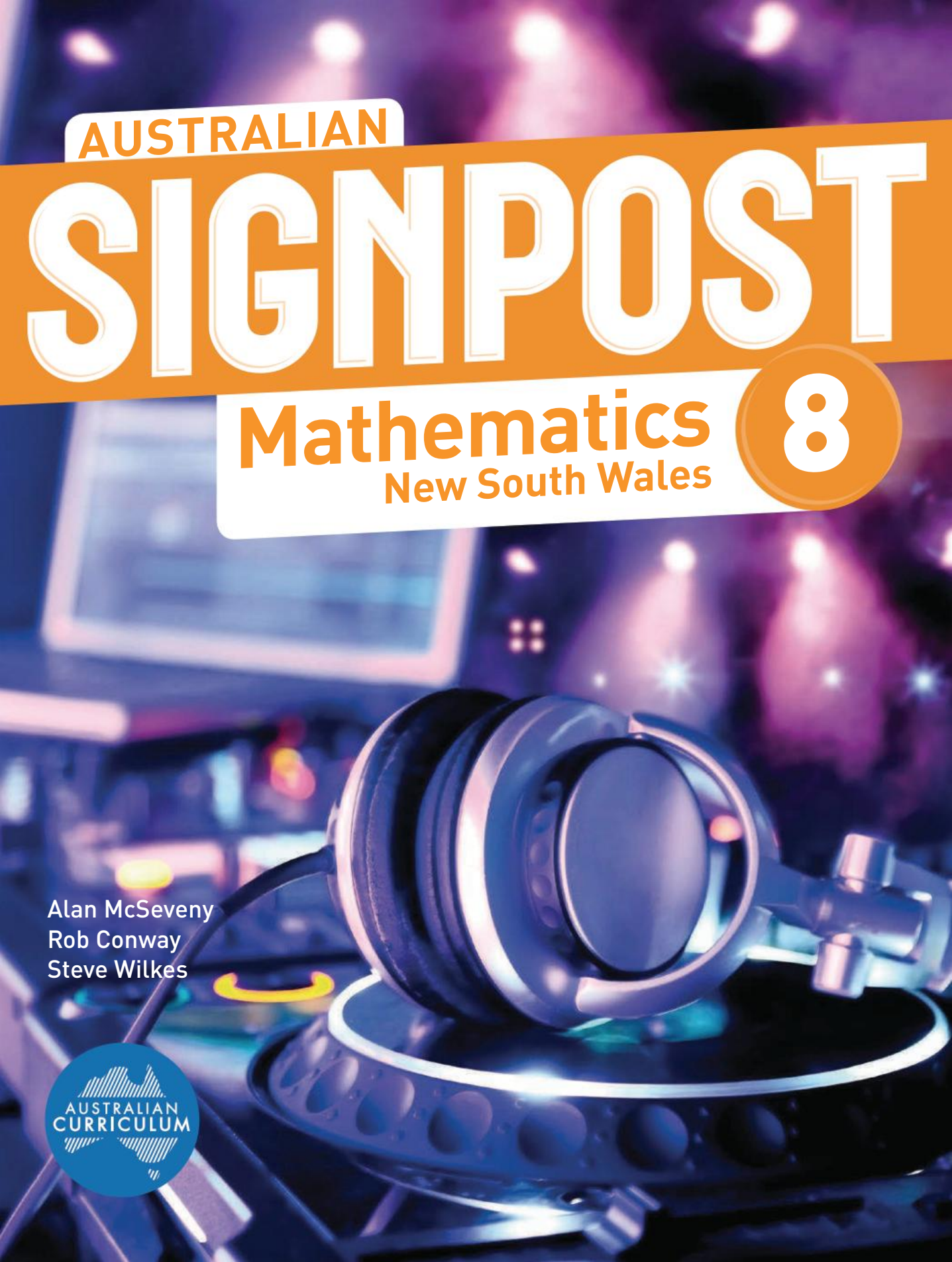
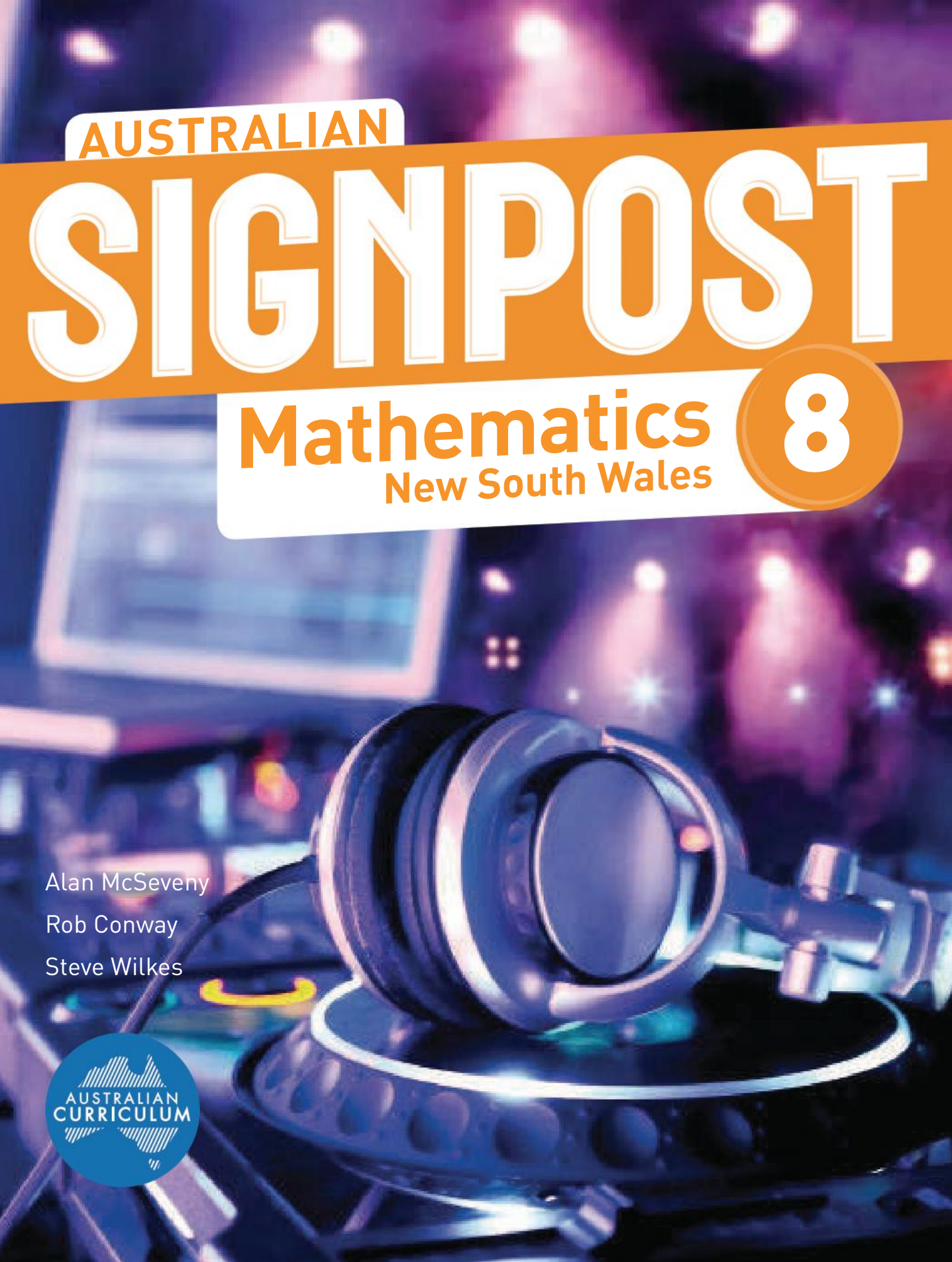

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It is better—much better—to have wisdom
and knowledge than gold and silver.

Proverbs 16:16

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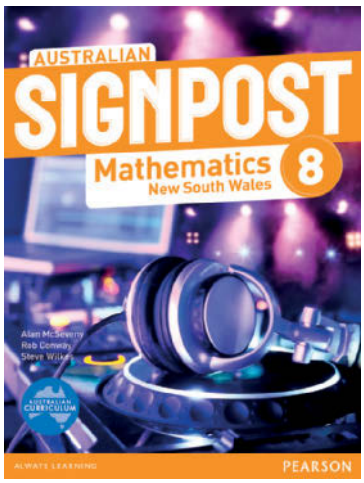
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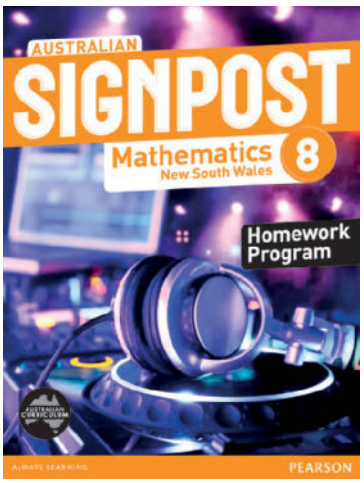
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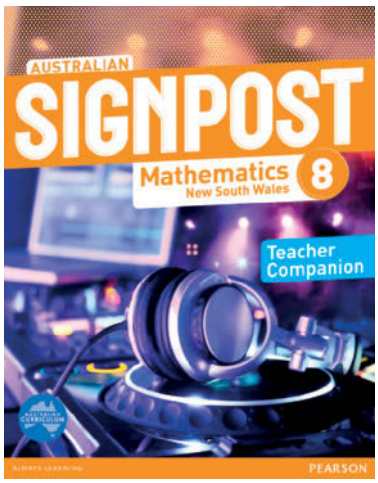
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The *Australian Signpost Mathematics New South Wales 7* and *Australian Signpost Mathematics New South Wales 8* learning package addresses Stage 4 of the NSW Syllabus for the Australian Curriculum. This new series continues the Signpost tradition of expanding students' ability in a broad range of mathematical skills while emphasising problem-solving and working mathematically.



Homework Program

The Homework Program consists of tear-out worksheets that mirror and supplement the content of the student book, providing opportunity for further practice and application of key skills.



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The Teacher Companion is an invaluable resource that provides guidance and support to teachers using the **Australian Signpost Mathematics New South Wales** package. It is a practical resource featuring teaching strategies, activities, reference to Stage 4 outcomes and content statements, and links to digital learning materials.

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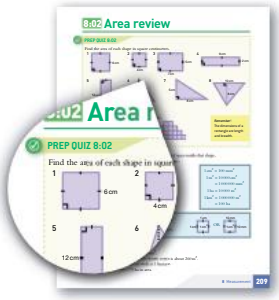
Features of the Student Book

- a complete year's work with full coverage of the NSW Syllabus for the Australian Curriculum
- a flexible structure with carefully graded exercises, colour-coded to indicate level of difficulty
- technological applications with GeoGebra activities and Pearson eBook 3.0 resources
- integration of a broad range of mathematical skills, expanding students' ability to solve problems, work mathematically and investigate
- diagnostic tests, chapter reviews and cumulative revision components for every chapter

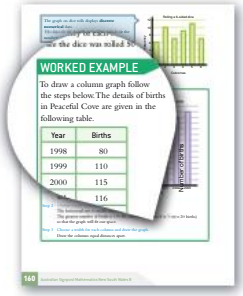


Chapter opener pages

summarise the key content and present the content statements addressed in each chapter. Features within each exercise such as GeoGebra activities, Fun spots and Investigations are listed here.

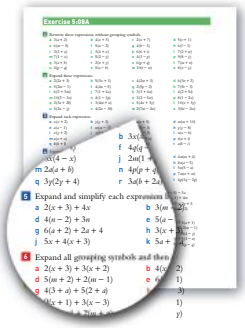


Prep quizzes review important skills and knowledge that students will need to complete an exercise.



Worked examples

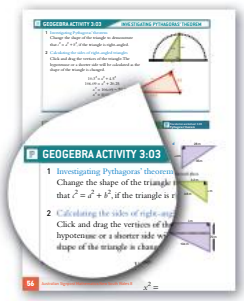
take students through key processes with step-by-step demonstrations of questions similar to those they will encounter.



Well-graded exercises

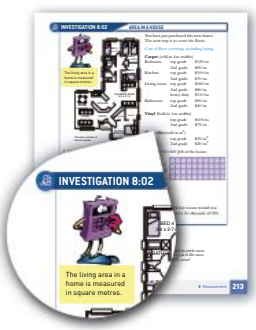
indicate each question's level of difficulty.

- 1 Foundation
- 3 Stage 4 level
- 6 Extension



GeoGebra activities

provide a deeper understanding of concepts covered within the exercise while providing the opportunity to integrate ICT into the classroom.



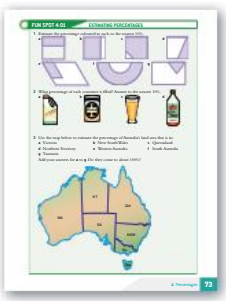
Investigations

encourage students to seek knowledge and develop research skills. The investigative tasks provide opportunities for students to interpret, question, model and communicate mathematically.

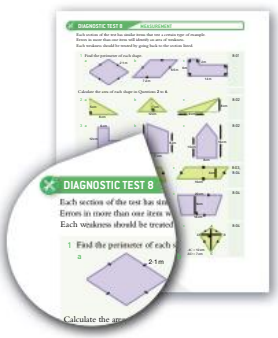


Challenge activities

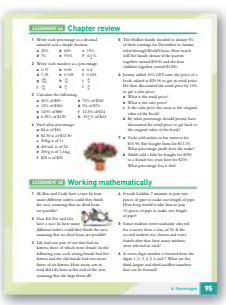
provide more difficult questions and investigations. They can be used to help students with a stronger understanding extend their knowledge.



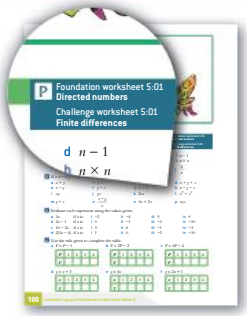
Fun spots provide amusement and interest while reinforcing concepts learnt within the chapter. They encourage creativity and divergent thinking, and show that mathematics is enjoyable.



Diagnostic tests at the end of each chapter test students' achievement of outcomes. Importantly, they also indicate student weaknesses that need to be addressed and provide links to the relevant sections in the text or digital resources.



Assignments are provided at the end of each chapter. They concentrate on revising the content of previous chapters and developing students' ability to work mathematically.

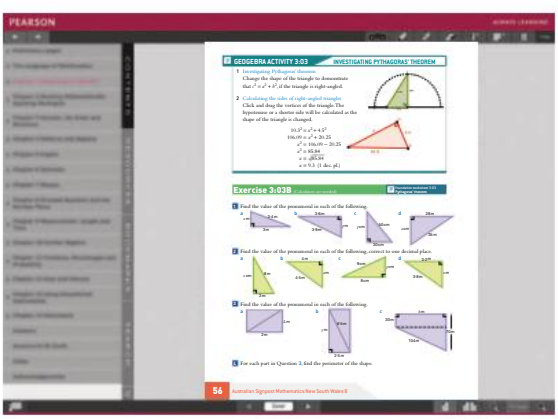


Foundation and Challenge worksheets

are indicated in the student book wherever a worksheet is available on the eBook. Foundation worksheets provide alternative exercises for consolidation of earlier stages. Challenge worksheets help students apply their learning in more advanced activities.

Pearson digital resources

A vast range of technology applications are available with the eBook to complement the content presented in the student book. Digital resources include interactive activities, practice and chapter tests, and Foundation and Challenge worksheets. From the eBook, students can also link directly to the GeoGebra applets referred to in the student book exercises.



The eBook aims to create a classroom environment that embraces mathematical technologies in a way that consolidates and extends student learning.

TREATMENT OF SYLLABUS CONTENT

The NSW Syllabus for the Australian Curriculum outlines the expected outcomes of students in Mathematics. The syllabus outcomes relevant to Stage 4 are listed below.

Working Mathematically (encompassing five components: Communicating, Problem Solving, Reasoning, Understanding and Fluency) is an integral part of the structure of each chapter. Special attention is given to Working Mathematically in Chapter 2 of the Year 7 text, in Chapter 1 of the Year 8 text and in the second assignment of every chapter of both texts.

Working Mathematically	
A student:	
> communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols	**MA4-1WM**
> applies appropriate mathematical techniques to solve problems	**MA4-2WM**
> recognises and explains mathematical relationships using reasoning	**MA4-3WM**
Number and Algebra	
A student:	
> compares, orders and calculates with integers, applying a range of strategies to aid computation	**MA4-4NA**
> operates with fractions, decimals and percentages	**MA4-5NA**
> solves financial problems involving purchasing goods	**MA4-6NA**
> operates with ratios and rates, and explores their graphical representation	**MA4-7NA**
> generalises number properties to operate with algebraic expressions	**MA4-8NA**
> operates with positive-integer and zero indices of numerical bases	**MA4-9NA**
> uses algebraic techniques to solve simple linear and quadratic equations	**MA4-10NA**
> creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane	**MA4-11NA**
Measurement and Geometry	
A student:	
> calculates the perimeters of plane shapes and the circumference of circles	**MA4-12MG**
> uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area	**MA4-13MG**
> uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume	**MA4-14MG**
> performs calculations of time that involve mixed units, and interprets time zones	**MA4-15MG**
> applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems	**MA4-16MG**
> classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles	**MA4-17MG**
> identifies and uses angle relationships, including those related to transversals on sets of parallel lines	**MA4-18MG**
Statistics and Probability	
A student:	
> collects, represents and interprets single sets of data, using appropriate statistical displays	**MA4-19SP**
> analyses single sets of data using measures of location, and range	**MA4-20SP**
> represents probabilities of simple and compound events	**MA4-21SP**

Stage 4 content statements

The Stage 4 content statements are divided into three strands:

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Content statements are listed below with their Australian Curriculum code or NSW syllabus reference. The chapters that address each content statement are shown in the right-hand column.

Number and Algebra	Reference	Chapters
Computation with Integers		
• Apply the associative, commutative and distributive laws to aid mental and written computation.	ACMNA151 (Stage 4)	1 (Y7) 1 (Y8)
• Compare, order, add and subtract integers.	ACMNA280 (Stage 4)	7 (Y7) 1, 2 (Y8)
• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies.	ACMNA183 (Stage 4)	7 (Y7) 2 (Y8)
Fractions, Decimals and Percentages		
• Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line.	ACMNA152 (Stage 4)	9 (Y7) 1 (Y8)
• Solve problems involving addition and subtraction of fractions, including those with unrelated denominators.	ACMNA153 (Stage 4)	9 (Y7) 1 (Y8)
• Multiply and divide fractions and decimals using efficient written strategies and digital technologies.	ACMNA154 (Stage 4)	6, 9 (Y7) 1 (Y8)
• Express one quantity as a fraction of another, with and without the use of digital technologies.	ACMNA155 (Stage 4)	9 (Y7)
• Round decimals to a specified number of decimal places.	ACMNA156 (Stage 4)	6 (Y7) 1 (Y8)
• Investigate terminating and recurring decimals.	ACMNA184 (Stage 4)	6 (Y7)
• Connect fractions, decimal and percentages and carry out simple conversions.	ACMNA157 (Stage 4)	6, 11 (Y7) 1 (Y8)
• Investigate the concept of irrational numbers, including π.	ACMNA186 (Stage 4)	3 (Y8)
• Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies.	ACMNA158 (Stage 4)	11 (Y7) 1, 4 (Y8)
• Solve problems involving the use of percentages, including percentage increases and decreases, with and without the use of digital technologies.	ACMNA187 (Stage 4)	4 (Y8)
Financial Mathematics		
• Investigate and calculate the Goods and Services Tax (GST), with and without the use of digital technologies.	NSW (Stage 4)	4 (Y8)
• Investigate and calculate 'best buys', with and without the use of digital technologies.	ACMNA174 (Stage 4)	9 (Y7)
• Solve problems involving profit and loss, with and without the use of digital technologies.	ACMNA189 (Stage 4)	4 (Y8)
Ratios and Rates		
• Recognise and solve problems involving simple ratios.	ACMNA173 (Stage 4)	9 (Y7) 1, 10 (Y8)
• Solve a range of problems involving ratios and rates, with and without the use of digital technologies.	ACMNA188 (Stage 4)	10 (Y8)
• Investigate, interpret and analyse graphs from authentic data.	ACMNA180 (Stage 4)	14 (Y7) 10 (Y8)
Algebraic Techniques 1		
• Introduce the concept of variables as a way of representing numbers using letters.	ACMNA175 (Stage 4)	4 (Y7)
• Extend and apply the laws and properties of arithmetic to algebraic terms and expressions.	ACMNA177 (Stage 4)	4 (Y7) 1, 5 (Y8)
• Simplify algebraic expressions involving the four operations.	ACMNA192 (Stage 4)	4 (Y7) 1, 2, 5 (Y8)

Algebraic Techniques 2		
• Create algebraic expressions and evaluate them by substituting a given value for each variable.	ACMNA176 (Stage 4)	4 (Y7) 1, 2, 5 (Y8)
• Extend and apply the distributive law to the expansion of algebraic expressions.	ACMNA190 (Stage 4)	4 (Y7) 1, 5 (Y8)
• Factorise algebraic expressions by identifying numerical factors.	ACMNA191 (Stage 4)	5 (Y8)
• Factorise algebraic expressions by identifying algebraic factors.	NSW (Stage 4)	5 (Y8)
Indices		
• Investigate index notation and represent whole numbers as products of powers of prime numbers.	ACMNA149 (Stage 4)	3 (Y7) 1 (Y8)
• Investigate and use square roots of perfect square numbers.	ACMNA150 (Stage 4)	3 (Y7) 1 (Y8)
• Use index notation with numbers to establish the index laws with positive-integer indices and the zero index.	ACMNA182 (Stage 4)	2 (Y8)
Equations		
• Solve simple linear equations.	ACMNA179 (Stage 4)	13 (Y7) 1, 9 (Y8)
• Solve linear equations using algebraic techniques and verify solutions by substitution.	ACMNA194 (Stage 4)	13 (Y7) 9 (Y8)
• Solve simple quadratic equations.	NSW (Stage 4)	9 (Y8)
Linear Relationships		
• Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point.	ACMNA178 (Stage 4)	7 (Y7) 1 (Y8)
• Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates.	ACMMG181 (Stage 4)	15 (Y7)
• Plot linear relationships on the Cartesian plane, with and without the use of digital technologies.	ACMNA193 (Stage 4)	11 (Y8)
• Solve linear equations using graphical techniques.	ACMNA194 (Stage 4)	11 (Y8)

Measurement and Geometry	Reference	Chapters
Length		
• Find perimeters of parallelograms, trapeziums, rhombuses and kites.	ACMMG196 (Stage 4)	10 (Y7) 8 (Y8)
• Investigate the concept of irrational numbers, including π.	ACMNA186 (Stage 4)	12 (Y8)
• Investigate the relationship between features of circles, such as the circumference, radius and diameter; use formulas to solve problems involving circumference.	ACMMG197 (Stage 4)	12 (Y8)
Area		
• Choose appropriate units of measurement for area and convert from one unit to another.	ACMMG195 (Stage 4)	8 (Y8)
• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.	ACMMG159 (Stage 4)	10 (Y7) 8 (Y8)
• Find areas of trapeziums, rhombuses and kites.	ACMMG196 (Stage 4)	8 (Y8)
• Investigate the relationship between features of circles, such as the area and the radius; use formulas to solve problems involving area.	ACMMG197 (Stage 4)	12 (Y8)
Volume		
• Draw different views of prisms and solids formed from combinations of prisms.	ACMMG161 (Stage 4)	8 (Y7)
• Choose appropriate units of measurement for volume and convert from one unit to another.	ACMMG195 (Stage 4)	10 (Y7) 8 (Y8)
• Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume.	ACMMG198 (Stage 4)	10 (Y7) 8 (Y8)
• Calculate the volumes of cylinders and solve related problems.	ACMMG217 (Stage 4)	12 (Y8)
Time		
• Solve problems involving duration, including using 12-hour and 24-hour time within a single time zone.	ACMMG199 (Stage 4)	8 (Y8)
• Solve problems involving international time zones.	NSW (Stage 4)	8 (Y8)
Right-Angled Triangles (Pythagoras)		
• Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.	ACMMG222 (Stage 4)	3 (Y8)
• Investigate the concept of irrational numbers.	ACMNA186 (Stage 4)	3 (Y8)

Properties of Geometrical Figures 1		
• Classify triangles according to their side and angle properties and describe quadrilaterals.	ACMMG165 (Stage 4)	8 (Y7)
• Identify line and rotational symmetries.	ACMMG181 (Stage 4)	15 (Y7)
• Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral.	ACMMG166 (Stage 4)	8 (Y7) 15 (Y8)
• Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning.	NSW (Stage 4)	5, 8 (Y7) 15 (Y8)
Properties of Geometrical Figures 2		
• Define congruence of plane shapes using transformations.	ACMMG200 (Stage 4)	13 (Y8)
• Develop the conditions for congruence of triangles.	ACMMG201 (Stage 4)	13 (Y8)
• Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.	ACMMG202 (Stage 4)	13 (Y8)
Angle Relationships		
• Use the language, notation and conventions of geometry.	NSW (Stage 4)	5 (Y7)
• Recognise the geometrical properties of angles at a point.	NSW (Stage 4)	5 (Y7) 15 (Y8)
• Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal.	ACMMG163 (Stage 4)	5 (Y7) 15 (Y8)
• Investigate conditions for two lines to be parallel.	ACMMG164 (Stage 4)	5 (Y7) 15 (Y8)
• Solve simple numerical problems using reasoning.	ACMMG164 (Stage 4)	5 (Y7) 15 (Y8)

Statistics and Probability	**Reference**	**Chapters**
Data Collection and Representation		
• Investigate techniques for collecting data, including census, sampling and observation.	ACMSP284 (Stage 4)	14 (Y7) 14 (Y8)
• Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.	ACMSP206 (Stage 4)	14 (Y8)
• Identify and investigate issues involving numerical data collected from primary and secondary sources.	ACMSP169 (Stage 4)	14 (Y8)
• Construct and compare a range of data displays, including stem-and-leaf plots and dot plots.	ACMSP170 (Stage 4)	14 (Y7) 14 (Y8)
Single Variable Data Analysis		
• Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data.	ACMSP171 (Stage 4)	14 (Y7)
• Investigate the effect of individual data values, including outliers, on the mean and median.	ACMSP207 (Stage 4)	14 (Y8)
• Describe and interpret data displays using mean, median and range.	ACMSP172 (Stage 4)	14 (Y7) 14 (Y8)
• Explore the variation of means and proportions of random samples drawn from the same population.	ACMSP293 (Stage 4)	14 (Y8)
Probability 1		
• Construct sample spaces for single-step experiments with equally likely outcomes.	ACMSP167 (Stage 4)	12 (Y7)
• Assign probabilities to the outcomes of events and determine probabilities for events.	ACMSP168 (Stage 4)	12 (Y7)
• Identify complementary events and use the sum of probabilities to solve problems.	ACMSP204 (Stage 4)	12 (Y7) 6 (Y8)
Probability 2		
• Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.	ACMSP205 (Stage 4)	6 (Y8)
• Represent events in two-way tables and Venn diagrams and solve related problems.	ACMSP292 (Stage 4)	6 (Y8)

Metric equivalents

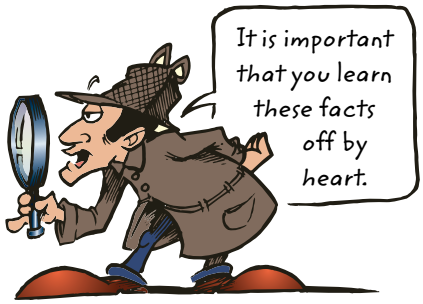
Length
1 m = 1000 mm = 100 cm = 10 dm 1 cm = 10 mm 1 km = 1000 m
Area
$1\ m^2 = 10\,000\ cm^2$ 1 ha = $10\,000\ m^2$ $1\ km^2$ = 100 ha
Mass
1 kg = 1000 g 1 t = 1000 kg 1 g = 1000 mg
Volume
$1\ m^3 = 1\,000\,000\ cm^3$ = $1000\ dm^3$ 1 L = 1000 mL 1 kL = 1000 L $1\ m^3$ = 1 kL $1\ cm^3$ = 1 mL $1000\ cm^3$ = 1 L
Time
1 min = 60 s 1 h = 60 min 1 day = 24 h 1 year = 365 days 1 leap year = 366 days

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

Seasons

Summer: December, January, February
Autumn: March, April, May
Winter: June, July, August
Spring: September, October, November





The language of mathematics

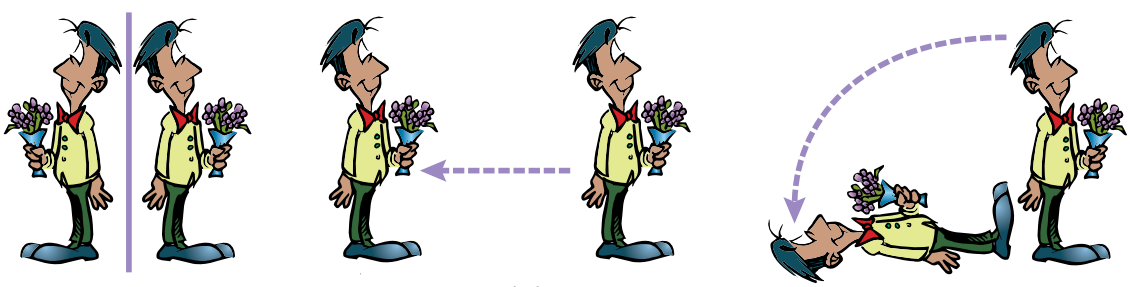
You should regularly test your knowledge by identifying the items on each card.

ID Card 1 (Metric units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

For answers see page 470.

ID Card 2 (Symbols)			
1 +	2 −	3 ×	4 ÷
5 =	6 ≒ or ≈	7 ≠	8 <
9 ≤	10 ≮	11 >	12 ≥
13 ≯	14 ≱	15 4^2	16 4^3
17 $\sqrt{25}$	18 $\sqrt[3]{27}$	19 %	20 ∴
21 e.g.	22 i.e.	23 $\frac{3}{4}$ n	24 $\frac{3}{4}$ d

For answers see page 470.



reflection (or flip)

translation (or slide)

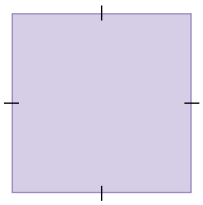
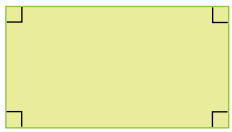
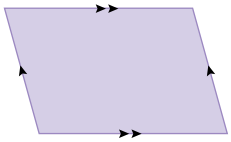
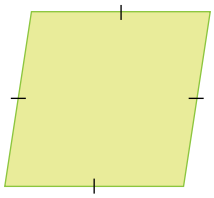
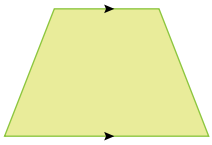
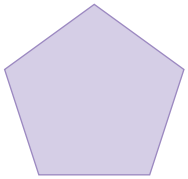
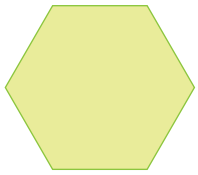
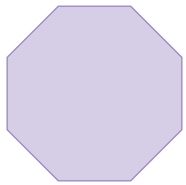
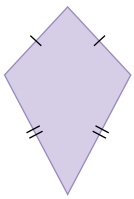
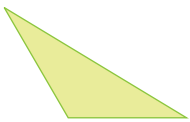
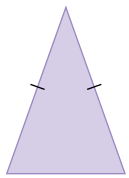
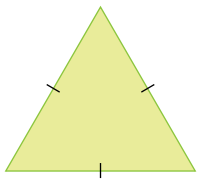
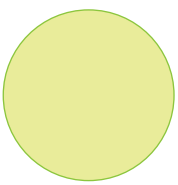
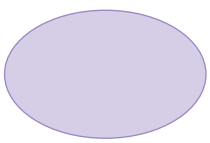
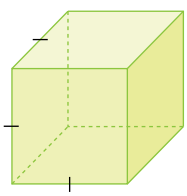
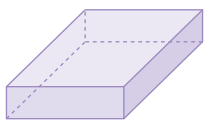
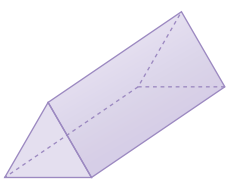
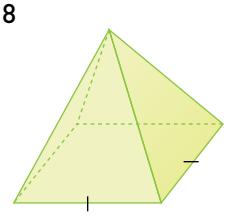
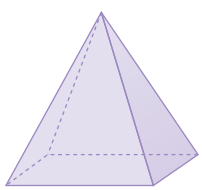
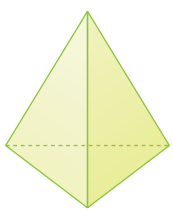
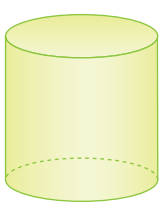
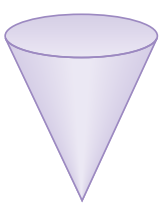
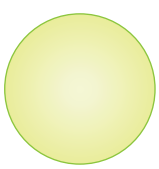
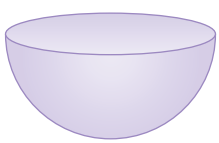
rotation (or turn)

ID Card 3 (Language)			
1 6 minus 2	**2** the sum of 6 and 2	**3** divide 6 by 2	**4** subtract 2 from 6
5 the quotient of 6 and 2	**6** $2\overline{)6}$ the divisor is	**7** $2\overline{)6}$ the dividend is	**8** 6 lots of 2
9 decrease 6 by 2	**10** the product of 6 and 2	**11** 6 more than 2	**12** 2 less than 6
13 6 squared	**14** the square root of 36	**15** 6 take away 2	**16** multiply 6 by 2
17 average of 6 and 2	**18** add 6 and 2	**19** 6 to the power of 2	**20** 6 less 2
21 the difference between 6 and 2	**22** increase 6 by 2	**23** share 6 between 2	**24** the total of 6 and 2

For answers see page 470.



ID Card 4 (Language)

1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different.	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

For answers see page 470.

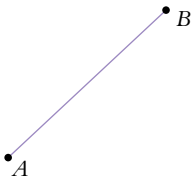
ID Card 5 (Language)

1



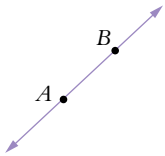
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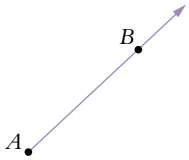
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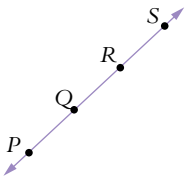
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4



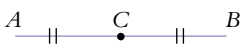
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5



..... points

6



C is the .....

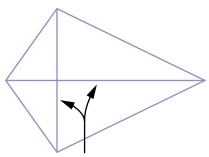
7



.....

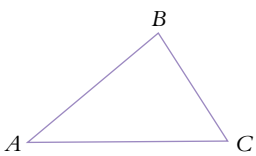
.....

8



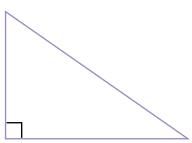
.....

9



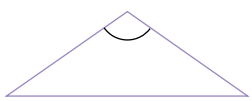
all angles less than 90°

10



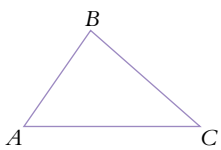
one angle 90°

11



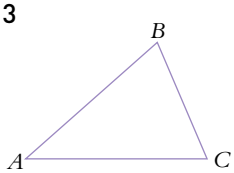
one angle greater than 90°

12



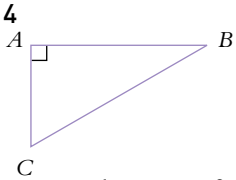
A, *B* and *C* are ..... of the triangle.

13



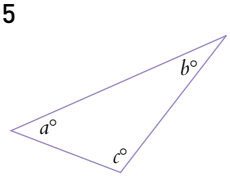
Use the vertices to name the Δ.

14



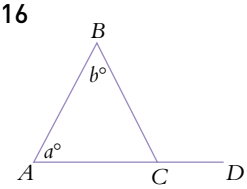
BC is the ..... of the right-angled Δ.

15



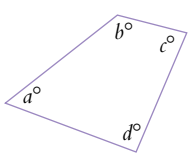
$a° + b° + c°$ = ...

16



$\angle BCD$ = ...

17



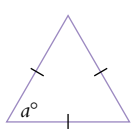
$a° + b° + c° + d°$ = ...

18



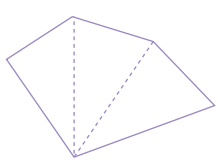
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



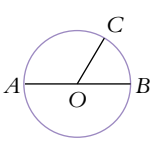
$a°$ = ...

20



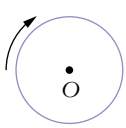
Angle sum = ...

21



AB is a .....
OC is a .....

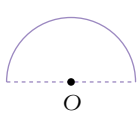
22



Name of distance around the circle.

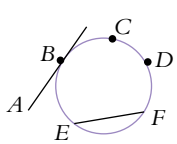
.....

23



.....

24

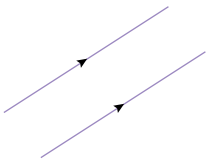


AB is a .....
CD is an .....
EF is a .....

For answers see page 470.

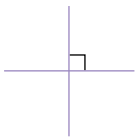
ID Card 6 (Language)

1



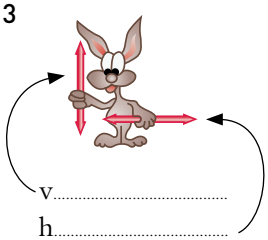
..... lines

2



..... lines

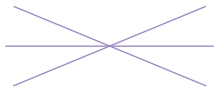
3



v.....

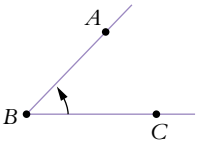
h.....

4



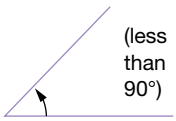
..... lines

5



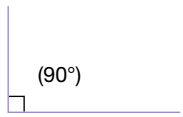
angle .....

6



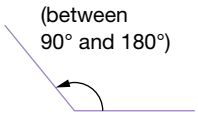
..... angle

7



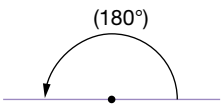
..... angle

8



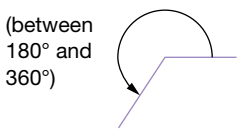
..... angle

9



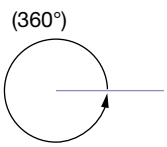
..... angle

10



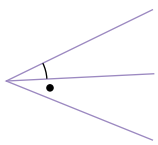
..... angle

11



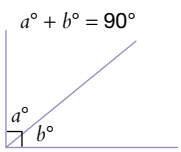
.....

12



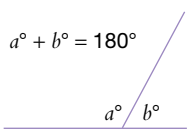
..... angles

13



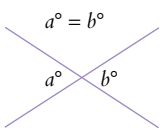
..... angles

14



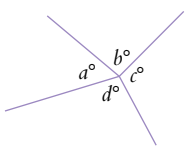
..... angles

15



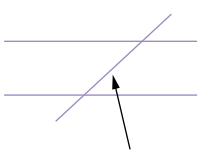
..... angles

16



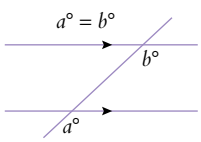
$a° + b° + c° + d° =$ ....

17



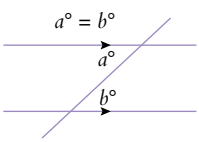
.....

18



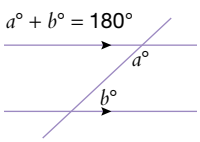
..... angles

19



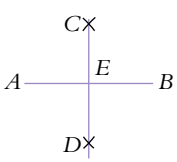
..... angles

20



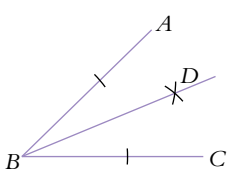
..... angles

21



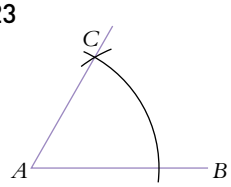
b..... an interval

22



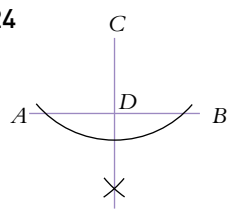
b..... an angle

23



$\angle CAB =$ .....

24



CD is p..... to AB.

For answers see page 471.

ID Card 7 (Language)

1

CE

C..... E.....

AD

A..... D.....

2

BCE

B..... C..... E.....

BC

B..... C.....

3

am

a..... m.....

means

b..... m.....

4

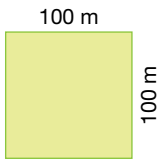
pm

p..... m.....

means

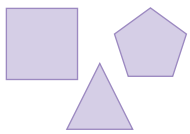
a..... m.....

5



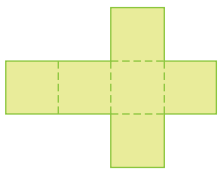
area is 1 .....

6



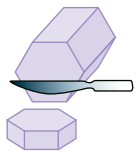
r..... shapes

7



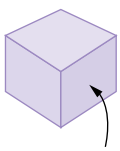
..... of a cube

8



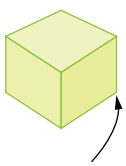
c.....-s.....

9



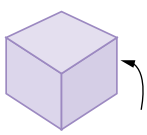
f.....

10



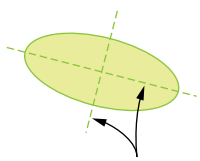
v.....

11



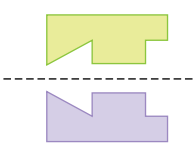
e.....

12



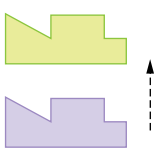
axes of .....

13



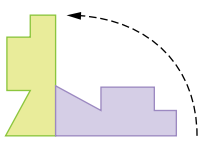
r.....

14



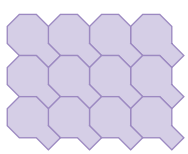
t.....

15



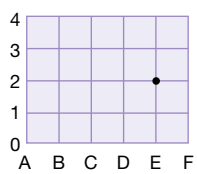
r.....

16



t.....

17



The c.....

of the dot are E2.

18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

t.....

19

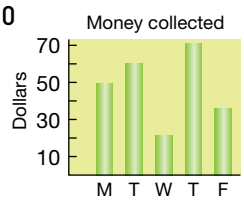
Money collected

Mon	● ● ● ●
Tues	● ● ● ◕
Wed	● ◔
Thurs	● ● ● ●
Fri	● ● ◔

● Stands for $10

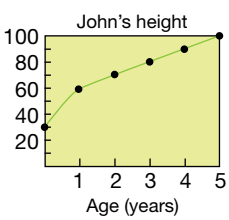
p..... graph

20



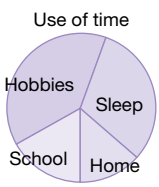
c..... graph

21



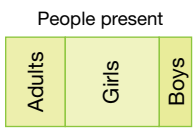
l..... graph

22



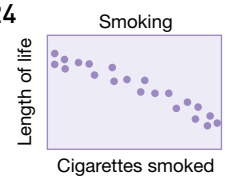
s..... graph

23



b..... graph

24



s..... d.....

For answers see page 471.

Arithmetic and Algebra cards

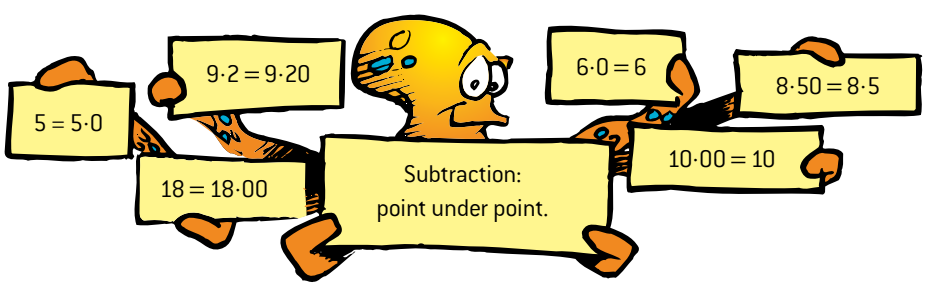
Arithmetic card

	A	B	C	D	E	F	G	H	I	J	K	L
1	2	5	$\frac{1}{2}$	100	0·2	5%	$\frac{2}{3}$	90	0·9	15%	13	56
2	3	8	$\frac{1}{3}$	90	0·1	40%	$\frac{1}{7}$	30	0·5	35%	18	36
3	9	7	$\frac{3}{4}$	80	0·7	20%	$\frac{1}{5}$	20	0·3	1%	15	21
4	5	12	$\frac{1}{5}$	20	0	60%	$\frac{1}{6}$	80	0·1	44%	10	96
5	4	15	$\frac{3}{5}$	80	0·5	25%	$\frac{1}{8}$	100	0·6	95%	19	88
6	1	18	$\frac{2}{3}$	30	0·4	10%	$\frac{1}{4}$	60	0·2	65%	12	24
7	0	4	$\frac{4}{5}$	60	0·8	50%	$\frac{1}{2}$	10	0·4	90%	20	43
8	11	3	$\frac{1}{4}$	40	0·3	80%	$\frac{4}{5}$	60	0·2	100%	14	62
9	10	10	$\frac{3}{10}$	70	0·6	30%	$\frac{2}{5}$	30	0·8	$33\frac{1}{3}$%	18	75
10	7	9	$\frac{2}{5}$	50	0·9	75%	$\frac{1}{3}$	70	0·7	85%	16	45

How to use these cards

As an example, if the instruction given for the Arithmetic card is 'column B + column K', then you would write answers for the following problems:

1 5 + 13 **2** 8 + 18 **3** 7 + 15 **4** 12 + 10 **5** 15 + 19
6 18 + 12 **7** 4 + 20 **8** 3 + 14 **9** 10 + 18 **10** 9 + 16



Algebra card

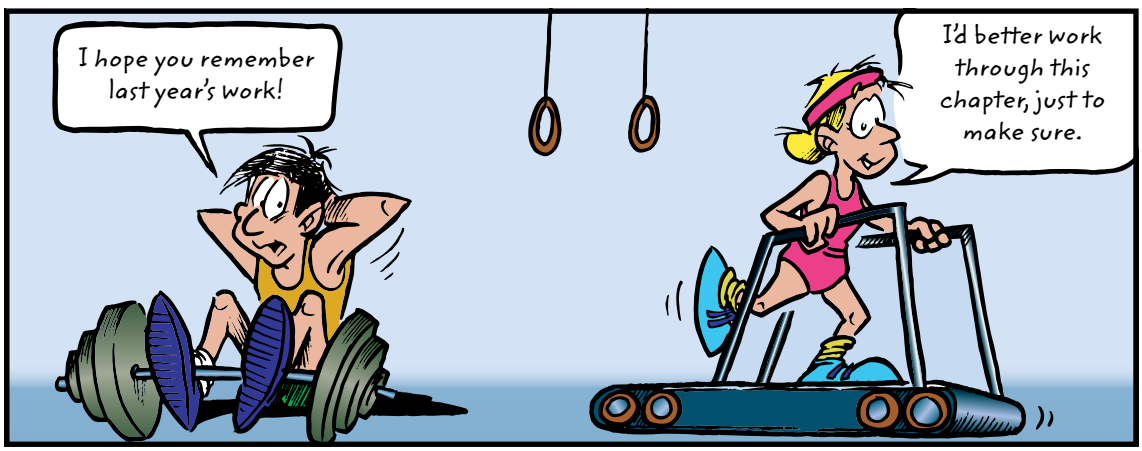
	A	B	C	D	E	F	G	H	I	J	K	L
1	4	-7	-11	x	$2y$	$-3x$	$-6y$	x^2	$2m$	x^7	$-5y$	-4
2	3	-6	7	$5x$	$7y$	$-x$	$-5y$	x^5	$3d$	x^2	$8y$	-3
3	5	-8	-9	$2x$	$10y$	$4x$	$4y$	x^3	$-2c$	x^8	$-12y$	-2
4	8	-2	-7	$3x$	$4y$	$-9x$	$-9y$	x^6	$-8h$	x^4	$0y$	-1
5	7	-10	-8	$10x$	$5y$	$-5x$	$-2y$	x	$9w$	x^3	$-7y$	0
6	2	-12	6	$7x$	$15y$	$-7x$	$-3y$	x^8	$-4d$	x^5	$0y$	1
7	0	-15	-5	$4x$	$6y$	$5x$	$-8y$	x^4	$-9n$	x	$-17y$	2
8	1	-5	9	$11x$	$3y$	$8x$	$-y$	x^7	$3x$	x^9	$-9y$	3
9	9	-3	8	$6x$	y	$-4x$	$-12y$	x^3	$-2w$	x^6	$5y$	4
10	10	-4	-3	$8x$	$9y$	$-8x$	$-7y$	x^4	$-7m$	x^{10}	$-8y$	5

For convenience, you might stick copies of these pages in your book.



WORKING MATHEMATICALLY AND YEAR 7 REVIEW

1



Contents

1:01 Working mathematically
1:02 Problem-solving strategies
Investigation 1:02 The left-hand rule for mazes
1:03 Beginnings in number
1:04 Number: Its order and structure
1:05 Fractions
1:06 Decimals
1:07 Percentages
1:08 Angles
1:09 2D space
1:10 3D space
Investigation 1:10 Identifying 3D Solids
1:11 Measurement
1:12 Directed numbers
1:13 The number plane
1:14 Algebra
1:15 Probability
1:16 Statistics
Assignments

Syllabus references (See page x for details.)

The **Working Mathematically** strand is interwoven throughout the content strands **Number and Algebra**, **Measurement and Geometry**, **Statistics and Probability**.

- Communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols.
- Applies appropriate mathematical techniques to solve problems.
- Recognises and explains mathematical relationships using reasoning.
- Work covered in Year 7 on numbers, fractions, decimals, percentages, angles, 2D space, 3D space, measurement, directed numbers, the number plane, algebra, probability and statistics is reviewed in this chapter.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

1:01 Working mathematically

Working mathematically in a real-life situation may require us to ask questions, apply problem-solving strategies, communicate mathematical ideas, apply reasoning, and think about the mathematical ideas that apply.

Exercise 1:01

P Challenge worksheet 1:01
The bridges of Königsberg

1 Imagine that you are working on this assembly line. As you work, you begin to think mathematically.



a Communicating

i Explain how this assembly line works.

ii Make up three questions to do with this assembly line that you could explore.

b Problem solving

i Each carton contains 8 boxes of chocolates. Each person on the assembly line places 5 chocolates into each box. In every hour of a 10-hour working day, 32 cartons are filled. How many chocolates are packed in one working day?

ii There is a red marker on the conveyor belt. Estimate how far the marker travels in one working day if the width of a box of chocolates is 20 cm. (If you need help, read the hints at the bottom of this page.)

c Reasoning

i Which are the most important parts of the process?

ii What are the problems that would need to be addressed to keep the assembly line going?

iii If the engine running the assembly line broke down, how would you organise the workers so that the packing could continue in an efficient way?

iv What would you do to improve this assembly line?

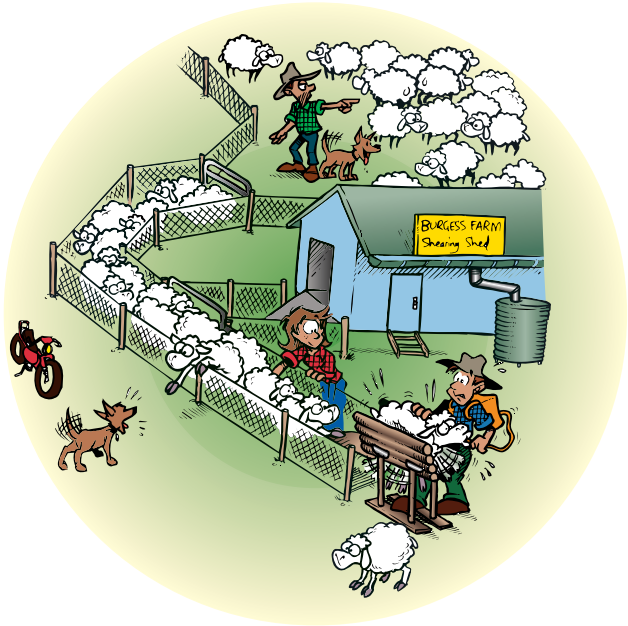
Hints for Question **b ii**:

1 It appears that there is a gap after each five boxes, so the belt moves about 8·5 cm in the picture to allow five boxes to be filled.

2 Use a ruler to measure the width of a box in the picture. This is equal to 20 cm in real life.

3 2560 boxes are filled each day.

2 Alan drenched 800 sheep. This involved squirting between 8 and 10 mL of a mineral supplement into the mouth of each sheep to keep them healthy. Using farmhands and dogs, David, the farmer, herded the sheep along the chute to a conveyor belt that Alan operated by foot.



a Communicating

i How has David organised the drenching to reduce the time taken to finish the task?

ii Explain how he has planned to complete the whole task of drenching the sheep.

iii What questions would you ask the farmer so that you understand all of the preparations he has made for the sheep drenching? Write at least three questions.

b Problem solving

i How many sheep can be drenched in 3 hours if one sheep is drenched every 30 seconds?

ii If one sheep in 60 misses treatment, how many of the 800 sheep would miss treatment? Of the 800 sheep, how many would have been treated?

iii During a drought, a bale of hay cost $380. In a normal year, the cost was $120. What was the increase in price? What percentage increase is this over the original price?

iv Following the drought it was decided to sell the 800 sheep. The farmer estimates that by the time of the sale, 18 sheep will have died. Each truck has 12 pens and 23 sheep fit into each pen. How many truckloads will be needed to take the sheep to market? If each sheep was sold for $57, how much money was received for the sale?

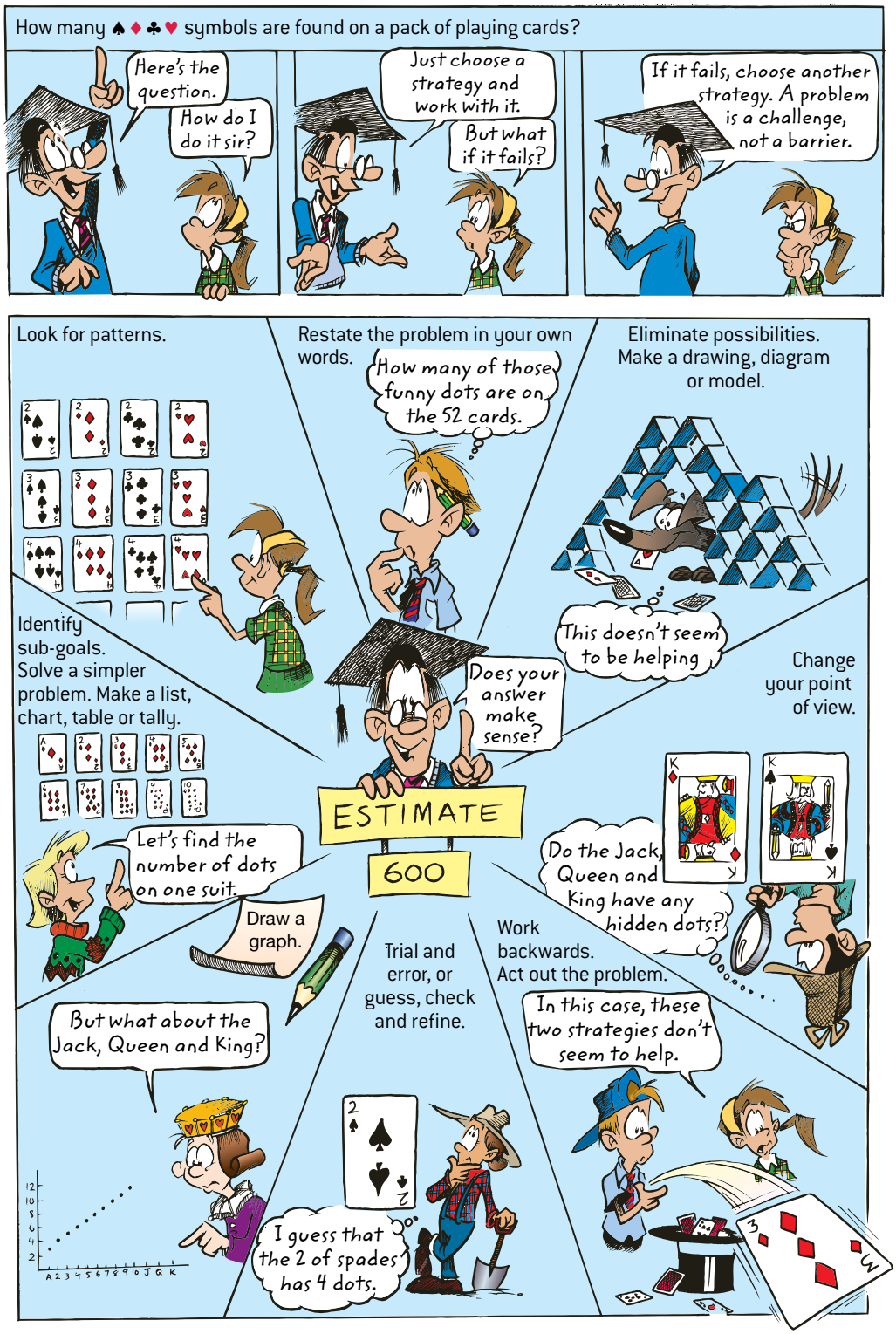
c Reasoning

i If you had twice as many dogs and three times as many people to help drench the 800 sheep, how much time is likely to be saved?

ii The sheep are harder to handle when they can see shadows as this makes them more fearful. How would you use this information in the planning of the drenching of sheep?

iii Could you improve the process shown in the picture? How could it be done in a different way? The farmer receives money from the sale of his sheep. What costs would the farmer have in order to earn this money? List as many as you can.

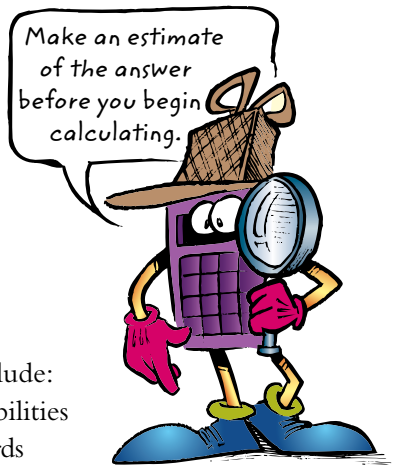
1:02 Problem-solving strategies



In Year 7 you were introduced to a variety of problem-solving strategies. These, along with strategies you have been using all your life, are the skills you bring to a problem. The problem may be as simple as finding your way from home to a shopping centre or as complex as trying to balance your budget.

Steps for solving problems

Step 1 Read the question carefully.
Step 2 Decide what you are asked to find.
Step 3 Look for information that might be helpful.
Step 4 Decide on the method you will use.
Step 5 Set out your solution clearly.
Step 6 Make sure that your answer makes sense.



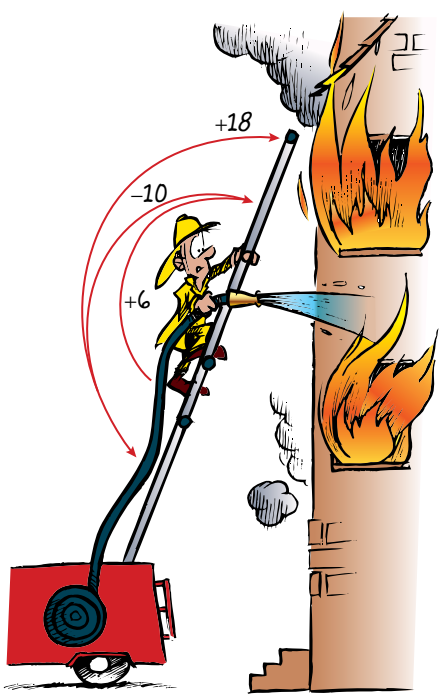
To estimate, ask:
- Will the answer be big or small?
- How big? How small?
- Will the answer be a whole number?
- Does my estimate make sense?
- Is it reasonable?

Useful strategies include:
- Eliminating possibilities
- Working backwards
- Acting it out
- Looking for patterns
- Solving a simpler problem
- Guess, check and refine
- Making a drawing, diagram or model
- Making a list, chart, table or tally

WORKED EXAMPLE 1

Make a drawing, diagram or model

A firefighter stood on the middle rung of a ladder, spraying water on a burning house. He then climbed up 6 rungs before the heat of the flames caused him to come down 10 rungs. After some minutes he was able to climb 18 rungs to the very top rung of the ladder. How many rungs did the ladder have?



Solution

First draw a picture of the ladder.
Let the middle rung be the zero position.
To get to the top rung the firefighter has gone up 6, then down 10, then up 18.

$6 - 10 + 18 = 14$

$\therefore$ there are 14 rungs above the middle rung.
There are also 14 rungs below the middle rung.
$14 + 1 + 14 = 29$
There are 29 rungs on the ladder.

WORKED EXAMPLE 2

Guess, check and refine

Jia is told to build 22 stools. Each stool must have either 3 or 4 legs and all 81 legs supplied must be used. How many of each type will she build?

Solution



The total number of stools is 22.

Guess 1: 10 three-legged stools and 12 four-legged stools.

Check 1: Number of legs = $(10 \times 3) + (12 \times 4)$
$= 30 + 48$
$= 78$

This is 3 legs too few.
(We need to use 3 more legs.)

Guess 2: 7 three-legged stools and 15 four-legged stools.

Check 2: Number of legs = $(7 \times 3) + (15 \times 4)$
$= 21 + 60$
$= 81$

This is the correct number of legs.
Jia will build 7 three-legged stools and 15 four-legged stools.

Exercise 1:02

P Foundation worksheet 1:02
Solving problems
Challenge worksheet 1:02
This puzzle is good 'training'
Appendix E:01
Fun spot: The Tower of Hanoi

1 Use the suggested strategy to solve each problem.

a **Making a list**

Three girls, Naomi, Elizabeth and Anna, were being considered for girl's captain and vice-captain, and two boys, Luke and Kuan, were being considered for boy's captain and vice-captain. In how many different ways could the captains and vice-captains be chosen?
One choice would be: Naomi (c), Elizabeth (vc), Luke (c), Kuan (vc).

b **Eliminating possibilities**

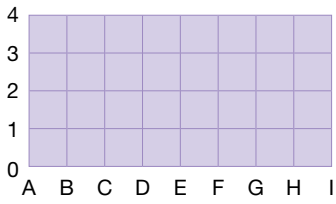


Each group in our class selected six bricks and found their total mass. Each brick selected by my group had a mass between 1·3 kg and 1·8 kg. The results from the groups were:

Group A	Group B	Group C	Group D
6·3 kg	7·8 kg	8·6 kg	11·1 kg

Which was my group?

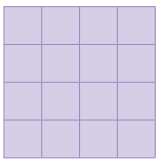
c Working backwards





A scientist attached a radio transmitter to a kangaroo to record its movements. She recorded the starting and finishing points on this grid. The length of each small square represents 3 km. After travelling 6 km east, 3 km south, 9 km west, 6 km north and then 12 km west, the kangaroo finished up at A2. What was the kangaroo's starting point?

d Acting it out

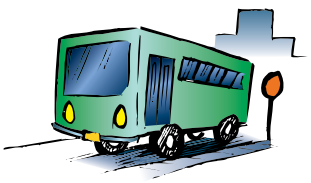


Tia and Rajiv decided to play noughts and crosses using a larger grid. To win, there must be four noughts or crosses in a line, either across, down or diagonally.

i How many ways are there to get four in a row?

ii How many ways are there to get three in a row with no empty squares between them?

e Looking for patterns



A bus that started with no passengers stops at five bus stops. At the first bus stop, eight people get on, and at each stop afterwards two fewer people get on than at the previous stop. Whenever possible, two passengers get off at each stop. How many people were on the bus when it left the fifth stop?

f Solving a simpler problem



How many faces would be showing if 102 cubes were placed on a table end to end, in a straight line. (To help find a solution, first consider the case of three cubes.)

g Guess, check and refine

For our school play, tickets cost $18 for adults and $15 for children. I was handed $297 as the payment for 17 tickets. How many of these tickets were for children?

h Making a drawing, diagram or model

Eight tennis players are to play in a round robin, where each player must play all other players in one set of singles. How many sets must be played?

2 Use any strategy you like to solve these problems.

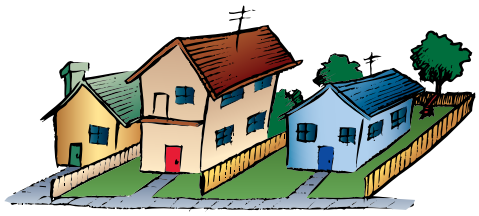


a A coin bought for $36 doubled its value each year for 5 years. What was its value after the 5 years?

b A plant that was bought for $9.50 had a height of 27 cm. In the first month it grew 2 cm in height. The growth for each of the next four months was double the growth of the previous month. How high was the plant after 5 months?

c Milo kept a cube made of heavy wire in his room. Its side lengths were 15 cm. A fly walked along the edges of the cube, from one vertex (or corner) to the opposite vertex of the cube. How far did the fly walk?

d The Bills family, the Collison family and the Foster family live in three houses side by side. The family in the middle house have no children and the family on the left do not own a fox terrier. Jennifer is proud of her whippet. Jennifer Collison and Damien Bills go to the same school. The whippet and the fox terrier are not neighbours. Who lives in each house?



e Taronga Park Zoo spent $250 000 on lions and tigers. They bought at least four lions and two tigers. If a lion costs $20 000 and a tiger costs $30 000, what possible purchases could they have made?

f In how many ways can 50 cents change be given if only 5 cent, 10 cent and 50 cent coins can be used?

g Mr French, Mr English, Mr Kraft and Mr Musik are four teachers who teach French, English, Craft and Music, however, none of them teaches a subject that sounds like his name. Mr French does not teach Music. When Mr French and Mr Kraft beat the other two at tennis, the English teacher is a bad loser but the music teacher is a gracious winner. What subject does each teacher teach?

h As the horses raced away, we noticed that there were 15 more legs than tails. How many horses were there?

i Sailors from a shipwreck were lost at sea. Search parties were sent out to find them. On the first day $\frac{1}{6}$ of the sailors were found. On the second day $\frac{1}{5}$ of those still lost were found. On the third day $\frac{1}{4}$ of those then lost were found. $\frac{1}{3}$ of those still lost were found on the fourth day. On the fifth day $\frac{1}{2}$ of those still lost were found and on the sixth day the last three sailors were found. How many sailors were lost originally?

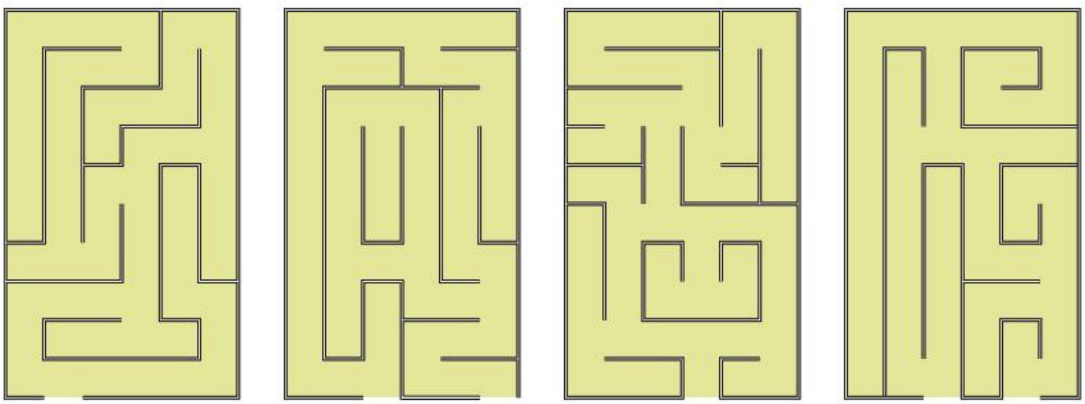


INVESTIGATION 1:02 THE LEFT-HAND RULE FOR MAZES

Use a pointer to travel through each of the following mazes in order to investigate these questions.

Can you find your way through these mazes by putting your hand on the left wall and walking forward?

Using this method, in which cases do you visit every part of the maze? For what kind of mazes would you visit every part?



By changing these mazes slightly and by drawing others, test your conclusions and ideas.

1:03 Beginnings in number

Exercises 1:03 to 1:16 are diagnostic check-ups. Complete one row at a time and then check your answers at the back of the book. If you make mistakes in any row, go to the eBook and click on the icon next to that row. This will link you to an Appendix page with an explanation and practice exercise.

Exercise 1:03

P Appendix A A:01A–A:01F

1 Write these Roman numerals as basic numerals in our number system.

a LX	**b** XL	**c** XXXIV	**d** CXVIII
e MDCCLXXXVIII	**f** MCMLXXXVIII	**g** $\overline{V}$CCCXXI	**h** MDCXV

2 Write these numerals as Roman numerals.

a 630	**b** 847	**c** 1308	**d** 3240
e 390	**f** 199	**g** 10 000	**h** 1773

3 Write the basic numeral for:

a six million, ninety thousand

b one hundred and forty thousand, six hundred

c $(8 \times 10\,000) + (4 \times 1000) + (7 \times 100) + (0 \times 10) + (5 \times 1)$

d $(7 \times 10^4) + (4 \times 10^3) + (3 \times 10^2) + (9 \times 10) + (8 \times 1)$

4 Write each of these in expanded form and write the basic numeral.

a 5^2 **b** 10^4 **c** 2^3 **d** 2^5

5 Write $6 \times 6 \times 6 \times 6$ as a power of 6.

6 Write the basic numeral for each of the following.

a 8×10^4 **b** 6×10^3 **c** 9×10^5 **d** 2×10^2

7 Use leading digit estimation to find an estimate for each of the following.

a $618 + 337 + 159$ **b** $38\,346 - 16\,097$ **c** $3250 \times 11{\cdot}4$

d $1987 \div 4$ **e** $38{\cdot}6 \times 19{\cdot}5$ **f** $84\,963 \div 3{\cdot}8$

1:04 Number: Its order and structure

Exercise 1:04

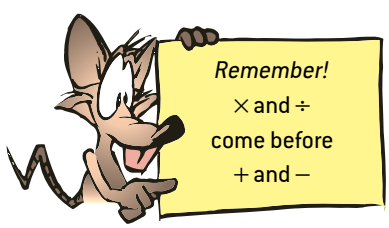
P Appendix A A:02A–A:02I

1 Simplify:

a $6 \times 2 + 4 \times 5$ **b** $12 - 6 \times 2$

c $4 + 20 \div (4 + 1)$ **d** $(6 + 7 + 2) \times 4$

e $50 - (25 - 5)$ **f** $50 - (25 - [3 + 19])$



2 Simplify:

a 347×1 **b** 84×0 **c** $36 + 0$

d $3842 + 0$ **e** $1 \times 30\,406$ **f** $864 \times 17 \times 0$

3 Decide if each of the following is true or false.

a $879 + 463 = 463 + 879$ **b** $76 \times 9 = 9 \times 76$

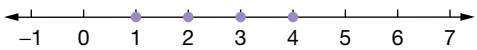
c $4 + 169 + 96 = (4 + 96) + 169$ **d** $4 \times 83 \times 25 = (4 \times 25) \times 83$

e $8 \times (17 + 3) = 8 \times 17 + 8 \times 3$ **f** $4 \times (100 - 3) = 4 \times 100 - 4 \times 3$

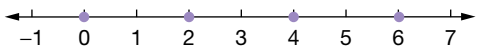
g $7 \times 99 = 7 \times 100 - 7 \times 1$ **h** $17 \times 102 = 17 \times 100 + 17 \times 2$

4 List the set of numbers graphed on each of the number lines below.

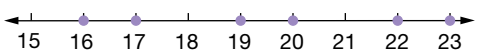
a



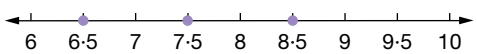
b



c



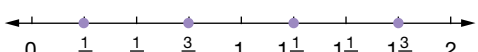
d



e



f



5 Use the number lines in Question **4** to decide whether each of the following is true or false.

a $3 < 5$ **b** $-1 < 0$ **c** $7 > 4$ **d** $0 > -1$

e $7{\cdot}5 < 9$ **f** $0 > 0{\cdot}7$ **g** $\frac{1}{4} < \frac{1}{2}$ **h** $1\frac{1}{4} > \frac{3}{4}$

6 Which of the numbers in the set $\{0, 3, 4, 6, 11, 16, 19, 20\}$ are:

a cardinal numbers **b** counting numbers **c** even numbers

d odd numbers **e** square numbers **f** triangular numbers?

7 List all factors of:

a 12 **b** 102 **c** 64 **d** 140

8 List the first four multiples of:

a 7 **b** 5 **c** 12 **d** 13

9 Find the highest common factor (HCF) of:

a 10 and 15 **b** 102 and 153 **c** 64 and 144 **d** 294 and 210

10 Find the lowest common multiple (LCM) of:

a 6 and 8 **b** 15 and 9 **c** 25 and 20 **d** 36 and 24

11 Write these using index notation.

a $10 \times 10 \times 10 \times 10$ **b** $7 \times 7 \times 7$ **c** $2 \times 2 \times 2 \times 2 \times 2$

Write these as basic numerals.

d 5^3 **e** 10^4 **f** 7×10^3 **g** 2×10^5

12 **a** List all of the prime numbers that are less than 30.

b List all of the composite numbers that are between 30 and 40.

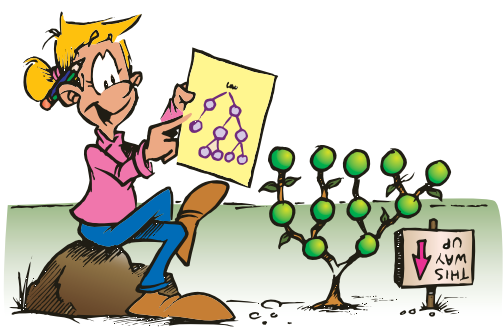
13 **a** Use a factor tree to write 252 as a product of prime factors.

b Write 400 as a product of prime factors.

c Write 1080 as a product of prime factors.

d Find the highest common factor (HCF) of 400 and 1080.

e Find the lowest common multiple (LCM) of 400 and 1080.



14 Find the smallest number that is greater than 2000 and is divisible by:

a 2 **b** 3 **c** 4 **d** 5 **e** 6 **f** 8

g 9 **h** 10 **i** 11 **j** 25 **k** 100 **l** 12

15 Complete the following.

a If $15^2 = 225$, then $\sqrt{225}$ = _____ **b** If $8^3 = 512$, then $\sqrt[3]{512}$ = _____

c If $13^2 = 169$, then $\sqrt{169}$ = _____ **d** If $4^3 = 64$, then $\sqrt[3]{64}$ = _____

1:05 Fractions

Exercise 1:05

P Appendix A A:03B–A:03N

1 Change each fraction to a whole or mixed numeral.

a $\frac{10}{2}$ b $\frac{9}{4}$

c $\frac{87}{10}$ d $\frac{11}{8}$



2 Change each mixed numeral to an improper fraction.

a $3\frac{1}{2}$ b $5\frac{3}{10}$

c $1\frac{3}{4}$ d $3\frac{1}{7}$

3 Simplify each fraction.

a $\frac{8}{10}$ b $\frac{20}{50}$

c $\frac{15}{100}$ d $\frac{18}{24}$

4 Complete the following equivalent fractions.

a $\frac{2}{5} = \frac{\square}{10}$ b $\frac{3}{4} = \frac{\square}{100}$ c $\frac{4}{1} = \frac{\square}{10}$ d $\frac{1}{3} = \frac{\square}{120}$

5 a $\frac{3}{10} + \frac{4}{10}$ b $\frac{19}{100} - \frac{6}{100}$ c $\frac{3}{8} + \frac{7}{8}$ d $\frac{5}{12} - \frac{1}{12}$

6 a $\frac{3}{5} + \frac{3}{10}$ b $\frac{9}{10} - \frac{3}{4}$ c $\frac{3}{4} + \frac{2}{5}$ d $\frac{31}{100} - \frac{1}{5}$

7 a Which fraction is the smaller: $\frac{3}{4}$ or $\frac{6}{10}$?

b Which fraction is the larger: $\frac{2}{5}$ or $\frac{1}{3}$?

c Arrange in order, from smallest to largest: $\{\frac{8}{10}, \frac{17}{20}, \frac{3}{4}\}$.

d Arrange in order, from largest to smallest: $\{\frac{1}{2}, \frac{3}{5}, \frac{1}{4}, \frac{2}{3}\}$.

8 a $3\frac{2}{5} + 1\frac{3}{4}$ b $10\frac{7}{8} - 3\frac{1}{2}$ c $5 - 2\frac{3}{5}$ d $10\frac{1}{5} - 1\frac{1}{2}$

9 a $\frac{2}{5} \times \frac{3}{4}$ b $\frac{3}{10} \times \frac{7}{10}$ c $\frac{3}{5} \times \frac{2}{3}$ d $\frac{9}{10} \times \frac{15}{16}$

10 a $4 \times \frac{3}{5}$ b $1\frac{7}{8} \times 3$ c $3\frac{3}{4} \times 1\frac{1}{3}$ d $2\frac{1}{2} \times 1\frac{4}{5}$

11 a $\frac{9}{10} \div \frac{1}{2}$ b $\frac{3}{8} \div \frac{3}{5}$ c $4 \div \frac{3}{5}$ d $1\frac{1}{2} \div 3\frac{3}{4}$

12 a Find $\frac{3}{5}$ of 2 km.

b What fraction of 2 m is 40 cm?

13 A jar contains 20 blue counters and 30 red counters. What is the:

a ratio of blue counters to red counters

b ratio of red counters to blue counters

c fraction of blue counters in the jar?

1:06 Decimals

Exercise 1:06

P Appendix A A:04A–A:04J

1 **a** Write $(1 \times 10) + (7 \times 1) + (5 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$ as a decimal.

b Write $(6 \times 100) + (8 \times 10) + (4 \times 1) + (0 \times \frac{1}{10}) + (2 \times \frac{1}{100})$ as a decimal.

2 Change each decimal to a fraction or mixed numeral in simplest form.

a 0·7 **b** 2·13 **c** 0·009 **d** 5·3

e 0·85 **f** 0·025 **g** 1·8 **h** 9·04

3 Change each fraction or mixed numeral to a decimal.

a $\frac{9}{10}$ **b** $\frac{13}{100}$ **c** $1\frac{1}{2}$ **d** $2\frac{99}{100}$

e $\frac{3}{5}$ **f** $\frac{33}{200}$ **g** $\frac{5}{8}$ **h** $\frac{3}{11}$

4 Write in ascending order (smallest to largest).

a {0·3, 0·33, 0·303} **b** {2, 0·5, 3·1} **c** {0·505, 0·055, 5·5}

Do not use a calculator for the following questions.

5 **a** 3·7 + 1·52 **b** 63·85 − 2·5 **c** 8 + 1·625 **d** 8 − 1·625

6 **a** 0·006 × 0·5 **b** 38·2 × 0·11 **c** $(0{\cdot}05)^2$ **d** 1·3 × 19·1

7 **a** 0·6 × 100 **b** 0·075 × 10 **c** 81·6 ÷ 100 **d** 0·045 ÷ 10

8 **a** 48·9 ÷ 3 **b** 1·5 ÷ 5 **c** 8·304 ÷ 8 **d** 0·123 ÷ 4

e 3·8 ÷ 0·2 **f** 0·8136 ÷ 0·04 **g** 875 ÷ 0·05 **h** 3·612 ÷ 1·2

9 **a** \$362 + \$3.42 **b** \$100 − \$41.63 **c** \$8.37 × 8 **d** \$90 ÷ 8

10 **a** Round 96 700 000 to the nearest million.

b Round 0·085 to the nearest hundredth.

c Round 86·149 to one decimal place.

d Write $0\dot{6}$ correct to two decimal places.

When you round, you are making an approximation.

1:07 Percentages

Exercise 1:07

P Appendix A A:05A–A:05F

1 Write each percentage as a fraction or mixed numeral in simplest form.

a 9% **b** 64% **c** 125% **d** $14\frac{1}{2}$%

2 Write each fraction or mixed numeral as a percentage.

a $\frac{3}{4}$ **b** $1\frac{3}{8}$ **c** $\frac{37}{300}$ **d** $4\frac{3}{5}$

3 Write each percentage as a decimal.

a 47% **b** 4% **c** 325% **d** 300%
e 50% **f** 104% **g** 12·7% **h** 0·3%

4 Change each decimal to a percentage.

a 0·87 **b** 1·3 **c** 5 **d** 0·825

5 **a** Find 8% of 560 L. **b** Find 70% of 680 g.
c Find 5% of $800. **d** Find 10% of 17·9 m.
e Jon scored 24 marks out of 32. What percentage did she score?
f 250 g of sugar is mixed with 750 g of flour. What percentage of the mixture is sugar?



Fractions, decimals and percentages are called rational numbers as they can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

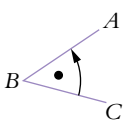
1:08 Angles

Exercise 1:08

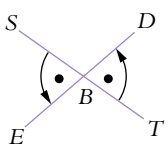
Appendix A A:06A–A:06D

1 Name each angle marked with a dot, using the letters in the diagrams.

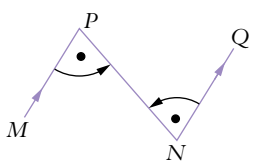
a



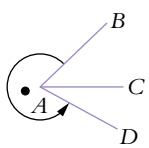
b



c

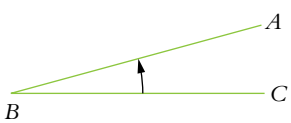


d

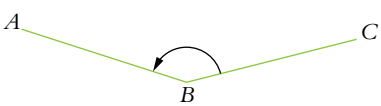


2 Use a protractor to measure $\angle ABC$.

a



b

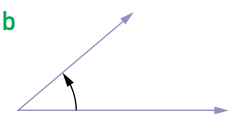


3 Classify each angle using one of these terms: acute, right, obtuse, straight, reflex, revolution.

a



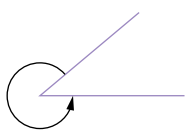
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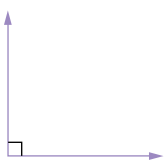
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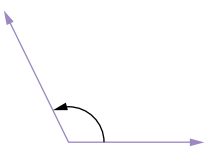
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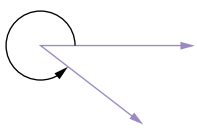
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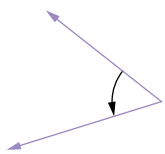
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g



h

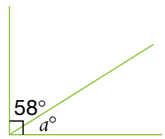


4 Draw a pair of:

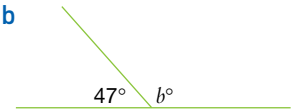
a adjacent complementary angles
b vertically opposite angles
c alternate angles
d adjacent supplementary angles.

5 Find the value of the pronumeral in each.

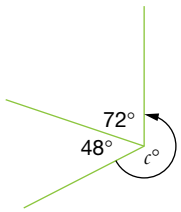
a



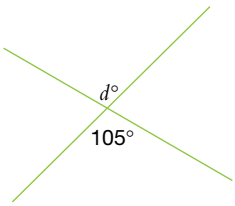
b



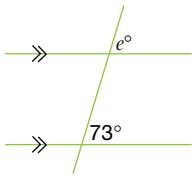
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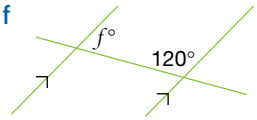
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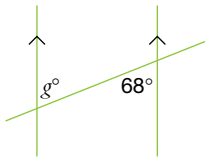
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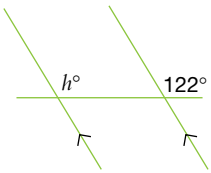
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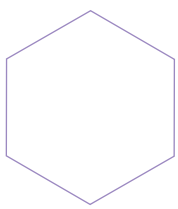


1:09 2D space

Exercise 1:09

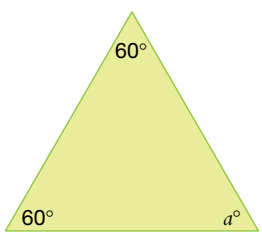
P Appendix A
A:07A, A:07D, A:07F, A:07G

1 **a** What is the name of the shape on the right?
b How many vertices has this shape?
c How many sides has this shape?
d How many angles has this shape?
e How many diagonals has this shape?

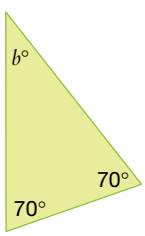


2 Choose two of the following names for each triangle and find the value of the pronumeral: equilateral, isosceles, scalene, acute-angled, right-angled, obtuse-angled

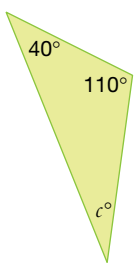
a



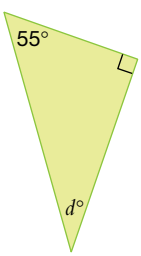
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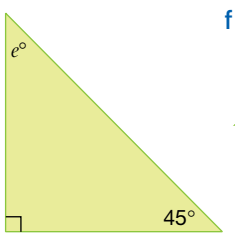
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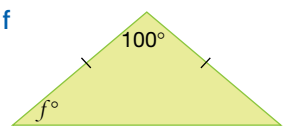
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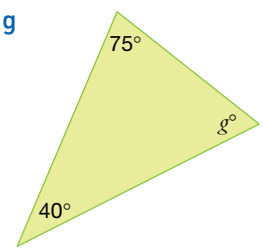
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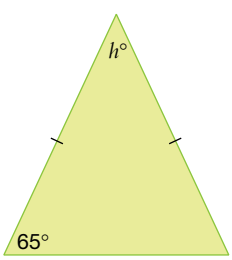
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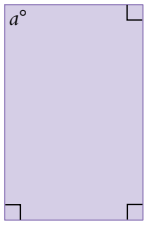


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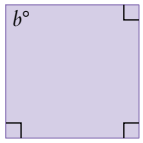


3 Calculate the value of the pronumeral in each quadrilateral.

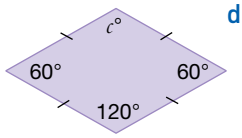
a



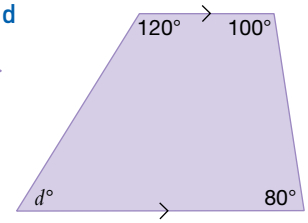
b



c



d



4 **a** Give the special name of each shape in Question 3.

b Which of the shapes in Question 3 have:

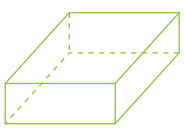
- **i** opposite sides equal
- **ii** all sides equal
- **iii** two pairs of parallel sides
- **iv** only one pair of parallel sides
- **v** diagonals meeting at right angles?

1:10 3D space

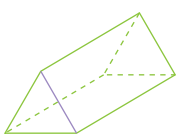
Exercise 1:10

P Appendix A A:08A–A:08E

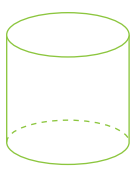
A



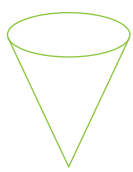
B



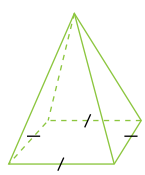
C



D



E



1 **a** Give the name of each solid above.

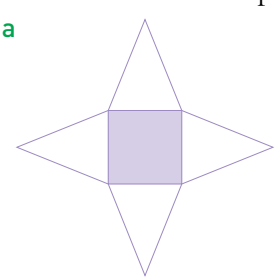
b Which of these solids have curved surfaces?

c For solid B, find:

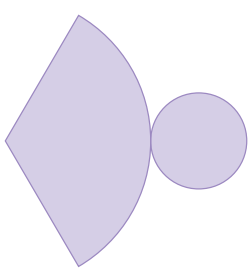
- **i** the number of faces (F)
- **ii** the number of vertices (V)
- **iii** the number of edges (E)
- **iv** number of edges + 2 ($E + 2$)
- **v** number of faces + number of vertices ($F + V$)

2 Name the solid corresponding to each net.

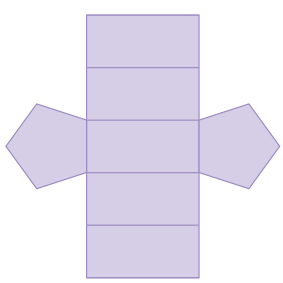
a



b



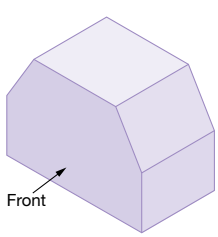
c



3 **a** Draw the front view of this prism.

b Draw the side view of this prism.

c Draw the top view of this prism.



INVESTIGATION 1:10 IDENTIFYING 3D SOLIDS

List the three-dimensional solids that you can find in this picture.

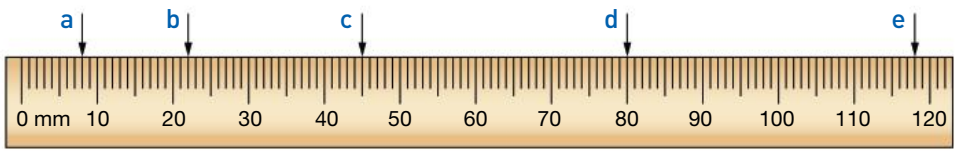


1:11 Measurement

Exercise 1:11

P Appendix A A:09B–A:09J

1 Write down each measurement in centimetres, giving answers correct to one decimal place.



2 Complete each of these conversions.

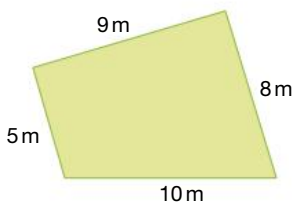
a 3000 mm = ____ cm b 2500 mL = ____ L c 630 mg = ____ g

d 7 km = ____ m e 7·8 kg = ____ g f 2·5 m = ____ cm

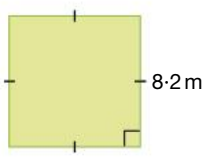
3 An interval is 8·4 cm long. It must be divided into 12 equal parts. How many millimetres would be in each part?

4 Find the perimeter of each of these shapes.

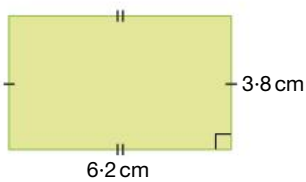
a



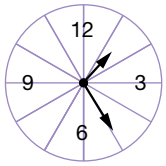
b

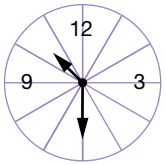


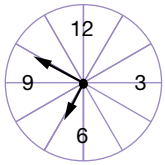
c

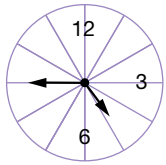


5 Write the time on each clock in both conventional and digital time.

a 

b 

c 

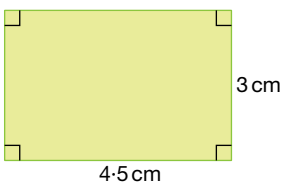
d 

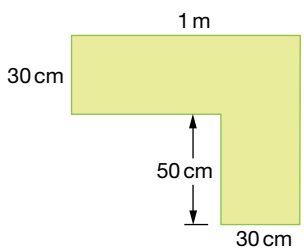
6 a Rajiv ran at a speed of 5 m/s for 20 s. How far did he run?

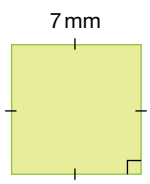
b Taya walked at a constant speed for 50 s. During this time she travelled 150 m. What was her speed?

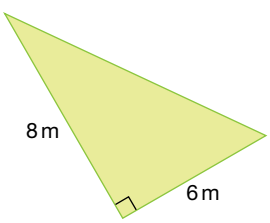
c A train travelling with a speed of 30 km/h travelled a distance of 120 km. How long did this take?

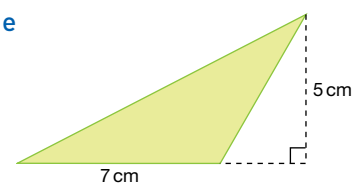
7 Find the area of each shape below.

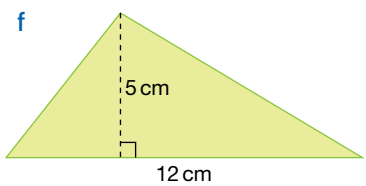
a 

b 

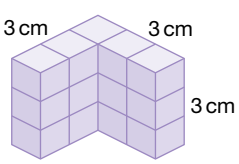
c 

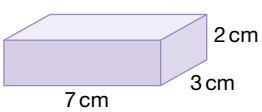
d 

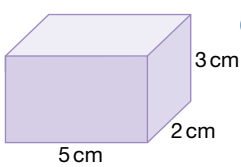
e 

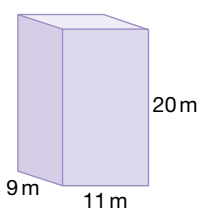
f 

8 Find the volume of each prism.

a 

b 

c 

d 

9 Select the most likely answer.

a A teaspoon of water would contain:	**A** 5 L	**B** 5 mL	**C** 5 kL
b A milk carton would contain:	**A** 1 mL	**B** 1 kL	**C** 1 L
c 1 mL is the same volume as:	**A** 1 m^3	**B** 1 cm^3	**C** 1 mm^3
d A small packet of peas has a mass of:	**A** 5 kg	**B** 50 g	**C** 5 g
e A bag of potatoes has a mass of:	**A** 20 kg	**B** 20 g	**C** 20 mg

10 The gross mass, net mass and container's mass of a product are required. Find the missing mass.

a Gross mass = 500 g, net mass = 400 g, container's mass = _____ g.

b Net mass = 800 g, container's mass = 170 g, gross mass = _____ g.

1:12 Directed numbers

Exercise 1:12

P Appendix A A:10A–A:10D

1 Which members of the following set are integers: $\{-3, \frac{1}{2}, -1{\cdot}5, 4, 0, -10\}$?

2 Give the basic numeral for each of the following.

a $-7 + 11$	**b** $-3 + 15$	**c** $-9 + 2$	**d** $-25 + 5$
e $2 - 13$	**f** $7 - 10$	**g** $-7 - 5$	**h** $-10 - 3$
i $6 - (-10)$	**j** $14 - (-1)$	**k** $3 + (+7)$	**l** $15 + (+1)$
m $10 - (3 - 9)$	**n** $15 - (2 - 5)$	**o** $3 + (-7 + 11)$	**p** $11 + (-5 + 18)$

3 Simplify:

a -4×-3	**b** -8×-2	**c** $-0{\cdot}2 \times -3$	**d** $-0{\cdot}1 \times -15$
e -4×14	**f** -5×8	**g** $7 \times (-1{\cdot}1)$	**h** $6 \times (-12)$
i $-35 \div (-5)$	**j** $(-40) \div (-10)$	**k** $60 \div -6$	**l** $14 \div -7$
m $\frac{-21}{-3}$	**n** $\frac{-24}{-4}$	**o** $\frac{-48}{6}$	**p** $\frac{-1{\cdot}8}{2}$

4 Write down the basic numeral for:

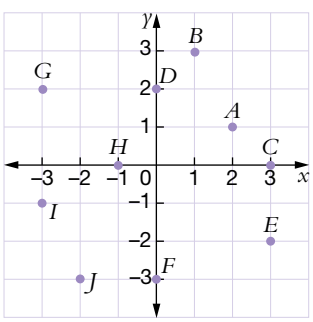
a $-3 + 6 \times 2$	**b** $-4 - 8 \times 2$	**c** $6 - 4 \times 4$	**d** $-30 + 2 \times 10$
e $-8 + 6 \times -3$	**f** $10 + 5 \times -2$	**g** $(2 - 20) \div 3$	**h** $(8 - 38) \div -3$
i $8 \times \$1.15 - 18 \times \$1.15$		**j** $35° + 2 \times 15° - 4 \times 20°$	

1:13 The number plane

Exercise 1:13

P Appendix A A:11B, A:11C

1 Find the coordinates of each of the points A to J.





The negatives are on the left on the x-axis.
The negatives are at the bottom on the y-axis.

2 On a number plane like the one in Question 1, plot the following points. Join them in the order in which they are given, to draw a picture.
$(2, 0)$ $(3, 0)$ $(3, -1)$ $(1, -1)$ $(1, -1{\cdot}5)$ $(2, -1{\cdot}5)$ $(2, -1)$ $(-2, -1)$ $(-2, -1{\cdot}5)$
$(-1, -1{\cdot}5)$ $(-1, -1)$ $(-3, -1)$ $(-3, 0)$ $(-2, 0)$ $(-1, 1)$ $(1, 1)$ $(2, 0)$ $(-2, 0)$

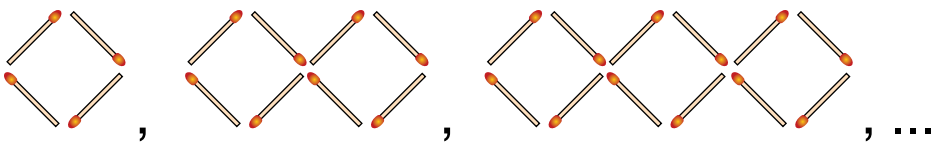
1:14 Algebra

Exercise 1:14

P Appendix A
A:11A, A:11B, A:12A–A:12I

1 If s represents the number of squares formed and m is the number of matches used, find a rule to describe each pattern and use it to complete the table.

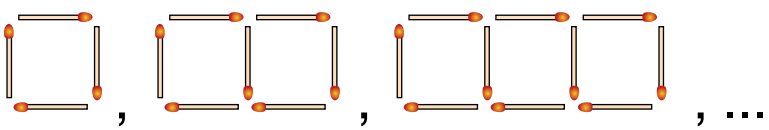
a



$m =$ _____

s	1	2	3	4	10	20	30	100
m								

b



$m =$ _____

s	1	2	3	4	10	20	30	100
m								

2 Rewrite each of these without the use of × or ÷ signs.

a $5 \times h + 2$ **b** $a + 3 \times y$ **c** $6 \times (a + 7)$ **d** $5 \times a \div 7$

3 Rewrite each of these, showing all multiplication and division signs.

a $3a + 8$ **b** $5p - 6q$ **c** $4(x + 2)$ **d** $\frac{a+7}{3}$

4 Given that $x = 3$, find the value of each expression.

a $6x$ **b** $2(x + 5)$ **c** $5x^2$ **d** $10 - 3x$

5 If $a = 2$ and $b = 5$, find the value of each expression.

a $3a + 7b$ **b** $\frac{10a}{b}$ **c** $4a(b - a)$ **d** $a^2 + b^2$

6 If $m = 2t + 1$, find the value of m when $t = 100$.

7 Discover the rule connecting x and y in each table.

a

x	1	2	3	4	5
y	7	10	13	16	19

b

x	1	2	3	4	5
y	11	15	19	23	27

8 Simplify:

a $1m$ **b** $1 \times a$ **c** $4 \times y$ **d** $y + y + y + y$
e $f \times 5$ **f** $a \times b$ **g** $5 \times k$ **h** $5 \times a \times b$
i $8x \times 0$ **j** $4y \times 0$ **k** $6m + 0$ **l** $3a \times 1$
m $7a + 5a$ **n** $10a + a$ **o** $7b - b$ **p** $114a - 64a$
q $m - 3m$ **r** $4b - 6b$ **s** $4x^2 + 3x^2$ **t** $6ab - 5ab$

9 Simplify:

a $3 \times 5a$ **b** $6 \times 10b$ **c** $7m \times 3p$ **d** $8x \times 4y$
e $a \times 4b$ **f** $6m \times 5$ **g** $-3k \times -5$ **h** $-6y \times 3$
i $12t \div 3$ **j** $30t \div 3$ **k** $6m \div 2a$ **l** $10a \div 5b$
m $15r \div 10$ **n** $8m \div 6$ **o** $3ab \times 7a$ **p** $5ab \times 4b$

10 Simplify:

a $5m + 7m - 10m$ **b** $8x - 6x - x$ **c** $5x + 2y + 7y$
d $12a + 3b - 2a$ **e** $7p + 2q + 3p + q$ **f** $3r + 2A + 3A + 5r$
g $6a + 7b - 2a + 5b$ **h** $4m + 3 - 2m + 1$ **i** $8m + 2a - 2m - 8a$
j $7a^2 - 4a + 2a^2$ **k** $2x^2 + 3x + 2x$ **l** $2x^2 + 3x + 2x + 3$

11 Expand each expression by removing the grouping symbols.

a $3(a + 9)$ **b** $5(x + 2)$ **c** $10(m - 4)$ **d** $9(2a - 3)$ **e** $6(4t + 3)$
f $5(2 + 4x)$ **g** $m(m + 7)$ **h** $a(a - 3)$ **i** $a(6 + a)$

12 Simplify:

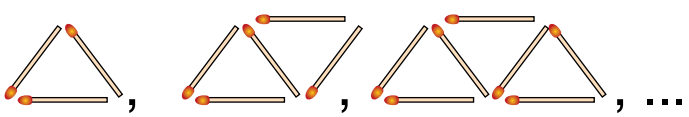
a $a \times a \times a \times a$ **b** $p \times p \times p \times p \times p \times p \times p$ **c** $b \times b$
d $m \times m \times m \times b \times b \times b \times b$ **e** $5 \times t \times t \times 7 \times t$

13 Solve these equations.

a $x + 5 = 9$ **b** $x + 4 = 28$ **c** $12 - a = 5$ **d** $6 - a = -1$
e $6m = 42$ **f** $5m = 100$ **g** $m + 7 = 2$ **h** $m - 1 = -5$

14 The sum of two consecutive numbers is 91. What are the numbers?

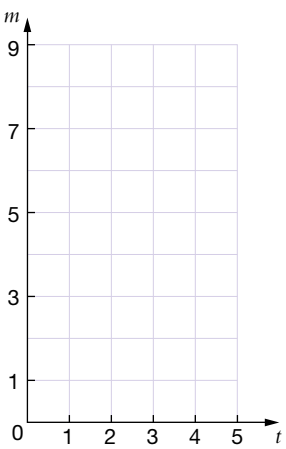
15



This pattern of triangles formed from matches gives the following table.

Number of triangles (t)	1	2	3	4
Number of matches (m)	3	5	7	9

Plot these ordered pairs on a number plane like the one to the right.



1:15 Probability

Exercise 1:15

Appendix A A:13A–A:13D

1 Match each term to its meaning and its percentage chance of occurring.

	impossible	
0%	certain	The event is more likely to happen than not.
50%	unlikely	The event is more likely not to happen.
between 50% and 100%	likely	The event will always happen.
100%	even chance	The event will never happen.
between 0% and 50%	fifty-fifty chance	The event is as likely to happen as not.
	no chance	

2 Write the sample space for each of these experiments.

a I will drop a cone (3D solid) onto the floor to find how it lands.

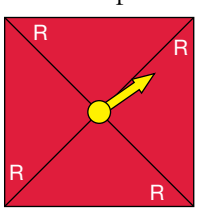
b I will toss a normal dice to see what number shows on the top face.

3 Rohan throws a dice. Write, as a fraction, the probability that he will throw:

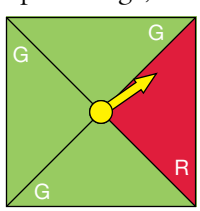
a a 5 **b** a 0 **c** an odd number

4 What is the probability, as a percentage, that the spinner will stop on red?

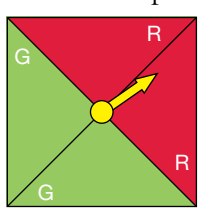
a



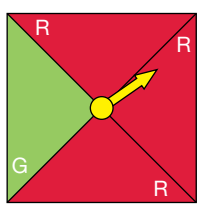
b



c



d



5 My last ten test scores have been 47, 58, 63, 65, 68, 72, 77, 81, 89 and 95. If one of these scores is chosen at random, what is the probability that it will be:

a 58 **b** greater than 75 **c** less than 45?

6 I tossed two coins 20 times. I tossed two heads (HH) 6 times, a head and a tail (HT) 10 times and two tails (TT) 4 times. Use these results to give the experimental probability of tossing:

a two heads **b** a head and a tail **c** two tails.

1:16 Statistics

Exercise 1:16

P Appendix A A:14A–A:14D

1 Organise the data below into a frequency distribution table.

Spelling test results
10, 8, 9, 8, 9, 7, 10, 10, 9, 10, 10, 9, 9, 7, 8, 8, 8, 6, 8, 9, 10, 10, 10, 8, 9, 9, 10, 10, 9, 10

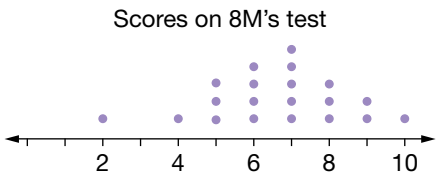
Score	Tally	Frequency
6		
7		
8		
9		
10		
	Total	

2 Draw a frequency histogram and polygon for the information in Question 1.

3 Find the range, mode, median and mean of the scores in Question 1.

4 Is the dot plot shown a correct graph of the list of scores given?
Use the dot plot to find the range, mode, median and mean.

Scores on 8M's test						
4	6	8	7	6	5	2
7	7	8	9	7	5	9
5	6	7	6	10	8	



5 For this stem-and-leaf plot:

a list all scores in order from smallest score to largest

b find the range of the scores

c find the mode and median.

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

tens | ones

ASSIGNMENT 1A Chapter review

1 Change these Roman numerals into our own numerals.

a XXXIV **b** MDCXL

c MMCDXIII **d** MCMXCII

2 Write 3^4 in expanded form and write its basic numeral.

3 What is the basic numeral for 9×10^5?

4 Simplify:

a $15 - 4 \times 3$

b $(6 + 4) \div (15 - 11)$

c $50 - (16 - 10)$

5 Plot $\{0, 3, 4\}$ on the number line.

6 Answer true or false.

a $0 < -4$

b $(-3)^2 > 2^2$

c $\frac{1}{5} < \frac{1}{4}$

7 List all of the factors of 24.

8 Simplify:

a $\frac{20}{50}$ **b** $\frac{40}{5}$

c $\frac{9}{4}$ **d** $\frac{3}{5} - \frac{2}{10}$

e $\frac{3}{10} \times \frac{7}{10}$ **f** $\frac{4}{5} \div \frac{1}{5}$

9 Answer true or false.

a $0{\cdot}7 > 0{\cdot}65$

b $(0{\cdot}05)^2 = 0{\cdot}0025$

c $(\frac{1}{2})^2 = \frac{1}{4}$

10 **a** Change 9% into a decimal.

b Write 60% as a fraction in its simplest form.

c Change $\frac{3}{4}$ to a percentage.

d Find 8% of 560 km.

11 Classify the following angles as acute, obtuse or reflex.

a 78° **b** 155° **c** 98°

d 185° **e** 278°

12 Complete the following conversions.

a 5 m = _____ mm

b 2·5 m = _____ cm

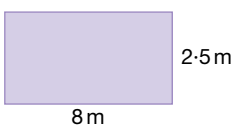
c 5·2 g = _____ mg

d 560 kg = _____ t

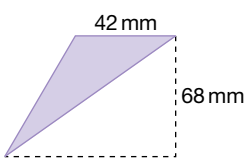
e 3300 mL = _____ L

13 Calculate the areas of the shapes shown.

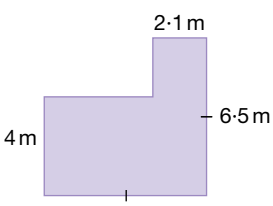
a



b



c



ASSIGNMENT 1B Working mathematically

1 Refer to **ID Card 7** on page xx to identify the figures numbered:

a 5 **b** 7 **c** 8
d 17 **e** 18 **f** 19
g 20 **h** 21 **i** 22
j 23



2 Use a calculator to find:

a three consecutive numbers that have a sum of 822
b three consecutive numbers that have a sum of 1998
c three consecutive numbers that have a sum of 24 852
d three consecutive numbers that have a product of 336
e three consecutive numbers that have a product of 2184
f three consecutive numbers that have a product of 15 600.

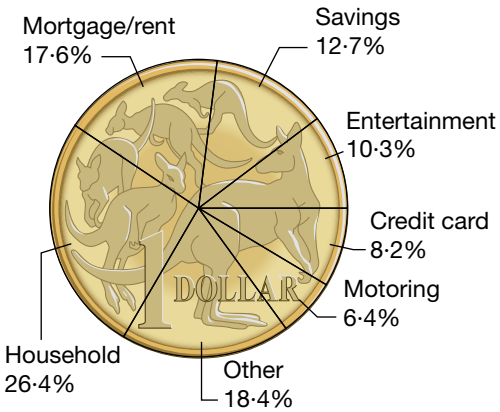
3 Which counting number, when squared, is closest to:

a 210 **b** 3187 **c** 2·6?

4 After the twin towers of the World Trade Centre fell in New York on 11 September 2001, it took 3.1 million hours of labour to remove 1 590 227 tons of debris. If this was completed on 10 May 2002, find:

a the number of days that passed until all debris was removed
b the average mass of debris that was removed in each hour of labour
c the number of tonnes of debris removed if 1 ton = 907 kilograms. (*Remember:* 1 tonne = 1000 kg.)

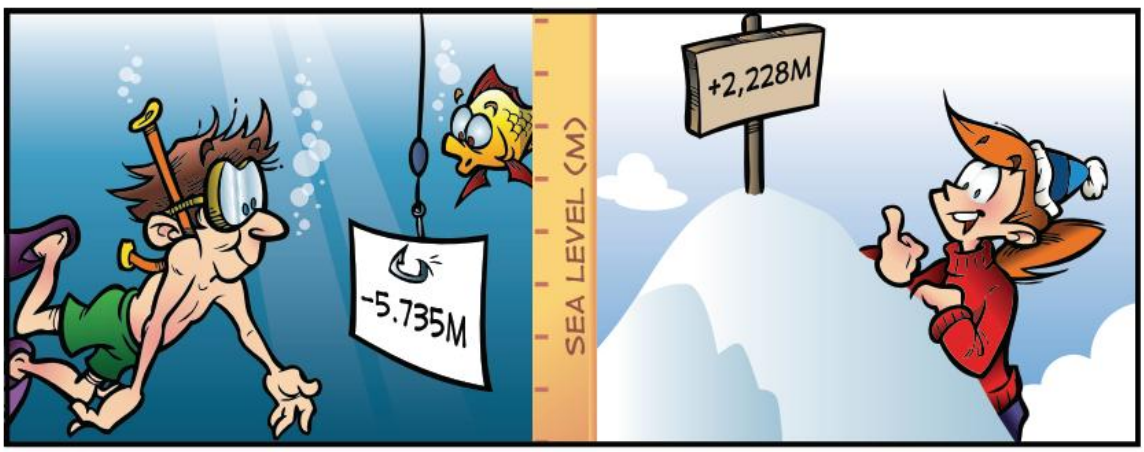
5 This graph appeared in the *Sunday Telegraph* where a survey carried out by the Commonwealth Bank of Australia suggested that Australians are not saving enough of their income.



a If the wage represented here is $960 per week, how much is allocated to:
i savings **ii** mortgage/rent
iii credit card **iv** motoring?
b What do you think was the most popular reason for saving?
c The three top reasons for saving were given in the article. Try to guess these three reasons.

NUMBER AND ALGEBRA

2



Contents

2:01 Operations with directed numbers
Fun spot 2:01 Where do sick ponies go?
2:02 Directed numbers and the calculator
Challenge 2:02 Speed and accuracy
2:03 Directed numbers and algebra
2:04 The index laws
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

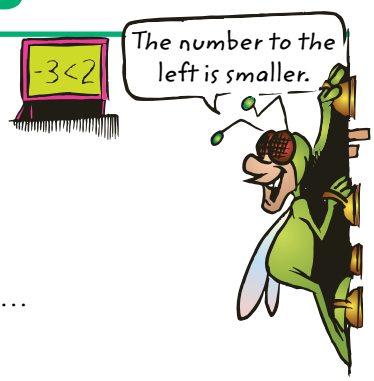
Selections from ***Computation with Integers***, ***Algebraic Techniques 1***, ***Algebraic Techniques 2***, ***Indices, Equations*** [Stage 4]

- Compare, order, add and subtract integers. (ACMNA280)
- Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies. (ACMNA183)
- Simplify algebraic expressions involving the four operations. (ACMNA192)
- Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)
- Use index notation with numbers to establish the index laws with positive-integer indices and the zero index. (ACMNA182)
- Solve simple quadratic equations.

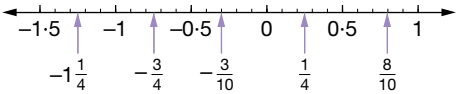
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

2:01 Operations with directed numbers



Directed numbers have size and direction. We can plot them on a number line.



Integers are directed whole numbers: … −4, −3, −2, −1, 0, 1, 2, 3, 4, …

- −7 is read as 'minus 7' or 'negative 7'.
- +3 (i.e. 'positive 3') can be written as 3.

Operation	Examples
Addition and subtraction	
(1) A negative number (or the minus sign) can represent a loss. A positive number (or the plus sign) can represent a gain.	(1) $-5 + 3 = -2$ A loss of 5 and a gain of 3 gives a loss of 2.
(2) $-5 + -3 = -5 + (-3) = -5 - 3$	(2) $-5 - 3 = -8$ A loss of 5 and a loss of 3 gives a loss of 8.
Subtracting a negative number	
(1) To subtract a directed number, add its opposite. • To take away −6, add 6.	(1) $-7 - -6 = (-7) - (-6) = -7 + 6$
(2) Two minus signs, side by side become a plus.	(2) $5 - -3 = 5 + 3$
(3) The opposite of minus 24 is plus 24.	(3) $- -24 = 24$
Multiplication	
(1) Multiplying two numbers with unlike signs gives a minus. • A minus times a plus gives a minus.	(1) $7 \times -6 = -42$
(2) Multiplying two numbers with like signs gives a plus. • A minus times a minus gives a plus.	(2) $-5 \times -3 = 15$
(3) Multiplying an even number of minus signs gives a plus.	(3) $(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$
(4) Multiplying an odd number of minus signs gives a minus.	(4) $(-2)^3 = -2 \times -2 \times -2 = -8$

Operation	Examples
Division	
(1) Division with two numbers of unlike signs gives a minus. • A minus divided by a plus gives a minus.	(1) $-42 \div 7 = -6$ $15 \div -3 = -5$
(2) Division with two numbers of like signs gives a plus. • A minus divided by a minus gives a plus.	(2) $-28 \div -7 = 4$ $-70 \div -10 = 7$
Multiplication and division	
(1) Two like signs give a plus.	(1) $-18 \div -6 = 3$
(2) Two unlike signs give a minus.	(2) $-5 \times 3 = -15$
(3) The order of operations applies to directed numbers: grouping symbols ( ) then × and ÷ then + and −	(3) $5 - (6 - 24)$ $= 5 - -18$ $= 23$

WORKED EXAMPLES

1 $-9 + 120 = 111$
A loss of 9 and a gain of 120 is a gain of 111.

2 $-120 + 9 = -111$
A loss of 120 and a gain of 9 is a loss of 111.

3 $-120 - 9 = -129$
A loss of 120 and a loss of 9 gives a loss of 129.

4 $120 - -9 = 129$
To take away −9, we add 9.
This is the same as $120 - (-9)$.

5 $-30 \times -5 = 150$
A minus times a minus gives a plus.

6 $-30 \times 5 = -150$
A minus times a plus gives a minus.

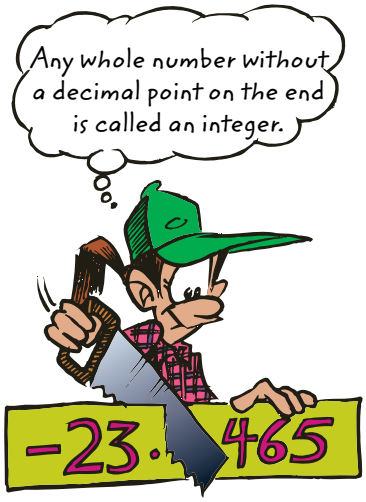
7 $-30 \div -5 = 6$
A minus divided by a minus gives a plus.

8 $30 \div -5 = -6$
A plus divided by a minus gives a minus.

9 $-30 - 5 \div -5$
$= -30 - -1$
$= -30 + 1$
$= -29$
Division is performed before subtraction, using the order of operations.

10 $(-3)^4$
$= -3 \times -3 \times -3 \times -3$
$= 81$
Multiplying an even number of minus signs gives a plus.

Exercise 2:01



1 Calculate each of the following.

a 9 less than 3	**b** 5 less than 1
c 8 less than 0	**d** 3 minus 7
e 4 minus 6	**f** 1 minus 9
g take 9 from 8	**h** take 12 from 8
i take 10 from 2	**j** 10 less than 4
k 9 minus 12	**l** take 6 from 10

2 When my bank account is overdrawn, the balance is shown as a negative number. Find my balance if I wrote a cheque for each of the following amounts from an account that held $20.

a $12	**b** $30	**c** $200
d $1335	**e** $10	**f** $100

3 Plot each set of directed numbers on a separate number line.

a $\{-1, 0, 2\}$	**b** $\{-5, -2, -1\}$	**c** $\{-1{\cdot}5, -0{\cdot}5, 0\}$	**d** $\{-7{\cdot}5, -5{\cdot}5, -3{\cdot}5\}$
e $\{-1\frac{1}{4}, -\frac{3}{4}, \frac{1}{2}\}$	**f** $\{-3\frac{1}{2}, -3, -2\frac{3}{4}\}$	**g** $\{-9\frac{1}{2}, -9\frac{1}{4}, -9\}$	**h** $\{-1\frac{1}{10}, -\frac{9}{10}, -\frac{5}{10}\}$

In Questions **4** to **6**, perform each calculation using the correct operation.

4

a $-6 + 12$	**b** $-4 + 9$	**c** $-8 + 11$	**d** $-7 + 37$
e $-15 + 4$	**f** $-33 + 2$	**g** $-56 + 8$	**h** $-65 + 15$
i $-10 - 12$	**j** $-60 - 14$	**k** $-12 - 32$	**l** $-7 - 21$
m $6 - 13$	**n** $4 - 7$	**o** $8 - 20$	**p** $9 - 16$
q $-7 - -10$	**r** $-4 - -9$	**s** $12 - -8$	**t** $9 - -20$
u $-8 + -3$	**v** $4 - +7$	**w** $5 + -14$	**x** $-10 + -6$

5

a -4×3	**b** -4×-3	**c** 4×-3	**d** $(-4) \times (-3)$
e -6×5	**f** -3×9	**g** $(-6) \times 7$	**h** -5×10
i 8×-3	**j** $9 \times (-9)$	**k** 5×-7	**l** 4×-7
m -6×-6	**n** -5×-4	**o** -9×-9	**p** $(-4) \times (-4)$
q $-1 \times -1 \times -1$	**r** $-2 \times -2 \times -2$	**s** $-1 \times -1 \times 1$	**t** $-2 \times -2 \times 2$
u $(-10)^2$	**v** $(-10)^3$	**w** $(-10)^4$	**x** $(-10)^5$

6

a $-12 \div 3$	**b** $-12 \div -3$	**c** $12 \div -3$	**d** $(-12) \div (-3)$
e $-15 \div 5$	**f** $-32 \div 8$	**g** $(-28) \div 4$	**h** $-28 \div 7$
i $35 \div -7$	**j** $16 \div (-4)$	**k** $25 \div -5$	**l** $42 \div -6$
m $-9 \div -9$	**n** $-21 \div -3$	**o** $-18 \div -6$	**p** $(-36) \div (-9)$
q $-6 \div -1$	**r** $6 \div -1$	**s** $-8 \div -1$	**t** $8 \div -1$

7 Calculate each of the following using the correct order of operations.

a $-30 \div 3 + 7$	**b** $-20 \div (-5 + 15)$	**c** $(14 - -6) \div 5$	**d** $(-3) \div (-3) - 1$
e $-4 \times 6 + 12$	**f** $2 \times 8 - 17$	**g** $-16 + (4 \times 4)$	**h** $-21 - 7 \times 7$
i $10 \div -5 + 7$	**j** $15 \div (-4 + 7)$	**k** $27 \div -3 + 6$	**l** $13 + -6 \div 2$
m $10 + -10 \div 5$	**n** $-80 \times -2 + 7$	**o** $-32 + -32 \times 2$	**p** $-36 + (-9) \times 2$
q $-1 \times -1 \div -1$	**r** $-24 + -18 \div 6$	**s** $8 - 1 \times 7 - 1$	**t** $-2 + -6 \times 2$

FUN SPOT 2:01 WHERE DO SICK PONIES GO?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

R	$-5 + 19$	**S**	$-4 - 91$	**H**	$14 - 30$
P	$-15 + 4$	**L**	$-14 - 5$	**T**	$-7 + 18$
O	$-33 + 2$	**E**	$-56 + 8$	**H**	-8×-5
A	5×-20	**T**	-9×-9	**O**	-9×4
E	$-48 \div 8$	**T**	$-28 \div -7$	**I**	$100 \div -20$



11	−31	81	40	−6	−16	−36	14	−95	−48	−11	−5	4	−100	−19

2:02 Directed numbers and the calculator

PREP QUIZ 2:02

1 $-3 + 1$	**2** $-7 - 3$	**3** $2 - 10$	**4** $-4 - 8 + 3$	**5** $0 - 7$
6 $-10 \div 2$	**7** -3×8	**8** $(-4) \times (-3)$	**9** $(-4)^2$	**10** $\frac{-12}{-3}$

Check your calculator to find out whether it has order of operations built in. Some do not. Many calculators (like the Casio fx-82AU PLUS), allow you to enter directed numbers in the written order and have a display that shows the question on its screen.

WORKED EXAMPLE 1

a $16 - -7$

16 [−] [−] 7 [=]

Answer = 23

b $(-11)^2$

[(] [−] 11 [)] [x^2] [=]

Answer = 121

c $-8 - 3 \times -11$

[−] 8 [−] 3 [×] [−] 11 [=]

Answer = 25

Other calculators use a sign change key [+/−] to enter negative numbers.

WORKED EXAMPLE 2

a $16 - (-7)$

16 [−] 7 [+/−] [=]

Answer = 23

b $(-11)^2$

11 [+/−] [x^2]

Answer = 121

c $-8 - 3 \times -11$

8 [+/−] [−] 3 [×] 11 [+/−] [=]

Answer = 25

Exercise 2:02

1 Complete the first column without a calculator and the second column with a calculator.

Question	Without calculator	With calculator
$-3 - 8$	-11	
$-9 - 2$		
$-7 + 2$		
$-15 + 6$		
$-7 + 23$		
$-2 + 15$		
$-2 - -3$		
$8 - -2$		

Question	Without calculator	With calculator
-3×4		
-6×3		
-7×-2		
-9×-5		
4×-7		
10×-6		
$20 \div (-5)$		
$-18 \div -3$		

Do the answers you found with a calculator agree with your answers found without a calculator? If not, do those questions again.

2 Use a calculator to find each of the following.

a -17×34 **b** $-123 - 479$ **c** $-78 + 372$

d $1567 - 3842$ **e** $-2160 \div 16$ **f** $797\,544 \div -318$

g $-841 - -793$ **h** $14 - 2107$ **i** $-14 - 2107$

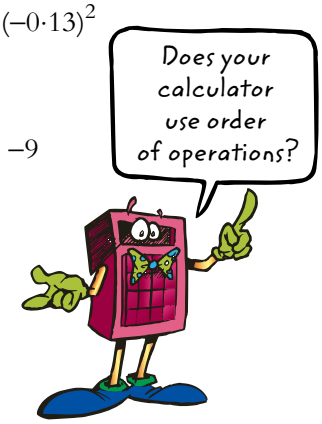
3 Use a calculator to find each of the following.

a $(-407)^2$ **b** $(-2{\cdot}7)^2$ **c** $(-0{\cdot}13)^2$

4 Use a calculator to find each of the following.

a $-6 - 7 \times -11$ **b** $-23 \times -6 + 91$ **c** $8 - 6{\cdot}1 \times -8{\cdot}4$

d $0{\cdot}5 - -1{\cdot}86 \times 0{\cdot}7$ **e** $-105 - 23 \times -2$ **f** $-5 \times -8 \times -3 \times -9$



5 Use a calculator to find each of the following. Operations above and below the fraction bar should be placed in grouping symbols.

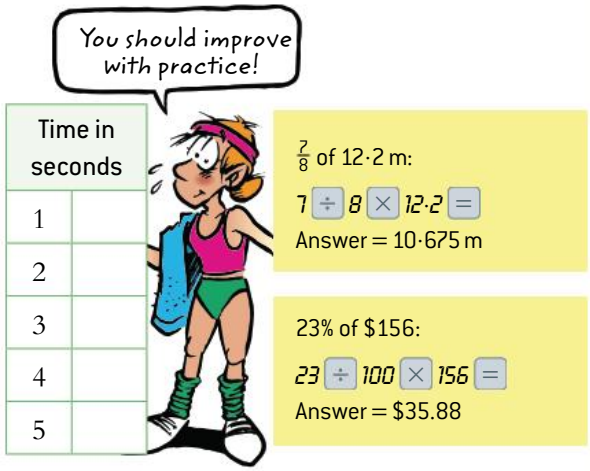
a $\sqrt{13^2 - 12^2}$ **b** $\sqrt{(13-12)^2}$ **c** $\sqrt{(12-13)^2}$

d $\dfrac{6-51}{-7+2}$ **e** $\dfrac{6-51}{-7-2}$ **f** $\dfrac{7-37}{-18-2}$

CHALLENGE 2:02 SPEED AND ACCURACY

These sets of questions are similar. The challenge is to improve your score as you do each set. Time how quickly you can complete each set of questions. Add 10 seconds for each mistake.

1 **a** $0{\cdot}87 - 9{\cdot}6$
b $1{\cdot}9^3$
c $8874 \div 29$
d $-5{\cdot}9 + 3{\cdot}8$
e How many 20-cent pencils can you buy for $11.45?
f One litre of petrol costs 158·5 cents. What is the cost of 28 L?
g Find the value of $\frac{1}{3} \times 3{\cdot}14 \times 9^2$.
h Change $\frac{11}{32}$ to a decimal.
i Find $\frac{7}{8}$ of 6·8 m.
j What is 13% of $876?



2 **a** $3{\cdot}42 - 6{\cdot}7$
b $1{\cdot}5^3$
c $8604 \div 18$
d $-9{\cdot}1 + 7{\cdot}3$
e How many 25-cent pencils can you buy for $15.85?
f One litre of petrol costs 161·5 cents. What is the cost of 24 L?
g Find the value of $\frac{1}{3} \times 3{\cdot}14 \times 7^2$, correct to three decimal places.
h Change $\frac{19}{32}$ to a decimal.
i Find $\frac{3}{8}$ of 7·4 m.
j What is 17% of $687?

3 **a** $1{\cdot}86 - 5{\cdot}4$
b $1{\cdot}7^3$
c $7326 \div 37$
d $-6{\cdot}2 + 3{\cdot}8$
e How many 30-cent pencils can you buy for $16.85?
f One litre of petrol costs 157·2 cents. What is the cost of 45 L?
g Find the value of $\frac{1}{3} \times 3{\cdot}14 \times 8^2$, correct to three decimal places.
h Change $\frac{21}{32}$ to a decimal.
i Find $\frac{5}{8}$ of 5·6 m.
j What is 19% of $753?

4 **a** $7{\cdot}86 - 8{\cdot}3$
b $1{\cdot}6^3$
c $8970 \div 26$
d $-8{\cdot}8 + 1{\cdot}9$
e How many 35-cent pencils can you buy for $22.55?
f One litre of petrol costs 162·6 cents. What is the cost of 35 L?
g Find the value of $\frac{1}{3} \times 3{\cdot}14 \times 6^2$.
h Change $\frac{13}{32}$ to a decimal.
i Find $\frac{5}{6}$ of 4·5 m.
j What is 21% of $496?

5 **a** $4{\cdot}72 - 6{\cdot}1$
b $2{\cdot}1^3$
c $7475 \div 25$
d $-7{\cdot}4 + 2{\cdot}7$
e How many 40-cent pencils can you buy for $27.65?
f One litre of petrol costs 160·8 cents. What is the cost of 35 L?
g Find the value of $\frac{1}{3} \times 3{\cdot}14 \times 5^2$, correct to three decimal places.
h Change $\frac{27}{32}$ to a decimal.
i Find $\frac{3}{4}$ of 3·9 m.
j What is 27% of $388?

2:03 Directed numbers and algebra

PREP QUIZ 2:03

Write the basic numeral for:

1 $-2 + 11$ **2** $1 - 10$ **3** $-5 - 5$

4 -3×7 **5** 4×-6 **6** -7×-2

7 $-14 \div 7$ **8** $32 \div -8$ **9** $-18 \div -3$

10 $-9 \times -3 + 6 \div -3 - 3 \times 2$, that is $(-9 \times -3) + (6 \div -3) - (3 \times 2)$



Pronumerals can represent negative numbers as well as positive numbers.

Simplifying algebraic expressions

The rules for directed numbers also apply to algebra.

e.g. $-2x - 10x = -12x$, $-10y \times 7 = -70y$, $15y \div -3 = -5y$

Substitution

When we substitute a negative number for a pronumeral, it may help our understanding if we place grouping symbols around the negative numbers. This is necessary when the number is being squared.

e.g. If $x = -3$, $x^2 = (-3)^2 = 9$ (a minus times a minus gives a plus)

If $x = 3$, $x^2 = (3)^2 = 9$ (note that $(-3)^2 = 3^2$)

WORKED EXAMPLE 1

Simplify each expression.

a $2x - 10x$
$= (2 - 10)x$
$= -8x$

b $-2x - -10x$
$= (-2 + 10)x$
$= 8x$

c $-5y \times -7$
$= (-5 \times -7)y$
$= 35y$

d $-18m \div 6$
$= (-18 \div 6)m$
$= -3m$



Brahmagupta, an Indian mathematician (c. 598–665 CE), introduced zero and negative numbers.

WORKED EXAMPLE 2

If $x = -10$, $y = -6$ and $z = 5$, find the value of each expression.

a $x - y$
$= -10 - (-6)$
$= -10 + 6$
$= -4$

b $x \div z$
$= -10 \div 5$
$= -2$

c $x + y + z$
$= -10 + (-6) + 5$
$= -10 - 6 + 5$
$= -11$

d $4x - 7$
$= 4 \times (-10) - 7$
$= -40 - 7$
$= -47$

WORKED EXAMPLE 3

If $m = -4$, $p = -9$ and $t = 9$, find the value of each expression.

a $p + t$
$= -9 + 9$
$= 0$

b $3m \div 6$
$= 3 \times (-4) \div 6$
$= -12 \div 6$
$= -2$

c $-4t + 3m$
$= -4 \times 9 + 3 \times (-4)$
$= -36 + -12$
$= -36 - 12$
$= -48$

d $-5(8 - 2t)$
$= -5 \times (8 - 2 \times 9)$
$= -5 \times (8 - 18)$
$= -5 \times (-10)$
$= 50$

Exercise 2:03

P Foundation worksheet 2:03
Number line graphs

1 Simplify:

a $3a + 5a$	**b** $-3a + 5a$	**c** $3a - 5a$	**d** $-3a - 5a$
e $- -x$	**f** $x - -x$	**g** $5x - -3x$	**h** $-4x - -9x$
i $-7m - 10m$	**j** $-7m + 8m$	**k** $10m - m$	**l** $m - 5m$
m $-3a \times 5$	**n** $-2 \times -a$	**o** $9 \times -6a$	**p** $-8a \times -10$
q $-14y \div 7$	**r** $-12y \div -2$	**s** $20y \div -5$	**t** $-32y \div -8$



2 Find the value of each expression if $n = -8$ and $t = -2$.

a $2n$	**b** $3t$	**c** $-n$	**d** $-3n$
e $n + t$	**f** $n - t$	**g** $t - n$	**h** $5 - t$
i $t - 8$	**j** $t + 8$	**k** $t + 5$	**l** $t - 5$
m $10n + 50$	**n** $7t + 14$	**o** $7t - 14$	**p** $6 - 5t$

3 Find the value of $2m + 8$ if m is equal to:

a 3 **b** 12 **c** -4 **d** -5

4 Find the value of $12 - 4x$ if x is equal to:

a 4 **b** 5 **c** -2 **d** -3

5 Find the value of y^2 if y is equal to:

a 5 **b** -10 **c** -6 **d** -1

6 Find the value of $\frac{24}{n}$ if y is equal to:

a 8 **b** -4 **c** -3 **d** -1

$\frac{24}{n}$ means $24 \div n$.

7 If $a = -6$, $b = -12$ and $c = -3$, find the value of:

a $a + b$	**b** $a + 2b$	**c** $2a + b$	**d** $2a - b$	**e** $-6a$	**f** $-3c$
g $-5b$	**h** ac	**i** $-12 \div a$	**j** $-b \div 12$	**k** $-12 \div c$	**l** $12 \div a$
m a^2	**n** $-a^2$	**o** $(-a)^2$	**p** $40 - a^2$	**q** $a + b + c$	**r** $a + b - c$
s $a - b - c$	**t** $a - b + c$				

8 If $x = -10$, $y = -5$ and $n = 8$, find the value of:

a $x - y$	**b** $5n + 2y$	**c** $3x - 2y$	**d** $x - y + 5$
e $(5 + x) \div 5$	**f** $(n + 32) \div x$	**g** $(x + y) \div 3$	**h** $(2 - 4n) \div 5$
i $5(8 - 2n)$	**j** $-2(3n - 4)$	**k** $6(10 - n)$	**l** $-3(5 - n)$
m $5 - y^2$	**n** $y^2 + n^2$	**o** $y^2 - x^2$	**p** $n^2 - y^2$
q $x + y + n$	**r** $x - y - n$	**s** $x - y + n$	**t** $-x + y - n$

9 Are the following expressions true if $x = -2$ and $y = 2$?

a $(x + y)^2 = x^2 + 2xy + y^2$ **b** $(x - y)^2 = x^2 - 2xy + y^2$

10 Substitute several values for x and y to give your opinion on whether these statements are true.

a $5x + 3 = 8x$	**b** $5(x + y) = 5x + y$
c $2x + 1 = 2(x + 1)$	**d** $(x + y)^2 = x^2 + y^2$
e $(x + y) \times (x - y) = x^2 - y^2$	**f** $\frac{1}{2} \times x = x \div 2$

2:04 The index laws

PREP QUIZ 2:04

Write the basic numeral for:

1 $3 \times 3 \times 3$ **2** 3^3 **3** $2 \times 2 \times 2$ **4** 2^3

Write in index form (as a power).

5 $10 \times 10 \times 10 \times 10 \times 10$
6 $10 \times 10 \times 10 \times 10$
7 $7 \times 7 \times 7$
8 $5 \times 5 \times 5 \times 5$
9 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
10 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$3 = 3^1$

3 has been used as a factor once only.

$3 \times 3 \times 3 \times 3 = 3^4$

3 has been used as a factor four times.

- Index laws explain the properties of powers of numbers.
- We will discover these laws by investigating powers of small positive numbers.
- We can use a calculator to find a power of a number.

Multiplying powers of the same number

$$\begin{aligned} 3^5 \times 3^4 &= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^{5+4} \\ &= 3^9 \end{aligned}$$

Use a calculator to show that $3^5 \times 3^4 = 3^9$.

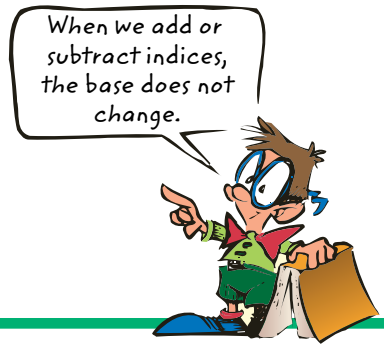
Conclusion: When multiplying powers of the same number, add the indices.

WORKED EXAMPLES

1 $2^5 \times 2^3 = 2^{5+3}$
$= 2^8$

2 $10^2 \times 10^3 = 10^{2+3}$
$= 10^5$

3 $3^4 \times 3^1 = 3^{4+1}$
$= 3^5$



Dividing powers of the same number

$3^7 \div 3^2 = (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3) \div (3 \times 3)$

$= \dfrac{{}^{1}\cancel{3} \times {}^{1}\cancel{3} \times 3 \times 3 \times 3 \times 3 \times 3}{{}^{1}\cancel{3} \times {}^{1}\cancel{3}}$

$= 3^{7-2}$

$= 3^5$

Use a calculator to show that $3^7 \div 3^2 = 3^5$.

Conclusion: When dividing powers of the same number, subtract the indices.

WORKED EXAMPLES

1 $2^8 \div 2^2 = 2^{8-2}$
$= 2^6$

2 $10^5 \div 10^2 = 10^{5-2}$
$= 10^3$

3 $3^4 \div 3^1 = 3^{4-1}$
$= 3^3$



Raising a power to a power

$(3^2)^4 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^{2 \times 4}$

$= 3^8$

Use a calculator to show that $(3^2)^4 = 3^8$.

Conclusion: When raising a power to a power, multiply the indices.

WORKED EXAMPLES

1 $(3^5)^2 = 3^{5 \times 2}$
$= 3^{10}$

2 $(2^4)^3 = 2^{4 \times 3}$
$= 2^{12}$

3 $(10^2)^3 = 10^{2 \times 3}$
$= 10^6$

$(2^4)^3$ is read '2 to the power of 4, all to the power of 3'.



The zero index

$3^4 \div 3^4 = (3 \times 3 \times 3 \times 3) \div (3 \times 3 \times 3 \times 3)$

$= \dfrac{{}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3}}{{}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3}}$ (cancelling)

$= 1$

$= 3^4 \div 3^4$ (dividing powers of the same number)

$= 3^0$

Use a calculator to show that $3^4 \div 3^4 = 1 = 3^0$.

Conclusion: Any number to the power of zero is equal to 1.

WORKED EXAMPLES

1 $2^0 = 1$

2 $10^2 \div 10^2 = 10^{2-2}$
$10^0 = 1$

3 $12^0 \times 3^2 = 1 \times 9$
$= 9$

Exercise 2:04

1 **a** Use the table below to write each of the following as a basic numeral.

i 2^5 **ii** 2^1 **iii** 2^7 **iv** 2^0 **v** 2^{10}

b Write each answer as a power of 2 (using the law for multiplication of indices), then use the table below to write as a basic numeral.

i $2^3 \times 2^3$ **ii** $2^5 \times 2^2$ **iii** $2^6 \times 2^4$ **iv** $2^1 \times 2^7$ **v** $2^5 \times 2^4$

c Use the table below to write each question as the product of two numbers, then use a calculator to find the answer. Compare these with the answers for part **b**. (They should be the same.)

i $2^3 \times 2^3$ **ii** $2^5 \times 2^2$ **iii** $2^6 \times 2^4$ **iv** $2^1 \times 2^7$ **v** $2^5 \times 2^4$

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
1	2	4	8	16	32	64	128	256	512	1024

2 **a** Use the table below to write each of the following as a basic numeral.

i 3^0 **ii** 3^4 **iii** 3^9 **iv** 3^5 **v** 3^8

b Write each answer as a power of 3 (using the law for division of indices), then use the table below to write the answer as a basic numeral.

i $3^3 \div 3^3$ **ii** $3^{10} \div 3^2$ **iii** $3^8 \div 3^4$ **iv** $3^7 \div 3^5$ **v** $3^9 \div 3^6$

c Use the table to write the question as a division of one number by another, then use a calculator to find the answer. Compare these answers with part **b**. (They should be the same.)

i $3^3 \div 3^3$ **ii** $3^{10} \div 3^2$ **iii** $3^8 \div 3^4$ **iv** $3^7 \div 3^5$ **v** $3^9 \div 3^6$

3^0	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
1	3	9	27	81	243	729	2187	6561	19 683	59 049

3 **a** Write each answer as a power (using the index law for raising a power to a power), and then use the tables in Questions 1 and 2 to write each answer as a basic numeral.

i $(3^5)^2$ **ii** $(2^4)^2$ **iii** $(3^2)^3$ **iv** $(2^2)^4$ **v** $(3^3)^3$

b Use the tables to write each question as the power of one number, then use a calculator to find the answer. Compare these answers with part **b**. (They should be the same.)

i $(3^5)^2$ **ii** $(2^4)^2$ **iii** $(3^2)^3$ **iv** $(2^2)^4$ **v** $(3^3)^3$

4 Use the index laws to write true or false for each of the following.

a $3^5 \times 3^3 = 9^8$ **b** $3^5 \times 3^3 = 3^8$ **c** $3^5 \times 3^3 = 3^{15}$ **d** $3^8 \times 3^4 = 9^{15}$

e $2^8 \div 2^4 = 1^4$ **f** $2^8 \div 2^4 = 2^4$ **g** $2^8 \div 2^4 = 2^2$ **h** $2^8 \div 2^4 = 1^2$

i $(3^5)^2 = 15^2$ **j** $(3^5)^2 = 3^{10}$ **k** $(3^5)^2 = 3^7$ **l** $(3^5)^2 = 15^2$

m $2^9 \div 2^3 = 2^6$ **n** $2^7 \div 2 = 2^6$ **o** $2^6 \div 2^6 = 2^0$ **p** $2^7 \div 2^6 = 2^1$

5 Simplify, writing answers in index form.

a $10^2 \times 10^3$ **b** $10^4 \times 10^2$ **c** $10^3 \times 10^3$ **d** $10^6 \times 10$

e $5^0 \times 5^4$ **f** $5^3 \times 5^3$ **g** $5^2 \times 5$ **h** $5^5 \times 5^4$

i $2^3 \times 2^2 \times 2^4$ **j** $2^6 \times 2^6 \times 2^6$ **k** $2^3 \times 2 \times 2^2$ **l** $2^5 \times 2^3 \times 2^3$

m $10^4 \times 10^6 \times 10^2$ **n** $10^2 \times 10^2 \times 10^2$ **o** $10^2 \times 10 \times 10^3$ **p** $10 \times 10 \times 10^5$

6 Simplify, writing answers in index form.

a $10^7 \div 10^3$ **b** $10^4 \div 10^2$ **c** $10^9 \div 10^6$ **d** $10^8 \div 10$

e $5^4 \div 5^4$ **f** $5^{10} \div 5^5$ **g** $5^3 \div 5$ **h** $5^3 \div 5^2$

i $2^8 \div 2^2 \div 2^2$ **j** $2^4 \div 2^2 \times 2^2$ **k** $2^{10} \div 2 \div 2^3$ **l** $2^7 \times 2^3 \div 2^2$

m $10^6 \div 10 \div 10$ **n** $10^5 \div 10^2 \div 10^2$ **o** $10^9 \div 10 \div 10^3$ **p** $10^7 \div 10 \div 10^3$

7 Simplify, writing answers in index form.

a $(3^{10})^2$ **b** $(2^3)^3$ **c** $(5^4)^3$ **d** $(7^2)^4$

e $(10^2)^4$ **f** $(10^4)^3$ **g** $(10^5)^6$ **h** $(10^3)^2$

i $(3^4 \times 3^3)^2$ **j** $(3^2 \times 3)^3$ **k** $(2^7 \div 2^3)^4$ **l** $(2^{10} \div 2^3)^2$

m $(10^3)^2 \div 10^6$ **n** $(10^2)^2 \div 10^2$ **o** $(10^4)^2 \div 10^3$ **p** $(10^5)^2 \div 10$

8 **a** 1 000 000 bytes = 1 megabyte

Write the number of bytes in a megabyte as a power of ten.

b 1 000 000 000 bytes = 1 gigabyte

Write the number of bytes in a gigabyte as a power of ten.

c 1 000 000 000 000 bytes = 1 terabyte

Write the number of bytes in a terabyte as a power of ten.



Investigate the storage capacity of modern computers.

MATHS TERMS 2

cubed
- used three times as a factor,
  e.g. $a^3 = a \times a \times a$

directed numbers
- numbers that have both size and direction (+ or −)
- positive
  greater than zero
- negative
  less than zero
- integer
  any whole number
- sign (of a number)
  whether a number is positive or negative
- opposite
  the number of the same size but opposite sign,
  e.g. The opposite of 3 is −3.

expression
- algebraic expressions are made up of one or more terms joined by operation signs,
  e.g. $2x - 7$, $5x + 2y + 1$, $a^2 - 2a + 1$

index (indices)
- the raised symbol when a number is written as a power

integer
- whole number,
  e.g. …, −3, −2, −1, 0, 1, 2, 3, …

like terms
- these have the same pronumeral parts,
  e.g. $3xy$ and $7xy$, $8x^2$ and $-x^2$
- unlike terms have different pronumeral parts,
  e.g. x^2 and x, $5xy$ and $2y$

power
- repeated as a factor,
  e.g. 3 to the power of 5
  $= 3^5$
  $= 3 \times 3 \times 3 \times 3 \times 3$

pronumeral
- a symbol (usually a letter) that takes the place of a numeral,
  e.g. x or θ

simplify
- to give the shortest or simplest answer

squared
- multiplied by itself,
  e.g. $5^2 = 5 \times 5$
- used twice as a factor

substitute (substitution)
- 'to put in place of'
- usually a numeral replaces a pronumeral,
  e.g. Substitute $x = 7$ in $2x + 3$.

Each section of the test has similar items that test a certain type of example.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Graph each set on a separate number line. 2:01

a $\{-2, 0, 3\}$ b $\{-2\frac{1}{2}, -2, -1\frac{3}{4}\}$ c $\{-2{\cdot}5, -1{\cdot}5, 0\}$

2 Write the basic numeral for each of the following. 2:01

a $-7 + 11$ b $-4 - 9$ c $-6 - 16$
d $-4 - -9$ e $7 - -2$ f $-3 + -11$

3 Write the basic numeral for each of the following. 2:01

a -5×7 b -2×-12 c 6×-4
d $(-9) \times (-3)$ e $(-8)^2$ f $-2 \times -2 \times -2$

4 Write the basic numeral for each of the following. 2:01

a $-90 \div 10$ b $-21 \div -7$ c $55 \div -5$
d $(-18) \div (-6)$ e $15 \div -1$ f $27 \div -3 \div -3$
g $-30 \div (-5 + 15)$ h $10 + -20 \div 5$ i $-6 - 6 \times 2$

5 Simplify. 2:03

a $-2m + 8m$ b $15x - 4x$ c $-12y - 3y$
d $n - -9n$ e $-3m \times 10$ f $-16x \div -4$

6 If $x = -3$, $y = -12$ and $n = 10$, find the value of: 2:03

a $10 - 3n$ b $x - y$ c $3y + 6$
d $3n + x$ e $3x - 2y$ f $12 - x^2$

7 Simplify, writing answers in index form. 2:04

a $2^5 \times 2^3$ b $10^2 \times 10^3$ c $3^3 \times 3$
d $2^8 \div 2^2$ e $10^5 \div 10^2$ f $3^4 \div 3$
g $(3^5)^2$ h $(2^4)^3$ i $(10^2)^3$

ASSIGNMENT 2A Chapter review

1 **a** The top of Mount Everest is approximately 8000 m above sea level and the surface of the Dead Sea is 430 m below sea level. What is the difference in height between the top of Mount Everest and the surface of the Dead Sea?

b The temperature inside the freezer was −14°C. Outside the freezer it was 28°C. What was the difference between these temperatures?



2 Complete these statements.

a A negative number can represent a l_____ and a positive number can represent a g_____ .

b The opposite of minus 24 is _____ .

c To subtract a directed number, add its _____.

d Two minus signs, side by side become a _____ .

e Multiplying two numbers with unlike signs gives a _____ .

f Multiplying two numbers with like signs gives a _____ .

g A minus times a minus gives a _____ .

h Multiplying an even number of minus signs gives a _____ .

i Multiplying an odd number of minus signs gives a _____ .

j Division with two numbers of unlike signs gives a _____ .

k A minus divided by a plus gives a _____ .

l Division with two numbers of like signs gives a _____ .

m A minus divided by a minus gives a _____ .

3 Use a calculator to answer these.

a $18\,965 - 19\,088$

b $(-11)^3$

c $91 - 8{\cdot}75 \times -3{\cdot}62$

d $-32\,005 \div -37$

e $(0{\cdot}75 - 12{\cdot}3)^2$

f $(38 - -12) \times (2865 - 2976)$

4 **a** Find the value of $3x - 8$ when $x = -3$.

b Find the value of $6x - 5y$ when $x = 3$ and $y = -10$.

c Find the value of $6m - 5n$ when $m = -9$ and $n = 7$.

5 Simplify these expressions.

a $132m - 140m$

b $(-y)^2$

c $15y - -5y$

d $-18x \div -9$

e $-11m \times -7$

f $-18x \div -(17 - 20)$

6 Write each answer as a power of 3.

a $3^{10} \div 3^2$

b $(3^4)^2$

c $3^6 \times 3^4$

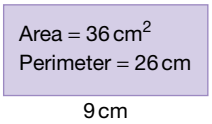
d 3×3^7

e $(3^3)^5$

f $3^6 \div 3^4$

ASSIGNMENT 2B Working mathematically

1 The rectangle below has an area of 36 cm^2. Give the dimensions (length and breadth) of four other rectangles that have the same area. Find the dimensions of the rectangle with area 36 cm^2 that has the smallest perimeter.



2 A machine was used to measure the heartbeats of various animals. It was found that the horse's heart was beating 30 times each minute, the rabbit's heart was beating 125 times each minute and the dog's heart was beating 92 times each minute. If the machine recorded the horse's heart for 10 minutes, the rabbit's heart for 5 minutes and the dog's for 4 minutes, what was the total number of heartbeats recorded?

3 Heather is told to build 24 stools. Each stool must have either 3 or 4 legs and all 87 legs supplied must be used. How many of each type will she build?



4 A firefighter stood on the middle rung of a ladder, spraying water on a burning house. He then climbed up 9 rungs before the heat of the flames caused him to come down 14 rungs. After some minutes he was able to climb 20 rungs to the very top of the ladder. How many rungs did the ladder have?

5 How many different groups of four people could be chosen to play tennis from the six people who have come to play?

6 Sandy bought two Australian finches: a gouldian finch and a diamond firetail finch. If the gouldian cost $16 more than the diamond firetail, find the cost of each bird if Sandy paid $72 altogether.

7 Write the question and answer for **1** to **24** on **ID Card 3**, page xvi.

8 When Vikings attacked, the local people would use a ladder to enter the door and then pull up the ladder behind them. Estimate the height of the tower and the height of the door above the ground.



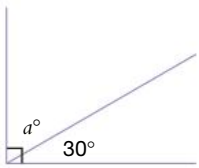
One of the two round towers at Clonmacnoise, County Offaly, Ireland.

ASSIGNMENT 2C Cumulative revision

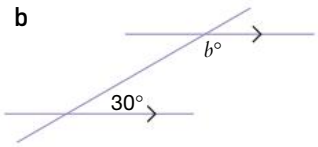
1 **a** Draw a pair of vertically opposite angles. 1:08
b What is the angle sum of adjacent supplementary angles?

2 Find the value of the pronumeral in each of the following diagrams. 1:08

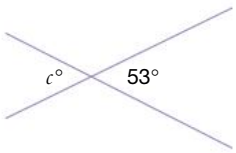
a



b

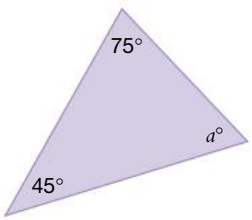


c

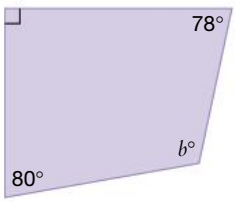


3 Find the value of the pronumeral in each of the following diagrams. 1:09

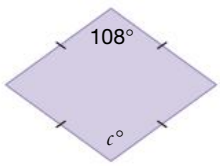
a



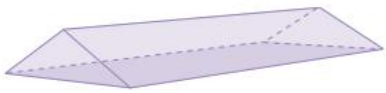
b



c



4 Find the number of faces plus the number of vertices minus the number of edges for this solid ($F + V - E$). 1:10



5 Convert: 1:11
a 6275 mm to cm **b** 7500 mg to g **c** 6500 mL to L **d** 8300 m to km

6 **a** Find the area of a square that has side lengths of 3·8 cm. 1:11
b Find the area of a triangle that has a base length of 14 m and a height of 8 m.
c Find the volume of a rectangular prism that has length 20 cm, breadth 8 cm and height 5 cm.

7 **a** $-8 - 3$ **b** -3×-11 **c** $6 - \frac{16}{2}$ 1:12
d $-3 - 5 \times -10$ **e** $-40 \div (-2 - 6)$ **f** $(-12 - 6) \div 6$

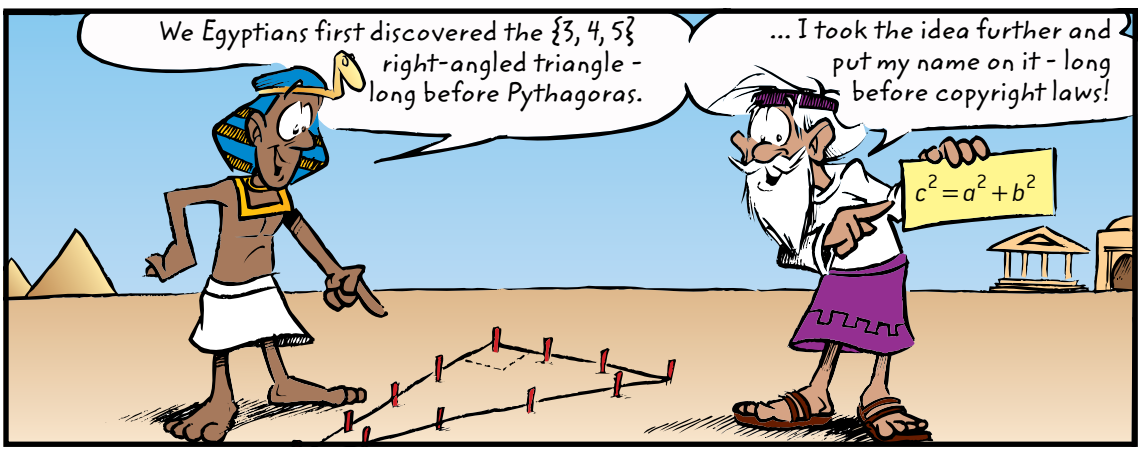
8 **a** Write $3 \times y + 5$ without a multiplication sign. 1:14
b If $y = 5$, find the value of $20 - 3y$.
c If $T = 3n + 7$, find the value of T when $n = 4$.

9 Simplify: 1:14
a $1y$ **b** $7m \times 0$ **c** $6 \times 8n$
d $6a + 10a$ **e** $9b - b$ **f** $6(a + 1) - 6$

10 Suri rolled a dice. What is the probability that the dice will show: 1:15
a a 5 **b** an odd number **c** a number greater than 2?

PYTHAGORAS' THEOREM

3



Contents

3:01 Investigating right-angled triangles
Investigation 3:01A Right-angled triangles
Investigation 3:01B Pythagoras' theorem
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3:02 Pythagoras' theorem: Calculating the hypotenuse
3:03 Pythagoras' theorem: Calculating one of the shorter sides
GeoGebra activity 3:03 Investigation Pythagoras' theorem
3:04 Applications of Pythagoras' theorem
3:05 Pythagorean triads and right-angled triangles
Fun spot 3:05 What happens when ducks fly upside down?
3:06 Irrational numbers (Surds)
Investigation 3:06 Pythagoras and speed
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Measurement and Geometry, **Number and Algebra**

Selections from ***Right-Angled Triangles (Pythagoras)***, ***Equations*** [Stage 4]

- Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles. (ACMMG222)
- Investigate the concept of irrational numbers. (ACMNA186)
- Solve simple quadratic equations.

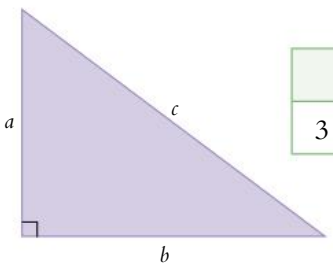
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

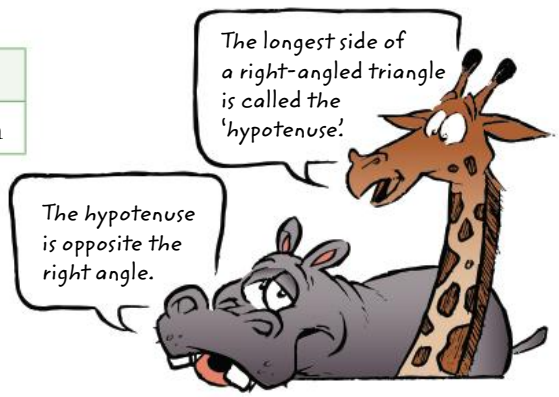
3:01 Investigating right-angled triangles

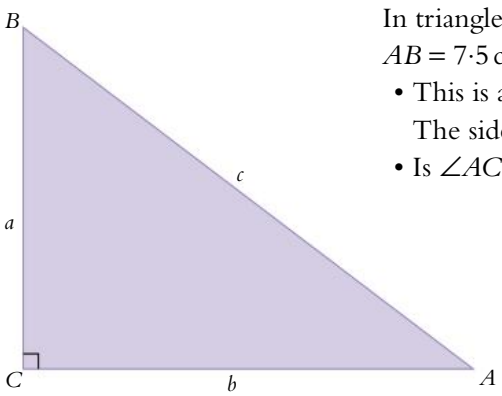
The Egyptians made use of the fact that a triangle that had sides 3 units, 4 units and 5 units in length was always a right-angled triangle. However, around 550 BCE a religious group led by the Greek philosopher Pythagoras discovered the relationship between the sides of all right-angled triangles.

As we investigate right-angled triangles we will call the side lengths a, b and c, where c will always stand for the length of the longest side. We call the longest side of a right-angled triangle the **hypotenuse**.



a	b	c
3 cm	4 cm	5 cm





In triangle ABC (ΔABC), $BC = 4{\cdot}5$ cm, $CA = 6$ cm and $AB = 7{\cdot}5$ cm.

- This is also a $\{3, 4, 5\}$ triangle if we let each unit be $1{\cdot}5$ cm. The side lengths are: $3 \times 1{\cdot}5$ cm, $4 \times 1{\cdot}5$ cm and $5 \times 1{\cdot}5$ cm.
- Is $\angle ACB$ a right angle? Which side is the hypotenuse?

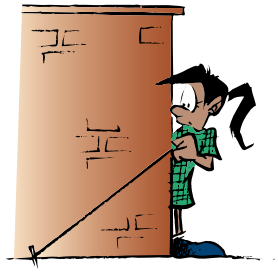
a	b	c
$4{\cdot}5$ cm	6 cm	$7{\cdot}5$ cm
$3 \times 1{\cdot}5$ cm	$4 \times 1{\cdot}5$ cm	$5 \times 1{\cdot}5$ cm

INVESTIGATION 3:01A RIGHT-ANGLED TRIANGLES

1 Take three lengths of rope or string 3 m, 4 m and 5 m in length. Use them to form a triangle, and measure the largest angle. Is it a right angle? (You could use a piece of string 12 m long and use marks to show lengths of 3 m, 4 m, and 5 m.)



2 Measure 1·5 m up and 2 m along the bottom of a wall that stands on flat ground. How long is the distance between these two points? Use string to measure this distance.
Is the triangle a {3, 4, 5} triangle?



3 Draw or find some right-angled triangles in the school grounds. Make the sides any length you wish. For each triangle, measure the side lengths a, b and c (where c is the longest side) and enter these in a table like the one below. Make sure you include the three sets of measurements in the table.

a	3	6	5			
b	4	8	12			
c						



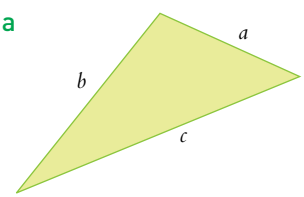
Exercise 3:01

P Foundation worksheet 3:01
Pythagoras' theorem A

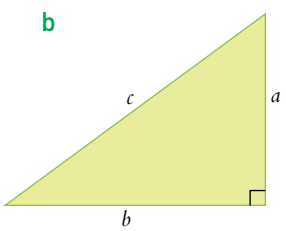
1 For each of the triangles drawn below, measure the sides to the nearest millimetre and complete the table.

a	b	c	a^2	b^2	c^2	$a^2 + b^2$

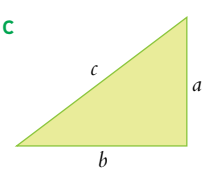
a



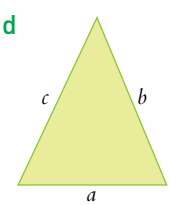
b



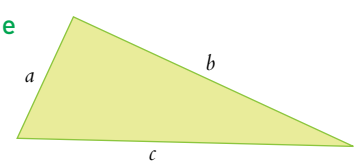
c

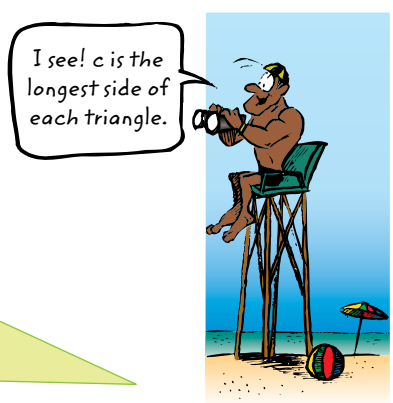


d



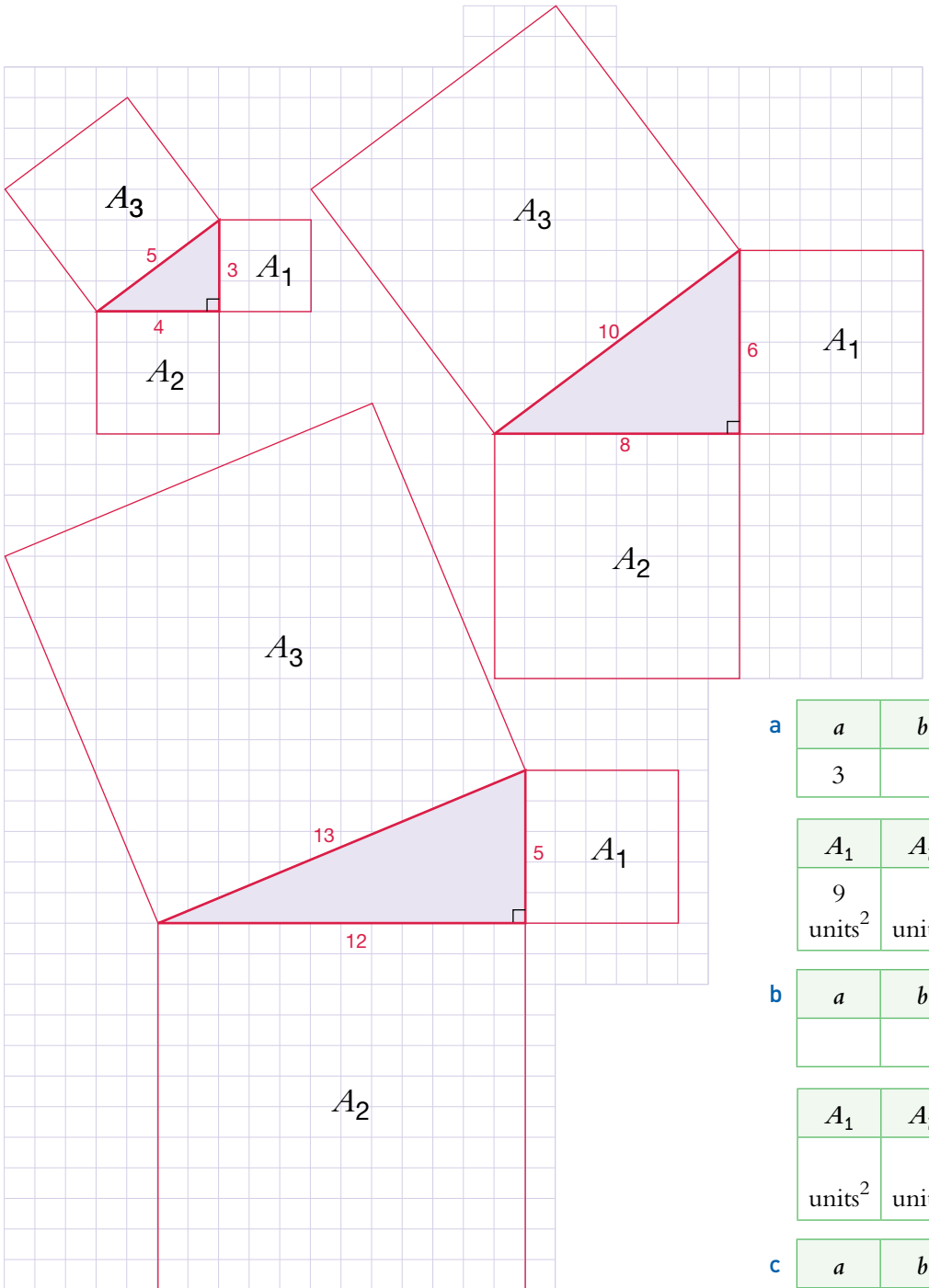
e





2 In which of the triangles in Question 1 does $c^2 = a^2 + b^2$? What type of triangles are they?

3 A square has been drawn on each of the sides of these right-angled triangles. Complete the tables for each triangle. Can you find a rule that links a, b and c?



a

a	b	c
3		

A_1	A_2	A_3
9 units^2	units^2	units^2

b

a	b	c

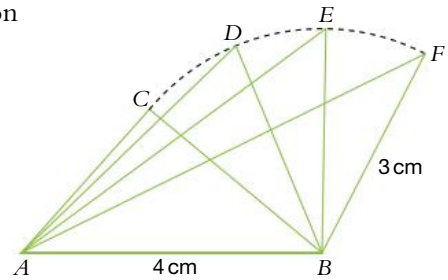
A_1	A_2	A_3
units^2	units^2	units^2

c

a	b	c

A_1	A_2	A_3
units^2	units^2	units^2

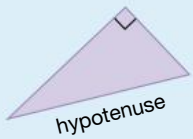
4 A sequence of triangles has been formed with a common base AB by gradually increasing the size of $\angle B$ while keeping the side BC constant in length. The vertex C moves along a circle to the positions D, E and F. This concept is demonstrated in GeoGebra activity 3:03 (1).



a Complete the table for the triangles named.

Triangle	AB^2	(right side)^2	(left side)^2
ΔABC			
ΔABD			
ΔABE			
ΔABF			

The longest side of a right-angled triangle is called the hypotenuse.



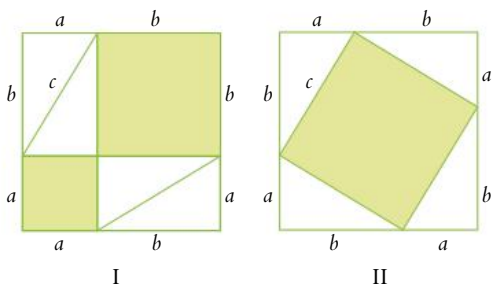
b For which triangle does AB^2 + (right side)^2 = (left side)^2?

c What is the size of $\angle B$ for the triangle in **b**?

While answering the questions above you should have discovered, like the Pythagoreans, the relationship between the sides of a right-angled triangle.

INVESTIGATION 3:01B PYTHAGORAS' THEOREM

Construct two identical squares as shown in the diagram. Use any value you like for a and b (e.g. $a = 6$ cm, $b = 8$ cm). The coloured squares are built on the three sides of a right-angled triangle that has side lengths a, b and c.



a Is the area of square I equal to the area of square II?

b Do the triangles in square I take up the same areas as the triangles in square II?

c If the triangles are removed from squares I and II, are the remaining parts equal in area?

d If the triangles are removed from square I, what is the area of the two remaining squares?

e If the triangles are removed from square II, what is the area of the remaining square?

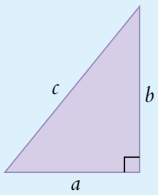
f Does $c^2 = a^2 + b^2$?

Complete: In any right-angled triangle the square on the _____ side is equal to the _____ of the squares on the other two _____ .

Pythagoras' theorem

- In any right-angled triangle the square of the longest side is equal to the sum of the squares of the two shorter sides. For the triangle shown, the theorem can be written as:

$c^2 = a^2 + b^2$



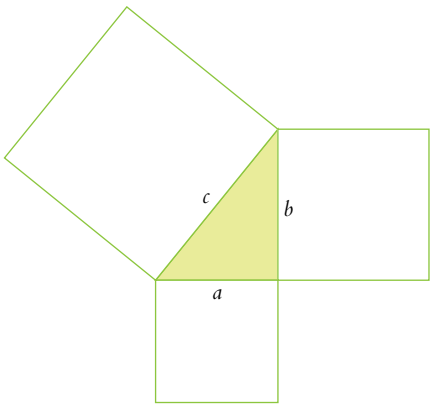
Converse

- If a triangle obeys the equation above, it must be right-angled.

Class activity

Pythagoras' theorem can also be illustrated by the following geometric method.

Construct a right-angled triangle and construct squares on each side of the triangle as shown in the diagram.



We have seen that for all right-angled triangles:

$c^2 = a^2 + b^2$

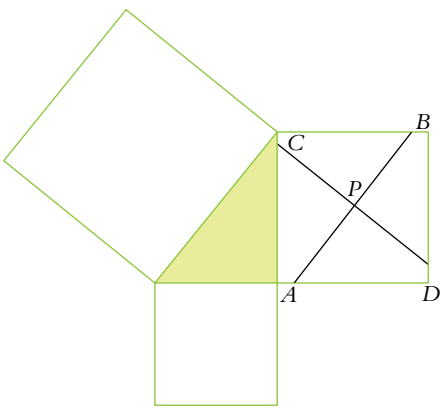
The areas of the squares constructed on the sides are a^2, b^2 and c^2 respectively, so the theorem can be restated as:

The square on the hypotenuse is equal to the sum of the squares on the two smaller sides.

The larger square can be made from the two smaller squares by dissecting the middle square and rearranging these pieces with the small square. The dissection is shown in the following diagram. Copy the dissection and find how the pieces can be arranged to form the large square.



- P is the centre of the square.
- AB is drawn through P, parallel to the hypotenuse.
- CD is drawn through P, perpendicular to the hypotenuse.



P GEOGEBRA ACTIVITY 3:01 PERIGAL'S DISSECTION

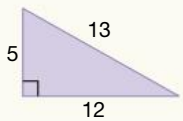
Complete this GeoGebra activity on Perigal's dissection to further your investigations on Pythagoras' theorem.

3:02 Pythagoras' theorem: Calculating the hypotenuse

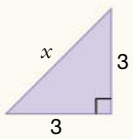
PREP QUIZ 3:02

For the triangles given in **1** to **3**, select the correct statements from **A**, **B** or **C**.

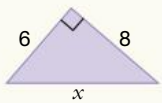
1 **A** $5^2 + 12^2 = 13^2$
B $(5 + 12)^2 = 13^2$
C $13^2 + 5^2 = 12^2$



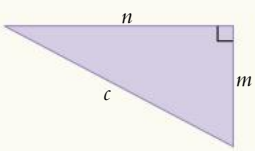
2 **A** $3 + 3 = x$
B $3^2 + 3^2 = x^2$
C $(3 + 3)^2 = x^2$



3 **A** $8^2 = 6^2 + x^2$
B $x^2 = 6^2 + 8^2$
C $x = 6 + 8$



4 Write Pythagoras' theorem for the triangle given.



The side length of a triangle must be positive.

5 If $4^2 = 16$, then $\sqrt{16}$ = _____

6 If $25^2 = 625$, then $\sqrt{625}$ = _____

7 If $\sqrt{25} = 5$, then 5^2 = _____

8 $3^2 + 4^2$ = _____

9 $\sqrt{36}$ = _____

10 Can 2, 3 and 4 be the side lengths of a right-angled triangle?

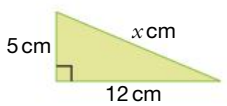
Pythagoras' theorem is used to calculate the lengths of the sides of right-angled triangles. To calculate one side, the other two sides must be known.

- $x^2 = 81$ has two solutions, $x = 9$ and $x = -9$. As side lengths must be positive we ignore the negative solution when finding a length.
- If $x^2 = c$ (where $x > 0$ and $c > 0$), then $x = \sqrt{n}$.

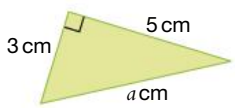
WORKED EXAMPLES

Calculate the length of the hypotenuse in each of the following triangles.

1



2

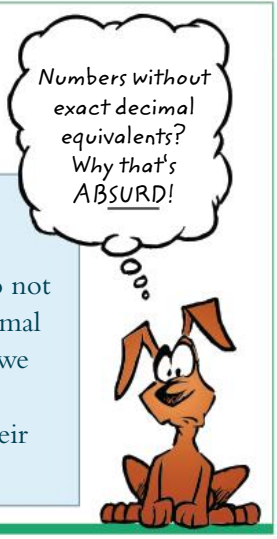


Solutions

1 $x^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
$x = \sqrt{169}$
$= 13$
Hypotenuse is 13 cm.

2 $a^2 = 3^2 + 5^2$
$= 9 + 25$
$= 34$
$a = \sqrt{34}$
$a \doteq 5{\cdot}8$ (1 dec. pl.)
Hypotenuse is $\sqrt{34}$ cm.

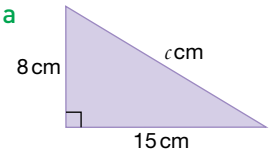
Numbers like $\sqrt{34}$ are called surds. **Surds** do not have exact decimal equivalents, so we use decimals to approximate their value.



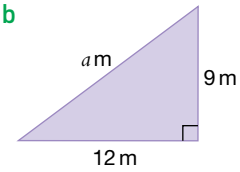
Exercise 3:02A (Calculators are not needed)

1 Use Pythagoras' theorem to write an equation that shows how the sides of each triangle are related to each other.

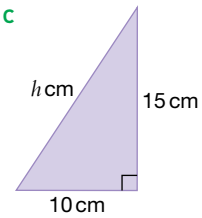
a



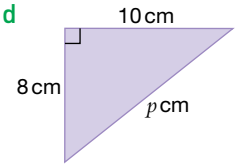
b



c



d



2 Complete the following table.

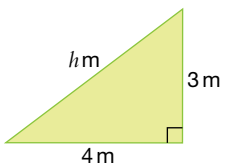
Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25																

3 Use the table in Question **2** to find:

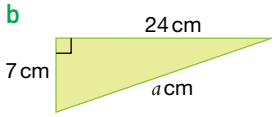
a $\sqrt{25}$ **b** $\sqrt{100}$ **c** $\sqrt{169}$ **d** $\sqrt{225}$ **e** $\sqrt{900}$ **f** $\sqrt{625}$ **g** $\sqrt{289}$

4 Find the length of the hypotenuse in each of the following. (Use the table in Question **2**.)

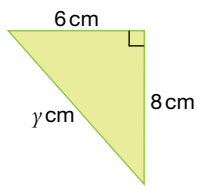
a



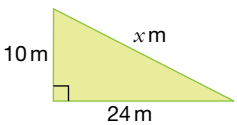
b



c



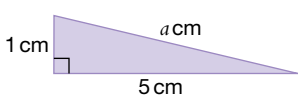
d



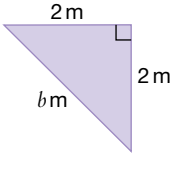
5 Find the length of the hypotenuse in each of the following. (Use the table below to evaluate the surds, correct to one decimal place.)

Square root	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{18}$	$\sqrt{20}$	$\sqrt{26}$	$\sqrt{29}$	$\sqrt{32}$	$\sqrt{34}$
Approximate value	2·2	2·8	3·2	3·6	4·1	4·2	4·5	5·1	5·4	5·7	5·8

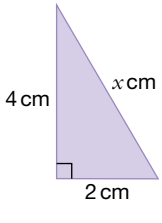
a



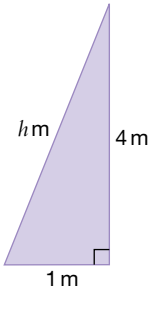
b



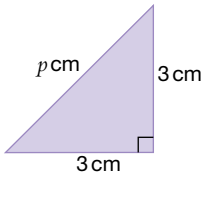
c



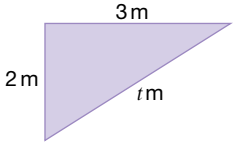
d



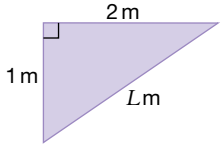
e



f

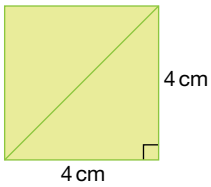


g

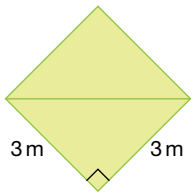


6 Use Pythagoras' theorem and the tables in Questions 2 and 5 to find the length of the diagonal of each square or rectangle.

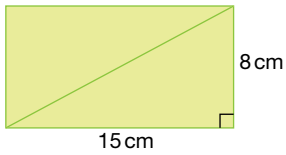
a



b

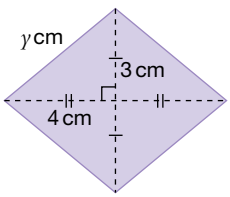


c

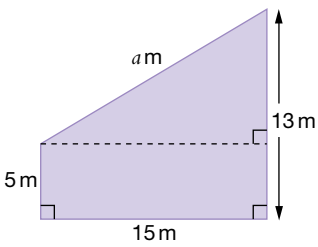


7 Use Pythagoras' theorem to find the value of pronumerals in each of the following. (You will need to use the tables in Questions 2 and 5.)

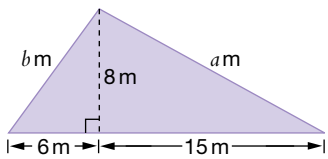
a



b



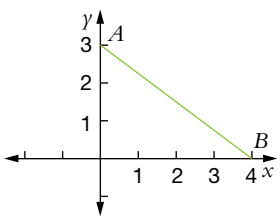
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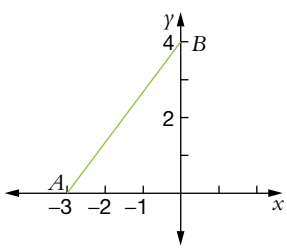
8 Find the perimeter of the shape in each part of Question 7.

9 Find the length of the interval AB in each of the following.

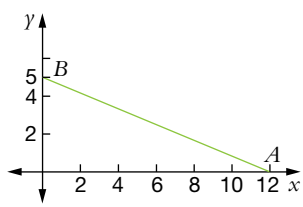
a



b



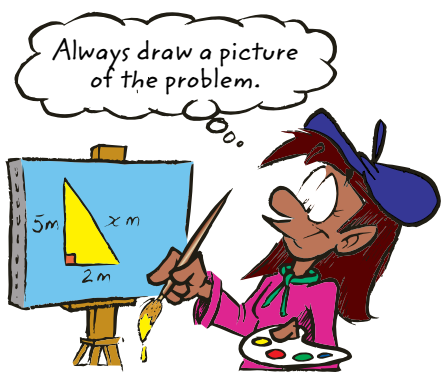
c



10 a The foot of a ladder is 2 m from a vertical wall. If the ladder reaches 5 m up the wall, find the length of the ladder, to the nearest centimetre.

b Two roads meet at right angles. One person walked 6 km along one road while another person walked 8 km along the other road. How far apart are they now?

c A radio mast is supported by three wires. Each wire is attached to the mast 5 m from the bottom. The wires meet the ground at a point 3 m from the bottom of the mast. Find the length of each wire to the nearest tenth of a metre.



Exercise 3:02B (Calculators are needed)

P Foundation worksheet 3:02 Pythagoras' theorem B

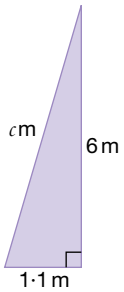
1 Use a calculator to simplify each of the following.

a 16^2 **b** $2 \cdot 5^2$ **c** $6 \cdot 8^2$ **d** $\sqrt{289}$

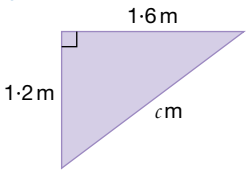
e $\sqrt{1681}$ **f** $\sqrt{33.64}$ **g** $\sqrt{96}$ (1 dec. pl.) **h** $\sqrt{148}$ (2 dec. pl.)

2 Use Pythagoras' theorem to calculate the length of the hypotenuse in each of the following.

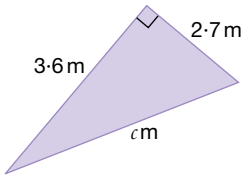
a



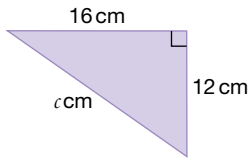
b



c

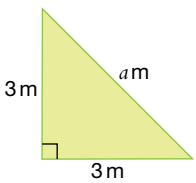


d

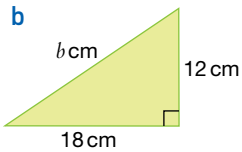


3 Find the value of the pronumeral in each of the following, correct to one decimal place.

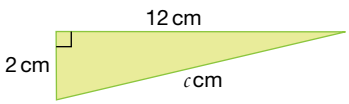
a



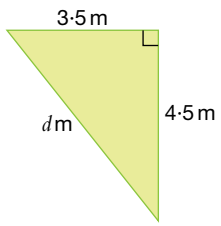
b



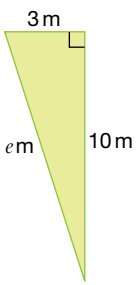
c



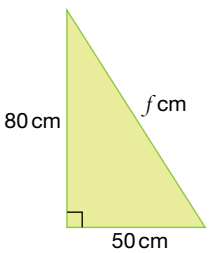
d



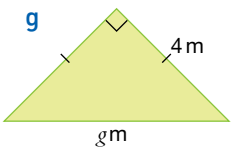
e

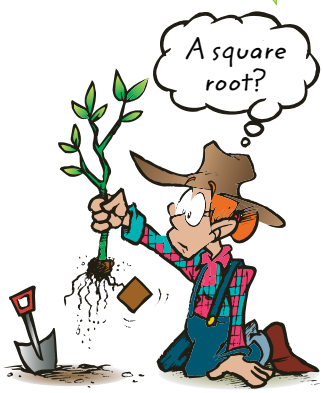


f



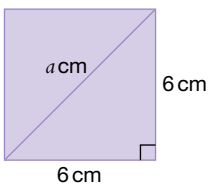
g



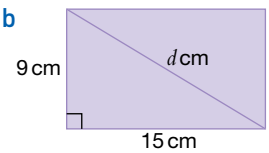


4 Find the value of the pronumeral in each of the following.
Give answers correct to one decimal place, where necessary.

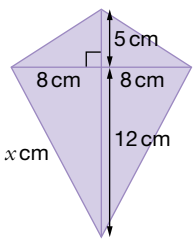
a



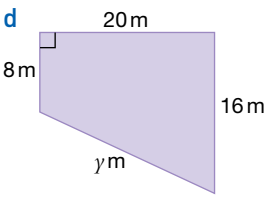
b



c



d



e Find the perimeter of the kite in **c**.

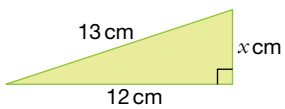
3:03 Pythagoras' theorem: Calculating one of the shorter sides

Pythagoras' theorem can be used to calculate the length of either short side.

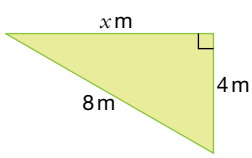
WORKED EXAMPLES

Find the value of x in each of the following.

1



2





Solutions

1

$13^2 = 12^2 + x^2$

$169 = 144 + x^2$

$(169 - 144 = 25)$

$25 = x^2$

$\therefore\ x = \sqrt{25}$

$= 5$

2

$8^2 = x^2 + 4^2$

$64 = x^2 + 16$

$(64 - 16 = 48)$

$48 = x^2$

$\sqrt{48} = x$

$x = \sqrt{48}$

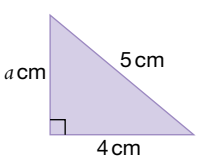
$\div 6{\cdot}93$ (2 dec. pl.)

$\div$ and $\approx$ mean 'is approximately equal to'.

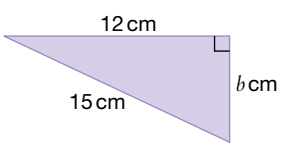
Exercise 3:03A (Calculators are not needed)

1 Use Pythagoras' theorem to write an equation that shows how the sides of each triangle are related to each other.

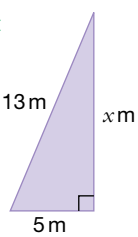
a



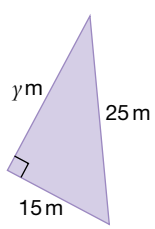
b



c



d



2 Use the following table to answer the questions below.

Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25	36	49	64	81	100	144	169	225	256	289	324	400	576	625	676	900

a $\sqrt{9}$ **b** $\sqrt{144}$ **c** $\sqrt{400}$ **d** $\sqrt{324}$ **e** $\sqrt{64}$ **f** $\sqrt{81}$

3 Complete the working for each of the following. Use the table from Question 2 in all exercises.

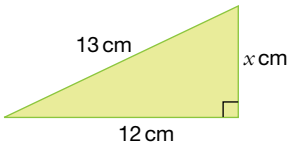
a $13^2 = 12^2 + x^2$

$169 = 144 + x^2$

$x^2 =$ _____

$x = \sqrt{}$

$x =$ _____



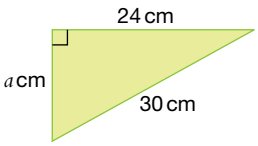
b $30^2 = 24^2 + a^2$

$900 = 576 + a^2$

$a^2 =$ _____

$a = \sqrt{}$

$a =$ _____

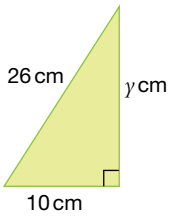


c $26^2 = y^2 + 10^2$

_____ $= y^2 +$ _____

$y^2 =$ _____

$y =$ _____

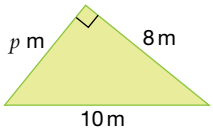


d $10^2 = p^2 + 8^2$

_____ $= p^2 +$ _____

$p^2 =$ _____

$p =$ _____



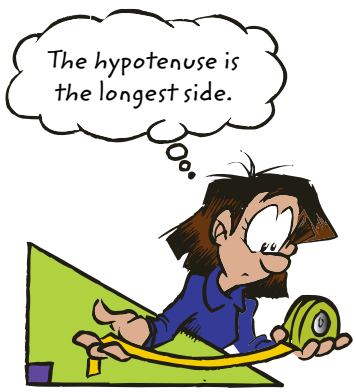
4 Using the setting out shown in Question 3, find the values of the pronumerals in Question 1.

5 **a** The hypotenuse of a right-angled triangle is 20 cm long. If one of the short sides is 12 cm long, what is the length of the other side?

b The longest side of a right-angled triangle is 17 cm. If one of the other sides is 8 cm in length, find the length of the third side.

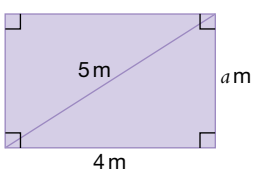
c The two longer sides of a right-angled triangle are 26 cm and 24 cm in length. How long is the shortest side?

d In a right-angled triangle the longest side is 15 cm long. The shortest side is 9 cm long. How long is the third side?

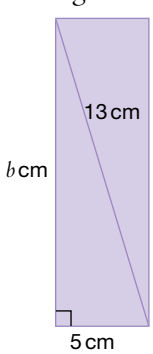


6 Find the value of the pronumeral in each of the following.

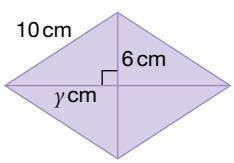
a



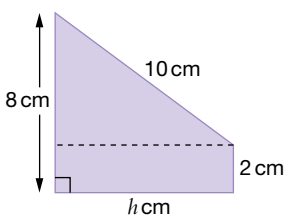
b



c



d

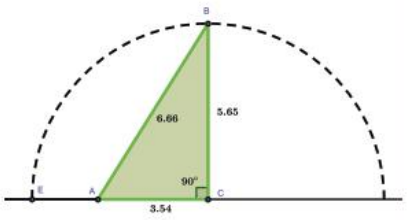


e Find the perimeter of the rectangle in **b**.

P GEOGEBRA ACTIVITY 3:03 INVESTIGATING PYTHAGORAS' THEOREM

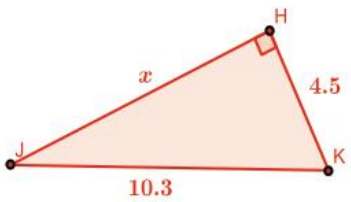
1 Investigating Pythagoras' theorem

Change the shape of the triangle to demonstrate that $c^2 = a^2 + b^2$, if the triangle is right-angled.



2 Calculating the sides of right-angled triangles

Click and drag the vertices of the triangle. The hypotenuse or a shorter side will be calculated as the shape of the triangle is changed.



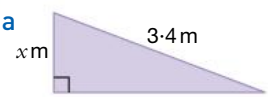
$$10.3^2 = x^2 + 4.5^2$$
$$106.09 = x^2 + 20.25$$
$$x^2 = 106.09 - 20.25$$
$$x^2 = 85.84$$
$$x = \sqrt{85.84}$$
$$x = 9.3 \text{ (1 dec. pl.)}$$

Exercise 3:03B (Calculators are needed)

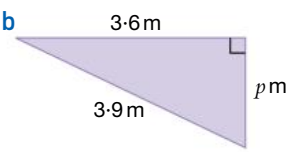
P Foundation worksheet 3:03 Pythagoras' theorem

1 Find the value of the pronumeral in each of the following.

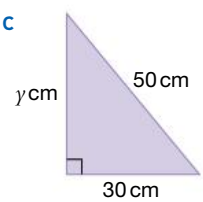
a



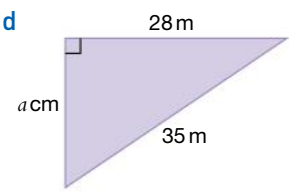
b



c

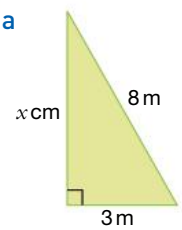


d

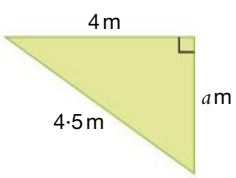


2 Find the value of the pronumeral in each of the following, correct to one decimal place.

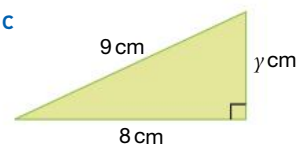
a



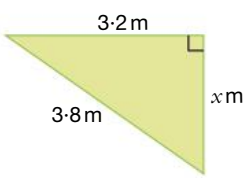
b



c

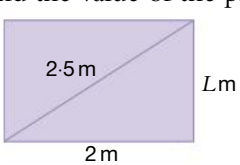


d

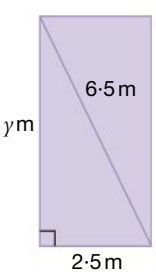


3 Find the value of the pronumeral in each of the following.

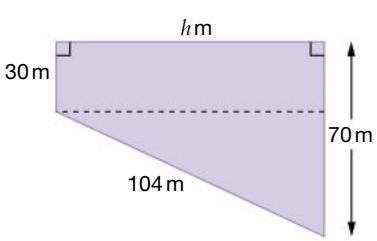
a



b

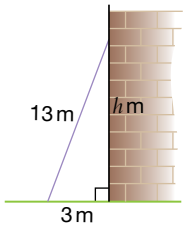


c

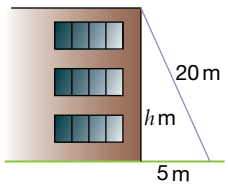


4 For each part in Question 3, find the perimeter of the shape.

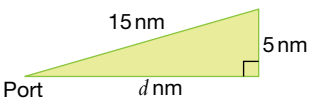
5 **a** A ladder that is 13 m long is placed so that its foot is 3 m from the base of a vertical wall. How far does the ladder reach up the wall?



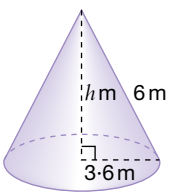
b A piece of wire 20 m long has one end attached to the top of a vertical building. The other end is fixed to the ground at a distance of 5 m from the building. Find the height of the building, correct to two decimal places.



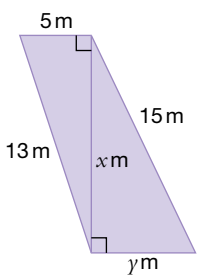
c A ship is 15 nautical miles (nm) from port. After sailing 5 nm due south, it is directly east of the port. How far is the ship then from the port?



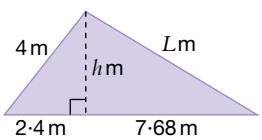
6 **a** The diagram shows a cone with a radius of 3·6 m and a slant height of 6 m. Calculate the height, h, of the cone.



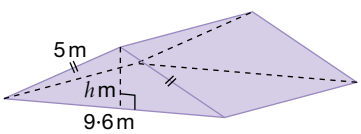
b Calculate x and y.



c Find the values of h and L.



d Find the value of h.



7 **a** The perimeter of a square is 16 cm. Find the length of its diagonal, correct to one decimal place.

b A rectangle's length is twice its breadth. Find the length of its diagonal, correct to one decimal place, if its perimeter is 90 cm.

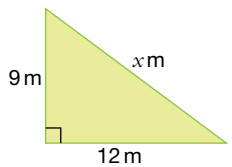
3:04 Applications of Pythagoras' theorem

The following exercise consists of a mixture of questions in which Pythagoras' theorem will be needed. In some questions, you will be asked to calculate the length of the hypotenuse. In other questions, you will need to calculate the length of the shorter side.

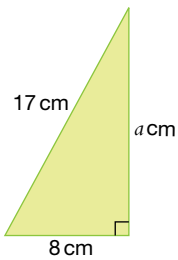
Exercise 3:04 (Calculators are needed)

1 Find the value of the pronumeral in each of the following.

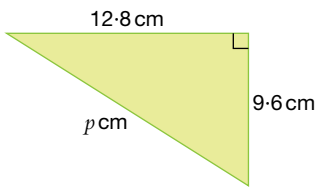
a



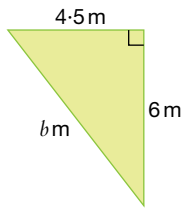
b



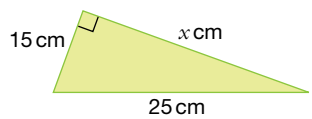
c



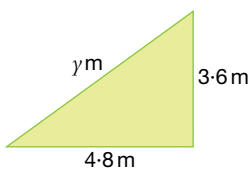
d



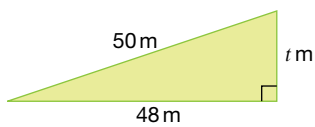
e



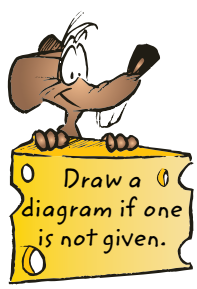
f



g

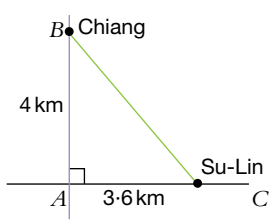


2 **a** Fiona measured the two shorter sides of a right-angled triangle and found them to be 16 cm and 30 cm. She then measured the longest side. What should its length have been?

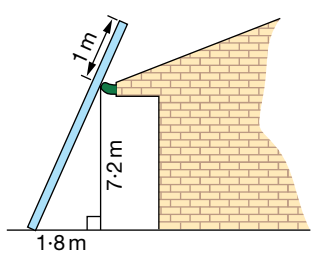


b Gino had an extension ladder that was 12 m long. If he placed the ladder against a wall so that the foot was 2 m from the building, how far up the wall would it reach, to the nearest centimetre?

c Chiang and Su-Lin start at A. Chiang walks towards B while Su-Lin walks towards C. How far apart are they when Chiang has walked 4 km and Su-Lin 3·6 km? Answer correct to one decimal place.

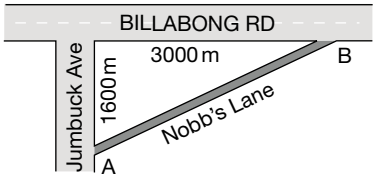


d Larry Bagnell needs to repair the guttering on his house. From the diagram calculate the length of the smallest ladder he needs to hire, if the ladder has to extend at least 1 m past the gutter.



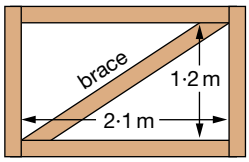
e Angelika is building a barbecue on a rectangular concrete slab. The slab is to be 2·4 m long and 1·8 m wide. What should the length of the diagonal be?

f Billabong Rd and Jumbuck Ave meet at right angles as shown in the diagram. Peter Perfect wants to go from A to B. What distance does he save by taking the shortcut down Nobb's Lane?



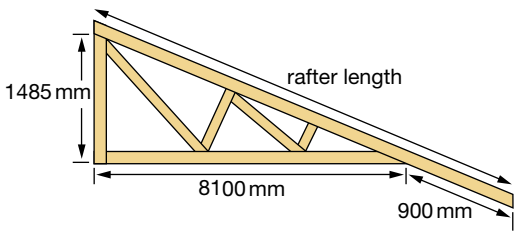
g Is a triangle with sides 3·6 m, 10·5 m and 11·1 m in length right-angled?

h The diagram shows a gate. Calculate the length of timber that would need to be bought to make the brace.

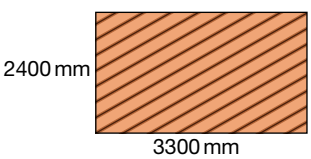


i Two roads intersect at right angles and continue in straight lines. Two drivers, Bill and Bev, leave the intersection at the same time. Bill drives down one road at an average speed of 60 km/h while Bev drives down the other road at an average speed of 80 km/h. Find how far each person has driven in half an hour and how far apart they are at that time.

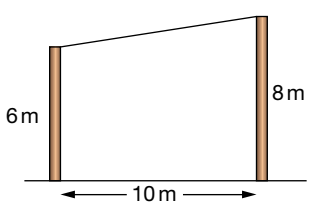
j Part of a roof truss is shown in the diagram. Calculate the length of the rafter to the nearest ten millimetres.



k Jasper and Jade decide to cover their bathroom wall with cedar boards as shown in the diagram. What is the longest length board that needs to be bought so that there would be no joins?

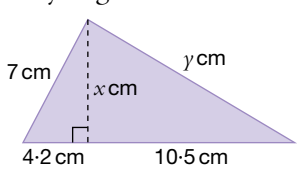


l Two vertical posts are 6 m tall and 8 m tall. If these posts are 10 m apart on level ground and a string is stretched from the top of one post to the top of the other, find the length of the string, to the nearest centimetre.

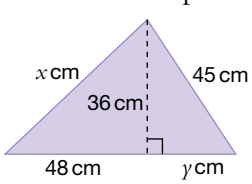


3 Use Pythagoras' theorem to find the values of the pronumerals in each of the following.

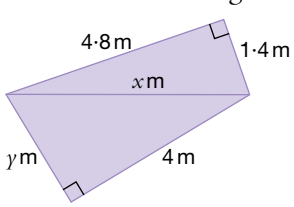
a



b



c



Start with a triangle in which two sides are known.

4 **a** Find the perpendicular distance from the corner of the triangle to the opposite side if the side is 25 cm long.



b Find the length of the diagonal if the length of the window is 48 cm and its breadth is 35 cm.



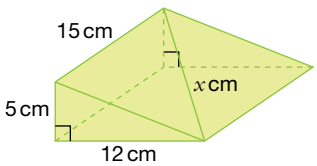
c A hat is made from a cone with a diameter of 18 cm and a slant height of 21 cm. What is the height of the hat?



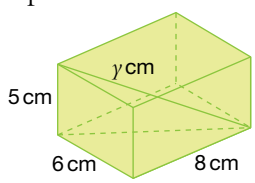
d The legs of this ladder are of equal length and are 1 m apart. The slant height of the ladder is 2·4 m. How high above the ground does the ladder reach?



5 **a** This solid is a triangular prism. Find the value of x, correct to two decimal places.



b This solid is a rectangular prism. Find the value of y, correct to two decimal places.



3:05 Pythagorean triads and right-angled triangles

A triangle has a right angle if the converse of Pythagoras' theorem is true.

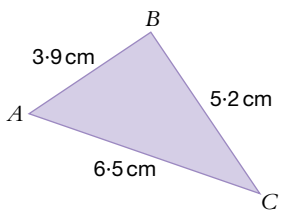
If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is right-angled.

If we are given the three sides of a triangle, we can discover whether it has a right angle by squaring the side lengths.

WORKED EXAMPLES

Test each triangle to see whether it is a right-angled triangle.

1

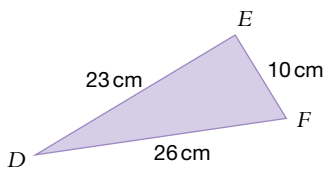


If ΔABC is a right-angled triangle then $6{\cdot}5^2 = 3{\cdot}9^2 + 5{\cdot}2^2$.

This is so because $42{\cdot}25 = 15{\cdot}21 + 27{\cdot}04$.

$\therefore \Delta ABC$ is a right-angled triangle.

2



If ΔDEF is a right-angled triangle then $26^2 = 23^2 + 10^2$.

This is not so because $676 \neq 529 + 100$.

$\therefore \Delta DEF$ is not a right-angled triangle.

Pythagorean triads

The three numbers of a Pythagorean triad correspond to the lengths of a right-angled triangle, where the third number represents the length of the hypotenuse.

- $\{5, 12, 13\}$ is a Pythagorean triad because $5^2 + 12^2 = 13^2$.
- For $\{a, b, c\}$ to be a Pythagorean triad, a, b and c must obey Pythagoras' theorem ($a^2 + b^2 = c^2$).
- If $\{a, b, c\}$ is a Pythagorean triad then $\{2a, 2b, 2c\}$ will be a Pythagorean triad because:

$$\begin{aligned}(2a)^2 + (2b)^2 &= (2a \times 2a) + (2b \times 2b)\\ &= 4a^2 + 4b^2\\ &= 4(a^2 + b^2)\\ &= 4c^2 \quad [\text{as } a^2 + b^2 = c^2]\\ &= (2c)^2\end{aligned}$$

A **Pythagorean triad** is a set of three numbers such that the sum of the squares of the first two equals the square of the third. $\{3, 4, 5\}$ is a Pythagorean triad.

You can make more triads by simply multiplying an existing one by a constant, such as:
$\{3, 4, 5\} \times 3 \Rightarrow \{9, 12, 15\}$

WORKED EXAMPLES

1 Is $\{6, 8, 10\}$ a Pythagorean triad?

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$10^2 = 100$

So $\{6, 8, 10\}$ is a Pythagorean triad.

2 Complete the Pythagorean triad $\{5, 12, ____\}$.

$5^2 + 12^2 = 25 + 144 = 169$

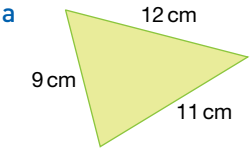
$13^2 = 169$

So $\{5, 12, 13\}$ is the Pythagorean triad.

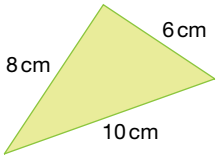
Exercise 3:05

1 Use Pythagoras' theorem to discover whether the triangle is right-angled or not.

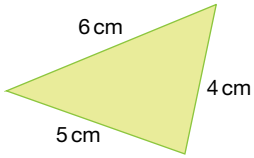
a



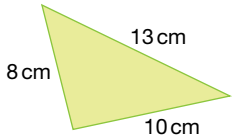
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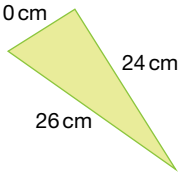
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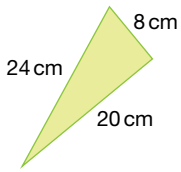
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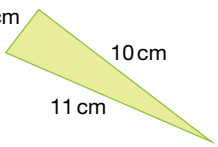
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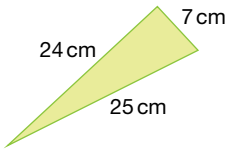
f



g

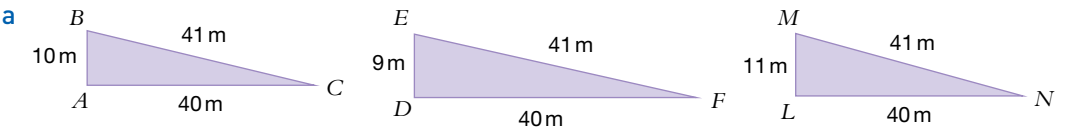


h

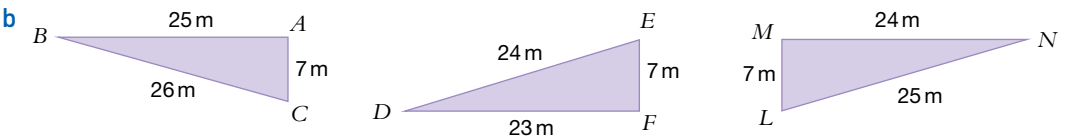


2 Which triangle in each part is the right-angled triangle?

a



b



3 Is each of the following a Pythagorean triad: yes or no?

a $\{3, 4, 5\}$ **b** $\{9, 12, 15\}$ **c** $\{5, 12, 13\}$ **d** $\{3, 5, 7\}$

e $\{24, 7, 25\}$ **f** $\{4, 3, 5\}$ **g** $\{8, 4, 9\}$ **h** $\{27, 36, 45\}$

4 The formula, $\{3x, 4x, 5x\}$ will produce Pythagorean triads for any counting number x. Use this formula to complete these Pythagorean triads.

a $\{6, 8, ____\}$ **b** $\{____, 12, 15\}$ **c** $\{12, ____, 20\}$ **d** $\{9, 12, ____\}$

e $\{30, 40, ____\}$ **f** $\{____, 36, 45\}$ **g** $\{300, ____, 500\}$ **h** $\{36, ____, 60\}$

5 The formula $\{n, \frac{n^2-1}{2}, \frac{n^2+1}{2}\}$ will also produce Pythagorean triads if we substitute odd numbers for n, starting at 3. Write the Pythagorean triad that is produced when we substitute:

a 3 **b** 5 **c** 7 **d** 9 **e** 11

6 Substitute further odd numbers for n, and record some more Pythagorean triads. Check that they actually work by testing them with Pythagoras' theorem.

7 Make a list of Pythagorean triads made up of three numbers that are all less than 100.

FUN SPOT 3:05 WHAT HAPPENS WHEN DUCKS FLY UPSIDE DOWN?

Answer each question and write the letter for that question in the box above the correct answer.

The Pythagorean triad {3, 4, 5} has been multiplied by a constant to produce these triads. Find the missing number in each triad.



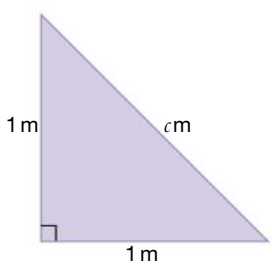
Q	6, 8, _____	**A**	_____, 12, 15	**T**	12, _____, 20
C	2·1, 2·8, _____	**P**	1·8, _____, 3	**E**	_____, 2, 2·5
U	33, _____, 55	**Y**	42, 56, _____	**U**	27, _____, 45
H	_____, 8·8, 11	**K**	3·75, _____, 6·25		

16	6·6	1·5	70	10	36	9	3·5	5	44	2·4

3:06 Irrational numbers (Surds)

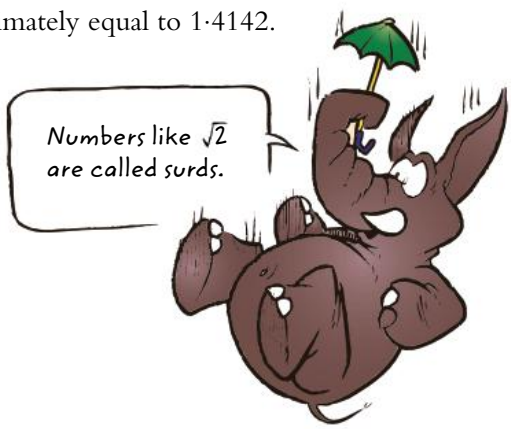
An irrational number is a number that cannot be written as a fraction or decimal. Examples are $\sqrt{5}$, $\sqrt{2}$, $\sqrt{2}+3$ and π. Irrational numbers such as $\sqrt{2}$ or $\sqrt{10}$ are called surds.

To find c in this triangle, we use Pythagoras' theorem.



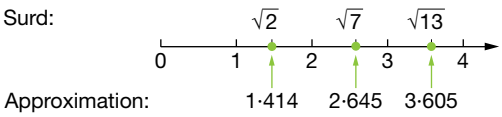
$$c^2 = a^2 + b^2$$
$$= 1^2 + 1^2$$
$$= 2$$
$$c = \sqrt{2}$$

$\sqrt{2}$ is the exact answer.
The surd $\sqrt{2}$ is approximately equal to 1·4142.



Surds on the number line

To place a surd on the number line, use a calculator to find an approximation for the surd, and then plot the surd at that point.



- A rational number can be written as a fraction $\frac{a}{b}$ (where a and b are integers) or as a terminating or recurring decimal.
- An irrational number cannot be written as a fraction $\frac{a}{b}$ where a and b are integers.

Note:

- $\sqrt{4}$ is not a surd as it can be written as 2 or $\frac{2}{1}$.
- $\sqrt{3}$ and $\sqrt{15}$ are surds. They are irrational numbers and they cannot be written as a decimal.
- The calculator can only give approximations for surds, as the decimal value of a surd continues without terminating or recurring.
  $\sqrt{3} \approx 1{\cdot}732$ (3 dec. pl.) or $1{\cdot}7320508$ (7 dec. pl.) or
  $\sqrt{3} \approx 1{\cdot}732050807568877293527446341505872366942805253810380628055806979451933016908800037081146186$ (90 dec. pl.)
  Notice that the decimal part does not terminate or repeat.
- If we wanted to show the exact answer to a problem, we would leave the answer in surd form.

Exercise 3:06

1 For each number, write *rational* or *irrational*. (Using the square root button on a calculator might help you decide.)



a $\sqrt{5}$	**b** $0{\cdot}75$	**c** $\sqrt{3}$	**d** $0{\cdot}9$
e $\sqrt{144}$	**f** $\frac{5}{8}$	**g** $\sqrt{7}$	**h** $\frac{1}{125}$
i $\sqrt{20}$	**j** $3{\cdot}42$	**k** $\sqrt{169}$	**l** 18
m $\sqrt{16}$	**n** $\sqrt{2}$	**o** $\sqrt{256}$	**p** $\sqrt{496}$
q $\frac{1}{3}$	**r** $\sqrt{14}$	**s** $\frac{7}{9}$	**t** $\sqrt{444}$

2 Which parts in Question 1 show a *surd*?

3 Between which two consecutive whole numbers does each of these surds lie?

a $\sqrt{3}$ **b** $\sqrt{5}$ **c** $\sqrt{8}$ **d** $\sqrt{11}$ **e** $\sqrt{15}$ **f** $\sqrt{18}$

4 Plot these surds on a number line.

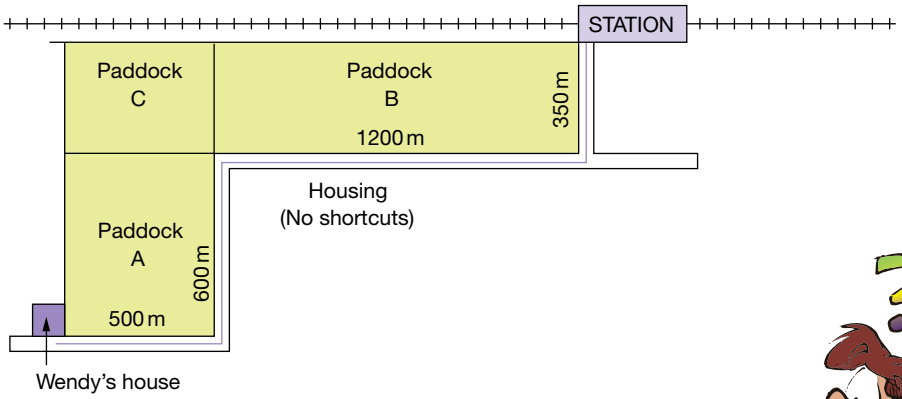
$\sqrt{3}, \sqrt{5}, \sqrt{8}, \sqrt{11}, \sqrt{15}, \sqrt{18}$

5 Arrange each set of numbers in order, from smallest to largest.

a $\sqrt{3}, 1\frac{1}{2}, 1{\cdot}2$ **b** $2{\cdot}2, \sqrt{5}, \sqrt{7}$ **c** $\sqrt{8}, 3, 2{\cdot}8$ **d** $4, \sqrt{11}, \sqrt{15}$

INVESTIGATION 3:06 PYTHAGORAS AND SPEED

Wendy usually walks to the railway station along roads shown in the diagram below.





Recently, after sleeping in, she needed to get to the station in a hurry and decided to take shortcuts through the paddocks. Wendy can run at 10 km/h and walk at 5 km/h. If she has only 18 minutes before the train leaves, should she:

a walk (arriving on time and unflustered)
b run (because she will be late otherwise)
c walk part of the way and run the rest
d change her plans (as she just can't make it)?

Explain your answer in sentences.

MATHS TERMS 3

hypotenuse
- the longest side of a right-angled triangle

irrational number
- a number that cannot be written as a fraction or decimal,
  e.g. $\sqrt{5}$, $\sqrt{3}+3$ and π are irrational.

Pythagoras' theorem
- in any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides

Pythagorean triad
- a set of three numbers such that the sum of the squares of the first two equals the square of the third,
  $\{3, 4, 5\}$ is a Pythagorean triad

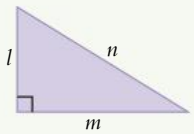
rational number
- a number that can be written as a fraction, or as a terminating or recurring decimal

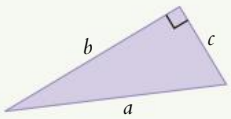
surd
- an irrational number like $\sqrt{2}$ or $\sqrt{10}$

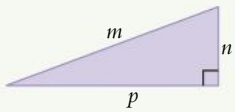
DIAGNOSTIC TEST 3 PYTHAGORAS' THEOREM

Each section of the test has similar items that test a certain type of example.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

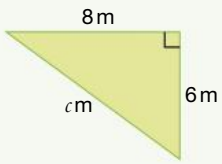
1 In each case, which side is the hypotenuse? 3:01

a 

b 

c 

2 Use Pythagoras' theorem to write an equation that shows how the sides of each triangle are related to each other. 3:02

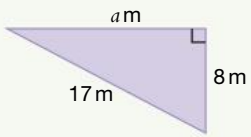
a 

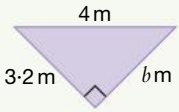
b 

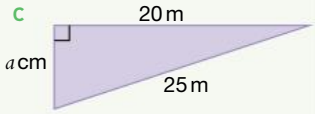
c 

3 Find the length of the unknown side in each of the triangles in Question 2. 3:02

4 Use Pythagoras' theorem to write an equation that shows how the sides of each triangle are related to each other. 3:03

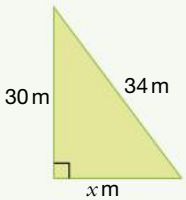
a 

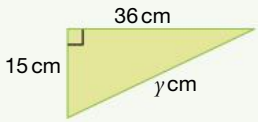
b 

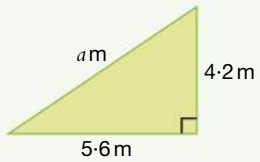
c 

5 Find the length of the unknown side in each of the triangles in Question 4. 3:03

6 Find the length of the unknown side in each of the triangles below. 3:04

a 

b 

c 

7 Complete each of the following Pythagorean triads. 3:05

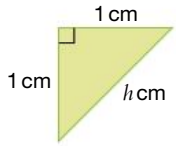
a $\{5, 12, ____\}$ b $\{6, ____, 10\}$ c $\{____, 24, 25\}$

8 Which of $\frac{3}{8}$, $\sqrt{7}$ and $0.\dot{3}$ is irrational? 3:06

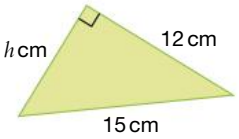
ASSIGNMENT 3A Chapter review

1 Use Pythagoras' theorem to find the value of the pronumeral in each case, correct to three decimal places.

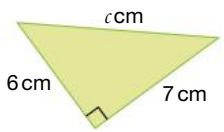
a



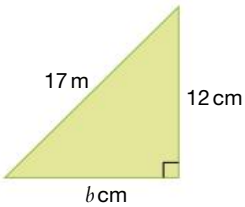
b



c

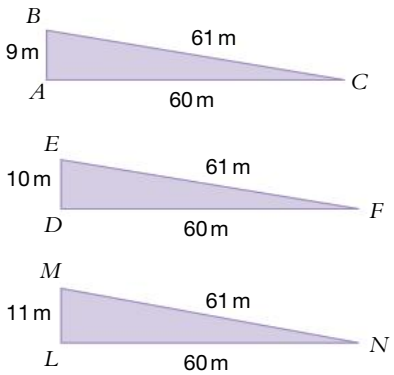


d

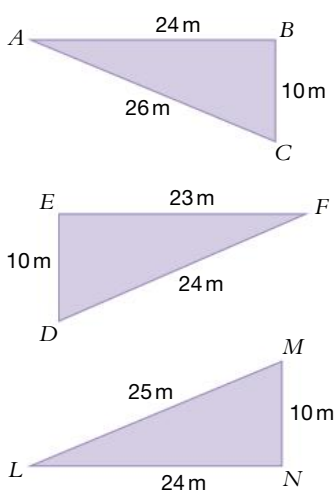


2 Which triangle in each part is the right-angled triangle?

a



b



3 Which of the following is a Pythagorean triad: {3, 4, 6}, {10, 12, 15}, {15, 36, 39} or {13, 15, 17}?

4 Which of each group of numbers are surds?

a $0{\cdot}25, \sqrt{2}, \sqrt{111}, 1{\cdot}3333$

b $\sqrt{169}, \sqrt{16}, \sqrt{3}, 17$

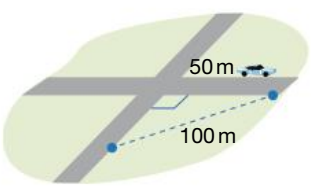
c $\sqrt{36}, 7, 6{\cdot}6, \sqrt{14}, \sqrt{20}$

5 Arrange the numbers in each part of Question **4**, in ascending order.

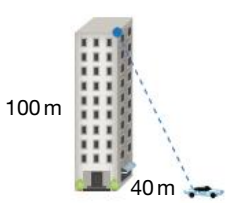
6 Give answers as a surd and then round to two decimal places.

a How far up a wall would a 4 m ladder reach if the foot of the ladder is positioned 2 m from the wall?

b Two roads meet at right angles. Starting at the intersection of the roads, one person travels 50 m along one road, and then stops. Another person starts from the intersection and travels along the other road? How far would this person need to travel so that he is 100 m from the first person?



c From the top of a 100 m building, a woman sees her car parked 40 m from the base of the building. How far is she from her car?



ASSIGNMENT 3B Working mathematically

1 Eight dots are drawn on a page. If no three dots are in the same straight line, how many intervals can be drawn that join any two dots?

2 Fred Shipshape built a sailing ship to mark Australia's 200th anniversary of European settlement. He wanted to use two words in naming it, but both words could not be the same. How many names were possible if the first name had to be either Endeavour, Australia, National, Young or Shining, and the second name had to be either Endeavour, Australia, Pride, Supreme, Courage or Star?



3 Five times Kate's age plus twice Sarah's age is equal to 25. List the possible ages for these two girls.

4 On a digital clock, how many times between 12 noon and 12 midnight will the readout be a palindromic number? (Use a dictionary to find a definition of a palindrome.)

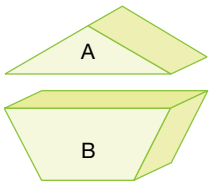


5 How many integers are less than 5 units from the number two? (An integer can be a positive whole number, a negative whole number, or zero.)

6 Drinks cost $1.20 each and ice-creams 80c each. The drinks and ice-creams for our group cost $12.40. If 12 items were purchased, how many drinks did we buy?

ASSIGNMENT 3C Cumulative revision

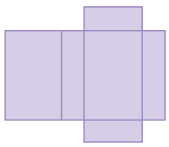
1 **a** Name solid A. **b** Name solid B.
c Calculate the number of faces, edges and vertices in solid B.
d Name the cross-sectional shape of the solid formed by placing solid A on top of solid B.



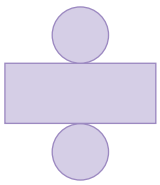
1:10

2 The nets of three solids are given below. Name the solid formed from each net.

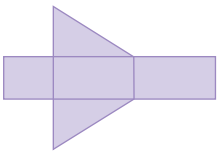
a



b

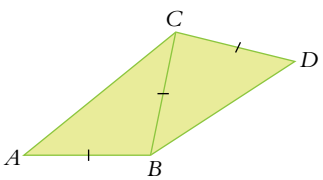


c



1:10

3 **a** What type of angle is $\angle ABC$?
b What type of angle is $\angle BAC$?
c By estimating, choose the angle that is closest in size to a right angle.
d Name an obtuse-angled isosceles triangle.



1:08

4 Change these Roman numerals into our own numerals.
a XLVIII **b** CIX **c** MDLXV **d** MCMXCIII

1:03

5 A property is rectangular in shape. It is 110 m long and 22 m wide. Calculate the cost of fencing the block if fencing is \$8.40 a metre.

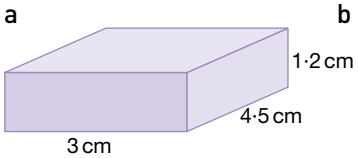
1:11

6 Write each time below as a digital time.
a 20 past 11 in the morning **b** a quarter to 8 in the evening

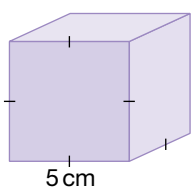
1:11

7 Calculate the volume of the shapes shown.

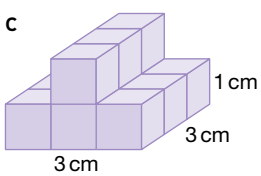
a



b



c



1:11

8 Write the basic numeral for each.
a $-9 + 5$ **b** $-8 - 7$ **c** $6 - -11$
d -6×-5 **e** 8×-3 **f** $-20 \div 4$

2:01

9 Simplify.
a $-3m + 7m$ **b** $-4a - 3a$ **c** $b - -9b$
d $-9y \times 4$ **e** $(-10)^2$ **f** $-120 \div -10$

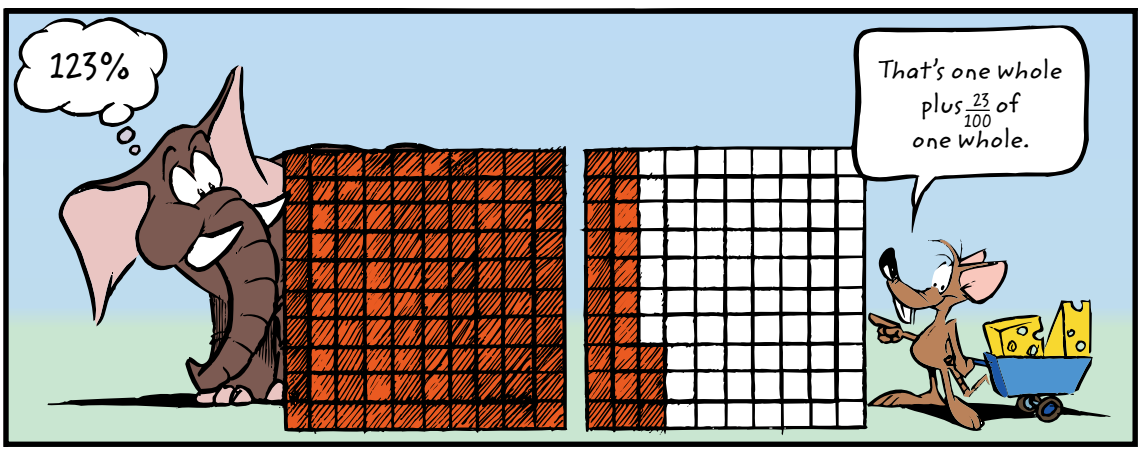
2:03

10 Simplify, writing answers in index form.
a $2^3 \times 2^4$ **b** $2^{10} \div 2^4$ **c** $(2^5)^2$

2:04

PERCENTAGES

4



Contents

4:01 Review of percentages
Fun spot 4:01 Estimating percentages
4:02 Harder conversions
Investigation 4:02 Conversions
4:03 Finding a percentage of a quantity
4:04 Percentage change
4:05 Percentage composition
4:06 Applications of percentages
Investigation 4:06 Are percentages ever misused?
4:07 GST (Goods and Services Tax)
Investigation 4:07 Shopper dockets
4:08 Profit and loss
4:09 Finding a quantity when a percentage is known (Unitary method)
Investigation 4:09 Earth percentages
Fun spot 4:09 Why did the hen feed her chickens concrete?
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

Selections from ***Fractions***, ***Decimals and Percentages***, ***Financial Mathematics*** [Stage 4]

- Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies. (ACMNA158)
- Solve problems involving the use of percentages, including percentage increases and decreases, with and without the use of digital technologies. (ACMNA187)
- Investigate and calculate the Goods and Services Tax (GST), with and without the use of digital technologies.
- Solve problems involving profit and loss, with and without the use of digital technologies. (ACMNA189)

Working Mathematically

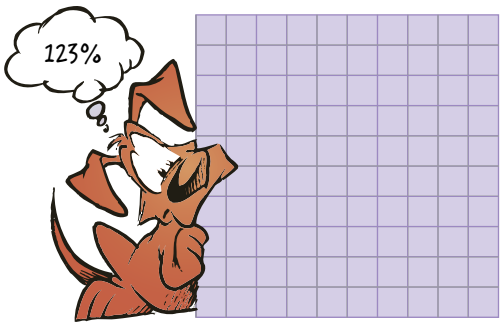
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

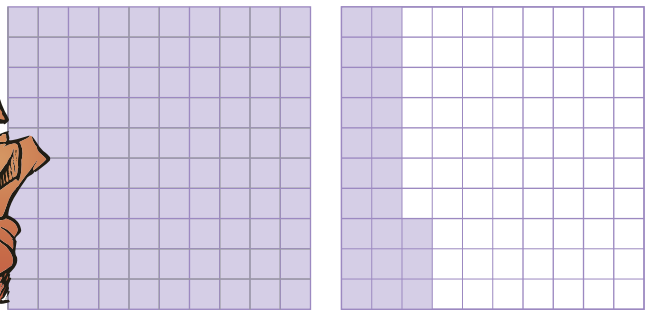
4:01 Review of percentages

Percentages are used in all walks of life. We all, therefore, need to understand percentages and to be able to perform calculations involving them.

WORKED EXAMPLES

1 Write 123% as a fraction and a mixed number.





Each whole has 100 parts

$100\% + 23\% = 123\%$

$1 + \frac{23}{100} = 1\frac{23}{100}$

$1 + 0{\cdot}23 = 1{\cdot}23$

2 Change the following percentages to decimals.

$91\% = \frac{91}{100} = 0{\cdot}91$

$4\% = \frac{4}{100} = 0{\cdot}04$

$40\% = \frac{40}{100} = 0{\cdot}4$

$400\% = \frac{400}{100} = 4$

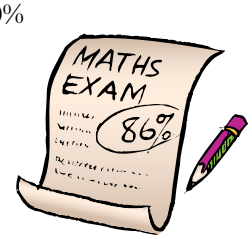
$106\% = \frac{106}{100} = 1{\cdot}06$

$350\% = \frac{350}{100} = 3{\cdot}5$

3 Change the following decimals to percentages.

$0{\cdot}22 = \frac{22}{100} = 22\%$

$0{\cdot}9 = 0{\cdot}90 = \frac{90}{100} = 90\%$



Exercise 4:01

1 Change these percentages to fractions.

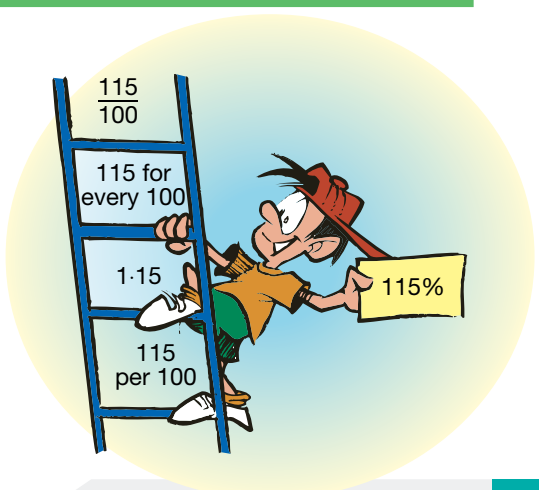
a 1% **b** 9% **c** 3% **d** 57%

2 Change each percentage to its equivalent basic fraction.

a 20% **b** 50% **c** 60% **d** 5%

3 Change each percentage to a whole or mixed number.

a 100% **b** 115% **c** 350% **d** 400%



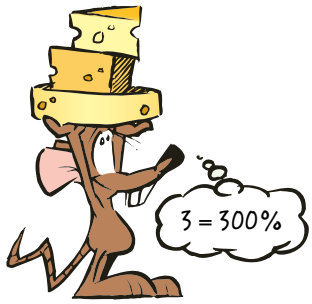
4 Change each fraction to a percentage.

a $\frac{1}{100}$ b $\frac{3}{100}$ c $\frac{7}{100}$ d $\frac{10}{100}$

5 Write each as a percentage. Change to hundredths first.

a $\frac{3}{10}$ b $\frac{7}{50}$ c $\frac{11}{20}$ d $\frac{2}{5}$ e $\frac{9}{25}$

f $3\frac{7}{100}$ g $1\frac{1}{2}$ h $2\frac{1}{10}$ i $3\frac{4}{5}$ j 2



To change a fraction to a percentage 'multiply by $\frac{100}{1}\%$'.

6 Write each fraction as a percentage by multiplying by $\frac{100}{1}\%$.

a $\frac{7}{40}$ b $\frac{3}{8}$ c $\frac{4}{5}$ d $\frac{1}{3}$ e $\frac{5}{6}$

7 Change each to an improper fraction, then write each as a percentage.

a $1\frac{1}{8}$ b $3\frac{1}{7}$ c $4\frac{1}{12}$ d $2\frac{1}{4}$ e $10\frac{3}{5}$

8 Write each percentage as a decimal.

a 14% b 63% c 85% d 32% e 11%

f 3% g 9% h 1% i 2% j 6%

k 10% l 30% m 50% n 90% o 70%

9 Write each decimal numeral as a percentage.

a 0·05 b 0·01 c 0·08 d 0·66 e 0·17

f 0·5 g 0·4 h 0·1 i 0·3 j 0·8

To change a decimal to a percentage, multiply by 100%.

e.g. $3{\cdot}7 = 3{\cdot}70 \times 100\%$
$= 370\%$

10 Write each set of numbers in order from smallest to largest.

a $\frac{1}{4}$, 100%, 0·5 b $\frac{1}{10}$, 1%, $\frac{1}{2}$

c $\frac{13}{100}$, 9%, $\frac{3}{4}$ d 1, 5%, $\frac{99}{100}$

11 Express each as a whole number or decimal.

a 200% b 600% c 100% d 300% e 500%

f 120% g 209% h 197% i 250% j 390%

12 Change each decimal to a percentage.

a 1·45 b 3·65 c 1·25 d 2·35 e 1·85

f 2·9 g 1·1 h 3·6 i 0·34 j 0·7

13 Use a calculator (or divide by 100 mentally) to change each percentage to a decimal.

a 147% b 113% c 124%

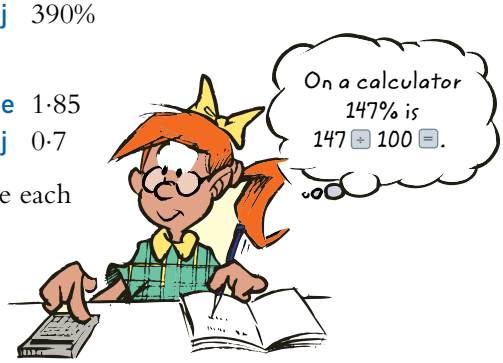
d 68% e 35% f 43%

g 9% h 6% i 4%

j 248% k 2% l 18%

m 13·6% n 9·2% o 16·3%

p 12·25% q 10·75% r 83·625%



147% = 147 ÷ 100

14 Write each set of numbers in order from smallest to largest.

a $\frac{3}{5}$, 15%, 0·51 b 1·8, 18%, $1\frac{1}{8}$ c 152%, 15·2, $1\frac{1}{2}$ d $\frac{2}{3}$, 60%, 0·65

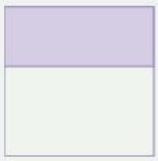
15 Explain why $\frac{2}{8} + \frac{1}{4}$ is not equal to $\frac{3}{12}$.



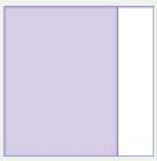
FUN SPOT 4:01 ESTIMATING PERCENTAGES

1 Estimate the percentage coloured in each shape, to the nearest 10%.

a



b



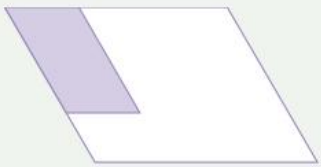
c



d



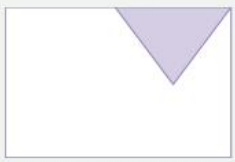
e



f



g



2 What percentage of each container is filled? Answer to the nearest 10%.

a



b



c



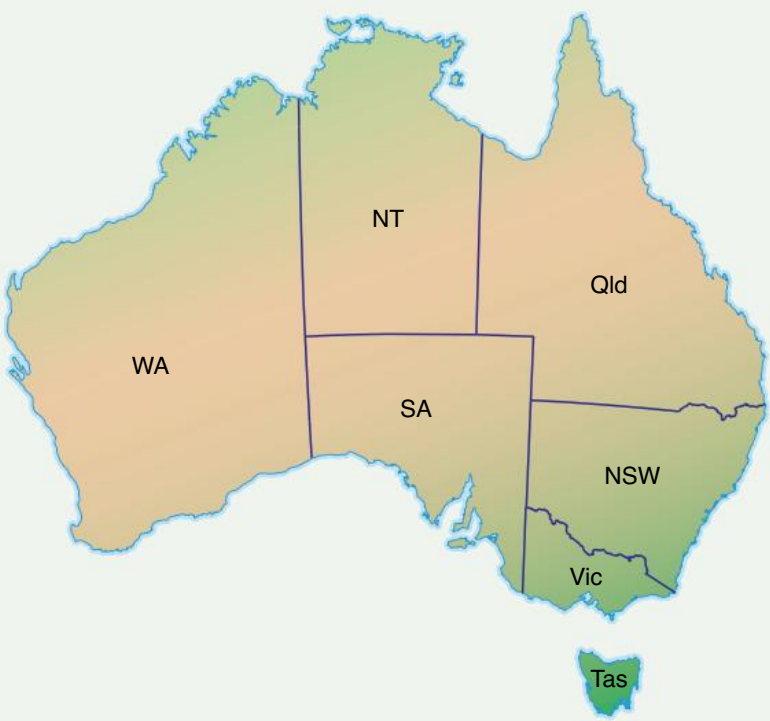
d



3 Use the map below to estimate the percentage of Australia's land area that is in:

a Victoria **b** New South Wales **c** Queensland
d Northern Territory **e** Western Australia **f** South Australia
g Tasmania

Add your answers for **a** to **g**. Do they add to about 100%?



4:02 Harder conversions

PREP QUIZ 4:02

Write as decimals:

1 $12\frac{1}{2}$ **2** $6\frac{1}{4}$ **3** $11\frac{3}{4}$ **4** $109\frac{1}{2}$

Simplify:

5 $0{\cdot}625 \times 100$ **6** $0{\cdot}33\dot{3} \times 100$ **7** $0{\cdot}\dot{3} \times 100$ **8** $0{\cdot}1\dot{6} \times 100$

Which two are equivalent?

9 1%, $\frac{1}{100}$, 0·1 **10** 10%, $\frac{1}{10}$, 0·01

WORKED EXAMPLES

1 Change 0·125 to a percentage.

2 Write $\frac{1}{6}$ as a percentage, correct to two decimal places.

3 Convert 13·8% to a decimal.

4 Convert $6\frac{1}{4}$% to a decimal.

Solutions

1 $0{\cdot}125 = 0{\cdot}125 \times 100\%$
$= 12{\cdot}5\%$

2 $\frac{1}{6} = (1 \div 6) \times 100\%$
$= 0{\cdot}1666\ldots \times 100\%$
$= 16{\cdot}67\%$

3 $13{\cdot}8\% = 13{\cdot}8 \div 100$
$= 0{\cdot}138$

4 $6\frac{1}{4}\% = 6{\cdot}25\%$
$= 6{\cdot}25 \div 100$
$= 0{\cdot}0625$

Using a calculator

1 0.125 × 100 = 12.5

2 1 ÷ 6 × 100 = 16.666667

3 13.8 ÷ 100 = 0.138

4 6.25 ÷ 100 = 0.0625

Exercise 4:02

P Foundation worksheet 4:02 Percentage conversions

1 Change these decimals to percentages.

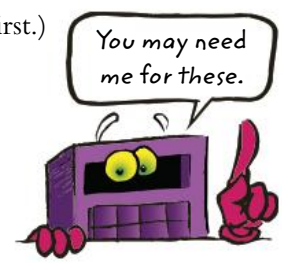
a 0·375 b 0·086 c 0·174 d 1·245 e 6·125
f 2·375 g 2·3 h 0·015 i 0·005

2 Convert each percentage to a decimal.

a 14·5% b 27·9% c 95·4% d 2·1% e 9·6%
f 23·85% g 87·35% h 112·5% i 287·5%

3 Convert each to a decimal. (Change the fraction part to a decimal first.)

a $12\frac{1}{2}$% b $87\frac{1}{2}$% c $13\frac{1}{2}$% d $7\frac{1}{4}$% e $10\frac{3}{4}$%
f $15\frac{1}{4}$% g $106\frac{3}{4}$% h $254\frac{1}{2}$% i $131\frac{1}{4}$%



4 Convert each to a percentage, correct to two decimal places.

a $\frac{2}{7}$ b $\frac{5}{6}$ c $\frac{7}{8}$ d $\frac{15}{16}$
e $\frac{2}{3}$ f $\frac{3}{40}$ g $1\frac{1}{6}$ h $2\frac{5}{6}$

INVESTIGATION 4:02 CONVERSIONS

Fraction to decimal	Fraction to percentage	Decimal to percentage
$\frac{3}{8} = 3 \div 8$ $= 8\overline{)3{\cdot}000}$ $0{\cdot}375$	$\frac{3}{8} = \frac{3}{8} \times \frac{100}{1}\%$ $= 37\frac{1}{2}\%$	$0{\cdot}375 = 0{\cdot}375 \times 100\%$ $= 37{\cdot}5\%$
Decimal to fraction	**Percentage to fraction**	**Percentage to decimal**
$0{\cdot}375 = \frac{375}{1000}$ $= \frac{3}{8}$	$37\frac{1}{2}\% = \frac{37.5}{100}$ $= \frac{375}{1000}$ $= \frac{3}{8}$	$37{\cdot}5\% = 37{\cdot}5 \div 100$ $= 0{\cdot}375$

Using words, explain how you would convert:

1. a fraction to a decimal
2. a decimal to a fraction
3. a fraction to a percentage
4. a percentage to a fraction
5. a decimal to a percentage
6. a percentage to a decimal.



Can you see connections?

4:03 Finding a percentage of a quantity

PREP QUIZ 4:03

1. 25% of a square is shaded red, 13% is shaded blue. What percentage of the square is neither red nor blue?

2. $\frac{1}{3} = 33\frac{1}{3}\%$ $\therefore \frac{2}{3} =$ _____%

3. $\frac{2}{4} = 50\%$ $\therefore \frac{1}{4} =$ _____%

4. $\frac{2}{8} = 25\%$ $\therefore \frac{1}{8} =$ _____%

5. $\frac{2}{16} = 12\frac{1}{2}\%$ $\therefore \frac{1}{16} =$ _____%

Change to a fraction in its simplest form:

6. 25%

7. 20%

Change to a decimal:

8. 24%

9. 6%

10. $14\frac{1}{2}\%$

To find a percentage of a quantity, write the percentage as a decimal (or fraction) and multiply by the quantity.

WORKED EXAMPLES

1 9% of 1600 g
(by fractions)
$= \frac{9}{\cancel{100}_{1}} \times \frac{\cancel{1600}^{16}}{1}$
$= \frac{144}{1}$
= 144 g

2 8% of 1600 g
(by decimals)
= (8 ÷ 100) × 1600
= (0·08 × 1600)
= 128·0
= 128 g

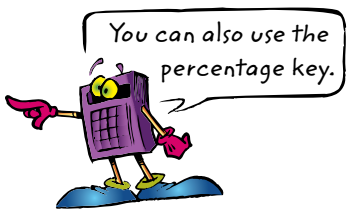
3 47% of 185 m
= (47 ÷ 100) × 185
= 0·47 × 185
= 86·95 m

OR 47 ÷ 100 × 185 =
OR 47 SHIFT % × 185 =

4 3% of 7 km
= 3% of 7000 m
= (3 ÷ 100) × 7000
= 0·03 × 7000
= 210·0
= 210 m

5 120% of 12 L
= (120 ÷ 100) × 12
= 1·2 × 12
= 14·4 L

OR 120 ÷ 100 × 12 =
OR 120 SHIFT % × 12 =



Exercise 4:03

P Foundation worksheet 4:03
Finding a percentage of a quantity

1 Replace the percentage with a decimal and then evaluate each of the following.

a 5% of 1600 g
b 9% of 2500 m
c 6% of $850
d 8% of 2000 L
e 83% of 40 cm
f 70% of 410 mL
g 40% of 85 min
h 67% of 20 000 t
i 17% of $300
j 24% of $36.50

48% of 600 g
= (48 ÷ 100) × 600
= 0·48 × 600
= 288 g

2 Replace the percentage with a decimal and then evaluate each of the following.

a 15% of $8
b 30% of 150 L
c 12% of $40
d 8% of 1800 g
e 2% of 3000 t
f 5% of 180 min
g 3% of 800 km
h 4% of 100 min
i 7% of 5000 L
j 105% of 800 m
k 115% of $2000
l 102% of 10 000 km
m 120% of 140 kL
n 379% of 162 km
o 280% of 365 days

3 Evaluate these by replacing the percentage with a decimal, or by using a calculator.

a 3% of 945 L
b 6% of 2425 m
c 84% of 10 t
d 76% of 8 km
e 35% of 9·6 m
f 120% of 2·8 g
g 245% of 0·9 t
h 1·2% of 840 mm
i 13·6% of 50 s
j 0·6% of 350 mL
k 0·7% of 150 000
l 0·08% of 8000 g
m 1·5% of 10 km
n 8·4% of 5 L
o $7\frac{1}{2}$% of $60
p $12\frac{1}{2}$% of 320 g
q $8\frac{1}{4}$% of 8900 kg
r $8\frac{3}{4}$% of $1000

1 3% of 794·8 t
= (3 ÷ 100) × 794·8 t
3 ÷ 100 × 794.8 = 23.844
OR 3 SHIFT % × 794.8 =

2 $106\frac{1}{4}$% of 1·35 km
$= (106 + \frac{1}{4}) \div 100 \times 1{\cdot}35$ km
106 + 1 ÷ 4 = ÷ 100 × 1.35 = 1.434
OR SHIFT $\square\frac{\square}{\square}$ 106 ▶ 1 ▼ 4 ▶ SHIFT % × 1.35 =

4 Advertising for the show *Nicholas Nickleby* accounted for 7% of the production costs of $1 500 000. How much was spent on advertising?

5 Random breath tests are used to discover whether a driver has drunk too much alcohol to drive safely. On New Year's Eve 2% of all drivers tested were above the legal limit. If 8000 were tested, how many were above the legal limit?

6 In 1974, only 2500 computers were in use in Australia: 62% by private business and 38% by the government. How many computers did the government use?

7 Skye needed a deposit of 28% of the cost of the home to obtain a loan from her bank. If the home cost $425 000, how much deposit did she need?

8 In Australia, about 10% of people suffer from asthma and 4% of the population are considered to have severe asthma. If Australia's population is 23 million, how many people in Australia have severe asthma?

9 Corey gained 93% of a possible 600 marks. How many more marks would he need to have scored to gain 96% of the marks?

10 Javier can run 400 m in 50 s. He needs to reduce his time by 13% to equal the Olympic record. What is the Olympic record for 400 m?

11 Following a survey of 2500 men of age 40 years and 2500 men of age 50 years, it was found that 43% of the 40-year-old men were substantially bald, whereas 58% of the 50-year-old men were substantially bald. How many more of the 50-year-old men were bald than of the 40-year-old men?

12 Her Majesty's Theatre has a capacity of 1000 people. On the opening night the theatre was 90% full. On the second night it was 85% full. How many people attended the theatre on the first two nights?

4:04 Percentage change

PREP QUIZ 4:04

1 Decrease 100% by 40%.
2 Decrease 100% by 13%.
3 Increase 100% by 18%.
4 Increase 100% by 300%.

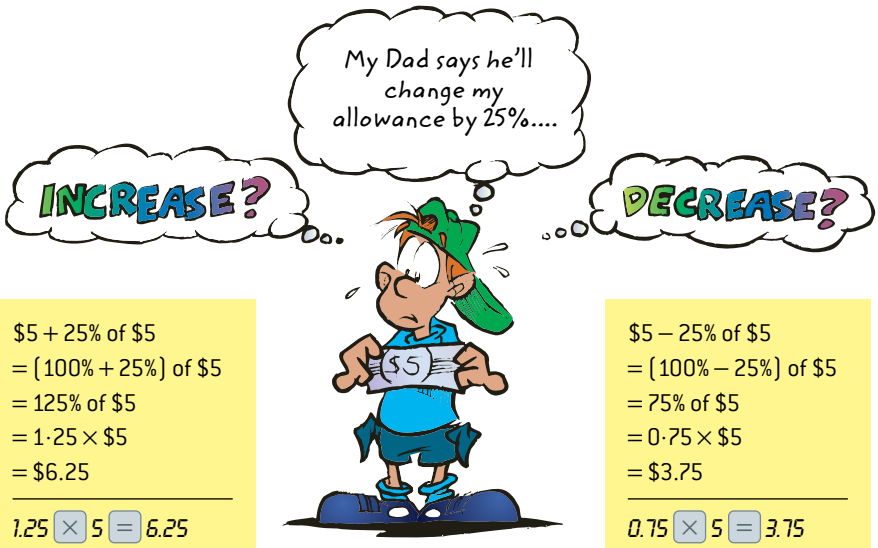
Find the following percentages.

5 100% – 20%
6 100% + 125%
7 100% + 6%
8 100% – 4%

Find the following quantities.

9 85% of 200
10 115% of 200

- 'Decrease' means to make smaller.
- 'Increase' means to make larger.



$5 + 25% of $5
= (100% + 25%) of $5
= 125% of $5
= 1·25 × $5
= $6.25

1.25 × 5 = 6.25

$5 − 25% of $5
= (100% − 25%) of $5
= 75% of $5
= 0·75 × $5
= $3.75

0.75 × 5 = 3.75

To increase an amount by 15%, find 115% of the amount (i.e. 100% + 15% = 115%).
To decrease an amount by 15%, find 85% of the amount (i.e. 100% − 15% = 85%).

Exercise 4:04

P Foundation worksheet 4:04
Percentage change

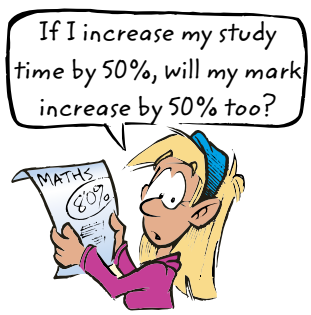
1 What percentage of an amount must I find to increase it by:

a 25% **b** 15% **c** 10% **d** 9%
e 100% **f** 300% **g** 155% **h** 132%?

2 What percentage of an amount must I find to decrease it by:

a 25% **b** 15% **c** 10% **d** 9%
e 1% **f** 95% **g** 37% **h** 30%?

3 **a** Increase $800 by 10%. **b** Increase $300 by 20%.
c Increase 5 m by 16%. **d** Increase 4 L by 12%.
e Increase 60 g by 85%. **f** Increase 70 t by 42%.
g Increase 10 kg by 4%. **h** Increase 30 ha by 6%.
i Increase 56 h by 100%. **j** Increase 81 s by 200%.



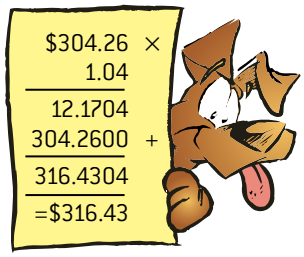
4 **a** Decrease $60 by 20%. **b** Decrease $360 by 80%.
c Decrease 8 m by 14%. **d** Decrease 3 m by 37%.
e Decrease 3·25 m by 40%. **f** Decrease 9·5 L by 30%.
g Decrease 20 kg by 5%. **h** Decrease 800 mm by 4%.
i Decrease 4 t by 23%. **j** Decrease 588 L by 90%.

5 To determine the selling price (retail price) of items sold in her shop, June increased the cost price by 45%. Find the selling price for each of the following articles.

a jacket: cost price $90 **b** scarf: cost price $8
c hat: cost price $30 **d** belt: cost price $12
e dress: cost price $200 **f** blouse: cost price $57

6 If wages have increased by 4%, find, to the nearest cent, the new wage paid to a person who was earning:

a $400 b $360
c $2111 d $304.21
e $856.22 f $1041.80



7 In a sale, all marked prices were reduced by 30%. Find the sale price for each of these items.

a



Was $475

b



Was $860

c



Was $595

8 Last year a new Aussiemobile cost $24 800 but inflation has increased its price by 11%. What is the new price?

9 Jane and Darcy purchased timber to build the frame of their house. They purchased 342 metres of timber. If wastage was 8%, what length of timber was actually used in the frame?

10 To relieve famine in Africa, 53 000 tonnes of food was sent by ship. If 35% of the food was spoilt at sea, how much food could be used for famine relief?

11 The population of a city in 2003 was 149 600. It was expected then that by the year 2009 the population would have increased by 19%. What was the expected population of the city for the year 2009?

12 During a drought, a low-flow showerhead was given to most homes in a town. This increased the number of homes that had a low-flow showerhead by 83%. If 8000 homes had a low-flow showerhead beforehand, how many homes had one afterwards?

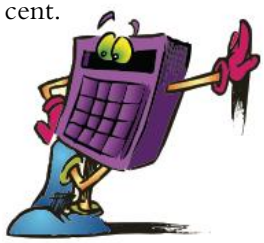
13 The value of my car depreciated 18% this year. If its value at the start of the year was $8640, what was its value at the end of the year?

14 Max wished to buy books to the value of $87.50. Because he is a regular customer, he was given a discount (reduction) of 16%. How much did he pay?



15 Lia purchased $1200 worth of shares. Within a month their value had dropped by 20%. During the following month there was an increase in the value of the shares by 20%. What was the value of the shares at the end of the second month? Why was this not $1200?

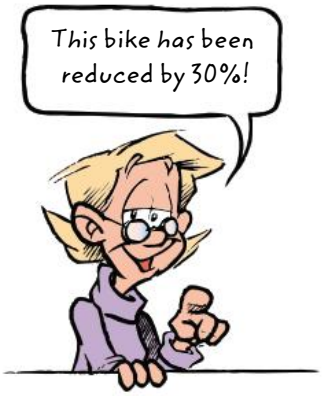
16 Use a calculator for each of the following, giving answers to the nearest cent.



a Increase $248.60 by 84%.
b Increase $946.85 by 74%.
c Increase $99.85 by 29%.
d Increase $465.35 by 147%.
e Increase $88.95 by 12.5%.
f Increase $475.64 by $135\frac{1}{2}$%.
g Decrease $6.75 by 18%.
h Decrease $73.45 by 81%.
i Decrease $0.94 by 35%.
j Decrease $138.97 by 68%.
k Decrease $199.98 by 7·5%.
l Decrease $384.66 by $18\frac{1}{2}$%.

17 Josh bought a car for $9000 but sold it to Sienna at a loss of 89% after it was involved in an accident. Sienna repaired the car and sold it to Nelson for 350% more than she paid for it. What price did Nelson pay for the car?

18 A betting agency takes 15% of all bets made on a race, so punters could expect to lose 15% of their money. Mixed-up Maurice wanted to boast that he backed the winner of every race on the program, so he bet on every horse in each of the seven races. For the first race he invested $1000, dividing this up so that he placed the same amount on each horse. He invested his return on the second race in the same way and continued this in all seven races. Calculate how much money he had left at the end of seven races if he lost 15% of his investment on each race? (Use a calculator and answer to the nearest dollar.)







4:05 Percentage composition

PREP QUIZ 4:05

What is the first quantity as a fraction of the second quantity?

1 50 g, 100 g **2** $2, $8 **3** 15 t, 20 t **4** 8c, 80c

Change the following fractions to percentages.

5 $\frac{1}{2}$ **6** $\frac{1}{4}$ **7** $\frac{3}{4}$ **8** $\frac{1}{10}$

9 Change $\frac{7}{8}$ to a percentage. **10** What fraction is 200 g of 2 kg?

To express one quantity as a percentage of another, write the first quantity as a fraction of the second and then convert to a percentage by multiplying by $\frac{100}{1}$%.

WORKED EXAMPLES

1 Express 4 g as a percentage of 25 g.

2 What percentage of 2 min is 51 s?

3 A tin of fishcake mix contains 204 g of Australian salmon, 156 g of potato, 24 g of seasoning and 16 g of cornflour. Find the percentages by weight of salmon and of potato.

Solutions

1 4 g as a fraction of 25 g

$= \frac{4}{25}$

$= \frac{4}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1}\%$

$= 16\%$

$\therefore$ 4 g is 16% of 25 g.

2 51 s as a fraction of 2 min

$= \frac{51}{120}$ (2 min = 120 s)

$= \frac{\cancel{51}^{17}}{\cancel{120}_{2\,\cancel{6}}} \times \frac{\cancel{100}^{5}}{1}\%$

$= \frac{85}{2}\%$ or $42\frac{1}{2}\%$

$\therefore$ 51 s is $42\frac{1}{2}\%$ of 2 min.

3 204 + 156 + 24 + 16 = 400

Weight of contents is 400 g

204 g of salmon as a fraction of 400 g.

$= \frac{204}{400}$

$= \frac{\overset{51}{\cancel{204}}}{\underset{1\,\cancel{4}}{\cancel{400}}} \times \frac{\cancel{100}^{1}}{1}\%$

$= \frac{51}{1}\%$

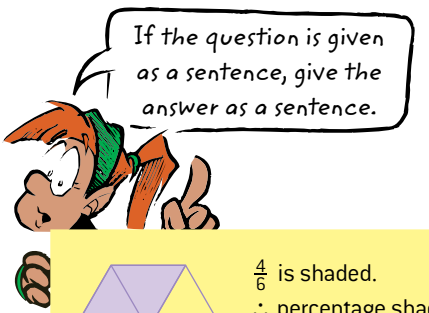
$\therefore$ 51% of the ingredients is salmon.

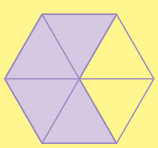
156 g of potato as a percentage of 400 g

$= \frac{\overset{39}{\cancel{156}}}{\underset{1\,\cancel{4}}{\cancel{400}}} \times \frac{\cancel{100}^{1}}{1}\%$

$= 39\%$

$\therefore$ 39% of the ingredients is potato.





$\frac{4}{6}$ is shaded.

$\therefore$ percentage shaded

$= \frac{\cancel{4}^{2}}{\cancel{6}_{3}} \times \frac{100}{1}\%$

$= \frac{200}{3}\%$

$= 66\frac{2}{3}\%$ or $66.\dot{6}\%$

By calculator: $\frac{4}{6} \times 100\%$

4 ÷ 6 × 100 = 66.666667

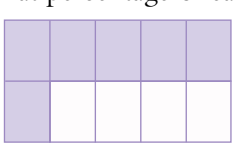
OR 4 ÷ 6 SHIFT % = 66.66666667

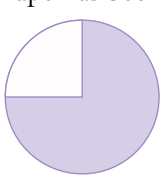
Percentage shaded is $66.\dot{6}\%$.

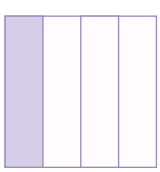
Exercise 4:05

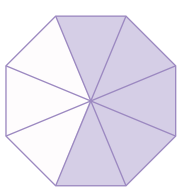
P Foundation worksheet 4:05 Percentages composition

1 What percentage of each shape has been shaded?

a 

b 

c 

d 

2 In each case, express the first quantity as a percentage of the second.

a 2 g, 20 g	**b** 5 m, 20 m	**c** 2 L, 200 L
d 15 mL, 60 mL	**e** 24 s, 120 s	**f** 372 ha, 600 ha
g 3 m, 40 m	**h** 14 kg, 80 kg	**i** 930 cm^2, 1200 cm^2
j 150 g, 450 g	**k** 27 m, 500 m	**l** 70 min, 105 min
m 6 L, 2 L	**n** 80 m, 64 m	**o** 440 g, 400 g

If the first quantity is greater than the second, then the percentage is greater than 100%.

3 What percentage is the first quantity of the second?

a 42 s, 1 min	**b** 3 min, 1 h	**c** 18 h, 1 day
d 27 h, 1 day	**e** 14 cm, 2 m	**f** 650 mm, 80 cm
g 2 m, 800 cm	**h** \$3, 40c	**i** 20 men, 10 men
j \$1.80, \$6	**k** 7000 g, 2 kg	**l** 35 m, 400 cm
m 7·2 m, 800 cm	**n** 26 kg, 1·3 t	**o** 40 g, 32 kg

In each case, the units must be made the same.

4 **a** A tin of fishcake mix contains 204 g of Australian salmon, 156 g of potato, 24 g of seasoning and 16 g of cornflour. Find the percentage by weight of seasoning and of cornflour.

b My Doggie dog food contains a minimum of 8·0% crude protein, a maximum of 0·5% crude fibre and 0·3% salt. If the tin contains 380 g (the net mass):

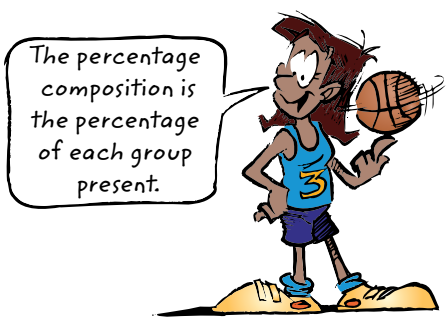
- **i** what is the mass of salt in the tin
- **ii** what is the least amount of crude protein that might be in the tin
- **iii** what is the greatest amount of crude fibre that might be in the tin?

c A 420 g (net) tin of corn kernels contains sweet corn, water, salt, and 4·2 g of added sugar. What percentage of the contents of the tin is added sugar?

d An 800 g tin of mushroom soup contains 6·2 g of protein, 20·7 g of fat, 34 g of carbohydrates, 0·4 g of sugars, 3·43 g of sodium and 150 mg of potassium. Find the percentage in the tin, correct to two decimal places of a per cent, of:

i carbohydrates **ii** fat **iii** sugars

e Due to safety regulations, only 600 people were permitted into the basketball stadium for the final between Midlands Secondary College and Sebastopol Secondary College. Team members (including coaches and umpires) accounted for 24 of these people and there were 321 female spectators. The rest of the crowd were male spectators. Find the percentage composition of those present.



f A school principal interviewed 40 of the students who left last year. Only 7 did not have a job. Find the percentage:

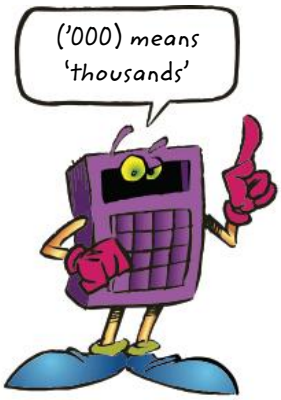
i who did not have a job **ii** who did have a job.

g Interest of \$840 was charged on a loan of \$6000. What percentage of the loan is the interest?

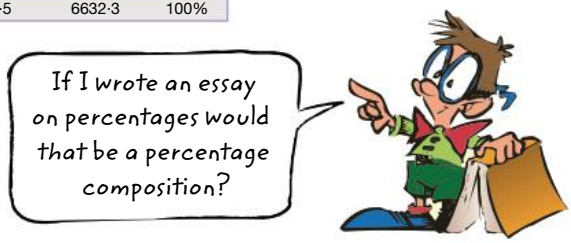
h Taylor sold his home for \$750 000 and paid the estate agent \$26 250 in fees. What percentage is this of the selling price?

i A time-and-motion study was carried out to determine how a secretary spent 8 hours at work in one day. The results were: typing 3 h, filing $1\frac{1}{2}$ h, sorting mail 30 min, telephone 15 min, meetings 45 min, other activities 2 h. Find the percentage composition for the activities of this 8-hour period.

j Use the information below to find the total number of people employed in each industry. Find the percentage of the total number employed in each industry. (Give percentages correct to one decimal place.)



EMPLOYED PERSONS BY INDUSTRY				
	Number ('000)			
Industry	Males	Females	Persons	% of total
Agriculture, forestry, fishing and hunting	284·7	109·0	393·7	5·9
Mining	86·5	7·2		
Manufacturing	841·5	295·5		
Food beverages and tobacco	123·6	51·7		
Metal products	175·2	28·0		
Other manufacturing	542·7	215·7		
Electricity, gas and water	121·9	11·1		
Construction	422·8	62·2		
Wholesale and retail trade	738·8	579·7		
Transport and storage	308·7	57·1		
Communication	107·0	39·8		
Finance, property and business services	351·2	301·0		
Public administration and defence	202·9	121·0		
Community services	425·3	715·6		
Recreation, personal and other services	192·4	249·4		
Total	4083·8	2548·5	6632·3	100%



4:06 Applications of percentages

PREP QUIZ 4:06

1 52% of students are boys. What percentage are girls?

2 0·7 = 70%. True or false?

3 82% of $1

4 $\frac{a}{50} = 14\%$ $\therefore a =$ _____

5 $\frac{x}{10} = 40\%$ $\therefore x =$ _____

6 Write 32% as a decimal.

7 Increase $30 by 100%.

8 Reduce $30 by 100%.

9 What fraction of 8 m is 8 cm?

10 What percentage of 8 m is 8 cm?

Percentages can be used to compare costs and make decisions about purchasing goods. Retailers use discounts to attract customers, but buyers should be wary because percentages can be misused to create an impression or to sell a product.

Exercise 4:06

P Foundation worksheet 4:06
Applications of percentages

1 One store advertises golf clubs for '20% off'. Another advertises the same clubs for '30% off'. Do we know which shop is cheaper? Why or why not?

2 At a football match 80% of people interviewed said that they were in favour of capital punishment. If only 5 people were interviewed, how many of these were in favour of capital punishment? Had enough people been asked to find out what Australians think about this?

3 Shifty Discounts had trouble selling their watches for $6, so they increased the price to $10 and advertised the watches at '30% off their marked price'. What was the new selling price of the watches?

4 Two stores advertised the same range of books. Honest Anne offered '20% off the publisher's recommended price'. Fair Freddie offered 'up to 50% off the publisher's recommended price'.

a At Honest Annie's I bought books with a total marked price of $120. What price did I pay after getting the 20% discount?

b At Fair Freddie's I bought the same books and was given 10% off on $100, and 50% off on $20. How much did I pay for the books altogether?

c Which shop was the cheapest and by how much?

A *discount* is a reduction from the normal selling price.



5 Screen doors marked at $98.50 were discounted by 40% if five or more doors were purchased. Isabella needed one screen door but couldn't resist this bargain. She bought five doors. How much did they cost? Did she really save money?

6 The book *Maths is Wonderful* was advertised at '$15.80 with a 10% discount' or '$20.40 with a 35% discount'. Which of these is the cheaper, and by how much?

7 A TV commercial claimed that 'Only 4% of people disliked our drink Fizz'. It did not give the information in the table below.

Reaction to Fizz	Number
disliked	24
not impressed	540
liked	36

a What percentage liked Fizz?

b What percentage were not impressed with Fizz?

c Why didn't the advertiser use the percentages in **a** and **b**?

8 A discount of $84 was given on a Blu-ray player with a marked price of $280. Find:

a the price paid

b the discount as a percentage of the marked price.

Discount as a percentage of marked price

$$= \frac{\text{discount}}{\text{marked up}} \times \frac{100}{1}$$

9 **a** In 2010, 14·30 million people in Australia lived in capital city districts. Of these 7·21 million were female. What percentage of the city population was female?

b In 2010, of the 8·03 million people that lived outside city districts, 4·03 million were males. What percentage does this represent?

c From the information above can you determine the percentage of Australia's total population that was female in 2010?

10 At 2 pm on 30 March 2012, Australia's population was 22 863 149. If the average growth rate of Australia's population is 1·8% per year, find an estimate for:

a the population on 30 March 2013

b the population on 30 March 2014 (Hint: You need to use your answer to part **a**.)

c the population on 30 March 2020

d the percentage increase from 2012 to 2020.

INVESTIGATION 4:06 ARE PERCENTAGES EVER MISUSED?

1 Collect three examples of media reports or advertising claims that use percentages.

a Comment on the reliability of each percentage.

b State whether the truth of the claims can be tested or assessed and, if so, explain how this could be done.

c Has useful information been omitted? List details that could have been included but were not.

2 Make a collection of examples in which percentages have been misused. Share your results with other students.

3 Find the interest earned for different investments in banks, building societies and elsewhere. Find the interest charged by banks, building societies, finance companies and elsewhere. How do these compare?



4:07 GST (Goods and Services Tax)

PREP QUIZ 4:07

Find 10% of:	**1** \$20	**2** \$3.50	**3** \$7.10	**4** \$14.36
Calculate:	**5** \$66 ÷ 11	**6** \$9.24 ÷ 11	**7** \$5.6 × 1·1	**8** \$127 × 1·1
	9 \$66 × $\frac{10}{11}$	**10** \$9.24 × $\frac{10}{11}$		

GST is a broad-based tax of 10% on most goods and services you buy. GST is included in the price you pay. However, because no GST is applied to some items such as basic food goods, a bill or shopping docket may itemise each product showing whether the GST was charged and how much GST was included in the bill.

To calculate the GST to add on to a price, simply find 10% of the price.

It is simple, however, to calculate the GST included in a price by dividing by 11, because the base price has been increased by 10% or $\frac{1}{10}$.

To find the GST included in a price, divide the price by 11.

WORKED EXAMPLES

1 Find the GST that needs to be applied to a price of $325.

2 What is the retail price of a DVD player worth $325 after the GST has been applied?

3 How much GST is contained in a price of $357.50?

4 What was the price of an item retailing at $357.50 before the GST was applied?

Solutions

1 The GST is 10% of the price.
$\therefore$ GST = $325 × 10%
= $32.50

2 The GST is added on to get the retail price.
$\therefore$ Retail price = $325 + $32.50
= $357.50

Note: The retail price can also be calculated by multiplying the original price by 110% (or 1·1) because 10% is added on.
Retail price = $325 × 1·1
= $357.50

3 To find the GST contained in a price, we divide it by 11. If the original price is increased by $\frac{1}{10}$, then the retail price, including the GST, is $\frac{11}{10}$ of the original price.
$\therefore$ GST = $357.50 ÷ 11
= $32.50

4 To find the original price, simply subtract the GST from the retail price.
$\therefore$ Original price = $357.50 − $32.50
= $325

Note: The original price can also be found by multiplying the retail price by $\frac{10}{11}$.
Original price = $357.50 × $\frac{10}{11}$
= $325

Exercise 4:07

P Foundation worksheet 4:07
Goods and services tax

1 Determine the GST that needs to be added to original prices of:

a $70	**b** $120	**c** $46.20	**d** $59.70	**e** $863
f $12 740	**g** $97.63	**h** $142.37	**i** $124.55	**j** $742.15

2 Determine the retail price after the GST of 10% is added to the following.

a $60	**b** $150	**c** $220	**d** $740	**e** $75	**f** $112
g $343	**h** $947	**i** $8.60	**j** $27.40	**k** $56.45	**l** $19.63

3 How much GST is contained in the following retail prices?

a $22	**b** $110	**c** $165	**d** $286	**e** $61.60	**f** $57.20
g $13.20	**h** $860.20	**i** $46.75	**j** $349.80	**k** $1004.30	**l** $1237.50

4 Find the total GST paid on the camping gear shown if its retail price is $1130.



5 What was the original price before the GST of 10% was added to each of these retail prices?

a $77	**b** $132	**c** $198	**d** $275	**e** $15.40	**f** $28.60
g $106.70	**h** $126.50	**i** $13.86	**j** $41.25	**k** $237.05	**l** $1238.38

6 When retailers sell a mixture of goods where the GST is only applied to some, they normally indicate which items on the bill include the GST. This bill indicates the items that include the GST with an asterisk (*). Determine the amount of GST that would be included in the total on this bill.

SHOPQUICK

Sauce	$ 1.20
Biscuits*	$ 1.35
Bread	$ 3.60
Toothbrush*	$ 3.10
Soap*	$ 1.93
Total	$ 11.18

GST incl. = ?

7 This retailer uses a percentage sign (%) to indicate which items on the bill include GST. Determine the GST included in each of the following bills.

a

% Magazine	$ 2.85
% Deodorant	$ 4.00
Milk	$ 5.60
Muffins	$ 2.60
% Biscuits	$ 2.17
Total	$ 17.22

GST incl. = ?

b

Bread	$ 2.70
Flour	$ 0.90
% Ice-cream	$ 4.48
% Soap	$ 0.98
% Tissues	$ 5.10
Total	$ 14.16

GST incl. = ?

c

Bananas	$ 3.50
% Soap	$ 3.92
% Detergent	$ 3.31
Tea bags	$ 4.46
Eggs	$ 2.89
% Bath salts	$ 6.05
Rice	$ 1.54
% Biscuits	$ 2.56
Total	$ 28.23

GST incl. = ?

INVESTIGATION 4:07 SHOPPER DOCKETS

1 Collect 'shopper dockets' from different supermarkets. Examine them to see how they indicate the GST charges.

2 Draw up a list of all the items you can that *do not* include GST.

3 When was the GST introduced in Australia?



4:08 Profit and loss

People who are self-employed may charge a fee for their services or sell goods for a profit. We all may, from time to time, be concerned about profit and loss. This may be in our employed work, in our hobbies, or in our work for charities and organisations. When earning and investing our money, we need to consider whether our investment of time, money and effort is justified by the results.

WORKED EXAMPLES

1 Aisha bought a painting for \$600 and sold it for \$648. Find the profit as a percentage of the cost.



2 A bicycle has a selling price of \$420, which includes a 20% profit on the cost price. What was the cost price?



Solutions

1 Profit = selling price − cost price
= \$648 − \$600
= \$48

Profit as a % of cost

$$= \frac{\text{profit}}{\text{cost price}} \times \frac{100}{1}\%$$

$$= \frac{\overset{8}{\cancel{48}}}{\underset{1}{\cancel{6}}\cancel{00}} \times \frac{\cancel{100}^{1}}{1}\%$$

= 8%

∴ profit is 8% of cost.

2 Cost price + 20% of cost price = selling price
120% of cost price = selling price
1.2 × cost price = \$420
cost price = $\frac{420}{1{\cdot}2}$
= \$350

∴ The cost price of the bicycle was \$350.

Exercise 4:08

1 Complete the table below.

	Cost price	Profit or loss	Selling price
a	$176	$8.80 profit	
b	$460	$92 loss	
c	$1.80	$0.63 profit	
d		$5000 loss	$25 000
e		$4.20 profit	$42
f		$2560 loss	$8000
g	$1680		$4800

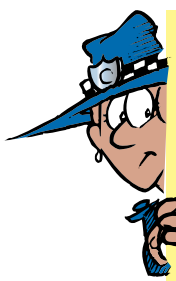


2 For parts **a** to **d** of Question **9**, find the profit or loss as a percentage of the cost price.

3 For parts **d** to **g** of Question **9**, find the profit or loss as a percentage of the selling price.

4 Cooper bought an old coin for $5 and later sold it for $10. Find:
- **a** his profit from the sale
- **b** the profit as a percentage of the cost price
- **c** the profit as a percentage of the selling price.

5 Harper paid $640 for her wedding dress. After the wedding, she sold the dress to her friend Willow for $400. Find:
- **a** Harper's loss from the sale
- **b** the loss as a percentage of the cost price
- **c** the loss as a percentage of the selling price.



Beware!
If percentage profit or loss is mentioned, the person might choose to use the cost or selling price, depending on which suits his or her purpose.

6 Liam bought some land for $40 000 and later sold it, making a profit of 20% on his cost. Find:
- **a** the profit
- **b** the selling price
- **c** the profit as a percentage of the selling price.

7 A fashion store buys a dress for $36 and wishes to make a profit of 80% on its re-sale.
- **a** How much profit is this?
- **b** What would be the selling price of the dress?
- **c** What is the profit as a percentage of the selling price?

8 Charlie bought a DVD for $28 and then sold it to his mate for $20.
- **a** What is the loss as a percentage of the price it cost Charlie to buy it?
- **b** What is the loss as a percentage of the price he charged his mate?

9 Find the cost price for each selling price below, given that the percentage profit was calculated on the cost price.

- **a** selling price = $300, profit = 50%
- **b** selling price = $100, profit = 25%
- **c** selling price = $250, profit = 25%
- **d** selling price = $44, profit = 10%
- **e** selling price = $420, profit = 5%
- **f** selling price = $65, profit = 30%
- **g** selling price = $300, profit = 20%
- **h** selling price = $96, profit = 60%

10 Milla bought a pair of shoes for $118.80. This included a profit of 65% on the cost price for the retailer.

- **a** What was the cost price?
- **b** How much was the profit?

11 A machine for a factory was purchased for $25 200. This included a profit of 40% on the cost price.

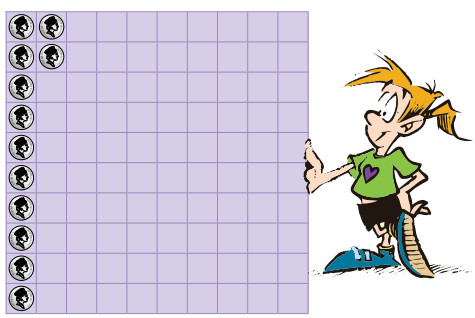
- **a** What was the cost price?
- **b** How much was the profit?
- **c** If this machine was resold for the cost price, what percentage loss would this be on the original purchase price? Answer to the nearest whole per cent.

4:09 Finding a quantity when a percentage is known (Unitary method)

Discussion

The same kind of coin has been placed on 12 of the 100 squares on this percentage chart. If the value of each coin is equivalent to 50 cents, how much money would be needed to cover every square with the same kind of coin?

- What is the value of 12 coins?
- What is the value of 1 coin?
- What is the value of 100 coins?



If a percentage of a quantity is known, we can find the whole quantity. First, we find 1% (or 10%), and then multiply to find 100% (the whole quantity). This is called the **unitary method.**

WORKED EXAMPLES

1 I spent 8% of my money on a new graphite tennis racquet that cost $108. How much money did I have before the purchase?

2 In a one-day cricket match a player made 102 runs. If this was 34% of the team's runs, how many runs did the team make?

Solutions

1
8% of my money = $108
∴ 1% of my money = $108 ÷ 8
= $13.50
∴ 100% of my money = $13.50 × 100
= $1350
∴ I had $1350 before the purchase.

2
34% of the runs = 102
∴ 1% of the runs = 102 ÷ 34
= 3
∴ 100% of the runs = 3 × 100
= 300
∴ The team scored 300 runs.

Exercise 4:09

P Foundation worksheet 4:10
Finding the quantity if a percentage is known

1 Determine the whole quantity if:

a 4% of it is 20 m
b 3% of it is 813 ha
c 30% of it is 27 kg
d 80% of it is 2800 t
e 28% of it is $29.12
f 94% of it is $564
g 120% of it is 840 mm
h 112% of it is $115.36

2 I spent 12% of my money on a surfboard that cost $468. How much money did I have?

3 If I. Beltem scored 32 runs in a one-day cricket match and this was 16% of his team's runs, how many runs did his team make?

4 The 132 injuries occurring on sports fields in December represented 11% of the year's total injuries on sports fields. How many injuries occurred in the year?

5 Ava raised $238.55 for a youth group. If this represented 13% of all money raised in a year, how much was raised that year?

INVESTIGATION 4:09 EARTH PERCENTAGES

Estimate the percentage of the Earth's surface that is habitable.

Estimate the percentage of the Earth's surface that is able to be cultivated.

Now research the answer. How close were your estimates?



FUN SPOT 4:09 WHY DID THE HEN FEED HER CHICKENS CONCRETE?

Work out the answer to each question and write the letter for that part in any box that is above the correct answer.

A Write 1% as a decimal.

B Write 10% as a fraction.

C Write $\frac{4}{5}$ as a percentage.

D Write $1\frac{1}{4}$ as a percentage.

E Write 1 as a percentage.

H Write 205% as a decimal.

I 1% of 100

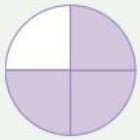
K 200% of 1

L If $16\frac{2}{3}\% = \frac{1}{6}$, what fraction is $8\frac{1}{3}\%$?

M How many metres is 5% of 2 km?

N 5 men, 3 women and 2 children are in a bus. What is the percentage of women?

O What percentage has been coloured?



R The cost price is \$100. The profit is 10% of the cost price. What is the selling price?

S The cost price is \$100. The loss is 10% of the cost price. What is the selling price?

T \$200 was invested at 10% p.a. simple interest for 3 years. How much interest was earned?

W 5% + 8%

Y 100% − 17%



\$90	2·05	100%	13%	0·01	30%	\$60	100%	125%	\$60	2·05	100%	100	\$60	75%

$\frac{1}{10}$	100%	80%	75%	100	100%	$\frac{1}{10}$	\$110	1	80%	2	$\frac{1}{12}$	0·01	83%	100%	\$110	\$90

MATHS TERMS 4

cost price
- the money paid for an item before reselling

discount
- to reduce the price of goods sold
- the amount or percentage a price is reduced
- a deduction from the normal selling price

GST
- Goods and Services Tax
- 10% of a base price is added on to the cost of most goods and services and included in the advertised retail price

loss
- occurs when the selling price is less than the cost price

marked price
- the price shown on the tag or advertising
- the original selling price

per annum (or p.a.)
- each year,
  e.g. 25% p.a. means 25% each year

percentage
- out of 100
  e.g. 17% means 17 out of 100

percentage composition
- the percentages of parts that make up the whole

percentage decrease
- the percentage by which an amount has decreased

percentage increase
- the percentage by which an amount has increased

profit (or gain)
- occurs when the selling price is more than the cost price

selling price
- The price paid when an item is purchased.



DIAGNOSTIC TEST 4 PERCENTAGES

Each section of the test has similar items that test a certain type of example.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1	Change to fractions or mixed numerals in simplest form: a 9% b 48% c 350%	4:01
2	Change to percentages: a $\frac{173}{100}$ b $\frac{11}{25}$ c $3\frac{1}{7}$	4:01
3	Change to decimals: a 17% b 4% c 106%	4:01
4	Change to percentages: a 0·22 b 0·9 c 3·7	4:01
5	Change to decimals: a 13·8% b $6\frac{1}{4}$% c $87\frac{1}{2}$%	4:02
6	Change to percentages: a 0·125 b 1·625 c $\frac{1}{6}$	4:02
7	Find: a 8% of 1600 g b 3% of 7 km c 120% of 12 L	4:03
8	a Increase \$5 by 15%. b Decrease \$5 by 15%. c Increase \$70 by 42%	4:04
9	a Express 4 g as a percentage of 25 g. b What percentage is 51 s of 2 min? c A tin of fishcake mix contains 204 g of Australian salmon, 156 g of potato, 24 g of seasoning and 16 g of cornflour. Find the percentage by weight of salmon.	4:06
10	a Determine the GST that needs to be added to a base price of \$73.70. b Determine the retail price after 10% GST is added to a base price of \$53.90. c How much GST is contained in a retail price of \$32.45? d What was the base price before 10% GST was added to give a retail price of \$21.45?	4:07
11	a A painting is bought for \$8000 and sold for \$10 000. What is the profit as a percentage of: i the cost price ii the selling price. b A car is bought for \$18 000 and sold 3 years later for \$13 500. What is the percentage loss on: i the cost price ii the selling price.	4:08
12	a A crowd included 5400 males. This was 60% of the crowd. What was the size of the total crowd? b 18% of a bag of lollies were red. If there were 27 red lollies, how many lollies in total were in the bag?	4:09

ASSIGNMENT 4A Chapter review

1 Write each percentage as a decimal numeral and a simple fraction.

a 25% **b** 60% **c** 15%
d 7% **e** 5·5% **f** $6\frac{1}{4}\%$

2 Write each number as a percentage.

a 0·37 **b** 0·08 **c** 0·4
d 1·35 **e** 0·125 **f** 0·025
g $\frac{17}{100}$ **h** $\frac{7}{10}$ **i** $\frac{4}{5}$
j $\frac{7}{20}$ **k** $\frac{2}{3}$ **l** $\frac{5}{8}$

3 Calculate the following.

a 20% of \$50 **b** 75% of \$240
c 12% of \$360 **d** 9% of \$70
e 140% of \$80 **f** 12.5% of \$24
g 6.25% of \$120 **h** $33\frac{1}{3}\%$ of \$42

4 Find what percentage:

a \$4 is of \$16
b \$2.50 is of \$12.50
c 50 kg is of 1 t
d 400 mL is of 2 L
e 200 g is of 1.6 kg
f \$32 is of \$25



5 The Hollier family decided to donate 5% of their earnings for December to famine relief. How much will the family donate if the parents together earned \$9100 and the four children together earned \$1250.

6 Jeremy added 10% GST onto the price of a book valued at \$29.90 to get its retail price. He then discounted the retail price by 10% to get a sales price.

a What is the retail price?
b What is the sales price?
c Is the sales price the same as the original value of the book?
d By what percentage should Jeremy have discounted the retail price to get back to the original value of the book?

7 **a** Vicki sold azaleas in her nursery for \$15.90. She bought them for \$11.35. What percentage profit does she make?
b Abdel sold a bike he bought for \$350 to a friend two years later for \$230. What percentage loss is this?

ASSIGNMENT 4B Working mathematically

1 Al, Ben and Cody have a race. In how many different orders could they finish the race, assuming that no dead heats are possible?



2 Dan, Ed, Fee and Gia have a race. In how many different orders could they finish the race, assuming that no dead heats are possible?

3 Lily had one pair of cats that had six kittens, three of which were female. In the following year, each young female had five kittens and the old female had two more litters of six kittens. How many cats in total did Lily have at the end of the year, assuming that she kept them all?

4 It took Lachlan 7 minutes to join two pieces of pipe to make one length of pipe. How long would it take him to join 10 pieces of pipe to make one length of pipe?

5 Some students were randomly selected for a survey from a class of 30. If the second student was chosen and every fourth after that, how many students were selected in total?

6 A seven-digit number is formed from the digits 1, 2, 3, 4, 5, 6 and 7. What are the third-largest and third-smallest numbers that can be formed?

ASSIGNMENT 4C Cumulative revision

1 Perform the following conversions. 1:11

a 7·5 L to mL **b** 400 L to mL **c** 14 000 mL to L **d** 2100 L to kL

2 Simplify: 2:01

a $4 - 10$ **b** $-2 + 3$ **c** $-4 - 6$ **d** $-4 \div -2$ **e** 10×-2

3 Draw a number plane with scales from −5 to 5 on both axes. Plot the points (0, 5); (2, −3); (−2, −3); (−4, 1); (1, 4). 1:13

4 Write the following, showing all multiplication and division signs. 1:14

a $3a$ **b** $4m + 2n$ **c** $5(x + y)$ **d** $\frac{a}{5}$ **e** $\frac{3m}{2}$

5 If $a = 5$ and $b = 3$, evaluate: 1:14

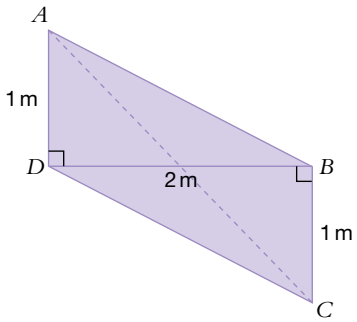
a $4a$ **b** $5b + 3$ **c** $\frac{(a+b)}{2}$ **d** $a^2 + b^2$

6 If $21 \times 72 = 1512$, write down the answers to: 1:06

a $2{\cdot}1 \times 7{\cdot}2$ **b** $21 \times 7{\cdot}2$ **c** $0{\cdot}21 \times 7{\cdot}2$

7 **a** A right-angled triangle has one side that is 6 cm long and one that is 8 cm long. What could the length of the third side be? 3:04

b A parallelogram is formed by joining two identical right-angled triangles as shown below. What is the length of the diagonal AC? Give your answer correct to two decimal places.



8 Write each of the following in index notation. 2:04

a $a \times a \times a \times a \times a$ **b** $10 \times 10 \times 10$

c $a \times a \times a \times b \times b$ **d** $4 \times p \times p \times 3 \times p$

9 Expand each of the following. 1:14

a $5(a + 4)$ **b** $2(2a + 3)$ **c** $4(m - 5)$ **d** $a(2b - 3)$

10 **a** Write down the factors of 36. 1:04

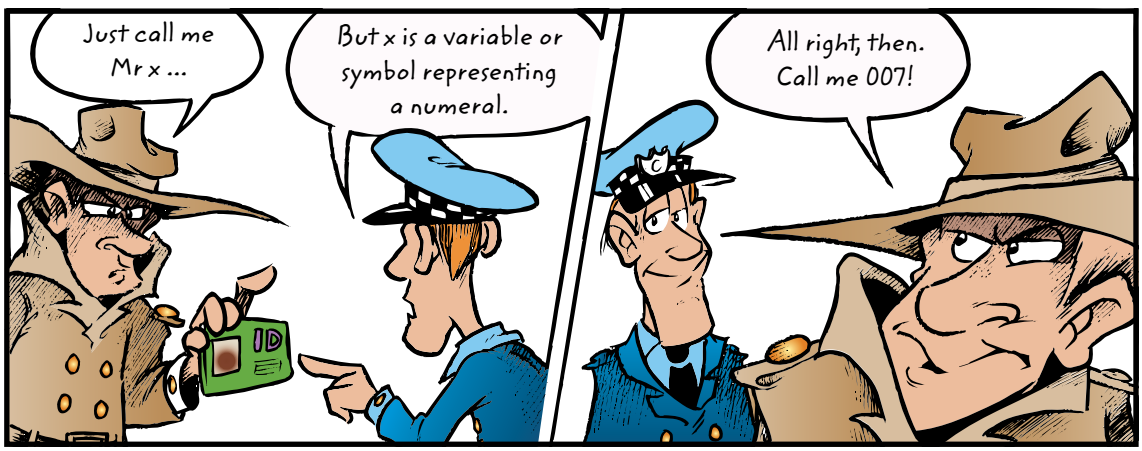
b Write down the factors of 54.

c What is the highest common factor of 36 and 54?

d Write down the first four multiples of 36 and 54, and then write down the lowest common multiple of 36 and 54.

ALGEBRA TECHNIQUES

5



Contents

5:01 Patterns and pronumerals
Investigation 5:01 Making sense of algebra
5:02 Addition and subtraction of like terms
5:03 Multiplication of pronumerals
Fun spot 5:03 Why do people laugh up their sleeves?
5:04 Division of pronumerals
5:05 Multiplication and division of pronumerals
Fun spot 5:05 Why didn't the boy drink his milk after his bath?
5:06 Using algebra
5:07 Index notation
5:08 Grouping symbols
Challenge 5:08 Expanding binomial products
5:09 Factorising
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

Selections from ***Algebraic Techniques 1***, ***Algebraic Techniques 2*** [Stage 4]

- Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)
- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions. (ACMNA177)
- Extend and apply the distributive law to the expansion of algebraic expressions. (ACMNA190)
- Factorise algebraic expressions by identifying numerical factors. (ACMNA191)
- Simplify algebraic expressions involving the four operations. (ACMNA192)
- Factorise algebraic expressions by identifying algebraic factors.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

5:01 Patterns and pronumerals

PREP QUIZ 5:01

If $a = 6$, find the value of:

1 $a + a$ **2** $2 \times a$ **3** $2a$ **4** $3a - a$

5 $7 - 11$ **6** $9 - (-3)$ **7** $(-4) \times (-3)$ **8** $12 \div (-2)$

Use the rule given to complete the table.

9 $M = 2n + 1$

n	0	1	2	3
M				

10 $y = 10 - x$

x	4	5	6	7
y				

Discussion



Examine the number of sides used for each group of triangles.

1 Complete a table of values that shows how *the number of sides* used is related to *the number of triangles*.

Number of triangles (x)	1	2	3	4	5
Number of sides (y)	3	5			

2 Describe the pattern in words and symbols. The number of sides increases by 2 each time, so the number of sides is 2 times the number of triangles plus 1 black side.
$y = 2x + 1$

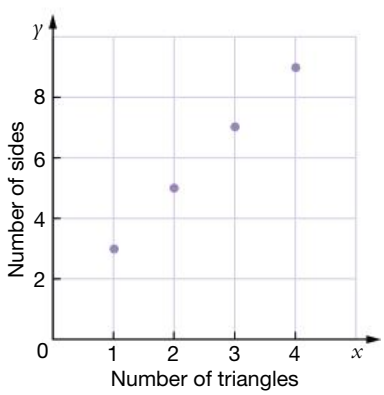
3 We can simplify this table as:
$y = 2x + 1$

x	1	2	3	4
y	3	5	7	9

4 How would you describe, in words, the pattern of points we have drawn?

5 What other points will belong to this pattern? Does this graph represent an increasing or decreasing number pattern?

- Negative numbers can also be substituted for pronumerals.
- Use parentheses when substituting directed numbers into algebraic expressions.



e.g. When $x = -7$

$x^2 = (-7)^2$

$= -7 \times -7$

$= 49$

$-7^2 \neq (-7)^2$

$-7^2 = -(7 \times 7)$

$= -49$

Only the 7 is squared in -7^2.

WORKED EXAMPLE 1

If $x = -2$, $y = -3$ and $z = 5$, find the value of:

a $x + y$ **b** xy **c** $z - x$ **d** $\dfrac{y+z}{x}$

Solutions

a $x + y = -2 + (-3)$
$= -5$

b $xy = -2 \times -3$
$= 6$

c $z - x = 5 - (-2)$
$= 7$

d $\dfrac{y+z}{x} = \dfrac{-3+5}{-2}$
$= -1$

WORKED EXAMPLE 2

Complete each table, using the rule given, and graph the result on a number plane.

a $y = 2x$

x	-2	-1	0	1
y				

b $b = 5 + a$

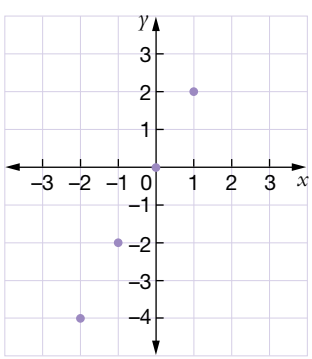
a	-4	-2	0	2
b				

Solutions

a When $x = -2$, $y = 2 \times -2$
$= -4$
When $x = -1$, $y = 2 \times -1$
$= -2$
When $x = 0$, $y = 2 \times 0$
$= 0$
When $x = 1$, $y = 2 \times 1$
$= 2$

x	-2	-1	0	1
y	-4	-2	0	2

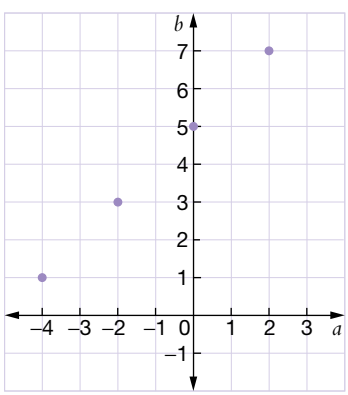
The points to be plotted are:
$(-2, 4)$, $(-1, -2)$, $(0, 0)$ and $(1, 2)$.



b When $a = -4$, $b = 5 + -4$
$= 1$
When $a = -2$, $b = 5 + -2$
$= 3$
When $a = 0$, $b = 5 + 0$
$= 5$
When $a = 2$, $b = 5 + 2$
$= 7$

a	-4	-2	0	2
b	1	3	5	7

The points to be plotted are:
$(-4, 1)$, $(-2, 3)$, $(0, 5)$ and $(2, 7)$.



WORKED EXAMPLE 3

Find a rule to describe the pattern shown in this table.

x	2	3	4	6	12
y	6	4	3	2	1

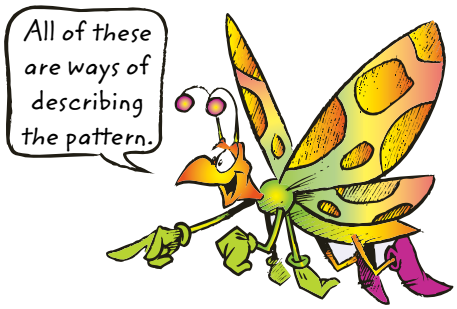
Solution

As x increases, y decreases. In this case, the two variables always have a product of 12.

Here $x \times y = 12$ or $xy = 12$

or $y = 12 \div x$ or $y = \frac{12}{x}$

or $x = 12 \div y$ or $x = \frac{12}{y}$



Exercise 5:01

P Foundation worksheet 5:01 Directed numbers

Challenge worksheet 5:01 Finite differences

1 Find the value of each expression if $n = -3$.

a $2n$ **b** $7n$ **c** $5 + n$ **d** $n - 1$

e $10 - n$ **f** $-2n$ **g** $n + n$ **h** $n \times n$

i $2n + 1$ **j** $10 + 3n$ **k** $5n + 15$ **l** $\frac{6}{n}$

2 If $x = -3$, $y = -4$, $z = 2$, evaluate:

a $x + y$ **b** $y + z$ **c** $z + x$ **d** $x + y + z$

e $x - y$ **f** $y - z$ **g** $x - z$ **h** $x - y - z$

i xy **j** yz **k** $2xz$ **l** $x^2 + z^2$

m $y \div z$ **n** $\frac{z - y}{x}$ **o** $3x + 2z$ **p** xyz

3 Evaluate each expression using the values given.

a	$3a$	if a is:	**i** -2	**ii** -4	**iii** 5	**iv** 0
b	$2a - 1$	if a is:	**i** 0	**ii** -1	**iii** -3	**iv** -10
c	$10 - 2a$	if a is:	**i** 3	**ii** 6	**iii** -1	**iv** -3
d	$2(3a - 4)$	if a is:	**i** 1	**ii** 0	**iii** -2	**iv** -10

4 Use the rule given to complete the table.

a $F = P - 1$

P	1	2	3	4
F				

b $F = 2P - 2$

P	1	2	3	4
F				

c $F = 4P - 4$

P	1	2	3	4
F				

d $y = x + 3$

x	1	2	3	4
y				

e $y = 3x$

x	1	2	3	4
y				

f $y = 2x + 1$

x	1	2	3	4
y				

5 Complete each table of values and then graph each on a separate number plane.

a $y = 1 + x$

x	−2	−1	0	1
y				

b $b = 2a$

a	−2	−1	0	1
b				

c $q = 3 - p$

p	2	3	4	5
q				

d $t = s - 5$

s	1	2	3	4
t				

e $y = 2x + 1$

x	-2	-1	0	1
y				

f $n = 3 - 2m$

m	0	1	2	3
n				

6 Complete each rule to describe the pattern in the table.

a $y = \square - x$

x	4	5	6	7
y	3	2	1	0

b $m = \square \div d$

d	1	2	3	4
m	24	12	8	6

c $p = \square w + 7$

w	0	1	2	3
p	7	17	27	37

d $y = \square x - 5$

x	0	1	2	3
y	−5	−2	1	4

e $b = a^2 + \square$

a	−4	−3	−2	−1
b	23	16	11	8

f $p = 50 - \square n$

n	0	1	2	3
p	50	42	34	26

7 Find the rule to describe each pattern.

a

x	0	1	2	3
y	0	7	14	21

b

d	8	9	10	11
b	3	4	5	6

c

v	4	5	6	7
M	9	11	13	15

d

a	0	1	2	3
M	0	1	4	9

e

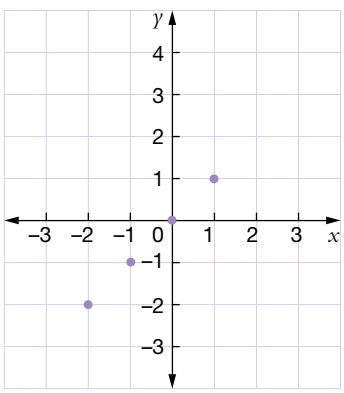
d	2	3	4	5
e	11	10	9	8

f

x	1	2	3	4
y	12	6	4	3

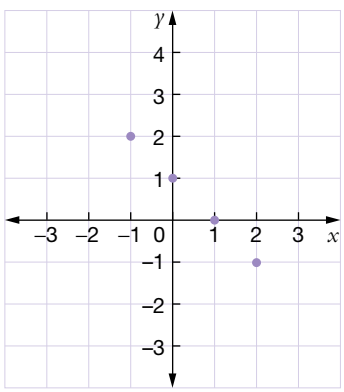
8 Complete the table of values with each graph and match it with the correct pattern rule.

A $y = 2x - 1$



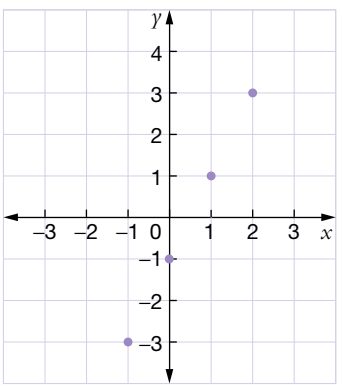
x	−2	−1	0	1
y				

B $y = x$



x	−1	0	1	2
y				

C $x + y = 1$



x	−1	0	1	2
y				

9 Milla and Kylie noticed that a large number of colourful flags were flown from wire joining poles along the main street during a celebration. They drew up the following table showing the number of poles (p) and the number of flags (f) as they walked along the street.

Poles (p)	1	2	3	4	5
Flags (f)	0	3	6	9	12



a Find a rule connecting the number of flags (f) and the number of poles (p).

b If the city of Bathurst had 96 flags, how many poles would they need?

c If the poles were 10 m apart, how far would the colourful display extend?

10 Sandy paid $70 to enter a tennis competition. She had to play eight other players and received $25 prize money for each player she beat. She worked out that her total profit (P) could be written in terms of the number of sets won (S) as $P = 25S - 70$.

a Use the rule to complete this table.

S	3	4	5	6	7	8
P						

b What happens to P if no sets are won?

c How many sets must Sandy win to be sure of winning money rather than losing money overall?

11 Sofia plays a quiz game in which a right answer scores 10 points but a wrong answer scores −5 points. Therefore, if r = number of right answers, and w = number of wrong answers, her score S would be given by the rule:

$S = 10r - 5w$



a What would Sofia's score be if she had:

i 3 right and 1 wrong

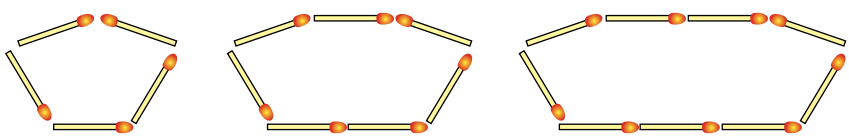
ii 4 right and 8 wrong

iii 2 right and 5 wrong

iv 5 right and 14 wrong?

b If Sofia answered 20 questions and her score was −10, how many questions did she answer correctly?

12 The following pattern has been made with matchsticks.



Base length (L)	1	2	3
Perimeter (P)	5	7	9

The rule for the pattern above is $P = 2L + 3$.

a Explain why the rule works.

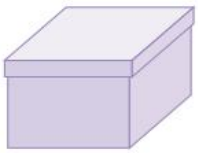
b Find the perimeter when the base length is:

i 4 ii 5 iii 10 iv 100 v 68

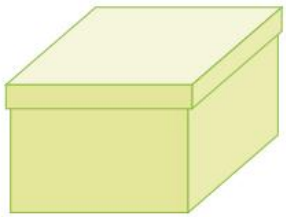
c Can the base length ever be zero? Explain your answer.

INVESTIGATION 5:01 MAKING SENSE OF ALGEBRA

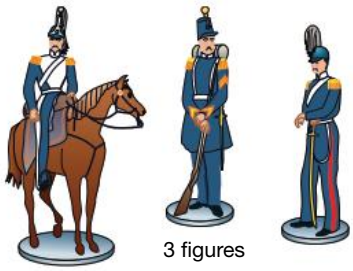
From Tasmania I was sent small and large boxes of toy figures. If each small box contains x figures and each large box contains y figures, we could write an algebraic expression for the number of figures present in a group.



x figures



y figures

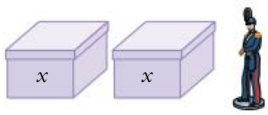


3 figures

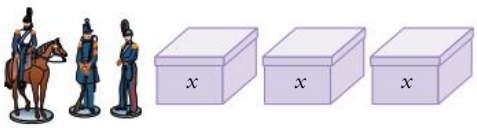
We do not know how many figures are in each box. That is why we can say that they contain x and y figures respectively. In the picture above we have $x + y + 3$ figures.

1 Use an algebraic expression to write the number of toy figures present.

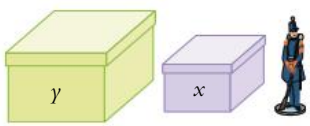
a



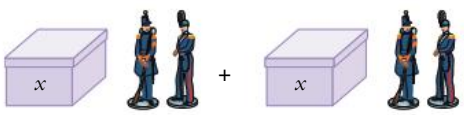
b



c



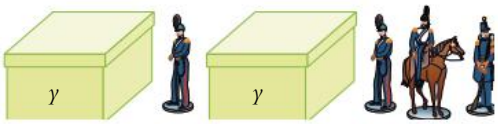
d



e



f



g



2 Find the number of toy figures present for each part of Question **1**:

A if the purple boxes contain 10 figures and the green boxes contain 16

B if the purple boxes contain 7 figures and the green boxes contain 11.

3 Draw some pictures of your own, like those above, to show:

a $x + 2$ toy figures **b** $3x + 1$ toy figures **c** $x + y$ toy figures.

5:02 Addition and subtraction of like terms

PREP QUIZ 5:02

Simplify each expression:

1 $5x + 2x$	**2** $10a - 7a$	**3** $3p + 7p + 2p$	**4** $9m + m$
5 $6m - m$	**6** $3a^2 + 5a^2$	**7** $2mn + 6mn$	**8** $9ab + 2ba$
9 $19x - 10x$	**10** $6p - 5p$		

In Year 7 you were introduced to algebra. This Prep quiz should have reminded you of the types of expressions you were asked to simplify.

Collecting like terms

WORKED EXAMPLES

1 $5x \mid -2y \mid -3x \mid +7y$

$= 5x \mid -3x \mid -2y \mid +7y$

$= 2x + 5y$

2 $4a + 6 - 5 - a$

$= 4a - a + 6 - 5$

$= 3a + 1$

3 $7ab - 3bc + 2ab$

$= 7ab + 2ab - 3bc$

$= 9ab - 3bc$

4 $8x^2 + 2x - 6x^2 - 5x$

$= 8x^2 - 6x^2 + 2x - 5x$

$= 2x^2 - 3x$

5 $5a - 7 + a + 5b$

$= 6a - 7 + 5b$

6 $6x + 7y - 9x + 2y$

$= -3x + 9y$ OR $9y - 3x$

Remember!
When adding or subtracting terms in algebra, the sign belongs to the term that follows.



Only *like terms* can be added or subtracted together.

Exercise 5:02A (Easier types)

P Foundation worksheet 5:02 Addition and subtraction

1 Simplify each expression.

a $5x + 3x$	**b** $2y + 7y$	**c** $6a + a$	**d** $b + 7b$
e $7m - 2m$	**f** $9n - 4n$	**g** $10a - a$	**h** $x - x$
i $4l^2 + 9l^2$	**j** $21x^2 + 9x^2$	**k** $5xy + 7xy$	**l** $8pq + 6pq$

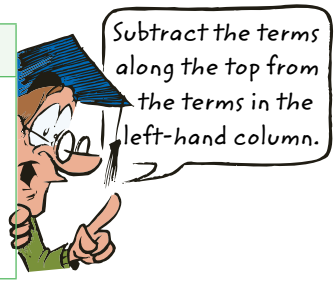
2 Complete these two tables.

a

+	a	$2a$	$5a$	$6a$	$10a$
$3a$					
$5a$				$11a$	
a					
$6a$					
$10a$					

b

−	$3a$	a	$2a$	$5a$	$4a$
$5a$					
$7a$			$5a$		
$12a$					
$8a$					
$10a$					



3 Simplify each expression.

a $5a + 2a + 3a$ **b** $10y - 3y + 2y$ **c** $5m^2 + 2m^2 + 4m^2$

d $6x^2 - 3x^2 + 2x^2$ **e** $5pq + 2pq - 3pq$ **f** $7p - 4p - 2p$

g $7a + 3a + 2a + 6a$ **h** $6k + 3k - 2k + k$ **i** $7q^2 - 2q^2 - 3q^2 + q^2$

4 Simplify each expression, and then substitute $a = 3$ to find its value.

a $5a + 2a$ **b** $3a + 2a$ **c** $5a + a$

d $9a - 7a$ **e** $6a - 5a$ **f** $16a - 11a$

g $2a^2 + 3a^2$ **h** $a^2 + 7a^2$ **i** $5a^2 - 3a^2$

$8a$ means $8 \times a$, so if $a = 3$, then $8a = 8 \times 3$.

5 Simplify these expressions by collecting like terms.

a $2x + 3y + 4x$ **b** $6m + 2n + 3m$ **c** $a + 8a + a$

d $5x + 4y - 2x$ **e** $7p - p + 4m$ **f** $5t - 4t + 3w$

g $3a + 2b + 4a + 2b$ **h** $8t + 4w - 6t + 2w$ **i** $9p + 3q - 7p + 2q$

6 Simplify each expression, and then let $x = 3$ and $y = 5$ to find its value.

a $2x + y + 3x$ **b** $3y + 4y + 10$ **c** $5x + 4y - 35$

d $3x + 2y + 2x + y$ **e** $5x - 2x + 3y + y$ **f** $4x + 3y - 2x - y$

Exercise 5:02B (Harder types)

1 Simplify these expressions, which may involve negative numbers.

a $10a - 3a$ **b** $3a - 10a$ **c** $6a - 7a$

d $-6x + 8x$ **e** $-3p + 2p$ **f** $-4y + 9y$

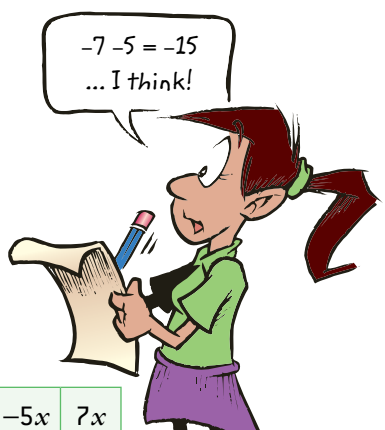
g $6m - 9m$ **h** $a - 5a$ **i** $2y - 12y$

j $-3x - 2x$ **k** $-2y - 5y$ **l** $-4p - 6p$

m $8x - (-3x)$ **n** $6y - (-2y)$ **o** $5t - (-2t)$

p $-3ab + 5ab$ **q** $9xy - 15xy$ **r** $-5x^2 - 7x^2$

s $-3x - (-2x)$ **t** $-7x - (-10x)$ **u** $-4p - (-4p)$



2 Complete these tables.

a

+	$-4x$	$-x$	$3x$	$-5x$	$7x$
$3x$					
$5x$			$8x$		
x					
$-2x$					
$-7x$					

b

−	$-4x$	$-x$	$3x$	$-5x$	$7x$
$3x$					
$5x$			$2x$		
x					
$-2x$					
$-7x$					

In Question **2b** subtract the terms at the top from the terms in the left-hand column.

3 Simplify these expressions by collecting the like terms.

a $5x + 2y - 4x$ b $9m + 4n - 5n$ c $8p + 3q - 9p$
d $5x - 7 - 3x$ e $10 - 4p + 2p$ f $9q - 3p + 7p$
g $9a - 3b - 9a$ h $-6m + 2n + 4m$ i $-3n + 2q + 7n$
j $-3x + 2x - 5y$ k $-5t - 2t + 3w$ l $m - 7m - 2n$
m $-5a - 2b - a$ n $6xy - 3x + 2xy$ o $5ab - 7ab + 6$
p $7x^2 + 9x - 8x^2$ q $-8p + 7pq - 2p$ r $5 - 2x^2 + x^2$

4 Simplify the following expressions.

a $2a + 4b + 6a + 2b$ b $7x + 3y - 3x + 4y$ c $9m - 2n + 7n + 2m$
d $9p - 7 + 6p - 5$ e $8a + 10 - 5a + 2$ f $12 - 6q - 5q + 3$
g $9k + 6l - 7k - 4l$ h $3m - 5m + 9l - 7l$ i $x + 7y - 5y + 2x$
j $3ab + 4a + 2ab + 3a$ k $6xy + 2 - 4xy - 5$ l $3x^2 + 2x + 7x^2 - x$
m $5a + 7 - 2a + 3a$ n $10 + 13a - 7a + 2a$ o $9y + 7y + 4x - 10y$
p $3a + 2b + a - 4b$ q $9l - 2q - 6l + 4q$ r $10t + 2w - 11t - 6w$
s $5ab - 4bc + 2ab + 3bc$ t $6mn - 3m + 4mn - 6m$ u $6 - 4kl - 9 - 3kl$
v $7x^2 + 6x - 6x^2 - 9x$ w $9q - 6q^2 + 5q^2 - 7q$ x $7 + 4h - 10h - 16$

5 Simplify the following expressions.

a $5xy - 2yx$ b $4ab - 10ba + a$
c $-2mn + 8nm + 6$ d $10 - ba - 5ab$
e $5m^2 + 7m + 3 - 2m^2 - 5m$ f $6a^2 + 9a - 1 + 5a^2 - 10a$
g $2xy - 5yz + 3yx - 7yz$ h $2a^2 + 3ab + a^2 - 2ba$
i $6 + 2mn - 5 + \mathrm{m} + 3nm$ j $9k^2 - 8kt + 7t^2 + 8tk$
k $10ab - 9ba + 3a - 4b + 6a$ l $9m^2 + 6n^2 - \mathrm{m}^2 - 10n - n$

$a \times b = b \times a$
so $ab = ba$

5:03 Multiplication of pronumerals

PREP QUIZ 5:03

Simplify these products by writing them without the multiplication signs.

1 $5 \times a$ **2** $p \times q$ **3** $3 \times a \times b$
4 $5 \times x \times 3 \times y$ **5** $2 \times x \times x$

Find the value of each expression if $a = 3$ and $b = 4$.

6 ab **7** $3a$ **8** $2a + 4b$ **9** b^2

10 Insert $\times$ signs to show the meaning of the expression mv^2.

The Prep quiz should have reminded you that a term like mn means $m \times n$. When we multiply, we multiply the numbers together and then the pronumerals without any multiplication signs.

e.g. $3a \times 6b = (3 \times a) \times (6 \times a)$
$= 3 \times 6 \times a \times b$
$= 18ab$



WORKED EXAMPLES

1 **a** $6 \times 4a = 24a$ **b** $3m \times 2n = 6mn$
c $3a \times bc = 3abc$ **d** $4 \times 2x \times 3x = 24x^2$

2 **a** $-3 \times 2y = -6y$ **b** $-5p \times -4q = 20pq$
c $3a \times -3a = -9a^2$ **d** $-2p \times 3q \times -4p = 24p^2q$

Multiply the numbers first, then the pronumerals.

Exercise 5:03

P Foundation worksheet 5:03 Multiplication of pronumerals

1 Simplify each product.

a $5 \times 2a$	**b** $7 \times 3x$	**c** $8 \times 2m$	**d** $10 \times 6y$
e $8p \times 7$	**f** $6n \times 5$	**g** $3q \times 7$	**h** $4 \times 11k$
i $3b \times 5$	**j** $6 \times 2a$	**k** $3 \times 6y$	**l** $12a \times 4$
m $\frac{1}{2} \times 6a$	**n** $12b \times \frac{1}{4}$	**o** $\frac{1}{3} \times 15m$	**p** $100y \times \frac{1}{100}$

2 Simplify:

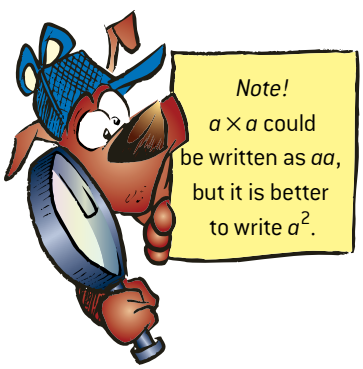
a $3m \times 2n$	**b** $6x \times 2y$	**c** $5t \times 5w$	**d** $7a \times 3b$
e $7a \times b$	**f** $p \times 5q$	**g** $3k \times 4l$	**h** $3b \times a$
i $4m \times 10n$	**j** $6x \times 3x$	**k** $9a \times 4y$	**l** $5x \times x$
m $3y \times 5y$	**n** $6m \times 7m$	**o** $k \times 7k$	**p** $\frac{1}{2}y \times 4y$

3 Simplify these products.

a $5ab \times 4n$	**b** $3 \times 2xy$	**c** $6 \times 3mn$
d $7 \times 2a \times 3$	**e** $3 \times 4b \times 2$	**f** $2 \times 2a \times b$
g $a \times b \times 5$	**h** $3 \times b \times 2c$	**i** $m \times 5 \times n$
j $3a \times 2b \times 4$	**k** $a \times 5b \times 6c$	**l** $x \times 4y \times 3$

4 Simplify:

a $a \times ab$	**b** $mn \times m$	**c** $3x \times ax$
d $2ab \times 3a$	**e** $4pq \times 3q$	**f** $5k \times 3kl$
g $3a \times 4bc$	**h** $9ab \times 10bc$	**i** $4xy \times 5xz$
j $3a \times 4b \times 2c$	**k** $mn \times np \times mp$	**l** $2ab \times b \times 3a$



5 Complete these multiplication tables.

a

×	6	a	$2b$	$9m$	$3ab$
$2a$					
$3b$					
7					
a					
ab					

b

×	x	$2b$	xy	$5x$	$2xz$
$4x$					
y					
$5y$					
$2xy$					
yz					

6 Simplify these products.

a $(-3x) \times 5$ **b** $7 \times (-2m)$ **c** $(-4) \times (-3a)$

d $9m \times (-3)$ **e** $(-5) \times (-6k)$ **f** $(-7a) \times (-2)$

g $(-a) \times 2b$ **h** $(-p) \times (-q)$ **i** $5n \times (-2m)$

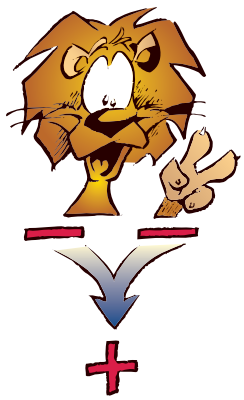
j $(-6t) \times (-w)$ **k** $(-8x) \times 7y$ **l** $10m \times (-3m)$

m $(-6y) \times (-6y)$ **n** $3a \times (-4a)$ **o** $(-3x) \times (-x)$

p $6 \times 2a \times (-5)$ **q** $5 \times (-3y) \times (-2)$ **r** $6a \times (-4) \times a$

s $9m \times 2n \times (-3)$ **t** $(-8k) \times 4 \times k$ **u** $(-3p) \times 4 \times (-2p)$

v $5xy \times (-2x)$ **w** $(-4a) \times (-6ab)$ **x** $(-5mn) \times mp$



7 Complete these tables.

a

×	-3	5	$2x$	$-4x$	$-y$
7					
-2					
$-x$					
$4x$					
$-3y$					

b

×	$2a$	$3b$	$-a$	$-4b$	$-ab$
$6a$					
$-3b$					
$-b$					
$3a$					
$2ab$					

8 Simplify these miscellaneous products.

	A	B	C	D
a	$5x \times 3y$	$-2 \times 4m$	$6x \times (-3x)$	$2y \times y$
b	$6 \times 5ab$	$a \times b \times c$	$-3a \times (-2b)$	$mn \times m$
c	$2a \times 4b$	$-4p \times 3q$	$3xy \times 2y$	$5 \times 2a \times 3a$
d	$-4m \times (-4m)$	$\frac{1}{2} \times 10ab$	$5mn \times 6mn$	$ab \times 4b$
e	$2x^2 \times 3y$	$3t \times 5w$	$a \times (-3a)$	$3mn \times 2n \times 4$
f	$(-6k) \times 3k$	$(-2l) \times (-3l)$	$2a \times 5b \times 10c$	$7xy \times 5$
g	$7ab \times 2a \times 0$	$x \times (-x)$	$-4h \times 2k$	$9a \times (-3a) \times 2$
h	$(-3x) \times (-3) \times 2x$	$-a \times 2a \times 4b$	$7mn \times 4mp$	$(-2x) \times (-3) \times (-4y)$

FUN SPOT 5:03 — WHY DO PEOPLE LAUGH UP THEIR SLEEVES?

Simplify each part and write the letter for that part in the box above the correct answer.

A	$6a + a$	**A**	$9x - x$	**E**	$9x - 5y - 2x$				
B	$15m - m$	**B**	$14x \div 2$	**E**	$9p + 3q - 7p - 2q$				
C	$8 \times 3k$	**E**	$3k - 2t - t$	**I**	$16m - m - m - m$				
E	$10 - x - x$	**N**	$6a \times a$	**K**	$8m + 3b - 2m + b$				
E	$7a \times 0$	**Y**	$6a - 1$	**N**	$6a - 7a$				
E	$4x + 5 + 2x$	**S**	$7 + x - 7$	**O**	$10x - 8x$				
F	$3a \times 2b$	**T**	$\frac{1}{3} \times 9a$	**P**	$b \times 7b$				
H	$16a \div 2$	**T**	$4p + 6p + 7$	**S**	$10x + 8 - 10x$	**R**	$7a - 6a$		
T	$a \times ab$	**T**	$3x - (-2x)$	**N**	$7x - 3y + 2x - 4y$	**S**	$6ab \times 4$		
E	$-5a + 2a$	**E**	$-4x - x$	**W**	$3x^2 \times 1$	**E**	$8a \times \frac{1}{2}$		
H	$(2a)^2$	**H**	$-4 \times k$	**Y**	$3a^2 + 5a^2$	**H**	$xy + yx$		
U	$101x - x$	**U**	$x - x + 1$	**R**	$6ab + b - 2ba + b$				



$14m$	0	$24k$	$7a$	$100x$	x	$6x + 5$	$3a$	$8a$	$8x$	a^2b	$24ab$	$3x^2$	$2xy$	$4a$	a	$-3a$	$5x$	$-4k$	$-5x$	$8a^2$	

$6m + 4b$	$2p + q$	$3k - 3t$	$7b^2$	$10p + 7$	$4a^2$	$7x - 5y$	$13m$	$4ab + 2b$	$6ab$	1	$-a$	$9x - 7y$	$6a - 1$	$7x$	$2x$	$6a^2$	$10 - 2x$	8	

5:04 Division of pronumerals

PREP QUIZ 5:04

Simplify these divisions.

1 $12x \div 6$	**2** $20y \div 5$	**3** $10a \div 2$	**4** $100m \div 5$
5 $\frac{12x}{6}$	**6** $\frac{12y}{5}$	**7** $\frac{10a}{2}$	**8** $\frac{100m}{5}$

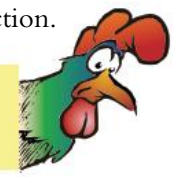
If $m = 20$ and $n = 5$, find the value of:

9 $m \div n$ **10** $\frac{m}{n}$

The Prep quiz should have reminded you that a division could be written as a fraction.

$$a \div b = \frac{a}{b}$$

$$x \div x = \frac{x}{x} = 1$$



If a division is written as one term 'over' the other, we simply divide the numbers and then cancel the pronumerals that are common in the 'top' and 'bottom'. (When we cancel, we are really dividing top and bottom by that pronumeral.)

WORKED EXAMPLES

1 a $\dfrac{20x}{5} = \dfrac{{}^{4}\cancel{20}x}{\cancel{5}_{1}}$

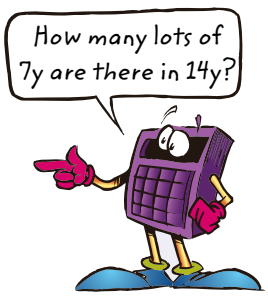
$= 4x$

b $\dfrac{15x}{3x} = \dfrac{{}^{5}\cancel{15}x^{1}}{{}_{1}\cancel{3}x_{1}}$

$= 5$

c $\dfrac{12ab}{6a^2} = \dfrac{{}^{2}\cancel{12}^{1}\cancel{ab}}{{}_{1}\cancel{6a}_{1}a}$

$= \dfrac{2b}{a}$



2 a $14y \div 7 = 2y$

Divide 14y into 7 parts.

b $14y \div 7y = 2$

3 a $9x^2 \div 3x$

$\dfrac{{}^{3}\cancel{9}\,\cancel{x}x}{{}_{1}\cancel{3}\,\cancel{x}} =$

$= 3x$

b $ab^2 \div ab$

$= \dfrac{{}^{1}\cancel{ab}^{1}b}{{}_{1}\cancel{ab}_{1}}$

$= b$

c $3xy \div 6y$

$= \dfrac{{}^{1}\cancel{3}x\,\cancel{y}^{1}}{{}_{2}6\,\cancel{y}_{1}}$

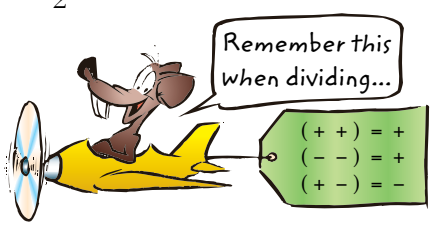
$= \dfrac{x}{2}$ (or $\dfrac{1}{2}x$ or $0{\cdot}5x$)

Divide the number first, then the pronumerals.

4 a $\dfrac{-6x}{2x} = -3$

b $\dfrac{-12ab}{-4b} = 3a$

c $18m \div (-6) = -3m$



Exercise 5:04

P Foundation worksheet 5:04 Division of pronumerals

1 Simplify each division.

a $\dfrac{6a}{3}$ **b** $\dfrac{10m}{5}$ **c** $\dfrac{16p}{4}$ **d** $\dfrac{12x}{x}$ **e** $\dfrac{18k}{6k}$ **f** $\dfrac{20a}{10a}$

g $\dfrac{9mn}{3m}$ **h** $\dfrac{6ab}{a}$ **i** $\dfrac{8pq}{2q}$ **j** $\dfrac{5x}{10}$ **k** $\dfrac{6a}{10}$ **l** $\dfrac{4m}{20}$

m $\dfrac{4n}{12}$ **n** $\dfrac{10m}{20m}$ **o** $\dfrac{6p}{12p}$ **p** $\dfrac{7x}{21x}$ **q** $\dfrac{9m}{12m}$ **r** $\dfrac{12mn}{6m}$

s $\dfrac{8ab}{4b}$ **t** $\dfrac{6x^2}{3x}$ **u** $\dfrac{15a^2}{5a}$

2 Simplify:

a $12m \div 6$	**b** $10x \div 2$	**c** $20p \div 4$	**d** $16a \div 8$
e $15n \div 5$	**f** $24p \div 12$	**g** $30l \div 10$	**h** $25k \div 5$
i $6m \div 3m$	**j** $20a \div 10a$	**k** $12m \div 2m$	**l** $18l \div 3l$
m $24p \div 8p$	**n** $35x \div 7x$	**o** $28k \div 7k$	**p** $14t \div 7t$
q $5ab \div a$	**r** $6mn \div 2m$	**s** $8pq \div pq$	**t** $6tw \div 3t$
u $10mn \div 10n$	**v** $8x^2 \div 8x$	**w** $12xy \div xy$	**x** $18a^2 \div 9a$

3 Simplify these divisions that involve negative terms.

a $\frac{-10a}{5}$	**b** $\frac{-6m}{2}$	**c** $\frac{-8q}{-4}$	**d** $\frac{-12q}{-4}$
e $\frac{20p}{-10p}$	**f** $\frac{-15p}{5p}$	**g** $\frac{-18n}{-9n}$	**h** $\frac{-24x}{-8x}$
i $\frac{-6xy}{3x}$	**j** $\frac{12mn}{-3n}$	**k** $\frac{-9kl}{-3l}$	**l** $\frac{-20mn}{-5}$
m $12x \div (-3)$	**n** $15m \div (-5)$	**o** $-20t \div 10$	**p** $-24n \div (-8)$
q $10m \div (-5m)$	**r** $-14x \div 2x$	**s** $-9w \div (-3w)$	**t** $-18p \div (-9p)$
u $6ab \div (-3b)$	**v** $-16mn \div 8$	**w** $-12ab \div (-3a)$	**x** $-ab \div (-a)$

4 Simplify these miscellaneous divisions.

	A	B	C	D
a	$25x \div 5x$	$10pq \div p$	$-3m \div 3$	$6p \div (-3)$
b	$60t \div 20$	$-5a \div 5a$	$12pq \div 4pq$	$-30m \div (-6m)$
c	$\frac{8ab}{4b}$	$\frac{3m}{2m}$	$\frac{7ab}{3a}$	$\frac{2x^2}{x}$
d	$\frac{12xy}{6xy}$	$-15a^2 \div 5a$	$6m \div (-6)$	$24pq \div 12q$

5:05 Multiplication and division of pronumerals

PREP QUIZ 5:05

Simplify each expression.

1 $6m \times 3$	**2** $6m \div 3$	**3** $8a \times 2a$	**4** $8a \div 2a$	**5** $10p \times 5q$
6 $2ab \times 4a$	**7** $12p \div p$	**8** $\frac{10a}{5a}$	**9** $\frac{6mn}{6n}$	**10** $\frac{9x^2}{3x}$

In this section you will practise multiplying and dividing algebraic expressions. First, examine the following examples.

WORKED EXAMPLES

1 $6m \times 3 \div 9m$
$= 18m \div 9m$
$= 2$

2 $16x^2 \div 8x \div 2$
$= 2x \div 2$
$= x$

3 $\dfrac{3a \times 2b}{6a}$
$= \dfrac{6ab}{6a}$
$= b$

Exercise 5:05

P Foundation worksheet 5:05
Multiplication and division of pronumerals

1 Simplify these expressions that involve three terms.

a $2 \times 3x \times 5y$
b $4a \times 5b \times 2c$
c $6x \times 2 \div 4x$
d $6 \times 5m \div 10$
e $20p \div 4 \times 3q$
f $10a \div 2a \times 3a$
g $15t \div 5t \div 3$
h $24pq \div 6p \div 2q$
i $24m \div 6m \times m$
j $6ab \times 2a \div 4b$
k $ab \times 7a \times 3b$
l $28tw \div 7w \times 2t$

2 Simplify each expression.

	A	B	C
a	$5m \times 2m$	$6p \div 3p$	$16p \div 8$
b	$12ab \div 6b$	$3pq \times 2p$	$6p \div 6p$
c	$-3p \times 2q$	$24m \div (-6)$	$3xy \times x$
d	$(3m)^2$	$(5a)^2$	$(4x)^2$
e	$(-2m) \times (-3n)$	$-24l \div (-6)$	$a \times 3a$
f	$6pq \times 2p$	$-5x \times 2xy$	$48mn \div 12m$
g	$(3pq)^2$	$(7mn)^2$	$(2tw)^2$
h	$3p \times 2 \times 4q$	$2a \times 3b \times 4c$	$9 \times 2p \times m$
i	$7ab \times 0 \times 5b$	$a \times 3x \times 2a$	$-3t \times 2t \times 5$
j	$-25xy \div 5x$	$9ab \div (-9ab)$	$-18x^2 \div (-9x)$
k	$12a \div 6 \times 3a$	$9p \div 3p \times 2p$	$24x^2 \div 6x \times 2$
l	$\dfrac{6a \times 3b}{9ab}$	$\dfrac{5x \times 4x}{10x}$	$\dfrac{4p \times 5p}{10p}$
m	$\dfrac{2m \times 6n}{3n \times 4m}$	$\dfrac{10x \times 3x}{6x \times 5}$	$\dfrac{(4ab)^2}{8ab^2}$

$(5xy)^2$
$= 5xy \times 5xy$
$= 25x^2y^2$



FUN SPOT 5:05 — WHY DIDN'T THE BOY DRINK HIS MILK AFTER HIS BATH?

Simplify each part and write the letter for that part in the box above the correct answer.

F $10x + 7x$	**H** $6m - 5m$	**E** $3a + 2a + 6a$
B $2p \times 9$	**M** $20b \div b$	**R** $2a \times a$
T $6m - 10m$	**I** $(-3) \times 6q$	**E** $(-40n) \div (-10)$
H $12k - 6l + 2k$	**O** $8t \times 7t$	**T** $36pq \div 9q$
N $15y \div (-3y)$	**A** $5x^2 + 2x - 3x^2$	**D** $7m - (-3m)$
H $-12k + 10k$	**A** $9a - 7a - a$	**T** $9a \times 7b$
E $(-18ab) \div 9b$	**F** $ab \times ac$	**S** $2x \times \frac{x}{2}$
H $3x - 7x$	**I** $-3m + 10m$	**K** $9p - 2q - 7p$
A $16mn \div 8$	**M** $(-6a) \times (-3b)$	**O** $7m - 3m + 2n$
H $8x \times 3xy$	**R** $14m^2 \div (-7m)$	**G** $5ab + 7ba$
D $8pq + 7pq - 3p$	**N** $4m \times 2n \times 5$	**R** $16x \div 4x \times 2x$
O $13k \times 2k \div k^2$	**R** $3m^2 + m^2 - 6m^2$	**T** $3x + 7y - 2y + x$
I $-m + n + m - n$	**F** $3a \times 4b \times 5c$	**I** $100bc \div 20b$
K $(-2x) \times 5x \times (-3)$	**L** $24ab \div 6a \times 2b$	**O** $6y^2 + 7y - 8y^2 + 2y$
E $(-6m) \times (-6m)$	**L** $5ab + (6a \times 2b)$	**N** $(32x^2 \div 4x) - 7x$



a	a^2bc	$-4m$	$-2a$	$-2m^2$	$10m$	$-2m$	$-18q$	x	$30x^2$	$7m$	-5	$12ab$	$-4x$	0	x^2	$18p$	$2mn$	$4p$	m	$-2k$	$11a$

$14k - 6l$	$2x^2 + 2x$	$15pq - 3p$	$40mn$	$-2y^2 + 9y$	$2a^2$	$56t^2$	$4m + 2n$	$18ab$	$8b^2$	$36m^2$	$60abc$	$4x + 5y$	$17x$	26	$8x$	$63ab$	$24x^2y$	$4n$	20	$5c$	$17ab$	$2p - 2q$



5:06 Using algebra

PREP QUIZ 5:06

Simplify each expression.

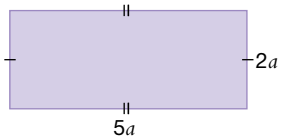
1 $5x + 2x + 3x$	**2** $2a + 7b + 6a$	**3** $9m - 12m$	**4** $6x^2 + 3x - 4x^2 - 4x$
5 $3 \times 7x$	**6** $12p \div 6p$	**7** $-10x \times 3x$	**8** $3ab \times 5a$
9 $\frac{10x^2}{5x}$	**10** $\frac{24ab}{6b}$		

Algebra is useful when solving problems. We have to learn to use a tool properly before we can put it to good use.

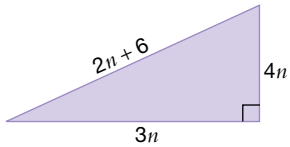
WORKED EXAMPLES

1 Write expressions for the perimeter and area of each shape.

a



b



2 If I bought 6 apples that cost k cents each and 4 pears that cost $2k$ cents each, find an expression for the total cost of the fruit.

Solutions

1 a Perimeter $= 5a + 2a + 5a + 2a$
$= 14a$ units

Area $= 5a \times 2a$
$= 10a^2$ square units

b Perimeter $= 3n + 4n + 2n + 6$
$= 9n + 6$ units

Area $= \frac{1}{2} \times 3n \times 4n$
$= 6n^2$ square units

2 Cost of apples $= 6 \times k$
$= 6k$ cents

Cost of pears $= 4 \times 2k$
$= 8k$ cents

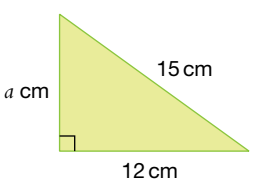
$\therefore$ Total cost $= 6k + 8k$
$= 14k$ cents

Exercise 5:06

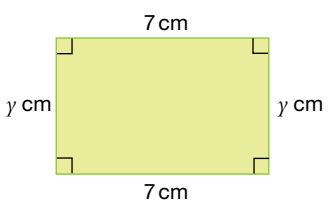
P Foundation worksheet 5:06
Using algebra

1 Write an expression for each perimeter.

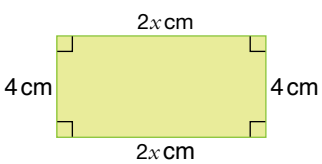
a



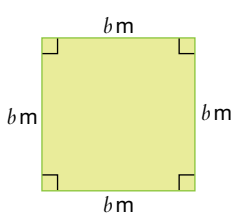
b



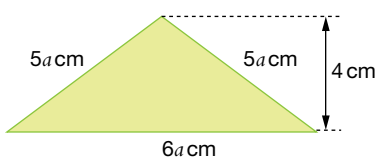
c



d



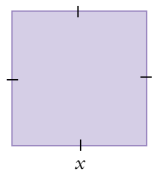
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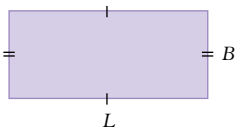
2 Write an expression for the area of each part of Question 1.

3 Write down an expression, in its simplest form, for the perimeter of each shape.
All measurements are in millimetres. (Similar markings on sides of a shape mean that their lengths are equal.)

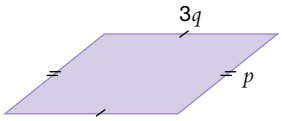
a



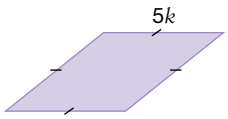
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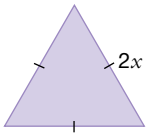
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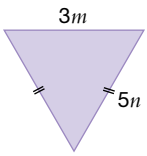
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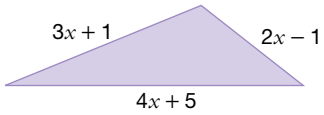
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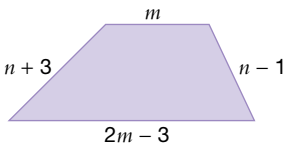
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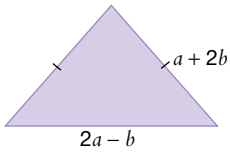
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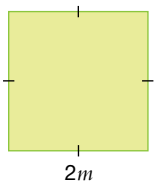


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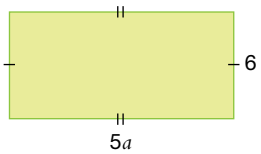


4 Find an expression for the area of each shape below. All measurements are in centimetres.

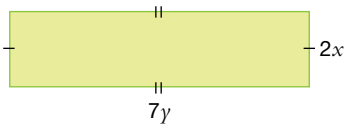
a



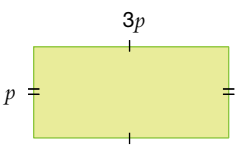
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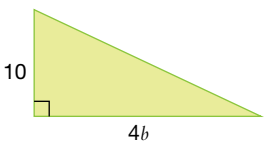
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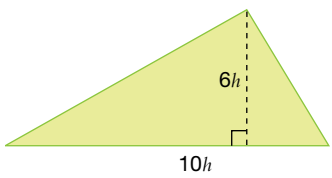
d



e



f



5 Find the simplest expression for each of the following problems.

a If Naji bought p pens and $3q$ pencils and Galan bought $2p$ pens and q pencils, how many pens and pencils did they buy in total?

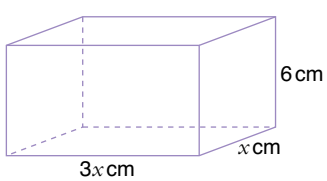
b Charlotte had $10a$ pears, but she ate $3a$ of them, and then bought $7a$ more. How many does she have now?

c Three different types of lollies cost $5x$, $3x$ and $2x$ cents each. If I buy 4 of each type, what will be the total cost?

d Box A contains m chocolates and $2n$ lollies. Box B contains $3m$ chocolates and n lollies. If Billy bought three of box A and two of box B, how many lollies did he have in total?

6 **a** Write down an expression for the volume of this prism.

b By thinking about the net of this prism, find an expression for the area of cardboard needed to construct it.



5:07 Index notation

PREP QUIZ 5:07

Evaluate:

1 2^2 **2** 5^2 **3** 10^4 **4** 2^3 **5** 3^3

Write these as powers.

6 $3 \times 3 \times 3$ **7** $a \times a \times a \times a \times a$ **8** $10 \times 10 \times 10 \times 10$

9 $x \times x \times x \times x \times x \times x \times x$ **10** $y \times y \times y \times y \times y \times y \times y \times y$

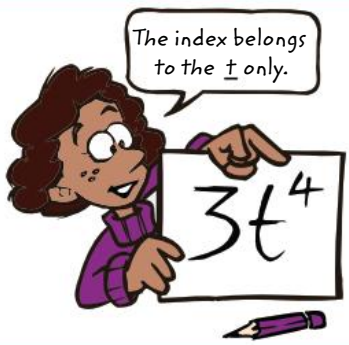
When a pronumeral is multiplied by itself a number of times we can simplify the expression using index notation.

WORKED EXAMPLES

1 $x \times x \times x \times x \times x \times x \times x \times x = x^8$
(eight xs multiplied together)

2 Evaluate $10a^2b^3$ if $a = 4$ and $b = 5$.

$10a^2b^3 = 10 \times a \times a \times b \times b \times b$
$= 10 \times 4 \times 4 \times 5 \times 5 \times 5$
$= 20\,000$



Exercise 5:07

1 Simplify each of the following, using index notation.

a $m \times m$ **b** $a \times a \times a \times a \times a \times a$ **c** $x \times x \times x \times x \times x$

d $5 \times n \times n \times n$ **e** $7 \times a \times a \times 3$ **f** $8 \times y \times y \times y \times 2$

g $p \times p \times q$ **h** $8 \times x \times 7 \times y \times y$ **i** $4 \times m \times m \times m \times n \times n$

2 Rewrite in expanded form (e.g. $2y^4 = 2 \times y \times y \times y \times y$).

a x^3 **b** $8b^2$ **c** $7m^6$ **d** y^5 **e** $3a^4$

f x^2y^2 **g** am^4 **h** x^3y^3 **i** a^3b **j** ap^3

k $2ap^2$ **l** $4ap^3$ **m** $6x^2y^2$ **n** $5a^5b$ **o** $10m^3n^4$

3 Find the value of each if $a = 2$, $b = 4$, $c = -5$, $x = 10$ and $y = 3$.

a b^2 **b** c^2 **c** a^4 **d** x^6 **e** y^3

f $3x^2$ **g** $2a^3$ **h** $4y^2$ **i** $2y^4$ **j** $5x^3$

k x^2a^2 **l** $3b^2c$ **m** $4a^2y^3$ **n** $2a^2x^2$ **o** $3c^3$

5:08 Grouping symbols

PREP QUIZ 5:08

Evaluate:

1 $4 \times (2 + 3)$ **2** $4 \times 2 + 4 \times 3$ **3** $3 \times (5 - 3)$

4 $3 \times 5 - 3 \times 3$ **5** $-2 \times (4 + 3)$ **6** $(-2) \times 4 + (-2) \times 3$

Simplify:

7 $5 \times a + 5 \times 2$ **8** $3 \times 2x + 3 \times 5y$ **9** $2x \times 3x + 2x \times 5$ **10** $3a \times 4a - 3a \times 2b$

In Year 7 we saw that an expression like $3(2 + 5)$ meant $3 \times (2 + 5)$, which in turn is equal to $3 \times 2 + 3 \times 5$. Similarly, in algebra:

$$a(b + c) = ab + ac \quad \text{OR} \quad a(b - c) = ab - ac$$

Grouping symbols

- parentheses ( )
- brackets [ ]
- braces { }

To write an expression without grouping symbols, we multiply each term inside the grouping symbols by the term outside.

WORKED EXAMPLES

1 a $5(a + 2) = 5 \times a + 5 \times 2$
$= 5a + 10$

b $6(2x - 3y) = 6 \times 2x - 6 \times 3y$
$= 12x - 18y$

c $x(x + 7) = x^2 + 7x$

d $3a(5 - 2a) = 15a - 6a^2$



e $-3(x + 5) = -3 \times x + (-3) \times 5$
$= -3x - 15$

f $-2x(3 - 5x) = -2x \times 3 - (-2x) \times 5x$
$= -6x + 10x^2$

Watch out for expressions that have negative signs outside the grouping symbols.

g An expression like $-(x + 3)$ is the same as $-1(x + 3)$.
$-1(x + 3) = -1 \times x + (-1) \times 3$
$= -x - 3$

2 Expand and simplify.

a $5(x + 3) + 2x$
$= 5x + 15 + 2x$
$= 7x + 15$

b $3(a + 4) + 2(5 - a)$
$= 3a + 12 + 10 - 2a$
$= a + 22$

c $2(5x + 3) - (4x - 3)$
$= 10x + 6 - 4x + 3$
$= 6x + 9$

d $x(2x - 3) - 4(x + 5)$
$= 2x^2 - 3x - 4x - 20$
$= 2x^2 - 7x - 20$

'Expand' means to rewrite without grouping symbols.



Exercise 5:08A

1 Rewrite these expressions without grouping symbols.

a $3(a+2)$	**b** $4(x+3)$	**c** $2(x+7)$	**d** $5(v+1)$
e $6(m-5)$	**f** $5(n-2)$	**g** $4(b-3)$	**h** $6(l-1)$
i $2(3+a)$	**j** $3(4+x)$	**k** $6(6+t)$	**l** $7(2+w)$
m $7(1-x)$	**n** $5(2-y)$	**o** $4(3-p)$	**p** $9(8-y)$
q $3(a+b)$	**r** $2(x+y)$	**s** $6(p+q)$	**t** $7(m+n)$
u $4(p-q)$	**v** $5(a-b)$	**w** $10(t-w)$	**x** $8(x-y)$

2 Expand these expressions.

a $2(2a+3)$	**b** $5(3x+1)$	**c** $4(2m+3)$	**d** $6(3n+2)$
e $5(2m-1)$	**f** $4(4n-5)$	**g** $2(5p-2)$	**h** $7(5t-3)$
i $6(2+3m)$	**j** $7(1+4x)$	**k** $3(3+4n)$	**l** $4(2+5t)$
m $10(2-3x)$	**n** $4(1-3p)$	**o** $3(2-5w)$	**p** $8(1-2x)$
q $2(3a+2b)$	**r** $3(4m+n)$	**s** $5(4x+3y)$	**t** $10(x+3y)$
u $5(2x-y)$	**v** $4(2a-3b)$	**w** $2(7m-2n)$	**x** $3(6t-2w)$

3 Expand each expression.

a $x(x+2)$	**b** $y(y+3)$	**c** $a(a+5)$	**d** $m(m+10)$
e $a(a-1)$	**f** $m(m-5)$	**g** $n(n-3)$	**h** $y(y-8)$
i $x(y+2)$	**j** $a(b+2)$	**k** $m(t-3)$	**l** $n(x-6)$
m $a(x+a)$	**n** $x(x+y)$	**o** $m(m+n)$	**p** $t(w+t)$
q $k(k+l)$	**r** $p(q-p)$	**s** $t(x+y)$	**t** $a(b-c)$

4 Expand each expression.

a $2a(a+2)$	**b** $3x(x+5)$	**c** $5n(n+2)$	**d** $4m(m+4)$
e $3p(p-4)$	**f** $4q(q-1)$	**g** $10m(m-7)$	**h** $8a(a-5)$
i $6x(4-x)$	**j** $2m(1+m)$	**k** $9t(t+2)$	**l** $5n(5-n)$
m $2a(a+b)$	**n** $4p(p+q)$	**o** $3x(x+y)$	**p** $7m(n+m)$
q $3y(2y+4)$	**r** $3a(b+2a)$	**s** $3m(2m+5n)$	**t** $4p(3q-2p)$

5 Expand and simplify each expression by collecting like terms.

a $2(x+3)+4x$	**b** $3(m+2)+4m$	**c** $5(a+5)-3a$
d $4(n-2)+3n$	**e** $5(a-4)-2a$	**f** $3(m-1)+4m$
g $6(a+2)+2a+4$	**h** $3(x+3)+2x-7$	**i** $4(y+3)-2y+5$
j $5x+4(x+3)$	**k** $5a+3(a-2)$	**l** $8m+4(m-2)$

6 Expand all grouping symbols and then simplify each expression.

a $2(x+3)+3(x+2)$	**b** $4(x+2)+2(x+3)$	**c** $5(a+2)+3(a+1)$
d $5(m+2)+2(m-1)$	**e** $6(y+1)+3(y-2)$	**f** $5(m+1)+2(m-1)$
g $4(3+a)+5(2+a)$	**h** $7(m+3)+2(2-m)$	**i** $6(2+y)+3(4-y)$
j $9(x+1)+3(x-3)$	**k** $8(p+1)+3(2+p)$	**l** $7(a+2)+5(3-a)$
m $2(m+n)+2(m+n)$	**n** $5(x+y)+5(x-y)$	**o** $3(p+q)+2(p-q)$

Exercise 5:08B (This exercise involves expanding with negative terms.)

1 Expand each of the following expressions.

a $-2(x+3)$ **b** $-3(a+5)$
c $-5(y+1)$ **d** $-9(k+2)$
e $-4(a-2)$ **f** $-2(p-3)$
g $-7(t-6)$ **h** $-5(x-1)$
i $-3(6+x)$ **j** $-9(7-q)$
k $-2(10+y)$ **l** $-6(8-h)$

Remember:
$-3(x-4) = -3 \times x - (-3) \times 4$
$= -3x + 12$
$-5(2y+7) = -5 \times 2y + (-5) \times 7$
$= -10y - 35$

2 Rewrite the following without grouping symbols.

a $-(a+2)$ **b** $-(x+3)$
c $-(p+7)$ **d** $-(q+1)$
e $-(x-3)$ **f** $-(t-2)$
g $-(w-6)$ **h** $-(y-1)$
i $-(2x+7)$ **j** $-(3x-2)$
k $-(4x+5)$ **l** $-(7y-3)$

Remember:
$-(x+7)$ is the same as $-1(x+7)$

3 Expand the following.

a $-x(x+2)$ **b** $-a(a+3)$ **c** $-m(m-4)$
d $-n(n-1)$ **e** $-a(b+3)$ **f** $-p(q+5)$
g $-t(x-3)$ **h** $-m(n-10)$ **i** $-2m(m+4)$
j $-5a(2a+5)$ **k** $-6x(x-7)$ **l** $-4t(5t+3)$

4 Expand and then collect like terms.

a $2(a+4)-4a$ **b** $6(p-4)+19$ **c** $5(a-1)-6a$
d $2a+3(1-a)$ **e** $6x+2(3-4x)$ **f** $10+6(a-2)$
g $6-4(a+2)$ **h** $5x-2(x+3)$ **i** $6m-2(m-1)$
j $10x-(x+3)$ **k** $16-(x+5)$ **l** $5y-(2+3y)$
m $6m-3(m+2)+5$ **n** $7a+6-2(a+4)$ **o** $20-(10+3a)+5a$

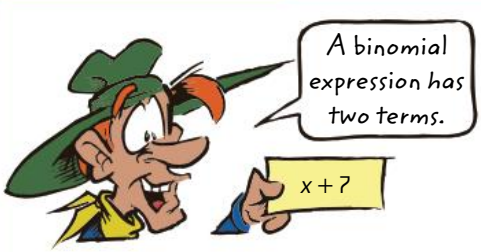
5 Expand all grouping symbols, and then simplify each expression.

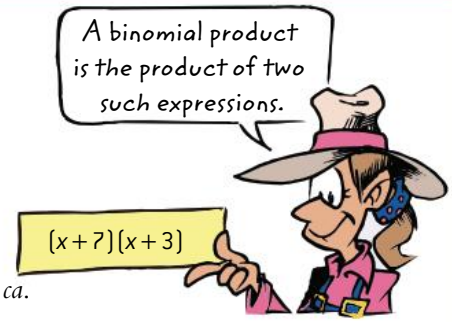
a $3(x+2)-2(x+3)$ **b** $4(a+2)-2(a+3)$ **c** $6(y+4)-5(y+1)$
d $5(a+4)-4(a-1)$ **e** $6(m+5)-2(m-2)$ **f** $8(t+3)-6(t-4)$
g $2(m-2)-3(m+2)$ **h** $5(y-2)-6(y+4)$ **i** $3(p-6)-2(p+2)$
j $5(p-2)-3(p-1)$ **k** $6(t-1)-4(t-2)$ **l** $8(q-3)-6(q-1)$
m $3(p+7)-(p-7)$ **n** $6(q+2)-(q+3)$ **o** $2(m-5)-(4-m)$

6 Expand all grouping symbols and then collect like term to simplify these expressions. Be careful!

a $3(x+5)+4(x+2)-5(x+1)$ **b** $6(m-2)-3(m+2)+5(2-m)$
c $5(2a+4)-(6-3a)-2(a+1)$ **d** $10(a-4)-6(4-a)-(a+4)$
e $x(x+7)-5(x-7)+2(1-x)$ **f** $a(2a+3b)+b(3a-2b)$
g $3y(y-2)+4y(x-7)-2x(y+4)$ **h** $3a(2a+4b)-3b(4a+c)-3c(3a-2b)$

CHALLENGE 5:08 EXPANDING BINOMIAL PRODUCTS





Note that if $(b + c)a$ is expanded the result would be $ba + ca$.

If a was replaced by another binomial expression, say $x + 3$, we would have:

$$(b + c)(x + 3) = b(x + 3) + c(x + 3)$$
$$= bx + 3b + cx + 3c$$

A binomial product, when expanded, gives an expression with four terms. Sometimes two of these may be added together.

Examples

1 $(x + 2)(x + 3) = x(x + 3) + 2(x + 3)$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

2 $(a + 5)(a - 3) = a(a - 3) + 5(a - 3)$
$= a^2 - 3a + 5a - 15$
$= a^2 + 2a - 15$

3 $(m + 3)^2 = (m + 3)(m + 3)$
$= m(m + 3) + 3(m + 3)$
$= m^2 + 3m + 3m + 9$
$= m^2 + 6m + 9$

4 $(a + b)(c + d) = a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

Remember:
There are no problems, just challenges!



Exercises

1 Expand the following.

a $(a + 2)(a + 4)$	**b** $(x + 2)(x + 5)$	**c** $(y + 1)(y + 2)$	**d** $(p + 7)(p + 2)$
e $(x + 3)(x - 1)$	**f** $(q + 4)(q - 2)$	**g** $(a + 3)(a - 4)$	**h** $(m + 5)(m - 2)$
i $(y - 3)(y + 2)$	**j** $(x - 1)(x + 3)$	**k** $(n - 5)(n + 4)$	**l** $(a - 3)(a + 3)$
m $(x - 1)(x - 2)$	**n** $(a - 4)(a - 4)$	**o** $(t - 4)(t - 7)$	**p** $(x - 5)(x - 6)$

2 Expand:

a $(a + 2)^2$	**b** $(m + 5)^2$	**c** $(x + 7)^2$	**d** $(m + 10)^2$
e $(x - 1)^2$	**f** $(a - 3)^2$	**g** $(n - 4)^2$	**h** $(y - 8)^2$

3 Expand and simplify:

a $(a + x)(b + y)$	**b** $(p + q)(x + y)$	**c** $(a + b)(c - d)$	**d** $(m - n)(x - y)$
e $(3 + x)(5 + y)$	**f** $(a + 4)(b + 5)$	**g** $(x + 7)(y - 3)$	**h** $(t - 5)(w - 3)$

5:09 Factorising

PREP QUIZ 5:09

Expand each of the following.

1 $2(x + 3)$ **2** $5(a - 7)$ **3** $p(p + 4)$ **4** $2m(m - n)$

Write down the factors of:

5 6 **6** 12 **7** 20

What is the highest common factor (HCF) of:

8 6 and 9 **9** 8 and 12 **10** 20 and 25?

Factors are numbers that will divide into a given number, leaving no remainder.
e.g. {1, 2, 3, 6, 9, 18} is the set of factors of 18.

If we expand the expression $3(x + 4)$, we get $3x + 12$.

To factorise $3x + 12$ we reverse this procedure. We need to see that 3 is the highest common factor of $3x$ and 12, and then write it outside the grouping symbols. The remainder is then written inside the grouping symbols.

WORKED EXAMPLES

1 $3x + 12 = 3 \times x + 3 \times 4$
$= 3(x + 4)$

2 $6m + 9 = 3 \times 2m + 3 \times 3$ (HCF is 3)
$= 3(2m + 3)$

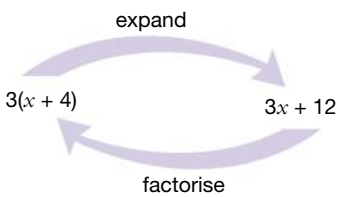
3 $5x - 20y = 5 \times x - 5 \times 4y$ (HCF is 5)
$= 5(x - 4y)$

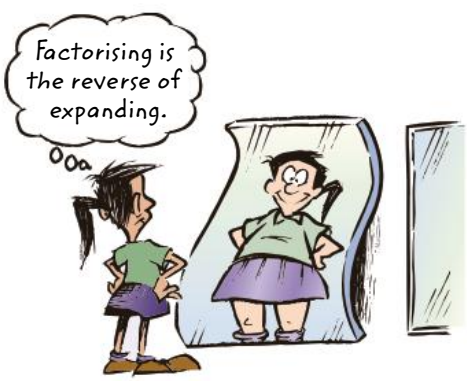
4 $2m^2 + 3m = m \times 2m + m \times 3$ (HCF is m)
$= m(2m + 3)$

5 $8ab - 6a = 2a \times 4b - 2a \times 3$ (HCF is $2a$)
$= 2a(4b - 3)$

6 $ax + a = a \times x + a \times 1$ (HCF is a)
$= a(x + 1)$

7 $2a + 4b - 6c = 2 \times a + 2 \times 2b - 2 \times 3c$ (HCF is 2)
$= 2(a + 2b - 3c)$





$ab + ac = a(b + c)$ or $ab - ac = a(b - c)$

Exercise 5:09

P Foundation worksheet 5:09 Factorising

1 Write the missing number in each case.

a $2x + 4 = ___(x + 2)$ b $5a + 10 = ___(a + 2)$ c $6y - 3 = ___(2y - 1)$

d $10p - 5 = ___(2p - 1)$ e $4n + 6 = ___(2n + 3)$ f $6 - 8m = ___(3 - 4m)$

g $ab + a = ___(b + 1)$ h $xy + 3x = ___(y + 3)$ i $p^2 - 2p = ___(p - 2)$

j $a^2 + ab = ___(a + b)$

2 Write the factor inside the grouping symbols.

a $2m + 10 = 2(\quad)$ b $5x + 10 = 5(\quad)$ c $4a + 4 = 4(\quad)$

d $3y - 9 = 3(\quad)$ e $7k - 14 = 7(\quad)$ f $10m - 10 = 10(\quad)$

g $4x + 6 = 2(\quad)$ h $6n + 9 = 3(\quad)$ i $8k + 6 = 2(\quad)$

j $6y - 4 = 2(\quad)$ k $4x - 10 = 2(\quad)$ l $10a - 15 = 5(\quad)$

m $ax + 3a = a(\quad)$ n $mn + 5n = n(\quad)$ o $4y + xy = y(\quad)$

p $ab - bc = b(\quad)$ q $xy - y = y(\quad)$ r $p - pq = p(\quad)$

s $x^2 + 3x = x(\quad)$ t $5a + a^2 = a(\quad)$ u $5y - 10xy = 5y(\quad)$

3 Factorise the following by taking out the HCF.

a $2y + 6$ b $3x + 6$ c $5m + 20$ d $6k + 12$

e $9n - 18$ f $6a - 3$ g $4y - 12$ h $14p - 7$

i $10 + 5a$ j $6 + 4p$ k $10 + 6g$ l $8 + 8y$

m $20 - 10m$ n $12 - 4a$ o $15 - 3q$ p $7 - 7p$

q $6m + 6n$ r $5x + 10y$ s $6n - 12m$ t $8p - 8q$

u $4p - 6$ v $9a + 12$ w $8 + 14x$ x $10 - 8k$

4 Factorise each of the following.

a $ax + 2x$ b $xy + 5y$ c $ab + 3b$ d $9x + xy$

e $5m - mn$ f $pq - 3q$ g $10x - ax$ h $tw - w$

i $ax + bx$ j $pq + qr$ k $mn - np$ l $ab - bc$

m $x^2 + 3x$ n $y^2 + 2y$ o $m^2 + m$ p $t^2 + tx$

q $p^2 - 5p$ r $6m - m^2$ s $2x^2 - 3x$ t $9l - 7l^2$

u $5m + 2mn$ v $7xy - 3x$ w $3ab + 5bc$ x $8pq - 7pr$

5 Factorise completely.

a $5x + 10xy$ b $4m + 8mn$ c $6p + 4pq$ d $10t + 15tw$

e $9mn - 3m$ f $6pq - 12q$ g $8t - 4tw$ h $6a - 3ab$

i $2x^2 + 4x$ j $9m^2 - 6m$ k $10n + 5n^2$ l $8pq - 4q^2$

m $3mn + 6nr$ n $4pq + 8pr$ o $6ab + 3bc$ p $8xy + 2xz$

q $9xy - 3xz$ r $6ab - 12ac$ s $5mn - 15mp$ t $8tw - 4xw$

u $a^2b - ab$ v $5x^2 - 10xy$ w $9mn + 6m^2$ x $6pq - 20q^2$

6 Factorise these by taking out the factor common to all three terms.

a $3x + 3y + 3z$ b $6a + 6b - 6c$ c $5m + 5n + 5$

d $5a + 10b - 15c$ e $6p - 3q + 9r$ f $12x + 6y - 9$

g $3a^2 + 3a + 3$ h $5m^2 + 10m + 15$ i $4p^2 - 2p + 8$

j $ax + ay + az$ k $xp + xq + xr$ l $5t + tw - tx$

m $5x^2 - 10x + 15xy$ n $6m + 15mn + 9mp$ o $10a^2 + 15ab - 5a$

7 Factorise the following by taking out a negative common factor.

a $-2x - 4$	**b** $-6m - 12$	**c** $-3p - 9$
d $-5p + 10$	**e** $-3q + 3$	**f** $-7n + 21$
g $-20 - 10a$	**h** $-12 - 4p$	**i** $-9 + 6q$
j $-6a - 12b$	**k** $-5m + 15n$	**l** $-10a + 2b$
m $-x^2 + 3x$	**n** $-ax - 5a$	**o** $-6ax + 3bx$

Remember:
$-2(x+3) = -2x - 6$
$-x(x-3) = -x^2 + 3x$

MATHS TERMS 5

algebra (algebraic)
- the use of pronumerals in solving problems and expressing ideas

cubed
- used three times as a factor, e.g. $a^3 = aaa$

directed numbers
- numbers that have size and direction (+ or −)

evaluate
- to find the value of an expression

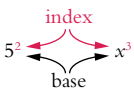
expand
- to remove parentheses, e.g. $3(2x - 7) = 6x - 21$

expression
- algebraic expressions are made up of one or more terms joined by + or − signs, e.g. $2x - 7$, $5x + 2y + 1$, $a^2 - 2a + 1$

formula
- a number sentence (or equation) written using pronumerals and numerals, e.g. $C = 50M + 6$, $A = L \times B$

grouping symbols
- these group numbers or terms. e.g. parentheses ( ), brackets [ ], braces { }, fraction bar $\frac{x+1}{2}$, square root sign $\sqrt{4x+1}$

index (indices)
- the raised symbol when a number is written as a power



index notation
- the use of powers to write a number or algebraic expression

like terms
- these have the same pronumeral parts, e.g. $3xy$ and $7xy$, $8x^2$ and $-x^2$
- unlike terms have different pronumeral parts, e.g. x^2 and x, $5xy$ and $2y$

power
- repeated as a factor, e.g. 3 to the power of $5 = 3^5$
  $= 3 \times 3 \times 3 \times 3 \times 3$

pronumeral
- a symbol (usually a letter) that takes the place of a numeral, e.g. x or θ

rule
- an instruction or formula that explains how the numbers in a pattern are related

simplify
- to give the shortest or simplest answer

squared
- multiplied by itself, e.g. $5^2 = 5 \times 5$
- used twice as a factor

substitute (substitution)
- to replace something
- usually a numeral replaces a pronumeral, e.g. substitute $x = 7$ in $2x + 3$.

value
- the number represented by a pronumeral, e.g. for the rule $y = x + 7$, find the value of y if x is given the value of 4

DIAGNOSTIC TEST 5 ALGEBRA TECHNIQUES

Each section of the test has similar items that test a certain type of example.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Use the rule given to complete the table. 5:01

a $P = 2S + 4$

S	0	2	4	6
P				

b $F = 3P - 3$

P	1	2	3	4
F				

2 Find the rule that describes each pattern. 5:01

a

x	0	1	2	3
y	0	4	8	12

b

a	5	6	7	8
b	12	14	16	18

3 Simplify: 5:02

a $5x + 3x$ **b** $2xy + 7xy$ **c** $3x^2 + 6x^2$

4 Simplify: 5:02

a $9m - \text{m}$ **b** $5ab - 3ab$ **c** $7a^2 - 6a^2$

5 Simplify: 5:02

a $3x + 4x + 2x$ **b** $7a + 5a - 3a$ **c** $10m - 7m + 2m$

6 Simplify: 5:02

a $6a - 9a$ **b** $-3m + 4m$ **c** $-2x - 3x$

7 Simplify by collecting the like terms. 5:02

a $5x + 2y + 3x$ **b** $7m - 3m + 2n$ **c** $6a + 3b + 4a - 2b$

8 Simplify by collecting the like terms. 5:02

a $7m + 6n - 9n$ **b** $6a + 3b - 7a$ **c** $5m + 2n - 3m - 4n$

9 Simplify: 5:03

a $6 \times 3m$ **b** $5x \times 3$ **c** $4 \times 7k$

10 Simplify: 5:03

a $3m \times n$ **b** $6a \times 3b$ **c** $5x \times 7y$

11 Simplify: 5:03

a $-2x \times 7$ **b** $5m \times (-3)$ **c** $-3x \times (-2y)$

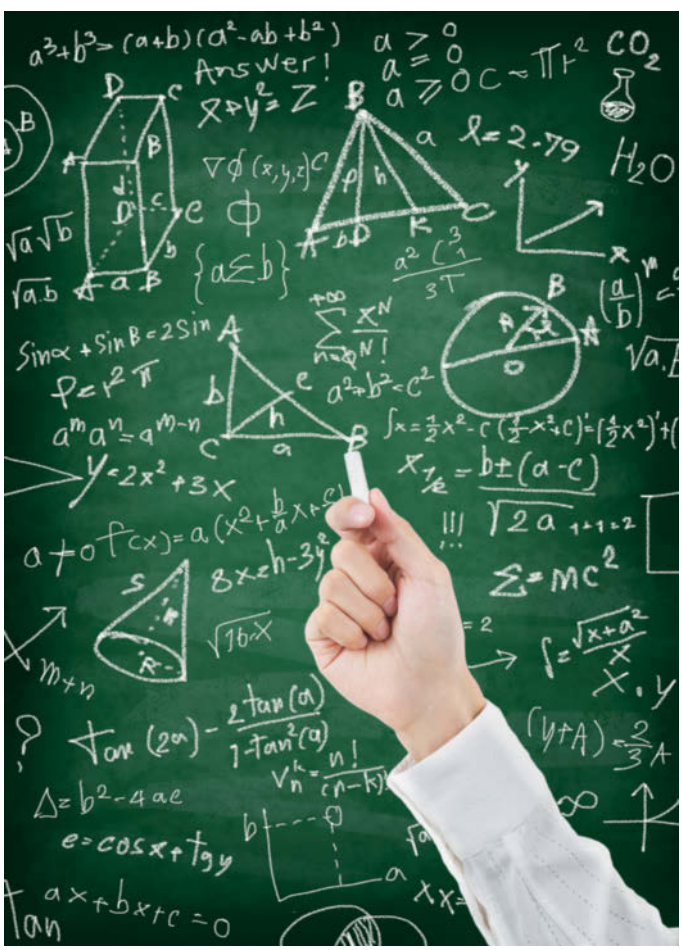
12 Simplify: 5:03

a $\dfrac{10x}{5}$ **b** $\dfrac{15m}{5m}$ **c** $12x \div 4$

13 Simplify: 5:04

a $\dfrac{ab}{b}$ **b** $\dfrac{4pq}{2q}$ **c** $9xy \div 3x$

14 Simplify, using index notation.		5:07
a $a \times a \times a \times a \times a$	**b** $x \times x \times x \times y$	**c** $8 \times m \times m \times m \times m$
15 Rewrite in expanded form.		5:07
a x^4	**b** x^3y^5	**c** $6a^6b$
16 Expand:		5:08
a $3(x + 2)$	**b** $5(a - 7)$	**c** $a(a + 3)$
17 Expand and simplify:		5:08
a $3(m + 3) - 2m$	**b** $4(x - 2) + 3x + 5$	**c** $2(x + 4) + 3(x - 1)$
18 Expand:		5:08
a $-3(x + 3)$	**b** $-2(a - 5)$	**c** $-x(x + 7)$
19 Expand and simplify:		5:08
a $5(x + 3) - 2(x + 1)$	**b** $4(a + 3) - 2(a - 1)$	**c** $3(x - 4) - 2(x + 1)$
20 Factorise:		5:09
a $2x + 8$	**b** $5y - 5$	**c** $6 + 12m$
21 Factorise:		5:09
a $ax + 3x$	**b** $6m - mn$	**c** $y^2 + 3y$



Algebra can be used to solve many problems.

ASSIGNMENT 5A Chapter review

1 Complete these tables of values.

a $y = 2x + 5$

x	0	1	2	3
y				

b $y = 2x - 5$

x	0	1	2	3
y				

c $y = 7 - 3x$

x	0	1	2	3
y				

d $y = x^2 + x - 1$

x	0	1	2	3
y				

2 Simplify the following expressions.

a $5m + 4m + m$
b $10n - n$
c $7y - 5y + y$
d $6t - 7t$
e $5p + 4q + p - 3q$
f $8xy + 3yx$
g $3a^2 - 4a^2 + 5a^2$
h $2m^2 + 4m + m^2 - m$
i $-7x - 5x$
j $10m - (-5m)$

3 Simplify the following expressions.

a $5m \times 4$
b $10n \times 2m$
c $7y \times 5y$
d $28t \div 7$
e $15n \div 3n$
f $8x^2 \div 4x$
g $30a \div 6a \times 5a$
h $30a \times 6a \div 5a$
i $12x - 5x \times 2$
j $10m \times 2 + 8m \div 4$

4 Simplify the following fractions.

a $\frac{12x}{3}$ **b** $\frac{12x}{8}$ **c** $\frac{2x}{6}$
d $\frac{6x}{9}$ **e** $\frac{8x}{2x}$ **f** $\frac{10xy}{6xz}$

5 Expand the following expressions.

a $5(a + 7)$ **b** $7(y - 4)$
c $m(m + 3)$ **d** $y(6 - a)$
e $5x(a + 1)$ **f** $9y(2x - 3y)$

6 Expand:

a $-5(a + 2)$
b $-7(z - 7)$
c $-m(m - 3)$

7 Expand and simplify.

a $3(n + 2) + 5n$
b $7y + 4(y - 1)$
c $10 - 5(a + 4)$
d $3(a + 4) + 5(a + 2)$
e $8(y + 3) + 2(y - 5)$
f $m(m + 2) + 5(m + 3)$

8 Expand and simplify.

a $3(n + 2) - 5n$
b $7y - 4(y - 1)$
c $10 - 5(a - 4)$
d $3(a + 4) - 5(a + 2)$
e $8(y + 3) - 2(y - 5)$
f $m(m + 2) - 5(m + 3)$

9 Factorise.

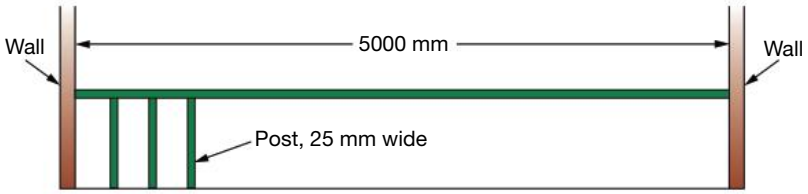
a $4m + 12$ **b** $15y - 10$
c $12 + 6n$ **d** $8x - 4y$
e $a^2 + 3a$ **f** $5k^2 - 10ak$
g $12x^2y + 8xz$

10 Factorise by taking out a negative factor.

a $-4m - 12$
b $-15y + 10$
c $-12 + 6n$
d $-8x - 4y$

ASSIGNMENT 5B Working mathematically

1 Larry has to make a handrail by fixing 40 vertical posts, which are 25 mm wide, between two walls that are 5000 mm apart. If the posts are to be equally spaced, how far apart should the posts be? (Answer to the nearest millimetre.)

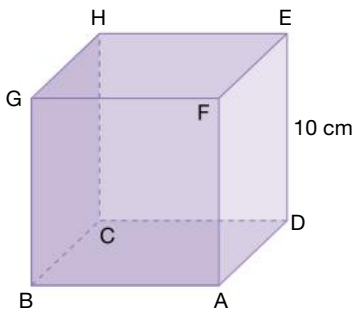


2 Four teams are to play in a soccer competition. The competition is a double round robin, where each team plays each other team twice on a home-and-away basis. Make up a draw for this competition, calling the teams 1, 2, 3 and 4, and making the first-mentioned team in each match the home team.

3 In the cube shown you can move from corner to corner along the edges of the cube only. You wish to go from A to H.

a What is the shortest possible path? How many different paths are there?

b If you cannot pass through any corner more than once, what is the length of the longest path from A to H?

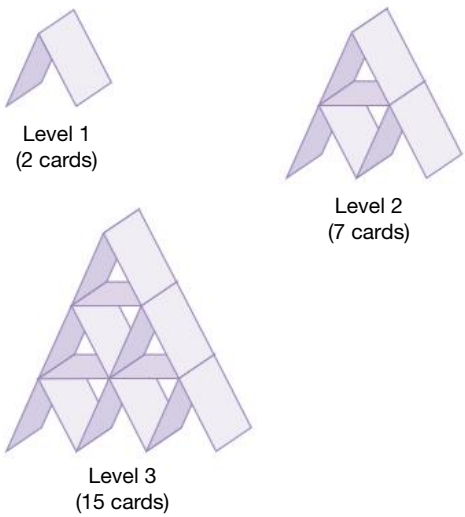


4 Five positive whole numbers add up to 15.

a What is the largest of the five numbers if it is known that the numbers are all different?

b What is the largest possible number if the numbers do not have to be all different?

5 Card towers can be made from playing cards as shown in the diagram below.



How many cards would be needed to build a:

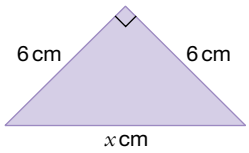
a Level 4 tower

b Level 10 tower?

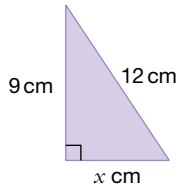
ASSIGNMENT 5C Cumulative revision

1 Find the value of the pronumeral in each of the following. Answer correct to one decimal place. 3:04

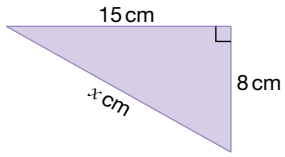
a



b



c



2 Change each of the following to its basic fraction. 4:01

a 20% **b** 25% **c** 60% **d** 72%

3 Change each fraction to a percentage. 4:01

a $\frac{5}{100}$ **b** $\frac{27}{100}$ **c** $\frac{100}{100}$ **d** $\frac{110}{100}$

4 Write each fraction as a percentage. 4:01

a $\frac{3}{10}$ **b** $\frac{9}{25}$ **c** $\frac{7}{20}$ **d** $\frac{11}{40}$

5 Write each percentage as a decimal. 4:01

a 15% **b** 7% **c** 108% **d** 74%

6 **a** Change 0·375 to a percentage. **b** Change 0·025 to a percentage. 4:01

c Change $\frac{15}{16}$ to a percentage. **d** Change $10\frac{3}{4}\%$ to a decimal.

7 Find: 4:03

a 15% of \$28 **b** 3% of 920 L

c 150% of 10 500 **d** 4·5% of \$21 500

8 **a** Don Nash scored 75 runs out of a total of 225. What percentage of the total did he score? 4:06

b Farmer Fay sells 3% of her flock of 5200 sheep. How many sheep does she sell?

c Cooper runs 1 km in 4 min 40 s. After one month of practice he improves his time to 4 min 15 s. What is his percentage improvement?

d An alloy is made of 7 kg of silver, 18 kg of lead and 15 kg of zinc. What percentage of the metal is silver?

9 State whether the following statements are true or false. 2:04

a $2^2 \times 2^3 = 2^6$ **b** $3^2 \times 3^3 = 3^6$ **c** $2^2 \times 3^3 = 6^5$

d $2^2 \times 3^2 = 6^2$ **e** $2^3 \times 2^3 = 2^6$ **f** $2^6 \div 2^3 = 2^3$

g $3^6 \div 3^3 = 3^2$ **h** $6^2 \div 3^2 = 2^2$ **i** $6^4 \div 2^2 = 3^2$

10 Write these fractions as terminating or recurring decimals. 1:06

a $\frac{3}{8}$ **b** $\frac{2}{3}$ **c** $\frac{1}{6}$ **d** $\frac{3}{20}$ **e** $\frac{2}{9}$ **f** $\frac{3}{7}$

PROBABILITY

6



Contents

6:01 Describing your chances
6:02 Finding the probability of an event
Fun spot 6:02 What chance of success?
6:03 Complementary events
Fun spot 6:03 Probability in games
6:04 Problem solving using Venn diagrams
Investigation 6:04 Counting the rings
6:05 Problem solving using two-way tables
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Statistics and Probability

Selections from ***Probability 1***, ***Probability 2*** [Stage 4]

- Identify complementary events and use the sum of probabilities to solve problems. (ACMSP204)
- Describe events using language of 'at least', exclusive 'or' (*A* or *B* but not both), inclusive 'or' (*A* or *B* or both) and 'and'. (ACMSP205)
- Represent events in two-way tables and Venn diagrams and solve related problems. (ACMSP292)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

6:01 Describing your chances

Probability terms you should know

- **sample space** — The set of all possible outcomes. e.g. 1, 2, 3, 4, 5, 6 on a normal dice.
- **chance experiment** — An experiment where the outcomes are random.
- **chance event** — A category in a chance experiment. e.g. rolling a 3 or 4 on a dice.
- **probability** — The chance of an event happening. e.g. The chance of a coin landing on heads is 50%.
- **trial** — One case of carrying out a chance experiment. e.g. spinning a spinner.
- **equally likely outcomes** — A situation in which all outcomes have the same chance of occurring.

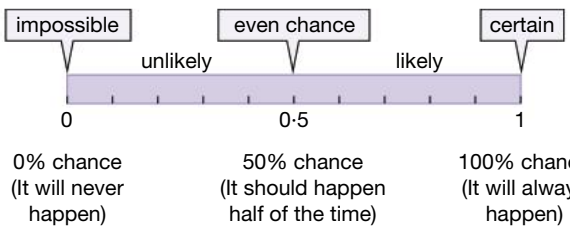
PREP QUIZ 6:01

State whether you think the probability of each of the following events has:
A an even chance **B** less than an even chance **C** more than an even chance.

1. If I toss a coin, I will get a tail.
2. The next person I see driving a car will be over 20.
3. If I take a card at random from a standard pack, it will be a club.
4. If I throw a dice, I will get an even number.
5. The next child born in the hospital will be female.
6. I will see a thylacine on the way home from school.
7. If I choose a class in Year 9, at least three of the students will have a computer in their home.
8. If I throw two dice, their sum will be greater than 9.
9. If I throw two coins, I will have either two heads or two tails.
10. In my next maths test, I will score over 80%.

The probability of an event is its chance of happening.

- Probability can be expressed as words, e.g. 'There is an even chance that we will win.'
- Probability can be expressed as a fraction, e.g. 'Our chance of winning is $\frac{1}{2}$.'
- Probability can be expressed as a decimal, e.g. 'Our chance of winning is 0·5.'
- Probability can be expressed as a percentage, e.g. 'We have a 50% chance of winning.'



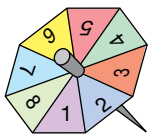
- An unlikely event has a probability of between 0% and 50%.
- A likely event has a probability of between 50% and 100%.

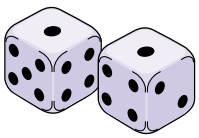
Subjective probability

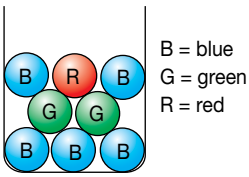
- Subjective probabilities are based on opinions or judgements. We make the best prediction we can, based on our past experiences and the known facts. We can make a prediction about the weather such as: 'There is a high chance of rain tomorrow', even though we are not a meteorologist with all the facts on hand.
- Remember that the most likely outcome will not always occur.

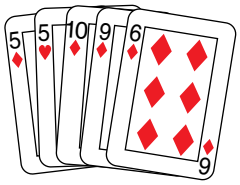


Exercise 6:01









1 Using the diagrams above, list the sample space (all possible outcomes) if:
- **a** the spinner is spun once
- **b** the two dice are rolled and the numbers on the upper faces added
- **c** two balls are drawn from the container at random
- **d** two cards are chosen at random.

2 The spinner above is spun once. Choose the label to the right that best describes the probability that the number will be:

a 1 **b** not 8 **c** more than 5
d odd **e** 0 **f** less than 10.

3 The dice above are rolled. Choose the label that best describes the probability that the sum of the top numbers will be:

a 12 **b** less than 12 **c** less than 6
d even **e** 1 **f** less than 15.

4 A coloured ball is drawn at random from the container above. Which label best describes the probability that the ball is:

a blue **b** green
c red **d** coloured?

Write the events in parts **a** to **d** in order, from least likely to most likely.

5 Two cards are drawn at random from the five above. Which label best describes the probability that the cards:

a are both fives **b** have a sum equal to 15
c have a sum greater than 9 **d** are both larger than 9?

Write the events in parts **a** to **d** in order, from least likely to most likely.

Chance
impossible
very unlikely
unlikely
even chance
likely
very likely
certain

6 Australia is playing cricket against England at the Sydney Cricket Ground.

a From the list below, select the five factors that you think would most influence the outcome of the match. Give a reason for each choice.

- The colour of the uniforms
- Who wins the toss
- The recent form of the players
- The number of tickets sold
- The country where it is played
- The condition of the wicket
- The weather
- The choice of umpires
- The length of the grass
- The ability of the captains



b Who do you think would win? Give a reason for your answer.

c If Australia has never lost in Sydney, is it possible that Australia will lose?

d Are the only possible outcomes 'Australia wins' and 'Australia loses'? Explain your answer.

7 Is each statement reasonable or not? Discuss the reasoning in each case.

a On my last four attempts, I have failed to lift this weight. I'm sure to succeed this time.

b My diamond ring is in one of these three jewellery boxes. I have searched two and it is not in them. It must be in the third one.

c We almost pushed the car out of the bog. Now that Arnie is here to help, I'm sure we will get it out this time.

d The first three puppies were female. The next is sure to be male.

e I've turned 48 cards of this pack and have turned only one Ace. The next card I turn is sure to be an Ace.



f I've just heard that a fire storm has passed through our street. I fear our home has been destroyed.

g I'm glad it's raining today because that means it won't rain for tennis tomorrow.

8 Discuss the contradictory nature of these statements.

a The horse was a certainty beaten.

b I was so close to winning the lottery. My number was 908 146. The winning number was 808 146.

c The winner came from an impossible position.

9 Comment critically on the statement: 'There are 26 letters in the English alphabet, so the probability that a person's name starts with X is 1 in 26'.

6:02 Finding the probability of an event

If all outcomes are equally likely, the probability of an event A occurring is given by:

$$P(A) = \frac{\text{number of favourable outcomes}}{n}$$

where n is the total number of outcomes in the sample space.

e.g. Find the probability of rolling a number greater than 4 on a normal dice (i.e. a 5 or a 6).
A dice has 6 sides.
P(rolling a 5 or a 6) $= \frac{2}{6}$

Useful terms

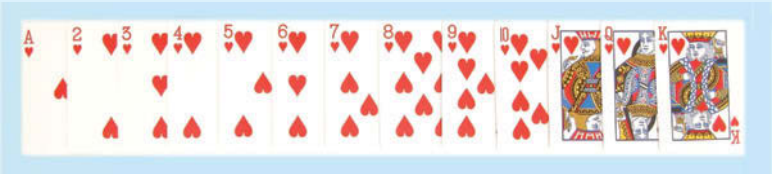
- **Mutually exclusive events** — These events have no elements in common.
  e.g. rolling an odd number and rolling an even number on a dice.
- **Non-mutually exclusive events** — These events have some elements in common.
  e.g. rolling an odd number and rolling a number less than 4.

Understanding the meaning when compound events are described

- Rolling at least 4 on a dice (4, 5 or 6).
- Rolling at most 4 on a dice (1, 2, 3 or 4).
- Rolling a number that is not 4 on a dice (1, 2, 3, 5 or 6).
- Rolling a number that is even and larger than 2. Both attributes must be present (4 or 6).
- Choosing someone rich or famous but not both: The person cannot be both rich and famous.
- Choosing someone rich or famous or both: The person could be rich, famous or both rich and famous.

Investigate the effect of the use of 'and' and 'or' when using internet search engines.

A standard pack of cards has four suits: hearts (♥), diamonds (♦), clubs (♣) and spades (♠).



The first of these cards is called an Ace.
The last three cards are the *picture* or *court* cards: Jack, Queen and King.

Exercise 6:02

P Foundation worksheet 6:02A
Probability
Foundation worksheet 6:02B
Using probability

1 Write 'mutually exclusive' (no outcomes in common) or 'non-mutually exclusive' (at least one outcome in common) for the two events. In each case, the events relate to choosing a card from a standard pack.

a A diamond will be chosen. A spade will be chosen.
b A red card will be chosen. A picture card will be chosen.
c A black card will be chosen. A club will be chosen.
d A King or Queen will be chosen. An Ace will be chosen.
e A diamond will be chosen. A number card will be chosen.
f The three of diamonds will be chosen. A red card will be chosen.
g A ten will be chosen. A heart will be chosen.

2 We can show the cards from a standard pack as, for example, 4H (4 of hearts), JD (Jack of diamonds), AS (Ace of spades). If we are using a standard pack of cards, list the cards that make up each of these sets.

a a diamond that is between 2 and 7
b a red card between 2 and 7
c a card between 2 and 7
d a King or a Queen
e a red picture card
f a card that is black and a King
g a card that is neither red nor a picture card
h a card that is neither red nor a spade.

3 One card is chosen at random from a standard pack of 52 cards. Find the probability of choosing:

a a diamond that is between 2 and 7
b a red card between 2 and 7
c a card between 2 and 7
d a King or a Queen
e a red picture card
f a card that is black and a King
g a card that is neither red nor a picture card
h a card that is neither red nor a spade.

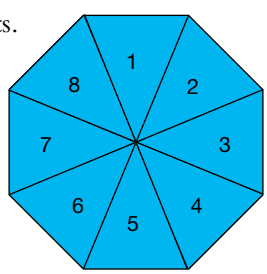
4 A normal six-sided dice is to be rolled once. Find the probability that the outcome will be:

a a 5
b a 5 or a 6
c a 7
d a number less than 7
e at least 5
f at most 5
g not a 5
h even and less than 5
i a number less than 3 or a number more than 3
j a number that is even or a number that is odd.



5 An eight-sided spinner is spun. List the outcomes in each of these events.

a a number higher than 3
b at least 3
c a 7
d a number no higher than 3
e not an even number
f not an even or odd number
g not even or higher than 4
h an even number that is less than 5
i a number less than 5 or a number more than 5
j a number that is less than 4 or greater than 6.



6 An eight-sided spinner is spun. Find the probability that the outcome will be:

a higher than 3 **b** at least 3
c a 7 **d** no higher than 3
e not an even number **f** not an even or odd number
g odd or higher than 4 or both **h** even and less than 5
i a number less than 5 or a number greater than 5
j a number that is less than 4 or greater than 6.

7 In a raffle, tickets numbered 1 to 100 were sold. Find the probability that the winning ticket was:

a an even number higher than 82 **b** an odd number higher than 83
c a number between 37 and 39 **d** a number between 37 and 38
e a number between 37 and 47 **f** a number not between 37 and 47.

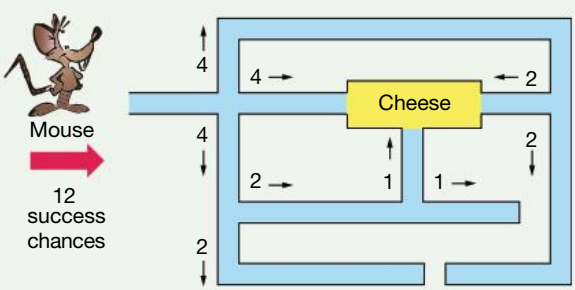
FUN SPOT 6:02 WHAT CHANCE OF SUCCESS?



- Luke is running his mouse through a variety of mazes. When the mouse comes to a point where there is more than one path to choose from, it is just as likely to choose any one of them. The mouse never turns back of its own accord.
- In each case below, the mouse begins with a number of 'success chances'. We will work out the probability that the mouse reaches the cheese without doubling back, by working out what fraction of the 'success chances' lead to cheese. (This fraction can then be expressed as a percentage.)

Example

Starting with 12 success chances, work out the probability that the mouse reaches the cheese without retracing its steps. At each fork in the maze, the 'success chances' must be shared equally among the new paths.



From the diagram, we can see that 7 out of 12 'success chances' reach the cheese.

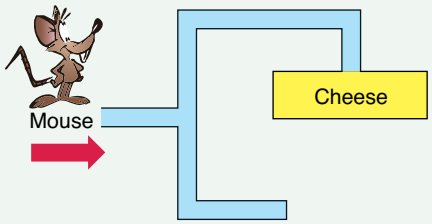
$$\therefore P(\text{reaching cheese}) = \frac{7}{12}$$
$$= \frac{7}{12} \times \frac{100}{1}$$
$$= 58\tfrac{1}{3}\%$$

$\therefore$ The mouse has $58\frac{1}{3}\%$ chance of reaching the cheese without having to turn back.

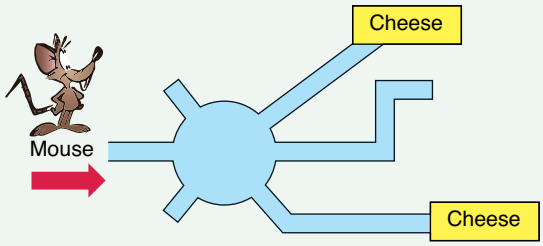
Questions

In each case below, find the probability as a percentage or as a fraction that the mouse will find the cheese without having to turn back.

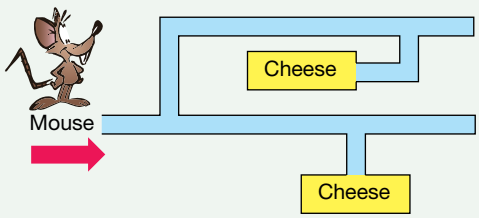
1 Start with 100 chances.



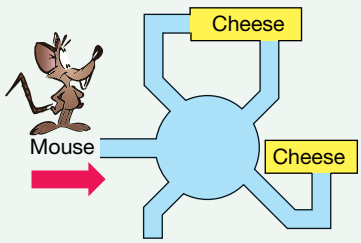
2 Start with 100 chances.



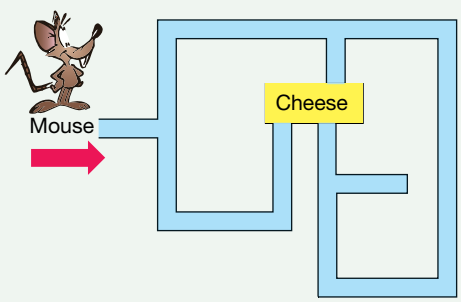
3 Start with 100 chances.



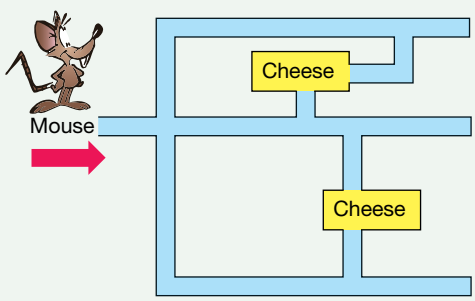
4 Start with 100 chances.



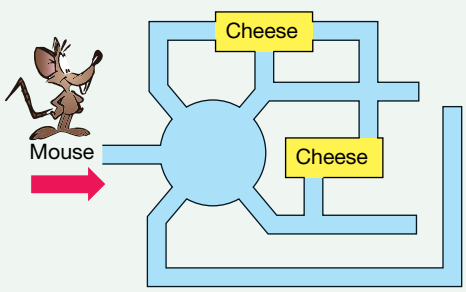
5 Start with 100 chances.



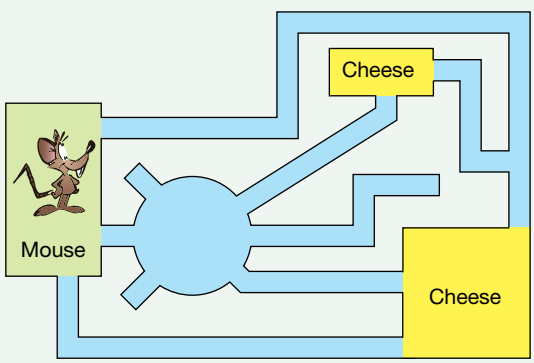
6 Start with 12 chances.



7 Start with 24 chances.



8 Start with 30 chances.



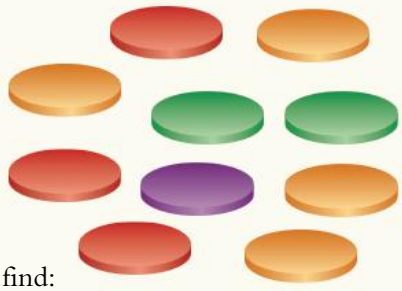
• You might like to make up some mazes of your own.

6:03 Complementary events

PREP QUIZ 6:03

From these 10 counters, one is to be chosen at random.
What is the probability that the counter chosen will be:

1 purple
2 not purple
3 red
4 not red
5 green
6 not green
7 orange
8 not orange
9 yellow
10 not yellow?



11 If $P(\text{red})$ means the probability of choosing a red counter, find:
$P(\text{red}) + P(\text{orange}) + P(\text{purple}) + P(\text{green})$.

- Question **11** of the Prep quiz shows us that the sum of the probabilities of all possible outcomes of an event is always 1 or 100%.

Tossing a coin

$P(\text{head}) = \frac{1}{2}$ $P(\text{tail}) = \frac{1}{2}$ $P(\text{head}) + P(\text{tail}) = 1$



Rolling a dice

$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

The sum of the probabilities of the possible outcomes is 1 or 100%.



- Questions **1** to **10** of the Prep quiz show us that:

$P(\text{purple}) + P(\text{not purple}) = \frac{1}{10} + \frac{9}{10} = 1$ $\therefore P(\text{not purple}) = 1 - P(\text{purple})$

$P(\text{orange}) + P(\text{not orange}) = \frac{4}{10} + \frac{6}{10} = 1$ $\therefore P(\text{not orange}) = 1 - P(\text{orange})$

$P(\text{yellow}) + P(\text{not yellow}) = \frac{0}{10} + \frac{10}{10} = 1$ $\therefore P(\text{not yellow}) = 1 - P(\text{yellow})$

These pairs of events, $P(E)$ and $P(\text{not } E)$, are complementary events.
The sum of their probabilities is 1 or 100%.

WORKED EXAMPLES

1 Luke's chance of clearing the high jump is $\frac{7}{10}$.
Luke's chance of *not clearing the high jump*
$= 1 - \frac{7}{10} = \frac{3}{10}$

2 The probability of throwing a sum of 8 when two dice are thrown is $\frac{5}{36}$.
The probability of *not throwing a sum of 8*
$= 1 - \frac{5}{36} = \frac{31}{36}$



- The sum of the probabilities of all possible outcomes to a simple event is 1 or 100%.
  e.g. If there is a 97% chance that something will happen, then there is a 3% chance that it will not happen.

$$P(E') = 1 - P(E)$$

  where $P(E')$ is the probability of E not occurring.
- E' is set notation for the complement of E.
  The complement of E is the set of possible outcomes outside of E.
  e.g. The complement of *catching less than 3* balls out of 5 would be *catching 3 or more* balls out of 5.

Exercise 6:03

P Challenge worksheet 6:03
Probability: An unusual case

1 Match each event below with its complementary event from the list on the right.

a It will rain tomorrow.
b I will win the race.
c A six will be rolled.
d I will not catch a cold next year.
e That team won't beat us.
f Alice will be school captain.

Choose one of these.
- A six will not be rolled.
- I will catch a cold next year.
- Alice will not be school captain.
- It won't rain tomorrow.
- That team will beat us.
- I won't win the race.

2 Write the complement of each event.

a The egg will break.
b The plate will not break.
c We will score 200 runs
d We will toss at least one head.
e I will win the raffle.
f We will win the soccer match.

3 In each case the probability of an event $P(E)$ is given. You are asked to find the probability of its complementary event $P(E')$.

a P(I will get a flat tyre) = 2%
P(I won't get a flat tyre) = ☐
b P(I will throw a 4 or less) = $\frac{2}{3}$
P(I will throw higher than 4) = ☐
c P(My allowance will be increased) = 40%
P(My allowance will not be increased) = ☐
d P(I will be given a present) = 0·99
P(I will not be given a present) = ☐
e P(You will select a yellow ball) = 1
P(You won't select a yellow ball) = ☐
f P(Lisa will learn to swim) = $\frac{7}{10}$
P(Lisa won't learn to swim) = ☐

4 In a standard pack of cards there are two red suits (hearts and diamonds) and two black suits (clubs and spades). When a card is chosen at random, the probability of selecting a red card is $\frac{1}{2}$. What is the probability of selecting:

a a black card
b a card that is not red
c either a black card or a red card
d neither a black card nor a red card?

5 A stamp is to be selected at random from this collection. We know some probabilities already.
$P(\text{Australian}) = \frac{3}{10}$, $P(\text{Polish}) = \frac{1}{17}$, $P(\text{French}) = \frac{37}{170}$.
Find the probability that the stamp selected is:

a not Polish **b** not Australian **c** not French.



6 From this coin collection, a coin is to be chosen at random. We know some probabilities already.
$P(\text{Australian}) = \frac{1}{4}$, $P(\text{Chinese}) = \frac{3}{800}$, $P(\text{British}) = \frac{1}{8}$
$P(\text{Singaporean}) = 2\%$, $P(\text{African}) = \frac{3}{20}$, $P(\text{American}) = 21\%$.
Find the probability that the coin chosen is:

a not Australian **b** not British
c not African **d** not Singaporean
e not American **f** not Chinese.



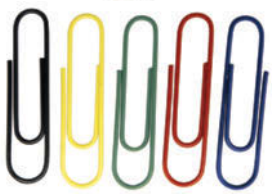
7 One pen is chosen at random from a cardboard box that has 37 red pens, 120 black pens and 43 blue pens. Find the probability that the pen is:

a red **b** not red **c** blue
d not blue **e** black **f** not black
g either red, black or blue.



8 One paperclip is chosen at random from the five shown. Find the probability that the paperclip chosen would be:

a yellow **b** not yellow
c either yellow or red **d** either yellow, green or red.



9 This pasta has pieces of different colour. 20% of the pieces are black, 23% are yellow and 49% are red. Some pieces are of another colour. One piece is to be taken at random. Find the probability that the piece will be:

a either black or yellow **b** either yellow or red
c not black **d** not red
e neither black nor yellow **f** neither yellow nor red.



Probabilities can be written as fractions, percentages or decimals.

10 In a raffle, the probabilities of the winning ticket being certain colours are:
$P(\text{red}) = 0{\cdot}375$, $P(\text{green}) = 0{\cdot}125$, and $P(\text{blue}) = 0{\cdot}25$.

a Are there tickets of any other colour? How do you know?
b Find, as a decimal, the probability that the winning ticket is:
i not red **ii** not green **iii** not blue **iv** neither red, green nor blue.

11 From a standard pack of 52 cards, one card is taken at random. Find the probability that the card will be:

a a diamond **b** not a diamond **c** a 4
d not a 4 **e** a picture card **f** not a picture card
g between 3 and 7 **h** not between 3 and 7 **i** either red or black.

12 A card is selected at random from a standard pack of cards plus a Joker. The probability of selecting a red card is $\frac{26}{53}$.

a What is the probability of selecting a card that is not red?

b What is the probability of selecting the Joker?

13 Using the probabilities given in Question **5**, find the probability that the stamp chosen will be:

a either Polish or Australian

b neither Polish nor Australian

c either Australian or French

d neither Australian nor French.

e Give two possible answers for the total number of stamps in this collection.
Give a reason for your answer.

14 Using the probabilities given in Question **6**, find the probability that the coin chosen will be:

a either Chinese, British or African

b neither Chinese, British nor African

c either Australian or American

d neither Australian nor American.

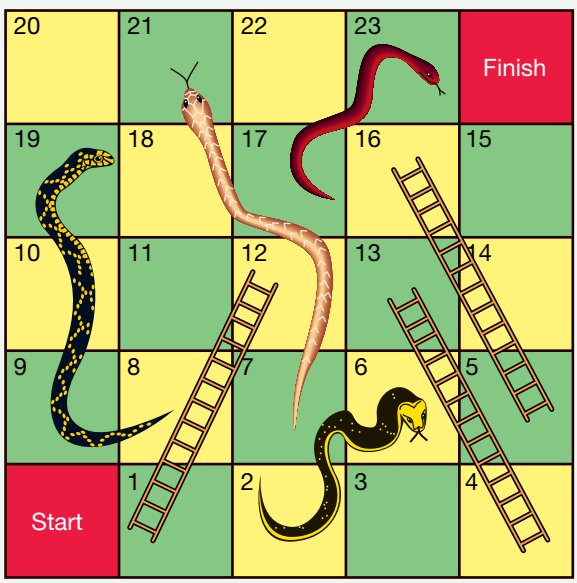
e Give two possible answers for the total number of coins in this collection.
Give a reason for your answer.

FUN SPOT 6:03 PROBABILITY IN GAMES

Game 1: Snakes and ladders

Place your counter at the start.
Roll a dice and move forward, following these rules.

- If you land on the head of a snake, you must move down to its tail.
- If you land on the foot of a ladder, you must move up to its top.



1 Find the probability that after one roll your counter will be on number:

a 3

b 20

c 13

d 2

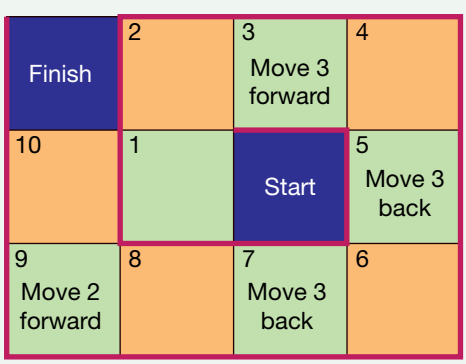
2 Find the probability that after one roll your counter will be on a number that is:

a odd

b even

Game 2: Spiral

Place your counter at the start.
Roll a dice and move forward in the order of the numbers, following the directions on the board.



3 Find the probability that after one roll your counter will be on number:

a 4

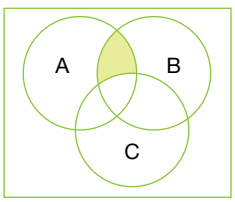
b 2

c 5

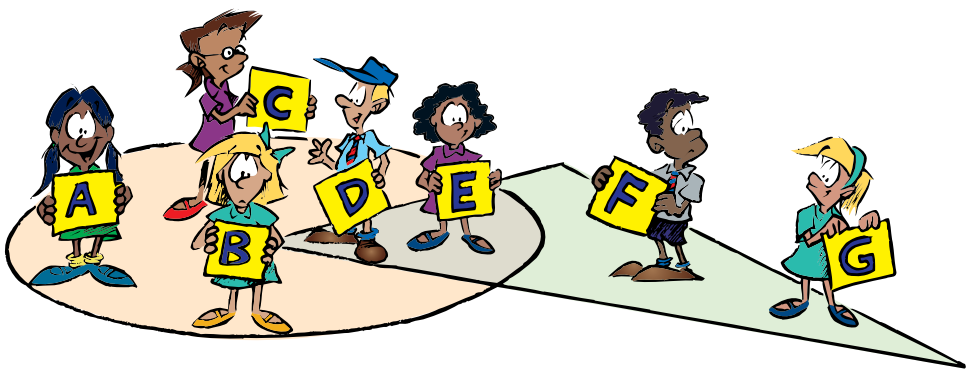
d 6

6:04 Problem solving using Venn diagrams

Sets of things are often referred to in problem solving, and **Venn diagrams** are the most popular method of showing the relationship between sets. A set may have many members, like the set of even numbers less than 70. On the other hand, a set may have no members at all, like the set of people who can run 100 metres in 5 seconds.

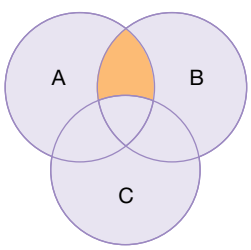


- A *set* is a collection of things.
- We can describe or list the members of a set using braces.
  e.g. {students who have dogs} is described
  {0, 1, 2} is listed
- A Venn diagram is used to represent sets so that we can solve problems more easily. Each set is represented by a *region* (e.g. a circle).

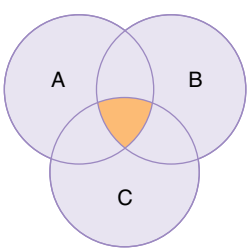


The students in the diagram are standing in the shapes on the playground.

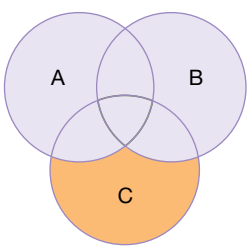
- The set of students inside the oval is {A, B, C, D, E}.
- The set of students inside the triangle is {D, E, F, G}.
- The set of students inside both shapes is {D, E}.
  (This is known as the **intersection** of the two larger sets.)
- The set of students in the oval but not in the triangle is {A, B, C}.
- The set of students in the triangle but not in the oval is {F, G}.



The orange part is in A and B but not C.



The orange part is in A, B and C.



The orange part is in C but not in A or B.

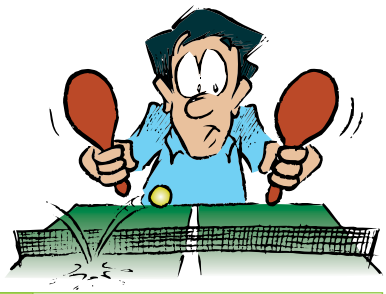
WORKED EXAMPLE 1

Of 30 table tennis players, 12 can play with the left hand but two can play with both the left hand and the right hand.

a How many can only play left-handed?

b How many can play only with the right hand?

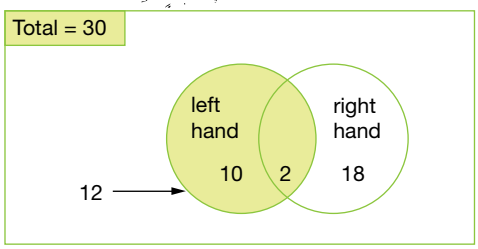
c How many can play right-handed?



Solutions

Use a Venn diagram to illustrate the problem.

- The left circle represents those who can play with the left hand (12).
- The overlap of the circles represents those who can play with either hand (2).



a Those who can only play left-handed = 12 − 2
as two of the 12 can play with the right hand as well.
∴ 10 can only play left-handed.

b The part of the right circle not coloured represents those who can play only with the right hand. Those who can only play right-handed = 30 − 12
∴ 18 can only play right-handed.

c We add the 2 who can play with both hands to the 18 who can only use their right hand.
Those who can play right-handed = 2 + 18
∴ 20 can play right-handed.

WORKED EXAMPLE 2

42 copies of the *Tribute* and 62 copies of the *NSW News* were delivered to 80 homes.

a How many homes received two papers?

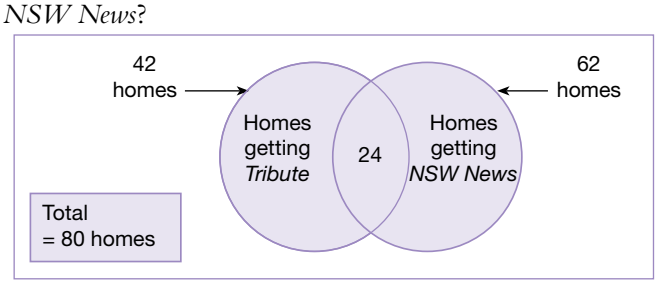
b How many homes received only the *Tribute*?

c How many homes received only the *NSW News*?

Solutions

Use a Venn diagram to illustrate the problem.

- We can assume that each home received at least one paper.
- We also assume that no home received more than one of each paper.

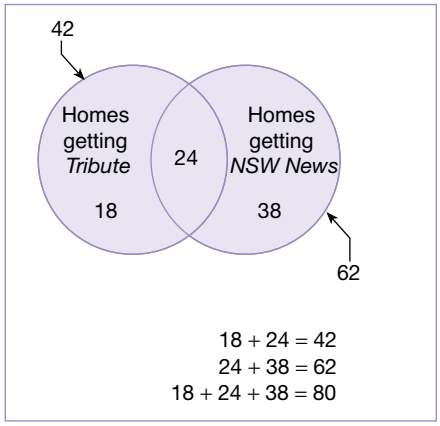




a Total number of papers sold = 42 + 62
= 104
If one paper were delivered to each of the 80 homes 104 − 80 (i.e. 24) papers would remain.
∴ 24 homes received two papers.

b 42 homes received the *Tribute* but 24 of these also got the *NSW News*.
42 – 24 (i.e. 18) homes received only the *Tribute*.
∴ 18 homes received only the *Tribute*.

c 62 homes received the *NSW News* but 24 of these also got the *Tribute*.
62 – 24 (i.e. 38) homes received only the *NSW News*.
∴ 38 homes received only the *NSW News*.

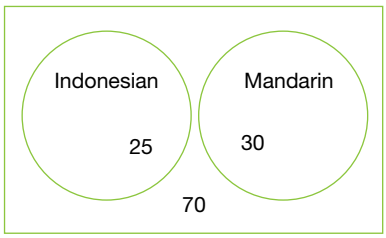


Exercise 6:04

1 The Venn diagram represents all Year 8 students. It shows the numbers studying each language. How many students:

a study Indonesian
b study Mandarin
c study neither language
d do not study Indonesian
e do not study Mandarin
f study both languages
g study either Indonesian or Mandarin?

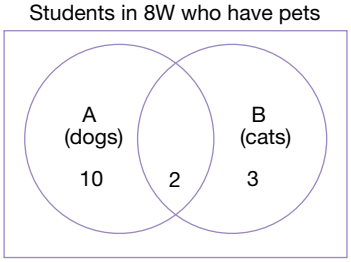
Students in Year 8 who study a language



2 The diagram shows the number of students in 8W who have dogs or cats as pets.
A = {students who have dogs} B = {students who have cats}

a How many students have both dogs and cats?
b How many students have dogs but not cats?
c How many students have dogs?

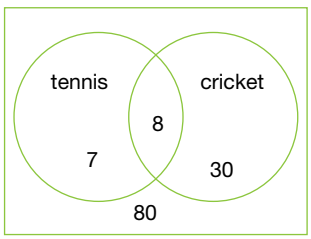
Students in 8W who have pets

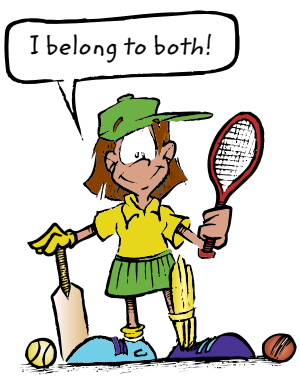


3 The Venn diagram represents two sports played by Year 8 students. How many students:

a play tennis
b play cricket
c play neither sport
d play cricket but not tennis
e play cricket or tennis or both
f play cricket or tennis but not both?

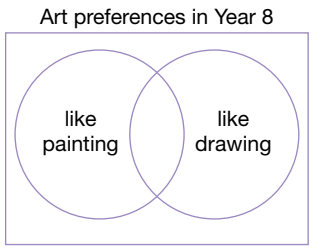
Students in Year 8 who play tennis or cricket





4 Copy this Venn diagram and write the number of students in each part, using the following information.

- 22 students like painting but not drawing
- 37 students like drawing but not painting
- 25 students like both drawing and painting
- 47 students like painting and 62 like drawing
- 41 students like neither painting nor drawing



5 Of 65 table tennis players, 8 can play with the left hand but three of those can play with both the left and the right hand.

a How many can only play left-handed?
b How many can play only with the right hand?
c How many can play right-handed?

6 32 copies of the *Tribute* and 70 copies of the *NSW News* were delivered to 85 homes.

a How many homes received two papers?
b How many homes received only the *Tribute*?
c How many homes received only the *NSW News*?

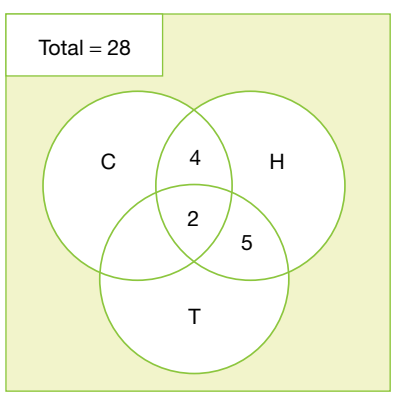
7 The tennis club and the cricket club joined to form one club. There were 40 people in the tennis club and 75 people in the cricket club but only 97 people in the combined club.

a Explain why there were not 115 people in the combined club.
b How many people belonged to both the tennis and the cricket club?
c How many people in the cricket club did not belong to the tennis club?

8 Sandy's cake shop sells white bread and brown bread. During the morning 65 people bought bread. 51 people bought white bread and 37 people bought brown bread. How many people bought both white bread and brown bread?

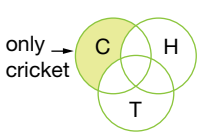
9 The Venn diagram on the right shows three intersecting sets: those who play cricket (C), those who play hockey (H) and those who play tennis (T). A number in one part shows the number of people in that part.

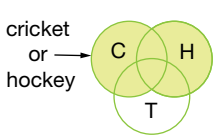
- Our class has 28 students and every student plays at least one of these sports.
- 2 students play all three sports, 4 play only cricket and hockey, 5 play hockey and tennis but not cricket, a total of 5 play cricket and tennis, a total of 12 play hockey, and altogether 15 play cricket.



How many play:

a only hockey
b only cricket and tennis
c only cricket
d only tennis
e tennis
f both cricket and hockey
g cricket or hockey
h tennis or hockey?





10 In our cricket team 5 players like to bowl, 8 players like to bat and one player does not like batting or bowling. If there are 11 players in the team, how many:

a like both batting and bowling
b like batting but not bowling
c like bowling but not batting?

11 100 people were asked what type of TV programs they like out of news (N), movies (M) and documentaries (D). 64 like news, 80 like movies, 2 like only documentaries, 23 like movies and documentaries, 51 like movies and news, 34 like only movies and news, and 10 like only news and documentaries.



a How many of these people like:

i all three types **ii** only news

iii news and documentaries **iv** only movies and documentaries

v only movies **vi** none of these types?

b How many of these people do not like:

i news **ii** movies **iii** documentaries?

12 In order to raise money for a charity, Year 8 were asked to buy cakes, drinks and sandwiches. 58 students bought cakes, 70 bought drinks, 46 bought sandwiches, 10 bought all three, 23 bought both cakes and drinks, 35 bought both drinks and sandwiches, 24 bought only cakes and 14 students bought nothing.

How many students:

a bought cakes and drinks but not sandwiches

b bought drinks and sandwiches but not cakes

c bought cakes and sandwiches but not drinks

d bought cakes and sandwiches

e bought only sandwiches

f bought only drinks

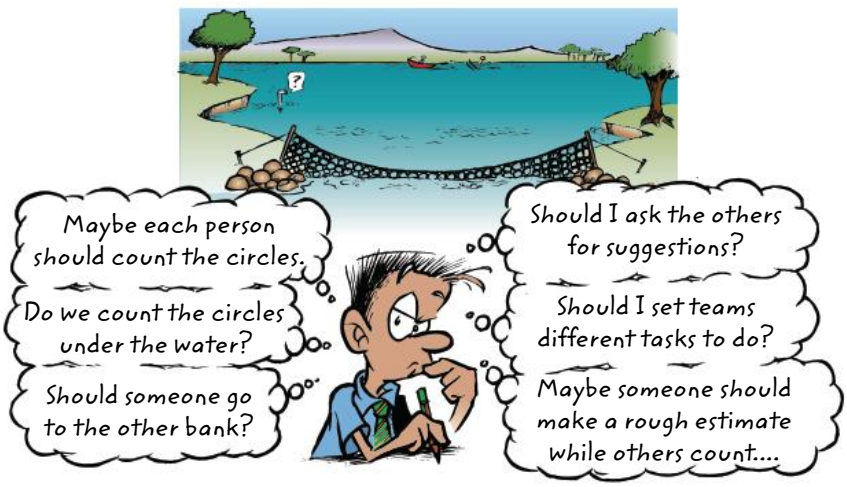
g were in Year 8?



INVESTIGATION 6:04 COUNTING THE RINGS

Mr McSeveny took his students to a cove frequently visited by sharks. He grouped the students in teams of 12 and asked each team to work out a way of getting the best possible estimate of the number of metal rings that can be seen in the shark net below. Some rings are visible under the water, which is 2 m deep at the centre of the 40 m wide net. The teams of students have only 20 minutes to find the number of rings, as accurately as possible.

- You are the captain of a team and must direct your team members in the task.
- Write down your plan and the directions you would give to the other 11 students.

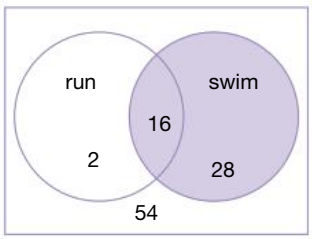


6:05 Problem solving using two-way tables

Venn diagrams and two-way tables have a lot in common.

A survey was given to 100 students to find out how many run or swim regularly. The results are shown below in a Venn diagram and a two-way table.

Students in Year 8 who run or swim



	swim	do not swim	total
run	16	2	18
do not run	28	54	82
total	44	56	100

The same information is found in both diagrams, however the two-way table shows totals. These show that:

- 100 have been surveyed
- 16 students run **and** swim
- 54 **neither** run **nor** swim
- 2 run **but** do not swim
- 28 swim **but** do not run
- 54 do not run **and** do not swim
- 44 swim (16 + 28 = 44)
- 56 do not swim (2 + 54 = 56)
- 18 run (16 + 2 = 18)
- 82 do not run (28 + 54 = 82)
- 46 run **or** swim (inclusive) (2 + 28 + 16 = 46)
- 30 run **or** swim but **not both** (2 + 28 = 30)
- 46 run **or** swim **or both** (2 + 28 + 16 = 46)



Note the use of the words 'and', 'or', 'both', 'neither … nor', 'but' and 'not both'.

Exercise 6:05

1 Year 8 students were surveyed to compare gender (male or female) and handedness (right or left).

a How many students were:

i male ii female

iii left-handed iv right-handed

	left-handed	right-handed	total
female	6	47	53
male	8	64	72
total	14	111	125

v female and left-handed
vi not a left-handed female
vii male and right-handed
viii not a right-handed male
ix neither female nor left-handed
x neither male nor right-handed
xi male or right-handed or both
xii female or left-handed or both?

b From these results, does it appear that males are more or less likely to be left-handed?

c Would 125 people be a large enough sample to form the conclusion that boys are more likely to be right-handed than girls?

2 Use the information below to complete the two-way table that shows those who exercise by running.

- 24 females run
- 25 females do not run
- 31 males run
- 20 males do not run

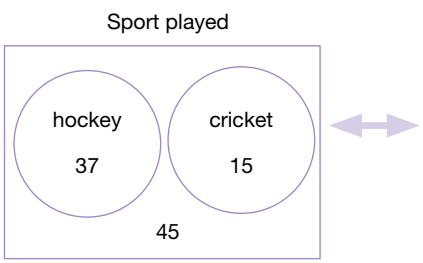
	run	do not run	total
female			
male			
total			

Of these people, how many:

a are female
b are male
c run
d do not run
e run who are not female
f run who are not male?

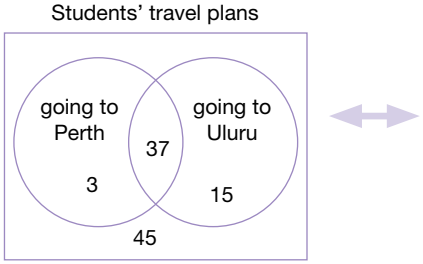
3 Use the Venn diagram to complete the two-way table.

a



	play cricket	do not play cricket	total
play hockey			
do not play hockey			
total			

b



	going to Uluru	not going to Uluru	total
going to Perth			
not going to Perth			
total			

4 Students were surveyed to compare mobile phone ownership to whether they had a part-time job.

	have a part-time job	does not have a part-time job	total
owns a mobile phone	16	2	18
does not own a mobile phone	28	54	82
total	44	56	100

How many students:

a own a phone **and** have a job
b **neither** own a phone **nor** have a job
c have a job **but** do not own a phone
d own a phone **but** do not have a job
e were involved in the survey
f have a job
g do not have a job
h own a phone
i do not own a phone
j do not have a phone **and** do not have a part-time job
k have a phone **or** have a job
l have a phone **or** have a job **but not both**?

If a student in this study is chosen at random, what is the probability that the person chosen:

m owns a mobile phone
n has a part-time job
o owns a mobile phone but does not have a part-time job?

5 Use the information below to complete the two-way table that shows the number of senior students that play the guitar and study mathematics.

- 24 play the guitar and study mathematics
- 25 play the guitar but do not study mathematics
- 55 study mathematics
- 100 students were surveyed in total

	study mathematics	do not study mathematics	total
play guitar			
do not play guitar			
total			

Of these students, how many:

a play the guitar
b do not play the guitar
c do not study mathematics
d study mathematics but do not play the guitar?

6 Construct a two-way table to represent a group of 50 women to show that 30 have at least one son, 35 have at least one daughter and 5 have no children. (Hint: Start by filling in the totals.)

How many women have:

a at least one boy and one girl
b at least one daughter but no sons?

	have a son	do not have a son	total
have a daughter			
do not have a daughter			
total			

MATHS TERMS 6

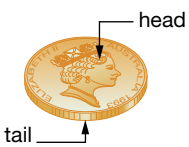
at random
- a way of choosing so that each outcome is equally likely to be chosen

complementary events
- the complement of an event is all possible outcomes outside that event,
  e.g. The complementary event to *throwing a six* is *not throwing a six*
- the probabilities of the event and its complement add up to 1
- If $P(E)$ is the probability that E will occur, then the probability that E won't occur, $P(E')$, is $1 - P(E)$,
  i.e. $P(E') = 1 - P(E)$

equally likely outcomes
- outcomes that have the same chance of happening

even chance (fifty-fifty chance)
- equal chance of happening or not happening,
  e.g. tossing a head in a coin toss
- 50% chance of happening and 50% chance of not happening

heads and tails
- each coin has two sides: a 'head' and a 'tail'



impossible
- cannot happen, not possible
- having a probability of 0 (i.e. 0%)

mutually exclusive events
- events that have no outcomes that are the same (i.e. no common elements)

probability
- the chance of an event happening
- has a range of 0 to 1 (i.e. 0% to 100%)
- for a simple event:

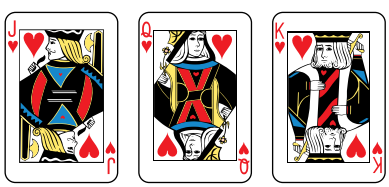
$$P(E) = \frac{n(E)}{n(S)} \text{ where}$$

$n(E)$ = number of outcomes in event E
$n(S)$ = number of possible outcomes (the number in the sample space)

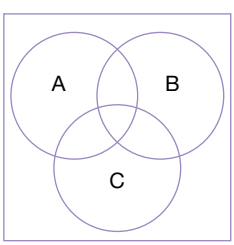
sample space (S)
- all of the possible outcomes,
  e.g. The sample space for tossing a coin is *head*, *tail*

simple event
- an event where each outcome is equally likely

standard pack of cards
- four suits:
  ♥ ♦ ♣ ♠
  hearts diamonds clubs spades
- 52 cards (13 in each of the four suits):
  Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10,
  Jack, Queen and King in each suit
- picture cards (or court cards):
  Jacks, Queens and Kings (12 in total)



Venn diagram
- a diagram that uses circles to show the relationship between events

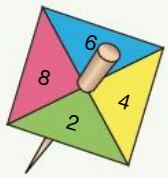


DIAGNOSTIC TEST 6 PROBABILITY

Each section of the test has similar items that test a certain type of question.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

6:01

1 List the sample space for each experiment.
   a A dice is rolled.
   b The spinner to the right is spun.
   c The product of the numbers when this spinner is spun twice.



6:02

2 Write 'mutually exclusive' or 'non-mutually exclusive' for the two events.
   a An even number will be rolled on a dice.
   A number less than 4 will be rolled on a dice.
   b A 'heart' will be chosen from a standard pack of cards.
   A 'black' card will be chosen from a standard pack.

6:02

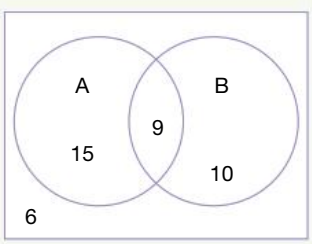
3 If a normal dice is to be rolled once, find the probability of rolling:
   a a number greater than 2
   b a number less than 5 but more than 2.

6:03

4 a A dice is rolled. What is the sum of the probabilities of all possible outcomes?
   b What is the complementary event of 'Year 8 will win the competition'.
   c If there is a 5% chance that we will reach home today, what is the chance that we will not reach home today?
   d A card is to be chosen at random from a standard pack. What is the probability that it will not be a diamond?

6:04

5 At the Year 8 camp, 9 students ate both apples and bananas, 15 ate only apples and 10 ate only bananas. If 40 students were present, how many students:
   a ate apples
   b ate bananas
   c ate either apples or bananas or both
   d ate neither apples nor bananas?



6:05

6 This table refers to a group of tourists that visited Australia. How many of these tourists:
   a were not from the UK
   b did not visit Sydney
   c from the UK visited Sydney
   d who visited Sydney were not from the UK
   e were there in total?

	from UK	not from UK	total
visited Sydney	33	42	75
did not visit Sydney	28	35	63
total	61	77	138

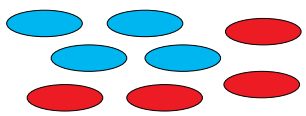
ASSIGNMENT 6A Chapter review

1 Choose one of these labels for the probability of each event described below.

impossible	very unlikely	unlikely
	even chance	
likely	very unlikely	certain

a I will roll a 6 on a normal dice.
b My dog will turn into a butterfly.
c When I toss a 20c coin, it will be a head.
d When two normal dice are rolled, I will obtain a total of 12.

2 Jack placed four red counters and four blue counters in a hat. He will choose a counter at random and place this counter on the desk in front of him before choosing another counter from the hat. He will do this five times until he has five counters on the desk before him.



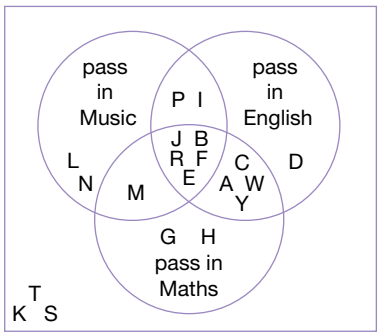
a What is the probability that the first counter chosen will be red?
b If the first counter was red, what is the probability that the second counter will be red?
c If the first two counters were red, what is the probability that the third counter will be red?
d If the first three counters were red, what is the probability that the fourth counter will be red?
e If the first four counters were red, what is the probability that the fifth counter will be red?

3 A card is chosen at random from a standard pack of 52 cards. What is the probability that the card will be:
a a 4
b anything but a 4
c an even number
d an even number that is red
e a picture card
f a picture card that is a club?

4 A Joker is added to a standard pack of cards so that the pack now contains 53 cards. A card is chosen at random from this pack. What is the probability that the card will:
a be the Joker
b not be the Joker
c be a red card
d be a red card that is even
e be a picture card
f be a picture card that is a club?

5 **a** Write the complementary event for 'Year 8 will win its next game'.
b The chance of spinning red on my spinner is 20%. What is the chance of not spinning red?
c The probability that I will win the raffle is $\frac{3}{500}$. What is the chance that I will not win?
d The chance of a light globe made in our factory being faulty is 0·001. What is the chance that one of our light globes, chosen at random, will not be faulty?

6 Each of 20 students wrote the first letter of their name in the part of the Venn diagram that described their test results.



How many students passed in:

a Music **b** English **c** Maths

d all three subjects

e both English and Maths

f only music

g only Music and English

h English and Maths but not Music?

How many students did not pass in:

i Music **j** English **k** Maths

l any of the three subjects?

7 Year 8 students were surveyed on whether they lived near a beach and if they could swim. Of the 57 who lived near a beach, 51 could swim. Of the 63 who did not live near a beach, 4 could not swim. Complete the following table and use it to find the number of students who:

a can swim

b cannot swim

c are in Year 8

d live near a beach but cannot swim.

	can swim	cannot swim	total
live near a beach			
do not live near a beach			
total			

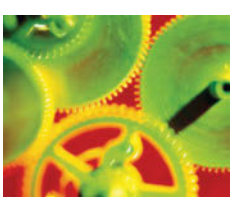
ASSIGNMENT 6B Working mathematically

1 The cost of buying a number of sound systems is the marked price multiplied by the number of systems.

a How many sound systems are involved if the cost is $5683.89? (Try *guess and check.)*

b What is the cost of one system?

2 **a** Explain how the turning of one wheel can affect other wheels.



b Estimate the number of cogs (teeth) that are on the large wheel at the top left of the photograph.

3 A toy soldier is approximately $\frac{1}{32}$ of a real soldier's height. Estimate the height of the toy soldier.

4 In the game *Take Two*, 100 *Scrabble* tiles are used. Each player starts with four tiles. When someone completes their crossword using all tiles they have taken, that person says 'Take Two!' and each person draws two more tiles. This continues until insufficient tiles remain for each player to take two. Scoring is similar to *Scrabble*. If three people played in the game, how many times would each person draw two tiles?

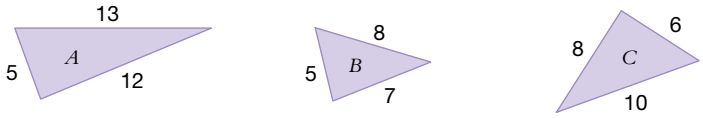
5 How many different sizes of rectangle can be made using exactly 24 tiles? (A rectangle 4 tiles long and 6 tiles wide is the same as a rectangle 6 tiles long and 4 tiles wide.)

6 Each side of a 4 cm square is increased by 7 cm. What has been the increase in:

a perimeter **b** area?

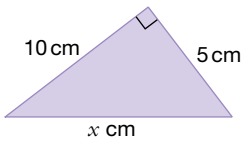
ASSIGNMENT 6C Cumulative revision

1 Which of these triangles would be right-angled? 3:01

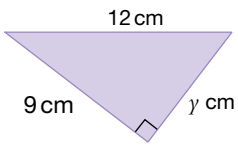


2 Find the value of the pronumerals, correct to two decimal places. 3:02, 3:03

a

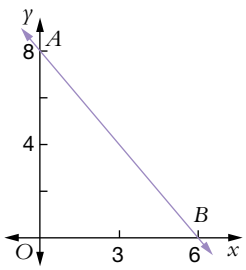


b



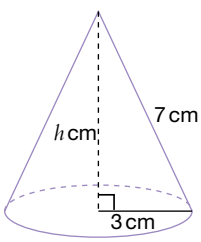
3 **a** Is ΔOAB a right-angled triangle? 3:02

b Find the length of AB.



4 Would a triangle that had sides of length 10 cm, 24 cm and 26 cm be a right-angled triangle? If so, then would $\{10, 24, 26\}$ be a Pythagorean triad? 3:05

5 The radius of this cone is 3 cm and its slant height is 7 cm. Find the height, h, of the cone. 3:04



6 Change to a percentage. 4:02

a 0·375 **b** $\frac{7}{8}$

7 Convert to a decimal. 4:02

a 4·5% **b** $12\frac{1}{2}\%$

8 **a** Find 7% of 1400 g. 4:03, 4:04, 4:05

b What fraction is 24 seconds of 3 minutes?

c A sandwich contains 10 g of meat, 50 g of bread and 4 g of cheese. Find the percentage, by mass, of cheese.

d Increase \$12 by 15%.

e Decrease \$12 by 15%.

9 Declan sells books for a living. He is given a commission of 6% on the first \$15 000 of sales made and 4% on sales in excess of \$15 000. How much commission would he make on sales of \$45 000? 4:06

10 Use the rule $y = 4x + 11$ to complete this table. 5:01

x	0	1	2	15
y				

11 What rule connecting x and y is shown in this table? 5:01

x	0	1	2	3
y	1	11	21	31

12 Simplify: 5:02, 5:03, 5:04

a $7m - 5m$ **b** $8x - x$ **c** $ab + 3ab$ **d** $6x^2 + 2x^2 - 3x^2$

e $2a + b + 7a$ **f** $6a - 13a$ **g** $-3 \times 6y$ **h** $7a \times 4b$

i $24m \div 3m$ **j** $6a^2 \div 2a$ **k** $8a \div 8 \times a$ **l** $2ma \times 3a \div m$

13 Evaluate each expression if $a = 10$, $b = 7$ and $c = 4$. 5:01

a $ab - 4$ **b** $a^2 + b^2$ **c** $c - 2a + b$ **d** $bc - a^2$

14 Simplify: 5:07, 5:08

a $m \times m \times m \times m$ **b** $5 \times a \times a \times a$ **c** $4(3x + 1) + 8$ **d** $2(9m - 5) - 3$

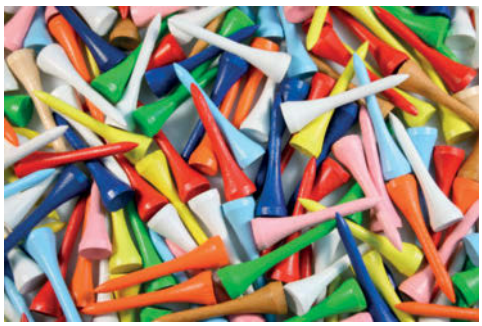
e $\frac{9x}{10} - \frac{6x}{10}$ **f** $\frac{m}{3} + \frac{m}{2}$ **g** $\frac{a}{5} \times \frac{2a}{3}$ **h** $\frac{5x}{6} \div \frac{2}{3}$

15 Factorise: 5:09

a $2x + 14$ **b** $m^2 - 3m$ **c** $22 - 33y$ **d** $9ab + 12a^2$



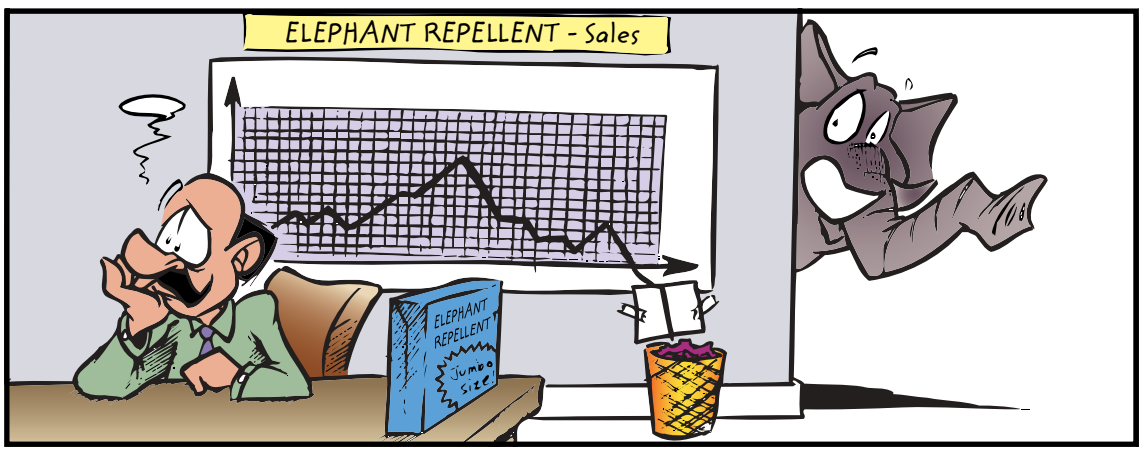
Find the probability of selecting each colour at random.
What is the sum of all of these probabilities?



Estimate the probability that a green tee would be chosen at random.

GRAPHS

7



Contents

7:01 The picture graph or pictogram
Fun spot 7:01 A number hex
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7:03 The divided bar graph
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7:05 The line graph
Investigation 7:05 Graphs can be sneaky!
7:06 The conversion graph
Investigation 7:06 Collecting graphs
7:07 The step graph
Investigation 7:07 Spreadsheet graphs
7:08 Miscellaneous and unusual graphs
Maths Terms, Diagnostic Test, Assignments

Syllabus references (See page x for details.)

Statistics and Probability

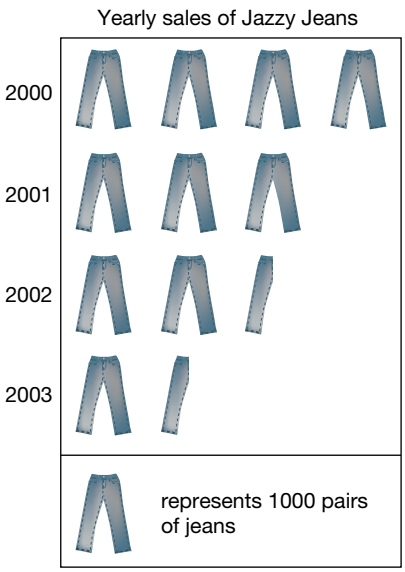
Selection from ***Data Collection and Representation*** [Stage 4]

- Construct and compare a range of data displays including stem-and-leaf plots and dot plots. (ACMSP170)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

7:01 The picture graph or pictogram





A picture graph can be used to display categorical data. The categories for this graph are the years on the vertical axis.



Features

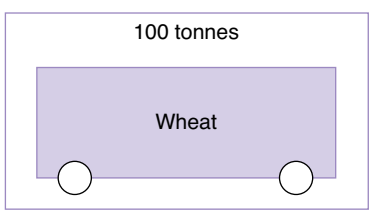
1. The images in the pictogram catch the attention of the reader.
2. The pictogram creates an impression rather than giving detailed information.
3. The pictogram allows you to compare the size of each category.
4. It is easy to understand.

Details

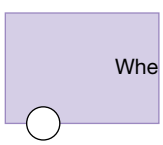
1. The pictogram must have a title or heading.
2. Only one axis is necessary.
3. Each symbol stands for a definite number of items. This information will be shown on the graph.
4. The axis is given a label (or name) unless the meaning is obvious.

Exercise 7:01

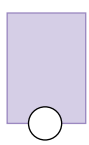
1 In picture graphs a symbol, or picture, can be divided into parts. What mass of wheat would be represented by each picture below?



a

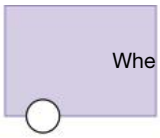


b



c





d



2 This graph shows the population of the mainland states of Australia.

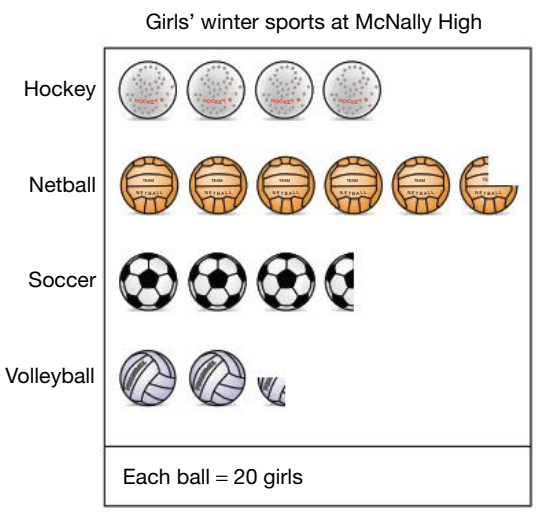


a The graph gives a clear picture of the relative population of each state compared to another.
   i Which state has the greatest population?
   ii Which state has the least population?

b The graph can only give an approximate representation of the actual population of each state. Determine how many people you think live in each state as shown by this picture graph. Compare your answers to the actual figures in the table below.

State	Pop (× 1000)
NSW	7318
Vic	5641
Qld	4599
SA	1660
WA	2367

3 This graph shows the winter sports played by girls at McNally High School.



a How many girls play each sport, according to this pictogram?

b How many more girls play soccer than volleyball?

c How many girls are enrolled at the school if every girl plays one sport?

d How many girls need to switch from netball to hockey, so that there are more girls playing hockey than netball?



4 This table shows the results of a survey taken of 200 people when asked about their favourite music. Complete the picture graph below using a CD symbol to represent 10 people.

Music	Number of people
Rock	
Country	
Pop	
Folk	
Jazz	
Classical	
	1 CD = 10 people

Music type	Number of people
Rock	45
Country	30
Pop	48
Folk	25
Jazz	32
Classical	20

5 The following table shows the new car sales for March 2012 in Australia for the top six manufacturers, to the nearest 100 cars.

a If a car symbol, like the one shown, is used to represent sales of 2000 cars, about how many symbols are needed to show Toyota's sales?

b Draw a picture graph showing this categorical data, which is divided into the categories of car manufacturers.



Australian new car sales	
Toyota	18 500
Holden	10 200
Mazda	9 300
Nissan	8 300
Hyundai	7 800
Ford	7 500

6 120 students were surveyed to find which type of fruit they preferred between apples, oranges, bananas and grapes. The results are shown in the pictogram below. Each fruit symbol represents 10 students.

Fruit	Number
Apples	
Oranges	
Bananas	
Grapes	
	1 fruit symbol = 10 students



a How many students selected each type of fruit, according to this picture graph?

b Arrange the types of fruit in order of popularity.

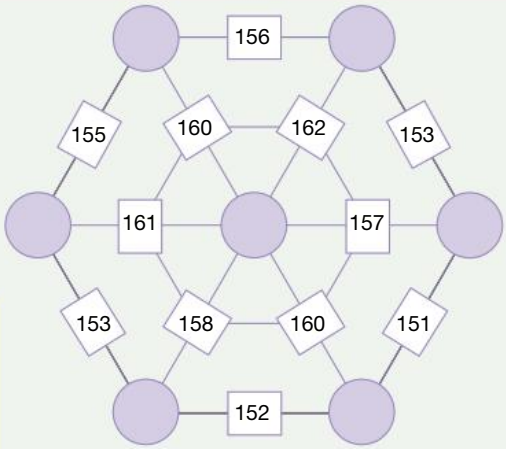
c Does the appearance of the graph match with your answer to part b?

d What false impression might this graph give to its readers?

FUN SPOT 7:01 A NUMBER HEX

The numbers 74, 75, 77, 77, 78, 79 and 83 must be placed in the coloured circles so that the number in each square is the sum of the numbers in the circles on either side of it.

Which of these numbers must be placed in the centre circle?





Guess a number for the centre circle. To check, work out the numbers that then go in the other circles.

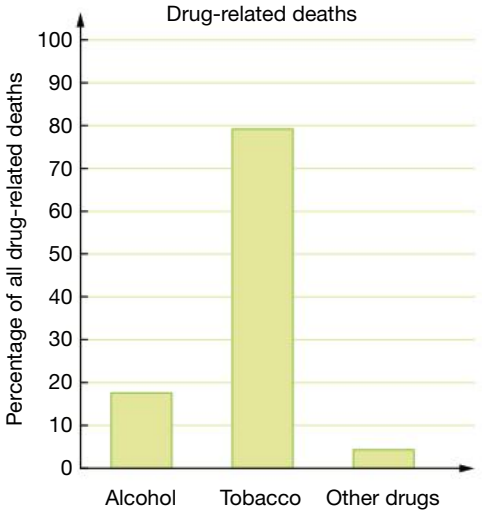
7:02 The column graph

Features

1. The column graph is easy to read and to understand.
2. It is easy to draw.
3. The graph allows comparisons to be made at a glance.
4. The column graph shows more detail than a picture graph.

Details

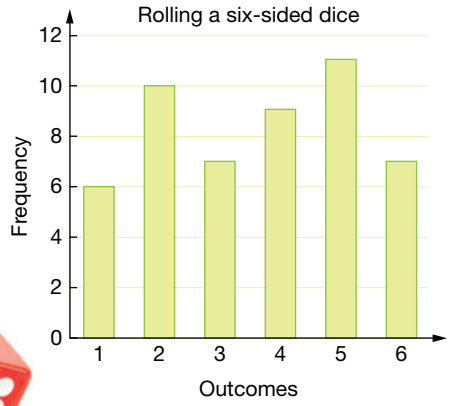
1. The column graph must have a title or heading.
2. Both vertical and horizontal axes are used.
3. Axes must be given labels (or names) unless the meaning is obvious.
4. The columns are rectangles, although vertical lines are sometimes used as columns.
5. An axis may represent non-numerical data.
6. Where numbers are used, care must be taken to choose a suitable scale for the axis.



This represents 18·7% of all deaths recorded, or 7352 of 39 316.
Source: Australian Bureau of Statistics

This graph on drug-related deaths displays **categorical** data.
The categories are alcohol, tobacco and other drugs.

The graph on dice rolls displays **discrete numerical** data.
The data are the outcomes, which are the numbers on a dice {1, 2, 3, 4, 5, 6}.



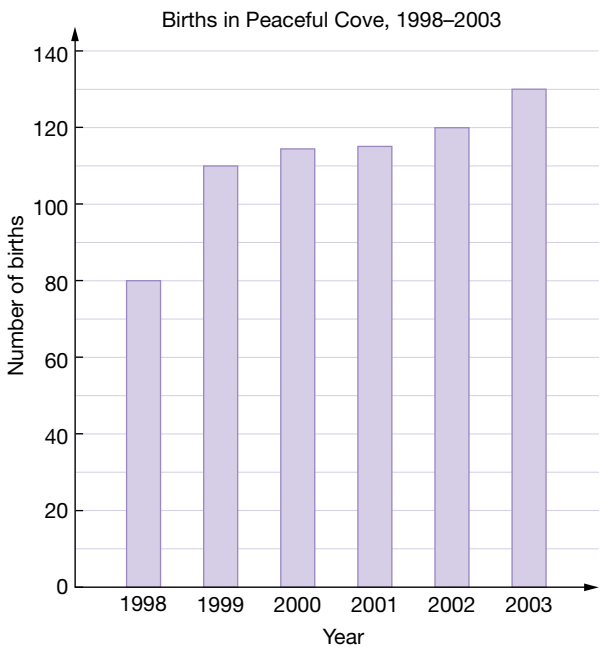
- The results of rolling the dice are clear to see. The outcome that occurred most was '5' and the outcome that occurred least was '1'.
- The total number of times the dice was rolled can be found by adding together the frequency of each column. Check you can see the dice was rolled 50 times.



WORKED EXAMPLE

To draw a column graph, follow the steps below. The details of births in Peaceful Cove are given in the following table.

Year	Births
1998	80
1999	110
2000	115
2001	116
2002	120
2003	130



Solution

Step 1 Choose a title.
We could use 'Births in Peaceful Cove 1998–2003'.

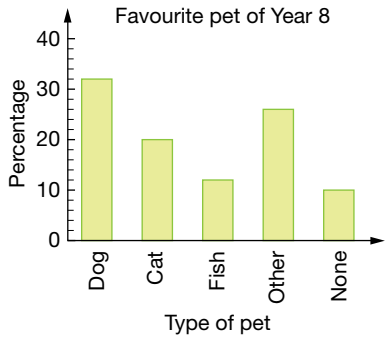
Step 2 Choose a scale and name for each axis.
The horizontal axis is usually used for time.
The greatest number of births is 130. We choose a scale (such as 1 cm = 20 births) so that the graph will fit our space.

Step 3 Choose a width for each column and draw the graph.
Draw the columns equal distances apart.

Exercise 7:02

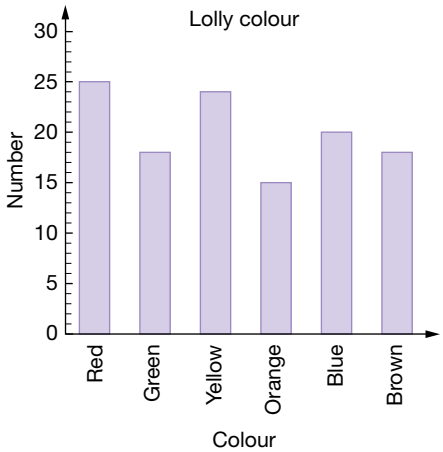
Foundation worksheet
Picture graphs and column graphs

1 This column graph shows the results of a survey taken of Year 8 students when asked about their favourite pet.



- **a** What is measured by the vertical axis?
- **b** Which type of pet was the most popular?
- **c** Can you tell which type of pet was the second most popular?
- **d** If there were 250 students in Year 8, how many preferred cats?
- **e** If 6 students preferred fish, how many students altogether must be in Year 8?

2 A packet of lollies was opened and the number of each colour was counted. The results are shown by this column graph.



- **a** Use the graph to complete the following table.

Colour	Red	Green	Yellow	Orange	Blue	Brown
Number	25					

- **b** Which two colours had the same number of lollies in the packet?
- **c** How many orange lollies were there?
- **d** How many lollies were in the packet?
- **e** What percentage of the lollies were green?

3 This table shows the number of children born to women in a suburb of Sydney.

Number of children	Frequency	Percentage (%)
None	315	10
1	327	11
2	1159	38
3	785	25
4	315	10
5	115	4
6 or more	66	2
Total	3082	100

Draw a column graph showing the percentages for each category (i.e. number of children born). Before you start, answer these questions.

- **a** How many columns are needed?
- **b** How will you label each axis?
- **c** What is the highest number needed on the vertical axis?
- **d** What scale will you use on the vertical axis?
- **e** Now construct your column graph.

4 Draw a column graph for each of the following sets of data.

a These are the numbers of road accidents in Peaceful Cove from 1999 to 2003.

Year	1999	2000	2001	2002	2003
Number of accidents	100	130	140	170	200

b These are the favourite colours of Year 8 students at Peaceful Cove College.

Colour preference	Green	Pink	Yellow	Blue	Red	Other
Number of students	228	210	90	200	114	150

5 Patty decided to start a pastry shop. At first it was unsuccessful but, as word got around that her pastries were delicious, business improved. The graph shows her profit or loss for the first six months of operation.

a In which month was the greatest profit made? What was the profit then?

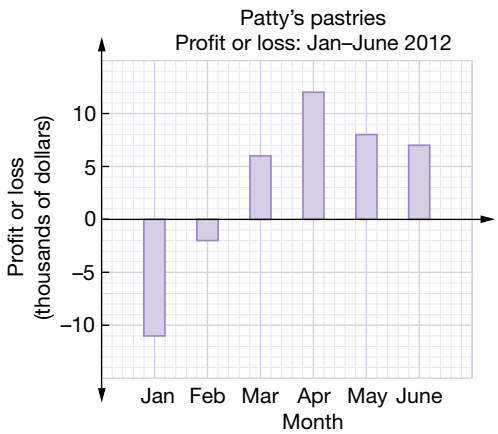
b In which month did the shop make a profit of $6000?

c What was the profit or loss for January?

d What was the reason for the continued improvement from January to April?

e In which month was the greatest improvement made on the previous month?

f For the six months January to June, what was the total profit made?

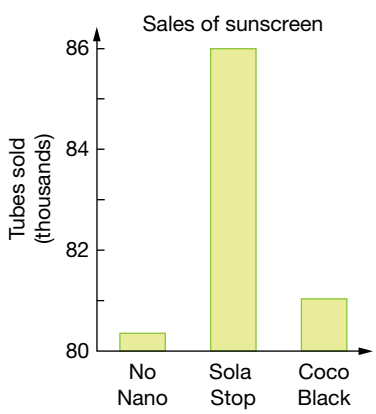


6 This column graph shows the sales of 75 g tubes of three brands of sunscreen for one summer.

a 'Sola Stop' appears to have significantly more sales than the other brands. Is this true? Give a reason for your answer.

b How could this graph be drawn to give a more accurate representation of the sales of sunscreens?

c Is this graph showing false information or is it showing the information falsely?

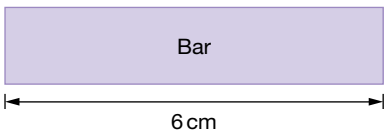


7:03 The divided bar graph

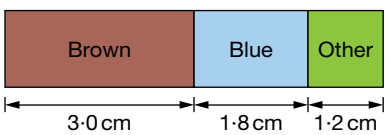
Features

1 The divided bar graph makes use of a rectangle.
2 The divided bar graph is uncomplicated in its appearance and takes up little space. (The measurements shown here are not usually given.)
3 The divided bar graph shows how the whole is divided according to each category. The length of each category represents the percentage of that category to the whole.
4 You can compare the sizes of categories easily.
5 The divided bar graph doesn't show details except the percentages of the parts, which may be given on the graph.

Colour	Brown	Blue	Other
Percentage	50	30	20

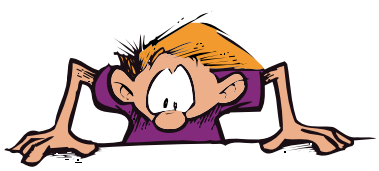


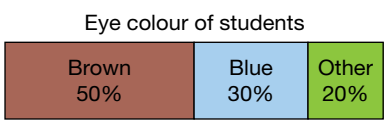
Note: 50% of 6 cm = 3.0 cm
30% of 6 cm = 1.8 cm
20% of 6 cm = 1.2 cm



Details

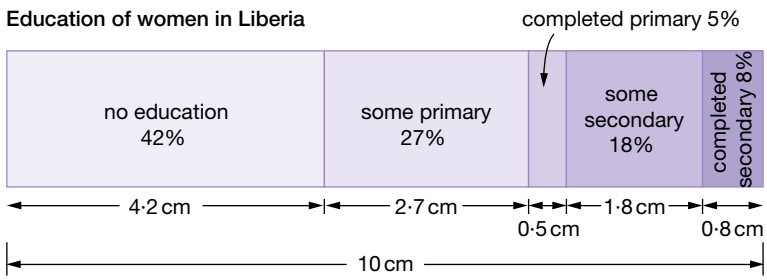
1 The divided bar graph must have a title or heading.
2 No axes are necessary.
3 When drawing the graph, decide on a length for the bar (10 cm is ideal). Find what fraction or percentage each category is of the whole, then divide the length of the bar into these percentages. Name each part of the bar with the category.
4 Measurements of length are not usually placed on the graph. The percentage of each category is often included.





WORKED EXAMPLE 1

This divided bar graph shows the education attained by women aged 15 to 49 in Liberia according to the Liberia Demographic Health Survey (2007).



Note: 42% of 10 cm = 4.2 cm
27% of 10 cm = 2.7 cm
5% of 10 cm = 0.5 cm
18% of 10 cm = 1.8 cm
8% of 10 cm = 0.8 cm

a What is the title of the graph?
b If the population of Liberia is 3 500 000, how many women aged 15 to 49 completed seconday school?
c What percentage on the graph is represented by 1 cm?
d What length is represented by 70%?

Solutions

a The title is 'Education of women in Liberia'.
b Number who complete secondary school
= 8% of 3 500 000
= 280 000
c Percentage represented by 1 cm
= 0·1 × 100
= 10%
d Length represented by 70%
= 70% of 10 cm
= 7 cm or 70 mm



WORKED EXAMPLE 2

Draw a divided bar graph to show the sport preferences of Year 8 students at Eureka High School.

Sport	Tennis	Cricket	Softball	Swimming	Total
Number of students	42	46	52	70	210

Solution

To draw a divided bar graph, follow these steps.

Step 1 Choose a title.
We could use 'Year 8 Sport Eureka High'.

Step 2 Choose an appropriate length for the bar.
Usually, the number in each category is converted to a percentage of the whole and a bar of length 10 cm is used. This allows each millimetre to represent 1% of students.

Tennis = $\frac{42}{210} \times 100\% = 20\%$ Cricket = $\frac{46}{210} \times 100\% \div 22\%$

Softball = $\frac{52}{210} \times 100\% \div 25\%$ Swimming = $\frac{70}{210} \times 100\% \div 33\%$

Step 3 Draw the graph.
Use a ruler to draw a bar of length 10 cm and then divide it into the following parts.
Tennis = 20% of 100 mm = 2.0 cm Cricket = 22% of 100 mm = 2.2 cm
Softball = 25% of 100 mm = 2.5 cm Swimming = 33% of 100 mm = 3.3 cm

Year 8 sport Eureka High



Exercise 7:03

1 This simple divided bar graph shows the numbers of wallabies and koalas in a wildlife reserve.

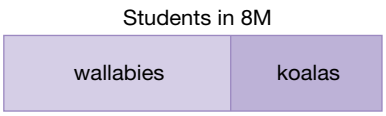
a Are there more wallabies or koalas represented by the graph?

b What is the length of the whole bar?

c What is the length of the bar that represents wallabies?

d What percentage of the bar represents wallabies?

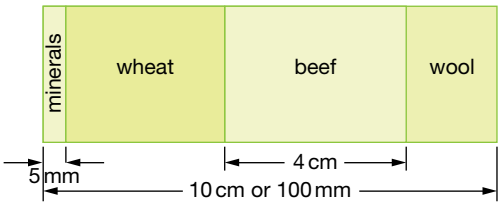
e If there are 25 animals in total, how many are wallabies?



2 A particular country's exports are shown in this divided bar graph. Find what percentage of the country's exports is taken up by:

a beef

b minerals.



3 This table shows how the students of Gibbsville get to school each day. Answer the following questions and then draw a divided bar graph to show this information.

Mode of transport	Walk	Car	Bicycle
Number of students	300	120	60

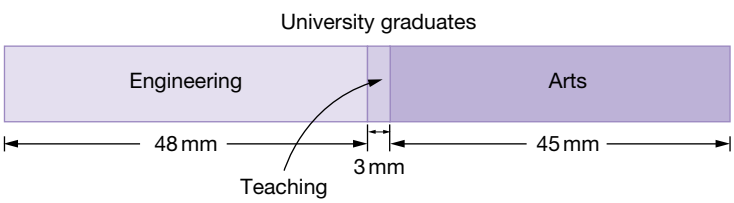
a How many students live in Gibbsville?

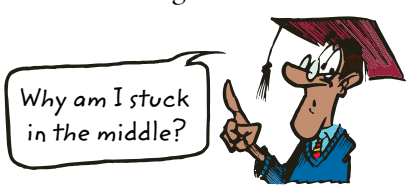
b Calculate the percentage of students that use each mode of transport.

c If a bar of length 8 cm is used to draw the graph, how long would the section be for each mode of transport?

d Draw a divided bar graph of length 8 cm. Write the percentages on the graph and give the graph a title.

4 A survey of university graduates included some engineering and arts graduates, and one from a teaching course. The length of the bar representing the graduate from teaching is 3 mm.





a How long is:

i the whole bar **ii** the bar representing Engineering **iii** the bar representing Arts?

b How many teachers are in the group?

c How many times as long as the teacher's section is the engineers' section?

d How many engineering graduates are in the group?

e How many times as long as the teacher's section is the arts graduates' section?

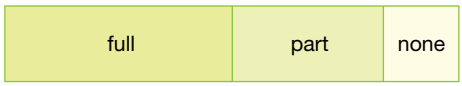
f What percentage of the group:

i are engineers **ii** is the teacher **iii** are arts graduates?

5 Graphs are sometimes drawn incorrectly or are skewed to influence readers. We need to be able to notice errors in graphs. For each part below, select the graph that correctly matches the information given.

a Of 60 children on the bus, 30 were in full uniform, 20 were partly in uniform and 10 were not in uniform.

A



Uniform

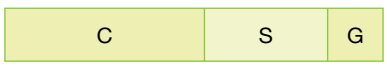
B



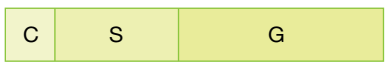
Uniform

b Concrete is made by mixing 1 part of cement (C) with 3 parts of sand (S) and 5 parts of gravel (G).

A



B



6 The divided bar graph gives the breakdown of a country's imports.



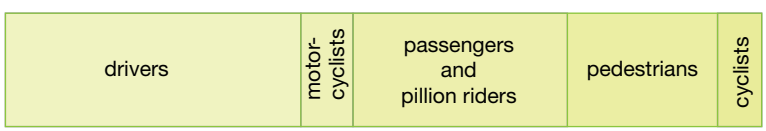
a Measure the length of the bar. This length represents 100% of the country's imports. What length represents 1%?

b Does this graph allow us to calculate the monetary value of any of the country's imports?

c What is the largest import?

d What percentage of the imports is made up by 'foodstuffs'?

7 Road traffic fatalities by type of road user.



a What percentage of fatalities were drivers?

b What percentage of fatalities were motorcyclists?

c Fewer motorcyclists were killed than drivers, so can you say that it is safer to be a motorcyclist than a driver? Why or why not?

8 Draw a divided bar graph for each of the following sets of data.

a These show the favourite pets of 200 children at Eureka High School.

Favourite pet	Guinea pig	Cat	Dog	Bird	Other
Number of children	40	30	84	20	26

b Year 8 students were asked to choose one of four subjects as a course for Year 9. Their responses are shown below.

Course chosen	Photography	Music	Art	Computing
Number of students	15	36	42	57

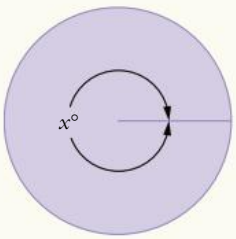
7:04 The sector graph or pie graph

PREP QUIZ 7:04

Convert the following fractions to percentages.

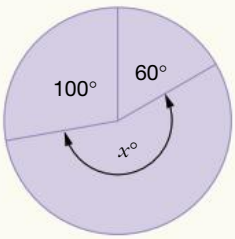
1 $\frac{7}{10}$ **2** $\frac{2}{5}$ **3** $\frac{3}{8}$

4



$x =$ _____

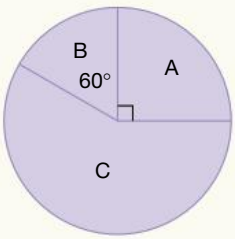
5



$x =$ _____

In the diagram, what fraction of the circle is:

6 sector A? **7** sector B? **8** sector C?



9 Find 25% of 360. **10** Simplify $\frac{210}{360}$.

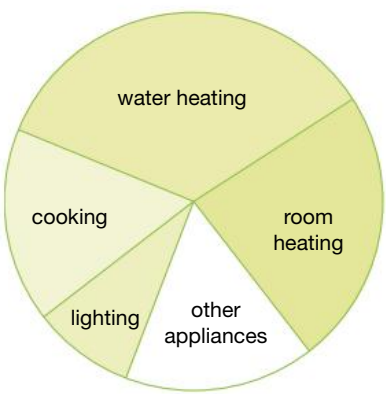
In a sector graph, a circle represents the whole and the parts are represented by sectors, or 'slices of pie'. For this reason, sector graphs are often referred to as pie charts or **pie graphs**.

Features

1. The sector graph is uncomplicated and appeals to people because it's like cutting up a pie.
2. Like the divided bar graphs, one whole (360° at the centre) is divided into parts according to the size of the categories.
3. Sizes of categories are easily compared.
4. The sector graph is attractive yet takes up little space.

Details

1. The sector graph must have a title or heading.
2. No axes are necessary.
3. Use mathematics to find the centre angle for each category. That is, find what fraction or percentage a category is of the whole. Give that fraction or percentage of the 360° at the centre to that category.

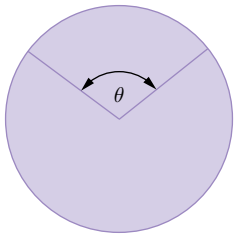


Household energy usage

This sector graph shows the usage of electricity in the average household.

WORKED EXAMPLE 1

Look at the sector graph for household energy usage above. The size of each sector is determined by the sector angle. The larger the sector angle, the larger the sector, and hence the larger that part is of the whole. What fraction and percentage of household energy is used to heat water, if the sector angle is 126°?



θ is the sector angle. If the complete circle was shaded, the sector angle would be 360°. ∴ to find the fraction of the whole, form the fraction $\frac{\theta}{360°}$.

Solution

$$\text{Fraction of household energy usage} = \frac{\cancel{126}^{63}{}^{7}}{\cancel{360}_{180}{}_{20}}$$

$$= \frac{7}{20}$$

$$\text{Percentage of household energy usage} = \frac{7}{20} \times \frac{100}{1}\%$$

$$= 35\%$$

WORKED EXAMPLE 2

Details of the make-up of Goanna Central School are shown in the table. Draw a sector graph to represent the information.

Infants	Primary	Secondary	Adults	Total
30	44	60	10	144

Solution

Follow these steps to draw the sector graph.

Step 1 Choose a title.
We could use 'Make-up of Goanna Central School'.

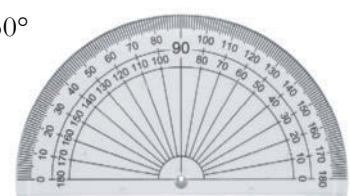
Step 2 Work out the angles at the centre.
Ask: 'What fraction is this category of the whole?' Then find this fraction of 360° to calculate the angle at the centre.

Infants = $\frac{30}{144} \times 360° = 75°$

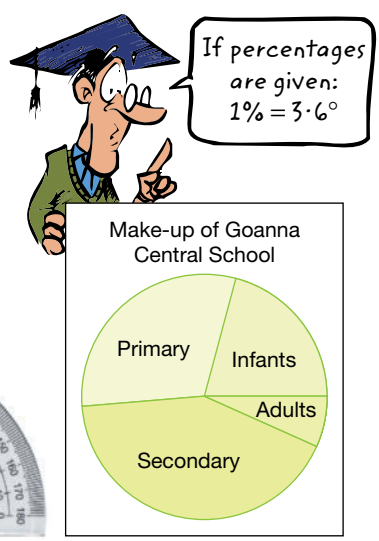
Primary = $\frac{44}{144} \times 360° = 110°$

Secondary = $\frac{60}{144} \times 360° = 150°$

Adults = $\frac{10}{144} \times 360° = 25°$

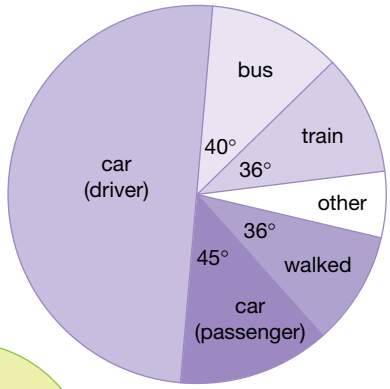


Step 3 Draw the graph.
Choose a suitable radius for the circle.



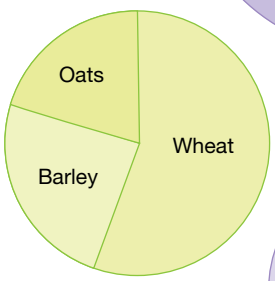
Exercise 7:04

1 The sector graph shows the method of travelling to work for all persons.



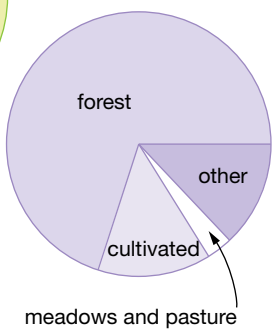
- **a** What percentage of the workforce caught a train to work?
- **b** What percentage of the workforce was driven to work by car?
- **c** What is the size of the sector angle for other means of transport?
- **d** What percentage of the workforce used a car to get to work?

2 This sector graph shows the area of crops planted in a particular region.



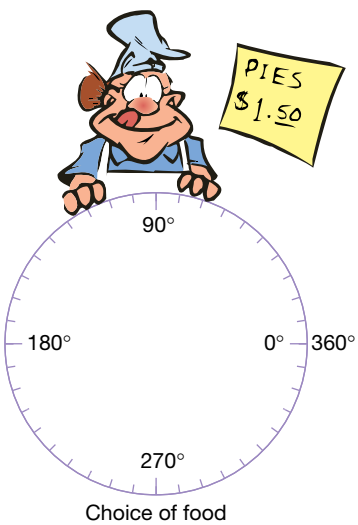
- **a** Measure the angle at the centre of the oats sector.
- **b** What percentage of the crop area is oats?
- **c** Find the area planted with oats if the total area is 300 000 hectares.

3 This sector graph shows land use in a rural community.



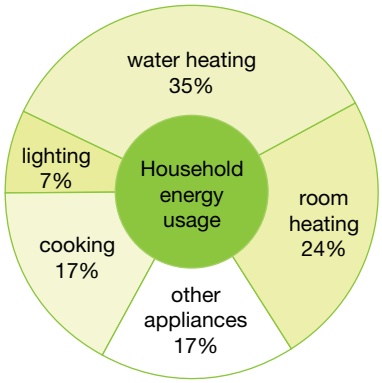
- **a** Measure the angle at the centre of the forest sector.
- **b** What percentage of land is forest?
- **c** What percentage of the land is not forest?
- **d** What percentage of land is cultivated? Round your answer to the nearest whole percentage.

4 Sandwiches, pies and sausage rolls were sold at the canteen. Of 36 people who bought lunch, 18 bought sandwiches, 9 bought pies and 9 bought sausage rolls.



- **a** When drawing a sector graph, 360° at the centre will represent 36 choices. How many degrees at the centre would represent one choice?
- **b** What fraction of the people ordered:
  - **i** sandwiches **ii** pies **iii** sausage rolls?
- **c** What angle at the centre must be used to represent the choice of:
  - **i** sandwiches **ii** pies **iii** sausage rolls?
- **d** Draw a sector graph to represent the choice of food.

5 This pie graph shows household energy use.



- **a** What percentage of the use is taken up by room heating?
- **b** Calculate the angle at the centre of the circle for each sector, to the nearest degree.
- **c** If the annual electricity bill is $2640, how much of this bill is for lighting?

6 Draw a sector graph for each of the following sets of data.

- **a** Of the $72 Rhonda spent on her trip to Sydney, $8 was spent on fares, $32 on food, $22 on entertainment and $10 on a gift.
- **b** Percentages (by mass) of ingredients I used to make concrete are given below.

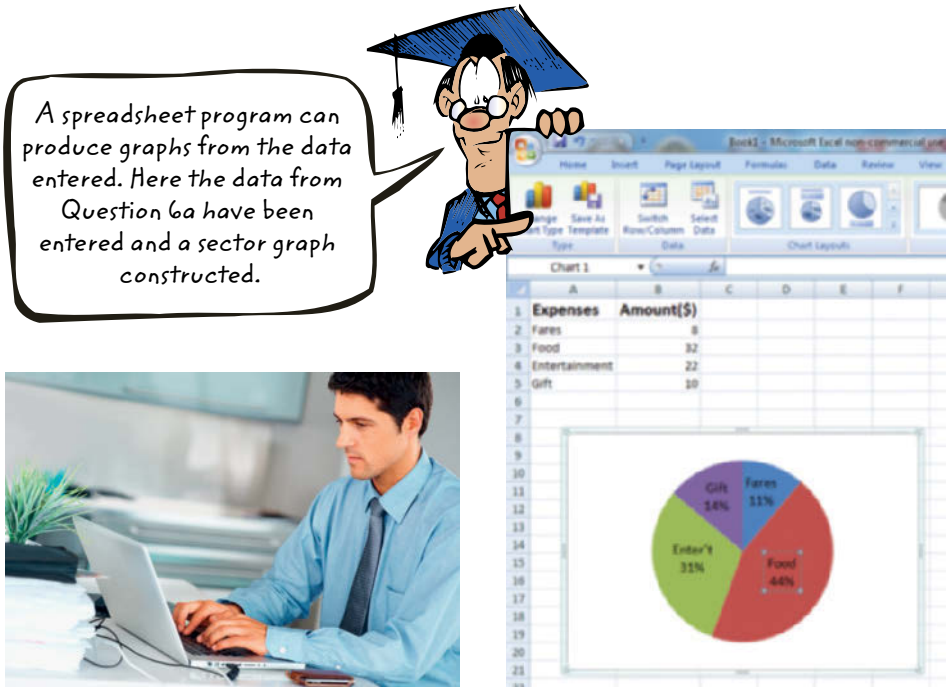
Type of ingredient	Gravel	Sand	Cement
Percentage used	45	36	19

- **c** The methods of travel to school of 90 students are given below.

Methods of travel	Bus	Car	Walk	Train
Number	40	22	20	8

- **d** The table below gives the favourite swimming strokes of 80 swimmers.

Favourite stroke	Freestyle	Breaststroke	Backstroke	Butterfly
Number	40	20	14	6



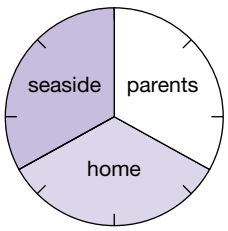
7 This table shows the distribution of blood types for the Australian population. Calculate the angles needed to match each percentage and construct a sector graph showing this categorical data.

Australian blood types	
Blood type	Percentage
O	49
A	38
B	10
AB	3

8 For each part below, select the graph that correctly matches the information given.

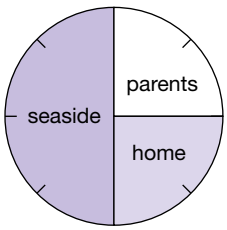
a Jan spent 10 days of her holiday at the seaside, 4 days visiting her parents and 2 days at home.

A



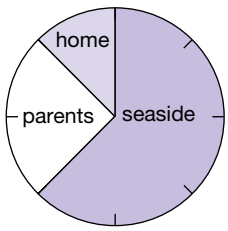
Jan's holiday

B



Jan's holiday

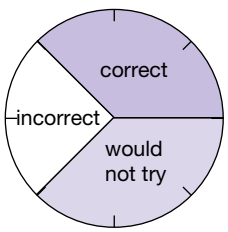
C



Jan's holiday

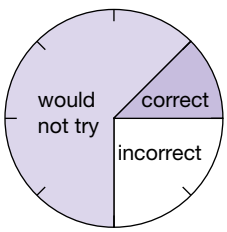
b 100 people were asked to spell 'hypotenuse'. 40 spelt it correctly, 35 would not try and 25 spelt it incorrectly.

A



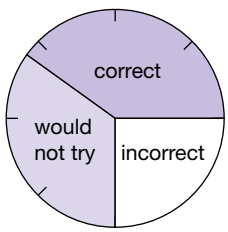
Spelling

B



Spelling

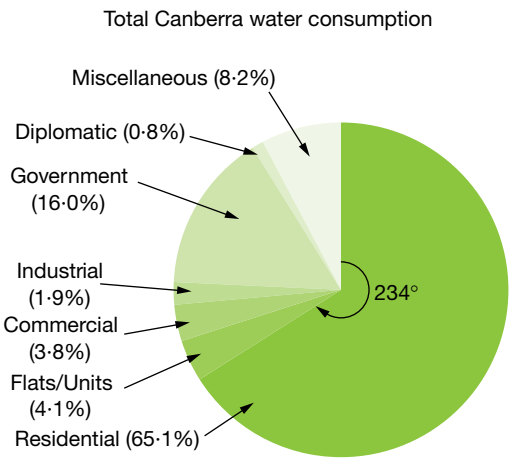
C



Spelling

9 This graph shows the breakdown of Canberra's water consumption.





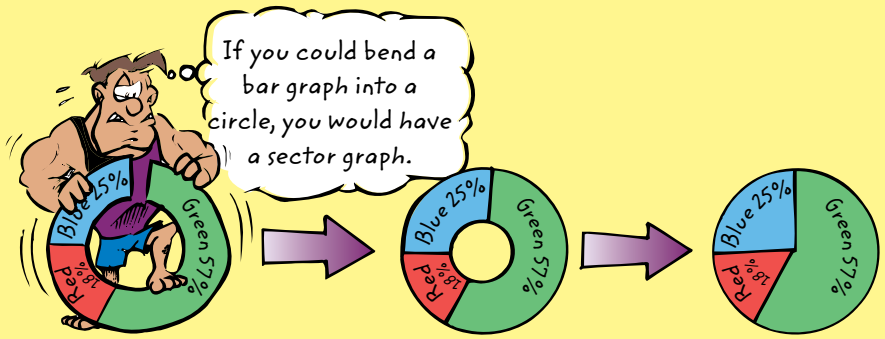
a According to the graph, who used the least amount of water?

b Are you able to say how much water was used for residential purposes? Why or why not?

c What angle at the centre should be allocated to the category 'Government'? Has it been? Is the graph accurate?

d What angle at the centre should be allocated to the category 'Diplomatic'?

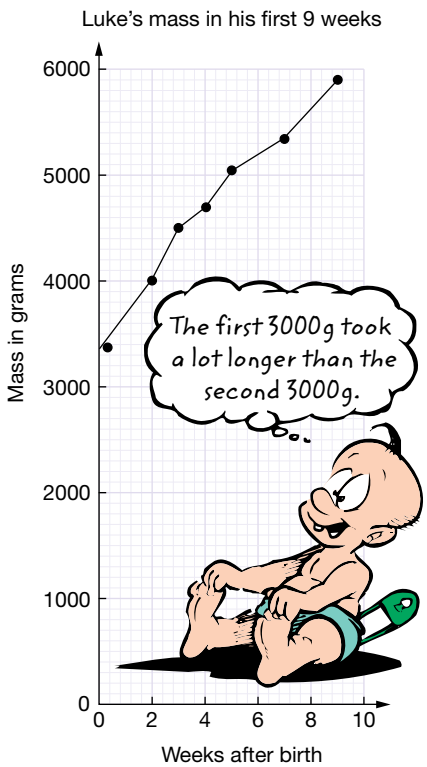
This sector graph shows the favourite colours of a group of students.
If 25% of the students chose blue, then $\frac{1}{4}$ of the circle (90° at the centre) should be used for this category.



7:05 The line graph

Features

1. The line graph conveys more information than most graphs, and is often quite detailed.
2. It shows trends and relationships clearly.
3. The line graph has a large variety of forms and applications. (Examples include travel graphs, conversion graphs, graphs describing physical phenomena, step graphs, lines of best fit, number plane graphs, etc.)
4. The graphs are read in the same way that number plane graphs are read, in that the value of x is linked to the value of y.
5. The 'lines' on a line graph may be curved.



Details

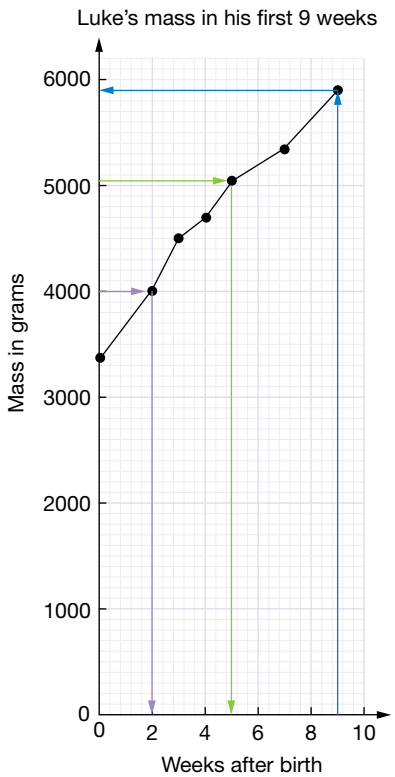
1. The line graph must have a title or heading.
2. Both axes must be labelled.
3. Different scales may need to be used on the two axes. Each axis may need a different range of numbers or a different type of unit.

4 Each point on the graph represents a pair of readings, one on the horizontal axis and one on the vertical axis.
5 When given information has been used to draw a line graph, other useful information can be taken from the graph. For example, even though Luke's mass was not taken at 1 week old, we can estimate his mass to have been 3650 g.
6 Line graphs should be used for continuous numerical data. This means that points between the dots on the graph also have meaning.

WORKED EXAMPLE 1

Refer to the line graph of Luke's mass in his first 9 weeks.

a What was Luke's mass at the end of 9 weeks?
b At what age did Luke have a mass of 4000 g?
c Within which week did Luke's mass first pass 5000 g?
d Use the graph to estimate what Luke's mass might have been at the end of the tenth week.



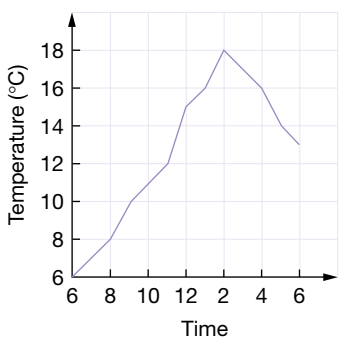
Solutions

a The blue arrows from the ninth week on the horizontal axis show the mass was 5900.
b The purple arrows from 4000 g on the vertical axis show the age was 2 weeks.
c The green arrows show that Luke's mass first passed 5000 g in the fifth week.
d If the black line on the graph was extended with the same slope, the mass would be over 6000 g, perhaps 6200 g.

WORKED EXAMPLE 2

Refer to the temperature line graph.

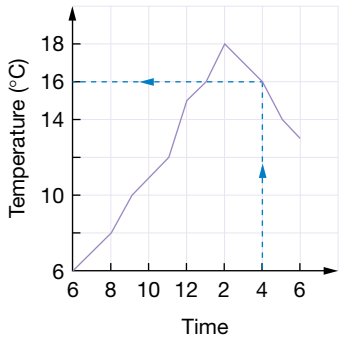
a What was the temperature at 4 pm?
b When was the temperature 11°C?
c When was the temperature above 16°C?
d Estimate the temperature at 4.30 pm.



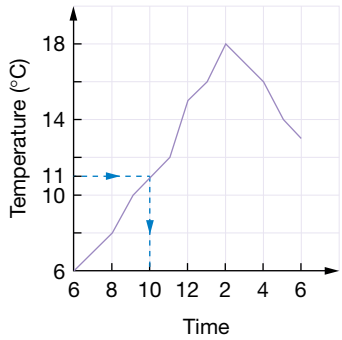
The vertical axis does not start at 0.
The section from 0 to 6 has been 'cut off' for convenience. We say the axis has been **truncated**.

Solutions

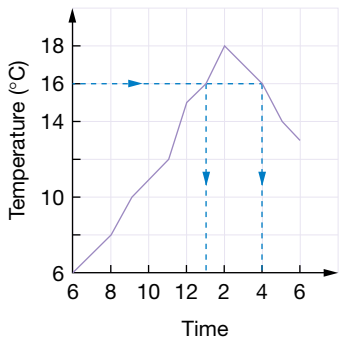
a At 4 pm the temperature was 16°C.



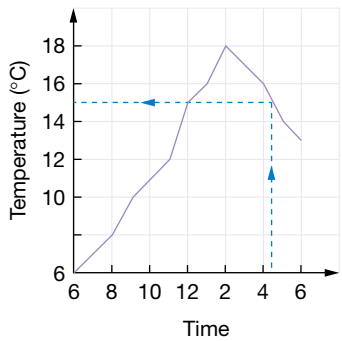
b When 11°C the time was 10 am.



c The temperature was above 16°C between 1 pm and 4 pm.



d The temperature was about 15°C at 4:30 pm.



WORKED EXAMPLE 3

While in hospital, Rocco's temperature was taken every 4 hours. The details are shown below. A normal temperature is 37°C. Draw a line graph to show the information.

Time	4 pm	8 pm	12 pm	4 am	8 am	12 noon	4 pm	8 pm
Temperature (°C)	38·2	39·4	39·8	40	39	37·6	36·6	37
	Thursday			Friday				

Solution

Follow these steps to draw the line graph.

Step 1 Choose a title.

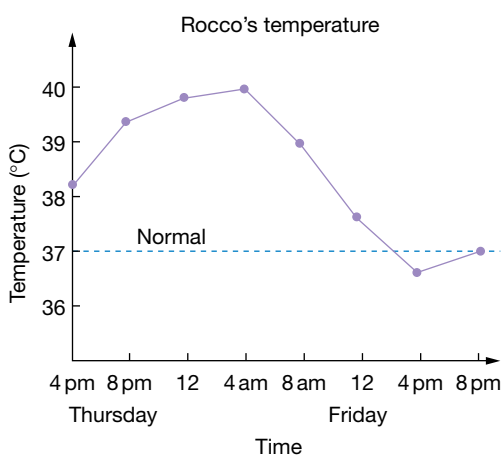
We could use 'Rocco's temperature'.

Step 2 Choose a scale and name for each axis.

We will need eight divisions on the time axis. We could name the vertical axis 'Temperature (°C)' and use five divisions, one for each degree °C.

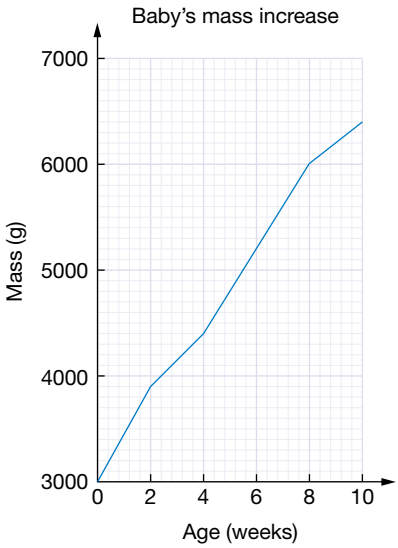
Step 3 Draw the graph.

Plot the points and join them with a line.



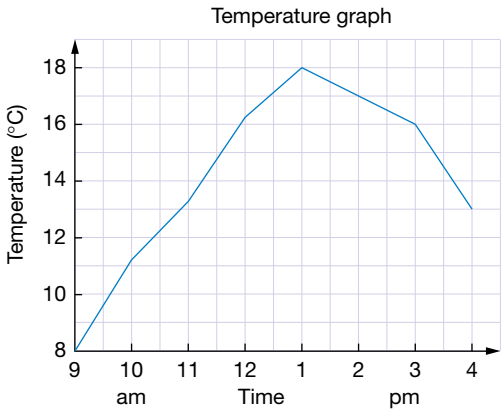
Exercise 7:05

1 This graph shows the mass of a baby, in grams, for 10 weeks after birth.



- **a** What did the baby mass at birth?
- **b** What was the baby's mass 4 weeks of age?
- **c** By how much did the baby's mass increase in the first two weeks of age?
- **d** By how much did the baby's mass increase from 2 weeks of age to 4 weeks of age?
- **e** Considering your answers to parts **c** and **d**, in which period, (0–2) or (2–4) was the baby's rate of growth the greatest?

2 This graph shows the temperature, in °C, over the course of one day.

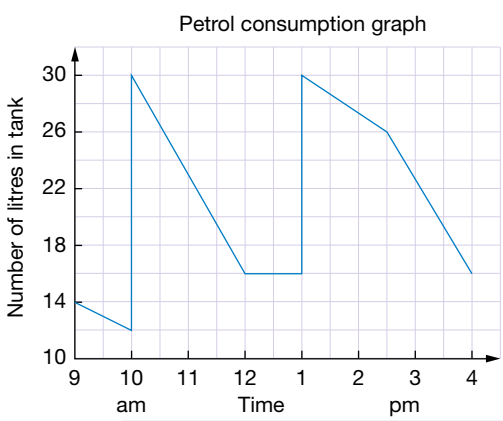


- **a** What was the temperature at 9 am?
- **b** At approximately what time in the morning did the temperature reach 14°C?
- **c** What was the approximate temperature at 2:30 pm?
- **d** For how long was the temperature above 16°C?
- **e** In what hourly periods did the greatest increase in temperature occur?

3 This table gives the temperature, in °C, taken at hourly intervals. Draw a line graph for this information using scales of 1 cm = 5°C on the vertical axis and 1 cm = 1 hour on the horizontal axis.

Time	7 am	8 am	9 am	10 am	11 am	12 noon	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm	7 pm
Temperature (°C)	17°	20°	22°	28°	30°	33°	38°	38°	37°	35°	32°	30°	25°

4 This graph shows how much petrol, in litres, remained in the tank of a car during a day.



- **a** How much petrol was in the tank at 9 am?
- **b** How much petrol was used from 9 am to 10 am?
- **c** When did the driver fill the tank?
- **d** How much petrol was used from 12 noon to 1 pm? What does this tell us about the car?
- **e** How much petrol was used in total?

5 This table shows the distance from home of a motorist, in km, at hourly intervals. Construct a line graph for the data.

Time	Distance (km)
10 am	0
11 am	70
noon	170
1 pm	250
2 pm	250
3 pm	310
4 pm	370
5 pm	470
6 pm	550



Remember:
When graphing the information in a table:
- Determine how big you want the graph to be.
- Look at the range of numbers on both axes.
- Determine the scales needed.

6 The table shows the distance of a cyclist from home each hour. Draw a line graph for this table using a scale of 1 cm = 1 km on the vertical axis and 1 cm = 1 hour on the horizontal axis.

Time	7 am	8 am	9 am	10 am	11 am	noon	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm
Distance (km)	0	14	30	42	57	57	69	55	42	26	10	0



7 The graph shows the height of an object, in m, after it is fired into the air.

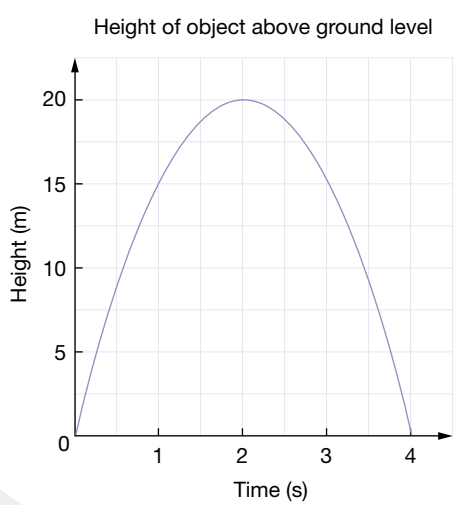
a How high is the object above ground level after 1 second?

b At what times is the object 10 m above ground level?

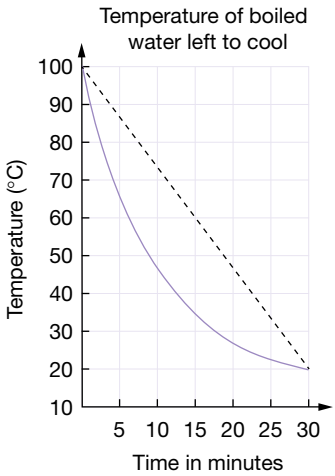
c What is the greatest height reached by the object?

d How long does it take for the object to fall from its maximum height to ground level?

e For how long is the object above a height of 17·5 m?



8 The graph shows the temperature of boiled water when left to cool.



- **a** How long does it take the water to reach a temperature of 80°C?
- **b** What is the temperature of the water after 10 min?
- **c** What is the temperature of the water after 20 min?
- **d** From the graph it can be seen that the water doesn't cool at a constant rate. The dotted line represents a constant cooling rate. If the water had cooled at a constant rate, what would its temperature have been after 10 min?

9 Draw a line graph for each of the following sets of data.

- **a** While in hospital, Angelina's temperature was taken every 4 hours. The details are shown below. A normal temperature is 37°C.

Time	4 pm	8 pm	12 am	4 am	8 am	12 pm	4 pm	8 pm
Temperature (°C)	39	39·6	40	39·2	38·4	37·8	37	37·2

- **b** Using an electronic device, Tracey measured the distance of a penguin from its burrow. It would search for food in the ocean all day then return at night to the burrow to feed its young. The details are shown below for an 18-hour period.

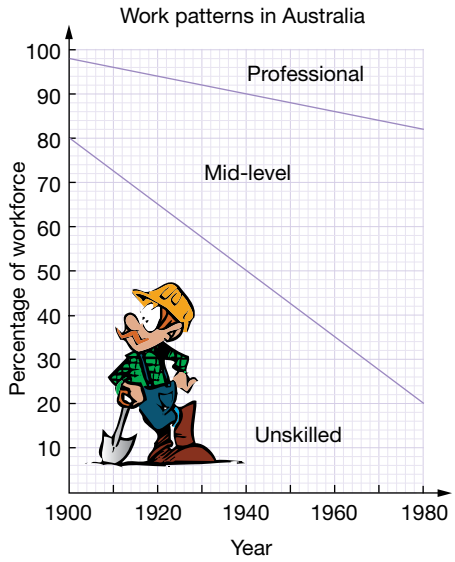
Time	4 am	6 am	8 am	10 am	noon	2 pm	4 pm	6 pm	8 pm	10 pm
Distance (km)	0	2	8	15	20	18	16	10	8	0

- **c**
  - At 7 pm Fred put the plug in the bath, turned on the water and let it run steadily for 6 minutes.
  - He turned the tap off and then took 3 minutes to prepare to get into his bath.
  - Before he got in the bath, the water was 15 cm deep.
  - After he sat in the water, the depth was 18 cm.
  - He bathed for 9 minutes before getting out of the bath.
  - He immediately removed the plug, allowing the water to escape. The bath was empty in 3 minutes.



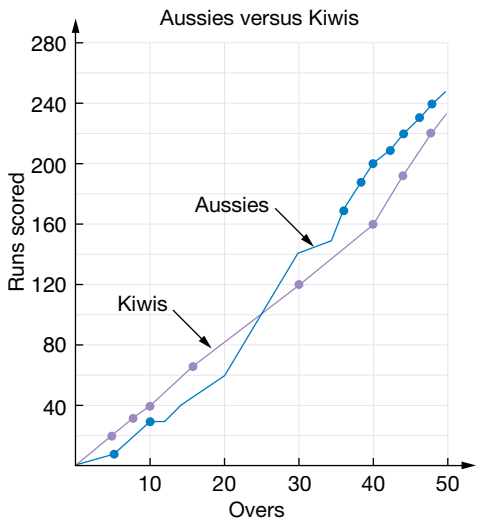
Draw a graph that shows the depth of water in the bath from the time that Fred turned on the water until the bath was empty again.
On the graph, let each centimetre on the vertical axis represent 5 cm of water, and each centimetre on the horizontal axis represent 3 minutes.

10 Over the years, machinery and technology have done more of the work previously carried out by unskilled labour. This graph, based on data from the Committee of Inquiry into Education and Training, 1979, shows the changes in the Australian workforce.



- **a** What kind of graph is this?
- **b** What percentage of workers in 1900 were professional?
- **c** What percentage of workers in 1900 were unskilled?
- **d** What percentage of workers in 1900 were in the mid-level category?
- **e** What percentage of each category was present in 1920?
- **f** What percentage of each category was present in 1980?
- **g** Do you think the percentages for each of the years mentioned on the horizontal axis have been researched? Why or why not?
- **h** If the trend of this graph continued to the year 2010, what would be the percentage of unskilled workers? If possible, find the actual data for 2010 and compare your results.
- **i** If this trend of this graph continued beyond the year 2010, do you think it is important for young people to develop workplace skills? Where can these skills be developed?

11 These line graphs show how the Aussies and the Kiwis performed in a one-day cricket match. The Aussies were the first to bat, and after batting for 50 overs (6 balls are bowled in each over) the Kiwis batted for 50 overs. When a person batting is out, a heavy dot is placed on the graph at that point.



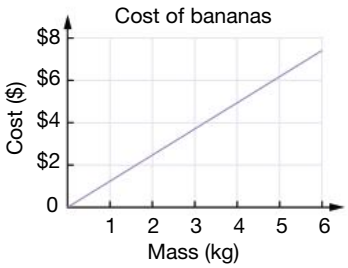
- **a** How many runs had the Aussies scored after 20 overs?
- **b** How many runs had the Kiwis scored after 20 overs?
- **c** How many of the Kiwis were out after 20 overs?
- **d** How many Aussies got out altogether?
- **e** How many Kiwis got out altogether?
- **f** What was the Aussies' total score?
- **g** What was the Kiwis' total score?
- **h** Who won the game?
- **i** Which team had scored more runs after 30 overs, and how many more had that team scored than the other team?
- **j** How many runs did the Aussies score in the 11th over?
- **k** During which over did the Aussies reach 200?
- **l** In which over did the Kiwis lose their 5th wicket?

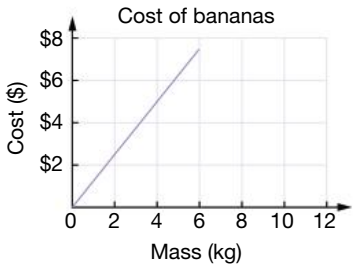
INVESTIGATION 7:05 GRAPHS CAN BE SNEAKY!

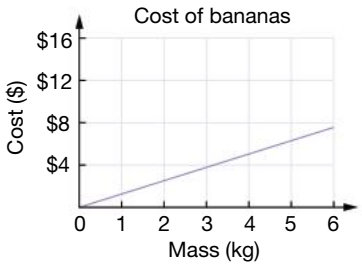
Graphs can give you various impressions, depending on how they are constructed.

> The choice of scales will determine the appearance of a line graph.

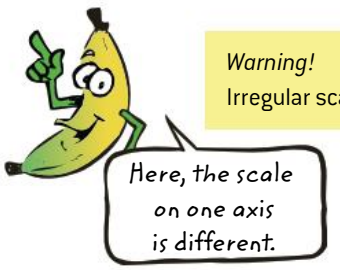
1 The same information is represented in all three of these graphs.







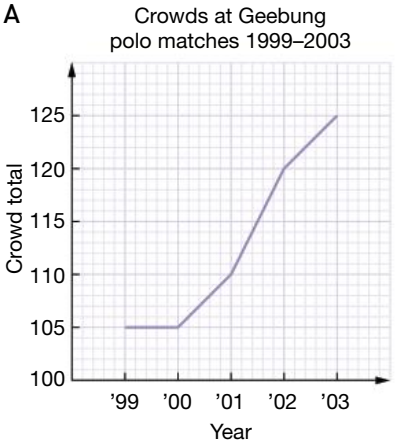
a Use the graphs to find the cost of 2 kg of bananas.
b Do all three graphs show the same cost for 2 kg?
c Does each graph agree on the mass of bananas that could be bought for $15? What is this mass?
d If the graphs show the same information, why do they look different?



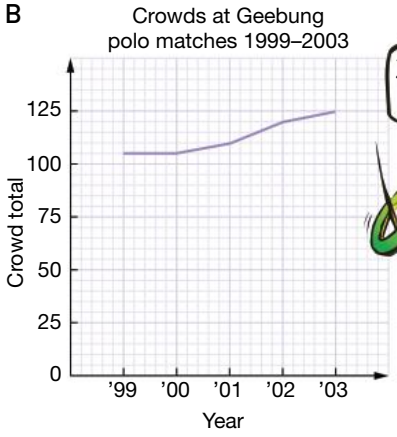
Warning!
Irregular scales

2 **a** On graph A, at what number does the vertical axis start?
b On graph B, at what number does the vertical axis start?
c On graph A, what does one small unit on the vertical axis represent?
d On graph B, what does one small unit on the vertical axis represent?
e Which graph suggests that the crowds are increasing rapidly?
f Do the graphs represent exactly the same information?

A



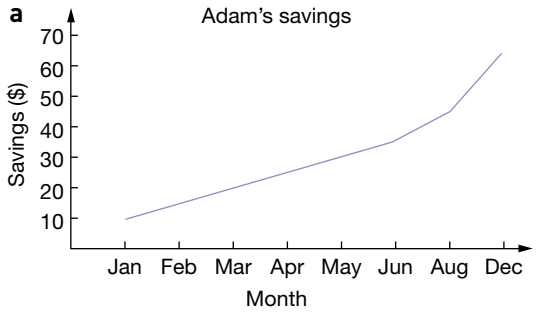
B



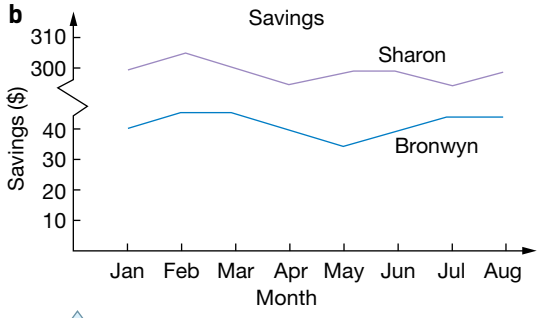


3 How do the following graphs mislead?

a



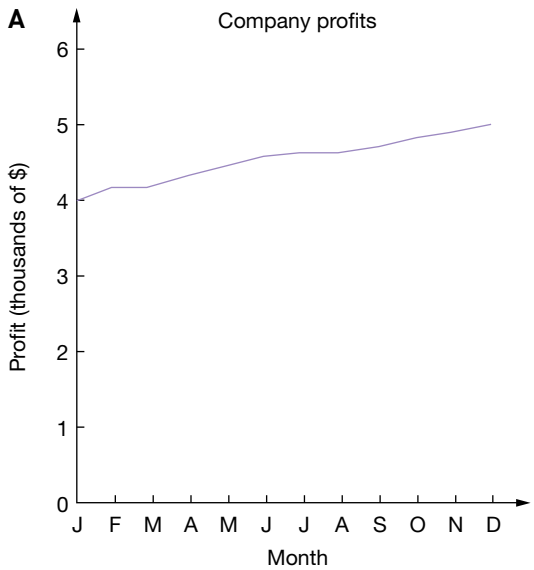
b



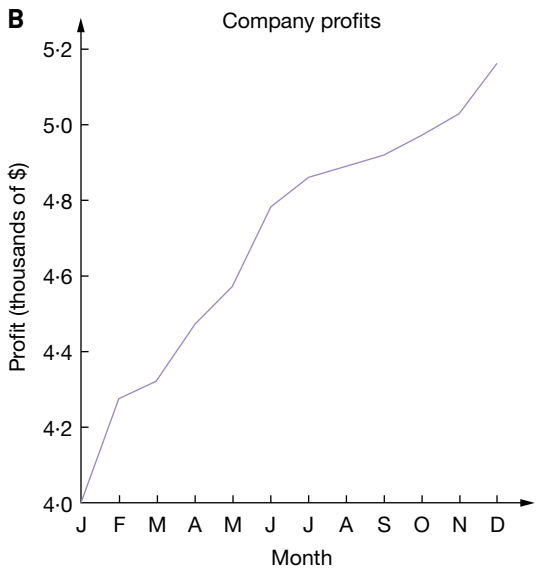
When part of an axis is not shown, as in the Savings graph above, it is said to be 'truncated'.

4 These graphs show different representations of the same data.

A



B



If an axis does not start from zero, the graph can be misleading.

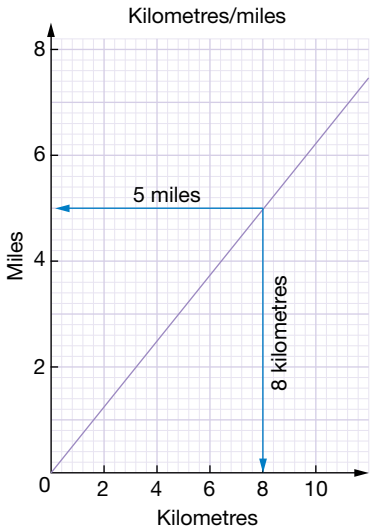
a Which graph suggests the more spectacular rise in profits?

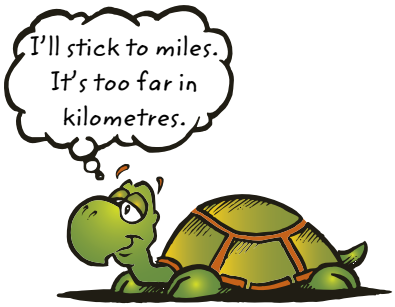
b Which graph gives the fairest representation of the relative sizes of the numbers?

c If you were the managing director in danger of losing your job, which graph would you show to the board of directors?

7:06 The conversion graph

This is a simple type of line graph.





Features

1. The conversion graph allows us to convert from one measurement to another.
2. The graph is read in the same way as other line graphs.
   e.g. 8 kilometres = 5 miles

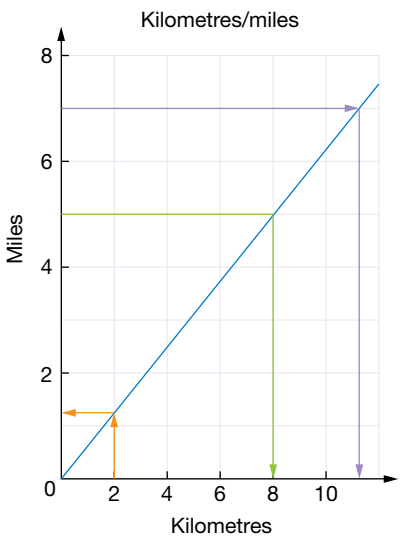
Details

1. The details listed above for line graphs apply to conversion graphs.
2. Conversion graphs are straight lines.

WORKED EXAMPLE 1

This graph allows us to convert miles into kilometres.
Use the graph to convert the following:

a 5 miles into kilometres
b 7 miles into kilometres
c 2 km into miles.



Solutions

a The green arrows show that 5 miles is equivalent to 8 km.
b The purple arrows show that 7 miles is approximately equal to 11.2 km.
c The orange arrows show that 2 km is approximately equal to 1.2 miles.

WORKED EXAMPLE 2

Construct a conversion graph to convert acres and hectares, given that 20 acres = 8·1 hectares.

Solution

Follow these steps to construct the conversion graph.

Step 1 Choose a title.

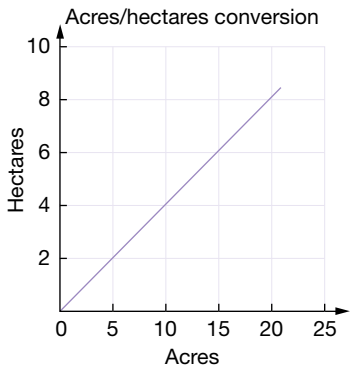
'Acres/hectares conversion' would be a good title.

Step 2 Choose a scale and name for each axis.

We need to go up to 20 on the acres axis and 8·1 on the hectares axis. The scales used could be:
1 cm = 5 acres and 1 cm = 2 hectares.

Step 3 Draw the graph.

We know that 0 acres = 0 hectares and 20 acres = 8·1 hectares.
Plot the points (0, 0) and (20, 8·1).
Join these two points.





Exercise 7:06

1 **a** Convert the following marks out of 60 into percentages. Give your answer to the nearest whole per cent.

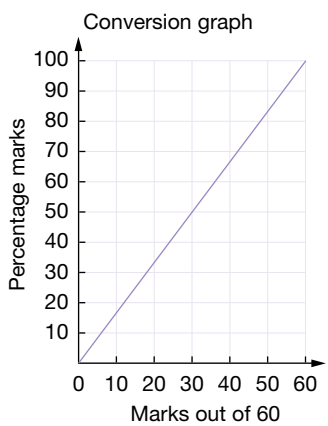
i 30 **ii** 40 **iii** 12 **iv** 16

b Convert the following percentages to marks out of 60. Give your answer to the nearest whole number.

i 80% **ii** 90% **iii** 56% **iv** 38%

c Two students score 30 and 40 marks out of 60. What is the difference in their percentage marks?

d Tony scores 80% and Mahir scores 60% in the same test. If the test was marked out of 60, by how many marks did Tony outscore Mahir?



2 This conversion graph converts between degrees Celsius and degrees Fahrenheit.

Change these temperatures from °C to °F.

a 0°C **b** 100°C

c 25°C **d** 50°C

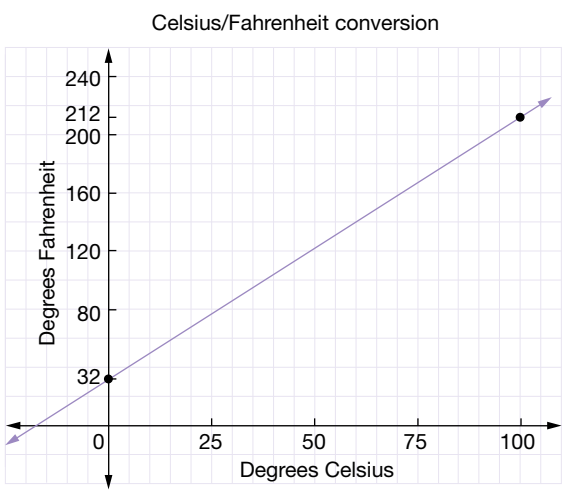
e 15°C **f** 110°C

Change these temperatures from °F to °C.

g 32°F **h** 176°F

i 160°F **j** 104°F

k 16°F **l** 0°F

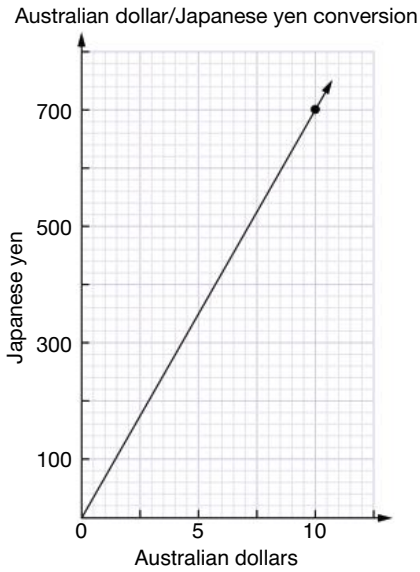


3 What is the value in yen of:

a $10 **b** $5
c $3 **d** $6.50
e $30 **f** $75
g $300 **h** $750?

What is the value in Australian dollars of:

i 300 yen **j** 400 yen
k 600 yen **l** 240 yen
m 450 yen **n** 6000 yen
o 2500 yen **p** 20 000 yen?



4 Use the facts that 0 kilograms = 0 pounds and 5 kg = 11 pounds to draw a conversion graph that converts between kilograms (kg) and pounds (lb).

5 The exchange rate between Australian dollars (A$) and British pounds (£) is A$1 = £0.65. This means 100 Australian cents = 65 British pence. Draw a conversion graph that will convert these two currencies, up to A$100.



INVESTIGATION 7:06 COLLECTING GRAPHS

Collect as many different types of graphs as you can and have them available for class discussion. Make sure that you have at least three graphs. Graphs may be found on the internet or in:

- newspapers
- *Choice* magazine
- car, science, financial, geographical or research magazines
- an atlas or encyclopaedia
- medical journals
- household energy bills.

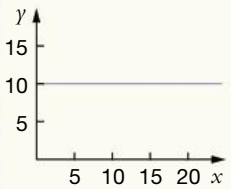


On the back of each graph write the source and date of publication. With each graph, include the article in which it appeared.

7:07 The step graph

A step graph is another type of line graph. It has this name because it resembles a set of steps.

PREP QUIZ 7:07



For the graph above:

1 find y when $x = 5$

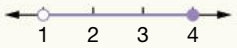
2 find y when $x = 10$

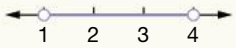
3 find y when $x = 15$.

If parking charges are \$1.80 per hour or part thereof, how much will it cost to park your car for:

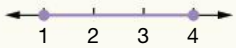
4 2 h? **5** $1\frac{1}{2}$ h? **6** $2\frac{1}{4}$ h?

Name the integers that belong to the following sets:

7 

8 

9 

10 

The step graph on the right shows the parking costs in a city street for different periods of time.



Features

1 The step graph is made up of several horizontal intervals or steps.

2 The end of one horizontal step and the start of the next one are on the same vertical line.

3 Each horizontal step has an open circle ○ at its start and a coloured-in circle ● at its end. An open circle excludes that point from the step. A coloured-in circle includes that point in the step. So, after 2 hours, the cost is \$20 not \$30.

Note: When a gap or jump appears in a graph we say that the graph is *discontinuous*.



Details

1 The details listed for line graphs apply to step graphs.

2 Every point on the graph has meaning in that it has a value on both axes. The data are continuous.

3 The ends of steps should be considered carefully: *Is this end included or not?*

WORKED EXAMPLE 1

The step graph gives the postal charges for sending a letter to the United Kingdom.

a Find the cost of posting a letter with mass:
  i 40 g **ii** 100 g

b What is the heaviest letter that can be posted for $4?

c What range of letters, in mass, can be posted for $7?

d Two separate letters with masses of 10 g and 15 g are to be posted to the same address. If the letters were combined and posted as one letter, would it cost more or less than if they were sent separately?





Solutions

a **i** From the 40 g mark on the mass axis, move up vertically until the step graph is met. Then move horizontally until the cost axis is met at $2.50.
The cost of posting a 40 g letter is $2.50.

ii The vertical line at 100 g cuts the step graph in two places. However, the closed circle means that 100 g is included in the $4 step.
The cost of posting a 100 g letter is $4.

b For $4, letters with a mass up to 100 g can be posted.

c Any letter with a mass 250 g or less can be posted for $7.

d A 10 g and 15 g letter each cost $1.50 to post or $3 in total.
A combined letter with a mass of 25 g would cost $2.50.
A combined letter is cheaper by 50c.



WORKED EXAMPLE 2

When constructing a step graph, follow the steps in this example.

Travelling by boat costs:

- $3 for up to 5 km
- $10 for more than 5 km and up to 10 km
- $18 for travelling more than 10 km.

Step 1 Choose a title.

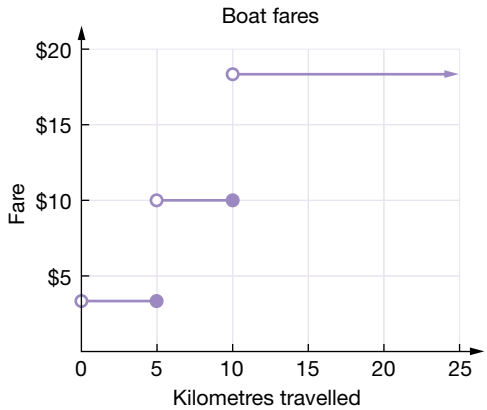
'Boat Fares' would be a good title.

Step 2 Choose a scale and a name for each axis.

Kilometres travelled, using 1 cm = 5 km, could be used on the horizontal axis. Fare (up to $20) could be used on the vertical axis with a scale of 1 cm = $5.

Step 3 Draw the graph.

Realise that there are three different fares. Check that for each the empty circle is on the left. Note that the top step continues on.





Exercise 7:07

1 The parking charges for El Cheapo Parking are shown here.

a What is the cost for 2 hours?

b For how long can you park for $6?

c What is the maximum charge shown here?

d What is the cost for half an hour?

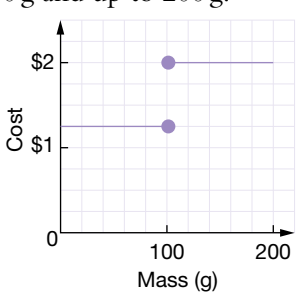
e Why do you think that this type of graph is called a step graph?



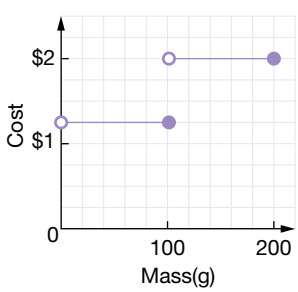
2 Which graph below correctly matches the information given?

To send a package, it costs $1.25 for a mass of up to 100 g and $2.00 for a mass more than 100 g and up to 200 g.

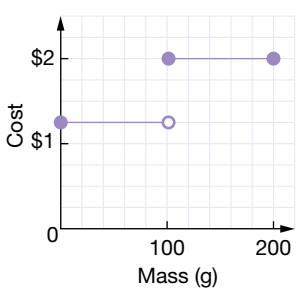
A



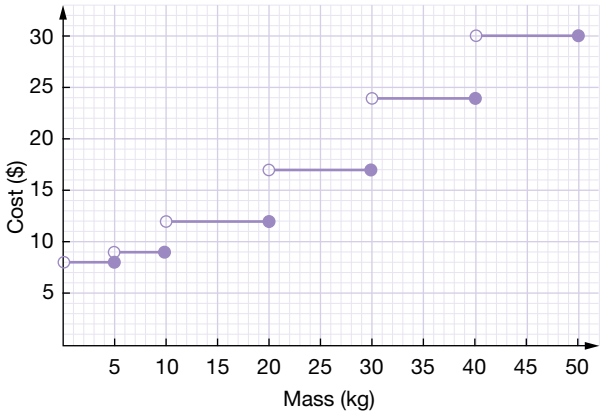
B



C



3 A carrier's charges for delivering a parcel are set out in the graph shown.



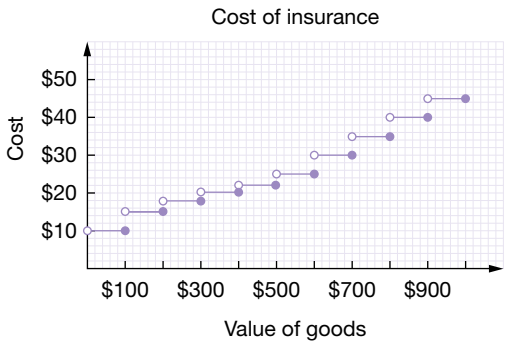
a What does it cost to deliver a parcel with a mass of:
   i 25 kg ii 8 kg iii 40 kg

b What is the heaviest parcel that can be delivered for:
   i $8 ii $17 iii $24

For each additional 10 kg, or part thereof, in excess of 50 kg an extra $2.40 is charged.

c What would be the cost of sending a parcel with a mass of 75 kg?

4 The graph shows the cost of insuring goods that are to be delivered.



a How much would it cost to insure goods that have a value of:
   i $80 ii $350 iii $700?

b What value of goods can be insured for $40?

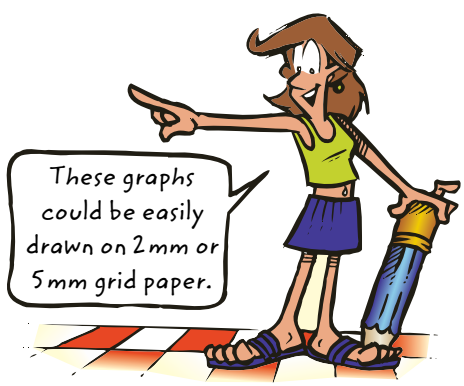
c Is it possible to calculate the cost of insuring goods valued at $1500 from this graph?

5 Paint is sold in tins of standard sizes. The table below shows the cost of paint required for coverage of areas in square metres. If two charges are possible, use the cheaper option. Draw a step graph to show the information.

Area (m^2)	0–5	5–10	10–20	20–30	30–40	40–50
Cost	$11	$14	$17	$20	$25	$30

6 Draw a step graph of these parking costs.

Time	Cost
1 hour or less	$10
More than 1 h and up to 2 h	$15
More than 2 h and up to 3 h	$20
More than 3 h and up to 4 h	$22
More than 4 h and up to 5 h	$25
More than 5 h	$27



INVESTIGATION 7:07 SPREADSHEET GRAPHS

The data contained in a spreadsheet can be presented in the form of a graph or chart.

A baby's mass in grams was recorded every two weeks. The results for the first ten weeks were entered in a spreadsheet as shown.

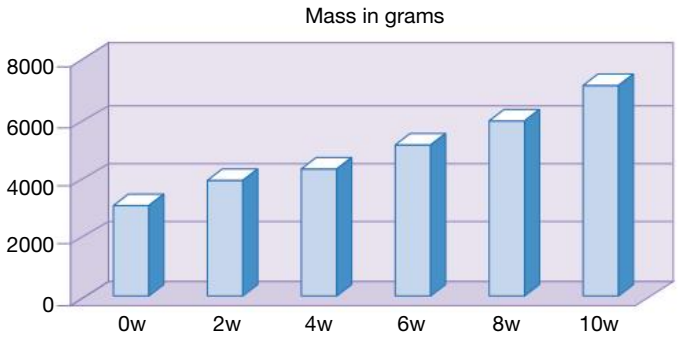
This data was then displayed as a graph by 'inserting a chart' on the spreadsheet. Three different samples are shown here.

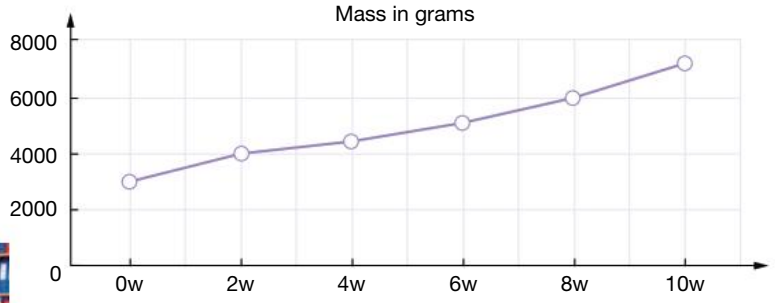
- Which graph do you think shows the data most clearly?
- Investigate the 'chart' option in a spreadsheet program such as Microsoft® Excel® by using this data or other data of your own.

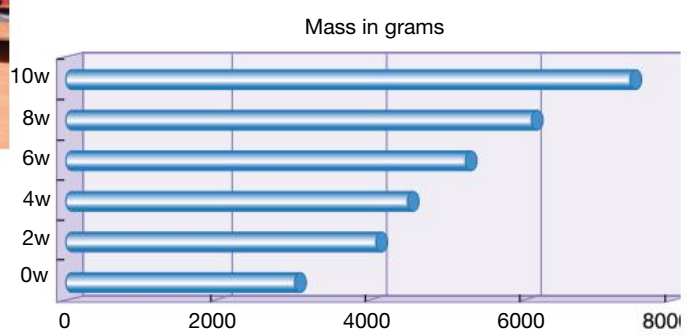
F14 *fx*

	A	B	C	D	E
1	age	mass			
2	0w	3000			
3	2w	4000			
4	4w	4500			
5	6w	5200			
6	8w	6100			
7	10w	7400			
8					
9					









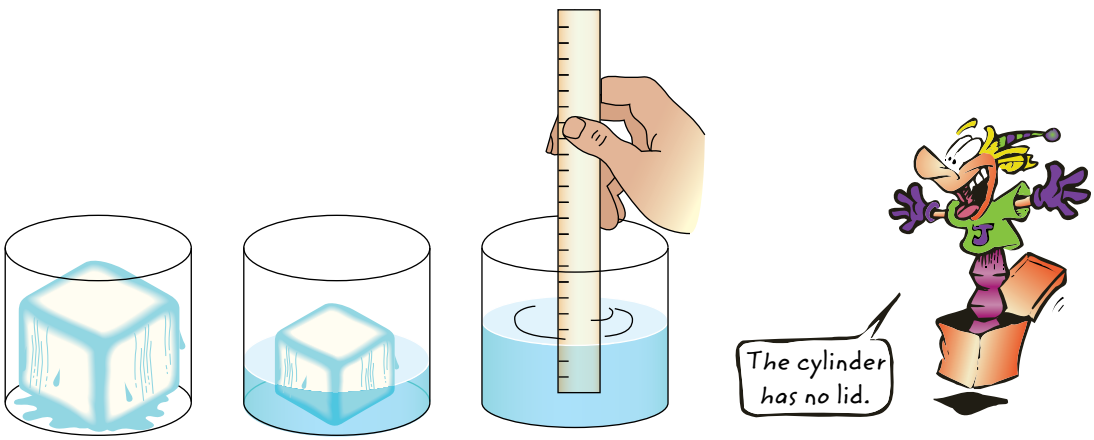
7:08 Miscellaneous and unusual graphs

This section will present a variety of graphs representing different types of data. Graphs are used to convey information in many fields of study such as science, geography and economics. Some newspapers and magazines use unusual ways of presenting graphs. Sometimes they use pictures or illustrations that resemble graphs but may lack accuracy or some of the features of a graph.

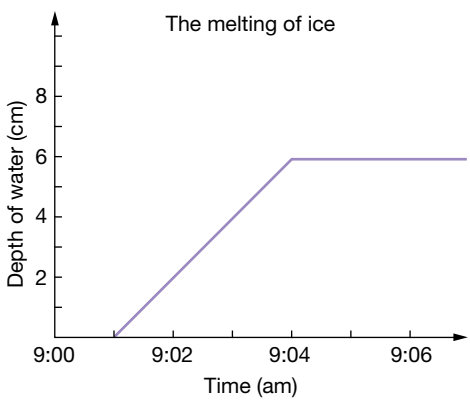
Exercise 7:08

P Foundation worksheet 7:08
Reading graphs

1 A block of ice was placed in a large cylinder and the level of water was recorded as the cylinder was heated.

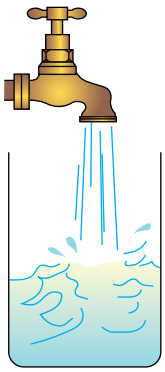


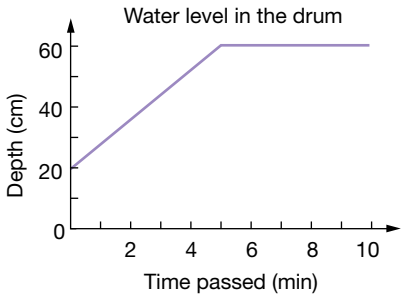
The results of this experiment are shown on the graph.



a When did the experiment begin?
b What was the depth of water at 9:02 am?
c When was the depth of water 4 cm?
d What was the greatest depth of water during the experiment?
e At what time did the last of the ice melt?
f If the experiment continued, allowing the water to boil for some time, what would happen to the water level in the cylinder? How would this be shown on the graph?

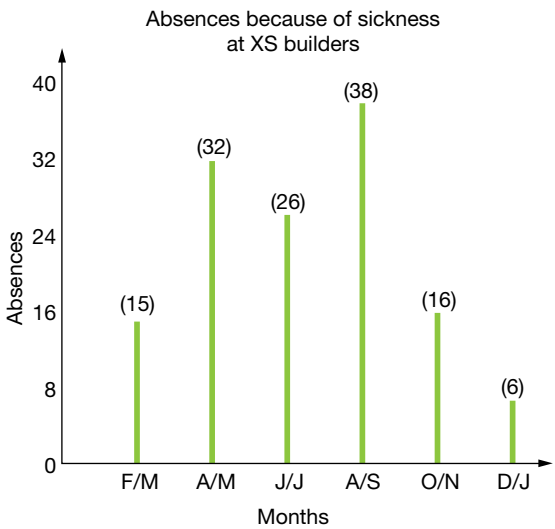
2 Water from a tap flowed into a drum that had been partly filled with water. The depth of water was measured as time passed and the results were graphed. The tap was turned off after 10 minutes.





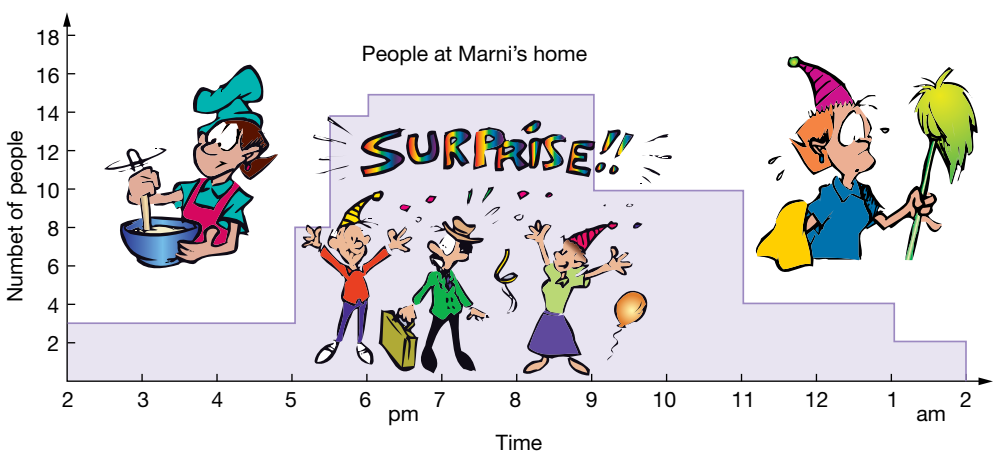
- **a** What depth of water was in the drum at the beginning of the experiment?
- **b** What depth of water was in the drum after 1 minute?
- **c** After how many minutes was the depth of water 40 cm?
- **d** Why does the graph level out after the 5th minute?
- **e** When was the tap turned off?
- **f** By how much did the water level rise in the first 5 minutes?

3



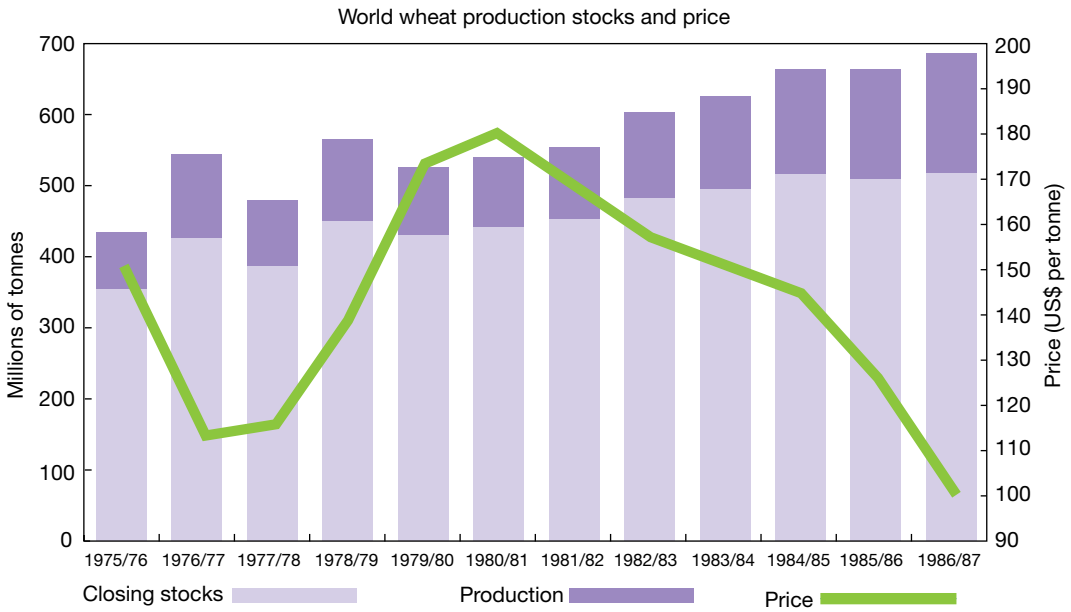
- **a** What kind of graph is this?
- **b** What does J/J stand for on the horizontal axis?
- **c** How many absences occurred in April/May?
- **d** How many absences occurred in October/November?
- **e** What was the greatest number of absences and in what time period did this occur?
- **f** Why do you think the absences were so low in December/January?
- **g** What was the total number of absences in these 12 months?

4 Marni and her parents wanted to throw a surprise party for Marni's husband. During the day these three prepared food and decorated the living room. The friends, two carloads full, arrived before James, the guest of honour, arrived. After the party, Marni's parents stayed for a while to help clean up the house.



a At what time did the first guests arrive? How many came then?
b How much time passed before the other guests arrived?
c How many guests were there altogether?
d When did James arrive home? How many people were at the party then?
e When did the first guests leave? How many left then?
f At what time did Marni's parents leave?
g See if you can make up a different story that fits this graph. The name of the graph and the titles of the axes may change if you wish.

5



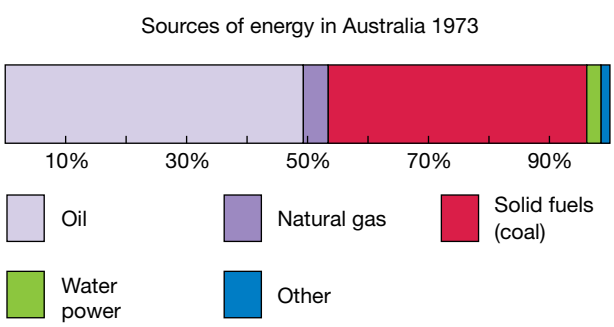
Source: Adapted from Economics, National Australia Bank

Often one graph is drawn on top of another so that comparisons can be made. Here, 'Production' refers to the wheat grown in that year. 'Closing stocks' refers to the total amount of wheat available at the end of the period, including what was unused in the previous time period.

a In which time period was the price of wheat at its highest?
b How many tonnes, to the nearest 50 million, were produced in the period:
   i 1975/76? ii 1977/78? iii 1982/83? iv 1986/87?
c In which time period was the price of wheat at its lowest? What was the price then?

6

a What kind of graph is this?
b What was the greatest source of energy in 1973?
c What percentage of energy came from oil?
d What percentage of energy came from natural gas?
e List the sources of energy in order of size, from largest to smallest.



f Would you expect the sources of energy now to be in the same percentages as in 1973? Why or why not?
g In the 2000s, what sources of power might be included in the category 'Other'?

7 **a** Which graph does this most resemble?

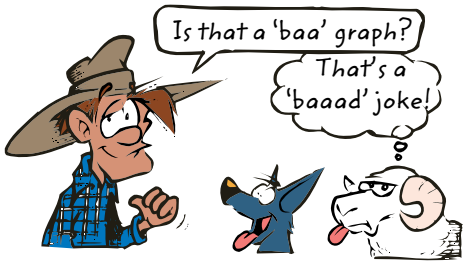
b Why have they chosen to divide a picture rather than a bar or rectangle?

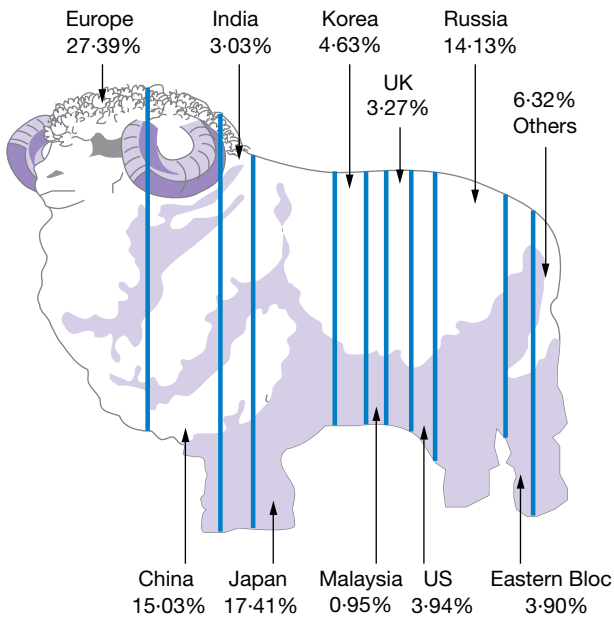
c What percentage of the Australian wool sold was purchased by Japan?

d How much more (as a percentage of the wool sold) did Europe buy than China?

e What percentage of the Australian wool clip was exported?

f What is the title of this graph?





8 It took teenager Steffi Graf $3\frac{1}{2}$ years to lift her world tennis computer ranking from 100 to 1. At 18 years of age she was the No. 1 seed for the 1988 Ford Australian Open.

a What kind of graph is this?

b What does the symbol # stand for?

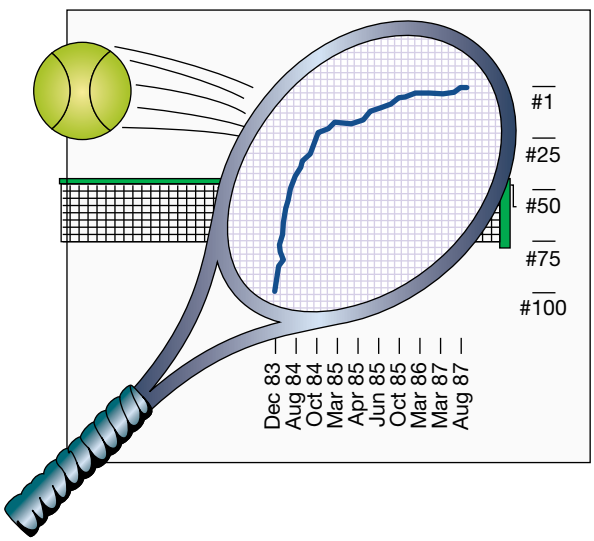
c Is this graph easy to read?

d Is the graph meant to give detailed information or is it meant to create an impression?

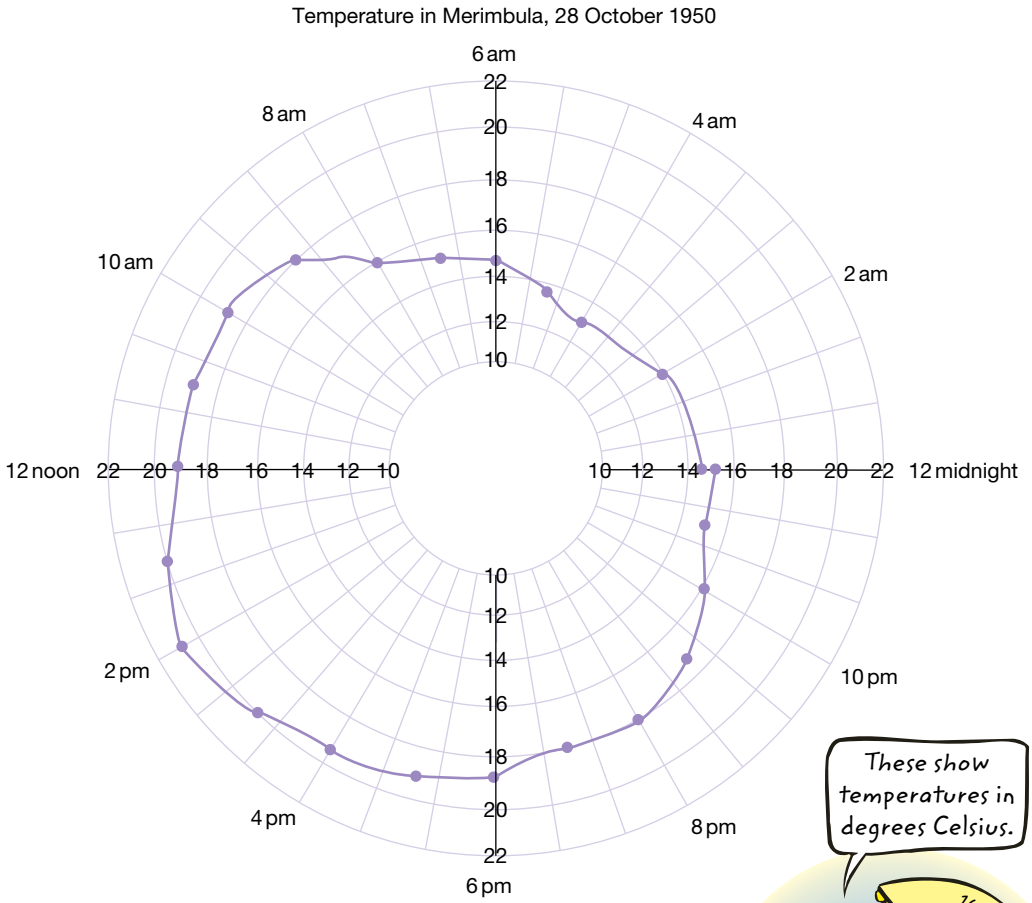
e What was Steffi's ranking in December 1983?

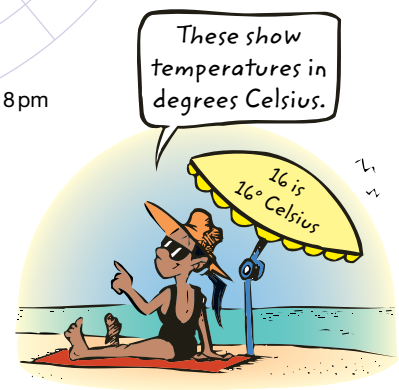
f What was her ranking in August 1984?





9 This unusual graph shows the temperature over a 24-hour period on the day Rhonda was born.





a How old was Rhonda on:
   i 2 August 1976
   ii 3 November 1999?

b What type of graph is this?

c What angle at the centre is used to represent:
   i 6 hours ii 1 hour?

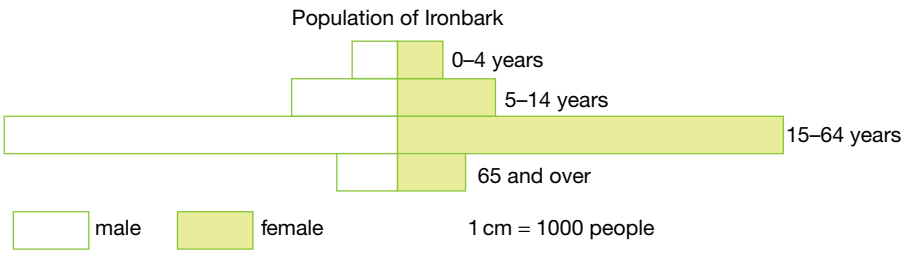
d What are the least and greatest temperatures that could be shown on this graph?

e What was the temperature at:
   i 9 am ii 3 am iii 3 pm?

f At what time was the temperature 16°C?

g On 28 October 1950, what were the least and greatest temperatures?

10 This graph resembles a column graph where each column is a divided bar graph.



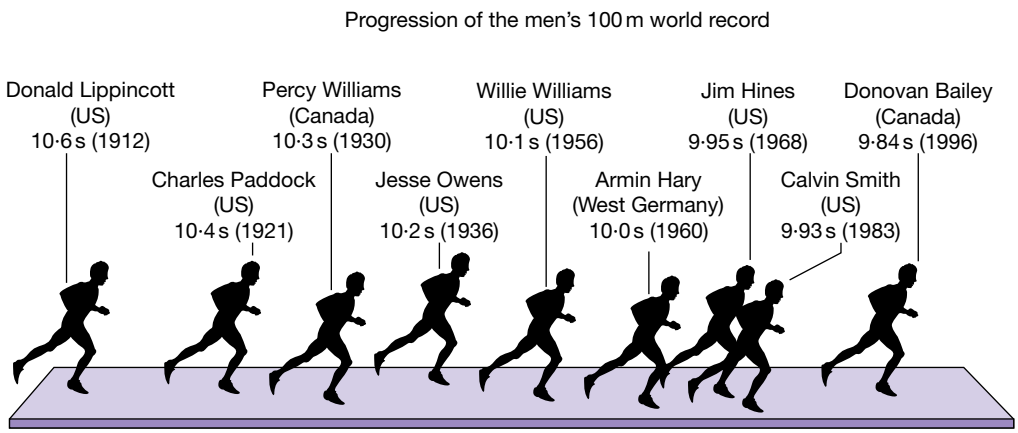
a Which of the population groups shown has:
i the smallest size **ii** the largest size?

b By measuring each column (or bar) using the scale given, find the number of people aged:
i 0–4 **ii** 5–14 **iii** 15–64
iv 65 and over. (Give answers to the nearest 50.)

c Use your answers to part **b** to find the population of Ironbark.

d What is the only category in which the females outnumber the males? Why is this?

11



a Why is this an impressive way to show the information gathered?

b Which of the graphs does this most resemble? Why?

c Who was the first man to run 100 metres in less than 10 seconds?

d By how much did Donovan Bailey break the world record when he won the final of the 100 m at the Atlanta Olympics in 1996?

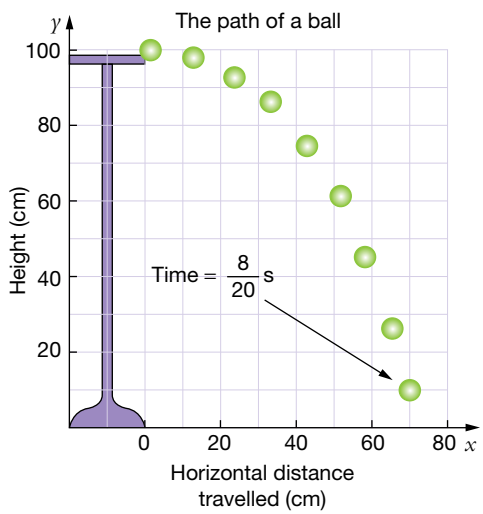
e How many athletes from the US held this record from 1912 to 1996?

f How much faster did Donovan Bailey run than the other Canadian, Percy Williams?

g What was the smallest margin by which the 100 m record was broken from 1912 to 1996?

h Research the progression of this world record after 1999.

12 A science class carried out an experiment using a stroboscope and a camera to take pictures of a falling ball as it rolled off the top of a bench. One picture showed the position of the ball in each $\frac{1}{20}$ of a second. A number plane was drawn on the photograph to produce the graph shown.



a What was the height of the ball at the beginning of the experiment?

b What horizontal distance had the ball travelled in $\frac{8}{20}$ of a second?

c What was the height of the ball when $\frac{8}{20}$ of a second had passed?

d How far had the ball fallen in $\frac{8}{20}$ s?

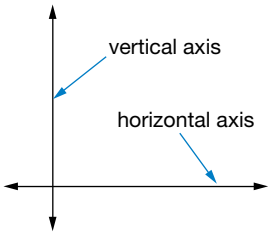
e What was the height of the ball when it had moved through a horizontal distance of 50 cm?

f How far had the ball moved in the horizontal direction when its height was 75 cm?

MATHS TERMS 7

axis (axes)

- a fixed line adopted for reference
- scales on the axes allow us to *read* the graph

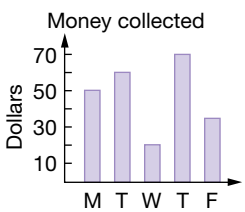


data

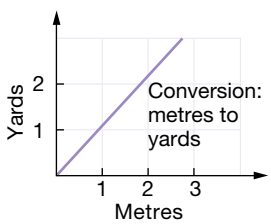
- the information to be examined
- categorical: data that uses non-numerical categories,
  e.g. man, woman, boy, girl
- numerical: data that uses numbers to show *how much*
  - *continuous* numerical data can have any reasonable numerical value,
    e.g. height.
  - *discrete* numerical data uses can have some of the numerical values,
    e.g. age in years or number of points scored.

graphs

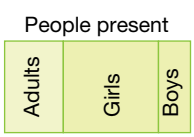
- column graph
  the heights of columns are compared



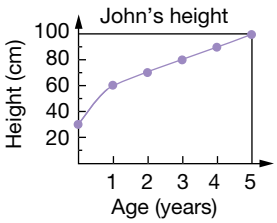
- conversion graph
  a line graph that allows us to change from one unit to another



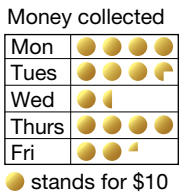
- divided bar graph
  a bar is divided to show the make up of the data



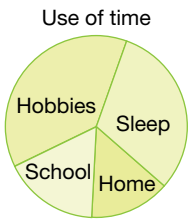
- line graph
  a continuous line shows the connection between the variables



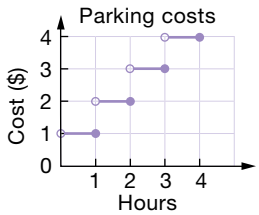
- picture graph
  a picture is used as a unit to show *how many*



- sector graph
  a circle is cut into sectors to show the parts of a whole



- step graph
  several sections or *steps* are used. This line graph is not continuous



scale

- the value and size of units used on each axis
- on the horizontal axis the scale might be 1 cm represents 1 hour

table

- in a table, rows and columns are used to group data into categories, which can then be compared easily

title

- the name or heading of a graph
- each graph should be given a title that describes its content

DIAGNOSTIC TEST 7 GRAPHS

Each section of the test has similar items that test a certain type of question.
Failure in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Draw a column graph to display the following data. 7:02

Wallaby population, Corinna National Park

Year	1995	1996	1997	1998	1999	2000
Number of wallabies	100	120	80	100	120	130

2 Draw a divided bar graph to display the following information. 7:03

Choice of breakfast cereal

Cereal	Weet Brits	Vita Flakes	Corn Bix	Total
Number of students	50	120	30	200

3 Draw a sector graph (pie graph) to display the following information. 7:04

Choice of flavour

Flavour	Strawberry	Chocolate	Vanilla	Others
Number of people	80	60	36	24

4 The temperature was measured in degrees Celsius every 4 hours. Draw a line graph to display the following information. 7:05

Temperature over a 24-hour period

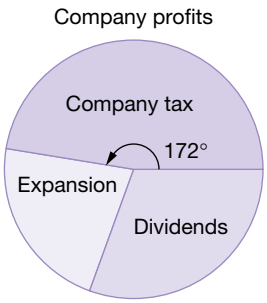
Time	midnight	4 am	8 am	noon	4 pm	8 pm	midnight
Temperature	15°C	12°C	16°C	19°C	20°C	18°C	16°C

5 Draw a conversion graph using the fact that: 7:06
20 Bangladeshi takas = 12·4 Indian rupees

6 Draw a step graph using this information: 7:07
- For up to 1 hour the cost is $6.
- For more than 1 hour and up to 2, the cost is $10.
- For more than 2 hours and up to 3, the cost is $14.
- For more than 3 hours the cost is $18.

ASSIGNMENT 7A Chapter review

1 Company profits are used to pay company tax, to expand the company's assets (through the modernisation of plant, expansion of productive capacity and provision of additional employment) and to pay dividends (that are again taxed through personal income tax). The angle at the centre for 'Company tax' is 172° and the angle at the centre for 'Expansion' is 79°.



a What kind of graph is this?

b What category receives the smallest part of the profits?

c What angle at the centre is used to represent 'Dividends'?

d Find the percentage of company profits that is allocated to 'Expansion'.

2 This graph shows the percentage of sales of new large cars for a particular month.

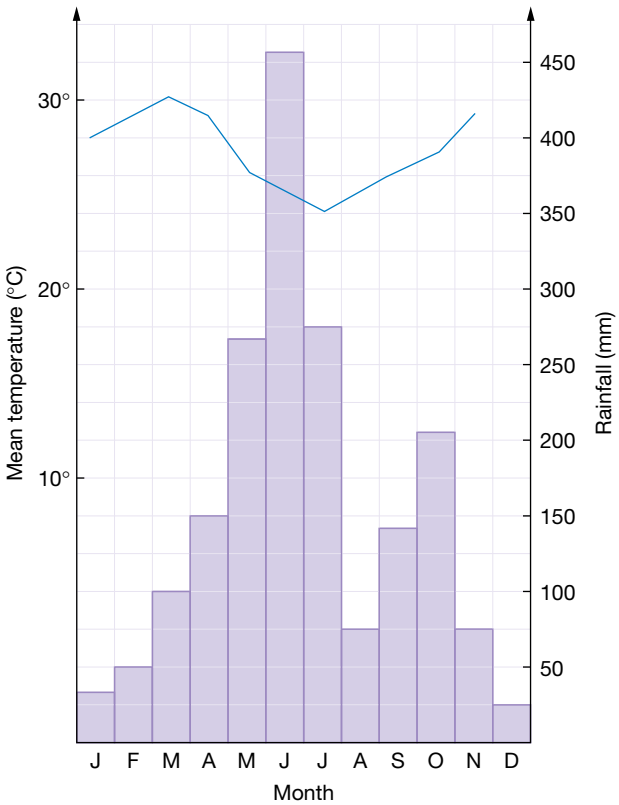


a Find, by measurement, the percentage represented by the graph for:

i Holden **ii** Ford **iii** Toyota **iv** Others

b If the total new car sales for the month was 4925 cars, how many Holden cars were sold?

3 This graph is a combination line and column graph often used in Geography. There are two vertical axes. The left-hand temperature axis refers to the line graph. The right-hand rainfall axis refers to the column graph.



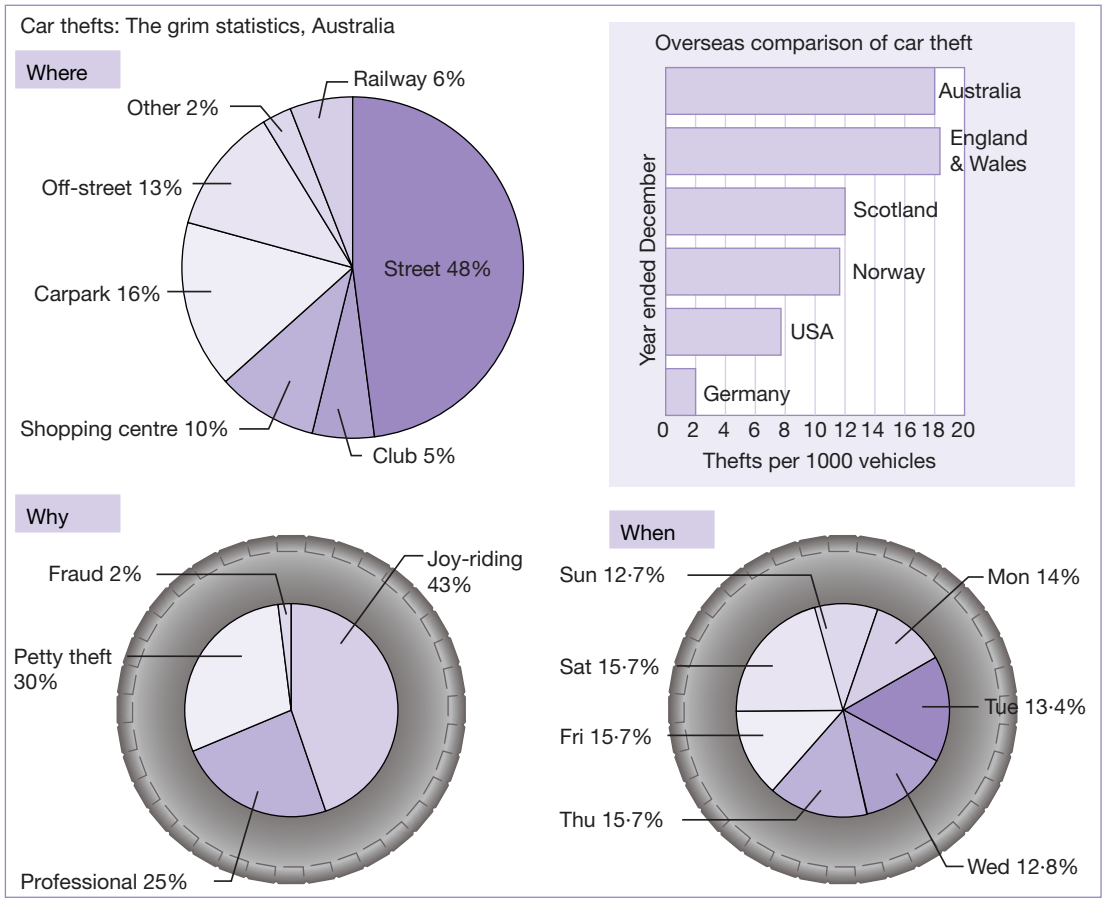
a In which month was the highest rainfall recorded?

b Which month had the lowest mean temperature? What was this temperature?

c How many months had a rainfall greater than 200 mm?

d The scale on the rainfall axis is 1 cm = 60 mm. Use this fact to find the rainfall for September.

4 Use the graphs to answer the following questions.



a In Australia, what percentage of cars were stolen from:

i railway stations **ii** car parks **iii** shopping centres

iv clubs **v** on the street **vi** off the street?

b What other places could fit the category 'Other' on the graph showing where thefts occur?

c What percentage of stolen cars were taken by joy riders?

d What fraction of all thefts were carried out by professional thieves?

e On which day was the least number of cars stolen?

f Why do you think two of the sector graphs are drawn as tyres?

g How many cars per thousand (to the nearest whole car) were stolen in:

i Australia **ii** Norway **iii** Germany

iv the USA **v** Scotland **vi** England and Wales?

h Are we told when these statistics were collected?

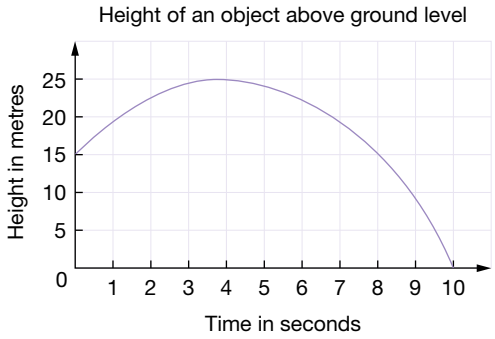
5 The graph shows the height of an object above the ground over a period of time.

a How high was the object above ground level at the start of timing? When did it pass this height again?

b When did the object reach its maximum height?

c When did the object reach the ground?

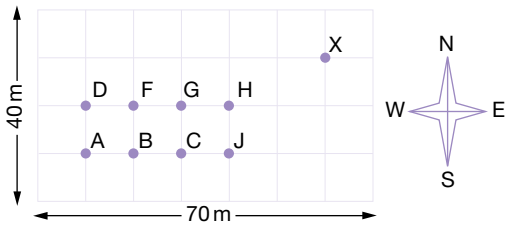
d How long did the object take to fall the last 15 m?



ASSIGNMENT 7B Working mathematically

1 Tom is eight years older than Clare. The sum of their ages is 54. How old is Tom?

2 Pirate Pete was drawing up a treasure map.



a He arrived at X after walking 20 m north, then 30 m east, then 30 m south, then 10 m east and then 30 m north. From which point did he start?

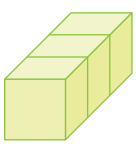
b Later he arrived at X after walking 40 m east, then 10 m south, then 20 m west, then 20 m north and then 10 m east. From which point did he start?

c Finally he arrived at X after walking 20 m south, then 40 m west, then 40 m north and then 60 m east. From which point did he start this time?

3 What six coins could be used to give a total of $3.40? Give two solutions.



4 Danny placed three of his building blocks in a row on the floor with ends touching.



a How many of the faces would be visible?

b How many faces would not be visible?

c How many faces would be visible if there were 70 blocks?

d How many faces would not be visible if there were 100 blocks?

5 A farmer has chickens and horses. If there are 16 heads and 40 legs on these altogether, how many horses are there?

ASSIGNMENT 7C Cumulative revision

1 On a number line, graph the set of counting numbers between 2 and 6. 1:04

2 Write these Roman numerals as basic numerals in our number system. 1:03
a CXXIII **b** XLIV **c** MDCIX **d** MCMLXXXVIII

3 Simplify: 1:04
a $7 \times 2 + 8 \times 10$ **b** $14 - (15 - 1)$
c $7 - 36 \div 4$ **d** $100 \div (80 \div [20 - 16])$
e $(7 \times 10\,000) + (2 \times 1000) + (1 \times 100) + (8 \times 10) + (5 \times 1)$

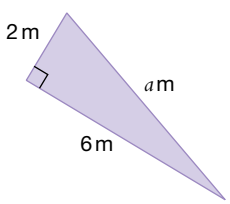
4 **a** $\frac{7}{10} + \frac{9}{10}$ **b** $\frac{3}{8} + \frac{2}{5}$ **c** $\frac{9}{10} - \frac{1}{4}$ **d** $\frac{3}{5} \times \frac{3}{4}$ 1:05
e $\frac{17}{100} \times \frac{1}{2}$ **f** $4 \times \frac{3}{8}$ **g** $\frac{9}{10} \div \frac{3}{4}$ **h** $\frac{3}{4} \div 5$

5 **a** $18{\cdot}65 + 4{\cdot}4$ **b** $10 - 3{\cdot}15$ **c** $41{\cdot}5 \div 100$ **d** $1{\cdot}3 \times 0{\cdot}5$ 1:06
e $4{\cdot}5 \div 3$ **f** $8{\cdot}26 \div 8$ **g** $4{\cdot}6 \div 0{\cdot}02$ **h** $(0{\cdot}05)^2$
i Round $0.\dot{6}$ to two decimal places.

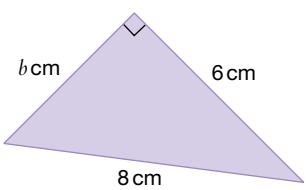
6 Write as a percentage: 1:07
a $\frac{3}{4}$ **b** $1\frac{2}{5}$ **c** $0{\cdot}145$ **d** $2{\cdot}05$

7 **a** Find 8% of \$43 000. **b** Find 13% of 800 km. 1:07
c Increase \$1600 by 5%. **d** Decrease \$1680 by 5%.
e 20% of my wages is \$130. How much are my wages?

8 Find the value of each pronumeral, correct to one decimal place. 3:04
a



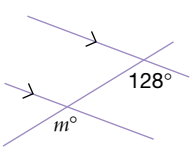
b



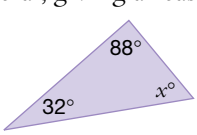
9 Simplify: 5:02, 5:05
a $8x + 3x$ **b** $10a - 9a$ **c** $3x^2 + 5x^2$ **d** $6m - m$
e $10 \times 4y$ **f** $20b \div 2$ **g** $\frac{1}{4} \times 16a$ **h** $2a \times 7b$
i $\frac{x}{5} + \frac{x}{5}$ **j** $\frac{15m}{20} - \frac{2m}{20}$ **k** $\frac{2x}{7} \times \frac{2y}{5}$ **l** $\frac{m}{4} \div \frac{m}{5}$

10 **a** Of 200 tickets sold in a raffle, Ronan bought 10, Naomi bought 8 and Bijal bought 2. A ticket was chosen at random to determine who won the prize. What is the probability that the prize was won by: 6:02, 6:03
i Ronan **ii** Naomi **iii** Bijal?
b The chance of our boat sinking as we cross the river is $\frac{3}{20}$. What is the chance that our boat will not sink?

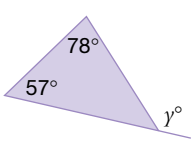
11 Find the value of each pronumeral, giving a reason for your answer. 1:08
a



b

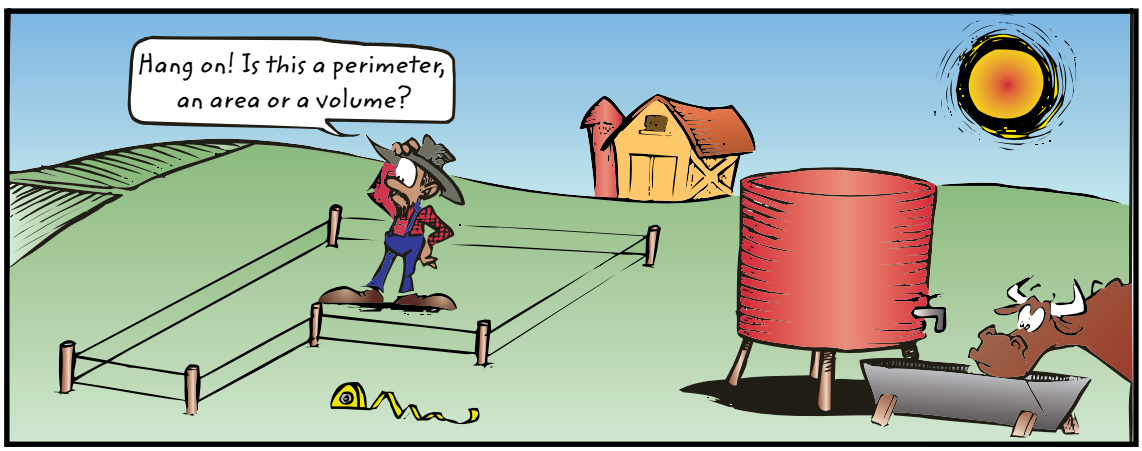


c



MEASUREMENT

8



Contents

8:01 Perimeter
Investigation 8:01 Skirting board and perimeter
8:02 Area review
Investigation 8:02 Area in a house
8:03 Areas of special quadrilaterals
GeoGebra activity 8:03 Finding areas
Investigation 8:03 Car parking areas
8:04 Formulas for the areas of special quadrilaterals
8:05 Units of volume and capacity
8:06 Volumes of prisms
GeoGebra activity 8:06 Volumes of prisms
Investigation 8:06A Estimating volume
Investigation 8:06B Volume of skyscrapers
Fun spot 8:06 Check your vision for good measure!
8:07 Measuring time
8:08 Adding and subtracting times
Fun spot 8:08 Why are gardeners funny?
Investigation 8:08 Excel spreadsheets and time
8:09 International time zones
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Measurement and Geometry

Selections from ***Length***, ***Area***, ***Volume***, ***Time*** [Stage 4]

- Find perimeters of parallelograms, trapeziums, rhombuses and kites. (ACMMG196)
- Find areas of trapeziums, rhombuses and kites. (ACMMG196)
- Choose appropriate units for area and convert from one unit to another. (ACMMG195)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving. (ACMMG159)
- Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving the volume. (ACMMG198)
- Solve problems involving duration, including using 12-hour and 24-hour time within a single time zone. (ACMMG199)
- Solve problems involving international time zones.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

8:01 Perimeter

The ability to calculate perimeters is necessary when calculating the stagger in staggered starts.

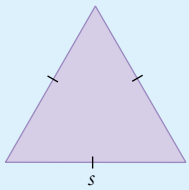


The perimeter of a plane shape is the length of its boundary.

To calculate the perimeter:
- find the lengths of all the sides
- add the lengths together.

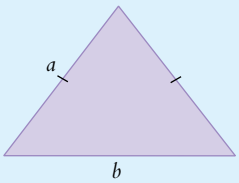
The geometric properties of some shapes allow the perimeter to be calculated using a simple formula.

Equilateral triangle



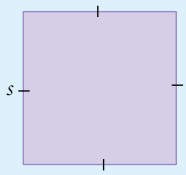
$P = 3s$

Isosceles triangle



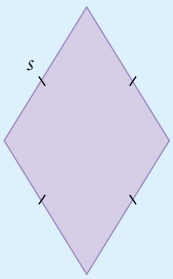
$P = 2a + b$

Square



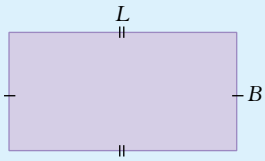
$P = 4s$

Rhombus



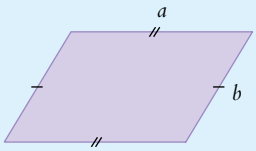
$P = 4s$

Rectangle



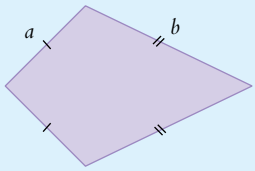
$P = 2L + 2B$
$= 2(L + B)$

Parallelogram



$P = 2a + 2b$
$= 2(a + b)$

Kite



$P = 2a + 2b$
$= 2(a + b)$

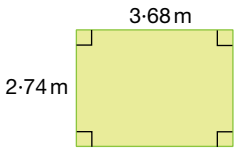
To find the perimeter of a trapezium, add the lengths of the four sides.

Measurements
- All measurements are approximations.
- They are as accurate as the measuring instruments used to obtain them.
- The greatest possible error in a correct measurement is half of the smallest unit used.

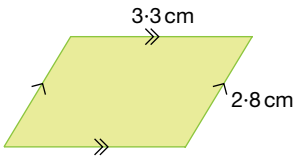
WORKED EXAMPLES

1 Find the perimeter of each of the following shapes.

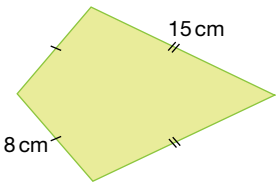
a



b



c



Solutions

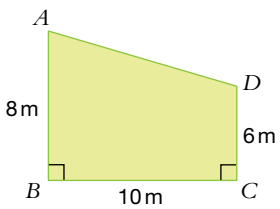
a Perimeter
$= 2 \times (3{\cdot}68 + 2{\cdot}74)$
$= 2 \times 6{\cdot}42$
$= 12{\cdot}84$ m

b Perimeter
$= 2 \times (3{\cdot}3 + 2{\cdot}8)$
$= 2 \times 6{\cdot}1$
$= 12{\cdot}2$ cm

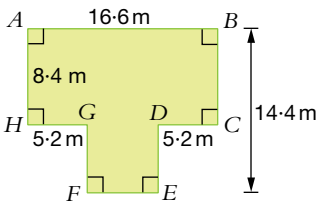
c Perimeter
$= 2 \times (8 + 15)$
$= 2 \times 23$
$= 46$ cm

2 Find the perimeter of each of the following shapes.

a



b

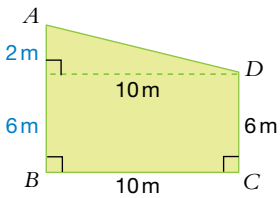


Remember:

1. Copy the diagram.
2. Work out the lengths of all the sides.
3. When adding up sides, be careful not to miss any sides.

Solutions

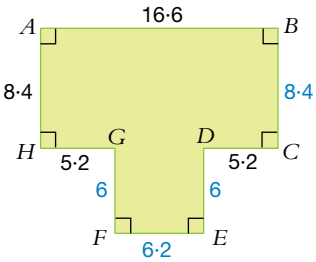
a



$AD^2 = 2^2 + 10^2$
$= 104$
$\therefore AD = \sqrt{104}$

Perimeter
$= 6 + 10 + 8 + \sqrt{104}$
$= 34{\cdot}20$ m (2 dec. pl.)

b



$5{\cdot}2 + FE + 5{\cdot}2 = 16{\cdot}6$
$FE = 6{\cdot}2$

$8{\cdot}4 + GF = 14{\cdot}4$
$\therefore GF = 6$
$BC = 8{\cdot}4$
$DE = 6$

Perimeter
= sum of horizontal sides + sum of vertical sides

Sum of horizontal sides
$= 16{\cdot}6 + 5{\cdot}2 + 5{\cdot}2 + 6{\cdot}2$
$= 33{\cdot}2$

Sum of vertical sides
$= 8{\cdot}4 + 6 + 8{\cdot}4 + 6$
$= 28{\cdot}8$

$\therefore$ Perimeter
$= 33{\cdot}2 + 28{\cdot}8$
$= 62$ m

Length and perimeter benchmarks

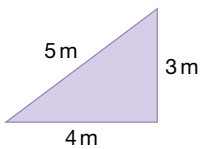
Gather some personal benchmarks so that you understand measurements as they are used.

- A metre is one long adult pace.
- The length of the Sydney Harbour Bridge is 1149 m (a little more than 1 km).
- The perimeter of a basketball court is 86 m.

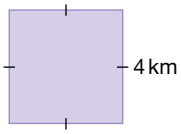
Exercise 8:01

1 Find the perimeter of each shape.

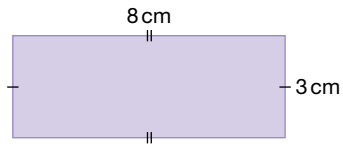
a



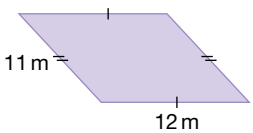
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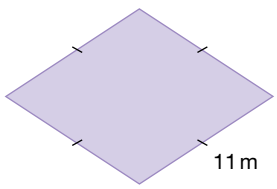
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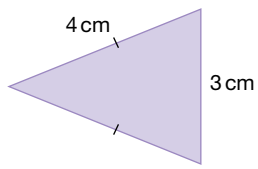
d



e

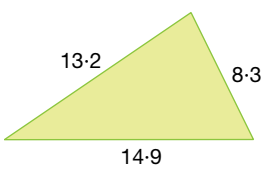


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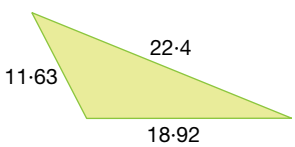


2 Find the perimeter of each of the following shapes. All measurements are in metres unless indicated otherwise.

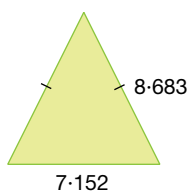
a



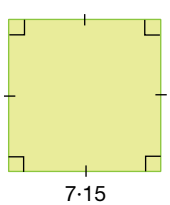
b



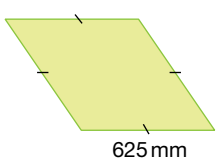
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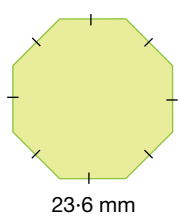
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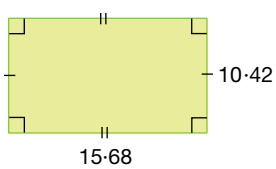
e



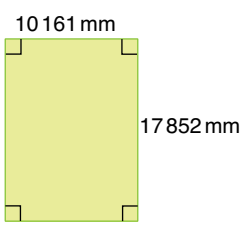
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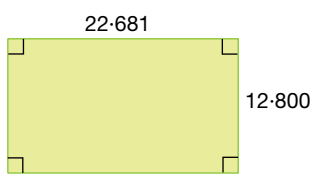
g



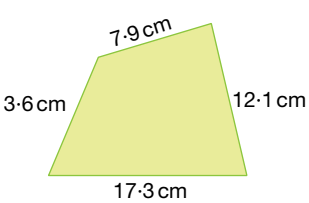
h



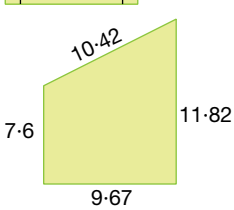
i

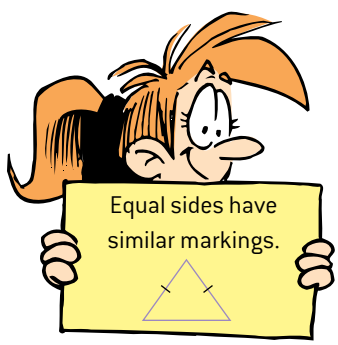


j



k

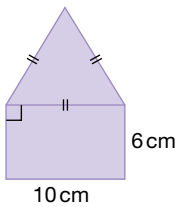




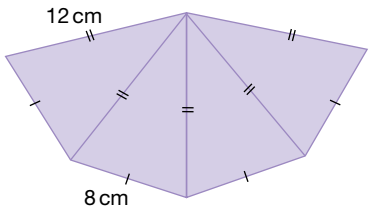
3 a An equilateral triangle has a side of length 4·68 m. What is its perimeter?
b Find the perimeter of a square that has a side of 563 mm.
c A rectangle is measured and found to be 45·6 cm long and 23·4 cm wide. Find its perimeter.
d A rectangle is 7162 mm long and 4168 mm wide. Find its perimeter.
e A rectangle is measured and its dimensions are: length 5·162 m and breadth 2·610 m. Find its perimeter.

4 a How does the perimeter of a rectangle change if you double the side lengths?
b How does the perimeter of a rectangle change if you treble the side lengths?
c The base of an isosceles triangle is 10 m and its perimeter is 34 m. Write the measurements of its three sides.

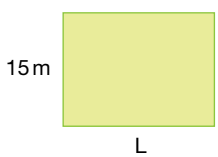
5 a A pentagon is made by placing an equilateral triangle on top of a rectangle. What is the perimeter of the pentagon?



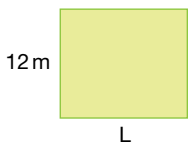
b A hexagon is made by joining four isosceles triangles as shown in the diagram. What is the perimeter of the hexagon?



6 Find the length of each rectangle.

a 

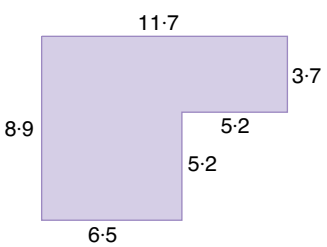
Perimeter = 70 m
Breadth = 15 m
Length = _____

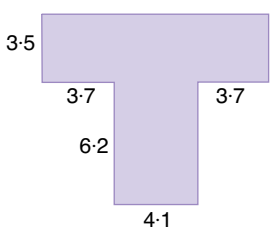
b 

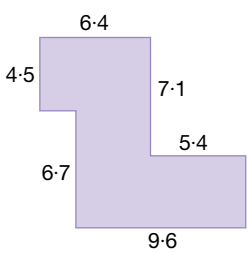
Perimeter = 52 m
Breadth = 12 m
Length = _____

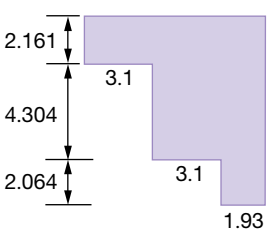
c The perimeter of a square is 26 m. What is the length of one side?

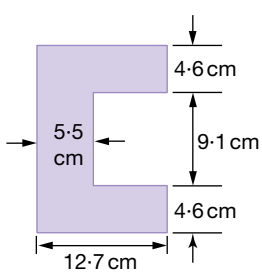
7 Calculate the perimeter of the following shapes. All angles are right angles and all measurements are in metres unless indicated otherwise.

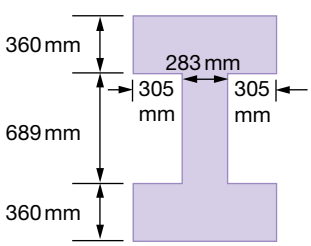
a 

b 

c 

d 

e 

f 

Remember:
Work out the lengths of all missing sides first.

8 **a** A farmer has a paddock 450 m long and 255 m wide. Find the perimeter of this paddock. How much would it cost to fence this paddock at 50c per metre?

b A jogger runs around a rectangular park that is 420 m long and 95 m wide. How far will he jog if he runs around the park 10 times?

c A building block is rectangular in shape. It is 26·4 m wide and 50·4 m deep. The block is to be fenced using welded fence panels. If the panels only come in lengths of 2·4 m, how many will be needed to fence the block?

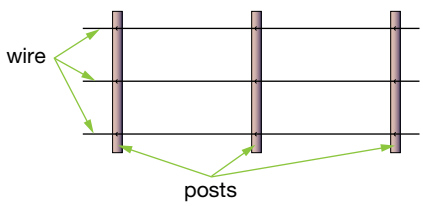
9 **a** The wheel of a steamroller has a diameter of 2·5 m. How far will the steamroller move if this wheel makes one revolution? Give your answer correct to one decimal place.

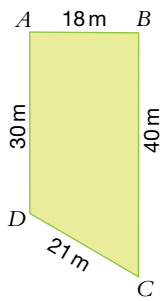
b The diameter of a car wheel is 35 cm. How many complete revolutions will this wheel need to make to travel 1 km? (Hint: Keep all units of length the same.)

c A circular bicycle track has a diameter of 125 m. Find how far a cyclist covers in completing one lap of the track. Give your answer to the nearest metre.

d If a circular running track has a diameter of 140 m, find how many complete laps must be run to cover at least 10 000 m.

10 A block of land, as shown in the diagram, is to be fenced using posts and three lines of wire.





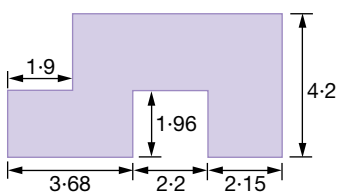
a Calculate the amount of wire needed to complete the fence.

b If posts are to be placed at 3 m intervals (as a maximum), calculate the least number of posts needed for the fence.

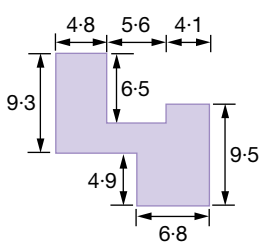
c Calculate the total cost of the fence if the posts are $8.50 each and the wire is 95 cents a metre.

11 Find the perimeter of the following shapes. All angles are right angles and all measurements are in metres.

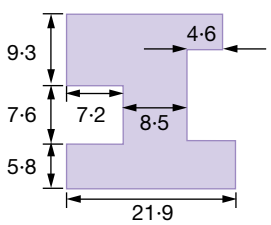
a



b



c





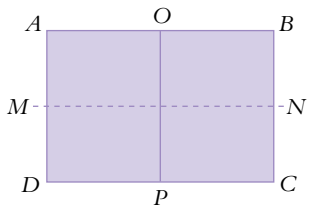
The photo shows a floor plan like those above. The timber formwork is erected ready for the pouring of the concrete.

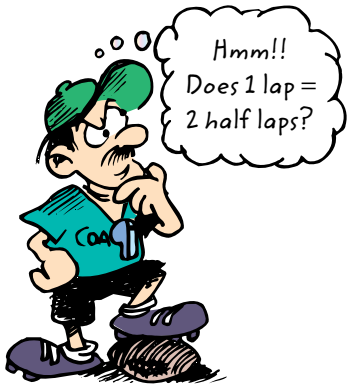
d A rugby field is rectangular in shape. It has a length of 90 m and a breadth of 60 m.
A coach told his team to run around the field 10 times to warm up. The team complained that this was too much, so the coach relented and told them to run around half of the field 15 times.
Some decided to run the half of the field *AOPD*.
The others decided to run the half *ABNM*. Find:

i the distance covered by 15 laps of *AOPD*

ii the distance covered by 15 laps of *ABNM*

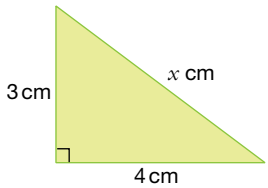
iii the distance they would have run in 10 laps of the field *ABCD*.



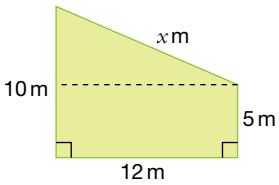


12 Use Pythagoras' theorem to find the value of x and then calculate the perimeter.

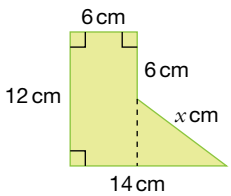
a



b

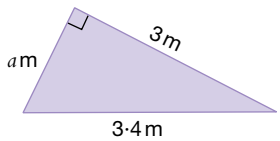


c

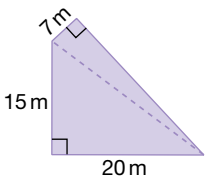


13 Calculate the perimeter of each of the following shapes. (You will need to use Pythagoras' theorem to find a missing side.)

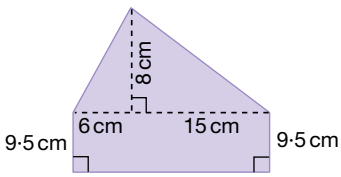
a



b

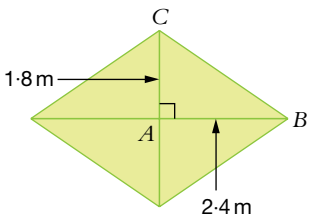


c



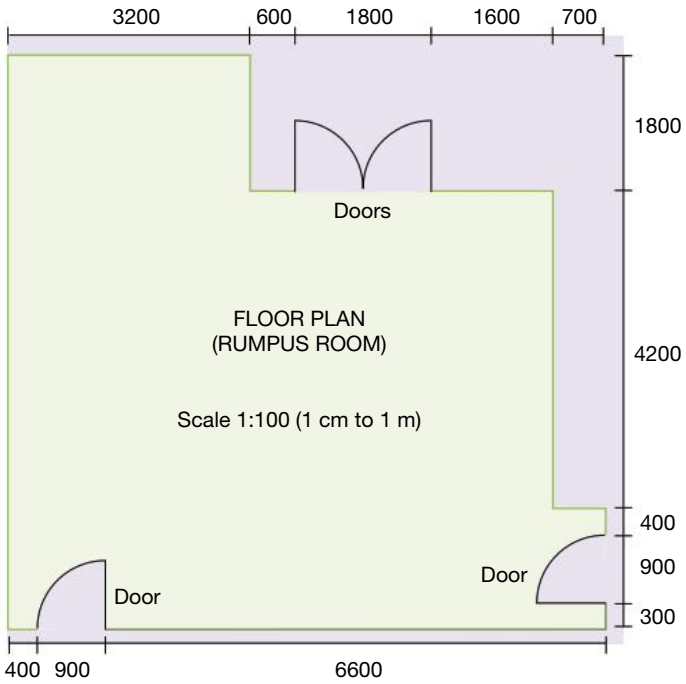
14 **a** The rhombus shown is made from four identical right-angled triangles. What is its perimeter?

b A rhombus has diagonals that are 14·4 cm and 19·2 cm long. What is its perimeter?



INVESTIGATION 8:01 SKIRTING BOARD AND PERIMETER

A scale drawing of a rumpus room in a house is shown. All measurements are in millimetres.



- Rob Young, a local builder, has been asked to fit skirting board to the room. (Skirting board is used to cover the gap between the wall and the floor of a building.)
- The skirting board is to be placed around the perimeter of the room except for the doorways. It can be ordered in lengths from 300 mm to 6·6 m at increments of 300 mm (i.e. 300 mm, 600 mm, 900 mm, 1·2 m and so on, up to 6·6 m).

1 Rob has been asked to do the job without any joins (except at corners). Is this possible? Give reasons for your answer.

2 What is the total length of skirting board required?

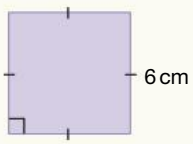
3 Rob has nine 3·3 m lengths of skirting left from earlier jobs. Show how he could use these to do the job. What is the smallest number of joins he could have?

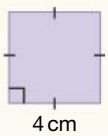
4 If Rob has no skirting board, what would he need to order to complete the job with the smallest number of joins and the smallest amount of waste?

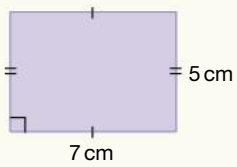
8:02 Area review

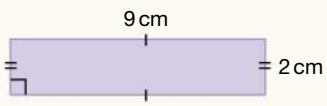
PREP QUIZ 8:02

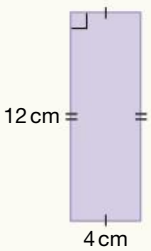
Find the area of each shape in square centimetres.

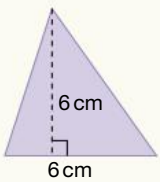
1


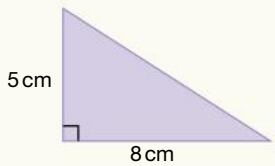
2


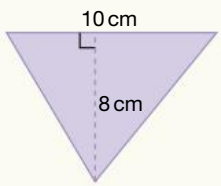
3


4


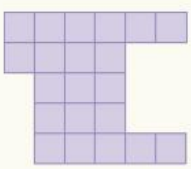
5


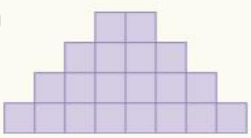
6


7


8


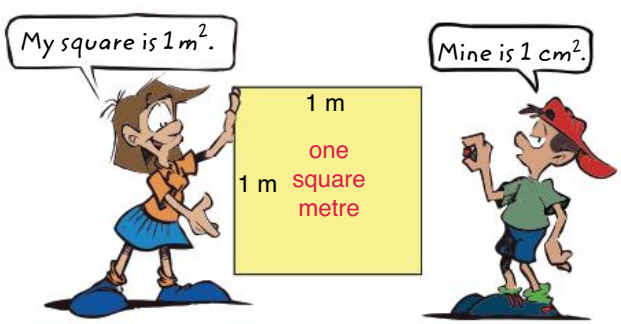
Find the areas of these shapes in square units.

9


10


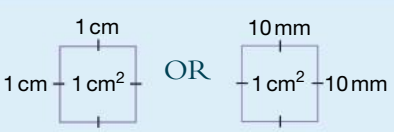
Remember:
The dimensions of a rectangle are length and breadth.

The area of a two-dimensional shape is the amount of space inside that shape.
We measure the number of square units inside the shape.



$1\text{ cm}^2 = 100\text{ mm}^2$
$1\text{ m}^2 = 10\,000\text{ cm}^2$
$= 1\,000\,000\text{ mm}^2$
$1\text{ ha} = 10\,000\text{ m}^2$
$1\text{ km}^2 = 1\,000\,000\text{ m}^2$
$= 100\text{ ha}$

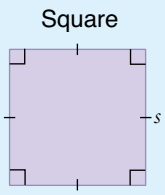
1 cm^2 is the area within a square that has 1 cm sides.
1 m^2 is the area within a square that has 1 m sides.
1 ha is the area within a square that has 100 m sides.



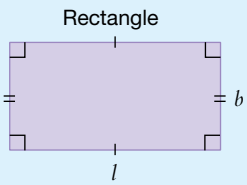
Area benchmarks

- The cover of this book is about 500 cm^2.
- The area of the playing surface of a tennis court is about 260 m^2.
- An area 100 m long and 100 m wide is 1 hectare.
- A soccer field is about 0.75 ha in area.

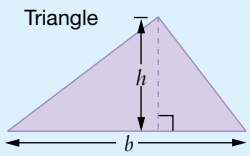
In Year 7 you learnt how to find the areas of squares, rectangles and triangles. The formulas for these shapes are shown below.



$A = s^2$



$A = lb$

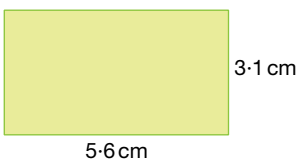


$A = \frac{1}{2}bh$ or $\frac{bh}{2}$

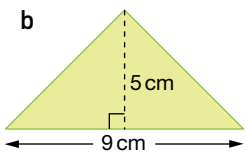
WORKED EXAMPLE 1

Find the area of each shape below.

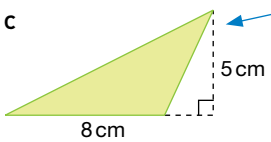
a



b



c



This is the vertical height of the triangle.

Solutions

a $A = lb$
$= 5{\cdot}6 \times 3{\cdot}1$
$= 17{\cdot}36$
∴ Area of rectangle $= 17{\cdot}36\ \text{cm}^2$

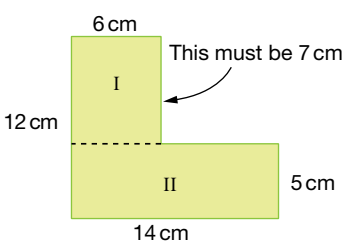
b $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 9 \times 5$
$= 22\frac{1}{2}$
∴ Area of triangle $= 22\frac{1}{2}\ \text{cm}^2$

c $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 8 \times 5$
$= 20$
∴ Area of triangle $= 20\ \text{cm}^2$

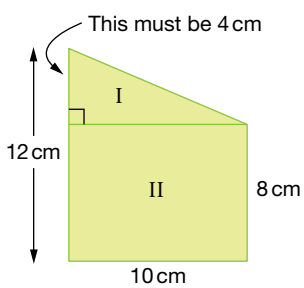
WORKED EXAMPLE 2

Divide each of the following into basic shapes and then find the area.

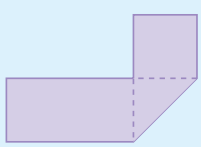
a



b



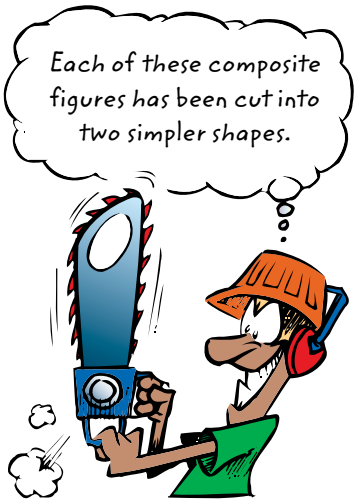
Sometimes a shape needs to be cut into more than one of the basic shapes before its area can be found. Such a shape is called a **composite figure**.



Solutions

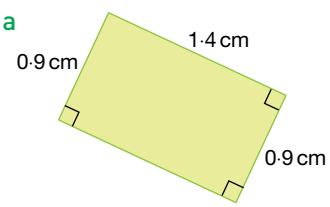
a Area of rectangle I
$= 6 \times 7$
$= 42\,\text{cm}^2$
Area of rectangle II
$= 14 \times 5$
$= 70\,\text{cm}^2$
$\therefore$ Total area
$= 42 + 70$
$= 112\,\text{cm}^2$

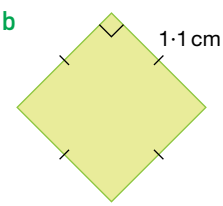
b Area of triangle I
$= \frac{1}{2} \times 10 \times 4$
$= 20\,\text{cm}^2$
Area of rectangle II
$= 10 \times 8$
$= 80\,\text{cm}^2$
$\therefore$ Total area
$= 20 + 80$
$= 100\,\text{cm}^2$

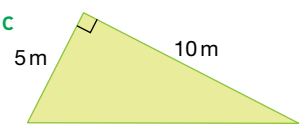


Exercise 8:02

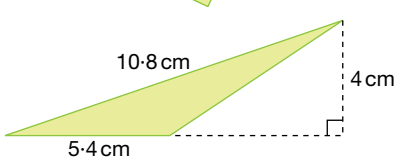
1 Calculate the area of each shape.

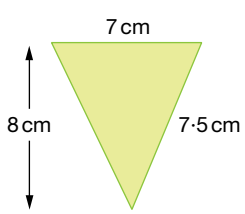
a


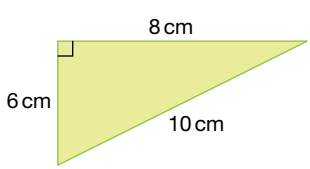
b


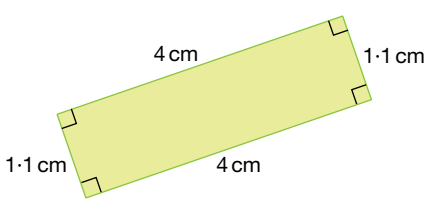
c


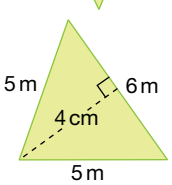
Note:
The 'base' of the triangle may not be on the bottom.

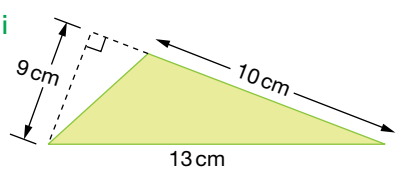
d


e


f


g


h


i


2 a A classroom is approximately 6 m long and 8 m wide. What is the approximate area?

b A football field is approximately 100 m long and 50 m wide. What is its area in:

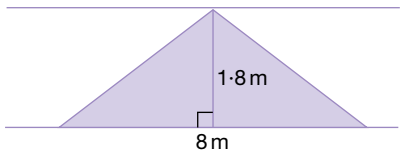
i square metres ii hectares?

c Calculate the number of square metres of paving bricks that should be bought to cover a rectangular area with length 6·5 m and breadth 2·4 m.

d A farmer has a rectangular paddock that is 620 m long and 280 m wide. What is the area of this paddock in:

i square metres ii hectares?

e A painter has to paint part of a wall as shown on the right. What is the area of the section he has to paint?



3 Find the area, to the nearest cm^2, of each of the following shapes:

a a rectangle with length and breadth of 5·6 cm and 3 cm respectively.

b a rectangle with dimensions of 3·7 cm and 9 cm

c a square with a side length of 2·5 cm

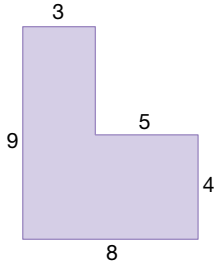
d a triangle with a base of 7·7 cm and a height of 9 cm

e a right-angled triangle whose side lengths are 2·5 m, 6 m and 6·5 m.

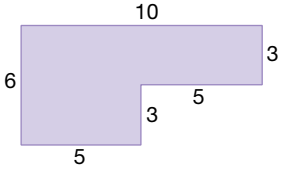
4 Write the dimensions (length and breadth), in whole numbers, of three different rectangles that have an area of 12 cm^2. (These will match pairs of factors of 12.)

5 Divide each composite shape into rectangles or triangles and then find its area. All measurements are in centimetres.

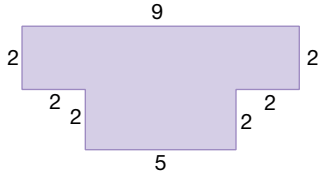
a



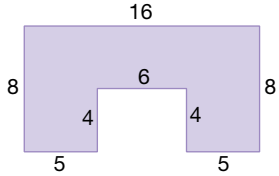
b



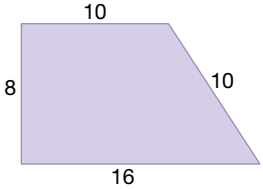
c



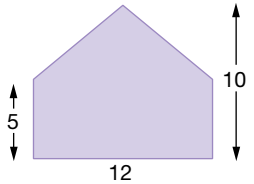
d



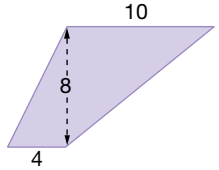
e



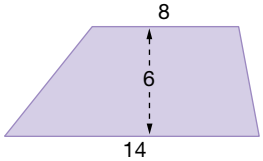
f



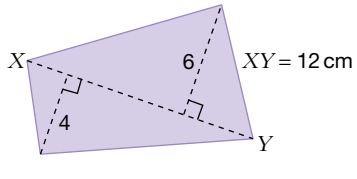
g



h

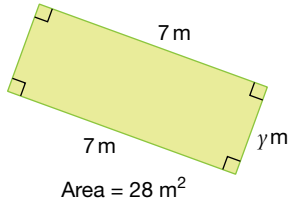


i



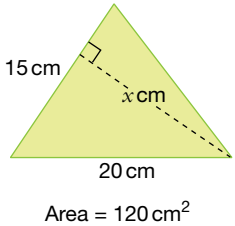
6 Under each shape the area is given. Use this to find the value of the pronumeral.

a



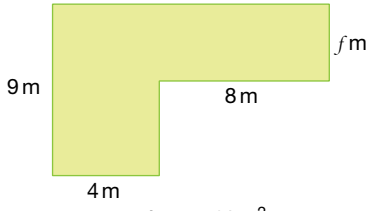
Area = 28 m^2

b



Area = 120 cm^2

c

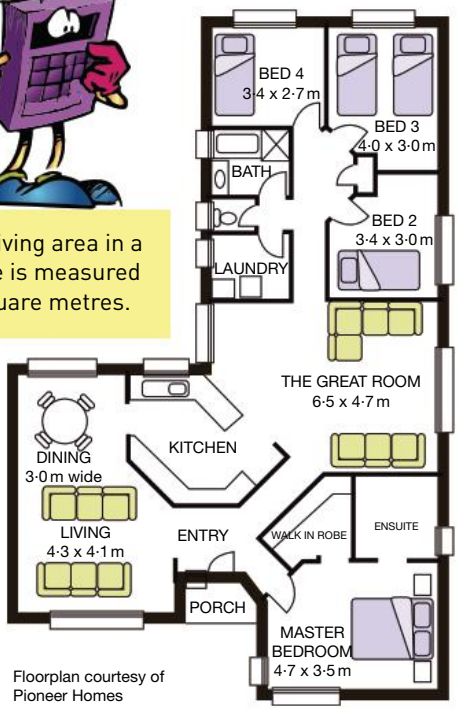


Area = 60 m^2

INVESTIGATION 8:02 AREA IN A HOUSE



The living area in a home is measured in square metres.



Floorplan courtesy of Pioneer Homes

You have just purchased this new home. The next step is to cover the floors.

Cost of floor coverings, including laying

Carpet (sold in 4 m widths)

Bedroom:	top grade	\$120/m
	2nd grade	\$90/m
Kitchen:	top grade	\$100/m
	2nd grade	\$70/m
Living room:	top grade	\$180/m
	2nd grade	\$80/m
	heavy duty	\$110/m
Bathroom:	top grade	\$50/m
	2nd grade	\$40/m

Vinyl (Sold in 4 m widths)

top grade	\$105/m
2nd grade	\$75/m

Floor tiles (sold in m^2)

top grade	\$35/m^2
2nd grade	\$20/m^2

1 Decide what type of floor covering you would choose for each part of the house.

2 Estimate the area of each type of covering needed. (For irregular areas, estimate a little more than otherwise.) Copy the grid on the right onto tracing paper. Each small square represents one square metre on the house plan. Check each of your estimates of area by placing your tracing paper over the plan of the house.



3 Estimate the cost if the materials were all:
a top grade **b** 2nd grade.

4 If you could afford top grade materials in just two rooms, which two rooms would you choose? Work out the total cost of floor coverings using this choice if a discount of 10% is given.





Which would be worth more: a square metre of 20c coins or 1 kg of \$1 coins?

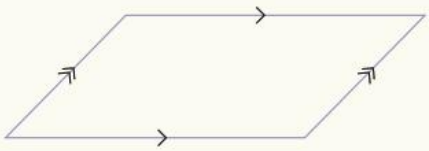
8:03 Areas of special quadrilaterals

PREP QUIZ 8:03

Refer to **ID Card 4** on page xvii to identify the figures numbered:

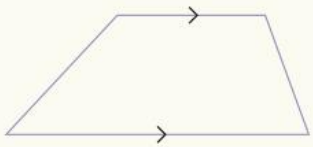
1 1 **2** 2 **3** 3 **4** 4 **5** 5 **6** 15

7 Copy the parallelogram below and draw lines to show how it can be made from a rectangle and two triangles.



Parallelogram

8 Copy the trapezium below and add lines to show how it can be made from a rectangle and two triangles.



Trapezium

9 Divide the trapezium into a rectangle and a triangle.

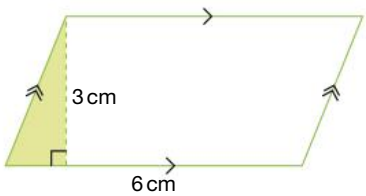


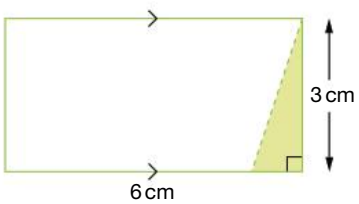
10 Divide the trapeziums into a rectangle and a triangle.



If you know how to find the areas of rectangles and triangles, you can calculate the areas of many other plane shapes.

Parallelogram





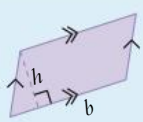


Cutting a triangle off one end of a parallelogram and adding it to the other end creates a rectangle. The area of the shape, of course, does not change.

Area of rectangle = 6 cm × 3 cm
= 18 cm^2

∴ Area of parallelogram = 18 cm^2

The area of a parallelogram
= base × perpendicular height
$A = bh$

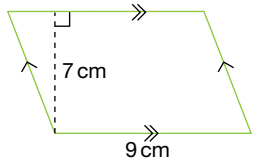


Exercise 8:03

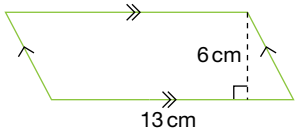
P Foundation worksheet 8:03
Finding the area

1 Find the area of each parallelogram by forming a rectangle.

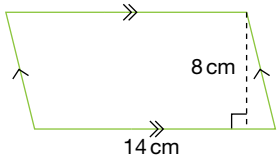
a



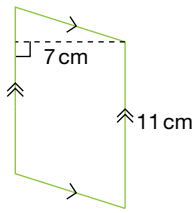
b



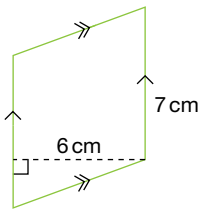
c



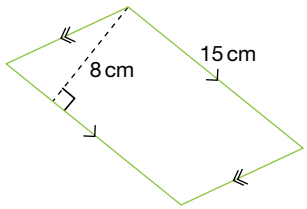
d



e



f

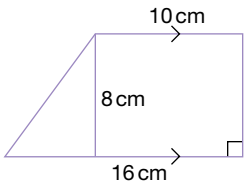


2 This trapezium has been divided into a rectangle and a triangle.

a What is the area of the rectangle?

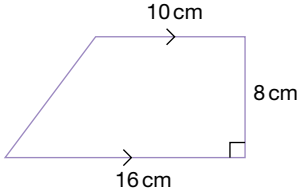
b What is the area of the triangle?

c What is the total area of the trapezium?

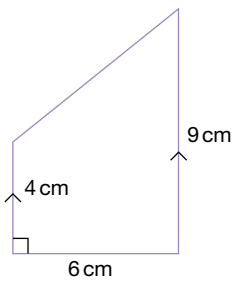


3 Find the area of each right trapezium by following the steps in Question 2.

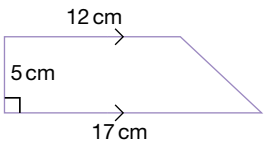
a

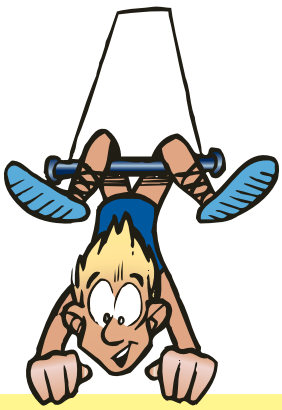


b



c





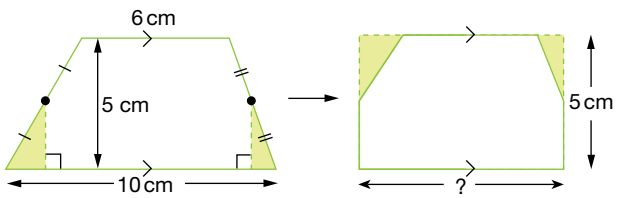
Note:
If a trapezium has a right angle, it is called a **right trapezium**.

4 The trapezium on the left has been formed into a rectangle by moving the two triangles shown.

a What must be the length of the rectangle?

b What is the area of the rectangle?

c What is the area of the trapezium?



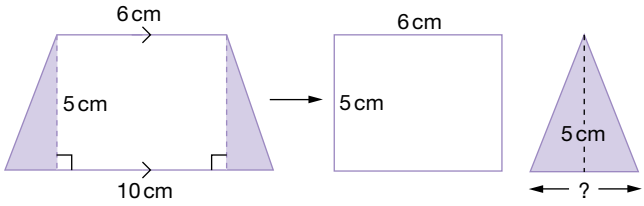
5 A trapezium is divided into a rectangle and a triangle made from the two ends.

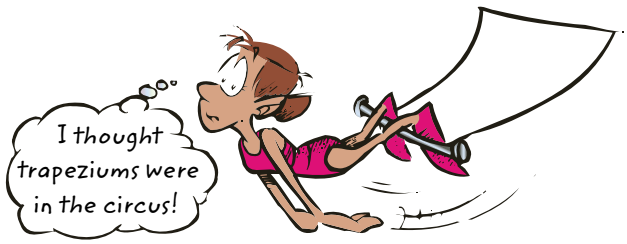
a What is the length of the base of the triangle?

b What is the area of the triangle?

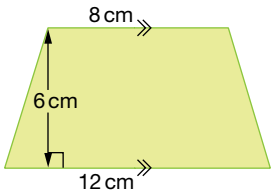
c What is the area of the rectangle?

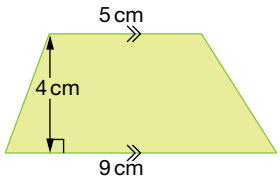
d What is the area of the trapezium?

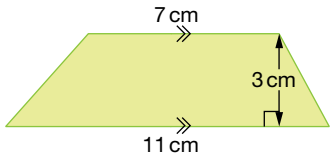


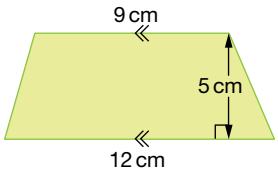


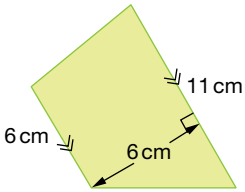
6 Using the method from either Question **4** or **5**, find the area of each of the following trapeziums.

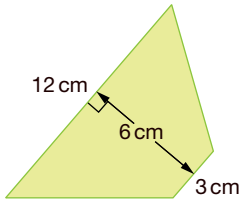
a 

b 

c 

d 

e 

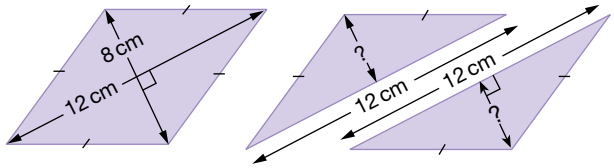
f 

7 This rhombus has been split into two identical triangles.

a What is the height of each triangle?

b What is the area of each triangle?

c What is the area of the rhombus?



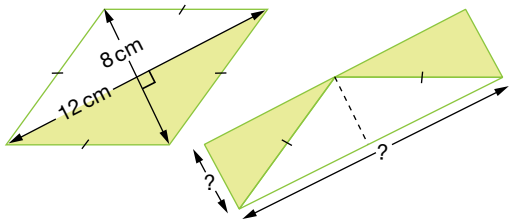
8 The rhombus in Question **7** can also be divided and pieced together to form a rectangle.

a What is the length of the rectangle?

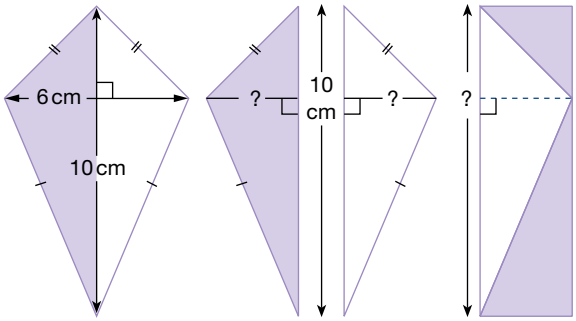
b What is the breadth of the rectangle?

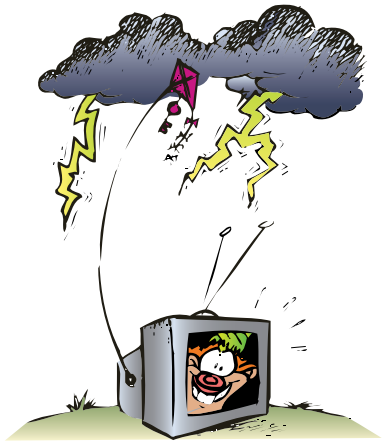
c What is the area of the rectangle?

d What is the area of the rhombus?



9





A kite can be split like the rhombus in Question 8.

a If it is split into two triangles, what is the 'height' of each triangle?

b What is the combined area of the two triangles?

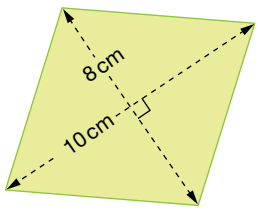
c If a rectangle is formed, what are the dimensions of the rectangle?

d What is the area of the rectangle?

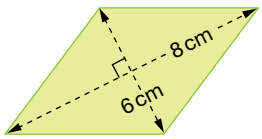
e What is the area of the kite?

10 Calculate the area of each kite or rhombus.

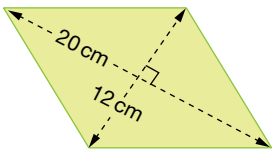
a



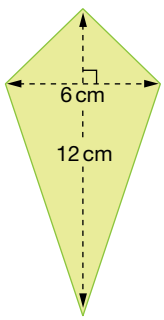
b



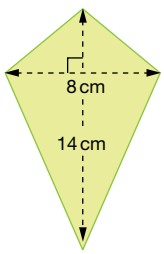
c



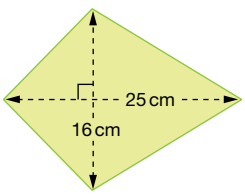
d



e



f





Estimate the area of the sail.

1 Area of a rectangle

Click and drag points B and D to change the length and breadth of the rectangle.

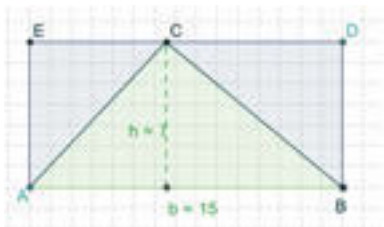
$A = L \times B$

Length = 8
Breadth = 4
Area = $8 \times 4 = 32$ units

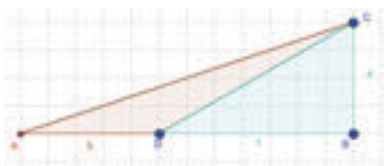


2 Area of a triangle

a Click and drag the points A, B and C to change the positions of the vertices of the triangle. Watch how the area of the triangle changes.
Complete this sentence:
The area of the triangle is _____ the area of a rectangle with the same _____ and _____.

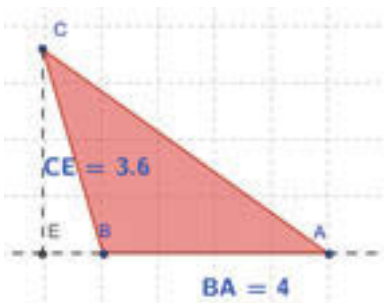


b Click and drag the points B, C and D to show that for an obtuse-angled triangle the area is also half the base times the height.



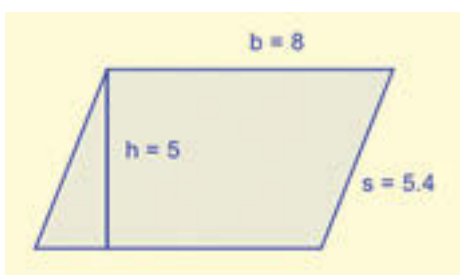
c Click and drag the points A, B and C to show that for any type of triangle, the area is given by the formula:

$A = \frac{1}{2}b \times h$ or $A = \frac{b \times h}{2}$

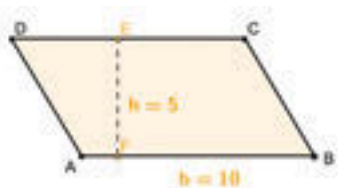


3 Area of a parallelogram

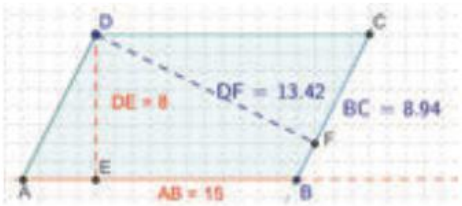
a Vary the dimensions of the parallelogram to change its area. What is the formula for the area of a parallelogram?



b Find the area of the red parallelogram by dragging the blue quadrilateral onto it, and using its given area. What did you discover?

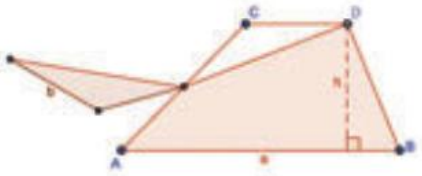


c Click and drag the points B and D to see that any side can be used as a base. In each case, the height is perpendicular to the base.

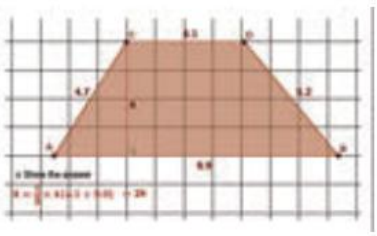


4 Area of a trapezium

a The formula for the area of a trapezium is demonstrated by changing the trapezium into a triangle of the same area.

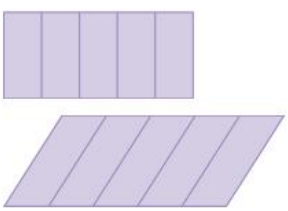


b The formula is used to calculate the area of a trapezium formed.



INVESTIGATION 8:03 CAR PARKING AREAS

Car parking bays are based on either rectangles or parallelograms as shown.





- What dimensions of each shape would be needed to provide a suitable size for parking a car? (Remember to allow for clearance between adjacent cars.)
- Which arrangement uses the greater area?
- What are the advantages and disadvantages of each arrangement?

Visit at least two different types of car parks. Can you see why the parking spaces were organised the way they are? Is there a more suitable arrangement? What mathematical skills would you need to use in your investigation? What traffic characteristics needed to be considered (e.g. number of entrances, turning circles of vehicles)?

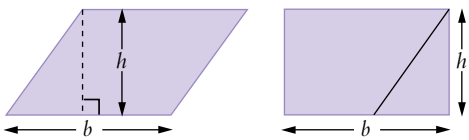
8:04 Formulas for the areas of special quadrilaterals

In the previous section, you found the areas of shapes by dividing them into rectangles or triangles. Sometimes it is quicker and easier to use a formula that will give the area for a particular shape.



Parallelogram

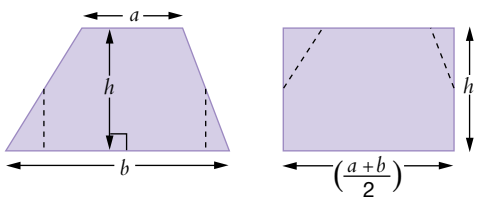
The **height** of the parallelogram becomes the **breadth** of the rectangle.
Area = $b \times h$ or $A = bh$, where b = base and h = height.



Trapezium

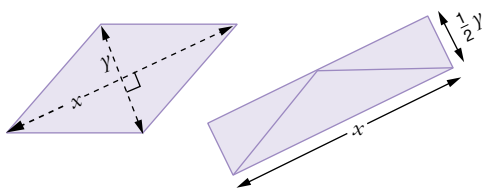
The length of the rectangle is the average of the two parallel sides, $l = \dfrac{a+b}{2}$.

Area = $h \times \dfrac{a+b}{2}$ or A = $\frac{1}{2}h(a+b)$, where a and b are the lengths of the parallel sides and h is the height.

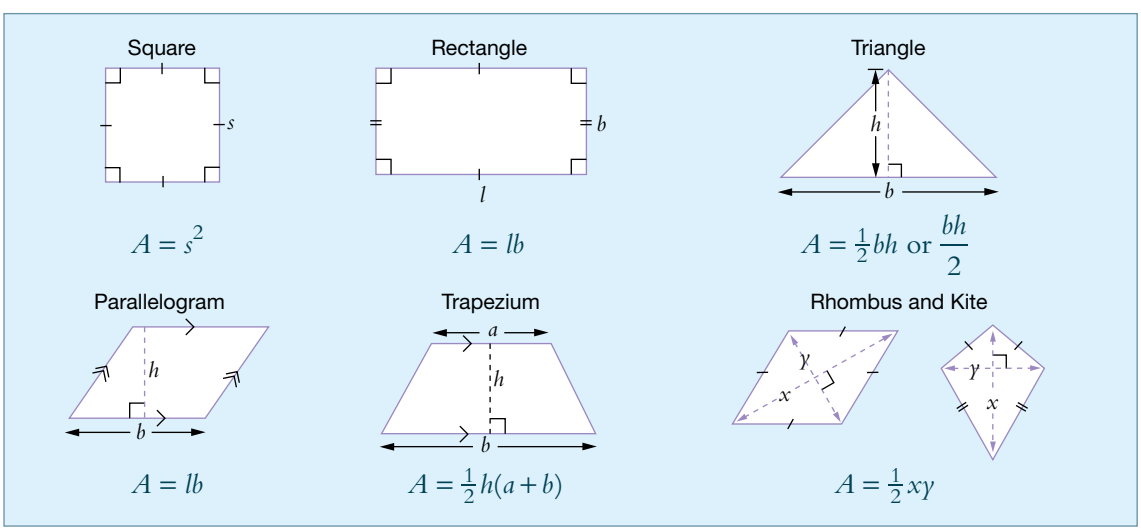


Rhombus and kite

For both of these shapes the length of the rectangle is the length of one diagonal, x. The breadth is half of the other diagonal, $\frac{1}{2}y$.
Area = $x \times \frac{1}{2}y$ or $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals.



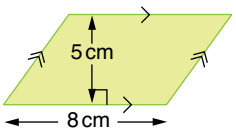
A summary of all the area formulas met so far is shown below.



WORKED EXAMPLES

Use a formula to find the area of each shape.

1



This is a parallelogram

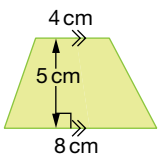
so $A = bh$

$= 8 \times 5$

$= 40\,\text{cm}^2$

$\therefore$ Area $= 40\,\text{cm}^2$

2



This is a trapezium

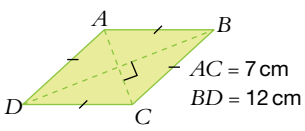
so $A = \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times 5 \times (4+8)$

$= 30\,\text{cm}^2$

$\therefore$ Area $= 30\,\text{cm}^2$

3



This is a rhombus

so $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 7 \times 12$

$= 42\,\text{cm}^2$

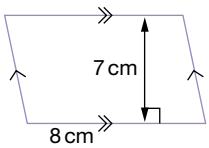
$\therefore$ Area $= 42\,\text{cm}^2$

Exercise 8:04

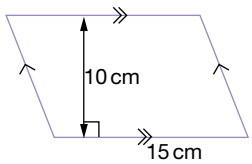
P Challenge worksheet 8:04
Practical applications of surface area

1 Use the formula $A = bh$ to find the area of each parallelogram.

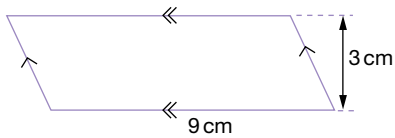
a



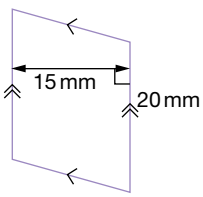
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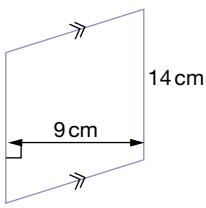
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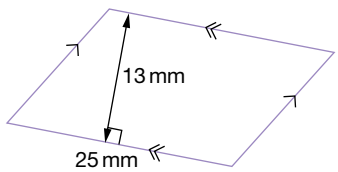
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e

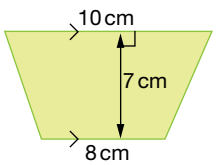


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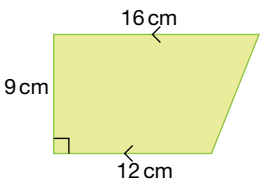


2 Use the formula $A = \frac{1}{2}h(a+b)$ to find the area of each trapezium.

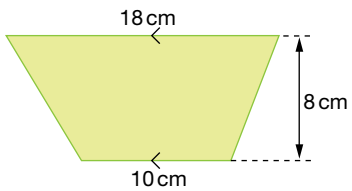
a



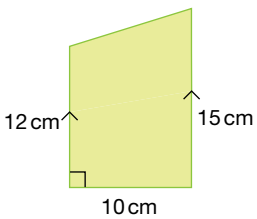
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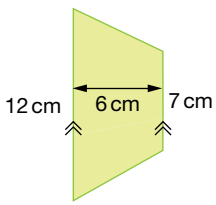
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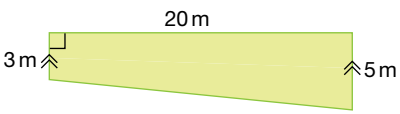
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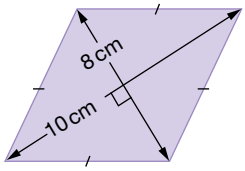
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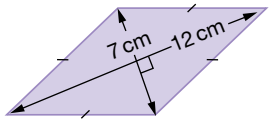


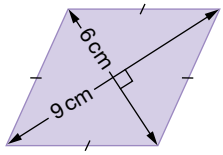
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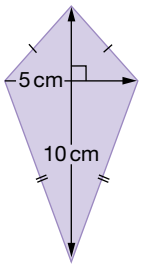


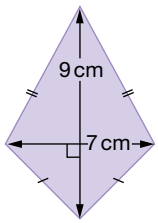
3 Use the formula $A = \frac{1}{2}xy$ to find the area of each rhombus or kite.

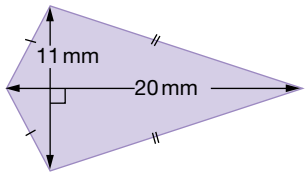
a 

b 

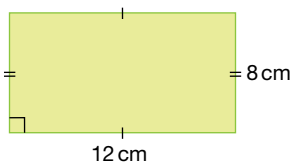
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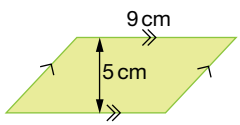
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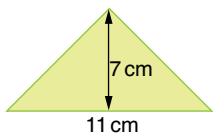
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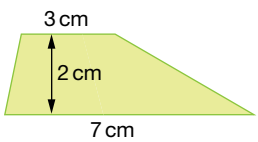
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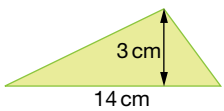
4 Use a formula to find the area of each shape.

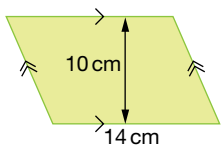
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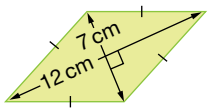
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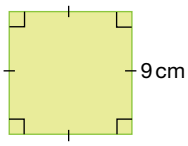
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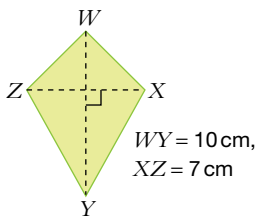
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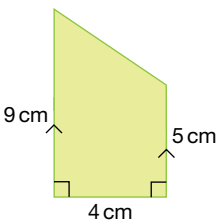
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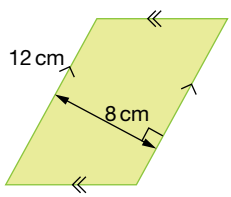
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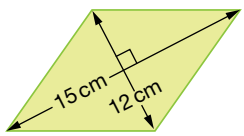
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h 

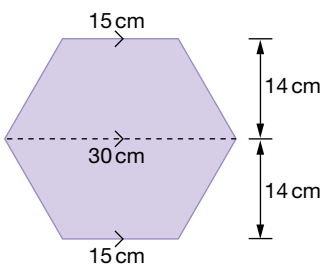
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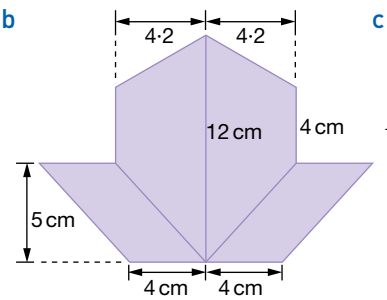
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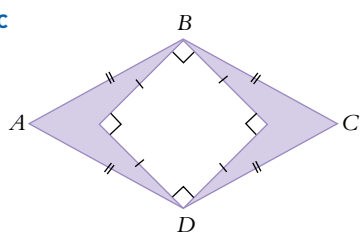
k 

l 

5 Each of the following shapes has been formed from the special quadrilaterals. Calculate the area shaded purple.

a 

b 

c 

6 The area of a rhombus is 96 cm^2. If the longer diagonal is 16 cm, find its perimeter.

8:05 Units of volume and capacity

The **volume** of an object is the space it occupies.
The **capacity** of a container is the volume of liquid it can hold.

PREP QUIZ 8:05

1 4 cm = ____ mm
2 2·5 cm = ____ mm
3 500 mm = ____ cm
4 3 m = ____ cm
5 5·75 m = ____ cm
6 8 km = ____ m
7 9·7 km = ____ m
8 7 000 mL = ____ L
9 23 000 L = ____ kL
10 2·5 kL = ____ L

Volume and capacity benchmarks

You should be gathering personal benchmarks so that you understand measurements as they are used.

- A place-value ones block has a volume of 1 cm^3 and a capacity of 1 mL.
- A 2 L container of milk demonstrates 2 full litres.
- An Olympic swimming pool holds 2·5 ML of water.



Volume	Unit	Conversions
cubic millimetres	mm^3	
cubic millimetres	cm^3	1 cm^3 = 1000 mm^3
cubic metres	m^3	1 m^3 = 1 000 000 cm^3
millilitre	mL	1 mL = 1 cm^3
litres	L	1 L = 1000 mL = 1000 cm^3
kilolitres	kL	1 kL = 1000 L = 1 m^3
megalitres	ML	1 ML = 1000 kL = 1 000 000 L

WORKED EXAMPLES

1 Convert 5 L into mL and into cm^3.
1 L = 1000 mL = 1000 cm^3
so 5 L = 5000 mL = 5000 cm^3

2 Convert 25 cm^3 into mm^3.
1 cm^3 = 1000 mm^3
so 25 cm^3 = 25 000 mm^3

3 Convert 54 m^3 into kL.
1 m^3 = 1 kL
so 54 m^3 = 54 kL

4 Convert 2·5 ML into kL and into m^3.
1 ML = 1000 kL = 1000 m^3
so 2·5 ML = 2500 kL = 2500 m^3

Exercise 8:05

1 Convert the following into mL and into cm^3.

a 3 L **b** 7 L **c** 0·5 L **d** 9 L

2 Convert into mm^3.

a $8\,cm^3$ **b** $20\,cm^3$ **c** $4\,cm^3$ **d** $16\,cm^3$

3 Convert into kL.

a 8 000 L **b** 2500 L **c** $8\,m^3$ **d** $12\,m^3$

4 Convert into kL and into m^3.

a 7 ML **b** 5·2 ML **c** 8 ML **d** 1·2 ML

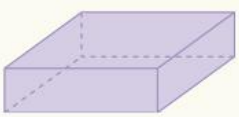
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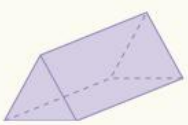
a Convert $3000\,cm^3$ into mL.
b Convert 3000 mL into L.
c Convert $2000\,cm^3$ into L.
d Convert 3 ML into L.
e Convert 73 000 000 L into ML.
f Convert 65 kL into m^3.
g Convert 6000 kL into ML.
h Convert 32 000 kL into ML.
i Convert 500 kL into ML.
j Convert 2 ML into kL.
k Convert 0·25 ML into kL.
l Convert 0·25 ML into L.

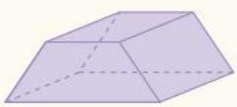
8:06 Volumes of prisms

PREP QUIZ 8:06

Name the following solids.

1 

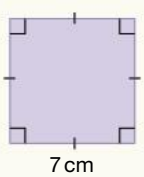
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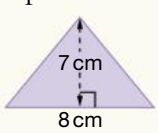
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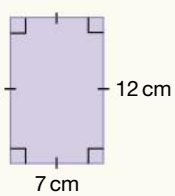
What shape is the cross-section of the object in:

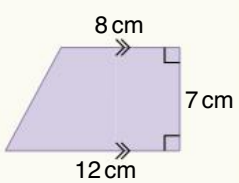
4 Question 1 **5** Question 2 **6** Question 3?

Calculate the area of each shape.

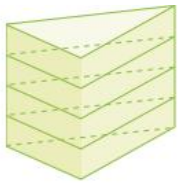
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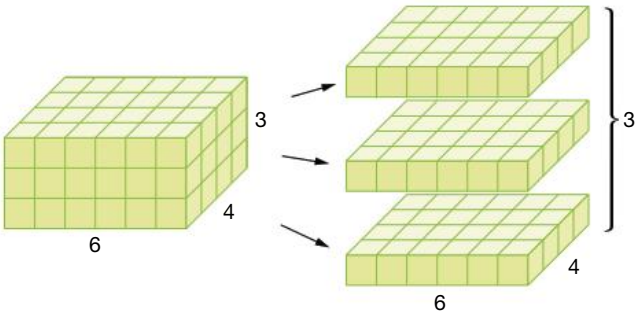
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9 

10 

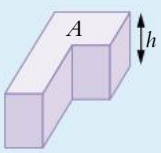
- A prism has a special pair of parallel faces called **bases**. If we cut the prism parallel to its base, the cross-section formed is always the same.
- The number of cubic units in a prism is equal to the number of units in the base times the number of layers.



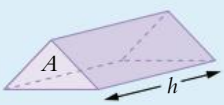


Note:
The number of cubes in the base is equal to the area of the base.

Number of cubic units in the base = 6×4
Number of layers = 3
Volume of prism = $6 \times 4 \times 3$
= 72 cubic units

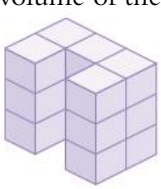


Volume of a prism
$V = Ah$
where A is the area of the cross-section or base and h is the height of the prism.



WORKED EXAMPLE 1

Find the volume of the prism.

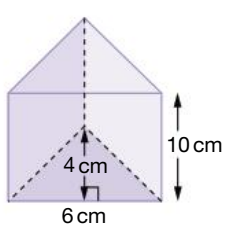


The area of the cross-section (or one layer) = 5 square units.
The height of the units prism (or number of layers) = 3 units
∴ Volume = 5×3
= 15 cubic units

WORKED EXAMPLE 2

Find the volume of each of these prisms.

a



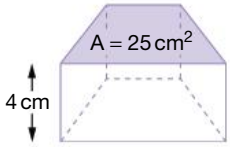
Area of a triangular cross-section
$= \frac{1}{2} \times 6 \times 4$
$= 12\,\text{cm}^2$

Height = 10 cm

∴ Volume = 12×10
$= 120\,\text{cm}^3$

Remember:
Volume is measured in cubic units, such as cm^3 or m^3.

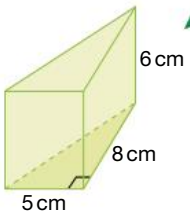
b



The cross-sectional area of this prism is $25\,\text{cm}^2$.

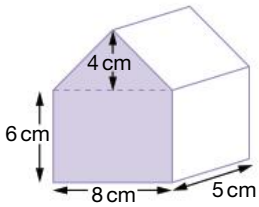
$V = Ah$
$= 25 \times 4$
$= 100\,\text{cm}^3$

Volume of prism is $100\,\text{cm}^3$





c



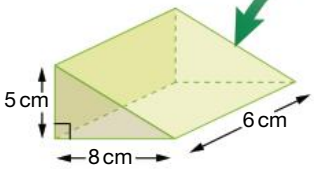
The cross-section is a triangle plus a rectangle.

$A = \frac{1}{2} \times 8 \times 4 + 8 \times 6$
$= 16 + 48$
$= 64\,\text{cm}^2$

$\therefore\ V = Ah$
$= 64 \times 5$
$= 320\,\text{cm}^3$

$\therefore$ Volume of prism is $320\,\text{cm}^3$.



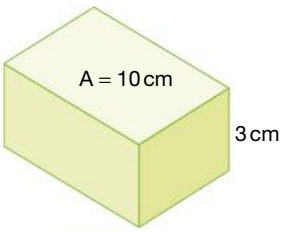


Exercise 8:06

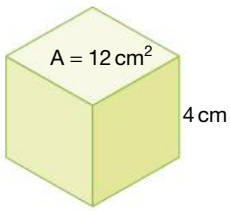
P Foundation worksheet 8:06
Volumes of prisms

1 Calculate the volume of each prism, given the cross-sectional area A and the height.

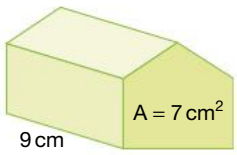
a



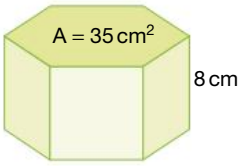
b



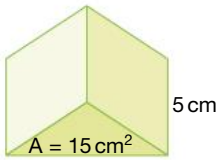
c



d

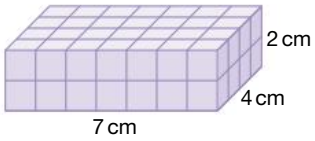


e

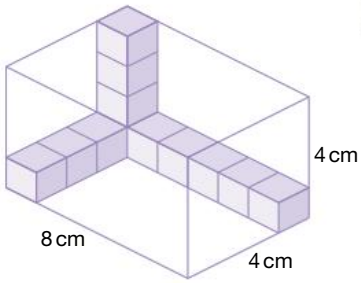


2 Calculate the volume of each rectangular prism.

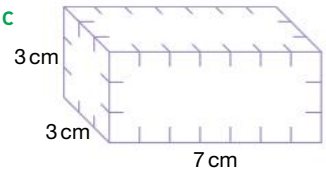
a



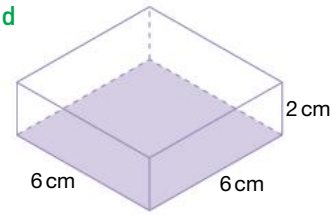
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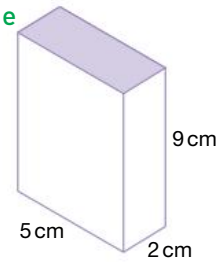
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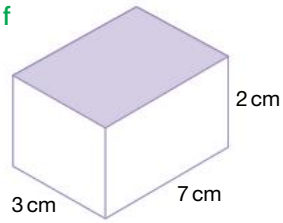
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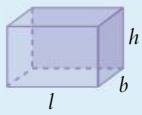


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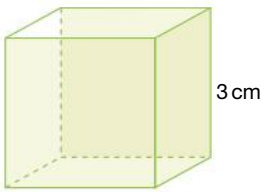


Another formula for the volume of a rectangular prism is:
$V = lbh$

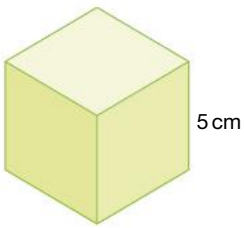


3 A cube is a rectangular prism where all the dimensions are equal. Find the volumes of these cubes.

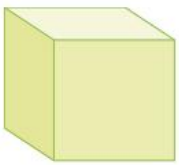
a



b



c



4 a What volume of soil will be needed to fill a rectangular garden bed 4·2 m long by 1·8 m wide to a height of 35 cm. (Hint: Watch your units!) Answer correct to one decimal place.

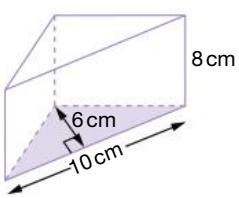
b Jan's rose garden has an area of 20 m^2. She wishes to cover it with leaf mulch to a depth of 10 cm. What volume of mulch should she buy?

c Adam's trailer is in the shape of a rectangular prism with a length of 1·8 m, a breadth of 1·2 m and a height of 0·3 m. What is the maximum volume of soil it can carry?

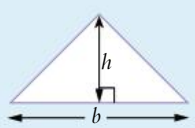
d What volume of concrete would Larry need to order for a house footing that is 450 mm wide, 300 mm deep and 10 m long. The footing is a rectangular prism.

5 a What is the area of the triangular cross-section?

b What is the volume of the prism?

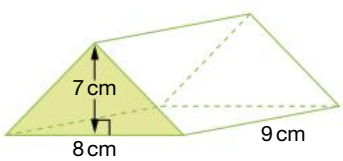


$A = \frac{1}{2}b \times h$



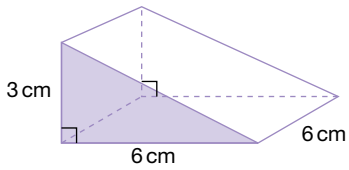
6 a What is the area of the cross-section of this triangular prism?

b What is the volume of the prism?

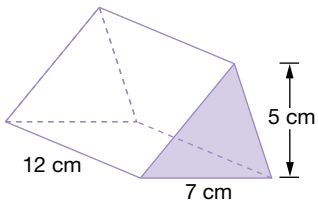


7 Calculate the volume of each of these prisms.

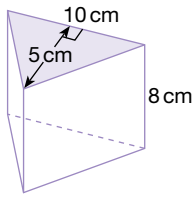
a



b

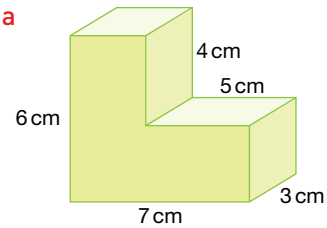


c

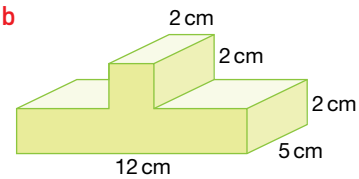


8 By first finding the area of the shaded cross-section, find the volume of each solid.

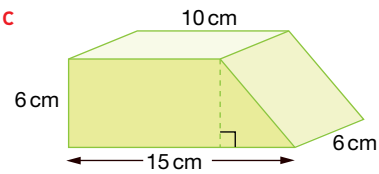
a



b

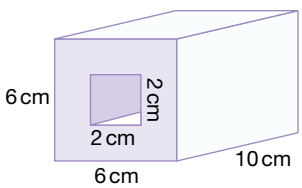


c

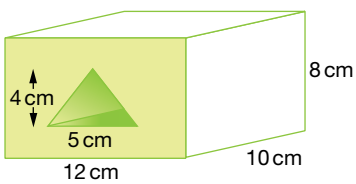


9 a Calculate the area of the shaded cross-section.

b Now calculate the volume of this prism. (The solid is still a prism even though it has a hole in it.)



10 Calculate the volume of this prism, which has a triangular hole cut out of it.



11 Calculate the volume of each of the following prisms, using the given information.

a A rectangular prism with dimensions of 9 cm, 7 cm and 4 cm.

b A rectangular prism with a length of 10 cm, a breadth of 7 cm and a height of 8 cm.

c A cube with a side length of 6 cm.

d A triangular prism where the triangular base has a height of 6 cm and a base length of 4 cm, and the prism has a height of 3 cm.

e A hexagonal prism where the area of the base is 14 cm^2 and the height is 5 cm.

f A pentagonal prism with a height of 12 cm and a cross-sectional area of 15 cm^2.

12 a If the volume of a prism is 36 cm^3 and its height is 4 cm, what must be the area of the base?

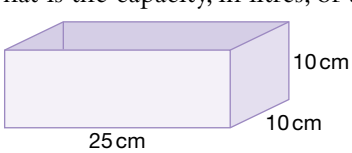
b If the cross-sectional area of a prism is 18 cm^2 and its volume is 90 cm^3, what must be its height?

c The dimensions of the base of a rectangular prism are 6 cm by 5 cm. If its volume is 180 cm^3, what must be its height?

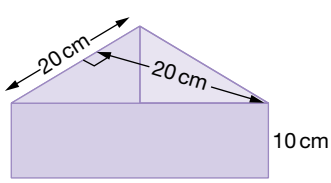


13 What is the capacity, in litres, of these tanks?

a



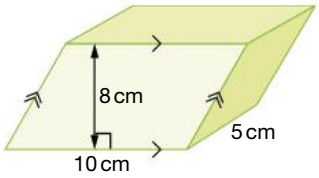
b



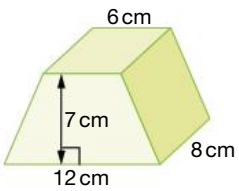
1 L = 1000 cm^3

14 These prisms have a parallelogram, trapezium and rhombus as their cross-sections. By calculating the area of the cross-section, find the volume of each prism.

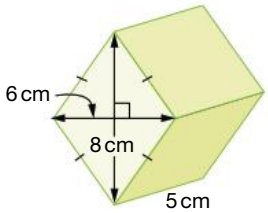
a



b



c

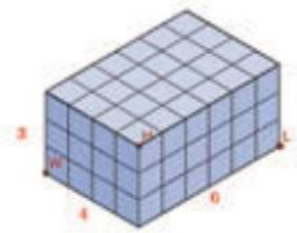


15 Vince has been asked to put a path 1 m wide around a rectangular lawn, which is 22 m long and 14 m wide. If the depth of the concrete is to be 10 cm, what volume of concrete does Vince need to order?

P GEOGEBRA ACTIVITY 8:06 VOLUMES OF PRISMS

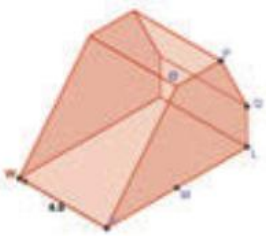
1 Volume of a rectangular prism

- Change the dimensions of a prism and find its volume.



2 Volume of a prism

- Change the cross-section and height of the object to produce different prisms.



INVESTIGATION 8:06A ESTIMATING VOLUME





Janette's family is moving to Nauru. They must take all their furniture with them, because they are moving into an unfurnished house. The family consists of two adults and two children. One container is available to them. It measures 6 m × 3 m × 4 m.

- Make a list of the essential items you think the family would take and then estimate the volume of each item. Once you have listed the necessities, fill the rest of the container with luxury items.
- Find out how furniture removalists estimate the volume of goods to be moved.





INVESTIGATION 8:06B VOLUME OF SKYSCRAPERS

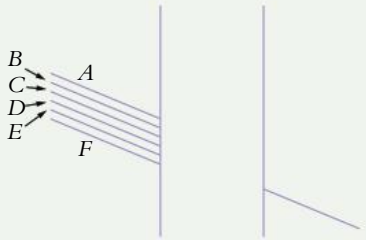
Look at the skyscrapers in this photograph. If you were told the height of one of the buildings on the right, how would you find its volume?



FUN SPOT 8:06 CHECK YOUR VISION FOR GOOD MEASURE!

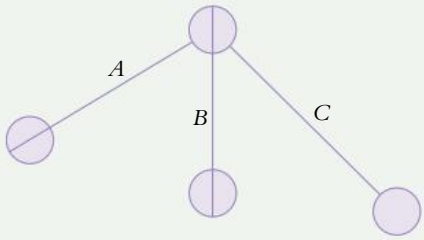
Often things are not what they seem. Sometimes our eyes may deceive us. Diagrams like those below are often called **optical illusions**. Answer each question, and then check by measurement.

1



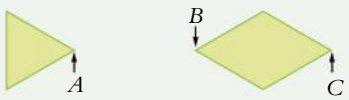
Which of the six lines on the left has been extended past the vertical lines?

2



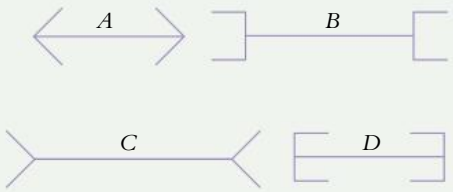
Which of the straight lines are the same length?

3



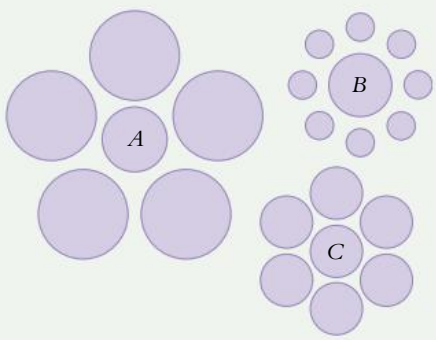
Which is the greater distance, from point A to point B, or from point B to point C?

4



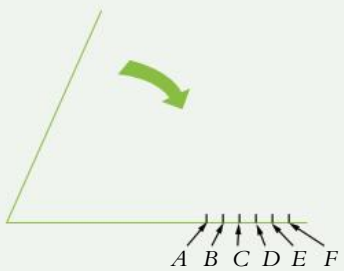
Which two of the above four intervals are equal in length?

5



Which centre circles are the same size?

6



To which point would the upper arm reach if it were lowered onto the lower arm?

8:07 Measuring time

The following list shows the relationships between the different units of time. It is important to be able to convert from one unit to another in order to perform different time calculations.

60 seconds (s) = 1 minute (min)
60 minutes = 1 hour (h)
24 hours = 1 day (d)
365 days = 1 year
366 days = 1 leap year
12 months = 1 year

The number of days in the months of the year can be remembered by the following rule.

30 days has September, April, June and November.
All the rest have 31, except February, which has 28 days clear and 29 each leap year.



The time of day is commonly measured in hours and minutes past the hour.

- 12-hour time uses a number from 1 to 12 to show hours and a number from 00 to 59 to show minutes. This is often followed by am or pm.
  e.g. 5:30 am, 11:48 pm

am means between 12 midnight and 12 noon
pm means between 12 noon and 12 midnight

- 24-hour time is always given as a four-digit number. The first two digits indicate the hours after midnight and the second two digits indicate the minutes past the hour. A colon is often placed between the second and third digits.
  e.g. 05:30 or 0530, 23:48 or 2348

- To convert 12-hour time to 24-hour follow these rules.
  For am times, leave the times the same except for single digit hours when a '0' is written in front.
  e.g. 3:15 am becomes 03:15.
  For pm times, add 12 to the hour digits.
  e.g. 3:15 pm becomes 15:15.

To change pm times to 24-hour times, add 12 hours to the pm times.
e.g. 1:20 pm = 13:20

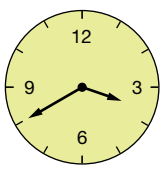
Exercise 8:07

1 How many:

a days in 3 weeks
b months in 5 years
c days in 2 normal years
d days in a fortnight
e complete weeks in a year
f seconds in 2 minutes
g hours in 2 days
h hours in a week
i minutes in a day?

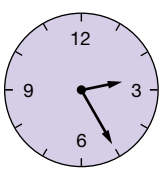
2 Complete the following.

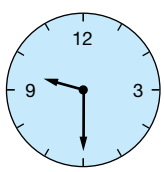
a 125 min = __ h __ min
b 95 s = __ min __ s
c 60 days = __ weeks __ days
d 1 h 18 min = __ min
e 1 min 10.8 s = __ s
f 4 days 4 h = __ h

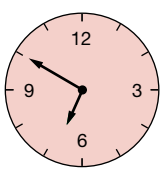
3 

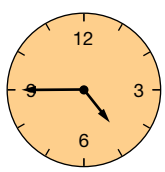
The time shown by this clock is usually given as '3:40' (meaning 40 minutes past 3 o'clock) or '20 to 4' (meaning 20 minutes before 4 o'clock).

Write down two ways of expressing the time shown by the clocks below.

a 

b 

c 

d 

e 

f 

g 

h 

4 The following are written in 24-hour time. Rewrite these times in standard 12-hour time, indicating whether they are am or pm.

a 0520 **b** 1030 **c** 1310 **d** 1600 **e** 2240 **f** 0915
g 1205 **h** 0855 **i** 00:20 **j** 23:59 **k** 14:43 **l** 11:01

5 Write these times as they would appear on a 24-hour clock.

a 5:20 am
b 10:50 am
c 3:15 pm
d 9:20 pm
e half past 2 in the morning
f 10 past 5 in the afternoon
g a quarter to 3 in the morning
h 12 noon

6 The timers on appliances often have a 24-hour display. The table below shows the starting and finishing times for various programs, and the timer settings that need to be made to record them. Complete the table.

Program	Timer settings
10:30 pm to 11:30 pm	22:30 to _____
9:15 am to 10:45 am	_____ to _____
7:45 pm to 9:10 pm	_____ to _____
_____ to _____	05:30 to 06:40
_____ to _____	12:00 to 13:30
_____ to _____	17:40 to 19:20



Some appliances use 12-hour timers while others use 24-hour timers.

8:08 Adding and subtracting times

60 min = 1 h
60 s = 1 min
The ° ′ ″ key can be used as h, min, s.

The calculator used in the examples is a Casio fx-82AU PLUS.

WORKED EXAMPLE 1

I spent 2 h 40 min studying on Monday, 1 h 55 min on Tuesday and 3 h 40 min on Saturday. How long did I study altogether?

With pencil and paper

2 h 40 min
1 h 55 min
3 h 40 min
6 h 135 min

135 min = 2 h 15 min
6 h + 2 h + 15 min = 8 h 15 min

With a calculator

2 ° ′ ″ 40 ° ′ ″ + 1 ° ′ ″ 55 ° ′ ″
+ 3 ° ′ ″ 40 ° ′ ″ =

The answer is 8·25.
∴ I studied for 8 hours 15 minutes.

WORKED EXAMPLE 2

a I've rented a boat for $6\frac{1}{2}$ hours. I've used it for 1 h 35 min. How much time is left?

With pencil and paper

$$\begin{array}{rrr} & \overset{5}{\cancel{6}}\text{ h} & \overset{90}{\cancel{30}}\text{ min} \\ - & 1\text{ h} & 35\text{ min} \\ \hline & 4\text{ h} & 55\text{ min} \end{array}$$

Trade 1 h for 60 min, then subtract.

With a calculator

6 ° ’ ” 30 ° ’ ” − 1 ° ’ ” 35 ° ’ ” =

The answer is 4 h 55 min.

∴ I have 4 hours 55 minutes left.

b If it is 6:40 am now, how long before it is 8:15 pm?

6:40 am	to	7:00 am	is		20 min
7:00 am	to	noon	is	5 h	
noon	to	8 pm	is	8 h	
8 pm	to	8:15 pm	is		15 min
		Total	is	13 h	35 min

In 13 h and 35 min after 6:40 am it will be 8:15 pm.

The method shown above is known as the bridging strategy and is usually performed mentally.



Exercise 8:08

1 Add these times together, expressing each answer in hours and minutes.

a 2 h 20 min + 1 h 30 min
b 5 h 19 min + 3 h 37 min
c 1 h 40 min + 2 h 30 min
d 7 h 34 min + 5 h 47 min

Remember: 60 min = 1 h

2 Subtract these times.

a 5 h 30 min − 2 h 10 min
b 3 h 55 min − 1 h 32 min
c 3 h 10 min − 1 h 40 min
d 9 h 20 min − 5 h 50 min
e 4 h 25 min − 1 h 47 min
f 7 h 13 min − 6 h 45 min

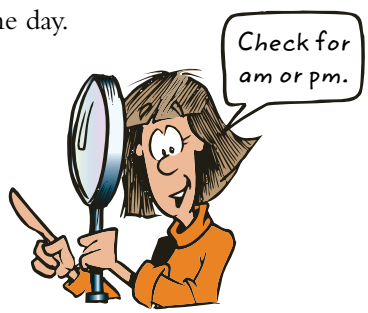
To subtract 1 h 40 min from 3 h 10 min, change 3 h 10 min to 2 h 70 min:

$$\begin{array}{rr} & 2\text{ h } 70\text{ min} \\ - & 1\text{ h } 40\text{ min} \\ \hline & 1\text{ h } 30\text{ min} \end{array}$$



3 Calculate the difference between each pair of times on the same day.

a 2:30 pm, 3:20 pm
b 5:20 am, 7:10 am
c 4:15 am, 7:30 pm
d 3:40 am, 9:50 pm
e 1030, 1330
f 0910, 1430
g 05:20, 16:15
h 03:40, 22:50



4 What is the length of time, in hours and minutes, from:

a 7:00 pm Tues to 5:00 am Wed
b 9:15 pm Sat to 6:30 am Sun
c 1:20 pm Fri to 12 noon Sat
d 10:30 Wed to 13:10 Thurs
e 1445 Tues to 0530 Thurs?

It is easier to find time differences if you change to 24-hour time.

5 What is the time:

a $1\frac{1}{2}$ hours after 7:30 pm
b $3\frac{1}{2}$ hours after 10:00 am
c 1 h 20 min after 6:45 am
d 2 h 40 min after 11:40 pm
e $3\frac{1}{2}$ hours before 5:15 am
f $1\frac{1}{4}$ hours before 12:30 pm
g 2 h 15 min after a quarter to 7 in the morning
h 7 h 45 min after 20 past 10 in the evening?

6 Round these answers correct to the nearest **i** hour **ii** minute.

a 6·5 h
b 3·75 h
c 4·25 h
d 7·333 h
e 8·6667 h
f 2·4 h
g 1·9 h
h 7·13 h
i 12·77 h
j 11·835 h

7 **a** Sharon rides from home to work in 1 h 16 min 42 s. She returns in 1 h 18 min 55 s. How long did she take for the round trip?

b Max swims a 200 m medley race consisting of one lap each of butterfly, backstroke, breaststroke and freestyle. If his times for each lap are 1 min 10.5 s, 1 min 18.7 s, 1 min 25.2 s and 51 s, what is his time for the complete race?

c In a triathlon, Kerani completed the three legs in the following times: Swim 26 min 53 s, Ride 1 h 11 min 8 s, Run 50 min 51 s. What was her overall time?

d A train leaves Wentworth Falls at 5:39 am and arrives at Strathfield station at 7:16 am. How long does the trip take?

e A normal soccer match lasts for 90 minutes. On a TV broadcast of a soccer match the elapsed time is shown on the top of the screen. How long remains in the match if the elapsed time is 69 min 20 s?

f Antje's air conditioner uses a 24-hour timer and she wants to set it to come on 40 minutes before she arrives home. For what time must she set it if she normally arrives home at 7:10 pm?

8 Use the bus timetable to answers the following questions.

BUS TIMETABLE

MONDAY TO FRIDAY

DEPARTS	ARRIVES	ARRIVES	ARRIVES	ARRIVES	ARRIVES
Blacktown Station	Cnr Varadys Rd and Garden St	Kings Langley Shops	Lalor Park School	Cnr Kennedy Pde and Burke Rd	Seven Hills Station
2:30 B	2:47	2:52	2:58	3:01	3:04
3:20	3:29	3:34	3:40	3:43	3:46
3:48	3:57	4:02	4:08	4:11	4:14
4:10	4:19	4:24	4:30	4:33	4:36
4:35	4:44	4:49	4:55	4:58	5:01
5:08	5:17	5:22	5:28	5:31	5:34

B – On school days, diverts to Blacktown High School

a How long is the bus ride from:
   i Blacktown Station to the corner of Kennedy Parade and Burke Road
   ii the Kings Langley shops to Seven Hills station
   iii the corner of Varadys Road and Garden Street to Lalor Park School?

b If the bus is 8 minutes late at what time would the school bus reach the Kings Langley shops?

c If an evening bus leaves Blacktown Station at 6:45 pm, at what time would it reach Lalor Park School?

d A woman leaves her home on Varadys Road at 3:45 pm and takes 9 minutes to walk to the bus stop. At what time could she expect to reach Seven Hills Station?

9 The tides for a period of three days are given in this table in 24-hour time.

Day	Time	Tide (m)
1	0414	0·3
	1012	1·4
	1607	0·4
	2227	1·7
2	0458	0·4
	1052	1·3
	1637	0·5
	2304	1·7
3	0544	0·5
	1135	1·2
	1709	0·6
	2343	1·6

a What is the lowest tide and at what time does it occur?

b How long is it between high tides on the first day?

c How long is it between low tides on the second day?

d How long is it between the first low tide and the first high tide on Day 3?

e What is the time difference between the first low tide on Day 1 and the first low tide on Day 2? Is this time difference the same between Day 2 and Day 3?

f At what time during the three days did the highest tide occur?

g Barney believes the best fishing occurs half an hour after high tide. If this is so, what would be the best fishing times on Day 3?

10 If today's date is 8 January 2015, what will the date be in:

a 10 days' time b 30 days' time c 20 weeks' time?

11 If today's date is 8 January 2015, what was the date:

a 2 days ago b 3 weeks ago c 5 months ago?

12 a Marika was born on 26 July 2014. On what date will she be 18 months old?

b After buying a home theatre system I agree to make 42 monthly payments. If the first payment is made in March 2013 when will the last payment be made?

c Giofranco signs a contract on 18 March. He has 30 days to make his first payment. When is the last day that this can be done?

d Sven leaves Sydney on a 15-night cruise on 21 May. On what day will he return?



The length of luxury cruises is usually measured in days or nights.

e The Norwegian explorer Roald Amundsen started his trip to the South Pole on 19 October 1911 and reached the pole on the 14th December. How many days did the trip take?

f The great Indian cricketer Sachin Tendulkar was born on 24 April 1973. He made his Test Cricket debut against Pakistan on 15 November 1989. How old was he in years and days?

FUN SPOT 8:08 WHY ARE GARDENERS FUNNY?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.



How many days are there in:

T January **S** September?

How many days are there from:

A 17 March to 17 May **F** 17 July to 17 September?

Today is Tuesday. What day of the week will it be in:

H 3 weeks' time **N** 4 days' time **U** 37 days' time?

If today is 16 October, what will the date be in:

H 3 weeks' time **S** 4 days' time **M** 37 days' time?

Write as 12-hour times.

E 15:45 **O** 20:45 **U** 10:45

What is the time:

V 12 hours after 8:45 pm **S** 12 hours before 3:45 pm?

How long is it from the first time to the second time?

Y 8 am, 1 pm **A** 7:35 am, 11:15 am **E** 09:20, 20:35

Answer each of the following in hours and minutes.

E 3 h 27 min + 2 h 45 min **E** 6 h − 2 h 15 min **H** 5 h 20 min − 35 min

31	Tuesday	3:45 pm	5 h	6 November	3 h 40 min	8:45 am	11 h 15 min	61	30	3 h 45 min	Saturday	20 October	6 h 12 min	8:45 pm	62	4 h 45 min	Thursday	22 November	10:45 am	3:45 am

INVESTIGATION 8:08 EXCEL SPREADSHEETS AND TIME

Complete this investigation using the ready-made Excel spreadsheet on the eBook.

- How many days old are you?
- When will you be 10 000 days old?
- How long have you been running if you started at 06:25 and finished at 11:06?

Answer these questions or make up your own.
This spreadsheet activity uses formatted cells to accept times.
Enter dates, times and durations into the blue cells using the format given.
The answer is shown in the green cell.

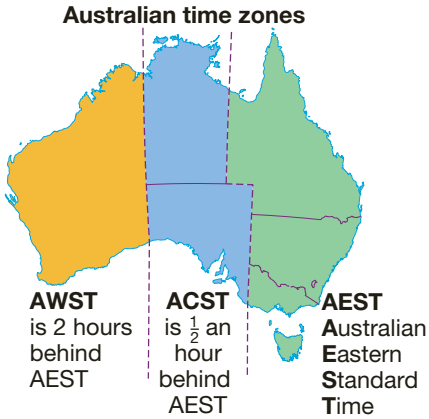
8:09 International time zones

The Earth rotates on its axis from the west to the east. Each rotation takes one day. This rotation causes night and day, and leads to time differences between different places on the Earth.

The time difference between two places is related to the difference in their longitudes. Places with the same longitude have the same time. If time were based entirely on longitude, then places that were close together but on different lines of longitude would have different times. This would lead to ridiculous and confusing situations (e.g. you would need to reset your watch if you drove from one side of a large city to the other side). To avoid such problems, the Earth is divided into **time zones** and places within each zone use the same time. This is called **Standard Time** and it is based on only one longitude.

Australia has three time zones.

- Australian Eastern Standard Time (AEST), based on the 150°E meridian
- Australian Central Standard Time (ACST), half an hour behind AEST
- Australian Western Standard Time (AWST), 2 hours behind AEST



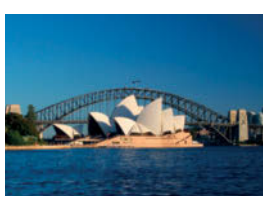
WORKED EXAMPLE 1

The time in Sydney is 10 hours ahead of the time in London. What is the time in:

a Sydney, when it is 9 am in London

b London, when it is 9 am in Sydney?


Time = 9 am


Time = ?

Solutions

a Sydney time = London time + 10 h
= 9 am + 10 h (or 09:00 + 10 h)
= 7 pm (or 19:00 h)

b London time = Sydney time − 10 h
= 9 am − 10 h (or 09:00 − 10 h)
= 11 pm (or 23:00) (the night before)



WORKED EXAMPLE 2

When it is 4:30 pm in Sydney it is 10:30 pm the day before in San Francisco.

a What is the time difference between Sydney and San Francisco?

b You are in San Francisco and you want to ring your friend in Sydney at 8 am before he leaves for school. At what time should you ring?



Solutions

a Time difference = time from 10:30 pm to 4:30 pm the next day
= 1 h 30 min + 16 h 30 min
= 18 h
∴ Sydney time is 18 h ahead of San Francisco time.

b Time in San Francisco = 8 am − 18 h
= 2 pm (the previous day)

WORKED EXAMPLE 3

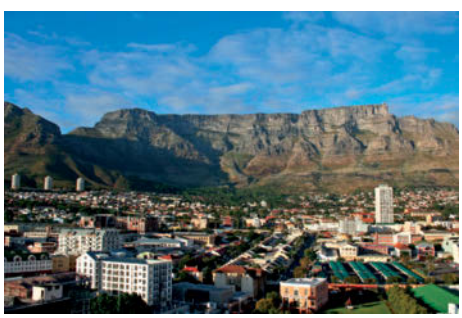
a Australia is playing a cricket match in Cape Town, South Africa. If the match starts at 10:30 am in Cape Town at what time will the televised match start in Perth? (The time in Perth is 6 hours ahead of the time in Cape Town.)

b A plane leaves London at 2:30 pm for an 11-hour flight to Los Angeles. If the time in Los Angeles is 7 hours behind the time in London, what time should the plane land in Los Angeles?

Solutions

a Starting time in Perth = 10:30 am + 6 h
= 1030 + 6 h
= 1630 (or 4:30 pm)

b Time in Los Angeles (when plane leaves London)
= 2:30 pm − 7 h
= 7:30 am
∴ Time in Los Angeles (on arrival) = 7:30 am + 11 h
= 6:30 pm
OR
Time in Los Angeles = 2:30 pm + 11 h − 7 h
= 2:30 pm + 4 h
= 6:30 pm



Exercise 8:09

1 What is the time:

a 6 hours after 2 pm
b 6 hours before 2 pm
c 10 hours after 1 pm
d 10 hours before 1 pm
e 13 hours after 4 am
f 13 hours before 4 am?

2 Find the time difference (on the same day) between A and B, if it is:

a 3 pm in A and 12 noon in B
b 8 am in A and 11 am in B
c 5 pm in A and 5 am in B
d 5 am in A and 3 pm in B
e 11 pm in A and 4 am in B
f midnight in A and noon at B.

3 The time in A is 12 noon. Find the time in B if it:

a 5 hours ahead of A **b** 5 hours behind A **c** 10 hours ahead of A

d 10 hours behind A **e** 15 hours ahead of A **f** 15 hours behind A.

4 **a** How long is it from:

i 08:00 to 14:00 **ii** 03:00 to 20:00?

b What time is it 8 hours after:

i 04:00 **ii** 14:00?

c What is the time 5 hours before:

i 1600 **ii** 0300?

5 The times in six cities are:

New York (10:09 am)
London (3:09 pm)
Moscow (6:09 pm)
Paris (4:09 pm)
Tokyo (11:09 pm)
Sydney (12:09 am the next day)



Use this information to state the time difference between the following pairs of cities. State which city's time is ahead and by how much.

a New York and London **b** London and Tokyo **c** Sydney and Moscow

d Paris and Moscow **e** New York and Sydney

6 When it is 0500 in Paris it is 1200 in Tokyo. What is the time in:

a Tokyo when it is 0700 in Paris **b** Tokyo when it is 1800 in Paris

c Paris when it is 1000 in Tokyo **d** Paris when it is 2200 in Tokyo?

7 Use the Australian time zone map on page 238 to answer the following questions. What would the time be in:

a WA, if it is 3:00 pm in NSW **b** WA, if it is 1 am in Qld

c SA, if it is 5:15 am in Vic **d** SA, if it is 12:10 pm in NSW

e NSW, if it is 1:20 am in WA **f** Vic, if it is 9:45 pm in SA

g WA, if it is 3:50 pm in SA **h** SA, if it is 5:35 am in WA?

8 **a** The time in Wellington is 2 hours ahead of the time in Sydney. What is the time in:

i Sydney when it is 9 am in Wellington

ii Wellington when it is 9 am in Sydney?

b The time in London is 7 hours behind the time in Beijing. What is the time in:

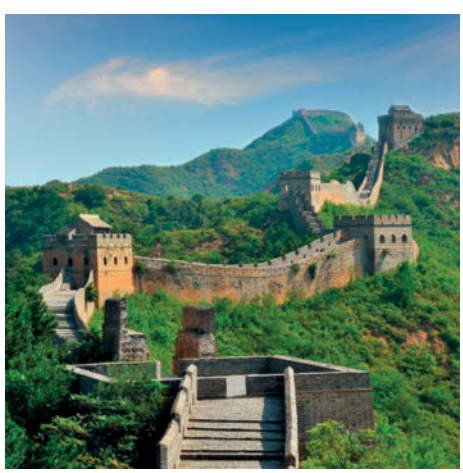
i London when it is 1600 in Beijing

ii Beijing when it is 0500 in London?

c The time in Sydney is 18 hours ahead of the time in Los Angeles. What is the time in:

i Sydney, when it is 1 am in Los Angeles

ii Los Angeles, when it is 11 am in Sydney?

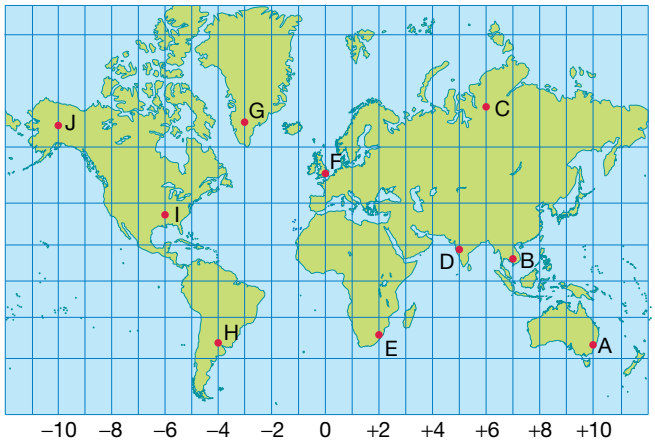


9 The map shows how time differs around the world.

- B is 3 hours behind A (Sydney), as B is at +7 and Sydney is at +10
- H is 6 hours ahead of J, as H is at −4 and J is at −10.

Use the map to find the time at each of the following places when it is noon in Sydney (at A).

a B **b** C **c** E
d I **e** J



10 **a** Use the map to find the time in Sydney (A) when it is 3 pm in London (F).
b What time is it in Nashville (I) when it is 11:30 pm in Sydney (A)?
c What time is it in Johannesburg (E) when it is 8 am in Buenos Aires (H)?
d What time is it in Buenos Aires (H) when it is 6 am in Sydney (A)?

11 **a** A cricket match starts in London at 11 am. Local time in Perth and Sydney is 8 hours ahead and 10 hours ahead, respectively, of the time in London. What time will the match start in:
i Perth **ii** Sydney?



b The President of the USA is to make an important speech at 1900 h (Washington time). At what time will the speech be heard in:
i Sydney (15 hours ahead of Washington time)
ii Beijing (12 hours ahead of Washington time)
iii Los Angeles (3 hours behind Washington time)
iv Honolulu (6 hours behind Washington time)?

c My friend Matthew lives in France near the Swiss border, which is 9 hours ahead of my time.

i At what time should I call him, if he expects a call at 9 pm his time?

ii At what time should he call me, if I expect a call at 9 pm my time?

d A flight from Singapore to Sydney is expected to take 7 hours. Sydney airport has a curfew, which means that no planes can land between 11 pm and 6 am. What is the latest time that the plane could leave Singapore to make the curfew? (Sydney's time is 3 hours ahead of Singapore's time.)



e An international football match is to be played between the USA and Japan in Berlin. The match will start at 19:00 (Berlin time). Tokyo time is 8 hours ahead of Berlin time, whereas San Francisco time is 9 hours behind Berlin time. At what time will the match start in:

i San Francisco ii Tokyo?

f Ray and Ken are two friends who both leave Sydney at 20:00 on different flights. Ken takes a flight to Los Angeles. Ray flies to Dubai. Both flights are expected to take 13 hours. Sydney time is 18 hours ahead of Los Angeles and 7 hours ahead of Dubai.
What will the time be in:

i Dubai, when Ray's plane lands

ii Los Angeles, when Ken's plane lands?

g The Athletics events at the 2012 Olympics in London were organised in two daily sessions.

Session 1 0900–1600	Session 2 1900–2200

In Australia, prime-time TV is from 7 to 10 pm, whereas prime-time radio is from 6 to 10 am and 3 to 7 pm. During the Olympics, time in Eastern Australia was 9 hours ahead of London time.

i Convert the athletics session times to Eastern Australian time.

ii How many hours of live athletics could be televised on prime-time TV?

iii How many hours of live athletics could be broadcast during prime-time radio?

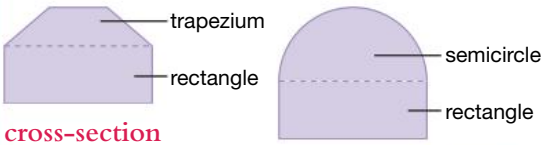
MATHS TERMS 8

24-hour time
- time given as a four-digit number; the first two digits indicating the hour, the second two indicating minutes,
  e.g. 13:20 (or 1320) means 20 minutes past 1 in the afternoon

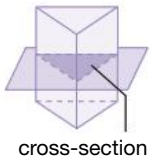
am
- times between 12 midnight and 12 noon

area
- the amount of space (square units) inside a two-dimensional shape
- units of area:
  square millimetre (mm^2)
  square centimetre (cm^2)
  square metre (m^2)
  hectare (ha)
  square kilometre (km^2)
- formulas are used to calculate the area of the common plane shapes

composite figure
- a figure that is formed by joining simple shapes



cross-section
- the shape on the face where a solid has been sliced



cube
- a prism that has six square faces

dimensions
- important measurements of shapes, e.g. length, breadth and height

hectare
- an area of 10 000 m^2
- the area of a square with side length 100 m

leap year
- a year containing 366 days
- occurs every fourth year (except for century years whose number is not divisible by 400).

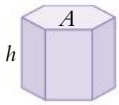
midnight
- 12 midnight, the 'middle' of the night

noon
- 12 noon, the 'middle' of the day

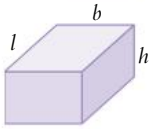
perimeter
- the distance around the boundary of a 2D shape

pm
- times between 12 noon and 12 midnight

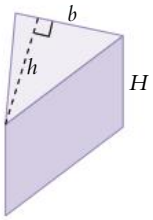
prism
- a solid with two identical ends joined by rectangular faces
- volume = area of base × height
  $V = Ah$



- rectangular prism
  - the ends are rectangles
  - volume
    = length × breadth × height
    $V = lbh$



- triangular prism
  - the ends are triangles
  - volume
    = area of base × height
    $V = (\frac{1}{2}bh) \times H$



second, minute, hour, day, week, fortnight, month, year
- commonly used units of time
- see Section 8:07 for relationships between units

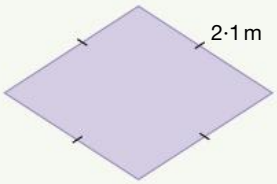
time zones
- areas of the world that use the same time

volume
- the amount of space (cubic units) inside a 3D object
- units of volume and capacity
  cubic millimetre (mm^3)
  cubic centimetre (cm^3)
  cubic metre (m^3)
  kilolitre (kL, 1 kL = 1000 L)
  megalitre (ML, 1 ML = 1000 kL)

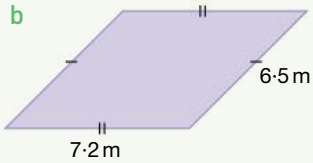
Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Find the perimeter of each shape. 8:01

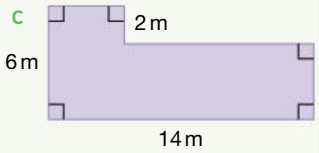
a



b

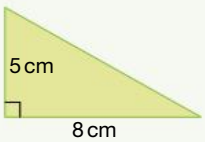


c



Calculate the area of each shape in Questions 2 to 6.

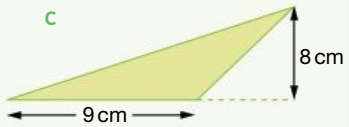
2 **a** 8:02



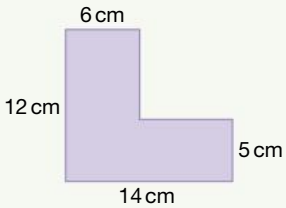
b



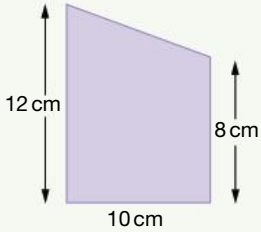
c



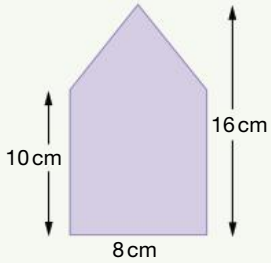
3 **a** 8:02



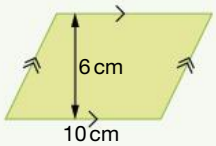
b



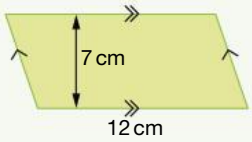
c



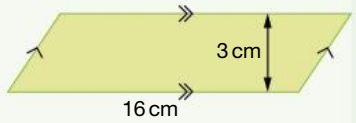
4 **a** 8:03, 8:04



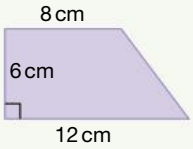
b



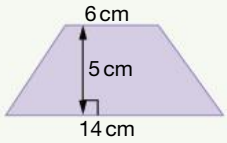
c



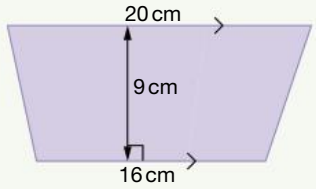
5 **a** 8:04



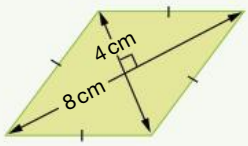
b



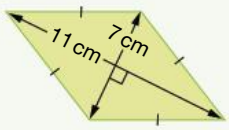
c



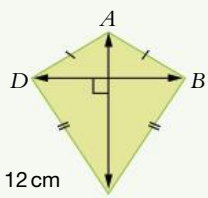
6 **a** 8:04



b



c

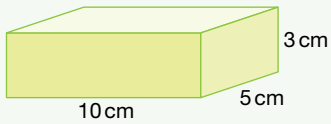


$AC = 12$ cm
$BD = 7$ cm

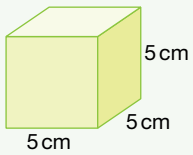
7 **a** Convert 7 ML to L. **b** Convert $2\,m^3$ to cm^3. 8:05
c Convert $5\,cm^3$ to mL. **d** Convert $5\,m^3$ to L.

In Questions **8** to **10** calculate the volume of each prism. 8:06

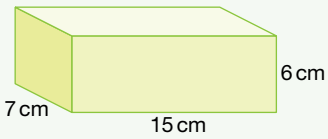
8 **a**



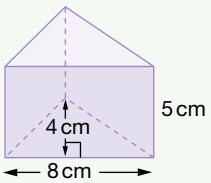
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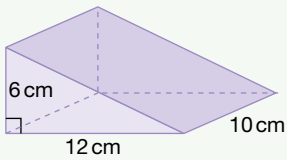
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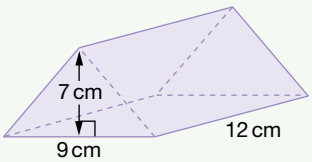
9 **a** 8:06



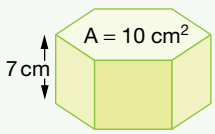
b



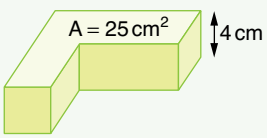
c



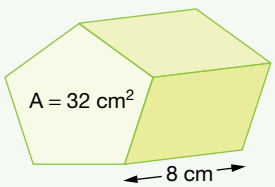
10 **a** 8:06



b



c



11 Write these times in 24-hour time. 8:07
a 5:30 am **b** 1:20 pm **c** 7:57 pm **d** 12:20 am

12 Write these times in 12-hour time. 8:07
a 1620 **b** 0840 **c** 2130 **d** 0545

13 Find the difference from the first time to the next. 8:08
a 8:30 am to 10:20 pm **b** 2:45 pm to 7:57 pm
c 10:12 am to 1:05 pm **d** 6:47 pm to 12:05 am

14 Write each of the following in hours and minutes. 8:08
a 3·5 h **b** 1·25 h **c** 10·6 h **d** 7·85 h

15 **a** 3 h 45 min + 2 h 25 min **b** 2 h 40 min + 1 h 55 min 8:08

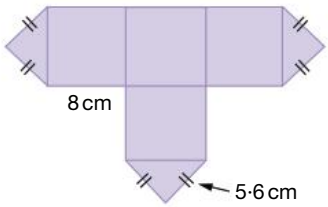
16 **a** 6 h 30 min − 1 h 35 min **b** 3 h 40 min − 2 h 55 min 8:08

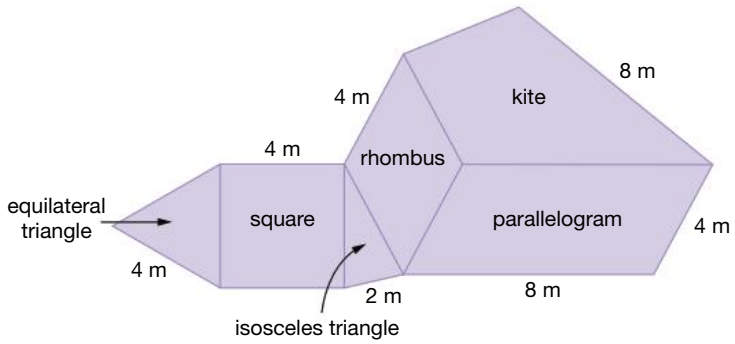
17 The time in Perth is 2 hours behind the time in Sydney and 8 hours ahead of the time in London. What is the time in: 8:09
a Perth when it is 9 am in Sydney **b** Sydney when it is 9 am in Perth
c London when it is 2 pm in Perth **d** Perth when it is 9 pm in London?

ASSIGNMENT 8A Chapter review

1 **a** This shape is made up of four squares and three identical isosceles triangles. Find the perimeter of the shape.

b Find the perimeter of this shape.





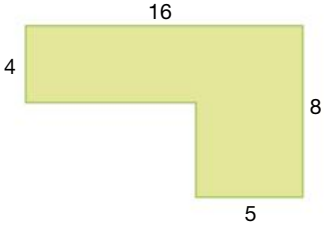
2 **a** Find the area of the surface of the water in an Olympic swimming pool 50 m long and 25 m wide.

b A book is 19 cm wide and 25 cm long. What is the area of its front cover?

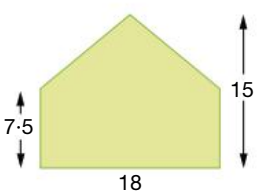
c The front of a house is 8 m high and has the shape of an isosceles triangle. If the breadth of the front (the base of the triangle) is 4 m, what is the area of the front view of the house?

3 Find the area of these composite shapes. (Measurements are in metres.)

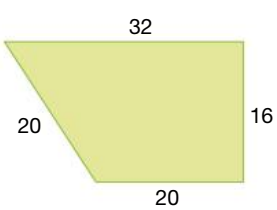
a



b

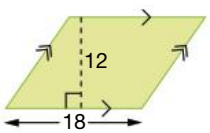


c



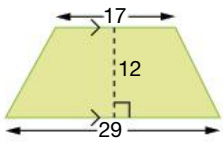
4 Use the formulas to find the area of each shape. (Measurements are in centimetres.)

a



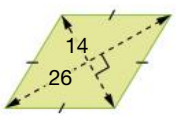
$A = bh$

b



$A = \frac{1}{2}h(a+b)$

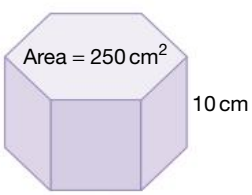
c



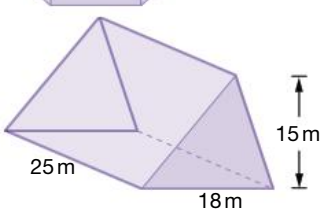
$A = \frac{1}{2}xy$

5 Find the volume of these prisms.

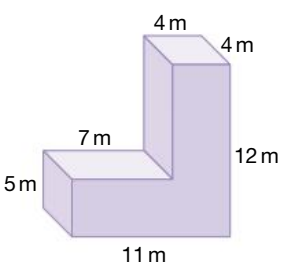
a



b



c



6 a A train leaves Sydney at 10:05 and arrives in Armidale at 18:12. How long did the trip take in hours and minutes?

b My finishing time in a 1500 m swim was 28 min 12 s. The winner completed the course in 16 min 15 s. How far behind the winner did I finish, in minutes and seconds?

c My birthday in on 26 September, whereas my brother's is on 17 December? How far apart are our birthdays in weeks and days?

7 a Raj's friends Signe and Anna are on an overseas holiday. He wants to phone them in Rome so that they get the call at 7:30 am. If he is ringing from Sydney, which is 8 hours ahead of Rome, at what time should he ring?

b The American Football Superbowl XVLI was held in Indianapolis starting at 6:30 pm. At what time did the game start in:

i San Francisco (3 hours behind)

ii London (5 hours ahead)

iii Sydney (16 hours ahead)?

c When it is 8 am in Los Angeles it is 2 am the next day in Melbourne. What is the time in:

i Melbourne when it is 3 pm in Los Angeles

ii Los Angeles when it is 6 am in Melbourne?

8 Roisin wanted to walk from Sydney to Melbourne, a distance of 960 km. If she left on 20 April and was able to maintain an average distance of 40 km per day, on what day would she complete the journey?



ASSIGNMENT 8B Working mathematically

1 If each of the vowels a, e, i, o, u could be followed by any consonant (by letters that are not vowels), how many pairs of letters could be formed?

2 Mai and Ronin married and had seven children. Six of the children married and had four children each. Assuming that everyone is still alive, how many people are now in this extended family in total?

3 Five students shared a secret. Each one shared the secret with four others. Each of these shared the secret with three others. Each of these shared the secret with two others and each of these shared the secret with one other. If no one person was told the secret twice, how many people all together now know the secret?

4 How many three-digit numbers can be formed using the digits 3, 2, 1 and 0 if each digit can be used only once in each number?

5 How many three-digit numbers can be formed using the digits 3, 2, 1 and 0 if each digit can be used as often as you like in each number?

6 A grower sent 240 cases of fruit to market. At the market, 80% of the cases were classed 1st quality, while the remainder were classed as 2nd quality. If the grower is paid $12 a case for 1st quality and $7 a case for 2nd quality, how much does the grower receive for his fruit?



The lean of the Leaning Tower of Pisa was:

- 1·43 m in 1372
- 3·77 m in 1550
- 4·75 m in 1817
- 4·8 m in 1935
- 5·2 m in 1997

What do you think could be done to save the building? Search the internet to discover what has been done.

ASSIGNMENT 8C Cumulative revision

1 Simplify:

a $5a + 2a - a$ **b** $2b \times 4 + b$ **c** $12a^2 \div 6a$ **d** $(4a)^2$ 5:02–5:04

2 **a** Express 4 cm as a percentage of 2 m. **b** Increase \$25 by 20%.

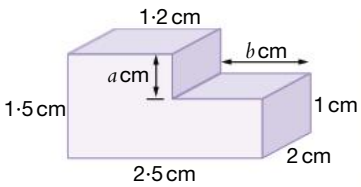
c Change 0·165 to a percentage. **d** Find $16\frac{1}{2}\%$ of \$75. 4:02–4:05

3 **a** The two shorter sides of a right-angled triangle are 2·4 m and 3·2 m. Find the length of the longest side.

b A ladder that is 11 m long is leaning against a wall. If the foot of the ladder is on level ground and is 1·5 m from the wall, how far up the wall does the ladder reach? 3:02–3:04

4 **a** Find the values of a and b.

b Draw the front view, top view and right-side view of the solid. 1:10



5 Write the following numbers as basic numerals:

a five thousand and twenty-six

b sixty thousand, seven hundred

c seven million, twenty thousand and six. 1:03

6 If $m = -4$, $p = -9$ and $t = 9$, find the value of each expression:

a $p + t$ **b** $3m \div 6$ **c** $-4t + 3m$ **d** $-5(8 - 2t)$ 2:03

7 Draw a picture of:

a a right-angled isosceles triangle

b an obtuse-angled scalene triangle

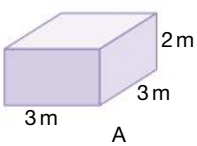
c an acute-angled isosceles triangle. 1:09

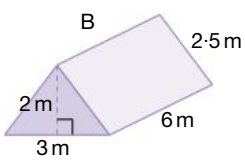
8 Your counter is at the start of this game. You must throw one dice and move forward following the directions until the turn is finished. Find the probability that after one turn your counter will be on square number:

a 1 **b** 2 **c** 3 **d** 11 6:02

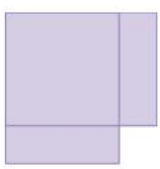


9 **a** What is the volume of A? **b** What is the volume of B? **c** Do prisms A and B have the same volume? 8:06





10 The figure shown is made from two identical overlapping rectangles which are 20 cm long and 15 cm wide. Calculate the area of the shape. 8:03



EQUATIONS

9



Contents

9:01 Inverse operations
9:02 Solving equations
Fun spot 9:02 Why did the tooth get dressed up?
9:03 Equations with pronumerals on both sides
GeoGebra activity 9:03 Equations with pronumerals on both sides
9:04 Equations with grouping symbols
GeoGebra activity 9:04 Equations with grouping symbols
9:05 Solving simple quadratic equations
9:06 Solving problems using equations
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

Selections from ***Equations*** [Stage 4]

- Solve simple linear equations. (ACMNA179)
- Solve linear equations using algebraic techniques and verify solutions by substitution. (ACMNA194)
- Solve simple quadratic equations.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

In Year 7 you were introduced to equations. An equation is a number sentence in which one (or more) of the numbers is unknown and is represented by a pronumeral, usually a letter of the alphabet.

e.g. $x + 5 = 12$ or $10 - 3a = 4$

Equations are sometimes called algebraic sentences.

We **solve** an equation to find the value of the pronumeral.

To solve equations it is necessary to understand inverse operations.

9:01 Inverse operations

PREP QUIZ 9:01

Write an algebraic expression for each of the following sentences.

1 Add 6 to x. **2** Multiply a by 8. **3** Divide y by 7. **4** Take 3 away from p.

5 Multiply x by 3 and then add 5.

6 Subtract 4 from y and then multiply the result by 9.

What is the opposite of:

7 adding 7 **8** subtracting 8 **9** multiplying by 9 **10** dividing by 10?

We start with a pronumeral when building algebraic expressions.

Algebraic expression	Meaning	Arrow diagram
$x + 5$	x has 5 added to it.	$x \xrightarrow{+5} x + 5$
$x - 5$	x has 5 subtracted from it.	$x \xrightarrow{-5} x - 5$
$5x$	x is multiplied by 5.	$x \xrightarrow{\times 5} 5x$
$\frac{x}{5}$	x is divided by 5.	$x \xrightarrow{\div 5} \frac{x}{5}$

- Arrow diagrams can be used to show how the expression has been made.
- The operation that has caused the change is written above the arrow.

- When solving equations, we need to reverse the steps to get back to the pronumeral. In other words, we need to perform the **inverse** operations.

$x \xrightarrow{+5} x+5$ then $x+5 \xrightarrow{-5} x$

$x \xrightarrow{-5} x-5$ then $x-5 \xrightarrow{+5} x$

$x \xrightarrow{\times 5} 5x$ then $5x \xrightarrow{\div 5} x$

$x \xrightarrow{\div 5} \frac{x}{5}$ then $\frac{x}{5} \xrightarrow{\times 5} x$



Inverse means *opposite*.

$2 \xrightarrow{+5} 7$

$7 \xrightarrow{-5} 2$

Adding 5 and subtracting 5 are inverse operations.

This process is called **backtracking**.

WORKED EXAMPLES

1 Complete the following arrow diagrams by writing the correct operation above each arrow.

a $x \rightarrow x+6$

b $y \rightarrow 5y \rightarrow 5y+7$

c $a \rightarrow a-4 \rightarrow 2(a-4)$

d $a \rightarrow a-8 \rightarrow \frac{a-8}{9}$

2 Use the inverse operations to get back to the pronumeral.

a $x+6 \xrightarrow{-6} \square$

b $5y+7 \xrightarrow{-7} \square \xrightarrow{\div 5} \square$

c $2(a-4) \xrightarrow{\div 2} \square \xrightarrow{+4} \square$

d $\frac{a-8}{9} \xrightarrow{\times 9} \square \xrightarrow{+8} \square$

Solutions

1 a $x \xrightarrow{+6} x+6$

b $y \xrightarrow{\times 5} 5y \xrightarrow{+7} 5y+7$

c $a \xrightarrow{-4} a-4 \xrightarrow{\times 2} 2(a-4)$

d $a \xrightarrow{-8} a-8 \xrightarrow{\div 9} \frac{a-8}{9}$



2 a $x+6 \xrightarrow{-6} x$

b $5y+7 \xrightarrow{-7} 5y \xrightarrow{\div 5} y$

c $2(a-4) \xrightarrow{\div 2} a-4 \xrightarrow{+4} a$

d $\frac{a-8}{9} \xrightarrow{\times 9} a-8 \xrightarrow{+8} a$

Exercise 9:01

P Foundation worksheet 9:01
Algebra review

1 Complete the following arrow diagrams by writing the correct operation above each arrow.

a $x \xrightarrow{\quad} x + 6$

b $m \xrightarrow{\quad} m - 5$

c $n \xrightarrow{\quad} 7n$

d $a \xrightarrow{\quad} \frac{a}{2}$

e $y \xrightarrow{\quad} y + 1 \xrightarrow{\quad} 3(y + 1)$

f $p \xrightarrow{\quad} p - 6 \xrightarrow{\quad} 9(p - 6)$

g $x \xrightarrow{\quad} x + 1 \xrightarrow{\quad} \frac{x+1}{2}$

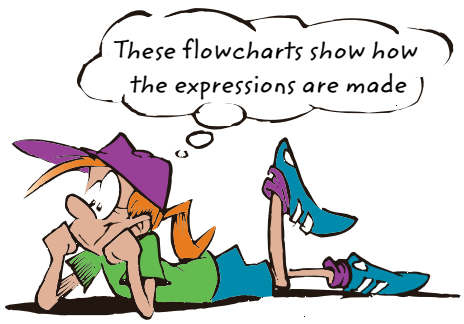
h $t \xrightarrow{\quad} t - 7 \xrightarrow{\quad} \frac{t-7}{10}$

i $q \xrightarrow{\quad} \frac{q}{2} \xrightarrow{\quad} \frac{q}{2} + 3$

j $a \xrightarrow{\quad} -4a \xrightarrow{\quad} -4a + 8$

k $m \xrightarrow{\quad} \frac{m}{-6} \xrightarrow{\quad} \frac{m}{-6} - 8$

l $a \xrightarrow{\quad} -5a \xrightarrow{\quad} 2 - 5a$



2 Draw arrow diagrams to show how the following expressions were made from each pronumeral.

a $9a$ **b** $y + 3$ **c** $q - 9$ **d** $\frac{x}{2}$

e $5n + 6$ **f** $7m - 1$ **g** $4p + 3$ **h** $9y - 5$

i $\frac{x}{3} + 2$ **j** $3 + \frac{n}{5}$ **k** $\frac{x+7}{2}$ **l** $\frac{a-1}{4}$

m $5(n + 2)$ **n** $8(x - 1)$ **o** $2(q + 9)$ **p** $10(t - 1)$

q $5 - 6x$ **r** $-7v + 1$ **s** $\frac{7-a}{2}$ **t** $9 - \frac{x}{6}$

3 Write the operations that must be performed on x to produce the following expressions.

a $7x$ **b** $x - 7$ **c** $x + 1$ **d** $\frac{x}{8}$

e $3x + 5$ **f** $7x + 2$ **g** $4x - 9$ **h** $6 + 4x$

i $\frac{x}{5} - 7$ **j** $10 + \frac{x}{5}$ **k** $\frac{x-7}{3}$ **l** $\frac{x+1}{2}$

m $3(x + 7)$ **n** $9(x - 4)$ **o** $-7x + 1$ **p** $12 - x$

4 Complete the following arrow diagrams.

a $\boxed{a} \xrightarrow{-2} \boxed{}$ b $\boxed{x} \xrightarrow{\times 4} \boxed{}$ c $\boxed{m} \xrightarrow{\div 3} \boxed{}$ d $\boxed{t} \xrightarrow{+9} \boxed{}$

e $\boxed{p} \xrightarrow{\times 5} \boxed{} \xrightarrow{+2} \boxed{}$ f $\boxed{x} \xrightarrow{\times 6} \boxed{} \xrightarrow{-4} \boxed{}$

g $\boxed{y} \xrightarrow{+1} \boxed{} \xrightarrow{\times 7} \boxed{}$ h $\boxed{b} \xrightarrow{\div 4} \boxed{} \xrightarrow{+2} \boxed{}$

i $\boxed{q} \xrightarrow{-3} \boxed{} \xrightarrow{\div 3} \boxed{}$ j $\boxed{n} \xrightarrow{\times(-2)} \boxed{} \xrightarrow{+6} \boxed{}$

k $\boxed{b+10} \xrightarrow{-10} \boxed{}$ l $\boxed{a-7} \xrightarrow{+7} \boxed{}$

m $\boxed{5m} \xrightarrow{\div 5} \boxed{}$ n $\boxed{\frac{n}{4}} \xrightarrow{\times 4} \boxed{x}$

o $\boxed{2x+3} \xrightarrow{-3} \boxed{} \xrightarrow{\div 2} \boxed{}$ p $\boxed{5p-4} \xrightarrow{+4} \boxed{} \xrightarrow{\div 5} \boxed{}$

q $\boxed{6+7t} \xrightarrow{-6} \boxed{} \xrightarrow{\div 7} \boxed{}$ r $\boxed{\frac{m+5}{4}} \xrightarrow{\times 4} \boxed{} \xrightarrow{-5} \boxed{}$

s $\boxed{\frac{a}{3}-5} \xrightarrow{+5} \boxed{} \xrightarrow{\times 3} \boxed{}$ t $\boxed{5-2x} \xrightarrow{-5} \boxed{} \xrightarrow{\div(-2)} \boxed{}$

5 Complete these arrow diagrams, showing how to get back to the pronumeral.

Backtrack to the pronumeral.

a $\boxed{3m} \rightarrow \boxed{m}$ b $\boxed{\frac{x}{2}} \rightarrow \boxed{x}$

c $\boxed{n+5} \rightarrow \boxed{n}$ d $\boxed{p-7} \rightarrow \boxed{p}$

e $\boxed{10x} \rightarrow \boxed{x}$ f $\boxed{\frac{a}{3}} \rightarrow \boxed{a}$

g $\boxed{8+t} \rightarrow \boxed{t}$ h $\boxed{q-5} \rightarrow \boxed{q}$

i $\boxed{2m+1} \rightarrow \boxed{2m} \rightarrow \boxed{m}$ j $\boxed{5p-2} \rightarrow \boxed{5p} \rightarrow \boxed{p}$

k $\boxed{7x+3} \rightarrow \boxed{7x} \rightarrow \boxed{x}$ l $\boxed{9q-4} \rightarrow \boxed{9q} \rightarrow \boxed{q}$

m $\boxed{6+3x} \rightarrow \boxed{3x} \rightarrow \boxed{x}$ n $\boxed{7+\frac{m}{2}} \rightarrow \boxed{\frac{m}{2}} \rightarrow \boxed{m}$

o $\boxed{\frac{x}{5}-2} \rightarrow \boxed{\frac{x}{5}} \rightarrow \boxed{x}$ p $\boxed{\frac{a+7}{3}} \rightarrow \boxed{a+7} \rightarrow \boxed{a}$

q $\boxed{\frac{b-2}{5}} \rightarrow \boxed{b-2} \rightarrow \boxed{b}$ r $\boxed{5-3x} \rightarrow \boxed{-3x} \rightarrow \boxed{x}$

s $\boxed{8-2p} \rightarrow \boxed{-2p} \rightarrow \boxed{p}$ t $\boxed{5-\frac{x}{3}} \rightarrow \boxed{\frac{-x}{3}} \rightarrow \boxed{x}$

6 What operations must be performed on these expressions to backtrack to a?

a $5a$	**b** $a-4$	**c** $a+9$	**d** $\frac{a}{8}$
e $2a+1$	**f** $7a+5$	**g** $8a-3$	**h** $6+5a$
i $\frac{a}{2}-5$	**j** $12+\frac{a}{5}$	**k** $\frac{a-7}{10}$	**l** $\frac{a+1}{3}$
m $8(a+7)$	**n** $2(a-11)$	**o** $-3a+1$	**p** $4-a$

7 What order of inverse operations must be performed to get back to the pronumeral?

a $9(2a-5)$	**b** $5(8y+7)$	**c** $3(2+9q)$	**d** $\frac{3x+5}{2}$
e $\frac{5x}{3}-2$	**f** $3+\frac{4n}{5}$	**g** $\frac{7+3x}{2}$	**h** $\frac{9a-1}{4}$
i $4(5-6x)$	**j** $-7(2v+1)$	**k** $\frac{7-3a}{2}$	**l** $9-\frac{x}{6}$

9:02 Solving equations

PREP QUIZ 9:02

Simplify each of the following.

1 $7x \div 7$ **2** $x+4-4$ **3** $a-7+7$ **4** $\frac{m}{5}\times 5$

What is the inverse of:

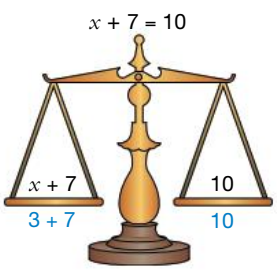
5 multiplying by 3 **6** adding 7 **7** subtracting 1 **8** dividing by 5?

Complete these arrowing diagrams, putting operations above each arrow.

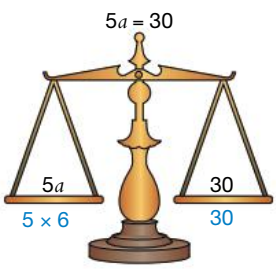
9 $\boxed{5m+3} \rightarrow \boxed{5m} \rightarrow \boxed{m}$ **10** $\boxed{8a-2} \rightarrow \boxed{8a} \rightarrow \boxed{a}$

- Solving equations is like balancing scales.
- With equations, we know that one side is equal to the other.
- The solution of the equation therefore is the value of the pronumeral that 'balances' the equation.





$x = 3$ balances the scale
so $x = 3$ is the solution.

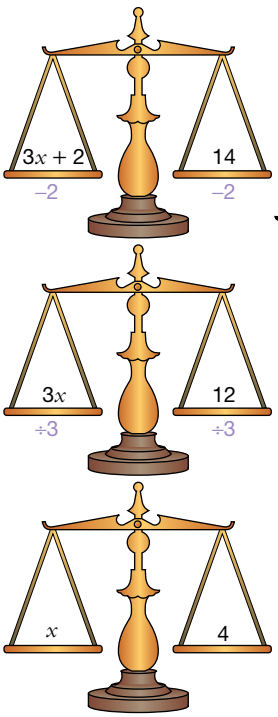


$a = 6$ balances the scale
so $a = 6$ is the solution.

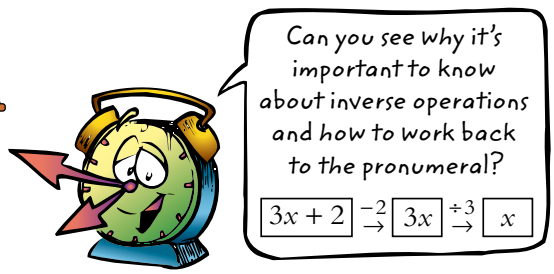
- Often, solving an equation requires us to change the equation into a simpler one. To do this we add (+), subtract (−), multiply (×) or divide (÷) both sides of the equation by the same number.
- Look at the solutions of these two equations, noting that both sides remain balanced because the same operation is done to both sides.

We need to perform operations that will leave only the pronumeral on one side of the equation.

$3x + 2 = 14$

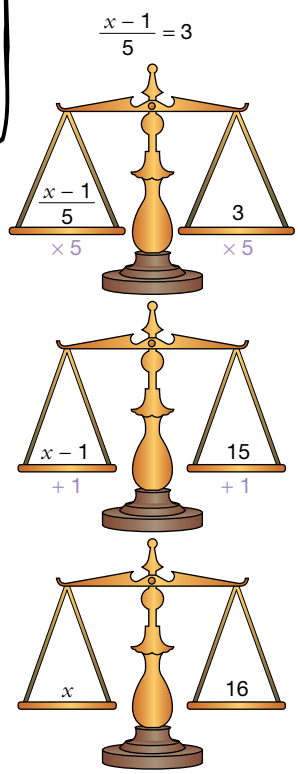


$x = 4$ is the solution





$\frac{x-1}{5} = 3$



$x = 16$ is the solution

Note here that the inverse operations are:

$\boxed{\frac{x-1}{5}} \xrightarrow{\times 5} \boxed{x-1} \xrightarrow{+1} \boxed{x}$



WORKED EXAMPLE 1

Solve each of these one-step equations.

a $m + 27 = 59$
$-27 \quad -27$
$m = 59 - 27$
$\therefore m = 32$

b $a - 16 = 9$
$+16 \quad +16$
$a = 9 + 16$
$\therefore a = 25$

c $3p = 23$
$\div 3 \quad \div 3$
$p = \frac{23}{3}$
$\therefore p = 7\frac{2}{3}$

d $\frac{x}{7} = 12$
$\times 7 \quad \times 7$
$x = 7 \times 12$
$\therefore x = 84$

WORKED EXAMPLE 2

Solve each of these two-step equations.

a $2a + 5 = 7$

$-5 \quad -5$

$2a = 2$

$\div 2 \quad \div 2$

$\therefore a = 1$

b $\dfrac{x+3}{5} = 2$

$\times 5 \quad \times 5$

$x + 3 = 10$

$-3 \quad -3$

$\therefore x = 7$

c $\dfrac{p}{3} - 6 = 15$

$+6 \quad +6$

$\dfrac{p}{3} = 21$

$\times 3 \quad \times 3$

$\therefore p = 63$

d $10 - 3m = 25$

$-10 \quad -10$

$-3m = 15$

$\div -3 \quad \div -3$

$\therefore m = -5$

Exercise 9:02

P Foundation worksheet 9:02 Solving equations 1

1 Solve each of these one-step equations.

a $x + 3 = 12$ **b** $q + 9 = 13$ **c** $y + 13 = 21$

d $9 + n = 15$ **e** $8 + p = 14$ **f** $20 + t = 33$

g $x - 7 = 2$ **h** $w - 8 = 8$ **i** $m - 5 = 11$

j $5a = 20$ **k** $7y = 35$ **l** $4y = 28$

m $\dfrac{x}{2} = 5$ **n** $\dfrac{a}{4} = 3$ **o** $\dfrac{m}{6} = 6$

2 Solve these one-step equations that involve negative integers.

a $x + 3 = 1$ **b** $m + 5 = 3$ **c** $a + 13 = 8$

d $2 + n = -5$ **e** $5 + p = -1$ **f** $7 + y = 0$

g $n - 7 = -2$ **h** $t - 8 = -3$ **i** $q - 5 = -5$

j $3a = -12$ **k** $7y = -21$ **l** $5x = -55$

m $-2a = -8$ **n** $-8y = -24$ **o** $-6x = -42$

p $\dfrac{x}{2} = -4$ **q** $\dfrac{w}{4} = -1$ **r** $\dfrac{v}{-3} = -5$

s $x - (-7) = 12$ **t** $t - (-8) = 10$ **u** $q - (-5) = -5$

3 Substitute the solution into each equation below and state whether the solution is correct or not.

a $x + 17 = 39$

$x = 56$

b $9 + m = 15$

$m = 7$

c $q - 13 = 16$

$q = 29$

d $n - 28 = 17$

$n = 11$

e $17 - d = 9$

$d = 8$

f $9 - p = 12$

$p = 3$

g $3y = 9$

$y = 27$

h $5n = 35$

$n = 7$

i $-4n = -20$

$n = -5$

j $\dfrac{x}{2} = 8$

$x = 4$

k $\dfrac{a}{4} = 2$

$k = 8$

l $\dfrac{n}{7} = 7$

$n = 1$

4 Solve these two-step equations. Clearly show each step in your working. (All the answers are integers.)

a $2x + 3 = 7$ b $3n + 5 = 8$ c $5y + 1 = 16$

d $4n - 3 = 9$ e $6k - 5 = 7$ f $9q - 4 = 23$

g $2 - 3w = 11$ h $3 + 5p = 3$ i $12 + 7s = 26$

j $12 - 3x = 6$ k $16 - 2j = 6$ l $20 - 9g = 2$

m $14 = 2v + 6$ n $30 = 8 + 2y$ o $10 = 4 - 2a$

p $6 - x = -2$ q $-5 - p = -3$ r $15 = 6 - 3h$

5 Solve these two-step equations. (The solutions are all positive integers.)

a $\frac{5x}{2} = 5$ b $\frac{3x}{4} = 6$ c $\frac{2t}{5} = 4$

d $\frac{x+1}{2} = 3$ e $\frac{a+5}{3} = 3$ f $\frac{m+8}{4} = 2$

g $\frac{n-1}{5} = 2$ h $\frac{y-7}{2} = 5$ i $\frac{x-5}{8} = 3$

j $\frac{x}{2} + 1 = 5$ k $\frac{h}{4} + 3 = 7$ l $\frac{a}{5} - 1 = 5$

m $5 + \frac{x}{2} = 9$ n $3 + \frac{q}{4} = 8$ o $9 - \frac{n}{2} = 5$

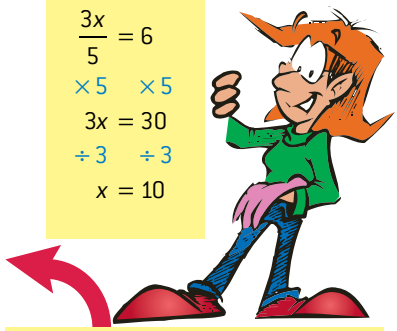
$\frac{3x}{5} = 6$

$\times 5 \quad \times 5$

$3x = 30$

$\div 3 \quad \div 3$

$x = 10$



Multiply each term on *both sides* by the denominator.

6 Find the solutions to each of the following equations.

a $2x + 3 = 4$ b $3n + 6 = 8$ c $5y + 1 = 3$

d $4n - 1 = 9$ e $6k - 4 = 7$ f $6q - 4 = 2$

g $2 + 3w = 10$ h $3 + 5p = 4$ i $12 + 7s = 20$

j $12 - 3x = 8$ k $15 - 2j = 6$ l $20 - 7g = 2$

m $14 = 2v + 3$ n $21 = 8 + 2y$ o $10 = 5 - 3y$

p $6 - 3x = -2$ q $-5 - 2p = -4$ r $15 = 10 - 3h$

$3a + 2 = 6$

$-2 \quad -2$

$3a = 4$

$\div 3 \quad \div 3$

$a = \frac{4}{3}$ or $1\frac{1}{3}$

7 Solve this set of equations, which are either one- or two-step types.

a $2x = 7$ b $n + 5 = 8$ c $5y + 2 = 12$

d $5 - y = 6$ e $7x + 3 = 17$ f $y - 5 = -3$

g $y + 4 = 1$ h $2y - 1 = 13$ i $-5x = 20$

j $4 + 5m = 8$ k $5y = 9$ l $5 - 2y = 9$

m $\frac{a}{2} = 5$ n $\frac{3x}{5} = 9$ o $\frac{t-3}{4} = 5$

p $\frac{q}{2} - 1 = 5$ q $\frac{w}{4} + 1 = 6$ r $\frac{3a}{8} = 2$

s $\frac{y+7}{2} = 3$ t $\frac{6-a}{3} = 3$ u $11 - 3x = 7$

v $25 + 2x = 11$ w $\frac{t+5}{2} = -4$ x $\frac{5-p}{4} = 3$



FUN SPOT 9:02 WHY DID THE TOOTH GET DRESSED UP?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

Solve these equations.

T $x + 8 = 15$	**A** $a = -4 - 6$	**A** $a = (\frac{1}{2})^3$
T $3 + x = 1$	**T** $8m = -56$	**D** $11x = 121$
A $x + x = 1$	**T** $75 \div a = 15$	**E** $0{\cdot}2x = -1$
E $8a = 0$	**S** $8 + x = 12$	**H** $1 - x = 4$
U $60 - n = 0$	**O** $35 \div n = 1$	**N** $5(x - 9) = 0$
T $n = 20 - n$	**U** $7 - x = 8$	**O** $\frac{100}{m} = 5$
O $6 \div x = 2$	**W** $\frac{1}{4} + n = 1$	**T** $7a = 7$
I $-3 - 3 = a$	**K** $1 - x = -1$	**S** $4x = 1$
B $-a = -8$	**T** $a - 8 = 2a$	**I** $\frac{x}{3} = 2$
T $10 - n = -2$	**E** $\frac{m}{20} = 5$	



5	−3	0	11	100	9	7	−6	4	−2	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	−10	8	3	60	1

−8	20	−7	$\frac{1}{2}$	2	−5	6	10	35	−1	12



9:03 Equation with pronumerals on both sides

PREP QUIZ 9:03

Simplify the following: **1** $3x - 2x$ **2** $5a + a$ **3** $4y - 4y$ **4** $-6p + 6p$

Which operation should be written above each arrow?

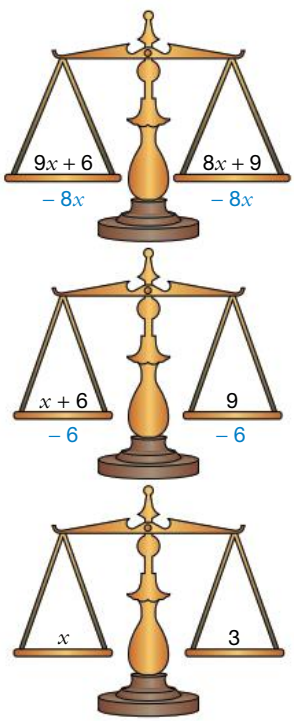
5 $3x + 5 \rightarrow 3a$ **6** $2a - 4 \rightarrow 2a$ **7** $\frac{x+3}{5} \rightarrow x + 3$

Complete these arrow diagrams using the operation above each arrow.

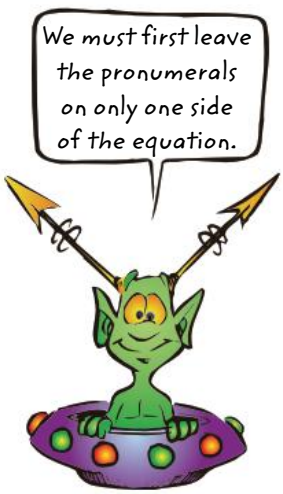
8 $5 + 2x \xrightarrow{-2x} \square$ **9** $3x + 1 \xrightarrow{-3x} \square$ **10** $5 - 2x \xrightarrow{+2x} \square$

To solve these equations we may have to add or subtract pronumerals as well as numerals. Follow the steps in each diagram.

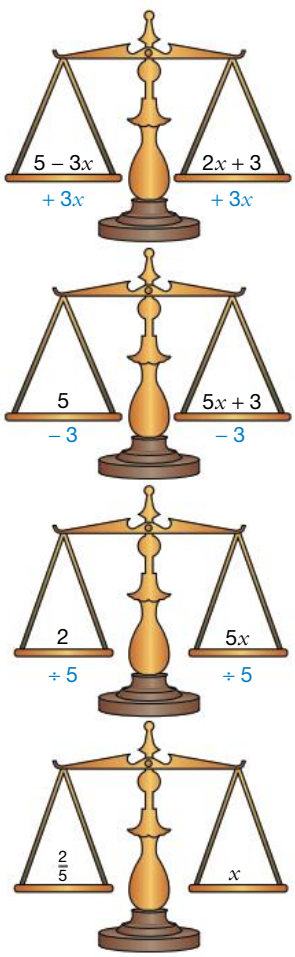
$9x + 6 = 8x + 9$



So $x = 3$ is the solution.



$5 - 3x = 2x + 3$



So $x = \frac{2}{5}$ is the solution.

Remember:
Leave the pronumeral on the side where the term will be positive.

WORKED EXAMPLES

1 Solve each of the following equations. Subtract the term with the smaller positive pronumeral from both sides so that the remaining pronumeral is positive.

a $3a + 5 = 2a + 7$ **b** $2x - 15 = 5x - 3$

2 Solve the following equations. Add the term with the greater negative pronumeral so that the remaining pronumeral is positive.

a $14 - 2x = 8 + x$ **b** $6 - a = 4 - 5a$

Solutions

1 a
$$\begin{aligned} 3a + 5 &= 2a + 7 \\ -2a & \quad -2a \\ a + 5 &= 7 \\ -5 & \quad -5 \\ \therefore\ a &= 2 \end{aligned}$$

b
$$\begin{aligned} 2x - 15 &= 5x - 3 \\ -2x & \quad -2x \\ -15 &= 3x - 3 \\ +3 & \quad +3 \\ -12 &= 3x \\ \div 3 & \quad \div 3 \\ -4 &= x \\ \therefore\ x &= -4 \end{aligned}$$

Therefore! $\therefore$

2 a
$$\begin{aligned} 14 - 2x &= 8 + x \\ +2x & \quad +2x \\ 14 &= 8 + 3x \\ -8 & \quad -8 \\ 6 &= 3x \\ \div 3 & \quad \div 3 \\ 2 &= x \\ \therefore\ x &= 2 \end{aligned}$$

b
$$\begin{aligned} 6 - a &= 4 - 5a \\ +5a & \quad +5a \\ 6 + 4a &= 4 \\ -6 & \quad -6 \\ 4a &= -2 \\ \div 4 & \quad \div 4 \\ \therefore\ a &= -\tfrac{1}{2} \end{aligned}$$

Exercise 9:03

P Foundation worksheet 9:03 Solving equations 2

1 Solve the following equations. (The answers are all integers.)

a $2a + 5 = a + 7$ **b** $3x + 1 = 2x + 8$ **c** $7p - 2 = 6p + 1$
d $10q - 3 = 9q + 5$ **e** $7n = 6n + 5$ **f** $8x = 7x - 3$
g $5m + 2 = 4m - 1$ **h** $3a + 7 = 2a + 5$ **i** $3x + 1 = x + 7$
j $5m + 2 = 3m + 8$ **k** $5y - 4 = y + 12$ **l** $3x + 7 = x + 1$
m $2x + 5 = 3x - 1$ **n** $4m + 2 = 6m + 10$ **o** $x + 7 = 5x - 1$
p $3x + 2 = 7 - 2x$ **q** $6m - 1 = 15 - 2m$ **r** $5 - 3p = p + 5$

2 Substitute the solution into each equation below and state whether it is correct or not.

a $5m - 2 = 4m + 1$, $m = 3$ **b** $3p + 7 = 2p + 12$, $p = 5$ **c** $10q + 3 = 11q + 1$, $q = -2$
d $7a + 3 = 5a + 1$, $a = 2$ **e** $8q - 7 = 5q + 5$, $q = 4$ **f** $x + 10 = 5x - 2$, $x = 2$

g $3n + 1 = n - 7$
$n = -3$

h $2m + 5 = 5m - 1$
$m = 2$

i $8a + 1 = 3a - 4$
$a = -1$

j $5x = 2x + 6$
$x = 2$

k $5a + 8 = 3a$
$a = 4$

l $3x = 10 + 2x$
$x = 2$

m $3 + 2x = 7 + 3x$
$x = 4$

n $5 - 2m = 6 + m$
$m = -1$

o $5p - 2 = 5 - 2p$
$p = 1$

3 Solve each of these equations.

a $3x + 1 = x + 2$
b $5m + 2 = 3m + 5$
c $4p + 2 = p + 4$
d $5m = 2m + 1$
e $3a = a - 1$
f $6n + 2 = n + 8$
g $5a - 1 = 3a + 4$
h $12n - 9 = 9n + 1$
i $8q - 5 = 5q + 3$
j $6n + 4 = 4n + 3$
k $8a + 5 = 5a + 7$
l $4k - 2 = k - 6$
m $2a + 5 = 4a + 2$
n $x + 6 = 4x + 2$
o $2q + 7 = 6q - 2$
p $8t = 5t + 8$
q $3m = 6m - 2$
r $6x = 5 - 2x$
s $3a + 2 = 5a + 6$
t $w - 3 = 7w + 5$
u $2m - 3 = 5m + 1$
v $5 - 2y = 3 + y$
w $10 + 3x = 9 - 5x$
x $3 - p = 16 - 4p$

P GEOGEBRA ACTIVITY 9:03 EQUATIONS WITH PRONUMERALS ON BOTH SIDES

Solve equations with varying degrees of difficulty.

Type 1	Type 2	Type 3
$4x + 3 = 3x + 2$	$4x - 3 = 3x + 2$	$4x - 3 = 3x - 2$
$1x + 3 = 2$	$1x - 3 = 2$	$1x - 3 = -2$
$1x = -1$	$1x = 5$	$1x = 1$
$x = -1$	$x = 5$	$x = 1$

9:04 Equations with grouping symbols

PREP QUIZ 9:04

Rewrite these expressions without grouping symbols.

1 $2(x + 3)$ **2** $3(a - 5)$ **3** $2(4a + 6)$ **4** $5(2p - 1)$ **5** $7(3 - 2q)$

Solve these one-step equations.

6 $x + 7 = 12$ **7** $x + 7 = 2$ **8** $5p = 10$ **9** $10p = 5$ **10** $8 - x = -3$

If you remember how to expand grouping symbols, these equations are no harder than those you have already solved.

WORKED EXAMPLES

1 Expand the grouping symbols and then solve each equation.

a $2(x + 4) = 10$ **b** $5(a - 3) = 3$ **c** $3(2m - 4) = 4m - 6$

2 Expand each set of grouping symbols and then solve the equations.

a $3(a + 7) = 4(a - 2)$ **b** $3(x + 4) + 2(x - 5) = 4$

Solutions

1 a
$$2(x + 4) = 10$$
$$2x + 8 = 10$$
$$-8 \quad -8$$
$$2x = 2$$
$$\div 2 \quad \div 2$$
$$\therefore x = 1$$

b
$$5(a - 3) = 3$$
$$5a - 15 = 3$$
$$+15 \quad +15$$
$$5a = 18$$
$$\div 5 \quad \div 5$$
$$\therefore a = \frac{18}{5} \text{ or } 3\frac{3}{5}$$

c
$$3(2m - 4) = 4m - 6$$
$$6m - 12 = 4m - 6$$
$$-4m \qquad -4m$$
$$2m - 12 = -6$$
$$+12 \quad +12$$
$$2m = 6$$
$$\div 2 \quad \div 2$$
$$\therefore m = 3$$

2 a
$$3(a + 7) = 4(a - 2)$$
$$3a + 21 = 4a - 8$$
$$-3a \qquad -3a$$
$$21 = a - 8$$
$$+8 \quad +8$$
$$29 = a$$
$$\therefore a = 29$$

b
$$3(x + 4) + 2(x - 5) = 4$$
$$3x + 12 + 2x - 10 = 4$$

Collect like terms.

$$5x + 2 = 4$$
$$-2 \quad -2$$
$$5x = 2$$
$$\div 5 \quad \div 5$$
$$x = \frac{2}{5}$$

Exercise 9:04

P Foundation worksheet 9:04 Solving equations 3

1 Expand the grouping symbols and then solve each equation.

a $2(x + 3) = 4$ **b** $3(a + 2) = 12$ **c** $7(n + 2) = 14$
d $6(p + 2) = 24$ **e** $3(a - 2) = 6$ **f** $2(m - 3) = 8$
g $9(x - 1) = 18$ **h** $5(q - 2) = 10$ **i** $2(a + 3) = 7$
j $3(x + 1) = 8$ **k** $2(x - 2) = 1$ **l** $5(t - 1) = 2$
m $2(3 - a) = 4$ **n** $4(7 - x) = 12$

2 Solve each equation by first expanding the grouping symbols. (Answers are all integers.)

a $3(2x + 1) = 9$ **b** $2(3a + 4) = 20$ **c** $5(2m + 1) = 25$
d $4(3a - 1) = 8$ **e** $5(2p - 3) = 5$ **f** $4(3q - 2) = 16$
g $2(x + 2) = x + 5$ **h** $3(a + 3) = 2a + 12$ **i** $5(m + 2) = 3m + 12$
j $3(m - 1) = 2m - 1$ **k** $4(n - 2) = 2n + 4$ **l** $3(2a + 1) = 4a + 7$
m $2m + 5 = 3(m - 1)$ **n** $6x + 3 = 3(x + 2)$ **o** $5 - 2x = 3(x - 5)$

3 Solve each equation.

a $2(x + 1) = 5$	**b** $3(a + 3) = -10$	**c** $5(p + 3) = 2$
d $3(a - 2) = 0$	**e** $5(q - 1) = -3$	**f** $7(t - 2) = 5$
g $2(3x + 1) = 5$	**h** $3(2p + 3) = 12$	**i** $5(2w + 3) = -20$
j $3(x - 1) = x + 2$	**k** $2(q - 3) = 4q - 9$	**l** $5(k + 2) = 2k + 11$
m $3m + 2 = 5(m - 1)$	**n** $7x = 5(2x - 1)$	**o** $3n - 7 = 6(2n - 2)$

4 Find the solution to each equation by first expanding all grouping symbols.

a $3(a + 2) = 2(a + 1)$	**b** $5(x - 1) = 4(x + 2)$
c $5(p - 2) = 4(p + 2)$	**d** $3(q + 2) = 2(q + 5)$
e $3(m + 1) = 5(m - 1)$	**f** $6(x + 2) = 4(x + 6)$
g $2(a - 7) = 5(a - 4)$	**h** $7(t + 2) = 4(t + 5)$
i $3(2a + 1) = 5(a + 2)$	**j** $4(3p - 1) = 5(2p + 1)$
k $6(t + 7) = 4(t + 10) + 8$	**l** $5(2a - 1) = 3(a + 6) - 7$
m $3(2 + m) = 5(2 - m) + 6m$	**n** $6(p + 3) = 5(2 - p) + 7p - 12$

5 Solve each equation. (See Worked Example 2b.)

a $3(a + 2) + a + 5 = 15$	**b** $5(m - 1) + 2m = 2$
c $2(m + 3) + 5(m + 2) = 23$	**d** $3(x + 2) + 2(x - 3) = 10$
e $5(p + 1) + 2(p + 4) = 20$	**f** $4(t - 2) + 2(t + 5) = 14$
g $4(2a + 3) + 2(a - 5) = 22$	**h** $2(2m + 3) + 3(m - 5) = 5$
i $5(a - 3) + 3(2 + 3a) = 19$	**j** $7(a + 5) + 2(6 - 3a) = 1$

6 Try solving these equations, but first read the warning.

a $3(a + 2) - 2(a + 1) = 6$
b $5(m + 3) - 4(m + 2) = 10$
c $5(n + 4) - 3(n - 2) = 30$
d $6(a + 2) - 4(a - 1) = 20$
e $4(a + 3) - (a + 2) = 13$
f $4(p + 5) - (p + 3) = 23$
g $5(2a + 1) - 2(a - 4) = 2$
h $6(2x + 5) - 5(3x + 2) = 10$

Warning!

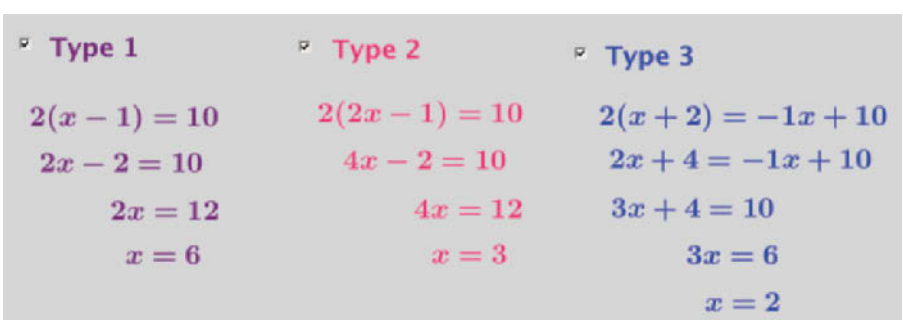
Remember how to expand with a negative term:

$-2(x + 4) = -2x - 8$

$-3(a - 1) = -3a + 3$

P GEOGEBRA ACTIVITY 9:04 EQUATIONS WITH GROUPING SYMBOLS

Solve equations with varying degrees of difficulty.



9:05 Solving simple quadratic equations

PREP QUIZ 9:05

Simplify:

1 $\sqrt{100}$ **2** $\sqrt{9}$ **3** $\sqrt{25}$ **4** $\sqrt{16}$ **5** $\sqrt{64}$

Write true or false:

6 $-3 \times -3 = 9$ **7** $-4 \times -4 = -16$ **8** $-4 \times -4 = 16$ **9** $-7 \times -7 = 49$

10 If $6 \times 6 = 36$ then the solution to $x^2 = 36$ is $x = 6$ or $x = -6$.

11 If $1{\cdot}5 \times 1{\cdot}5 = 2{\cdot}25$ and $-1{\cdot}5 \times -1{\cdot}5 = 2{\cdot}25$ then the solution to $x^2 = 2{\cdot}25$ is $x = 1{\cdot}5$ or $x = -1{\cdot}5$.

A quadratic equation has a pronumeral that is raised to the power of 2.
$x^2 = 64$ is a simple quadratic equation.

The term 'quadratic' comes from *quadratus*, which is the Latin word for 'squared'.

The Prep quiz has reminded us that:

- If $c > 0$, there are two values of x that solve a simple quadratic equation of the form $x^2 = c$. One value will be positive and the other will be negative.

  If $x^2 = 12$
  then $x = \sqrt{12}$ or $-\sqrt{12}$ (exact solutions)
  and $x = 3{\cdot}46$ or $-3{\cdot}46$ (2 dec. pl.)

- Note that $x^2 = c$ has no solution if c is a negative number.
  If $x^2 = -64$ then there is no solution.
  This is because no number times itself can give a negative answer.

Exercise 9:05

P Foundation worksheet 9:05 Formulas

1 Solve each equation, giving the two solutions.

a $x^2 = 4$ **b** $x^2 = 9$ **c** $x^2 = 36$ **d** $x^2 = 100$
e $c^2 = 1$ **f** $a^2 = 81$ **g** $m^2 = 16$ **h** $y^2 = 49$

2 In each case, give the answers as surds (that is, as exact values).

a $x^2 = 6$ **b** $x^2 = 11$ **c** $x^2 = 45$ **d** $x^2 = 2$
e $c^2 = 109$ **f** $m^2 = 72$ **g** $b^2 = 321$ **h** $a^2 = 8$

3 Write the solutions to each part of Question 2, correct to two decimal places.

4 Give all solutions as exact values in their simplest form.

a $x^2 = 25$ **b** $x^2 = 5$ **c** $x^2 = -5$ **d** $x^2 = 4$
e $y^2 = 144$ **f** $c^2 = 145$ **g** $n^2 = 121$ **h** $a^2 = 169$
i $x^2 = -36$ **j** $x^2 = 27$ **k** $x^2 = -27$ **l** $x^2 = 49$
m $y^2 = 12{\cdot}25$ **n** $y^2 = 72{\cdot}25$ **o** $c^2 = 26{\cdot}01$ **p** $b^2 = 1{\cdot}21$

9:06 Solving problems using equations

PREP QUIZ 9:06

Solve these simple equations.

1 $7x = 56$ **2** $9 + a = 6$ **3** $\frac{m}{3} = 6$ **4** $n + 6 = 4$ **5** $3x = -12$

Write expressions for:

6 the sum of a and b

7 the product of 5 and y

8 the number 5 bigger than x

9 the number 6 less than m

10 the next even number after n, if n is an even number.

Equations can be used to solve problems. To do this, we must translate or rewrite a worded problem into an equation, where the pronumeral represents the number we are trying to find.

Solving problems

Step 1 Introduce a pronumeral.
Step 2 Write down an equation.
Step 3 Solve the equation.
Step 4 Answer the problem.



WORKED EXAMPLES

1 The product of a certain number and 6 is 72. What is the number?

Solution

Let the number be n. ← Step 1 →
$6n = 72$ ← Step 2 →
$n = 12$ ← Step 3 →
∴ The number is 12. ← Step 4 →

2 If a number is multiplied by 6, and 7 is subtracted from the product, the answer is 23. What is the number?

Solution

Let the number be x.
$6x - 7 = 23$
$6x = 30$
$x = 5$
∴ The number is 5.

3 The sum of three consecutive integers is 30. What are the integers?

Solution

Let the smallest integer be n.
So $n + (n + 1) + (n + 2) = 30$
$3n + 3 = 30$
$3n = 27$
$n = 9$
∴ The integers are 9, 10 and 11.

'Consecutive' means 'one after the other', so the next consecutive integer after n would be $n + 1$.



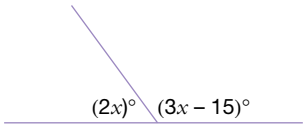
4 A rectangle is twice as long as it is wide. If the perimeter is 144 cm, what are its dimensions?



Solution

Let the width of the rectangle be x.
Then the length will be $2x$.
So $2x + x + 2x + x = 144$
i.e. $6x = 144$
$x = 24$
∴ The width of the rectangle is 24 units.
The length of the rectangle is 48 units.

5 Find the value of x in this diagram.



Solution

To solve this problem you need to remember that the two angles add to give 180° (angles on a straight line).

So $2x + (3x - 15) = 180$
i.e. $5x - 15 = 180$
$5x = 195$
$x = 39$
∴ The value of x is 39.

Exercise 9:06

P Challenge worksheet 9:06 Oranges for sale

1 Solve these number problems by first writing an equation.

Let the number be n.

- **a** The sum of a certain number and 11 is 23. What is the number?
- **b** A certain number minus 9 is equal to 13. What is the number?
- **c** 15 minus a certain number is equal to 8. What is the number?
- **d** The product of 9 and a certain number is 108. What is the number?
- **e** The sum of a certain number and 3 is −5. What is the number?
- **f** A certain number is bigger than 10 and the difference from 10 is 7. What is the number?

2 **a** If a number is multiplied by 3, and 5 is added to the product, the result is 17. What is the number?

These will be two-step equations!
Also, you may need grouping symbols.

- **b** When 6 is subtracted from the product of 4 and a certain number, the answer is 14. What is the number?
- **c** The sum of a certain number and 3 is then multiplied by 7. If the answer is 63, what is the number?
- **d** The product of a certain number and 5 is subtracted from 90. If the answer is 25, what is the number?
- **e** 9 is subtracted from a certain number and the result is multiplied by 4. If the answer is 24, what is the number?
- **f** The sum of two consecutive integers is 23. What are the integers?
- **g** The sum of three consecutive integers is 51. What are the integers?

If n is an even number, then the next even number will be $n + 2$.



3 The equation for each of these problems will have the pronumeral on both sides.

a The result of adding 12 to a certain number is the same as multiplying the number by 4. Find the number.

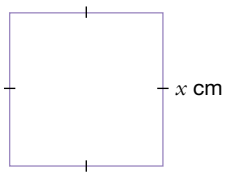
b Twice a number plus seven is the same as three times the number plus one. What is the number?

c Five times a number less seven is the same as three times the number plus five. Find the number.

d If the sum of a number and 3 is multiplied by 4, the answer is the same as twice the number plus 16. Find the number.

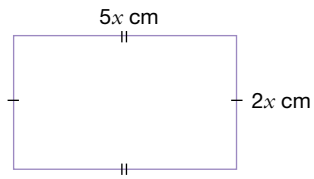
4 Use the information given with each shape to form an equation, and then solve it to find the value of the pronumeral.

a



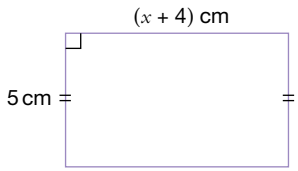
Perimeter = 64 cm

b



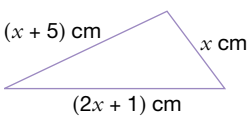
Perimeter = 98 cm

c



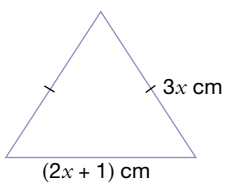
Area = 45 cm^2

d



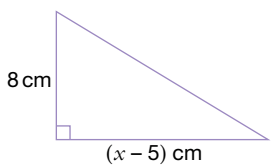
Perimeter = 22 cm

e



Perimeter = 57 cm

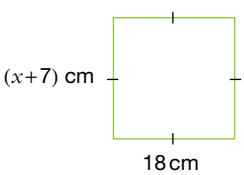
f



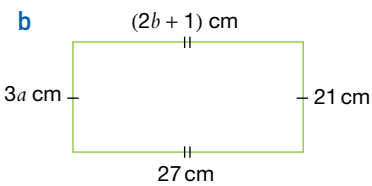
Area = 24 cm^2

5 From the properties of the shapes, write down equations and solve them for each pronumeral.

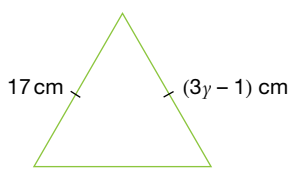
a



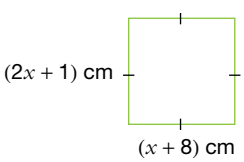
b



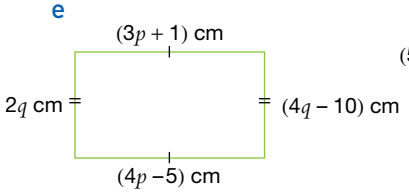
c



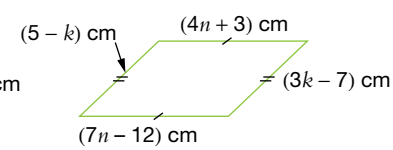
d



e

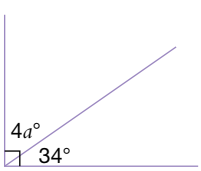


f

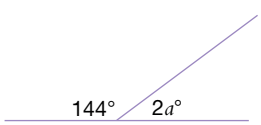


6 Find the value of a in these shapes. You will need to remember some geometry.

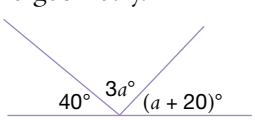
a



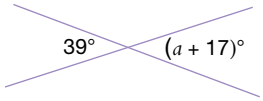
b



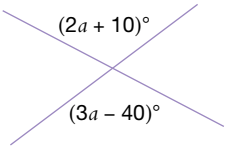
c



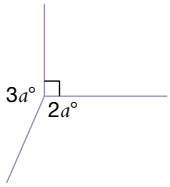
d



e



f



7 Try solving these assorted problems by first forming an equation.

a My mother is three times as old as I am. If she is 24 years older than me, what are our ages? (Hint: Let my age be x.)

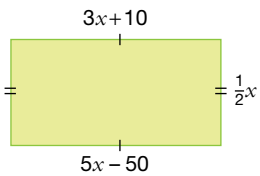
b Allyson and Kylie have \$35 between them. If Kylie has 4 times as much as Allyson, how much does each girl have? (Hint: Let Allyson's amount be a.)



c Four pens and five pencils cost a total of \$3.70. If a pen costs 25 cents more than a pencil, find the cost of each item.

d The perimeter of a rectangle is 44 cm. If its length is three times its width, find the dimensions of the rectangle. Also find its area.

e Determine the dimensions of this rectangle and hence its area.

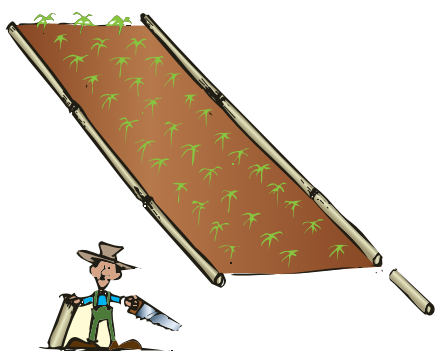




f Gregory has \$6 more than Penny, and Phillip has \$13 less than Gregory. How much does each person have if they have \$29 altogether? (Hint: Let Gregory's amount be x.)

g When George was 12 his father was three times his age. Now his father is only twice George's age. How old are they now? (Let George's age, now, be x.)

8 Two sides of a garden bed are to be lined with logs as shown. Three logs are needed for each side but 1 m is cut off the third log to make sure that the logs don't extend beyond the garden. The width of the garden is 3 m and its area is 42 m^2. How long is a whole log? (Hint: Let x be the length of a whole log.)



MATHS TERMS 9

algebraic expression

- an algebraic expression consists of one or more terms joined together by operation signs,
  e.g. $a + 5, x^2 - x + 4, \frac{3m-1}{7}$
- an expression does *not* have an 'equals' sign like an equation

equation

- a number sentence where one or more of the numbers is missing or unknown
- the unknown number is represented by a pronumeral,
  e.g. $x + 5 = 8, \frac{3x+1}{7} = \frac{x-5}{2}$

evaluate

- to find the value of an expression,
  e.g. Find the value of $3a + 7$ if $a = 3$
- to find the answer (usually after substitution)

grouping symbols

- the most common types are:
  parentheses ( )
  brackets [ ]
  braces { }
- used to 'group' a number of terms together in an expression,
  e.g. $5(x + 3)$

inverse operation

- the operation that will reverse or 'undo' a previous operation,
  e.g. Addition is the inverse operation of subtraction; division is the inverse operation of multiplication

pronumeral

- a symbol used to represent a number
- usually a letter such as x

quadratic equation

- quadratic equations have a squared pronumeral term and usually have two solutions
  e.g. $x^2 = 49$ has two solutions:
  $x = 7$ or $x = -7$

solution

- the answer to a problem
  OR
  the method of finding the answer to a problem
- the solution to an equation is the number that makes it a true sentence
  e.g. $x = 3$ is the solution to $x + 2 = 5$

solve

- find the *solution* or answer to a problem or equation

substitute

- to replace a pronumeral with a numeral
  e.g. To substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2$
  $= 12 - 2$
  $= 10$

DIAGNOSTIC TEST 9 EQUATIONS

Each section of the test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Solve: 9:02

a $x + 11 = 27$ b $p - 5 = 2$ c $5 + a = 3$ d $y - 7 = -2$

2 Solve: 9:02

a $6m = 54$ b $3x = 5$ c $\frac{n}{4} = 8$ d $\frac{a}{3} = -2$

3 Solve these two-step equations. (Answers are integers.) 9:02

a $2x + 3 = 7$ b $3m - 1 = 8$ c $6 + 4y = 22$ d $4 - 2a = 6$

4 The answers to these equations involve fractions. 9:02

a $2x + 1 = 6$ b $4y - 3 = 4$ c $7 + 5k = 3$ d $9 - 2t = 6$

5 Solve these two-step equations. 9:02

a $\frac{5m}{4} = 10$ b $\frac{m+2}{3} = 4$ c $\frac{x}{2} + 4 = 6$ d $\frac{w-2}{5} = 2$

6 Solve: 9:03

a $2a + 1 = a + 7$ b $3m + 2 = m - 6$
c $4k - 1 = 5 - 2k$ d $7x = 5x + 9$

7 Solve: 9:04

a $2(a + 4) = 8$ b $6(m - 3) = 12$
c $2(2y + 1) = 6$ d $3(5m - 1) = 27$

8 Solve: 9:04

a $3(n + 2) = 2n + 7$ b $9p + 1 = 7(p + 3)$
c $2(x + 1) = 5(x - 5)$ d $3(2x - 1) = 4(3x - 1)$

9 Solve: 9:05

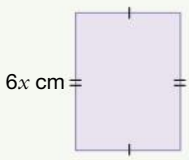
a $x^2 = 16$ b $c^2 = 9$ c $m^2 = 100$ d $b^2 = 3$

10 Form an equation for each number problem, letting the number be n, and solve it. 9:06

a The product of a number and 7 is 63. Find the number.
b The sum of a number and 5 is 2. What is the number?

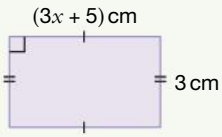
11 Form an equation from the information with each shape, and solve it to find x. 9:06

a



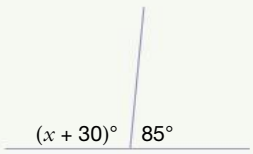
Perimeter = 80 cm

b



Area = 60 cm^2

c



ASSIGNMENT 9A Chapter review

1 Complete these arrow diagrams by filling in the boxes, showing the 'backtracking' steps.

a $5(a-4)$ $\xrightarrow{\div 5}$ $\square$ $\xrightarrow{+4}$ $\square$

b $9y+7$ $\xrightarrow{-7}$ $\square$ $\xrightarrow{\div 9}$ $\square$

c $\frac{a-8}{3}$ $\xrightarrow{\times 3}$ $\square$ $\xrightarrow{+8}$ $\square$

2 Solve these simple one-step equations

a $x+13=21$ **b** $m-5=3$
c $12-n=2$ **d** $t-9=-9$
e $6y=-24$ **f** $5x=12$
g $-8a=-8$ **h** $-3y=10$
i $\frac{x}{2}=6$ **j** $\frac{w}{4}=-8$
k $x-(-3)=6$ **l** $t-(-8)=-2$
m $x^2=36$ **n** $c^2=81$
o $m^2=7$ **p** $b^2=11$

3 Solve these two-step equations.

a $2x+3=5$ **b** $3n+2=8$
c $4n-3=17$ **d** $2k-5=7$
e $9q-4=12$ **f** $2+3w=7$
g $3+9p=3$ **h** $12+5s=26$
i $12-3x=7$ **j** $16-5j=6$
k $20-8g=2$ **l** $14=9v+6$
m $27=3+2y$ **n** $13=4-2a$
o $6-x=-6$ **p** $-5-p=4$
q $\frac{n-3}{5}=4$ **r** $\frac{y+7}{6}=1$
s $\frac{x}{2}-7=5$ **t** $\frac{h}{4}+7=5$

4 Solve these equations that have the pronumeral on both sides of the equation.

a $2x+5=x+7$ **b** $4y-3=2y+5$
c $7w+1=9w-3$ **d** $2+3p=p+8$
e $4-3x=2x+9$ **f** $7k+5=9-3k$

5 Solve:

a $2(x+3)=6$ **b** $3(n+5)=12$
c $4(n-1)=20$ **d** $2(3k-4)=4$
e $5(2q+4)=10$ **f** $2(5+3w)=16$
g $6(x-2)=9$ **h** $3(4x+5)=9$

6 Solve these equations.

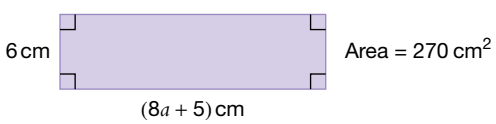
a $2(x+3)=x+8$
b $5a+9=6(a+1)$
c $4(y-3)=2(y+5)$
d $2(1+3p)=5(p+8)$
e $5(2+q)=3(7-3q)$
f $2(4-3x)=3(2x+9)$

7 Write an equation for each problem and solve it. Let the number be x.

a When 5 is added to the product of a number and 7 the answer is 61. Find the number.

b If 9 is subtracted from a number and the result is multiplied by 7 the answer is -21. Find the number.

8 Form an equation and solve it to find a.



ASSIGNMENT 9B Working mathematically

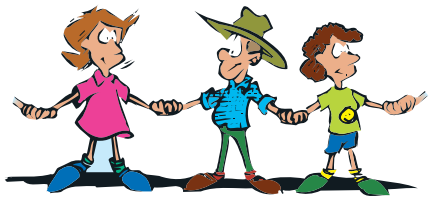
1 I purchased 15 boxes of matches. If each box contains between 47 and 54 matches, choose which of the following answers could be the total number of matches.
A 615 **B** 705 **C** 763 **D** 810

2 One can of food has a mass of 250 g, whereas another has a mass of 450 g. A third can has a mass of 675 g and a fourth can has the second largest mass of these four cans. The total mass of the four cans is:
A 1675 g **B** 1825 g **C** 1826 g **D** 2125 g

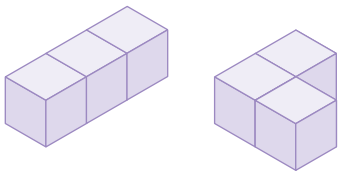
3 To celebrate the birth of a new nation, 3400 people link hands to reach across the city of Sydney. If the space between bodies is 1·3 m and the width of each body is 0·5 m, how far will the people stretch?

4 The total number of votes cast in three electorates was 648 493. If in one of these electorates 249 306 votes were cast and in another 198 486 votes were cast, how many votes were cast in the third electorate?

5 Sophia is 28 years younger than Alec. Roman is one-quarter the age of Sophia. Maria is 18 years older than Roman. If Maria is 28, how old is Alec?



6 When three cubes are joined at their faces they can only form two different solids. These are shown here. How many different solids can be formed by joining four cubes?





Some of the solids mentioned in Question 6 above can be interlocked to form this famous puzzle called the Soma Cube. Find out which ones they are.

ASSIGNMENT 9C Cumulative revision

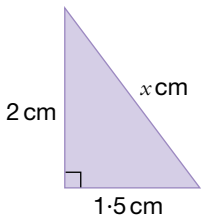
1 Before they played, Sandy had 15 marbles and Luke had 10. Luke lost 5 marbles to Sandy. 4:06

a What percentage of his marbles did Luke lose?

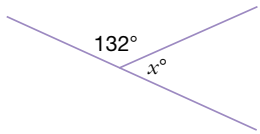
b By what percentage did the number of Sandy's marbles increase?

2 Find the value of x in each case. 3:02, 1:08

a



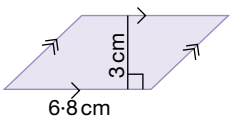
b



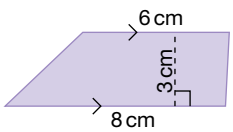
3 The probability that I will be allowed to leave before 10 am is 0·35. What is the probability that I won't be allowed to leave before 10 am? 6:03

4 Calculate the area of each of these shapes. 8:04

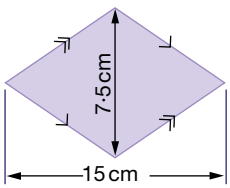
a



b



c



5 **a** Expand $6(x - 3)$. **b** Factorise $12a - 20$. 5:08, 5:09

c Expand $4a(a + 2)$. **d** Factorise $6ab + 12b$.

6 **a** Convert 17 000 t to kg. **b** Change 7800 m to km. 1:11

c Write 800 min in hours and minutes.

7 Simplify: 1:12

a $-3 - 4 - 2$ **b** $12 - 6 - 8$ **c** $-3 \times 2 \times -4$ **d** $(-4 + 6) \times -2$

8 **a** If 40% of a fertiliser is superphosphate, how many kilograms of superphosphate are needed for 1 tonne of fertiliser? 4:06

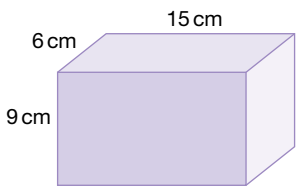
b A real estate agent receives $4\frac{1}{2}\%$ commission on the value of each house that she sells. How much does she receive for the sale of a house valued at \$525 000?

9 **a** What is the time difference from 13:20 on Tuesday to 09:40 on Thursday? 8:08

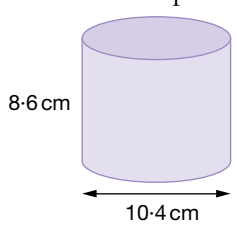
b If I began a journey at 07:25 that took 13 hours and 40 minutes to complete, at what time did I arrive at my destination?

10 Calculate the volume of these solids, correct to two decimal places. 8:06

a

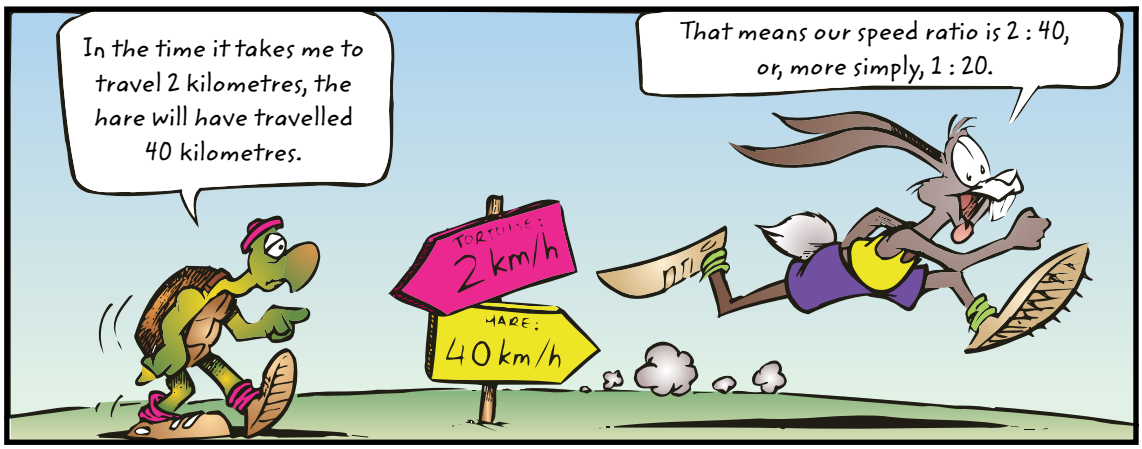


b



RATIOS AND RATES

10



Contents

10:01 Ratios
10:02 Equivalent ratios
Challenge 10:02 How are you at mixing paint?
10:03 Using ratios to solve problems
Investigation 10:03 Using ratios
10:04 Dividing a quantity in a given ratio
Fun spot 10:04 Multiplication can be fun
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10:05 Scale drawing
Investigation 10:05 Scale my room!
Challenge 10:05 Cutting rectangles
10:06 Rates
Fun spot 10:06 Why didn't the piglets listen to their father?
10:07 Distance–time graphs
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

Selections from ***Ratios and Rates*** [Stage 4]

- Recognise and solve problems involving simple ratios. (ACMNA173)
- Solve a range of problems involving ratios and rates, with and without the use of digital technologies. (ACMNA188)
- Investigate, interpret and analyse graphs from authentic data. (ACMNA180)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

10:01 Ratios

Examples of ratio are found in many real-life situations, such as adding water to a juice concentrate to make a fruit drink.

The instructions on this juice packet say: 'Mix 1 part juice with 3 parts water'.





This example illustrates many of the important aspects of ratio.

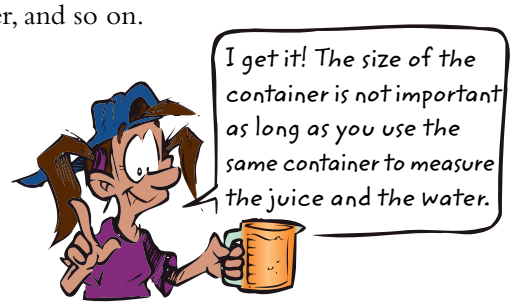
- A ratio is a comparison of two like quantities. In this example we are comparing the **volume** of juice that needs to be mixed with the **volume** of water. The relative sizes of the numbers give the correct sweetness to the drink. Varying the numbers in the ratio will give either a sweeter drink or a watered-down drink.
- The order of the numbers in the comparison is important. The instructions '3 parts juice with 1 part water', are not be the same as '1 part juice with 3 parts water'.
- Notice that there is no mention of units. The word 'parts' is used to indicate that whatever units are used to measure the juice must also be used to measure the water. The size of the unit or measure is not important.
- 1 L of juice would need 3 L of water
- 500 mL of juice would need 3 × 500 mL of water
- 100 mL of juice would need 3 × 100 mL of water
- 1 bucket of juice would need 3 × 1 buckets of water, and so on.

To write this ratio down we could write:

- ratio of juice to water = 1 : 3
- ratio of water to juice = 3 : 1

Alternatively we could write it in fraction form:

- ratio of juice to water = $\frac{1}{3}$
- ratio of water to juice = $\frac{3}{1}$



A *ratio* is a comparison of numbers in a definite order. The numbers are expressed in the same units and are called the *terms* of the ratio.

The ratio can be written in the form $a : b$ or $\frac{a}{b}$ or *a to b*.

WORKED EXAMPLES

1 Jill says to Jack: 'My fish is six times as long as your fish'.
Jill is comparing the length of her fish to the length of Jack's fish.

a Write down the ratio of:

i the length of Jill's fish to the length of Jack's fish

ii the length of Jack's fish to the length of Jill's fish.

b Does Jill's statement tell us the actual length of either fish?



Solutions

a If Jack's fish is 1 unit long, then Jill's fish is 6 units long.
So the ratio of:

i the length of Jill's fish to the length of Jack's fish is 6 : 1

ii the length of Jack's fish to the length of Jill's fish is 1 : 6.

b Jill's statement does not tell us anything about the actual lengths of the two fish.
All it says is that whatever the length of Jack's fish, her fish is six times longer.
If we know the length of either fish we could work out the length of the other.

2 Bronte buys a guinea pig for $2 and later sells it for $3. Find the ratio of her:

a selling price to cost price

b selling price to profit

c cost price to profit

d cost price to profit to selling price.

Solutions

Now, selling price = cost price + profit.

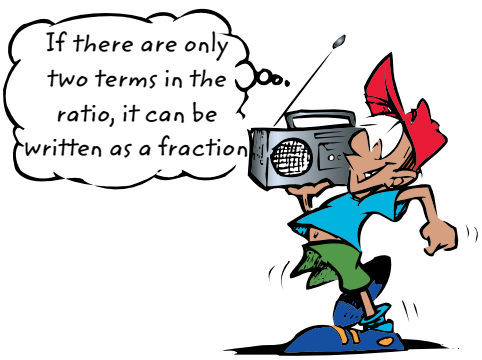
∴ cost price = $2
profit = $1
selling price = $3

a selling price : cost price = 3 : 2 or $\frac{3}{2}$

b selling price : profit = 3 : 1 or $\frac{3}{1}$

c cost price : profit = 2 : 1 or $\frac{2}{1}$

d cost price : profit : selling price = 2 : 1 : 3



We can use ratios to compare more than two numbers.
e.g. I earned $\$A$ in week 1, $\$B$ in week 2 and $\$C$ in week 3.
Ratio of earnings of week 1 : week 2 : week 3 = $A : B : C$

Exercise 10:01

1 Liam has four times as much money as Bronwyn.

a What is the ratio of:

i Liam's amount of money to Bronwyn's amount of money

ii Bronwyn's amount of money to Liam's amount of money?

b Do we know the amount of money that Liam or Bronwyn actually has?

2 'I can lift twice as much as you', says Milla to Dominic.

a Do we actually know how much either person can lift?

b What is the ratio of:

i the amount that Milla can lift to the amount that Dominic can lift

ii the amount that Dominic can lift to the amount that Milla can lift?

3 The ratio of cement to sand in a mortar mix is 1 : 4 (1 part cement to 4 parts sand).

a What fraction of the mortar mix is cement? b What fraction of the mortar mix is sand?

4 A particular paint colour, 'Sunset', is made by mixing red and yellow paints.

a If $\frac{3}{5}$ of 'Sunset' is red, what fraction of 'Sunset' is yellow?

b What is the ratio of red paint to yellow paint in 'Sunset'?

5 Kylie buys chickens for $3 and sells them for $5. Find the ratio of:

a selling price to cost price

b selling price to profit

c cost price to profit

d profit to selling price

e cost price to profit to selling price.

6 A class contains 14 boys and 16 girls.

a What is the ratio of boys to girls?

b What is the ratio of girls to boys?

c What is the ratio of boys to the total number of students in the class?

d What fraction of the class is boys?

e What is the ratio of girls to the total number of students in the class?

f What fraction of the class is girls?

7 A breeder of budgerigars notes that the ratio of yellow birds : blue birds : other coloured birds is 5 : 3 : 2.

a What colour is the most common?

b What colour is the least common?

c What fraction of the birds is yellow?

d What fraction of the birds is blue?

e What is the ratio of blue birds to the total number of birds?



Ratios can have more than two terms.

8 Concrete mix is made by mixing cement, sand and gravel in the ratio 1 : 2 : 4.

a What fraction of concrete mix is cement? b What fraction of concrete mix is gravel?

c What is the ratio of cement to gravel? d What is the ratio of sand to gravel?

e If we use 1 kg of cement, how much gravel should we use?

9 What is the ratio of:

a a centimetre to a millimetre

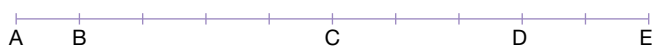
b a metre to a centimetre

c a kilogram to a gram

d a litre to a millilitre

e a minute to a second?

10 The line below is divided into ten equal parts.



Find the ratios:

a $\frac{AB}{AE}$ b $\frac{AB}{BC}$ c $\frac{AE}{AB}$ d $\frac{CD}{CB}$ e $\frac{CD}{AE}$

11 One-third of a class is girls.

a What fraction of the class is boys?

b What is the ratio of boys to girls?

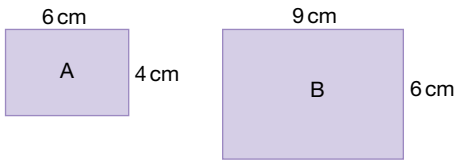
c What is the ratio of girls to boys?

12 What is the ratio of:

a length of rectangle A to length of rectangle B

b width of rectangle A to width of rectangle B

c area of rectangle A to area of rectangle B?



13 The correct sweetness for a drink is obtained by mixing cordial and water in the ratio of 1 : 3.

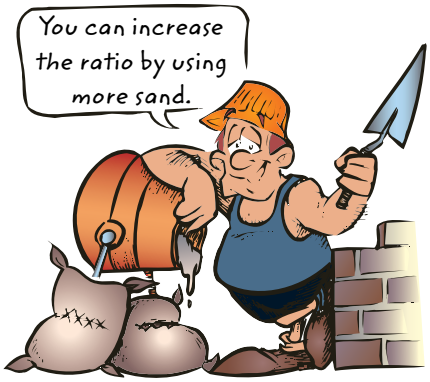
a If the ratio of cordial to water were 1 : 4, would the drink be sweeter than if it were mixed properly?

b Bill mixes water and cordial in the ratio 2 : 1. Would his drink be sweeter or watered down compared to the proper mixture?



14 a A pink-coloured paint is made by mixing red and white in the ratio of 1 : 5. If the red and white paints were mixed in the ratio 1 : 8, would the paint be lighter or darker than the original?

b Mortar mix is made by mixing sand and cement in the ratio 5 : 2. Stronger mortar is made by increasing the fraction of cement in the mixture. If the ratio of sand to cement in a mix were 7 : 2, would this be stronger or weaker than the mix mentioned above?



10:02 Equivalent ratios

In comparing two quantities we often do not know the actual size of either. For example, the statement 'My fish is twice as long as yours' only gives us the **relative** sizes of the two fish. There are an infinite number of actual ratios that could describe this situation, such as:

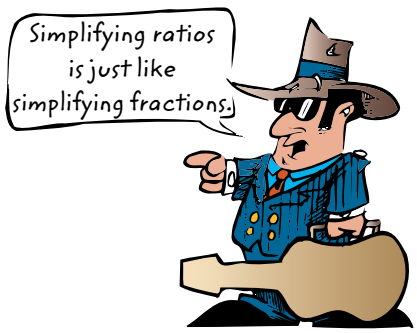
a 16 cm : 8 cm **b** 10 cm : 5 cm **c** 2 m : 1 m



In all of these, the first term is twice the second term, and the simplest ratio to express this relationship is 2 : 1. In this way, the set of ratios behaves like the set of equivalent fractions $\frac{16}{8} = \frac{10}{5} = \frac{2}{1}$, so we can simplify ratios in the same way that we simplify fractions.

If the terms of a ratio are multiplied or divided by any number (except zero), an *equivalent ratio* is formed.

WORKED EXAMPLES



1 Are the ratios 3 : 1 and 12 : 10 equivalent?

2 Simplify these ratios:

a 15 : 12 **b** 3·5 : 1·5 **c** $3 : \frac{1}{2}$

d $2\frac{1}{3} : 1$ **e** 1 m : 20 cm

3 A school has 650 girls and 500 boys.
What is the ratio of boys to girls?

4 Express 7 : 5 in the form of $a : 1$ by dividing both terms by the second term.

5 Express 6 : 8 in the form $1 : b$ by dividing both terms by the first term.

Solutions

1 3 : 1 and 12 : 10

$\frac{3}{1} = \frac{3 \times 4}{1 \times 4} = \frac{12}{4} \neq \frac{12}{10}$

$\therefore$ 3 : 1 and 12 : 10 are not equivalent.

2 **a** 15 : 12

$= \frac{15}{3} : \frac{12}{3}$

$= 5 : 4$

b 3·5 : 1·5

$= \frac{3{\cdot}5 \times 10}{1{\cdot}5 \times 10}$

$= \frac{35}{15}$

$= \frac{7}{3}$

$= 7 : 3$

c $3 : \frac{1}{2}$

$= 3 \times 2 : \frac{1}{2} \times 2$

$= 6 : 1$

d $2\frac{1}{3} : 1$

$= \frac{7}{3} \times 3 : \frac{3}{3} \times 3$

$= 7 : 3$

e 1 m : 20 cm

change to the same units

= 100 cm : 20 cm

$= \frac{100}{20} : \frac{20}{20}$

$= 5 : 1$

3 boys : girls = 500 : 650

$= \frac{500}{50} : \frac{650}{50}$

= 10 : 13

4 7 : 5

$= \frac{7}{5} : \frac{5}{5}$

= 1·4 : 1

5 6 : 8

$= \frac{6}{6} : \frac{8}{6}$

$= 1 : 1\frac{1}{3}$

Exercise 10:02

1 Copy and complete the following.

a 2 : 1 = 6 : ☐ **b** 15 : 1 = 30 : ☐ **c** 1 : 4 = 20 : ☐ **d** 2 : 3 = 6 : ☐

e 9 : 5 = ☐ : 15 **f** 3 : 13 = ☐ : 52 **g** 3 : 4 = ☐ : 12 **h** 6 : 5 = ☐ : 20

i 6 : 4 = 3 : ☐ **j** 9 : 3 = 3 : ☐ **k** 12 : 8 = 3 : ☐ **l** 20 : 6 = 10 : ☐

m 10 : 15 = ☐ : 3 **n** 24 : 36 = ☐ : 3 **o** 48 : 100 = ☐ : 20 **p** 35 : 120 = ☐ : 24

2 Simplify the following ratios.

a 15 : 5 **b** 10 : 30 **c** 24 : 16 **d** 16 : 30 **e** 25 : 75

f 120 : 8 **g** 100 : 10 **h** 100 : 1000 **i** 66 : 11 **j** 12 : 72

k 8 : 20 **l** 320 : 48 **m** 8 : 96 **n** 14 : 49 **o** 30 : 42

p 15 : 200 **q** 27 : 18 **r** 40 : 72 **s** $\frac{5}{15}$ **t** $\frac{20}{5}$

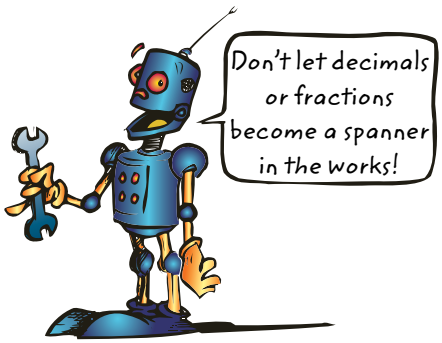
u $\frac{16}{40}$ **v** $\frac{100}{45}$ **w** $\frac{24}{36}$ **x** $\frac{42}{12}$

3 Simplify the following ratios after first changing to the same units.

a 3 cm : 1 mm **b** 5 cm : 20 mm **c** 1 m : 50 cm **d** 25 m : 1 km
e 1·5 km : 200 m **f** 3·5 L : 500 mL **g** $1\frac{1}{2}$ h : 30 min **h** 200 kg : 1 t
i $1\frac{1}{4}$ min : 15 s **j** 2 days : 6 h **k** \$5 : 50c : \$5.50

4 Simplify the following ratios.

a 1·2 : 1 **b** 3·6 : 1·2 **c** 4·5 : 6
d 6·25 : 1 **e** 1·2 : 6 **f** $\frac{1}{2} : \frac{1}{10}$
g $\frac{1}{10} : \frac{1}{5}$ **h** $\frac{2}{3} : 1$ **i** $1 : \frac{3}{4}$
j 20 : 70 : 50 **k** 8 : 4 : 12 **l** 33 : 9 : 12
m 100 : 65 : 35



5 **a** In 8M there are 16 boys and 14 girls.
What is the ratio of:
i boys to girls **ii** girls to boys
iii boys to the total number in the class
iv boys to girls to the total number in the class?

b Mitsuya sells a pot for \$60 and makes a profit of \$20. Find the following ratios.
i profit : cost price **ii** selling price : cost price
iii profit : selling price **iv** cost : profit : selling price

c On a scale drawing 1 cm represents 1 m. Write this as a ratio.

d A store has a discount sale. A suit marked at \$250 is sold for \$200.
i What is the ratio of the discount to the marked price?
ii What is the ratio of the marked price to the discount to the selling price?

A discount is a reduction in price.

e Two business partners, Clancy and Skye, invest \$4500 and \$2500 each in a business. What is the ratio of Clancy's share to Skye's share?

f In an election, Bagnell scored 225 votes, Farrant 175 votes and McGregor 125 votes. Find the ratio of: Bagnell's votes : Farrant's votes : McGregor's votes. (Express this ratio in its simplest form.)

g An alloy is made by combining 2·5 kg of metal A with 1·5 kg of metal B and 2 kg of metal C. Find the ratio of the weights of:
i metal A to metal B **ii** metal B to metal C
iii metal A to metal C **iv** metal A to the alloy
v metal A to metal B to metal C.

Note:
2·5 : 1·5 : 2
Multiply by 10
= 25 : 15 : 20

h On a map, a distance of 1·5 cm represents 5 km. What is the ratio of the distance on the map to the real distance?

6 Express each of the following ratios in the form $a : 1$ by dividing both terms by the second term.

a 16 : 5 **b** 12 : 8 **c** 9 : 12 **d** 1·6 : 1·2 **e** 0·2 : 0·8

7 Express each of the following ratios in the form $1 : b$ by dividing both terms by the first term.

a 20 : 12 **b** 16 : 8 **c** 8 : 15 **d** 1·5 : 6 **e** 2·4 : 8·4

CHALLENGE 10:02 HOW ARE YOU AT MIXING PAINT?

Green paint is mixed with white paint to make a light green colour. The more white you use, the lighter the green.



A B

- If the paints in each picture were mixed, which mixture would give the darker green? Give a reason for your choice.
- Which of the reasons below are correct?

a Batch **B** is a darker green because more green paint is used.

b They are the same because you get **B** if you add one tin of green paint and one tin of white to batch **A**.

c In **A**, for every green tin there are $1\frac{1}{2}$ white tins. In **B**, for every green tin there are $1\frac{1}{3}$ white tins. There is less white in **B** so it's a darker green.

d $\frac{2}{5}$ of batch A is green. $\frac{3}{7}$ of batch **B** is green. $\frac{3}{7}$ is greater than $\frac{2}{5}$ so **B** is the darker green.

e To batch **A** add half as much paint again. We then have 3 green and $4\frac{1}{2}$ white. In batch **B** we have 3 green and 4 white so **B** gives a darker green.

f There are 4 white tins in Batch **B** and only 3 in batch **A** so batch **A** gives the darker green.

g If I took 3 lots of batch **A** and 2 lots of batch **B** both lots would have 6 tins of green paint. The 3 lots of batch **A** would have 9 tins of white paint, whereas the 2 lots of batch **B** would have only 8, so batch **B** would give a darker green.

10:03 Using ratios to solve problems

PREP QUIZ 10:03

'My fish is three times longer than yours.'
Find the length of the bigger fish if the smaller fish is:

1 10 cm long **2** 30 cm long **3** 1 m long.

Find the length of the smaller fish if the bigger fish is:

4 30 cm long **5** 1·2 m long.

Complete the following:

6 $\frac{3}{4} = \frac{\square}{20}$ **7** $\frac{15}{8} = \frac{\square}{24}$ **8** $\frac{9}{16} = \frac{27}{\square}$

Solve the equations:

9 $\frac{x}{8} = \frac{5}{4}$ **10** $\frac{x}{5} = \frac{4}{3}$

- We have seen that statements such as 'My fish is three times longer than yours' do not give the actual size of either fish. However, once the length of either fish is known, the length of the other can be calculated.
- In many problems we are told the ratio of two quantities. We are then given one quantity and asked to calculate the other.

WORKED EXAMPLES

1 Lara mixes sand and cement in the ratio 5 : 2 by volume. If Lara uses five buckets of cement, how much sand should she use?



Solution

Method 1 (the unitary method)

2 buckets of cement to 5 buckets of sand

1 bucket of cement to $\frac{5}{2}$ buckets of sand

5 buckets of cement to $\frac{5}{2} \times 5$ buckets of sand

$\therefore$ 5 buckets of cement needs 12·5 buckets of sand.

Method 2 (the ratio method)

Let the number of buckets of sand be x.

$$\frac{\text{buckets of sand}}{\text{buckets of cement}} = \frac{5}{2}$$

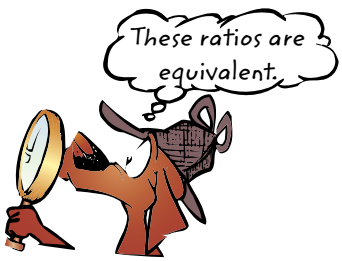
$$\frac{x}{5} = \frac{5}{2}$$

$$x = \frac{5}{2} \times 5$$

$$= 12.5$$

$\therefore$ 5 buckets of cement needs 12·5 buckets of sand.

$$2:5$$
$$= 2 \text{ parts} : 5 \text{ parts}$$
$$= 1 \text{ part} : \tfrac{5}{2} \text{ parts}$$
$$= 5 \text{ parts} : \tfrac{25}{2} \text{ parts}$$



2 Two partners, Ashlee and Brody, decide to divide the profits from a business venture in the same ratio as their investments. If Ashlee invests $9000 and Brody invests $5000, how much should Brody receive if Ashlee receives $3600?

Solution

Method 1

Ashlee's share : Brody's share = 9 : 5

9 shares earn $3600

1 share earns $3600 ÷ 9 = $400

5 shares earn $400 × 5 = $2000

$\therefore$ Brody receives $2000.

Method 2

Let Brody's share be $\$x$.

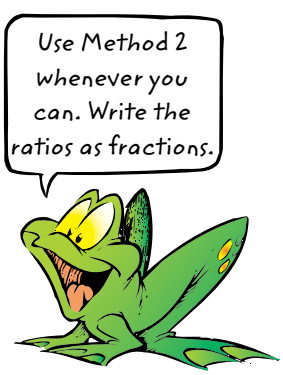
$$\frac{\text{Brody's share}}{\text{Ashlee's share}} = \frac{5}{9}$$

$$\frac{x}{3600} = \frac{5}{9}$$

$$x = \frac{5}{9} \times 3600$$

$$= \$2000$$

Brody receives $2000.



Exercise 10:03

P Foundation worksheet 10:03
Ratio problems

1 Solve the following equations.

a $\frac{x}{5} = \frac{7}{4}$ **b** $\frac{x}{3} = \frac{7}{2}$ **c** $\frac{x}{5} = \frac{5}{8}$ **d** $\frac{x}{4} = \frac{5{\cdot}6}{3}$

e $y : 4 = 7 : 2$ **f** $m : 10 = 4 : 12$ **g** $3 : 2 = 7 : a$ **h** $3 : b = 4 : 5$

2 **a** The ratio of boys to girls in the room is 2 : 1. If there are 8 girls, how many boys are there?

$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{2}{1}$

b Aki and Rachel share some money in the ratio 5 : 3. If Rachel receives $204, how much does Aki receive?

$\frac{\text{Aki's share}}{\text{Rachel's share}} = \frac{5}{3}$

c The ratio of children to adults on a bushwalk is 10 : 3. If there are 12 adults, how many children are there?

d Liam and Bronte divided some money in the ratio 2 : 3. If Liam has $5.40, how much does Bronte have? How much money was divided?

e A mortar mix is made up of sand and cement in the ratio 4 : 1. If 12 parts of sand are used, how many parts of cement are needed?

f A two-stroke fuel mixture is made by mixing petrol and oil in the ratio of 25 : 1. How many litres of petrol need to be added to 200 mL of oil to make this mixture?

3 **a** The ratio of a boy's mass to that of his father is $\frac{3}{8}$. If the boy weighs 27 kg, what does his father weigh?

b A type of solder is made by mixing lead and tin in the ratio of 2 : 3.

i How much lead would need to be mixed with 5·4 kg of tin to make the solder?

ii How much tin would need to be mixed with 285 g of lead to make the solder?

c A model aeroplane is built to a scale of 1 : 80. If the wingspan of the model is 85 cm, what is the wingspan of the aeroplane?

d A kindergarten works on the basis of two teachers for every 19 children. How many teachers would be needed for 38 children?

e The ratio of the selling price of an item to the cost price is 11 : 10. Find the cost price if the selling price is $59.51.

4 **a** The scale on a map is 1 : 25 000. What actual distance would be represented by a length of 3·5 cm on the map?

b A concrete mix is made by mixing gravel, sand and cement in the ratio of 5 : 4 : 2. How much gravel and sand would be added to 5 parts of cement?

c An alloy is made by mixing copper, tin and zinc in the ratio of 10 : 12 : 9. If only 28 kg of copper is used, how much tin and zinc is needed to make the alloy?

d A sum of money was divided among three people in the ratio 8 : 3 : 5. If the smallest share was $720, what were the other shares and how much money was there to be divided?

5 9-carat gold is a mixture of 3 parts gold to 5 parts other metals (to add hardness).

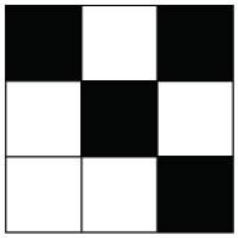
a What is the ratio of gold to other metals?

b If 24 g of gold is used to make 9-carat gold, how much of the other metal is used?

c If 60 g of other metals was used, how much gold would be needed?

INVESTIGATION 10:03 USING RATIOS

1 Square tiles of length 20 cm are arranged in the pattern on the right. This square of nine tiles, some black and some white, is repeated to cover a rectangular floor of length 6 m and breadth 3 m.



a How many tiles are needed to cover the floor?

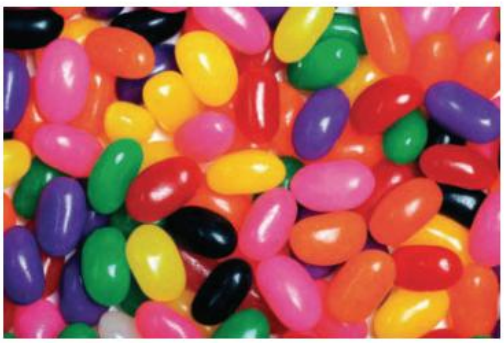
b How many black tiles are needed?

c How many of each tile needs to be ordered, allowing for 10% breakages?

d What would be the total cost if each black tile costs $1.85 and each white tile costs $2.40?

2 Investigate which measurements for rectangular floors would allow this pattern to fit exactly on the floor (with only complete patterns used).

3 Investigate the various gears of a 10-speed racing bicycle.



Estimate the ratio of lollies in the photograph on the left.

- red lollies : green lollies
- red lollies : green lollies : black lollies

10:04 Dividing a quantity in a given ratio

This type of ratio problem involves dividing or sharing a quantity into a number of parts, using a given ratio.



Discussion

Ravi and Jane put their money together to buy a car. Ravi has $3000 and Jane has $5000. Two years later they sell the car for $5600.

- If only the money invested is considered, how much should each receive from the sale?
- What other information do you think could influence the size of the shares in the car when it is sold? Discuss with others in your class.

WORKED EXAMPLE 1

Project Elephant in India was introduced to stop the poaching of bull elephants for tusks. As a result, the ratio of males to females, in some areas, has increased from 1 : 100 back to the normal ratio of 1 : 12. If there are 585 elephants in a wildlife reserve, how many males and females would be present if the ratio was back to normal?

Solutions

Method 1

There are 585 elephants.
Ratio of males : females = 1 : 12
The total number of parts is 13.
Each part = $\frac{585}{13} = 45$
$\therefore$ Number of males = 45
$\therefore$ Number of females = 12×45
$= 540$

Method 2

There are 585 elephants.
Males = $\frac{1}{13}$ of population
Females = $\frac{12}{13}$ of population
$\therefore$ Number of males = $\frac{1}{13} \times 585$
$= 45$
$\therefore$ Number of females = $\frac{12}{13} \times 585$
$= 540$

WORKED EXAMPLE 2

Cliff and Richard divide $1890 between them in the ratio 5 : 4. How much does each receive?

Solutions

Method 1

$1890 is to be divided into 9 shares.
Cliff takes 5 shares.
Richard will take 4 shares.
9 shares = $1890
$\frac{1890}{9} = 210$
$\therefore$ 1 share = $210
$210 \times 4 = $840
$\therefore$ 4 shares = $840
$210 \times 5 = $1050
$\therefore$ 5 shares = $1050
Cliff receives $1050 and Richard receives $840.

Method 2

$1890 is to be divided into 9 shares.
Cliff takes $\frac{5}{9}$ shares.
Richard takes $\frac{4}{9}$ shares.
Cliff's share = $\frac{5}{9} \times 1890
$= 1050
Richard's share = $\frac{4}{9} \times 1890
$= 840
Cliff receives $1050 and Richard receives $840.

Exercise 10:04

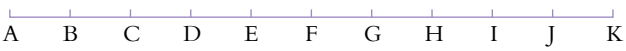
1 **a** Divide 16 in the ratio 5 : 3.
b Divide 200 in the ratio 1 : 4.
c Divide 420 in the ratio 7 : 5.
d Divide 1510 in the ratio 3 : 2.
e Divide 585 in the ratio 1 : 3 : 5.
f Divide 41 400 in the ratio 6 : 4 : 5.

2 **a** Pewter is made by mixing tin and lead in the ratio 4 : 1. How much of each metal would there be in 20 kg of pewter?



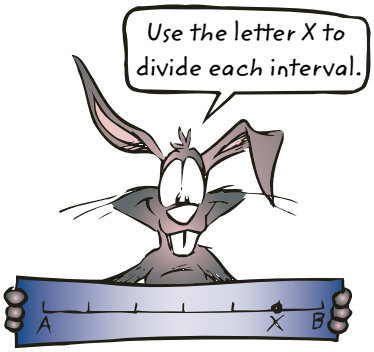
b In a town the ratio of adults to children is 2 : 5. How many children are there in the town if the population is 8400?
c A tin of two-stroke fuel mixture holds 3·9 L. If the mixture is made from petrol and oil in the ratio 25 : 1, find the amount of oil in the mixture.
d A fruit drink is made by mixing juice and water in the ratio 3 : 1. How much juice would be needed to make 1200 mL of drink?
e A brand of fertiliser is made by mixing nitrates, potash and phosphates in the ratio 2 : 3 : 3. How many kilograms of each are in a 50 kg bag of fertiliser?
f A certain range of houses are made using three different types of bricks in the ratio 4 : 1 : 3. If 28 000 bricks are needed to build a house, how many of each type will be needed?
g A concrete mix is made by mixing gravel, sand and cement in the ratio 6 : 4 : 1 by mass. Find the mass of each required to make 66 kg of the cement mix.
h The lengths of the sides of a triangle are in the ratio 1 : 2 : 3. Find the lengths of the sides if the triangle has a perimeter of 42·6 cm.

3 **a**



Which point divides the interval AK in the ratio:
i 9 : 1 **ii** 7 : 3 **iii** 3 : 7
iv 4 : 1 **v** 1 : 4?

b Draw an interval *AB*, 12 cm long, and divide it into 6 equal parts. Then find the point that divides the line in the ratio 5 : 1.
c Draw an interval 8 cm long and divide it in the ratio 5 : 3.
d Draw an interval 6 cm long and divide it in the ratio 1 : 4.



4 **a** A rectangle has a perimeter of 64 cm. If the ratio of its length to its breadth is 5 : 3, find the length of the rectangle.
b The ratio of the populations of town A and town B is 3 : 8, whereas the ratio of the populations of towns B and C is 5 : 2. If the total population of the three towns is 85 342, find the population of each town.
c A saline solution is diluted with water in the ratio 1 : 24. This solution is then taken and mixed with water in the ratio 1 : 3. How much of the original saline solution would there be in 200 mL of this final solution?

FUN SPOT 10:04 MULTIPLICATION CAN BE FUN

Long multiplication sums can be a pain. But have you tried doing them like this?
For a sum like 58×25, follow the steps given below.

Numbers in this column are doubled.

58	×	25
~~116~~		~~12~~
~~232~~		~~6~~
464		3
928		1
1450		

Numbers in this column are halved, ignoring any remainder. If the number in this column is even, the row is crossed out.

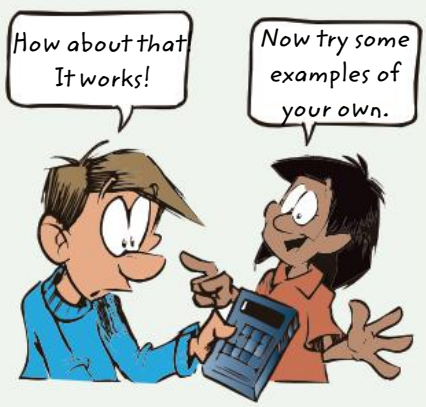
To find the answer, add up the remaining numbers in this column.

$58 \times 25 = 1450$

Here's another example for you to follow:

35	×	21
~~70~~		~~10~~
140		5
~~280~~		~~2~~
560		1
735		

$\therefore 35 \times 21 = 735$



Now try some questions of your own. Check your answers with a calculator.

INVESTIGATION 10:04 RATIO RESEARCH

1 Collect information from containers of products that need to be mixed. In each case write the instructions as a ratio.

2 Make a list of cases you discovered where different units are used in the same ratio, such as three teabags to one teapot of boiling water.

3 Make a list of appliances and machines used for mixing ingredients in a given ratio, such as a bread maker.

4 Discuss your findings with others in the class.



10:05 Scale drawing

PREP QUIZ 10:05



Complete the following:

1 1 m = … cm **2** 1 cm = … mm **3** 1 m = … mm

Convert:

4 2000 cm to m **5** 250 000 mm to m

Convert these to simple number ratios, without units:

6 1 cm : 1 m **7** 1 mm : 1 m

Complete the following:

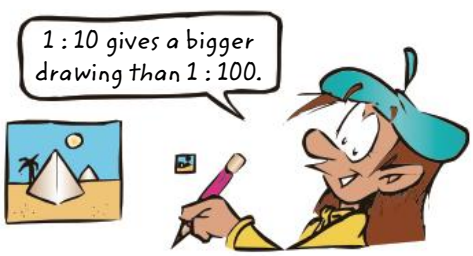
8 1 : 100 = 5 : ☐ **9** 1 : 1000 = 6·8 : ☐ **10** 1 cm : 1 m = 5 cm : ☐

Scale drawings are used often in everyday life. People such as architects and engineers make scale drawings of buildings and bridges, and people such as builders must be able to read them so that these things can be built.

- Scale drawings have the same shapes as the objects that they represent but they are of different sizes.
- The scale of a drawing is given as length on drawing : real length. So a scale of 1 : 100 means that the real length is 100 times the length on the drawing.
- The scale determines the size of the drawing. The closer the size of terms in the scale ratio, the closer the drawing is to real size. So a 1 : 10 scale drawing would be bigger than a 1 : 100 scale drawing of the same object.

There are two types of problems that involve scale drawings.

- Making a scale drawing of an object.
- Calculating the real size of an object.



A scale drawing of an object is the same shape as the object but a different size.

Scale is length on drawing : real length

A scale can be written in two ways (e.g. 1 cm : 1 m or 1 : 100).

If the scale drawing is larger than the real length, then the scale drawing is called an *enlargement* and the first term of the scale would be the larger (e.g. 5 : 1).

WORKED EXAMPLES

1 Write these scales without units.

a 1 mm : 1 m **b** 1 cm : 2 m **c** 5 mm : 1 km

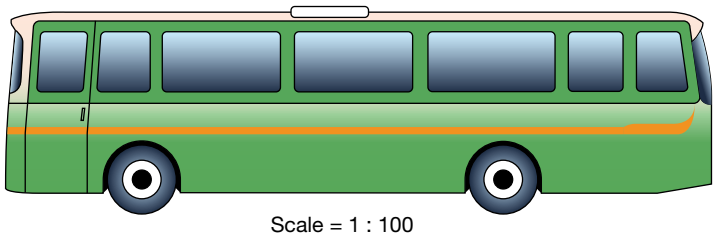
2 **a** The scale on a drawing is 1 : 100. What is the real distance between two points that are 5 cm apart on the drawing?

b Two trees are 50 m apart. How far apart would they be on a drawing with a scale of 1 : 500?

3 From the drawing, by measurement and calculation, find:

a the maximum length of the bus

b the maximum height of the bus.



Scale = 1 : 100

Solutions

1 **a** 1 mm : 1 m
= 1 mm : 1000 mm
= 1 : 1000

b 1 cm : 2 m
= 1 cm : 200 cm
= 1 : 200

c 5 mm : 1 km
= 5 mm : 1000 m
= 5 mm : 1 000 000 mm
= 1 : 200 000

2 **a** Scale = 1 : 100

$\therefore \frac{1}{100} = \frac{5 \text{ cm}}{\text{real distance}}$

$\therefore$ Real distance = 5 × 100 cm
= 500 cm
= 5 m

b Scale = 1 : 500

$\therefore \frac{1}{500} = \frac{\text{distance on drawing}}{50 \text{ m}}$

$\therefore$ 500 × dist. on drawing = 50 m

$\therefore$ Distance on drawing $= \frac{50 \text{ m}}{500}$

$= \frac{1}{10}$ m

= 10 cm

3 **a** Length of drawing = 93 mm

$\therefore$ Real length = 100 × 9·3 cm
= 930 cm
= 9·3 m

b Height on drawing = 27 mm

$\therefore$ Real height = 100 × 2·7 cm
= 270 cm
= 2·7 m



The accuracy of answers in scale drawing work is determined by the scale and the limitations of our measuring instruments.

Exercise 10:05

P Foundation worksheet 10:05
Scale drawing

1 Copy and complete each of the following.

a 1 : 100 = 1 cm : … **b** 1 : 1000 = 1 cm : … **c** 1 : 1000 = 1 m : …

2 Write each of the following scales in ratio form.

a 1 cm to 1 m **b** 1 cm to 10 m **c** 1 mm to 10 m **d** 1 mm to 5 m
e 2 cm to 1 m **f** 5 mm to 1 m **g** 5 cm to 1 m **h** 5 cm : 1 km

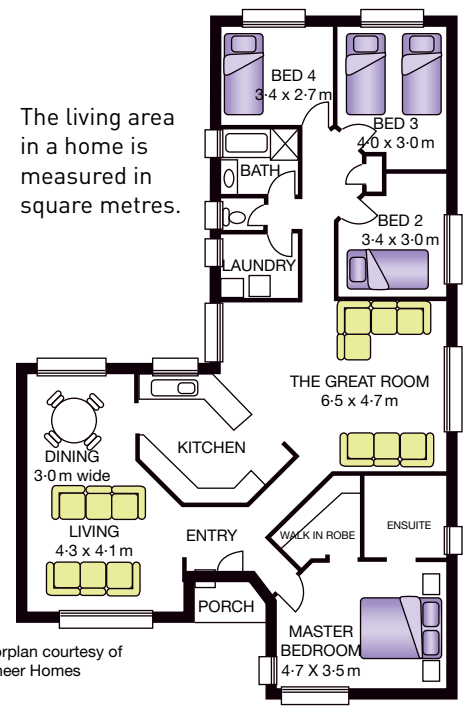
3 Calculate the real distance between points that are the following distances apart on a map, if the scale is 1 : 1000.

a 5 cm **b** 3·5 cm **c** 10·6 cm **d** 8·1 cm
e 1 mm **f** 8 mm **g** 26 mm **h** 0·5 mm

4 The scale used in this house plan is 1 : 200.

a What real distance is represented by 1 cm?
b Find the real length of:
  i the bathroom
  ii bedroom 3
  iii the house
  iv the inner kitchen wall.
c Find the real area of bedroom 3.
d Find the inside area of the house to the nearest square metre.
e Find the area of the great room.
f How much would it cost to tile the floor of the great room at $38.50 per square metre?

The living area in a home is measured in square metres.



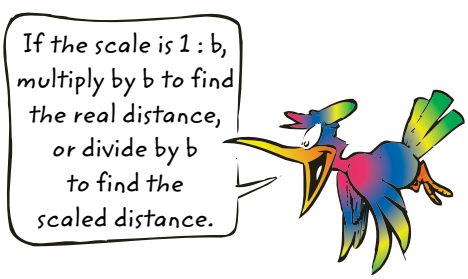
Floorplan courtesy of Pioneer Homes

5 The scale on a drawing is 1 cm to 5 m. Calculate the real distance between two points if they are the following distances apart on the drawing.

a 2 cm **b** 5 cm **c** 6·4 cm **d** 4·2 cm
e 1 mm **f** 42 mm **g** 27 mm **h** 59 mm

6 A scale drawing is to be made using a scale of 1 : 100. What lengths on the drawing would represent the following real distances?

a 100 cm **b** 300 cm
c 750 cm **d** 4·3 m
e 63 m **f** 42 m
g 15·6 m **h** 2·4 m



7 What lengths would represent the following real distances on a scale drawing with a scale of 1 cm : 20 m?

a 40 m	**b** 60 m	**c** 200 m	**d** 30 m
e 8 m	**f** 25 m	**g** 11 m	**h** 315 m

8 **a** The plan of a house is drawn to a scale of 1 : 100. If a room measures 36 mm by 42 mm on the plan, how big is the room in real life?

b A model aeroplane is built to a scale of 1 : 72. What would the real wingspan be if the wingspan of the model is 30 cm?

c A map has a scale of 1 : 250 000. What real distance would be represented by a scaled distance of:

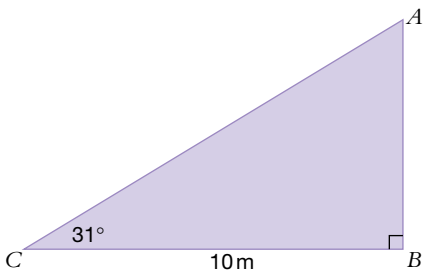
i 1 mm **ii** 35 mm **iii** 1 cm **iv** 5·2 cm?

d A girl made this scale drawing to calculate the height, AB, of a triangular wall.

i Measure the distance, CB, on the scale drawing and calculate the scale of the drawing.

ii By measurement and calculation, find the height, AB.

iii Find the area of this triangular wall.



9 **a** An oval is 85 m long and 58 m wide. What would these lengths be on a scale drawing if the scale was:

i 1 cm : 10 m **ii** 1 cm : 5 m **iii** 1 : 200?

b A house fits inside a rectangle 23 300 mm long and 12 200 mm wide. What is the smallest size of paper on which a scale drawing could be made if the scale is 1 : 100?

c A 1 : 50 scale model of a yacht is to be built. If the yacht is 25 m long, how long will the model yacht be?

d A scale drawing is to be made of a car. If the car is 6·25 m long and the model is to be 25 cm long, what should the scale be?

e A kitchen design company is asked to design a kitchen for a new house. The dimensions of the kitchen are 5200 mm by 3800 mm. What would the scaled size of the kitchen be if the scale is 1 : 100? Calculate the scaled size of the refrigerator (real size 600 mm by 600 mm) and the sink (real size 1200 mm by 500 mm).

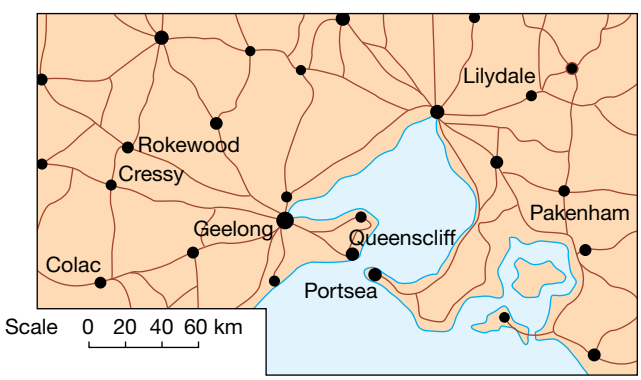
10 Use the scale on the map to find the distance by road from:

a Geelong to Cressy

b Portsea to Queenscliff

c Lilydale to Colac

d Rokewood to Pakenham.

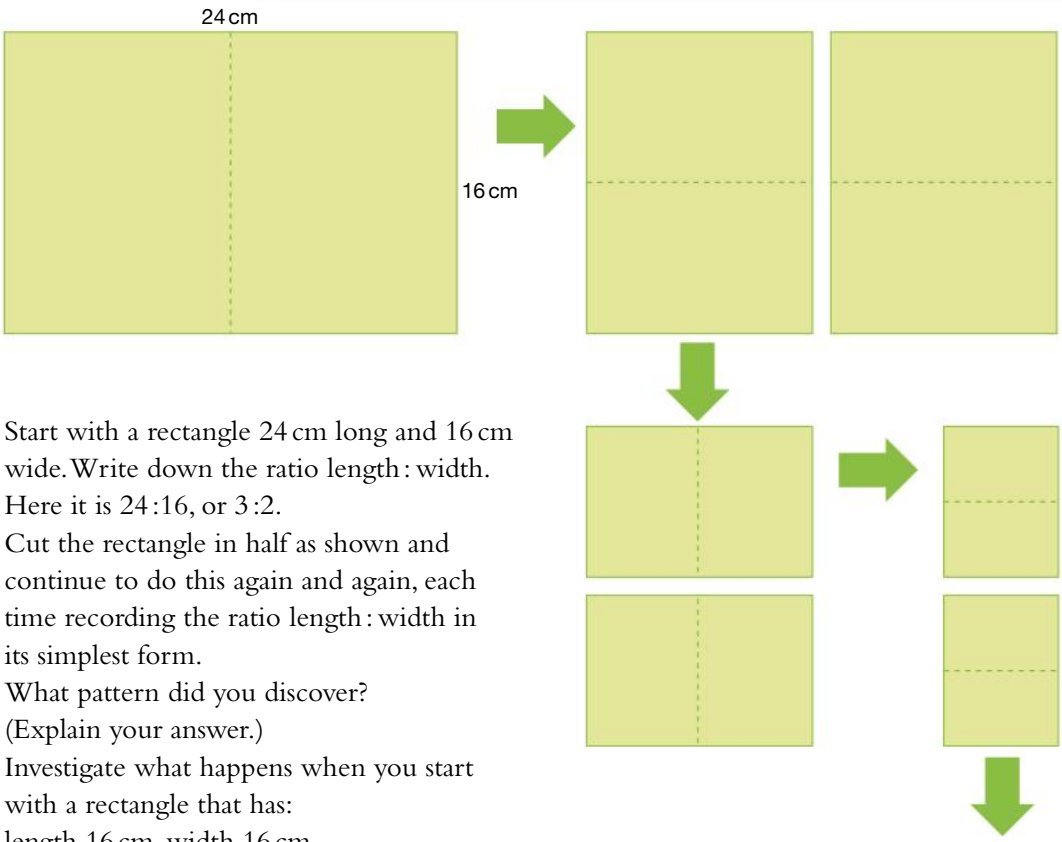


INVESTIGATION 10:05 SCALE MY ROOM!

- Make a scale drawing of a room in your house.
- Calculate the scaled sizes of the furniture in the room and mark the position of each piece on the drawing.
- Make paper cut-outs to represent the furniture.
- Use paper cut-outs to rearrange the furniture in the room. Can you find a better arrangement?



CHALLENGE 10:05 CUTTING RECTANGLES



- Start with a rectangle 24 cm long and 16 cm wide. Write down the ratio length : width. Here it is 24 : 16, or 3 : 2.
- Cut the rectangle in half as shown and continue to do this again and again, each time recording the ratio length : width in its simplest form.
- What pattern did you discover? (Explain your answer.)
- Investigate what happens when you start with a rectangle that has:

a length 16 cm, width 16 cm

b length 24 cm, width 8 cm.

10:06 Rates

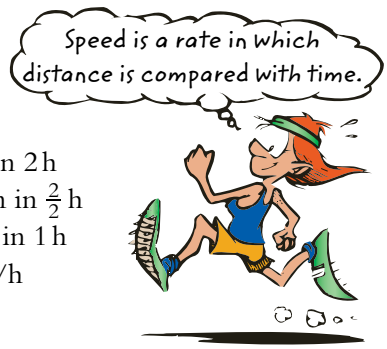
You need to be able to compare quantities that have different units.

- 4 teaspoons of coffee : 2 mugs of water
- 1 cap of detergent : 5 litres of water
- 6 small buttons : 8 large buttons
- 90 kilometres : 2 hours

- A *rate* is a comparison of unlike quantities.
- The most commonly used rate is speed, which compares *distance travelled* to *time taken*.
  e.g. 20 km : 2 hours = 10 km : 1 hour = 10 km/h

- To make rates easier to work with, we usually change the rate to an equivalent rate where we write down how many of the first quantity correspond to one of the second quantity. Rates can be changed (as with ratios) by multiplying or dividing both quantities by the same number.

e.g. 150 km in 2 h
$= \frac{150}{2}$ km in $\frac{2}{2}$ h
= 75 km in 1 h
= 75 km/h



- To simplify rates we can multiply or divide both terms by the same number, just as we do for ratios.
- When using a rate we must show the units.
- Rates are usually expressed by writing down how many of the first quantity correspond to one of the second.
  e.g. 6000 mL for 10 bottles = 600 mL per 1 bottle
  = 600 mL/bottle



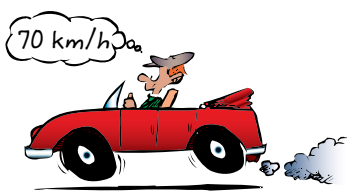
- The solution of rate problems is also similar to the solution of ratio problems. The two basic problems are shown in Worked Examples **2** and **3**.

WORKED EXAMPLE 1

Express each of the following as a rate in its simplest form.

a A woman travels 210 km in 3 hours.

b 500 sheep are grazed on 200 hectares.



Solutions

a 210 km in 3 hours
divide each term by 3
= 70 km in 1 hour
= 70 km/h

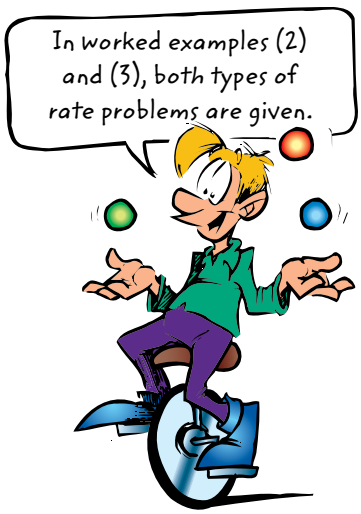
b 500 sheep on 200 hectares
divide by 200
$= \frac{500}{200}$ sheep on 1 hectare
= 2·5 sheep/hectare

WORKED EXAMPLE 2

A mine supplies ore at the rate of 1500 tonnes per day.

a How many tonnes can the mine supply in 4 weeks?

b How long will it take to supply 60 000 tonnes?



Solutions

a 1500 t/day

$= \frac{1500\text{ t} \times 28}{1\text{ day} \times 28}$

$= \frac{42\,000\text{ t}}{4\text{ weeks}}$

4 weeks = 28 days

∴ 42 000 t can be supplied in 4 weeks.

b 1500 t/day

$= \frac{1500\text{ t} \times 40}{1\text{ day} \times 40}$

$= \frac{60\,000\text{ t}}{40\text{ days}}$

60 000 ÷ 1500 = 40

60 000 t/40 days

∴ 60 000 t can be supplied in 40 days.

WORKED EXAMPLE 3

A shearer sheared 70 sheep in 2 hours. If he could continue at this rate, find:

a how long he would take to shear 200 sheep, to the nearest hour

b how many sheep could be sheared in 8 hours.

Solutions

a 70 sheep in 2 hours

= 35 sheep in 1 hour

= 35 sheep/hour

35 sheep in 1 hour

= 1 sheep in $\frac{1}{35}$ hour

= 200 sheep in $200 \times \frac{1}{35}$ hour

= 200 sheep in $5\frac{5}{7}$ hours

∴ 200 sheep would take 6 hours to shear, to the nearest hour.

b 35 sheep in 1 hour

= 35 × 8 sheep in 8 hours

= 280 sheep in 8 hours

∴ 280 sheep could be sheared in 8 hours.



Notice:
In **a** the time to shear 1 sheep is found.
In **b** the number of sheep sheared in 1 hour is found.

Exercise 10:06

P Foundation worksheet 10:06 Rates

1 Write each of the following as a rate in simplest form.

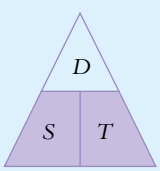
a 8 km in 2 h
b 10 kg for $5
c 5 km in 20 min
d 120 L in 4 h
e 500c for 20 g
f $315 for 7 days
g 120 children for 4 teachers
h 20 degrees in 5 min
i 60 sheep in 3 hours
j 40 kg over 8 m^2
k 70 g for 10 cm^3
l 60 km in $1\frac{1}{2}$ h
m 150 L in $1\frac{1}{2}$ h
n 18 min for 4·5 km

2 Complete the equivalent rates.

a 2 km/min = … km/h
b 4 kg/min = … kg/h
c 0·02 kg/m^2 = … kg/ha
d 5 L/h = … L/day
e \$5/g = \$…/kg
f 5 mL/min = … mL/h
g 4 km/h = … m/h
h 2 L/h = … mL/h
i 1·2 t/d = … kg/d
j 20c/min = \$… /min
k \$2.45/kg = … c/kg
l 15 mL/min = … L/min
m 3 m/s = … km/h
n 25 mL/s = … L/h
o 15 t/h = … t/day
p 20 g/m^2 = … kg/ha
q 10 km/L = … m/mL
r 7 g/cm^3 = … t/m^3

3 **a** I walk at 5 km/h. How far can I walk in 3 hours?
b How far will a car travelling at 80 km/h travel in $2\frac{1}{2}$ hours?
c A jet plane is travelling at 600 km/h. How far will it travel in 25 minutes?
d A girl cycled at 18 km/h for $4\frac{1}{2}$ h. How far did she cycle?
e How long will it take to travel 200 km at 80 km/h?
f How long should it take to run 14 km at an average speed of 4 min/km?
g A train travels 960 km at an average speed of 80 km/h. How long will the trip take?
h How long will it take a spacecraft travelling at 1700 km/h to travel 238 000 km?
i A train left town A at 9 am and arrived at town B at 4 pm. What was its average speed if it travelled 665 km?
j A rocket takes $1\frac{1}{2}$ hours to orbit the Earth. If one orbit is 30 000 km, what is its speed? How far would it travel in 10 minutes at this speed?
k A woman runs the marathon in 2 hours 25 minutes. If she covers 42·2 km, calculate her average speed.

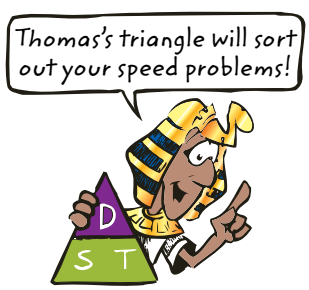




D = distance, S = speed, T = time

If you cover the quantity you are trying to calculate, this triangle will tell you what to do.

$$D = S \times T,\ S = \frac{D}{T},\ T = \frac{D}{S}$$



4 **a** Nails cost \$4.60 per kg. What is the cost of 20 kg?
b A cricket team scored at an average of 5 runs/over. How many runs did they score off 17 overs?
c A shearer could shear 15 sheep/hour. How many sheep could he shear in 8 hours?
d The density of iron is 7·5 g/cm^3. What is the mass of 500 cm^3?
e A heart is beating at 90 beats/min. How many beats will it make in 1 hour?
f Perfume costs \$12 for 100 mL. How much would 1 L cost?
g A mine supplies 1200 t of ore per hour. How much would it supply in 1 week?
h If AUD\$1 = US\$0·8988, what is the value in US dollars of \$400 Australian?
i Fertiliser is spread at a rate of 1 kg/40 m^2. What area can be covered by 25 kg?
j Water is dripping from a tap at a rate of 5 L/h. How much water will leak in 1 day?

5 a Nails cost $4.60/kg. How many kilograms can be bought for $20?

b If I can save $40/week, how long should it take to save $1000?

c A bulldozer is moving soil at the rate of 15 t/h. How long will it take to move 400 t?

d A tank that holds 1000 L is leaking at a rate of 2·5 L/min. How long will it take to lose 400 L at this rate?

e Blue metal is moved at a rate of 22 t/truck. How many trucks would be needed to move 480 t of metal?



f A car uses petrol at the rate of 9·5 L/100 km. How many litres would be used in travelling 350 km?

g A machine fills bottles at the rate of 120 bottles/min. How long would it take to fill 1000 bottles at this rate?

h Fertiliser is to be spread at a rate of 20 kg/ha. How many hectares can be covered by 50 kg?

i If it takes 25 tiles to cover 1 m^2, how many square metres will be covered by 175 tiles?

j Runs were scored at a rate of 5 runs/over in a cricket match. How many overs did it take to score 90 runs?

6 a Light travels at 300 000 km/s.

i How far does it travel in 1 minute?

ii How long would it take light to travel from the sun, which is 148 800 000 km away?

b The density of iron is 7·5 g/cm^3. Find the mass in kilograms of a rectangular block of iron 100 cm long, 100 cm wide and 2 cm thick.

c The fastest man in the world can run 100 m in just under 10 s. If he could continue at this rate, what would his speed be in kilometres per hour?

d A racing car travels at a speed of 180 km/h. How long would it take to travel 100 m?

e Land rates are paid at 4.95 cents per dollar of the land's value.

i How much will the rates be for land with a value of $15 000?

ii What would be the value of land for which the rates are $544.50?

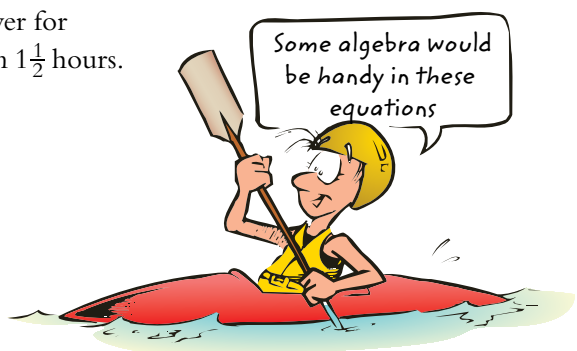
7 a Pump A can fill a tank in 5 minutes, whereas pump B can fill a tank in 10 minutes. How long will it take to fill the tank if both are working together?

b In making one revolution, a car wheel travels 200 cm. How fast are the wheels spinning, in revolutions per minute, when the car is travelling at 60 km/h? (Answer to the nearest revolution per minute.)

c Sally paddled her kayak upstream in a river for $2\frac{1}{2}$ hours. She returned that afternoon in $1\frac{1}{2}$ hours. If the river is flowing at $2\frac{1}{2}$ km/h, find:

i her paddling speed in still water

ii the distance she travelled upstream.



FUN SPOT 10:06 WHY DIDN'T THE PIGLETS LISTEN TO THEIR FATHER?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

U Simplify the ratio 12 : 8.

C Find x if $12 : 8 = x : 40$.

S Find m if $\frac{m}{3} = \frac{20}{6}$.

E Simplify the rate 18 L in 6 m^2.

Find the value of y if:

B 12 L/s = y L/min

A 0·6 kg/h = y g/h



This rectangle is a scale drawing of a garden.

Find the ratio of:

W length : breadth

R length : diagonal

O diagonal : breadth

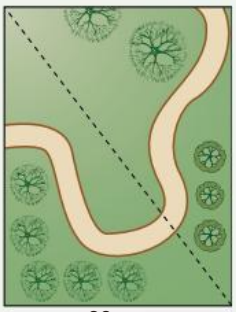
A How many metres long is the garden?

A What is the scale used in this drawing?

E What is the area of the scale drawing?

H What is the area of the garden?

B What is the ratio of the area of the scale drawing to the area of the garden?



150 newspapers are divided in the ratio 7 : 3.

E How many newspapers are in the smaller share?

S How many newspapers are in the larger share?

A What fraction of the newspapers is the smaller share?

720	3 L/m^2	60	600	3 : 2	10	45	1200 m^2	12 m^2	4 : 3	1 : 10^3	105	40	1 : 10^6	5 : 3	$\frac{3}{10}$	4 : 5

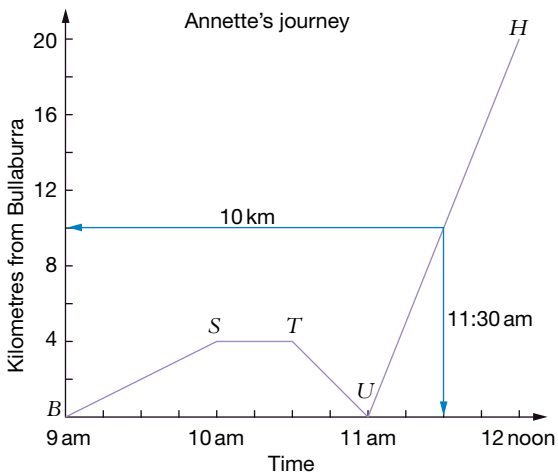
10:07 Distance–time graphs

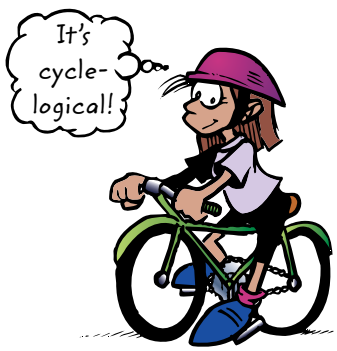
- A distance–time graph (or travel graph) is a line graph that relates distance and time.
- *Time* is shown on the horizontal axis.
- *Distance from a particular place* is shown on the vertical axis.
- The steeper the graph, the greater the speed.
- If the graph is horizontal, the person has stopped (i.e. speed is zero), as it is assumed that travel is along a straight road.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \qquad S = \frac{D}{T} \quad \begin{cases} T = \dfrac{D}{S} \\ D = S \times T \end{cases}$$

WORKED EXAMPLE

Annette travelled from Bullaburra to her home along the Western Highway. Her distance from Bullaburra as time passed is given by the graph.



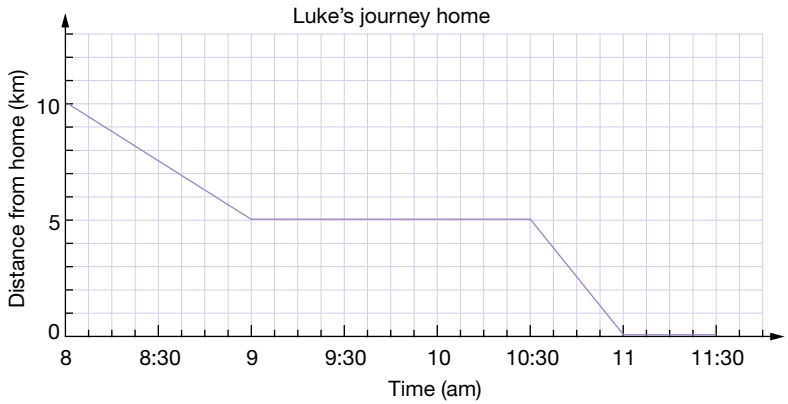


- During the trip Annette stopped for a while, then returned to Bullaburra, then travelled to her home where she arrived at 12 noon.
- Annette travelled 4 km from Bullaburra (BS) before she stopped. She had travelled slowly (4 km/h), stopping at 10 am.
- Annette waited for half an hour (ST), then returned to Bullaburra more quickly than she had come away (TU).
- She then very quickly (20 km in one hour) went home (UH), arriving at 12 noon.
- The distance from Bullaburra is shown on the vertical axis, so when Annette reaches home (H) she is 20 km from Bullaburra.
- When the graph slopes up as time passes, Annette is travelling away from Bullaburra. When the graph slopes down, Annette is moving towards Bullaburra.
- The steeper the line the greater is her speed.
- When the line is horizontal she has stopped.
- Annette is 10 km from Bullaburra at 11:30 am.

Exercise 10:07

Foundation worksheet 10:07
Travel graphs

1



a How far from home was Luke at:

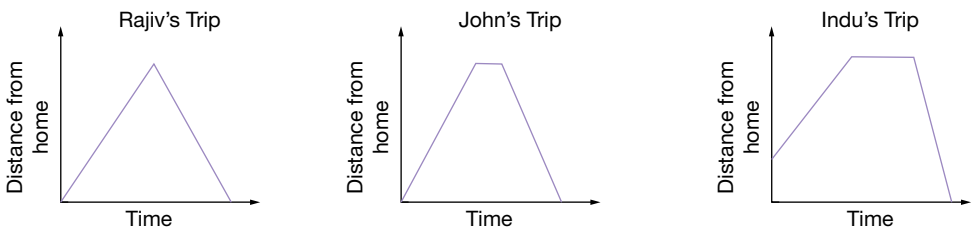
i 8 am **ii** 9 am **iii** 10 am **iv** 11 am

v 9:30 am **vi** 11:30 am **vii** 10:30 am **viii** 8:30 am

b During what times was Luke:

i resting (or stationary) **ii** travelling towards home?

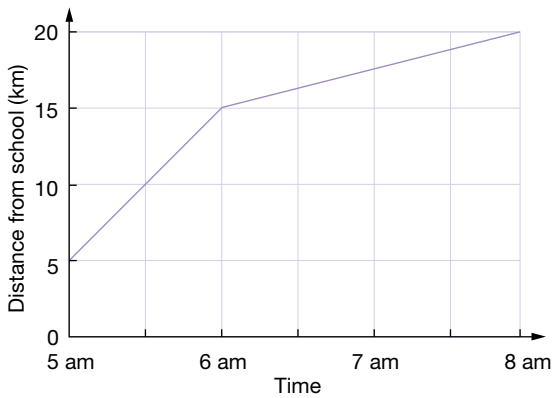
2



From these three graphs:

a who did not rest **b** who did not start from home?

3 Alana left home at 5 am and travelled out of town by bicycle until it got a flat tyre. She had to push the bike to her uncle's home, which she reached at 8 am.



a How far from the school is:

i Alana's home

ii her uncle's home?

b How far did Alana travel between:

i 5 am and 6 am

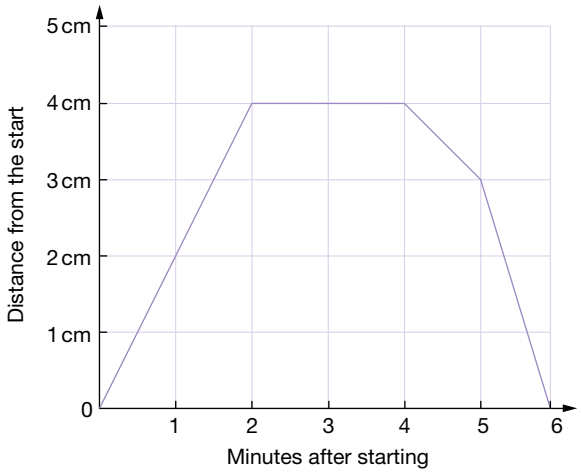
ii 6 am and 8 am?

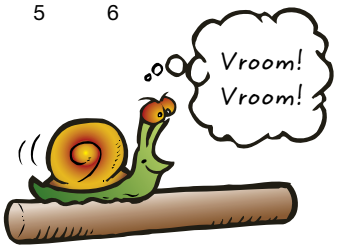
c Was Alana travelling faster between 5 am and 6 am or between 6 am and 8 am?

d When did Alana's bicycle get a flat tyre?

e How far did Alana travel in total?

4 A snail was placed at one end of a 5 cm long stick. This graph shows the progress of the snail.



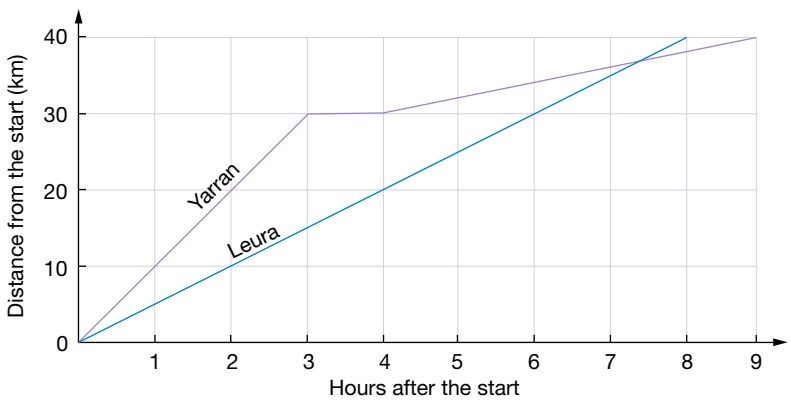


a How far from the start was the snail after:
- **i** 2 minutes
- **ii** 4 minutes
- **iii** 1 minute
- **iv** 6 minutes?

b When was the snail:
- **i** 3 cm from the start
- **ii** 4 cm from the start
- **iii** not moving?

c How far did the snail travel:
- **i** in the first minute
- **ii** in the first 6 minutes
- **iii** in the sixth minute?

5 Leura and Yarran are competing in a 40 km bicycle race over rough country.



a How far did Yarran travel in the first 2 hours? What was his speed?

b How far did Leura travel in the first 2 hours? What was her speed?

c When did Yarran have an accident? How long did it take him to fix his bicycle? What was his speed in the last 5 hours?

d Who won the race? How long afterwards did the second person finish?

e How many hours after the start (to the nearest half hour) did Leura pass Yarran?

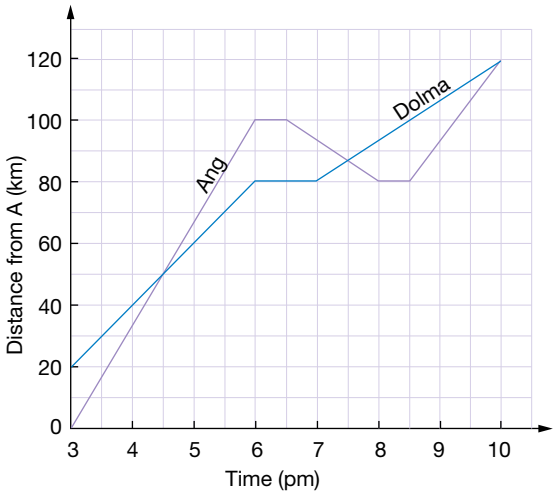
6 Draw a distance-time graph to describe this story. The hare and the tortoise agreed to race over 4 km. In the first $\frac{1}{2}$ hour, the tortoise travelled 500 m. It continued at this speed until it had finished the race. The hare, who had run 2 km in the first $\frac{1}{2}$ hour, looked back and couldn't see the tortoise so it sat down to rest and fell asleep. It woke up just as the tortoise crossed the finish line.

a What was the tortoise's average speed and when did it pass the hare?

b What was the hare's average speed:
- **i** in the first $\frac{1}{2}$ hour
- **ii** in the first 4 hours?

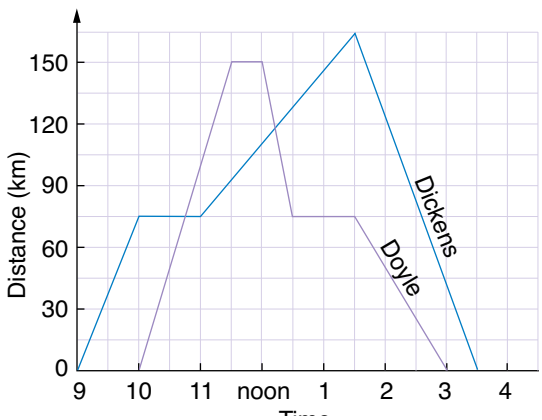
7 Two hikers, Ang and Dolma, set out to travel from town A to town B. They travel along the same road and in the same direction.

- **a** State when and where each hiker begins his trip.
- **b** At what times do their paths cross?
- **c** Both of them stop at 6 pm. How far apart are they then?
- **d** At what time does Ang begin to move back towards A?
- **e** How far does each hiker travel?



8 The graph shows the journeys of two motorists, and gives their distances from a town.

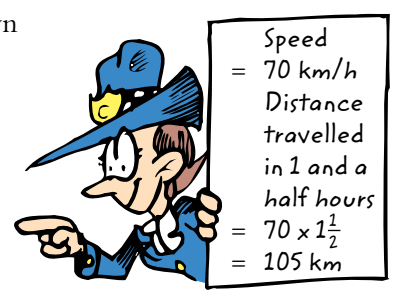
- **a** How far is Dickens from the town when Doyle starts his journey?
- **b** At approximately what times do their paths cross?
- **c** At what time does Doyle begin his return journey?
- **d** How far is Doyle from town when Dickens begins his return journey?
- **e** Who returns to town first? What time elapses before the other motorist arrives?



9 A group of people go on a bus tour. The bus leaves their town at 9:00 am. The bus travels at an average speed of 80 km/h and reaches town B in 1 hour. After spending 30 minutes for morning tea, the bus continues on to town C which it reaches after $1\frac{1}{2}$ hours. Its average speed for this part of the journey is 70 km/h.

The bus spends 1 hour at town C before returning to town A. The trip from town C to town A is completed without any stops and it takes 3 hours.

By dividing this journey into its separate parts, make a table comparing distance from home with time, and use it to complete a distance-time graph for this journey.



MATHS TERMS 10

rate

- a comparison of unlike quantities written in a definite order, e.g. 100 km in 5 h or 20 km/h
- units are used in this comparison, e.g. 8 *people* per *car*

ratio

- a comparison of quantities or numbers written in a definite order
- equivalent ratio (an equal ratio), which is formed when the terms of a ratio are multiplied or divided by the same number

scale

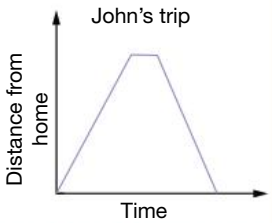
- the ratio of the length that a drawing has to the original length
- if a map has a scale of 1 : 10 000, then 1 cm on the map represents 10 000 cm (i.e. 100 m) in real terms

scale drawing

- a scale drawing has the same shape as the object it represents but a different size
- the scale of a drawing = length on drawing : real lengths

distance–time graph

- this line graph relates distance and time to show progress on a trip



DIAGNOSTIC TEST 10 RATIOS AND RATES

Each section of the test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Concrete mix is made by mixing 1 part cement with 2 parts sand and 4 parts gravel. Write down the ratio of: 10:01

a cement to sand
b cement to gravel
c cement to sand to gravel
d gravel to cement to sand.

2 Simplify these ratios. 10:02

a 10 : 4
b 16 : 4
c 1·5 : 6
d $1\frac{1}{2} : 2\frac{1}{4}$

3 Complete the following. 10:03

a 5 : 2 = 10 : …
b 1 : 3 = 6 : …
c 4 : 3 = … : 18
d 2 : 5 = … : 20

4 Solve the following. 10:03

a $\frac{x}{3} = \frac{5}{8}$
b $\frac{x}{5} = \frac{10}{3}$
c $\frac{x}{4} = \frac{1{\cdot}8}{5}$
d $\frac{2.5}{6} = \frac{x}{4}$

5 a The ratio of boys to girls is 6 : 5. If there are 15 girls, how many boys are there? 10:03
b Sand and cement are mixed in the ratio 7 : 2. If 3 kg of cement are used, how much sand is used?
c Liam and Bronte share money in the ratio 2 : 3. If Liam's share is $1.80, what is Bronte's share?
d An alloy is made by mixing metals A, B and C in the ratio 2 : 5 : 3. If 9 kg of C are used, how much of B is used?

6 a Divide 15 in the ratio 3 : 2. b Divide 45 in the ratio 2 : 7. 10:04
c Divide 420 in the ratio 11 : 10. d Divide 120 in the ratio 1 : 2 : 7.

7 A drawing has a scale 1 : 100. Convert the following scaled distances to real distances. 10:05
a 3 cm b 5·2 cm c 1 mm d 42 mm

8 A drawing has a scale 1 cm : 5 m. Convert the following scaled distances to real distances. 10:05
a 3 cm b 5·2 cm c 1 mm d 42 mm

9 A drawing has a scale of 1 : 100. Convert the following real distances to scaled distances. 10:05
a 5 m b 43 m c 75 cm d 4·8 m

10 Repeat Question **9** using a scale of 1 cm : 2 m. 10:05

11 Simplify these rates. 10:06
a 50 km in 2 hours b 600 t in 4 hours
c 100 g for 8 cm^3 d 4 kg over 10 m^2

12 Complete the following. 10:06
a 1 m/s = … m/h b 200 kg/h = … t/day
c $15/g = $… /kg d 6 L/h = … mL/min

13 a Ore is supplied at a rate of 200 t/day. How much is supplied in 2 weeks? 10:06
b A car travels at a speed of 100 km/h. How far will it travel in $3\frac{1}{2}$ h?
c Water is pumped at a rate of 100 L/h. How long will it take to pump 550 L?
d A car uses petrol at a rate of 9·6 km/L. How many litres will it use to travel 200 km?

14 Draw a distance–time graph to show Heather's journey. She left home at 7 am to walk to Naomi's place, 10 km away. After walking steadily for 2 hours she had covered 8 km. She then stopped to rest for 30 minutes before continuing to walk steadily for 1 more hour until she reached her goal. 10:07

ASSIGNMENT 10A Chapter review

1 Simplify these ratios.

a 3 : 12 **b** 15 : 10 **c** 1·2 : 6·0

d 2·5 : 0·25 **e** $\frac{1}{3} : \frac{5}{6}$ **f** $2\frac{1}{4} : 1\frac{5}{8}$

2 Solve the following.

a $\frac{x}{3} = \frac{6}{9}$ **b** $\frac{a}{16} = \frac{3}{4}$ **c** $\frac{m}{5} = \frac{3}{4}$ **d** $\frac{8}{15} = \frac{n}{6}$

3 **a** The ratio of boys to girls is 7 : 6. There are 56 boys. How many girls are there?

b The ratio of cars to wheels is 1 : 5. There are 35 cars. How many wheels are there?

c The ratio of cost price to selling price is 7 : 9. If the selling price is $13.50 what is the cost price?

4 **a** Greg and Gill divide $55 in the ratio 7 : 4. How much does each receive?

b The length and width of a rectangle are in the ratio 8 : 5. If the perimeter of the rectangle is 78 cm, what is the length of the rectangle?

c Rae, Rina and Rod are given pocket money in the ratio of their ages, 7, 10 and 11. If the total of their pocket money is $42, how much does Rod receive?

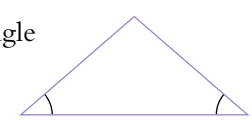
d In a sample of 195 vehicles, the ratio of cars to buses to motor cycles was 9 : 4 : 2. How many buses were there?

5 A map has a scale of 1 : 1 000 000. What distance in km is represented on the map by a length of:

a 6 mm **b** 2 cm **c** 3·7 cm

6 **a** The sizes of the angles in a triangle are in the ratio 2 : 3 : 4. What is the size of the largest angle?

b In an isosceles triangle the apex angle is three times the size of each base angle. What is the size of each angle in the triangle?



7 A car uses 15 litres of petrol to travel 210 km.

a Convert this to a rate in km/L.

b How many litres would be needed at this rate to travel 371 km?

c How many millilitres of petrol would be needed to travel 1 km, correct to the nearest mL?

8 The road distance from Sydney to Brisbane is 1010 km.

a If a motorist leaves Sydney at 5 am and drives at an average speed of 75 km/h, at what time will she arrive?

b If she wants to arrive by 5 pm, what must be her average speed for the trip, correct to the nearest km/h?

9 A car is travelling at 60 km/h. How far will it travel in the 6 seconds it takes you to cross the road?

10 The graph shows the trips of Ms Chew and Ms Travers and gives their distance from town *A* at different times.

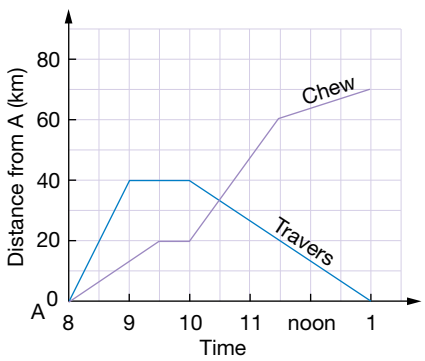
a At 10 am how far is Ms Chew from A?

b At 10 am how far is Ms Travers from A?

c At what time are they the same distance from A?

d Which of the two completes the trip by returning to A?

e How far is Ms Chew from A when Ms Travers is 20 km from A? (There are two possible answers.)



ASSIGNMENT 10B Working mathematically

1 Refer to **ID Card 5** on page xviii to identify the figures numbered:

a 2 **b** 3 **c** 4 **d** 5
e 8 **f** 16 **g** 17

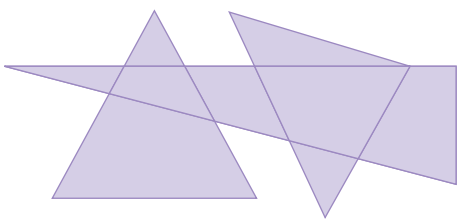
2 Find two numbers that have a sum of 12 and a product of 35.

3 All 10 girls in 8B play tennis or hockey. Some play both sports. If 8 of the girls play tennis and 8 of the girls play hockey, how many play both sports?

4 If it's true that 'all dogs bark', and that 'Nero is a dog', which of the following must be true?

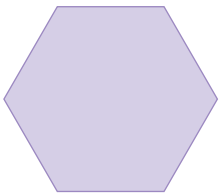
a Nero does not bark.
b Nero is not a dog.
c Nero barks.
d All dogs are called Nero.

5 How many triangles are there in this shape? (Be careful, some may be hidden.)



6 Find a number less than 100 that is a perfect square and the sum of its two digits is even.

7 This is a regular hexagon.



a Divide this shape into two pieces that can be joined together to form a parallelogram.
b Divide the hexagon into three pieces that can be joined to form a rhombus.

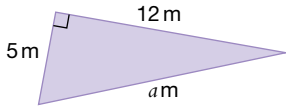


Each domino can use numbers from zero to six on each part and no two dominoes are alike. How many dominoes are in a set?

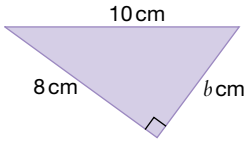
ASSIGNMENT 10C Cumulative revision

1 Use Pythagoras' theorem to find the value of each pronumeral. 3:04

a



b



2 Simplify: 5:02, 5:08

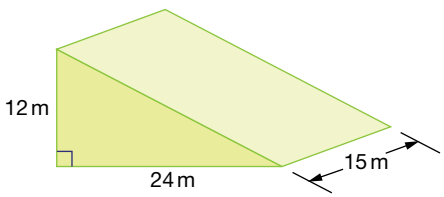
a $6m + 3n + 8 + 2n$ **b** $4(3a + 1) + 6(3 - a)$

3 Which of each set has the smallest value? 4:01

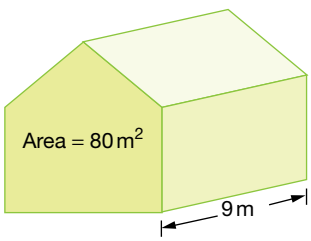
a $\{0{\cdot}75,\ 1{\cdot}125\%,\ \frac{1}{4}\}$ **b** $\{99{\cdot}1\%,\ \frac{5}{2},\ 1\frac{1}{4}\}$

4 Find the volume of each prism. 8:06

a



b

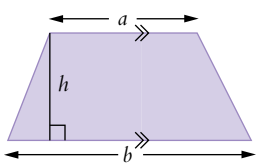


5 **a** How much will I pay for a coat costing \$120 at a 10% discount sale? 4:04, 4:07

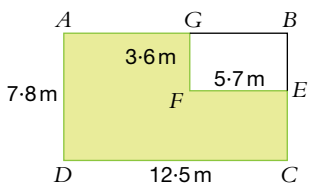
b How much GST is included in a price of \$120?

6 What is the length of time, in hours and minutes, from 17:40 on Tuesday to 13:25 on Thursday in the same week? 8:07

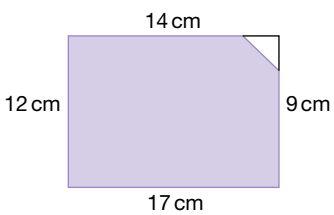
7 The formula for the area of a trapezium is $A = \frac{1}{2}h(a+b)$. 8:04
Find A when $h = 3{\cdot}6$, $a = 2{\cdot}4$ and $b = 6{\cdot}8$.



8 **a** Find the area of the rectangle $ABCD$. 8:02
b Find the area of the rectangle $GBEF$.
c Use your answers from parts **a** and **b** to find the area of the coloured shape.



9 Find the area of the coloured shape. 8:02



10 Solve the equations: 9:02, 9:03

a $5a = 16$ **b** $\frac{3m}{4} = 5$ **c** $p - 6 = 2p + 4$

COORDINATE GEOMETRY

11



Contents

11:01 Introducing straight line graphs
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GeoGebra activity 11:04 Using graphs to solve linear equations
11:05 Non-linear graphs
Fun spot 11:05 The game of sprouts
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Number and Algebra

Selections from ***Linear Relationships*** [Stage 4]

- Plot linear relationships on the Cartesian plane with and without the use of digital technologies. (ACMNA193)
- Solve linear equations using graphical techniques. (ACMNA194)

Working Mathematically

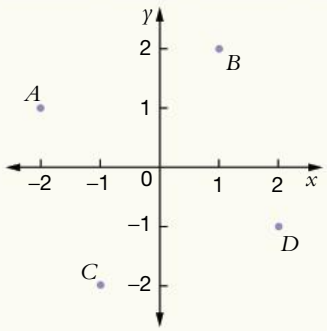
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

11:01 Introducing straight line graphs

PREP QUIZ 11:01

What are the coordinates of the following points?

1 A **2** B **3** C **4** D



If $x = 3$, what is the value of:

5 $4x$ **6** $2x + 1$ **7** $1 - x$?

8 What value has $x - 5$ when $x = 4$?

9 If $x = 0$, what is the value of $2x - 3$?

10 Solve $4 + y = 9$.

- Most of the equations we have met have had only one solution. For example, the solution to $y + 7 = 10$ is $y = 3$.
- When an equation has two different pronumerals, more than one solution usually exists.

Equations with two pronumerals

Equations such as $x + y = 7$ have more than one unique solution.

We know that $3 + 4 = 7$, so one solution is $x = 3$, $y = 4$.
This could be written as $(3, 4)$.

We can substitute other values of x to find other solutions.

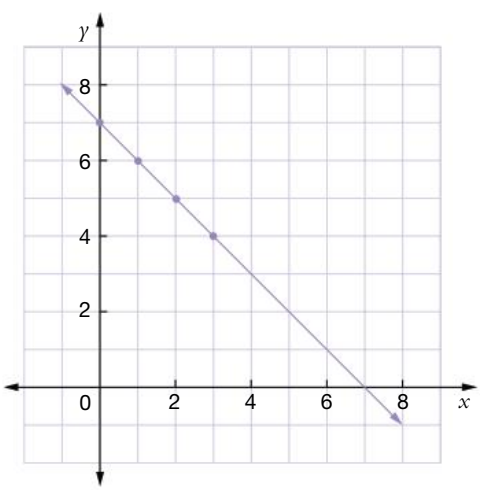


When $x = 0$, $y = 7$. The point is $(0, 7)$.
When $x = 1$, $y = 6$. The point is $(1, 6)$.
When $x = 2$, $y = 5$. The point is $(2, 5)$.
When $x = 3$, $y = 4$. The point is $(3, 4)$.

These points can be written in a table of values.

x	0	1	2	3
y	7	6	5	4

If we plot these points on the number plane they all fall on the same straight line. Test the other solutions you have found to see if they lie on this same straight line.



This is the graph of the straight line with equation $x + y = 7$.

The values that are substituted do not have to be integers (whole numbers).

- Can you find solutions to $x + y = 7$ that are not whole numbers?
- How many solutions are there?
- Do these solutions also lie on the straight line?

An equation with two different pronumerals like $x + y = 7$ has an infinite number of solutions. Ordered pairs can represent these solutions.

e.g. $(1, 6)$ represents the solution $x = 1, y = 6$
$(3{\cdot}5, 3{\cdot}5)$ represents the solution $x = 3{\cdot}5, y = 3{\cdot}5$.

If these ordered pairs are plotted on a number plane, a graph of the equation is produced. Using the graph of an equation, solutions can be found by writing down the coordinates of any point that lies on the graph.

Exercise 11:01

1 In each case substitute the value of x to find the value of y and fill in the table.

a $x + y = 4$

If $x = 0, y = 4$
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y	4				

b $y = 2x$

If $x = 0, y = 0$
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y	0				

c $y = 3x - 2$

If $x = 0, y = -2$
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y	−2				

d $y = 10x + 36$

If $x = 0, y = 36$
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y	36				

e $y = 8 - x$

If $x = 0, y =$ _____
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y					

f $2x + y = 16$

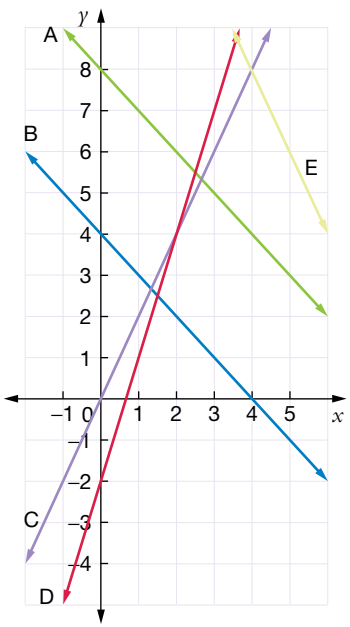
If $x = 0, y =$ _____
$x = 1, y =$ _____
$x = 2, y =$ _____
$x = 3, y =$ _____
$x = 4, y =$ _____

x	0	1	2	3	4
y					

2 Five of the six sets of points from Question 1 have been plotted on the number plane.

a Use the tables from Question 1 to find the equation of the lines A to E.

b What is the equation of the line that is not shown on the graph?



> When the graph of an equation produces a straight line it is called a linear equation and the relationship between x and y is said to be a linear relationship.

3 State whether the values given for x and y make the equation *true* or *false*.

a $y = 4x$
$x = 5, y = 20$

b $x + y = 6$
$x = 4, y = -2$

c $x + 2y = 12$
$x = -4, y = 8$

d $2x + y = 7$
$x = 3, y = 1$

e $5x + 10y = 105$
$x = 9, y = 6$

f $y = 2x + 8$
$x = 0, y = 6$

g $y = 36x - 10$
$x = 1, y = 26$

h $3x - 2y - 8 = 0$
$x = 10, y = 11$

i $8x - y + 10 = 0$
$x = 4, y = 12$

4 The x-values used in a table can be any numbers you like. Complete each of the following tables using the given equation (rule).

a $y = 2x + 5$

x	0	1	27	−8
y				

b $y = 4 - 2x$

x	−2	4	30	52
y				

c $y = 3x - 7$

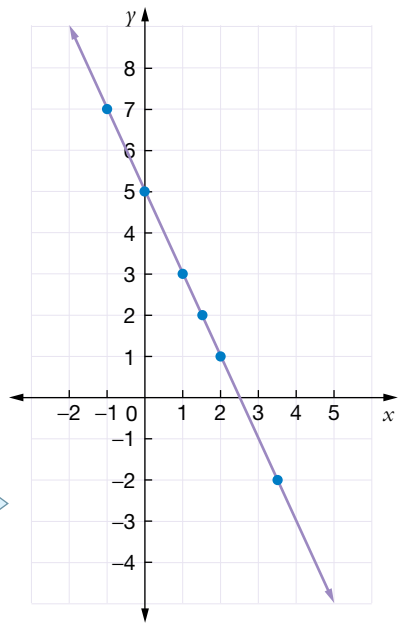
x	0	1	−1	4·5
y				

d Why do you think the numbers 0, 1 and 2 are most frequently chosen as x-values in a table of values?

5 **a** Annabelle was asked to graph the line $y = \frac{x}{3} + 4$. Rather than use the x-values 0,1 and 2 she decided to use 0, 3 and 6. Why do you think she chose these numbers? What table of values should she have obtained?

b Hannah was asked to graph the line $x + 4y = 8$. Rather than choose x-values she decided to choose y-values. She chose the values of $y = 0$, $y = 1$ and $y = 2$. Calculate the three x-values that Hannah should have obtained.

6 The line with the equation $y = 5 - 2x$ has been graphed on the number plane. Use the graph or the equation to complete the following solutions.



a $x = -1, y =$ _____
b $x = 0, y =$ _____
c $x = 1, y =$ _____
d $x = 2, y =$ _____
e $x = 1\frac{1}{2}, y =$ _____
f $x = 3\frac{1}{2}, y =$ _____

7 Use the equation in Question **6** to find the y-coordinates of the points that have an x-coordinate of:

a 10 b -10 c 14·5 d $-2{\cdot}6$

The line is continuous (or unbroken) to indicate that there are an infinite number of solutions.

8 Find an equation linking x and y and use it to find the value of y when x is 100.

a

x	0	1	2	3	4
y	0	3	6	9	12

b

x	0	1	2	3	4
y	2	3	4	5	6

c

x	0	1	2	3	4
y	–5	–4	–3	–2	–1

d

x	0	1	2	3	4
y	8	7	6	5	4

e

x	0	1	2	3	4
y	1	11	21	31	41

f

x	0	1	2	3	4
y	8	12	16	20	24

9 Rules and equations can be written in more than one way. For example, $y = x - 1$ produces the same table of values as $x - y = 1$. State which of the tables A, B, C or D would be produced by each of the following equations.

a $y = x - 5$
b $y = x + 5$
c $y = 5 - x$
d $x - y = 5$
e $x - y = -5$
f $x + y = 5$
g $x - y + 5 = 0$
h $y = -x + 5$

A

x	0	1	2
y	5	4	3

B

x	0	1	2
y	5	6	7

C

x	0	1	2
y	–5	–4	–3

D

x	0	1	2
y	–5	0	5

10 a For each of the following equations write down the solutions given by the coordinates of the three coloured points.

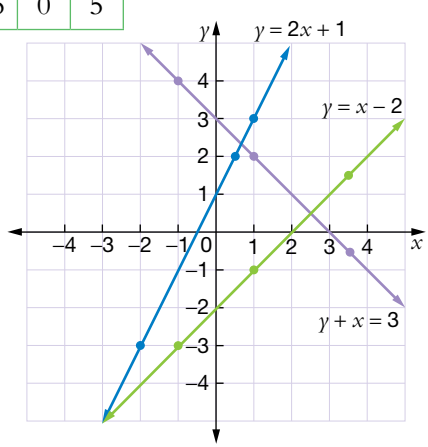
i $x + y = 3$ ii $y = 2x + 1$ iii $y = x - 2$

b On which lines do the following points lie?

i $(0, -2)$ ii $(-1, -1)$ iii $(2, 1)$

c For which equations are the following solutions?

i $x = -2, y = -3$ ii $x = 1, y = -1$
iii $x = \frac{1}{2}, y = 2\frac{1}{2}$ iv $x = -1\frac{1}{2}, y = -2$



11:02 Graphing straight lines

- To draw a straight line, we need to know *two* points through which it passes.
- When we graph the equation of a straight line we find *three* points, just in case we make a mistake with one of them.

Graphing straight lines

1. In this section, each linear equation represents a straight line. All the solutions to that equation will lie on the line.
2. Two points are needed to draw a straight line, but we usually find three points so that we can check the answers.
3. If the three points are not in line, at least one answer is wrong.
4. Any other point on the straight line will also be a solution to the equation.
5. We may choose any three values for x, but we usually let $x = 0$, 1 and 2.

WORKED EXAMPLE 1

Draw the graph of each straight line.

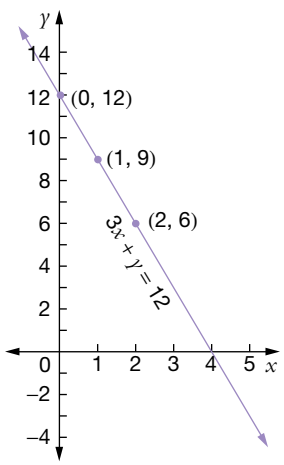
a $3x + y = 12$ **b** $y = 2x + 5$ **c** $y = 3x - 2$

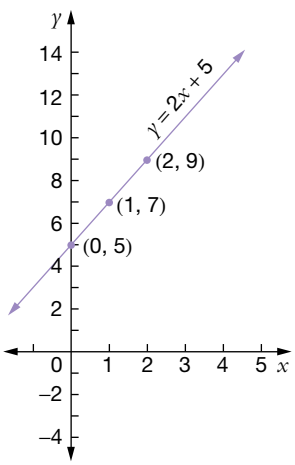
Solutions

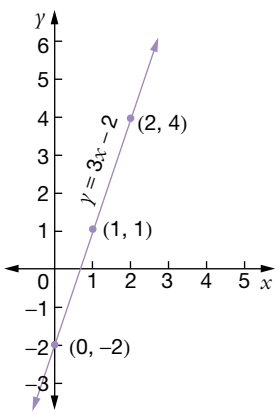
a When $x = 0$, $0 + y = 12$
$\therefore y = 12$
$x = 1$, $3 + y = 12$
$\therefore y = 9$
$x = 2$, $6 + y = 12$
$\therefore y = 6$

b When $x = 0$, $y = 0 + 5$
$\therefore y = 5$
$x = 1$, $y = 2 + 5$
$\therefore y = 7$
$x = 2$, $y = 4 + 5$
$\therefore y = 9$

c When $x = 0$, $y = 0 - 2$
$\therefore y = -2$
$x = 1$, $y = 3 - 2$
$\therefore y = 1$
$x = 2$, $y = 6 - 2$
$\therefore y = 4$







WORKED EXAMPLE 2

Does the point (3, 3) lie on $3x + y = 12$?

Solution

To see if a point lies on a graph, we substitute the coordinates into its equation.

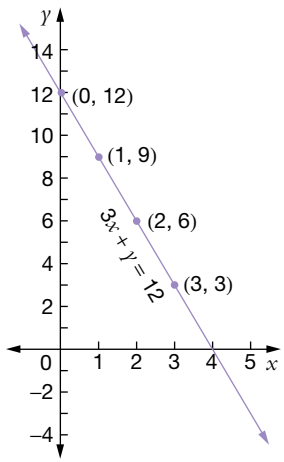
$$3x + y = 12$$

$x = 3$, $\quad 3 \times 3 + y = 12$

$$9 + y = 12$$

$$\therefore\ y = 3$$

The point (3, 3) lies on $3x + y = 12$.



Straight line equations are called *linear equations*.

Exercise 11:02

1 Complete each table of values and then graph the equation on a number plane.

a $y = x + 3$

x	−1	0	1	2
y				

b $y = x - 1$

x	−1	0	1	2
y				

c $y = \frac{1}{2}x$

x	−2	0	2	4
y				

d $y = -x$

x	−1	0	1	2
y				

e $y = 5 - x$

x	−1	0	1	2
y				

f $y = 4 - 2x$

x	−1	0	1	2
y				

2 Graph each equation on a separate number plane by first completing a table of values.

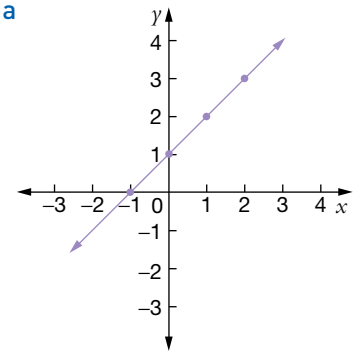
a $y = x + 1$ **b** $y = x - 2$
c $y = 3 - x$ **d** $y = -1 - x$
e $y = 2x - 3$ **f** $y = 3x + 1$
g $x + y = 4$ **h** $x - y = 2$



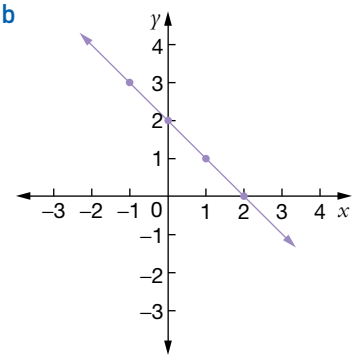
3 Complete the table of values for each graph below, and also work out what its equation must be. They can all be written in the form $y = \square x + \triangle$.

x	-1	0	1	2
y				

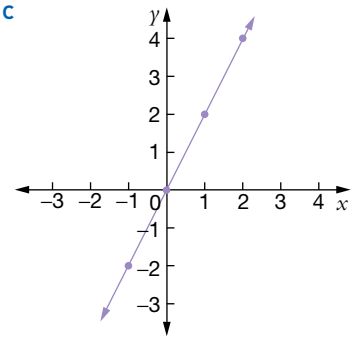
a



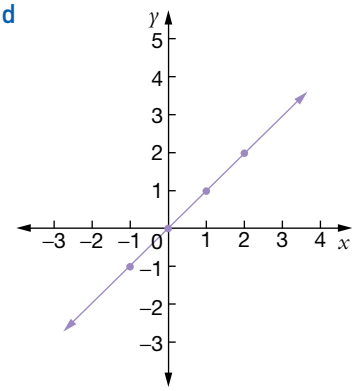
b



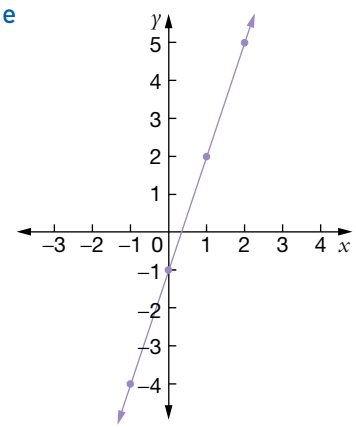
c



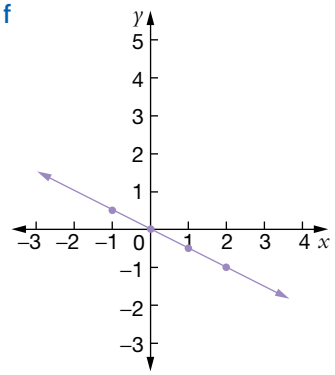
d



e



f



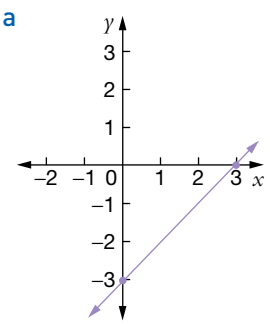
4 State whether each of the following points lies on the graph of the given equation.

a	$(1, 2)$	$y = 2x - 1$	**b**	$(3, 1)$	$y = x - 1$	**c**	$(1, 3)$	$y = x + 2$
d	$(5, 0)$	$y = 2x + 3$	**e**	$(0, 2)$	$y = 2x + 2$	**f**	$(4, 1)$	$x + y = 3$
g	$(-1, 2)$	$y = 2x - 5$	**h**	$(1, -2)$	$y = x + 1$	**i**	$(-1, -2)$	$y = 3 - x$
j	$(-3, 2)$	$y = 2x - 1$	**k**	$(4, -3)$	$x + y = 1$	**l**	$(-3, -4)$	$y = 1 + x$

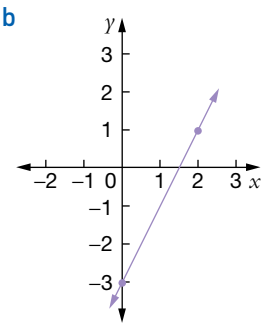
5 By completing tables of values or just by examining the points on each graph below, match each graph with the correct equation from the list below.

A $x + y = 3$ **B** $y = 2x - 3$ **C** $x - y = 3$ **D** $y = \frac{1}{2}x + 2$

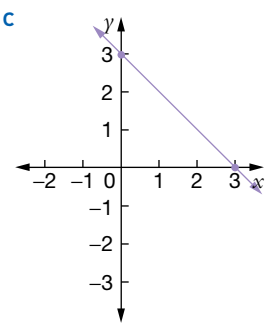
a



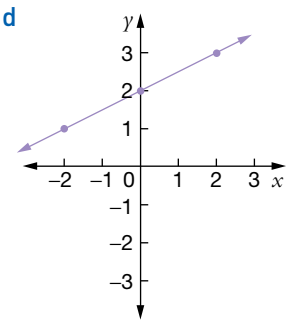
b



c



d



6 It is usual to use the numbers 0, 1 and 2 as x-values, but sometimes it is better to use other x-values or even y-values. Graph each of the following using the table of values given.

a $y = \frac{x}{3} + 2$

x	0	3	6
y			

b $y = 5 - \frac{x}{4}$

x	0	4	8
y			

c $x + 3y = 4$

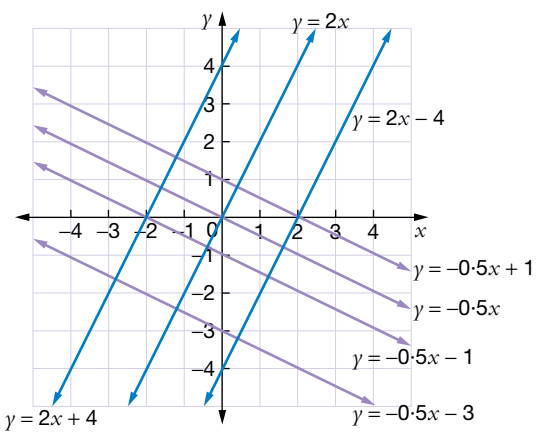
x			
y	0	1	2

Equations of straight lines

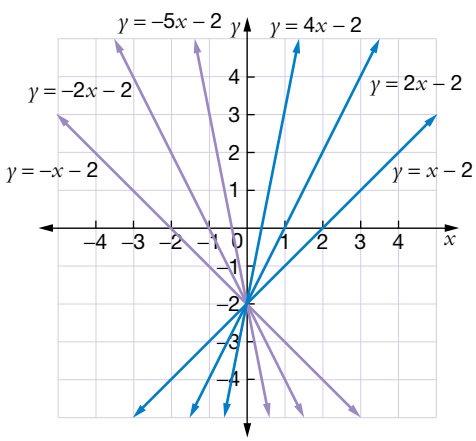
A relationship exists between the equation of a line and the position and slope of its graph.
The numbers in the equation determine the features of a line.
Two numbers are important: the coefficient of x, which is the number written before the x (i.e. multiplying), and the constant term.
The equation of a line can be written in different ways.
Some are written in the form $y = mx + b$.
e.g. in the equation $y = 7x + 5$:

- the coefficient of x is 7
- the constant term is 5.

7 **a** **i** How are the purple lines alike?
ii How are the equations of the purple lines alike?
b **i** How are the blue lines alike?
ii How are the equations of the blue lines alike?
c **i** If you were to graph $y = -0.5x + 4$, which lines would it be parallel to?
ii If you were to graph $y = 2x + 1$, which lines would it be parallel to?



8 **a** How are all the lines alike?
b How are all the equations alike?
c Where do all the lines meet the y-axis?
d Is the line $y = 4x - 2$ steeper than the line $y = 2x - 2$?
e Is the line $y = -5x - 2$ steeper than the line $y = -2x - 2$?
f What part of the equation is controlling the slope of the line?



9 **a** Do all lines that have a negative coefficient (e.g. $y = -0.5x + 4$) decrease (slope down) or increase (slope up) as x increases in value?

b Would all the lines that have a positive coefficient (e.g. $y = 2x - 4$) decrease (slope down) or increase (slope up) as x increases in value?

c By investigating all the graphs, can you find a connection between the constant b in each equation (e.g. the 4 in $y = 2x + 4$) and the graph of the line?

10 Study the equations below to determine their similarities and differences. Predict the features of each set of equations.

- Will they be uphill or downhill?
- Will they have a steep slope or a gentle one?
- Will they be parallel?
- Where will they cross the y-axis?
- Will they all pass through the same point?

Use graphing software such as GeoGebra to graph each set of lines to test your predictions.

a $y = 4x, y = 4x + 2, y = 4x - 2$

b $y = 2x + 1, y = 4x + 1, y = 5x + 1$

c $y = -4x, y = -2x, y = -\frac{1}{2}x$

d $y = x + 2, y = 2x, y = -2x + 8$

P GEOGEBRA ACTIVITY 11:02 GRAPHING STRAIGHT LINES

1 Graphing lines by plotting points

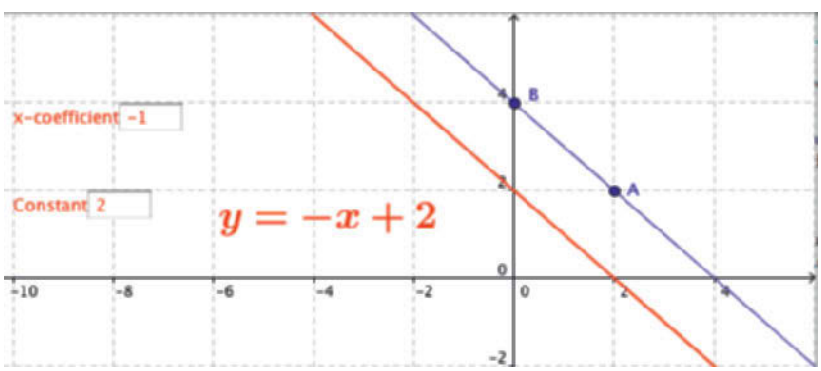
Use the equation to plot two points on a hidden line. If they are correct the line appears.

2 Straight lines and their equations

Compare four different lines and examine their similarities and differences. The equations of all the lines can be varied. Predict the effect of the change. Explore the effects of the coefficient of x and the constant term in the equation on the shape of the graph.

3 Interpreting the equation of a line

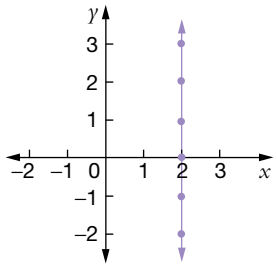
Find the equation of a blue line by superimposing a red line on a blue line. The equation of the red line is given. Use the input boxes to change the coefficient of x and the constant term in the equation of the red line.



11:03 Lines parallel to the axes

This line is parallel to the y-axis.

x	2	2	2	2	2	2
y	−2	−1	0	1	2	3

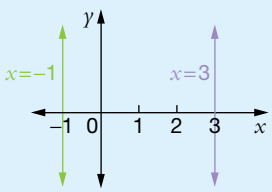


- The table of values shows that x is always 2, no matter what value y has.
  So the equation is $x = 2$.

Lines parallel to the y-axis have equations of the form

$x = a$

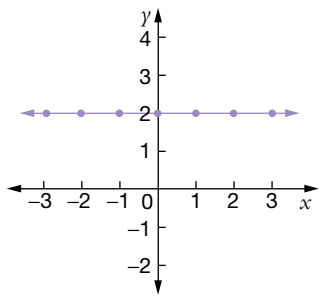
where a is the point at which the line cuts the x-axis.





This line is parallel to the x-axis.

x	−2	−1	0	1	2	3
y	2	2	2	2	2	2

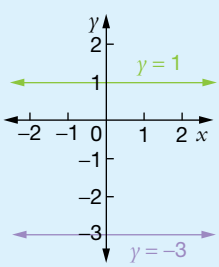


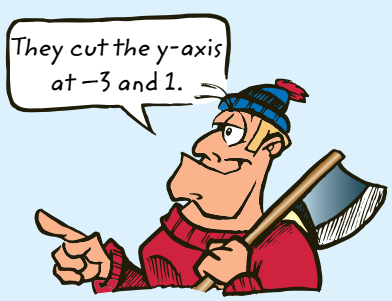
- The table of values shows that y is always 2, no matter what value x has.
  So the equation is $y = 2$.

Lines parallel to the x-axis have equations of the form

$y = b$

where b is the point at which the line cuts the y-axis.

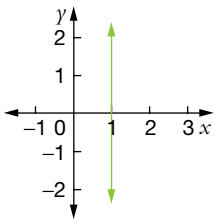




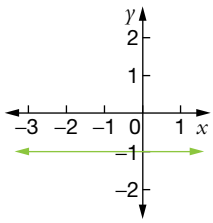
Exercise 11:03

1 Write down the equation of each line.

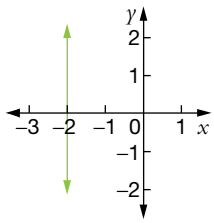
a



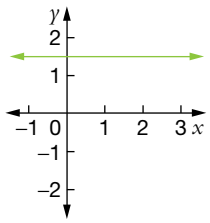
b



c

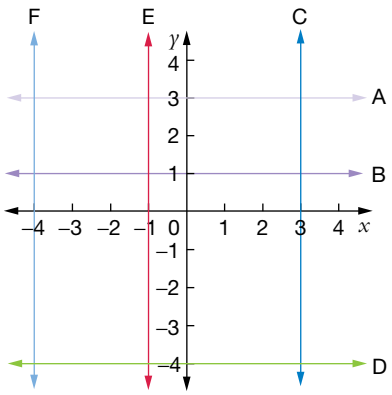


d

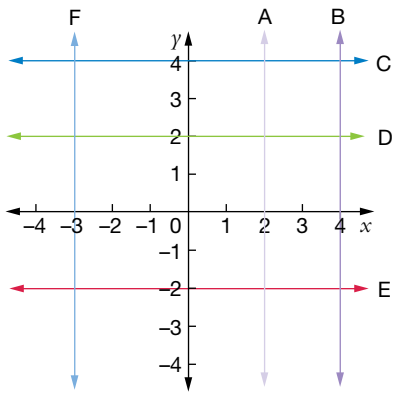


2 For each number plane write down the equations of the lines A to F.

a



b



3 **a** What is the equation of the y-axis?

b What is the equation of the x-axis?

4 Using values from −5 to 5 on each axis, draw the graphs of the following straight lines.

a $y = 4, x = 5, y = -1, x = 0$

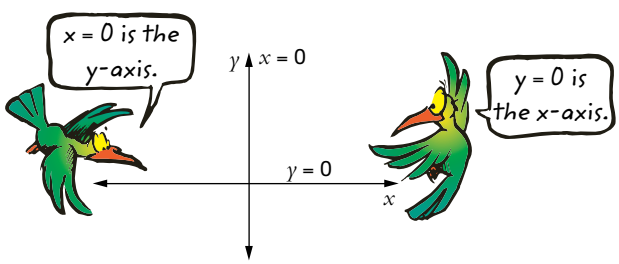
b $x = 1, y = 0, x = 2, y = 3$

c $y = 4, x = 2, y = -2, x = -4$

d $x = 5, y = -5, x = 2, y = 2$

e $y = -2, y = 0, x = 0, x = 3$

Which of these encloses a square region?



5 Give the point where each pair of lines cross. (You can use the graphs in Question 2.)

a $x = 3$ and $y = 1$

b $x = -4$ and $y = 3$

c $x = -1$ and $y = -4$

d $x = 2$ and $y = -2$

e $x = -3$ and $y = 2$

f $x = 4$ and $y = 4$

g $x = 0$ and $y = 3$

h $x = 2$ and $y = 0$

i $x = 0$ and $y = 0$

FUN SPOT 11:03 — WHY SHOULD YOU NEVER TELL SECRETS IN A VEGETABLE GARDEN?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

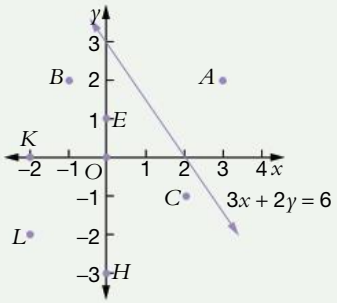


If $x = 3$, find the value of:

A $2x$ **B** $1 - x$ **C** x^2
D $5x - 4$ **E** $-x$

Give the coordinates of:

A A **B** B
C C **E** E
H H **K** K
L L **O** O



Where does the line $3x + 2y = 6$ cut:

A the x-axis
E the y-axis
U What is the equation of the x-axis
S What is the equation of the y-axis?

Simplify:

A $5 \times x$ **A** $y + y$ **A** $y \times y$ **E** $x + 2x$ **R** $2 \times 5x$
R $4y \times 3$ **N** $8x - x$ **N** $2y \div 2$ **N** $2x - 2x$ **S** $8x \div x$
S $2x \div 2x$ **S** $3x - 3$ **T** $5x - 3y + 3$ if $x = 5$ and $y = 7$

–2	(0, 3)	9	6	$y = 0$	$x = 0$	(0, 1)	(2, –1)	(0, 0)	$12y$	0	(0, –3)	(3, 2)	1	$3x$	(2, 0)	$10x$	$3x - 3$

$5x$	$7x$	11	(–1, 2)	-3	$2y$	y	8	7	y^2	(–2, -2)	(–2, 0)

11:04 Using straight lines to solve equations

Straight line graphs can be used to solve two different types of problems involving equations.

Using straight lines to solve an equation involving *x*

Consider the line with equation $y = 2x - 5$.

When we substitute different values for y, different equations are obtained.

When $y = 11$, we obtain $11 = 2x - 5$.

When $y = 3$, we obtain $3 = 2x - 5$.

When $y = -9$, we obtain $-9 = 2x - 5$.

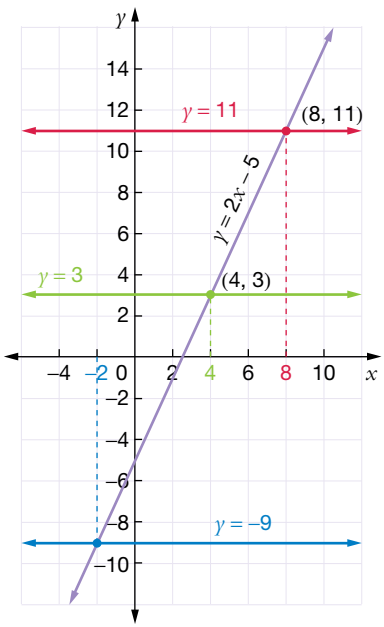
To solve $11 = 2x - 5$ we find the point where $y = 11$ meets the line $y = 2x - 5$.

We then draw a vertical line from this point to the x-axis. This line meets the x-axis at 8.

So the solution to $11 = 2x - 5$ is $x = 8$.

Similarly, the solution to:
$3 = 2x - 5$ is $x = 4$
$-9 = 2x - 5$ is $x = -2$.

Note that equations can be written in different forms:
$11 = 2x - 5$ is the same as $2x - 5 = 11$



To solve $2x - 5 = 11$ (or $11 = 2x - 5$) find the point on $y = 2x - 5$ that has a y-coordinate of 11. The x-coordinate of this point is the solution.

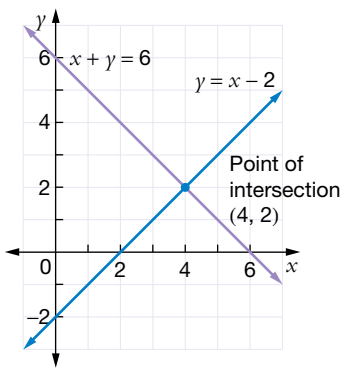
Using straight lines to solve equations involving *x* and *y*

In the diagram, the lines $x + y = 6$ and $y = x - 2$ intersect at the point (4, 2).

$4 + 2 = 6$ and $2 = 4 - 2$, so it is clear that (4, 2) is a solution to both equations.

Hence, $x = 4$, $y = 2$ is a solution to both equations.

When two lines cross, the point of intersection gives the only solution to both equations.



Exercise 11:04

P Foundation worksheet 11:04
Graphing straight lines

1 Give the coordinates of the point of intersection for each pair of lines.

a $x = 3$, $y = 2$

b $x = 3$, $y = x - 1$

c $x + y = 1$, $y = 2$

d $x = 3$, $y = -3$

e $y = -3$, $y = 3x + 3$

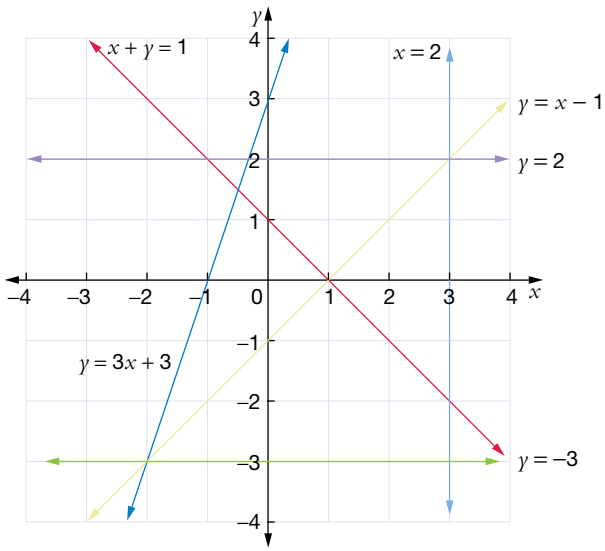
f $x + y = 1$, $y = x - 1$

g $x + y = 1$, $x = 0$

h $y = 3x + 3$, $y = x - 1$

i $y = x - 1$, $y = 0$

j $y = 2$, $y = x - 1$



Substitute the coordinates into both equations to check graphical solutions.



2 Graph each pair of lines and find their point of intersection.

a $x = -1$, $y = 1$

b $x = 2$, $y = 3$

c $x = -3$, $y = x$

d $y = x + 1$, $y = -3$

e $y = x$, $y = 2x$

f $y = x - 2$, $y = 2 - x$

g $y = 2x - 1$, $y = 5 - x$

h $x + y = 4$, $y = 3x - 4$

3 The graphs of four lines have been drawn on the number plane. Use the graphs to find the solution to each pair of equations.

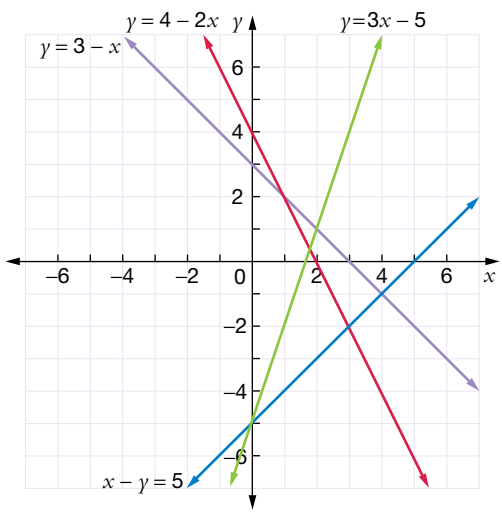
a $y = 3 - x$ and $y = 4 - 2x$

b $y = 3x - 5$ and $y = 3 - x$

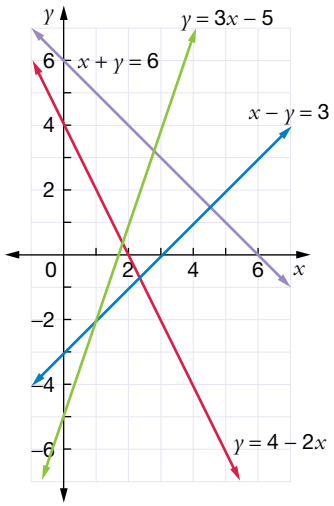
c $x - y = 5$ and $y = 4 - 2x$

d $y = 3 - x$ and $x - y = 5$

e $y = 3x - 5$ and $x - y = 5$



4 Use the graph provided to estimate the coordinates of the point of intersection of the following pairs of lines. Test the accuracy of your estimates by substituting the values into both equations.



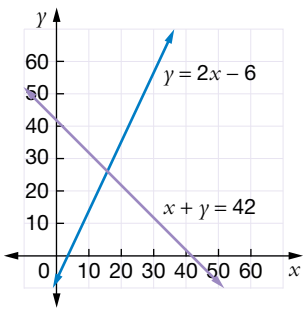
- **a** $x + y = 6$ and $x - y = 3$
- **b** $y = 3x - 5$ and $y = 4 - 2x$
- **c** $x - y = 3$ and $y = 4 - 2x$
- **d** $y = 3x - 5$ and $x + y = 6$

5 Graph each pair of lines and find the pair of numbers that is a solution to both equations.

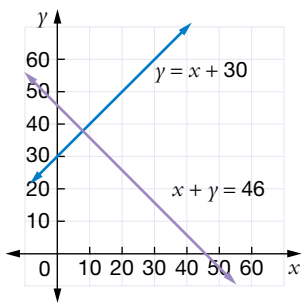
- **a** $x + y = 5$ and $y = 2x - 7$
- **b** $y = 3 - 3x$ and $x + y = 5$
- **c** $x - y = 2$ and $x + y = 8$

6 In each of the following use the graph to estimate the solution to the equations. Check your solution by substituting the values into each equation.

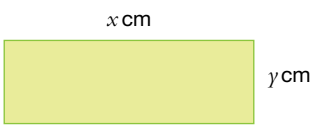
a

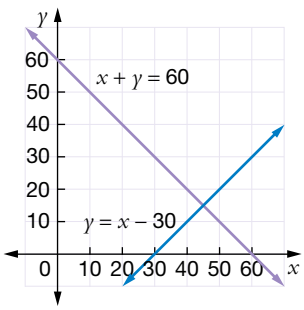


b



7 Alan was trying to find the length and breadth of a rectangle.

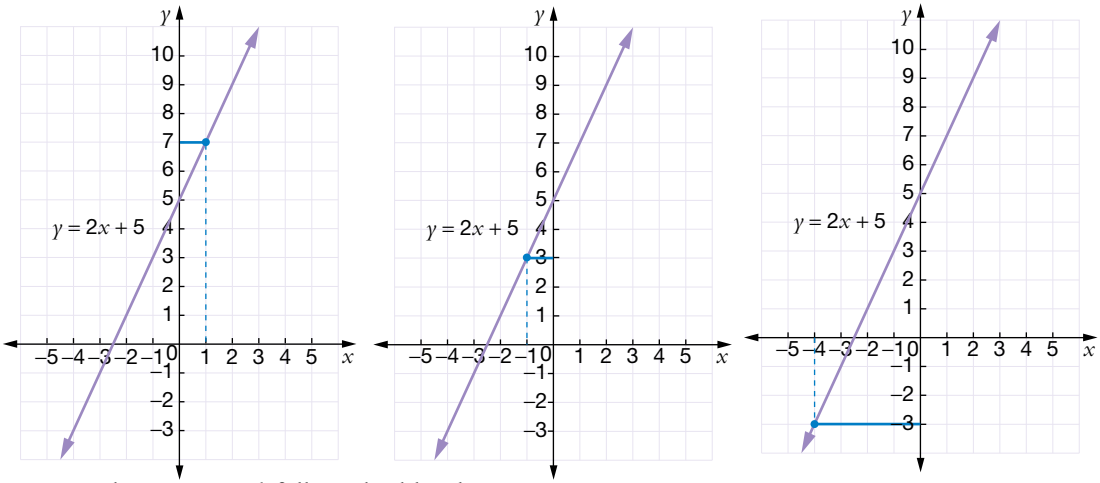




He knew that the perimeter of the rectangle was 120 cm and that it was 30 cm longer than it was wide. Rather than guess the answer he thought he would use graphs to find the length and breadth. He wrote the equations $x + y = 60$ and $y = x - 30$ to describe what he knew about the rectangle and then graphed them to find the point of intersection. Where did his graphs intersect and what were the dimensions of the rectangle?

8 The graph of $y = 2x + 5$ is shown. Start on the y-axis and follow the blue lines to find the solution to each equation.

a $7 = 2x + 5$ **b** $3 = 2x + 5$ **c** $-3 = 2x + 5$

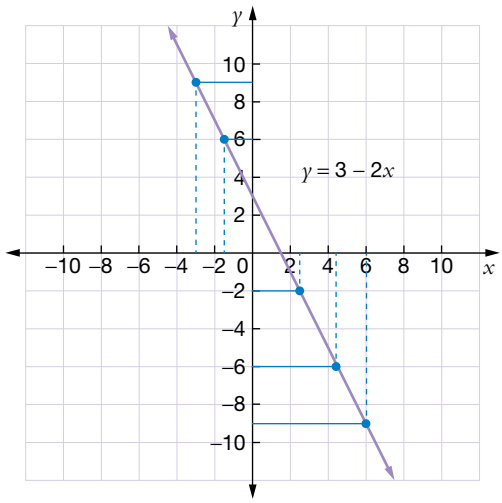


9 Start on the y-axis and follow the blue lines to find the solution to each of the following equations.

a $3 - 2x = 9$ **b** $3 - 2x = 6$

c $3 - 2x = -2$ **d** $3 - 2x = -6$

e $3 - 2x = -9$



Remember:
Equations can be written in different forms.
$3 - 2x = 9$ is the same as $9 = 3 - 2x$.

10 Use the graph in Question **9** to solve the following equations. (Note that the lines have not been marked.)

a $7 = 3 - 2x$ **b** $-5 = 3 - 2x$

c $3 - 2x = 10$ **d** $3 - 2x = -7$

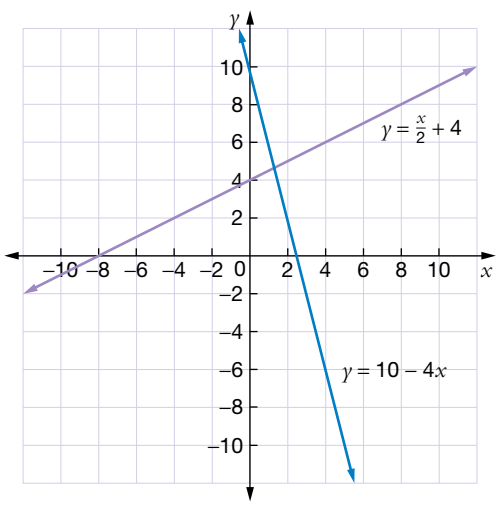
11 Use the graphs to find the solutions to the following equations.

a $6 = 10 - 4x$ **b** $2 = 10 - 4x$

c $10 - 4x = 0$ **d** $10 - 4x = -8$

e $8 = \frac{x}{2} + 4$ **f** $2 = \frac{x}{2} + 4$

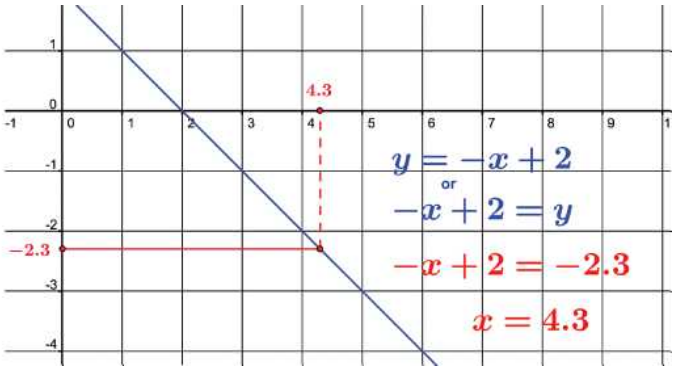
g $\frac{x}{2} + 4 = 3$ **h** $\frac{x}{2} + 4 = -1$



P GEOGEBRA ACTIVITY 11:04 USING GRAPHS TO SOLVE LINEAR EQUATIONS

This activity demonstrates the graphical method of solving equations. Vary the equation and then see the solution using input boxes and checkboxes.

The coordinates of a variable point are substituted into two equations. Observe the changes to the equations as the point is moved from one line to the other and to the intersection of the lines.



11:05 Non-linear graphs

The equations that we have met so far have produced straight lines when graphed. These are called linear graphs (or linear relationships). Not all graphs are straight lines. If the equation $y = x^2$ is plotted a curve is obtained. This curve is one of the many non-linear graphs.

Non-linear graphs:

- are plotted using a table of values
- need more than three points to draw an accurate graph
- can be drawn using graphing software.

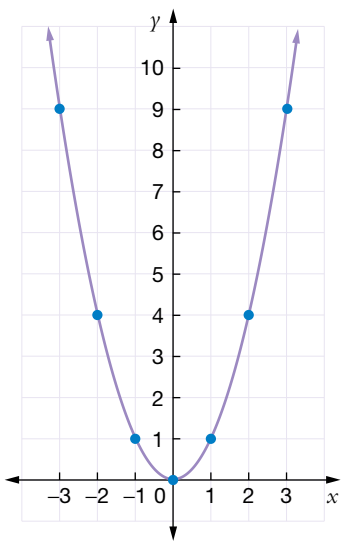
Exercise 11:05

1 **a** The graph of $y = x^2$ is shown. Complete the table of values for $y = x^2$.

x	−3	−2	−1	0	1	2	3
y							

b What would be the value of y when:

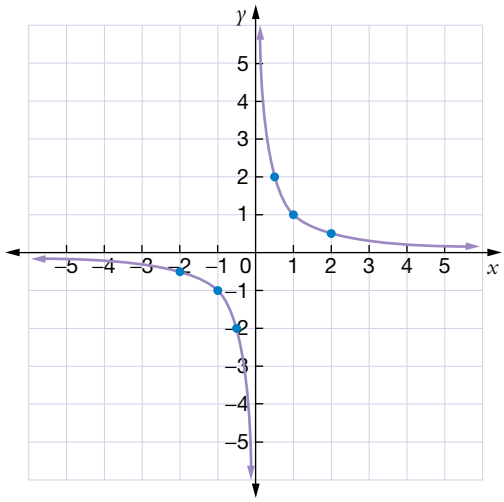
- **i** $x = 10$
- **ii** $x = -10$
- **iii** $x = 2{\cdot}5$
- **iv** $x = -2{\cdot}5$?



2 **a** The graph of $y = \frac{1}{x}$ is shown.

Complete the table of values for $y = \frac{1}{x}$.

x	-2	-1	$-0{\cdot}5$	0	$0{\cdot}5$	1	2
y							



Notice you can't have a y-value for $x = 0$. Do you know why?

b What would be the value of y when:

i $x = 4$

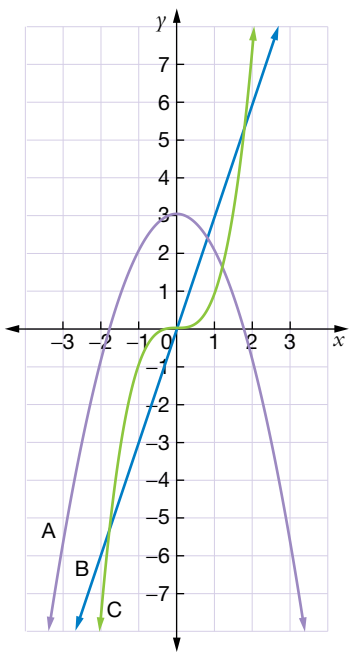
ii $x = -4$?

c Use a calculator to estimate the value of y, correct to one decimal place, when:

i $x = 1.5$

ii $x = -1.5$?

3 The graphs of $y = 3x$, $y = x^3$ and $y = 3 - x^2$ are shown.



a Complete the following tables of values.

i $y = 3x$

x	0	1
y		

ii $y = x^3$

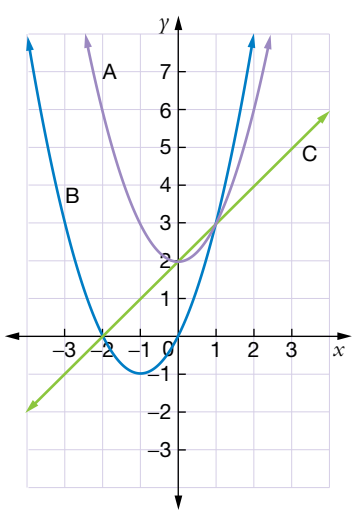
x	0	1
y		

iii $y = 3 - x^2$

x	0	1
y		

b Use your answers from part **a** to match each of graphs A, B and C to the equation that produced it.

4 The graphs of equations $y = x + 2$, $y = x^2 + 2$, and $y = x^2 + 2x$ are shown on the number plane.



a If $x = -1$, find the value of y when:

i $y = x^2 + 2x$

ii $y = x^2 + 2$

iii $y = x + 2$

b Use the values obtained in part **a** to find which equation produced each of the following graphs.

i Graph A

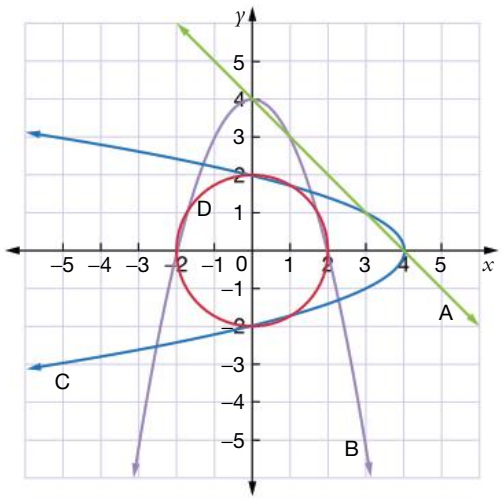
ii Graph B

iii Graph C

5 Similar-looking equations can produce very different-looking graphs. Match each of the equations below to its graph.

a $x + y = 4$ **b** $x^2 + y^2 = 4$

c $x + y^2 = 4$ **d** $x^2 + y = 4$



6 Select the equation of the straight line from each of the following sets of equations.

a $y = x^3, y = x - 3, y = x^2 - 3$ **b** $y = x^3, y = x^2, y = x$

c $x + y = 1, x^2 + y^2 = 1, x + y^2 = 1$

FUN SPOT 11:05 THE GAME OF SPROUTS

This game is played by two people. All you need is a pen and a piece of paper. Before the game begins, a number of crosses are drawn on the paper.

- The first player then joins any two crosses and places another cross somewhere on the line that has just been drawn.
- The second player then does the same.
- Each player then takes it in turns until one player can't go.

The two rules that must be observed are:

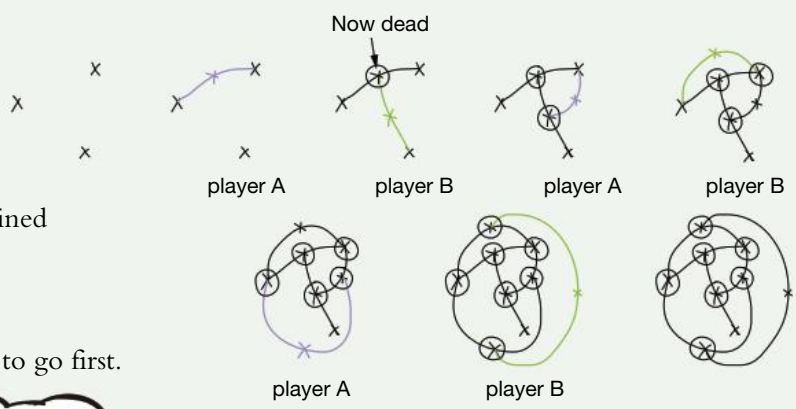
- A line cannot be drawn so that it cuts another line.
- If a cross has three lines coming from it, then it can't be used again because it is 'dead'.

A sample game is shown below, starting with three crosses.

It is now player A's turn. Although there are two crosses left, they can't be joined without crossing a line.

∴ Player B has won.

You should take it in turns to go first.





MATHS TERMS 11

axis (axes)
- the number lines on the number plane, used to give the position of points
- x-axis: the horizontal axis
- y-axis: the vertical axis

coordinates
- a pair of numbers that gives the position of a point on a number plane relative to the origin
- x-coordinate: the first of the coordinates. It tells how far right (or left) the point is from the origin
- y-coordinate: the second of the coordinates. It tells how far the point is above (or below) the origin

graph
- a line or curve drawn on the number plane
- to draw a line or curve onto a number plane using points that are solutions of an equation

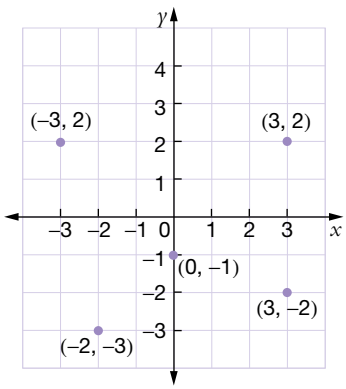
intercept
- the point where a graph cuts an axis
- x-intercept: the point where a graph cuts the x-axis
- y-intercept: the point where a graph cuts the y-axis

intersection
- the point where two graphs cross

linear equation
- the equation of a straight line, e.g. $y = 3x - 4$, $x + y = 5$
- coefficient: the number placed before (i.e. multiplying) a pronumeral (or pronumerals),
  e.g. 3 is the coefficient of $3x$ in $y = 3x - 4$
- constant term: the term containing no pronumerals,
  e.g. -4 is the constant term in $y = 3x - 4$.

non-linear equation
- an equation that produces a curve when plotted

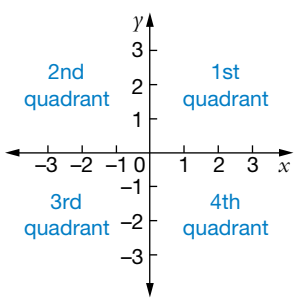
number plane
- a rectangular grid that allows the position of points in a plane to be identified by an ordered pair of numbers



origin
- the origin of a number plane is the point $(0, 0)$. This is the point of intersection of the x- and y-axes

plot
- to mark the position of a point on the number plane

quadrants
- the four quarters that the number plane is divided into by the x- and y-axes



DIAGNOSTIC TEST 11 COORDINATE GEOMETRY

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Complete these tables of values. 11:01

a $y = x + 2$

x	–1	0	1	2
y				

b $y = 4 - x$

x	–1	0	1	2
y				

c $y = 2x - 3$

x	1	2	3	4
y				

2 Graph each of the equations in Question 1. 11:02

3 Work out an equation for each table of values. 11:01

x	–1	0	1	2
y	3	4	5	6

x	0	1	2	3
y	5	4	3	2

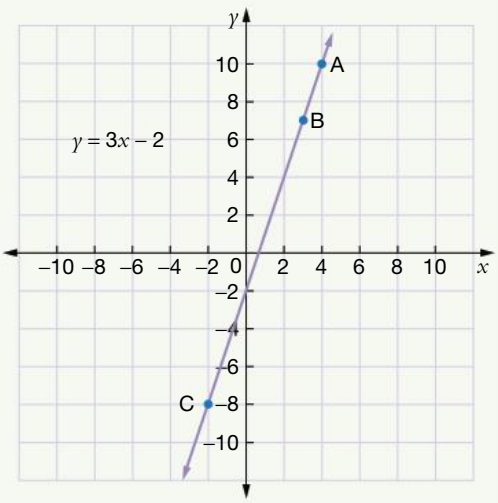
x	–2	0	2	4
y	–1	0	1	2

4 State whether the values given for x and y make the equation *true* or *false*. 11:01

a $y = 2x - 1$
$x = 2, y = 3$

b $y = 1 - 2x$
$x = 2, y = 3$

c $2x - y = 8$
$x = -5, y = -18$



5 Write down the solutions to the equation $y = 3x - 2$ given by the coordinates of the three coloured points in the graph above. 11:01

a A **b** B **c** C

6 Complete the following solutions for the equation $y = 3x - 2$. 11:01

a $x = 8, y =$ _____ **b** $x = -10, y =$ _____ **c** $x = 1{\cdot}8, y =$ _____

7 State what would be similar about the graphs of each set of lines. 11:02

a $y = 2x$, $y = 3x - 1$, $y = 5x - 4$

b $y = -3x$, $y = -2x + 4$, $y = -x + 5$

c $y = 4x$, $y = 4x + 5$, $y = 4x - 3$

8 Graph each of the following lines. 11:03

a $y = 2$ **b** $x = 3$ **c** $y = -1$

11:03

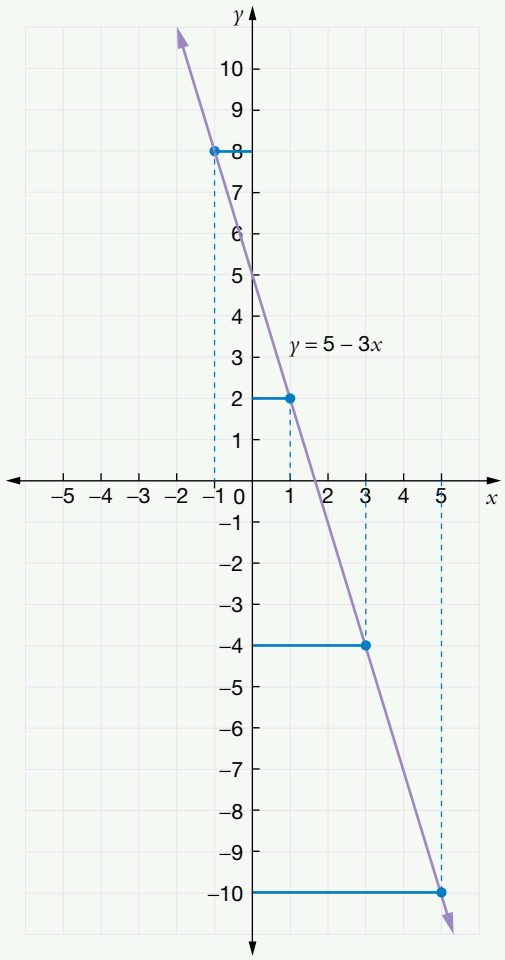
9 Use the graph to write down the solution to the following equations.

a $8 = 5 - 3x$

b $2 = 5 - 3x$

c $5 - 3x = -4$

d $5 - 3x = -10$



11:04

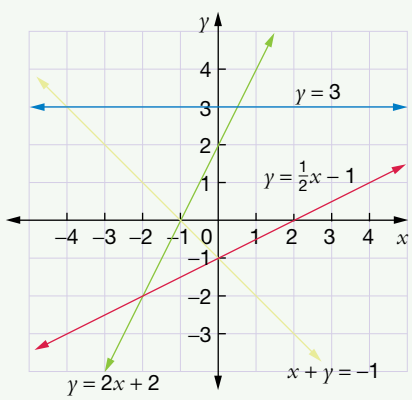
10 Write down the coordinates of the point of intersection of each of the following pairs of lines.

a $y = 3$
$x + y = -1$

b $x + y = -1$
$y = 2x + 2$

c $y = 2x + 2$
$y = \frac{1}{2}x - 1$

d $x + y = -1$
$y = \frac{1}{2}x - 1$



11:05

11 Select the equation of the straight line from each of the following sets of equations.

a $y = 3x$, $y = x^2 + 2$, $y = 2x^3$

b $y - x^3 = 3$, $y = -x^2$, $y = x$

c $x^2 + y^2 = 2$, $x + y = 0$, $x - y^2 = 1$

ASSIGNMENT 11A Chapter review

1 Complete each table of values using the equation indicated.

a $y = 2x + 1$

x	0	1	2
y			

b $y = \frac{x}{3} - 4$

x	0	1	2
y			

2 Complete these solutions to the equation $y = 8 - x$.

a $x = 3, y =$ _____

b $x = -10, y =$ _____

c $x = 15, y =$ _____

d $x = 3{\cdot}4, y =$ _____

3 Find the equation linking x and y.

a

x	1	2	3	4
y	10	20	30	40

b

x	1	2	3	4
y	6	11	16	21

4 Graph the equations $y = 2x - 3$ and $y = 6 - x$ on the same number plane. Find the x- and y-value that is a solution of both equations.

5 From the following list of equations choose the line or lines that:

a increase, as x increases in value

b decrease, as x increase in value

c has the steepest slope

d the flattest slope

e are parallel

f all pass through the point $(0, -3)$.

$y = 2x - 3$ $\quad$ $y = -x + 6$

$y = -2x + 3$ $\quad$ $y = 4x - 3$

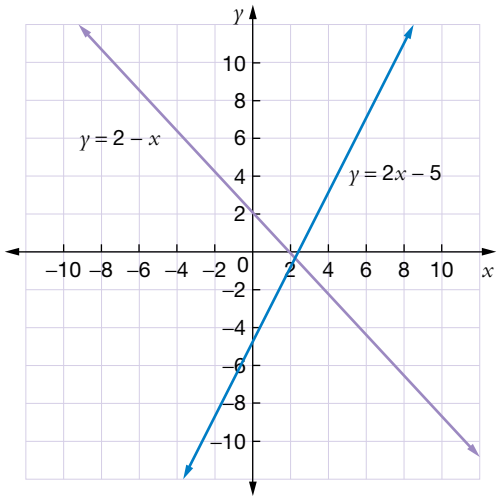
$y = 5x - 2$ $\quad$ $y = -3x + 2$

$y = 4x$ $\quad$ $y = 3x - 3$

6 Use the graphs to solve:

a $2 - x = -6$ $\quad$ **b** $2 - x = 4$

c $2x - 5 = 8$ $\quad$ **d** $2x - 5 = -7$

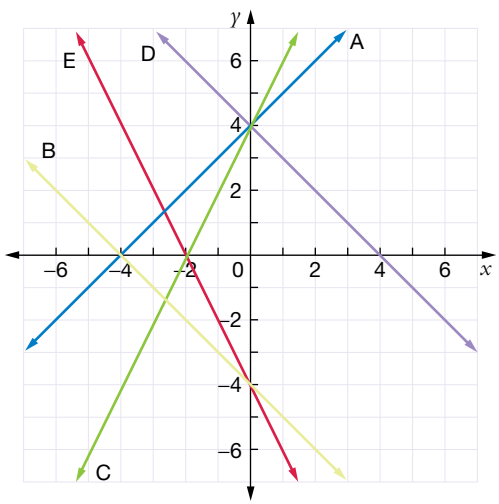


7 Match the graphs A to E to each of the following equations.

a $y = x + 4$ $\quad$ **b** $y = 4 - x$

c $y = -x - 4$ $\quad$ **d** $y = 2x + 4$

e $y = -2x - 4$



ASSIGNMENT 11B Working mathematically

1 Refer to **ID Card 6** on page xix to identify the figures numbered:

a 8 **b** 10 **c** 11
d 12 **e** 14 **f** 15
g 16

2 A 7 m rope is cut in half and one half is kept. This is cut in half and, again, one half is kept. The remaining piece of rope is cut into quarters and one of the quarters is kept. What length of the rope has finally been kept? (Give your answer in millimetres.)

3 A cake was cut into slices of equal size. Three-quarters of the cake was eaten and eight slices remained. Into how many slices was the cake originally cut?

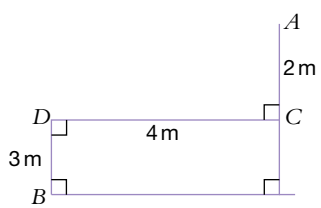
4 All boxes of Bright Tip toothpicks hold between 47 and 55 toothpicks. Heather, my two-year-old daughter, emptied a number of unused boxes of toothpicks. These she put in a pile. If there were 386 toothpicks, how many boxes had Heather emptied?

5 A garden is rectangular. Its perimeter is 15 m and it is known to be four times longer than it is wide. Find its length.

6 Eight people volunteered for a medical experiment involving two people. How many different pairs of volunteers could be chosen?

ASSIGNMENT 11C Cumulative revision

1 Use Pythagoras' theorem to calculate the distance from A to B, correct to one decimal place.



3:02–3:04

2 **a** The council rates on a property are 0·38% of its value. What are the rates on a property valued at $680 000?

b The owner of a shop sells goods for their cost plus 60%. How much will he sell a cup for if he pays $12 for it?

4:03, 4:04

3 **a** A triangle has angles whose sizes are in the ratio 1 : 2 : 3. Find the sizes of the angles.

b Sketch the triangle in part **a**.

10:03

4 The probability of throwing a total of 5 or less using three dice is $\frac{5}{108}$. What is the probability of throwing a total that is more than 5?

6:03

5 Simplify:

a $6(a + b) - b$ **b** $3x \times 8x$ **c** $6(7 - 2b) - 12b$ **d** $6x \times -3$

5:02, 5:03, 5:08

6 Find:

a $\sqrt{13^2 - 12^2}$ **b** $\sqrt{(13-12)^2}$ **c** $\dfrac{6-51}{-7+2}$ **d** $\dfrac{6-51}{-7-2}$

2:02

7 **a** A rectangle that is 20 cm long has an area the same as a square of side 15 cm. What is the breadth of the rectangle?

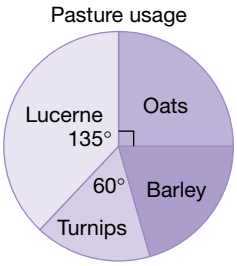
b A rectangle has an area of 120 cm^2. If its length is 15 cm, what is its breadth?

8:02

8 **a** What percentage of the total pasture is taken up by oats?

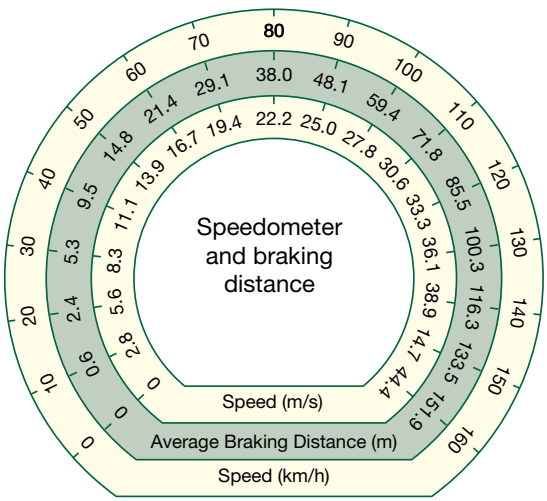
b Calculate the percentage of the total pasture that is taken up by barley.

c If the farm has a total size of 150 hectares, how many hectares are planted with turnips?



7:04

9 The speedometer shows an estimate of the braking distance of a car for speeds up to 160 km/h. The innermost scale shows the speed in metres per second.



a What is the average braking distance if the speed is:

i 30 km/h

ii 100 km/h

iii 25 m/m

iv 2·8 m/m

b At what speed (in km/h) is the braking distance:

i 21·4 m

ii 48·1 m

iii 85·5 m

iv 38·0 m

c At what speed (in m/s) is the braking distance:

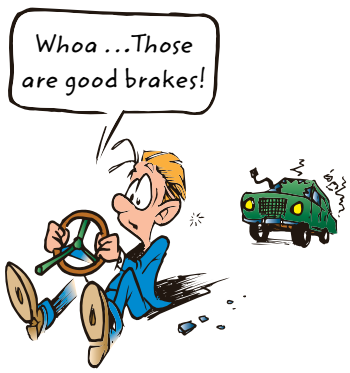
i 2·4 m **ii** 14·8 m

iii 9·5 m **iv** 151·9 m

d Use the speedometer to change 60 km/h into m/s.

e Use the speedometer to change 22·2 m/s into km/h.

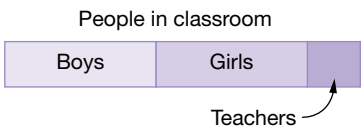
10:06



10 **a** What kind of graph is this?

b What percentage of people in the classroom are boys, correct to the nearest per cent? (Hint: Measure the graph.)

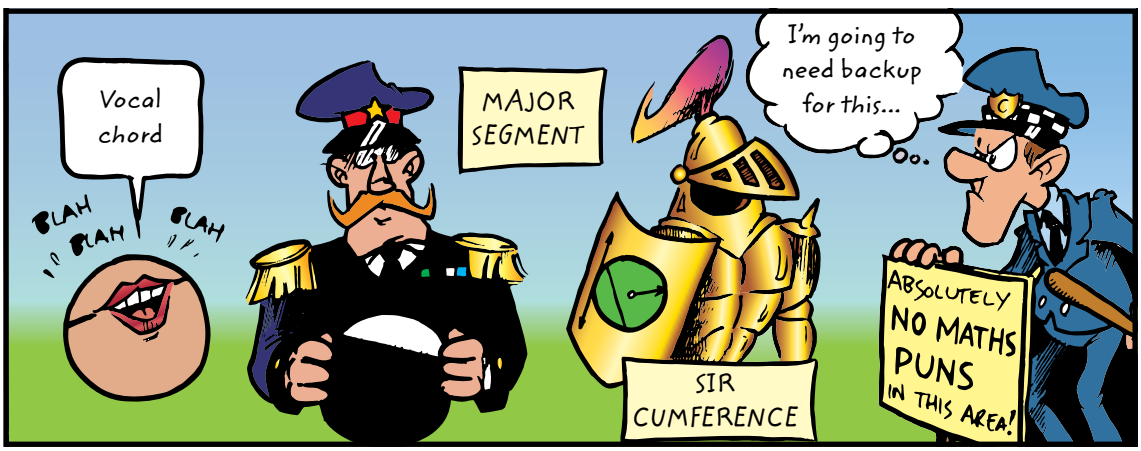
c How many teachers are in the classroom if there are 40 students?



7:03

CIRCLES AND CYLINDERS

12



Contents

12:01 Parts of a circle
12:02 Circumference of a circle
Investigation 12:02 The circumference of a circle
12:03 Solving problems involving circumference
12:04 Area of a circle
Investigation 12:04 The area of a circle
Fun spot 12:04 Do fewer bumps mean a smoother ride?
12:05 Area of a sector
12:06 Volume of a cylinder
Investigation 12:06 The cylinder
Challenge 12:06 Pythagoras' theorem and circles
Fun spot 12:06 What did the dirt say when it saw the clouds?
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Measurement and Geometry

Selections from ***Length***, ***Area***, ***Volume*** [Stage 4]

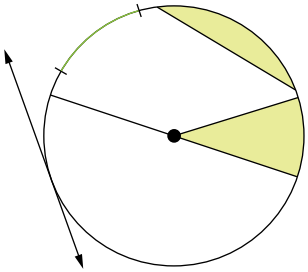
- Investigate the concept of irrational numbers, including π. (ACMNA186)
- Investigate the relationship between features of circles, such as the area and the radius; use formulas to solve problems involving area. (ACMMG197)
- Calculate the volumes of cylinders and solve related problems. (ACMMG217)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

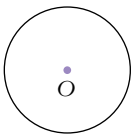
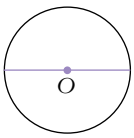
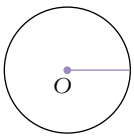
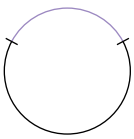
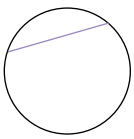
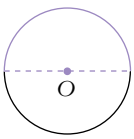
From earliest times, the circle has fascinated people around the world. The Sun and Moon, which both appear as circles from the Earth, have been worshipped. The circle has been studied since early Greek times and we now use circles to make sense of our world. Positions on the Earth's surface are described in terms of intersecting circles.

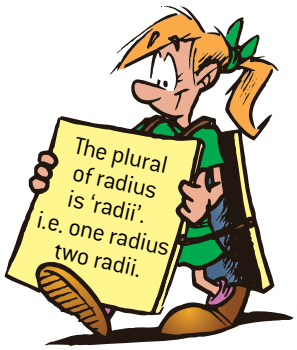
How many parts of this circle can you name?



12:01 Parts of a circle

It is important to know what the different parts of a circle are called. They are summarised in the table below.

Diagram	Name	Definition
1 	Circle	The set of all points that are a fixed distance from a point called the **centre** O.
2 	Diameter	An interval that divides a circle in half and passes through the centre.
3 	Radius	An interval drawn from the centre to the circle. It is half a diameter.
4 	Arc	Part of a circle.
5 	Chord	An interval joining two points on the circle.
6 	Semicircle	Half of a circle.



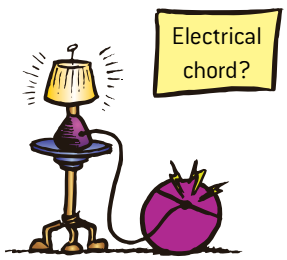
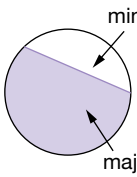
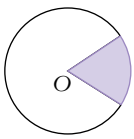
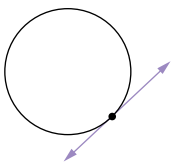
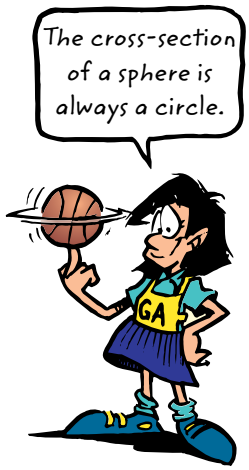
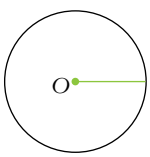


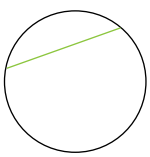
Diagram	Name	Definition
7 	Segment	A chord cuts a circular region into two segments: a minor segment and a major segment.
8 	Sector	The section bounded by two radii and the included arc of a circle.
9 	Tangent	A line that touches a circle at only one point.

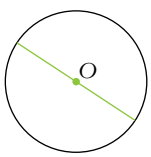


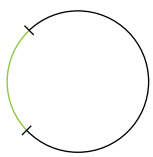
Exercise 12:01

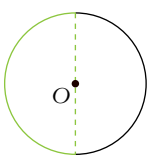
1 Name the feature shown in green, on each circle.

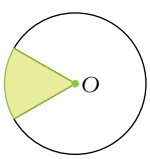
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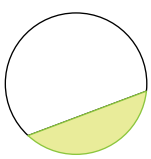
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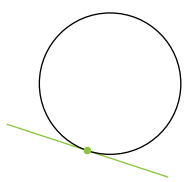
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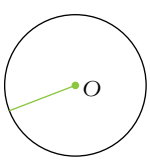
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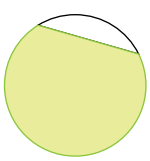
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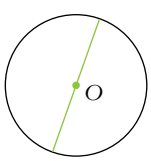
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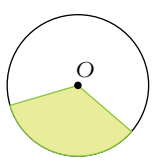
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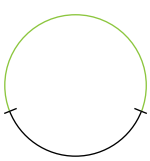
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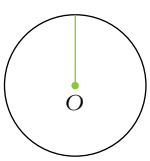
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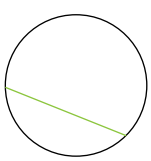
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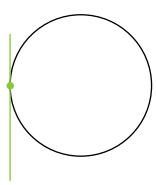
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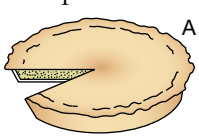
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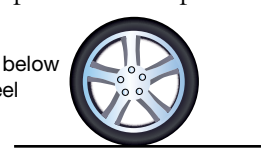
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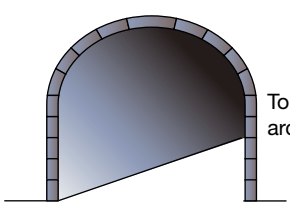
o 

p 

2 Which part of a circle is shown in these practical examples?

a  A piece of pie

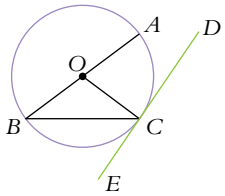
b Road below a wheel 

c  Top of an archway

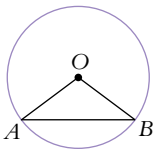
3 Name the feature shown by interval:

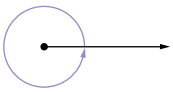
a AB **b** OC

c BC **d** DE



4 If O is the centre of the circle, what type of triangle must ΔAOB be? Why?

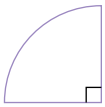




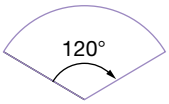


5 What fraction of the area of a complete circle is each of these sectors?

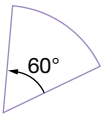
a



b

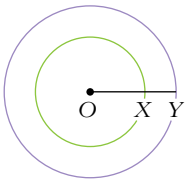


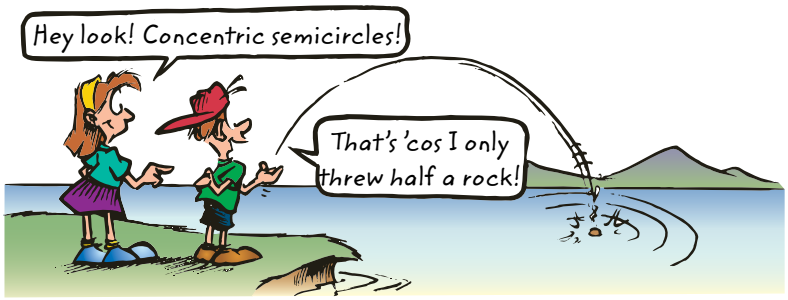
c



$$\text{Fraction} = \frac{\text{angle}}{360^\circ}$$

6 If two or more circles are drawn with the same centre but different radii, they are called **concentric** circles. If radius $OX = 5$ cm and radius $OY = 8$ cm, what must be the distance XY between the two circles?

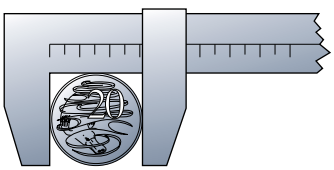


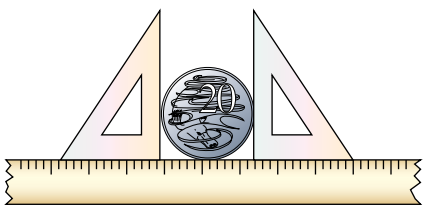


12:02 Circumference of a circle

The perimeter of a circle is called the *circumference*.

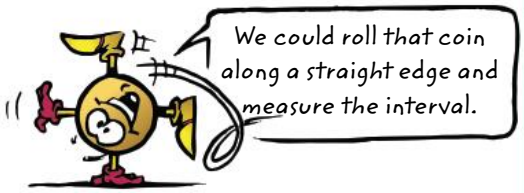
It is usually easier to measure the diameter of a circle than its circumference, using either a special gauge or a method like that shown below with a ruler and set squares.





WORKED EXAMPLES

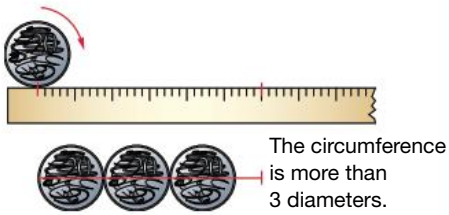
1 Use a practical method to measure the circumference of a 20-cent coin.



Step 1
Mark a point on the circumference of a 20-cent coin.

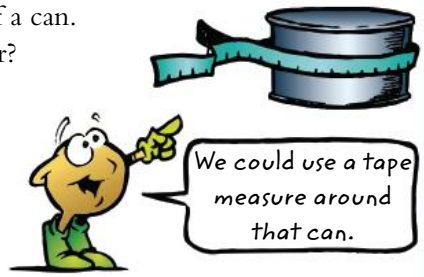
Step 2
Place the coin beside a ruler with the mark pointing to the scale.

Step 3
Roll the coin along the ruler until the mark returns again to the scale. Make sure that the coin does not slip.



Step 4
Measure the circumference.
Circumference ≑ 8·8 cm

2 Use a practical method to measure the circumference of a can. Is the circumference more than three times the diameter?



Step 1
Measure the circumference with a tape measure. Also measure the diameter.

Step 2
Divide the circumference by the diameter.
Circumference > 3 × the diameter

The larger the diameter of a circle, the greater is its circumference. The following investigation will help you to discover the special relationship that exists between the circumference of a circle and its diameter.

INVESTIGATION 12:02 THE CIRCUMFERENCE OF A CIRCLE

1 Step 1 Tie a piece of chalk to the end of a long string.

Step 2 Cut the string so that the chalk is 1 m away from the other end of the string when stretched.

Step 3 In the playground, have a friend hold one end while you stretch the string and draw a circle.



Step 4 Lay string around the circle you have drawn and then measure the length of the string to find the length of the circumference.

Step 5 How many diameters would fit around the circumference?

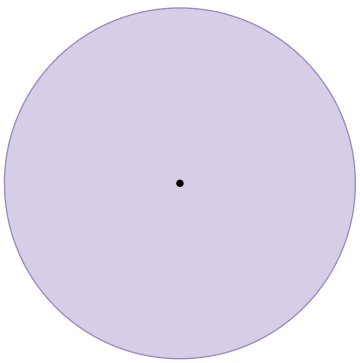
2 The diameter and circumference of five circular items were measured and the results recorded in the table below. Complete the third column of the table, giving your answers correct to one decimal place.

Diameter (d)	Circumference (C)	Circumference ÷ Diameter ($C \div d$)
2 cm	6·2 cm	
5 cm	15·7 cm	
3·2 cm	10 cm	
8 cm	25 cm	

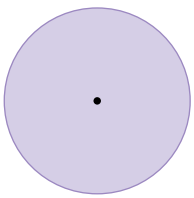


3 Use string or cotton to trace around each circle to measure its circumference. Also measure the diameter of each circle and complete the table given. (Give measurements to the nearest millimetre.)

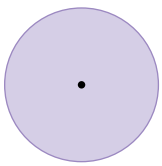
a



b



c



Diameter (d)	Circumference (C)	Circumference ÷ Diameter ($C \div d$)
a		
b		
c		

4 Find some large circles in the environment (tree trunks, water tanks, etc.), estimate their circumference, record these and then check by measurement.

The number π (pi)

- From Investigation 12:02 we can conclude that for all circles:

$$\frac{\text{length of circumference } (C)}{\text{length of diametre } (d)} \div 3{\cdot}14$$

- The value of this ratio is 3·141 592 654 ...
- The decimal continues forever, without ever repeating.
- This ratio is used in many calculations, so it is given the symbol π (the Greek letter *pi*). $\frac{C}{d} = \pi$
- The formula is usually written as $C = \pi d$.
- Sometimes the formula for the circumference is given in terms of the radius, r.

  $C = \pi d$

  $d = 2r$

  $C = \pi \times 2r$ or $C = 2\pi r$



Remember:
÷ means 'is approximately equal to'.

- The number π cannot be given an exact decimal or fraction value. It is therefore called an irrational number. (Other examples of irrational numbers are roots, such as $\sqrt{2}, \sqrt{3}, \sqrt[3]{5}$, etc.) In calculations involving π we must use an approximation.
- There is a π button on your calculator. A 10-figure calculator will give $\pi \div 3{\cdot}141\,592\,654$.



$\pi \div 3{\cdot}14$ or $3{\cdot}142$ (as a decimal)
OR
$\pi \div 3\frac{1}{7}$ or $\frac{22}{7}$ (as a fraction)

The definition of π

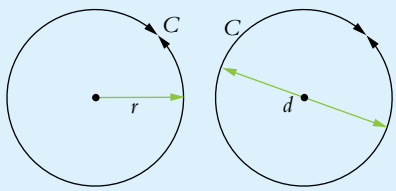
The number π is the ratio of the circumference to the diameter of any circle.

$$\pi = \frac{C}{d}$$

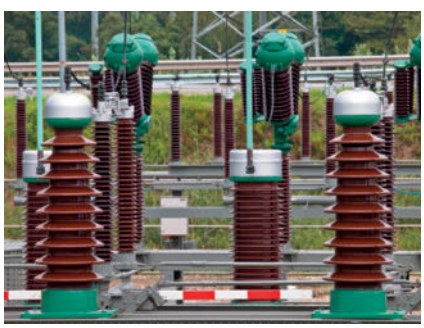
The circumference C of a circle is given by the formula:

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$

where d is the diameter and r is the radius.



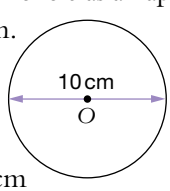
- The value of $\pi \div 3{\cdot}141\,592\,654$.
- π is the Greek letter equivalent to 'p'. It is the first letter in the Greek word *perimetron*, which means perimeter.
- If an exact answer is required, we leave π in the answer (e.g. 7π cm).



WORKED EXAMPLES

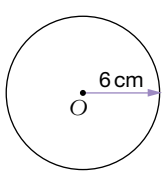
1 Find the circumference of each circle as an approximation using $\pi = 3{\cdot}14$ and as an exact answer.

a Here, the diameter is given.



$C = \pi d$
$= 3{\cdot}14 \times 10$
$= 3{\cdot}14$
$\therefore$ Circumference $= 31{\cdot}4$ cm
The exact answer is 10π cm.

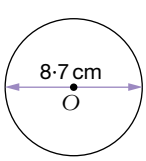
b Here, the radius is given.



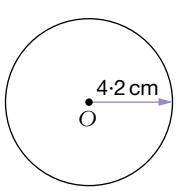
$C = 2\pi r$
$= 2 \times 3{\cdot}14 \times 6$
$= 37{\cdot}68$
$\therefore$ Circumference $= 37{\cdot}68$ cm
The exact answer is 12π cm.

2 In most cases, it is much easier to use a calculator, especially one with a π button.

a $C = \pi d$
$= \pi \times 8{\cdot}7$
Press: [π] [×] 8.7 [=]
$= 27{\cdot}331\,856\,09$
$= 27{\cdot}3$ cm (1 dec. pl.)

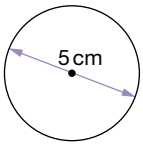


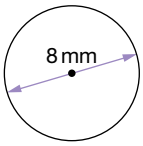
b $C = 2\pi r$
$= 2 \times \pi \times 4{\cdot}2$
Press: 2 [×] [π] [×] 4.2 [=]
$= 26{\cdot}389\,378\,29$
$= 26{\cdot}4$ cm (1 dec. pl.)

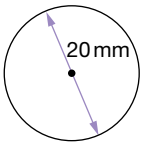


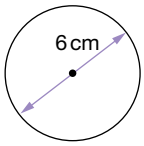
Exercise 12:02

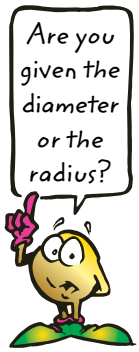
1 Find the circumference of each circle as an approximation using $\pi = 3{\cdot}14$, and as an exact answer.

a 

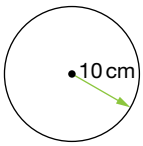
b 

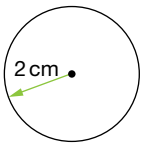
c 

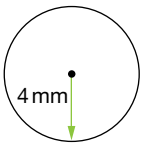
d 

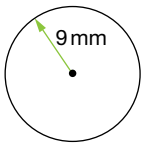


2 Calculate the circumference of each of these circles, using $\pi = 3{\cdot}14$.

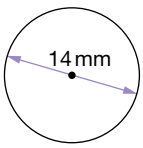
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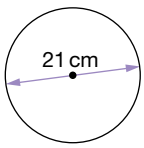
b 

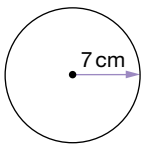
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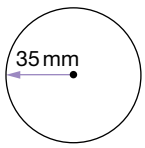
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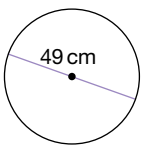
3 Using the approximation $\pi = \frac{22}{7}$, find the circumference of each circle.

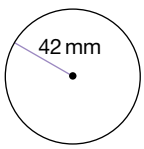
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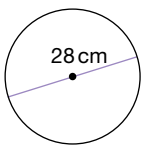
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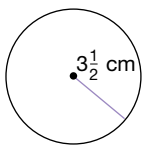
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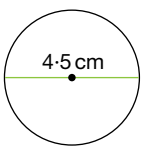
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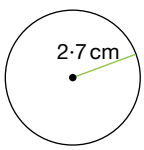
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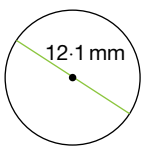
g 

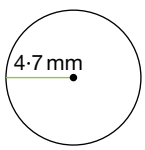
h 

4 Using a calculator with a π button, calculate the circumference of each of these circles, correct to one decimal place.

a 

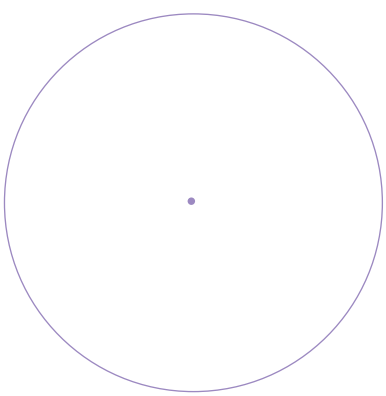
b 

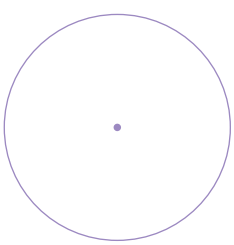
c 

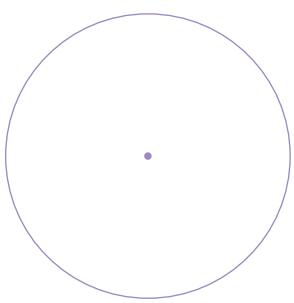
d 

e Diameter = 8·7 cm
f Diameter = 25 mm
g Diameter = 32·7 cm
h Radius = 1·9 cm
i Radius = 3·8 mm
j Radius = 15·6 cm

5 Measure the diameter of each of these circles and then calculate the circumference, to the nearest centimetre.

a 

b 

c 

The length of an arc

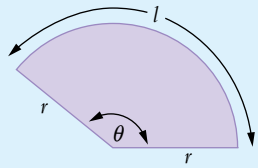
To find the arc length of a sector, l, first find what fraction the sector is of the circle by dividing the sector angle θ by 360°.

Then find this fraction of the circumference.

e.g. If a sector has radius 15 cm and the angle at the centre is 10°, then:

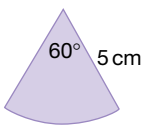
arc length $= \frac{\theta}{360°} \times 2\pi r$
$= 2{\cdot}62$ cm (2 dec. pl.)

$l = \frac{\theta}{360°} \times 2\pi r$

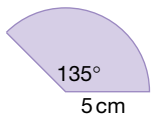


6 Use the formula $l = \frac{\theta}{360°} \times 2\pi r$ to find the arc length of each sector. Answer correct to two decimal places.

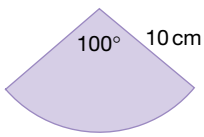
a



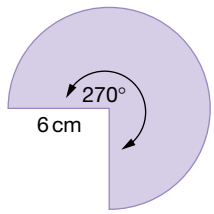
b



c



d



7 **a** A car tyre has a diameter of 60 cm. How far will the wheel travel in one revolution?

b How far, in metres, will it travel in 1000 revolutions?

c How many complete revolutions are needed to travel 1 km?



8 If the radius of the Earth is about 6000 km, what is its circumference at the equator, to the nearest 1000 km?

If the circumference is about 3 times the diameter, then the diameter is about $\frac{1}{3}$ of the circumference.

Note: If $C = \pi d$, then $d = \frac{C}{\pi}$ or $d = C \div \pi$.

9 Find the diameter of a circle, correct to one decimal place, which has a circumference of:

a 20 cm **b** 50 mm **c** 35 cm **d** 15·2 mm

10 The circumference of a tree trunk (its girth) was measured to be 5·2 m. What would be the diameter and radius of the cross-section at this point? Answer in metres, correct to one decimal place.

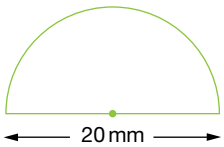
11 Ryan wanted to calculate the thickness of a metal light pole. Using a tape measure, he found that the circumference was 42 cm. What was the thickness to the nearest centimetre?

12:03 Solving problems involving circumference

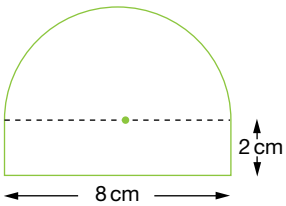
WORKED EXAMPLES

Determine the perimeters of these shapes, correct to one decimal place.

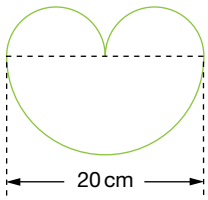
1



2



3



Solutions

1 Perimeter $= \frac{1}{2}$ circle + diameter

$\therefore P = \frac{1}{2}(\pi d) + 20$

$= \frac{1}{2}(\pi \times 20) + 20$

$= 31{\cdot}415\ldots + 20$

$= 51{\cdot}4$ mm (1 dec. pl.)

2 Perimeter = circle + 3 sides of rectangle

$\therefore P = \frac{1}{2}(\pi d) + 2 + 8 + 2$

$= \frac{1}{2}(\pi \times 8) + 12$

$\frac{1}{2} = 12{\cdot}566\ldots + 12$

$= 24{\cdot}6$ cm (1 dec. pl.)

3 The perimeter of this shape is made up of three semicircles, two with a diameter (d_1) of 10 cm and one with a diameter (d_2) of 20 cm.

$P = 2(\frac{1}{2}\pi d_1) + \frac{1}{2}\pi d_2$

$= \pi d_1 + \frac{1}{2}\pi d_2$

$= (\pi \times 10) + (0{\cdot}5 \times \pi \times 20)$ cm

$= 62{\cdot}8$ cm (1 dec. pl.)

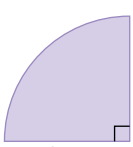


Exercise 12:03

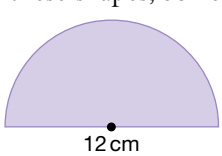
P Foundation worksheet 12:03 Solving circle problems

1 Calculate the perimeter of each of these shapes, correct to the nearest centimetre.

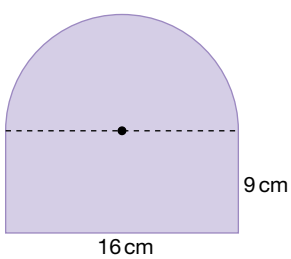
a



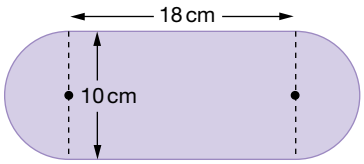
b



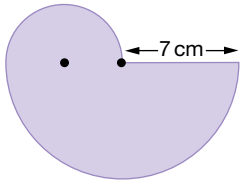
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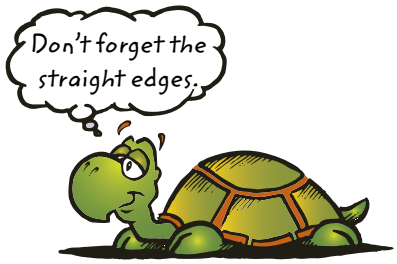
d



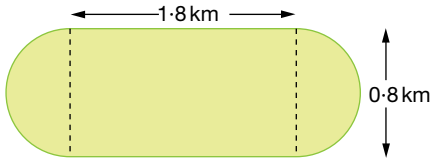
e



Use a calculator or $\pi = 3{\cdot}14$ in Questions **2** to **5**.



2 Cars race around a track with the measurements shown on the diagram. Find the length of a lap and the distance, to the nearest km, of a 20-lap race.



3 The minute hand of a clock is 9·5 cm long. How far does the tip of the hand travel in one day, correct to the nearest metre?

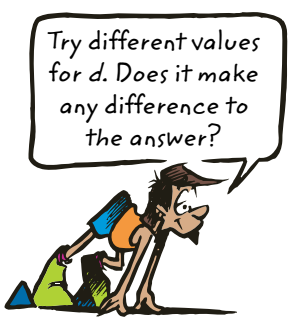
4 **a** A vinyl record has a diameter of 30 cm. How far would a point on the outside rim of the record travel in one revolution? Answer to the nearest centimetre.

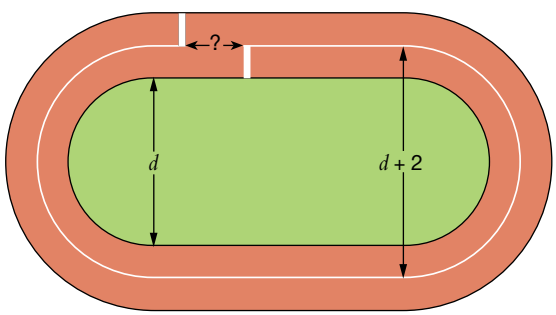
b When playing, the record revolves at a rate of 33 revolutions per minute (rpm). How far would the point travel in 10 minutes? Answer to the nearest metre.

5 If a trundle wheel has a circumference of 1 m, find its diameter, correct to the nearest mm.



6 When athletes run around a track with circular ends, they have a 'staggered start', since the perimeter of the outer lanes is greater. If the width of a lane is 1 metre, how much start should a runner in lane 1 give to the runner in lane 2, if the runner in lane 1 is to complete exactly one lap of the field?







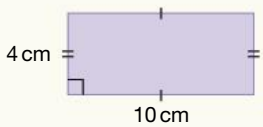
- Can you find out how much start the inside runner appears to give the outside runner on an official Olympic track for a 400 m event?

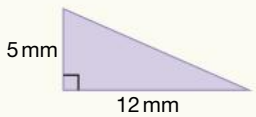
12:04 Area of a circle

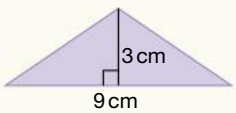
PREP QUIZ 12:04

1 If the diameter of a circle is 18 cm, what is its radius?
2 If the radius of a circle is 4·5 cm, what is its diameter?
3 What is the value of π, correct to one decimal place?
4 What is the formula for the circumference of a circle?

What is the area of each of these shapes?

5


6


7


What fraction of the area of a circle would each of these sectors be?

8


9

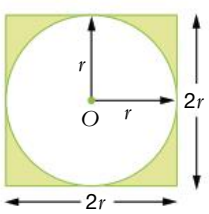

10


When we find the area of a shape, we divide it up into square units, such as square centimetres. This is easy for a shape like a rectangle, which has straight sides, but how can we find the area of a circle in terms of square units? By working through the following Investigation, you will discover a formula you can use to find the area of a circle.

INVESTIGATION 12:04 THE AREA OF A CIRCLE

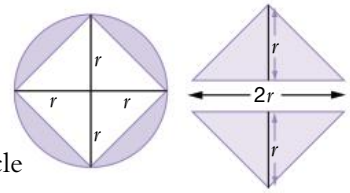
1 **a** This circle with a radius of r units has been drawn inside a square. Can you see that the sides of the square must be $2r$ units long? What is the area of the square?

b Which is larger, the area of the square or the area of the circle?



2 **a** Inside this circle with a radius of r units, a square has been drawn. The area of the square will be the same as two triangles that each has a base of $2r$ units and a height of r units. What is the area of the square?

b Which is larger, the area of the square or the area of the circle



From Questions **1** and **2** you should see that the area of a circle lies between $4r^2$ and $2r^2$.

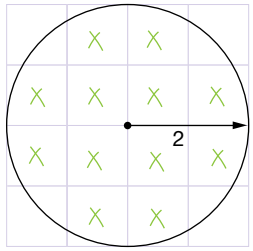
$2r^2 <$ area of a circle $< 4r^2$

So a reasonable approximation for the area of a circle might be:

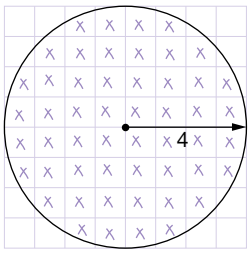
$A \div 3r^2$ square units



3 To gain an approximate value for the area inside a circle, we could 'count squares', including only those for which more than half of the square lies inside the circle. This circle has a radius of 2 units. If we count the squares marked, what is an approximate value for the area of this circle? (How does this compare with the approximate formula $A \div 3r^2$?)

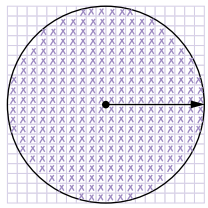


4 The method in Question **3** can be made more accurate by using smaller square units.



a Carefully count the squares marked in the circle on the left. What is the approximate area of this circle in square units? How does this compare with $A \div 3r^2$?

b The number of marked squares in the circle on the right, with a radius of 10 units, is 316. If $A \div 3r^2$, what is the value of A when $r = 10$?



5 This way of investigating the area of a circle is to slice it into sectors (Figure 1) and then arrange them as in Figure 2.

Figure 1

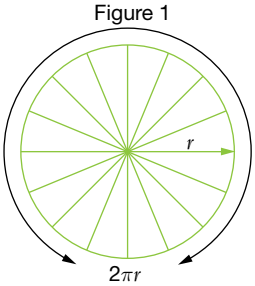


Figure 2

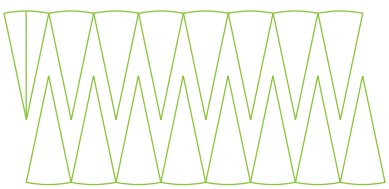
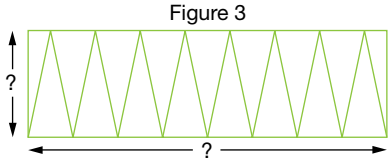


Figure 3



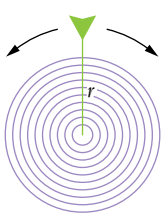
By taking half of the sector on one end and placing it on the other end, we obtain a figure that looks very much like a rectangle, as in Figure 3.

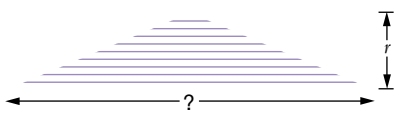
a Now the length of this rectangle would be half the circumference of the circle. What is this length?

b What would be the breadth of the rectangle?

c Since the area of a rectangle is length × breadth, what would be the area of this rectangle (which, of course, would be the same as the area of the circle)?

6 Imagine the area inside a circle to be a series of rubber rings. If the rings were cut along a radius and allowed to fold out flat, the layers would form a triangle.







a What is the formula for the area of a triangle?

b The height of the triangle would be r, the radius of the circle. What would be the length of the base of the triangle?

c What then would be the area of the triangle?

d What would be the area of the circle?

In this investigation, we discovered that the area of a circle is approximately 3 × radius squared.

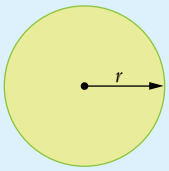
$A \doteq 3r^2$

Questions **5** and **6** led to the discovery of the following formula.



The area of a circle with a radius of r units is $A = \pi r^2$.

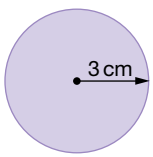
The radius of a circle, if the area is given, is $r^2 = \frac{A}{\pi}$.



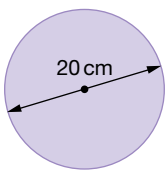
WORKED EXAMPLE 1

Find the areas of these circles as an approximation using $\pi = 3{\cdot}14$, and as an exact answer.

a $A = \pi r^2$
$= 3{\cdot}14 \times 3^2$
$= 3{\cdot}14 \times 9$
$= 28{\cdot}26$
$\therefore$ Area $\doteq 28{\cdot}26\,\text{cm}^2$ (2 dec. pl.)
Exact answer is $9\pi\ \text{cm}^2$.



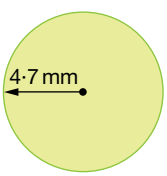
b Here, radius = 10 cm
$A = \pi r^2$
$= 3{\cdot}14 \times 10^2$
$= 3{\cdot}14 \times 100$
$= 314$
$\therefore$ Area $\doteq 314\,\text{cm}^2$
Exact answer is $100\pi\,\text{cm}^2$.



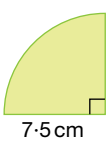
WORKED EXAMPLE 2

Using a calculator, find the areas of these shapes, correct to one decimal place.

a $A = \pi r^2$
$= \pi \times (4{\cdot}7)^2$
Press: 4.7 x^2 × π =
$\therefore$ Area $\doteq 69{\cdot}4\,\text{mm}^2$
(1 dec. pl.)



b For a full circle,
$A = \pi r^2$
$= \pi \times (7{\cdot}5)^2$
This is $\frac{1}{4}$ or a circle, so
$A = \frac{\pi \times (7{\cdot}5)^2}{4}$
Press: 7.5 x^2 × π ÷ 4 =
Area $\doteq 44{\cdot}2\,\text{cm}^2$ (1 dec. pl.)



WORKED EXAMPLE 3

Find the radius of a circle of area $100\,\text{m}^2$.

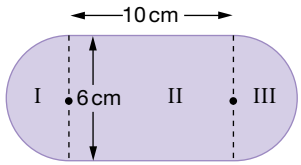
$r^2 = \frac{A}{\pi}$
$= \frac{100}{\pi}$
$\therefore r^2 \doteq 31{\cdot}831$
$r \doteq \sqrt{31{\cdot}831}$
$\therefore r \doteq 5{\cdot}64$ m (2 dec. pl.)

Units of area
$1\,\text{cm}^2 = 100\,\text{mm}^2$
$1\,\text{m}^2 = 10\,000\,\text{cm}^2$
$1\,\text{ha} = 10\,000\,\text{m}^2$
$1\,\text{km}^2 = 1\,000\,000\,\text{m}^2 = 100\,\text{ha}$

WORKED EXAMPLE 4

Calculate the areas of these composite shapes, correct to one decimal place.

a



Area of rectangle $= lb$

$\therefore$ Area II $= 10 \times 6$

$= 60\,\text{cm}^2$

Area of I + III $= \pi r^2$

(where $r = 3\,\text{cm}$)

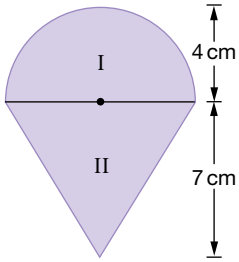
$\therefore$ Area of I + III $= \pi \times 3^2$

$= 28{\cdot}3\,\text{cm}^2$

$\therefore$ Total area $\div 60 + 28{\cdot}3$

$\div 88{\cdot}3\,\text{cm}^2$ (1 dec. pl.)

b



Area of I $= \frac{1}{2}\pi r^2$ (semicircle)

$= \frac{1}{2} \times \pi \times 4^2$

$= 25{\cdot}1\,\text{cm}^2$

Area of II $= \frac{1}{2}bh$ (Can you see that $h = 7$ cm, $b = 8$ cm?)

$= \frac{1}{2} \times 8 \times 7$

$= 28\,\text{cm}^2$

$\therefore$ Total area $\div 25{\cdot}1 + 28$

$\div 53{\cdot}1\,\text{cm}^2$ (1 dec. pl.)

Just for interest

Sometimes a different formula is used to find the area of a circle.

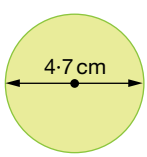
The radius is half the diameter, so $r = \frac{d}{2}$. If we now change the formula $A = \pi r^2$ by substituting $\frac{d}{2}$ for r, we now have the formula $A = \frac{\pi d^2}{4}$.



We can calculate the area of this circle using this formula.

$A = \frac{\pi d^2}{4}$

$= \frac{\pi \times (4{\cdot}7)^2}{4}$



Press: 4.7 x^2 × π ÷ 4 =

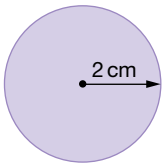
Area $\div 17{\cdot}3\,\text{cm}^2$ (1 dec. pl.)

$A = \pi r^2$ and $A = \frac{\pi d^2}{4}$ both give the area of a circle.

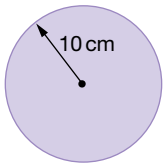
Exercise 12:04

1 Calculate the area of each of these circles. Give the exact answer in terms of π and as an approximation using $\pi = 3{\cdot}14$.

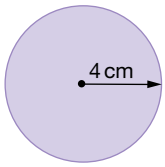
a



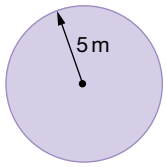
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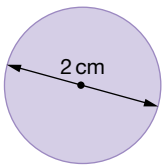
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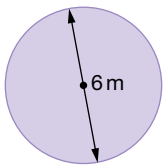
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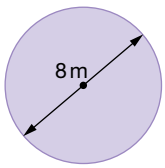
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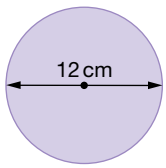
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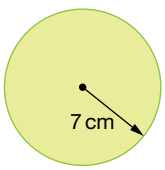


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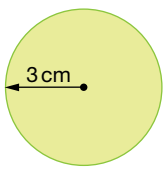


2 Calculate the area of each of these circles. Use the approximation $\pi = \frac{22}{7}$.

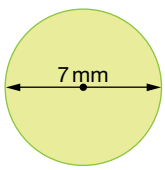
a



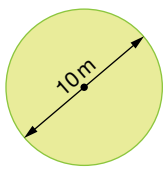
b



c



d

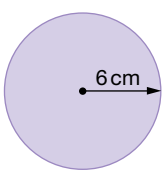


3 Use the formula $r^2 = \frac{A}{\pi}$ to find the radius of a circle, to the nearest centimetre, if its area is:

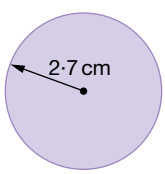
a $200\,\text{cm}^2$ **b** $57\,\text{cm}^2$ **c** $750\,\text{cm}^2$ **d** $1\,\text{m}^2$

4 Use a calculator to find the area of each of these circles, correct to one decimal place.

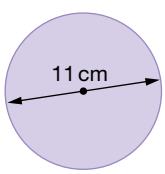
a



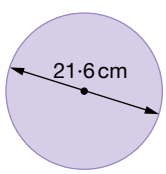
b



c



d



e Radius = 16 cm **f** Radius = 30 cm **g** Radius = 7·8 mm

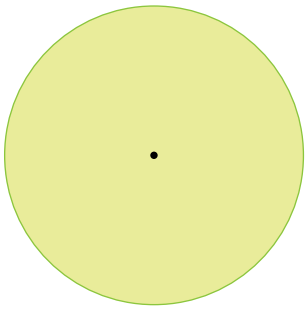
h Diameter = 18 cm **i** Diameter = 36 cm **j** Diameter = 17·6 mm

5 Use the formula $A = \frac{\pi d^2}{4}$ to calculate the area, correct to the nearest square centimetre, of a circle with a diameter of:

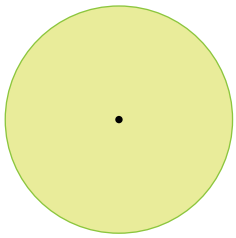
a 8 cm **b** 12 cm **c** 5·6 cm **d** 14·8 cm

6 Measure the diameters of these circles, correct to the nearest millimetre, and calculate the area of each circle.

a

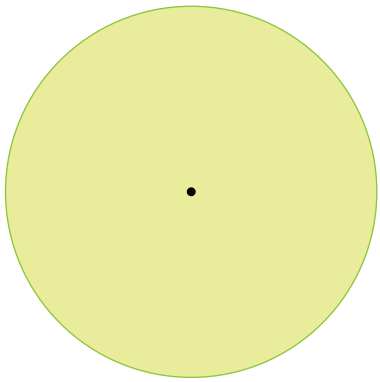


b



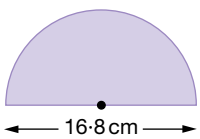
1 cm = 10 mm

c

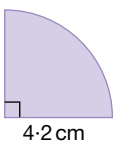


7 Determine what fraction of a circle each shape must be and then calculate its area, correct to one decimal place.

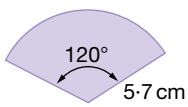
a



b



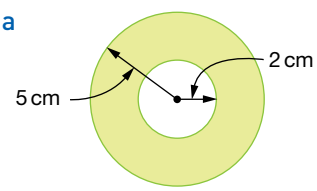
c



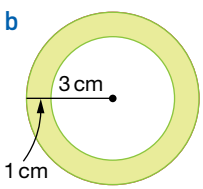
Part **c** is one-third of a circle.

8 Determine the area of the shaded region between each pair of circles, correct to one decimal place.

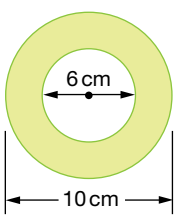
a



b



c



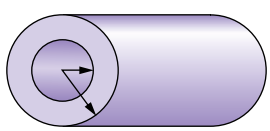
The region between two circles with the same centre is called an *annulus*.

9 **a** A plate has a diameter of 30 cm. What is the area, to the nearest square centimetre?

b A circular rug has a radius of 2·5 m. What is its area, correct to one decimal place?

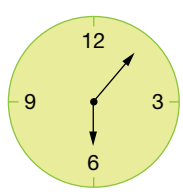
c The lid of a can is a circle with a radius of 5·5 cm. What is its area, correct to the nearest square centimetre?

d The cross-section of a pipe is an annulus where the internal radius is 3 cm and the external radius is 4 cm. Find the area of the cross-section, correct to one decimal place.



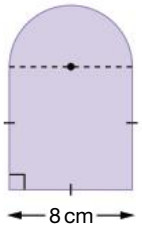
e The minute hand on a clock is 9·6 cm long. What area, correct to one decimal place, will this hand sweep out in:

i 1 hour **ii** 30 minutes **iii** 40 minutes?

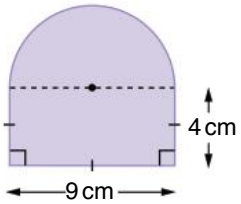


10 Determine the area enclosed by these composite shapes. Give answers correct to the nearest square centimetre.

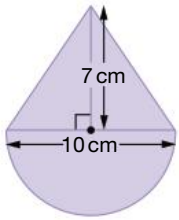
a



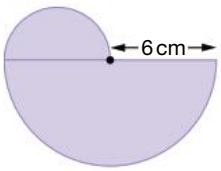
b



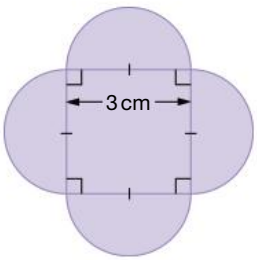
c



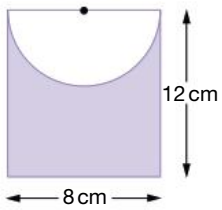
d



e



f

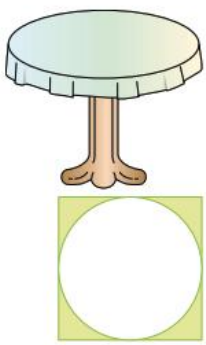


11 a A tablecloth is to cover a circular table that has a diameter of 1·9 m. What area of cloth is needed if the tablecloth is to hang 15 cm over the edge of the table? Answer correct to the nearest square centimetre.

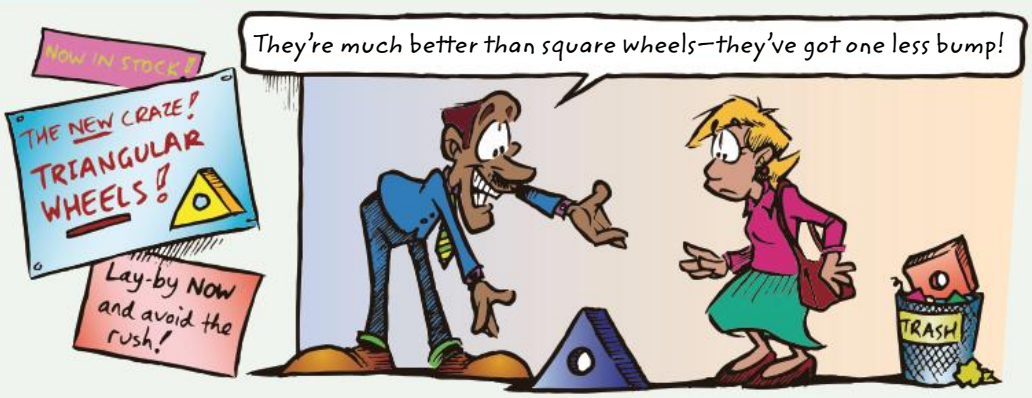
b A circle was cut out of the square as shown in the diagram. Using $\pi = 3{\cdot}14$, the area of the circle was found to be $28{\cdot}26\,cm^2$.

i What was the side length of the square?

ii What area of cardboard was left after the circle had been removed?



FUN SPOT 12:04 — DO FEWER BUMPS MEAN A SMOOTHER RIDE?



Consider and discuss the cartoon above.

- Would three bumps per revolution give you a smoother ride than four bumps per revolution?
- Would ten bumps per revolution (wheels in the shape of decagons) give you a smoother ride than four bumps per revolution (square wheels)?

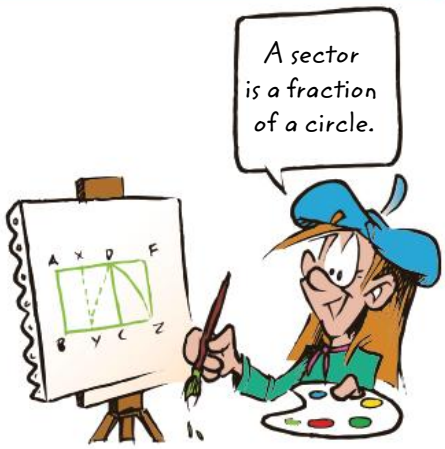
12:05 Area of a sector

PREP QUIZ 12:05

1 What fraction of a circle is a semicircle?
2 What fraction of a circle is a quadrant?

What fraction of the circle is a sector that has a centre angle of:

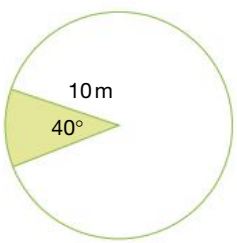
3	90°	**4**	180°	**5**	45°	**6**	30°
7	10°	**8**	1°	**9**	17°	**10**	330°?



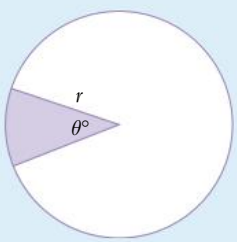
There are 360° at the centre of a circle.

If a sector has an angle of 40° at the centre, its area would be $\frac{40}{360}$ (or $\frac{1}{9}$) of the area of the circle. The area of this sector would be $\frac{40}{360} \times \pi r^2$.

Area of the sector $= \frac{1}{9} \times \pi \times 10^2$
$= \frac{100}{9}\pi$ square metres
$= 34{\cdot}91\text{ m}^2$ (2 dec. pl.)



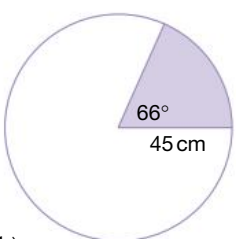
Area of a sector $= \frac{\theta}{360} \times \pi r^2$, where θ is the size (in degees) of the angle at the centre, and r is the radius of the circle.



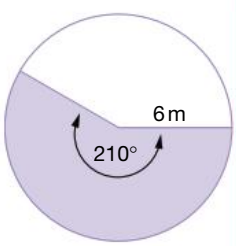
WORKED EXAMPLES

Find the area of each sector in terms of θ, and as a decimal correct to two decimal places.

1 Area of a sector
$= \frac{\theta}{360} \times \pi r^2$
$= \frac{66}{360} \times \pi \times 45^2$
$= 371.25\pi$ cm
$= 1166{\cdot}32$ cm (2 dec. pl.)



2 Area of a sector
$= \frac{\theta}{360} \times \pi r^2$
$= \frac{210}{360} \times \pi \times 6^2$
$= 21\pi$ m
$= 65{\cdot}97$ m (2 dec. pl.)



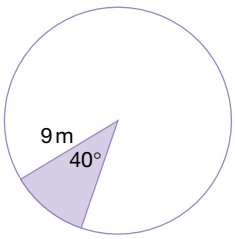
Exercise 12:05

1 What fraction of 360° is:

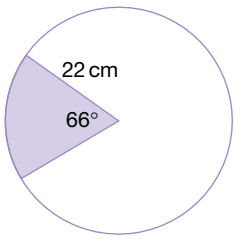
a 180° **b** 90° **c** 30° **d** 45° **e** 18°?

2 What fraction of the circle is coloured?

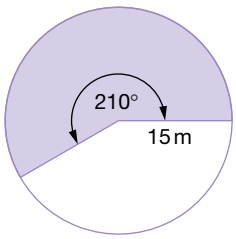
a



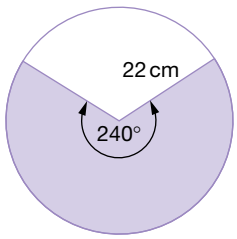
b



c



d

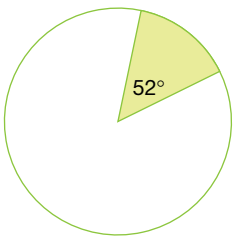


3 Find the area of each circle in Question **2**, correct to one decimal place.

4 Find the area of each coloured sector in Question **2**, correct to one decimal place.

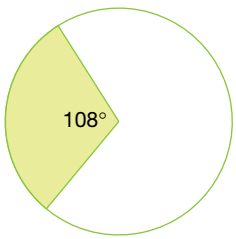
5 Find the area of each of these sectors in terms of π, and then correct to two decimal places.

a



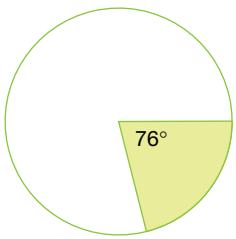
radius = 27 m

b



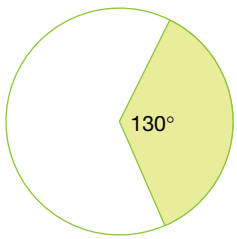
radius = 81 cm

c



radius = 18 mm

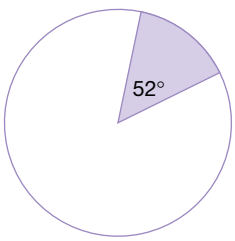
d



radius = 9 m

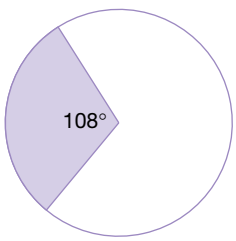
6 Find the area of each of these sectors in terms of π, and then correct to two decimal places.

a



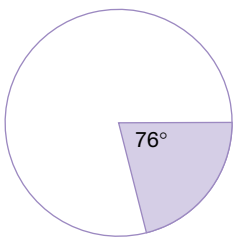
diameter = 27 m

b



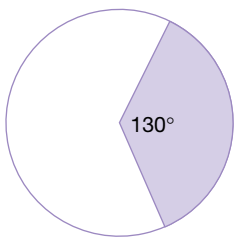
diameter = 81 cm

c



diameter = 18 mm

d



diameter = 9 m

7 Find the area, correct to two decimal places, of a sector that has:

a centre angle 112°, radius 39 mm

b centre angle 37°, radius 45·7 m

c centre angle 84°, diameter 38 cm

d centre angle 7°, diameter 47 m

8 **a** Find the area of a sector that makes an angle of 43° at the centre of a circle of radius 8 cm.

b Find the area of a sector that makes an angle of 204° at the centre of a circle of diameter 7 m.

9 If a sector with centre angle 15° is cut from a circle of diameter 14 cm, what area of the circle will remain?

10 Consider the answers to Questions **5** and **6**. How does the area of a circle change when you halve the radius?

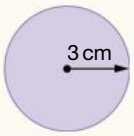
12:06 Volume of a cylinder

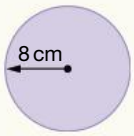
PREP QUIZ 12:06

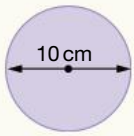
1 What is the formula for the circumference of a circle? $C =$ _____

2 What is the formula for the area of a circle? $A =$ _____

What is the area of each circle, correct to the nearest square centimetre?

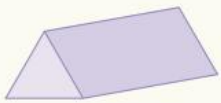
3 

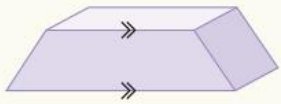
4 

5 

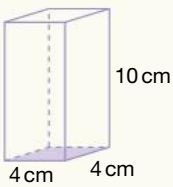
Name these solids.

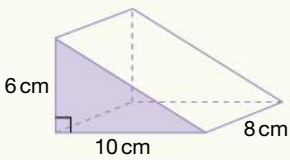
6 

7 

8 

Calculate the volumes of these solids.

9 

10 

In Chapter 8, we saw that the volume of any prism could be calculated using the formula:

$V = Ah$,

where A = cross-sectional area

h = height of the prism

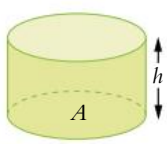


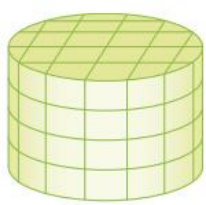
- A cylinder can be thought of as a prism with a circular cross-section.
- We know how to find the area of a circle, so we can use the above formula to find the volume of a cylinder.

$V = Ah$

$A = \pi r^2$

$\therefore\ V = \pi r^2 h$

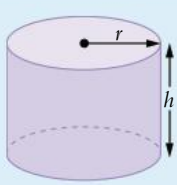




The volume of a cylinder is given by the formula:

$$V = \pi r^2 h,$$

where r = radius of the cylinder h = height of the cylinder.



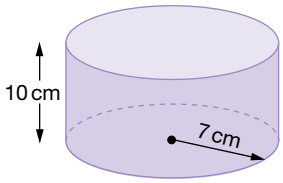
Remember from Chapter 8:

- The volume of an object is the space that it occupies.
- The capacity of a container is the volume of liquid it can hold.

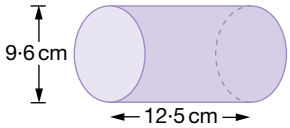
WORKED EXAMPLE 1

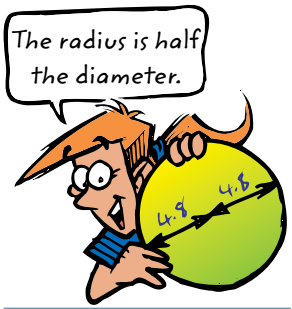
Find the volumes of these cylinders.

a



b





Diameter = 9·6 cm
∴ Radius = 4·8 cm

Solutions

a Use $\pi \div \frac{22}{7}$.

$V = \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540$

∴ Volume = $1540\,\text{cm}^3$

b Use π on a calculator.

$V = \pi r^2 h$

$= \pi \times (4{\cdot}8)^2 \times 12{\cdot}5$

Press: 4·8 x^2 × π × 12·5 =

Answer = 904·8 (1 dec. pl.)

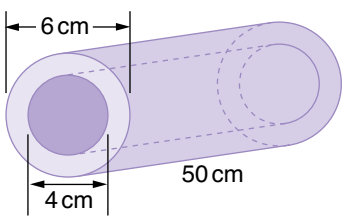
∴ Volume ÷ $904{\cdot}8\,\text{cm}^3$

WORKED EXAMPLE 2

A metal pipe has the dimensions as shown.

a Calculate the volume of metal in the pipe.

b Calculate the capacity of the pipe if it were full of water.



Solutions

a Volume of pipe = volume of outer cylinder − volume of inner cylinder

$V = \pi R^2 h - \pi r^2 h$

$= \pi \times 3^2 \times 50 - \pi \times 2^2 \times 50$

$= 1413{\cdot}7 - 628{\cdot}3$

∴ $V = 785{\cdot}4$ (1 dec. pl.)

∴ Volume of metal in pipe is $785{\cdot}4\,\text{cm}^3$.

b The water would be held in the inner cylinder.

Volume of inner cylinder

$V = \pi r^2 h$

$= \pi \times 2^2 \times 50$

$= 628{\cdot}3\,\text{cm}^3$ (1 dec. pl.)

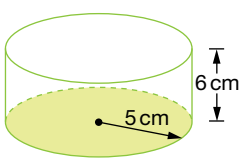
$= 628{\cdot}3\,\text{mL}$

Remember:
$1\,\text{cm}^3 = 1\,\text{mL}$

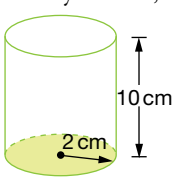
Exercise 12:06

1 Calculate the volume of each of these cylinders, correct to one decimal place.

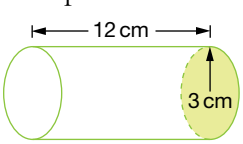
a



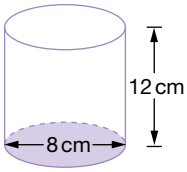
b

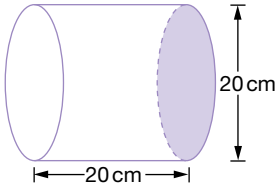


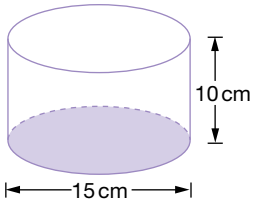
c



2 Determine the volume of each of these cylinders to the nearest cubic centimetre.

a 

b 

c 

3 Find the volume of a cylinder, to the nearest cubic centimetre, whose:

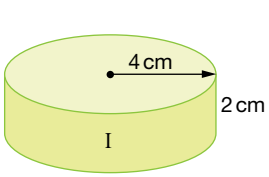
a radius = 7 cm, height = 5 cm

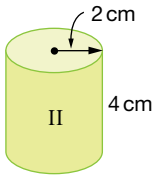
b radius = 6 cm, height = 15 cm

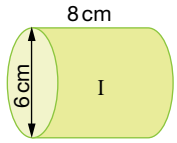
c diameter = 16 cm, height = 9 cm

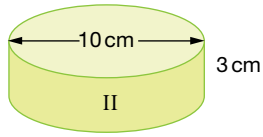
d diameter = 2 cm, height = 8 cm

4 Which of each pair of cylinders has the greater volume, I or II?

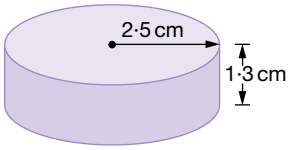
a 

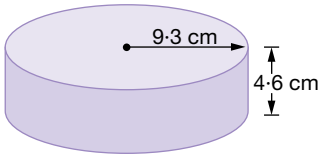


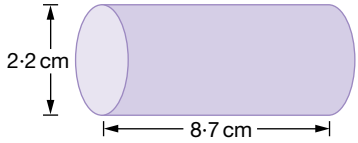
b 



5 Calculate the volumes of these cylinders, correct to one decimal place.

a 

b 

c 

6 A circular swimming pool has a diameter of 4 m and a depth of 1·5 m. Calculate its volume and determine the number of litres of water it would take to fill it.

$1\text{ mL} = 1\text{ cm}^3$

7 A cylindrical glass is 10 cm tall and has a diameter of 5 cm.
If the glass is full, how many millilitres of water would it hold?

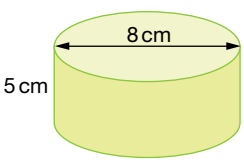
WARNING!
Don't mix units.

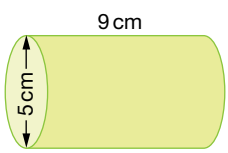
8 A telegraph pole is 10 m high and has a diameter of 36 cm.
Calculate the volume of the pole in cubic metres of timber.

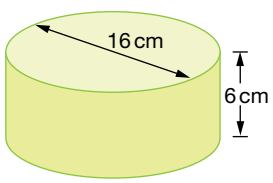
9 a The cylinder of a car engine has a diameter of 9 cm. Calculate its capacity if its height is 10 cm. Answer correct to one decimal place.

b If the engine has 6 cylinders each with a diameter of 9 cm and a height of 10 cm, calculate the capacity of the engine? Answer correct to the nearest 100 cm^3?

10 Another formula for the area of a circle is $A = \frac{\pi d^2}{4}$ so the formula for volume of a cylinder would be $V = \frac{\pi d^2}{4}h$. Use this formula to find the volumes of these cylinders to the nearest cubic centimetre.

a 

b 

c 

Another formula for the volume of a cylinder:

$$V = \frac{\pi d^2}{4} h$$

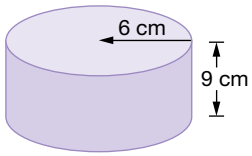
where d = diameter and h = height

11 **a** What is the volume of this cylindrical can?

b Will it hold 1 litre of water?

c If the height were doubled to 18 cm, how many complete litres would it hold?

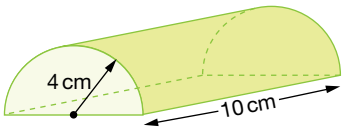
d If the height were 9 cm, but the radius were doubled to 12 cm, how many complete litres would the can hold?



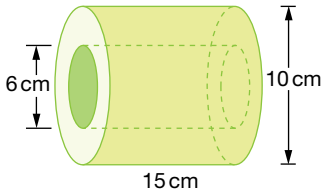
Remember:
$1\text{ L} = 1000\text{ cm}^3$

12 Calculate the volume, to the nearest cubic centimetre, of each solid.

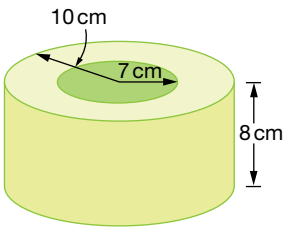
a



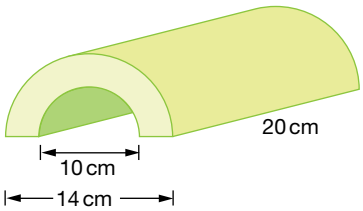
b



c

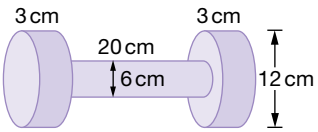


d

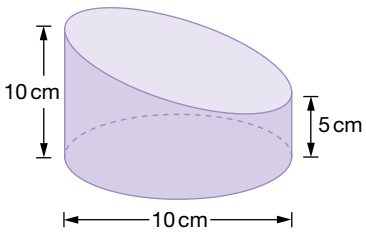


13 **a** Determine the volume of each of these solids, correct to the nearest cubic centimetre.

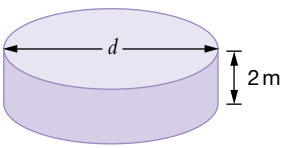
i



ii



b This water tank holds 2000 litres of water when full. What must be its diameter, to the nearest centimetre?





How could you estimate the volume of one of these barrels?

INVESTIGATION 12:06 THE CYLINDER

1 At a recent agricultural show the Cool-Cola company displayed a strange can of drink. This can held the same amount of liquid as a normal can, 375 cm^3, but was only 1 cm high. Can you work out the diameter of the top?



- Use *guess, check and refine* to find the diameter correct to the nearest centimetre. This method is also called **iteration**.
- You know that the height is 1 cm; guess a value for the diameter, halve this to find the radius, and then use $V = \pi r^2 h$ to find the volume. If this is too small, choose a larger value for the diameter and find the new volume. Continue in this way until you find the diameter, correct to the nearest centimetre, that gives a volume closest to 375 cm^3.

Diameter choice (cm)	Radius (cm)	Volume (cm^3)
(1)		
(2)		
(3)		
(4)		

$V = \pi r^2 h$, where r is in cm.
$h = 1$ cm
$V = \pi r^2$ cm^3

2 Calculate the amount of toothpaste that has been squeezed onto this toothbrush from the tube shown. If Rachel uses this much toothpaste twice a day, how many days would one tube of toothpaste last? (1 g of toothpaste has a volume of 0·8 cm^3.)



3 Estimation

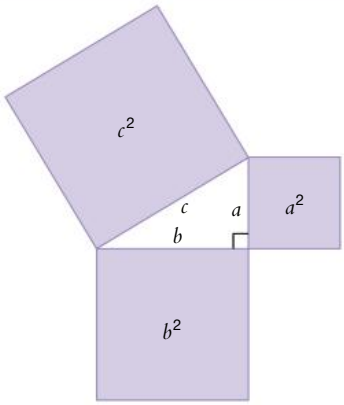
Think of your arm as a long tube (cylinder). Estimate the volume of your arm. Repeat this for your legs, body, neck and head to arrive at an estimate of the volume of your body. Can you think of a way to *measure* the volume of your body?

CHALLENGE 12:06 PYTHAGORAS' THEOREM AND CIRCLES

Pythagoras' theorem was covered in Chapter 3.

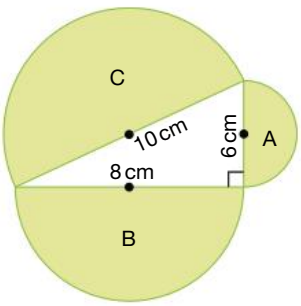
'The square on the hypotenuse is equal to the sum of the squares on the other two sides'

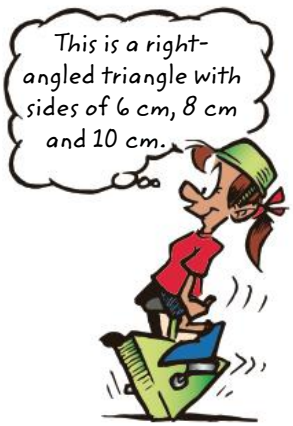
$c^2 = a^2 + b^2$



You might be amazed to discover an extension of this theorem.

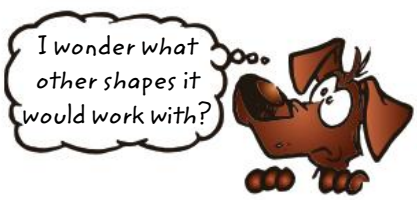
Answer the following questions about this diagram.





1 **a** What is the radius of semicircle A?
b What is the area of semicircle A?
c What is the area of semicircle B?
d What is the area of semicircle C?

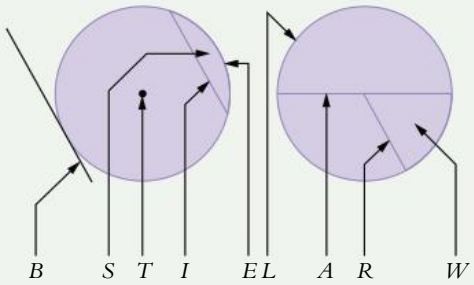
2 Add your answers to **b** and **c** above, and compare the result with the answer to **d**.
You should find the following result:
The area of the semicircle on the hypotenuse is equal to the sum of the areas of the semicircles on the other two sides.



FUN SPOT 12:06 WHAT DID THE DIRT SAY WHEN IT SAW THE CLOUDS?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

- What is the formula for:
  - **M** the diameter of a circle
  - **I** the area of a circle
  - **N** the circumference of a circle
  - **A** the volume of a cylinder?
- What is the name of each feature shown below?





E	$0{\cdot}2^2$	**Y**	$0{\cdot}3 \times 0{\cdot}2$	**M**	$3{\cdot}6 \div 6$	**I**	$0{\cdot}3 + 0{\cdot}06$
F	$1 - 0{\cdot}01$	**I**	$0{\cdot}36 \times 10$	**N**	$0{\cdot}3 + 0{\cdot}7 \times 2$	**M**	$0{\cdot}5 \div 2 - 0{\cdot}25$
L	$0{\cdot}5^2 - 0{\cdot}4^2$	**U**	$\sqrt{0{\cdot}81}$	**D**	Which is the largest, 0·30, 0·009 or 0·1		

0·36	0·99	3·6	centre	radius	diameter	πr^2	1·7	segment	$2r$	0·06	πd	$\pi r^2 h$	0·6	arc	sector	chord	0·09	semicircle	tangent	0·04	0	0·9	0·30	



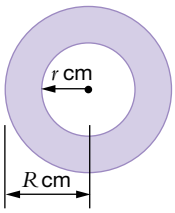
The Big Banana, Coffs Harbour. Estimate the volume of the banana, assuming the deck railing is 1.2 m high.



MATHS TERMS 12

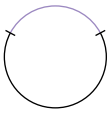
annulus

- the area bounded by two circles that have the same centre
- area of an annulus $= \pi R^2 - \pi r^2$



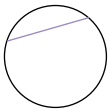
arc

- part of a circle



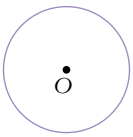
chord

- an interval joining two points on a circle



circle

- the set of all points in a plane that are a fixed distance from a point called the centre (O)



circumference

- the perimeter of a circle
- the length of the boundary of a circle

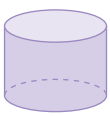
concentric circles

- a group of circles that have the same centre



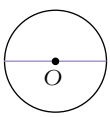
cylinder

- a cylinder could be thought of as a prism whose cross-section is a circle



diameter

- an interval that passes through the centre of a circle and divides it in half

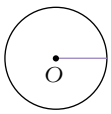


pi (π)

- the ratio of the circumference to the diameter of any circle
- an irrational number approximately equal to 3·14 (2 dec. pl.), 3·141 592 (7 dec. pl.) or $\frac{22}{7}$ (as a fraction)

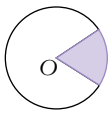
radius (radii is the plural)

- an interval drawn from the centre of a circle to a point on the circle
- half of the diameter



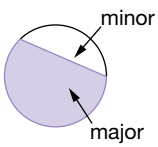
sector

- the area bounded by two radii and the included arc of a circle



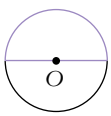
segment

- a section of a circle bounded by an arc and its chord
- a chord cuts a circular region into two segments: a minor segment and a major segment



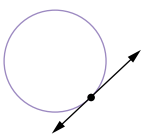
semicircle

- half of a circle



tangent

- a line that touches a circle at only one point



Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

12:01

1 Name the part of the circle shown in each diagram.

a  b  c  d  e 

12:02

2 Calculate the circumference of each of these circles, correct to one decimal place.

a 

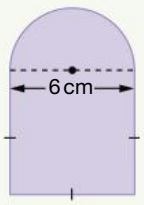
b 

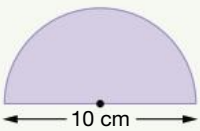
c 

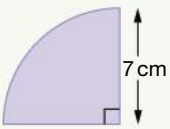
d 

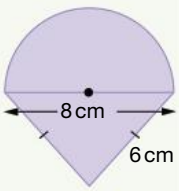
12:03

3 Find the perimeter of each of these shapes, correct to the nearest centimetre.

a 

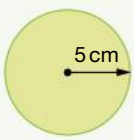
b 

c 

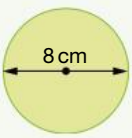
d 

12:04

4 Calculate the area of each of these circles, correct to the nearest square centimetre.

a 

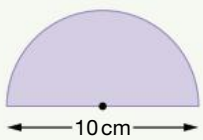
b 

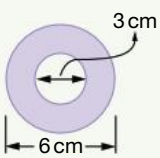
c 

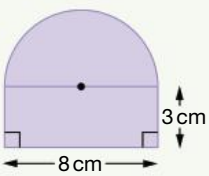
d 

12:05

5 Calculate the area of each shaded region, correct to the nearest square centimetre.

a 

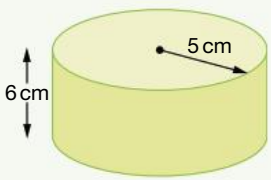
b 

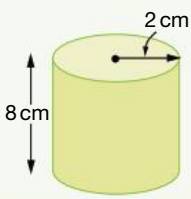
c 

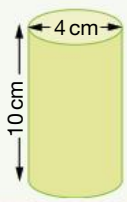
d 

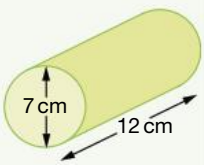
12:06

6 Calculate the volume of each of these cylinders, correct to the nearest cubic centimetre.

a 

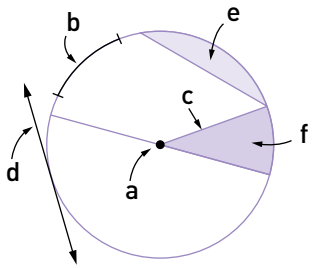
b 

c 

d 

ASSIGNMENT 12A Chapter review

1 Write the names of each labelled part of the circle.



2 **a** Is the diameter of a circle an axis of symmetry?
b What fraction of a circle is a quadrant?
c What are concentric circles? (See Question **6** in Exercise 12:01.)
d What is the approximate value of π, correct to the nearest whole number?

3 What is the exact circumference (in terms of π) of a circle that has a:
a diameter of 12 cm
b radius of 5 m?

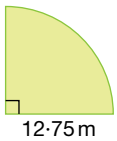
4 What is the circumference, correct to the nearest centimetre, of a circle that has a:
a diameter of 15·6 cm
b radius of 2·7 m?

5 What is the exact area (in terms of π) of a circle that has a:
a diameter 26 cm
b radius 3 m?

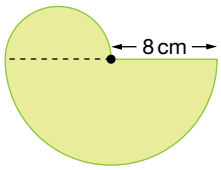
6 What is the area of a circle, correct to one decimal place, that has a:
a diameter of 28·5 cm
b radius of 1·8 m?

7 Calculate the area of these shapes, correct to one decimal place.

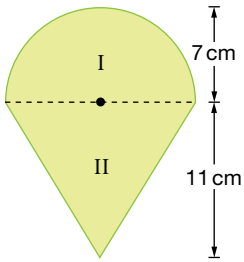
a



b

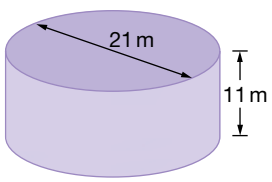


c

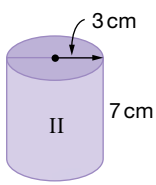


8 Find the volume of each of these objects, correct to one decimal place.

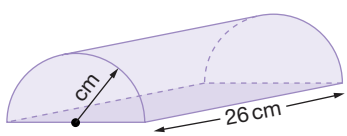
a



b



c





Sectors are used to define the landing areas for shot put and discus.

ASSIGNMENT 12B Working mathematically

1 Given the rule $\square^2 + 26 = 15 \times \square$:
   a find all the positive integers between 1 and 5 which, when placed in the boxes, make this a false statement. (Integers are whole numbers. The same integer must go into both boxes.)
   b find all the integers which, when placed into the boxes, turn the rule into a true statement. (There are two such integers.)

2 Lachlan had 4 shirts, 2 ties and 3 belts. How many different combinations of a shirt, a tie and a belt can he wear?

3 There are six people in the Hollier family. Only two of them can fit in their neighbour's car. How many different pairs could be chosen to fill these two seats?

4 Fewer than 40 students are on a bus. One-quarter of them come from Kincumber while two-fifths of them come from Erina. How many students are there?

5 A length of piping is cut 15 times. How many pieces would result?

6 A pack of 52 cards contains 13 hearts, 13 diamonds, 13 clubs and 13 spades. How many cards must you take unseen and one at a time to be sure that you have at least one of each type?

7 Estimate the diameter of one of the wheels on the truck below and work out its circumference.



ASSIGNMENT 12C Cumulative revision

1 Simplify:

a $5a \times 6$ **b** $7a^2 \times a$ **c** $6 - 2(a + 2)$ **d** $6m \div m$

5:05, 5:08

2 **a** Bill has three times as many stamps as Bob. If they have 1260 stamps in total, how many does each boy have?

b Jill has \$3.50 more than Jai. If they have \$19.10 in total, how much does each girl have?

9:05

3 **a** A netball team has played 60 games. If the ratio of wins to losses to draws is 26 : 3 : 1, how many games have they won?

b In a recipe, 2 eggs are needed for 6 cups of flour. If 3 eggs are used, how much flour must be used?

10:03, 10:04

4 **a** Find the value of F if $m = 1{\cdot}6$ and $a = 9{\cdot}8$, given that $F = ma$.

b If $S = \sqrt{ab}$, find S when $a = 12{\cdot}5$ and $b = 11{\cdot}52$.

9:05

5 Calculate the area of:

a a square of side 6·5 cm

b a rectangle 5·6 cm long and 2·4 cm wide

c a kite with diagonals of lengths 16 cm and 34 cm

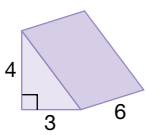
d a parallelogram that has a pair of sides 8 cm long and 5·5 cm apart.

8:04

6 The probability that our team will win their next soccer game is $\frac{5}{7}$. What is the probability that they won't win?

6:03

7 Find the volume of this prism. Measurements are in centimetres.



8:06

8 Solve these equations:

a $4(2a - 3) = 15$ **b** $4x + 3(2x - 3) = 4$ **c** $4(1 - x) = 3(2x + 7)$

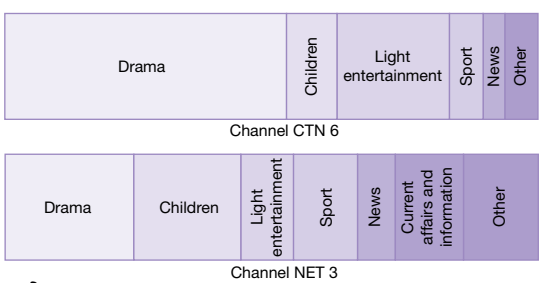
9:04

9 **a** A scale drawing is made of a field 120 m long and 50 m wide, using a scale of 1 : 100. What are the dimensions of the scale drawing?

b On a drawing with a scale of 1 : 1000, the distance between two points is 2·3 cm. How far apart are these points in real life?

10:05

10 An analysis of television programs was carried out on two channels and these graphs were produced.



a Which channel gave a greater percentage of time to children's programs?

b What percentage of CTN 6's time was devoted to drama programs?

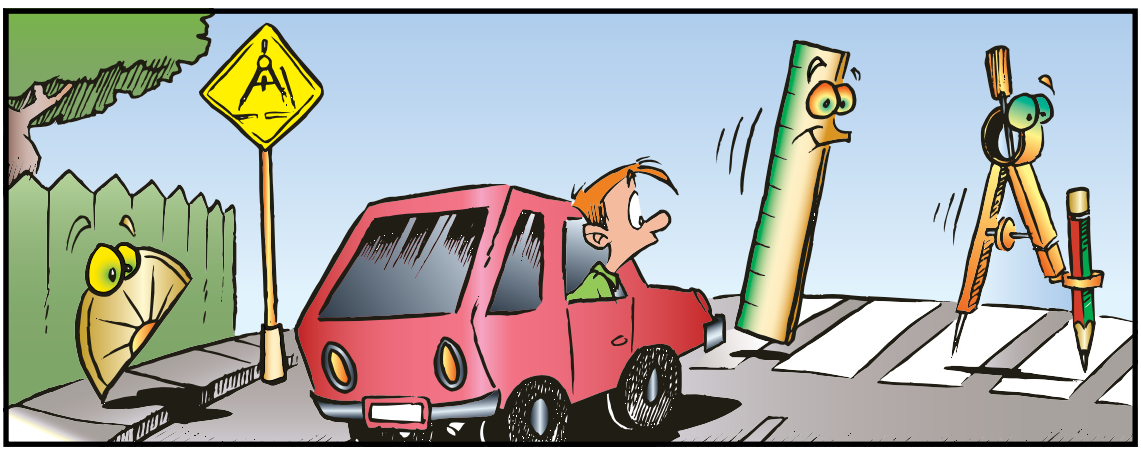
c CTN 6 spent 3% of its time on current affairs and information. What percentage more time did NET 3 spend on this than CTN 6?

d What percentage of each channel's time was spent on light entertainment?

7:03

CONGRUENCE

13



Contents

13:01 Congruence and transformations
Investigation 13:01 Congruent figures and tessellations
13:02 Congruent figures
GeoGebra activity 13:02 Congruent figures
Fun spot 13:02 What is a stupid ant?
13:03 The minimum conditions for congruent triangles
GeoGebra activity 13:03A Are triangles with matching angles equal always congruent?
GeoGebra activity 13:03B Conditions for congruent triangles
13:04 Applying the congruency tests
Fun spot 13:04 What do you call a man with a shovel?
13:05 Using congruent triangles to find sides and angles
13:06 Deducing properties of the special quadrilaterals
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Measurement and Geometry

Selections from ***Properties of Geometrical Figures 2*** [Stage 4]

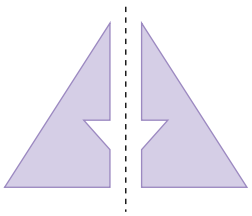
- Define congruence of plane shapes using transformations. (ACMMG200)
- Develop the conditions for congruence of triangles. (ACMMG201)
- Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning. (ACMMG202)

Working Mathematically

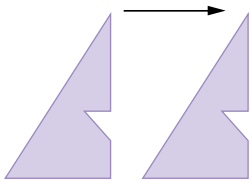
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

13:01 Congruence and transformations

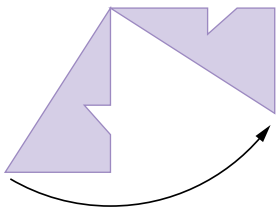
- In past years you should have encountered reflections, translations and rotations, or, to give them their more familiar names, flips, slides and turns.



Flip the shape.



Slide the shape.

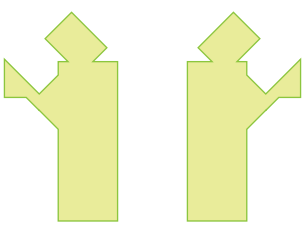


Turn the shape around.

- When a figure undergoes one of these transformations it is unchanged in shape and size. The only thing that changes is its position.

> Congruent figures are the same shape and size. When one is placed on top of the other they coincide exactly.

- Congruent figures are produced by flips, slides and turns.
- When congruent figures are placed on top of each other (or superimposed) so that they coincide exactly, the matching sides and angles are obviously equal.



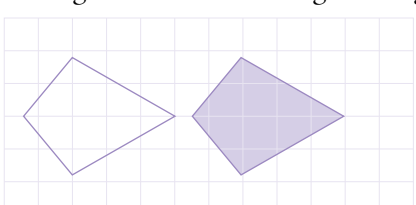
These two shapes are congruent.
Can you identify the matching sides and angles?

Exercise 13:01

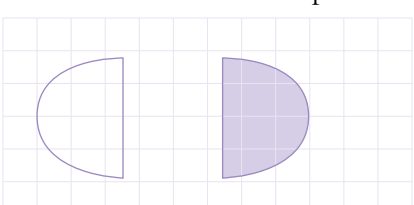
P Foundation worksheet 13:01 Congruent shapes

1 Each diagram shows two congruent figures. What transformations have taken place?

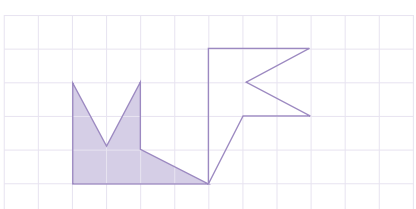
a



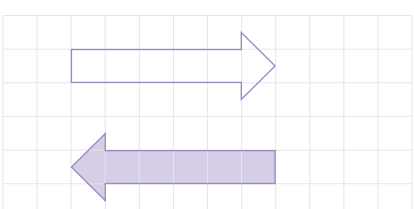
b



c

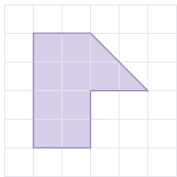


d



2 Copy the shape shown onto grid paper and draw a shape that is congruent to it using:

a a translation **b** a reflection **c** a rotation.



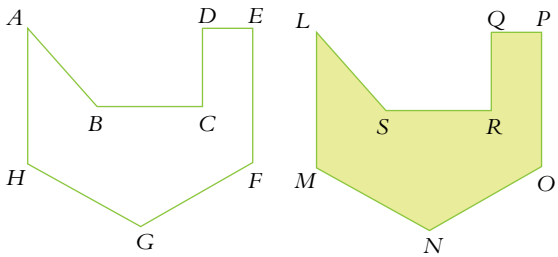
3 **a** Which angle matches with:

i $\angle A$ **ii** $\angle C$ **iii** $\angle M$?

b Which side matches with:

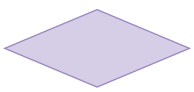
i BC **ii** MN **iii** PO?

c Do the shapes have the same area?



4 Match each of the following shapes to a congruent figure below. (Measure the shapes carefully.)

a



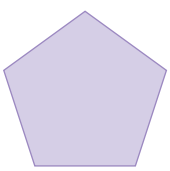
b



c



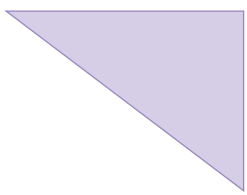
d

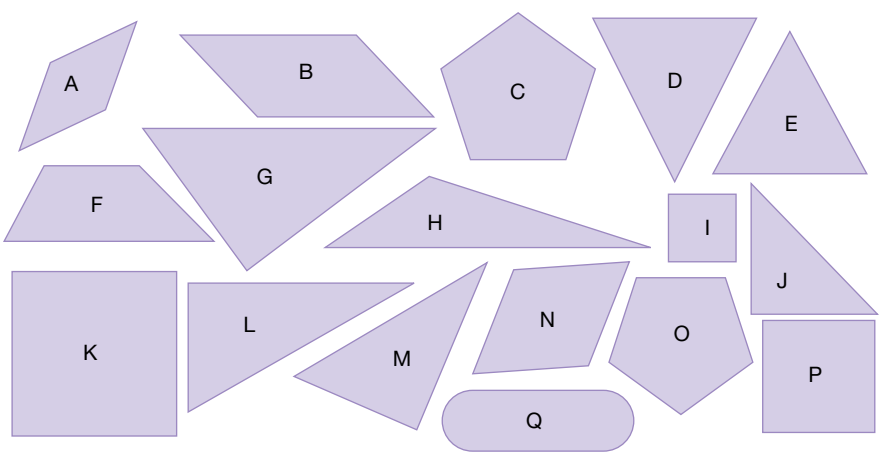


e



f



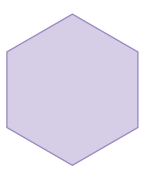


5 Show how the regular hexagon can be divided into:

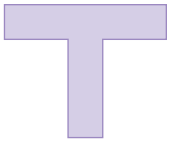
a 2 congruent figures

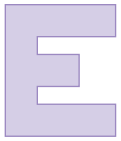
b 3 congruent figures

c 4 congruent figures.



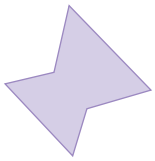
6 Copy the following shapes. Add one line to divide each shape into two congruent figures.

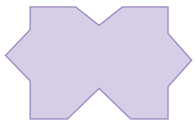
a 

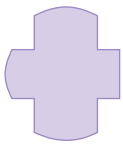
b 

c 

d 

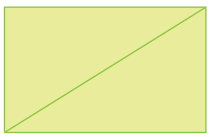
e 

f 

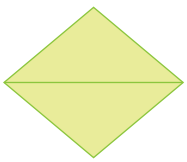
g 

h 

7 A diagonal has been drawn on each of the following quadrilaterals. State whether or not the two triangles obtained are congruent.

a 
Rectangle

b 
Parallelogram

c 
Rhombus

8 The two figures shown are congruent.

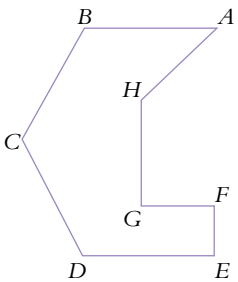
a What transformations have produced the shapes?

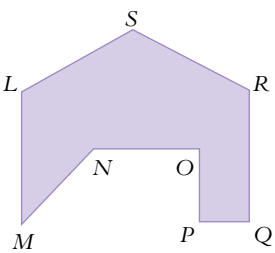
b What angle matches with:

i $\angle C$ ii $\angle A$ iii $\angle R$?

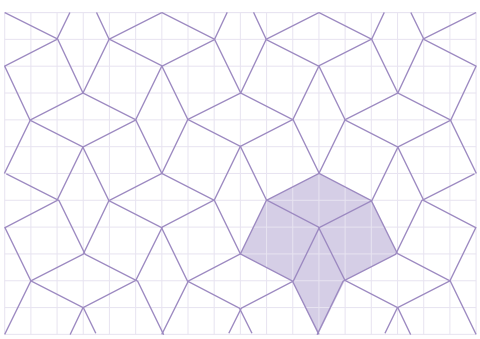
c What side matches with:

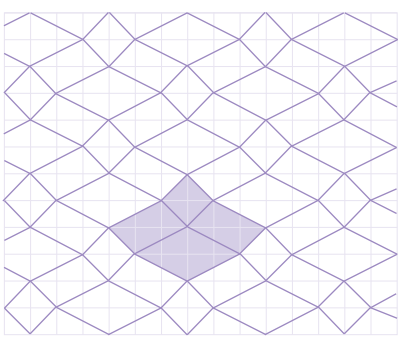
i HG ii DE iii MN?





9 Tiling patterns are based on congruent shapes. In the tile patterns shown, four small sections have been joined to form a 'tile'. Redraw the patterns and then colour them in using only the new 'tile' shape.

a 

b 

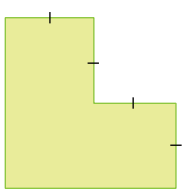
c Do congruent figures have the same area?

10 Show how the figure shown can be divided into:

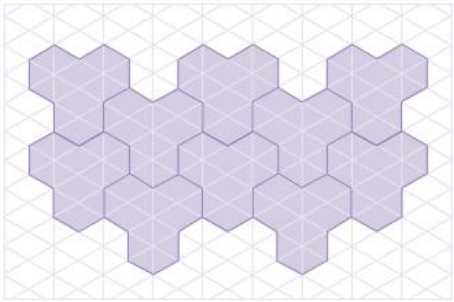
a 2 congruent figures

b 3 congruent figures

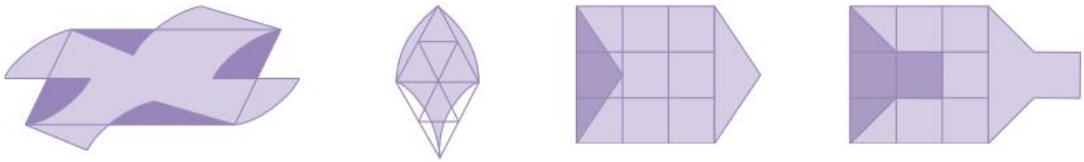
c 4 congruent figures.



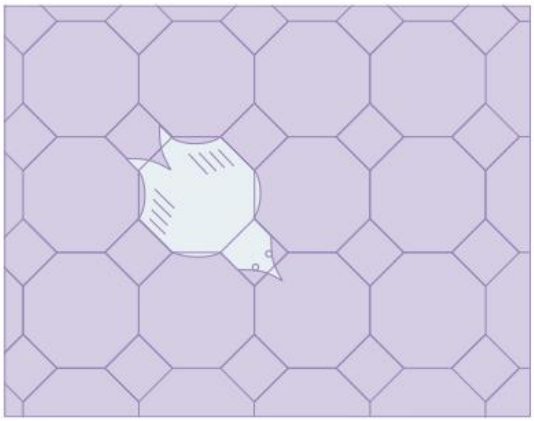
A tiling pattern is called a **tessellation**. When you look at a tessellation that only has tiles of one shape, it is obvious that all of the tiles must be congruent, as in the example shown.

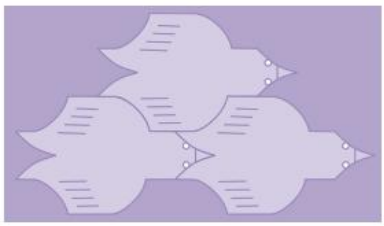


- There are different mathematical methods that can be used to produce a tile shape that will tessellate (form a tiling pattern). One such method involves *translating opposite sides*. The examples below illustrate this method.
  1. Start with a base shape that will tessellate on its own. (Simple examples include an equilateral triangle, square, regular hexagon.)
  2. Whatever you do to one side must be translated to the other side.



A more complicated example is shown below.





Much use is made of tessellation patterns in many art designs, and some artists use the geometry of tessellations to great effect. One such artist was a Dutchman named Maurits Escher. See if you can find out more about Escher and his designs.



13:02 Congruent figures

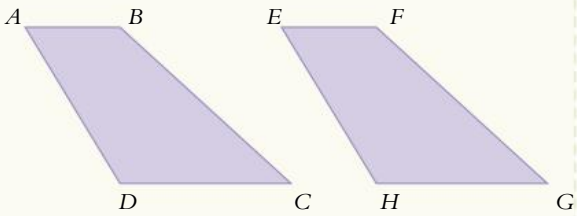
PREP QUIZ 13:02

Quadrilateral $ABCD$ is congruent to quadrilateral $EFGH$. Which is the matching side to:

1 AB **2** EH **3** DC?

Which is the matching angle to:

4 $\angle ABC$ **5** $\angle EHG$?

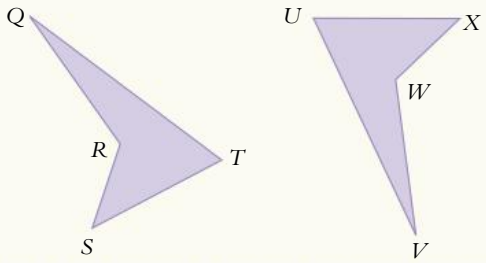


Quadrilateral $QRST$ is congruent to quadrilateral $VWXU$. Which is the matching side to:

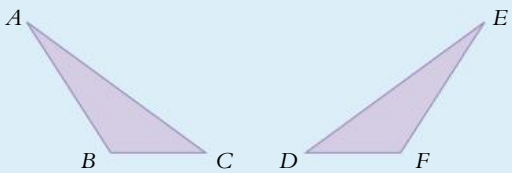
6 RS **7** UX **8** QT?

Which angle matches:

9 $\angle VUX$ **10** $\angle RQT$?



Congruent figures have the same shape and size.



$\Delta ABC \equiv \Delta EFG$ means ΔABC is congruent to ΔEFG.

If two shapes are congruent, then:

- matching sides are equal
- matching angles are equal
- the shapes have the same area.

When writing a congruence statement, name the vertices in matching order.
e.g. $\Delta ABC \equiv \Delta EFD$

If $\Delta ABC \equiv \Delta EFD$ then:

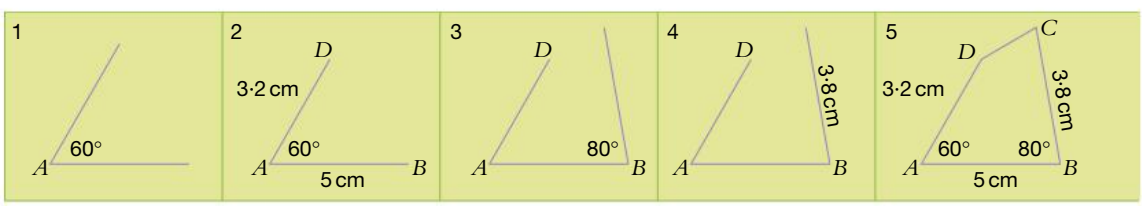
$AB = EF$ $\quad \angle ABC = \angle EFD$

$AC = ED$ $\quad \angle BCA = \angle FDE$

$BC = FD$ $\quad \angle BAC = \angle FED$



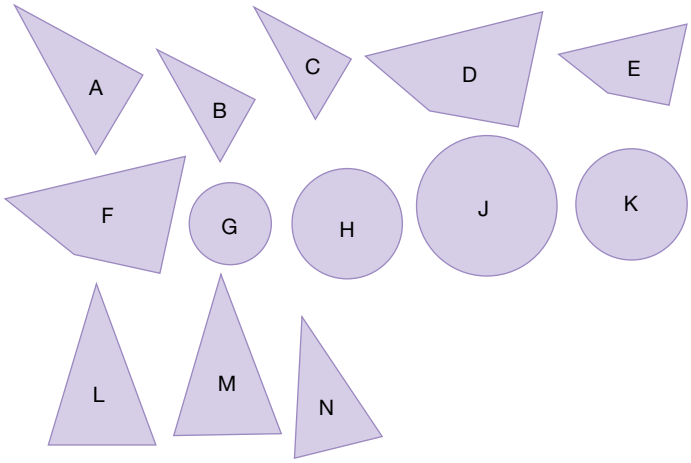
To construct congruent figures we can use geometrical instruments. We simply use the same steps for both shapes or copy the original shape.



Exercise 13:02

P Foundation worksheet 13:02
Congruent figures

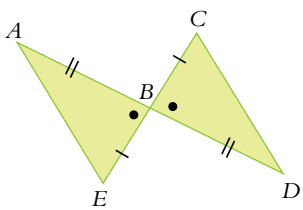
1 Which shape is congruent to:
- **a** figure B
- **b** figure D
- **c** figure K
- **d** figure M?



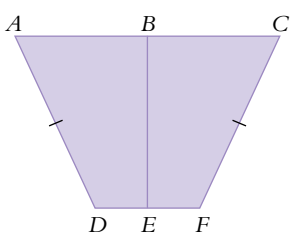
2 Use the figures in Question 1 to complete these congruence statements.
- **a** $\Delta B \equiv \ldots$
- **b** Figure D $\equiv \ldots$
- **c** $\Delta M \equiv \ldots$

3 Rotating either triangle about B produces the other triangle.
Hence, $\Delta ABE \equiv \Delta DBC$.
Which side matches:
- **a** AB
- **b** EB
- **c** AE?

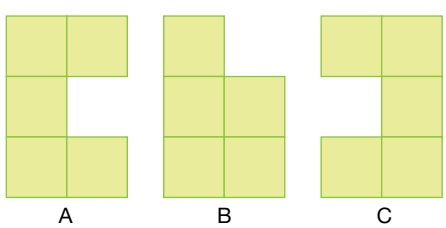
Which angle matches:
- **d** $\angle EAB$
- **e** $\angle ABE$
- **f** $\angle BEA$?



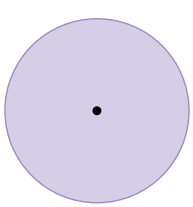
4 The trapezium $ACFD$ has been divided into two congruent figures by the axis of symmetry BE.
Which side matches side:
- **a** AB
- **b** AD
- **c** BE?
- **d** Complete the congruence statement: $ABED \equiv \ldots$
- **e** Would these congruent figures have the same area?
- **f** Would all congruent figures have the same area.



5
- **a** Which two shapes are congruent?
- **b** Do the congruent figures have the same area?
- **c** Do shapes B and C have the same area?
- **d** Why aren't B and C congruent?

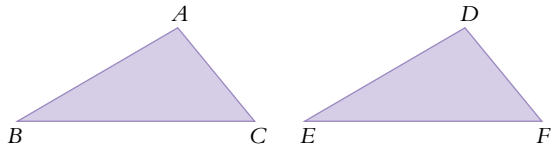


6 What is the condition for two circles to be congruent?

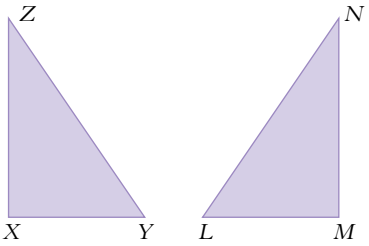


7 Each of the following pairs of triangles is congruent.
Write down the pairs of matching sides and matching angles for each part.

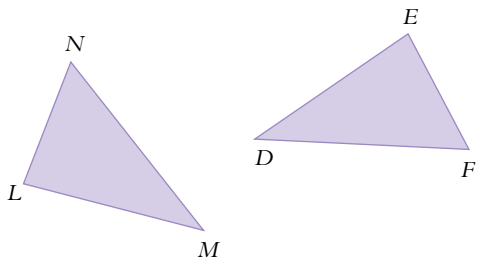
a



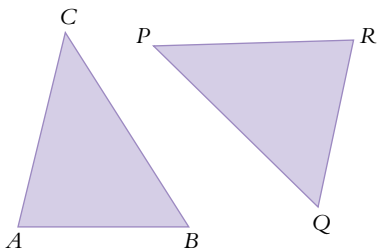
b



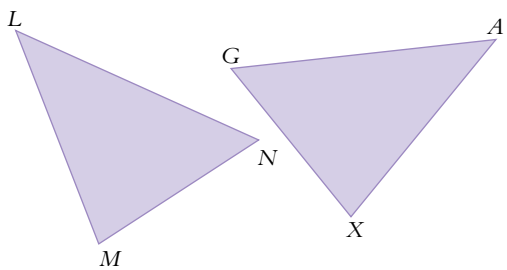
c

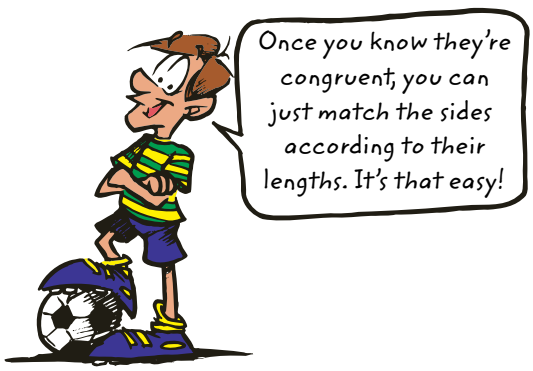


d



e

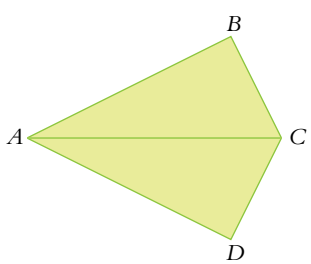




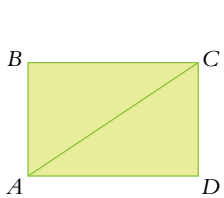
8 Write a congruence statement for each of the pairs of triangles in Question **7**. Make sure to name the vertices in matching order.

9 In each of the following, the shapes have been made by joining or overlapping two congruent triangles. Complete the following congruence statement for each shape. Make sure that the vertices are written in matching order.
$\Delta ABC \equiv \Delta \ldots$

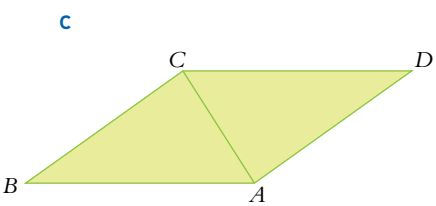
a



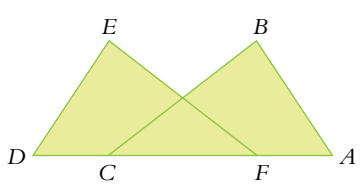
b



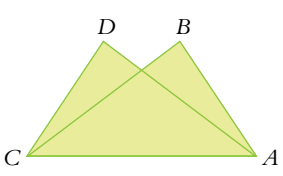
c



d



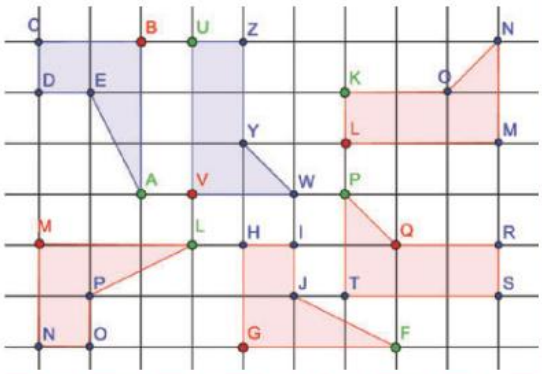
e



GEOGEBRA ACTIVITY 13:02 CONGRUENT FIGURES

Congruent figures 1
Use translation and rotation to find congruent triangles and find pairs of matching angles and sides.

Congruent figures 2
Use reflection to produce congruent figures.

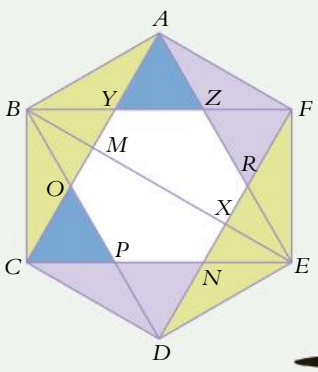


FUN SPOT 13:02 WHAT IS A STUPID ANT?

Work out the answer to each question and write the letter for that part in the box above the correct answer.

The diagram shows a regular hexagon on which some of its diagonals have been drawn. Because of the symmetry of the hexagon, the diagonals have formed many congruent triangles.

Find the triangle that is congruent to each of the following.





N ΔBFE **N** ΔCDN | **G** ΔBMC **R** ΔBFX | **N** ΔAFY **A** ΔBCP | **A** ΔABC **O** ΔAYB | **I** ΔBMY **T** ΔCOP

ΔFEN	ΔECB	ΔEXN	ΔBMA	ΔDCN	ΔDNE	ΔBDX	ΔFED	ΔAFR	ΔAZY



Explain how this playground shows congruent figures.

13:03 The minimum conditions for congruent triangles

We have seen that if two shapes are congruent then one shape can be superimposed on the other so that they coincide exactly.

For a triangle, this would mean three pairs of matching sides and three pairs of matching angles.

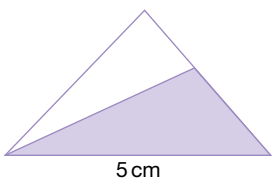
- What do we need to know about two triangles before we can say they are congruent?
- Do we always need to check that there are three pairs of matching sides and angles, or is there a shorter way?

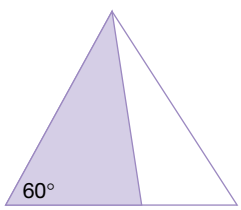
In the next exercise, we will investigate the **minimum conditions** for congruent triangles. A minimum condition is the smallest amount of information that we need to know about two triangles before we can say that they are congruent.

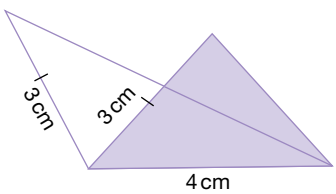
Note: It will be necessary to construct or trace triangles in this exercise.

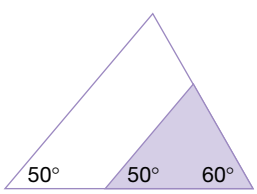
Exercise 13:03

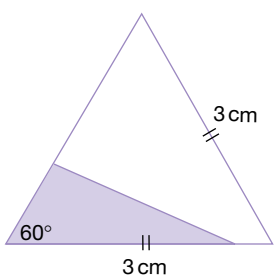
1 Each of the five diagrams below shows one triangle superimposed on another.













Using the triangles, decide whether two triangles are congruent if they:

a have only one side equal
b have only one angle equal
c have only two sides equal
d have two angles equal
e have one side and one angle equal.

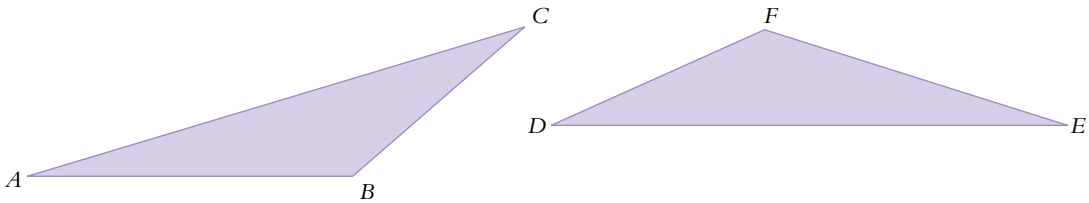
2 Can we be sure that two triangles are congruent if we can compare only two pieces of information on each one? Use the triangles in Question 1 to answer this question.

3 **a** When a photograph is enlarged, are:
- **i** the angles in the photo and enlargement the same
- **ii** the photo and the enlargement congruent?

b If two triangles have their three angles equal, does it mean they are congruent?

To compare three pieces of information to ensure that two triangles are congruent, we could compare:
- three sides
- two sides and the included angle
- one side and two angles
- three angles.

4 Copy one of the following triangles onto paper and cut it out.

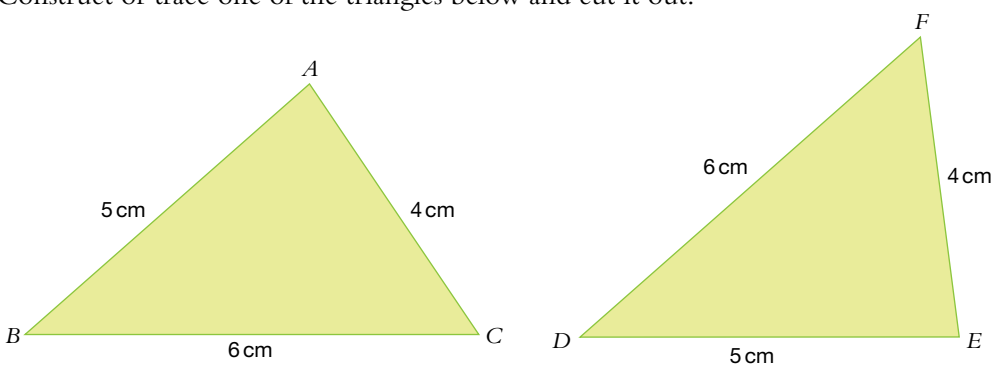


a Do the triangles have their sides equal in length?

b By superimposing the cut-out triangle over the other triangle, see if the triangles are congruent.

c Do the triangles have the same-sized angles?

5 Construct or trace one of the triangles below and cut it out.

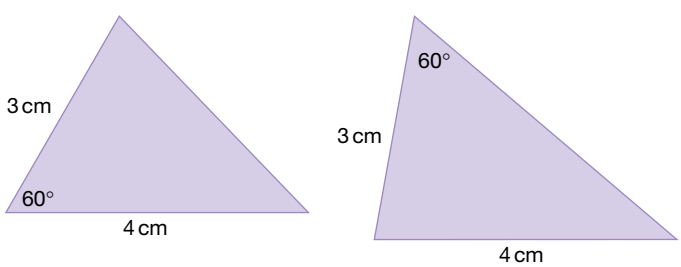


a By placing the cut-out triangle over the other triangle, find out if the two triangles are congruent.

b Name the pairs of matching (or corresponding) angles.

In Questions **4** and **5**, the triangles with matching sides of equal length were congruent. We say that three pairs of sides equal is a minimum condition for congruent triangles. This is the first minimum condition, abbreviated as SSS.

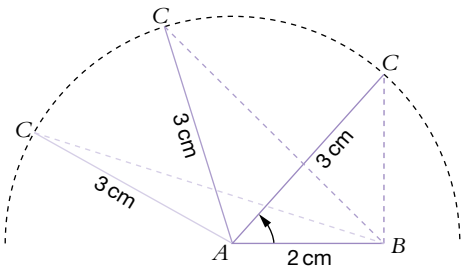
6



a Do the triangles at left have two sides and one angle equal?

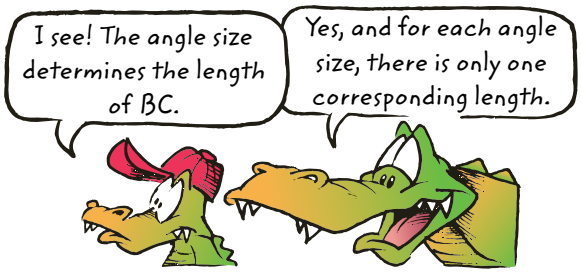
b Are the triangles congruent? Check by measuring the third side.

7 The diagram shows a triangle with sides AB and AC of given lengths. The angle CAB is allowed to vary in size so that C moves on a circle with radius AC and centre A.



a What happens to the length of BC as $\angle BAC$ gets bigger?

b If you fix $\angle BAC$ at a certain size (e.g. 30°) is it possible to get two different lengths for BC?

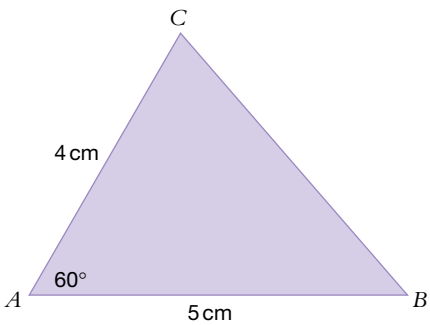


8 Construct or trace the triangle shown on a piece of paper.

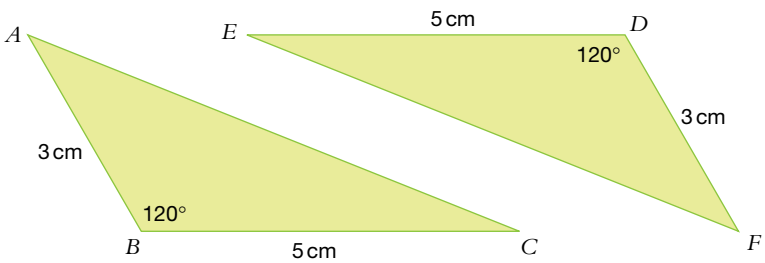
a Measure BC to the nearest millimetre.

b Is it possible to get more than one triangle from this information?

c Cut your triangle out and compare it with the triangles of other students in your class. Are they all congruent?



9 Construct or trace one of the triangles below and cut it out.



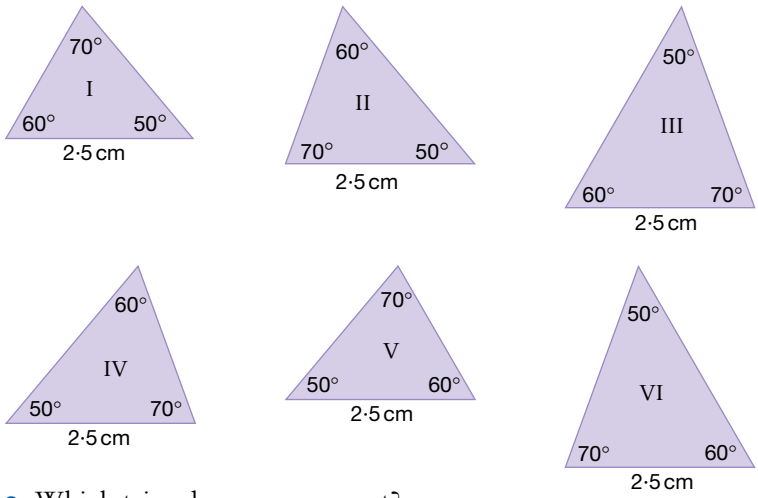
a Is $AC = EF$?

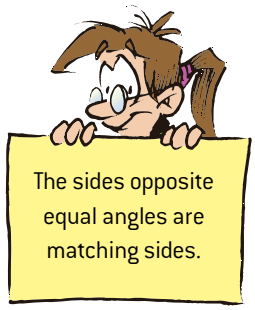
b By superimposing, find whether ΔABC is congruent to ΔDEF.

In Questions **6** to **9**, we found that when two sides and the angle between them in one triangle are equal to two sides and the angle between them in the other triangle, then the triangles are congruent.

It is important that the angle is *included by* (between) the two sides. This is the second minimum condition, abbreviated as SAS.

10 In each of the following triangles, the angles match and one side in each has the same length. Construct or copy each triangle and cut it out.

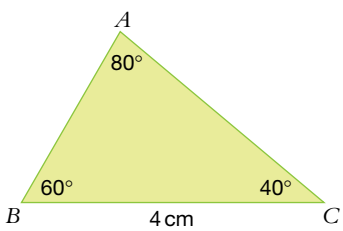


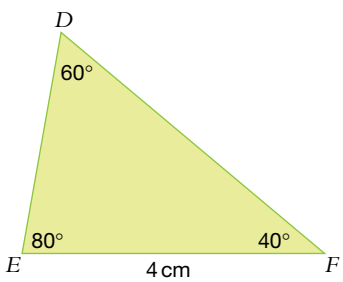


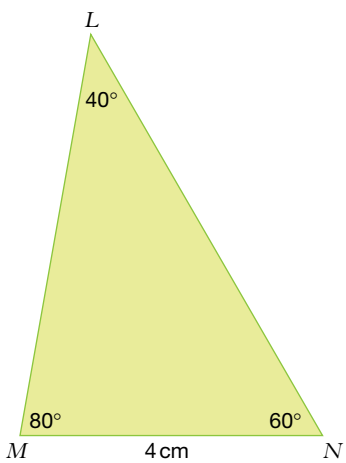
a Which triangles are congruent?

b For each pair of congruent triangles, how could you describe the position of the 2.5 cm side?

11 Construct or copy each triangle and cut it out.





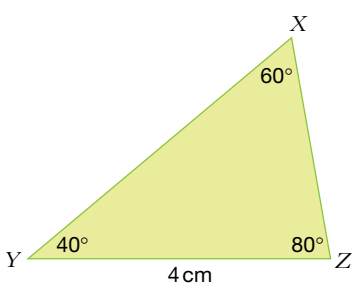


a Which angle is the 4 cm side opposite in:

i ΔABC **ii** ΔDEF **iii** ΔLMN?

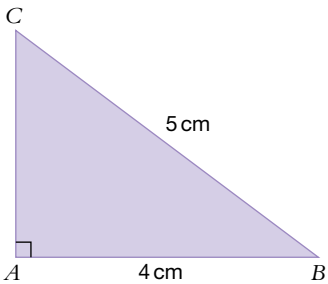
b Which of the triangles shown is congruent to ΔXYZ?

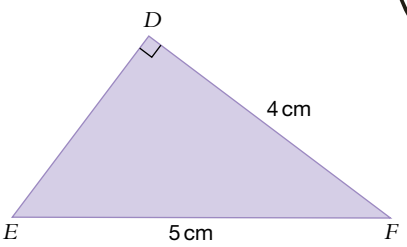
c How could you describe the position of the 4 cm side in each of the congruent triangles?

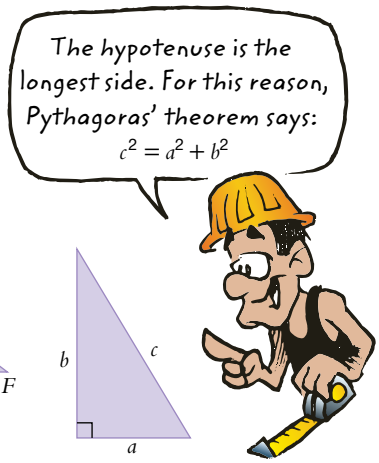


Questions **10** and **11** have shown us that if the angles of one triangle are equal to the angles of another triangle, and a side in one is equal to a side in the same position of the other, then the triangles are congruent. This is the third minimum condition, abbreviated as AAS.

12 The fourth set of minimum conditions is restricted to right-angled triangles only.
Copy one of the triangles below and cut it out.
Does your cut-out match both triangles?

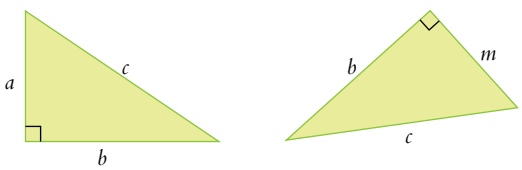






a Write down the pairs of matching sides.
b Are the triangles congruent?

13 The two right-angled triangles shown have their hypotenuse and one side equal in length.

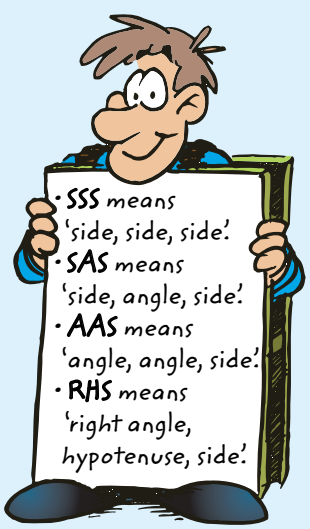


a Write down Pythagoras' theorem for each triangle.
b By rearranging the formula, show that $a = m$.
c Are the triangles congruent?

Questions **12** and **13** show that two right-angled triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle. This is the fourth minimum condition, abbreviated as RHS.

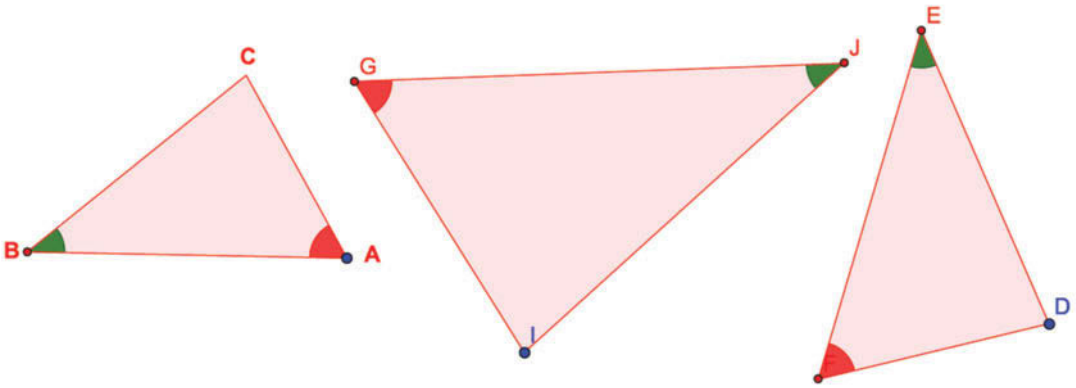
Summary

1. Two triangles are congruent if three sides of one triangle are equal to three sides of the other. (SSS)
2. Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other. (SAS)
3. Two triangles are congruent if two angles and a side of one triangle are equal to two angles and the matching side of the other. (AAS)
4. Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle. (RHS)



P GEOGEBRA ACTIVITY 13:03A ARE TRIANGLES WITH MATCHING ANGLES EQUAL ALWAYS CONGRUENT?

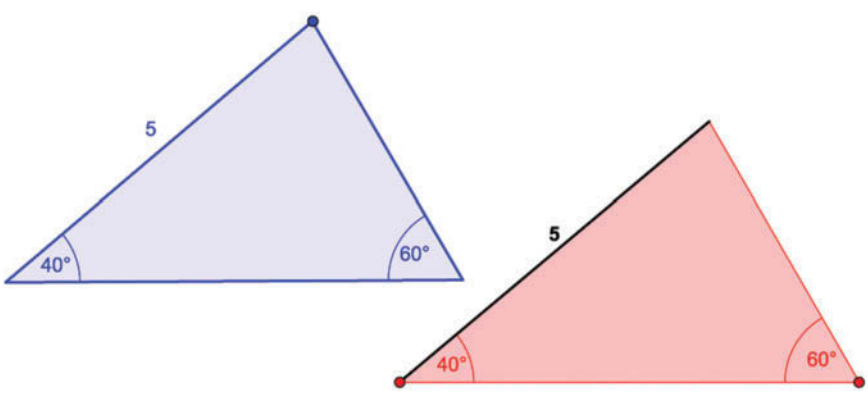
In this activity, triangles in which all pairs of matching angles are equal are tested for congruency.



P GEOGEBRA ACTIVITY 13:03B CONDITIONS FOR CONGRUENT TRIANGLES

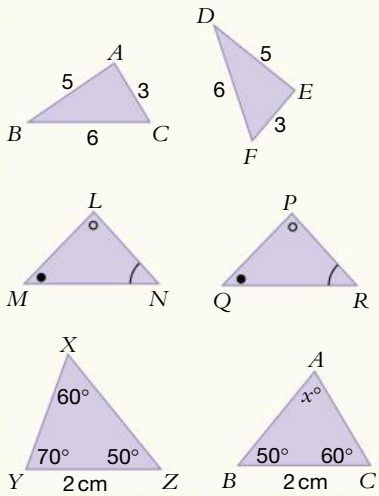
In this set of five activities, two triangles are constructed under different conditions and these triangles are then superimposed to see if they are congruent. These activities demonstrate clearly the minimum conditions that are necessary for two triangles to be congruent.

The activities look at the conditions SSS, SAS, AAS and RHS and whether triangles with equal angles are congruent.



13:04 Applying the congruency tests

PREP QUIZ 13:04



Name the side that matches:

1 AC **2** AB **3** BC

Name the angle that matches:

4 $\angle A$ **5** $\angle B$ **6** $\angle C$

Name the side that matches:

7 LM **8** MN

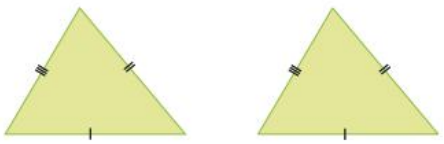
9 Find the value of x.

10 Are the 2 cm sides matching?

The minimum conditions deduced in the last section are used to prove that two triangles are congruent. Special care must be taken in exercises that involve overlapping triangles.

Congruency tests for triangles

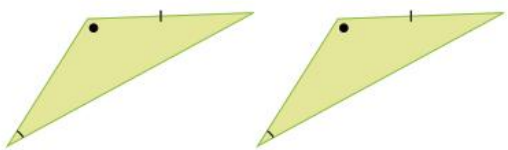
- Two triangles are congruent if all three matching sides are equal in length. (SSS)



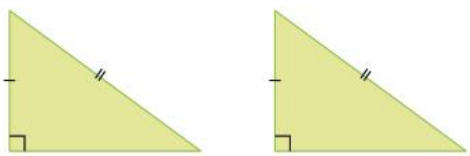
- Two triangles are congruent if two pairs of matching sides are equal in length and the included angles are equal. (SAS)



- Two triangles are congruent if two matching angles are equal and one pair of matching sides is equal in length. (AAS)

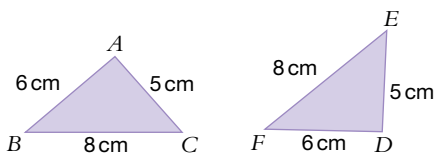


- Two right-angled triangles are congruent if the hypotenuse and one pair of matching sides are equal in length. (RHS)



WORKED EXAMPLES

1 Why is ΔABC congruent to ΔDEF?

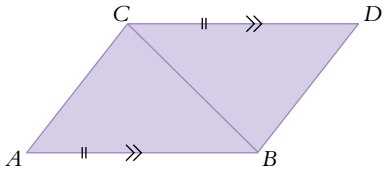


All the matching sides in ΔABC and ΔDEF are equal in size.
$\therefore \Delta ABC \equiv \Delta DEF$ (SSS)

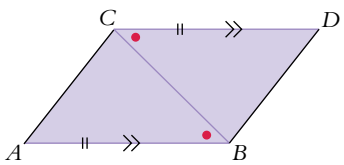
Note:
$\equiv$ means
'is congruent to'



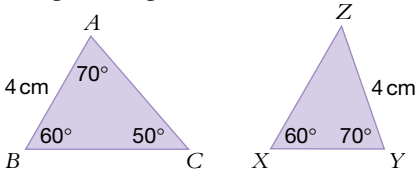
2 Which of the congruency tests can be used to show that ΔABC is congruent to ΔDCB?



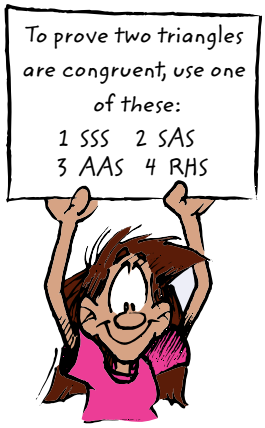
The markings show the equal sides and equal angle. ΔABC and ΔDCB have two sides and an included angle equal.
$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)



3 Are these two triangles congruent?



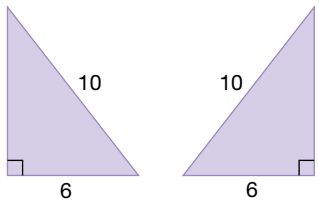
- Only one pair of side lengths are given, so we cannot use SSS, SAS or RHS. Hence, we can only test for AAS.
- All three matching angles in the triangles are equal. The missing angle in ΔXYZ must be 50° because the angle sum of a triangle is 180°.
- The 4 cm side is opposite the 50° angle in ΔABC and opposite the 60° angle in ΔXYZ. Hence, the sides are not matching.
- Therefore, the triangles are not congruent.



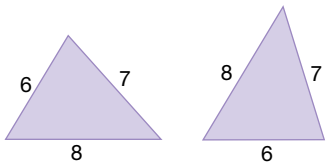
Exercise 13:04

1 State the congruency test used to establish that the following pairs of triangles are congruent. All side lengths are in the same units.

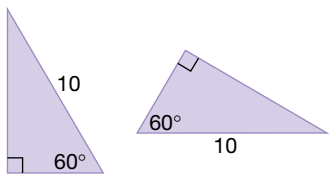
a



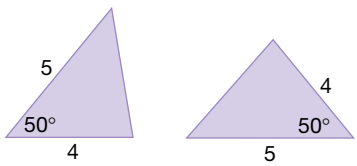
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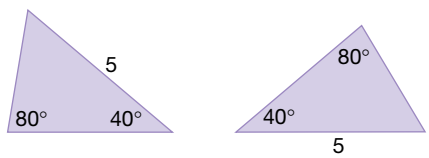
c



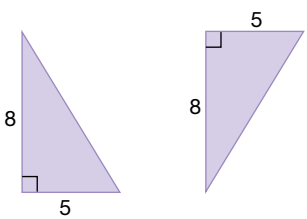
d



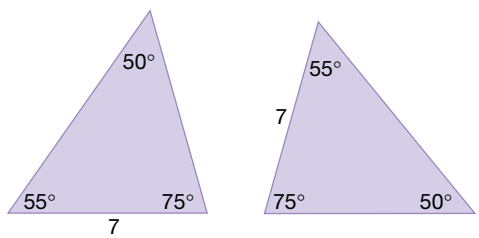
e



f



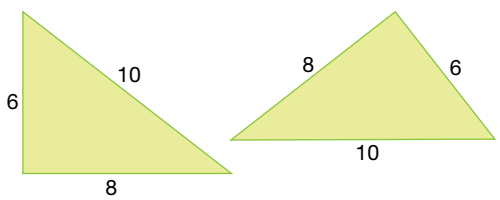
g



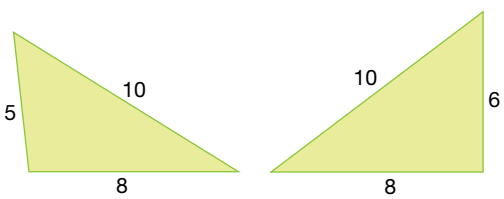
Note: When the two triangles both have a right angle, don't assume that the congruency test is RHS.

2 State whether the triangles in the following pairs are congruent. For those that are, state the congruency test used to establish congruence.

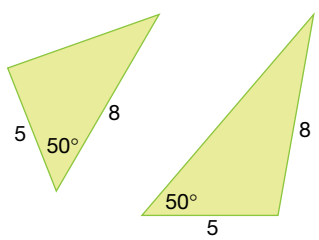
a



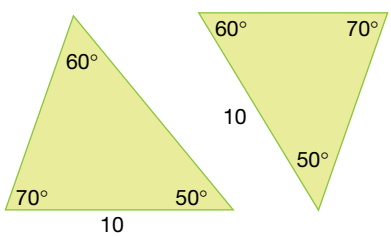
b



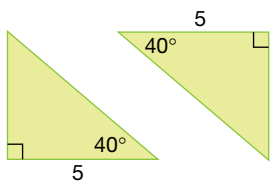
c



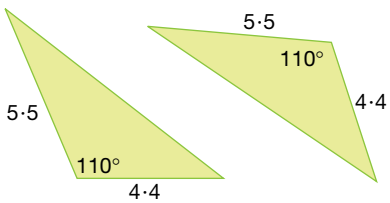
d



e

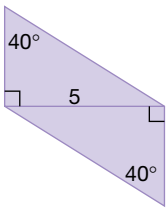


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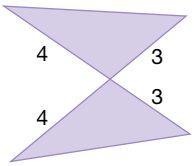


3 The following pairs of triangles are congruent. State the congruency test used to establish the congruence.

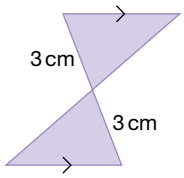
a



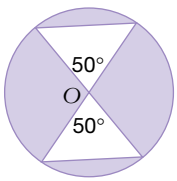
b



c

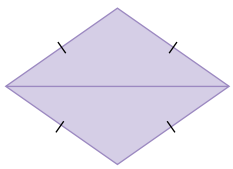


d

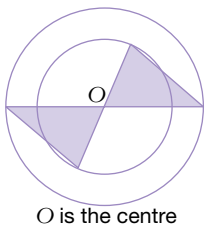


O is the centre of the circle.

e

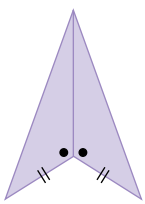


f

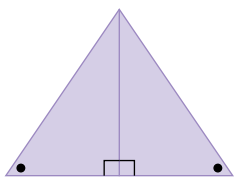


O is the centre of both circles.

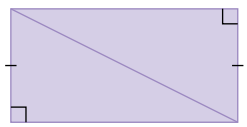
g



h

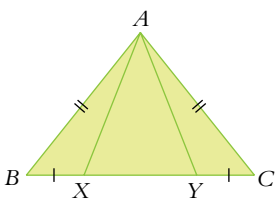


i

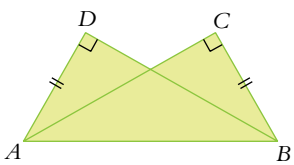


4 Show why each of the following statements is true.

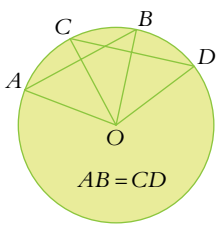
a $\Delta ACY \equiv \Delta ABX$



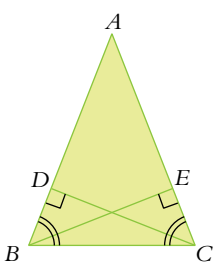
b $\Delta ADB \equiv \Delta BCA$



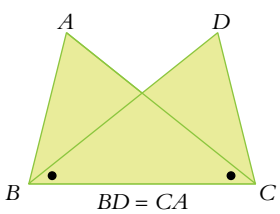
c $\Delta AOB \equiv \Delta COD$



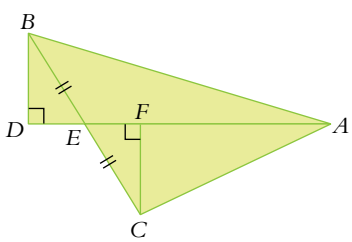
d $\Delta DBC \equiv \Delta ECB$



e $\Delta ABC \equiv \Delta DCB$



f $\Delta BDE \equiv \Delta CFE$



FUN SPOT 13:04 WHAT DO YOU CALL A MAN WITH A SHOVEL?

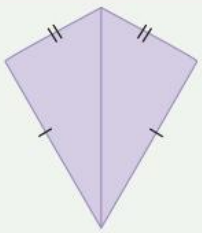
Work out the answer to each part and write the letter for that part in the box above the correct answer.

Which congruence test can be used to state why the triangles in each pair are congruent?



SSS	RHS	SAS	AAS

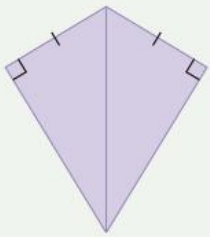
D



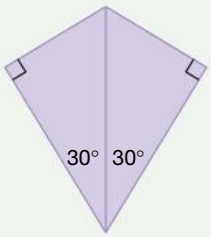
U



O



G



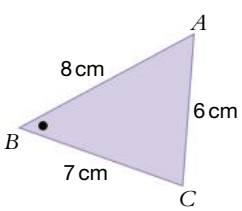
13:05 Using congruent triangles to find sides and angles

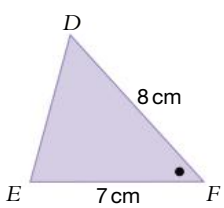
If two triangles can be shown to be congruent, then all corresponding sides and angles are equal.

Using congruent triangles to find the values of unknown angles and sides or to prove relationships is an important skill in geometry.

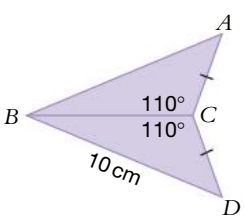
WORKED EXAMPLES

1 Show that $\Delta ABC \equiv \Delta DFE$ and hence, find the length of DE.





2 State why $\Delta ACB \equiv \Delta DCB$ and hence, find the value of AB.



Solutions

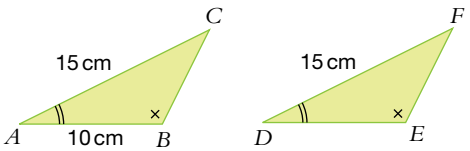
1 $\Delta ABC \equiv \Delta DFE$ (SAS)
$\therefore DE = AC$
(matching sides of congruent Δs)
$\therefore DE = 6$ cm

2 ΔACB and ΔDCB have two matching sides and an included angle equal.
$\therefore \Delta ACB \equiv \Delta DCB$ (SAS)
$\therefore AB = DB$ (matching sides of congruent Δs)
$\therefore AB = 10$ cm

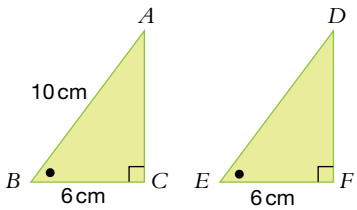
Exercise 13:05

1 In each of the following, state why $\Delta ABC \equiv \Delta DEF$ and hence, find the value of DE.

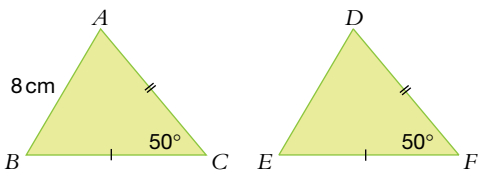
a



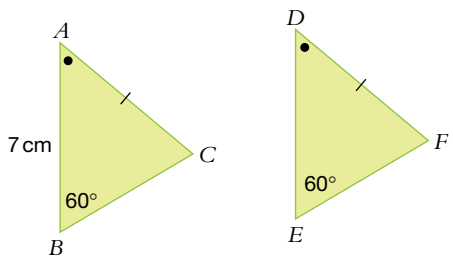
b



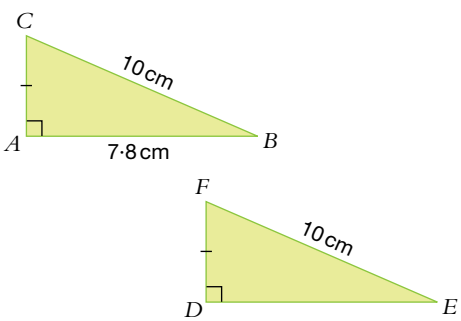
c



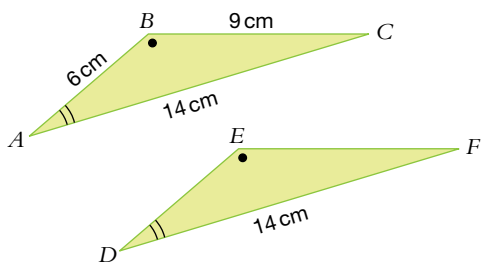
d



e

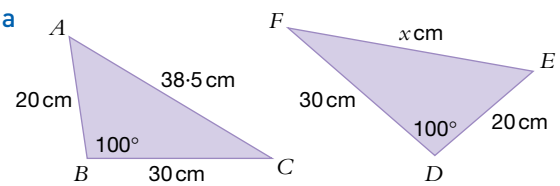


f

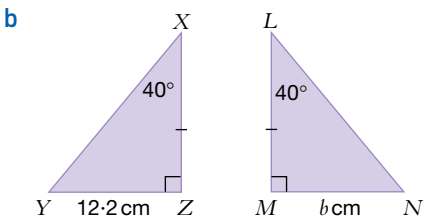


2 Find the value of the pronumeral in each of the following, giving reasons for your answer.

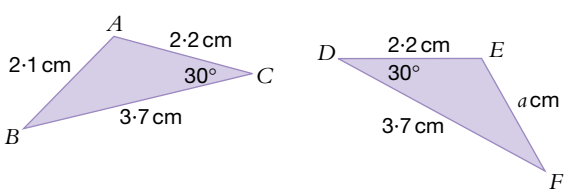
a



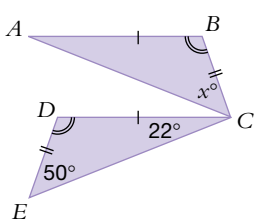
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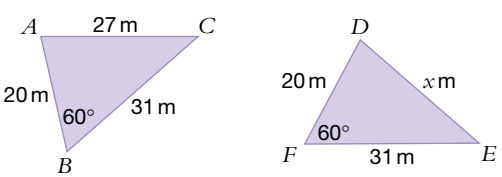
c



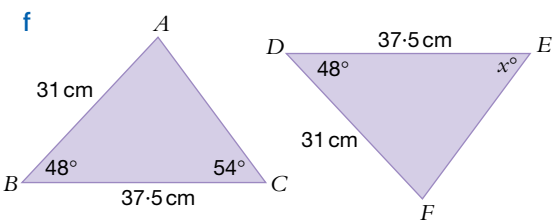
d



e

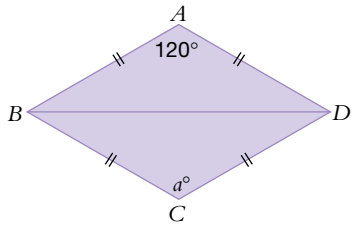


f

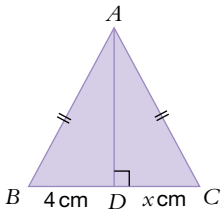


3 Find the value of the pronumeral in each of the following, giving reasons for your answer.

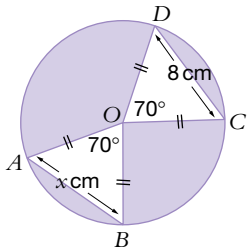
a



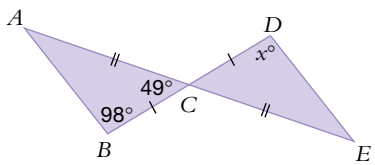
b



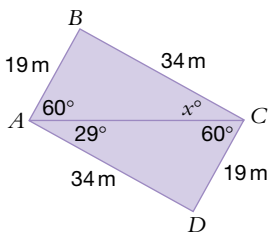
c



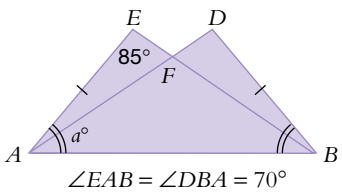
d



e



f



$\angle EAB = \angle DBA = 70°$

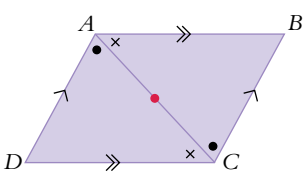
13:06 Deducing properties of the special quadrilaterals

Many of the properties of the special quadrilaterals can be established using congruent triangles combined with geometric proofs. The following exercise investigates the following properties of quadrilaterals.

- A rectangle has four right angles.
- The diagonals of a rectangle are equal in length.
- A square is a rectangle with a pair of equal adjacent sides.
- The opposite sides and opposite angles of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.
- A rhombus is a parallelogram with all sides equal.
- The diagonals of a rhombus bisect each other at right angles.
- The diagonals of a rhombus bisect the angles through which they pass.

Exercise 13:06

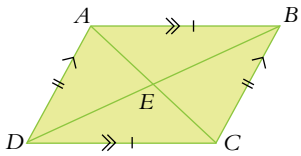
1 $ABCD$ is a parallelogram.



a Why does $\angle BAC$ equal $\angle DCA$?
b Why does $\angle BCA$ equal $\angle DAC$?
c Does $\angle BAD$ equal $\angle DCB$?
d Why is $\Delta ABC \equiv \Delta CDA$?
e ΔABC is rotated about the midpoint of AC (red dot).
  i Does $\angle ABC = \angle CDA$?
  ii Does $AB = DC$ and $BC = AD$?

This question shows that the opposite sides and opposite angles of a parallelogram are equal.

2 **a** Using the fact that the opposite sides of a parallelogram are equal, state which congruency test can be used to show that $\Delta ABE \equiv \Delta CDE$.



b ΔABE is rotated about the point E.

i Does $AE = EC$?

ii Does $BE = ED$?

This question shows that the diagonals of a parallelogram bisect each other.

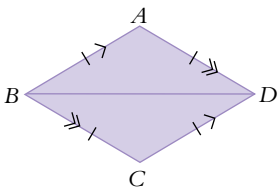
3 A rhombus is a parallelogram with all its sides equal.

a Which congruency test can be used to show that $\Delta ABD \equiv \Delta CBD$?

b ΔABD is reflected in the line BD.

i Why is $\angle ABD = \angle CBD$?

ii Why is $\angle ADB = \angle CDB$?

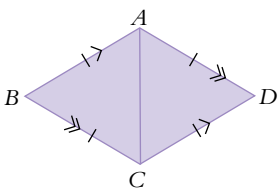


c Which congruency test can be used to show that $\Delta ABC \equiv \Delta ADC$?

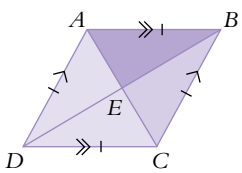
d ΔADC is reflected in the line AC.

i Why is $\angle BAC = \angle DAC$?

ii Why is $\angle BCA = \angle DCA$?



4



a Why does $AE = EC$?

b State why $\Delta ABE \equiv \Delta CBE$.

c ΔABE is reflected in the line BE.

d State why $\angle AEB = \angle CEB = 90°$.

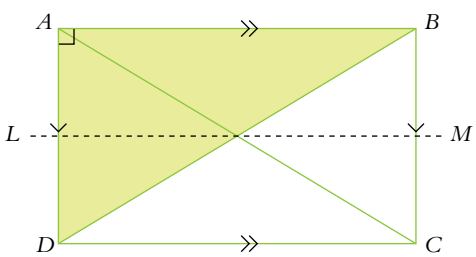
The last two questions have shown that the diagonals of a rhombus bisect each other at right angles, and that they bisect the angles through which they pass.

5 A rectangle is a parallelogram with a right angle.

a Prove that all the angles must be right angles.

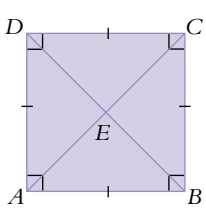
b Assuming the answer to part **a** and the properties of a parallelogram, state why $\Delta ABD \equiv \Delta DCA$.

c ΔABD is reflected in the horizontal axis of symmetry LM. State why $AC = DB$.



This question proves that all the angles of a rectangle are right angles and that its diagonals are equal in length.

6 A square is a rectangle with a pair of adjacent sides equal. Using this definition and the properties of the rectangle, write down as many properties of the square as you can.



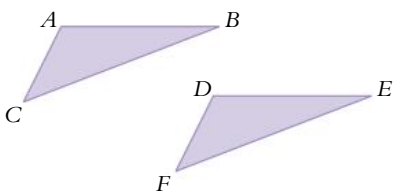
MATHS TERMS 13

congruency tests
- a set of four tests used to decide whether two triangles are congruent
- the tests are usually referred to by the abbreviations SSS, SAS, AAS and RHS

congruent figures
- shapes that are identical in shape and size

congruent triangles
- triangles that are identical in shape and size

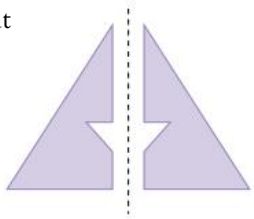
matching angles (or sides)
- angles or sides that are in the same or corresponding positions in congruent figures



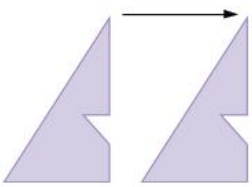
- matching angles in the triangles above are $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$
- matching sides are AC and DF, AB and DE, BC and EF

superimpose
- to place one shape on top of another

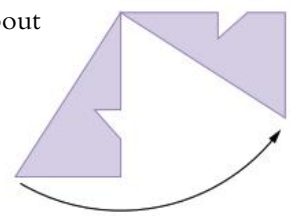
transformations (congruence transformations)
- the movement of a shape to a new position; the shape in the new position would be congruent to the shape in the original position
- reflection: a flip about a line



- translation: a slide



- rotation: a turn about a point

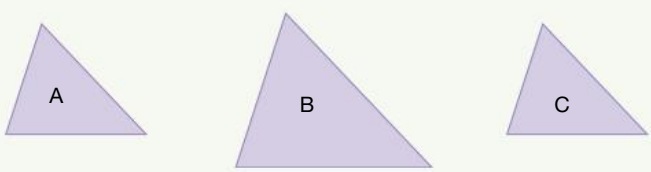


DIAGNOSTIC TEST 13 CONGRUENCE

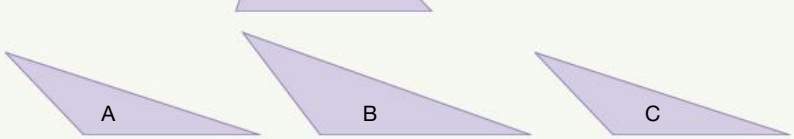
Each section of the test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.
- You will need a ruler and compasses.

1 Identify the congruent triangles in each of the following. 13:01

a



b



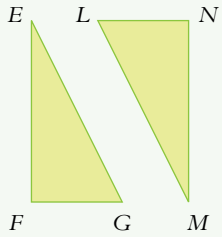
2 $\Delta EFG \equiv \Delta MNL$

Which side matches:

a EF **b** EG **c** FG?

Which angle matches:

d $\angle EFG$ **e** $\angle FGE$ **f** $\angle LMN$?



13:01, 13:02

3 BD is an axis of symmetry of ΔACD.

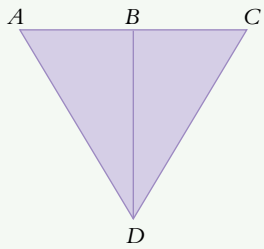
a Complete: $\Delta ABD \equiv \ldots$

b Which side matches AB?

c Which side matches CD?

d Which angle matches $\angle BAD$?

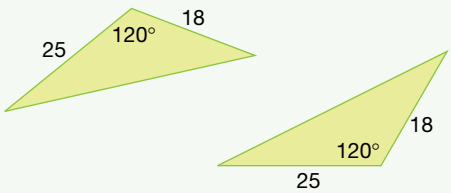
e Which angle matches $\angle BDA$?



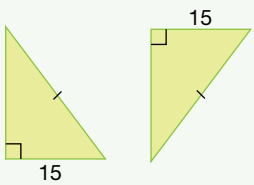
13:01, 13:02

4 State the congruency test that can be used to show that the pairs of triangles are congruent.

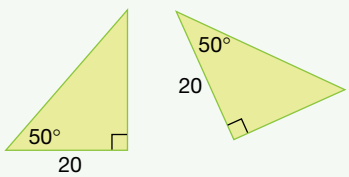
a



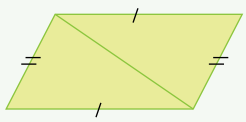
b



c



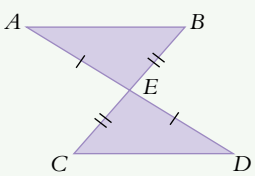
d



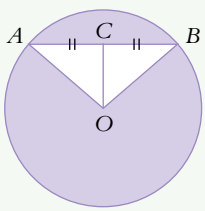
13:04

5 State why the two triangles are congruent.

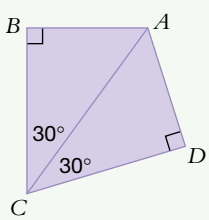
a



b



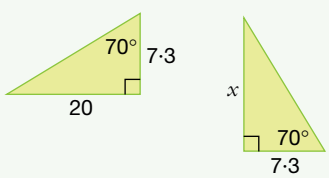
c



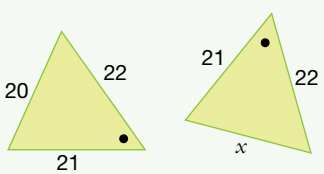
13:04

6 Find the value of the pronumeral in each of the following.

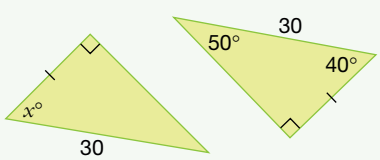
a



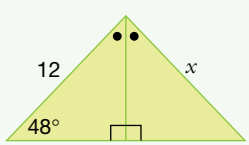
b



c



d



13:05

ASSIGNMENT 13A Chapter review

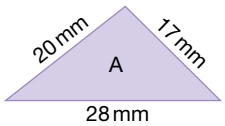
1 From the triangles below, select the triangle that is congruent to:

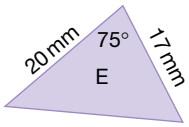
a triangle A

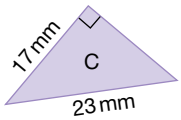
b triangle B

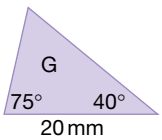
c triangle C

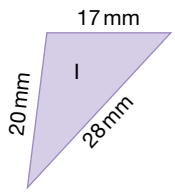
d triangle D.

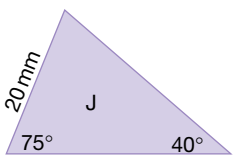


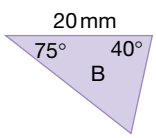


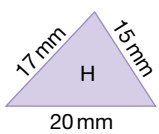


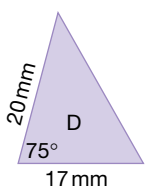


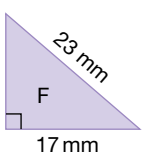


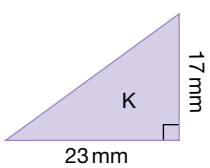


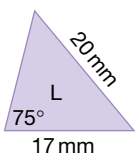






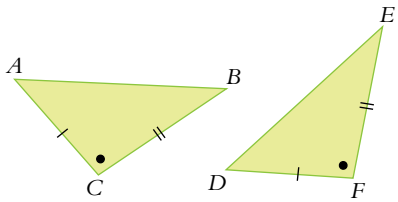






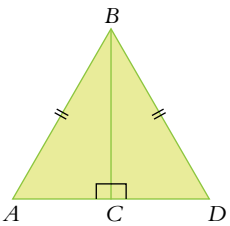
2 In each of the following, state the congruency test that can be used to say why the triangles are congruent.

a



$\Delta ABC \equiv \Delta DEF$

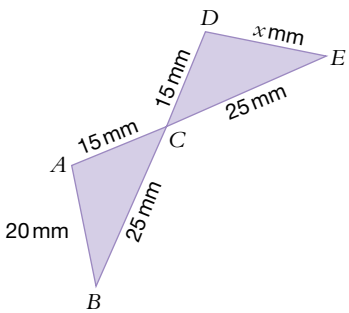
b



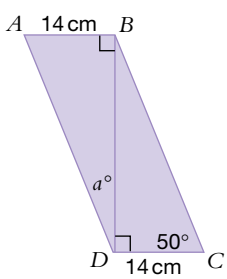
$\Delta ABC \equiv \Delta DBC$

3 Find the value of each pronumeral, giving reasons for your answer.

a



b



4 Figure 1 has been rotated to produce Figure 2.

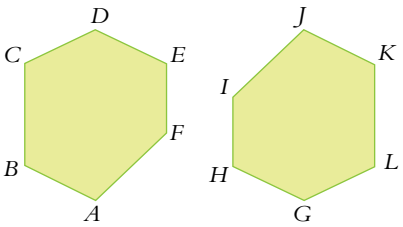


Figure 1 Figure 2

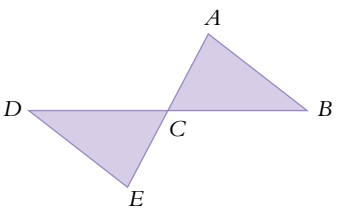
Which angle matches:

a $\angle A$ **b** $\angle E$ **c** $\angle I$?

Which side matches:

d BC **e** DE **f** GL?

5

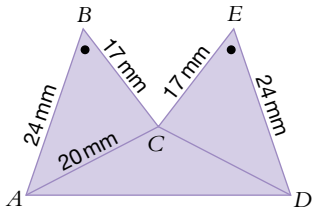


ΔABC is rotated about the point C to produce ΔEDC. Write down the pairs of:

a matching angles

b matching sides.

6

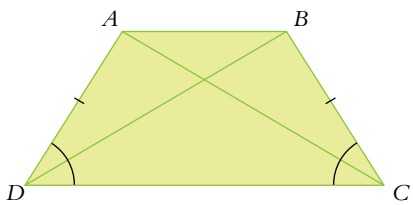


a Which congruency test allows us to say that $\Delta ABC \equiv \Delta DEC$?

b What is the length of CD?

c Which angle in ΔCED is equal to ΔBAC?

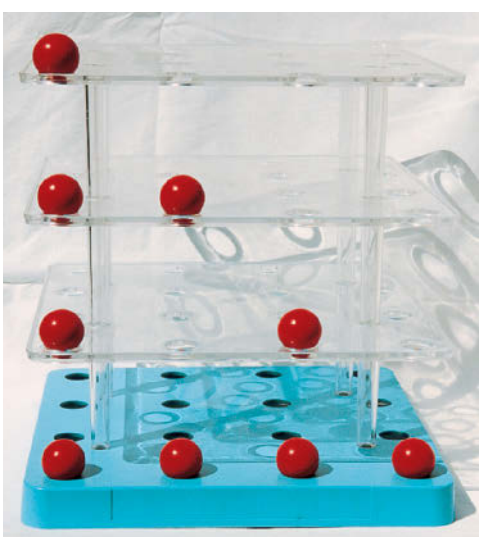
7



a Which congruency test can be used to show that $\Delta ADC \equiv \Delta BCD$?

b If ΔADC is reflected to produce ΔBCD, state why $BD = AC$.

c Which angle is equal to $\angle DAC$?



The distance between the centres of adjacent balls on the horizontal line is 7 cm. The distance between the centres of adjacent balls on the vertical line is 8 cm. What is the distance between adjacent balls on the hypotenuse?

ASSIGNMENT 13B Working mathematically

1 Complete each of the following by finding the missing digits.

a
$$\begin{array}{r} 3\ 8 \\ +\ \square\ \square \\ \hline 7\ 4 \\ \hline \end{array}$$

b
$$\begin{array}{r} 9\ 0\ 4 \\ -\ \square\ \square \\ \hline \square\ 1 \\ \hline \end{array}$$

c
$$\begin{array}{r} 5\ \square\ 4 \\ +\ \square\ 3\ \square \\ \hline \square\ 1\ 0\ 1 \\ \hline \end{array}$$

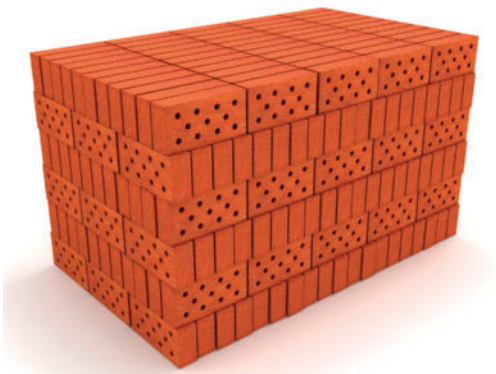
d
$$\begin{array}{r} \square\ 6\ 3\ 2 \\ -\ 9\ \square\ \square \\ \hline \square\ 0\ 7 \\ \hline \\ \hline \end{array}$$

2 The petrol tank of my car holds 65 litres. I drove into a petrol station and filled the tank. It cost me $40.60.

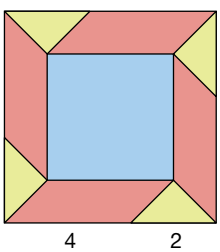
a If the petrol cost $1.45 per litre, how many litres did I buy?

b How much was in the tank before it was filled?

3 The layers of bricks in this stack are not the same. How many bricks are in the stack?

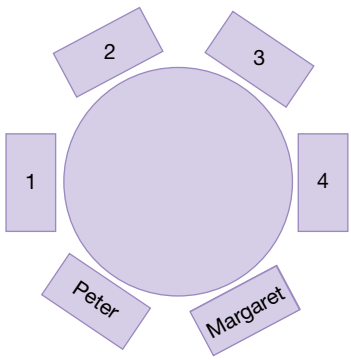


4 This design has been made from a square, four identical triangles and four identical parallelograms. What is the area of the parallelogram? (First find the difference between areas of the small and large square.)



5 Solar panels are installed in a house. The electricity generated by the panels is used in the house or, if not required, it is fed into the electricity grid. The homeowner receives 6c per kilowatt-hour (kWh) for every kWh that goes back into the grid. If the solar panels do not generate enough electricity to cover the house's needs, it must be bought at a cost of 21.85 cents per kWh. In a 99-day billing period, the solar panels generated 1107 kWh. Of these, 709 kWh were used in the house. The remainder was fed back into the grid. How much did the solar panels save the owner?

6 Sarah, Kate, Ben and Emma had dinner in a restaurant with their parents Margaret and Peter. Emma sat next to one of her parents. Ben sat between Sarah and Kate, and opposite his father. Sarah sat between Ben and Emma. Name the person who sat in each of the seats 1, 2, 3 and 4.

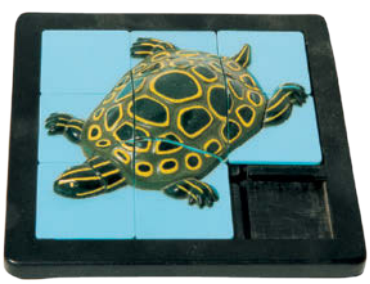


7 In the turtle puzzle below, the tiles can be moved, one at a time, by sliding into the empty space. What is the least number of moves that would leave the empty space:

a in the centre

b on the top left

c on the top middle?



ASSIGNMENT 13C Cumulative revision

1 Simplify the following:
a $4x + 3y + 6 + 2x + 4y$ **b** $4x^2 - 3x + 2x^2 + 8x - 6$ 5:02

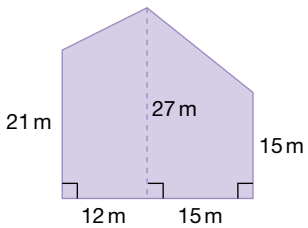
2 Expand and simplify:
a $x(2x - 1) + 3x(1 - x)$ **b** $4a(a - 3 + b) - b(a - 2)$ 5:08

3 Solve:
a $3p = 2p - 7$ **b** $1 - 6a = 3a + 8$ **c** $-4 - 6b = 12 - 3b$ 9:03

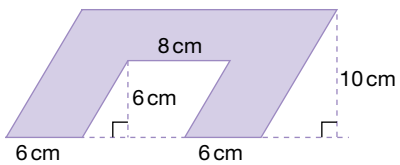
4 **a** If $x = -3$, evaluate $3x^2 - 4x - 5$. **b** If $a = 4$, $b = -2$, evaluate $\dfrac{(a-b)^3}{a+2b-4}$. 2:03

5 Find the area of the following shapes. 8:03

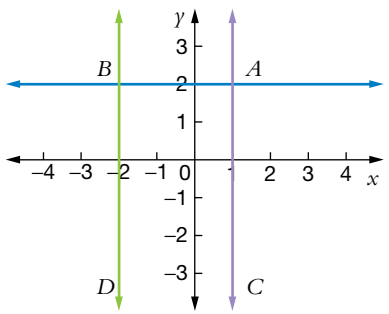
a



b

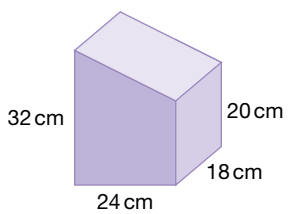


6 The lines cross at the points A and B. What is the equation of the line:
a AB
b AC
c BD? 11:03



7 **a** If a fair dice is thrown the probability of rolling a 6 is $\frac{1}{6}$. What is the probability of not rolling a six? 6:03
b It two fair dice are rolled the probability of rolling two sixes is $\frac{1}{36}$. What is the probability of not rolling two sixes?

8 What is the volume of this trapezoidal prism? 8:06

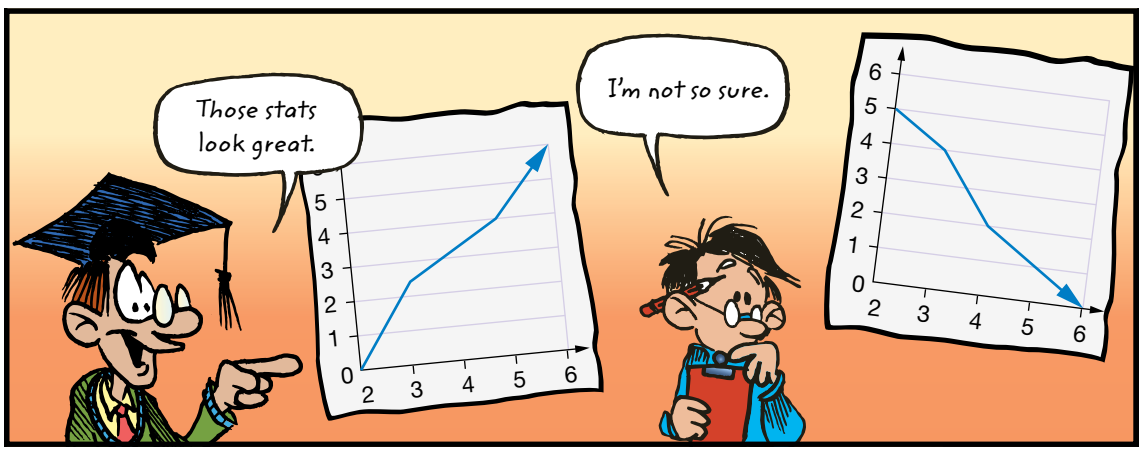


9 **a** The ratio of the weights of two boxes is 3 : 2. If the total weight of the boxes is 58 kg, find the weight of the lighter box. 10:04
b In making up a paint, two colours A and B are mixed in the ratio 1·5 : 1. How much of each paint is needed to make 4 L of paint?

10 **a** A bronze alloy consists of 80% copper, 12% tin and 8% zinc. How much tin will be in 70 kg of this bronze? 4:03, 4:04, 4:05
b When heated, a metal girder expands from 15 m in length to 15·06 m. What is the percentage increase in the length of the girder?

STATISTICS

14



Contents

14:01 Collecting data
Investigation 14:01 Internet sources
14:02 Sampling
Challenge 14:02 Catch and release
14:03 Analysing data
14:04 Statistical graphs
Investigation 14:04 Misuse of graphs
14:05 Grouped data
14:06 Back-to-back stem-and-leaf plots
Challenge 14:06 Wedding anniversaries
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details)

Statistics and Probability

Selections from ***Data Collection and Representation***, ***Single Variable Data Analysis*** [Stage 4]

- Identify and investigate issues involving numerical data collected from primary and secondary sources. (ACMSP169)
- Construct and compare a range of data displays including stem-and-leaf plots and dot plots. (ACMSP170)
- Describe and interpret data displays using mean, median and range. (ACMSP172)
- Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. (ACMSP206)
- Investigate the effect of individual data values, including outliers, on the mean and median. (ACMSP207)
- Explore the variation of means and proportions of random samples drawn from the same population. (ACMSP293)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

14:01 Collecting data

In Year 7 we saw that three main ways of collecting data were by a census, a sample or direct observation. In collecting this data it may come from one of two sources: a **primary** source or a **secondary** source.

Primary source data has been collected yourself. This may have been done by writing and distributing a survey or by interviewing others and recording their responses to questions. Primary data may also be collected by direct observation, such as by counting or measurement. Such observations can be also made over time by recording data in a spreadsheet or log. Primary data is usually reliable because you know where the data came from and how it was collected.

Secondary source data has been obtained from an external source such as sites on the internet, TV, newspapers or other people's research information. Secondary source data is usually easier, and often cheaper, to acquire than primary data, but it may not be as reliable. Questions such as who collected it, how was it collected or was there any bias, need to be addressed.



The governing body of a nation often takes a census. The Bible records that, during the Roman Empire, Caesar Augustus issued a decree that a census should be taken of the entire Roman world. In Australia today, a census is taken every five years. The first three in this century were in 2001, 2006 and 2011.

Census data can be found on the ABS (Australian Bureau of Statistics) website.

Regardless of whether the source is primary or secondary, it is important to avoid a **bias** in the data. For example, if people at a screening of a science fiction movie were asked about their favourite type of movie, they would be more likely to choose science fiction movies than people chosen at random. Likewise, when a TV news program asks their viewers if they think the government is doing enough about crime, after just reporting on several criminal acts, the response to such a survey question is more likely to attract a negative response. Sometimes, a biased result is deliberately obtained by a poor survey targeted at a particular sample of people that is not a fair representation of a wider population.

WORKED EXAMPLE 1

Georgia saw an advertisement in the newspaper that said 4 out of 5 people used 'Whitey-O' toothpaste. However, when Georgia surveyed her class at school only 10 out of 25 used it. Which data source do you think is the most reliable, and why?

Solution

Georgia can be more confident of the data she received from her classmates because she collected it herself, from a primary source. The data in the advertisement, a secondary source, may be correct but Georgia doesn't know how it was collected, who was surveyed or if there was any bias involved in the process. Georgia's class may not be typical of a wider population so this data does not mean that the data in the advertisement is unreliable.

WORKED EXAMPLE 2

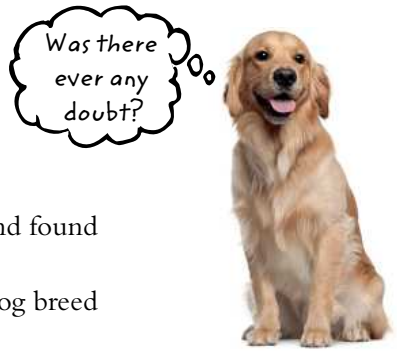
State whether each set of data has come from a primary or secondary source.

a Allyson read in a magazine that the most popular dog breed in NSW was a Golden Retriever.

b Paul interviewed students in his class and found their most popular dog breed was a Golden Retriever.

c Jenny handed out a survey at the local shopping centre and found that a Golden Retriever was the most popular dog breed.

d Rob searched the internet and found the most popular dog breed was a Golden Retriever.



Solutions

a Secondary source: The magazine is an external source.

b Primary source: Paul collected the data himself by interviewing other students.

c Primary source: Jenny collected the data herself.

d Secondary source: Rob searched for the data, but he used an external source. He didn't actually collect the data himself.

Exercise 14:01

1 State whether each source of data is a primary source or a secondary source.

a Alan counted his DVDs and found he had 10 western, 12 sci-fi and 15 comedy movies.

b Bella read in the newspaper that her favourite football team had 8 wins, 5 losses and 7 draws for the season.

c Conrad walked his street and noticed the pets consisted of 11 dogs and 7 cats.

d Donald surveyed his classmates and found collectively they had 25 brothers and 18 sisters.

e Elise saw a politician on the TV say the unemployment rate in Australia was 5·3%.

f Frankie looked up car sales on the internet and found the biggest-selling medium-sized car was a Mazda.

g Greg weighed himself each day for a week after exercising and found his weight fell from 85·7 kg to 83·5 kg.

h Huang went to a shopping centre and found there were 8 shoe stores, 13 clothing stores and 4 hair dressers.

i Isaac asked his friends what their favourite song was and found 7 out of 12 agreed with him.

j Jodie went to the local music store and found her favourite song was rated third on the Top 10 list.



2 For each of the following surveys, state whether or not you think it will give an unbiased set of data.

- **a** People are selected from a phone book and asked about the number of people who live in their house.
- **b** People at a football match are asked about their favourite sport.
- **c** Survey forms are sent to rate payers by the council to ask opinions about traffic lights.
- **d** Bicycle riders are asked about the need for more bike lanes on the roads.
- **e** Patrons at a rock concert were surveyed about the need for an opera house.
- **f** A business questions every fifth customer about customer service satisfaction.

3 For which of the following do you think you could find data from a secondary source and which would you have to collect the data yourself as a primary source?

- **a** Heights of students in your class.
- **b** Population of the states of Australia.
- **c** A-league football team with the biggest crowds.
- **d** Most popular football team with the students at your school.
- **e** Biggest-selling breakfast cereal.
- **f** The number of students in a class that have cereal for breakfast.
- **g** Most popular music group with Year 8 students.
- **h** Biggest-selling music groups in Australia.



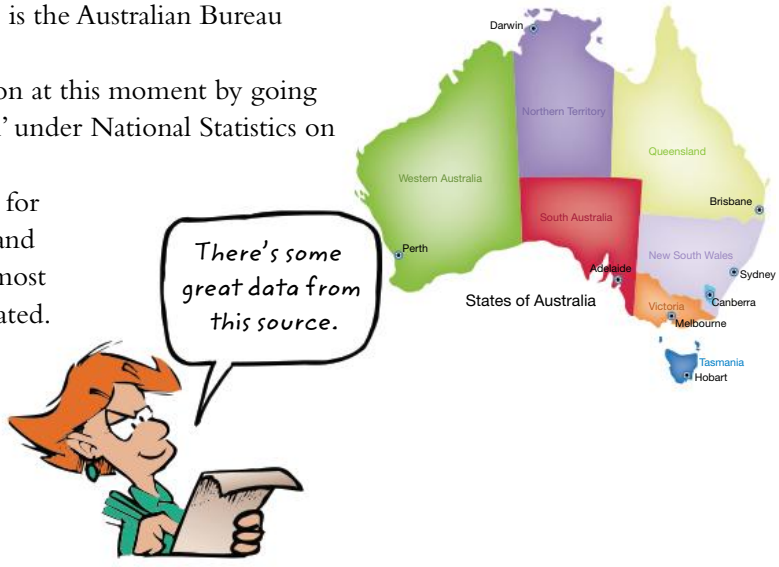
4 What could be wrong with these claims?

- **a** Dog food company declares 'Munchies' preferred by 4 out of 5 dogs.
- **b** Australian sports writers say Shane Warne is the best bowler in the world.
- **c** Workers' union says the best party to vote for is Labor.
- **d** Employees surveyed said they would rather work for an older boss.
- **e** 85% of workers think that unemployment benefits are too generous.

5 You carry out a survey at school by questioning a sample of students in the playground and find that the most popular Rugby League team is the same team that you support. You suspect there may have been an unintentional bias in the results. What factors might have caused a bias in the results?

6 A reliable secondary source is the Australian Bureau of Statistics (ABS).

- **a** Find Australia's population at this moment by going to 'Australia's Population' under National Statistics on the ABS website.
- **b** Look for the population for each state and territory and list them in order from most populated to least populated.



INVESTIGATION 14:01 INTERNET SOURCES



The internet is a popular source of information these days. People use technology when they want information, and they 'google' it. How do we know that information from such a secondary source is reliable? Any individual or organisation can create a website.

Here are some helpful tips.

1 **Check the domain**
Web addresses that include **.gov** are government sites so they should be reliable. Those that contain **.edu** are recognised educational institutions and should be accurate and professional. When looking for Australian data then an Australian website ending in **.au** is preferable.

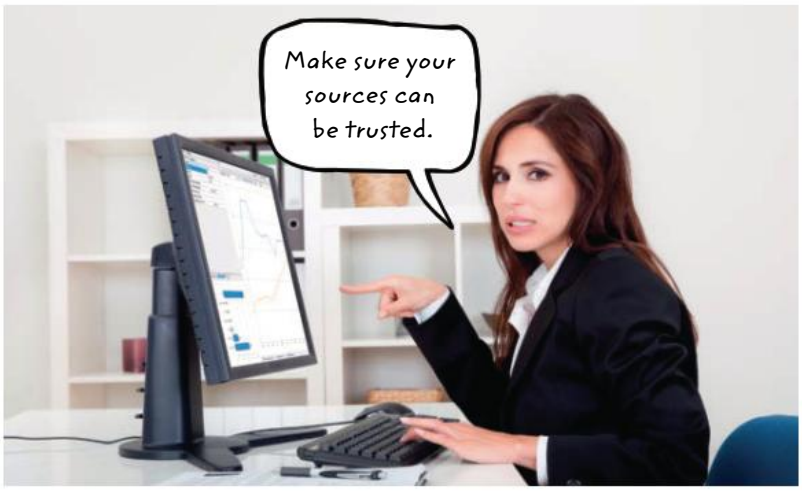
2 **Check the date**
Reliable sites should include the date they were created or last updated. This helps to know if the data it contains is up to date.

3 **Check the sources**
A reliable website will indicate its sources of information. That is, other websites, publications, authors, etc.

4 **Check their contact information**
A website that has no contact or feedback details is less likely to be trustworthy.

5 **Check the accuracy**
A site that appears to be poorly proofread and checked, which has spelling and punctuation errors, would cast doubt upon how carefully and accurately the data has been collected and compiled.

6 **Check any vested interests**
Are there reasons to suspect that any data is biased? For example, a car maker's website reviewing the quality of various cars is not likely to give a poor review of its own models, but may be more critical of a competitor's car.



14:02 Sampling

To collect accurate data about a population, a **census** provides a high degree of accuracy because every member of the population is surveyed. Information about subgroups can also be obtained but a census may be difficult to carry out. It may take a long time and be expensive.



A sample should represent the whole.

A sample is usually taken when it is impractical to survey a whole population. This is easier to manage and the data can be collected by personal interview, by telephone, by mail or by delivery and later pick-up. The data may also be compiled by direct observation. It is important, however, that a sample is a reliable representation of the whole population. Bias can occur when the method of selection for the sample favours one part of the group being studied. For example, the age, gender, race, education and political beliefs of people in the sample should reflect that of the population. Three basic methods of sampling are described below.

Random sampling

When a sample is chosen randomly, every member of the population has the same chance of being selected. No subgroups in the population are favoured by the selection process. This limits the chance of bias occurring. However, there is the possibility that the sample does not accurately represent the whole population, particularly if the sample chosen is small. For example, for a population that has equal numbers of males and females, if a small sample of 10 is selected it will be highly unlikely that there will be exactly 5 males and 5 females. Various techniques can be used to obtain a random sample, such as drawing names from a hat or by generating random numbers using a calculator or computer and assigning these numbers to members of the population.



Random numbers can be obtained by pressing the Ran# key (above the decimal point). They are given as decimal numbers (3 dec. pl.).
Press: SHIFT Ran# =
Press = repeatedly to generate new random numbers.

Systematic sampling

A systemic sample is selected according to an ordering scheme or rule. For example, every 10th member of the population is selected or every 100th product on a production line. It is important to not necessarily start with the first member of a population but with a random starting point. Also it is important to avoid any regular patterns present in the population being sampled. If every 10th house along a road is picked, for example, then the sample would have houses from only one side of the street, because one side has odd numbered addresses and the other has even numbered addresses.



Stratified sampling



A stratified sample is divided into subgroups or 'strata'. The sample is chosen in the same proportion or percentage of each subgroup. For example, if there are 60 men and 40 women in a club and a sample of 10 is to be taken, then it should consist of 6 men and 4 women, the same proportion as the total membership. There are several potential advantages to stratified sampling. Firstly, data about individual subgroups might also prove useful. Secondly, the sample is likely to be more representative of the whole population. Thirdly, different methods of selection may be used in different subgroups.

However, this method is only useful when there are obvious subgroups, such as Year groups in a school.

If we choose members from each subgroup randomly, then we have taken a **stratified random sample**.

WORKED EXAMPLE 1

Indicate whether each of the following is an example of random sampling, systematic sampling or stratified sampling.

In a school of 600 students, a sample of 60 was to be chosen.

a From an alphabetical list of names, starting with the 5th name, every 10th name after that was chosen.

b The names of all 600 students were placed in a hat and 60 were drawn out.

c Ten students from each of Years 7 to 12 were selected.

Solutions

a This is systematic sampling as the selection followed a predetermined rule of every 10th name.

b This is a random sample as each student was equally likely to be picked and the selection followed no pattern. It was completely random.

c This is a stratified sample as the separate groups were identified in the school population and selections were made so that each year was proportionally represented in the total sample.

WORKED EXAMPLE 2

Use the random number generator on your calculator to select a sample of 5 people out of 40.

Solution

Assign a unique number from 1 to 40 at random to each person in the sample. The calculator will generate a random number from 0·000 to 0·999. Multiplying such a decimal number by 40 and ignoring the digits after the decimal point will give a number from 0 to 39. If we add 1 to this number we will generate a number from 1 to 40.

e.g. Press: 40 SHIFT Ran# + 1 =

Answer = 36·64

So ignoring the decimal part:

Answer = 36

To get the next number just press =.

Doing this four more times will give five numbers to select the sample from the group of 40.

e.g. Four more random numbers of: 0·843, 0·763, 0·309, 0·025 would result in the numbers: 34, 31, 13 and 2. So, the required sample would consist, in order, of those in the group numbered 2, 13, 31, 34 and 36.

Note: This example has been generated from the random number 0.891. A different number results each time this calculation is performed.

If a number occurs twice, just generate another number.

WORKED EXAMPLE 3

Of the 560 students who completed a fun run, 146 were Year 9, 106 were Year 8 and 308 were Year 7. A sample of 80 is to be surveyed to comment on the race. How many of each group should be surveyed in order to have a stratified sample that represents those who participated?

Solution

The proportion each group in the total 560 should be represented in the sample of 80.

146 Year 9 students
$= \frac{146}{560} \times 80$
= 20·857…
= 21 (nearest whole number)

106 Year 8 students
$= \frac{106}{560} \times 80$
= 15·142…
= 15 (nearest whole number)

308 Year 7 students
$= \frac{308}{560} \times 80$
= 44

So the sample of 80 should consist of 21 Year 9, 15 Year 8 and 44 Year 7 students.

Exercise 14:02

1 State whether each of the following is an example of random, systematic or stratified sampling.
To complete a survey about how many people lived in each house in the area:

- **a** Josh visited every fourth house in his street
- **b** Joanna selected one house from each street in her neighbourhood
- **c** Jack put the street names and numbers in a hat and drew out 10 to visit.

2 Of 4000 batteries produced, we tested every 50th battery.

a What type of sample is this?

b How many batteries were tested?

3 A crowd consists of 30% men and 70% women. A sample is to be stratified to equally represent both groups. If the sample is to have a total of 60, how many men and women need to be surveyed?

4 Three students were asked to conduct a survey to determine the percentage of students in the school that had been to a concert this year.

- Hyrani asked four of her friends and found that three had been to a concert this year. She reported that 75% of the school population were likely to have been to a concert this year.
- Luka asked three boys from each class in the school and found that only 19% had been to a concert. As he felt more comfortable talking to the boys, he did not ask any girls.
- Anthea surveyed every 4th boy and every 4th girl in each class in the school. She concluded that 37% of the school had been to a concert this year.

a Which of the samples would give the most reliable information?

b What is wrong with Hyrani's sample?

c What is wrong with Luka's sample?

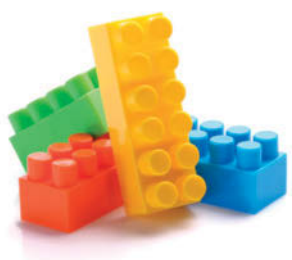
d What methods of sampling has Anthea used?

5 A stratified random sample of 144 students is to be taken from Years 9 to 12 to investigate 'country of birth'. The table shows the number of students in each Year. There are 432 in total. How many students should be chosen from each Year?

Year	Number
9	135
10	117
11	99
12	81

6 Of those participating in a fundraising event, 134 were men, 159 were women and 207 were children. A stratified random sample of 50 is to be taken. Calculate the number in each group that should be chosen for the sample.

7 A bucket contained 500 blocks. When a sample of 25 was taken out of the bucket, 8 were red, 7 were blue, 6 were yellow and 4 were green. If there were no other colours in the bucket, estimate the total number of each colour in the bucket.



8 Gizmos are produced on an assembly line. The manufacturer wants to make sure that no more than 3% are faulty.

a If a sample of 200 is selected, up to how many can be faulty for the manufacturer to be satisfied?

b If a sample of 10 includes only 1 faulty gizmo, should the manufacturer be worried?

9 A random sample of five people is to be selected from a list of 30. Use the random number generator on your calculator to obtain five numbers from 1 to 30 inclusive. (See Worked Example 2.)

10 If a high school was surveyed to determine the number of hours of homework students were doing each night, what groups would have to be identified within the school in order to obtain a reliable stratified sample?

11 We want to carry out a survey on the use of a lift in a 12-storey apartment building. A survey will be left at selected apartments on floors 3 to 12. The apartments will be chosen using stratified sampling of the floors. There are 100 apartments on each of the floors 3 to 8 and 50 on each of the floors 9 to 12. We will distribute 80 surveys altogether.
How many surveys will be distributed on:

a floor 2 **b** floor 6 **c** floor 10?

12 Open a page in a novel to carry out a survey to identify the percentage of words used that contain the letter 'e'.

- Select the first word of every line on the page for 20 lines.
- Close your eyes and randomly point to a word with your finger for say, 20 words.
- Compare the results of the two methods of selecting a sample. How might you increase the reliability of the results?

CHALLENGE 14:02 CATCH AND RELEASE

A method of estimating the size of a population is one often referred to as 'catch and release'. For example, 50 trout in a lake are caught and tagged and then released back into the lake. Then some time later 40 trout are caught and it is found that 10 of them are tagged. If the proportion of tagged trout in this second sample is taken as representative of the whole trout population in the lake, then an estimate for the whole population can be found.

$$\frac{\text{Tagged trout in second sample}}{\text{Number in second sample}} = \frac{\text{Tagged trout in first sample}}{\text{Number in population}}$$

$$\text{Number in population } (P) = \frac{\text{Tagged trout in first sample}}{\text{Tagged trout in second sample}} \times \text{Number in second sample}$$

$$P = \frac{50}{10} \times 40$$

$$P = 200$$

So the estimated population of trout in the lake is 200.

1 A sample of 30 kookaburras are caught and tagged and released back into a wildlife park. A week later, 50 kookaburras were observed in the park but only 10 of them were tagged. Estimate the size of the kookaburra population in the park.

2 To estimate the number of fish in a lake, 100 were caught, tagged and released. A few days later 60 were caught and it was found that only 18 of them were tagged. Estimate the fish population of the lake.

3 Elroy noted that there were no black jelly beans in a large jar of jelly beans. So to estimate the number of jelly beans in the jar, Elroy put 20 black jelly beans in the jar. He shook the jar repeatedly and then tipped out thirty jelly beans. He found that only 4 of them were black.



a Estimate the number of jelly beans in the jar, including the black ones.

b How many jelly beans were in the jar before Elroy put in the black ones?





Tagging wildlife helps to monitor their populations and ensure their protection.

14:03 Analysing data

PREP QUIZ 14:03

For the scores 3, 4, 9, 3, 6 find the following values.

1 range **2** mode

3 median **4** mean

For the scores in the frequency distribution table:

5 complete the *fx* column.

6 find the totals of the *f* and *fx* columns.

Score (x)	Frequency (f)	fx
2	1	
4	4	
6	8	
8	5	
10	2	

For the scores in the frequency distribution table find the following values.

7 range **8** mode

9 median **10** mean

In Year 7 we used a measure of spread called the range and measures of centre (or measures of location) called the mode, mean and median to analyse sets of data. They were defined as follows.

Range = highest score − lowest score

Mode = score that occurs most often
i.e. the score with the highest frequency

Mean = the sum of the scores divided by the number of scores
i.e. the common definition of 'average'

Median = the 'middle' score when the scores are placed in order.
If there is an even number of scores, the median is the average of the middle two scores.



We will now use these statistical measures to analyse various samples of data and determine the effects of sample size and individual scores.

WORKED EXAMPLE 1

These marks were gained by a class of 25 students on a test out of 20. The scores have been placed in order.

4, 5, 7, 7, 8, 9, 9, 10, 10, 11, 12, 12, 13, 13, 14, 15, 15, 15, 15 16, 17, 17, 18, 19, 20

a Find the mean and median of a sample of 5 scores by taking:

- **i** the first score and then every fifth score (i.e. the 1st, 6th, 11th, 16th and 21st scores)
- **ii** the fifth score and then every fifth score (i.e. the 5th, 10th, 15th, 20th and 25th scores)

b Find the mean and median of the 25 scores and compare it with the samples.

c How could a sample be selected that would be more representative of the whole class?

Solutions

a **i** The five sample scores are
4, 9, 12, 15 and 17.
Sum of the scores = 57
Mean = $\frac{57}{5}$
= 11·8
Median (3rd score) = 12

ii The five sample scores are
8, 11, 14, 16 and 20.
Sum of the scores = 69
Mean = $\frac{69}{5}$
= 13·8
Median (3rd score) = 14

b Sum of all 25 scores = 311
Mean = $\frac{311}{25}$
= 12·44
Median (13th score) = 13
The mean and median of the first sample are less than those for the whole group. The mean and median for the second group are higher. This is because the scores are in ascending order and the second sample does not include the first four low scores.

c A more representative systematic sample would have resulted by starting the count of five from the middle of the first group. Of course if the scores were not in order and a simple random sample was selected, then a more representative sample should be obtained by increasing the size of the sample.

WORKED EXAMPLE 2

A group of 10 students completed a test and their scores, as percentages, are shown below.
82, 75, 92, 88, 78, 69, 84, 90, 80 and 32

a Find the mean and median for this set of scores.

b The student who scored 32 had been absent prior to the test and had not been prepared. It was argued that this score should not be counted for the group. If this score is excluded, find the mean and median for the other nine scores.

c Which measure was affected the most by including the low score of 32 in the group?

Solutions

a Sum of 10 scores = 770
Mean = $\frac{770}{10}$
= 77
32 69 75 78 80 82 84 88 90 92
Median (average of 5th and 6th scores)
= $\frac{80+82}{2}$ = 81

b Sum of 9 scores = 738
Mean = $\frac{738}{9}$
= 82
69 75 78 80 82 84 88 90 92
Median (5th score) = 82

c When the low score was included, the median was only slightly affected, decreasing from 82 to 81, however, the mean fell significantly from 82 to 77. Scores that vary significantly from the rest of the group are called **outliers**. There may be reasons to not include outliers, such as the '32' in this data set.

Outliers may occur for various reasons. They may indicate a fault with the data or the manner in which it was collected. There may be an identifiable explanation for an outlier as in the previous example. However, particularly in large sets of data, a small number of outliers can often be expected. The effect of outliers can be greater for smaller samples of data.

Outliers can have a significant effect on the mean, particularly when they only occur at one 'end' of the sample. The median may be a better measure of 'centre' for the sample.



Outliers are scores that differ significantly from other data in the sample.

Exercise 14:03

P Challenge worksheet 14:03 Working with data

1 The number of points the local rugby team scored in 12 games follows.
13, 10, 22, 17, 30, 25, 9, 19, 35, 27, 13, 20

a Arrange these scores in order from smallest to largest.

b From the ordered scores calculate the mean and median for samples containing the:

i 1st, 4th, 7th and 10th scores

ii 2nd, 5th, 8th and 11th scores

iii 3rd, 6th, 9th and 12th scores.

c The mean of all 12 scores is 20. The mean of which sample in part **b** is closest to this?

d The median of all 12 scores is 19·5. The median of which sample in part **b** is closest to this?

2 The heights of 50 students are measured and recorded **in order** from shortest to tallest. If a systematic sample of 10 heights is used to estimate the mean and median, which numbered height should be the first chosen?

3 A sample was taken from 400 scores so that the mean of the population could be estimated. Unsure of how large the sample should be, Kim continued to select scores at random from a hat. These are the first 24 scores in order.

44 62 81 50 59 62 67 62 44 40 72 77
60 55 57 64 68 59 49 66 61 85 41 79

a Find the mean, correct to the nearest whole number, of the first:
i 2 scores **ii** 4 scores **iii** 8 scores **iv** 16 scores

b The mean of the 24 scores is 61. Which sample above gave this result?

4 Using your calculator, generate a random number less than 100 by writing down the first two digits of the three-digit decimal number. That is, for 0·349, simply write down 34. For 0·073, write down 07, or simply 7.

a Do this five times to get five two-digit numbers and find their mean.
b Do this another five times and find the mean of the 10 numbers.
c If you increased the size of the sample, what do you think will happen to the value of the mean?
d For a large sample, what do you think should be the value of the median?

5 A six-sided dice was rolled 10 times and the results were recorded. This was done six times, as can be seen by the results shown in the table.

Number	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Total
1	2	1	1	3	2	1	10
2	1	3	2	1	3	1	11
3	0	2	1	3	1	2	9
4	3	0	2	0	3	2	10
5	2	1	3	2	1	2	11
6	2	3	1	1	0	2	9
Mean	3·8						



Smaller samples can be joined together to give more reliable data.

The mean for the first sample of 10 throws has been calculated. Check to see you can get this answer.

a Calculate the mean for each sample.
b What would you expect the mean to be if the dice was rolled a large number of times?
c The mean for each small sample may vary from your expected result but if the six small samples are taken together to form a larger sample, a more reliable result might be expected. Calculate the mean for the total column, correct to one decimal place. How does this result compare with your answer to part **b**?

6 Clint made the following runs in eight innings of cricket:
34, 45, 29, 59, 43, 67, 38 and 61.

Remember:
Write the scores in order to find the median.

a Calculate the mean and median for these scores.

Clint made only 2 runs in the ninth innings.

b Calculate the mean and median of the nine scores when this innings is included.

c Which of the measures of centre, the mean or the median, is least affected by including the low score?

d If the outlier score of 2 is included, which measure, the mean or the median, do you think is a better indicator of Clint's ability?

7 **a** For each set of data, find the:

i range **ii** mode **iii** median **iv** mean (1 dec. pl.)

Sample A:	1	9	9	10	11	12	12	12	13
Sample B:	8	9	9	10	11	12	12	12	13

b The samples differ by only one score. How does this one score affect the answers in part **a**?

c Which score would you consider to be an outlier?

8 Five properties are for sale in a particular street. Four are flats and one is a double-storey home. The market prices are $330 000, $350 000, $360 000, $410 000 and $1 900 000.

Real estate agents usually refer to the 'median house price' when talking about house values. Sometimes they call this the 'average' house price.

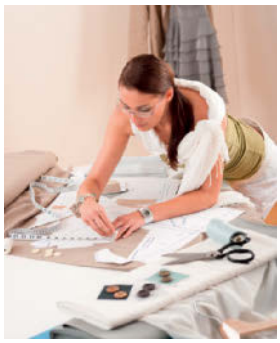
a What is the mean of these prices?

b What is the median house price?

c Which measure, the mean or median, do you think best represents the property prices in this street?

9 The annual salaries of eight people working in a company are $190 000, $47 000, $55 000, $58 000, $61 000, $47 000, $69 000 and $59 000. Which of the mode, mean and median provides the most accurate indication of the salary of the 'average' employee?

10 Tazy is a clothes designer and needs to know the sizes of dresses sold in NSW. Which measure of centre would be most useful to a dress manufacturer: the mode, the median or the mean? Give reasons for your answer.



11 A student wants to use a measure of centre to show how he has performed in eight maths tests. His marks are listed here.
56, 56, 58, 58, 85, 97, 56, 55
Which should he use: the mode, the median or the mean?
Give reasons for your answer.

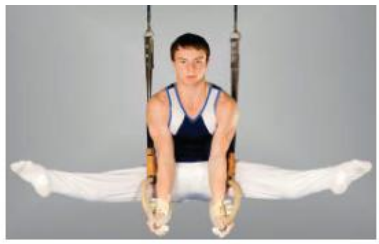
12 Alan has the following scores in 10 rounds of golf.
78 79 82 77 78 95 82 79 79 82
Calculate the mode, mean and median of these scores. Which is the best measure of Alan's performance?

13 **a** Calculate the mean for each of the following sets of marks. How many marks in each set are larger than the mean?

i 75, 75, 75, 75, 70 **ii** 75, 75, 75, 75, 80

b In a test the mean was 60. Does this mean there were as many scores over 60 as there were below 60? Give reasons for your answer.

14 In a gymnastics competition, each judge is asked to calculate a deduction. The mean of the two middle deductions is called the 'Judges' Deduction'. The gymnast's score is calculated by subtracting the Judges' Deduction from 8·8. What would a gymnast's score be if four judges calculated deductions of 0·62, 0·54, 0·37 and 0·62?



15 In a diving competition the five judges gave scores of 8, 7·5, 8·5, 9 and 8.

a Delete the highest and lowest scores and find the mean of the remaining three scores. Multiply this by the Degree of Difficulty of 2·8 to get the score for the dive.

b Repeat the process in part **a** to find the score for a dive with a Degree of Difficulty of 3·2 if the judges' scores are 6, 6·5, 6, 7, 7·5.

14:04 Statistical graphs

PREP QUIZ 14:04

This data shows the number of pets in the families of students in a class.

1 3 2 2 4 1 5 2 1 1
6 4 1 2 5 2 1 4 1 2



For this data, what is the:

1 number of outcomes **2** mode
3 median **4** mean
5 number of children who had three pets
6 number of children who had more than two pets?

This frequency distribution table shows the number of children sponsored by the families in a class. Complete the table and find how many families:

7 did not sponsor a child
8 sponsored two children
9 sponsored fewer than two children
10 there were altogether.

Outcome	Tally	Frequency
0	||||	
1	𝍸 ||||	
2	𝍸 ||||	
3	|	

Total: …

Dot plots, frequency histograms and polygons

A frequency distribution table is very good for collating and organising data. But, when analysing the data, it is often more desirable to have the information presented in the form of a diagram or graph such as a dot plot, frequency histogram or polygon.



WORKED EXAMPLE 1

For a class of 26 students the following marks out of 10 were obtained in a test.

5 6 6 4 9 3 5 10 8 9 7 3 6 7 5 6 5 4 3 6 7 10 9 8 7 4

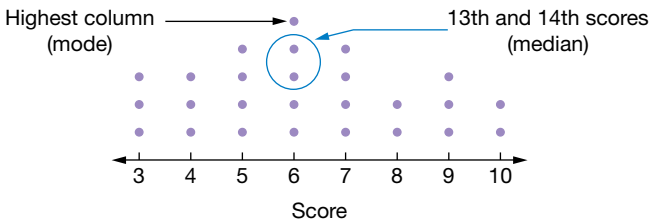
1 Draw a dot plot to represent the data.

2 Use the dot plot to find the:
a range **b** mode **c** median

3 Organise the data into a frequency distribution table.

Solutions

1 The dot plot is shown.



2 **a** The range is the difference between the highest score and the lowest score.
Range = 10 − 3
= 7

b The mode is the most frequent score. This is easy to identify by the highest column (or columns) of dots.
Mode = 6

c The median, the middle score, is also easy to identify because the scores are in order. This set of data has 26 scores, so the median will be the average of the 13th and 14th scores. Here, they are the same score of 6.
Median = 6

3 The frequency distribution table is shown below.

Score	Tally	Frequency
3	|||	3
4	|||	3
5	||||	4
6	~~||||~~	5
7	||||	4
8	||	2
9	|||	3
10	||	2
	Total:	26

WORKED EXAMPLE 2

Use the test results for the 26 students to complete the following tasks.

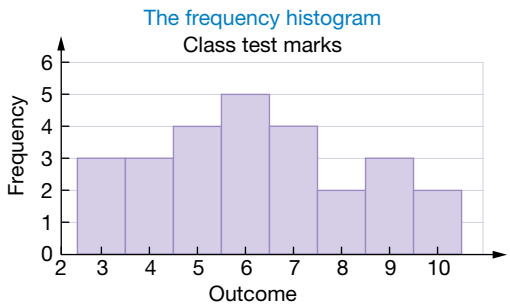
5 6 6 4 9 3 5 10 8 9 7 3 6 7 5 6 5 4 3 6 7 10 9 8 7 4

a Draw a frequency histogram to represent the data.
b Draw a frequency polygon to represent the data.
c Draw a frequency histogram and polygon on one set of axes.

Solutions

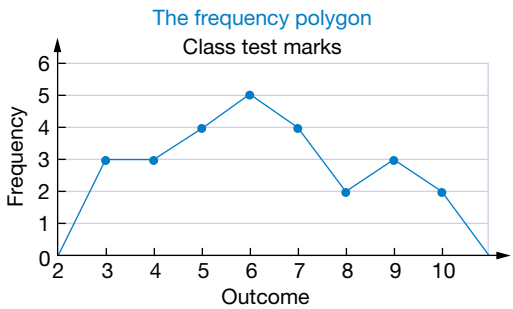
a The frequency histogram is shown on the right.

- The frequency histogram is a column graph.
- The graph has a title and the axes are labelled.
- The first column begins one-half of a column width in from the vertical axis.

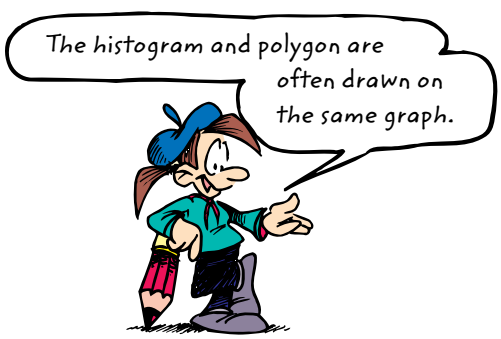


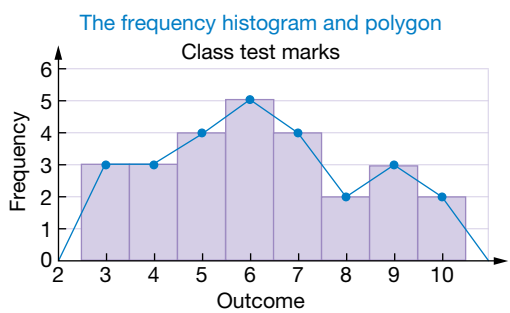
b The frequency polygon is shown on the right.

- The frequency polygon is a line graph.
- The graph has a title and the axes are labelled.
- The first non-zero dot is one unit in from the vertical axis.
- The dots showing the data are joined by straight lines and are joined to the horizontal axis as shown.



c The frequency histogram and polygon is shown on the right.





Note:
The range is easily determined from the graph by finding the difference between the highest and lowest scores, or outcomes, on the horizontal axis.
The mode is identified by the highest column (or columns).
The median is a little harder to determine from the graph because the heights of each column need to be added progressively until the middle score(s) is found.

WORKED EXAMPLE 3

Use the test results for the 26 students to complete the following tasks.

5 6 6 4 9 3 5 10 8 9 7 3 6 7 5 6 5 4 3 6 7 10 9 8 7 4

a Find the mean of the scores.

b Show the mean on the:

i dot plot **ii** frequency histogram and polygon.

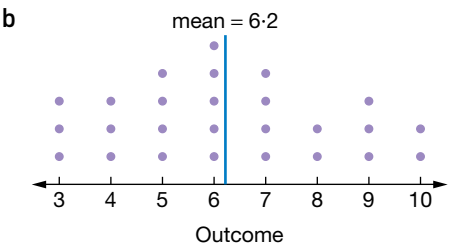
Solutions

a The mean is not easily identified from a frequency histogram and polygon, so the frequency table can be used. Add a frequency times score (or fx) column and then calculate the mean by dividing the total of the fx column by the total of the frequency column.

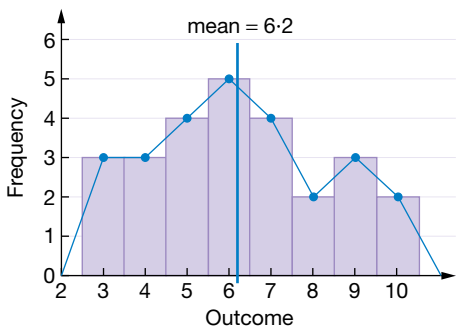
$$\text{Mean} = \frac{162}{26}$$
$$= 6{\cdot}2 \text{ (1 dec. pl.)}$$

Score	Frequency	fx
3	3	9
4	3	12
5	4	20
6	5	30
7	4	28
8	2	16
9	3	27
10	2	20
Total:	26	162

b



c



Once the mean has been calculated its position could be shown on a dot plot or frequency graph as shown here. Its position could then be compared with the mode or the median. In this case, the median and mode are both 6. Because the mode, mean and median are all measures of centre, for most large sets of data they are close in value.

Exercise 14:04

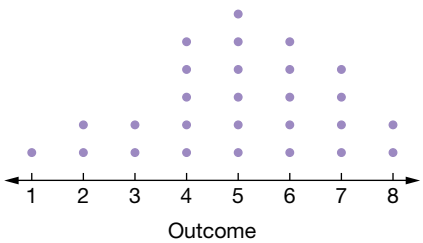
P Foundation worksheet 14:04
Dot plots and column graphs

1 a From this dot plot find:
- i the total frequency (How many dots are there?)
- ii the outcome that is the mode
- iii the median outcome.

b Complete the frequency distribution table for this set of data.

c Calculate the mean.

d What do you notice about the mode, median and mean for this set of data?



Outcome (x)	Frequency (f)	fx
1	1	1
2	2	4
3	2	6
4	5	20
5		
6		
7		
8		

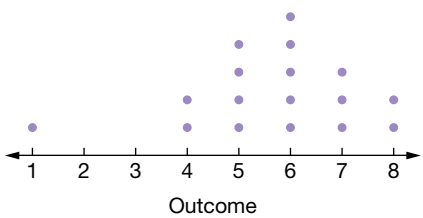
2 a From this dot plot find:
- i the total frequency
- ii the outcome that is the mode
- iii the median outcome.

b By completing a frequency distribution table, or otherwise, calculate the mean.

c Sketch this dot plot and mark on it the positions of the mode, median and mean.

d How does this set of data differ from the data in Question 1?

e How has the outlier score of 1 affected the mode, median or mean?



Find the mean by adding the product of each outcome and its number of dots and then divide this by the total number of dots.

3 Six coins were tossed 30 times and the number of heads recorded. The results are shown in this frequency histogram.

a How many times were four heads thrown?

b What was the most frequent result?

c If, for a particular throw, there were two heads, how many tails were there?

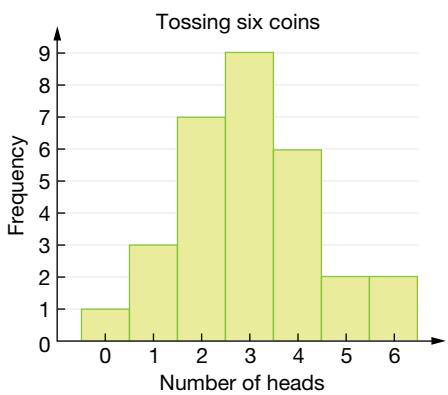
d On how many occasions did five tails result?

e Determine the median number of heads.

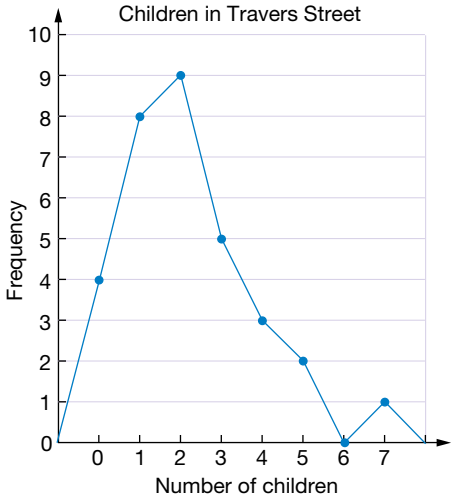
f Multiply each outcome by its frequency to find the total number of heads thrown.

g Use your answer to part f to calculate the mean.

h What can you say about the mode, mean and median?

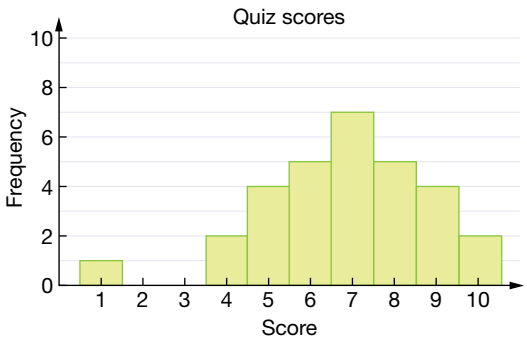


4 In Travers Street, each family was surveyed to determine its number of children. The results are shown in this frequency polygon.



- **a** How many families had no children?
- **b** What was the most common number of children?
- **c** How many families had six children?
- **d** How many families had two children or less?
- **e** How many families were surveyed altogether?
- **f** How many children were there altogether?
- **g** Use your answers to parts **e** and **f** to calculate the mean number of children per household.
- **h** Determine the median for this data.
- **i** Which is greater, the mean or the median?

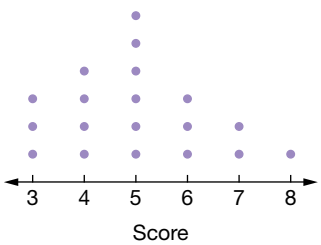
5 This column graph shows the scores on a spelling test for a class of 30 students.



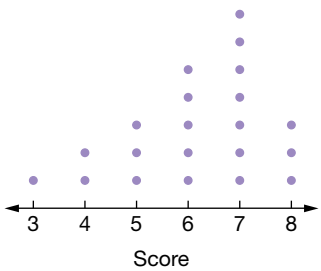
- **a** What is the mode for these scores?
- **b** What is the median score?
- **c** Calculate the mean score for the class.
- **d** The student who scored 1 on the quiz had been absent prior to the test and was not prepared. Hence, his score seems be an outlier. Find the mean and the median of the other 29 scores if the score of 1 is not counted.
- **e** What affect did the outlier have?

6 Determine the mode, mean and median for each set of data represented by these graphs.

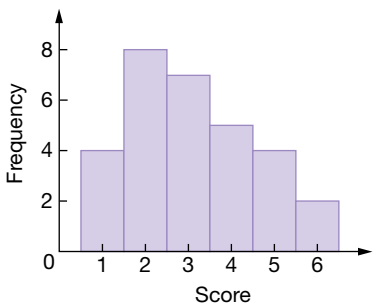
a



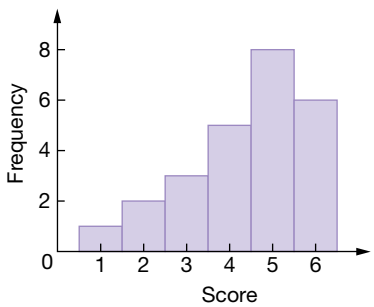
b



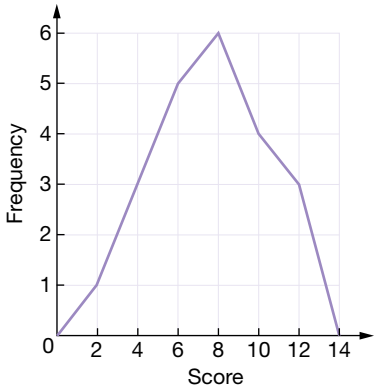
c



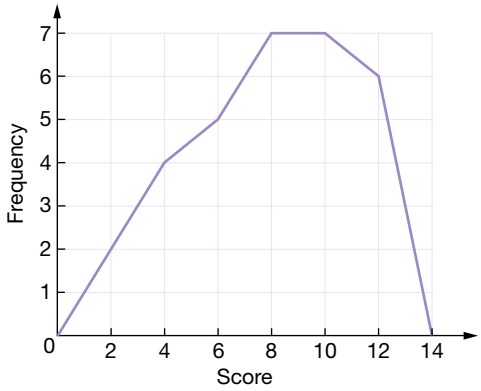
d



e



f

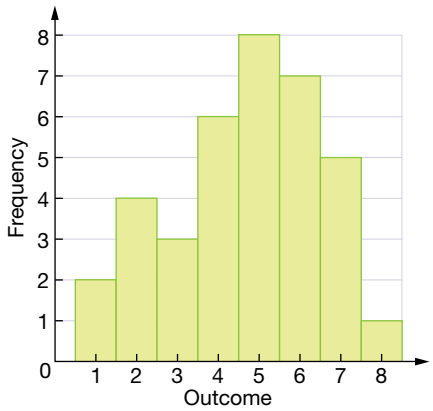


7 Use the histogram on the right to find:

- **a** the range of scores
- **b** the mode of the scores
- **c** the median of the scores
- **d** the mean of the scores.

Another 4 scores of 8 are added to the data.

- **e** What is the mean now?
- **f** How have these extra scores affected the mode and median?



8 A goal kicker for a football team kicked the following number of goals in his 24 games this season.

2	3	0	2	2	1	1	4
6	0	1	3	3	2	1	4
5	1	5	2	0	2	4	1

- **a** Draw a dot plot for this set of data.
- **b** Indicate on the diagram the mode and the median.
- **c** Calculate the mean and also show its position on the dot plot.



9 The scores of 70 students for an English language test with 20 questions are recorded below.

19	13	18	14	15	20	19	19	17	12
10	14	8	12	11	17	18	16	16	15
14	18	14	13	19	20	10	13	11	17
16	15	13	19	12	17	17	20	10	11
14	13	15	16	19	12	12	20	10	17
11	13	18	19	20	18	18	11	12	15
16	15	14	14	17	16	19	18	17	13

A set of scores may have more than one mode.

- **a** Determine the range for this set of scores and collate them in a frequency distribution table.
- **b** Hence, determine the mode and median and calculate the mean.
- **c** Construct a frequency histogram and polygon and show the positions of the median and mean.

10 The trees in each backyard of Eggleton Street were counted and the number recorded. The data is shown below.

0	4	8	7	2	1	3	1	4	2	12	6	5	1	4	3	5
7	4	6	2	3	9	8	3	4	2	2	5	1	6	3	1	4
5	5	2	3	10	4	7	5	4	6	2	1	0	3	5	5	

a What are the highest and lowest scores in this data?
b Organise this data into a frequency distribution table including a score × frequency (fx) column.
c Determine the mode and median and calculate the mean.
d Construct a frequency histogram and indicate on it the median and mean.



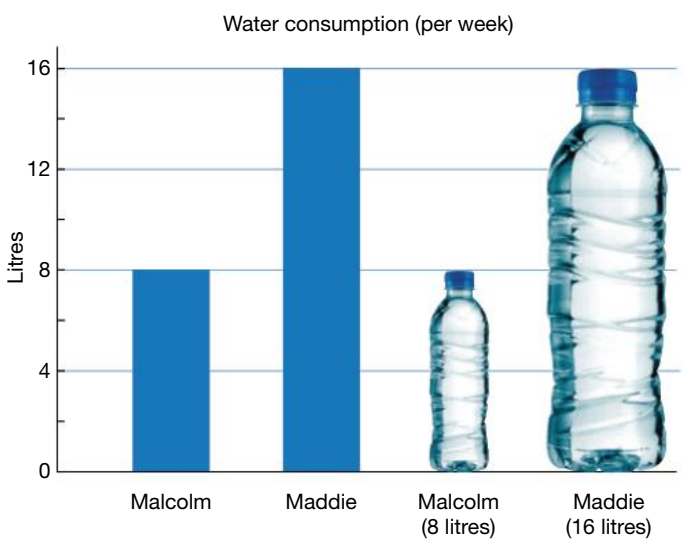
11 Two dice were thrown and the number of times that each total appeared was recorded.

Outcome	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	6	8	11	14	17	15	12	7	5	2

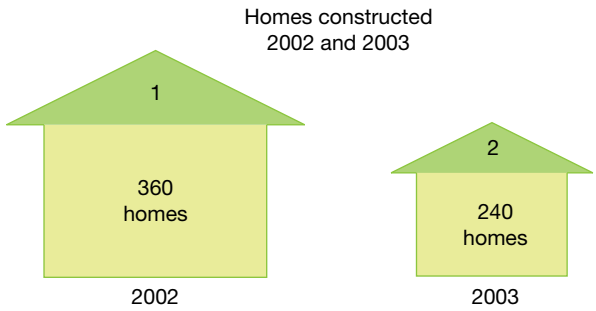
a How many times were the dice thrown?
b What is the mode?
c Calculate the mean for this data.
d Determine the median.
e Construct a frequency histogram and indicate the positions of the mean and median.
f If the dice were rolled a large number of times, what would you expect the mean to be?

INVESTIGATION 14:04 MISUSE OF GRAPHS

Graph can be designed to deceive. Some of these methods were mentioned in Chapter 8 on graphs. One type of graph needs special mention. This is the type where diagrams are used to convey the relative size of quantities.
If you wished to compare two quantities, one twice the size of the other, you could draw two pictures, one twice as high as the other. But to keep it in proportion it would also need to be twice as wide. However, the larger figure is now four times the area of the smaller figure and the impression given by the areas is that the larger figure is four times as big as the smaller figure. An example is shown here.



1 Here, no axes are given, although the number of homes is given for each year.



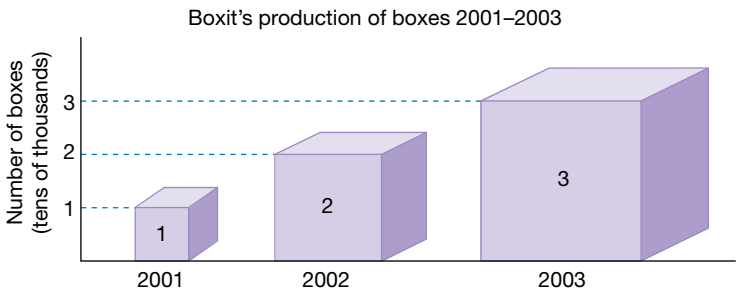
a What fraction of the height of Figure 1 is the height of Figure 2?

b What fraction of the number of homes constructed in 2002 is the number constructed in 2003?

c The area of Figure 2 is $\frac{4}{9}$ of the area of Figure 1. Would this graph tend to give a false impression?

d Draw a column graph to correctly represent this information.

2



a How many times greater is the height of box 2 than the height of box 1?

b How many times greater is the area of the face of box 2 than the area of the face of box 1?

c How many times greater is the volume of box 2 than the volume of box 1?

d How many times greater is the height of box 3 than the height of box 1?

e How many times greater is the area of the face of box 3 than the area of the face of box 1?

f How many times greater is the volume of box 3 than the volume of box 1?

g Draw a column graph that shows the same data using simple columns of equal width.

- Box 2 is meant to represent twice what box 1 represents, but it appears to be 8 times as big.
- Box 3 is meant to represent 3 times what box 1 represents, but it appears to be 27 times as big.

14:05 Grouped data

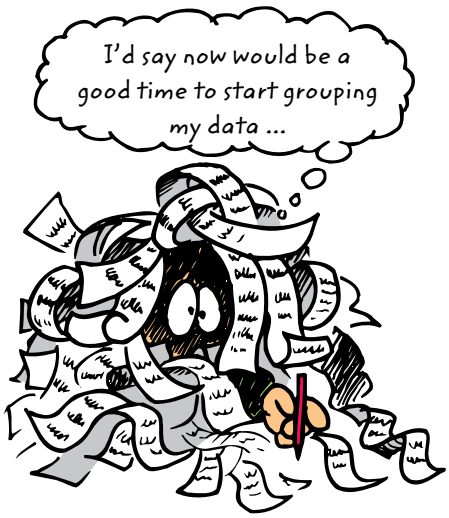
In cases where there are a large number of possible outcomes, it is usually more convenient to group the outcomes into classes.

For example, if test marks out of 100 were being tallied, the range could be as large as 100, so a more convenient frequency distribution table might be constructed using class intervals of 10 marks, such as 1–10, 11–20, 21–30, etc.

A class interval could also be used when the data is continuous rather than discrete.

In **continuous data**, the data are measurements that can assume any value within a class, such as heights of people or rainfall figures.

In **discrete data**, the data are a set of exact outcomes with no values in between, such as numbers rolled on a dice (1, 2, 3, 4, 5 or 6).



We usually choose a class size so that a set of data is divided up into between 5 and 10 intervals.

Measures of centre in grouped data

- The mode becomes the **modal class**.
- The **mean** is estimated by using the class centre as a representative figure for each class. So the mean is given by:

$$\text{Mean} = \frac{\text{Total of } f \times c.c. \text{ column}}{\text{Total of frequency column}}$$

- The median becomes the **median class.**

WORKED EXAMPLE 1

The percentage results for 60 students in an examination are shown below, correct to the nearest whole number..

78	63	89	55	92	74	62	69	43	90	91	83	49	37	58
73	78	65	62	87	95	77	69	82	71	60	61	53	59	42
43	33	98	88	73	82	75	63	67	59	57	48	50	51	66
73	68	46	69	70	91	83	62	47	39	63	67	74	52	78

a Organise the data into a frequency distribution table.

b Use the table to find the measures of centre.

i mode **ii** mean **iii** median

c Draw a frequency histogram and polygon to represent the data.

Solutions

a The frequency distribution table for this set of data would look like this.

Class	Class centre (*c.c.*)	Tally	Frequency (f)	$f \times c.c.$
29–37	33	||	2	66
38–46	42	𝍸	5	210
47–55	51	𝍸 |||	8	408
56–64	60	𝍸 𝍸 ||	12	720
65–73	69	𝍸 𝍸 ||||	14	966
74–82	78	𝍸 ||||	9	702
83–91	87	𝍸 ||	7	609
92–100	96	|||	3	288
		Total:	60	3969

Counting down the frequency column, the 30th and 31st scores are in this class.

Note:

- The *class centre* (*c.c.*) column has been added.
- The *frequency* × *score* (*fx*) column has been changed to the *frequency* × *class centre* ($f \times c.c.$) column.

b **i** The modal class is 65–73.

ii The mean is estimated by using the class centre for each class.

$$\text{Mean} = \frac{\text{Total of } f \times c.c. \text{ column}}{\text{Total of frequency column}}$$

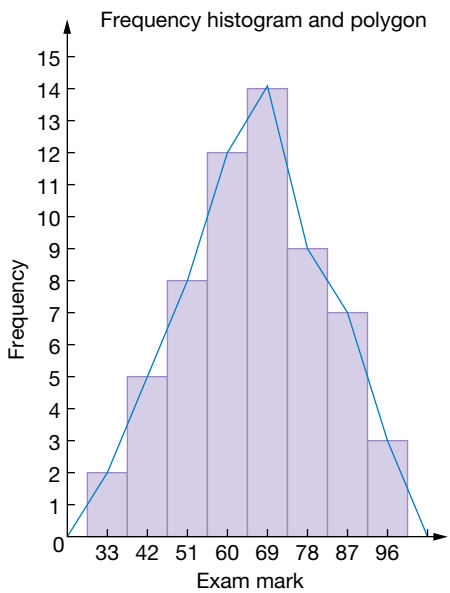
$$= \frac{3969}{60}$$

$$= 66{\cdot}15$$

iii There are 60 scores, so the middle score is the average of the 30th and 31st scores. Both lie in the 65–73 class. The median class is 65–73.

c When constructing frequency diagrams for grouped data, the columns are indicated on the horizontal axis by the class centres.

- The modal class 65-73 is represented by the class centre 69.
- The frequency polygon can be drawn by joining the midpoints of the tops of columns. To complete the polygon, assume that the classes on either side of the columns have zero members.



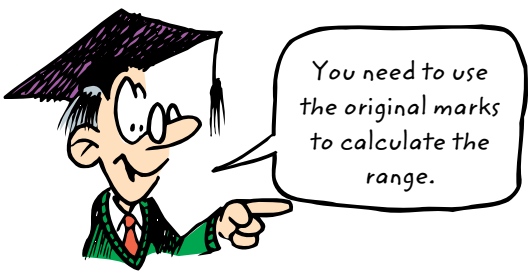
WORKED EXAMPLE 2

The heights of 50 students were measured and organised into a frequency distribution table with classes of 140–<150, 150–<160, etc. This means any height from 140 cm to less than 150 cm is included in the first class. A height of 149·8 cm is in this class. The class centre is taken as half-way between 140 and 150 (i.e. 145). The data is continuous so there are no gaps between the classes.

Class	Class centre	Tally	Frequency	$f \times c.c.$
140–<150	145	|||	3	435
150–<160	155	𝍸 𝍸 |	11	1705
160–<170	165	𝍸 𝍸 𝍸 𝍸 ||||	19	3135
170–<180	175	𝍸 𝍸 |||	13	2275
180–<190	185	||||	4	740
		Total:	50	8290

a Use the frequency distribution table to find the following.

i modal class **ii** mean **iii** median class



b Draw a frequency histogram to represent the data.

Solutions

a **i** The modal class is 160–<170.

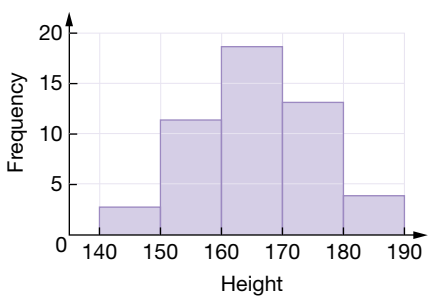
ii $$\text{Mean} = \frac{\text{Total of } f \times c.c. \text{ column}}{\text{Total of frequency column}}$$

$$= \frac{8290}{50}$$

$$= 165{\cdot}8$$

iii The median class (average of the 25th and 26th scores) is 160–<170.

b The histogram has been constructed with a continuous scale on the horizontal axis.



Exercise 14:05

P Foundation worksheet 14:05
Frequency distribution tables

1 Complete the frequency table using the scores below.

51	40	63	81	73	73	35
49	55	61	31	60	44	54
87	56	97	50	37	53	60
58	84	77	63	80	60	53
42	38	52	43	50	72	80

Class	Tally	Frequency
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		

Total: …

2 What is the class centre of these classes?

a 6–10 **b** 15–21 **c** 31–39 **d** 5–8

3 Complete this frequency distribution table and find the modal class, mean and median class.

Class	*c.c.*	Tally	Frequency	$f \times c.c.$
1–5		|||		
6–10		𝍸 ||		
11–15		𝍸 𝍸		
16–20		𝍸 |||		
21–25		||		

Total: … …

4 **a** Use the classes 0–2, 3–5, 6–8, 9–11, 12–14 and 15–17 to complete a frequency distribution table for the scores:

11	9	13	8	7	2	4	11	0	15	10	4
5	10	5	11	14	17	1	3	8	4	9	6
4	8	4	7	11	10	4	0	15	4	8	7
6	9	6	4	10	9	12	3	1	8	10	7

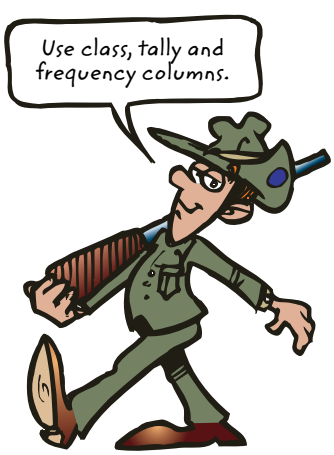
b What is the modal class?

c What is the median class?

d Use the totals of the frequency and $f \times c.c.$ columns to calculate the mean.

e Does the mean lie in the median class?

f Construct a frequency histogram for this grouped data.



5 The heights of 40 basketball players were measured to the nearest centimetre.

160	149	153	143	179	159	152	161
169	157	167	163	148	175	173	150
175	160	170	162	145	158	163	164
172	178	169	171	153	165	152	161
170	174	166	165	158	157	152	148



a Complete this frequency distribution table for this continuous data.

Class	*c.c.*	Tally	Frequency	$f \times c.c.$
140–<150	145			
150–<160				
160–<170				
170–<180				

b Determine the modal class and median class for this data.

c Calculate a value for the mean from the table.

d How could a more reliable value for the mean be found?

e Draw a frequency histogram and polygon for this data.

6 A total of 30 people were given a driving test and each received a score out of 10. Use the scores below to complete each of the two frequency distribution tables.

5	6	2	5	7	7	8	5	4	7
6	8	10	6	4	7	6	8	7	6
4	7	5	6	7	5	9	7	8	6

Class	*c.c.*	Tally	Frequency	$f \times c.c.$
2–4				
5–7				
8–10				
		Total:	…	…

Outcome	Tally	Frequency	fx
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
	Total:	…	…

a In which of the tables is the value of the individual scores lost?

b Calculate the mean for both tables. Which one is the more accurate?

7 Use the completed frequency distribution table to complete the grouped frequency distribution table.

Outcome	0	1	2	3	4	5	6	7	8
Frequency	1	2	22	10	8	4	5	6	2

Class	Class centre	Frequency
0–2		
3–5		
6–8		

a Do you think that three classes is enough to show the spread of scores realistically?

b Normally between 5 and 10 classes are used. Why would it be difficult to use more than 5 classes in this case?

8 Draw a frequency histogram and a frequency polygon for each of these grouped frequency distribution tables.

a

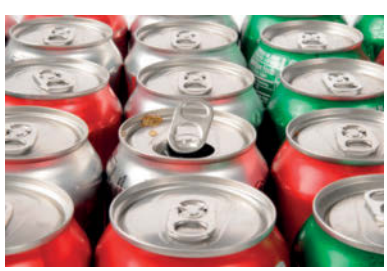
Class	Class centre	Frequency
16–20	18	5
21–25	23	7
26–30	28	10
31–35	33	13
36–40	38	15
41–45	43	6
46–50	48	4

b

Class	Class centre	Frequency
3–<6	4·5	1
6–<9	7·5	4
9–<12	10·5	6
12–<15	13·5	8
15–<18	16·5	12
18–<21	19·5	3
21–<24	22·5	2

9 The number of cans of drink sold by a shop each day was as follows. The highest and lowest scores are circled.

30	28	42	21	(54)	47	36	41	29	39
37	22	(18)	25	26	43	50	29	37	38
23	29	30	19	28	20	40	47	51	28
33	35	31	27	42	26	44	46	36	36
53	50	29	20	32	41	36	25	27	19
51	46	37	42	27	28	31	47	36	35
29	32	41	36	32	41	35	33	28	40

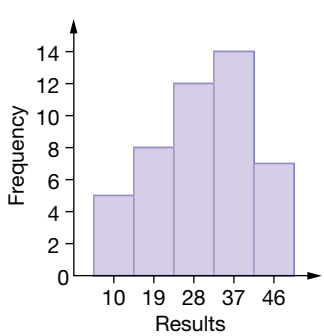


a Complete a frequency distribution table for the results using intervals of 16–22, 23–29, 30–36, 37–43, 44–50, 51–57.

b For how many weeks was the survey conducted?

c What was the mean number of cans sold?

10 a From this frequency histogram, determine what the class groupings must have been, given it was discrete data of whole numbers.



b Complete a frequency distribution table for this data.

c Find the:

i modal class ii median class iii mean

11 The exam results for Year 8 students were as follows.

69	32	75	51	67	36	88	3	71	7
95	81	40	99	20	51	16	67	75	20
82	62	28	15	87	76	27	46	32	13
14	49	17	10	28	83	25	55	24	73
41	22	37	39	25	16	37	72	21	49
37	90	34	14	81	52	69	66	89	66
8	84	29	14	69	28	75	36	7	50
40	51	62	13	64	24	47	47	86	77
50	56	47	42	39	97	16	65	62	45
52	89	71	43	61	98	74	63	12	10
96	46	52	90	15	59	67	69	54	74
28	55	70	43	56	80	73	63	26	21
86	41	49	44	69	30	93	21	28	72
16	31	52	88	53	56	27	32	10	35
34	68	56	88	62	82	5	34	41	49
71	95	92	86	76	48	85	59	25	10



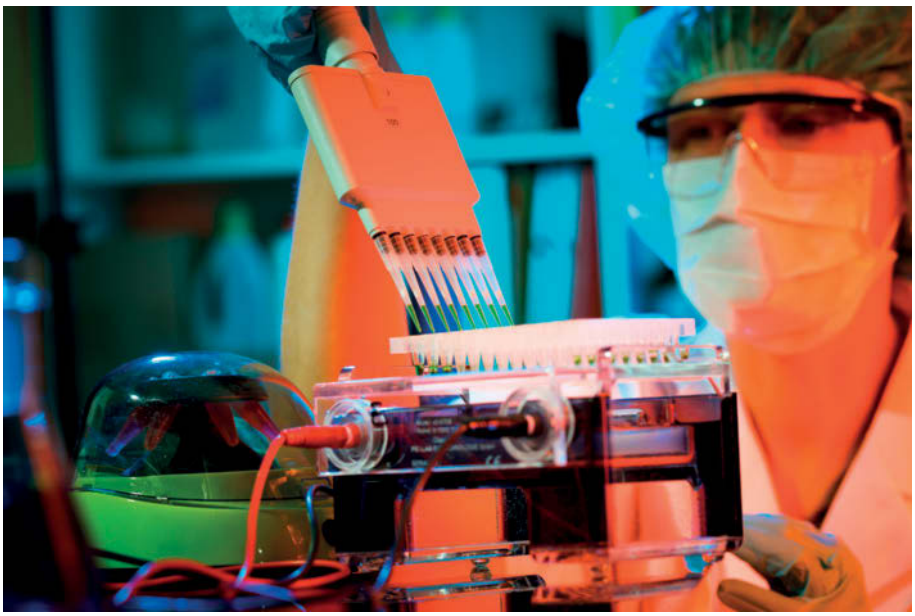
a Using classes of 1–10, 11–20, etc., construct a frequency distribution table.

b Determine the:

i range **ii** modal class **iii** median

c Calculate a value for the mean.

d Compare the location of the modal class, the median class and the mean.

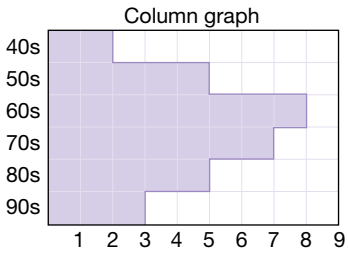


The careful handling of statistics is essential in any research laboratory.

14:06 Back-to-back stem-and-leaf plots

In Year 7 you were introduced to stem-and leaf plots.

- A stem-and-leaf plot retains the original data to at least two significant digits, where the data has been grouped.
- The great advantage is that the identity of individual scores is not lost.



Now we will examine 'back-to-back' stem-and-leaf plots

- The back-to-back stem-and-leaf plot allows us to display two sets of data and compare them at a glance.

WORKED EXAMPLE

Our class was given a topic test in which we performed poorly. Our teacher decided to give a similar test one week later, after a thorough revision of the topic. The results are shown on this back-to-back stem-and-leaf plot as percentages. (This is an ordered display.)

Test results

(4 | 1 represents 41)

First test Leaf	Stem	Second test Leaf
9 8 6 6 0	3	
9 7 7 3 1 1 1	4	3 6 6 8
8 8 5 3 3 0	5	1 7 9 9
9 8 7 5 3	6	3 8 9
	7	0 5 5 5 8 9
	8	2 6 7 7
0	9	0 0 1 3

This is a score of 69

This is a score of 79



- The improvement in the second test is easy to see. The results in the first test are grouped at the top of the stem, with scores from 30% to 70%. In the second test, after revision, the scores are more evenly spread, with about half the scores above 70%.
- The mode, median and range are easily compared.

Test 1		Test 2	
Mode	= 41	Mode	= 75
Median	= 50	Median	= 75
Range	= 90 − 30	Range	= 93 − 43
	= 60		= 50

Check to see that you can find all these values.

- Outliers are easily identified. In the first test the score 90 is an *outlier*. In the second test there are no outliers.

Exercise 14:06

1 The scores for two groups of students on a quiz are shown below and are also displayed as an ordered back-to-back stem-and-leaf plot.

Group A: 26 32 15 44 19 35 27 28 39 35

Group B: 29 20 37 41 11 13 25 36 23 25

a For each group find the:

i range **ii** mode

iii median **iv** mean

b Which two measures of centre are the same for:

i Group A **ii** Group B

Group A / Group B

Leaf	Stem	Leaf
3 1	1	5 9
9 5 5 3 0	2	6 7 8
7 6	3	2 5 5 9
1	4	4

2 This back-to-back stem-and-leaf plot shows the number of seconds taken by each member of a class to complete their 3×Times tables and their 6×Times tables.

a Which table seems to be the better known?

b From the scores for 3×Tables, find the mode, the median and the range.

c From the scores for 6×Tables, find the mode, the median and the range.

d Do any outliers exist in either set of scores? Explain why or why not.

Time in seconds

3×Tables / 6×Tables

Leaf	Stem	Leaf
9 8 8	0	
7 4 3 2 2 2 0	1	6 6 7 9
9 8 8 7 7 3 1	2	0 4 8
9 6 6 4 4	3	0 2 2 2 6 9
3 2 2	4	1 5 5 6 7 8 8 9
5	5	2 9 9 9
	6	0

3 The scores of students and parents on a sports trivia quiz are recorded below.

Students

44	15	30	35	20	24	18	40	37	34	28
39	41	19	23	30	30	41	47	20	35	28
45	22	33	26	42	25	30	21	42	19	

Parents

18	32	20	24	11	35	31	42	38	27	16
26	16	6	40	23	37	31	26	30	42	21
18	41	42	22	25	32					

Students / Parents

Leaf	Stem	Leaf
	0	
	1	
	2	
	3	
	4	

a Prepare an ordered back-to-back stem-and-leaf plot for this data.

b For each set of scores, find the mode, the median and the range.

c Which of the measures in **b** is most useful in comparing the two groups?

d Find the mean of each set of scores, correct to two decimal places.

4 The pulse rates of 24 adults and 24 children were measured and recorded below.

Adults

12	64	59	60	72	61	68	55	76
76	80	81	58	64	72	60	92	56
57	68	80	68	72	64			

Students

86	67	85	98	77	81	85	102	72
62	88	92	86	82	94	78	94	85
95	99	86	77	99	100			

a Use an ordered back-to-back stem-and-leaf plot to organise this data.
b What outliers are present in this data?
c Can any of these outliers be ignored? Why or why not?
d Find the median and mode of each set of scores.
e Describe each set of scores and compare them.

CHALLENGE 14:06 WEDDING ANNIVERSARIES



Are there some months or parts of a month that are more popular for weddings than others? Do many people get married on Easter Sunday or Christmas Day?

- Collect as many wedding anniversary dates as you can. (Each member of a group should collect at least 20.) You will need to ask for the day and the month. The year is not needed. The more dates you collect, the more likely it is that you will discover trends.
- Organise your results as follows.

1 Make a list of data (e.g. 2 March, 10 June, 29 July).

2 Draw an ordered stem-and-leaf display, treating each month as two halves (e.g. 1 Jan to 15 Jan, 16 Jan to end of month).

For example:

Wedding anniversaries

Stem (month)	Leaf (day)	Frequency
$1^{(1)}$	1 2 4 4 8 8 9 11 11 15	10
$1^{(16)}$	16 20 27 27 28 31	6
$2^{(1)}$	4 7 7 8 12 12 14	7
$2^{(16)}$	17 20 21 21 28	5
$3^{(1)}$	3 3 8 10	4
.	.	.
.	.	.
.	.	.
$12^{(1)}$	3 8 10 10 11 11 13 13 14 14 15	11
$12^{(16)}$	16 16 17 17 18 19 20 21 21 21	10

3 Draw statistical graphs of the data collected.

4 Report on the trends and discoveries you have made. Suggest reasons for the trends present and include possible limitations to your experiment.
- Could the same couple have been asked more than once by the group?
- Would climate cause the trends to be different for those married in Australia to those married elsewhere?
- Should Australian weddings be considered separately?
- Did the length of month influence the results?

5 Use your imagination to make an interesting extension project.

MATHS TERMS 14

census

- a survey of a whole population

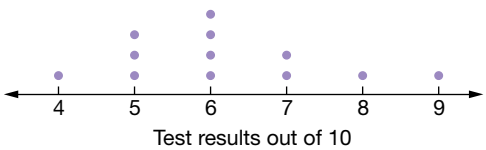
data

- the scores or information collected
- discrete data: data that is restricted to certain limited values,
  e.g. age in whole years
- continuous data: data that can take on any value within reason,
  e.g. height

dot plot

- a graph that uses one axis and a number of dots above the axis

e.g.



frequency

- the number of times an outcome occurs in the data,
  e.g. for the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 the outcome 5 has a frequency of 3

frequency distribution table

- a table that shows all the possible outcomes and their frequencies (it is often extended by adding other columns, such as a frequency × score column),

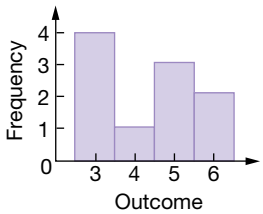
e.g.

Outcome	Frequency
3	4
4	1
5	3
6	2

frequency histogram

- a type of column graph showing the outcomes and their frequencies

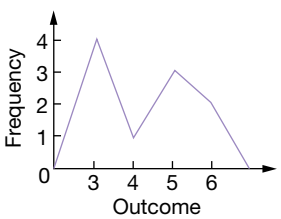
e.g.



frequency polygon

- a line graph showing outcomes and their frequencies. To complete the polygon, the outcomes immediately above and below those present are used. The height of these columns is zero,

e.g.



grouped data

- data that is organised into groups or classes
- class interval: the size of the groups into which the data is organised,
  e.g. 1–5 (5 scores); 11–20 (10 scores)
- class centre: the middle outcome of a class
  e.g. the class 1–5 has a class centre of 3

mean

- the number obtained by 'evening out' all the scores until they are equal,
  e.g. if the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 were 'evened out' the number obtained would be 4·3
- the mean is the arithmetic average

$$\text{Mean} = \frac{\text{Sum of the scores}}{\text{Number of scores}}$$

median

- the middle score for an odd number of scores or the mean of the middle two scores for an even number of scores
- median class
  in grouped data, the class that contains the median

mode (modal class)

- the outcome (or class) that occurs most often

outcome

- a possible value of the data

outlier

- a score that is separated from the main body of scores

range

- the difference between the highest and lowest scores

sample

- a part, usually a small part, of a large population
- random sample: a sample taken so that each member of the population has the same chance of being included
- systematic sample: a sample selected according to some ordering scheme, e.g. every tenth member
- stratified sample: a sample taken proportionally from each subgroup in a population

sources of data

- primary: the data has been collected yourself
- secondary: the data has come from an external source,
  e.g. newspapers, internet, etc.

statistics

- the collection, organisation and interpretation of data

stem-and-leaf plot

- a graph that shows the spread of scores without losing the identity of the data,
  e.g. **Test results**
  (6|8 represents 68)

Stem	Leaf
4	1 8
5	9 3 7 0 4
6	8 5 6 7 2 4 8 6
7	3 7 3 1 9 3 5
8	5 4 8 0 4
9	3 7 1

- ordered stem-and-leaf plot
  the *leaves* have been placed in order,
  e.g.

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

- back-to-back stem-and-leaf plot
  this can be used to compare two sets of scores with one set on each side,
  e.g. **Test results**
  (4|1 represents 41)

First test		Second test
Leaf	Stem	Leaf
9 8 6 6 0	3	
9 7 7 3 1 1 1	4	3 6 6 8
8 8 5 3 3 0	5	1 7 9 9
9 8 7 5 3	6	3 8 9
	7	0 5 5 5 8 9
0	8	2 6 7 7
	9	0 0 1 3

DIAGNOSTIC TEST 14 STATISTICS

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

14:01

1 State whether each set of data has come from a primary or secondary source.
   a Carrie catalogued her music collection and found that 72% were pop, 18% were rock and 10% were country music.
   b Kate read in a sports magazine that football crowds were up 14% this year.
   c The weather channel reported that the maximum temperatures for this week were 27, 26, 28, 27, 25, 24 and 23.
   d Maddie counted the traffic outside her house and found that 15% of the vehicles that passed were trucks.

14:02

2 State whether each set of data has been gathered as a stratified sample, a systematic sample or simply a random sample.
   a Joel stood at the school gate and surveyed every tenth student that walked past.
   b Jimmy wandered the playground and surveyed those he bumped into.
   c Jemma surveyed 10 boys and 10 girls from each year in the school.

14:02

3 A stratified random sample of 120 students is to be taken from Years 7 to 10 to investigate 'favourite hobbies'. The table shows the number of students in each Year. There are 450 in total. How many students should be chosen from:
   a Year 7 b Year 8
   c Year 9 d Year 10?

Year	Number
7	135
8	120
9	105
10	90

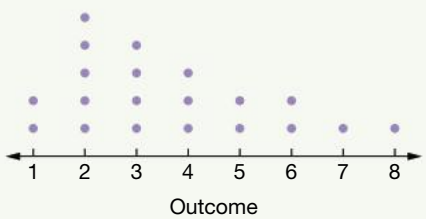
14:03

4 a Find the mean and median for the following 10 scores.
   3 13 15 15 15 16 16 17 18 19
   b Now remove the outlier score of 3 and re-evaluate the mean and median.
   c Did the outlier have a greater effect on the mean or the median?
   d Which is a more representative measure of the centre of the original 10 scores, the mean or the median?

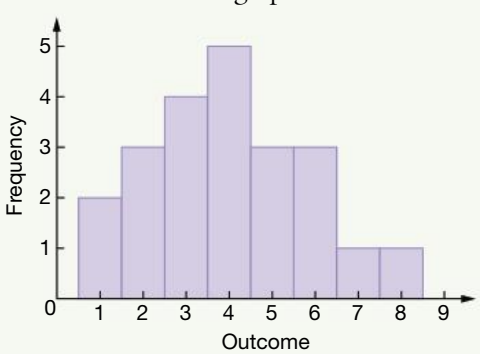
14:04

5 Find the mode, mean and median for the data shown in each graph.

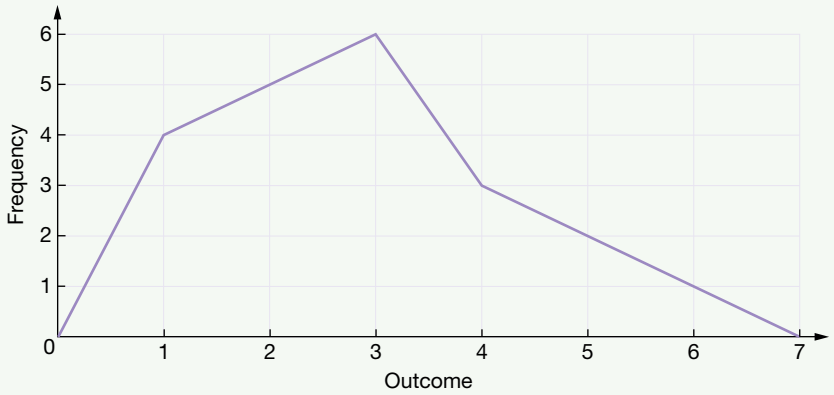
a



b



c



6 Complete each frequency distribution table and find the modal class, mean and median class. 14:05

a

Class	*c.c.*	Tally	*f*	$f \times c.c.$
10–<20		𝍸 ||		
20–<30		𝍸 𝍸		
30–<40		𝍸 𝍸 ||		
40–<50		|||		
50–<60		|		

b

Class	*c.c.*	*f*	$f \times c.c.$
1–5		11	
6–10		18	
11–15		22	
16–20		10	
21–25		7	

7 Year 3 and Year 4 students were tested on their knowledge of multiplication tables. The results are shown in this back-to-back stem-and-leaf display. 14:06

a List any outliers in these results.

b Find the mode, median and range for Year 3.

c Find the mode, median and range for Year 4.

Test results

(5 | 1 represents 51)

Year 3 results		Year 4 results
Leaf	Stem	Leaf
9 8 3 0	3	5
8 8 7 4 2 2 2	4	
8 7 7 4 0	5	1 4 6 6 9
7 6 3	6	0 2 2 5 8
1	7	3 6 8 8 9 9 9
	8	0 0

ASSIGNMENT 14A Chapter review

1 In a large manufacturing company, 100 samples are selected each day, for 40 days, and tested. The results show the number of faulty products.

3	5	4	4	6	7	3	2	1	2
2	3	4	5	5	3	4	2	2	2
1	4	2	3	5	1	1	3	3	4
4	2	3	5	4	5	3	4	3	1

a Organise this data into a frequency distribution table.

b Calculate the mode, mean and median.

2 A group of students sat for an English language test and the results were entered in this stem-and-leaf display.

a How many scored in the fifties?

b What was the mode of the results?

c How many students scored less than 50?

Test results

(7 | 3 represents 73)

Stem	Leaf
3	1 8 2
4	6 3 3 1 3
5	4 1 1 5
6	2 2 8 4 7 7 1
7	6 3 4 6 5 5 1 4
8	2 6 6 7 3 3 1
9	2 9 9 1 0 0

d How many students were tested?

e What was the median of the results?

f If 65 and above is considered to be a passing grade, how many passed?

g Make a grouped frequency table for this data using classes of 30–39, 40–49, etc.

3

Class	Class centre ($c.c.$)	Frequency (f)	$f \times c.c.$
1–5		3	
6–10		7	
11–15		12	
16–20		8	
21–25		6	
26–30		5	

a Use the table to calculate:

i the modal class

ii the mean.

b Construct a frequency polygon.

c Is it possible to work out the range from this table?

d What is the median class?

4 A survey of families was conducted to find out how many children were in the family. Use the information in the table to answer the following questions.

a How many families were surveyed?

b How many families had 1 or 2 children?

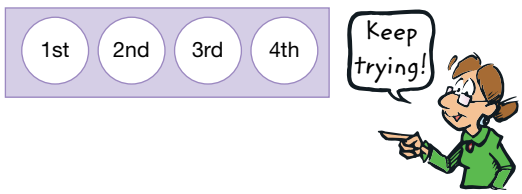
c Draw a frequency histogram to illustrate the data.

No. of children	Frequency
1	14
2	38
3	22
4	8
5	0
6	2

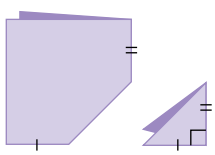
ASSIGNMENT 14B Working mathematically

1 Plastic numbers are needed for the doors on the 3rd floor of a large hotel. If all of the numbers 301 to 346 are needed, how many of each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 will be needed?

2 The second digit of a four-digit number is twice as big as the first digit. The third digit is twice as big as the second digit and the fourth digit is twice as big as the third digit. What is the four-digit number?



3 A rectangular piece of paper 8 cm long and 4 cm wide was folded. This was then cut (as shown) from midpoint to midpoint of the sides shown. The smaller part was then unfolded and laid flat. What was the area of this triangle?



4 Pete was tracing his family tree. How many great-great-grandparents would he have? (The order goes: parents, grandparents, great-grandparents, then great-great-grandparents.)

5 If I use only 5c, 10c and 20c coins, how many different ways can I make up 40 cents?

6 Use the temperature graph below to answer the following questions.

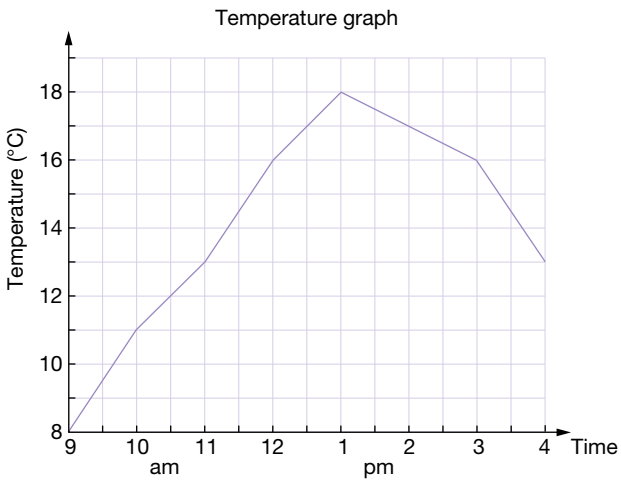
a What was the temperature at 9:00 am?

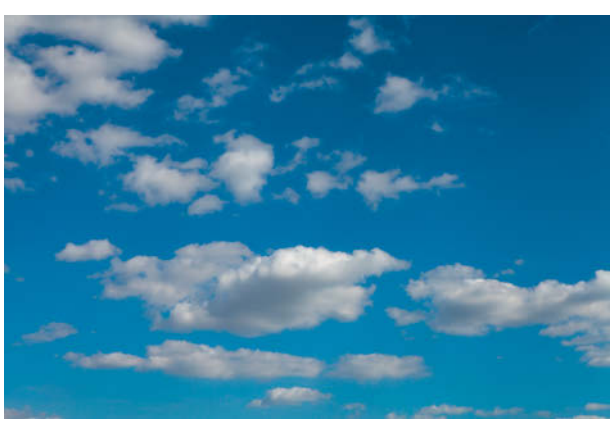
b At approximately what time in the morning did the temperature reach 14°C?

c What was the approximate temperature at 2:30 pm?

d For how long was the temperature above 16°C?

e In what hourly periods did the greatest increase in temperature occur?





Weather bureaus around the world keep statistics on many aspects of weather. Average rainfall and average temperature are often quoted in weather broadcasts.

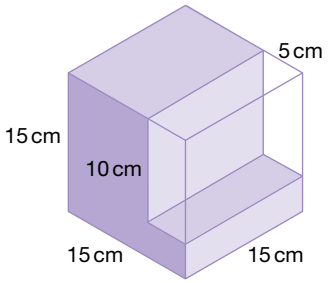
ASSIGNMENT 14C Cumulative revision

1 The diagram shows a cube with a part of it removed. The removed piece has the shape of a rectangular prism. Calculate:

a the volume of the cube

b the volume of the rectangular prism before the part was removed

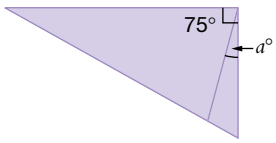
c the volume of the remaining shape.



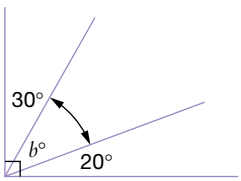
8:06

2 Find the values of the pronumerals, giving reasons for your answers.

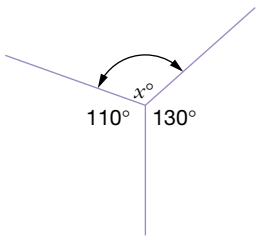
a



b



c



1:08

3 A map has a scale of 1 : 5000. What distance is represented by a length of 1 mm on the map? If the distance between two points in real life is 350 m, how far apart would they be on the map?

10:05

4 Tristan walks 1 km in 8 minutes. At this rate how long would it take him to walk 400 m? How far could he walk in 30 seconds?

10:06

5 Samarra runs 100 m in 12 seconds. How fast is this in km/h?

10:06

6 **a** A property is 20 m wide and 150 m long. Assuming that it is a rectangular shape, find its area in hectares.

b A cube has a volume of 250 cm^3. What is its side length, to the nearest whole number?

8:02, 8:06

7 Find the rule connecting y and x in each case.

a

x	4	5	6	7
y	8	10	12	14

b

x	2	3	4	5
y	7	10	13	16

c

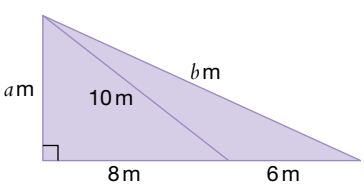
x	1	2	3	4
y	1	4	9	16

5:01

8 **a** What percentage of 1 kg is 6 g? **b** What percentage of 2·4 m is 60 cm?

4:05

9 Find the value of a and then b in the diagram.



3:04

10 Simplify:

a $-3 - 4 \times 2$ **b** $12 - 6 \times 5$ **c** $-4 - 3 + 6 - 2 + 8$ **d** $\dfrac{12-4}{4-8}$

2:01

REASONING IN GEOMETRY

15



Contents

15:01 Adjacent angles
15:02 Angles at a point and vertically opposite angles
15:03 Angles associated with parallel lines
Fun spot 15:03 What did the canary say when its cage fell apart?
GeoGebra activity 15:03 Discovering more about angles
15:04 Angle sum of a triangle
15:05 Angle sum of a quadrilateral
GeoGebra activity 15:05 Angle sum of a quadrilateral
15:06 Isosceles and equilateral triangles
GeoGebra activity 15:06 Investigating triangles
15:07 More involved numerical problems
GeoGebra activity 15:07 Geometric reasoning
Challenge 15:07 Can you explain why?
Fun spot 15:07 Numbered boxes
Maths terms, Diagnostic test, Assignments

Syllabus references (See page x for details.)

Measurement and Geometry

Selection from ***Angle Relationships***, ***Properties of Geometrical Figures 1*** [Stage 4]

- Recognise the geometrical properties of angles at a point.
- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal. (ACMMG163)
- Investigate conditions for two lines to be parallel. (ACMMG164)
- Solve simple numerical problems using reasoning. (ACMMG164)
- Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral. (ACMMG166)
- Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

15:01 Adjacent angles

In Year 7 many basic geometrical facts were discovered. These facts and others will be used to develop logical reasoning. It is important, therefore, that you revise past work.

Refer to **ID Card 6** on page xix and identify parts **1** to **15**. Learn the terms you do not know.

PREP QUIZ 15:01

1 What is the size of a right angle?

2 $30 + m = 90$. Find the value of m.

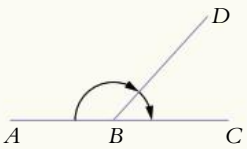
3 $x + 42 = 90$. Find the value of x.

4 What is the size of a straight angle?

5 $a + 50 = 180$. Find the value of a.

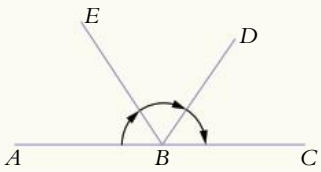
6 $135 + b = 180$. Find the value of b.

7



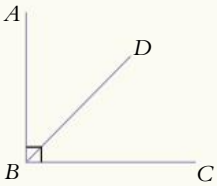
$\angle ABC$ is a straight angle. What can you say about $\angle ABD + \angle DBC$?

8



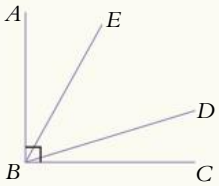
$\angle ABC$ is a straight angle. What can you say about $\angle ABE + \angle EBD + \angle DBC$?

9



$\angle ABC$ is a right angle. What can you say about $\angle ABD + \angle DBC$?

10



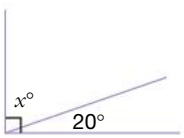
$\angle ABC$ is a right angle. What can you say about $\angle ABE + \angle EBD + \angle DBC$?

The Prep quiz illustrates two important basic facts.

1. If two adjacent angles make a right angle, then the sum of these angles is 90°.
2. If two adjacent angles make a straight angle, then the sum of these angles is 180°.

WORKED EXAMPLE 1

Find the value of the pronumeral, giving a reason for your answer.



Solution

$x + 20 = 90$ (adj. comp. ∠s)

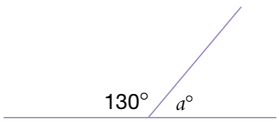
$\therefore x = 70$ OR (∠s make a right angle)

When the degree symbol is given in the question, $x°$, a degree symbol is not required in the answer.

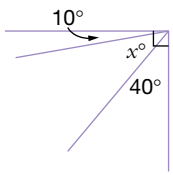
WORKED EXAMPLE 2

Find the value of the pronumeral in each of the following, giving a reason for your answer.

a



b



Word	Abbreviation
adjacent	adj.
angles	∠s
complementary	comp.
supplementary	supp.
straight	str.
point	pt

Solutions

a $a + 130 = 180$ (adj. supp. ∠s)
$\therefore\ a = 50$ OR (∠s make a str. ∠)

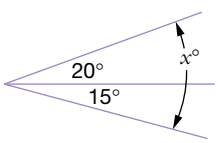
b $10 + x + 40 = 90$ (adj. comp. ∠s)
$\therefore\ x = 40$



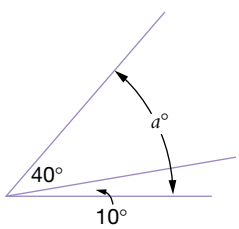
Exercise 15:01

1 Write down the value of the pronumeral in each of the following.

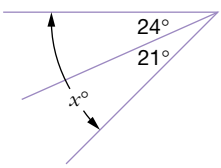
a



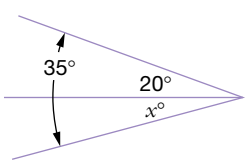
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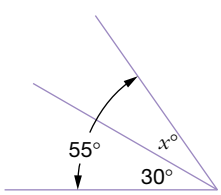
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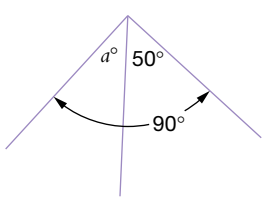
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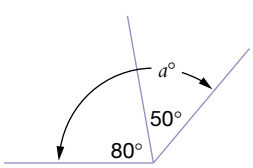
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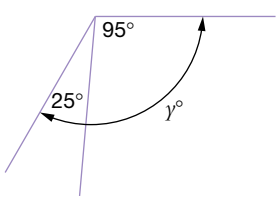
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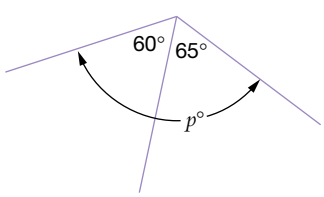
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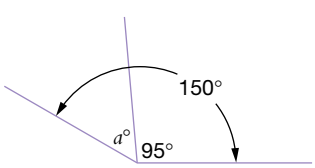
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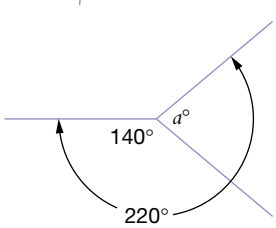
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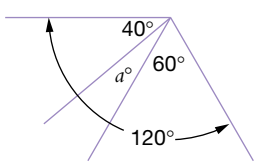
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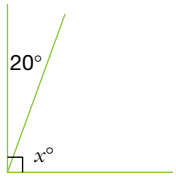


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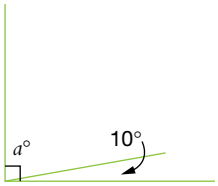


2 Find the value of the pronumeral in each of the following. Give a reason for your answer.

a

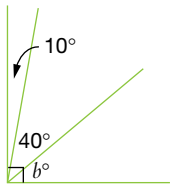


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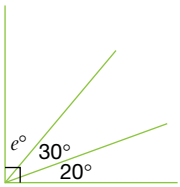




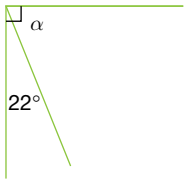
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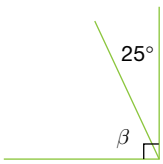
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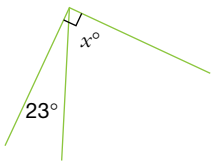
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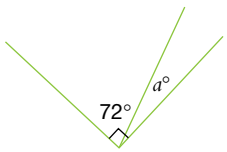
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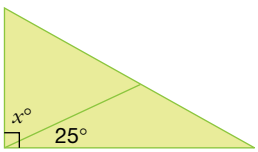
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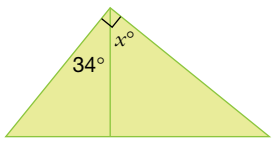
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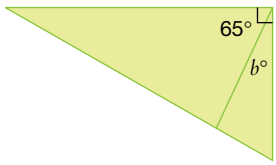
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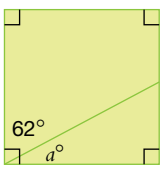
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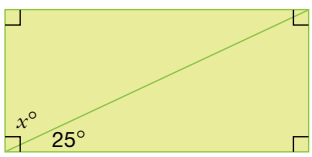
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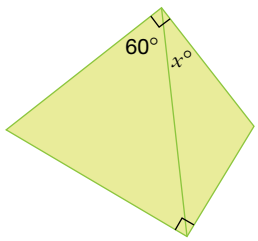
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m



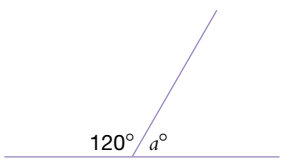
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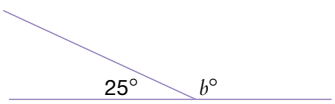
3 Find the value of the pronumeral in each of the following. Give a reason for your answer.



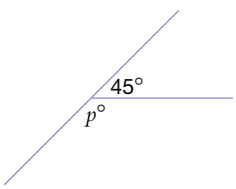
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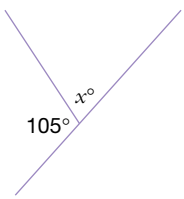
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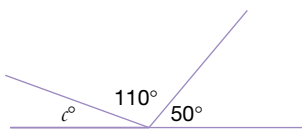
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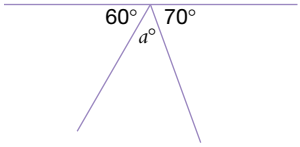
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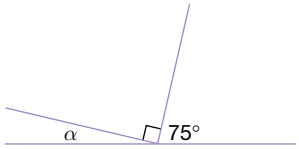
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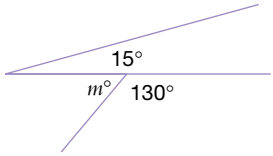
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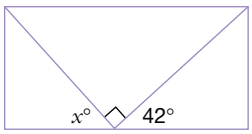
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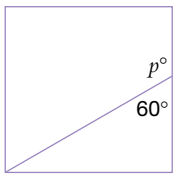
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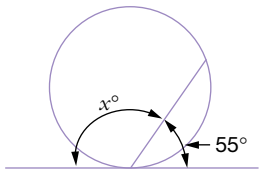
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j

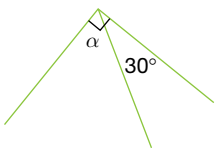


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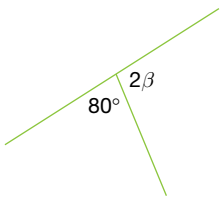


4 α (alpha) and β (beta) are Greek letters that stand for the measure of an angle. Write an equation for each of the following and solve it to find the values of α and β.

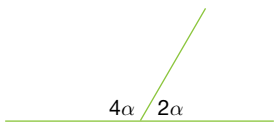
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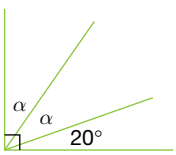
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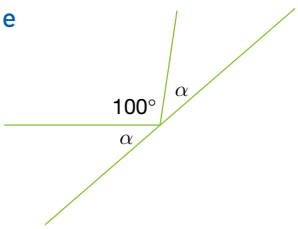
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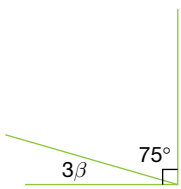
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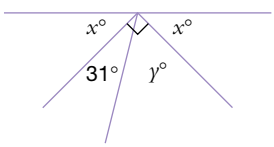


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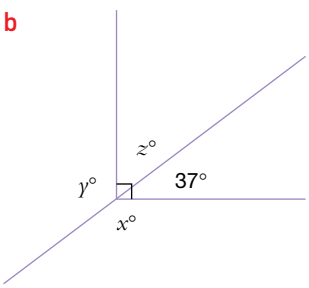


5 Find the value of the pronumerals in each of the following. Give reasons for your answers.

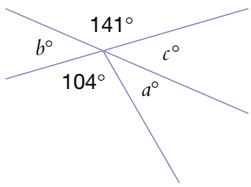
a



b



c



15:02 Angles at a point and vertically opposite angles

The Greek letters α, β, γ, δ and θ are used to stand for the *measure* or *size* of an angle.
- This means that the *degrees* symbol is not used with them.

Angles at a point

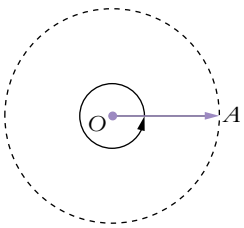


Figure 1

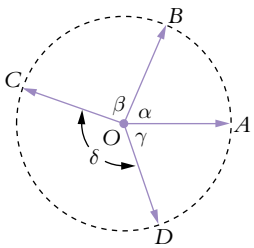
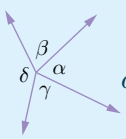


Figure 2

Here the arm OA can move around the fixed point O.

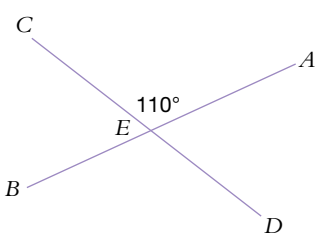
In Figure 1 the arm OA completes one revolution and so moves through an angle of 360°. In Figure 2 the arm OA stops at B, C and D before continuing to its starting position. Despite these stops the arm OA has still completed one revolution. Hence, we can see that:



$$\alpha + \beta + \gamma + \delta = 360°$$

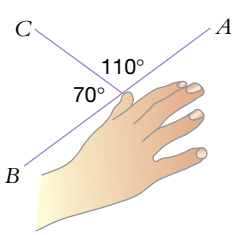


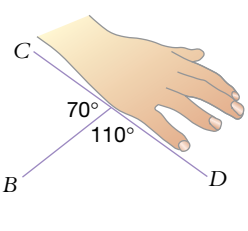
Vertically opposite angles

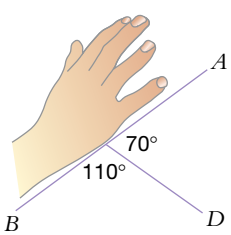


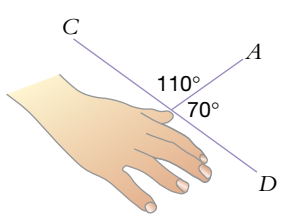
AB and CD are straight lines and $\angle CEA = 110°$. Using the knowledge gained in the last section, we can calculate the size of the other three angles.

By progressively covering up each of the arms ED, AE, CE and BE, we get the series of diagrams below. Each of these diagrams shows a pair of adjacent supplementary angles.

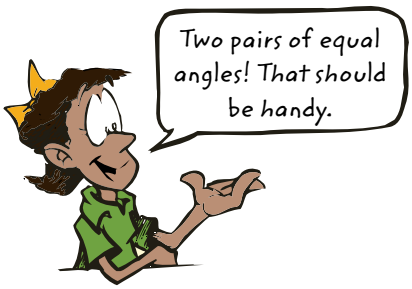


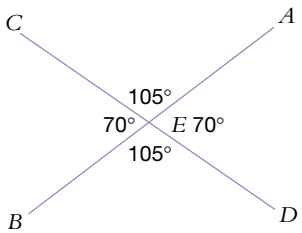




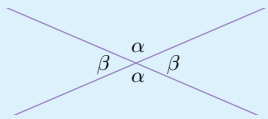


• From these results, we can see that when two lines cross, two pairs of equal angles are formed. These angles, which are opposite each other, are called **vertically opposite angles**.





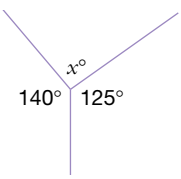
When two straight lines cross, two pairs of vertically opposite angles are formed. Vertically opposite angles are equal.



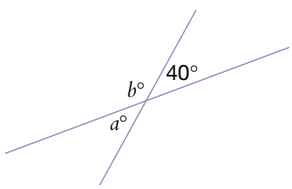
WORKED EXAMPLES

Find the values of the pronumerals in each of the following.

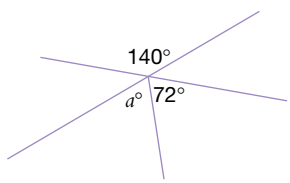
1



2



3



Solutions

1 $x + 140 + 125 = 360$ ($\angle$s at a pt)
$\therefore\ x = 95$

2 $a = 40$ (vert. opp. $\angle$s)
$b + 40 = 180$ (adj. supp. $\angle$s)
$\therefore\ b = 140$

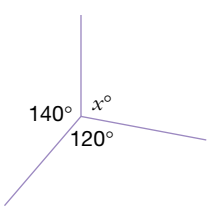
3 $a + 72 = 140$
(vert. opp. $\angle$s)
$\therefore\ a = 68$

Exercise 15:02

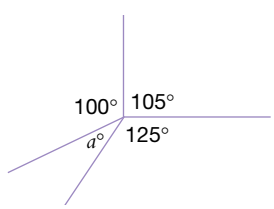
P Foundation worksheet 15:02 Angle relationships

1 Find the value of the pronumeral in each of the following. Give reasons for your answers.

a

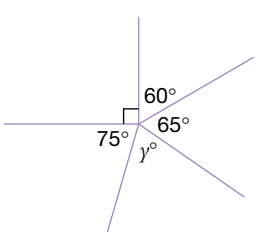


b

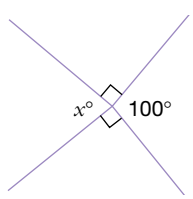




c

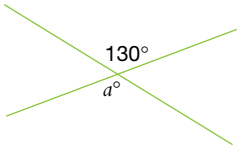


d

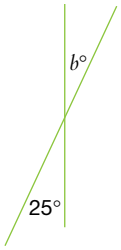


2 Find the value of the pronumeral in each of the following. Give reasons for your answers.

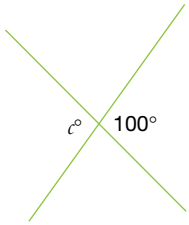
a



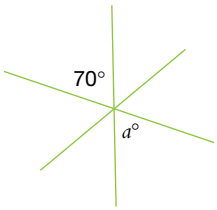
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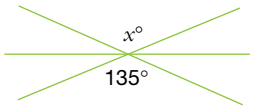
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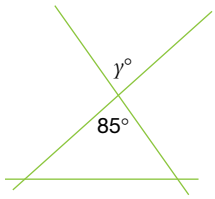
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e



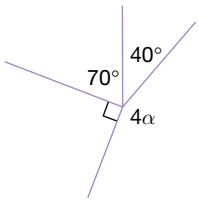
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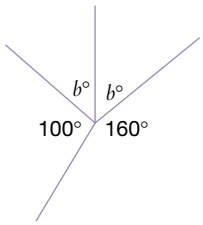
3 Find the values of the pronumerals in each of the following. Give reasons for your answers.



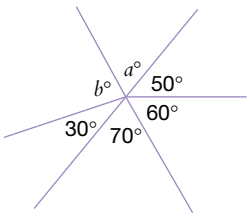
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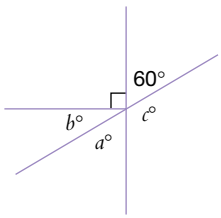
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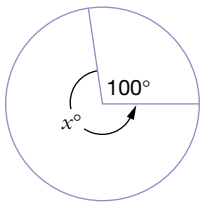
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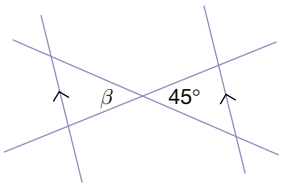
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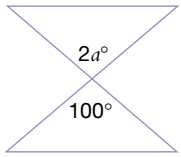
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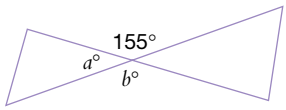
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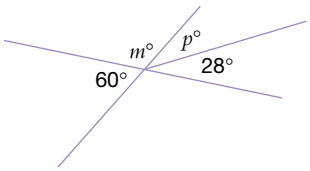
g



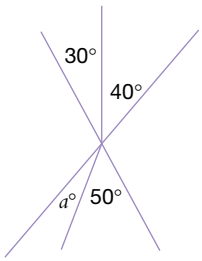
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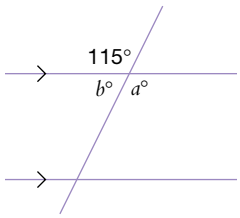
i



j

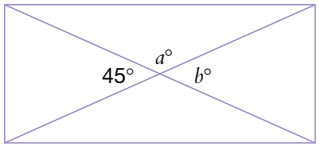


k

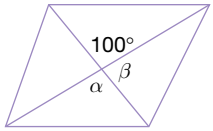


4 Find the values of the pronumerals in each of the following. Give reasons for your answers.

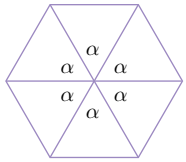
a



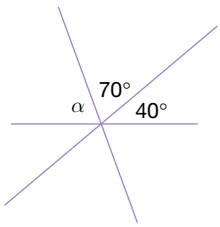
b



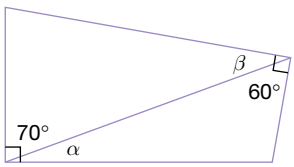
c



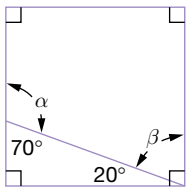
d



e

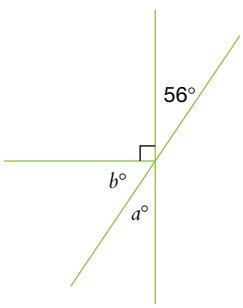


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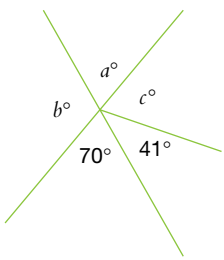


5 Find the values of the pronumerals in each of the following. Give reasons for your answers.

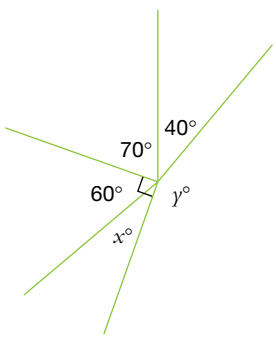
a

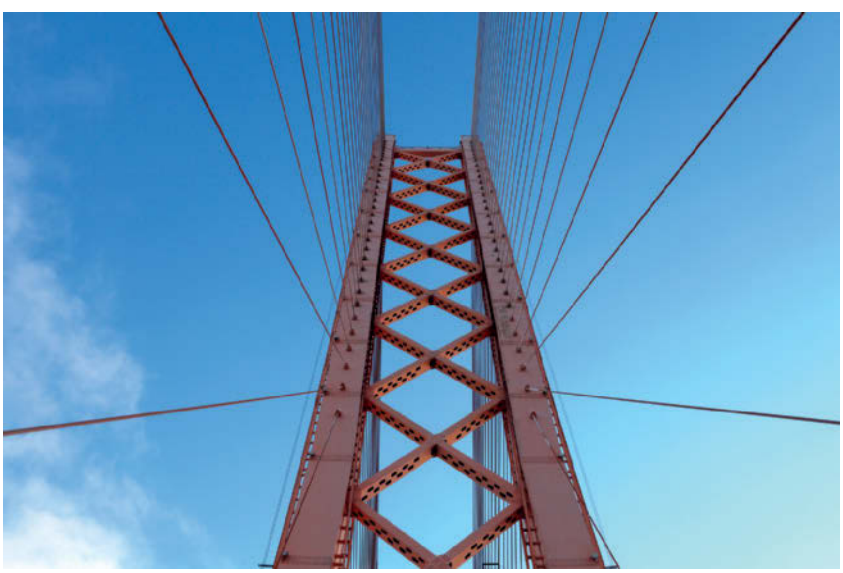


b



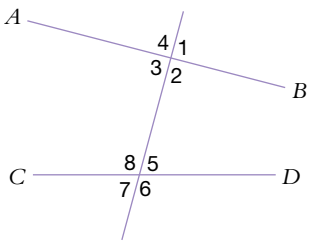
c





Many pairs of vertically opposite angles can be seen in this bridge support.

15:03 Angles associated with parallel lines

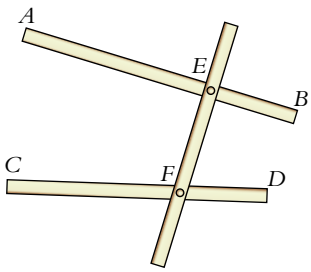


When two lines AB and CD are cut by a third line, called a **transversal**, eight angles are formed, as shown. From what we have learnt about vertically opposite angles, we can see that:

angle 1 = angle 3 angle 5 = angle 7

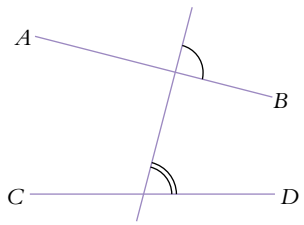
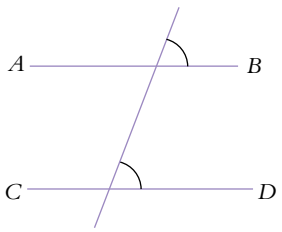
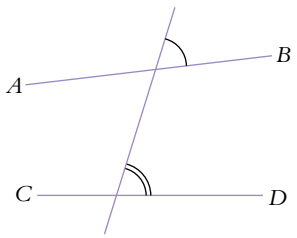
angle 2 = angle 4 angle 6 = angle 8

A model of this situation can be made from rulers or cardboard strips, as shown.





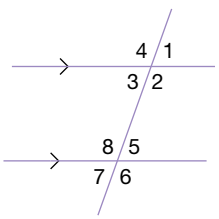
- The rulers need to be pivoted at E and F.
- By moving the ruler AB into different positions, the relationships between the angles can be investigated.
- Of particular importance is the case when the lines AB and CD are parallel. When the lines are parallel, the angles that the lines make with the transversal are equal. This can be seen from the sequence of diagrams below.

		
The top angle is bigger than the bottom angle. The lines will cross on the right of the transversal.	The top angle is equal to the bottom angle. The lines are parallel.	The top angle is smaller than the bottom angle. The lines will cross on the left of the transversal.

When two parallel lines are cut by a transversal, the following pairs of angles are equal.

angle 1 = angle 5 angle 4 = angle 8

angle 2 = angle 6 angle 3 = angle 7



This means that:

- angles 1, 3, 5, 7 are equal
- angles 2, 4, 6, 8 are equal.

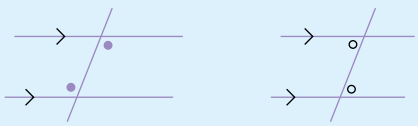


The special angle relationships that occur when two lines are parallel are set out in the following rule box.

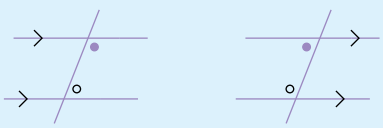
- When a pair of parallel lines is cut by a transversal, four pairs of equal **corresponding** angles are formed.



- When a pair of parallel lines is cut by a transversal, two pairs of equal **alternate** angles are formed.

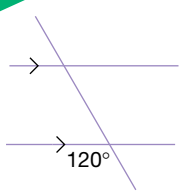


- When a pair of parallel lines is cut by a transversal, two pairs of **co-interior** angles are formed. Co-interior angles are supplementary.

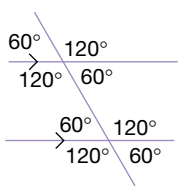


WORKED EXAMPLE 1

Copy the diagram and write in the values of all the other angles in the figure.



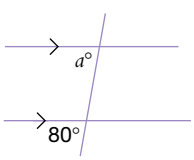
Solution



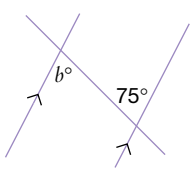
WORKED EXAMPLE 2

Find the value of the pronumeral in each of the following, giving reasons for your answers.

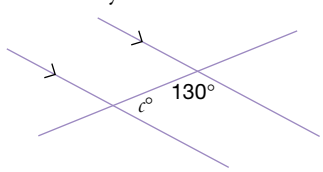
a



b

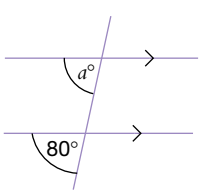


c



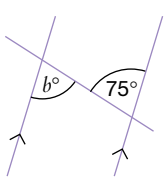
Solutions

a



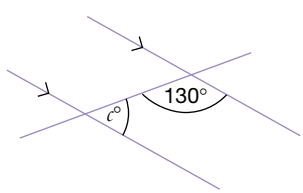
$a = 80$
(corres. ∠s and parallel lines)

b



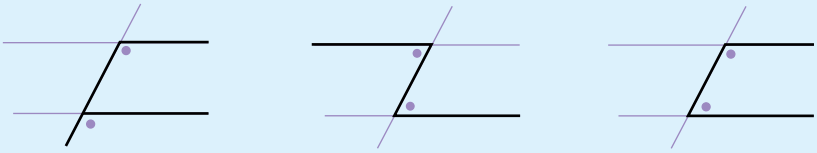
$b = 75$
(alt. ∠s and parallel lines)

c



$c = 50$
(co-int. ∠s and parallel lines)

The diagrams below show a summary of the three types of angles formed when a pair of parallel lines is cut by a transversal.



Corresponding angles (or 'F' angles)

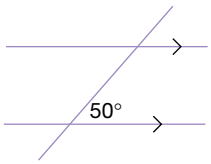
Alternate angles (or 'Z' angles)

Co-interior angles (or 'C' angles)

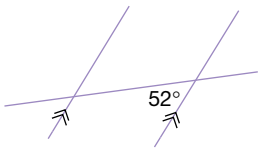
Exercise 15:03

1 Copy each of the diagrams and then write down the sizes of all the angles.

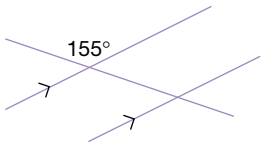
a



b

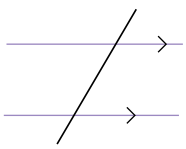


c

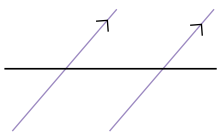


2 Copy each of the diagrams below and mark in the four pairs of equal corresponding angles using the symbols ×, •, ○, and ☽).

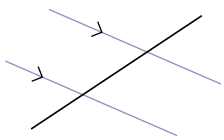
a



b

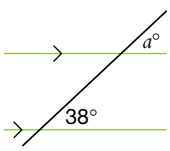


c

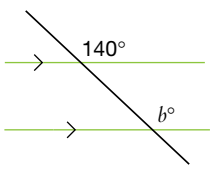


3 Find the values of the pronumerals in each of the following, giving a reason for each answer.

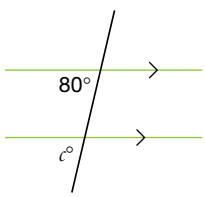
a



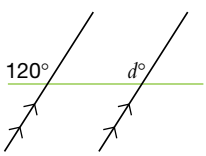
b



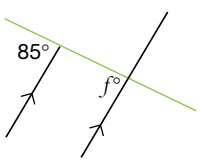
c



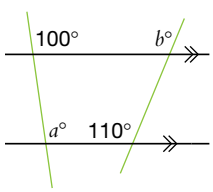
d



e



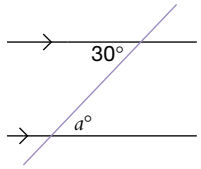
f



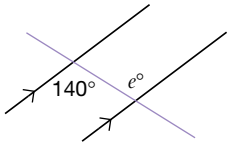
4 Copy each of the diagrams in Question 2 and mark in the two pairs of alternate angles, using the symbols × and •.

5 Find the value of the pronumeral in each of the following, giving a reason for each answer.

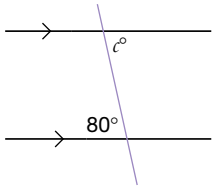
a



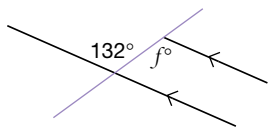
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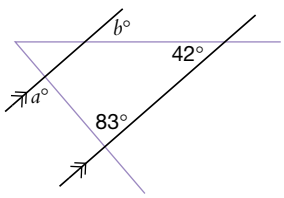
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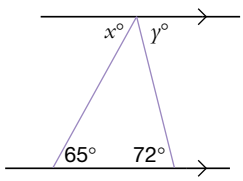
d



e



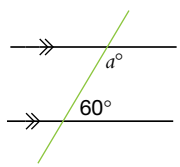
f



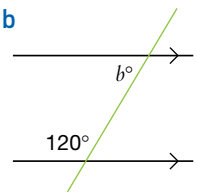
6 Copy each of the diagrams in Question 2 and mark in the two pairs of co-interior angles.

7 Find the values of the pronumerals in each of the following, giving a reason for each answer.

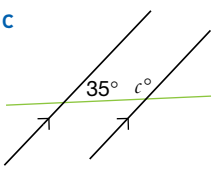
a



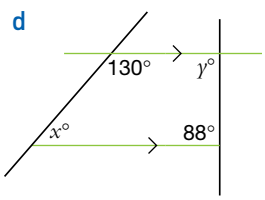
b



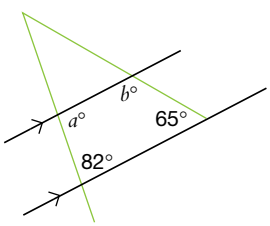
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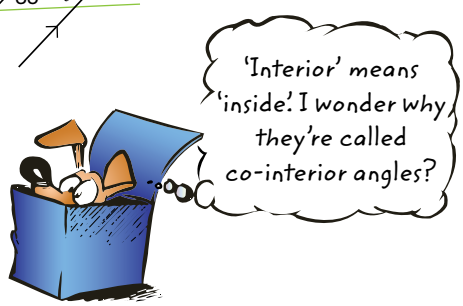


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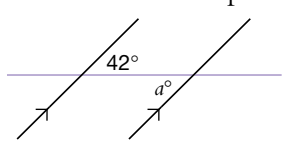
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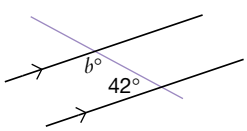


8 Find the value of the pronumeral in each of the following. Give a reason for each answer.

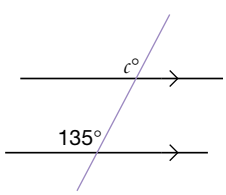
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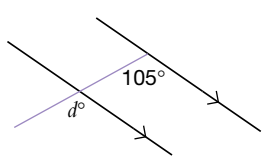
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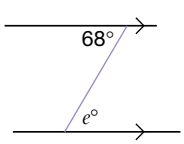
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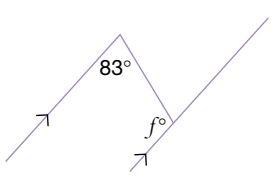
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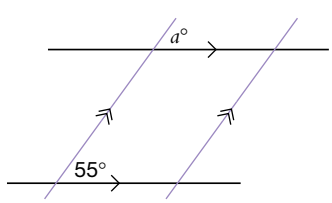
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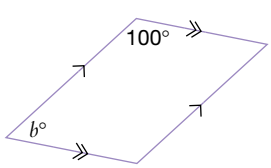
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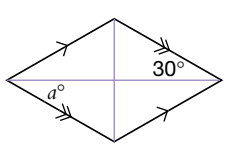
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h

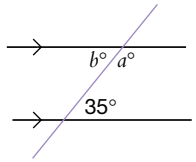


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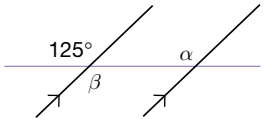


9 Find the values of the pronumerals in each of the following. Give a reason for each answer.

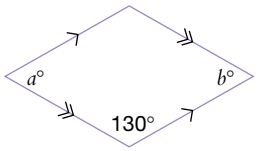
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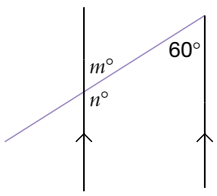
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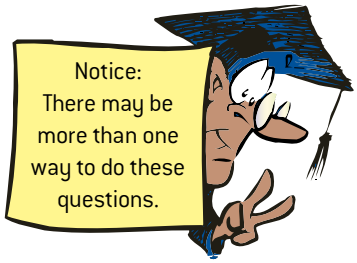


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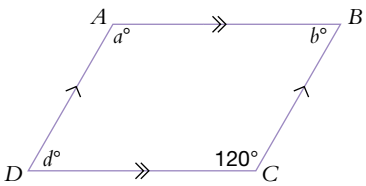


d



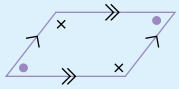


e



Parts **c** and **e** show this important fact:

The opposite angles of a parallelogram are equal.



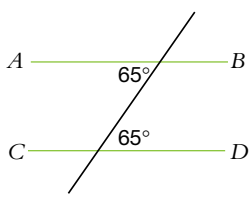
In the questions so far, we have seen that if two lines are parallel then corresponding angles are equal, alternate angles are equal, and co-interior angles are supplementary. The reverse is also true.

A pair of lines are parallel, if, when cut by a transversal:

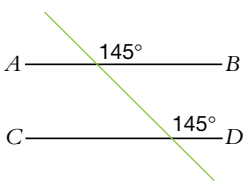
- corresponding angles are equal
- alternate angles are equal
- co-interior angles are supplementary.

10 Give a reason why $AB \parallel CD$ in each case.

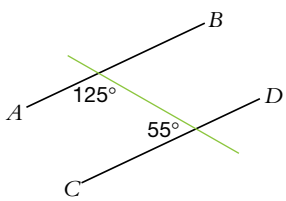
a



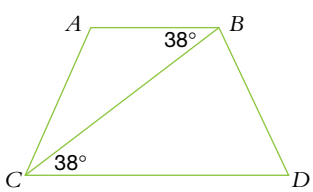
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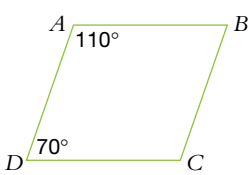
c



d



e



The symbol || means 'is parallel to'.

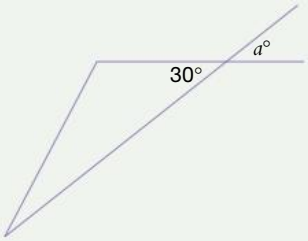


FUN SPOT 15:03 WHAT DID THE CANARY SAY WHEN ITS CAGE FELL APART?

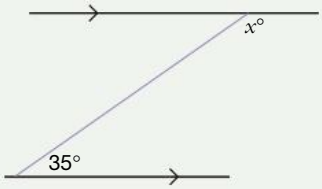
Answer each question and write the letter for that question in the box above the correct answer.

Find the value of the pronumeral in each of the following.

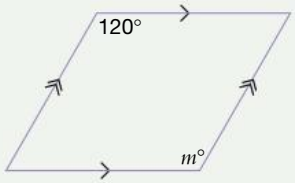
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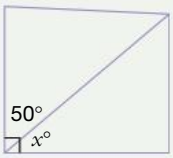
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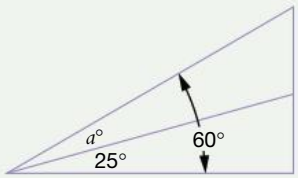
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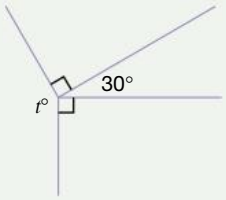
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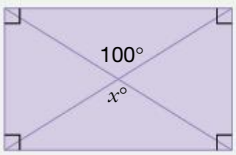
H



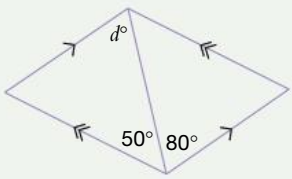
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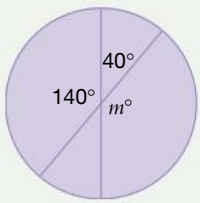
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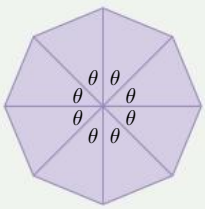
P



H



C



100	140	150	30	80	45°	35	40	120	145

P GEOGEBRA ACTIVITY 15:03 DISCOVERING MORE ABOUT ANGLES

Use these seven activities to revise each of the topics in an interactive way. Instructions are included in each activity

1 **Adjacent angles**

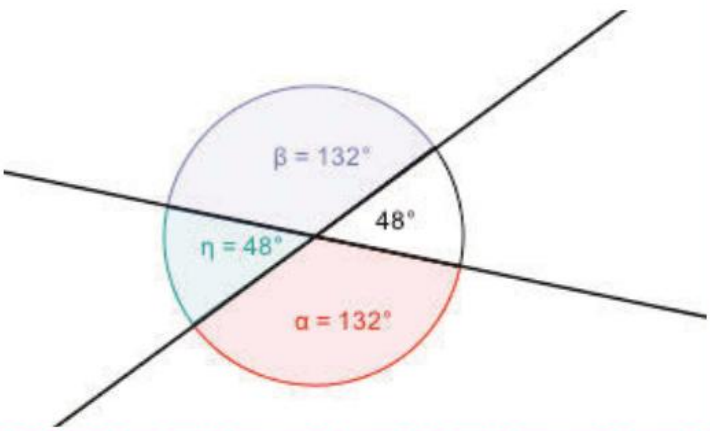
2 **Angles at a point**

3 **Vertically opposite angles**

4 **Angles at intersecting lines**

5 **Angles and parallel lines** (3 activities)

.



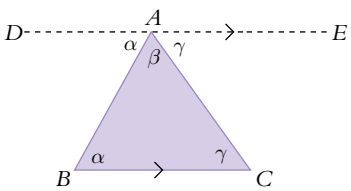
15:04 Angle sum of a triangle

In Year 7 it was discovered that the angles of a triangle add up to 180°. This was done by cutting the angles of a triangle and rearranging them to form a straight angle.



The angle sum of a triangle

Using our knowledge of angles on parallel lines, we can prove that the angles of a triangle add up to 180°.



Draw a line DE through A, parallel to BC.

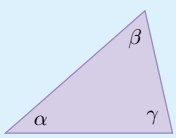
$\angle BAD = \alpha$ (alternate to $\angle ABC$, $DE \parallel BC$)
$\angle CAE = \gamma$ (alternate to $\angle ACB$, $DE \parallel BC$)

So, $\alpha + \beta + \gamma = 180°$ ($\angle DAE$ is a straight angle)

$\therefore$ the angle sum of a triangle is 180°.

The angle sum of a triangle is 180°.

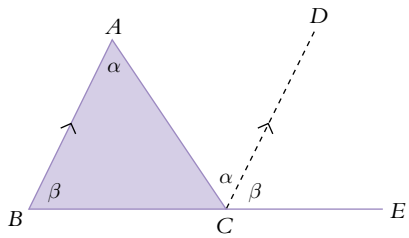
$$\alpha + \beta + \gamma = 180°$$





The exterior angle of a triangle

Using our knowledge of angles on parallel lines, we can find a relationship between the exterior angle ACE and the angles inside the triangle.



Draw a line CD through C, parallel to BA.

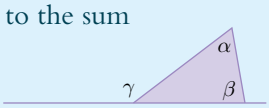
$\angle ACD = \alpha$ (alternate to $\angle BAC$, $BA \parallel CD$)
$\angle DCE = \beta$ (corresponding to $\angle ABC$, $BA \parallel CD$)

So, $\angle ACE = \alpha + \beta$ ($\angle ACE = \angle ACD + \angle DCE$)

$\therefore$ the exterior angle of a triangle is equal to the sum of the interior (or remote) opposite angles.

The exterior angle of a triangle is equal to the sum of the two remote interior angles.

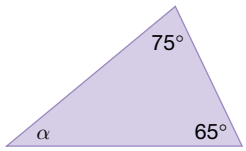
$$\gamma = \alpha + \beta$$



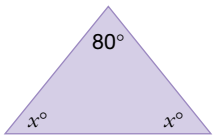
WORKED EXAMPLES

Find the values of the pronumerals, giving reasons for your answers.

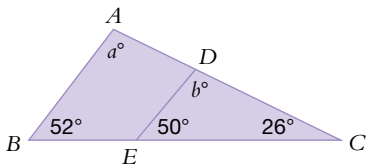
1



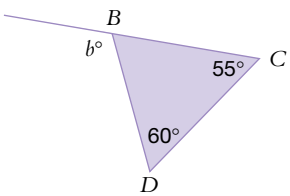
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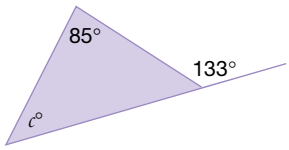
3



4



5



Solutions

1 $\alpha + 65° + 75° = 180°$ (angle sum of a Δ)

$\therefore \alpha = 40°$

Note:
When the degree symbol is not given in the question, a degree symbol is required in the answer.

2 $x + x + 80 = 180$ (angle sum of a Δ)

$\therefore 2x + 80 = 180$

$\therefore 2x = 100$

$\therefore x = 50$

3 $b + 50 + 26 = 180$ (angle sum of ΔDEC)

$\therefore b = 104$

$a + 52 + 26 = 180$ (angle sum of ΔABC)

$\therefore a = 102$

4 $b = 55 + 60$ (exterior angle of ΔBCD)

$\therefore b = 115$

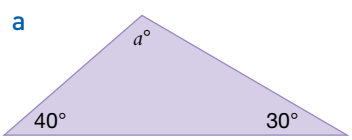
5 $c + 85 = 133$ (exterior angle of a Δ)

$\therefore c = 48$

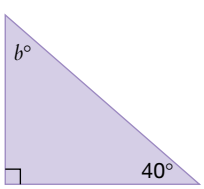
Exercise 15:04

1 Find the value of the pronumeral in each of the following. Give a reason for each answer.

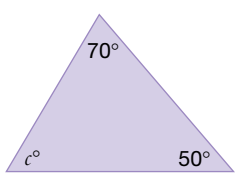
a



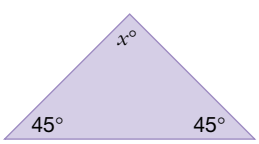
b



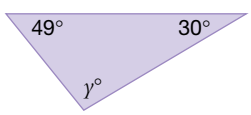
c



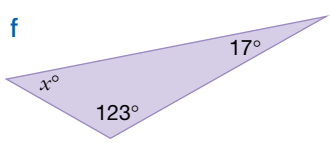
d



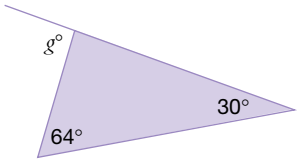
e



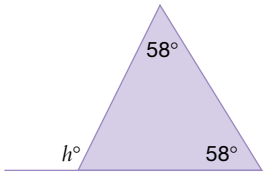
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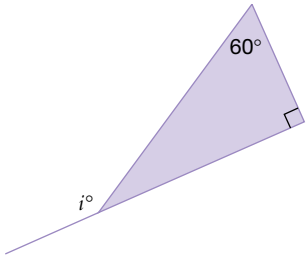
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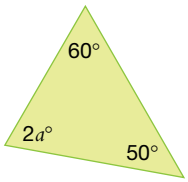


i

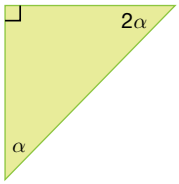


2 Write an equation for each of the following and solve it to find the value of the pronumeral.

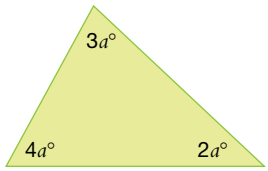
a



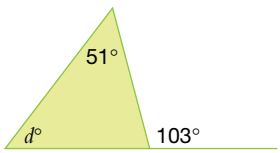
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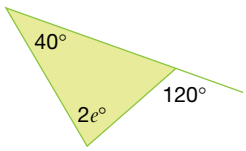
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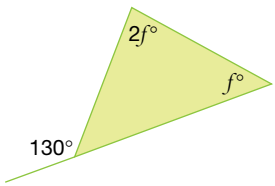
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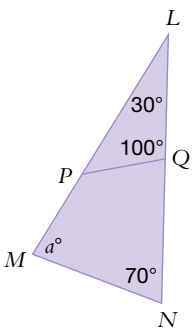


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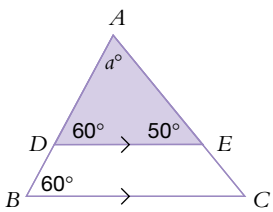


3 In each of the following there is more than one triangle. By finding the angle sum of the shaded triangle, find the value of the pronumeral.

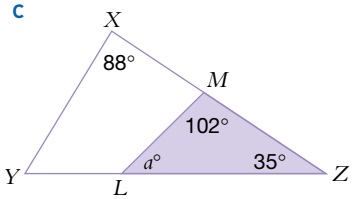
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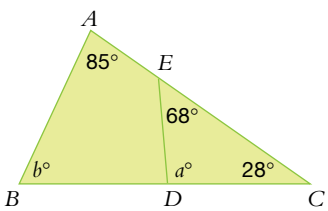


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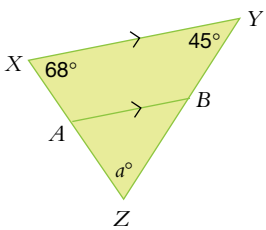


4 Examine each shape carefully and, by using the correct triangle, find the values of the pronumerals.

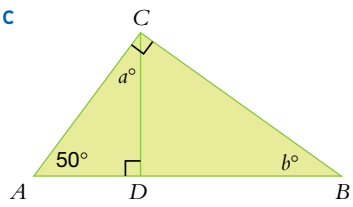
a



b

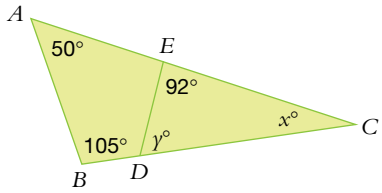


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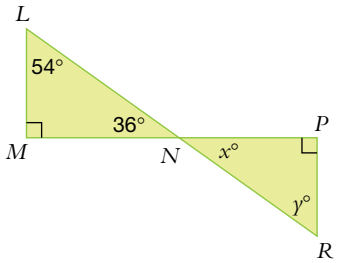


5 In each of the following find the value of x and then use this to find the value of y. Give a reason for each statement in your answer.

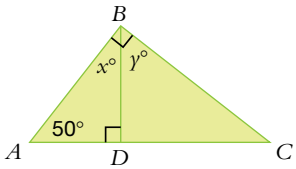
a



b

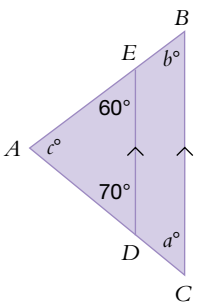


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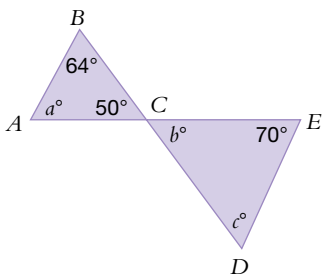


6 Find the values of the pronumerals in each of the following, giving reasons for your answers.

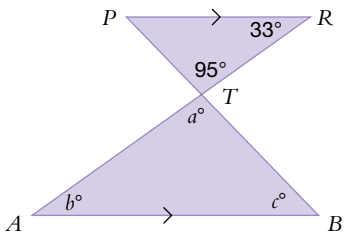
a



b



c



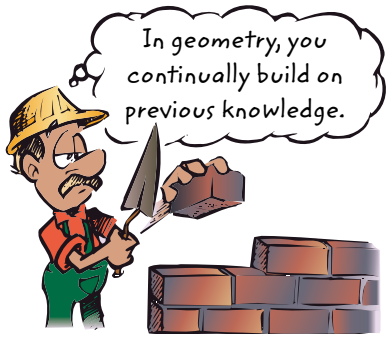


Carpenters need an understanding of parallel lines in roof construction.

15:05 Angle sum of a quadrilateral

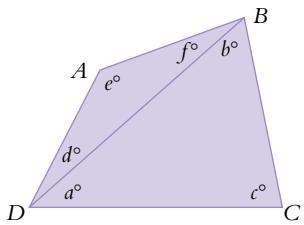
In the last section we proved that the angles of a triangle have a sum of 180°.

All quadrilaterals can be divided into two triangles, so we can conclude that the angle sum of a quadrilateral is $2 \times 180°$ or 360°.



The angle sum of a quadrilateral

Using our knowledge of angles in triangles, we can prove that the angle sum of a quadrilateral is 360°.

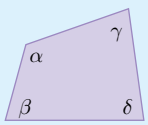


Angle sum of $ABCD$
$= \angle DAB + \angle ABC + \angle BCD + \angle CDA$
$= e + (f + b) + c + (a + d)$
$= (a + b + c) + (d + e + f)$
= angle sum of ΔDBC + angle sum of ΔDAB
$= 180° + 180°$
$= 360°$

$\therefore$ the angle sum of a quadrilateral is 360°.

The angles of a quadrilateral add up to 360°.

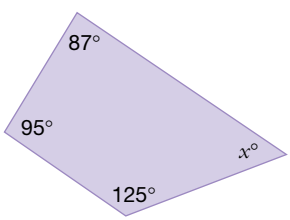
$\therefore \alpha + \beta + \gamma + \delta = 360°$



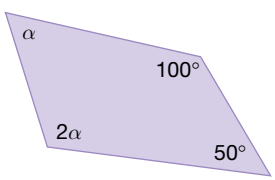
WORKED EXAMPLES

Find the value of the pronumeral in each of the following.

1



2



Solutions

1 $x + 125 + 95 + 87 = 360$ (angle sum of a quad.)
$\therefore x + 307 = 360$
$\therefore x = 53$

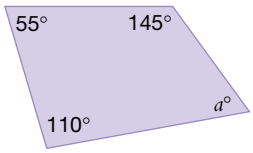
2 $2\alpha + \alpha + 100° + 50° = 360°$ (angle sum of a quad.)
$\therefore 3\alpha + 150° = 360°$
$\therefore 3\alpha = 210°$
$\therefore \alpha = 70°$

Exercise 15:05

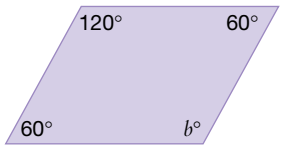
Foundation worksheet 15:05
Triangles and quadrilaterals

1 Find the value of the pronumeral in each of the following, giving a reason in each case.

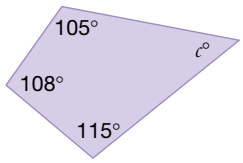
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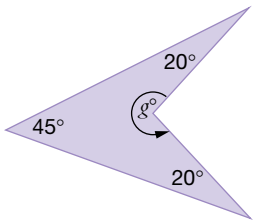
b



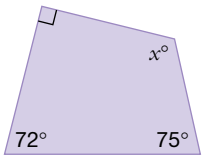
c



d

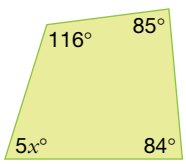


e

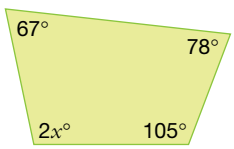


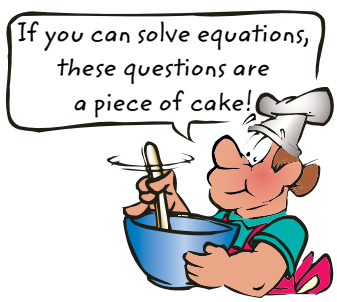
2 Write an equation for the angle sum of each of the following and then solve it to find the value of the pronumeral.

a

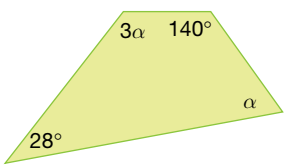


b

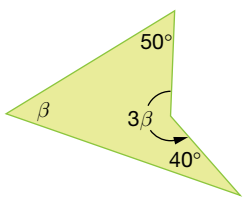




c

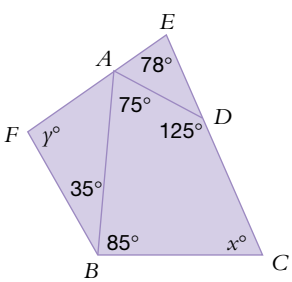


d

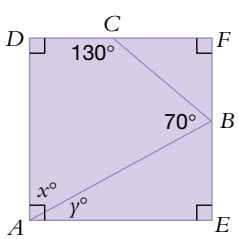


3 Use the angle sum of the quadrilateral $ABCD$ to find the value of x and then find the value of y.

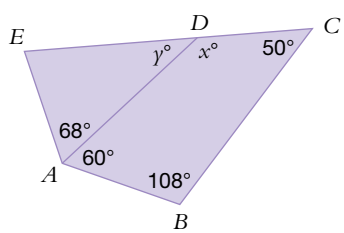
a



b

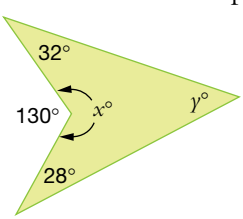


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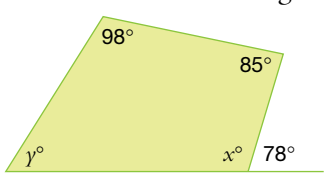


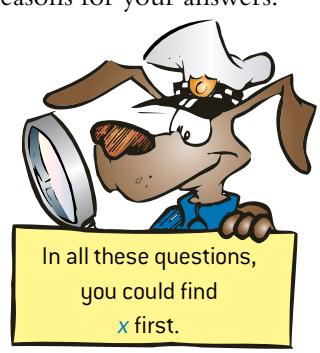
4 Find the values of the pronumerals in each of the following. Give reasons for your answers.

a

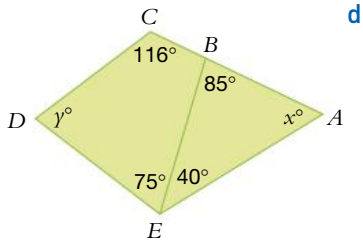


b

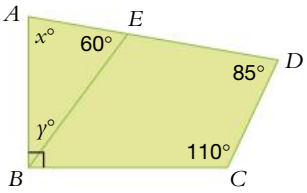




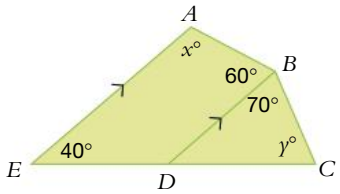
c



d



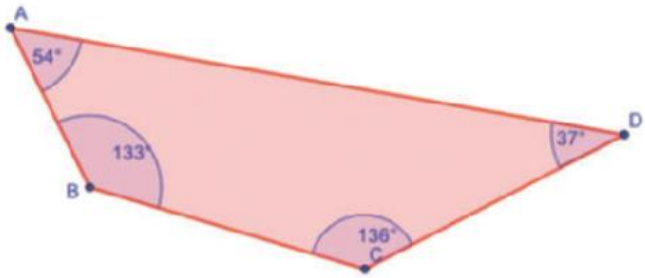
e



P GEOGEBRA ACTIVITY 15:05 ANGLE SUM OF A QUADRILATERAL

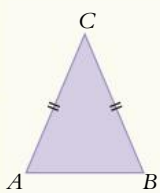
1 **Angle sum of a quadrilateral**
Change the shape of the quadrilateral to investigate the angle sum of a quadrilateral.

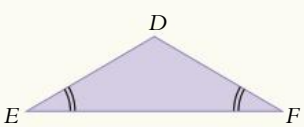
2 **The parallelogram family**
Change a parallelogram into a rectangle, rhombus or square to investigate how the sides, angles and diagonals change as the shape changes.

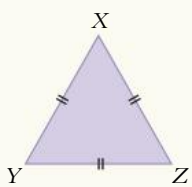


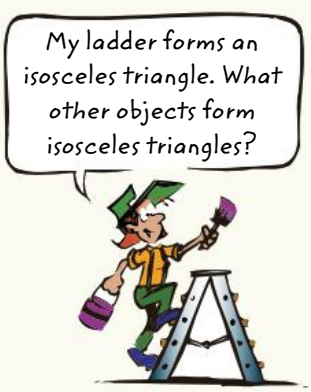
15:06 Isosceles and equilateral triangles

PREP QUIZ 15:06









1 Which of the triangles above are isosceles?
2 Which of the triangles above is equilateral?
3 In ΔABC, which angles are equal?
4 In ΔDEF, which sides are equal?

Complete the following:

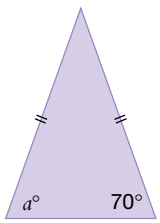
5 If a triangle has two sides equal, then it has ____ _____ _____.
6 If a triangle has two angles equal, then it has ____ _____ _____.
7 In ΔXYZ all angles are _____.
8 If a triangle has three equal angles, the sides are ____ _____.
9 If a triangle has three sides equal, then its angles are ____ _____.
10 If a triangle has three equal sides, then each of its angles will measure ____.

This Prep quiz should have reminded you of some properties of isosceles and equilateral triangles.

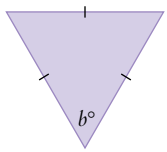
WORKED EXAMPLES

Find the values of the pronumerals in each of the following, giving reasons for your answers.

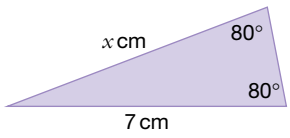
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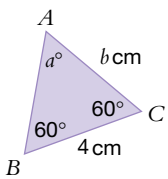
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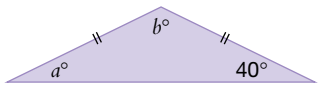
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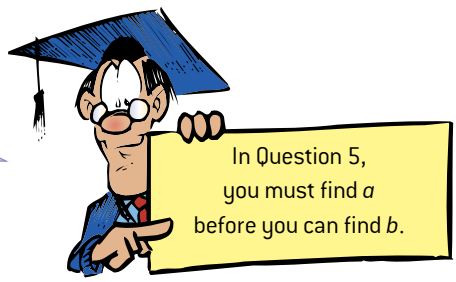


4



5





Solutions

1 $a = 70$ (equal angles of isosceles triangle)
(equal ∠s of isos. Δ)

2 $b = 60$ (angle of an equilateral triangle)
(∠ of equal. Δ)

3 $x = 7$ (equal sides of isosceles triangle)
(equal sides of isos. Δ)

4 $a + 60 + 60 = 180$ (angle sum of a triangle)
$\therefore a = 60$ (∠ sum of Δ)
$\therefore \Delta ABC$ is equilateral
$\therefore b = 4$ (sides of equilateral triangle)
(sides of equil. Δ)

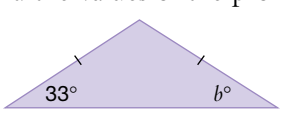
5 $a = 40$ (equal ∠s of isos. Δ)
$a + b + 40 = 180$ (∠ sum of Δ)
$40 + b + 40 = 180$
$\therefore b = 100$



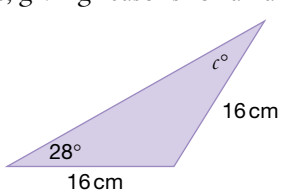
Exercise 15:06

1 Find the values of the pronumerals, giving reasons for all answers.

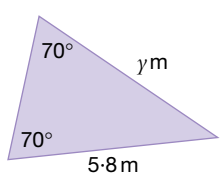
a



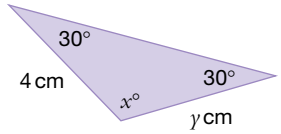
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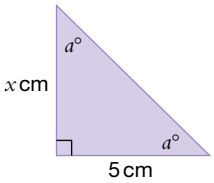
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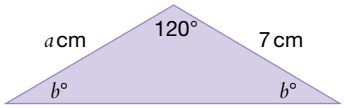
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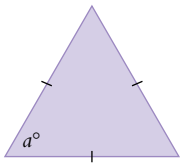
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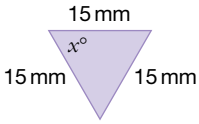
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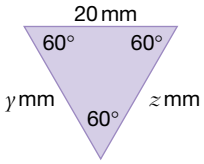
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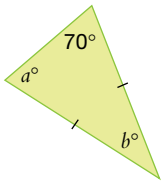


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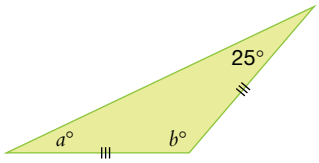


2 Find the value of a, then b, in each of the following. Give reasons for your answers.

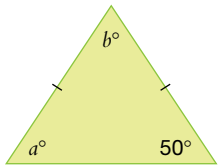
a



b

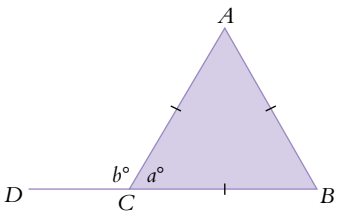


c



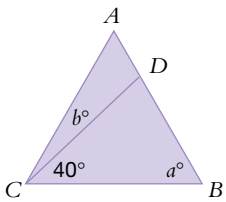
3 Find the value of a, then b, in each of the following. Give reasons for your answers.

a



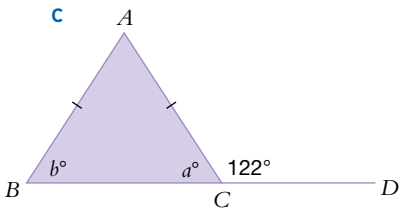
ΔABC is equilateral.

b

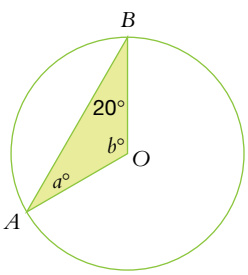


ΔABC is equilateral.

c

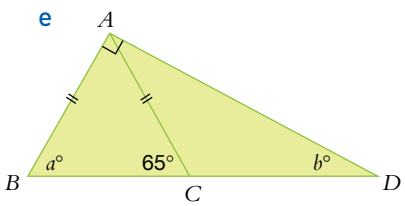


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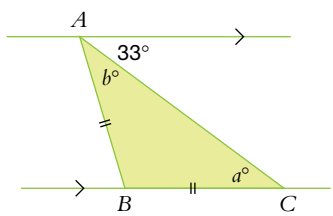


O is the centre of the circle.

e



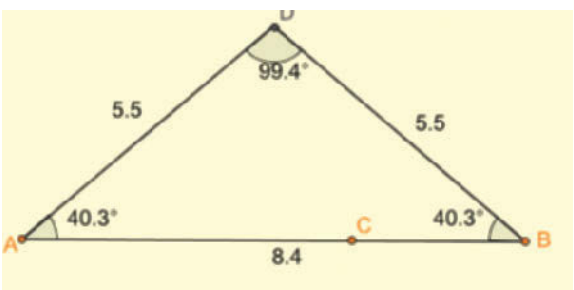
f



P GEOGEBRA ACTIVITY 15:06 INVESTIGATING TRIANGLES

1 **Side and angle properties of triangles**
Change the positions of the vertices of the triangles to investigate their side and angle properties.

2 **Isosceles and equilateral triangles**
Change the positions of the vertices to explore the properties of isosceles and equilateral triangles.

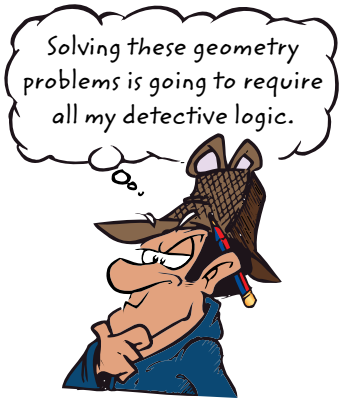


15:07 More involved numerical problems

Many problems involve more than one step of reasoning. More than one geometrical fact may be used, or the same fact may be used in different situations.

When solving these problems, we usually work backwards, asking questions such as:

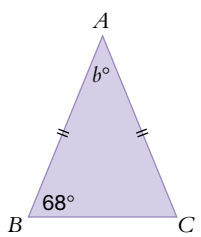
- What am I trying to find?
- What information do I need to know to find it?
- Can I find this other information by relating to it what I already know?



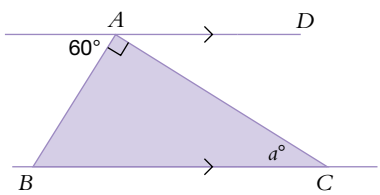
WORKED EXAMPLES

Find the values of the pronumerals, giving reasons for your answers.

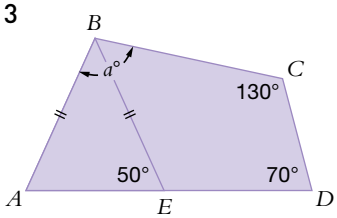
1



2



3



Solutions

1 $\angle ACB = 68°$ (equal $\angle$s of isos. Δ)

$\therefore\ b + 68 + 68 = 180$ (angle sum of Δ)

$\therefore\ b = 44$

2 $\angle ABC = 60°$ (alt. $\angle$s, $AD \parallel BC$)

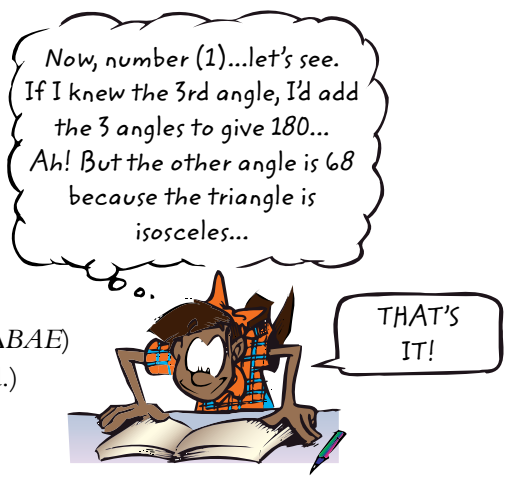
$\therefore\ a + 60 + 90 = 180$ (angle sum of Δ)

$\therefore\ a = 30$

3 $\angle BAE = 50°$ (equal $\angle$s of isos. ΔBAE)

$\therefore\ a + 130 + 70 + 50 = 360$ (angle sum of quad.)

$\therefore\ a = 110$

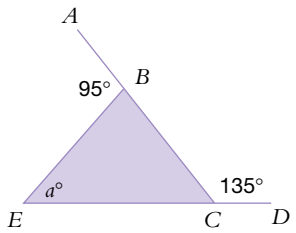


Exercise 15:07

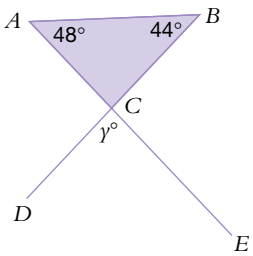
P Foundation worksheet 15:07
Finding the angles
Challenge worksheet 15:07
The logical prisoner

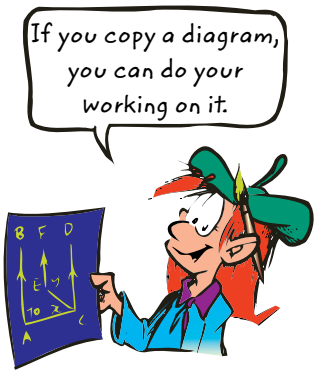
1 Find the value of the pronumeral in each of the following. Give reasons for your answers.

a

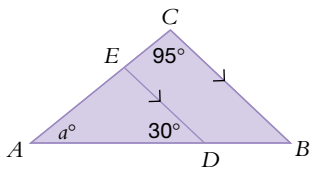


b

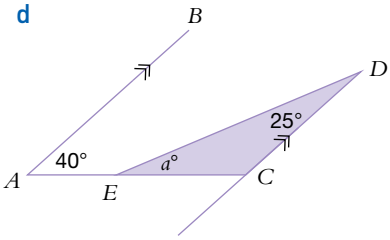




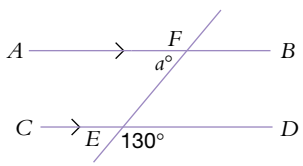
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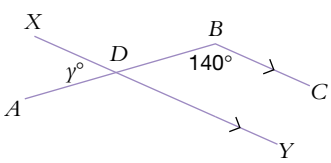
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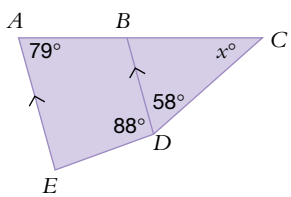
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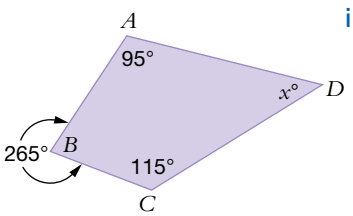
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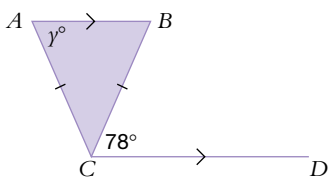
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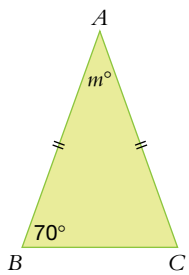


i

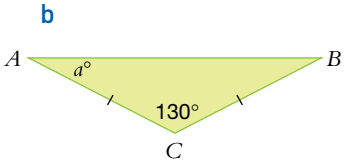


2 Find the values of the pronumerals in each of the following. Give reasons for your answers.

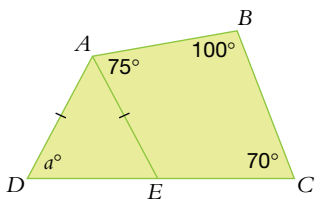
a



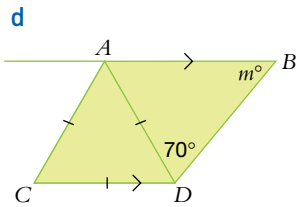
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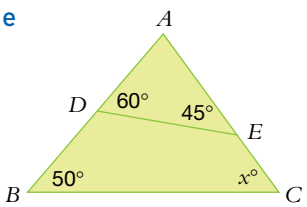
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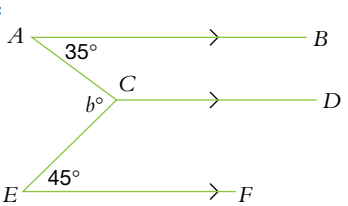
d



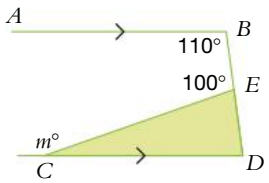
e



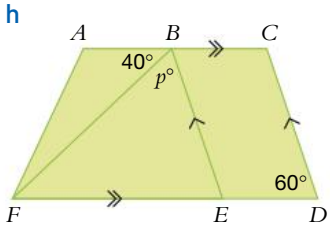
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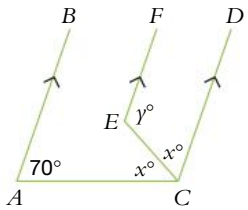
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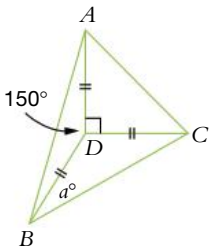


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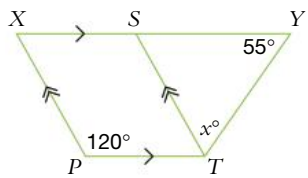


3 Find the value of the pronumeral in each of the following. Give reasons for your answers.

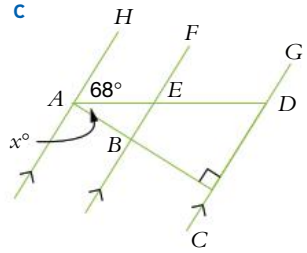
a



b

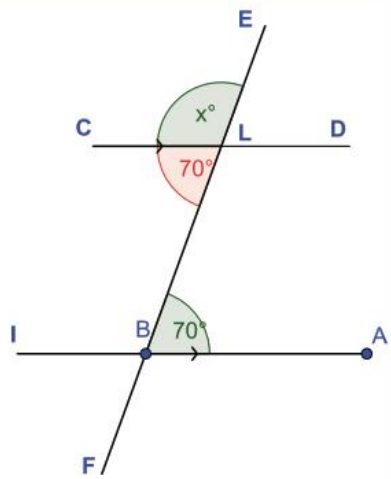


c



P GEOGEBRA ACTIVITY 15:07 GEOMETRIC REASONING

These activities look at the different ways of writing simple proofs involving parallel lines, and the angle sums of a triangle and quadrilateral.



CHALLENGE 15:07 CAN YOU EXPLAIN WHY?

A real test of your knowledge is to explain something in your own words.

For each part, read the explanation in this chapter and then try to explain it in your own words.

1 Why do angles at a point have a sum of 360°?

2 Why are vertically opposite angles equal?

3 Why are the opposite angles of a parallelogram equal?

4 Why is the angle sum of a triangle 180°?

5 Why is the exterior angle of a triangle equal to the sum of the two remote interior angles?

6 Why is the angle sum of a quadrilateral 360°.

FUN SPOT 15:07 NUMBERED BOXES

This is a variation on the well-known game of Boxes.

Construct a lattice of dots as shown below and scatter numbers from 1 to 5 throughout the lattice.



2	2	3	1	2	5	1	4	2	3
3	5	2	4	5	4	2	3	5	4
5	3	1	4	1	5	1	4	3	1
3	2	4	1	2	3	5	4	2	2
4	1	5	2	3	4	5	1	5	3

Players then take it in turns to join two dots together. If, when joining two dots, a square is completed, that player scores the points in that square and has another turn.

The game continues until all the squares, or boxes, have been formed. The points are then totalled and the player with the highest score wins.



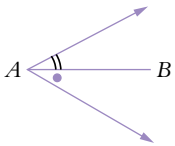
2	–1	3	2
1	–5	4	–3
–4	2	1	3



MATHS TERMS 15

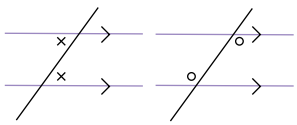
adjacent angles

- angles that have the same vertex and a common arm. They lie on opposite sides of the common arm



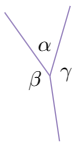
alternate angles

- angles that lie 'inside' the parallel lines and on opposite sides of the transversal
- they are equal if the lines are parallel



angles at a point

- the sum of all angles at a point is 360°
  $\alpha + \beta + \gamma = 360°$

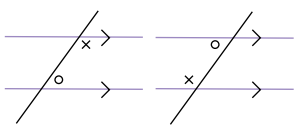


angle sum

- angle sum of a triangle is 180°
- angle sum of a quadrilateral is 360°

co-interior angles

- angles that lie 'inside' the parallel lines and on the same side of the transversal
- they are supplementary if the lines are parallel



complement

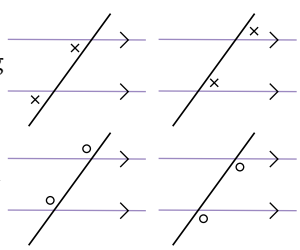
- the angle that needs to be added to a given angle to give 90°, e.g. the complement of 60° is 30°

complementary angles

- two angles that add up to 90°

corresponding angles

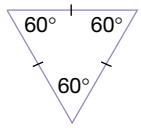
- angles that are in corresponding or matching positions relative to the transversal and a parallel line



- they are equal if the lines are parallel
- there are four pairs of corresponding angles in the diagrams above

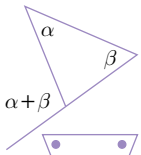
equilateral triangle

- all sides are equal
- all angles are 60°



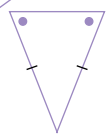
exterior angle of a triangle

- it equals the sum of the two interior remote angles



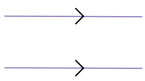
isosceles triangle

- two sides are equal
- base angles are equal



parallel lines

- straight lines, in the same plane, that do not meet
- lines are parallell if they have:
  - a pair of equal corresponding angles
  - a pair of equal alternate angles
  - a pair of co-interior angles that are supplementary



supplement

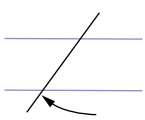
- the angle that needs to be added to a given angle to give 180°, e.g. the supplement of 120° is 60°

supplementary angles

- two angles that add up to 180°

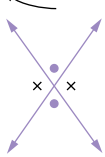
transversal

- a line that crosses two or more other lines



vertically opposite angles

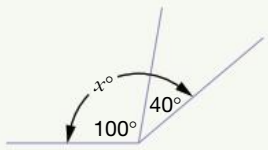
- these are equal angles, formed when two straight lines intersect

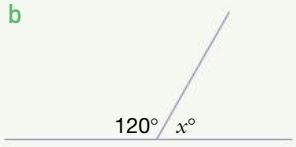


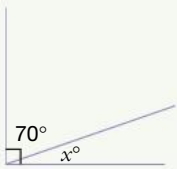
DIAGNOSTIC TEST 15 REASONING IN GEOMETRY

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

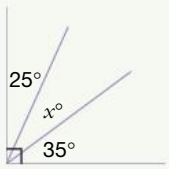
1 Find the value of the pronumeral in each of the following. 15:01

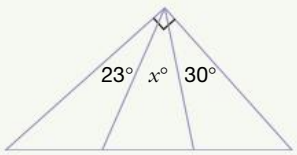
a 

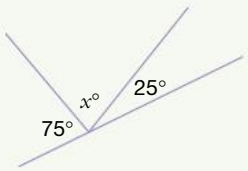
b 

c 

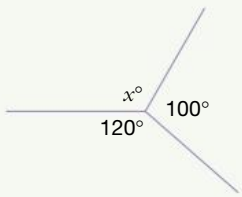
2 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:01

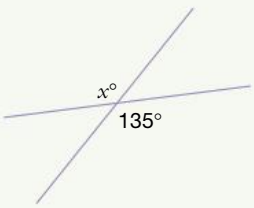
a 

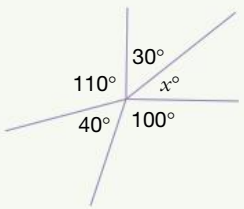
b 

c 

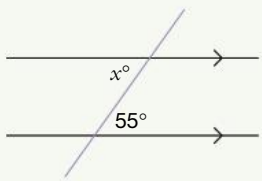
3 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:02

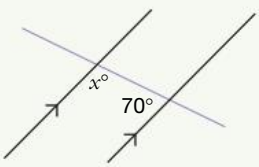
a 

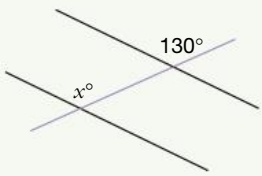
b 

c 

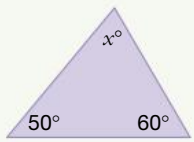
4 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:03

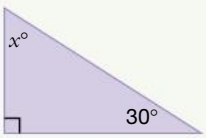
a 

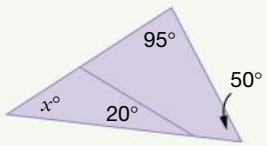
b 

c 

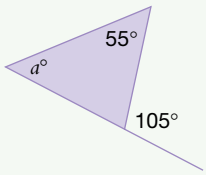
5 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:04

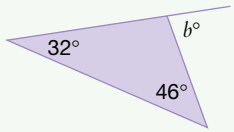
a 

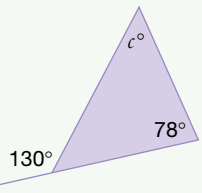
b 

c 

6 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:04

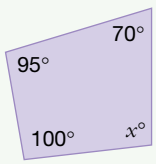
a 

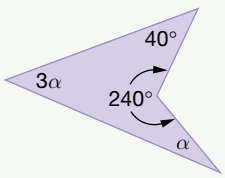
b 

c 

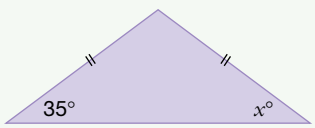
7 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:05

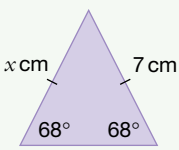
a 

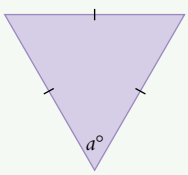
b 

c 

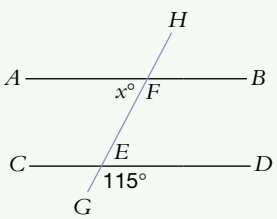
8 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:06

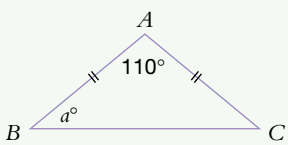
a 

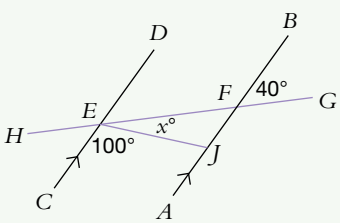
b 

c 

9 Find the value of the pronumeral in each of the following, giving reasons for your answers. 15:07

a 

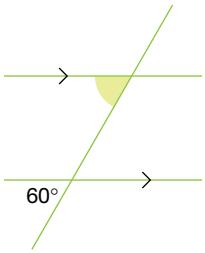
b 

c 

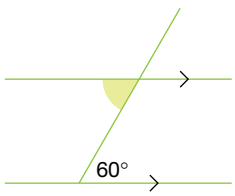
ASSIGNMENT 15A Chapter review

1 In each of the following diagrams, the marked angle is equal to 60°. Give the reason for this.

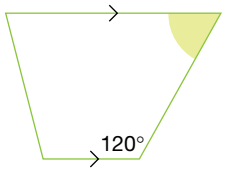
a



b



c



2 **a** A quadrilateral has three angles that measure 60°, 90° and 120°. What is the size of the fourth angle?

b A quadrilateral has two equal angles. The other two angles measure 70° and 30°. What is the size of the equal angles?

c An isosceles triangle has at least one angle of 30°. What are the sizes of the other two angles?

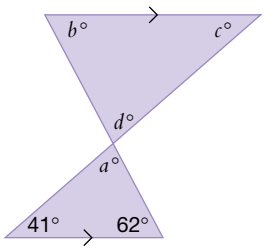
3 From the diagram, write the reason for each of the following statements.

a $b = 62$

b $c = 41$

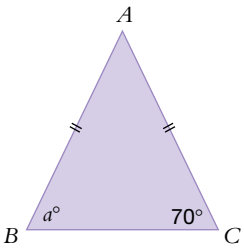
c $a + 41 + 62 = 180$

d $d = a$

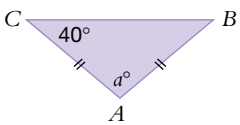


4 Find the value of the pronumeral in each of the following. Give a reason for each answer.

a

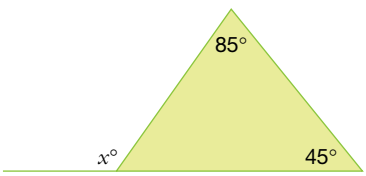


b

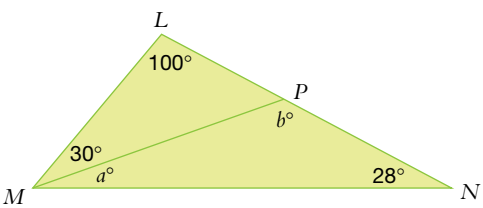


5 Find the values of the pronumerals in each of the following. Give a reason for each answer.

a



b



ASSIGNMENT 15B Working mathematically

1 Lyn has collected 48 different stamps and I have collected 75 different stamps. If 34 of Lyn's stamps are the same as 34 of mine, how many different stamps have we collected in total?



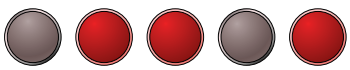
2 How many counting numbers between 16 and 94 are even?

3 Five people meet and shake hands with one another. How many handshakes are necessary if each person shakes hands with every other person only once?



4 In order to play mixed doubles at tennis last week, we had to choose two out of three girls and two out of four boys. How many different groups of four could be chosen to play?

5 A machine can give a signal by using five lights in a row. Each light can be either on or off. How many different signals are possible?



6 Write down the value of 3^1, 3^2, 3^3, 3^4, 3^5 and 3^6. By examining the answers, find the last digit of 3^{2000}.

7



a At half past 10 what is the angle between the hands of a clock?

b What would be the angle between the hands 4 hours later?



Estimate the total number of people that could stand inside the trunk of this tree.

ASSIGNMENT 15C Cumulative revision

1 Simplify:

a $2x \times 3x$ **b** $4ab \times 3ab$ **c** $12a \div 4a$ **d** $16ab \div 8a$

5:03, 5:04

2 Factorise:

a $4a - 16$ **b** $a^2 - 6a$ **c** $2m^2 - 12m$ **d** $12ab - 6a$

5:09

3 Use a calculator where necessary to evaluate the following, correct to one decimal place.

a $\sqrt{256 + 289}$ **b** $\sqrt{256} + \sqrt{289}$ **c** $\sqrt{256 \times 25}$ **d** $\sqrt{256} \times \sqrt{25}$

2:02

4 **a** Michelle earns \$320 per week. She receives a wage rise of \$14.40 per week. Express this rise as a percentage of her wage.

b Roberto invests \$2000 and at the end of a year he receives an interest payment of \$150. What is the percentage interest rate of his investment?

4:04

5 If I take three cards at random from a standard pack, without replacement, the probability that I would select three picture cards is $\frac{11}{1105}$. What is the probability that I will not select three picture cards?

6:03

6 In our classroom there are 3 men, 2 women, 15 girls and 10 boys. If one person is selected at random, what is the probability that the person is:

a a woman **b** a boy **c** a female **d** a baby **e** a human being?

6:02

7 Use Pythagoras' theorem to find the perpendicular height (as a surd) of an equilateral triangle with a side length of 4 cm. What is the area of the triangle, correct to one decimal place?

3:03, 3:04

8 A cylindrical tank has a diameter of 2·4 m and a height of 8·6 m. Calculate the:

a volume in cubic metres, correct to one decimal place

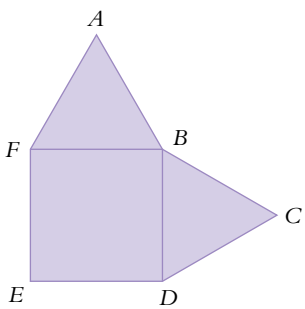
b the capacity in litres, correct to the nearest 100 litres.



12:05

9 A square and two equilateral triangles are joined as shown in the diagram. Find the size of the obtuse angles:

a $\angle AFE$ **b** $\angle ABC$



15:07

10 If in Question **9** a line were drawn from A to C, what would be the size of $\angle BAC$?

15:07

ANSWERS

ID Cards

ID Card 1 (Metric units)

1 metres	**2** decimetres	**3** centimetres	**4** millimetres
5 kilometres	**6** square metres	**7** square centimetres	**8** square kilometres
9 hectares	**10** cubic metres	**11** cubic centimetres	**12** seconds
13 minutes	**14** hours	**15** metres per second	**16** kilometres per hour
17 grams	**18** milligrams	**19** kilograms	**20** tonnes
21 litres	**22** millilitres	**23** kilolitres	**24** degrees Celsius

ID Card 2 (Symbols)

1 add (or plus)	**2** subtract (or minus)	**3** multiplied by (or times)
4 divided by	**5** is equal to	**6** is approximately equal to
7 is not equal to	**8** is less than	**9** is less than or equal to
10 is not less than	**11** is greater than	**12** is greater than or equal to
13 is not greater than	**14** is not greater than or equal to	**15** 4 squared
16 4 cubed	**17** the square root of 25	**18** the cube root of 27
19 per cent	**20** therefore	**21** for example
22 that is	**23** numerator	**24** denominator

ID Card 3 (Language)

1 $6-2=4$	**2** $6+2=8$	**3** $6\div 2=3$	**4** $6-2=4$
5 $6\div 2=3$	**6** 2	**7** 6	**8** $6\times 2=12$
9 $6-2=4$	**10** $6\times 2=12$	**11** $2+6=8$	**12** $6-2=4$
13 $6^2=36$	**14** $\sqrt{36}=6$	**15** $6-2=4$	**16** $6\times 2=12$
17 $(6+2)\div 2=4$	**18** $6+2=8$	**19** $6^2=36$	**20** $6-2=4$
21 $6-2=4$	**22** $6+2=8$	**23** $6\div 2=3$	**24** $6+2=8$

ID Card 4 (Language)

1 square	**2** rectangle	**3** parallelogram	**4** rhombus (or diamond)
5 trapezium	**6** regular pentagon	**7** regular hexagon	**8** regular octagon
9 kite	**10** scalene triangle	**11** isosceles triangle	**12** equilateral triangle
13 circle	**14** oval (or ellipse)	**15** cube	**16** rectangular prism
17 triangular prism	**18** square pyramid	**19** rectangular pyramid	**20** triangular pyramid
21 cylinder	**22** cone	**23** sphere	**24** hemisphere

ID Card 5 (Language)

1 point A	**2** interval AB	**3** line AB
4 ray AB	**5** collinear points	**6** midpoint
7 number line	**8** diagonals	**9** acute-angled triangle
10 right-angled triangle	**11** obtuse-angled triangle	**12** vertices
13 ΔABC	**14** hypotenuse	**15** $180°$
16 $(a+b)°$	**17** $360°$	**18** $a°=b°$
19 $a°=60°$	**20** $3\times 180°=540°$	**21** AB is a diameter, OC is a radius
22 circumference	**23** semicircle	**24** AB is a tangent, CD is an arc, EF is a chord

ID Card 6 (Language)

1 parallel lines
2 perpendicular lines
3 vertical, horizontal
4 concurrent lines
5 angle ABC or CBA
6 acute angle
7 right angle
8 obtuse angle
9 straight angle
10 reflex angle
11 revolution
12 adjacent angles
13 complementary angles
14 supplementary angles
15 vertically opposite angles
16 360°
17 transversal
18 corresponding angles
19 alternate angles
20 co-interior angles
21 bisecting an interval
22 bisecting an angle
23 $\angle CAD = 60°$
24 CD is perpendicular to AB

ID Card 7 (Language)

1 Common Era, Anno Domini
2 Before Common Era, Before Christ
3 ante meridiem (before midday)
4 post meridiem (after midday)
5 hectare
6 regular shapes
7 net of a cube
8 cross-section
9 face
10 vertex
11 edge
12 axes of symmetry
13 reflection (or flip)
14 translation (or slide)
15 rotation (or turn)
16 tessellation
17 coordinates
18 tally
19 picture graph
20 column graph
21 line graph
22 sector (or pie) graph
23 bar graph
24 scatter diagram

Chapter 1

Exercise 1:01

1 a Answers will vary.

b i 64 000 (*Note*: There are 5 people on the assembly line.)

ii Estimates should be about 1·5 km.
Method: Since 0·6 cm on the picture represents 20 cm, the scale is $1 : 33\frac{1}{3}$.
To fill 5 boxes the belt moves $8{\cdot}5 \times 33\frac{1}{3}$ cm (i.e. $283\frac{1}{3}$ cm). The average belt movement for one box is $283\frac{1}{3} \div 5$ cm (i.e. $56\frac{2}{3}$ cm).
So to fill 2560 boxes, the belt moves $2560 \times 56\frac{2}{3}$ cm (i.e. $145\,066\frac{2}{3}$ cm). This is about 1·45 km.

c i Your answer might include the efficient working of the machinery, the speed of the packing at the end of the line, ensuring that there are sufficient chocolates to pack in boxes, ensuring sufficient workers etc.

ii Your answer might include maintenance of machinery, sufficient supply of boxes, adequate working conditions and pay for employees, what to do with large cartons when packed etc.

iii Answers will vary.

2 a i Answers will vary, but note that everything is done to ensure the sheep are in the right place ready to be drenched when the drencher is ready.

ii Answers will vary, but might include that the sheep and Alan have been organised in advance, how the sheep were brought to the pen, and that Alan has arrived with adequate drench for all sheep.

iii Sample questions:
How many sheep do you have?
How much drench is required?
How long do you estimate the drenching will take?
How much does Alan charge per hour?
How much does the drench cost?

b i 360 ii 13, 787 iii \$260, $216\frac{2}{3}$%
iv 3 truckloads, \$44 574

c i Answers will vary; however, using this equipment you can only drench one sheep at a time so not much time can be saved.

ii Answers will vary; however, the person drenching sheep and those herding the sheep should stand so their shadows don't fall on the path of the sheep.

iii Answers will vary. A different way could be: instead of using the machine at the end of the process to feed one sheep at a time past the drencher, the sheep could be crowded into the last stall and the drencher could move from sheep to sheep.

iv Answers will vary in terms of *costs*, but will include the purchase of the lambs, feed and water, drenching, shearing, and farm maintenance, such as pens and fences.

Exercise 1:02

1 **a** 12 **b** C **c** F1
d **i** 10 **ii** 24
e 13 (if we include the driver) **f** 308
g 3 **h** 28

2 **a** $1152 **b** 89 cm **c** at least 45 cm
d Collison—yellow, Foster—cream, Bills—blue
e 5 lions and 5 tigers *or* 8 lions and 3 tigers **f** 7
g Mr French—Craft, Mr English—French, Mr Kraft—Music, Mr Musik—English
h 5 horses **i** 18

Exercise 1:03

1 **a** 60 **b** 40 **c** 34 **d** 118
e 1788 **f** 1988 **g** 5321 **h** 1615

2 **a** DCXXX **b** DCCCXLVII
c MCCCVIII **d** MMMCCXL
e CCCXC **f** CXCIX **g** $\overline{\text{X}}$
h MDCCLXXIII

3 **a** 6 090 000 **b** 140 600 **c** 84 705
d 74 398

4 **a** $5 \times 5; 25$ **b** $10 \times 10 \times 10 \times 10; 10\,000$
c $2 \times 2 \times 2; 8$ **d** $2 \times 2 \times 2 \times 2 \times 2; 32$

5 6^4

6 **a** 80 000 **b** 6000 **c** 900 000 **d** 200

7 **a** 1100 **b** 20 000 **c** 30 000 **d** 500
e 800 **f** 20 000

Exercise 1:04

1 **a** 32 **b** 0 **c** 8 **d** 60
e 30 **f** 47

2 **a** 347 **b** 0 **c** 36 **d** 3842
e 30 406 **f** 0

3 **a** true **b** true **c** true **d** true
e true **f** true **g** true **h** true

4 **a** {1, 2, 3, 4} **b** {0, 2, 4, 6}
c {16, 17, 19, 20, 22, 23} **d** {6·5, 7·5, 8·5}
e {0·1, 0·3, 0·4, 0·5, 0·8} **f** $\{\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}\}$

5 **a** true **b** true **c** true **d** true
e true **f** false **g** true **h** true

6 **a** 0, 3, 4, 6, 11, 16, 19, 20
b 3, 4, 6, 11, 16, 19, 20 **c** 4, 6, 16, 20
d 3, 11, 19 **e** 4, 16 **f** 3, 6

7 **a** {1, 12, 2, 6, 3, 4}
b {1, 102, 2, 51, 3, 34, 6, 17}
c {1, 64, 2, 32, 4, 16, 8}
d {1, 140, 2, 70, 4, 35, 5, 28, 7, 20, 10, 14}

8 **a** {7, 14, 21, 28} **b** {5, 10, 15, 20}
c {12, 24, 36, 48} **d** {13, 26, 39, 52}

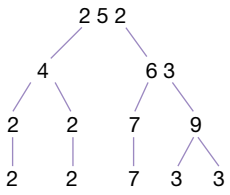
9 **a** 5 **b** 51 **c** 16 **d** 42

10 **a** 24 **b** 45 **c** 100 **d** 72

11 **a** 10^4 **b** 7^3 **c** 2^5 **d** 125
e 10 000 **f** 7000 **g** 200 000

12 **a** {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
b {32, 33, 34, 35, 36, 38, 39}

13 **a**



$252 = 2^2 \times 7 \times 3^2$
b $2 \times 2 \times 2 \times 2 \times 5 \times 5$
c $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$ **d** 40 **e** 10 800

14 **a** 2002 **b** 2001 **c** 2004 **d** 2005
e 2004 **f** 2008 **g** 2007 **h** 2010
i 2002 **j** 2025 **k** 2100 **l** 2004

15 **a** 15 **b** 8 **c** 13 **d** 4

Exercise 1:05

1 **a** 5 **b** $2\frac{1}{4}$ **c** $8\frac{7}{10}$ **d** $1\frac{3}{8}$

2 **a** $\frac{7}{2}$ **b** $\frac{53}{10}$ **c** $\frac{7}{4}$ **d** $\frac{22}{7}$

3 **a** $\frac{4}{5}$ **b** $\frac{2}{5}$ **c** $\frac{3}{20}$ **d** $\frac{3}{4}$

4 **a** $\boxed{\frac{4}{10}}$ **b** $\boxed{\frac{75}{100}}$ **c** $\boxed{\frac{40}{10}}$ **d** $\boxed{\frac{40}{120}}$

5 **a** $\frac{7}{10}$ **b** $\frac{13}{100}$ **c** $\frac{4}{12}$ or $\frac{1}{3}$ **d** $\frac{7}{24}$

6 **a** $\frac{9}{10}$ **b** $\frac{3}{20}$ **c** $1\frac{3}{20}$ **d** $\frac{11}{100}$

7 **a** $\frac{6}{10}$ **b** $\frac{2}{5}$ **c** $\{\frac{3}{4}, \frac{8}{10}, \frac{17}{20}\}$
d $\{\frac{2}{3}, \frac{3}{5}, \frac{1}{2}, \frac{1}{4}\}$

8 **a** $5\frac{3}{20}$ **b** $7\frac{3}{8}$ **c** $2\frac{2}{5}$ **d** $8\frac{7}{10}$

9 **a** $\frac{3}{10}$ **b** $\frac{21}{100}$ **c** $\frac{2}{5}$ **d** $\frac{27}{32}$

10 **a** $2\frac{2}{5}$ **b** $5\frac{5}{8}$ **c** 5 **d** $4\frac{1}{2}$

11 **a** $1\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $6\frac{2}{3}$ **d** $\frac{2}{5}$

12 **a** 1200 m or $1\frac{1}{5}$ km **b** $\frac{1}{5}$

13 **a** 2 : 3 **b** 3 : 2 **c** $\frac{2}{5}$

Exercise 1:06

1 **a** 17·537 **b** 684·02

2 **a** $\frac{7}{10}$ **b** $2\frac{13}{100}$ **c** $\frac{9}{1000}$ **d** $5\frac{3}{10}$
e $\frac{17}{20}$ **f** $\frac{1}{40}$ **g** $1\frac{4}{5}$ **h** $9\frac{1}{25}$

3 **a** 0·9 **b** 0·13 **c** 1·5 **d** 2·99
e 0·6 **f** 0·165 **g** 0·625 **h** $0.2\dot{7}$

4 **a** {0·3, 0·303, 0·33} **b** {0·5, 2, 3·1}
c {0·055, 0·505, 5·5}

5 **a** 5·22 **b** 61·35 **c** 9·625 **d** 6·375

6 **a** 0·003 **b** 4·202 **c** 0·0025 **d** 24·83

7 **a** 60 **b** 0·75 **c** 0·816 **d** 0·0045

8 **a** 16·3 **b** 0·3 **c** 1·038 **d** 0·030 75
e 19 **f** 20·34 **g** 17 500 **h** 3·01

9 **a** $365.42 **b** $58.37 **c** $66.96 **d** $11.25

10 **a** 97 000 000 **b** 0·09
c 86·1 **d** 0·67

Exercise 1:07

1 **a** $\frac{9}{100}$ **b** $\frac{16}{25}$ **c** $1\frac{1}{4}$ **d** $\frac{29}{200}$

2 **a** 75% **b** $137\frac{1}{2}\%$ **c** $12\frac{1}{3}\%$ **d** 460%

3 **a** 0·47 **b** 0·04 **c** 3·25 **d** 3
e 0·5 **f** 1·04 **g** 0·127 **h** 0·003

4 **a** 87% **b** 130% **c** 500% **d** 82·5%

5 **a** 44·8 L **b** 476 g **c** \$40 **d** 1·79 m
e 75% **f** 25%

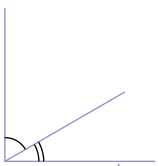
Exercise 1:08

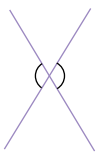
1 **a** $\angle ABC$ **b** $\angle SBE, \angle DBT$
c $\angle MPN, \angle PNQ$ **d** reflex $\angle BAD$

2 **a** 15° **b** 148°

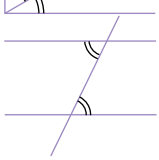
3 **a** revolution **b** acute angle **c** straight angle
d reflex angle **e** right angle **f** obtuse angle
g reflex angle **h** acute angle

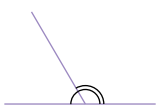
4 **a** **b**





c **d**





5 **a** 32 **b** 133 **c** 240 **d** 105
e 73 **f** 60 **g** 68 **h** 122

Exercise 1:09

1 **a** regular hexagon **b** 6 **c** 6 **d** 6
e 9

2 **a** equilateral, acute-angled; $a = 60$
b isosceles, acute-angled; $b = 40$
c scalene, obtuse-angled; $c = 30$
d scalene, right-angled; $d = 35$
e isosceles, right-angled; $e = 45$
f isosceles, obtuse-angled; $f = 40$
g scalene, acute-angled; $g = 65$
h isosceles, acute-angled; $h = 50$

3 **a** $a = 90$ **b** $b = 90$ **c** $c = 120$ **d** $d = 60$

4 **a** A: rectangle, B: square, C: rhombus, D: trapezium
b **i** A, B, C **ii** B, C **iii** A, B, C
iv D **v** B, C

Exercise 1:10

1 **a** A: rectangular prism, B: triangular prism, C: cylinder, D: cone, E: square pyramid
b C, D
c **i** $F = 5$ **ii** $V = 6$ **iii** $E = 9$
iv $E + 2 = 11$ **v** $F + V = 11$

2 **a** square pyramid **b** cone
c pentagonal prism

3 **a** **b** **c**







Exercise 1:11

1 **a** 0·8 cm **b** 2·2 cm **c** 4·5 cm **d** 8 cm
e 11·8 cm

2 **a** 300 cm **b** 2·5 L **c** 0·63 g **d** 7000 m
e 7800 g **f** 250 cm

3 7 mm

4 **a** 32 m **b** 32·8 m **c** 20 cm

5 **a** 25 minutes past 1; 1:25 **b** Half past 10; 10:30
c 10 minutes to 7; 6:50 **d** A quarter to 5; 4:45

6 **a** 100 m **b** 3 m/s **c** 4 hours

7 **a** $13{\cdot}5\text{ cm}^2$ **b** 4500 cm^2 **c** 49 mm^2
d 24 m^2 **e** $17{\cdot}5\text{ cm}^2$ **f** 30 cm^2

8 **a** 15 cm^3 **b** 42 cm^3 **c** 30 cm^3
d 1980 m^3

9 **a** B, 5 mL **b** C, 1 L **c** B, 1 cm^2
d B, 50 g **e** A, 20 kg

10 **a** 100 g **b** 970 g

Exercise 1:12

1 −3, 4, 0, −10

2 **a** 4 **b** 12 **c** −7 **d** −20 **e** −11
f −3 **g** −12 **h** −13 **i** 16 **j** 15
k 10 **l** 16 **m** 16 **n** 18 **o** 7
p 24

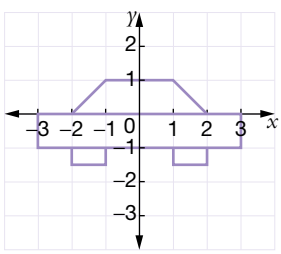
3 **a** 12 **b** 16 **c** 0·6 **d** 1·5 **e** −56
f −40 **g** −7·7 **h** −72 **i** 7 **j** 4
k −10 **l** −2 **m** 7 **n** 6 **o** −8
p −0·9

4 **a** 9 **b** −20 **c** −10 **d** −10 **e** −26
f 0 **g** −6 **h** 10 **i** −\$11.50 **j** −15°

Exercise 1:13

1 $A(2,1)$, $B(1,3)$, $C(3,0)$, $D(0,2)$, $E(3,-2)$, $F(0,-3)$, $G(-3,2)$, $H(-1,0)$, $I(-3,-1)$, $J(-2,-3)$

2



Exercise 1:14

1 **a** $m = 4s$

s	1	2	3	4	10	20	30	100
m	4	8	12	16	40	80	120	400

b $m = 3s + 1$

s	1	2	3	4	10	20	30	100
m	4	7	10	13	31	61	91	301

2 **a** $5h + 2$ **b** $a + 3y$ **c** $6(a + 7)$ **d** $\frac{5a}{7}$

3 **a** $3 \times a + 8$ **b** $5 \times p - 6 \times q$ **c** $4 \times (x + 2)$
d $(a + 7) \div 3$

4 **a** 18 **b** 16 **c** 45 **d** 1

5 **a** 41 **b** 4 **c** 24 **d** 29

6 201

7 **a** $y = 3x + 4$ **b** $y = 4x + 7$

8 **a** m **b** a **c** $4y$ **d** $4y$ **e** $5f$
f ab **g** $5k$ **h** $5ab$ **i** 0 **j** 0
k $6m$ **l** $3a$ **m** $12a$ **n** $11a$ **o** $6b$
p $50a$ **q** $-2m$ **r** $-2b$ **s** $7x^2$ **t** ab

9 **a** $15a$ **b** $60b$ **c** $21mp$ **d** $32xy$
e $4ab$ **f** $30m$ **g** $15k$ **h** $-18y$
i $4t$ **j** $10t$ **k** $\frac{3m}{a}$ **l** $\frac{2a}{b}$
m $\frac{3r}{2}$ **n** $\frac{4m}{3}$ **o** $21a^2b$ **p** $20ab^2$

10 **a** $2m$ **b** x **c** $5x + 9y$
d $10a + 3b$ **e** $10p + 3q$ **f** $8r + 5A$
g $4a + 12b$ **h** $2m + 4$ **i** $6m - 6a$
j $9a^2 - 4a$ **k** $2x^2 + 5x$ **l** $2x^2 + 5x + 3$

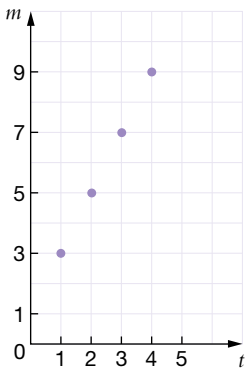
11 **a** $3a + 27$ **b** $5x + 10$ **c** $10m - 40$
d $18a - 27$ **e** $24t + 18$ **f** $10 + 20x$
g $m^2 + 7m$ **h** $a^2 - 3a$ **i** $6a + a^2$

12 **a** a^4 **b** p^7 **c** b^2 **d** m^3b^5 **e** $35t^3$

13 **a** $x = 4$ **b** $x = 24$ **c** $a = 7$ **d** $a = 7$
e $m = 7$ **f** $m = 20$ **g** $m = -5$ **h** $m = -4$

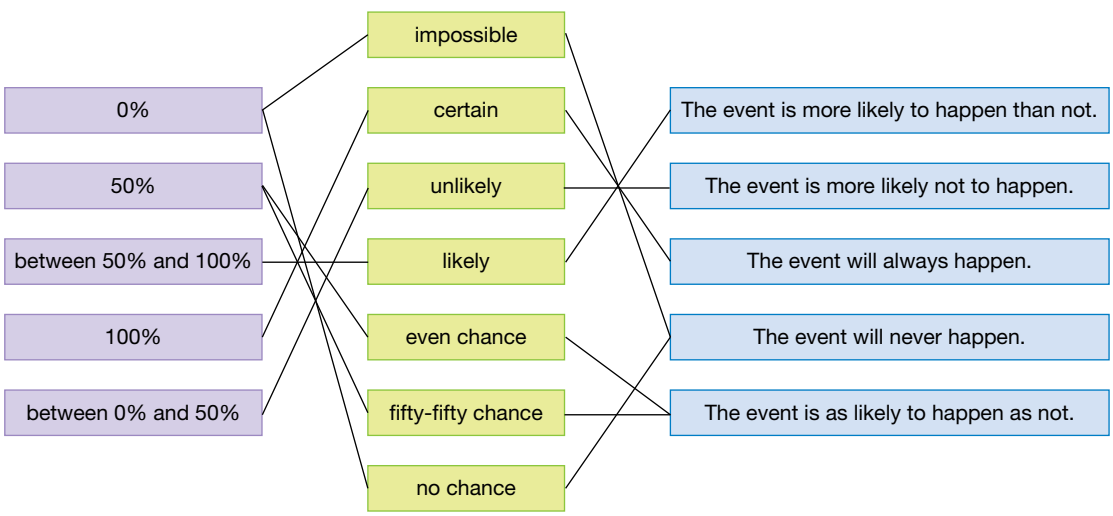
14 45, 46

15



Exercise 1:15

1



2 **a** sitting on the flat surface, lying on its side
b 1, 2, 3, 4, 5, 6

3 **a** $\frac{1}{6}$ **b** 0 **c** $\frac{1}{2}$

4 **a** 100% **b** 25% **c** 50% **d** 75%

5 **a** $\frac{1}{10}$ (or 10%) **b** $\frac{4}{10}$ (or $\frac{2}{5}$ or 40%)
c 0 (or 0%)

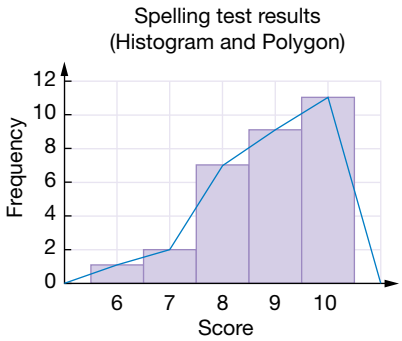
6 **a** P(two heads) = $\frac{6}{20}$ (or $\frac{3}{10}$ or 30%)
b P(a head and a tail) = $\frac{10}{20}$ (or $\frac{1}{2}$ or 50%)
c P(two tails) = $\frac{4}{20}$ (or $\frac{1}{5}$ or 20%)

Exercise 1:16

1

Score	Tally	Frequency
6	|	1
7	||	2
8	𝍸 ||	7
9	𝍸 ||||	9
10	𝍸 𝍸 |	11
	Total:	30

2



3 Range = 10 − 6 = 4, Mode = 10, Median = 9, Mean = 8·9

4 Yes, Range = 10 − 2 = 8, Mode = 7, Median = 7, Mean = 6·6,

5 **a** 41, 48, 50, 53, 54, 57, 59, 62 ,64, 65, 66, 66, 67, 68, 68, 71, 73, 73, 73, 75, 77, 79, 80, 84, 84, 85, 88, 91, 93, 97

b Range = 97 − 41 = 56

c Mode = 73, Median = $\frac{68+71}{2}$ = 69·5

ASSIGNMENT 1A

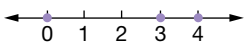
1 **a** 34 **b** 1640 **c** 2413 **d** 1992

2 3 × 3 × 3 × 3 = 81

3 900 000

4 **a** 3 **b** $2\frac{1}{2}$ **c** 44

5



6 **a** F **b** T **c** T

7 1, 2, 3, 4, 6, 8, 12, 24

8 **a** $\frac{2}{5}$ **b** 8 **c** $2\frac{1}{4}$ **d** $\frac{2}{5}$ **e** $\frac{21}{100}$ **f** 4

9 **a** T **b** T **c** T

10 **a** 0·09 **b** $\frac{3}{5}$ **c** 75% **d** 44·8 km

11 **a** acute **b** obtuse **c** obtuse **d** reflex **e** reflex

12 **a** 5000 **b** 250 **c** 5200 **d** 0·56 **e** 3·3

13 **a** 20 m^2 **b** 1428 mm^2 **c** 31·25 m^2

ASSIGNMENT1B

1 **a** hectare **b** net of a cube **c** cross-section **d** coordinates **e** tally **f** picture graph **g** column graph **h** line graph **i** sector (or pie) graph **j** bar graph

2 **a** 273, 274, 275 **b** 665, 666, 667 **c** 8283, 8284, 8285 **d** 6, 7, 8 **e** 12, 13, 14 **f** 24, 25, 26

3 **a** 14 **b** 56 **c** 2

4 **a** 241 days **b** 0·5 tonnes/hour **c** 1 442 335·9 tonnes

5 **a** **i** \$121.92 **ii** \$168.96 **iii** \$78.72 **iv** \$61.44

b Answers will vary.

c The top three reasons given were: holiday (23%), retirement 14%, home deposit 10%

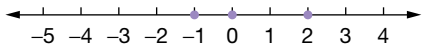
Chapter 2

Exercise 2:01

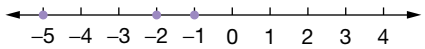
1 **a** −6 **b** −4 **c** −8 **d** −4 **e** −2 **f** −8 **g** −1 **h** −4 **i** −8 **j** −6 **k** −3 **l** −4

2 **a** \$8 **b** −\$10 **c** −\$180 **d** −\$1315 **e** \$10 **f** −\$80

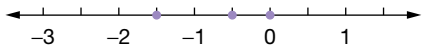
3 **a**



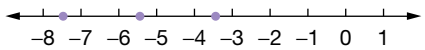
b



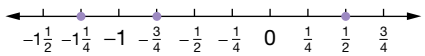
c



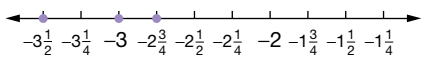
d



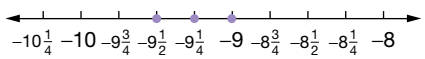
e



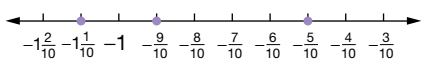
f



g



h



4 **a** 6 **b** 5 **c** 3 **d** 30 **e** −11 **f** −31 **g** −48 **h** −50 **i** −22 **j** −74 **k** −44 **l** −28 **m** −7 **n** −3 **o** −12 **p** −7 **q** 3 **r** 5 **s** 20 **t** 29 **u** −11 **v** −3 **w** −9 **x** −16

5 **a** −12 **b** 12 **c** −12 **d** 12 **e** −30 **f** −27 **g** −42 **h** −50 **i** −24 **j** −81 **k** −35 **l** −28 **m** 36 **n** 20 **o** 81 **p** 16 **q** −1 **r** −8 **s** 1 **t** 8 **u** 100 **v** −1000 **w** 10 000 **x** −100 000

6 **a** −4 **b** 4 **c** −4 **d** 4 **e** −3 **f** −4 **g** −7 **h** −4 **i** −5 **j** −4 **k** −5 **l** −7 **m** 1 **n** 7 **o** 3 **p** 4 **q** 6 **r** −6 **s** 8 **t** −8

7 **a** −3 **b** −2 **c** 4 **d** 0 **e** −12 **f** −1 **g** 0 **h** −70 **i** 5 **j** 5 **k** −3 **l** 10 **m** 8 **n** 167 **o** −96 **p** −54 **q** −1 **r** −27 **s** 0 **t** −14

Prep quiz 2:02

1 −2 **2** −10 **3** −8 **4** −9 **5** −7 **6** −5 **7** −24 **8** 12 **9** 16 **10** 4

Exercise 2:02

Question	Without calculator	With calculator
$-3 - 8$	-11	-11
$-9 - 2$	-11	-11
$-7 + 2$	-5	-5
$-15 + 6$	-9	-9
$-7 + 23$	16	16
$-2 + 15$	13	13
$-2 - -3$	1	1
$8 - -2$	10	10
-3×4	-12	-12
-6×3	-18	-18
-7×-2	14	14
-9×-5	45	45
4×-7	-28	-28
10×-6	-60	-60
$20 \div (-5)$	-4	-4
$-18 \div -3$	6	6

2 **a** -578 **b** -602 **c** 294 **d** -2275
e -135 **f** -2508 **g** -48 **h** -2093
i -2121
3 **a** 165 649 **b** 7·29 **c** 0·0169
4 **a** 71 **b** 229 **c** 59·24 **d** 1·802
e -59 **f** 1080
5 **a** 5 **b** 1 **c** 1 **d** 9 **e** 5 **f** 1·5

Challenge 2:02

1 **a** $-8·73$ **b** 6·859 **c** 306 **d** $-2·1$
e 57 **f** \$44.38 **g** 84·78 **h** 0·34375
i 5·95 m **j** \$113.88
2 **a** $-3·28$ **b** 3·375 **c** 478 **d** $-1·8$
e 63 **f** \$38.76 **g** 51·287 **h** 0·59375
i 2·775 m **j** \$116.79
3 **a** $-3·54$ **b** 4·913 **c** 198 **d** $-2·4$
e 56 **f** \$70.74 **g** 66·987 **h** 0·65625
i 3·5 m **j** \$143.07
4 **a** $-0·44$ **b** 4·096 **c** 345 **d** $-6·9$
e 64 **f** \$56.91 **g** 37·68 **h** 0·40625
i 3·75 m **j** \$104.16
5 **a** $-1·38$ **b** 9·261 **c** 299 **d** $-4·7$
e 69 **f** \$56.28 **g** 26·167 **h** 0·84375
i 2·925 m **j** \$104.76

Prep quiz 2:03

1 9 **2** -9 **3** -10 **4** -21 **5** -24
6 14 **7** -2 **8** -4 **9** 6 **10** 19

Exercise 2:03

1 **a** $8a$ **b** $2a$ **c** $-2a$ **d** $-8a$ **e** x
f $2x$ **g** $8x$ **h** $5x$ **i** $-17m$ **j** m
k $9m$ **l** $-4m$ **m** $-15a$ **n** $2a$
o $-54a$ **p** $80a$ **q** $-2y$ **r** $6y$
s $-4y$ **t** $4y$
2 **a** -16 **b** -6 **c** 8 **d** 24 **e** -10
f -6 **g** 6 **h** 7 **i** -10 **j** 6
k 3 **l** -7 **m** -30 **n** 0 **o** -28
p 16
3 **a** 14 **b** 32 **c** 0 **d** -2
4 **a** -4 **b** -8 **c** 20 **d** 24
5 **a** 25 **b** 100 **c** 36 **d** 1
6 **a** 3 **b** -6 **c** -8 **d** -24
7 **a** -18 **b** -30 **c** -24 **d** 0 **e** 36
f 9 **g** 60 **h** 18 **i** 2 **j** 1
k 4 **l** -2 **m** 36 **n** -36 **o** 36
p 4 **q** -21 **r** -15 **s** 9 **t** 3
8 **a** -5 **b** 30 **c** -20 **d** 0 **e** -1
f -4 **g** -5 **h** -6 **i** -40 **j** -40
k 12 **l** 9 **m** -20 **n** 89 **o** -75
p 39 **q** -7 **r** -13 **s** 3 **t** -3
9 **a** yes **b** yes
10 **a** false **b** false **c** false **d** false
e true **f** true

Prep quiz 2:04

1 27 **2** 27 **3** 8 **4** 8 **5** 10^5
6 10^4 **7** 7^3 **8** 5^4 **9** 3^8 **10** 3^{10}

Exercise 2:04

1 **a** **i** 32 **ii** 2 **iii** 128 **iv** 1 **v** 1024
b **i** 2^6, 64 **ii** 2^7, 128 **iii** 2^{10}, 1024
iv 2^8, 256 **v** 2^9, 512
c **i** 8×8, 64 **ii** 32×4, 128
iii 64×16, 1024 **iv** 2×128, 256
v 32×16, 512
The answers to part **b** are the same as the answers to part **c**.
2 **a** **i** 1 **ii** 81 **iii** 19 683 **iv** 243
v 6561
b **i** 3^0, 1 **ii** 3^8, 6561 **iii** 3^4, 81
iv 3^2, 9 **v** 3^3, 27
c **i** $27 \div 27$, 1 **ii** $59\,049 \div 9$, 6561
iii $6561 \div 81$, 81 **iv** $2187 \div 243$, 9
v $19\,683 \div 729$, 27
3 **a** **i** 3^{10}, 59 049 **ii** 2^8, 256 **iii** 3^6, 729
iv 2^8, 256 **v** 3^9, 1683
b **i** 243^2, 59 049 **ii** 16^2, 256 **iii** 9^3, 729
iv 4^4, 256 **v** 27^3, 19 683
4 **a** false **b** true **c** false **d** false
e false **f** true **g** false **h** false
i false **j** true **k** false **l** false
m true **n** true **o** true **p** true
5 **a** 10^5 **b** 10^6 **c** 10^6 **d** 10^7
e 5^4 **f** 5^6 **g** 5^3 **h** 5^9
i 2^9 **j** 2^{18} **k** 2^6 **l** 2^{11}
m 10^{12} **n** 10^6 **o** 10^6 **p** 10^7

6 **a** 10^4 **b** 10^2 **c** 10^3 **d** 10^7
e 5^0 (or 1) **f** 5^5 **g** 5^2 **h** 5^1 (or 5)
i 2^4 **j** 2^4 **k** 2^7 **l** 2^8
m 10^4 **n** 10^1 (or 10) **o** 10^5 **p** 10^3

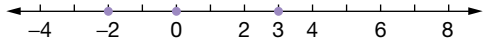
7 **a** 3^{20} **b** 2^9 **c** 5^{12} **d** 7^8
e 10^8 **f** 10^{12} **g** 10^{30} **h** 10^6
i 3^{14} **j** 3^9 **k** 2^{16} **l** 2^{14}
m 10^0 (or 1) **n** 10^2 **o** 10^5 **p** 10^9

8 **a** 10^6 **b** 10^9 **c** 10^{12}

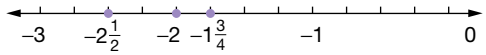
Diagnostic test 2

1 Graph each set on a separate number line.

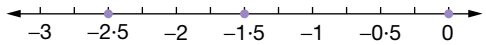
a



b



c



2 **a** 4 **b** −13 **c** −22 **d** 5 **e** 9
f −14

3 **a** −35 **b** 24 **c** −24 **d** 27 **e** 64
f −8

4 **a** −9 **b** 3 **c** −11 **d** 3 **e** −15
f 3 **g** −3 **h** 6 **i** −18

5 **a** $6m$ **b** $11x$ **c** $-15y$ **d** $10n$ **e** $-30m$
f $4x$

6 **a** −20 **b** 9 **c** −30 **d** 27 **e** 15
f 3

7 **a** 2^8 **b** 10^5 **c** 3^4 **d** 2^6 **e** 10^3
f 3^3 **g** 3^{10} **h** 2^{12} **i** 10^6

ASSIGNMENT 2A

1 **a** 8430 m **b** 42°C

2 **a** loss, gain **b** −24 **c** opposite **d** plus
e minus **f** plus **g** plus **h** plus
i minus **j** minus **k** minus **l** plus
m plus

3 **a** −123 **b** −1331 **c** 122·675 **d** 865
e 133·4025 **f** −5550

4 **a** −17 **b** 68 **c** −89

5 **a** $-8m$ **b** y^2 **c** $20y$ **d** $2x$
e $77m$ **f** $-6x$

6 **a** 3^8 **b** 3^8 **c** 3^{10} **d** 3^8
e 3^{15} **f** 3^2

ASSIGNMENT 2B

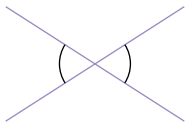
1 Answers may vary. Four cases are: [length 36 cm, breadth 1 cm], [length 18 cm, breadth 2 cm], [length 6 cm, breadth 6 cm] and [length 72 cm, breadth 0·5 cm]
The rectangle with the smallest perimeter has dimensions; length 6 cm, breadth 6 cm.

2 1293 **3** 9 with 3 legs, 15 with 4 legs
4 31 **5** 15 **6** $28, $44

7 **1** $6-2=4$ **2** $6+2=8$ **3** $6\div2=3$
4 $6-2=4$ **5** $6\div2=3$ **6** 2
7 6 **8** $6\times2=12$ **9** $6-2=4$
10 $6\times2=12$ **11** $2+6=8$ **12** $6-2=4$
13 $6^2=36$ **14** $\sqrt{36}=6$ **15** $6-2=4$
16 $6\times2=12$ **17** $(6+2)\div2=4$
18 $6+2=8$ **19** $6^2=36$ **20** $6-2=4$
21 $6-2=4$ **22** $6+2=8$ **23** $6\div2=3$
24 $6+2=8$

ASSIGNMENT 2C

1 **a**



b 180°

2 **a** $a=60$ **b** $b=150$ **c** $c=53$

3 **a** $a=60$ **b** $b=112$ **c** $c=108$

4 $5+6-9=2$

5 **a** 627·5 cm **b** 7·5 g **c** 6·5 L **d** 8·3 km

6 **a** 15·2 cm **b** 56 m^2 **c** 800 cm^3

7 **a** −11 **b** 33 **c** −2 **d** 47
e 5 **f** −3

8 **a** $3y+5$ **b** 5 **c** $T=19$

9 **a** y **b** 0 **c** $48n$ **d** $16a$
e $8b$ **f** $6a$

10 **a** $\frac{1}{6}$ **b** $\frac{3}{6}$ (or $\frac{1}{2}$ or 50%) **c** $\frac{4}{6}$ (or $\frac{2}{3}$)

Chapter 3

Exercise 3:01

1

	a	b	c	a^2	b^2	c^2	a^2+b^2
a	2	3	4	4	9	16	13
b	3	4	5	9	16	25	25
c	2·4	3·2	4	5·76	10·24	16	16
d	2·5	3	3	6·25	9	9	15·25
e	2	4·8	5·2	4	23·04	27·04	27·04

2 **b**, **d**, **f**, right-angled triangles

3 **a**

a	b	c
3	4	5

A_1	A_2	A_3
9	16	25

b

a	b	c
6	8	10

A_1	A_2	A_3
36	64	100

c

a	b	c
5	12	13

A_1	A_2	A_3
25	144	169

4 **a**

Triangle	AB^2	(right side)^2	(left side)^2
ΔABC	16	9	$(2{\cdot}5)^2 = 6{\cdot}25$
ΔABD	16	9	16
ΔABE	16	9	25
ΔABF	16	9	36

b ΔABE **c** 90°

Investigation 3:01B

a yes **b** yes **c** yes **d** $a^2 + b^2$ **e** c^2
f yes
In any right-angled triangle the square on the longest side (or hypotenuse) is equal to the sum of the squares on the other two sides.

Prep quiz 3:02

1 A **2** B **3** B **4** $c^2 = n^2 + m^2$ **5** 4
6 25 **7** 25 **8** 25 **9** 6 **10** no

Exercise 3:02A

1 **a** $c^2 = 8^2 + 15^2$ **b** $a^2 = 9^2 + 12^2$
c $h^2 = 10^2 + 15^2$ **d** $p^2 = 8^2 + 10^2$

2

Number	3	4	5	6	7	8	9	10	12
Square	9	16	25	36	49	64	81	100	144
Number	13	15	16	17	18	20	24	25	26
Square	169	225	256	289	324	400	576	625	676

3 **a** 5 **b** 10 **c** 13 **d** 15
e 30 **f** 25 **g** 17
4 **a** 5 m **b** 25 cm **c** 10 cm **d** 26 m
5 **a** 5·1 cm **b** 2·8 m **c** 4·5 cm **d** 4·1 m
e 4·2 cm **f** 3·6 m **g** 2·2 m
6 **a** 5·7 cm **b** 4·2 m **c** 17 cm
7 **a** 5 **b** 17 **c** $a = 17; b = 10$
8 **a** 20 cm **b** 50 m **c** 48 m
9 **a** 5 units **b** 5 units **c** 13 units
10 **a** 5·39 m (or 539 cm) **b** 10 km **c** 5·8 m

Exercise 3:02B

1 **a** 256 **b** 6·25 **c** 46·24 **d** 17
e 41 **f** 5·8 **g** 9·8 **h** 12·17
2 **a** 6·1 m **b** 2 m **c** 4·5 m **d** 20 cm
3 **a** 4·2 m **b** 21·6 cm **c** 12·2 cm **d** 5·7 m
e 10·4 m **f** 94·3 cm **g** 5·7 m
4 **a** 8·5 cm **b** 17·5 cm **c** 14·4 cm
d 21·5 m **e** 47·7 cm (or 47·6 cm)

Exercise 3:03A

1 **a** $5^2 = a^2 + 4^2$ **b** $15^2 = b^2 + 12^2$
c $13^2 = 5^2 + x^2$ **d** $25^2 = y^2 + 15^2$
2 **a** 3 **b** 12 **c** 20 **d** 18 **e** 8 **f** 9
3 **a** $13^2 = 12^2 + x^2$
$169 = 144 + x^2$
$x^2 = 25$
$x = \sqrt{25}$
$x = 5$
b $30^2 = 24^2 + a^2$
$900 = 576 + a^2$
$a^2 = 324$
$a = \sqrt{324}$
$a = 18$
c $26^2 = y^2 + 10^2$
$676 = y^2 + 100$
$y^2 = 576$
$y = 24$
d $10^2 = p^2 + 8^2$
$100 = p^2 + 64$
$p^2 = 36$
$p = 6$
4 **a** 3 **b** 9 **c** 12 **d** 20
5 **a** 16 cm **b** 15 cm **c** 10 cm **d** 12 cm
6 **a** 3 **b** 12 **c** 8 **d** 8 **e** 34 cm

Exercise 3:03B

1 **a** 1·6 **b** 1·5 **c** 40 **d** 21
2 **a** 7·4 **b** 2·1 **c** 4·1 **d** 2·0
3 **a** 1·5 **b** 6 **c** 96
4 **a** 7 m **b** 17 m **c** 300 m
5 **a** 12·65 m (2 dec. pl.) **b** 19·36 m (2 dec. pl.)
c 14·14 m (2 dec. pl.)
6 **a** 4·8 m **b** $x = 12; y = 9$
c $h = 3·2; L = 8·32$ **d** $h = 1·4$
7 **a** 5·7 cm **b** 33·5 cm

Exercise 3:04

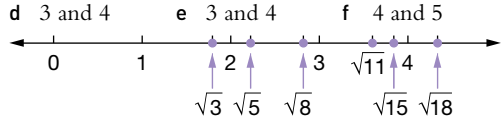
1 **a** 15 **b** 15 **c** 16 **d** 7·5 **e** 20
f 6 **g** 14
2 **a** 34 cm **b** 11·83 m **c** 5·4 km
d 8·42 m **e** 3 m **f** 1200 m
g yes **h** 2·42
i Bill, 30 km; Bev, 40 km; 50 km
j 9140 mm **k** 4081 mm **l** 10·20 m
3 **a** $x = 5·6; y = 11·9$ **b** $x = 60; y = 27$
c $x = 5; y = 3$
4 **a** 21·7 cm **b** 59·4 cm **c** 19·0 cm **d** 2·3 m
5 **a** 19·85 cm **b** 11·18 cm

Exercise 3:05

1 **a** no **b** yes **c** no **d** no **e** yes
f no **g** no **h** yes
2 **a** ΔDEF **b** ΔLMN
3 **a** yes **b** yes **c** yes **d** no **e** yes
f yes **g** no **h** yes
4 **a** {6, 8, 10} **b** {5, 12, 13} **c** {12, 16, 20}
d {9, 12, 15} **e** {30, 40, 50} **f** {27, 36, 45}
g {300, 400, 500} **h** {36, 48, 60}
5 **a** {3, 4, 5} **b** {5, 12, 13} **c** {7, 24, 25}
d {9, 40, 41} **e** {11, 60, 61}
6 Answers will vary.
7 Answers will vary.

Exercise 3:06

1 **a** irrational **b** rational **c** irrational
d rational **e** rational **f** rational
g irrational **h** rational **i** irrational
j rational **k** rational **l** rational
m rational **n** irrational **o** rational
p irrational **q** rational **r** irrational
s rational **t** irrational
2 a, c, g, i, n, p, r and t
3 **a** 1 and 2 **b** 2 and 3 **c** 2 and 3
d 3 and 4 **e** 3 and 4 **f** 4 and 5
4



5 **a** $1{\cdot}2, 1\frac{1}{2}, \sqrt{3}$ **b** $2{\cdot}2, \sqrt{5}, \sqrt{7}$
c $2{\cdot}8, \sqrt{8}, 3$ **d** $\sqrt{11}, \sqrt{15}, 4$

Investigation 3:06

a Wendy will take 24·4 min walking (missing train)
b 12·2 minutes running (early but flustered).
c Wendy should run part of the way and walk the rest.

Diagnostic test 3

1 **a** n **b** a **c** m
2 **a** $c^2 = 6^2 + 8^2$ **b** $l^2 = 3^2 + 6^2$
c $x^2 = 5{\cdot}6^2 + 4{\cdot}2^2$
3 **a** 10 m **b** 6·7 m (1 dec. pl.) **c** 7 m
4 **a** $17^2 = a^2 + 8^2$ **b** $4^2 = b^2 + 3{\cdot}2^2$
c $25^2 = a^2 + 20^2$
5 **a** 15 **b** 2·4 **c** 15
6 **a** 16 **b** 39 **c** 7
7 **a** {5, 12, 13} **b** {6, 8, 10} **c** {7, 24, 25}
8 $\sqrt{7}$

ASSIGNMENT 3A

1 **a** $h = \sqrt{2} \div 1{\cdot}414$ **b** $a = 9$
c $\sqrt{85} \div 9{\cdot}220$ **d** $b = \sqrt{145} \div 12{\cdot}042$
2 **a** ΔLMN is right-angled
b ΔABC is right-angled
3 {15, 36, 39}
4 **a** $\sqrt{2}, \sqrt{111}$ **b** $\sqrt{3}$ **c** $\sqrt{14}, \sqrt{20}$
5 **a** $0{\cdot}25, 1{\cdot}3333, \sqrt{2}, \sqrt{111}$
b $\sqrt{3}, \sqrt{16}, \sqrt{169}, 17$
c $\sqrt{14}, \sqrt{20}, \sqrt{36}, 6{\cdot}6, 7$
6 **a** $\sqrt{12}$ m, 3·46 m **b** $\sqrt{7500}$ m, 86·60 m
c $\sqrt{11600}$ m, 107·70 m

ASSIGNMENT 3B

1 28 **2** 28
3 (Kate 1, Sarah 10), (Kate 3, Sarah 5), (Kate 5, Sarah 0)
4 57 **5** 9 **6** 7 drinks

ASSIGNMENT 3C

1 **a** triangular prism **b** trapezoidal prism
c 6 faces, 12 edges, 8 vertices **d** pentagon
2 **a** rectangular prism **b** cylinder
c triangular prism
3 **a** obtuse **b** acute **c** $\angle BCD$ **d** ΔABC
4 **a** 48 **b** 109 **c** 1565 **d** 1993
5 $2217.60
6 **a** 11:20 am **b** 7:45 pm
7 **a** $16{\cdot}2\,\text{cm}^3$ **b** $125\,\text{cm}^3$ **c** $12\,\text{cm}^3$
8 **a** −4 **b** 56 **c** 17 **d** 30 **e** −24 **f** −5
9 **a** $4m$ **b** $-7a$ **c** $10b$
d $-36y$ **e** 100 **f** 12
10 **a** 2^7 **b** 2^6 **c** 2^{10}

Chapter 4

Exercise 4:01

1 **a** $\frac{1}{100}$ **b** $\frac{9}{100}$ **c** $\frac{3}{100}$ **d** $\frac{57}{100}$
2 **a** $\frac{1}{5}$ **b** $\frac{1}{2}$ **c** $\frac{3}{5}$ **d** $\frac{1}{20}$
3 **a** 1 **b** $1\frac{3}{20}$ **c** $3\frac{1}{2}$ **d** 4
4 **a** 1% **b** 3% **c** 7% **d** 10%
5 **a** 30% **b** 14% **c** 55% **d** 40%
e 36% **f** 307% **g** 150% **h** 210%
i 380% **j** 200%
6 **a** $17\frac{1}{2}\%$ **b** $37\frac{1}{2}\%$ **c** 80% **d** $33\frac{1}{3}\%$
e $83\frac{1}{3}\%$
7 **a** $112\frac{1}{2}\%$ **b** $314\frac{2}{7}\%$ **c** $408\frac{1}{3}\%$ **d** 225%
e 1060%
8 **a** 0·14 **b** 0·63 **c** 0·85 **d** 0·32
e 0·11 **f** 0·03 **g** 0·09 **h** 0·01
i 0·02 **j** 0·06 **k** 0·10 **l** 0·30
m 0·50 **n** 0·90 **o** 0·70
9 **a** 5% **b** 1% **c** 8% **d** 66% **e** 17%
f 50% **g** 40% **h** 10% **i** 30% **j** 80%
10 **a** $\frac{1}{4}, 0{\cdot}5, 100\%$ **b** $1\%, \frac{1}{10}, \frac{1}{2}$
c $9\%, \frac{13}{100}, \frac{3}{4}$ **d** $5\%, \frac{99}{100}, 1$
11 **a** 2 **b** 6 **c** 1 **d** 3 **e** 5
f 1·2 **g** 2·09 **h** 1·97 **i** 2·5 **j** 3·9
12 **a** 145% **b** 365% **c** 125% **d** 235%
e 185% **f** 290% **g** 110% **h** 360%
i 34% **j** 70%
13 **a** 1·47 **b** 1·13 **c** 1·24 **d** 0·68
e 0·35 **f** 0·43 **g** 0·09 **h** 0·06
i 0·04 **j** 2·48 **k** 0·02 **l** 0·18
m 0·136 **n** 0·092 **o** 0·163 **p** 0·1225
q 0·1075 **r** 0·836 25
14 **a** $15\%, 0{\cdot}51, \frac{3}{5}$ **b** $18\%, 1\frac{1}{8}, 1{\cdot}8$
c $1\frac{1}{2}, 152\%, 15{\cdot}2$ **d** $60\%, 0{\cdot}65, \frac{2}{3}$
15 $\frac{2}{8} + \frac{1}{4} = \frac{2}{8} + \frac{2}{8} = \frac{4}{8} = \frac{1}{2}$
$\frac{3}{12} = \frac{1}{4}$

Fun spot 4:01

1 **a** 40% **b** 80% **c** 90% **d** 30%
e 20% **f** 80% **g** 10%
2 **a** 10% **b** 50% **c** 60% **d** 20%
3 **a** 3% **b** 10% **c** 22% **d** 18%
e 33% **f** 13% **g** 1%

Prep quiz 4:02

1 12·5 **2** 6·25 **3** 11·75 **4** 109·5
5 62·5 **6** $33.\dot{3}$ **7** $33.\dot{3}$ **8** $16.\dot{6}$
9 $1\%, \frac{1}{100}$ **10** $10\%, \frac{1}{10}$

Exercise 4:02

1 **a** 37·5% **b** 8·6% **c** 17·4% **d** 124·5%
e 612·5% **f** 237·5% **g** 230% **h** 1·5%
i 0·5%
2 **a** 0·145 **b** 0·279 **c** 0·954 **d** 0·021
e 0·096 **f** 0·2385 **g** 0·8735 **h** 1·125
i 2·875
3 **a** 0·125 **b** 0·875 **c** 0·135 **d** 0·0725
e 0·1075 **f** 0·1525 **g** 1·0675 **h** 2·545
i 1·3125
4 **a** 28·57% **b** 83·33% **c** 87·5% **d** 93·75%
e 66·67% **f** 7·5% **g** 116·67% **h** 283·33%

Prep quiz 4:03

1 62% **2** $66\frac{2}{3}$% **3** 25% **4** $12\frac{1}{2}$%
5 $6\frac{1}{4}$% **6** $\frac{1}{4}$ **7** $\frac{1}{5}$ **8** 0·24
9 0·06 **10** 0·145

Exercise 4:03

1 **a** 80 g **b** 225 m **c** $51 **d** 160 L
e $33\frac{1}{3}$ cm or 33·2 cm **f** 287 mL **g** 34 min
h 13 400 t **i** $51 **j** $8.76
2 **a** $1.20 **b** 45 L **c** $4.80 **d** 144 g
e 60 t **f** 9 min **g** 24 km **h** 4 min
i 350 L **j** 840 m **k** $2300 **l** 10 200 km
m 168 kL **n** 613·98 km **o** 1022 days
3 **a** 28·35 L **b** 145·5 m **c** 8·4 t **d** 6·08 km
e 3·36 m **f** 3·36 g **g** 2·205 t **h** 10·08 mm
i 6·8 s **j** 2·1 mL **k** 1050 **l** 6·4 g
m 0·15 km **n** 0·42 L **o** $4.50 **p** 40 g
q 734·25 kg **r** $87.50
4 $105 000 **5** 160 **6** 950
7 $119 000 **8** 912 000 **9** 18
10 43·5 s **11** 375 **12** 1750

Prep quiz 4:04

1 60% **2** 87% **3** 118% **4** 400% **5** 80%
6 225% **7** 106% **8** 96% **9** 170 **10** 230

Exercise 4:04

1 **a** 125% **b** 115% **c** 110% **d** 109%
e 200% **f** 400% **g** 255% **h** 232%
2 **a** 75% **b** 85% **c** 90% **d** 91%
e 99% **f** 5% **g** 63% **h** 70%
3 **a** $880 **b** $360 **c** 5·8 m **d** 4·48 L
e 111 g **f** 99·4 t **g** 10·4 kg **h** 31·8 ha
i 112 h **j** 243 s or 4 min 3 s
4 **a** $48 **b** $72 **c** 6·88 m **d** 1·89 m
e 1·95 m **f** 6·65 L **g** 19 kg **h** 768 mm
i 3·08 t **j** 58·8 L
5 **a** $130.50 **b** $11.60 **c** $43.50
d $17.40 **e** $290 **f** $82.65
6 **a** $416 **b** $374.40 **c** $2195.44
d $316.38 **e** $890.47 **f** $1083.47
7 **a** $332.50 **b** $602 **c** $416.50
8 $27 528 **9** 314·64 m **10** 34 450 t
11 178 024 **12** 14 640 **13** $7084.80
14 $73.50 **15** $1152
16 **a** $457.42 **b** $1647.52 **c** $128.81
d $1149.41 **e** $100.07 **f** $1120.13
g $5.54 **h** $13.96 **i** $0.61
j $44.47 **k** $184.98 **l** $313.50
17 $4455 **18** $321

Prep quiz 4:05

1 $\frac{1}{2}$ **2** $\frac{1}{4}$ **3** $\frac{3}{4}$ **4** $\frac{1}{10}$ **5** 50%
6 25% **7** 75% **8** 10% **9** 87·5% **10** $\frac{1}{10}$

Exercise 4:05

1 **a** 60% **b** 75% **c** 25% **d** $62\frac{1}{2}$%
2 **a** 10% **b** 25% **c** 1% **d** 25%
e 20% **f** 62% **g** $7\frac{1}{2}$% **h** $17\frac{1}{2}$%
i $77\frac{1}{2}$% **j** $33\frac{1}{3}$% **k** $5\frac{2}{5}$% or 5·4%
l $66\frac{2}{3}$% **m** 300% **n** 125% **o** 110%
3 **a** 70% **b** 5% **c** 75% **d** $112\frac{1}{2}$%
e 7% **f** $81\frac{1}{4}$% **g** 25% **h** 750%
i 200% **j** 30% **k** 350% **l** 875%
m 90% **n** 2% **o** 1·25%
4 **a** seasoning = 6%, cornflour = 4%
b **i** 1·14 g **ii** 30·4 g **iii** 1·9 g
c 1%
d **i** 4·25% **ii** 2·59% **iii** 0·05%
e team members = 4%, female spectators = 53·5%, male spectators = 42·5%
f **i** 17·5% **ii** 82·5%
g 14% **h** 3·5%
i typing = 37·5%, filing = 18·75%, sorting mail = 6·25%, telephone = 3·125%, meetings = 9·375%, other activities = 25%
j Mining 93 700, 1·4%;
Food Beverages 175 300, 2·6%;
Metal Products 203 200, 3·1%;
Other 758 400, 11·5%;
Electricity, gas and water 133 000, 2·0%;
Construction 485 000, 7·3%;
Wholesale 1 318 500, 19·9%;
Transport 365 800, 5·5%;
Communication 146 800, 2·2%;
Finance 652 200, 9·8%;
Public Administration 323 900, 4·9%;
Community Services 1 140 900, 17·2%;
Recreation 441 800, 6·7%.

Prep quiz 4:06

1 48% **2** true **3** 82 cents **4** $a = 7$
5 $x = 4$ **6** 0·32 **7** $60 **8** $0
9 $\frac{1}{100}$ **10** 1%

Exercise 4:06

1 We can't know which shop is cheaper because we don't know the original price at each store.
2 4, no **3** $7
4 **a** $96 **b** $100 **c** Honest Anne's, $4
5 $295.50, no
6 The one with a 35% discount is cheaper by $0.96
7 **a** 6% **b** 90%
c They would not impress people.
8 **a** $196 **b** 30%
9 **a** 50·4% (1 d.p.) **b** 50·2% (1 d.p.)
c 50·2% (1 d.p.)
10 **a** 23 274 685 **b** 23 693 630 **c** 26 370 494
d 15·3% (1 d.p.)

Prep quiz 4:07

1 $2 **2** 35c **3** 71c **4** $1.44
5 $6 **6** 84c **7** $6.16 **8** $139.70
9 $60 **10** $8.40

Exercise 4:07

1 **a** $7 **b** $12 **c** $4.62 **d** $5.97
e $86.30 **f** $1274 **g** $9.76 **h** $14.24
i $12.46 **j** $74.22
2 **a** $66 **b** $165 **c** $242
d $814 **e** $82.50 **f** $123.20
g $377.30 **h** $1041.70 **i** $9.46
j $30.14 **k** $62.10 **l** $21.59
3 **a** $2 **b** $10 **c** $15 **d** $26
e $5.60 **f** $5.20 **g** $1.20 **h** $78.20
i $4.25 **j** $31.80 **k** $91.30 **l** $112.50
4 $102.73
5 **a** $70 **b** $120 **c** $180
d $250 **e** $14 **f** $26
g $97 **h** $115 **i** $12.60
j $37.50 **k** $215.50 **l** $1125.80
6 $0.58
7 **a** $0.82 **b** $0.96 **c** $1.44

Exercise 4:08

1 **a** $184.80 **b** $368 **c** $2.43 **d** $30 000
e $37.80 **f** $10 560 **g** $3120 profit
2 **a** 5% **b** 20% **c** 35% **d** $16\frac{2}{3}$%
3 **d** 20% **e** 10% **f** 32% **g** 65%
4 **a** $5 **b** 100% **c** 50%
5 **a** $240 **b** 37·5% **c** 60%
6 **a** $8000 **b** $48 000 **c** $16\frac{2}{3}$%
7 **a** $28.80 **b** $64.80 **c** $44\frac{4}{9}$%
8 **a** 28·6% (1 d.p.) **b** 40%
9 **a** $200 **b** $80 **c** $200 **d** $40
e $400 **f** $50 **g** $250 **h** $60
10 **a** $72 **b** $46.80
11 **a** $18 000 **b** $7200 **c** 29%

Exercise 4:09

1 **a** 500 m **b** 27 100 ha **c** 90 kg **d** 3500 t
e $104 **f** $600 **g** 700 mm **h** $103
2 $3900 **3** 200 runs **4** 1200 **5** $1835

Diagnostic test 4

1 **a** $\frac{9}{100}$ **b** $\frac{12}{25}$ **c** $3\frac{1}{2}$
2 **a** 173% **b** 44% **c** 314%
3 **a** 0·17 **b** 0·04 **c** 1·06
4 **a** 22% **b** 90% **c** 370%
5 **a** 0·138 **b** 0·0625 **c** 0·875
6 **a** $12\frac{1}{2}$% **b** $162\frac{1}{2}$% **c** $16\frac{2}{3}$%
7 **a** 128 g **b** 0·21 km **c** 14·4 L
8 **a** $5.75 **b** $4.25 **c** $99.40
9 **a** 16% **b** 42·5% **c** 51%
10 **a** $7.37 **b** $59.29 **c** $2.95 **d** $19.50
11 **a** **i** 25% **ii** 20%
b **i** 25% **ii** $33\frac{1}{3}$%
12 **a** 9000 **b** 150

ASSIGNMENT 4A

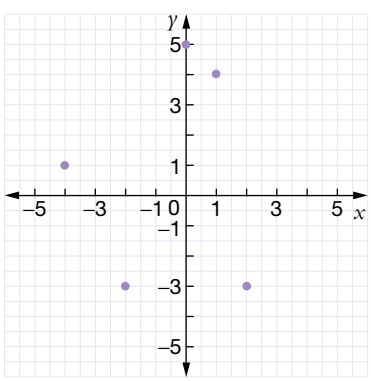
1 **a** 0·25, $\frac{1}{4}$ **b** 0·6, $\frac{3}{5}$ **c** 0·15, $\frac{3}{20}$
d 0·07, $\frac{7}{100}$ **e** 0·055, $\frac{11}{200}$ **f** 0·0625, $\frac{1}{16}$
2 **a** 37% **b** 8% **c** 40% **d** 135%
e 12·5% **f** 2·5% **g** 17% **h** 70%
i 80% **j** 35% **k** $66\frac{2}{3}$% **l** 62·5%
3 **a** $10 **b** $180 **c** $43.20 **d** $6.30
e $112 **f** $3 **g** 7·50 **h** $14
4 **a** 25% **b** 20% **c** 5% **d** 20%
e 12·5% **f** 128%
5 $517.50
6 **a** $32.89 **b** $29.60 **c** No **d** approx. 9·1%
7 **a** 40% **b** approx. 34%

ASSIGNMENT 4B

1 6 **2** 24 **3** 35 **4** 63 min **5** 8
6 Third largest number is 7 654 231; third smallest number is 1 234 657.

ASSIGNMENT 4C

1 **a** 7500 mL **b** 400 000 mL **c** 14 L **d** 2·1 kL
2 **a** −6 **b** 1 **c** −10 **d** 2 **e** −20
3



4 **a** $3 \times a$ **b** $4 \times m + 2 \times n$
c $5 \times (x + y)$ **d** $a \div 5$
e $3 \times m \div 2$ or $(3 \times m) \div 2$
5 **a** 20 **b** 18 **c** 4 **d** 34
6 **a** 15·12 **b** 151·2 **c** 1·512
7 **a** 10 cm or $\sqrt{28} \div 5{\cdot}3$ cm **b** 2·83 m
8 **a** a^5 **b** 10^3 **c** a^3b^2 **d** $12p^3$
9 **a** $5a + 20$ **b** $4a + 6$ **c** $4m - 20$ **d** $2ab - 3a$
10 **a** 1, 36, 2, 18, 3, 12, 4, 9, 6
b 1, 54, 2, 27, 3, 18, 6, 9 **c** 18
d 36, 72, 108, 144; 54, 108, 162, 216; 108

Chapter 5

Prep quiz 5:01

1 12 **2** 12 **3** 12 **4** 12 **5** −4
6 12 **7** 12 **8** −6

9

n	0	1	2	3
M	1	3	5	7

10

x	4	5	6	7
y	6	5	4	3

Exercise 5:01

1 **a** −6 **b** −21 **c** 2 **d** −4 **e** 13
f 6 **g** −6 **h** 9 **i** −5 **j** 1
k 0 **l** −2
2 **a** −7 **b** −2 **c** −1 **d** −5 **e** 1
f −6 **g** −5 **h** −1 **i** 12 **j** −8
k −12 **l** 13 **m** −2 **n** −2 **o** −5
p 24
3 **a** **i** −6 **ii** −12 **iii** 15 **iv** 0
b **i** −1 **ii** −3 **iii** −7 **iv** −21
c **i** 4 **ii** −2 **iii** 12 **iv** 16
d **i** −2 **ii** −8 **iii** −20 **iv** −68

4 **a**

P	1	2	3	4
F	0	1	2	3

b

P	1	2	3	4
F	0	2	4	6

c

P	1	2	3	4
F	0	4	8	12

d

x	1	2	3	4
y	4	5	6	7

e

x	1	2	3	4
y	3	6	9	12

f

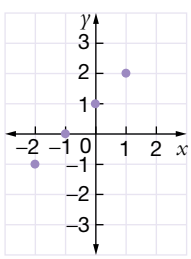
x	1	2	3	4
y	3	5	7	9

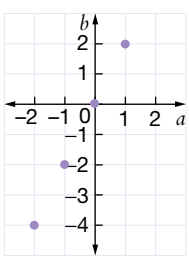
5 **a**

x	−2	−1	0	1
y	−1	0	1	2

b

a	−2	−1	0	1
b	4	−2	0	2



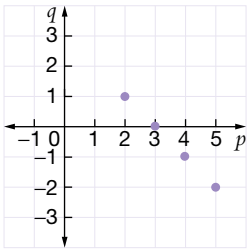


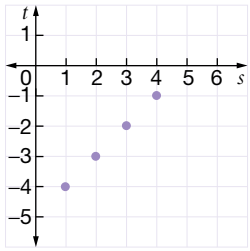
c

p	2	3	4	5
q	1	0	−1	−2

d

s	1	2	3	4
t	−4	−3	−2	−1



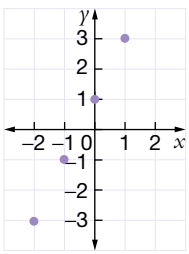


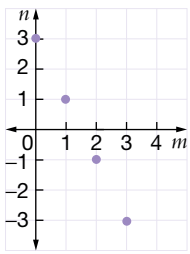
e

x	−2	−1	0	1
y	−3	−1	1	3

f

m	0	1	2	3
n	3	1	−1	−3





6 **a** $y = 7 - x$ **b** $m = 24 \div d$
c $p = 10w + 7$ **d** $y = 3x - 5$
e $b = a^2 + 7$ **f** $p = 50 - 8n$
7 **a** $y = 7x$ **b** $b = d - 5$
c $M = 2v + 1$ **d** $M = a^2$
e $e = 13 - d$ **f** $x \times y = 12$

8 **a**

x	−2	−1	0	1
y	−2	−1	0	1

, B

b

x	−1	0	1	2
y	2	1	0	−1

, C

c

x	−1	0	1	2
y	−3	−1	1	3

, A

9 **a** $f = 3p - 3$ **b** 33 **c** 320 m
10 **a**

S	3	4	5	6	7	8
P	5	30	55	80	105	130

b $P = -70$ **c** 3
11 **a** **i** 25 **ii** 0 **iii** −5 **iv** −20 **b** 6
12 **a** $P = L + 2 + 2 + (L - 1) = 2L + 3$
b **i** 11 **ii** 13 **iii** 23 **iv** 203
v 139
c No, because the basic shape cannot be made with a base length of zero.

Investigation 5:01

1 **a** $2x + 1$ **b** $3 + 3x$ **c** $y + x + 1$
d $(x + 2) + (x + 2) = 2x + 4$
e $1 + x + 1 + x + 1 = 2x + 3$
f $y + 1 + y + 3 = 2y + 4$
g $(3x + 3) + (2x + 4) = 5x + 7$

2 **A** **a** 21 **b** 33 **c** 27 **d** 24
e 23 **f** 36 **g** 57
B **a** 15 **b** 24 **c** 19 **d** 18
e 17 **f** 26 **g** 42

Prep quiz 5:02

1 $7x$ **2** $3a$ **3** $12p$ **4** $10m$ **5** $5m$
6 $8a^2$ **7** $8mn$ **8** $11ab$ **9** $9x$ **10** p

Exercise 5:02A

1 **a** $8x$ **b** $9y$ **c** $7a$ **d** $8b$ **e** $5m$
f $5n$ **g** $9a$ **h** 0 **i** $13l^2$ **j** $30x^2$
k $12xy$ **l** $14pq$

2 **a**

+	a	$2a$	$5a$	$6a$	$10a$
$3a$	$4a$	$5a$	$8a$	$9a$	$13a$
$5a$	$6a$	$7a$	$10a$	$11a$	$15a$
a	$2a$	$3a$	$6a$	$7a$	$11a$
$6a$	$7a$	$8a$	$11a$	$12a$	$16a$
$10a$	$11a$	$12a$	$15a$	$16a$	$20a$

b

–	$3a$	a	$2a$	$5a$	$4a$
$5a$	$2a$	$4a$	$3a$	0	a
$7a$	$4a$	$6a$	$5a$	$2a$	$3a$
$12a$	$9a$	$11a$	$10a$	$7a$	$8a$
$8a$	$5a$	$7a$	$6a$	$3a$	$4a$
$10a$	$7a$	$9a$	$8a$	$5a$	$6a$

3 **a** $10a$ **b** $9y$ **c** $11m^2$ **d** $5x^2$
e $4pq$ **f** p **g** $18a$ **h** $8k$
i $3q^2$
4 **a** $7a, 21$ **b** $5a, 15$ **c** $6a, 18$ **d** $2a, 6$
e $a, 3$ **f** $5a, 15$ **g** $5a^2, 45$ **h** $8a^2, 72$
i $2a^2, 18$
5 **a** $6x + 3y$ **b** $9m + 2n$ **c** $10a$
d $3x + 4y$ **e** $6p + 4m$ **f** $t + 3w$
g $7a + 4b$ **h** $2t + 6w$ **i** $2p + 5q$
6 **a** $5x + y, 20$ **b** $7y + 10, 45$
c $5x + 4y - 35, 0$ **d** $5x + 3y, 30$
e $3x + 4y, 29$ **f** $2x + 2y, 16$

Exercise 5:02B

1 **a** $7a$ **b** $-7a$ **c** $-a$ **d** $2x$
e $-p$ **f** $5y$ **g** $-3m$ **h** $-4a$
i $-10y$ **j** $-5x$ **k** $-7y$ **l** $-10p$
m $11x$ **n** $8y$ **o** $7t$ **p** $2ab$
q $-6xy$ **r** $-12x^2$ **s** $-x$ **t** $3x$
u 0

2 **a**

+	$-4x$	$-x$	$3x$	$-5x$	$7x$
$3x$	$-x$	$2x$	$6x$	$-2x$	$10x$
$5x$	x	$4x$	$8x$	0	$12x$
x	$-3x$	0	$4x$	$-4x$	$8x$
$-2x$	$-6x$	$-3x$	x	$-7x$	$5x$
$-7x$	$-11x$	$-8x$	$-4x$	$-12x$	0

b

–	$-4x$	$-x$	$3x$	$-5x$	$7x$
$3x$	$7x$	$4x$	0	$8x$	$-4x$
$5x$	$9x$	$6x$	$2x$	$10x$	$-2x$
x	$5x$	$2x$	$-2x$	$6x$	$-6x$
$-2x$	$2x$	$-x$	$-5x$	$3x$	$-9x$
$-7x$	$-3x$	$-6x$	$-10x$	$-2x$	$-14x$

3 **a** $x + 2y$ **b** $9m - n$ **c** $3q - p$
d $2x - 7$ **e** $10 - 2p$ **f** $9q + 4p$
g $-3b$ **h** $2n - 2m$ **i** $4n + 2q$
j $-x - 5y$ **k** $-7t + 3w$ **l** $-6m - 2n$
m $-6a - 2b$ **n** $8xy - 3x$ **o** $-2ab + 6$
p $9x - x^2$ **q** $7pq - 10p$ **r** $5 - x^2$
4 **a** $8a + 6b$ **b** $4x + 7y$ **c** $11m + 5n$
d $15p - 12$ **e** $3a + 12$ **f** $15 - 11q$
g $2k + 2l$ **h** $2l - 2m$ **i** $3x + 2y$
j $5ab + 7a$ **k** $2xy - 3$ **l** $10x^2 + x$
m $6a + 7$ **n** $10 + 8a$ **o** $6y + 4x$
p $4a - 2b$ **q** $3l + 2q$ **r** $-t - 4w$
s $7ab - bc$ **t** $10mn - 9m$ **u** $-3 - 7kl$
v $x^2 - 3x$ **w** $2q - q^2$ **x** $-9 - 6h$
5 **a** $3xy$ **b** $-6ab + a$ **c** $6mn + 6$
d $10 - 6ab$ **e** $3m^2 + 2m + 3$ **f** $11a^2 - a - 1$
g $5xy - 12yz$ **h** $3a^2 + ab$ **i** $1 + 5mn + m$
j $9k^2 + 7t^2$ **k** $ab + 9a - 4b$ **l** $8m^2 + 6n^2 - 11n$

Prep quiz 5:03

1 $5a$ **2** pq **3** $3ab$ **4** $15xy$ **5** $2x^2$
6 12 **7** 9 **8** 22 **9** 16 **10** $m \times v \times v$

Exercise 5:03

1 **a** $10a$ **b** $21x$ **c** $16m$ **d** $60y$
e $56p$ **f** $30n$ **g** $21q$ **h** $44k$
i $15b$ **j** $12a$ **k** $18y$ **l** $48a$
m $3a$ **n** $3b$ **o** $5m$ **p** y
2 **a** $6mn$ **b** $12xy$ **c** $25tw$ **d** $21ab$
e $7ab$ **f** $5pq$ **g** $12kl$ **h** $3ab$
i $40mn$ **j** $18x^2$ **k** $36ay$ **l** $5x^2$
m $15y^2$ **n** $42m^2$ **o** $7k^2$ **p** $2y^2$
3 **a** $20an$ **b** $6xy$ **c** $18mn$ **d** $42a$
e $24b$ **f** $4ab$ **g** $5ab$ **h** $6bc$
i $5mn$ **j** $24ab$ **k** $30abc$ **l** $12xy$
4 **a** a^2b **b** m^2n **c** $3ax^2$ **d** $6a^2b$
e $12pq^2$ **f** $15k^2l$ **g** $12abc$ **h** $90ab^2c$
i $20x^2yz$ **j** $24abc$ **k** $m^2n^2p^2$ **l** $6a^2b^2$

5 **a**

×	6	a	$2b$	$9m$	$3ab$
$2a$	$12a$	$2a^2$	$4ab$	$18am$	$6a^2b$
$3b$	$18b$	$3ab$	$6b^2$	$27bm$	$9ab^2$
7	42	$7a$	$14b$	$63m$	$21ab$
a	$6a$	a^2	$2ab$	$9am$	$3a^2b$
ab	$6ab$	a^2b	$2ab^2$	$9abm$	$3a^2b^2$

6 b

×	x	$2y$	xy	$5x$	$2xz$
$4x$	$4x^2$	$8xy$	$4x^2y$	$20x^2$	$8x^2z$
y	xy	$2y^2$	xy^2	$5xy$	$2xyz$
$5y$	$5xy$	$10y^2$	$5xy^2$	$25xy$	$10xyz$
$2xy$	$2x^2y$	$4xy^2$	$2x^2y^2$	$10x^2y$	$4x^2yz$
yz	xyz	$2y^2z$	xy^2z	$5xyz$	$2xyz^2$

6 a $-15x$ b $-14m$ c $12a$ d $-27m$
e $30k$ f $14a$ g $-2ab$ h pq
i $-10mn$ j $6tw$ k $-56xy$ l $-30m^2$
m $36y^2$ n $-12a^2$ o $3x^2$ p $-60a$
q $30y$ r $-24a^2$ s $-54mn$ t $-32k^2$
u $24p^2$ v $-10x^2y$ w $24a^2b$ x $-5m^2np$

7 a

×	-3	5	$2x$	$-4x$	$-y$
7	-21	35	$14x$	$-28x$	$-7y$
-2	6	-10	$-4x$	$8x$	$2y$
$-x$	$3x$	$-5x$	$-2x^2$	$4x^2$	xy
$4x$	$-12x$	$20x$	$8x^2$	$-16x^2$	$-4xy$
$-3y$	$9y$	$-15y$	$-6xy$	$12xy$	$3y^2$

b

×	$2a$	$3b$	$-a$	$-4b$	$-ab$
$6a$	$12a^2$	$18ab$	$-6a^2$	$-24ab$	$-6a^2b$
$-3b$	$-6ab$	$-9b^2$	$3ab$	$12b^2$	$3ab^2$
$-b$	$-2ab$	$-3b^2$	ab	$4b^2$	ab^2
$3a$	$6a^2$	$9ab$	$-3a^2$	$-12ab$	$-3a^2b$
$2ab$	$4a^2b$	$6ab^2$	$-2a^2b$	$-8ab^2$	$-2a^2b^2$

8

	A	B	C	D
a	$15xy$	$-8m$	$-18x^2$	$2y^2$
b	$30ab$	abc	$6ab$	m^2n
c	$8ab$	$-12pq$	$6xy^2$	$30a^2$
d	$16m^2$	$5ab$	$30m^2n^2$	$4ab^2$
e	$6x^2y$	$15tw$	$-3a^2$	$24mn^2$
f	$-18k^2$	$6l^2$	$100abc$	$35xy$
g	0	$-x^2$	$-8hk$	$-54a^2$
h	$18x^2$	$-8a^2b$	$28m^2np$	$-24xy$

Prep quiz 5:04

1 $2x$ **2** $4y$ **3** $5a$ **4** $20m$ **5** $2x$
6 $\frac{12y}{5}$ **7** $5a$ **8** $20m$ **9** 4 **10** 4

Exercise 5:04

1 a $2a$ b $2m$ c $4p$ d 12 e 3
f 2 g $3n$ h $6b$ i $4p$ j $\frac{x}{2}$
k $\frac{3a}{5}$ l $\frac{m}{5}$ m $\frac{n}{3}$ n $\frac{1}{2}$ o $\frac{1}{2}$
p $\frac{1}{3}$ q $\frac{3}{4}$ r $2n$ s $2a$ t $2x$
u $3a$

2 a $2m$ b $5x$ c $5p$ d $2a$ e $3n$
f $2p$ g $3l$ h $5k$ i 2 j 2
k 6 l 6 m 3 n 5 o 4
p 2 q $5b$ r $3n$ s 8 t $2w$
u m v x w 12 x $2a$

3 a $-2a$ b $-3m$ c $2q$ d $3p$ e -2
f -3 g 2 h 3 i $-2y$ j $-4m$
k $3k$ l $4mn$ m $-4x$ n $-3m$ o $-2t$
p $3n$ q -2 r -7 s 3 t 2
u $-2a$ v $-2mn$ w $4b$ x b

4

	A	B	C	D
a	5	$10q$	$-m$	$-2p$
b	$3t$	-1	3	5
c	$2a$	$\frac{3}{2}$	$\frac{7b}{3}$	$2x$
d	2	$-3a$	$-m$	$2p$

Prep quiz 5:05

1 $18m$ **2** $2m$ **3** $16a^2$ **4** 4 **5** $50pq$
6 $8a^2b$ **7** 12 **8** 2 **9** m **10** $3x$

Exercise 5:05

1 a $30xy$ b $40abc$ c 3 d $3m$
e $15pq$ f $15a$ g 1 h 2
i $4m$ j $3a^2$ k $21a^2b^2$ l $8t^2$

2

	A	B	C
a	$10m^2$	2	$2p$
b	$2a$	$6p^2q$	1
c	$-6pq$	$-4m$	$3x^2y$
d	$9m^2$	$25a^2$	$16x^2$
e	$6mn$	$4l$	$3a^2$
f	$12p^2q$	$-10x^2y$	$4n$
g	$9p^2q^2$	$49m^2n^2$	$4t^2w^2$
h	$24pq$	$24abc$	$18mp$
i	0	$6a^2x$	$-30t^2$
j	$-5y$	-1	$2x$
k	$6a^2$	$6p$	$8x$
l	2	$2x$	$2q$
m	1	x	$2a$

Prep quiz 5:06

1 $10x$ **2** $8a + 7b$ **3** $-3m$ **4** $2x^2 - x$
5 $21x$ **6** 2 **7** $-30x^2$ **8** $15a^2b$
9 $2x$ **10** $4a$

Exercise 5:06

1 a $(a + 27)$ cm b $(2y + 14)$ cm
c $(8 + 4x)$ cm d $4b$ m e $16a$ cm
2 a $6a\,\text{cm}^2$ b $7y\,\text{cm}^2$ c $8x\,\text{cm}^2$
d $b^2\,\text{m}^2$ e $12a\,\text{cm}^2$
3 a $4x$ mm b $2L + 2B$ mm c $2p + 6q$ mm
d $20k$ mm e $6x$ mm f $10n + 3m$ mm
g $9x + 5$ mm h $3m + 2n - 1$ mm
i $4a + 3b$ mm
4 a $4m^2\,\text{cm}^2$ b $30a\,\text{cm}^2$ c $14xy\,\text{cm}^2$
d $3p^2\,\text{cm}^2$ e $20b\,\text{cm}^2$ f $30h^2\,\text{cm}^2$
5 a $3p + 4q$ b $14a$ c $40x$ cents
d $9m + 8n$
6 a $18x^2\,\text{cm}^3$ b $6x^2 + 48x\,\text{cm}^2$

Prep quiz 5:07

1 4 **2** 25 **3** 10 000 **4** 8 **5** 27
6 3^3 **7** a^5 **8** 10^4 **9** x^6 **10** y^7

Exercise 5:07

1 **a** m^2 **b** a^6 **c** x^4 **d** $5n^3$ **e** $21a^2$
f $16y^3$ **g** p^2q **h** $56xy^2$ **i** $4m^3n^2$

2 **a** $x \times x \times x$ **b** $8 \times b \times b$
c $7 \times m \times m \times m \times m \times m \times m$
d $y \times y \times y \times y \times y$ **e** $3 \times a \times a \times a \times a$
f $x \times x \times y \times y$ **g** $a \times m \times m \times m \times m$
h $x \times x \times x \times x \times y \times y \times y$ **i** $a \times a \times a \times b$
j $a \times p \times p \times p$ **k** $2 \times a \times p \times p$
l $4 \times a \times p \times p \times p$ **m** $6 \times x \times x \times x \times y \times y$
n $5 \times a \times a \times a \times a \times a \times b$
o $10 \times m \times m \times m \times n \times n \times n \times n$

3 **a** 16 **b** 25 **c** 16 **d** 1 000 000
e 27 **f** 300 **g** 16 **h** 36
i 162 **j** 5000 **k** 400 **l** −240
m 432 **n** 800 **o** −375

Prep quiz 5:08

1 20 **2** 20 **3** 6 **4** 6 **5** −14
6 −14 **7** $5a + 10$ **8** $6x + 15y$
9 $6x^2 + 10x$ **10** $12a^2 - 6ab$

Exercise 5:08A

1 **a** $3a + 6$ **b** $4x + 12$ **c** $2x + 14$
d $5v + 5$ **e** $6m - 30$ **f** $5n - 10$
g $4b - 12$ **h** $6l - 6$ **i** $6 + 2a$
j $12 + 3x$ **k** $36 + 6t$ **l** $14 + 7w$
m $7 - 7x$ **n** $10 - 5y$ **o** $12 - 4p$
p $72 - 9y$ **q** $3a + 3b$ **r** $2x + 2y$
s $6p + 6q$ **t** $7m + 7n$ **u** $4p - 4q$
v $5a - 5b$ **w** $10t - 10w$ **x** $8x - 8y$

2 **a** $4a + 6$ **b** $15x + 5$ **c** $8m + 12$
d $18n + 12$ **e** $10m - 5$ **f** $16n - 20$
g $10p - 4$ **h** $35t - 21$ **i** $12 + 18m$
j $7 + 28x$ **k** $9 + 12n$ **l** $8 + 20t$
m $20 - 30x$ **n** $4 - 12p$ **o** $6 - 15w$
p $8 - 16x$ **q** $6a + 4b$ **r** $12m + 3n$
s $20x + 15y$ **t** $10x + 30y$ **u** $10x - 5y$
v $8a - 12b$ **w** $14m - 4n$ **x** $18t - 6w$

3 **a** $x^2 + 2x$ **b** $y^2 + 3y$ **c** $a^2 + 5a$
d $m^2 + 10m$ **e** $a^2 - a$ **f** $m^2 - 5m$
g $n^2 - 3n$ **h** $y^2 - 8y$ **i** $xy + 2x$
j $ab + 2a$ **k** $mt - 3m$ **l** $nx - 6n$
m $ax + a^2$ **n** $x^2 + xy$ **o** $m^2 + mn$
p $tw + t^2$ **q** $k^2 + kl$ **r** $pq - p^2$
s $tw + ty$ **t** $ab - ac$

4 **a** $2a^2 + 4a$ **b** $3x^2 + 15x$ **c** $5n^2 + 10n$
d $4m^2 + 16m$ **e** $3p^2 - 12p$ **f** $4q^2 - 4q$
g $10m^2 - 70m$ **h** $8a^2 - 40a$ **i** $24x - 6x^2$
j $2m + 2m^2$ **k** $9t^2 + 18t$ **l** $25n - 5n^2$
m $2a^2 + 2ab$ **n** $4p^2 + 4pq$ **o** $3x^2 + 3xy$
p $7mn + 7m^2$ **q** $6y^2 + 12y$ **r** $3ab + 6a^2$
s $6m^2 + 15mn$ **t** $12pq - 8p^2$

5 **a** $6x + 6$ **b** $7m + 6$ **c** $2a + 25$
d $7n - 8$ **e** $3a - 20$ **f** $7m - 3$
g $8a + 16$ **h** $5x + 2$ **i** $2y + 17$
j $9x + 12$ **k** $8a - 6$ **l** $12m - 8$

6 **a** $5x + 12$ **b** $6x + 14$ **c** $8a + 13$
d $7m + 8$ **e** $9y$ **f** $7m + 3$
g $22 + 9a$ **h** $5m + 25$ **i** $24 + 3y$
j $12x$ **k** $11p + 14$ **l** $2a + 29$
m $4m + 4n$ **n** $10x$ **o** $5p + q$

Exercise 5:08B

1 **a** $-2x - 6$ **b** $-3a - 15$ **c** $-5y - 5$
d $-9k - 18$ **e** $-4a + 8$ **f** $-2p + 6$
g $-7t + 42$ **h** $-5x + 5$ **i** $-18 - 3x$
j $-63 + 9q$ **k** $-20 - 2y$ **l** $-48 + 6h$

2 **a** $-a - 2$ **b** $-x - 3$ **c** $-p - 7$
d $-q - 1$ **e** $-x + 3$ **f** $-t + 2$
g $-w + 6$ **h** $-y + 1$ **i** $-2x - 7$
j $-3x + 2$ **k** $-4x - 5$ **l** $-7y + 3$

3 **a** $-x^2 - 2x$ **b** $-a^2 - 3a$ **c** $-m^2 + 4m$
d $-n^2 + n$ **e** $-ab - 3a$ **f** $-pq - 5p$
g $-tx + 3t$ **h** $-mn + 10m$ **i** $-2m^2 - 8m$
j $-10a^2 - 25a$ **k** $-6x^2 + 42x$ **l** $-20t^2 - 12t$

4 **a** $8 - 2a$ **b** $6p - 5$ **c** $-a - 5$ **d** $3 - a$
e $6 - 2x$ **f** $6a - 2$ **g** $-2 - 4a$ **h** $3x - 6$
i $4m + 2$ **j** $9x - 3$ **k** $11 - x$ **l** $2y - 2$
m $3m - 1$ **n** $5a - 2$ **o** $10 + 2a$

5 **a** x **b** $2a + 2$ **c** $y + 19$ **d** $a + 24$
e $4m + 34$ **f** $2t + 48$ **g** $-m - 10$ **h** $-y - 34$
i $p - 22$ **j** $2p - 7$ **k** $2t + 2$ **l** $2q - 18$
m $2p + 28$ **n** $5q + 9$ **o** $3m - 14$

6 **a** $2x + 18$ **b** $-2m - 8$ **c** $11a + 12$
d $15a - 68$ **e** $x^2 + 37$ **f** $2a^2 + 6ab - 2b^2$
g $3y^2 - 34y + 2xy - 8x$ **h** $6a^2 + 3bc - 9ac$

Challenge 5:08

1 **a** $a^2 + 6a + 8$ **b** $x^2 + 7x + 10$
c $y^2 + 3y + 2$ **d** $p^2 + 9p + 14$
e $x^2 + 2x - 3$ **f** $q^2 + 2q - 8$
g $a^2 - a - 12$ **h** $m^2 + 3m - 10$
i $y^2 - y - 6$ **j** $x^2 + 2x - 3$
k $n^2 - n - 20$ **l** $a^2 - 9$
m $x^2 - 3x + 2$ **n** $a^2 - 8a + 16$
o $t^2 - 11t + 28$ **p** $x^2 - 11x + 30$

2 **a** $a^2 + 4a + 4$ **b** $m^2 + 10m + 25$
c $x^2 + 14x + 49$ **d** $m^2 + 20m + 100$
e $x^2 - 2x + 1$ **f** $a^2 - 6a + 9$
g $n^2 - 8n + 16$ **h** $y^2 - 16y + 64$

3 **a** $ab + ay + bx + xy$ **b** $px + py + qx + qy$
c $ac - ad + bc - bd$ **d** $mx - my - nx + ny$
e $15 + 3y + 5x + xy$ **f** $ab + 5a + 4b + 20$
g $xy - 3x + 7y - 21$ **h** $tw - 3t - 5w + 15$

Prep quiz 5:09

1 $2x + 6$ **2** $5a - 35$ **3** $p^2 + 4p$
4 $2m^2 - 2mn$ **5** 1, 2, 3, 6 **6** 1, 2, 3, 4, 6, 12
7 1, 2, 4, 5, 10, 20 **8** 3 **9** 4 **10** 5

Exercise 5:09

1 **a** 2 **b** 5 **c** 3 **d** 5 **e** 2
f 2 **g** a **h** x **i** p **j** a

2 **a** $m + 5$ **b** $x + 2$ **c** $a + 1$ **d** $y - 3$
e $k - 2$ **f** $m - 1$ **g** $2x + 3$ **h** $2n + 3$
i $4k + 3$ **j** $3y - 2$ **k** $2x - 5$ **l** $2a - 3$
m $x + 3$ **n** $m + 5$ **o** $4 + x$ **p** $a - c$
q $x - 1$ **r** $1 - q$ **s** $x + 3$ **t** $5 + a$
u $1 - 2x$

3 **a** $2(y + 3)$ **b** $3(x + 2)$ **c** $5(m + 4)$
d $6(k + 2)$ **e** $9(n - 2)$ **f** $3(2a - 1)$
g $4(y - 3)$ **h** $7(2p - 1)$ **i** $5(2 + a)$
j $2(3 + 2p)$ **k** $2(5 + 3q)$ **l** $8(1 + y)$
m $10(2 - m)$ **n** $4(3 - a)$ **o** $3(5 - q)$
p $7(1 - p)$ **q** $6(m + n)$ **r** $5(x + 2y)$
s $6(n - 2m)$ **t** $8(p - q)$ **u** $2(2p - 3)$
v $3(3a + 4)$ **w** $2(4 + 7x)$ **x** $2(5 - 4k)$

4 **a** $x(a + 2)$ **b** $y(x + 5)$ **c** $b(a + 3)$
d $x(9 + y)$ **e** $m(5 - n)$ **f** $q(p - 3)$
g $x(10 - a)$ **h** $w(t - 1)$ **i** $x(a + b)$
j $q(p + r)$ **k** $n(m - p)$ **l** $b(a - c)$
m $x(x + 3)$ **n** $y(y + 2)$ **o** $m(m + 1)$
p $t(t + x)$ **q** $p(p - 5)$ **r** $m(6 - m)$
s $x(2x - 3)$ **t** $l(9 - 7l)$ **u** $m(5 + 2n)$
v $x(7y - 3)$ **w** $b(3a + 5c)$ **x** $p(8q - 7r)$

5 **a** $5x(1 + 2y)$ **b** $4m(1 + 2n)$ **c** $2p(3 + 2q)$
d $5t(2 + 3w)$ **e** $3m(3n - 1)$ **f** $6q(p - 2)$
g $4t(2 - w)$ **h** $3a(2 - b)$ **i** $2x(x + 2)$
j $3m(3m - 2)$ **k** $5n(2 + n)$ **l** $4q(2p - q)$
m $3n(m + 2r)$ **n** $4p(q + 2r)$ **o** $3b(2a + c)$
p $2x(4y + z)$ **q** $3x(3y - z)$ **r** $6a(b - 2c)$
s $5m(n - 3p)$ **t** $4w(2t - x)$ **u** $ab(a - 1)$
v $5x(x - 2y)$ **w** $3m(3n + 2m)$ **x** $4q(p - 5q)$

6 **a** $3(x + y + z)$ **b** $6(a + b - c)$
c $5(m + n + 1)$ **d** $5(a + 2b - 3c)$
e $3(2p - q + 3r)$ **f** $3(4x + 2y - 3)$
g $3(a^2 + a + 1)$ **h** $5(m^2 + 2m + 3)$
i $2(2p^2 - p + 4)$ **j** $a(x + y + z)$
k $x(p + q + r)$ **l** $t(5 + w - x)$
m $5x(x - 2 + 3y)$ **n** $3m(2 + 5n + 3p)$
o $5a(2a + 3b - 1)$

7 **a** $-2(x + 2)$ **b** $-6(m + 2)$ **c** $-3(p + 3)$
d $-5(p - 2)$ **e** $-3(q - 1)$ **f** $-7(n - 3)$
g $-10(2 + a)$ **h** $-4(3 + p)$ **i** $-3(3 - 2q)$
j $-6(a + 2b)$ **k** $-5(m - 3n)$ **l** $-2(5a - b)$
m $-x(x - 3)$ **n** $-a(x + 5)$ **o** $-3x(2a - b)$

Diagnostic test 5

1 **a**

S	0	2	4	6
P	4	8	12	16

b

P	1	2	3	4
F	0	3	6	9

2 **a** $y = 4x$ **b** $b = 2a + 2$
3 **a** $8x$ **b** $9xy$ **c** $9x^2$
4 **a** $8m$ **b** $2ab$ **c** a^2
5 **a** $9x$ **b** $9a$ **c** $5m$
6 **a** $-3a$ **b** m **c** $-5x$
7 **a** $8x + 2y$ **b** $4m + 2n$ **c** $10a + b$
8 **a** $7m - 3n$ **b** $-a + 3b$ **c** $2m - 2n$
9 **a** $18m$ **b** $15x$ **c** $28k$
10 **a** $3mn$ **b** $18ab$ **c** $35xy$
11 **a** $-14x$ **b** $-15m$ **c** $6xy$
12 **a** $2x$ **b** 3 **c** $3x$
13 **a** a **b** $2p$ **c** $3y$
14 **a** a^5 **b** x^3y **c** $8m^4$
15 **a** $x \times x \times x \times x \times x$
b $x \times x \times x \times x \times y \times y \times y \times y \times y$
c $6 \times a \times a \times a \times a \times a \times a \times b$
16 **a** $3x + 6$ **b** $5a - 35$ **c** $a^2 + 3a$
17 **a** $m + 9$ **b** $7x - 3$ **c** $5x + 5$
18 **a** $-3x - 9$ **b** $-2a + 10$ **c** $-x^2 - 7x$
19 **a** $3x + 13$ **b** $2a + 14$ **c** $x - 14$
20 **a** $2(x + 4)$ **b** $5(y - 1)$ **c** $6(1 + 2m)$
21 **a** $x(a + 3)$ **b** $m(6 - n)$ **c** $y(y + 3)$

ASSIGNMENT 5A

1 **a** $y = 2x + 5$

x	0	1	2	3
y	5	7	9	11

b $y = 2x - 5$

x	0	1	2	3
y	−5	−3	−1	1

c $y = 7 - 3x$

x	0	1	2	3
y	7	4	1	−2

d $y = x^2 + x - 1$

x	0	1	2	3
y	−1	1	5	11

2 **a** $10m$ **b** $9n$ **c** $3y$ **d** $-t$
e $6p + q$ **f** $11xy$ **g** $4a^2$ **h** $3m^2 + 3m$
i $-12x$ **j** $15m$
3 **a** $20m$ **b** $20mn$ **c** $35y^2$ **d** $4t$ **e** 5
f $2x$ **g** $25a$ **h** $36a$ **i** $2x$ **j** $22m$
4 **a** $4x$ **b** $\frac{3x}{2}$ **c** $\frac{x}{3}$ **d** $\frac{2x}{3}$ **e** 4
f $\frac{5y}{3z}$
5 **a** $5a + 35$ **b** $7y - 28$ **c** $m^2 + 3m$
d $6y - ay$ **e** $5ax + 5x$ **f** $18xy - 27y^2$
6 **a** $-5a - 10$ **b** $-7z + 49$ **c** $-m^2 + 3m$
7 **a** $8n + 6$ **b** $11y - 4$ **c** $-5a - 10$
d $8a + 22$ **e** $10y + 14$ **f** $m^2 + 7m + 15$
8 **a** $6 - 2n$ **b** $3y + 4$ **c** $30 - 5a$
d $2 - 2a$ **e** $6y + 34$ **f** $m^2 - 3m - 15$

9 **a** $4(m+3)$ **b** $5(3y-2)$ **c** $6(2+n)$
d $4(2x-y)$ **e** $a(a+3)$ **f** $5k(k-2a)$
g $4x(3xy+2z)$
10 **a** $-4(m+3)$ **b** $-5(3y-2)$ **c** $-6(2-n)$
d $-4(2x+y)$

ASSIGNMENT 5B

1 98 mm
2 More than one answer is possible. One solution is:

Round 1	1 v 2, 3 v 4
Round 2	1 v 3, 2 v 4
Round 3	1 v 4, 2 v 3
Round 4	2 v 1, 4 v 3
Round 5	3 v 1, 4 v 2
Round 6	4 v 1, 3 v 2

3 **a** 30 cm; 4 **b** 70 cm
4 **a** 5 **b** 11 **5** **a** 26 **b** 155

ASSIGNMENT 5C

1 **a** 8·5 **b** 7·9 **c** 17
2 **a** $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$ **d** $\frac{18}{25}$
3 **a** 5% **b** 27% **c** 100% **d** 110%
4 **a** 30% **b** 36% **c** 35% **d** $27\frac{1}{2}\%$
5 **a** 0·15 **b** 0·07 **c** 1·08 **d** 0·74
6 **a** 37·5% **b** 2·5% **c** 93·75% **d** 0·1075
7 **a** \$4.20 **b** 27·6 L **c** 15 750 **d** \$967.50
8 **a** $33\frac{1}{3}\%$ **b** 156 **c** 8·9% **d** 17·5%
9 **a** False **b** False **c** False **d** True
e True **f** True **g** False **h** True
i False
10 **a** 0·375 **b** $0\cdot\dot{6}$ **c** $0\cdot1\dot{6}$ **d** 0·15
e $0\cdot\dot{2}$ **f** $0\cdot\dot{4}28571\dot{4}$

Chapter 6

Prep quiz 6:01

1 A **2** C **3** B **4** A **5** A **6** B
7 In most classes, C **8** B **9** A
10 Answers will vary.

Exercise 6:01

1 **a** 1, 2, 3, 4, 5, 6, 7, 8
b 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
c (blue and blue), (blue and green), (blue and red), (green and red), (green and green)
d (5 of diamonds and 5 of hearts), (5 of diamonds and 10 of diamonds), (5 of diamonds and 9 of diamonds), (5 of diamonds and 6 of diamonds), (5 of hearts and 10 of diamonds), (5 of hearts and 9 of diamonds), (5 of hearts and 6 of diamonds), (10 of diamonds and 9 of diamonds), (10 of diamonds and 6 of diamonds), (9 of diamonds and 6 of diamonds)
2 **a** very unlikely **b** very likely **c** unlikely
d even chance **e** impossible **f** certain
3 **a** very unlikely **b** very likely
c unlikely (occurs about 28% of the time)
d likely **e** impossible **f** certain
4 **a** likely **b** unlikely **c** very unlikely
d certain; c(red), b(green), a(blue), d(coloured)
5 **a** very unlikely
b unlikely (occurs 3 out of 10) **c** certain
d impossible; *d*(both are larger than 9), *a*(are both fives), *b*(have a sum equal to 15), *c*(have a sum greater than 9)
6 **a** Answers will vary; however, any 5 from the following might apply: who wins toss; recent form of the players; the country where it is played; the condition of the wicket; the weather; the ability of the captains.
b Answers will vary.
c Yes **d** No, because it may be a draw.
7 **a** Unreasonable **b** Reasonable
c Reasonable **d** Unreasonable
e Unreasonable **f** Reasonable
g Unreasonable
8 **a** If the horse was truly a certainty, then it could not have been beaten.
b Difference between numbers is 100 000! (Mine was not the next ticket sold.) 'Closeness' to winning a lottery does not equate with proximity of numbers, but rather to physical position of balls or tickets in the barrel, etc.
c If it was a truly impossible position, then winning would not have been possible.
Note: All of these expressions are not meant to be interpreted literally. They involve emotive exaggerations (i.e. poetic licence) in order to stress a point of view.
9 This statement is false because some letters (such as the X) are rarely used to begin names.

Exercise 6:02

1 **a** mutually exclusive **b** non-mutually exclusive
c non-mutually exclusive **d** mutually exclusive
e non-mutually exclusive
f non-mutually exclusive
g non-mutually exclusive
2 **a** 3D, 4D, 5D, 6D
b 3H, 4H, 5H, 6H, 3D, 4D, 5D, 6D
c 3H, 4H, 5H, 6H, 3D, 4D, 5D, 6D, 3C, 4C, 5C, 6C, 3S, 4S, 5S, 6S
d KH, KD, KC, KS, QH, QD, QC, QS
e JH, JD, QH, QD, KH, KD **f** KC, KS
g AC, 2C, 3C, 4C, 5C, 6C, 7C, 8C, 9C, 10C, AS, 2S, 3S, 4S, 5S, 6S, 7S, 8S, 9S, 10S
h AC, 2C, 3C, 4C, 5C, 6C, 7C, 8C, 9C, 10C, JC, QC, KC

3 **a** $\frac{4}{52}$ **b** $\frac{8}{52}$ **c** $\frac{16}{52}$ **d** $\frac{8}{52}$
e $\frac{6}{52}$ **f** $\frac{2}{52}$ **g** $\frac{20}{52}$ **h** $\frac{13}{52}$

4 **a** $\frac{1}{6}$ **b** $\frac{2}{6}$ (or $\frac{1}{3}$) **c** 0 **d** 1 (or $\frac{6}{6}$)
e $\frac{2}{6}$ (or $\frac{1}{3}$) **f** $\frac{5}{6}$ **g** $\frac{5}{6}$ **h** $\frac{2}{6}$ (or $\frac{1}{3}$)
i $\frac{5}{6}$ **j** 1 (or $\frac{6}{6}$)

5 **a** 4, 5, 6, 7, 8 **b** 3, 4, 5, 6, 7, 8 **c** 7
d 1, 2, 3 **e** 1, 3, 5, 7
f There are no outcomes in this event. It is an empty set.
g 1, 3 **h** 2, 4 **i** 1, 2, 3, 4, 6, 7, 8
j 1, 2, 3, 7, 8

6 **a** $\frac{5}{8}$ **b** $\frac{6}{8}$ (or $\frac{3}{4}$) **c** $\frac{1}{8}$ **d** $\frac{3}{8}$
e $\frac{4}{8}$ (or $\frac{1}{2}$) **f** $\frac{0}{8}$ (or 0) **g** $\frac{6}{8}$ (or $\frac{3}{4}$)
h $\frac{2}{8}$ (or $\frac{1}{4}$) **i** $\frac{7}{8}$ **j** $\frac{5}{8}$

7 **a** $\frac{9}{100}$ **b** $\frac{8}{100}$ **c** $\frac{1}{100}$
d $\frac{0}{100}$ (or 0) **e** $\frac{9}{100}$ **f** $\frac{92}{100}$

Fun spot 6:02

1 50% (or $\frac{1}{2}$) **2** 40% (or $\frac{2}{5}$)
3 50% (or $\frac{1}{2}$) **4** 75% (or $\frac{3}{4}$)
5 $87\frac{1}{2}$% (or $\frac{175}{200}$ or $\frac{7}{8}$) **6** $\frac{7}{12}$ (or $58\frac{1}{3}$%)
7 $\frac{13}{24}$ (or $54\frac{1}{6}$%) **8** $\frac{24}{30}$ (or $\frac{4}{5}$ or 80%)

Prep quiz 6:03

1 $\frac{1}{10}$ **2** $\frac{9}{10}$ **3** $\frac{3}{10}$ **4** $\frac{7}{10}$
5 $\frac{2}{10}$ (or $\frac{1}{5}$) **6** $\frac{8}{10}$ (or $\frac{4}{5}$) **7** $\frac{4}{10}$ (or $\frac{2}{5}$)
8 $\frac{6}{10}$ (or $\frac{3}{5}$) **9** 0 **10** 1 **11** 1

Exercise 6:03

1 **a** It won't rain tomorrow.
b I won't win the race.
c A six will not be rolled.
d I will catch a cold next year.
e That team will beat us.
f Alice will not be school captain.

2 **a** The egg will not break.
b The plate will break.
c We will not score 200 runs.
d We will toss no heads.
e I will not win the raffle.
f We will not win the soccer match.

3 **a** 98% **b** $\frac{1}{3}$ **c** 60%
d 0·01 **e** 0 **f** $\frac{3}{10}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 1 **d** 0

5 **a** $\frac{16}{17}$ **b** $\frac{7}{10}$ **c** $\frac{133}{170}$

6 **a** $\frac{3}{4}$ **b** $\frac{7}{8}$ **c** $\frac{17}{20}$ **d** 98%
e 79% **f** $\frac{797}{800}$

7 **a** $\frac{37}{200}$ **b** $\frac{163}{200}$ **c** $\frac{43}{200}$ **d** $\frac{157}{200}$
e $\frac{3}{5}$ **f** $\frac{2}{5}$ **g** 1

8 **a** $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{2}{5}$ **d** $\frac{3}{5}$

9 **a** $\frac{43}{100}$ or 43% **b** $\frac{18}{25}$ or 72%
c $\frac{4}{5}$ or 80% **d** $\frac{51}{100}$ or 51%
e $\frac{57}{100}$ or 57% **f** $\frac{7}{25}$ or 28%

10 **a** Yes. The probabilities of the given ticket colours combined do not add up to 1.
b **i** 0·625 **ii** 0·875 **iii** 0·75 **iv** 0·25

11 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{13}$ **d** $\frac{12}{13}$ **e** $\frac{3}{13}$
f $\frac{10}{13}$ **g** $\frac{3}{13}$ **h** $\frac{10}{13}$ **i** 1

12 **a** $\frac{27}{53}$ **b** $\frac{1}{53}$

13 **a** $\frac{61}{170}$ **b** $\frac{109}{170}$ **c** $\frac{44}{85}$ **d** $\frac{41}{85}$
e 170 or 340 stamps, etc. The total number of stamps in this collection must be some multiple of 170 as there have to be 37 French stamps in every 170 stamps present.

14 **a** $\frac{223}{800}$ **b** $\frac{577}{800}$ **c** 46% or $\frac{23}{50}$ **d** 54% $\frac{27}{50}$
e 800 or 1600. The total number of coins in this collection must be a multiple of 800 since the probability with the largest denominator is $P(\text{Chinese}) = \frac{3}{800}$, so there must be 3 Chinese coins in the collection for every 800 coins present.

Fun spot 6:03

1 **a** $\frac{1}{6}$ **b** 0 **c** $\frac{1}{6}$ **d** $\frac{1}{3}$

2 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** 0 **d** $\frac{1}{3}$

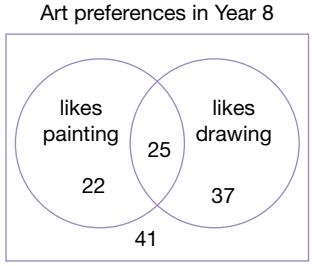
Exercise 6:04

1 **a** 25 **b** 30 **c** 70 **d** 100
e 95 **f** 0 **g** 55

2 **a** 2 **b** 10 **c** 12

3 **a** 15 **b** 38 **c** 80 **d** 30
e 45 **f** 37

4



5 **a** 5 **b** 57 **c** 60

6 **a** 17 **b** 15 **c** 53

7 **a** Some people belonged to both clubs.
b 18 **c** 57

8 23

9 **a** 1 **b** 3 **c** 6 **d** 7
e 17 **f** 6 **g** 21 **h** 22
10 **a** 3 **b** 5 **c** 2
11 **a** **i** 17 **ii** 3 **iii** 27 **iv** 6
v 23 **vi** 5
b **i** 36 **ii** 20 **iii** 65
12 **a** 13 **b** 25 **c** 11 **d** 21
e 0 **f** 22 **g** 119

Exercise 6:05

1 **a** **i** 72 **ii** 53 **iii** 14 **iv** 111
v 6 **vi** 119 **vii** 64 **viii** 61
ix 64 **x** 6 **xi** 119 **xii** 61
b No. About 11% of males and 11% of females are left-handed in this table.
c No. 125 people is not a large enough study to form a conclusion.

2 **a** 49 **b** 51 **c** 55 **d** 45
e 31 **f** 24

	run	do not run	total
female	24	25	49
male	31	20	51
total	55	45	100

3 **a**

	play cricket	do not play cricket	total
play hockey	0	37	37
do not play hockey	15	45	60
total	15	82	97

b

	going to Uluru	not going to Uluru	total
going to Perth	37	3	40
not going to Perth	15	45	60
total	52	48	100

4 **a** 16 **b** 54 **c** 28 **d** 2 **e** 100
f 44 **g** 56 **h** 18 **i** 82 **j** 54
k 46 **l** 30 **m** $\frac{18}{100}$ (or 18%)
n $\frac{44}{100}$ (or 44%) **o** $\frac{2}{100}$ (or 2%)

5 **a** 49 **b** 51 **c** 45 **d** 31

	study mathematics	do not study mathematics	total
play guitar	24	25	49
do not play guitar	31	20	51
total	55	45	100

6

	have a son	do not have a son	total
have a daughter	20	15	35
do not have a daughter	10	5	15
total	30	20	50

a 20 **b** 15

Diagnostic test 6

1 **a** 1, 2, 3, 4, 5, 6 **b** 2, 4, 6, 8
c 4, 8, 12, 16, 24, 32, 36, 48, 64
2 **a** non-mutually exclusive **b** mutually exclusive
3 **a** $\frac{4}{6}$ (or $\frac{2}{3}$) **b** $\frac{2}{6}$ (or $\frac{1}{3}$)
4 **a** 1 **b** Year 8 will not win the competition.
c 95% **d** $\frac{3}{4}$ (or 75% or 0·75)
5 **a** 24 **b** 19 **c** 34 **d** 6
6 **a** 77 **b** 63 **c** 33 **d** 42 **e** 138

ASSIGNMENT 6A

1 **a** unlikely (or very unlikely) **b** impossible
c even chance **d** very unlikely
2 **a** $\frac{4}{8}$ (or $\frac{1}{2}$ or 50%) **b** $\frac{3}{7}$ **c** $\frac{2}{6}$ (or $\frac{1}{3}$)
d $\frac{1}{5}$ (or 20%) **e** 0 (or $\frac{0}{4}$)
3 **a** $\frac{1}{13}$ (since there are four 4s) **b** $\frac{12}{13}$
c $\frac{5}{13}$ **d** $\frac{10}{52}$ (or $\frac{5}{26}$) **e** $\frac{3}{13}$ **f** $\frac{3}{52}$
4 **a** $\frac{1}{53}$ **b** $\frac{52}{53}$ **c** $\frac{26}{53}$ **d** $\frac{10}{53}$
e $\frac{12}{53}$ (not counting the Joker) **f** $\frac{3}{53}$
5 **a** 'Year 8 will not win its next game.' **b** 80%
c $\frac{497}{500}$ **d** 0·999 (or 99·9%)
6 **a** 10 **b** 12 **c** 12 **d** 5 **e** 9
f 2 **g** 2 **h** 4 **i** 10 **j** 8
k 8 **l** 3

7

Year 8	can swim	cannot swim	total
live near beach	51	6	57
do not live near beach	59	4	63
total	110	10	120

a 110 **b** 10 **c** 120 **d** 6

ASSIGNMENT 6B

1 a The only reasonable answer is 3 sound systems because \$5683.89 divided by 3 is equal to \$1894.63. Dividing \$5683.89 by any other reasonable number does not give a whole number of cents.
b \$1894.63
2 a As one wheel turns it forces the other wheels to turn because the wheels are interlocked. If a wheel is turning clockwise, it will force the interlocked wheel to turn anticlockwise.
b There would be about 90 cogs on this wheel. The method used would be to count the number of cogs on a quarter of the wheel and multiply by 4.
3 about 5·6 cm (assuming that a real soldier has a height of about 180 cm)
4 14 times
5 4 (1 by 24, 2 by 12, 3 by 8 and 4 by 6)
6 a 28 cm **b** 105 cm^2

ASSIGNMENT 6C

1 A and C **2 a** 11·18 **b** 7·94
3 a yes **b** 10 **4** yes, yes
5 6·3 cm (1 dec. pl.)
6 a 37·5% **b** 87·5%
7 a 0·045 **b** 0·125
8 a 98 g **b** $\frac{2}{15}$ **c** 6·25% **d** \$13.80
e \$10.20
9 \$2100

10

x	0	1	2	15
y	11	15	19	71

11 $y = 10x + 1$
12 a 2 m **b** $7x$ **c** $4ab$ **d** $5x^2$
e $9a + b$ **f** $-7a$ **g** $-18y$ **h** $28ab$
i 8 **j** $3a$ **k** a^2 **l** $6a^2$
13 a 66 **b** 149 **c** −9 **d** −72
14 a m^4 **b** $5a^3$ **c** $12x + 12$ **d** $18m - 13$
e $\frac{3x}{10}$ **f** $\frac{5m}{6}$ **g** $\frac{2a^2}{15}$ **h** $\frac{5x}{4}$
15 a $2(x + 7)$ **b** $m(m - 3)$ **c** $11(2 - 3y)$
d $3a(3b + 4a)$

Chapter 7

Exercise 7:01

1 a 50t **b** 25t **c** 150t **d** 75t
2 a i NSW **ii** SA
b compare answers with table
3 a hockey = 80, netball = 115, soccer = 70, volleyball = 45
b 25 **c** 210 **d** 18

4

Music	Number (1 disc = 10 people)
Rock	
Country	
Pop	
Folk	
Jazz	
Classical	

5 a $9\frac{1}{4}$
b Australian New Car Sales Figures

Toyota	
Holden	
Mazda	
Nissan	
Hyundai	
Ford	

6 a apples = 45, oranges = 25, bananas = 20, grapes = 30
b apples, grapes, oranges, bananas
c no
d Because the size of the fruit symbols is different it might give the impression that bananas are the second most popular fruit and that grapes are as popular as apples.

Fun spot 7:01

The centre number should be 83.

Exercise 7:02

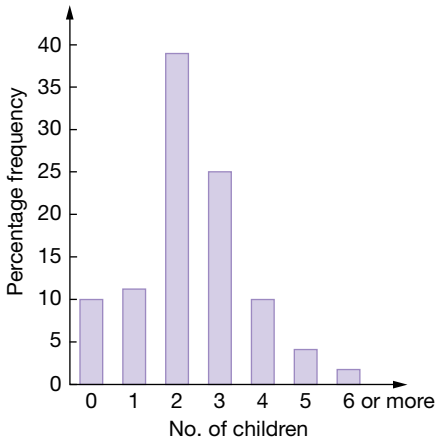
1 a Percentage of class that preferred each pet.
b Dog
c No. There could be one pet in the 'Other' category that is more popular than cats.
d 50 **e** 60

2 a

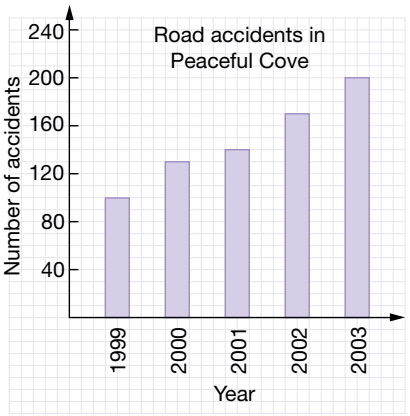
Colour	Red	Green	Yellow	Orange	Blue	Brown
Number	25	18	24	15	20	18

b Green and Brown **c** 15 **d** 120 **e** 15%
3 a 7
b 'Number of Children' and 'Percentage Frequency'
c 38 **d** say, 1 unit = 5%

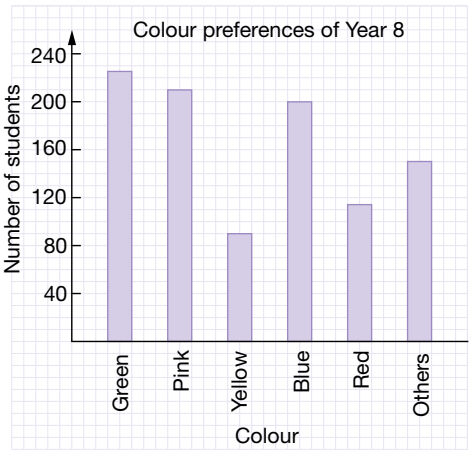
e



4 a



b



5 a April, $12 000 b March
c loss of $11 000
d Word spread that her pastries were delicious.
e February f $20 000

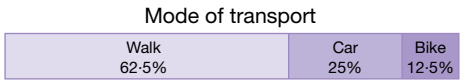
6 a It is the best but not 'by far'. The vertical axis does not start at zero and so gives an exaggerated impression.
b Start the vertical axis at zero.
c It is showing the information falsely.

Exercise 7:03

1 a more wallabies b 5 cm c 3 cm
d 60% e 15

2 a 40% b 5%

3 a 480
b walk = 62·5%, car = 25%, bicycle = 12·5%
c walk = 5 cm, car = 2 cm, bicycle = 1 cm
d



4 a i 96 mm ii 48 mm iii 45 mm
b 1 c 16 times d 16 e 15 times
f i 50% ii $3\frac{1}{8}$% iii $46\frac{7}{8}$%

5 a A b B

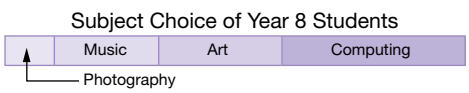
6 a 1 mm b no c foodstuffs and machinery
d 15%

7 a 39% b 7%
c You cannot say it is safer to be a motor cyclist because the graph gives no idea of the relative numbers of drivers and motor cyclists. As there are many more motorists than motor cyclists, it is possible that the fatality rate of drivers is considerably less than that for motor cyclists.

8 a



b



Prep quiz 7:04

1 70% 2 40% 3 37·5% 4 360°
5 200° 6 $\frac{1}{4}$ 7 $\frac{1}{6}$ 8 $\frac{7}{12}$
9 90 10 $\frac{7}{12}$

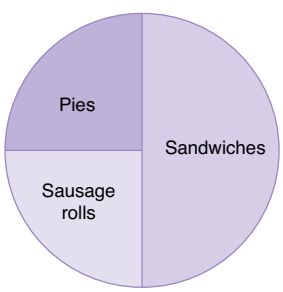
Exercise 7:04

1 a 10% b 12·5% c 23° d 62·5%

2 a 72° b 20% c 60 000 hectares

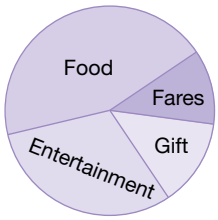
3 a 252° b 70% c 30% d 13%

4 a i 10° b i $\frac{1}{2}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$
c i 180° ii 90° iii 90°
d

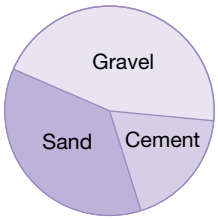


5 a 24%
b Water heating = 126°, Room heating = 86°, Other = 61°, Cooking = 61°, Lighting = 25°
c $184.80

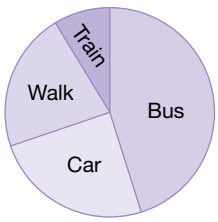
6 **a** Rhonda's expenses



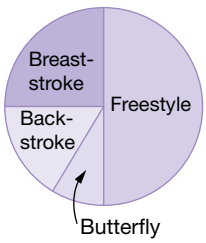
b Composition of concrete mix



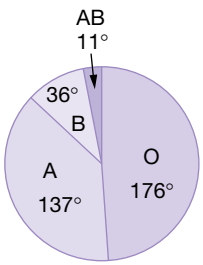
c Method of travel to school



d Favourite swimming stroke



7



8 **a** C **b** C

9 **a** Diplomatic

b No. The amount of water is not given, only the percentages.

c 57·6° Yes, as accurate as can be expected.

d 2·88°

Exercise 7:05

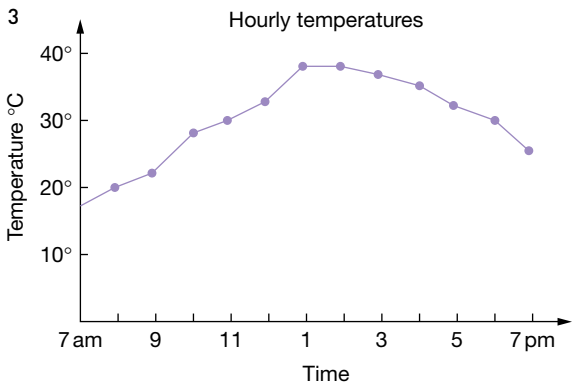
1 **a** 3000 g **b** 4400 g **c** 900 g **d** 500 g

e 0–2 weeks

2 **a** 8° **b** 11·15 **c** 16·5° **d** 3 h

e 9 am to 10 am; 11 am to 12 noon

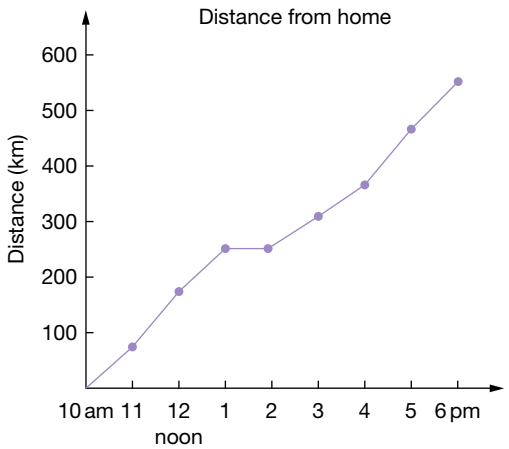
3



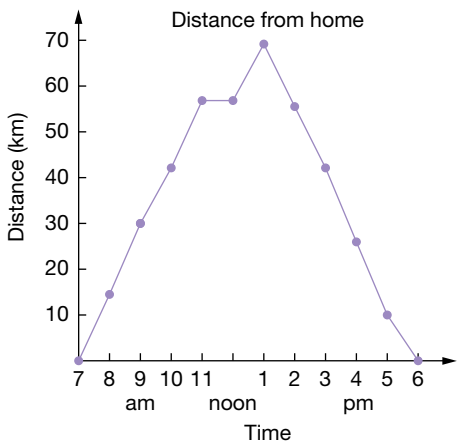
4 **a** 14 L **b** 2 L **c** 10 am and 1 pm

d None. The car was stopped. **e** 30 L

5



6

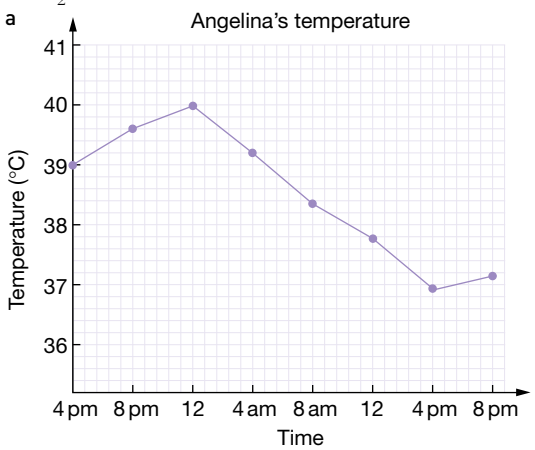


7 **a** 15 m **b** 0·6 s and 3·4 s **c** 20 m **d** 2 s

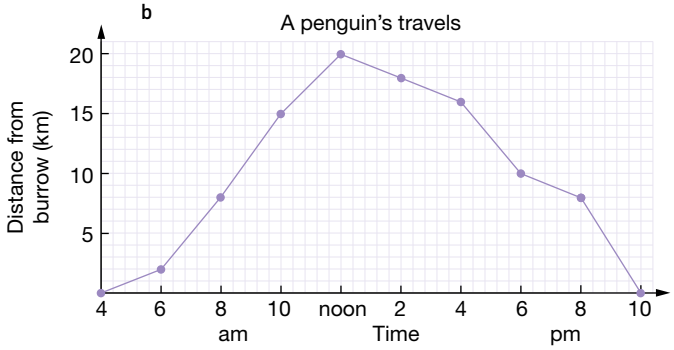
e 1·4 s

8 **a** $2\frac{1}{2}$ min **b** 45°C **c** 28°C **d** 73°C

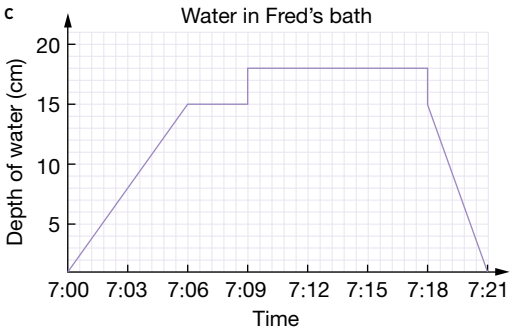
9 **a**



b



c



10 **a** line graphs **b** 2% **c** 80% **d** 18%
e unskilled, 65%; mid-level, 29%; professional, 6%
f unskilled, 20%; mid-level, 62%; professional, 18%
g No. You would not expect the lines joining the points for 1900 and 1980 to be perfectly straight.
h 0%
i Yes; at schools, colleges of advanced education, and universities, to name a few.

11 **a** 60 **b** 80 (or 81) **c** 4 **d** 9 **e** 8
f about 250 **g** about 235 **h** Aussies
i Aussies, 20 runs **j** 0 runs **k** 40th
l 30th

Investigation 7:05

1 **a** $5 **b** yes **c** yes, 6 kg
d They have different scales on at least one axis.

2 **a** 100 **b** 0 **c** 1 person **d** 5 people
e graph A **f** yes

3 **a** July, Sept, Oct, Nov are left off the horizontal axis and space is not left for them. This makes it look like Adam's savings are increasing faster than they really are.
b The vertical axis is broken between 40 and 300. This makes it look like there is not much difference between Sharon's savings and Bronwyn's savings.

4 **a** Graph B **b** Graph A **c** Graph B

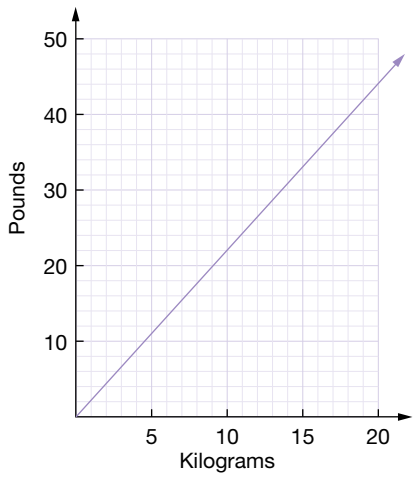
Exercise 7:06

1 **a** **i** 50 **ii** 67 **iii** 20 **iv** 27
b **i** 48 **ii** 54 **iii** 34 **iv** 23
c 17 **d** 12

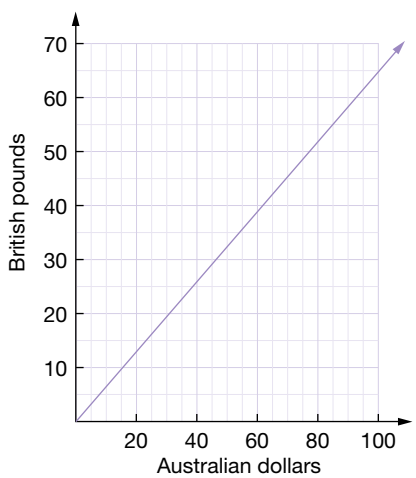
2 **a** 32°F **b** 212°F **c** 77°F **d** 122°F
e 59°F **f** 230°F **g** 0°C **h** 80°C
i 71°C **j** 40°C **k** −9°C **l** −18°C

3 **a** 700 yen **b** 350 yen **c** 210 yen
d 460 yen **e** 2100 yen **f** 5300 yen
g 21 000 yen **h** 53 000 yen **i** $4
j $6 **k** $9 **l** $3
m $6 **n** $90 **o** $40
p $300

4



5



Prep quiz 7:07

1 10 **2** 10 **3** 10 **4** $3.60 **5** $3.60
6 $5.40 **7** {2, 3, 4} **8** {2, 3}
9 {1, 2, 3} **10** {1, 2, 3, 4}

Exercise 7:07

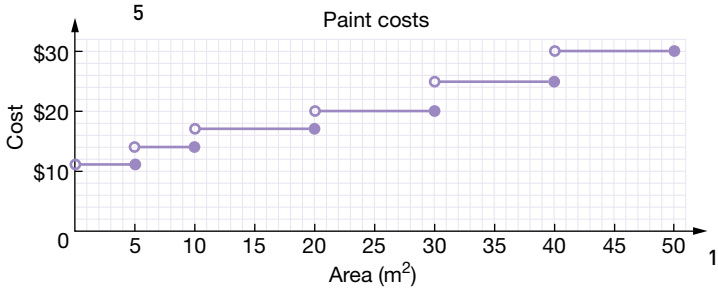
1 **a** $4 **b** 3 hours **c** $8 **d** $2
e The graph is made up of step-like intervals.

2 B

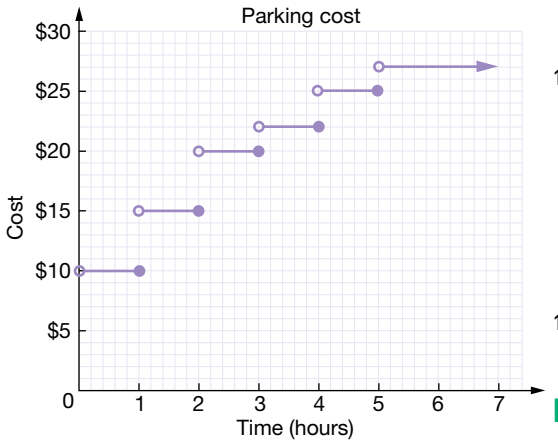
3 **a** **i** $17 **ii** $9 **iii** $24
b **i** 5 kg **ii** 30 kg **iii** 40 kg
c $37.20

4 **a** **i** $10 **ii** $20 **iii** $30
b up to $900 **c** no

5



6



Exercise 7:08

1 a 9:01 am b 2 cm c 9:03 am d 6 cm
e 9:04 am
f It would drop. The line would begin to drop.

2 a 20 cm b 28 cm c 3 min
d Because the drum is full and running over.
e after 10 minutes f 40 cm

3 a column graph b June/July c 32
d 16 e 38, August/September
f Some workers may have been on holidays.
g 133

4 a 5 pm, 5 b 30 min c 11
d 6 pm, 15 e 9 pm, 5 f 1 am
g answers vary

5 a 1980/81
b i 450 million tonnes ii 500 million tonnes
iii 600 million tonnes iv 700 million tonnes
c 1986/87, US$112

6 a divided bar graph b oil c 49%
d about 5%
e oil, solid fuels, natural gas, water power, other
f No; other forms of energy have been developed, e.g. solar. g solar and perhaps nuclear

7 a divided bar graph
b The picture catches our interest and tells that sheep are involved.
c 17·41% d 12·36% e 95%
f Australia's Wool Export

8 a line graph b number c no
d It's meant to create an impression.
e #100 f about #45

9 a i 25 ii 49 b line graph
c i 90° ii 15° d 10°C, 22°C
e i 18°C ii 13°C iii 20°C
f 8 am, 10 pm
g 13°C, 21°C

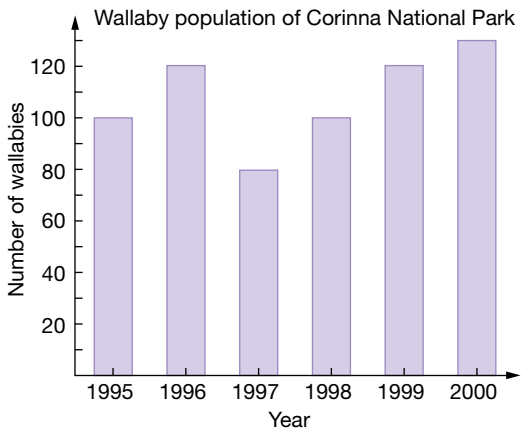
10 a i 0–4 years ii 15–64 years
b i 1200 ii 2700 iii 10 300 iv 1700
c 15 900
d 65 and over. Females tend to live longer than males.

11 a You can see what the figures mean in terms of how far ahead in a race each new record holder would be compared to the previous record.
b A horizontal column graph in which the time taken is the length of each column and the left-hand side of the graph has been cut off.
c Jim Hines d 0·09 s e 6
f 0·46 s g 0·01 s h research

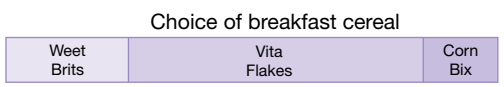
12 a 100 cm b 70 cm c 10 cm d 90 cm
e about 65 cm f about 44 cm

Diagnostic test 7: Graphs and tables

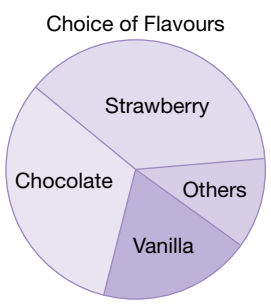
1



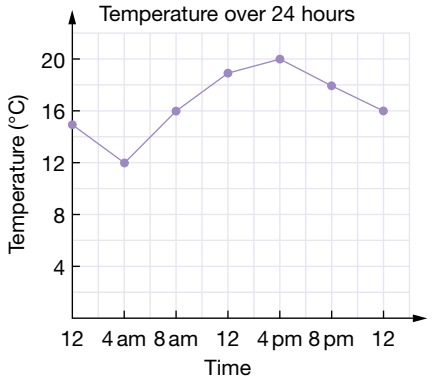
2



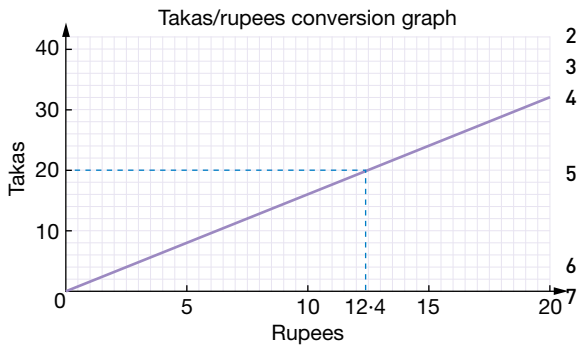
3



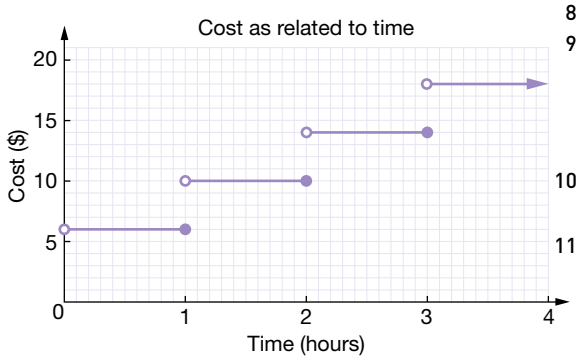
4



5



6



ASSIGNMENT 7A

1 **a** sector graph **b** expansion **c** 109°
d 21 $\frac{17}{18}$ % or 21·94%

2 **a** **i** 60% **ii** 24% **iii** 6% **iv** 10%
b 2955

3 **a** June **b** July; 24°C **c** 4 **d** 144 mm

4 **a** **i** 6% **ii** 16% **iii** 10% **iv** 5%
v 48% **vi** 13%
b airports, motels, sporting events etc.
c 43% **d** $\frac{1}{4}$ **e** Sunday
f Clever way to show sector graphs, reminding reader of cars.
g **i** 18 **ii** 12 **iii** 2 **iv** 8 **v** 12
vi 18
h no

5 **a** 15 m, after 8 s **b** after 4 s **c** after 10 s
d 2 seconds

ASSIGNMENT 7B

1 31

2 **a** B **b** G **c** J

3 ($1, $1, $1, 20c, 10c, 10c) or
($1, $1, 50c, 50c, 20c, 20c) or
($2, $1, 20c, 10c, 5c, 5c) or
($2, 50c, 50c, 20c, 10c, 10c)

4 **a** 11 **b** 7 **c** $(70 \times 3) + 2 = 212$ **d** 302

5 4 horses

ASSIGNMENT 7C

1



2 **a** 123 **b** 44 **c** 1609 **d** 1988

3 **a** 94 **b** 0 **c** −2 **d** 5 **e** 72 185

4 **a** $1\frac{6}{10}$ or $1\frac{3}{5}$ **b** $\frac{31}{40}$ **c** $\frac{13}{20}$ **d** $\frac{9}{20}$
e $\frac{17}{200}$ **f** $1\frac{1}{2}$ **g** $1\frac{1}{5}$ **h** $\frac{3}{20}$

5 **a** 23·05 **b** 6·85 **c** 0·415 **d** 0·65
e 1·5 **f** 1·0325 **g** 230 **h** 0·0025
i 0·67

6 **a** 75% **b** 140% **c** 14·5% **d** 205%

7 **a** $3440 **b** 104 km **c** $1680 **d** $1596
e $650

8 **a** 6·3 **b** 5·3

9 **a** $11x$ **b** a **c** $8x^2$ **d** 5 m
e $40y$ **f** $10b$ **g** $4a$ **h** $14ab$
i $\frac{2x}{5}$ **j** $\frac{13m}{20}$ **k** $\frac{4xy}{35}$ **l** $\frac{5}{4}$

10 **a** **i** $\frac{1}{20}$ **ii** $\frac{1}{25}$ **iii** $\frac{1}{100}$
b $\frac{17}{20}$

11 **a** $m = 128$, corresponding angles and parallel lines
b $x = 60$, angle sum of a triangle
c $y = 135$, exterior angle of a triangle

Chapter 8

Exercise 8:01

1 **a** 12 m **b** 16 km **c** 22 cm
d 46 m **e** 44 m **f** 11 cm

2 **a** 36·4 m **b** 52·95 m **c** 24·518 m
d 28·6 m **e** 2500 mm **f** 188·8 mm
g 52·2 m **h** 56 026 mm **i** 70·962 m
j 40·9 cm **k** 39·51 m

3 **a** 14·04 m **b** 2252 mm **c** 138 cm
d 22 660 mm **e** 15·544 m

4 **a** The perimeter is doubled.
b The perimeter is trebled.
c A triangle will be drawn with sides 10 m, 12 m, and 12 m.

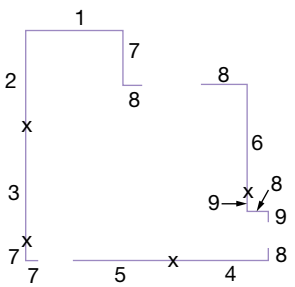
5 **a** 42 cm **b** 56 cm

6 **a** 20 m **b** 14 m **c** 6·5 m

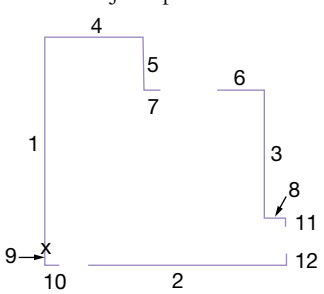
7 **a** 41·2 m **b** 42·4 m **c** 46 m
d 33·318 m **e** 76·4 cm **f** 5824 mm
8 **a** perimeter = 1410 m; cost = $705
b 10·3 km **c** 64
9 **a** 7·9 m **b** 910 rev **c** 393 m **d** 23 laps
10 **a** 327 m **b** 36 **c** $616.65
11 **a** 28·38 m **b** 61 m **c** 112·8 m
d **i** 3150 m **ii** 3600 m **iii** 3000 m
12 **a** $x = 5$; perimeter = 12 cm
b $x = 13$; perimeter = 40 m
c $x = 10$; perimeter = 48 cm
13 **a** $a = 1{\cdot}6$; perimeter = 8 m **b** 66 m
c 67 cm
14 **a** 12 m **b** 48 cm

Investigation 8:01

1 No. One wall is 7600 mm long which is longer than the largest length of skirting board available (6·6 m).
2 Total length required = 27 400 (27·4 m)
3



Note: The following is only one possible solution. The numbers show how the 9 skirting boards are used. Joins are indicated by '×'.
Board 1 is 3200 mm long (100 mm waste).
Boards 2, 3, 4, 5, 6 are 3300 mm long.
Board 7 is cut into 3 pieces (1000 mm, 400 mm, 1800 mm).
Board 8 is cut into 4 pieces (600 mm, 1600 mm, 700 mm, 300 mm).
Board 9 is cut into 2 pieces (900 mm, 400 mm).
If joins at corners are not counted the smallest number of joins possible is 4.
4



Boards 1 and 2 (6·6 m); Board 3 (4·2 m);
Board 4 (3·3 m); Boards 5 and 6 (1·8 m);
Board 7 (600 mm); Board 8 (900 mm);
Board 9 (1·2 m); Boards 10 and 11 (600 mm);
Board 12 (300 mm)

He would have to order: 2 × 6·6 m, 1 × 4·2 m, 1 × 3·3 m, 1 × 1·8 m, 1 × 1·2 m, 1 × 900 mm, 3 × 600 mm, 1 × 300 mm.
This would only require 1 join and give 1·1 m of waste.

Prep quiz 8:02

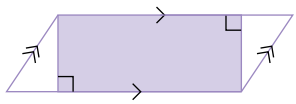
1 36 cm^2 **2** 16 cm^2 **3** 35 cm^2
4 18 cm^2 **5** 48 cm^2 **6** 18 cm^2
7 20 cm^2 **8** 40 cm^2 **9** 21 $units^2$
10 20 $units^2$

Exercise 8:02

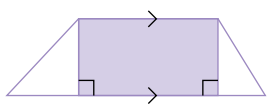
1 **a** 1·26 cm^2 **b** 1·21 cm^2 **c** 25 m^2
d 10·8 cm^2 **e** 28 cm^2 **f** 24 cm^2
g 4·4 cm^2 **h** 12 m^2 **i** 45 cm^2
2 **a** 48 m^2
b **i** 5000 m^2 **ii** 0·5 ha
c 15·6 m^2
d **i** 173 600 m^2 **ii** 17·36 ha
e 7·2 m^2
3 **a** 17 cm^2 **b** 33 cm^2 **c** 6 cm^2
d 35 cm^2 **e** 8 m^2
4 Rectangles with dimensions: 1 cm × 12 cm, 2 cm × 6 cm, 3 cm × 4 cm
5 **a** 47 cm^2 **b** 45 cm^2 **c** 28 cm^2
d 104 cm^2 **e** 104 cm^2 **f** 90 cm^2
g 56 cm^2 **h** 66 cm^2 **i** 60 cm^2
6 **a** 4 **b** 16 **c** 3

Prep quiz 8:03

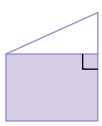
1 square **2** rectangle **3** parallelogram
4 rhombus (or diamond) **5** trapezium
6 cube
7



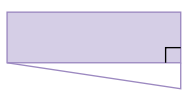
8



9



10



Exercise 8:03

1 **a** 63 cm^2 **b** 78 cm^2 **c** 112 cm^2
d 77 cm^2 **e** 42 cm^2 **f** 120 cm^2
2 **a** 80 cm^2 **b** 24 cm^2 **c** 104 cm^2
3 **a** 104 cm^2 **b** 39 cm^2 **c** 72·5 cm^2
4 **a** 8 cm **b** 40 cm^2 **c** 40 cm^2
5 **a** 4 cm **b** 10 cm^2 **c** 30 cm^2 **d** 40 cm^2

6 **a** $60\,cm^2$ **b** $28\,cm^2$ **c** $27\,cm^2$
d $52{\cdot}5\,cm^2$ **e** $51\,cm^2$ **f** $45\,cm^2$
7 **a** 4 cm **b** $24\,cm^2$ **c** $48\,cm^2$
8 **a** 12 cm **b** 4 cm **c** $48\,cm^2$ **d** $48\,cm^2$
9 **a** 3 cm **b** $30\,cm^2$ **c** 3 cm × 10 cm
d $30\,cm^2$ **e** $30\,cm^2$
10 **a** $40\,cm^2$ **b** $24\,cm^2$ **c** $120\,cm^2$
d $36\,cm^2$ **e** $56\,cm^2$ **f** $200\,cm^2$

Exercise 8:04

1 **a** $56\,cm^2$ **b** $150\,cm^2$ **c** $27\,cm^2$
d $300\,mm^2$ **e** $126\,cm^2$ **f** $325\,mm^2$
2 **a** $63\,cm^2$ **b** $126\,cm^2$ **c** $112\,cm^2$
d $135\,cm^2$ **e** $57\,cm^2$ **f** $80\,m^2$
3 **a** $40\,cm^2$ **b** $42\,cm^2$ **c** $27\,cm^2$
d $25\,cm^2$ **e** $31{\cdot}5\,cm^2$ **f** $110\,mm^2$
4 **a** $96\,cm^2$ **b** $45\,cm^2$ **c** $38{\cdot}5\,cm^2$
d $10\,cm^2$ **e** $21\,cm^2$ **f** $140\,cm^2$
g $42\,cm^2$ **h** $81\,cm^2$ **i** $35\,cm^2$
j $28\,cm^2$ **k** $96\,cm^2$ **l** $90\,cm^2$
5 **a** $630\,cm^2$ **b** $107{\cdot}2\,cm^2$ **c** $24\,cm^2$
6 40 cm

Prep quiz 8:05

1 40 mm **2** 25 mm **3** 50 cm **4** 300 cm
5 575 cm **6** 8000 m **7** 9700 m **8** 7 L
9 23 kL **10** 2500 L

Exercise 8:05

1 **a** 3000 mL, $3000\,cm^3$ **b** 7000 mL, $7000\,cm^3$
c 500 mL, $500\,cm^3$ **d** 9000 mL, $9000\,cm^3$
2 **a** $8000\,mm^3$ **b** $20\,000\,mm^3$
c $4000\,mm^3$ **d** $16\,000\,mm^3$
3 **a** 8 kL **b** 2·5 kL **c** 8 kL **d** 12 kL
4 **a** 7000 kL, $7000\,m^3$ **b** 5200 kL, $5200\,m^3$
c 8000 kL, $8000\,m^3$ **d** 1200 kL, $1200\,m^3$
5 **a** 3000 mL **b** 3 L **c** 2 L
d 3 000 000 L **e** 73 ML **f** $65\,m^3$
g 6 ML **h** 32 ML **i** 0·5 ML
j 2000 kL **k** 250 kL **l** 250 000 L

Prep quiz 8:06

1 rectangular prism **2** triangular prism
3 trapezoidal prism **4** rectangle
5 triangle **6** trapezium **7** $49\,cm^2$
8 $28\,cm^2$ **9** $84\,cm^2$ **10** $70\,cm^2$

Exercise 8:06

1 **a** $30\,cm^3$ **b** $48\,cm^3$ **c** $63\,cm^3$
d $280\,cm^3$ **e** $75\,cm^3$
2 **a** $56\,cm^3$ **b** $128\,cm^3$ **c** $63\,cm^3$
d $72\,cm^3$ **e** $90\,cm^3$ **f** $42\,cm^3$
3 **a** $27\,cm^3$ **b** $125\,cm^3$ **c** $729\,cm^3$
4 **a** $2{\cdot}6\,m^3$ **b** $2\,m^3$ **c** $0{\cdot}648\,m^3$
d $1{\cdot}35\,m^3$
5 **a** $30\,cm^2$ **b** $240\,cm^3$
6 **a** $28\,cm^2$ **b** $252\,cm^3$
7 **a** $54\,cm^3$ **b** $210\,cm^3$ **c** $200\,cm^3$
8 **a** $66\,cm^3$ **b** $140\,cm^3$ **c** $450\,cm^3$
9 **a** $32\,cm^2$ **b** $320\,cm^3$
10 $860\,cm^3$
11 **a** $252\,cm^3$ **b** $560\,cm^3$ **c** $216\,cm^3$
d $36\,cm^3$ **e** $70\,cm^3$ **f** $180\,cm^3$
12 **a** $9\,cm^2$ **b** 5 cm **c** 6 cm
13 **a** 2·5 L **b** 2 L
14 **a** $400\,cm^3$ **b** $504\,cm^3$ **c** $120\,cm^3$
15 $7{\cdot}6\,cm^3$

Exercise 8:07

1 **a** 21 **b** 60 **c** 730 **d** 14 **e** 52
f 120 **g** 48 **h** 168 **i** 1440
2 **a** 2 h 5 min **b** 1 min 35 s
c 8 weeks 4 days **d** 78 min
e 70·8 s **f** 100 h
3 **a** 2:25 or 25 past 2 **b** 9:30 or half past 9
c 6:50 or 10 to 7 **d** 4:45 or a quarter to 5
e 10:15 or a quarter past 10 **f** 8:36 or 24 to 9
g 11:20 or 20 past 11 **h** 7:55 or 5 to 8
4 **a** 5:20 am **b** 10:30 am **c** 1:10 pm
d 4:00 pm **e** 10:40 pm **f** 9:15 am
g 12:05 pm **h** 8:55 am **i** 12:20 am
j 11:59 pm **k** 2:43 pm **l** 11:01 am
5 **a** 0520 **b** 1050 **c** 1515 **d** 2120
e 0230 **f** 1710 **g** 0245 **h** 1200
6

Program	Timer settings
10:30 pm to 11:30 pm	22:30 to 23:30
9:15 am to 10:45 am	09:15 to 10:45
7:45 pm to 9:10 pm	19:45 to 21:10
5:30 am to 6:40 am	05:30 to 06:40
12 noon to 1:30 pm	12:00 to 13:30
5:40 pm to 7:20 pm	17:40 to 19:20

Exercise 8:08

1 **a** 3 h 50 min **b** 8 h 56 min **c** 4 h 10 min
d 13 h 21 min
2 **a** 3 h 20 min **b** 2 h 23 min **c** 1 h 30 min
d 3 h 30 min **e** 2 h 38 min **f** 0 h 28 min
3 **a** 50 min **b** 1 h 50 min **c** 15 h 15 min
d 18 h 10 min **e** 3 h **f** 5 h 20 min
g 10 h 55 min **h** 19 h 10 min
4 **a** 10 h **b** 9 h 15 min **c** 22 h 40 min
d 26 h 40 min **e** 38 h 45 min
5 **a** 9 pm **b** 1:30 pm **c** 8:05 am
d 2:20 am **e** 1:45 am **f** 11:15 am
g 9 am **h** 6:05 am
6 **a** **i** 7 h **ii** 6 h 30 min
b **i** 4 h **ii** 3 h 45 min
c **i** 4 h **ii** 4 h 15 min
d **i** 7 h **ii** 7 h 20 min

e i 9 h ii 8 h 40 min
f i 2 h ii 2 h 24 min
g i 2 h ii 1 h 54 min
h i 7 h ii 7 h 8 min
i i 13 h ii 12 h 46 min
j i 12 h ii 11 h 50 min

7 **a** 2 h 35 min 37 s **b** 4 min 45·4 s
c 2 h 28 min 52 s **d** 1 h 37 min
e 20 min 40 s **f** 6:30 pm

8 **a** i 23 min ii 12 min iii 11 min
b 3:00 pm **c** 7:05 pm **d** 4:14 pm

9 **a** 0·3 m, 4:14 am (0414) on day 1 **b** 12 h 15 min
c 11 h 39 min **d** 5 h 51 min **e** 44 min; No
f 10:27 pm (2227) on day 1 and again at 11:04 pm (2304) on day 2
g 12:05 pm (1205) [and 12:13 am (0013) day 4, but this is not in day 3]

10 **a** 18 Jan 2015 **b** 7 Feb 2015
c 28 May 2015

11 **a** 6 Jan 2015 **b** 18 Dec 2014
c 8 Aug 2014

12 **a** 26 Jan 2016 **b** Aug 2016
c 17 April **d** 4 June **e** 56 days
f 16 years 205 days

Exercise 8:09

1 **a** 8 pm **b** 8 am **c** 11 pm **d** 3 am
e 5 pm **f** 3 pm

2 **a** 3 h **b** 3 h **c** 12 h **d** 10 h
e 5 h **f** 12 h

3 **a** 5 pm (17:00) **b** 7 am (07:00)
c 10 pm (22:00) **d** 2 am (02:00)
e 3 am (03:00) **f** 9 pm (21:00)

4 **a** i 6 h ii 17 h **b** i 12:00 ii 22:00
c i 1100 ii 2200

5 **a** London by 5 h **b** Tokyo by 8 h
c Sydney by 6 h **d** Moscow by 2 h
e Sydney by 14 h

6 **a** 1400 **b** 0100 (the next day) **c** 0300
d 1500

7 **a** 1 pm **b** 11 pm **c** 4:45 am
d 11:40 am **e** 3:20 am
f 10:15 pm **g** 2:20 pm **h** 7:05 am

8 **a** i 7 am ii 11 am
b i 0900 ii 1200 **c** i 7 pm ii 5 pm

9 **a** 9 am **b** 8 am **c** 4 am
d 8 pm (the previous day)
e 4 pm (the previous day)

10 **a** 1 am (the next day)
b 7:30 am (the previous day) **c** 2 pm
d 4 pm (the previous day)

11 **a** i 7 pm ii 9 pm
b i 1000 (the next day)
ii 0700 (the next day)
iii 1600 iv 1300
c i 12 noon ii 6 am
d 1 pm (1300)
e i 10:00 ii 03:00 (the next day)
f i 0200 ii 1500
g i Session 1—1800 to 0100,
Session 2—0400 to 0700
ii 3 h iii 2 h

Diagnostic test 8

1 **a** 8·4 m **b** 27·4 m **c** 40 m

2 **a** 20 cm^2 **b** 36 cm^2 **c** 36 cm^2

3 **a** 112 cm^2 **b** 100 cm^2 **c** 104 cm^2

4 **a** 60 cm^2 **b** 84 cm^2 **c** 48 cm^2

5 **a** 60 cm^2 **b** 50 cm^2 **c** 162 cm^2

6 **a** 16 cm^2 **b** 38·5 cm^2 **c** 42 cm^2

7 **a** 7 000 000 L **b** 2 000 000 cm^3
c 5 mL **d** 5000 L

8 **a** 150 cm^3 **b** 125 cm^3 **c** 630 cm^3

9 **a** 80 cm^3 **b** 360 cm^3 **c** 378 cm^3

10 **a** 70 cm^3 **b** 100 cm^3 **c** 256 cm^3

11 **a** 05:30 **b** 13:20 **c** 19:57 **d** 00:20

12 **a** 4:20 pm **b** 8:40 am **c** 9:30 pm **d** 5:45 am

13 **a** 13 h 50 min **b** 5 h 12 min **c** 2 h 53 min
d 5 h 18 min

14 **a** 3 h 30 min **b** 1 h 15 min
c 10 36 min **d** 7h 51 min

15 **a** 6 h 10 min **b** 4 h 35 min

16 **a** 4 h 55 min **b** 45 min

17 **a** 7 am **b** 11 am **c** 6 am **d** 5 am

ASSIGNMENT 8A

1 **a** 89·6 cm **b** 46 m

2 **a** 1250 m^2 **b** 475 cm^2 **c** 16 m^2

3 **a** 84 m^2 **b** 202·5 m^2 **c** 416 m^2

4 **a** 216 cm^2 **b** 276 cm^2 **c** 182 cm^2

5 **a** 2500 cm^3 **b** 3375 m^3 **c** 332 m^3

6 **a** 6 h 7 min **b** 11 min 57 s **c** 82

7 **a** 3:30 pm (15:30)
b i 3:30 pm ii 11:30 pm iii 10:30 am
c i 9 am ii 12 midday

8 13 May

ASSIGNMENT 8B

1 105 **2** 39 **3** 325 **4** 18 **5** 48
6 $2640

ASSIGNMENT 8C

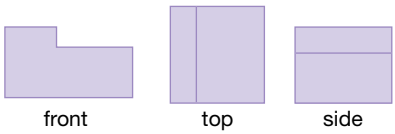
1 **a** $6a$ **b** $9b$ **c** $2a$ **d** $16a^2$

2 **a** 2% **b** $30 **c** 16·5% **d** $12.38

3 **a** 4 m **b** 10·90 m

4 **a** $a = 0{\cdot}5, b = 1{\cdot}3$

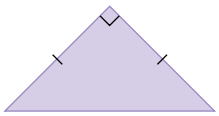
b



5 **a** 5026 **b** 60 700 **c** 7 020 006

6 **a** 0 **b** −2 **c** −48 **d** 50

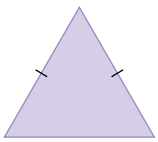
7 **a**



b



c



8 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** 0 **d** $\frac{1}{3}$

9 **a** $18\,m^3$ **b** $18\,m^3$ **c** yes

10 $375\,cm^2$

Chapter 9

Prep quiz 9:01

1 $x + 6$ **2** $8a$ **3** $\frac{y}{7}$ **4** $p - 3$

5 $3x + 5$ **6** $9(y - 4)$ **7** subtracting 7

8 adding 8 **9** dividing by 9

10 multiplying by 10

Exercise 9:01

1 **a** $+6$ **b** -5 **c** $\times 7$

d $\div 2$ **e** $+1, \times 3$ **f** $-6, \times 9$

g $+1, \div 2$ **h** $-7, \div 10$ **i** $\div 2, +3$

j $\times(-4), +8$ **k** $\div(-6), -8$ **l** $\times(-5), +2$

2 **a** $a \xrightarrow{\times 9} 9a$

b $y \xrightarrow{+3} y + 3$

c $q \xrightarrow{-9} q - 9$

d $x \xrightarrow{\div 2} \frac{x}{2}$

e $n \xrightarrow{\times 5} 5n \xrightarrow{+6} 5n + 6$

f $m \xrightarrow{\times 7} 7m \xrightarrow{-1} 7m - 1$

g $p \xrightarrow{\times 4} 4p \xrightarrow{+3} 4p + 3$

h $y \xrightarrow{\times 9} 9y \xrightarrow{-5} 9y - 5$

i $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+2} \frac{x}{3} + 2$

j $n \xrightarrow{\div 5} \frac{n}{5} \xrightarrow{+3} 3 + \frac{n}{5}$

k $x \xrightarrow{+7} x + 7 \xrightarrow{\div 2} \frac{x+7}{2}$

l $a \xrightarrow{-1} a - 1 \xrightarrow{\div 4} \frac{a-1}{4}$

m $n \xrightarrow{+2} n + 2 \xrightarrow{\times 5} 5(n + 2)$

n $x \xrightarrow{-1} x - 1 \xrightarrow{\times 8} 8(x - 1)$

o $q \xrightarrow{+9} q + 9 \xrightarrow{\times 2} 2(q + 9)$

p $t \xrightarrow{-1} t - 1 \xrightarrow{\times 10} 10(t - 1)$

q $x \xrightarrow{\times(-6)} -6x \xrightarrow{+5} 5 - 6x$

r $v \xrightarrow{\times(-7)} -7v \xrightarrow{+1} -7v + 1$

s $a \xrightarrow{\times(-1)} -a \xrightarrow{+7} 7 - a \xrightarrow{\div 2} \frac{7-a}{2}$

t $x \xrightarrow{\div(-6)} -\frac{x}{6} \xrightarrow{+9} 9 - \frac{x}{6}$

3 **a** $\times 7$ **b** -7 **c** $+1$

d $\div 8$ **e** $\times 3, +5$ **f** $\times 7, +2$

g $\times 4, -9$ **h** $\times 4, +6$ **i** $\div 5, -7$

j $\div 5, +10$ **k** $-7, \div 3$ **l** $+1, \div 2$

m $+7, \times 3$ **n** $-4, \times 9$ **o** $\times(-7), +1$

p $\times(-1), +12$

4 **a** $a - 2$ **b** $4x$ **c** $\frac{m}{3}$

d $t + 9$ **e** $5p, 5p + 2$ **f** $6x, 6x - 4$

g $y + 1, 7(y + 1)$ **h** $\frac{b}{4}, \frac{b}{4} + 2$

i $q - 3, \frac{q-3}{3}$ **j** $-2n, -2n + 6$

k b **l** a **m** m

n n **o** $2x, x$ **p** $5p, p$

q $7t, t$ **r** $m + 5, m$ **s** $\frac{a}{3}, a$

t $-2x, x$

5 **a** $\div 3$ **b** $\times 2$ **c** -5

d $+7$ **e** $\div 10$ **f** $\times 3$

g -8 **h** $+5$ **i** $-1, \div 2$

j $+2, \div 5$ **k** $-3, \div 7$ **l** $+4, \div 9$

m $-6, \div 3$ **n** $-7, \times 2$ **o** $+2, \times 5$

p $\times 3, -7$ **q** $\times 5, +2$ **r** $-5, \div(-3)$

s $-8, \div(-2)$ **t** $-5, \times(-3)$

6 **a** $\div 5$ **b** $+4$ **c** -9

d $\times 8$ **e** $-1, \div 2$ **f** $-5, \div 7$

g $+3, \div 8$ **h** $-6, \div 5$ **i** $+5, \times 2$

j $-12, \times 5$ **k** $\times 10, +7$ **l** $\times 3, -1$

m $\div 8, -7$ **n** $\div 2, +11$ **o** $-1, \div(-3)$

p $-4, \div(-1)$

7 **a** $\div 9, +5, \div 2$ **b** $\div 5, -7, \div 8$
c $\div 3, -2, \div 9$ **d** $\times 2, -5, \div 3$
e $+2, \times 3, \div 5$ **f** $-3, \times 5, \div 4$
g $\times 2, -7, \div 3$ **h** $\times 4, +1, \div 9$
i $\div 4, -5, \div (-6)$ **j** $\div (-7), -1, \div 2$
k $\times 2, -7, \div (-3)$ **l** $-9, \times (-6)$

Prep quiz 9:02

1 x **2** x **3** a **4** m
5 dividing by 3 **6** subtracting 7
7 adding 1 **8** multiplying by 5
9 $-3, \div 5$ **10** $+2, \div 8$

Exercise 9:02

1 **a** $x = 9$ **b** $q = 4$ **c** $y = 8$ **d** $n = 6$
e $p = 6$ **f** $t = 13$ **g** $x = 9$ **h** $w = 16$
i $m = 16$ **j** $a = 4$ **k** $y = 5$ **l** $y = 7$
m $x = 10$ **n** $a = 12$ **o** $m = 36$
2 **a** $x = -2$ **b** $m = -2$ **c** $a = -5$
d $n = -7$ **e** $p = -6$ **f** $y = -7$ **g** $n = 5$
h $t = 5$ **i** $q = 0$ **j** $a = -4$ **k** $y = -3$
l $x = -11$ **m** $a = 4$ **n** $y = 3$ **o** $x = 7$
p $x = -8$ **q** $w = -4$ **r** $v = 15$
s $x = 5$ **t** $t = 2$ **u** $q = -10$
3 **a** incorrect **b** incorrect **c** correct
d incorrect **e** correct **f** incorrect
g incorrect **h** correct **i** incorrect
j incorrect **k** correct **l** incorrect
4 **a** $x = 2$ **b** $n = 1$ **c** $y = 3$ **d** $n = 3$
e $k = 2$ **f** $q = 3$ **g** $w = -3$ **h** $p = 0$
i $s = 2$ **j** $x = 2$ **k** $j = 5$ **l** $g = 2$
m $v = 4$ **n** $y = 11$ **o** $a = -3$ **p** $x = 8$
q $p = -2$ **r** $h = -3$
5 **a** $x = 2$ **b** $x = 8$ **c** $t = 10$ **d** $x = 5$
e $a = 4$ **f** $m = 0$ **g** $n = 11$
h $y = 17$ **i** $x = 29$ **j** $x = 8$ **k** $k = 16$
l $a = 30$ **m** $x = 8$ **n** $q = 20$ **o** $n = 8$
6 **a** $x = \frac{1}{2}$ **b** $n = \frac{2}{3}$ **c** $y = \frac{2}{5}$ **d** $n = 2\frac{1}{2}$
e $k = 1\frac{5}{6}$ **f** $q = 1$ **g** $w = 2\frac{2}{3}$ **h** $p = \frac{1}{5}$
i $s = 1\frac{1}{7}$ **j** $x = 1\frac{1}{3}$ **k** $j = 4\frac{1}{2}$ **l** $g = 2\frac{4}{7}$
m $v = 5\frac{1}{2}$ **n** $y = 6\frac{1}{2}$ **o** $a = -1\frac{2}{3}$ **p** $x = 2\frac{2}{3}$
q $p = -\frac{1}{2}$ **r** $h = -1\frac{2}{3}$
7 **a** $x = 3\frac{1}{2}$ **b** $n = 3$ **c** $y = 2$ **d** $y = -1$
e $x = 2$ **f** $y = 2$ **g** $y = -3$ **h** $y = 7$
i $x = -4$ **j** $m = \frac{4}{5}$ **k** $y = 1\frac{4}{5}$ **l** $y = -2$
m $a = 10$ **n** $x = 15$ **o** $t = 23$ **p** $q = 12$
q $w = 20$ **r** $a = 5\frac{1}{3}$ **s** $y = -1$ **t** $a = -3$
u $x = 1\frac{1}{3}$ **v** $x = -7$ **w** $t = -13$ **x** $p = -7$

Prep quiz 9:03

1 x **2** $6a$ **3** 0 **4** 0 **5** -5
6 $+4$ **7** $\times 5$ **8** 5 **9** 1 **10** 5

Exercise 9:03

1 **a** $a = 2$ **b** $x = 7$ **c** $p = 3$ **d** $q = 8$
e $n = 5$ **f** $x = -3$ **g** $m = -3$ **h** $a = -2$
i $x = 3$ **j** $m = 3$ **k** $y = 4$ **l** $x = -3$
m $x = 6$ **n** $m = -4$ **o** $x = 2$ **p** $x = 1$
q $m = 2$ **r** $p = 0$
2 **a** correct **b** correct **c** incorrect
d incorrect **e** correct **f** incorrect
g incorrect **h** correct **i** correct
j correct **k** incorrect **l** incorrect
m incorrect **n** incorrect **o** correct
3 **a** $x = \frac{1}{2}$ **b** $m = 1\frac{1}{2}$ **c** $p = \frac{2}{3}$
d $m = \frac{1}{3}$ **e** $a = -\frac{1}{2}$ **f** $n = 1\frac{1}{5}$
g $a = 2\frac{1}{2}$ **h** $n = 3\frac{1}{3}$ **i** $q = 2\frac{2}{3}$
j $n = -\frac{1}{2}$ **k** $a = \frac{2}{3}$ **l** $k = -1\frac{1}{3}$
m $a = 1\frac{1}{2}$ **n** $x = 1\frac{1}{3}$ **o** $q = 2\frac{1}{4}$
p $t = 2\frac{2}{3}$ **q** $m = \frac{2}{3}$ **r** $x = \frac{5}{8}$
s $a = -1\frac{1}{3}$ **t** $w = -1\frac{1}{3}$ **u** $m = -1\frac{1}{3}$
v $y = \frac{2}{3}$ **w** $x = -\frac{1}{8}$ **x** $p = 4\frac{1}{3}$

Prep quiz 9:04

1 $2x + 6$ **2** $3a - 15$ **3** $8a + 12$
4 $10p - 5$ **5** $21 - 14q$ **6** $x = 5$
7 $x = -5$ **8** $p = 2$ **9** $p = \frac{1}{2}$ **10** $x = 11$

Exercise 9:04

1 **a** $x = -1$ **b** $a = 2$ **c** $n = 0$ **d** $p = 2$
e $a = 4$ **f** $m = 7$ **g** $x = 3$ **h** $q = 4$
i $a = \frac{1}{2}$ **j** $x = 1\frac{2}{3}$ **k** $x = 2\frac{1}{2}$ **l** $t = 1\frac{2}{5}$
m $a = 1$ **n** $x = 4$
2 **a** $x = 1$ **b** $a = 2$ **c** $m = 2$ **d** $a = 1$
e $p = 2$ **f** $q = 2$ **g** $x = 1$ **h** $a = 3$
i $m = 1$ **j** $m = 2$ **k** $n = 6$ **l** $a = 2$
m $m = 8$ **n** $x = 1$ **o** $x = 4$
3 **a** $x = 1\frac{1}{2}$ **b** $a = -6\frac{1}{3}$ **c** $p = -2\frac{3}{5}$
d $a = 2$ **e** $q = \frac{2}{5}$ **f** $t = 2\frac{5}{7}$
g $x = \frac{1}{2}$ **h** $p = \frac{1}{2}$ **i** $w = -3\frac{1}{2}$
j $x = 2\frac{1}{2}$ **k** $q = 1\frac{1}{2}$ **l** $k = \frac{1}{3}$
m $m = 3\frac{1}{2}$ **n** $x = 1\frac{2}{3}$ **o** $n = \frac{5}{9}$
4 **a** $a = -4$ **b** $x = 13$ **c** $p = 18$ **d** $q = 4$
e $m = 4$ **f** $x = 6$ **g** $a = 2$ **h** $t = 2$
i $a = 7$ **j** $p = 4\frac{1}{2}$ **k** $t = 3$ **l** $a = 2\frac{2}{7}$
m $m = 2$ **n** $p = -5$
5 **a** $a = 1$ **b** $m = 1$ **c** $m = 1$ **d** $x = 2$
e $p = 1$ **f** $t = 2$ **g** $a = 2$ **h** $m = 2$
i $a = 2$ **j** $a = -46$
6 **a** $a = 2$ **b** $m = 3$ **c** $n = 2$ **d** $a = 2$
e $a = 1$ **f** $p = 2$ **g** $a = -1\frac{3}{8}$ **h** $x = 3\frac{1}{3}$

Prep quiz 9:05

1 10 **2** 3 **3** 5 **4** 4 **5** 8
6 true **7** false **8** true **9** true **10** true
11 true

Exercise 9:05

1 **a** $x = 2$ or -2 **b** $x = 3$ or -3
c $x = 6$ or -6 **d** $x = 10$ or -10
e $c = 1$ or -1 **f** $a = 9$ or -9
g $m = 4$ or -4 **h** $y = 7$ or -7
2 **a** $x = \sqrt{6}$ or $-\sqrt{6}$ **b** $x = \sqrt{11}$ or $-\sqrt{11}$
c $x = \sqrt{45}$ or $-\sqrt{45}$ **d** $x = \sqrt{2}$ or $-\sqrt{2}$
e $c = \sqrt{109}$ or $-\sqrt{109}$ **f** $m = \sqrt{72}$ or $-\sqrt{72}$
g $b = \sqrt{321}$ or $-\sqrt{321}$ **h** $a = \sqrt{8}$ or $-\sqrt{8}$
3 **a** $x = 2{\cdot}54$ or $-2{\cdot}54$ **b** $x = 3{\cdot}32$ or $-3{\cdot}32$
c $x = 6{\cdot}71$ or $-6{\cdot}71$ **d** $x = 1{\cdot}41$ or $-1{\cdot}41$
e $c = 10{\cdot}44$ or $-10{\cdot}44$ **f** $m = 8{\cdot}49$ or $-8{\cdot}49$
g $n = 17{\cdot}92$ or $-17{\cdot}92$ **h** $a = 2{\cdot}83$ or $-2{\cdot}83$
4 **a** $x = 5$ or -5 **b** $x = \sqrt{5}$ or $-\sqrt{5}$
c No real solution exists. **d** $x = 2$ or -2
e $y = 12$ or -12 **f** $c = \sqrt{145}$ or $-\sqrt{145}$
g $n = 11$ or -11 **h** $a = 13$ or -13
i No real solution exists. **j** $x = \sqrt{27}$ or $-\sqrt{27}$
k No real solution exists. **l** $x = 7$ or -7
m $y = 3{\cdot}5$ or $-3{\cdot}5$ **n** $y = 8{\cdot}5$ or $-8{\cdot}5$
o $c = 5{\cdot}1$ or $-5{\cdot}1$ **p** $b = 1{\cdot}1$ or $-1{\cdot}1$

Prep quiz 9:06

1 $x = 8$ **2** $a = -3$ **3** $m = 18$ **4** $n = -2$
5 $x = -4$ **6** $a + b$ **7** $5y$ **8** $x + 5$
9 $m - 6$ **10** $n + 2$

Exercise 9:06

1 **a** 12 **b** 22 **c** 7 **d** 12 **e** -8
f 17
2 **a** 4 **b** 5 **c** 6 **d** 13 **e** 15
f 11, 12 **g** 16, 17, 18
3 **a** 4 **b** 6 **c** 6 **d** 2
4 **a** 16 **b** 7 **c** 5 **d** 4 **e** 7
f 11
5 **a** $x = 11$ **b** $a = 7, b = 13$ **c** $y = 6$
d $x = 7$ **e** $p = 6, q = 5$ **f** $n = 5, k = 3$
6 **a** 14 **b** 18 **c** 30 **d** 22 **e** 50
f 54
7 **a** 12, 36 **b** \$7, \$28
c pen = 55c; pencil = 30c
d $l = 16{\cdot}5$ cm, $w = 5{\cdot}5$ cm; area = $90{\cdot}75$ cm^2
e 100 units × 15 units, area = 1500 units^2
f Gregory = \$16, Penny = \$10, Phillip = \$3
g father = 48, George = 24
8 A whole log is 5 m long.

Diagnostic test 9

1 **a** $x = 16$ **b** $p = 7$ **c** $a = -2$ **d** $y = 5$
2 **a** $m = 9$ **b** $x = \frac{5}{3}$ **c** $n = 32$ **d** $a = -6$
3 **a** $x = 2$ **b** $m = 3$ **c** $y = 4$ **d** $a = -1$
4 **a** $x = 2\frac{1}{2}$ **b** $y = 1\frac{3}{4}$ **c** $k = -\frac{4}{5}$
d $t = 1\frac{1}{2}$
5 **a** $m = 8$ **b** $m = 10$ **c** $x = 4$ **d** $w = 12$
6 **a** $a = 6$ **b** $m = -4$ **c** $k = 1$ **d** $x = 4\frac{1}{2}$
7 **a** $a = 0$ **b** $m = 5$ **c** $y = 1$ **d** $m = 2$
8 **a** $n = 1$ **b** $p = 10$ **c** $x = 9$ **d** $x = \frac{1}{6}$
9 **a** $x = 4$ or -4 **b** $c = 3$ or -3
c $m = 10$ or -10 **d** $b = \sqrt{3}$ or $-\sqrt{3}$
10 **a** 9 **b** -3
11 **a** 4 **b** 5 **c** 65

ASSIGNMENT 9A

1 **a** $a - 4 \rightarrow a$ **b** $9y \rightarrow y$ **c** $a - 8 \rightarrow a$
2 **a** $x = 8$ **b** $m = 8$ **c** $n = 10$
d $t = 0$ **e** $y = -4$ **f** $x = 2\frac{2}{5}$
g $a = 1$ **h** $y = -3\frac{1}{3}$ **i** $x = 12$
j $w = -32$ **k** $x = 3$ **l** $t = -10$
m $x = 6$ or -6 **n** $c = 9$ or -9
o $m = \sqrt{7}$ or $-\sqrt{7}$ **p** $b = \sqrt{11}$ or $-\sqrt{11}$
3 **a** $x = 1$ **b** $n = 2$ **c** $n = 5$ **d** $k = 6$
e $q = 1\frac{7}{9}$ **f** $w = 1\frac{2}{3}$ **g** $p = 0$ **h** $s = 2\frac{4}{5}$
i $x = 1\frac{2}{3}$ **j** $j = 2$ **k** $g = 2\frac{1}{4}$ **l** $v = \frac{8}{9}$
m $y = 12$ **n** $a = -4\frac{1}{2}$ **o** $x = 12$ **p** $p = -9$
q $n = 23$ **r** $y = -1$ **s** $x = 24$ **t** $h = -8$
4 **a** $x = 2$ **b** $y = 4$ **c** $w = 2$ **d** $p = 3$
e $x = -1$ **f** $k = \frac{2}{5}$
5 **a** $x = 0$ **b** $n = -1$ **c** $n = 6$ **d** $k = 2$
e $q = -1$ **f** $w = 1$ **g** $x = 3\frac{1}{2}$ **h** $x = -\frac{1}{2}$
6 **a** $x = 2$ **b** $a = 3$ **c** $y = 11$ **d** $p = 38$
e $q = \frac{11}{14}$ **f** $x = -1\frac{3}{4}$
7 **a** $7x + 5 = 61, x = 8$ **b** $7(x - 9) = -21, x = 6$
8 $6(8a + 5) = 270, a = 5$

ASSIGNMENT 9B

1 C **2** C **3** 6118·7 m **4** 200 701 votes
5 68 **6** 5

ASSIGNMENT 9C

1 **a** 50% **b** $33\frac{1}{3}$%
2 **a** 2·5 **b** 48
3 0·65
4 **a** 20·4 cm^2 **b** 21 cm^2 **c** 56·25 cm^2
5 **a** $6x - 18$ **b** $4(3a - 5)$ **c** $4a^2 + 8a$
d $6b(a + 2)$
6 **a** 17 000 000 kg **b** 7·8 km **c** 13 h 20 min
7 **a** -9 **b** -2 **c** 24 **d** -4
8 **a** 400 kg **b** \$23 625
9 **a** 44 hours and 20 minutes **b** 21:05
10 **a** 810 cm^3 **b** 2922·23 cm^3

Chapter 10

Exercise 10:01

1 **a** **i** 4:1 **ii** 1:4 **b** no
2 **a** no **b** **i** 2:1 **ii** 1:2
3 **a** $\frac{1}{5}$ **b** $\frac{4}{5}$
4 **a** $\frac{2}{5}$ **b** 3:2
5 **a** 5:3 **b** 5:2 **c** 3:2 **d** 2:5 **e** 3:2:5
6 **a** 14:16 **b** 16:14 **c** 14:30 **d** $\frac{14}{30}$
e 16:30 **f** $\frac{16}{30}$
7 **a** yellow
b We can't tell. It would be one of the other colours.
c $\frac{1}{2}$ **d** $\frac{3}{10}$ **e** 3:10
8 **a** $\frac{1}{7}$ **b** $\frac{4}{7}$ **c** 1:4 **d** 1:2
e 4 kg
9 **a** 10:1 **b** 100:1 **c** 1000:1 **d** 1000:1
e 60:1
10 **a** 1:10 **b** 1:4 **c** 10:1 **d** 3:4
e 3:10
11 **a** $\frac{2}{3}$ **b** 2:1 **c** 1:2
12 **a** 6:9 **b** 4:6 **c** 24:54
13 **a** no **b** sweeter
14 **a** lighter **b** weaker

Exercise 10:02

1 **a** 3 **b** 2 **c** 80 **d** 9 **e** 27
f 12 **g** 9 **h** 24 **i** 2 **j** 1
k 2 **l** 3 **m** 2 **n** 2 **o** 9·6
p 7
2 **a** 3:1 **b** 1:3 **c** 3:2 **d** 8:15
e 1:3 **f** 15:1 **g** 10:1 **h** 1:10
i 6:1 **j** 1:6 **k** 2:5 **l** 20:3
m 1:12 **n** 2:7 **o** 5:7 **p** 3:40
q 3:2 **r** 5:9 **s** $\frac{1}{3}$ **t** $\frac{4}{1}$
u $\frac{2}{5}$ **v** $\frac{20}{9}$ **w** $\frac{2}{3}$ **x** $\frac{7}{2}$
3 **a** 30:1 **b** 5:2 **c** 2:1 **d** 1:40
e 15:2 **f** 7:1 **g** 3:1 **h** 1:5
i 5:1 **j** 8:1 **k** 10:1:11
4 **a** 6:5 **b** 3:1 **c** 3:4 **d** 25:4
e 1:5 **f** 5:1 **g** 1:2 **h** 2:3
i 4:3 **j** 2:7:5 **k** 2:1:3 **l** 11:3:4
m 20:13:7
5 **a** **i** 8:7 **ii** 7:8 **iii** 8:15 **iv** 8:7:15
b **i** 1:2 **ii** 3:2 **iii** 1:3 **iv** 2:1:3
c 1:100 **d** **i** 1:5 **ii** 5:1:4
e 9:5 **f** 9:7:5
g **i** 5:3 **ii** 3:4 **iii** 5:4 **iv** 5:12
v 5:3:4
h 3:1 000 000
6 **a** 3·2:1 **b** 1·5:1 **c** 0·75:1 **d** $1{:}\dot{3}:1$
e 0·25:1
7 **a** 1:0·6 **b** 1:0·5 **c** 1:1·875 **d** 1:4
e 1:3·5

Challenge 10:02

c, **d**, **e** and **g** are the only correct reasons given.

Prep quiz 10:03

1 30 cm **2** 90 cm **3** 3 m **4** 10 cm
5 0·4 m **6** 15 **7** 45 **8** 48
9 10 **10** $6\frac{2}{3}$

Exercise 10:03

1 **a** $x = 8\frac{3}{4}$ **b** $x = 10\frac{1}{2}$ **c** $x = 3\frac{1}{8}$
d $x = 7{\cdot}47$ (2 dec. pl.) **e** $y = 14$ **f** $m = 3\frac{1}{3}$
g $a = 4\frac{2}{3}$ **h** $b = 3\frac{3}{4}$
2 **a** 16 **b** \$340 **c** 40
d \$8.10, \$13.50 **e** 3 parts **f** 5 L
3 **a** 72 kg **b** **i** 3·6 kg **ii** 427·5 g **c** 68 m
d 4 teachers **e** \$54.10
4 **a** 875 m
b 12·5 parts of gravel and 10 parts of sand.
c 33·6 kg of tin and 25·2 kg of zinc.
d \$1200 and \$1920; total divided = \$3840
5 **a** 3:5 **b** 40 g **c** 36 g

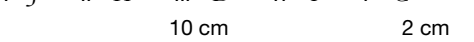
Investigation 10:03

1 **a** 450 tiles **b** 200 **c** 220 black, 275 white
d \$1067

Exercise 10:04

1 **a** 10, 6 **b** 40, 160 **c** 245, 175
d 906, 604 **e** 65, 195 : 325
f 16 560, 11 040, 13 800
2 **a** 16 kg tin, 4 kg lead **b** 6000 **c** 150 mL
d 900 mL
e 12·5 kg nitrates, 18·75 kg each of potash and phosphates
f 14 000, 3500, 10 500
g 36 kg gravel, 24 kg sand, 6 kg cement
h 7·1 cm, 14·2 cm, 21·3 cm

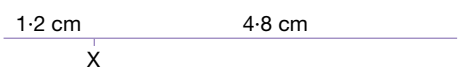
3 **a** **i** *J* **ii** *H* **iii** *D* **iv** *I* **v** *C*
b



c



d



4 **a** 20 cm
b town A: 18 030; town B: 48 080; town C: 19 232
c 2 mL

Prep quiz 10:05

1 100 **2** 10 **3** 1000 **4** 20
5 250 **6** 1 : 100 **7** 1 : 1000 **8** 500
9 6800 **10** 5 m

Exercise 10:05

1 **a** 1 m **b** 10 m **c** 1 km
2 **a** 1 : 100 **b** 1 : 1000 **c** 1 : 10 000
d 1 : 5000 **e** 1 : 50 **f** 1 : 200
g 1 : 20 **h** 1 : 20 000
3 **a** 5000 cm = 50 m **b** 3500 cm = 35 m
c 10 600 cm = 106 m **d** 8100 cm = 81 m
e 1000 mm = 1 m **f** 8000 mm = 8 m
g 26 000 mm = 26 m **h** 500 mm = 50 cm
4 **a** 2 m
b **i** 2 m **ii** 4 m **iii** 18 m **iv** 6·4 m
c 10 m^2 **d** 148 m^2 **e** 24 m^2 **f** $924
5 **a** 10 m **b** 25 m **c** 32 m **d** 21 m
e 0·5 m **f** 21 m **g** 13·5 m **h** 29·5 m
6 **a** 1 cm **b** 3 cm **c** 7·5 cm
d 0·043 m = 4·3 cm **e** 0·63 m = 63 cm
f 0·42 m = 42 cm **g** 0·156 m = 15·6 cm
h 0·024 m = 2·4 cm
7 **a** 2 cm **b** 3 cm **c** 10 cm **d** 1·5 cm
e 0·4 cm = 4 mm **f** 1·25 cm **g** 0·55 cm
h 15·75 cm
8 **a** 3·6 m by 4·2 m **b** 2160 cm = 21·6 m
c **i** 250 m **ii** 8750 m **iii** 2500 m
iv 13 000 m = 13 km
d **i** 5 cm : 10 m = 1 : 200 **ii** 6·0 m **iii** 30 m^2
9 **a** **i** 8·5 cm by 5·8 cm **ii** 17 cm by 11·6 cm
iii 42·5 cm by 29 cm
b 233 mm by 122 mm **c** 0·5 m = 50 cm
d 1 : 25
e 52 mm by 38 mm; refrigerator 6 mm square; sink 12 mm by 5 mm
10 **a** 100 km **b** 300 km **c** 320 km **d** 260 km

Challenge 10:05

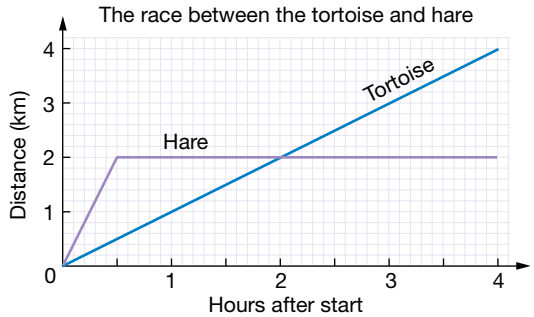
The ratio length : width alternates between 3 : 2 and 3 : 4.
a The ratio length : width alternates between 1 : 1 and 1 : 2
b The ratio length : width alternates between 3 : 1 and 3 : 2

Exercise 10:06

1 **a** 4 km/h **b** 2 kg/$ **c** 0·25 km/min
d 30 L/h **e** 25c/g **f** $45/day
g 30 children/teacher **h** 4 degrees/min
i 20 sheep/h **j** 5 kg/m^2 **k** 7 g/cm^3
l 40 km/h **m** 100 L/h **n** 4 min/km
2 **a** 120 km/h **b** 240 kg/h **c** 200 kg/ha
d 120 L/day **e** $5000/kg **f** 300 mL/h
g 4000 m/h **h** 2000 mL/h **i** 1200 kg/d
j $0.20/min **k** 245c/kg **l** 0·015 L/min
m 10·8 km/h **n** 90 L/h **o** 360 t/day
p 200 kg/ha **q** 10 m/mL **r** 7 t/m^3
3 **a** 15 km **b** 200 km **c** 250 km
d 81 km **e** $2\frac{1}{2}$ h **f** 56 min
g 12 h **h** 140 h (5 days 20 h)
i 95 km/h **j** 20 000 km/h; $3333\frac{1}{3}$ km
k 17·46 km/h
4 **a** $92 **b** 85 **c** 120 **d** 3750 g
e 5400 **f** $120 **g** 201 600 t **h** $359.50
i 1000 m^2 **j** 120 L
5 **a** 4·35 kg **b** 25 **c** $26\frac{2}{3}$ h **d** 160 min
e 22 trucks (21 full loads, 1 load of 18 t)
f 33·25
g $8\frac{1}{3}$ min **h** 2·5 **i** 7 **j** 18
6 **a** **i** 18 000 000 km **ii** 496 s = 8 min 16 s
b 150 kg **c** 36 km/h **d** 2 s
e **i** $742.50 **ii** $11 000
7 **a** $3\frac{1}{3}$ min **b** 500 revs/min
c **i** 10 km/h **ii** 18·75 km

Exercise 10:07

1 **a** **i** 10 km **ii** 5 km **iii** 5 km
iv 0 km. Luke is at home at 11 am.
v 5 km **vi** 0 km. Luke is at home.
vii 5 km **viii** 7·5 km
b **i** Luke was resting from 9 am to 10:30 am and from 11 am to 11:30 am.
ii From 8 am to 9 am, and from 10:30 am to 11 am.
2 **a** Rajiv **b** Indu
3 **a** **i** 5 km **ii** 20 km
b **i** 10 km **ii** 5 km
c Between 5 am and 6 am
d 6 am **e** 15 km
4 **a** **i** 4 cm **ii** 4 cm **iii** 2 cm
iv 0 cm. The snail had returned to the starting position.
b **i** $1\frac{1}{2}$ minutes after starting and 5 minutes after starting.
ii The snail was 4 cm from where it started from the 2nd to the 4th minute.
iii The snail was not moving from 2 minutes after starting until 4 minutes after starting.
c **i** 2 cm **ii** 8 cm **iii** 3 cm
5 **a** 20 km, 10 km/h **b** 10 km, 5 km/h
c 3 hours after the start.
1 hour to fix his bicycle. 2 km/h
d Leura, 1 hour **e** $7\frac{1}{2}$ hours
6

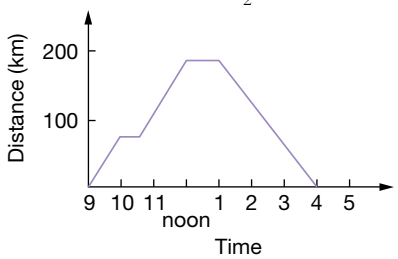


a 1 km/h. The tortoise passed the hare after 2 hours.
b **i** 4 km/h **ii** 0·5 km/h

7 **a** Ang starts from A at 3 pm. Dolma starts 20 km from A at 3 pm.
b 4:30 pm; 7:30 pm **c** 20 km **d** 6:30 pm
e Ang travels 160 km. Dolma travels 100 km.

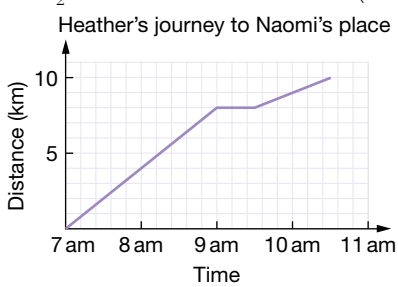
8 **a** 75 km **b** 10:45 am and 12:15 pm **c** noon
d 75 km **e** Doyle; $\frac{1}{2}$ h

9



Diagnostic test 10

1 **a** 1:2 **b** 1:4 **c** 1:2:4 **d** 4:1:2
2 **a** 5:2 **b** 4:1 **c** 1:4 **d** 2:3
3 **a** 4 **b** 18 **c** 24 **d** 8
4 **a** 1·875 **b** 16·6 **c** 1·44 **d** 1·6
5 **a** 18 **b** 10·5 **c** \$2·70 **d** 15 kg
6 **a** 10:5 **b** 10:35 **c** 220:200 **d** 12:24:84
7 **a** 300 cm = 3 m **b** 520 cm = 5·2 m
c 100 mm = 0·1 m **d** 4200 mm = 4·2 m
8 **a** 15 m **b** 26 m **c** 0·5 m **d** 21 m
9 **a** 0·05 m = 5 cm **b** 0·43 m = 43 cm
c 0·75 cm **d** 0·048 m = 4·8 cm
10 **a** 2·5 cm **b** 21·5 cm
c 0·375 cm **d** 2·4 cm
11 **a** 25 km/h **b** 150 t/h
c 12·5 g/cm^3 **d** 0·4 kg/m^2
12 **a** 3600 m/h **b** 4·8 t/d
c \$15 000/kg **d** 100 mL/min
13 **a** 2800 t **b** 350 km
c $5\frac{1}{2}$ h **d** 20·8 L (1 dec. pl.)
14



ASSIGNMENT 10A

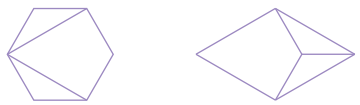
1 **a** 1:4 **b** 3:2 **c** 1:5 **d** 10:1
e 2:5 **f** 18:13
2 **a** $x = 2$ **b** $a = 12$ **c** $m = 3\frac{3}{4}$ **d** $n = 3\frac{1}{5}$
3 **a** 48 **b** 175 **c** \$10.50
4 **a** Greg = \$35, Gill = \$20
b length = 24 cm, width= 15 cm
c \$16.50 **d** 60
5 **a** 6 km **b** 20 km **c** 37 km
6 a 80° **b** 36°, 36°, 108°
7 **a** 14 km/L **b** 26·5 L **c** 71 mL
8 **a** 6:28 pm **b** 84 km/h
9 100 metres
10 **a** 20 km **b** 40 km **c** 10:30 am
d Travers **e** 5 km and 60 km

ASSIGNMENT 10B

1 **a** interval AB **b** line AB **c** ray AB
d collinear points **e** diagonals
f $(a + b)°$ **g** 360°
2 5 and 7 **3** 6 **4** C **5** 12 **6** 64
7 **a**



b



ASSIGNMENT 10C

1 **a** 13 m **b** 6 cm
2 **a** $6m + 5n + 8$ **b** $6a + 22$
3 **a** 1·125% **b** 99·1%
4 **a** 2160 m^3 **b** 720 m^3
5 **a** \$108 **b** \$10.91
6 43 hours and 45 minutes
7 16·56 units^2
8 **a** 97·5 m^2 **b** 20·52 m^2 **c** 76·98 m^2
9 199·5 cm^2
10 **a** $a = 3·2$ **b** $m = 6\frac{2}{3}$ **c** $p = -10$

Chapter 11

Prep quiz 11:01

1 (−2,1) **2** (1,2) **3** (−1,−2) **4** (2,−1)
5 12 **6** 7 **7** −2 **8** −1
9 −3 **10** $y = 5$

Exercise 11:01

1 **a** $x + y = 4$

x	0	1	2	3	4
y	4	3	2	1	0

b $y = 2x$

x	0	1	2	3	4
y	0	2	4	6	8

c $y = 3x - 2$

x	0	1	2	3	4
y	−2	1	4	7	10

d $y = 10x + 36$

x	0	1	2	3	4
y	36	46	56	66	76

e $y = 8 - x$

x	0	1	2	3	4
y	8	7	6	5	4

f $2x + y = 16$

x	0	1	2	3	4
y	16	14	12	10	8

2 a A ($y = 8 - x$) B ($x + y = 4$) C ($y = 2x$)
D ($y = 3x - 2$) E ($2x + y = 16$)

b $y = 10x + 36$

3 a true **b** false **c** true **d** true **e** true
f false **g** true **h** true **i** false

4 a $y = 2x + 5$

x	0	1	27	−8
y	5	7	59	−11

b $y = 4 - 2x$

x	−2	4	30	52
y	8	−4	−56	−100

c $y = 3x - 7$

x	0	1	−1	4·5
y	−7	−4	−10	6·5

d They are small integers and are easier to compute with than large numbers or fractions. They are also close to the origin and thus will provide points which are easier to plot.

5 a They are all divisible by 3 and so the y-value will work out to be an integer.

x	0	3	6
y	4	5	6

b

x	8	4	0
y	0	1	2

6 a 7 **b** 5 **c** 3 **d** 1 **e** 2 **f** −2

7 a −15 **b** 25 **c** -24 **d** 10·2

8 a $y = 3x$ **b** $y = x + 2$ **c** $y = x - 5$
d $y = 8 - x$ or $y = -x + 8$ **e** $y = 10x + 1$
f $y = 4x + 8$

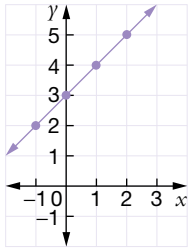
9 a C **b** B **c** A **d** C **e** B
f A **g** B **h** A

10 a i $(-1, 4), (1, 2), (3\frac{1}{2}, -\frac{1}{2})$
ii $(-2, -3), (\frac{1}{2}, 2), (1, 3)$
iii $(-1, -3), (1, -1), (3\frac{1}{2}, 1\frac{1}{2})$
b i $y = x - 2$ **ii** $y = 2x + 1$ **iii** $x + y = 3$
c i $y = 2x + 1$ **ii** $y = x - 2$ **iii** $x + y = 3$
iv $y = 2x + 1$

Exercise 11:02

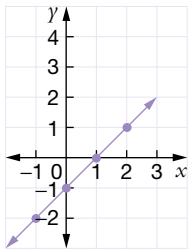
1 a $y = x + 3$

x	−1	0	1	2
y	2	3	4	5



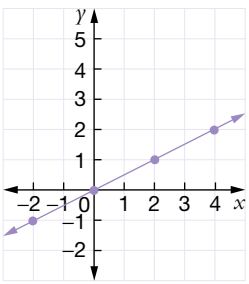
b $y = x - 1$

x	−1	0	1	2
y	−2	−1	0	1



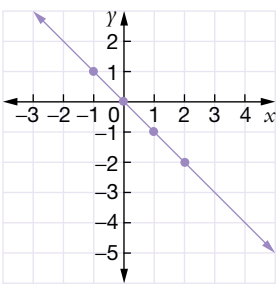
c $y = \frac{1}{2}x$

x	−2	0	2	4
y	−1	0	1	2



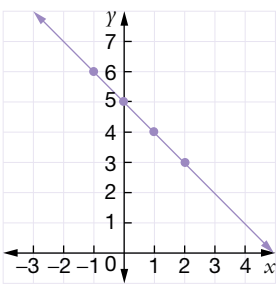
d $y = -x$

x	−1	0	1	2
y	1	0	−1	−2



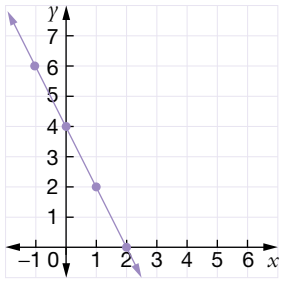
e $y = 5 - x$

x	−1	0	1	2
y	6	5	4	3

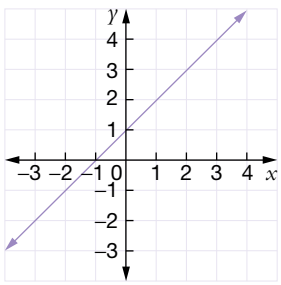


f $y = 4 - 2x$

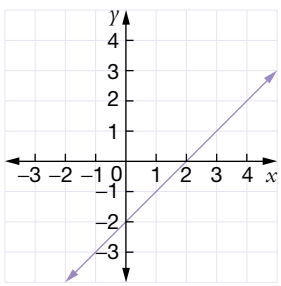
x	−1	0	1	2
y	6	4	2	0



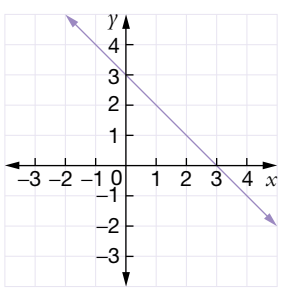
2 a



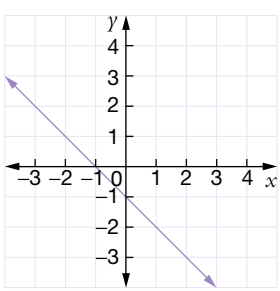
b



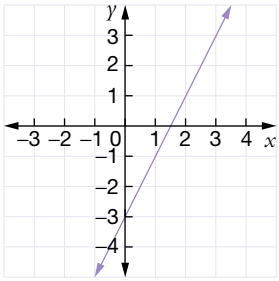
c



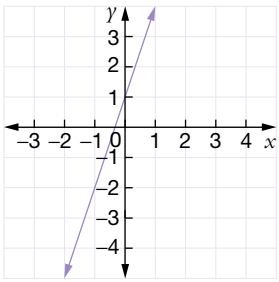
d



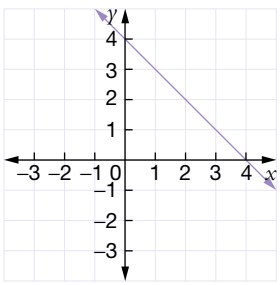
e



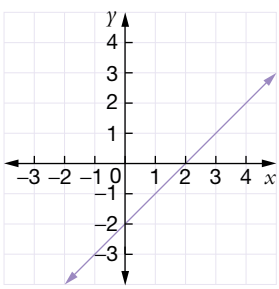
f



g



h



3 a $y = x + 1$

x	−1	0	1	2
y	0	1	2	3

b $y = 2 - x$

x	−1	0	1	2
y	3	2	1	0

c $y = 2x$

x	−1	0	1	2
y	−2	0	2	4

d $y = x$

x	−1	0	1	2
y	−1	0	1	2

e $y = 3x - 1$

x	−1	0	1	2
y	−4	−1	2	5

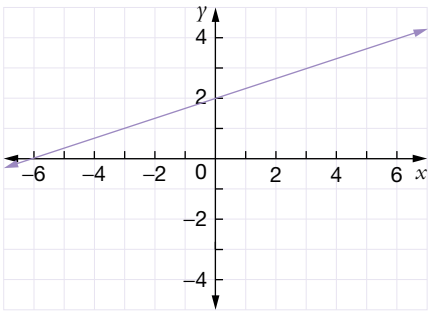
f $y = -\frac{1}{2}x$

x	−1	0	1	2
y	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1

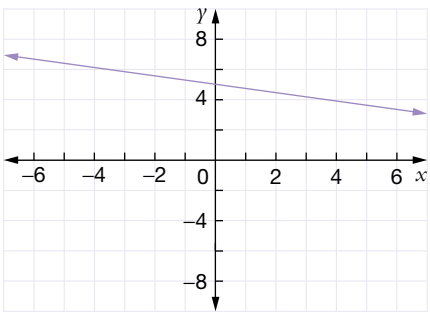
4 **a** no **b** no **c** yes **d** no **e** yes
f no **g** no **h** no **i** no **j** no
k yes **l** no

5 **a** C **b** B **c** A **d** D

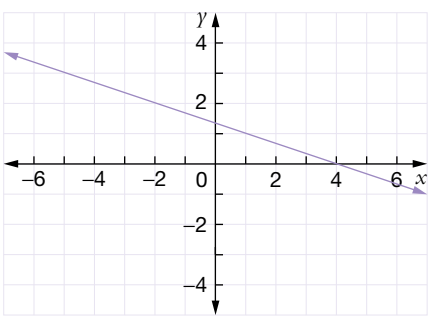
6 **a**



b



c



7 **a** **i** The purple lines are parallel.
ii The purple equations all have −0·5 as the coefficient of x in the equation.
b **i** The blue lines are parallel.
ii The blue equations all have 2 as the coefficient of x in the equation.
c **i** It would be parallel to the purple lines
ii It would be parallel to the blue lines.

8 **a** They all pass through the point (0,−2).
b They all have a constant term of −2.
c They all meet the y-axis at −2.
d yes **e** yes
f The larger the coefficient of x the steeper the line.

9 **a** Decrease **b** Increase
c The constant term is the y-intercept of the line. It indicates where the line will cross the y-axis.

10 **a** These lines are parallel and they will increase (slope up) as x increases in value.
b These lines all cross the y-axis at 1. They also are all increasing. The line $y = 5x + 1$ is the steepest of the lines while the line $y = 2x + 1$ is least steep of the three lines.
c These lines all cross the y-axis at 0. They are also decreasing (sloping downwards) as x increases in value.
d These lines all meet at the point (2,4).

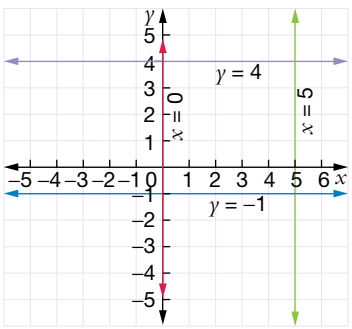
Exercise 11:03

1 **a** $x = 1$ **b** $y = -1$
c $x = -2$ **d** $y = 1\frac{1}{2}$

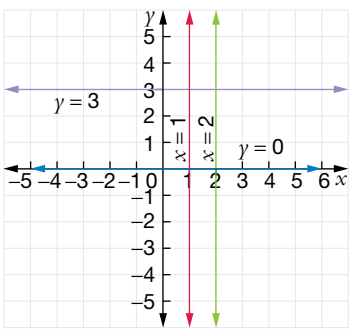
2 **a** A: $y = 3$, B: $y = 1$, C: $x = 3$, D: $y = -4$, E: $x = -1$, F: $x = -4$
b A: $x = 2$, B: $x = 4$, C: $y = 4$, D: $y = 2$, E: $y = -2$, F: $x = -3$

3 **a** $x = 0$ **b** $y = 0$

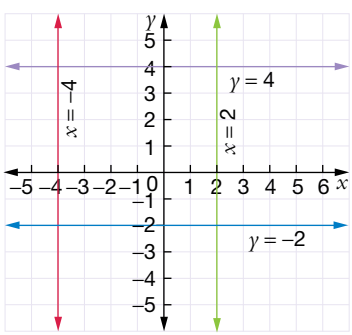
4 **a**



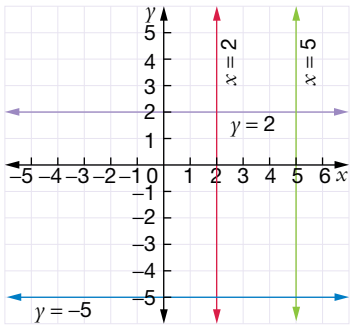
b



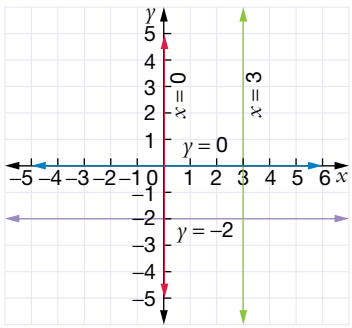
c



d



e



The lines in **a** and **c** enclose a square region.

5 **a** (3, 1) **b** (−4, 3) **c** (−1, −4) **d** (2, −2)
e (−3, 2) **f** (4, 4) **g** (0, 3) **h** (2, 0)
i (0, 0)

Exercise 11:04

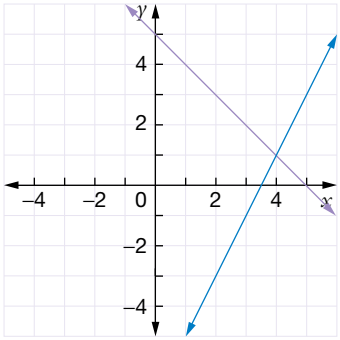
1 **a** (3, 2) **b** (3, 2) **c** (−1, 2) **d** (3, −3)
e (−2, −3) **f** (1, 0) **g** (0, 1) **h** (−2, −3)
i (1, 0) **j** (3, 2)

2 **a** (−1, 1) **b** (2, 3) **c** (−3, −3) **d** (−4, −3)
e (0, 0) **f** (2, 0) **g** (2, 3) **h** (2, 2)

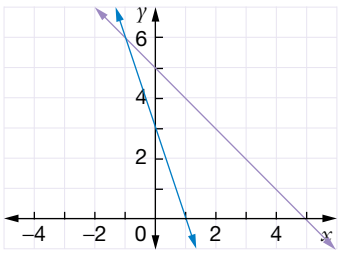
3 **a** $x = 1, y = 2$ **b** $x = 2, y = 1$
c $x = 3, y = -2$ **d** $x = 4, y = 1$
e $x = 0, y = -5$

4 **a** $x = -4{\cdot}5, y = 1{\cdot}5$. Substitution shows the solution is exact.
b $x = 1{\cdot}8, y = 0{\cdot}4$. Substitution shows the solution is exact.
c $x = 2{\cdot}3, y = -0{\cdot}7$. Substitution shows the solution is exact for $x - y = 3$ but not for $y = 4 - 2x$. The solution is a good approximation.
d $x = 2{\cdot}7, y = 3{\cdot}3$. Substitution shows the solution is exact for $x + y = 6$ but not for $y = 3x - 5$. The solution is a good approximation.

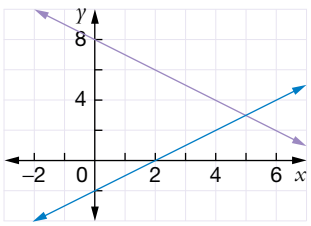
5 **a** Solution is $x = 4, y = 1$



b Solution is $x = -1, y = 6$



c Solution is $x = 5, y = 3$



6 **a** $x = 16, y = 26$. Substitution shows the solution is exact.
b $x = 8, y = 38$. Substitution shows the solution is exact.

7 (45,15). The rectangle was 45 cm long and 15 cm wide.

8 **a** $x = 1$ **b** $x = -1$ **c** $x = -4$

9 **a** $x = -3$ **b** $x = -1{\cdot}5$ **c** $x = 2{\cdot}5$
d $x = 4{\cdot}5$ **e** $x = -6$

10 **a** $x = -2$ **b** $x = 4$ **c** $x = -3{\cdot}5$ **d** $x = 5$

11 **a** $x = 1$ **b** $x = 2$ **c** $x = 2{\cdot}5$ **d** $x = 4{\cdot}5$
e $x = 8$ **f** $x = -4$ **g** $x = -2$ **h** $x = -10$

Exercise 11:05

1 **a**

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b **i** 100 **ii** 100 **iii** $6\frac{1}{4}$ **iv** $6\frac{1}{4}$

2 **a**

x	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	−0·5	−1	−2		2	1	0·5

b **i** 0·25 **ii** -0·25
c **i** 0·7 **ii** -0·7

3 a i

x	0	1
y	0	3

ii

x	0	1
y	0	1

iii

x	0	1
y	3	2

b Graph A is $y = 3 - x^2$, graph B is $y = 3x$, graph C is $y = x^3$

4 a i -1 ii 3 iii 1

b i $y = x^2 + 2$ ii $y = x^2 + 2x$ iii $y = x + 2$

5 a Graph A **b** Graph D **c** Graph C **d** Graph B

6 a $y = x + 3$ **b** $y = x$ **c** $x + y = 1$

Diagnostic test 11

1 a

x	-1	0	1	2
y	1	2	3	4

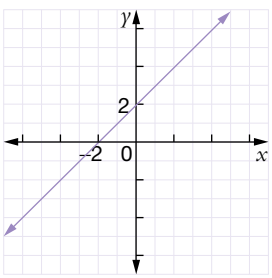
b

x	-1	0	1	2
y	5	4	3	2

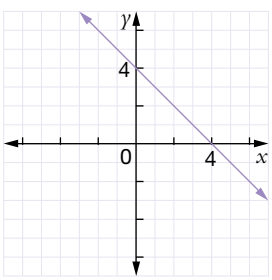
c

x	1	2	3	4
y	-1	1	3	5

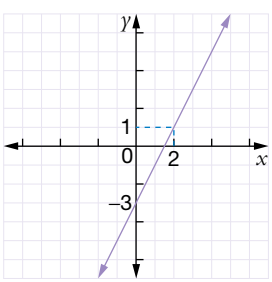
2 a



b



c



3 a $y = x - 4$ **b** $x + y = 5$ **c** $y = \frac{1}{2}x$

4 a True **b** False **c** True

5 a $x = 4, y = 10$ **b** $x = 3, y = 7$

c $x = -2, y = -8$

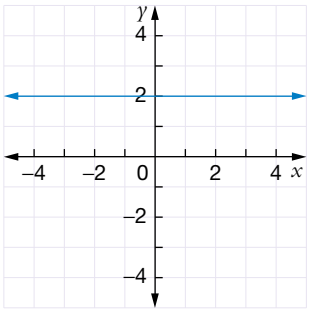
6 a $y = 22$ **b** $y = -32$ **c** $y = 3{\cdot}4$

7 a The lines all are increasing (sloping up) as x increases in value.

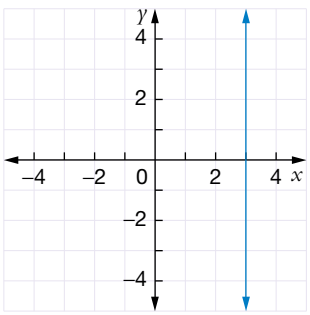
b The graphs are all decreasing (sloping downwards) as x increases in value.

c The lines are parallel and are all increasing as x increases.

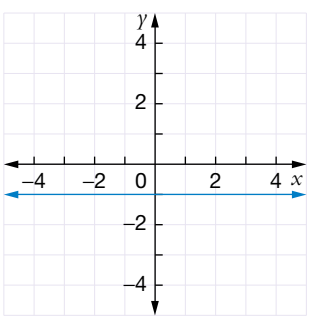
8 a



b



c



9 a -1 **b** 1 **c** -3 **d** 5

10 a $(-4,3)$ **b** $(-1,0)$ **c** $(-2,-2)$ **d** $(0,-1)$

11 a $y = 3x$ **b** $y = 3x$ **c** $x + y = 0$

ASSIGNMENT 11A

1 a $y = 2x + 1$

x	0	1	2
y	1	3	5

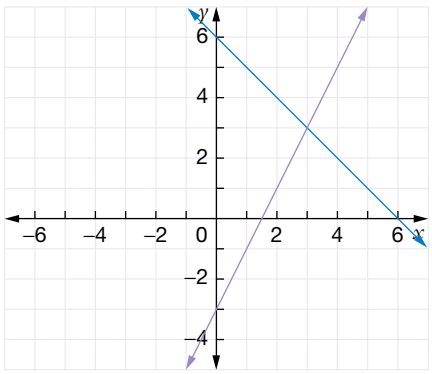
b $y = \frac{x}{3} - 4$

x	0	3	6
y	-4	-3	-2

2 a $y = 5$ **b** $y = 18$ **c** $y = -7$ **d** $y = 4{\cdot}6$

3 a $y = 10x$ **b** $y = 5x + 1$

4



The point of intersection is the solution. So $x = 3$ and $y = 3$.

5 **a** $y = 2x - 3$, $y = 4x - 3$, $y = 5x - 2$, $y = 4x$, $y = 3x - 3$
b $y = -x + 6$, $y = -2x + 3$, $y = -3x + 2$
c $y = 5x - 2$ **d** $y = -x + 6$
e $y = 4x - 3$, $y = 4x$
f $y = 2x - 3$, $y = 4x - 3$, $y = 3x - 3$

6 **a** $x = 8$ **b** $x = -2$ **c** $x = 6{\cdot}5$ **d** $x = -1$

7 **a** Graph A **b** Graph D **c** Graph B
d Graph C **e** Graph E

ASSIGNMENT 11B

1 **a** *obtuse* angle **b** *reflex* angle
c *revolution* **d** *adjacent* angles
e *supplementary* angles **f** *vertically opposite* angles
g 360°

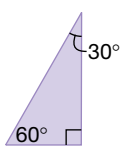
2 437·5 mm **3** 32 **4** 8 **5** 6 m **6** 28

ASSIGNMENT 11C

1 6·4 m

2 **a** \$2584 **b** \$19.20

3 **a** 30°, 60°, 90° **b**



4 $\frac{103}{108}$

5 **a** $6a + 5b$ **b** $24x^2$ **c** $42 - 24b$ **d** $-18x$

6 **a** 5 **b** 1 **c** 9 **d** 5

7 **a** 11·25 cm **b** 8 cm

8 **a** 25% **b** 20·8$\dot{3}$% **c** 25 ha

9 **a** **i** 5·3 m **ii** 59·4 m **iii** 48·1 m **iv** 0·6 m
b **i** 60 km/h **ii** 90 km/h **iii** 120 km/h **iv** 80 km/h
c **i** 5·6 m/s **ii** 13·9 m/s **iii** 11·1 m/s **iv** 44·4 m/s
d 16·7 m/s **e** 80 km/h

10 **a** divided bar graph **b** 43% **c** 7 teachers

Chapter 12

Exercise 12:01

1 **a** radius **b** chord **c** diameter
d arc **e** semicircle **f** sector
g minor segment **h** tangent **i** radius
j major segment **k** diameter **l** sector
m arc **n** radius **o** chord
p tangent

2 **a** sector **b** tangent **c** semicircle
d tangent **e** semicircle

3 **a** diameter **b** radius **c** chord
d tangent

4 isosceles. Two sides are equal radii.

5 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **6** 3 cm

Exercise 12:02

1 **a** 5π cm, 15·7 cm **b** 8π mm, 25·12 mm
c 20π mm, 62·8 mm **d** 6π cm, 18·84 cm

2 **a** 62·8 cm **b** 12·56 cm
c 25·12 mm **d** 56·52 mm

3 **a** 44 mm **b** 66 cm **c** 44 cm **d** 220 mm
e 154 cm **f** 264 mm **g** 88 cm **h** 22 cm

4 **a** 14·1 cm **b** 17·0 cm **c** 38·0 mm
d 29·5 mm **e** 27·3 cm **f** 78·5 cm
g 102·7 cm **h** 11·9 cm **i** 23·9 mm
j 98·0 cm

5 **a** 16 cm **b** 9 cm **c** 12 cm

6 **a** 5·24 cm **b** 11·78 cm **c** 17·45 cm
d 28·27 cm

7 **a** 188·5 cm (1 dec. pl.)
b 1885 m (nearest m)
c 531 revolutions (nearest rev.)

8 38 000 km

9 **a** 6·4 cm **b** 15·9 mm **c** 11·1 cm
d 4·8 cm

10 Diameter = 1·7 m (1 dec. pl.), Radius = 0·8 m (1 dec. pl.)

11 Thickness = 13 cm (nearest cm)

Exercise 12:03

1 **a** 29 cm **b** 31 cm **c** 59 cm **d** 67 cm
e 40 cm

2 6·1 km; 122 km **3** 14 m

4 **a** 94 cm **b** 310 m

5 318 mm **6** 2π metres, i.e. ≈ 6·3 m

Prep quiz 12:04

1 9 cm **2** 9·0 cm **3** 3·1

4 $C = \pi d$ or $C = 2\pi r$ **5** 40 cm^2 **6** 30 cm^2

7 13·5 cm^2 **8** $\frac{1}{4}$ **9** $\frac{3}{4}$ **10** $\frac{1}{3}$

Investigation 12:04

1 **a** $4r^2$ square units **b** square
2 **a** $2r^2$ square units **b** circle
3 12 units^2
4 **a** 52 units^2 **b** 300
5 **a** πr **b** r **c** πr^2 square units
6 **a** $A = \frac{1}{2}bh$ **b** $2\pi r$ **c** πr^2 **d** πr^2

Exercise 12:04

1 **a** 4π cm^2, 12·56 cm^2 **b** 100π cm^2, 314 cm^2
c $16\ \pi$ cm^2, 50·24 cm^2 **d** 25π m^2, 78·5 m^2
e π cm^2, 3·14 cm^2 **f** $9\ \pi$ m^2, 28·26 m^2
g $16\ \pi$ m^2, 50·24 m^2 **h** 36π cm^2, 113·04 cm^2
2 **a** 154 cm^2 **b** $28\frac{2}{7}$ cm^2 **c** $38\frac{1}{2}$ mm^2
d $78\frac{4}{7}$ cm^2
3 **a** 8 cm **b** 4 cm **c** 15 cm
d 56 cm (1 m$^2 = 10\,000$ cm^2)
4 **a** 113·1 cm^2 **b** 22·9 cm^2 **c** 95·0 cm^2
d 366·4 cm^2 **e** 804·2 cm^2 **f** 2827·4 cm^2
g 191·1 mm^2 **h** 254·5 cm^2 **i** 1017·9 cm^2
j 243·3 mm^2
5 **a** 50 cm^2 **b** 113 cm^2 **c** 25 cm^2
d 172 cm^2
6 **a** 1257 mm^2 **b** 707 mm^2 **c** 1886 mm^2
7 **a** 110·8 cm^2 **b** 13·9 cm^2 **c** 34·0 cm^2
8 **a** 66·0 cm^2 **b** 22·0 cm^2 **c** 50·3 cm^2
9 **a** 707 cm^2 **b** 19·6 m^2 **c** 95 cm^2
d 22·0 cm^2
e **i** 289·5 cm^2 **ii** 144·8 cm^2 **iii** 193·0 cm^2
10 **a** 89 cm^2 **b** 68 cm^2 **c** 74 cm^2
d 71 cm^2 **e** 23 cm^2 **f** 71 cm^2
11 **a** 3·8013 m^2 **b** **i** 6 cm **ii** 7·74 cm^2

Prep quiz 12:05

1 $\frac{1}{2}$ **2** $\frac{1}{4}$ **3** $\frac{1}{4}$ **4** $\frac{1}{2}$ **5** $\frac{1}{8}$
6 $\frac{1}{12}$ **7** $\frac{1}{36}$ **8** $\frac{1}{360}$ **9** $\frac{17}{360}$ **10** $\frac{11}{12}$

Exercise 12:05

1 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{12}$ **d** $\frac{1}{8}$ **e** $\frac{1}{20}$
2 **a** $\frac{1}{9}$ **b** $\frac{11}{60}$ **c** $\frac{7}{12}$ **d** $\frac{2}{3}$
3 **a** 254·5 m^2 **b** 1520·5 cm^2 **c** 706·9 m^2
d 1520·5 cm^2
4 **a** 28·3 m^2 **b** 278·8 cm^2 **c** 412·3 m^2
d 1013·7 cm^2
5 **a** $\frac{1053\pi}{10}$, 330·81 m^2 **b** $\frac{19683\pi}{10}$, 6183·60 cm^2
c $\frac{342\pi}{5}$, 214·89 mm^2 **d** $\frac{117\pi}{4}$, 91·89 m^2
6 **a** $\frac{1053\pi}{40}$, 82·70 m^2 **b** $\frac{19683\pi}{40}$, 1545·90 cm^2
c $\frac{171\pi}{10}$, 53·72 mm^2 **d** $\frac{117\pi}{16}$, 22·97 m^2
7 **a** 1486·60 mm^2 **b** 674·34 m^2
c 264·63 cm^2 **d** 33·74 m^2
8 **a** 24·02 cm^2 **b** 21·81 m^2
9 147·52 cm^2
10 If you halve the radius, you divide the area by 4.The answers to Question **6** are one-quarter of the answers to Question **5**.

Prep quiz 12:06

1 $C = \pi d$ or $C = 2\pi r$ **2** $A = \pi r^2$ **3** 28 cm^2
4 201 cm^2 **5** 79 cm^2 **6** rectangular prism
7 triangular prism **8** trapezoidal prism
9 160 cm^3 **10** 240 cm^3

Exercise 12:06

1 **a** 471·2 cm^3 **b** 125·7 cm^3 **c** 339·3 cm^3
2 **a** 603 cm^3 **b** 6283 cm^3 **c** 1767 cm^3
3 **a** 770 cm^3 **b** 1696 cm^3 **c** 1810 cm^3
d 25 cm^3
4 **a** I **b** II
5 **a** 25·5 cm^3 **b** 1249·9 cm^3 **c** 33·1 cm^3
6 18·850 m^3; 18 850 L
7 196·3 mL
8 1·02 m^3 (2 dec. pl.)
9 **a** 636·2 cm^3 **b** 3800 mL or 3·8 L
10 **a** 251 cm^3 **b** 177 cm^3 **c** 1206 cm^3
11 **a** 1017·9 cm^3 (1 dec. pl.)
b yes **c** 2 **d** 4
12 **a** 251 cm^3 **b** 754 cm^3 **c** 1282 cm^3
d 754 cm^3
13 **a** **i** 1244 cm^3 **ii** 589 cm^3 **b** 1128 cm
14 About 468 m^3.
(Base diameter – 5·6 m, height – 19 m.)

Challenge 12:06

1 **a** 3 cm **b** 14·1 cm^2 (1 dec. pl.)
c 25·1 cm^2 (1 dec. pl.) **d** 39·3 cm^2 (1 dec. pl.)

Diagnostic test 12

1 **a** diameter **b** radius **c** arc
d semicircle **e** sector
2 **a** 25·1 cm **b** 15·7 cm **c** 12·6 cm **d** 56·5 cm
3 **a** 26 cm **b** 27 cm **c** 25 cm **d** 25 cm
4 **a** 79 cm^2 **b** 28 cm^2 **c** 50 cm^2 **d** 154 cm^2
5 **a** 39 cm^2 **b** 21 cm^2 **c** 49 cm^2 **d** 8 cm^2
6 **a** 471 cm^3 **b** 101 cm^3 **c** 126 cm^3 **d** 462 cm^3

ASSIGNMENT 12A

1 **a** centre **b** arc **c** radius
d tangent **e** segment **f** sector
2 **a** yes **b** one-quarter
c circles that have the same centre **d** 3
3 **a** 12π cm **b** 10π cm
4 **a** 49 cm **b** 1696 cm (or 16·96 m)
5 **a** 169π cm^2 **b** 9π m^2
6 **a** 637·9 cm^2 **b** 10·2 m^2
7 **a** 127·7 m^2 **b** 125·7 cm^2 **c** 154·0 cm^2
8 **a** 3810·0 m^3 **b** 197·9 cm^3 **c** 6616 cm^3

ASSIGNMENT 12B

1 **a** 2, 5 **b** 3, 4
2 24 **3** 6
4 20 (as 20 is a multiple of both 4 and 5)
5 16 **6** 40

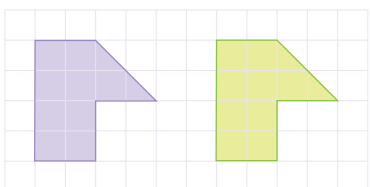
ASSIGNMENT 12C

1 **a** $30a$ **b** $7a^3$ **c** $2 - 2a$ **d** 6
2 **a** Bill has 945 stamps. Jeff has 315 stamps
b Frances has \$7.80. Jill has \$11.30.
3 **a** 52 **b** 9 cups
4 **a** 15·68 **b** 12
5 **a** $42{\cdot}25\,\text{cm}^2$ **b** $13{\cdot}44\,\text{cm}^2$
c $272\,\text{cm}^2$ **d** $44\,\text{cm}^2$
6 $\frac{2}{7}$
7 $36\,\text{cm}^3$
8 **a** $a = 3{\cdot}375$ or $3\frac{3}{8}$ **b** $x = 1{\cdot}3$ **c** $x = -1{\cdot}7$
9 **a** 1·2 m by 0·5 m **b** 23 m
10 **a** NET 3 **b** 53% **c** 10% more
d CTN 6, 21%; NET 3, 10%

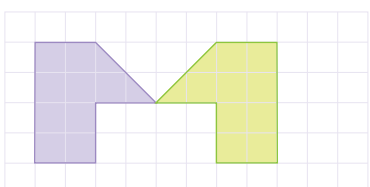
Chapter 13

Exercise 13:01

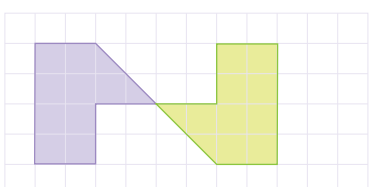
1 **a** translation **b** reflection **c** rotation
d rotation
2 **a** Sample answers:



b

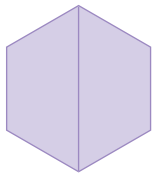


c

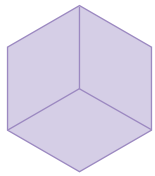


3 **a** i $\angle L$ ii $\angle R$ iii $\angle H$
b i SR ii HG iii EF **c** Yes
4 **a** A **b** P **c** F **d** O **e** H **f** G

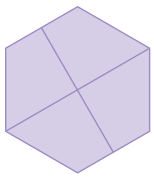
5 **a**



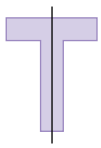
b



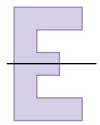
c



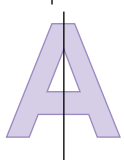
6 **a**



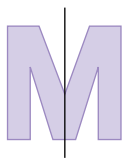
b



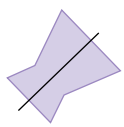
c



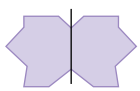
d



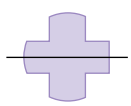
e



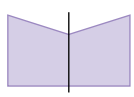
f



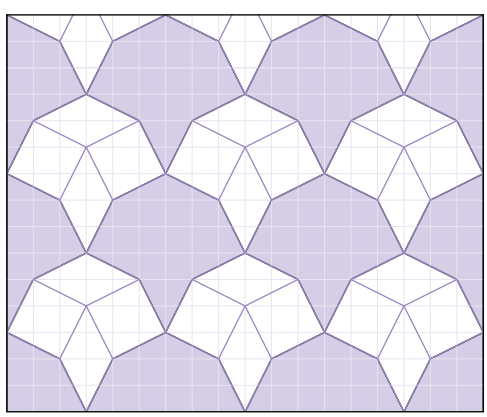
g



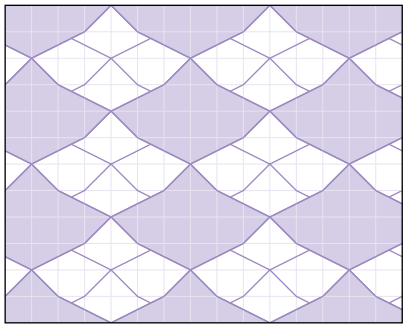
h



7 **a** yes **b** yes **c** yes
8 **a** The figure has been produced by a combination of rotation, reflection and translation. It can also be produced by just rotation and reflection.
b i $\angle S$ ii $\angle M$ iii $\angle D$
c i NO ii RQ iii AH
9 **a**



b

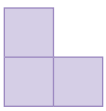


c yes

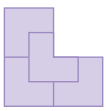
10 a



b



c



Prep quiz 13:02

1 EF **2** AD **3** HG **4** $\angle EFG$
5 $\angle ADC$ **6** WX **7** TS **8** VU
9 $\angle QTS$ **10** $\angle WVU$

Exercise 13:02

1 a shape C **b** shape F **c** shape H
d shape L
2 a $\Delta B \equiv \Delta C$ **b** shape D ≡ shape F
c $\Delta M \equiv \Delta L$
3 a DB **b** CB **c** DC
d $\angle CDB$ **e** $\angle DBC$ **f** $\angle BCD$
4 a CB **b** CF **c** BE
d $ABED \equiv CBEF$ **e** yes **f** yes
5 a A and C **b** yes **c** yes
d Shapes B and C are not congruent because sides do not match. They are not the same shape.
6 Two circles will be congruent if they have the same radius.
7 a angles: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$
sides: AB and DE, BC and EF, AC and DF
b angles $\angle X$ and $\angle M$, $\angle Y$ and $\angle L$, $\angle Z$ and $\angle N$
sides: XY and ML, XZ and MN, YZ and LN
c angles: $\angle L$ and $\angle E$, $\angle M$ and $\angle D$, $\angle N$ and $\angle F$
sides: LM and ED, LN and EF, MN and DF
d angles: $\angle A$ and $\angle R$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle P$
sides: AB and RQ, AC and RP, BC and QP
e angles: $\angle M$ and $\angle X$, $\angle N$ and $\angle G$, $\angle L$ and $\angle A$
sides: LM and AX, LN and AG, MN and XG
8 a $\Delta ABC \equiv \Delta DEF$ **b** $\Delta XYZ \equiv \Delta MLN$
c $\Delta LMN \equiv \Delta EDF$ **d** $\Delta ABC \equiv \Delta RQP$
e $\Delta LMN \equiv \Delta AXG$
9 a $\Delta ABC \equiv \Delta ADC$ **b** $\Delta ABC \equiv \Delta CDA$
c $\Delta ABC \equiv \Delta CDA$ **d** $\Delta ABC \equiv \Delta DEF$
e $\Delta ABC \equiv \Delta CDA$

Exercise 13:03

1 No in all cases.
2 We cannot say that the triangles are definitely congruent.
3 a i yes **ii** no **b** no
4 a yes **b** They are congruent. **c** yes
5 a They are congruent.
b $\angle A$ and $\angle E$, $\angle B$ and $\angle D$, $\angle C$ and $\angle F$
6 a yes **b** no
7 a BC gets longer. **b** no
8 a 4·6 cm **b** no **c** yes
9 a yes **b** The triangles are congruent.
10 a I and V, II and IV, III and VI
b The 2·5 cm side is opposite the same-sized angle.
11 a i 80° **ii** 60 **iii** 40°
b ΔDEF
c The 4 cm side is opposite the 60° angle in both triangles.
12 a BC and FE, AB and DF, AC and DE **b** yes
13 a $c^2 = a^2 + b^2$; $c^2 = m^2 + b^2$
b $a^2 = c^2 - b^2$ and $m^2 = c^2 - b^2$ ∴ $a^2 = m^2$
∴ $a = m$ (as both are positive)
c yes

Prep quiz 13:04

1 EF **2** ED **3** DF **4** $\angle E$ **5** $\angle D$
6 $\angle F$ **7** PQ **8** QR **9** 70 **10** no

Exercise 13:04

1 a RHS **b** SSS **c** AAS **d** SAS
e AAS **f** SAS **g** AAS
2 a yes (SSS) **b** no **c** no **d** no
e yes (AAS) **f** yes (SAS)
3 a AAS **b** SAS **c** AAS **d** SAS
e SSS **f** SAS **g** SAS **h** AAS
i RHS
4 a In Δs ACY and ABX
i $AB = AC$ (data)
ii $BX = CY$ (data)
iii $\angle ABX = \angle ACY$
(base angles of isosceles ΔABC)
∴ $\Delta ACY \equiv \Delta ABX$ (SAS)
b In Δs ADB and BCA
i AB is common
ii $AD = BC$ (data)
iii $\angle ADB = \angle ACB = 90°$ (data)
∴ $\Delta ADB \equiv \Delta BCA$ (RHS)
c In Δs AOB and ODC
i $AO = CO$ (radii of circle)
ii $BO = DO$ (radii of circle)
iii $AB = CD$ (data)
∴ $\Delta AOB \equiv \Delta ODC$ (SSS)

d In Δs DBC and ECB
- **i** $\angle ABC = \angle ACB$ (data)
- **ii** $\angle BDC = \angle CEB$ (data)
- **iii** BC is common.

$\therefore \Delta DBC \equiv \Delta ECB$ (AAS)

e In Δs ABC and DCB
- **i** $\angle ACB = \angle DBC$ (data)
- **ii** $CA = BD$ (data)
- **iii** BC is common.

$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)

f In Δs BDE and CFE
- **i** $\angle BDE = \angle CFE$ (data)
- **ii** $\angle BED = \angle CEF$ (vert. opp. ∠s)
- **iii** $BE = CE$ (data)

$\therefore \Delta BDE \equiv \Delta CFE$ (AAS)

Exercise 13:05

1 a (AAS); $DE = 10$ cm **b** (AAS); $DE = 10$ cm
c (SAS); $DE = 8$ cm **d** (AAS); $DE = 7$ cm
e (RHS); $DE = 7{\cdot}8$ cm **f** (AAS); $DE = 6$ cm

2 a $\Delta ABC \equiv \Delta EDF$ (SAS)
$\therefore x = 38{\cdot}5$ (matching sides of congruent Δs)
b $\Delta XYZ \equiv \Delta LNM$ (AAS)
$\therefore b = 12{\cdot}2$ (matching sides of congruent Δs)
c $\Delta ABC \equiv \Delta DEF$ (SAS)
$\therefore a = 2{\cdot}1$ (matching sides of congruent Δs)
d $\Delta ABC \equiv \Delta CDE$ (SAS)
$\therefore x = 50$ (matching ∠s of congruent Δs)
e $\Delta BAC \equiv \Delta FDE$ (SAS)
$\therefore x = 27$ (matching sides of congruent Δs)
f $\Delta ABC \equiv \Delta FDE$ (SAS)
$\therefore x = 54$ (matching ∠s of congruent Δs)

3 a $\Delta ABD \equiv \Delta CBD$ (SSS)
$\therefore a = 120°$ (matching angles of congruent Δs)
b $ABD \equiv \Delta ACD$ (RHS)
$\therefore x = 4$ (matching sides of congruent Δs)
c $\Delta AOB \equiv \Delta COD$ (SAS)
$\therefore x = 8$ (matching sides of congruent Δs)
d $\Delta ABC \equiv \Delta EDC$ (SAS)
$\therefore x = 98$ (matching ∠s of congruent Δs)
e $\Delta ABC \equiv \Delta CDA$ (SSS)
$\therefore x = 29$ (matching ∠s of congruent Δs)
f $\Delta AEB \equiv \Delta BDA$ (SAS)
$\therefore a° = \angle AEB$ (matching ∠s of congruent Δs)
$\angle AEB = 25°$ (∠ sum of ΔAEB)
$\therefore a = 25$

Exercise 13:06

1 a alt. ∠s and || lines **b** alt. ∠s and || lines
c yes **d** AAS
e i yes **ii** yes

2 a AAS **b i** yes **ii** yes

3 a SSS **b** matching ∠s of congruent Δs
c SSS **d** matching ∠s of congruent Δs

4 a Diagonals of a parallelogram bisect each other.
b SSS
c matching ∠s of congruent Δs
d $\angle AEB = \angle CEB$ and $\angle AEB + \angle CEB = 180°$.
Hence, two equal angles are adding to give 180°.
This means they both must be equal to 90°.

5 a $\angle ADC = 90°$ (co-int. ∠s, $AB \parallel DC$)
$\angle ABC = 90°$ (co-int. ∠s, $AD \parallel BC$)
$\angle BCD = 90°$ (co-int. ∠s, $AB \parallel DC$)
b SSS or RHS **c** matching ∠s of congruent Δs

6 All angles are right angles.
All diagonals are equal.
All sides are equal.
Diagonals bisect each other at right angles.
Opposite sides are parallel.
Diagonals bisect the angles at the vertices.

Diagnostic test 13

1 a A and C **b** A and C
2 a MN **b** ML **c** NL
d $\angle MNL$ **e** $\angle NLM$ **f** $\angle GEF$
3 a $\Delta ABD \equiv \Delta CBD$ **b** CB **c** AD
d $\angle BCD$ **e** $\angle BDC$
4 a SAS **b** RHS **c** AAS **d** SSS
5 a SAS **b** SSS **c** AAS
6 a $x = 20$ **b** $x = 20$ **c** $x = 40$ **d** $x = 12$

ASSIGNMENT 13A

1 a ΔI **b** ΔG **c** ΔF **d** ΔE
2 a SAS **b** RHS
3 a $x = 20$ (matching sides of congruent Δs)
b $\angle DBC = a°$ (matching ∠s of congruent Δs)
$a + 50 = 90$
$a = 40$
4 a $\angle J$ **b** $\angle H$ **c** $\angle F$ **d** KL **e** GH
f DC
5 a $\angle DCE$ and $\angle BCA$, $\angle CDE$ and $\angle CBA$, $\angle DEC$ and $\angle BAC$
6 a SAS **b** 20 **c** $\angle EDC$
7 a SAS **b** matching sides of congruent triangles
c $\angle CBD$

ASSIGNMENT 13B

1 a
$$\begin{array}{r} 38 \\ +\ 36 \\ \hline 74 \end{array}$$
b
$$\begin{array}{r} 904 \\ -\ 63 \\ \hline 841 \end{array}$$
c
$$\begin{array}{r} 564 \\ +\ 537 \\ \hline 1101 \end{array}$$
d
$$\begin{array}{r} 1632 \\ -\ 925 \\ \hline 707 \end{array}$$

2 I bought 28 L. The tank had 37 L in it before it was filled.
3 No. of bricks $= 3 \times 57 + 3 \times 55 = 336$
4 4 unit^2

5 $178.80 (nearest cent)
6 1, Emma; 2, Sarah; 3, Ben: 4, Kate
7 **a** 2 **b** 4 **c** 3

ASSIGNMENT **13C**

1 **a** $6x + 7y + 6$ **b** $6x^2 + 5x - 6$
2 **a** $-x^2 + 2x$ **b** $4a^2 - 12a + 3ab + 2b$
3 **a** $p = -7$ **b** $a = -\frac{7}{9}$ **c** $b = -5\frac{1}{3}$
4 **a** 34 **b** −54
5 **a** 603 m^2 **b** 152 cm^2
6 **a** $y = 2$ **b** $x = 1$ **c** $x = -2$
7 **a** $\frac{5}{6}$ **b** $\frac{35}{36}$
8 11 232 cm^3
9 **a** 23·2 kg **b** A 2·4 L, B 1·6 L
10 **a** 8·4 kg **b** 0·4%

Chapter 14

Exercise 14:01

1 **a** primary **b** secondary **c** primary
d primary **e** secondary **f** secondary
g primary **h** primary **i** primary
j secondary
2 **a** unbiased **b** biased **c** unbiased
d biased **e** biased **f** unbiased
3 **a** primary **b** secondary **c** secondary
d primary **e** secondary **f** primary
g primary **h** secondary
4 **a** Preferred to what isn't said.
b Writers could be biased because Shane Warne is Australian.
c More union leaders could be members of the Labor party.
d We don't know what bosses the employees have worked for before; how wide is their experience.
e Workers are not unemployed and may not have had the experience of unemployment to make an informed comment.
5 You might have only approached your friends or other students you get along with and so could be more likely to have the same opinion as you. Also, how was the question asked? 'I like the Tigers; what team do you support?' would be suggesting the response you wanted.
6 **a** Answer will vary.
(Approx. population in 2013 = 22·8 million)
b Unless a drastic change has occurred since writing this text, the order will be: NSW, VIC, QLD, WA, SA, TAS, ACT, NT

Exercise 14:02

1 **a** systematic **b** stratified **c** random
2 **a** systematic **b** 80
3 18 men, 42 women
4 **a** Anthea's sample
b Sample was to small and biased.
c Biased, as he only talked to boys.
d Systematic (every 4th boy and girl) and stratified (each class).
5 Year 9 = 45, Year 10 = 39, Year 11 = 33, Year 12 = 27
6 13 men, 16 women, 21 children
7 160 red, 140 blue, 120 yellow, 80 green
8 **a** 6 **b** no, because the sample is too small
9 Answers will vary.
10 Each year group, each class in the year if they are graded classes, possibly boys and girls.
11 **a** none **b** ten **c** five
12 Repeat using a different page, the last word on each line or simply by counting more words.

Challenge 14:02

1 150 2 Approx. 333 3 **a** 150 **b** 130

Prep quiz 14:03

1 6 2 3 3 4 4 5
5 *fx* column is: 2, 16, 48, 40, 20 6 20, 126
7 8 8 6 9 6 10 6·3

Exercise 14:03

1 **a** 9, 10, 13, 13, 17, 19, 20, 22, 25, 27, 30, 35
b **i** mean = 17·25, median = 16·5
ii mean = 19·75, median = 19·5
iii mean = 23, median = 22
c sample **ii** **d** sample **ii**
2 The fifth height.
3 **a** **i** 53 **ii** 59·25 **iii** 60·875 **iv** 59·75
b sample iii
4 **a** Answers will vary.
b Answers will vary.
c It will vary less and be closer to the centre of the numbers possible.
d Very close to 50 (or 49·5).
5 **a**

No.	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Total
Mean	3·8	3·6	3·7	3·0	2·8	3·9	

b 3·5 **c** 3·5, same as part b
6 **a** mean = 47, median = 44
b mean = 42, median = 43
c the median
d the median
7 **a** Sample A: **i** 12 **ii** 12 **iii** 11 **iv** 9·9
Sample B: **i** 5 **ii** 12 **iii** 11 **iv** 10·7
b For Sample A the range is greater and the mean is lower.
c The score of 1 in sample A.
8 **a** $670 000 **b** $360 000 **c** the median
9 mode = $47 000, mean = $73 250, median = $58 500.
The median is the most representative wage.

10 The mode is most useful to the dress manufacturer as this indicates the size most often purchased.

11 He should use the mean because this takes into account every mark and he is concerned with overall performance.

12 mode = 79, 82 mean = 81·1 median = 79. The median is the best measure of Alan's performance.

13 **a** **i** mean = 74; 4 **ii** mean = 76; 1

b not necessarily

14 8·22

15 **a** mean = 8·17 (2 dec.pl.)
score = 22·88 (2 dec.pl.)

b 20·8

Prep quiz 14:04

1 20 **2** 1 **3** 2 **4** 2·5 **5** 1
6 7 **7** 4 **8** 9 **9** 13 **10** 23

Exercise 14:04

1 **a** **i** 27 **ii** 5 **iii** 5

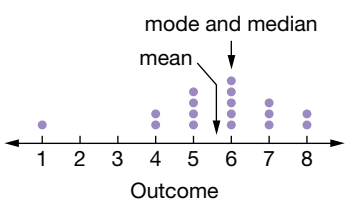
b

Outcome (x)	Frequency (f)	fx
1	1	1
2	2	4
3	2	6
4	5	20
5	6	30
6	5	30
7	4	28
8	2	16
Total	27	135

c 5 **d** They are all 5.

2 **a** **i** 17 **ii** 6 **iii** 6 **b** 5·6 (1 dec.pl.)

c



d the measures of centre are not all equal

e it has caused the mean to be lower than the median.

3 **a** 6 **b** 3 **c** 4 **d** 3 **e** 3 **f** 90
g 3 **h** they are all equal to 3

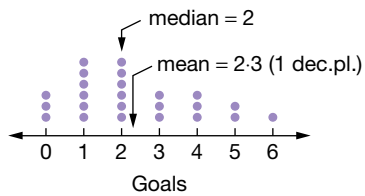
4 **a** 4 **b** 2 **c** 0 **d** 21 **e** 32 **f** 70
g 2·2 (1 dec.pl.) **h** 2 **i** the mean

5 **a** 7 **b** 7 **c** 6·8
d mean = 7, median = 7 **e** it lowered the mean

6 **a** mode = 5, mean = 5, median = 5
b mode = 7, mean = 6·14, median = 6
c mode = 2, mean = 3·1, median = 3
d mode = 5, mean = 4·4, median = 5
e mode = 8, mean = 7·6, median = 8
f mode = 8–10, mean = 8, median = 8

7 **a** 7 **b** 5 **c** 5 **d** 4·7 (1 dec. pl.)
e 5 **f** no change to either

8 **a–b** mode = 1 and 2



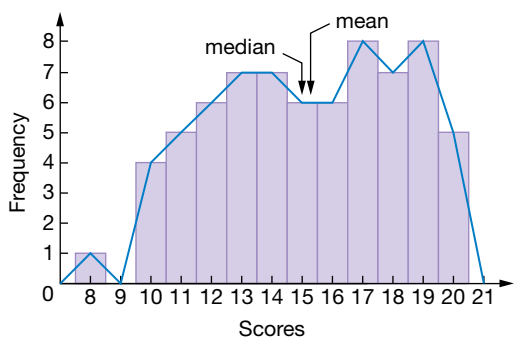
c mean = 2·3 (1 dec.pl.)

9 **a**

Outcome	Freqency (f)	fx
8	1	8
9	0	0
10	4	40
11	5	55
12	6	72
13	7	91
14	7	98
15	6	90
16	6	96
17	8	136
18	7	126
19	8	152
20	5	100
Total	70	1064

b mode = 17 and 19, median = 15, mean = 15·2

c



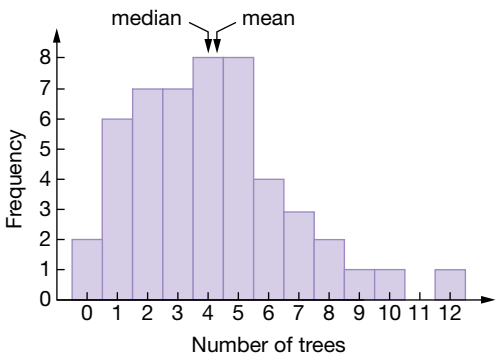
10 **a** 12, 0

b

Outcome	Freqency	fx
0	2	0
1	6	6
2	7	14
3	7	21
4	8	32
5	8	40
6	4	24
7	3	21
8	2	16
9	1	9
10	1	10
11	0	0
12	1	12
Total	50	205

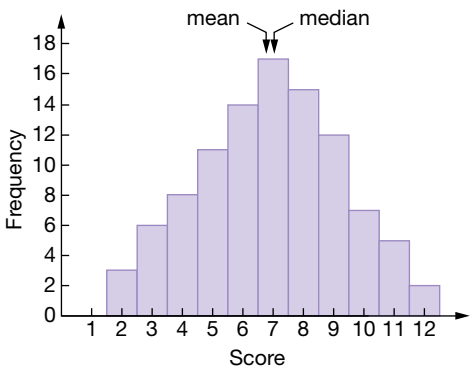
c mode = 4 and 5, median = 4, mean = 4·1

d



11 **a** 100 **b** 7 **c** 6·91 **d** 7

e

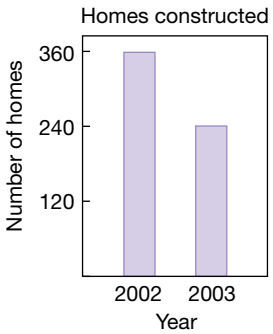


f 7

Investigation 14:04

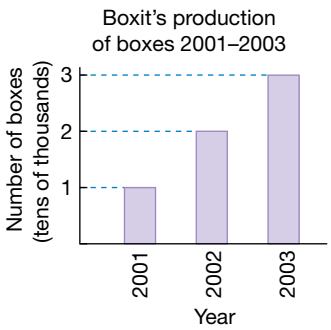
1 **a** $\frac{2}{3}$ **b** $\frac{2}{3}$ **c** yes

d



2 **a** 2 times **b** 4 times **c** 8 times
d 3 times **e** 9 times **f** 27 times

g



Exercise 14:05

1

Class	Tally	Frequency
31–40	𝍸	5
41–50	𝍸 |	6
51–60	𝍸 𝍸 |	11
61–70	|||	3
71–80	𝍸 |	6
81–90	|||	3
91–100	|	1
	Total	35

2 **a** 8 **b** 18 **c** 35 **d** 6·5

3

Class	*c.c.*	Frequency	$f \times c.c.$
1–5	3	3	9
6–10	8	7	56
11–15	13	10	130
16–20	18	8	144
21–25	23	2	46
	Total	30	385

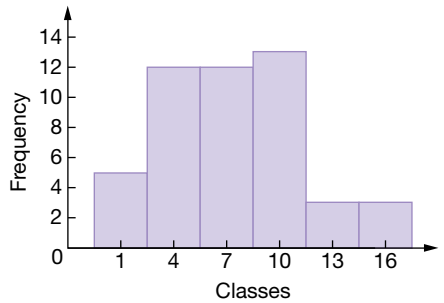
modal class = 11–15
mean = 12·8 (1 dec.pl.)
median class = 11–15

4 a

Class	*c.c.*	Frequency	$f \times c.c.$
0–2	1	5	5
3–5	4	12	48
6–8	7	12	84
9–11	10	13	130
12–14	13	3	39
15–17	16	3	48
	Total	48	354

b 9–11 **c** 6–8 **d** 7·375 **e** yes

f



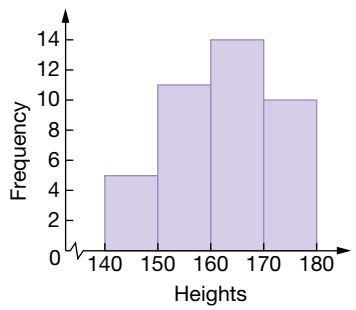
5 a

Class	*c.c.*	Tally	Frequency (f)	$f \times c.c.$
140–<150	145	𝍸	5	725
150–<160	155	𝍸 𝍸 |	11	1705
160–<170	165	𝍸 𝍸 ||||	14	2310
170–<180	175	𝍸 𝍸	10	1750
		Total	40	6490

b 160–<170, 160–<170 **c** 162·25

d use more, smaller classes

e



6

Outcome	Frequency	fx
1	0	0
2	1	2
3	0	0
4	3	12
5	5	25
6	7	42
7	8	56
8	4	32
9	1	9
10	1	10
Total	30	188

Class	*c.c.*	Frequency	fx
2–4	3	4	12
5–7	6	20	120
8–10	9	6	54
	Total	30	186

a The value of the individual scores is lost in the grouped table.

b ungrouped mean = $\frac{188}{30}$ = 6·27 (2 dec. pl.)
grouped mean = $\frac{186}{30}$ = 6·2
ungrouped mean is more accurate.

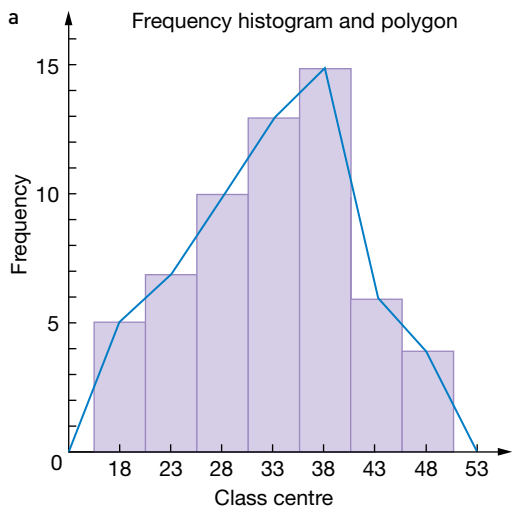
7

Class	Class centre	Frequency
0–2	1	25
3–5	4	22
6–8	7	13
	Total	60

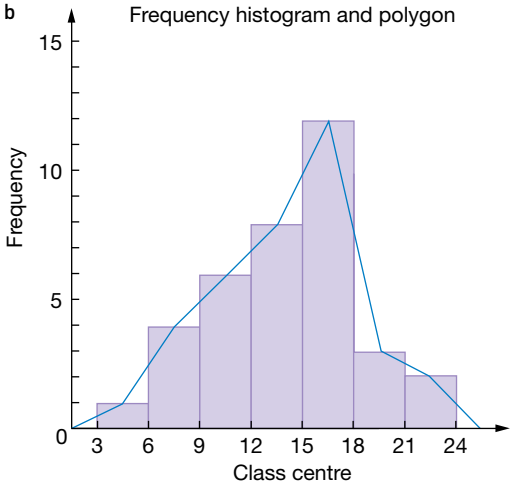
a Using three classes does not show the spread of scores realistically as it makes the first two classes appear very similar. If the classes 0–1, 2–3, 4–5 etc. had been used, it would show how different they are.

b It would be difficult to use more than 5 classes in this case since there are only 9 outcomes. Having groups with the same number of outcomes would not be possible if more than 5 outcomes are used.

8 a



b



9 a

No. of cans sold (x)	$c.c.$	Frequency (f)	$f \times c.c.$
16–22	19	7	133
23–29	26	18	468
30–36	33	18	594
37–43	40	15	600
44–50	47	8	376
51–57	54	4	216
	Total	70	2387

b 10 weeks **c** mean $= \frac{2387}{70} = 34{\cdot}1$ cans per day

10 a 6–14, 15–23, 24–32, 33–41, 42–50

b

Frequency (f)	$f \times c.c.$
5	50
8	152
12	336
4	518
7	322
Total 46	1378

c modal class 33–41
median class 24-32
mean $\frac{1378}{46} = 29{\cdot}96$

11 a

Class	$c.c.$	Frequency (f)	$f \times c.c.$
1–10	5·5	9	49·5
11–20	15·5	15	232·5
21–30	25·5	19	484·5
31–40	35·5	17	603·5
41–50	45·5	20	910
51–60	55·5	17	943·5
61–70	65·5	21	1375·5
71–80	75·5	16	1208
81–90	85·5	18	1539
91–100	95·5	8	764
	Total	160	8110

b i $99 - 3 = 96$ **ii** 61–70
iii median falls between 41–50 and 51 –60, as the 80th score is in the 41–50 class and the 81st score is in the 51–60 class.

c mean $= \frac{8110}{160} = 50{\cdot}7$ (1 dec.pl.)

d The mean and median are approximately equal, but less than the modal class.

Exercise 14:06

1 a Group A: **i** 29 **ii** 35 **iii** 30 **iv** 30
Group B: **i** 30 **ii** 25 **iii** 25 **iv** 26

b i median and mean **ii** mode and median

2 a 3 × tables

b mode = 12 s, median = 27 s, range = 47 s

c There are two modes, 32 s and 59 s,
median = (39 + 41) ÷ 2 = 40 s, range = 44 s.

d No outlier is present in the data for either table as no score stands alone at one end of the data.

3 a

Students		Parents
Leaf	Stem	Leaf
	0	6
9 9 8 5	1	1 6 6 8 8
8 8 6 5 4 3 2 1 0 0	2	0 1 2 3 4 5 6 6 7
9 7 5 5 4 3 0 0 0 0	3	0 1 1 2 2 5 7 8
7 5 4 2 2 1 1 0	4	0 1 2 2 2

b Students: mode = 30, median = 30, range = 32
Parents: mode = 42, median = 26·5, range = 36

c the median

d Students' mean ≑ 30·72
Parents' mean ≑ 27·57

4 a

Adults		Children
Leaf	Stem	Leaf
2	1	
	2	
	3	
	4	
9 8 8 6 5	5	
8 8 4 4 4 1 0 0	6	2 7
6 6 2 2 2	7	2 7 7 8
1 0 0	8	1 2 5 5 5 6 6 6 8
2	9	2 4 4 5 8 9 9
	10	0 2

b For the adults' pulse rates, 12 and 92 are outliers. For the children's pulse rates, 62 might be considered an outlier (or might not).

c The outlier 12 can be ignored unless you know that the adult was dying at the time. It is outside the range of human pulse rates and so must be an error in measurement.

d *Adults:* median = (64 + 68) ÷ 2 = 66 (If the score 12 is ignored, the median for adults is 68.) There are three modes: 64, 68 and 72.
Children: median = 86. There are two modes: 85 and 86.

e The adult pulse rates range from 55 to 92 (a range of 37), have a median of 68, modes of 64, 68 and 72. (For this description the score 12 has been ignored.) The child pulse rates range from 62 to 102 (a range of 40), have a median of 86 and modes of 85 and 86. Overall, the pulse rates of these children tend to be about 18 beats per minute more than those of these adults.

Diagnostic test 14

1 **a** primary **b** secondary **c** secondary **d** primary

2 **a** systematic **b** random **c** stratified

3 **a** 36 **b** 32 **c** 28 **d** 24

4 **a** mean = 14·7, median = 15·5

b mean = 16, median = 15 **c** mean **d** median

5 **a** mode = 2, mean = 3·65, median = 3

b mode = 4, mean = 4, median = 4

c mode = 3, mean = 2·86, median = 3

6 **a**

Class	*c.c.*	Tally	*f*	*f*×*c.c.*
10–<20	15	𝍸 ||	7	105
20–<30	25	𝍸 𝍸	10	250
30–<40	35	𝍸 𝍸 ||	12	420
40–<50	45	|||	3	135
50–<60	55	|	1	55
		Total	33	965

modal class = 30–<40, mean = 29·2 (1 dec. pl.), median class = 20–<30

b

Class	*c.c.*	*f*	*f*×*c.c.*
1–5	3	11	33
6–10	8	18	144
11–15	13	22	286
16–20	18	10	180
21–25	23	7	161
Totals		68	804

Modal class = 11–15
Mean = 11·82 (2 dec. pl.)
Median class = 11–15

7 **a** 35 in the Year 4 results is an outlier.

b mode = 42, median = 48, range = 41

c mode = 79, median = (65 + 68) ÷ 2 = 66·5, range = 45

ASSIGNMENT 14A

1 **a**

Number of faulty products	Tally	Frequency	*fx*
1	𝍸	5	5
2	𝍸 |||	8	16
3	𝍸 𝍸	10	30
4	𝍸 ||||	9	36
5	𝍸 |	6	30
6	|	1	6
7	|	1	7

b Mode = 3; Mean = 3·25; Median = 3

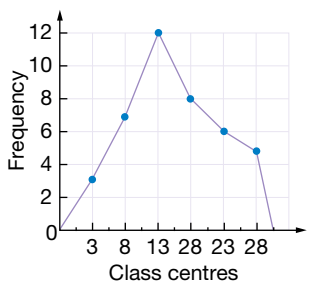
2 **a** 4 **b** 43 **c** 8 **d** 40 **e** 72 **f** 24

g

Class	*c.c.*	*f*
30–39	34·5	3
40–49	44·5	5
50–59	54·5	4
60–69	64·5	7
70–79	74·5	8
80–89	84·5	7
90–99	94·5	6

3 **a** **i** 11–15 **ii** 15·68

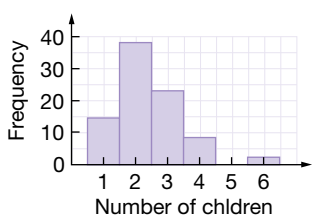
b



c No **d** 11–15

4 **a** 84 **b** 52

c



ASSIGNMENT 14B

1

Digit	0	1	2	3	4	5	6	7	8	9
Number needed	13	15	15	61	12	5	5	4	4	4

2 1248

3 4 cm^2

4 16 (assuming that all are different people).

5 9

6 **a** 8° **b** 11:20 am **c** 16·5° **d** 3 h

e 9–10 am, 11–12 am

ASSIGNMENT 14C

1 **a** $3375\,cm^3$ **b** $750\,cm^3$ **c** $2625\,cm^3$
2 **a** $a = 15$ ($\angle$s in a right $\angle$)
b $b = 40$ ($\angle$s in a right $\angle$)
c $x = 120$ ($\angle$s at a point)
3 5 m; 70 mm
4 3 min, 12 sec; 62·5 m
5 30 km/h
6 **a** 0·3 ha **b** 6 cm
7 **a** $y = 2x$ **b** $y = 3x + 1$ **c** $y = x^2$
8 **a** 0·6% **b** 25%
9 $a = 6$, $b = 15{\cdot}2$ (1 dec. pl.)
10 **a** -11 **b** -18 **c** 5 **d** -2

Chapter 15

Prep quiz 15:01

1 90° **2** $m = 60$ **3** $x = 48$ **4** 180°
5 $a = 130$ **6** $b = 45$
7 They add up to 180°.
8 They add up to 180°.
9 They add up to 90°.
10 They add up to 90°.

Exercise 15:01

1 **a** $x = 35$ **b** $a = 50$ **c** $x = 45$ **d** $x = 15$
e $x = 25$ **f** $a = 40$ **g** $a = 130$ **h** $y = 120$
i $p = 125$ **j** $a = 55$ **k** $a = 80$ **l** $a = 20$
2 All are adjacent complementary angles.
a $x = 70$ **b** $a = 80$ **c** $b = 40$ **d** $e = 40$
e $\alpha = 68°$ **f** $\beta = 65°$ **g** $x = 67$ **h** $a = 18$
i $x = 65$ **j** $x = 56$ **k** $b = 25$ **l** $a = 28$
m $x = 65$ **n** $x = 30$
3 All are adjacent supplementary angles.
a $a = 60$ **b** $b = 155$ **c** $p = 135$ **d** $x = 75$
e $c = 20$ **f** $a = 50$ **g** $\alpha = 15°$ **h** $m = 50$
i $x = 48$ **j** $p = 120$ **k** $x = 125$
4 **a** $\alpha + 30° = 90°$
$\alpha = 60°$
b $2\beta + 80° = 180$
$\beta = 50°$
c $4\alpha + 2\alpha = 180°$
$6\alpha = 180°$
$\alpha = 30°$
d $\alpha + \alpha + 20° = 90°$
$2\alpha = 70°$
$\alpha = 35°$
e $\alpha + 100° + \alpha = 180°$
$2\alpha = 80°$
$\alpha = 40°$
f $3\beta + 75° = 90°$
$3\beta = 15°$
$\beta = 5°$
5 **a** $y + 31 = 90$ (adj. comp. $\angle$s)
$\therefore y = 59$
$x + x + 90 = 180$ (adj. supp. $\angle$s)
$2x = 90$
$\therefore x = 45$
b $x + 37 = 180$ (adj. supp. $\angle$s)
$\therefore x = 143$
$z + 37 = 90$ (adj. comp. $\angle$s)
$\therefore z = 53$
$y + 53 = 180$ (adj. supp. $\angle$s)
$\therefore y = 127$
c $c + 141 = 180$ (adj. supp. $\angle$s)
$\therefore c = 39$
$b + 141 = 180$ (adj. supp. $\angle$s)
$\therefore b = 39$
$a + 39 + 104 = 180$ (adj. supp. $\angle$s)
$\therefore a = 37$

Exercise 15:02

1 All are angles at a point.
a $x = 100$ **b** $a = 30$ **c** $y = 70$ **d** $x = 80$
2 All are vertically opposite angles.
a $a = 130$ **b** $b = 25$ **c** $c = 100$
d $a = 70$ **e** $x = 135$ **f** $y = 85$
3 **a** $4\alpha + 40° + 70° + 90° = 360°$ (angles at a point)
$4\alpha = 160°$
$\alpha = 40°$
b $b + b + 160 + 100 = 360$ (angles at a point)
$2b = 100$
$b = 50$
c $a = 70$ (vert. opp. $\angle$s)
$a + b + 30 = 180$ (adj. supp. $\angle$s)
$70 + b + 30 = 180$
$b = 80$
d $a = 60$ (vert. opp. $\angle$s)
$c + 60 = 180$ (adj. supp. $\angle$s)
$c = 120$
$b + 60 + 90 = 180$ (adj. supp. $\angle$s)
$b = 30$
e $x = 260$ (angles at a point)
f $\beta = 45°$
(vert. opp. $\angle$s)
g $2a = 100$ (vert. opp. $\angle$s)
$a = 50$
h $b = 155$ (vert. opp. $\angle$s)
$a = 25$ (adj. supp. $\angle$s)
i $m = 120$ (adj. supp. $\angle$s)
$120 + p + 28 = 180$ (adj. supp. $\angle$s)
$p = 32$
j $a + 50 = 30 + 40$ (vert. opp. $\angle$s)
$a = 20$
k $a = 115$ (vert. opp. $\angle$s)
$b = 65$ (adj. supp. $\angle$s)

4 **a** $b = 45$ (vert. opp. $\angle$s)
$a = 135$ (adj. supp. $\angle$s)

b $\alpha = 100°$ (vert. opp. $\angle$s)
$\beta = 80°$ (adj. supp. $\angle$s)

c $6\alpha = 360°$ (angles at a point)
$\alpha = 60°$

d $\alpha + 70° + 40° = 180°$ (adj. supp. $\angle$s)
$\alpha = 70°$

e $\alpha + 70° = 90°$ (adj. comp. $\angle$s)
$\alpha = 20°$
$\beta + 60° = 90°$ (adj. comp. $\angle$s)
$\beta = 30°$

f $\alpha + 70° = 180°$ (adj. supp. $\angle$s)
$\alpha = 110°$
$\beta + 20° = 90°$ (adj. comp. $\angle$s)
$\beta = 70°$

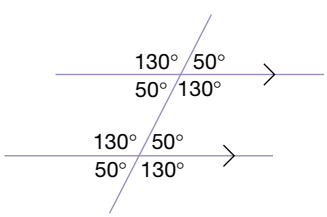
5 **a** $a = 56$(vert. opp. $\angle$s)
$b + 90 + 56 = 180$ (adj. supp. $\angle$s)
$\therefore b = 34$

b $a = 70$ (vert. opp. $\angle$s)
$b + 70 = 180$ (adj. supp. $\angle$s)
$\therefore b = 110$
$c + 41 = b$ (vert. opp. $\angle$s)
$c + 41 = 110$
$\therefore c = 69$

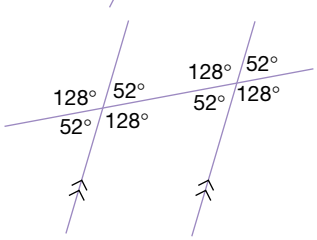
c $x + 60 = 90$ (adj. comp. $\angle$s)
$\therefore x = 30$
$y + 40 + 70 + 90 = 360$ (angles at a point)
$\therefore y = 160$

Exercise 15:03

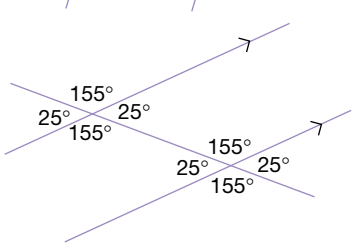
1 **a**



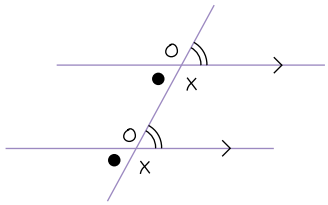
b



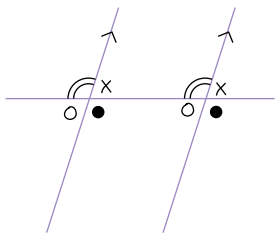
c



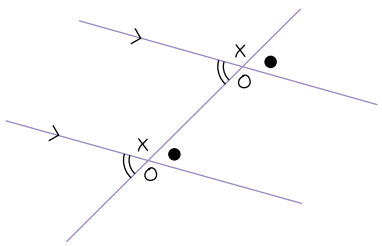
2 **a**



b



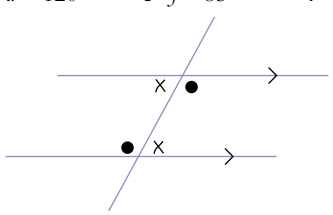
c



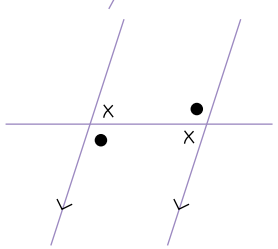
3 All reasons are: corresponding angles and parallel lines.

a $a = 38$ **b** $b = 140$ **c** $c = 80$
d $d = 120$ **e** $f = 85$ **f** $a = 100, b = 110$

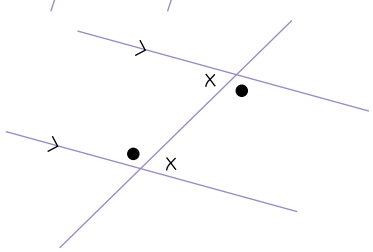
4 **a**



b



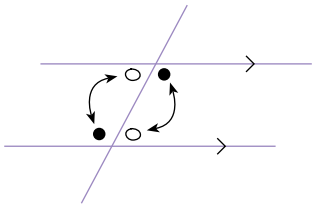
c



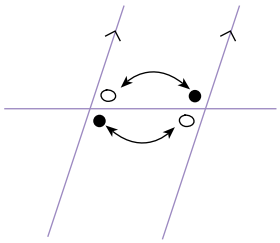
5 All reasons are: alternate angles and parallel lines.

a $a = 30$ **b** $e = 140$ **c** $c = 80$
d $f = 132$ **e** $a = 83, b = 42$ **f** $x = 65, y = 72$

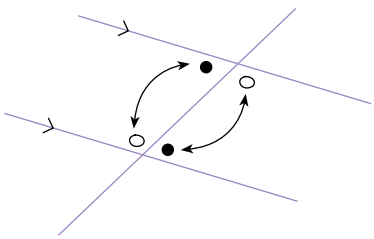
6 **a**



b



c



The pairs are shown by arrows.

7 All answers are: cointerior angles and parallel lines.

a $a = 120$ **b** $b = 60$ **c** $c = 145$

d $x = 50, y = 92$ **e** $a = 98, b = 115$

8 **a** $a = 42$ (altern. angles, || lines)

b $b = 138$ (co-int. angles, || lines)

c $c = 135$ (corresp. angles, || lines)

d $d = 105$ (corresp. angles, || lines)

e $e = 68$ (altern. angles, || lines)

f $f = 97$ (co-int. angles, || lines)

g $a = 55$ (corresp. angles, || lines)

h $b = 80$ (co-int. angles, || lines)

i $a = 30$ (altern. angles, || lines)

9 **a** $a = 145$ (co-int. angles, || lines),
$b = 35$ (altern. angles, || lines)

b $\alpha = 125°$ (corresp. angles, || lines),
$\beta = 125°$ (vert. opp. angles)

c $a = 50$ (co-int. angles, || lines,
$b = 50$ (co-int. angles, || lines)

d $m = 60$ (altern. angles, || lines),
$n = 120$ (co-int. angles, || lines)

e $a = 120$ (opp. angles of a parallelogram equal),
$b = 60$ (co-int. angles, || lines),
$d = 60$ (co-int. angles, || lines)

10 **a** Alternate angles are equal.

b Corresponding angles are equal.

c Cointerior angles are supplementary.

d Alternate angles are equal.

e Cointerior angles are supplementary.

Exercise 15:04

1 **a** $a = 110$ ($\angle$ sum of a triangle)

b $b = 50$ ($\angle$ sum of Δ)

c $c = 60$ ($\angle$ sum of Δ)

d $x = 90$ ($\angle$ sum of Δ)

e $y = 101$ ($\angle$ sum of Δ)

f $x = 40$ ($\angle$ sum of Δ)

g $g = 94$ (exterior $\angle$ of Δ)

h $h = 116$ (exterior $\angle$ of Δ)

i $i = 150$ (exterior $\angle$ of Δ)

2 **a** $2a + 60 + 50 = 180$
$2a = 70$
$a = 35$

b $2\alpha + \alpha + 90° = 180°$
$3\alpha = 90°$
$\alpha = 30°$

c $3a + 2a + 4a = 180$
$9a = 180$
$a = 20$

d $d + 51 = 103$
$d = 52$

e $40 + 2e = 120$
$2e = 80$
$e = 40$

f $2f + f = 130$
$3f = 130$
$f = 43\frac{1}{3}$

3 **a** $a = 80$ ($\angle$ sum of ΔLMN)

b $a = 70$ ($\angle$ sum of ΔADE)

c $a = 43$ ($\angle$ sum of ΔLMZ)

4 **a** $a = 84; b = 67$ **b** $a = 67$ **c** $a = 40; b = 40$

5 **a** $x + 50 + 105 = 180$ ($\angle$ sum of ΔABC)
$x = 25$
$25 + 92 + y = 180$ ($\angle$ sum of ΔEDC)
$y = 63$

b $x = 36$ (vert. opp. $\angle$s)
$y + 36 + 90 = 180$ ($\angle$ sum of ΔPNR)
$y = 54$

c $x = 40$ ($\angle$ sum of ΔABD)
$40 + y = 90$ (comp. $\angle$s)
$y = 50$

6 **a** $c = 50$ ($\angle$ sum of ΔADE);
$a = 70$ (corresp. $\angle$s, || lines);
$b = 60$ (corresp. $\angle$s, || lines)

b $b = 50$ (vert. opp. $\angle$s);
$a = 66$ ($\angle$ sum of ΔABC);
$c = 60$ ($\angle$ sum of ΔCDE)

c $a = 95$ (vert. opp. $\angle$s);
$b = 33$ (alt. $\angle$s, || lines);
$c = 52$ ($\angle$ sum of ΔABT)

Exercise 15:05

1 The reason for all is: angle sum of a quadrilateral.

a $a = 50$ **b** $b = 120$ **c** $c = 32$

d $g = 275$ **e** $x = 123$

2 a $5x + 84 + 85 + 116 = 360$

$5x + 285 = 360$

$5x = 75$

$x = 15$

b $2x + 67 + 78 + 105 = 360$

$2x + 250 = 360$

$2x = 110$

$x = 55$

c $3\alpha + 140° + \alpha + 28° = 360°$

$4\alpha + 168° = 360°$

$4\alpha = 192°$

$\alpha = 48°$

d $3\beta + 50° + \beta + 40° = 360°$

$4\beta + 90° = 360°$

$4\beta = 270°$

$\beta = 67\frac{1}{2}°$

3 a $x = 75$; $y = 87$ **b** $x = 70$; $y = 20$

c $x = 142$; $y = 38$

4 a $x = 230$ (angles at a point);
$y = 70$ ($\angle$ sum of quad.)

b $x = 102$ (adj. supp. $\angle$s); $y = 75$ ($\angle$ sum of quad.)

c $x = 55$ ($\angle$ sum of Δ); $y = 74$ ($\angle$ sum of quad.)

d $x = 75$ ($\angle$ sum of quad.); $y = 45$ ($\angle$ sum of Δ)

e $x = 120$ (co-int. $\angle$s, $\|$ lines);
$y = 70$ ($\angle$ sum of quad. $ABCE$)

Prep quiz 15:06

1 all of them **2** ΔXYZ

3 $\angle CAB$ and $\angle CBA$ **4** DE and EF

5 two angles equal **6** two sides equal

7 equal **8** all equal **9** all equal **10** 60°

Exercise 15:06

1 a $b = 33$ (base $\angle$s of isos. Δ)

b $c = 28$ (base $\angle$s of isos. Δ)

c $y = 5{\cdot}8$ (equal sides of isos. Δ)

d $x = 120$ ($\angle$ sum of Δ); $y = 4$ (equal sides of isos. Δ)

e $x = 5$ (equal sides of isos. Δ); $a = 45$ ($\angle$ sum of Δ)

f $a = 7$ (equal sides of isos. Δ); $b = 30$ ($\angle$ sum of Δ)

g $a = 60$ (equilateral Δ)

h $x = 60$ (equilateral Δ)

i $y = 20$; $z = 20$ (equilateral Δ)

2 a $a = 70$ (base $\angle$s of isos. Δ); $b = 40$ ($\angle$ sum of Δ)

b $a = 25$ (base $\angle$s of isos. Δ); $b = 130$ ($\angle$ sum of Δ)

c $a = 50$ (base $\angle$s of isos. Δ); $b = 80$ ($\angle$ sum of Δ)

3 a $a = 60$ (angle of equil. Δ); $b = 120$ (adj. supp. $\angle$s)

b $a = 60$ (angle of equil. Δ);
$b + 40 = 60$ (angle of equil. Δ), $\therefore$ $b = 20$

c $a = 58$ (adj. supp. $\angle$s); $b = 58$ (equal $\angle$s of isos. Δ)

d $a = 20$ (equal $\angle$s of isos. Δ); $b = 140$ ($\angle$ sum of Δ)

e $a = 65$ (equal $\angle$s of isos. Δ);
$b = 25$ ($\angle$ sum of ΔABD)

f $a = 33$ (alt. $\angle$s, $\|$ lines); $b = 33$ (equal $\angle$s of isos. Δ)

Exercise 15:07

1 a $\angle EBC = 85°$ (adj. supp. $\angle$s)
$\angle ECB = 45°$ (adj. supp. $\angle$s)
$\therefore$ $a = 50$ (angle sum of Δ)

b $\angle ACB + 48° + 44° = 180°$ ($\angle$ sum of Δ)
$\angle ACB = 88°$
$y = 88$ (vert. opp. $\angle$s)

c $\angle AED = 95°$ (corresp. $\angle$s, $\|$ lines)
$\therefore$ $a + 95 + 30 = 180$ ($\angle$ sum of ΔAED)
$\therefore$ $a = 55$

d $\angle ECD = 140°$ (co-int. $\angle$s, $\|$ lines)
$\therefore$ $a + 140 + 25 = 180$ ($\angle$ sum of Δ)
$\therefore$ $a = 15$

e $\angle FEC = 130°$ (vert. opp. $\angle$s)
$\therefore$ $a + 130 = 180°$ (co-int. $\angle$s, $\|$ lines)
$\therefore$ $a = 50$

f $\angle ADY = 140°$ (corresp. $\angle$s, $\|$ lines)
$\therefore$ $y + 140 = 180$ (adj. supp. $\angle$s)
$\therefore$ $y = 40$

g $\angle DBC = 79°$ (corresp. $\angle$s, $\|$ lines)
$\therefore$ $x + 58 + 79 = 180$ ($\angle$ sum of Δ)
$\therefore$ $x = 43$

h $\angle ABC = 95°$ ($\angle$s at a point)
$\therefore$ $x + 115 + 95 + 95 = 360$ ($\angle$ sum of a quad.)
$\therefore$ $x = 55$

i $\angle ABC = 78°$ (alt. $\angle$s, $\|$ lines)
$\therefore$ $y = 78$ (equal $\angle$s of isos. Δ)

2 a $\angle ACB = 70°$ (equal $\angle$s of isos. Δ)
$\therefore$ $m + 70 + 70 = 180$ ($\angle$ sum of Δ)
$\therefore$ $m = 40$

b $\angle ABC = a°$ (equal $\angle$s of isos. Δ)
$a + a + 130 = 180$ ($\angle$ sum of Δ)
$\therefore$ $a = 25$

c $\angle AEC + 75° + 100° + 70° = 360°$
($\angle$ sum of quad. $ABCE$)
$\angle AEC = 115°$
$\therefore$ $a + 115 = 180$ (adj. supp. $\angle$s)
$\therefore$ $a = 65$

d $\angle ADC = 60°$ (angle of equil. Δ)
$\therefore$ $\angle CDB = 60° + 70°$
($\angle CDB = \angle ADC + \angle ADB$)
$= 130°$
$\therefore$ $m + 130 = 180$ (co-int. $\angle$s, $\|$ lines)
$\therefore$ $m = 50$

e $\angle DAE + 60° + 45° = 180°$ ($\angle$ sum of ΔADE)
$\angle DAE = 75°$
$\therefore$ $50 + 75 + x = 180$ ($\angle$ sum of ΔABC)
$\therefore$ $x = 55$

f $\angle ACD = 145°$ (co-int. $\angle$s, $AB \| CD$)
$\angle ECD = 135°$ (co-int. $\angle$s, $EF \| CD$)
$\therefore$ $b + 145 + 135 = 360$ ($\angle$s at a point)
$\therefore$ $b = 80$

g $\angle CDB = 70°$ (co-int. $\angle$s and $AB \parallel CD$)
$\angle CED = 80°$ (adj. supp. $\angle$s)
$\therefore \angle ECD + 70° + 80° = 180°$ ($\angle$ sum of ΔECD)
$\angle ECD = 30°$
$\therefore m = 150$ (adj. supp. $\angle$s)

h $\angle FEB = 60°$ (corresp. $\angle$s, $BE \parallel CD$)
$\angle ABE = 120°$ (co-int. $\angle$s, $AB \parallel FE$)
$\therefore p + 40 = 120$ ($\angle EBF + \angle FBA = \angle ABE$)
$\therefore p = 80$

i $\angle ACD = 110°$ (co-int. $\angle$s, $AB \parallel CD$)
$\therefore 2x = 110$ ($\angle ACE + \angle ECD = \angle ACD$)
$x = 55$
$\therefore y + 55 = 180$ (co-int. $\angle$s, $CD \parallel EF$)
$\therefore y = 125$

3 a $\angle BDC + 150° + 90 = 360°$ ($\angle$s at a point)
$\angle BDC = 120°$
$\angle BCD = a°$ (equal $\angle$s of isos. Δ)
$\therefore a + a + 120 = 180$ ($\angle$ sum of ΔBDC)
$\therefore a = 30$

b $\angle PTS = 60°$ (co-int. $\angle$s, $PX \parallel TS$)
$\angle PTY + 55° = 180°$ (co-int. $\angle$s, $AD \parallel BC$)
$\therefore \angle PTY = 125°$
$\therefore 125 = x + 60$
($\angle PTY = \angle PTS + \angle STY$)
$\therefore x = 65$

c $\angle HAC = 90°$ (co-int. $\angle$s, $HA \parallel GC$)
$\therefore x + 68 = 90$ ($\angle CAD + \angle DAH = \angle CAH$)
$\therefore x = 22$

Diagnostic test 15

1 a 140 **b** 60 **c** 20
2 a $x = 30$ (adj. comp. $\angle$s)
b $x = 37$ (adj. comp. $\angle$s)
c $x = 80$ (adj. supp. $\angle$s)
3 a $x = 140$ ($\angle$s at a point)
b $x = 135$ (vert. opp. $\angle$s)
c $x = 80$ ($\angle$s at a point)
4 a $x = 55$ (alt. $\angle$s, $\parallel$ lines)
b $x = 110$ (co-int. $\angle$s, $\parallel$ lines)
c $x = 130$ (corresp. $\angle$s, $\parallel$ lines)
5 All reasons: $\angle$ sum of a triangle.
a $x = 70$ **b** $x = 60$ **c** $x = 35$
6 a $a = 50$ (ext. $\angle$ of Δ)
b $b = 78$ (ext. $\angle$ of Δ) **c** $c = 52$ (ext. $\angle$ of Δ)
7 All reasons: $\angle$ sum of a quad.
a $x = 65$ **b** $x = 95$ **c** $\alpha = 20°$
8 a $x = 35$ (equal $\angle$s of isos. Δ)
b $x = 7$ (equal sides of isos. Δ)
c $a = 60$ ($\angle$ of equil. Δ)
9 a $\angle EFB = 115°$ (corresp. $\angle$s, $AB \parallel CD$)
$x + 115 = 180$ (adj. supp. $\angle$s)
$\therefore x = 65$

b $\angle BCA = a°$ (equal $\angle$s of isos. Δ)
$a + a + 110 = 180$ ($\angle$ sum of Δ)
$\therefore a = 35$

c $\angle AFE = 40°$ (vert. opp. $\angle$s)
$\angle CEF = 140°$ (co-int. $\angle$s, $AB \parallel CD$)
$\therefore x + 100 = 140$
$\therefore x = 40$

ASSIGNMENT 15A

1 a corresp. $\angle$s, $\parallel$ lines **b** alt. $\angle$s, $\parallel$ lines
c co-int. $\angle$s, $\parallel$ lines
2 a 90° **b** 130°
c 30° and 120° or 75° and 75°
3 a alt. $\angle$s, $\parallel$ lines **b** alt. $\angle$s, $\parallel$ lines
c $\angle$ sum of Δ **d** vert. opp. $\angle$s
4 a $a = 70$ (equal $\angle$s, of isos. Δ)
b $\angle CBA = 40$
$a + 40 + 40 = 180$ (angle sum of triangle)
$a = 100$
5 a $x = 130$ (ext. $\angle$ of a Δ)
b $a + 30 + 100 + 28 = 180$ ($\angle$ sum of ΔLMN)
$\therefore a = 22$
$a + b + 28 = 180$ ($\angle$ sum of ΔMNP)
$22 + b + 28 = 180$
$\therefore b = 130$

ASSIGNMENT 15B

1 89 **2** 38 **3** 10 **4** 18 **5** 32
6 $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729$; last digit of 3^{2000} is 1.
7 a 135° **b** 105°

ASSIGNMENT 15C

1 a $6x^2$ **b** $12a^2b^2$ **c** 3 **d** $2b$
2 a $4(a - 4)$ **b** $a(a - 6)$
c $2m(m - 6)$ **d** $6a(2b - 1)$
3 a 23·3 **b** 33 **c** 80 **d** 80
4 a 4·5% **b** 7·5%
5 $\frac{1094}{1105}$
6 a $\frac{1}{15}$ **b** $\frac{1}{3}$ **c** $\frac{17}{30}$ **d** 0 **e** 1
7 Perpendicular height $= \sqrt{12}$ cm; Area 6·9 cm^2
8 a 38·9 m^3 **b** Capacity = 38 900 L
9 a 150° **b** 150°
10 15°