88
28				

a One of the formulas Jill used is =B10*$C$5. What does this formula calculate?

b Why did Jill use $ signs in the formula?

c How much interest did Jill's investment earn in the 14th month?

d What formula could Jill use to make the spreadsheet calculate the total amount of interest earned in the first 12 months?

e What was Jill's investment worth at the end of 18 months?

f Construct a spreadsheet similar to Jill's that shows her investment compounding weekly instead of monthly.

g Use your spreadsheet to determine the extra interest Jill will earn in 6 months if the investment compounds weekly instead of monthly.

11 EXERCISE 1.6 Samir invested $3000 at 4.8% p.a. monthly compounding interest for 3 years.

a What was the monthly interest rate?

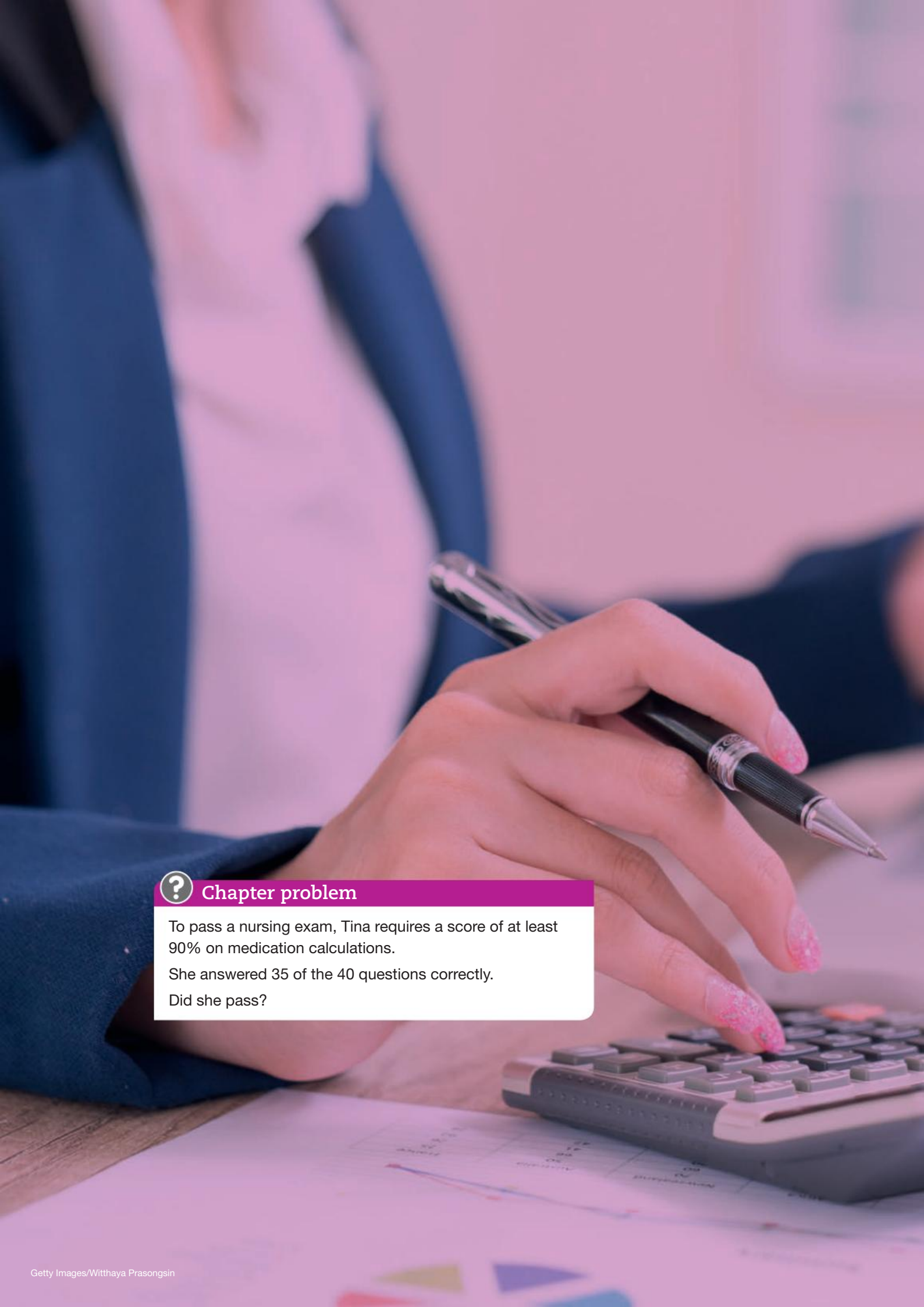
b Use technology to determine how much interest Samir will earn.

12 EXERCISE 1.6 Pedro has $10 000 to invest for 6 years. Use technology to determine which of the following is the best investment. Select **A**, **B** or **C**.

A 4% p.a. monthly compounding

B 4.1% p.a. annually compounding

C 4.4% p.a. simple interest



Chapter problem

To pass a nursing exam, Tina requires a score of at least 90% on medication calculations.

She answered 35 of the 40 questions correctly.

Did she pass?

Getty Images/Witthaya Prasongsin



CHAPTER 2

ALGEBRA, NUMBER AND STRUCTURE

CRUNCHING NUMBERS

2.1 Order of operations
2.2 Fraction problems
2.3 Decimal problems
2.4 Percentage problems
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Practice basic number facts
- Use fractions, decimals and percentages to solve problems

How are we ever going to use this?

- Shopping
- Working
- Everyday living

Study design coverage

AREA OF STUDY 1: ALGEBRA, NUMBER AND STRUCTURE

- mathematical conventions notations for numbers and number operations
- estimation and approximation including rounding.

VCE Mathematics Study Design 2023-2027 p. 73, © VCAA 2022

Videos (1):
2.1 Order of operations

Worksheets (3):
2.4 Percentage problems • Percentage power • Working with percentages

Puzzles (2):
2.1 Order of operations
Keyword activity Crunching numbers crossword

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

2.1 Order of operations

Video Order of operations

Puzzle Order of operations

Some people use the memory aid '**BIDMAS**' to help them remember the order in which we make calculations with mixed operations.

Order of operations

B is for brackets – do what's inside the brackets first.

I is for indices – calculate any **indices** (powers) next.

DM together – do **division** and **multiplication** working from left to right.

AS together – do **addition** and **subtraction** working from left to right.

EXAMPLE 1

Evaluate $120 - 2 \times (4 + 3)^2$

Steps		Working
B	add 4 + 3 inside the brackets first	$120 - 2 \times (4 + 3)^2$
		$= 120 - 2 \times (7)^2$
I	evaluate indices (i.e. 7^2) next	$= 120 - 2 \times 49$
DM	do the multiplication 2×49 next	$= 120 - 98$
AS	finish with the subtraction	$= 22$

EXERCISE 2.1 Order of operations

ANSWERS p. 417

1 How quickly can you complete these addition and subtraction number fact statements? Time yourself. Can you beat 25 seconds for answering these questions correctly?

a $3 + 2 =$ _____ **b** $7 - 4 =$ _____ **c** $8 + 3 =$ _____

d $9 - 4 =$ _____ **e** $10 - 5 =$ _____ **f** $4 +$ _____ $= 12$

g _____ $- 6 = 5$ **h** _____ $+ 5 = 13$ **i** $18 -$ _____ $= 12$

j $13 +$ _____ $= 20$ **k** $11 -$ _____ $= 4$ **l** $8 +$ _____ $= 19$

2 Are you just as quick with multiplication and division number facts? Time yourself. How long did it take you to get all 10 statements completed correctly?

a $4 \times 5 =$ _____ **b** $9 \times 3 =$ _____ **c** $7 \times 5 =$ _____

d $24 \div 3 =$ _____ **e** $80 \div 10 =$ _____ **f** $6 \times$ _____ $= 42$

g _____ $\times 8 = 56$ **h** $50 \div$ _____ $= 10$ **i** _____ $\div 8 = 12$

j $9 \times$ _____ $= 45$

3 EXAMPLE 1 Evaluate each mixed expression.

a $16 - (2 + 5)$ **b** $(24 + 6) \div 3$ **c** $(18 + 2) \div (11 - 6)$

d $5 \times (4 - 1)^2$ **e** $3 \times (4 + 6)^2$ **f** $200 - 2 \times (15 + 5)$

g $20 + 2 \times (2 + 3)^2$ **h** $12 \div 2 + 5 \times (3 + 7)$ **i** $(52 + 8) \div (12 - 7)$

j $4 \times (7 + 5) - 4 \times 9$

4 Test your skills in everyday number-crunching by solving each problem below. How many can you do without using a calculator?

a Keira needs 12 pens for a workshop she is running. There are 6 pens in a packet. How many packets does she need?

b Sam's restaurant bill is $42 and he paid with $50. How much change should he get?

c A cake needs to cook for 50 minutes. It's been in the oven for 44 minutes. How much longer does it need in the oven?

d During a football game, the East Tigers scored five 6-point goals and kicked three 1-point behinds. How many points did the team score?

e Every morning Lara swims 40 laps of the Olympic pool at swimming training. This morning she has completed 23 laps so far. How many more laps has she still to swim?

f Lara's personal best (PB) for breaststroke is 48 seconds. In the last time trial, she improved her PB by 2 seconds. What was her previous PB?

g When Noah was driving in a 40 km/h school zone, his speed was recorded by a speed camera as 65 km/h. How many km/h over the school zone speed limit was he driving?

h Mae paid for her canteen purchase with six 20c coins, a $1 coin and two $2 coins. How much did she pay?

i Samantha's monthly child care bill is increasing by $55. Currently, she pays $425 per month. Calculate her new monthly charge.

j Anastacia has a newspaper delivered every weekday and 2 each day of the weekend. How many newspapers does she have delivered per week?

k In Damien's wallet there are two $50 notes, four $20 notes, a $10 note and a $5 note. What is the total value of the notes in his wallet?



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Practical activity Make this number!

Make and cut out individual cards for 1, 2, 3, 4, 5, +, –, ×, ÷, (and), as shown.

1	2	3	4
5	+	–	×
÷	(	)	

Joe arranged the cards as below to make the answer 43.

(	1	2	4	+	5	)	÷	3

Your challenge is to arrange the cards to make the following amounts:

- a number divisible by 10
- a negative number
- the largest number possible
- a number between 150 and 200
- a decimal that ends in .25
- the smallest positive number possible.

You don't have to use all the cards but you can only use each card once per amount.

2.2 Fraction problems

The purity of gold is measured in carats. Pure gold is 24 carats. Pure gold is too soft for most applications so a metal alloy is mixed with the gold to give it strength. The fraction of gold in a piece of gold jewellery is written as a fraction over 24. For example, a 9-carat ring contains $\frac{9}{24}$ pure gold.



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EXAMPLE 2

Aisha has an 18-carat gold necklace that weighs 300 g.

a What fraction of the weight of the necklace is pure gold?

b Calculate the weight of pure gold in the necklace.

Steps	Working
a The fraction is the number of carats (18) over 24.	fraction $= \frac{18}{24}$
We can simplify this fraction by dividing the **numerator** (top) and the **denominator** (bottom) by 6, or by using the fraction key on your calculator.	$\frac{18}{24} = \frac{18 \div 6}{24 \div 6}$ $= \frac{3}{4}$
	$\frac{3}{4}$ of the weight of Aisha's necklace is pure gold.
b Calculate $\frac{3}{4}$ of 300 g.	$\frac{3}{4} \times 300\text{ g} = 225\text{ g}$

EXERCISE 2.2 Fraction problems

ANSWERS p. 418

1 EXAMPLE 2a Calculate the fraction of gold in a piece of jewellery with each carat rating. Answer as a simplified fraction.

a 12 carat **b** 16 carat **c** 10 carat **d** 22 carat **e** 14 carat

2 EXAMPLE 2b Imran's 72 g ring is 12-carat gold. How many grams of pure gold are in his ring?

3 Caleb is a jewellery manufacturer. He is making an 18-carat gold necklace that will weigh 200 g. How much gold and alloy will he mix?

4 Anya is planning a trip to the city to see an exhibition. It will take her a quarter of an hour to walk to the station, the train trip will take half an hour, and then it will take her 10 minutes to walk to the exhibition.

a How long will it take Anya to travel to the exhibition?

b Anya wants to arrive at the exhibition at half-past ten. At what time will she need to leave?

5 When Ziad surveyed the 240 students in Years 11–12 at his school, he discovered that $\frac{3}{5}$ of them have not decided what they are going to do after Year 12. How many students have decided already what they are going to do after Year 12?

6 Ellie put a box of 18 eggs into her supermarket trolley, but she dropped them when she tried to put them on the check-out. The check-out operator said, 'I'll have to replace the box. A third of the eggs are broken'. How many of the eggs were not broken?

7 Nanette is preparing a dessert for a dinner party. The recipe makes dessert for 4 people but she needs to make enough for 6 people. How much of each ingredient will she require?

Ingredients
2 tablespoons egg substitute
100 g butter
220 g almond meal
120 g sultanas
80 g sugar
200 mL milk

8 Shan has an underactive thyroid gland. Her doctor prescribed $\frac{2}{5}$ mg of thyroxine each morning. Each tablet contains $\frac{1}{5}$ mg thyroxine.

a How many tablets should Shan take each morning?

b The bottle contains 60 tablets. How many days' supply of tablets are in the bottle?

9 Film camera shutter speeds are measured in fractions of a second. When the shutter speed is $\frac{1}{15}$, the shutter is open for $\frac{1}{15}$ of a second. Digital cameras also use shutter speeds, but the effect is achieved by digitally restricting the amount of light received rather than by using a shutter.

a Which shutter speed is the faster, $\frac{1}{60}$ or $\frac{1}{125}$? Give a reason for your answer.

b How many times longer is the shutter open at $\frac{1}{125}$ than at $\frac{1}{500}$?



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10 Dylan has been admitted to the hospital's emergency department with a stroke. A stroke is caused by a blood clot or a blood vessel bursting or bleeding in the brain. The amount in milligrams (mg) of emergency medication a stroke patient is given is equivalent to $\frac{9}{10}$ of the patient's mass in kg. Dylan's mass is 80 kg.

a Use a calculation to show that his emergency dose of medication is 72 mg.

b Dylan will be given $\frac{1}{10}$ of the emergency medication immediately and the remainder over the next 24 hours. How much will Dylan be given immediately?

2.3 Decimal problems

EXAMPLE 3

Express each number, correct to 2 decimal places.

a 14.3743 **b** 14.375

Steps	Working
a We have to decide whether 14.3743 is closer to 14.37 or 14.38. Because the first digit after the 7 is less than 5, we can just leave off the extra digits.	$14.3743 \approx 14.37$
b When we're rounding 14.375, the first number after the 7 is a 5. This means we can't just remove it. We need to round the number up.	$14.375 \approx 14.38$

EXAMPLE 4

Calculate the average of the numbers 14.6, 22.8, 19.7 and 23.5, correct to 1 decimal place.

Steps	Working
To calculate the average of 4 numbers, add them together and divide by 4.	$14.6 + 22.8 + 19.7 + 23.5 = 80.6$ average $= 80.6 \div 4$ $= 20.15$ ≈ 20.2
Check the answer is reasonable. The average should be around the middle of the 4 numbers. Two of the numbers are lower than 20.2 and 2 are higher. The answer is reasonable.	

EXERCISE 2.3 Decimal problems

ANSWERS p. 418

1 EXAMPLE 3 Express each number correct to 2 decimal places.

a 14.236 **b** 21.628 **c** 112.349

d 134.2345 **e** 3.925 **f** 0.1349

2 Evaluate each expression, correct to 1 decimal place.

a $12.348 + 5.216$ **b** $17.129 - 6.142$ **c** 3.2×4.5 **d** $16.8 \div 4.1$

3 Karl correctly rounded a number to 1 decimal place. His answer was 28.7. What could the number have been?

4 What is the smallest number that rounds to 34.8, correct to 1 decimal place?

5 EXAMPLE 4 Calculate the average of each list of numbers, correct to 1 decimal place.

a 17.4, 19.8, 22.7, 21.9 **b** 24.8, 36.4, 19.2, 32.8, 29.7, 30.7

6 Holly and Beck are competing against each other in a skateboard competition. The winner will be the person whose total score for 3 routines is the higher. The points awarded by the judges for the 3 routines are given below.

Holly's scores		
Routine 1	Routine 2	Routine 3
54.82	61.34	72.35
50.33	66.75	81.64
55.68	62.98	79.54
58.20	67.88	78.43
53.65	64.56	77.25

Beck's scores		
Routine 1	Routine 2	Routine 3
76.20	51.34	66.89
71.30	60.28	64.77
77.42	49.35	62.15
81.40	55.25	60.50
80.23	52.66	64.54

a Calculate the scores awarded to each person for each of their 3 routines.

b Calculate each competitor's total score.

c Who won the competition?



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6 In skateboard competitions, scores are rounded to 2 decimal places. The steps below show how a contestant's score is determined.

Step 1: Each of the 5 judges awards a score from 0.00 to 100.

Step 2: The lowest and the highest scores are removed.

Step 3: The average of the 3 remaining scores is calculated, correct to 2 decimal places.

Here are the scores the judges gave Mason for his first routine:

	J1	J2	J3	J4	J5
Scores	65.82	71.56	66.98	73.56	69.88

a Which 2 scores are not used to calculate Mason's score?

b Calculate his score, correct to 2 decimal places.

8 To legally drive a car, a driver on a full licence must have a blood alcohol content (BAC) of less than 0.05.

a When Olivia was tested by police, her BAC was three times the legal limit. What was Olivia's BAC?

b In a roadside test, Ethan returned a BAC of 0.025. He was charged because it was claimed that his BAC was 5 times what it was allowed to be. How could Ethan explain that his BAC was *lower* than the legal limit?

9 When doctors prescribe most medications, they use decimals rather than fractions. Zoe was prescribed 75.5 mg/day of a medication for her heart condition, taken in 3 doses. Calculate the size of each dose, correct to 1 decimal place.

10 In a diving competition, a diver's score is multiplied by the degree of difficulty of the dive to determine the total score.

Chan is competing in a diving competition. He can't decide whether to do a dive with a degree of difficulty of 2.9, for which judges usually give him a score of 7.1, or to attempt a dive with a 2.1 degree of difficulty, for which he usually receives a score of 8.5. What do you recommend he should do?

2.4 Percentage problems

Percentages are everywhere! When we borrow or invest money the interest rate is a percentage, and shops advertise **discounts** as percentages.

A percentage is a convenient way of writing a fraction with a denominator of 100. The expression 23% is another way of writing the fraction $\frac{23}{100}$.

EXAMPLE 5

Anand scored 80% in his maths assessment. There were 40 questions in the test. How many did he answer correctly?

Steps	Working
When we are finding a percentage of an amount, we multiply the amount by the percentage. We can choose to write the percentage as a fraction or as a decimal.	correct questions $= 80\% \times 40$ $= \frac{80}{100} \times 40$ $= 32$ or correct questions $= 80\% \times 40$ $= 0.80 \times 40$ $= 32$

EXAMPLE 6

What percentage of \$1200 is \$900?

Steps	Working
Express the two amounts as a fraction, writing the amount following the word 'of' as the denominator. Then multiply the fraction by 100%.	fraction $= \frac{900}{1200}$ percentage $= \frac{900}{1200} \times 100\%$ $= 75\%$

EXAMPLE 7

Decrease \$75 by 20%.

Steps	Working
There are two ways to solve this problem. The first way is to calculate 20%, then subtract it from \$75.	20% of \$75 $= 0.20 \times \$75$ $= \$15$ final answer $= \$75 - 15$ $= \$60$
Alternatively, when we reduce an amount by 20%, we only want 100% – 20% = 80%. Calculate 80% of \$75.	80% of \$75 $= 0.80 \times \$75$ $= \$60$

Worksheet Percentage problems

Worksheet Percentage power

Worksheet Working with percentages

EXERCISE 2.4 Percentage problems

ANSWERS p. 418

1 EXAMPLE 5 It rained on 25% of the 12 days Kristie was in Fiji. On how many days did it rain?

2 Connor went on a diet. Before the diet he weighed 86 kg, then he lost 12% of his body mass on the diet.

a How much weight did Connor lose on the diet? Answer correct to the nearest kg.

b Calculate Connor's weight after the diet.

3 EXAMPLE 6 Express each fraction as a percentage.

a $\frac{7}{8}$ **b** $\frac{11}{20}$ **c** $\frac{19}{25}$

4 Edmond earns $85 per week in his part-time job. He spends 40% of his earnings on sport. How much does Edmond spend on sport each week?

5 There are 2500 passengers on a cruise ship, of whom 8% are vegetarian. How many passengers are vegetarian?

6 EXAMPLE 7 Phoebe receives a 5% staff discount at the supermarket where she works part time. How much will Phoebe have to pay for a trolley of food that normally costs $124?

7 The local bookshop gives schools a 15% discount. The school purchased books with a normal price of $2800. How much did the school have to pay?

8 Eloise's naturopath gives patients who pay by EFTPOS a 10% discount. If she uses EFTPOS, how much will Eloise have to pay for a consultation normally costing $98?

9 In Australia, the price of all goods and services is increased by 10% for GST to determine the final price. Calculate the final price of a car that is priced at $52 000 before GST.

10 In an end-of-summer sale, the price of all swimwear was reduced by 45%. Calculate the sale price of some swimmers originally priced at $85.



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11 The value of an apartment increased from $450 000 to $510 000. What percentage increase is this? Answer correct to 1 decimal place.

12 During a bushfire, 66% of a 356 000-hectare national park was burnt. How many hectares were burnt?

13 Students of the fashion college are given a 5% discount when they buy materials to use in their course. Yasmina buys 5 metres of fabric to use at college. The fabric is normally priced at $24 per metre. How much will Yasmina have to pay for the fabric?

14 The office store reduced the price of calculators from $25 to $20. What percentage reduction is this?

15 Super Burger employees are going to be given a 3.5% pay rise. Currently, Smeera earns $18.75 per hour. How much will she earn per hour after the pay rise?



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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

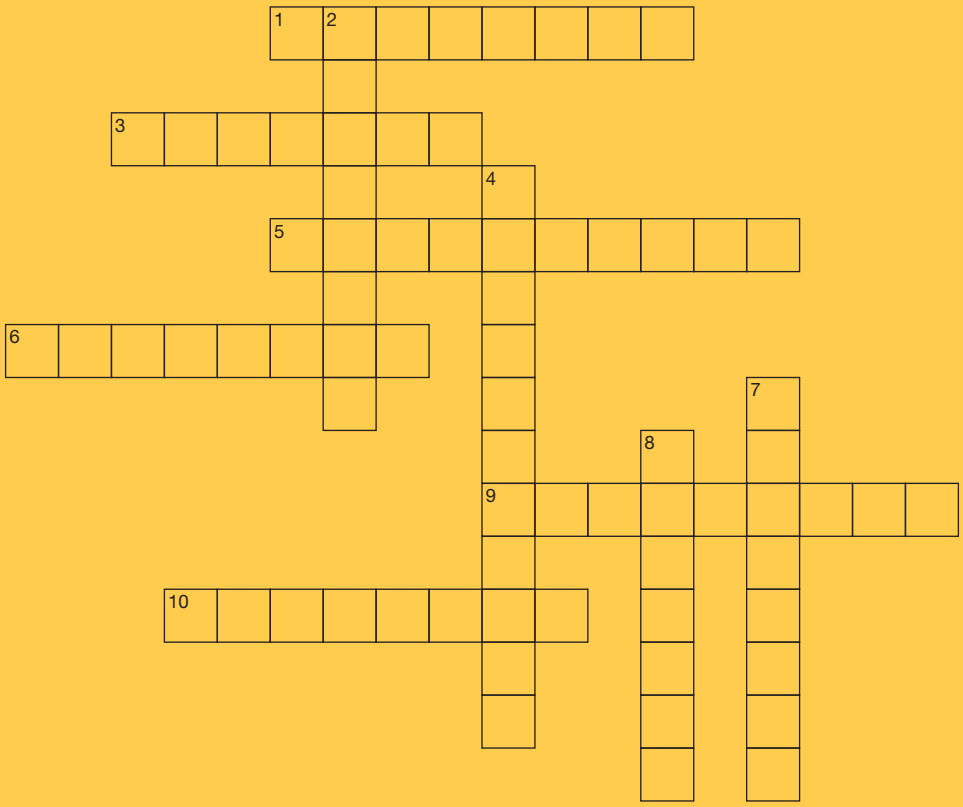
Keyword activity

CLUELESS CROSSWORD

ANSWERS p. 418

Copy or download this crossword and position the keywords from this chapter in the crossword. Then write a set of clues for your crossword.

BRACKETS	DECIMAL	DECREASE	DENOMINATOR	DISCOUNT
FRACTION	INCREASE	INDICES	NUMERATOR	PERCENTAGE



Puzzle
Crunching numbers crossword

Solution to the chapter problem

Problem

To pass a nursing exam, Tina requires a score of at least 90% on medication calculations. She answered 35 of the 40 questions correctly.

Did she pass?

Solution

Tina scored 35 out of 40, or $\frac{35}{40}$.

Multiply the fraction by 100% to change it into a percentage.

$$\text{Tina's mark as a percentage} = \frac{35}{40} \times 100\%$$
$$= 87.5\%$$

Tina scored less than 90%. She did not pass the exam.

2 Test yourself

Crunching numbers

ANSWERS p. 418

1 EXERCISE 2.1 Copy and complete each basic number fact.

a $8 + 5 = ____$ **b** $4 \times ____ = 44$ **c** $72 \div ____ = 9$

2 EXERCISE 2.1 Use the correct order of operations to evaluate each expression.

a $24 \div 4 \times 2$ **b** $3 \times (8 - 3)^2$ **c** $(12 - 8) \times (6 + 3)$ **d** $96 - 3 \times (2 + 3 \times 4)$

3 EXERCISE 2.2 An 18-carat gold chain is made from 18 parts pure gold to every 6 parts alloy.

a What fraction of the chain is pure gold?

b The chain weighs 120 g. How many grams of pure gold does it contain?

4 EXERCISE 2.2 Five people shared a meal in a restaurant. Ryan is paying $\frac{1}{5}$ of the cost and Silvia is paying the remainder. The bill came to $165.

a What fraction of the bill will Silvia pay?

b How much will Silvia pay?

5 EXERCISE 2.3 Evaluate each expression, correct to 1 decimal place.

a 21.5×18.3 **b** $62.75 \div 12$ **c** $75.3 + 12.2 \times 1.5$

6 EXERCISE 2.3 Jess is bottle-feeding a baby lamb. The lamb requires 800 mL of formula each day spread over 6 feeds. How many millilitres (mL) should Jess put in each bottle, correct to 1 decimal place?

7 EXERCISE 2.4 The number of children in a day care centre increased from 60 to 75. What percentage increase is this?

8 EXERCISE 2.4 In the last 2 months, 40% of the 25 children in Year 1 have had head lice. How many of the Year 1 children have had head lice in the last 2 months?

9 EXERCISE 2.4 The cost of private health insurance is about to increase by 8.5%. Currently Emma pays $80 per fortnight for her health insurance. How much will it cost after the increase?

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Chapter problem

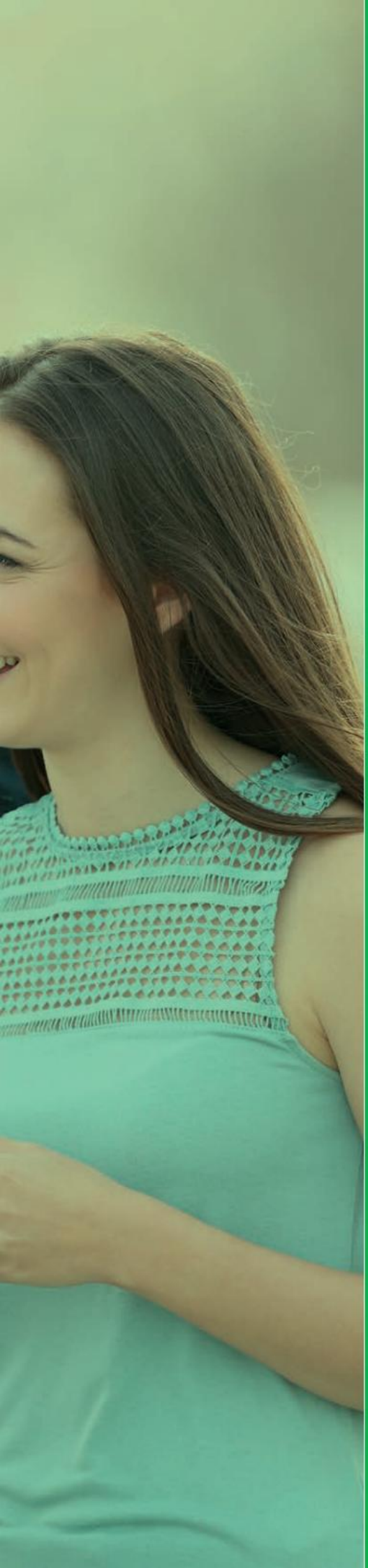
Anya is a market researcher. She has been asked to research political views in her community.

One Wednesday morning, Anya stands at an intersection on the main street of town and asks people passing, 'Are you going to vote for Liberal or Labor in the next election?'

Will Anya's data be reliable and unbiased?

If not, how can she improve her methods?

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CHAPTER 3

DATA ANALYSIS, PROBABILITY AND STATISTICS

THAT'S BIASED

3.1 Statistical investigation
3.2 Types of samples
3.3 Bias in sampling
3.4 Questionnaires
3.5 Bias in questionnaires
3.6 Errors in surveys
3.7 Misleading graphs
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Follow the process of statistical enquiry
- Examine various types of samples and how to choose them
- Design a questionnaire correctly
- Detect bias and errors in questionnaires, samples and statistical data
- Identify misleading graphs

How are we ever going to use this?

- For understanding how data is collected and presented
- When we undertake a survey
- To evaluate the results of surveys
- When we read and interpret statistical information in the media

Study design coverage

AREA OF STUDY 2: DATA ANALYSIS, PROBABILITY AND STATISTICS

- development and specification of data collection requirements and methods, including consideration of audience and purpose of data collection, errors and misrepresentations in statistics.

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Worksheets (3):

3.4 Student survey form • Census questions

3.7 Misleading graphs

Puzzle sheets (2):

Keyword activity Questionnaires word match • Statistical scramble

Weblinks (5):

3.1 Australian Bureau of Statistics • United Nations • World Health Organisation

3.2 Roy Morgan • YouGov

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

3.1 Statistical investigation

Information is collected by a variety of groups to enable people to make informed decisions. This process is called **statistical investigation** and involves the following steps:

- **Posing questions**: identifying the problem and deciding what information is needed and in what form
- **Collecting data**: identifying the target **population**, choosing between using a **census** or a **sample** of the population, deciding how to collect information and then collecting the information
- **Organising data**: organising the collected information for clearer analysis, such as using a frequency table or spreadsheet, with either grouped or ungrouped data
- **Displaying data**: presenting the information in a way that makes it easy to follow and understand, such as with graphs or tables
- **Analysing data**: calculating summary statistics, such as mean, median and mode, then looking for patterns and relationships in the data
- **Writing a report**: communicating the results of the investigation in a clear way, with the conclusions supported by the statistics

EXERCISE 3.1 Statistical investigation

ANSWERS p. 418

1 In your own words, summarise the steps in the process of statistical investigation.

2 Classify each activity as one of these steps:

- posing questions (PQ)
- collecting data (CD)
- organising data (OD)
- displaying data (DD)
- analysing data and drawing conclusions (AD)
- writing a report (WR)

a Liong asks fellow students in Year 12 how they travel to school
b Jane draws a frequency histogram of her data
c Anna finds the mean house price for her suburb
d Managing Director Theo decides he needs to know potential customers' favourite car colour
e Kieran makes recommendations in his final report
f Emilia puts her information on favourite holiday destinations into a frequency table
g Lee concludes that coffee is her friends' favourite daytime drink
h Simon visits the houses in Smith Street to collect their Census forms
i Madeline decides to research the drinking habits of 18-year-old people
j Liam displays data about pocket money received by Year 12 students in a sector graph
k Phoebe writes a report on her data about popular sports
l Kim goes through a pile of surveys and records the responses to a specific question

3 Choose a topic to research and describe how you would implement each of the 6 steps of a statistical inquiry.

4 Different organisations collect information on a large scale. Visit the website of each organisation below and write down at least 5 things they collect information about.
a Australian Bureau of Statistics (ABS): www.abs.gov.au
b United Nations (UN): www.un.org/en/
c World Health Organisation (WHO): www.who.int/en/

5 The privacy of collected information can be important. Visit the **ABS** website and click on the **Census** tab. Scroll down and click on **Privacy**. Describe in your own words how the ABS ensures the privacy of the information we provide in the Census.

Weblink Australian Bureau of Statistics

Weblink United Nations

Weblink World Health Organisation



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3.2 Types of samples

In Year 11, we learned that we could collect information using a **census** or a **sample**. Which we choose depends on why the information is being collected.

Sample	Census
• surveys a selected group of people or items	• surveys all people or items in the population
• gives approximate information about the population	• gives exact information about the population
• simple and inexpensive	• complex and expensive
• can be done quickly	• takes a lot of time to collect and process the information

If we decide to use a sample for a survey, there are 4 types of samples we can use.

In a **random sample**, every member of the population has an equal chance of being included in the sample. For example, when a computer selects a customer of a phone company at random to be surveyed about the company's performance, every customer of that company has an equal chance of being selected.

In a **stratified sample**, different categories in a population are represented according to their proportion of the population and then members of each category are selected randomly. For example, a school's population may be made up of 72% junior students and 28% senior students. A stratified sample of school students must also be made up of 72% junior students and 28% senior students.

In a **systematic sample**, selections are made on a regular basis. For example, testing every 500th battery produced in a factory to check that the machines producing the batteries are working properly.

In a **self-selected sample**, whoever wishes to participate answers the questions asked. For example, when a news website asks people to vote on who should be Australia's next prime minister, anyone can participate.

EXAMPLE 1

The United Football Club has 14 100 members. This table gives a breakdown of members by age group.

Age group	Number of members
Under 30 years	2100
30 to 39 years	2700
40 to 49 years	5100
50 years and over	4200

Andrew is considering the 4 different ways listed below of choosing a sample for his market research. For each method, state the type of sample used.

a Every 100th member on the alphabetical membership list

b Names selected randomly by a computer

c Putting up a sign at the entrance of the club asking members to volunteer

d Selecting 21 members who are under 30 years old, 27 members 30 to 39 years old, 51 members 40 to 49 years old, and 42 members aged 50 years and over.

Steps	Working
a Selections are made on a regular basis, in this case, every 100th member.	Systematic sample
b Every member of the population has an equal chance of being included and the computer chooses randomly.	Random sample
c Any member who volunteers can participate.	Self-selected sample
d A specific number of members are chosen based on the number of members in each age group.	Stratified sample

When we use a **stratified sample**, we need to calculate how many people from each category should be included in the sample.

EXAMPLE 2

Selina decided to use a stratified sample to survey members at her local gym. There are 970 gym members, of which 590 are female and 380 are male. She plans to survey 10% of the members.

a How many members should Selina survey?

b How many female members should she survey?

c How many male members should she survey?



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Steps	Working
a Calculate 10% of the membership.	$10\% \text{ of } 970 = \frac{10}{100} \times 970$ $= 97$ Selina should survey 97 members.
b Female members make up 590 out of 970 members. Find the fraction of the number of members to be surveyed.	$\frac{590}{970} \times 97 = 59$ Selina should survey 59 female members.
c Male members make up 380 out of 970 members. or From the total to be surveyed, subtract the number of females.	$\frac{380}{970} \times 97 = 38$ 97 members – 59 females = 38 males Selina should survey 38 males.

EXERCISE 3.2 Types of samples

ANSWERS p. 418

1 EXAMPLE 1 Which type of sampling is described in each case?

a Selecting every 120th name from a customer database

b The names of all students in Years 11–12 are placed in a hat and 2 are drawn out to represent the school at a council function

c A current affairs TV show asks viewers to respond to a Yes/No question

d Selecting an appropriate number of students from each year level at a local high school

e Subscribers are sent an email asking for their opinion of the play they have just seen

f Names of employees at a bank are drawn out of a barrel

g The audience at a concert finds prize tickets under every 40th seat

h A company sends out an email to customers asking for their opinions

i Employees are sorted from tallest to shortest and every 5th employee completes a questionnaire

j 20 females and 28 males were surveyed out of a group of athletes with 100 females and 140 males

k An airport customs officer searches every 10th passenger

l A medical researcher advertises for people to participate in a health survey

m 5 cards are selected from a pack of cards without looking

n An import business employs 125 women and 250 men, and 17 women and 34 men are surveyed about their work hours

o A computer selects every 1000th name from the electoral (voters') roll

p A bank surveys 5% of its customers in each age group

2 Children from 400 families attend the local primary school. The parent council has raised money for new play equipment for the playground and wants to interview parents about their ideas. The committee would like to survey 40 families. Suggest how they might select the 40 families using a:

a random sample

b stratified sample

c systematic sample

d self-selected sample

3 For the survey above, which method of sampling is the best to use? Explain your answer.

4 Maria wishes to survey the members of the fan club she belongs to. Suggest a method she could use to find a random sample of members to survey.

5 Mr Carrozza, the principal at Nelson Valley College, wants to survey a sample of students' parents. Suggest how he could find a systematic sample of parents.

6 In a self-selected sample, people volunteer to complete a survey. Suppose you need to collect data on the usage of the school canteen. Suggest 3 places in your school where you might make questionnaires available for those who want to fill them in.

7 EXAMPLE 2 Amia is going to use a stratified sample to survey the parents of her local netball club. There are 540 children playing netball. There are 380 primary school students and 160 high school students. Amia is going to survey 15% of parents.

a How many parents should complete the survey?

b How many parents of primary school students should complete the survey?

c How many parents of high school students should complete the survey?

8 Kieran wishes to survey Year 8 students about their opinions of the school. There are 96 boys and 69 girls in Year 8. He aims to survey 20% of students.

a How many students should complete the survey?

b How many boys should complete the survey?

9 Joanna is going to ask a sample of her online friends about their favourite sport. She has 376 single friends and 416 friends in a relationship. She is going to survey 12.5% of her friends.

a How many friends will she survey?

b How many single friends will be in the survey?

c How many friends in a relationship will be in the survey?

10 Global Communications employs 750 people: 479 males and 271 females. The company intends to survey 75 employees about their working conditions.

a How many males should be surveyed?

b How many females should be surveyed?

11 James is researching political views in Victoria. He stands on Collins Street in central Melbourne and surveys every 10th person who walks past.

a Is this a representative sample of the population of Victoria? Why or why not?

b If not, describe how James could achieve a representative sample.

Investigation Sample sizes in polls

In this investigation we will look at the sizes of samples polling organisations use.

Visit the **Roy Morgan** market research company website.

1 Click on the **Findings** tab and select one of the polls in Australia that interests you.

2 Write down what the poll is about and the size of the sample (n).

3 Use the internet to find the current population of Australia.

4 What percentage is the sample of Australia's population?

Remember: $\frac{\text{sample}}{\text{population}} \times 100\%$

Visit the **YouGov Australia** website.

1 Select a recent national or state poll and write down the sample size they used.

2 Use the internet to find the population of Australia or the state for the poll you have chosen.

3 What percentage is the sample of the population?

4 Write 2 to 3 sentences describing what you have found.

5 How well do you think the samples reflect the opinions of the population?



Alamy Stock Photo/Jeff Greenberg

Weblink Roy Morgan

Weblink YouGov

3.3 Bias in sampling

Bias is an unwanted influence that favours a particular section of the population unfairly. Bias produces unreliable results because the sample is not truly representative of the population.

Bias can happen if a sample is not selected properly.

EXAMPLE 3

For his survey, Andrew is considering finding a fair sample of United Football Club members by asking 100 people:

a as they come through the door on a Friday night

b playing Bingo on a Wednesday morning.

In what way is each sample biased? How could you ensure that an accurate sample is found?



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Steps	Working
a The people at the club on a Friday night are more likely to be younger people.	All age groups would not be represented fairly.
b The people at the club on a Wednesday morning are more likely to be retired or not working.	Working members would be under-represented in the survey.
	For a more accurate sample, Andrew could find a list of all club members and choose every, say, 10th member (other answers are possible).

EXERCISE 3.3 Bias in sampling

ANSWERS p. 419

1 EXAMPLE 3 For each sample given below, state how:

- **i** the sample is biased
- **ii** you could ensure that an accurate sample is found.

a Asking people in the crowd at an AFL match for their favourite sport

b Calling every 6th phone number on a page in the White Pages phone database

c Surveying people about their preferred supermarket brand in a shopping centre with only one supermarket

d Asking people in the inner-city for their view on drought support by the Government

e Asking the teachers what music should be played at the Year 12 formal

f Surveying the Maths staff on what books should be bought for the school library

g Finding out what people think of a new design for a football club jersey by asking people who walk past the office door

h Surveying taxi drivers on who they will vote for in the next election

i Emailing a survey to club members to complete

j Asking McDonalds customers to name their favourite coffee shop

2 Ms Benedict's class is completing a statistical investigation project as part of their assessment. The students need to decide who they will ask to answer their questions.

For each investigation, suggest:

- **i** what kind of group should be surveyed to get useful results
- **ii** what kind of group is likely to give less useful or biased results.

a New tourist attractions that could be built in Adelaide

b Favourite holiday destinations of children

c Most popular teacher at Nelson Valley College

d Most popular sport in Victoria

e Changes to the school uniform

f Exercise equipment to be installed in outdoor areas

g Types of stores to include in a new shopping centre

h Entertainment provided in a chain of retirement homes

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

3.4 Questionnaires

Worksheet Student survey form

Worksheet Census questions

Using **questionnaires** is one of the most common ways to collect information.

A good questionnaire:

- uses simple language
- has clear and precise questions
- meets privacy requirements, such as prevention of unauthorised people accessing our information
- is free from bias.

EXERCISE 3.4 Questionnaires

ANSWERS p. 419

Andrew has designed a questionnaire to survey members about the entertainment that the United Football Club provides, including trips away. Read the questionnaire (but do not answer its questions).

Help us help you!

UNITED FOOTBALL CLUB
UFC

Please answer this short questionnaire to help the United Football Club better serve you.

1. How old are you?

- ☐ Under 30 years
- ☐ From 31 to 40 years
- ☐ From 41 to 50 years
- ☐ Over 50 years

2. Are you male or female?

- ☐ Male
- ☐ Female
- ☐ Prefer not to say

3. Which of the Club's entertainments do you attend?

4. How often do you attend the following club activities?

Concerts Films Dance nights Trips away

5. Do you think the entertainment offered is reasonably priced?

6. Should the club offer more types of entertainment and, if so, what should it offer?

7. Are there any other comments you would like to make about the Club's entertainment program?

Thank you for taking the time to complete this questionnaire.
Please hand it in at the bar.

There are some problems with the way this questionnaire has been designed and written.

1 **Closed questions** give a number of options for the answer. How many questions in the questionnaire are closed?

2 **Open-ended questions** provide no options and require the person to write their own answer. How many questions in the questionnaire are open-ended?

3 The first question about age has a problem with the options given. What age(s) has Andrew left out? Rewrite the options to this question to fix this error.

4 When Andrew asks, 'Which of the club's entertainments do you attend?' it is not clear exactly what he means. Club members could be unsure which of the club's activities are classified as entertainment. Rewrite the question to make it clear.

5 When Andrew asks, 'How often do you attend the following club activities?' he will get a range of answers. Analysing the data will be easier if Andrew provides some options. Suggest a series of tick boxes that Andrew could provide for people to record their answers.

6 Would all club members agree on what 'reasonably priced' means? Suggest how Andrew could change question **5** to make it clearly understood.

7 Write another question about the cost of entertainment for this questionnaire.

8 There is a risk that answers won't remain private if the questionnaires are handed in at the bar. Suggest a more secure way to collect the questionnaire forms.

9 Do you think Andrew has missed anything in his questionnaire? (Hint: How would people who don't attend entertainment events express their views?) Write 3 additional questions Andrew could use.

10 Using questionnaires is not the only way to collect data. What other ways can be used to collect information?



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3.5 Bias in questionnaires

The way a question is worded in a questionnaire can introduce bias. The wording of the questions can lead people towards a particular view. This can unfairly influence the way people answer the question.

EXAMPLE 4

Andrew has written some questions to ask club members about its restaurant.

a How often do you eat at our fabulous restaurant?

b Rate your last meal: Excellent Yummy Quite nice

In what way are these questions biased? Rewrite them so they are not biased.

Steps

a Using the word 'Fabulous' encourages the person answering the question to give a positive answer.
It would be better to ask: 'How often do you eat at the restaurant?'

b This question doesn't give any negative options.
The question could be:
'Rate your last meal: Poor Average Good'

EXERCISE 3.5 Bias in questionnaires

ANSWERS p. 419

1 EXAMPLE 4 For each question below,

i state how it is biased

ii rewrite it so that it is not biased.

a Do you prefer to holiday in fascinating Sydney or dusty Alice Springs?

b Do you agree that we should increase the tax on those disgusting cigarettes?

c Rate the last movie you saw: OK Good Engaging

d Do you prefer exciting soccer or boring old football?

e Do you think the club is open long enough or should we be able to rage on into the early morning?

f Rate your Health fund: Useless Not very good Reasonable

g Are there enough events for young people in this quiet town?

h Isn't this just the greatest music video you've ever seen?

i Are you one of those lame people who walk to school?

j Rate the gym you attend: Best ever Very good OK

2 You have decided to survey your fellow students about the sports offered at your school.
Make up 3 examples of biased questions. Then give 3 unbiased versions of these questions.

3.6 Errors in surveys

There are different ways in which error or bias can be part of the data collection process. Errors can result from:

- the questioner not asking the question correctly
- the respondent interpreting the question incorrectly
- the respondent answering the questionnaire incorrectly
- the questioner recording the answers incorrectly.

When collecting data, the aim is to avoid or minimise these errors.

Information that is collected can also be interpreted carelessly to favour a particular point of view.

EXAMPLE 5

Alex stood at the school gate one morning and asked 10 students if they liked the school uniform. Five students said something negative about the uniform. Alex submitted a report to the School Principal. In the report, he stated that half of the students hate the uniform. Explain 3 things that are wrong with this conclusion.

Steps

1. Alex only asked 10 students and it is not a random selection. The sample is unlikely to be representative of all students at the school.
2. The question is vague and it is not clear what students don't like. It may not be the whole uniform that students don't like, but some particular item.
3. Alex exaggerated his conclusion when he used the word 'hate'. We don't know how strongly students felt about the uniform.



AAP Image/Dan Peled

EXERCISE 3.6 Errors in surveys

ANSWERS p. 419

1 Each situation listed below can affect the way a questionnaire is answered. Explain how it could affect the usefulness of the survey results.

- **a** The questionnaire is very long.
- **b** The presentation of the questionnaire is messy and hard to follow.
- **c** The choice of categories in some questions don't cover all the options.
- **d** The time allowed to answer the questionnaire is too short.

2 There are different ways to collect data. For each method listed, describe one possible error to avoid if using that method.

- **a** Sending a questionnaire by post.
- **b** Asking people to complete a diary each day recording their behaviour.
- **c** Calling up households and asking questions over the phone.
- **d** Having an interviewer ask a person the questions and completing the questionnaire.
- **e** Direct observation (for example, recording how many vehicles travel through an intersection at a given time of day).

3 EXAMPLE 5 Rose asks 5 Year 12 students if they like the food served at the school canteen and 3 of them said they don't like it. Jane writes an article for the school newspaper with the headline, '60% of students hate canteen food.' Explain 3 things that are wrong with this headline.

4 The Hightop Hotel emails its customers to complete a survey about its service. It receives 25 replies, each of which makes some positive comments about the company's service. An advertisement is published saying, 'Customers 100% happy with our hotel.' Explain what is wrong with this claim.

5 A newspaper headline reads, 'Government program for disadvantaged students a disaster'. In the article, it is revealed that 35 students out of 1000 students receiving government support have dropped out of school. Explain why this headline is misleading.

6 Use the internet to find some news or magazine articles that are based on statistics. Read them carefully to check if they have presented the information properly. Write a report on 3 of the articles you find, highlighting their use of statistics, either correctly or incorrectly.

Investigation Statistics in the media

We live in a world of 24-hour news, through TV, news websites, Facebook, Twitter and blogs. To detect bias, we need to consider where the news comes from and what samples were used. Is the information supplied by a journalist, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, a large sample, an unrepresentative sample, or an online/SMS poll? Each may have a particular bias that may influence how the story is reported.

Find 3 examples of news items or surveys reported in a newspaper or online and investigate the following questions:

1. Where did it come from?
2. What could influence how the story was reported?
3. Who wrote the story?
4. Does it show any bias?
5. Can it be supported by other news providers?
6. What type of sampling was used? Was it representative?
7. How many people were questioned or surveyed?
8. When was the survey conducted?



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3.7 Misleading graphs

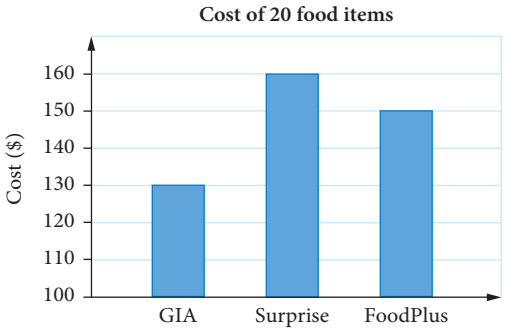
Sometimes, graphs are deliberately drawn to give a false impression, like the one below. This can be done by:

- not having a scale on the vertical axis
- showing only part of the scale or an irregular scale
- not showing 0 on the vertical scale or only showing a small part of the vertical axis
- using pictures or 3D figures on the graph to exaggerate the differences.

Worksheet
Misleading graphs

The graph shows the cost of 20 basic food items at 3 different supermarkets. If you view the graph quickly, it appears that the GIA supermarket is much cheaper than the other 2 supermarkets, but a closer look will show that there really isn't a big difference in cost between the 3 supermarkets. While the information on the graph is correct, it gives a **misleading** impression.

Only showing a small part of the vertical axis on a graph is the most common way to create a misleading impression. As you work through the exercise questions, you will learn about other ways to create misleading impressions.



EXERCISE 3.7 Misleading graphs

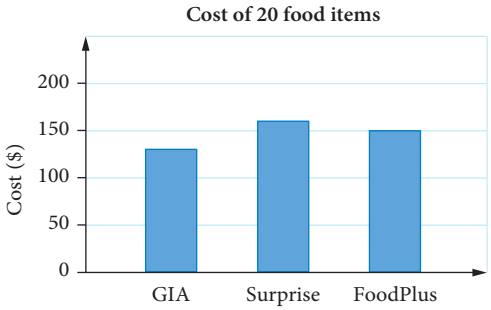
ANSWERS p. 419

1 Use the misleading graph on the previous page to answer the following questions.

a How much do the 20 food items cost at each supermarket?

b How much cheaper is this cost at GIA than Surprise?

c This graph represents the same information more accurately.

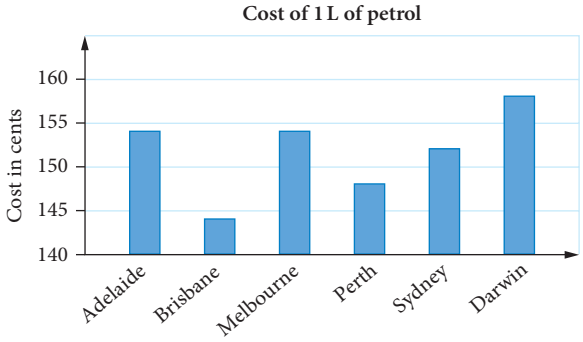


In which of the 2 graphs does the cost of the items at GIA look much cheaper than the cost at the other 2 supermarkets?

d Write a sentence to describe how the scales on the vertical axes are different on the 2 graphs.

e Which company out of GIA and Surprise do you think produced the graph shown in part **c**? Give a reason for your answer.

2 Candice drew a column graph to show the average cost of a litre of petrol in 6 capital cities.



a In which city is the price of petrol the cheapest?

b Which city has the highest petrol price?

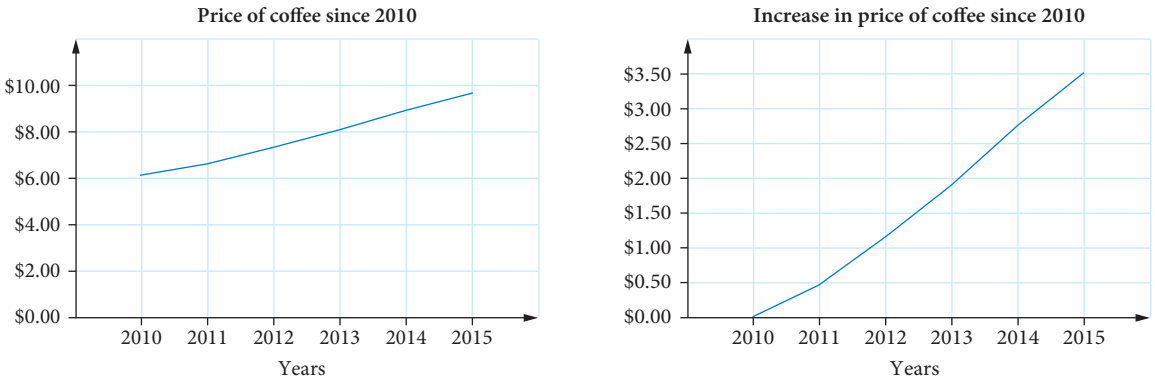
c Use the graph to estimate the price of petrol in Brisbane.

d Approximately, how many cents cheaper is the price of petrol in Brisbane than in Darwin?

e Candice included this graph in an advertisement for motoring holidays in Brisbane. She claimed the graph shows that petrol prices are much lower in Brisbane than they are in other capital cities. Explain what Candice did to the graph to make the price of petrol look a lot cheaper in Brisbane.

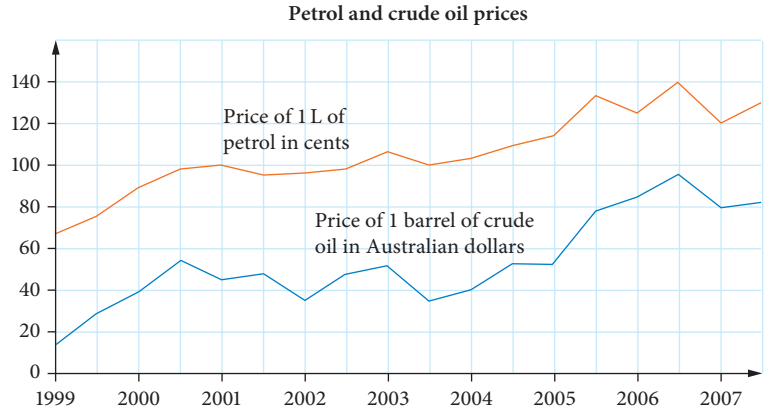
3 This table shows the price of a brand of coffee from 2010 to 2015, with the data graphed in 2 different ways below.

Year	2010	2011	2012	2013	2014	2015
Price of coffee	$6.15	$6.60	$7.30	$8.05	$8.90	$9.65
Increase since 2010	$0	$0.45	$1.15	$1.90	$2.75	$3.50



The 2 graphs illustrate how the vertical scale can make increases look small or big. Imagine that you want to draw a graph to show how the population in your area has increased. Describe how you could make the increase look small and how you could make it look big.

4 Daniel thinks that oil companies have been ripping people off. He decided to investigate the historical prices of petrol and the cost of crude oil.



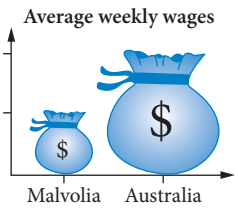
a What was the price of 1 L of petrol in 2007?

b How much did a barrel of crude oil cost in 2007?

c Profit = selling price – cost price. The graph looks like it shows the oil cost at 56 c and the petrol cost at 104 c in 2003, giving the impression that the profit is 104 – 56 = 48 c. Explain why this impression is completely false.

d What relationship does the graph show between the price of petrol and the price of crude oil?

5 This graph compares the average weekly wages in Malvolia ($900) and Australia ($1800).



a What impression does this graph give?

b Explain 2 things that are misleading about this graph.

c What could be used to represent the weekly wages on the graph instead of pictures?

d Redraw this graph so that the average weekly wages are represented correctly.

Puzzle
Questionnaires word match

Puzzle
Statistical scramble

Keyword activity

WORD MATCH

ANSWERS p. 419

Match each word with its definition.

Words	Definitions
1 bias	**A** Your right not to share personal information
2 census	**B** A representative group selected for a survey
3 population	**C** A sample for which every member of the population has an equal chance of being selected
4 privacy	**D** A sample in which the different categories in a population are represented according to the numbers of people or items in each category
5 questionnaire	**E** An unwanted influence in data collection that makes results unreliable
6 random	**F** The process of collecting and analysing data to help people to make informed decisions
7 sample	**G** A list of survey questions
8 self-selected	**H** A sample for which selections are made on a regular basis, such as every 100th item
9 stratified	**I** The group of people from which a sample is chosen
10 statistical investigation	**J** A sample in which whoever wishes to participate answers the questions asked
11 systematic	**K** A survey of the entire population

Solution to the chapter problem

Problem

Anya is a market researcher. She has been asked to research political views in her community.

One Wednesday morning, Anya stands at an intersection on the main street of town and asks people passing, 'Are you going to vote for Liberal or Labor in the next election?'

Will Anya's data be reliable and unbiased?

If not, how can she improve her methods?

Solution

There are 2 problems with Anya's collection of data.

The sample:
On a Wednesday morning she will miss people who are working or shopping elsewhere. She will mainly be asking older people who have retired and unemployed people. Her sample is NOT representative of her community.

The question:
There are alternatives to voting for Liberal or Labor – other political parties or independent candidates. Anya's question does not cover all the options. Also, she has not specified *which* election – local council, state or federal governments.

Improvements:
Anya could choose her sample from the electoral roll of the community.

The question could be asked with a tick box option for answering or it could be an open-ended question. It should specify what level of government the question is referring to. For example, 'Which party or person will you vote for in the next federal election?'

3 Test yourself

That's biased

ANSWERS p. 420

1 EXERCISE 3.1 List the steps of a statistical investigation in your own words.

2 EXERCISE 3.2 Should a census or a sample be used to find the most popular TV program in Australia? Give a reason for your answer.

3 EXERCISE 3.2 VICmaths Engineering employs 430 people, with 295 male employees and 135 female employees. The CEO wishes to survey the employees about working conditions at the company. He would like to survey 86 employees. Suggest how the firm contracted to do the research might select the 86 employees using a:

a random sample
b stratified sample
c systematic sample
d self-selected sample

4 EXERCISE 3.2 Lindsay is going to use a stratified sample to survey the parents of the 375 students in her dance school. The dance students are made up of 295 girls and 80 boys. Lindsay is going to survey 20% of parents.

a How many parents should be selected to complete the survey?
b How many parents of girls should be selected?
c How many parents of boys should be selected?

5 EXERCISE 3.3 Jeremy is collecting data about the quality of coffee in his town. He is asking people, 'Which coffee shop makes the best coffee?' For this survey:

a give an example of a sample that would be biased
b state how you could ensure that an accurate sample is found.

6 EXERCISE 3.4 Roxanne wrote the following question as part of a questionnaire she was giving to Year 12 students about their consumption of alcohol: 'How often do you drink?'

a What problem might people have answering this question?
b Rewrite the question by providing some tick box options.

7 EXERCISE 3.5 Tranh is surveying students about the quality of the food sold at the school canteen.

a Write an example of a biased question he could use if he wanted to show how great the food is.
b Rewrite the question you wrote in part **a** so that it is NOT biased.

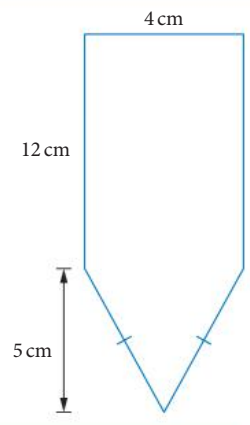
8 EXERCISE 3.6 Kendall interviewed her Year 11 class about changing the time the school day starts. 11 of the 20 students said they wouldn't mind if school started a bit earlier. Kendall presents a report to the school principal with the heading, 'Students support starting school half an hour earlier.'
Explain 3 things that are wrong with this heading.

9 EXERCISE 3.7 Redraw the misleading graph in Exercise 3.7 question **2** so that it is not misleading.

Chapter problem

Lucy is making bookmarks in the shape shown on the right. She is going to edge them with ribbon.

How much ribbon will she need for each bookmark? Give your answer correct to 2 significant figures.



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CHAPTER 4

ALGEBRA, NUMBER AND STRUCTURE; SPACE AND MEASUREMENT

MEASUREMENT AND ACCURACY

4.1 Error in measurement
4.2 Percentage error
4.3 Significant figures
4.4 Circumference and area of a circle
4.5 Pythagoras' theorem
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Determine precision, absolute error and limits of accuracy
- Calculate the percentage error involved in a measurement
- Understand significant figures
- Use significant figures with circles and Pythagoras' theorem

How are we ever going to use this?

- To develop measurement skills, an essential component of many trades; for example, shopfitting, carpentry, upholstery, building, interior decorating and dressmaking
- To take measurements and make calculations when building or renovating a home; for example, pool fencing, garden edging, making curtains and replacing gutters

Study design coverage

AREA OF STUDY 1: ALGEBRA, NUMBER AND STRUCTURE

- rational numbers and irrational numbers related to measurement
- estimation and approximation including interval estimates, rounding, significant figures, leading-digit approximations, floor and ceiling values and percentage error.

AREA OF STUDY 4: SPACE AND MEASUREMENT

- conventions, properties and measurement of perimeter
- calibration and error in measurement, including tolerance, accuracy and precision.

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Videos (4):

4.3 Significant figures
4.4 Circumference of a circle • Area of a circle
4.5 Pythagoras' theorem 1

Skillsheets (1):

4.5 Pythagoras' theorem

Worksheets (7):

4.1 Accuracy in measurement
4.3 Significant figures
4.5 Pythagoras' leopard • Pythagoras' problems • Pythagoras' theorem time trial • Pythagorean two-step problems • Applications of Pythagoras' theorem

Puzzles (1):

4.5 Pythagoras' puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

4.1 Error in measurement

Worksheet
Accuracy in measurement

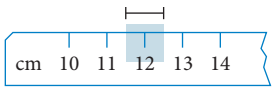
We make many types of errors when measuring.

- **Parallax error** results from looking at the measuring scale from an angle instead of from directly in front.
- **Zero error** happens if the measuring device is not properly set to 0 beforehand or if the 0 mark on a ruler is not placed exactly at the start of the object being measured.
- **Reaction error** occurs when measuring time with a stopwatch, for example, when we take a short time to react at the start of the measurement.

We can avoid most of these errors with a bit of care. Another way we can reduce their effect is to take a number of measurements and average them.

Even if we reduce these **human errors**, our measurements are only as good as the precision of the measuring device we are using.

The ruler shown here has a precision of 1 cm (the size of one unit), so a measurement anywhere in the shaded zone will be recorded as 12 cm but can involve an error of up to $\frac{1}{2}$ cm either side of the 12-cm mark.



The type of error due to the precision of the measuring device is called the absolute error.

Absolute error and limits of accuracy

The **precision** of a measuring device is the smallest unit on its scale.

The **absolute error** is half of the precision of the measuring device being used. We record the absolute error as half of one unit.

The **limits of accuracy** of a measurement are the lower and upper limits of the range in which the actual measurement lies:

lower limit = measurement − absolute error

upper limit = measurement + absolute error

EXAMPLE 1

While repairing her boundary fence, Shannon measured the distance between the strands of wire to be 24 cm using the ruler shown.



a What is the precision shown on the ruler's scale?

b What is the absolute error of Shannon's measurement?

c What are the limits of accuracy of Shannon's measurement?

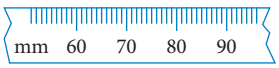
Steps	Working
a There are 5 units between 0 and 10 cm on the ruler, so each unit is 10 ÷ 5 = 2 cm.	precision = 2 cm
b The absolute error is half of the precision.	absolute error = 0.5 × 2 cm = 1 cm
c The shortest length the measurement could be is 24 − 1 = 23 cm (the lower limit). The longest length the measurement could be is 24 + 1 = 25 cm (the upper limit). The actual distance is between 23 cm and 25 cm.	The limits of accuracy are 23 cm and 25 cm.

EXERCISE 4.1 Error in measurement

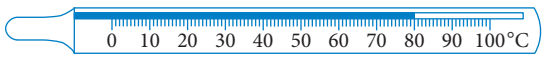
ANSWERS p. 420

1 For each measuring instrument below, state the precision of the instrument.

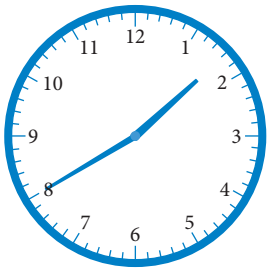
a



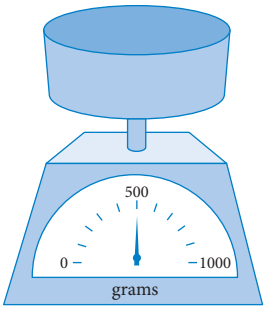
b



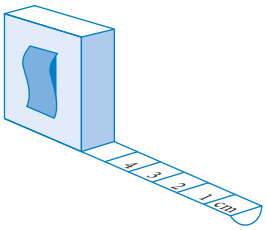
c



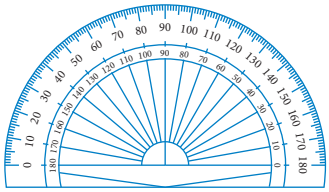
d



e



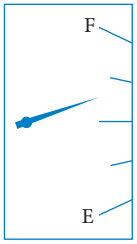
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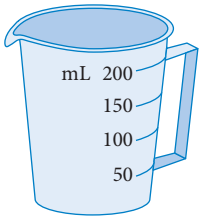
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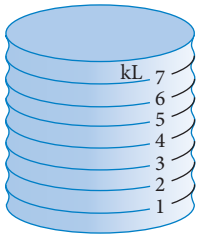
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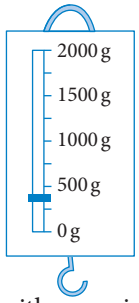
i



j



k



2 Which of the following lengths could be measured using a measuring tape with a precision of 1 cm? Select the correct answer **A**, **B**, **C** or **D**.

A 8 mm **B** 16.45 cm **C** 7 cm **D** 125 mm

3 Which of the following angle sizes is incorrectly recorded if using a protractor marked in degrees? Select **A**, **B**, **C** or **D**.

A 32° **B** 88.4° **C** 145° **D** 170°

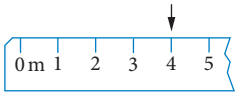
4 EXAMPLE 1 For each measuring instrument shown, state:

i its precision

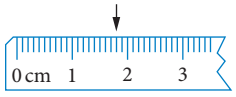
ii the measurement reading to the nearest unit

iii the absolute error of the measurement.

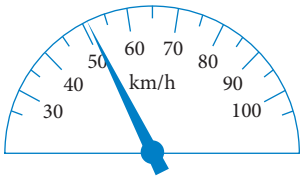
a



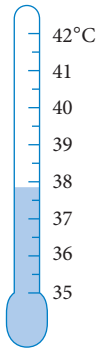
b



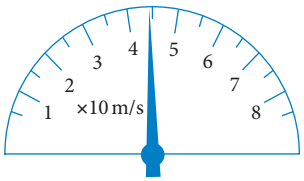
c



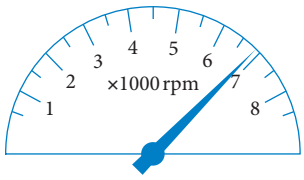
d



e



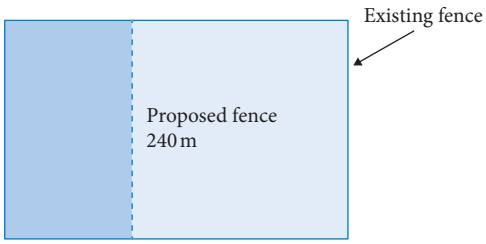
f



5 Copy the table below and fill in the missing values.

	Measurement	Absolute error	Lower limit of accuracy	Upper limit of accuracy
a	18 L correct to the nearest L			
b	78 cm correct to the nearest cm			
c	35 kg correct to the nearest 5 kg			
d	9.4 s correct to 1 decimal place			
e	730 km correct to the nearest 10 km			

6 Santo has decided to divide one of his existing paddocks into two by erecting a new fence. He has measured the width of the field to be 240 metres (to the nearest 10 metres), as shown.



- **a** What is the absolute error of the measured width?
- **b** What are the limits of accuracy of the width?
- **c** What length of fencing wire should Santo buy to ensure he has enough to do the job?

7 We can obtain a more reliable measurement by taking repeated measurements and averaging them. Find the average value of each set of repeated measurements.

We find the average by adding all the measurements and dividing this result by the number of measurements.

- **a** Length of a shed: 12.3 m, 11.9 m, 12.2 m, 12.3 m, 12.1 m
- **b** Mass of a carton of fruit: 15.9 kg, 15.7 kg, 16.1 kg, 15.5 kg
- **c** Capacity of a water trough: 75 L, 77 L, 75 L, 72 L, 79 L, 77 L, 76 L

8 Jolene measured the width of her bedroom 4 times and got the following results: 2964 mm, 3021 mm, 2938 mm and 2899 mm. To minimise any error, she calculated the average of these measurements correct to the nearest millimetre. What was her answer?

4.2 Percentage error

We use percentage error to compare the size of the error in different measurements.

Percentage error

The **percentage error** of a measurement is the absolute error as a percentage of the measurement.

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

EXAMPLE 2

Angelina measured her mass on a set of bathroom scales to be 58 kg.

- **a** What are the limits of accuracy of this measurement?
- **b** What is the percentage error, correct to 2 decimal places?

Steps	Working
a Because the measurement is given to the nearest kilogram, we assume that the precision of the scales is 1 kg. Use this to find the absolute error and the limits of accuracy.	absolute error $= 0.5 \times 1$ kg $= 0.5$ kg lower limit $= 58 - 0.5$ $= 57.5$ kg upper limit $= 58 + 0.5$ $= 58.5$ kg The limits of accuracy are 57.5 kg and 58.5 kg.
b The percentage error is the absolute error as a percentage of the measurement.	$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$ $= \frac{0.5}{58} \times 100\%$ $= 0.8620\ldots\%$ $\approx 0.86\%$

EXERCISE 4.2 Percentage error

ANSWERS p. 420

1 EXAMPLE 2 The length of an A4 sheet of paper is 30 cm, correct to the nearest centimetre.

- **a** What is the absolute error of this measurement?
- **b** What are the limits of accuracy of this measurement?
- **c** What is the percentage error, correct to 2 decimal places, of this measurement?

2 The time it takes Saxon to drive his tractor from one end of his property to the other was measured as 7 minutes, correct to the nearest minute.

- **a** What is the absolute error for this measurement?
- **b** What is this error expressed in seconds?
- **c** What is the percentage error, correct to 2 decimal places?



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3 In the table below, Jayden has made a list of some of the materials he has in his farm shed. Copy and complete the table. Give your answer correct to 2 decimal places where appropriate.

	Item	Quantity	Recorded to the nearest	Absolute error	Limits of accuracy	Percentage error
a	Fuel	280 L	5 L			
b	Fence posts	350	50			
c	Fencing wire	540 m	20 m			
d	Fertiliser	870 kg	10 kg			

4 A truck weighs 3.4 tonne to the nearest 0.1 tonne What is the percentage error of this measurement, correct to 2 decimal places?

5 The Earth has a diameter of 12 742 km to the nearest km. What is the percentage error of this measurement, correct to 4 decimal places?

6 During the course of an experiment, Andrew took the following measurements: 15.2 mg, 800 g (to the nearest 10 g) and 0.15 g.

- **a** Find the percentage error of each measurement, correct to 2 decimal places.
- **b** Which of the measurements was the most accurate?

The measurement with the smallest percentage error is the most accurate.

7 The length of one city block is measured to be 509 m. The length of a basketball court is measured to be 26 m.

- **a** Find the percentage error of each measurement, correct to 3 decimal places.
- **b** Which of the 2 measurements is more accurate?

Practical activity Taking accurate measurements

This group activity involves measuring some common items around the school and will require the following instruments:

- 30 cm ruler
- tape measure
- trundle wheel
- stopwatch
- kitchen scales that measure to the nearest gram
- a 1 L measuring cylinder.

Each group will measure:

a the length of the classroom

b the width of a classroom desk

c the thickness of a $2 coin

d the length of the longest building in the school

e the mass of 4 pencils

f the mass of 5 textbooks

g the volume of water in a glass tumbler

h the time it takes for a ball to hit the ground when dropped from the second floor.

Before dividing into up to 5 groups to do the measurements, complete steps 1 and 2 below.

1 Copy the table below.

				Results of measurement					
	Instrument	Units	Tolerance	Group 1	Group 2	Group 3	Group 4	Group 5	Average
a									
b									
c									
d									
e									
f									
g									
h									

2 As a class, agree on:

- the instrument that should be used for each measurement
- the units (for example, cm, mL) that should be used for the measurement
- the absolute error for the measurement.

These should be entered into your table.

3 In your groups, make the measurements. Take care to avoid parallax and zero errors. Write your group's result for each measurement in your table.

4 When all measuring is complete, obtain the results of the other groups and enter them in your table as well.

5 Using the data of all groups, calculate the average for each measurement and enter this in the last column.

6 Discuss the results as a class and answer the following questions.

- **a** What problems did you experience in making the measurements?
- **b** Did you feel averaging all the groups' results gave a more reliable result?
- **c** How sure were you that your results were as accurate as they could have been?
- **d** What errors other than absolute error may have occurred in any of the measurements?



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These digital calipers measure lengths to the nearest 0.01 mm. The display says 31.79 mm.

4.3 Significant figures

Video Significant figures

Worksheet Significant figures

When we're talking about large quantities, we often use an approximate amount. For example, the media could report that a bushfire burned out 3 000 000 hectares, or that 20 000 people were stranded because of floods. Just as numbers can be rounded to decimal places, they can also be rounded to significant figures. When the media reports in this way, it is reporting the quantities using **significant figures**.

With whole numbers, we start counting significant figures from the first digit. With small decimals, we start counting significant figures from the first digit that is not 0. For example, when we round 28 219 to 28 000, we are using the 2 most important numbers to approximate 28 219. In this case, we are using 2 significant figures.

EXAMPLE 3

Round each number correct to 2 significant figures.

a 28 315 008 **b** 32 724 **c** 0.006 952

Steps	Working
a The first 2 digits are 2 and 8. Decide whether 28 315 008 is closer to 28 000 000 or 29 000 000. The third digit is 3, which is less than 5, so round down.	28 315 008 ≈ 28 000 000
b The first 2 digits are 32. Decide whether 32 724 is closer to 32 000 or 33 000. The third digit is 7, which is bigger than 5, so round up.	32 724 ≈ 33 000
c With 0.006 952, the first 2 significant figures are 69 because the 0s are not significant. Decide whether 0.006 952 is closer to 0.0069 or 0.0070. The third digit is 5, so we round up.	0.006 952 ≈ 0.0070

We must write the 0 after the 7 to show we have rounded to 2 significant figures – we can't leave it out!

Significant figures

- When we round an amount to 2 decimal places, we have 2 digits following the decimal point.
- When we round to 2 significant figures, we only put the 2 most important digits in the amount.

EXERCISE 4.3 Significant figures

ANSWERS p. 420

1 EXAMPLE 3 The city of Nelson Springs has a population of 372 413. Write this population correct to 2 significant figures.

2 Ewan's superannuation account contains $105 496. Round this amount to:

a 2 significant figures **b** 1 significant figure **c** 3 significant figures

3 Express $482 356 correct to:

a 1 significant figure **b** 2 significant figures **c** 3 significant figures

4 Express each decimal correct to 2 significant figures.

a 0.004 623 **b** 0.025 73 **c** 0.019 623 **d** 0.000 466

5 **a** In 2016, the population of Melbourne was 4 485 211. Express this amount correct to 3 significant figures.

b In 2021, Melbourne's population grew to 5 060 000 (to 3 significant figures).
Write a possible population that can be rounded to 5 060 000.

c Melbourne's population is predicted to reach 7 739 000 by 2051. By how much is Melbourne's population predicted to increase between 2016 and 2051? Express your answer correct to 3 significant figures.

6 Every 3 months, each home owner in Nelson Springs pays an average of $723 in council rates. There are 673 522 home owners in the area.

a Calculate the total amount the council receives in rates every 3 months. Express your answer correct to 3 significant figures.

b How much does the council receive in rates each year? Express your answer correct to 2 significant figures.

7 In 2021, the population of Victoria was 6 648 624. Round this number to 3 significant figures.

8 Write each number correct to 1 significant figure.

a 2914.23 **b** 3.2548 **c** 11 950

d 0.005 134 **e** 20.46 **f** 0.6517

9 Evaluate each expression correct to 2 significant figures.

a $0.2 \div 0.3$ **b** $11 \div 1990$ **c** $16 \div 12$

d $\sqrt{0.0075}$ **e** $9\,300\,000 \times 0.085$ **f** 2.7^2

g $\sqrt{560}$ **h** $\sqrt{5.6}$ **i** 3.4×9.9

10 Express each measurement correct to 3 significant figures.

a Mass of an aeroplane, 351 540 kg

b Average depth of the Pacific Ocean, 4188 m

c Speed of a greyhound, 67.14 km/h

d Capacity of a tablespoon, 14.79 mL

e Distance from Earth to the Sun, 149 573 881 km



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4.4 Circumference and area of a circle

When we calculate the **circumference** (perimeter) or **area** of a circle, we use the number π.

$\pi = 3.141\,592\,653\,589\,793\ldots$

We can estimate our answers by using $\pi \approx 3$ or check our answer will be in a certain interval by using $\pi \approx 3$ and $\pi \approx 4$.

Because π is not a terminating decimal, we always have to round our answers. We round our answers to a given number of significant figures depending on the context of the question.

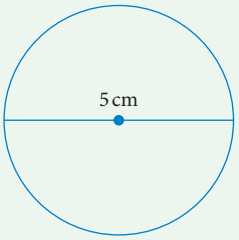
Circumference and area of a circle

Circumference: $C = \pi d$ or $C = 2\pi r$

Area: $A = \pi r^2$

EXAMPLE 4

This circle has a diameter of 5 cm.



a Estimate its circumference using $\pi \approx 3$.

b Find the 2 values its exact circumference will be between, using $\pi \approx 3$ and $\pi \approx 4$.

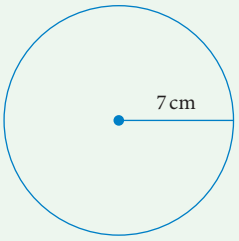
c Find the circumference of this circle in terms of π.

d Find the circumference of this circle, correct to 3 significant figures.

Steps	Working
a Use the circumference formula with the diameter and let $\pi = 3$.	$C = \pi d$ $\approx 3 \times 5$ $= 15$ cm
b Substitute $\pi = 3$ and $\pi = 4$.	$C = \pi d$ $\quad$ $C = \pi d$ $\approx 3 \times 5$ $\quad$ $\approx 4 \times 5$ $= 15$ cm $\quad$ $= 20$ cm The exact circumference is between 15 and 20 cm.
c Don't substitute any number for π, and leave π in the answer.	$C = \pi d$ $= \pi \times 5$ $= 5\pi$ cm
d Use the [π] key on the calculator when substituting for π. Write the answer to the correct number of significant figures.	$C = \pi d$ $= \pi \times 5$ $= 15.707\,963\ldots$ ≈ 15.7 cm

EXAMPLE 5

This circle has a **radius** of 7 cm.



a Estimate its area using $\pi \approx 3$.

b Find the 2 values its exact area will be between, using $\pi \approx 3$ and $\pi \approx 4$.

c Find the area of this circle in terms of π.

d Find the area of this circle, correct to 3 significant figures.

Steps	Working
a Use the area formula and let $\pi = 3$.	$A = \pi r^2$ $\approx 3 \times 7^2$ $= 147$ cm^2
b Substitute $\pi = 3$ and $\pi = 4$.	$A = \pi r^2$ $\quad$ $A = \pi r^2$ $\approx 3 \times 7^2$ $\quad$ $\approx 4 \times 7^2$ $= 147$ cm^2 $\quad$ $= 196$ cm^2 The exact area is between 147 and 196 cm^2.
c Don't substitute any number for π, and leave π in the answer.	$A = \pi r^2$ $= \pi \times 7^2$ $= 49\pi$ cm^2
d Use the [π] key on the calculator when substituting for π. Write the answer to the correct number of significant figures.	$A = \pi r^2$ $= \pi \times 7^2$ $= 153.938\,04\ldots$ ≈ 154 cm^2

Video
Circumference of a circle

Video
Area of a circle

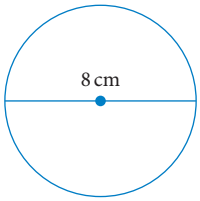
EXERCISE 4.4 Circumference and area of a circle

ANSWERS p. 421

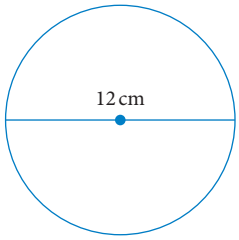
1 EXAMPLE 4 For each circle:

- **i** estimate its circumference, using $\pi \approx 3$
- **ii** find the 2 values the exact circumference will be between, using $\pi \approx 3$ and $\pi \approx 4$
- **iii** find its circumference in terms of π
- **iv** find its circumference, correct to 3 significant figures.

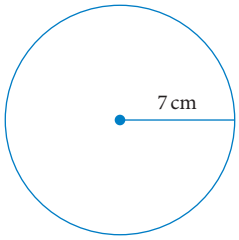
a



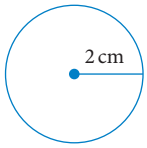
b



c



d

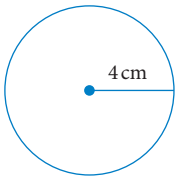


2 EXAMPLE 5 For each circle:

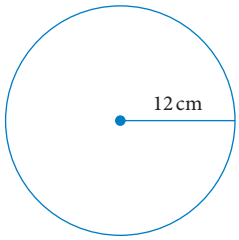
- **i** estimate its area using $\pi \approx 3$
- **ii** find the 2 values the exact area will be between, using $\pi \approx 3$ and $\pi \approx 4$
- **iii** find its area in terms of π
- **iv** find its area, correct to 3 significant figures.

Remember to find the radius of the circle before doing area calculations.

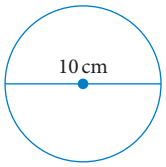
a



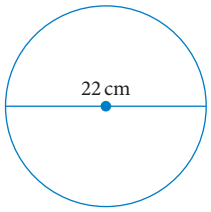
b



c



d



3 A child's inflatable swimming pool has a diameter of 1.4 m. Find its circumference:

a in terms of π **b** correct to 2 significant figures.

4 A sprinkler on the school playing fields sprays water in a circle of radius 13.1 m. Find the area being watered:

a in terms of π **b** correct to 2 significant figures.

5 The radius of the equator of Earth is 6378 km. Calculate the length of the equator, correct to 2 significant figures.

6 Kevlar® is a very strong, light plastic used to make solid bicycle wheels. Find the area of Kevlar® needed for a wheel with a diameter of 686 mm. Give your answer correct to 3 significant figures.

7 Tina's bicycle has wheels with diameter of 60 cm.

a How far does the bicycle move when the wheel turns around once? Give your answer correct to 2 significant figures.

b Tina cycles 900 m to school. Approximately how many complete turns does the bicycle wheel make?

8 The radius of a 20-cent coin is 16 mm. Calculate, correct to 4 significant figures:

a its circumference **b** the area of its circular face.

4.5 Pythagoras' theorem

Pythagoras' theorem

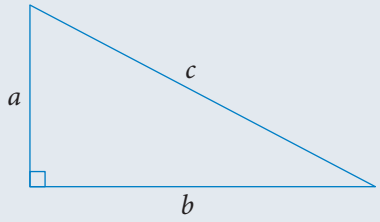
In a right-angled triangle, the square of the **hypotenuse** is equal to the sum of the squares of the other 2 sides.

$(\text{hypotenuse})^2 = (\text{side 1})^2 + (\text{side 2})^2$

If c is the length of the hypotenuse, and a and b are the lengths of the other 2 sides, then

$c^2 = a^2 + b^2$

The **hypotenuse** is the longest side of a right-angled triangle.



When using **Pythagoras' theorem** to calculate the length of a side in a right-angled triangle, we usually have an answer that is a **surd**, in **square root** form, for example, $\sqrt{27}$.

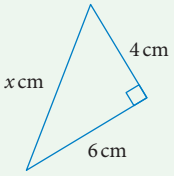
Because surds are NOT terminating decimals, we have to round our answers to a given number of significant figures depending on the context of the question.

EXAMPLE 6

For this triangle, find the value of x:

a as a surd

b correct to 2 significant figures.



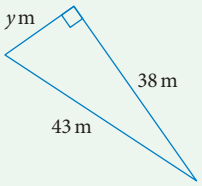
Steps	Working
a Use the formula for Pythagoras' theorem.	$x^2 = 4^2 + 6^2$ $= 52$ $x = \sqrt{52}$
b Use the $\sqrt{\ }$ key on the calculator. Write the answer to the required number of significant figures.	$x = \sqrt{52}$ $= 7.2111\ldots$ ≈ 7.2

EXAMPLE 7

Find the value of y in this triangle:

a as a surd

b correct to 3 significant figures.



Steps	Working
a Use the formula for Pythagoras' theorem.	$43^2 = 38^2 + y^2$ $y^2 = 43^2 - 38^2$ $= 405$ $y = \sqrt{405}$
b Use the $\sqrt{\ }$ key on the calculator. Write the answer to the required number of significant figures.	$y = \sqrt{405}$ $= 20.1246\ldots$ ≈ 20.1

Skillsheet Pythagoras' theorem
Worksheet Pythagoras' leopard
Worksheet Pythagoras' problems
Worksheet Pythagoras' theorem time trial
Worksheet Pythagorean two-step problems
Worksheet Applications of Pythagoras' theorem
Puzzle Pythagoras' puzzle
Video Pythagoras' theorem 1

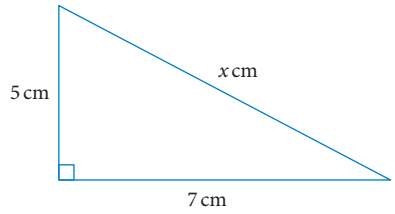
EXERCISE 4.5 Pythagoras' theorem

ANSWERS p. 421

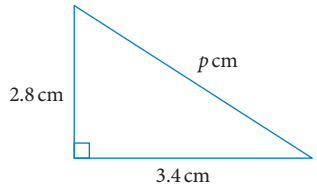
1 EXAMPLE 6 For each triangle, find the value of the **variable** (length of the unknown side):

i as a surd **ii** correct to 2 significant figures.

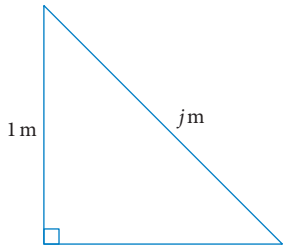
a



b



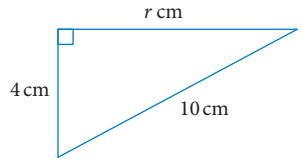
c



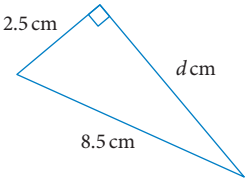
2 EXAMPLE 7 For each triangle, find the value of the variable (length of the unknown side):

i as a surd **ii** correct to 2 significant figures.

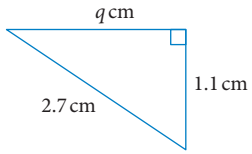
a



b



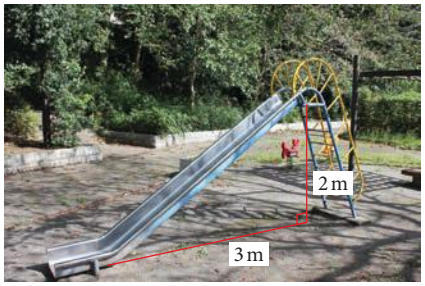
c



3 Find the length of this playground slide:

a as a surd

b correct to 2 significant figures.



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4 Find the height of a TV screen with diagonal 140 cm and length 122 cm, correct to 3 significant figures.



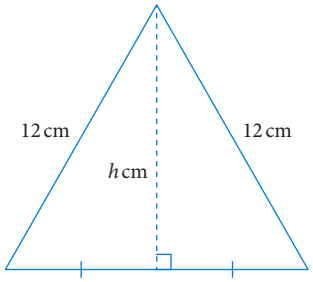
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5 A firefighter places a ladder against a house. The top of the ladder is 3.2 m above the ground. The foot of the ladder is 1.6 m from the wall. How long is the ladder, correct to 3 significant figures?

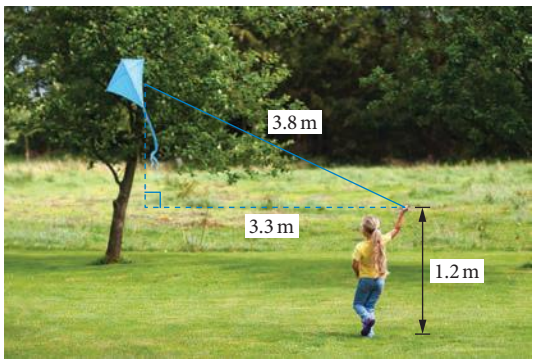


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6 An equilateral triangle has sides of 12 cm. Find its perpendicular height, h cm, correct to 4 significant figures.

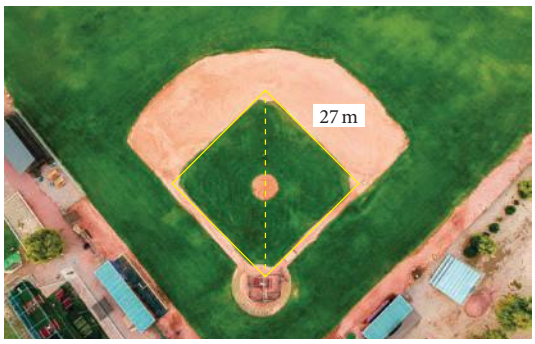


7 Olympia holds a kite string 1.2 m above the ground, and the kite flies at an angle as shown in the diagram. How high is the kite above the ground, rounded to 2 significant figures?



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8 A baseball diamond is a square shape of side length 27 m, as shown in the diagram. What is the distance from the home plate to second base, shown by the dotted yellow line, rounded to 2 significant figures?



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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Keyword activity

DESCRIBING AND DEFINING

ANSWERS p. 421

1 Describe the different ways in which a measurement can be inaccurate.

2 Write the meaning of each term in your own words.

precision absolute error limits of accuracy percentage error

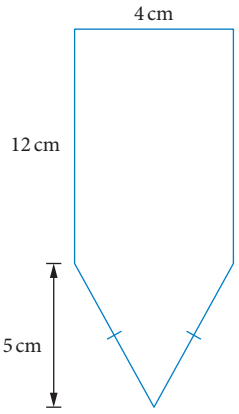
3 Explain how to round a number to 2 significant figures.

Solution to the chapter problem

Problem

Lucy is making bookmarks in the shape shown on the right. She is going to edge them with ribbon.

How much ribbon will she need for each bookmark? Give your answer correct to 2 significant figures.

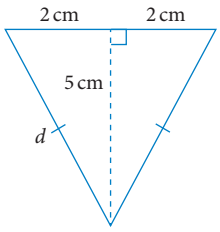


Steps

To calculate the perimeter of the bookmark, we will need to find the lengths of the 2 unknown sides (d) of the triangle.

Since the triangle is isosceles, we can form 2 right-angled triangles.

Working



Using Pythagoras' theorem:

$d^2 = 2^2 + 5^2$

$= 29$

$d = \sqrt{29}$

$= 5.385\,164\ldots$ cm (do not round yet)

To find the perimeter, add together all 5 sides of the shape.

$P = 4 + 12 + 12 + 2 \times 5.385\,164\ldots$

$= 38.770\,32\ldots$ cm

Round your answer to 2 significant figures.

$P \approx 39$ cm

Lucy needs 39 cm of ribbon for each bookmark she makes.

4 Test yourself

Measurement and accuracy

ANSWERS p. 421

1 EXERCISE 4.1 Aidan is making bread and uses electronic scales to measure 180 g of flour. These scales measure correct to the nearest gram. What is:

a the precision of Aidan's scales?

b the absolute error involved in this measurement?

2 EXERCISE 4.1 Bella's food processor measures ingredients correct to the nearest 5 g. The processor showed that Bella added 65 g of sugar to the mixture. What are the limits of accuracy of this measurement?

3 EXERCISE 4.2 What is the percentage error for Bella's measurement of 65 g of sugar in question **2**, correct to 2 decimal places?

4 EXERCISE 4.2 The tallest building in the world is the Burj Khalifa in Dubai. Its height is given as 830 m, correct to the nearest 10 m. Calculate the percentage error of this measurement, correct to 2 decimal places.

5 EXERCISE 4.3 Write each number correct to 2 significant figures.

a 741 600 **b** 8 370 000 000 **c** 4030

d 0.000 722 **e** 0.000 003 495 **f** 0.000 206

6 EXERCISE 4.3 In 2021, the population of Geelong was 264 929. Round this to 3 significant figures.

7 EXERCISE 4.4 The radius of Earth is 6400 km, correct to 2 significant figures. A satellite travels around Earth 1700 km above the surface. How far does the satellite travel in one orbit of Earth? Give your answer correct to 2 significant figures.

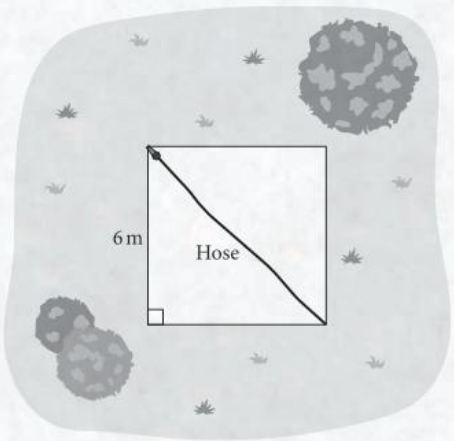
8 EXERCISE 4.4 Jack is making a cushion from two circles of material with radius 18 cm. Find the area of material Jack needs. Give your answer correct to 3 significant figures.

9 EXERCISE 4.5 Magda is going to lay a hose along the diagonal of a square garden bed with side length 6 m.

Calculate the length of the hose:

a as a surd

b correct to 2 significant figures.



Practice set 1

ANSWERS p. 421

- You will need to use an online **compound interest calculator** for Section B, Questions **2** and **14**.

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 2.3 Which of the following decimals can be rounded to 6.74?

A 6.732 **B** 6.7349 **C** 6.744 **D** 6.7452

2 EXERCISE 2.4 A department store increases the cost price of clothing by 200%. The store buys a vest for $12. What will be its selling price after the increase?

A $24 **B** $36 **C** $212 **D** $224

3 EXERCISE 1.1 Calculate the simple interest earned on $800 at 4% p.a. for 3 years.

A $12 **B** $24 **C** $32 **D** $96

4 EXERCISE 1.3 Lesley owns an antique gold necklace that is appreciating at 8% p.a. Today, it is worth $860. Predict by how much its value will increase in the next 3 years, correct to the nearest dollar.

A $206 **B** $223 **C** $1066 **D** $1083

5 EXERCISE 2.4 The monthly service fee at a retirement village is equivalent to 41% of the monthly Age Pension. The *fortnightly* Age Pension is $868.30. How much is the monthly service fee, correct to the nearest dollar?

A $356 **B** $712 **C** $771 **D** $1881

6 EXERCISE 4.1 Which of the following lengths is incorrectly recorded from using a ruler marked in centimetres?

A 32.4 cm **B** 88 cm **C** 145 cm **D** 170 cm

7 EXERCISE 1.2 Amy invests $4500 at 3.5% p.a. compounding annually for 8 years. Calculate the amount of interest she earns, correct to the nearest dollar.

A $1260 **B** $1426 **C** $5926 **D** $9000

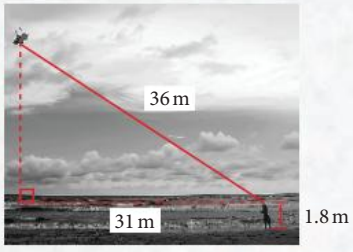
8 EXERCISE 4.3 Express 45 493.456 correct to 2 significant figures.

A 45 000 **B** 45 493.45 **C** 45 493.46 **D** 46 000

9 EXERCISE 4.4 The radius of the Earth is 6400 km and the radius of the moon is 1740 km. How much longer is the Earth's circumference than the moon's circumference, correct to 2 significant figures?

A 11 000 km **B** 15 000 km **C** 29 000 km **D** 40 000 km

10 EXERCISE 4.5



iStock.com/Герман Бикаев

Julia is flying a kite. How high, correct to 1 decimal place, is the kite above the ground?

A 18.3 m **B** 20.1 m **C** 47.5 m **D** 49.3 m

Section B Short-answer questions

1 EXERCISE 2.4 Calculate:

a 2% of 250 kg

b 75% of $150 000

c 80% of 4 hours in minutes

d 18% of 36 L in mL

2 EXERCISE 1.4 Alexei invested his annual bonus of $5300 for 4 years at 2.7% p.a. compounding annually.

a Use an online **compound interest calculator** to calculate the amount his investment will be worth in 4 years.

b How much interest will the investment earn?

3 EXERCISE 2.4 What percentage is:

a 69 out of 75?

b 9 mm out of 300 mm?

c 300 g of 2 kg?

d 6 hours of 1 day?

4 EXERCISE 2.2 Marna has a job interview in the city. Leaving from home, it's going to take her half an hour to walk to the station, the train trip will take $\frac{3}{4}$ of an hour, then it will take 5 minutes to walk to the building.

a How long will it take Marna to travel to the interview?

b Marna needs to arrive at the building by 11.30 am. At what time will she need to leave home?

5 EXERCISE 3.4 Iskander's survey asks, 'How often do you go out to dinner?' Suggest a series of tick boxes that he could provide for people to record their answers.

6 EXERCISE 4.4 Calculate the area of a circle with a diameter of 16 cm. Express your answer correct to 3 significant figures.

7 EXERCISE 3.2 Melinda decides to use a stratified sample of 10% of the students at her school. She has the following data from the school office:

Male	390
Female	450
Total students	840

Year 7	165
Year 8	128
Year 9	171
Year 10	155
Year 11	127
Year 12	94

a How many students should complete the survey?

b How many female students should be part of the survey?

c How many Year 8 students should be surveyed?

8 EXERCISE 2.2 When Katie looked at ticket sales to the Year 12 formal, she found that of the 72 students in Year 12, $\frac{2}{3}$ had already bought their tickets. How many Year 12 students have not yet bought tickets?

9 EXERCISE 3.3 Henri is conducting a survey for a Victorian company. He goes to Bendigo Marketplace and asks every 10th person who walks past to complete a short questionnaire.

a In what way is this sample biased?

b How could Henri ensure that an accurate sample of all Victorians is found?

10 EXERCISE 2.1 Evaluate each expression.

a $2 + (13 - 6) \times 11$ **b** $\frac{8 - 3}{100 \div 4}$ **c** $14 - 5 \times (-3)$ **d** $\frac{40}{16 \div (-2)}$ **e** 2×5^2

11 EXERCISE 3.5 Joanna's survey included the question:

'Rate your meal at the fabulous Diggies' Steak House:

Great Delicious Quite good

a In what way is this question biased?

b Rewrite the question so that it is not biased.

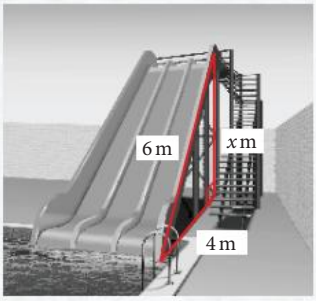
12 EXERCISE 1.3 Ethan earned a salary of $72 300. His salary increased each year by the inflation rate of 2.1% p.a. Calculate Ethan's salary in 5 years, correct to the nearest dollar.

13 EXERCISE 2.4 Terry and Andrea purchased a block of land for $133 000. Six years later they sold it for $164 000. Calculate their percentage profit, correct to 2 decimal places.

14 EXERCISE 1.6 Use an online **compound interest calculator** to answer this question.

For an investment of $30 000 earning 6% p.a. over 10 years, how much more interest will it earn when interest is compounding monthly compared to interest compounding annually?

15 EXERCISE 4.5 Use Pythagoras' theorem to calculate the height, x m, of the top of the slide. Answer in metres correct to 2 significant figures.



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16 EXERCISE 3.2

a Give an example of an investigation where you would use a census to collect the information.

b What would be your target population for your investigation?



Chapter problem

Seth is making an afternoon tea slice, but he's a bit confused by the recipe. It calls for a pinch of salt, a tad of mixed spice and a dollop of cream.

How much of each item does he need?

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CHAPTER 5

SPACE AND MEASUREMENT

MEASURING AT HOME

5.1 Tiny units of cooking
5.2 Miles, feet and inches
5.3 Pints and gallons
5.4 Perimeter problems
5.5 Area problems
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Use non-metric units for cooking
- Calculate with non-metric units for overseas purchases
- Calculate the perimeter of shapes including circles
- Use area formulas
- Solve problems involving measurement, perimeter and area

How are we ever going to use this?

- Cooking
- Internet shopping
- Home renovations
- In the building and/or construction trades

Study design coverage

AREA OF STUDY 4: SPACE AND MEASUREMENT

- measurements and related quantities including derived quantities, metric and relevant non-metric measures
- conventions, properties and measurement of perimeter, area of compound shapes and objects.

VCE Mathematics Study Design 2023–2027 p. 75, © VCAA 2022

Videos (2):

5.4 Circumference of a circle

5.5 Area of a circle

Worksheets (5):

5.4 A page of composite shapes • A page of circles

5.5 Area ID • Circumference and area • A page of composite shapes • A page of circles

Puzzles (2):

5.5 Area group clues • Areas of quadrilaterals and triangles

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

5.1 Tiny units of cooking

In Australia, we generally use the **metric system**; however, there are still some non-metric units in common use, such as in cooking recipes. Measures such as **pinch**, **smidgen**, **dash**, **dollop**, **slather**, **sprinkle**, **drizzle**, **tad**, **drop** and **dash** all appear in Australian cookbooks. While there are no officially recognised sizes for these small quantities, chefs have a common understanding of their relative sizes. In most cases, it doesn't matter if our estimate of a small quantity is not accurate, but in some situations like cheese- and yoghurt-making, it's important to measure them accurately.



Alamy Stock Photo/Patti McConville/Stockimo

Description	Size
Teaspoon	5 mL
Tad	$\frac{1}{4}$ teaspoon
Dash	$\frac{1}{8}$ teaspoon
Pinch	$\frac{1}{16}$ teaspoon
Smidgen	$\frac{1}{32}$ teaspoon
Drop	$\frac{1}{64}$ teaspoon

EXAMPLE 1

Calculate the size of a tad in mL.

Steps	Working
1 tad is $\frac{1}{4}$ of a teaspoon and 1 teaspoon = 5 mL.	1 tad = $\frac{1}{4} \times 5$ mL
Calculate $\frac{1}{4}$ of 5 mL.	= 1.25 mL

EXAMPLE 2

How many pinches are equivalent to a tad?

Steps	Working
1 pinch = $\frac{1}{16}$ teaspoon 1 tad = $\frac{1}{4}$ teaspoon To find the number of pinches in a tad, divide the tad by the pinch.	number of pinches in a tad = $\frac{1}{4} \div \frac{1}{16}$ $= \frac{1}{4} \times \frac{16}{1}$ $= 4$

EXERCISE 5.1 Tiny units of cooking

ANSWERS p. 421

*Use the information in the table before Example 1 to answer questions **1** to **6**.*

1 EXAMPLE 1 Calculate the size of each measure in millilitres (mL), correct to 3 decimal places.

a Smidgen **b** Dash **c** Drop

2 EXAMPLE 2 How many drops are in a smidgen?

3 How many drops are in a teaspoon?

4 What is the relationship between a smidgen and a pinch?

5 A dash is approximately the amount delivered by 8 shakes of a pepper container. How many shakes of a pepper container would deliver a tad?

6 A pinch is also the quantity you could hold between your thumb and forefinger. How many pinches are equivalent to a tad?

*Tablespoon measurements: use this guide to answer questions **7** and **8**.*



Level tablespoon:
flat top



Rounded tablespoon:
a scoop, double the size
of a level tablespoon



Heaped tablespoon:
as much content as possible,
3 times the size of a level tablespoon
(4 times for fine powders)

Nikki M Group

7 An Australian level tablespoon holds 20 mL. Cocoa is a fine powder. How many mL of cocoa powder are in a heaped tablespoon?

8 A dollop of cream is a rounded tablespoon. How many dollops of cream are in a 300 mL container?

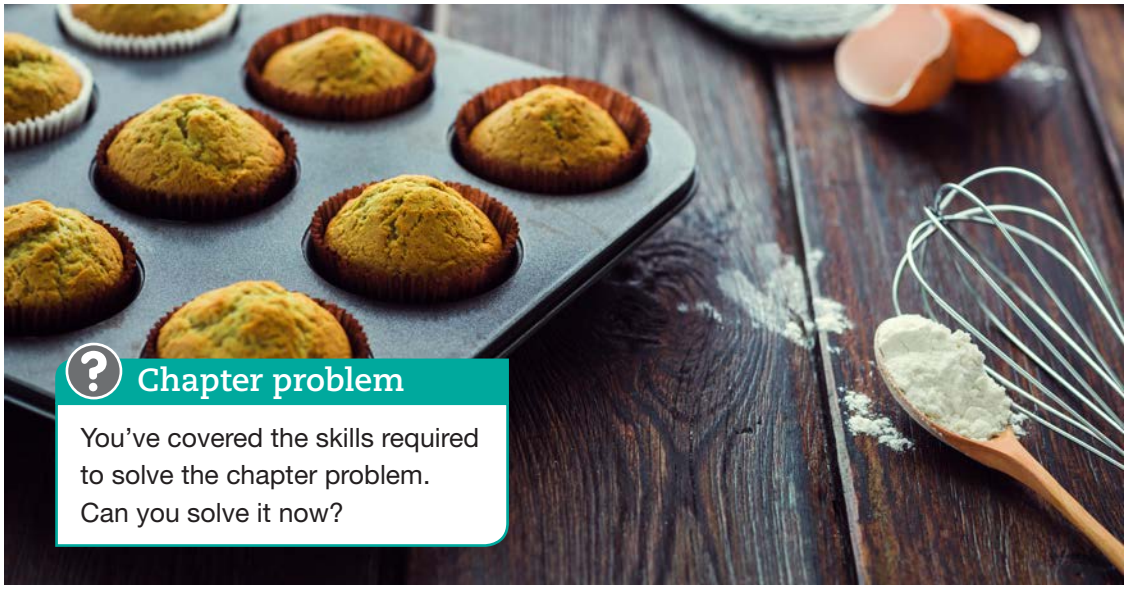
9 A measuring cup's volume is 250 mL and will hold 200 g of sugar. Jack needs 175 g of sugar for a cake he is making. How many mL does he need? Answer to the nearest 10 mL.

10 One cup of sultanas weighs 180 g. Jack requires 45 g of sultanas for his cake. What fraction of a cup does he need?

11 Muffins can be baked in 4 different sizes.

Mini	30 mL
Regular	80 mL
Large	125 mL
Super-size	180 mL

Caitlin bought 2 different trays for baking muffins but she doesn't know what size muffins they are for. One tray holds 12 muffins and the other holds 8 muffins. Caitlin filled both trays with water to measure their capacities. The 12-hole muffin tray held 360 mL and the 8-hole muffin tray held 1 L. What sizes of muffins are Caitlin's trays used for?



Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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5.2 Miles, feet and inches

Most of the world uses the metric system today. The only countries that still use the imperial system are the USA and Myanmar (previously called Burma). The UK and former British Commonwealth countries like Australia and Canada still use imperial units occasionally.

The mile, a unit of distance, originated in ancient Rome. It was the length covered when soldiers walked 1000 paces (in Latin, *mille passus*). Today, miles still appear on British road signs, and nautical miles are used in air and sea navigation.

Non-metric distance	Metric equivalent
1 mile	1.6093 km
1 foot	30.48 cm
1 inch	2.54 cm
1 nautical mile	1.852 km

EXAMPLE 3

Kenrick is planning a trip to the USA. He wants to drive the famous Route 66 that starts in Chicago and ends in Los Angeles. Route 66 is 2400 miles long. How far is this to the nearest kilometre?



Shutterstock.com/BCFC

Steps	Working
The conversion is 1 mile = 1.6093 km.	2400 miles = 2400 × 1.6093 km
Multiply the number of miles by 1.6093 km.	= 3862.32 km
	≈ 3862 km
Write the conclusion.	Route 66 is 3862 km long.

EXAMPLE 4

A supply ship, the *Aurora Australis*, takes food and equipment to Australian research scientists in Antarctica. The trip from Hobart to Mawson is 5475 km. How far is this in nautical miles, to the nearest whole number?



Shutterstock.com/FiledIMAGE

Steps	Working
The conversion is:	5475 km = 5475 ÷ 1.852
1 nautical mile = 1.852 km	= 2956.263…
The number of nautical miles will be smaller than the number of kilometres.	≈ 2956 nautical miles
Divide by the conversion factor.	
Write the conclusion.	5475 km is 2956 nautical miles.

1 nautical mile = 1.852 km
If we're going from left (nautical miles) to right (km), we multiply by the conversion factor.
To go in the other direction, we divide by the conversion factor.

EXAMPLE 5

Convert 250 mm into inches, correct to 2 decimal places.

Steps	Working
There isn't a conversion between millimetres and inches in the table. Convert mm to cm first.	250 mm = 250 ÷ 10 = 25 cm
The conversion is 1 inch = 2.54 cm.	25 cm = 25 ÷ 2.54
To go from right (cm) to left (inches), divide by the conversion factor.	= 9.8425… ≈ 9.84 inches

EXAMPLE 6

Tash sailed from Cuba to Miami, a distance of 240 nautical miles, at an average speed of 6 knots. (A knot is 1 nautical mile per hour).

a How long did the trip take?

b Tash left Cuba at 3 am on Monday. What day and time did she arrive in Miami?

Steps	Working
a Time taken = distance covered ÷ average speed Divide the distance by the speed.	time = 240 ÷ 6 = 40 hours
b Add 40 hours to 3 am Monday.	3 am Monday + 40 h = 3 am Monday + 24 h + 16 h = 3 am Tuesday + 16 h = 3 am Tuesday + 12 h + 4 h = 3 pm Tuesday + 4 h = 7 pm Tuesday Tash arrived at 7 pm on Tuesday.

EXERCISE 5.2 Miles, feet and inches

ANSWERS p. 422

1 EXAMPLE 3 How many kilometres are equivalent to 3 miles? Give your answer correct to 2 decimal places.

2 EXAMPLE 4 Convert 250 km to nautical miles, correct to 2 decimal places.

3 EXAMPLE 5 Use the information in the conversion table on page 88 to copy and complete each conversion, correct to 2 decimal places.

a 8 miles = _____ km

b 12 km = _____ miles

c 510 feet = _____ m

d 16 inches = _____ cm

e 125 mm = _____ inches

f 64 nautical miles = _____ km

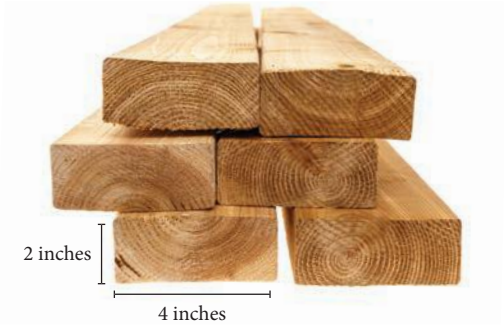
g 108 km = _____ nautical miles

h 10 inches = _____ cm

4 A plane is flying at a height of 3560 feet. How high is this in metres, correct to the nearest metre?

5 In London in 1921, the length of the marathon running race was standardised at 26.2 miles. Calculate the length of the marathon in kilometres, correct to 1 decimal place.

6 A piece of timber like the ones in the photo is commonly known as a piece of '4 by 2'. It gets its name because the end is 4 inches by 2 inches. Calculate the dimensions of the end of a 4 by 2 in millimetres, correct to the nearest 10 mm.



Dreamstime.com/Elena Elisseeva

7 EXAMPLE 6 Anastacia is going to crew on a ship sailing from Papeete in Tahiti to Hilo in Hawaii, a distance of 2402 nautical miles.

- **a** How far is it from Papeete to Hilo in kilometres, correct to the nearest km?
- **b** The ship will average 12 knots for the trip. How long will the trip take, correct to the nearest hour?
- **c** Express your answer to part **b** in days and hours.
- **d** The ship will leave Papeete at 2 am on July 4. On what day and at what time will it arrive in Hilo?

8 Brother Malachy flew from the monastery at La Paz in Bolivia to Caracas in Venezuela, a distance of 1500 nautical miles.

- **a** How far was the flight in kilometres?
- **b** The flight took 9 hours and 45 minutes. Calculate the speed of the plane in km/h and knots, correct to 1 decimal place.

9 Many middle-distance runners aspire to run a 4-minute mile – that is, running a 1-mile race in 4 minutes or less.

- **a** Calculate the speed in km/h, correct to 2 decimal places, required to achieve a 4-minute mile.
- **b** The current world record for completing a 1-mile race is 3 minutes 43 seconds. Calculate the average speed required to run 1 mile in 3 minutes and 43 seconds. Give your answer in km/h, correct to 2 decimal places.

10 A pound is an imperial measure of mass. 1 kg = 2.205 pounds.

- **a** The average Australian man weighs 87 kg. What is this to the nearest pound?
- **b** The average American man weighs 198 pounds. What is this to the nearest kg?
- **c** Convert 1 pound to kilograms, correct to 3 decimal places.
- **d** A Quarter Pounder hamburger contains a meat patty that weighs a $\frac{1}{4}$ of a pound. What is this mass to the nearest gram?

5.3 Pints and gallons

For the metric units of liquid capacity, we use millilitres, **litres**, kilolitres and megalitres. The imperial system uses pints, quarts, **gallons** and barrels. These are **non-metric units**. However, the sizes of these measures differ between the USA and the UK.

Unit	Size	Example
Pints British imperial pint	568.261 mL (approximately $\frac{1}{2}$ L)	A pint of beer
US pint	473.176 mL	
Quarts British imperial quart	2 pints, $\frac{1}{4}$ gallon 1.137 L (approximately 1 L)	
US quart	2 pints, $\frac{1}{4}$ gallon 0.946 L	A quart of milk
Gallons British imperial gallon	8 pints 4.546 L	A gallon of petrol
US gallon	8 pints 3.785 L	
Barrels Oil barrel	42 US gallons 159.0 L	

In South Australia, a pint of beer is 425 mL!

The world oil trade uses the measurement of US barrels. The price is quoted per barrel but is calculated in the United Arab Emirates (UAE) by the litre. Oil is no longer shipped in actual physical barrels, but the unit has remained.

EXAMPLE 7

Hamish is travelling in an American recreational vehicle. The fresh-water tank holds 45 US gallons. Calculate the tank's capacity in litres, correct to the nearest litre.

Steps	Working
1 US gallon = 3.785 L. Moving from left to right from gallons to litres, multiply by the conversion factor.	45 US gallons = 45 × 3.785 = 170.325 ≈ 170 L

EXERCISE 5.3 Pints and gallons

ANSWERS p. 422

Use the information in the table above to answer questions in this exercise.

1 EXAMPLE 7 Charlotte's car has a petrol tank with capacity of 15 US gallons. Calculate the capacity of the tank correct to the nearest litre.

2 How many British pints make an imperial gallon?

3 By what percentage of a US pint is a British pint larger? Give your answer to the nearest whole percentage.

4 What percentage of a British gallon is a US gallon? Give your answer correct to 1 decimal place.

5 Calculate, correct to the nearest whole number, the capacity of a US oil barrel in:

a litres **b** imperial gallons.

6 Sienna is buying a 2-quart tub of imported American ice cream. How many litres does it contain? Give your answer correct to 1 decimal place.

7 Blood donors often speak of donating 'a pint of blood', but a pint of blood is only 470 mL, which is smaller than an imperial pint and more like a US pint.

a How much smaller is a pint of blood than an imperial pint? Answer to the nearest mL.

b By what percentage of an imperial pint is a pint of blood smaller than an imperial pint? Give your answer to the nearest whole percentage.

8 Calculate the amount of each product that is made from a 42-gallon barrel of refined crude oil, using the diagram on the right. Give each answer correct to the nearest litre.

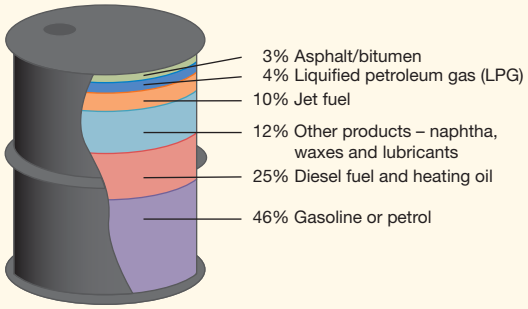
a petrol

b diesel fuel and heating oil

c jet fuel

d LPG

Typical products made from a 42-gallon barrel of refined crude oil



9 A bulk petrol tanker holds the equivalent of 2.1 million US barrels of oil. Calculate the tanker's capacity in:

a US gallons **b** litres, correct to the nearest million litres.

Investigation An old pie recipe

Rachel loves the meat pies her great grandmother makes and she is going to try to adapt the old family recipe to make some pies herself. She needs to adapt 1 ingredient and 3 instructions.

Ingredients

- about 1 pound of chuck steak

Instructions

- Cut the steak into cubes with sides half an inch long.
- Make $\frac{1}{4}$ pint of meat gravy.
- Cook in an oven set to 325°F.

1 How much chuck steak is required in kilograms?

2 What length in centimetres should the steak cubes be?

3 Approximately how many millilitres of meat gravy are needed?

4 At what temperature in degrees Celsius should the oven be set?

Video
Circumference of a circle

Worksheet
A page of composite shapes

Worksheet
A page of circles

5.4 Perimeter problems

The **perimeter** is the distance around the outside of a shape. Fencing and security are common perimeter applications.

EXAMPLE 8

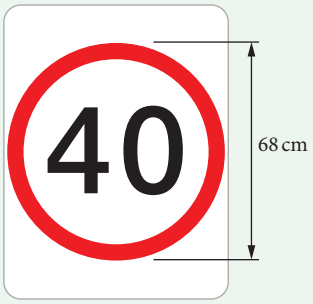
What is the perimeter of this stop sign?



Steps	Working
Add all the lengths of the sides together. As all 8 sides have the same length, it's easier to multiply the length of a side by 8.	perimeter = 8 × 45 cm = 360 cm = 3.6 m
Write the conclusion.	The perimeter is 360 cm or 3.6 m.

EXAMPLE 9

What is the circumference of the red circle in this speed limit sign, correct to 1 decimal place?



Steps	Working
Use the circumference formula $C = \pi d$, where $d = 68$. Write the answer to the required number of decimal places.	$C = \pi d$ $= \pi \times 68$ $= 213.6283\ldots$ ≈ 213.6 cm

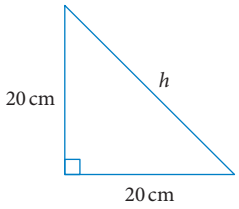
We learnt how to find the circumference of a circle in Chapter 4 on page 72.

9780170453943

EXAMPLE 10

What is the perimeter of the road sign, correct to 1 decimal place?



Steps	Working
Before we can find the perimeter, we need to calculate the length of the 2 sides that make the arrow.	
We can use Pythagoras' theorem to calculate the missing lengths. 	$h^2 = 20^2 + 20^2$ $= 400 + 400$ $= 800$ $h = \sqrt{800}$ $= 28.2842\ldots$ cm
Now that we know the lengths of all the sides, we can add them.	perimeter $= 160 + 40 + 160 + 2 \times 28.2842\ldots$ $= 416.5685\ldots$ ≈ 416.6 cm

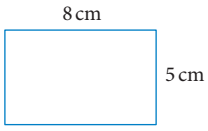
Remember! Don't round your working in the middle of the problem. Round at the end only.

EXERCISE 5.4 Perimeter problems

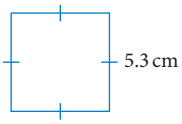
ANSWERS p. 422

1 EXAMPLE 8 Calculate the perimeter of each shape.

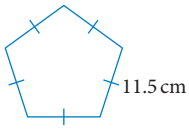
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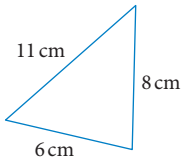
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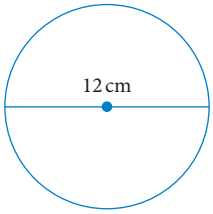


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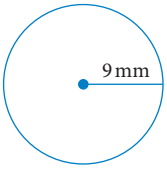


2 EXAMPLE 9 Calculate the circumference of each circle, correct to 1 decimal place.

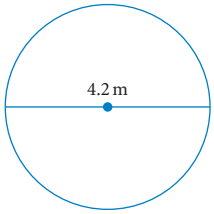
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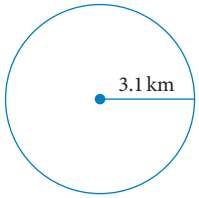
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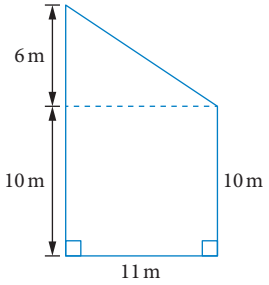


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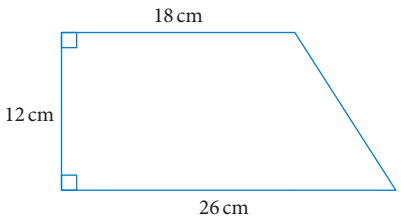


3 EXAMPLE 10 Use Pythagoras' theorem to calculate the length of the unknown side, then calculate the perimeter of the shape, correct to 1 decimal place.

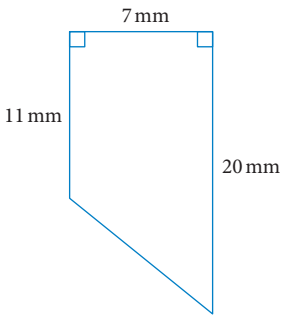
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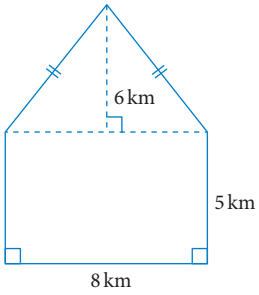
b



c



d



4 Calculate the perimeter of each shape, to the nearest centimetre.

a



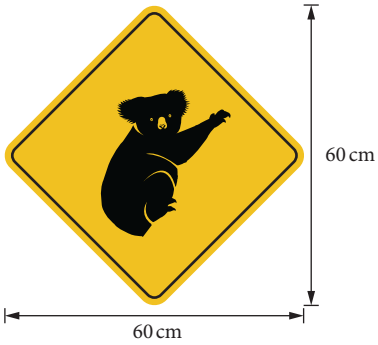
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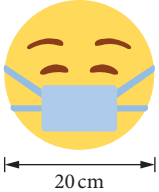
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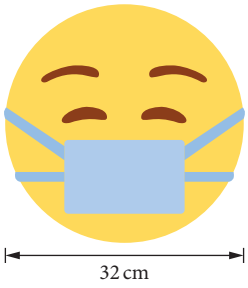


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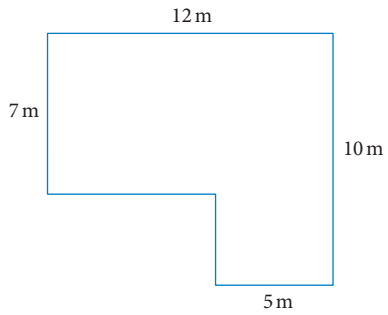


5 During the COVID-19 pandemic, the medical centre displayed emojis to remind patients to wear masks. How much longer is the circumference of the larger emoji than the circumference of the smaller?





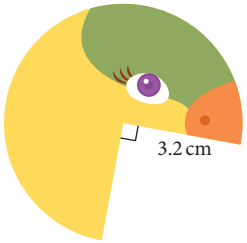
6 The diagram shows the external dimensions of Madeline's house. She is going to install an inground termite protection barrier around the perimeter.



a How long will the barrier be?

b The barrier will cost $21 per metre to have it fully installed and charged ready for use. Calculate the cost of Madeline's termite barrier.

7 Isabella is designing an emoji based on a circle with a quadrant removed. Calculate the perimeter of her emoji, correct to 1 decimal place.



8 The perimeter of a rectangle is 64 mm. The shorter sides are 15 mm long. What is the length of the longer sides?

9 Bees make storage units for their honey in the shape of regular hexagons. The perimeter of one of the hexagons is 29.4 mm. Calculate the length of 1 side of a hexagon.



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Worksheet
Area ID

Worksheet
Circumference and area

Worksheet
A page of composite shapes

Worksheet
A page of circles

Puzzle
Area group clues

Puzzle
Areas of quadrilaterals and triangles

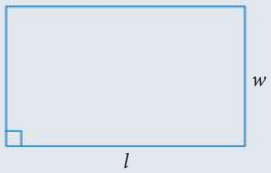
5.5 Area problems

This summary shows formulas for the areas of common shapes.

Area of a rectangle

Area = length × width

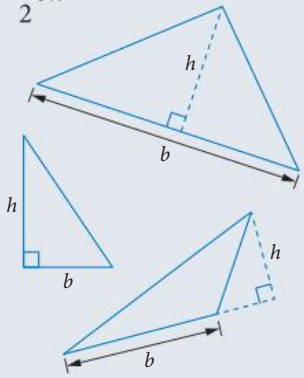
$A = l \times w$



Area of a triangle

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

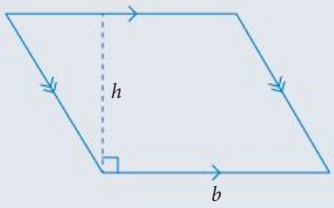
$A = \frac{1}{2}bh$



Area of a parallelogram

Area = base × perpendicular height

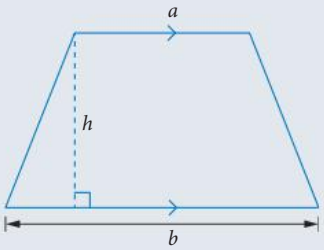
$A = bh$



Area of a trapezium

$\text{Area} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{perpendicular height}$

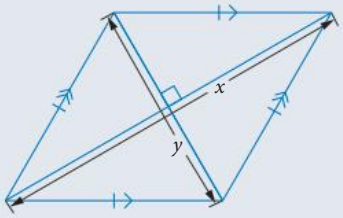
$A = \frac{1}{2}(a + b)h$



Area of a rhombus

$\text{Area} = \frac{1}{2} \times (\text{diagonal}) \times (\text{other diagonal})$

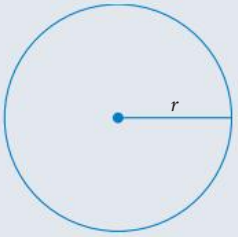
$A = \frac{1}{2}xy$



Area of a circle

$\text{Area} = \pi \times (\text{radius})^2$

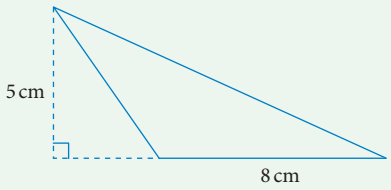
$A = \pi r^2$



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EXAMPLE 11

Calculate the area of this triangle.



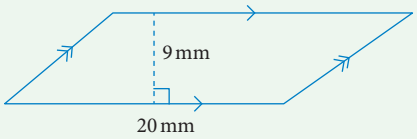
Steps	Working
Use the formula $A = \frac{1}{2}bh$ with $b = 8$ and $h = 5$.	$A = \frac{1}{2} \times 8 \times 5$ $= 20\,\text{cm}^2$

Note: In the area of a triangle formula there is always a right angle between the b and the h.

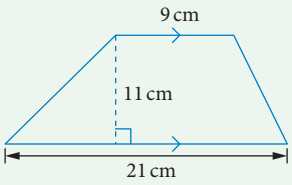
EXAMPLE 12

Calculate the area of each quadrilateral.

a



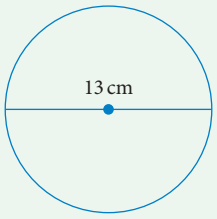
b



Steps	Working
a The formula for the area of a parallelogram is $A = bh$ In the diagram, $b = 20\,\text{mm}$ and $h = 9\,\text{mm}$.	$A = 20 \times 9$ $= 180\,\text{mm}^2$
b The formula for the area of a trapezium is $A = \frac{1}{2}(a + b)h$ In the diagram, $h = 11\,\text{cm}$, $a = 9\,\text{cm}$, $b = 21\,\text{cm}$.	$A = \frac{1}{2} \times (9 + 21) \times 11$ $= 165\,\text{cm}^2$

EXAMPLE 13

Calculate the area of the circle, correct to 1 decimal place.

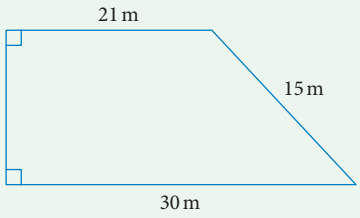


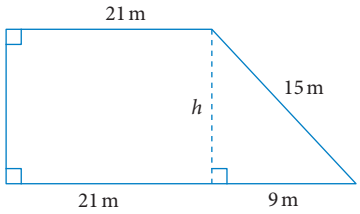
Steps	Working
The formula for the area of a circle is $A = \pi r^2$. In the diagram, the diameter is 13 cm, so the radius $r = \frac{1}{2} \times 13\,\text{cm} = 6.5\,\text{cm}$.	$A = \pi \times 6.5^2$ $= 132.7322\ldots$ $\approx 132.7\,\text{cm}^2$

Video
Area of a circle

EXAMPLE 14

Calculate the area of this **composite** shape.



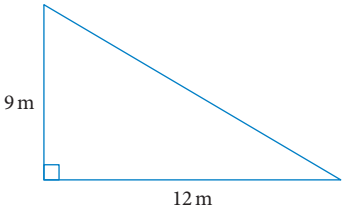
Steps	Working
Separate the shape into a rectangle and a triangle, then use Pythagoras' theorem to calculate the height of the triangle.	 $15^2 = h^2 + 9^2$ $h^2 = 15^2 - 9^2$ $= 144$ $h = \sqrt{144}$ $= 12\,\text{m}$
Calculate the area of the rectangle and the triangle and add them together. (Or use the formula for the area of a trapezium with $a = 21$, $b = 21 + 9 = 30$, $h = 12$.)	area of the rectangle $= 21 \times 12$ $= 252\,\text{m}^2$ area of the triangle $= \frac{1}{2} \times 12 \times 9$ $= 54\,\text{m}^2$ total area $= 252 + 54\,\text{m}^2$ $= 306\,\text{m}^2$

EXERCISE 5.5 Area problems

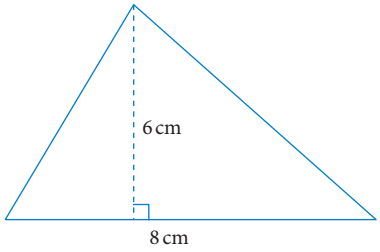
ANSWERS p. 422

1 EXAMPLE 11 Find the area of each triangle.

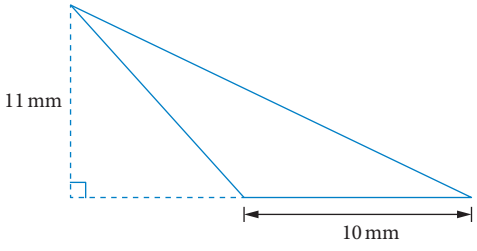
a



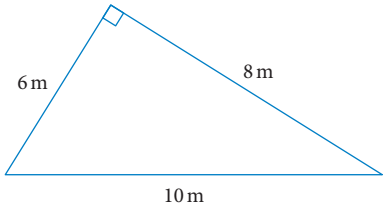
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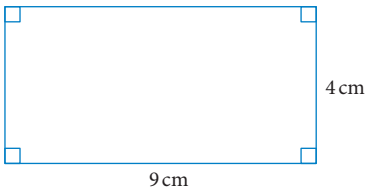
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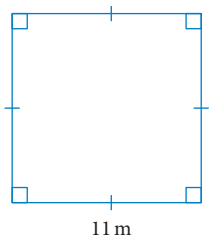
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2 EXAMPLE 12 Find the area of each quadrilateral.

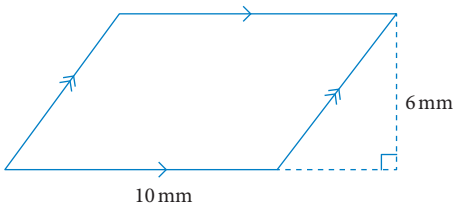
a



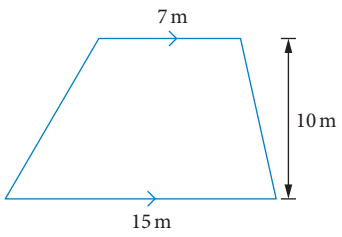
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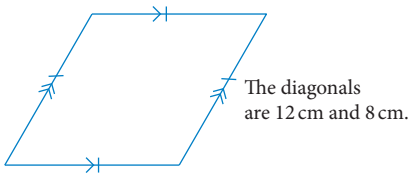
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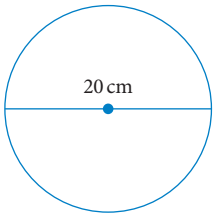


e

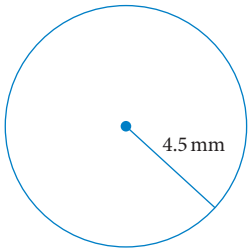


3 EXAMPLE 13 Find the area of each shape, correct to 2 decimal places.

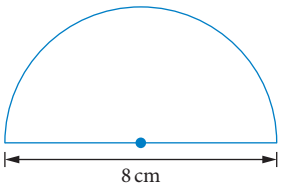
a



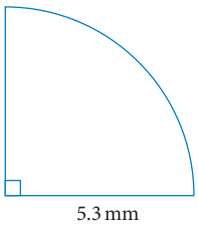
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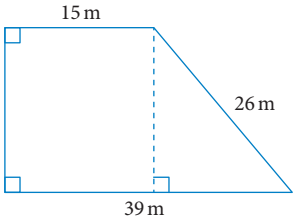


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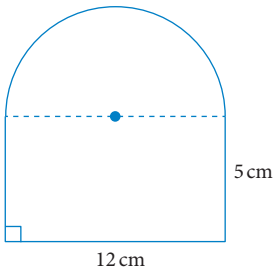


4 EXAMPLE 14 Calculate the area of each composite shape, correct to 2 decimal places where necessary.

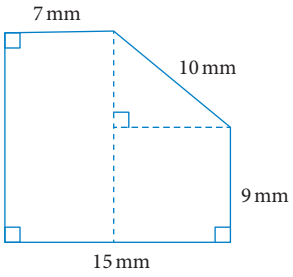
a



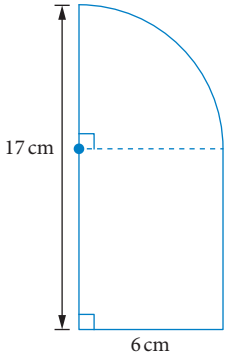
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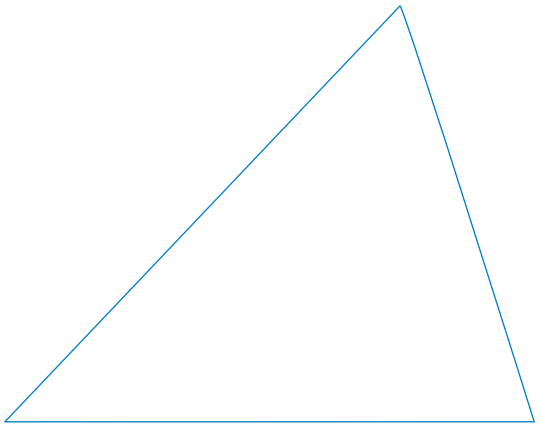


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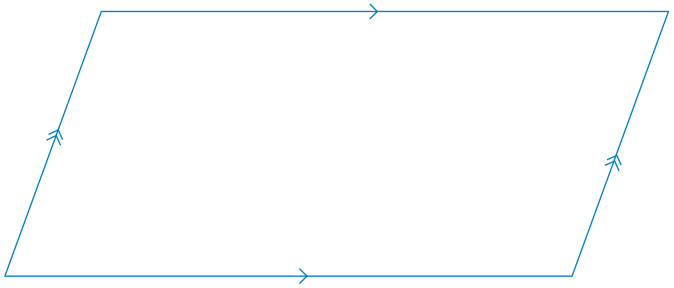


5 Accurately measure the appropriate parts of each shape in millimetres and then calculate its area.

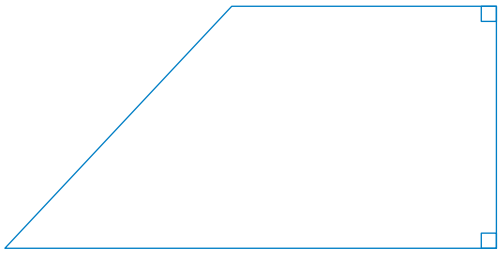
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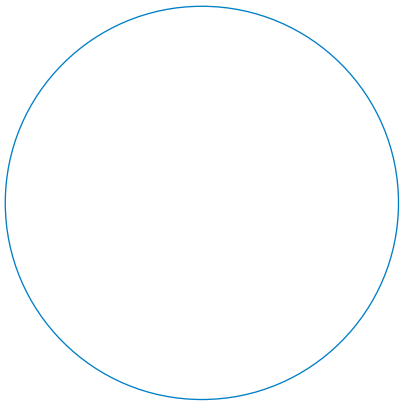
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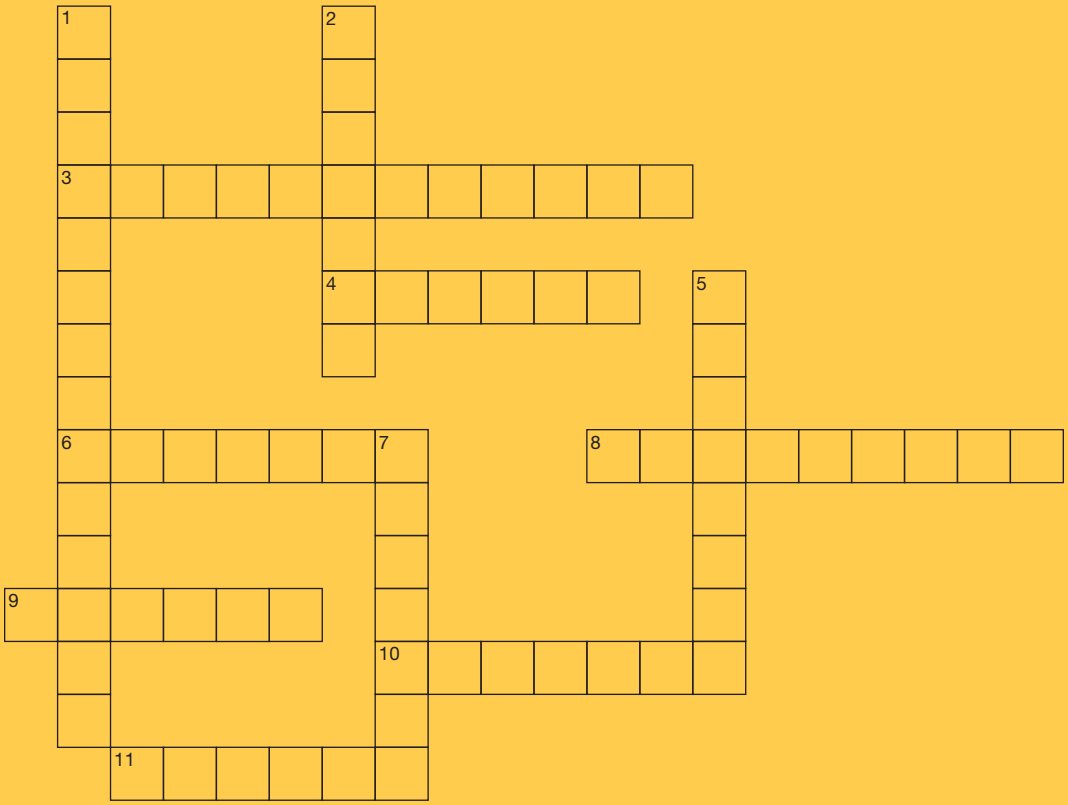
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Keyword activity

CROSSWORD SUMMARY

ANSWERS p. 422

Copy and complete the crossword using the chapter summary below.



Australia uses the metric system for $_3$_____, but some countries, particularly the USA, use $_1$_____ (2 words). We use units for $_6$_____ such as pinch, dollop and tad. The international oil trade measures amounts of oil in $_7$_____ and $_2$_____, whereas in Australia it is measured in $_4$_____.

The perimeter is a measure of the distance around the $_{10}$_____ of a shape and the area measures the amount of surface a shape $_{11}$_____. We can use formulas to calculate area. This table summarises the formulas.

Shape	Area formula
$_5$_____	$A = \frac{1}{2}bh$
Rectangle	$A = lw$
Parallelogram	$A = bh$
$_8$_____	$A = \frac{1}{2}(a + b)h$
Rhombus	$A = \frac{1}{2}xy$
$_9$_____	$A = \pi r^2$

Solution to the chapter problem

Problem

Seth is making an afternoon tea slice, but he's a bit confused by the recipe. It calls for a pinch of salt, a tad of mixed spice and a dollop of cream. How much of each item does he need?

Solution

A pinch is $\frac{1}{16}$ of a 5 mL teaspoon, which is equivalent to 0.3125 mL.

A tad is $\frac{1}{4}$ of a 5 mL teaspoon, or 1.25 mL.

A dollop of cream is a rounded tablespoon. A level tablespoon holds 20 mL and a rounded tablespoon holds twice as much. Seth will need 2 × 20 mL, which is 40 mL of cream.

5 Test yourself

Measuring at home

ANSWERS p. 422

1 EXERCISE 5.1 A flat tablespoon is equivalent to 3 teaspoons. A heaped tablespoon holds 4 times the quantity of a flat tablespoon. One teaspoon holds 5 mL. How much does a heaped tablespoon hold?

2 EXERCISE 5.1 One cup of coarse sugar holds 210 g or 7 oz.

a How many grams is equivalent to 1 oz of coarse sugar?

b Georgia needs 50 g of coarse sugar for the cake she is making.

Approximately what fraction of a cup does she need: $\frac{1}{4}$, $\frac{1}{3}$ or $\frac{1}{2}$?

3 EXERCISE 5.2 Use the information in the table to complete the following conversions, correct to 2 decimal places.

1 mile	1.6093 km
1 nautical mile	1.852 km
1 inch	25.4 mm

a 27 miles = _____ km

b 63 km = _____ nautical miles

c 12 inches = _____ cm

4 EXERCISE 5.2 A rescue helicopter is flying to the site of a boating accident at a speed of 36 knots. The site is 24 nautical miles away.

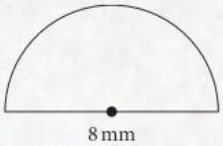
a Calculate the distance to the site in km, correct to 1 decimal place.

b How long will it take the helicopter to arrive?

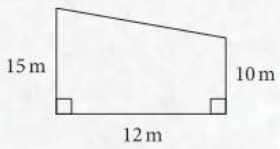
5 EXERCISE 5.3 An imperial pint holds 568 mL and a US pint holds 473 mL. How many US pints are equivalent to 20 imperial pints? Answer to the nearest pint.

6 EXERCISE 5.4 Calculate the perimeter of each shape. For part **a**, give your answer correct to 1 decimal place.

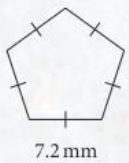
a



b



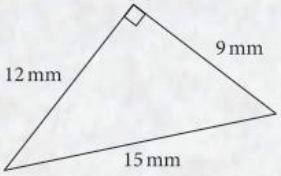
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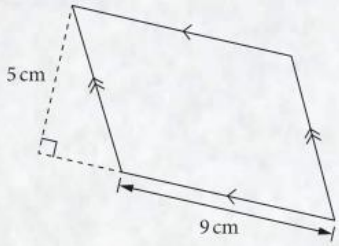
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7 EXERCISE 5.5 Calculate the area of each shape. Give your answer correct to the nearest whole number.

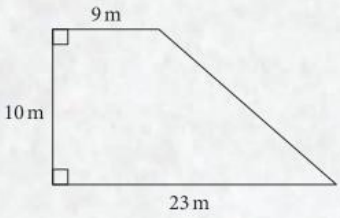
a



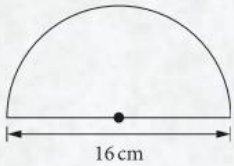
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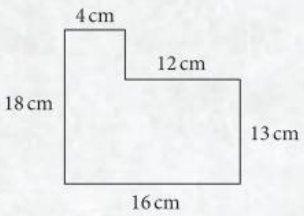
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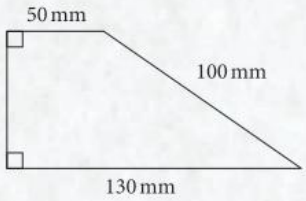
d



e



f





Chapter problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money in total if she takes out the loan for 15 years or 30 years?

What is the difference in the total amount she would repay?

b Suggest a reason Lee might decide to take the loan over 30 years.

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CHAPTER 6

DISCRETE MATHEMATICS:
FINANCIAL AND CONSUMER MATHEMATICS

BORROWING MONEY

6.1 Reducing balance loans
6.2 Loan spreadsheets
6.3 Online loan calculators
6.4 Investigating loans
6.5 Buy or rent?
6.6 Organising your debts
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Use technology to investigate the progress of a loan
- Investigate how the interest rate and repayment size affect the time taken to repay a loan and the total interest paid
- Compare buying and renting a home
- Compare strategies to consolidate debt

How are we ever going to use this?

- To compare loans
- To calculate the total amount we will repay for a loan
- To make decisions about the best loans for our individual circumstances

Study design coverage

AREA OF STUDY 3: DISCRETE MATHEMATICS

Financial and consumer mathematics

- money management including investments and loans, credit and debit, comparing mortgages versus rental costs and debt consolidation.

VCE Mathematics Study Design 2023-2027 p. 74, © VCAA 2022

Videos (1):
6.1 Reducing balance loans

Worksheets (2):
6.2 Reducing balance loan: spreadsheet
6.5 Can you afford to buy a home?

Spreadsheets (1):
6.2 Reducible loans
Test yourself Reducible loans

Weblinks (1):
6.3, 6.4, 6.5, 6.6 MoneySmart

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

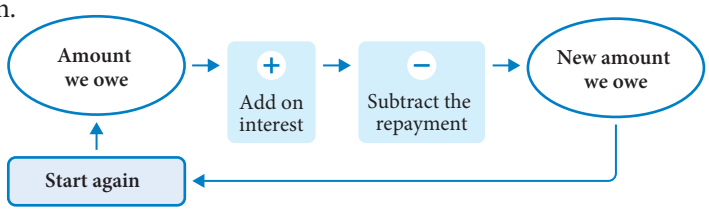
6.1 Reducing balance loans

When we borrow money from a financial institution such as a bank, **credit union** or finance company, we are charged **interest**. The interest rate depends on the type of financial institution and the reason we are borrowing the money. Wise consumers shop around for the best value before they commit themselves.

There are 2 types of interest on loans.

In Chapter 1, we looked at **simple interest**, also known as **flat rate interest**. With simple interest, we pay the same amount of interest throughout the loan.

With **reducible interest**, we only pay interest on the amount of money *still owing*. As the amount we owe decreases, the amount of interest decreases.



The table below shows the progress of a **reducing balance loan**. The **principal** is $15 000, borrowed at 9% p.a. monthly reducing interest, with monthly **repayments** of $800. All amounts are rounded to the nearest cent.

Reducing balance loan
Amount borrowed: $15 000
Interest rate: 9% p.a. monthly reducible
Monthly repayment (R): $800

1st month's interest
$0.09 \div 12 \times \$15\,000$

$\$15\,000 + \$112.50$

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1st	$15 000.00	$112.50	$15 112.50	$14 312.50
2nd	$14 312.50	$107.34	$14 419.84	$13 619.84
3rd	$13 619.84	$102.15	$13 721.99	$12 921.99
4th	$12 921.99	$96.91	$13 018.90	$12 218.90

$\$15\,112.50 - \$800$

$\$14\,419.84 - \$800$

The principal at the start of the 2nd row is the same as the amount at the end of the 1st row.

2nd month's interest
$0.09 \div 12 \times \$14\,312.50$

$\$14\,312.50 + \$107.34$

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EXAMPLE 1

When Ethan borrowed $10 000 from the bank to buy a small car, the bank charged 9% p.a. reducible interest on a secured loan and his monthly repayments were $240. Find the values of **a** to **h** in the table, then find how much Ethan will owe after he has made 3 repayments, and the total interest he will pay in the first 3 months. Round all values to the nearest cent.

With a **secured loan**, the car is used as security. If Ethan doesn't repay the loan, the bank can take the car.

Ethan's reducing balance car loan
Amount borrowed: $10 000
Interest rate: 9% p.a. monthly reducible
Monthly repayments: $240

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$10 000	$75	$10 075	$9835
2	**a**	**b**	**c**	**d**
3	**e**	**f**	**g**	**h**



Video
Reducing balance loans

Steps	Working
a The principal for the 2nd month is the same as the amount owing at the end of the 1st month.	$9835
b Calculate 1 month's interest at 9% p.a. on the amount Ethan owes.	0.09 ÷ 12 × $9835 = $73.76
c $P + I$ is the sum of **a** and **b**.	$9835 + $73.76 = $9908.76
d The amount owing at the end of the 2nd month is **c** minus the monthly repayment of $240.	$9908.76 − $240 = $9668.76
e The principal for the 3rd month is the same as the amount owing at the end of the 2nd month.	$9668.76
f Calculate 1 month's interest on $9668.76.	0.09 ÷ 12 × $9668.76 = $72.52
g $P + I$ is the sum of **e** and **f**.	$9668.78 + $72.52 = $9741.30
h The amount owing at the end of the 3rd month is **g** minus $240.	$9741.30 − $240 = $9501.30

Ethan's reducing balance car loan
Amount borrowed: $10 000
Interest rate: 9% p.a. monthly reducing
Monthly repayments: $240

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1st	$10 000.00	$75.00	$10 075.00	$9835.00
2nd	$9835.00	$73.76	$9908.76	$9668.76
3rd	$9668.76	$72.52	$9741.30	$9501.30

Notice that the interest decreases as the amount owing decreases.

The amount Ethan still owes after he has made 3 repayments is the amount owing after the 3rd month. — $9501.30

The interest Ethan has paid is the sum of the values in the interest column. — interest = $75 + $73.76 + $72.52 = $221.28

EXERCISE 6.1 Reducing balance loans

ANSWERS p. 422

1 EXAMPLE 1 Chantelle borrowed $7000 from the bank at 9% p.a. monthly reducible interest to buy her first car. Her monthly instalments are $320.

a Use a calculation to show that Chantelle will pay $52.50 for interest in the 1st month.

b Use another calculation to show that Chantelle will owe $7052.50 at the end of the 1st month immediately before she makes her 1st repayment.

c Show by a calculation that Chantelle will owe $6732.50 immediately after she makes her 1st repayment.

d Find the values of **A** to **H** in Chantelle's repayment table.

Chantelle's reducing balance car loan
Amount borrowed: $7000
Interest rate: 9% p.a. monthly reducible
Monthly repayments: $320

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$7000	$52.50	$7052.50	$6732.50
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

e How much will Chantelle owe immediately after she makes her 3rd repayment?

f How much interest will Chantelle pay in the first 3 months of the loan?

g How much less interest will she pay in the 3rd month than in the 1st month?

h Why is the amount of interest getting smaller each month?



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2 a Copy and complete the first 4 rows of Ryan's personal loan repayment table.

Ryan's reducing balance loan
Amount borrowed: $16 000
Interest rate: 7.2% p.a. monthly reducing
Monthly repayments: $400

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$16 000	$96	$16 096	$15 696
2				
3				
4				

b How much will Ryan owe immediately after he has made his 4th repayment?

c What percentage of Ryan's 4th repayment was interest? Give your answer to 1 decimal place.

3 Bianca borrowed $18 000 at 8% p.a. reducible interest, calculated every 6 months, to expand her business. She agreed to repay the loan over 3 years with half-yearly repayments of $3434.

a What is the amount of interest that Bianca will pay in her first half-year?

b Find the values of **A** to **E** in the table.

Bianca's business loan
Amount borrowed: $18 000
Interest rate: 8% p.a. calculated every half-year
Half-yearly repayments: $3434

Half-years	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$18 000.00	$720.00	$18 720.00	$15 286.00
2	**A**	**B**	$15 897.44	**C**
3	$12 463.44	**D**	$12 961.98	$9527.98
4	$9527.98	$381.12	$9909.10	$6475.10
5	$6475.10	$259.00	**E**	$3300.10
6	$3300.10	$132.00	$3432.10	0

c After how long will Bianca have repaid more than half the loan?

4 Jackson borrowed $12 800 in a secured loan at 7.56% p.a. monthly reducible interest to buy a boat.

a Copy and complete the first 3 rows of this table.

Jackson's loan
Amount borrowed: $12 800
Interest rate: 7.56% p.a. monthly reducing
Monthly repayments: $900

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$12 800			
2				
3				

b Calculate the total amount Jackson will repay in the first 3 months.

c How much will Jackson repay off the principal in the first 3 months of the loan?

d How much interest will he pay in the first 3 months?

e If Jackson doesn't complete his repayments, what does he risk happening?

5 a Create a spreadsheet to calculate the progress of a $10 000 loan at 6% p.a. monthly reducible interest with repayments of $260 per month. Use this spreadsheet outline to get started.

	A	B	C	D	E
1	Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	1	$10,000			
3	2				
4	3				
5	4				
6	5				

b Use your spreadsheet to calculate the amount still owing on the loan immediately after the 5th repayment.

6.2 Loan spreadsheets

Worksheet
Reducing balance loan: spreadsheet

Spreadsheet
Reducible loans

Home loans are reducing balance loans that last for a long time – for example, 30 years. Before calculators were available, banks had to calculate loan and interest amounts using pen and paper! The table for a 30-year loan would require 360 rows, one for each month.

Today, however, we have technology such as spreadsheets to do the 'number-crunching' involved.

EXAMPLE 2

To answer this question, use the link to download the **Reducible loans** spreadsheet.

Jordan is borrowing $20 000 at 6% p.a. reducible interest and his monthly repayments are $810.

a How long will it take Jordan to repay the loan?

b How much interest will he pay?

Steps

In the blue cells, enter 20 000 for the loan amount, 0.06 for the interest rate and 810 for the monthly repayment.

	A	B	C	D	E
1	Reducible loans				
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.				
3					
4	Loan amount	$20,000.00			
5	Interest rate as a decimal	0.06			
6	Monthly repayment	$810.00			
7					
8		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment
9	1st month	$20,000.00	$100.00	$20,100.00	$19,290.00
10	2nd month	$19,290.00	$96.45	$19,386.45	$18,576.45
11	3rd month	$18,576.45	$92.88	$18,669.33	$17,859.33
12	4th month	$17,859.33	$89.30	$17,948.63	$17,138.63
13	5th month	$17,138.63	$85.69	$17,224.32	$16,414.32
14	6th month	$16,414.32	$82.07	$16,496.39	$15,686.30
15	7th month	$15,686.39	$78.43	$15,764.83	$14,954.83
16	8th month	$14,954.83	$74.77	$15,029.60	$14,219.60
17	9th month	$14,219.60	$71.10	$14,290.70	$13,480.70
18	10th month	$13,480.70	$67.40	$13,548.10	$12,738.10
19	11th month	$12,738.10	$63.69	$12,801.79	$11,991.79
20	12th month	$11,991.79	$59.96	$12,051.75	$11,241.75
21	13th month	$11,241.75	$56.21	$11,297.96	$10,487.96
22	14th month	$10,487.96	$52.44	$10,540.40	$9,730.40
23	15th month	$9,730.40	$48.65	$9,779.05	$8,969.05
24	16th month	$8,969.05	$44.85	$9,013.90	$8,203.90
25	17th month	$8,203.90	$41.02	$8,244.92	$7,434.92
26	18th month	$7,434.92	$37.17	$7,472.09	$6,662.09
27	19th month	$6,662.09	$33.31	$6,695.40	$5,885.40
28	20th month	$5,885.40	$29.43	$5,914.83	$5,104.83
29	21st month	$5,104.83	$25.52	$5,130.35	$4,320.35
30	22nd month	$4,320.35	$21.60	$4,341.95	$3,531.95
31	23rd month	$3,531.95	$17.66	$3,549.61	$2,739.61
32	24th month	$2,739.61	$13.70	$2,753.31	$1,943.31
33	25th month	$1,943.31	$9.72	$1,953.03	$1,143.03
34	26th month	$1,143.03	$5.72	$1,148.74	$338.74
35	27th month	$338.74	$1.69	$340.44	-$469.56
36	28th month	-$469.56	-$2.35	-$471.91	-$1,281.91
37	29th month	-$1,281.91	-$6.41	-$1,288.32	-$2,098.32
38	30th month	-$2,098.32	-$10.49	-$2,108.81	-$2,918.81
39	31st month	-$2,918.81	-$14.59	-$2,933.41	-$3,743.41
40	32nd month	-$3,743.41	-$18.72	-$3,762.12	-$4,572.12
41	33rd month	-$4,572.12	-$22.86	-$4,594.98	-$5,404.98
42	34th month	-$5,404.98	-$27.02	-$5,432.01	-$6,242.01
43	35th month	-$6,242.01	-$31.21	-$6,273.22	-$7,083.22
44	36th month	-$7,083.22	-$35.42	-$7,118.63	-$7,928.63

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Steps	Working
a Look at the values in the last column, E. When the amount owing becomes negative, the loan has been paid off. The first negative amount owing is in cell E35, which corresponds to the 27th payment month.	Jordan will repay the loan after 27 months (which is 2 years, 3 months). Jordan will make 26 monthly payments of $810, but his final payment only needs to be $340.44.
b Add all the values in the interest column up to the 27th month. Entering a formula into the spreadsheet will be the easiest way. The formula is = SUM(C9:C35).	Jordan will pay $1400.44 interest.

EXERCISE 6.2 Loan spreadsheets

ANSWERS p. 423

*Use the link to download the **Reducible loans** spreadsheet for this exercise.*

Spreadsheet
Reducible loans

1 EXAMPLE 2 Samantha borrowed $5000 at 7% p.a. reducible interest to buy a motor scooter. Her monthly repayments are $350.



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- **a** How much interest will Samantha pay in the 1st month?
- **b** How much of her 2nd repayment will be interest?
- **c** How much will she owe immediately after she has made her 12th repayment?
- **d** How long will it take Samantha to repay the loan?
- **e** What is the value of Samantha's last repayment?
- **f** How much interest will Samantha pay on the loan?
- **g** Calculate the total amount Samantha will repay.

2 Trinh borrowed $6000 at 4.25% p.a. reducible interest to buy some new computer equipment. Each month Trinh repays $360.

- **a** How long will it take Trinh to repay the loan?
- **b** How much interest will he pay?
- **c** How much more than he borrowed will Trinh repay?

3 Jacinta's grandmother lends her $7000 interest-free to buy her first car, provided that she repays her $400 each month.

- **a** How long will it take Jacinta to repay the interest-free loan?
- **b** If she had to borrow the money from the bank, Jacinta would be charged 9% p.a. reducible interest. How much interest is she saving with her grandmother's interest-free loan?

4 Ashok wants to borrow $9000. The bank has offered him a reducible interest loan at 7.4% p.a. and suggested that he repay $400 each month.

- **a** How long will it take Ashok to repay the loan with monthly repayments of $400?
- **b** Ashok thinks he can afford to repay $550 per month. How long will it take him to repay the loan at $550 per month?
- **c** Will Ashok save any money by paying the loan off more quickly? If so, how much would he save?
- **d** What general conclusion can you make about the advantage of repaying a reducible loan quickly?
- **e** Is the same conclusion true for flat rate loans? Explain your answer.

5 a On the spreadsheet, what formulas are in cells B9, C9, D9 and E9?

b Why do the formulas in C9 and E9 contain $ signs but the formulas in B9 and D9 don't?

6 Lily made a spreadsheet of her reducing balance loan.

	A	B	C	D	E
1	Reducible loans				
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.				
3					
4	Loan amount	$4,000.00			
5	Interest rate as a decimal	0.06			
6	Monthly repayment	$120.00			
7					
8		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment
9	1st month	$4,000.00	$20.00	$4,020.00	$3,900.00
10	2nd month	$3,900.00	$19.50	$3,919.50	$3,799.50
11	3rd month	$3,799.50	$19.00	$3,818.50	$3,698.50
12	4th month	$3,698.50	$18.49	$3,716.99	$3,596.99
13	5th month	$3,596.99	$17.98	$3,614.97	$3,494.97
14	6th month	$3,494.97	$17.47	$3,512.45	$3,392.45
15	7th month	$3,392.45	$16.96	$3,409.41	$3,289.41
16	8th month	$3,289.41	$16.45	$3,305.86	$3,185.86
17	9th month	$3,185.86	$15.93	$3,201.79	$3,081.79
18	10th month	$3,081.79	$15.41	$3,097.20	$2,977.20
19	11th month	$2,977.20	$14.89	$2,992.08	$2,872.08
20	12th month	$2,872.08	$14.36	$2,886.44	$2,766.44
21	13th month	$2,766.44	$13.83	$2,780.28	$2,660.28
22	14th month	$2,660.28	$13.30	$2,673.58	$2,553.58
23	15th month	$2,553.58	$12.77	$2,566.35	$2,446.35
24	16th month	$2,446.35	$12.23	$2,458.58	$2,338.58
25	17th month	$2,338.58	$11.69	$2,350.27	$2,230.27
26	18th month	$2,230.27	$11.15	$2,241.42	$2,121.42
27	19th month	$2,121.42	$10.61	$2,132.03	$2,012.03
28	20th month	$2,012.03	$10.06	$2,022.09	$1,902.09
29	21st month	$1,902.09	$9.51	$1,911.60	$1,791.60
30	22nd month	$1,791.60	$8.96	$1,800.56	$1,680.56
31	23rd month	$1,680.56	$8.40	$1,688.96	$1,568.96
32	24th month	$1,568.96	$7.84	$1,576.80	$1,456.80
33	25th month	$1,456.80	$7.28	$1,464.09	$1,344.09
34	26th month	$1,344.09	$6.72	$1,350.81	$1,230.81
35	27th month	$1,230.81	$6.15	$1,236.96	$1,116.96
36	28th month	$1,116.96	$5.58	$1,122.55	$1,002.55
37	29th month	$1,002.55	$5.01	$1,007.56	$887.56
38	30th month	$887.56	$4.44	$892.00	$772.00
39	31st month	$772.00	$3.86	$775.86	$655.86
40	32nd month	$655.86	$3.28	$659.14	$539.14
41	33rd month	$539.14	$2.70	$541.83	$421.83
42	34th month	$421.83	$2.11	$423.94	$303.94
43	35th month	$303.94	$1.52	$305.46	$185.46
44	36th month	$185.46	$0.93	$186.39	$66.39

What formulas did Lily enter in cells B9, C9, D9, E9 and B10?

7 Gabriel borrowed $30 000 from the bank to buy a car. The bank is charging him 7.5% p.a. monthly reducible finance, and each month Gabriel repays $950.

Use a spreadsheet to determine the number of monthly repayments Gabriel will make and the total amount of interest he will pay on the loan.



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8 Monique borrowed $6400 at 5.8% p.a. monthly reducing interest to go on a holiday to Japan. Her monthly repayments are $520.

a How long will it take Monique to repay the loan?

b How much interest will Monique pay?



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9 Use the spreadsheet to investigate how increasing the interest rate affects the time taken to repay a loan if the monthly repayment stays the same. Copy and complete the statement:

When interest rates go up, it takes us _____ to repay the loan and we pay _____ interest.

10 Use the spreadsheet to investigate how increasing the monthly repayment amount affects the time taken to repay a loan, if the interest rate stays the same. Copy and complete the statement:

If we increase our monthly repayments, it takes us _____ to repay the loan and we pay _____ interest.

6.3 Online loan calculators

Most people use online calculators to help them investigate and manage reducing balance loans. Find an online loan calculator to use in this section – for example, the **mortgage calculator** on the **MoneySmart** website.

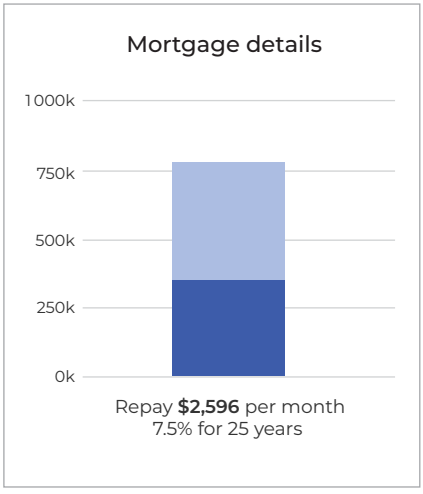
Weblink
MoneySmart

EXAMPLE 3

Elle borrows $350 000 to buy an apartment to live in. She will repay the loan in equal monthly instalments over 25 years at 7.5% p.a. monthly reducible interest. In addition, she will be charged a $10 monthly fee.

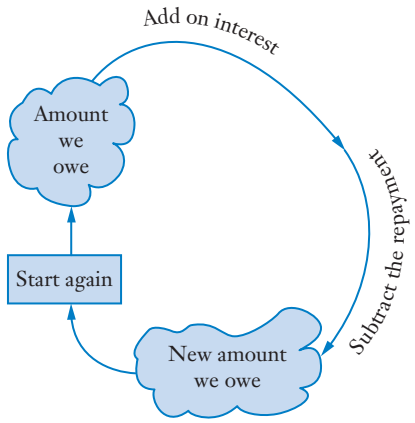
a How much are Elle's monthly repayments?

b How much will she pay in interest and fees?

Steps	Working
a Enter the values into the **mortgage calculator**.	

ASIC's MoneySmart website mortgage calculator (https://www.moneysmart.gov.au/tools-and-resources/calculators-and-tools/mortgage-calculator). Date accessed March 2017

Elle's monthly repayments are $2596.



b She will make monthly repayments for 25 years. Find the total she will repay.	number of months = 25 × 12 = 300 total repaid = repayment × number of months in 25 years = $2596 × 300 = $778 800
Calculate the interest and fees.	interest and fees = total repaid – amount borrowed = $778 800 – $350 000 = $428 800
Write your answer.	Elle will pay $428 800 in interest and fees.

If the online calculator's answer is a little different to yours, that's because it shows the repayment amount ($2566) **correct to the nearest dollar**, but when it does its calculations it uses a more precise (unrounded) value.

Weblink
MoneySmart

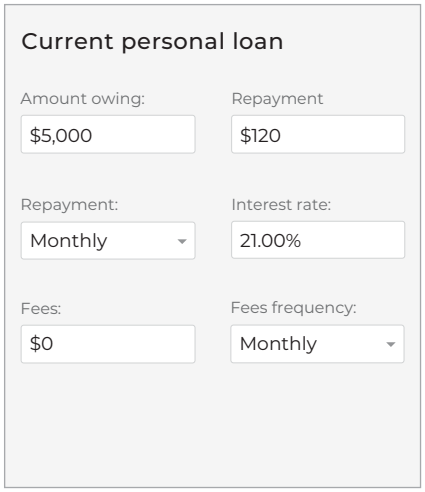
EXAMPLE 4

Owen overspent on his credit card and bought $5000 worth of items with it. His credit card charges 21% p.a. monthly reducible interest. He can only afford to repay $120 per month.

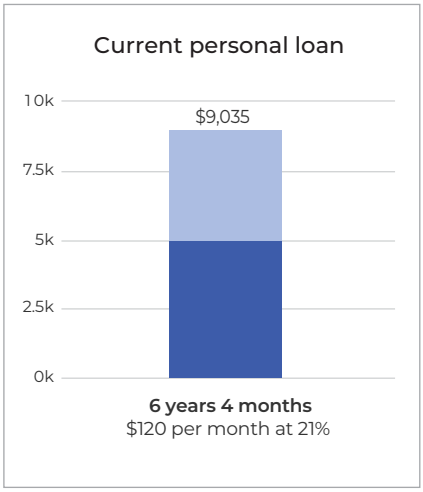
a How long will it take Owen to repay the $5000 credit card bill if he makes $120 repayments each month?

b How much in total will it cost Owen to pay off his $5000 credit card bill?

Steps	Working
a Enter the values into the **personal loan calculator** on moneysmart.gov.au. Use the calculator titled **How can I repay my loan sooner?** Set fees to $0.	



ASIC's MoneySmart website personal loan calculator (https://www.moneysmart.gov.au/tools-and-resources/calculators-and-tools/personal-loan-calculator). Date accessed March 2017



The calculator says that it will take Owen 6 years 4 months to pay off the loan.

b Calculate the number of repayments.	number of repayments = 6 × 12 + 4 = 76
Calculate the total to repay.	total to repay = repayment × number of months = $120 × 76 = $9120
Write your answer.	It will cost Owen $9120 to pay off his $5000 credit card bill.

In reality, Owen's final repayment will be less than $120 to pay off the loan, so the online calculator says the total repayments are actually $9035.

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EXERCISE 6.3 Online loan calculators

ANSWERS p. 423

You will need access to an online calculator for this exercise.
*Use the **personal loan calculator** to answer questions **1** to **3**.*

1 EXAMPLE 3 Shan is borrowing $15 000 at 9.8% p.a. monthly reducible interest to start a business. She plans to repay the loan in monthly repayments over 5 years. In addition, the bank will charge a $10 monthly fee.

- **a** How much are her monthly repayments?
- **b** Calculate the total amount Shan will repay over the 5 years.
- **c** How much interest will she pay during the 5 years?

2 Juan wants to borrow $40 000 to buy an SUV. He can get the money from a finance company at 17% p.a. reducible interest.

- **a** Calculate the monthly repayments if Juan takes the loan over 15 years.
- **b** How much interest will he pay if he takes the loan over 15 years?
- **c** Explain why 15 years isn't a suitable term for a car loan.
- **d** If you were Juan, what would you do?

3 EXAMPLE 4 Hoa wants to borrow $10 000 to go on a trip to Canada. Her bank will lend her the money at 8.75% p.a. reducible interest with no monthly account fees. Hoa can afford to repay $650 per month.

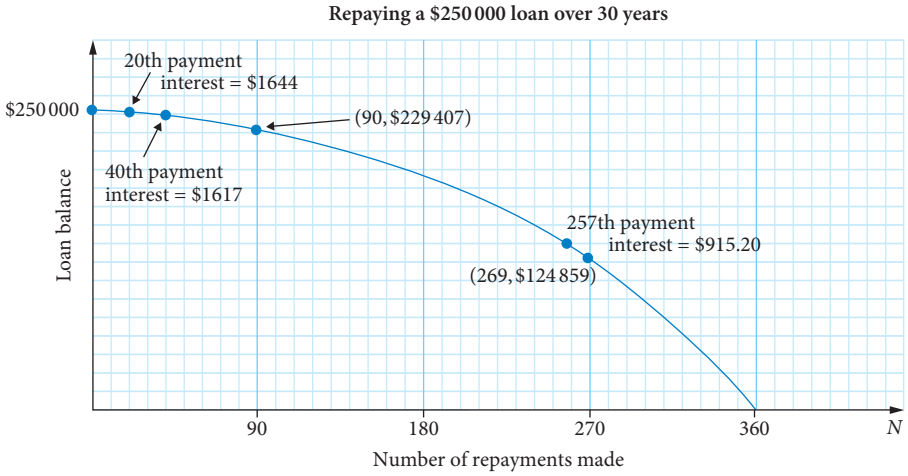
- **a** Use the **How can I repay my loan sooner?** section on the **personal loan calculator** to calculate the number of monthly repayments Hoa will need to make to repay the loan.
- **b** How much interest will Hoa pay?



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4 When Chris bought his townhouse, he borrowed $250 000 at 8% p.a. monthly reducing interest over 30 years and his monthly repayments were $1834.40. The graph shows the balance he owed on the loan after N months.



a The graph says that in the 20th month, the interest paid was $1644. However, the monthly repayment was $1834.40. How much of the repayment was not interest but a decrease in the loan?

b The graph says that in the 40th month, the interest paid was $1617. For a reducing balance loan, why is the interest in the 40th repayment less than the interest in the 20th repayment?

c Use the graph to determine the amount Chris still owed on the loan after 90 months.

d Calculate the total monthly repayments Chris made during the 90 months.

e In the 257th month, how much of the repayment:

i was interest? **ii** was a decrease in the loan?

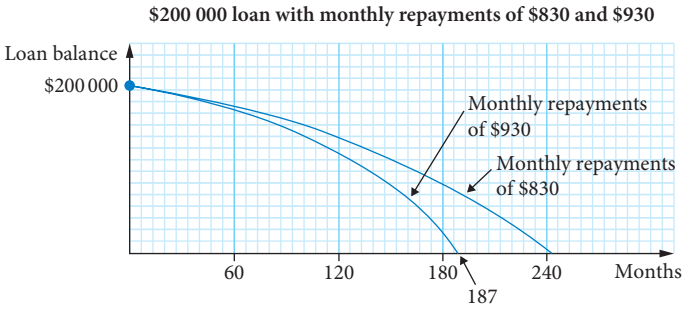
f What percentage (correct to 1 decimal place) of the loan was still owing after 269 months?

g How many years and months is 269 months?

h It took Chris 269 months to repay roughly the first half of the loan. How long did it take him to repay the second half of the loan?

i For a reducing balance loan, when do we pay the most interest: at the beginning of the loan or at the end of the loan?

5 When Lara borrowed $200 000 to buy an apartment, the bank told her the repayments would be $830 per month. Lara decided that she could afford to repay $930 per month. The graph shows the balance of a $200 000 loan with monthly repayments of $830 and $930.



a How long will Lara take to repay the loan if she makes monthly repayments of:

i $830? **ii** $930?

b How much less time will Lara take to pay off the loan by making monthly repayments of $930?

c How much money will Lara save in repayments by making monthly repayments of $930?

d Why do you think some people may choose to take the smaller monthly repayments even though they will have to pay more in the long run?

*Use the **mortgage calculator** to answer the remaining questions in this exercise.*

6 Bree borrowed $260 000 at 6.75% p.a. over 15 years.

a How much were her monthly repayments?

b Interest rates fell to 6.5% p.a. How much cheaper were Bree's monthly repayments after the fall in interest rates?

7 When interest rates go up, loan repayments increase. When Jai borrowed $170 000, the interest rate was 6.75% p.a. and he took out the loan over 10 years.

a How much was Jai's monthly repayment when he took out the loan?

b By how much did Jai's monthly repayments increase when the interest rate went up to 7% p.a.?

c How much more did Jai have to repay each year at 7% p.a. than at the original interest rate?

8 Zhi is buying a house. He is going to borrow $240 000 at 8% p.a. He is trying to decide whether to take the loan over 20 or 30 years.

a How much more are the monthly repayments over 20 years than over 30 years?

b How much more interest will he pay if he takes the loan over 30 years than over 20 years?

c If you were Zhi, would you take the loan over 20 or 30 years? Give reasons for your answer.

9 During the 1980s, interest rates rose as high as 18% p.a. How much more were the monthly repayments on a $150 000 loan over 20 years in the 1980s compared to the same loan at 5.7% p.a. in the late 2010s?

10 Sarah is deciding between two different home loans. She is borrowing $140 000 over 25 years. Sarah summarised the terms and conditions of the two loans in a table.

	Interest rate p.a.	Loan establishment fee	Annual loan fee	Mortgage discharge fee
Big bank	7.1%	$320	$248	$228
Small mortgage company	7.0%	$598	$76	$314

a Calculate the value of the monthly repayments for each loan.

b Copy and complete this table to help Sarah determine which loan is better.

	Big bank	Small mortgage company
Loan establishment fee		
Mortgage discharge fee		
Total annual loan fee over 25 years		
Total monthly repayments		
Total cost of the loan		

c Which 2 features of a loan – the interest rate, loan establishment fee, annual loan fee or mortgage discharge fee – most influence the total cost of a loan?

d Which of the 2 loans do you recommend Sarah take? Why?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

6.4 Investigating loans

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When we borrow a large amount of money, does a change in the interest rate affect how long it takes to pay off the loan? Does increasing the monthly repayment amount save any money in the long run? In the following exercise, you are going to investigate these 2 questions yourself. You will need to visit the **MoneySmart** website and search for **mortgage calculators**. Select **How can I repay my loan sooner?**

EXERCISE 6.4 Investigating loans

ANSWERS p. 423

Work in groups for each investigation.

Changes in interest rates

1 Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $200 000		Monthly repayment $1500 Monthly fee $10	
Interest rate	Time to repay the loan	Total repaid	Interest
5.00%	16 years 6 months	$295 582	$95 582
5.50%			
5.75%			
6.00%			
7.00%			

2 Change the amount borrowed in the calculator, then copy and complete this table.

Amount borrowed $_____		Monthly repayment $1500 Monthly fee $10	
Interest rate	Time to repay the loan	Total repaid	Interest
5.00%			
5.50%			
5.75%			
6.00%			
7.00%			

3 What happens to the amount of interest you pay on a $150 000 loan with monthly repayments of $1000 if the interest rate drops from 4% p.a. to 3.75% p.a.?

4 Write a sentence to describe how changes in interest rates affect the length of time it takes to repay a loan and the total interest involved. Assume the monthly repayment remains the same.

Changes in monthly repayments

5 Copy this table and use the online calculator to complete it. The middle row has been completed as an example.

Amount borrowed $200 000		Interest rate 6% p.a. Monthly fee $10	
Monthly repayment	Time to repay the loan	Total repaid	Interest
$1250			
$1400			
$1600	16 years 7 months	$318 030	$118 030
$2000			
$2400			

6 How does increasing the monthly repayment amount affect the total interest paid? What effect does decreasing the monthly repayments have?

7 How does increasing the monthly repayment amount affect the time required to pay off the loan? What effect does decreasing the monthly repayments have?

8 Does doubling the monthly repayments halve the time it takes to pay off the loan?

9 Suppose you are going to borrow money to buy a place to live in. What advice can you give yourself about interest rates and repayments?

Investigation Making smart repayments

Let's examine the effects of making loan repayments more frequently and increasing the size of each repayment.
Visit the **MoneySmart** website and search for **mortgage calculators**. Select **How can I repay my loan sooner?**

1 Darius has a $250 000 loan at 8% p.a. and he can afford to repay $3000 per month or $1500 per fortnight. Does paying off a home loan fortnightly instead of monthly make any difference?

Use the online calculator to help you copy and complete the missing values in the table.

	Monthly repayments	Fortnightly repayments
Value of the repayment	$3000	$1500
Time to pay off the loan		
Total repayments		
Total interest		

2 Repeat question **1** for a $300 000 loan at 12% p.a. with a monthly repayment of $5000 and a fortnightly repayment of $2500.

3 Investigate some other loan amounts (principals). Make sure the monthly repayment is twice the value of the fortnightly repayment.

4 What conclusions can you make? Does repaying half the monthly repayment each fortnight make any difference to the loan?

5 Madison borrowed $250 000 at 8% p.a. with a monthly repayment of $2000. Use the section **How can I repay my loan sooner?** Use the calculator to find the term (length of time) of Madison's loan. Record the total amount and the interest paid by Madison.

6 Does increasing the monthly repayments by $20 make any difference? Change the repayment from $2000 to $2020. Record the new term of the loan, as well as the new total amount and interest paid.

7 Repeat question **6** for a larger monthly repayment.

8 What conclusions can you make? Does repaying a larger amount make much difference to the loan?

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6.5 Buy or rent?

For years, owning a house on a quarter-acre (1000 m^2) block of land has been considered the Australian dream, but with a growing population and an increasing demand for housing, this is no longer affordable for many Australians. So, is it better to own or rent a home? It depends on a person's or family's situation and preference, and both options have their advantages and disadvantages. Buying a property is expensive and the owner is responsible for all of the costs (e.g. rates, maintenance and building insurance), but the property is theirs. In contrast, when renting a property, the only expenses are the rent, bond, utility bills (gas, water and electricity) and contents insurance.

EXAMPLE 5

Monthly rent is calculated as between 0.8% and 1.1% of the value of a property. How much can you expect to pay monthly and weekly in rent for a property valued at $465 000? Express both answers as a minimum/maximum range.

Steps	Working
Minimum monthly rent is 0.8% or 0.008 of $465 000.	minimum monthly rent = 0.008 × $465 000 = $3720
Maximum monthly rent is 1.1% or 0.011 of $465 000.	maximum monthly rent = 0.011 × $465 000 = $5115
Write the answer.	The monthly rent will be from $3720 to $5115.
To calculate weekly rent, first multiply the monthly rent by 12 to get the annual rent, then divide by 52 to get the weekly rent.	minimum weekly rent = $3720 × 12 ÷ 52 = $858.4615… ≈ $858.46 maximum weekly rent = $5115 × 12 ÷ 52 = $1180.3846… ≈ $1180.38
Write the answer.	The weekly rent will be between $858.46 and $1180.38.

EXERCISE 6.5 Buy or rent?

ANSWERS p. 423

1 Who is responsible for each of these costs for a rental property? Is it the owner or the tenant (renter)?

a Rates and council charges
b Building insurance
c Contents insurance
d Fixing broken items – for example, plumbing
e Rent
f Mortgage (home loan) repayments
g Electricity bills
h Repainting
i Bond
j Pest inspection for termites

2 EXAMPLE 5 What is the minimum and maximum monthly rent for properties with the following values?

	Description	Value	Minimum monthly rent at 0.8%	Maximum monthly rent at 1.1%
a	Inner-city unit	$620 000		
b	Inner-city luxury apartment	$2.1 million		
c	Suburban unit	$420 000		
d	Outer suburban house	$575 000		

3 Calculate the weekly rent for the following properties, correct to the nearest dollar.

a monthly rent of $1600

b monthly rent of $2100

c annual rent of $24 500

d monthly rent $1429

4 To buy any of the properties in question **2**, you would need a minimum of 20% of the property's value as a deposit. Calculate the deposit you would need for each property.

5 Ella lives with her parents but would like to buy an apartment to live in one day. She is thinking of buying an apartment and renting it out at first to cover the mortgage. The apartment is priced at $595 000 and she has saved $150 000 to use as a deposit.

a Is $150 000 sufficient for a 20% deposit? Use a calculation to justify your answer.

b How much will Ella need to borrow?

c At 0.8%, how much rent can Ella expect to earn per month?

d Calculate the annual rent Ella can expect.

e The table shows the expenses Ella is expecting.

Calculate the total *annual* expenses Ella will face in owning and renting out this apartment.

Mortgage repayments	$2611 per month
Rental agent fees	$70 per month
Body corporate fees	$2500 per year
Insurance	$490 per year
Personal savings to expenses	$200 per month
Rates	$370 per quarter

f Do you think Ella can afford to buy this apartment and rent it out? Use calculations to justify your answer.

6 Carrie is looking at two similar houses – one for sale and one for rent. She has saved $125 000 to use as a deposit if she decides to buy. The table below shows costs involved for Carrie if she rented or bought one of the properties.





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Item	Buying cost	Renting cost	Annual buying cost	Annual renting cost
Rent	0	$1430 per week		
Mortgage repayments	$3399 per month for 20 years	0		
Council rates	$1600	0		
Insurance	$1359	$242		
Maintenance	$500	0		

a Complete the two columns on the right.

b Calculate the cost over 1 year of owning and renting the properties.

c How much interest would Carrie pay on the mortgage over the 20-year loan?

d If you were Carrie, would you buy or rent? Justify your answer.

7 Xavier moves house a lot with his work. He's thinking of renting for life and investing the difference between the rent he needs to pay and what his mortgage repayments would have been. He's been saving for a few years and has $84 000, which he was going to use as a house deposit.



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a Use a calculation to show that $84 000 is sufficient for a 20% deposit for a property priced at $420 000.

b After paying a 20% deposit, how much would Xavier need to borrow to buy the property in part **a**?

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c Use the **mortgage calculator** on the **MoneySmart** website to calculate the monthly repayments on Xavier's loan at 2.5% p.a. for 25 years.

d Calculate the difference between the monthly rent and mortgage repayments.

e Xavier is going to invest the difference between the rent and mortgage repayments in a managed fund. He is confident he will get interest of at least 5% p.a. with this investment. Use the **compound interest calculator** on the **MoneySmart** website to calculate the future value of Xavier's $84 000 investment with his regular monthly savings (answer to part **d**).

f Calculate the total amount of rent Xavier would pay over 25 years, assuming the rent stays the same.

g Do you think Xavier's strategy is a good one? Why or why not?

Investigation Advantages of renting and buying

Here are some common statements people make about renting versus buying:

- 'Rent is dead money.'
- 'I can move more easily if I rent.'
- 'It's too hard to save a deposit to buy a place.'

Research other lifestyle considerations involved in the decision to rent or buy. Discuss each consideration in your group.

Worksheet
Can you afford to buy a home?

Investigation Can you afford to buy a home?

Could you buy a home at age 25? Copy the table below or download it using the link. Complete it by answering the questions on the next page.

Name	
1 Future job	
2 Annual gross pay	
3 Monthly gross pay	
4 Maximum monthly repayment	
5 Savings interest rate	
6 Deposit amount you could save	
7 Mortgage interest rate	

Period of mortgage	15 years	20 years	25 years	30 years
8 Maximum loan amount				
9 Monthly repayment				
10 Total amount repaid				
11 Total amount repaid if paid fortnightly				
12 Amount I can afford to spend on a property				

13 Property I can afford in a suitable location. Paste photo here.	Address	
	Price	
14 Net monthly income after deducting income tax and mortgage repayments		
15 Five more expenses from my net monthly income		
16 Strategies for making buying property more affordable		



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1 Write the type of career you plan to have when you are 25 – for example, panel beater, vet nurse, childcare worker.

2 Research the annual gross pay you will receive in this occupation. Do not include any overtime.

3 Calculate your monthly gross pay.

4 Calculate 30% of your monthly pay. This is the maximum amount your monthly repayment can be.

5 Research the current savings interest rate.

6 Suppose you are going to save 30% of your income each month for 5 years at the current savings interest rate. Use the **MoneySmart monthly savings calculator** to determine your savings after 5 years. This amount is your deposit.

7 Research the current mortgage interest rate.

8 Use the **MoneySmart How much can I borrow? mortgage calculator** to determine the maximum amount you can borrow over 15, 20, 25 and 30 years.

9 Record the monthly repayments in row 9 of the table.

10 Record the total you will repay in row 10 of the table.

11 Use the online calculator to determine the total you will repay if you make half the monthly repayment each fortnight. Record these amounts in row 11 of the table.

12 Calculate the amount you can afford to spend on a property by adding the amount you can borrow to the deposit you can save.

13 Visit a real estate website and search for a property you can afford to buy in a suitable location. Paste a photograph of the property in the table. Record the address of the property and the selling price.

14 Assume that you will pay 25% of your gross pay in income tax. Subtract this tax and your monthly mortgage repayment from your gross pay. This amount represents the **net pay** you will have left each month. Write it in row 14 of the table.

15 List 5 different things you will have to pay out of the amount remaining in row 15.

16 Buying your first home is financially challenging. In your group, discuss strategies you could use to make it easier. Record the strategies in row 16 of the table.

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6.6 Organising your debts

Throughout their lives, people may have a number of loans at the same time: home loan, car loan, credit card debt, student loan and short-term loans. Finance companies often advertise that they can help consolidate your debt so that you combine your loans into one big loan. While this sounds like a good idea, it can cost you more money in the long run and it can potentially increase your debt. Before you engage a company to help you get out of debt, check the company carefully on the **ASIC** website to make sure the company is registered in Australia. If it is not Australian-registered, do not trust it!

If you are struggling to pay your mortgage or household bills, always talk to the loan provider or utility company first, because they usually have support for people in financial distress. Ask to speak with the hardship team. They will be able to help or provide free financial assistance and advice.

Comparison interest rates

To stop banks and other financial institutions from advertising artificially low interest rates and charging high undisclosed fees, the government introduced the **comparison rate** to help people compare rates from different institutions. The comparison rate helps borrowers to know the true cost of a loan. It is compulsory for institutions to calculate the mortgage comparison rate based on a $150 000 loan over 25 years, and it includes specified fees and charges, but not all of them.

EXAMPLE 6

Ashleigh is shopping around for the best interest rate for her home loan. She is considering these banks.

	Advertised interest rate p.a.	Comparison rate p.a.
Conn Bank	1.84%	3.38%
Best Pack Bank	2.41%	2.88%
Grab Bank	3.11%	3.11%

a Which bank has the best rate for borrowers?

b Which of the banks doesn't have monthly fees added to their repayments?

Steps	Working
a When we're borrowing money, we look for the lowest comparison rate, not the lowest advertised rate. When we're investing money, we want the highest, safe rate of interest.	The best rate for borrowing money is 2.88% with Best Pack Bank.
b Monthly fees increase the advertised interest rate to give a higher comparison rate.	Grab Bank doesn't have monthly fees added to the loan repayments.

EXAMPLE 7

Finn owes $3200 on his credit card and his bank charges him 21.5% p.a. interest.

a How much interest will the bank charge for 1 month?

b Finn owed $3200 for a month, and he repaid $625 at the end of the month. How much does he still owe?

Steps	Working
a Calculate 21.5% of $3200, then divide by 12 to work out the monthly interest.	interest = $3200 × 0.215 ÷ 12 = $57.333 33… ≈ $57.33
b The interest is added to the loan, then Finn's repayment is subtracted from it.	amount owing = $3200 + $57.33 – $625 = $2632.33

EXAMPLE 8

Jake owes 3 different amounts. This table shows his debts and the interest he is being charged.

	Debt	Interest rate p.a.	Monthly repayment	Monthly interest charge
Car loan	$32 500	9.5%	$840	$257.29
Credit card	$8240	22.0%	$198	$151.07
Store credit card	$5310	18.0%	$122	$79.65

Jake can consolidate the three loans into one bank loan at 9% p.a. flat rate interest with monthly repayments of $1304.75 for 4 years.

a Calculate the value of his new single bank loan.

b Calculate the value of the monthly interest on his consolidation loan.

c How much will he save in interest per month with the loan?

d How much will he pay altogether to repay the loan?

Steps	Working
a Add the values of the 3 loans.	single loan = $32 500 + $8240 + $5310 = $46 050
b Find 9% of $46 050 divided by 12.	monthly interest = 0.09 × 46 050 ÷ 12 = $345.375 ≈ $345.38
c Add the 3 monthly interest charges, then subtract the answer to part **b**.	interest on 3 loans = $257.29 + $151.07 + $79.65 = $488.01 interest saving = $488.01 – $345.38 = $142.63
d Multiply the monthly payment by the number of months. There are 48 months in 4 years.	total Jake pays = $1304.75 × 48 = $62 628

EXERCISE 6.6 Organising your debts

ANSWERS p. 424

1 EXAMPLE 6 Oliver has several loans totalling $18 435. He is considering consolidating the loans into one personal bank loan. He has options with 3 different banks.

	Advertised interest rate for personal loan	Comparison rate
River Bank	9.99% p.a.	11.08% p.a.
Vamp Bank	9.95% p.a.	11.25% p.a.
South Bank	10.50% p.a.	10.65% p.a.

a Which bank has the best borrowing rate for personal loans?

b How much more would Oliver pay in 1 year with the River Bank personal loan at the advertised interest rate compared to the comparison rate?

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2 Paba borrowed $21 500 at 9.7% p.a. reducible finance for 6 years, but her bank included a $20 monthly service fee she wasn't expecting.

Use the **mortgage calculator** on the **MoneySmart** website to answer the following questions.

a Determine the value of Paba's monthly repayments without a monthly service charge.

b How much will her monthly repayments be including the service fee?

c How much more will Paba have to repay with the service fee than without?

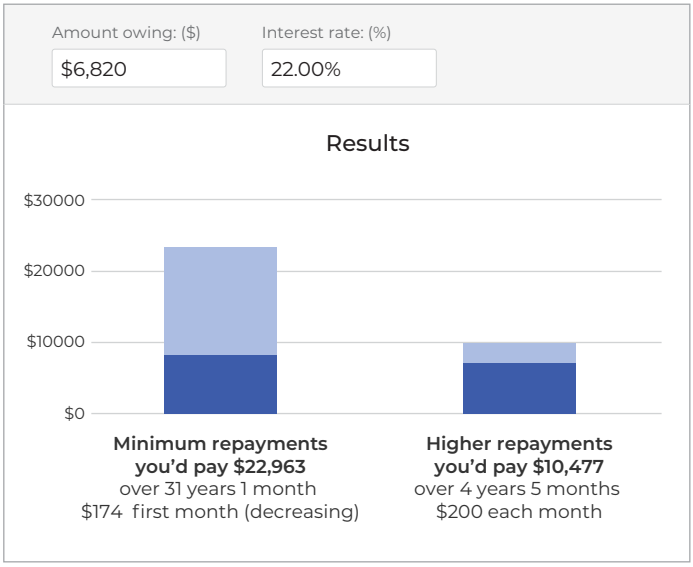
3 EXAMPLE 7 Jamie owes $6820 on his credit card, and he is being charged 22% p.a. interest on the amount owing. He repays $200 on the card each month.

a Calculate the monthly interest Jamie is charged.

b After interest, how much will he still owe at the end of 1 month?

c If Jamie makes $200 monthly repayments, how long would it take to repay $6820 with zero interest charges?

d Jamie's current bank statement shows the following information.



How long would it take Jamie to repay the loan if he only makes the minimum repayments?

e How long will it take Jamie to repay this loan with monthly repayments of $200?

9780170453943

f Jamie is considering changing banks for his credit card.

	Initial interest rate (%)	Standard rate (%)	Annual card fee
Sir Charge Bank	0% for 28 months	13.99	$55
FAQ Bank	0% for 20 months	12.49	$0 for the 1st year $58 thereafter

How much would Jamie still owe at the end of the interest-free (0%) periods with the Sir Charge and FAQ credit cards he is considering? Ignore the annual card fees.

g Calculate the monthly interest charge for Sir Charge and FAQ for the 1st month after the interest-free period.

h Do you recommend Jamie switch to Sir Charge Bank with the longer interest-free period or to FAQ Bank with the lower interest rate? Give a reason for your answer.

4 Claire overindulged with her credit card at the Black Friday online sales. She now owes $5376, and her bank is charging her 21% p.a. interest.

Use the **credit card calculator** on the **MoneySmart** website to answer the following questions.

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a Claire's bank states that her minimum repayment is $137 for the 1st month but then the repayments will decrease each month. How long will it take her to repay the loan at the minimum amount?

b How much total interest will she pay if she only makes the minimum repayments?

c Claire thinks she can make higher repayments. Scroll down to **higher repayments**, then describe the effect of making $400 monthly repayments.

d What advice can you give Claire?

5 Jon has 4 credit cards, all of which have interest rates between 20% and 22% p.a. He owes a total of $12 500 on the cards.

a Approximately how much interest is Jon paying on the cards per month?

b Jon spoke to his bank and arranged to consolidate all the cards into one account at 15% p.a. Calculate the monthly interest charge for the single account in the 1st month.

c Use the **credit card calculator** on the **MoneySmart** website to determine the minimum amount he will have to pay for the 1st month.

d Jon can afford to repay $800 per month. How long will it take him to repay the loan?

e How much interest will Jon save by paying $800 per month compared to paying the minimum repayments?

6 EXAMPLE 8 Mitch currently makes fortnightly repayments on 4 loans, as shown in the table.

	Amount owing	Fortnightly repayments	Number of repayments left	Total value of future repayments
Car loan	$11 260	$183.40	78	$183.40 × 78 = $14 305.20
Credit card	$3745	$176.00	26	
Personal loan	$8830	$199.00	52	
Centrelink debt	$2850	$150.00	19	
Total	$26 785			

a How much does Mitch pay each fortnight to repay his loans?

b Copy and complete the last column in the table. The first entry has been completed as an example.

c Mitch spoke with a debt consolidation company who offered him a $28 600 consolidation loan, which will cover the amount he owes plus the consultancy fee. How much is the consultancy fee?

d The new repayments will only be $375 per fortnight for 4 years. How much less will the new loan repayment be per fortnight than he is paying at present?

e How much more in total will Mitch repay if he changes to the consolidation loan compared to repaying the 4 loans separately?

f Why does repaying a smaller amount fortnightly for the consolidation loan result in a much greater total amount repaid?

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7 Johanna has several unsecured loans with a total value of $25 600. Because the loans are unsecured, she is being charged a higher rate of 18% p.a. interest. The bank offered her a loan at 9% p.a. reducible interest over 5 or 10 years provided she offered her home unit as security. The loan will be the equivalent of a mortgage and have a $10 monthly fee.

a Use the **mortgage calculator** on the **MoneySmart** website to help you to complete this table for the new loan offer.

Length of loan	Monthly repayments	Total amount to be repaid	Interest charges
5 years			
10 years			

b Currently, Johanna is repaying $936 per month, and she has 36 months remaining on her loan. How much interest will Johanna pay if she stays with her current loans?

c If she swaps to a secured loan, what risk does she run?

d What would you recommend Johanna do?

8 Natalie bought her home several years ago when interest rates were much higher. She also has a $15 360 personal loan. Currently her mortgage of $265 000 is on a fixed rate of 5.2% p.a. interest, but she is considering taking out a new mortgage so she can add her personal loan to it. She did some research and obtained the following information about loan charges.

Break fee (to break a fixed-contract loan)	$198
Loan termination fee (to end a loan)	$210
Loan application fee (to apply for a new loan)	$265
Loan switching fee (to change from one loan to another at the same financial institution)	$140

a Natalie's bank is going to charge all the fees listed above except the loan termination fee. Her new loan will be to cover the remaining total of her original mortgage, her personal loan and the fees. Calculate the value of her new loan.

b Natalie summarised her current and future repayments in the table below.

Loan	Details
Current mortgage	Monthly repayments of $2123 for 15 years.
Current personal loan	Monthly repayments of $793.60 for 24 months
Proposed new mortgage	At 2.8% p.a. comparison rate, repayments of $1913 per month for 15 years

If Natalie goes ahead with the consolidation loan, will she be better or worse off? How much less or more will the change cost her?

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Investigation How can you stay on top of your credit card?

- Visit the **MoneySmart** website (moneysmart.gov.au) and find 6 things you can do to stay on top of your credit card.
- Create a poster to display this information.
- Display copies of your poster in areas frequented by people who use credit cards.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 424

Use the terms in this list to complete the chapter summary below.

credit union	decreases	flat	fortnightly	huge
interest	minimum	money	real estate	reducible
repayment	same	secured		

When we borrow money from a bank, $_1$_____ (2 words) or finance company, we have to pay $_2$_____.

If the loan is $_3$_____ and we don't repay the money, the financial institution can take the property or car we listed as security to recoup the money owed to them.

Often finance companies charge simple or $_4$_____ rate interest, where we pay the $_5$_____ amount of interest every year, based on the principal borrowed. Reducing balance loans charge $_6$_____ interest, where the amount of interest $_7$_____ as it is based on the loan amount still owing.

When we borrow a lot of money over a long time – for example, to buy $_8$_____ (2 words) – the total amount we repay can be $_9$_____. Even small changes in interest rates can make a big difference to the size of each $_{10}$_____. If we can pay more than the $_{11}$_____ amount required each month or repay half the monthly repayment $_{12}$_____, we can save a lot of $_{13}$_____ in interest.

Solution to the chapter problem

Problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money in total if she takes out the loan for 15 years or 30 years? What is the difference in the total amount she would repay?

b Suggest a reason Lee might decide to take the loan over 30 years.

Solution

a total amount repaid = monthly repayment × number of months

number of months = number of years × 12

Loan over 15 years

total amount repaid = $2265.95 × 15 × 12
= $407 871

Loan over 30 years

total amount repaid = $1727.70 × 30 × 12
= $621 972

The total amount to repay is higher for the 30-year loan.

difference = $621 972 – $407 871
= $214 101

b Even though Lee will have to repay $214 101 more, she might choose the 30-year loan because she can't afford the monthly repayments of $2265.95 for the 15-year loan.

6 Test yourself

Borrowing money

ANSWERS p. 424

1 EXERCISE 6.1 Dee borrowed $12 000 to buy some equipment for her photography business. The bank charges Dee 7.2% p.a. monthly reducible interest with monthly repayments of $370.

a Find the values of **A** to **H** in Dee's loan table.

Dee's loan
Amount borrowed: $12 000
Interest rate: 7.2% p.a. monthly reducible
Monthly repayments: $370

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$12 000	$72	$12 072	$11 702
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

b How much will Dee owe immediately after she makes her 3rd repayment?

c How much interest will Dee pay in the first 3 months?



Spreadsheet
Reducible loans

2 EXERCISE 6.2 Use the **Reducible loans** spreadsheet for this question.

Chelsea is borrowing $28 000 at 8% p.a. reducible interest to buy a new ute, and her monthly repayments will be $950.

a How long will it take Chelsea to repay the loan?

b How much interest will she pay in total?

3 EXERCISE 6.3 Sam is borrowing $270 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 20 years at 7.2% p.a. monthly reducible interest. In addition, he will be charged a $10 monthly account-keeping fee. Use an online **mortgage calculator** for the following questions.

a How much are Sam's monthly repayments?

b How much will he pay in interest and fees?

c Sam can afford to repay $2400 per month. How long will it take Sam to repay the loan if he pays $2400 per month?

d How much will Sam save in interest if he pays $2400 per month?

4 EXERCISE 6.4

a How does an increase in interest rates affect the value of the repayments on a loan?

b How does an increase in the size of each loan repayment affect the time it takes to pay off the loan and the total amount paid on the loan in total?

5 EXERCISE 6.5 Kelly's monthly rent is $2650. Calculate her weekly rent, correct to the nearest dollar.

6 EXERCISE 6.5 The monthly rent of a property ranges from 0.8% to 1.1% of its market value. Calculate the monthly rent of a property valued at $675 000.

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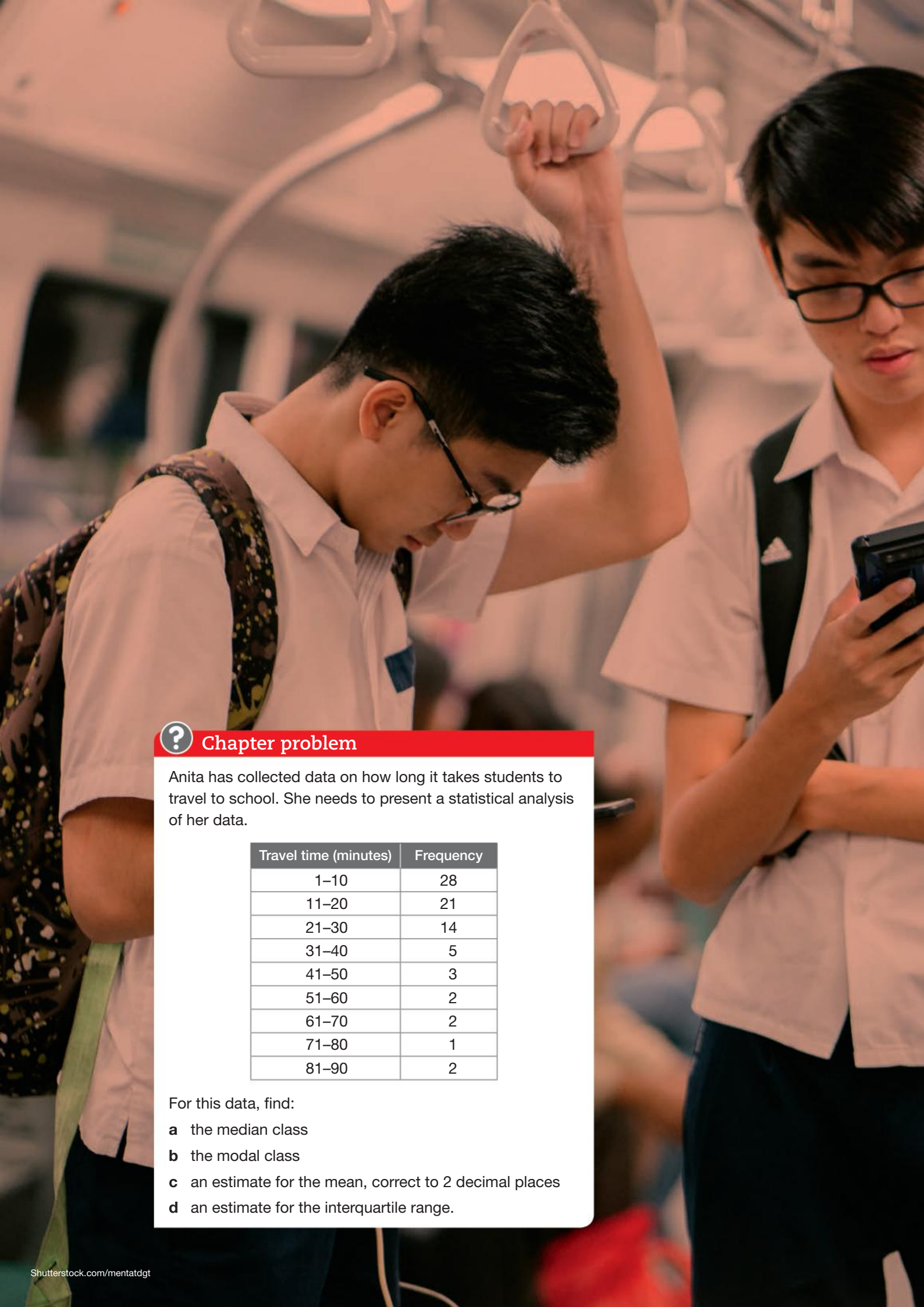
7 EXERCISE 6.6 Cooper saw an ad stating that if he changed his home loan to St Dragon Bank, he would receive a $4000 cashback bonus. Currently he owes $275 000 on his home loan, and his bank would charge him $298 to terminate his contract and $570 to discharge the mortgage if he switched to St Dragon Bank. This table shows his repayments under both loans.

	Monthly repayments	Length of the loan
Current bank	$1521	20 years
St Dragon Bank	$1314	25 years

a Calculate the total amount Cooper will repay if he stays with his current bank.

b Calculate the total value of the repayments with St Dragon Bank.

c Including the $4000 bonus and the fees involved, is it better for Cooper to stay with his current bank or swap to St Dragon Bank? How much better or worse off would he be?



Chapter problem

Anita has collected data on how long it takes students to travel to school. She needs to present a statistical analysis of her data.

Travel time (minutes)	Frequency
1–10	28
11–20	21
21–30	14
31–40	5
41–50	3
51–60	2
61–70	2
71–80	1
81–90	2

For this data, find:

a the median class

b the modal class

c an estimate for the mean, correct to 2 decimal places

d an estimate for the interquartile range.

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CHAPTER 7

DATA ANALYSIS, PROBABILITY AND STATISTICS

ANALYSING DATA

7.1 Measures of central tendency and spread
7.2 Outliers
7.3 The shape of a distribution
7.4 Data in class intervals
7.5 Analysing frequency tables
7.6 Percentiles
7.7 Standard deviation
7.8 Data spreadsheets
Extended investigation: Will your team win?
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Review measures of central tendency and spread
- Identify outliers in a data set and their effects on the mean and median
- Organise data into frequency tables with class intervals
- Find measures of central tendency and spread for data grouped in class intervals
- Use cumulative frequency histograms and polygons to estimate the quartiles, the median and the interquartile range
- Interpret percentiles with large groups of data
- Calculate the standard deviation as a measure of spread
- Use a spreadsheet to analyse data

How are we ever going to use this?

- When we read information in the media involving statistics
- When deciding whether information has been analysed correctly to support opinions
- If we have to collect, organise and analyse data for school, work or elsewhere

Study design coverage

AREA OF STUDY 2: DATA ANALYSIS, PROBABILITY AND STATISTICS

- collection and modelling of data, including the construction of tables or spreadsheets and graphs to represent data and correct representations
- comparing and interpreting data sets and graphs, including using measures of central tendency and spread (percentiles and standard deviation) and cumulative frequency.

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Videos (2):
7.1 The mode, median and mean • Interquartile range

Weblinks (1):
7.8 Bureau of Meteorology

Puzzles (1):
7.1 Above average code puzzle

Worksheets (5):
7.1 Mean, median, mode 1 • Mean, median, mode 2 • Interquartile range
7.7 Statistical calculations • Statistics review
7.8 Investigation components

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

7.1 Measures of central tendency and spread

Video
The mode, median and mean

Video
Interquartile range

Worksheet
Mean, median, mode 1

Worksheet
Mean, median, mode 2

Worksheet
Interquartile range

Puzzle
Above average code puzzle

Measures of central tendency

Mean

The sum of the data values divided by the number of values.

The mean is the best **measure of central tendency** but it can be affected by outliers (extreme values) because it is calculated using every data value.

Median

The middle value (for an odd number of values) or the average of the 2 middle values (for an even number of values) when the data are sorted in order.

The median is the better measure to use if there are outliers because it is not affected by them.

Mode

The most common or frequent value or category.

The mode can be found for both categorical and numerical data.

There can be more than one mode in a data set, or no mode, and it is not affected by outliers.

Measures of spread

Range

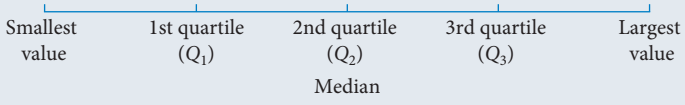
range = largest value – smallest value

Quartiles

Q_1 = lower quartile, Q_2 = median, Q_3 = upper quartile.

These divide the data into 4 equal groups.

25% of the data lies between each quartile.



Interquartile range (IQR)

IQR = upper quartile (Q_3) – lower quartile (Q_1)

50% of the data lies within the interquartile range.

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EXAMPLE 1

The daily maximum temperatures (°C) in Cairns for 16 days in July were:

32, 30, 31, 32, 31, 30, 31, 31, 31, 31, 29, 25, 28, 27, 29, 28

For this set of data, find:

a the median **b** the mode

c the mean **d** the range.

Steps	Working
a Put the data values in order.	25, 27, 28, 28, 29, 29, 30, 30, 31, 31, 31, 31, 31, 31, 32, 32
There are 16 temperatures, so the median will be the average of the 8th and 9th temperatures.	median $= \frac{30 + 31}{2}$ $= 30.5$ The median is 30.5.
b The value 31 occurs 6 times, more often than any other value.	The mode is 31.
c mean $= \frac{\text{sum of values}}{\text{number of values}}$	mean $= \frac{476}{16}$ $= 29.75$
d range = largest value − smallest value	range = 32 − 25 = 7

EXAMPLE 2

The number of runs scored by Jonas' cricket team in their Twenty20 competition were:

78, 142, 126, 174, 142, 120, 137, 104, 161, 173, 149, 154, 180, 137, 174

a For this data, find:

i the 3 quartiles **ii** the interquartile range **iii** the range.

b Which is the better **measure of spread** for this data? Why?

Steps	Working
First, put the scores in order.	78, 104, 120, 126, 137, 137, 142, 142, 149, 154, 161, 173, 174, 174, 180
a i There are 15 scores, so the median or second quartile will be the 8th value. The first and third quartiles will be the middle value (or median) of each half.	$Q_1 = 126$ $Q_2 = 142$ $Q_3 = 173$
ii interquartile range $= Q_3 - Q_1$	interquartile range = 173 − 126 = 47
iii range = largest value − smallest value	range = 180 − 78 = 102
b	As this set of data has an outlier (extreme value) of 78, the interquartile range is a better measure of spread because the outlier has no effect on it. The outlier does affect the range, making it very big, which does not reflect most of the data.

EXERCISE 7.1 Measures of central tendency and spread

ANSWERS p. 424

1 EXAMPLE 1 Find the mean (correct to 1 decimal place), the median, the mode and the range for each set of data.

a The distance (in kilometres) a salesperson travels each working day for 2 weeks:

45, 80, 65, 50, 45, 41, 50, 40, 45, 50

b The number of seats filled on an aircraft over 7 journeys:

56, 104, 78, 100, 89, 125, 100

c The ages of the Maths teachers in a school:

49, 39, 37, 41, 27, 25, 41, 47, 39

2 For each set of data in question **1**, which measure of central tendency (mean, median or mode) best represents the data? Explain your answer.

3 In Year 10, students are awarded a grade (A, B, C, D or E) for their achievement in English. One class obtained the following results:

Grade	A	B	C	D	E
Number of students	4	9	8	6	3

a What is the mode of the class grades?

b Is it possible to find the mean or the median for these grades? Explain your answer.

4 EXAMPLE 2 Find the 3 quartiles and the interquartile range for each set of data.

a The number of orders taken per hour for Crusty Pizza on one Saturday:

12, 3, 5, 15, 6, 9, 10, 18, 15, 8, 4, 1, 5, 7, 9

b The points scored by an AFL club in matches over 1 year:

84, 77, 82, 159, 52, 149, 83, 86, 46, 101, 1

149, 118, 63, 113, 78, 47, 58, 62, 110, 112, 104

c The average monthly rainfall, in mm, over 16 months in Geelong:

74, 83, 32, 85, 29, 21, 19, 61, 85, 81, 63, 9, 85, 6, 29, 16

5 Zina asked the students in her class how much pocket money they were given each week. The amounts are in dollars.

20, 32, 32, 40, 23, 32, 29, 75, 25, 35, 28

a Calculate the mean amount of pocket money.

b Find the median amount of pocket money.

c Which measure (mean or median) is the better reflection of a typical amount of pocket money for this group? Explain your answer.

d Zina wants to convince her parents to give her more pocket money. Which measure would she use in her argument? Why?

e Find the upper and lower quartiles for this data.

f What is the interquartile range for this data?

g Which is the better measure of spread for this data – the range or the interquartile range? Justify your answer.

6 During a severe storm the local emergency services centre recorded the number of minutes it took to reach people needing assistance. The results were:

18, 15, 10, 8, 22, 90, 12, 1, 15, 3, 15

a How many emergency calls did the centre receive?

b Calculate the mean.

c What is the mode of this data?

d Find the median.

e The mean, mode and median are meant to measure the centre of the data. Which one best represents this data? Why?

f One call was from a rural property on the other side of a washed-out bridge. Which of the data values is likely to be the length of time it took the emergency services to reach this property? Justify your answer.

7 Nelson Electronics has a workforce of 45 people, as shown in the table below.

Position	Number of staff	Annual salary ($)
CEO	1	1 700 000
Department managers	4	750 000
Executive assistants	8	300 000
Team members	32	120 000

a Determine the mean and the median salary for this firm.

b When the firm is trying to attract quality employees, what measure do you think they will use for publicising the 'average salary'? Explain your answer.

c Find the upper and lower quartiles for these salaries.

d What is the interquartile range for this data?

e Which is the better measure of spread for this data – the range or the interquartile range? Justify your answer.

8 A netball coach kept a record of the number of goals scored by Christina and Mahla during the first half of the season. The results are shown below.

Christina: 12, 23, 25, 19, 10, 35, 20, 32, 19, 25, 4, 27
Mahla: 13, 30, 20, 24, 17, 25, 17, 34, 20, 29, 19, 15

a For each player, calculate the:

i mean **ii** mode **iii** median **iv** range.

b Who scored the most goals over the 12 games?

c Find the upper and lower quartiles for each player.

d What is the interquartile range for each player?

e Which is the better measure of spread for comparing these players – the range or the interquartile range? Justify your answer.

f Who do you think is the better player? Why?

7.2 Outliers

An **outlier** is a value that is very different from the rest of the data. It can be either much larger than the other values or much smaller than them. An outlier can have a significant effect on the mean, but it has no impact on the median. In the following exercise, you will see the difference an outlier can make.

EXERCISE 7.2 Outliers

ANSWERS p. 425

1 This data shows the ages of members of the Binns and Thomson families.

Ages of the Binns family: 19, 31, 21, 3, 6, 14, 19, 24, 11
Ages of the Thomson family: 19, 31, 21, 3, 6, 14, 19, 24, 91

a Calculate the mean age of each family. Give your answer correct to 1 decimal place.

b What is the difference between these two sets of data?

c What effect does the difference identified in part **b** have on the mean?

2 Eleven houses have been sold in Keswick Street over the last 2 years. The selling prices are listed below:

$620 000, $625 000, $700 500, $738 000, $625 000, $1 800 000
$598 000, $612 000, $696 500, $720 000, $705 000

a Find the median sale price for the houses.

b Find the mean sale price, to the nearest dollar.

c Which measure of central tendency best describes the price of houses in Keswick Street? Justify your answer.

d Which price is the outlier in this data?

e Calculate the mean of the remaining prices when this outlier is removed. Is this mean closer to the median you found in part **a**?

3 Mark and Steve's batting scores for 6 innings of cricket are as follows.

Mark: 45, 48, 53, 38, 32, 40, 51
Steve: 23, 57, 6, 125, 65, 5, 37

a Calculate the mean score for each batter, correct to 1 decimal place.

b Which batter is better if you use the mean to judge them?

c Find the median score for each player.

d Which batter is better if you use the median to judge them?

e Which player would you rather have in your cricket team? Why?

4 Twelve people work in a small business that sells electronics equipment. Their annual salaries are:

$71 000, $67 000, $76 000, $64 000, $61 000, $63 000
$72 000, $66 000, $73 000, $70 000, $275 000, $890 000

a Calculate the mean salary for this company. Give your answer correct to the nearest dollar.

b Find the median salary for this company.

c Which measure (mean or median) is the better reflection of a typical salary for this company? Justify your answer.

d If you wanted to attract new employees to the company, which would you advertise as the average salary? Why?

e Which values are outliers in this data?

f Calculate the mean salary of the remaining values if the outliers are removed.



Alamy Stock Photo/Hero Images Inc.

5 Rivka is a market gardener. In the first 5 years of her business, her annual profits were $32 000, $67 000, $71 000, $72 000 and $75 000

a Find the mean and the median for this data.

b Which measure of central tendency gives the more accurate impression of Rivka's usual annual profit? Explain your answer.

c In what situation might Rivka use the other measure of central tendency to describe her usual annual profit?

d Which score is the outlier in this data?

e Calculate the mean and the median of the remaining scores if the outlier is removed.

6 An obstetrician specialises in providing care for expectant mothers and delivering their babies. The data below shows the number of caesarean deliveries performed by a sample of 20 Australian obstetricians in a 12-month period:

22, 38, 15, 204, 3, 16, 21, 13, 24, 32, 19, 24, 22, 31, 8, 21, 14, 37, 28, 21

a What is the value of the outlier in this set of data?

b Some people would accuse a doctor who performed a lot of caesarean deliveries of doing unnecessary operations. Why might one obstetrician need to perform a lot more caesarean deliveries than the other doctors?

c Calculate the mean and the median of the 20 values.

d Calculate the mean and the median without the outlier.

e What do your answers to parts **c** and **d** show?

7 A property developer has 40 new apartments for sale.

Levels 1–5	20 apartments, $330 000 each
Levels 6 and 7	8 apartments, $380 000 each
Levels 8 and 9	8 apartments, $425 000 each
Level 10	3 apartments, $835 000 each
Level 11	Penthouse apartment, $1.7 million

a Determine the median price of the apartments.

b Calculate the mean price of the apartments.

c When the developer is advertising the apartments for sale, which 'average' would the developer use? Explain your answer.

d The developer will be speaking to potential investors in his company. What 'average' might he use to make his company look more profitable? Explain your answer.

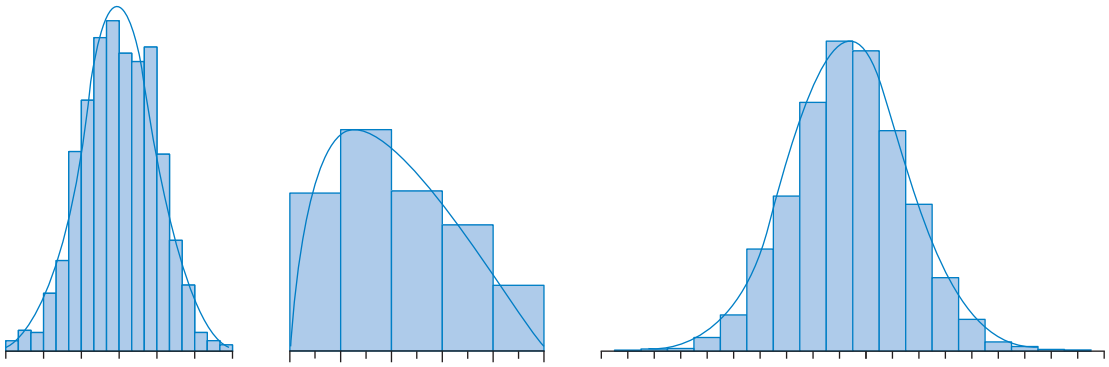
e Which price(s) is an outlier in this data?

f Calculate the mean after removing the outlier(s).

7.3 The shape of a distribution

Numerical data are often graphed on a histogram. When there is an overall pattern to the data in a histogram, we can draw a smooth curve around the histogram to represent the data. We can also draw a curve around dot plots and stem-and-leaf plots to see the shape of the data.

Here are 3 examples of smooth curves that represent the general **shape of a distribution**.



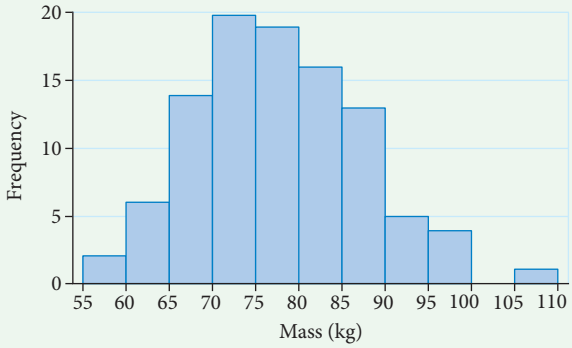
Symmetrical distributions

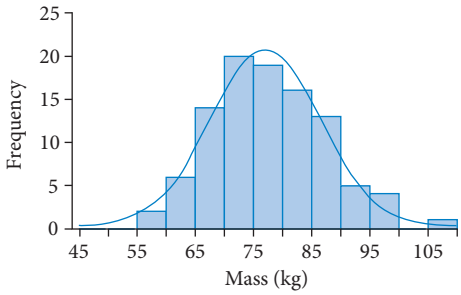
The first and third curves above show **symmetrical distributions**. One half is the mirror-image of the other half. You could fold it down the middle and the two sides would match.

A symmetrical curve could represent the masses of all high school students in Australia.

EXAMPLE 3

Construct a smooth symmetrical curve over this histogram.



Steps	Working
Locate the centre and use the heights of the columns to form a smooth hump for the top of the curve. Then symmetrically flatten out the curve on both sides, slightly above the horizontal axis.	

Histograms rarely have a perfectly symmetrical shape. Sometimes the peak is pushed sideways or the histogram has more than one peak (high point). Statisticians have names for the different types of curves and distributions.

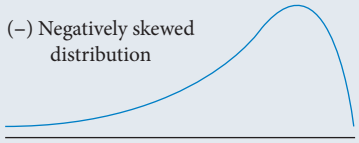
Skewed distributions

When data are not symmetrical but pushed to one side, it is said to be a **skewed distribution** (which means 'slanted').

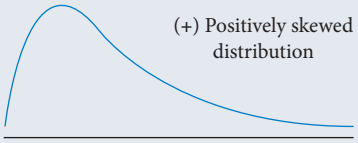
We can identify the type of skew by looking at the 'tail' of the curve.

Skewed distributions

When the tail is on the left, the data are **negatively skewed**.



When the tail is on the right, the data are **positively skewed**.



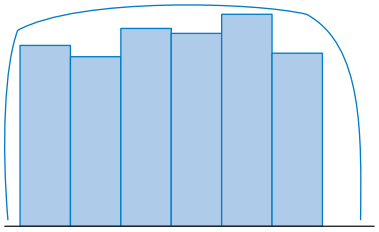
One way of remembering the direction of skewness is to note that on a number line, negative numbers are on the left and positive numbers are on the right.

A **negatively skewed distribution** could represent the marks scored by students on an easy test. Most students achieved high marks and comparatively few scored low marks.

A **positively skewed distribution** could represent house prices in a small country town. There are lots of moderately priced houses and comparatively few highly priced bigger houses and farms.

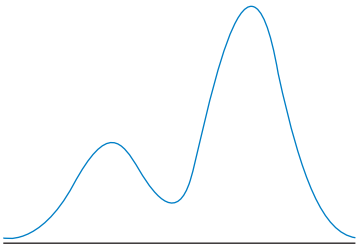
Uniform distributions

If we roll a normal die many times, each of the 6 possible numbers should appear roughly the same number of times, giving a **uniform distribution**. Its graph is close to a big rectangle. Uniform means 'the same', and in a uniform distribution, the scores are evenly distributed.



Bi-modal distributions

This graph represents the amount of traffic crossing a bridge over a day. There are 2 peaks on the curve, corresponding to the morning and afternoon busy times. Because it has 2 peaks, this curve represents a **bi-modal distribution**.



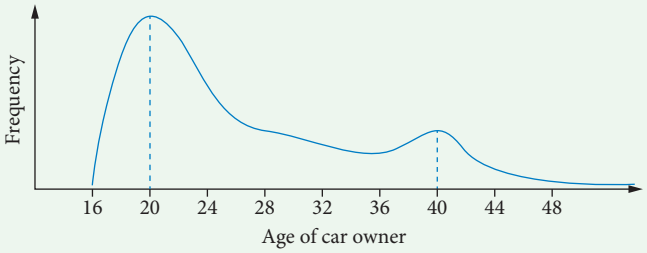
A curve with only one peak represents a **unimodal distribution**.



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EXAMPLE 4

This graph shows the number of car owners by age who claim on their car insurance. Describe the shape of the distribution.



Steps	Working
The distribution is not symmetrical, so it is skewed. The tail points to the right, so it is positively skewed. There are peaks at age 20 and 40, so the distribution is bi-modal.	The distribution is positively skewed and bi-modal.

EXERCISE 7.3 The shape of a distribution

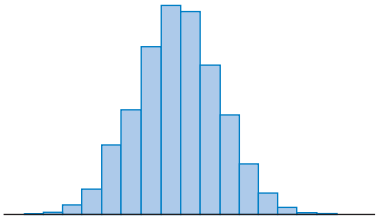
ANSWERS p. 425

1 EXAMPLE 3 Copy each of the following histograms.

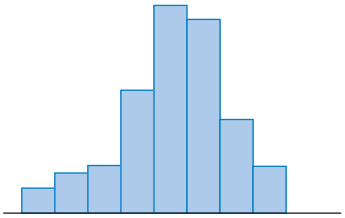
a Sketch a smooth curve over each one.

b State whether the curve is symmetrical or not.

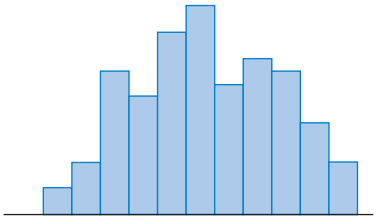
i



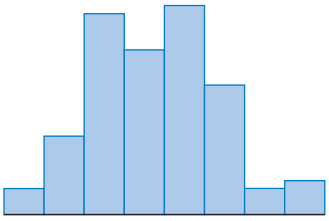
ii



iii

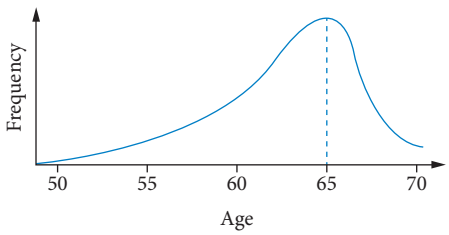


iv

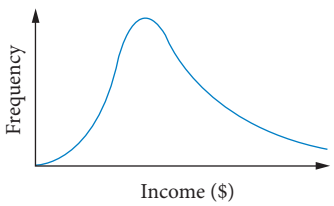


2 EXAMPLE 4 What type of distribution is shown in each graph? Choose from symmetrical, positively skewed, negatively skewed and bi-modal.

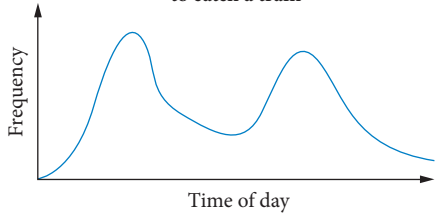
a Age at retirement



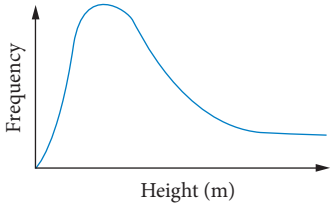
b Australian family income



c Number of people waiting to catch a train



d Height of trees in a forest



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3 Which graphs in question **2** are unimodal (have one peak)?

4 Sketch a graph to represent each distribution described below. Remember to label the axes.

a The age at which a person dies is negatively skewed, with a mode of 80 years.

b The percentage marks students scored on a difficult exam has a bi-modal, positively skewed distribution. A small group scored 90%, but the majority scored around 40%.

c A bus carrying passengers to a concert includes a large group of dancers and a small group of rugby players. The distribution of the masses (weights) of the passengers is bi-modal.

d The heights of 2000 randomly selected, 17-year-old girls are distributed symmetrically.

e When a die is rolled 100 times, the numbers that come up have a uniform distribution.

f The number of minutes visitors to a museum spend looking at a display that includes a 5-minute video is bi-modal. Most visitors spend less than 1 minute looking at the display, but some also watch the entire video and the display. Very few people look at the display for between 1 and 5 minutes.



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5 How can you tell that the data in this box-and-whisker plot are positively skewed?



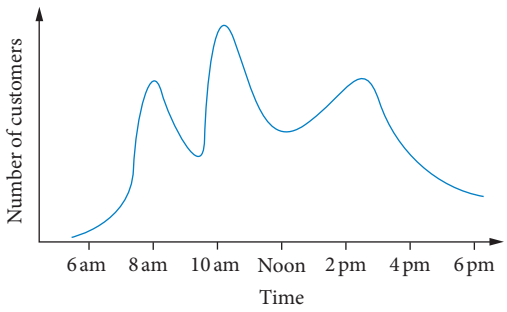
6 Construct a box-and-whisker plot that shows data that are negatively skewed.

7 Construct a dot plot with scores from 1 to 8 that has a bi-modal, symmetrical distribution.

8 This 'tri-modal' distribution represents the number of customers at a city business at different times of the day.

a Why is it called a tri-modal distribution?

b What type of business do you think it could be? Give a reason for your answer.



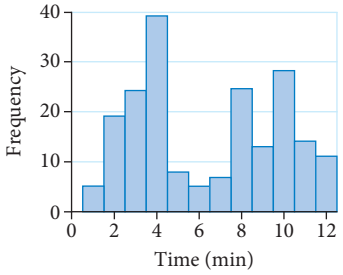
9 This stem-and-leaf plot shows the exam marks of a group of History students.

Stem	Leaf
5	3 6 7 8 9
6	2 5 4
7	1 2
8	2
9	0

a Describe the shape of the distribution.

b Change 4 of the exam marks to make the distribution more like a symmetrical distribution. For example, you could change the 59 to 79.

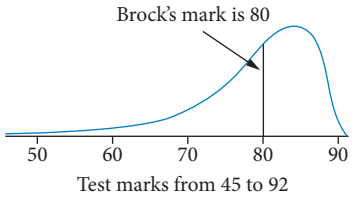
10 This histogram shows the amount of time customers at an appliance store had to wait for items to be brought in from the warehouse.



a Describe the shape of the distribution.

b Suggest a possible reason for the shape of the distribution.

11 Sometimes, test marks in student reports are presented as a graph showing the position of the mark relative to the distribution of the marks of all students. The graph below shows Brock's test mark for mathematics compared to the rest of his class.



a Describe the shape of the distribution.

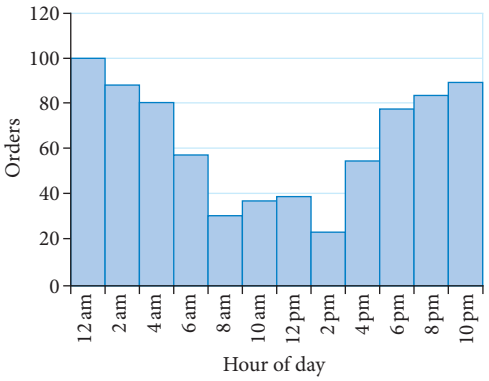
b Was the mathematics test easy or hard? Justify your answer.

c Estimate the mode mark.

d How did Brock perform in the test compared to the rest of the students?

Investigation Is it really bi-modal?

This histogram shows the number of online orders a company receives per 2-hour interval over a 24-hour period. The data appears to be bi-modal.



1 Construct another graph to display the same data, but start the horizontal axis at 12 pm.

2 Describe the shape of the distribution displayed on the graph you constructed.

3 When you are displaying cyclic data (for example, hourly, daily or monthly data) describe how you can construct a graph that makes the data appear to be bi-modal when it is really unimodal.

7.4 Data in class intervals

Numerical data can be either discrete or continuous.

Discrete data are counted and can only take on separate, distinct values. For example, the number of children in a family is discrete because it can only be whole values: 0, 1, 2 …

Continuous data are measured on a smooth scale and can take on a full range of values. For example, a town's maximum daily temperature can be 29°C or 29.2°C or 29.23°C, etc. The level of accuracy of the measurement depends on the measuring instrument.

For a large number of data values, particularly continuous data, it is not practical to use a frequency table that includes every value. Instead, we can group the data into **class intervals**, then use a grouped frequency table to count the **frequency** of each class interval.

When we have a frequency table with class intervals, we can add extra columns to help with analysing the data. We add columns for:

- the **cumulative frequency** (a running total of frequency) to help estimate the median
- the **class centre** (cc) for each class interval (the average of the first and last values in the interval)
- the frequency multiplied by the class centre ($f \times$ cc) to help calculate an estimate for the mean.

EXAMPLE 5

Classify each type of numerical data as continuous or discrete.

a Number of goals scored in a soccer match

b Speed of vehicles on a highway

Steps	Working
a The values are whole numbers.	discrete data
b Speed is measured on a smooth scale with no gaps.	continuous data

EXAMPLE 6

The daily maximum temperatures (°C) for January and February for the Goulburn Valley are given below.

January

28.4, 26.4, 24.9, 22.5, 27.6, 28.5, 31.2, 33.1, 35.0, 32.1, 35.9
32.5, 39.1, 29.5, 29.1, 34.6, 38.5, 38.3, 23.1, 22.0, 30.0, 32.1
36.1, 31.0, 25.7, 29.0, 30.2, 35.1, 36.2, 38.3, 36.6

February

21.6, 27.6, 31.3, 35.4, 35.9, 34.0, 20.2, 23.5, 34.7, 39.0, 41.2
32.0, 26.0, 25.1, 29.5, 34.0, 35.5, 26.5, 23.4, 21.0, 26.4, 32.5
35.0, 30.4, 20.1, 21.8, 25.6, 26.1

Arrange this data in a grouped frequency table, using class intervals of 10–14.9, 15–19.9, 20–24.9, 25–29.9, 30–34.9, 35–39.9 and 40–44.9. Include a cumulative frequency column.

Steps **Working**

Temperature (°C)	Tally	Frequency	Cumulative frequency
10–14.9		0	0
15–19.9		0	0 + 0 = 0
20–24.9	𝍸 𝍸 |	11	0 + 11 = 11
25–29.9	𝍸 𝍸 𝍸 |	16	11 + 16 = 27
30–34.9	𝍸 𝍸 𝍸 |	16	27 + 16 = 43
35–39.9	𝍸 𝍸 𝍸	15	43 + 15 = 58
40–44.9	|	1	58 + 1 = 59
	Total	59	

EXERCISE 7.4 Data in class intervals

ANSWERS p. 426

1 EXAMPLE 5 Classify each type of numerical data as continuous (C) or discrete (D).

- **a** Distances jumped by a team of long jumpers
- **b** Reaction times of car drivers
- **c** Number of points scored by Essendon in the first 7 games of the season
- **d** Number of cups of coffee people drink each day
- **e** Number of pages in your favourite books
- **f** Weights of pieces of luggage on a flight to England
- **g** Amounts of water used by a household each month
- **h** Shirt sizes of Year 11 students
- **i** Class sizes at your school
- **j** Times taken to run 200 metres



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2 EXAMPLE 6 A PE teacher measured the heights of 25 Year 11 students, recording the results to the nearest centimetre:

151, 167, 181, 172, 179, 155, 159, 162, 169, 174, 178, 180, 158
166, 171, 168, 157, 160, 175, 172, 150, 169, 163, 170, 176

- **a** Arrange this data in a grouped frequency table, using class intervals of 150–154, 155–159, 160–164, 165–169, 170–174, 175–179 and 180–184. Include a cumulative frequency column.
- **b** Is this data discrete or continuous?

3 The numbers of incidents of malicious damage to property per month in the inner city were recorded over 3 years:

54, 41, 55, 49, 37, 38, 37, 48, 51, 44, 52, 44, 58, 70, 60, 46, 63, 54
45, 43, 46, 55, 55, 67, 49, 66, 90, 45, 66, 62, 51, 51, 53, 53, 38, 52

- **a** Arrange this data into a frequency table using class intervals of 36–40, 41–45, 46–50, 51–55, etc, up to 86–90. Include a cumulative frequency column.
- **b** Are there any outliers in this data set? If so, what are they?
- **c** Is this discrete or continuous data?

4 This table shows the speeds of cars passing a school during 1 hour.

Speed (km/h)	Frequency
51–60	21
61–70	13
71–80	4
81–90	2

- **a** How many cars passed the school in this hour?
- **b** How many cars followed the speed limit of 60 km/h?
- **c** The police decided to book anyone travelling at more than 10 km/h over the speed limit. How many drivers did they book?
- **d** Is this data discrete or continuous?
- **e** Copy the table and add a cumulative frequency column.

5 Customers of a coffee shop were asked how many cups of coffee they drank each week.

Number of cups of coffee	Frequency
1–5	3
6–10	13
11–15	15
16–20	7
21–25	12

- **a** How many people were surveyed?
- **b** How many people drank 10 or fewer cups of coffee each week?
- **c** How many people drank an average of more than 20 cups of coffee each week?
- **d** Is this discrete or continuous data?
- **e** Copy the table and add a cumulative frequency column.

7.5 Analysing frequency tables

We can estimate measures of central tendency and spread from a frequency table with class intervals.

EXAMPLE 7

This is the data on coffee drinking from question **5** in Exercise 7.4.

Number of cups of coffee	Frequency
1–5	3
6–10	13
11–15	15
16–20	7
21–25	12

a For this set of data, find:

- **i** the median class
- **ii** the modal class
- **iii** an estimate for the mean.

b Is it possible to find the range for grouped data? Explain your answer.

Steps **Working**

a i Add a cumulative frequency column.

There are 50 values so the median (middle) score will be the average of the 25th and the 26th values. Reading from the cumulative frequency column, both of these values are in the 11–15 class.

Number of cups of coffee	Frequency	Cumulative frequency
1–5	3	3
6–10	13	16
11–15	15	31
16–20	7	38
21–25	12	50

The median class is the 11–15 class.

ii The class with the highest frequency is 11–15, with a frequency of 15.

The modal class is the 11–15 class.

iii For an estimate of the mean, we need to add columns for class centre (cc) and frequency × class centre ($f \times$ cc).

Number of cups of coffee	Class centre, cc	Frequency, f	$f \times$ cc
1–5	3	3	9
6–10	8	13	104
11–15	13	15	195
16–20	18	7	126
21–25	23	12	276
	Total	50	710

This is only an *estimate* for the mean because we don't have the exact data values.

$$\text{mean} \approx \frac{710}{50} = 14.2$$

b It is not possible to find the range for grouped data because we only have the class intervals and do not know the actual values.

Cumulative frequency histograms and polygons

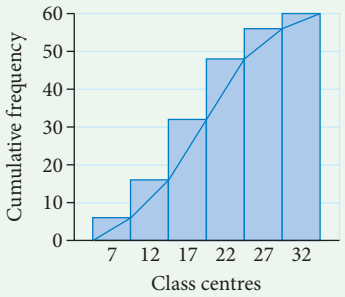
A **cumulative frequency histogram** is a column graph of cumulative frequency.

A **cumulative frequency polygon** is a line graph of cumulative frequency, drawn by joining the **top right-hand corner** of each column of a cumulative frequency histogram.

A cumulative frequency histogram and polygon can be used to estimate the median, the quartiles and the interquartile range.

EXAMPLE 8

The ages of people swimming at an indoor pool were recorded and grouped into 6 class intervals, then the data were graphed on a cumulative frequency histogram and polygon.



a Copy and complete this table.

Ages	Class centre	Frequency	Cumulative frequency
5–9		6	6
10–14			16
15–19			32
20–24			48
25–29			56
30–34			60

b Use the cumulative frequency polygon to estimate:

i the quartiles **ii** the median **iii** the interquartile range.

Steps

a For each class interval, the class centre is the average of the first and last values in the interval.

For the interval 5–9, the class centre is $\frac{5+9}{2} = \frac{14}{2} = 7$, halfway between 5 and 9.

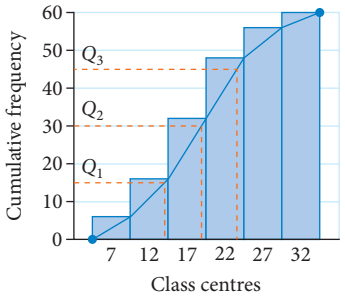
To complete the frequency column, subtract pairs of cumulative frequencies: 16 – 6 = 10, 32 – 16 = 16, and so on.

Working

Ages	Class centre	Frequency	Cumulative frequency
5–9	7	6	6
10–14	12	10	16
15–19	17	16	32
20–24	22	16	48
25–29	27	8	56
30–34	32	4	60

b There are 60 values in the data set.

i To find the quartiles, divide the 'Cumulative frequency' axis of the graph into 4 equal parts (60 ÷ 4 = 15) and draw lines for Q_1, Q_2 and Q_3 (from 15, 30 and 45 respectively) across to the polygon, then down to the 'Class centres' axis. The centre of each column is the class centre.



Estimating values on the 'Class centres' axis:

$Q_1 \approx 14$ $Q_2 \approx 18$ $Q_3 \approx 23$

ii The median is Q_2.

The median is 18.

iii The interquartile range (IQR) is $Q_3 - Q_1$.

IQR ≈ 23 – 14
= 9

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EXERCISE 7.5 Analysing frequency tables

ANSWERS p. 426

1 EXAMPLE 7 Jordan is looking for a good used car. This table shows the prices of the cars she has considered.

Price ($)	Frequency
5000–<6000	3
6000–<7000	5
7000–<8000	6
8000–<9000	12
9000–<10 000	8

a How many cars has Jordan looked at?

b Copy this table and add columns for class centre, cumulative frequency and frequency × class centre.

c Use the table to find:

i the median class

ii the modal class

iii an estimate for the mean.

2 Max surveyed the amount of money his friends spent on fuel in the last week.

Fuel cost ($)	Frequency
10–<20	9
20–<30	11
30–<40	5
40–<50	3
50–<60	2
60–<70	1
70–<80	2
80–<90	3

a How many people did Max survey?

b Copy this table and add columns for class centre, cumulative frequency and frequency × class centre.

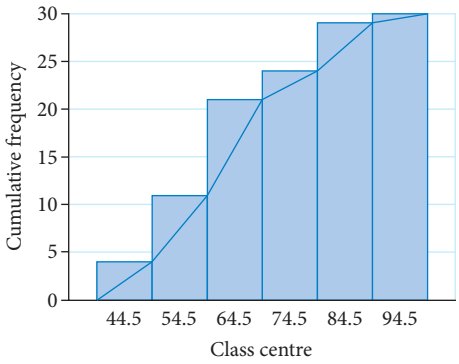
c Use the table to find:

i the median class

ii the modal class

iii an estimate for the mean (correct to 1 decimal place).

3 The percentage marks of a mathematics assessment task were recorded and grouped into 6 class intervals, and a cumulative frequency graph was produced.



a How many students are in this class?

b Copy and complete this frequency table for the data.

Marks (%)	Class centre (cc)	Frequency (f)	Cumulative frequency	Frequency × class centre ($f \times$ cc)
40–49	44.5		4	
50–59	54.5		11	
	64.5		21	
	74.5		24	
	84.5		29	
	94.5		30	
	Total			

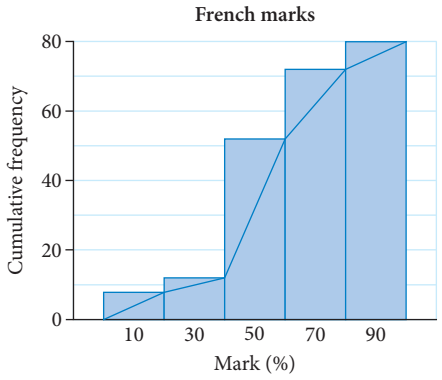
c For this set of data, find:

i the median class

ii the modal class

iii an estimate for the mean (correct to 1 decimal place).

4 EXAMPLE 8 The test marks of 80 students in a French exam were grouped into 5 class intervals and summarised in this cumulative frequency graph.



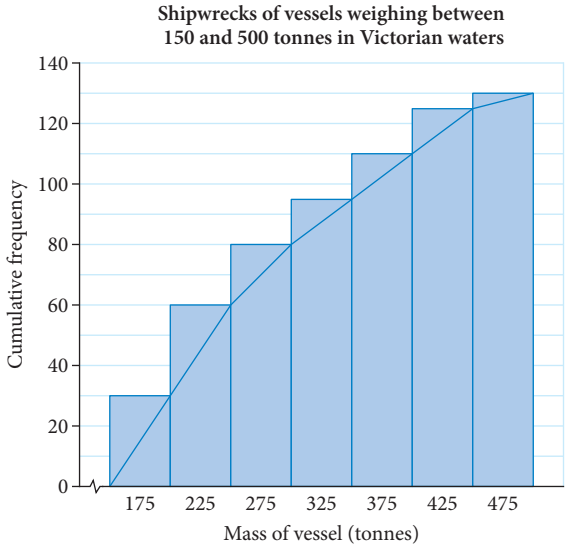
a Copy and complete this table.

Class centre	Frequency	Cumulative frequency
10		8
30		12
50		52
70		72
90		80

b Find an estimate for the median.

c Find an estimate for the interquartile range.

5 The waters around the southern coast of Australia can be very dangerous. This graph shows information about the masses of ships that have been wrecked on the Victorian coast, grouped into class intervals with the class centres shown on the 'Mass of vessel' axis.



a Copy and complete this table.

Class centre	Frequency	Cumulative frequency
175		30
225		
275		
325		
375		
425		
475		130

b Use the graph to estimate the median.

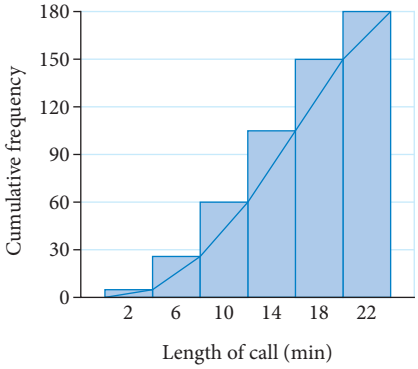
c Use the graph to estimate the interquartile range.

6 Customers complained that they had difficulty contacting the Chatphone company because its helpline was often busy. The company hired Francine to investigate the time its operators took to complete each call. This cumulative frequency graph summarises this data during a 1-hour period.

Find an estimate for:

a the median

b the interquartile range.



7 Divya recorded the typical number of kilometres her friends drive each day.

60, 54, 21, 32, 18, 15, 0, 25, 60, 32, 12, 22, 32, 45, 27, 21, 15, 18
44, 51, 60, 16, 31, 19, 15, 23, 39, 41, 50, 26, 10, 62, 17, 0, 11, 20

a Copy and complete the frequency distribution table.

Class	Class centre (cc)	Tally	Frequency (f)	Cumulative frequency	$f \times$ cc
0–9	4.5				
10–19					
20–29					
30–39					
40–49					
50–59					
60–69					
		Total			

b For this set of data, find:

i the median class **ii** the modal class

iii an estimate for the mean (correct to 1 decimal place).

c Draw a cumulative frequency histogram and polygon for this data.

d Use your graphs to find an estimate for:

i the median **ii** the interquartile range.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Investigation Percentiles

Parts of words give us hints about what they mean. Words like 'quad' and 'quart' have something to do with 4. A quadrilateral is a shape with 4 sides, a quad bike has 4 wheels and when a mother has 4 babies at the same time, they are called quadruplets. A quarter is 1 of 4 equal parts, and when we divide an ordered group into 4 equal subgroups, we are making **quartile** groups.

The word 'cent' is related to 100; for example, there are 100 cents in a dollar. Per cent (%) is a value 'out of 100' and a century is 100 years or 100 runs in cricket. Percentile groups are what we get when we sort a large group in order and make 100 equal subgroups.

Now imagine that all the students in your school were lined up on the school oval from the shortest to the tallest. Find out how many students there are in your school.

1 If the students were divided into 100 approximately equal groups, how many would be in each group?

2 Each of the 100 groups contains 1% of the whole school.

- The height of the tallest person in the shortest group is called the 1st percentile.
- The height of the tallest person in the second-shortest group is called the 2nd percentile.
- The height of the tallest person in the tallest group is called the 100th percentile.

In which group do you think you would be?

3 In which groups would you expect most of the Year 8 students to fall?

4 In which groups would you expect most Year 12 students to fall?

5 How many percentile groups are in one quartile group?

6 Why is the 25th percentile the same as the 1st quartile?

7 What percentile is the same as the 3rd quartile?

8 What percentile is the same as the median?

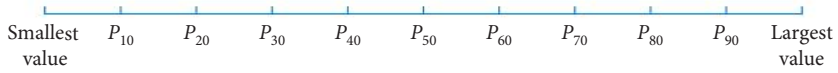


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7.6 Percentiles

When we divide data into 4 equal parts, they are separated by **quartiles**: Q_1, Q_2, Q_3. Another way of sorting large sets of data into groups is to divide it using **percentiles**.

Percentiles (P_1, P_2, P_3 … P_{99}) are values that divide the data into 100 equal groups.

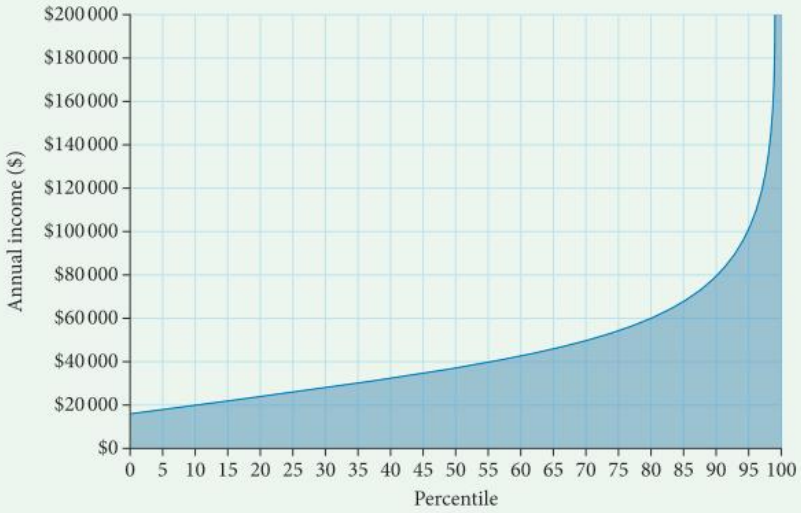


- P_{24} cuts off the lowest 24% of scores
- P_{60} cuts off the lowest 60% of scores
- P_{87} cuts off the lowest 87% of scores (or the top 13% of scores)

Percentiles are only meaningful when analysing large sets of data.

EXAMPLE 9

This graph shows the annual income percentiles for last year.



a What income is at the 20th percentile?

b Georgina earns $40 000 per year. What percentile is this?

c What is the approximate difference in income between the 80th and 90th percentiles?

d The graph only goes up to the 99th percentile. Suggest a possible reason for this.

Steps	Working
a Find the 20th percentile on the horizontal axis and read off the value on the vertical axis.	The income is about $24 000.
b Find $40 000 on the vertical axis and read off the value on the horizontal axis.	It is approximately the 54th percentile.
c Find the incomes for the 80th and 90th percentiles.	80th percentile: $60 000 90th percentile: $79 000
Find the difference between incomes for the 2 percentiles.	difference: 79 000 – 60 000 = 19 000 The difference in income is approximately $19 000.
d The 100th percentile would have very high incomes.	Including very high incomes makes it difficult, or nearly impossible, to have an appropriate scale on the vertical axis.

EXAMPLE 10

The following information is based on population data for the heights of girls aged 16 years.

- The 5th percentile, $P_5 = 152$ cm
- The median, the 50th percentile, $P_{50} = 163$ cm
- The 75th percentile, $P_{75} = 167$ cm
- The 90th percentile, $P_{90} = 171$ cm
- The 97th percentile, $P_{97} = 175$ cm

In the following questions, all of the girls mentioned are aged 16.

a Holly's height is 175 cm. Is she tall for her age? What percentage of 16-year-old girls are taller than her?

b Olga is taller than 90% of girls her age. What is her height?

c $\frac{1}{4}$ of girls Verity's age are taller than her. How tall is she?

d What height separates the bottom 5% of heights from the top 95%?

e What percentile is a height of 163 cm?

Steps	Working
a 175 cm is the 97th percentile.	A height at the 97th percentile means that Holly is taller than 97% of girls her age. Holly is tall for her age, with only 3% of girls aged 16 taller than her.
b 90% is the 90th percentile.	Olga's height = $P_{90} = 171$ cm
c $\frac{1}{4} = 25\%$, so 25% of girls are taller than Verity. This puts her height at the 75th percentile.	Verity's height = $P_{75} = 167$ cm
d 5% is the 5th percentile.	$P_5 = 152$ cm
e	163 cm = P_{50} P_{50} is also the median.

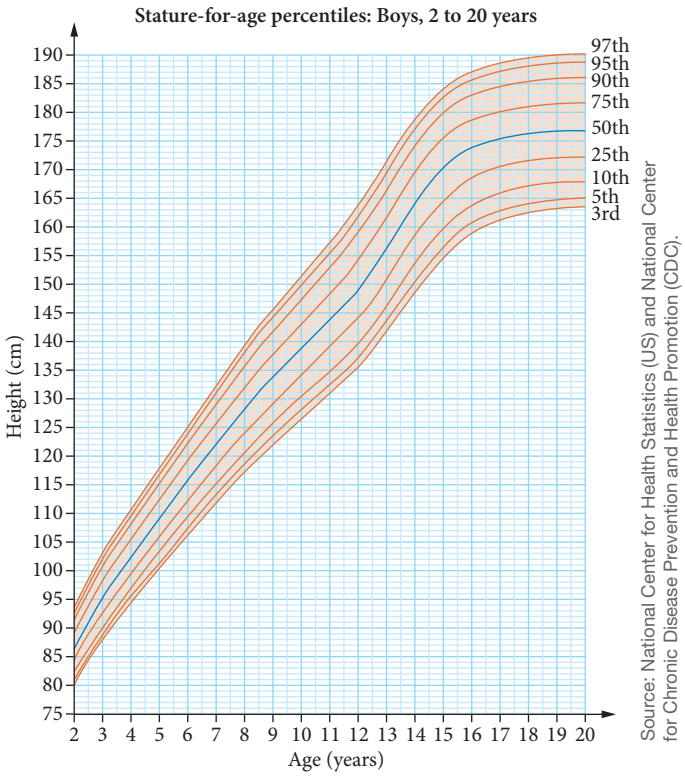
EXERCISE 7.6 Percentiles

ANSWERS p. 427

1 EXAMPLE 9 Use the graph in Example 9 to answer the following questions.

- **a** What income is at the 40th percentile?
- **b** Sue earns $60 000. What percentile is this?
- **c** What is the approximate difference in income between the 30th and 50th percentiles?
- **d** What is another name for the 50th percentile?
- **e** Approximately what percentage of the population earns more than $80 000?

2 This percentiles chart shows the range of heights for boys aged 2 to 20 in the US.



- **a** Izak is aged 9 and 129 cm tall. What percentage of boys his age are shorter than him?
- **b** **i** Justin is 11 years old and 155 cm tall. What percentage of boys his age are shorter than him?
  - **ii** A boy's height usually follows the same percentile level as he grows up. How tall should Justin be when he turns 18?
- **c** Liong is 103 cm tall, which is at the 10th percentile for boys his age.

  How old is he?
- **d** Asam is 16 and his height is at the 3rd quartile.
  - **i** What is Asam's height?
  - **ii** Predict what Asam's height will be when he turns 20.

3 EXAMPLE 10 The following information is based on population data for the body mass indices (BMI, in kg/m^2) of boys aged 16 years.

- The 10th percentile, $P_{10} = 17.6$
- The 25th percentile, $P_{25} = 18.8$
- The 50th percentile, $P_{50} = 20.6$
- The 90th percentile, $P_{90} = 25.4$
- The 97th percentile, $P_{97} = 29.4$

In the questions below, all of the boys mentioned are aged 16.

- **a** Sanjay has the median BMI for boys his age. What is his BMI?
- **b** Michael has a BMI of 18.8. Is this high for his age? What percentage of boys aged 16 have a BMI lower than him?
- **c** 10% of boys aged 16 have a higher BMI than Harley. What is his BMI?
- **d** Adrian has a BMI of 29.4. Is this high for his age? What percentage of boys aged 16 have a BMI higher than him?
- **e** 10% of boys aged 16 have a lower BMI than Lachy. What is his BMI?

4 This table shows the percentiles for the heights (cm) of girls aged 2 to 5 years, according to the child growth standards of the World Health Organization (WHO).

Age (years)	P_5	P_{25}	P_{50}	P_{75}	P_{85}	P_{99}
2.0	80.4	83.5	85.7	87.9	89.1	93.2
2.5	84.9	88.3	90.7	93.1	94.3	98.9
3.0	88.8	92.5	95.1	97.6	99.0	103.9
3.5	92.4	96.3	99.0	101.8	103.3	108.5
4.0	95.6	99.8	102.7	105.6	107.2	112.8
4.5	98.7	103.1	106.2	109.2	110.9	116.7
5.0	101.6	106.2	109.4	112.6	114.4	120.5

© WHO 2017

a What is the median height of a 4-year-old girl?

b **i** Libby is aged 2.5 years and is 88.3 cm tall. Is she tall for her age? What percentage of girls her age are shorter than her?

ii What is Libby's expected height when she turns 5 years old?

c Only 15% of girls Renee's age are taller than her. How tall is she if she is 3.5 years old?

d Mikayla is 2 years old and 93.2 cm tall. Is she short for her age? What percentage of girls her age are taller than her?

e Mia is aged 3 years and her height is at the 3rd quartile. What is her height now and what will it be in 18 months' time?

7.7 Standard deviation

The **standard deviation** is another measure of spread, like the range and the interquartile range. It describes how far each data value is from the mean. The larger the standard deviation, the more spread out the values are; the lower the standard deviation, the more consistent the values are.

The symbol for standard deviation is 'σ', which is the Greek letter sigma. The formula for standard deviation is quite complicated and is beyond the scope of this course. Instead, you can use your calculator's statistics mode to calculate it.

Worksheet Statistical calculations

Worksheet Statistics review

EXAMPLE 11

The net weekly wages of 8 casual workers are:

$730, $490, $600, $440, $490, $370, $700, $580

Use your calculator to find the mean and standard deviation.

Steps **Working**

Follow the instructions for the statistics mode (SD or STAT) of your calculator:

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 730 = 490 =, etc. to enter in column AC to leave table	730 M+ 490 M+, etc.
Calculate the mean. ($\overline{x}$ = 550) The symbol for mean is $\overline{x}$.	SHIFT 1 Var $\overline{x}$ =	RCL $\overline{X}$
Calculate the standard deviation. (σ = 117.260…)	SHIFT 1 Var $x\sigma_n$ =	RCL σx
Return to normal (COMP) mode.	MODE COMP	MODE 0

Calculate the mean. mean, $\overline{x}$ = $550

Calculate the standard deviation. standard deviation, $\sigma \approx$ $117.26

EXAMPLE 12

Twenty echidnas from Booderee National Park were tagged and returned to their habitat. Rangers later captured several samples of 10 echidnas and recorded the number tagged in each sample.

Find, correct to 2 decimal places:

a the mean number of tagged echidnas per sample

b the standard deviation of tagged echidnas.

Echidnas tagged per sample	Frequency
0	8
1	11
2	5
3	4
4	2
5	1



Shutterstock.com/Stephen Reynolds

Steps **Working**

For the data presented in the frequency table, follow the instructions for your calculator as shown.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 0 = 1 =, etc. to enter in x column 8 = 11 =, etc. to enter in FREQ column AC to leave table	0 2ndF STO 8 M+ 1 2ndF STO 11 M+ etc.
Calculate the mean. ($\bar{x}$ = 1.4838…)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{X}$
Calculate the standard deviation. (σ = 1.340…)	SHIFT 1 Var $x\sigma_n$ =	RCL σx

a Calculate the mean. mean, $\bar{x} \approx 1.48$

b Calculate the standard deviation. standard deviation, $\sigma_n \approx 1.34$

EXAMPLE 13

Indoor cricket selectors are trying to choose between two pairs of players (Sanjeev and Angus vs Christian and Tyler) for the state team. The combined numbers of runs they scored in the previous 6 matches are as follows:

Sanjeev and Angus: 34, 30, 36, 35, 29, 34

Christian and Tyler: 41, 26, 37, 35, 25, 34

Which pair is the more consistent?

Steps	Working
Use your calculator to find the standard deviation for each pair's scores.	Sanjeev and Angus: $\sigma \approx 2.58$ Christian and Tyler: $\sigma \approx 5.74$
The more consistent pair is the one with the smaller standard deviation.	Sanjeev and Angus are the more consistent pair of cricketers.

EXERCISE 7.7 Standard deviation

ANSWERS p. 427

1 EXAMPLE 11 Find the mean and standard deviation of each set of data. Give your answers correct to 2 decimal places.

a 20, 24, 17, 21, 19, 26, 21, 23, 16

b 60, 51, 82, 65, 27, 38, 32, 64, 18, 20, 74

c 14, 13, 17, 14, 13, 16, 16, 17, 16, 12, 12

d 67, 58, 89, 72, 34, 45, 39, 71, 25, 27, 81

2 Martin is looking for a job in the construction industry. He collected information on the wages of apprentices at a large construction company. The following amounts are the weekly wages of 10 apprentices.

$542, $728, $884, $601, $774, $586, $801, $675, $758, $627

a Find the mean wage.

b Find the standard deviation of this data. Give your answer correct to 1 decimal place.

c Give a reason for the high standard deviation for this data.

3 Jana is looking for a unit to rent in Highgate Hill. She used a website to find the following data about the weekly rent for a number of units in Highgate Hill.

$400, $450, $600, $530, $529, $430, $430, $390, $550, $450, $350, $420, $540, $495, $480

a Find the mean rental price.

b Find the standard deviation for this data. Give your answer correct to 2 decimal places.

c Why might places in the same area have different weekly rents?

d If Jana can afford to pay $420 per week, how difficult will it be for her to find a place to rent?

4 EXAMPLE 12 Students' marks out of 10 in a quiz are shown in this frequency table.

Mark	Frequency
5	4
6	3
7	8
8	4
Total	19

Find, correct to 1 decimal place:

a the mean

b the standard deviation.

5 Andrea surveyed 50 students on how many text messages they sent within the last 2 hours.

Number of texts	Frequency
0	8
1	4
2	10
3	10
4	15
5	3
Total	50

For this data, find, correct to 2 decimal places:

a the mean

b the standard deviation.

6 Centrelink officers recorded how many jobs each person applied for before they received their first job interview.

For this data, find, correct to 2 decimal places:

a the mean

b the standard deviation.

Number of jobs applied for	Frequency
1	2
2	5
3	10
4	6
5	11
6	13
7	3

7 EXAMPLE 13 Nikita is looking for her ideal place to live. She likes a consistent, warm climate. Nikita is considering 2 places: Port Paradise and Palm Tree Cove. This table shows the mean monthly temperatures (°C) of each location.

	J	F	M	A	M	J	J	A	S	O	N	D
Port Paradise	30	28	26	25	24	24	24	24	26	27	28	30
Palm Tree Cove	32	34	28	26	25	22	20	21	25	29	31	32

a Calculate, to 2 decimal places, the standard deviations of the temperatures of each place.

b Use your answers to part **a** to advise Nikita which place would suit her better.

8 Brad, Aryn and Kim are keen computer gamers. Each week they play 12 competition games against players of equal ability. The table shows the number of games each of them won during the last 6 weeks.

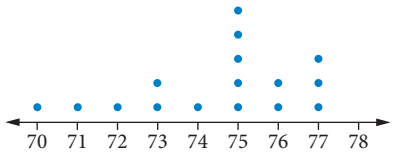
Brad	8	9	7	10	5	11
Aryn	2	4	6	3	7	5
Kim	10	5	8	3	9	4

a Calculate the mean number of games won out of 12 and the standard deviation of the weekly results for each player. Give your answer correct to 1 decimal place.

b Which player is the most consistent? Explain your answer.

c You have to select one player to compete at the state contest. Who would you choose and why?

9 Paul's golf scores for 16 rounds of golf are shown on this dot plot.



a From smallest to largest, write out a list of Paul's scores for the 16 rounds of golf.

b Calculate the mean and the standard deviation for this data.

c Is Paul a consistent player? Explain your answer.



iStock.com/GlobalStock

10 This stem-and-leaf plot shows Carly's pulse rate every 2 minutes while she was exercising on the walking machine at the gym.

Stem	Leaf
12	7 9
13	0 3 5
14	3 3 3 6
15	4 5 7

a Would you expect a high or low standard deviation for this data? Explain your answer.

b Calculate the standard deviation for this data.

c Why do we not want a consistent pulse rate in this situation?



iStock.com/Imgorthand

11 During the Christmas season, highway billboards displayed the number of drivers booked for speeding each day, from December 24 to January 4.

146, 46, 305, 102, 183, 194, 210, 98, 102, 208, 168, 110

a Arrange the data from smallest to largest.

b Determine the value of the mean, median, mode and range.

c Calculate the interquartile range of the data.

d Calculate the standard deviation for the data. Give your answer correct to 1 decimal place.

e A reporter for the local newspaper is writing an article about the number of motorists booked for speeding each day.

i Which value do you think she should use for the 'average'? Give a reason for your answer.

ii Which value do you think she should use for describing the spread of scores? Give a reason for your answer.

Practical activity Comparing the size of maths textbooks with English novels

1 Ask your Maths teacher to bring in a copy of every maths textbook they have, and your English teacher to lend you a copy of every novel used in senior English classes.

2 Record the number of pages in each book for each of the 2 groups.

3 Calculate the mean and standard deviation for the number of pages in the maths textbooks.

4 Calculate the mean and standard deviation for the number of pages in the English novels.

5 'Maths textbooks have more pages than English novels but there is a greater variation in the number of pages in English novels.' Do you agree with this statement? Does your data support this? Justify your answer.

7.8 Data spreadsheets

Spreadsheets have statistical functions to calculate some of the measures of central tendency and spread. In this exercise, we will start with a small set of data to familiarise you with the functions, and then we will analyse some larger datasets.

EXERCISE 7.8 Data spreadsheets

ANSWERS p. 428

1 a Enter into a spreadsheet the data shown about the daily maximum temperatures in Alice Springs for one week.

	A	B	C	D	E
1	Day	Temperature (°C)			
2	Sunday	29		Mean	
3	Monday	31		Mode	
4	Tuesday	30		Median	
5	Wednesday	33		Maximum	
6	Thursday	29		Minimum	
7	Friday	28		Range	
8	Saturday	35		Interquartile range	
9				Standard deviation	
10					

b Copy each formula into the given cells to find the statistical information listed in column D.

Cell E2: = average(B2:B8)

Cell E3: = mode(B2:B8)

Cell E4: = median(B2:B8)

Cell E5: = max(B2:B8)

Cell E6: = min(B2:B8)

Cell E7: = E5–E6

Cell E8: = quartile(B2:B8,3)-quartile(B2:B8,1)

Cell E9: = stdev.p(B2:B8)

Sometimes when you type the first couple of letters, the spreadsheet will suggest the correct word.

c Save your results.

2 a Repeat question **1** using data for the town or city where you live. Go to the **Bureau of Meteorology** website (www.bom.gov.au) to find the data.

b Find data for 1 month instead of 1 week. Repeat question **1** for this new data. You will need to adjust the formulas you enter for the statistics.

Weblink
Bureau of Meteorology



iStock.com/bennymarty

3 a Enter into a spreadsheet the following data showing the monthly number of motor vehicle thefts in a capital city over 2 years.

	A	B	C	D	E	F
1	Motor vehicle theft in a capital city					
2						
3	Jan 2019	20		Jan 2020	15	
4	Feb 2019	19		Feb 2020	19	
5	Mar 2019	26		Mar 2020	17	
6	Apr 2019	17		Apr 2020	21	
7	May 2019	27		May 2020	20	
8	Jun 2019	13		Jun 2020	18	
9	Jul 2019	17		Jul 2020	13	
10	Aug 2019	17		Aug 2020	13	
11	Sep 2019	20		Sep 2020	14	
12	Oct 2019	18		Oct 2020	14	
13	Nov 2019	12		Nov 2020	18	
14	Dec 2019	24		Dec 2020	20	
15						
16	Mean			Mean		
17	Median			Median		
18	Mode			Mode		
19	SD			SD		
20						

b Copy the following formulas into the given cells.

Cell B16: = average(B3:B14)

Cell B17: = median(B3:B14)

Cell B18: = mode(B3:B14)

Cell B19: = stdev.p(B3:B14)

Cell E16: = average(E3:E14)

Cell E17: = median(E3:E14)

Cell E18: = mode(E3:E14)

Cell E19: = stdev.p(E3:E14)

The data for 2020 has many modes, but the spreadsheet only lists one of them.

c Save your results.

d Comment on the differences between the 2 years.

4 Repeat question **3** for Melbourne or your own town. You will need to use the internet to find the data. Try searching 'crime statistics and motor vehicle theft'.

5 a Create a spreadsheet similar to that in question **1** for the following data on the number of burgers sold between 11 am and 2 pm in Hungry Macs for one week.

Day of week	Number of burgers sold
Monday	60
Tuesday	71
Wednesday	63
Thursday	69
Friday	78
Saturday	90
Sunday	86

b Find the mean, the median and the standard deviation for this data and save your spreadsheet.

c Calculate the same statistics for the *weekdays* only.

d Describe the differences between the data for the whole week and the weekday-only data.
Why do you think these differences occur?

6 This data shows the heights of 30 students' navels above the ground in centimetres.

107, 101, 100, 81, 97, 98, 68, 96, 104, 93, 90, 94, 95, 86, 92
81, 85, 98, 88, 90, 99, 81, 100, 81, 90, 104, 100, 93, 98, 113

a Create a spreadsheet similar to that in question **1** for this data.

b Find the mean, median, mode and standard deviation for this data and save your spreadsheet.

c Delete the outlier of 68 cm. Calculate the same statistics for the remaining data and save this new spreadsheet.

d Describe the differences between the 2 sets of data. Why do you think these differences occur?

7 a The students of a class were asked how many children were in their family. Enter these results into a spreadsheet.

	A	B	C
1	Number of children in family (x)	Frequency (f)	fx
2			
3	1	4	
4	2	8	
5	3	7	
6	4	3	
7	5	1	
8	6	2	
9	Total		
10			
11	Mean		
12			

b To calculate the total number of students, enter the formula **=sum(B3:B8)** into cell B9.

c To calculate the entries for the fx column, enter the formula **=A3*B3** into cell C3.

d Use **Fill Down** to fill in the formulas for the rest of that column (C4 to C8).

e To calculate the total for the fx column, enter the formula **=sum(C3:C8)** into cell C9.

f To calculate the mean, enter the formula **=C9/B9** into cell B11. What is the mean?

g Repeat this exercise for another frequency table you choose yourself.



Worksheet
Investigation components

Extended Investigation — Will your team win?

To complete this investigation, you need access to some long-term sports data.

Research your team's win/loss record for home games for at least the past 3 years. Analyse the data using the statistical methods you have learned in this chapter, then predict the chances that your team will:

- win a home game
- score more than 90 points in a game
- lose fewer than 6 points in a game.

Write a report describing your results, suitable for publishing in your team's online newsletter.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 428

bi-modal	central tendency	class centre
class intervals	cumulative frequency	cumulative frequency histogram
cumulative frequency polygon	interquartile range	mean
median	mode	negatively
outliers	percentiles	positively
quartiles	range	spread
standard deviation	uniform	

Copy and complete each sentence using a word from the list. When you have completed this, you will have a summary of the chapter.

In analysing data, we use measures of $_1$_____ _____ (2 words) and measures of $_2$_____.

The $_3$_____ is the sum of all the data values divided by the number of values. The $_4$_____ is the middle value when the values are arranged from smallest to largest. The $_5$_____ is the data value with the highest frequency. The largest value minus the smallest value is called the $_6$_____. The 3 values that divide a set of data into 4 equal parts are called $_7$_____. We use these to calculate the $_8$_____ _____ (2 words). Another measure of spread is the $_9$_____ _____ (2 words), which we use our calculators to find.

When we have large amounts of numerical data, we can group them into a frequency table using $_{10}$_____ _____ (2 words). In order to analyse this type of frequency table, we add a $_{11}$_____ _____ (2 words) column (cc), an $f \times$ cc column and a $_{12}$_____ _____ (2 words) column. To estimate the median and the quartiles for grouped data, we draw 2 graphs: the $_{13}$_____ _____ _____ (3 words) and the $_1$_____ _____ _____ (3 words).

The values that divide a set of data into 100 equal parts are called $_{15}$_____. Usually this is data about large populations, like the heights of the people of Victoria.

When analysing data, we should look out for $_{16}$_____, data values that are very different from the rest of the values, because they will affect the mean and the standard deviation.

We can also consider the shape of the data – whether it is symmetrical, $_{17}$_____ skewed or $_{18}$_____ skewed. Sometimes the distribution will be $_{19}$_____, where the scores are evenly distributed. We can also consider whether the distribution is $_{20}$_____ (has 2 peaks).

Solution to the chapter problem

Problem

Anita has collected data on how long it takes students to travel to school. She needs to present a statistical analysis of her data.

For this data, find:

a the median class

b the modal class

c an estimate for the mean, correct to 2 decimal places

d an estimate for the interquartile range.

Travel time (minutes)	Frequency
1–10	28
11–20	21
21–30	14
31–40	5
41–50	3
51–60	2
61–70	2
71–80	1
81–90	2

Solution

Add and complete columns for class centre, $f \times$ cc and cumulative frequency.

Travel time (minutes)	Class centre cc	Frequency f	$f \times$ cc	Cumulative frequency
1–10	5.5	28	154.0	28
11–20	15.5	21	325.5	49
21–30	25.5	14	357.0	63
31–40	35.5	5	177.5	68
41–50	45.5	3	136.5	71
51–60	55.5	2	111.0	73
61–70	65.5	2	131.0	75
71–80	75.5	1	75.5	76
81–90	85.5	2	171.0	78
	Total	78	1639.0	

a There are 78 scores, so the middle scores are the 39th and the 40th scores. Reading from the cumulative frequency column, these are in the 11–20 class.

The median class is 11–20.

b The modal class is the class with the highest frequency, 28.

The modal class is 1–10.

c $\text{mean} \approx \dfrac{1639}{78}$

$= 21.0128\ldots$

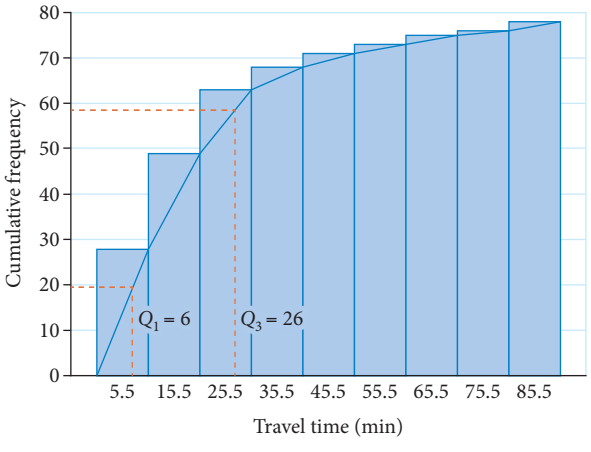
≈ 21.01

d We can approximate the interquartile range using a cumulative frequency histogram and polygon.

For Q_1, we estimate the $\frac{1}{4} \times 78 = 19.5$th score, which, reading from the polygon, is approximately 6.

For Q_3, we estimate the $\frac{3}{4} \times 78 = 58.5$th score, which, reading from the polygon, is approximately 26.

IQR $\approx 26 - 6 = 20$



9780170453943

7 Test yourself

Analysing data

ANSWERS p. 428

1 EXERCISE 7.1 In a gymnastics competition, the judges awarded the following scores:

6.2, 7.0, 8.2, 8.8, 7.0, 9.1, 7.0, 8.1, 8.0

For these scores, find the:

a mean, correct to 2 decimal places
b mode
c median
d range
e quartiles
f interquartile range.

2 EXERCISE 7.1 Frannie's Online Music Store recorded the daily sales figures of Joe Hendry's latest album for the first 18 days after it was released.

29, 18, 15, 13, 15, 14, 14, 18, 12, 14, 14, 12, 18, 14, 12, 18, 13, 14

a Calculate the mean sales for this data, correct to 1 decimal place.
b Find the median sales for this data.
c Which measure (mean or median) is the better reflection of the sales figures? Explain your answer.
d Find the range for this data.
e Find the upper and lower quartiles for this data and the interquartile range.
f Which is the better measure of spread for this data – the range or the interquartile range? Justify your answer.

3 EXERCISE 7.2 Jacinta collected the following data on computer frauds per year for the last 5 years:

808, 1126, 1003, 913, 300

a What is the outlier for this data?
b Calculate the mean of the data:
  i with all scores included
  ii without the outlier included.
c What effect does the outlier have on the mean?

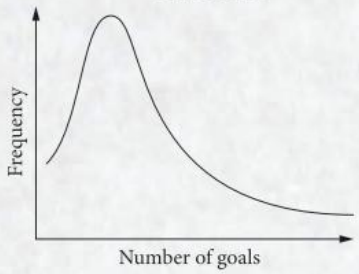
4 EXERCISE 7.2 Bassam collected the following data on monthly rainfall at Swan Hill in millimetres for 1 year:

9, 24, 14, 100, 19, 19, 22, 5, 23, 2, 20, 5

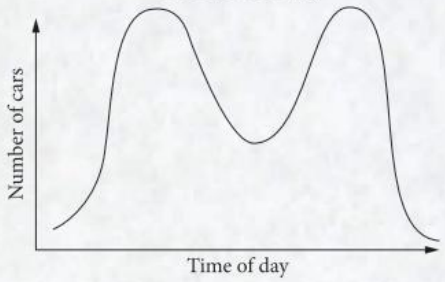
a What is the outlier for this data?
b Find the mean and the median for the data:
  i with all scores included
  ii without the outlier included.
c Compare the mean with all scores included to the mean without the outlier included. What effect does the outlier have on the mean?
d Compare the median with all scores included to the median without the outlier included. What effect does the outlier have on the median?
e Which measure of central tendency best describes the rainfall in Swan Hill? Justify your answer.

5 EXERCISE 7.3 What type of distribution is shown in each graph?
Choose from symmetrical, positively skewed, negatively skewed, uniform and bi-modal.

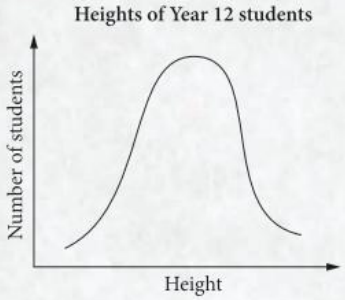
a



b



c



6 EXERCISE 7.3 Sketch a graph for each distribution described below. Remember to label the axes.

a The percentage marks students scored on an easy assessment task are negatively skewed, with most scoring between 80 and 90%.

b The masses of all Year 11 students in Australia are distributed symmetrically.

c When a card is chosen randomly from a deck of cards 500 times, the suit of the card (diamonds, hearts, clubs, spades) has a uniform distribution.

7 EXERCISES 7.4, 7.5 Flower seeds were sown in boxes. This table shows the number of seedlings that sprouted in each box.

Number of seedlings	Frequency
1–5	3
6–10	6
11–15	6
16–20	9
21–25	7
26–30	5
31–35	4

a Copy this table and add columns for class centre, cumulative frequency and frequency × class centre.

b For this set of data find:

i the median class **ii** the modal class **iii** an estimate for the mean.

8 EXERCISE 7.5 Draw a cumulative frequency histogram and polygon for the data in question **7**.
Use these graphs to find an estimate for:

a the median

b the interquartile range.

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9 EXERCISE 7.6 Use the graph in Example **9** to answer the following questions.

a What income is at the 60th percentile?

b Freddie earns $120 000 per year. What percentile is this?

c What is the approximate difference in income between the 10th and 55th percentiles?

d Approximately what percentage of the population earns more than $100 000?

10 EXERCISE 7.7 Use your calculator to find, correct to 2 decimal places, the standard deviation of these temperatures at Cloncurry in Queensland (°C):

25, 24, 23, 20, 16, 12, 11, 12, 17, 20, 23, 25

11 EXERCISE 7.7 Carmelina recorded the number of drinks bought by people visiting her shop during one hour. The results are shown in the table.

Number of drinks	Frequency
0	7
1	5
2	13
3	4
4	1
5	2
6	1
Total	33

For this data, find correct to 2 decimal places:

a the mean

b the standard deviation.

12 EXERCISE 7.8 The following data shows the overnight minimum temperatures in Horsham for 1 week in May. Enter the data in a spreadsheet like the spreadsheet in Exercise 7.8, question **1** and complete the formulas correctly to fill cells E2 to E8.

Day	Temperature (°C)
Sunday	8
Monday	12
Tuesday	7
Wednesday	5
Thursday	1
Friday	5
Saturday	9



Chapter problem

Celine is planning to make a gift box for the birthday present she is giving to her friend Jenny. It will be a rectangular box with a lid. She is going to cut out the box from a flat sheet of thick cardboard.

What shape will she need to cut out for the box and what shape for the lid?

Shutterstock.com/5 second Studio



CHAPTER 8

SPACE AND MEASUREMENT

CREATING DESIGNS

8.1 2D shapes
8.2 It's symmetrical
8.3 Slide, flip or spin?
8.4 Similar figures
8.5 Tile patterns
Extended investigation: Researching and making tessellations
8.6 3D shapes
8.7 From 3D to 2D
8.8 Perspective drawings
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Review what we know about common 2D and 3D shapes
- Recognise line and rotational symmetry
- Identify different types of tessellations and transformations
- Calculate unknowns in similar figures
- Interpret 2D representations of 3D objects
- Create perspective in drawings

How are we ever going to use this?

- Whenever we use shapes or solids in design
- To know how to make models of 3D objects
- To make decorative patterns for house renovations, craft activities and paving
- In an occupation where design is an important element, for example graphic design

Study design coverage

AREA OF STUDY 4: SPACE AND MEASUREMENT

- spatial and geometric constructions including transformations, similarity, symmetry and projections.

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Videos (1):
8.4 Finding an unknown side in similar figures

Skillsheets (1):
8.4 Finding sides in similar triangles

Worksheets (11):
8.1 2D shapes
8.3 5 mm grid paper
8.4 A page of similar figures • Enlargements and reductions
8.5 Tessellations • Investigation components
8.6 Sorting solids • 3D solids
8.7 Making solids • Sketching solids
8.8 Perspective drawings
Test yourself 5 mm grid paper

Puzzles (1):
8.1 Polygon puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

8.1 2D shapes

Worksheet 2D shapes

Puzzle Polygon puzzle

2D shapes are flat and have 2 dimensions: length and width.

3D shapes are solids and have 3 dimensions: length, width and depth.

EXERCISE 8.1 2D shapes

ANSWERS p. 429

1 For each triangle:
- **i** sketch the triangle, marking any equal angles or sides
- **ii** write down 2 properties of the triangle.

a right-angled triangle **b** scalene triangle **c** acute-angled triangle
d equilateral triangle **e** obtuse-angled triangle **f** isosceles triangle

2 **a** How many sides does a quadrilateral have?
b What is an 8-sided shape called?
c How many sides does a dodecagon have?
d What is the name given to a polygon that has all sides equal and all angles equal?

3 For each quadrilateral:
- **i** sketch the quadrilateral, marking any equal angles or sides
- **ii** write down 3 properties of the shape.

a square **b** kite **c** parallelogram
d rhombus **e** rectangle **f** trapezium

4 **a** What is the sum of the 3 angles in a triangle?
b What is the sum of the 4 angles in a quadrilateral?

5 Draw a 5-sided shape. Write its correct name and draw in its diagonals.

A **diagonal** is a line that joins any 2 vertices of a given shape or solid that are not next to each other.

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6 Name all the 2D shapes you can see in these photos. How many shapes did you name?

a



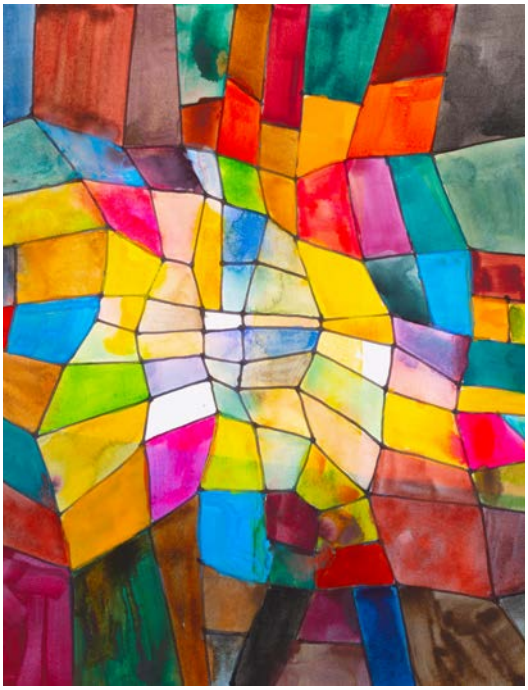
Shutterstock.com/Zyn Chakrapong

b



Shutterstock.com/one pony

c



Shutterstock.com/clivewa

d



Shutterstock.com/Aleksandar Todorovic

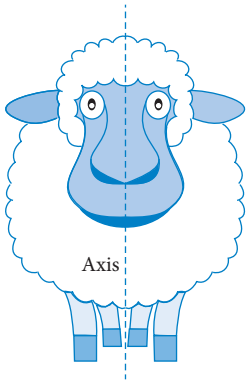
7 Is each statement true (T) or false (F)?

- **a** An isosceles triangle must be acute-angled.
- **b** A square is a regular quadrilateral.
- **c** An obtuse-angled triangle must also have acute angles.
- **d** A rhombus is a special type of parallelogram.
- **e** An acute-angled triangle cannot be isosceles.
- **f** A rectangle is a special type of trapezium.
- **g** An equilateral triangle must be acute-angled.
- **h** A trapezium must have 2 equal sides.

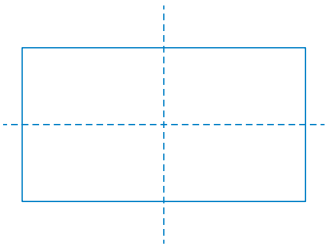
8.2 It's symmetrical

Symmetry is often used in design. Architects use symmetry in the design of buildings to make them more attractive. There are 2 types of symmetry.

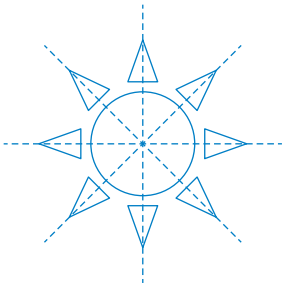
A shape has **line symmetry** if it can be divided by a line into 2 identical parts that are mirror images of one another. The line that divides the halves is called the **axis of symmetry** (or sometimes just the axis).



1 axis of symmetry

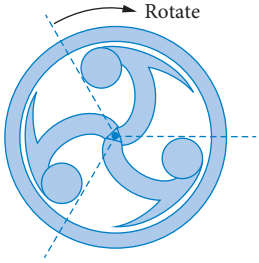


2 axes of symmetry

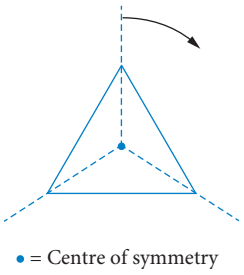


4 axes of symmetry

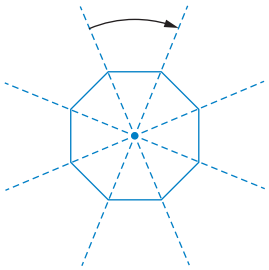
A shape has **rotational symmetry** if it can be spun about a central point so that it fits onto itself more than once in a 360° rotation. The point is called the **centre of symmetry**. The number of times the shape repeats itself in 360° is called the **order of rotational symmetry**.



Order of rotational symmetry 3



Order of rotational symmetry 3



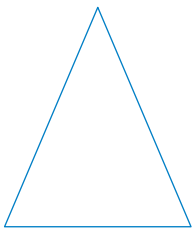
Order of rotational symmetry 8

EXERCISE 8.2 It's symmetrical

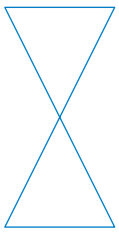
ANSWERS p. 429

1 Copy each shape, draw in any axes of symmetry and state how many there are.

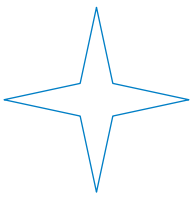
a



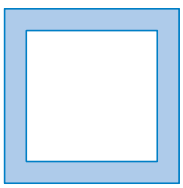
b



c



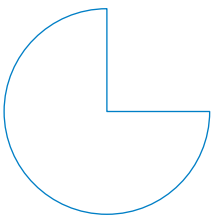
d



e

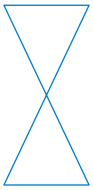


f

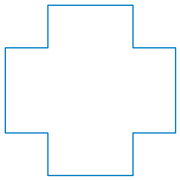


2 Copy each shape, mark the centre of rotational symmetry and state the order of rotational symmetry.

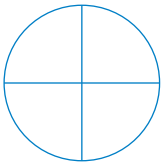
a



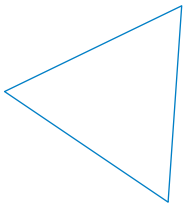
b



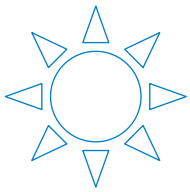
c



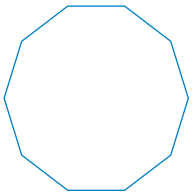
d



e



f



3 For each car tyre, state the order of rotational symmetry (ignore any bolts or holes for bolts).

a



b



c



d



e

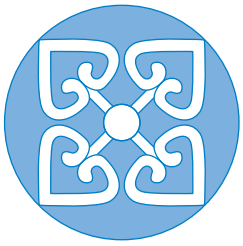


f



4 Classify each design as having line symmetry, rotational symmetry, both types of symmetry or no symmetry.

a



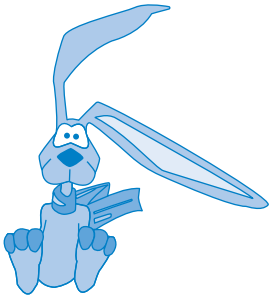
b



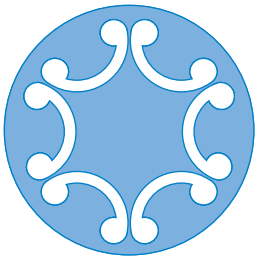
c



d



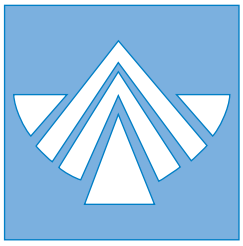
e



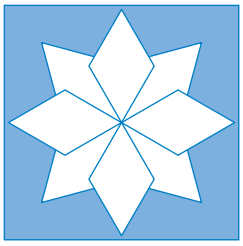
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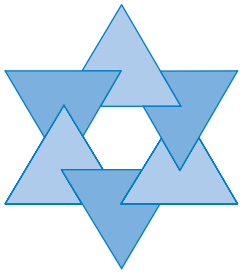
g



h



i



Investigation Symmetry in art and architecture



Alamy Stock Photo/Jonathan Welch



Shutterstock.com/Dmitry Strizhakov

1 Use the internet to find examples of both line symmetry and rotational symmetry in art and architecture, similar to those shown above. The symmetry may be in part of the artwork or in the design or decoration of the building.

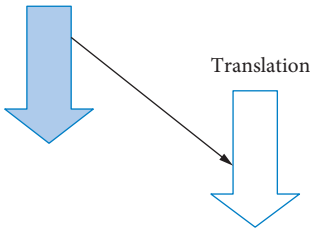
2 Print out one image of a work of art and one image of a building showing:

a line symmetry **b** rotational symmetry.

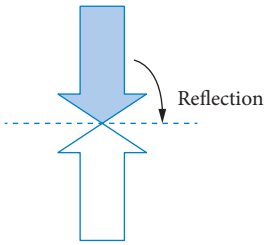
3 On each image, draw the lines of symmetry, mark the centre of rotational symmetry and state the order of rotational symmetry.

8.3 Slide, flip or spin?

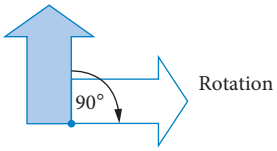
Patterns in tiles, wallpaper and paving are usually made by taking a basic shape and repeating it. A pattern can be created by **sliding**, **flipping** or **spinning** the shape. These movements have special names.



A 'slide' is called a **translation**.



A 'flip' is called a **reflection**.



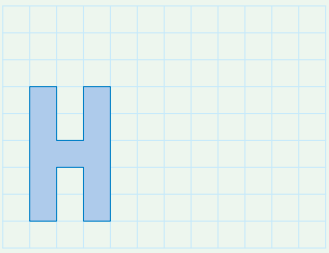
A 'spin' is called a **rotation**.

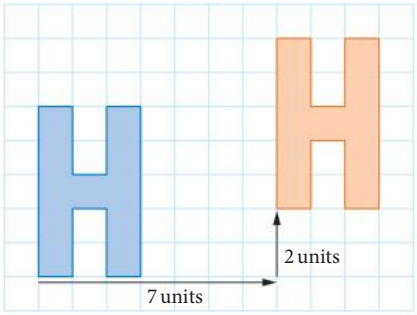
Each of these movements is a type of **transformation**. Transformation means change.

- The original shape is called the **original**.
- The transformed shape is called the **image.**

EXAMPLE 1

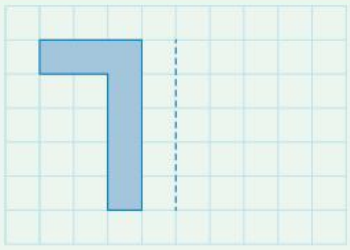
Translate the H-shape 7 units right and 2 units up.



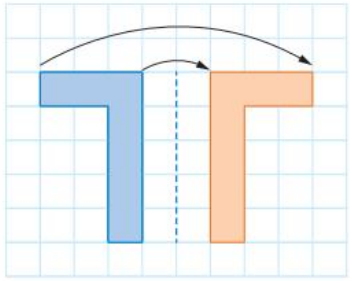
Steps	Working
Slide each point 7 squares across to the right and 2 squares up. Join the points to draw the whole shape.	

EXAMPLE 2

Reflect the L-shape across the dotted line.

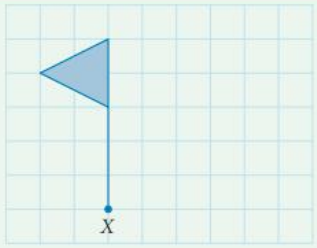


The dotted line is called the **line of reflection.**

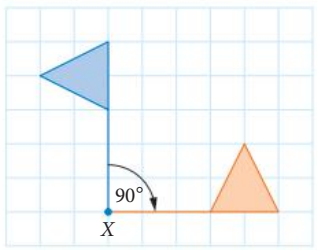
Steps	Working
Flip to make the mirror image of the original, back-to-front and the same distance from the line of reflection.	

EXAMPLE 3

Rotate the flag shape 90° clockwise about point X.



Point X is the **centre of rotation.**

Steps	Working
Spin clockwise around X to make the image. The original and image are the same distance from the centre of rotation, so point X does not move.	

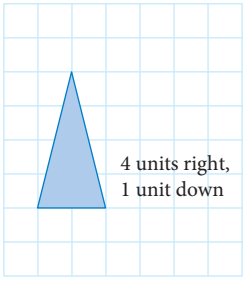
EXERCISE 8.3 Slide, flip or spin?

ANSWERS p. 429

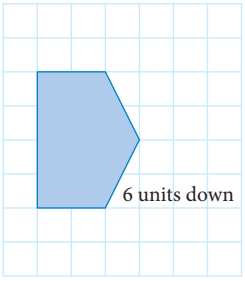
Worksheet
5 mm grid paper

1 EXAMPLE 1 Copy each shape on grid paper and draw its translated image according to the directions given.

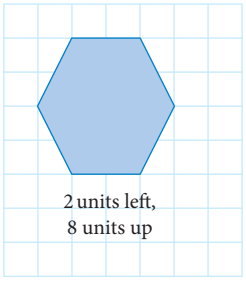
a



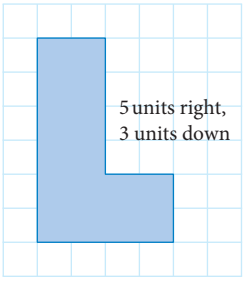
b



c

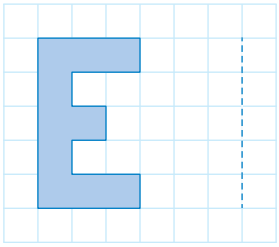


d

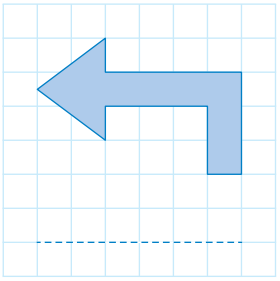


2 EXAMPLE 2 Copy each shape on grid paper and draw its image reflected across the dotted line.

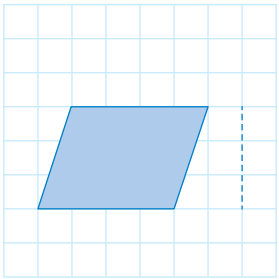
a



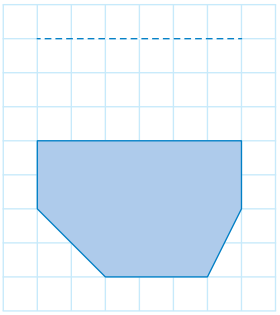
b



c

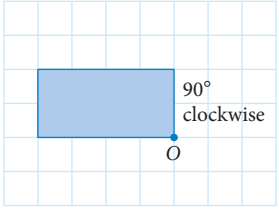


d

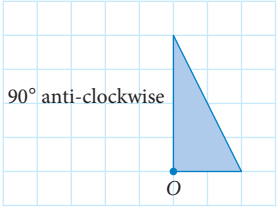


3 EXAMPLE 3 Copy each shape on grid paper and draw its image by rotating each shape about the point O by the given angle.

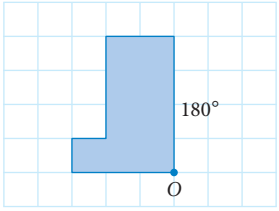
a



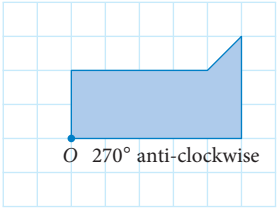
b



c



d

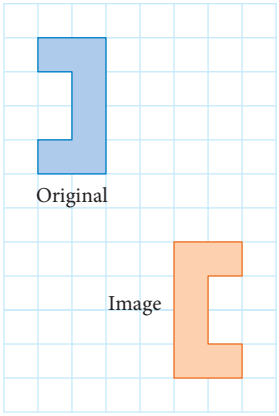


9780170453943

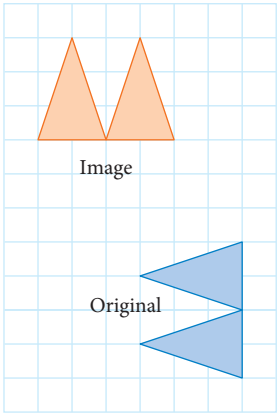
4 Why don't rotations of 180° need the label 'clockwise' or 'anti-clockwise'?

5 Translations, reflections and rotations can be applied one after the other in combination. For each diagram below, state the combination of 2 transformations that has been used on the original figure to create the image.

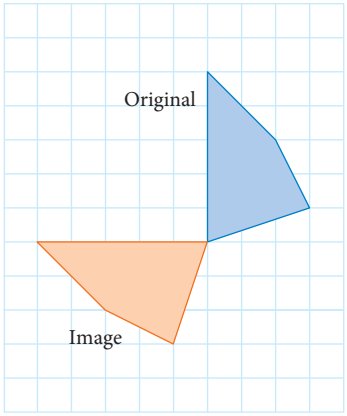
a



b

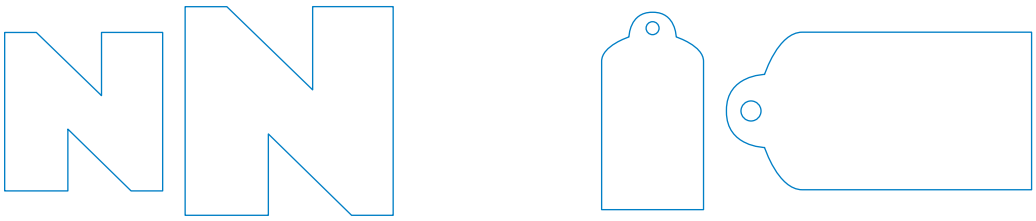


c



8.4 Similar figures

Figures that have the *same shape* but not necessarily the same size are said to be **similar figures**. One is an enlargement or reduction of the other. These are pairs of similar figures:



A photograph and its enlargement are similar figures.



Original

Image

iStock.com/Marcus Lindstrom

Video Finding an unknown side in similar figures

Skillsheet Finding sides in similar triangles

Worksheet A page of similar figures

Worksheet Enlargements and reductions

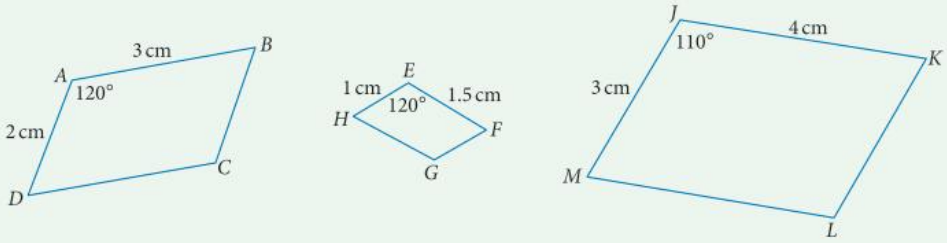
Similar figures

Similar figures have the same shape.

Their matching angles are equal, and their matching sides are in the same **ratio**.

EXAMPLE 4

Which 2 of these parallelograms are similar?

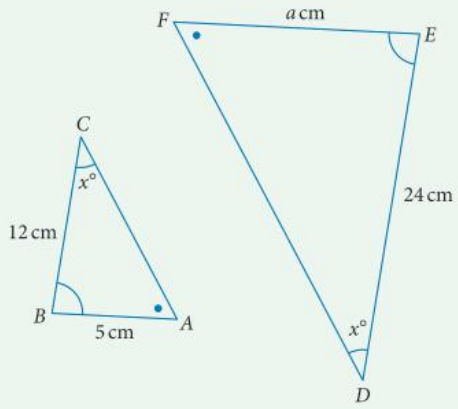


Steps	Working
For similar figures, matching angles must be equal and matching sides must be in the same ratio.	$\angle A = 120°$ and $\angle E = 120°$ but $\angle J = 110°$.
Check the ratios of matching sides in *ABCD* and *EFGH*.	$\frac{EF}{AB} = \frac{1.5}{3} = \frac{1}{2}$ $\frac{EH}{AD} = \frac{1}{2}$ $\therefore \frac{EF}{AB} = \frac{EH}{AD}$
You can tell which sides are matching. They join matching angles.	
Conclusion.	The matching sides are in the same ratio. So the 2 similar parallelograms are *ABCD* and *EFGH*.

EXAMPLE 5

These 2 triangles are similar.

Find the value of a.



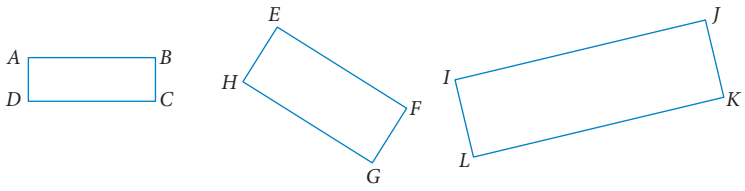
Steps	Working
Write an equation involving matching sides, where a is in the numerator of the first fraction.	Since matching sides are in the same ratio:
The sides marked a cm and 5 cm are matching sides.	$\frac{a}{5} = \frac{24}{12}$
The sides marked 24 cm and 12 cm are matching sides.	$a = \frac{24}{12} \times 5$ $= 10$
Alternative method: Use the scale factor.	scale factor $= \frac{24}{12} = 2$ $a = 5 \times 2 = 10$

EXERCISE 8.4 Similar figures

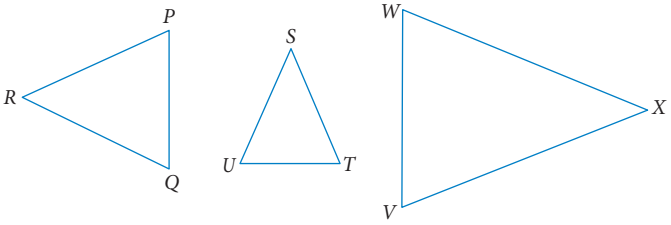
ANSWERS p. 430

1 EXAMPLE 4 Find the pair of similar figures in each set of shapes below.

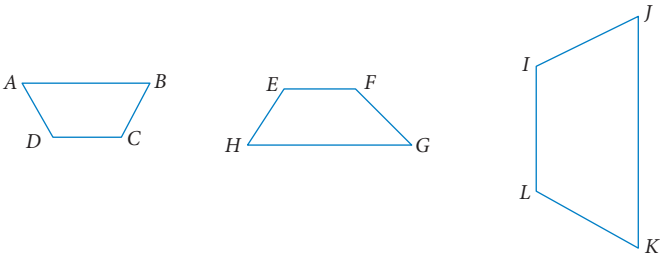
a



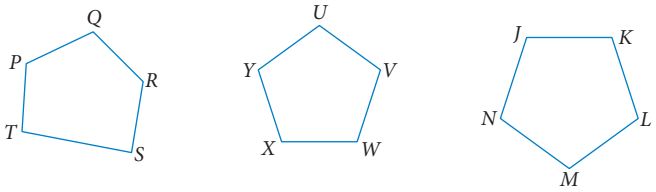
b



c



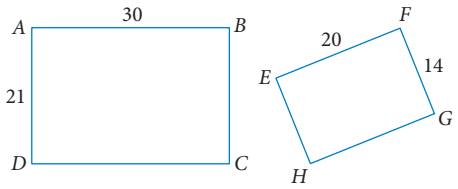
d



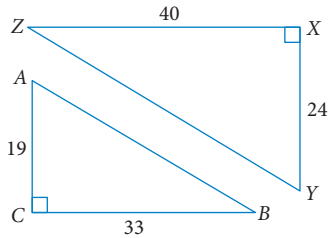
2 Decide whether the following pairs of figures are similar. If they are similar:

- **i** name a pair of matching angles
- **ii** name a pair of matching sides
- **iii** find the ratio of the matching sides.

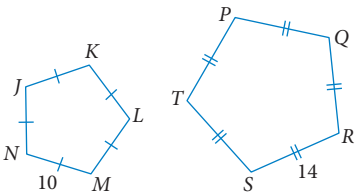
a



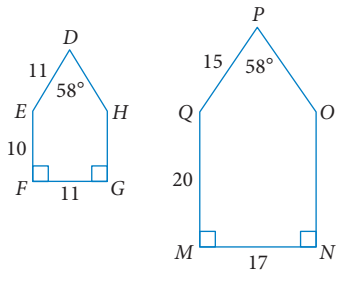
b



c

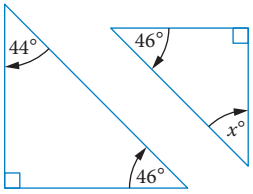


d

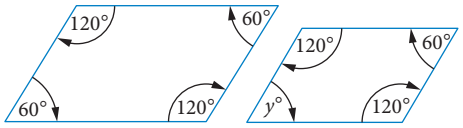


3 Find the value of the variable in each pair of similar figures.

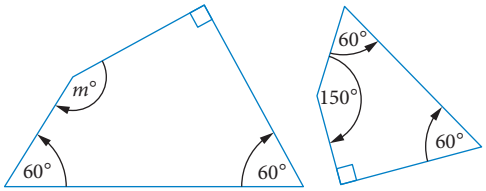
a



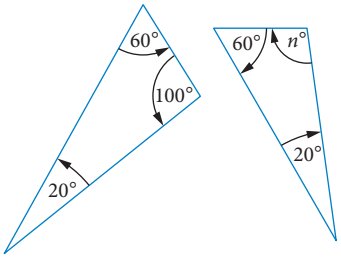
b



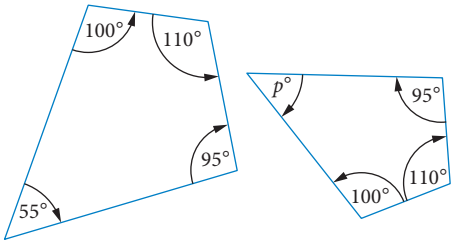
c



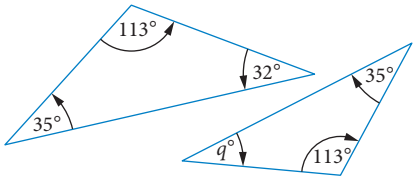
d



e

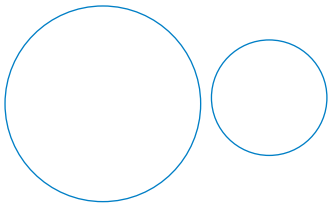


f

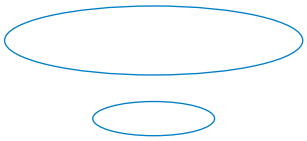


4 a State whether each pair of figures is similar.

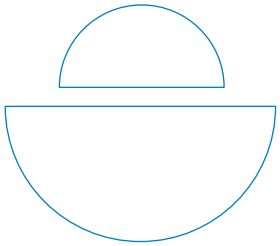
i



ii



iii



b How did you decide on your answer for part **a**?

5 a Draw 2 squares that are similar. State the ratio of the matching sides.

b Draw 2 circles that are similar.

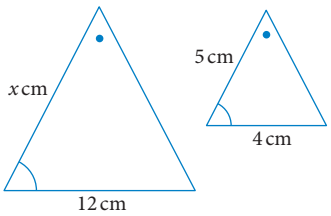
c Draw 2 rectangles that are similar. State the ratio of the matching sides.

d Draw 2 similar right-angled triangles. State the ratio of the matching sides.

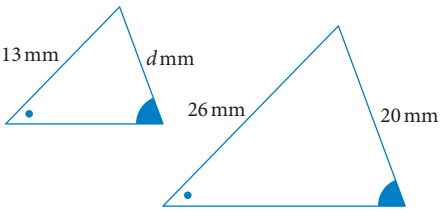
e Make up your own shape and draw it. Draw another that is similar to it.

6 EXAMPLE 5 Find the value of the variable in each pair of similar triangles.

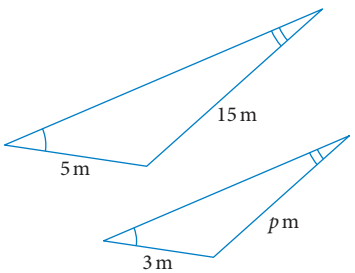
a



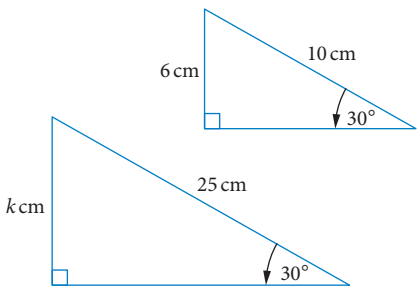
b



c



d



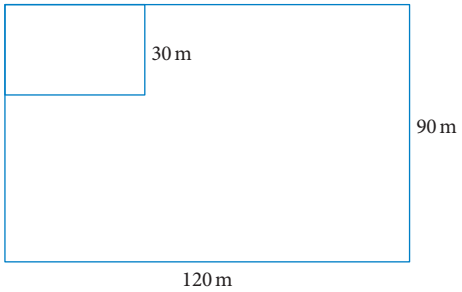
9780170453943

7 A large rectangular paddock, of length 120 m and width 90 m, has a small rectangular pen of width 30 m in one corner. The paddock and the pen are similar figures.

What is the length of the pen? Select the correct answer **A**, **B**, **C** or **D**.

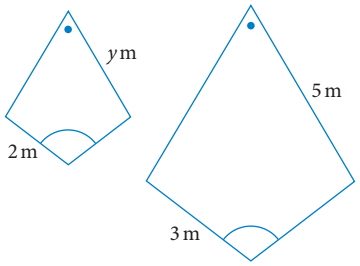
A 22.5 m **B** 37.5 m

C 40 m **D** 60 m

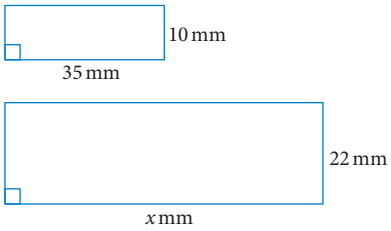


8 Find the value of the variable in each pair of similar figures.

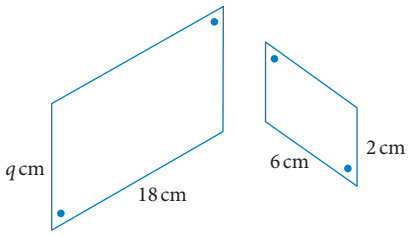
a



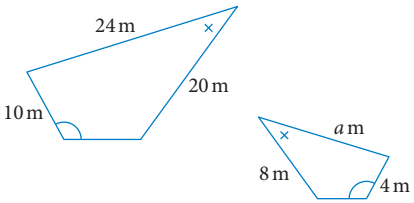
b



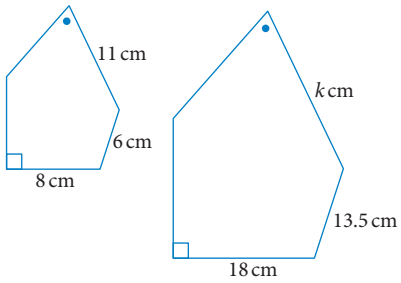
c



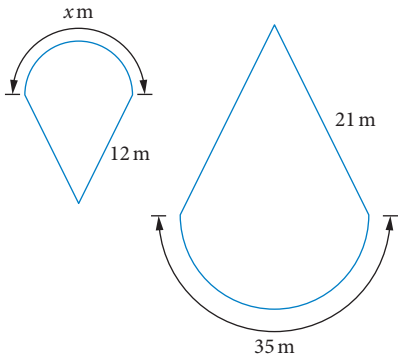
d



e

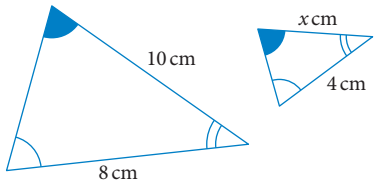


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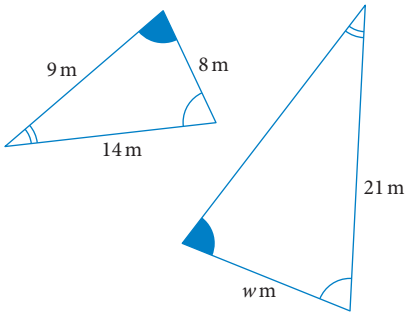


9 For each pair of similar triangles, find the value of the variable.

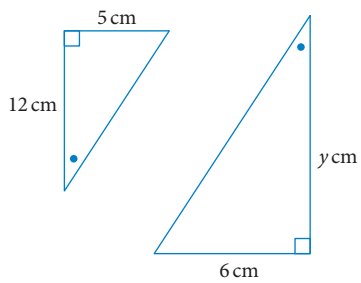
a



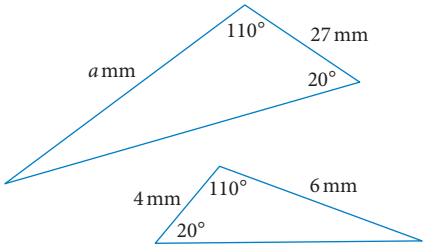
b



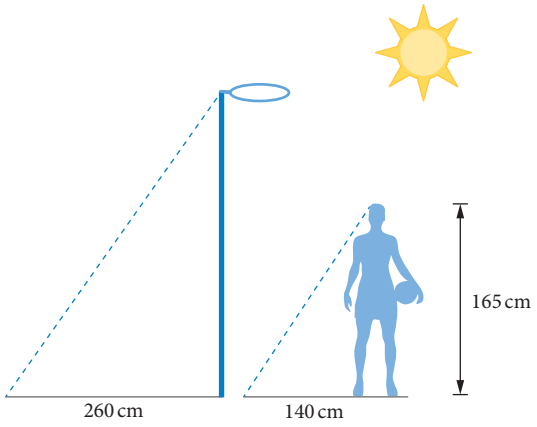
c



d

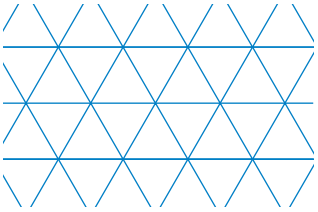


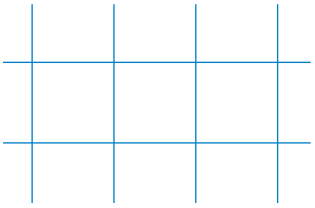
10 On a sunny day, Sandra stands next to a pole supporting a netball ring. She is 165 cm tall and casts a shadow of 140 cm. The pole casts a shadow of 260 cm. How high is the pole, correct to the nearest centimetre?

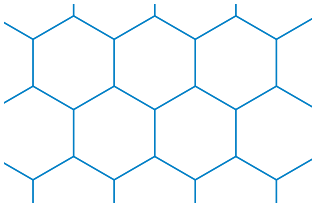


8.5 Tile patterns

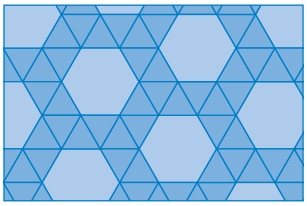
A 'tile pattern' of repeated shapes covering a flat surface so that there is no overlapping or gaps is called a **tessellation**. A **regular tessellation** uses only one shape, a **regular polygon**. There are only 3 regular tessellations, shown below.

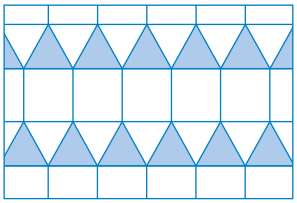

Equilateral triangles


Squares

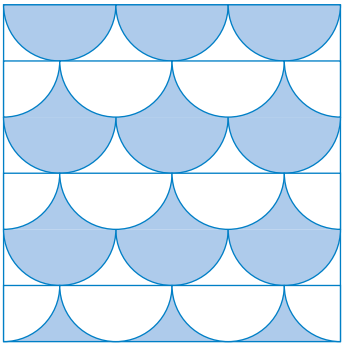

Regular hexagons

A **semi-regular tessellation** uses 2 or more regular polygons. There are only 8 semi-regular tessellations. Here are 2 examples.


Hexagon and triangle


Square and triangle

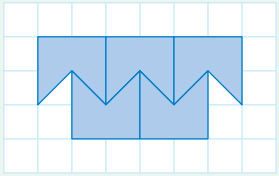
A **non-regular tessellation** uses shapes that are not regular polygons.

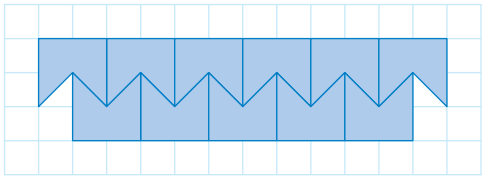


Tessellations are created by using transformations of a shape: translating, reflecting, rotating, or a combination of these transformations.

EXAMPLE 6

Extend this tessellation by repeating the pattern.



Steps	Working
Repeat the pentagon in the upper row and in the lower row to extend the pattern.	

EXERCISE 8.5 Tile patterns

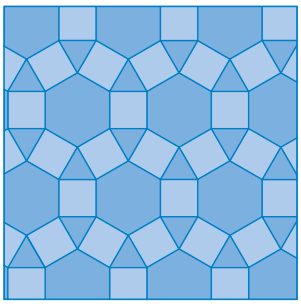
ANSWERS p. 430

1 Which 3 shapes are used in drawing regular tessellations?

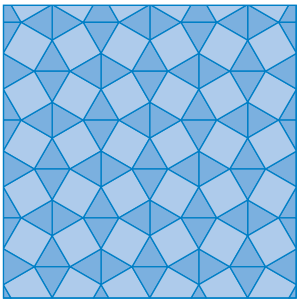
2 Search for semi-regular tessellations on the internet. Each one is made up of 2 or more shapes. Write down which shapes are used in each of the 8 semi-regular tessellations.

3 What regular polygons have been used to make each tessellation shown?

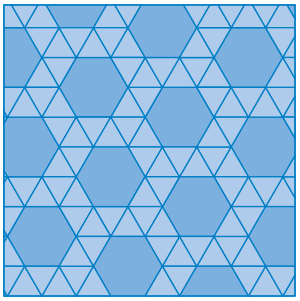
a



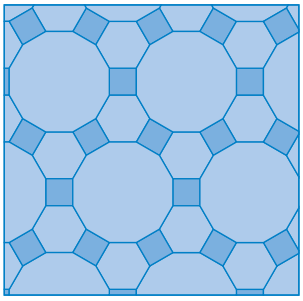
b



c



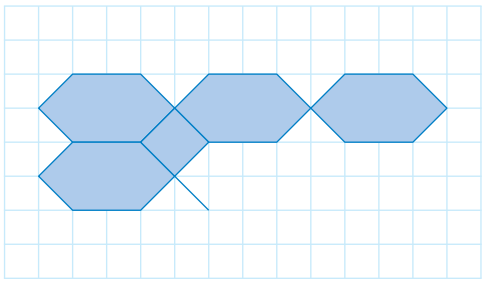
d



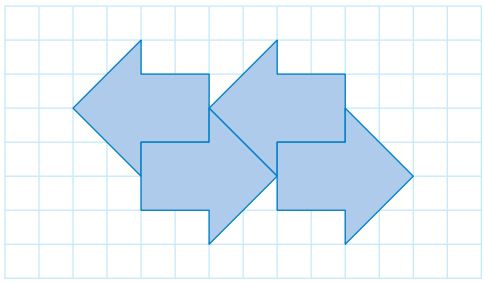
Worksheet
Tessellations

4 EXAMPLE 6 Copy and complete each tessellation on grid paper. Colour them to show the different shapes used. (These tessellations can also be found on the **Tessellations** worksheet, which can be downloaded.)

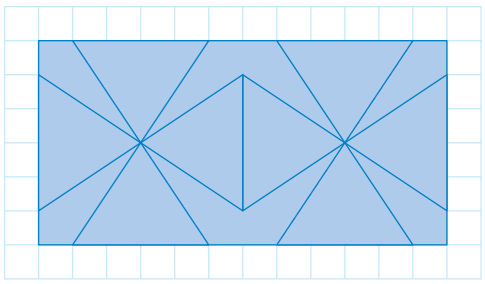
a



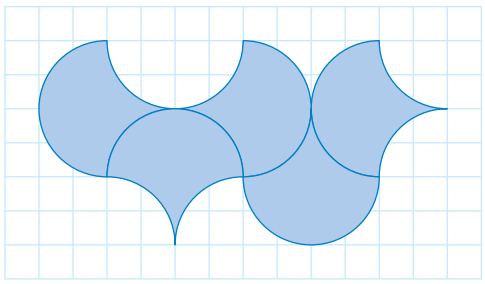
b



c



d



5 Redraw a tessellation that you like or create one of your own, using any combination of the transformations. If your first shape doesn't work out, keep trying different shapes.

6 Andrew and Catriona run an interior design business. They have a client who wants a bathroom renovation. They need to present the client with a choice of designs for a border at the top of the wall tiles. What are some possible designs? The tiles can be all the same shape or different shapes.

Worksheet
Investigation components

Extended investigation — Researching and making tessellations

Part A: Choose one of the 3 following topics and research the questions given for that topic.

Escher

1 Who was Escher? Why is he famous?

2 How did he use tessellations?

3 Print or reproduce an example of his work using tessellations.

Quilting patterns

1 How do quilting patterns use tessellations?

2 Find 4 examples of quilting patterns using tessellations. Print or reproduce them.

3 Name the shapes that are used in each one.

Islamic art

1 Why do you think tessellations are used so often in Islamic art?

2 Find 4 examples of Islamic art using tessellations. Print or reproduce them.

3 Name the shapes that are used in each one.

Part B: Make your own tessellations

There are many websites that will show you how to create an original tessellation. Search for an appropriate website and make 2 original tessellations: one based on standard geometrical shapes and another based on a shape you construct yourself.

For example, search for the **YouTube** video: **How to make a tessellation** by Studio Jocelyn.

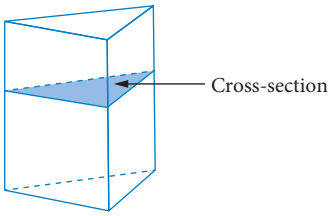
Your portfolio should include:

- answers to the questions in the topic you selected in Part A
- the 2 original tessellations you produced in Part B, including the website you used and a description of any challenges you overcame.

8.6 3D shapes

3D shapes are **three-dimensional** or solid shapes.

A **cross-section** of a solid shape is a 'slice' of the solid cut across it rather than along it.



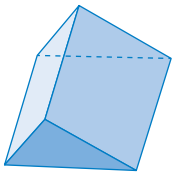
EXERCISE 8.6 3D shapes

ANSWERS p. 430

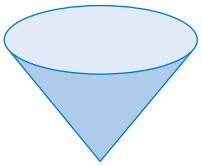
1 For each solid shown:

- **i** name the solid
- **ii** draw a neat diagram of a cross-section of the solid.

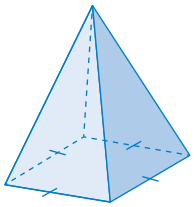
a



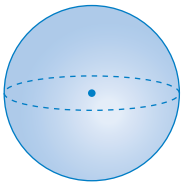
b



c



d



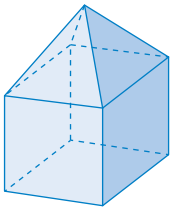
2 **a** How many faces does a cube have?

b How many edges does a square pyramid have?

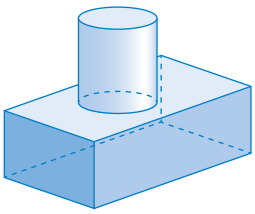
c How many vertices does a rectangular prism have?

3 Each composite solid below is made up of 2 basic solids. Name the 2 solids that have been combined.

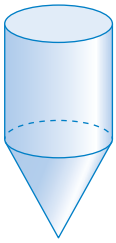
a



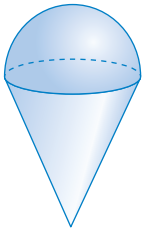
b



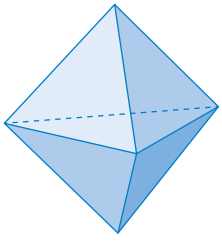
c



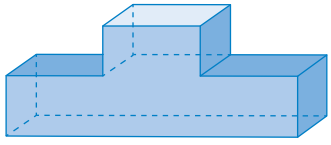
d



e



f



4 Is each statement true (T) or false (F)?

- **a** The cross-section of a triangular prism is a triangle.
- **b** A triangular pyramid has 4 vertices.
- **c** A rectangular prism is a hexahedron (a solid with 6 faces).
- **d** A cone has 2 faces.
- **e** Both a cube and a square pyramid have 6 faces.
- **f** A pyramid has the same cross-section from bottom to top.

Worksheet
Sorting solids

Worksheet
3D solids

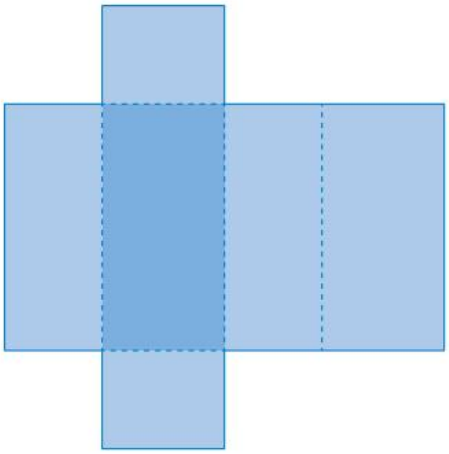
8.7 From 3D to 2D

Worksheet
Making solids

Worksheet
Sketching solids

Nets of solids

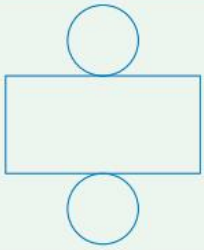
A cardboard box starts out as a flat piece of cardboard. We cut out a particular shape and fold it to make the box. The 2D shape we cut out is called the **net** of a solid. For a closed box, the net would look like this:



Some cardboard boxes have tabs that can be glued to hold the box together. Tabs are not included in this net.

EXAMPLE 7

What solid would this net make?



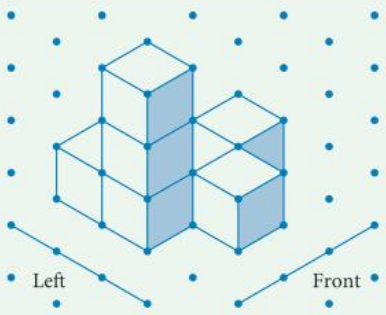
Steps	Working
The rectangle would curve around to match the circles. 	This is the net of a cylinder.

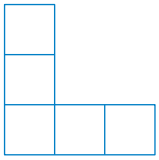
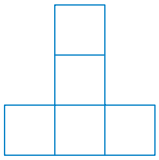
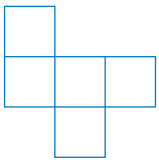
Different views of solids

A solid can be drawn from different perspectives (points of view). It can be drawn as it appears when we look from the front, or from the top, or from one of the sides.

EXAMPLE 8

This solid is made with 7 cubes.
Draw the front, left and top views.



Steps	Working
Imagine you are standing in front of the solid – this is what you would see.	 Front view
Imagine you are standing on the left of the solid – this is what you would see.	 Left view
Looking down at the solid from above – this is what you would see.	 Top view

EXERCISE 8.7 From 3D to 2D

ANSWERS p. 431

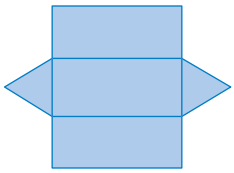
1 Draw a neat sketch of each solid.

a rectangular prism **b** triangular prism **c** cylinder

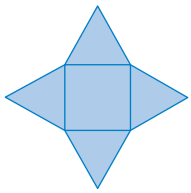
d square pyramid **e** cone

2 EXAMPLE 7 Match each net with the solid it makes in parts **A** to **F**. Write the name of each solid.

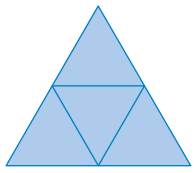
a



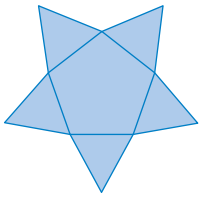
b



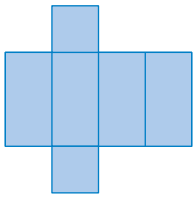
c



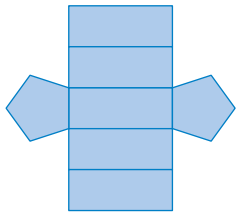
d



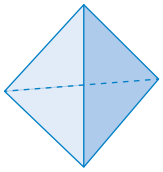
e



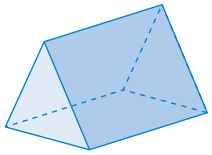
f



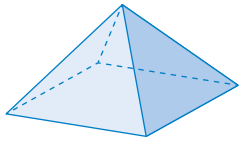
A



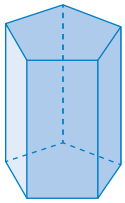
B



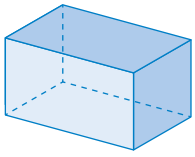
C



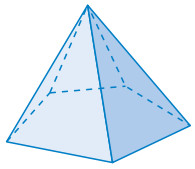
D



E

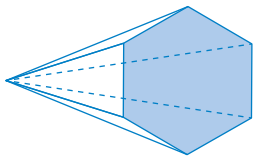


F

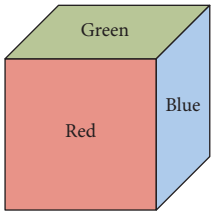


3 **a** What is the name of this solid?

b Draw the net of this solid.

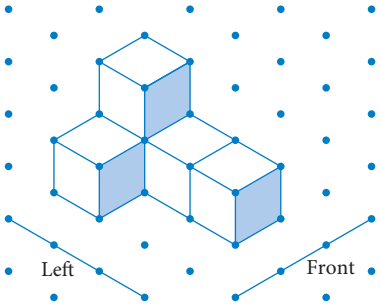


4 Draw the net of this cube, showing the correct positions of the 3 coloured faces shown.



5 EXAMPLE 8 For each solid, draw each view.

a

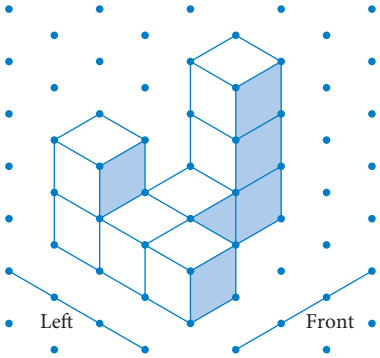


i front view

ii left view

iii top view

b

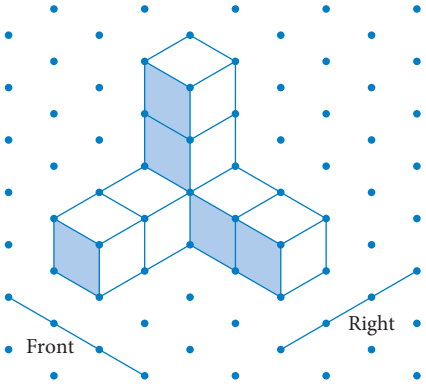


i left view

ii back view

iii top view

c

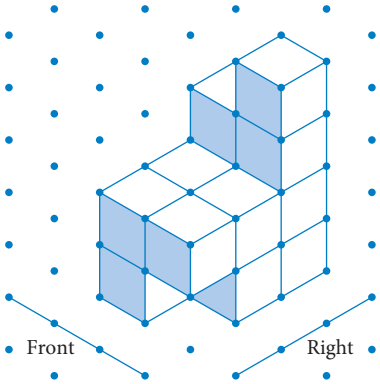


i front view

ii right view

iii top view

d



i back view

ii right view

iii top view

9780170453943

6 Draw the top and front view of each object.

a



Shutterstock.com/ Maxx-Studio

b



Shutterstock.com/ Vereshchagin Dmitry

c



Shutterstock.com/ Olga Kovalenko

d



Shutterstock.com/ Africa Studio

7 Sketch the front, left and right view of this children's cubby house.





Photos courtesy of Sue Thomson

8 Sketch the front, left and 'bird's eye' view of this small country church.





Photos courtesy of Sue Thomson



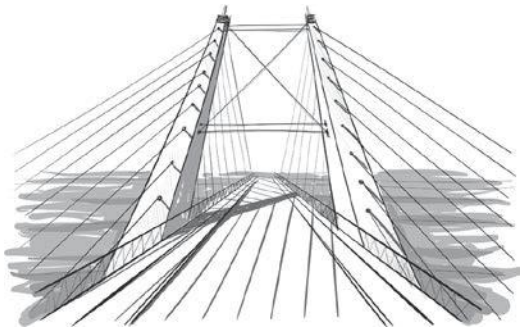
'Bird's eye view' means the top view.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

8.8 Perspective drawings

In **perspective drawings**, objects that are further away are drawn smaller than objects that are up close. If a solid shape is drawn in perspective, it will have a **vanishing point**. A vanishing point is a point on the horizon where lines in a perspective drawing meet. Sometimes pictures can have 2 or 3 vanishing points.



EXERCISE 8.8 Perspective drawings

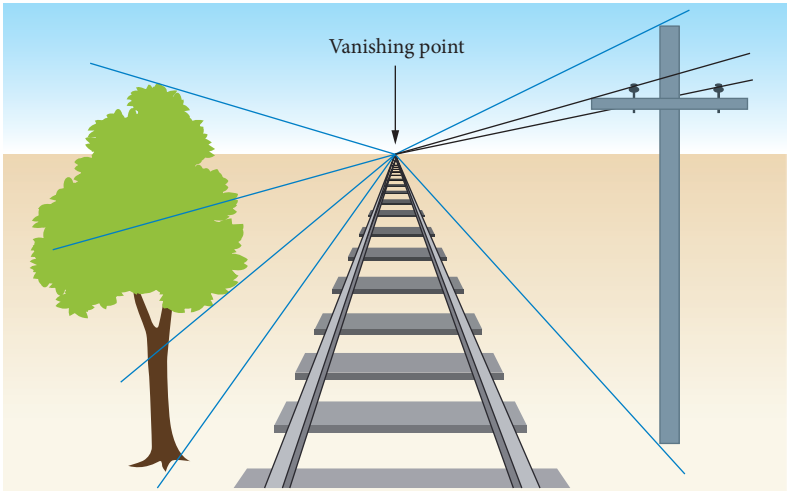
ANSWERS p. 431

Worksheet
Perspective drawings

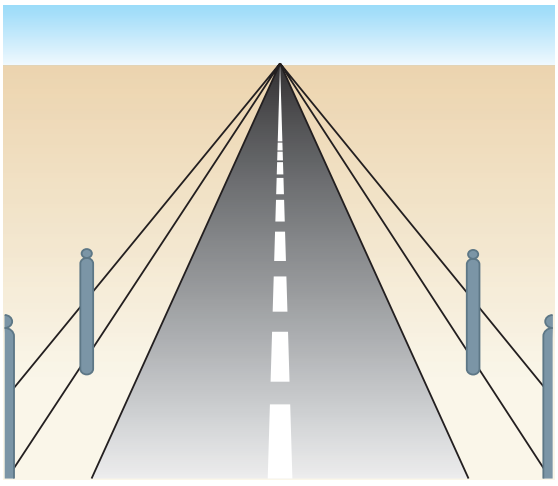
*This exercise can also be downloaded as the **Perspective drawings** worksheet.*

1 This straight railway track is drawn in perspective. The point on the horizon where the lines meet and the railway seems to disappear is the vanishing point. Notice that the sleepers (rectangular planks) appear to be smaller and closer together near the vanishing point.

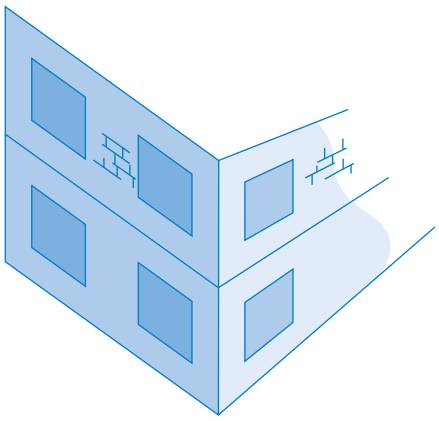
Draw a row of trees beyond the first one, on the left of the track, and a row of telegraph poles beyond the first pole, on the right.



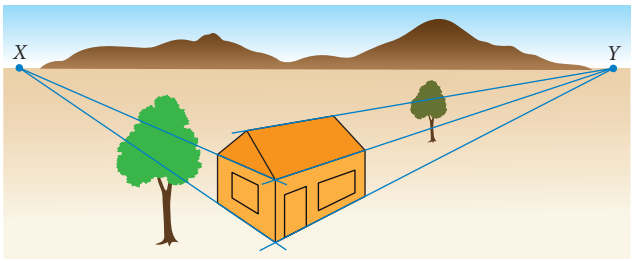
2 Copy this diagram and draw in the rest of the fence posts.



3 Copy and complete this perspective drawing of a building by drawing lines to the vanishing point.

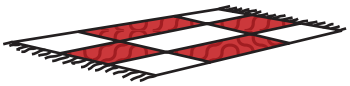


4 This perspective drawing has 2 vanishing points, X and Y.

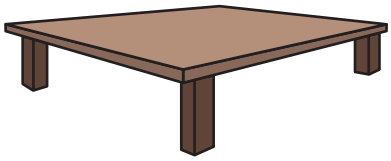


Copy each perspective drawing and find the 2 vanishing points for each one.

a

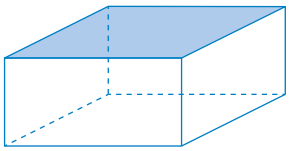


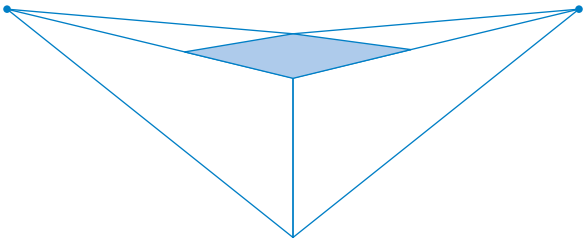
b



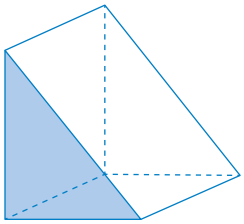
5 Copy and complete the perspective drawing of each prism using the lines and vanishing points shown.

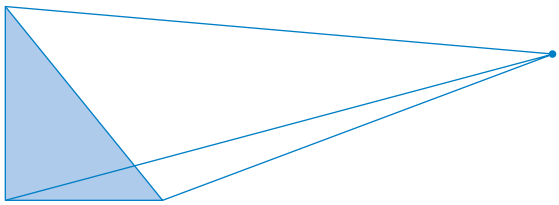
a



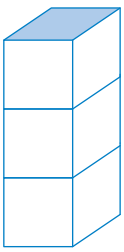


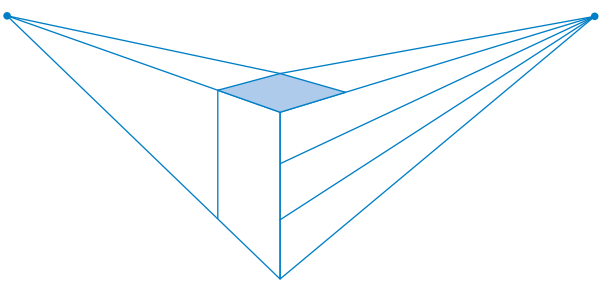
b





c





Investigation Perspective drawing in art

1 Search the internet for examples of perspective drawing in art. Find examples that have 1 vanishing point and 2 vanishing points.

2 Print copies of 2 pictures with:

- **a** 1 vanishing point
- **b** 2 vanishing points.

3 On each example, draw in the lines to the vanishing points.

4 Try drawing your own picture with vanishing points.



Shutterstock.com/Sleeping cat

Keyword activity

WORD MATCH

ANSWERS p. 431

Match each word with its definition.

Words

1 line symmetry
2 net
3 perspective drawing
4 regular polygon
5 rotational symmetry
6 reflection
7 rotation
8 similar figures
9 tessellation
10 three-dimensional
11 transformation
12 translation
13 two-dimensional

Definitions

A A flat shape with equal sides and equal angles
B Describes an object that has length, width and depth
C A pattern of repeated shapes that cover a flat surface with no overlapping or gaps
D When one half of a shape is the mirror image of the other half
E Sliding a shape to a new position
F Describes an object that has length and width only
G A way of drawing where objects further away are drawn smaller than objects up close
H Translation, reflection and rotation are examples of this
I Spinning a shape to a new position
J A shape has this feature if it can be spun about a point so that it repeats its shape
K Two figures that are the same shape but not the same size
L Flipping a shape to a new position
M What you get when you 'flatten' a solid into its faces

Solution to the chapter problem

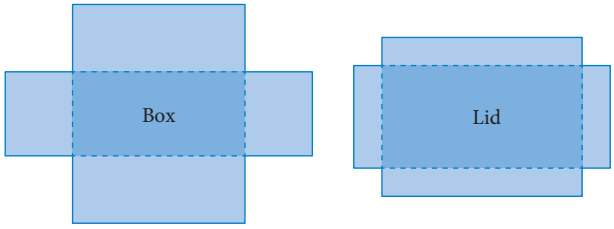
Problem

Celine is planning to make a gift box for the birthday present she is giving her friend Jenny. It will be a rectangular box with a lid. She is going to cut out the box from a flat sheet of thick cardboard. What shape will she need to cut out for the box and what shape for the lid?

Solution

Celine will need the net of the box and its top. Each one is an open rectangular prism. Each of these nets would need to have its sides taped together.

We could also add tabs for gluing the box together if required.



8 Test yourself

Creating designs

ANSWERS p. 431

1 EXERCISE 8.1 Draw a neat sketch of each 2D shape.

a An isosceles triangle with 1 obtuse angle

b A pentagon

c A quadrilateral whose diagonals cross at right angles

2 EXERCISE 8.1 What polygon am I? There may be more than one answer.

a I have 3 sides and all of my angles are equal.

b I am a quadrilateral with 4 equal sides.

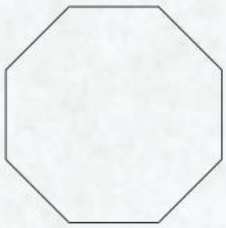
c I have 8 sides.

3 EXERCISE 8.2 State the number of axes of symmetry of each shape.

a



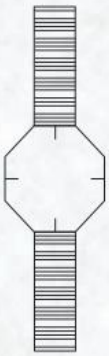
b



c



d



4 EXERCISE 8.2 Decide whether each shape has rotational symmetry. If it does, state the order of rotational symmetry.

a



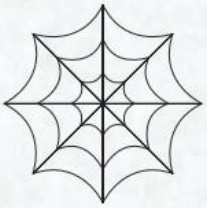
b



c

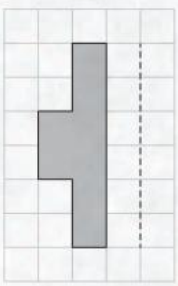


d



5 EXERCISE 8.3 Use grid paper for this question.

a Copy and reflect the figure across the dotted line.

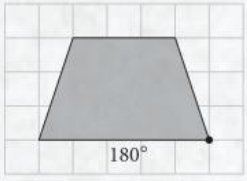


Worksheet
5 mm grid paper

b Copy and translate the figure 4 units right, 6 units up.



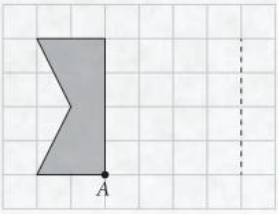
c Copy and rotate the figure 180° about the given point.





Worksheet
5 mm grid paper

6 EXERCISE 8.3 Copy the shape below on grid paper, then translate it 4 units to the right and 2 units up. Then rotate it 90° anti-clockwise about vertex A and reflect it across the given dotted line.



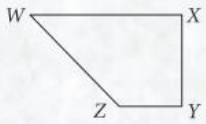
7 EXERCISE 8.4

For this pair of similar figures:

a name a pair of matching sides

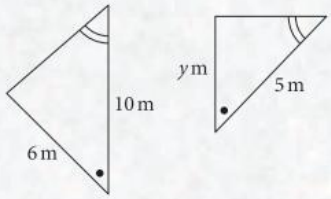
b name a pair of matching angles.



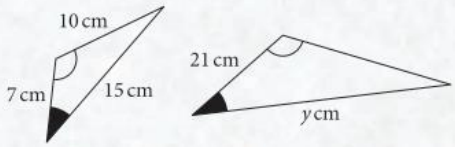


8 EXERCISE 8.4 Find the value of y in each pair of similar figures.

a



b

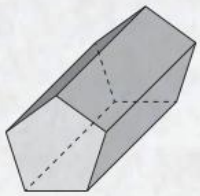


9 EXERCISE 8.5 Imagine you are renovating your bathroom. You want to have a feature wall with tiles that are *not* square and that have an unusual pattern. You can use tiles that are all the same or combine different shapes. What shapes could the tiles be and how would you arrange them? Create some designs for your feature wall.

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10 EXERCISE 8.6 Name each solid.

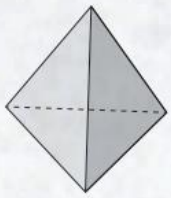
a



b



c

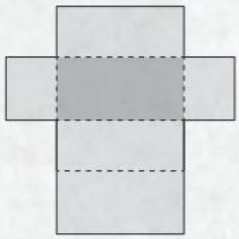


11 EXERCISE 8.7 Draw a neat sketch of:

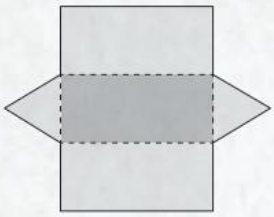
a a rectangular prism **b** a cone **c** a square pyramid.

12 EXERCISE 8.7 Name the solid that can be made from each net.

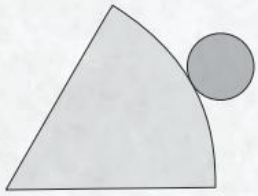
a



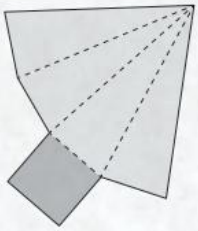
b



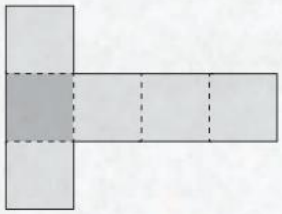
c



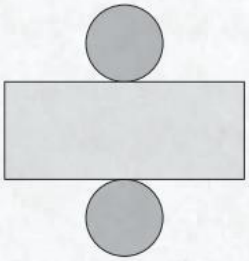
d



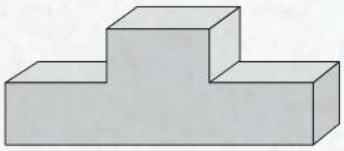
e



f



13 EXERCISE 8.7 This diagram shows a podium for competitors to be awarded medals at a sports tournament.



Front

For the podium, draw:

a the right view **b** the front view **c** the top view.

14 EXERCISE 8.8 Draw a perspective drawing showing a road or railway lines looking smaller as they extend into the distance.

Practice set 2

ANSWERS p. 432

Spreadsheet
Reducible loans

Weblink
MoneySmart

- You will need to download the **Reducible loans** spreadsheet for questions **2** and **11** of Section B.
- You will need to use the **mortgage calculator** on the **MoneySmart** website for question **5** of Section B.

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 5.1 In cooking, a pinch is $\frac{1}{16}$ of a teaspoon and a dash is $\frac{1}{8}$ of a teaspoon. How many pinches are equivalent to a dash?

A $\frac{1}{8}$ **B** $\frac{1}{2}$ **C** 2 **D** 8

2 EXERCISE 5.2 A drone is flying at a height of 2400 feet. Use the conversion 1 foot = 30.48 cm to calculate the height of the drone in metres, correct to the nearest metre.

A 732 m **B** 787 m **C** 7315 m **D** 7874 m

3 EXERCISE 7.1 Theresa recorded the favourite party food of a group of 3-year-old children. What is the only statistical measure that can be found for this data?

A mean **B** median **C** mode **D** range

4 EXERCISE 5.5 Which calculation could be used to find the area of a circle with a diameter of 3 m?

A $\pi \times 3$ **B** $\pi \times 3^2$ **C** $2 \times \pi \times 1.5$ **D** $\pi \times 1.5^2$

5 EXERCISE 7.7 Faizal recorded his golf scores for the last 10 weeks.

75, 80, 85, 73, 77, 81, 76, 73, 84, 76

Calculate the standard deviation for the scores, correct to 1 decimal place.

A 4.1 **B** 4.3 **C** 10 **D** 78

6 EXERCISE 6.1 Nina is repaying a loan. The progress of the loan is shown in this table.

Amount borrowed: $12 000
Interest rate: 6% p.a. reducible
Half-yearly repayments: $1070

Half-year	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$12 000.00	$360.00	$12 360.00	$11 290.00
2	$11 290.00	$338.70	$11 628.70	$10 558.70
3	$10 558.70	$316.76	$10 875.46	$9805.46
4	$9805.46	$294.16	$10 099.62	$9029.62
5	$9029.62	$270.89	$9300.51	$8230.51
6	$8230.51	$246.92	$8477.43	$7407.43

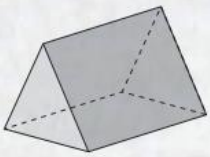
How much does Nina owe immediately after she made her 3rd repayment?

A $9029.62 **B** $9805.46 **C** $10 558.70 **D** $10 875.46

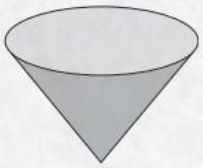
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7 EXERCISE 8.6 Which solid shown is a prism?

A



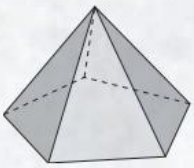
B



C



D



8 EXERCISE 5.5 Calculate, in square metres, the area of a rectangular window that is 2 metres long and 90 centimetres wide.

A $0.18\,m^2$ **B** $1.8\,m^2$ **C** $18\,m^2$ **D** $180\,m^2$

9 EXERCISE 6.5 Seth owns a house worth \$780 000. The monthly rent is 1% of the property's value. How much do Seth's tenants pay per fortnight to rent his house?

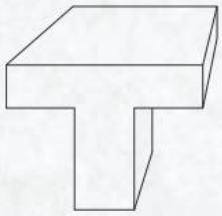
A \$1800 **B** \$3600 **C** \$3900 **D** \$7800

10 EXERCISE 8.1 I am a quadrilateral with opposite sides of equal length. My diagonals bisect each other at right angles. What am I?

A parallelogram **B** trapezium **C** rectangle **D** rhombus

Section B Short-answer questions

1 EXERCISE 8.8 Copy the 1-point perspective drawing of the 3D letter T below and locate the vanishing point on your drawing.



2 EXERCISE 6.2 Use the **Reducible loans** spreadsheet for this question.

Anton borrowed \$13 000 at 5% p.a. reducible interest to buy a car. His monthly repayments are \$470.

a How much interest will Anton pay in the first month?

b How much will he owe immediately after the 15th monthly repayment?

c How long will it take Anton to repay the loan?

d What is the value of his last repayment?

e How much interest will Anton pay on the loan?

3 EXERCISE 7.4 This table shows the speeds of trail bikes as they passed a checkpoint in a dirt bike competition. The speeds were measured correct to the nearest km/h.

Speed (km/h)	Frequency
21–30	19
31–40	15
41–50	5
51–60	1

a How many bikes passed the checkpoint?

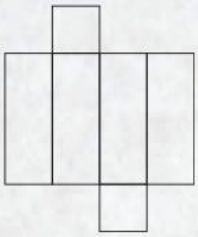
b How many bikes were travelling faster than the recommended safety speed of 40 km/h?

c What is the modal class?

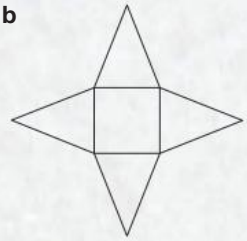
d In what class is the median?

4 EXERCISE 8.7 The diagrams show the net of 4 different solids. Name the solid shape that can be made with each net?

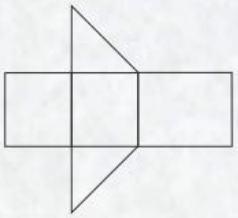
a



b



c



d



5 EXERCISE 6.3 For this question, use the **mortgage calculator** on the **MoneySmart** website.

Joshua borrows $450 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 30 years at 5.5% p.a. reducible interest. He will also be charged a $5 monthly account-keeping fee.

a How much are Joshua's monthly repayments?

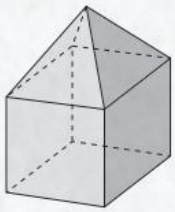
b How much will Joshua pay in interest and fees?

6 EXERCISE 5.2 Charlie flew from Hawaii to Tahiti, a distance of 2384 nautical miles.

a Use the conversion 1 nautical mile = 1852 metres to calculate the length of the flight, correct to the nearest kilometre.

b The flight took 8 hours and 30 minutes. Calculate the speed of the plane in km/h and knots.

7 EXERCISE 8.6 How many vertices, edges and faces does this solid have?



8 EXERCISES 5.4, 5.5 Calculate the perimeter and area of this road sign.



9 EXERCISE 5.3 Jake's motorhome has a freshwater tank that holds 50 US gallons. Use the conversion 1 US gallon = 3.785 L to calculate the tank's capacity in litres.

10 EXERCISE 6.1 When Elizabeth borrowed $20 000 from the bank to buy a car, the bank charged 9% p.a. reducible interest on a secured loan and her monthly repayments were $440.

Elizabeth's reducing balance car loan
Amount borrowed: $20 000
Interest rate: 9% p.a. monthly reducible
Monthly repayments: $440

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$20 000	$150	$20 150	$19 710
2	**a**	**b**	**c**	**d**
3	**e**	**f**	**g**	**h**

Calculate the values of **a** to **h** in the table, correct to the nearest cent.

i How much will Elizabeth owe after she has made 3 repayments?

j Calculate the total interest she will pay in the first 3 months.

11 EXERCISE 6.2 Use the **Reducible loans** spreadsheet for this question.

Noah is borrowing $30 000 at 5% p.a. reducible interest and his monthly repayments are $910.

a How long will it take Noah to repay the loan?

b How much interest will Noah pay?

12 EXERCISE 7.1 This data shows the number of minutes of exercise Chelsea did each day for 2 weeks.

60, 45, 80, 65, 75, 50, 45, 41, 50, 50, 40, 45, 50, 75

Find the mean, median, mode and range for Chelsea's data.

13 EXERCISE 6.4 What happens to the amount of interest you pay on a $236 000 loan with monthly repayments of $1300 if the interest rate drops from 3.9% p.a. to 3.25% p.a.?

14 EXERCISE 7.1 The number of goals scored by Bella's netball team in the games in the summer competition were:

8, 22, 16, 34, 12, 20, 17, 4, 31, 7, 19, 14, 30, 27, 15

For this data, find:

a each quartile **b** the interquartile range **c** the range.

15 EXERCISE 7.4 Classify each type of numerical data as continuous (C) or discrete (D).

a The distances thrown by javelin contestants at a sports carnival

b The reaction time of kick boxers

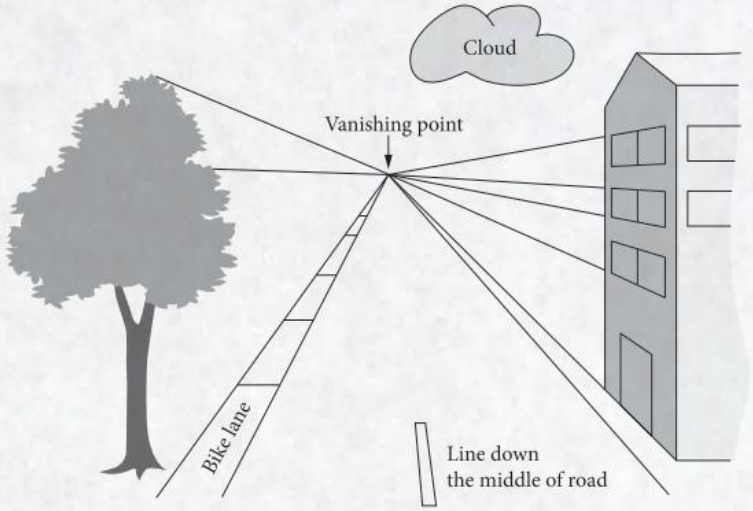
c The number of goals scored by the school soccer team in the winter competition

d The number of emails people send each day

e The number of contacts in your mobile phone

f The height of each student in your maths class

16 EXERCISE 8.8 Lara is making a 1-point perspective drawing of a road disappearing into a tunnel. She put some of the perspective lines in the picture to help her. Copy the drawing and complete it by adding the following features.



a Put a tunnel at the vanishing point.

b Add a footpath along the right-hand side of the road.

c Add a third window to the building facing the front of the drawing.

d Add an additional building closer to the vanishing point.

e Put two more gum trees on the left-hand side of the road.

f Add another cloud.

g Put a broken line down the enter of the road.

17 EXERCISE 7.5 Erika recorded the number of minutes of exercise her friends do each day.

60, 52, 31, 36, 28, 15, 0, 25, 60, 42, 52, 29, 41, 41, 17, 16, 31, 39,
15, 27, 39, 41, 50, 27, 37, 55, 27, 45, 15, 28, 63, 62, 27, 0, 11, 20

a Copy and complete the frequency distribution table.

Class	Class centre cc	Tally	Frequency f	Cumulative frequency	$f \times cc$
0–<10	5				
10–<20					
20–<30					
30–<40					
40–<50					
50–<60					
60–<70					
		Total			

b For this set of data find:

i the median class **ii** the modal class

iii an estimate for the mean (correct to 1 decimal place).

c Calculate the range of the data.

d Draw a cumulative frequency histogram and polygon for this data.

e Use these graphs to find an estimate for:

i the median **ii** the interquartile range.

18 EXERCISE 7.6 The following information is based on population data for the body mass indices (BMI kg/m^2) of girls aged 18 years.

- The 10th percentile $P_{10} = 18.2$
- The 25th percentile $P_{25} = 19.4$
- The 50th percentile $P_{50} = 21.2$
- The 75th percentile $P_{75} = 23.8$
- The 90th percentile $P_{90} = 27$
- The 95th percentile $P_{95} = 30$

In the questions below, all of the girls mentioned are aged 18.

a Rina has the median BMI for girls her age. What is her BMI?

b Ava's BMI is at the lower quartile. What is her BMI?

c Suki's BMI is 28.4. Is this a high value for her age?

d What percentage of girls have a BMI higher than 23.8?

e What percentage of 18-year-old girls have a BMI greater than 19.4?

19 EXERCISE 7.7 Evan recorded the number of magazines bought by customers at his newsagency during 1 hour. The results are shown in the table.

For this data, find correct to 2 decimal places:

a the mean

b the standard deviation.

Number of magazines	Frequency
0	6
1	7
2	11
3	3
4	1
5	1
6	1
Total	30

20 EXERCISES 8.1, 8.2, 8.4 Is each statement true or false?

a An obtuse-angled triangle can only have 1 obtuse angle.

b A rectangle is a regular quadrilateral.

c A square has only 2 axes of symmetry.

d A square is a special rhombus.

e An acute-angled triangle cannot be equilateral.

f A trapezium must have 2 equal angles.

g Each pair of sides in 2 similar figures is in the same ratio.

21 EXERCISE 8.7 Draw a neat sketch of each of these solids and their net.

a rectangular prism

b triangular prism

c cylinder

d square pyramid

e cone

22 EXERCISE 7.3 Draw a graph that shows:

a a symmetrical distribution

b a negatively skewed distribution

c a bi-modal distribution

d a positively skewed distribution.



Chapter problem

Paul owns a factory that makes sails. It costs $9000 per month to pay for rent, electricity and wages in the factory. Each sail costs $250 to make and sells for $850.

a How many sails does Paul need to make and sell per month to break even?

b What profit or loss will he make in a month if he makes and sells 20 sails?

Getty Images/Ben Welsh



CHAPTER 9

ALGEBRA, NUMBER AND STRUCTURE

EQUATIONS AND FORMULAS

9.1 Algebraic expressions
9.2 Solving equations
9.3 Formulas and equations
9.4 Formulas and graphs
9.5 Intersecting lines
9.6 Points of intersection
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Manipulate algebraic expressions and substitute into algebraic expressions
- Solve equations
- Use equations and formulas in real-life situations
- Graph linear equations
- Solve simultaneous equations using graphs and algebra

How are we ever going to use this?

- As part of the work in many trades and workplaces
- When looking at health and fitness; for example, to calculate BMI (body mass index) and heart rate

Study design coverage

AREA OF STUDY 1: ALGEBRA, NUMBER AND STRUCTURE

- symbolic expressions, equations and formulas
- graphical and algebraic analysis of relations including transposition of formulas and finding a break-even point using simultaneous equations.

VCE Mathematics Study Design 2023–2027 p. 73, © VCAA 2022

Videos (3):
9.3 Formulas • Formulas and equations
9.4 Linear modelling

Worksheets (4):
9.1 Collecting like terms • Expanding algebra
9.3 Working with formulas
9.5 Intersection of lines

Puzzles (2):
9.2 Equations code puzzle
9.3 Substitution code puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

9.1 Algebraic expressions

Worksheet Collecting like terms

Worksheet Expanding algebra

EXAMPLE 1

Simplify each expression.

a $7d + 3e + 3d + e$ **b** $6m - 8 + 3m + 6$ **c** $5d \times 6d \times 4$ **d** $12a^2b \div 3ac$

Steps	Working
a Group like terms together.	$7d + 3e + 3d + e = 7d + 3d + 3e + e$ $= 10d + 4e$
b Group like terms together. Remember the + or – sign goes with the term after it.	$6m - 8 + 3m + 6 = 6m + 3m - 8 + 6$ $= 9m - 2$
c Multiply numbers and variables separately.	$5d \times 6d \times 4 = 5 \times 6 \times 4 \times d \times d$ $= 120d^2$
d Write the division as a fraction and then simplify.	$12a^2b \div 3ac = \frac{12a^2b}{3ac}$ $= \frac{4ab}{c}$

EXAMPLE 2

Expand each expression.

a $7(p + 4)$ **b** $9k - 4(k - 3)$

Steps	Working
a Multiply each term inside the brackets by the number or variable outside the brackets.	$7(p + 4) = 7 \times p + 7 \times 4$ $= 7p + 28$
b Expand the brackets first and then simplify.	$9k - 4(k - 3) = 9k - 4 \times k - 4 \times (-3)$ $= 9k - 4k + 12$ $= 5k + 12$

Be extra careful with '–' signs in front of brackets!

EXAMPLE 3

If $p = 3$ and $q = 14$, evaluate each expression.

a $2p + q$ **b** $\frac{pq + 2}{4}$

Steps	Working
a Replace p and q with the given values.	$2p + q = 2 \times 3 + 14$ $= 20$
b	$\frac{pq + 2}{4} = \frac{3 \times 14 + 2}{4}$ $= \frac{44}{4}$ $= 11$

EXERCISE 9.1 Algebraic expressions

ANSWERS p. 433

1 EXAMPLE 1 Simplify each expression.

a $10ab + 2 - ab$ **b** $7ab + 2bc - 3ab + bc$ **c** $4k^2 + 3 + 5k^2 + 8$

d $7 - 2x - 3$ **e** $x^2 - 6x - x + 7$ **f** $p^2 + q^2 + 3p^2 - q^2$

g $3 + 4y + 8y + 11$ **h** $11r - 3s - 4r + 6s$ **i** $7g + 8b - b - g$

j $6x^2 - 4x - 9x + 6$ **k** $20a + 11m + 32a - 11m$ **l** $3q - 7 - 8q + 12q$

m $5 - 2y - 5 - 4y$ **n** $s^2 + 4s^2 + 12 - 5s^2$

2 EXAMPLE 1 Simplify each expression.

a $3d \times 4d$ **b** $5 \times 4m \times 6n$ **c** $2y \times 3y \times 8$

d $3r \times 3 \times 3r$ **e** $3ac \times 6cd$ **f** $hjk \times hj \times jk$

g $4q^2 \times 6p^2$ **h** $2m \times 3n \times mn$ **i** $14m^2 \div 7$

j $6e^2 \div (-2e)$ **k** $-5p^2 \div p$ **l** $\frac{14x^2}{2x}$

m $\frac{15d^2e^2}{3de}$ **n** $\frac{3a^2b^2c}{3ab^2c}$ **o** $\frac{-24acb}{3bc}$

p $\frac{-10m^2n}{5mn}$

3 EXAMPLE 2 Expand each expression.

a $2(4m + 5n)$ **b** $5k(3k - 2)$ **c** $7j(3j + k)$

d $4n(2n + 5m)$ **e** $8 + 6(p - 5)$ **f** $3(q - 3m) + 5m$

g $31 - 7(t + 2)$ **h** $2(9n - 2) + 8$ **i** $3(a + 4) + 2(a + 1)$

j $2(x + 5) + 6(x - 1)$ **k** $3(n + 5) - 2(n + 4)$ **l** $8(4y + 1) - 5(2y - 3)$

4 EXAMPLE 3 If $d = 10$, $e = -3$ and $f = 5$, evaluate each expression.

a def **b** f^2 **c** e^3

d $d \div f$ **e** $de \div 2$ **f** $d^2 - f^2$

g $7e$ **h** $3e + 2d - f$ **i** $f + d - e$

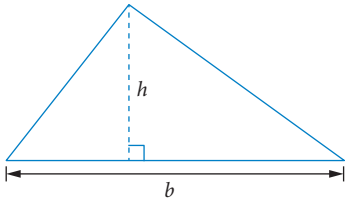
j $\frac{d}{2} - 3f$ **k** $6ef$ **l** $\frac{e + f}{4}$

5 The formula for the area of a triangle is

$$A = \frac{1}{2}bh$$

where A is the area of the triangle, b is the length of the base and h is the height.

Find the area of a triangle with base length 12 m and height 7 m.



6 A plumber charges according to the formula $C = 42h + 65$, where C is the charge in dollars and h is the number of hours worked. How much does the plumber charge for working for $3\frac{1}{2}$ hours?

7 The formula for converting Celsius temperatures to Fahrenheit temperatures is

$$F = \frac{9}{5}C + 32$$

where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Use the formula to convert 26°C to degrees Fahrenheit.

8 The number of hours of sleep per day recommended for children has the formula

$$H = 14 - \frac{A}{2}$$

where H is the recommended number of hours of sleep and A is the age of the child.

Find the number of hours of sleep recommended per day for a 6-year-old child.

9.2 Solving equations

Puzzle
Equations code puzzle

Solving equations

When solving an equation:

- aim to have the variable (such as x) on one side of the equation and a number on the other side, so the equation is in the form '$x =$ _____'
- do the same inverse operation to both sides of the equation
- check your solution by substituting back into the original equation.

EXAMPLE 4

Solve these one-step equations.

a $k + 9 = 20$ **b** $m - 5 = 4$ **c** $7p = 63$ **d** $\frac{u}{12} = 4$

Steps	Working
a Subtract 9 from both sides.	$k + 9 = 20$ $k + 9 - 9 = 20 - 9$ $k = 11$
b Add 5 to both sides.	$m - 5 = 4$ $m - 5 + 5 = 4 + 5$ $m = 9$

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c Divide both sides by 7.	$7p = 63$ $\frac{7p}{7} = \frac{63}{7}$ $p = 9$
d Multiply both sides by 12.	$\frac{u}{12} = 4$ $\frac{u}{12} \times 12 = 4 \times 12$ $u = 48$

EXAMPLE 5

Solve these two-step equations.

a $5w + 8 = 33$ **b** $\frac{m}{6} - 5 = 7$ **c** $\frac{2h}{3} = 4$ **d** $\frac{x+1}{2} = 7$

Steps	Working
a Subtract 8 from both sides.	$5w + 8 = 33$ $5w + 8 - 8 = 33 - 8$ $5w = 25$
Divide both sides by 5.	$\frac{5w}{5} = \frac{25}{5}$ $w = 5$
b Add 5 to both sides.	$\frac{m}{6} - 5 = 7$ $\frac{m}{6} - 5 + 5 = 7 + 5$ $\frac{m}{6} = 12$
Multiply both sides by 6.	$\frac{m}{6} \times 6 = 12 \times 6$ $m = 72$
c Multiply both sides by 3.	$\frac{2h}{3} = 4$ $\frac{2h}{3} \times 3 = 4 \times 3$ $2h = 12$
Divide both sides by 2.	$\frac{2h}{2} = \frac{12}{2}$ $h = 6$
d Multiply both sides by 2.	$\frac{x+1}{2} = 7$ $\frac{x+1}{2} \times 2 = 7 \times 2$ $x + 1 = 14$
Subtract 1 from both sides.	$x + 1 - 1 = 14 - 1$ $x = 13$

EXAMPLE 6

Solve $2x + 14 = 3x + 7$.

Steps	Working
Subtract $2x$ from both sides.	$2x + 14 = 3x + 7$ $2x + 14 - 2x = 3x + 7 - 2x$ $14 = x + 7$
Subtract 7 from both sides.	$14 - 7 = x + 7 - 7$ $7 = x$
Write it with x on the left-hand side.	$x = 7$

EXERCISE 9.2 Solving equations

ANSWERS p. 433

1 EXAMPLE 4 Solve each equation.

a $a + 7 = 20$ **b** $x - 12 = 7$ **c** $4p = 56$ **d** $\frac{h}{11} = 4$

e $q - 15 = 35$ **f** $16n = 96$ **g** $\frac{y}{8} = 17$ **h** $w + 23 = 84$

2 EXAMPLE 5 Solve each equation.

a $8a - 4 = 420$ **b** $3x + 12 = 27$ **c** $2p - 18 = 48$ **d** $3m - 5 = 22$

e $\frac{y}{10} - 5 = 3$ **f** $\frac{g}{3} - 1 = 11$ **g** $\frac{t}{5} + 4 = 13$ **h** $\frac{d}{8} + 19 = 42$

i $\frac{3x}{2} = 9$ **j** $\frac{15x}{9} = 15$ **k** $\frac{2x}{3} = 8$ **l** $\frac{3N}{5} = 3$

m $\frac{x+1}{3} = 2$ **n** $\frac{x-3}{2} = 3$ **o** $\frac{N+2}{5} = 1$ **p** $\frac{N-3}{4} = 5$

3 EXAMPLE 6 Solve each equation.

a $m + 8 = 2m + 4$ **b** $2n + 17 = 6n + 5$ **c** $5y + 14 = 3y + 24$

d $8p - 15 = 3p + 10$ **e** $d + 24 = 3d$ **f** $5t = 2t + 42$

Investigation The solution is $y = 3$

1 Create 10 different equations that have the solution $y = 3$.

Your 10 equations must include at least one with each operation and one with the variable on both sides.

2 Change the following equation to make its solution $y = 3$. Show a number of different ways to do this.

$12y + 3(y - 5) = 45$

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9.3 Formulas and equations

Sometimes when we substitute the known information into a **formula**, we are left with an equation to solve. Equation solving is one of the mathematical tools that help us to find the answers to problems.

EXAMPLE 7

After age 30, a person's height decreases by 0.06 cm per year. This can be modelled by the formula

$h = T - 0.06(a - 30)$

where

h = height in cm
T = height in cm at age 30
a = age in years.

Grandma was 168 cm tall on her 100th birthday. How tall was she when she was 30 years old?

Steps	Working
Grandma is 100 years old, $a = 100$, and her height now is $h = 168$. We want to find T, her height when she was 30.	$a = 100$, $h = 168$
Substitute the values into the formula.	$h = T - 0.06(a - 30)$ $168 = T - 0.06(100 - 30)$
Simplify the equation.	$168 = T - 4.2$
Solve the equation.	$168 + 4.2 = T - 4.2 + 4.2$ $172.2 = T$ $T = 172.2$
Answer the question.	Grandma was 172.2 cm tall when she was 30 years old.



Video Formulas

Video Formulas and equations

Worksheet Working with formulas

Puzzle Substitution code puzzle

EXERCISE 9.3 Formulas and equations

ANSWERS p. 433

1 EXAMPLE 7 The femur bone is found in the thigh. The formula that relates an adult female's height, h cm, to the length of her femur bone, F cm, is $h = 61.41 + 2.32F$.

Dagma is 174 cm tall. How long is her femur bone, correct to 1 decimal place?

2 Use the formula $v^2 = u^2 + 2as$ to determine the value of a when $s = 4$, $u = 5$ and $v = 7$.

3 The cost, C dollars, of a taxi trip over a distance of d kilometres is given by

$C = 2 + 1.2d$

What distance did James travel when his taxi trip cost him $32?

4 The perimeter P of a rectangle is given by the formula

$P = 2L + 2W$

where L is the length and W is the width of the rectangle.

The perimeter of a rectangle is 71 cm and its width is 17 cm. Calculate the length of the rectangle.

5 The formula $P = \frac{V}{I}$ relates the power P in watts, the voltage V and the current I in amperes in an electric circuit. Calculate the voltage in a circuit when the power is 6 watts and the current is 25 amperes.

6 Fried's formula for calculating an infant's dose of an adult's medicine is

$$D = \frac{MA}{150}$$

where

D = the infant dose in millilitres
A = the adult dose in millilitres
M = the infant's age in months.

16-month-old Bella had a bad cough. During the night, her mother gave her a 3.2 mL dose of an adult cough medicine to help ease her cough. How much of the medicine can an adult take?

7 The number of times a cricket chirps in a minute is related to the temperature by the formula

$$T = \frac{n}{8} + 5$$

where n is the number of cricket chirps and T (in °C) is the temperature.

a On a warm day, a cricket chirped 200 times per minute. What was the temperature?

b How many chirps per minute will a cricket make when the temperature is 20°C?

8 The distance between Earth and Mars changes depending on their orbits around the Sun. Approximately every 2 years, the path a rocket would follow on a trip from Earth to Mars is 55 000 000 km (55 million km) long.

a In 1964, the first spacecraft to travel to Mars took 228 days to get there. Use the distance, speed and time formula, $D = S \times T$, to calculate the average speed of the rocket in km/day correct to the nearest whole number.

b To minimise health risks to astronauts from cancer-causing cosmic rays, future trips to Mars need to be completed in 150 days. Express 150 days in hours then calculate the average speed required for manned flights to Mars in km/h.



NASA

9 People living on rural properties often have their own sewerage treatment systems.

The formula $V = DF$ calculates the recommended volume, V litres, of a sewerage tank, based on the number of days, D, the sewerage stays in the tank, and the rate at which sewerage enters the tank, F, in litres per day.

Zack's sewerage tank holds 12 000 L. If the household produces 520 L of sewerage per day, how many days will the sewerage stay in the treatment tank?

10 In indoor cricket, a batter's score can be calculated using the formula

$S = R - 5n$

where R is the number of runs the batter made and n is the number of times they were given out.

a In one match, Rose made 60 runs and her score was 45. How many times was she given out?

b Ibrahim batted well to score 31. He was given out twice. How many runs did he make?

11 The value $\$V$ of an item when it is n years old is given by the formula

$V = P(1 - r)^n$

where P is the original value of the item and r is the annual rate of **depreciation** (decrease) as a decimal; for example, 5% = 0.05.

a Calculate the value of a 6-year-old lounge that was originally valued at \$4500. Its annual rate of depreciation is 18%.

b After 5 years, Joel's car was worth \$12 000. If it has been depreciating at an annual rate of 15%, what was the value of the car when it was new?

12 The speed a car travels during a driver's reaction time is given by the formula

$$D = \frac{5vt}{18}$$

where

D = the distance in metres
v = the speed of the car in km/h
t = the driver's reaction time in seconds.

When Pascal was travelling at 90 km/h, his car travelled 45 m during his reaction time. What is Pascal's reaction time?

13 Police use a formula to determine, by looking at the skid marks, how fast a car was travelling when it braked. The formula is

$$l = S \times \frac{210 + 97R}{1000}$$

where

l = the length of the skid marks in metres
S = the speed of the car in km/h
R = the road surface index.

A set of skid marks on a dry asphalt road was 32 m long. The road surface index for dry asphalt is 1.3.

a Show that the above formula simplifies to the equation $32 = 0.3361S$ after substituting the correct values.

b Determine, correct to 1 decimal place, the speed at which the car was travelling.

9.4 Formulas and graphs

Formulas can be graphed on the number plane. In Year 11, we learnt how to plot points and draw straight lines on the number plane. We will now apply those skills to graphing formulas.

Video
Linear modelling

EXAMPLE 8

The concentration of a drug in a person's body decreases as time passes. This is represented by the graph shown, which has the equation $C = -25t + 200$. Note that the variables x and y have been replaced by t and C respectively.

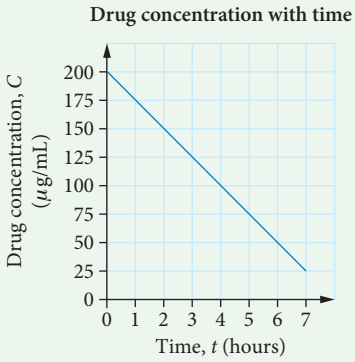
a What is the gradient of the line?

b What does the gradient represent?

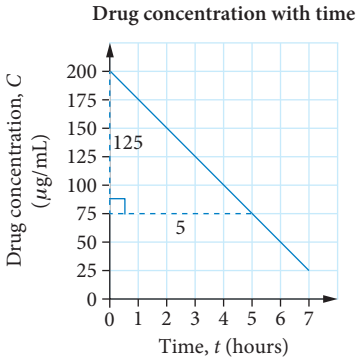
c Find the vertical intercept of the line.

d What does the vertical intercept represent?

e When will there be no drug remaining in the body?



μg means microgram or one-millionth of a gram.

Steps	Working
a Draw a triangle below the line to calculate the gradient. The line goes down 125 units for every 5 units across. The gradient will be negative because the line goes down rather than up, representing the decreasing drug concentration. or Read the gradient from the equation of the line. It is the number in front of the t.	 $m = -\frac{125}{5}$ $= -\frac{25}{1}$ $= -25$
b The gradient represents the decrease in drug concentration per hour. It is negative because the concentration is decreasing.	The drug concentration per hour goes down 25 μg/mL every hour.
c The vertical intercept is the general name for the y-intercept. It is the value where the line crosses the vertical axis and is the constant in $C = -25t + 200$.	The vertical intercept is 200. We don't say 'y-intercept' because the equation involves t and C, not x and y.
d The vertical intercept is the drug concentration when $t = 0$.	The vertical intercept represents the initial concentration of the drug: 200 μg/mL.
e There is no drug remaining when $C = 0$. We can either extend the line to see where it crosses the horizontal axis, or substitute $C = 0$ into the equation and solve the equation.	When $C = 0$, $0 = -25t + 200$ $25t = 200$ $\frac{25t}{25} = \frac{200}{25}$ $t = 8$. There will be no drug in the body after 8 hours.

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EXAMPLE 9

Carly has a window-cleaning business. Each day it costs her an average of $75 for fuel and car maintenance and $8 per job.

a Let C = Carly's daily costs in dollars and n = the number of jobs she does per day. Write an equation for Carly's daily costs.

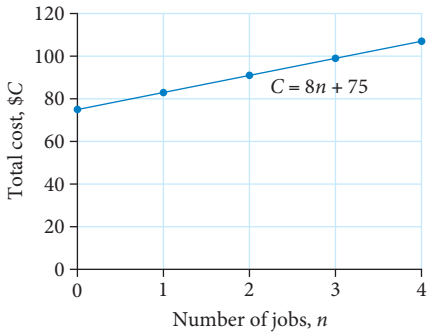
b Construct a graph to illustrate Carly's daily costs.

c What is the gradient and vertical intercept of the graph and what do these values represent?

Steps	Working
a Carly's daily costs = \$75 car costs + \$8 per job	$C = 75 + 8 \times n$ $C = 8n + 75$
b Complete a table of values for $C = 8n + 75$.	

n	0	1	2	3	4
C	75	83	91	99	107

Use the table of values to graph a line on a number plane.



c

The gradient measures the rate of change of Carly's costs as the number of jobs increases by 1. It is positive because the cost is increasing.

For $C = 8n + 75$, the gradient is 8 and the vertical intercept is 75.

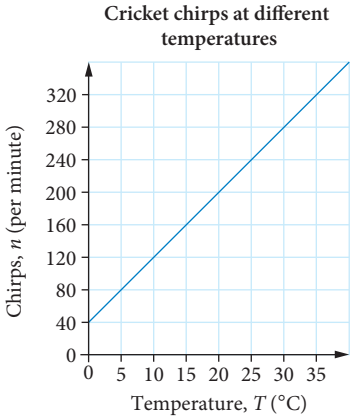
The gradient represents Carly's costs per job (\$8).

The vertical intercept represents Carly's fixed daily costs (\$75); the cost without doing any jobs.

EXERCISE 9.4 Formulas and graphs

ANSWERS p. 434

1 EXAMPLE 8 The number of times a cricket chirps in a minute, n, is related to the temperature, T (°C), by the equation $n = 8T + 40$. This is shown on the graph.





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a What is the graph's vertical intercept?

b What does the vertical intercept represent in this situation?

c Calculate the gradient of the line.

d In this context, what does the gradient represent?

e How many times per minute does a cricket chirp when it is 32°C?

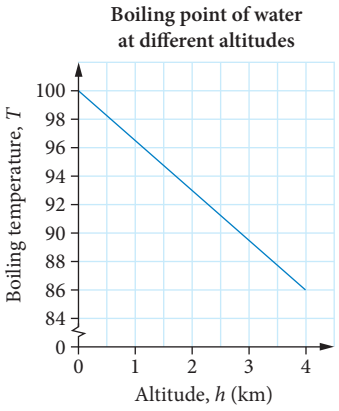
f At what temperature do crickets chirp 160 times per minute?

g Using the formula, calculate how many times per minute a cricket chirps at 100°C. Is this realistic?

2 At sea level, water boils at 100°C. At different altitudes, the boiling temperature changes.

'Altitude' is another word for 'height'.

This graph shows the relationship between altitude, h km, and the temperature, T°C, at which water boils. The equation of the line is $T = -3.5h + 100$.



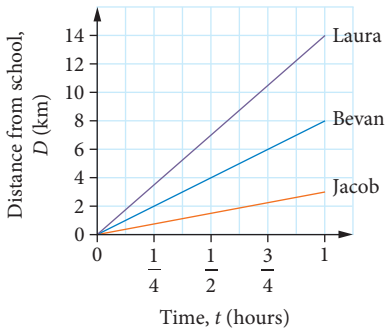
a Water boils at 100°C at sea level. What altitude is sea level?

b Calculate the gradient of the line.

c What physical quantity does the gradient represent?

d Use the graph to determine the boiling point of water at an altitude of 2.8 km.

3 After school, 3 students went home by walking, jogging or riding a bike. Each of them travelled at a constant speed and took 1 hour to reach home. The graph shows the distance they travelled.



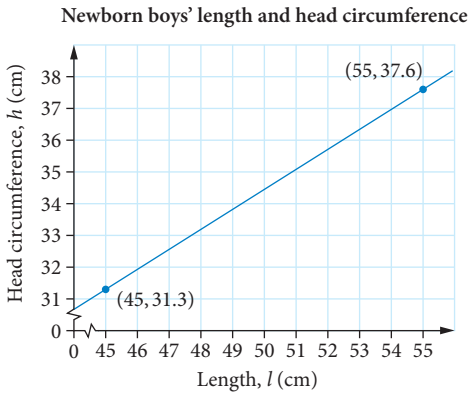
a Calculate the gradient of each line.

b In this context, what does the gradient represent?

c Which student travelled at the slowest speed? Give a reason for your answer.

d Match Laura, Bevan and Jacob to their method of travelling home, that is, walking, jogging or riding a bike.

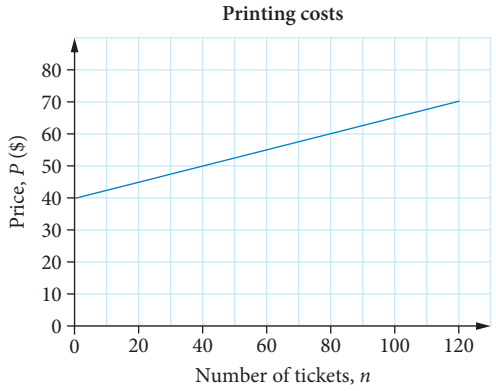
4 This graph shows the length and head circumference of average newborn boys.



Newborn baby Jason is 51 cm long and his head circumference is 35.1 cm.
Do you think Jason's parents should be worried about his growth? Give a reason for your answer.

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5 Anastasia operates a printing business. When she quotes a price for printing tickets, she charges an initial fee to cover the cost of design and an additional fee of $0.25 per ticket printed. The price she charges for printing tickets is shown on the graph.



a How much is Anastasia's initial charge for the design?

b What is the gradient of the line?

c What physical quantity does the gradient represent?

d How much does she charge to design and print 500 tickets?

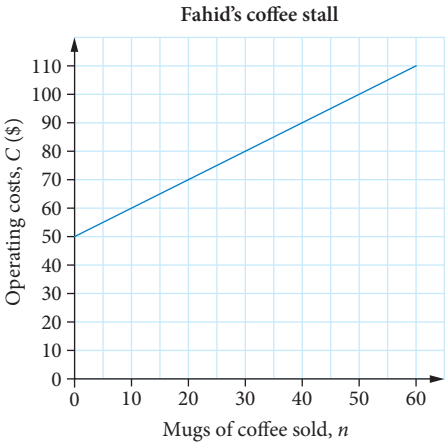
e When Anastasia designs and prints elaborate wedding invitations, she charges an initial fee of $110 and 50 c per invitation. Write a formula to determine the price, in D dollars, for designing and printing n elaborate wedding invitations.

6 Fahid runs a coffee stall at sporting events. The graph shows how much it costs Fahid to set up the stall and make mugs of coffee.

a How much does it cost Fahid to set up the stall?

b How much does it cost him to make each mug of coffee?

c If n is the number of mugs sold and C is Fahid's costs in dollars, what is the equation of the line on the graph?



7 EXAMPLE 9 Belinda's hobby is skydiving. This table of values shows, for a recent jump, her height above the ground t seconds after she opened her parachute.

Time, t seconds	0	60	90	120
Height, h metres	600	300	150	0

a How high above the ground was Belinda when she opened her parachute?

b How many metres did Belinda fall every second?

c Explain why the equation $h = 600 - 5t$ represents this situation.

d Graph this equation.



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8 Gabriel is investing $2000 at a simple interest rate of 3.5% p.a. This table shows the interest earned, using the formula $I = Prn$.

Number of years (n)	0	1	2	3	4	5
Interest earned (I)	0	$70	$140	$210	$280	$350

a Graph this function.

b Find the gradient of the line and explain what this value represents.

c Find the vertical intercept and explain what this value represents.

d Use the equation to find the interest earned if $2000 is invested at 3.5% p.a. for 7 years.

e Gabriel withdraws his money after 2 years. How much money in total does he receive?

9 The cost of filling a petrol tank is related to the number of litres it holds, as shown in this table.

Petrol amount (L litres)	0	10	20	30	40	50
Cost ($C)	0	14	28	42	56	70

a Graph the relationship between petrol amount and cost.

b What is the gradient of the graph?

c Calculate the cost of buying 75 L of petrol.

10 Amy has some old dressmaking patterns that show material measurements in yards, an imperial measure, which she needs to convert to metres.

a Use this table of values to construct a conversion graph between yards and metres.

Yards	0	1	3	5	10
Metres	0	0.9	2.7	4.5	9.1

b Use your graph to estimate the number of metres of material Amy needs when a pattern asks for $2\frac{3}{4}$ yards.

c Amy has 4.2 m of material left for making a dress. The pattern says she needs 4 yards of material. Does Amy have enough material to make the dress? Justify your answer.



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9.5 Intersecting lines

We can graph 2 lines on the same number plane to help us solve problems.

EXAMPLE 10

Cherie sells scones for $3 each. It costs her $240 for the necessary equipment and $1 to make each scone.

a Find an equation for C, the cost in dollars for making n scones.

b Find an equation for I, the income in dollars from selling n scones.

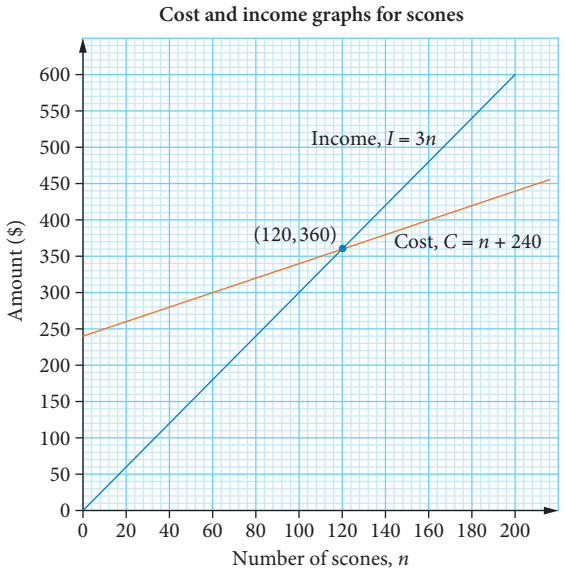
c Graph both equations on the same axes, for values of n from 0 to 200.

d How many scones does Cherie need to sell to break even?

e How much profit will Cherie make if she sells 390 scones?



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Steps	Working
a It costs $240 plus $1 for each scone she makes.	Cost = $240 + n × $1 $C = 240 + n$
b Income is $3 for each scone.	Income = n × $3 $I = 3n$
c Graph both lines together.	
d 'Break even' means that costs and income are the same. This occurs on the graph where the 2 lines intersect, at (120, 360). At the **break-even point**, 120 represents the number of scones.	Cherie needs to sell 120 scones to break even.

Worksheet
Intersection of lines

e The profit is the amount made once the cost is subtracted from the income. First, substitute $n = 390$ into both the Income and Cost equations.

profit = income − cost

$I = 3n$
$= 3 \times 390$
$= \$1170$

For 390 scones, the income will be $1170.

$C = n + 240$
$= 390 + 240$
$= \$630$

For making 390 scones, the cost will be $630.

Profit = $1170 − $630
= $540

Cherie will make a $540 profit when she makes and sells 390 scones.

Break-even point

In the previous example, when $n = 120$, $C = 360$ and $I = 360$, the cost and income are the same. This means that when Cherie makes 120 scones, it costs her $360 and she receives $360 from the sales. From the graphs, we can see that:

- if she sells more than 120 scones, she will make a **profit** because her income is greater than her costs
- if she sells fewer than 120 scones, she will make a **loss** because her income is less than her costs.

The value of 120, when costs and income are equal, is called the **break-even point**.

If we graph the equations of a business' costs and its income, then the break-even point is the point where the 2 lines intersect. Below the break-even point, the costs are greater than the income and the business makes a loss. Above the break-even point, the income is greater than the costs and the business makes a profit.

EXERCISE 9.5 Intersecting lines

ANSWERS p. 434

1 EXAMPLE 10 Khalid bakes and sells muffins. The equipment used to bake the muffins costs $200 and the ingredients for each muffin cost $1. Khalid sells the muffins for $2 each.

Let n = the number of muffins, $\$C$ = the total cost to make n muffins and $\$I$ = the income from selling n muffins.

a Explain why $C = n + 200$ and $I = 2n$.

b Graph $C = n + 200$ and $I = 2n$ on the same axes for values of n from 0 to 400.

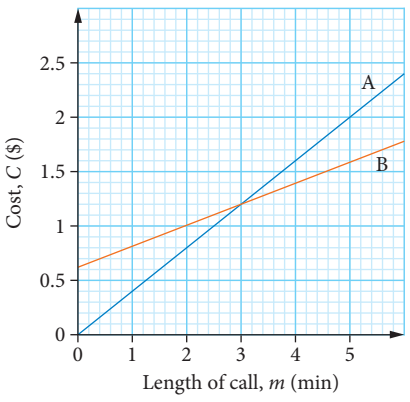
c What are the coordinates of the point where the 2 lines intersect?

d What does this point represent?

e Will Khalid make a profit if he sells 120 muffins? Explain your answer.

f How much profit will Khalid make if he sells 230 muffins?

2 Chloe is deciding between 2 mobile phone plans. Plan A offers her no connection fee and charges of 40 cents per minute for each call. Calls under plan B cost 20 cents per minute with a 60-cent connection fee. The graphs show the costs of phone calls under each plan for calls up to 6 minutes.

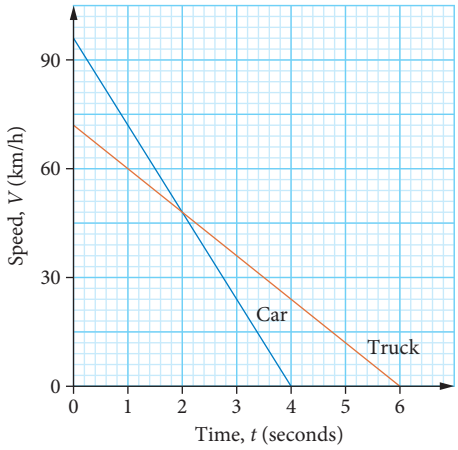


a Which plan (A or B) has the equation $C = 0.6 + 0.2m$? Explain the reason for your choice.

b For what length of phone call is the cost the same under both plans?

c Which plan is the cheaper for longer calls?

3 A truck and a car were travelling on the freeway. When an accident occurred ahead of them, both vehicles had to stop quickly. The graphs show the speed each vehicle was travelling at t seconds after they applied their brakes.



a How fast were the car and the truck travelling in km/h before the accident?

b How many seconds after applying their brakes were the vehicles travelling at the same speed?

c How much longer did the truck take to stop compared to the car?

d Write an equation for V, the speed in km/h, for the speeds of each vehicle t seconds after applying their brakes.

e Explain why the gradients of both lines are negative.

f Suggest a reason why the truck took longer to stop than the car, even though the car was travelling faster initially.

4 Allana sells flowers at the market for $10 per bunch.

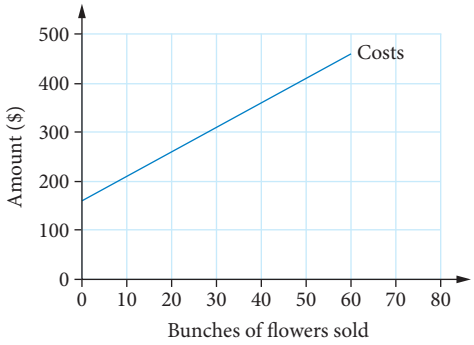
a Copy and complete this table.

Bunches of flowers sold	0	10	20	30	40	50	60
Income received ($)							

b The graph shows Allana's costs when she sells n bunches of flowers.

Make a copy of the graph using these values.

n	0	10	20	30	40	50	60
C	160	210	260	310	360	410	460



Use your answers to part **a** to graph Allana's income on the same axes.

c Explain how you know that Allana will break even when she sells 32 bunches of flowers.

d How many bunches of flowers will Allana need to sell to make a profit?



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5 Each week a toy car factory's fixed costs total $1500. Its costs increase by $15 for every toy car it produces. Each item produced sells for $35.

Let n = the number of toy cars the factory produces each week, $\$C$ = the total costs to produce n items, and $\$I$ = the income the factory receives from selling n items.

a Does the expression $35n$ represent the income from selling n items or the cost to produce n items?

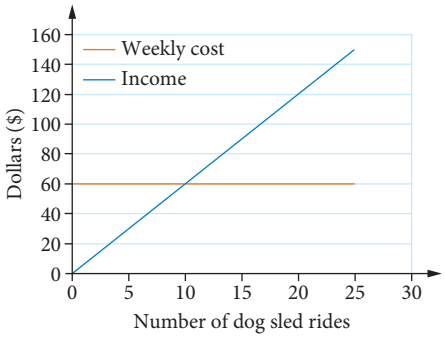
b What does the expression $15n + 1500$ represent?

c Graph the lines $I = 35n$ and $C = 15n + 1500$ on the same set of axes for values of n from 0 to 200.

d How many toy cars does the factory need to produce each week to break even?

e How much profit will the company make if it sells 100 toy cars in a week?

6 Jon earns money taking tourists on dog sled rides in the snow. Jon has displayed his expenses and income on a graph.



a How much are Jon's weekly costs?

b How many dog sled rides does Jon require each week to break even?

c How much does Jon charge for a dog sled ride?

d Suggest a reason why Jon's cost line is horizontal.

7 To advertise the school musical, the drama class is making and selling promotional T-shirts. The set-up costs to make the T-shirts is $160 and each shirt will cost $10 to make. The class will sell the T-shirts for $20 each.

a Find an equation for C, the cost in dollars for making n T-shirts.

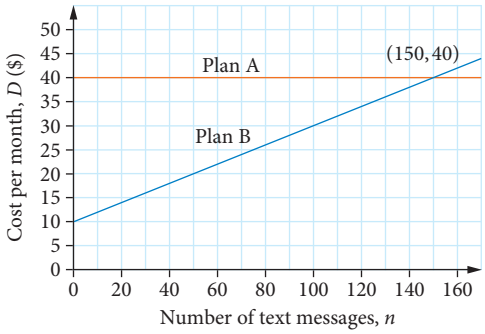
b Find an equation for I, the income in dollars from selling n T-shirts.

c Graph both equations on the same axes, for values of n from 0 to 50.

d How many T-shirts need to be sold to break even?

e The class estimates it will sell 90 T-shirts. How much profit can they expect to make?

8 Grant is hard of hearing and uses his mobile phone for texting only. The graph shows the monthly cost, $\$D$, for n texts. Plan A is a fixed price per month with unlimited texts while Plan B has a monthly charge and a cost per text.





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a What is the monthly cost for Plan A?

b What is the vertical intercept of the graph of Plan B?

c What does the vertical intercept in Plan B represent?

d How much does each text cost to send on Plan B?

e What is the equation of the line representing Plan B?

f For what number of calls per month is Plan A the same cost as Plan B?

g Grant sends an average of 5 or 6 texts per day. Which plan would you recommend he chooses? Use calculations to justify your recommendation.

Practical activity Costing the Year 12 formal

In this investigation, we will model the costs and income for running the Year 12 formal.

1 You will need to collect the following information for where you live:

Costs

Hire of venue _____

Decorations _____

Band/DJ _____

Other costs _____

Total costs _____

These are the fixed costs for the formal.

The committee decides to charge \$15 per person to cover the fixed costs.

2 On the same number plane:

- draw a graph for $I = 15n$ for n from 0 to 200.
- draw a graph for $F =$ _____ for the fixed costs. This will be a horizontal line.

3 a Where do the 2 lines intersect? How many people need to attend the formal to cover the fixed costs?

b How many people do you expect to attend the formal? Will you cover the fixed costs if you charge \$15 per person?

c Depending on what you have found, adjust the amount you need to charge per person to cover the fixed costs. This could be more or less than \$15. Draw the income graph for this new amount.

d How many people now need to attend the formal to cover the fixed costs?

4 Find out from the venue how much they charge per person for catering and calculate what you need to charge for your formal tickets.

9.6 Points of intersection

Graphing 2 equations on the number plane is not the only way to find the point of intersection. We can use algebra to find the point of intersection, that is, the values of the variables that satisfy **both** equations. Because we solve the 2 equations together, we call it solving **simultaneous equations**. Simultaneous is another word for 'at the same time'.

EXAMPLE 11

Solve these 2 equations simultaneously.

$y = x + 2$ and $y = 2x - 3$

When we solve a pair of equations simultaneously, we are finding the coordinates of the points where the 2 graphs intersect on the number plane.

Steps	Working
At the point of intersection, the y from each equation has the same value.	$x + 2 = 2x - 3$
Subtract x from both sides.	$x + 2 - x = 2x - 3 - x$ $2 = x - 3$
Add 3 to both sides.	$2 + 3 = x - 3 + 3$ $5 = x$ $x = 5$
Then we calculate the value of y. Substitute the value we have found for x (5) into either one of the equations.	$y = x + 2$ or $y = 2x - 3$ $= 5 + 2$ $= 2 \times 5 - 3$ $= 7$ $= 7$ The solution is $x = 5$ and $y = 7$.

We can now use algebra as an alternative way to find Cherie's break-even point from Example 10.

EXAMPLE 12

Cherie's costs for making scones can be calculated using the equation $C = n + 240$.

Cherie's income from selling scones can be calculated using the equation $I = 3n$.

How many scones does Cherie need to sell to break even?

Steps	Working
At the break-even point, costs (C) and income (I) are equal.	$n + 240 = 3n$
We find n first.	
Subtract n from both sides.	$n + 240 - n = 3n - n$ $240 = 2n$
Divide both sides by 2.	$\frac{240}{2} = \frac{2n}{2}$ $n = 120$
Then we find C or I. Substitute the value you have found for n (120) into either one of the equations.	$C = n + 240$ or $I = 3n$ $= 120 + 240$ $= 3 \times 120$ $= 360$ $= 360$
Answer the problem.	Cherie needs to make 120 scones to break even. (Both her costs and her income will be $360 when she makes 120 scones.)

EXERCISE 9.6 Points of intersection

ANSWERS p. 435

1 EXAMPLE 11 Solve each pair of equations simultaneously.

a $y = 2x - 1$ and $y = x + 4$

b $y = 2x + 4$ and $y = 3x$

c $y = x - 4$ and $y = 3x + 2$

d $y = -4x$ and $y = 2x - 12$

2 EXAMPLE 12 Francesca is organising a morning tea to raise funds for a refugee charity. She is hiring a marquee for $160 and catering for the event at a cost of $7 per person. She will charge each person $15 to attend.

Francesca's costs for the morning tea can be calculated using the equation $C = 7n + 160$.

Francesca's income from the morning tea can be calculated using the equation $I = 15n$.

To break even, how many people does Francesca need to attend the morning tea?

3 Marko sells hot chips at his fast-food store. Each day, the cooking oil costs $48 and each container of chips costs him $0.80 to make. He sells the containers of chips for $4 each.

Marko's costs for making chips can be calculated using the equation $C = 0.8n + 48$.

Marko's income from selling chips is calculated by the equation $I = 4n$.

How many containers of chips does Marko need to sell in one day to break even?

4 On the Easter weekend, Dane and Mitchell headed north in different cars to go camping. Dane took mainly backroads and averaged 60 km/h. Mitchell left an hour later, took a different route and averaged 90 km/h.

The distance (D) Dane has travelled after t hours can be calculated by $D = 60t$.

The distance (D) Mitchell travelled after t hours can be calculated by $D = 90t - 60$.

From the time of Dane's departure, how long is it until they have travelled the same distance?

5 Camilla is a mechanic who charges $55 per hour. Alex is also a mechanic. He charges a basic fee of $50 and then $45 per hour.

a Explain why the equation $C = 55h$ represents the amount Camilla charges.

b Explain why Alex's charge is given by the equation $C = 50 + 45h$.

c Calculate the number of hours worked for which Camilla and Alex would charge the same amount.

6 Lawrence and Trudie are landscape gardeners. Lawrence charges a flat rate of $70 per hour. Trudie charges a call-out fee of $90 and then $55 per hour.

a Write the equation that represents the amount Lawrence charges.

b What equation represents the amount Trudie charges?

c For how many hours of work would Trudie and Lawrence charge the same amount?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 435

Use the listed words to copy and complete the summary of this chapter below. Some of the words are used more than once.

algebraic	break-even	costs	depreciation
equations	expanding	formulas	income
interest	intersecting	loss	model
profit	simultaneous	substitution	subtracting

In this chapter we reviewed our algebra skills from Year 11: adding and $_{1}$_____ like terms, multiplying and dividing terms, simplifying and evaluating $_{2}$_____ expressions by $_{3}$_____ brackets or $_{4}$_____, respectively.

We revised solving $_{5}$_____ and substituting values into $_{6}$_____.

We used straight-line graphs to $_{7}$_____ real-life contexts such as business costs, simple $_{8}$_____ and $_{9}$_____.

We learnt that $_{10}$_____ lines can be used to solve $_{11}$_____ equations, and applied this idea to practical problems such as business, travelling and comparing phone plans.

In business, we draw a graph for the money coming in, called $_{12}$_____, and the money being spent, called $_{13}$_____. The point of intersection of these 2 lines is the $_{14}$_____ point. At this point, the business owner does not make a $_{15}$_____ or a $_{16}$_____. We also learnt to find this point using algebra.

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Solution to the chapter problem

Problem

Paul owns a factory that makes sails. It costs $9000 per month to pay for rent, electricity and wages in the factory. Each sail costs $250 to make and sells for $850.

a How many sails does Paul need to make and sell per month to break even?

b What profit or loss will he make in a month if he makes and sells 20 sails?

Solution

a Let n = the number of sails, C = the cost to produce n sails per month, and I = the income from selling n sails.

The equations are:

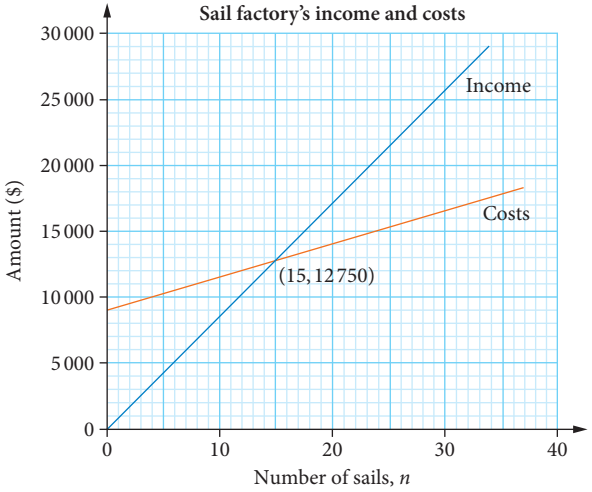
Costs: $C = 250n + 9000$

Income: $I = 850n$

We can find the break-even point by graphing the 2 equations or by solving them algebraically.

Using the graphs:

Graph the 2 straight lines on the same number plane.



The lines intersect at (15, 12 750) which means the break-even point is $n = 15$.

Paul needs to sell 15 sails per month to break even.

Using algebra:

At the break-even point, Paul's costs and income are the same.

$$250n + 9000 = 850n$$

$250n + 9000 - 250n = 850n - 250n$ Subtract $250n$ from both sides

$$9000 = 600n$$

$\frac{9000}{600} = \frac{600n}{600}$ Divide both sides by 600

$$15 = n$$

$$n = 15$$

Paul needs to sell 15 sails per month to break even.

b When $n = 20$:

$$\text{cost} = \$250 \times 20 + \$9000$$
$$= \$14\,000$$

$$\text{income} = \$850 \times 20$$
$$= \$17\,000$$

Income is greater than cost, so a profit will be made.

$$\text{profit} = \$17\,000 - \$14\,000$$
$$= \$3000$$

Paul will make $3000 profit when he makes and sells 20 sails in a month.

9 Test yourself

Equations and formulas

ANSWERS p. 435

1 EXERCISE 9.1 Simplify each expression.

a $13m - 3m + 8m$ **b** $7x + 3y + 3x + 7y$ **c** $33h + 6g - 5h + g$ **d** $4 \times 2w \times 4w$

e $3k \times 6j \times 2$ **f** $4d \times 3e \times 5de$ **g** $66m \div 6$ **h** $\frac{15p}{3p}$

i $14d^2 \div 2$ **j** $13b^2 \div b$

2 EXERCISE 9.1 Expand each expression and simplify where possible.

a $6(n + 2)$ **b** $-5(2b + 7)$ **c** $j(9j - 3)$

d $10 + 2(n + 4)$ **e** $22v - 3(v + 5)$ **f** $5(y + 2) + 4(y - 7)$

3 EXERCISE 9.1 If $x = 5$ and $y = -2$, evaluate each expression.

a $x + y$ **b** xy **c** $3x - 2y$

d $6y^2$ **e** $6x \div y$ **f** $x^2 + y^2$

4 EXERCISE 9.2 Solve each equation.

a $k + 6 = 13$ **b** $x - 3 = 8$ **c** $10f = 120$

d $\frac{m}{4} = 7$ **e** $4p + 3 = 23$ **f** $2x - 12 = 18$

g $\frac{h}{7} + 5 = 16$ **h** $\frac{n}{2} - 8 = 12$ **i** $x + 6 = 3x + 4$

j $2u + 6 = 5u - 3$ **k** $12h - 8 = 8h + 4$ **l** $2x + 9 = 7x - 16$

5 EXERCISE 9.3 Kellani is paid \$45 for every scarf she knits. Her income is given by the equation $I = 45n$, where n is the number of scarves she knits. How many scarves did Kellani knit in a day when she was paid \$270?

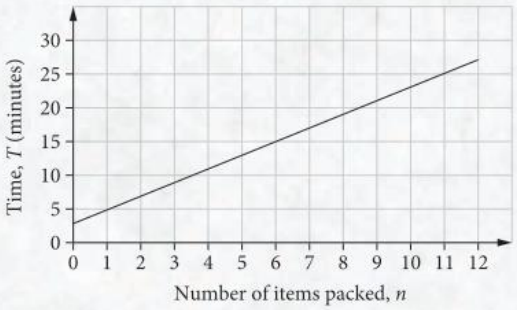
6 EXERCISE 9.3 The sum, S, of the angles in a polygon in degrees is given by $S = 180(n - 2)$, where n is the number of sides of the polygon.

a Find the sum of the angles when the polygon has 9 sides.

b How many sides does a polygon with an angle sum of 1080° have?

7 EXERCISE 9.4 Goran packs glass items in boxes.

He has to assemble the box before he can pack it. The graph shows the time, T minutes, it takes him to assemble one box and pack n glass items in it.



a How long does it take Goran to assemble a box before he starts to pack it?

b Calculate the gradient of the line.

c What physical amount does the gradient represent?

d Use the graph to find how many items Goran can pack in 15 minutes.

e Calculate the time it takes Goran to assemble a box and pack 15 items in it.

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8 EXERCISE 9.4 Adrian's Awesome Appetisers charges $10 per person plus an $80 fixed charge. This table shows Adrian's charges.

Number of people, n	0	50	100	150	200	250
Charge, C ($)	80	580	1080	1580	2080	2580

a Construct a graph showing Adrian's charges for n people.

b Use the graph to find the cost for 170 people.

c Adrian charged NVM Bank $830 for catering for a party. How many people were catered for?

d Find the vertical intercept of the line and explain what this value represents.

e Find the gradient and explain what this value represents.

9 EXERCISE 9.5 The Year 12 fundraising committee plans to sell school souvenir USB drives. The manufacturer quoted an 'initial charge of $126 plus $5 per unit' and the committee thinks that it will sell the units for $12 each.

a Find an equation for $C, the cost of making n USB drives.

b Find an equation for $I, the income from selling n USB drives.

c Graph both equations on the same axes for values of n from 0 to 20.

d How many USB drives does the committee need to sell to break even?

e There are 140 students in Year 12. If half of them buy a USB drive each, how much profit can the committee expect to make?

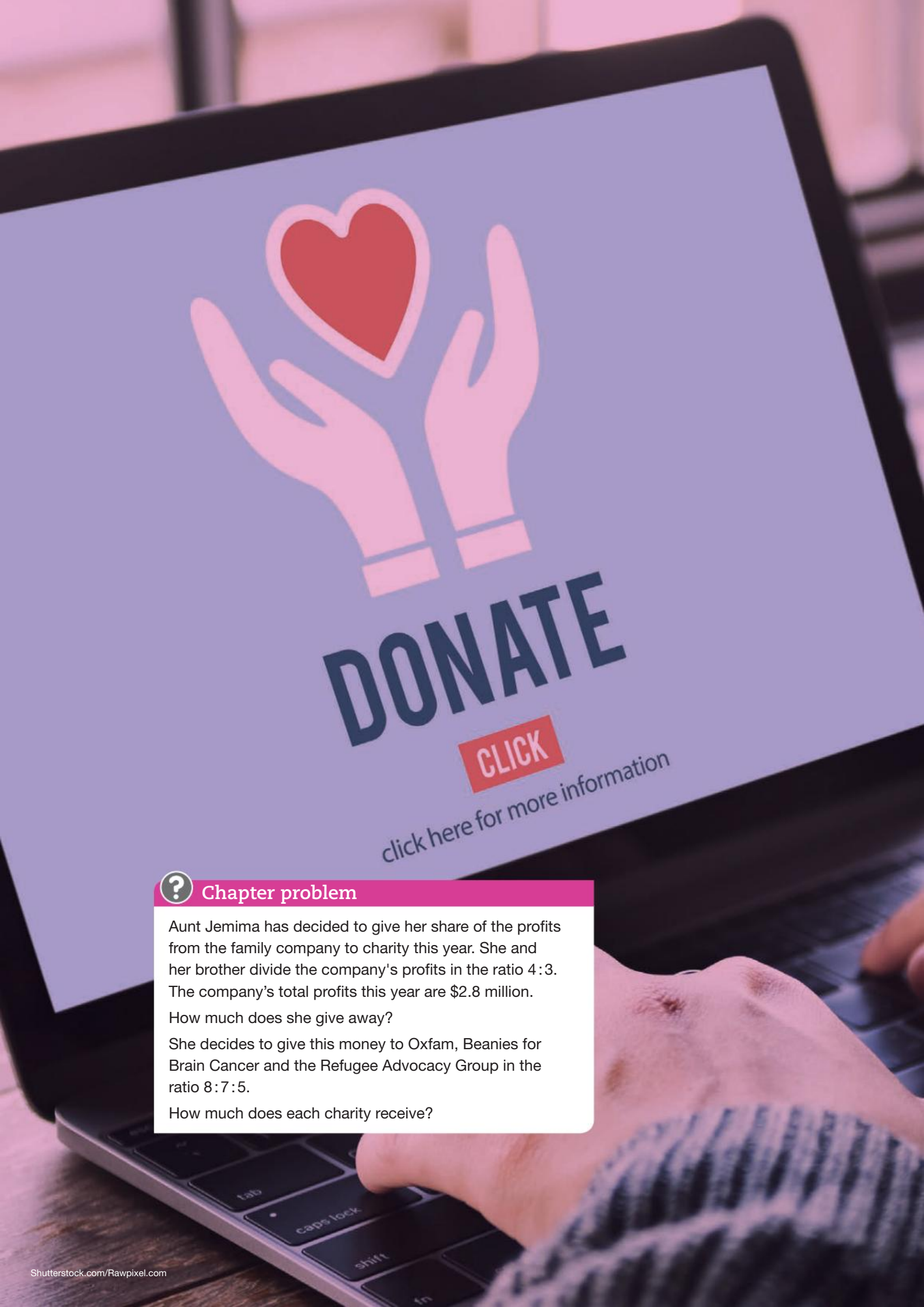
10 EXERCISE 9.6 Solve this pair of equations simultaneously.

$y = 2x + 17$
$y = 5x + 8$

11 EXERCISE 9.6 The equations for question **9** were:

Costs: $C = 5n + 126$
Income: $I = 12n$

Using algebra, find the number of USBs sold at the break-even point.



Chapter problem

Aunt Jemima has decided to give her share of the profits from the family company to charity this year. She and her brother divide the company's profits in the ratio 4 : 3. The company's total profits this year are $2.8 million.

How much does she give away?

She decides to give this money to Oxfam, Beanies for Brain Cancer and the Refugee Advocacy Group in the ratio 8 : 7 : 5.

How much does each charity receive?

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CHAPTER 10

ALGEBRA, NUMBER AND STRUCTURE

RATIO AND VARIATION

10.1 Ratios
10.2 Scales on maps
10.3 Ratio problems
10.4 Direct variation
10.5 Inverse variation
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Review ratio skills
- Use ratios in calculations with maps and plans
- Use ratios in everyday contexts
- Calculate quantities using direct and inverse variation

How are we ever going to use this?

- When making map calculations
- When reading house plans
- When mixing ingredients where there is a relationship between the quantities such as recipes, concrete, fertilisers, pesticides or other concentrates

Study design coverage

AREA OF STUDY 1: ALGEBRA, NUMBER AND STRUCTURE

- ratios and proportions in a practical context
- direct and indirect variation.

VCE Mathematics Study Design 2023–2027 p. 73, © VCAA 2022

Videos (7):

10.1 Simplifying ratios • Ratio problems • Dividing a quantity in a given ratio
10.4 Direct linear variation • Direct proportion
10.5 Inverse variation • Inverse proportion

Skillsheets (1):

10.1 Ratios

Worksheets (9):

10.1 Ratio calculations • Ratio match
10.2 Map scales • Map of Adelaide
10.3 Ratio problems • Ratios in other contexts • Calculating quantities in ratios
10.5 Direct and inverse proportion • Variation problems

Puzzles (3):

10.1 Simplifying ratios • Ratio match-up puzzle • Ratios code puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

10.1 Ratios

Video Simplifying ratios

Video Ratio problems

Video Dividing a quantity in a given ratio

Skillsheet Ratios

Worksheet Ratio calculations

Worksheet Ratio match

Puzzle Simplifying ratios

Puzzle Ratio match-up puzzle

Puzzle Ratios code puzzle

EXAMPLE 1

Simplify each ratio.

a 25 : 40 **b** 24 : 16 **c** 18 : 30 : 15

Steps	Working
a Divide each term of the ratio by a common factor, in this case 5.	25 ÷ 5 : 40 ÷ 5 = 5 : 8
b Common factor is 8.	24 ÷ 8 : 16 ÷ 8 = 3 : 2
c Common factor is 3.	18 ÷ 3 : 30 ÷ 3 : 15 ÷ 3 = 6 : 10 : 5

EXAMPLE 2

To make concrete, a builder mixes sand and cement in the ratio 5 : 4. If a mix of concrete contains 20 kg of cement, find:

a the amount of sand in the mix **b** the total mass of the mix.

Steps	Working
a Sand : cement = 5 : 4	4 parts (cement) = 20 kg 1 part = 20 ÷ 4 = 5 kg 5 parts (sand) = 5 × 5 = 25 kg There is 25 kg of sand in the concrete.
b Add mass of sand and cement.	total mass = 20 + 25 = 45 kg There is 45 kg of concrete.

EXAMPLE 3

Nick, Hong and Rob agreed to divide the profits of their business in the ratio 2:3:5.
Their profit this year was $45 000. Find the size of each person's share of the profits.

Steps	Working
Find the total number of parts and then calculate each share.	total number of parts = 2 + 3 + 5 = 10 Nick's share = $\frac{2}{10} \times \$45\,000$ = $9000 Hong's share = $\frac{3}{10} \times \$45\,000$ = $13 500 Rob's share = $\frac{5}{10} \times \$45\,000$ = $22 500 Check: $9000 + $13 500 + $22 500 = $45 000

EXERCISE 10.1 Ratios

ANSWERS p. 436

1 EXAMPLE 1 Simplify each ratio.

a 18:12 **b** 56:24 **c** 27:9:36 **d** 14:35:21:49

2 Zoe buys goods for $30 and sells them for $38. Find, in simplest form, the ratio of:

a the cost price to the selling price **b** the selling price to the cost price

c the profit to the selling price **d** the cost price to the profit.

3 EXAMPLE 2 An alloy contains copper and iron in the ratio 2:5. A quantity of alloy contains 20 kg of copper. What mass of iron does it contain?

4 To make superglue, the contents of Tube A and Tube B are mixed in the ratio 3:1.

a Dave used 21 mL of Tube A. How much of Tube B did he need?

b How much glue will Dave make altogether when he uses 8 mL of Tube B?

5 EXAMPLE 3 Divide 450 kg in the ratio:

a 4:5 **b** 3:2

6 Geoff and Amal share the weekly rent of $780 in the ratio 7:6. What is Amal's share?

7 A store has 30 gas heaters and 20 electric heaters in its warehouse. Find, in simplest form, the ratio of:

a gas heaters to electric heaters **b** gas heaters to all heaters.

8 Toudi, Ash and Felicity share a Lotto prize in the ratio 11:8:6. Ash received $720.

a How much did Toudi and Felicity each receive?

b What was the total prize money shared?

9 A 20 m cable is cut into 3 sections in the ratio 2:3:5. Find the length of each section.

10 Ed's Farmers' Market buys fruit and vegetables in the ratio 7:9. Ed ordered 14.4 tonnes of vegetables. What was the total amount of produce he ordered?

11 At Nelson High, the ratio of students who speak a second language to students who speak only English is 5 : 8. There are 923 students at the school. How many students speak only English?

12 In a study of 225 000 people, it was found that the ratio of right-handed people to left-handed people was 13 : 2.

a How many left-handed people were there?

b How many more right-handed people than left-handed people were there?

10.2 Scales on maps

We can use a map's **scale** to calculate lengths on the map.

Worksheet
Map scales

Worksheet
Map of Adelaide

EXAMPLE 4

The scale on a bushwalking map is 1 : 50 000. On the map, a walking trail is 4.5 cm long.
How long is the trail in real life?

Steps	Working
The scale 1 : 50 000 means that 1 unit on the map means 50 000 units in real life. Multiply the scaled length on the map by 50 000 to find the real length.	real length = 4.5 × 50 000 = 225 000 cm
Convert to metres.	= 225 000 ÷ 100 = 2250 m
Write your answer.	The trail is 2250 m (or 2.25 km) long.



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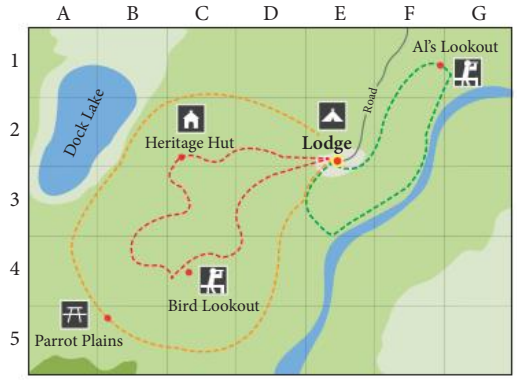
EXERCISE 10.2 Scales on maps

ANSWERS p. 436

1 EXAMPLE 4 A map's scale is 1 : 2000. Calculate the real lengths of each feature.

a lake 1.5 cm wide **b** walking track 5.8 cm long **c** creek 9.5 cm long

2 A map shows 3 walking trails that start at the Lodge.



Scale 1 : 10 000

Walking key
- Easy
- Moderate
- Challenging

Remember! You can use a piece of string to help you measure lengths on a map.

a What is the scale?

b What length in real life is represented by 10 cm on the map?

c How long is the section of river?

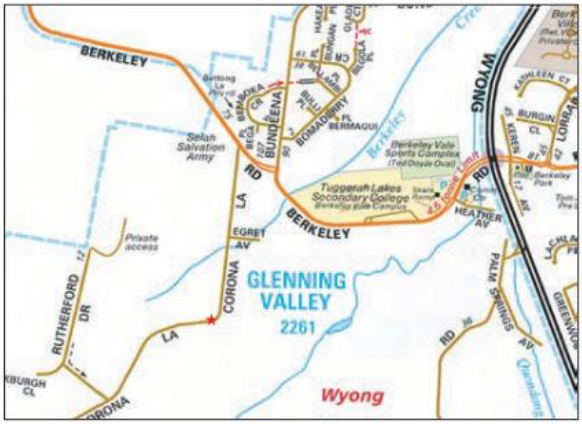
d Estimate the real-life length of the moderate (outer/orange) walking track.

e Jude started walking along the moderate walking track at 11 am. She walked at a speed of 3 km/h. At what time did Jude finish walking the track?

3 Zack lives in Corona Lane. The red star on the map shows the position of his house.

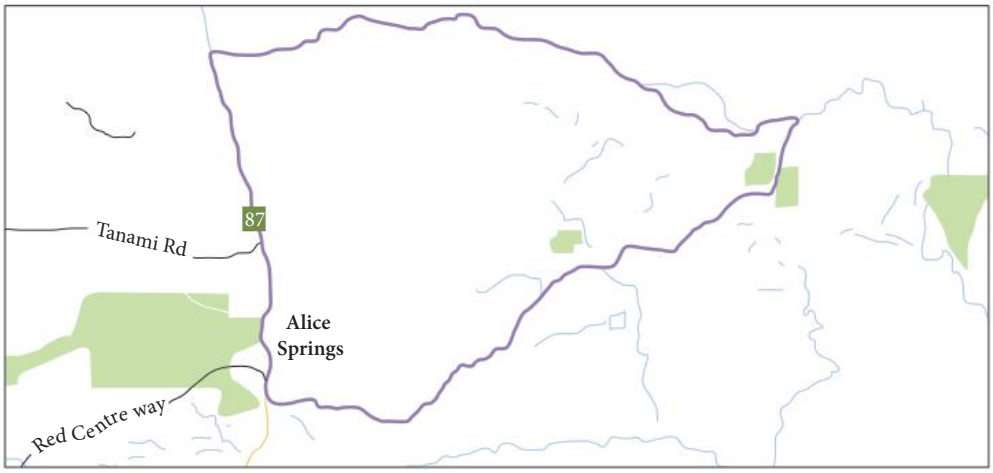
a Use the scale 1 : 18 000 to calculate the distance Zack walks to Tuggerah Lakes Secondary College each day.

b Zack walks to school at a speed of 1 m/s. How long does it take him to walk to school?



Map proudly reproduced with permission from Sydway Street Directory

4 This map shows the road around East MacDonnell National Parks in Central Australia.



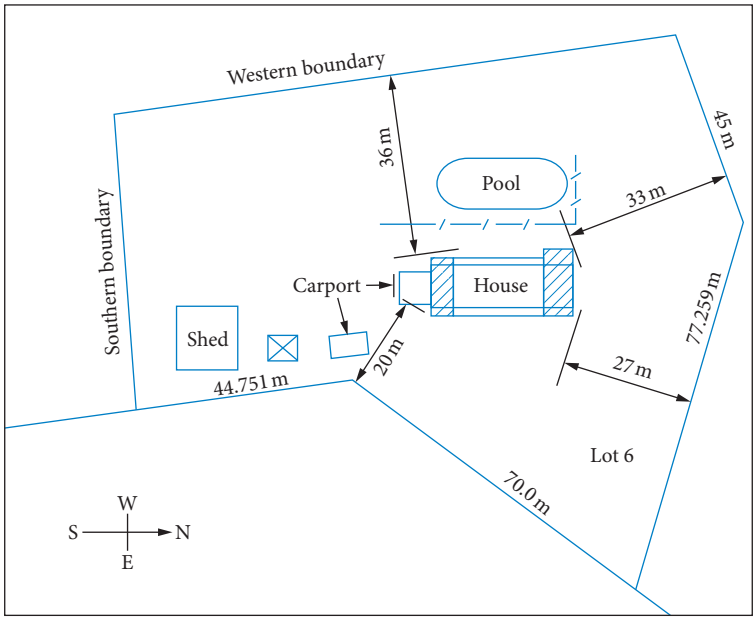
Scale 1 : 1 000 000

a Use the scale to calculate the length of the road (the circuit outlined in purple).

b The suggested time to drive one circuit of this road is $5\frac{1}{2}$ hours. At what average speed do drivers need to travel to complete the circuit in $5\frac{1}{2}$ hours? Express your answer correct to the nearest km/h.

c What do you think the surface of the road is like? State a reason for your answer.

5 **Cadastral maps** show the boundaries of properties. This part of a cadastral map shows the dimensions of a block of land and the buildings on the land.



a Approximately how close to the house is the nearest boundary?

b The scale is 1 : 1200. Calculate the length of the southern and western boundaries.

c The owners are going to replace the fence on the southern and western boundaries. The fencing costs $28 per metre for materials and labour. Calculate the cost of replacing the fencing.

6 The town of Gympie in south-east Queensland is prone to flooding. The map shows the areas affected when the depth of the Mary River is 18 m and 23.5 m (the 1-in-50 year flood level). The scale is 1 : 2260.



a Which properties in Mary Street flood when the Mary River reaches 18 m?

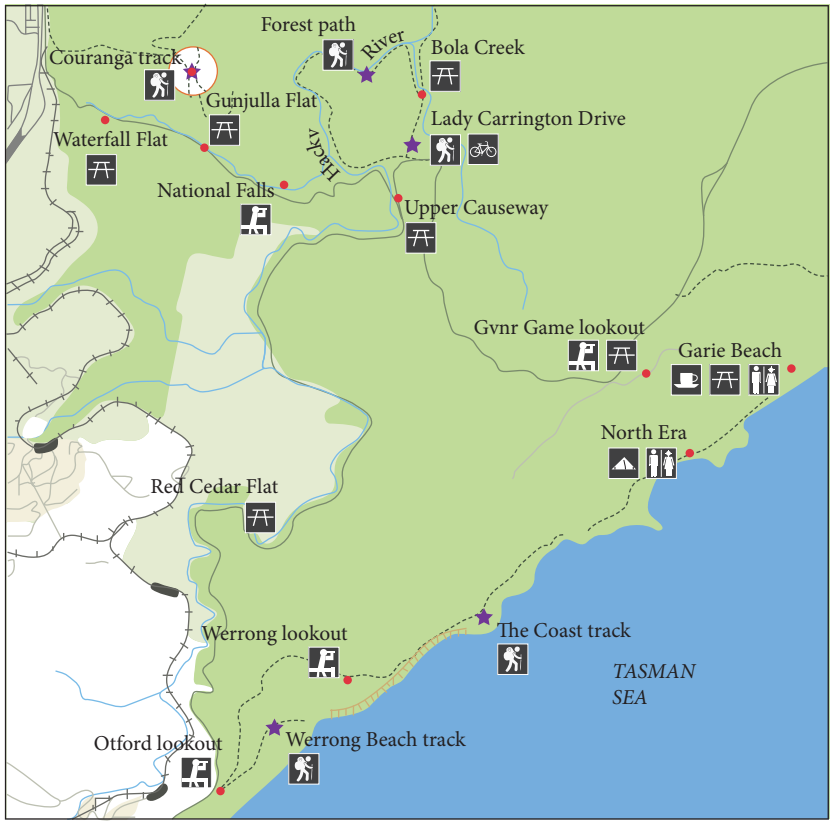
b Calculate, correct to the nearest metre, the length of Reef Street that is under water when the Mary River is at 18 m.

c Calculate the length of Mary Street that is under water during a 1-in-50 year flood.

d Estimate the depth of the Mary River when the shop at 187 Mary Street floods.

e Rhys is considering buying the hotel at 135 Mary Street. What recommendation could you give him with regard to flooding?

7 Jose, Isaac and Chase were hiking along the Coast track in a national park. They left their bikes at Otford lookout and set out for Garie Beach.



a The map scale is 1 : 127 500. To the nearest 0.1 km, how long is the walk from the Otford lookout to Garie Beach?

b The boys hike at an average speed of 4 km/h. How long should the hike take them?

c When they had been hiking for 2 hours, Chase fell and broke his leg. Isaac phoned emergency services for help. Approximately where on the track should he tell the rescue helicopter they are?



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Practical activity A cadastral map of your school

A cadastral map shows information about the boundaries of a block of land and the blocks that surround it. (See an example of a cadastral map in question **5** on page 236.) In this activity, you are going to make a cadastral map of your school and surrounding properties. Make the map suitable for display in the school's front office foyer.

On the map you should include a compass rose, a scale, the lengths of all boundaries and the positions of buildings.

Start your project by downloading a map and a picture of the terrain from **Google Earth**.

10.3 Ratio problems

Worksheet Ratio problems

Worksheet Ratios in other contexts

Worksheet Calculating quantities in ratios

EXERCISE 10.3 Ratio problems

ANSWERS p. 436

1 Gus and Judy share a lottery prize in the ratio 7 : 5. The prize is $240 000. How much does each receive?

2 Adam needs to make 800 g of short-crust pastry. Flour and butter are needed in the ratio 3 : 1. How much flour is needed?

3 Simon and Joshua's heights are in the ratio 8 : 9. Joshua is 1.71 m tall. How tall is Simon?

4 Market research found that cars are purchased in the following ratio of colours:

white : blue : other colours = 12 : 10 : 13

Dynamo Car Mart orders 42 white cars. How many blue cars should they order?

5 Phuong, Janelle and Ahmet bought a Lotto ticket for $12. Phuong put in $3, Janelle put in $3 and Ahmet put in $6. They won $4000.

a What is the ratio of their contributions to the ticket price?

b They agree to split the winnings in the same ratio as their contributions to the ticket price. How much prize money does each person get?

6 James and Eleanor invest in a card-making business in the ratio 3 : 5. Eleanor's investment is $520 000.

a How much did James invest?

b In their first year they make a profit of $62 000. How much does each of them receive for their share of the profits?

7 An insecticide concentrate is mixed with water before it is sprayed on vegetable crops. The mixing ratio depends on the type of vegetable, as the table shows. Jennifer is going to make 3 L of each type of spray.

Type of crop	Ratio of concentrate to water	Volume of concentrate required (mL)	Volume of water required (mL)
Broccoli	1 : 14		
Cauliflower	1 : 19		
Leafy vegetables	1 : 9		

a Express 3 L in millilitres.

b Copy and complete the table to show the number of millilitres of concentrate and water Jennifer needs to use for each.

8 Gold used in jewellery is classified according to its gold content. The ratio of gold to other metals is given in carats. Pure gold is 24 carats, and 5-carat gold has gold mixed with other metals in the ratio 5 : 19.

a Write the ratio of gold to other metals in a 9-carat necklace.

b Vanessa purchased an 18-carat gold bracelet with a mass of 60 g. How much gold is in the bracelet?

9 Tyson makes and sells saddles and other leather goods. Before he sells them, he treats them with a leather dressing made from lanolin and neatsfoot oil mixed in the ratio 2 : 3.

- **a** How much neatsfoot oil should Tyson mix with 40 mL of lanolin to make his leather dressing?
- **b** How much leather dressing will this make?
- **c** Tyson makes 200 mL jars of leather dressing to sell to customers of a riding school. How much of each ingredient does he use for each jar?

10 Nadim makes garden compost by mixing his vegetable and plant waste with straw and soil in the ratio 12 : 3 : 2.

- **a** How much straw will he need if he has 480 L of vegetable and plant waste?
- **b** How much soil should he add to a mixture containing 12 L of straw?
- **c** How much compost will he make by mixing 60 L of straw with the other ingredients?

11 A recipe for making pizza bases suggests mixing plain flour, water and yeast in the ratio of 16 : 10 : 1 by weight.



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- **a** Jasmine is making pizzas at home and is going to use 400 g of flour. How many grams of water and yeast will Jasmine need to add to make the pizza dough?
- **b** Spiro runs a small pizza shop. He makes all the dough for his pizza bases in the morning before he opens. He calculates that he needs to mix 2.5 kg of water with the other ingredients. How much flour does he use?
- **c** A commercial enterprise making pizza bases uses 3 kg of yeast each day. How much flour and water does it use?

12 Filomena is a hairdresser. She has been asked to dye a customer's hair. Her colour chart shows that the colour she wants can be obtained by mixing 2 dyes (dye A and dye B) in the ratio 5 : 3. Filomena estimates that she will need to make 120 mL of the mixture. How much of each dye will she need to use?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Investigation Variation

Part A

1 a James buys cans of soft drink at $1.50 per can. This table shows the number of cans in the top row. Copy and complete the table by showing the cost in the second row.

Number of cans, n	0	1	2	3	4	5	6	7	8
Total cost, C ($)	0								

b Copy and complete:

As James buys more cans, the cost _____.

c Calculate:

i the ratio of the cost of 1 can to the cost of 2 cans in simplest form

ii the ratio of the cost of 2 cans to the cost of 4 cans in simplest form

iii the ratio of the cost of 2 cans to the cost of 6 cans in simplest form.

d Copy and complete:

i When James doubles the number of cans he buys, the cost _____.

ii When James triples the number of cans he buys, the cost _____.

e When James buys half the number of cans, what happens to the cost?

2 a At Jane's part-time job, she earns $85 for 5 hours work. Copy and complete the table below.

Number of hours, h	0	1	2	3	4	5	8	10	15
Total cost, P ($)	0								

b Copy and complete: The more hours Jane works, the _____ she is paid.

c Calculate:

i the ratio of 1 hour of pay to 2 hours of pay in simplest form

ii the ratio of 4 hours of pay to 8 hours of pay in simplest form

iii the ratio of 5 hours of pay to 15 hours of pay in simplest form.

d Copy and complete:

i When Jane works double the number of hours, her total pay _____.

ii When Jane works triple the number of hours, her total pay _____.

e When Jane works half the number of hours, what happens to her pay?

Part B

The situations in Part A are examples of **direct variation** or **direct proportion**. When two variables vary directly or are in proportion, there is a constant ratio between them.

1 a Copy and complete the following for Part A, question **1**:

total cost of the cans = _____ × the number of cans purchased

b Rewrite the above statement algebraically using variables.

C = _____

1.50 is called the constant of variation. It's the value that links the variables.

2 a Copy and complete the following for Part A, question **2**:

total pay = _____ × the number of hours of work

b Rewrite the above statement algebraically using variables.

P = _____

17 is called the **constant of variation**. It's the value that links the variables.

10.4 Direct variation

Cans of soft drink are priced at $1.50 per can. This table shows the number of cans and the cost.

Number of cans, n	0	1	2	3	4	5	6	7	8
Total cost, C	0	$1.50	$3.00	$4.50	$6.00	$7.50	$9.00	$10.50	$12.00

As we buy more cans, the cost increases.

When we double the number of cans we buy, the cost doubles.

When we triple the number of cans we buy, the cost triples.

When we halve the number of cans, the cost halves.

This situation is an example of 'proportion' or **direct variation**. When 2 variables are in proportion, one variable is a constant multiplier of the other variable.

total cost of the cans = the number of cans × $1.50

or $C = 1.50 \times n$

Any relationship between 2 quantities where there is a constant ratio between them can be written in the form

$y = kx$

where k is a number called the **constant of variation** (or **constant of proportionality**).

We say that the cost of the cans **varies directly with** the number of cans, or the cost is **directly proportional to** the number of cans.

Video
Direct linear variation

Video
Direct proportion

EXAMPLE 5

A car travels at an average speed of 90 km/h. The distance it travels and the time it takes are related by direct variation. This is shown in the table below.

Time (t hours)	0	1	2	3	4	5
Distance (d km)	0	90	180	270	360	450

a Write an equation relating time (t) to distance (d).

b Find the constant of variation.

c Complete this sentence: For every _____ , the car travels _____ km.

d Use the equation to find the distance the car travels in 10 hours.

Steps	Working
a The variation equation is in the form $y = kx$.	In this case, $d = kt$.
b To find k, substitute one of the points into the equation.	Substitute (3, 270):
Any point from the table should give the same value of k.	$d = kt$ $270 = k \times 3$ $3k = 270$ $k = 90$
c Complete the sentence.	For every hour, the car travels 90 km.
d Substitute $t = 10$ into the equation.	$d = 90t$ When $t = 10$, $d = 90 \times 10$ $= 900$ The car travels 900 km in 10 hours.

Note: Variation can be described in words, given as an equation or shown on a straight-line graph.

EXERCISE 10.4 Direct variation

ANSWERS p. 436

1 EXAMPLE 5 The cost of filling your petrol tank is directly proportional to the number of litres it takes. This is shown in the table below.

Petrol amount (L litres)	0	10	20	30	40	50
Cost ($\$C$)	0	14	28	42	56	70

a Write the direct variation equation relating the amount of petrol (L) to the cost (C).

b Find the constant of variation.

c Write the equation for C in terms of L.

d Use the equation to calculate the cost of buying 75 L of petrol.

2 The weight of an object on the Moon (M kg) is $\frac{1}{6}$ the weight of the same object on Earth (E kg).

a Sam weighs 66 kg on Earth. How much does he weigh on the Moon?

b Write an equation relating weight on the Moon (M) to weight on Earth (E).

c Find the constant of variation.

d Write the equation for M in terms of E.

e Using the equation, calculate the weight on the Moon of an astronaut who weighs 84 kg on Earth.

f Calculate the weight on Earth of a machine weighing 44 kg on the Moon.

3 Angus is a store manager for Speedy Pizzas. He is paid according to the formula

$W = 21.2h$

where W = wages in dollars and h = hours worked.

a How much is Angus paid for 7 hours' work?

b What is the constant of variation in this variation equation?

c What does the constant of variation represent in this context?

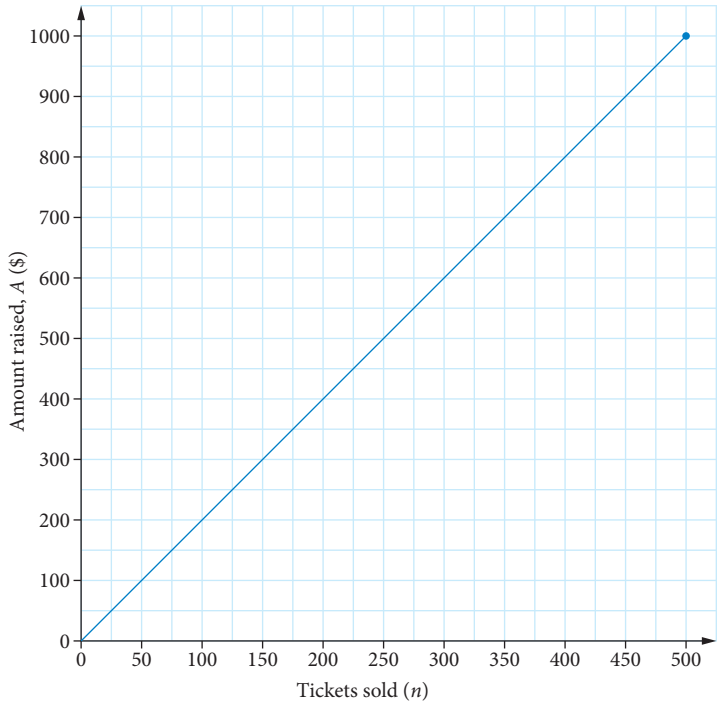
d How much is Angus paid for a week in which he works 37 hours?

e How many hours does Angus need to work to be paid $360.40?



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4 Chantelle is organising a raffle for a local mental health charity. Prizes have been donated by local businesses so that all the funds raised will go to the charity. The graph below shows that the amount raised, A in dollars, varies directly with the number of raffle tickets sold (n).



a How much is raised if they sell 200 tickets?

b How many tickets have to be sold to raise $900?

c How much does 1 ticket cost?

d Write the variation equation for A (amount raised) in terms of n (number of tickets sold).

e How much will they raise if they sell 1375 tickets?

f The charity needs office supplies to the value of $3200. How many tickets need to be sold to raise this amount?

5 A small car has a fuel consumption of 12.5 km/L. This means the car travels 12.5 kilometres for every 1 litre of fuel used.

a Write the direct variation equation relating distance expected to travel (D) to amount of fuel used (L).

b Find the constant of variation.

c Write the equation for D in terms of L.

d Use the equation to calculate the distance travelled on 75 L of fuel.

e How many litres are expected to be used to travel 250 km?

f How realistic is this model? What other factors may affect how much fuel a car uses?

10.5 Inverse variation

Video
Inverse variation

Video
Inverse proportion

Worksheet
Direct and inverse proportion

Worksheet
Variation problems

With direct variation, as one variable increases, the other variable increases in the same ratio.

Sometimes when one variable increases, the other variable *decreases*.

So, what if:

- when we double the value of x, the value of y halves
- when we triple the value of x, the value of y is divided by 3
- when we halve the value of x, the value of y doubles.

When 2 variables change in this way, we call it **inverse variation** (or **indirect variation** or **inverse proportion**).

This relationship can be written in the form

$$y = \frac{k}{x}$$

where k is a number called the **constant of variation** (or **constant of proportionality**), as before.

Some examples of inverse variation are:

- The number of students in a class and the time the teacher can spend with each student: the more students in the class, the less time the teacher has to spend with each student.
- Time and speed of a journey: if you halve your speed, you will double the time taken to complete your journey.

  (Remember: time $= \frac{\text{distance}}{\text{speed}}$)

We say that the time taken **varies inversely** (or **indirectly**) **with** the speed, or the time taken is **inversely proportional to** the speed.

EXAMPLE 6

Jackets R Us sell warm winter jackets. As the average daily temperature decreases, they sell more jackets. The number of jackets sold is inversely proportional to the temperature. On a day when the temperature was 2°C, they sold 160 jackets.

a Write an equation for the relationship between the temperature (T) and the number of jackets sold (J).

b Find the constant of variation for this equation.

c How many jackets would Jackets R Us expect to sell on a day when the temperature is 8°C?

d On a day when the store sells 50 jackets, what might the temperature be?

Steps	Working
a The inverse variation equation is of the form $y = \frac{k}{x}$.	$J = \frac{k}{T}$
b When $T = 2$, $J = 160$.	$160 = \frac{k}{2}$ $k = 160 \times 2$ $= 320$ Therefore, $k = 320$.

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c Use the equation with $T = 8$.	$J = \frac{k}{T}$ $J = \frac{320}{T}$ $= \frac{320}{8}$ $= 40$ When the temperature is 8°C, the store would expect to sell 40 jackets.
d Use the equation with $J = 50$.	$50 = \frac{320}{T}$ $50T = 320$ $T = \frac{320}{50}$ $= 6.4$ When 50 jackets are sold, the temperature may have been 6.4°C.

EXERCISE 10.5 Inverse variation

ANSWERS p. 436

1 EXAMPLE 6 Joe is hiring a bus to take himself and some friends to a concert. The cost per person varies inversely with the number of people on the bus. If there are 25 people on the bus, each person will pay $30.

- **a** What is the total cost for the bus?
- **b** Write an equation for the inverse variation between the cost per person (C in dollars) and the number of people on the bus (N).
- **c** Find the constant of variation for this equation.
- **d** What is the cost per person for the trip if 10 people are on the bus?
- **e** Joe calculates the cost per person is $25. How many people are travelling to the concert?

2 The Thomson family are travelling to Melbourne. The faster they travel, the less time it takes. When they have an average speed of 80 km/h, it takes them 6 hours to travel to Melbourne.

- **a** What is the distance from their home to Melbourne?
- **b** Write the inverse variation equation for the relationship between the time taken (T) and the speed (S).
- **c** Find the constant of variation for this equation.
- **d** What is the time taken when the Thomsons can average 100 km/h? Answer in hours and minutes.
- **e** Due to roadworks in several places, the trip takes 8 hours. What was the family's average speed?

3 Dimitri owns a construction business. The time it takes to complete a job varies indirectly with the number of people working on the job. When he sends 15 workers to complete a job, it takes 20 days to complete.

- **a** Write an equation for the relationship between the number of workers on the job (N) and the time taken to complete the job (T).
- **b** Find the constant of variation for this equation.
- **c** Due to illness, Dimitri can only send 12 workers. How long will it take the 12 workers to complete the job?
- **d** Another job of the same size is completed in 7.5 days. How many workers were on this job?

4 The time taken to download a file varies indirectly with the download speed of an internet service. The faster the download speed, the shorter the time it takes. At work, Anita has a download speed of 100 Mbps and it takes 15 seconds to download a legal file she is consulting.

- **a** Write an equation for the relationship between the download speed (S) and the time taken (T).
- **b** Find the constant of variation for this equation.
- **c** At home, Anita has a slower download speed of 25 Mbps. How long will it take to download the same file at home?
- **d** Anita sometimes works over coffee in a local café. The same file takes 30 seconds to download at the café. What is the download speed of the internet at the café?

5 June and Jim have a selection of bulbs to plant but are undecided about the size of the garden bed they want to plant them in. They know that a garden with length 8 m and width 2 m will be big enough for all the bulbs.



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- **a** What area does the garden need to be?
- **b** Write an equation for the inverse variation between the length of the garden bed (L) and the width of the garden bed (W).
- **c** Find the constant of variation for this equation.
- **d** They are considering making the garden bed 4 m wide. How long would it need to be?
- **e** June likes the idea of a long skinny garden bed only 1 m wide. How long would it need to be to plant all the bulbs?

Keyword activity

DESCRIBE AND EXPLAIN

ANSWERS p. 436

1. In your own words, describe how to divide a quantity in a ratio. Include an example in your explanation.
2. Imagine your friend doesn't know how to use the scale on a map. Write a short paragraph explaining how they can use it. Include an example.
3. What is a cadastral map?
4. Where do you use ratios, either at school or in your home life? Name 2 examples.
5. Name an example of direct variation.
6. Describe inverse variation (also called indirect variation) in your own words.

Solution to the chapter problem

Problem

Aunt Jemima has decided to give her share of the profits from the family company to charity this year. She and her brother divide the company's profits in the ratio 4 : 3. The company's total profits this year are $2.8 million.

How much does she give away?

She decides to give this money to Oxfam, Beanies for Brain Cancer and the Refugee Advocacy Group in the ratio 8 : 7 : 5.

How much does each charity receive?

Solution

Divide $2.8 million in the ratio 4 : 3.

Total number of parts: $4 + 3 = 7$

Aunt Jemima's share: $\frac{4}{7} \times 2\,800\,000 = 1\,600\,000$

Aunt Jemima has $1.6 million to give away.

Divide $1.6 million in the ratio 8 : 7 : 5:

Total number of parts: $8 + 7 + 5 = 20$

Oxfam: $\frac{8}{20} \times 1\,600\,000 = 640\,000$

Beanies for Brain Cancer: $\frac{7}{20} \times 1\,600\,000 = 560\,000$

Refugee Advocacy Group: $\frac{5}{20} \times 1\,600\,000 = 400\,000$

Oxfam receives $640 000, Beanies for Brain Cancer receives $560 000 and the Refugee Advocacy Group receives $400 000.

10 Test yourself

Ratio and variation

ANSWERS p. 436

1 EXERCISE 10.1 Joe's second-hand bookshop sold the following books in one day:

Crime novels	48
Science fiction novels	20
Non-fiction books	35
Romance novels	28

Express each of the following as a ratio in simplest form:

a crime novels to romance novels

b science fiction novels to non-fiction books

c non-fiction books to romance novels

d crime novels to science fiction novels.

2 EXERCISE 10.1 A triangle has sides in the ratio 2 : 5 : 4. If the shortest side is 18 cm, find:

a the lengths of the other 2 sides

b the perimeter of the triangle.

3 EXERCISE 10.1 The ratio of chocolate to vanilla ice creams in a supermarket freezer is 3 : 2. There are 125 ice creams altogether. How many more chocolate ice creams than vanilla ice creams are there?

4 EXERCISE 10.2 Use the map in Exercise 10.2, question **3** on page 235 to answer these questions.

a Estimate the distance by road from Zack's home to the Berkeley Vale Sports Complex.

b Estimate the length of Quondong Creek that is shown on the map.

5 EXERCISE 10.2 Use the map in Exercise 10.2, question **7** on page 237 to answer this question.

Estimate the distance by road from Otford lookout to the Red Cedar Flat picnic area, to the nearest 0.1 km.

6 EXERCISE 10.3 Gemma has a vegetable garden and a small fruit orchard. To fertilise them she makes mixtures containing amounts of nitrogen (N), phosphorus (P) and potassium (K), but the ratios for the vegetable garden and the orchard are different. The table shows the ratios Gemma uses.

Copy and complete the table to show the weights of different ingredients Gemma must use to make her fertilisers.

Plant type	$N:P:K$	Nitrogen (kg)	Phosphorus (kg)	Potassium (kg)	Total mixture (kg)
Vegetables	6 : 4 : 5	54			
				35	
Fruit trees	7 : 1 : 2		15		
		140			

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7 EXERCISE 10.3 Martin mixes a fertiliser combining nitrogen, phosphorus and potassium in the ratio 5 : 1 : 2. How much of each ingredient will he use to make 120 kg of the fertiliser?

8 EXERCISE 10.4 The monthly cost of renting retail space is directly proportional with the area of the floor space. A space of 60 m^2 rents for $1800 per month.

- **a** What is the monthly cost to rent a space of 120 m^2?
- **b** Write an equation relating the cost per month (R) to the area of floor space (S).
- **c** Find the constant of variation.
- **d** Write the equation for R in terms of S.
- **e** Calculate the monthly cost of renting a retail space of 1850 m^2.
- **f** Jamilla pays $7800 per month in rent. How many square metres of retail space does she rent?

9 EXERCISE 10.5 The more students there are in a class, the less time the teacher can spend with each individual student. Mr Yen's class usually has 24 students and he can spend 2.5 minutes with each one.

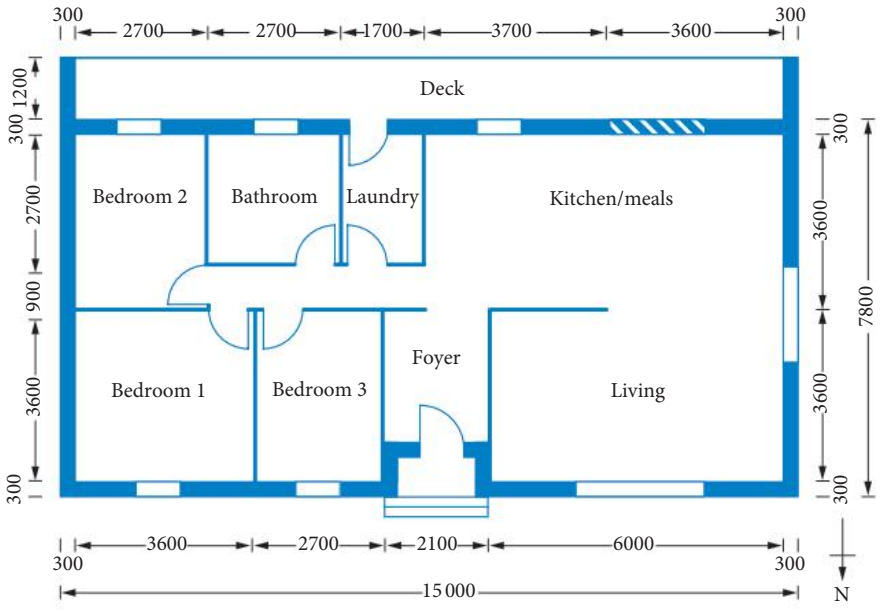
- **a** Write an inverse variation equation for the relationship between the number of students in the class (N) and the time Mr Yen can spend with each student (T), with k as the constant of variation.
- **b** Find the constant of variation for this equation.
- **c** On Thursday, 9 students are away on an excursion. How much time can Mr Yen spend with each of the remaining students in the lesson on Thursday?
- **d** Mrs Thomson is away and some of the students from her class are added to Mr Yen's class. He finds he can only spend 2 minutes with each student that day. How many students were in the class?

Chapter problem

Frankie and Charlie are building a house according to this plan. They need to order tiles for the bathroom and laundry.

How many square metres of tiles will they need for both rooms? Answer to the nearest square metre.

The builder is preparing to fit the interior doors. How many interior doors will they need to order for all rooms?



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CHAPTER 11

SPACE AND MEASUREMENT

FROM PAPER TO REALITY

11.1 House plans and elevations
11.2 Scales on house plans
Extended investigation: Design your family holiday home
11.3 Reading house plans
11.4 Renovate my house
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Draw plans and elevations for buildings
- Interpret house plans, including commonly used symbols and abbreviations
- Find actual measurements from house plans, such as lengths and areas
- Calculate renovation costs for remodelling houses

How are we ever going to use this?

- When designing, building, renovating or buying a home
- When we draw or interpret plans in the building industry
- To make calculations of length, area and costs when working in a trade, such as building, painting and floor covering.

Study design coverage

AREA OF STUDY 4: SPACE AND MEASUREMENT

- calculations of enlargement and reduction using scaling techniques for two-dimensional and three-dimensional plans, diagrams and models
- conventions, properties and measurement of area of compound shapes.

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Worksheets (2):

11.2 Interpreting an office plan • Investigation components

Weblinks (1):

11.2 SketchUp

Nelson MindTap

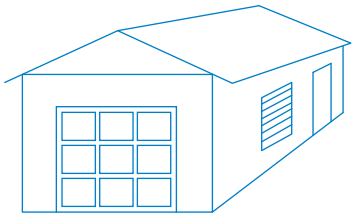
To access resources above, visit **cengage.com.au/nelsonmindtap**

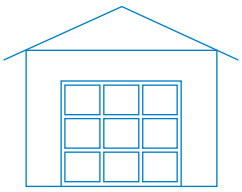
11.1 House plans and elevations

A draftsperson drawing a design for a new building includes different views of the building, such as:

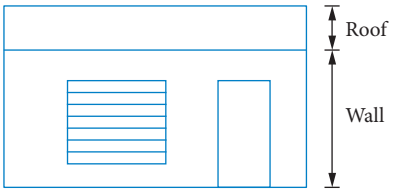
- a **plan**: a diagram of the floor plan, viewed from above
- the **elevation views**: the views of the front, back, left side and right side of the building.

These diagrams show a garage with the front and right-side elevations, and the view from above (plan):





Front elevation



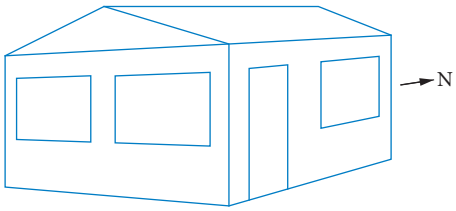
Right elevation

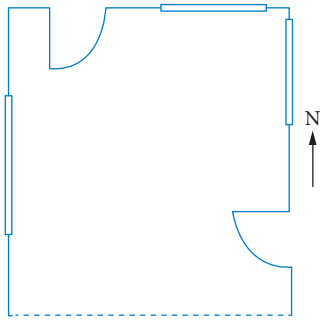


EXERCISE 11.1 House plans and elevations

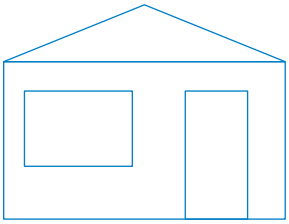
ANSWERS p. 437

1 The plan and 2 elevations for Erik's workshed are shown below.

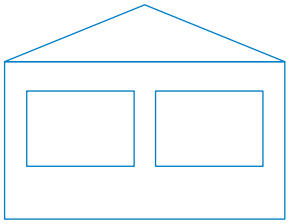




Plan



Northern elevation



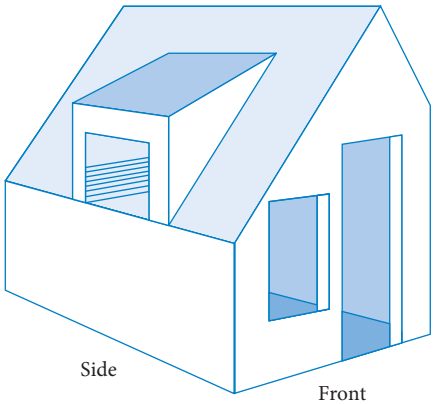
Southern elevation

Remember: the eastern elevation is the view from the eastern side.

a The dotted line shows that the plan is incomplete. Copy and complete the plan.

b Draw the eastern and western elevations.

2 For the house shown, sketch the front and side elevations. You do not need to draw the elevations to scale, but use a ruler and be as accurate as you can.



Side

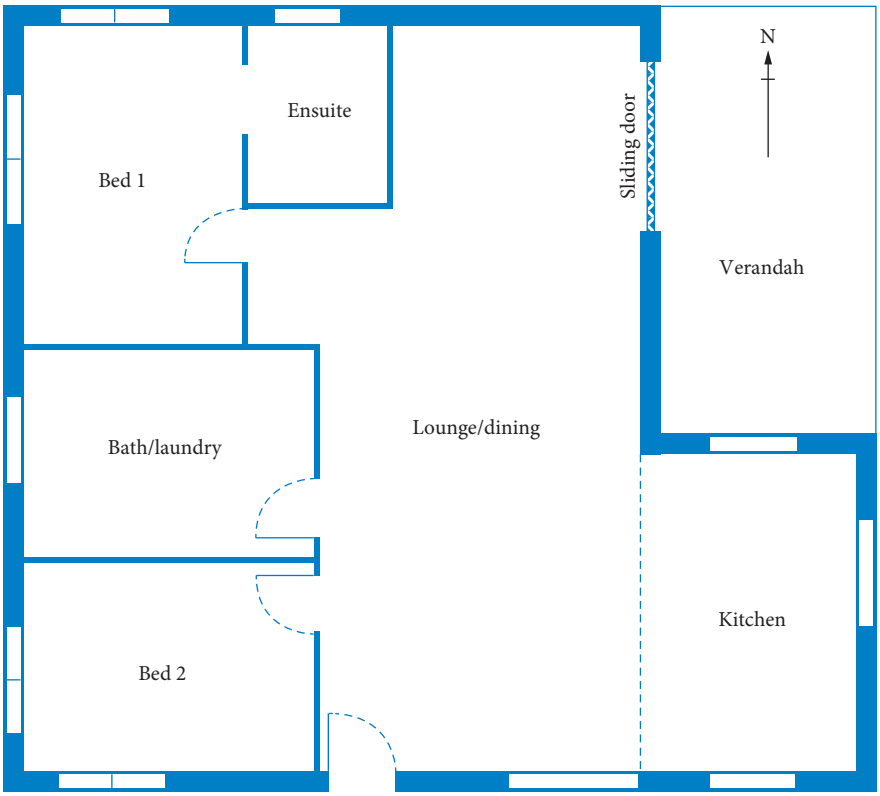
Front

3 For the house shown, sketch the front and possible side elevations. You do not need to draw them to scale, but use a ruler and be as accurate as you can.



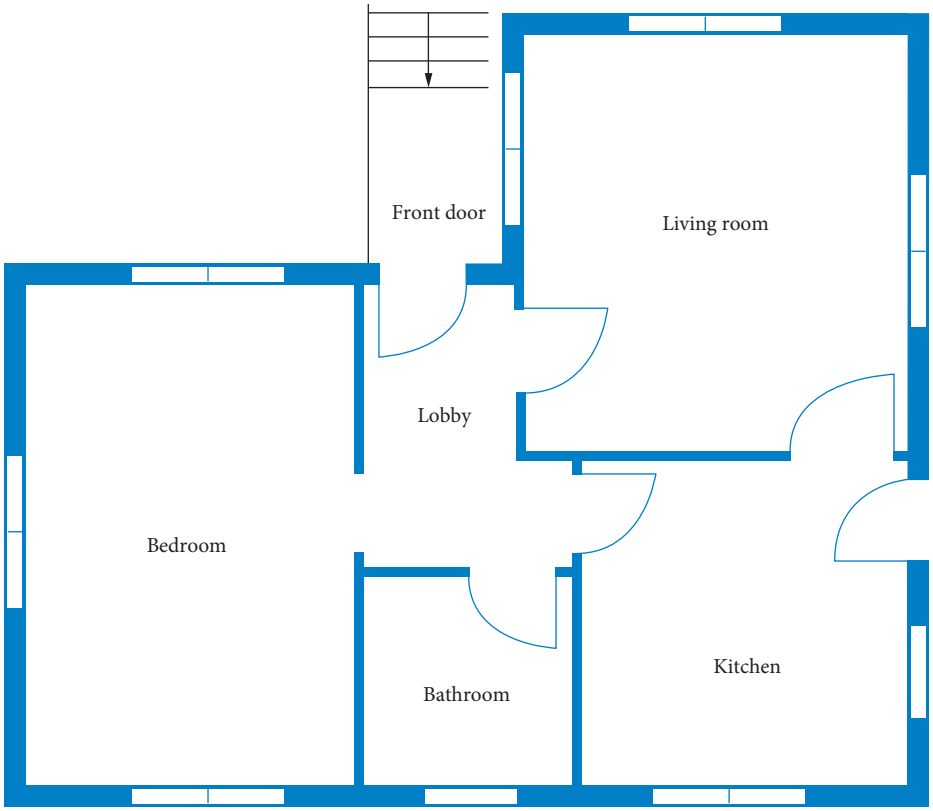
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4 This is the plan of Grigor and Silvana's holiday house.



Sketch the northern and western elevations. You do not need to draw them to scale, but use a ruler and be as accurate as you can.

5 This is the plan for a one-bedroom apartment, the front door of which faces north. Draw elevation views of each side of the apartment. Remember to identify the direction of each elevation.



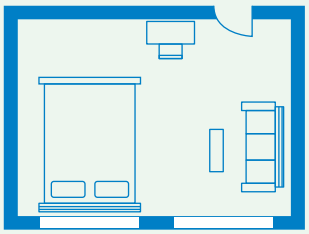
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11.2 Scales on house plans

Scales on house plans are usually written in ratio form, for example, 1 : 20 or 1 : 100. The first term refers to the scaled length and the second term refers to the real length. For a scale of 1 : 20, any object on the plan has a real size that is 20 times larger.

EXAMPLE 1

This bedroom is drawn to a scale of 1 : 130. Calculate the internal dimensions of the room in metres.



Steps	Working
Measure the length of the room on the diagram and find the real length by multiplying by the scale.	scaled length = 36 mm real length = 36 × 130 = 4680 mm = 4.68 m
Measure the width of the room on the diagram and find the real width by multiplying by the scale.	scaled width = 26 mm real width = 26 × 130 = 3380 mm = 3.38 m
Write the answer.	The dimensions of the room are 4.68 m by 3.38 m.

Worksheet
Interpreting an office plan

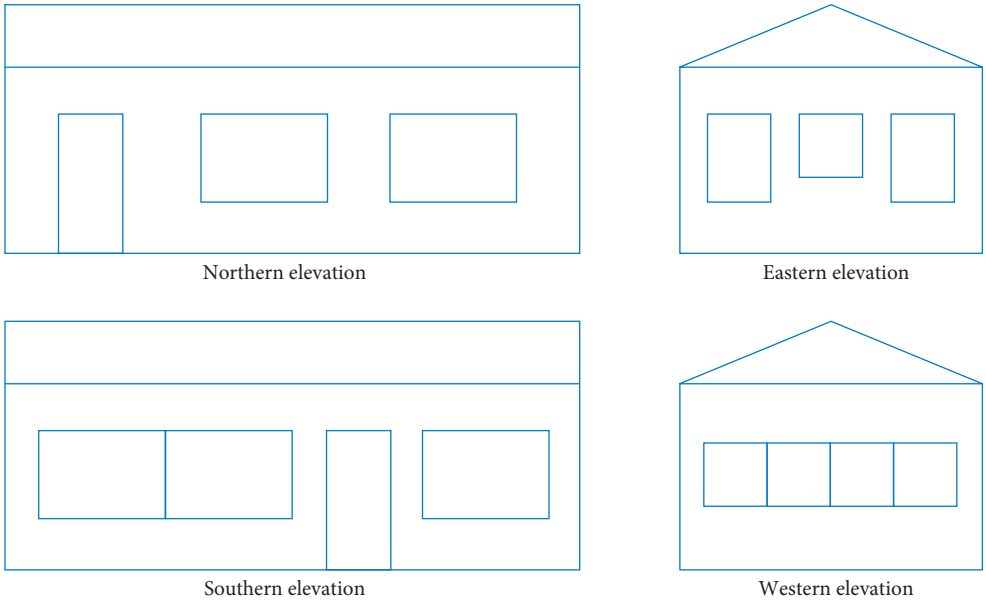
EXERCISE 11.2 Scales on house plans

ANSWERS p. 437

1 In the table below, you are given a scale and the length of an object drawn with that scale. Copy and complete the table by finding the real size of each object. The grey cells do not need to be completed.

Scale	Scaled length (cm)	Actual size		
		mm	m	km
1 : 100	1.0			
1 : 100	8.0			
1 : 200	2.5			
1 : 200 000	3.2			
1 : 500 000	15.0			
1 : 1 000 000	4.5			
1 : 2 500 000	9.2			

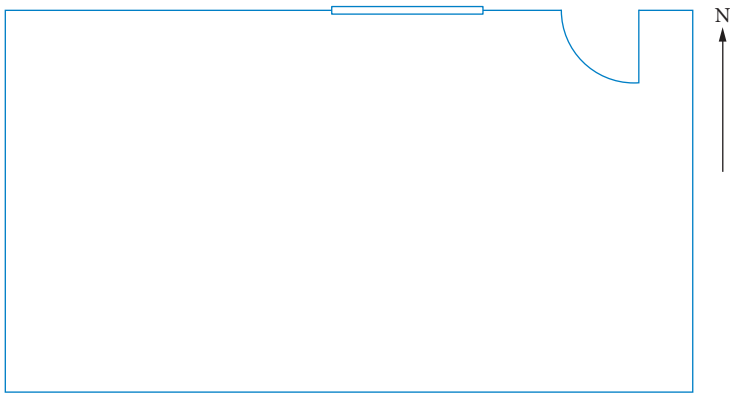
2 EXAMPLE 1 In these elevations of Charlotte's house, the scale is 1 : 100.



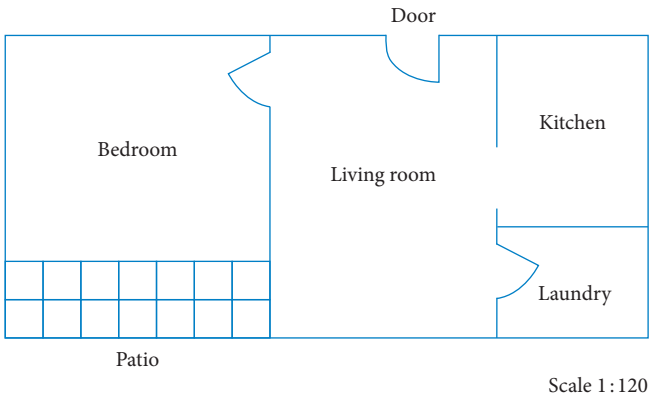
a What is the actual length of the northern elevation?

b What is the actual length of the eastern elevation?

c Copy this floor plan and complete it by marking all windows and doors shown in the elevation drawings.



3 The plan of a cabin is drawn to a scale of 1 : 120.



By measuring and using the scale, find the actual measurements, in metres, of.

a the dimensions of the bedroom

b the dimensions of the living room

c the dimensions of the kitchen

d the length of the patio

e the width of the door (in centimetres).

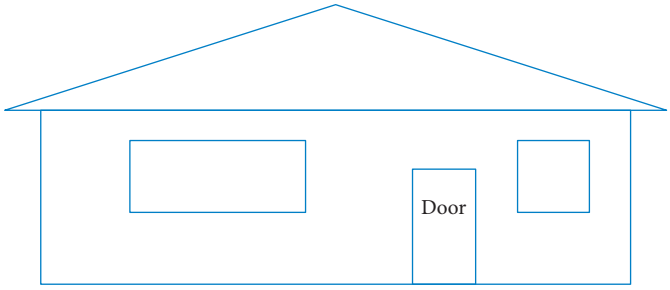
4 Here is the front view of a house.

a How wide is the house (in metres)?

b What is the height of the walls?

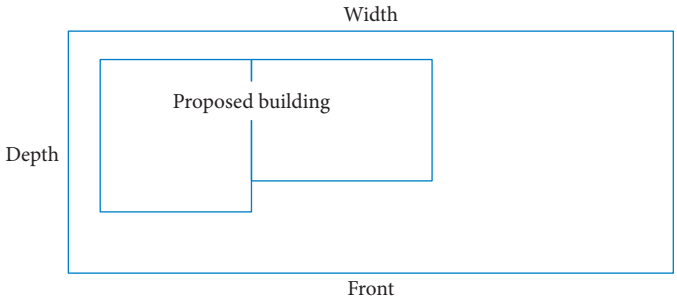
c What is the height of the house from the floor to the highest point of the roof?

d How high is the door (in metres)?



Scale 1 : 150

5 Darryl and Christine are building a house on a new block of land. Write all answers to the following questions in metres.



Scale 1 : 400

a What is the width of the block of land?

b What is the depth of the block of land?

c How far is the building from the left-hand boundary?

d What is the length of the longest side of the proposed building?

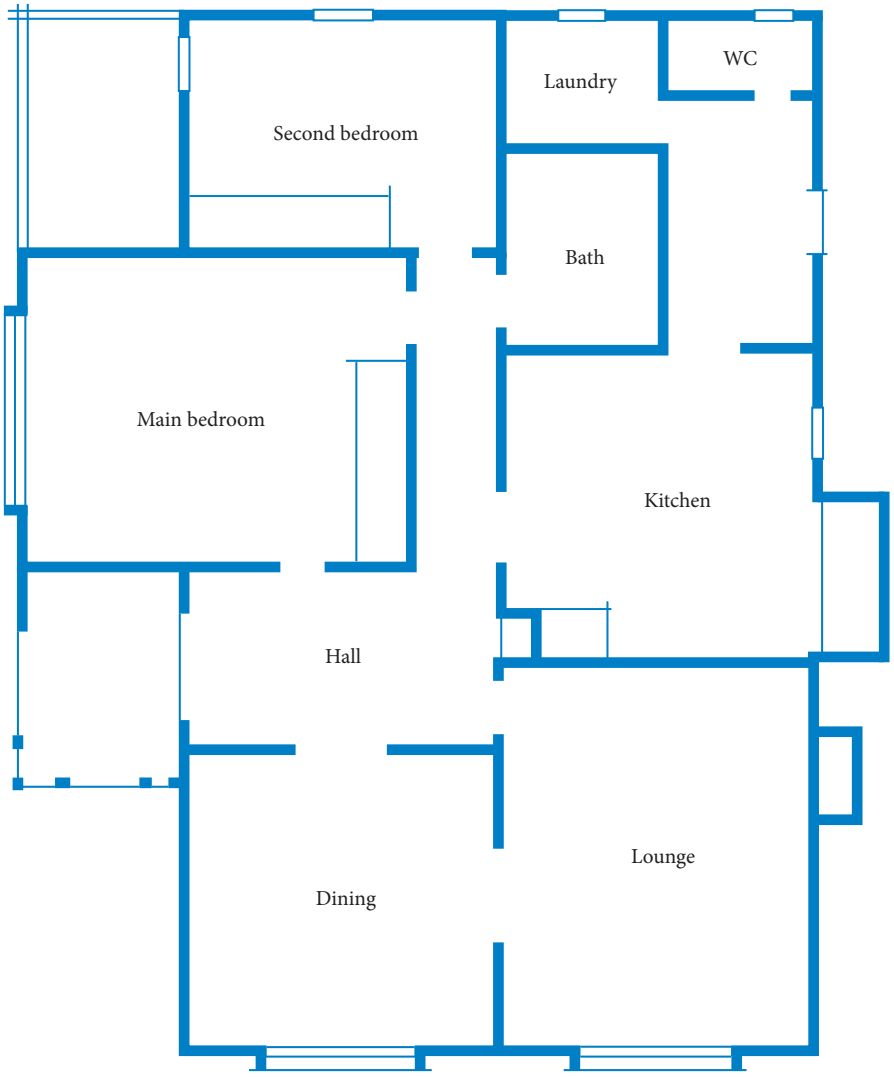
e What is the distance of the building from the right-hand boundary of the land?

f What is the distance of the building from the front boundary of the land?

g What is the area of the block of land in square metres?

6 This house plan has a scale of 1 : 100. Measure and calculate:

- **a** the length (from door to window) of the main bedroom
- **b** the length of the window in the main bedroom
- **c** the length of the laundry
- **d** the area of the bathroom floor
- **e** the longer side of the lounge room
- **f** the area of the dining room floor.



Scale 1 : 100



Alamy Stock Photo/Fir Mamat

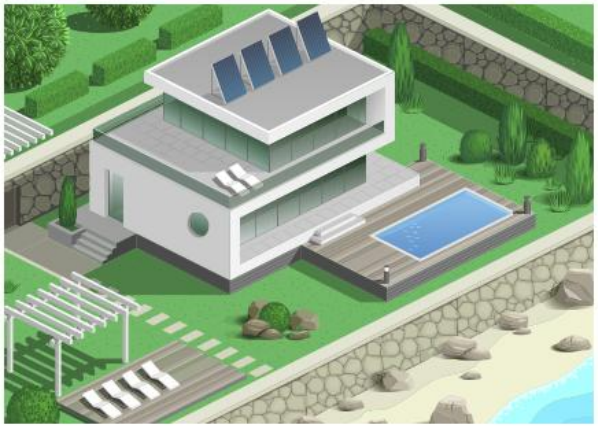
Extended investigation | Design your family holiday home

In this investigation, you will use 3D modelling software to design a building. You can use SketchUp or any other software. A free version of SketchUp, **SketchUp Make**, can be downloaded from www.sketchup.com. You can find tutorials on the internet if you don't know how to use SketchUp.

Your parents own a block of land near a lake and want to build a holiday home on it. They have asked you to design the house and prepare the plans and elevations for the builder.

Design a house with the following features:

- 2 bedrooms
- 1 bathroom
- 1 kitchen
- 1 lounge room that is big enough to hold a dining room table
- verandah across the front facing the road and a view of the lake.



iStock.com/denisik 11

Include the following items in your design portfolio:

- your initial sketches for the plans
- floor plan, elevations and a 3D view of the house produced with computer software
- a scale drawing showing the house on the block of land. The drawing should also include the driveway, the position of a large cylindrical water tank and the position of a future swimming pool
- ideas for landscaping.

Weblink SketchUp

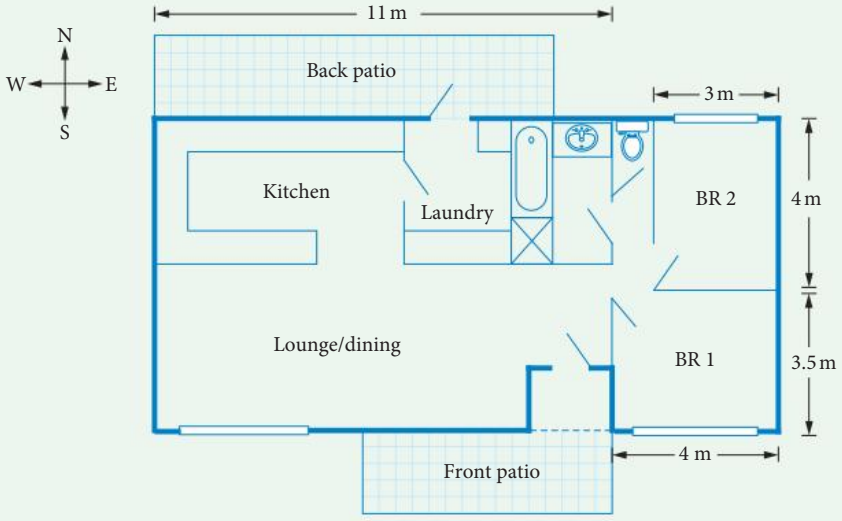
Worksheet Investigation components

11.3 Reading house plans

Before a house or building can be built, a plan must be drawn up. House plans use many different symbols and abbreviations. They are either drawn to scale or have measurements written on them. Often, measurements are shown in **millimetres** to avoid the use of decimal points, which can lead to errors in printing and reading.

EXAMPLE 2

This is the floor plan for Grant's house.



a What are the dimensions of bedroom 1?

b What is the area of bedroom 1?

c What is the length across the front of the house?

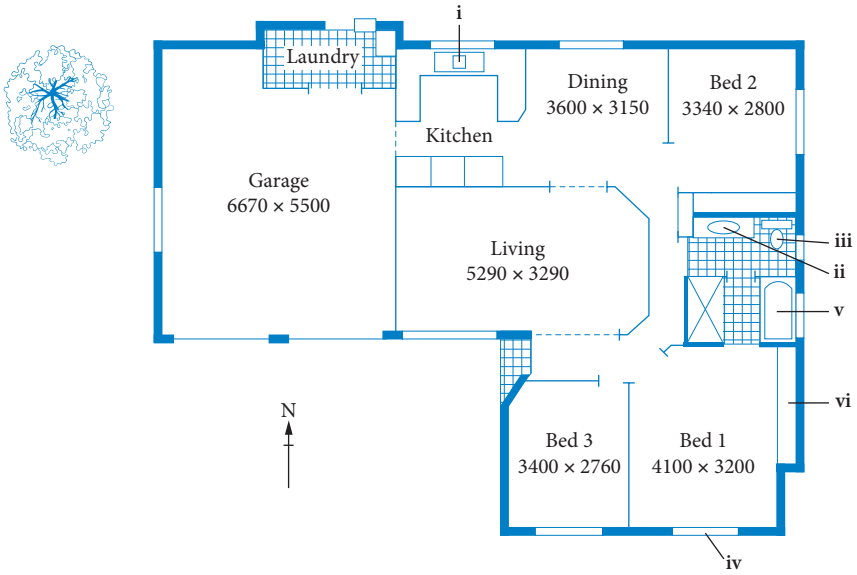
d What is the width of the house excluding the patios?

Steps	Working
a Find bedroom 1 on the plan. The dimensions are the length and the width.	The dimensions of bedroom 1 are 4 m by 3.5 m.
b Area = lw	area = 4×3.5 $= 14\,m^2$
c Find the full length from the plan.	length = $11 + 4$ $= 15\,m$
d Find the full width from the plan.	width = $4 + 3.5$ $= 7.5\,m$

EXERCISE 11.3 Reading house plans

ANSWERS p. 437

1 This is the plan for Menhal's new house.



a What does each symbol used on the plan represent?

i  **ii**  **iii** 

iv **v**  **vi** 

b Three areas on the plan are shaded with this pattern. 
What does this mean?

c How many bedrooms does the house have?

d Which bedrooms have built-in wardrobes?

e What are the dimensions of bedroom 1?

f Where is the laundry?

g Where is the back door?

h How many bathrooms does the house have?

i How many toilets are there?

j How many doors lead into the bathroom?

k Which room has dimensions 5.29 m by 3.29 m?

l What are the dimensions of the garage?

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m If Menhal looks out of the window of each room mentioned below, in which direction is she facing?

i living room **ii** dining room **iii** bedroom 1

iv bedroom 2 **v** garage

n Which direction do the garage doors face?

o From which room can Menhal see people arriving at the front door?

p If Menhal stands in the front hallway, list the 4 rooms she can see into.

q Menhal is working in the kitchen. Other than the garage, list the 3 rooms she can see into.

2 a How many bedrooms does this house have?

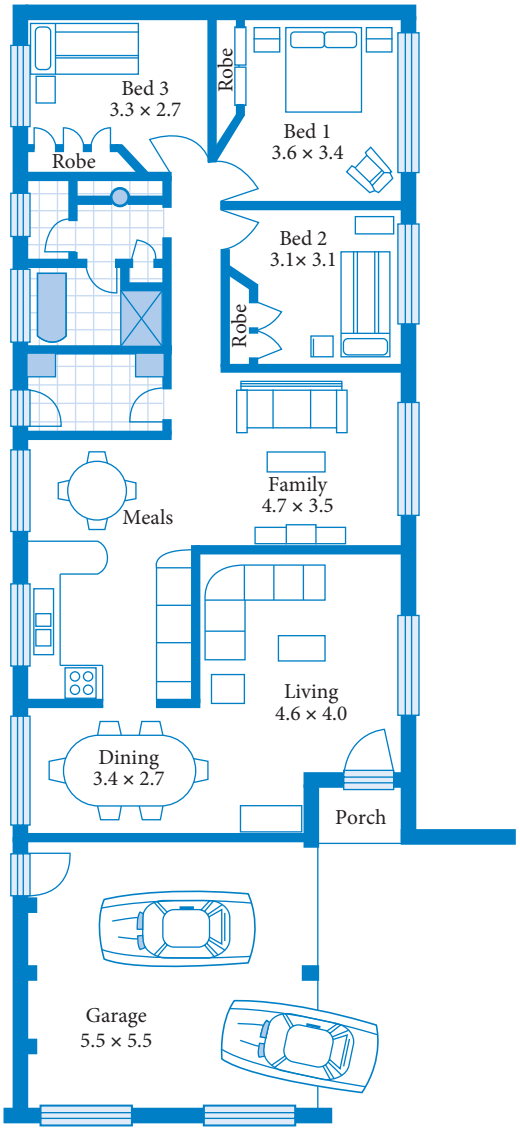
b How many of the bedrooms have built-in wardrobes?

c Is there an ensuite bathroom?

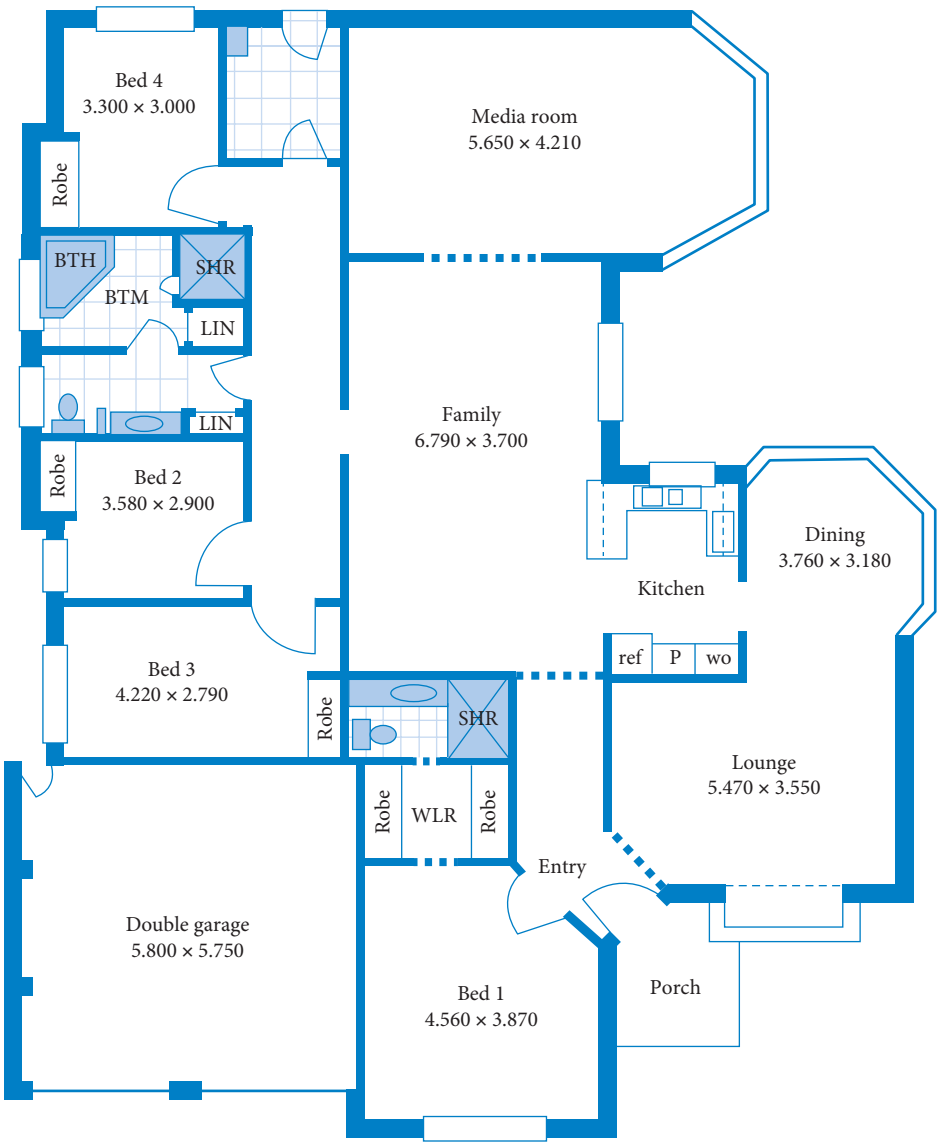
d What are the dimensions of the family room?

e Which bedroom has the largest floor area?

f The floor area of the house is 128 m^2. Calculate the cost of building this house at a rate of $672 per square metre.



3 EXAMPLE 2 All measurements on this plan are in metres.



a What are the dimensions of the family room, in millimetres?

b How many toilets are there?

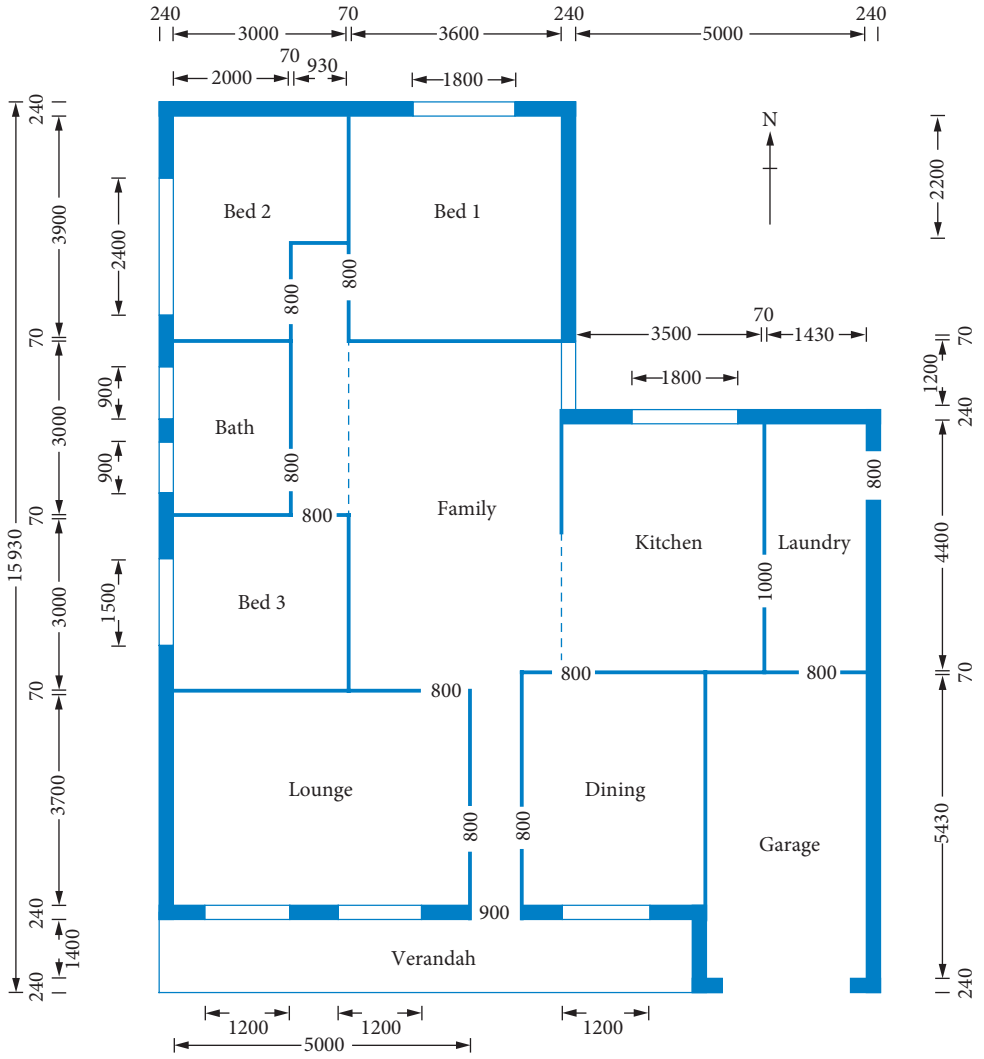
c Where are the 2 linen presses?

d What does 'SHR' stand for?

e Which bedroom has the smallest floor area?

f Which has the larger floor area: the family room or the media room? By how many square metres?

4 Judy and Keith are buying a house with this floor plan.



a What is the thickness of the external walls of this house in millimetres (mm)?

b What is the thickness of the internal walls?

c What is the width of the house at the front, in metres?

d What is the length of the left side of the house?

e What are the dimensions of the family room?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Investigation My future home

There are lots of websites that contain house floor plans. Search for 'project homes' on the internet and choose some floor plans for houses that you like.

1 For each of the plans:

- **a** if possible, obtain an estimate for the cost of building the house
- **b** calculate the building cost per square metre.

2 Compare and contrast the features of each house.

3 Based on the information you have found, decide which house you prefer, and present your findings.



Shutterstock.com/Lev Kropotov

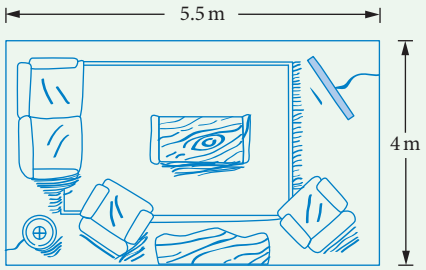
11.4 Renovate my house

We use house plans to calculate the materials required for a job and to estimate costs.

EXAMPLE 3

This is a diagram of Jackie's family room.

Jackie is laying cork tiles on the family room floor. The tiles cost $37.50 per square metre. How much will the cork tiles cost?

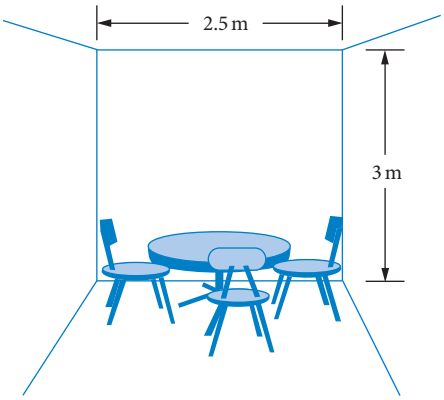


Steps	Working
First find the area of the floor.	area = 5.5 × 4 = 22 m^2
Multiply the area by the price.	cost = 22 × $37.50 = $825

EXERCISE 11.4 Renovate my house

ANSWERS p. 437

1 EXAMPLE 3 The dining room in Nicole's unit is very small. To make the room look larger, she plans to cover one wall with mirror tiles. The wall she plans to cover is 3 m high and 2.5 m wide.

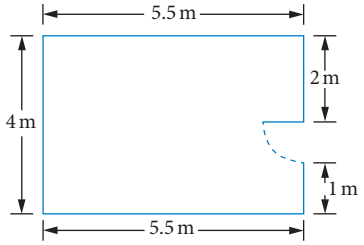


a What is the area of the wall?

b The mirror tiles Nicole has chosen cost $36.50 per square metre. What will be the total cost of the mirror tiles for the wall in Nicole's dining room?

2 Luis is replacing the skirting boards in his lounge room.

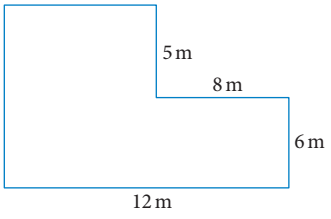
Skirting boards are the wooden boards that run along the base of the walls of a room.



a How many metres of skirting boards will he need?

b The skirting boards Luis has chosen cost $7.60 per metre. Calculate the total cost of the skirting boards he will need.

3 The floor area of Jesse's new house is shown.



a What is the floor area of this house?

b Using a cost of $1630 per square metre, calculate the amount that his building firm, Affordable Homes, charges to build this house.

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*Use the floor plan for Grant's house in Example 2 on page 259 to answer questions **4** to **7**.*

4 Grant is renovating bedroom 2. The skirting boards need replacing.

a Calculate the perimeter of bedroom 2.

b The door is 1 m wide. How many metres of skirting board are needed? [Note: Skirting board is required under the window.]

c Skirting boards cost approximately $5.70 per metre. What will be the cost of the skirting boards for bedroom 2?

5 Grant is also replacing the carpet in bedroom 2.

a How many square metres of carpet will he need?

b Carpet comes in rolls 3.6 m wide. What length of carpet will Grant need?

c The carpet he likes costs $156 per metre. How much will the carpet cost in total?

6 **a** Grant needs to paint the ceiling of bedroom 1. What is the area of the ceiling?

b The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2 of the ceiling.

i Calculate the number of litres of paint Grant needs.

ii The ceiling paint costs $34.90 for a 4-L can. Calculate the cost of paint for the ceiling.

7 **a** In which order should Grant do the renovations from questions **4**, **5** and **6**? Give reasons for your answer.

b What is the approximate total cost of these renovations?

*Use the floor plan for Judy and Keith's house from page 263 to answer questions **8** to **11**.*

8 Judy and Keith have decided to replace all the floor coverings before they move into the house. They are going to lay wood parquet flooring in the lounge room. The flooring costs $68 per square metre. How much will the flooring cost?

9 Other rooms need new carpet. Judy has chosen a carpet that is 3.66 m wide. How many metres of carpet are required for each of these rooms?

a bedroom 1 **b** bedroom 2 **c** bedroom 3

10 The kitchen and laundry are to have new tiled floors. The tiles cost $41.10 per square metre, allowing for breakage and wastage. How much will it cost to buy tiles for these 2 floors? Answer correct to the nearest dollar.

11 Judy and Keith are expecting a baby and they are going to make bedroom 2 the baby's room. Judy wants to decorate the room with a thin frieze pattern around the walls. It will go above the door and the windows.

a How many metres long will the frieze be?

b Each roll of frieze is 5 m long. How many rolls of frieze will Judy need to buy?

A frieze is a thin wallpaper pattern that goes around the walls.

12 Judy and Keith were surprised by the estimated cost of their renovations. Before signing off on the purchase of this house, for comparison they checked out the cost of building a project home of similar style and with the same floor area as this house. The cost of the project home was approximately $1270 per square metre. Calculate the estimated cost of building this project home.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 437

Write a summary of this chapter in your own words.

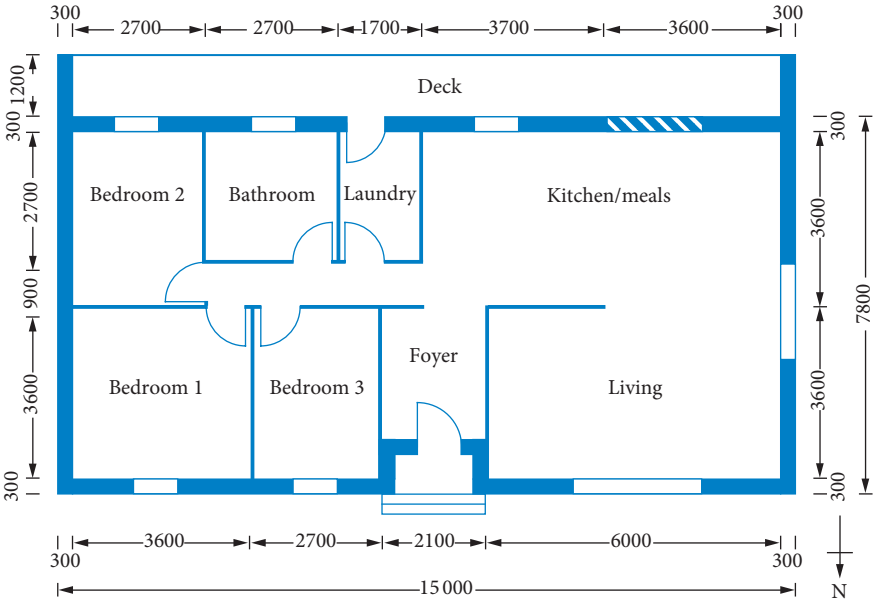
- Include the meanings of plan, elevation and scale.
- List the important things to know when reading house plans.
- Write a paragraph about what you learned about renovating.

Solution to the chapter problem

Problem

Frankie and Charlie are building a house according to this plan. They need to order tiles for the bathroom and laundry. How many square metres of tiles will they need for both rooms? Answer to the nearest square metre.

The builder is preparing to fit the interior doors. How many interior doors will they need to order for all rooms?



Solution

Convert the measurements from millimetres to metres.

2700 mm = 2.7 m

1700 mm = 1.7 m

Find the areas of the 2 rooms:

Area of bathroom = 2.7 m × 2.7 m
= 7.29 m^2

Area of laundry = 1.7 m × 2.7 m
= 4.59 m^2

Total area = 7.29 + 4.59
= 11.88 m^2
= 12 m^2 (to the nearest square metre)

Count the interior doors. Do not include the front door and the back door.

There are 5 interior doors.

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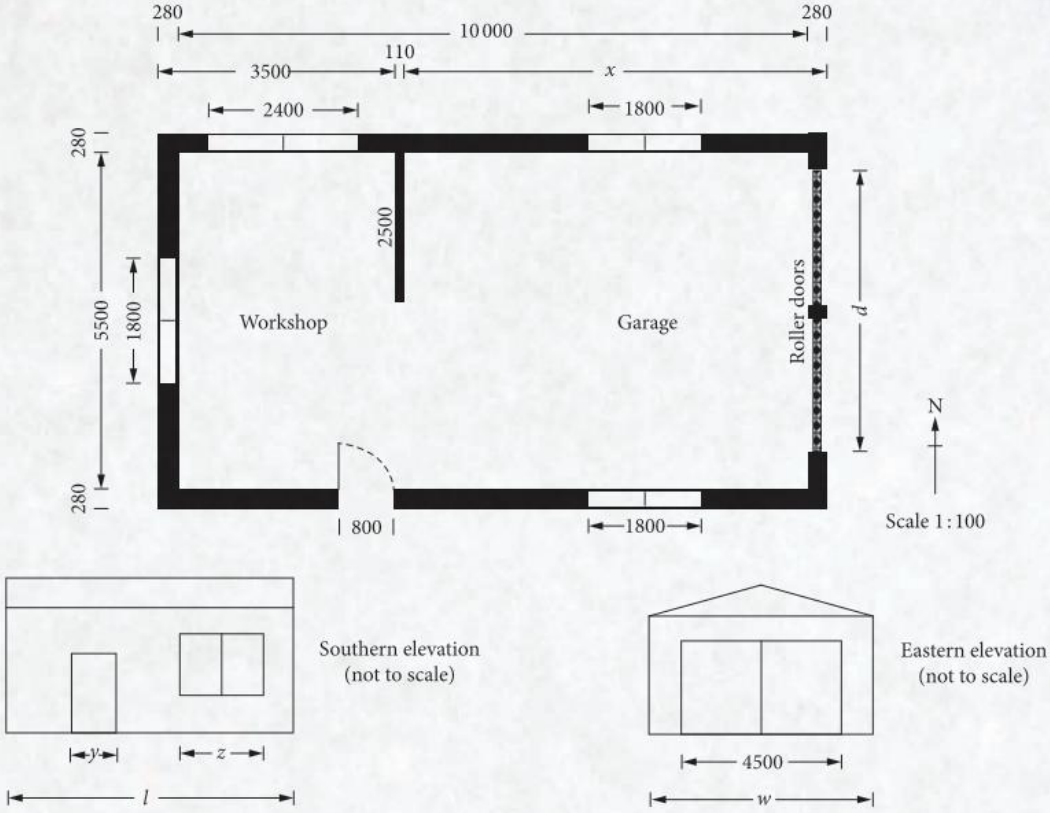
11 Test yourself

From paper to reality

ANSWERS p. 437

1 EXERCISE 11.1

a Draw elevation views of the northern and western sides of this building. You are not required to draw them to scale, but use a ruler and be as accurate as possible.



b What is the length of x on the plan?

c Find the lengths of y, z, l, w and d.

2 EXERCISE 11.2 Use the elevations in Exercise 11.2 question **2** on page 256 to answer these questions.

a What is the actual height of the walls?

b What is the height of the roof above the ceiling?

3 EXERCISE 11.2 Use the plan in Exercise 11.2 question **3** on page 256 to answer these questions.

By measuring and using the scale, find each actual measurement, in metres, of the following:

a the dimensions of the laundry

b the length of the cabin

4 EXERCISE 11.3 Use the plan in question **1** above to answer the following questions.

a What is the width of the external walls of the building?

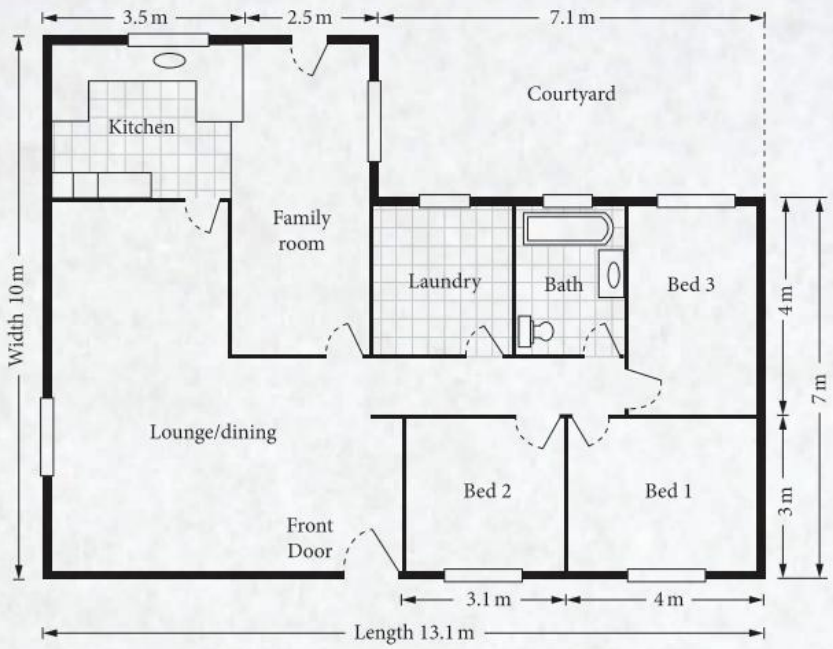
b What is the width of the wall between the workshop and the garage?

c Find the internal width of the building.

d What are the dimensions of the workshop?

e What are the dimensions of the garage?

5 EXERCISES 11.3, 11.4 This is the plan for the house that Sam and Nic are buying.



a How many doors are shown on this plan?

b What are the dimensions of the house?

c Calculate the width of the kitchen.

d **i** Find the floor area of the house.

ii An estimate for the cost to build a house is $1065 per square metre. Estimate the cost of building this house to the nearest dollar.

e What percentage (correct to 1 decimal place) of the house's floor area is taken up by bedroom 1?

f A builder is going to tile the courtyard. What is the area of the courtyard?

g To allow for cutting and fitting, a builder always buys 10% more tiles than required to cover a floor area.

i How many square metres of tiles should the builder buy for the courtyard, including the extra 10%?

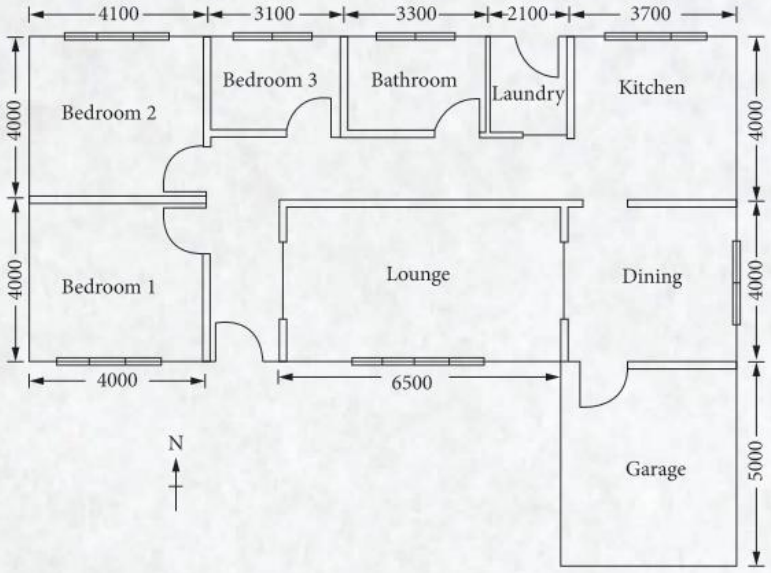
ii If each tile measures 32 cm by 32 cm, how many tiles are needed in total?

h The guttering across the front of the house needs replacing.

i How much guttering is needed?

ii The guttering company charges a fee of $50 plus $24.75 per metre to supply and install the guttering. How much will they charge to gutter the front of the house?

6 EXERCISE 11.4 This is the floor plan for Harry and Meg's house. They have decided to renovate some areas of the house.



a Harry and Meg are going to lay wood parquet flooring in the lounge room. The flooring costs $68 per square metre. How much will the flooring cost?

b They are going to replace the skirting boards in bedroom 1. Assume the door is 820 mm wide.

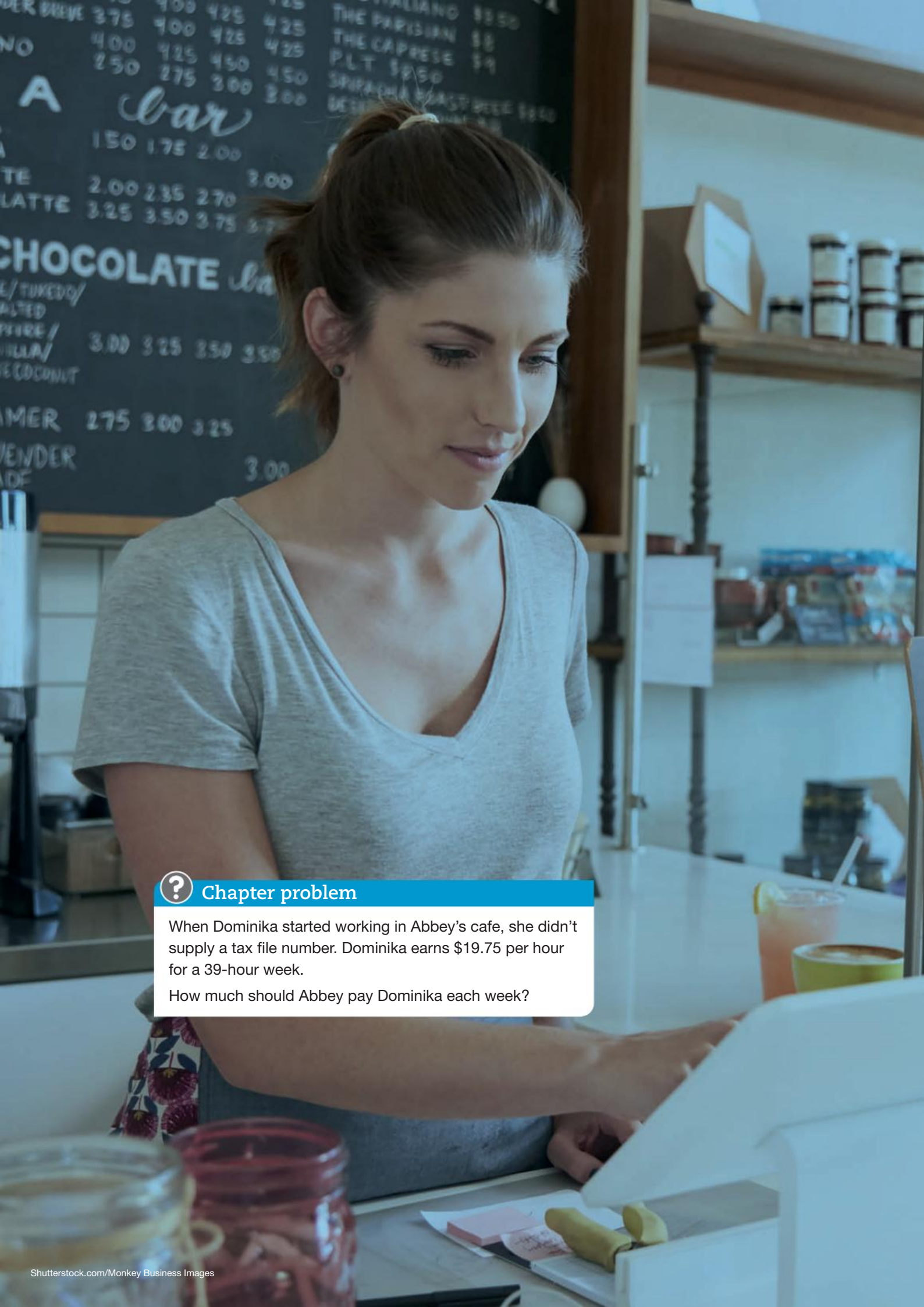
- **i** How many metres of skirting board are needed?
- **ii** Skirting boards cost approximately $15 per metre. What will be the cost of the skirting boards?

c The tiles on the floor of the bathroom and laundry need replacing. The width of these rooms is 2300 mm.

- **i** Calculate the total floor area of the bathroom and laundry.
- **ii** Each tile is 200 mm by 200 mm. How many tiles are needed?
- **iii** The tiles cost $41.10 per square metre (ignore wastage). How much will the new tiles cost?

d The ceiling of the lounge room needs to be repainted.

- **i** What is the area of the ceiling?
- **ii** The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2. How many litres of paint will be required for the ceiling?
- **iii** Ceiling paint costs $34.90 for a 4-L can. Estimate the cost of painting the ceiling.



Chapter problem

When Dominika started working in Abbey's cafe, she didn't supply a tax file number. Dominika earns $19.75 per hour for a 39-hour week.

How much should Abbey pay Dominika each week?

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CHAPTER 12

DISCRETE MATHEMATICS:
FINANCIAL AND CONSUMER MATHEMATICS

RUNNING A BUSINESS

12.1 Doing your research
12.2 Fixed and variable costs
12.3 Will we break even?
12.4 Setting a price
12.5 GST
12.6 Paying staff
12.7 Comparing sales
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Make calculations to do with business finance
- Analyse and find the break-even point
- Calculate and set prices
- Calculate GST
- Calculate staff wages, taxation, superannuation and other costs
- Work out insurance costs

How are we ever going to use this?

- To understand and apply finance requirements if we decide to start a business
- If we make items to sell, we will calculate prices
- When taking out insurance, a major part of financial security

Study design coverage

AREA OF STUDY 3: DISCRETE MATHEMATICS

Financial and consumer mathematics

- taxation systems at the business level
- income and expenditure calculations such as GST, invoicing and BAS
- comparison of financial products and services such as insurance.

VCE Mathematics Study Design 2023–2027 p. 74, © VCAA 2022

Videos (2):
12.5 GST • Discounts and GST

Spreadsheets (2):
12.3 Abbey's cafe
12.7 Comp calculations

Weblinks (1):
12.1, 12.2 MoneySmart

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

12.1 Doing your research

Good research is the key to a successful business. Before starting a business, an entrepreneur (business owner) will:

- investigate the demand for the business in the area
- investigate the **costs** for renting, furniture, finance, staff, electricity, technology, insurance, transport and raw materials
- speak with an accountant
- develop a business plan
- apply for an **Australian business number (ABN)** with the Australian Taxation Office
- obtain legal advice.

EXAMPLE 1

Abbey is planning to open a cafe in a shopping centre. Her **rent** will be $312 per square metre of floor area per month. The floor area of the cafe is 26 m^2. Calculate the annual rent for the cafe.



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Steps	Working
Calculate the monthly rent by multiplying the cost per square metre by the floor area.	monthly rent = 26 × $312 = $8112
Calculate the annual rent by multiplying by 12.	annual rent = $8112 × 12 = $97 344

EXERCISE 12.1 Doing your research

ANSWERS p. 438

1 EXAMPLE 1 The management of a shopping village quoted $298 per square metre of floor area per month for the rent of a take-away food kiosk. The kiosk has a floor area of 22 m^2.

- **a** Calculate the monthly rent for the kiosk.
- **b** What is the annual rent?
- **c** We can calculate the daily rent by dividing the monthly rent by the number of days in the month. How much cheaper is the daily rent in May than in June?

2 A cafe needs a good espresso machine. Abbey can buy a new machine for $8600 or rent it for $66 per week, provided she signs a 2-year contract.

- **a** How much will it cost Abbey to rent the machine for 2 years?
- **b** If Abbey rents the machine, at the end of the 2 years she will be able to buy the machine for 50% of its value when new. Calculate the total amount Abbey will pay for the coffee machine if she rents it for 2 years then buys it.

3 Abbey needs an industrial dishwasher. She can buy a suitable dishwasher for $6000 or **lease** one for 36 months at $230 per month.

- **a** How much will the lease cost over the 36 months?
- **b** The lease conditions state that if the lease is terminated early, the customer will be required to pay 75% of the remaining lease. Calculate the total amount Abbey will have to pay for the lease if she terminates the contract immediately after she has had the dishwasher for 24 months.
- **c** Abbey can buy the dishwasher on terms of 10% deposit and the balance at 4.5% p.a. flat rate interest over 3 years. Calculate the total amount Abbey would pay for the dishwasher on these terms.
- **d** Would you recommend Abbey lease or buy the dishwasher? Justify your answer.

4 As part of her market research, Abbey spoke with many cafe owners in shopping centres within 10 km of where she plans to open. She found that the average amount spent per customer ranged between $5.30 and $7.10.

- **a** How much can Abbey expect to receive from customers if she has 450 customers per day?
- **b** Approximately how many customers will she need per day to receive more than $2500?

5 Abbey researched the **wholesale** cost of different coffee beans.

Type of coffee bean	250-g packet	5-kg packet	10-kg packet
Colombian beans	$13.50	$175	$360
Mocha beans	$13.90	$192	$380
Premium bean blend	$14.25	$285	$565



Shutterstock.com/Freedom Life

- **a** How much cheaper is it per kilogram to buy Colombian beans in 5-kg packets than in 10-kg packets?
- **b** What is the cheapest way per kilogram to buy Mocha beans?
- **c** Abbey doesn't think she'll be able to use all of a 10-kg pack of Premium bean blend before the beans go stale. Will it be cheaper for her to buy Premium beans in 250-g or 5-kg packets? Explain your answer.

6 Abbey has heard that **workers compensation** insurance is expensive. In her research, Abbey discovers that this insurance costs $1.70 for every $100 she pays in **wages** each year.

a Use a calculation to show that $1.70 for every $100 is the same as 1.7%.

b Calculate Abbey's annual workers compensation insurance **premium** if her annual wages bill is:

i $25 000

ii $160 000

iii $350 000

7 Abbey has also been advised to take out some income protection insurance for herself. What is income protection insurance? What are the advantages of having income protection insurance? Visit the **MoneySmart** website and look for **Plan for your future**, **Life insurance** and **Income protection insurance**.

Weblink
MoneySmart

Investigation Types of insurance

In this course we've looked at some different types of insurance: car insurance, health insurance, home and content insurance, and workers compensation insurance. What other types of insurance are there? What is the benefit of each type? Research other types of insurance, who uses them and why.

Some examples are:

- travel insurance
- domestic workers insurance
- professional indemnity insurance
- landlord insurance.

Create a poster to display the information.

12.2 Fixed and variable costs

Every business has costs. There are 2 types of costs:

- **overheads** or **fixed costs**
- **variable costs**.

As the name suggests, fixed costs stay the same regardless of the business's productivity. Variable costs change as the business's productivity increases or decreases.

In her cafe, rent is one of Abbey's fixed costs because it doesn't change when she sells more coffee. In contrast, the cost of coffee beans is a variable cost because it changes. The more coffee Abbey sells, the more she will need to spend on coffee beans.

Abbey needs $300 000 to start up the cafe, as well as some extra funds to buy her supplies and pay her bills. The bank adviser recommended that she take 2 loans over a 5-year period to cover the $300 000.

- a $200 000 loan at 4.5% p.a. monthly reducing finance, with monthly repayments of $3739
- an interest-only loan at 5.2% p.a.

In addition, the bank recommended that Abbey should have a business overdraft of $50 000 at 6.1% p.a. interest.

A **business overdraft** is credit on a business account that helps the business to pay its bills where the account balance goes below $0.

EXERCISE 12.2 Fixed and variable costs

ANSWERS p. 438

Use the information on page 274 about business loans for Abbey's cafe to answer questions **1** *to* **4**.

1 **a** How much will Abbey need to borrow on the interest-only loan?

b With an interest-only loan, only the interest is paid each month (the loan amount does not reduce). What monthly amount will she have to pay on this loan?

c Calculate the total amount she will have to pay per month on these 2 loans.

2 Use the **mortgage calculator** on the **MoneySmart** website to answer these questions.

a How much interest will Abbey pay on the $200 000 loan over 5 years?

b Calculate the total amount Abbey will repay on the $200 000 loan.



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Weblink
MoneySmart

3 The $50 000 overdraft will cost 6.1% p.a. interest on money used and 0.2% p.a. interest on the part of the overdraft that is *not* used. During the first month of operating, Abbey thinks she will need to use $12 000 of the overdraft.

a Calculate the interest for 1 month on the $12 000 that Abbey plans to use.

b How much of the overdraft won't be used?

c Calculate the monthly interest on the unused part of the overdraft.

d Calculate the total amount of interest Abbey will have to pay for the overdraft in the first month.

4 Calculate the total amount of interest Abbey will have to pay for the overdraft for a month in which she used $22 000 of the overdraft.

5 Categorise the following costs into fixed and variable costs.

a Repayments on the business loan

b Cleaning costs

c Cost of milk and milk alternatives

d Cost of the phone and internet

e Cost of insurance

f Cost of staff wages

g Cost of the ingredients for cakes and muffins

h Cost of staff **superannuation** payments

i Cost of leasing tables and chairs

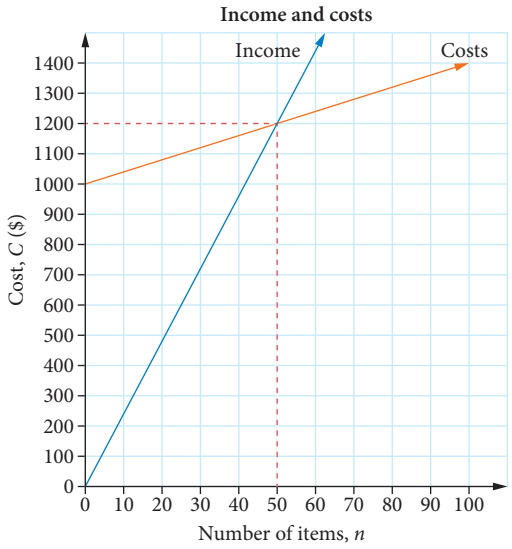
j Cost of electricity

12.3 Will we break even?

Any business needs to sell enough items in order to have enough income to cover the fixed costs and make a profit. The **break-even point** is where a business's costs are the same as the money it receives from sales. Selling above the break-even point is essential to making a business succeed. If a business isn't making a profit, it won't last long!

EXAMPLE 2

Kit owns a small business that makes mobile phone covers. The total fixed costs are $1000 per week and the variable costs are $4 per phone cover produced. Each phone cover produced is sold for $24. How many items does the business need to sell each week to break even and how much income will the business receive?

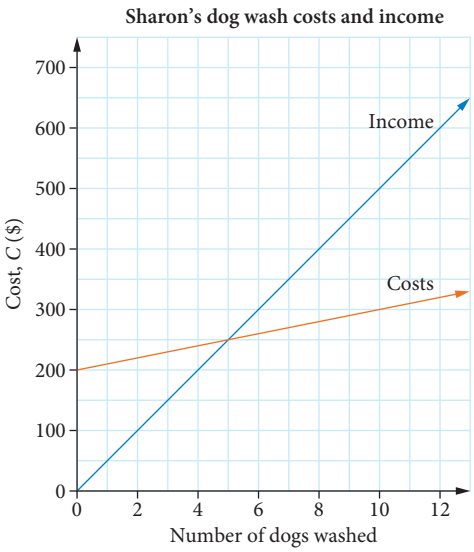
Steps	Working
In Chapter 9, *Equations and formulas*, we learned that constructing a graph is an easy way to determine a break-even point. In this context we can use the equations: Cost, $C = 1000 + 4n$ Income, $I = 24n$ where n = the number of phone covers. The orange line shows the business's costs, C, for producing different numbers of items. The blue line shows the income, I, that the business receives from selling different numbers of items.	
The point where the lines cross is the break-even point (50, 1200), representing 50 items and an income (and costs) of $1200.	break-even point = (50, 1200) $n = 50$ items $I = \$1200$
or	
Solve the equations simultaneously by making the costs equal the income.	$1000 + 4n = 24n$
Subtract $4n$ from both sides.	$1000 = 20n$
Divide both sides by 20.	$n = 50$
Substitute into the I equation to calculate the income.	$I = 24 \times 50$ $= 1200$
Write the answer.	The business needs to sell 50 items per week to break even, and its income will be $1200.

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EXERCISE 12.3 Will we break even?

ANSWERS p. 438

1 EXAMPLE 2 Sharon has a dog-washing business. The graph shows her weekly **expenses** and income.



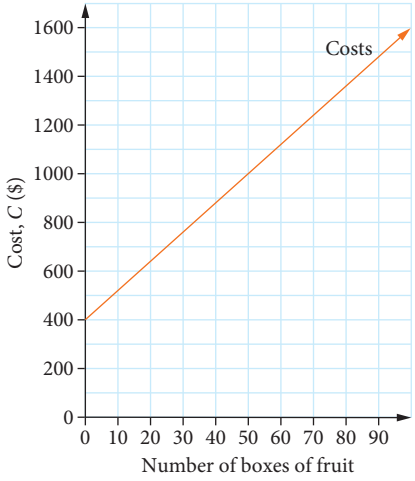
- **a** Explain how you know that Sharon's fixed costs are $200 per week.
- **b** How much does Sharon charge for washing each dog?
- **c** How many dogs does Sharon have to wash each week to break even?
- **d** How much profit does Sharon make in a week if she washes 10 dogs?

2 Holly sells boxes of fruit from her street stall. She sells each box for $20.

- **a** Copy and complete this table to show the money she will receive from selling different numbers of boxes of fruit.

Boxes of fruit sold	0	10	20	50	75
Money received					

- **b** The graph shows Holly's weekly costs for selling fruit. Copy the graph and show Holly's income line on it.

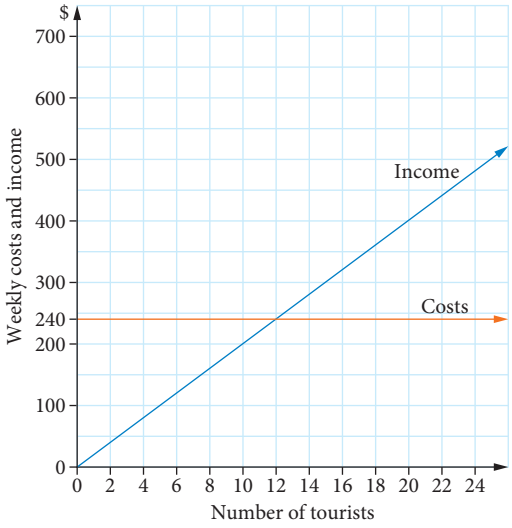


- **c** How many boxes of fruit does Holly need to sell each week to break even?
- **d** How much are Holly's fixed expenses each week?
- **e** How much does Holly's cost increase for each extra box of fruit she sells?
- **f** Calculate Holly's profit when she sells 90 boxes of fruit in a week.



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3 During the summer tourist season, Afah shows tourists the sights of his area in a horse-drawn carriage. This graph shows his weekly costs and income.



a How much does Afah charge each tourist he takes in his carriage?

b How many tourists does he need to drive each week to break even?

c Suggest a reason why Afah's cost line is horizontal.

4 Download the spreadsheet **Abbey's cafe** to complete this activity.

Spreadsheet
Abbey's cafe

	A	B	C	D
1	Abbey's cafe			
2	Only enter or change values in cells shaded green			
3				
4				
5	**Expenses**			
6	Fixed costs			
7	Rent	$6,500.00		
8	Other	$2,450.00		
9	Total	$8,950.00		
10				
11	Variable costs			
12	1 cup of coffee	$0.45		
13	1 muffin or cakes	$0.55		
14				
15	Wages	Pay per hour	Hours worked per month	Monthly pay
16	Employee 1	$19.00	100	$1.900.00
17	Employee 2	$18.00	120	$2,160.00
18	Employee 3	$20.00	110	$2,200.00
19	Employee 4	$20.00	130	$2,600.00
20			Total	$8,860.00
21				
22	Total monthly costs	$21,753.00		
23				
24	Superannuation	Rate as a percentage	Amount to be paid	
25		5	$443.00	
26				
27	**Income**			
28	Price of coffee and cake	$7.00		
29	Number of units sold	3500		
30	Total monthly income	$24,500.00		
31				
32				
33	Abbey's break-even sales	3108		
34	Abbey's monthly profit	$2,747.00		
35				

Abbey's cafe will be open 7 days per week and for late-night shopping. Her monthly rent will be $8250 and her other fixed costs will total $2875. She plans to work in the shop herself and have 4 employees who will each work an average of 120 hours per month and earn $21 per hour.

a Adjust the values in Abbey's spreadsheet to match Abbey's plans.

b Superannuation costs 10% of wages. How much superannuation will Abbey have to pay per month on her monthly labour costs?

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c Abbey thinks it will cost her 54 c for each cup of coffee and 59 c for each cake or muffin she makes. She plans to have an opening price of $6.20 for a coffee with a cake or muffin. How many coffees with cake or muffin will she need to sell to break even?

d How much profit will Abbey make if she sells 4390 coffees with cake or muffins monthly?

e How many extra sales of coffee with cake or muffin will Abbey need to make each month to break even if she has to increase each of her workers to 130 hours per month?

f Predict what will happen to Abbey's gross profit in each of these situations, then use the spreadsheet to check the accuracy of your predictions. Assume that all other values stay the same, unless otherwise instructed.

- **i** Her rent increases.
- **ii** The number of coffees with cake or muffins she sells increases.
- **iii** The average amount she has to pay her staff increases.
- **iv** The government increases the rate at which she has to pay superannuation.
- **v** Abbey increases the price of coffee with cake or muffin.
- **vi** Abbey finds a cheaper source of ingredients for the coffee.
- **vii** One of Abbey's full-time workers resigns and she is able to employ 15-year-old students after school and on weekends to replace the full-time worker. The students will have a lower rate of pay.

12.4 Setting a price

Setting a price for items or services is tricky. The price has to be high enough to cover costs, while at the same time being low enough to make people want to buy the product or service. If there isn't much competition, the business will probably be able to charge more than if there are other similar businesses in the area.

There is a variety of ways to determine prices.

Type of pricing	What it means
Direct costs margin	The profit as a percentage of direct costs
Break-even margin	The profit as a percentage of the break-even price
Cost-plus pricing	The profit as a fixed amount that is usually related to the fixed costs and is unrelated to the cost of the product
Perceived value pricing	Charging a price that the business thinks people will pay
Going rate pricing	Charging the same price as close competitors

EXAMPLE 3

Hannah uses direct costs margin when she determines prices in her clothing shop. Her direct costs margin is 75%. How much will she charge for a jumper with a **cost price** of $60?

Steps	Working
Hannah's profit is 75% of her cost price.	profit = 75% × $60 = $45
Add the profit to the cost price.	selling price = $60 + $45 = $105

EXAMPLE 4

Imran sells imported clothing. A pair of jeans cost him $36 to import. The break-even price is $44 and Imran wants to make a 50% profit on the break-even price. What price should he charge for the jeans?

Steps	Working
Imran's profit is 50% of his break-even price.	profit = 50% × $44 = $22
Add the profit to the break-even price.	selling price = $44 + $22 = $66



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EXERCISE 12.4 Setting a price

ANSWERS p. 438

1 EXAMPLE 3 In his furniture business, Sanjay uses direct costs margin pricing. He determines the prices of items he sells by adding 125% of the cost of the items. Calculate the price Sanjay will charge for each of the following items.

- **a** Bookcase that cost $210
- **b** Bed that cost $310
- **c** Leather lounge that cost $2680
- **d** Dining table that cost $520

2 EXAMPLE 4 Margo sets the prices in her gift shop to make 40% on her break-even costs. How much will Margo charge for each item?

- **a** Silk flower arrangement with a break-even price of $64
- **b** Set of wine glasses with a break-even price of $40
- **c** Wallet with a break-even price of $52
- **d** Glass necklace with a break-even price of $28

3 Steven has a food van at local markets. He is considering several ways to price items he sells.

Item	Steven's cost price	Price by going rate pricing	Price by cost-plus pricing	Price by direct costs margin (at 95%)
Hot chips	95 c	$3.00	$2.95	
Hot dog	$1.10	$2.00		
Hamburger	$2.90	$4.50		
Meat pie	$1.50	$3.60		
Sandwich	$1.80	$4.00		

a How much do Steven's competitors charge for a hamburger?

b When Steven determines his prices by cost-plus pricing, how much does he add to his cost price to determine his selling price?

c Complete the Price by cost-plus pricing column.

d Use a calculation to show that Steven will charge $3.51 or $3.50 for a sandwich when he determines the price by direct costs margin.

e Complete the last column in the table.

f Steven isn't sure which pricing system he should use. What do you recommend? Justify your answer.



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4 Jackie sells floor coverings using a direct costs margin of 100%.

a It costs her $38 per metre to buy carpet. How much will she charge for the carpet?

b Jackie sells floating timber flooring at $78 per metre. How much does it cost Jackie to buy the timber flooring?

c During the 'End of Financial Year' sale, Jackie reduced the prices of floor rugs by 20%. Calculate the sale price of a floor rug that cost Jackie $80.

12.5 GST

Video
GST

Video
Discounts and GST

Most businesses must add a 10% **goods and services tax (GST)** to the prices of the goods and services they provide. Essential items such as basic foods and some medical supplies are exempt from GST. Businesses must pay the GST they collect to the Federal Government when they lodge their **business activity statement (BAS)**, usually quarterly. To help manage information about their transactions and other business activities, Australian businesses must obtain an **Australian business number (ABN)** from the Australian Taxation Office before they begin trading.

EXAMPLE 5

Abbey wants to receive $4.20 from the sale of muffins.

a How much GST will she have to add to the price of a muffin?

b At what price will Abbey sell the muffins?

Steps	Working
a Calculate 10% of $4.20.	GST = 10% × $4.20 = 42 c
b Add the GST.	selling price = $4.20 + $0.42 = $4.62 Abbey would round the price of the muffins to $4.60 or $4.65.

EXAMPLE 6

Abbey sells melting moment biscuits for $3 each. How much GST is included in the price?

Steps	Working
GST is $\frac{1}{11}$th of the price because full price = 100% + 10% = 110% and GST is 10%. To calculate the GST, divide the price by 11.	GST = $3 ÷ 11 = $0.272 727… = 27 c (correct to the nearest cent)

EXAMPLE 7

When Abbey completed her BAS, she found that her GST-inclusive sales totalled $395 600. She also calculated that the amount of GST the government will refund to her for GST on her purchases was $2655. How much GST will Abbey have to pay with her BAS?

Steps	Working
Divide the GST-inclusive sales by 11.	GST collected = $395 600 ÷ 11 = $35 963.64
Abbey has to pay the difference between the GST she has collected in her sales and the GST she paid in her costs.	GST to pay = GST collected – GST paid = $35 963.64 – $2655 = $33 308.64
The tax office ignores cents at the end of calculations.	Abbey will have to pay $33 308.

EXERCISE 12.5 GST

ANSWERS p. 438

1 EXAMPLE 5 How much GST should be added to each price?

a $89 pair of shoes
b $44 pair of shorts
c $28 underwear
d $22 hair cut
e $15 book
f $8.50 box of chocolates

2 EXAMPLE 5 The prices of the following items don't include GST. Determine the GST-inclusive prices.

a $5 coffee and cake
b $3.60 sandwich
c $3.20 muffin
d $7.25 meal of pie, peas and chips

3 EXAMPLE 6 Each of the following prices is GST-inclusive. Calculate the amount of GST included in each price. Express your answer correct to the nearest cent.

a $7.80 roll of deodorant
b $18 bag of dog food
c $76 dog grooming
d $42 length of sewing fabric
e $19 packet of macadamia nuts
f $15 bottle of shampoo

4 Abbey's business electricity bill is $2560 GST-inclusive. How much GST is included in the bill?

5 EXAMPLE 7 In her BAS, Abbey recorded that her GST-inclusive sales were $231 400 and her GST-exempt sales totalled $11 325.

a How much GST is included in Abbey's sales?
b Abbey has paid a total of $16 850 in GST that she can claim back from the government in her BAS. Will she receive a refund or have to pay more GST?
c Calculate the value of the refund or extra payment amount.

6 Businesses that lodge their BAS late or are late paying their GST can be fined and they can be charged interest on any outstanding GST payments. The annual interest rate is 3.01% and the daily rate is 0.008 246 57%.

This table shows the fine for a late BAS.

Days overdue	Fine
28 days or less	$222
29–56 days	$444
57–84 days	$666
85–112 days	$888
113 days or more	$1110

a Andrew was late completing his BAS. He should have completed it by 28 July but he didn't send it to the tax office until 10 September. What fine can he expect to get?
b Andrew owed $36 700 in GST. How much interest will he be charged?

Investigation What items don't have a GST charged?

Find a copy of your family's supermarket docket.

1 Classify the items on the docket into items that have a GST charged and those that don't.

Items with GST	Items without GST

2 What is the difference between the items that have a GST added and those that don't?

3 Decide whether each item would have a GST charged or not.

fresh apples	packet of chips	magazines
BBQ chicken	raw chicken	carton of milk
loaf of bread	bunch of flowers	tub of ice cream



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12.6 Paying staff

A business pays more than just its staff's wages. It also pays **workers compensation insurance**, **superannuation** on behalf of its employees, and **annual leave loading** to staff. It also deducts its employees' **PAYG** (pay as you go) **income tax** from their pay.

EXAMPLE 8

Lucy earns $18.65 per hour for a 35-hour week. She is entitled to 4 weeks' paid leave each year with no annual leave loading. How much should her employer save each week to cover Lucy's **holiday pay**?

Steps	Working
Calculate Lucy's holiday pay. **Annual leave loading** is an extra amount that some businesses pay to employees when they take their holidays.	holiday pay for 1 week = $18.65 × 35 = $652.75 holiday pay for 4 weeks = $652.75 × 4 = $2611
There are 52 − 4 = 48 weeks where Lucy is paid while she is working. Divide the holiday pay by 48.	amount employer must save each week = $2611 ÷ 48 = $54.3958… ≈ $54.40

EXAMPLE 9

Lucy's employer must pay an amount equivalent to 10% of her gross pay into superannuation. The payments must be deposited into the employee's superannuation account every quarter. How much does Lucy's employer pay into her superannuation account every quarter?

Steps	Working
A quarter (of a year) is 13 weeks long. Calculate Lucy's pay for 13 weeks.	Lucy's pay for 13 weeks = $18.65 × 35 × 13 = $8485.75
Calculate 10% of the pay for 13 weeks.	superannuation per quarter = 10% × $8485.75 = $848.58

A business's **labour percentage** compares the cost of paying staff with the amount of sales (income). It is the labour costs as a percentage of the total sales without GST.

EXAMPLE 10

On a day when the total amount of sales, including GST, was $2244, Abbey's labour costs were $315. What was her labour percentage?

Steps	Working
First calculate the GST from the day's sales.	GST = $2244 ÷ 11 = $204
Calculate the sales without GST.	sales without GST = $2244 – $204 = $2040
Calculate the labour costs as a percentage of the sales without GST.	labour percentage = $\frac{315}{2040} \times 100$ = 15.4%

EXERCISE 12.6 Paying staff

ANSWERS p. 438

1 EXAMPLE 8 Rhianna earns $19.25 per hour for a 36-hour week. She is entitled to 4 weeks of paid leave each year with no annual leave loading. How much should her employer put away each week to cover Rhianna's holiday pay?

2 Hunter earns $1230 per week and he is entitled to 6 weeks of paid leave each year with no annual leave loading.

- **a** Calculate Hunter's holiday pay.
- **b** How many weeks per year does Hunter work?
- **c** How much should his employer save per week to cover Hunter's holiday pay?

3 Jake earns $21 per hour for a 32-hour week. He is entitled to 4 weeks of paid annual leave, including a 17.5% annual leave loading of 4 weeks' pay.

- **a** How much does Jake earn per week when he is working?
- **b** Calculate Jake's pay for 4 weeks of holiday pay, including the 17.5% annual leave loading.
- **c** How much should his employer save per week during the 48 weeks Jake is working, to cover his holiday pay with annual leave loading?

4 EXAMPLE 9 Sam earns $19.86 per hour for a 40-hour week. His employer must pay an amount equivalent to 10% of his pay into a superannuation account.

- **a** How much does Sam earn per week?
- **b** How much superannuation must his employer pay per week on Sam's behalf?
- **c** The employer pays the superannuation into Sam's account every quarter. How much is the quarterly payment?

5 Calculate the quarterly superannuation payments, at a rate of 10%, for:

- **a** Kye, who earns $1008 per week
- **b** Scarlett, who earns $24.10 per hour for a 38-hour week
- **c** Camilla, who worked for 60 days in the quarter and earned $197 per day.

6 Employees who don't have a **tax file number (TFN)**, or an exemption from quoting a TFN, pay higher tax. Because Daksh doesn't want to give Abbey his TFN, Abbey must deduct 47% of Daksh's wage as tax. Daksh earns $924 per week.

a How much tax does Abbey have to deduct from Daksh's weekly pay?

b How much does Daksh receive in his weekly pay?

7 EXAMPLE 10 Calculate the labour percentage for each day, correct to 1 decimal place.

	Day	Labour costs	Sales without GST
a	Monday	$480	$2100
b	Thursday	$698	$3500
c	Friday	$520	$2600
d	Sunday	$300	$1200

8 Today, Abbey's total sales, including GST, were $1408 and her labour costs were $260. Calculate today's labour percentage, correct to 1 decimal place.

9 Abbey wants to keep labour costs to under 20%. Tomorrow, she anticipates that her total sales, including GST, will be $1804.

a Calculate the amount of GST included in tomorrow's anticipated sales.

b What is the maximum amount Abbey wants to pay for labour tomorrow?

c Labour costs an average of $19 per hour. For how many hours should Abbey employ staff tomorrow to stay under 20% labour costs?

10 People who are not Australian residents pay different tax rates from Australian citizens. This table shows the amount of tax that employers must take out of the pay of non-resident workers if the employee provides a TFN.

Weekly earnings	Weekly tax
$0 to $ 2307	32.5 c for each $1
$2308 to $3461	$750 plus 37 c for each $1 over $2308
$3461 and over	$1177 plus 45 c for each $1 over $3461

Calculate the tax that employers must deduct from the weekly pay of a non-resident worker if their pay is:

a $920 **b** $1360 **c** $2550 **d** $4000

11 An Australian citizen who earns $2000 per week pays $467 in income tax each week. Use the table in question **10** to determine how much more tax per week a foreign worker pays on the same income.

12 This is the table Abbey uses to calculate the amount of tax to take out of an employee's **fortnightly** wage ($\$x$) for employees who are Australian residents and have supplied a TFN.

Fortnightly income (x)	Formula for employer tax deductions
Up to \$700	Nil
\$701 to \$1730	$0.19 \times (x - 700)$
\$1731 to \$4615	$196 + 0.325 \times (x - 1731)$
\$4616 to \$6923	$1133 + 0.37 \times (x - 4615)$

Calculate the fortnightly tax Abbey should deduct from the pay of an employee who is an Australian resident with a TFN who is on a fortnightly wage of:

a \$1640 **b** \$540 **c** \$1950 **d** \$5200

13 Abbey paid \$1014 into Daniel's superannuation account for the last quarter of the year, which was at a rate of 10%. How much does Daniel earn per week?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

12.7 Comparing sales

Every day, Abbey does a comp (comparison) calculation to track how well the day's business went compared to the same day the previous year. This calculation helps her to know how effective her strategies to increase sales and decrease costs have been.

Comparison formula

$$\text{today's comp} = \frac{\text{today's sales} - \text{same day last year's sales}}{\text{same day last year's sales}} \times 100\%$$

Abbey does her comp calculations on the value of her sales without GST.

EXAMPLE 11

Today, Abbey's sales totalled \$1950, all of which included GST. Her 'without GST' sales on the same day last year were \$1260. Calculate Abbey's comp for today, correct to 1 decimal place.

Steps	Working
Calculate the GST in today's sales.	GST = \$1950 ÷ 11 = \$177.2727… ≈ \$177.27
Calculate today's sales without GST.	today's sales without GST = \$1950 – \$177.27 = \$1772.73
Use the formula to find the comp. Last year's sales = \$1260	$\text{comp} = \frac{1772.73 - 1260}{1260} \times 100\%$ = 40.6928… ≈ 40.7%
Write the answer.	Abbey's sales today are 40.7% better than on the same day last year.

EXERCISE 12.7 Comparing sales

ANSWERS p. 438

1 EXAMPLE 11 All of the sales in this table are 'not including GST'. Calculate the comp for each day, correct to 1 decimal place.

Today's sales	Sales same day last year	Today's comp
$1942	$1356	**a**
$2949	$1568	**b**
$1605	$1945	**c**
$2648	$2948	**d**

2 Two of the answers in question **1** were negative.

- **a** Write a sentence to explain what a negative comp means.
- **b** Do you think Abbey should be worried about negative comps?
- **c** Suggest some possible reasons for a negative comp.

3 On the 24th of June, Abbey's total sales were $1890, including $160 for items that don't include GST.

- **a** Calculate the value of Abbey's total sales without GST.
- **b** On the same day last year, all of Abbey's sales of $1469 included GST. Calculate the value of these sales without GST.
- **c** Calculate Abbey's comp for the 24th of June.

4 Explain what a comp of 100% means.

5 This year, on New Year's Day, Abbey had a 320% comp. Her New Year's Day sales (without GST) the year before were $1135.

- **a** Use the 320% comp to calculate the value of Abbey's sales this year, not including GST.
- **b** Because it was very hot on New Year's Day this year, Abbey's sales included $312 for iced water. No GST is charged on iced water, but the remainder of Abbey's sales on the day included GST. Calculate the value of the GST Abbey collected on New Year's Day.



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6 Group discussion questions

Indicate whether you think that the following situations will result in a positive or a negative comp at Abbey's cafe.

a Today the department store in the shopping centre is having a '25% off everything' sale.

b The forecast for today is for cold, wet weather.

c The other food shops in the centre will be closed today.

d A new cafe will be opening in the centre today.

e School holidays start today.

Investigation GST and comp calculations

Is the value of comp calculations much different if GST is included?

To complete this investigation, you can either use the formula and do all the calculations manually or you can download and use a copy of the **Comp calculations** spreadsheet from Nelson MindTap.



Spreadsheet
Comp calculations

	A	B	C
1	**Comp calculations**		
2			
3	Only enter or change values in cells shaded green		
4			
5	**Today's sales**		
6	Sales including GST =	$1,200.00	
7	GST	$109.09	
8	Sales ex GST	$1,090.91	
9	GST fee sales =	$300.00	
10	Total sales ex GST	$1,390.91	
11	Total sales including GST	$1,500.00	
12			
13	**Last year's sales**		
14	Sales including GST =	$1,000.00	
15	GST =	$90.91	
16	Sales ex GST	$909.09	
17	GST fee sales =	$0.00	
18	Total sales ex GST	$909.09	
19	Total sales including GST	$1,000.00	
20			
21	**Today's comp value without including GST**	53	%
22	**Today's comp value including GST**	50	%
23			

In your investigation:

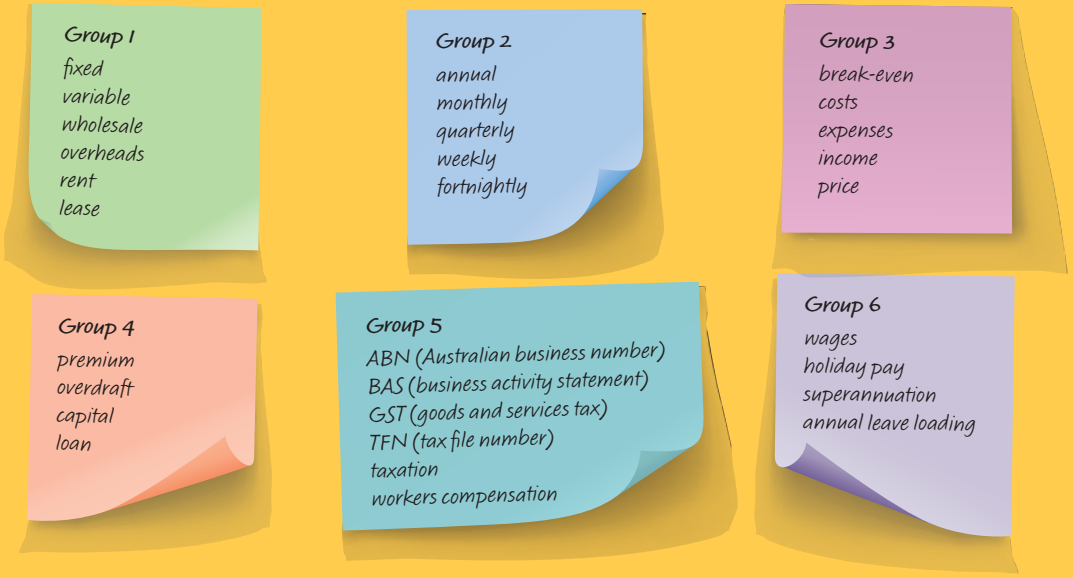
- calculate comps with and without GST, including situations where most of the sales are GST free and others with no GST-free sales
- try to find some situations where the comp values are the same with and without considering GST
- try to find some situations where the comp values are very different with and without considering GST
- write a conclusion indicating when you think it is important to take GST into consideration in comp calculations.

Keyword activity

WORD CATEGORIES

ANSWERS p. 439

The keywords from this chapter have been arranged into 6 groups. Your task is to determine the criteria used to assign the words to their group then write a sentence to explain why the words belong in the group.



Solution to the chapter problem

Problem

When Dominika started working in Abbey's cafe, she didn't supply a tax file number. Dominika earns $19.75 per hour for a 39-hour week.

How much should Abbey pay Dominika each week?

Solution

Each week, Dominika earns $19.75 × 39 = $770.25.

Because she hasn't supplied a tax file number, Abbey is required to deduct 47% from Dominika's wage and send it to the tax office.

Amount to be deducted = 47% of $770.25
= $362.02

Amount Abbey will pay Dominika = $770.25 – $362.02
= $408.23

12 Test yourself

Running a business

ANSWERS p. 439

1 EXERCISE 12.1 Monthly rent at Nelson Plaza is $224 per square metre of floor area.

a How much can a shop owner expect to pay in rent if their floor area is 56 m^2?

b Another shop's annual rent is $84 224. What is its floor area?

2 EXERCISE 12.1 Packs of Mocha Jamaica coffee beans are available in 4 sizes:

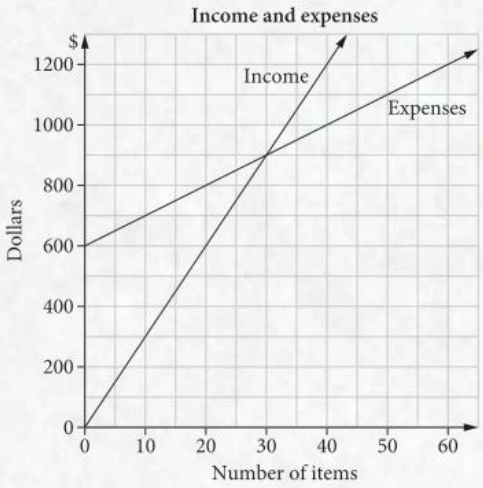
500 g: $22.20 2 kg: $88.50 5 kg: $205 10 kg: $412

a Which price represents the cheapest price per kilogram?

b A restaurant only uses 6 kg of coffee per month. Any more than 6 kg is wasted because it goes stale. What is the cheapest way for the restaurant to buy its monthly coffee supplies?

3 EXERCISE 12.2 When she started a business, Dana borrowed $210 000 in an interest-only loan at 4.6% p.a. How much interest does she have to pay each month?

4 EXERCISE 12.3 This graph shows Alison's weekly costs and income from selling items she produces.



a How many items does Alison need to sell each week to break even?

b At what price does Alison sell the items she produces?

c How much are Alison's fixed prices per week?

5 EXERCISE 12.3 Each week, Sam's costs are $4500 and he sells items for $65 each. Will he make a profit or a loss in a week when he sells 70 items? Use a calculation to justify your answer.

6 EXERCISE 12.4 To calculate selling prices, Hamish adds 40% to his break-even price. How much will he charge for a mobile phone with a break-even price of $340?

7 EXERCISE 12.4 Sienna uses a direct costs margin of 75% when she determines her selling prices. How much will she charge for an item that cost her $124?

8 EXERCISE 12.5 The price of a freezer including GST is $1606. What is the price of the freezer without GST?

9 EXERCISE 12.5 Abbey wants to receive $4.40 from the sale of muffins after GST has been deducted. At what price should she sell the muffins to cover the 10% GST?

10 EXERCISE 12.6 Luca earns $24 an hour for a 34-hour week. He is an Australian citizen who has supplied a tax file number. Luca is entitled to 4 weeks of paid annual leave, with no annual leave loading.

a Calculate Luca's fortnightly pay.

b Use the table in Exercise 12.6 question **12** on page 287 to calculate the tax Luca's employer is required to deduct per fortnight.

c At 10%, how much superannuation does Luca's employer have to pay on his behalf per fortnight? Answer to the nearest dollar.

d How much should Luca's employer save per fortnight to cover the pay for Luca's 4-week holiday?

11 EXERCISE 12.6 The prices of all items in a shop include GST. Last week, the total sales were $14 568 and the labour costs were $2350. Calculate the labour percentage, to 1 decimal place.

12 EXERCISE 12.7 Yesterday, Abbey's sales totalled $1980, all of which included GST. Her sales without GST on the same day last year were $1725.

a Complete a comp calculation for Abbey's sales yesterday, rounding to 1 decimal place.

b How did Abbey's sales yesterday compare to sales on the same day last year?

Practice set 3

ANSWERS p. 439

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 10.1 Jane works in a plant nursery. One of her tasks is to grow seedlings from seeds. She uses 16 types of flower seeds, 12 types of grass seeds and 8 types of vegetable seeds. What is the ratio of the number of types of grass seeds to the number of types of vegetable seeds to the number of types of flower seeds?

A 2:3:4 **B** 2:4:3 **C** 3:2:4 **D** 3:4:2

2 EXERCISE 12.1 Packs of coffee beans are available in 4 sizes:

500 g: $20

2 kg: $84

5 kg: $198

10 kg: $412

Which size represents the cheapest price per kg?

A 500 g **B** 2 kg **C** 5 kg **D** 10 kg

3 EXERCISE 10.2 The scale on the local National Park walking trail map is 1 : 10 000. On the map, the distance between the car park and the picnic area is 60 mm. What is the actual distance?

A 6 m **B** 60 m **C** 600 m **D** 6 km

4 EXERCISE 10.4 The total cost $\$C$ of buying N packets of seeds is directly related to the number bought according to the formula $C = kN$, where k is the constant of variation.

When I buy 20 packets, the total cost is $5. What is the value of the constant of variation?

A $\frac{1}{15}$ **B** $\frac{1}{4}$ **C** 4 **D** 15

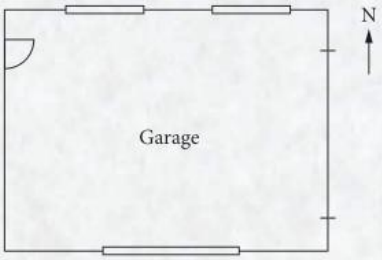
5 EXERCISE 12.5 The price of a commercial drink chiller is $1595 including 10% GST. What is the price of the drink chiller without GST?

A $145 **B** $159.50 **C** $1435.50 **D** $1450

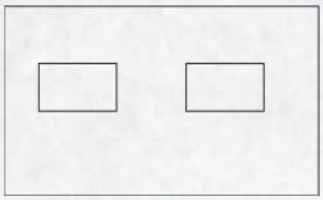
6 EXERCISE 10.1 Stella and Kai share their business profits in the ratio of 3 : 2. Calculate Kai's share of a $3000 profit.

A $600 **B** $1200 **C** $1800 **D** $2000

7 EXERCISE 11.1 Which diagram shows the northern elevation for the garage shown on the plan?



A



B



C



D



8 EXERCISE 10.4 Liam makes a soil additive to improve the soil in his vegetable patch by mixing compost, top soil and vermiculite in the ratio of 3 : 6 : 1. How much top soil will he use to make 240 kg of the soil additive?

A 24 kg **B** 72 kg **C** 120 kg **D** 144 kg

9 EXERCISE 11.2 The scale on a house plan is 1 : 80. The real verandah length is 8 m. How long is the verandah on the plan?

A 10 mm **B** 10 cm **C** 64 mm **D** 64 cm

10 EXERCISE 12.6 The prices of all items in a shop include GST. Last week, the total sales were $10 494 and the labour costs were $1980. Calculate the labour percentage, correct to 1 decimal place.

A 5.3% **B** 17.2% **C** 18.9% **D** 20.8%

Section B Short-answer questions

1 EXERCISE 11.2 The scale on Hannah's house plan is 1 : 100. What does each 1 cm on the plan represent in real life?

2 EXERCISE 10.1 Guy and Jake share the rent from a unit they own in the ratio 4 : 5. The annual rent is $14 040. How much does each person receive?

3 EXERCISE 9.2 Solve each equation.

a $10k - 8 = 8k + 4$

b $9x + 3 = 7x - 5$

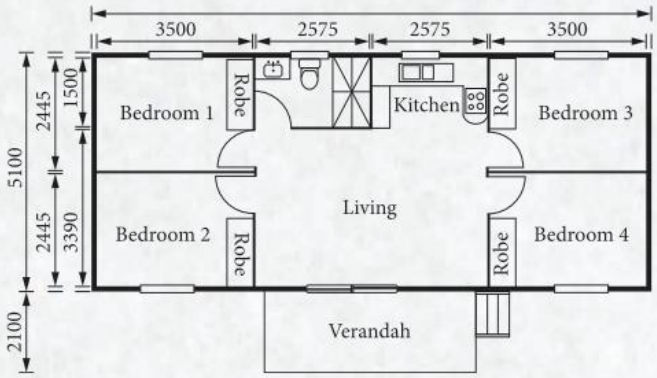
4 EXERCISE 10.3 The scale on a plan is 1 : 100.

a On the plan the height of the walls is 5 cm. What is the actual height of the walls?

b In real life the width of the verandah is 3 m. What is the width of the verandah on the plan?

5 EXERCISE 9.3 The formula $S = ut + \frac{1}{2}at^2$ can be used to calculate S, the distance an object travels during t seconds, where u = initial speed and a = acceleration.
Calculate the value of S when $a = 6\,\text{m/s}^2$, $t = 5\,\text{s}$ and $u = 4\,\text{m/s}$.

6 EXERCISE 11.3 This is the floor plan of the holiday house Keith and Judy are building.



Use the plan to answer the following questions.

a How many internal doors are shown on the plan?

b What type of door is the front door from the verandah into the living room?

c How many windows are there?

d What are the dimensions of bedroom 1?

e Determine the total length and depth of the house, including the verandah.

f The 4 bedrooms are going to have carpet on the floor. How many square metres of carpet will be required? Round your answer up to the nearest square metre.

g Judy wants to put a floating timber floor in the living room. The flooring comes in packets that hold $4\,\text{m}^2$. How many packets of flooring will Keith and Judy need?

7 EXERCISE 10.1 A quadrilateral has sides in the ratio 2 : 3 : 5 : 4. The shortest side is 30 cm.

a Calculate the lengths of the other 3 sides.

b What is the perimeter of the quadrilateral?

8 EXERCISE 10.4 The monthly cost of renting space in a warehouse varies directly with the area of the floor space. A space of $50\,\text{m}^2$ rents for $2400 per month.

a What is the monthly cost to rent a space of $100\,\text{m}^2$?

b Copy and complete the table of values for this relationship.

Floor area, F (m^2)	50	100	500	1000	2000
Monthly rent, R					

c Calculate the constant of variation.

d What does the constant of variation represent in this context?

e Determine an equation that relates R and F.

f Calculate the monthly cost of renting warehouse space of $1650\,\text{m}^2$.

9 EXERCISE 12.1 Monthly rent in a shopping centre is $234 per square metre of floor area. A shop's annual rent is $87 750. What is its floor area?

10 EXERCISE 10.5 The value of a car, $\$V$, varies inversely with its age in years A. When the car is 1 year old, it is worth \$24 000. The equation $V = \frac{k}{A}$ represents the relationship between the car's age and its value.

a Calculate the value of k.

b Determine the value of the car when it is 3 years old.

11 EXERCISE 9.6 Solve this pair of equations simultaneously.

$y = 2x + 11$

$y = 3x + 8$

12 EXERCISE 11.4 Tim and Kim bought a house with the floor plan shown in Exercise 11.3 question **2** on page 261. Refer to that plan to answer the following questions.

a What are the dimensions of the garage?

b Tim and Kim decide to concrete the garage floor.

i How many square metres of concrete will they need?

ii Concrete costs \$75/square metre for a depth of 100 mm. How much will the garage floor cost?

c How many windows are there in this house?

d The carpet in bedroom 1 needs to be replaced.

i How many square metres of carpet will they need?

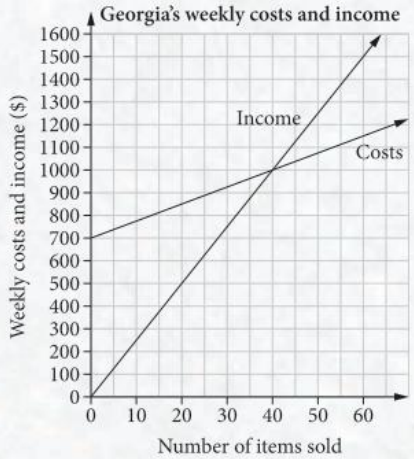
ii Carpet costs \$85 per square metre. How much will it cost to replace the carpet?

e The ceilings in all 3 bedrooms need to be repainted with 2 coats of paint.

i How many square metres of ceiling need to be repainted?

ii One litre of ceiling paint covers 12 m^2. White ceiling paint costs \$49.90 for a 4-litre tin. Calculate the cost of repainting the ceilings with 2 coats.

13 EXERCISE 12.3 This graph shows Georgia's weekly costs and income from selling the knitted items she makes.



a How many knitted items does Georgia need to sell each week to break even?

b At what price does Georgia sell the items she produces?

c How much are Georgia's fixed costs per week?

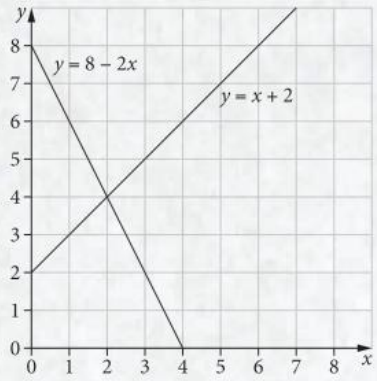
d Calculate Georgia's profit or loss in a week when she sells 60 knitted items.

14 EXERCISE 12.4 To calculate selling prices, Luke adds 50% to his break-even price.

How much will he charge for a mobile phone with a break-even price of \$440?

15 EXERCISE 12.5 Emma owns a cake shop. She wants to receive $4.20 from the sale of cupcakes after GST has been deducted. At what price should she sell the cupcakes to cover the 10% GST?

16 EXERCISES 9.5, 9.6 The diagram shows the graphs of the lines $y = 8 - 2x$ and $y = x + 2$.



a Use the graph to write down the simultaneous solution of the equations $y = 8 - 2x$ and $y = x + 2$.

b Solve the simultaneous equations $y = 8 - 2x$ and $y = x + 2$ algebraically.

17 EXERCISE 10.5 Mrs Kelly is a Year 1 teacher. She has a large supply of coloured pencils to share among the 20 children in her class. When all the students are present, they can have 6 pencils each. Let C = the number of children present and P = the number of pencils they can each have.

a Explain how you know that the number of pencils each child can have is an example of inverse variation.

b Write an equation linking C, P and a constant k.

c Determine the value of k.

d One winter's day, 5 children were away from school because they were sick. How many pencils was each child in the class able to have on that day?

18 EXERCISE 12.2 When she started a business, Ava borrowed $320 000 in an interest-only loan at 5.6% p.a. reducible interest calculated monthly. How much interest does she have to pay each month? Answer correct to the nearest dollar.

19 EXERCISE 9.3 The value $\$V$ of an item when it is n years old is given by the formula $V = P(1 - r)^n$, where P is the original value of the item and r is the annual rate of depreciation as a decimal, for example, 8% = 0.08 and 20% = 0.20.

a Calculate the value of a 4-year-old laptop that was originally valued at $1600. Its annual rate of depreciation is 20%. Answer correct to the nearest dollar.

b After 5 years, Andrew's golf buggy was worth $6000. If it has been depreciating at an average annual rate of 12%, what was the value of the golf buggy when it was new? Answer correct to the nearest hundred dollars.

20 EXERCISE 12.6 Noah earns $25 per hour for a 30-hour week. He is an Australian citizen who has supplied a tax file number. Noah is entitled to 4 weeks of leave each year with no annual leave loading.

a Calculate Noah's fortnightly pay.

b Use the table in Exercise 12.6 question **12** on page 287 to calculate the tax Noah's employer is required to deduct per fortnight.

c At 10%, how much superannuation does Noah's employer have to pay on his behalf per fortnight? Answer to the nearest dollar.

d How much should Noah's employer save per fortnight to cover Noah's 4-week leave pay?



Chapter problem

James collected the following data from 10 Year 12 students in his Maths class.

Student	Height (cm)	Right foot length (cm)
Amy	176	26
Joe	178	23
Annika	151	29
Janine	168	25
Stephen	186	30
Thanh	187	28
Gillian	149	22
Vamsee	174	28
Lalaja	172	26
Darryl	177	23

Two students were absent from class when James collected the data: Lyn, whose right foot length is 24 cm, and Byron, whose height is 181 cm. Use James' data to estimate Lyn's height and Byron's foot length.

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CHAPTER 13

DATA ANALYSIS, PROBABILITY AND STATISTICS

FITTING THE DATA

13.1 Scatterplots
13.2 Lines of fit
13.3 Using lines of fit
13.4 Interpolation and extrapolation
13.5 Bubble graphs
13.6 Stacked area graphs
13.7 Radar charts
13.8 Heat maps
13.9 Pictograms, Mekko graphs and sunburst graphs
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Draw scatterplots and lines of fit
- Use a line of fit to make predictions, both by interpolation and extrapolation
- Recognise the limitations of interpolation and extrapolation
- Interpret a range of different graphs
- Use technology to draw a variety of different graphs

How are we ever going to use this?

- When we make predictions about what data shows
- When we interpret graphs in the media
- If we have to present data to support a proposal or argument

Study design coverage

AREA OF STUDY 2: DATA ANALYSIS, PROBABILITY AND STATISTICS

- contemporary representations of data and graphs derived from technology including reviewing appropriateness of graphical representations, including pictograms, bubble, Mekko, radar, sunburst, heat map and stacked area charts
- interpolation and extrapolation of data, predictions, limitations, inferences and conclusions comparing data sets and graphs.

VCE Mathematics Study Design 2023–2027 p. 74, © VCAA 2022

Videos (1):
13.3 Line of best fit

Worksheets (6):
13.1 A page of scatterplots • Height vs shoe size • Body measurements • 2 mm grid paper

13.2 Scatterplots
13.3 Lines of fit

Weblinks (2):
13.5 , 13.7 Bureau of Meteorology
13.8 Bureau of Meteorology • Trailforks

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

13.1 Scatterplots

Worksheet A page of scatterplots

Worksheet Height vs shoe size

Worksheet Body measurements

Bivariate data are data with 2 variables. For example, you might collect data on the height and weight of people. Bivariate data can be graphed on a **scatterplot**.

The first variable is called the **explanatory variable** and is graphed on the **horizontal axis**.

The second variable is called the **response variable** and is graphed on the **vertical axis**.

We can look at the scatterplot to see if it has any of the following features:

- there is a pattern to the points
- as one variable increases, the other variable increases (or decreases)
- there are groups of points
- most of the points are together but a few are out on their own.



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EXAMPLE 1

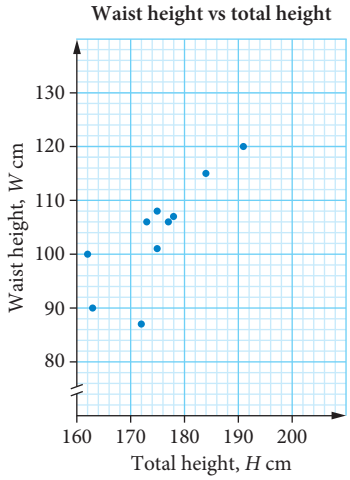
Ten students had their height and the height of their waist above the floor measured and recorded.

Total height, H cm	175	177	178	184	162	172	173	191	163	175
Waist height, W cm	101	106	107	115	100	87	106	120	90	108

a Graph this bivariate data in a scatterplot.

b Which is the explanatory variable and which is the response variable?

c Comment on any patterns in the scatterplot.

Steps	Working
a The total height goes on the horizontal axis. The waist height goes on the vertical axis.	
b The explanatory variable is on the horizontal axis, the response variable is on the vertical axis.	The explanatory variable is total height. The response variable is waist height.
c Describe any patterns.	As total height increases, the waist height increases.

EXERCISE 13.1 Scatterplots

ANSWERS p. 439

Grid paper is required for this exercise. Keep your scatterplots to use in later exercises.

Worksheet
2mm grid paper

1 EXAMPLE 1 This table shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

a Present this bivariate data in a scatterplot.

b Which is the explanatory variable and which is the response variable?

c Comment on any patterns in the scatterplot.



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2 Matthew asked a group of 11 students how many hours per week they spent playing sport and playing video games.

Sport (hours per week)	6	2	10	4	7	6	10	4	7	5	3
Video games (hours per week)	10	1	0	5	2	12	0	1	3	2	4

a Present this bivariate data in a scatterplot.

b Which is the response variable and which is the explanatory variable?

c Comment on any patterns in the scatterplot.

3 Simone measured the heights and arm spans of a group of 10 senior students.

Height, H (cm)	170	195	181	181	166	200	163	162	183	167
Arm span, A (cm)	171	186	187	178	165	160	147	143	115	169

a Construct a scatterplot to show this bivariate data.

b Which is the explanatory variable and which is the response variable?

c What does the scatterplot show?

4 This table shows the normal resting pulse of a sample of students and the time it takes each of them to swim 50 m.

Resting pulse (beats/min)	42	70	64	62	55	60	50	72	80
Swimming time (s)	30	48	50	43	40	45	36	49	59

a Construct a scatterplot for this set of data.

b Comment on the features of the scatterplot.

5 This table shows the sales of CD albums and digital albums over 9 years during the 2010s. The data are in thousands.

CD albums	46 174	49 818	44 045	38 659	39 529	33 114	30 223	27 356	14 226
Digital albums	91	418	788	1322	2279	3301	4818	6838	7377

a Present this bivariate data in a scatterplot.

b Describe what the scatterplot shows.

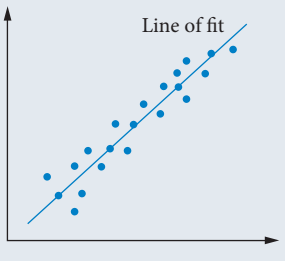
13.2 Lines of fit

If the data on a scatterplot shows a pattern approximating a straight line, we can draw a **line of fit** through the points.

Lines of fit

A line of fit:

- represents most or all the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible.



EXAMPLE 2

This table from Exercise 13.1 question **1** on page 301 shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

Graph this bivariate data on a scatterplot and draw a line of fit for it.

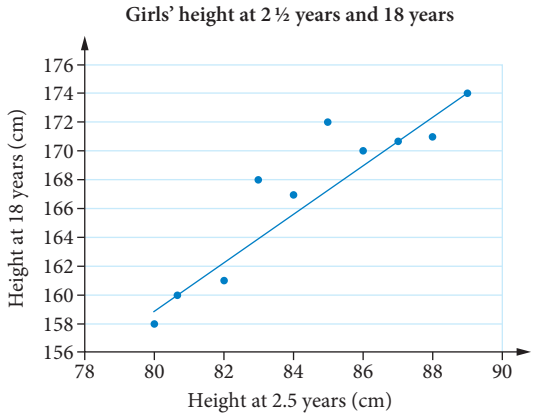
Steps

Graph the data first.

Then draw a line through the middle of the points with roughly the same number of points above and below it.

In this case, 3 points are on the line, 4 points are above the line and 3 points are below the line.

Working

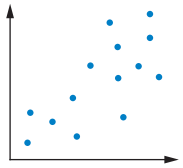


EXERCISE 13.2 Lines of fit

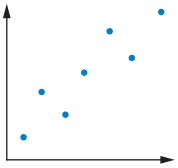
ANSWERS p. 440

1 EXAMPLE 2 Copy or print each scatterplot and draw a line of fit. You can print a copy of the graphs from the worksheet **Scatterplots**.

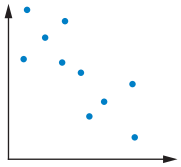
a



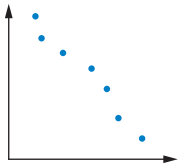
b



c



d



Worksheet
Scatterplots

2 This table shows the birth rate and the female life expectancy for a number of countries. The birth rate is births per year per 1000 population. Life expectancy is measured in years.

a Draw a scatterplot of birth rate against life expectancy.

b Draw in a line of fit for this data.

Country	Birth rate (per 1000 population)	Female life expectancy (years)
Australia	13.3	85.5
Brazil	14.7	79.8
Canada	11.2	84.6
Fiji	21.4	76.0
Germany	8.1	83.5
Iraq	31.0	72.0
Kenya	36.1	61.0
Laos	28.0	69.5
Nepal	24.3	70.0
Niger	50.0	57.0
Rwanda	42.1	61.0

3 This table shows the height above sea level and the average annual rainfall for some places in Australia.

City	Height above sea level (m)	Mean annual rainfall (mm)
Alice Springs	581	282
Ballarat	432	694
Hobart	24	616
Kalgoorlie	387	266
Mount Isa	365	463
Norfolk Island	73	1017
Perth	15	736
Winton	188	382

a Draw a scatterplot of height above sea level against mean annual rainfall.

b Draw a line of fit for this data.

4 This table shows information about the amounts of energy, carbohydrate and fat contained in 100 g of some take-away foods.

Food	Energy (kJ)	Carbohydrate (g)	Fat (g)
Hamburger	1030	26.6	9.1
Cheeseburger	1070	23.9	11.2
Chicken burger	921	20.2	10.1
Fish burger	988	24.5	10.2
Grilled chicken wrap	771	13.4	10.2
Chicken salad	325	5.5	3.4
Egg and bacon wrap	767	15.3	9.1
Hash browns	1150	26.4	17.2
Chips	1480	39.7	19.0

a Draw a scatterplot of energy against carbohydrate.

b Draw a line of fit for this data.

c Draw a scatterplot of energy against fat and a line of fit for this data.

Investigation Lines of fit using a spreadsheet

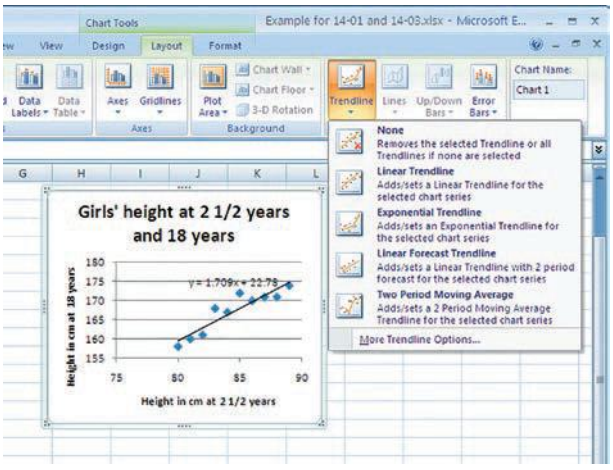
In this Investigation, use the data from Example 2.

1 Enter the data into the spreadsheet, then graph them on a scatterplot.

2 Select the graph and select **Trendline** from the **Layout** menu.

3 Select **Linear Trendline** and the line will appear on the graph. The spreadsheet equation for the line of fit is $y = 1.709x + 22.78$.

4 Enter the data for questions **2**, **3** and **4** of Exercise 13.2 and draw the line of fit for each. Are they the same as, or close to, what you drew?



Used with permission from Microsoft

Note: The spreadsheet uses a formula to calculate the line of fit, but when we draw a line of fit by hand, we have to position the line 'by eye'. It's unlikely that we will position the line in exactly the same place that the technology does. This means that our line will be a little different from the line drawn using a spreadsheet.

Practical activity My body parts

1 Measure and record the body part stated below for each student in your class, correct to the nearest centimetre:
   - **a** wrist circumference
   - **b** neck circumference
   - **c** forearm length (wrist to elbow).

2 Construct scatterplots for:
   - **a** wrist circumference and neck circumference
   - **b** wrist circumference and forearm length.

3 Add a line of fit to each of the scatterplots.

4 What do the graphs show you about the relationship between the body parts?

13.3 Using lines of fit

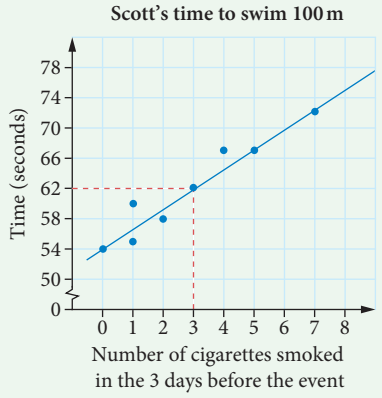
Lines of fit show the association between variables. Sometimes, they are used to make predictions within the range of the data. This is called **interpolation**.

Video
Line of best fit

Worksheet
Lines of fit

EXAMPLE 3

Scott is a competitive swimmer. He noticed that his times for the 100-m butterfly event are affected by his smoking habit. He graphed his times for his last 8 events and the numbers of cigarettes he smoked in the 3 days before on a scatterplot. He drew a line of fit through the data.





Shutterstock.com/Master1305

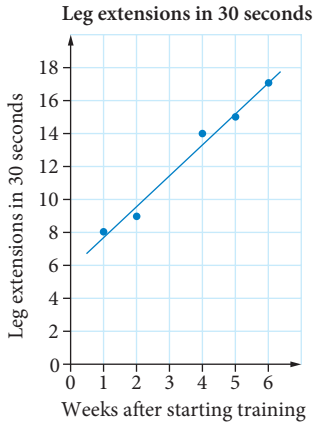
- **a** What is the explanatory variable?
- **b** What is the response variable?
- **c** What does the line of fit show about the relationship between these 2 variables?
- **d** Use the line of fit to predict Scott's time for the 100-m butterfly event when he has smoked 3 cigarettes in the 3 days before the event.

Steps	Working
a Explanatory variable is on the horizontal axis.	The explanatory variable is the number of cigarettes smoked.
b Response variable is on the vertical axis.	The response variable is the swimming time.
c Describe the pattern.	As the number of cigarettes smoked increases, the time for the 100 m increases.
d Find 3 on the horizontal axis, go up to the line and then across to the time.	Scott's time will be approximately 62 seconds.

EXERCISE 13.3 Using lines of fit

ANSWERS p. 440

1 EXAMPLE 3 Anita is trying to strengthen her quadriceps muscles. This graph shows the number of leg extensions she was able to complete in 30 seconds each week after she started training.





Shutterstock.com/antonio diaz

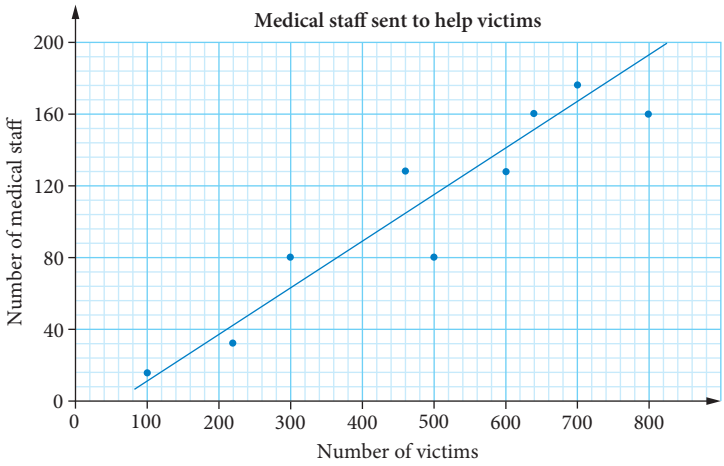
a What is the explanatory variable?

b What is the response variable?

c What does the line of fit show about the relationship between these 2 variables?

d Anita forgot to record the number of leg extensions she could do in week 3. Use the line of fit to predict this number.

2 Humanitarian agencies send people to assist when a natural disaster occurs. This scatterplot shows the number of medical staff one small agency sent to assist after earthquakes and floods occurred in different parts of the world and the estimated number of victims in each disaster.



a What is the explanatory variable?

b What is the response variable?

c Use the line of fit to estimate the number of medical staff the agency will send to a disaster with an estimated 200 victims.

d The agency sent 180 medical staff to assist in a disaster. Approximately how many victims were affected by the disaster?

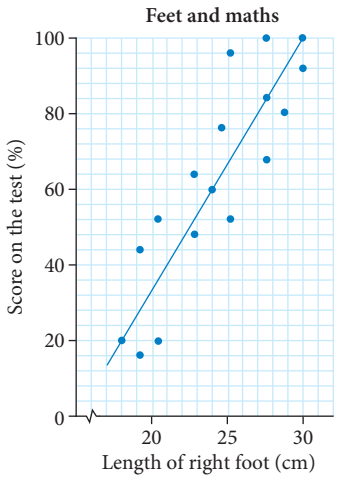


Getty Images/Stringer

e Describe the relationship between the number of medical staff and the number of victims.

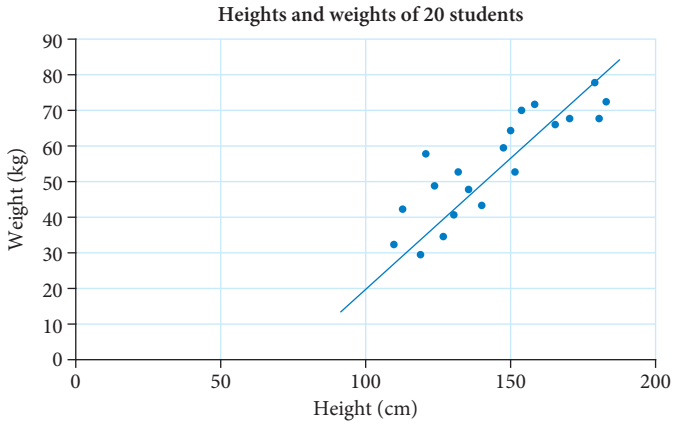
f Kyle noted that the greater the number of medical staff, the more victims there were requiring help. He argued that if the agency decreased the number of medical staff, there would be fewer people needing help. What is wrong with Kyle's argument?

3 Billy announced, 'People with big feet are better at maths than people with small feet'. He had given a maths test to a large sample of students in the school library and measured the length of each person's right foot. He displayed the results on this scatterplot.



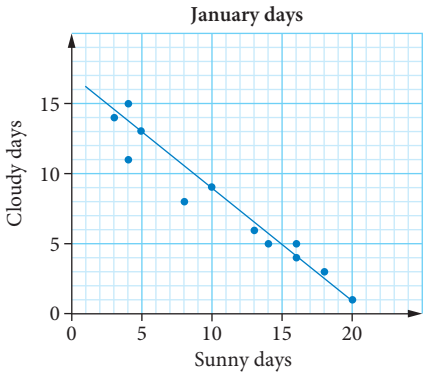
- **a** According to Billy's line of fit, what is the right foot length of a person who scored 58 on the maths test?
- **b** Describe the relationship between the length of the right foot and the score on the test as shown by this line of fit.
- **c** Big feet don't cause high maths scores. How can Billy's results be explained?

4 This graph shows the scatterplot for the heights and weights of 20 students, with a line of fit drawn on it.



- **a** Describe the relationship between the height and weight of students as shown by this line of fit.
- **b** Use the line of fit to estimate the weight of a student who is 150 cm tall.
- **c** Does an increase in height *cause* an increase in weight? Why or why not?

5 Dinesh graphed the number of sunny days in January against the number of dry, cloudy days in January for 12 different Australian places. He then drew a line of fit on his scatterplot.



- **a** Use the line of fit to estimate how many dry cloudy days there would be if there were 7 sunny days.
- **b** Describe the relationship between the number of sunny days and the number of cloudy days in January.
- **c** Suggest a reason for this relationship.

13.4 Interpolation and extrapolation

A line of fit helps us to make **predictions**, as we did in some of the questions in Exercise 13.3. If we make a prediction on information that lies *within* the data, this is called **interpolation**. If we make a prediction about information that lies *outside* the data, this is called **extrapolation**.

However, we need to be careful when we are making predictions from sets of data.

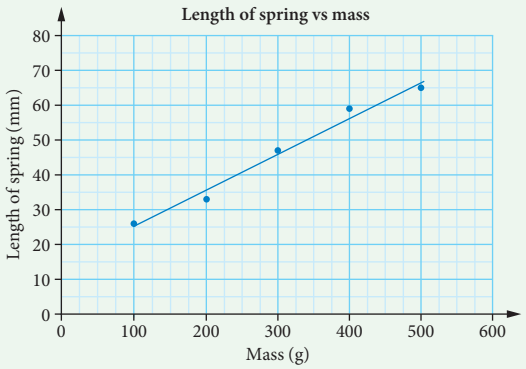
- Predictions within the data can be unreliable if the points on the scatterplot are not very close to the line of fit. If points on the scatterplot are close to the line of fit, then the predictions are more reliable.
- Predictions that are outside the given data can be unreliable because we can't be sure that the relationship continues as a line beyond the data.

EXAMPLE 4

In a science experiment, Clare and Dilani attached different weights to a spring. They recorded the length of the spring for each weight.

Mass (g)	100	200	300	400	500
Length of spring (mm)	26	33	47	59	65

They graphed the results on a scatterplot and drew a line of fit.



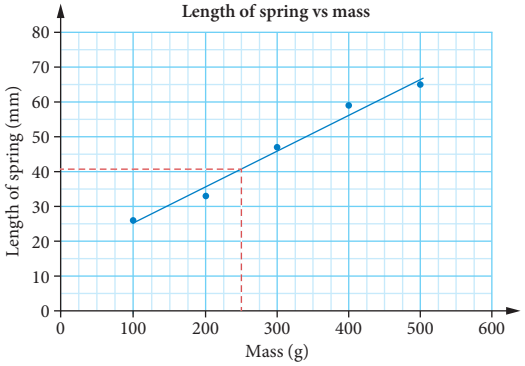
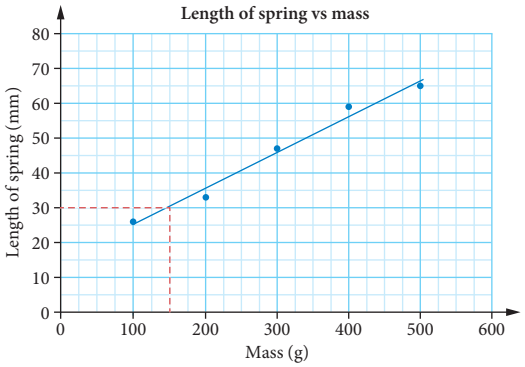
a Use the line to find:

- **i** the length of the spring for a mass of 250 g
- **ii** the mass when the length of the spring is 30 mm.

b How reliable are these predictions?

c The equation of the line is $L = 0.104m + 14.8$. Use the equation to predict the length of the spring when the attached mass is 800 g.

d How reliable is this prediction?

Steps	Working
a i Find 250 g on the horizontal axis, go up to the graph and across to length.	 The length of the spring is approximately 41 mm.
ii Find 30 mm on the vertical axis, go across to the line and down to mass.	 The mass is approximately 150 g.
b These predictions are within the given data.	These predictions are reasonably reliable as the points on the scatterplot are close to the line of fit.
c Substitute $m = 800$ into the formula.	$L = 0.104m + 14.8$ $= 0.104 \times 800 + 14.8$ $= 98$ The length of the spring with 800 g attached is approximately 98 mm.

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d A mass of 800 g is outside the data shown on the graph, so it is an example of **extrapolation**.

The prediction is not necessarily reliable as we don't know if the relationship continues to be a line. It is also possible that with a heavy weight attached, the spring might break.

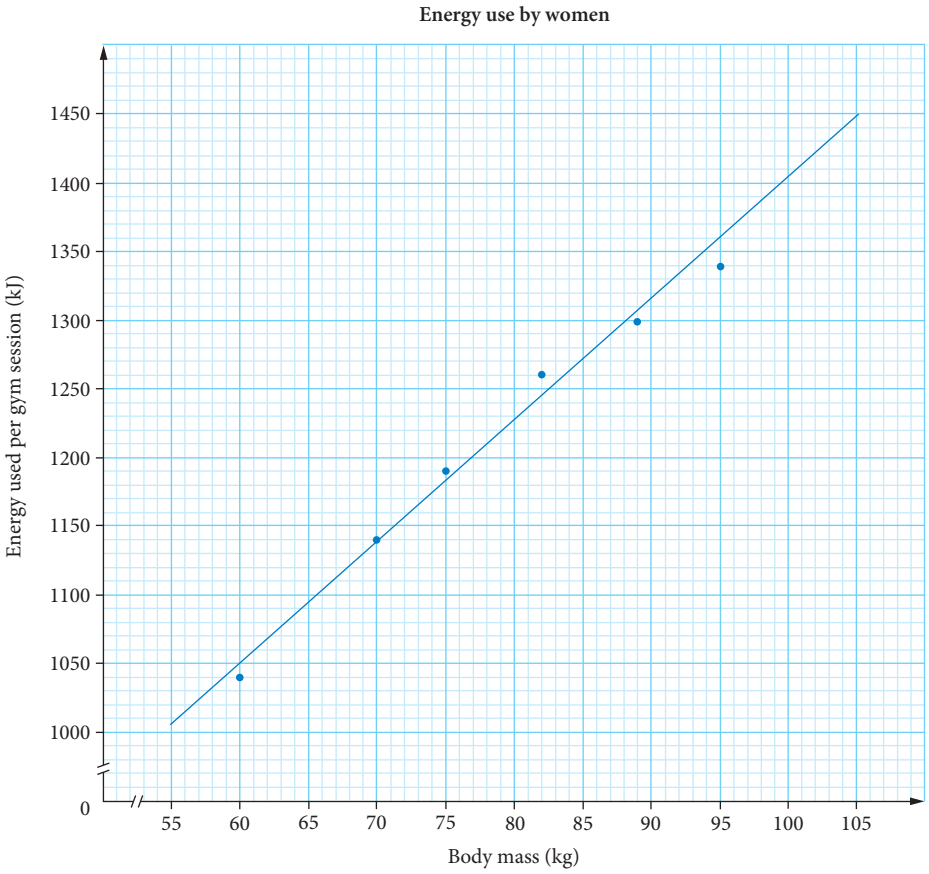
EXERCISE 13.4 Interpolation and extrapolation

ANSWERS p. 441

1 EXAMPLE 4 Jasmine recorded the body mass of a sample of women at the gym and the amount of energy they use per gym session.

Body mass (kg)	60	70	75	82	89	95
Energy used (kJ)	1040	1140	1190	1260	1300	1340

She graphed the data on a scatterplot and drew a line of fit.



a Use the line of fit to find:

i the amount of energy used by a woman of mass 65 kg

ii the body mass of a woman who uses 1450 kJ of energy per gym session.

b How reliable are these predictions? Justify your answer.

c The equation of the line is

$E = 16.9B - 143$

where B is the body mass and E is the energy used. Use it to predict the energy used by a woman of mass 120 kg.

d How reliable is this prediction? Justify your answer.

2 In Exercise 13.2 question **2** on page 303, you drew a scatterplot of the birth rate (B) and female life expectancy (E) in several countries. A line of fit for this data has the equation $E = -0.725B + 91.2$.

a Predict the female life expectancy in a country where the birth rate is 40.

b Determine the birth rate in a country where the female life expectancy is 65 years.

c How reliable are these predictions? Justify your answer.

d What would be the female life expectancy in a country where the birth rate is 60?

e How reliable do you think estimates outside this data would be? Justify your answer.

3 In Exercise 13.2 question **3** on page 304, you drew a scatterplot for the height above sea level (H) and the mean annual rainfall (R) for several cities. A line of fit for this data has the equation $R = -0.769H + 755.5$.

a Predict the mean annual rainfall for a city 300 m above sea level.

b Determine the height above sea level for a city that receives 400 mm of rain per year.

c How reliable are these predictions? Justify your answer.

d Calculate the mean annual rainfall for a city 800 m above sea level.

e Use the equation to predict the mean annual rainfall for a city 1000 m above sea level. Is this possible? What is wrong with this prediction?

4 Ms Cranston has a set of results for her class on 2 Maths tests.

Student	Bill	Ruth	Mary	Ella	Greg	Jim	Meg	Tara	Bob	Clem	Bree	Amy
Algebra test	60	38	65	??	75	48	67	23	82	16	92	80
Data test	35	21	47	31	56	40	54	11	59	20	62	??

Ella and Amy missed a test and Ms Cranston wants to give them an estimate.

a Draw a scatterplot for this data (except for Ella and Amy) and draw a line of fit.

b Predict the results that Ella and Amy might have received in the test they missed.

c How reliable do you think these predictions are? Justify your answer.

5 Ryan investigated the heights (x cm) of a group of his friends and their hand spans (y cm, the maximum distance between the tips of their thumb and little finger).

Height, x (cm)	170	178	160	183	168	145	155
Hand span, y (cm)	20	21	19	22	20	17	19

a Construct a scatterplot for the data and draw a line of fit on the plot.

b A line of fit has equation $y = 0.12x - 0.02$. Use the equation to predict the hand span of Izak, who is 165 cm tall. Answer to the nearest centimetre.

c Robert Wadlow was the tallest man in the world. His height was 272 cm (see photo on the page opposite of his actual-size statue at London's Ripley's 'Believe it or Not' Museum). Use the equation of the line of fit to predict his hand span, correct to the nearest centimetre, and explain why this measurement is unlikely to be correct.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

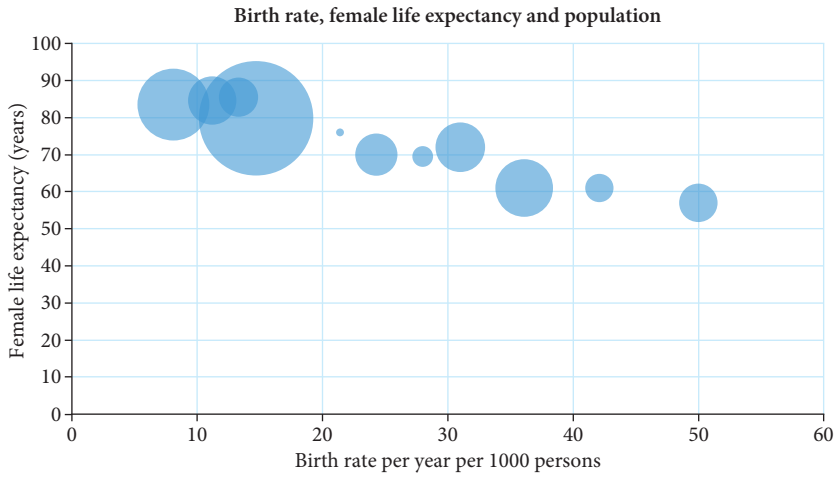


Alamy Stock Photo/Gary Wilkinson

13.5 Bubble graphs

Scatterplots show the relationship between 2 variables. **Bubble graphs** allow us to add a third variable to the graph by making the points different sizes. The area of the bubble shows the size of the third variable – the larger the circle, the larger the third variable. The bubbles can also be different colours to give more information.

This data comes from Exercise 13.2 question **2** on page 303, with the addition of population information. The area of each circle indicates the population of the country.



EXERCISE 13.5 Bubble graphs

ANSWERS p. 441

1 Use the bubble graph on the previous page to answer these questions.

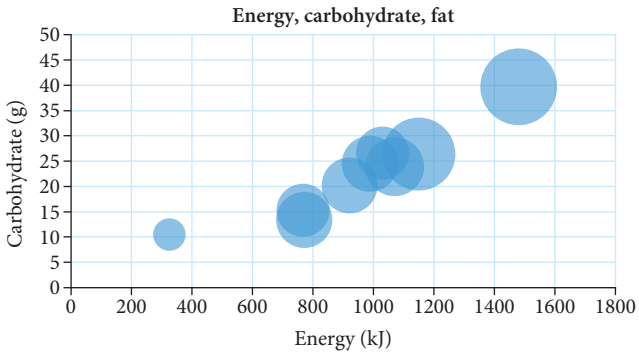
Country	Birth rate (per 1000 population)	Female life expectancy (years)
Australia	13.3	85.5
Brazil	14.7	79.8
Canada	11.2	84.6
Fiji	21.4	76.0
Germany	8.1	83.5
Iraq	31.0	72.0
Kenya	36.1	61.0
Laos	28.0	69.5
Nepal	24.3	70.0
Niger	50.0	57.0
Rwanda	42.1	61.0

a Which country has the largest population (represented by the biggest bubble)?

b Which country has the smallest population?

c Does there appear to be any link between a country's population and the other 2 variables?

2 The data from Exercise 13.2 question **4** on page 304 has been graphed in this bubble graph. In Exercise 13.2, we could only graph 2 of the 3 variables at a time. Now we can graph all 3 on the same graph. In this graph, the area of the circles indicates the amount of fat (in grams).



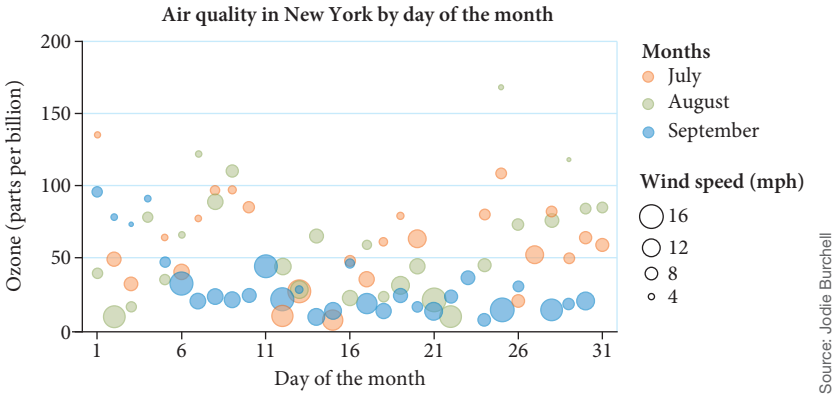
Food	Energy (kJ)	Carbohydrate (g)	Fat (g)
Hamburger	1030	26.6	9.1
Cheeseburger	1070	23.9	11.2
Chicken burger	921	20.2	10.1
Fish burger	988	24.5	10.2
Grilled chicken wrap	771	13.4	10.2
Chicken salad	325	5.5	3.4
Egg and bacon wrap	767	15.3	9.1
Hash browns	1150	26.4	17.2
Chips	1480	39.7	19.0

a Which food has the lowest amount of all 3 quantities?

b Which food gives you the most energy? Does it also have more fat than the other foods?

c Is it easy to read exact values from this type of graph? Explain your answer.

3 Some bubble graphs use colour to make them easier to read. This graph shows the air quality in New York over 3 months. The different colours show which month the information is from. The size of the bubbles shows the wind speed.



a Low ozone levels mean high air quality. What month had the best overall air quality?

b When is the air quality at its best: when it's very windy or when it's not very windy?

c Good air quality is considered to be an ozone level of 50 parts per billion or below. On how many days was the air quality poorer than this?

4 This data shows temperature information for Melbourne for 2 weeks in May.

Use a spreadsheet or other computer software to draw a bubble graph of this data:

- enter the data into the spreadsheet
- highlight the data
- go to **Insert** (at the top)
- select the bubble graph option in **Charts**, **Scatter**.

Day of the month	Maximum temperature (°C)	Minimum temperature (°C)
11	16	11
12	19	7
13	18	8
14	16	8
15	13	7
16	16	8
17	15	10
18	16	10
19	19	9
20	21	12
21	15	6
22	17	4
23	20	6
24	20	7

5 Go to the **Bureau of Meteorology** website at www.bom.gov.au and record the temperature data for the previous 2 weeks for the town or suburb you live in. Draw a bubble graph of the data.

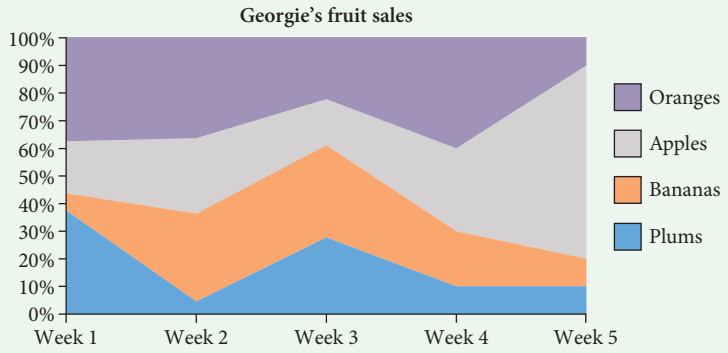
Weblink
Bureau of Meteorology

13.6 Stacked area graphs

A **stacked area graph** shows the sizes of different categories, usually over time, and what percentage each category contributes to the overall total.

EXAMPLE 5

This stacked area graph shows the sales of 4 different fruits over 5 weeks.



a Which fruit had the lowest sales in Week 2?

b In which week did apples have the highest sales amongst the fruits?

Steps	Working
a In week 2, the blue part of the graph is smaller than the other colours.	Plums had the lowest sales in week 2.
b In week 5, the grey part of the graph is larger than the other colours.	Apples had the highest sales in week 5.

EXERCISE 13.6 Stacked area graphs

ANSWERS p. 441

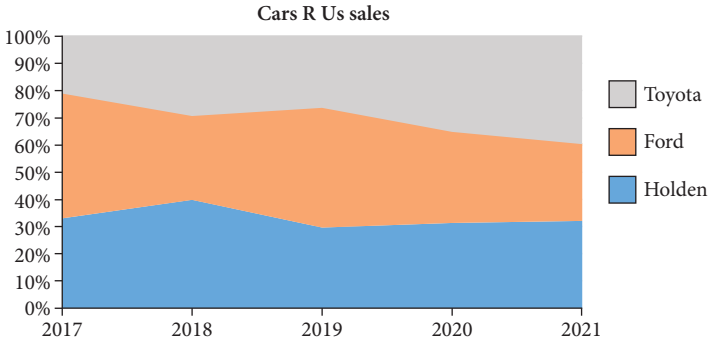
1 EXAMPLE 5 Use the graph of Georgie's fruit sales above to answer these questions.

a Which fruit(s) decreased their share of total sales over these weeks?

b In which weeks were banana sales the greatest percentage of sales?

c Suggest a reason why apple sales (as a percentage) might have shown a large increase in week 5.

2 This graph shows the sales of 3 brands at Cars R Us from 2017 to 2021.



a In 2017, which brand sold the highest percentage of cars?

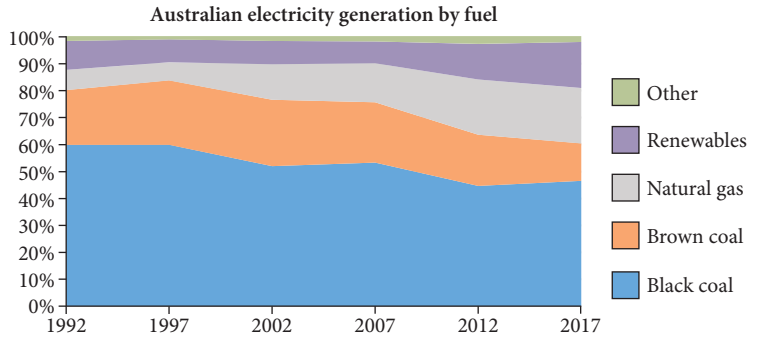
b For each brand of car, find the year of the highest sales percentage.

c Which brand has increased its share of the market from 2017 to 2021?

d Can you find how many cars were sold in 2021 from this graph? Why or why not?

e Is one brand clearly more popular than the others? Explain your answer.

3 This graph shows the contribution of different fuels to the generation of electricity in Australia from 1992 to 2017.



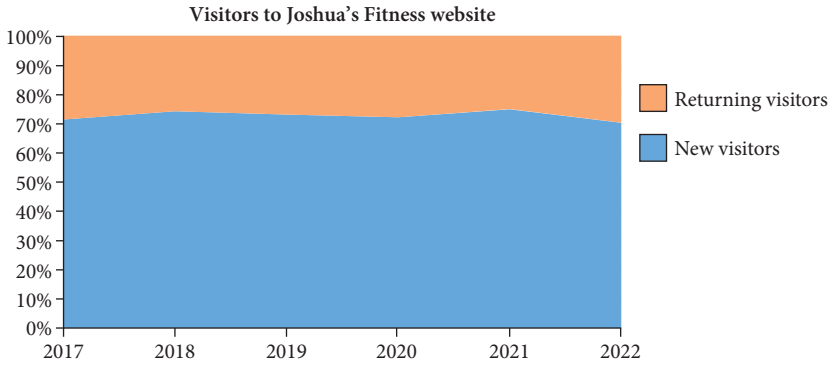
Department of Industry, Science, Energy and Resources, Australian Energy Statistics, Table O, September 2021, as viewed at https://www.energy.gov.au/publications/australian-energy-update-2021

a Which fuel type has contributed about the same percentage to electricity generation over this time?

b What types of fuel use decreased from 1992 to 2017?

c What types of fuel use increased from 1992 to 2017?

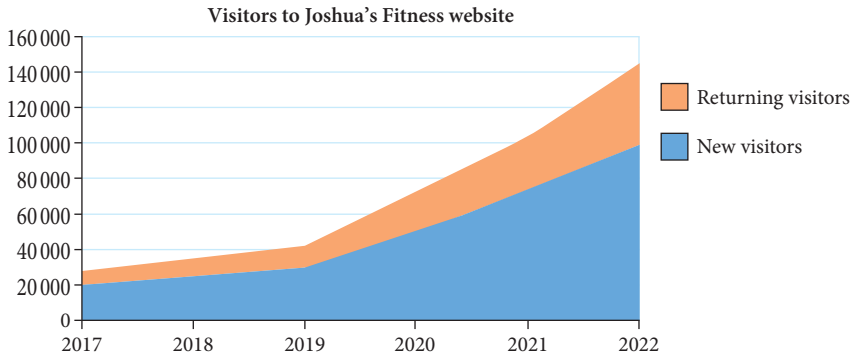
4 This graph shows the number of visitors to the Joshua's Fitness website over several years.



a Has there been much change in the percentage of returning visitors from 2017 to 2022? Explain your answer.

b The same information is shown on the area graph below.

The graph above is a stacked area graph. The graph below is an area graph. Notice the difference?



What does this graph tell us that we couldn't tell from the stacked area graph?

5 This data is from the Australian national census on the population (in millions) of the 3 east-coast states in the given years.

	1996	2001	2006	2011	2016
Victoria	4.5	4.7	5.0	5.5	6.0
NSW	6.2	6.5	6.8	7.2	7.8
Queensland	3.3	3.6	3.9	4.3	4.7

a Use a spreadsheet or other computer software to draw a stacked area graph of the this data:
- enter the data into the spreadsheet
- highlight the data
- go to **Insert** (at the top)
- select the stacked area graph option in **Charts**, **Line**, **2-D area**. Choose the third option (the first 2 are area graphs).

b What does this graph show about the populations of these states?

13.7 Radar charts

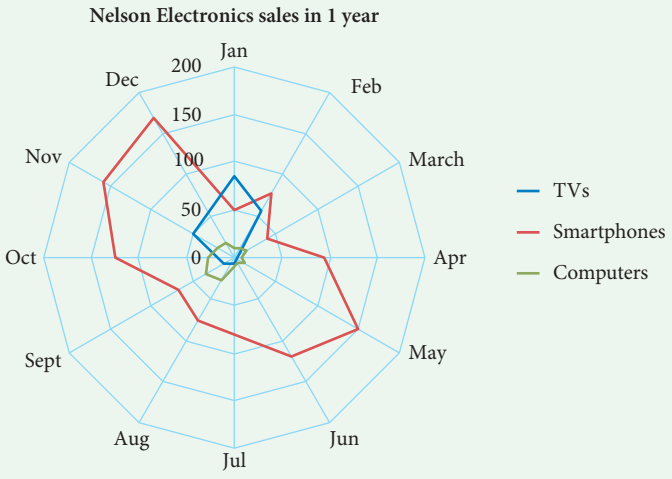
A **radar chart** is a circular version of a line graph and is used for data that is recorded over time.

EXAMPLE 6

This graph shows the monthly sales of 3 products at Nelson Electronics over 1 year.

a What is the most popular product sold at Nelson Electronics?

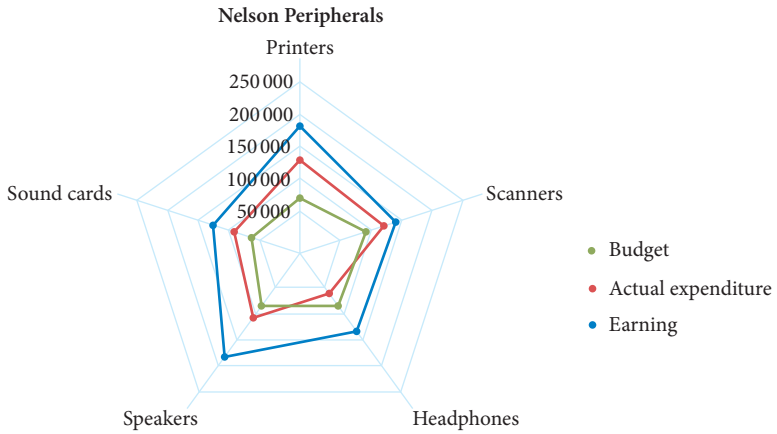
b In what month was the answer to part **a** NOT the highest-selling product? What product was?



Steps	Working
a The red line is above the other lines for most of the year	The smartphone is the most popular product.
b The blue line is above the red line in January.	The smartphone was not the highest-selling product in January. TVs were the highest-selling product in January.

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A radar chart can also be used to graph data, as shown. In this case, different products made by Nelson Peripherals are on each axis and the same information (budget, actual expenditure and earning) for each one is graphed.



EXERCISE 13.7 Radar charts

ANSWERS p. 441

1 EXAMPLE 6 Use the 'Nelson Electronics sales in 1 year' radar chart in Example 6 to answer these questions.

a How many smartphones were sold in May?

b In what month were 50 TVs sold?

c In what month were the most computers sold?

d In what month was the total number of products sold the greatest?

2 Use the 'Nelson Peripherals' radar chart to answer these questions.

a Calculate the difference between the budget and the actual expenditure for each product.

b Which product had the greatest difference between the budget and the actual expenditure?

c Which product had the highest earnings?

d The formula to calculate the profitability of a product is

$$\frac{\text{earnings}}{\text{actual expenditure}} \times 100.$$

Calculate the profitability of each product.

e Use your answer to part **d** to determine which product is the most profitable for the company.

3 The following table shows climate data for Darwin and Melbourne. It is the maximum average monthly temperatures in degrees Celsius (°C).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Darwin	33	33	33	33	32	32	32	33	34	34	35	33
Melbourne	27	24	23	19	16	15	14	15	18	19	25	23

Use a spreadsheet or other computer software to draw a radar chart of this data:

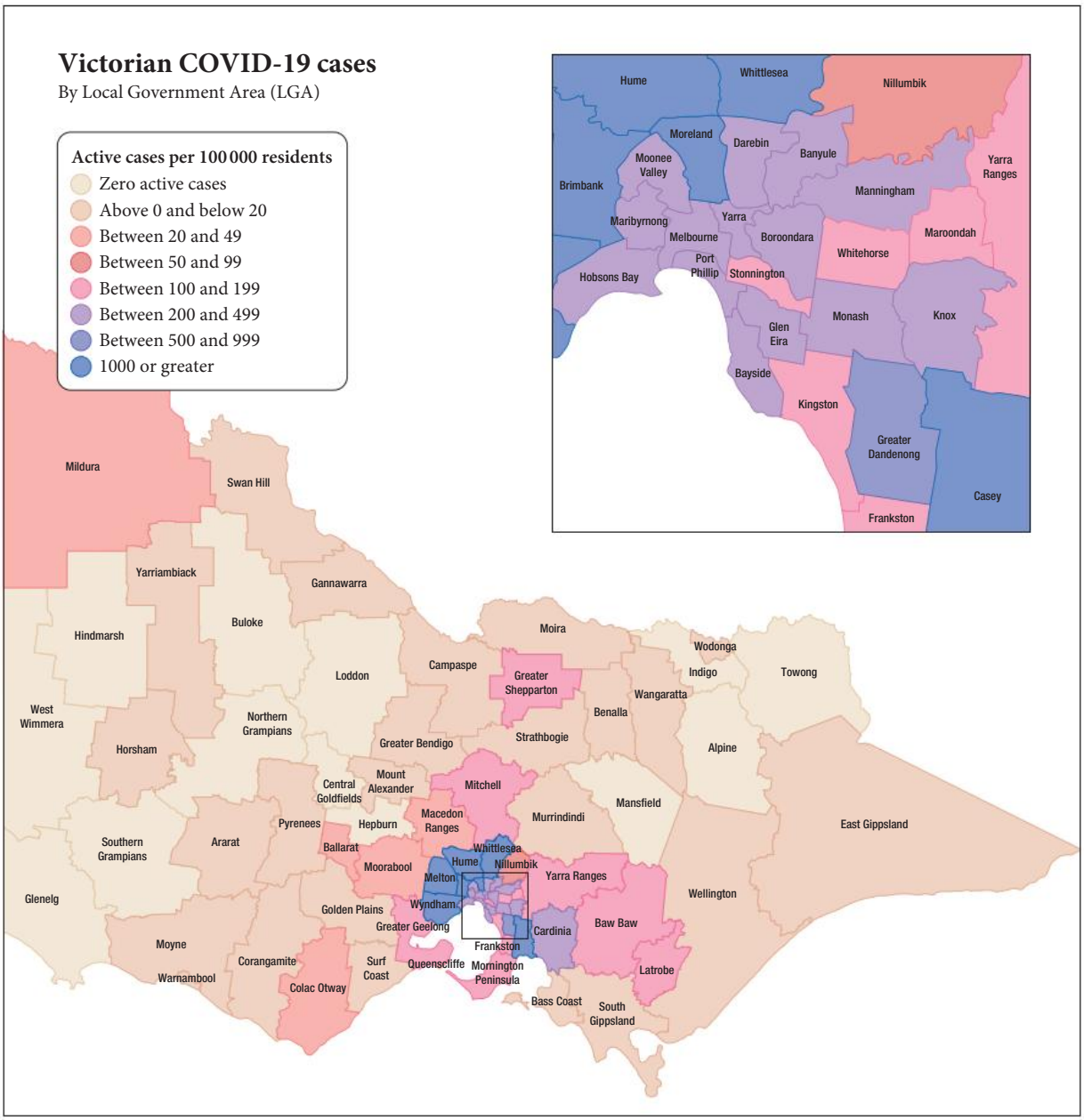
- enter the data into the spreadsheet
- highlight the data
- go to **Insert** (at the top)
- select the radar chart option in **Charts**, **Waterfall**, **Radar**.

4 Go to the **Bureau of Meteorology** website and find the same data as in question **3** for where you live. Add the data to your spreadsheet (or other software) and draw a radar chart with all 3 places on it.

Weblink
Bureau of Meteorology

13.8 Heat maps

A **heat map** is a graph where the sizes of data values on a map are shown by colour. Heat maps represent data visually, making it easier to interpret complex data, track changes and identify trends. They can also draw attention to 'hot spots' as high values are often shown by intense or darker colours, with the colour slowly changing to lighter colours for low values. A heat map will usually have a key to show what the colours are representing. The map below shows the case numbers of COVID-19 in Victoria on one day in October 2021.



Victorian case numbers by location by Local Government Area, https://www.coronavirus.vic.gov.au/victorian-coronavirus-covid-19-data.
© Copyright State Government of Victoria © OpenStreetMap contributors, © CARTO

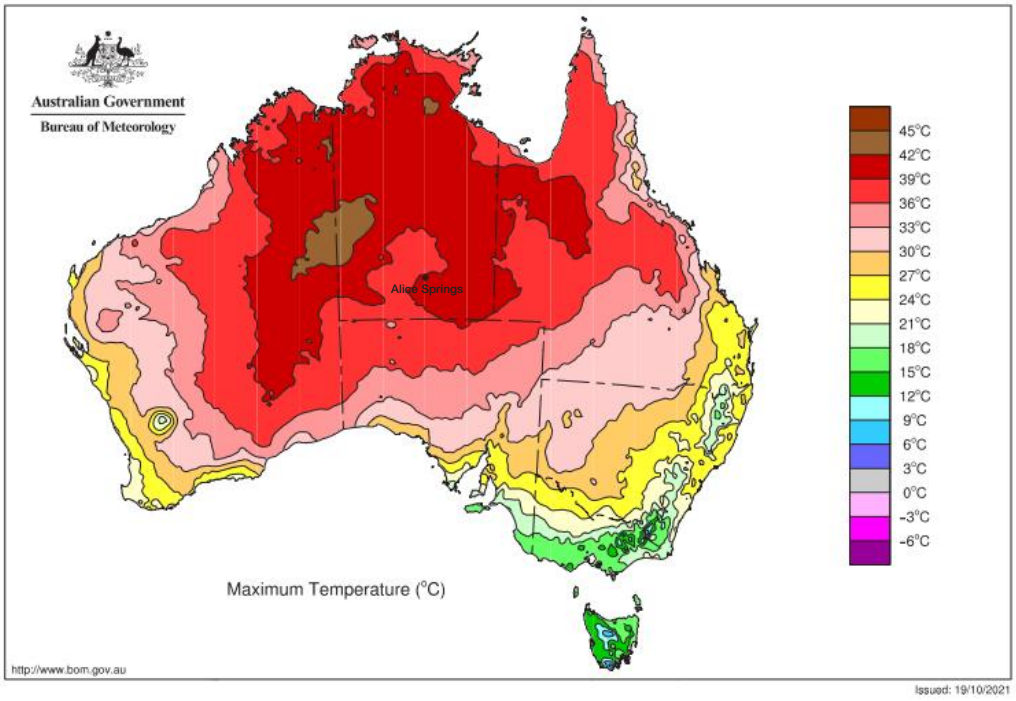
EXERCISE 13.8 Heat maps

ANSWERS p. 441

1 Use the heat map above to answer these questions.

- **a** What do the light red areas like Mildura show?
- **b** What do the tan areas like East Gippsland show?
- **c** What are the case numbers in the Casey local government area?
- **d** What are the case numbers in the Mitchell local government area?
- **e** What is one disadvantage of using a heat map to present data?

2 This map shows the maximum temperatures across Australia on one day.



Daily maximum temperature for Australia, Product Code: IDCKAXDA90, Bureau of Meteorology, © Copyright Commonwealth of Australia 2021 Licensed under Creative Commons Attribution Australia Licence

a Which state/territory had temperatures all below 21°C?

b Which state/territory had temperatures all above 30°C?

c True or false: The warmest temperatures are in the northern states.

d What was the temperature range in Victoria on this day?

e Within what range was the temperature at Alice Springs on this day?

f Search the **Bureau of Meteorology** website to find other heat maps. Save 3 different examples and explain what is being represented and the location being represented.

Weblink
Bureau of Meteorology

3 Using Google, search for 'heat maps on Strava'. Focus on the Global Heatmap.

a What are these maps about?

b Which continent has the most activity?

c Zoom in on Australia. Where are the regions with the most activity?

d Using the menu, change the activity type.

i What differences are there between the heat map for running and the heat map for cycling? Why is this?

ii What differences are there between the heat map for running and the heat map for swimming? Why is this?

e Using the menu, change the map style. What is the difference between satellite and hybrid?

f Copy and save 3 maps of interest to you. Explain what is being represented and the location being represented.

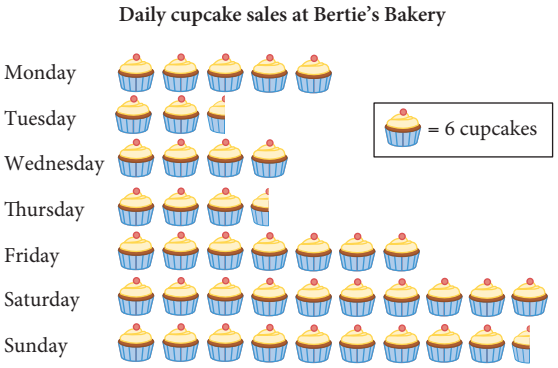
4 Trailforks.com is a website that shows heat maps of mountain biking trails. Name 3 sites of popular trails in Victoria.

Weblink
Trailforks

5 Heat maps are used to represent data in many areas, such as sport, real estate, power usage, website usage and climate. Find on the internet at least 3 different examples of heat maps in a document, including information about the heat map.

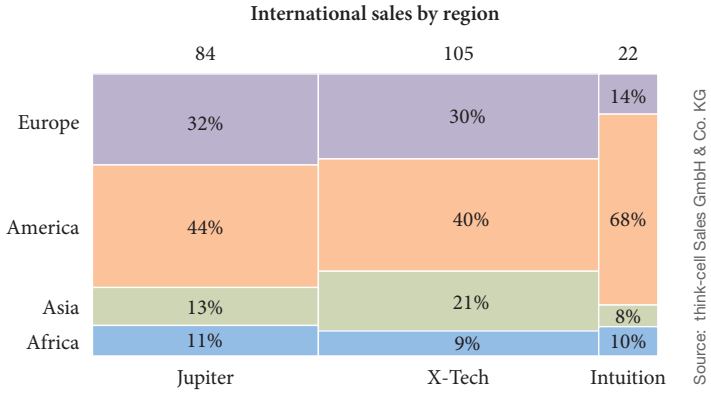
13.9 Pictograms, Mekko graphs and sunburst graphs

Pictograms or **picture graphs** use simple icons to represent data, making the information easy to read and visually interesting.

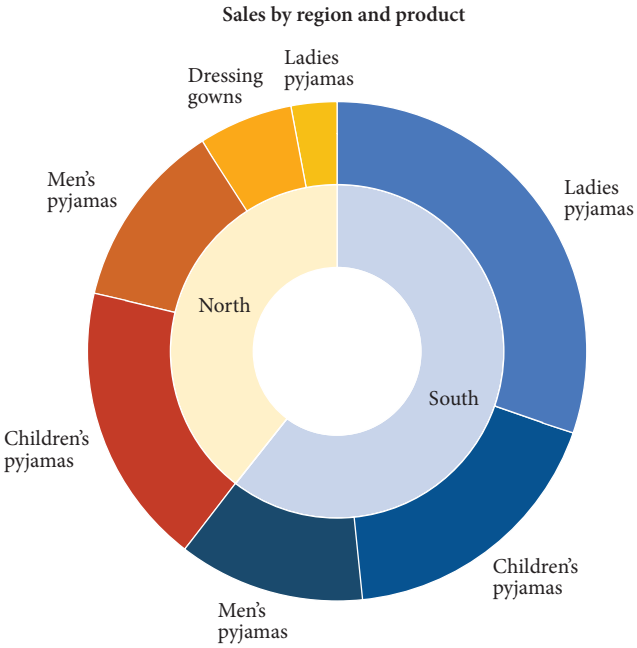




A **Mekko graph** has 2 dimensions. Column widths vary and each column has segments of different heights. In this example, the horizontal divisions shows the total sales of 3 technology companies relative to each other and the vertical divisions show the sales in each world region.



A **sunburst graph** displays hierarchical data. Each level of the hierarchy is represented by one ring or circle, with the innermost circle as the top of the hierarchy. In this example, the inner circle shows 2 regions where this company operates. The outer circle shows the sales of different products in those regions.



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EXERCISE 13.9 Pictograms, Mekko graphs and sunburst graphs

ANSWERS p. 442

1 Use the pictogram 'Daily cupcake sales at Bertie's Bakery' on page 320 to answer these questions.

- **a** How many cupcakes were sold on Saturday?
- **b** On which day were 42 cupcakes sold?
- **c** True or false: More cupcakes are sold on the weekend than over the rest of the week.
- **d** How many cupcakes were sold on Tuesday?
- **e** What is one disadvantage of using a pictogram to present data? Consider how you would show a day when 31 cupcakes were sold.

2 Draw a pictogram like the 'People living in extreme poverty' example on page 320 to illustrate the statement:

In 2007, 4 in every 10 people in Australia were not connected to the internet.

3 Use the Mekko graph 'International sales by region' on page 320 to answer these questions.

- **a** Which company had the most sales overall?
- **b** Which company had the highest percentage of its sales in America?
- **c** In which region of the world do each of the companies sell approximately the same percentage of their products?

4 Use the sunburst graph 'Sales by region and product' on page 320 to answer these questions.

- **a** Which region had the most sales overall?
- **b** Which product sold the most in the South region?
- **c** Which product sold the most in the North region?
- **d** What was the lowest-selling product overall?
- **e** Which product was sold in only one of the regions?

5 Use the internet to find more examples of each of these types of graphs. Save at least 3 examples of each type in a document.

6 Each of the types of graphs in this section can be drawn using a spreadsheet or other graphing software. Use an online tutorial and the software of your choice to draw a graph for each of the following sets of data.

Pictogram: Annie's Organic Veg

Day	Tue	Wed	Thu	Fri	Sat	Sun
Sales of strawberries (punnets)	130	160	145	185	195	170

Mekko: Population make-up of 3 countries

Country	Population	Male	Female
Australia	25 000 000	12 200 000	12 800 000
Canada	37 400 000	18 600 000	18 800 000
New Zealand	4 900 000	2 400 000	2 500 000

Sunburst: Population make-up of 3 countries

Use the same information as for the Mekko graph. Country should be at the centre and Male/Female in the second ring.



iStock.com/edb3_16

Investigation Using technology to draw graphs

You have now used technology to draw a wide variety of graphs. Within the software you use, there are options available for colours, background, size, font type, gridlines, adding legends and many other aspects of the presentation of each graph. In some software, you can change between different graph types.

Choose 5 of the graphs you have drawn in this chapter.

For each graph:

- print the graph as you have drawn it already
- change 3 aspects of the graph, for example, the colours used, the background and the font
- print the new graph you have created.

Take some time to play around with all the different features of graphs available in the software you're using. Use any data from this chapter. Print your favourites.

Create a display of the graphs you have printed.

Keyword activity

WORD MATCH

ANSWERS p. 442

Match each word or phrase (numbered 1 to 12) with the correct definition (A to L).

Words		Definitions	
1	Bivariate data	**A**	A graph showing data with 2 variables
2	Bubble graph	**B**	A line drawn on a scatterplot
3	Extrapolation	**C**	A graph like a circular version of a line graph
4	Heat map	**D**	A graph using simple icons to represent data
5	Interpolation	**E**	A graph like a scatterplot but showing the relationship between 3 variables
6	Line of fit	**F**	A graph showing the sizes of different categories, usually over time, and what percentage each category contributes to the overall total.
7	Mekko graph	**G**	When we make a prediction on information that lies within the data
8	Pictogram	**H**	When we make a prediction on information that lies outside the data
9	Radar chart	**I**	A graph with varying column widths and each column having segments of different heights
10	Scatterplot	**J**	A graph that displays hierarchical data
11	Stacked area graph	**K**	Data with 2 variables
12	Sunburst graph	**L**	A graph that shows the sizes of data values in different regions using a colour code

Solution to the chapter problem

Problem

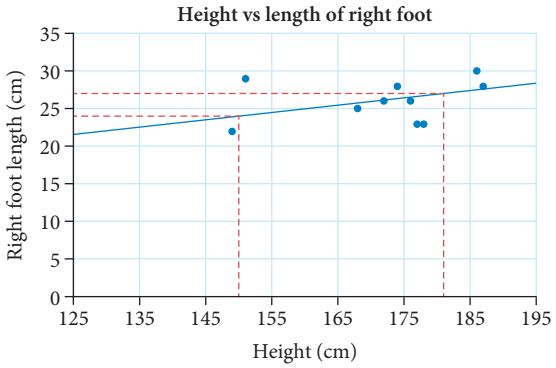
James collected the following data from 10 Year 12 students in his Maths class.

Student	Height (cm)	Right foot length (cm)
Amy	176	26
Joe	178	23
Annika	151	29
Janine	168	25
Stephen	186	30
Thanh	187	28
Gillian	149	22
Vamsee	174	28
Lalaja	172	26
Darryl	177	23

Two students were absent from class when James collected the data: Lyn, whose right foot length is 24 cm, and Byron, whose height is 181 cm. Use James' data to estimate Lyn's height and Byron's foot length.

Solution

Plot a scatterplot for James' data and add a line of fit. You can do this by hand or using a spreadsheet.



To estimate Lyn's height, draw a line from 24 cm on the Right foot length axis across to the line of fit and then down to the Height axis.

An estimate for Lyn's height is 150 cm.

To estimate Byron's right foot length, draw a line from 181 cm on the Height axis up to the line of fit and then across to the Right foot length axis.

An estimate for Byron's right foot length is 27 cm.

13 Test yourself

Fitting the data

ANSWERS p. 442

1 EXERCISE 13.1 This table contains data about time spent on homework and watching TV for 15 Year 12 students.

Student	Time on homework per week (hours)	Time watching TV per week (hours)
Amy	6	2
Joe	6	10
Annika	23	4
Janine	7	14
Stephen	23	9
Thanh	2	5
Gillian	17	4
Vamsee	1	3
Lalaja	16	20
Darryl	10	10
Lyn	15	0
Jeremy	9	4
Ben	0	17
Abdul	3	9
Miriam	4	1

Draw a scatterplot for this data.

2 EXERCISES 13.2, 13.3 Eliza works in a coffee shop. She thinks there is a relationship between the daily average temperature and hot chocolate sales.

Average temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Number of hot chocolates sold	42	15	37	24	9	20	35	20	10	30	30	4

a Graph Eliza's bivariate data in a scatterplot and draw a line of fit for this data.

b How many hot chocolates could Eliza expect to sell on a day with a temperature of 20°C?

c Predict the temperature on a day when Eliza sells 25 hot chocolates.

d Do you think there is a relationship between the daily average temperature and hot chocolate sales? If so, what is it? If not, why not?

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3 EXERCISE 13.4 A group of 12 students measured their heights and the lengths of their stride in centimetres. A stride is the biggest step a person can take from a standing position, measured from toe to toe.

Height, H (cm)	165	140	180	176	160	164	178	170	148	157	150	162
Length of stride, L (cm)	105	85	111	104	98	95	108	102	92	97	88	100

a Draw a scatterplot for this data and draw a line of fit for this data.

b Use your line of fit to predict:

i the length of stride of Jo, who is 150 cm tall

ii the height of Emir, who has a stride length of 95 cm.

c How reliable are these predictions? Explain your answer.

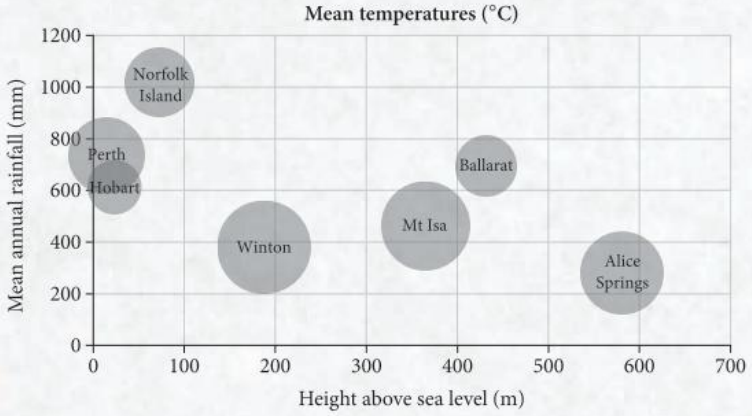
d The equation for the line of fit is $L = 0.596H + 1.85$. Use the equation to predict:

i the length of stride of Zhang who is 200 cm tall

ii the height of Janet who has a stride length of 80 cm.

e How reliable are these predictions? Explain your answer.

4 EXERCISE 13.5 This bubble graph and table show climate data for 7 different places in Australia. The area of the bubble indicates the mean temperature.



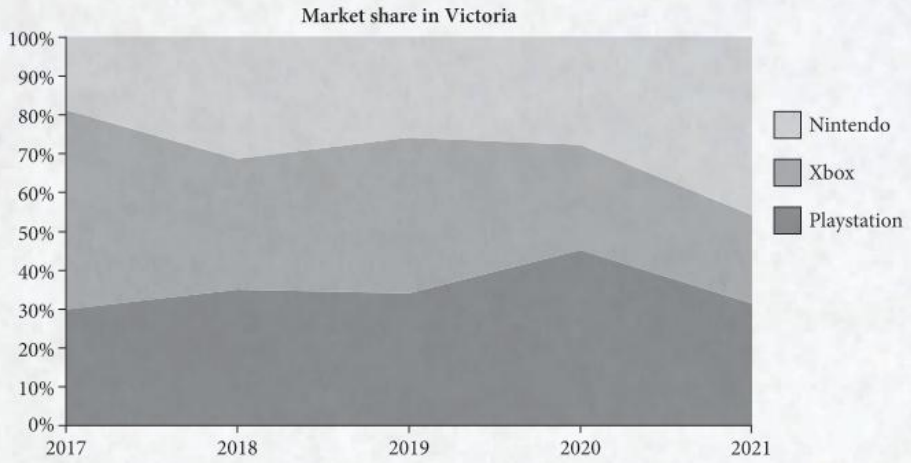
a Which 3 cities have the highest mean temperature?

b Which city has the lowest mean temperature?

c Arrange the cities in order from lowest mean temperature to highest mean temperature.

	Height above sea level (m)	Mean annual rainfall (mm)
Alice Springs	581	282
Ballarat	432	694
Hobart	24	616
Mount Isa	365	463
Norfolk Island	73	1017
Perth	15	736
Winton	188	382

5 EXERCISE 13.6 This stacked area graph shows market share for these gaming consoles in Victoria from 2017 to 2021.



a In 2017, which brand had the highest market share?

b For each brand, find the year of highest market share.

c Which brand has increased its share of the market from 2017 to 2021?

d Can you find how many gaming consoles were sold in 2021 from this graph? Why or why not?

e Is one brand clearly more popular than the others? Explain your answer.

6 EXERCISE 13.7 Use the 'Nelson Electronics sales in 1 year' radar chart on page 316 to answer these questions.

a How many TVs were sold in January?

b In what month were 25 000 computers sold?

c In what month were the most smartphones sold?

d What month had the lowest total number of products sold?

7 EXERCISE 13.8 Use the heat maps on pages 318–319 to answer these questions.

a Refer to the map showing the case numbers of COVID-19 in Victoria.

i What do the light cream areas like Hepburn show?

ii What areas of Victoria are shaded the darkest colour?

iii What colour is the area where you live? What does this mean about case numbers in October 2021?

b Refer to the map showing maximum temperatures across Australia.

i Within what range was the maximum temperature in Brisbane on this day?

ii What was the maximum temperature range in South Australia on this day?

iii Which state/territory had the largest range of maximum temperatures on this day?

8 EXERCISE 13.9 Use the graphs on page 320 to answer these questions.

a Refer to the pictogram 'Daily cupcake sales at Bertie's Bakery'.

- **i** How many cupcakes were sold on Wednesday?
- **ii** On which day were 15 cupcakes sold?
- **iii** True or false: No 2 days had the same cupcake sales.
- **iv** How many cupcakes were sold on Friday?

b Refer to the Mekko graph 'International sales by region'.

- **i** Which company had the least sales overall?
- **ii** Which company had the lowest percentage of sales in America?
- **iii** Which company had the highest percentage of sales in Europe?
- **iv** In which continent did all companies have their highest percentage of sales?

c Refer to the sunburst graph 'Sales by region and product'.

- **i** Which region had the lowest sales overall?
- **ii** Which product sold the least in the South region?
- **iii** Which product sold the least in the North region?
- **iv** What was the highest selling product overall?



Chapter problem

Holly receives an annual pension of $85 000 from her superannuation account. She also receives $920 in dividends from her share portfolio and $6000 from her term deposits.

How much income tax and Medicare levy does Holly have to pay each year?

Shutterstock.com/William Potter



CHAPTER 14

DISCRETE MATHEMATICS:
FINANCIAL AND CONSUMER MATHEMATICS

PAYING TAX AND SUPER

14.1 Income tax and Medicare levy
14.2 Minimising your tax
14.3 The cost of having children
14.4 Paying child support
Extended investigation: Child support for Yolantha
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Calculate income tax and the Medicare levy
- Investigate ways to minimise personal income tax
- Make calculations involving the cost of having children
- Calculate the value of child support payments

How are we ever going to use this?

- Checking that we are paying the correct amount of tax
- Minimising the tax we have to pay
- Knowing our financial responsibilities when we have children

Study design coverage

AREA OF STUDY 3: DISCRETE MATHEMATICS

Financial and consumer mathematics

- taxation systems at the personal level.

VCE Mathematics Study Design 2023–2027 p. 74, © VCAA 2022

Videos (1):
14.1 Income tax

Worksheets (3):
14.1 Income tax tables

14.4 Child support grid • Investigation components

Weblinks (1):
14.2 Australian Taxation Office

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

14.1 Income tax and Medicare levy

Video
Income tax

Running a country is an expensive business. The Australian Government provides the community with public healthcare, social services, border security, education, highways and much more. **Income tax** is one of the taxes the government uses to pay for these things.

PAYG (pay as you go) tax is deducted from your pay and is only an estimate of the income tax you need to pay. The actual amount of tax that you're supposed to pay is calculated at the end of the **financial year** (30 June) and is based on your total **income** over the previous 12 months.

After 30 June, you are required to complete a **tax return** to tell the Australian Taxation Office (ATO) how much money you've earned and how much PAYG tax you've paid. The ATO then calculates the actual income tax payable and decides whether you've paid too much tax or not enough. If you've paid too much, you will receive a **tax refund**. If you haven't paid enough, you will be sent a bill to pay the **tax debt**.

Worksheet
Income tax tables

This table shows the tax rates for different income brackets.

Taxable income	Tax on this income
Up to $18 200	Nil
$18 201 to $45 000	19 c for each $1 over $18 200
$45 001 to $120 000	$5092 plus 32.5 c for each $1 over $45 000
$120 001 to $180 000	$29 467 plus 37 c for each $1 over $120 000
$180 001 and over	$51 667 plus 45 c for each $1 over $180 000

Individual income tax rates - Resident tax rates 2020–21, https://www.ato.gov.au/rates/individual-income-tax-rates/ © Australian Taxation Office for the Commonwealth

EXAMPLE 1

Calculate the tax payable on each taxable income.

a $35 200 **b** $76 835 **c** $213 450

Steps	Working
a $35 200 is in the range $18 201 to $45 000. Tax = 19 c for each $1 over $18 200 (which means 19% or 0.19).	tax = 0.19 × ($35 200 − $18 200) = $3230
b $76 835 is in the range $45 001 to $120 000. Tax = $5092 + 32.5 c for each $1 above $45 000 (which means 32.5% or 0.325).	tax = $5092 + 0.325 × ($76 835 − $45 000) = $15 438.38
c $213 450 is in the range $180 001 and over. Tax = $51 667 plus 45 c for each $1 over $180 000 (which means 45% or 0.45).	tax = $51 667 + 0.45 × ($213 450 − $180 000) = $66 719.50

There is also a tax that goes towards funding Medicare (the public health system) called the **Medicare levy**. For most people, the Medicare levy is 2% of their taxable income.

Higher income earners who do not have private health insurance are charged a higher Medicare levy.

Medicare levy on annual taxable income

		Higher income with no private health insurance		
	Basic	Tier 1	Tier 2	Tier 3
Singles	$23 226–$90 000	$90 001–$105 000	$105 001–$140 000	$140 001 or more
Families	$36 705–$180 000	$180 001–$210 000	$210 001–$280 000	$280 001 or more
Medicare levy	2%	3%	3.25%	3.5%

EXAMPLE 2

Liam's taxable income is $124 500 and he has private health insurance.

a Calculate his Medicare levy.

b How much more would his Medicare levy be if he didn't have health insurance?

Steps	Working
a Liam has health insurance so his Medicare levy is 2% or 0.02.	Medicare levy = 0.02 × $124 500 = $2490
b Without health insurance, Liam's Medicare levy would be 3.25% or 0.0325. Subtract from the answer to part **a**.	Medicare levy= 0.0325 × $124 500 = $4046.25 difference in Medicare levy = $4046.25 − $2490 = $1556.25 Liam would have to pay $1556.25 more if he did not have health insurance.

Allowable tax deductions

If you donate some of your income to charities or spend it on work-related items and activities, then you do not have to pay tax on those amounts. These amounts are 'tax deductible' and are called **allowable tax deductions**. We deduct these amounts from our annual income and then income tax is calculated on the remaining amount, called our **taxable income**.

Taxable income

taxable income = total income – allowable tax deductions

EXAMPLE 3

Declan works as a shop fitter. His gross income this year will be $63 240 and he thinks he will receive $210 interest from the bank. His allowable tax deductions will be $410 union fees, a $100 donation to the Red Cross and $2200 for the cost of tools and equipment he bought to use in his job. During the year he paid $12 540 in PAYG tax.



iStock.com/milanvirijevic

a Calculate:

- **i** Declan's total income
- **ii** his total deductions
- **iii** his taxable income
- **iv** the tax he should pay on his taxable income
- **v** his Medicare levy.

b Will Declan receive a refund from the tax office? If yes, how much? If no, how much more tax will he have to pay?

Steps	Working
a i Declan's total income is the sum of his pay and bank interest.	total income = $63 240 + $210 = $63 450
ii Deductions include union fees, costs of tools and donations to charity.	total deductions = $410 + $2200 + $100 = $2710
iii Taxable income = total income – allowable deductions	taxable income = $63 450 – $2710 = $60 740
iv The tax rate for Declan's income is $5092 + 32.5 c for each dollar above $45 000.	tax = $5092 + 0.325 × ($60 740 – $45 000) = $10 207.50
v Declan's Medicare levy is 2%.	Medicare levy = 0.02 × $60 740 = $1209.40
b Compare the amount Declan paid in PAYG tax to the sum of the tax and Medicare levy.	tax + Medicare levy = $10 207.50 + $1209.40 = $11 416.90
Declan paid $12 540 in PAYG tax, which is more than he needed to pay. He will get a tax refund.	Declan's tax refund = $12 540 – $11 416.90 = $1123.10

EXERCISE 14.1 Income tax and Medicare levy

ANSWERS p. 442

1 EXAMPLE 1 Use the income tax table on page 330 to calculate the tax payable on each taxable income.

a $23 780 **b** $97 342 **c** $166 400 **d** $221 400

e $15 450 **f** $72 345

2 EXAMPLE 2 Calculate the Medicare levy on each taxable income.

a $72 300

b $104 600 (single person without health insurance)

c $145 000 (single person with health insurance)

3 EXAMPLE 3 Terry works in a golf pro shop. His taxable income is $78 900.

a Calculate the amount of tax he is required to pay.

b What is his Medicare levy?

c His employer deducted $15 467 in PAYG tax. Will Terry receive a refund from the tax office?

4 Chantelle is a signwriter. Her weekly taxable income is $1659.

a How much is Chantelle's annual taxable income?

b How much tax should she pay each year?

c What is her Medicare levy?

d How much tax should her employer deduct from her pay each week?

5 Dom's commission on book sales totalled $89 400 last year. The total value of his allowable deductions was $895. During the year, Dom paid a total of $19 900 in PAYG tax instalments. Will he receive a tax refund, and if so, how much? Use calculations to justify your answer and remember to include his basic Medicare levy.

6 Kelly is a nursing supervisor. After she received a $1600 pay rise, her annual salary became $105 400. How much of her $1600 pay rise was she required to pay in extra tax?



iStock.com/LumiNola

7 EXAMPLE 3 Last year, Eliza's gross weekly pay was $1720 and she received $523 in bank interest. Her allowable tax deductions totalled $1832.

- **a** What was Eliza's taxable income?
- **b** Calculate the amount of tax and the Medicare levy she has to pay.
- **c** Each week $273 was deducted from Eliza's pay under the PAYG system. How much was her tax refund?

8 Paul's taxable income is $64 000 and his friend Goran's taxable income is $128 000. Goran said to Paul, 'Because I earn twice as much as you, I pay twice as much tax as you do'. Is Goran correct? Justify your answer.

9 Som earned $87 420 last year from his full-time job as a carpenter. He also earned an additional $29 842 from private jobs. His work-related deductions totalled $1350. Som does not have private health insurance.

- **a** How much did Som have to pay last year in tax and Medicare levy?
- **b** Som's full-time employer deducted a total of $19 720 in PAYG tax. How much extra will Som have to pay the tax office?

10 Jay is a low-income earner and does not have health insurance. Last year, his Medicare levy was $960. What was Jay's taxable income?

11 Taylor owns a townhouse. Her tenant pays rent of $760 per week. Last year, Taylor's expenses were:

Agent management fee = 7% of the rent collected
Postage and administration = $9 per month
Yearly insurance premium = $512
Council and water rates = $925
Maintenance levy = $370 per quarter

- **a** Calculate the annual rent collected for Taylor's townhouse.
- **b** All Taylor's expenses involved in owning the townhouse are allowable tax deductions. Calculate the total amount Taylor can claim as allowable deductions for the townhouse.
- **c** Taylor's annual gross income from her job was $109 340 and she had no further deductions. Calculate her total taxable income.
- **d** How much was Taylor's tax and Medicare levy?

14.2 Minimising your tax

Many people pay more tax than they need to because they are unaware of how their tax can be minimised legally. Donations to charity, for example, are a good way to minimise your tax while helping others. Knowing what deductions can be claimed in your job, keeping good records and receipts, as well as making personal superannuation contributions are all effective strategies for minimising tax.

Your union, your employer and tax agents are all good sources of information about items that can be claimed in your employment category.

Superannuation

There are 3 types of contributions into superannuation:

- employer contributions – currently 10% of the employee's **gross pay**
- personal concessional contributions – payments into your own superannuation account that become tax deductions; the contribution tax is 15%
- personal non-concessional contributions – payments into your own superannuation account that don't become tax deductions.

Payments into a superannuation account have a double tax benefit. If you make concessional contributions, you can reduce your taxable income and hence, your personal tax. In addition, when you retire, the income you receive from your superannuation account is tax-free.

EXAMPLE 4

When Sally retired, she converted her superannuation account into a private pension account. Each year her pension pays her $74 000. In addition, she receives $4500 income from shares and $5278 in bank interest. She also makes a $1000 donation to a registered charity that she can claim as a deduction.

a Calculate Sally's total annual income.

b How much is Sally's taxable income?

c How much is Sally's income tax and Medicare levy?

Steps	Working
a Add Sally's pension, income from shares and bank interest.	total income = $74 000 + $4500 + $5278 = $83 778
b The private pension doesn't count as taxable income. Add the income from shares and bank interest and subtract the donations to charity.	taxable income = $4500 + $5278 – $1000 = $8778
c Sally's income is in the nil tax and no Medicare levy category.	Sally's tax and Medicare levy = $0.

EXAMPLE 5

Ethan is self-employed. Before he contributed to superannuation, his taxable income was $105 600. He decided to make the maximum $27 500 concessional contribution to superannuation.

a How much income tax would Ethan have had to pay without the contribution to superannuation?

b Calculate Ethan's income tax following the $27 500 superannuation concessional contribution.

c Including the 15% contribution tax to superannuation, how much less tax did Ethan have to pay as a result of his superannuation concessional contribution?

Steps	Working
a $105 600 is in the range $5092 + 32.5 c for each $1 over $45 000	tax = $5092 + 0.325 × ($105 600 – $45 000) = $24 787
b Calculate the new taxable income and tax.	new taxable income = $105 600 – $27 500 = $78 100 tax after the concessional contribution = $5092 + 0.325 × ($78 100 – $45 000) = $15 849.50
c Calculate 15% of $27 500 and add it to the answer to part **b**.	super contribution tax = 0.15 × $27 500 = $4125 total tax Ethan pays = $4125 + $15 849.50 = $19 974.50
Subtract the total tax he pays from the answer to part **a**.	tax saving = $24 787 – $19 974.50 = $4812.50

EXERCISE 14.2 Minimising your tax

ANSWERS p. 443

Use the information in the income tax table on page 330 to answer these questions.

1 EXAMPLE 4 Each year Mia receives a $94 000 annual pension from her superannuation fund. She also receives $3256 from shares and $7450 in term deposit interest. During the year, she donated $850 to charity. This donation was tax-deductible.

a Calculate Mia's taxable income.

b How much income tax does she pay?

2 EXAMPLE 5c Calculate the contribution tax payable on the following superannuation concessional contributions. The contribution tax is 15%.

a $24 000 **b** $15 000 **c** $7000

3 Georgia earns $72 400 p.a. from her job in a department store. Each fortnight, her employer deducts $648 for pay as you go (PAYG) tax.

a Calculate her fortnightly take-home pay (after tax).

Georgia decided that she could afford to pay $250 per fortnight into her superannuation account as a concessional contribution and a further $20 to charity.

b What is Georgia's taxable income (annual pay) after her superannuation contribution and charity donations?

c How much income tax and Medicare levy will she have to pay on her income in part **b**?

d Calculate Georgia's fortnightly take-home pay after she makes the $250 contribution to superannuation and $20 donation to charity.

e How much less is Georgia's take-home pay per fortnight when she makes the $250 contribution to superannuation and $20 donation to charity?

f Calculate the total amount Georgia will contribute to superannuation annually.

g How much will she pay annually in superannuation contribution tax?

h Before making the extra superannuation payments and donations to charity, Georgia had an annual tax and Medicare levy bill of $15 445. How much less combined tax and Medicare levy will Georgia pay by making these concessional contributions to superannuation and donations to charity?

i Georgia's employer has to pay 10% of her gross wage into superannuation. This amount is not taxed. Calculate the total amount that will be added to Georgia's superannuation account during the year from her employer and her personal concessional contributions.

4 When you have 2 jobs you can claim the travel between the jobs as a tax deduction, but you have to keep a diary to justify your claim.

Jane is a nurse in a GP clinic. Four days per week, she does a shift at the local nursing home after she finishes at the GP clinic. It's 14 km from the clinic to the nursing home.

a Calculate the number of kilometres Jane travels annually between the GP clinic and the nursing home, assuming she makes the trips for 48 weeks of the year.

b Jane can claim 75 c for each kilometre she drives between the jobs. Calculate the annual tax deduction she can claim for this travel.

c Jane is required to wear a nurse's uniform and she can claim $1 per load of uniforms she washes. On average, Jane does 3 loads of uniform washing per fortnight. How much can she claim per year (based on 48 weeks) for uniform laundry?

Jane should remember to keep a diary where she records the dates she washes her uniforms.

5 During the COVID-19 pandemic, the tax office introduced a temporary tax deduction for working from home. This rule allowed people who had to work from home to claim an 80 c per hour working-from-home deduction.

During the pandemic, Brianna worked from home for 48 weeks of the financial year instead of catching the train into the city on each of the 5 working days. She was employed to work a 40-hour week.

a How much can Brianna claim at 80 c per hour for working from home?

b Brianna's annual gross pay is $72 320. Assuming her gross pay is her taxable income, how much income tax would she have to pay?

c How much income tax will Brianna save by claiming the COVID-19 working-from-home tax deduction at 80 c per hour?

d Brianna normally had to pay $38 per week for train travel to work. Calculate the total amount she saved during the pandemic from the 80 c per hour tax deduction and train travel she didn't require.

To claim the working-from-home tax deduction, Brianna had to keep a work diary where she recorded the hours she worked from home each day.

6 Lanis is a self-employed tradesman. Because his registered office is in the house where he lives, he can claim travel to and from jobs, provided all the kilometres are work related. This table shows Lanis' travel diary for the first week of June.

	Odometer at start of trip	Odometer at end of trip	Kilometres travelled	Reason for trip	Work related?
Monday	42 312	42 349		Home to job site	
	42 349	42 371		Job site to supplies shop	
	42 371	42 385		Supply shop to football practice	
	42 385	42 403		Football practice to home	
Tuesday	42 403	42 440		Home to job site	
	42 440	42 477		Job site to home	
Wednesday				No travel, completing paperwork and placing orders at home	
Thursday	42 477	42 533		Home to job site via timber supply yard	
	42 533	42 580		Job site to home	
Friday	42 580	42 618		Home to job site via post office to post invoices	
	42 618	42 660		Job site to home via the pub	

a Copy and complete the table.

b How many kilometres can Lanis claim as work related during the first week of June?

c Calculate the value of these kilometres at 75 c per km.

7 Suzie works from home and her home office is used exclusively for her work. Suzie's home floor area is 275 m^2 and her office is a rectangle of 4.5 m by 3 m.

a What percentage of Suzie's home's floor area is the area of her office? Answer correct to 1 decimal place.

b Suzie's electricity bill is $364. Use the same percentage from part **a** to calculate how much of the electricity bill Suzie can claim as a business expense. Answer correct to the nearest dollar.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Weblink Australian Taxation Office

Investigation My workplace deductions

In this investigation, you are going to research a possible career after Year 12. The allowable tax deductions are not the same for all jobs.

Log onto the **Australian Taxation Office** website (www.ato.gov.au/individuals) and search for **Occupation and industry-specific guides**.

Select a job that you may do after Year 12 and research:

- the income and allowances associated with the job
- the allowable tax deductions
- the records you are required to keep
- how much you can expect to earn per week after applying allowable tax deductions and considering tax rates.

14.3 The cost of having children

Babies can be cute and cuddly, but they can also be very expensive. Both parents are responsible for supporting their children financially for a very long time. It's not just the actual costs associated with a child, such as food, nappies, clothes, or pram; having a child can decrease the employment options for parents, and the costs of childcare are high for a parent returning to work.

Depending on the type of care, the parent's income and what government assistance is available, the cost of having a child in childcare while the parents work can be anything between $30 and $150 per day.

EXAMPLE 6

Priyanka pays $72 per day for childcare for her 2-year-old son. She works 5 days per week and her take-home pay is $820.

a How much is her weekly childcare bill?

b What percentage of her take-home pay does Priyanka spend on childcare? Give your answer correct to 1 decimal place.

Steps	Working
a Multiply the cost per day ($72) by the number of days Priyanka works per week (5).	Priyanka's weekly childcare cost = 5 × $72 = $360
b Write $360 over $820 then multiply by 100.	Percentage of take-home pay Priyanka spends on childcare $= \frac{360}{820} \times 100\%$ $\approx 43.9\%$

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Services Australia has calculated the annual cost of raising a child depending on the combined parents' income. It uses this information to determine the amount each parent must pay after divorce or separation towards child support.

The annual cost of a child under 12 years of age

Parents combined child support income	Cost for 1 child
Up to $39 479	17 c for each $1
$39 480 to $78 957	$6711 plus 15 c for each $1 over $39 479
$78 958 to $118 436	$12 633 plus 12 c for each $1 over $78 957
$118 437 to $157 914	$17 370 plus 10 c for each $1 over $118 436
$157 915 to $197 393	$21 318 plus 7 c for each $1 over $157 914
Over $197 393	$24 082

Costs of the Children Table 2021, Child Support Guide, Version 4.58 - Released 20 September 2021, Department of Social Services
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EXAMPLE 7

Erin and Peter have a daughter aged 10. Their combined income is $136 600. Calculate the annual cost of raising their daughter.

Steps	Working
$136 600 falls in the range $118 437 to $157 914.	annual cost
Annual cost = $17 370 + 10 c for each $1 over $118 436	= $17 370 + 0.10 × ($136 600 − $118 436) = $19 186.40
Write the answer.	It costs Erin and Peter $19 186.40 each year to raise their daughter.

EXERCISE 14.3 The cost of having children

ANSWERS p. 443

1 EXAMPLE 6 Caitlin works 4 days per week and has a net pay of $628 per week. Her baby attends long daycare while Caitlin works, costing $80 per day. What proportion of her take-home pay, as a percentage correct to the nearest per cent, is her childcare cost?

2 As a disabled worker, Farris receives government help to pay childcare. Farris' 2 boys attend childcare at a subsidised cost of $24 per child per day. Farris works seven 6-hour days per fortnight and he earns $22 per hour after tax. How much remains from his fortnightly take-home pay after he pays the childcare bill?

3 Before the birth of their first child, Mark and Melissa's electricity bill was $345 per quarter. Their electricity bill increased by 7% after the birth of their child. What was the cost of the extra electricity they used per quarter after the birth of their child?

4 Shaw and Maryanne take their 2 girls to after-school activities. Mia, the younger daughter, has a 1-hour music lesson each week, 1 swimming lesson per week and a karate class on Saturday. Tegan, the older daughter, belongs to swim squad and trains most afternoons. These costs are shown in the price list.

Shaw and Maryanne pay for all the activities a term in advance. A term is 3 months (13 weeks) long. How much does it cost for the girls' after-school activities per term?

After-school activities price list

Music lessons	$35 per half hour
Swimming lessons	$20.40 per week
Karate	$18 per lesson
Swim squad	$137 per month

5 Levi and Falia's 3 children go to the local independent primary school. The fees per year are:

First child	$1350
Second child	$1100
Third child	$950

a What percentage of the first child's cost is the cost of the second and third child? Give your answers correct to the nearest per cent.

b Levi and Falia pay the fees in 4 equal instalments over the course of a year. How much is each instalment?

6 Jasmine's children attend moderately priced private schools. This table shows the school fees per year.

Calculate the total cost for one child to attend the school from Foundation to Year 12 inclusive.

Year	Cost per year
Foundation	$2300
Years 1 to 3	$4530
Years 4 to 6	$5500
Years 7 to 10	$7200
Years 11 and 12	$9560

7 Ian's son is in Year 12 at a prestigious private school. His school fees for Year 12 are $38 000. The school wants Ian to join the staff. They offered him a deal on his son's school fees. He thought they said they would reduce the school fees *to* 95%, but in fact they said they would reduce the school fees *by* 95%. What difference do the little words 'to' and 'by' make to the amount Ian would have to pay for his son's school fees?

8 EXAMPLE 7 Use the information in the table before Example 7 on page 339 to determine the annual cost for a child when the parents have the following combined child support incomes. Express each value correct to the nearest dollar.

a $175 000 **b** $82 650 **c** $210 000 **d** $35 200

9 According to Services Australia, the annual cost of Peter and Michelle's 8-year-old daughter is $21 330. How much is the parent's combined child support income?

14.4 Paying child support

Unfortunately, relationships can break down and families can separate. One parent might have the responsibility of looking after the children, or the parents can agree to share the role. Whatever the arrangements, both parents must contribute towards the costs involved. The amount of child support paid depends on the parents' individual incomes and the number of nights per year each parent cares for the child/ren. Services Australia uses the following system to determine costs.

Child support calculation grid

Steps	Parent 1 name	Parent 2 name
S1: Child support income Taxable income less $26 319 for supporting themselves.		
S2: Total child support income Add the parents' incomes.		
S3: Percentage of total child support income Find S1 as a percentage of S2 for each parent, rounded to the nearest whole number (they should add to 100%).		
S4: Percentage of annual care Calculate what percentage of the year each parent cares for the child, rounded to the nearest whole number.		
S5: Cost percentage based on S4 (from Table A) This is S4 converted to the cost percentage of raising a child.		
S6: Percentage of income less cost percentage (S3 – S5) Both parents pay child support but parent with positive % owes parent with negative %.		
S7: Annual cost of raising child (from Table B)		
S8: Annual amount parent with positive % needs to pay Positive % × S7		

Table A

Care and cost percentages table

Care percentage	Cost percentage
0–13%	0%
14–34%	24%
35–47%	25% plus 2% for every percentage point over 35% of care
48–52%	50%
53–65%	51% plus 2% for every percentage point over 53% of care
66–86%	76%
87–100%	100%

Source: Services Australia © Commonwealth of Australia, 2021

Table B

The annual cost of a child under 12 years of age

Parents combined child support income	Cost for 1 child
Up to $39 479	17 c for each $1
$39 480 to $78 957	$6711 plus 15 c for each $1 over $39 479
$78 958 to $118 436	$12 633 plus 12 c for each $1 over $78 957
$118 437 to $157 914	$17 370 plus 10 c for each $1 over $118 436
$157 915 to $197 393	$21 318 plus 7 c for each $1 over $157 914
Over $197 393	$24 082

Source: Costs of the Children Table 2021, Child Support Guide, Version 4.58 - Released 20 September 2021, Department of Social Services © Commonwealth of Australia. Licensed under Creative Commons 3.0 Attribution.

EXAMPLE 8

Luke and Maria, who are now divorced, have a 4-year-old son, James. Luke's taxable income is $72 300 and Maria's is $75 000. James is in Luke's care on Fridays and Saturdays every second week and in Maria's care the remaining days. How much should Luke pay Maria in child support?

Refer to the 'Child support calculation grid' on the next page.

Steps	Working
Let Maria be Parent 1 and Luke be Parent 2. S1: Calculate the child support income for each parent.	Maria's income = $75 000 – $26 319 = $48 681 Luke's income = $72 300 – $26 319 = $45 981
S2: Calculate the total child support income.	total = $48 681 + $45 981 = $94 662
S3: Percentage of total child support income.	Maria's percentage of the child support income: $= \frac{48\,681}{94\,662} \times 100\%$ ≈ 51% Luke's percentage of the child support income: $= \frac{45\,981}{94\,662} \times 100\%$ ≈ 49% Note that 51% + 49% = 100%.
S4: Percentage of annual care. Luke cares for James 2 days per fortnight, so Maria cares for James 14 – 2 = 12 days per fortnight. There are 26 fortnights in a year, so multiply 12 by 26, then divide by 365 days in a year, then multiply by 100% to make it a percentage.	Maria's percentage of care: $= \frac{12 \times 26}{365} \times 100\%$ ≈ 85% Luke's percentage care: $= \frac{2 \times 26}{365} \times 100\%$ ≈ 14% Note: The percentages do not add to 100% because of rounding.
S5: Cost percentage based on 85% and 14% using Table A.	Maria's cost percentage = 76% Luke's cost percentage = 24%
S6: Percentage of income less cost percentage (S3 – S5). Maria's percentage is negative (and Luke owes her child support) because she spends more time caring for James.	Maria = 51% – 85% = –34% Luke = 49% – 24% = 25%
The parent with the positive percentage has to pay child support to the other parent.	Luke has to pay child support.
S7: Annual cost of raising child (from Table B).	cost = $12 633 + 0.12 × ($94 662 – $78 957) = $14 517.60
S8: Amount Luke needs to pay.	Luke's payment = 25% of $14 517.60 = $3629.40 Luke will have to pay Maria $3629.40 annually.

Here is the calculations grid with the completed values.

Child support calculation grid

Steps	Parent 1 name	Parent 2 name
	Maria	Luke
S1: Child support income Taxable income less $26 319 for supporting themselves.	$48 681	$45 981
S2: Total child support income Add the parents' incomes.	$94 662	
S3: Percentage of total child support income Find S1 as a percentage of S2 for each parent, rounded to the nearest whole number (they should add to 100%).	51%	49%
S4: Percentage of annual care Calculate what percentage of the year each parent cares for the child, rounded to the nearest whole number.	85%	14%
S5: Cost percentage based on S4 (from Table A) This is S4 converted to the cost percentage of raising a child.	76%	24%
S6: Percentage of income less cost percentage (S3 – S5) Both parents pay child support but parent with positive % owes parent with negative %.	–34%	25% Luke owes
S7: Annual cost of raising child (from Table B)	$14 517.60	
S8: Annual amount parent with positive % needs to pay Positive % × S7	Luke pays $3629.40	

EXERCISE 14.4 Paying child support

ANSWERS p. 443

*Download the **Child support grid** to answer these questions.*

Worksheet
Child support grid

1 EXAMPLE 8 Olivia cares for her 3-year-old daughter, Bekkie, whose father, Jack, lives interstate. Jack's taxable income is $125 400 and Olivia's is $62 000.



iStock.com/M-image

a Calculate both parents' child support incomes by subtracting $26 319 from their taxable incomes. Write the amounts in the grid.

b What is their combined total child support income? Record this amount in the grid.

c Calculate their percentages of the total child support income correct to whole percentages. Record these amounts in the grid.

d Record 100% for Olivia's percentage of annual care and 0% for Jack.
Determine their cost percentages from Table A.

e Calculate the percentage of income less cost percentage for each parent.

f The parent with the positive percentage has to pay child support. Who has to pay in this situation?

g Calculate the annual cost of raising Bekkie using Table B.

h Calculate the amount of the annual payment.

2 Noah and Emma have a 5-year-old son, Seth, who lives 1 week with Noah and then the following week with Emma. Emma's taxable income is $95 650 and Noah's is $84 570. Which parent will pay the other child support? How much will it be?

3 Riley and Zoe are the parents of 2-year-old Xander. Zoe cares for Xander on weekdays and Riley cares for Xander on weekends. Zoe's taxable income is $75 400 and Riley's is $96 300. How much child support will Riley have to pay Zoe annually?

Worksheet Child support grid

Worksheet Investigation components

Extended investigation — Child support for Yolantha

Yolantha's parents are Dylan and Lucy. She lives with Dylan 3 days per week and with Lucy 4 days a week. Dylan and Lucy both have taxable incomes of $85 000.

Dylan has been offered a new job in a remote location. The job will mean that he has to move away from where he currently lives, and he won't be able to care for Yolantha, except for a 14-day period each year when he would be back for holidays. His taxable income at the new job would be $130 000.

How would Dylan's child support payments be affected if he took the new job?

Hints

- Use the child support grid to calculate how much child support Dylan pays in his current job.
- Calculate the child support Dylan would need to pay if he took the new job.
- Compare the two payments.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 443

The keywords from this chapter are missing in the following chapter summary. Use the listed words to copy and complete the summary of this chapter below.

deductions	gross	health	income	Medicare
minimise	net	PAYG	percentage	receipts
retired	superannuation	support	taxable	

In this chapter, we calculated income tax. Most people pay income tax under the $_1$_____ system, where employers deduct tax from an employee's $_2$_____ income before paying them their $_3$_____ income. Personal $_4$_____ tax is calculated on our $_5$_____ income. We work out our taxable income by subtracting our allowable $_6$_____ from our gross income.

The 2% $_7$_____ levy is also based on our taxable income. Higher income earners who don't have private $_8$_____ insurance pay a higher percentage.

We can $_9$_____ the tax we have to pay if we keep $_{10}$_____ and records of our deductible items. Making concessional contributions to $_{11}$_____ reduces our taxable income too and adds to our tax-free income when we are $_{12}$_____.

Children are expensive, and both parents are responsible for the cost of raising their children. Child $_{13}$_____ payments are also based on the parents' taxable income less a personal support amount. The value of the payments is affected by each parent's income and the $_{14}$_____ of nights per year that each parent cares for the child.

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Solution to the chapter problem

Problem

Holly receives an annual pension of $85 000 from her superannuation account. She also receives $920 in dividends from her share portfolio and $6000 interest from her term deposits.

How much income tax and Medicare levy does Holly have to pay each year?

Solution

Holly's only assessable income for tax purposes is from her share dividends and the interest from her term deposits.

Holly's taxable income = $920 + $6000
= $6920

This income is below the minimum level for taxation and the Medicare levy. Holly doesn't have to pay anything.

14 Test yourself

Paying tax and super

ANSWERS p. 443

*Use the tables on pages 330–331 to answer questions **1** to **4**, and **6**.*

1 EXERCISE 14.1 Calculate the tax payable on each taxable income.

a $73 240 **b** $154 300 **c** $265 500

2 EXERCISE 14.1 Calculate the Medicare levy on each taxable income.

a $59 600

b $121 300 individual income with health insurance

c $245 500 family income with no health insurance

3 EXERCISE 14.1 Mustaf's taxable income is $61 300.

a Calculate the tax and Medicare levy he has to pay.

b His employer deducts $450 each fortnight from his pay for PAYG tax. How much will his tax refund be?

4 EXERCISE 14.1 Last year, Dante's gross weekly income was $720, and he received $157 interest from his bank. His allowable taxation deductions were union membership of $215, a donation of $50 to the Red Cross and uniform cleaning costs totalling $420.

a What was Dante's yearly gross income?

b Calculate Dante's taxable income.

c How much income tax is he required to pay?

d Calculate Dante's Medicare levy.

e Each week, Dante's employer deducts $90 from his pay for PAYG tax. Calculate the value of Dante's tax refund.

5 EXERCISE 14.2 Sarah kept a travel diary of work-related kilometres for her travel between her 2 jobs. In it she can prove that she has travelled 820 km. At a rate of 75 c per km, how much can she claim as a work-related travel expense?

6 EXERCISE 14.2 Without claiming a tax deduction for contributions to superannuation, Jon has a taxable income of $145 000. As a self-employed person, Jon also makes a $25 000 concessional contribution to his superannuation to minimise his tax. The concessional contribution will be taxed at 15%.

a Calculate Jon's income tax without the concessional contribution to his superannuation.

b Calculate Jon's income tax after he makes the concessional contribution to his superannuation.

c How much less tax will he pay, including the contribution tax, if he makes the contribution to his superannuation?

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7 EXERCISE 14.3 Amelia is going back to work after the birth of her first baby. The quote for the baby's childcare was $95 per day until the baby is 6 months old. Amelia works 9 days per fortnight, earning $1625 gross.

a What percentage of her gross pay is her childcare cost? Give your answer correct to the nearest whole per cent.

b Amelia's mother offered to care for the baby 5 days per fortnight for free. How much per fortnight will this offer save Amelia?

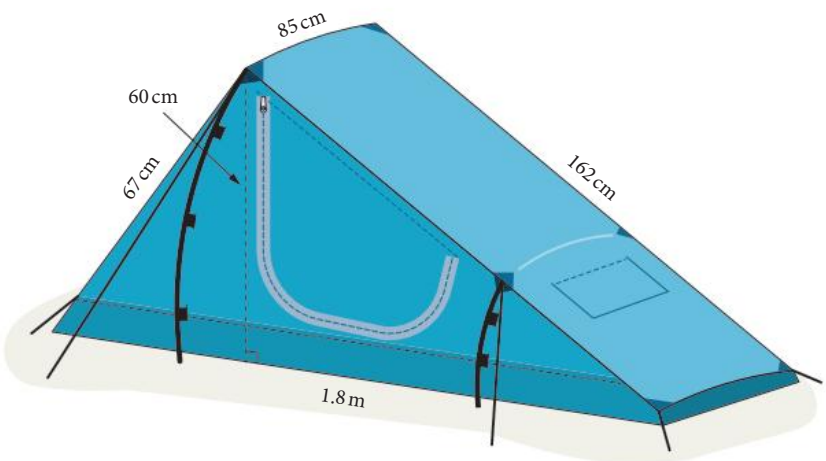
8 EXERCISE 14.3 Use the table titled 'The annual cost of a child under 12 years of age' on page 339 to calculate the annual cost for a 9-year-old child whose parents have a combined child support income of $155 600.

9 EXERCISE 14.4 James and Charlotte, who have recently separated, have a baby daughter, Rose. Charlotte is the primary carer of Rose, with James spending time with Rose on Saturday and Sunday afternoons. James's taxable income is $141 200 and Charlotte's is $42 000. Charlotte's income will change when the baby is 1 year old as she plans to return to work after her maternity leave. How much child support does James have to pay Charlotte for the first year of Rose's life?

Chapter problem

Rory is looking to buy a lightweight tent to take on overnight bush walks. The tent needs to weigh less than 1.5 kg without including the weight of any poles or ropes. He is considering the triangular tent (shown) made from a polyester fabric that weighs 450 g per square metre.

Rory fits comfortably inside the tent, but is it light enough?



Shutterstock.com/trek6500



CHAPTER 15

SPACE AND MEASUREMENT

SURFACE AREA AND VOLUME

15.1 Composite areas
15.2 Surface areas of prisms and cylinders
15.3 Surface areas of pyramids, cones and spheres
15.4 Volumes and capacities of prisms and cylinders
15.5 Volumes of pyramids, cones and spheres
Keyword activity
Solution to the chapter problem
Test yourself

What will we do in this chapter?

- Calculate the area of shapes including composite shapes
- Find the surface area of prisms, cylinders, pyramids, cones and spheres including composite solids
- Find the volume of prisms, cylinders, pyramids, cones and spheres including composite solids
- Find the capacity of solid objects

How are we ever going to use this?

- When we are covering furniture, doing craft or making clothes and need to work out how much material is required
- When calculating the quantity of materials for a job, for example, the amount of soil or mulch we need in the garden
- Many trades require knowledge of area, surface area and volume, including carpenters, plumbers and landscape gardeners

Study design coverage

AREA OF STUDY 4: SPACE AND MEASUREMENT

- conventions, properties and measurement of surface area and volume of compound shapes and objects.

VCE Mathematics Study Design 2023-2027 p. 75, © VCAA 2022

Videos (5):

15.2 Surface area of a prism
• Surface area of a cylinder

15.3 Surface area of a pyramid
• Surface area of a cone and sphere

15.4 Volumes of prisms and cylinders

Skillsheets (1):

15.4 What is volume?

Worksheets (8):

15.1 Composite areas

15.2 Nets of solids

15.4 A page of prisms and cylinders
• Volumes of solids • Volume and capacity

15.5 Measurement formulas chart
• A page of solid shapes
• Sweet areas and volumes

Puzzles (5):

15.1 Area code puzzle

15.2 Surface area

15.4 Formula matching game

15.5 Volume code puzzle • Officer Cubic

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

15.1 Composite areas

Worksheet Composite areas

Puzzle Area code puzzle

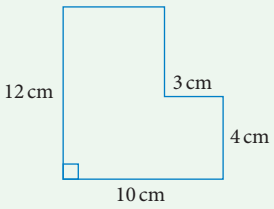
Composite shapes are made up of smaller, simpler shapes. We can find the **area** of a composite shape by adding or subtracting the areas of the simpler shapes.

The area formulas can be found in Chapter 5 on page 98.

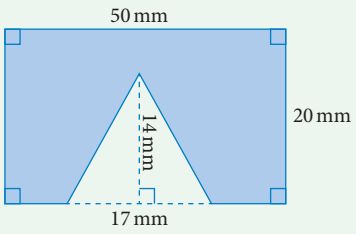
EXAMPLE 1

Find the area of each shape.

a



b



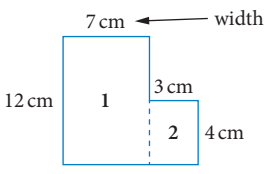
Steps

a Divide the shape into 2 rectangles, as shown, and determine unknown lengths.

There is more than one way to divide this shape. All methods result in the same answer.

Calculate the area of each rectangle and add them together.

Working



width of rectangle 1 = 10 − 3
= 7 cm

area of rectangle 1 = 7 × 12
= 84 cm^2

area of rectangle 2 = 3 × 4
= 12 cm^2

total area = 84 + 12
= 96 cm^2

b Find the area of the rectangle and subtract the area of the triangle.

Sometimes, it is easier to find the area by subtraction.

$$\text{area of rectangle} = 50 \times 20 = 1000\,\text{mm}^2$$

$$\text{area of triangle} = \frac{1}{2} \times 17 \times 14 = 119\,\text{mm}^2$$

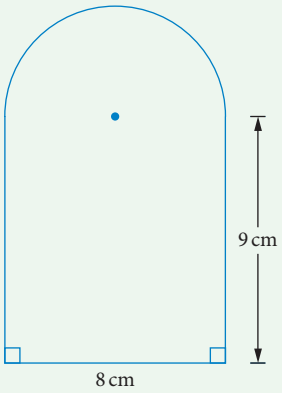
$$\text{total area} = 1000 - 119 = 881\,\text{mm}^2$$

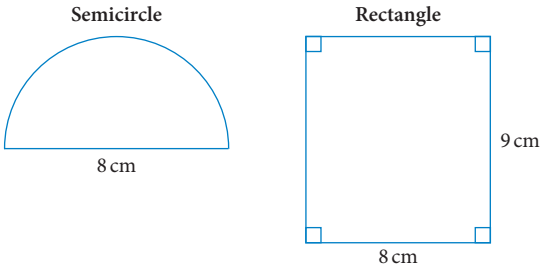


Shutterstock.com/Fotomay

EXAMPLE 2

Find the area of this shape, correct to 2 decimal places.



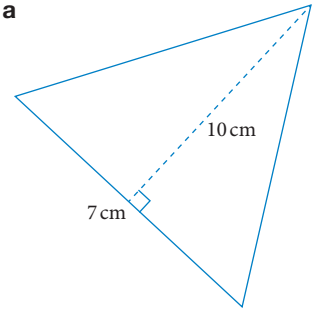
Steps	Working
Divide the shape into a semicircle and a rectangle.	
Find the area of the semicircle, determining the length of the radius first.	$\text{radius, } r = \frac{1}{2} \times 8 = 4$ $\text{area of semicircle} = \frac{1}{2} \times \pi r^2$ $= \frac{1}{2} \times \pi \times 4^2$ $= 25.1327\ldots$
Find the area of the rectangle.	$\text{area of rectangle} = lw$ $= 8 \times 9$ $= 72$
Add both areas together and round to 2 decimal places.	$\text{total area} = 25.1327\ldots + 72$ $= 97.1327\ldots$ $\approx 97.13\,\text{cm}^2$

EXERCISE 15.1 Composite areas

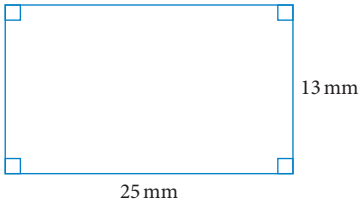
ANSWERS p. 443

1 Find the area of each shape, correct to 1 decimal place where necessary.

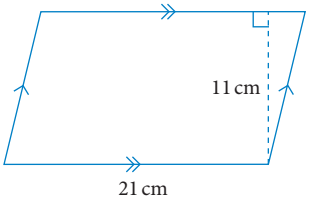
a



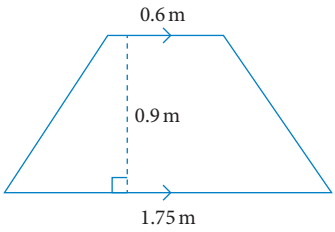
b



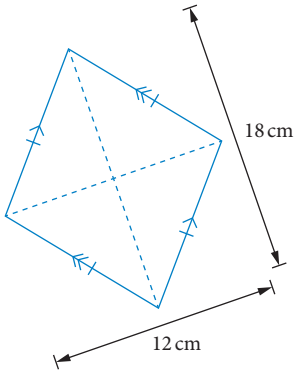
c



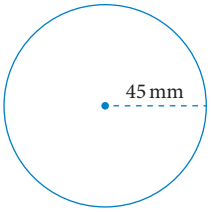
d



e

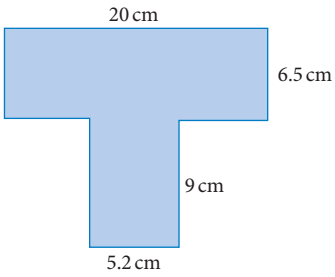


f

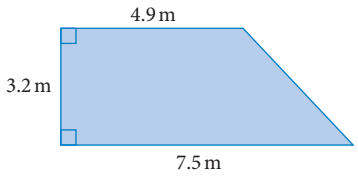


2 EXAMPLE 1 Find the area of each shape, correct to 2 decimal places where necessary.

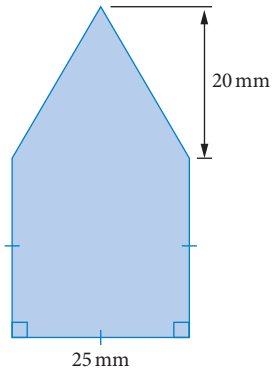
a



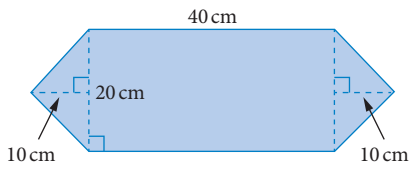
b



c



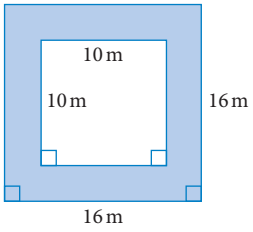
d



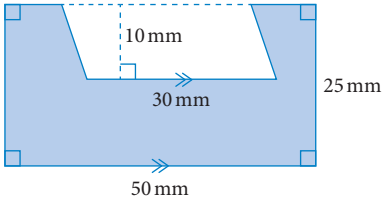
9780170453943

3 Calculate the area of each shaded region.

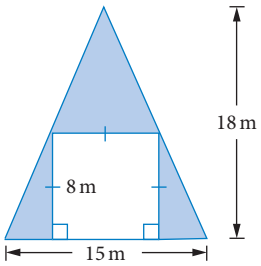
a



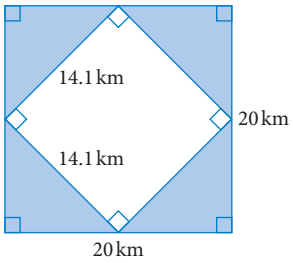
b



c

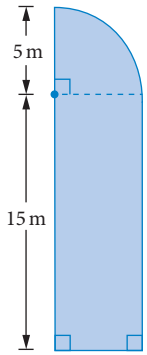


d

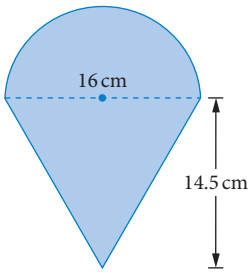


4 EXAMPLE 2 Calculate the area of each shaded region, correct to 2 decimal places.

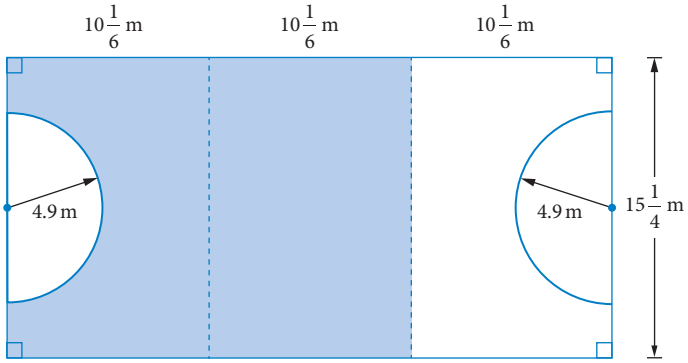
a



b



5 This diagram shows a netball court.



a Calculate, correct to 2 decimal places, the area of one of the goal semicircles.

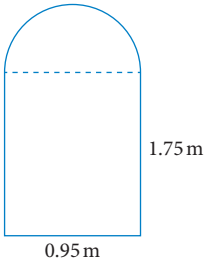
b Calculate, correct to 1 decimal place, the size of the shaded region where the 'wing attack' player can play.

6 Rebekah and Ewan are putting 3 arch windows in the front room of their house. The diagram shows one of the windows with its dimensions.

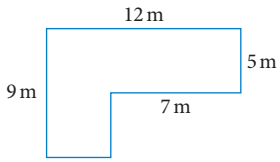
a Find the area of one arch window, correct to 2 decimal places.

b Find the area of all 3 arch windows.

c 10% of the area is NOT glass. Find the area of glass used.



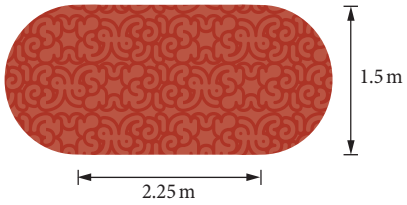
7 A children's playground has an L-shape as shown.



a What is the area of the playground?

b The playground is to be covered with soft-fall rubber for safety. How much will the soft-fall rubber cost if it is sold for $7.20 per square metre?

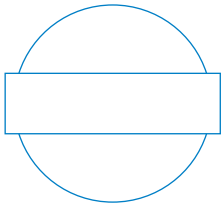
8 Angelina buys a new rug for her living room. The rug is composed of one rectangle and two semicircles as shown.



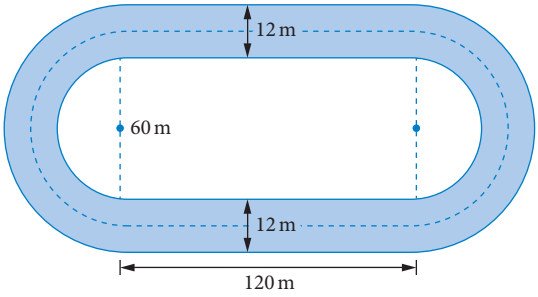
a Calculate the area of the rug, correct to 2 decimal places.

b Angelina's living room floor is a rectangle 4 m by 4.5 m. What area of the floor will **not** be covered by the rug?

9 This badge is made up of a metal disc overlaid with a rectangle. The metal disc has a radius of 3.5 cm. The rectangle is 1 cm longer than the diameter of the circle and 1.5 cm wide. Calculate the total area of the disc and the rectangle, correct to 2 decimal places.



10 Gotham City has a multi-purpose football field surrounded by an athletics track, as shown. Calculate the area of the athletics track, correct to 2 decimal places.



15.2 Surface areas of prisms and cylinders

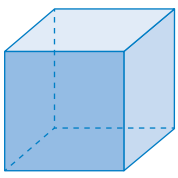
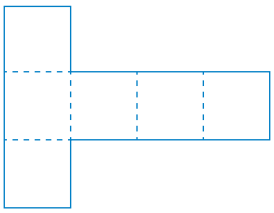
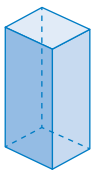
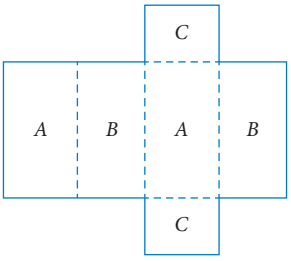
In Year 11, we found the **surface area** of cubes, rectangular **prisms**, triangular prisms and cylinders.

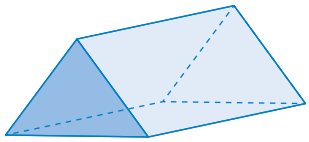
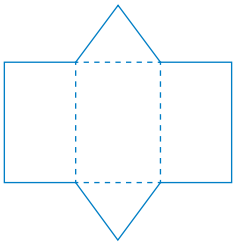
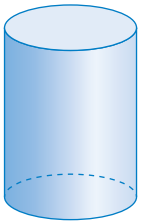
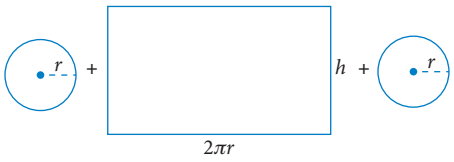
Video Surface area of a prism

Video Surface area of a cylinder

Worksheet Nets of solids

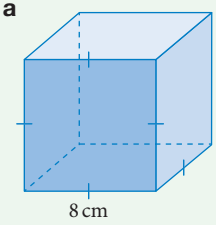
Puzzle Surface area

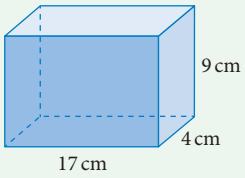
Solid	Net	Surface area
Cube 		The 6 faces of the cube are identical squares. SA = 6 × area of square SA = $6s^2$ (s = side length)
Rectangular prism 		The faces of a rectangular prism are matching pairs of identical rectangles. SA = 2 × top (C) + 2 × front (A) + 2 × side (B)

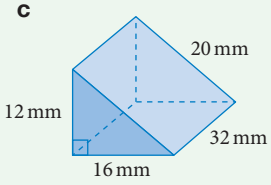
Solid	Net	Surface area
Triangular prism 		5 faces: 2 triangles and 3 rectangles. The ends of the triangular prism are identical triangles. SA = 2 × triangle + areas of 3 different rectangles
Cylinder 		SA = 2 × circles + rectangle SA = $2\pi r^2 + 2\pi rh$ (r = radius, h = height)

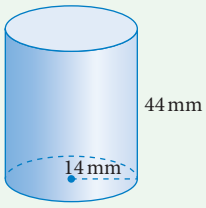
EXAMPLE 3

Find the surface area of each solid (correct to 2 decimal places for the cylinder).

a 

b 

c 

d 

Steps	Working
a This is a cube with 6 identical square faces.	surface area $= 6 \times 8^2$ $= 384\,\text{cm}^2$
b This is a rectangular prism with 3 matching pairs of faces.	surface area $= 2 \times (4 \times 17) + 2 \times (9 \times 17) + 2 \times (4 \times 9)$ $= 514\,\text{cm}^2$
c This is a triangular prism with identical triangles and 3 rectangles.	surface area $= 2 \times \left(\frac{1}{2} \times 16 \times 12\right) + 12 \times 32 + 16 \times 32 + 20 \times 32$ $= 1728\,\text{mm}^2$
d This is a cylinder.	surface area $= 2\pi r^2 + 2\pi rh$ $= 2 \times \pi \times 14^2 + 2 \times \pi \times 14 \times 44$ $= 5101.946\ldots$ $\approx 5101.95\,\text{mm}^2$

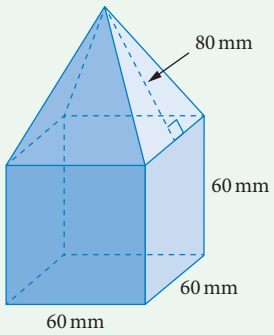
Surface area of a solid

To calculate the surface area of a solid:

- calculate the area of each face
- add the areas together to calculate the total surface area.

EXAMPLE 4

Calculate the surface area of this composite solid made from a square pyramid and a cube.



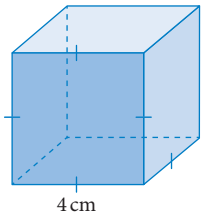
Steps	Working
This solid has 4 triangular faces and 5 square faces. Find the area of the 4 triangles.	area of triangular faces $= 4 \times \frac{1}{2} \times 60 \times 80$ $= 9600\ \text{mm}^2$
Find the area of the 5 squares.	area of square faces $= 5 \times 60 \times 60$ $= 18\,000\ \text{mm}^2$
Add the areas together.	total surface area $= 9600 + 18\,000$ $= 27\,600\ \text{mm}^2$

EXERCISE 15.2 Surface areas of prisms and cylinders

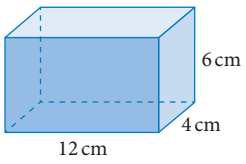
ANSWERS p. 444

1 EXAMPLE 3 Find the surface area of each solid (correct to 2 decimal places for the cylinder).

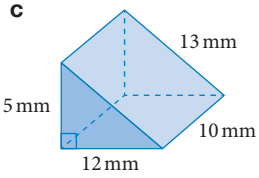
a



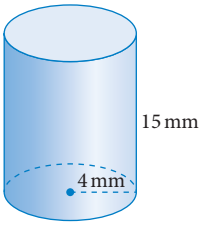
b



c



d



2 This metal water tank supplies water to steam trains. It has the shape of a square prism, where each side face is a rectangle 3.2 m wide by 1.4 m high. Find the surface area of metal in the water tank.

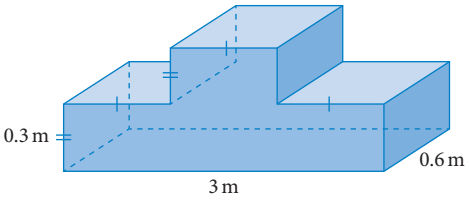


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3 Find the surface area of this can.
Answer correct to 1 decimal place.



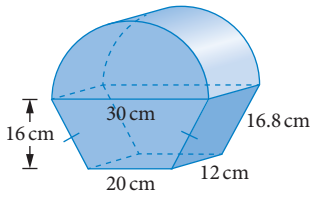
4 EXAMPLE 4 The local athletics club has a portable podium for medal presentations. Every surface, including the base, needs repainting. Calculate the surface area to be repainted.



5 Lisa is making a jewellery box, shaped as shown.
She is going to cover it with special material.

a Find the surface area of the jewellery box. Give your answer correct to the nearest square centimetre.

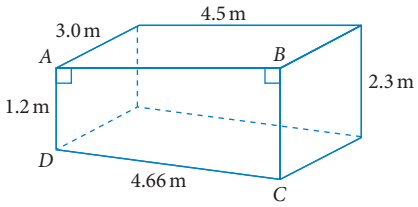
b Lisa allows an extra 10% of material for the edges and wastage. How much material should Lisa buy? Give your answer correct to the nearest square centimetre.



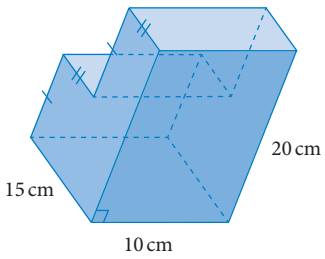
6 Bruno needs to resurface the interior of his pool.
(All measurements are in metres.)

a Calculate the area of the trapezium *ABCD*.

b Calculate the total surface area of the 5 faces that Bruno needs to resurface, correct to 2 decimal places.



7 Ginny has made a pair of wooden bookends in an L-shape, as shown. She is going to stain the bookends before she uses them. Find the total surface area of both bookends.



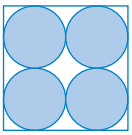
Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

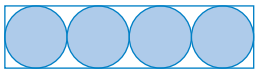
Practical activity Christmas bauble boxes

Irena has been asked to design a new cardboard box for a set of 4 spherical Christmas baubles with a diameter of 6.5 cm. She presents 2 designs:

An open square box



An open rectangular box



An open box has no top.

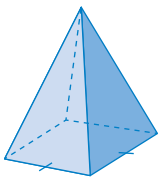
1. Calculate the area of cardboard required for the open square box. Allow for an extra centimetre on each dimension (length, width, height) to have space for interior packaging. Add 10% extra area for overlapping sections to glue the box together.
2. Calculate the area of cardboard required for the open rectangular box. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
3. Is there another possible design to hold the 4 baubles? If so, provide a sketch and calculate the area of cardboard required for it. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
4. Which design would you recommend? Justify your decision.



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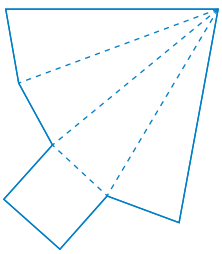
15.3 Surface areas of pyramids, cones and spheres

A **pyramid** has a triangle, **quadrilateral** or other **polygon** as its **base**, and triangular faces that meet at a point called the **apex**. We name pyramids by the shape of their base. For example, this is a square pyramid.

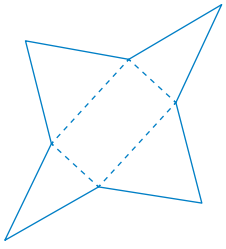


The **nets** of some pyramids are shown below. Knowing the net will help us find the surface areas.

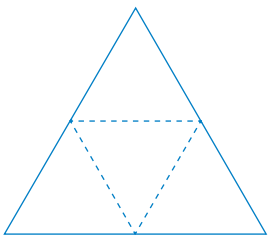
Square pyramid



Rectangular pyramid



Triangular pyramid

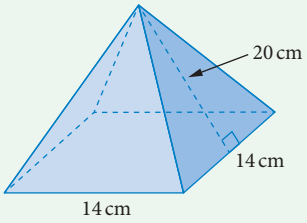


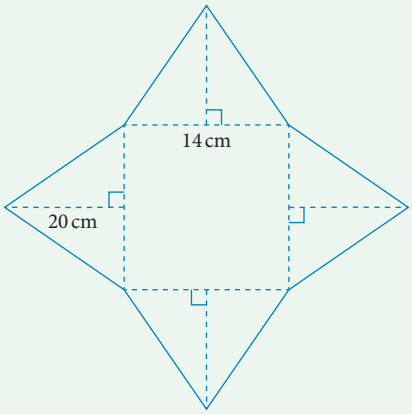
Surface area of a pyramid

Surface area = area of base + area of all triangular side faces

EXAMPLE 5

A square pyramid and its net are shown. Calculate the surface area of the pyramid.

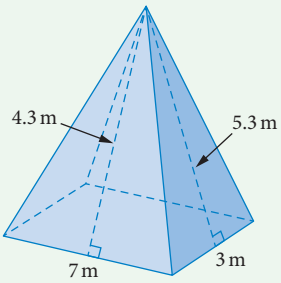




Steps	Working
Calculate the area of the square base.	area $= 14^2$ $= 196\,\text{cm}^2$
Calculate the area of the 4 triangles. They are all the same because the base is a square.	area $= 4 \times \left(\frac{1}{2} \times 14 \times 20\right)$ $= 560\,\text{cm}^2$
Add the areas together.	total surface area $= 196 + 560$ $= 756\,\text{cm}^2$

EXAMPLE 6

Find the surface area of this rectangular pyramid.



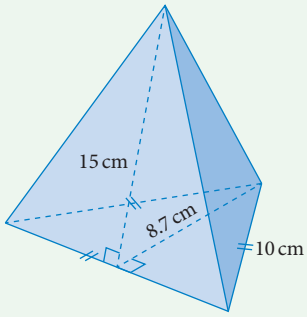
Steps	Working
Find the area of the rectangular base.	area $= 7 \times 3$ $= 21\,\text{m}^2$
Opposite triangles are identical. Find the area of the left/right triangles.	area $= 2 \times \left(\frac{1}{2} \times 3 \times 5.3\right)$ $= 15.9\,\text{m}^2$
Find the area of the front/back triangles.	area $= 2 \times \left(\frac{1}{2} \times 7 \times 4.3\right)$ $= 30.1\,\text{m}^2$
Add the areas together.	total surface area $= 15.9 + 30.1 + 21$ $= 67\,\text{m}^2$

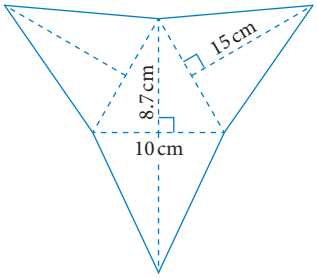
Video
Surface area of a pyramid

EXAMPLE 7

Find the surface area of this triangular pyramid.

Note that its base is an equilateral triangle.

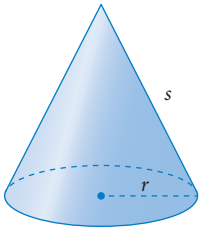
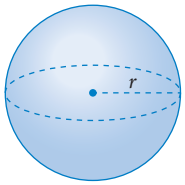


Steps	Working
The net of this pyramid is shown: the base is an equilateral triangle and the 3 side faces are identical triangles.	
Find the area of the 3 side faces.	$\text{area} = 3 \times \left(\frac{1}{2} \times 10 \times 15\right)$ $= 225\,\text{cm}^2$
Find the area of the base triangle.	$\text{area} = \frac{1}{2} \times 10 \times 8.7$ $= 43.5\,\text{cm}^2$
Add the areas together.	$\text{total surface area} = 225 + 43.5$ $= 268.5\,\text{cm}^2$



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There are special formulas for the surface areas of cones and spheres.

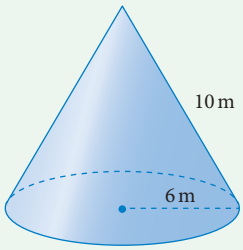
Shape	Surface area formula
Cone 	SA = area of circle + area of curved surface $SA = \pi r^2 + \pi rs$ where r = radius and s = slant height
Sphere 	$SA = 4 \times \pi \times (\text{radius})^2$ $SA = 4\pi r^2$ where r = radius

Video
Surface area of a cone and sphere

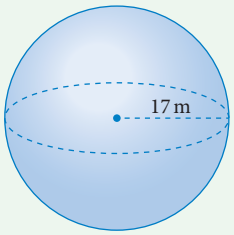
EXAMPLE 8

Find the surface area of each solid, correct to 2 decimal places.

a



b



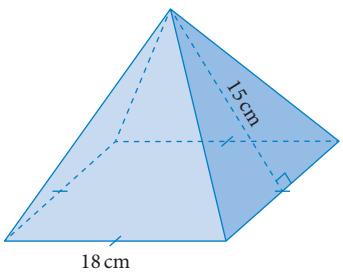
Steps	Working
a Use the formula for the surface area of a cone. $SA = \pi r^2 + \pi rs$ where $r = 6$ and $s = 10$.	$SA = \pi \times 6^2 + \pi \times 6 \times 10$ $= 301.5928\ldots$ $\approx 301.59\ \text{m}^2$
b Use the formula for the surface area of a sphere. $SA = 4\pi r^2$ where $r = 17$.	$SA = 4 \times \pi \times 17^2$ $= 3631.6811\ldots$ $\approx 3631.68\ \text{m}^2$

EXERCISE 15.3 Surface areas of pyramids, cones and spheres

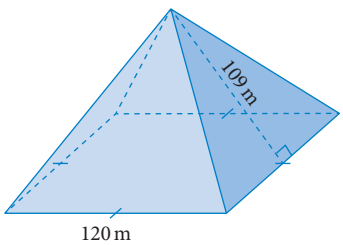
ANSWERS p. 444

1 EXAMPLE 5 Find the surface area of each square pyramid.

a

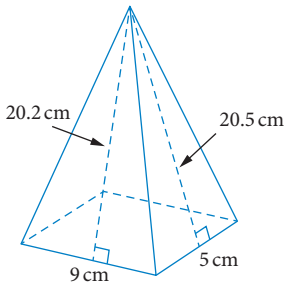


b

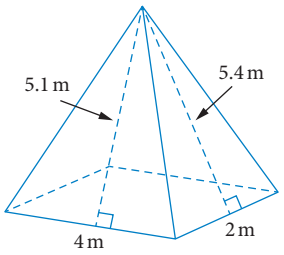


2 EXAMPLE 6 Calculate the surface area of each rectangular pyramid.

a

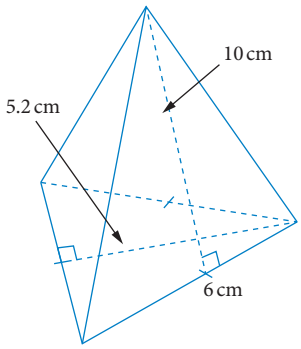


b

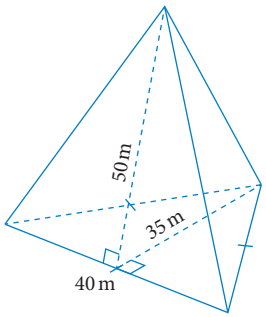


3 EXAMPLE 7 Calculate the surface area of each triangular pyramid. Each base is an equilateral triangle.

a

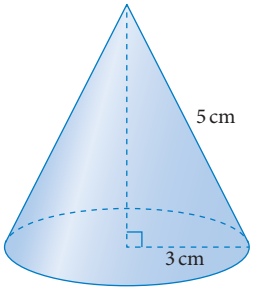


b

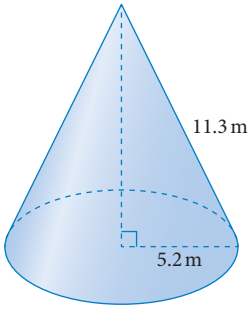


4 EXAMPLE 8 Calculate the surface area of each cone, correct to 1 decimal place.

a

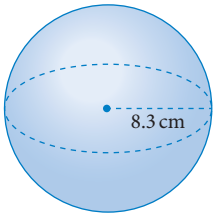


b

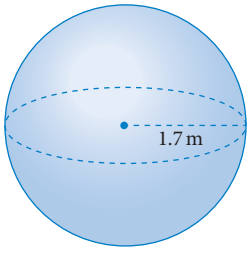


5 Find the surface area of each sphere, correct to 2 decimal places.

a



b

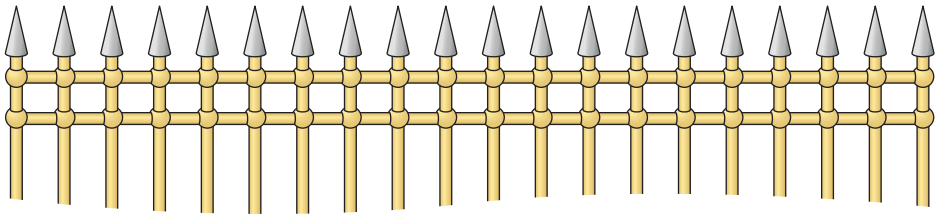


6 The Great Pyramid in Egypt is a square pyramid. The length of its base is 230 m and the slant height of the triangular faces is 300 m. Calculate the surface area of the Great Pyramid, including the base.

7 Edward wins the public speaking competition for his state. He receives a crystal glass trophy in the shape of a triangular pyramid. The base is an equilateral triangle with sides 4 cm and perpendicular height 3.5 cm. The slant height of the sides of the trophy is 22 cm. Calculate the surface area of the glass.

9780170453943

8 Dale is dipping cone-shaped spikes into a silver electroplating solution before he installs the spikes on top of a fence. The curved surface and the circular base of each cone will be electroplated. A total of 20 cone-shaped spikes are needed to complete the fence. The spikes have a slant height of 15 cm and a radius of 2 cm. Calculate, to the nearest square centimetre, the total surface area to be electroplated by Dale.



9 Natalie has a traffic cone with a radius of 12 cm and a slant height of 40 cm. She must dip the cone into orange paint to cover the sides but not the circular base. What area will Natalie cover with the paint? Give your answer to the nearest square centimetre.

10 A tennis ball has a diameter of 7 cm.

a Calculate the surface area of the tennis ball, correct to the nearest square centimetre.

b If the ball is made of two identical pieces, what is the area of each piece?

11 Earth is approximately a sphere with a radius of 6400 km.

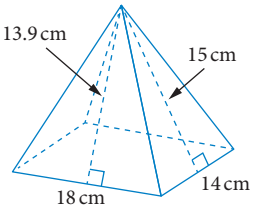
a Calculate the surface area of Earth, correct to the nearest 10 square kilometres.

b Approximately 70% of Earth's surface is water. What area of Earth's surface is covered with water?

c What area of Earth's surface is land?

12 Andrea is making a stained glass lampshade in the shape of a square pyramid. The bottom edges are 20 cm long and the slant height of the triangles is 24 cm. Find the total area of glass that Andrea will need to make the lampshade. Note: Remember a lampshade does not have a base!

13 Harry believes his seedlings grow best when they are planted in glass pyramids. He uses a glass cover in the shape of a rectangular pyramid, which fits over a glass base with dimensions of 18 cm by 14 cm, as shown. Calculate the total surface area of the pyramid and the glass base.

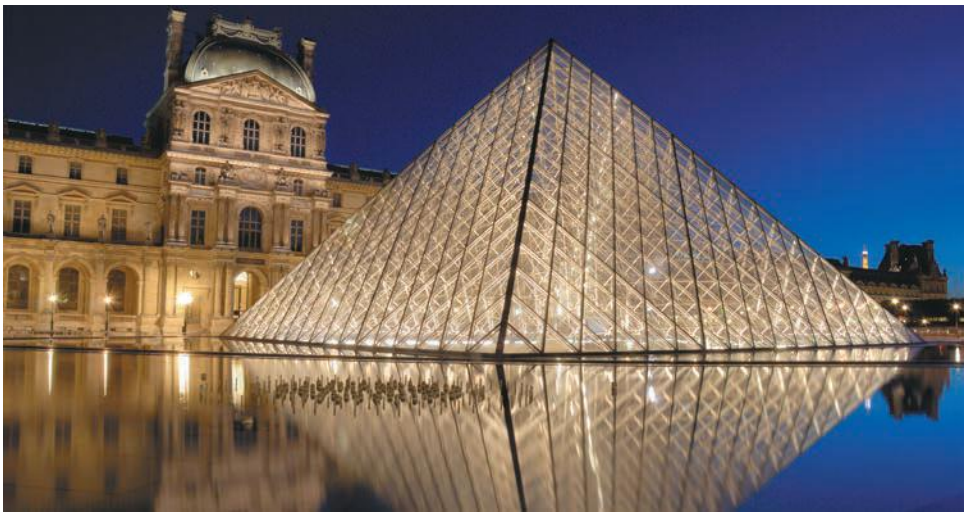


14 A mirror ball is 33 cm in diameter. It is covered with small mirror tiles which have an area of 1.44 cm^2 each.

a Find the surface area of the sphere, correct to 2 decimal places.

b How many tiles are used to cover the mirror ball?

15 The entrance to the Louvre Museum in Paris is a large square pyramid made of glass. The base of the pyramid is 35 m long. The slant height of the sides of the pyramid is 27.8 m.



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a Calculate the area of the 4 sides of the pyramid.

b The sides are made from approximately 640 glass pieces in the shape of a rhombus. What is the approximate area of one glass piece, to the nearest square metre?

16 This truck delivers LPG to homes for use in cooking and heating. The tank is comprised of a cylinder of length 3 m with a hemisphere of diameter 2 m at each end.

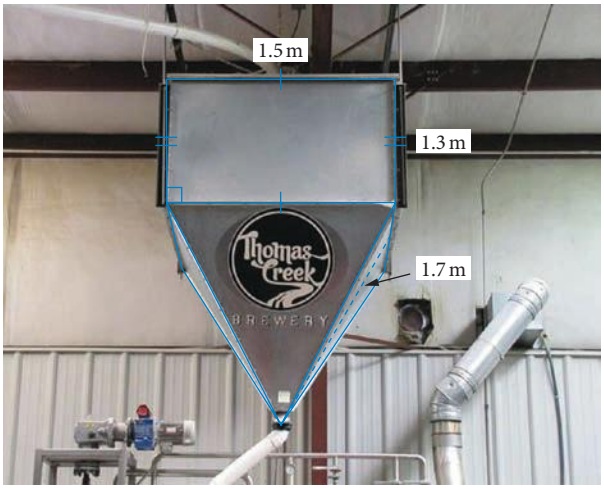


Alamy Stock Photo/Justin Kasezniinez

a Calculate the surface area of the LPG tank, correct to 1 decimal place.

b 1 L of paint covers 14 m^2 of the tank. How much paint will Bill need to cover the tank with 2 coats? Answer in litres, correct to 1 decimal place.

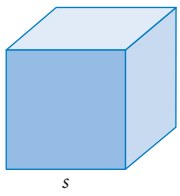
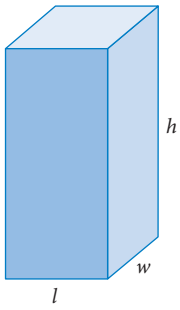
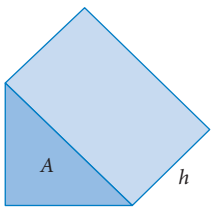
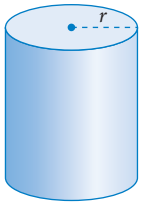
17 Jeff uses sheet metal to make this hopper to use in the beer making process. The hopper is in the shape of a square pyramid under a square prism, without a top. How much sheet metal, to the nearest square metre, did Jeff use to make the hopper?



GreerToday.com Photo

15.4 Volumes and capacities of prisms and cylinders

In Year 11, we found the **volumes** of cubes, rectangular prisms, triangular prisms and cylinders.

Solid	Diagram	Volume
Cube		$V = s \times s \times s$ $= s^3$
Rectangular prism		$V = l \times w \times h$ $= lwh$
Any prism		$V = A \times h$ $= Ah$ A is the area of the end or **cross-section**.
Cylinder		$V =$ area of circle $\times h$ $V = \pi r^2 h$

Video
Volumes of prisms and cylinders

Skillsheet
What is volume?

Worksheet
A page of prisms and cylinders

Worksheet
Volumes of solids

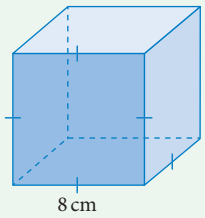
Worksheet
Volume and capacity

Puzzle
Formula matching game

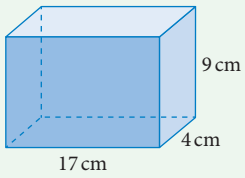
EXAMPLE 9

Find the volume of each solid (correct to 2 decimal places for the cylinder).

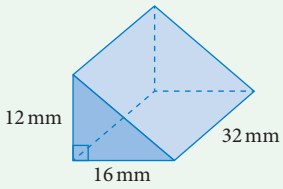
a



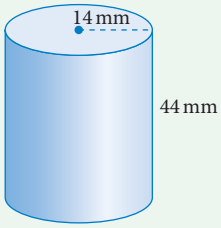
b



c



d



Steps	Working
a This is a cube with all lengths 8 cm.	volume $= 8^3$ $= 512\text{ cm}^3$
b This is a rectangular prism.	volume $= 4 \times 9 \times 17$ $= 612\text{ cm}^3$
c This is a triangular prism with the base of the triangle 16 mm and the height of the triangle 12 mm.	volume $= \frac{1}{2} \times 16 \times 12 \times 32$ $= 3072\text{ mm}^3$
d This is a cylinder.	volume $= \pi \times 14^2 \times 44$ $= 27\,093.0950\ldots$ $\approx 27\,093.10\text{ mm}^3$

Capacity is the amount of liquid or gas a container will hold and is measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

Volume and capacity

$1\text{ cm}^3 = 1\text{ mL}$

$1\text{ m}^3 = 1000\text{ L} = 1\text{ kL}$

EXAMPLE 10

A bucket in the shape of a rectangular prism has a base of 21 cm by 30 cm and is 22 cm high.

a Calculate the volume of the bucket in cubic centimetres (cm^3).

b How many whole litres of water can the bucket hold?

Steps	Working
a Volume of a rectangular prism = lwh	volume $= 21 \times 30 \times 22$ $= 13\,860\text{ cm}^3$
b First change the number of cm^3 to mL. Change the millilitres to litres and round **down** to the nearest litre.	$13\,860\text{ cm}^3 = 13\,860\text{ mL}$ $13\,860 \div 1000 = 13.86\text{ L}$ $\approx 13\text{ L}$

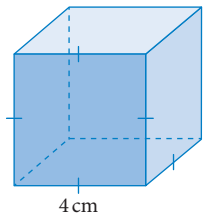
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EXERCISE 15.4 Volumes and capacities of prisms and cylinders

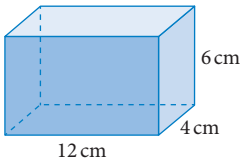
ANSWERS p. 444

1 EXAMPLE 9 Find the volume of each solid (correct to 2 decimal places for the cylinder).

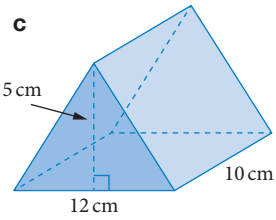
a



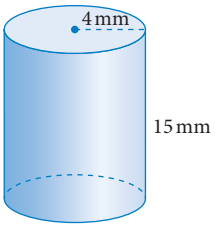
b



c



d



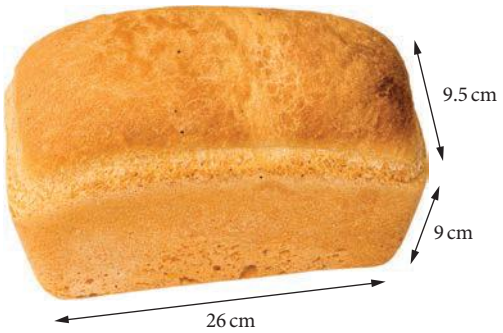
2 EXAMPLE 10 For each solid in question **1**, find its capacity in millilitres.

3 The end of this loaf of bread is a rectangle 9 cm by 9.5 cm.

a What is the area of the end of the loaf of bread?

b The loaf is 26 cm long. Calculate the volume of the loaf of bread.

c The loaf contains 22 slices of bread. Calculate the average volume of a slice of bread, correct to the nearest cubic centimetre.

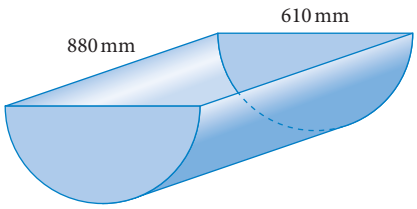


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4 Gianni makes soft drinks cans. He is experimenting with a new size of can. Which of these options provides the larger volume? Justify your answer with calculations.

- Can *A* has a radius of 3.95 cm and a height of 11.8 cm.
- Can *B* has a diameter of 6.1 cm and a height of 14 cm.

5 Silvana runs a farm. She uses half-cylinder water troughs as shown for the animals.



a Calculate the volume of one water trough in **cubic metres (m^3)**, correct to 2 decimal places. Hint: Convert measurements to metres first.

b How much water is needed to fill one water trough?

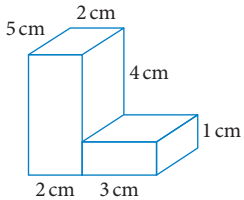
c There are 25 water troughs on the farm. How much water is needed to fill all the troughs?

d Silvana uses a cylindrical tank on a trailer to fill the water troughs. Calculate the volume of the tank, correct to 2 decimal places, if its diameter is 1560 mm and its height is 1210 mm.

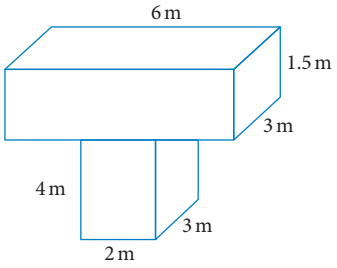
e Can this tank hold enough water to fill all the troughs? If not, how many trips will Silvana have to make?

6 Calculate the volume of each prism.

a



b



7 Find the capacity of each solid in question **6**. State your answer in the most appropriate unit.

Investigation Density

Could you lift a cubic metre of air?

Could you lift a cubic metre of feathers?

Could you lift a cubic metre of cement?

Each of the above has the same volume but not the same mass.

- Find the meaning of **density** with regard to solid objects. Write down the definition.
- What units are used to measure density?
- The following items have different densities. Find the density of each object (in kg/m^3) and place the objects in order from the lowest density to the highest density.

chalk	concrete	dry peat
fine sand	hard wood	sandstone
soft wood	topsoil	water



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15.5 Volumes of pyramids, cones and spheres

There are special formulas for calculating the volumes of pyramids, cones and spheres.

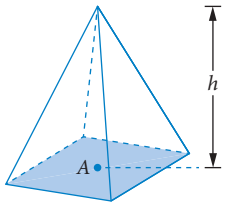
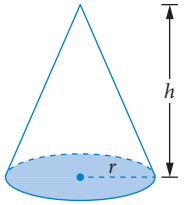
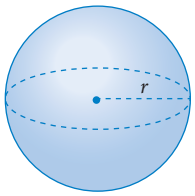
Worksheet Measurement formulas chart

Worksheet A page of solid shapes

Worksheet Sweet areas and volumes

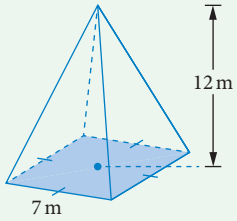
Puzzle Volume code puzzle

Puzzle Officer Cubic

Solid	Diagram	Volume
Pyramid		$V = \frac{1}{3} \times$ area of base $\times$ height $= \frac{1}{3}Ah$ A is the area of the base
Cone		$V = \frac{1}{3} \times$ area of circular base $\times$ height $V = \frac{1}{3}\pi r^2 h$
Sphere		$V = \frac{4}{3} \times \pi \times \text{radius}^3$ $V = \frac{4}{3}\pi r^3$

EXAMPLE 11

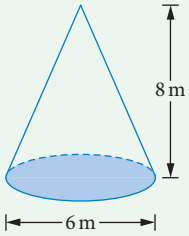
Calculate the volume of this pyramid.



Steps	Working
The base, A, is a square. $A = s^2$ where $s = 7$.	$A = 7^2$ $= 49\,\text{m}^2$
$V = \frac{1}{3}Ah$ where $h = 12$.	$V = \frac{1}{3} \times 49 \times 12$ $= 196\,\text{m}^3$

EXAMPLE 12

Find the volume of this cone, correct to the nearest cubic metre.

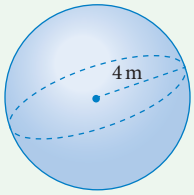


Steps	Working
Volume of a cone: $V = \frac{1}{3}\pi r^2 h$ where $r = \frac{1}{2} \times 6 = 3$ and $h = 8$.	$V = \frac{1}{3} \times \pi \times 3^2 \times 8$ $= 75.398\,22\ldots$ $\approx 75\,\text{m}^3$

Remember to halve the diameter to find the radius.

EXAMPLE 13

Find the volume of this sphere, correct to 2 decimal places.



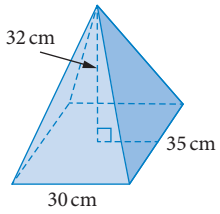
Steps	Working
Volume of a sphere: $V = \frac{4}{3}\pi r^3$ where $r = 4$.	$V = \frac{4}{3} \times \pi \times 4^3$ $= 268.0825\ldots$ $\approx 268.08\,\text{m}^3$

EXERCISE 15.5 Volumes of pyramids, cones and spheres

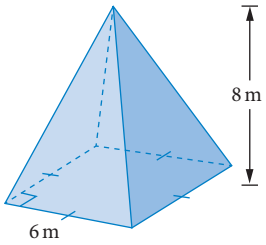
ANSWERS p. 444

1 EXAMPLE 11 Find the volume of each pyramid.

a

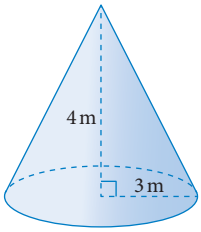


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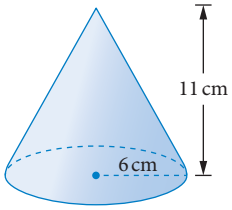


2 EXAMPLE 12 Find, correct to 1 decimal place, the volume of each cone.

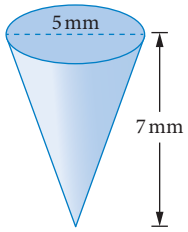
a



b

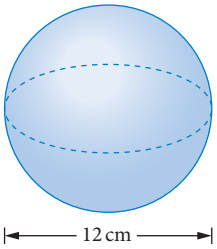


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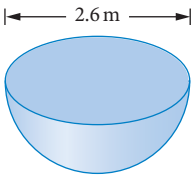


3 EXAMPLE 13 Find, correct to 1 decimal place, the volume of the sphere and hemisphere ('half sphere').

a



b



4 The Great Pyramid of Khufu, which was built around 2550 BCE, is in the shape of a square pyramid. The sides of the square are 230 m long, and the original height was 146 m. Calculate the original volume of the pyramid, correct to the nearest cubic metre.



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5 An ice-cream cone is full of ice-cream, and there is a hemisphere of ice-cream on top of the cone.



Shutterstock.com/Stockcreations

a What is the radius of the hemisphere?

b What is the radius of the top of the cone?

c Determine the amount of ice-cream that is contained in the cone and the hemisphere. Express your answer correct to the nearest millilitre (mL).

6 A large sphere with a radius of 5.5 m is used to store gas in an oil refinery.

a Calculate the volume of the oil refinery sphere in cubic metres. Give your answer correct to 3 decimal places.

b What is the volume in litres?

7 The bell tower of this church has the shape of a square pyramid on top of a square prism.



Alamy Stock Photo/Nick Hawkes

a Find the height of the square pyramid.

b Calculate the volume of the pyramid, correct to 1 decimal place.

c Calculate the volume of the bell tower, correct to 1 decimal place.

8 Traffic cones are often used by road workers to close off a section of the road. These cones are 66 cm tall and have a diameter of 31 cm.



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a What is the volume of one cone? Give your answer correct to the nearest cubic centimetre.

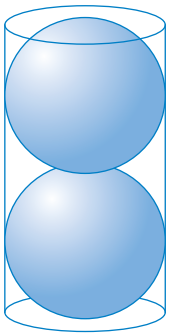
b If these cones were solid plastic, they would be very heavy. In fact, 85% of their volume is hollow space inside a thin plastic cone. What is the volume of the plastic used in making one of these cones?

9 The radius of each sphere in the diagram is 6 cm.

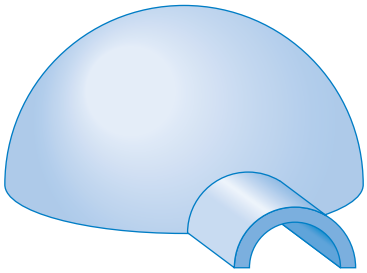
a Calculate the total volume of the 2 spheres.

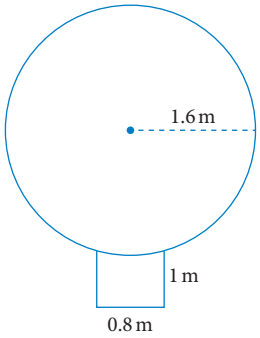
b What is the radius and height of the cylinder?

c Calculate the volume of the cylinder.



10 The native Canadian and Greenland Inuit people built houses made of snow called igloos as temporary shelters. The main section was a hemisphere, and the entrance was half a cylinder. The diagram shows the internal dimensions of an igloo. How much air was inside an empty igloo, correct to 1 decimal place?





Investigation Finding pyramid buildings

Search the internet for buildings or structures that are pyramids.

1 Write down the locations of at least 10 pyramid buildings/structures. Include pictures where possible.

2 Find the dimensions of up to 5 of the buildings you have found.

3 For each building, find:

a its surface area

b its volume.

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Keyword activity

CHAPTER AND FORMULA SUMMARY

ANSWERS p. 444

Choose from the word list and formula list to copy and complete the summary of this chapter given below.

Word list

capacity	composite	cones	cubic	cylinders	net
prisms	pyramids	square	spheres	surface area	volume

Formula list

$SA = 4\pi r^2$ $SA = \pi r^2 + \pi rs$ $SA = 2\pi r^2 + 2\pi rh$

SA = area of base + area of all triangular side faces

$V = \pi r^2 h$ $V = \frac{1}{3}\pi r^2 h$ $V = \frac{4}{3}\pi r^3$ $V = \frac{1}{3}Ah$

In this chapter, we reviewed finding area and calculating the surface area, volume and capacity of solid objects.

We found the areas of both simple and [1]_____ shapes, which are shapes made up of simpler shapes. To find the areas of composite shapes, we either add or subtract the areas of simple shapes. Area is measured in [2]_____ units.

The [3]_____ _____ of a solid object is the sum of the areas of each surface of the solid. Drawing the [4]_____ of the solid helps us determine the geometrical shapes involved in the calculation of the surface area. Combining any shapes that are the same makes the calculations simpler.

The [5]_____ of a solid object is the space it takes up. Volume is measured in [6]_____ units.

[7]_____ is the amount of liquid or gas that an object contains. We can convert from cubic centimetres (cm^3) to millilitres (mL) or from cubic metres (m^3) to litres (L).

The solid objects we used were [8]_____, [9]_____ and solids with curved surfaces. These included rectangular prisms, triangular prisms, cubes, [10]_____, pyramids with different bases, [11]_____ and [12]_____.

These are some formulas we use for calculating surface area and volume.

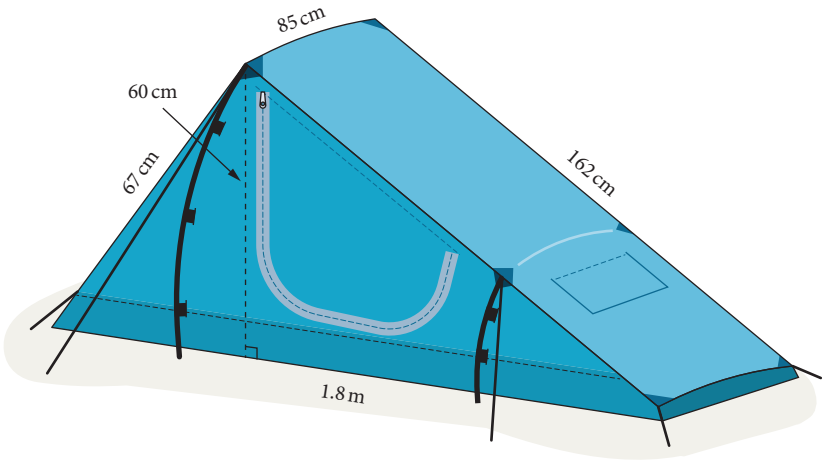
	Formula
Surface area of a cylinder	[13]
Surface area of a cone	[14]
Surface area of a sphere	[15]
Surface area of a pyramid	[16]
Volume of a cylinder	[17]
Volume of a pyramid	[18]
Volume of a cone	[19]
Volume of a sphere	[20]

Solution to the chapter problem

Problem

Rory is looking to buy a lightweight tent to take on overnight bush walks. The tent needs to weigh less than 1.5 kg without including the weight of any poles or ropes. He is considering the triangular tent (shown) made from a polyester fabric that weighs 450 g per square metre.

Rory fits comfortably inside the tent, but is it light enough?



Solution

Express all measurements in metres and calculate the tent's surface area.

Piece		Area in m^2
Floor	$A = 1.8 \times 0.85$	1.53
Triangular front and back	$A = 2 \times \left(\frac{1}{2} \times 1.8 \times 0.6\right)$	1.08
Short roof piece	$A = 0.67 \times 0.85$	0.5695
Long roof piece	$A = 1.62 \times 0.85$	1.377
	Total	4.5565

The tent uses approximately 4.6 m^2 of fabric.

weight of the fabric = 4.6 × 450 g
= 2070 g
= 2.07 kg

Based on these calculations, the tent is NOT light enough for Rory. It weighs 0.57 kg more than he would like.

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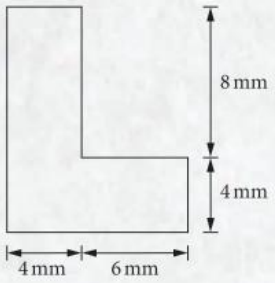
15 Test yourself

Surface area and volume

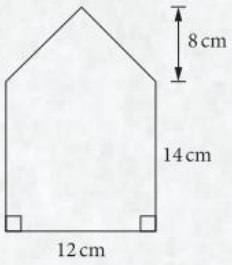
ANSWERS p. 444

1 EXERCISE 15.1 Calculate the area of each composite shape, correct to 1 decimal place where appropriate.

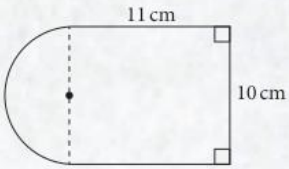
a



b

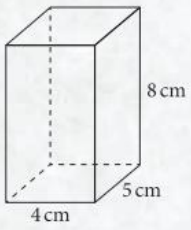


c

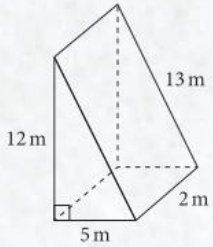


2 EXERCISE 15.2 Calculate the surface area of each solid.

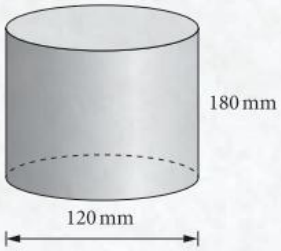
a



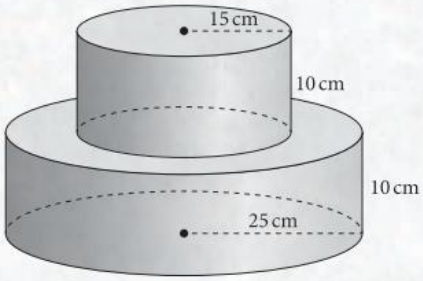
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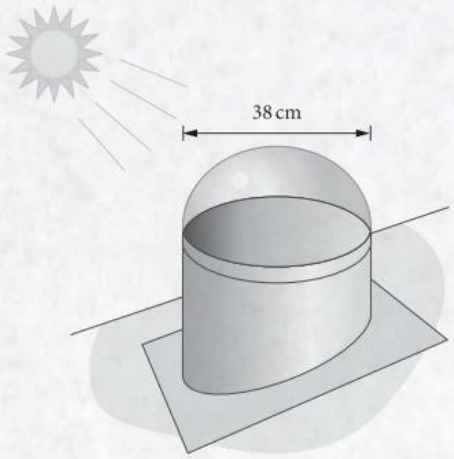
3 EXERCISE 15.2 Calculate the curved surface area of this cylinder, correct to 1 decimal place.



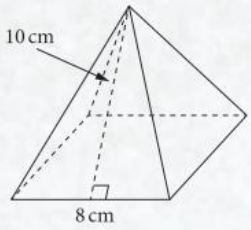
4 EXERCISE 15.2 Mia is making an 18th birthday cake. It is made up of 2 cylinders placed on top of one another as shown. Mia will ice the top and sides of the top cake and the ring and sides of the bottom cake. Calculate the area to be iced. Answer correct to the nearest cubic centimetre.



5 EXERCISE 15.3 The diameter of a glass hemisphere which lets light into Logan's bathroom is 38 cm. What is the external surface area of this glass dome, correct to 1 decimal place?



6 EXERCISE 15.3 Grace has a small, wooden, square pyramid that she plans to paint. The length of the base is 8 cm and the slant height of each of the triangular faces is 10 cm. Calculate the area Grace needs to paint.



7 EXERCISE 15.3 An open cone for holding confetti has a radius of 4.5 cm and a slant height of 18 cm. Calculate the amount of cardboard needed to make one cone. Round your answer up to the nearest cm^2.

8 EXERCISE 15.4 Michael has a small aquarium for tropical fish. It is a rectangular prism with length 40 cm, width 18 cm and height 24 cm.

a What is the volume of Michael's fish tank in cm^3?

b What is the total capacity of the fish tank in litres?

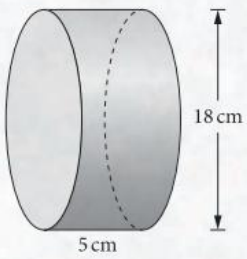
c This fish tank is only filled to 5 cm below the top. What is the volume of water in the tank in litres?

9 EXERCISE 15.4 Inga has bought a set of 9 storage cubes to hold books. Each cube has a side length of 35 cm.

a What is the volume of one storage cube?

b What is the total volume of storage space in the whole set of cubes?

10 EXERCISE 15.4 Calculate the volume of this cylinder, correct to the nearest mm^3.

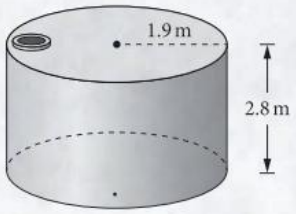


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11 EXERCISE 15.4

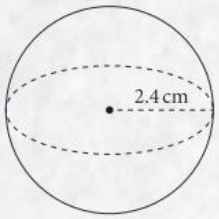
a Calculate the volume of the water tank, correct to the nearest m^3.

b What is the capacity of the tank in litres?

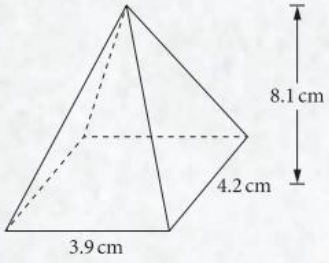


12 EXERCISE 15.5 Calculate the volume of each solid, correct to the nearest cm^3.

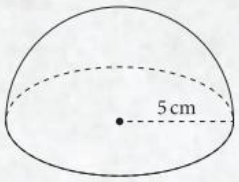
a



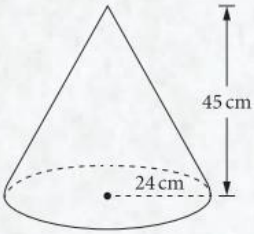
b



c



d



13 EXERCISE 15.5 Lara is making chocolate truffles in the shape of spheres. Each of the truffles has a diameter of 3 cm.

a Calculate the volume of each truffle, correct to 1 decimal place.

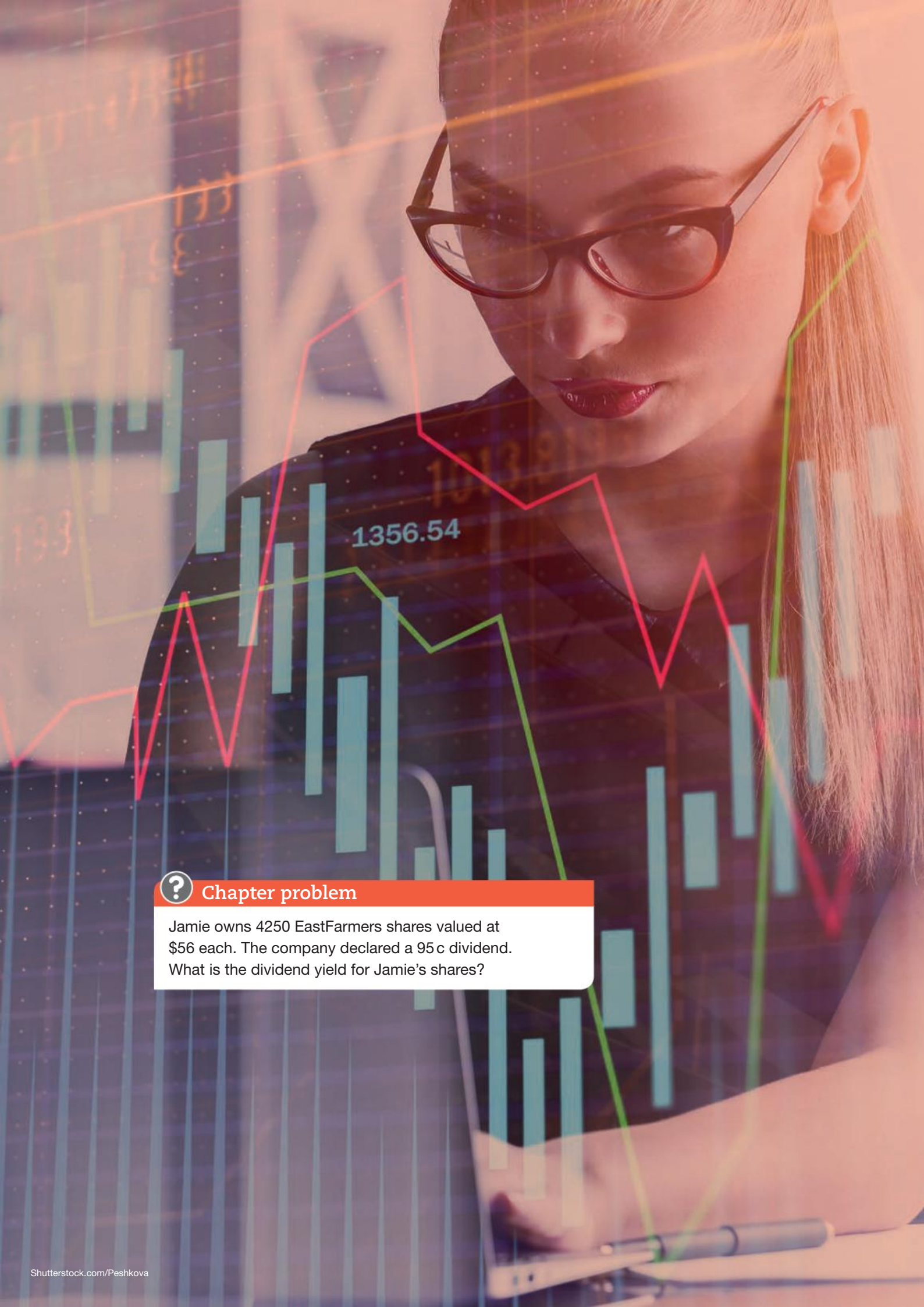
b How many truffles can she make from 1 L of the truffle mixture?

14 EXERCISE 15.5 Anh is producing decorative metal cones for gardens. Each cone is 66 cm high and has a base radius of 8 cm. Neil orders 25 cones for his sculpture garden. Find the volume of the 25 cones, correct to the nearest cm^3.

15 EXERCISE 15.5 The Palace of Peace and Reconciliation in Nur-Sultan, Kazakhstan, is in the shape of a square pyramid. Its base is a square with sides 62 m and the height of the pyramid is also 62 m. Calculate the volume of this pyramid, correct to the nearest m^3.



istock.com/Diego Fiore



Chapter problem

Jamie owns 4250 EastFarmers shares valued at $56 each. The company declared a 95 c dividend. What is the dividend yield for Jamie's shares?

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CHAPTER 16

DATA ANALYSIS, PROBABILITY AND STATISTICS; DISCRETE MATHEMATICS: FINANCIAL AND CONSUMER MATHEMATICS

AUSTRALIA AND THE WORLD

16.1 The COVID-19 pandemic
16.2 The pandemic and our economy
16.3 Supply, demand and prices
16.4 Exchange rates
16.5 The share market
16.6 Inflation and recession
16.7 The gender pay gap
Extended investigation: My investment portfolio 2
Solution to the chapter problem
Keyword activity
Test yourself

What will we do in this chapter?

- Consider how things that happen overseas affect life in Australia
- Create graphs from tables of values to understand pandemic numbers
- Understand the difference between an exponential and logistic curve
- Learn how an economy works
- Use exchange rates to calculate overseas prices in Australian dollars
- Make calculations involving shares
- Interpret data about gender pay

How are we ever going to use this?

- To understand how repayments, such as a mortgage, are influenced by interest rates change
- When we want to invest in or buy and sell shares
- When we buy (import) or sell (export) items overseas

STUDY DESIGN COVERAGE

AREA OF STUDY 2: DATA ANALYSIS, PROBABILITY AND STATISTICS

- long-term data and relative frequencies in practical situation such as in relation to epidemics, climate, environment, sport and marketing.

AREA OF STUDY 3: DISCRETE MATHEMATICS

Financial and consumer mathematics

- informal consideration of financial risk at the national and global level (short, medium and long term)
- analysis and interpretation of financial information and data sets, trends and economic indicators and their impact (at the personal, community, national or global level) such as gender pay gap, career trends and interruption, currency fluctuations and inflation, stock market movements and recessions.

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Worksheets (1):
16.7 Investigation components

Puzzles (1):
Keyword activity Australia and the world crossword

Weblinks (2):
16.3 Easy calculation
16.6 MoneySmart

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

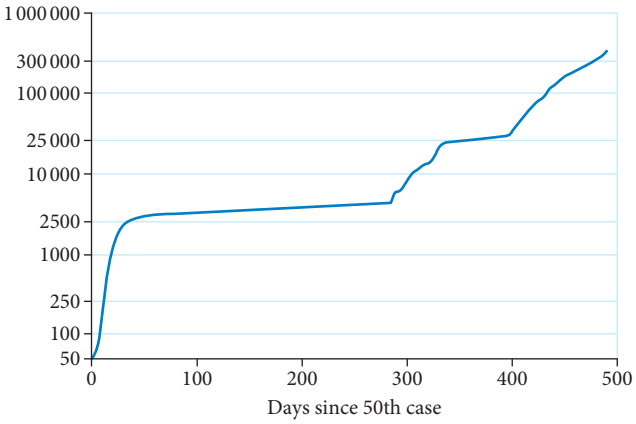
16.1 The COVID-19 pandemic

The COVID-19 **pandemic** that spread across the world rapidly in 2020–21 made us all realise that what happens overseas affects every Australian in some way. Lockdowns, job losses, reduced income, face masks and even remote school learning all became features of a new way of life. While Australia's geographic isolation helped protect us from the worst of the pandemic, what happened overseas affected us all.

The spread of the virus is an example of **exponential** growth, where the rate of increase in cases is determined by the number of existing cases. The **doubling time**, or the time it takes for the number of cases to double, is one way to measure how fast the virus is spreading through communities.

This graph shows the number of new daily COVID-19 cases in Thailand for 500 days. After 15 July 2021 (day 300), the number of known cases is doubling every 4 weeks.

COVID-19 cases in Thailand 2020–21: the first 500 days



Note that the scale on this vertical axis is uneven in order to show the dramatic growth in the number of COVID-19 cases.

EXAMPLE 1

There were 9300 new cases of COVID-19 in Thailand on 15 July. The new case numbers are doubling every 4 weeks.

a Construct a table of values showing the expected number of new daily COVID-19 cases in Thailand every 4 weeks following 15 July, until 2 December.

b Use a graph to display the predicted new daily COVID-19 cases in Thailand from 15 July to 2 December.

c Comment on the likelihood that the numbers in the table will be correct.

Steps

a The new cases are doubling every 4 weeks. The key dates are every 4 weeks after 15 July; that is 12 August, 9 September, 7 October, 4 November and 2 December.

Working

15 July: 9300 new cases

12 August: $9300 \times 2 = 18\,600$

9 September: $18\,600 \times 2 = 37\,200$

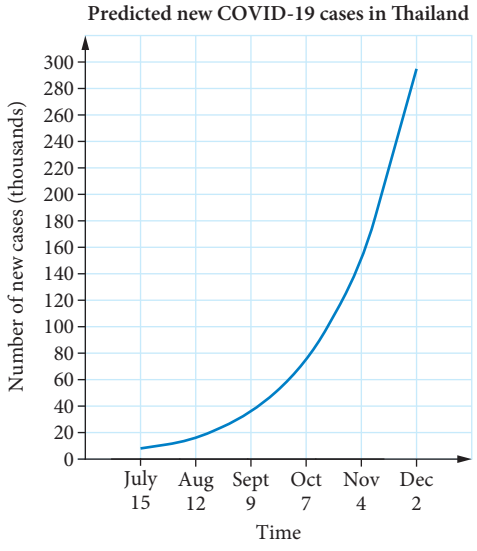
7 October: $37\,200 \times 2 = 74\,400$

4 November: $74\,400 \times 2 = 148\,800$

2 December: $148\,800 \times 2 = 297\,600$

Date	Predicted number of new COVID-19 cases
15 July	9300
12 August	18 600
9 September	37 200
7 October	74 400
4 November	148 800
2 December	297 600

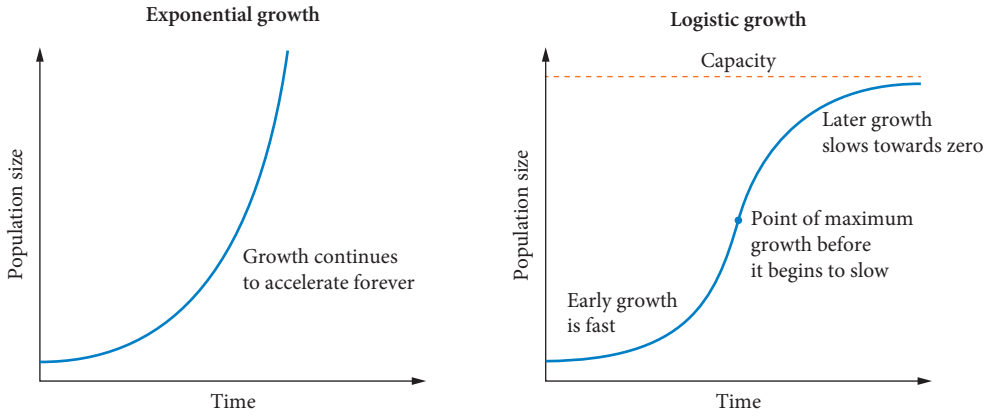
b Construct a graph with dates on the horizontal axis and case numbers from 0 to 300 000 on the vertical axis.



c In the short term, the numbers are likely to be a good prediction unless there is a significant change in the doubling time resulting from lockdowns or vaccinations. In the long term, the cases cannot continue to double indefinitely. If it did, there would be nobody left in the population to be infected with the virus.

Exponential and logistic growth

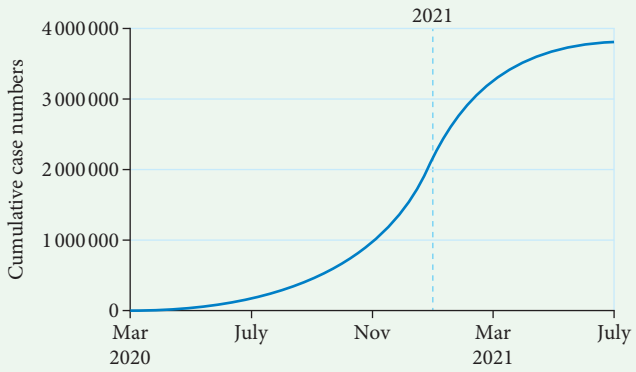
In the real world, the number of living things can't keep increasing exponentially forever because they run out of food and space. Many living things increase exponentially in the beginning before their growth reaches a maximum rate, then tapers off. In this situation, the growth follows a **logistic curve**.



- Exponential graphs are in the shape of the letter J.
- Logistic curves are in the shape of the letter S.

EXAMPLE 2

The graph shows the cumulative number of COVID-19 cases in California from March 2020 to July 2021.



a For what period of time was the cumulative number of COVID-19 cases in California following an exponential curve?

b What type of curve was the graph following after this time?

Steps	Working
a Look for the time where the curve is starting and in a J shape. This happens until the start of 2021.	The curve is exponential from March 2020 to January 2021.
b The curve isn't increasing quickly in a J shape. It is following an S shape.	The curve is a logistic curve from January 2021.

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EXERCISE 16.1 The COVID-19 pandemic

ANSWERS p. 444

1 EXAMPLE 1 On 14 July, there were 585 new cases of COVID-19 in Fiji, with new daily case numbers doubling every 2 weeks.

a Copy and complete this table of values showing the predicted number of new daily COVID-19 cases in Fiji from 14 July to 8 September.

Date	14 July	28 July	11 August	25 August	8 September
Predicted new number of cases					

b Construct a graph to show the predicted number of new daily COVID-19 cases in Fiji from 14 July to 8 September.

c The population of Fiji is 900 000. What percentage of the population is predicted to have a new case of COVID-19 on 8 September?

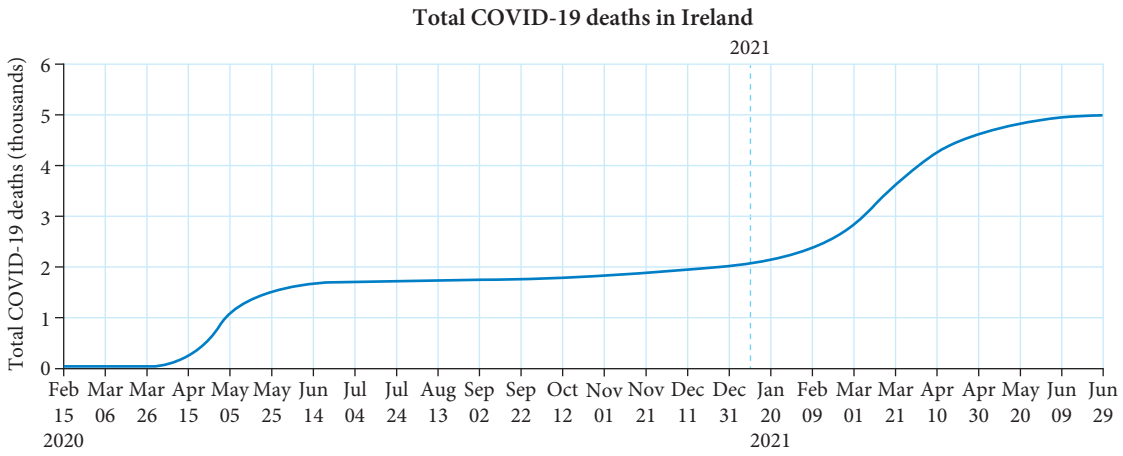
2 EXAMPLE 2 On 11 March 2020, Ireland had 43 new cases of COVID-19 and the number of cases was doubling every 8 days.

a Construct a table of values to show the predicted number of new daily COVID-19 cases in Ireland from 11 March to the end of April 2020.

b Construct a graph of the number of new cases per day from 11 March 2021 to the end of April 2021.

c Is the graph exponential or logistic? Justify your answer.

3 This graph shows the total number of deaths from COVID-19 in Ireland from February 2020 to June 2021.



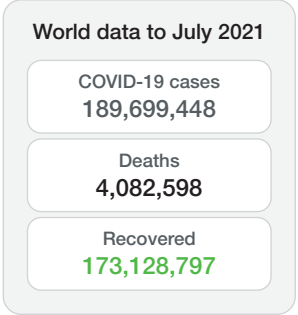
a Approximately how many people died of COVID-19 in Ireland by the end of March 2021?

b By what approximate date had 1000 people in Ireland died from COVID-19?

c There are 2 time spans on the curve when the curve was following an exponential path. Approximately when were they?

4 **a** What percentage of worldwide cases of COVID-19 recovered, as of July 2021? Answer correct to 1 decimal place.

b What percentage of people worldwide who contracted COVID-19 died by July 2021? Express your answer correct to 2 decimal places.

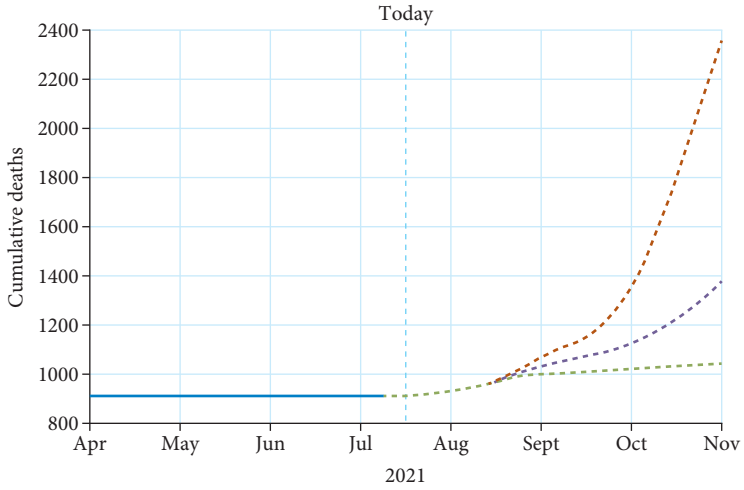


5 In July 2021, the number of COVID-19 cases in Australia increased and both Melbourne and Sydney were locked down at the same time. This graph published at the end of July shows the predicted cumulative number of deaths in 3 different scenarios.

Green – everyone wearing masks

Purple – less than 95% of the population wearing masks

Brown – people who are vaccinated begin moving around at pre-pandemic rates



Use the graph and its predictions to answer the following questions.

a Approximately how many Australian deaths occurred prior to July 2021?

b Approximately how many deaths were predicted from July to September?

c Describe what was predicted to happen if people who have been vaccinated started to move around at pre-pandemic rates.

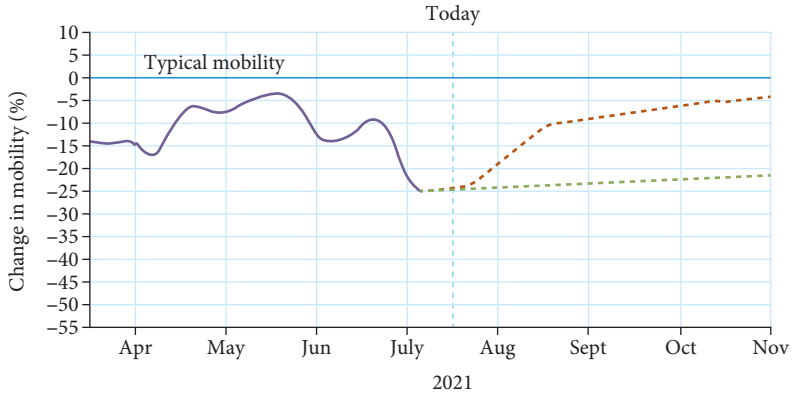
6 How much the Australian population is moving about can be tracked by the movement of their mobile phones. In this graph from July 2021, the **purple** line shows how much people are moving around compared to pre-pandemic times.

The **brown** line was the predicted amount the population will be able to move around as the number of vaccinated people increases.

The **green** line was the predicted amount the population will be able to move around if masks are the only protection against the virus.



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a In April, how much less than the typical amount were Australians moving about?

b Describe Australian mobility in July.

c What were the experts predicting would happen to our mobility as more Australians became vaccinated?

7 One way to compare the effect of COVID-19 on different communities is to look at case and death rates compared to the total population. This table shows case and death information for several countries to July 2021.

Country	Total cases	Total deaths	Population in millions	Cases per million of population	Deaths per million of population
USA	34.9 million	624 189	333	34 900 000 ÷ 333 = 104 804	624 189 ÷ 333 = 1874
India	31.0 million	412 563	1394		
Brazil	19.2 million	539 050	214		
UK	5.2 million	128 593	68		
Australia	31 513	912	26		
New Zealand	2794	26	5		

a Copy the table and complete the last 2 columns. The first row has been completed as an example. Round all values down to the nearest whole number.

b Which one of the countries in the table had a large number of cases but a relatively small number of deaths per million of population?

c Which 2 countries had the smallest number of deaths per million of population? Suggest 3 features that these two countries have in common that may have contributed to their relatively low death rates.

d Which country in the table had the highest number of COVID-19 deaths?

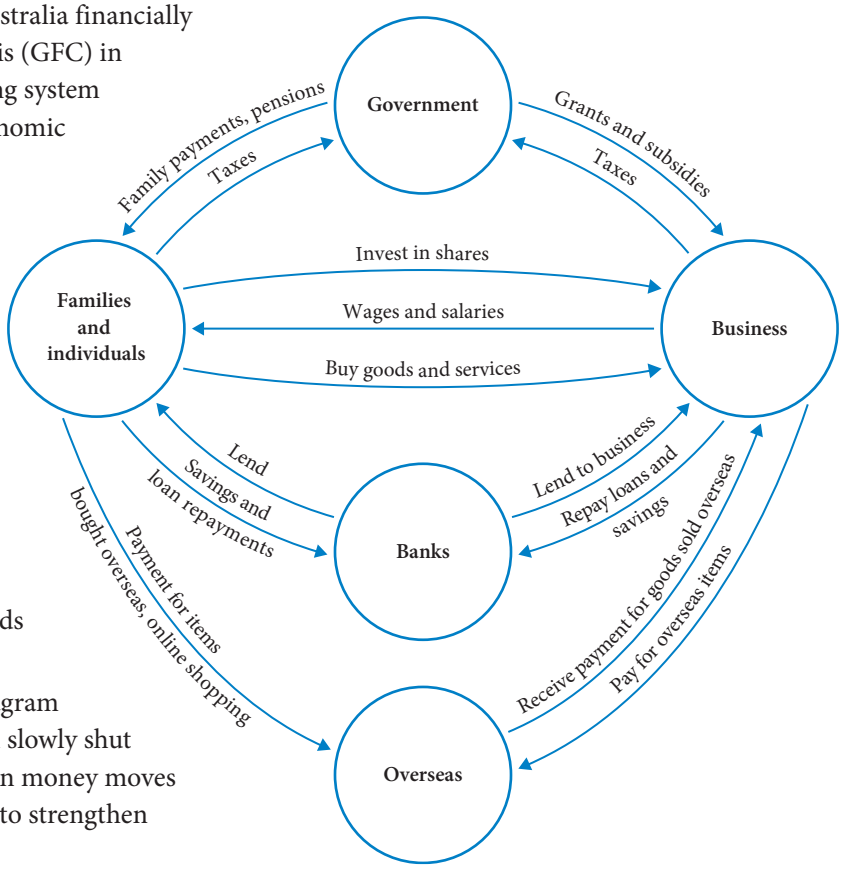
e Which country of those in the table had the worst death rate per million of population?

16.2 The pandemic and our economy

What happens overseas affects Australia financially as well. The Global Financial Crisis (GFC) in 2007 that started in the US banking system plunged the whole world into economic stress. Through appropriate and timely economic stimulus packages, Australia was the only developed country to survive the GFC without losing a single bank, although many citizens lost money following the stock market fall.

The key to a good economy is having money continually circling through it. This diagram shows a simplified version of how an economy works. The arrowheads show the direction money flows.

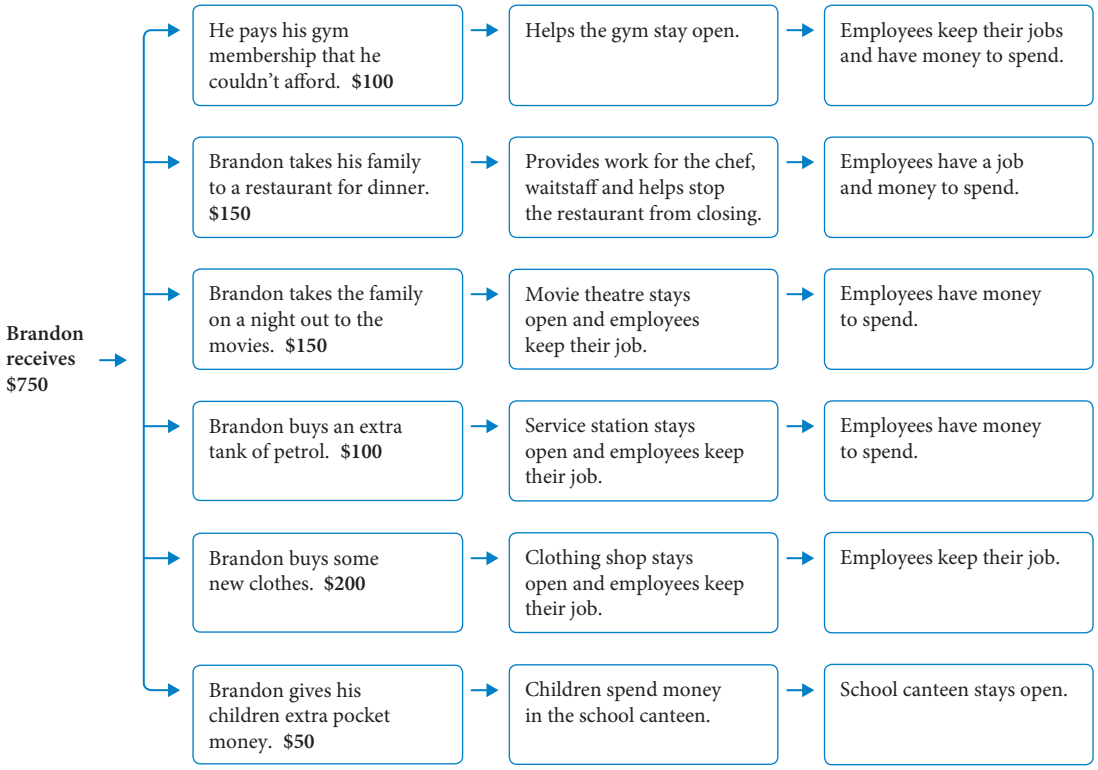
When any of the arrows in the diagram are not working, the economy can slowly shut down and create a recession. When money moves slowly, the **Reserve Bank** steps in to strengthen the links.



A **stimulus package** is one strategy where the government gives people extra money to spend, to start money circling again.

The government makes stimulus payments to people on lower incomes because they are more likely to spend the extra money straight away.

For example, this is what Brandon did with his $750 stimulus payment:





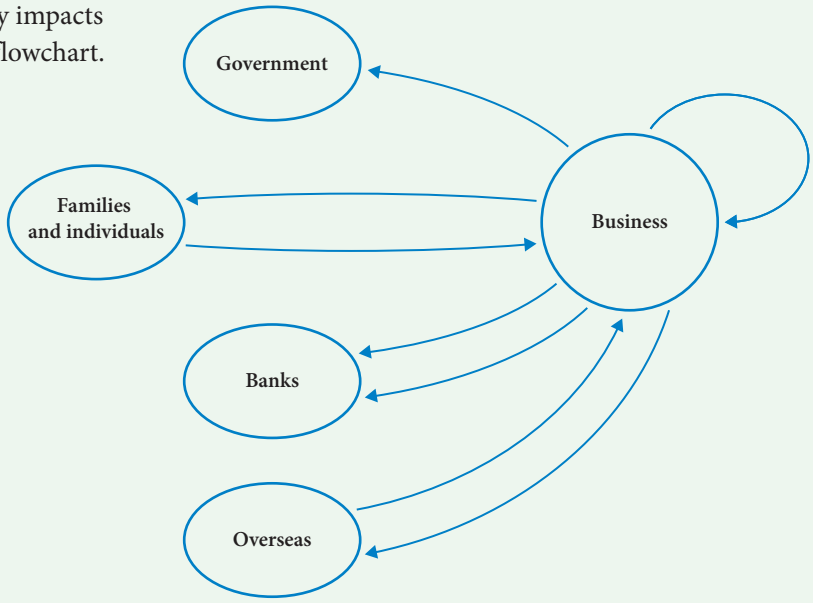
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EXAMPLE 3

Peter is the owner of a small family business that makes items to sell on the Australian market and overseas. This summary shows last week's cash flow. Surplus cash was saved in the bank for future bills.

Income	Expenses
$98 000 Australian sales from individuals and families	$16 400 wages
$43 500 overseas sales	$3245 tax bill
	$21 000 to Australian businesses for raw materials
	$19 650 to overseas business for raw materials
	$9200 bank loan repayment

Show how the business's money impacts the economy on the economy flowchart.



Steps

Working

Remember that money flows in the direction of the arrow.

From business to:

Government: $3245 tax

Families: $16 400 in wages

Bank: $9200 loan repayment

Overseas: $19 650 for raw materials

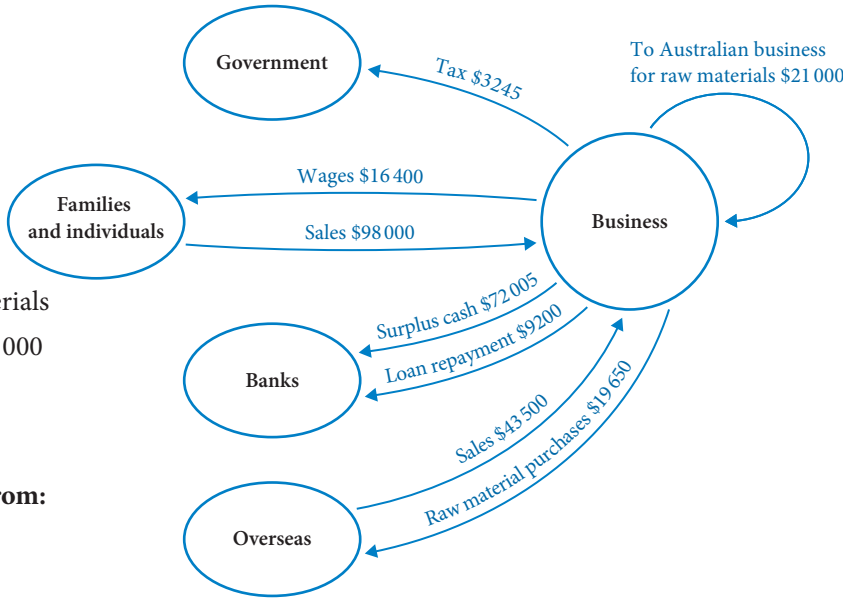
Other Australian business: $21 000 for raw materials

Bank: $72 005 surplus cash

Money back to the business from:

Families: $98 000 from sales

Overseas: $43 500 for sales



EXAMPLE 4

During the 2008 global financial crisis (GFC), the Australian government spent $2900 per person on stimulus packages. The population of Australia was 21.25 million.

a How much did the government spend on stimulus during the GFC? Answer in scientific notation.

b Write your answer without scientific notation.

c Write this amount in words.

Steps	Working
a Write 21.25 million as 21 250 000. Multiply the number of people by the amount per person.	amount spent = 21 250 000 × $2900 = $6.1625 × 10^{10}
b Move the decimal point in the number 6.1625 ten places to the right.	$61 625 000 000
c The 61 is in the billions position. The 625 is in the millions position.	sixty-one billion, six hundred and twenty-five million dollars

EXERCISE 16.2 The pandemic and our economy

ANSWERS p. 445

1 Before the community went into the first lockdown to stop the spread of COVID-19, Jessica was working 19 hours per week, earning $21.60 per hour. Her employer was forced to shut.

a Calculate Jessica's fortnightly wage before the lockdown.

b In the first lockdown, all workers in Jessica's situation received a $1500 government payment per fortnight. How much better off was Jessica with this payment compared to working?

c In later lockdowns, the payment rules were changed and she only received $750 per fortnight. Was Jessica better or worse off during the second lockdown compared to when she was working?

2 In the first few months of 2021, the Australian Government paid businesses affected by COVID-19 an allowance to keep their employees on the payroll. The payment rate was:

$1000 per fortnight for employees who worked more than 20 hours per week

$650 for employees who worked less than 20 hours per week

Angelo's restaurant was severely affected by COVID-19 restrictions. He employs 12 people for more than 20 hours per week and 15 for less than 20 hours per week.

How much did Angelo's business receive in government support for the first 12 weeks of 2021?

3 Amelia received a $750 stimulus package during the COVID-19 pandemic.

a What could Amelia do with the $750 that would **not** help the economy?

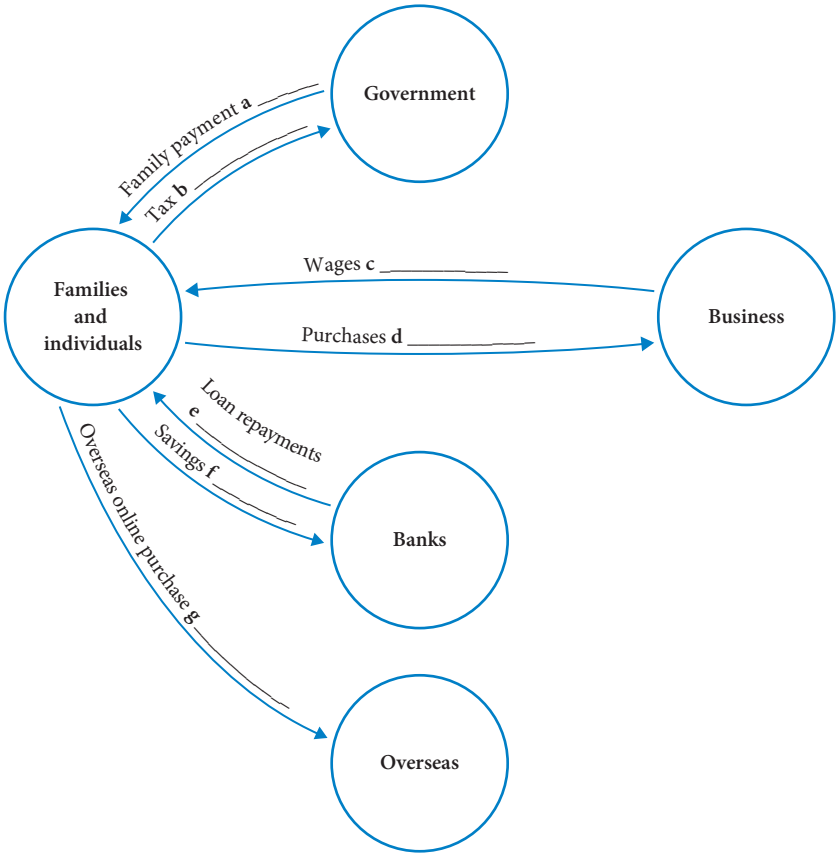
b Suggest 3 ways she could use the money to help the economy and explain how each way helps.

4 EXAMPLE 3 Amy earned $810 last week and she paid $206 in tax. She also received $23 in family assistance from the government. This is how she spent the money:

$260 on food for her family
$35 online overseas shopping
$42 on petrol
$105 loan repayment

She saved the remaining money for future bills.

Copy the money flow diagram and show how Amy's money flows through the economy.



5 EXAMPLE 4 In 2021, the Australian population was approximately 26 million. By July, the Australian Government had spent $7600 per person to save people's incomes, jobs and the economy.

a Calculate the amount the government has spent to save jobs, incomes and the economy to July 2021. Give your answer in scientific notation.

b Write your answer to part **a** without scientific notation.

c Write this amount in words.

d The Australian Government spent $61 625 000 000 on economic stimulus in the GFC. How much more has the Australian Government spent on stimulus during the COVID-19 pandemic?

6 To make sure the building industry stayed strong, the government introduced an incentive for people to build new homes or renovate their current residence. The government paid $25 000 towards the cost of building or renovations provided the owners met eligibility criteria.

a Complete the missing values in the home builder grant applications table.

Home builder grants: Applications to April 9 2021

	New build	Renovations	Total
NSW	16 266	6909	23 175
VIC	29 198	**i**	35 169
QLD	21 871	4422	26 293
WA	16 391	1361	17 752
SA	**ii**	2247	12 693
TAS	2568	494	3062
ACT	2176	667	2843
NT	366	10	376
Total	99 253	22 110	**iii**



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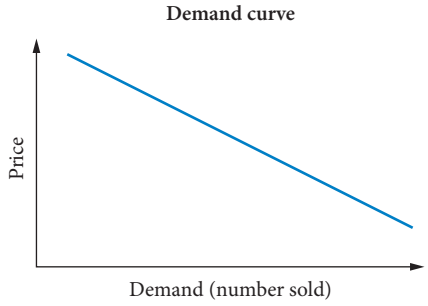
b 96% of the applications were approved. How many home builder grants were approved? Answer correct to the nearest whole number.

c Calculate the total value of the home builder grants at $25 000 per grant.

d Write the amount in words.

16.3 Supply, demand and prices

The demand graph below shows the relationship between the price of an item and its demand, the number of the items consumers buy. As the price goes up, the number of items consumers buy goes down. When the price of bananas is very high, the supermarket doesn't sell many bananas.



We learnt about business break-even points in Chapter 12: *Running a business*. Prices need to be sufficiently high for a business to make a profit, but sufficiently low to encourage consumers to buy. The more essential a product is, the less impact price changes have on quantities sold.

EXAMPLE 5

When avocados are in season, there's plenty available and their price is low. This table shows a supermarket's records of daily avocado sales.

Price per avocado	$1	$2	$3	$4	$5
Daily sales	780	420	310	220	150

a Calculate the daily revenue (income) the supermarket receives from selling the avocados at different prices.

b Plot the points representing price and daily sales on a scatterplot, then draw a line of fit through the points.

c What does the graph show?

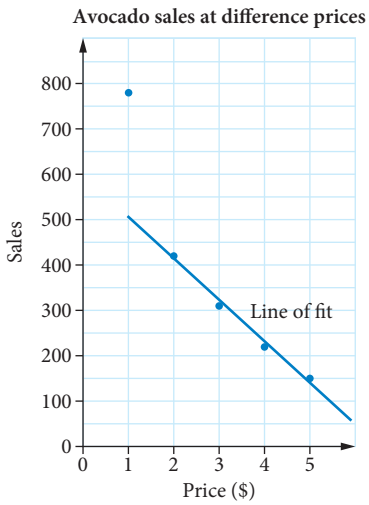
d The avocado grower receives the equivalent of 30% of the value of the sales. Calculate the amount the grower receives at each price. Comment on the values.

Steps | **Working**

a Multiply the price by the daily sales, for example, $5 × 150 = $750.

Price per avocado	$1	$2	$3	$4	$5
Daily sales	780	420	310	220	150
Daily revenue	$780	$840	$930	$880	$750

b Put price on the horizontal axis and sales on the vertical axis. Plot the points from the table then rule a line of fit.



c Comment about the point that is quite different from the others, and the relationship the line shows.

The number of sales when the price was $1 is an outlier.

The graph shows that as the price increases, the number of avocados sold decreases.

d Calculate 30% of each value in the bottom row of the table in part **a**. For example:

30% of $780 = 0.30 × $780
= $234

Daily revenue	$780	$840	$930	$880	$750
Grower's revenue	$234	$252	$279	$264	$225

The amount the grower receives is similar across all price and sales ranges.

Depreciation and discounting are other factors that can affect the value of an item.

Weblink
Easy calculation

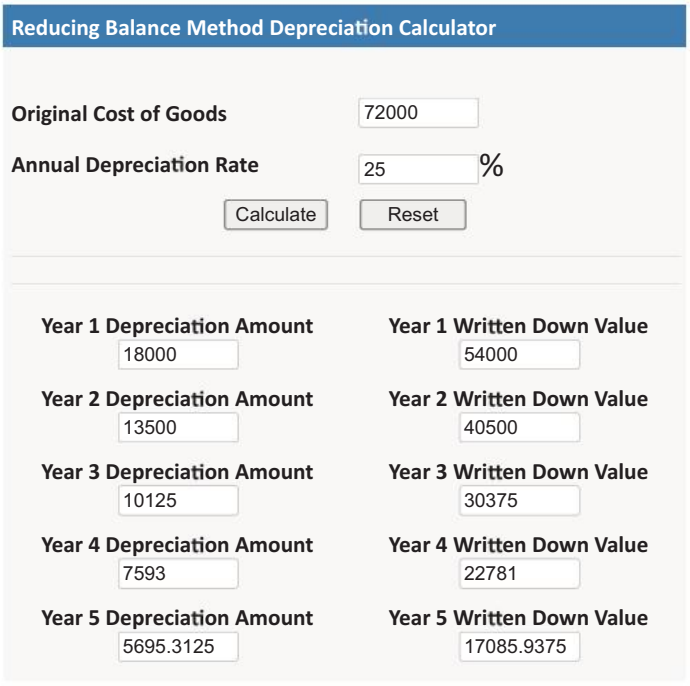
EXAMPLE 6

Jett's caravan cost $72 000 new. It is depreciating at 25% p.a.

a How much will the van be worth when it is 3 years old?

b How much value will the caravan lose during the first 3 years?

To solve this problem, you will need access to the **depreciation calculator** at www.easycalculation.com/finance/rbm-depreciation.php.

Steps	Working
a Enter 72 000 for the original cost and 25 for the annual depreciation rate.	
	When Jett's caravan is 3 years old, its written down value will be $30 375.
b There are 2 ways we can calculate the depreciation. We can subtract the value at 3 years from the new value	lost value = $72 000 − $30 375 = $41 625
or	or
We can add the 3 separate amounts for the annual depreciation.	lost value = $18 000 + $13 500 + $10 125 = $41 625

EXERCISE 16.3 Supply, demand and prices

ANSWERS p. 446

1 EXAMPLE 5 The table shows a supermarket's daily sales of punnets of blueberries at different prices.

Price	$2	$3	$4	$5	$6
Daily sales	50	30	20	15	12

a Construct a scatterplot and draw a line of fit.

b What does the graph show?

c Use the graph to predict daily sales if the price of a punnet increases to $7.

d The supermarket would sell a lot more punnets of blueberries if the price was only $1. Why aren't the berries sold for $1?

2 When there is an oversupply of items, stores lower prices to encourage more sales. A shop sells balls of wool for $15 each. They have a large supply of 1200 balls and there is not much demand for the wool. The store reduced the price by 20% and sold 90% of their stock.

- **a** At what price did they sell the balls of wool?
- **b** How many balls of wool did they sell?
- **c** How much money did they receive from the sales?

3 Michael owns a convenience store. He buys biscuits for $2.10. If he prices them at $5 per packet, he sells 60 packets per week, but if he prices them at $4, he sells 100 packets per week.

- **a** How much weekly profit does Michael make from selling the biscuits at $5 per packet?
- **b** Calculate his weekly biscuit profit when he prices the biscuits at $4.
- **c** What price do you recommend he put on a packet of biscuits? Justify your answer.

4 Towards the end of winter, Deepak noted that he had a large number of winter shoes in his shoe store. He needs to clear the stock to pay for new summer stock. He has 840 pairs of winter shoes with an average selling price of $195 and an average cost price to him of $100.



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- **a** Calculate the amount Deepak would receive if he sold all his winter shoes at an average price of $195.
- **b** How much did he pay, in total, for the remaining winter shoes?
- **c** From past experience, Deepak knows that with a '30% off sale', he will sell 95% of the shoes. How many pairs of shoes would he expect to sell with a '30% off sale'?
- **d** Calculate the value of the shoes he expects to sell in the sale.
- **e** Will Deepak make a profit from selling the shoes at 30% off? What will his profit or loss be?

5 As a result of COVID-19 state border closures, people became cautious about booking holidays. The demand for holiday accommodation fell. One luxury city hotel introduced a cheap weekend package rate to try to encourage locals to spend the weekend at the hotel. Normally the price of a room is $450 per night. The weekend special price for a 2-night stay is only $420. What percentage discount is the hotel offering? Express your answer to the nearest whole percentage.

6 EXAMPLE 6 During the pandemic, fewer new cars were imported into Australia. This led to an increase in demand for second-hand cars, particularly work vehicles. Jack bought a new tradies ute just before the pandemic. He paid $56 000 for it.

- **a** Use the **online depreciation calculator** at www.easycalculation.com/finance/rbm-depreciation.php to calculate to the nearest dollar the value of Jack's ute when it was 3 years old, using an annual depreciation rate of 16%.
- **b** As a result of reduced imports from overseas, Jack was surprised to learn that his ute had only depreciated at 2.5% p.a. Calculate the value of Jack's ute when it was 3 years old as a result of the pandemic.
- **c** How much value had Jack's ute lost in the 3 years?

Weblink
Easy calculation

7 During the pandemic, many retired Australians bought caravans and motor homes to travel within their state because they couldn't go on cruises. The price of second-hand motor homes increased by 18% to 24%. Within what price range would you expect a $156 000 second-hand motor home to cost during the pandemic?

16.4 Exchange rates

When we want to buy items from overseas, we need overseas money to pay for the goods. Similarly, when overseas companies or individuals want to buy Australian items, they need to pay in Australian dollars.

The amount of overseas currency we can get for 1 Australian dollar (AUD) is called the **exchange rate**. We know how demand and supply affects prices. The same thing happens with the value of the Australian dollar, whose value changes depending on demand.

This table shows what 1 Australian dollar (AUD) is worth in some countries (2021 figures).

Country	Exchange rate (currency abbreviation)
USA	0.7411 US dollars (USD)
China	4.6821 renminbi (RMB)
Most of Europe	0.6229 euro (€, EUR)
Hong Kong	5.68 HK dollars (HKD)
India	54.13 Indian rupee (INR)
Japan	80.78 yen (¥, JPY)
Indonesia	10 345 Indonesian rupiah (IDR)
New Zealand	1.0465 NZ dollars (NZD)
United Kingdom	0.5431 pounds (£, UKP)

EXAMPLE 7

Use the exchange rate 1 AUD = 0.74 USD to convert:

a $2000 Australian to US dollars

b US$850 to Australian dollars.

Steps	Working
a Each $1 Australian buys US$0.74. Multiply 2000 by 0.74.	A$2000 = US$2000 × 0.74 = US$1480
b To convert back to Australian dollars from US dollars, divide by 0.74.	US$850 = A$850 ÷ 0.74 ≈ $1148.65

EXAMPLE 8

The price of a barrel of crude oil is US$72 and the exchange rate is A$1 = US$0.75. Answer the following questions to calculate the price of 1 L of petrol at an Australian petrol station.

a Convert the price of a barrel of crude oil to Australian dollars.

b If 1 barrel = 159 L, calculate the cost of 1 L of petrol in cents, correct to 2 decimal places.

c Add to this cost per litre the excise (import tax) of 38.14 cents, production, transport and marketing costs of 13.25 cents, and oil company profit of 2.2 cents.

d Add 10% GST to find the final price of 1 L of petrol, correct to the nearest cent.

Steps	Working
a A$1 = US$0.75. Convert to Australian dollars by dividing by the exchange rate.	cost of a barrel = US$72 ÷ 0.75 = A$96
b Dividing the barrel price by 159.	cost of 1 L of crude oil = $96 ÷ 159 = $0.603 77… ≈ 60.38 c

c Add extra costs.	cost per litre = 60.38 + 38.14 + 13.25 + 2.2 = 113.97 c
d Add 10% GST.	10% of 113.97 = 11.397 c 113.97 + 11.397 = 125.367 c = $1.253 67 ≈ $1.25

EXERCISE 16.4 Exchange rates

ANSWERS p. 446

1 EXAMPLE 7 Use the exchange rate A$1 = US$0.74 to copy and complete each statement.

a A$900 = US$ _____

b A$1325 = US$ _____

c A$ _____ = US$2350

d A$ _____ = US$650

2 Use the exchange rate A$1 = £0.54 (UK pounds) to copy and complete each statement.

a A$200 = £ _____

b A$5400 = £ _____

c A$ _____ = £468

d A$ _____ = £5560

3 Claire buys a book online from the US using a debit card for US$24.

a Find the price in Australian dollars when the exchange rate is A$1 = US$0.75.

b Claire's bank charges a foreign currency fee, equivalent to 3% of the Australian dollar value, for any online overseas purchases. Calculate the total amount in Australian dollars that Claire will pay for the book.

4 Jonty regularly imports materials from America for his leadlight business. The supplies costs US$420. The price Jonty pays for his supplies in Australian dollars changes as the exchange rate changes.

a Copy and complete this table to show the cost of Jonty's supplies at different exchange rates. The first line has been completed as an example.

Exchange rate	Calculation	Cost of the supplies to the nearest dollar
A$1 = US$0.55	420 ÷ 0.55 = 763.64	$764
A$1 = US$0.65		
A$1 = US$0.75		
A$1 = US$0.85		
A$1 = US$1.05		

b What is the relationship between the value of the Australian dollar and the price Jonty has to pay for his supplies?

c Predict the amount Jonty's supplies will cost if the exchange rate becomes A$1 = US$1.10. Use your calculator to test the accuracy of your prediction.

5 The price we pay for petrol is affected by the supply of crude oil and our exchange rate. The majority of our crude oil comes through the Singapore market and is priced in American dollars. This table shows the costs involved.

The cost of petrol	
Crude oil	50%
Taxes and GST	38%
Production, transport and marketing costs	10%
Oil company profit	2%

Alex paid $1.60 per litre for petrol. How much of the $1.60 went to pay for:

a crude oil

b taxes and GST

c production, transport and marketing costs

d oil company profit

6 EXAMPLE 8 The price of a barrel of crude oil is US$65 and the exchange rate is A$1 = US$0.73. Answer the following questions to calculate the price of 1 L of petrol at an Australian petrol station.

a Convert the price of a barrel of crude oil to Australian dollars.

b If 1 barrel = 159 L, calculate the cost of 1 L of petrol in cents, correct to 2 decimal places.

c Add to this cost per litre the excise (import tax) of 38.14 cents, production, transport and marketing costs of 13.25 cents and oil company profit of 2.2 cents.

d Add 10% GST to find the final price of 1 L of petrol, correct to the nearest cent.

7 Make some predictions. What do you think will happen to the price of 1 L of petrol in the following situations?

a The exchange rate stays the same and the price of crude oil increases.

b The price of crude oil stays the same and the exchange rate goes up, i.e. the value of the Australian dollar increases.

c The price of crude oil falls and the value of the Australian dollar increases.

d The price of crude oil increases and the value of the Australian dollar falls.

e The major oil producing countries decide to limit the amount of oil they supply to the world market.

8 Check the accuracy of your predictions in question **7** by calculating the price of 1 L of petrol in the following situations.

	Price of a barrel of crude	Exchange rate
a	US$75	A$1 = US$0.73
b	US$65	A$1 = US$0.85
c	US$60	A$1 = US$0.80
d	US$80	A$1 = US$0.70

Investigation What happens to import and export prices as the exchange rate changes?

Daniel's family owns a seafood business in Australia. They have a contract to sell lobsters to the Japanese market at ¥1296/kg. (1296 Japanese yen per kilogram). How does the family's income from their Japanese lobster sales increase or decrease when the value of the Australian dollar changes? Use calculations to justify your answer.

Find the number of Japanese yen equivalent to A$1. Then make calculations using this value and a few larger and smaller values.



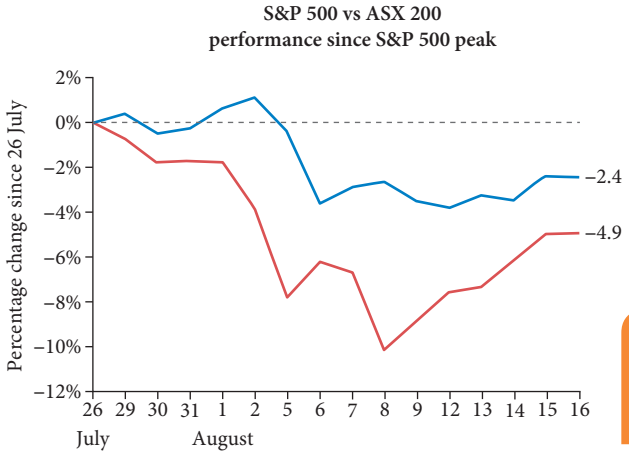
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16.5 The share market

A person with money to invest can buy a part, or **share**, of a company, and then receive a share of the company's profits, called a **dividend**. The **share market** (or stock market) is the place where shares in companies are bought and sold, mostly online these days.

The financial report on a daily news program describes how different shares performed, whether their prices rose or fell. The stock markets around the world usually have an influence on each other. For example, the New York Stock Exchange is a world leader because of its large size. Gains and falls on this exchange are usually followed by similar changes in other share markets.

This graph shows the changes in the Australian ASX 200 (blue graph, statistics based on the average price of Australian shares) and the US S&P 500 (red graph, statistics based on the average price of US shares) over 3 weeks. The Australian market followed the US market but not to the same extent.



The 'missing' days are weekend days, when the share market is closed.

EXAMPLE 9

Ali owns 2650 shares in Primary Aviation.

a The company paid a dividend of $1.23. How much did Ali receive?

b Ali bought the shares for $8.24 then sold them for $11.50 each. What percentage profit did Ali make, correct to 1 decimal place? Ignore any fees Ali may have been charged on the sale.

Steps	Working
a Multiply the dividend per share by the number of shares.	the amount Ali received = 2650 × $1.23 = $3259.50
b Calculate Ali's profit by subtracting the amount Ali paid for the shares from the amount he receives from selling them. Use the formula: $\frac{\text{profit}}{\text{original price}} \times 100\%$	total amount Ali paid for the shares = 2650 × $8.24 = $21 836 total Ali received from selling his shares = 2650 × $11.50 = $30 475 Ali's profit = $30 475 − $21 836 = $8639 $\text{percentage profit} = \frac{8639}{21836} \times 100\%$ = 39.5631…% ≈ 39.6%

The **dividend yield** of a share is the dividend as a percentage of its market price. It is like an interest rate for shares and is used by investors to compare the performance of a share.

Dividend yield

$$\text{dividend yield} = \frac{\text{dividend}}{\text{share price}} \times 100\%$$

EXAMPLE 10

The market value of Owen's mining shares is $38.75 and they paid a dividend of $1.96. Calculate its dividend yield, correct to 1 decimal place.

Steps	Working
$\text{dividend yield} = \frac{\text{dividend}}{\text{share price}} \times 100\%$	$\text{dividend yield} = \frac{1.96}{38.75} \times 100\%$ ≈ 5.1%

EXAMPLE 11

This is the statement that came with Sarah's dividend statement.

Dividend	$1400
Franking credit at 30%	$600

A franking credit means that the company has paid that amount of tax already on behalf of the share owner.

When Sarah completed her tax return, she calculated that she owed the tax office $100, without considering the franking credit. How much will she have to pay if she uses the credit?

Steps	Working
Sarah can use the $600 franking credit to pay the tax, and keep the rest of the credit.	Sarah owes $100 but she has a $600 credit. Sarah will get a refund of $600 − $100 = $500.

EXERCISE 16.5 The share market

ANSWERS p. 446

1 EXAMPLE 9 Rachel has 4200 shares in NVM Limited.

- **a** The company paid a dividend of $0.65 per share. How much did Rachel receive?
- **b** Rachel paid $5.30 each for the shares. How much did the package cost her?
- **c** Rachel sold the shares for $6.95 each. How much profit did she make?
- **d** Calculate Rachel's percentage profit, correct to 1 decimal place.

2 Last week, Cengage Bank shares were worth $101.50 each. Today they are only worth $97.80. Calculate the percentage loss in value, correct to 1 decimal place.

3 EXAMPLE 10 Calculate the dividend yield for each dividend, correct to 2 decimal places.

	Share value	Dividend
a	$12.60	$1.05
b	$103.40	$3.55
c	$89.72	$2.10
d	$50.30	$0.96
e	$12.80	75 c

4 Eva had $48 000 to invest. Term deposit rates were very low at only 0.75% p.a. so she decided to invest only half in a term deposit and the remainder in shares.

- **a** The shares she selected cost $3.20 each. How many shares did she buy?
- **b** At the end of the year, the shares paid a 55 c dividend. Calculate the total value of Eva's dividend payment.
- **c** Calculate the dividend yield, correct to 1 decimal place.
- **d** Calculate the interest Eva earned for the year on the term deposit.
- **e** Which was the better investment and by how much?

5 Jai's superannuation is invested in the growth option. This means that his whole account is invested in stocks and shares. The value of his account can be calculated with the formula:

$$\text{account balance} = (\text{number of units}) \times \frac{\text{ASX}\,200}{100}$$

where ASX 200 is a value that is based on the average price of Australian shares.

- **a** Jai has 473.2 units in superannuation. Copy and complete the table. The first row has been completed as an example.

ASX 200	Value of Jai's superannuation account
4450	$473.2 \times \frac{4450}{100} = \$21\,057.40$
7200	
3650	
6610	
3200	

- **b** Growth funds have the potential to make big, quick profits but they can lose value quickly. When the ASX was at 7200, a careless remark by a CEO of a US tech company caused world markets to fall and the ASX fell to 6650. By how much did Jai's superannuation account balance fall as a result?

6 EXAMPLE 11 Zac has a $5200 franking credit on his shares and without the franking credit, his tax bill is $2350.

a After considering Zac's franking credit, will he get a tax refund or have to pay more tax?

b How much will the amount be?

7 Gemma's shares had a $2120 franking credit. Without the franking credit, her tax bill was $2760. How much tax did she have to pay?

8 The dividend from Jon's shares is fully franked. Without considering the franking credit, Jon's tax bill would have been $3900. After including the franking credit his tax bill reduced to $1450. How much was his franking credit?



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ASX = Australian Stock Exchange

9 Boun owns shares. This financial year, his franking credits totalled $6570 and he received a tax refund of $2130 at the end of the year. Without this franking credit, how much would his tax bill have been?

10 Nikita owns 7500 shares with a market value of $42 each.

a Calculate the value of Nikita's shares.

b The shares have a dividend yield of 3.2%. Calculate the dividend from 1 share, correct to the nearest cent.

The shares are fully franked, which means that 30% of the dividend is paid directly to the tax office and 70% is paid to the share owner.

c Calculate the amount that Nikita receives and the amount sent to the tax office.

d Nikita pays 18 c tax for every dollar she earns from shares. Explain why her earnings from the shares are $10 050.

e How much tax does Nikita have to pay on her share earnings?

f How much tax refund will Nikita receive from the franking credit?

11 Investors have to pay capital gains tax on any profit they make from selling their shares. Kate has to pay 45% of any capital gain she makes in tax. She bought 1000 shares at the bottom of the market for $2.90. When the market bounced back, she sold them for $5.10 each.

a How much profit (capital gain) did Kate make?

b How much tax will she have to pay?

c How much of the capital gain will Kate get to keep?

Chapter problem

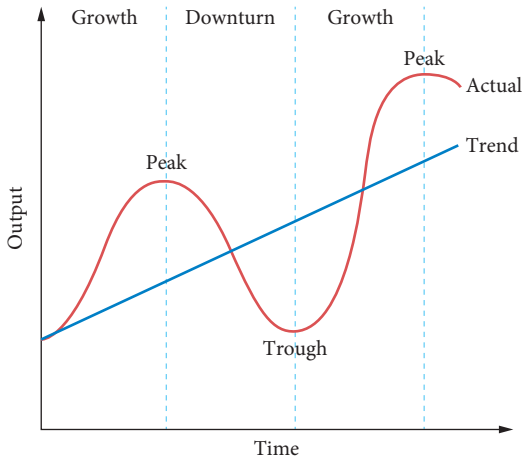
You've covered the skills required to solve the chapter problem. Can you solve it now?

16.6 Inflation and recession

Our economy goes through cycles of ups and downs. We see growth when consumers are spending money, which results in price increases and inflation. We see downturns where there's more unemployment, less spending, and difficulties in repaying loans. If the downturn lasts for 6 months or longer, it is called a **recession**.

This graph shows the business and economy cycle.

Fortunately, periods of growth are usually longer than periods of downturn. Our Reserve Bank monitors the economy and makes changes to minimise downturns to avoid a recession, such as lowering interest rates to make borrowing and spending easier.



EXERCISE 16.6 Inflation and recession

ANSWERS p. 446

1 This table shows interest rates for loaning money in developed ('rich') economies in July 2021 during the COVID-19 pandemic.

a Which countries had the highest interest rate?

b When an interest rate is negative, banks pay borrowers interest instead of charging them interest. During the pandemic, which countries paid interest to borrowers?

Interest rates in developed economies

Bank	Rate
Australia: Reserve Bank	0.1%
Canada: Bank of Canada	0.25%
England: Bank of England	0.1%
Europe: European Central Bank	0%
Japan: Bank of Japan	–0.1%
New Zealand: Reserve Bank	0.25%
Switzerland: Swiss National Bank	–0.75%
USA: Federal Reserve	0.25%

2 This table shows interest rates for loaning money in emerging ('growing') economies in July 2021 during the COVID-19 pandemic.

a Calculate the mean interest rate of the 4 countries in the table.

b How does the interest rates compare between the emerging and developed economies?

c Interest rates are related to inflation and price increases. In which country, India or Australia, are prices more likely to be rising?

Interest rates in emerging economies

Bank	Rate
Brazil: Central Bank	4.25%
China: People's Bank of China	3.85%
India: Bank of India	4%
Russia: Bank of Russia	5.5%

*Central banks change interest rates to affect confidence in the economy and change economic activity, but a change in rates affects people differently. Complete questions **3** to **6** to see some of the effects.*

3 Julia is retired. She has $450 000 invested and she uses the interest to pay her bills.

- **a** How much interest does Julia earn at 5% p.a.?
- **b** How much interest will she earn when the interest rate falls to 1% p.a.?
- **c** How does a fall in the interest rate affect Julia?

4 Conrad is a business owner who has a $10 000 interest-only loan to help his business's cash flow.

- **a** Calculate the monthly interest charge Conrad pays at an interest rate of 4.5% p.a.
- **b** What is the monthly interest when the rate drops to 2% p.a.?
- **c** How much will Conrad save per month with the lower interest rate?



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Weblink
MoneySmart

*Use a **mortgage calculator** such as the one at moneysmart.gov.au to answer questions **5** and **6**.*

5 Steve and Maria have a $350 000 mortgage on their home. They are making monthly repayments over 25 years.

- **a** How much are their monthly repayments when interest rates are 5% p.a.?
- **b** How much less per month do they have to pay when interest rates are only 2% p.a.?
- **c** Over the 25-year loan, how much less interest will Steve and Maria pay at 2% compared to 5% interest?

6 Use **How much can I borrow?** on the **MoneySmart mortgage calculator** to answer this question.

Finn wants to buy an apartment to live in. He can afford monthly repayments of $2000 over 25 years. His bank does not charge a monthly fee on his account.

- **a** How much is Finn able to borrow when interest rates are 5% p.a.?
- **b** How much more can he borrow when interest rates fall to 2% p.a.?
- **c** Write a sentence to explain how a change in interest rates affects the amount Finn is able to borrow.

7 The COVID-19 pandemic had an indirect impact on Australian real estate prices. People learned that they could work from home and no longer needed to live near their workplace. Low interest rates encouraged many people to move house, which caused house prices to rise. Victorian house prices rose by 13%.

Jana's house was valued at $456 900. Calculate its value after a 13% price increase.

8 During the pandemic, Seth decided to take advantage of the low interest rates and move out of Melbourne. He bought a rural property for $940 000.

- **a** Use the table to calculate the amount of stamp duty he had to pay for the new property.

Dutiable value is the value on which the government charges stamp duty. Usually it is the property's sale price.

Dutiable value range	Duty payable
Up to $25 000	1.4% of the dutiable value of the property
$25 001 to $130 000	$350 plus 2.4% of the dutiable value in excess of $25 000
$130 001 to $960 000	$2870 plus 6% of the dutiable value in excess of $130 000
$960 001 and above	5.5% of the dutiable value

b The loan repayments on Seth's city apartment were $2600 per month. He used the equity in his city apartment and borrowed $140 000 at 2.4% p.a. over 15 years to buy his new rural property. How much more money did Seth have to spare per month as a result of his reduced mortgage repayments? Use the **MoneySmart mortgage calculator** to answer this question.

Hint: Calculate his repayments on the $140 000, then calculate how much less his monthly repayment is now compared to what he was paying for his inner-city apartment.

Practical activity Calculating inflation 2

On page 13 of Chapter 1, *Investing money*, you recorded the prices of a range of items at the beginning of this year. Now we will compare them to this month's prices.

- Record today's price of each item in the table. (Remember to check the same brand and size as before for each item.)
- Calculate the total price for the items now (N) and for the items previously (P).
- Determine the number of days (D) from the previous pricing to today.

1 Use the formula below to calculate the annual rate of inflation, correct to 1 decimal place.

$$\text{Annual rate of inflation} = \frac{N - P}{P} \times 100\% \times \frac{365}{D}$$

2 Write a paragraph to explain why the formula works.

3 Compare your value to the government's official rate of inflation. Can you explain any difference between them?

16.7 The gender pay gap

The Australian Government adopted the principle of 'equal pay for equal work' for men and women in 1969. Fifty years later, however, women still earn considerably less than men, and the difference is called the 'gender pay gap'.

Gender pay gap

$$\text{gender pay gap} = \frac{\text{average male earnings} - \text{average female earnings}}{\text{average male earnings}} \times 100\%$$

EXAMPLE 12

In 2020, the average weekly wage for an Australian male working full time was $1804, compared to $1562 for an Australian female working full time. Calculate the gender pay gap, correct to 1 decimal place.

Steps	Working
Substitute into the formula: average male earnings = $1804 average female earnings = $1562	$\text{gender pay gap} = \frac{\$1804 - \$1562}{\$1804} \times 100\%$ $= 13.4146\ldots$ $\approx 13.4\%$

EXERCISE 16.7 The gender pay gap

ANSWERS p. 447

1 EXAMPLE 12 This table shows the average annual male and female earnings in several Australian industries. Calculate the gender pay gap in each industry.

	Industry	Male	Female
a	Financial sector	$165 455	$119 955
b	Real estate and property rentals	$143 087	$107 029
c	Media and telecommunications	$148 000	$117 216
d	Home healthcare and assistance	$106 369	$89 669
e	Education and training	$125 674	$114 112
f	Retail	$89 225	$76 912
g	Manufacturing	$108 725	$95 678

2 a In only 2 Australian industries in the table in question **1** is the gender pay gap less than the Australian average of 13.4%. What industries are they?

b In which 2 industries is the gender pay gap the greatest?

3 In Australia, women earn 98% of what men with identical skills and experience earn. The men in Keira's workplace with the same skills as hers earn $25.60 per hour. How much does Keira earn per hour?

4 This table shows the percentage of Australian women in different types of management positions.

Type of position	Percentage of positions held by women
CEO or head of the business	18.3%
Key management	32.2%
Other executive positions	33.4%
Senior managers	36.5%
Other managers	43.0%
Australian workforce	50.5%

a What percentage of the Australian workforce is male?

b What percentage of CEO or head of the business are men?

c Of Australia's top 200 companies, only 9.5% have a female CEO. How many of our top 200 companies have a female CEO?

d What is the relationship between the level of seniority of the job and the number of women employed at that level?

e What could explain the reason for the gender pay gap? Discover some possible reasons as you work through the following questions.

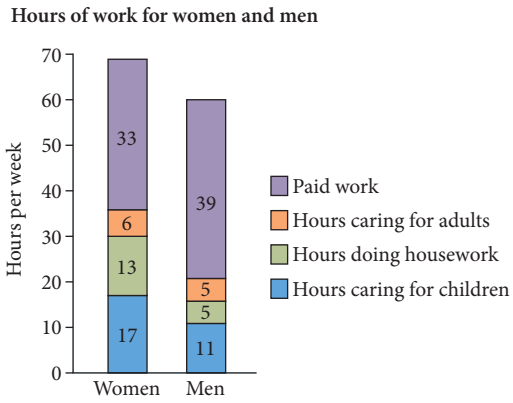
5 Managing a home and looking after children takes a lot of time. Women tend to do more of this work than men do. This graph shows the hours per week on average men and women around the world spend on paid and unpaid work.

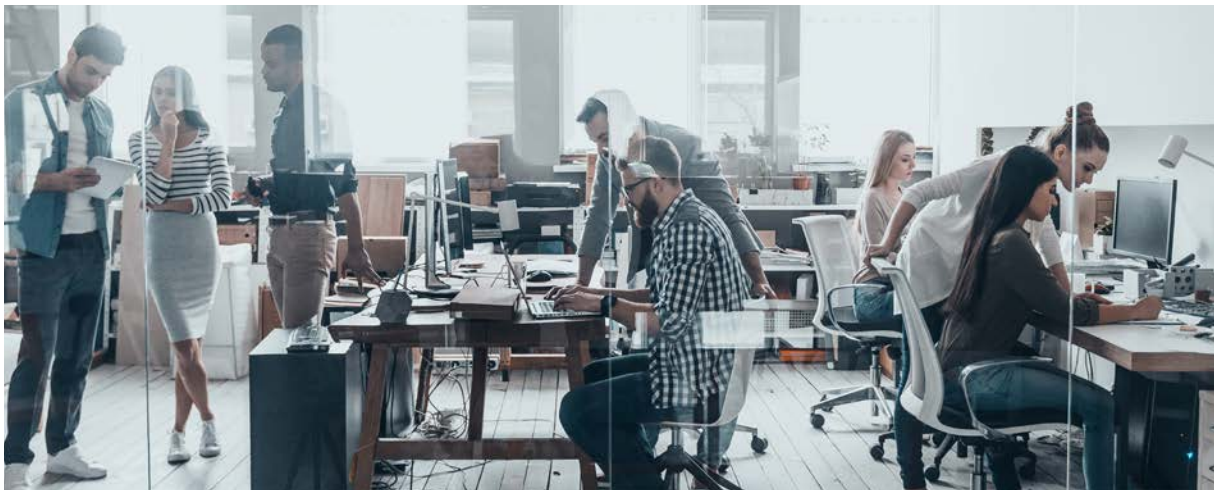
a How many hours of work on average do women do per week?

b How many more hours per week on average do women do unpaid work compared to men?

c How many more hours per week on average of housework do women do than men?

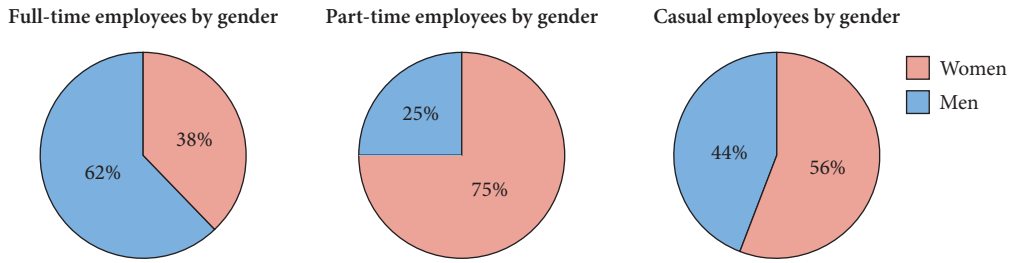
d How many fewer hours per week of paid work on average do women do than men?





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6 Full-time, part-time and casual wage rates affect total pay. These graphs show the differences between men and women's type of employment.



a In what type of employment is there 3 times as many women as men?

b Wages in full-time employment are higher than in casual or part-time employment. What gender is more involved in full-time employment?

c Casual employees are the most likely to have their work hours reduced or to lose their job in an economic downturn. Are men or women more affected by this problem?

d For the same job, pay rates for part-time work is 10% less than full-time work. Zac and Lily both work as cleaners. Zac works full time and is paid $24 per hour. Lily works part-time. How much less does Lily earn for working 8 hours than Zac?

7 Some economists say that on average a mother's wage reduces by 7% for each child she has. Before Sophie had children, she earned $2840 per week. How much can she expect to earn if she returns to the workforce after having 3 children?

8 The gender pay gap is surrounded by fallacies (false beliefs). Match each of the following fallacies in parts **a** to **e** with the truth in **i** to **v**.

a Women earn less because they choose lower paying jobs.

b Everyone with the skills and experience has the same chance of getting a top job.

c Men are better educated so they should be paid more.

d Childcare costs are a woman's responsibility.

e More women work part-time so they should be paid less.

i 3 out of every 5 university graduates are female.

ii Working less hours should mean less total pay, not less pay per hour.

iii Australian law requires both parents to provide for their children.

iv It doesn't matter if it's a highly skilled job or an unskilled job, women are paid less per hour than men.

v Most of the CEOs of Australia's top 200 companies are white, middle-class males. CEOs are more likely to have the name Peter than to be a woman.



Worksheet
Investigation components

Extended investigation — My investment portfolio 2

In the extended investigation in Chapter 1: *Earning money*, you role-played investing money in a term deposit and bought some shares. You will need your records of these investments to complete this investigation.

Write a report of your portfolio's performance.

Include:

- a description of your investments
- any dividend you received
- a calculation showing a dividend yield
- any interest you received on your term deposit
- the capital gain you made on your shares
- an evaluation of the best and worst performing parts of your portfolio.

Describe what you would do differently should you have the money to invest in this way again in the future.



Solution to the chapter problem

Problem

Jamie owns 4250 EastFarmers shares valued at $56 each. The company declared a 95 c dividend. What is the dividend yield for Jamie's shares?

Solution

The value of Jamie's shares $= 4250 \times \$56$

$= \$238\,000$

The dividend Jamie receives $= 4250 \times 95\,\text{c}$

$= \$4037.50$

$$\text{dividend yield} = \frac{\text{dividend}}{\text{share price}} \times 100\%$$

$$= \frac{4037.50}{238\,000} \times 100$$

$= 1.7\%$

The dividend yield for Jamie's shares is 1.7%.

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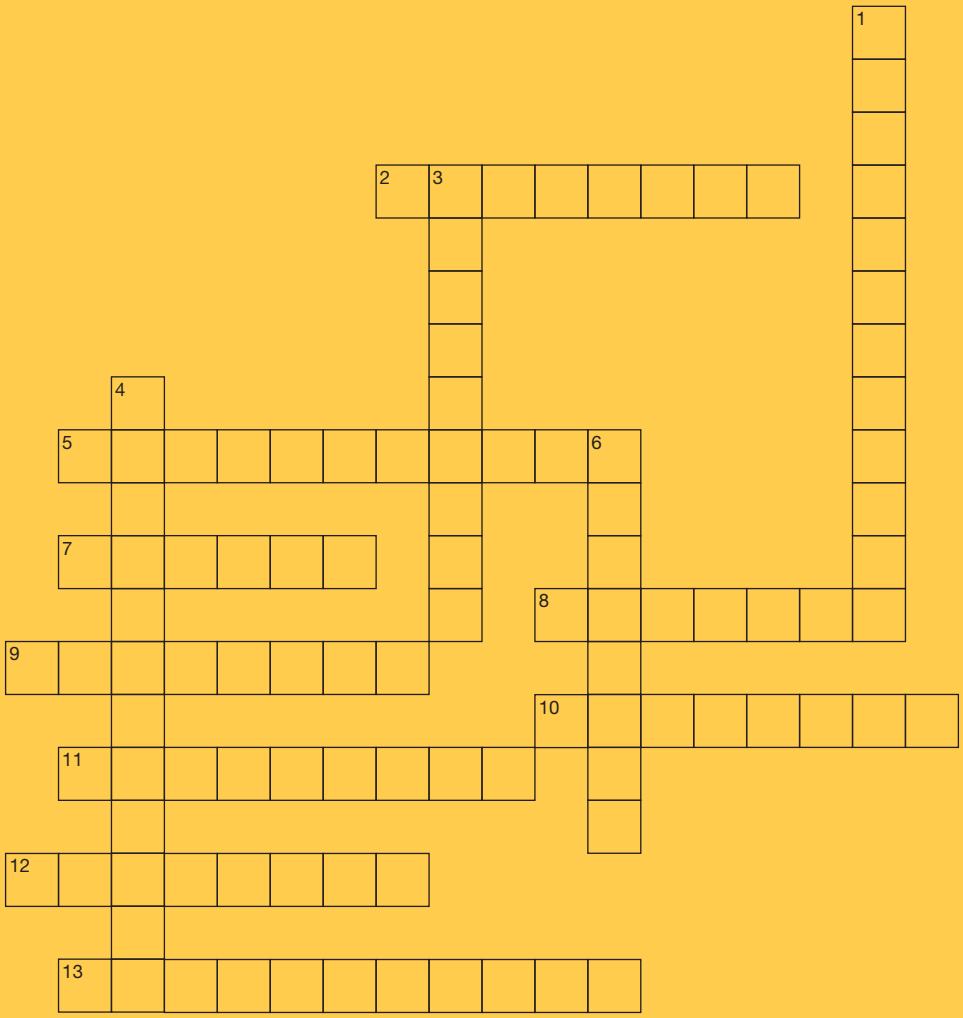
Puzzle
Australia and the world crossword

Keyword activity

CROSSWORD

ANSWERS p. 447

Copy (or download) this crossword puzzle and complete it using the clues below.



Across

2 A payment made to share holders

5 A type of relationship or graph that gets very big quickly

7 If you own some of these, you own part of a business

8 A number 1 followed by 9 zeros

9 A worldwide spread of a disease

10 A type of package provided by the government designed to help the economy avoid a recession

11 High unemployment and little growth in the economy

12 A type of tax credit that can be part of a dividend payment

13 Australia's central bank

Down

1 When an item loses value

3 Prices increase

4 The relationship between money from different countries

6 A type of graph that results when an exponential graph cannot continue indefinitely

16 Test yourself

Australia and the world

ANSWERS p. 447

1 EXERCISE 16.1 COVID-19 is spreading on an island in the Pacific. New cases are doubling every 3 weeks.

a Copy and complete this table.

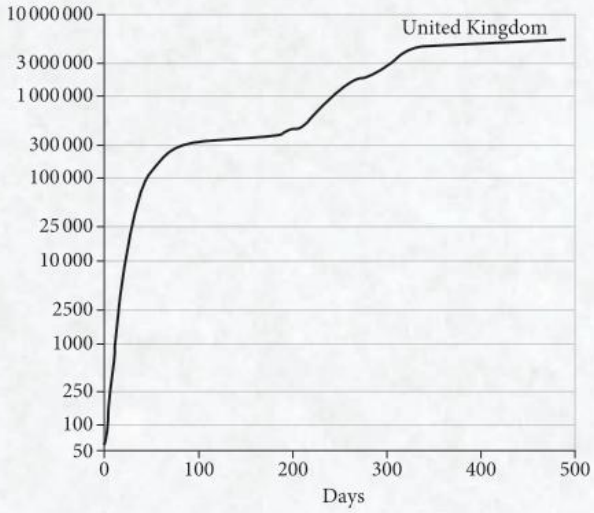
Date	August 2	August 23	September 13	October 4	October 25
Cases	87				

b Construct a graph to show the data in the table.

c What type of graph is it?

2 EXERCISE 16.1 This graph shows the number of COVID-19 cases in the UK days after the first 50 cases were detected.

Note the uneven scale to show the rapid rise in cases.



a Approximately how many days after the first 100 cases were detected were there 1 million cases in the UK?

b What type of graph is this?

3 EXERCISE 16.3 Peter owns a plant nursery. He decided to have a sale to reduce the amount of stock he had. He has 380 mature shrubs valued at $44 each and 810 small plants worth $12 each. He reduced the prices by 25% and sold 85% of both types of plants.

a At what price did he sell the plants?

b How many plants did he sell?

c How much money did he receive from the sales?

4 EXERCISE 16.4 The exchange rate between Australian and New Zealand dollars is A$1 = NZ$1.07.

a What is the value of A$500 in New Zealand dollars?

b Change NZ$80 to Australian dollars.

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5 EXERCISE 16.4 Complete this statement.

When the value of the Australian dollar increases, we pay (a) _____ for items we buy from overseas and we receive (b) _____ for items we sell overseas.

6 EXERCISE 16.5 Jason bought 2000 mining shares for $28 each.

a How much did he pay for the shares?

b The shares paid a dividend of $1.99 per share. What dividend did he receive?

c Calculate the dividend yield, correct to 1 decimal place.

d Jason sold the shares for $31 each. What capital gain did he make?

7 EXERCISE 16.6 In times of high inflation, the Reserve Bank increases interest rates to dampen the economy. Scott has $55 000 to invest in a term deposit. He was going to invest it at 4% p.a. on Monday afternoon but he didn't get to the bank in time. On Tuesday morning, the Reserve Bank increased rates to 4.5% p.a.

a How much interest would Scott have earned for the 6-month investment if he had invested on the Monday afternoon?

b Calculate the interest the investment will make for 6 months at the Tuesday rate.

c How much extra will Scott earn for a 6-month term deposit at the higher interest rate?

8 EXERCISE 16.7 In a small island community, the average male earnings are $65 920 and the average female earnings are $58 880.

Use the gender pay gap formula to calculate the gender pay gap in the community, correct to 1 decimal place.

$$\text{gender pay gap} = \frac{\text{average male earnings} - \text{average female earnings}}{\text{average male earnings}} \times 100\%$$

Practice set 4

ANSWERS p. 447

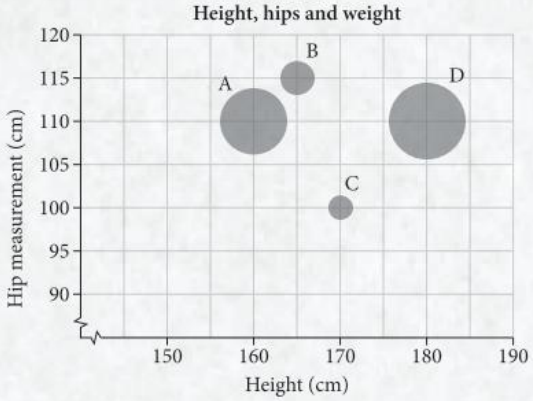
Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 14.1 Jason's income from his job is $78 925 and he earned $3569 from shares and interest. His allowable taxation deductions are $2840. What is Jason's taxable income?

A $76 085 **B** $79 654 **C** $82 494 **D** $85 334

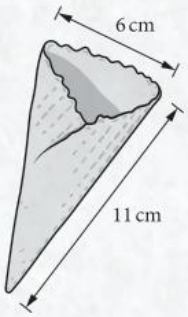
2 EXERCISE 13.5 This bubble graph shows the heights, hip measurements and weights of 4 people.



Stella is tall and thin. Which circle on the graph represents Stella?

A A **B** B **C** C **D** D

3 EXERCISE 15.3 Ella makes ice-cream cones from circular waffles. How many cm^2 of waffles does Ella use to make 12 cones? [Reminder: The formula for the surface area of a cone is $SA = \pi r^2 + \pi rs$.]



A 104 cm^2 **B** 207 cm^2 **C** 1244 cm^2 **D** 2488 cm^2

4 EXERCISE 14.1 Use the tax table to calculate the income tax payable on a taxable income of $132 500.

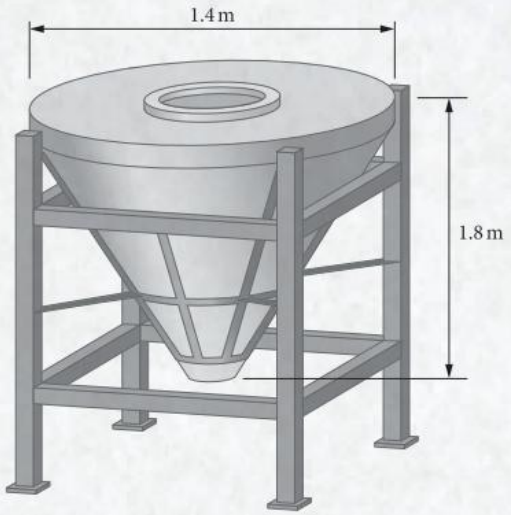
Taxable income	Tax
Up to $18 200	Nil
$18 201 to $45 000	19 c for each dollar above $18 200
$45 001 to $120 000	$5092 + 32.5 c for each dollar above $45 000
$120 001 to $180 000	$29 467 + 37 c for each dollar above $120 000
$180 001 and over	$51 667 + 45 c for each dollar over $180 000

A $78 492 **B** $73 992 **C** $49 197 **D** $34 092

Individual income tax rates - Resident tax rates 2020–21, https://www.ato.gov.au/rates/individual-income-tax-rates/ © Australian Taxation Office for the Commonwealth

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5 EXERCISE 15.5 Alexander's water storage unit is in the shape of a cone. The diameter of the top is 1.4 m and it is 1.8 m high. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



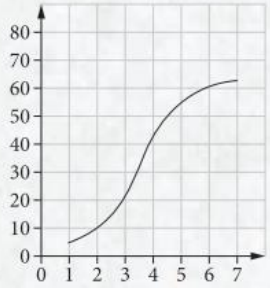
What is the capacity of the water storage container in litres, correct to the nearest litre?

A 924 L **B** 2770 L **C** 9236 L **D** 27 709 L

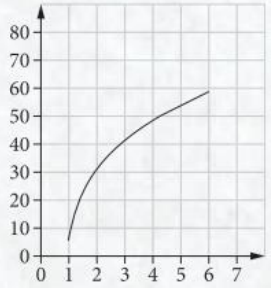
6 EXERCISE 16.1

Which of the graphs best illustrates exponential growth?

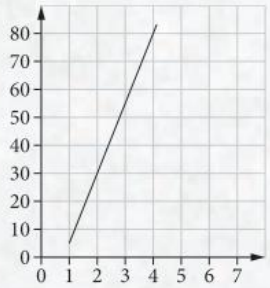
A



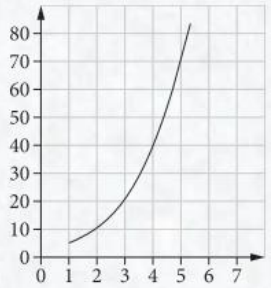
B



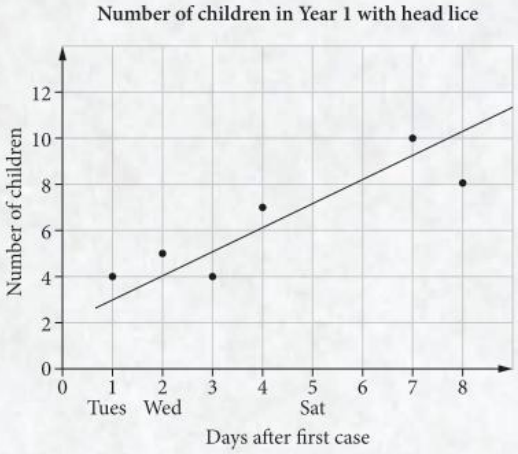
C



D



7 EXERCISE 13.4 Cases of head lice have been reported in the children in Year 1. The first case was discovered on Monday. The graph shows the number of children with head lice each day after Monday. There is no data for the weekend.



Which phrase below describes an estimate of the number of children with head lice on Saturday?

A Reliable estimate of 7 children

B Unreliable estimate of 7 children

C Reliable estimate of 8 children

D Unreliable estimate of 8 children

8 EXERCISE 14.1 Max has an annual taxable income of $120 000. How much more is his annual Medicare levy because he doesn't have health insurance?

Medicare levy on taxable income

		Higher income with no private health insurance		
	Basic	Tier 1	Tier 2	Tier 3
Singles		$90 001–$105 000	$105 001–$140 000	$140 001 or more
Medicare levy	2%	3%	3.25%	3.5%

A $1200 **B** $1500 **C** $2400 **D** $3900

Section B Short-answer questions

1 EXERCISE 14.1 Use the tax table in question **4** on page 410 and the Medicare levy table above to answer this question.

Last year, Lucy's gross weekly income was $820 and she received $256 interest from her bank. Her allowable taxation deductions were: Nurse's association fees of $615, a donation of $240 to the Red Cross and uniform cleaning costs totalling $420.

a What was Lucy's yearly gross income?

b Calculate Lucy's taxable income.

c How much income tax is she required to pay?

d Calculate Lucy's Medicare levy.

e Each week, Lucy's employer deducts $120 from her pay for PAYG tax. Calculate the value of Lucy's tax refund.

2 EXERCISE 16.5 Kuldip bought 1500 shares in a health company for $16 each.

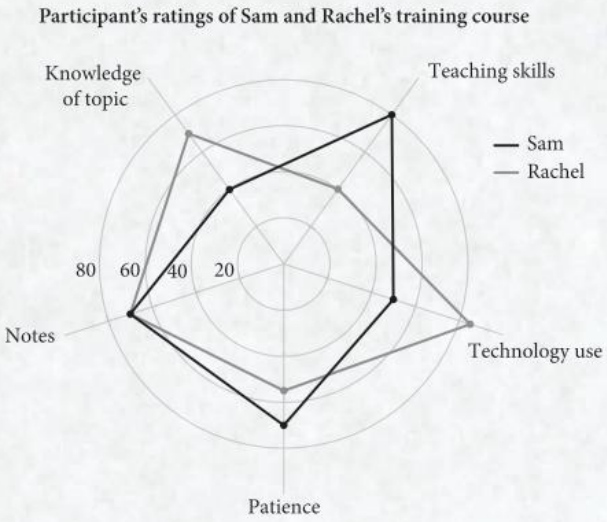
a How much did she pay for the shares?

b The shares paid a dividend of $1.05 per share. What dividend did she receive?

c Calculate the dividend yield, correct to 1 decimal place.

d Kuldip sold the shares for $29 each. What capital gain did she make?

3 EXERCISE 13.7 Sam and Rachel are both first-aid trainers. This graph shows the evaluations from the participants of the last training course they ran.



a Was Sam or Rachel rated as having more knowledge of first aid?

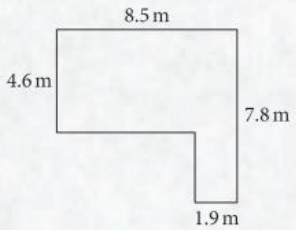
b For what feature of the training were they rated equally?

c Sam and Rachel's supervisor wants them to each improve one aspect of their training skills. What aspects do you recommend they each concentrate on improving?

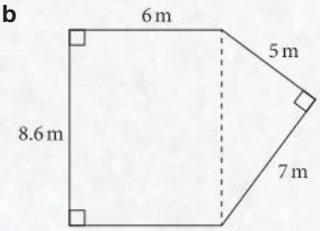
4 EXERCISE 14.2 Mia kept a travel diary of work-related travel kilometres for her travel between her 2 jobs.
In it she can prove that she has travelled 960 km. At a rate of 75 c/km, how much can she claim on her tax as a work-related travel expense?

5 EXERCISE 15.1 Calculate the area of each composite shape, correct to 1 decimal place.

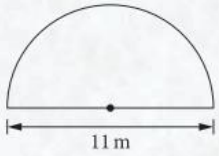
a



b



c

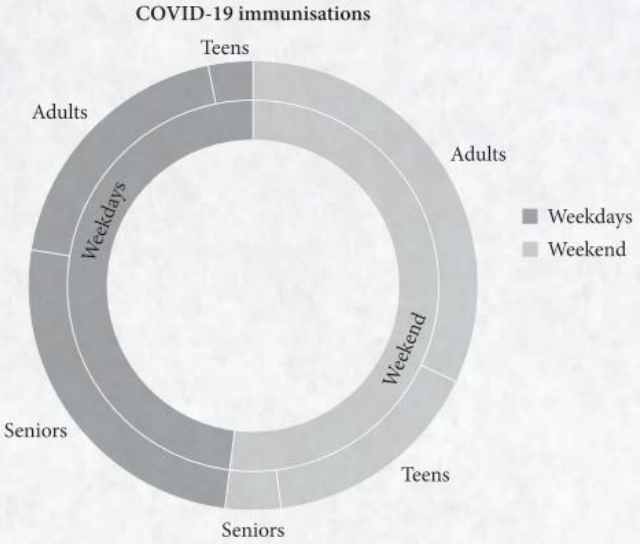


6 EXERCISE 16.4 The exchange rate between Australian dollars and euros, the European currency, is A$1 = 0.6188 EUR.

a What is the value of A$800 in Euros?

b Change 100 euros into Australian dollars.

7 EXERCISE 13.8 A COVID-19 mass vaccination centre used a graph to illustrate the different age groups that were vaccinated on weekdays and weekends.



a What type of graph is this?

b When were most seniors vaccinated?

c Approximately, what was the ratio of adults to teens vaccinated on the weekend?

8 EXERCISE 14.3 Use the table to calculate the annual cost for a 10-year-old child whose parents have a combined child support income of $175 600.

The annual cost of a child under 12 years of age

Parents' combined child support income	Cost for 1 child
Up to $39 479	17 c for each $1
$39 480 to $78 957	$6711 + 15 c for each $1 over $39 479
$78 958 to $118 436	$12 633 + 12 c for each $1 over $78 957
$118 437 to $157 914	$17 370 + 10 c for each $1 over $118 436
$157 915 to $197 393	$21 318 + 7 c for each $1 over $157 914
Incomes over $197 393	$24 082

Costs of the Children Table 2021, Child Support Guide, Version 4.58 - Released 20 September 2021, Department of Social Services © Commonwealth of Australia. Licensed under Creative Commons 3.0 Attribution.

9 EXERCISE 16.6 Max is retired. He uses the interest on his investments to pay his bills. He has $250 000 invested.

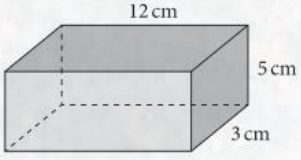
a Calculate the interest Max earned for a 6-month investment when interest rates were 2.5% p.a.

b The Reserve Bank dropped the interest rate to 2% p.a. How much interest will Max earn in 6 months on his investment?

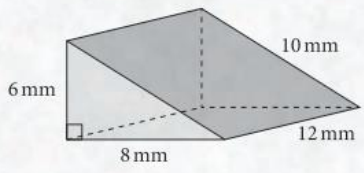
c How much less will Max get for a 6-month investment at the lower rate of interest?

10 EXERCISE 15.2 Calculate the surface area of each prism.

a

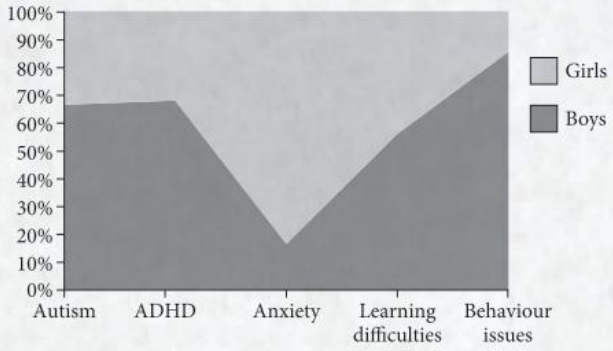


b



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11 EXERCISE 13.6 Ms Taggart, the school counsellor, prepared a graph showing psychological issues in the first 3 years of schooling.



a What type of graph did Ms Taggart construct?

b In Ms Taggart's school, is ADHD more common in boys or girls?

c What is the approximate ratio of boys with autism to girls with autism in the school?

d Do more boys or girls in the school have behavioural issues?

e What type of psychological issue is much more common in girls than boys?

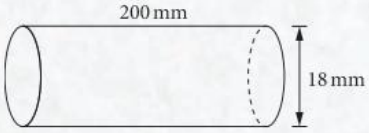
12 EXERCISES 15.3, 15.5 A size 5 soccer ball has a diameter of 23 cm. Using the formulas $SA = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$, calculate the surface area and volume of a size 5 soccer ball.



13 EXERCISE 16.4 Copy the following sentence and insert 'more' or 'less' into the gaps to make the statement true.

When the value of the Australian dollar falls, we pay (a) _____ for items we buy from overseas and we receive (b) _____ for items we sell overseas.

14 EXERCISE 15.2 This pipe is in the shape of an open cylinder. Calculate its curved surface area, correct to 1 decimal place.



15 EXERCISE 16.7 In a rural community, the average annual male earnings are $73 240 and the average female earnings are $9000 less than that of males.

Use the gender pay gap formula to calculate the gender pay gap in the community, correct to 1 decimal place.

$$\text{gender pay gap} = \frac{\text{average male earnings} - \text{average female earnings}}{\text{average male earnings}} \times 100\%$$

16 EXERCISE 14.2 Use the income tax table in question **4** on page 410 to complete this question.

Without a claim for contributions to superannuation, Tash has a taxable income of $195 000. As a self-employed person, she can make a $25 000 concessional contribution to superannuation to minimise her tax. The concessional contribution will be taxed at 15%. How much less tax will she pay, including the contributions tax, if she makes the contribution to superannuation?

Answers

CHAPTER 1

EXERCISE 1.1

1 **a** $115.20 **b** $243.36 **c** $866.25 **d** $350.63

2 **a** $44.10 **b** $193.60 **c** $54.27 **d** $4.58

3 0.38%

4 **a** 0.26% **b** 0.06% **c** 1.56% **d** 0.0085%
e 0.12% **f** 0.78%

5 **a** $1088 **b** $4288

6 **a** $15.86 **b** $880.86

7 $59 743.75

8 **a** 0.048% **b** $20 736

9 **a** $250 **b** $21 500
c The interest rate was too high and she didn't check whether the investment company was known to be a scam risk.

10 **a** $1600 **b** $600 **c** approx. $250
d It's the original investment (principal).

EXERCISE 1.2

1

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$480$	$\$8000 + \$480$ $= \$8480$
End of the 2nd year	$I = Prn$ $= \$8480 \times 0.06 \times 1$ $= \$508.80$	$\$8480 + \$508.80$ $= \$8988.80$
End of the 3rd year	$I = Prn$ $= \$8988.80 \times 0.06 \times 1$ $= \$539.33$	$\$8988.80 + \$539.33$ $= \$9528.13$

2 **a** $480 **b** $12 480 **c** $12 979.20
d $13 498.37 **e** $1498.37

3 **a** $4260.10 **b** $260.10 **c** $4.10

4 **a**

	Interest	Balance
End of the 1st month	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$98$	$\$14\,000 + \$98$ $= \$14\,098$
End of the 2nd month	$I = Prn$ $= \$14\,098 \times 0.007 \times 1$ $= \$98.686$	$\$14\,098 + \$98.686$ $= \$14\,196.686$
End of the 3rd month	$I = Prn$ $= \$14\,196.686 \times 0.007 \times 1$ $= \$99.3768$	$\$14\,196.686 + \$99.3768$ $= \$14\,296.06$

b $296.06

5 **a** $9600 **b** 8% **c** $1597.44

6 **a** $5208.75
b Scam indicators include a:
- cold phone call from someone Zac doesn't know
- push from the 'advisor' to act quickly
- very high interest rate
- claim that the opportunity is 'risk-free'
- minimum investment of $10 000.

7 **a** Because the interest increases each year, a compound graph curves up. Simple interest graphs are straight lines.
b 7 years

8 **a** $400 **b** X is Nick, Y is Adam
c Nick's by $35.45

EXERCISE 1.3

1 **a** 1.08 **b** 1.06 **c** 1.035 **d** 1.028
e 1.0175

2 **a** 7% **b** 6.5% **c** 1.5% **d** 12%
e 10%

3 B **4** $50.84 **5** $20 248 **6** $840

7 **a** $18.13 **b** $24.58

8 $25.21 **9** 24.56, 75.40, 231.48, 710.63, 2181.63

10 **a** 100% + 11.17% = 111.17% = 1.1117
b 15 schillings

11 $1700 **12** 2840

13 **a** 30 201 000 **b** 2043

14 **a** $174 000 **b** $23 000 **c** $21 000
d $776 000 **e** $3000 **f** $57 881 000

EXERCISE 1.4

1 **a** $3974.44 **b** $974.44
c Teacher to check

2 **a** $6529.12 **b** $1353.20
c $13 170.10 **d** $46 960.25

3 Year 1: $1311
Year 2: $1348.36
Year 3: $1386.79
Total: $4046.16

4 **a** $77 318.13 **b** $27 318.13
c Teacher to check

5 $9177.22

6 The future values are Option 1 $23 300 and Option 2 $22 823.32. Option 1 is the better investment because she ends up with more money.

EXERCISE 1.5

1 **a** $1215.51 **b** $3218.44
c $43 630.30 **d** $55 475.18

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2 a $649.96 **b** $716.50
c $2466.86 **d** $966.95

3 a $5948.93 **b** $9948.93

4 a B8: =B4
C8: =B8*$B$5
D8: =B8 + C8
A9: =A8 + 1
B9: =D8
b 0.045
c $48 536.88 **d** $24 536.88

5 a Teacher to check **b** approximately 18 years

EXERCISE 1.6

1 a $1966.81 **b** $517.13
c $359.36 **d** $366.16

2 a $7193.75 **b** $1393.75

3 $1744.93 **4** $4.19 **5** $464

6 The monthly compounding gives the better return (final values of $511 274.92 for monthly compounding compared to $510 880.61 for annually compounding)

7 The managed fund gives the better return (final values of $11 533.75 for the managed fund compared to $10 133.50 for the finance company).

KEYWORD ACTIVITY

1 H **2** J **3** E **4** A
5 B **6** G **7** I **8** C
9 F **10** K **11** D

TEST YOURSELF 1

1 $1312

2 a

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8400 \times 0.04 \times 1$ $= \$336$	$\$8400 + \$336$ $= \$8736$
End of the 2nd year	$I = Prn$ $= \$8736 \times 0.04 \times 1$ $= \$349.44$	$\$8736 + \$349.44$ $= \$9085.44$
End of the 3rd year	$I = Prn$ $= \$9085.44 \times 0.04 \times 1$ $= \$363.42$	$\$9085.44 + \$363.42$ $= \$9448.86$

b $1048.86

3 1.07 **4** $60.62 **5** $833.49

6 $394.15 **7** $8607.59

8 Investment B is better ($172.06 interest earned compared to $163.20)

9 Graph 1 shows compound interest.
Graph 2 shows simple interest.

10 a the interest for the first month
b to keep the cell reference the same when she copied down
c $436.09
d either =SUM(C10:C27) or =D27-C4
e $67 622.88
f

	A	B	C	D
1	Jill's spreadsheet			
2				
3				
4		Initial investment C4	$60,000.00	
5		Annual interest rate as a decimal C5	0.08	
6		Monthly interest rate C6	0.006666667	
7		Weekly interest rate	0.001538462	
8				
9	Number of weeks	Value of the investment at the beginning of the week	Weekly interest	Value of the investment at the end of the week
10	1	$60,000.00	$92.31	$60,092.31
11	2	$60,092.31	$92.45	$60,184.76
12	3	$60,184.76	$92.59	$60,277.35
13	4	$60,277.35	$92.73	$60,370.08
14	5	$60,370.08	$92.88	$60,462.96
15	6	$60,462.96	$93.02	$60,555.98
16	7	$60,555.98	$93.16	$60,649.14
17	8	$60,649.14	$93.31	$60,742.45
18	9	$60,742.45	$93.45	$60,835.90
19	10	$60,835.90	$93.59	$60,929.49
20	11	$60,929.49	$93.74	$61,023.23
21	12	$61,023.23	$93.88	$61,117.11
22	13	$61,117.11	$94.03	$61,211.14
23	14	$61,211.14	$94.17	$61,305.31
24	15	$61,305.31	$94.32	$61,399.63
25	16	$61,399.63	$94.46	$61,494.09
26	17	$61,494.09	$94.61	$61,588.69
27	18	$61,588.69	$94.75	$61,683.45
28	19	$61,683.45	$94.90	$61,778.34
29	20	$61,778.34	$95.04	$61,873.39
30	21	$61,873.39	$95.19	$61,968.58
31	22	$61,968.58	$95.34	$62,063.91
32	23	$62,063.91	$95.48	$62,159.40
33	24	$62,159.40	$95.63	$62,255.03
34	25	$62,255.03	$95.78	$62,350.80
35	26	$62,350.80	$95.92	$62,446.73

g $6.37

11 a 0.4% **b** $463.66

12 B, the annually compound interest ($12 726.37 compared to $12 707.42 and $12 640)

CHAPTER 2

EXERCISE 2.1

1 a 5 **b** 3 **c** 11 **d** 5
e 5 **f** 8 **g** 11 **h** 8
i 6 **j** 7 **k** 7 **l** 11

2 a 20 **b** 27 **c** 35 **d** 8
e 8 **f** 7 **g** 7 **h** 5
i 96 **j** 5

3 a 9 **b** 10 **c** 4 **d** 45
e 300 **f** 160 **g** 70 **h** 56
i 12 **j** 12

4 a 2 **b** $8 **c** 6 minutes
d 33 **e** 17 **f** 50 seconds
g 25 **h** $6.20 **i** $480
j 9 **k** $195

EXERCISE 2.2

1 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{5}{12}$
d $\frac{11}{12}$ **e** $\frac{7}{12}$

2 36 g

3 150 g gold, 50 g alloy

4 **a** 55 minutes

b 9.35 am, plus time to wait for the train to arrive, if needed

5 96

6 12

7 3 tablespoons egg substitute
150 g butter
330 g almond meal
180 g sultanas
120 g sugar
300 mL milk

8 **a** 2 **b** 30 days

9 **a** $\frac{1}{125}$

Both fractions have the same numerator, so the one with the larger denominator is the smaller fraction.

b 4 times longer

10 **a** $\frac{9}{10} \times 80 = 72$ mg **b** 7.2 mg

EXERCISE 2.3

1 **a** 14.24 **b** 21.63 **c** 112.35 **d** 134.23
e 3.93 **f** 0.13

2 **a** 17.6 **b** 11.0 **c** 14.4 **d** 4.1

3 Many answers are possible between 28.65 and 28.749.

4 34.75

5 **a** 20.5 **b** 28.9

6 **a** Holly: 54.72, 64.76, 78.41
Beck: 77.95, 53.08, 63.82

b Holly: 197.89, Beck: 194.85

c Holly

7 **a** 65.82 and 73.56 **b** 69.47

8 **a** 0.15

b 0.05 is 0.050, which is more than 0.025. ($5 \times 0.05 = 0.25$).

9 25.2 mg per dose

10 He should do the dive with a degree of difficulty of 2.9 to achieve a possible score of 20.59. The other dive may only score 17.85.

EXERCISE 2.4

1 3 days

2 **a** 10 kg **b** 75.68 kg

3 **a** 87.5% **b** 55% **c** 76%

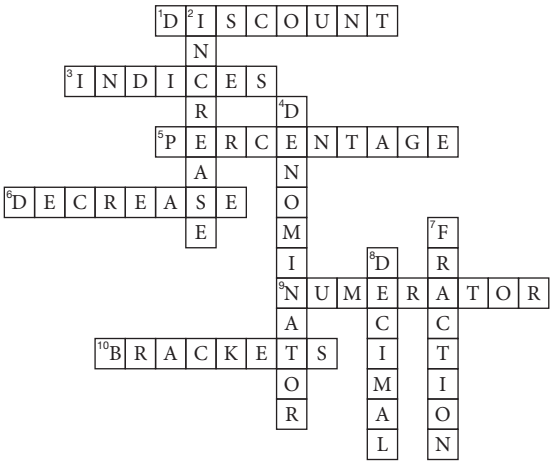
4 $34 **5** 200 **6** $117.80

7 $2380 **8** $88.20 **9** $57 200

10 $46.75 **11** 13.3% **12** 234 960 ha

13 $114 **14** 20% **15** $19.41

KEYWORD ACTIVITY



TEST YOURSELF 2

1 **a** 13 **b** 11 **c** 8

2 **a** 12 **b** 75 **c** 36 **d** 54

3 **a** $\frac{3}{4}$ **b** 90 g

4 **a** $\frac{4}{5}$ **b** $132

5 **a** 393.5 **b** 5.2 **c** 93.6

6 133.3 mL **7** 25% **8** 10 **9** $86.80

CHAPTER 3

EXERCISE 3.1

1 Teacher to check

2 **a** CD **b** DD **c** AD **d** PQ
e WR **f** OD **g** AD **h** CD
i PQ **j** DD **k** WR **l** OD

3 Teacher to check

4 Teacher to check

5 Encryption of data, restricted access to data, deletion of identifiable information when no longer required, staff and third-party providers are legally bound not to share information.

Other answers are possible.

EXERCISE 3.2

1 **a** systematic **b** random **c** self-selected
d stratified **e** self-selected **f** random
g systematic **h** self-selected **i** systematic
j stratified **k** systematic **l** self-selected
m random **n** stratified **o** systematic
p stratified

2 Teacher to check

3 Teacher to discuss answers with the class

4–6 Teacher to check

7 **a** 81 **b** 57 **c** 24

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8 **a** 33 **b** 19

9 **a** 99 **b** 47 **c** 52

10 **a** 48 **b** 27

11 **a** No. He only surveyed people who were in central Melbourne. He has excluded people from regional Victoria and people who work in other parts of the city.

b Teacher to check

EXERCISE 3.3

Teacher to check

EXERCISE 3.4

1 3 **2** 4

3 A 30-year-old member would have nowhere to tick. Better option:

- ☐ 30 years or under
- ☐ From 31 to 40 years
- ☐ From 41 to 50 years
- ☐ Over 50 years

Other answers are possible.

4 Which of the following do you attend?

- ☐ Concerts ☐ Special events
- ☐ Films ☐ Trips away
- ☐ Fundraisers ☐ Other

Other answers are possible.

5 ☐ Weekly

- ☐ Once a month
- ☐ 2–3 times per month
- ☐ Over 3 times per month

Other answers are possible.

6 For the activities you attend, the cost is:

- ☐ Cheap
- ☐ OK
- ☐ Expensive
- ☐ I don't attend activities because they are too expensive

Other answers are possible.

7 Teacher to check

8 A box could be provided. Other answers are possible.

9 Teacher to check

10 phone survey, observation, taking measurements, online data. Other answers are possible.

EXERCISE 3.5

1 **i** **a** use of 'fascinating' and 'dusty'

b use of 'disgusting'

c no negative options

d use of 'exciting' and 'boring'

e second half of question shouldn't be used

f no positive options

g use of 'quiet'

h use of 'greatest', yes/no response only

i use of 'one of those' and 'lame'

j no negative options

ii Teacher to check

2 Teacher to check

EXERCISE 3.6

Teacher to check

EXERCISE 3.7

1 **a** GIA $130, Surprise $160, FoodPlus $150

b $30

c the first graph

d The scale on the second graph starts at 0; the scale on the first graph only goes from $100 to $160.

e GIA to make their prices seem cheaper than their competitors but not too cheap.

2 **a** Brisbane **b** Darwin

c 144 c **d** about 14 c

e Candice only used 135 c to 155 c on the vertical axis and this makes the differences look bigger.

3 To make the increase look small: graph the actual population; To make the increase look big: graph the increase in the population.

4 **a** 120 c **b** $80

c The cost of petrol is in *cents*, the cost per barrel of crude oil is in *dollars*: vertical scale is different for each graph.

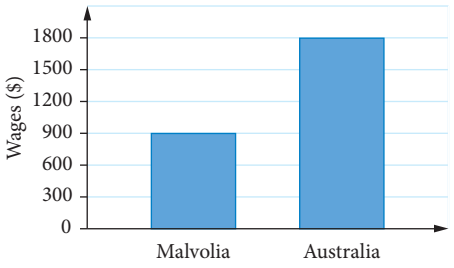
d The 2 graphs move up and down in a similar fashion: this suggests the cost of petrol and crude oil are related.

5 **a** That Australia's average weekly wages are *much* greater (4 times) than in Malvolia.

b No vertical scale, using a 3D picture instead of a column.

c columns

d



KEYWORD ACTIVITY

1 E	**2** K	**3** I	**4** A
5 G	**6** C	**7** B	**8** J
9 D	**10** F	**11** H	

TEST YOURSELF 3

1 Teacher to check

2 Sample, teacher to check reason

3 Teacher to check

4 **a** 75 **b** 59 **c** 16

5 Teacher to check

6 **a** knowing what time frame to consider

Other answers are possible.

b Make the question, 'How often do you drink each week?'

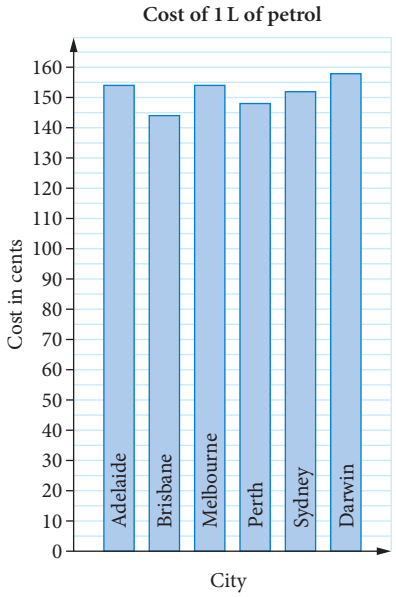
- ☐ once a week
- ☐ 2 to 3 times a week
- ☐ more than 3 times a week
- ☐ I don't drink

Other answers are possible.

7 Teacher to check

8 11 out of 20 students is not a big proportion to suggest strong support, those 11 students said they 'wouldn't mind' if school started earlier, not that they 'support', there is no mention of 'half an hour earlier' in the survey.

9



CHAPTER 4

EXERCISE 4.1

1 **a** 1 mm **b** 1°C **c** 1 minute
d 100 g **e** 1 cm **f** 1°
g 5 km/h **h** $\frac{1}{4}$ of a tank **i** 50 mL
j 1 kL **k** 250 g

2 C

3 B

4 **a** **i** 1 m **ii** 4 m **iii** 0.5 m
b **i** 1 mm **ii** 18 mm **iii** 0.5 mm
c **i** 5 km/h **ii** 51 km/h **iii** 2.5 km/h
d **i** 0.5°C **ii** 37.8°C **iii** 0.25°C
e **i** 5 m/s **ii** 44 m/s **iii** 2.5 m/s
f **i** 500 rpm **ii** 6900 rpm **iii** 250 rpm

5

	Measurement	Absolute error	Lower limit of accuracy	Upper limit of accuracy
a	18 L correct to the nearest L	0.5 L	17.5 L	18.5 L
b	78 cm correct to the nearest cm	0.5 cm	77.5 cm	78.5 cm
c	35 kg correct to the nearest 5 kg	2.5 kg	32.5 kg	37.5 kg
d	9.4 s correct to 1 decimal place	0.05 s	9.35 s	9.45 s
e	730 km correct to the nearest 10 km	5 km	725 km	735 km

6 **a** 5 m **b** 235 m and 245 m **c** 245 m

7 **a** 12.16 m **b** 15.8 kg **c** 75.86 L

8 2956 mm

EXERCISE 4.2

1 **a** 0.5 cm **b** 29.5 cm and 30.5 cm **c** 1.67%

2 **a** 0.5 min **b** 30 s **c** 7.14%

3

	Item	Absolute error	Limits of accuracy	Percentage error
a	Fuel	2.5 L	277.5 L and 282.5 L	0.89%
b	Fence posts	25 posts	325 and 375 posts	7.14%
c	Fencing wire	10 m	530 m and 550 m	1.85%
d	Fertiliser	5 kg	865 kg and 875 kg	0.57%

4 1.47% **5** 0.0039%

6 **a** 15.2 mg – 0.33%, 800 g – 0.63%, 0.15 g – 3.33%

b 15.2 mg

7 **a** city block – 0.098%, basketball court – 1.923%

b The city block is more accurate.

EXERCISE 4.3

1 370 000

2 **a** $110 000 **b** $100 000 **c** $105 000

3 **a** $500 000 **b** $480 000 **c** $482 000

4 **a** 0.0046 **b** 0.026 **c** 0.020
d 0.000 47

5 **a** 4 490 000

b Teacher to check: can be any value from 5 055 000 to 5 064 999

c $3\,253\,789 \approx 3\,250\,000$

6 **a** $487 000 000 **b** $1 900 000 000

7 6 650 000

8 **a** 3000 **b** 3 **c** 10 000
d 0.005 **e** 20 **f** 0.7

9 **a** 0.67 **b** 0.0055 **c** 1.3
d 0.087 **e** 790 000 **f** 7.3
g 24 **h** 2.4 **i** 34

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10 **a** 352 000 kg **b** 4190 m
c 67.1 km/h **d** 14.8 mL
e 150 000 000 km

EXERCISE 4.4

1 **a** **i** 24 cm **ii** 24 cm and 32 cm
iii 8π cm **iv** 25.1 cm
b **i** 36 cm **ii** 36 cm and 48 cm
iii 12π cm **iv** 37.7 cm
c **i** 42 cm **ii** 42 cm and 56 cm
iii 14π cm **iv** 44.0 cm
d **i** 12 cm **ii** 12 cm and 16 cm
iii 4π cm **iv** 12.6 cm

2 **a** **i** 48 cm^2 **ii** 48 cm^2 and 64 cm^2
iii 16π cm^2 **iv** 50.3 cm^2
b **i** 432 cm^2 **ii** 432 cm^2 and 576 cm^2
iii 144π cm^2 **iv** 452 cm^2
c **i** 75 cm^2 **ii** 75 cm^2 and 100 cm^2
iii 25π cm^2 **iv** 78.5 cm^2
d **i** 363 cm^2 **ii** 363 cm^2 and 484 cm^2
iii 121π cm^2 **iv** 380 cm^2

3 **a** 1.4π m **b** 4.4 m

4 **a** 171.61π m^2 **b** 540 m^2

5 20 000 km

6 370 000 mm^2

7 **a** 190 cm **b** ≈ 477 turns

8 **a** 100.5 mm **b** 804.2 mm^2

EXERCISE 4.5

1 **a** **i** $\sqrt{74}$ cm **ii** 8.6 cm
b **i** $\sqrt{19.4}$ cm **ii** 4.4 cm
c **i** $\sqrt{2}$ m **ii** 1.4 m

2 **a** **i** $\sqrt{84}$ cm **ii** 9.2 cm
b **i** $\sqrt{66}$ cm **ii** 8.1 cm
c **i** $\sqrt{6.08}$ cm **ii** 2.5 cm

3 **a** $\sqrt{13}$ m **b** 3.6 m

4 68.7 cm **5** 3.58 m **6** 10.39 cm

7 3.1 m **8** 38 m

KEYWORD ACTIVITY

Teacher to check

TEST YOURSELF 4

1 **a** 1 g **b** 0.5 g

2 62.5 g and 67.5 g

3 3.85% **4** 0.60%

5 **a** 740 000 **b** 8 400 000 000 **c** 4000
d 0.000 72 **e** 0.000 003 5 **f** 0.000 21

6 265 000 **7** 51 000 km **8** 2040 cm^2

9 **a** $\sqrt{72}$ m **b** 8.5 m

PRACTICE SET 1

SECTION A

1 C **2** B **3** D **4** B
5 C **6** A **7** B **8** A
9 C **10** B

SECTION B

1 **a** 5 kg **b** $112 500
c 192 minutes **d** 6480 mL

2 **a** $5896 **b** $596

3 **a** 92% **b** 3%
c 15% **d** 25%

4 **a** 80 minutes **b** 10.10 am

5 Many answers possible.
Sample answer:
☐ Never
☐ Once per month
☐ 2 or 3 times per month
☐ 1 or more times per week

6 201 cm^2

7 **a** 84 **b** 45 **c** 13

8 24

9 **a** It only includes data for people in Bendigo.
b Many answers are possible. For example, he could interview every 1000th person on the electoral role.

10 **a** 79 **b** $\frac{1}{5}$ **c** 29
d −5 **e** 50

11 **a** It uses emotive language and only has favourable alternatives.
b Many answers possible. For example:
Rate your meal at Diggies' Steak House:
☐ Poor
☐ Satisfactory
☐ Good

12 $80 217 **13** 23.31%

14 $856.47 **15** 4.5 m

16 Teacher to check

CHAPTER 5

EXERCISE 5.1

1 **a** 0.156 mL **b** 0.625 mL **c** 0.078 mL

2 2 **3** 64

4 A smidgen is half a pinch.

5 16 **6** 4 **7** 80 mL

8 7.5 **9** 220 mL **10** $\frac{1}{4}$ cup

11 The 12-hole muffin tray is for mini muffins.
The 8-hole muffin tray is for large muffins.

EXERCISE 5.2

1 4.83 km **2** 134.99 nautical miles

3 **a** 12.87 **b** 7.46 **c** 155.45
d 40.64 **e** 4.92 **f** 118.53
g 58.32 **h** 25.40

4 1085 m **5** 42.2 km

6 100 mm by 50 mm

7 **a** 4448 km **b** 200 hours
c 8 days 8 hours **d** 10 am on July 12

8 **a** 2778 km **b** 284.9 km/h, 153.8 knots

9 **a** 24.14 km/h **b** 25.98 km/h

10 **a** 192 pounds **b** 90 kg
c 0.454 kg **d** 114 g

EXERCISE 5.3

1 57 L **2** 8 **3** 20% **4** 83.3%

5 **a** 159 L **b** 35 imperial gallons

6 1.9 L

7 **a** 98 mL **b** 17%

8 **a** 73 L **b** 40 L **c** 16 L **d** 6 L

9 **a** 88.2 million gallons **b** 334 million litres

EXERCISE 5.4

1 **a** 26 cm **b** 21.2 cm **c** 57.5 cm **d** 25 cm

2 **a** 37.7 cm **b** 56.5 mm **c** 13.2 m **d** 19.5 km

3 **a** 49.5 m **b** 70.4 cm **c** 49.4 mm **d** 32.4 km

4 **a** 144 cm **b** 400 cm **c** 201 cm **d** 170 cm
e 231 cm

5 37.7 cm

6 **a** 44 m **b** $924

7 21.5 cm **8** 17 mm **9** 4.9 mm

EXERCISE 5.5

1 **a** 54 m^2 **b** 24 cm^2
c 55 mm^2 **d** 24 m^2

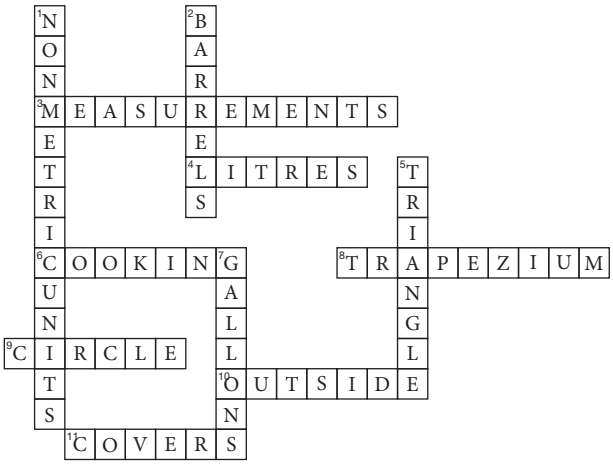
2 **a** 36 cm^2 **b** 121 m^2
c 60 mm^2 **d** 110 m^2
e 48 cm^2

3 **a** 314.16 cm^2 **b** 63.62 mm^2
c 25.13 cm^2 **d** 22.06 mm^2

4 **a** 270 m^2 **b** 116.55 cm^2
c 201 mm^2 **d** 94.27 cm^2

5 **a** 1925 mm^2 **b** 2625 mm^2
c 1600 mm^2 **d** 2123.7 mm^2

KEYWORD ACTIVITY



TEST YOURSELF 5

1 60 mL

2 **a** 30 g **b** approximately $\frac{1}{4}$

3 **a** 43.45 **b** 34.02 **c** 30.48

4 **a** 44.4 km **b** 40 minutes

5 24

6 **a** 20.6 mm **b** 50 m **c** 36 mm

7 **a** 54 mm^2 **b** 45 cm^2 **c** 160 m^2
d 101 cm^2 **e** 228 cm^2 **f** 5400 mm^2

CHAPTER 6

EXERCISE 6.1

1 **a** 0.09 ÷ 12 × $7000 = $52.50

b $7000 + $52.50 = $7052.50

c $7052.50 − $320 = $6732.50

d **A** $6732.50 **B** $50.49 **C** $6782.99
D $6462.99 **E** $6462.99 **F** $48.47
G $6511.46 **H** $6191.46

e $6191.46 **f** $151.46 **g** $4.03

h She owes less, so the interest is calculated on a smaller amount.

2 **a**

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	$15 696.00	$94.18	$15 790.18	$15 390.18
3	$15 390.18	$92.34	$15 482.52	$15 082.52
4	$15 082.52	$90.50	$15 173.02	$14 773.02

b $14 773.02 **c** 22.6%

3 **a** $720

b **A** $15 286 **B** $611.44 **C** $12 463.44
D $498.54 **E** $6734.10

c 4 months

9780170453943

4 a

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$12 800	$80.64	$12 880.64	$11 980.64
2	$11 980.64	$75.48	$12 056.12	$11 156.12
3	$11 156.12	$70.28	$11 226.40	$10 326.40

b $2700 **c** $2473.60 **d** $226.40

e The financial institution can sell the boat to recover the money owed to them.

5 a

	A	B	C	D	E
1	Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	1	10,000	=B2*0.06/12	=B2+C2	=D2-260
3	2	=E2	Copy down	Copy down	Copy down
4	3	Copy down			

b $9153.68

EXERCISE 6.2

1 a $29.17 **b** $27.30 **c** $1024.05

d 15 months **e** $335.93 **f** $235.93

g $5235.93

2 a 18 months **b** $195.43 **c** $195.43

3 a 18 months **b** $532.31

4 a 25 months **b** 18 months

c Yes, he will save $128.75.

d The quicker you repay a reducing balance loan, the less interest you will pay.

e No, the interest is a fixed amount.

5 a =B4, =B9*$B$5/12, =B9+C9, =D9–$B$6

b The dollar signs keep the interest rate and the repayment amount the same when we copy down.

6 =B3, =B9*$B$5/12, =B9+C9, =D9–$B$6, =E9

7 The diagram shows the final rows of the spreadsheet.

	A	B	C	D	E
39	31st month	$4,925.75	$30.79	$4,956.54	$4,006.54
40	32nd month	$4,006.54	$25.04	$4,031.58	$3,081.58
41	33rd month	$3,081.58	$19.26	$3,100.84	$2,150.84
42	34th month	$2,150.84	$13.44	$2,164.28	$1,214.28
43	35th month	$1,214.28	$7.59	$1,221.87	$271.87
44	36th month	271.87	$1.70	$273.57	–$676.43
45					
46					
47		Sum of interest = SUM(C9:C44			
48			$3,523.57		
49					

The spreadsheet shows that it will take 36 months for Gabriel to repay the loan. The interest is in column C. The formula =SUM(C7:C42) will produce the sum of the interest.

Gabriel will make 36 repayments and pay a total of $3523.57 in interest.

8 a 13 months **b** $214.45

9 longer/more time, more **10** shorter, less

EXERCISE 6.3

1 a $327 **b** $19 634 **c** $4634

2 a $616 **b** $70 808

c The car will be worn out before it is paid for.

d Save up for the car or buy a cheaper car.

3 a 17 **b** $645

4 a $190.40

b The loan is less by the 40th month as reducible interest is calculated on the amount owing.

c $229 407 **d** $165 096

e **i** $915.20 **ii** $919.20

f 49.9% **g** 22 years 5 months

h 91 months (or 7 years 7 months)

i at the beginning, when the loan amount is highest

5 a **i** 240 months **ii** 187 months

b 53 months **c** $25 290

d They can't afford to pay higher monthly repayments or they don't want to.

6 a $2301 **b** $36

7 a $1952 **b** $22 **c** $264

8 a $246 **b** $152 184

c Answers will vary. A sample answer is: If I could afford the extra $246 per month, I would pay it. If I couldn't afford the whole $246 per month, I would pay as much extra as I could afford.

9 $1266. Payments in the 1980s were more than double the late-2010s value.

10 a big bank $998, small mortgage company $989

b

	Big bank	Small mortgage company
Loan establishment fee	$320	$598
Mortgage discharge fee	$228	$314
Total annual loan fee over 25 years	$6200	$1900
Total monthly repayments	$299 400	$296 700
Total cost of the loan	**$306 148**	**$299 512**

c the interest rate and the annual loan fee

d The loan from the small mortgage company because she will save $6636.

EXERCISE 6.4

Teacher to discuss answers with the class.

EXERCISE 6.5

1 owner: a, b, d, f, h, j

tenant: c, e, g, i

2 a $4960, $6820 **b** $16 800, $23 100

c $3360, $4620 **d** $4600, $6325

3 a $369 **b** $485 **c** $471 **d** $330

4 a $124 000 **b** $420 000 **c** $64 000 **d** $85 000

5 a yes, 0.2 × $595 000 is less than $150 000

b $445 000 **c** $4760 per month

d $57 120 **e** $39 042

f Yes, her expected income from rent is greater than her expenses.

6 a

Item	Cost for buying	Cost for renting	Annual buying cost	Annual renting cost
Rent	0	$1430 per week	0	$74 360
Mortgage repayments	$3399 per month for 20 years	0	$40 788	0
Council rates	$1600	0	$1600	0
Insurance	$1359	$242	$1359	$242
Maintenance	$500	0	$500	0

b owning $44 247, renting $74 602

c $320 760

d Teacher to discuss

7 a 0.2 × $420 000 = $84 000 **b** $336 000

c $1507 **d** $2593

e $1 836 585 **f** $1 230 000

g Teacher to discuss

EXERCISE 6.6

1 a South Bank **b** $200.94

2 a $395 **b** $415

c $1440

3 a $125.03 **b** $6745.03

c 34.1 months **d** 31 years 1 month

e 4 years 5 months

f Sir Charge $1220, FAQ $2820

g Sir Charge $14.22, FAQ $29.35

h The longer zero-interest period is the better deal. He'll owe $1600 less and there isn't much difference in the interest rates.

4 a 25 years 9 months **b** $10 676

c It would only take her 16 months to pay off the card and she'd save $9991.

d She should pay as much each month as she possibly can.

5 a approximately $220 **b** $156.25

c $254 **d** 18 months

e $17 826

6 a $708.40

b $4576, $10 348, $2850, total $32 079.20

c $1815 **d** $333.40

e $6920.80

f The longer loan time results in more repayments.

7 a

Length of the loan	Monthly repayments	Total amount to be repaid	Interest charges
5 years	$541	$32 485	$6 885
10 years	$334	$40 115	$14 515

b $8096

c She could lose her home unit if she doesn't make all the repayments.

d If she changes to a 5-year loan the monthly repayments will be much lower, but she'll pay an extra $1211 in interest. There is also the risk of losing her home with the new loan if she doesn't make all the repayments. The current loan is cheaper and safer in the long run provided she can pay the higher repayments.

8 a $280 963

b If she keeps her current loans, she will repay a total of $401 186.40. If she takes out a consolidation loan, she will repay a total of $344 340, which will make her better off by $56 846.40.

KEYWORD ACTIVITY

1 credit union **2** interest **3** secured

4 flat **5** same **6** reducible

7 decreases **8** real estate **9** huge

10 repayment **11** minimum **12** fortnightly

13 money

TEST YOURSELF 6

1 a **A** $11 702 **B** $70.21

C $11 772.21 **D** $11 402.21

E $11 402.21 **F** $68.41

G $11 470.62 **H** $11 100.62

b $11 100.62 **c** $210.62

2 a 2 years 9 months **b** $3278.34

3 a $2136 **b** $512 602

c 15 years 10 months **d** $57 681

4 a It increases the value of the repayments.

b It decreases the time taken to pay off the loan and the total amount to repay.

5 $612 **6** $5400 to $7425

7 a $365 040 **b** $394 200

c He would be $26 028 worse off if he changed to St Dragon Bank.

CHAPTER 7

EXERCISE 7.1

1 a mean = 51.1 median = 47.5

modes = 45, 50 range = 40

b mean = 93.1 median = 100

mode = 100 range = 69

c mean = 38.3 median = 39

modes = 39, 41 range = 24

2 a The median, because it's not affected by outliers.

b The median or mode, because they're not affected by outliers.

c The modes, because they represent the most common values, or the mean because there are no outliers.

3 a B

b No, because the data are not numerical.

4 a $Q_1 = 5$, $Q_2 = 8$, $Q_3 = 12$, IQR = 7

b $Q_1 = 62$, $Q_2 = 83.5$, $Q_3 = 112$, IQR = 50

c $Q_1 = 20$, $Q_2 = 46.5$, $Q_3 = 82$, IQR = 62

5 a mean = $33.73

b median = $32.00

c median, teacher to check

d mean, teacher to check

e lower quartile, $Q_1 = 25$, upper quartile, $Q_3 = 35$

f IQR = 10

g The interquartile range, because most of the scores are close together.

6 a 11 calls **b** mean = 19 min

c 15 min **d** 15 min

e The mode or median is the best measure because they are not affected by the outlier of 90.

f 90 mins, because it would take a long time to get there.

7 a mean = $243 111.11, median = $120 000

b mean; Teacher to check

c lower quartile, Q_1 = $120 000; upper quartile, Q_3 = $300 000

d $180 000

e interquartile range; Teacher to check

8 a

	i Mean	ii Mode	iii Median	iv Range
Christina	20.9	19, 25	21.5	31
Mahla	21.9	17, 20	20	21

b Mahla

c Christina: $Q_1 = 15.5$, $Q_3 = 26$

Mahla: $Q_1 = 17$, $Q_3 = 27$

d Christina: IQR = 10.5

Mahla: IQR = 10

e, f Teacher to check

EXERCISE 7.2

1 a Binns: 16.4, Thomson: 25.3

b the last age in each list

c The much larger age of 91 makes the mean higher.

2 a $6 960 500 **b** $767 273

c The median, because it is more typical of most of the prices.

d $1 800 000

e $664 000, which is much closer to the median

3 a Mark: 43.9, Steve: 45.4 **b** Steve

c Mark: 45, Steve: 37 **d** Mark

e Teacher to check

4 a $154 000 **b** $70 500

c The median, because 10 out of 12 scores are close to the median.

d The mean, because it's higher.

e $275 000, $890 000 **f** $68 300

5 a mean = $63 400, median = $71 000

b The median, because 4 out of 5 years are close to this value.

c For example, if she wanted to minimise the tax she paid.

d $32 000

e mean = $71 250, median = $71 500

6 a 204 deliveries

b This doctor could be a specialist in difficult pregnancies and deliveries.

c mean = 30.65, median = 21.5

d mean = 21.5, median = 21

e The outlier has a big effect on the mean, but only a very small effect on the median.

7 a $355 000 **b** $431 125

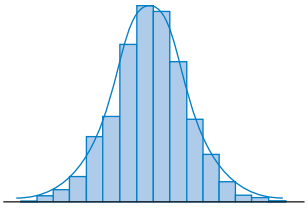
c The median, because it is lower.

d The mean, because it is higher.

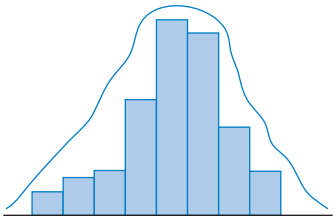
e $835 000, $1 700 000 **f** $362 222

EXERCISE 7.3

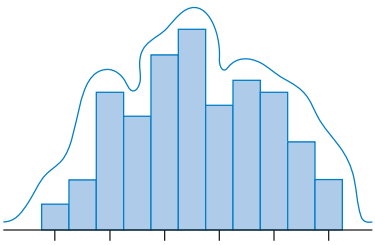
1 i



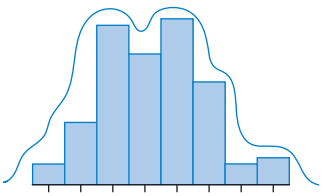
ii



iii



iv



b i symmetrical **ii** not symmetrical

iii not symmetrical **iv** not symmetrical

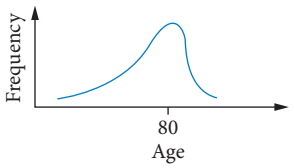
2 a negatively skewed **b** positively skewed

c bi-modal **d** positively skewed

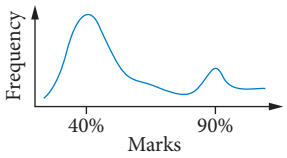
3 a, b and d

4 Many answers are possible. Teacher to discuss.

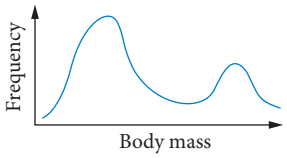
a



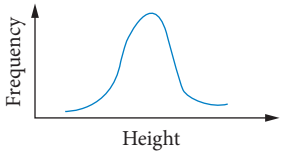
b



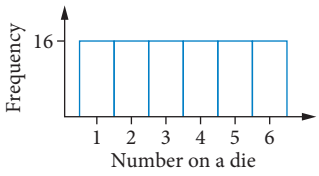
c



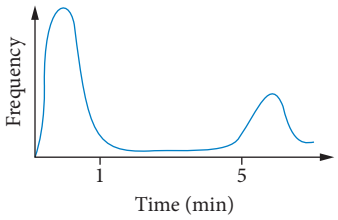
d



e

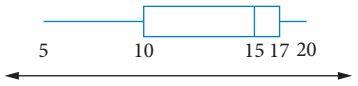


f



5 The tail extends on the right to 29, and half of the data lies from 5 to 9.

6 Teacher to discuss. The diagram shows a sample answer.



7 Many answers are possible. Teacher to check

8 **a** It has 3 peaks.

b Many answers are possible. It could be a coffee shop. The peak times correspond to people going to work, plus morning and afternoon tea times.

9 **a** positively skewed

b Remove scores 56, 57, 58, 59. Make 2 scores in the 80s and an extra 2 scores in the 70s.

10 **a** bi-modal

b Many possible reasons, for example:

- There are 2 warehouse employees and one works faster than the other.
- The information is from 2 different times (for example, weekdays and weekends). The weekend employees are only part-time and can't locate items as quickly as the full-time employees.
- Some items are more difficult to access than others.

11 **a** negatively skewed

b Easy, because most people got high marks.

c around 85

d Brook scored just below average and below the mode.

EXERCISE 7.4

1 **a** C **b** C **c** D **d** D
e D **f** C **g** C **h** D
i D **j** C

2 **a**

Heights	Frequency	Cumulative frequency
150–154	2	2
155–159	4	6
160–164	3	9
165–169	5	14
170–174	5	19
175–179	4	23
180–184	2	25
Total	25	

b continuous

3 **a**

No. of incidents	Frequency	Cumulative frequency
36–40	4	4
41–45	6	10
46–50	5	15
51–55	12	27
56–60	2	29
61–65	2	31
66–70	4	35
71–75	0	35
76–80	0	35
81–85	0	35
86–90	1	36
Total	36	

b yes, 90

c discrete

4 **a** 40 **b** 21 **c** 6 **d** discrete
e cumulative frequency: 21, 34, 38, 40

5 **a** 50 **b** 16 **c** 12 **d** discrete
e cumulative frequency: 3, 16, 31, 38, 50

EXERCISE 7.5

1 **a** 34

b

Price ($)	Class centre (cc)	Frequency (f)	Cumulative frequency	$f \times$ cc
5000–<6000	5500	3	3	16 500
6000–<7000	6500	5	8	32 500
7000–<8000	7500	6	14	45 000
8000–<9000	8500	12	26	102 000
9000–<10 000	9500	8	34	76 000
	Total	34		272 000

c **i** 8000–<9000 **ii** 8000–<9000 **iii** 8000

2 **a** 36

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b

Fuel cost ($)	Class centre (cc)	Frequency (f)	Cumulative frequency	$f \times$ cc
10–<20	15	9	9	135
20–<30	25	11	20	275
30–<40	35	5	25	175
40–<50	45	3	28	135
50–<60	55	2	30	110
60–<70	65	1	31	65
70–<80	75	2	33	150
80–<90	85	3	36	255
	Total	36		1300

c **i** 20–<30 **ii** 20–<30 **iii** 36.1

3 a 30

b

Marks (%)	Class centre (cc)	Frequency (f)	Cumulative frequency	$f \times$ cc
40–49	44.5	4	4	178.0
50–59	54.5	7	11	381.5
60–69	64.5	10	21	645.0
70–79	74.5	3	24	223.5
80–89	84.5	5	29	422.5
90–99	94.5	1	30	94.5
	Total	30		1945

c **i** 60–69 **ii** 60–69 **iii** 64.8

4 a

Class centre	Frequency	Cumulative frequency
10	8	8
30	4	12
50	40	52
70	20	72
90	8	80

b 54 **c** 70 – 44 = 26

5 a

Class centre	Frequency	Cumulative frequency
175	30	30
225	30	60
275	20	80
325	15	95
375	15	110
425	5	125
475	5	130

b 260 **c** 365 – 210 = 155

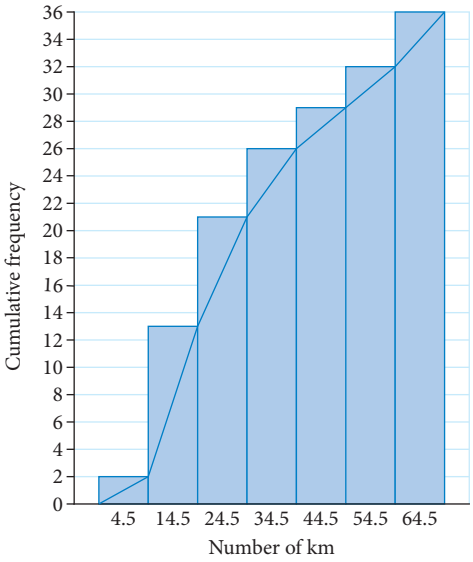
6 a 15 **b** 19 – 10 = 9

7 a

Class	Class centre (cc)	Tally	Frequency (f)	Cumulative frequency	$f \times$ cc
0–9	4.5	||	2	2	9.0
10–19	14.5	𝍸 ||	11	13	159.5
20–29	24.5	𝍸 |||	8	21	196.0
30–39	34.5	𝍸	5	26	172.5
40–49	44.5	|||	3	29	133.5
50–59	54.5	|||	3	32	163.5
60–69	64.5	||||	4	36	258.0
		Total	36		1092

b **i** 20–29 **ii** 10–19 **iii** 30.3

c



d **i** 25.5 **ii** 44 – 16 = 28

EXERCISE 7.6

1 a $30 000 **b** 80th percentile
c $10 000 **d** median
e 9%

2 a 25%
b **i** 95% **ii** 188 cm **c** 5 years
d **i** 178 cm **ii** 182 cm

3 a 20.6 **b** no, 25%
c 25.4 **d** yes, 3%
e 17.6

4 a 102.7 cm
b **i** no, 25% **ii** 106.2 cm
c 103.3 cm **d** no, 1% **e** 97.6 cm, 109.2 cm

EXERCISE 7.7

1 a mean = 20.78, σ = 3.05
b mean = 48.27, σ = 21.34
c mean = 14.55, σ = 1.83
d mean = 55.27, σ = 21.34

2 a $697.60 **b** 103.6 **c** Teacher to check

3 a $469.60 **b** 67.43 **c** Teacher to check
d Quite difficult, because the mean is higher than $420. Other answers are possible.

4 a 6.6 **b** 1.0

5 a 2.58 **b** 1.52

6 a 4.40 **b** 1.61

7 a Port Paradise: σ = 2.17, Palm Tree Cove: σ = 4.46
b Port Paradise, because the temperatures are more consistent (there is a smaller standard deviation).

8 a Brad: mean = 8.3, σ = 2.0
Aryn: mean = 4.5, σ = 1.7
Kim: mean = 6.5, σ = 2.6
b Aryn, because he has the lowest standard deviation.
c Brad, because he has the highest mean and a low standard deviation.

9 a 70, 71, 72, 73, 73, 74, 75, 75, 75, 75, 75, 76, 76, 77, 77, 77

b mean = 74.4, $\sigma = 2.06$

c Paul is a consistent player because his scores show a small standard deviation.

10 a You would expect a high standard deviation because the data are spread from 127 to 157.

b 10.08

c Teacher to check

11 a 46, 98, 102, 102, 110, 146, 168, 183, 194, 208, 210, 305

b mean = 156 median = 157
mode = 102 range = 259

c IQR = 201 – 102 = 99 **d** $\sigma = 66.9$

e i Either the mean or the median, because they are about the centre of the data.

ii The IQR, because this is not affected as much by the outliers at the top and bottom of the ordered data.

EXERCISE 7.8

1 mean = 30.71, mode = 29, median = 30, maximum = 35, minimum = 28, range = 7, interquartile range = 33 – 29 = 4, standard deviation = 2.31

2 Teacher to check

3

	Mean	Median	Mode	σ
2019	19.17	18.5	17	4.45
2020	16.83	17.5	20	2.79

d Theft was higher in 2019 and the number of thefts were spread over a wider range in 2019. Other answers are possible.

4 Teacher to check

5 a Teacher to check

b mean = 73.9, median = 71, $\sigma = 10.5$

c mean = 68.2, median = 69, $\sigma = 6.3$

d Teacher to check

6 a Teacher to check

b mean = 93.4, median = 94.5, mode = 81, $\sigma = 9.22$

c mean = 94.31, median = 95, mode = 81, $\sigma = 8.06$

d Without the outlier, the spread is reduced and the mean increases. Other answers are possible.

7 f mean = 2.8

KEYWORD ACTIVITY

1 central tendency **2** spread
3 mean **4** median
5 mode **6** range
7 quartiles **8** interquartile range
9 standard deviation **10** class intervals
11 class centre **12** cumulative frequency
13 cumulative frequency histogram
14 cumulative frequency polygon
15 percentiles **16** outliers
17 positively **18** negatively
19 uniform **20** bi-modal

TEST YOURSELF 7

1 a 7.71 **b** 7.0 **c** 8.0 **d** 2.9

e $Q_1 = 7.0$, $Q_2 = 8.0$, $Q_3 = 8.5$ **f** IQR = 1.5

2 a 15.4

b 14

c The median is better because the outlier raises the mean.

d 17

e $Q_1 = 13$, $Q_3 = 18$, IQR = 5

f The IQR, because the outlier makes the range much bigger.

3 a 300

b i 830 **ii** 962.5

c The outlier reduces the mean.

4 a 100 mm

b i mean = 21.8 mm, median = 19 mm

ii mean = 14.7 mm, median = 19 mm

c The outlier raises the mean.

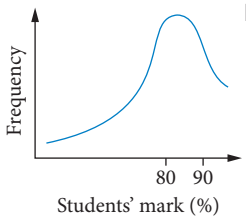
d The outlier has no effect on the median.

e The median, because it is unaffected by the outlier.

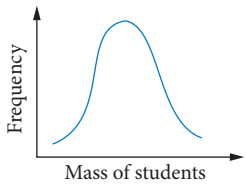
5 a positively skewed **b** bi-modal

c symmetrical

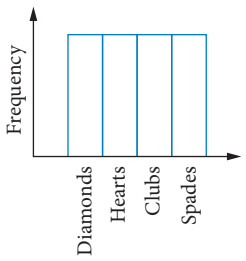
6 a



b



c



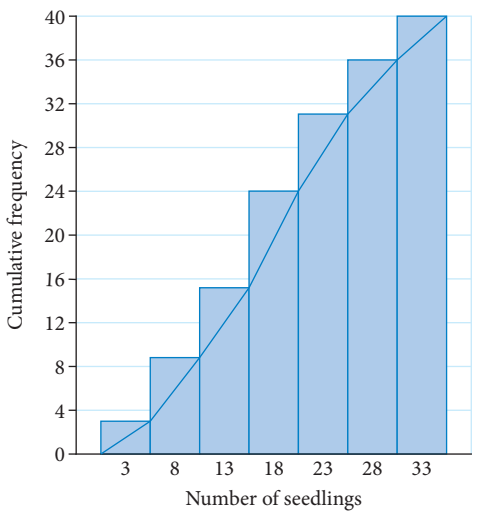
7 a

Number of seedlings	Class centre	Frequency	Cumulative frequency	$f \times cc$
1–5	3	3	3	9
6–10	8	6	9	48
11–15	13	6	15	78
16–20	18	9	24	162
21–25	23	7	31	161
26–30	28	5	36	140
31–35	33	4	40	132
	Total	40		730

b i 16–20 **ii** 16–20 **iii** 18.25

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8



a 18.5 **b** 25 – 11 = 14

9 **a** $42 000 **b** 97th **c** $20 000 **d** 6%

10 5.05

11 **a** 1.91 **b** 1.50

12 Teacher to check spreadsheet
mean = 6.71, mode = 5, median = 7, max = 12, min = 1, range = 11, IQR = 4, SD = 3.24

CHAPTER 8

EXERCISE 8.1

1 Teacher to check

2 **a** 4 **b** octagon **c** 12 **d** regular

3 Teacher to check

4 **a** 180° **b** 360°

5 Teacher to check drawing; pentagon

6 Teacher to check

7 **a** F **b** T **c** T **d** T
e F **f** F **g** T **h** F

EXERCISE 8.2

1 **a** 1 **b** 2 **c** 4 **d** 4
e 2 **f** 1

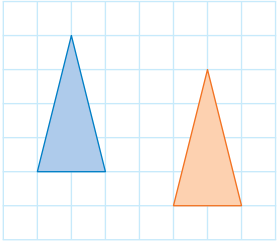
2 **a** 2 **b** 4 **c** 4 **d** 3
e 8 **f** 10

3 **a** 5 **b** 4 **c** 5 **d** 14
e 9 **f** 12

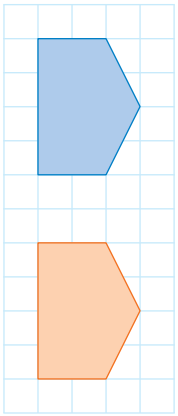
4 **a** both **b** no symmetry **c** line
d no symmetry **e** both **f** line
g line **h** both **i** rotational

EXERCISE 8.3

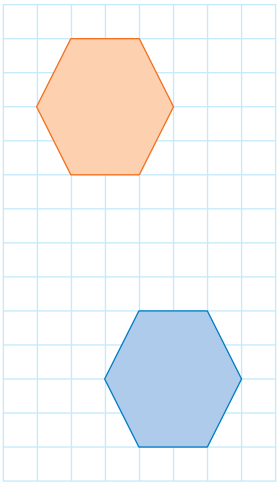
1 **a**



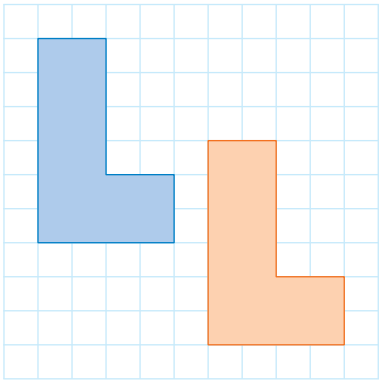
b



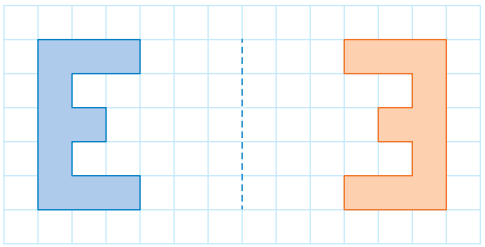
c



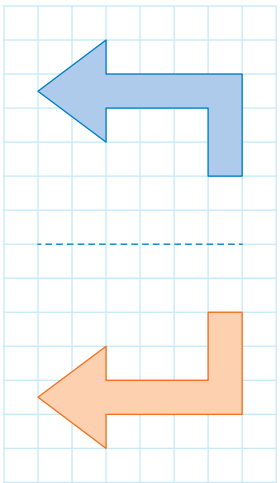
d



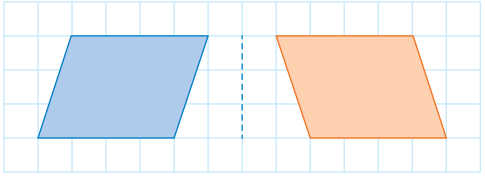
2 **a**



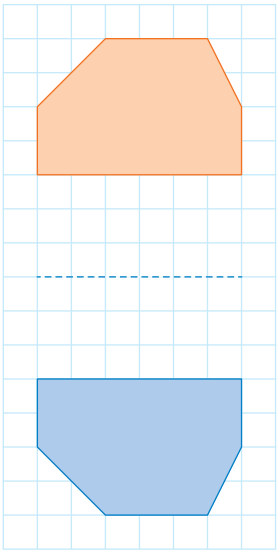
b



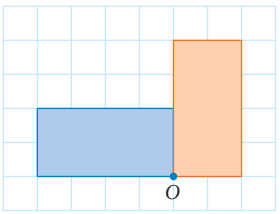
c



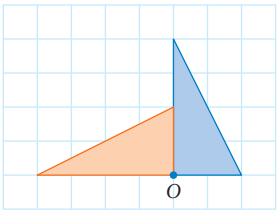
d



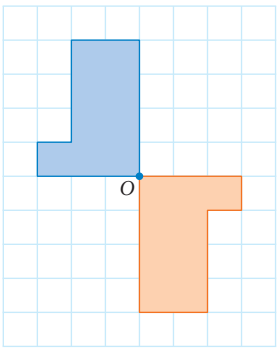
3 a



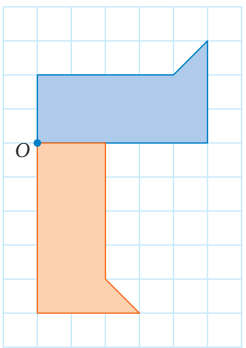
b



c



d



4 Rotation by 180° in either direction gives the same result.

5 a rotation and translation, or 2 reflections, or reflection and translation

b rotation and translation

c rotation and reflection, or reflection

EXERCISE 8.4

1 a *ABCD* and *IJKL* **b** ΔSTU and ΔXVW

c *ABCD* and *KJIL* **d** *UVWXY* and *JKLMN*

2 a similar

i Teacher to check **ii** Teacher to check

iii 3 : 2

b not similar

c similar

i Teacher to check **ii** Teacher to check

iii 5 : 7

d not similar

3 a $x = 44$ **b** $y = 60$ **c** $m = 150$

d $n = 100$ **e** $p = 55$ **f** $q = 32$

4 a i yes **ii** no **iii** yes

b i similar: same shape; one is an enlargement or reduction of the other

ii not similar: measured length and width, ratio is different

iii similar: same shape; one is an enlargement or reduction of the other

5 Teacher to check

6 a $x = 15$ **b** $d = 10$ **c** $p = 9$ **d** $k = 15$

7 C

8 a $y = 3\frac{1}{3}$ **b** $x = 77$ **c** $q = 6$

d $a = 9.6$ **e** $k = 24.75$ **f** $x = 20$

9 a $x = 5$ **b** $w = 12$ **c** $y = 14.4$ **d** $a = 40.5$

10 306 cm

EXERCISE 8.5

1 equilateral triangle, square, hexagon

2 equilateral triangle and square (2 types); hexagon and equilateral triangle (2 types); and octagon and square; dodecagon, square and hexagon; hexagon, square and equilateral triangle; dodecagon and equilateral triangle

3 a equilateral triangle, square, hexagon

b equilateral triangle, square

c hexagon, equilateral triangle

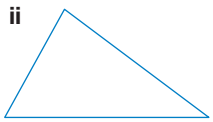
d dodecagon, hexagon, square

4–6 Teacher to check

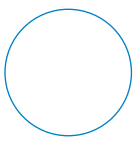
EXERCISE 8.6

1 a i triangular prism **b i** cone

ii



ii

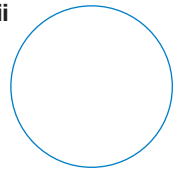


c i square pyramid **d i** sphere

ii



ii



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2 **a** 6 **b** 8 **c** 8

3 **a** cube, square pyramid

b rectangular prism, cylinder

c cylinder, cone

d cone, hemisphere

e 2 triangular pyramids

f 2 rectangular prisms

4 **a** T **b** T **c** T

d T **e** F **f** F

EXERCISE 8.7

1 Teacher to check

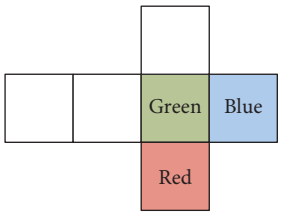
2 **a** B; triangular prism **b** C; square pyramid

c A; tetrahedron **d** F; pentagonal pyramid

e E; rectangular prism **f** D; pentagonal prism

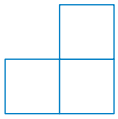
3 **a** hexagonal pyramid **b** Teacher to check

4

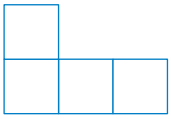


Other answers are possible.

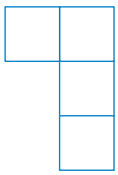
5 **a** **i**



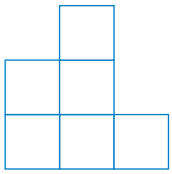
ii



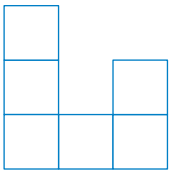
iii



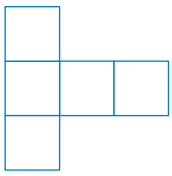
b **i**



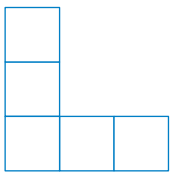
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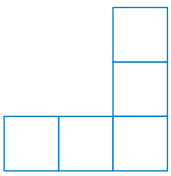
iii



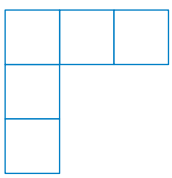
c **i**



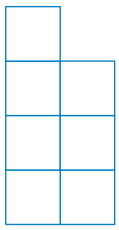
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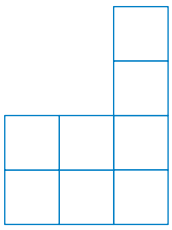
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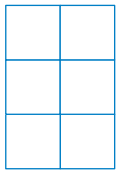
d **i**



ii

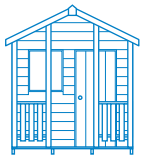


iii

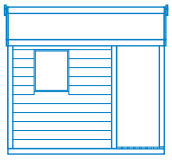


6 Teacher to check

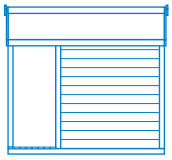
7



Front

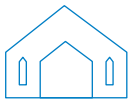


Left side

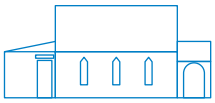


Right side

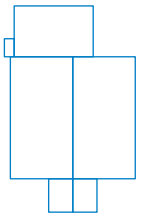
8



Front



Left side



Bird's eye

EXERCISE 8.8

Teacher to check

KEYWORD ACTIVITY

1 D **2** M **3** G **4** A

5 J **6** L **7** I **8** K

9 C **10** B **11** H **12** E

13 F

TEST YOURSELF 8

1 Teacher to check

2 **a** equilateral triangle

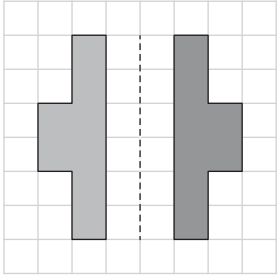
b square, rhombus

c octagon

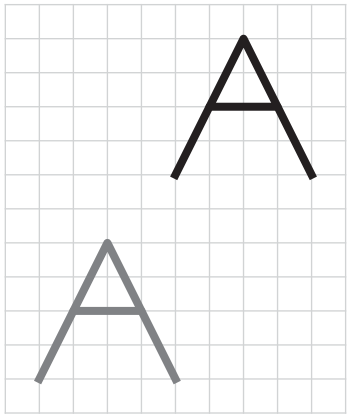
3 **a** 2 **b** 8 **c** 1 **d** 2

4 **a** yes, 4 **b** yes, 5 **c** yes, 2 **d** yes, 8

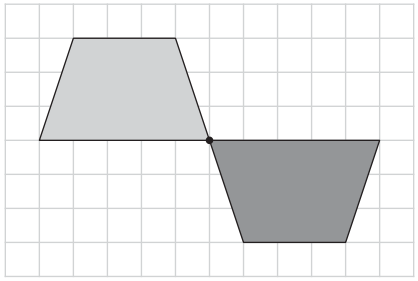
5 **a**



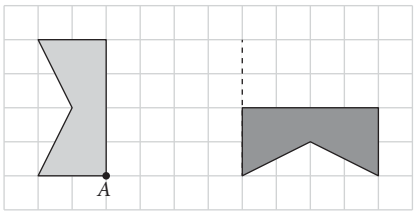
b



c



6



7 **a** PQ and YX, or PS and YZ, or SR and ZW, or QR and XW.

b $\angle P$ and $\angle Y$, or $\angle Q$ and $\angle X$, or $\angle R$ and $\angle W$, or $\angle S$ and $\angle Z$.

8 **a** $y = 3$ **b** $y = 45$

9 Teacher to check

10 **a** pentagonal prism **b** cylinder

c triangular pyramid

11 Teacher to check

12 **a** rectangular prism **b** triangular prism

c cone **d** square pyramid

e cube **f** cylinder

13 Teacher to check

14 Teacher to check

PRACTICE SET 2

SECTION A

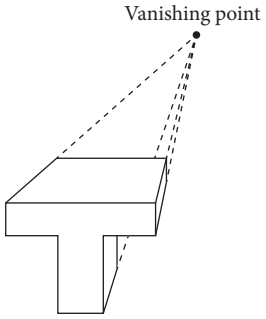
1 C **2** A **3** C **4** D

5 A **6** B **7** A **8** B

9 B **10** D

SECTION B

1



2 **a** \$54.17 **b** \$6577.25 **c** 30 months

d \$211.15 **e** \$841.15

3 **a** 40 **b** 6

c 21–30 **d** 31–40

4 **a** rectangular prism **b** square pyramid

c triangular prism **d** cylinder

5 **a** \$2560 **b** \$471 600

6 **a** 4415 km **b** 519.4 km/h, 280.5 knots

7 vertices = 9, edges = 16, faces = 9

8 perimeter = 3.44 m, area = 5300 cm^2 or 0.53 m^2

9 189.25 L

10 **a** \$19 710 **b** \$147.83

c \$19 857.83 **d** \$19 417.83

e \$19 417.83 **f** \$145.63

g \$19 563.46 **h** \$19 123.46

i \$19 123.46 **j** \$443.46

11 **a** 3 years **b** \$2338.63

12 median = 50, mode = 50, range = 40, mean = 55.1

13 The interest reduces.

14 **a** $Q_1 = 12$, $Q_2 = 17$, $Q_3 = 27$ **b** 15 **c** 30

15 **a** C **b** C **c** D **d** D

e D **f** C

16 Teacher to check. Sample drawing:



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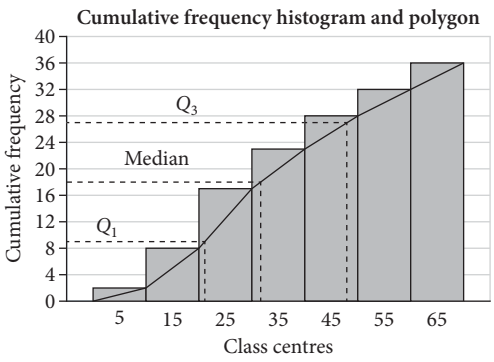
17 a

Class	Class centre cc	Tally	Frequency f	Cumulative frequency	$f \times$ cc
0–<10	5	||	2	2	10
10–<20	15	𝍸 |	6	8	90
20–<30	25	𝍸 ||||	9	17	225
30–<40	35	𝍸 |	6	23	210
40–<50	45	𝍸	5	28	225
50–<60	55	||||	4	32	220
60–<70	65	||||	4	36	260
		Total	36		1240

b **i** 30–<40 **ii** 20–<30 **iii** 34.4

c 63

d



e **i** ≈ 31 **ii** 48 – 21 = 27

18 a 21.2 **b** 19.4

c Yes, it's higher than more than 90% of the girls her age.

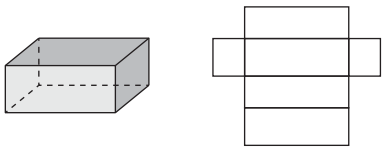
d 25% **e** 75%

19 a 1.77 **b** 1.43

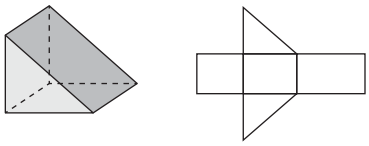
20 a true **b** false **c** false **d** true

e false **f** false **g** true

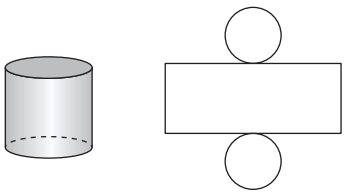
21 a



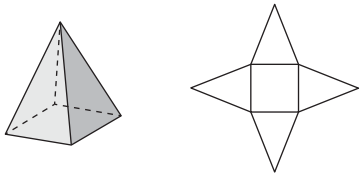
b Other answers possible.



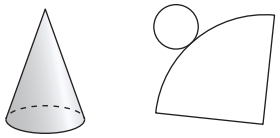
c



d



e



22 Teacher to check

CHAPTER 9

EXERCISE 9.1

1 a $9ab + 2$ **b** $4ab + 3bc$ **c** $9k^2 + 11$

d $4 - 2x$ **e** $x^2 - 7x + 7$ **f** $4p^2$

g $14 + 12y$ **h** $7r + 3s$ **i** $6g + 7b$

j $6x^2 - 13x + 6$ **k** $52a$ **l** $7q - 7$

m $-6y$ **n** 12

2 a $12d^2$ **b** $120mn$ **c** $48y^2$ **d** $27r^2$

e $18ac^2d$ **f** $h^2j^3k^2$ **g** $24p^2q^2$ **h** $6m^2n^2$

i $2m^2$ **j** $-3e$ **k** $-5p$ **l** $7x$

m $5de$ **n** a **o** $-8a$ **p** $-2m$

3 a $8m + 10n$ **b** $15k^2 - 10k$ **c** $21j^2 + 7jk$

d $8n^2 + 20mn$ **e** $6p - 22$ **f** $3q - 4m$

g $17 - 7t$ **h** $18n + 4$ **i** $5a + 14$

j $8x + 4$ **k** $n + 7$ **l** $22y + 23$

4 a −150 **b** 25 **c** −27 **d** 2

e −15 **f** 75 **g** −21 **h** 6

i 18 **j** −10 **k** −90 **l** $\frac{1}{2}$

5 42 m² **6** $212 **7** 78.8°F **8** 11 h

EXERCISE 9.2

1 a $a = 13$ **b** $x = 19$ **c** $p = 14$ **d** $h = 44$

e $q = 50$ **f** $n = 6$ **g** $y = 136$ **h** $w = 61$

2 a $a = 53$ **b** $x = 5$ **c** $p = 33$ **d** $m = 9$

e $y = 80$ **f** $g = 36$ **g** $t = 45$ **h** $d = 184$

i $x = 6$ **j** $x = 9$ **k** $x = 12$ **l** $N = 5$

m $x = 5$ **n** $x = 9$ **o** $N = 3$ **p** $N = 23$

3 a $m = 4$ **b** $n = 3$ **c** $y = 5$ **d** $p = 5$

e $d = 12$ **f** $t = 14$

EXERCISE 9.3

1 48.5 cm **2** $a = 3$ **3** 25 km

4 18.5 cm **5** 150 volts **6** 30 mL

7 a 30°C **b** 120

8 a 241 228 km/day **b** 15 278 km/h

9 23.1 days, to 1 decimal place

10 a 3 **b** 41

11 a $1368.03 **b** $27 045, to the nearest $

12 1.8 s

13 a $l = 32$, $R = 1.3$, $32 = S \times \frac{210 + 97 \times 1.3}{1000}$

$32 = S \times 0.3361$

b 95.2 km/h

EXERCISE 9.4

1 **a** 40

b The number of chirps when the temperature is 0°C.

c 8

d The increase in the number of chirps per minute when the temperature increases by 1°C.

e 296 **f** 15°C **g** 840, not realistic

2 **a** 0

b −3.5

c The number of degrees lower at which water boils for every extra kilometre in altitude.

d about 90°C

3 **a** Laura: 14, Bevan: 8, Jacob: 3

b speed

c Jacob: he has the smallest gradient, or he didn't travel as far as the others in 1 hour.

d Laura bike, Bevan jog, Jacob walk

4 No, the graph says the head circumference should be about 35 cm, which it is.

5 **a** \$40 **b** $\frac{1}{4}$ or 0.25

c additional cost per ticket **d** \$165

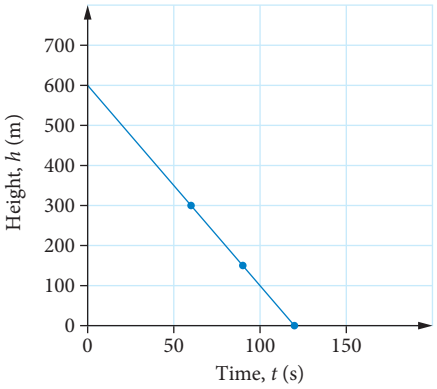
e $D = \frac{1}{2}n + 110$

6 **a** \$50 **b** \$1 **c** $C = n + 50$

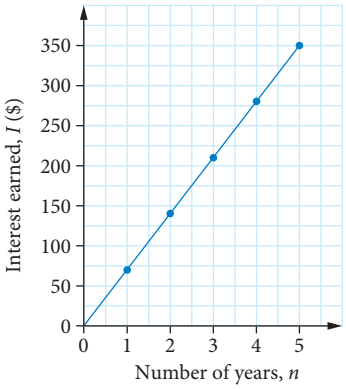
7 **a** 600 m **b** 5

c She started at 600 and fell 5 m every second (−5).

d



8 **a**

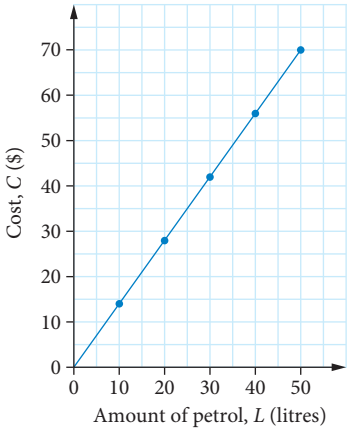


b 70, amount of interest each year

c 0, amount of interest at the start

d \$490 **e** \$2140

9 **a**



b 1.4 **c** \$105

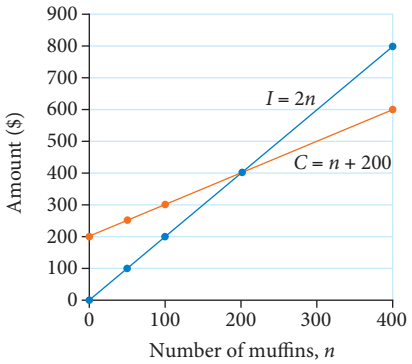
10 **a** Teacher to check **b** 2.5 m

c Yes, 4.2 m is more than 4 yards.

EXERCISE 9.5

1 **a** Teacher to check

b



c (200, 400) **d** break-even point

e No, he needs to sell more than 200 muffins to break even.

f \$30

2 **a** B **b** 3 minutes **c** B

3 **a** truck 72 km/h, car 96 km/h

b 2 **c** 2 seconds

d car: $m = 96 - 24t$, truck: $m = 72 - 12t$

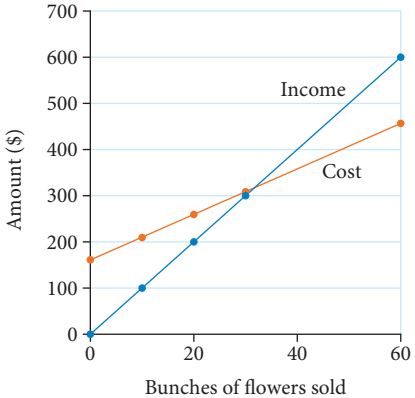
e Both vehicles were slowing down.

f The truck was heavier.

4 **a**

Bunches of flowers sold	0	10	20	30	40	50	60
Income received ($)	0	100	200	300	400	500	600

b



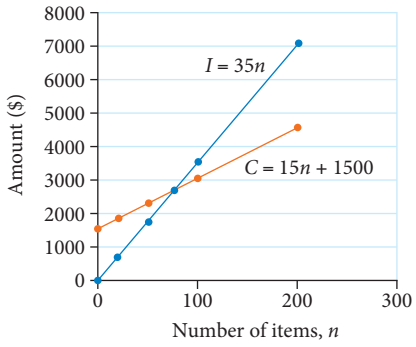
c The graphs intersect at (32, 320), which is the break-even point.

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d more than 32 bunches

5 **a** income **b** expenses

c



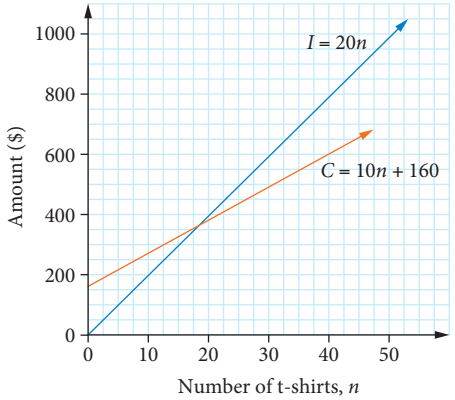
d 75 **e** $500

6 **a** $60 **b** 10 **c** $6

d It's the weekly cost of running the dog sleds, such as feeding the dogs. It doesn't change with the number of dog sled rides.

7 **a** $C = 10n + 160$ **b** $I = 20n$

c



d 16 **e** $740

8 **a** $40 **b** $10

c monthly service charge **d** 20 c ($0.20)

e $D = 0.2n + 10$ **f** 150

g Plan A

EXERCISE 9.6

1 **a** $x = 5, y = 9$ **b** $x = 4, y = 12$

c $x = -3, y = -7$ **d** $x = 2, y = -8$

2 20 people **3** 15

4 2 hours

5 **a** This is her hourly rate ($55) multiplied by the number of hours (h) she works.

b This is his $50 call-out fee, plus his hourly rate ($45) multiplied by the number of hours (h) he works.

c 5 hours

6 **a** $C = 70h$ **b** $C = 90 + 55h$ **c** 6 hours

KEYWORD ACTIVITY

1 subtracting **2** algebraic

3 expanding **4** substitution

5 equations **6** formulas

7 model **8** interest

9 depreciation **10** intersecting

11 simultaneous **12** income

13 costs **14** break-even

15 profit **16** loss

TEST YOURSELF 9

1 **a** $18m$ **b** $10x + 10y$ **c** $28h + 7g$

d $32w^2$ **e** $36kj$ **f** $60d^2e^2$

g $11m$ **h** 5 **i** $7d^2$

j $13b$

2 **a** $6n + 12$ **b** $-10b - 35$ **c** $9j^2 - 3j$

d $18 + 2n$ **e** $19v - 15$ **f** $9y - 18$

3 **a** 3 **b** −10 **c** 19

d 24 **e** −15 **f** 29

4 **a** $k = 7$ **b** $x = 11$ **c** $f = 12$

d $m = 28$ **e** $p = 5$ **f** $x = 15$

g $h = 77$ **h** $n = 40$ **i** $x = 1$

j $u = 3$ **k** $h = 3$ **l** $x = 5$

5 6 scarves

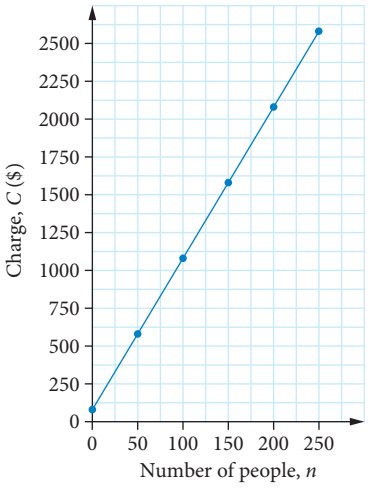
6 **a** 1260° **b** 8

7 **a** 3 minutes **b** 2

c time to pack 1 item **d** 6 items

e 33 minutes

8 **a**

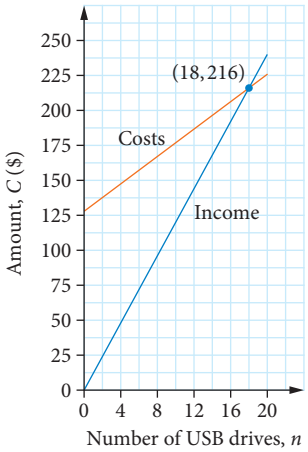


b $1780 **c** 75

d 80, the fixed charge **e** 10, the cost per person

9 **a** $C = 5n + 126$ **b** $I = 12n$

c



d 18 **e** $364

10 $x = 3, y = 23$ **11** 18 USBs

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CHAPTER 10

EXERCISE 10.1

1 **a** 3:2 **b** 7:3
c 3:1:4 **d** 2:5:3:7
2 **a** 15:19 **b** 19:15
c 4:19 **d** 15:4
3 50 kg
4 **a** 7 mL **b** 32 mL
5 **a** 200 kg, 250 kg **b** 270 kg, 180 kg
6 $360
7 **a** 3:2 **b** 3:5
8 **a** Toudi $990, Felicity $540 **b** $2250
9 4 m, 6 m, 10 m
10 25.6 tonnes **11** 568 students
12 **a** 30 000 **b** 165 000

EXERCISE 10.2

1 **a** 30 m **b** 116 m **c** 190 m
2 **a** 1:10 000 **b** 1 km **c** 580 m
d 1 km **e** 11.20 am
3 **a** 630 m (map length ≈ 3.5 cm)
b approximately 10.5 minutes
4 **a** 200 km **b** 36 km/h
c The average speed is low. The road is probably not sealed and could be rough in places.
5 **a** 20 m **b** 60 m and 114 m **c** $4872
6 **a** Even numbers from 156 to 190 and 175, 177, 181.
b 129 m **c** 307 m
d approximately 21 m
e It will flood during 1-in-50-year floods.
7 **a** 11.6 km **b** 2 h 54 min
c south-west of North Era

EXERCISE 10.3

1 Gus: $140 000, Judy: $100 000
2 600 g **3** 1.52 m **4** 35 blue cars
5 **a** 1:1:2
b Phuong: $1000, Janelle: $1000, Ahmet: $2000
6 **a** $312 000 **b** James: $23 250, Eleanor: $38 750
7 **a** 3000 mL
b

Concentrate required (mL)	Water required (mL)
200	2800
150	2850
300	2700

8 **a** 3:5 **b** 45 g
9 **a** 60 mL **b** 100 mL
c lanolin: 80 mL, neatsfoot oil: 120 mL
10 **a** 120 L **b** 8 L **c** 340 L
11 **a** water: 250 g, yeast: 25 g **b** 4 kg
c flour: 48 kg, water: 30 kg
12 dye A: 75 mL, dye B: 45 mL

EXERCISE 10.4

1 **a** $C = kL$ **b** $k = 1.4$ **c** $C = 1.4$ L
d $105
2 **a** 6 kg **b** $M = kE$ **c** $k = \frac{1}{6}$
d $M = \frac{1}{6}E$ **e** 14 kg **f** 264 kg
3 **a** $148.40 **b** 21.2 **c** pay rate per hour
d $784.40 **e** 17 hours
4 **a** $400 **b** 450 **c** $2
d $A = 2n$ **e** $2750 **f** 1600 tickets
5 **a** $D = kL$ **b** $k = 12.5$ **c** $D = 12.5$ L
d 937.5 km **e** 20 L **f** Teacher to check

EXERCISE 10.5

1 **a** $750 **b** $C = \frac{k}{N}$ **c** $k = 750$
d $75 **e** 30
2 **a** 480 km **b** $T = \frac{k}{S}$ **c** $k = 480$
d 4 h 48 min **e** 60 km/h
3 **a** $T = \frac{k}{N}$ **b** $k = 300$ **c** 25 days **d** 40
4 **a** $T = \frac{k}{S}$ **b** $k = 1500$ **c** 60 s **d** 50 Mbps
5 **a** 16 m^2 **b** $W = \frac{k}{L}$ **c** $k = 16$ **d** 4 m
e 16 m

KEYWORD ACTIVITY

Teacher to check

TEST YOURSELF 10

1 **a** 12:7 **b** 4:7 **c** 5:4 **d** 12:5
2 **a** 45 cm, 36 cm **b** 99 cm
3 25 more chocolate ice creams
4 **a** about 950 m **b** about 430 m
5 6.4 km
6

Nitrogen (kg)	Phosphorus (kg)	Potassium (kg)	Total mixture (kg)
54	36	45	135
42	28	35	105
105	15	30	150
140	20	40	200

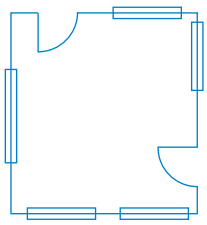
7 nitrogen 75 kg, phosphorus 15 kg, potassium 30 kg
8 **a** $3600 **b** $R = kS$ **c** 30
d $R = 30S$ **e** $55 500 **f** 260 m^2
9 **a** $T = \frac{k}{N}$ **b** $k = 60$
c 4 minutes **d** 30 students

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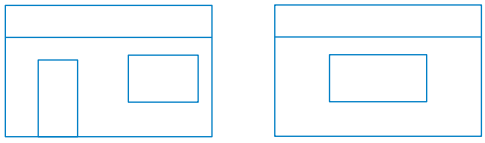
CHAPTER 11

EXERCISE 11.1

1 a



b



Eastern elevation | Western elevation

2–5 Teacher to check

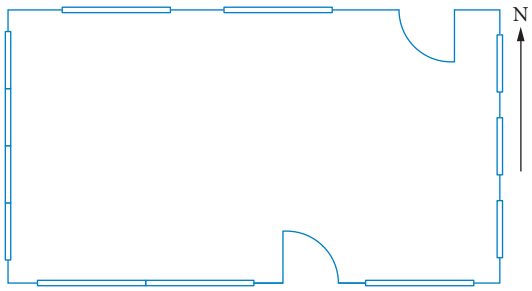
EXERCISE 11.2

1

Actual size		
mm	m	km
1000	1	
8000	8	
5000	5	
6 400 000	6 400	6.4
75 000 000	75 000	75.0
45 000 000	45 000	45.0
230 000 000	230 000	230.0

2 a 7.6 m **b** 4 m

c



3 a 4.2 m by 3.6 m **b** 3.6 m by 4.8 m
c 2.4 m by 3 m **d** 4.2 m
e 84 cm

4 a 10.5 m **b** 3 m **c** 4.95 m **d** 2.25 m

5 a 32 m **b** 12.8 m **c** 1.6 m **d** 17.6 m
e 12.8 m **f** 3.2 m **g** 409.6 m^2

6 a 5 m **b** 2.5 m **c** 2 m
d 2 m by 2.5 m = 5 m^2
e 5 m **f** 4 m by 3.8 m = 15.2 m^2

EXERCISE 11.3

1 a i kitchen sink **ii** vanity (or wash basin)
iii toilet **iv** window
v bath **vi** built-in wardrobe
b They are all tiled. **c** 3
d 1 and 2 **e** 4100 mm × 3200 mm
f in the back of the garage
g in the laundry **h** 1
i 1 **j** 2
k living room **l** 6670 mm × 5500 mm
m i south **ii** north **iii** south
iv east **v** west
n south **o** living room
p living room, dining room, bedroom 1, bedroom 3
q dining room, bedroom 2, living room

2 a 3 **b** 3
c no **d** 4.7 m × 3.5 m
e bedroom 1 **f** $86 016

3 a 6790 mm × 3700 mm **b** 2
c outside the bathroom **d** shower
e bedroom 4
f family room = 25.123 m^2, media room = 23.787 m^2
Family room is larger by 1.336 m^2.

4 a 240 mm **b** 70 mm **c** 12.39 m
d 15.69 m **e** 3.6 m × 6.07 m

EXERCISE 11.4

1 a 7.5 m^2 **b** $273.75

2 a 18 m **b** $136.80

3 a 92 m^2 **b** $149 960

4 a 14 m **b** 13 m **c** $74.10

5 a 12 m^2 **b** 4 m **c** $624

6 a 14 m^2

b i $2\frac{1}{3}$ L ≈ 3 L (rounded up) **ii** $34.90

7 a painting, skirting boards, carpet
b $733 (excluding labour or tools)

8 $1258

9 a 3.9 m **b** 3.9 m **c** 3 m

10 $892 **11 a** 11.53 m **b** 3

12 approx. $185 000 – $200 000

KEYWORD ACTIVITY

Teacher to check

TEST YOURSELF 11

1 a Teacher to check **b** x = 6950 mm
c y = 800 mm, z = 1800 mm, l = 10 560 mm, w = 6060 mm, d = 4500 mm

2 a 2.5 m **b** 0.8 m

3 a 2.4 m × 1.8 m **b** 10.2 m

4 a 280 mm **b** 110 mm
c 5500 mm = 5.5 m **d** 5.5 m × 3.22 m
e 6.67 m × 5.5 m

5 a 9 **b** 13.1 m × 10 m **c** 3 m
d i 109.7 m^2 **ii** approx. $116 831
e 10.9% **f** 21.3 m^2
g i 23.43 m^2 **ii** 229 tiles

h **i** 13.1 m **ii** $374.23

6 **a** $1768

b **i** 15.18 m **ii** $227.70

c **i** 12.42 m^2 **ii** 310.5 ≈ 311 tiles

iii $510.46 ≈ $511

d **i** 26 m^2 **ii** $4\frac{1}{3}$ L

iii $69.80 (2 cans required)

CHAPTER 12

EXERCISE 12.1

1 **a** $6556 **b** $78 672 **c** $7.05

2 **a** $6864 **b** $11 164

3 **a** $8280 **b** $7590 **c** $6729

d Buy if she can afford the deposit.

4 **a** $2385 to $3195

b approximately 403

5 **a** $1 **b** 10-kg packet

c It's the same price per kg in both sizes.

6 **a** Teacher to check

b **i** $425 **ii** $2720 **iii** $5950

7 It pays an income if a worker is sick or injured and can't work. Advantages: pays an income to pay rent, bills, buy food etc. when unable to work, helps support family members such as children, helps to pay medical bills if required.

EXERCISE 12.2

1 **a** $100 000 **b** $433.33 **c** $4172.33

2 **a** $24 316 **b** $224 316

3 **a** $61 **b** $38 000 **c** $6.33 **d** $67.33

4 $116.50

5 Fixed costs: a, b, d, e, i
Variable costs: c, f, g, h and j

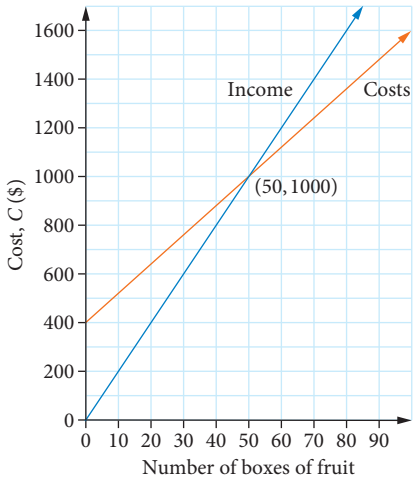
EXERCISE 12.3

1 **a** The cost graph shows $200 for no dogs washed.

b $50 **c** 5 **d** $200

2 **a** 0, $200, $400, $1000, $1500

b



c 50 **d** $400 **e** $12 **f** $320

3 **a** $20 **b** 12

c His only costs are food, stabling for the horses and insurance.

4 **a** Teacher to check

b $1008 **c** 4221 **d** 44.30 **e** 311

f **i** decrease **ii** increase

iii decrease **iv** decrease

v increase provided sales stay at the same amount

vi increase **vii** increase

EXERCISE 12.4

1 **a** $472.50 **b** $697.50 **c** $6030 **d** $1170

2 **a** $89.60 **b** $56 **c** $72.80 **d** $39.20

3 **a** $4.50 **b** $2

c $3.10, $4.90, $3.50, $3.80

d $1.80 × 0.95 + 1.80

e $1.85, $2.15, $5.66, $2.93, $3.51

f Teacher to discuss

4 **a** $76/m **b** $39/m **c** $128

EXERCISE 12.5

1 **a** $8.90 **b** $4.40 **c** $2.80 **d** $2.20

e $1.50 **f** 85 c

2 **a** $5.50 **b** $3.96 **c** $3.52 **d** $7.98

3 **a** 71 c **b** $1.64 **c** $6.91 **d** $3.82

e $1.73 **f** $1.36

4 $232.73

5 **a** $21 036.36 **b** pay more **c** $4186.36

6 **a** $444 **b** $130.14 for 43 days late

EXERCISE 12.6

1 $57.75

2 **a** $7380 **b** 46 **c** $160.43

3 **a** $672 **b** $3158.40 **c** $65.80

4 **a** $794.40 **b** $79.44 **c** $1032.72

5 **a** $1310.40 **b** $1190.54 **c** $1182

6 **a** $434.28 **b** $489.72

7 **a** 22.9% **b** 19.9% **c** 20.0% **d** 25.0%

8 20.3%

9 **a** $164 **b** $328 **c** less than 17.3 hours

10 **a** $299 **b** $442 **c** $839.54 **d** $1419.55

11 $183

12 **a** $178.60 **b** 0 **c** $267.18 **d** $1349.45

13 $780

EXERCISE 12.7

1 **a** 43.2% **b** 88.1% **c** −17.5% **d** −10.2%

2 **a** The sales on that day were lower than on the same day the previous year.

b yes

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c Many possible answers, including bad weather, COVID-19 outbreak, a big special event bringing increased shoppers last year

3 **a** $1732.73 **b** $1335.45 **c** 29.7%

4 double last year's takings

5 **a** $4767 **b** $445.50

6 positive comps: a, c, e
negative comps: b, d

KEYWORD ACTIVITY

Group 1 – types of costs

Group 2 – timing (for repayments)

Group 3 – aspects of costs and making a profit

Group 4 – finance and loans

Group 5 – government requirements

Group 6 – costs of employing staff

TEST YOURSELF 12

1 **a** $12 544 **b** 376 m^2

2 **a** 5 kg at $41/kg

b buy one 5-kg pack and two 500-g packs

3 $805

4 **a** 30 **b** $30 **c** $600

5 yes, profit of $50 **6** $476

7 $217 **8** $1460

9 $4.84 or $4.85

10 **a** $1632 **b** $177.08 **c** $163 **d** $136

11 17.7%

12 **a** 4.3%

b Her sales were 4.3% higher than on the same day last year.

PRACTICE SET 3

SECTION A

1 C **2** C **3** C **4** B

5 D **6** B **7** A **8** D

9 B **10** D

SECTION B

1 1 m

2 Guy: $6240, Jake: $7800

3 **a** $k = 6$ **b** $x = -4$

4 **a** 5 m **b** 3 cm

5 95 m

6 **a** 5 **b** sliding door

c 6 **d** 3500 mm by 2445 mm

e 12.15 m by 7.2 m **f** 35 m^2

g 5

7 **a** 45 cm, 75 cm, 60 cm **b** 210 cm

8 **a** $4800

b

Floor area F in m^2	50	100	500	1000	2000
Monthly rent, R	$2400	$4800	$24 000	$48 000	$96 000

c 48 **d** cost per m^2

e $R = 48F$ **f** $79 200

9 375 m^2

10 **a** 24 000 **b** $8000

11 $x = 3$ and $y = 17$

12 **a** 5.5 m by 5.5 m

b **i** 30.25 m^2 **ii** $2268.75

c 12

d **i** 12.24 m^2 **ii** $1040.40

e **i** 30.76 m^2 **ii** $99.80 (2 tins)

13 **a** 40 **b** $25 **c** $700 **d** $350

14 $660 **15** $4.62

16 **a**, **b** $x = 2$, $y = 4$

17 **a** As the number of children goes down, the number of pencils they can have goes up.

b $P = \frac{k}{C}$ **c** 120 **d** 8 pencils

18 $1493 **19** **a** $655 **b** $11 400

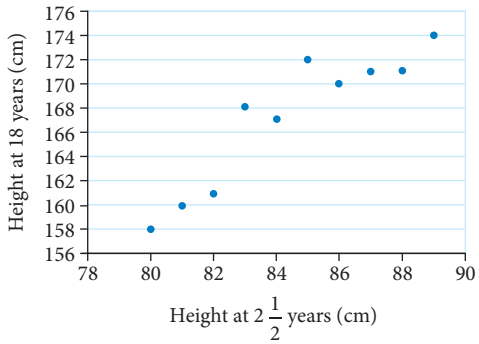
20 **a** $1500 **b** $152 **c** $150

d $68.75 (including superannuation)

CHAPTER 13

EXERCISE 13.1

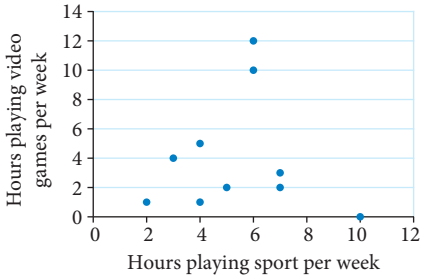
1 **a**



b Height at $2\frac{1}{2}$ years is the explanatory variable, height at 18 years is the response variable.

c As height at $2\frac{1}{2}$ years increases, height at 18 years increases.

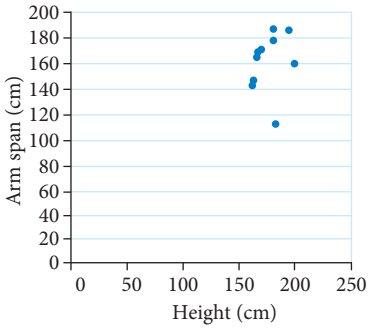
2 **a**



b Hours playing sport per week is the response variable, Hours playing video games per week is the explanatory variable.

c There is no pattern to this data.

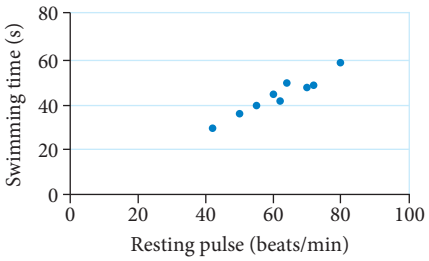
3 a



b Height is the explanatory variable and Arm span is the response variable.

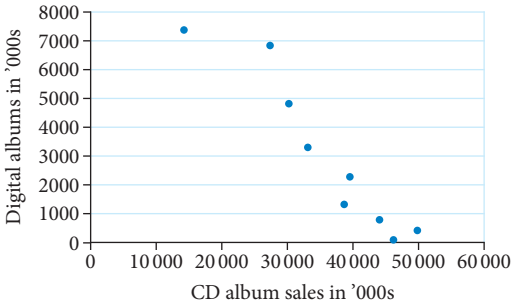
c There is one point out on its own. They are mostly in a bunch.

4 a



b As resting pulse increases, swimming time increases.

5 a

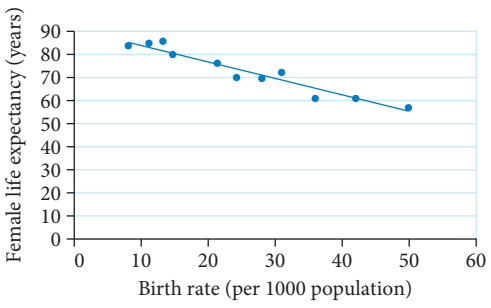


b As CD album sales increase, digital album sales decrease.

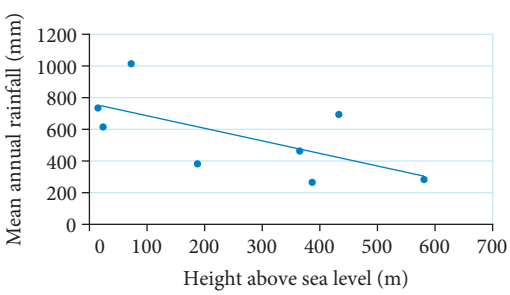
EXERCISE 13.2

1 Teacher to check

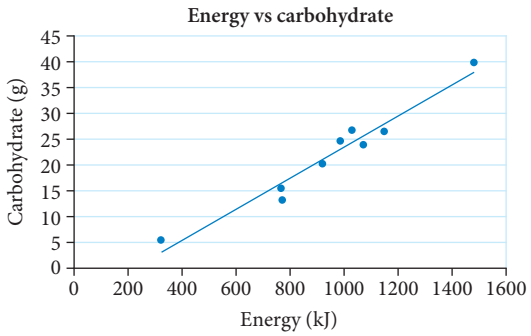
2 a, b



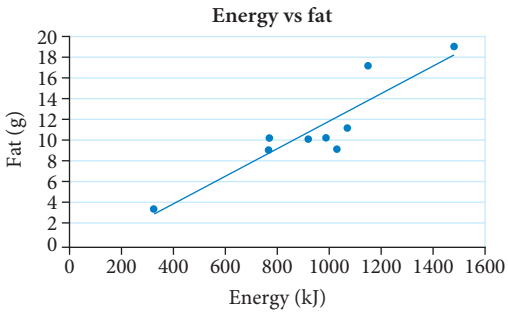
3 a, b



4 a, b



c



EXERCISE 13.3

1 a weeks after starting training

b leg extensions in 30 seconds

c The longer Anita has trained, the more leg extensions she can do in 30 seconds.

d 11 or 12

2 a number of victims

b number of medical staff

c approximately 40 staff

d approximately 750 victims

e The greater the number of victims, the greater the number of medical staff sent.

f The agency calculated the number of medical staff required based on the number of victims, not the reverse, which is what Kyle's statement implies.

3 a approximately 23.5 cm

b The longer the right foot, the higher the test score.

c The age of the students is relevant. Younger students who haven't learned as much maths generally have smaller feet and would score lower on the test.

4 a As the height of students increases, so does the weight of students.

b approximately 55 kg

c No. Teacher to check reasoning

5 a 10 or 11

b As the number of sunny days increases, the number of dry cloudy days decreases.

c The total number of sunny, dry cloudy plus wet days must be the same as the number of days in the month. As the number of sunny days increases, the number of dry cloudy or wet days must decrease to keep the total the same as the number of days in the month.

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EXERCISE 13.4

1 a **i** 1090 kJ **ii** 105 kg

b Reliable, because the points are close to a straight line

c 1885 kJ **d** Teacher to check

2 a approximately 62 years

b approximately 36 births

c Reliable, because the points are close to a straight line.

d approximately 48 years

e Teacher to check

3 a approximately 525 mm

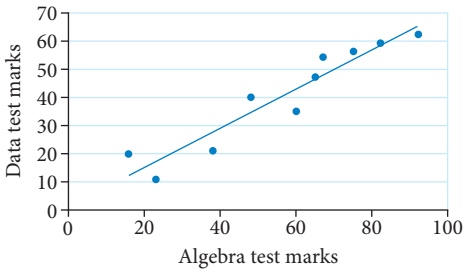
b approximately 460 m

c Not very reliable, because the points are quite spread out.

d approximately 140 mm

e −14 mm, not possible as you can't have negative rainfall

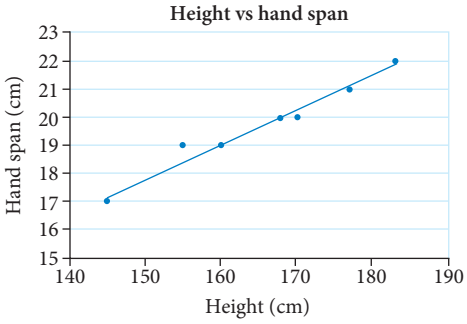
4 a



b Ella: 43 in Algebra test, Amy: 57 in Data test

c very reliable, because the points are close to a straight line

5 a



b 20 cm

c 33 cm. The value is very likely to be wrong because Robert Wadlow's height was much greater than the heights used in the scatterplot.

EXERCISE 13.5

1 a Brazil **b** Fiji

c No, because the circles are different sizes with no pattern.

2 a chicken salad **b** chips, yes

c No, because it is hard to read exact areas of circles.

3 a September

b windy (the large circles are at the bottom of the graph)

c 32 days

4, 5 Teacher to check

EXERCISE 13.6

1 a oranges **b** week 2 or 3

c Apples may be cheaper than other fruit that week. Other answers are possible.

2 a Ford **b** Holden: 2018, Ford: 2019, Toyota: 2021

c Toyota **d** No, only percentages are shown.

e No, they're all approximately the same.

3 a other **b** black coal, brown coal

c natural gas, renewables

4 a No, because there is almost a straight line between the two sections.

b Visits to the website have increased from 30 000 to 142 000 between 2017 and 2022.

5 a Teacher to check

b The graph shows the population of all 3 states increased by around the same amount between 1996 and 2006. The growth of Victoria's population was higher than the other states from 2006 to 2016, although the growth of NSW's population was the highest from 2011 to 2016.

EXERCISE 13.7

1 a 150 **b** November

c September **d** December

2 a printers – $50 000, scanners – $25 000, headphones – $20 000, speakers – $15 000, sound cards – $30 000

b printers **c** printers

d All answers are approximate: printers 136%, scanners 120%, headphones 215%, speakers 164%, sound cards 130%

e headphones

3, 4 Teacher to check

EXERCISE 13.8

1 a between 20 and 49 cases per 100 000 residents

b between 0 and 20 cases per 100 000 residents

c 1000 or greater cases per 100 000 residents

d between 100 and 199 cases per 100 000 residents

e It gives a general overview of data, but it is difficult to read precise information.

2 a Tasmania **b** Northern Territory

c true **d** from 12°C to 30°C approximately

e 39°C to 42°C **f** Teacher to check

3 a physical activity as recorded around the world using the Strava app

b Europe

c the east coast, capital cities

d **i** The running map shows activity in areas or 'dots', whereas the cycling activity is shown in 'lines'. This is because cycling is done, generally, for longer distances and along roads.

ii There is very little activity on the swimming map. This could be due to factors such as people not having waterproof devices to use Strava while swimming and it being cooler weather.

e The satellite map shows the terrain. The hybrid map shows the terrain, state borders and town names.

f Teacher to check

4,5 Teacher to check

EXERCISE 13.9

1 **a** 60 cupcakes **b** Friday
c false **d** 15 cupcakes
e Difficult to show exact or large numbers with symbols

2 Teacher to check

3 **a** X-Tech **b** Intuition **c** Africa

4 **a** South **b** ladies pyjamas
c children's pyjamas **d** dressing gowns
e dressing gowns

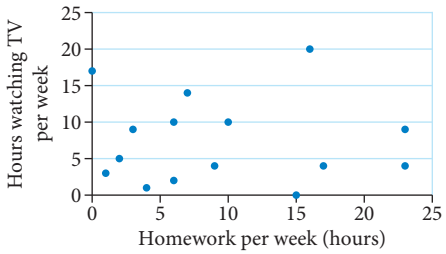
5 Teacher to check **6** Teacher to check

KEYWORD ACTIVITY

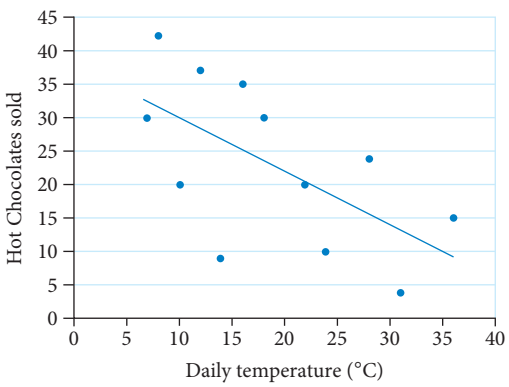
1	K
2	E
3	H
4	L
5	G
6	B
7	I
8	D
9	C
10	A
11	F
12	J

TEST YOURSELF 13

1

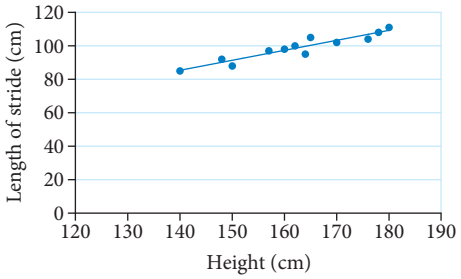


2 **a**



b 22 drinks **c** 16°C
d Yes, the higher the temperature, the fewer hot chocolates sold.

3 **a**



b **i** ≈ 90 cm **ii** ≈ 155 cm
c Very reliable, because the points are close to a straight line.
d **i** 121 cm **ii** 131 cm
e These predictions are outside the original data so may be unreliable. We don't know if the relationship continues.

4 **a** Mt Isa, Alice Springs, Winton
b Hobart
c Hobart, Ballarat, Norfolk Island, Perth, Alice Springs, Mt Isa, Winton

5 **a** Xbox
b PlayStation: 2020, Xbox: 2017, Nintendo: 2021
c Nintendo
d no, only percentages of the market given
e no, most popular varies from year to year

6 **a** approximately 80 000 **b** October
c December **d** March

7 **a** **i** zero active cases
ii Brimbank, Casey, Hume, Melton, Moreland, Whittlesea, Wyndham
iii Teacher to check
b **i** 27°C to 30°C **ii** 15°C to 42°C
iii Western Australia (18°C to 45°C)

8 **a** **i** 24 cupcakes **ii** Tuesday
iii true **iv** 42 cupcakes
b **i** Intuition **ii** X-Tech
iii Jupiter **iv** America
c **i** North **ii** men's pyjamas
iii ladies' pyjamas **iv** children's pyjamas

CHAPTER 14

EXERCISE 14.1

1 **a** $1060 **b** $22 103 **c** $46 635
d $70 297 **e** nil **f** $13 979

2 **a** $1446 **b** $3138 **c** $2900

3 **a** $16 109 **b** $1578
c No. He will get a bill for $2220.

4 **a** $86 268 **b** $18 504.10 **c** $1725.36 **d** $389.03

5 No, he still owes $1101.23 **6** $520

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7 **a** $88 131

b tax = $19 109.58, Medicare levy = $1762.62

c $3723.80

8 Incorrect. Paul's tax is $11 267, Goran's tax is $32 437. Goran's tax is almost 3 times Paul's.

9 **a** tax = $28 138, Medicare levy = $3767

b $12 185

10 $48 000

11 **a** $39 520 **b** $5791.40 **c** $143 068.60

d tax = $38 002.38, Medicare levy = $5007.40

EXERCISE 14.2

1 **a** $10 706 **b** nil

2 **a** $3600 **b** $2250 **c** $1050

3 **a** $2136 **b** $65 380

c tax = $11 715, Medicare levy = $1307

d $1866 **e** $270 **f** $6500

g $975 **h** $2423 **i** $13 740

4 **a** 2688 km **b** $2016 **c** $72

5 **a** $1536 **b** $13 971 **c** $499 **d** $2323

6 **a**

Monday	37 yes, 22 yes, 14 no, 18 no
Tuesday	37 yes, 37 yes
Thursday	56 yes, 47 yes
Friday	38 yes, 42 no

b 274 km **c** $205.50

7 **a** 4.9% **b** $18

EXERCISE 14.3

1 51% **2** $588 **3** $24.15 **4** $1820.20

5 **a** second child 81%, third child 70%

b $850

6 $80 310

7 'To' 95% means he would have to pay $36 100.
'By' 95% means he would have to pay only $1900.

8 **a** $22 514 **b** $13 076 **c** $24 082 **d** $5984

9 $158 085

EXERCISE 14.4

1 **a** Olivia: $35 681, Jack: $99 081

b $134 762 **c** Jack 74%, Olivia 26%

d Jack 0%, Olivia 100% **e** Jack 74%, Olivia −74%

f Jack **g** $19 002.60

h $14 061.92

2 Child support incomes $69 331, $58 251

Total = $127 582

Percentages of income 54%, 46%

Percentages of annual care 50%, 50%

Cost percentages 50%, 50%

Percentage differences 4%, −4%

Emma will pay child support.

Annual cost = $18 284.60

Final amount = $731.38 annually

3 Child support incomes $49 081, $69 981

Total = $119 062

Percentages of income 41%, 59%

Percentages of annual care 71%, 28%

Cost percentages 76%, 24%

Percentage differences −35%, 35%

Annual cost = $17 432.60

Riley will pay $6101.41 annually.

KEYWORD ACTIVITY

1 PAYG	**2** gross	**3** net
4 income	**5** taxable	**6** deductions
7 Medicare	**8** health	**9** minimise
10 receipts	**11** superannuation	**12** retired
13 support	**14** percentage	

TEST YOURSELF 14

1 **a** $14 270 **b** $42 158 **c** $90 142

2 **a** $1192 **b** $2426 **c** $7978

3 **a** tax = $10 389, Medicare levy = $1226

b $85

4 **a** $37 597 **b** $36 912 **c** $3555

d $738 **e** $387

5 $615

6 **a** $38 717 **b** $29 467 **c** $5500

7 **a** 53% **b** $475

8 $21 086.40

9 Child support incomes $15 681, $114 881

Total = $130 562

Percentages of income 12%, 88%

Percentages of annual care 100%, 0%

Cost percentages 100%, 0%

Percentage differences −88%, 88%

Annual cost = $18 582.60

James will pay $16 352.69.

CHAPTER 15

EXERCISE 15.1

1 **a** 35 cm^2 **b** 325 mm^2 **c** 231 cm^2

d 1.1 m^2 **e** 108 cm^2 **f** 6361.7 mm^2

2 **a** 176.8 cm^2 **b** 19.84 m^2 **c** 875 mm^2

d 1000 cm^2

3 **a** 156 m^2 **b** 950 mm^2 **c** 71 m^2

d 201.19 km^2

4 **a** 94.63 m^2 **b** 216.53 cm^2

5 **a** 37.71 m^2 **b** 272.4 m^2

6 **a** 2.02 m^2 **b** 6.06 m^2 **c** 5.45 m^2

7 **a** 80 m^2 **b** $576

8 **a** 5.14 m^2 **b** 12.86 m^2

9 50.48 cm^2 **10** 5594.34 m^2

EXERCISE 15.2

1 **a** 96 cm^2 **b** 288 cm^2 **c** 350 mm^2
d 477.52 mm^2
2 38.4 m^2 **3** 243.5 cm^2 **4** 6.72 m^2
5 **a** 2716 cm^2 **b** 2988 cm^2
6 **a** 7.875 m^2 **b** 40.23 m^2 **7** 2300 cm^2

EXERCISE 15.3

1 **a** 864 cm^2 **b** 40 560 m^2
2 **a** 329.3 cm^2 **b** 39.2 m^2
3 **a** 105.6 cm^2 **b** 3700 m^2
4 **a** 75.4 cm^2 **b** 269.5 m^2
5 **a** 865.70 cm^2 **b** 36.32 m^2
6 190 900 m^2 **7** 139 cm^2
8 2136 cm^2 **9** 1508 cm^2
10 **a** 154 cm^2 **b** 77 cm^2
11 **a** 514 718 540 km^2 **b** 360 302 978 km^2
c 154 415 562 km^2
12 960 cm^2 **13** 712.2 cm^2
14 **a** 3421.19 cm^2 **b** 2376 tiles
15 **a** 1946 m^2 **b** about 3 m^2
16 **a** 31.4 m^2 **b** 4.5 L
17 13 m^2

EXERCISE 15.4

1 **a** 64 cm^3 **b** 288 cm^3 **c** 300 cm^3
d 753.98 mm^3
2 **a** 64 mL **b** 288 mL **c** 300 mL **d** 0.75 mL
3 **a** 85.5 cm^2 **b** 2223 cm^3 **c** 101 cm^3
4 *A* is 454 cm^3, *B* is 409 cm^3;
Can *A* has a larger volume than Can *B*
5 **a** 0.13 m^3 **b** 130 L **c** 3250 L **d** 2.31 m^3
e No, it only holds 2310 L; 2 trips are required.
6 **a** 65 cm^3 **b** 51 m^3
7 **a** 65 mL **b** 51 000 L

EXERCISE 15.5

1 **a** 11 200 cm^3 **b** 96 m^3
2 **a** 37.7 m^3 **b** 414.7 cm^3 **c** 45.8 mm^3
3 **a** 904.8 cm^3 **b** 4.6 m^3
4 2 574 467 m^3
5 **a** 2.25 cm **b** 2.25 cm **c** 66 mL
6 **a** 696.910 m^3 **b** 696 910 L
7 **a** 1.1 m **b** 1.2 m^3 **c** 4.1 m^3
8 **a** 16 605 cm^3 **b** 2490.75 cm^3
9 **a** 1810 cm^3 **b** $r = 6$ cm, $h = 24$ cm
c 2714 cm^3
10 8.8 m^3

KEYWORD ACTIVITY

1	composite
2	square
3	surface area
4	net
5	volume
6	cubic
7	capacity
8	prisms
9	pyramids
10	cylinders
11	cones
12	spheres
Surface area of a cylinder	$SA = 2\pi r^2 + 2\pi rh$
Surface area of a cone	$SA = \pi r^2 + \pi rs$
Surface area of a sphere	$SA = 4\pi r^2$
Surface area of a pyramid	SA = area of base + area of all triangular side faces
Volume of a cylinder	$V = \pi r^2 h$
Volume of a pyramid	$V = \frac{1}{3}Ah$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$

TEST YOURSELF 15

1 **a** 72 mm^2 **b** 216 cm^2 **c** 149.3 cm^2
2 **a** 184 cm^2 **b** 120 m^2
3 67 858.4 mm^2 **4** 4477 cm^2
5 2268.2 cm^2 **6** 224 cm^2
7 255 cm^2
8 **a** 17 280 cm^3 **b** 17.28 L
c 13 680 cm^3; 13.68 L
9 **a** 42 875 cm^3 **b** 385 875 cm^3
10 1272 mm^3
11 **a** 32 m^3 **b** 32 000 L
12 **a** 57 906 cm^3 **b** 44 cm^3
c 262 cm^3 **d** 27 143 cm^3
13 **a** 14.1 cm^3 **b** 70 truffles
14 110 584 cm^3 **15** 79 443 m^3

CHAPTER 16

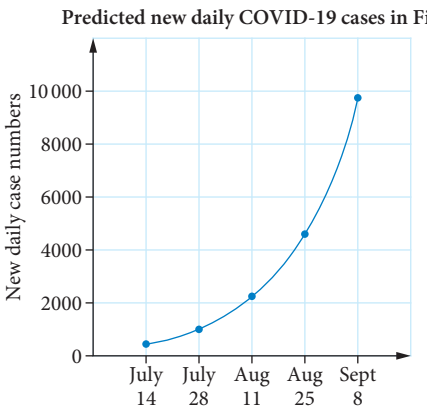
EXERCISE 16.1

1 **a**

Date	14 Jul	28 Jul	11 Aug	25 Aug	8 Sept
Predicted number of new cases	585	1170	2340	4680	9360

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b

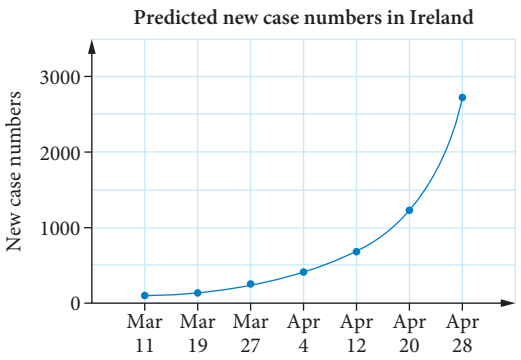


c 1.04%

2 a

Date	Predicted number of new cases
March 11	43
March 19	86
March 27	172
April 4	344
April 12	688
April 20	1376
April 28	2752

b



c exponential: it has a doubling time, the numbers are continuing to go up rapidly

3 a 4000 **b** May 1, 2020

c March to May 2020, January to March 2021

4 a 91.3% **b** 2.15%

5 a 900 **b** 100

c exponential growth in deaths

6 a 15% **b** 25% less than normal

c We will be able to move around more.

7 a

Country	Total cases	Total deaths	Population in millions	Cases per million of population	Deaths per million of population
USA	34.9 million	624 189	333	34 900 000 ÷ 333 = 104 804	624 189 ÷ 333 = 1874
India	31.0 million	412 563	1394	31 000 000 ÷ 1394 = 22 238	412 563 ÷ 1394 = 295
Brazil	19.2 million	539 050	214	19 200 000 ÷ 214 = 89 719	539 050 ÷ 214 = 2518
UK	5.2 million	128 593	68	5 200 000 ÷ 68 = 76 470	128 593 ÷ 68 = 1891
Australia	31 513	912	26	31 513 ÷ 26 = 1212	912 ÷ 26 = 35
New Zealand	2794	26	5	2794 ÷ 5 = 559	26 ÷ 5 = 5

b India

c Australia and New Zealand; There are several reasons, for example geographic isolation, low population density, strong border controls.

d USA

e Brazil

EXERCISE 16.2

1 a $820.80

b $679.20

c worse off by $70.80

2 $130 500

3 a save it

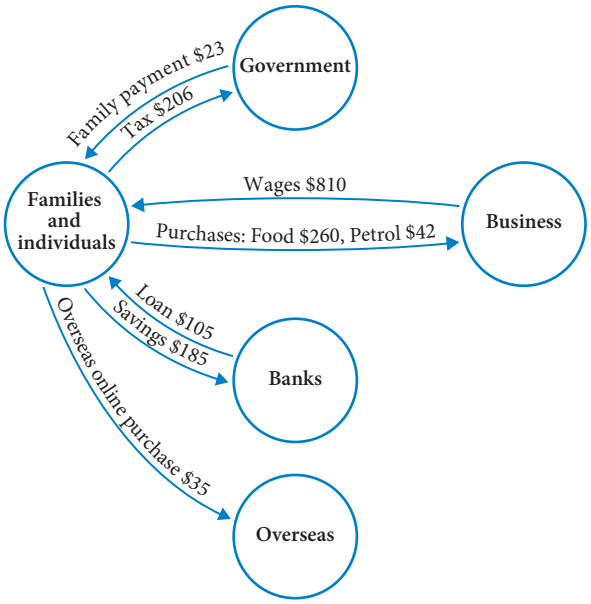
b Many answers possible, for example:

Eat at her favourite restaurant – provides employment for the staff

Buy clothes – helps keep the shop open

Get her car serviced – provides an income to the mechanic

4



5 a $\$1.976 \times 10^{11}$

b $197 600 000 000

c 197 billion 600 million dollars

d 135 billion 975 million dollars

6 a **i** 5971 **ii** 10 446 **iii** 121 363

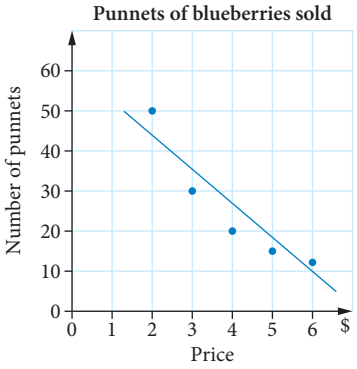
b 116 508

c $2 912 700 000

d 2 billion 912 million 7 hundred thousand dollars

EXERCISE 16.3

1 a



b As the price increases, the number sold decreases.

c 0

d The supermarket would make a loss as the cost of the product for the supermarket would exceed the sale price.

2 a $12 **b** 1080 **c** $12 960

3 a $174 **b** $190

c $4; he makes more profit

4 a $163 800 **b** $84 000 **c** 798

d $108 927 **e** profit of $29 127

5 53%

6 a $33 191 **b** $51 904 **c** $4096

7 $184 080 to $193 440

EXERCISE 16.4

1 a $666 **b** $980.50

c $3175.68 **d** $878.38

2 a £108 **b** £2916

c $866.67 **d** $10 296.30

3 a $24 \div 0.75 = 32$ **b** $32.96

4 a

Exchange rate	Calculation	Cost of the supplies to the nearest dollar
A$1 = US$0.55	$420 \div 0.55 = 763.64$	$763
A$1 = US$0.65	$420 \div 0.65 = 646.15$	$646
A$1 = US$0.75	$420 \div 0.75 = 560$	$560
A$1 = US$0.85	$420 \div 0.85 = 494.12$	$494
A$1 = US$1.05	$420 \div 1.05 = 400$	$400

b As the value of the Australian dollar increases, the amount he has to pay decreases.

c $382

5 a 80 c **b** 60.8 c **c** 16 c **d** 3.2 c

6 a $89.04 **b** 56 c

Calculating the price of 1 litre of petrol	
The cost of 1 barrel of oil in Australian dollars divided by 159. (There's 159 L of oil in a barrel.)	56 c
Excise (import tax)	38.14 c/L
Production, transport and marketing costs	13.25 c
Oil company profit	2.2 c
Total	109.59 c
GST on the total	10.959 c
Final cost of 1 litre of petrol	$120.55

c 109.59 c **d** 120.55 c

7 a price up **b** price down

c Either, price could stay the same or go down depending on the relative amounts.

d price up **e** price up

8 a $1.30 **b** $1.12 **c** $1.11 **d** $1.38

EXERCISE 16.5

1 a $2730 **b** $22 260 **c** $6930 **d** 31.1%

2 3.6%

3 a 8.33% **b** 3.43% **c** 2.34%

d 1.91% **e** 5.86%

4 a 7500 **b** $4125 **c** 17.2%

d $180 **e** shares by $3945

5 a

ASX200	Value of Jai's superannuation account
4450	$473.2 \times \frac{4450}{100} = \$21\,057.40$
7200	$473.2 \times \frac{7200}{100} = \$34\,070.40$
3650	$473.2 \times \frac{3650}{100} = \$17\,271.80$
6610	$473.2 \times \frac{6610}{100} = \$31\,278.52$
3200	$473.2 \times \frac{3200}{100} = \$15\,142.40$

b $2602.60

6 a refund **b** $2850

7 $640 **8** $2450 **9** $8700

10 a $315 000 **b** $1.34

c She receives $7035 and $3015 goes to tax.

d $1.34 \times 7500 = \$10\,050$

e $1809 **f** $1206

11 a $2200 **b** $990 **c** $1210

EXERCISE 16.6

1 a US, Canada and NZ

b banks in Switzerland and Japan

2 a 4.4% **b** much higher **c** India

3 a $22 500 **b** $4500

c She has less money to pay her bills.

4 a $37.50 **b** $16.67 **c** $20.83

5 a $2046 **b** $563 **c** $168 773

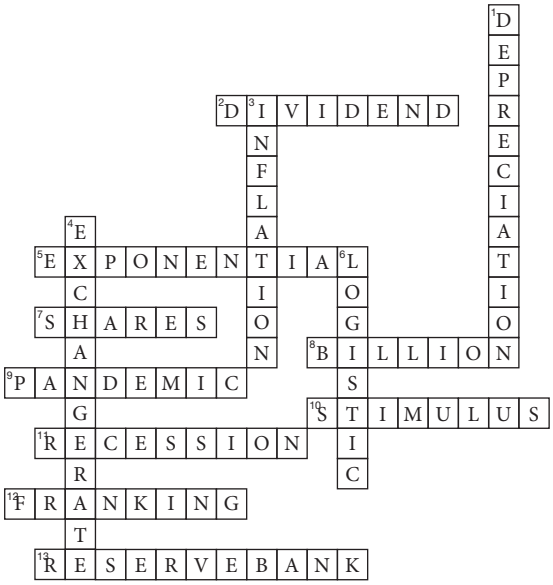
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6 **a** $342 120 **b** $129 740
c When rates go down, he can borrow more.
7 $516 297
8 **a** $51 470 **b** $1673

EXERCISE 16.7

1 **a** 27.5% **b** 25.2% **c** 20.8% **d** 15.7%
e 9.2% **f** 13.8% **g** 12%
2 **a** education and training, manufacturing
b financial sector, and real estate and property rentals
3 $25.09/h
4 **a** 49.5% **b** 81.7% **c** 19
d The higher the level, the lower the number of women.
e Teacher to check
5 **a** 69 **b** 15 **c** 8 **d** 6
6 **a** part-time **b** male **c** women **d** $19.20
7 $2243.60
8 **a** iv **b** v **c** i
d iii **e** ii

KEYWORD ACTIVITY

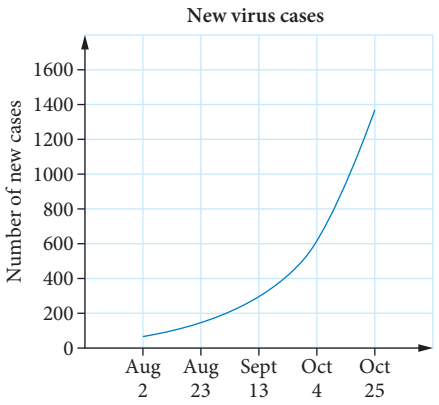


TEST YOURSELF 16

1 **a**

Date	2 Aug	23 Aug	13 Sept	4 Oct	25 Oct
COVID-19 cases	87	174	348	696	1392

b



c exponential
2 **a** 240 days **b** logistic
3 **a** $33 and $9
b 323 mature shrubs, 688 small plants
c $16 851
4 **a** $535 **b** $74.77
5 **a** less **b** more
6 **a** $56 000 **b** $3980 **c** 7.1% **d** $6000
7 **a** $1100 **b** $1237.50 **c** $137.50
8 10.7%

PRACTICE SET 4

SECTION 1

1 B **2** C **3** C **4** D
5 A **6** D **7** A **8** B

SECTION 2

1 **a** $42 896 **b** $41 621 **c** $4449
d $832.42 **e** $957
2 **a** $24 000 **b** $1575 **c** 6.6%
d $19 500
3 **a** Rachel **b** notes
c Rachel: teaching skills, Sam: knowledge of topic
4 $720
5 **a** 45.2 m^2 **b** 69.1 m^2 **c** 47.5 m^2
6 **a** 495 euros **b** A$161.60
7 **a** sunburst **b** weekdays **c** 2 : 1
8 $22 556
9 **a** $3125 **b** $2500 **c** $625
10 **a** 222 cm^2 **b** 336 mm^2
11 **a** stacked area graph **b** boys
c 2 : 1 **d** boys **e** anxiety
12 surface area = 1661.9 cm^2
volume = 6370.6 cm^3
13 **a** more **b** less
14 11 309.7 mm^2
15 12.3% **16** $6700

ANSWERS

Glossary and index

2D shapes Flat shapes that have 2 dimensions: length and width. (p. 172)

3D shapes Solid shapes that have 3 dimensions: length, width and depth. (p. 172)

absolute error The maximum possible error for a measurement, equal to half of the precision (smallest unit) of the scale of the measuring device used. (p. 65)

See also **precision** and **limits of accuracy**.

allowable tax deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. These deductions are subtracted from a person's annual income to determine their taxable income. (p. 332)

annual Per year. (p. 5)

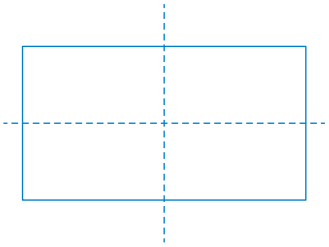
annual leave loading Extra pay, usually 17.5% of 4 weeks' pay, given by some employers to an employee during annual leave. (p. 284)

appreciation The increase in value of an item or asset over time. (p. 11)

area The amount of surface occupied by a flat shape, measured in square units. (p. 72)

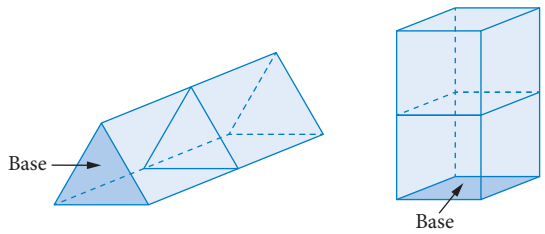
Australian business number (ABN) A business's ID number for tax purposes. (p. 272)

axis of symmetry A line that divides or folds a symmetrical shape into halves, so that one side is the mirror-image of the other side. (p. 174)



2 axes of symmetry

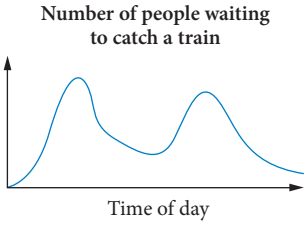
base (of a prism) One of the parallel end faces of a prism. (p. 358)



bias In statistics, an unwanted influence that stops the sample or survey from being representative of a population. (p. 50)

BIDMAS *See* **order of operations**.

bi-modal distribution A statistical distribution with 2 peaks. (p. 143)



bivariate data Data that relates 2 variables measured on the same group; for example, height and foot length of students. (p. 300)

break-even point The point where the costs and the expenses of a business are the same. Below the break-even point, the business makes a loss. Above the break-even point, the business makes a profit. (p. 219)

bubble graph A graph that shows the relationship between 3 variables using circles of different sizes. (p. 311)

business activity statement (BAS) A business's statement sent to the Australian Taxation Office. (p. 282)

capacity Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 366)

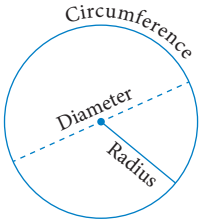
capital The money available for investing or starting a business. (p. 290)

categorical data Information represented as a category rather than a number, for example, the make of car or eye colour. (p. 136)

Differs from **numerical data**.

census A survey of all the items or people in a population. (p. 44)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.14159…), d is the diameter and r is the radius. (p. 72)



class centre The centre of a **class interval**. The average of the first and last values in the interval. For example, the class centre of the interval 1–10 is 5.5. (p. 146)

class interval In statistics, when there are many data values, they may be grouped into class intervals. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30, and so on. (p. 146)

comparison rate A more realistic interest rate that includes fees and charges, to help customers compare loans from different financial institutions. (p. 126)

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composite Made of more than one thing; for example, a composite shape is made up of 2 or more simpler shapes. (p. 100)

compound interest Interest paid on the principal invested as well as on any accumulated interest, so you get 'interest on your interest'. (p. 7)

Differs from **simple interest**.

compounding period How often interest is calculated when using compound interest, for example, monthly, quarterly or yearly. (p. 19)

constant of variation (constant of proportionality) The value of k in the direct variation equation $y = kx$ or the inverse variation equation $y = \frac{k}{x}$. (p. 241)

continuous data Numerical data that can be measured on a smooth scale without any 'gaps' and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without gaps, unlike discrete data. (p. 146)

cost In business, money spent to run the business. (p. 272)

cost price The price it costs a shop to buy a product from the factory or wholesaler. (p. 279)

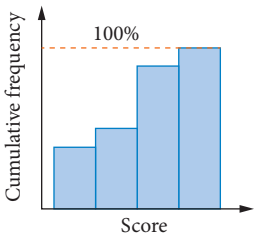
credit union A type of financial institution, like a bank. (p. 108)

cross-section A 'slice' of a solid shape cut across it rather than along it. (p.187)

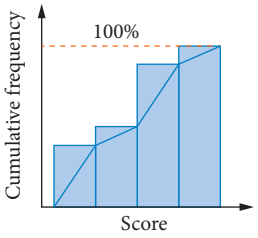
cubic metre (m^3) A unit of volume equal to the volume of a cube of length 1 m. $1\ m^3 = 1\,000\,000\ cm^3$. (p. 367)

cumulative frequency A running total of frequencies. (p. 146)

cumulative frequency histogram A histogram in which the height of each column represents the cumulative frequency of each value. (p. 149)



cumulative frequency polygon A line graph formed by joining the ends of the tops of the columns of the cumulative frequency histogram. (p. 149)



data Raw statistical information. (p. 44)

decrease To make smaller. (p. 37)

denominator The number at the bottom of a fraction. (p. 32)

depreciation Loss in value of an item over time. (p. 213)

direct variation (or **direct proportion**) The relationship between 2 variables (say x and y) given by an equation of the form $y = kx$, where k is the **constant of variation**. (p. 241)

See also **inverse variation**.

discount A reduction in the usual price. (p. 37)

discrete data Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2 …). Discrete data have a scale with 'gaps' or jumps, unlike continuous data. (p. 146)

dividend Profit paid to the owner of company shares. (p. 397)

dividend yield The dividend of a share as a percentage of its market price. (p. 398)

elevation views On a plan, the views of a building from the sides, front and back. (p. 252)

exchange rate The value of one country's money in terms of another country's money. (p. 394)

expense The cost of running a business. (p. 277)

explanatory variable For bivariate data, the variable whose value does not depend on the other variable. It is represented on the horizontal axis of a scatterplot. (p. 300)

See also **response variable**.

exponential Describes when the rate of growth in a quantity is determined by the size of the quantity; for example, cases are doubling every month. (p. 380)

extrapolation Using data to make a prediction outside the range of the original data. (p. 307)

See also **interpolation**.

financial year A period of one year used for tax and business purposes. In Australia, the financial year begins on 1 July and ends on 30 June. (p. 330)

fixed cost A regular cost that doesn't change over time. (p. 274)

Differs from **variable cost**.

flat rate interest *See* **simple interest**.

formula A rule written as an algebraic equation, using variables. For example, the formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 211)

fortnightly Every 2 weeks. (p. 287)

frequency The number of times a value or group of values occurs in a data set. (p. 146)

future value The value of an investment in the future. (p. 7)

gallon A non-metric liquid measure. 1 US gallon is approximately 3.785 L and 1 UK (imperial) gallon is approximately 4.546 L. (p. 92)

goods and services tax (GST) Good and services tax, usually 10% in Australia. (p. 282)

gross pay Pay before the tax is taken out. (p. 335)

See also **net pay**.

heat map A graph where the sizes of data values on a map are shown by colour. (p. 318)

holiday loading *See* **annual leave loading**.

holiday pay Total pay for holidays, including usual pay plus any annual leave loading. (p. 284)

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle. (p. 75)

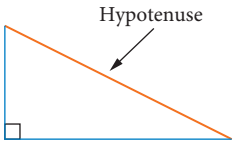


image A transformed shape after it has been translated, reflected or rotated. (p. 176)

income Money that is earned or gained (usually regularly). (p. 330)

income tax A personal tax that is calculated on the amount of income a person earns over the financial year. (p. 284)

increase To make bigger. (p. 10)

indices (plural of index) Powers; for example, in the expression $7x^2y^3$, the 2 and the 3 are indices. (p. 30)

indirect variation *See* **inverse variation**.

inflation The rate at which the overall cost of goods and services is increasing. (p. 11)

instalment *See* **repayment**.

interest Money earned on an investment, or money paid to a financial institution for borrowing. (p. 4)

interest rate The percentage of the investment or loan on which interest is calculated. (p. 4)

interpolation Making a prediction within the range of the original data. (p. 305)

See also **extrapolation**.

interquartile range The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data. (p. 136)

inverse variation (or **indirect variation** or **inverse proportion**) A relationship between 2 variables (say x and y) represented by the equation $y = \frac{k}{x}$, where k is the **constant of variation**. (p. 244)

See also **direct variation**.

lease A legal document setting out the conditions for a person to rent a property from someone else. (p. 273)

limits of accuracy The limits of a measured value; for example, the limits of accuracy of a measured height of 171 cm are 170.5 cm to 171.5 cm, meaning that the actual height lies within this range. The limits are found by adding to and subtracting from the measured value $\frac{1}{2}$ of the smallest unit marked on the measurement scale.
Lower limit = measurement – absolute error.
Upper limit = measurement – absolute error. (p. 65)

See also **absolute error** and **precision**.

line of fit A straight line that represents a set of points on a scatterplot, obtained through experiment or observation. (p. 302)

line symmetry Symmetry where one half of a shape is the mirror image of the other half. (p. 174)

See also **rotational symmetry**.

litre A measure of liquid. 1 L = 1000 mL The size of a tall carton of milk. (p. 92)

logistic curve A type of graph that increases very quickly at the start and then flattens out after time. (p. 382)

loss The amount of money lost by a shop or a business on an item; loss = cost price – selling price. (p. 220)

mean The average of a set of numerical data, calculated by adding all the values and dividing by the number of values. (p. 136)

measure of central tendency A statistical value, such as the mean, median or mode, that describes the centre or average of a set of data. (p. 136)

measure of spread A statistical value, such as the range, interquartile range or standard deviation, that describes the spread of a set of data. (p. 137)

median The middle score of a set of numerical data, or the average of the 2 middle values when the values are arranged in order. (p. 136)

Medicare levy A tax to cover the costs of the public health system, calculated as a percentage (usually 2%) of a person's income. (p. 331)

Mekko graph A graph with 2 dimensions. Column widths vary and each column has segments of different heights. (p. 320)

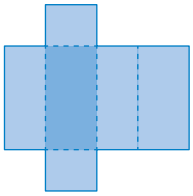
metric system A measurement system based on powers of 10. (p. 86)

mode The most common value(s) in a set of data. (p. 136)

mortgage A home loan or property loan. (p. 115)

negatively skewed distribution *See* **skewed distribution**.

net (of a solid) The faces of a solid shape laid out flat. For example, this is the net of a rectangular prism. (p. 188)



net pay The amount remaining after tax is deducted from gross pay. (p. 125) *See also* **gross pay**.

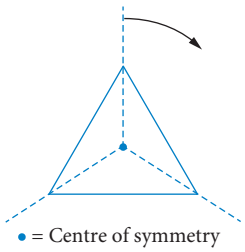
non-metric units Units of measurement that aren't based on the metric system, such as miles, feet, inches, gallons. (p. 92)

numerator The number at the top of a fraction. (p. 32)

numerical data Data that are numbers, such as heights or the number of cars owned. (p. 146)

order of operations The recognised order to calculate an expression with 2 or more operations: brackets, indices (powers), divide and multiply from left to right, add and subtract from left to right. This can be remembered using the initials BIDMAS. (p. 30)

order of rotational symmetry The number of times a shape fits onto itself during a 360° rotation. (p. 174)



Order of rotational symmetry 3

outlier An extreme (high or low) value in a data set that is very different from the other values. It affects the mean but not the mode or median. (p. 140)

overhead A fixed business cost that has to be paid regardless of the number of items produced, such as rent or insurance. (p. 274)

pandemic An infectious disease that has spread widely around the world. (p. 380)

PAYG income tax Pay as you go tax; income tax deducted from our pay in instalments each payday by our employer. (p. 284)

per annum (p.a.) Per year. (p. 4)

percentage error The absolute error of a measurement as a percentage of the measurement.

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\% \text{ (p. 67)}$$

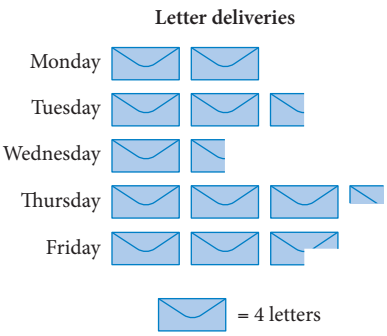
percentiles 99 values that divide a set of data into 100 equal parts when the data are arranged in order. (p. 154)

See also **quartiles**.

perimeter The distance around the outside of a shape. (p. 94)

perspective drawing A drawing where objects further away are drawn smaller than objects up close. (p. 192)

pictogram A type of graph where pictures or icons are used to indicate quantities. (p. 320)



plan Diagram of the floor of a house or building. (p. 252)

polygon A 2D shape with straight sides. (p. 358)

polyhedron A solid with all flat faces.

population In statistics, all of the items under investigation. (p. 44)

positively skewed distribution *See* **skewed distribution**.

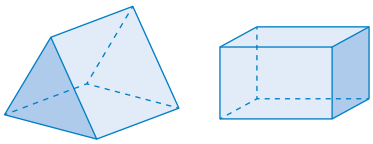
precision The smallest unit on the scale of a measuring device. For example, the precision of a ruler may be 1 millimetre. (p. 65)

See also **absolute error** and **limits of accuracy**.

premium The amount paid for an insurance policy. (p. 274)

principal The original amount of money invested or borrowed, upon which interest is calculated. (p. 4)

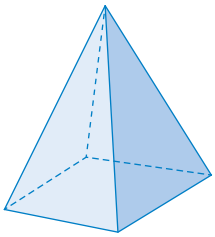
prism A solid shape with flat sides that has the same face at both ends. A triangular prism and square prism are shown. (p. 354)



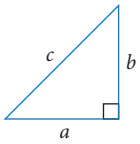
profit The amount of money gained or made by a business. (p. 220)

pronumeral *See* **variable**.

pyramid A solid shape that has a polygon as its base, with side faces that are triangles that meet at a point called the apex. The diagram shows a square pyramid. (p. 358)



Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides. $c^2 = a^2 + b^2$ (p. 75)



quadrilateral A 4-sided shape. (p. 358)

quarterly Four times per year, or every 3 months. (p. 6)

quartiles Three values that divide a set of data into 4 equal parts when the data are arranged in order. The 1st quartile (Q_1) is the lower quartile, the 2nd quartile (Q_2) is the median and the 3rd quartile (Q_3) is the upper quartile. (p. 136)

See also **interquartile range**, **percentiles**.

questionnaire A form with questions used to collect information, usually about people's habits or opinions. (p. 52)

radar chart A graph that is like a circular version of a line graph. (p. 316)

radius (plural: **radii**) The length of the interval joining the centre of a circle to the circumference (see diagram at **diameter**). The radius is half of the **diameter**. (p. 73)

random sample A sample for which every member of a population has an equal chance of selection. (p. 46)

range The difference between the highest and lowest values in a set of data. (p. 136)

ratio A comparison of 2 or more quantities with the same units, for example, 2 : 3. (p. 179)

real estate Property such as houses, units, factories, office. (p. 13)

reducible interest Interest that is charged on the amount still owing on the loan; the type of interest charged on a reducing balance loan. (p. 108)

reducing balance loan A loan where the interest charged is calculated on the balance owing on the loan after each repayment. (p. 108)

reflection A transformation that 'flips' a shape across a line to give a mirror image that is back to front. (p. 176)

regular polygon A polygon with all sides (and all angles) the same size. For example, a regular pentagon has 5 equal sides. (p. 184)

rent Money paid to the owner of a property for living in or using the property. (p. 272)

repayment (or **instalment**) Amount paid regularly to pay off a loan. (p. 108)

Reserve Bank The Australian Central Bank. (p. 385)

response variable For bivariate data, the variable whose value depends on the other variable. It is represented on the vertical axis of a scatterplot. (p. 300)

See also **explanatory variable**.

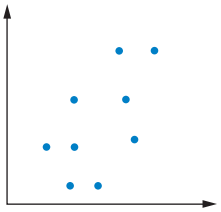
rotation A transformation that 'spins' or turns a shape about or around a fixed point in a certain angle and direction. (p. 176)

rotational symmetry A plane shape has rotational symmetry if it can be spun around a central point so that it fits onto itself more than once in a 360° rotation. (p. 174)

sample A group of items selected from a population to study or survey. (p. 44)

scale (on a map or diagram) The ratio of scaled length to actual length; for example, a scale of 1 : 500 means that lengths represented on the map or diagram are actually 500 times larger in real life. (p. 234)

scatterplot A graph of points on a number plane showing a relationship between 2 variables. (p. 300)



self-selected sample A sample in which people volunteer to be part of the sample, such as an SMS poll or a website survey, so it is not random. (p. 46)

shape of a distribution The way the data in a frequency distribution is spread; can be symmetrical, positively skewed or negatively skewed. (p. 142)

share A unit of investment in a company. (p. 397)

significant figures The meaningful digits in a number that show its level of accuracy; the first non-zero digits, for example, 31 487 000 has 5 significant figures. (p. 70)

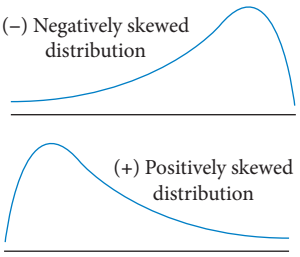
similar figures Figures that are the same shape but not necessarily the same size: one is an enlargement or reduction or the other. Corresponding angles are the same and corresponding sides are in the same ratio. (p. 179)

simple interest (or **flat rate interest**) Interest earned or charged only on the original amount of money (principal) invested or borrowed. (p. 4)

Differs from **compound interest**.

simultaneous equations Two (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$. (p. 224)

skewed distribution A statistical distribution where most of the data scores are either low (positively skewed) or high (negatively skewed). The tail indicates the direction of the skew. (p. 143)



square root (of a number) The positive value which, if squared, gives the number. For example, $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$. (p. 75)

stacked area graph A graph that shows the sizes of different categories, usually over time, and what percentage each category contributes to the overall total. (p. 314)

standard deviation (symbol σ) A statistical measure of the spread of a set of data. (p. 157)

stratified sample A sample consisting of a percentage of items from each 'strata' or 'layer' of a population. For example, a stratified sample from a population of 35% children and 65% adults should contain 35% children and 65% adults. (p. 46)

substitute To replace a variable in an expression or formula with a number. (p. 73)

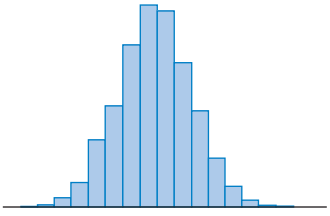
sunburst graph A graph that displays hierarchical data. Each level of the hierarchy is represented by one ring or circle, with the innermost circle as the top of the hierarchy. (p. 320)

superannuation A saving scheme designed to provide an income in retirement. (p. 275)

surd A square root (or other root) whose exact value cannot be found, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (p. 75)

surface area The total area of all the faces of a solid shape. (p. 354)

symmetrical distribution A statistical distribution where one half is the mirror-image of the other half. You can fold it down the middle and the 2 sides would match. (p. 142)



symmetry *See* **line symmetry** and **rotational symmetry**.

systematic sample A sample chosen by using a set pattern; for example, choosing every 10th number in the phone book. (p. 46)

taxable income A person's income after allowable tax deductions have been made, which is the amount used for calculating income tax. (p.332)

tax file number (TFN) A taxpayer's personal ID number. (p. 286)

tessellation A pattern of repeated shapes covering a flat surface so that there is no overlapping or gaps. (p. 184)

three-dimensional (3D) Having length, width and depth, describing a solid or object. (p. 187)

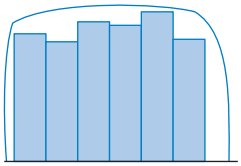
See also **3D shapes**.

transformation In geometry, the process of moving or changing a shape. (p. 176) *See also* **reflection**, **rotation** and **translation**.

translation A transformation that 'slides' a shape a certain distance and direction. (p. 176)

two-dimensional (2D) Having length and width, describing a flat shape or figure. (p. 172) *See also* **2D shapes**.

uniform distribution The shape of a statistical distribution where each value has the same or similar frequency. Its graph is close to one big rectangle as the scores are evenly distributed. (p. 143)



variable (or **pronumeral**) A letter or symbol such as x that stands in place of a number. (p. 76)

variable cost A cost that changes between periods. (p. 274)

Differs from **fixed cost**.

volume The amount of space occupied by a solid, measured in cubic units. (p. 365)

wage An amount paid for work as a rate in dollars per hour. (p. 274)

wholesale A type of business that sells large quantities at relatively low prices, usually to other businesses. (p. 273)

workers compensation A form of insurance that is paid to any worker who is injured or becomes unwell as a direct result of work. (p. 274)



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