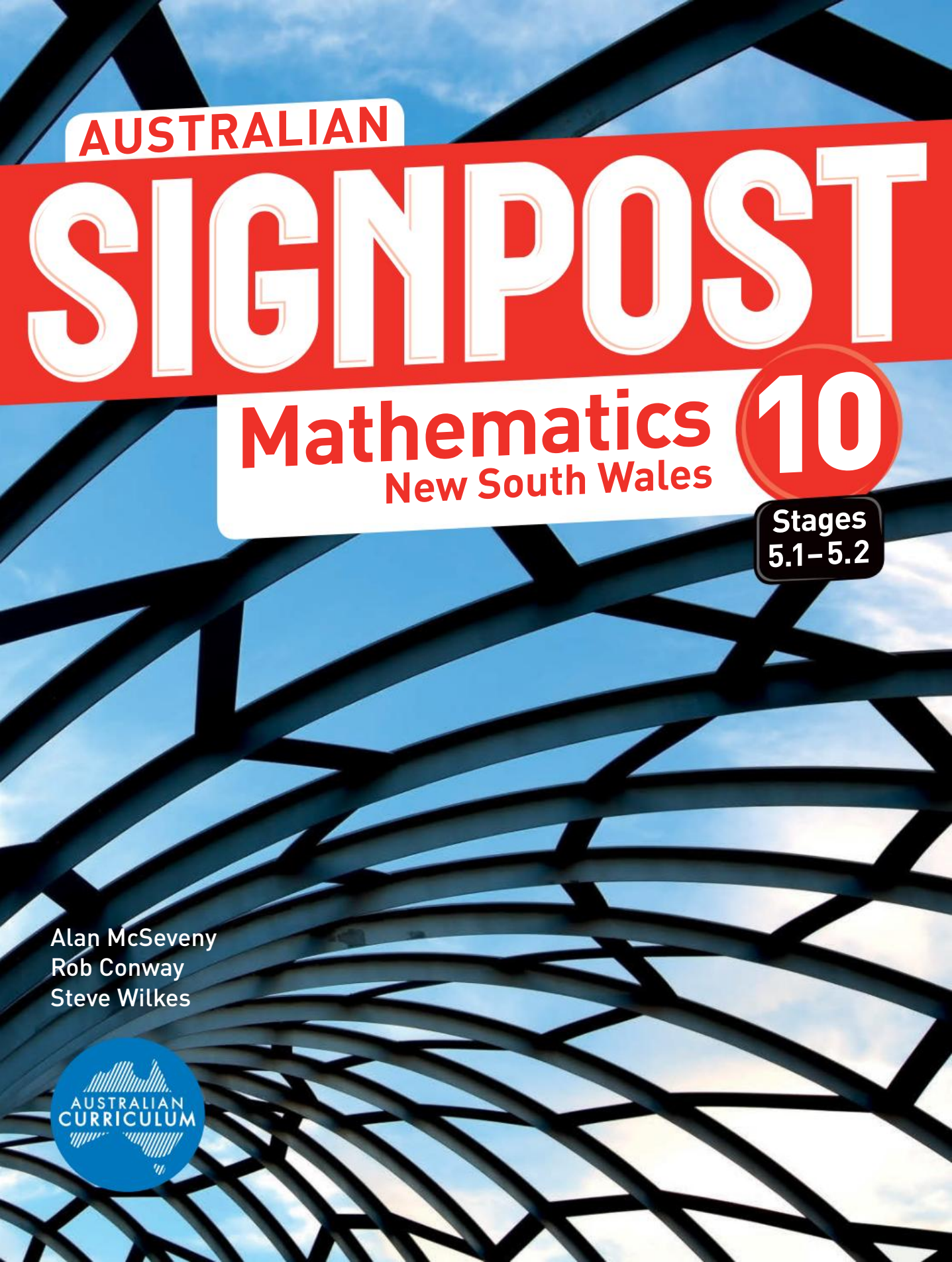
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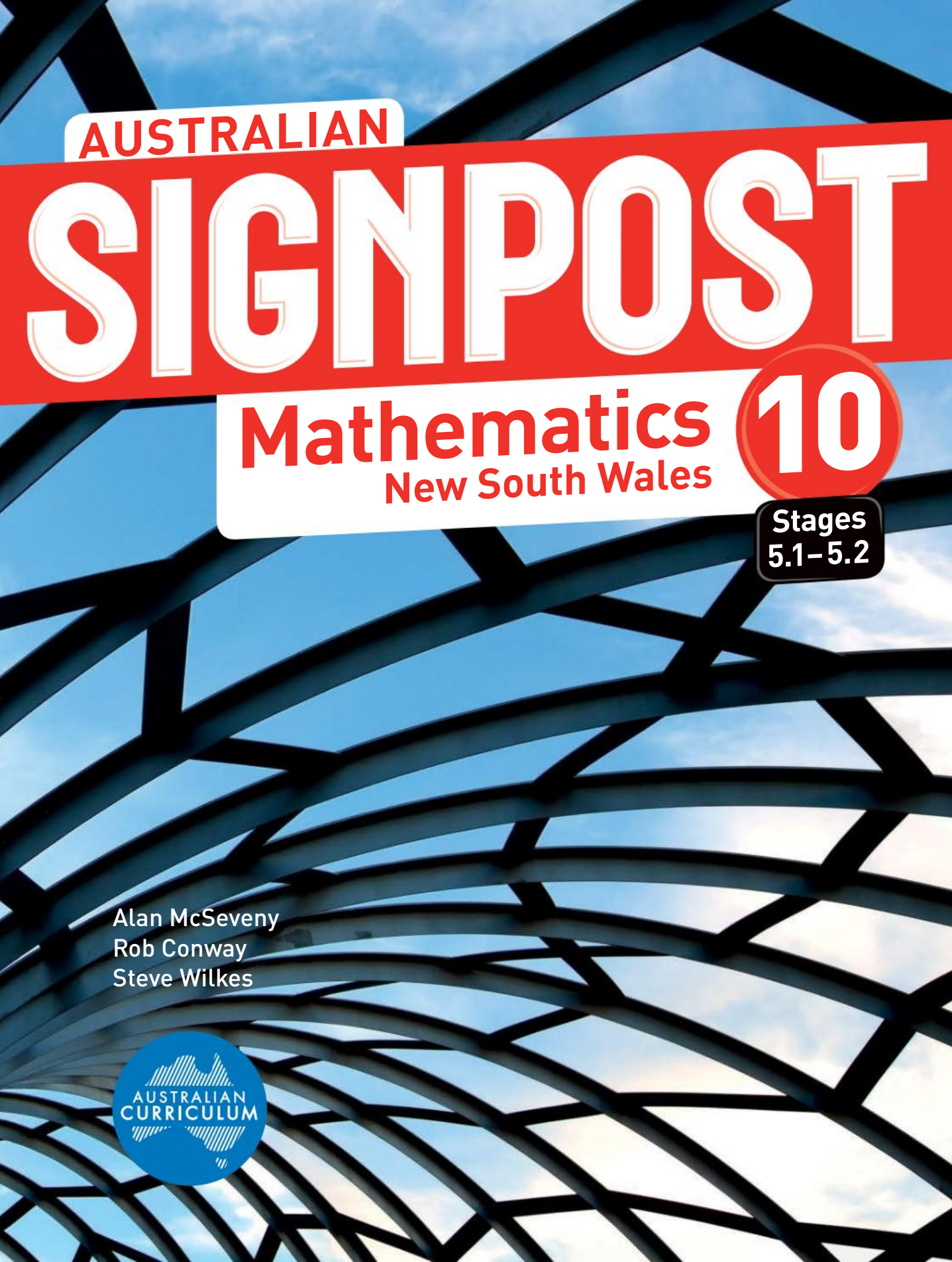
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Understanding is a fountain of life
to those who have it.

Proverbs 16:22

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Australian Signpost Mathematics New South Wales 10 Stages 5.1–5.2

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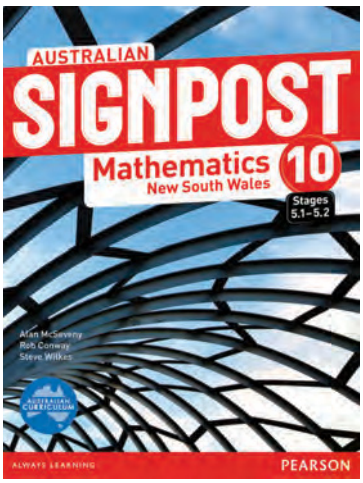

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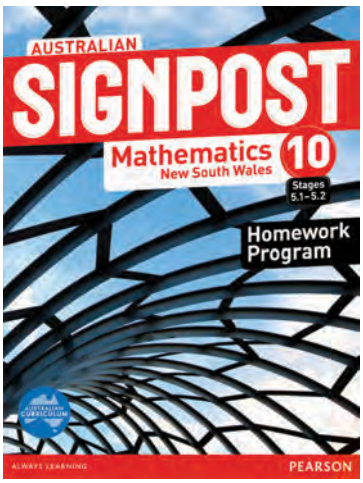
The *Australian Signpost Mathematics New South Wales 10 Stages 5.1–5.2* and *Australian Signpost Mathematics New South Wales 10 Stages 5.1–5.3* learning package addresses Stage 5 of the NSW Syllabus for the Australian Curriculum. This new series continues the Signpost tradition of expanding students' ability in a broad range of mathematical skills while emphasising problem-solving and working mathematically.



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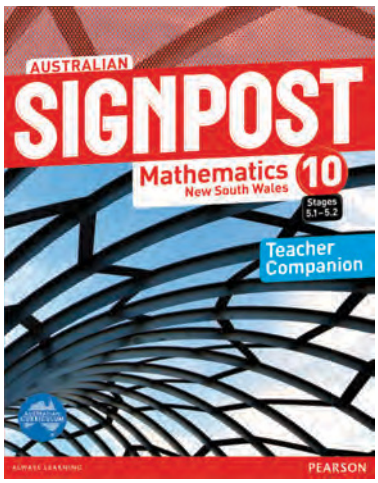
The Homework Program consists of tear-out worksheets that mirror and supplement the content of the student book, providing opportunity for further practice and application of key skills.



Teacher Companion

The Teacher Companion is an invaluable resource that provides guidance and support to teachers using the **Australian Signpost Mathematics New South Wales** package. It is a practical resource featuring teaching strategies, activities, reference to Stage 5 outcomes and content statements, and links to digital learning materials.

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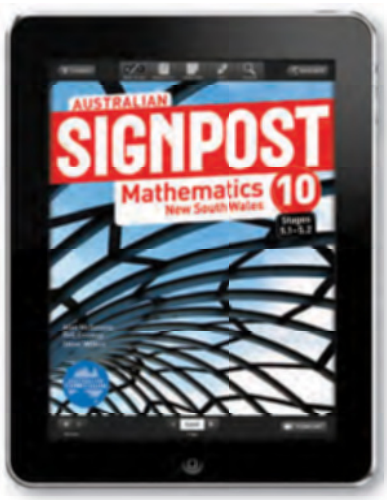

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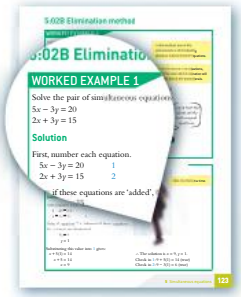
The **Australian Signpost Mathematics New South Wales 10 Stages 5.1–5.2** learning package covers Stage 5 outcomes of the NSW Syllabus for the Australian Curriculum.

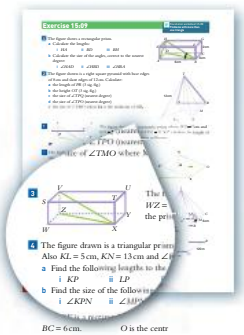
Features of the Student Book

- a complete year's work with full coverage of the NSW Syllabus for the Australian Curriculum
- a flexible structure with carefully graded exercises, colour-coded to indicate level of difficulty
- technological applications with GeoGebra activities and Pearson eBook 3.0 resources
- integration of a broad range of mathematical skills, expanding students' ability to solve problems, work mathematically and investigate
- diagnostic tests, chapter reviews and cumulative revision components for every chapter



Worked examples

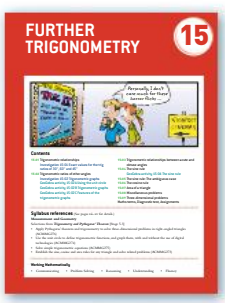
take students through key processes with step-by-step demonstrations of questions similar to those they will encounter.



Well-graded exercises

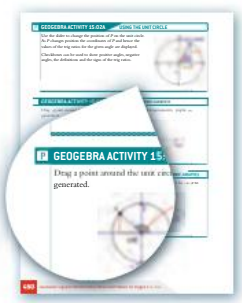
indicate each question's level of difficulty.

- 2 Foundation
- 7 Stages 5.1–5.2 level
- 12 Extension



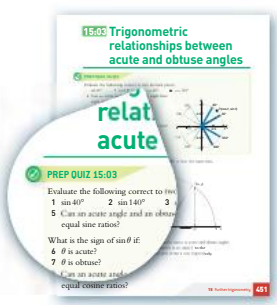
Chapter opener pages

summarise the key content and present the content statements addressed in each chapter. Features within each exercise such as GeoGebra activities, Fun spots and Investigations are listed here.



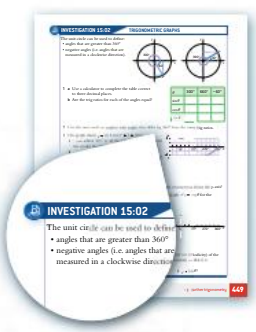
GeoGebra activities

provide a deeper understanding of concepts covered within the exercise while providing the opportunity to integrate ICT into the classroom.



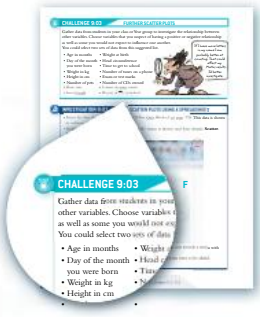
Prep quizzes

review important skills and knowledge that students will need to complete an exercise.



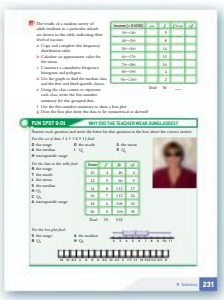
Investigations

encourage students to seek knowledge and develop research skills. The investigative tasks provide opportunities for students to interpret, question, model and communicate mathematically.

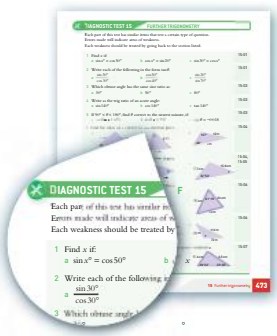


Challenge activities

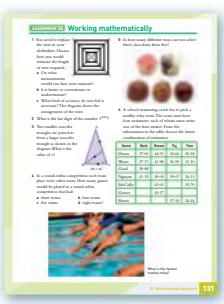
provide more difficult questions and investigations. They can be used to help students with a stronger understanding extend their knowledge.



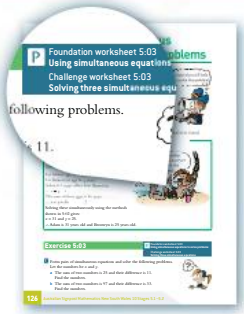
Fun spots provide amusement and interest while reinforcing concepts learnt within the chapter. They encourage creativity and divergent thinking, and show that mathematics is enjoyable.



Diagnostic tests at the end of each chapter test students' achievement of outcomes. Importantly, they also indicate student weaknesses that need to be addressed and provide links to the relevant sections in the text or digital resources.



Assignments are provided at the end of each chapter. They concentrate on revising the content of previous chapters and developing students' ability to work mathematically.

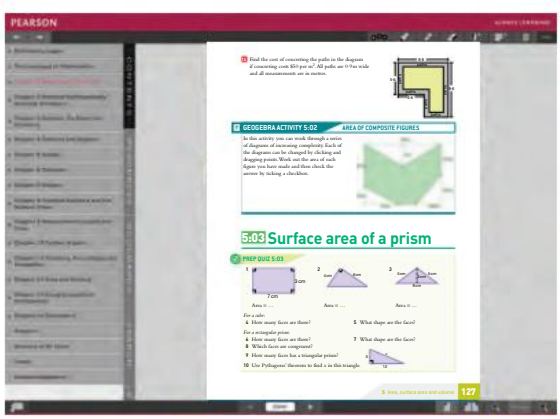


Foundation and Challenge worksheets

are indicated in the student book wherever a worksheet is available on the eBook. Foundation worksheets provide alternative exercises for consolidation of the fundamental content of the work being introduced. Challenge worksheets help students apply their learning in more advanced activities.

Pearson digital resources

A vast range of technology applications are available with the eBook and ProductLink to complement the content presented in the student book. Digital resources include interactive activities, Foundation and Challenge worksheets and appendices. From the eBook, students can also link directly to the GeoGebra activities referred to in the student book exercises.



The eBook aims to create a classroom environment that embraces mathematical technologies in a way that consolidates and extends student learning.

TREATMENT OF SYLLABUS CONTENT

The syllabus outcomes and statements relevant to Stages 5.1–5.2 of the NSW Syllabus for the Australian Curriculum are listed below. Chapters where the content is treated are shown.

Working Mathematically (encompassing five components: Communicating, Problem Solving, Reasoning, Understanding and Fluency) is an integral part of the structure of each chapter. Special attention is given to Working Mathematically in the second Assignment of every chapter in all Year 9 and 10 Student Books.

Working Mathematically	
A student:	
> uses appropriate terminology, diagrams and symbols in mathematical contexts	**MA5.1-1WM**
> selects and uses appropriate strategies to solve problems	**MA5.1-2WM**
> provides reasoning to support conclusions that are appropriate to the context	**MA5.1-3WM**
> selects appropriate notations and conventions to communicate mathematical ideas and solutions	**MA5.2-1WM**
> interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems	**MA5.2-2WM**
> constructs arguments to prove and justify results	**MA5.2-3WM**
Number and Algebra	
A student:	
> solves financial problems involving earning, spending and investing money	**MA5.1-4NA**
> operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases	**MA5.1-5NA**
> determines the midpoint, gradient and length of an interval, and graphs linear relationships	**MA5.1-6NA**
> graphs simple non-linear relationships	**MA5.1-7NA**
> solves financial problems involving compound interest	**MA5.2-4NA◊**
> recognises direct and indirect proportion, and solves problems involving direct proportion	**MA5.2-5NA**
> simplifies algebraic fractions, and expands and factorises quadratic expressions	**MA5.2-6NA**
> applies index laws to operate with algebraic expressions involving integer indices	**MA5.2-7NA**
> solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques	**MA5.2-8NA**
> uses the gradient-intercept form to interpret and graph linear relationships	**MA5.2-9NA**
> connects algebraic and graphical representations of simple non-linear relationships	**MA5.2-10NA◊**
Measurement and Geometry	
A student:	
> calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms	**MA5.1-8MG**
> interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures	**MA5.1-9MG**
> applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression	**MA5.1-10MG**
> describes and applies the properties of similar figures and scale drawings	**MA5.1-11MG**
> calculates the surface areas of right prisms, cylinders and related composite solids	**MA5.2-11MG**
> applies formulas to calculate the volumes of composite solids composed of right prisms and cylinders	**MA5.2-12MG**
> applies trigonometry to solve problems, including problems involving bearings	**MA5.2-13MG◊**
> calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar	**MA5.2-14MG**
Statistics and Probability	
A student:	
> uses statistical displays to compare sets of data, and evaluates statistical claims made in the media	**MA5.1-12SP**
> calculates relative frequencies to estimate probabilities of simple and compound events	**MA5.1-13SP**
> uses quartiles and box plots to compare sets of data, and evaluates sources of data	**MA5.2-15SP◊**
> investigates relationships between two statistical variables, including their relationship over time	**MA5.2-16SP**
> describes and calculates probabilities in multi-step chance experiments	**MA5.2-17SP**

The NSW Syllabus for the Australian Curriculum has indicated the minimum Stage 5 content recommendations in relation to current Stage 6 Mathematics Board Developed Courses.

◊ refers to minimum Stage 5 substrands to be covered by students planning to study Preliminary Mathematics General/HSC Mathematics General 2. All subtrands of Stage 5.1 are also part of the recommended minimum.

Stages 5.1–5.2 content statements

The Stage 5 content statements are divided into three strands:

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Content statements are listed below with their Australian Curriculum code or NSW syllabus reference. The chapters that address each content statement are shown in the right-hand column.

Number and Algebra (5.1)	Reference	Chapter
Financial Mathematics		
• Solve problems involving earning money.	NSW (Stage 5.1)	Ch 8 (Y9)
• Solve problems involving simple interest.	ACMNA211 (Stage 5.1)	Ch 8 (Y9) Ch 6 (Y10)
• Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.	ACMNA229 (Stage 5.1)	Ch 6 (Y10)
Indices		
• Extend and apply the index laws to variables, using positive-integer indices and the zero index.	ACMNA212 (Stage 5.1)	Ch 6 (Y9)
• Simplify algebraic products and quotients using index laws.	ACMNA231 (Stage 5.1)	Ch 6 (Y9)
• Apply index laws to numerical expressions with integer indices.	ACMNA209 (Stage 5.1)	Ch 6 (Y9)
Linear Relationships		
• Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.	ACMNA294 (Stage 5.1)	Ch 9 (Y9)
• Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software.	ACMNA214 (Stage 5.1)	Ch 9 (Y9)
• Sketch linear graphs using the coordinates of two points.	ACMNA215 (Stage 5.1)	Ch 7 (Y9) Ch 9 (Y9)
• Solve problems involving parallel lines.	ACMNA238 (Stage 5.1)	Ch 3 (Y10) Ch 7 (Y10)
Non-linear Relationships		
• Graph simple non-linear relations, with and without use of digital technologies.	ACMNA296 (Stage 5.1)	Ch 9 (Y9)
• Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technologies as appropriate.	ACMNA239 (Stage 5.1)	Ch 7 (Y10)

Measurement and Geometry (5.1)	Reference	Chapter
Area and Surface Area		
• Calculate the areas of composite shapes.	ACMMG216 (Stage 5.1)	Ch 5 (Y9)
• Solve problems involving the surface areas of right prisms.	ACMMG218 (Stage 5.1)	Ch 5 (Y9) Ch 8 (Y10)
Numbers of Any Magnitude		
• Investigate very small and very large time scales and intervals.	ACMMG219 (Stage 5.1)	Ch 1 (Y9)
• Express numbers in scientific notation.	ACMMG210 (Stage 5.1)	Ch 1 (Y9) Ch 6 (Y9)
Right-angled Triangles (Trigonometry)		
• Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles.	ACMMG223 (Stage 5.1)	Ch 12 (Y9)
• Apply trigonometry to solve right-angled triangle problems.	ACMMG224 (Stage 5.1)	Ch 12 (Y9)
• Solve right-angled triangle problems, including those involving angles of elevation and depression.	ACMMG245 (Stage 5.1)	Ch 12 (Y9)
Properties of Geometrical Figures		
• Use the enlargement transformation to explain similarity.	ACMMG220 (Stage 5.1)	Ch 10 (Y9)
• Solve problems using ratio and scale factors in similar figures.	ACMMG221 (Stage 5.1)	Ch 10 (Y9)

Statistics and Probability (5.1)	Reference	Chapter
Single Variable Data Analysis		
• Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources.	ACMSP228 (Stage 5.1)	Ch 9 (Y10)
• Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal'.	ACMSP282 (Stage 5.1)	Ch 13 (Y9)
• Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread.	ACMSP283 (Stage 5.1)	Ch 13 (Y9)
• Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data.	ACMSP253 (Stage 5.1)	Ch 9 (Y10)
Probability		
• Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.	ACMSP226 (Stage 5.1)	Ch 4 (Y9)

Number and Algebra (5.2)	Reference	Chapter
Financial Mathematics◊		
• Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.	ACMNA229 (Stage 5.2)	Ch 6 (Y10)
Ratios and Rates		
• Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 (Stage 5.2)	Ch 14 (Y9)
Algebraic Techniques		
• Apply the four operations to simple algebraic fractions with numerical denominators.	ACMNA232 (Stage 5.2)	Ch 3 (Y9)
• Apply the four operations to algebraic fractions with pronumerals in the denominator.	NSW (Stage 5.2)	Ch 2 (Y10)
• Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 (Stage 5.2)	Ch 3 (Y9)
• Factorise algebraic expressions by taking out a common algebraic factor.	ACMNA230 (Stage 5.2)	Ch 3 (Y9) Ch 2 (Y10)
• Expand binomial products and factorise monic quadratic expressions using a variety of strategies.	ACMNA233 (Stage 5.2)	Ch 2 (Y10)
Indices		
• Apply index laws to algebraic expressions involving integer indices.	NSW (Stage 5.2)	Ch 6 (Y9)
Equations		
• Solve linear equations.	ACMNA215 (Stage 5.2)	Ch 7 (Y9)
• Solve linear equations involving simple algebraic fractions.	ACMNA240 (Stage 5.2)	Ch 7 (Y9)
• Solve simple quadratic equations using a range of strategies.	ACMNA241 (Stage 5.2)	Ch 2 (Y10) Ch 11 (Y10)
• Substitute values into formulas to determine an unknown.	ACMNA234 (Stage 5.2)	Ch 11 (Y9)
• Solve problems involving linear equations, including those derived from formulas.	ACMNA235 (Stage 5.2)	Ch 11 (Y9)
• Solve linear inequalities and graph their solutions on a number line.	ACMNA236 (Stage 5.2)	Ch 7 (Y9)
• Solve linear simultaneous equations, using algebraic and graphical techniques, including with the use of digital technologies.	ACMNA237 (Stage 5.2)	Ch 5 (Y10)
Linear Relationships		
• Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line.	NSW (Stage 5.2)	Ch 9 (Y9)
• Solve problems involving parallel and perpendicular lines.	ACMNA238 (Stage 5.2)	Ch 7 (Y10)
Non-linear Relationships◊		
• Graph simple non-linear relationships, with and without use of digital technologies, and solve simple related equations.	ACMNA296 (Stage 5.2)	Ch 7 (Y10)
• Explore the connection between algebraic and graphical representations of relationships such as simple quadratics, circles and exponentials using digital technologies as appropriate.	ACMNA239 (Stage 5.2)	Ch 7 (Y10)

Measurement and Geometry (5.2)	Reference	Chapter
Area and Surface Area		
• Calculate the surface areas of cylinders and solve related problems.	ACMMG217 (Stage 5.2)	Ch 5 (Y9) Ch 8 (Y10)
• Solve problems involving surface area for a range of prisms, cylinders and composite solids.	ACMMG242 (Stage 5.2)	Ch 5 (Y9) Ch 8 (Y10)
Volume		
• Solve problems involving the volumes of right prisms.	ACMMG218 (Stage 5.2)	Ch 5 (Y9) Ch 8 (Y10)
• Solve problems involving volume for a range of prisms, cylinders and composite solids.	ACMMG242 (Stage 5.2)	Ch 8 (Y10)
Right-angled Triangles (Trigonometry)◊		
• Apply trigonometry to solve right-angled triangle problems.	ACMMG224 (Stage 5.2)	Ch 12 (Y9)
• Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.	ACMMG245 (Stage 5.2)	Ch 12 (Y9)
Properties of Geometrical Figures		
• Formulate proofs involving congruent triangles and angle properties.	ACMMG243 (Stage 5.2)	Ch 3 (Y10)
• Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar.	ACMMG220 (Stage 5.2)	Ch 10 (Y9)
• Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes.	ACMMG244 (Stage 5.2)	Ch 3 (Y10)

Statistics and Probability (5.2)	Reference	Chapter
Single Variable Data Analysis◊		
• Determine quartiles and interquartile range.	ACMSP248 (Stage 5.2)	Ch 13 (Y9)
• Construct and interpret box plots and use them to compare data sets.	ACMSP249 (Stage 5.2)	Ch 13 (Y9)
• Compare shapes of box plots to corresponding histograms and dot plots.	ACMSP250 (Stage 5.2)	Ch 13 (Y9)
• Investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians.	ACMSP227 (Stage 5.2)	Ch 9 (Y10)
Bivariate Data Analysis		
• Investigate and describe bivariate numerical data where the independent variable is time.	ACMSP252 (Stage 5.2)	Ch 9 (Y10)
• Use scatter plots to investigate and comment on relationships between two numerical variables.	ACMSP251 (Stage 5.2)	Ch 9 (Y10)
Probability		
• List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays: assign probabilities to outcomes and determine probabilities for events.	ACMSP225 (Stage 5.2)	Ch 4 (Y9) Ch 4 (Y10)
• Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence.	ACMSP246 (Stage 5.2)	Ch 4 (Y10)
• Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.	ACMSP247 (Stage 5.2)	Ch 4 (Y10)

5.3 Extension chapters (Ch 10–15)

If students intend to study the Mathematics ('2 Unit') course in Years 11 and 12, it is recommended that the following 5.3 substrands should also be covered.

- Algebraic Techniques (Ch 10)
- Surds and Indices (Ch 12)
- Equations (Ch 11)
- Linear Relationships (Ch 14)

It is also recommended that at least some of the content of the following 5.3 substrands should also be covered.

- Non-Linear Relationships (Ch 14)
- Trigonometry and Pythagoras' Theorem (Ch 15)
- Properties of Geometrical Figures (Ch 13)

Details of the content statements covered are provided at the beginning of each of these chapters.

P DIGITAL RESOURCES

Chapter	Drag-and-drop activities ProductLink only	Technology activities Excel (E)—ProductLink only GeoGebra (G)—eBook and ProductLink	Worksheets Foundation worksheets (F) Challenge worksheets (C) —eBook and ProductLink
1 Review of Year 9			Appendix summary of Year 9 work
2 Algebra and equations	Binomial products Factorising trinomials 1 Special products Grouping in pairs Factorising trinomials 2 Mixed factorisations Maths terms 2		2:01A Simplifying algebraic fractions (F) 2:01B Simplifying algebraic fractions (F) 2:02A Grouping symbols (F) 2:02B Common factors (F) 2:04 Factorising trinomials (F) 2:05 Quadratic equations (F)
3 Properties of geometrical figures	Angles and parallel lines Triangles Quadrilaterals Angle sum of polygons Maths terms 3	The parallelogram family (G) Convex and concave polygons (G) The angle sum of a polygon (G) The exterior angle sum of a convex polygon (G)	3:02 Congruent triangle proofs (F) 3:03 Using congruent triangles (F) 3:04 Formulas (F)
4 Probability	Theoretical probability Maths terms 4		4:01 Probability review (F)
5 Simultaneous equations	Maths terms 5 Simultaneous equations	Break-even analysis (E) Simultaneous equations: The graphical method (G) Simultaneous equations: The substitution method (G) Simultaneous equations: The elimination method (G) Petrol to LPG conversion (E)	5:01 Graphical method of solution (F) 5:02A The substitution method (F) 5:03 Using simultaneous equations to solve problems (F) 5:03 Solving three simultaneous equations (C)
6 Financial mathematics	Depreciation Compound interest Maths terms 6	Who wants to be a millionaire? (E) Compound interest (E)	6:02 Simple interest (F) 6:04 Compound interest (F) 6:06 Compound interest formula (F)
7 Linear and non-linear relationships	Parallel and perpendicular lines Parabolas Identifying graphs from equations Maths terms 7	Parallel lines (G) Perpendicular lines (G) Parabolas and curve sketching (E) Investigating parabolas (E) Parabolas of the form $y = ax^2$ (G) Parabolas of the form $y = ax^2 + c$ (G) Exponential graphs (G) Matching equations and graphs G)	7:02 Parallel lines (F) 7:03 Perpendicular lines (F) 7:07 Miscellaneous graphs (F)
8 Surface area and volume	Maths terms 8		8:03 Volume of a prism (F) 8:04 Volume of composite solids (F)
9 Statistics	Box-and-whisker plots Maths terms 9		9:01 Interquartile range (F)
10 Further algebra	Special products Grouping in pairs Factorising trinomials 1 Factorising trinomials 2 Mixed factorisations Maths terms 10		10:03 Common factors (F) 10:04 Grouping in pairs (F) 10:06 Factorising trinomials (F) 10:09 Simplifying algebraic fractions (F) 10:10 Addition and subtraction of algebraic fractions (F)
11 Further equations	Quadratic equations Completing the square Solving literal equations Maths terms 11	Literal equations (E)	11:01 Equations with fractions (F) 11:02 Quadratic equations (F) 11:04 The quadratic formula (F) 11:08 Solving literal equations (F)

Chapter	Drag-and-drop activities **ProductLink only**	Technology activities **Excel (E)—ProductLink only** **GeoGebra (G)—eBook and ProductLink**	Worksheets **Foundation worksheets (F)** **Challenge worksheets (C)** **—eBook and ProductLink**
12 Further number	Simplifying surds Operations with surds Maths terms 12	Magic squares with surds (E)	12:02 Surds (F) 12:03 Addition and subtraction of surds (F) 12:04 Multiplication and division of surds (F) 12:05 Binomial products: Surds (F)
13 Further geometry	Properties of quadrilaterals Maths terms 13		13:01 Non-numerical proofs (F) 13:02 Similar triangle proofs (F)
14 Further coordinate geometry	The midpoint, distance and gradient formulas General form of a line Transforming curves Maths terms 14	Distance, gradient and midpoint formulas (G) Parabolas of the form $y = (x + a)^2$ and $y = (x + a)^2 + k$ (G) Parabolas of the form $y = ax^2 + bx + c$ (G) Hyperbolas (G) Parabolas in real life (E) Curves involving powers (G) The intersection of graphs (G) Curves and their equations (G)	14:01 Substituting into formulas (F) 14:02 Point-gradient form (F) 14:04 The general form (F) 14:06 Coordinate geometry (F) 14:08 The parabola $y = ax^2 + bx + c$ (F) 14:13 Curves of the form $y = ax^n$ and $y = ax^n + d$ (F) 14:14 Curves of the form $y = ax^n$ and $y = a(x - r)^n$ (F) 14:15 Curves of the form $y = a(x - r)(x - s)(x - t)$ (F)
15 Further trigonometry	Sine rule Cosine rule Maths terms 15	Using the unit circle (G) Trigonometric graphs (G) Features of trigonometric graphs (G) The sine rule (G)	15:03 Trig ratios of obtuse angles (F) 15:04 The sine rule (F) 15:05 The sine rule: The ambiguous case (F) 15:06 The cosine rule (F) 15:08 Sine rule or cosine rule? (F) 15:09 Problems with more than one triangle (F)

Teachers also have access to other digital resources on the eBook and ProductLink, including:

- syllabus overview document
- editable teaching programs
- answers to all worksheets
- answers to Prep quizzes and Investigations
- answers to the Homework Program.

Metric equivalents

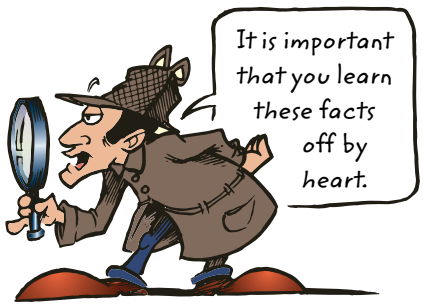
Length
1 m = 1000 mm = 100 cm = 10 dm 1 cm = 10 mm 1 km = 1000 m
Area
$1\ m^2 = 10\,000\ cm^2$ 1 ha = $10\,000\ m^2$ $1\ km^2$ = 100 ha
Mass
1 kg = 1000 g 1 t = 1000 kg 1 g = 1000 mg
Volume
$1\ m^3 = 1\,000\,000\ cm^3$ = $1000\ dm^3$ 1 L = 1000 mL 1 kL = 1000 L $1\ m^3$ = 1 kL $1\ cm^3$ = 1 mL $1000\ cm^3$ = 1 L
Time
1 min = 60 s 1 h = 60 min 1 day = 24 h 1 year = 365 days 1 leap year = 366 days

Months of the year

30 days each has September,
April, June and November.
All the rest have 31, except February alone,
Which has 28 days clear and 29 each leap year.

Seasons

Summer: December, January, February
Autumn: March, April, May
Winter: June, July, August
Spring: September, October, November



The language of mathematics

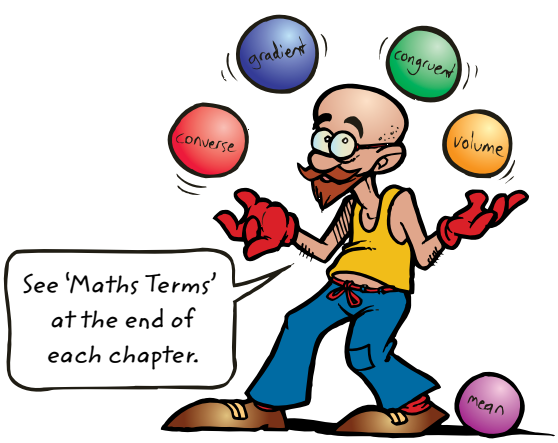
You should regularly test your knowledge by identifying the items on each card.

ID Card 1 (Metric units)			
1 m	2 dm	3 cm	4 mm
5 km	6 m^2	7 cm^2	8 km^2
9 ha	10 m^3	11 cm^3	12 s
13 min	14 h	15 m/s	16 km/h
17 g	18 mg	19 kg	20 t
21 L	22 mL	23 kL	24 °C

For answers see page 475.

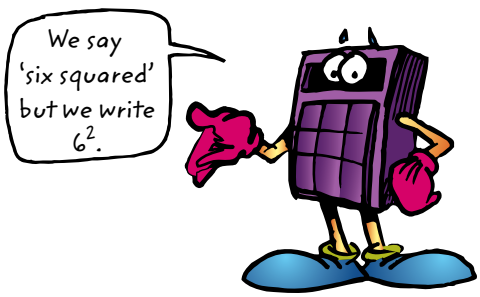
ID Card 2 (Symbols)			
1 $=$	2 $\div$ or $\approx$	3 $\neq$	4 $<$
5 $\leq$	6 $\not<$	7 $>$	8 $\geq$
9 4^2	10 4^3	11 $\sqrt{2}$	12 $\sqrt[3]{2}$
13	14 $\parallel$	15 $\equiv$	16 $\parallel\!\parallel$
17 %	18 $\therefore$	19 e.g.	20 i.e.
21 π	22 Σ	23 $\bar{x}$	24 $P(E)$

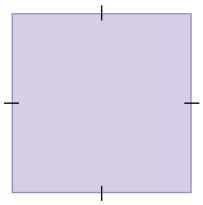
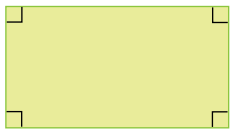
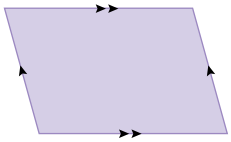
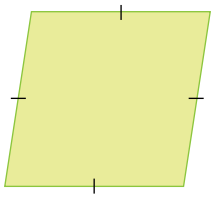
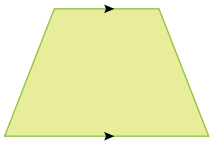
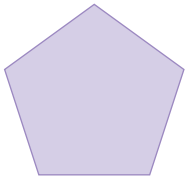
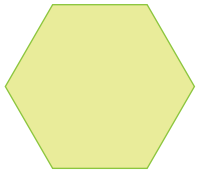
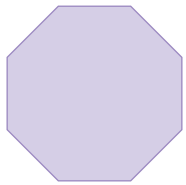
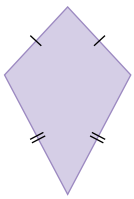
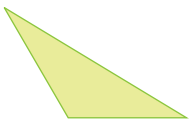
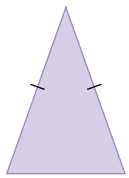
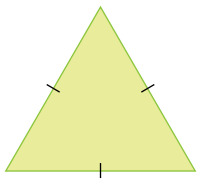
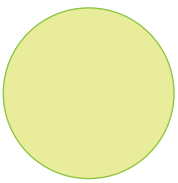
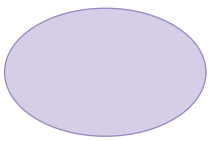
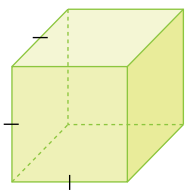
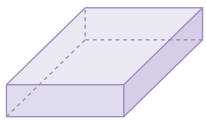
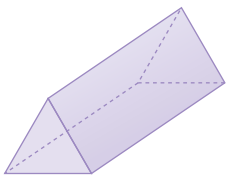
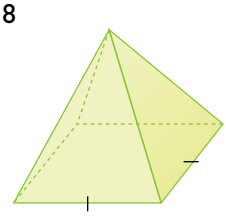
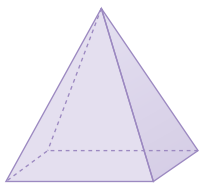
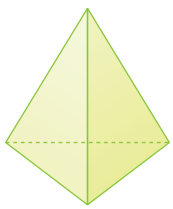
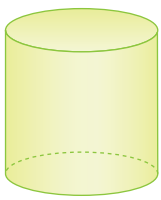
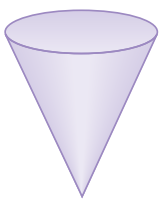
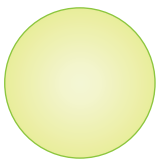
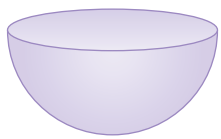
For answers see page 475.



ID Card 3 (Language)			
1 6 minus 2	**2** the sum of 6 and 2	**3** divide 6 by 2	**4** subtract 2 from 6
5 the quotient of 6 and 2	**6** $2\overline{)6}$ the divisor is	**7** $2\overline{)6}$ the dividend is	**8** 6 lots of 2
9 decrease 6 by 2	**10** the product of 6 and 2	**11** 6 more than 2	**12** 2 less than 6
13 6 squared	**14** the square root of 36	**15** 6 take away 2	**16** multiply 6 by 2
17 average of 6 and 2	**18** add 6 and 2	**19** 6 to the power of 2	**20** 6 less 2
21 the difference between 6 and 2	**22** increase 6 by 2	**23** share 6 between 2	**24** the total of 6 and 2

For answers see page 475.



ID Card 4 (Language)			
1 	2 	3 	4 
5 	6 	7 	8 
9 	10  All sides different	11 	12 
13 	14 	15 	16 
17 	18 	19 	20 
21 	22 	23 	24 

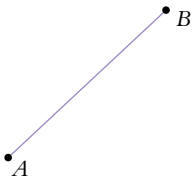
For answers see page 475.

ID Card 5 (Language)

1

.....

2



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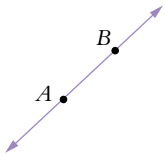

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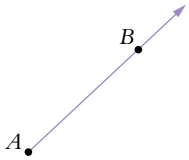

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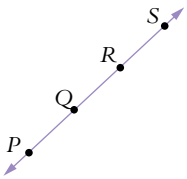


..... points

6

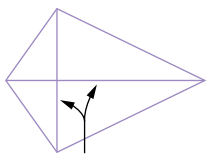
C is the .....

7

.....

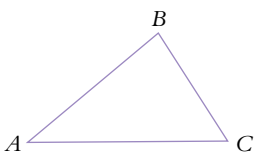
.....

8



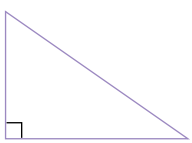
.....

9



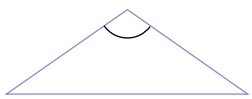
all angles less than 90°

10



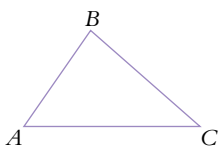
one angle 90°

11



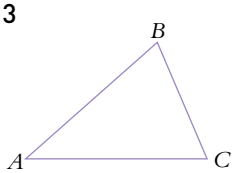
one angle greater than 90°

12



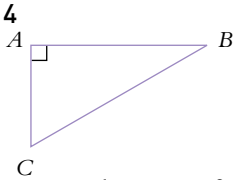
A, *B* and *C* are ..... of the triangle.

13



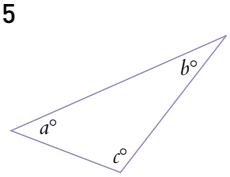
Use the vertices to name the Δ.

14



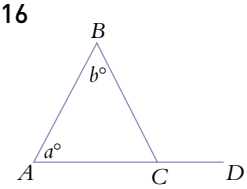
BC is the ..... of the right-angled Δ.

15



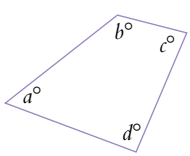
$a° + b° + c°$ = ...

16



$\angle BCD$ = ...

17

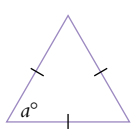


$a° + b° + c° + d°$ = ...

18

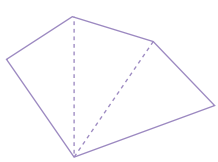
Which is true?
(a) $a° < b°$
(b) $a° = b°$
(c) $a° > b°$

19



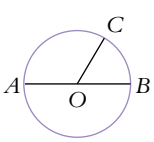
$a°$ = ...

20



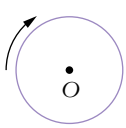
Angle sum = ...

21



AB is a .....
OC is a .....

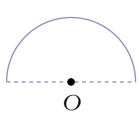
22



Name of distance around the circle.

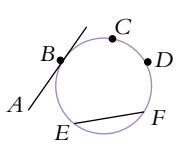
.....

23



s.....

24

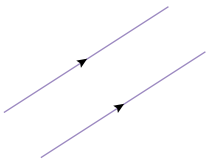


AB is a .....
CD is an .....
EF is a .....

For answers see page 475.

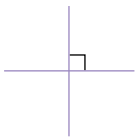
ID Card 6 (Language)

1



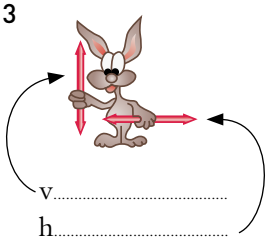
..... lines

2



..... lines

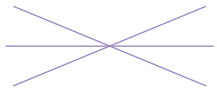
3



v.....

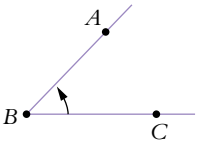
h.....

4



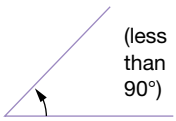
..... lines

5



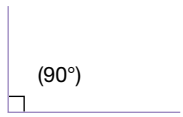
angle .....

6



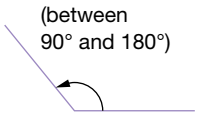
..... angle

7



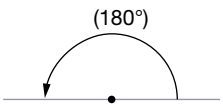
..... angle

8



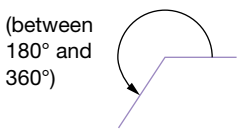
..... angle

9



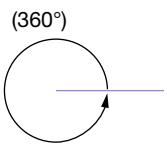
..... angle

10



..... angle

11



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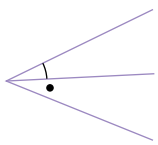
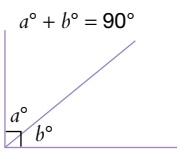
12



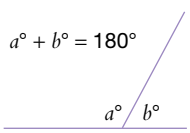
..... angles

13



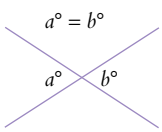
..... angles

14



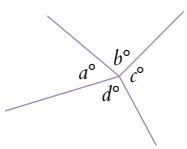
..... angles

15



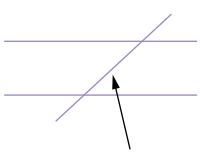
..... angles

16



$a° + b° + c° + d° =$ ....

17



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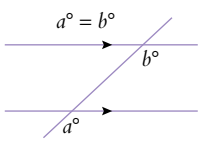
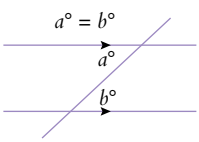
18



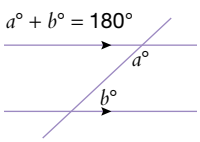
..... angles

19



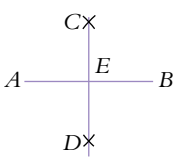
..... angles

20



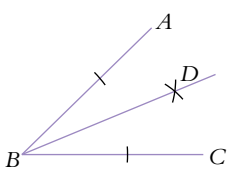
..... angles

21



b..... an interval

22



b..... an angle

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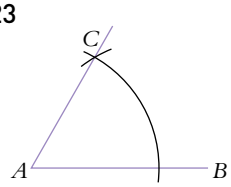
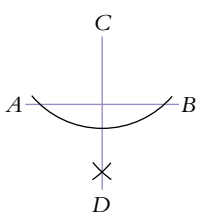


$\angle CAB =$ .....

24



CD is p..... to AB.

For answers see page 476.

ID Card 7 (Language)

1

CE

C..... E.....

AD

A..... D.....

2

BCE

B..... C..... E.....

BC

B..... C.....

3

am

a..... m.....

means

b..... m.....

4

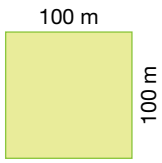
pm

p..... m.....

means

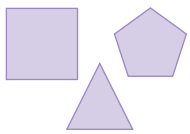
a..... m.....

5



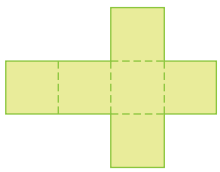
area is 1 .....

6



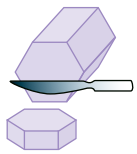
r..... shapes

7



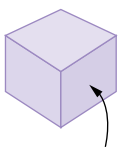
..... of a cube

8



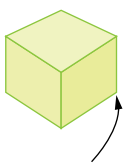
c.....-s.....

9



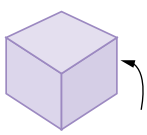
f.....

10



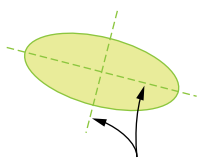
v.....

11



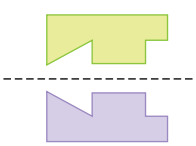
e.....

12



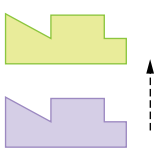
axes of .....

13



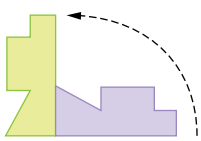
r.....

14



t.....

15



r.....

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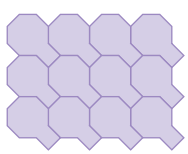

t.....

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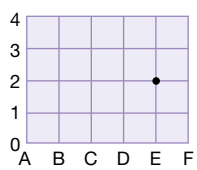


The c.....
of the dot are $(E, 2)$.

18

Cars sold

Mon	𝍸 I
Tues	𝍸 𝍸 𝍸 I
Wed	𝍸 𝍸 II
Thurs	𝍸 𝍸 𝍸
Fri	𝍸 𝍸 II

t.....

19

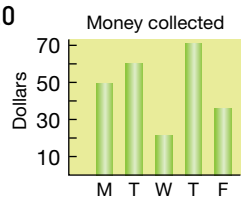
Money collected

Mon	● ● ● ●
Tues	● ● ● ◔
Wed	● ◔
Thurs	● ● ● ●
Fri	● ● ◔

● stands for $10

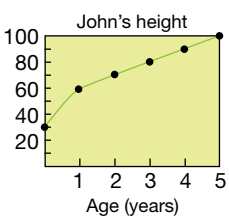
p..... graph

20



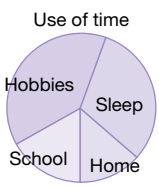
c..... graph

21



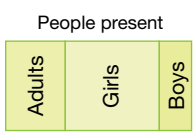
l..... graph

22



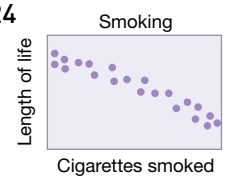
s..... graph

23



b..... graph

24



s..... d.....

For answers see page 476.

Algebra card

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	3	2·1	$\frac{1}{4}$	m	$\frac{2m}{3}$	$-3m$	$5m^2$	$-5x$	$\frac{x}{6}$	$-3x$	$-\frac{x}{2}$	$x + 2$	$x - 3$	$2x + 1$	$3x - 8$
2	−1	−0·4	$\frac{1}{8}$	$-4m$	$\frac{m}{4}$	$2m$	$-2m^3$	$3x$	$-\frac{x}{3}$	$5x^2$	$\frac{x}{4}$	$x + 7$	$x - 6$	$4x + 2$	$x - 1$
3	5	0·8	$\frac{1}{3}$	$10m$	$-\frac{m}{4}$	$-5m$	$8m^5$	$10x$	$-\frac{2x}{7}$	$-8x$	$\frac{2x}{5}$	$x + 5$	$x + 5$	$6x + 2$	$x - 5$
4	−2	1·5	$\frac{1}{20}$	$-8m$	$-\frac{3m}{2}$	$7m$	$6m^2$	$-15x$	$\frac{x}{10}$	$-4x^4$	$-\frac{x}{5}$	$x + 1$	$x - 9$	$3x + 3$	$2x + 4$
5	−8	−2·5	$\frac{3}{5}$	$2m$	$-\frac{m}{5}$	$10m$	m^2	$7x$	$\frac{2x}{3}$	$2x^3$	$\frac{x}{3}$	$x + 8$	$x + 2$	$3x + 8$	$3x + 1$
6	10	−0·7	$\frac{2}{7}$	$-5m$	$-\frac{3m}{7}$	$-6m$	$-9m^3$	$9x$	$-\frac{2x}{5}$	x^2	$\frac{3x}{5}$	$x + 4$	$x - 7$	$3x + 1$	$x + 7$
7	−6	−1·2	$\frac{3}{8}$	$8m$	$-\frac{m}{6}$	$9m$	$2m^6$	$-6x$	$\frac{5x}{6}$	$5x^2$	$\frac{2x}{3}$	$x + 6$	$x - 1$	$x + 8$	$2x - 5$
8	12	0·5	$\frac{9}{20}$	$20m$	$\frac{2m}{5}$	$-4m$	$-3m^3$	$-12x$	$\frac{3x}{4}$	$4x^3$	$-\frac{x}{7}$	$x + 10$	$x - 8$	$5x + 2$	$x - 10$
9	7	0·1	$\frac{3}{4}$	$5m$	$\frac{3m}{5}$	$-10m$	m^7	$5x$	$-\frac{3x}{7}$	$-3x^5$	$-\frac{3x}{7}$	$x + 2$	$x + 5$	$2x + 4$	$2x - 4$
10	−5	−0·6	$\frac{7}{10}$	$-9m$	$-\frac{4m}{5}$	$-7m$	$-8m^4$	$-3x$	$-\frac{x}{6}$	$-7x^5$	$\frac{2x}{9}$	$x + 1$	$x - 7$	$5x + 4$	$x + 7$
11	−11	−1·8	$\frac{1}{10}$	$-7m$	$\frac{m}{5}$	$-8m$	$-4m$	$-4x$	$\frac{x}{5}$	$-x^3$	$\frac{x}{3}$	$x + 9$	$x + 6$	$2x + 7$	$x - 6$
12	4	−1·4	$\frac{2}{5}$	$3m$	$\frac{m}{3}$	$12m$	$7m^2$	$-7x$	$-\frac{3x}{4}$	x^{10}	$\frac{x}{6}$	$x + 3$	$x - 10$	$2x + 3$	$2x + 3$

How to use this card

As an example, if the instruction given for the Algebra card is 'column D + column F' then you would write answers for the following problems.

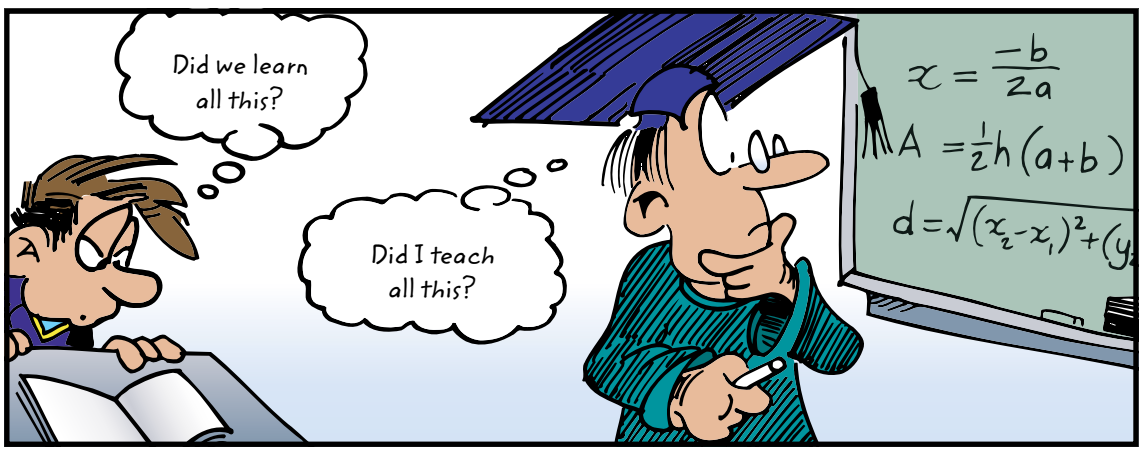
1 $m + (-3m)$ **2** $(-4m) + 2m$ **3** $10m + (-5m)$ **4** $(-8m) + 7m$

5 $2m + 10m$ **6** $(-5m) + (-6m)$ **7** $8m + 9m$ **8** $20m + (-4m)$

9 $5m + (-10m)$ **10** $(-9m) + (-7m)$ **11** $(-7m) + (-8m)$ **12** $3m + 12m$

REVIEW OF YEAR 9

1



Contents

1:01 Number and measurement
- 1:01A Order of operations
- 1:01B Fractions
- 1:01C Decimals
- 1:01D Percentages
- 1:01E Ratios
- 1:01F Rates
- 1:01G Significant figures
- 1:01H Approximations
- 1:01I Estimations
- 1:01J Using units of measurement
- 1:01K Accuracy of measurements

1:02 Algebra

1:03 Probability
Investigation 1:03 Blaise Pascal

1:04 Area, surface area and volume

1:05 Indices

1:06 Equations and inequalities

1:07 Financial mathematics

1:08 Coordinate geometry

1:09 Similarity and scale drawing

1:10 Formulas and problem solving

1:11 Trigonometry

1:12 Statistics

1:13 Proportion

Assignment

Syllabus references (See pages x–xiii for details.)

Number and Algebra, Measurement and Geometry, Statistics and Probability

- All content covered in Year 9 is reviewed.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

This chapter is a summary of the work covered in *Australian Signpost Mathematics 9 New South Wales Stages 5.1–5.2*. For an explanation of the work, refer to the relevant Appendix on the eBook.

1:01 Number and measurement

Rational numbers include integers, fractions, decimals and percentages (both positive and negative). All rational numbers can be written as terminating or recurring decimals. The following exercises revise the basic skills when working with rational numbers.

1:01A Order of operations

Exercise 1:01A

P Appendix A A:01A

Simplify these expressions without using a calculator.

1 **a** $4 - (5 - 3)$ **b** $6 - (9 - 4)$ **c** $-4 + (3 + 1)$
d $6 + 4 \times 2$ **e** $9 - 3 \times 4$ **f** $16 + 4 \div 4$
g $10 \times 4 - 4 \times 7$ **h** $30 \div 3 + 40 \div 2$ **i** $5 \times 8 + 6 \times 5$
j 5×2^2 **k** 3×10^2 **l** $3^2 + 4^2$
m $6 + 3 \times 4 + 1$ **n** $8 + 4 \div 2 + 1$ **o** $6 - (-6 - 6)$

2 **a** $6 \times (5 - 4) + 3$ **b** $27 \div (3 + 6) - 3$ **c** $16 - [10 - (6 - 2)]$
d $\dfrac{30+10}{30-10}$ **e** $\dfrac{15+45}{45+5}$ **f** $\dfrac{14}{14-7}$
g $(6 + 3)^2$ **h** $(10 + 4)^2$ **i** $(19 - 9)^2$

1:01B Fractions

Exercise 1:01B

P Appendix A A:01B1–A:01B9

1 Change to mixed numerals.
a $\frac{7}{4}$ **b** $\frac{49}{6}$ **c** $\frac{15}{4}$ **d** $\frac{11}{8}$

2 Change to improper fractions.
a $5\frac{1}{2}$ **b** $3\frac{1}{7}$ **c** $8\frac{3}{4}$ **d** $66\frac{2}{3}$

3 Simplify the following fractions.
a $\frac{48}{80}$ **b** $\frac{70}{150}$ **c** $\frac{200}{300}$ **d** $\frac{250}{450}$

4 Complete the following equivalent fractions.
a $\frac{3}{4} = \frac{\square}{24}$ **b** $\frac{2}{5} = \frac{\square}{50}$ **c** $\frac{2}{7} = \frac{\square}{28}$ **d** $\frac{1}{3} = \frac{\square}{120}$

Find the simplest possible answers without using a calculator.

5 **a** $\frac{7}{15}+\frac{1}{15}$ **b** $\frac{13}{20}-\frac{2}{5}$ **c** $\frac{5}{8}+\frac{3}{10}$ **d** $\frac{6}{7}-\frac{3}{5}$

6 **a** $6\frac{1}{2}+2\frac{3}{5}$ **b** $4\frac{3}{4}+2\frac{3}{10}$ **c** $4\frac{3}{4}+6\frac{1}{10}$ **d** $5\frac{3}{8}-1\frac{9}{10}$

7 **a** $\frac{3}{5}\times\frac{4}{7}$ **b** $\frac{18}{25}\times\frac{15}{16}$ **c** $\frac{4}{9}\times\frac{3}{10}$ **d** $\frac{7}{10}$ of $\frac{2}{3}$

8 **a** $6\times\frac{3}{4}$ **b** $2\frac{1}{2}\times1\frac{4}{5}$ **c** $1\frac{1}{3}\times15$ **d** $10\frac{1}{2}\times1\frac{3}{7}$

9 **a** $\frac{9}{10}\div\frac{2}{3}$ **b** $\frac{3}{8}\div\frac{3}{5}$ **c** $\frac{4}{5}\div6$ **d** $2\frac{3}{4}\div1\frac{1}{2}$

1:01C Decimals

Exercise 1:01C

P Appendix A A:01C1–A:01C11

1 Place in order from smallest to largest.

a 0·606, 0·6, 0·66, 0·066 **b** 1·53, 0·153, 1·053

c 0·7, 0·017, 7, 0·77 **d** 3·5, 3·45, 3·05, 3·4

Complete the following questions without using a calculator.

2 **a** 7·301 + 2 **b** 3·05 + 0·4 **c** 0·004 + 3·1 **d** 6 + 0·3 + 0·02

e 8·67 − 6·7 **f** 9·12 − 1·015 **g** 8 − 3·112 **h** 162·3 − 3

3 **a** 0·012 × 3 **b** 0·03 × 0·2 **c** 0·45 × 1·3 **d** $(0{\cdot}05)^2$

4 **a** 3·14 × 10 **b** 0·5 × 1000 **c** 0·0003 × 100 **d** $3{\cdot}8\times10^4$

5 **a** 0·15 ÷ 5 **b** 1·06 ÷ 4 **c** 15·35 ÷ 5 **d** 0·01 ÷ 4

6 **a** 1·3 ÷ 3 **b** 9·1 ÷ 11 **c** 14 ÷ 9 **d** 6 ÷ 7

7 **a** 48·04 ÷ 10 **b** 1·6 ÷ 100 **c** 0·9 ÷ 1000 **d** $6{\cdot}5\div10^4$

8 **a** 8·4 ÷ 0·4 **b** 0·836 ÷ 0·08 **c** 7·5 ÷ 0·005 **d** 1·4 ÷ 0·5

9 Express as a simplified fraction or mixed numeral.

a 3·017 **b** 0·04 **c** 0·86 **d** 16·005

10 Express as a decimal.

a $\frac{4}{5}$ **b** $\frac{7}{200}$ **c** $\frac{5}{8}$ **d** $\frac{8}{11}$

11 Express these recurring decimals as fractions.

a 0·5555… **b** 0·257 257 2…

c $0{\cdot}\dot{7}\dot{2}$ **d** $0{\cdot}\dot{6}4\dot{2}$

12 Express these recurring decimals as fractions.

a 0·8333… **b** 0·915 151 5… **c** $0{\cdot}43\dot{5}$ **d** $0{\cdot}8\dot{9}4\dot{2}$

1:01D Percentages

Exercise 1:01D

Appendix A A:01D1–A:01D7

1 Express as a fraction.

a 54% **b** 203% **c** $12\frac{1}{4}\%$ **d** 9·1%

2 Express as a percentage.

a $\frac{11}{20}$ **b** $\frac{4}{9}$ **c** $1\frac{1}{4}$ **d** $\frac{2}{3}$

3 Express as a decimal.

a 16% **b** 8·6% **c** 3% **d** $18\frac{1}{4}\%$

4 Express as a percentage.

a 0·47 **b** 0·06 **c** 0·375 **d** 1·3

5 **a** Find 36% of 400 m. **b** Find 9% of 84 g. **c** Find $8\frac{1}{2}\%$ of \$32.

d At the local church, the offertories for 2005 amounted to \$127 000. If 68% of this money was used to pay the salary of the two full-time ministers, how much was paid to the ministers?

6 **a** If 9% of Luke's money was spent on a \$5.40 fare, how much money did Luke have?

b 70% of Alana's weight is 17·5 kg. How much does Alana weigh?

c Lyn bought a book for a reduced price of 70 cents. This was 14% of the book's recommended retail price. What was the recommended retail price?

d A total of 54 minutes of mathematics lesson time was lost in one week because of other activities. If this represents 30% of the allocated weekly time for mathematics, what is this allocated time?

7 **a** Express 85 cents as a percentage of \$2.

b 4 kg of sugar, 9 kg of flour and 7 kg of mixed fruit were mixed. What is the percentage (by weight) of flour in the mixture?

c Of 32 birds in Rachel's aviary, 6 are canaries. What percentage of her birds are canaries?

d When Steve Waugh retired from test cricket in 2003, he had scored 32 centuries from 260 innings. In what percentage of his innings did he score centuries?

1:01E Ratios

Exercise 1:01E

Appendix A A:01E1–A:01E3

1 **a** Simplify each ratio.

i \$15 : \$25 **ii** 9 kg : 90 kg **iii** 75 m : 35 m **iv** $120\,m^2 : 40\,m^2$

b Find the ratio in simplest terms of 5·6 m to 40 cm.

c Naomi spent \$8 of \$20 she was given by her grandparents and saved the rest. What is the ratio of money spent to money saved?

d In a class, $\frac{3}{4}$ of the students walk to school and $\frac{1}{5}$ ride bicycles. Find the ratio of students who walk to students who ride bicycles.

e At the end of their test cricket careers, Steve Waugh had scored 50 fifties and 32 hundreds from 260 innings, whereas Mark Waugh had scored 47 fifties and 20 hundreds from 209 innings.
   i Find the ratio of the number of hundreds scored by Steve to the number scored by Mark.
   ii Find the ratio of the number of times Steve scored 50 or more to the number of innings.

f Express each ratio in the form $X:1$.
   i $3:5$ ii $2:7$ iii $10:3$ iv $25:4$

g Express each ratio in **f** in the form $1:Y$.

2 a If $x:15 = 10:3$, find the value of x.

b If the ratio of the populations of Africa and Europe is $3:2$, find the population of Africa if Europe's population is 732 million.

c The ratio of the average population density per km^2 of Asia to that of Australia is $60:1$. If the average in Asia is 152 people per km^2, what is the average in Australia?

d The ratio of the population of Sydney to the population of Melbourne is $8:7$. If 4·8 million people live in Sydney, how many people live in Melbourne?

3 a If 84 jelly beans are divided between Naomi and Luke in the ratio $4:3$, how many jelly beans does each receive?

b The sizes of the angles of a triangle are in the ratio $2:3:4$. Find the size of each angle.

c A total of 22 million people live in the cities of Tokyo and Moscow. If the ratio of the populations of Tokyo and Moscow is $6:5$, what is the population of each city?

d In 1914, 60 baptisms were recorded in Cobargo. The ratio of males to females who were baptised was $3:2$. How many of each were baptised?

1:01F Rates

Exercise 1:01F

P Appendix A A:01F

1 Complete these equivalent rates.

a 5 km/min = … km/h

b 8 km/L = … m/mL

c 600 kg/h = … t/day

d 2·075 cm^3/g = … cm^3/kg

2 a Find the average speed (to the nearest mile per hour) if it took 564 hours to travel 3396 miles by horse and trap.

b Over a period of 30 working days, Aaron earned $1386. Find his average daily rate of pay.

c Shareen marked 90 books in 7 hours. What rate is this in minutes per book?

d On a hot day, our family used an average of 36 L of water per hour. Change this rate to cm^3 per second (cm^3/s).

1:01G Significant figures

Exercise 1:01G

Appendix A A:01G

1 State the number of significant figures in each of the following.

a 21	**b** 4·6	**c** 2·52	**d** 0·616
e 16·32	**f** 106	**g** 3004	**h** 2·03
i 1·06	**j** 50·04	**k** 0·5	**l** 0·003
m 0·000 32	**n** 0·06	**o** 0·006	**p** 3·0
q 25·0	**r** 2·60	**s** 13·000	**t** 6·40
u 41 235	**v** 600 (to nearest hundred)		**w** 482 000 (to nearest thousand)
x 700 (to nearest ten)		**y** 1600	**z** 16 000

2 State the number of significant figures in each of the following.

a 3·0	**b** 3·00	**c** 0·3	**d** 0·03
e 0·030	**f** 0·0030	**g** 0·0300	**h** 3·0300

1:01H Approximations

Exercise 1:01H

Appendix A A:01H

1 Round each of the following correct to one decimal place.

a 4·63	**b** 0·81	**c** 3·17	**d** 0·062
e 15·176	**f** 8·099	**g** 0·99	**h** 121·62
i 0·119	**j** 47·417	**k** 0·35	**l** 2·75

2 Round each of the following correct to two decimal places.

a 0·537	**b** 2·613	**c** 7·134	**d** 1·169
e 12·0163	**f** 8·399	**g** 412·678	**h** 0·0756
i 0·4367	**j** 100·333	**k** 0·015	**l** 0·005

3 Round each number correct to: **i** 1 sig. fig. **ii** 2 sig. fig.

a 7·31	**b** 84·9	**c** 0·63	**d** 2·58
e 4·16	**f** 0·0073	**g** 0·0828	**h** 3·05
i 0·009 34	**j** 0·0098	**k** 7·52	**l** 0·0359

4 Round each of the following to the number of significant figures indicated.

a 2·3 (1 sig. fig.)	**b** 14·63 (3 sig. fig.)	**c** 2·15 (2 sig. fig.)
d 0·93 (1 sig. fig.)	**e** 4·07 (2 sig. fig.)	**f** 7·368 94 (3 sig. fig.)
g 0·724 138 (3 sig. fig.)	**h** 5·716 (1 sig. fig.)	**i** 31·685 (4 sig. fig.)
j 0·007 16 (1 sig. fig.)	**k** 0·78 (1 sig. fig.)	**l** 0·007 16 (2 sig. fig.)

5 Round each of the following to the number of decimal places indicated.

a 5·61 (1 dec. pl.)	**b** 0·16 (1 dec. pl.)	**c** 0·437 (2 dec. pl.)
d 15·37 (1 dec. pl.)	**e** 8·333 (2 dec. pl.)	**f** 413·789 (1 dec. pl.)
g 71·98 (1 dec. pl.)	**h** 3·0672 (3 dec. pl.)	**i** 9·99 (1 dec. pl.)
j 4·7998 (3 dec. pl.)	**k** 0·075 (2 dec. pl.)	**l** 0·0035 (3 dec. pl.)

1:01I Estimations

Exercise 1:01I

Appendix A A:01I

1 Give an estimate for each of the following.

a $12{\cdot}7 \times 5{\cdot}8$ **b** $0{\cdot}55 \times 210$ **c** $17{\cdot}8 \times 5{\cdot}1 \times 0{\cdot}336$

d $15{\cdot}6 \div 2{\cdot}165$ **e** $(4{\cdot}62 + 21{\cdot}7) \times 4{\cdot}21$ **f** $7{\cdot}8 \times 5{\cdot}2 + 21{\cdot}7 \times 0{\cdot}89$

g $(0{\cdot}93 + 1{\cdot}72)(8{\cdot}5 - 1{\cdot}7)$ **h** $\dfrac{43{\cdot}7 + 18{\cdot}2}{7{\cdot}8 + 2{\cdot}9}$ **i** $\dfrac{101{\cdot}6 - 51{\cdot}7}{21{\cdot}3 - 14{\cdot}8}$

j $\dfrac{0{\cdot}68 \times 51}{0{\cdot}25 \times 78}$ **k** $\dfrac{11{\cdot}6 - 3{\cdot}92}{12{\cdot}7 + 6{\cdot}58}$ **l** $3{\cdot}52^2 \times \sqrt{17{\cdot}9}$

m $\sqrt{41{\cdot}7 \times 5{\cdot}6}$ **n** $\sqrt{4{\cdot}26} \times \sqrt{105{\cdot}6}$ **o** $3{\cdot}1^3 \times 1{\cdot}8^4$

p $\dfrac{4{\cdot}1 \times \sqrt{48{\cdot}12}}{26{\cdot}23}$ **q** $\dfrac{15{\cdot}7^2}{11{\cdot}3 \times 3{\cdot}1}$ **r** $\dfrac{16{\cdot}7}{2{\cdot}15} + \dfrac{41{\cdot}6}{4{\cdot}7}$

s $\dfrac{0{\cdot}65}{0{\cdot}01} - \dfrac{0{\cdot}75 \times 3{\cdot}6}{0{\cdot}478}$

1:01J Using units of measurement

Exercise 1:01J

Appendix A A:01J

1 Convert the following.

a 12 kL to L **b** 5 t to kg **c** 3·85 g to mg

d 8 Mm to m **e** 7 m to μm **f** 2 m to nm

g 6 GB to B **h** 4 TB to GB **i** 3 TB to GB

j 25 mm to μm **k** 5·4 MB to kB **l** 500 000 ns to μs

2 Our computer has a capacity of 1·2 terabytes. We have 800 gigabytes of available space left. How much space have we used?

1:01K Accuracy of measurements

Exercise 1:01K

Appendix A A:01K

1 Find the absolute error (greatest possible error) for each of the following measurements.

a 8 g **b** 17·4 m **c** 27·625 L

d 32 cm **e** 1·8 t **f** 2·76 s

g 8000 L (measured to the nearest 1000 L)

2 **a** Alana's bathroom scales are marked in tenths of a kilogram. She measured her mass as 63·7 kg. Find the absolute error of the measurement and her smallest and largest possible mass.

b A metal prism is measured as 11·3 cm thick. In what range must the actual thickness of the prism lie?

1:02 Algebra

Exercise 1:02

P Appendix A A:02A–A:02E

1 Simplify these expressions.

a $a + 5a$ b $5y + 3y + 2y$ c $5m - 3m$

d $3x^2 - 2x^2$ e $7k + 3k - 2k$ f $20y - 10y - 5y$

g $3a + 2b + 5a$ h $7n + 10m - 3m$ i $6m - 5 + 5m + 7$

j $6x - y^2 + x + 2y^2$ k $3ab - 5a + 2ab$ l $7x^2 + 2xy - x^2 + 3xy$

m $5a + 7 - 2a + 4b$ n $9k + 5m - 3 - 2m$ o $8x + 2y - 3z - 7x$

2 Simplify these products.

a $5y \times 7$ b $\frac{1}{2} \times 10q$ c $2x \times 3y$ d $5a \times 4b$

e $9m \times n$ f $3m \times 2m$ g $5ab \times 2a$ h $5mn \times 2mn$

i $(-5x) \times 4$ j $(-4y) \times (-3)$ k $(-10pq) \times (-\frac{1}{2}pq)$ l $2m \times 3n \times 7$

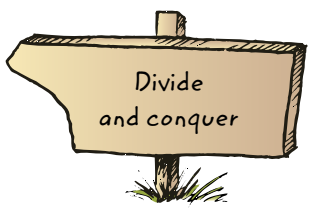
m $6a \times 2b \times 5a$ n $(-5p) \times 2q \times (-3r)$

3 Simplify these expressions.

a $12x \div 3$ b $15a \div 5$ c $10y \div 2y$ d $24k \div 8k$

e $18mn \div 9m$ f $16ab \div 4b$ g $15p \div 10p$ h $12q \div 10$

i $20a \div 10b$ j $25m \div 5n$ k $20ab \div 12ac$ l $24m^2 \div 16mn$



4 Expand and simplify these expressions.

a $5(x + 4)$ b $10(5 - y)$ c $2y(y + 5)$ d $5a(a - 3)$

e $7p(p + q)$ f $-3(x - 5)$ g $-5(a + 4)$ h $-2a(3x - y)$

i $6(x + 3) + 7$ j $5(y - 3) + 2y$ k $7(a - b) - 3b$ l $2(x + 3) + 5(x - 2)$

m $5(a - 2) + 3(a + 2)$ n $2y(y - 3) + y(y + 2)$ o $6(m + 3) - 3(m + 5)$ p $6(x - 5) - 3(2 - x)$

q $5p(p + 7) - 2p(2p - 1)$

5 Factorise by taking out the highest common factor.

a $5m + 10$ b $8k - 24$ c $8a + 6$ d $15n - 10$

e $18n - 24m$ f $ab + ac$ g $n^2 - mn$ h $pq - p$

i $3ab + 6a$ j $10mn - 5m$ k $8xy + 4yz$ l $abc - bcd$

m $3ac + 9ad$ n $6x^2y + 9x$ o $3a + 6b + 9c$ p $8x^2 + 4x - 12$

6 Factorise by taking out a negative common factor.

a $-4a - 8$ b $-15x - 10$ c $-6p + 9$ d $-2n + 16$

e $-x^2 - 2x$ f $-5a - 2ab$ g $-3a^2 + 6a$ h $-9yz + 6xy$

7 Simplify the following.

a $\frac{3a}{2} + \frac{a}{2}$ b $\frac{3x}{5} - \frac{2x}{5}$ c $\frac{a}{3} + \frac{4a}{3}$ d $\frac{9m}{10} - \frac{3m}{10}$

8 Reduce each of these expressions to its simplest form.

a $\frac{x}{3} + \frac{x}{5}$ b $\frac{y}{3} - \frac{y}{4}$ c $\frac{2a}{3} + \frac{a}{2}$ d $\frac{3n}{8} - \frac{n}{4}$

e $\frac{x}{4} + \frac{y}{3}$ f $\frac{2a}{3} - \frac{3b}{2}$ g $\frac{3m}{5} + \frac{n}{2}$ h $\frac{k}{6} - \frac{2l}{4}$

9 Simplify these products.

a $\frac{x}{2} \times \frac{y}{3}$ **b** $\frac{m}{2} \times \frac{m}{5}$ **c** $\frac{3}{a} \times \frac{4}{m}$ **d** $\frac{1}{n} \times \frac{1}{3n}$

e $\frac{p}{q} \times \frac{x}{y}$ **f** $\frac{2}{a} \times \frac{a}{4}$ **g** $\frac{m}{5} \times \frac{10}{n}$ **h** $\frac{3x}{5} \times \frac{2}{9x}$

10 Simplify these divisions.

a $\frac{m}{2} \div \frac{m}{4}$ **b** $\frac{5a}{3} \div \frac{2a}{9}$ **c** $\frac{5}{a} \div \frac{2}{a}$ **d** $\frac{a}{b} \div \frac{2a}{b}$

e $\frac{a}{b} \div \frac{x}{y}$ **f** $\frac{2p}{3q} \div \frac{8p}{9q}$ **g** $\frac{10k}{3n} \div \frac{2k}{9n}$ **h** $\frac{a}{2} \div \frac{a}{3}$

1:03 Probability

Exercise 1:03

P Appendix A A:03A–A:03E

1 Using the figures shown in the table, find the probability of selecting at random a matchbox containing:

a 50 matches **b** 48 matches

c more than 50 matches **d** at least 50 matches.

Number of matches	48	49	50	51	52
Number of boxes	3	6	10	7	4

2 A single dice is rolled. What is the probability of rolling:

a a five **b** less than 3 **c** an even number **d** less than 7?

3 A bag contains 3 red, 4 white and 5 blue marbles. If one is selected from the bag at random, find the probability that it is:

a white **b** red or white **c** *not* red **d** pink.

4 A standard pack of cards has four suits, hearts and diamonds (both red), and spades and clubs (both black). In each suit there are 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The Jack, Queen and King are called picture (court) cards.

A card is drawn from a standard pack. What is the probability that the card is:

a red **b** not red **c** a 6 **d** not a 6

e a picture card **f** a red Ace **g** a spade **h** a red 13

i either a red 5 or a 10 **j** either a heart or a black Ace

k either a blue 5 or a 7 **l** either a heart or a black card

(In each of these cases, the events may not be mutually exclusive.)

m either a picture card or a diamond

n either a number larger than 2 or a club

o either a heart or a 5

p either a Queen or a black picture card

q either a number between 2 and 8 or an even-numbered heart?

There are 4 suits with 13 cards in each suit so the number of cards in a standard pack is 52. (In some games a Joker is also used.)

5 The Venn diagram shows two sports played by 90 Year 10 students. What is the probability that a randomly chosen student plays:

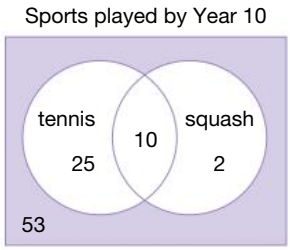
a both tennis and squash

b neither tennis nor squash

c tennis

d tennis or squash but not both?

(Give answers as a fraction and as a percentage correct to the nearest per cent.)



6 The table shows how many females and males among 85 Year 10 students are right-handed or left-handed. What is the probability that a randomly chosen student is:

a both female and left-handed

b right-handed

c male

d a right-handed male?

Year 10	left-handed	right-handed	
female	4	41	45
male	7	33	40
	11	74	85

(Give answers as a fraction and as a percentage correct to the nearest per cent.)

7 **a** There are one red, two green and three yellow counters in a container. Show all possible outcomes, as an array (a table), when two counters are taken from the container, one after the other, without replacing the first counter. (Call the green counters G1 and G2, and the yellow counters Y1, Y2 and Y3.)

b Three counters, one red and two yellow, are in a container. Show all possible outcomes, both as an array (a table) and as a tree diagram, when two counters are taken from the container, one after the other, returning the first choice to the container before making the second choice (i.e. with replacement).

INVESTIGATION 1:03 BLAISE PASCAL

Blaise Pascal (1623–1662) was indeed a genius. Mathematician, physicist, philosopher and theologian, he achieved fame in all these areas in his relatively short lifetime.

After discovering geometry at the age of 12, his ability became legendary. He invented the first calculating machine when he was 18, but is remembered most for establishing the foundation of probability theory (along with Fermat). He often related his philosophy to mathematics, as with the famous example called 'Pascal's wager'. Use the internet to discover more about Blaise Pascal.

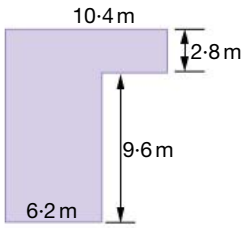
1:04 Area, surface area and volume

Exercise 1:04

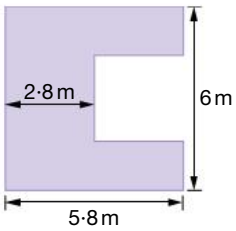
P Appendix A A:04A–A:04F

1 Find the perimeters of the following figures.

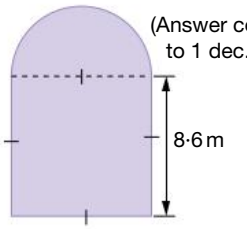
a



b



c

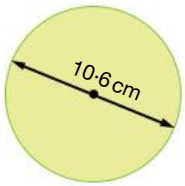


(Answer correct to 1 dec. pl.)

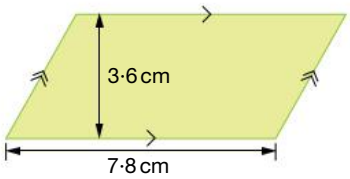
(Use $\pi = 3{\cdot}142$)

2 Find the area of each plane shape correct to two decimal places.

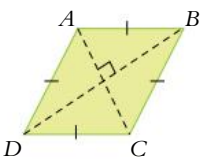
a



b



c

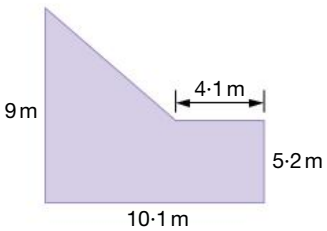


$AC = 3{\cdot}6$ cm

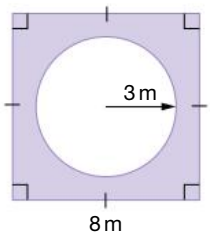
$BD = 6{\cdot}4$ cm

3 Find the areas of the following shaded figures correct to three significant figures.

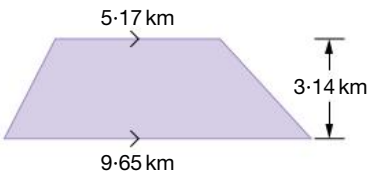
a



b

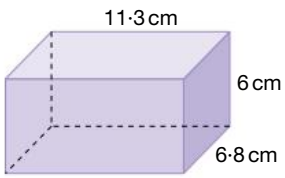


c



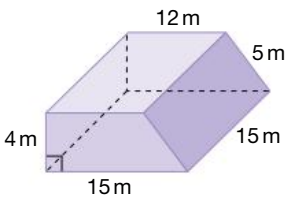
4 Find the surface areas of the following solids.

a



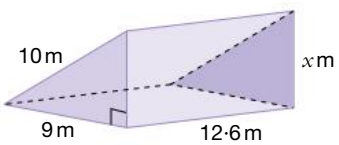
Rectangular prism

b



Trapezoidal prism

c



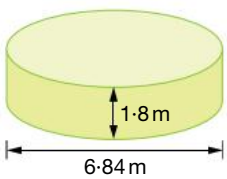
Triangular prism

(Use Pythagoras' theorem to find x.)

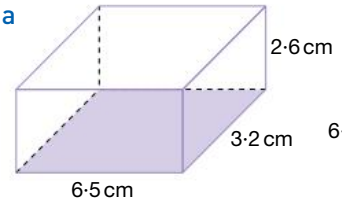
5 For the cylinder shown, find correct to two decimal places:

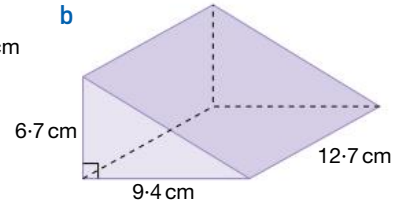
a the curved surface area

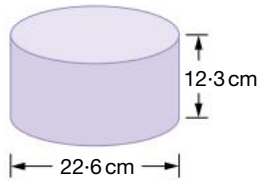
b the total surface area.

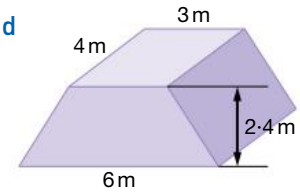


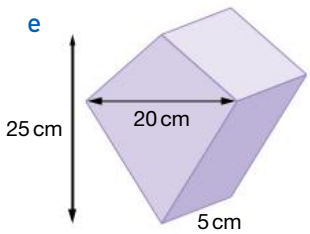
6 Calculate the volumes of these prisms and cylinders correct to one decimal place.

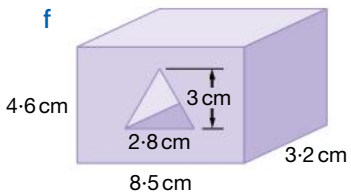
a 

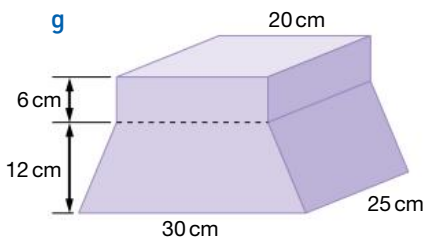
b 

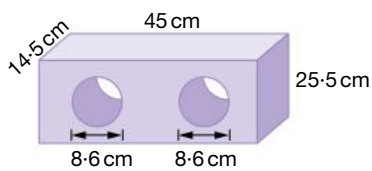
c 

d 

e 

f 

g 

h 

1:05 Indices

Exercise 1:05

P Appendix A A:05A–A:05E

1 Write in index form.

a $a \times a \times a$ b $2 \times 2 \times 2 \times 2$ c $n \times n \times n \times n \times n$ d $10 \times 10 \times 10$

2 Simplify, giving your answers in index form.

a $2^4 \times 2^5$ b $a^3 \times a^2$ c $m \times m^4$ d $10^6 \times 10^2$

e $a^{10} \div a^2$ f $y^4 \div y^3$ g $b^3 \div b$ h $10^5 \div 10^2$

i $(m^3)^4$ j $(a^2)^3$ k $(x^4)^2$ l $(10^5)^2$

m $a^0 \times 3$ n $b^0 + c^0$ o $6y^0$ p $e^6 \div e^6$

q $6a^2 \times 5$ r $6m^3 \div 3$ s $6a \times 5a$ t $(4x^4)^2$

3 Simplify.

a $6a^4 \times 5ab^3$ b $7a^2b^2 \times 8a^3b$ c $4a^2b^3 \times 6a^2b^4$ d $10a^7 \times a^3b^3$

e $(7x^3)^2$ f $(2m^2)^4$ g $(x^2y^3)^3$ h $(5xy^2)^4$

i $30a^5 \div 5a^3$ j $100x^4 \div 20x$ k $36a^3b^4 \div 12a^2b^4$ l $8y^7z^2 \div y^7z^2$

4 Rewrite without a negative index.

a 4^{-1} b 10^{-1} c x^{-1} d $2a^{-1}$

e 5^{-2} f 2^{-3} g m^{-3} h $5x^{-2}$

5 Rewrite each of the following with a negative index.

a $\frac{1}{3}$ **b** $\frac{1}{7}$ **c** $\frac{1}{a}$ **d** $\frac{3}{x}$

e $\frac{1}{2^4}$ **f** $\frac{1}{10^6}$ **g** $\frac{1}{y^4}$ **h** $\frac{5}{n^3}$

6 Write these numbers in scientific (or standard) notation.

a 148 000 000 **b** 68 000 **c** 0·000 15 **d** 0·000 001 65

7 Write these as basic numerals.

a $6{\cdot}2 \times 10^4$ **b** $1{\cdot}15 \times 10^6$ **c** $7{\cdot}4 \times 10^{-3}$ **d** $6{\cdot}91 \times 10^{-5}$

8 Write each as bytes (B), using scientific notation.

a 480 000 bytes (B) **b** 1·4 kilobytes (kB) **c** 3·4 terabytes (TB)

d 8 GB **e** 8·50 MB **f** 4·15 GB

9 Write each as seconds (s), using scientific notation.

a 5 ms **b** 4·2 µs **c** 7·5 ns **d** 125 µs **e** 65 ns **f** 80 ms

10 Use scientific notation to write:

a 10·5 megalitres as litres **b** 102 gigalitres as litres

c 850 milligrams as grams **d** 67 nanometres as metres

1:06 Equations and inequalities

Exercise 1:06

P Appendix A A:06A–A:06E

1 Solve the following.

a $a + 7 = 25$ **b** $m - 6 = -1$ **c** $5x = 75$

d $10 - y = 12$ **e** $3p = 7$ **f** $\frac{n}{4} = 3$

g $2x + 3 = 7$ **h** $3q - 5 = 1$ **i** $8m + 5 = 21$

j $5y + 2 = 3$ **k** $2x - 7 = 10$ **l** $9k - 1 = 5$

m $5 + 3x = 11$ **n** $10 - 7y = 3$ **o** $15 - 2q = 8$

2 Solve the following.

a $5m + 2 = 4m + 7$ **b** $3x - 7 = 2x - 3$ **c** $5x + 2 = 6x - 5$

d $4p + 2 = 2p + 10$ **e** $5n - 6 = 3n + 2$ **f** $7m - 3 = 2m + 7$

g $2a + 3 = 3a - 5$ **h** $3m - 2 = 5m - 10$ **i** $q + 7 = 8q + 14$

j $10 - 2x = x + 4$ **k** $2y + 5 = 2 - y$ **l** $3 + 8p = 29 - 5p$

m $2k - 3 = 4k + 8$ **n** $3z + 7 = z + 10$ **o** $6y - 3 = 2y + 8$

p $5n + 4 = 8n + 2$ **q** $10 - q = 9 + 3q$ **r** $13 - 2m = 9 - 5m$

3 Solve these equations involving grouping symbols.

a $5(a+1)=15$ b $4(x-3)=16$ c $4(5-x)=8$

d $3(2x+5)=33$ e $9(1+2m)=0$ f $2(2k-7)=15$

g $3(5-2a)=27$ h $4(3-2x)=36$ i $3(2m-5)=11$

j $3(a+2)+2(a+5)=36$ k $2(p+3)+p+1=31$ l $2(x+5)=x+16$

m $2(y-3)=3(y+1)$ n $4(2b+7)=2(3b-4)$ o $4(2y+3)+3(y-1)=2y$

p $3(m-4)-(m+2)=0$ q $2m-3(1-m)=22$ r $5(y-3)-3(1-2y)=4$

s $4(2x-1)-2(x+3)=5$

4 Solve these equations.

a $\frac{5x}{2}=10$ b $\frac{2a}{3}=6$ c $\frac{8m}{5}=4$

d $\frac{n+1}{5}=2$ e $\frac{x-4}{2}=1$ f $\frac{2p+5}{3}=1$

5 Solve these equations involving fractions.

a $\frac{a}{8}+\frac{a}{8}=4$ b $\frac{2x}{5}-\frac{x}{5}=3$ c $\frac{5p}{3}-\frac{p}{3}=8$

d $\frac{a}{2}+\frac{a}{3}=5$ e $\frac{3m}{4}+\frac{m}{3}=13$ f $\frac{n}{5}+\frac{3n}{2}=34$

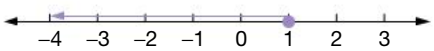
g $\frac{q}{2}-\frac{q}{3}=6$ h $\frac{2k}{3}-\frac{k}{4}=10$ i $\frac{3x}{4}-\frac{x}{2}=15$

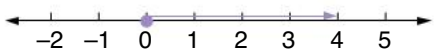
j $\frac{3y}{5}-\frac{2y}{3}=2$ k $\frac{5p}{4}+\frac{3p}{8}=3$ l $\frac{7a}{8}-\frac{2a}{3}=1$

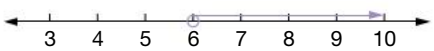
m $\frac{m+6}{3}=\frac{2m+4}{4}$ n $\frac{n-3}{2}=\frac{2n-6}{4}$ o $\frac{5x-1}{3}=\frac{3-x}{2}$

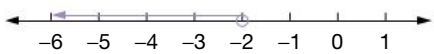
p $\frac{x+3}{2}+\frac{x+5}{5}=8$ q $\frac{m+2}{5}-\frac{m+3}{6}=1$ r $\frac{3a+4}{2}-\frac{a-1}{3}=\frac{2a+3}{4}$

6 Write the inequality for x shown on each number line below.

a 

b 

c 

d 

7 Solve these inequalities and show each solution on a number line.

a $x+7>11$ b $a-5<3$ c $10-y\geq 8$

d $3m\leq 21$ e $15<4x$ f $\frac{m}{4}\leq 1$

g $2x+1>5$ h $5m-3\geq 6$ i $7-3n\geq 4$

j $5x+6>x+18$ k $3x-5<x+6$ l $m+3\geq 2m-7$

m $3-a\leq 5-2a$ n $12-b\geq 2b+21$ o $3(m+4)<2(m+6)$

p $\frac{x}{2}+1<6$ q $\frac{p}{3}-1>4$ r $\frac{3x}{4}-5\geq 1$

s $5-\frac{2y}{3}\leq 6$ t $\frac{p-1}{4}<2$ u $\frac{2p+3}{2}>7$

v $\frac{4+x}{5}\leq 1$ w $\frac{x}{2}+\frac{x}{3}>5$ x $\frac{a}{4}+\frac{a}{2}\leq 6$

y $3b-\frac{2b}{3}<5$ z $\frac{x}{2}-\frac{2x}{3}\leq 3$ α $\frac{y}{2}-3y>4$

1:07 Financial mathematics

Exercise 1:07

P Appendix A A:07A–A:07F

1 **a** Michelle is paid $18.40 per hour and time-and-a-half for overtime. If a normal day's work is 7 hours, how much would she be paid for 10 hours work in one day?

b Jake receives a holiday loading of $17\frac{1}{2}\%$ on 4 weeks normal pay. If he works 37 hours in a normal week and is paid $9.20 per hour, how much money does he receive as his holiday loading?

c In a week, a saleswoman sells $6000 worth of equipment. If she is paid $150 plus 10% commission on sales in excess of $4000, how much does she earn?

d A waiter works from 5:00 pm until 1:30 am on four days in one week. His hourly rate of pay is $14.65 plus an average of $9.20 in tips per working night. Find his income for the week.

2 **a** Find the net pay for the week if Saransh earns $1115.40, is taxed $226.20, pays $111.50 in superannuation and his miscellaneous deductions total $76.34. What percentage of his gross pay did he pay in tax? (Answer correct to one decimal place.)

b Find the tax payable on a taxable income of $40 180 if the tax is $4650 plus 30 cents for each $1 in excess of $37 000.

c James received a salary of $68 300 and an income of $496 from investments. His total tax deductions were $3050. What is his taxable income?

d Tom's taxable income for the year was $13 860. Find the tax that must be paid if the rate is 15 cents for each $1 in excess of $6000.

3 **a** An item has a marked price of $87.60 in two shops. One offers a 15% discount, and the other a discount of $10.65. Which is the better buy, and by how much?

b Emma bought a new tyre for $100, Jade bought one for $85 and Diane bought a retread for $58. If Emma's tyre lasted 32 000 km, Jade's 27 500 km, and Diane's 16 000 km, which was the best buy? (Assume that safety and performance for the tyres are the same.)

c Alice wants to get the best value when buying tea. Which will she buy if Pet tea costs $1.23 for 250 g, Jet tea costs $5.50 for 1 kg and Yet tea costs $3.82 for 800 g?

4 **a** Find the GST (10%) that needs to be added to a base price of:

i $75 **ii** $6.80 **iii** $18.75

b For each of the prices in **a**, what would the retail price be? (Retail price includes the GST.)

c Find the GST contained in a retail price of:

i $220 **ii** $8.25 **iii** $19.80

5 a What is meant by the expression 'buying on terms'?

b Find the amount Jason will pay for a fishing line worth $87 if he pays $7 deposit and $5.69 per month for 24 months. How much extra does he pay in interest charges?

c Nicholas was given a discount of 10% on the marked price of a kitchen table. If the discount was $22, how much was the marked price?

d A factory's machinery depreciates at a rate of 15% per annum. If it is worth $642 000, what will be its value after one year?

e The price of a book was discounted by 20%. A regular customer was given a further discount of 15%. If the original price was $45, what was the final price of the book?

6 a The cost price of a DVD player was $180 and it was sold for $240. What was:

i the profit as a percentage of the cost price

ii the profit as a percentage of the selling price?

b A new car worth $32 000 was sold after two years for $24 000. What was:

i the loss as a percentage of the original cost price

ii the loss as a percentage of the final selling price?

1:08 Coordinate geometry

Exercise 1:08

P Appendix A A:08A–A:08I

1 a What are the coordinates of the origin?

b What is the name of the vertical axis?

c What is the name of the horizontal axis?

d In which quadrant does $(-3, -2)$ lie?

2 On the same number plane, graph these straight lines.

a $y = 3x - 2$ b $x + y = 4$ c $y = -2x + 3$

3 a Which of the following points lie on the line $y = 3x - 11$?
$(0, 0)$, $(3, -2)$, $(1, 4)$, $(5, 4)$, $(10, 41)$

b Which of the following lines passes through the point $(0, 2)$?
$y = 2x$, $y = 2x - 2$, $y = x + 2$, $y = 5x + 2$, $x + y = 2$

4 a Write the equation of the line that is:

i parallel to the x-axis and 2 units above it

ii parallel to the x-axis and 3 units below it

iii the x-axis

iv parallel to the y-axis and 2 units to the right of it

v parallel to the y-axis and 1 unit to the left of it

vi the y-axis.

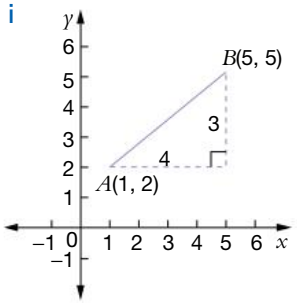
b On the same number plane, graph the following lines.

i $y = 3$ **ii** $y = -1$ **iii** $x = 4$ **iv** $x = -2$

5 **a** Write Pythagoras' theorem in words.

b Find the length of AB in each triangle as an exact value.

i



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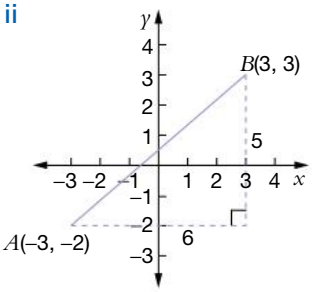

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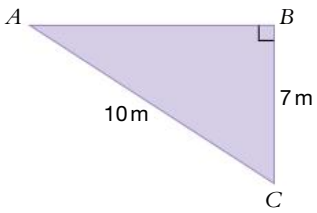


6 **a** Find the distance between (1, 4) and (5, 2).

b A is the point (–2, 1). B is the point (5, 3). Find the distance AB.

c Find the distance AB between $A(-5, 2)$ and $B(-3, -6)$.

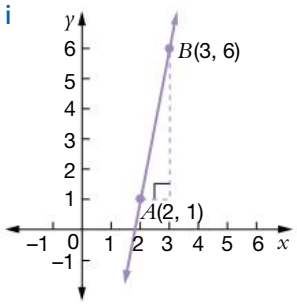
d Find the distance between the origin and (6, 8).

7 Find the midpoint of the interval joining the following points.

a (3, 6) and (9, 10) **b** (–1, 8) and (3, –2) **c** (0, 3) and (7, 8)

8 **a** Find the gradient of AB in each case.

i



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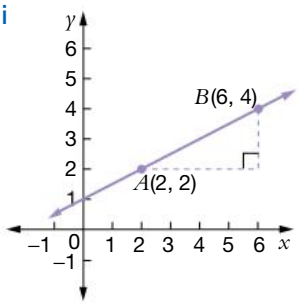

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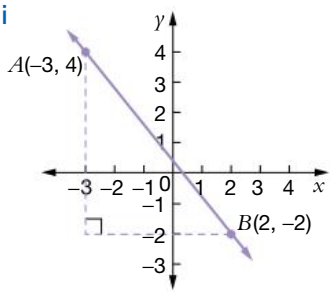


b Find the gradient of the line that passes through the following points.

i (2, 6) and (5, 7) **ii** (–4, 3) and (1, 4) **iii** (–4, 1) and (–2, –4)

9 Find the gradient and y-intercept of the following lines.

a $y = 4x + 7$ **b** $y = x - 3$ **c** $y = -2x + 5$

10 Find the equation of a line that has:

a a gradient of 4 and a y-intercept of 5

b a gradient of $-\frac{1}{2}$ and a y-intercept of 3

c a gradient of 2 and passes through the point (0, –4)

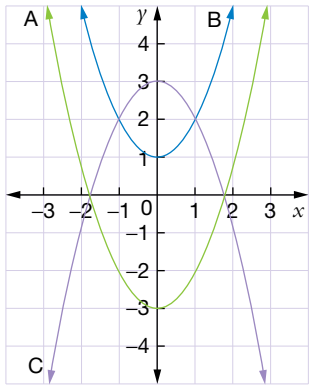
d a y-intercept of –3 and a gradient of –7.

11 Match each equation with the corresponding parabola in the diagram.

a $y = -x^2 + 3$

b $y = x^2 + 1$

c $y = x^2 - 3$

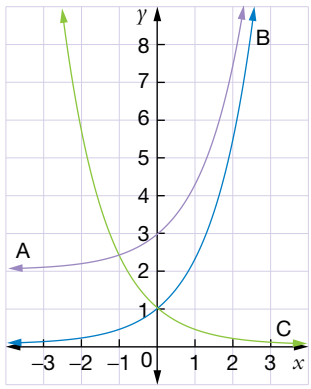


12 Match each equation with the corresponding exponential curve in the diagram.

a $y = 2^x$

b $y = 2^{-x}$

c $y = 2 + 2^x$



1:09 Similarity and scale drawing

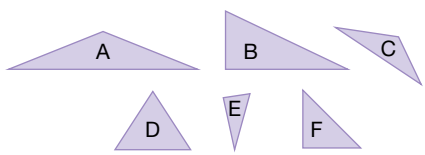
Exercise 1:09

P Appendix A A:09A–A:09B

1 **a** Explain the meaning of the term 'similar figures'.

b Which triangle is similar to A?

c Which triangle is similar to B?



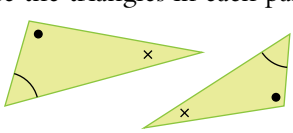
2 True or false?

a When two figures are similar the matching sides are proportional.

b When two figures are similar the matching angles are equal.

3 Are the triangles in each pair similar?

a



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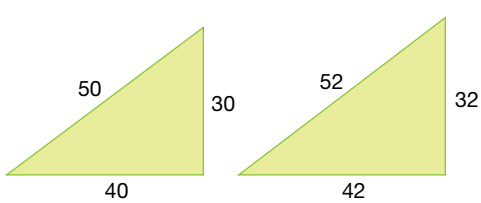
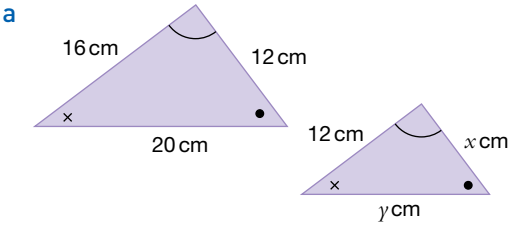


4 Find the value of the pronumerals in each pair of similar triangles.

a



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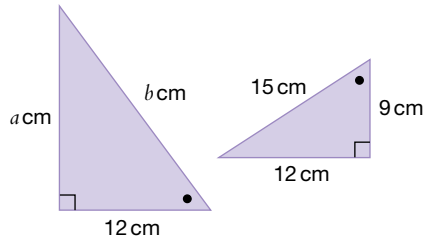

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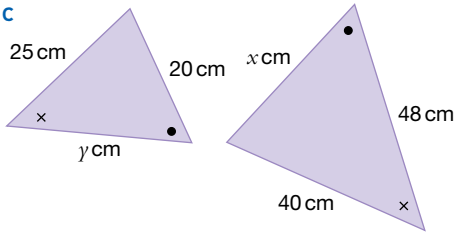

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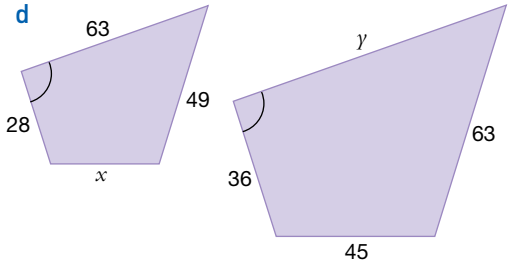


5 A basketball court is rectangular in shape with length 28 m and breadth 15 m. A scale drawing is to be made of the court.

a Calculate the dimensions of the scale drawing if the scale used is 1 : 200.

b What scale should be used if the length of the scale drawing must be 7 cm?

1:10 Formulas and problem solving

Exercise 1:10

P Appendix A A:10A–A:10E

1 Write expressions for:

a the sum of $3a$ and $4b$

b the product of $3a$ and $4b$

c the difference between k and m, if k is greater than m

d the average of x, y and z

e twice the sum of m and 5

f the square of the difference between a and b

g the square root of the sum of $5m$ and $4n$

h the sum of three consecutive integers, if the first is m.

2 If $a = 3$, $b = 5$ and $c = -6$, find the value of:

a $a + c$ **b** $2a + 3b$ **c** $a + b + c$

d $\frac{1}{2}c^2$ **e** $ac - b^2$ **f** $a^2 + c^2$

g $\dfrac{3b - c}{2a}$ **h** $\sqrt{ab + c}$ **i** $\sqrt{\dfrac{3c}{a - b}}$

3 If $x = 2{\cdot}1$, $y = 3{\cdot}5$ and $z = 2{\cdot}8$, evaluate P if:

a $P = 2x + 3y$ **b** $P = \dfrac{x+y}{z}$ **c** $P = \dfrac{x+y}{x-z}$

4 **a** If $s = ut + \frac{1}{2}at^2$, find s if $u = 9$, $t = 4$ and $a = 7$.

b Given $F = p + qr$, find F if $p = 2{\cdot}3$, $q = 3{\cdot}9$ and $r = 0{\cdot}9$.

c For the formula $T = a + (n-1)d$, find T if $a = 9{\cdot}2$, $n = 6$ and $d = 1{\cdot}3$.

5 **a** Given that $v = u + at$, find u if $v = 20$, $a = 9$ and $t = 2$.

b If $y = mx + b$, find b when $x = 2$, $y = 4$ and $m = -3$.

c Given that $A = B^2 + 2C$, find C when $A = 48$ and $B = 5$.

d For the formula $A = \dfrac{x+y}{n}$, evaluate y if $A = 6$, $n = 9$ and $x = 42$.

6 **a** Given that $V = lbh$, evaluate b when $V = 4{\cdot}32$, $l = 1{\cdot}2$ and $h = 0{\cdot}9$.

b It is known that $S = \dfrac{a}{1-r}$. Find a when $S = 25$ and $r = 0{\cdot}6$.

c Find C if $F = 32 + \dfrac{9C}{5}$ and $F = 77$.

d If $v^2 = u^2 + 2as$, find a if $v = 2{\cdot}1$, $u = 1{\cdot}6$ and $s = 0{\cdot}3$.

e Given that $T = a + (n-1)d$, find d if $T = 24{\cdot}6$, $a = 8{\cdot}8$ and $n = 4$.

7 **a** Translate the following into equations using n as the unknown number.

- **i** A certain number is multiplied by 8, then 11 is added and the result is 39.
- **ii** I think of a number, double it, add 7 and the result is 5.
- **iii** I think of a number, add 4 and then divide the result by 10. The answer is 7.

b Solve each of the following problems by first forming an equation.

- **i** If 5 is added to 3 times a certain number, the result is 38. What is the number?
- **ii** If one-quarter of a certain number is added to half the same number, the result is 6. What is the number?
- **iii** A rectangle is four times as long as it is wide. If it has a perimeter of 340 m, what are its dimensions?

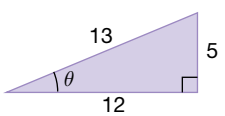
1:11 Trigonometry

Exercise 1:11

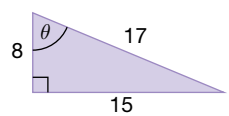
P Appendix A A:11A–A:11E

1 State the value of $\tan\theta$ as a fraction.

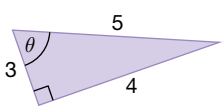
a



b

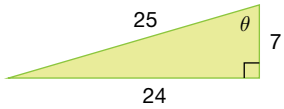


c



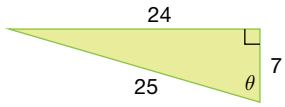
2 State the value of each ratio as a fraction.

a



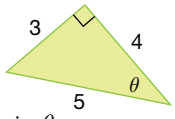
$\sin\theta = \ldots$

b



$\cos\theta = \ldots$

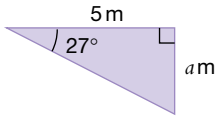
c



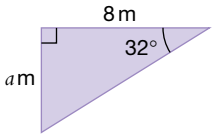
$\sin\theta = \ldots$

3 Find the value of a correct to one decimal place.

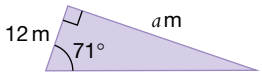
a



b

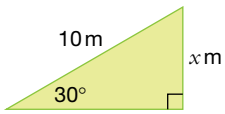


c



4 Find the value of x correct to one decimal place.

a



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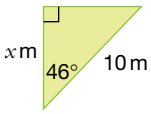
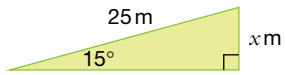
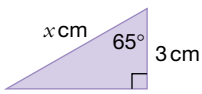


c



5 Find the value of x correct to one decimal place.

a



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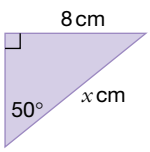
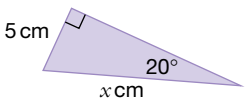
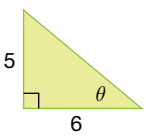


c



6 Find θ to the nearest degree.

a



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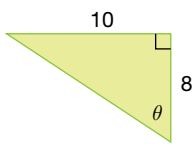
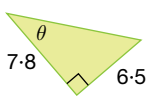
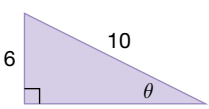


c

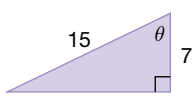


7 Find θ correct to the nearest minute.

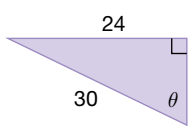
a



b



c



8 The angle of depression of an object on a level plane is observed to be 19° from the top of a 21 m tower. How far from the foot of the tower is the object? Answer correct to the nearest centimetre.

9 The angle of elevation of the top of a vertical cliff is observed to be 23° from a boat 180 m from the base of the cliff. What is the height of the cliff? Answer correct to one decimal place.

10 A ship sails south for 50 km, then N43°E until it is due east of its starting point. How far is the ship from its starting point to the nearest metre?

1:12 Statistics

Exercise 1:12

P Appendix A A:12A–A:12H

1 A dice was rolled 50 times and the results are shown below. Organise these results into a frequency distribution table and answer the questions.

5	4	1	3	2	6	2	1	4	5
5	1	3	2	6	3	2	4	4	1
6	2	5	1	6	6	6	5	3	2
6	3	4	2	4	1	4	2	4	4
2	3	1	5	4	2	2	3	2	1

a Which number on the dice was rolled most often?
b Which number had the lowest frequency?
c How often did a 3 appear?
d For how many throws was the result an odd number?
e On how many occasions was the result greater than 3?

2 Use the information in Question 1 to draw, on separate diagrams:
a a frequency histogram **b** a frequency polygon.

3 **a** For the scores 5, 1, 8, 4, 3, 5, 5, 2, 4, find:
i the range **ii** the mode **iii** the mean **iv** the median.
b Use your frequency distribution table in Question 1 to find:
i the range **ii** the mode **iii** the mean **iv** the median.
c Add a cumulative frequency column to your frequency distribution table in Question 1.
i What is the cumulative frequency of the score 4?
ii How many students scored 3 or less?
iii Does the last figure in your cumulative frequency column equal the total of the frequency column?

4 Use your cumulative frequency table in Question 3 to draw:
b a frequency histogram **b** a frequency polygon.

5 The number of cans of drink sold by a shop each day was as follows.

30	28	42	21	(54)	47	36
37	22	(18)	25	26	43	50
23	29	30	19	28	20	40
33	35	31	27	42	26	44
53	50	29	20	32	41	36
51	46	37	42	27	28	31
29	32	41	36	32	41	35
41	29	39	46	36	36	33
29	37	38	25	27	19	28
47	51	28	47	36	35	40

The highest and lowest scores have been circled.

a Tabulate these results using classes of 16–22, 23–29, 30–36, 37–43, 44–50, 51–57. Make up a table using these column headings: Class, Class centre, Tally, Frequency, Cumulative frequency.

b What was the mean number of cans sold?

c Construct a cumulative frequency histogram and cumulative frequency polygon (or ogive) and find the median class.

d What is the modal class?

e Over how many days was the survey held?

6 a Use the data from the first four rows of Question **5** to construct a stem-and-leaf plot.

b Use the answer to **a** to make this an ordered stem-and-leaf plot.

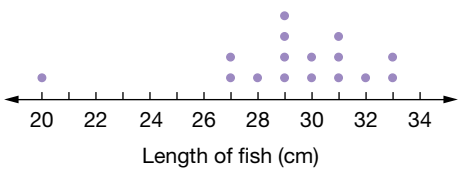
Note: See Chapter 9 for a review of interquartile range, box plots and comparing sets of data.

7 a Find the interquartile range of the scores:
1, 2, 2, 5, 7, 9, 10, 10, 11, 11, 11, 11

b Draw a cumulative frequency polygon using the frequency distribution table below and use it to find the interquartile range of the scores.

x	10	11	12	13	14	15	16	17	18	19
y	2	0	5	4	5	6	5	6	3	4

c The lengths of 16 fish caught were measured. The results are shown on this dot plot. What is the interquartile range?

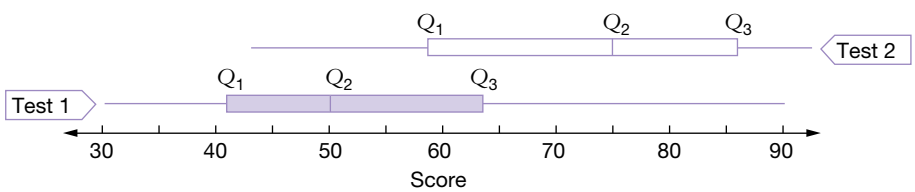


d What is the interquartile range of the times shown in the stem-and-leaf plot?

Time (in days)	
Stem	Leaf
3	8 8
4	0 3 4 5 8
5	1 3 4 4
6	0

8 Draw a box plot for the data in Question **7a**.

9 These box plots were drawn to compare the results of Year 10 on two tests.



a By how much was the median for Test 2 higher than the median of Test 1?

b What was the range and interquartile range of Test 1?

10

Test 1 scores					
12	17	19	12	15	10
9	22	24	11	18	8
25	15	18	20	18	18

Test 2 scores					
21	15	18	7	11	16
20	12	23	12	10	13
12	19	12	14	20	9

a Draw a dot plot for the scores in Test 1.
b Draw a back-to-back stem-and-leaf display to compare the scores in Tests 1 and 2.
c Draw a double box plot to compare the scores in Tests 1 and 2.

1:13 Proportion

Exercise 1:13

P Appendix A A:13A–A:13C

1 Write each rate in its simplest terms.
a 200 km in 4 hours = … km/h
b \$56 for 7 kg = \$…/kg
c 60 litres in $2\frac{1}{2}$ hours = … L/h
d 250 g per 10 cm^3 = … g/cm^3

2 Convert these rates.
a 6 km/h = … m/min
b 120 000 mL/min = … L/s
c 15 000 kg/day = … t/day
d \$100 per hectare = … c/m^2

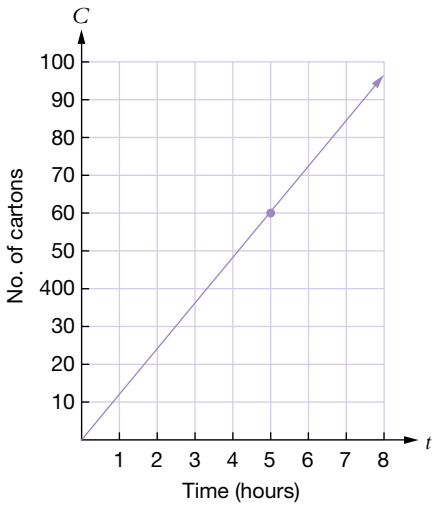
3 For each example below, state whether it is an example of direct or inverse (indirect) proportion.
a The amount of ingredients available and the number of pizzas that can be made.
b The number of people eating 10 pizzas and the time taken to eat them.
c The speed at which a car travels and the distance covered in a given time.
d The speed at which a car travels and the time taken to cover a certain distance.

4 M is directly proportional to t, and when $t = 6$, $M = 30$.
a Given that $M = kt$, find the value of k.
b Find the value of M when $t = 17$.
c Find the value of t when $M = 174$.

5 I need 8 eggs to make cakes for 3 people.
a How many eggs do I need to make cakes for 15 people?
b If I have 3 dozen eggs, for how many people can I make cakes?

6 Travelling at a constant average speed I can travel a distance D in a certain time T. If I travel 120 km in 5 hours:
a Determine an equation for this relationship in the form $D = kT$.
b What is the constant of proportionality?
c How far can I travel in 8 hours?
d How long would it take me to travel 228 km?

7 This graph shows the number of cartons a factory can produce in a given time.



- **a** By determining the gradient of the line, find the constant of proportionality for this example of direct proportion.
- **b** How many cartons could be produced in 22 hours?
- **c** How much time would be needed to produce 200 cartons? Give your answer in hours and minutes.

8 The time taken (t) to clean the windows of a building and the number of window cleaners employed (n) are inversely proportional. If 6 window cleaners can clean the windows in 8 hours:

- **a** Find an equation in the form $t = \frac{k}{n}$ for this relationship.
- **b** What is the constant of proportionality?
- **c** How long would it take 12 cleaners to do the same job?
- **d** How many cleaners are needed to do the job in 3 hours?

9 The time taken (T) for Henry to drive home is inversely proportional to the average speed (S) at which he drives. If his speed is 40 km/h he gets home in 3 hours.

- **a** What is the proportional constant for this relationship?
- **b** How long does he take if he increases his average speed to 50 km/h? (Give your answer in hours and minutes.)
- **c** At what speed must he travel to get home in 2 hours?

10 The frequency of a note produced on a string is inversely proportional to the length of the string. If a string of length 50 cm produces a frequency of 600 Hz:

- **a** what frequency is produced by a string of length 40 cm
- **b** what length of string would produce a frequency of 800 Hz?

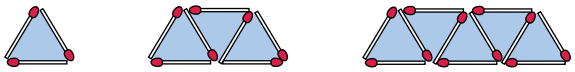
Estimate the total number of people that could stand inside this tree.

ASSIGNMENT 1A Working mathematically

P Appendix B

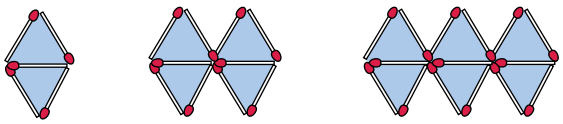
1 Complete a table of values for each matchstick pattern below, and hence find the rule for each, linking the number of coloured triangles (t) to the number of matches (m).

a



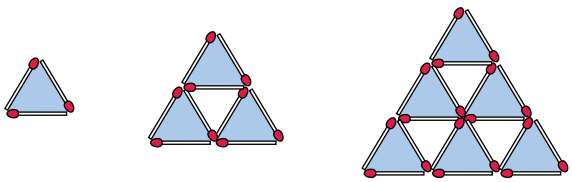
t	1	3	5
m			

b



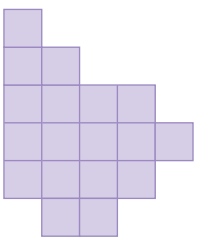
t	2	4	6
m			

c



t	1	3	6
m			

2 Divide this shape into three pieces that have the same shape.



3 Ryan answered all 50 questions in a maths competition in which he received 4 marks for each correct answer but lost one mark for each incorrect answer.

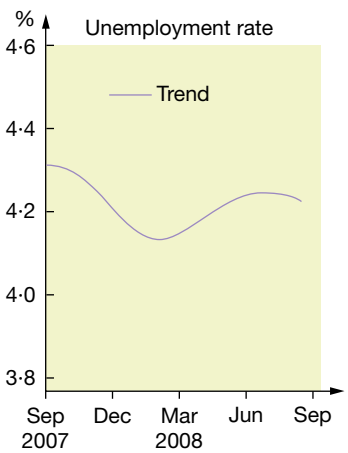
a What is Ryan's score if he answered 47 questions correctly?

b How many answers did he get right if his score was 135?

4 It takes 3 minutes and 15 seconds to join two pieces of pipe. How long would it take to join 6 pieces of pipe into one length?

5 A number of cards can be shared between 4 people with no remainder. When shared between 5 or 6 people, there are two cards left over. If there are fewer than 53 cards, how many are there?

6 From September 2007 to September 2008, the unemployment rate fell from 4·3% to 4·2%.



Source: Australian Bureau of Statistics, Labour Force, Australia, Sep. 2008

a If the number of unemployed in September 2008 was 474 100, what was the size of the total workforce? Answer correct to four significant figures.

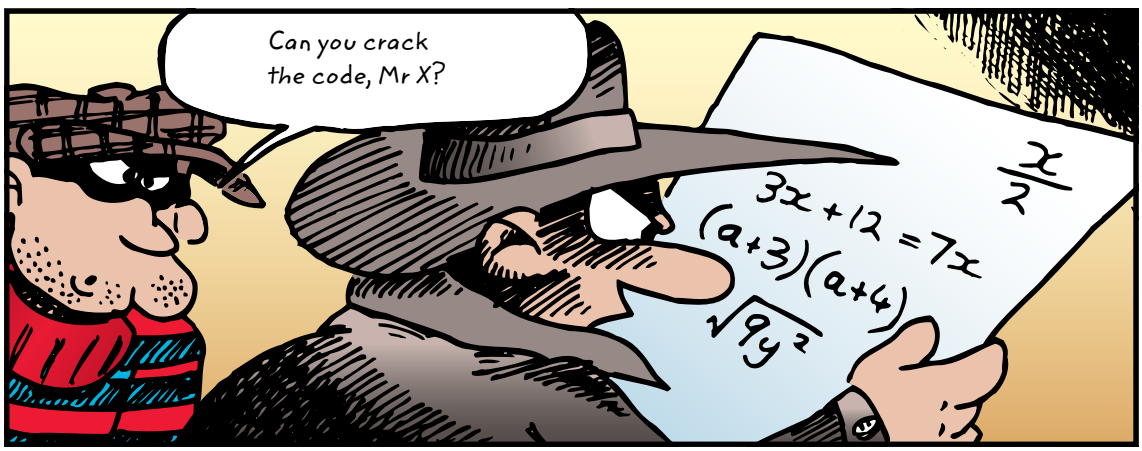
b How many people were unemployed in September 2007, assuming the workforce was the same size as in 2008? Answer correct to four significant figures.

c If the rate of 4·2% is only correct to one decimal place, the rate could really be from 4·15% to 4·25%. How many people does this approximate range of 0·1% represent?

d Find the current unemployment rate and compare with the data shown here.

ALGEBRA AND EQUATIONS

2



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Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** and ***Equations*** [Stage 5.2]

- Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)
- Apply the four operations to algebraic fractions with pronumerals in the denominator (NSW)
- Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)
- Solve simple quadratic equations using a range of strategies (ACMNA241)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

Before starting this chapter it would be beneficial to review the algebra and equations met in Year 9 by completing sections 1:02, 1:05, 1:06 and 1:10.

2:01 Further algebraic fractions

PREP QUIZ 2:01

Rewrite these fractions in their simplest form.

1 $\frac{6}{8}$ **2** $\frac{4a}{5a}$ **3** $\frac{10x}{5xy}$ **4** $\frac{np}{mn}$ **5** $\frac{9p^2}{12pq}$

Simplify these expressions.

6 $\frac{5x}{6} \times \frac{3}{x}$ **7** $\frac{ay}{m} \div \frac{y}{am}$ **8** $\frac{x}{5} + \frac{3x}{5}$ **9** $\frac{n}{6} + \frac{n}{4}$ **10** $\frac{2p}{3} - \frac{p}{2}$

In Year 9 you were shown how to simplify algebraic fractions as well as how to perform the four basic operations.

The Prep quiz above should have reminded you of these skills. In this section we will extend the addition and subtraction of fractions to those with pronumerals in the denominator.

Addition and subtraction

When adding or subtracting two fractions you should remember this rule.

> Rewrite each fraction as two equivalent fractions with a common denominator, then add or subtract the numerators.

WORKED EXAMPLE 1

If the denominators are the same, simply add or subtract the numerators.

a $\frac{2x}{5} + \frac{3x}{5} = \frac{2x + 3x}{5}$

$= \frac{{}^{1}\cancel{5}x}{{}_{1}\cancel{5}}$

$= x$

b $\frac{5}{a} - \frac{3}{a} = \frac{5 - 3}{a}$

$= \frac{2}{a}$

WORKED EXAMPLE 2

If the denominators are different, first find the lowest common multiple (LCM).

a $\frac{x}{4} + \frac{2x}{3} = \frac{x \times 3}{4 \times 3} + \frac{2x \times 4}{3 \times 4}$

$= \frac{3x}{12} + \frac{8x}{12}$

$= \frac{11x}{12}$

b $\frac{5}{2x} + \frac{2}{3x} = \frac{5 \times 3}{2x \times 3} - \frac{2 \times 2}{3x \times 2}$

$= \frac{15}{6x} - \frac{4}{6x}$

$= \frac{11}{6x}$

c $\frac{5n}{6} - \frac{3n}{4} = \frac{10n}{12} - \frac{9n}{12}$

$= \frac{n}{12}$

d $\frac{2a}{n} - \frac{3a}{4n} = \frac{8a}{4n} - \frac{3a}{4n}$

$= \frac{5a}{4n}$

Multiplication and division

When multiplying:
- cancel any common factors
- multiply the numerators together and multiply the denominators together.

When dividing:
- turn the second fraction upside down
- multiply as above (i.e. invert and multiply).

Remembering the index laws can also be useful.

When multiplying, add the indices.
e.g. $a^5 \times a^3 = a^{5+3} = a^8$

When dividing, subtract the indices.
e.g. $a^5 \div a^3 = a^{5-3} = a^2$

WORKED EXAMPLES

1 $\frac{2ab}{3} \times \frac{9}{4b} = \frac{{}^{1}\cancel{2}\cancel{a}^{1}\cancel{b}}{{}_{1}\cancel{3}} \times \frac{{}^{3}\cancel{9}}{{}_{2}\cancel{4}\cancel{b}_{1}}$

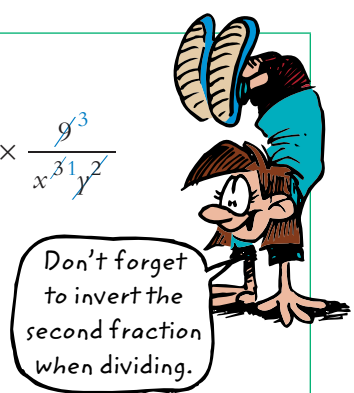
$= \frac{a \times 3}{1 \times 2}$

$= \frac{3a}{2}$

2 $\frac{x^2y^4}{6} \times \frac{9}{x^3y^2} = \frac{\cancel{x^2}\,y^{4\,2}}{{}_{2}\cancel{6}} \times \frac{\cancel{9}^{3}}{\cancel{x^3}^{1}\cancel{y^2}}$

$= \frac{y^2 \times 3}{2 \times x}$

$= \frac{3y^2}{2x}$



3 $\frac{2mp}{5n} \div \frac{8p}{15mn} = \frac{{}^{1}\cancel{2}m\,\cancel{p}^{1}}{{}_{1}\cancel{5}\cancel{n}_{1}} \times \frac{{}^{3}\cancel{15}m\,\cancel{n}^{1}}{{}_{4}\cancel{8}\cancel{p}_{1}}$

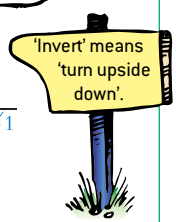
$= \frac{m \times 3m}{1 \times 4}$

$= \frac{3m^2}{4}$

4 $\frac{3x^2}{8y^5} \div \frac{15x^3}{4y} = \frac{{}^{1}\cancel{3}\,\cancel{x^2}}{{}_{2}\cancel{8}\,\cancel{y^5}^{4}} \times \frac{{}^{1}\cancel{4}\,\cancel{y}}{{}_{5}\cancel{15}\,\cancel{x^3}^{1}}$

$= \frac{1 \times 1}{2y^4 \times 5x}$

$= \frac{1}{10xy^4}$



Exercise 2:01

P Foundation worksheet 2:01A & B
Simplifying algebraic fractions

1 Simplify the following.

a $\frac{3a}{2}+\frac{a}{2}$ b $\frac{3x}{5}-\frac{2x}{5}$ c $\frac{a}{3}+\frac{4a}{3}$ d $\frac{9m}{10}-\frac{3m}{10}$

e $\frac{x}{4}+\frac{y}{4}$ f $\frac{5a}{3}-\frac{2b}{3}$ g $\frac{2}{a}+\frac{3}{a}$ h $\frac{7}{x}+\frac{1}{x}$

i $\frac{3}{y}-\frac{2}{y}$ j $\frac{9}{m}-\frac{1}{m}$ k $\frac{5a}{x}+\frac{2a}{x}$ l $\frac{2x}{y}-\frac{3x}{y}$

m $\frac{5}{3n}+\frac{7}{3n}$ n $\frac{3}{2x}-\frac{1}{2x}$ o $\frac{8a}{5b}+\frac{2a}{5b}$ p $\frac{7m}{4x}-\frac{3m}{4x}$

2 Reduce each of these expressions to its simplest form.

a $\frac{x}{3}+\frac{x}{5}$ b $\frac{a}{2}+\frac{a}{5}$ c $\frac{y}{3}-\frac{y}{4}$ d $\frac{m}{2}-\frac{m}{4}$

e $\frac{2a}{3}+\frac{a}{2}$ f $\frac{5x}{3}+\frac{2x}{4}$ g $\frac{3n}{8}-\frac{n}{4}$ h $\frac{4p}{5}-\frac{3p}{10}$

i $\frac{x}{4}+\frac{y}{3}$ j $\frac{2a}{3}-\frac{3b}{2}$ k $\frac{3m}{5}-\frac{n}{2}$ l $\frac{k}{6}-\frac{2l}{4}$

m $\frac{2}{x}+\frac{4}{3x}$ n $\frac{1}{3a}+\frac{2}{4a}$ o $\frac{7}{2m}-\frac{2}{5m}$ p $\frac{5}{8x}-\frac{1}{2x}$

q $\frac{2a}{3x}+\frac{3a}{2x}$ r $\frac{x}{3m}-\frac{2x}{m}$ s $\frac{5m}{2n}+\frac{3m}{4n}$ t $\frac{2x}{3a}+\frac{y}{4a}$

3 Simplify these products.

a $\frac{x}{2}\times\frac{y}{3}$ b $\frac{a}{4}\times\frac{b}{3}$ c $\frac{m}{2}\times\frac{m}{5}$ d $\frac{a}{4}\times\frac{a}{10}$

e $\frac{3}{a}\times\frac{4}{m}$ f $\frac{2}{x}\times\frac{1}{y}$ g $\frac{1}{p}\times\frac{4}{p}$ h $\frac{1}{n}\times\frac{1}{3n}$

i $\frac{p}{q}\times\frac{x}{y}$ j $\frac{2}{a}\times\frac{a}{4}$ k $\frac{m}{5}\times\frac{10}{n}$ l $\frac{3x}{5}\times\frac{2}{9x}$

m $\frac{ab}{3}\times\frac{2}{b}$ n $\frac{x}{y}\times\frac{y}{x}$ o $\frac{6m}{5a}\times\frac{15a}{2m}$ p $\frac{8x}{5p}\times\frac{2a}{3x}$

4 Simplify these divisions.

a $\frac{m}{2}\div\frac{m}{4}$ b $\frac{n}{3}\div\frac{n}{5}$ c $\frac{5a}{3}\div\frac{2a}{9}$ d $\frac{x}{5}\div\frac{3x}{10}$

e $\frac{5}{a}\div\frac{2}{a}$ f $\frac{3}{2m}\div\frac{1}{3m}$ g $\frac{a}{b}\div\frac{2a}{b}$ h $\frac{3x}{5y}\div\frac{x}{10y}$

i $\frac{a}{b}\div\frac{x}{y}$ j $\frac{2p}{3q}\div\frac{8p}{9q}$ k $\frac{10k}{3n}\div\frac{2k}{9n}$ l $\frac{a}{2}\div\frac{a}{3}$

m $\frac{xy}{2}\div\frac{y}{4}$ n $\frac{b}{2}\div\frac{ab}{6}$ o $\frac{xy}{c}\div\frac{y}{cx}$ p $\frac{9a}{b}\div\frac{4a}{3b}$

Simplify these expressions.

5 **a** $\frac{a}{3} \times \frac{12}{5a}$ **b** $\frac{2}{p} \times \frac{p}{3}$ **c** $\frac{15}{x} \div 5$ **d** $3b \div \frac{6}{b}$

e $\frac{xy}{z} \times \frac{2z}{x}$ **f** $\frac{ab}{c} \div \frac{a}{c}$ **g** $\frac{9m}{2} \times \frac{4m}{3}$ **h** $\frac{2x}{y} \div \frac{x}{2y}$

i $\frac{4}{pq} \times \frac{p}{q}$ **j** $\frac{3}{a} \times \frac{2}{b}$ **k** $\frac{4ab}{x} \times \frac{xy}{2ac}$ **l** $\frac{9bc}{2a} \div \frac{6b}{4a}$

m $\frac{2}{x} \times \frac{x}{3} \times \frac{9}{4}$ **n** $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b}$ **o** $\frac{8bc}{3a} \times \frac{9a}{b} \times \frac{1}{4c}$ **p** $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

6 **a** $\frac{y^5}{3} \times \frac{6}{y^2}$ **b** $\frac{a^4}{8} \times \frac{6}{a^6}$ **c** $\frac{m^2}{4n^5} \times \frac{10n^6}{5m^3}$

d $\frac{x^2y^3}{12} \times \frac{6}{x^3y^2}$ **e** $\frac{a^3b^2}{3a} \times \frac{6b}{b^5}$ **f** $\frac{5m^4}{3n^3} \times \frac{12n^6}{15m^2}$

g $\frac{4a^3}{x^2z^3} \times \frac{6x^5z^3}{3a^2}$ **h** $\frac{k^3n^2}{5k^5} \times \frac{10kn}{n^5}$ **i** $\frac{pq^2}{qr^3} \times \frac{pr^6}{p^2q^2r^2}$

7 **a** $\frac{3x^5}{2} \div \frac{9x^3}{8}$ **b** $\frac{5m^2}{6} \div \frac{10m^4}{9}$ **c** $\frac{3}{2y^3} \div \frac{5}{4y^6}$

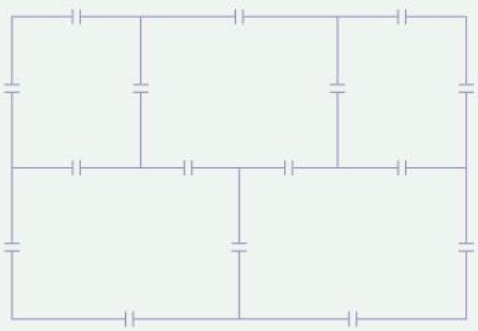
d $\frac{4x^6}{3y^5} \div \frac{5x^2}{6y}$ **e** $\frac{5a^3}{3b^4} \div \frac{10a^2}{6b}$ **f** $\frac{p^6}{4q^5} \div \frac{5p^2}{6q^4}$

g $\frac{x^6y^2}{z^3} \div \frac{x^4}{yz^5}$ **h** $\frac{fg^3}{3h^5} \div \frac{f^3g}{6h^2}$ **i** $\frac{5ax^3}{4by^5} \div \frac{15b^2x^2}{8ay^7}$

FUN SPOT 2:01 AN A-MAZE-ING HOUSE!

The diagram shows the plan of a house with five rooms. There is a doorway between adjacent rooms as well as nine doors opening to the outside.

Is it possible to walk through each and every door of the house without going through any door twice? You can start anywhere, inside or outside the house.



2:02 Expanding and factorising

PREP QUIZ 2:02

Expand: **1** $3(a + 5)$ **2** $m(m - 6)$ **3** $2y(3y + z)$ **4** $-5(4x - 1)$

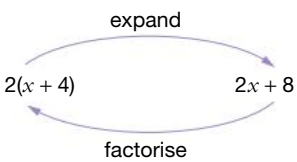
Simplify: **5** $p^3 \times p^4$ **6** $5x^2 \times 3x^5$ **7** $3m^2n^4 \times 4mn^3$

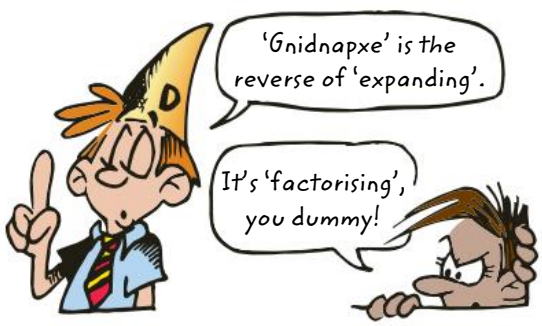
What is the highest common factor (HCF) of:

8 12 and 20 **9** $5ab$ and $10bc$ **10** x^2y^5 and x^3y^4

The Prep quiz above should have reminded you about expanding algebraic expressions. Each number inside the grouping symbols is multiplied by the term outside.

If we expand the expression $2(x + 4)$, we obtain $2x + 8$. To factorise $2x + 8$, we simply reverse this procedure. We notice that 2 is the highest common factor of $2x$ and 8, so 2 is written outside the parentheses and the remainder is written inside the parentheses: $2x + 8 = 2(x + 4)$.





This section is a review of these two skills that are needed throughout this chapter. The worked examples below also include expressions that involve skills used when multiplying with indices.

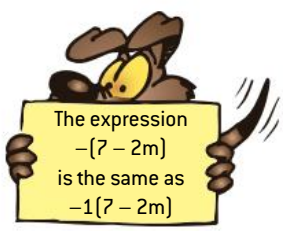
WORKED EXAMPLE 1

Expand, and simplify, each expression.

a $x(x + 5) = x \times x + x \times 5$
$= x^2 + 5x$

b $5a(3a - 2b) = 5a \times 3a - 5a \times 2b$
$= 15a^2 - 10ab$

c $3p(p + 4) - 5(2p - 3) = 3p \times p + 3p \times 4 - 5 \times 2p - 5 \times (-3)$
$= 3p^2 + 12p - 10p + 15$
$= 3p^2 + 2p + 15$



d $y^2(y^3 + 5) = y^5 + 5y^2$

e $4n^2m^3(2n^3 - 5m^4) = 8n^5m^3 - 20n^2m^7$

f $w^3(w^2 - 4) - (w^3 + 2) = w^5 - 4w^3 - w^3 - 2$
$= w^5 - 5w^3 - 2$

WORKED EXAMPLE 2

Factorise these expressions by taking out the highest common factor.

a $9a + 12b = 3 \times 3a + 3 \times 4b$ HCF = 3
$= 3(3a + 4b)$

b $10x^2 - 5xy = 5x \times 2x - 5x \times y$ HCF = $5x$
$= 5x(2x - y)$

c $6m^2 - 9m + 3mn = 3m(2m - 3 + n)$

d $-p^2 + 5pq = -p \times p - (-p) \times 5q$ HCF = $-p$
$= -p(p - 5q)$

e $x^5 + 6x^2 = x^2(x^3 + 6)$ HCF = x^2

f $4m^5n^3 - 6m^2n^6 = 2m^2n^3(2m^3 - 3n^3)$ HCF = $2m^2n^3$

g $3x^2y^3 - 12x^3y^4 + 9x^4y^2 = 3x^2y^2(y - 4xy^2 + 3x^2)$ HCF = $3x^2y^2$

Exercise 2:02

P Foundation worksheet 2:02A Grouping symbols
Foundation worksheet 2:02B Common factors

1 Expand the following.

a $a(a + 7)$	**b** $x(x - 1)$	**c** $y(2y + 7)$	**d** $5p(p - 4)$
e $6z(3 + 7z)$	**f** $2a(3a - 1)$	**g** $4k(2k + 3m)$	**h** $8m(n - 3m)$
i $-2(x + 5)$	**j** $-7(y - 5)$	**k** $-(m + 4)$	**l** $-4(3k - 1)$
m $-p(p + 1)$	**n** $-x(x - y)$	**o** $-2z(3z + 5)$	**p** $-ab(a - b)$

2 Simplify:

a $2(a + 3) + 5a + 2$	**b** $3(x + 5) + 7x - 8$	**c** $5(y - 2) + 3y + 7$
d $4(a - 1) + 6a - 5$	**e** $3(p + 2) - 2p + 4$	**f** $10(m + 3) - 11m - 15$
g $5a + 6 + 2(a + 7)$	**h** $2x + 7 + 5(x - 1)$	**i** $7n - 4 + 3(n - 1)$
j $4h - 1 + 7(h + 2)$	**k** $6x + 2(x + 1) + 5$	**l** $4y + 6(y + 2) - 10$
m $3a + 10 - 2(a + 1)$	**n** $10m + 4 - 5(m + 4)$	

3 Simplify each expression by expanding the grouping symbols and then collecting like terms.

a $3(x - 2) - 2(x + 1)$	**b** $5(y + 2) + 3(y + 4)$	**c** $2(a - 1) + 5(a - 1)$
d $8(m - 3) + 5(m + 2)$	**e** $4(3x + 2) + 5(x - 4)$	**f** $6(x + 7) + 2(2x - 1)$
g $5(x - 7) - 3(x - 4)$	**h** $6(m + 1) - 3(m + 2)$	**i** $9(a + 5) - 7(a - 3)$
j $5(n - 5) - 3(n + 7)$	**k** $x(x + 3) + 3(x + 1)$	**l** $a(a + 3) + 7(a - 3)$
m $m(m + 3) - 4(m + 3)$	**n** $t(t - 5) - 4(t - 5)$	**o** $a(a + 2b) + a(2a + b)$
p $x(x - y) + y(x + y)$		

4 Expand and simplify:

a $x^2(x^2 - 1)$	**b** $a^3(5 - a^2)$	**c** $a^2(5a - a^3)$
d $x(x^2 + y)$	**e** $m(7 - m^2)$	**f** $y(y^2 - xy)$
g $3a^2(2a^3 + 3a)$	**h** $5x(3x^2 - x)$	**i** $2m^3(n^2 - m^2)$
j $x(5x^2 - 3x + 7)$	**k** $x^2(2x^2 + 7x - 14)$	**l** $y(y^2 - 7y - 1)$
m $a^2b^3(a^3 - b^4)$	**n** $2x^2y(2x^3 - 3y^4)$	**o** $6nm^3(2n^3m - 5nm^4)$
p $x^3(x^2 - 3) + 4(x^3 + 2)$	**q** $m^3(m^3 - 1) - m^4(m^2 + 2)$	**r** $z^5(z^2 - 4) - (z^5 - 1)$

5 Factorise fully the following expressions.

a $9x + 6$	**b** $10 + 15a$	**c** $4m - 6n$
d $x^2 + 7x$	**e** $2a^2 - 3a$	**f** $12y - 6y^2$
g $ab - bx$	**h** $st - s$	**i** $4ab + 10bc$
j $-4m + 6n$	**k** $-x^2 - 3x$	**l** $-15a + 5ab$
m $3x + x^2 - ax$	**n** $ax + ay + az$	**o** $4m - 8n + 6p$
p $5ab - 15ac + 10ad$	**q** $x^2 - 7x + xy$	**r** $a(a + 3) - (a + 3)$

6 Using the rule for multiplying with indices, factorise the following expressions.

a $x^3 + 4x^2$	**b** $a^5 - 6a^3$	**c** $p^6 + p^4$	**d** $y^5 - 3y^2$
e $2x^3 + 6x^4$	**f** $5a^5 - 10a^4$	**g** $6p^6 + 3p$	**h** $5y - 3y^2$
i $x^3y^2 + 4x^2y$	**j** $a^5b^3 - a^3b^5$	**k** $p^6q^2 + p^4q^3$	**l** $y^5z - y^2z^4$
m $3m^3n + 9m^2n^2$	**n** $8a^5b^3 - 6a^3b$	**o** $9p^6q^5 + 12p^3q^5$	**p** $10xy^5 - 4x^4y^2$

7 Factorise these expressions.

a $x^2y^3 + x^3y^2 + x^4y^4$	**b** $3a^2b^3 - 6a^3b^5 + 9a^4b^4$	**c** $8m^3n^2 + 12m^5n + 4m^4n^2$
d $5x^2z^2 - 10x^3z^3 + 15x^4z^4$	**e** $x^2y^3z^4 - x^3y^4z^2 + x^4y^2z^3$	**f** $9a^2b^3c^5 + 3a^5b^4c^3 + 6a^4b^2c^4$

CHALLENGE 2:02 GROUPING IN PAIRS

For some algebraic expressions there may not be a factor common to every term. For example, there is no factor common to every term in the expression:

$3x + 3 + mx + m$

The first pair of terms have a common factor of 3 and the second pair of terms have a common factor of m. So:

$3x + 3 + mx + m = 3(x + 1) + m(x + 1)$

Now, it can be seen that $(x + 1)$ is a common factor for each term.

$3(x + 1) + m(x + 1) = (x + 1)(3 + m)$

Therefore:

$3x + 3 + mx + m = (x + 1)(3 + m)$

The original expression has been factorised by grouping the terms in pairs.

Exercises

1 Complete the factorisation of each expression.

a $5(a + 1) + b(a + 1)$	**b** $4(y - 3) + x(y - 3)$
c $p(q + 7) - 5(q + 7)$	**d** $m(p + q) + n(p + q)$
e $a(a - b) + b(a - b)$	**f** $2x(y + 3) - (y + 3)$

$ab + ac + bd + cd = a(b + c) + d(b + c)$
$= (b + c)(a + d)$

2 Factorise these expressions.

a $6x + 6 + ax + a$	**b** $8p - 8q + mp - mq$	**c** $ab + 3bc + 5a + 15c$
d $x^2 + xy + xz + yz$	**e** $ab + b + 4a + 4$	**f** $12m^2 + 16m + 3mn + 4n$
g $mn - m + n - 1$	**h** $x^3 + x^2 + x + 1$	**i** $a^2 + bc + ac + ab$

These will be treated further in Chapter 10.

2:03 Binomial products

PREP QUIZ 2:03

Simplify:
1 $5x + 7x$ **2** $2a - a$
3 $x^2 + 3x - 5x + 3$

Expand:
4 $2(x + 5)$ **5** $x(x - 2)$
6 $-3(a + 1)$ **7** $-y(5 - y)$

Expand and simplify:
8 $x(x + 1) + 3(x + 1)$ **9** $5(a + 5) - a(a + 5)$
10 $2x(3x - 2) - 5(3x + 2)$

A binomial expression contains two terms, e.g. $2x - 7$ or $a + b$.

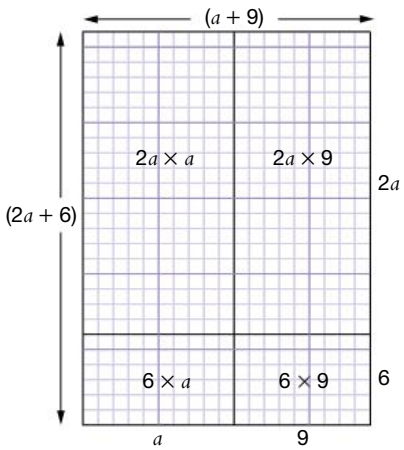
A binomial product is the product of two such expressions, e.g. $(2x - 7)(a + 5)$.

Multiplying binomial expressions

The expansion of binomial products may be demonstrated by considering the area of a rectangle with dimensions $(2a + 6)$ and $(a + 9)$.

- The area of the whole rectangle must be equal to the sum of the four smaller areas.
- Area $= (2a + 6)(a + 9)$
  $= 2a(a + 9) + 6(a + 9)$
  $= 2a^2 + 18a + 6a + 54$
  $= 2a^2 + 24a + 54$
- We can see that the product of two binomials has four terms. Often two of these may be added together to simplify the answer.

$(2a + 6)(a + 9)$ by areas



WORKED EXAMPLES

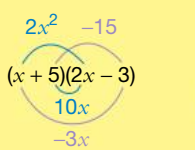
1 $(a + 2)(b + 4) = a(b + 4) + 2(b + 4)$
$= ab + 4a + 2b + 8$

2 $(a - 2)(a + 7) = a(a + 7) - 2(a + 7)$
$= a^2 + 7a - 2a - 14$
$= a^2 + 5a - 14$

3 $(x + 2y)(2x + y) = x(2x + y) + 2y(2x + y)$
$= 2x^2 + xy + 4xy + 2y^2$
$= 2x^2 + 5xy + 2y^2$

4 $(1 - x)(x - 3) = 1(x - 3) - x(x - 3)$
$= x - 3 - x^2 + 3x$
$= 4x - x^2 - 3$

You should notice that each term in the first binomial is multiplied by each term in the second.



$= 2x^2 + 10x - 3x - 15$
$= 2x^2 + 7x - 15$

$(a + b)(c + d) = a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

Exercise 2:03

1 Expand the following binomial products.

a $(a+2)(b+3)$	**b** $(x+1)(y+4)$	**c** $(m+7)(n+5)$	**d** $(a+3)(x+2)$
e $(p+5)(q+4)$	**f** $(2x+1)(y+3)$	**g** $(a+6)(3p+2)$	**h** $(4x+1)(2y+3)$
i $(3a+1)(2b-7)$	**j** $(7x+5)(2p+1)$	**k** $(5p+3)(x-4)$	**l** $(2x+y)(a+2b)$

2 Expand the following and collect the like terms.

a $(a+2)(a+3)$	**b** $(x+1)(x+5)$	**c** $(n+3)(n+4)$	**d** $(p+2)(p+5)$
e $(m+1)(m-3)$	**f** $(y+7)(y-2)$	**g** $(x+1)(x-6)$	**h** $(t+2)(t-4)$
i $(x-2)(x-4)$	**j** $(n-7)(n-1)$	**k** $(a-6)(a-3)$	**l** $(x-10)(x-9)$
m $(y-11)(y+7)$	**n** $(a-2)(a+1)$	**o** $(x-8)(x-8)$	**p** $(m-9)(m-2)$
q $(a-3)(a+3)$	**r** $(x-7)(x+3)$	**s** $(y+12)(y+5)$	**t** $(a-8)(a+8)$
u $(q+5)(q+5)$	**v** $(x-1)(x-9)$	**w** $(t+3)(t+10)$	**x** $(k-8)(k+11)$

3 Find these products and simplify.

a $(a+3)(2a+1)$	**b** $(2x+1)(x+2)$	**c** $(3m+2)(m+5)$	**d** $(y+3)(4y+1)$
e $(2x+1)(2x+3)$	**f** $(3n+2)(2n+1)$	**g** $(2x+3)(4x+3)$	**h** $(5t+2)(2t+3)$
i $(2x-2)(5x-1)$	**j** $(8p+1)(3p-2)$	**k** $(5m-2)(2m-5)$	**l** $(3q+1)(7q-2)$
m $(3x+2)(6x-2)$	**n** $(2n+3)(2n-3)$	**o** $(8y-1)(8y+1)$	**p** $(3k-2)(5k-3)$
q $(7p-1)(7p-1)$	**r** $(3x-1)(5x-3)$	**s** $(5x+4)(5x+4)$	**t** $(9y-4)(3y+2)$
u $(5p+2)(p-7)$	**v** $(10q-1)(q-10)$	**w** $(4a+3)(3a+4)$	**x** $(7p+5)(7p-5)$

4 Expand and simplify:

a $(3+x)(4+x)$	**b** $(5-a)(2-a)$	**c** $(7+m)(1-m)$
d $(3-n)(3+n)$	**e** $(4+y)(y+5)$	**f** $(x-7)(5-x)$
g $(9+k)(k+10)$	**h** $(2a+1)(3+a)$	**i** $(3n+1)(7-2n)$
j $(x+y)(x+2y)$	**k** $(2n+m)(n+2m)$	**l** $(a-b)(2a+3b)$
m $(2p-q)(2p+q)$	**n** $(3x+y)(2x-5y)$	**o** $(3a+2b)(2a+3b)$
p $(9w-5x)(9w-5x)$		

2:04 Factorising quadratic trinomials

PREP QUIZ 2:04

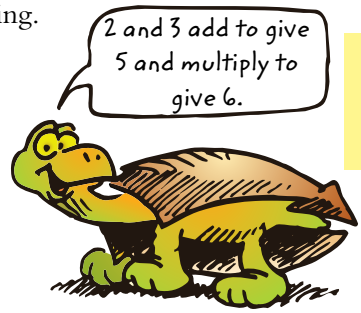
Expand:

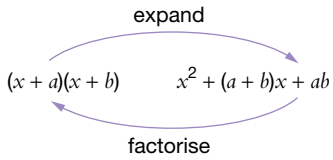
1 $(x+2)(x+3)$	**2** $(a-1)(a+3)$	**3** $(m-7)(m-2)$
4 $(x+5)^2$	**5** $(a-2)^2$	

Find two numbers a and b where:

6 $a+b=5$ and $ab=6$	**7** $a+b=9$ and $ab=20$
8 $a+b=-2$ and $ab=-15$	**9** $a+b=3$ and $ab=-4$
10 $a+b=7$ and $ab=-18$	

- An expression with three terms is called a *trinomial*.
- If the highest power of the pronumeral is 2, then an expression with three terms is called a *quadratic trinomial*.
  e.g. $x^2 + 3x - 4$
- Factorising is the reverse of expanding.
  $(x + a)(x + b) = x^2 + ax + bx + ab$
  $= x^2 + (a + b)x + ab$
  Using this result, to factorise $x^2 + 5x + 6$ we look for two values a and b, where $a + b = 5$ and $ab = 6$.
  These numbers are 2 and 3,
  so: $x^2 + 5x + 6 = (x + 2)(x + 3)$





$(x+2)(x+3) = (x+3)(x+2)$
because
$a \times b = b \times a$

WORKED EXAMPLES

Factorise:

1 $x^2 + 7x + 10$ **2** $m^2 - 6m + 8$ **3** $y^2 + y - 12$ **4** $x^2 - 9x - 36$ **5** $3y^2 + 15y - 72$

If $x^2 + 7x + 10 = (x + a)(x + b)$ then $a + b = 7$ and $ab = 10$.

Solutions

1 $2 + 5 = 7$
$2 \times 5 = 10$
$\therefore x^2 + 7x + 10$
$= (x + 2)(x + 5)$

2 $(-2) + (-4) = -6$
$(-2) \times (-4) = 8$
$\therefore m^2 - 6m + 8$
$= (m - 2)(m - 4)$

3 $(-3) + 4 = 1$
$(-3) \times 4 = -12$
$\therefore y^2 + y - 12$
$= (y - 3)(y + 4)$

4 $3 + (-12) = -9$
$3 \times (-12) = -36$
$\therefore x^2 - 9x - 36$
$= (x + 3)(x - 12)$

5 $3y^2 + 15y - 72$
$= 3(y^2 + 5y - 24)$
$(-3) + 8 = 5$
$(-3) \times 8 = -24$
$\therefore 3(y^2 + 5y - 24)$
$= 3(y - 3)(y + 8)$

Exercise 2:04

P Foundation worksheet 2:04
Factorising trinomials

1 Factorise each of these trinomials.

a $x^2 + 4x + 3$ **b** $x^2 + 3x + 2$ **c** $x^2 + 6x + 5$
d $x^2 + 7x + 6$ **e** $x^2 + 9x + 20$ **f** $x^2 + 10x + 25$
g $x^2 + 12x + 36$ **h** $x^2 + 10x + 21$ **i** $x^2 + 9x + 18$
j $x^2 + 14x + 40$ **k** $x^2 + 15x + 54$ **l** $x^2 + 13x + 36$
m $x^2 - 4x + 4$ **n** $x^2 - 12x + 36$ **o** $x^2 - 7x + 12$
p $x^2 - 9x + 20$ **q** $x^2 + 2x - 3$ **r** $x^2 + x - 12$
s $x^2 + 4x - 12$ **t** $x^2 + 7x - 30$ **u** $x^2 - x - 2$
v $x^2 - 10x - 24$ **w** $x^2 - 7x - 30$ **x** $x^2 - x - 56$

2 Factorise:

a $a^2 + 6a + 8$	**b** $m^2 + 9m + 18$	**c** $y^2 + 13y + 42$
d $p^2 + 7p + 12$	**e** $x^2 + 12x + 20$	**f** $n^2 + 17n + 42$
g $s^2 + 21s + 54$	**h** $a^2 + 18a + 56$	**i** $x^2 - 3x - 4$
j $a^2 - 2a - 8$	**k** $p^2 - 5p - 24$	**l** $y^2 + y - 6$
m $x^2 + 7x - 8$	**n** $q^2 + 5q - 24$	**o** $m^2 + 12m - 45$
p $a^2 + 18a - 63$	**q** $y^2 + 6y - 55$	**r** $x^2 - 2x + 1$
s $k^2 - 5k + 6$	**t** $x^2 - 13x + 36$	**u** $a^2 - 22a + 72$
v $p^2 + 22p + 96$	**w** $q^2 - 12q - 45$	**x** $m^2 - 4m - 77$

3 Choose the correct set of factors for each trinomial.

a $x^2 - 3x + 2$	**A** $(x + 2)(x + 1)$	**B** $(x + 2)(x - 1)$	**C** $(x - 2)(x + 1)$	**D** $(x - 2)(x - 1)$
b $x^2 - 5x - 6$	**A** $(x - 6)(x + 1)$	**B** $(x + 6)(x - 1)$	**C** $(x - 2)(x + 3)$	**D** $(x + 2)(x - 3)$
c $x^2 - 7x + 12$	**A** $(x - 2)(x - 6)$	**B** $(x - 12)(x - 1)$	**C** $(x - 4)(x - 3)$	**D** $(x + 4)(x + 3)$
d $x^2 - 5x - 24$	**A** $(x + 8)(x - 3)$	**B** $(x - 8)(x + 3)$	**C** $(x - 6)(x + 4)$	**D** $(x + 6)(x - 4)$

4 State whether each factorisation is correct or incorrect. If it is incorrect, give a reason for your answer.

a $x^2 - 6x - 8 = (x - 4)(x - 2)$	**b** $x^2 - x - 6 = (x - 3)(x + 2)$
c $x^2 - x + 6 = (x - 3)(x - 2)$	**d** $x^2 - 6x - 16 = (x - 2)(x + 8)$
e $x^2 + 7x - 60 = (x - 5)(x + 12)$	**f** $x^2 - 14x + 15 = (x - 15)(x - 1)$

5 Factorise by first taking out a common factor (see Worked Example **5**).

a $2x^2 + 6x + 4$	**b** $3x^2 - 6x - 9$	**c** $5x^2 - 10x - 40$
d $2x^2 + 16x + 32$	**e** $3x^2 - 30x - 33$	**f** $3x^2 + 21x + 36$
g $4a^2 - 12a - 40$	**h** $2n^2 + 8n + 6$	**i** $5x^2 - 30x + 40$
j $3x^2 - 21x + 36$	**k** $3a^2 - 15a - 108$	**l** $5x^2 + 15x - 350$

Further factorisation of quadratic expressions is covered in Chapter 10.

2:05 Quadratic equations

PREP QUIZ 2:05

Factorise:	**1** $x^2 - 3x$	**2** $x^2 + 7x$	**3** $x^2 + 3x + 2$	**4** $x^2 - 4x - 12$
	5 $x^2 + x - 20$	**6** $x^2 - 8x + 7$		
Solve for x:	**7** $3x = 12$	**8** $7x = 0$	**9** $x - 4 = 0$	**10** $x + 6 = 0$

In a quadratic equation the highest power of the pronumeral is 2.

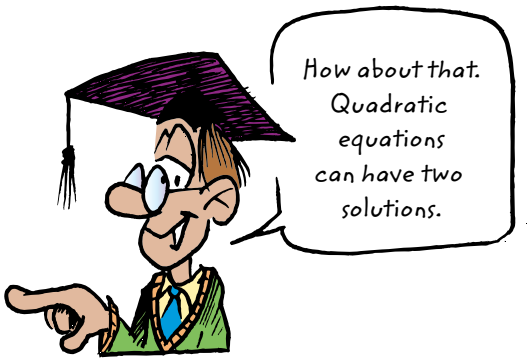
e.g. $x^2 = 9$ $5x^2 - 8 = 0$ $x^2 - 6x = 0$ $x^2 - 4x + 3 = 0$

The first two equations above can be solved directly but the second two require the expression to be factorised.

The term quadratic comes from *quadraticus*, which is the Latin word for square.

Equations of the form $ax^2 = c$

To solve $x^2 = 9$ we find the square root of both sides of the equation.
The square root of 9 is 3 or −3.



So if $x^2 = 9$, then $x = \pm 3$.
The equation has two solutions: $x = 3$ and $x = -3$

WORKED EXAMPLES

Solve these equations.

1 $x^2 - 16 = 0$ **2** $a^2 - 5 = 0$ **3** $3m^2 = 10$ **4** $k^2 + 4 = 0$

Solutions

1 $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm\sqrt{16}$
$\therefore x = \pm 4$

2 $a^2 - 5 = 0$
$a^2 = 5$
$\therefore a = \pm\sqrt{5}$
(as decimal approximations
$a = 2{\cdot}236$ or $-2{\cdot}236$)

3 $3m^2 = 10$
$m^2 = \frac{10}{3}$
$\therefore m = \pm\sqrt{\frac{10}{3}}$
(as decimal approximations
$m = 1{\cdot}826$ or $-1{\cdot}826$)

4 $k^2 + 4 = 0$
$k^2 = -4$
The square of a real number is positive.
So this equation has no real solutions!

Equations of the form $ax^2 + bx + c = 0$

To solve $x^2 - 4x + 3 = 0$ we need to factorise the algebraic expression and then use the Null Factor Law.

> If $p \times q = 0$, then at least one of p and q must be zero.

$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$

So either $x - 3 = 0$ or $x - 1 = 0$
$\therefore x = 3$ or $x = 1$
Substituting these values into the original equation will show that they are both solutions.

Note: For all the quadratic equations in this section, $a = 1$. These quadratic equations are said to be '**monic**'.
Equations with values for a other than 1 will be treated in Chapter 10.

WORKED EXAMPLES

1 This equation is factorised.

$(a-5)(a+8)=0$

$a-5=0$ or $a+8=0$

$\therefore a=5$ or $a=-8$

2 Factorise before solving.

$n^2+2n-15=0$

$(n+5)(n-3)=0$

$n+5=0$ or $n-3=0$

$\therefore n=-5$ or $n=3$

Solutions can be checked by substitution.

For Worked Example 2:

Substituting $n=-5$

$n^2+2n-15=0$

LHS $=(-5)^2+2(-5)-15$

$=25-10-15$

$=0$

Substituting $n=3$

$n^2+2n-15=0$

LHS $=(3)^2+2(3)-15$

$=9+6-15$

$=0$

(LHS = Left-hand side of the equation)

3 Factorise before solving.

$y^2-4y=0$

$y(y-4)=0$

$y=0$ or $y-4=0$

$\therefore y=0$ or $y=4$

4 Before solving, all the terms are gathered to one side of the equation, letting the other side be zero.

$x^2=x+6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x-3=0$ or $x+2=0$

$\therefore x=3$ or $x=-2$

To solve a quadtratic equation:

- gather all the terms to one side of the equation
- factorise
- solve the two resulting simple equations.

Exercise 2:05

P Foundation worksheet 2:05 Quadratic equations

1 Solve these equations.

a $x^2=25$ **b** $x^2=81$ **c** $x^2=1$ **d** $x^2=100$

e $x^2-36=0$ **f** $x^2-64=0$ **g** $x^2-121=0$ **h** $x^2-400=0$

2 Solve these equations, leaving your answers in square root form.

a $x^2=6$ **b** $n^2=3$ **c** $y^2-10=0$ **d** $k^2-15=0$

e $4x^2=8$ **f** $5p^2=3$ **g** $8q^2-10=0$ **h** $7z^2-5=0$

3 Solve these equations, evaluating your answers correct to three decimal places.

a $x^2=2$ **b** $2a^2=7$ **c** $c^2-12=0$ **d** $8g^2-3=0$

4 **a** Solve the equation $x^2+9=0$. **b** Give a reason for your answer in **a**.

5 Find the two solutions for each equation. Check your answers by substitution.

a $x(x-5)=0$ **b** $y(y+7)=0$ **c** $5n(n-9)=0$

d $(x-2)(x-1)=0$ **e** $(x-7)(x-3)=0$ **f** $(a-5)(a-2)=0$

g $(y+3)(y+4)=0$ **h** $(t+3)(t+2)=0$ **i** $(x+9)(x+5)=0$

j $(a-6)(a+6)=0$ **k** $(y+8)(y-7)=0$ **l** $(n+1)(n-1)=0$

6 After factorising the left-hand side of each equation, solve the following.

a $x^2 + 3x = 0$ b $m^2 - 5m = 0$ c $y^2 + 2y = 0$
d $x^2 + 3x + 2 = 0$ e $a^2 - 5a + 6 = 0$ f $y^2 + 12y + 35 = 0$
g $a^2 - 10a + 21 = 0$ h $x^2 - 10x + 16 = 0$ i $m^2 - 11m + 24 = 0$
j $h^2 + h - 20 = 0$ k $x^2 + 2x - 35 = 0$ l $a^2 - 4a - 45 = 0$
m $x^2 + x - 56 = 0$ n $y^2 - 8y + 7 = 0$ o $a^2 + 9a - 10 = 0$

7 a Solve the equation $x^2 - 4x + 4 = 0$.
b How many solutions are there for this quadratic equation?
c When will a quadratic equation have only one solution?

8 Solve:

a $x^2 + 4x + 4 = 0$ b $m^2 - 6m + 9 = 0$ c $z^2 + 2z + 1 = 0$
d $k^2 - 8k + 16 = 0$ e $a^2 - 10a + 25 = 0$ f $y^2 + 20y + 100 = 0$

9 Solve this mixed set of quadratic equations.

a $x^2 - 4 = 0$ b $x^2 - 4x = 0$ c $x^2 - 4x - 12 = 0$
d $m^2 - 9m - 10 = 0$ e $m^2 - 9 = 0$ f $m^2 - 9m = 0$
g $a^2 - 6a = 0$ h $a^2 - 6a - 16 = 0$ i $a^2 - 6 = 0$
j $h^2 + 16 = 0$ k $h^2 + 16h = 0$ l $h^2 + 16h - 17 = 0$

10 Gather all the terms to one side of the equation and then solve.

a $x^2 = 3x$ b $m^2 = 8m$ c $t^2 = -5t$
d $x^2 = 5x - 4$ e $a^2 = 2a + 15$ f $y^2 = 3y - 2$
g $m^2 = 9m - 18$ h $n^2 = 7n + 18$ i $h^2 = 4h + 32$
j $x^2 + x = 2$ k $y^2 + 2y = 3$ l $x^2 - 7x = -10$
m $y^2 + 3y = 18$ n $t^2 + 3t = 28$ o $y^2 + 2y = 15$
p $x^2 + 4x = x - 2$ q $a^2 - 2a = 3a - 6$ r $z^2 + 10 = z + 10$
s $q^2 + 5 = 1 - 5q$ t $x^2 + x - 10 = 25 - x$ u $60 - 15a = 10 - a^2$

FUN SPOT 2:05 — WHAT DID THE COMPUTER SAY AT LUNCHTIME?

Answer each question and write the letter for that question in the box above the correct answer.

Solve these quadratic equations for x.

B $x^2 - 49 = 0$
A $x^2 + 5x = 0$
I $x^2 + 3x - 18 = 0$
A $x^2 + x - 30 = 0$
Y $x^2 - 12x + 20 = 0$

N $(x - 3)(x - 1) = 0$
A $x^2 + 5x + 4 = 0$
T $x^2 - 5x + 6 = 0$
E $x^2 - 3x - 10 = 0$

E $(x + 8)(x + 7) = 0$
C $x^2 - 3x + 2 = 0$
H $x^2 + 5x - 14 = 0$
V $x^2 + 10x + 24 = 0$

1, 2	−6, 5	1, 3	−6, 3	−7, 2	0, −5	−6, −4	−7, −8	−1, −4	−7, 7	2, 10	2, 3	5, −2

MATHS TERMS 2

algebraic expression
- a group of terms that are joined by addition or subtraction signs

binomial
- an algebraic expression consisting of two terms,
  e.g. $2x + 4$, $3x - 2y$

cancel
- to simplify a fraction by dividing the numerator and denominator by a common factor,
  e.g. $\frac{{}^{3}\cancel{21}\,x\,y \div 7x}{{}_{2}\cancel{14}\,x \div 7x} = \frac{3y}{2}$

coefficient
- the number that multiplies a pronumeral in an algebraic expression,
  e.g. in $3x - 5y$, the coefficient of x is 3 and the coefficient of y is -5

denominator
- the bottom of a fraction

expand
- to remove grouping symbols by multiplying the terms inside the grouping symbols by the term or terms outside

factorise
- to write an expression as a product of its factors
- the reverse of expanding

grouping symbols
- brackets [ ]
- parentheses ( )

like terms
- terms that have identical pronumeral parts,
  e.g. $7x$ and $10x$
  $5a^2b$ and $-3a^2b$
- only like terms may be added or subtracted together. This is called 'collecting like terms'

numerator
- the 'top' of a fraction

product
- the result of multiplying terms or expressions together

quadratic equation
- an equation in which the highest power of the unknown pronumeral is 2,
  e.g. $x^2 - 16 = 0$, $x^2 + 5x + 6 = 0$
- a quadratic equation may have two, one or no solutions

quadratic trinomial
- expressions such as $x^2 + 4x + 3$, which can be factorised as $(x + 3)(x + 1)$
- the highest power of the variable is 2

substitution
- the replacement of a pronumeral with a numeral in a formula or expression

trinomial
- an algebraic expression consisting of three terms

DIAGNOSTIC TEST 2 ALGEBRA AND EQUATIONS

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Simplify these expressions. 2:01

a $\frac{2x}{3} + \frac{x}{3}$ b $\frac{6a}{7} - \frac{4a}{7}$ c $\frac{5}{x} + \frac{2}{x}$ d $\frac{9}{2m} - \frac{3}{2m}$

2 Simplify: 2:01

a $\frac{2x}{3} + \frac{x}{5}$ **b** $\frac{5a}{6} - \frac{3a}{4}$ **c** $\frac{5}{3x} + \frac{1}{4x}$ **d** $\frac{5}{2n} - \frac{2}{3n}$

3 Simplify these products. 2:01

a $\frac{2a}{3} \times \frac{b}{5}$ **b** $\frac{2x}{3} \times \frac{3x}{8}$ **c** $\frac{2n}{5} \times \frac{10}{3n}$ **d** $\frac{3p}{4q} \times \frac{8t}{9p}$

4 Simplify these divisions. 2:01

a $\frac{2x}{5} \div \frac{8x}{15}$ **b** $\frac{7}{5m} \div \frac{3}{10m}$ **c** $\frac{2x}{3y} \div \frac{4x}{9y}$ **d** $\frac{9t}{4w} \div \frac{6t}{w}$

5 Simplify these expressions that involve indices. 2:01

a $\frac{y^4}{6} \times \frac{3}{y^2}$ **b** $\frac{a^4b}{5} \times \frac{10}{a^3b^2}$ **c** $\frac{x^3y^4}{z^3} \times \frac{2z^2}{x^2y^5}$

6 Simplify: 2:01

a $\frac{3x^2}{2} \div \frac{15x^3}{4}$ **b** $\frac{3a^2}{5b^5} \div \frac{9a^3}{10b}$ **c** $\frac{3m^4n}{5p^6} \div \frac{6m^3n^2}{10p^4}$

7 Expand the following: 2:02

a $x(x + 7)$ **b** $5m(2m - 7)$ **c** $-4(3y + 5)$ **d** $-3t(4t - 5w)$

8 Expand and simplify: 2:02

a $5(n + 4) - 2n + 1$ **b** $2(3x - 5) + 3(x + 3)$ **c** $5(3a - 1) - 2(4a - 3)$

9 Expand: 2:02

a $x^2(x^4 + 3)$ **b** $-2y^2(y^3 - 4)$ **c** $3n^4m^3(4n^2 - m^4)$

10 Factorise: 2:02

a $8a + 10b$ **b** $6p^2 - 12pq$ **c** $-x^2 + 5xz$

11 Factorise completely: 2:02

a $w^5 + 5w^3$ **b** $8x^6 + 6x^8$ **c** $4m^4n^3 - 6m^3n^4$

12 Expand and simplify: 2:03

a $(x + 4)(x + 3)$ **b** $(a - 3)(2a - 1)$ **c** $(2 - y)(3 + y)$ **d** $(2x + y)(x - 3y)$

13 Factorise these trinomials. 2:04

a $x^2 + 7x + 12$ **b** $x^2 - 5x + 6$ **c** $a^2 - 3a - 10$ **d** $n^2 + n - 20$

14 Solve these equations, giving the answers correct to three decimal places if necessary. 2:05

a $x^2 = 49$ **b** $a^2 = 7$ **c** $m^2 - 16 = 0$ **d** $3y^2 - 5 = 0$

15 Solve: 2:05

a $(x - 4)(x + 5) = 0$ **b** $z(z - 3) = 0$ **c** $(a + 6)(a - 6) = 0$

16 Factorise the left-hand side of each equation and then solve. 2:05

a $t^2 - 5t = 0$ **b** $x^2 - 6x + 8 = 0$ **c** $y^2 + y - 56 = 0$

ASSIGNMENT 2A Chapter review

1 Simplify each expression.

a $\frac{2x}{5} + \frac{x}{5}$ **b** $\frac{2a}{3} - \frac{5a}{9}$

c $\frac{7x}{8} + \frac{x}{6}$ **d** $\frac{6}{n} - \frac{5}{n}$

e $\frac{2}{3y} + \frac{3}{2y}$ **f** $\frac{4a}{5w} - \frac{2a}{3w}$

g $\frac{2p}{3} \times \frac{q}{4}$ **h** $\frac{2a}{3b} \times \frac{6b}{5a}$

i $\frac{2x}{5} \div \frac{4x}{5}$ **j** $\frac{7n}{5m} \div \frac{3n}{10m}$

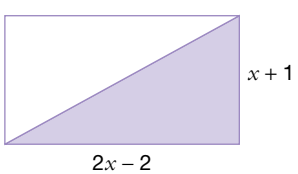
k $\frac{k^4t}{12} \times \frac{9}{k^3t^2}$ **l** $\frac{4a^3b^4}{c^3} \times \frac{2c^2}{3a^2b^5}$

m $\frac{4x^4}{5y^3} \div \frac{6x^3}{15y}$ **n** $\frac{8p^4q^7}{5q^4} \div \frac{6p^3q^2}{10p}$

2 Expand, and simplify where possible.

a $(x - 1)(x + 2)$
b $5x + 3(x - 1)$
c $2(x + 3) - 2x - 3$
d $(2x + 1)(x - 7)$
e $(x + 5)(x - 5)$
f $(3x + 2)^2$
g $x(x - 3) + 2(x + 1)$
h $(2 - x)(3 - x)$
i $(x + y)(y - x)$
j $(2x - y)^2$
k $x^3(x^3 - 1)$
l $3x^2y^4(2xy^3 + 5x^3y)$

3 Find an expression for the shaded area of this rectangle. Expand and simplify your answer.



4 Take out the highest common factor to factorise these expressions.

a $m^3 + 4m^2$
b $4n^5 - 6n$
c $a^3b^2 - a^2b^3$
d $9p^6q^5 + 6p^5q^6 - 12p^4q^4$

5 Factorise these expressions.

a $10x^2 - 5x$
b $a^2 + ab - ac$
c $y^2 + 9y + 20$
d $n^2 - n - 12$
e $8 - 6k + k^2$
f $a(a + 4) - 5(a + 4)$
g $x^3 + 5x^2 + 6x$

6 Solve these equations.

a $a^2 - 36 = 0$
b $4t^2 - 9 = 0$
c $m^2 + 25 = 0$
d $(x + 2)(x - 3) = 0$
e $(n - 5)(n + 5) = 0$
f $(x - 3)^2 = 0$
g $5w^2 - 10w = 0$
h $x^2 - 7x + 12 = 0$
i $y^2 - 4y - 45 = 0$
j $15 + 8p + p^2 = 0$

7 **a** Rearrange $10 + 3y - y^2 = 0$ in the form $0 = y^2 - 3y - 10$, and then solve the equation.

b Solve the equation:

$42 - m - m^2 = 0$

8 Rearrange each equation and solve.

a $y^2 = 4y + 21$
b $n^2 + 2n = 80$
c $z + 110 = z^2$
d $x^2 + 5x = 2x + 70$

ASSIGNMENT 2B Working mathematically

1

Move three dots in the diagram on the left to obtain the diagram on the right.

2 A ladder hangs over the side of a ship. The rungs in the ladder are each 2·5 cm thick and are 18 cm apart. The fifth rung from the bottom of the ladder is just above the water level. If the tide is rising at a rate of 15·5 cm per hour, how many rungs will be under water in 3 hours?

3 A set of Australian coins consists of a 10c, 20c, 50c, $1 and $2 coin. How many different sums of money can be obtained by taking any three of the coins?

4 Roger started a trip into the country between 8 am and 9 am, when the hands of the clock were together. He arrived at his destination between 2 pm and 3 pm, when the hands of the clock were exactly 180° apart. For how long did he travel?

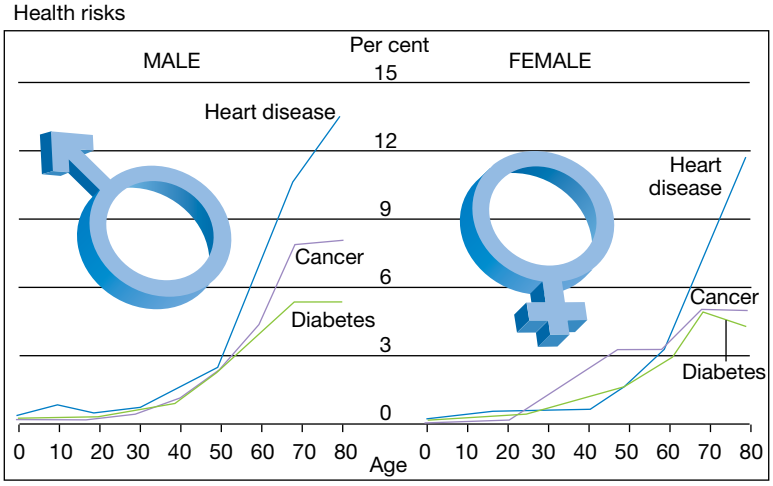
5 What is the smallest whole number that, when multiplied by 7, will give you an answer consisting entirely of 8s?

6 **a** From the data in the graph below, who has the greater chance of having heart disease: a 60-year-old woman or a 60-year-old man?

b Who has the greater chance of having cancer: a 50-year-old woman or a 50-year-old man?

c Which of the three diseases reveals the greatest gender difference for the age range of 20 to 50 years?

d Would the number of 80-year-old men suffering from heart disease be greater or less than the number of 80-year-old women suffering from heart disease? Give a reason for your answer.



Source: Australian Institute of Health and Welfare

ASSIGNMENT 2C Cumulative revision

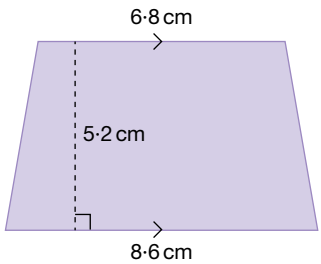
1 **a** Find 15% of \$125. 1:01D
b What percentage is \$25 of \$125?
c Decrease a price of \$125 by 30%.
d 15% discount of a price is equal to \$24. What is the full price?

2 Simplify these ratios. 1:01E
a $20 : 45$ **b** $3{\cdot}5 : 4{\cdot}2$ **c** $2\frac{1}{3} : 1\frac{1}{2}$

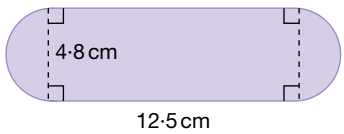
3 Simplify the following expressions. 1:02
a $5x - 2y - x + y$ **b** $6ab \times 3ac$ **c** $10ax \div 5a$ **d** $\frac{2a}{3} + \frac{3a}{5}$

4 A card is drawn from a standard pack of 52 playing cards. 1:03
What is the probability the card is:
a red **b** a club **c** a Jack **d** the 7 of spades?

5 Find the area of each shape. 1:04
a



b

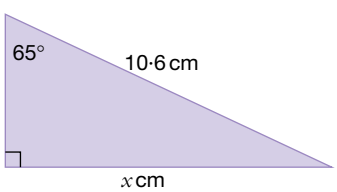


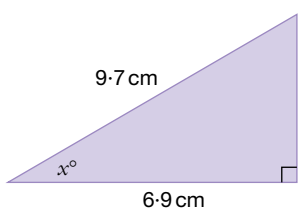
6 Evaluate: 1:05
a $5^2 \times 2^5$ **b** $4^5 \div 4^4$ **c** 4^{-2} **d** $(2^3)^{-1}$

7 Solve these equations: 1:06
a $9m - 4 = 14$ **b** $5x - 7 = 3x - 1$ **c** $6(2a + 3) = 2(5a + 11)$

8 Yvonne is paid an hourly rate of \$28.40 for a 36 hour week. The first 6 hours overtime are paid at time-and-a-half; after that extra hours worked are paid double-time. Find Yvonne's wage for a week in which she works 45 hours. 1:07

9 For these triangles, find: 1:11
a the value of x to one decimal place **b** the value of x to the nearest degree.

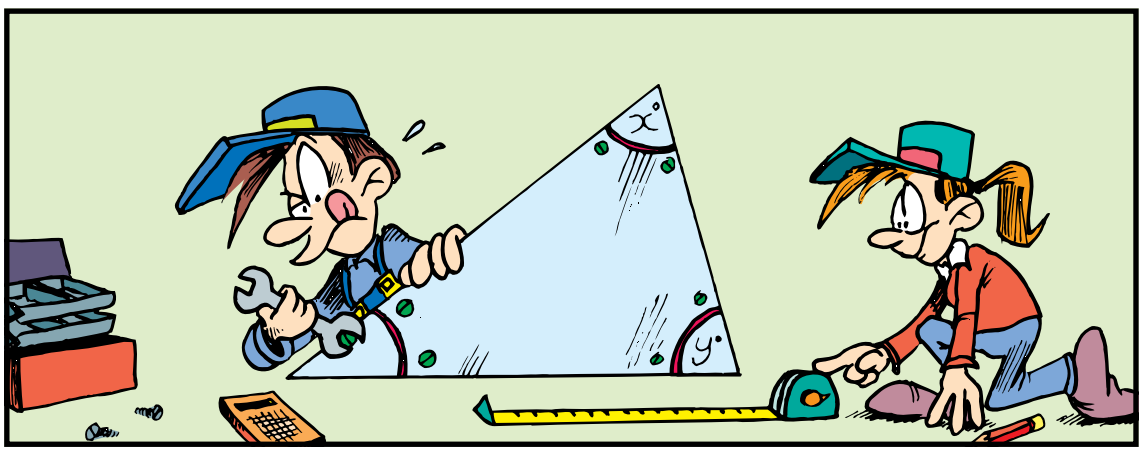




10 For the set of scores 3 5 4 7 5 4 8 3 4, find the: 1:12
a range **b** mode **c** median **d** mean **e** Q_1 **f** Q_3
g interquartile range.

PROPERTIES OF GEOMETRICAL FIGURES

3



Contents

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3:02 Congruent triangle proofs

3:03 Deducing properties of the special triangles and quadrilaterals

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3:04 Polygons

GeoGebra activity 3:04A Convex and concave polygons

Investigation 3:04A The angle sum of a polygon

GeoGebra activity 3:04B The angle sum of a polygon

Investigation 3:04B The exterior angle sum of a convex polygon

GeoGebra activity 3:04C The exterior angle sum of a convex polygon

Investigation 3:04C Regular polygons and tessellations

Fun spot 3:04 The game of Hex

Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Properties of Geometrical Figures*** [Stage 5.2]

- Formulate proofs involving congruent triangles and angle properties (ACMMG243)
- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

Working Mathematically

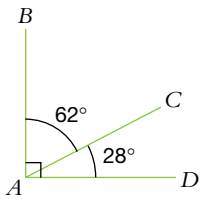
- Problem Solving
- Reasoning
- Understanding
- Fluency
- Communicating

3:01 Deductive reasoning in numerical exercises

Throughout Stage 4 many geometrical facts concerning angles, triangles and quadrilaterals were discovered. A summary of these facts follows.

Adjacent angles

- have a common vertex (or point), A
- have a common arm, AC
- lie on opposite sides of this common arm



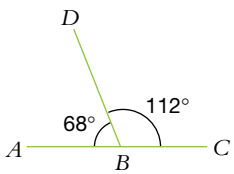
If two adjacent angles add to 90°, then together they form a right angle.

e.g. $28° + 62° = 90°$

The complementary angles are adjacent so $\angle BAD$ is a right angle.

If two adjacent angles add to 180°, then together they form a straight angle.

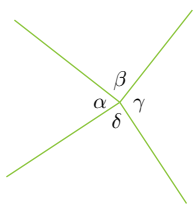
e.g. $68° + 112° = 180°$



The supplementary angles are adjacent so ABC is a straight line.

If a series of adjacent angles completes a revolution, the angles must add to 360°.

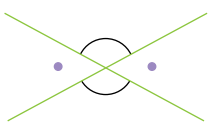
We say that the angle sum at the point is 360°.



Angle sum = 360°

Vertically opposite angles

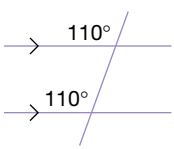
When two straight lines cross, two pairs of vertically opposite angles are formed. Vertically opposite angles are equal.

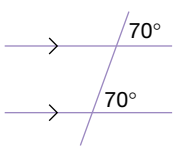


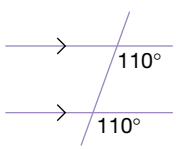
Vertically opposite angles are equal.

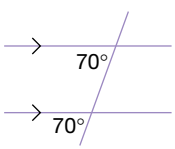
Corresponding angles

If a transversal cuts two parallel lines, then corresponding angles are formed. Corresponding angles are equal.







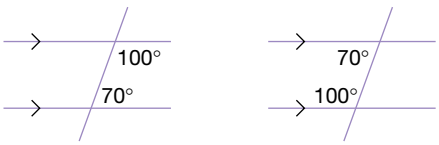


Alternate angles

If a transversal cuts two parallel lines, then alternate angles are formed. Alternate angles are equal.

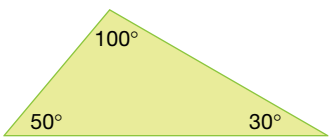
Co-interior angles

If a transversal cuts two parallel lines, then co-interior angles are formed. Co-interior angles are supplementary.



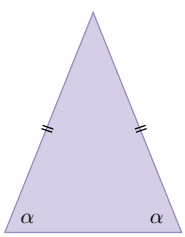
Angle sum of a triangle

In any triangle, the angles add to 180°.



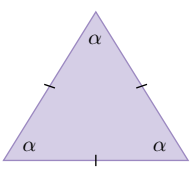
Isosceles triangles

- two equal sides
- two equal angles
- one axis of symmetry



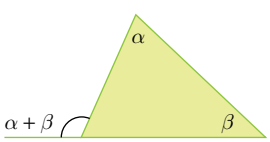
Equilateral triangles

- three equal sides
- three equal angles
- three axes of symmetry



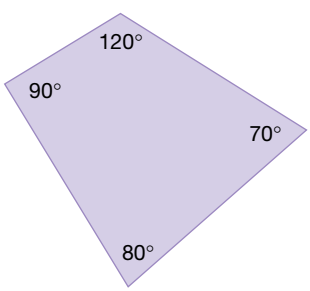
Exterior angle of a triangle

The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



Angle sum of a quadrilateral

In any quadrilateral, the angles add to 360°.



In the following exercises you will be asked to find the sizes of unknown sides and angles, and to give a reason for each statement that you make, using the correct terminology. Reasons are usually abbreviated but take care not to abbreviate them to such an extent that the meaning is lost.

Remember that problems can often be solved in more than one way and that usually one method is more efficient than another.

Some commonly used abbreviations and symbols are shown in the following tables.

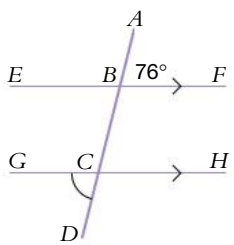
Term	
adjacent	adj.
alternate	alt.
angles	∠s
co-interior	co-int.
complementary	comp.
congruent	cong't
corresponding	corr.
is congruent to	≡
is simliar to	|||
is parallel to	||

Term	
is perpendicular to	⊥
isosceles	isos.
equilateral	equi.
triangle	Δ
parallelogram	par'm
point	pt
quadrilateral	quad.
straight	str.
supplementary	supp.
vertically opposite	vert. opp.

3:01A Exercises using parallel lines

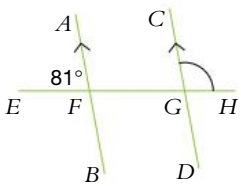
WORKED EXAMPLES

1



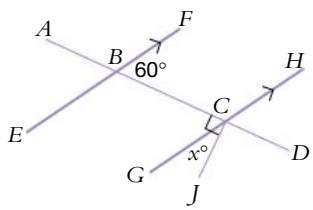
$EF \parallel GH$
Find the size of $\angle GCD$.
Give reasons.

2



$AB \parallel CD$
Find the size of $\angle CGH$.
Give reasons.

3



$EF \parallel GH$
Find the size of x.
Give reasons.

Solutions

1 $\angle EBC = 76°$ (vert. opp. ∠s)
$\angle GCD = 76°$ (corr. ∠s, $EF \parallel GH$)

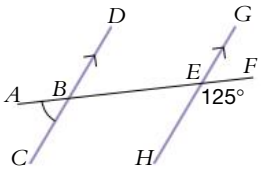
2 $\angle FGC = 81°$ (corr. ∠s, $AB \parallel CD$)
$\angle CGH = 99°$ (adj. supp. ∠s)

3 $\angle BCG = 60°$ (alt. ∠s, $EF \parallel GH$)
$x + 60 = 90$ (comp. ∠s)
$x = 30$

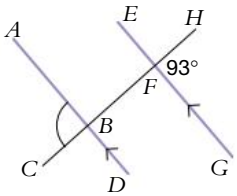
Exercise 3:01A

1 Find the size of $\angle ABC$ in each diagram. Give reasons.

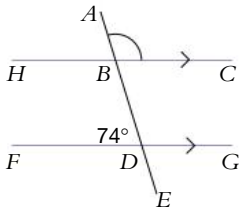
a



b

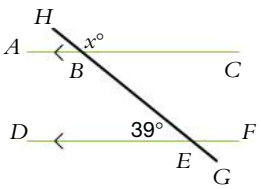


c

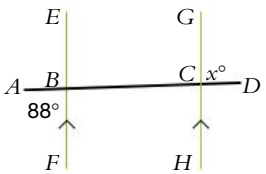


2 Find the value of x in each diagram. Give reasons.

a



b



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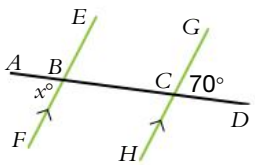

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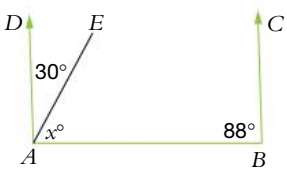

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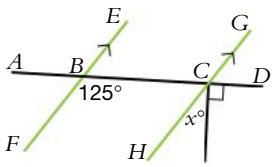

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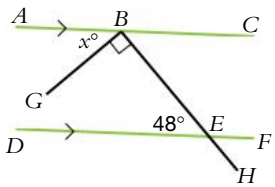
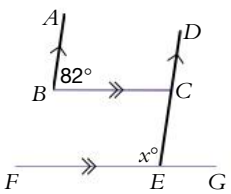


3 Find the value of x in each diagram. Give reasons.

a



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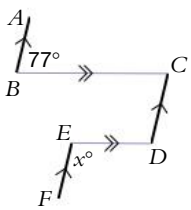

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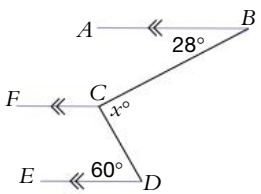

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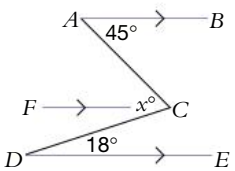

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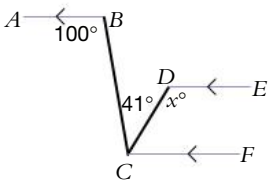

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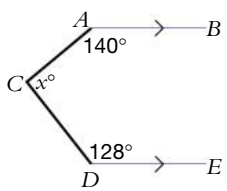

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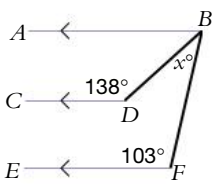

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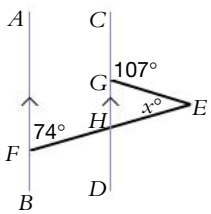

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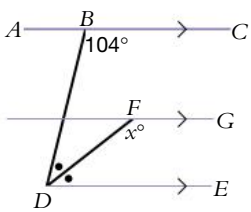
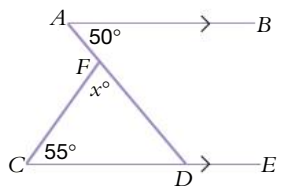


$\angle BDF = \angle FDE$

3:01B Exercises using triangles

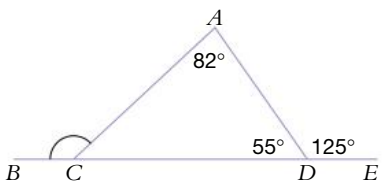
WORKED EXAMPLES

1



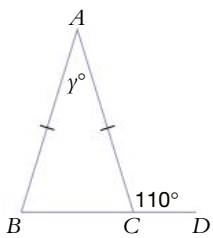
$AB \parallel CE$
Find the size of x.
Give reasons.

2



Find the size of $\angle BCA$.
Give reasons.

3



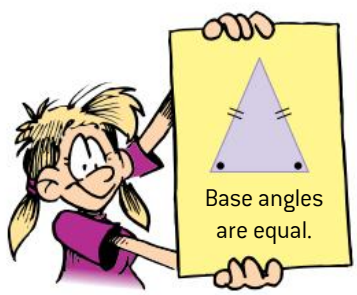
Find the value of y.

Solutions

1
$\angle FDC = 50°$ (alt. $\angle$s, $AB \parallel CE$)
$x + 55 + 50 = 180$ ($\angle$ sum of Δ)
$\therefore\ x = 75$

2
$\angle ADC = 55°$ (adj. supp. $\angle$s)
$\angle BCA = 82° + 55°$ (ext. $\angle$ of Δ)
$\therefore\ \angle BCA = 137°$

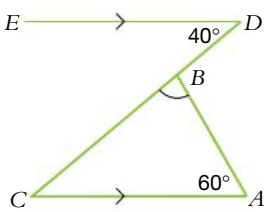
3
$\angle ACB = 70°$ (adj. supp. $\angle$s)
$\therefore\ \angle ABC = 70°$ (base $\angle$s of isos. Δ)
$y + 70 + 70 = 180$ (angle sum of Δ)
$\therefore\ y = 40$



Exercise 3:01B

1 Find the size of $\angle ABC$ in each diagram. Give reasons.

a



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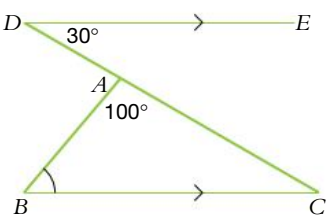

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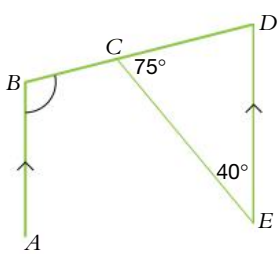
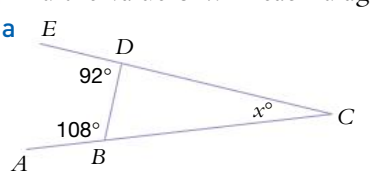


2 Find the value of x in each diagram. Give reasons.

a



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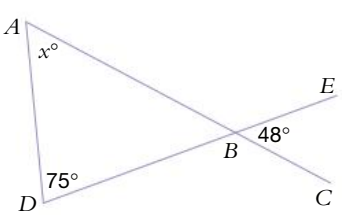

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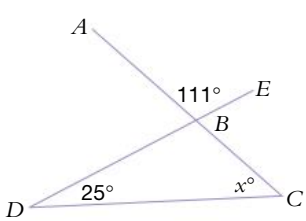
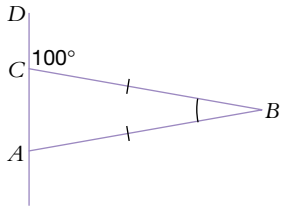


3 Find the size of $\angle ABC$ in each diagram. Give reasons.

a



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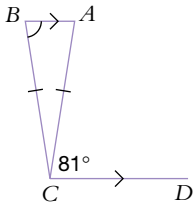

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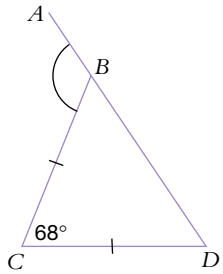
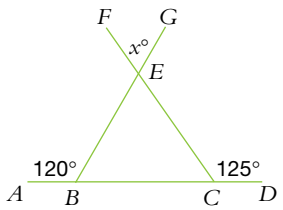


4 Find the value of x in each diagram. Give reasons.

a



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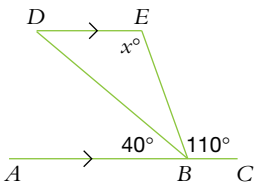

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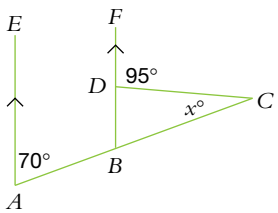

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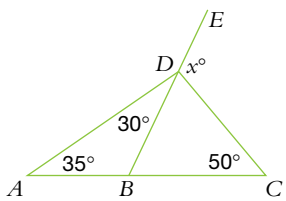

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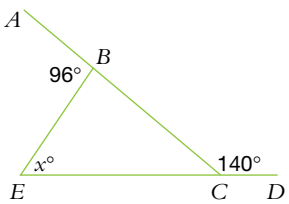

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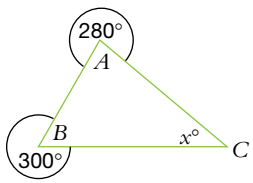
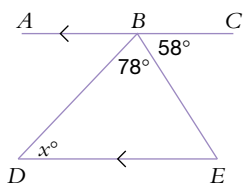


5 Find the value of x in each diagram. Give reasons.

- Copy the diagram.
- Write the sizes of as many angles as you can.
- Look for a connection.

a



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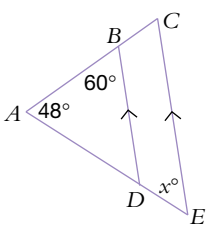

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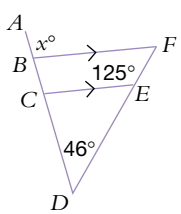

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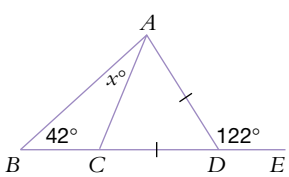

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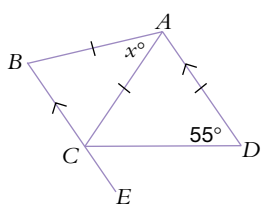
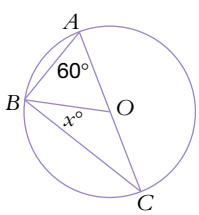


$AB = AC = AD$

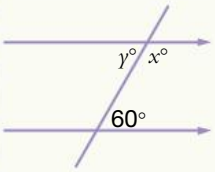
f



O is the centre.

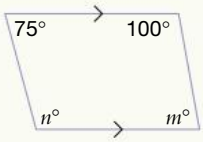
3:01C Exercises using quadrilaterals

PREP QUIZ 3:01C



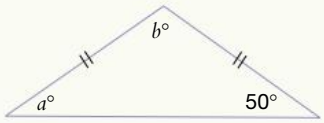
Find the value of:

1 x **2** y



Find the value of:

3 m **4** n

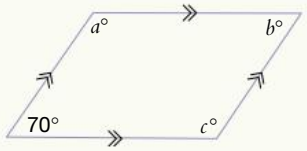


Find the value of:

5 a **6** b

For the parallelogram shown, find the value of:

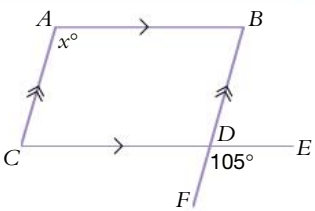
7 a **8** b **9** c



10 Complete: The angle sum of a quadrilateral is …

WORKED EXAMPLES

1

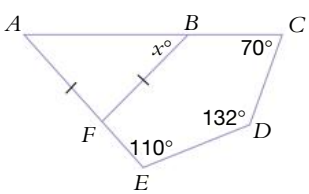


$ABDC$ is a parallelogram.
Find the size of x. Give reasons.

Solution

$\angle BDC = x°$ (opp. $\angle$s of a par'm)
$\angle BDC = 105°$ (vert. opp. $\angle$s)
$\therefore\ x = 105$

2

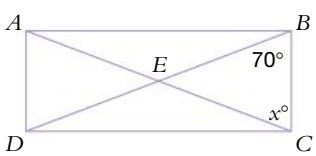


$AF = BF$
Find the size of x. Give reasons.

Solution

$\angle EAC = 360° - (70° + 132° + 110°)$ ($\angle$ sum of quad.)
$= 48°$
$\therefore\ x = 48$ (base $\angle$s of isos. Δ)

3



$ABCD$ is a rectangle.
Find the value of x. Give reasons.

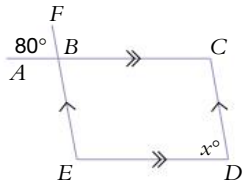
Solution

$EB = EC$ (diagonals of a rectangle bisect each other and are equal)
$\therefore\ x = 70$ (base $\angle$s of isos. ΔBEC)

Exercise 3:01C

1 Find the value of x in each diagram. Give reasons.

a



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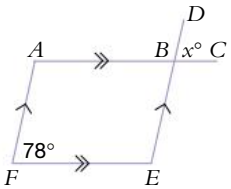

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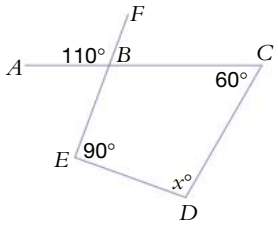

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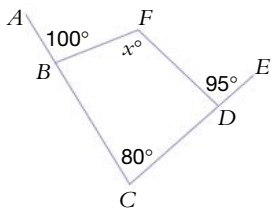

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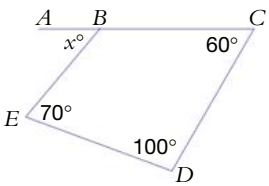

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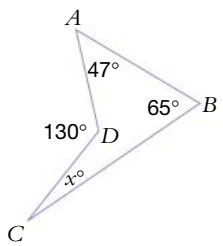
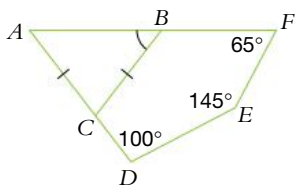


2 Find the value of $\angle ABC$ in each diagram. Give reasons.

a



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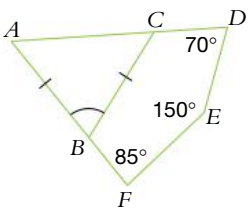

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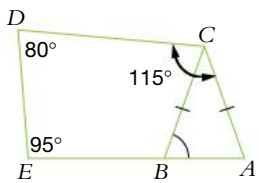

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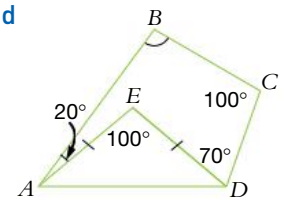

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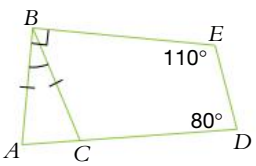

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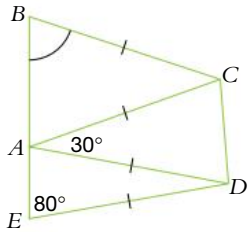
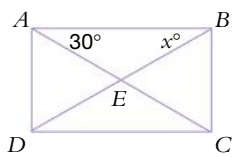


3 $ABCD$ is a rectangle. Find the value of x. Give reasons.

a



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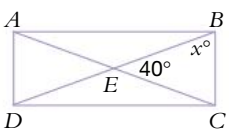

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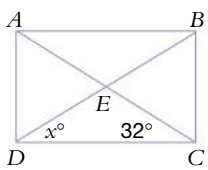
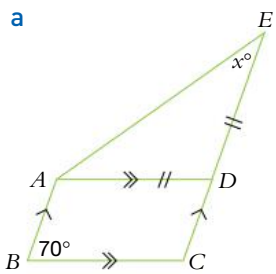


4 Find the value of x. Give reasons.

a



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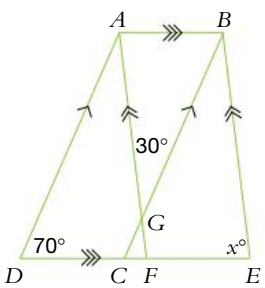

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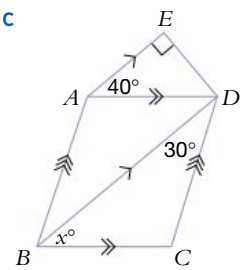
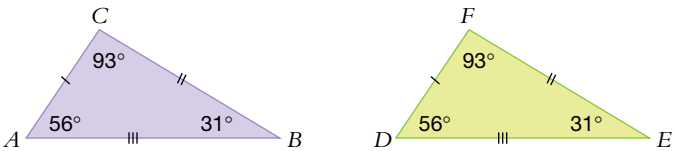


3:02 Congruent triangle proofs

Congruent triangles can be superimposed on one another using one or more translations, reflections or rotations.

When the angles and sides of congruent triangles are matched according to size, three equal pairs of matching sides and angles result.

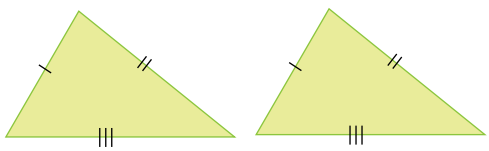


Matching angles: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$

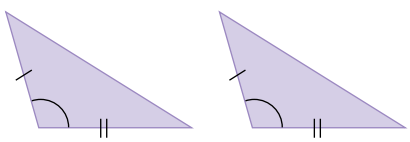
Matching sides: AB and DE, AC and DF, BC and EF

To prove that two triangles are congruent, one of four congruence tests is used. Each test requires that three pieces of information about the two triangles be compared.

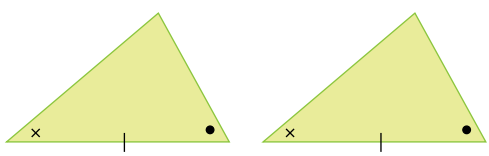
1 SSS (side, side, side)
Two triangles are congruent if three sides of one triangle are equal to three sides of the other.

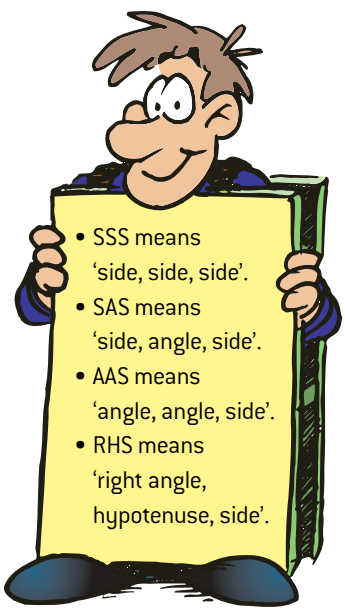


2 SAS (two sides and an included angle)
Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other.



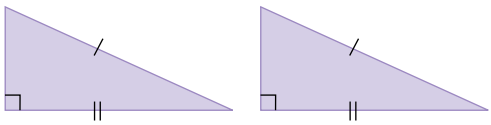
3 AAS (two angles and a matching side)
Two triangles are congruent if two angles and a side of one triangle are equal to two angles and the matching side of the other triangle.





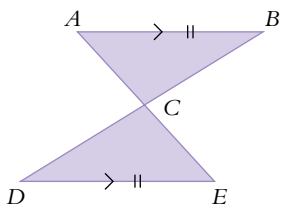
4 RHS (right-angle, hypotenuse and side)

Two right-angled triangles are congruent if the hypotenuse and one side of one right-angled triangle are equal to the hypotenuse and one side of the other.



Two sides (or angles) can be shown to be equal if they are the matching sides (or angles) of a pair of congruent triangles.

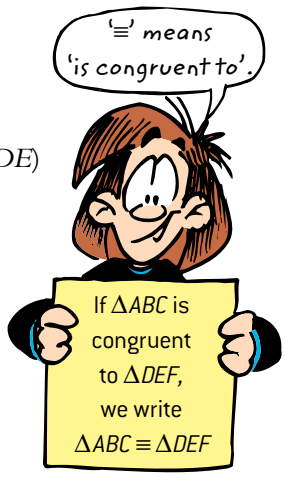
WORKED EXAMPLE 1



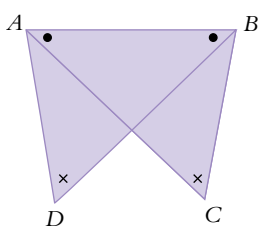
Prove that $\Delta ABC \equiv \Delta EDC$.

In Δs ABC and EDC:

1	$\angle ACB = \angle ECD$	(vert. opp. $\angle$s)
2	$\angle CAB = \angle CED$	(alt. $\angle$s, $AB \parallel DE$)
3	$AB = DE$	(given)
	$\therefore \Delta ABC \equiv \Delta EDC$	(AAS)



WORKED EXAMPLE 2



Prove that $\Delta ABC \equiv \Delta BAD$ and hence that $AD = BC$.

In Δs ABC and BAD:

1	AB is common.	
2	$\angle BAC = \angle ABD$	(given)
3	$\angle BCA = \angle ADB$	(given)
	$\therefore \Delta ABC \equiv \Delta BAD$	(AAS)
	$\therefore AD = BC$	(matching sides of cong't Δs)

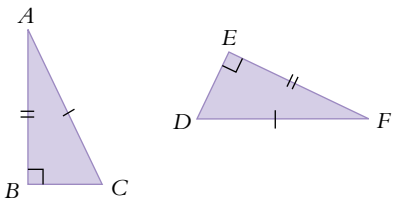
When writing congruent triangle proofs, write the vertices in matching order as shown in the examples.

Exercise 3:02

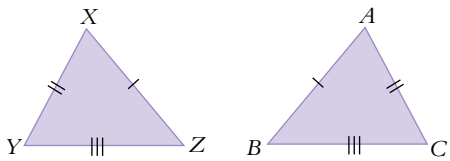
P Foundation worksheet 3:02
Congruent triangle proofs

1 Prove that the two triangles in each pair are congruent.

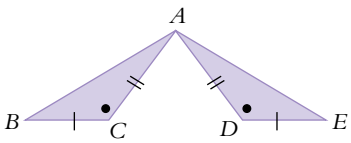
a



b



c



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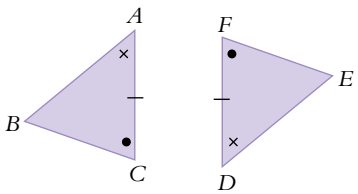

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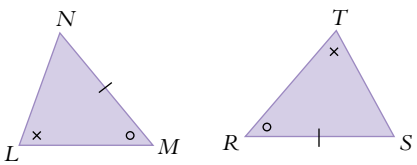
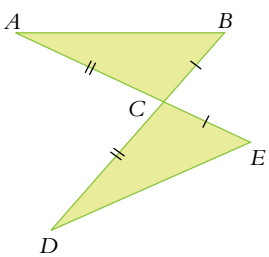
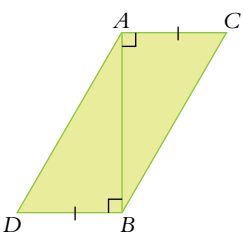


Setting out proofs
In Δs *ABC* and *DEF*:
1 *AB* = *DE* (given)
2 … = … (…)
3 … = … (…)
∴ Δ*ABC* ≡ Δ*DEF* (…)

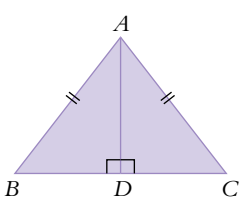
2 **a** Prove that $\Delta ABC \equiv \Delta DEC$.



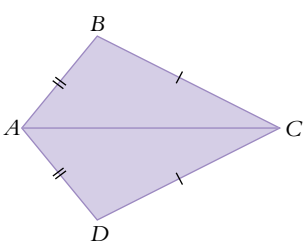
b Prove that $\Delta ABD \equiv \Delta BAC$.



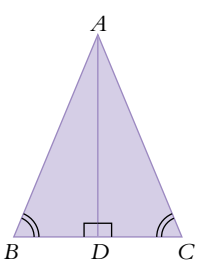
c Prove that $\Delta ABD \equiv \Delta ACD$.



d Prove that $\Delta ABC \equiv \Delta ADC$.



e Prove that $\Delta ABD \equiv \Delta ACD$.



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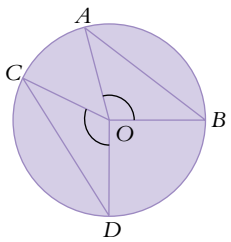
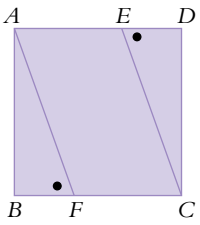


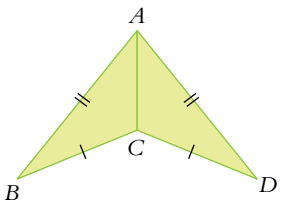
O is the centre of the circle.
Prove that $\Delta AOB \equiv \Delta COD$.

g



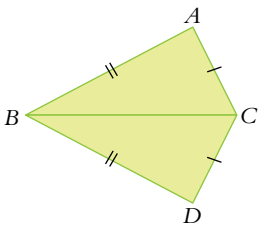
$ABCD$ is a square.
$\angle AFB = \angle CED$
Prove that $\Delta ABF \equiv \Delta CDE$.

3 a



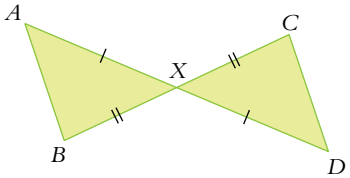
Prove that $\angle BAC = \angle DAC$.

b



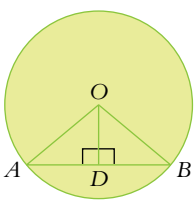
Prove that $\angle ABC = \angle CBD$.

c



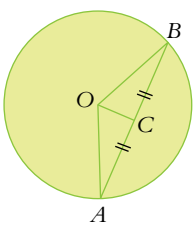
Prove that $\angle BAX = \angle CDX$, and hence that $AB \parallel CD$.

d



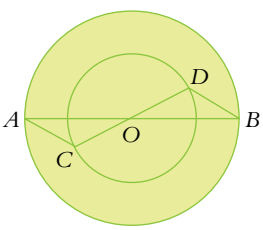
Prove that $AD = DB$.

e



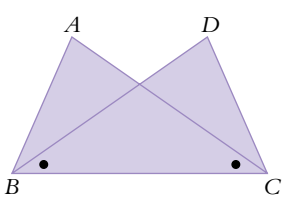
Prove that $\angle OCA = \angle OCB = 90°$.

f



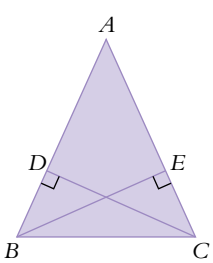
Prove that $AC = DB$ and $AC \parallel DB$.

4 a



$\angle DBC = \angle ACB$ and $BD = CA$.
Prove that $\Delta ABC \equiv \Delta DCB$.

b



$\angle ABC = \angle ACB$
Prove that $\Delta DBC \equiv \Delta ECB$.

Hints:

1. Write the three sides of each triangle.
2. Match up the ones that are equal.
3. Repeat the above for the angles.

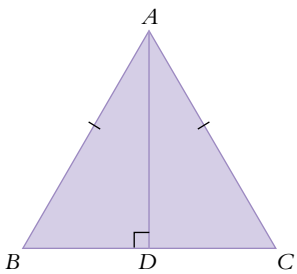
3:03 Deducing properties of the special triangles and quadrilaterals

In the following exercise congruent triangles are used to prove the properties of isosceles and equilateral triangles as well as some properties of the special quadrilaterals.

WORKED EXAMPLE 1

Use congruent triangles to prove that the angles opposite equal sides in an isosceles triangle are equal.

Solution



Data: ΔABC is isosceles with $AB = AC$.
Aim: To prove that $\angle ABC = \angle ACB$.
Construction: Draw AD perpendicular to BC, meeting BC at D.
Proof: In Δs ABD and ACD:

1 $AB = AC$ (given)

2 AD is common.

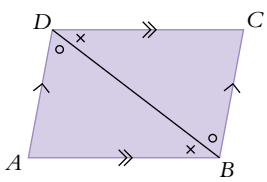
3 $\angle ADB = \angle ADC$ ($AD \perp BC$)
$\therefore \Delta ABD \equiv \Delta ACD$ (RHS)
$\therefore \angle ABD = \angle ACD$ (matching $\angle$s of cong't Δs)
$\therefore \angle ABC = \angle ACB$

WORKED EXAMPLE 2

Prove that the opposite sides of a parallelogram are equal, given that a parallelogram is a quadrilateral with opposite sides parallel.

Solution

Data: $ABCD$ is a parallelogram with $AB \parallel DC$ and $AD \parallel BC$.
Aim: To prove $AB = CD$ and $AD = CB$.
Construction: Join BD.
Proof: In Δs ABD and CDB



1 $\angle ABD = \angle CDB$ (alt. $\angle$s, $AB \parallel DC$)

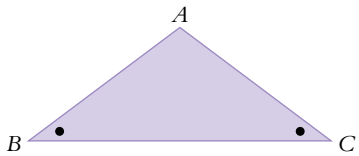
2 $\angle ADB = \angle CBD$ (alt. $\angle$s, $AD \parallel BC$)

3 BD is common.
$\therefore \Delta ABD \equiv \Delta CDB$ (AAS)
$\therefore AB = CD$ and $AD = CB$ (matching sides of cong't Δs)

Exercise 3:03

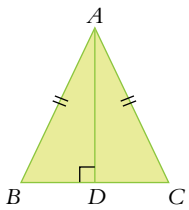
P Foundation worksheet 3:03
Using congruent triangles

1 ΔABC has two equal angles. Draw $AD \perp BC$ and use congruent triangles to prove that $AB = AC$.

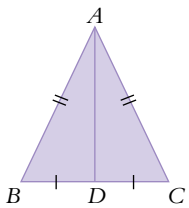


2 Use congruent triangles to prove the following properties of isosceles triangles.

a A line drawn at right angles to the base of an isosceles triangle through the third vertex bisects the base.

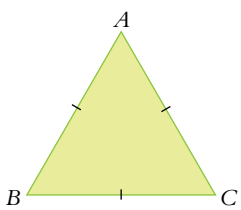


b A line drawn from the midpoint of the base of an isosceles triangle to the third vertex is perpendicular to the base.

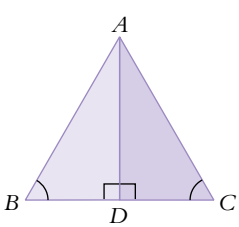


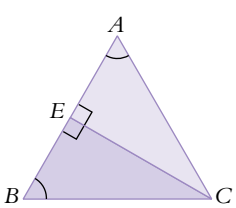
3 **a** ΔABC has all its sides equal. Use the result that if a triangle has two equal sides then the angles opposite the equal sides are equal to prove the following:
$\angle A = \angle B = \angle C$

b How can the result in **a** be used to show that each angle of an equilateral triangle is 60°?



4





ΔABC has three equal angles.
In the first diagram, a perpendicular has been drawn from A to BC.
In the second diagram, the perpendicular has been drawn from C to AB.
Explain why:

a $\Delta ABD \equiv \Delta ACD$

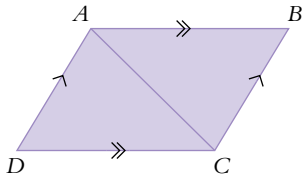
b $AB = AC$

c $\Delta CBE \equiv \Delta CAE$

d $BC = AC$

You have proved that if a triangle has three equal angles, then it has three equal sides.

5 $ABCD$ is a parallelogram.

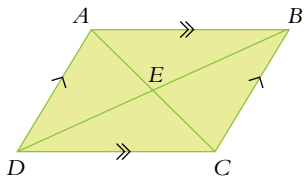


a Why does $\angle BAC$ equal $\angle DCA$?
b Why does $\angle BCA$ equal $\angle DAC$?
c Does $\angle BAD$ equal $\angle BCD$?
d Prove that $\Delta ABC \equiv \Delta CDA$. Hence prove that:
  i $\angle ABC = \angle CDA$
  ii $AB = DC$ and $BC = AD$.

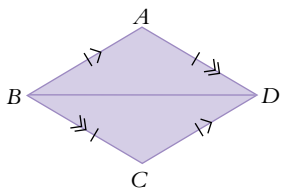
You have proved that the opposite sides and opposite angles of a parallelogram are equal.

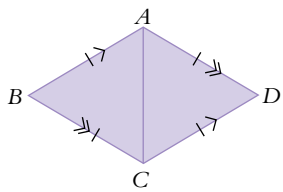
6 Using the fact that the opposite sides of a parallelogram are equal, prove that $\Delta ABE \equiv \Delta CDE$ and hence that $AE = EC$ and $EB = ED$.

You have proved that the diagonals of a parallelogram bisect each other.



7 A rhombus is a parallelogram with all its sides equal. Explain why:

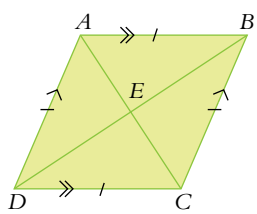




a $\Delta ABD \equiv \Delta CBD$
b $\angle ABD = \angle CBD$
c $\angle ADB = \angle CDB$
d $\Delta ABC \equiv \Delta ADC$
e $\angle BAC = \angle DAC$
f $\angle BCA = \angle DCA$.

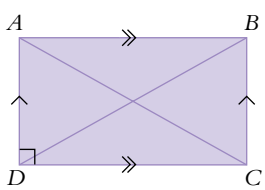
8 **a** In the rhombus shown on the right, why does $AE = EC$?
b Prove that $\Delta ABE \equiv \Delta CBE$ and hence that $\angle AEB = \angle CEB = 90°$.

In the last two questions you have proved that the diagonals of a rhombus bisect each other at right angles, and that they bisect the angles through which they pass.

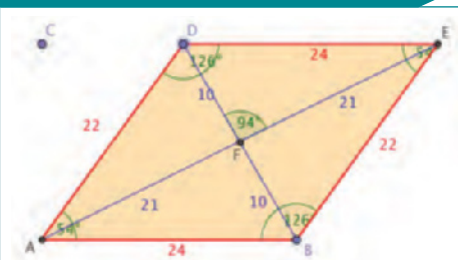


9 A rectangle is a parallelogram with a right angle.
a Prove that all the angles must be right angles.
b Assuming the answer to **a** and the properties of a parallelogram, prove that $\Delta ABD \equiv \Delta DCA$ and hence $AC = DB$.

You have proved that all the angles of a rectangle are right angles and that its diagonals are equal in length.



P GEOGEBRA ACTIVITY 3:03 THE PARALLELOGRAM FAMILY



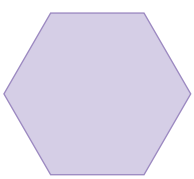
Click and drag the points to change a parallelogram into a rectangle, square or rhombus.

Reinforce your knowledge of the definitions and properties of the parallelogram family by using checkboxes to observe the changes to angles, sides and diagonals as the shapes change.

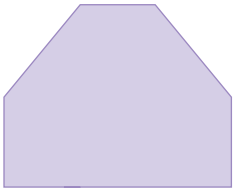
3:04 Polygons

The Pentagon is a famous polygonal building.

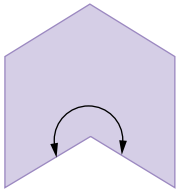
- A polygon is a plane figure with straight sides.
- A polygon is said to be regular if all of its sides and angles are equal. (If they are not, it is said to be irregular.)
- Some polygons are named according to the number of sides.



A regular hexagon



An irregular hexagon



A concave hexagon

- A polygon can be concave or *convex*.
- In a *convex* polygon all the angles are acute or obtuse.
  If a polygon has any reflex angles it is said to be concave.

P GEOGEBRA ACTIVITY 3:04A CONVEX AND CONCAVE POLYGONS

ABCD is a convex quadrilateral.
- All its angles are acute or obtuse.
- The diagonals lie inside the quadrilateral.

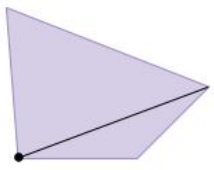
Click and drag the vertices of a quadrilateral or a hexagon and change it from convex to concave and vice versa. Then test your understanding by identifying a randomly generated polygon as either convex or concave.

Angle sum of a polygon

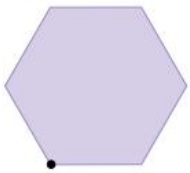
We can use the result for the angle sum of a triangle to work out the angle sum of a polygon because all polygons can be divided into triangles.

INVESTIGATION 3:04A THE ANGLE SUM OF A POLYGON

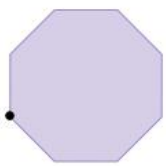
1 Divide each of these polygons into triangles by drawing diagonals from one vertex. The quadrilateral and pentagon have already been done.


Quadrilateral

Pentagon


Hexagon

Heptagon


Octagon

Nonagon

2 Sketch a decagon (10 sides) and a dodecagon (12 sides) and divide them into triangles.

3 Use the results of your work to complete this table.

Number of sides in polygon	3	4	5	6	7	8	9	10	11
Number of triangles	1	2	3						

4 What do you think is the relationship between the 'number of sides' and the 'number of triangles'?

5 If a polygon had n sides how many triangles would be formed?

6 If the angle sum of one triangle is 180°, what is the angle sum of an n-sided polygon?

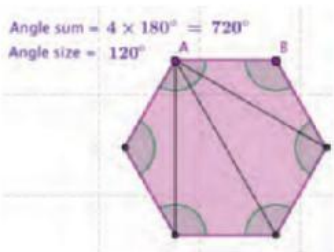
7 If the polygon is regular (all angles and sides equal), what would be the size of each angle in an n-sided polygon?

GEOGEBRA ACTIVITY 3:04B THE ANGLE SUM OF A POLYGON

Use checkboxes to choose either a regular or irregular polygon and use a slider to select the number of sides (from 3 to 12).

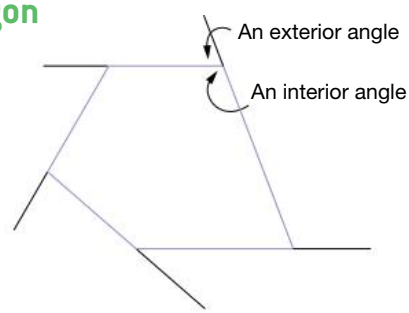
Divide the polygons up into triangles and explore the relationship between the number of sides and the angle sum of a polygon.

Work out the sizes of the internal angles in regular polygons.

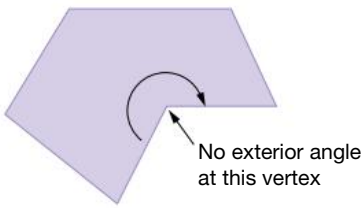


The exterior angle sum of a convex polygon

- An exterior angle of a polygon is an angle that forms a straight line with one of the interior angles.



- If the interior angle is a reflex angle then no exterior angle exists.

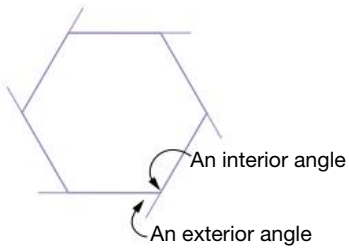


- The angles in a convex polygon are acute or obtuse so a pair of interior and exterior angles will exist at each vertex.

The sum of the exterior angles of a convex polygon is explored in the following investigation.

INVESTIGATION 3:04B THE EXTERIOR ANGLE SUM OF A CONVEX POLYGON

1 At each vertex of the hexagon, a side has been produced to form an exterior angle. One interior angle/exterior angle pair is formed at each vertex. How many interior angle/exterior angle pairs does a hexagon have?

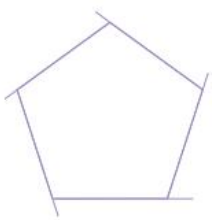


2 At each vertex the exterior angle and interior angle make a straight angle and so sum to 180°. What is the sum of all the interior and exterior angles in the hexagon?

3 What is the sum of all of the interior angles? (Use the result of Investigation 3:04A.)

4 Use the answers to Questions **2** and **3** to find the sum of all of the exterior angles.

5 Repeat Steps 1 to 4 above for the pentagon shown.



6 What do the results above suggest about the sum of the exterior angles of any polygon?

7 Read through the following.
Let the sum of the interior angles be I.
Let the sum of the exterior angles be E.
For an n-sided polygon:

$$I = (n-2) \times 180$$
$$I + E = 180n$$
$$E = 180n - (n-2) \times 180$$
$$= 180n - 180(n-2)$$
$$= 180n - 180n + 360$$
$$= 360$$

- The sum of the interior angles of a polygon of n sides is $(n-2) \times 180°$.
- The sum of the exterior angles of any convex polygon is 360°.

WORKED EXAMPLES

1 Find the sum of the interior angles of an octagon and the size of an interior angle, if the octagon is regular.

2 A regular polygon has an exterior angle of 20°. How many sides does the polygon have?

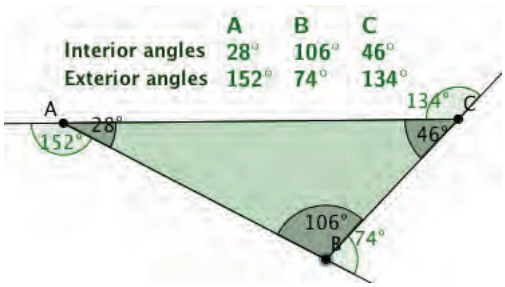
Solutions

1 Sum of interior angles = $(n - 2) \times 180°$
For an octagon, n is equal to 8.
Sum of interior angles = $(8 - 2) \times 180°$
$= 1080°$
If the octagon is regular, all angles are equal.
$\therefore$ Size of an interior angle = $1080° \div 8$
$= 135°$

2 Sum of exterior angles = 360°
For a polygon, the number of sides is equal to the number of exterior angles.
Number of angles = $\frac{360}{20}$
$= 18$
$\therefore$ Number of sides = 18

P GEOGEBRA ACTIVITY 3:04C THE EXTERIOR ANGLE SUM OF A CONVEX POLYGON

Click and drag the vertices of a polygon. Observe the relationship between the interior and exterior angle pairs. Find how the sum of the exterior angles is related to the sum of the interior angles.

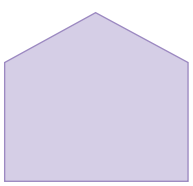
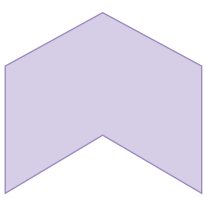
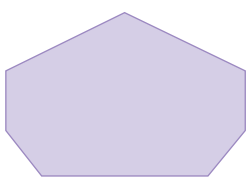


Exercise 3:04

P Foundation worksheet 3:04 Formulas

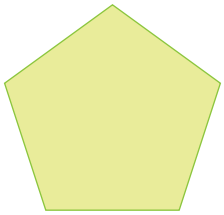
1 Evaluate $(n - 2) \times 180$ if n is:
a 6 b 10 c 15

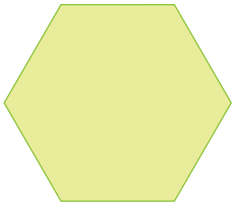
2 For each of the following polygons find:
i the number of sides ii the interior angle sum

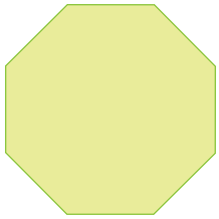
a 
b 
c 

3 Find the interior angle sum of:
a an octagon b a decagon c a heptagon

4 For each of the following regular polygons, find the size of the interior angle.

a 

b 

c 

d a decagon (10 sides)

e a nonagon (9 sides)

f a regular polygon with 17 sides

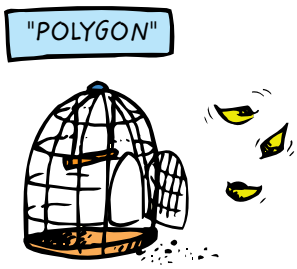
5 A polygon has 20 sides. Find:

a the sum of the interior angles

b the sum of the exterior angles

c the size of an interior angle if the polygon is regular

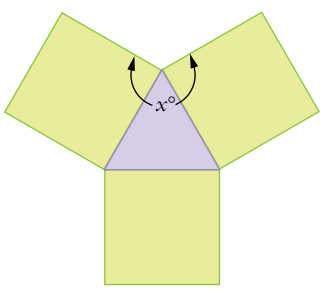
d the size of an exterior angle if the polygon is regular.

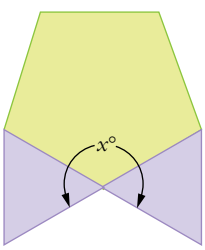


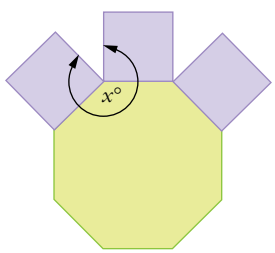
6 Find the number of sides in a regular polygon that has an exterior angle of:

a 60° b 40° c 10° d 15°

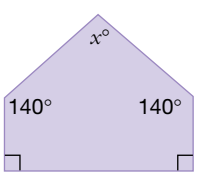
7 Tiles in the shapes of regular polygons have been used to make the following shapes. Find the value of the pronumerals.

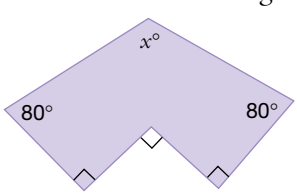
a 

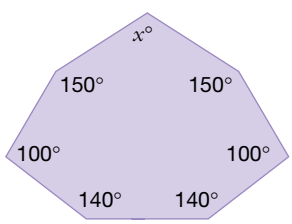
b 

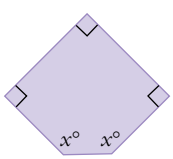
c 

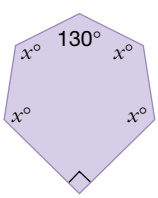
8 Find the value of the pronumeral in each of the following.

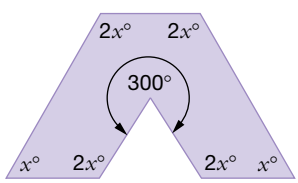
a 

b 

c 

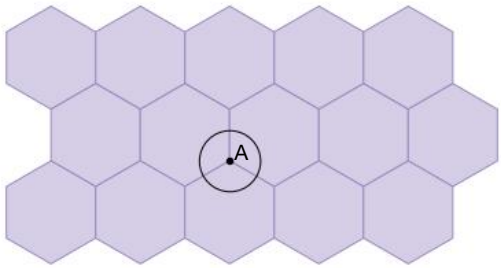
d 

e 

f 

INVESTIGATION 3:04C REGULAR POLYGONS AND TESSELLATIONS

A tessellation is a tiling pattern. If a shape can form a tile pattern on its own, it is said to tessellate. The diagram below is part of a tessellation based on a regular hexagon.



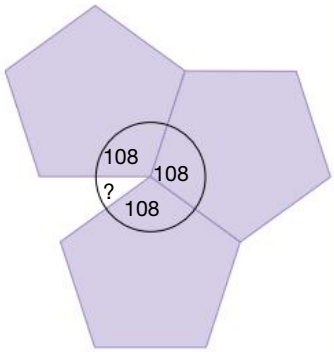
1 What is the size of an interior angle in a regular hexagon?

2 How many hexagons meet at the vertex, *A*?

3 If a shape tessellates, what must be the angle sum at each vertex?

4 Use the diagram on the right to explain why a regular pentagon will not tessellate.

5 What are the other two regular polygons that will tessellate?

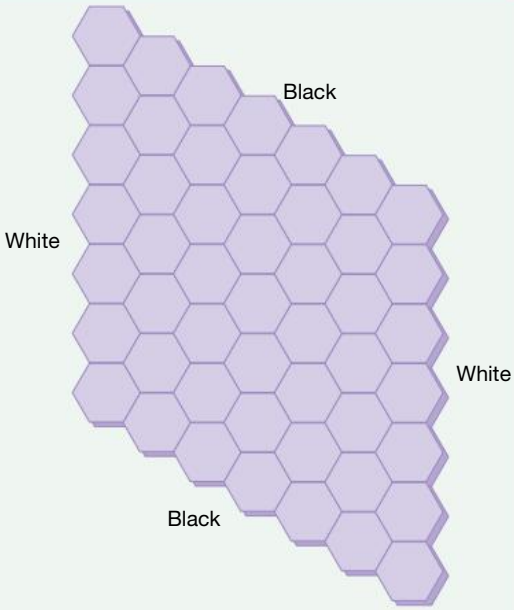


FUN SPOT 3:04 THE GAME OF HEX

Hex is played on a board of hexagons like the one shown. This board has seven hexagons along each edge, but bigger boards may be used.

The game is played between two players, one having a supply of white counters, the other a supply of black counters. Each takes it in turn to place a counter anywhere on the board, with the object being to form a continuous chain of one's own counters from one side of the board to the other. The player with the white counters must form a chain connecting the two 'white' edges of the board before black can join the two 'black' edges.

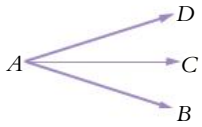
(This game was invented by Piet Hein in 1942 under the name 'Polygon'.)



MATHS TERMS 3

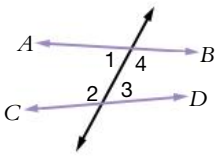
adjacent angles

- share a common arm and vertex
- lie on opposite sides of the common arm
- $\angle DAC$ and $\angle BAC$ are adjacent angles



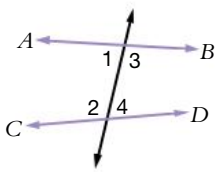
alternate angles

- a pair of angles on opposite sides of the transversal between the other two lines
- in the diagram, the alternate angles are 1 and 3, 2 and 4
- are equal when $AB \parallel CD$



co-interior angles

- a pair of angles on the same side of the transversal and between the other two lines
- in the diagram the co-interior angles are 1 and 2, 3 and 4
- are supplementary when $AB \parallel CD$



complementary angles

- angles that add to give 90°

congruent triangles

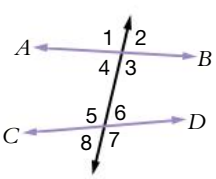
- triangles that are identical in shape and size

congruency tests

- a set of tests used to prove that two triangles are congruent
- the tests are usually referred to by the abbreviations SSS, SAS, AAS and RHS

corresponding angles

- angles that are in corresponding positions at each intersection
- in the diagram, the corresponding angles are 1 and 5, 2 and 6, 3 and 7, 4 and 8
- are equal when $AB \parallel CD$



deductive

- a system in which results called theorems are produced from a set of basic facts that are accepted to be true

exterior angle

- an angle formed when the side of a convex polygon is produced

matching angles (or sides)

- sides (or angles) that are in the same (or corresponding) positions in congruent figures

polygon

- a plane figure with straight sides
- regular polygons have all sides and angles equal
- convex polygons have all angles either acute or obtuse
- some polygons have special names

proof

- a series of steps that establishes the truth of a result

quadrilateral

- a polygon with four sides
- there are six special quadrilaterals

supplementary angles

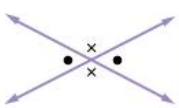
- angles that add to give 180°

triangle

- a polygon with three sides
- equilateral, isosceles and scalene triangles have three sides, two sides and no sides equal in length respectively
- acute-angled triangles have three acute angles
- right-angled triangles have one right angle
- obtuse-angled triangles have one obtuse angle

vertically opposite angles

- two pairs of equal angles formed when two straight lines cross

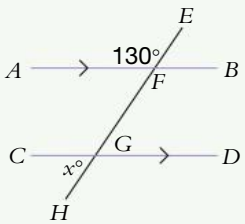


Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

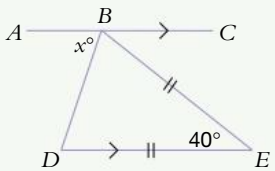
3:01

1 For each figure find the value of x, giving reasons.

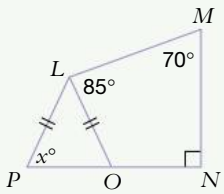
a



b



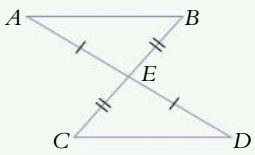
c



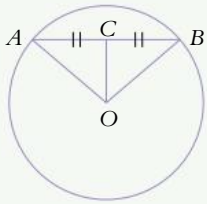
3:02

2 Prove that the two triangles in each pair are congruent.

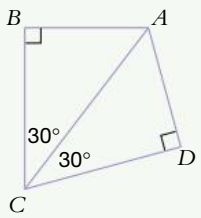
a



b

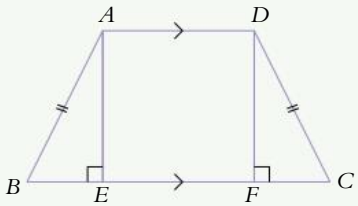


c

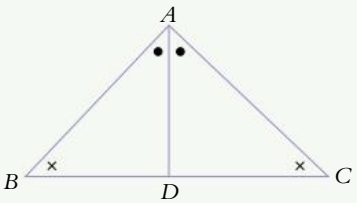


3:03

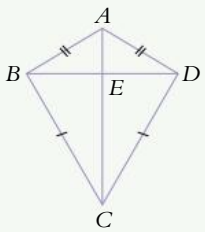
3 **a** Prove that $\angle ABE = \angle DCF$.



b Use congruent triangles to prove that $AD \perp BC$.



c Use congruent triangles to prove that $\angle CAB = \angle CAD$.



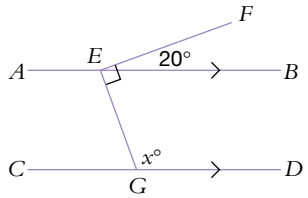
3:04

4 **a** Find the angle sum of a polygon with 15 sides.
b What is the size of the interior angle in a regular octagon?
c A regular polygon has an exterior angle of 20°. How many sides does it have?

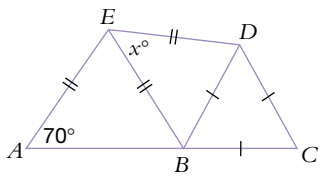
ASSIGNMENT 3A Chapter review

1 In each of the following, find the value of x. Give reasons for your answer.

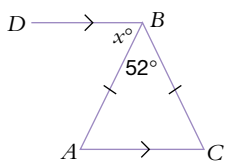
a



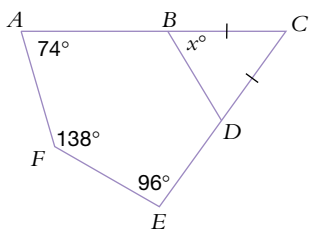
b



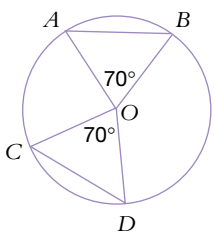
c



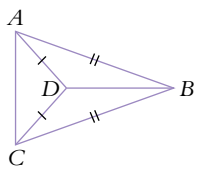
d



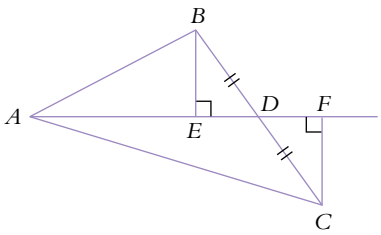
2 **a** Prove that $\Delta AOB \equiv \Delta COD$.



b In the figure $AD = CD$ and $AB = CB$. Use congruent triangles to prove that $\angle ADB = \angle CDB$.



3



ΔABC is any triangle. D is the midpoint of BC, and BE and CF are perpendiculars drawn to AD, produced if necessary. Prove that $BE = CF$.

4 A convex polygon has 20 sides.

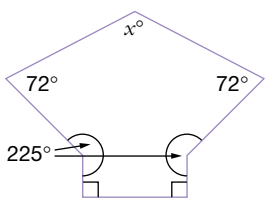
a Find the sum of the interior angles.

b If the polygon is regular (all its sides and angles are equal) find the size of:

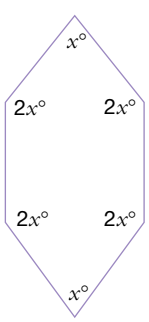
i an interior angle

ii an exterior angle.

5 Find the value of x.

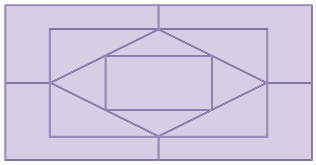


6 A special hexagon is made with four of its angles equal, and the remaining two angles are both half the size of the others. This is shown in the diagram. Find the size of the angles.

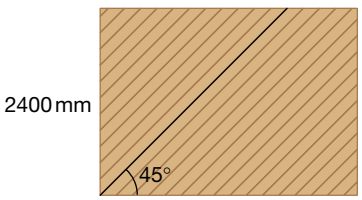


ASSIGNMENT 3B Working mathematically

1 What is the minimum number of colours needed to colour this diagram if no two adjacent regions may have the same colour?



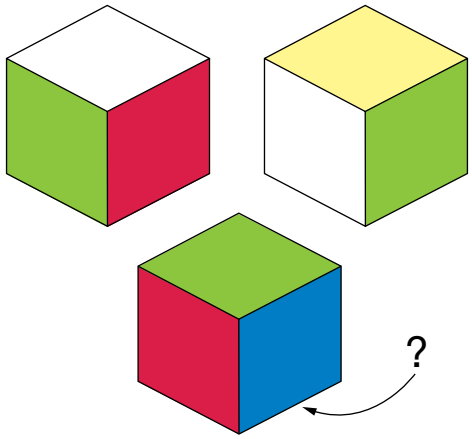
2 Giri and Todd decide to cover their bathroom wall with cedar boards as shown in the diagram. Each board is inclined at 45° to the horizontal. What is the longest length of board that needs to be bought so that there would be no joins?



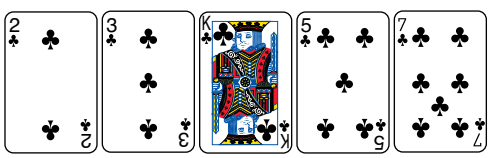
3 The holes on a golf course have to be rated in order of degree of difficulty. The scores of all the players who played in a recent tournament are recorded in the table. Explain what calculations you would use to rate the holes, and list the holes in order of degree of difficulty with 1 being the hardest and 4 the easiest. Remember that par is the number of strokes that are allocated to complete a hole and that in golf a lower score is better than a higher score.

Hole	Par	Golfers' scores					
		2	3	4	5	6	7
2	4	1	51	310	85	9	—
7	5	—	20	211	198	23	4
11	3	25	269	156	4	—	—
16	3	50	292	99	13	—	—

4 Three views of the same cube are given. Determine the colour of the face on the bottom of the third cube.



5 In a card game, two points are scored when any combination of cards total 15. (*Note:* An Ace counts as 1 and the Jack, Queen and King count as 10.) For example, for the hand below, the score is 6.



$$2 + 3 + K = 15$$
$$5 + K = 15$$
$$3 + 5 + 7 = 15$$

What would the score be for these hands?

a 10, 10, 10, 5, 5

b 4, 5, 6, 5, 6

ASSIGNMENT 3C Cumulative revision

1 Simplify: 2:01

a $\frac{a}{3} \times \frac{a}{2}$ **b** $\frac{a}{3} + \frac{a}{2}$ **c** $\frac{a}{3} \div \frac{a}{2}$ **d** $\frac{a}{3} - \frac{a}{2}$

2 Simplify: 2:01

a $\frac{3ab}{2} \times \frac{4}{9a}$ **b** $\frac{3ab}{4} \div \frac{9a}{2}$ **c** $\frac{m^2n^3}{6} \times \frac{15n}{2m^3}$ **d** $\frac{9a^2}{4y^4} \div \frac{18a^3}{2y^2}$

3 Expand and simplify: 2:02

a $2y(3y - 5) + 3(3y - 5)$ **b** $4x(x - 2) - 3(x - 2)$

4 Factorise: 2:02

a $4m^2 - 8m$ **b** $10xy^2 + 15x^2y^2$

5 Expand and simplify: 2:03

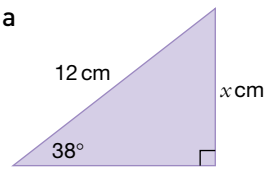
a $(x + 3)(x + 4)$ **b** $(a - 2)(a - 1)$ **c** $(m + 4)(m - 5)$ **d** $(m - 4)(m + 5)$

6 Factorise: 2:04

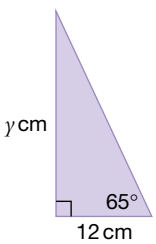
a $x^2 + 4x - 5$ **b** $x^2 - 4x - 5$ **c** $x^2 + 5x + 4$ **d** $x^2 - 5x + 4$

7 Use trigonometry to find the value of the pronumeral, correct to one decimal place. 1:11

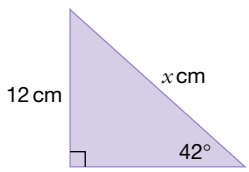
a



b



c



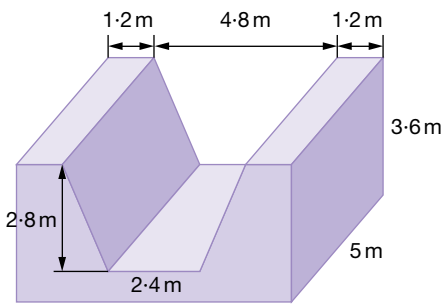
8 If the points $A(-2, 5)$ and $B(4, 7)$ are joined, find: 1:08

a the midpoint of AB **b** the length of AB **c** the gradient of AB.

9 For the prism shown calculate: 1:04

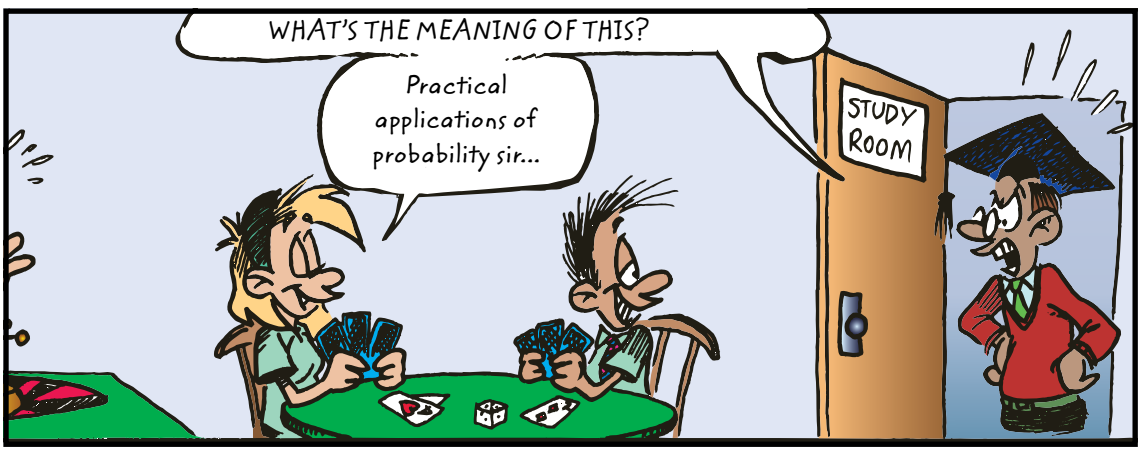
a the area of the cross-section

b the volume.



PROBABILITY

4



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4:02 Two-step chance experiments
Investigation 4:02 Experimental probability without replacement
4:03 Three-step chance experiments
4:04 The probability of two- and three-step events
Investigation 4:04 What is the chance of a boy and a girl?
4:05 The multiplication rule for multi-step events
Fun spot 4:05 The Monty Hall problem
4:06 The multiplication rule for dependent events
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Statistics and Probability

Selections from ***Probability*** [Stage 5.2]

- List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events (ACMSP225)
- Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence (ACMSP246)
- Use the language of 'if … then', 'given', 'of', 'knowing that' to investigate conditional statements and to identify common mistakes in interpreting such language (ACMSP247)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

4:01 Probability and language

- **Probability, $P(E)$,** is the chance of an event happening. This will be a number from 0 to 1 and can be expressed as a fraction, a decimal or a percentage.
  Theoretical probability is the likelihood of an event occurring under ideal circumstances.
  e.g. The probability of rolling a 6 on a normal dice is $\frac{1}{6}$.
  Experimental probability is the predicted chance of occurrence using the relative frequencies taken from a chance experiment.
  Subjective probabilities are probabilities based on opinions or judgements.
- The **sum of the probabilities** of all possible outcomes is always 1 or 100%.

The theoretical probability of an event, E, is the number of times the event can occur, divided by the total number of possible outcomes (as long as each outcome is equally likely to occur).

$$P(E) = \frac{n(E)}{n(S)}$$

S represents the sample space, which is the set of possible outcomes.

Complementary events

- The **complement** of 'an event happens' is 'the event does not happen'.
  The complement of $P(E)$ is written $P(E')$ or $P(\bar{E})$.
  Two events are complementary if the sum of their probabilities is 1.
  $P(E) + P(E') = 1$ or $P(E') = 1 - P(E)$

Other terms

- The **sample space** is the set of all possible outcomes, e.g. 1, 2, 3, 4, 5, 6 on a normal dice.
- Choosing at **random** is choosing in a way that does not affect the likelihood of any outcome.
- **A trial** is one case of carrying out a chance experiment, e.g. spinning a spinner.
- **Equally likely outcomes** are present where each of the outcomes has the same chance of occurring.

- **Mutually exclusive events** have no elements in common, e.g. tossing an odd number and tossing an even number on a dice.
- **Non-mutually exclusive events** have some elements in common, e.g. rolling an odd number and rolling a number less than 4.
- The **relative frequency** of an event is the fraction of times that event occurs. (This can also be written as a percentage.) This is the same as the experimental probability of the event occurring.

Conditional statements

- **Conditional statements** restrict the sample space by giving extra information regarding the event.

Example 1:

Given that an even number has been rolled on a normal six-sided dice, calculate the probability that a 4 has been rolled.

Sample space on a normal dice is $\{1, 2, 3, 4, 5, 6\}$.

The probability of rolling a 4 on a normal dice is $\frac{1}{6}$.

However the sample space has been reduced to $\{2, 4, 6\}$.

This knowledge reduces the question to: 'What is the chance of choosing a 4 from $\{2, 4, 6\}$?'

The probability that a 4 has been rolled is clearly $\frac{1}{3}$.

Example 2:

What is the chance that six heads have been tossed in a row on a fair coin, given that the first five tosses were heads?

The chance of tossing six heads in a row is $\frac{1}{64}$.

We know that the first five tosses are heads, so the only toss in question is the sixth toss. The chance of this being a head is $\frac{1}{2}$. The chance that six heads have been tossed in a row (given the extra information) is $\frac{1}{2}$.

WORKED EXAMPLES

A card is chosen at random from a standard pack of 52 playing cards (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King in each of four suits: hearts, diamonds, clubs and spades).

a What is the probability of choosing the 8 of hearts?

The 8 of hearts is one card out of the 52 in the sample space.

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(8\text{H}) = \frac{1}{52}$$

b What is the probability of not choosing the 8 of hearts?

This is the complement of choosing the 8 of hearts.

$$P(E') = 1 - P(E)$$

$$P(\text{not } 8\text{H}) = 1 - \frac{1}{52}$$

$$= \frac{51}{52}$$

Exercise 4:01

P Foundation worksheet 4:01 Probability review

1 Which of these experiments have equally likely outcomes?

- **A** spinning a tennis racquet to determine who serves
- **B** selecting the winner of a raffle
- **C** choosing a job applicant for an employment position from six people who have been interviewed

2 **a** Describe the chance of each of these events happening, using the terms *certain*, *likely*, *unlikely*, *impossible* and *even chance*.

A It will snow in Sydney some time during the next week.
B We will have rain tomorrow.
C There will be a holiday on 25 December.
D I can walk from home to the Sydney Harbour Bridge without resting.
E I will travel overseas next month.
F I will live to the age of 94.
G I will sit for a mathematics test during the next three months.
H My favourite sporting team will win at least one of its next three matches.

b Place the events **A** to **H** in order from least likely to most likely.

3 **a** Describe two events that have a probability of about $\frac{1}{2}$.
b Describe two events that have a probability of less than $\frac{1}{2}$.
c Describe two events that have a probability of more than $\frac{1}{2}$.

4

certain	probable	even chance	improbable	impossible

Which of the words above best describes an event that has a probability of:

a 0·5 **b** 2% **c** $\frac{17}{20}$ **d** 0 **e** 1 **f** 50%?

Write the answers to probability questions as fractions unless told otherwise.

5 A glass contains eight counters. One is white, three are yellow, two are green and two are red. A counter is drawn at random. What is the probability that the counter is:

a white **b** red **c** white or yellow
d not white **e** not red **f** black or blue?

6 A dice is rolled once. What is the probability of throwing:

a a 2 **b** a 6 **c** anything but a 6
d 2 or less **e** a 0 **f** a number less than 10?

7 A standard pack of cards has four suits: hearts and diamonds (both red), and spades and clubs (both black). In each suit there are 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The Jack, Queen and King are called picture (court) cards. A card is drawn from a standard pack. From a standard pack of cards, a card is chosen at random. What is the probability that the card is:

a black **b** not black **c** yellow
d a 5 **e** not a 5 **f** a picture card
g a club **h** not a club **i** an Ace or a King?

From the results of an experiment or sample, we can calculate the experimental probability. This is used as an estimate of the theoretical probability.

8 A factory tested a sample of 500 CDs and found four to be faulty. Use these results to estimate the probability that a CD produced by this factory is:

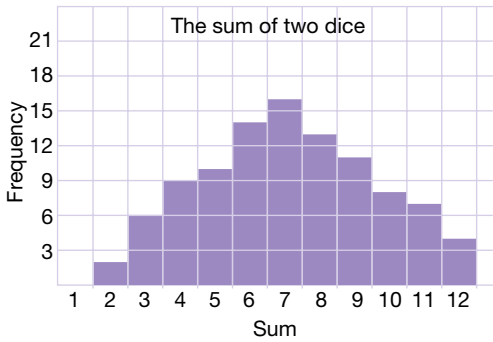
a faulty **b** not faulty.

9 Two dice were thrown 100 times and the sum of the upper faces was recorded. The results are shown on this graph. Use these results to find the experimental probability of throwing a total:

a of 7 **b** of 6

c greater than 7 **d** other than 6

e of 6 or 7 **f** that is even.



10 High school students were asked to vote on the predominant colour to be used on our Olympic Games uniform for the opening ceremony. The results are shown in the table on the right. If one of these people is chosen at random, what is the probability (as a percentage) that the person:

a is female

b chose gold

c is a male who chose orange

d is a female who chose green

e is not a female who chose green

f chose either green or gold

g chose neither green nor gold?

	Male	Female
Green	38	40
Gold	44	32
Blue	7	8
Orange	2	14
Fawn	7	3
Other	2	3
Total	100	100

Number of students = 200

11 A barrel contains 100 blue tickets numbered 1 to 100, 50 red tickets numbered 1 to 50, and 50 green tickets numbered 1 to 50. If a ticket is drawn at random from the barrel, what is the probability that it is:

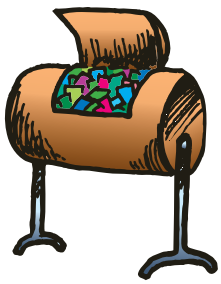
a green **b** green or red

c a 36 **d** a 72

e less than 51 **f** less than 60

g not a 50 **h** not less than 60

i either a 36 or a 72?

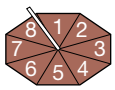


12 **a** If I have already tossed three 6s in a row on a normal dice, what is the chance that I will make it four 6s in a row with my next toss?

b If only heads have been tossed in the first three tosses of a coin, what is the chance that the first four tosses will all be heads?

c Two numbers are to be chosen at random from the numbers 1 to 100. The number 9 is the first chosen. From those remaining a second number must be chosen. What is the chance that the two numbers will be 9 and 10?

A

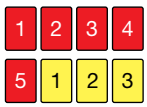


The spinner is spun.

B

The spinner is spun.

C



A card is chosen at random, then returned.

D

The spinner is spun.

13 **a** Using spinner A, I spin a 5 on my first spin. What is the chance that I will spin a 5 on the second spin?

b Each time a card is chosen at random from C above, it is replaced. If the last three choices have been red, what is the chance that the next choice will be red?

c Given that the result of spinning A above was an even number, what is the probability that the spin was a 6?

d Given that spinning D, above, produced a consonant, what is the probability that U was spun?

e Given that spinning D, above, produced a consonant, what is the probability that an R or an S was spun?

f Given that a 2 has been chosen from C, above, what is the chance that the card is red?

g Of the possible outcomes in spinning B, we know that 'orange', 'yellow' and 'white' were not spun. What is the chance that 'pink' was spun?

h Knowing that a number spun on A is greater than 5, what is the probability that the number spun was 7?

i Knowing that a card chosen from C, above, is red, what is the probability that it is a 3?

j Spinner A, above was spun. Knowing that the result was even, what is the probability that the number is 8?

14 **a** The weather prediction on Tuesday was 'for the weekend, there is a 50% chance that it will rain on Saturday, and a 60% chance that it will rain on Sunday'. It did not rain on Saturday. Discuss what effect this might have on the chance of it raining on Sunday.

b I have tossed a coin 20 times and it has landed 'heads' every time. Discuss this situation, and the chance that my next toss will be a head.

15 Three students, A, B and C, are placed in a line in random order. The possible orders are ABC, ACB, BAC, BCA, CAB and CBA. Use this list to find the probability that:

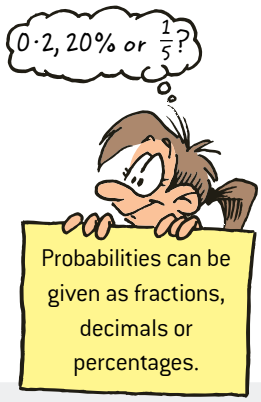
a A is placed before B in the order

b A is placed before both B and C

c A is placed between B and C

d A is placed before either B or C

e A is not placed before either B or C.



16 a The probability that two cards drawn from a standard pack of cards will both be hearts is $\frac{3}{51}$. What is the probability that the two cards drawn are *not* both hearts?

b The probability of throwing a sum of either 5 or 6 using two dice is 0·25. What is the probability that the sum is *neither* 5 *nor* 6?

c There is a 37% chance that I will be sent overseas next year. What is the chance that I will not be sent overseas?

d If 3 coins are tossed, the probability of getting 3 heads is 0·125. What is the probability of getting fewer than 3 heads?

e i The probability of my dying this year is 0·5%. What is the probability that I will not die this year?

ii To insure my life for $10 000 during this year would cost me $135. Does this represent good value? Explain your answer.

INVESTIGATION 4:01 PROBABILITIES GIVEN AS ODDS

The use of odds in gambling

- Odds such as 5/1, 11/4, even money, 2/1 and 2/1 on (or 1/2) are used to give the ratio of what could be gained to what could be lost.
- The fact that for odds of 5/1, $5 might be won while only $1 might be lost, suggests that the probability of success would be 1 chance in 6. This is rarely the case, however, as odds are usually set so that the gambler is at a disadvantage. Odds of 5/1 may be offered when the probability is really 1 chance in 10. This is how the bookmaker, betting agency or insurance company makes a profit.
  This means that the ordinary gambler is sure to lose over time.
- Odds offered on a horse race or a football match are subjective, as each of these events occurs only once. This is an application of subjective probability.

Discussion

1 What do odds of 10/1 mean?

2 Explain why the ordinary gambler must lose over time.

3 Is it healthy for a government to encourage people to gamble? Why, or why not?

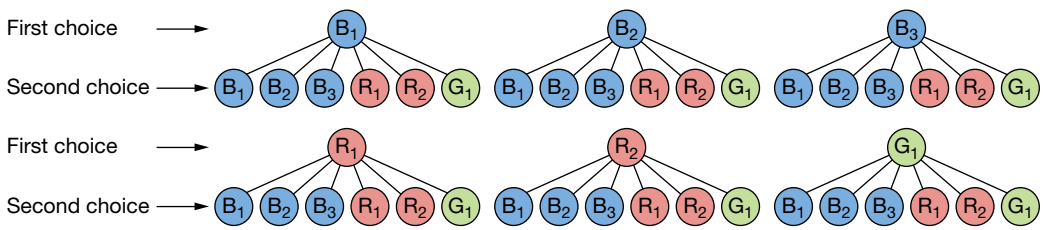
4:02 Two-step chance experiments

- For a chance experiment, listing all possibilities allows us to find **theoretical probabilities**.
- Carrying out the experiment allows us to find the **experimental probability** of each event.
- The experiments below involve **sampling**, the name given to the repeated drawing of items from a collection.

Experiment 1: Choosing two counters with replacement

A bag contains one green, two red and three blue counters.
A counter is chosen at random, *replaced*, and then a second counter is chosen.

Finding the theoretical probabilities (with replacement)
The tree diagram shows the possible outcomes for this two-step chance experiment.

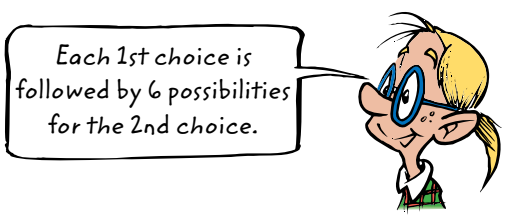


This table shows the same possible outcomes that are shown in the tree diagram above.

The outcomes for the two following cases are indicated by the coloured dots.

Second choice

First choice	B_1	B_2	B_3	R_1	R_2	G_1
B_1	BB	BB	BB	BR	BR	BG
B_2	BB	BB	BB	BR	BR	BG
B_3	BB	BB	BB	BR	BR	BG
R_1	RB	RB	RB	RR	RR	RG
R_2	RB	RB	RB	RR	RR	RG
G_1	GB	GB	GB	GR	GR	GG



Case 1

a There are 9 ways of choosing two blues out of the 36 possible outcomes.
$P(\text{two blues}) = \frac{9}{36}$ or 0·25 or 25%

b What is the effect of knowing the result of the first step of this two-step experiment?
If we know the first choice was blue, then only the 18 outcomes starting with 'blue' are possible. Of these, 9 give us two blues.
$P(\text{two blues, knowing the first choice is blue}) = \frac{9}{18}$ or 0·5 or 50%

Case 2

a There are 4 ways of choosing a red and a green out of 36 possible outcomes.
$P(\text{red and green}) = \frac{4}{36}$ or $0\dot{1}$ or $11\frac{1}{9}\%$

b What is the effect of knowing the result of the first step of this two-step experiment?
If we know that the first counter was red, then only the 12 outcomes starting with 'red' can occur. Of these, 2 produce the outcome 'red and green'.
$P(\text{red and green, knowing the first choice is red}) = \frac{2}{12}$ or $0\dot{1}6$ or $16\frac{2}{3}\%$

Knowing the result of the first step of a two-step experiment changes the probabilities.
If we know that the result of the first step was 'green', then 'two blues' can't occur.

Finding experimental probabilities (with replacement)

- If we carry out Experiment 1 a large number of times, we can find an experimental probability for each event.
- If we carry out Experiment 1 many times, we will get a good approximation of the theoretical probabilities.

Here 100 trials of Experiment 1 have been used, so we can express the experimental probabilities as percentages.

We can use the table to find the experimental probabilities of different events. For example:

$P(\text{two blues}) = \frac{26}{100}$ or 26%

$P(\text{red and green}) = \frac{11}{100}$ or 11%

$P(\text{at least one blue}) = \frac{26+32+16}{100}$ or 74%

$P(\text{exactly one blue}) = \frac{32+16}{100}$ or 48%

Here are the results of 100 trials of Experiment 1.

With replacement

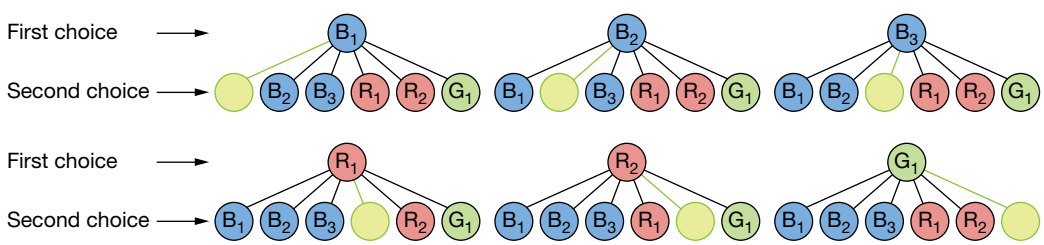
Outcome	Frequency
two blue	26
two red	12
two green	3
blue and red	32
blue and green	16
red and green	11
Total:	100

Experiment 2: Choosing two counters without replacement

A bag contains one green, two red and three blue counters.
A counter is chosen at random and *not replaced*. A second counter is then chosen.
(This is the same as taking two counters at the same time.)

Finding the theoretical probabilities (without replacement)

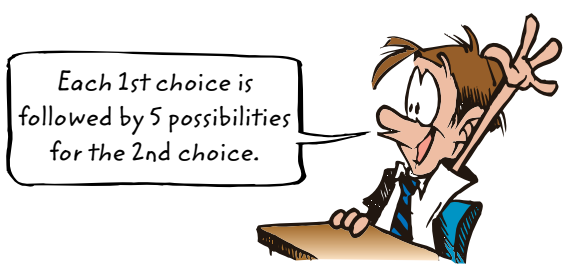
The tree diagram shows the outcomes of this two-step chance experiment.
The counter drawn in the first choice cannot be chosen as the second choice.



This table shows the same possible outcomes that are shown in the tree diagram above.

The outcomes for the two following cases are indicated by the coloured dots.

First choice \ Second choice	B_1	B_2	B_3	R_1	R_2	G_1
B_1		BB	BB	BR	BR	BG
B_2	BB		BB	BR	BR	BG
B_3	BB	BB		BR	BR	BG
R_1	RB	RB	RB		RR	RG
R_2	RB	RB	RB	RR		RG
G_1	GB	GB	GB	GR	GR	



Case 1

a There are now 6 ways of getting two blues out of the 30 possible outcomes.
$P(\text{two blues}) = \frac{6}{30}$ or $\frac{1}{5}$ (This is a 20% chance.)

b What is the effect of knowing the result of the first step of this two-step experiment?
If we know the first choice was blue, then now, only the 15 outcomes starting with 'blue' are possible. Of these, 6 are two blues.
$P(\text{two blues, knowing the first choice is blue}) = \frac{6}{15}$ or $\frac{2}{5}$ (This is a 40% chance.)

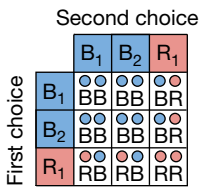
Case 2

a We can see that there are 4 ways of getting a red and a green out of 30 possible outcomes.
$P(\text{red and green}) = \frac{4}{30}$ or $\frac{2}{15}$ (This is a $13\frac{1}{3}$% chance.)

b What is the effect of knowing the result of the first step of this two-step experiment?
If we know that the first counter was red, then now, only the 10 outcomes starting with 'red' can occur. Of these, 2 are 'red and green'.
$P(\text{red and green, knowing the first choice is red}) = \frac{2}{10}$ or $\frac{1}{5}$ (This is a 20% chance.)

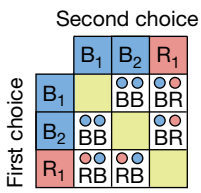
Exercise 4:02

1 Two counters are chosen at random, **with replacement**, from two blue counters and one red counter. The table shows all possible outcomes.
Find the probability of choosing:



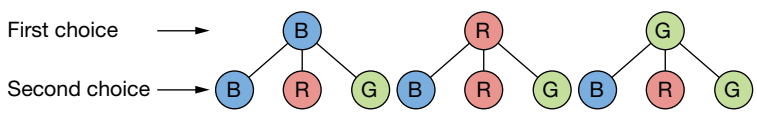
a two blue counters
b two red counters
c a blue counter and a red counter
d a blue then a red
e two counters that are not the same colour.

2 Two counters are chosen at random, **without replacement**, from two blue counters and one red counter. The table shows all possible outcomes.
Find the probability of choosing:



a two blue counters
b two red counters
c a blue counter and a red counter
d a blue then a red
e two counters that are not the same colour.

3 Two counters are chosen at random, **with replacement**, from one blue counter, one red counter and one green counter. This tree diagram shows all possible outcomes.



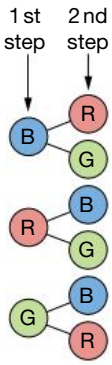
Find the probability of choosing:

a two blue counters
b two counters the same colour
c a blue and a green
d a blue then a red
e two counters that are not the same colour.

If we know that the first choice was blue, what is now the chance of choosing:

f two blue
g two red
h a blue and a green
i two counters of different colour?

4 Two counters are chosen at random, **without replacement**, from one blue, one red and one green. This tree diagram shows all possible outcomes.



Find the probability of choosing:

a two blue counters
b two counters the same colour
c a blue and a green
d a blue then a green
e a blue as either choice
f a blue as the second choice
g no green counter
h two that are not the same colour.

If we know that the first choice was blue, what is now the chance of choosing:

i two blue
j two red
k a blue and a green
l two counters of different colour?

5 100 green raffle tickets and 100 orange raffle tickets have been placed in a barrel. Each ticket displays one of the numbers from 00 to 99. The only ticket I bought was Green 23.

a Before the ticket was chosen, what was my chance of winning?
b It was announced that the winning ticket was green. What was then my chance of winning?
c It was announced that the winning ticket was green and had a first digit of 2. What was then my chance of winning?
d It was announced that the winning ticket was number 23 but no colour was given. What was then my chance of winning?
e If it was announced that the winning ticket was orange, what would then be my chance of winning?

6 Explain the effect on the probabilities of knowing the result of the first step in a two-step (or three-step) chance experiment. Use your answers to Question 5 to help with the explanation.

INVESTIGATION 4:02 EXPERIMENTAL PROBABILITY WITHOUT REPLACEMENT

Here we will find experimental probabilities for Experiment 2 on page 82.
3 blue, 2 red and 1 green counter are placed in a bag. One counter is taken at random. This is not replaced before the second counter is taken.

1 Use 100 trials so you can express the experimental probabilities as percentages. Complete the table on the right (after keeping a tally). Use your results to give the experimental probability of:

a two blues
b a red and a green
c the two counters having at least one blue
d the two counters having exactly one blue.

2 Compare your results for this experiment with the results in the table given for Experiment 1 on page 82.

a Explain why the probabilities are different for 'two greens'.
b List other outcomes that have significantly different experimental probabilities to those in Experiment 1.

Here are the results of 100 trials of Experiment 2.

With replacement

Outcome	Frequency
two blue	
two red	
two green	
blue and red	
blue and green	
red and green	
Total:	100

4:03 Three-step chance experiments

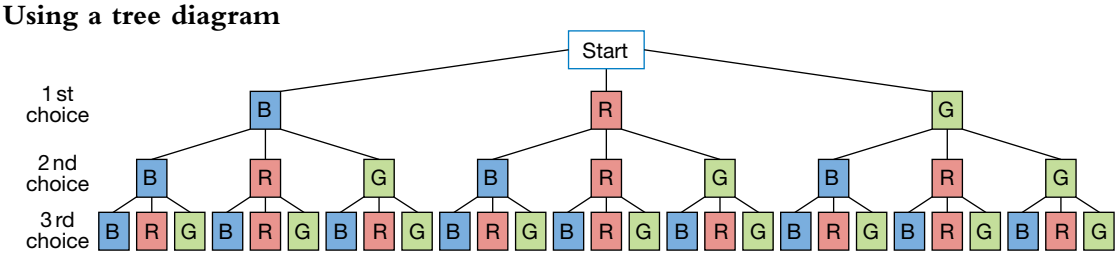
The methods we use here are an extension of the methods used in Exercise 4:02.

Choosing three tickets with replacement

Case 1

A bag contains one blue, one red and one green ticket.
Three tickets are to be chosen at random, with replacement after each choice is made. One is taken at random, replaced, and then a second is taken. This is replaced and then a third is taken.
The possible outcomes can be shown in a tree diagram or a list (or even a table).

Using a tree diagram



- There are 27 possible ordered outcomes. One outcome would be BRB (i.e. blue red blue).
- This form of tree diagram is of limited use, as often the number of possible outcomes is large.

Making a list

We could list the outcomes methodically (i.e. start with the first counter mentioned and list all possibilities for that counter then do the same for each other first choice).

The list would be:

BBB, BBR, BBG,	BRB, BRR, BRG,	BGB, BGR, BGG,
RBB, RBR, RBG,	RRB, RRR, RRG,	RGB, RGR, RGG,
GBB, GBR, GBG,	GRB, GRR, GRG,	GGB, GGR, GGG

Using a table

Where three choices are made, a table is hard to use. In this example we have combined the second and third choices so that a two-way table could be used.

	BB	BR	BG	RB	RR	RG	GB	GR	GG
B	BBB	BBR	BBG	BRB	BRR	BRG	BGB	BGR	BGG
R	RBB	RBR	RBG	RB	RRR	RRG	RGB	RGR	RGG
G	GBB	GBR	GBG	GRB	GRR	GRG	GGB	GGR	GGG

2nd and 3rd choice in order

We can use the tree diagram, the list or the table to write the probabilities of events for Case 1.

$P(\text{three blue tickets}) = \frac{1}{27}$ (BBB)

$P(\text{three tickets of the same colour}) = \frac{3}{27}$ (BBB, RRR and GGG)

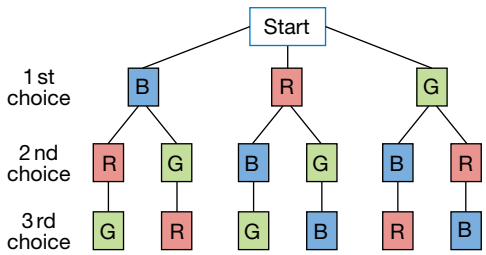
$P(\text{not all the same colour}) = 1 - \frac{3}{27} = \frac{24}{27}$ not (BBB, RRR and GGG)

Choosing three tickets without replacement

Case 2

A bag contains one blue, one red and one green ticket.
Three tickets are to be chosen at random without replacement after each choice is made.
One is taken at random, not replaced, and a second is taken. This is not replaced and a third is taken.

Using a tree diagram



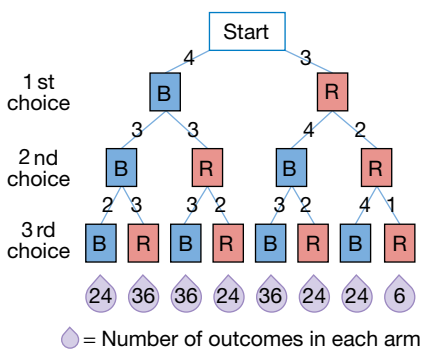
- The 6 possible ordered outcomes are BRG, BGR, RBG, RGB, GBR and GRB.

We can use the tree diagram above to write the probabilities of events for Case 2.
$P(\text{three blue tickets}) = \frac{0}{6} = 0$ (This event is impossible.)
$P(\text{three different colours}) = \frac{6}{6}$ (This event will always happen.)
$P(\text{the last choice is green}) = \frac{2}{6} = \frac{1}{3}$ (We need only consider the third choice.)

Case 3

A bag contains four blue and three red tickets.
A ticket is taken at random, not replaced, and then a second is taken. This is not replaced and then a third ticket is taken.

- As there are many identical choices possible, we can use numbers on the joining lines to show how many of that category could be chosen. By multiplying along the lines we can find the number of ways each event can occur.
- There are 7 ways of choosing the first ticket. This ticket is not returned, so there are only 6 ways of choosing the second. This ticket is also not returned, so there are only 5 ways of choosing the third.



The number of ways of choosing three tickets $= 7 \times 6 \times 5$
$= 210$

- We can use the tree diagram above to find the number of ways of choosing particular colours. There are 4 ways of choosing the first blue ticket, then 3 ways of choosing the second blue ticket and then 2 ways of choosing the third (once two blue tickets have been removed).

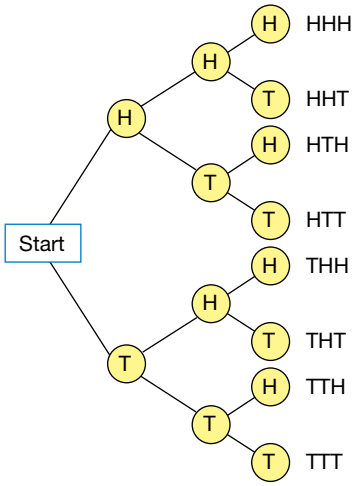
The number of ways of choosing three blue tickets $= 4 \times 3 \times 2$
$= 24$

- A blue and two red tickets can be chosen as BRR, RBR or RRB.

The number of ways of choosing a blue and two reds $= (4 \times 3 \times 2) + (3 \times 4 \times 2) + (3 \times 2 \times 4)$
$= 24 + 24 + 24$
$= 72$

Exercise 4:03

1 A coin is tossed three times. All possible outcomes are shown on this tree diagram.

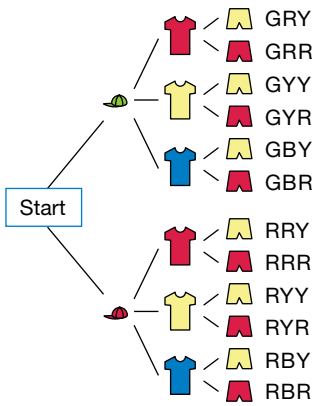


- **a** How many possible outcomes are there?
- **b** How many possibilities exist for each of the three tosses?
- **c** Is the number of possible outcomes $2 \times 2 \times 2$?
- **d** How many outcomes contain a head and two tails?
- **e** How many outcomes contain a tail and two heads?
- **f** How many outcomes contain three tails?

What is the probability that there will be:

- **g** three tails
- **h** three heads
- **i** a head and two tails
- **j** a tail and two heads
- **k** no heads
- **l** at least two heads?

2 Rohan chooses an outfit at random that consists of a hat, a shirt and a pair of pants. He chooses from two different hats, three different shirts and two different pairs of pants. This tree diagram shows all of the possible choices he can make.



In how many ways can he choose:

- **a** a hat
- **b** a shirt
- **c** a pair of pants
- **d** an entire outfit?
- **e** Is the number of possible outcomes $2 \times 3 \times 2$?

How many outfits have:

- **f** three red items
- **g** a green item
- **h** two yellow items
- **i** a red or green item
- **j** a blue item
- **k** exactly two red items?

What is the probability that the outfit chosen will have:

- **l** three red items
- **m** a green item
- **n** two yellow items
- **o** a red or green item
- **p** a blue item
- **q** exactly two red items?

3 Case 1 on page 85 displays a diagram, a list and a table that represent the choosing of three tickets, drawn at random (with replacement) from a bag containing one blue, one red and one green ticket. Use those displays to answer these questions.

After each choice the ticket is replaced.

- **a** How many possible outcomes are there?
- **b** How many outcomes have three green tickets?
- **c** How many outcomes have fewer than three green tickets?
- **d** How many outcomes have exactly one green ticket?

What is the probability that the tickets chosen will be:

- **e** three greens
- **f** three blues
- **g** a green and two reds
- **h** a red and two greens
- **i** no greens
- **j** at least two greens?

4 Case 2 on page 86 displays a diagram and a list that represent the choosing of three tickets, drawn at random (without replacement) from a bag. The bag contains one blue, one red and one green ticket. Use the displays to answer the questions.

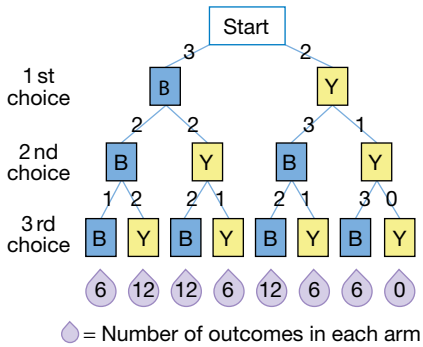
After each choice the ticket is **not** replaced.

a How many possible outcomes are there?
b How many outcomes have three green tickets?
c How many outcomes have one green ticket?
d How many outcomes have green as the first choice?

What is the probability that the tickets chosen will be:

e three greens **f** no greens **g** one green
h green as the first choice **i** red as the first or second choice?

5 This tree diagram represents the choosing of three tickets from a bag, at random (without replacement). The bag contains three blue and two yellow tickets.



Use this tree diagram to answer these questions.

a How many possible outcomes are there?
b Is the number of possible outcomes $5 \times 4 \times 3$?
c How many outcomes have three blue tickets?
d How many outcomes produce a blue and two yellow tickets?
e How many outcomes have one yellow ticket?
f How many outcomes have yellow as the first choice?
g Explain why it is impossible to choose three yellow tickets.

Note:
First choice could be any one of the 5 original tickets.
Second choice will be one of the remaining 4 tickets.
Third choice will be one of the remaining 3 tickets.
There are 18 ways of choosing 'a blue and 2 yellows':
- 6 in the order BYY
- 6 in the order YBY
- 6 in the order YYB.

What is the probability of choosing:

h three blues
i no yellows
j exactly one yellow
k yellow as the first choice
l blue as the first or second choice?

6 A bag contains four blue and three red tickets. A ticket is taken at random and not replaced. A second ticket is taken and is not replaced. A third ticket is taken and not replaced.
Use the diagram for Case 3 on page 86 to answer the following questions.

a How many possible outcomes are there?

How many ways are there of choosing:

b three blue tickets **c** a blue and two red tickets
d three red tickets **e** less than three red tickets
f a red ticket as the third choice **g** a red and two blue tickets?

What is the probability of choosing:

h three blue tickets **i** a blue and two red tickets
j three red tickets?

4:04 The probability of two- and three-step events

To find the total number of outcomes in a two-step or three-step experiment, multiply the numbers of possible choices in each step of the experiment. For example, if a coin is tossed and a dice is rolled, the 12 possible outcomes could be shown in a table.

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

We could find this result by multiplying the number outcomes for the coin toss (2) by the number outcomes for the dice roll (6).

Total number of possible outcomes $= 2 \times 6$
$= 12$

Discussion

Two of these counters are to be chosen at random, without replacement.

- To choose two blue counters, we must choose a blue counter at each choice (i.e. B then B).
  There are 5 ways of choosing a blue as the first choice.
  There are 4 ways of choosing a blue as the second choice once a blue is removed.
  n(blue then blue without replacement) $= 5 \times 4$
  This is: the number in choice 1 × the number in choice 2.
- We can choose a blue and a red counter by choosing a blue then a red or by choosing a red then a blue (i.e. B then R, or R then B).
  n(blue then red without replacement) $= 5 \times 2$
  n(red then blue without replacement) $= 2 \times 5$
  So the number of ways of choosing a blue and a red $= (5 \times 2) + (2 \times 5)$
  $= 20$
- If two coins are tossed at the same time, or two tickets are chosen at once, we can treat these as happening one after the other.

$n(E)$ means the number of ways of choosing event E.

PREP QUIZ 4:04

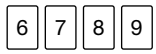
Use the diagram above showing one green, five blue and two red counters to find the number of ways of choosing:

1 any two counters
2 two red counters
3 two blue counters
4 a green then a blue counter
5 a blue then a green counter
6 a green and a blue counter
7 a green then a red counter
8 a blue then a red counter
9 a red then a blue counter
10 a blue and a red counter.

To find the number of ways an event involving two or three steps can occur, multiply together the number of successful ways of performing each of the steps involved.

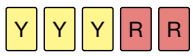
- There are are two square numbers (1 or 4) on a dice. Each roll of a dice has two possible successful outcomes for square numbers. If the dice is rolled three times, the number of ways of obtaining three square numbers = $2 \times 2 \times 2 = 8$

- Four cards marked 6, 7, 8 and 9 are used to show a three-digit number. Cards can be used only once in each number.



The number of ways of showing a three-digit number = $4 \times 3 \times 2 = 24$

- Two cards are chosen randomly from five cards; three are yellow and two are red. A yellow and a red ticket can be chosen in two ways (YR or RY).



Note that there may be more than one way to obtain a certain outcome.

The number of ways of choosing a red and yellow card = $(3 \times 2) + (2 \times 3) = 12$

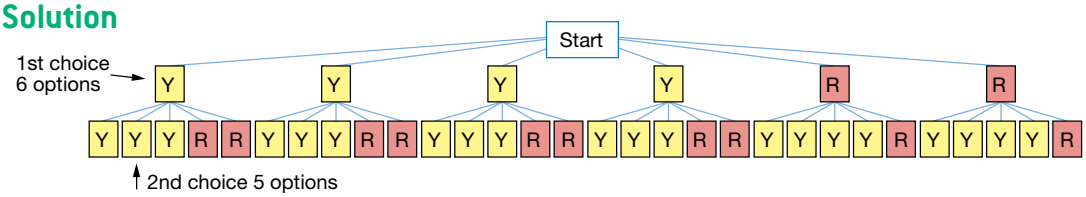
WORKED EXAMPLE 1

A box contains four yellow and two red tickets. Two tickets are chosen at random without replacement.

a What is the probability of choosing two yellow tickets?

b What is the probability that the two tickets will be different colours?

Solution



a There are 6 options for the first choice and 5 options for the second choice, after the first ticket has been removed. The total number of outcomes = 6×5.

To obtain two yellow tickets we must choose a yellow then a yellow.

We multiply the number of ways of choosing yellow as the first choice by the number of ways of choosing yellow as second choice, after the first yellow is removed.

The number of ways of selecting two yellow tickets = 4×3.

$$P(\text{choosing 2 yellow tickets}) = \frac{\text{number of ways of choosing two yellows}}{\text{total number of outcomes}}$$

$$= \frac{4 \times 3}{6 \times 5}$$

$$= \frac{2}{5} \text{ or } 40\%$$

$P(E) = \dfrac{n(E)}{n(S)}$

The probability of choosing two yellow tickets is $\frac{2}{5}$.

b We can choose one ticket of each colour by choosing yellow then red, or red then yellow.

So the number of ways of choosing one of each colour = $(4 \times 2) + (2 \times 4)$ or 16 ways.

$$P(\text{choosing a yellow and a red}) = \frac{\text{number of ways of choosing a yellow and a red}}{\text{total number of outcomes}}$$

$$= \frac{(4 \times 2) + (2 \times 4)}{6 \times 5}$$

$$= \frac{16}{30} \text{ or } \frac{8}{15}$$

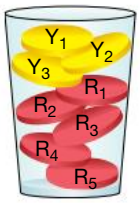
$P(E) = \dfrac{n(E)}{n(S)}$

WORKED EXAMPLE 2

A glass contains three yellow and five red counters. Three counters are to be chosen at random, without replacement.

a What is the probability of choosing three red counters?

b What is the probability of choosing two yellows and a red?



Solution

a There are $8 \times 7 \times 6$ ways of selecting three counters, in order, without replacement.

We multiply the 8 options for Choice 1 by the 7 options for Choice 2 and then by the 6 options for Choice 3.

There are $5 \times 4 \times 3$ ways of selecting three red counters, in order, without replacement.

$$P(\text{choosing 3 red counters}) = \frac{\text{number of ways of choosing 3 reds}}{\text{total number of outcomes}}$$

$$= \frac{5 \times 4 \times 3}{8 \times 7 \times 6}$$

$$= \frac{5}{28}$$

$P(E) = \dfrac{n(E)}{n(S)}$

The probability of choosing three red counters is $\frac{5}{28}$.

b We can choose two yellows and a red by choosing YYR, YRY or RYY.

For YYR (yellow, yellow, red) there are 3 ways of choosing the first yellow, 2 ways of choosing the second yellow and 4 ways of choosing the red. So there are $3 \times 2 \times 4$ ways of choosing YYR.

So the number of ways of choosing two yellows and a red

$= (3 \times 2 \times 4) + (3 \times 4 \times 2) + (4 \times 3 \times 2)$

$$P(\text{2 yellows and a red}) = \frac{\text{number of ways of choosing YYR, YRY, RYY}}{\text{total number of outcomes}}$$

$$= \frac{(3 \times 2 \times 4) + (3 \times 4 \times 2) + (4 \times 3 \times 2)}{8 \times 7 \times 6}$$

$$= \frac{72}{336} \text{ or } \frac{3}{14}$$

$P(E) = \dfrac{n(E)}{n(S)}$

Exercise 4:04

A	B	C	D
The spinner is spun.	The coin is tossed.	The dice is rolled.	A card is chosen at random.

1 Find the number of outcomes in the sample space if the experiment involves the two steps:

a A and B **b** A and C **c** A and D **d** B and C **e** B and D **f** C and D

Find the probability of:

g tossing a head and rolling a 6

h choosing the Ace of spades and tossing a tail

i spinning a D and rolling a 5

j spinning a C and rolling an even number.

2 Find the number of outcomes in the sample space if the experiment involves the three steps:

a A, B and C **b** A, B and D **c** A, C and D

d B, C and D **e** B, B and B **f** A, A and A

Find the probability of:

g tossing a head, rolling a 6 and choosing a red Ace

h spinning a C, tossing a head and rolling a 5.

3 Two cards are to be selected at random from one of the groups shown, **replacing the first card** before the second is chosen. Find the number of ways this can be done using the cards in:

a E **b** F **c** G

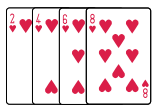
Find the probability of:

d choosing the 2 of hearts followed by the 4 of hearts using Group E

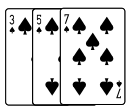
e choosing a Jack followed by a Queen using Group G

f choosing an odd number followed by a spade using Group F.

E



F



G

4 Two cards are to be selected at random from one of the groups above, **without replacing the first card**. Find the number of ways this can be done using the cards in:

a E **b** F **c** G

Find the probability of:

d choosing the 2 of hearts followed by the 4 of hearts from Group E

e choosing a Jack followed by a Queen from Group G

f choosing an odd number followed by a spade from Group F.

5 Three cards are to be selected at random from one group above, **without replacement**. Find the number of ways this can be done using the cards in:

a E **b** F **c** G

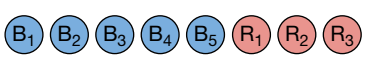
Find the probability of:

d choosing Jack, Queen and King in that order from Group G

e choosing a diamond then a club then a diamond from Group G

f choosing the 2, 4 and 6 in that order from Group E.

6 Two counters are to be chosen at random from this group of counters, **replacing the first counter** before the second is chosen.



a In how many ways can the two counters be chosen? Call this $n(S)$.

b **i** In how many ways can we choose two blue counters? Call this $n(E_1)$.

ii What is the probability that we will choose two blue counters?

c **i** In how many ways can we choose two red counters? Call this $n(E_2)$.

ii What is the probability that we will choose two red counters?

d **i** In how many ways can we choose a blue counter followed by a red counter? Call this $n(E_3)$.

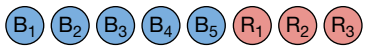
ii What is the probability that we will choose a blue counter followed by a red counter?

e **i** In how many ways can we choose a red counter followed by a blue counter? Call this $n(E_4)$.

ii What is the probability that we will choose a red counter followed by a blue counter?

f i In how many ways can we choose a blue and a red counter in any order? Call this $n(E_5)$.
ii What is the probability that we will choose a blue and a red counter in any order?

7 Two counters are to be chosen at random from this group of counters, **without replacement**.



a In how many ways can the two counters be chosen? Call this $n(S)$.
b i In how many ways can we choose two blue counters? Call this $n(E_1)$.
ii What is the probability that we will choose two blue counters?
c i In how many ways can we choose two red counters? Call this $n(E_2)$.
ii What is the probability that we will choose two red counters?
d i In how many ways can we choose a blue counter followed by a red counter? Call this $n(E_3)$.
ii What is the probability that we will choose a blue counter followed by a red counter?
e i In how many ways can we choose a red counter followed by a blue counter? Call this $n(E_4)$.
ii What is the probability that we will choose a red counter followed by a blue counter?
f i In how many ways can we choose a blue and a red counter in any order? Call this $n(E_5)$.
ii What is the probability that we will choose a blue and a red counter in any order?

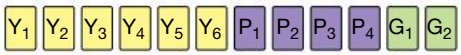
8 Two names are to be drawn from a hat at random to determine the captain and vice-captain of our team. Because the same person cannot be both captain and vice-captain, the first name is not replaced before the second is chosen. The names being considered are Hannah, Flynn, Hudson, Alana and Naomi.

a In how many ways can the two roles be chosen? Call this $n(S)$.
b What is the probability that we will choose Hannah as captain and Hudson as vice-captain?
c What is the probability that we will choose Hudson as captain and not Flynn as vice-captain?
d What is the probability that we will choose Flynn or Hudson as captain and Alana or Naomi as vice-captain?

9 Two tickets are to be chosen at random from this group of tickets, **with replacement**.

a In how many ways can the two tickets be chosen? Call this $n(S)$.
b i In how many ways can we choose two yellow tickets? Call this $n(E_1)$.
ii What is the probability that we will choose two yellow tickets?
c i In how many ways can we choose two red tickets? Call this $n(E_2)$.
ii What is the probability that we will choose two red tickets?
d i In how many ways can we choose a green ticket followed by a red ticket? Call this $n(E_3)$.
ii What is the probability that we will choose a green ticket followed by a red ticket?
e i In how many ways can we choose a red ticket followed by a green ticket? Call this $n(E_4)$.
ii What is the probability that we will choose a red ticket followed by a green ticket?
f i In how many ways can we choose a green and a red ticket in any order? Call this $n(E_5)$.
ii What is the probability that we will choose a green and a red ticket in any order?

10 Three counters are to be chosen at random from this group of counters, **without replacement**.



a In how many ways can the three counters be chosen? Call this $n(S)$.

b i In how many ways can we choose three purple counters? Call this $n(E_1)$.

ii What is the probability that we will choose three purple counters?

c i In how many ways can we choose three yellow counters? Call this $n(E_2)$.

ii What is the probability that we will choose three yellow counters?

d i In how many ways can we choose two green counters followed by a yellow counter? Call this $n(E_3)$.

ii What is the probability that we will choose two green counters followed by a yellow counter?

INVESTIGATION 4:04 WHAT IS THE CHANCE OF A BOY AND A GIRL?

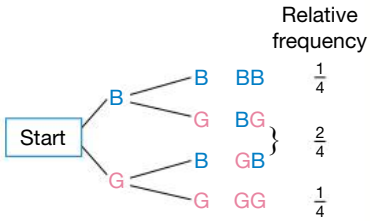
Consider that, overall, there is an even chance of a baby being born a girl or a boy.

We could use this tree diagram to predict that in a family of two children, there is a 25% chance of 2 boys, a 50% chance of a boy and a girl, and a 25% chance of 2 girls.

But is this true?

Could some couples tend to have more children of the same sex, even though about 50% of each sex is born overall?

The first two children in a family



This tree diagram shows all possible outcomes.

1 Survey as many families as you can, and record the sex of the first two children born.

The first two children	Tally	Number	Relative frequency (%)
2 boys (BB)			
a boy and a girl (BG or GB)			
2 girls (GG)			
Total:			

2 Combine your data with that of other students, trying not to count the same family more than once. Remember that the more cases we consider, the more accurate our conclusions should be.

- Based on the results of your investigation, do you think that there is a 50% chance that the first two children in the same family will be a boy and a girl?
- If not, what estimate would you give for the chance that the first two children in a family will be a boy and a girl?

3 Do you think you have collected sufficient data to be sure that your conclusion is correct? Explain why or why not.

4:05 The multiplication rule for multi-step events

Independent events

Two events are independent if the occurrence of the first event does not affect the chance of the occurrence of the second. The probability of the second will not be affected. (One happening or not happening does not influence the chance of the other happening.)

WORKED EXAMPLES

1 'Tossing a head' on the first toss of a coin does not affect the chance of 'tossing a head' on a second toss of the coin so these events are independent events.

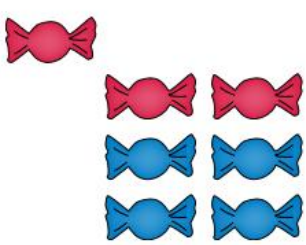
2 'Spinning a 5 on a spinner' and 'rolling a 5 on a dice' are independent events, because the occurrence of the first has no bearing on the occurrence of the second.

3 Three tickets are to be chosen at random, with replacement, from 4 blue and 4 red tickets. The probabilities involved in any choice will not be affected by the results of earlier choices, because tickets are replaced. 'Choosing a blue ticket as Choice 1', 'choosing a blue ticket as Choice 2' and 'choosing a red ticket as Choice 3' are therefore three independent events.

Dependent events

Two events are dependent events if the occurrence of the first event affects the chance of the occurrence of the second. The probability of the second will be affected by the outcome of the first. (One happening or not happening influences the chance of the other happening.)

WORKED EXAMPLE 1



A bag contains three red and four blue lollies. Two lollies are to be chosen at random, without replacement.

The probability of choosing a red lolly as the first choice is $\frac{3}{7}$.

If a red is chosen as the first choice and not replaced, only two red and four blue lollies remain for the the second choice.

The probability of choosing a red lolly as the second choice is $\frac{2}{6}$.

When the first choice is made, the number of lollies available for the second choice is reduced. The probabilities of the second choice are affected by the first choice, so they are dependent events.

WORKED EXAMPLE 2

A two-digit number is to be made by randomly choosing two cards in order, without replacement, from five cards marked 6, 6, 7, 7 and 8.

The probability of choosing a 7 as the tens digit is $\frac{2}{5}$.

If a 7 is chosen as the tens digit and not replaced, the fours cards remaining are 6, 6, 7 and 8.

The probability of choosing a 7 as the ones digit is $\frac{1}{4}$.

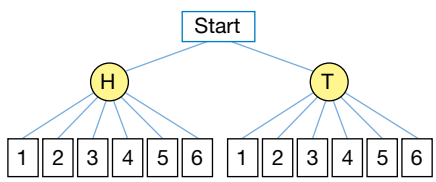
When the first choice is made, the number of cards available for the second choice is reduced. The probabilities of the second choice are affected by the first choice, so they are dependent events.

The multiplication rule for independent events

When two events A and B are independent, the probability of both occurring is:
$P(A \text{ and } B) = P(A) \times P(B)$

Explanation

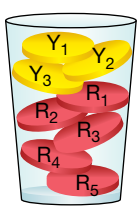
1 A coin is tossed and a dice is rolled. The tree diagram below shows all possible outcomes. We can see that there are 2×6 possible outcomes, and that there are 1×3 ways of obtaining a head and an odd number.



$$P(\text{head and an odd number}) = \frac{n(E)}{n(S)}$$
$$= \frac{1 \times 3}{2 \times 6}$$
$$= \frac{1}{2} \times \frac{3}{6}$$
$$= \left(\begin{array}{c}\text{chance of choosing} \\ \text{a head}\end{array}\right) \times \left(\begin{array}{c}\text{chance of choosing} \\ \text{an odd number}\end{array}\right)$$

We can see that the multiplication rule gives the same result as a tree diagram.

2 Three yellow and five red counters are in a glass. Two counters are chosen at random, with replacement. What is the probability of choosing 2 red counters?



There are 8 ways of choosing the first counter and there are 8 ways of choosing the second counter because the first counter chosen is replaced.
The total number of outcomes $n(S) = 8 \times 8 = 64$

There are 5 ways of choosing red as the first counter and 5 ways of choosing red as the second counter because the first counter chosen is replaced.

$$P(\text{2 red counters}) = \frac{\text{number of ways of choosing 2 reds}}{\text{total number of outcomes}}$$
$$= \frac{5 \times 5}{8 \times 8}$$
$$= \frac{5}{8} \times \frac{5}{8}$$
$$= \left(\begin{array}{c}\text{chance of choosing} \\ \text{a red counter}\end{array}\right) \times \left(\begin{array}{c}\text{chance of choosing} \\ \text{a red counter}\end{array}\right)$$

$$P(E) = \frac{n(E)}{n(S)}$$

We can see that the multiplication rule gives the same result as our reasoning.

WORKED EXAMPLE 1

I have two containers. Container 1 has three green tickets and seven blue tickets. Container 2 has two red tickets and three yellow tickets. If I choose a ticket from each container at random, what is the probability of choosing a green ticket and a yellow ticket?

Solution

$P(\text{green from container 1}) = \frac{3}{10}$, $P(\text{yellow from container 2}) = \frac{3}{5}$, $P(A \text{ and } B) = P(A) \times P(B)$

$P(\text{green and yellow}) = P(\text{green}) \times P(\text{yellow})$

$= \frac{3}{10} \times \frac{3}{5}$

$= \frac{9}{50}$

The probability of choosing a green ticket and a yellow ticket is $\frac{9}{50}$.

WORKED EXAMPLE 2

A normal dice is rolled, and a fair spinner showing the letters D, E, F and G is spun.
What is the chance of rolling an odd number on the dice and spinning a G on the spinner?

Solution

$P(\text{rolling an odd number}) = \frac{3}{6}$, $P(\text{spinning a G}) = \frac{1}{4}$, $P(A \text{ and } B) = P(A) \times P(B)$

$P(\text{odd number and a G}) = P(\text{odd number}) \times P(\text{spinning a G})$

$= \frac{3}{6} \times \frac{1}{4}$

$= \frac{3}{24}$ (or $\frac{1}{8}$)

The probability of rolling an odd number and spinning a G is $\frac{1}{8}$.

WORKED EXAMPLE 3

Two of the seven marbles in a bag are to be chosen at random, with replacement. Three marbles are red and four are blue. Find the probability of choosing:

a two red marbles

b a red marble, then a blue marble.

Solution

a $P(\text{red as 1st choice}) = \frac{3}{7}$

$P(\text{red as 2nd choice}) = \frac{3}{7}$

$P(A \text{ and } B) = P(A) \times P(B)$

$P(\text{2 red marbles}) = P(\text{red}) \times P(\text{red})$

$= \frac{3}{7} \times \frac{3}{7}$

$= \frac{9}{49}$

The probability of choosing two red marbles is $\frac{9}{49}$.

b $P(\text{red as 1st choice}) = \frac{3}{7}$

$P(\text{blue as 2nd choice}) = \frac{4}{7}$

$P(A \text{ and } B) = P(A) \times P(B)$

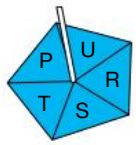
$P(\text{red then blue}) = P(\text{red}) \times P(\text{blue})$

$= \frac{3}{7} \times \frac{4}{7}$

$= \frac{12}{49}$

The probability of choosing a red marble, then a blue marble is $\frac{12}{49}$.

Exercise 4:05

A	B	C	D
The coin is tossed.	The dice is rolled.	A ticket is chosen at random.	 The spinner is spun.

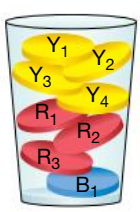
1 Refer to A to D above, and use the multiplication rule $P(A \text{ and } B) = P(A) \times P(B)$ for two-step events to answer these questions. Find the probability that:

- **a** a head is tossed and a 6 is rolled
- **b** a head is tossed and a red ticket is chosen
- **c** a head is tossed and a T is spun
- **d** a 6 is rolled and a red ticket is chosen
- **e** a 6 is rolled and a T is spun
- **f** a red ticket is chosen and a T is spun
- **g** a yellow ticket is chosen and a number less than 3 is rolled
- **h** a yellow ticket is chosen and a tail is tossed
- **i** a yellow ticket is chosen and P, U or R is spun.

2 Refer to A to D in Question 1, and use the multiplication rule $P(A \text{ and } B) = P(A) \times P(B)$ for two-step events to answer these questions. Find the probability that:

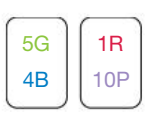
- **a** when a coin is tossed twice, the outcome will be two heads
- **b** when a dice is rolled twice, the outcome will be two sixes
- **c** two red tickets are chosen from two choices, with replacement
- **d** we would obtain two Ts from two spins of the spinner.

3 One blue, three red and four yellow counters are placed in a glass and two of these counters are chosen at random, with replacement. Find the probability of choosing:



- **a** two yellow counters
- **b** two red counters
- **c** a yellow then a red
- **d** a red then a yellow
- **e** a yellow then a blue
- **f** a blue then a yellow
- **g** a red then a blue
- **h** a blue then a red.

4 I have two containers. One has five green tickets and four blue tickets. The other has one red ticket and 10 purple tickets. If I choose a ticket from each container at random, what is the probability of my choosing:

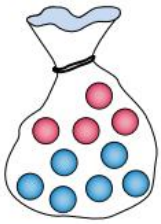


- **a** a green ticket and a purple ticket
- **b** a blue ticket and a purple ticket
- **c** a green ticket and a red ticket
- **d** a blue ticket and a red ticket?

5 A normal dice is rolled, and a fair spinner showing the letters B, C, D, E, F, G and H is spun. Find the chance of:

- **a** rolling a 2 on the dice and spinning a B on the spinner
- **b** rolling an even number on the dice and spinning a consonant (not a, e, i, o or u) on the spinner
- **c** rolling a 5 or a 6 on the dice and spinning a vowel (a, e, i, o or u) on the spinner.

6 Two of the 10 marbles in a bag are to be chosen at random, with replacement. Four of the marbles are red and six are blue. Find the probability of choosing:



a two red marbles
b a red marble, then a blue marble
c two blue marbles
d a blue marble then a red marble.

7 I have just finished my term tests. I estimate that the probability of my passing Mathematics is $\frac{4}{5}$, the probability of my passing English is $\frac{2}{3}$, the probability of my passing Science is $\frac{3}{5}$ and the probability of my passing Indonesian is $\frac{1}{2}$. Assuming that these estimates are accurate, find the probability that I will pass both:

a Mathematics and English
b Mathematics and Science
c Indonesian and English
d Science and Indonesian.

8 The probability that I will win a raffle is $\frac{3}{100}$. The probability that I will lose my raffle tickets is $\frac{3}{8}$. Find the probability that:

a I will win the raffle and lose my tickets
b I will win the raffle and not lose my tickets.

By considering the tree diagrams in Section 4:03, we can see that when three events A, B and C are independent, the probability of all three occurring together is:

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

9 The probability that Sam will pass in the term tests are: Mathematics $\frac{9}{10}$, English $\frac{4}{5}$, Science $\frac{1}{2}$, Indonesian $\frac{2}{3}$.

Find the probability that Sam will pass in:

a Mathematics, English and Science
b Mathematics, English and Indonesian
c English, Science and Indonesian
d Mathematics, Indonesian and Science.

A

The coin is tossed.

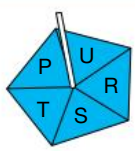
B

The dice is rolled.

C

A ticket is chosen at random.

D



The spinner is spun.

10 Refer to A to D above, to answer these questions. Find the probability of obtaining:

a 3 heads by tossing the coin three times
b 'a number less than 6' each time, when rolling the dice three times
c three red tickets by choosing three tickets, with replacement
d three yellow tickets by choosing three tickets, with replacement
e the letters S, U and P in that order by spinning the spinner three times
f a head, an even number and a red ticket using A, B and C
g a tail, a square number and a yellow ticket using A, B and C
h a head, a red ticket and a letter from the word 'SPUR' using A, C and D.

11 Kai-Lin, Su-Lin, Mark and Chris are all to compete in a different age race tomorrow. Kai-Lin has 1 chance in 4 of winning, Su-Lin has 3 chances in 5, Mark has 1 chance in 6 and Chris has 7 chances in 8. Find the probability that:

a Kai-Lin and Su-Lin both win their races
b Kai-Lin, Su-Lin and Mark all win their races
c Kai-Lin, Su-Lin and Chris all win their races
d Mark, Chris and Su-Lin all win their races
e Kai-Lin and Su-Lin win but Mark does not win
f Kai-Lin and Mark win but Chris does not win
g all four people win their races.

12 The probability of my spending Christmas Day in Sydney is $\frac{7}{10}$, the probability of my spending it in London is $\frac{2}{10}$ and the probability of my spending it elsewhere is $\frac{1}{10}$. Explain why the probability of my spending the day in both Sydney and London is not $\frac{7}{10} \times \frac{2}{10}$.

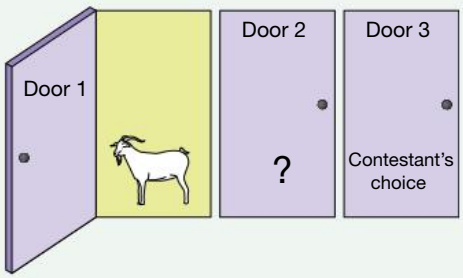
FUN SPOT 4:05 THE MONTY HALL PROBLEM

A quiz show host, Monty Hall, would give his contestants a choice of three doors. Behind one was a car; behind the other two were goats. After the contestant had named a door, Monty, who knew where the car was hidden, would open one of the other doors to show that it did not contain the car. He would then offer the contestant the opportunity to switch from their original choice to the other door not yet opened.

Questions

1 Would the contestant improve his or her chances of winning the car if they changed from their original choice?

2 Should the contestant change their choice or not? Give a reason for your answer.



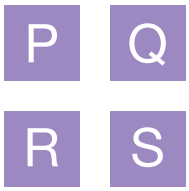
4:06 The multiplication rule for dependent events

If events are dependent we can still use the multiplication rule, provided that the second step, and any other steps that follow, are the probabilities of the events happening given that the earlier events have already happened. (These probabilities are **conditional probabilities**.)

> If the events A and B are dependent, the probability of both occurring is:
> $P(A \text{ and } B) = P(A) \times P(B)$, where $P(B)$ is the chance B occurs given that A has already occurred.

WORKED EXAMPLE 1

Two movies are to be chosen at random from the movies P, Q, R and S. The first movie chosen will be shown on Monday and not replaced (so the same movie is not shown on both nights). The second movie chosen will be shown on Tuesday. What is the probability that Movie P will be shown on Monday and Movie S will be shown on Tuesday?



Solution

Once the first movie has been chosen, there will only be three movies left from which to choose the second movie. (We are assuming that the same movie will not be shown on both nights.)

$P(A \text{ and } B) = P(A) \times P(B)$

$P(\text{P then S}) = P(\text{P}) \times P(\text{S if P has already been chosen})$

$= \frac{1}{4} \times \frac{1}{3}$ (Once the first is chosen, only 3 movies remain.)

$= \frac{1}{12}$

There is a 1 in 12 chance that Movie P will be shown on Monday and Movie S on Tuesday.

WORKED EXAMPLE 2

Two counters are chosen at random **without replacement** from three red and two green counters. Find the probability of choosing:

a two red counters **b** a red and a green in any order.

If three counters are chosen at random without replacement, find the probability of choosing:

c three red counters **d** two green then a red.

Solutions

a $P(A \text{ and } B) = P(A) \times P(B)$

$P(\text{two red}) = P(\text{red}) \times P(\text{red if a red has already been chosen})$

$= \frac{3}{5} \times \frac{2}{4}$ (Once the first red is chosen, 2 of the remaining 4 counters are red.)

$= \frac{6}{20}$ (or $\frac{3}{10}$)

b $P(\text{a red and a green in any order}) = P(\text{R then G}) + P(\text{G then R})$

$= (\frac{3}{5} \times \frac{2}{4}) + (\frac{2}{5} \times \frac{3}{4})$

$= \frac{6}{20} + \frac{6}{20}$

$= \frac{12}{20}$ (or $\frac{3}{5}$)

c $P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$

$P(3 \text{ red}) = P(\text{red}) \times P(\text{red if the first choice was red}) \times P(\text{red if the first two choices were red})$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$

$= \frac{6}{60}$ (or $\frac{1}{10}$)

d $P(\text{two green then a red})$

$= P(\text{G}) \times P(\text{G given G on the 1st choice}) \times P(\text{R given two greens chosen})$

$= \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}$

$= \frac{6}{60}$ (or $\frac{1}{10}$)

Using tree diagrams

Tree diagrams can be used to show the probabilities of events in two- and three-step experiments. We place the probability of each event on the arm leading to that event.

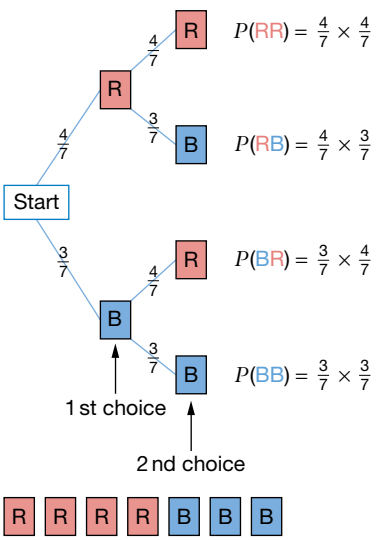
> If the probabilities are placed on the arms of a tree diagram, we can find the probability of a multi-step event by multiplying the probabilities on the arms of that event.
> This is also useful for compound events that can be achieved in more than one way.

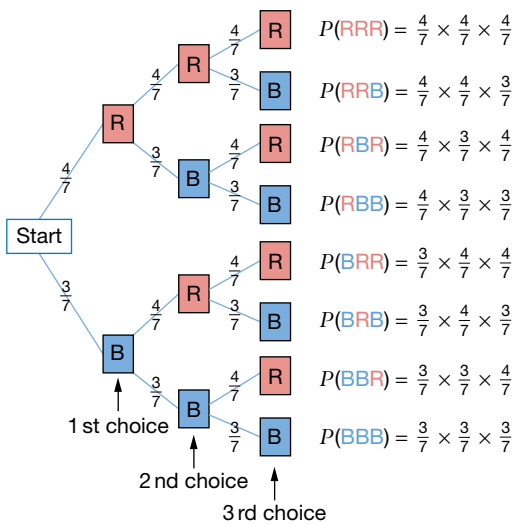
Note:

- On each arm of the tree diagram, the probability of that choice is shown. To find the probability of a particular outcome, we multiply the probabilities on the arms of that outcome.
- In **Case A** the choices are independent events, as the ticket chosen is replaced after each choice. The probabilities for each selection do not change from one choice to the next.
- In **Case B** the choices are dependent events, as the chosen ticket is not replaced after each choice. The probabilities for each selection change from one choice to the next as the number of tickets available to be chosen is decreasing. The probabilities for the second selection are not the same as the probabilities for the first.

Case A (Independent events)

Four red and three blue tickets are placed in a box and tickets are chosen at random **with replacement**. The probability tree diagrams for selecting two and three tickets are shown.





- If two tickets are chosen, a blue and a red ticket could be chosen in two ways.

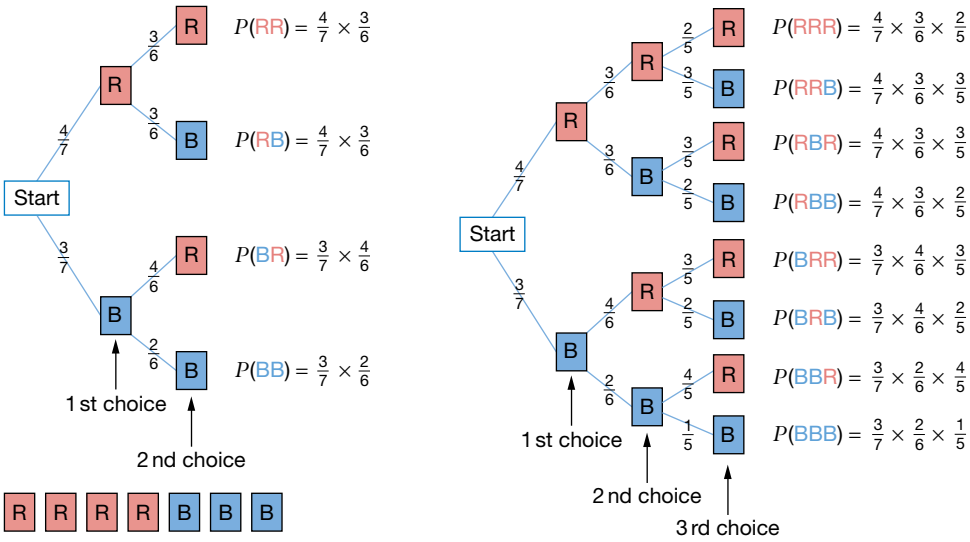
$$\begin{aligned} P(\text{blue and red}) &= P(\text{blue then red}) + P(\text{red then blue}) \\ &= P(\text{BR}) + P(\text{RB}) \\ &= \left(\tfrac{3}{7} \times \tfrac{4}{7}\right) + \left(\tfrac{4}{7} \times \tfrac{3}{7}\right) \\ &= \tfrac{24}{49} \end{aligned}$$

- If three tickets are chosen, two reds and a blue ticket could be chosen in three ways.

$$\begin{aligned} P(\text{2 reds and a blue}) &= P(\text{2 reds then blue}) + P(\text{red then blue then red}) + P(\text{blue then 2 reds}) \\ &= P(\text{RRB}) + P(\text{RBR}) + P(\text{BRR}) \\ &= \left(\tfrac{4}{7} \times \tfrac{4}{7} \times \tfrac{3}{7}\right) + \left(\tfrac{4}{7} \times \tfrac{3}{7} \times \tfrac{4}{7}\right) + \left(\tfrac{3}{7} \times \tfrac{4}{7} \times \tfrac{4}{7}\right) \\ &= \tfrac{144}{343} \end{aligned}$$

Case B (Dependent events)

Four red and three blue tickets are placed in a box and tickets are chosen at random **without replacement**. The probability tree diagrams for selecting two and three tickets are shown.



- If two tickets are chosen, a blue and a red ticket could be chosen in two ways.

P(blue and red) = P(blue then red) + P(red then blue)

$= P(BR) + P(RB)$

$= (\frac{3}{7} \times \frac{4}{6}) + (\frac{4}{7} \times \frac{3}{6})$

$= \frac{24}{42}$ (or $\frac{4}{7}$)

- If three tickets are chosen, two reds and a blue ticket could be chosen in three ways.

P(2 reds and a blue) = P(2 reds then blue) + P(red then blue then red) + P(blue then two reds)

$= P(RRB) + P(RBR) + P(BRR)$

$= (\frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}) + (\frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}) + (\frac{3}{7} \times \frac{4}{6} \times \frac{3}{5})$

$= \frac{108}{210}$ (or $\frac{18}{35}$)

Exercise 4:06

1 To select a captain and vice-captain from our team of five players we placed our five names in a hat and drew out two names at random, the first to be captain, the second to be vice-captain. What is the probability that:

a I will be captain and my team-mate, Jo, will be vice-captain

b Jo will be captain and I will not be vice-captain

c neither Jo nor I will be captain or vice-captain?

2 Two different given names are to be chosen at random from Hannah, Joy, Joanna and Belle. Hannah Joy is a different outcome to Joy Hannah. What is the probability that the outcome:

a will be Hannah Joy **b** will not be Hannah Joy **c** will start with Hannah?

3 Six cards, numbered 1 to 6, are placed in a hat. Two of these are chosen at random, without replacement, to make a two-digit number. The cards chosen are placed in order, the first becoming the tens digit. What is the probability that the number formed:

a will be 65 **b** will be 12 or 13 **c** will not be 12 or 13?

Three of these cards are chosen at random, without replacement, to make a three-digit number. The cards chosen are placed in order of choice, the first becoming the hundreds digit. What is the probability that the number formed:

d will be 123 **e** will be 123 or 124 **f** will not be 123 or 124?

4 Two tickets are chosen at random **without replacement**, from 20 tickets, marked 1 to 20. Find the probability of choosing:

a the 7 and then the 14 **b** two even numbers
c two odd numbers **d** two one-digit numbers
e two two-digit numbers **f** an odd then and even number
g two three-digit numbers **h** two multiples of 10.

If three of these tickets are chosen at random without replacement, find the probability of choosing:

i three one-digit numbers **j** the 5, the 10 and the 15 in that order
k three odd numbers **l** three numbers that are not all odd.

5 Two counters are chosen at random **without replacement**, from 2 red and 5 green counters. Find the probability of choosing counters that are:

a both red **b** not both red
c both green **d** a red and a green in either order.

If three counters are chosen at random without replacement, find the probability of choosing:

e three green counters **f** three red counters
g two green then a red **h** two green and a red in any order.

6 Tom's grandchildren each chose one jelly bean at random from a jar containing eight jelly beans. Six of these were green and two were red. Find the probability that the first two jelly beans chosen were:

a a green then a red
b both green
c both red
d not both red
e a red then a green
f a green and a red in any order.

Find the probability that the first three jelly beans chosen were:

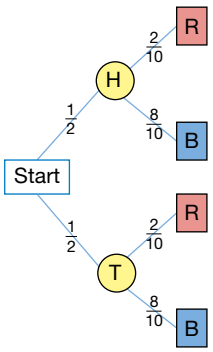
g two red and then a green **h** two green and then a red
i all red **j** not all red.

7 From a hat containing three yellow and five red tickets, three tickets are chosen at random **without replacement**.

a What is the probability of choosing three red tickets?
b What is the probability of choosing one red ticket and two yellow tickets in any order?

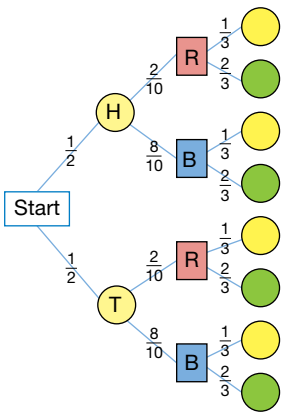
8 A coin is tossed and a ticket is chosen at random from a hat containing two red and eight blue tickets. The probability of each step has been placed on the appropriate branch of the tree diagram. Find the probability of obtaining:

a a head and a red ticket
b a head and a blue ticket
c a tail and a red ticket
d a tail and a blue ticket
e a tail and any colour of ticket
f a head and any colour of ticket
g a blue ticket with either a head or a tail
h a red ticket with either a head or a tail.



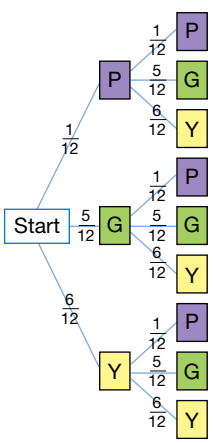
9 A coin is tossed, a ticket is chosen at random from a hat containing two red and eight blue tickets and a counter is chosen at random from one yellow and two green counters. The probability of each step has been placed on the appropriate branch of the tree diagram. Find the probability of obtaining:

a a head, a red ticket and a yellow counter
b a head, a blue ticket and a green counter
c a tail, a red ticket and a yellow counter
d a tail, a blue ticket and a yellow counter
e a head, a blue ticket and a yellow counter
f a tail, a red ticket and a green counter
g a head, any colour of ticket and a yellow counter
h a tail, any colour of ticket and a green counter.

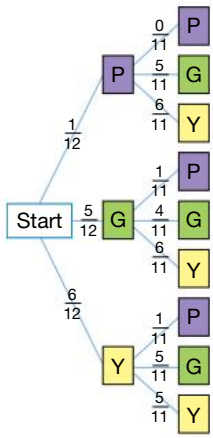


10 Two tickets are chosen at random from a hat containing one purple, five green and six yellow tickets. The first ticket **is replaced** before the second is chosen. The probability of each step has been placed on the appropriate branch of the tree diagram. Find the probability of choosing:

a two purple tickets
b two green tickets
c a green then a yellow
d a yellow then a green
e a green and a yellow in any order
f no yellow tickets
g at least one yellow
h two green tickets knowing that the first ticket was green
i two yellow tickets knowing that the first ticket was yellow
j two tickets that are not the same colour.

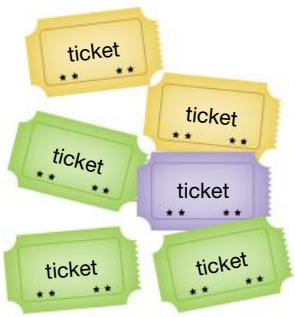


11 Two tickets are chosen at random from a hat containing one purple, five green and six yellow tickets. The first ticket **is not replaced** before the second is chosen. The probability of each step has been placed on the appropriate branch of the tree diagram. Find the probability of choosing:



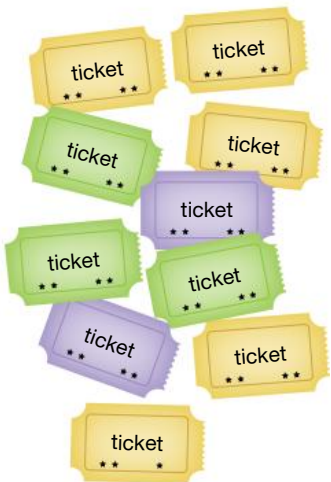
a two purple tickets
b two green tickets
c a green then a yellow
d a yellow then a green
e a green and a yellow in any order
f no yellow tickets
g at least one yellow
h two green tickets knowing that the first ticket was green
i two yellow tickets knowing that the first ticket was yellow
j two tickets that are not the same colour.

12 Two tickets are chosen at random from a hat containing one purple, two yellow and three green tickets. The first ticket **is replaced** before the second is chosen. Create a probability tree like the one in Question 10. Place the probability of each step on the appropriate branch of your tree diagram, and then use your diagram to find the probability of choosing:



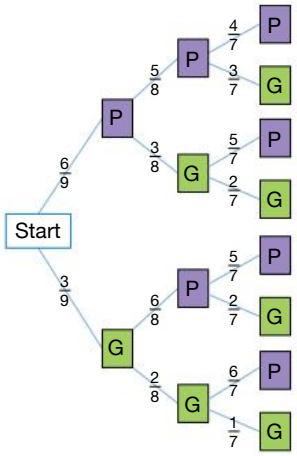
a two purple tickets
b two green tickets
c a green then a yellow
d a yellow then a green
e a green and a yellow in any order
f no yellow tickets
g at least one yellow ticket
h two green tickets knowing that the first ticket was green
i two yellow tickets knowing that the first ticket was yellow
j two tickets that are not the same colour.

13 Two tickets are chosen at random from a hat containing two purple, five yellow and three green tickets. The first ticket **is not replaced** before the second ticket is chosen. Create a probability tree like the one in Question 11. Place the probability of each step on the appropriate branch of your tree diagram, and then use your diagram to find the probability of choosing:



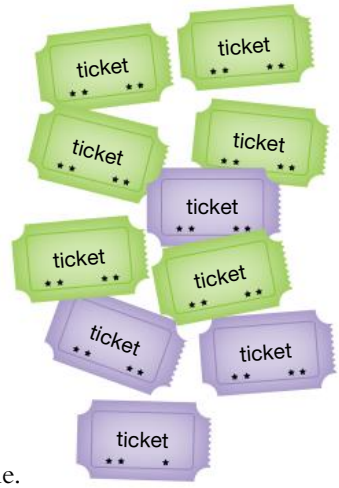
a two purple tickets
b two green tickets
c a green then a yellow
d a yellow then a green
e a green and a yellow in any order
f no yellow tickets
g at least one yellow
h two green tickets knowing that the first ticket was green
i two yellow tickets knowing that the first ticket was yellow
j two tickets that are not the same colour.

14 Three tickets are chosen at random **without replacement** from a hat containing six purple and three green tickets. The probability of each step has been placed on the appropriate branch of the tree diagram. Find the probability of choosing:



- **a** three purple tickets
- **b** fewer than three purple tickets
- **c** three green tickets
- **d** fewer than three green tickets
- **e** three tickets that are not all the same colour
- **f** a green then two purples
- **g** two purples then a green
- **h** a purple then a green then a purple
- **i** two greens and a purple in any order
- **j** any outcome where the last ticket chosen is green
- **k** any outcome where the last ticket chosen is not green
- **l** three green tickets knowing that the first ticket was green
- **m** three purple tickets knowing that the first ticket was purple.

15 Three tickets are chosen at random **without replacement** from a hat containing four purple and six green tickets. Create a probability tree like the one in Question 14. Place the probability of each step on the appropriate branch of your tree, and then use your diagram to find the probability of choosing:



- **a** three purple tickets
- **b** fewer than three purple tickets
- **c** three green tickets
- **d** fewer than three green tickets
- **e** three tickets that are not all the same colour
- **f** a green then two purples
- **g** two purples then a green
- **h** a purple then a green then a purple
- **i** two greens and a purple in any order
- **j** any outcome where the last ticket chosen is green
- **k** any outcome where the last ticket chosen is not green
- **l** three green tickets knowing that the first ticket was green
- **m** three purple tickets knowing that the first ticket was purple.

16 Find the probability of choosing three consecutive numbers at random, without replacement, from the numbers 1 to 9. (Hint: Use the multiplication rule for dependent events.)

MATHS TERMS 4

complementary event

- the complement of 'an event happening' is that 'the event does not happen',
  e.g. If the event is 'throwing a 6', the complementary event would be 'not throwing a 6'

dependent events

- the occurrence of the first event affects the chance of occurrence of the second event. The probability of the second event is affected by the outcome of the first

experimental probability

- the predicted chance of occurrence based on the relative frequencies taken from a chance experiment

$$\text{experimental probability} = \frac{\text{number of times event occurred}}{\text{total number of trials}}$$

independent events

- the occurrence of the first event does not affect the chance of occurrence of the second event. The probability of the second event is not affected by the outcome of the first

multiplication rule for multi-step events

- when two events are independent, the probability of both occurring is:
  $P(A \text{ and } B) = P(A) \times P(B)$
- when two events are dependent, the probability of both occurring is:
  $P(A \text{ and } B) = P(A) \times P(B)$, where $P(B)$ is the chance B occurs given that A has already occurred

mutually exclusive events

- events that cannot occur at the same time. They have no elements in common,
  e.g. tossing an odd number on a dice and tossing a 4

outcomes

- the possible results when a trial is to be performed,
  e.g. the possible outcomes when tossing a dice are 1, 2, 3, 4, 5, and 6

probability $P(E)$

- the chance of an event happening. This will be a number between 0 and 1 or equal to 0 or 1

random

- choosing at random does not affect the likelihood of any outcome

relative frequency

- this is the same as the experimental probability

$$\text{relative frequency} = \frac{\text{frequency of an event}}{\text{total frequency}}$$

sample space

- the set of all possible outcomes,
  e.g. 1, 2, 3, 4, 5 and 6 on a normal dice

theoretical probability

- the likelihood of an event occurring under ideal circumstances,
  e.g. the probability of rolling a 6 on a normal dice is $\frac{1}{6}$

trial

- one case of carrying out a chance experiment,
  e.g. one spin of a spinner

two- and three-step chance experiments

- each step is carried out in turn. The experiment may be undertaken with replacement or without replacement,
  e.g. draw two counters from a bag containing 2 blue and 3 red counters without replacement

DIAGNOSTIC TEST 4 PROBABILITY

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 a A card is chosen at random from a standard pack. Find the probability that the card is:
i a red King ii not a red King.
b Given that an odd number has been rolled on a normal dice, find the probability that the number is:
i 5 ii not 5. 4:01

2 Using a fair coin, I have tossed five heads in a row.
Is the probability of tossing a head on the next toss:
A less than 50% B 50% C more than 50%? 4:02

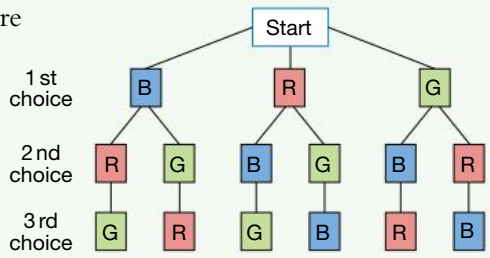
3 Three blue, two red and one green counter are placed in a bag. One is taken at random, replaced and then a second counter is chosen.
The table shows all possible outcomes.
a What is the probability that both counters are blue?
b What is the probability that both counters will be blue if we already know that the first counter chosen was blue?
c What is the probability that the two counters will be a red and a green (in any order)? 4:03

First choice \ Second choice	B_1	B_2	B_3	R_1	R_2	G_1
B_1	BB	BB	BB	BR	BR	BG
B_2	BB	BB	BB	BR	BR	BG
B_3	BB	BB	BB	BR	BR	BG
R_1	RB	RB	RB	RR	RR	RG
R_2	RB	RB	RB	RR	RR	RG
G_1	GB	GB	GB	GR	GR	GG

4 Three blue, two red and one green counter are placed in a bag. One is taken at random. This time, the first counter is not replaced before the second counter is chosen. The table shows all possible outcomes.
a What is the probability that both counters are blue?
b What is the probability that both counters will be blue if we already know that the first counter chosen was blue?
c What is the probability that the two counters will be a red and a green (in any order)? 4:03

First choice \ Second choice	B_1	B_2	B_3	R_1	R_2	G_1
B_1		BB	BB	BR	BR	BG
B_2	BB		BB	BR	BR	BG
B_3	BB	BB		BR	BR	BG
R_1	RB	RB	RB		RR	RG
R_2	RB	RB	RB	RR		RG
G_1	GB	GB	GB	GR	GR	

5 One blue, one red and one green ticket are placed in a bag. Three tickets are chosen without replacement after each choice is made. The tree diagram shows all possible outcomes.



Find the probability of choosing three tickets so that they are:
a red then blue then green b all blue
c three different colours. 4:03

6 Find the number of outcomes in the sample space if: 4:04
- a a fair dice is tossed twice
- b a coin is tossed three times
- c two tickets are chosen from five with replacement
- d two tickets are chosen from five without replacement
- e three tickets are chosen from five with replacement
- f three tickets are chosen from five without replacement.

7 Four yellow and two red tickets are in a box. Two tickets are to be chosen from the box at random without replacement. 4:04
- a What is the probability of choosing two yellow tickets?
- b What is the probability that the two tickets will be different colours?

8 Three yellow and five red counters are in a glass. Three counters are chosen at random, without replacement. 4:04
- a What is the probability of choosing three red counters?
- b What is the probability of choosing two yellows and a red?

9 a I have two containers. One has three green tickets and seven blue tickets. The other has two red tickets and three yellow tickets. If I choose a ticket from each container, what is the probability of my choosing a green ticket and a yellow ticket? 4:05

b A normal dice is rolled, and a fair spinner showing D, E, F and G is spun. What is the chance of rolling an odd number on the dice and spinning a G on the spinner?

10 Two movies are to be shown at random from the movies A, B, C, D and E. The first movie chosen will be shown on Monday and not replaced, and the second chosen will be shown on Tuesday. What is the probability that movie B will be shown on Monday and movie D will be shown on Tuesday? 4:06

11 a Two counters are chosen at random without replacement from three red and two green counters. Find the probability of choosing: 4:06
- i two red counters
- ii a red and a green in any order.

b If three counters are chosen at random without replacement from three red and two green counters, find the probability of choosing:
- i three red counters
- ii two green then a red.

ASSIGNMENT 4A Chapter review

1 From a standard pack of 52 cards, a card is chosen at random. What is the probability that the card chosen is:

a a number between 3 and 8
b a club that is a picture card
c not a red Jack
d not a club that is a picture card?

2 An unknown number of tickets were placed in a hat.
Two tickets were drawn from the hat without replacing the first ticket. This was repeated 100 times to obtain the results in the table. Use these results to find, as a percentage, the experimental probability of choosing:

a a blue and a green ticket
b two blue tickets
c fewer than two blue tickets
d exactly one blue ticket
e at least one blue ticket
f two tickets of the same colour.

Here is the result of 100 trials, taking two tickets from a hat without replacement.

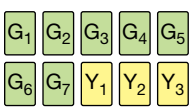
Outcome	Frequency
two blue	21
two red	12
two green	8
blue and red	30
blue and green	18
red and green	11
Total:	100

3 I bought 5 of the 100 tickets in a raffle. Three prizes are to be drawn in order.
Find the probability that:

a I will win the first prize drawn
b I will win the second prize drawn given that I did not win the first prize drawn
c I will win the third prize drawn given that I did not win either of the first two prizes
d I will win the third prize drawn given that I have already won one prize.

4 Use the pictures on the right to answer these questions. Find the probability of:

a spinning a C on the spinner then tossing a tail on a coin
b rolling a 5 on a dice then spinning a D on the spinner
c choosing a green ticket at random from these tickets then tossing a head on a coin
d spinning a vowel then tossing a head on a coin
e choosing a yellow ticket at random then spinning a letter of the word CAFE
f rolling an even number on a dice then spinning a consonant on the spinner.



ASSIGNMENT 4B Working mathematically

1 The travel graph shows the journeys of James and Callum between town A and town B. (They travel on the same road.)

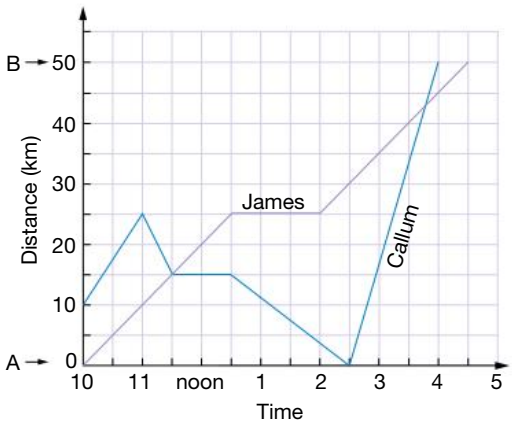
a How far from A is Callum when he commences his journey?

b How far is James from B at 2:30 pm?

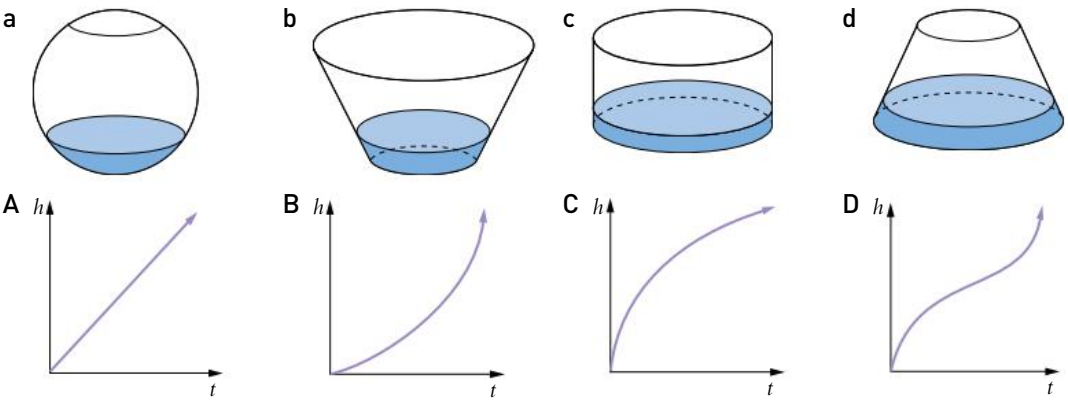
c When do James and Callum first meet?

d Who reaches town B first?

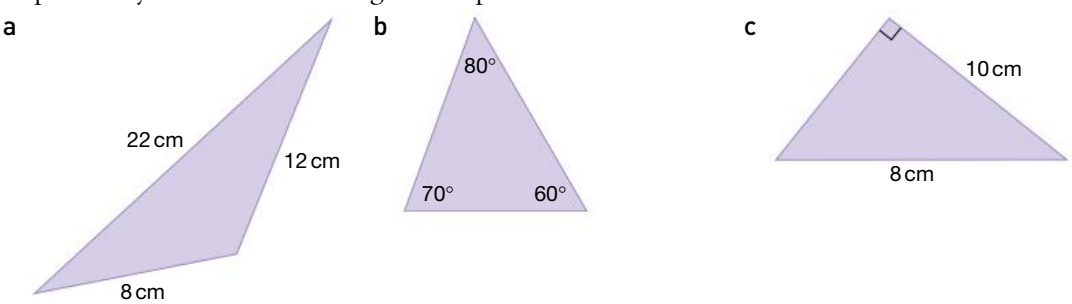
e At what time does Callum stop to rest?



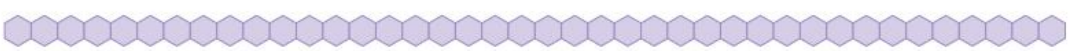
2 Water is poured into each container at a constant rate. The graphs indicate the height of the water in each container against time. Match each graph with the correct container.



3 Explain why each of these triangles is impossible.



4 How many sides are drawn when 40 regular hexagons are joined in a row, as as shown below?



5 How many two-digit numbers can be made from the digits 0, 1, 2, 3, 4 and 5 if the number is a multiple of 5, and no digit is used more than once in the same number?

ASSIGNMENT 4C Cumulative revision

1 State the number of significant figures in each of the following. 1:01G

a 3·05 **b** 0·025 **c** 10 000 (to the nearest thousand)

2 Convert the following. 1:01J

a 6 t to kg **b** 7 ML to L **c** 8000 ns to μs

3 Use scientific notation to write: 1:05

a 304 gigalitres as litres (L) **b** 1·8 terabytes as bytes (B)

4 Between which two measurements would the following lie? 1:1K

a 15·3 m **b** 6·25 s

5 Expand and simplify. 1:02

a $6(5m + 3) - 10$ **b** $5a - 3(a + 7)$ **c** $7(x - 5) + 3(5x - 2)$

6 For the scores 2, 3, 3, 3, 4, 8, 8, 9, 9 find: 1:12

a the range **b** the median **c** the mode

7 Factorise each trinomial. 2:04

a $x^2 + 3x - 28$ **b** $y^2 - 9y - 10$ **c** $x^2 + 11x + 24$

8 Solve: 2:05

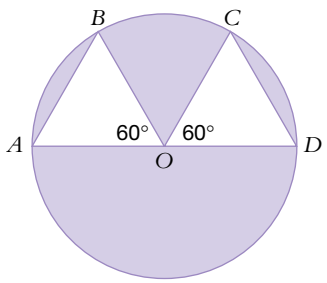
a $(y - 5)(y - 8) = 0$ **b** $(x + 3)(x - 4) = 0$ **c** $3m(m + 11) = 0$

9 Factorise the left-hand side of each equation and solve it. 2:05

a $t^2 + 8t = 0$ **b** $x^2 - 9 = 0$ **c** $y^2 + y - 12 = 0$

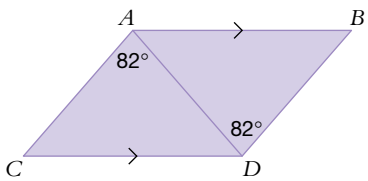
10 Prove that the two triangles are congruent, and that $AB = CD$. 3:02

a



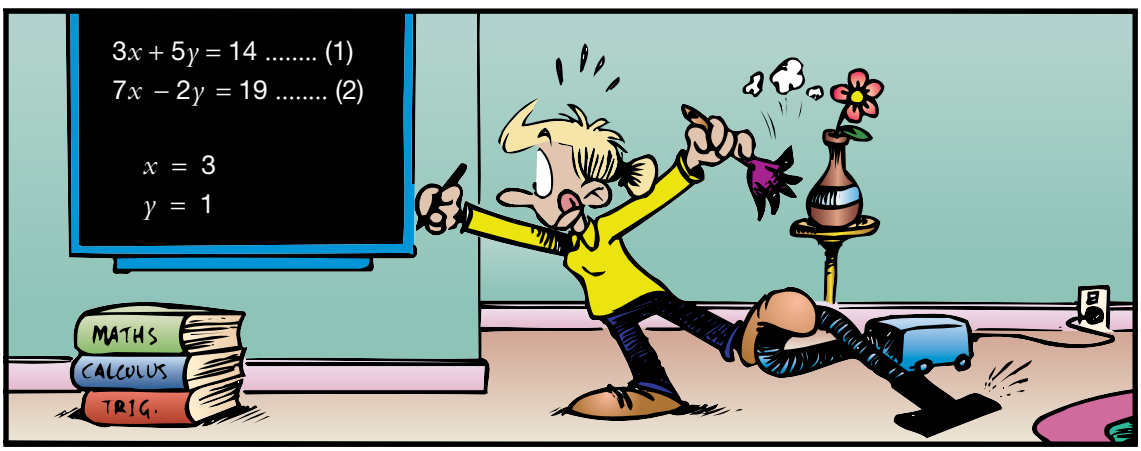
O is the centre.

b



SIMULTANEOUS EQUATIONS

5



Contents

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5:02B Elimination method
GeoGebra activity 5:02B Simultaneous equations: The elimination method

5:03 Using simultaneous equations to solve problems
Investigation 5:03 Solving problems by 'guess and check'

Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Sections from ***Equations*** [Stage 5.2]

- Solve linear simultaneous equations, using algebraic and graphical techniques, including with the use of digital technologies (ACMNA237)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

5:01 The graphical method of solution

PREP QUIZ 5:01

If $y = 2x - 1$, find y when:

1 $x = 1$ **2** $x = 0$ **3** $x = -1$ **4** $x = -5$

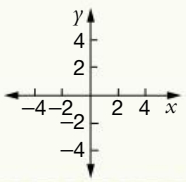
If $x - 2y = 5$, find y when:

5 $x = 0$ **6** $x = 1$ **7** $x = 2$ **8** $x = -4$

9 If $3x - y = 2$, complete the table below.

x	0	1	2
y			

10 Copy this number plane and graph the line $3x - y = 2$.



There are many situations in which we need to find when or where two conditions occur together. The following example illustrates this.

WORKED EXAMPLE 1

The sum of two numbers is 16. Their difference is 4. What are the numbers?

Graphical solution

We are looking for two numbers, which we will call x and y, that satisfy both conditions.

We write the two conditions as equations.

1 'The sum is 16' becomes $x + y = 16$.

2 'The difference is 4' becomes $x - y = 4$.

If we graph these two equations, we can find the point where the graphs cross.

> The values of x and y at the point of intersection satisfies both conditions, as the point lies on both lines.

These values of x and y are the two numbers we have been asked to find.

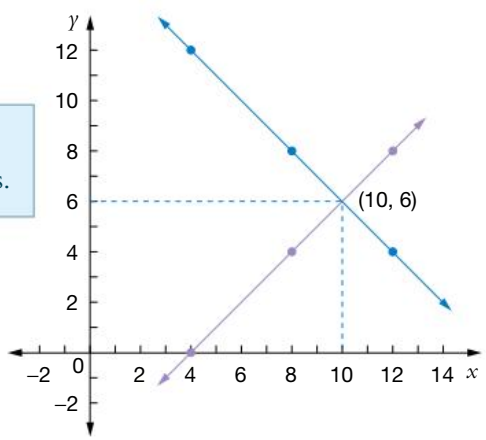
- From the graph, we can see that the lines cross at (10, 6), so the simultaneous solution is $x = 10$ and $y = 6$.
- Therefore, the numbers 10 and 6 have a sum of 16 and a difference of 4.

$x + y = 16$

x	4	8	12
y	12	8	4

$x - y = 4$

x	4	8	12
y	0	4	8



To solve problems involving two conditions, the given information can usually be written in the form of two equations. The equations are then graphed using a table of values (as shown in the previous example). The point of intersection of the graphs tells us when and where the two conditions occur together.

WORKED EXAMPLE 2

Solve the following equations simultaneously.

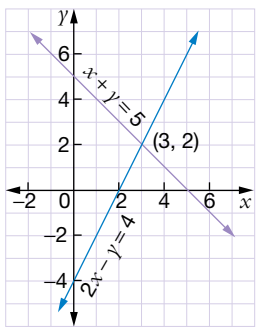
$x + y = 5$

$2x - y = 4$

Graphical solution

You will remember from your earlier work on coordinate geometry that when the solutions to an equation such as $x + y = 5$ are graphed on a number plane, they form a straight line.

Hence, to solve the equations $x + y = 5$ and $2x - y = 4$ simultaneously, we could simply graph each line and find the point of intersection. Since this point lies on both lines, its coordinates give the solution.



$x + y = 5$

x	0	1	2
y	5	4	3

$2x - y = 4$

x	0	1	2
y	−4	−2	0

- The lines $x + y = 5$ and $2x - y = 4$ intersect at $(3, 2)$.
  Therefore the solution is:
  $x = 3$
  $y = 2$

To solve a pair of simultaneous equations graphically, we graph each line.
The solution is given by the coordinates of the point of intersection of the lines.

It is sometimes difficult to graph lines accurately and it is often difficult to read the coordinates of the point of intersection.

Despite these problems, the graphical method remains an extremely useful technique for solving simultaneous equations.

Exercise 5:01

P Foundation worksheet 5:01
Graphical method of solution

1 Use the graph to write the solutions to the following pairs of simultaneous equations.

a $y = x + 1$
$x + y = 3$

b $y = x + 1$
$x + 2y = -4$

c $y = x + 3$
$3x + 5y = 7$

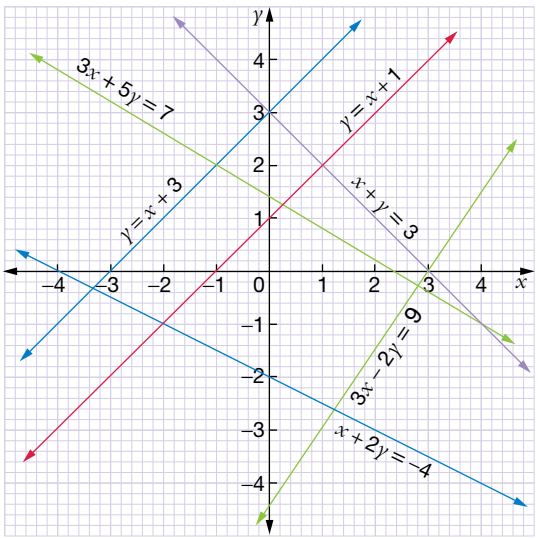
d $y = x + 3$
$x + y = 3$

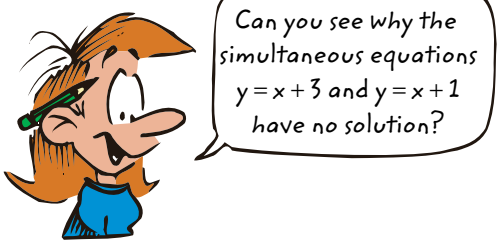
e $x + y = 3$
$3x + 5y = 7$

f $3x - 2y = 9$
$x + y = 3$

g $y = x + 3$
$y = x + 1$

h $y = x + 1$
$2y = 2x + 2$





2 Use the diagram in Question 1 to estimate, correct to one decimal place, the solutions of the following simultaneous equations.

a $y = x + 1$
$3x + 5y = 7$

b $y = x + 3$
$x + 2y = -4$

c $3x - 2y = 9$
$x + 2y = -4$

d $3x - 2y = 9$
$3x + 5y = 7$

3 Solve each of the following pairs of equations by graphical means. All solutions are integral (i.e. they are whole numbers).

a $x + y = 1$
$2x - y = 5$

b $2x + y = 3$
$x + y = 1$

c $x - y = 3$
$2x + y = 0$

d $3x - y - 2 = 0$
$x - y + 2 = 0$

e $3a - 2b = 1$
$a - b = 1$

f $p + 2q = 2$
$p - q = -4$

g $3a + 2b = 5$
$a = 1$

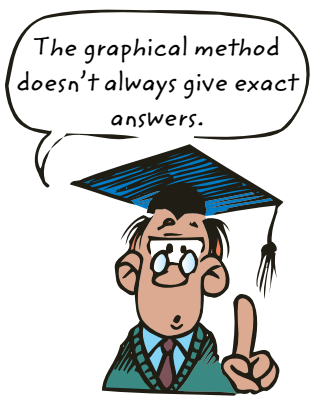
h $p = 6$
$p - q = 4$

4 Solve each pair of simultaneous equations by the graphical method. (Use a scale of 1 cm to 1 unit on each axis.)

a $y = 4x$
$x + y = 3$

b $3x - y = 1$
$x - y = 2$

c $x = 4y$
$x + y = 1$



5 Estimate the solution to each of the following pairs of simultaneous equations by graphing each, using a scale of 1 cm to 1 unit on each axis. Give the answers correct to one decimal place.

a $4x + 3y = 3$
$x - 2y = 1$

b $x - y = 2$
$8x + 4y = 7$

c $4a - 6b = 1$
$4a + 3b = 4$

6 The sum of two numbers is 15. Their difference is 3.
Use the graphical method of solving simultaneous equations to find the two numbers.

WORKED EXAMPLE 3 (EXTENSION)

A runner set off from a point and maintained a speed of 9 km/h. Another runner left the same point 10 minutes later, followed the same course, and maintained a speed of 12 km/h. When, and after what distance travelled, would the second runner have caught up to the first runner?

We have chosen to solve this question graphically.

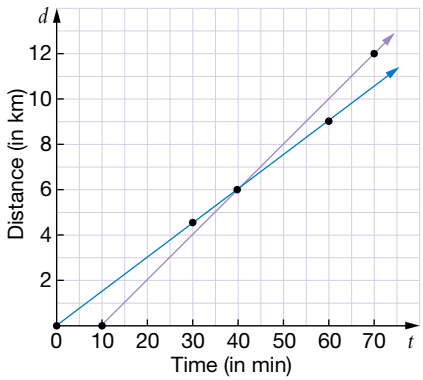
First runner

t	0	30	40	60
d	0	4·5	6	9

Second runner

t	10	30	40	70
d	0	4	6	12

When $t = 40$, the second runner has been running for 30 minutes and has travelled 6 km.



t = time in minutes after the first runner begins
d = distance travelled in kilometres

- From the graph, we can see that the lines cross at (40, 6).
- The simultaneous solution is $t = 40$, $d = 6$.
- The second runner caught the first runner 40 min after the first runner had started and when both runners had travelled 6 km.

7 A car passed a point on a course at exactly 12 noon and maintained a speed of 60 km/h. A second car passed the same point 1 hour later, followed the same course, and maintained a speed of 100 km/h. When, and after what distance from this point, would the second car have caught up to the first car? (Hint: Use the method shown in Worked Example 3 but leave the time in hours.)

8 Mary's salary consisted of a retainer of $480 a week plus $100 for each machine sold in that week. Bob had no retainer, but was paid $180 for each machine sold. Study the tables below, graph the lines, and use them to find the number, N, of machines Bob would have to sell to have a wage equal to Mary (assuming they both sell the same number of machines). What salary, S, would each receive for this number of sales?

Mary

N	0	4	8
S	480	880	1280

N = number of machines

Bob

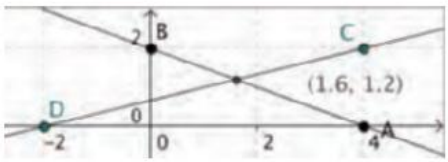
N	0	4	8
S	0	720	1440

S = salary

GEOGEBRA ACTIVITY 5:01 — SIMULTANEOUS EQUATIONS: THE GRAPHICAL METHOD

Two different activities explore the graphical solution.

1 Two lines and their equations are displayed. Use the point of intersection to estimate the solutions to the equations. Use a checkbox to test the accuracy of your solutions. Click and drag points on the lines to generate a new set of lines and equations.



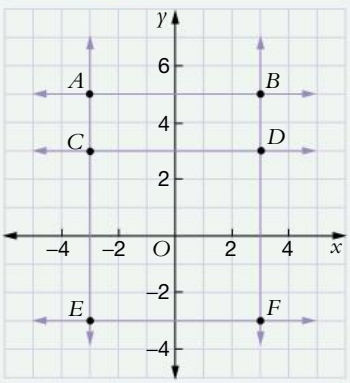
2 Two equations are generated randomly. Click and drag the two given lines to match the graphs of the given equations. The lines will change colour when correct. The solution to the equations can be read from the intersection of the lines.

FUN SPOT 5:01 — WHAT DID THE BOOK SAY TO THE LIBRARIAN?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Write the equation of:

A line AB	**C** line OB
U line BF	**A** line EB
I the y-axis	**O** line AF
U line OF	**K** line AE
E line CB	**T** the x-axis
T line EF	**N** line OD
Y line CD	**O** line OA



$y = \frac{5}{3}x$	$y = \frac{4}{3}x + 1$	$y = x$	$x = 0$	$y = 0$	$y = 5$	$x = -3$	$y = \frac{1}{3}x + 4$	$y = 3$	$y = -\frac{5}{3}x$	$x = 3$	$y = -\frac{4}{3}x + 1$	$y = -x$	$y = -3$

5:02 The algebraic method of solution

PREP QUIZ 5:02

Solve the equations:
1 $7x = 42$ **2** $5y = 12$ **3** $10x = 7$

Simplify:
4 $7x - 9x$ **5** $-5y + 3y$ **6** $4x - (-2x)$

Find the value of y if $x = 4$ in the following.
7 $x + y = 2$ **8** $2x + y = 7$ **9** $x + 3y = 13$ **10** $3x - 4y = 0$

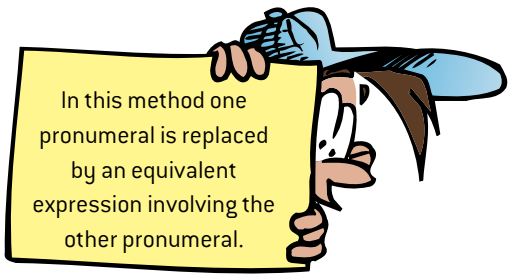
In the previous section, we noted that the graphical method of solving simultaneous equations lacked accuracy for many questions. Because of this, we need a method that gives the exact solution. There are two such algebraic methods:

- the substitution method
- the elimination method.

5:02A Substitution method

WORKED EXAMPLE 1

Solve the simultaneous equations:
$2x + y = 12$ and $y = 5x - 2$



Solution

When solving simultaneous equations, first 'number' the equations involved.

$2x + y = 12$ 1
$y = 5x - 2$ 2

Now from 2 we can see that $5x - 2$ is equal to y.
If we substitute this for y in equation 1, we have:

$$\begin{aligned} 2x + (5x - 2) &= 12 \\ 7x - 2 &= 12 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

So the value of x is 2. This value for x can now be substituted into either equation 1 or equation 2 to find the value for y.

In 1:
$2(2) + y = 12$
$4 + y = 12$
$y = 8$

In 2:
$y = 5(2) - 2$
$= 10 - 2$
$= 8$

So, the solution is:
$x = 2, y = 8$

To check this answer substitute into equations 1 and 2.

WORKED EXAMPLE 2

Solve the simultaneous equations:
$3a + 2b = 7$ and $4a - 3b = 2$

Solution

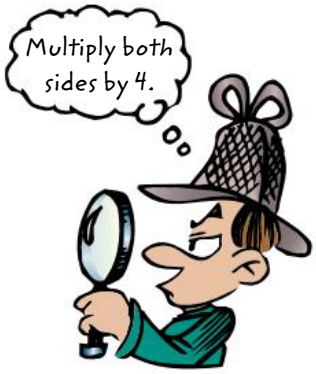
$3a + 2b = 7$ 1
$4a - 3b = 2$ 2

Making a the subject of 2 gives:

$$a = \frac{2 + 3b}{4}$$

If we substitute this expression for a into equation 1, we get:

$$3\left(\frac{2+3b}{4}\right) + 2b = 7$$
$$3(2 + 3b) + 8b = 28$$
$$6 + 9b + 8b = 28$$
$$17b = 22$$
$$b = \tfrac{22}{17}$$



Substituting this value for b into equation 2 gives:

$$4a - 3(\tfrac{22}{17}) = 2$$
$$4a - \tfrac{66}{17} = \tfrac{34}{17}$$
$$4a = \tfrac{100}{17}$$
$$a = \tfrac{25}{17}$$

So, the solution is:

$a = \frac{25}{17}, b = \frac{22}{17}$

To check your answer, substitute $a = \frac{25}{17}, b = \frac{22}{17}$ in equations 1 and 2.

Exercise 5:02A

P Foundation worksheet 5:02A
The substitution method

1 Solve the following pairs of equations using the substitution method. Check all solutions.

a $x + y = 3$ and $y = 4$
b $x + y = 7$ and $y = x + 3$
c $x + y = -3$ and $y = x + 1$
d $x - y = 5$ and $y = 1 - x$
e $2x + y = 9$ and $y = x - 3$
f $2x + y = 8$ and $y = x - 4$
g $2x - y = 10$ and $y = 10 - 3x$
h $x + 2y = 9$ and $y = 2x - 3$
i $2x + y = 14$ and $x = 6$
j $2x + y = 7$ and $x = y - 4$

2 Use one of each pair of equations to express y in terms of x. Then use the method of substitution to solve the equations. Check all solutions.

a $x + 2y = 4$
$x - y = 7$

b $2x - 3y = 4$
$2x + y = 6$

c $x + 2y = 8$
$x + y = -2$

d $x - y = 2$
$x + 2y = 11$

e $2x - y = -8$
$2x + y = 0$

f $x + y = 5$
$2x + y = 7$

g $x + 2y = 11$
$2x - y = 2$

h $3x + y = 13$
$x + 2y = 1$

i $3x + 2y = 2$
$2x - y = -8$

3 Solve the following simultaneous equations using the substitution method.

a $2x - y = 1$
$4x + 2y = 5$

b $3a + b = 6$
$9a + 2b = 1$

c $m - 2n = 3$
$5m + 2n = 2$

d $4x - 2y = 1$
$x + 3y = -1$

4 Solve the following pairs of simultaneous equations.

a $2a - 3b = 1$
$4a + 2b = 5$

b $7x - 2y = 2$
$3x + 4y = 8$

c $3m - 4n = 1$
$2m + 3n = 4$

d $2x - 3y = 10$
$5x - 3y = 3$

P GEOGEBRA ACTIVITY 5:02A — SIMULTANEOUS EQUATIONS: THE SUBSTITUTION METHOD

This activity explores the substitution method of solution. Choose from two different types of equations. Then try to solve the two randomly generated equations. Use a checkbox to obtain a worked solution.

$x - y = 2$ (1)
$x - 5y = 26$ (2)
$x = 2 + y$ (3)

$(2 + y) - 5y = 26$ Substitute (3) in (2)
$2 + y - 5y = 26$
$-4y = 24$
$y = -6$
$x = 2 + -6$ Substitute in (3)
$x = -4$

5:02B Elimination method

WORKED EXAMPLE 1

Solve the pair of simultaneous equations.
$5x - 3y = 20$
$2x + 3y = 15$

In this method, one of the pronumerals is eliminated by adding or subtracting the equations.

Solution

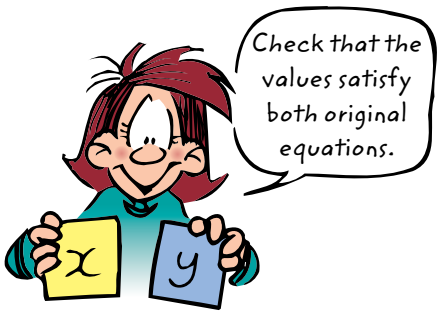
You add or subtract the equations, depending upon which operation will eliminate one of the pronumerals.

First, number each equation.
$5x - 3y = 20$ 1
$2x + 3y = 15$ 2

Now if these equations are 'added', the y terms will be eliminated, giving:
$7x = 35$
$x = 5$

Substituting this value into equation 1 gives:
$5(5) - 3y = 20$
$25 - 3y = 20$
$3y = 5$
$y = \frac{5}{3}$ or $1\frac{2}{3}$

$\therefore$ The solution is $x = 5$, $y = 1\frac{2}{3}$.
Check in 1: $5(5) - 3(1\frac{2}{3}) = 20$ (true)
Check in 2: $2(5) + 3(1\frac{2}{3}) = 15$ (true)



WORKED EXAMPLE 2

Solve the pair of simultaneous equations.
$x + 5y = 14$
$x - 3y = 6$

Take one step at a time.

Solution

First, number each equation.
$x + 5y = 14$ 1
$x - 3y = 6$ 2

Now, if equation 2 is 'subtracted' from equation 1, the x terms are eliminated.
$8y = 8$
$y = 1$

Substituting this value into 1 gives:
$x + 5(1) = 14$
$x + 5 = 14$
$x = 9$

$\therefore$ The solution is $x = 9$, $y = 1$.
Check in 1: $9 + 5(1) = 14$ (true)
Check in 2: $9 - 3(1) = 6$ (true)

WORKED EXAMPLE 3

Solve the pair of simultaneous equations.
$2x + 3y = 21$
$5x + 2y = 3$

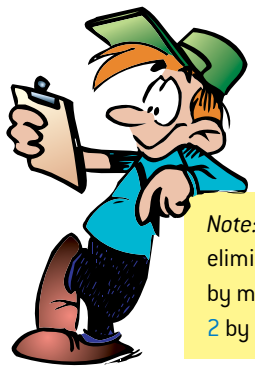
To eliminate a pronumeral, the size of the coefficients in each equation must be made the same by multiplying one or both equations by a constant.

Solution

First, number each equation.
$2x + 3y = 21$ 1
$5x + 2y = 3$ 2

Multiply equation 1 by 2 and equation 2 by 3.
$4x + 6y = 42$ 1★
$15x + 6y = 9$ 2★

Now, 2★ is subtracted from 1★, so the y terms are eliminated.
$-11x = 33$
$x = -3$



Note: x could have been eliminated instead of y, by multiplying 1 by 5 and 2 by 2.

Substituting this value into 1 gives:
$2(-3) + 3y = 21$
$-6 + 3y = 21$
$3y = 27$
$y = 9$

$\therefore$ The solution is $x = -3$, $y = 9$.
Check in 1: $2(-3) + 3(9) = 21$ (true)
Check in 2: $5(-3) + 2(9) = 3$ (true)

Exercise 5:02B

1 Add the equations to solve each pair of equations using the elimination method.

a $x + y = 9$
$x - y = 1$

b $x + y = 14$
$2x - y = 1$

c $2x + y = 7$
$x - y = 2$

d $x + 2y = 3$
$x - 2y = 7$

e $3x - 2y = 5$
$x + 2y = 7$

f $5x - 2y = 1$
$3x + 2y = 7$

g $x + 3y = 10$
$-x + y = 6$

h $-x + 2y = 12$
$x + 2y = -4$

i $3x + y = 11$
$-3x + 2y = 10$

j $2x + 7y = 5$
$x - 7y = 16$

k $5x - 2y = 0$
$4x + 2y = 9$

l $7x + 5y = -3$
$2x - 5y = 21$

2 Subtract the equations to solve each pair of equations using the elimination method.

a $2x + y = 5$
$x + y = 3$

b $5x + y = 7$
$3x + y = 1$

c $10x + 2y = 2$
$7x + 2y = -1$

d $3x - 2y = 0$
$x - 2y = 4$

e $5x - y = 14$
$2x - y = 2$

f $x - 3y = 1$
$2x - 3y = 5$

g $2x + y = 10$
$x + y = 7$

h $2x + 5y = 7$
$2x + y = 5$

i $5x - y = 16$
$5x - 3y = 8$

j $6x + y = 13$
$6x - y = 11$

k $2x + 5y = 20$
$3x + 5y = 17$

l $7x - 2y = 1$
$4x - 2y = 4$

3 Solve these simultaneous equations using the elimination method.

a $2x + y = 7$
$x - y = -4$

b $x + y = 5$
$2x - y = 1$

c $x - y = 12$
$2x + y = 3$

d $3x + 2y = 2$
$x - 2y = -10$

e $2x + 3y = 13$
$4x - 3y = -1$

f $3x + 4y = -1$
$3x - 2y = -10$

g $5x + 2y = 1$
$3x - 2y = 7$

h $7x - 3y = 31$
$7x + y = -1$

i $8x - 2y = 34$
$8x + 4y = 4$

4 After multiplying either or both of the equations by a constant, use the elimination method to solve each pair of equations.

a $x + y = 7$
$2x + 3y = 17$

b $2x + y = 7$
$x + 2y = 11$

c $5x + y = 12$
$3x + 2y = 10$

d $4x - y = 10$
$x + 3y = 9$

e $4x - y = 6$
$3x + 2y = -1$

f $5x - 2y = -16$
$x + 3y = 7$

g $12x - 3y = 18$
$4x + 2y = 0$

h $3x - 7y = 2$
$9x + 5y = 32$

i $2x + 3y = 8$
$3x + 2y = 7$

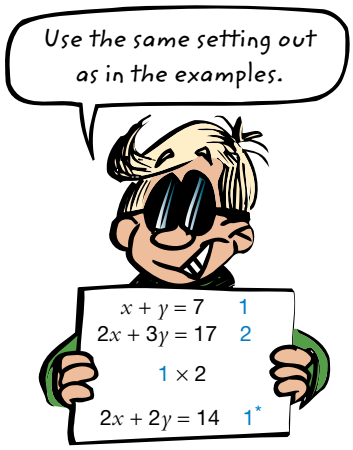
j $5x + 2y = 10$
$4x + 3y = 15$

k $5x + 2y = 28$
$3x + 5y = 51$

l $2x + 2y = -2$
$3x - 5y = -19$

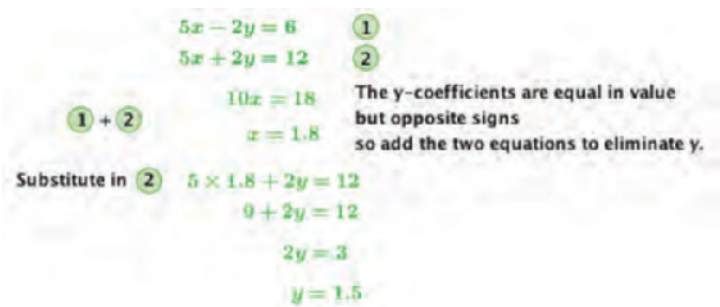
m $7x + 3y = 4$
$5x + 2y = 3$

n $2x - 4y = -2$
$3x + 5y = 45$



P GEOGEBRA ACTIVITY 5:02B
SIMULTANEOUS EQUATIONS: THE ELIMINATION METHOD

This activity explores the elimination method of solution. Choose from four different types of equations. Then try to solve the two randomly generated equations. Use a checkbox to obtain a worked solution.



5:03 Using simultaneous equations to solve problems

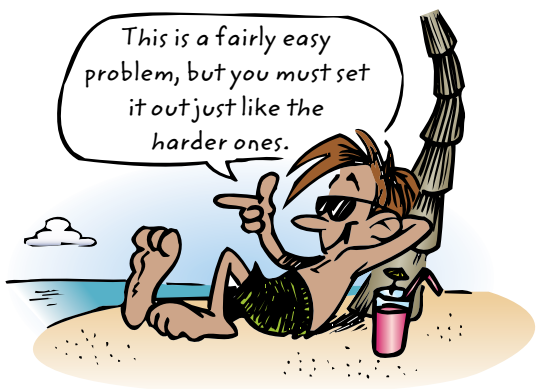
Simultaneous equations can be used to solve problems that involve two conditions, often in an easier way than with only one equation. The same techniques we use to solve single equations are used here.

Remember:

- Read the question carefully.
- Work out what the problem wants you to find.
- Use pronumerals to represent the unknown quantities.
- Translate the words of the question into mathematical expressions.
- Form equations by showing how different mathematical expressions are related.
- Solve the equations.
- Finish off with a sentence stating the value of the quantity or quantities that were found.

WORKED EXAMPLE

Adam is 6 years older than his sister, Bronwyn.
If the sum of their ages is 56 years, find their ages.



Solution

Let Adam's age be x years.
Let Bronwyn's age be y years.
Adam is 6 years older than Bronwyn.
$\therefore x = y + 6$ 1
The sum of their ages is 56 years.
$\therefore x + y = 56$ 2
Solving these simultaneously using the methods shown in 5:02 gives:
$x = 31$ and $y = 25$.
$\therefore$ Adam is 31 years old and Bronwyn is 25 years old.

Exercise 5:03

P Foundation worksheet 5:03
Using simultaneous equations to solve problems
Challenge worksheet 5:03
Solving three simultaneous equations

1 Form pairs of simultaneous equations and solve the following problems.
Let the numbers be x and y.

a The sum of two numbers is 25 and their difference is 11.
Find the numbers.

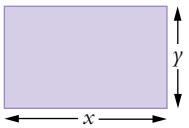
b The sum of two numbers is 97 and their difference is 33.
Find the numbers.

c The sum of two numbers is 12, and one of the numbers is three times the other. Find the numbers.

d The difference between two numbers is 9 and the smaller number plus twice the larger number is equal to 24. Find the numbers.

e The larger of two numbers is equal to 3 times the smaller number plus 7. Also, twice the larger number plus 5 times the smaller is equal to 69. Find the numbers.

2 Each of the following problems has two unknown quantities, and two pieces of information. Form two simultaneous equations and solve each problem.



a The length of a rectangle is 5 cm more than the width. If the perimeter of the rectangle is 22 cm, find the length and the width.

b One pen and one pencil cost $2. Two pens and three pencils cost $4.70. Find the cost of each.

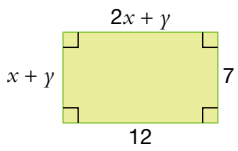
c If a student's maths mark exceeded her science mark by 15, and the total marks for both tests was 129, find each mark.

d Six chocolates and three drinks cost $14.70, whereas three chocolates and two drinks cost $8.60. Find the price of each.

e Bill has twice as much money as Jim. If I give Jim $2.50, he will have three times as much as Bill. How much did Bill and Jim have originally?

3 Form two equations from the information on each figure to find values for x and y.

a



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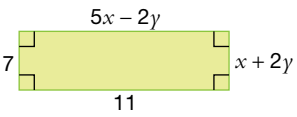

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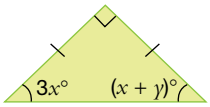
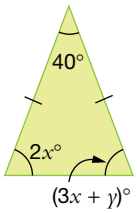
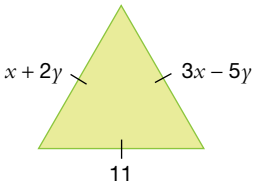


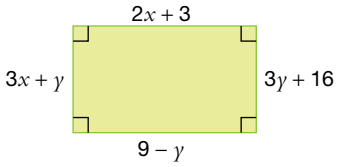
d



e



f



4 a John buys 100 plants for a total of $630. He buys either grevilleas for $3 each or waratahs for $8 each. How many of each plant does he buy?

b A truck is loaded with two different types of boxes. If 150 of box A and 115 of box B are loaded onto the truck, its capacity of 10 tonnes is reached. If 300 of box A are loaded, then the truck can only take 30 of box B before the capacity of 10 tonnes is reached. Find the weight of each box.

c A theatre has 2100 seats. All of the rows of seats in the theatre have either 45 seats or 40 seats. If there are three times as many rows with 45 seats than those with 40 seats, how many rows are there?

d A firm has five times as many junior workers as it does senior workers. If the weekly wage for a senior is $620 and for a junior is $460, find how many of each are employed if the total weekly wage bill is $43 800.

5 Use graphical methods to solve the following problems.

a Esther can buy scarves for $6 each. She bought a roll of material for $20 and gave it to a dressmaker, who then charged $3.50 to make each scarf. How many scarves would Esther need to have made for the cost to be the same as buying them for $6 each.

b Star Bicycles had produced 3000 bicycles and were producing 200 more per week. Prince Bicycles had produced 2500 bicycles and were producing 300 more each week. After how many weeks would they have produced the same number of bicycles?

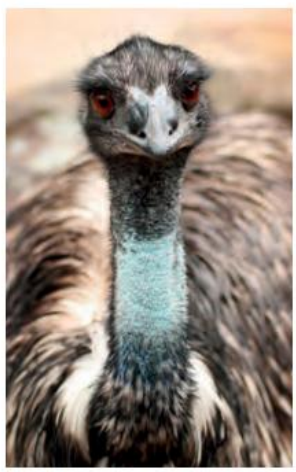
INVESTIGATION 5:03 SOLVING PROBLEMS BY 'GUESS AND CHECK'

Consider the following problem.

> A zoo enclosure contains wombats and emus. If there are 50 eyes and 80 legs, find the number of each type of animal.

We know that each animal has two eyes. A wombat has four legs and an emu has two legs. We could try to solve this problem by guessing a solution and then checking it.

Solution



If each animal has two eyes, then, because there are 50 eyes, I know there must be 25 animals.

If my first guess is 13 wombats and 12 emus, then the number of legs would be $13 \times 4 + 12 \times 2 = 76$.

Since there are more legs than 76, I need to increase the number of wombats to increase the number of legs to 80.

I would eventually arrive at the correct solution of 15 wombats and 10 emus, which gives the correct number of legs ($15 \times 4 + 10 \times 2 = 80$).

Try solving these problems by guessing and then checking various solutions.

1 Two numbers add to give 86 and subtract to give 18. What are the numbers?

2 At the school disco, there were 52 more girls than boys. If the total attendance was 420, how many boys and how many girls attended?

3 In scoring 200 runs, Max hit a total of 128 runs as boundaries. (A boundary is either 4 runs or 6 runs.) If he scored 29 boundaries in total, how many boundaries of each type did he score?

4 Sharon spent $5158 buying either Company A shares or Company B shares. These were valued at $10.50 and $6.80 respectively. If she bought 641 shares in total, how many of each did she buy?

• In this chapter you have learnt to solve questions like these using simultaneous equations. Solve each of these problems using simultaneous equations.

MATHS TERMS 5

elimination method
- solving simultaneous equations by adding or subtracting the equations together to eliminate one pronumeral

graphical solution
- the solution obtained by graphing two equations in the number plane and observing the point of intersection
- if the point of intersection is $(3, -2)$, then the solution is $x = 3$ and $y = -2$

guess and check
- a method of solving problems by guessing a solution and then checking to see if it works. Solutions are modified until the correct solution is found

simultaneous equations
- when two (or more) pieces of information about a problem can be represented by two (or more) equations
- these are then solved to find the common or simultaneous solution,
  e.g. the equations $x + y = 10$ and $x - y = 6$ have many solutions but the only simultaneous solution is $x = 8$ and $y = 2$

substitution method
- solving simultaneous equations by substituting an equivalent expression for one pronumeral in terms of another, obtained from another equation,
  e.g. if $y = x + 3$ and $x + y = 7$, then the second equation could be written as $x + (x + 3) = 7$ by substituting for y using the first equation

DIAGNOSTIC TEST 5 SIMULTANEOUS EQUATIONS

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

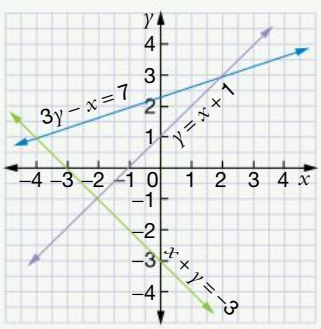
5:01

1 Use the graph to solve the following simultaneous equations.

a $x + y = -3$
$y = x + 1$

b $y = x + 1$
$3y - x = 7$

c $3y - x = 7$
$x + y = -3$



5:02A

2 Solve the following simultaneous equations using the substitution method.

a $y = x - 2$
$2x + y = 7$

b $x - y = 5$
$2x + 3y = 2$

c $4a - b = 3$
$2a + 3b = 11$

5:02B

3 Solve the following simultaneous equations using the elimination method.

a $2x - y = 3$
$3x + y = 7$

b $4x - 3y = 11$
$2x + y = 5$

c $2a - 3b = 4$
$3a - 2b = 6$

ASSIGNMENT 5A Chapter review

1 Solve the following simultaneous equations using the most suitable method.

a $x + y = 3$
$2x - y = 6$

b $4x - y = 3$
$2x + y = 5$

c $4a + b = 6$
$5a - 7b = 9$

d $6a - 3b = 4$
$4a - 3b = 8$

e $a - 3b = 5$
$5a + b = 6$

f $2x - 3y = 6$
$3x - 2y = 5$

g $p = 2q - 7$
$4p + 3q = 5$

h $4x - y = 3$
$4x - 3y = 7$

i $7m - 4n - 6 = 0$
$3m + n = 4$

2 A man is three times as old as his daughter. If the difference in their ages is 36 years, find the ages of father and daughter.

3 A theatre can hold 200 people. If the price of admission was \$5 per adult and \$2 per child, find the number of each present if the theatre was full and takings were \$577.

4 A man has 100 shares of stock A and 200 shares of stock B. The total value of the stocks is \$420. If he sells 50 shares of stock A and buys 60 shares of stock B, the value of his stocks is \$402. Find the price of each type of share.

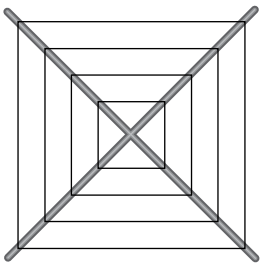
5 Rectangle A is three times as long as rectangle B and twice as wide. If the perimeters are 50 cm and 20 cm respectively, find the dimensions of the larger rectangle.

6 A rectangle has a perimeter of 40 cm. If the length is reduced by 5 cm and 5 cm is added to the width, it becomes a square. Find the dimensions of the rectangle.

7 A kayaker paddles at 16 km/h with the current and 8 km/h against the current. Find the speed of the current.

ASSIGNMENT 5B Working mathematically

1 You need to replace the wire in your clothesline. Discuss how you would estimate the length of wire required.



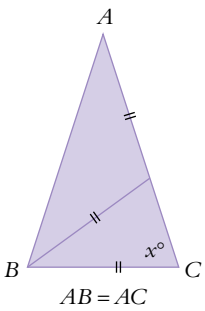
a On what measurements would you base your estimate?

b Is it better to overestimate or underestimate?

c What level of accuracy do you feel is necessary? The diagram shows the arrangement of the wire.

2 What is the last digit of the number 3^{2004}?

3 Two smaller isosceles triangles are joined to form a larger isosceles triangle as shown in the diagram. What is the value of x?



4 In a round-robin competition each team plays every other team. How many games would be played in a round-robin competition that had:

a three teams **b** four teams

c five teams **d** eight teams?

5 In how many different ways can you select three chocolates from five?

6 A school swimming coach has to pick a medley relay team. The team must have four swimmers, each of whom must swim one of the four strokes. From the information in the table choose the fastest combination of swimmers.

Name	Back	Breast	Fly	Free
Dixon	37·00	44·91	34·66	30·18
Wynn	37·17	41·98	36·59	31·10
Goad	38·88			
Nguyen	41·15	49·05	39·07	34·13
McCully		43·01		32·70
Grover		43·17		
Harris			37·34	34·44

What is the fastest medley relay?

ASSIGNMENT 5C Cumulative revision

1 **a** Find the net pay for a week if Akram earns \$728, is taxed \$136, pays \$74 as superannuation and has miscellaneous deductions of \$53. What percentage of his gross pay (to one decimal place) was paid in tax? 1:07

b Hannah was given a 14% discount on a lounge suite. What was the marked price if the discount was \$224?

2 Use the diagram on the right to find: 1:08

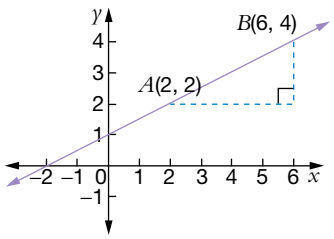
a the length of interval AB (1 dec. pl.)

b the midpoint of interval AB

c the gradient of line AB

d the y-intercept of line AB

e the x-intercept of line AB.

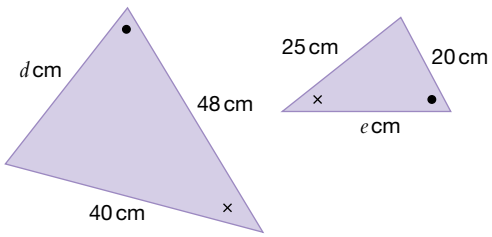


3 **a** Are these triangles similar? 1:09

b Find the value of d.

c Find the value of e.

d Calculate the length and breadth of a scale drawing of a rectangular space 28 m long and 15 m wide if a scale of 1 : 80 is used.



4 Given that $v = u + at$: 1:10

a find the value of v if $u = 20$, $a = 9{\cdot}8$ and $t = 4$

b find the value of u if $v = 76$, $a = 9{\cdot}8$ and $t = 5$

c find the value of t if $u = 30$, $a = 9{\cdot}8$ and $v = 49{\cdot}6$

5 Expand and simplify: 2:04

a $(m + 5)(m - 3)$ **b** $(a - 3)(a + 3)$ **c** $(2y + 3)(7y + 5)$

6 Students in Years 9 and 10 were asked to choose either History or Geography as an optional subject. The table shows their choices. 1:03

	History	Geography	Total
Year 9	63	54	117
Year 10	75	68	143
Total	138	122	260

What is the probability that a student, chosen at random:

a would be from Year 9

b would have chosen Geography

c would be a Year 10 student who chose History?

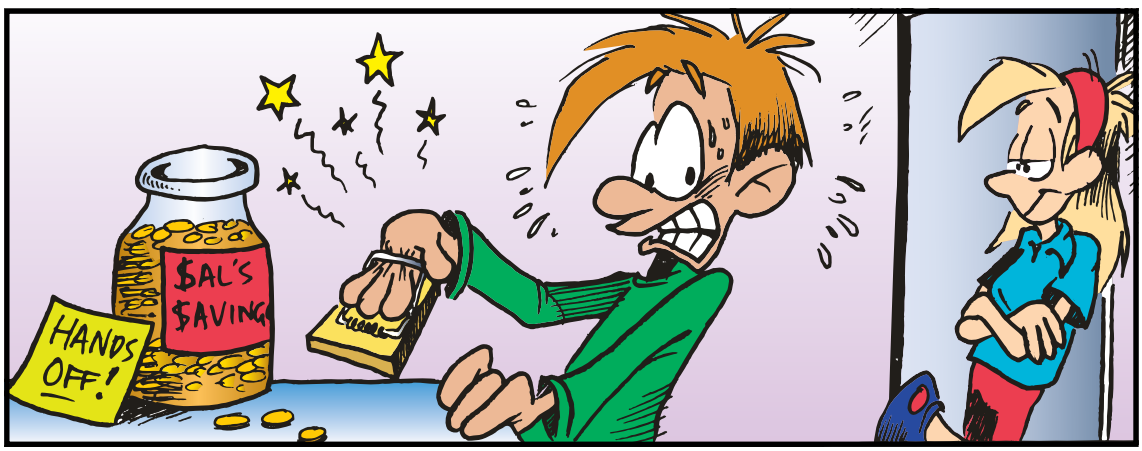
7 A six-sided dice is rolled and a coin is tossed. Find the probability that: 4:05

a a 3 is rolled and a head is tossed

b a tail is tossed and a number greater than 2 is rolled.

FINANCIAL MATHEMATICS

6



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Fun spot 6:04 What is the difference between a book and a bore?
6:05 Depreciation
6:06 Compound interest and depreciation formulas
Investigation 6:06 Compound interest tables
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Financial Mathematics*** [Stages 5.1, 5.2$^{\diamond}$]

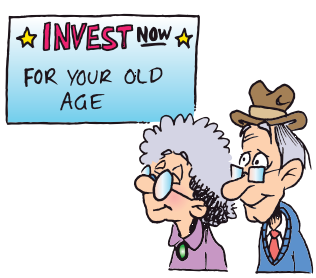
- Solve problems involving simple interest (ACMNA211)
- Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

In Year 9 we concentrated on aspects of earning and spending money. Now we will consider aspects of saving and investing money.

6:01 Saving money

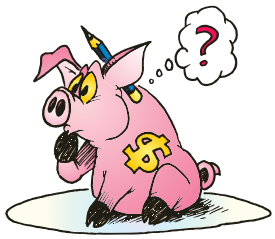
Savings accounts	Target and award saver accounts	Term deposit accounts
Meaning		
Usually a transaction card is used to deposit or withdraw your money.	A savings account from which you cannot withdraw money without forfeiting interest.	Your money is invested for a fixed period of time, usually at an agreed interest rate.
Advantages		
You can deposit and withdraw without notice. Safe, encourages saving and may help you get a loan.	Better interest rate than savings account. Encourages saving by reducing the temptation to withdraw your money.	Higher interest rates than savings accounts and this rate is usually fixed for the period of the investment. No extra bank charges.
Disadvantages		
Lower interest rates are offered for savings accounts. Bank fees apply.	If you withdraw your money early you do not receive the higher rate of interest, but receive a lower rate instead. This causes inconvenience if some of the money is needed urgently.	It usually requires the investment of a minimum amount. The fixed amount invested cannot be withdrawn before the end of the agreed period (or term) without reducing the interest rate for the investment.
		

When saving or investing money:

1. Compare rates of interest (they are continually changing).
2. Consider the terms of the investment carefully.
3. Ask questions about terms, services and safety.
4. Higher interest rates are often given where there is inconvenience or some risk to your money.

Exercise 6:01

1 Write answers in your own words. Refer to the table on the previous page if necessary.



- **a** What are the advantages of a normal *savings* account?
- **b** What is a *term deposit* account?
- **c** What are the advantages of a *term deposit* account?
- **d** What are the disadvantages of a *term deposit* account?
- **e** What are the disadvantages of a normal *savings* account?
- **f** What are the advantages of a *target savings* account?
- **g** What are the disadvantages of an *award saver* account?

2 Angus decided to open an *award saver* account. He chose to bank $200 each month, starting with a balance of $200. The money will earn 2·65% p.a. interest calculated daily and paid monthly as long as no withdrawals are made during that month. If a withdrawal is made, the interest rate for that month falls to 0·5% p.a.

- **a** What rate of interest applies if a withdrawal is made?
- **b** What rate of interest applies if no withdrawal is made?
- **c** How often is interest calculated?
- **d** How often is interest paid?
- **e** If 2·65% interest is given for 1 year (365 days), what percentage interest would be given for one day? (Give your answer correct to three decimal places.)

3 Tomeka was given the following information when asking about a *term deposit* account.

Term rates—Interest paid at maturity—Interest rates p.a.						
	3–<6 months	6–<8 months	8–<10 months	10–<12 months	12–<24 months	24 months
$500–$4999	4·10%	4·10%	4·15%	4·20%	4·25%	4·35%
$5000–$9999	4·60%	4·60%	4·65%	4·70%	4·75%	4·85%
$10 000–$19 999	4·60%	4·65%	4·70%	4·75%	4·85%	4·92%
$20 000–$49 999	4·65%	4·70%	4·75%	4·80%	4·87%	4·95%

- **a** If Tomeka has $4000 to invest, what is the shortest term necessary to receive 4·20% p.a. for the investment?
- **b** What interest rate is given on:
  - **i** $12 000 invested for 10 months
  - **ii** $9000 invested for 7 months
  - **iii** $45 000 invested for 18 months?

4 Emma opened a special cash management account that offered interest of 2·5% p.a. for a balance under $5000, 4·25% p.a. for a balance between $5000 and $10 000, 4·75% p.a. for a balance between $10 000 and $20 000, and 6% p.a. for a balance over $20 000. Her first five cash or personal cheque withdrawals each month are free of bank transaction fees. After that she pays $3 per transaction. She needed $5000 to open this account.

a How many transactions are allowed each month before bank transaction fees are charged? What are these transaction fees?

b What interest rate would Emma receive if her balance is:

i $8150 ii $1809 iii $17 080?

c How much did Emma need to open this account?

5 A major Australian bank was offering these interest rates for their ordinary savings accounts in 2012.

Balance	Interest rate
Balances from $0 to less than $10 000	0·01% p.a.
Balances between $10 000 and $19 999	0·01% p.a.
Balances between $20 000 and $49 999	0·01% p.a.
Balances between $50 000 and $99 999	0·05% p.a.
Balances over $100 000	0·20% p.a.

Note:
This is 0·01%
i.e. $\frac{1}{10\,000}$
[Not 1%]

For these savings accounts a monthly account-keeping fee of $4 applied, unless you satisfied a condition that granted an exemption. One such condition was being under 21 years of age.

a Calculate the interest a balance of $1000 would earn in an ordinary savings account for:

i 1 year ii 1 month (nearest cent)

b How much would have to be in the account each month to earn enough interest to cover the $4 account-keeping fee?

6 The bank in Question 5 also had special youth savings accounts for those under 18. These accounts paid 4·00% p.a. interest on top of the rates shown in the table. However, this extra interest is only paid at the end of each month if there is at least one deposit during the month and no withdrawals are made.

a What interest rate applies for a youth savings account with a balance of $1000?

b If the extra conditions did not apply, calculate the interest a balance of $1000 would earn in a 'Youth' savings account for:

i 1 year

ii 1 month (Give your answer to the nearest cent.)

c Henry opened an account on his 16th birthday on 31 July with a deposit of $5000 given to him by his grandparents. He deposited a further $100 on 1 September.

i How much interest did Henry earn for the month of August?

ii How much interest did Henry earn for the month of September?

INVESTIGATION 6:01 WHO CAN YOU BANK ON?

Most people have some type of savings account with a bank. They all offer a range of accounts that have different terms and conditions. The interest rates may vary as well as the fees and charges that often go unnoticed by customers when they sign up.

Go to the websites of the major banks and community banks, and look for the following information.

- The types of savings accounts on offer.
- Interest rates paid on deposits, including how often the interest is paid.
- Fees that are charged, including monthly fees, withdrawal fees, service fees and ATM fees.
- Term deposits on offer and the interest rates that apply.

You could also seek this information from other financial institutions such as credit unions and compare with the major banks.

6:02 Simple interest

PREP QUIZ 6:02

What fraction of a year is:

1 1 month **2** 5 months **3** 11 months

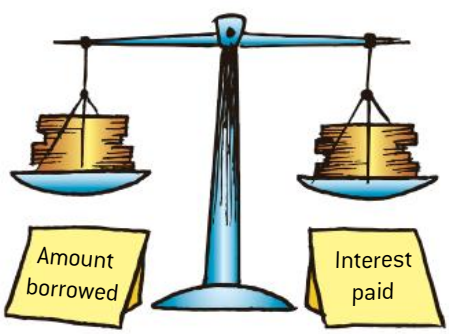
4 1 day **5** 7 days **6** 128 days?

To find 9·5% of \$800 press:
9·5 ÷ 100 × 800 =

Find:

7 9% of \$650 **8** 13·6% of \$800 **9** $37\frac{1}{2}$% of \$40 000 **10** 95% of \$7.80

- **Interest** is the payment made for the use of money invested (or borrowed).
- The money invested (or borrowed) is called the **principal**.
- **Simple interest** is interest paid on the original principal. The same interest is paid for each time period.



What is the simple interest on \$750 invested at 8% p.a. for 3 years?

Interest for 1 year is 8% of \$750 = \$60

For simple interest, the same amount of interest is earned each year.
Simple interest = (interest for 1 year) × (number of years)

p.a. means per annum or per year

Interest for 3 years = (8% of \$750) × 3
= \$60 × 3
= \$180

Interest on \$100 at a rate of 15% p.a. simple interest for 1 year
= \$100 × 15% × 1
= \$100 × 0·15 × 1 or $\$\frac{100}{1} \times \frac{15}{100} \times \frac{1}{1}$

Interest on \$100 at a rate of 15% p.a. simple interest for 7 months
= \$100 × 15% × $\frac{7}{12}$ (7 months = $\frac{7}{12}$ of a year)

Simple interest

$$I = PRN$$

- If R is given as % per annum, then N must be in years.
- If R is given as % per month, then N must be in months.

where I is the simple interest
P is the principal invested
R is the rate of interest
N is the number of time periods

WORKED EXAMPLES

1 Find the simple interest paid on \$860 invested at $6\frac{1}{2}$% p.a. for 5 years.

2 What is the simple interest on \$2400 at 8% p.a. for 5 months?

3 Find the simple interest on \$900 for 240 days at a rate of 11% p.a.

4 What is the simple interest paid on \$1950 invested for 7 months at 0·75% per month?

Solutions

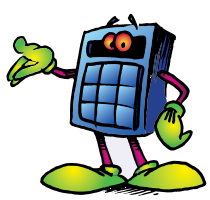
1 $I = PRN$
$= 860 \times 6\frac{1}{2}\% \times 5$
$= 860 \times 0{\cdot}065 \times 5$
$= \$279.50$
∴ The simple interest is \$279.50.

2 $I = PRN$
$= 2400 \times 8\% \times \frac{5}{12}$
$= 2400 \times 0{\cdot}08 \times 5 \div 12$
$= \$80$
∴ The simple interest is \$80.

5 months is $\frac{5}{12}$ years

3 $I = PRN$
$= 900 \times 11\% \times \frac{240}{365}$
$= 900 \times 0{\cdot}11 \times 240 \div 365$
$\doteqdot \$65.10$
∴ The simple interest is \$65.10

4 $I = PRN$
$= 1950 \times 0{\cdot}75\% \times 7$
$= 1950 \times (0{\cdot}75 \div 100) \times 7$
$= 1950 \times 0{\cdot}0075 \times 7$
$\doteqdot \$102.38$
∴ The simple interest is \$102.38.



To change 0·75% to a decimal, press ·75 ÷ 100 = on your calculator.
e.g. 1950 × 0·75% × 7 becomes 1950 × ·75 ÷ 100 × 7 =

Exercise 6:02

P Foundation worksheet 6:02
Simple interest

1 Find the simple interest charged for a loan of:

a $620 at 18% p.a. for 4 years
b $4500 at 26% p.a. for 5 years
c $15.60 at 15% p.a. for 3 years
d $391 at 9% p.a. for 7 years
e $90 000 at 7% p.a. for 25 years
f $90 000 at 11% p.a. for 25 years
g $500 for 2 years at 14% p.a.
h $25 000 for 6 years at 8% p.a.
i $16 860 for 10 years at 9% p.a.
j $2758 for 5 years at 10% p.a.

2 Find the simple interest earned on:

a $98 at 6·4% p.a. for 5 years
b $9500 at 7·2% p.a. for 3 years
c $120.80 at $6\frac{1}{2}$% p.a. for 2 years
d $8.40 at $5\frac{1}{2}$% p.a. for 7 years
e $600 at 5·8% p.a. for 4 years
f $850 at 7·1% p.a. for 6 years
g $138 at $8\frac{1}{4}$% p.a. for 2 years
h $140 at $10\frac{3}{4}$% p.a. for 14 years.

3 Find the simple interest charged for a loan of:

a $2000 for 3 months at 1·1% per month
b $8550 for 2 months at 1·6% per month
c $96 for 7 months at 0·9% per month
d $900 for 18 months at 0·7% per month
e $280 000 for 6 months at 1·2% per month
f $89.75 for 1 month at 0·9% per month
g $697.80 for 5 months at 1·25% per month
h $896.40 for 3 months at 0·25% per month
i $30 465 for 7 months at 1·3% per month
j $806 for 9 months at 1·125% per month.

4 Find the simple interest on:

11 months is $\frac{11}{12}$ years

a $6200 at 12% p.a. for 6 months
b $1150 at 10% p.a. for 1 month
c $1150 at 10% p.a. for 7 months
d $20 000 at 7% p.a. for 11 months
e $8400 at 17% p.a. for 9 months
f $9516 at 13% p.a. for 11 months
g $28.20 at 12% p.a. for 5 months
h $708 at 8% p.a. for 1 month
i $1848 for 5 months at 14% p.a.
j $540 for 7 months at 4% p.a.

5 Find the simple interest (to the nearest cent) earned on:

a $490 for 76 days at 16% p.a.
b $1096 for 207 days at 26% p.a.
c $740 for 9 days at 37% p.a.
d $9700 for 304 days at 9% p.a.
e $6000 for 40 days at $10\frac{1}{4}$% p.a.
f $50 000 for 30 days at $6\frac{1}{2}$% p.a.
g $3865 for 23 days at $9\frac{3}{4}$% p.a.
h $65 for 203 days at $3\frac{1}{2}$% p.a.
i $961.80 for 407 days at 11·8% p.a.
j $4150 for 15 days at 12·75% p.a.

$10\frac{1}{4}\% = 10{\cdot}25 \div 100$
$= 0{\cdot}1025$

6:03 Solving simple interest problems

PREP QUIZ 6:03

Write each of these percentages as a decimal.

1 18% **2** 5% **3** 5·4% **4** $5\frac{1}{2}\%$ **5** $5\frac{1}{4}\%$

If $I = PRN$:

6 find I when $P = 2000$, $R = 5\%$ and $N = 2$

7 find I when $P = 100$, $R = 0{\cdot}1$ and $N = 3$

8 find P when $I = 5$, $R = 0{\cdot}1$ and $N = 10$

9 find R when $I = 32$, $P = 80$ and $N = 4$

10 find N when $I = 120$, $P = 500$ and $R = 0{\cdot}08$.

Simple interest formula: $I = PRN$

WORKED EXAMPLE 1

Rhonda borrowed \$78 000 for 4 months at 12·75% p.a. simple interest to pay for her new house while her old one was being sold. How much interest did she pay?

Solution

$P = \$78\,000$

$N = 4$ months or $\frac{4}{12}$ years

$R = 12{\cdot}75\%$ or $0{\cdot}1275$

$I = \ldots$

$I = PRN$

$= 78\,000 \times 12{\cdot}75\% \times \frac{4}{12}$

$= 78\,000 \times 0{\cdot}1275 \times \frac{4}{12}$

$= \$3315$

$\therefore$ Rhonda paid \$3315 in interest.

WORKED EXAMPLE 2

I paid \$8000 to borrow \$50 000 for 2 years. What was the rate of simple interest charged?

Solution

$I = \$8000$

$P = \$50\,000$

$N = 2$ years

$R = \ldots$

$I = PRN$

$8000 = 50\,000 \times R \times 2$

$= 100\,000\ R$

Divide both sides by 100 000.

$0{\cdot}08 = R$

$R = 8\%$

$\therefore$ I was charged interest at a rate of 8% p.a.

WORKED EXAMPLE 3

Greg borrowed $1200 at 1·5% simple interest per month. Which is the best estimate of the interest charges for six months: $20, $100, $200 or $1800?

Solution

Here we are asked to make an estimate. Your reasoning could be as follows.

Each month Greg pays $1.5 for every $100 borrowed (i.e. 1·5%).
That's $15 for $1000, so it's more than $15 for $1200.
He'd pay 6 times as much for 6 months, so the interest must be more than $90.
The best of these estimates is $100.

Exercise 6:03

1 **a** Jack borrowed $14 695 at 8% p.a. simple interest for 5 years so that he could buy a car. At the end of the 5 years, both interest and loan had been paid.
- **i** How much interest was charged?
- **ii** How much was paid back altogether?

b Scott borrowed $4000 from Mona to be repaid after 3 years, along with 9% p.a. simple interest. How much will Mona receive after 3 years?

c Erika invested $8400 in the credit union for 11 months at 1·2% per month simple interest. How much interest did she earn?

d Jenny invests $4000 for 3 years at 6·5% p.a. simple interest, while Robert invests $4000 for 3 years at 5·8% p.a. simple interest. How much more interest does Jenny receive than Robert?

e Sam borrowed $60 000 and paid it back over 5 years. During this time he was charged 9·5% p.a. simple interest on the original amount borrowed. How much interest did he pay?

f What simple interest would $8000 earn in 3 months if the rate of interest is $11\frac{1}{4}\%$ p.a?

g A building society pays interest daily. If an amount of $15 640 were invested at 5·5% p.a, how much simple interest would be earned in:
- **i** 1 day
- **ii** 3 days
- **iii** 29 days?

2 **a** Each year 350 people are allowed to join a club. Of these an average of 46% are women. How many women would you expect to join the club in 4 years?

b Twenty-five loads of soil, each 9·6 tonnes when loaded, were taken to the sports complex to top-dress the new oval. An average of 3·5% of each load was lost in transit. How much soil was lost?

c When James got a job, he decided to give his parents 22% of his weekly net pay. This was his contribution to family expenses. If his weekly net pay was $316.80, how much did he give his parents in the first 15 weeks?

d Julia heard that a famine was causing great suffering to many people in Africa. She decided that for the next 7 months she would give 15% of her savings of $248 per month towards relief for famine victims. How much did she give?

Do not use a calculator for Question 3.

3 **a** Morgan borrowed \$7000 at 8·25% p.a. simple interest. Which is the best estimate of the interest charged for 6 years: \$35, \$350, \$550 or \$3500?

b Luke borrowed \$880 at 11% p.a. simple interest for 9 months. Which is the best estimate of the interest charged: \$8, \$80, \$800 or \$8000?

c Tess borrowed \$140 000 at 9·75% p.a. interest over 20 years. Which is the best estimate of the interest charged: \$2800, \$28 000, \$280 000 or \$2 800 000?

d Rajiv invested \$40 000 at 4·9% p.a. interest for 7 months. Which is the best estimate of the interest earned: \$100, \$1000, \$10 000 or \$100 000?

4 When I invest money, at the end of the time of investment I am given the principal invested plus the interest.

a How much will I receive after 3 years if I invest \$10 000 at 7% p.a. simple interest?

b How much will I receive after 18 months if I invest \$3700 at 10% p.a. simple interest?

c How much will I receive after 7 months if I invest \$30 000 at 1·1% p.a. per month simple interest?

d How much will I receive after 10 months if I invest \$14 600 at 9·5% p.a. simple interest?

5 When an amount of money is borrowed over a period of time, the amount to be repaid is equal to the amount borrowed plus the interest charged.

a How much must I pay back if I borrow \$10 000 for 3 years at 10% p.a. simple interest?

b How much must I pay back if I borrow \$108 000 for 25 years at a simple interest rate of 6·25% p.a?

c How much must I pay back if I borrow \$5000 at $8\frac{1}{2}\%$ simple interest for 6 years?

d How much must I pay back if I borrow \$30 000 at 12% p.a. simple interest for 3 months?

6 **a** Find the simple interest rate (to the nearest per cent) that will allow \$2500 to earn \$675 interest in 3 years.

b What sum of money would you have to invest for 5 years at 11% simple interest to produce interest at \$3080?

c For how many years would \$8000 have to be invested at $11\frac{1}{2}\%$ simple interest to produce \$5520 interest?

d What simple interest rate would allow \$6000 to grow to an amount of \$14 550 in 10 years?

INVESTIGATION 6:03 FINANCIAL SPREADSHEETS

Below are two versions of a spreadsheet.

- The first table shows the income earned by a student over 4 years.
- The numbers down the left and the letters across the top allow us to use coordinates to name any cell of the spreadsheet.
- The second table shows the formula used to obtain the cells in column F and row 10.

	A	B	C	D	E	F
1	**Income**	**2003 (Y7)**	**2004 (Y8)**	**2005 (Y9)**	**2006 (Y10)**	**Total**
2	Odd jobs	212	264	220	160	$856
3	Selling newspapers	364	380	0	0	$744
4	Mowing lawns	60	260	180	45	$545
5	Baby-sitting	0	140	235	380	$755
6	Typing	0	0	20	110	$130
7	Cashier	0	0	1654	1840	$3,494
8	Washing cars	104	260	86	52	$502
9						
10	**Total**	**$740**	**$1,304**	**$2,395**	**$2,587**	**$7,026**

	A	B	C	D	E	F
1	**Income**	**2003 (Y7)**	**2004 (Y8)**	**2005 (Y9)**	**2006 (Y10)**	**Total**
2	Odd jobs	212	264	220	160	=SUM(B2:E2)
3	Selling newspapers	364	380	0	0	=SUM(B3:E3)
4	Mowing lawns	60	260	180	45	=SUM(B4:E4)
5	Baby-sitting	0	140	235	380	=SUM(B5:E5)
6	Typing	0	0	20	110	=SUM(B6:E6)
7	Cashier	0	0	1654	1840	=SUM(B7:E7)
8	Washing cars	104	260	86	52	=SUM(B8:E8)
9						
10	**Total**	**=SUM(B2:B8)**	**=SUM(C2:C8)**	**=SUM(D2:D8)**	**=SUM(E2:E8)**	**=SUM(F2:F8)**

1 What is referred to in cell:
a D6 **b** B3 **c** A5 **d** F8 **e** C10?

2 What is meant by:
a =SUM(B2:E2) **b** =SUM(C2:C8)?

3 What is the value of '=SUM(B10:E10)'?

4 Why do '=SUM(B10:E10)' and '=SUM(F2:F8)' have the same value?
Make a spreadsheet of your own on a subject of your choice.

6:04 Compound interest

PREP QUIZ 6:04

What amount would be returned after 1 year if the rate of simple interest is 10% p.a. and the principal invested is:

1 \$10 000 **2** \$11 000 **3** \$12 100 **4** \$13 310 **5** \$14 641?

What amount would be returned after 1 month if the rate of simple interest is 12% p.a. and the principal invested is:

6 \$100 000 **7** \$101 000 **8** \$102 010 **9** \$103 030.10?

10 \$2000 is invested for 1 year at 10% p.a. simple interest. The money received is then invested for 1 year at 10% p.a. simple interest. How much is received at the end of the second year?

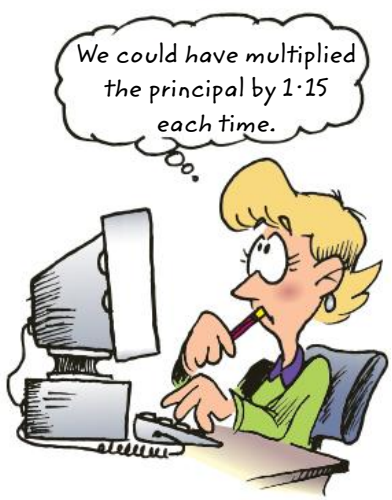
- When *simple interest* is applied, the interest earned in one year (or time period) does not itself earn interest in following years. The interest in any time period is calculated using the original principal and so is the same for each time period.
- When *compound interest* is applied, the interest earned in one year (or time period) itself earns interest in following years. The interest is added to the previous principal and so the amount of interest in each time period increases from one period to the next.

> For compound interest, interest earned in previous time periods also earns interest.

Consider what happens when \$40 000 is invested for 4 years at 15% p.a. compound interest. Find the total amount of interest earned.

Amount after 1 year = principal + interest

Amount after 1 year	$= \$40\,000 + 15\% \times \$40\,000$ $= \$40\,000 + \$6000$ $= \$46\,000$
Amount after 2 years	$= \$46\,000 + 15\% \times \$46\,000$ $= \$46\,000 + \$6900$ $= \$52\,900$
Amount after 3 years	$= \$52\,900 + 15\% \times \$52\,900$ $= \$52\,900 + \$7935$ $= \$60\,835$
Amount after 4 years	$= \$60\,835 + 15\% \times \$60\,835$ $= \$60\,835 + \$9125.25$ $= \$69\,960.25$



$\therefore$ Total interest = final amount − original principal
$= \$69\,960.25 - \$40\,000$
$= \$29\,960.25$

WORKED EXAMPLE 1

Find the compound interest earned if $9000 is invested for 3 years at 13% p.a. and interest is compounded yearly. Answer to the nearest cent.

Solution

In each year, the principal will increase by 13%.

Method 1

Amount after 1 year
= $9000 + 13% of $9000
= $10 170
Amount after 2 years
= $10 170 + 13% of $10 170
= $11 492.10
Amount after 3 years
= $11 492 + 13% of $11 492.10
≒ $12 986.07

Method 2

Amount after 1 year
= $9000 × 1·13
= $10 170
Amount after 2 years
= $10 170 × 1·13
= $11 492.10
Amount after 3 years
= $11 492.10 × 1·13
≒ $12 986.07

∴ Interest earned in 3 years = final amount − original principal
= $12 986.07 − $9000
= $3986.07

WORKED EXAMPLE 2

$12 000 is invested at a compound interest rate of 9% p.a. Interest, however, is compounded monthly. Calculate the amount by which the investment will grow in 2 months.

Solution

9% p.a. is the same as $\frac{9}{12}$% per month.
$\frac{9}{12}\% = \frac{3}{4}\% = 0{\cdot}75\%$

$0{\cdot}75\% = 0{\cdot}75 \div 100$
$= 0{\cdot}0075$

Amount after 1 month
= $12 000 + 0·75% of $12 000
= $12 090
Amount after 2 months
= $12 090 + 0·75% of $12 090
≒ $12 180.68

$A_1 = \$12\,000 \times 1{\cdot}0075$
$= \$12\,090$

$A_2 = \$12\,090 \times 1{\cdot}0075$
≒ $12 180.68

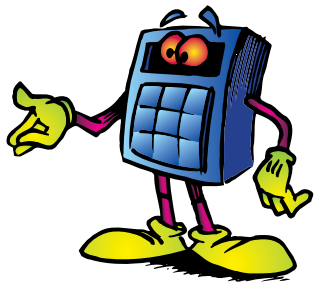
∴ Interest earned in 2 months = final amount − original principal
= $12 180.68 − $12 000
= $180.68

Finding compound interest by repeated multiplication using a calculator

In Method 2 in the previous examples, we can see that to calculate the amount accumulated after n time periods, we simply have to multiply the principal by (1 + interest rate) n times. The amount \$9000 accumulates to, at 13% p.a., after 3 years is given by:

$$\begin{aligned}\text{Amount} &= \$9000 \times 1{\cdot}13 \times 1{\cdot}13 \times 1{\cdot}13\\ &= \$12\,986.07\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Interest earned} &= \$12\,986.07 - \$9000\\ &= \$3986.07\end{aligned}$$



Exercise 6:04

P Foundation worksheet 6:04 Compound interest

1 Compare the money earned at simple interest and at compound interest in the tables below.

Rate	Simple interest on \$100 for:				
	2 years	4 years	6 years	8 years	10 years
10% p.a.	\$20	\$40	\$60	\$80	\$100

Rate	Compound interest (monthly) on \$100 for:				
	2 years	4 years	6 years	8 years	10 years
10% p.a.	\$22.04	\$48.94	\$81.76	\$121.82	\$170.70

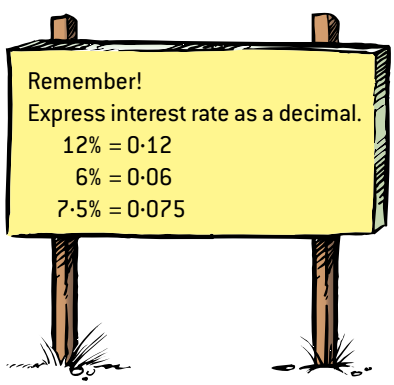
How much more interest is earned on \$100 invested at 10% p.a. compound interest (compounded monthly) than is earned on \$100 invested at 10% p.a. simple interest invested for a period of:

a 2 years **b** 6 years **c** 8 years **d** 10 years?

In Questions **2** to **5**, interest is compounded annually.

2 By repeated multiplication by (1 + interest rate) find how much each principal accumulates to, for the given number of years.

a \$2000 at 10% p.a. for 2 years
b \$5000 at 12% p.a. for 3 years
c \$10 000 at 7% p.a. for 2 years
d \$50 000 at $6\frac{1}{2}$% p.a. for 3 years
e \$25 000 at 7·5% p.a. for 4 years



3 Find the amount received and the compound interest earned if:

- **a** $400 is invested for 2 years at 12% p.a.
- **b** $1900 is invested for 3 years at 13% p.a.
- **c** $5350 is invested for 2 years at 9% p.a.
- **d** $100 is invested for 3 years at 8% p.a.
- **e** $2874 is invested for 2 years at 7·5% p.a.
- **f** $650 000 is invested for 2 years at 6% p.a.
- **g** $85 700 is invested for 3 years at $12\frac{1}{2}$ p.a.

4 Naomi invested $5000 for 4 years at a rate of 10% p.a. compound interest. To what amount did the investment grow in this time, and what was the total interest earned?

5 Joni was given 8% p.a. compound interest on an investment of $80 000 over 3 years. What interest did she earn altogether?

6 Find the amount of compound interest earned (to the nearest cent) if:

- **a** $400 is invested for 3 months at 1% per month, compounded monthly
- **b** $53 000 is invested for 2 months at 0·8% per month, compounded monthly
- **c** $8000 is invested at 12% p.a. for 2 months, compounded monthly
- **d** $2870 is invested at 6% p.a. for 3 months, compounded monthly.

7 Luke could invest $1000 at a simple interest rate of 11% p.a. for 4 years or at 10% p.a. compounded interest for the same period of time. Which is the better investment and by how much?

8 The population of Kangaroo Creek is 200 and is expected to increase each year by 80% of the previous year's population. What is the expected population in 3 years' time?

9 The value of a house has increased by 20% of the previous year's value in each of the last 4 years. If, 4 years ago, the value of the house was $100 000, what is its value now?

10 If we assume an inflation rate of 8% p.a, what would you expect to pay in 3 years' time for:

- **a** a garage that now costs $30 000
- **b** a container of ice-cream that now costs $4.50
- **c** a toothbrush that now costs $2.85?

FUN SPOT 6:04

WHAT IS THE DIFFERENCE BETWEEN A BOOK AND A BORE?

Answer each question and write the letter for that question in the box above the correct answer.

Write as a percentage:

H $\frac{1}{20}$ **O** $\frac{3}{4}$

T 0·85 **U** 0·02

Write as a decimal:

O 3% **U** 12·5%

Calculate:

A 1% of \$1 **B** 15% of \$1

C 4% of \$810 **E** $5\frac{1}{2}$% of \$8000

Find the simple interest earned by investing \$100 at 12% p.a. for:

H 1 year **K** 4 years **N** 6 months

Interest of 10·25% p.a. was charged on a loan, where interest was calculated daily. What percentage interest (given to three decimal places) would be charged for:

O 1 day

P the month of April? (Take 1 year to be 365 days.)

S Given that $I = PRN$, find the value of R if $I = 180$, $P = 600$ and $N = 3$.

A watch was bought for \$200. Find its value after 1 year if it depreciated:

T 20% **U** 11% **Y** 1·5%

\$197	0·03	2%	\$32·40	\$0·01	\$6	10%	\$12	0·125	85%	\$178	0·842%	\$160	5%	\$440	\$0·15	0·028%	75%	\$48

6:05 Depreciation

PREP QUIZ 6:05

1 100% − 20% **2** 100% − 7% **3** 20% + 80%

4 7% + 93% **5** Reduce \$350 by 20%. **6** Find 80% of \$350.

7 Reduce \$800 by 7%. **8** Find 93% of \$800.

9 Is reducing an amount by 20% the same as finding 80% of the amount?

10 Is reducing an amount by 7% the same as finding 93% of the amount?

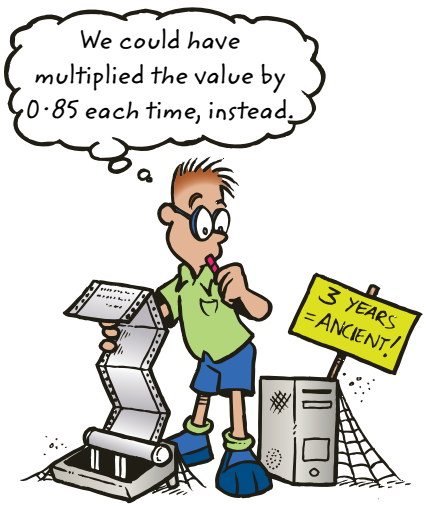
- When the value for an item decreases year by year it is said to be **depreciating**. Many of the things we own depreciate at more or less a constant rate. This may be caused by age or by the availability of new models.
- **Depreciation** is the loss in value of an object over a period of time.

Consider what happens when machinery worth $40 000 depreciates for 4 years at a rate of 15% p.a. Find the total amount of depreciation.

Value after 1 year = original value − depreciation for that year

Value after 1 year = $40 000 − 15% × $40 000 = $40 000 − $6000 = $34 000
Value after 2 years = $34 000 − 15% × $34 000 = $34 000 − $5100 = $28 900
Value after 3 years = $28 900 − 15% × $28 900 = $28 900 − $4335 = $24 565
Value after 4 years = $24 565 − 15% × $24 565 = $24 565 − $3684.75 = $20 880.25

∴ Total depreciation = original value − final value
= $40 000 − $20 880.25
= $19 119.75



WORKED EXAMPLE 1

A new computer depreciates by 30% per year. If it costs $4000 new, what would it be worth in 3 years?

Solution

Each year the value decreases by 30%.

Method 1

Value after 1 year
= $4000 − 30% of $4000
= $2800
Value after 2 years
= $2800 − 30% of $2800
= $1960
Value after 3 years
= $1960 − 30% of $1960
= $1372

Method 2

Value after 1 year
= 70% of $4000
= $2800
Value after 2 years
= 70% of $2800
= $1960
Value after 3 years
= 70% of $1960
= $1372

∴ The value of the computer after 3 years would be $1372.

WORKED EXAMPLE 2

If the population of Bilby Downs is decreasing by 20% every year, what would be the population in 2 years if it is now 700?

Solution

After a 20% decrease, 80% of the population remains.

Method 1

Population after 1 year
= 700 – 20% of 700
= 560
Population after 2 years
= 560 – 20% of 560
= 448

Method 2

Population after 1 year
= 80% of 700
= 560
Population after 2 years
= 80% of 560
= 448

∴ The population of Bilby Downs after 2 years is 448.

Exercise 6:05

1 **a** In the first year after purchase, a car costing $24 300 depreciated 18%. What was its value after 1 year?
b Find the depreciation during the first year if a ring bought for $2300 depreciated at a rate of 9·3% p.a.

2 **a** Find the value of a yacht after 2 years if its original value was $136 000 and the rate of depreciation is 12% p.a.
b A machine is purchased for $81 600. If it depreciates at a rate of 8% p.a, what will be its value after 2 years?
c Furniture purchased for $4350 depreciates at a rate of 11% p.a. What would be its value after 3 years?
d Find the value of a teacher's library after 3 years if its original value was $9800 and it depreciated at a rate of 10% p.a.
e If the wallaby population of our district is 840 and is dropping by 12% p.a, how many wallabies would you expect to be in the district in 2 years?
f The population of Kangaburra is dropping at a rate of 4% p.a. If its population in 2000 was 56 700, what was its likely population in the year 2003?

3 Find the depreciation (i.e. loss) for each of the examples in Question **2**.

4 **a** Elena bought a video camera for $1350. What would be the value of the video camera after 3 years if it depreciated at a rate of 16% p.a?
b If the $1350 had been invested for 3 years at 6% p.a. compound interest, what amount would she have received?

5 **a** Tim bought a second-hand car for $35 000. If it depreciates at a rate of 3% p.a, what would be its value in 2 years?

b If $35 000 is invested at 5% p.a. compound interest (just covering the inflation rate), what would it grow to in 2 years?

6 Julia and Lillian each invested $5000. Julia's investment depreciated at a rate of 3% p.a. for 3 years, while during that time Lillian's investment grew at a rate of 9% p.a. (compounded yearly). Find the difference in the value of their investments at the end of the 3 years.

7 Which of the rates 5% p.a, 6% p.a, 7% p.a. or 8% p.a. would cause a car valued at $6000 to drop in value to $4826 in 3 years?

8 How many years would it take to cause a machine valued at $11 000 to drop below $7000 in value if the rate of depreciation is 14% p.a?

6:06 Compound interest and depreciation formulas

PREP QUIZ 6:06

Write each percentage as a decimal.

1 9% **2** $6\frac{1}{2}\%$ **3** 11·25%

Give the rate per month correct to two decimal places:

4 9% p.a. **5** 8% p.a. **6** 6·5% p.a.

Evaluate:

7 $1 + R$, if $R = 0{\cdot}15$ **8** $1 - R$, if $R = 0{\cdot}15$

9 $P(1 + R)^n$, if $P = \$40\,000$, $R = 0{\cdot}15$ and $n = 4$

10 $P(1 - R)^n$, if $P = \$40\,000$, $R = 0{\cdot}15$ and $n = 4$

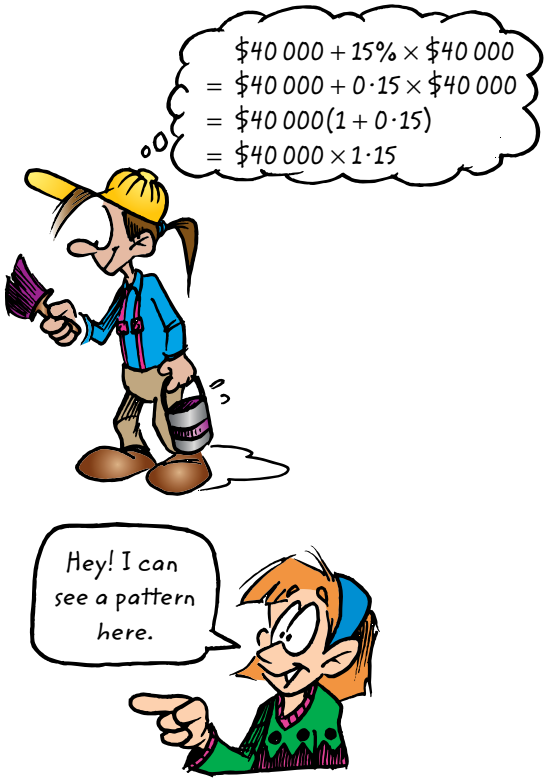
Compound interest

On page 144 we saw what happens when $40 000 is invested for 4 years at 15% p.a. compound interest.

We can find the amount to which the investment grows in each year by multiplying the amount of the previous year by 1·15 (i.e. 1 + 15%).

Amount after 1 year = principal + interest

Amount after 1 year	= \$40 000 + 15% of \$40 000 = \$40 000 × 1·15 = \$46 000
Amount after 2 years	= \$46 000 + 15% of \$46 000 = \$46 000 × 1·15 = \$52 900
Amount after 3 years	= \$52 900 + 15% of \$52 900 = \$52 900 × 1·15 = \$60 835
Amount after 4 years	= \$60 835 + 15% of \$60 835 = \$60 835 × 1·15 = \$69 960.25



The working above could have been shown as:

Amount after 1 year $= \$40\,000(1 + 0{\cdot}15)^1$
Amount after 2 years $= \$40\,000(1 + 0{\cdot}15)^2$
Amount after 3 years $= \$40\,000(1 + 0{\cdot}15)^3$
Amount after 4 years $= \$40\,000(1 + 0{\cdot}15)^4$
$= \$69\,960.25$

Clearly if \$$P$ were to be invested at a rate of r per year, then the amount A to which the investment would grow in n years would be given by $A = P(1 + R)^n$.

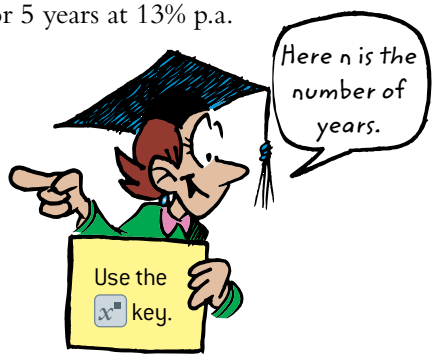
Compound interest formula

$$A = P(1 + R)^n$$

where P is the principal invested
n is the number of time periods
R is the rate of compound interest for one time period as a decimal
A is the amount after n time periods.

WORKED EXAMPLE 1

Find the compound interest earned if \$9000 is invested for 5 years at 13% p.a. (Answer to the nearest cent.)



Solution

$P = \$9000$
$n = 5$
$R = 13\%$
$= 0{\cdot}13$

$A = P(1 + R)^n$
$= \$9000(1 + 0{\cdot}13)^5$
$= \$9000(1{\cdot}13)^5$
$= \$16\,581.92$
$\therefore$ Interest $= \$16\,581.92 - \$9000$
$= \$7581.92$

$\therefore$ The compound interest earned in 5 years is \$7581.92.

WORKED EXAMPLE 2

$12 500 is invested at a compound interest rate of 9% p.a. Interest, however, is compounded monthly. Calculate the amount to which the investment will grow in 4 years.

Solution

Here:

$P = \$12\,500$

Time = 4 years

= 48 months

$\therefore n = 48$

Rate = 9% p.a.

= 9 ÷ 12% per month

= 0·75% per month

$\therefore R = 0{\cdot}0075$

$A = P(1 + R)^n$

$= \$12\,500(1 + 0{\cdot}0075)^{48}$

$= \$12\,500(1{\cdot}0075)^{48}$

$= \$17\,892.567$

$\div \$17\,892.57$

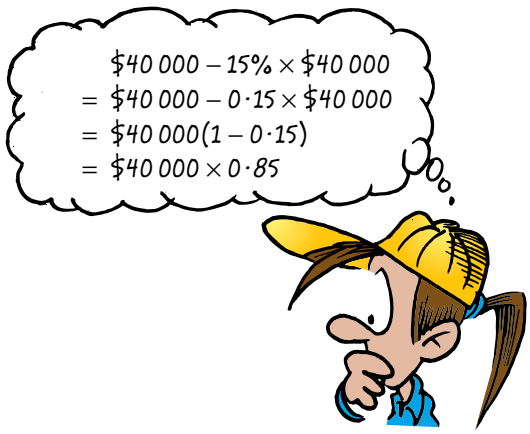
$\therefore$ The amount to which the investment will grow in 4 years is $17 892.57.

Depreciation formula

On page 149 we saw what happens to the value of machinery worth $40 000 when it depreciates for 4 years at a rate of 15% p.a. We found the value of the machinery each year by multiplying the value of the previous year by 0·85 (i.e. 1 − 15%).

Value after 1 year = original value − depreciation for that year

Value after 1 year	$= \$40\,000 - 15\%$ of $\$40\,000$ $= \$40\,000 \times 0{\cdot}85$ $= \$34\,000$
Value after 2 years	$= \$34\,000 - 15\%$ of $\$34\,000$ $= \$34\,000 \times 0{\cdot}85$ $= \$28\,900$
Value after 3 years	$= \$28\,900 - 15\%$ of $\$28\,900$ $= \$28\,900 \times 0{\cdot}85$ $= \$24\,565$
Value after 4 years	$= \$240\,565 - 15\%$ of $\$24\,565$ $= \$24\,565 \times 0{\cdot}85$ $= \$20\,880.25$



The working above could have been shown as:

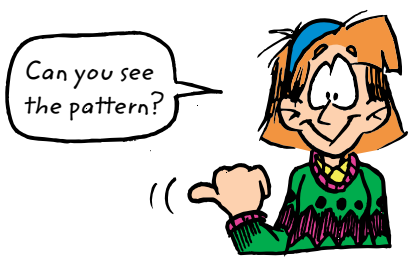
Value after 1 year $= \$40\,000(1 - 0{\cdot}15)^1$

Value after 2 years $= \$40\,000(1 - 0{\cdot}15)^2$

Value after 3 years $= \$40\,000(1 - 0{\cdot}15)^3$

Value after 4 years $= \$40\,000(1 - 0{\cdot}15)^4$

$= \$20\,880.25$



Clearly, if an item of value $\$P$ were to depreciate at a rate R per year, then the value A, after n years, would be given by $A = P(1 - R)^n$.

Depreciation formula

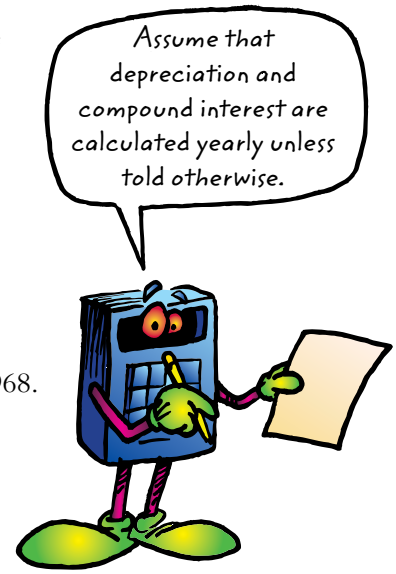
$$A = P(1 - R)^n$$

where P is the original value
n is the number of time periods
R is the rate of depreciation for one time period as a decimal
A is the value after n time periods.

WORKED EXAMPLES

1 Adam buys a second-hand car for $13 400. What will the car be worth in 5 years, to the nearest dollar, if each year it depreciates 18%?

2 Carol paid $2200 for a new computer. What would be the value of the unit in 6 years, to the nearest dollar, if the rate of depreciation were 28% p.a?



Solutions

1 $P = \$13\,400$
$n = 5$
$R = 18\%$
$= 0{\cdot}18$

$A = P(1 - R)^n$
$= 13\,400(1 - 0{\cdot}18)^5$
$= 13\,400(0{\cdot}82)^5$
$\fallingdotseq \$4967.91$

$\therefore$ The value of the car in 5 years is $4968.

2 $P = \$2200$
$n = 6$
$R = 28\%$
$= 0{\cdot}28$

$A = P(1 - R)^n$
$= \$2200(1 - 0{\cdot}28)^6$
$= \$2200(0{\cdot}72)^6$
$\fallingdotseq \$306.49$

$\therefore$ After 6 years, the value of the computer is $306.

Exercise 6:06

P Foundation worksheet 6:06
Compound interest formula

1 Use the formula $A = P(1 + R)^n$ to find the amount received if:
- **a** $4000 is invested for 5 years at 12% p.a.
- **b** $860 is invested for 7 years at 7% p.a.
- **c** $17 360 is invested for 20 years at 6·25% p.a.
- **d** $476.50 is invested for 6 years at $5\frac{1}{2}\%$ p.a.

2 Find the interest earned for each of the investments in Question 1.

3 Use the formula $A = P(1 - R)^n$ to find the value of an item worth:
- **a** $3000 after it depreciates 9% p.a. for 12 years
- **b** $35 000 after it depreciates 25% p.a. for 4 years
- **c** $465 after it depreciates 30% p.a. for 7 years.

4 Find the amount of compound interest earned (to the nearest cent) if:

a $400 is invested for 5 years at 12% p.a. compounded monthly

b $5350 is invested for 10 years at 9% p.a. compounded monthly

c $2874 is invested for 20 years at 7·5% p.a. compounded monthly

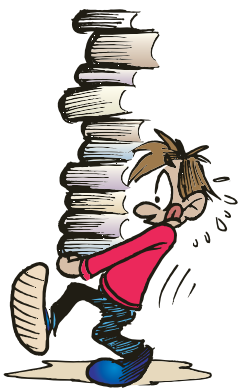
d $500 is invested for 6 years at 8% p.a. compounded half-yearly.

5 Jenny Chew discovered that she was to inherit the amount to which an investment of her great-grandfather had grown. He had invested 50 pounds at 9% p.a. compound interest, 100 years ago. Find how much she will receive if one pound is equal to two dollars. (Assume that interest is compounded annually.)

6 Luke Travers had to decide between investing his $1000 at a simple interest rate of 16% p.a. for 8 years or investing it at 11% p.a. compound interest for the same period of time. Which would be the better investment and by how much?

7 A library depreciates at a rate of 15% p.a. If its value now is $18 700, what will be its value in:

a 8 years **b** 15 years **c** 21 years?



8 Find the value of a $20 million ship after 10 years if its rate of depreciation is:

a 9% p.a. **b** 18% p.a. **c** $17\frac{1}{2}$% p.a.

9 After having depreciated at a rate of 12% p.a, a printing machine is now worth $2000. What was the machine's value 7 years ago?

10 A library now worth $17 600 has been depreciating at a rate of 9% p.a. for the last 8 years. What was its value 8 years ago?

11 $1000 is invested for 5 years at 12% p.a. compound interest. To what does this investment grow if interest is compounded:

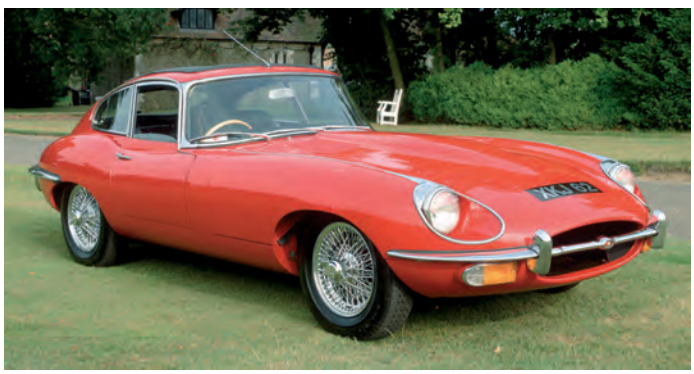
a yearly **b** 6 monthly **c** 3 monthly

d monthly **e** fortnightly **f** daily?

Is there a limit to which the investment can grow as we reduce the time period for compounding the interest? (Consider compounding the interest each hour, minute and second.)

12 Adam invests $10 000 at 9% p.a. simple interest while Su-Lin invests $10 000 at 9% p.a. compounded monthly. Find the difference in the value of their investments after:

a 1 year **b** 5 years **c** 10 years.



If this car depreciates at 15% a year, how long will it take to halve its price if its original price is $150 000?

INVESTIGATION 6:06 COMPOUND INTEREST TABLES

If a calculator with a power key or a computer is not available, a table could be used that shows the effects of compound interest. The sample table below shows the amount $1 will grow to for various interest rates and time periods.

Accumulated value of $1 (to four decimal places)									
Number of time periods	Interest rate								
	0·5%	1%	5%	6%	7%	8%	10%	12%	15%
1	1·0050	1·0100	1·0500	1·0600	1·0700	1·0800	1·1000	1·1200	1·1500
2	1·0100	1·0201	1·1025	1·1236	1·1449	1·1664	1·2100	1·2544	1·3225
3	1·0151	1·0303	1·1576	1·1910	1·2250	1·2597	1·3310	1·4049	1·5209
4	1·0202	1·0406	1·2155	1·2625	1·3108	1·3605	1·4641	1·5735	1·7490
5	1·0253	1·0510	1·2763	1·3382	1·4026	1·4693	1·6105	1·7623	2·0114
6	1·0304	1·0615	1·3401	1·4185	1·5007	1·5869	1·7716	1·9738	2·3131
7	1·0355	1·0721	1·4071	1·5036	1·6058	1·7138	1·9487	2·2107	2·6600
8	1·0407	1·0829	1·4775	1·5938	1·7182	1·8509	2·1436	2·4760	3·0590
9	1·0459	1·0937	1·5513	1·6895	1·8385	1·9990	2·3579	2·7731	3·5179
10	1·0511	1·1046	1·6289	1·7908	1·9672	2·1589	2·5937	3·1058	4·0456
11	1·0564	1·1157	1·7103	1·8983	2·1049	2·3316	2·8531	3·4785	4·6524
12	1·0617	1·1268	1·7959	2·0122	2·2522	2·5182	3·1384	3·8960	5·3503
18	1·0939	1·1961	2·4066	2·8543	3·3799	3·9960	5·5599	7·6900	12·3755
24	1·1272	1·2697	3·2251	4·0489	5·0724	6·3412	9·8497	15·1786	28·6252

Exercises

Find the accumulated value and hence the compound interest earned when:

1 $10 000 is invested for 10 years at 8% p.a. compounded annually

2 $50 000 is invested for 18 months at 12% p.a. compounded annually.

Solutions

1 If interest rate = 8% and time periods = 10, then $1 will grow to $2.1589.
∴ $10 000 will grow to:
$2.1589 × 10 000 = $21 589
∴ Interest = $21 589 − $10 000
= $11 589

2 If interest rate = 12% p.a, then interest rate per month = 1%.
So for 18 months, $1 will grow to $1.1961.
∴ $50 000 will grow to:
$1.1961 × 50 000 = $59 805
∴ Interest = $59 805 − $50 000
= $9805

Exercises

1 Find the accumulated amount and the interest earned when:
   a $20 000 is invested for 12 years at 7% p.a. compounded annually
   b $150 000 is invested for 24 years at 10% p.a. compounded annually
   c $7500 is invested for 8 years at 6% p.a. compounded annually.

2 Find an accumulated amount when $100 000 is invested at:
   a 12% p.a. for 2 years, interest compounded annually
   b 1% per month for 24 months, interest compounded monthly.

3 Why are the answers different for parts **a** and **b** of Question **2** when the interest rate and terms are the same? (2 years = 24 months, 12% p.a. = 1% per month)

MATHS TERMS 6

compound interest
- interest earned is added to the principal so that the interest earned for the next time period is calculated on this greater amount
- the interest earned in previous time periods also earns interest
- the formula for compound interest is:
  $A = P(1 + R)^n$
  where A = amount after n time periods
  P = principal invested
  n = number of time periods
  R = rate of interest for one time period as a decimal

depreciation
- the loss in value of an object over a period of time
- the formula for calculating the reduced value of an object is:
  $A = P(1 - R)^n$
  where A = value after n time periods
  P = original value
  n = number of time periods
  R = rate of depreciation for one time period as a decimal

flat rate
- the same as a simple interest rate

instalment
- a payment made to reduce the amount owing on a loan

interest
- the payment made for the use of money invested or borrowed
- usually expressed as a percentage of the amount borrowed

principal
- the amount of money invested or borrowed

repayment
- a payment made to reduce the amount owing on a loan

simple interest
- interest is paid on the original principal throughout the term of the loan
- the formula for calculating simple interest is:
  $I = PRN$
  where I = simple interest
  P = principal invested
  R = rate of interest
  N = number of time periods

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 a Find the simple interest charged on \$600 at 12% p.a. for 5 years. b What is the simple interest earned by \$14 260 invested for 3 years at 4% p.a? c Find the interest charged on a loan of \$85 000 taken over 15 years if a simple interest rate of 8% p.a. is charged.	6:02
2 a Find the simple interest charged on \$860 at $6\frac{1}{2}$% p.a. for 5 years. b What interest would be paid on \$25 000 invested for 6 years at a simple interest rate of 9·3% p.a? c Find the simple interest earned if \$9000 is invested for 3 years at a rate of $6\frac{3}{4}$% p.a.	6:02
3 a What is the simple interest on \$2400 at 8% p.a. for 5 months? b Find the simple interest on \$900 for 240 days at a rate of 11% p.a. c What is the simple interest paid on \$19 500 invested for 7 months at 0·75% per month?	6:02
4 a Jodie borrowed \$78 000 for 4 months at 12·75% p.a. simple interest to pay for her new house while her old one was being sold. How much interest did she pay? b I paid \$8000 to borrow \$50 000 for 2 years. What was the rate of simple interest charged? c Troy borrowed \$1200 at 1·5% per month simple interest. Which is the best estimate of the interest charges for 6 months: \$20, \$100, \$200 or \$1800?	6:03
5 a Find the compound interest earned if \$9000 is invested for 3 years at 13% p.a. if interest is compounded yearly. Answer correct to the nearest cent. b \$12 000 is invested at a compound interest rate of 9% p.a. Interest, however, is compounded monthly. Calculate the amount to which the investment will grow in 2 months. c \$12 500 is invested at a compound interest rate of 9% p.a. Interest, however, is compounded monthly. Use the formula to calculate the amount to which the investment will grow in 4 years.	6:04
6 a A new computer depreciates by 30% per year. It costs \$4000 new. What will it be worth in 4 years? b If the population of Bilby Downs is decreasing by 20% of its population every year, what would be the population in 2 years if it is now 800? c Carol paid \$2200 for a new bicycle. Use the depreciation formula to find the value of the bicycle in 6 years if its rate of depreciation is 15% p.a.	6:05

ASSIGNMENT 6A Chapter review

1 A bank pays 4·5% p.a. for a target savings account.
- **a** What would be the interest earned on a balance of $3240 for one month?
- **b** If this interest is added to the account, how much interest would be earned in the next month?

2 Annabelle invested $60 000 in a term deposit that paid interest at a rate of 4·35% p.a. for 6 months or 4·45% p.a. for 12 months.
- **a** Annabelle chose to invest for 6 months. How much interest did she earn?
- **b** At the end of the 6 months she added the interest to the original $60 000 and invested it again for another 6 months at the same rate. How much interest did she earn for this 6 months?
- **c** If Annabelle had chosen to invest the $60 000 initially for 12 months at the higher rate, would she have earned more or less interest and what would have been the difference?

3 Determine the amount of simple interest earned by the following:
- **a** $1500 at 7% p.a. over 5 years
- **b** $9500 at 9·5% p.a. over $3\frac{1}{2}$ years
- **c** $2200 at 8% p.a. over 5 months
- **d** $660 at 1·5% per month over 9 months

4 Kate's savings account pays interest of 5% p.a. Interest is calculated and added to the account every 6 months. If she starts with $2000 in the account and does not use the account for 2 years, what will her new balance be at the end of the 2 years?

5 $24 000 is invested for 2 years. Interest is paid at a rate of 6% p.a. and is compounded annually. Interest is not withdrawn. How much interest will have been earned after 2 years?

6 Machinery valued at $140 000 depreciates at a rate of 16% p.a. Find the value of the machinery after 3 years.

7 Terry invested $250 000 in an account that earned 6% p.a. compound interest.
- **a** How much would be in the account at the end of 10 years if:
  - **i** interest was calculated annually
  - **ii** interest was calculated every 3 months
  - **iii** interest was calculated monthly?
- **b** How much more interest is earned by calculating the interest monthly rather than annually?

8 Margaret decided to invest $50 at 5% p.a. compound interest for 200 years, with the interest to be added annually. Her nearest living relative after 200 years would inherit the proceeds.
- **a** Find the amount the relative should inherit.
- **b** Find the amount if the interest rate had been 5% p.a. simple (flat) interest.

9 A printing machine bought for $245 000 will depreciate at the rate of 17·5% p.a. When the value of the machine falls below $100 000, it is replaced by a new model. After how many years will the printing machine be replaced?

10 Bilbo invested $2000 in an account that will earn 5·5% p.a. compound interest calculated monthly. The interest is added to the balance each month. At the end of the year, for the next two years, he deposited another $2000 into the account. How much will Bilbo have in the account at the end of 3 years?

ASSIGNMENT 6B Working mathematically

1 Find the number of three-digit numbers possible if only the digits 0, 1, 2, 3, 4 and 5 may be used, the number must be a multiple of 5, and no digit may be used more than twice in the same number.

2 Of the 30 students in 10M, 13 love surfing, 15 love hiking and 7 love neither.
- **a** How many students love both surfing and hiking?
- **b** How many love surfing but not hiking?
- **c** How many love hiking but not surfing?

3 A man and his granddaughter share the same birthday. For six consecutive birthdays the man's age is an exact multiple of his granddaughter's age. How old is each of them at the 6th of these birthdays? Can you find another set of numbers that work, but is highly unlikely?

4 Roger started a trip to the country between 8 am and 9 am when the hands of the clock were together. He arrived at his destination between 2 pm and 3 pm, when the hands of the clock were exactly 180° apart. For how long did he travel?

5 $3! = 3 \times 2 \times 1$ (pronounced 3 factorial)
$5! = 5 \times 4 \times 3 \times 2 \times 1$
$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
How many zeros are there at the end of the answer to 20!? (A calculator will be of no help.)

6 **a** For Australia in 2007:
- **i** How many people were killed on the road?
- **ii** How many fatalities were there per 100 000 people?
- **iii** Therefore, what is an estimation of Australia's population according to these figures?

b Where did Australia rank among these 20 OECD countries in terms of least fatalities per 100 000 population?

c Compared to 2005, which country in 2007 had:
- **i** the greatest decrease in fatalities
- **ii** the greatest increase in fatalities?

Road safety comparisons: fatalities OECD countries 2007

	Number of fatalities in 2007		
	2007	% compared to 2005	Fatalities per 100000 persons
Australia	1603	−1·5%	7·6
Austria	691	−10·0%	8·3
Czech Republic	1222	−5·0%	11·9
Denmark	406	22·7%	7·4
Finland	380	0·0%	7·2
France	4620	−13·1%	7·5
Germany	4949	−7·7%	6·0
Hungary	1232	−3·6%	12·2
Iceland	15	−21·1%	4·9
Japan	6639	−16·3%	5·2
Netherlands	709	−5·5%	4·3
New Zealand	422	4·2%	10·0
Norway	233	4·0%	5·0
Poland	5583	2·6%	14·7
Portugal	974	−21·9%	9·2
Slovenia	293	13·6%	14·6
Spain	3823	−13·9%	8·6
Sweden	471	7·0%	5·2
Switzerland	384	−6·1%	5·1
United States	41 059	−5·5%	13·6
Total	75 708	−8·8%	7·8

ASSIGNMENT 6C Cumulative revision

1 Solve these equations to find the value of x. 1:06

a $7x - 5 = 12$ **b** $4x - 5 = 2x + 7$ **c** $7 - 3x = 10 - 5x$

2 **a** Find the GST added to a price of $43.50 before a sale. 1:07

b How much GST is included in a selling price of $57.20?

3 Simplify the following: 2:01

a $\frac{7x}{6} + \frac{5x}{6}$ **b** $\frac{4}{3x} - \frac{2}{5x}$ **c** $\frac{4xz}{3y} \times \frac{9yz}{2x}$ **d** $\frac{6b}{5a} \div \frac{3}{10ab}$

4 Expand and simplify: 2:02, 2:03

a $6(2m + 4) - 2(4m - 1)$ **b** $2m^2n^3(3m - 5n)$

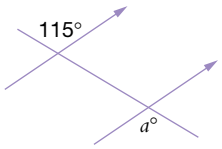
c $(x + 5)(x - 1)$ **d** $(2y - 5)(2y + 5)$

5 Solve these quadratic equations: 2:05

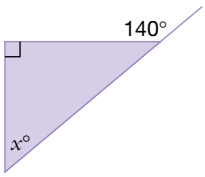
a $x^2 - 64 = 0$ **b** $(x - 9)(x + 2) = 0$ **c** $x^2 - x - 12 = 0$

6 Find the value of the pronumeral in each figure. 3:01

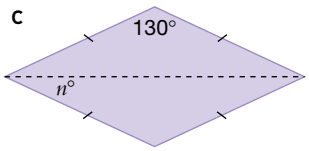
a



b



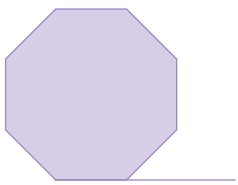
c



7 For a regular octagon, what is the size of each: 3:04

a interior angle

b exterior angle?



8 There are 10 red and 10 blue marbles in a bag. A marble is selected and its colour noted. This is done three times. What is the probability of the third marble chosen being blue if: 4:02

a after each selection the marble is put back in the bag

b each marble is not put back in the bag and the first two marbles are:

i blue **ii** red?

9 There are two boxes. In one box there are ten $10 notes and ten $20 notes. In the second box there are fifteen $10 notes and five $20 notes. 4:05

a If one note is selected at random from each box, what is the probability they will both be $20?

b If only one box is chosen at random and a note is selected, what is the probability of selecting a $20 note?

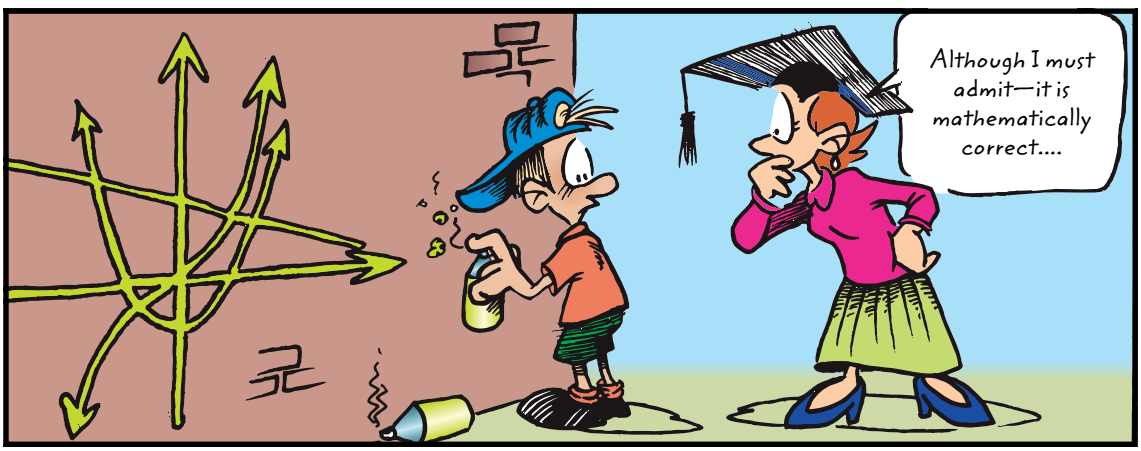
10 Solve each pair of simultaneous equations. 5:02

a $2x + 5y = 3$
$3x - 4y = 16$

b $5a - 3b = 2$
$b = 6a + 8$

LINEAR AND NON-LINEAR RELATIONSHIPS

7



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7:04 The parabolas $y = ax^2$ and $y = ax^2 + c$
GeoGebra activity 7:04A Parabolas of the form $y = ax^2$
GeoGebra activity 7:04B Parabolas of the form $y = ax^2 + c$
7:05 Exponential graphs
GeoGebra activity 7:05 Exponential graphs
Fun spot 7:05 The tower of Hanoi
7:06 The equation of a circle
Fun spot 7:06 Try to match this one!
7:07 Miscellaneous graphs
GeoGebra activity 7:07 Matching equations and graphs
Fun spot 7:07 What is HIJKLMNO?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Linear Relationships*** [Stages 5.1, 5.2] and ***Non-Linear Relationships*** [Stages 5.1, 5.2◊]

- Solve problems involving parallel and perpendicular lines (ACMNA238)
- Explore the connection between algebraic and graphical representations of relationships such as simple quadratics, circles and exponentials using digital technologies as appropriate (ACMNA239)
- Graph simple non-linear relationships, with and without the use of digital technologies, and solve simple related equations (ACMNA296)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

7:01 Straight line graphs: A review

PREP QUIZ 7:01

For the equation $y = 2x + 1$, find the value of y when x is equal to:

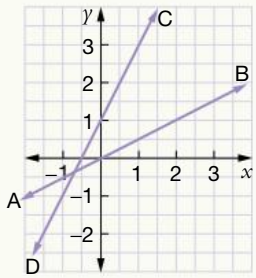
1 0
2 1
3 2
4 If $x + 2y = 7$, find y when $x = 3$.

For the line $y = 2x + 1$, write:

5 the gradient
6 the y-intercept.

From the diagram, which line has:

7 a y-intercept of 0
8 a y-intercept of 1
9 a gradient of 2
10 a gradient of $\frac{1}{2}$?



In Year 9 it was found that equations such as the following represent straight lines on the number plane.

$y = 2x + 1$, $y = -3x + 6$ gradient–intercept form

$x + 2y - 4 = 0$ general form

Equations in the form $y = mx + b$ are in gradient–intercept form where:

- m is the gradient of the line
- b is the y-intercept of the line.

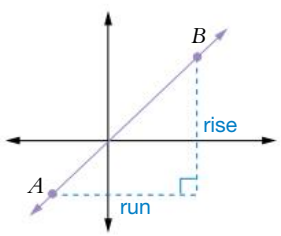
The equations of all straight lines include one or two pronumerals such as an x-term or a y-term (or both) and a constant. They cannot have any other terms.

Straight lines may be graphed by:

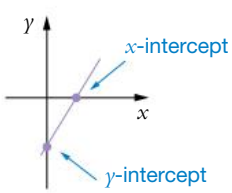
- completing a table of values and then plotting the points
- using the y-intercept and gradient that can be obtained from the equation.

The equation of a straight line can be found by calculating its gradient and noting the y-intercept.

- Gradient $= \dfrac{\text{rise}}{\text{run}}$



- The y-intercept is where the line cuts the y-axis.

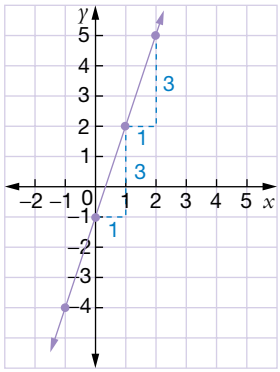


These methods are shown in the following examples.

WORKED EXAMPLE 1

Draw the graph of $y = 3x - 1$.
We can complete a table and then plot the points.

x	-1	0	1	2
y	-4	-1	2	5



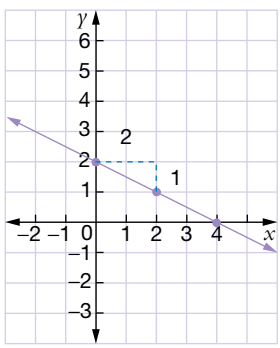
The equation is in gradient–intercept form so the y-intercept and the gradient can be used to draw the line.
$b = -1$ so start at $y = -1$.
$m = 3$ so 'count' the gradient
(a rise of 3 for every run of 1).

Remember!
For $y = mx + b$
m = gradient
b = y-intercept

WORKED EXAMPLE 2

For $x + 2y - 4 = 0$
Completing a table:

x	0	2	4
y	2	1	0

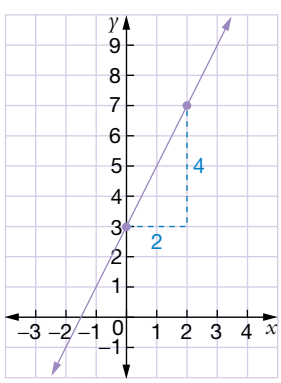


Rearranging the equation:
$x + 2y - 4 = 0$
$2y = -x + 4$
$y = -\frac{1}{2}x + 2$
$b = 2$ so start at $y = 2$.
$m = -\frac{1}{2}$ so 'count' the gradient (a drop of 1 for every run of 2).
The line slopes down to the right.

Remember!
A negative gradient always slopes down to the right.

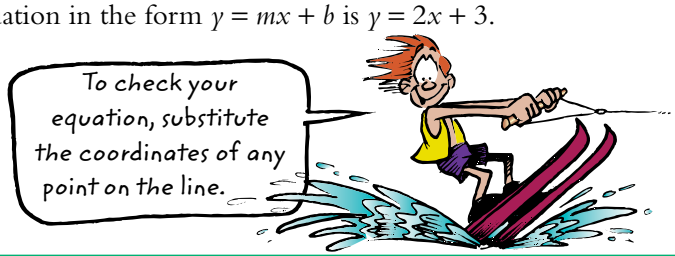
We can also find the equation of a straight line by determining its gradient and noting where it cuts the y-axis.

WORKED EXAMPLE 3



To find the equation of this line, we note:
- the y-intercept $b = 3$
- the gradient $m = \frac{4}{2} = 2$

$\therefore$ The equation in the form $y = mx + b$ is $y = 2x + 3$.



Exercise 7:01

1 Complete the table of values for each equation and then draw its graph.

a $y = 3x + 2$

x	0	1	2
y			

b $y = 4x - 1$

x	0	1	2
y			

c $y = 6x - 3$

x	0	1	2
y			

d $y = 2x + 5$

x	0	1	2
y			

e $x + y = 6$

x	0	1	2
y			

f $y = -2x - 1$

x	0	1	2
y			

2 Draw the graph of each line on a separate number plane by following these steps.

- Mark the y-intercept (the value of b) on the y-axis.
- 'Count' the gradient (the value of m) from this point, and mark a second point.
- Draw a line through this second point and the y-intercept.

a $y = 2x + 1$ **b** $y = 2x - 3$ **c** $y = -2x + 2$

d $y = x + 3$ **e** $y = -x - 1$ **f** $y = x$

g $y = \frac{1}{2}x + 1$ **h** $y = -\frac{1}{2}x - 1$ **i** $y = \frac{1}{3}x + 2$

3 Write the equation of the line that has:

a a gradient of 3 and a y-intercept of 4

b a gradient of -3 and a y-intercept of 4

c a gradient of 3 and a y-intercept of -4

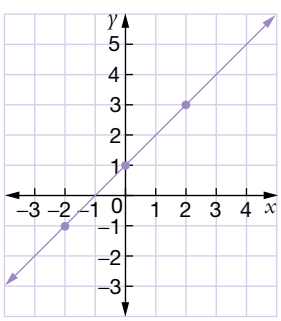
d a gradient of -3 and a y-intercept of -4

e a gradient of $-\frac{1}{2}$ and a y-intercept of -2

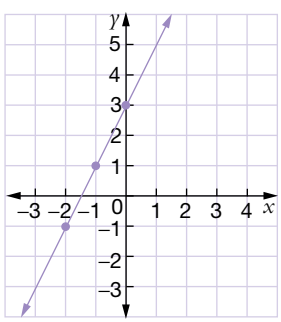
f a gradient of 2 and a y-intercept of $\frac{1}{2}$.

4 By noting the gradient and y-intercept, determine the equation of each line.

a



b



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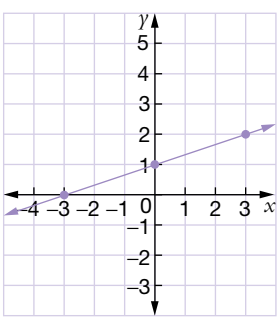

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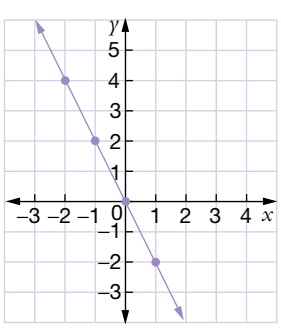

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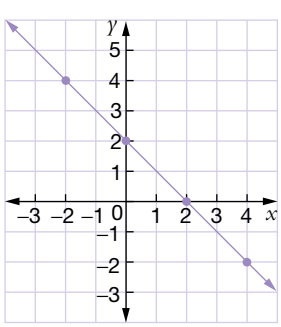

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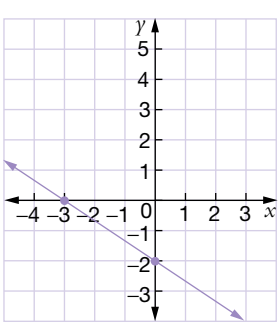
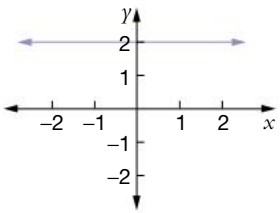


5 These lines are parallel to the axes. What are their equations?

a



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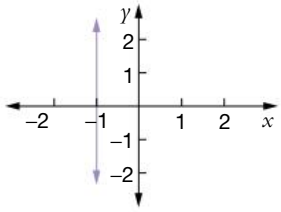

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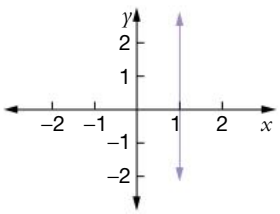

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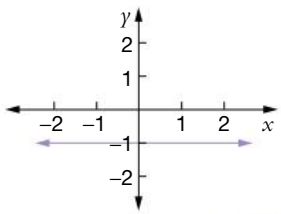
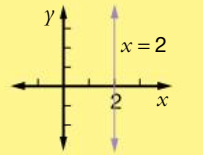
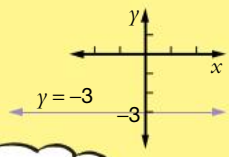


Remember!
Lines parallel to the axes have equations in the forms shown below.





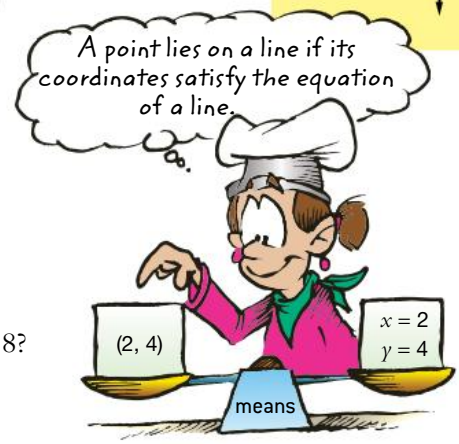
6 Does the point (2, 4) lie on each of the following lines?

a $y = 3x - 2$

b $x + y = 6$

c $2x + 4y = 0$

d $4x - 2y = 0$



(2, 4) lies on $y = 3x + 2$
because $4 = 3 \times 2 - 2$

7 Which of these points lie on the line $y = 5x - 8$?
(0, 8), (3, 0), (2, 2), (1, –3), (4, 12), (0, –8)

8 **a** The point (2, 3) lies on the line $y = 2x + b$. What is the value of b?

b The point (–1, 2) lies on the line $y = -x + b$. What is the value of b?

9 Match the lines A to G with the equations below.

a $y = x - 2$

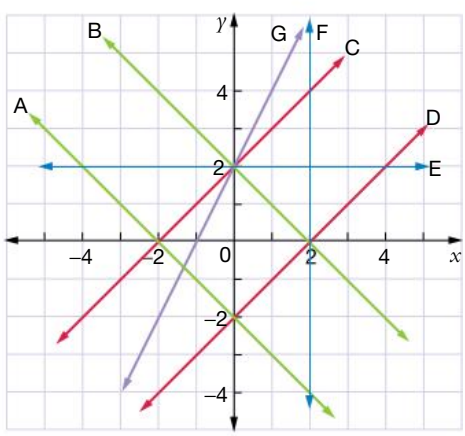
b $x = 2$

c $y = 2 - x$

d $x + y = -2$

e $y = 2$

f $x + y = 2$



10 For each of the following find the gradient and y-intercept of the line by rearranging the equation into gradient–intercept form.

a $x + y = 6$ **b** $2x + y = 3$

c $3x - y = 4$ **d** $x + 2y - 6 = 0$

e $2x - 3y - 4 = 0$ **f** $4x + 2y - 5 = 0$

11 Draw the graph of each of these lines.

a $x + 2y = 3$ **b** $2x + y = 3$ **c** $x - y = 3$

d $3x + y - 4 = 0$ **e** $2x - 3y - 1 = 0$ **f** $x - 2y + 1 = 0$

7:02 Parallel lines

PREP QUIZ 7:02

1 What are parallel lines?

p is the line $y = 2x$ and q is the line $y = 2x + 3$.

2 Find the gradient of p.

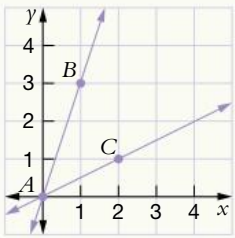
3 Find the gradient of q.

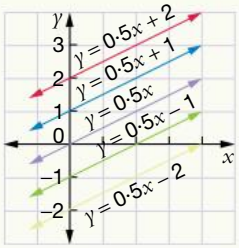
4 Do p and q have the same slope?

Find the gradient of the line:

5 AB

6 AC





From the graph:

7 What is the gradient of each line?

8 Are the lines parallel?

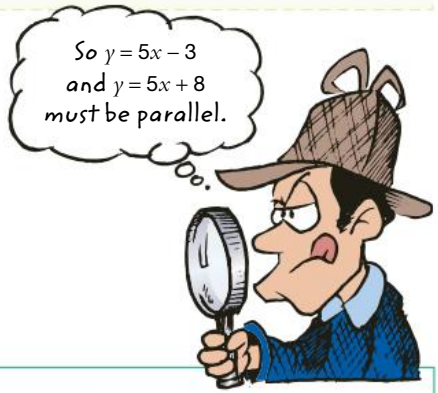
9 Does each line have the same y-intercept?

10 Would the lines $y = 0{\cdot}5x + 2$ and $y = 2x + 0{\cdot}5$ be parallel?

Two straight lines are **parallel** if their gradients are equal.
The gradients of two lines are equal if the lines are parallel.
If the gradients of the lines are m_1 and m_2, then $m_1 = m_2$.

For parallel lines: $m_1 = m_2$

The symbol for parallel lines is ||.



WORKED EXAMPLE 1

Which of the lines $y = 3x - 5$, $y = x - 5$, $y = 3x + 7$ and $6x - 2y + 1 = 0$ are parallel?

Solution

Lines with equal gradients will be parallel.
If the line is in the form $y = mx + b$, then m is the gradient.
The gradient of $y = 3x - 5$ is 3.
The gradient of $y = x - 5$ is 1.
The gradient of $y = 3x + 7$ is 3.
To find the gradient of $6x - 2y + 1 = 0$, make y the subject.

$6x - 2y + 1 = 0$
$\therefore\ 6x + 1 = 2y$
$\therefore\ 3x + \frac{1}{2} = y$
$\therefore\ y = 3x + \frac{1}{2}$

$6x - 2y + 1 = 0$ is in the general form.
$y = 3x + \frac{1}{2}$ is in gradient–intercept form.

The gradient of $6x - 2y + 1 = 0$ is 3.
$\therefore\ y = 3x - 5$, $y = 3x + 7$ and $6x - 2y + 1 = 0$ are parallel.

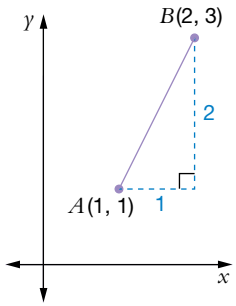
WORKED EXAMPLE 2

A is the point $(1, 1)$, B is $(2, 3)$, C is $(-6, -12)$ and D is $(8, 16)$. Are the lines AB and CD parallel?

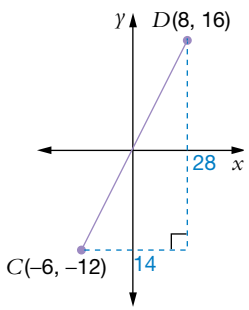
Solution

AB and CD will be parallel if their gradients are equal.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$



$\therefore$ Gradient of AB
$= \frac{2}{1}$
$= 2$

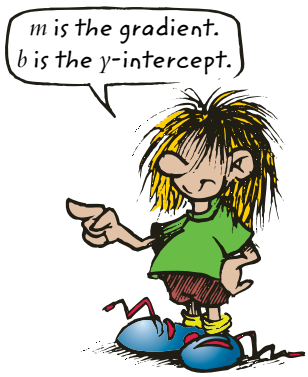


$\therefore$ Gradient of CD
$= \frac{28}{14}$
$= 2$

Hence $AB \parallel CD$.

WORKED EXAMPLE 3

Find the equation of the line that passes through the point $(1, 4)$ and is parallel to $y = 3x - 2$.



Solution

Let the equation of the line be $y = mx + b$.

$y = 3x - 2$ has gradient 3

$\therefore$ $m = 3$ parallel lines have equal gradients

$\therefore$ $y = 3x + b$

$4 = 3(1) + b$, $(1, 4)$ lies on the line

$\therefore$ $b = 1$

$\therefore$ The equation of the line is $y = 3x + 1$.

Exercise 7:02

P Foundation worksheet 7:02 Parallel lines

1 Write the slope of a line that is parallel to the line:

a $y = 2x + 6$ **b** $y = -x + 2$ **c** $y = -\frac{1}{2}x - 3$ **d** $y = \frac{4}{3}x - \frac{2}{3}$

e $y = 4 + x$ **f** $y = 1 - 4x$ **g** $y = 2 - \frac{1}{2}x$ **h** $y = -2 - 6x$

2 Are the following pairs of lines parallel?

a $y = 3x + 2$ and $y = 3x - 1$ **b** $y = 5x - 2$ and $y = 2x - 5$

c $y = x + 7$ and $y = x + 1$ **d** $y = x - 3$ and $y = 1x + 2$

e $y = 4 + x$ and $y = x + 6$ **f** $y = 1 - 4x$ and $y = 4x - 3$

g $y = 2 - \frac{1}{2}x$ and $y = -\frac{1}{2}x - 4$ **h** $y = -2 - 6x$ and $y = 4 - 6x$

3 Find the slope of a line that is parallel to the line:

a $x + y = 6$ **b** $x - y = -4$ **c** $x + 2y = 1$ **d** $2x - 3y + 6 = 0$

4 Are the following pairs of lines parallel?

a $y = 3x + 2$ and $2y = 6x - 3$ **b** $y = 2x + 1$ and $2x - y + 3 = 0$

c $3x + y - 5 = 0$ and $3x + y + 1 = 0$ **d** $x + y = 6$ and $x + y = 8$

5 **a** Which of the following lines are parallel to $y = 2x + 3$?

$y = 3x + 2$ $2x - y + 6 = 0$ $2y = x + 3$ $y = 2x - 3$

b Two of the following lines are parallel. Which are they?

$y = x - 3$ $x + y = 3$ $y = 3x$ $3y = x$ $y = -x + 8$

c A is the point $(1, 3)$, B is $(3, 4)$, C is $(6, 7)$ and D is $(8, 8)$. Which of the lines AB, BC, CD and DA are parallel?

d E, F, G and H are the points $(0, 0)$, $(2, 1)$, $(4, 1)$ and $(2, 0)$. Show that $EF \parallel GH$ and that $FG \parallel EH$.

e The line p has gradient 4 and y-intercept 3. The line q passes through the points $(-1, -2)$ and $(1, 6)$. Show that p is parallel to q.

6 **a** Find the equation of the line that has y-intercept 3 and is parallel to $y = 5x - 1$.

b Line AB is parallel to $y = 3x - 4$. Find the equation of AB if its y-intercept is -1.

c Line EF is parallel to $y = x + 5$. Its y-intercept is 3. What is the equation of EF?

d A line has a y-intercept of 10 and is parallel to the line $y = 4 - x$. What is the equation of this line?

e AB is a line that passes through the point $(2, 3)$. What is the equation of AB if it is parallel to $y = 5x + 2$?

f Find the equation of the line that passes through $(1, 0)$ and is parallel to $y = -3x - 1$.

g A is the point $(0, 0)$ and B is the point $(1, 3)$. Find the equation of the line that has y-intercept 5 and is parallel to AB.

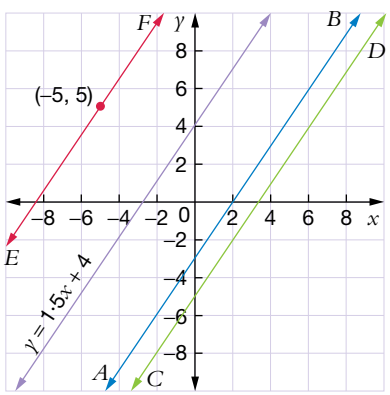
h Find the equation of the line that has y-intercept -3 and is parallel to the x-axis.

i What is the equation of the line that is parallel to the x-axis and passes through the point $(-2, -3)$?

j Two lines $y = -2x + 1$ and $y = mx - 7$ are parallel. Find the value of m.

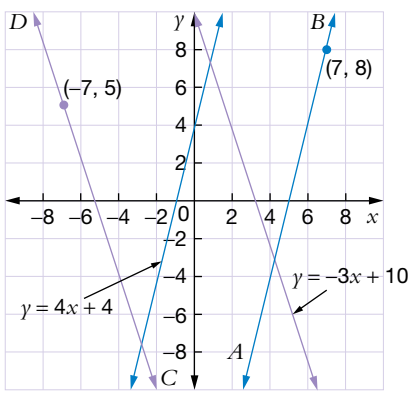
7 The lines AB, CD and EF are parallel to the line $y = 1{\cdot}5x + 4$. Find the equation of:

a AB **b** CD **c** EF



8 The diagram shows two sets of parallel lines. Find the equation of:

a AB **b** CD



9 a Find the equation of the line that passes through the point (1, 4) and is parallel to the line $2x + y = 2$.

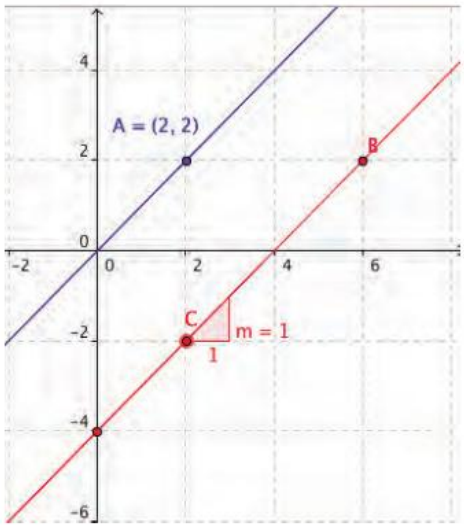
b A line is drawn through the point (−2, 5) parallel to the line $x - 3y + 4 = 0$. What is the equation of the line?

c A line is drawn through the point (−2, −4) parallel to the line $2x + 3y = 6$. What is the equation of this line and where will it cut the y-axis?

P GEOGEBRA ACTIVITY 7:02 PARALLEL LINES

1 Use the sliders to change the slopes of the lines and investigate when the lines will be parallel. How are the equations of parallel lines similar? How are they different?

2 This activity uses a variable line and point. Move the point and work out the equation of the line through the point parallel to the given line. Use the checkbox to see if you are correct.



7:03 Perpendicular lines

PREP QUIZ 7:03

1 $3 \times -\frac{1}{3}$ **2** $-\frac{1}{2} \times 2$ **3** $-\frac{3}{4} \times \frac{4}{3}$ **4** -1×-1

In the diagram, AB is perpendicular to BC, and DE is perpendicular to EF.

5 Find the gradient of AB. Call this m_1.

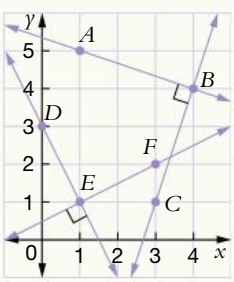
6 Find the gradient of BC. Call this m_2.

7 Using your answers to Questions **5** and **6**, find the product of the gradients, $m_1 m_2$.

8 Find the gradient of DE. Call this m_3.

9 Find the gradient of EF. Call this m_4.

10 Using your answers to Questions **8** and **9**, find the product of the gradients, $m_3 m_4$.



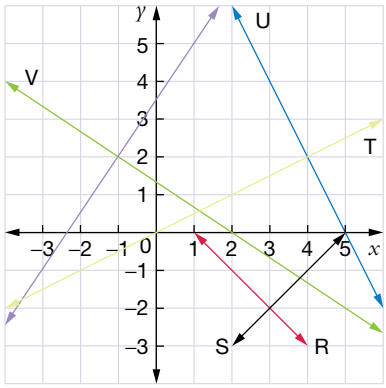
Questions **5** to **10** of the Prep quiz suggest that a condition for two lines to be perpendicular might be that the product of their gradients is equal to −1.

We do not intend to prove this here, but let us look at several pairs of lines where the product of the gradient is -1 to see if the angle between the lines is 90°.

A Line R has gradient -1 $\quad\therefore m_1 = -1$
Line S has gradient 1 $\quad\therefore m_2 = 1$
Note that $m_1 m_2 = -1$

B Line T has gradient $\frac{1}{2}$ $\quad\therefore m_1 = \frac{1}{2}$
Line U has gradient -2 $\quad\therefore m_2 = -2$
Note that $m_1 m_2 = -1$

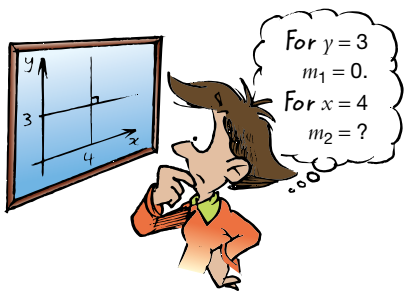
C Line V has gradient $-\frac{2}{3}$ $\quad\therefore m_1 = -\frac{2}{3}$
Line W has gradient $\frac{3}{2}$ $\quad\therefore m_2 = \frac{3}{2}$
Note that $m_1 m_2 = -1$



Using measurement or Pythagoras' theorem, we can show that the angle between each pair of lines is 90°.

- If two lines are perpendicular then the product of their gradients is -1.
- $m_1 m_2 = -1$ (where neither gradient is zero).
- If the product of the gradients of two lines is -1, then the lines are perpendicular.

For perpendicular lines: $m_1 m_2 = -1$
(or $m_1 = -\dfrac{1}{m_2}$) where neither m_1 nor m_2 can equal zero.
The symbol for perpendicular lines is $\perp$.



A line parallel to the y-axis must be perpendicular to a line parallel to the x-axis.

WORKED EXAMPLE 1

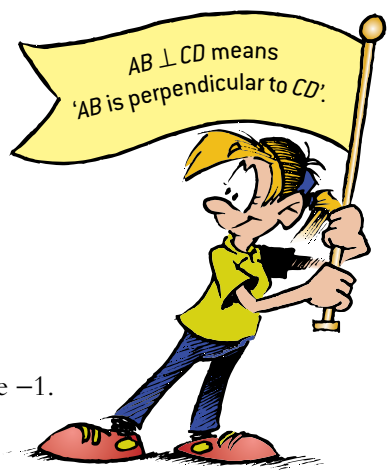
Which of the lines $y = 4x$, $y = 3x + 2$ and $y = x$ is perpendicular to $x + 4y + 2 = 0$?

Solution

Step 1: Find the gradient of $x + 4y + 2 = 0$.
Writing this in gradient form gives:
$y = -\frac{1}{4}x - 2$
$\therefore$ The gradient of this line is $-\frac{1}{4}$.

Step 2: Find the gradients of the other lines.
The gradient of $y = 4x$ is 4.
The gradient of $y = 3x + 2$ is 3.
The gradient of $y = x$ is 1.

Step 3: Find which gradient in step 2 will multiply $-\frac{1}{4}$ to give -1.
Conclusion: $-\frac{1}{4} \times 4 = -1$
$\therefore$ $x + 4y + 2 = 0$ is perpendicular to $y = 4x$.



WORKED EXAMPLE 2

Find the equation of the line that passes through the point (2, 4) and is perpendicular to $y = 3x - 2$.

Solution

Let the equation of the line be $y = mx + b$.
Now, the gradient of $y = 3x - 2$ is 3.

$\therefore\ m = -\frac{1}{3}$ $\quad -\frac{1}{3} \times 3 = -1$

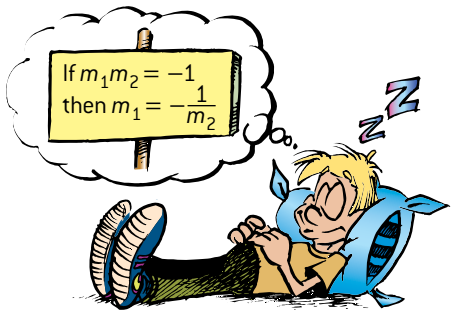
$\therefore\ y = -\frac{1}{3}x + b$

$4 = -\frac{1}{3}(2) + b$ $\quad$ (2, 4) lies on line

$4 = -\frac{2}{3} + b$

$\therefore\ b = 4\frac{2}{3}$

$\therefore$ The equation of the line is $y = -\frac{1}{3}x + 4\frac{2}{3}$.



Exercise 7:03

P Foundation worksheet 7:03
Perpendicular lines

1 Write the slope of the line that is perpendicular to the line with slope:

a 2 b -2 c 4 d $\frac{1}{4}$
e $-\frac{1}{2}$ f $\frac{5}{3}$ g $-\frac{2}{3}$ h $-1\frac{1}{2}$

2 Find the slope of the line that is perpendicular to the line:

a $y = x - 3$ b $y = -x + 4$ c $y = 5x - 2$ d $y = -4x + 1$
e $y = \frac{1}{2}x - 3$ f $y = -\frac{1}{4}x - 8$ g $y = \frac{3}{4}x + 3$ h $y = -\frac{5}{2}x - 3$

3 Are the following pairs of lines perpendicular?

a $y = \frac{1}{5}x + 3,\ y = -5x + 1$
b $y = 3x - 2,\ y = -\frac{1}{3}x + 7$
c $y = 2x - 1,\ y = -\frac{1}{2}x + 3$
d $y = \frac{2}{3}x + 4,\ y = -\frac{3}{2}x - 5$
e $y = 4x,\ y = \frac{1}{4}x - 3$
f $y = \frac{3}{4}x - 1,\ y = -\frac{4}{3}x$
g $y = 3x - 1,\ x + 3y + 4 = 0$
h $x + y = 6,\ x - y - 3 = 0$
i $x + 2y = 3,\ 2x + 3y = 0$
j $y = 0{\cdot}5x,\ y = -2x + 4$

4 a Which of the following lines are perpendicular to $y = 2x$?

$y = 3x$ $\quad y = 2x - 3$ $\quad x + 2y = 4$ $\quad y = -0{\cdot}5x + 5$

b Two of the following lines are perpendicular. Which are they?

$y = -1\frac{1}{2}x + 2$ $\quad y = \frac{1}{2}x - 1$ $\quad y = \frac{2}{3}x$

c A is the point $(2, -1)$, B is the point $(3, -2)$ and C is $(4, -1)$. Prove that $AB \perp BC$.

d E, F, G and H are the points (2, 6), (4, 2), (2, 1) and (0, 2) respectively. Prove that $EF \perp FG$ and $GH \perp HE$.

e The line p has gradient $-\frac{1}{4}$ and y-intercept 3. The line q passes through the points $(-1, -2)$ and (1, 6). Show that p is perpendicular to q.

5 a Find the equation of the line that has y-intercept 5 and is perpendicular to $y = -\frac{1}{3}x + 1$.

b The line AB is perpendicular to $y = -x + 4$. Its y-intercept is 3. What is the equation of AB?

c Find the equation of CD if CD is perpendicular to the line $y = -\frac{1}{2}x$ and has a y-intercept of 0.

d A line has a y-intercept of 1·5 and is perpendicular to the line $y = -2x + 1$. Find the equation of the line.

e If AB passes through the point (2, 3) and is perpendicular to $y = -x - 7$, find the equation of AB.

f Find the equation of the line that passes through (1, 0) and is perpendicular to $y = -3x - 1$.

g A is the point (0, 0) and B is the point (1, 3). Find the equation of the line that has y-intercept 5 and is perpendicular to AB.

h Find the equation of the line that has y-intercept -3 and is perpendicular to the y-axis.

i What is the equation of a line that is perpendicular to the x-axis and passes through $(3, -2)$?

j Two lines $y = -2x + 1$ and $y = mx - 3$ are perpendicular. Find the value of m.

6 Find the equation of the line that is:

a perpendicular to the line $y = x - 3$ and passes through the point (3, 4)

b perpendicular to the line $y = -x + 2$ and passes through the point $(-2, 5)$

c perpendicular to the line $y = 4x - 3$ and passes through the point (4, 3)

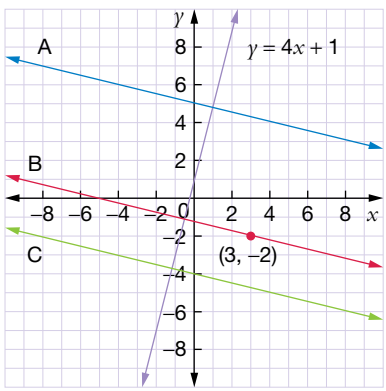
d perpendicular to the line $y = -\frac{1}{2}x + 5$ and passes through the point $(-3, 6)$.

7 The lines A, B and C are all perpendicular to $y = 4x + 1$.
Find the equation of:

a A

b B

c C



8 a A line is drawn perpendicular to the line $2x - y = 3$ through the point (6, 5).
What is the equation of the line?

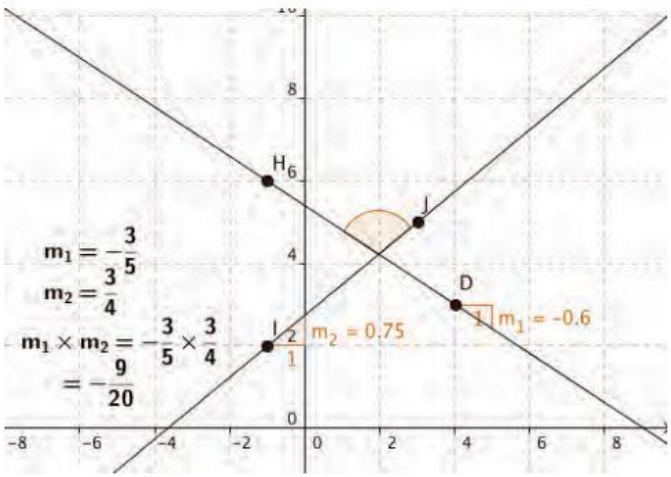
b A line is drawn from the point $(-3, 5)$ perpendicular to the line $3x - y - 3 = 0$.
What is the equation of the line?

c A line is drawn perpendicular to the line $2x + 3y = 15$ and passes through the point $(5, -3)$.
What is the equation of this line and where will it intersect the y-axis?

P GEOGEBRA ACTIVITY 7:03 PERPENDICULAR LINES

1 Compare the gradients of two lines in this activity to investigate the condition for two lines to be perpendicular.

2 This activity uses a variable line and point. Move the point and work out the equation of the line through the point perpendicular to the given line. Use the checkbox to see if you are correct.



7:04 The parabolas $y = ax^2$ and $y = ax^2 + c$

PREP QUIZ 7:04

Find the value of the following when $x = 1$:

1 $3x^2$ **2** $-2x^2$ **3** $0{\cdot}5x^2$

Solve the equations:

4 $x^2 = 4$ **5** $4x^2 = 9$

Find the value of y when $x = 2$ if:

6 $y = x^2$ **7** $y = x^2 + 4$ **8** $y = x^2 - 4$

Complete the table of values for the following:

9 $y = x^2 + 2$

x	-2	-1	0	1	2
y					

10 $y = -3x^2$

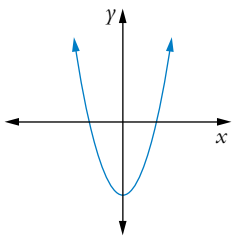
x	-2	-1	0	1	2
y					

In Year 9 two examples of non-linear relationships were encountered: the parabola and the exponential curve. As with straight lines these were graphed by using an equation to produce a table of x- and y-values. The points generated were then plotted to produce the graph.

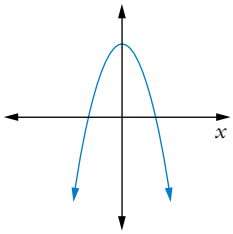
In the following exercise we will look at parabolas with equations of the form $y = ax^2$ and $y = ax^2 + c$ and investigate the effect that different values of a and c have on the graph of the parabola.

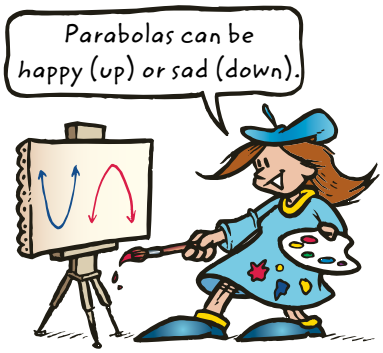
Parabolas have two basic shapes.

- 'concave up'
  open upwards



- 'concave down'
  open downwards





- All parabolas have a turning point that is called the vertex.

The parabolas studied in the following exercise are also symmetrical about the y-axis. This means that there are pairs of points with the same y-coordinate. To find the x-coordinate of a point on the parabola given its y-coordinate, a simple quadratic equation will need to be solved.

WORKED EXAMPLE 1

Complete a table of values from -2 to 2 at intervals of 0·5 to graph the following parabolas.

a $y = 2x^2$ **b** $y = 6 - x^2$

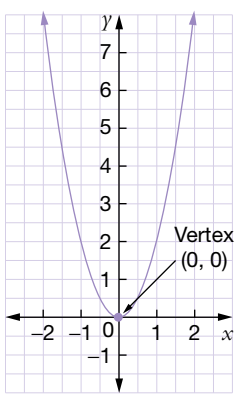
Label the vertex of each parabola and state whether the parabola is 'concave up' or 'concave down'.

Solution

a $y = 2x^2$

x	-2	$-1{\cdot}5$	-1	$-0{\cdot}5$	0	0·5	1	1·5	2
y	8	4·5	2	0·5	0	0·5	2	4·5	8

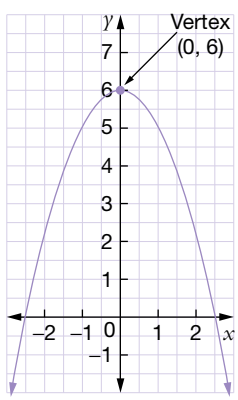
The parabola is concave up.



b $y = 6 - x^2$

x	-2	$-1{\cdot}5$	-1	$-0{\cdot}5$	0	0·5	1	1·5	2
y	2	3·75	5	5·75	6	5·75	5	3·75	2

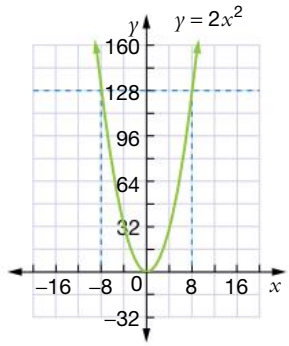
The parabola is concave down.



The vertex is the turning point of the parabola.

WORKED EXAMPLE 2

Find the points on the curve $y = 2x^2$ that have a y-coordinate of 128.



Solution

If $y = 2x^2$ and $y = 128$ then:

$128 = 2x^2$

$x^2 = 64$

$x = \pm\sqrt{64}$

$x = -8$ or 8

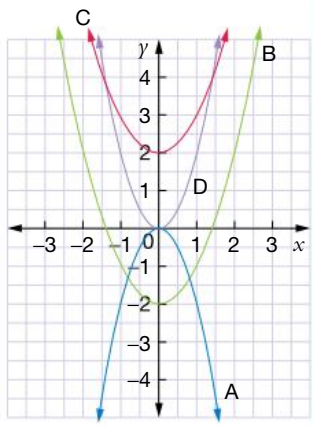
The points are $(-8, 128)$ and $(8, 128)$.

WORKED EXAMPLE 3

Match each of the parabolas A to D with the equations below.

a $y = 2x^2$ **b** $y = x^2 + 2$

c $y = -2x^2$ **d** $y = x^2 - 2$



Solution

a For $y = 2x^2$, when $x = 0$, $y = 0$. Graphs A and D pass through $(0, 0)$.
When $x = 1$, $y = 2$. Only graph D passes through $(1, 2)$.
Hence this is graph D.

b For $y = x^2 + 2$, when $x = 0$, $y = 2$.
The only graph passing through $(0, 2)$ is C.

c For $y = -2x^2$ when $x = 0$, $y = 0$. Hence it could be A or D.
When $x = 1$, $y = -2$. Hence this is graph A.

d For $y = x^2 - 2$, when $x = 0$, $y = -2$.
The only graph passing through $(0, -2)$ is B.

The shape of the parabola is clearly demonstrated by the water arcs of this fountain.

Exercise 7:04

1 **a** Complete the table below for the equation $y = x^2$.

x	−3	−2·5	−2	−1·5	−1	−0·5	0	0·5	1	1·5	2	2·5	3
y	9											6·25	

b Use a sheet of graph paper to graph the curve $y = x^2$, using your table.

Note: A graphics calculator or graphing software could be used in the following exercises as an alternative to plotting points.

2 Complete the following tables and then graph all four curves on one number plane.
Hint: On the y-axis, use values from 0 to 12.

a $y = x^2$

x	−3	−2	−1	0	1	2	3
y							

b $y = 2x^2$

x	−2	−1	−0·5	0	0·5	1	2
y							

c $y = 3x^2$

x	−2	−1	−0·5	0	0·5	1	2
y							

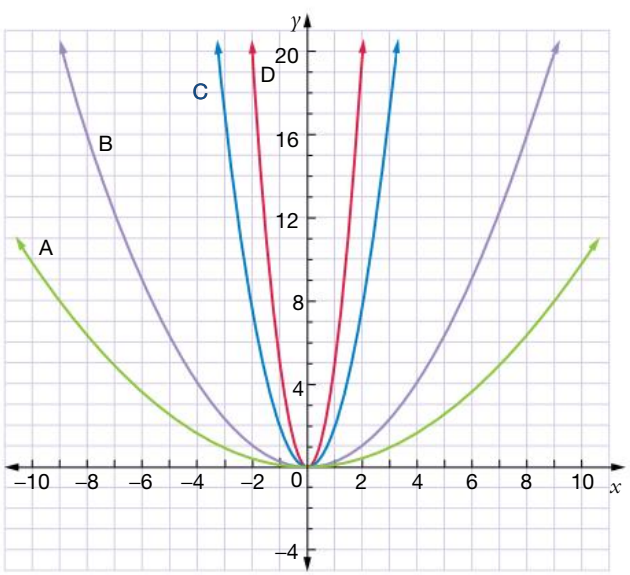
d $y = 0{\cdot}5x^2$

x	−3	−2	−1	0	1	2	3
y							

- For the equation $y = ax^2$, what is the effect on the graph of varying the value of a?

3 Match each of the parabolas A to D with the equations below.

a $y = 0{\cdot}25x^2$
b $y = 5x^2$
c $y = 2x^2$
d $y = 0{\cdot}1x^2$



4 Complete the following tables and then graph all four curves on one number plane.
Hint: On the y-axis, use values from -2 to 13.

a $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

b $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

c $y = x^2 + 4$

x	-3	-2	-1	0	1	2	3
y							

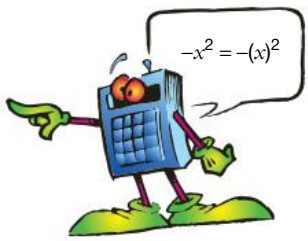
d $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							

- What is the difference in the curves $y = x^2$ and $y = x^2 + 2$?
- Can you see that the shape of the curve is the same in each case?
- For the equation $y = x^2 + c$, what is the effect on the graph of varying the value of c?

5 **a** Complete the table of values for $y = -x^2$.

x	-3	-2	-1	0	1	2	3
y							

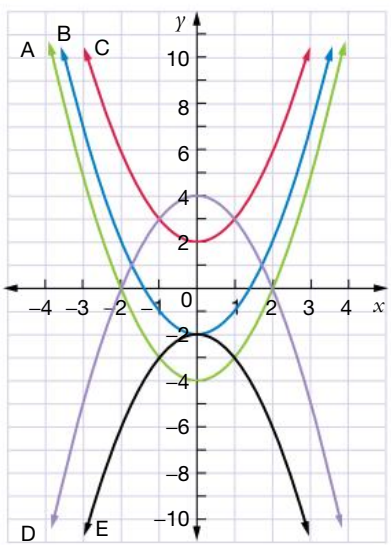


b Plot these values on a number plane. What is the effect of the minus sign in $y = -x^2$?

c On the same diagram, sketch the graphs of $y = -x^2 - 1$ and $y = 2 - x^2$.

6 Match each of the following equations with the graphs A to E.

a $y = -x^2 - 2$
b $y = x^2 + 2$
c $y = x^2 - 2$
d $y = x^2 - 4$
e $y = 4 - x^2$



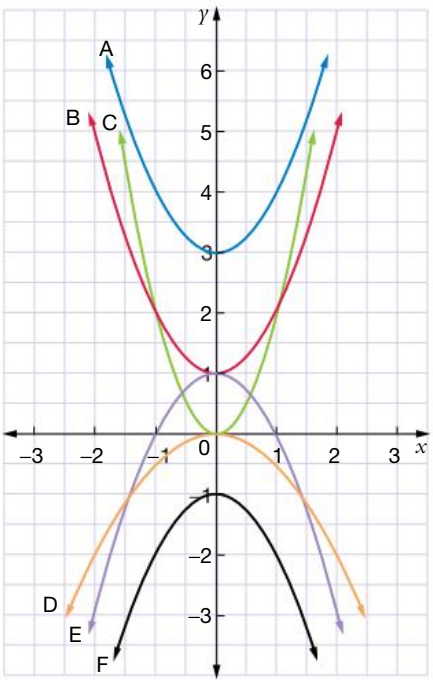
7 Find the equation of the parabola that results from performing the following transformations on the parabola $y = x^2$.

- **a** moving it up 2 units
- **b** moving it down 2 units
- **c** turning it upside down and then moving it up 4 units
- **d** turning it upside down and then moving it down 2 units
- **e** moving it up 2 units and then reflecting it in the x-axis

8 What transformations of the parabola $y = x^2$ would produce each of the parabolas A to E in Question **6**?

9 Match each equation with the corresponding parabola in the diagram.

- **a** $y = 2x^2$
- **b** $y = -\frac{1}{2}x^2$
- **c** $y = x^2 + 1$
- **d** $y = x^2 + 3$
- **e** $y = -x^2 - 1$
- **f** $y = 1 - x^2$



A point lies on a parabola if its coordinates satisfy the equation of the parabola: e.g. $(1, 3)$ lies on the parabola $y = 2x^2 + 1$ because $3 = 2 \times 1^2 + 1$

10 **a** On which of the following parabolas does the point $(2, 4)$ lie?
$y = x^2$, $y = 8 - x^2$, $y = 6 - 2x^2$, $y = 2x^2 - 4$

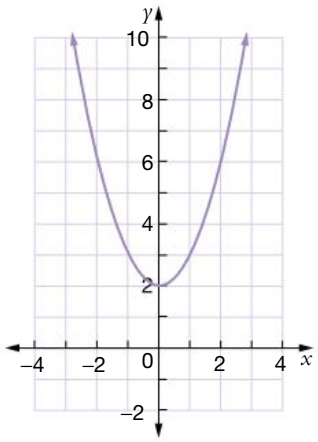
- **b** Which of these parabolas pass through the point $(1, 2)$?
$y = 4 - x^2$, $y = 2x^2$, $y = x^2 + 1$, $y = 3 - x^2$
- **c** The parabolas $y = x^2$, $y = 2x^2$ and $y = 3x^2$ all pass through the point $(0, 0)$. Which parabola also passes through the point $(3, 18)$?
- **d** The parabola $y = ax^2$ passes through the point $(-2, 16)$. What is the value of a?
- **e** The parabola $y = ax^2 + 2$ passes through the point $(4, 10)$. What must the value of a be?
- **f** Find the x-coordinates of the points that have a y-coordinate of:
  - **i** 100 on the parabola $y = x^2$
  - **ii** 12 on the parabola $y = x^2 - 4$
  - **iii** 28 on the parabola $y = 3x^2 + 1$
  - **iv** -94 on the parabola $y = 4 - 2x^2$

11 The equation of the parabola shown in the diagram is of the form $y = ax^2 + c$.

a From the graph, find the value of c and then complete the following: The form of the equation is $y = ax^2 + \ldots$

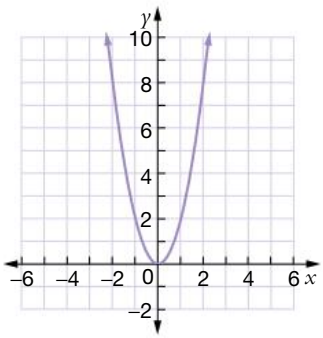
b From the graph, give one point that the parabola passes through except $(0, 2)$.

c Using the answers from parts **a** and **b**, find the equation of the parabola.

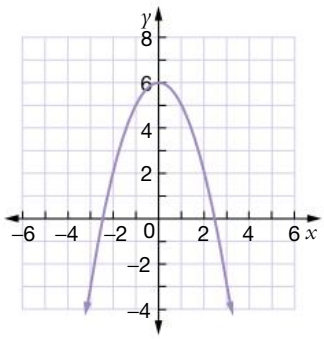


12 Find the equation of each of the following parabolas.

a



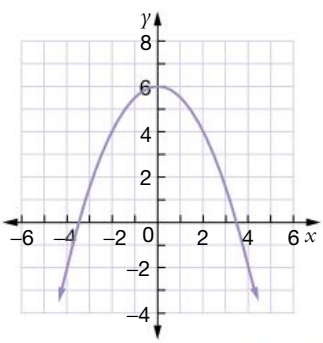
b



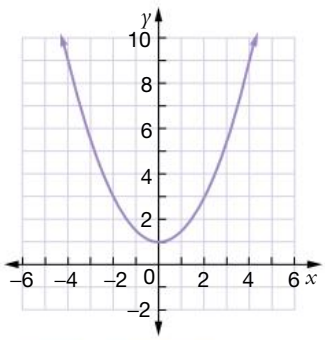
To find an equation of the form $y = ax^2 + c$:

1. Find c (the y-intercept).
2. Find a (by substituting the coordinates of any point into the resulting equation).

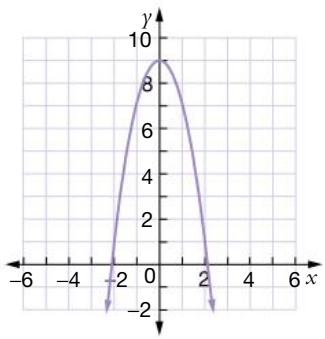
c



d



e

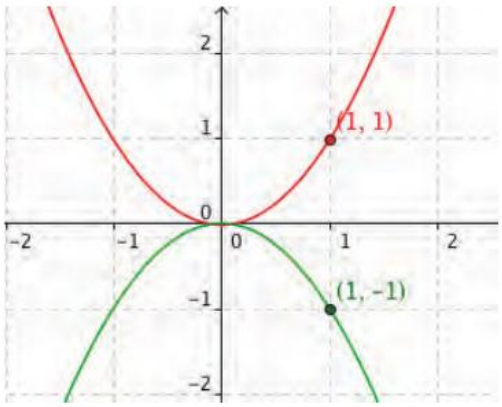


The curve formed by the chain is called a catenary. It resembles a parabola.

GEOGEBRA ACTIVITY 7:04A PARABOLAS OF THE FORM $y = ax^2$

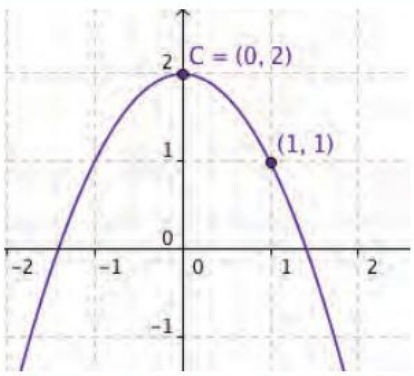
This activity uses sliders to change the value of a. You can see instantly what changing the value of a does to the shape of the graph.

You can then choose to find the equation of an unknown parabola that is randomly generated.



GEOGEBRA ACTIVITY 7:04B PARABOLAS OF THE FORM $y = ax^2 + c$

Investigate how changing the values of a and c change the shape of the parabola. Test your skills by finding the equation of a randomly generated parabola.



7:05 Exponential graphs

PREP QUIZ 7:05

Find the value of:

1 2^3 **2** 2^5 **3** 2^0 **4** 2^{-1} **5** 2^{-5}

If $y = 2^{-x}$, find y when x is:

6 1 **7** 3 **8** 2

Use your calculator to find, to one decimal place, the value of:

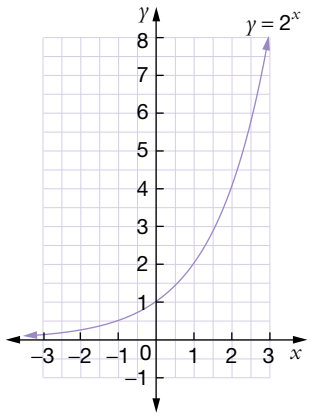
9 $2^{1.5}$ **10** $2^{-2.5}$

When equations such as $y = 2^x$, $y = -2^x$ and $y = 2^{-x} + 1$ are graphed the resulting curves are called exponential curves. Exponential equations involve calculating varying powers of a fixed positive number, often called the base. In the examples above the base is 2.

Exponential graphs can be drawn by completing a table of values and plotting the resulting points on a number plane. This is shown for the curves $y = 2^x$ and $y = 1 + 3^{-x}$.

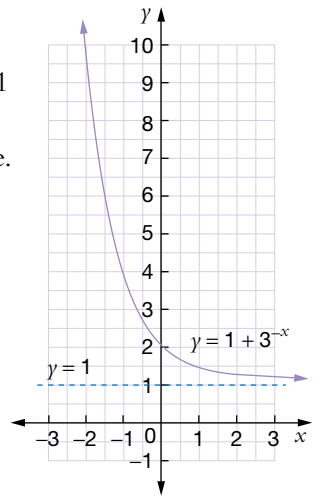
$y = 2^x$

- The curve increases as the x-values increase.
- The curve rises steeply for positive values of x.
- The curve flattens out for negative values of x and the smaller x becomes the closer the curve gets to the x-axis. The x-axis is an asymptote for this part of the curve.
- Because 2^x is always positive the curve is always above the x-axis.
- The curve passes through $(0, 1)$ on the y-axis because $2^0 = 1$. The y-intercept is 1.



$y = 1 + 3^{-x}$

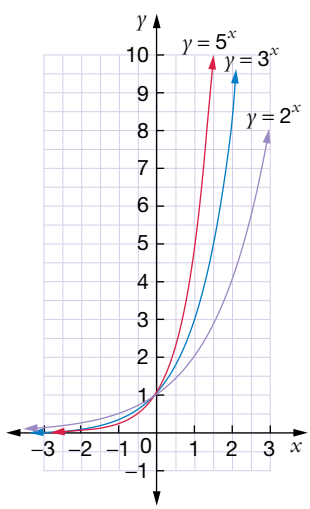
- The curve decreases as the x-values increase.
- As x increases the curve gets closer to the line $y = 1$. We say that $y = 1$ is an asymptote and that the curve is asymptotic.
- The curve is always above the line $y = 1$ because 3^{-x} is always positive.
- The curve passes through $(0, 2)$. The y-intercept is 2.



In the following exercise the similarities and differences between different curves will be investigated. Analysing the equation of the curve will enable you to predict the shape of the curve.

Exercise 7:05

1 The graphs of $y = 2^x$, $y = 3^x$ and $y = 5^x$ are shown on the number plane.

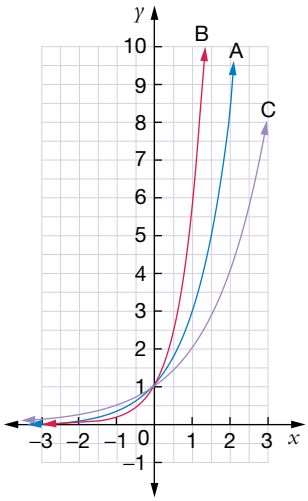


Compare the graphs.

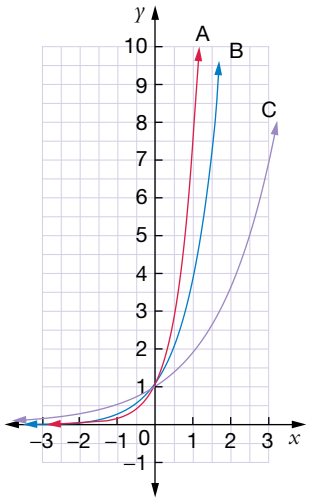
- **a** Where do the curves cross?
- **b** Is $y = 5^x$ always above $y = 3^x$?
- **c** Which graph increases the fastest?
  Which graph increases the slowest?
- **d** If you were asked to graph $y = 10^x$ on this graph, where would it be in relation to the other curves?
- **e** If you were asked to graph $y = 1{\cdot}8^x$ on this graph, where would it be in relation to the other curves?

2 In each of the following, match each curve with its equation.

a $y = 2^x$, $y = 3^x$, $y = 6^x$



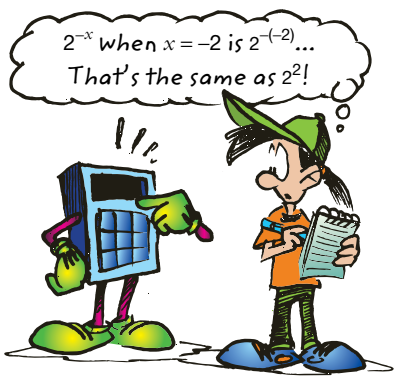
b $y = 4^x$, $y = 1{\cdot}9^x$, $y = 8^x$



3 **a** Complete the table of values for $y = 2^{-x}$ and graph the curve on a number plane.

x	−2	−1·5	−1	−0·5	0	0·5	1	1·5	2
y									

b Compare your graph with $y = 2^x$. What do you notice?



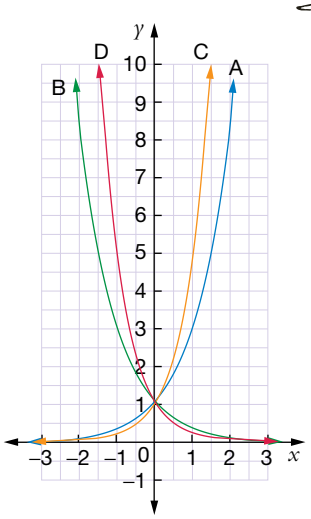
4 Match each of the graphs A to D to the following equations.

a $y = 3^x$

b $y = 5^x$

c $y = 5^{-x}$

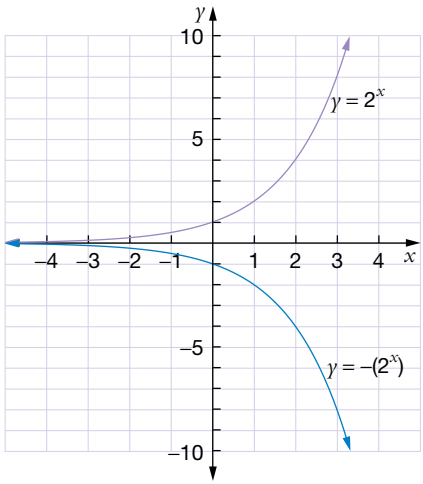
d $y = 3^{-x}$



5 If each of the following curves is reflected in the y-axis, what is the equation of the curve that is obtained?

a $y = 4^x$ **b** $y = 6^x$ **c** $y = 8^{-x}$ **d** $y = 1{\cdot}2^x$ **e** $y = 3{\cdot}6^{-x}$

6 The graphs $y = 2^x$ and $y = -2^x$ have been graphed on the number plane.



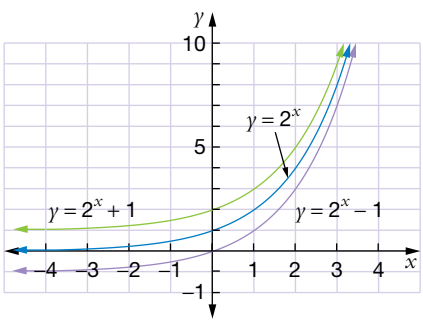
- **a** Look at each pair of points with the same x-coordinate e.g. $(1, 2)$ and $(1, -2)$. Is each pair of points equidistant from the x-axis?
- **b** Are these curves reflections of each other in the x-axis?
- **c** Draw on the same number plane the graphs of $y = 2^{-x}$ and $y = -(2^{-x})$? Are these curves reflections of each other in the x-axis?

7 If each of the following curves is reflected in the x-axis, what is the equation of the curve that is obtained?

- **a** $y = 3^x$
- **b** $y = 4^{-x}$
- **c** $y = 1 \cdot 8^{-x}$
- **d** $y = -(1 \cdot 2^x)$
- **e** $y = -6^{-x}$

8
- **a** How are the graphs of $y = a^x$ and $y = a^{-x}$ related?
- **b** How are the graphs of $y = a^x$ and $y = -a^x$ related?

9 Study the graphs of the curves $y = 2^x$, $y = 2^x + 1$ and $y = 2^x - 1$ on the diagram.

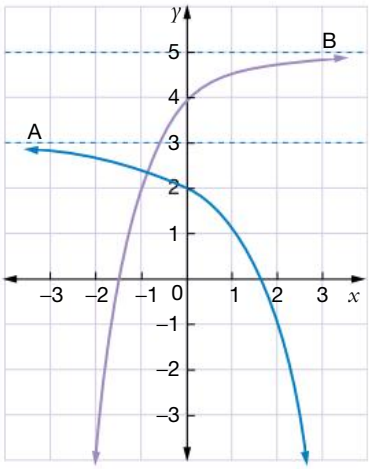


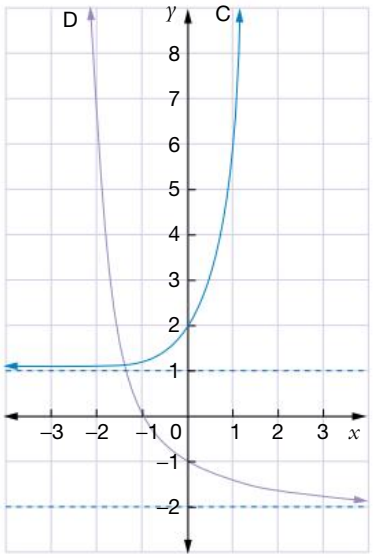
- **a** What transformation will change:
  - **i** $y = 2^x$ into $y = 2^x + 1$
  - **ii** $y = 2^x$ into $y = 2^x - 1$?
- **b** What is the equation of the curve that results if $y = 2^x$ is translated:
  - **i** 3 units vertically upwards
  - **ii** 4 units vertically downwards?
- **c** Sketch on the same number plane the curves $y = 2^x + 2$ and $y = 2^x - 3$.

10 Find the equation of the curve that results when the curve $y = 3^x$ is:

- **a** reflected in the x-axis
- **b** reflected in the y-axis
- **c** translated 1 unit vertically upwards
- **d** translated 1 unit vertically downwards
- **e** reflected in the y-axis and then translated 1 unit vertically downwards
- **f** reflected in the x-axis and then translated 1 unit vertically upwards.

11 For each of the curves A to D answer the following questions.





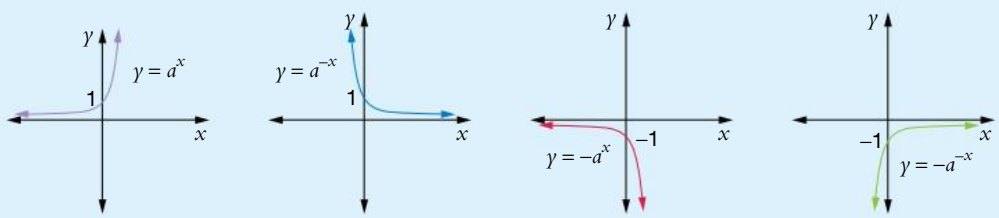
a Is the curve increasing or decreasing?
b Is the curve concave up or concave down?
c What happens to the y-values as the x-values become:
   i very large ii very small?
d What is the y-intercept?

12 From the list of equations below, select those that would fit into each of the categories **a** to **f**.

A $y = 3^x$	**B** $y = 2^{-x}$	**C** $y = -4^x$	**D** $y = 3^{-x}$
E $y = 2^x + 1$	**F** $y = 1 - 3^x$	**G** $y = 4^x$	**H** $y = -2^{-x}$
I $y = 1 - 2^x$	**J** $y = 4^{-x} + 3$	**K** $y = -3^x$	**L** $y = 4^x - 1$

a The curves that can be formed from $y = 3^x$ using reflections and/or translations.
b The curves that are increasing.
c The curves that are decreasing.
d The curves that are increasing and concave up.
e The curves that are increasing and concave down.
f The curves that have a y-intercept of:
   i 1 ii -1

The basic exponential curve shapes ($a > 1$)



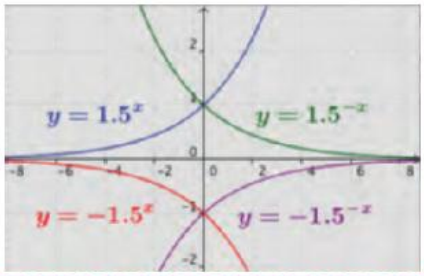
GEOGEBRA ACTIVITY 7:05 EXPONENTIAL GRAPHS

Investigate four basic types of exponential graphs:

$y = a^x + c$, $y = a^{-x} + c$, $y = -a^x + c$ and $y = -a^{-x} + c$

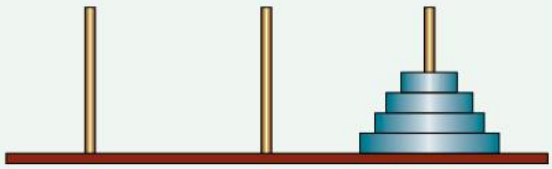
Change the values of a and c to observe the changes to the graph.

Test your skills by finding the equation of a randomly generated parabola.



FUN SPOT 7:05 THE TOWER OF HANOI

This famous puzzle consists of three vertical sticks and a series of discs of different radii that are placed on one stick to form a tower, as shown in the diagram. The aim of the puzzle is to move the discs so that the tower is on one of the other sticks. The rules are:

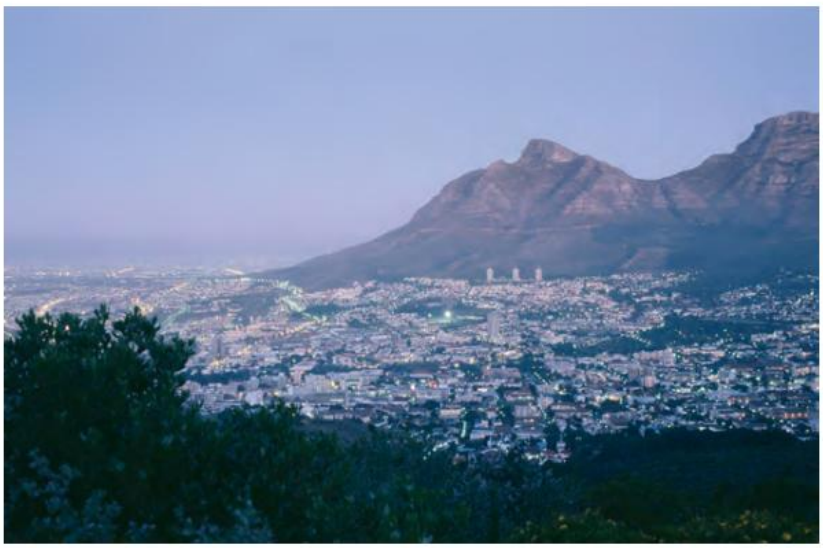


- only one disc can be moved at a time to another stick
- a larger disc can never be placed on top of a smaller one.

The puzzle can be made more difficult by having more discs.

Investigate the minimum number of moves needed if there are 2, 3 or 4 discs. Can you generalise your results?

Can you predict the minimum number of moves needed if there are, say, 8 discs? (Hint: An exponential relationship can be found!)



Exponential curves are used in the study of population growth.

7:06 The equation of a circle

The diagram shows a circle of radius r units with its centre at the origin, O.

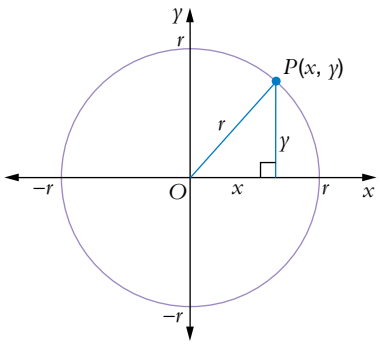
Now, every point on the circle is r units from O.

Let P be any point on the circle with coordinates (x, y).

$\therefore\ OP = r$

But $OP^2 = x^2 + y^2$ Pythagoras' theorem

$\therefore\ x^2 + y^2 = r^2$



> The equation of a circle with its centre at the origin and radius r is:
> $x^2 + y^2 = r^2$

WORKED EXAMPLES

1 A circle has its centre at the origin and a radius of 5 units. What is its equation?

2 A circle has an equation $x^2 + y^2 = 16$. What is the centre and radius of the circle?

Solution

1 $x^2 + y^2 = r^2$ equation of a circle centre (0, 0), radius r

$r = 5$

$\therefore\ x^2 + y^2 = 25$

2 $x^2 + y^2 = 16$

$r^2 = 16$

$r = 4$

$\therefore$ Circle has centre (0, 0) and radius 4 units.

Exercise 7:06

1 Which of the following represent the equations of circles? (You may need to rearrange some equations.)

A $x^2 - y^2 = 25$ **B** $x^2 + y^2 = 16$ **C** $y^2 - x^2 = 1$

D $x^2 + y^2 = 1$ **E** $x^2 + y^2 - 4 = 0$ **F** $y^2 = 4 - x^2$

G $x^2 + y = 1$ **H** $x^2 + y^2 = 9$ **I** $x + y^2 = 9$

2 Write the equation of a circle with its centre at the origin and a radius of:

a 1 unit **b** 3 units **c** $2\frac{1}{2}$ units.

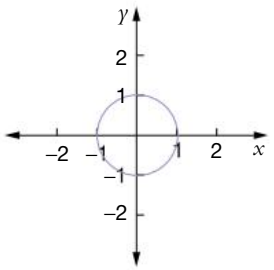
3 State the radius of the following circles.

a $x^2 + y^2 = 100$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 6{\cdot}25$

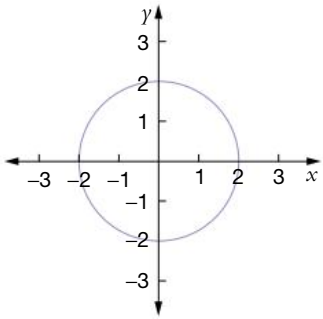
4 Graph each of the circles in Question 3 on separate number planes.

5 What is the equation of the following circles?

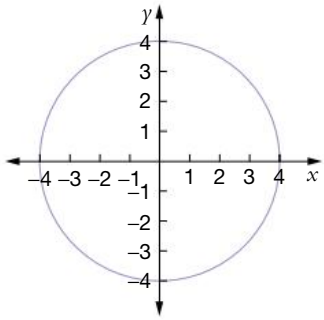
a



b



c

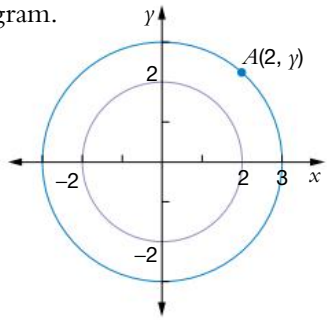


6 a Write the equations of both circles shown in the diagram.

b If the circle $x^2 + y^2 = 25$ was drawn on this diagram, how far would it be from the outer circle?

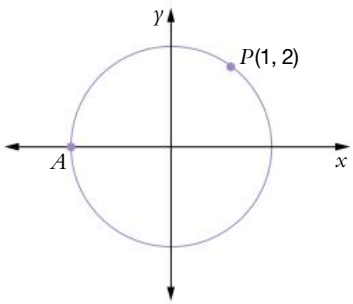
c On which of the circles does the point $(1, \sqrt{3})$ lie?

d Find the y-coordinate of the point A.



7 a The point $P(1, 2)$ lies on the circle shown. What is the equation of the circle?

b What are the coordinates of A?



8 Find the equation of the circle with its centre at the origin that passes through the point:

a $(-4, 3)$

b $(-2, -3)$

FUN SPOT 7:06 TRY TO MATCH THIS ONE!

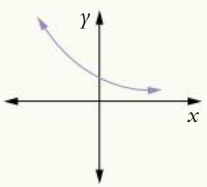
This is a simple game. All you need is 20 matches (or strokes on a piece of paper) and two people.

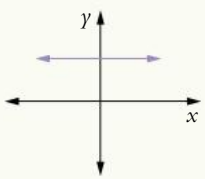
- Each person takes it in turns to remove only 1 or 2 of the matches remaining in front of them.
- The winner is the player who takes the last match.
- Take it in turns to go first.
- Try the game starting with a different number of matches.

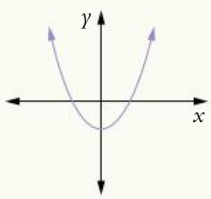
7:07 Miscellaneous graphs

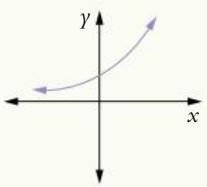
PREP QUIZ 7:07

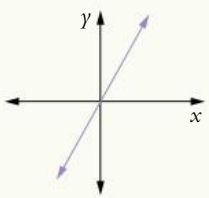
Match each graph with the best equation from the given list.

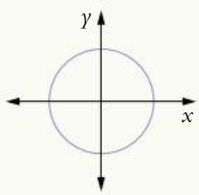
1 

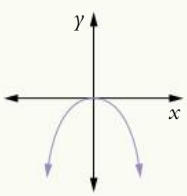
2 

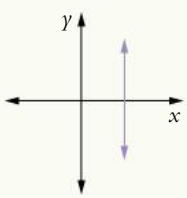
3 

4 

5 

6 

7 

8 

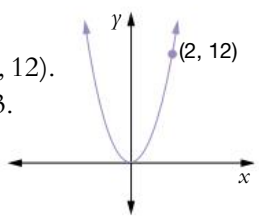
A $x = 2$
B $y = 2^x$
C $y = -x^2$
D $y = 2^{-x}$
E $y = 2$
F $y = 2x$
G $y = x^2 - 1$
H $x^2 + y^2 = 4$

9 What is the y-intercept of the curve $y = 3^x + 2$?
10 Find the value of a if the curve $y = ax^2 + 4$ passes through the point $(1, 6)$.

When matching equations to graphs or when finding the equation of a given line or curve it is important to:

1 Identify the type of curve.
2 Select the right form of equation to use. (For a parabola is it $y = ax^2$ or $y = ax^2 + c$?)
3 Remember that if a line or curve passes through a point then the coordinates of the point must satisfy the equation of the curve.

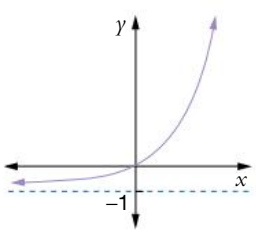
e.g. The parabola shown is of the form $y = ax^2$ and it passes through $(2, 12)$.
Substituting $x = 2$ and $y = 12$ and $y = ax^2$ gives $12 = a \times 2^2$ or $a = 3$.



The y-intercept, which is easily found by substituting $x = 0$, is one of the most useful points to use in matching equations and graphs.

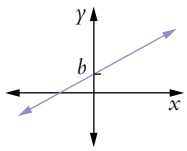
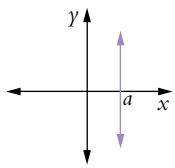
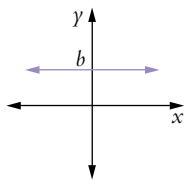
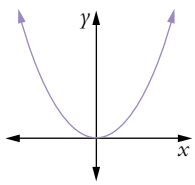
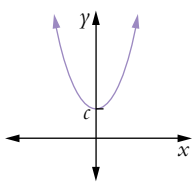
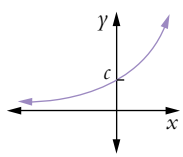
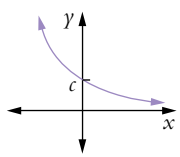
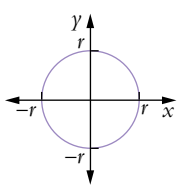
e.g. Is the curve shown $y = 2^x + 1$ or $y = 2^x - 1$?

The y-intercepts for the curves $y = 2^x + 1$ and $y = 2^x - 1$ are 2 and 0 respectively. Since the curve shown passes through the origin it must be $y = 2^x - 1$.



For exponential graphs the asymptote can also be used to match equations and graphs.

A summary of the graphs met so far and their equations are shown in the following table.

Type of graph	Equation	Graph
Straight line	$y = mx + b$ $ax + by + c = 0$	
Lines parallel to the axes	$x = a$ $y = b$	 
Parabola	$y = ax^2$ $y = ax^2 + c$	 
Exponential curve	$y = a^x$ $y = a^{-x}$ $y = a^x + c$ $y = a^{-x} + c$	 
Circle	$x^2 + y^2 = r^2$	

Exercise 7:07

P Foundation worksheet 7:07
Miscellaneous graphs

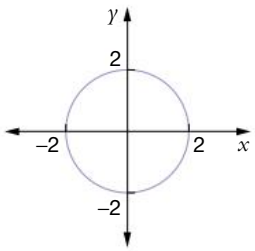
1 From the list of equations on the right, choose those that represent:

a a straight line
b a circle
c a parabola
d an exponential curve.

A $x^2 + y^2 = 16$	**B** $y = 6 - x^2$
C $y = 3$	**D** $y = 2^{-x}$
E $y = x^2 - 2$	**F** $y = 4^{-x}$
G $y = 3^x$	**H** $x^2 + y^2 = 1$
I $y = 2^x - 2$	**J** $2x + 4y = 3$

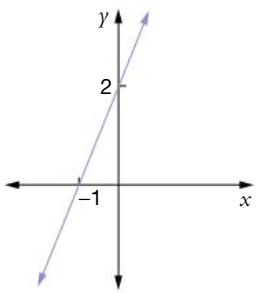
2 Match each graph with its equation from the given list.

a



A $y = 4 - 2^x$	**B** $y = 4 - x^2$
C $y = 5^x$	**D** $x^2 + y^2 = 4$
E $y = 2x + 2$	**F** $y = -2x + 2$
G $y = 2^x + 2$	**H** $y = 5$
I $y = x^2 - 4$	**J** $y = 2^{-x} + 2$

b



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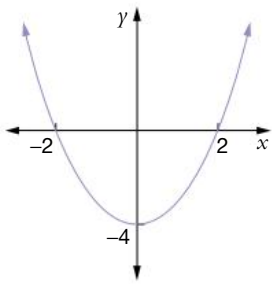

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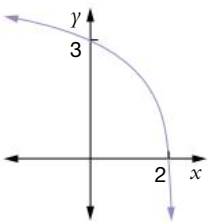

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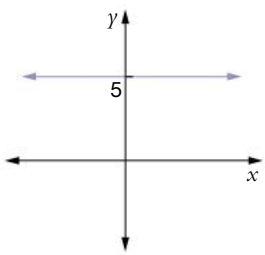

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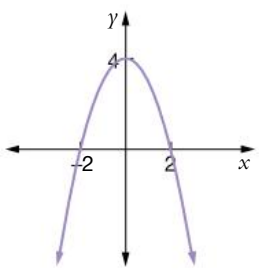

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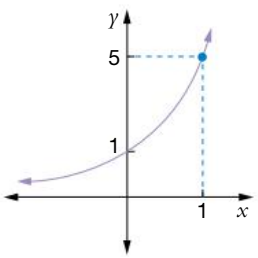

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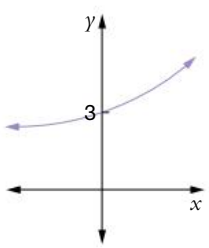

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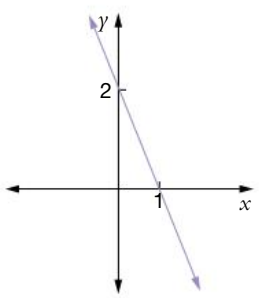

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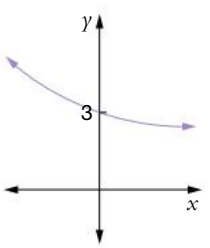
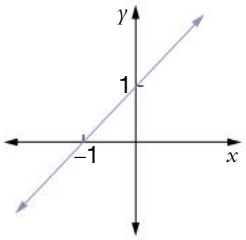


3 Using a table of values, or otherwise, sketch the graph of the following equations. Show where each one cuts the coordinate axes.

a $y = x - 1$	**b** $y = 4 - x$	**c** $y = 2x + 1$
d $x = -1$	**e** $y = 3$	**f** $x = 5$
g $y = x^2 + 2$	**h** $y = x^2 - 4$	**i** $y = 1 - x^2$
j $x^2 + y^2 = 16$	**k** $x^2 + y^2 = 36$	**l** $x^2 + y^2 = 10$
m $x + y = 3$	**n** $y = 2x^2$	**o** $y = 8 - 2x^2$
p $y = 4^x$	**q** $y = 2^{-x}$	**r** $y = 1 - 2^x$

4 Determine the equation of each graph.

a



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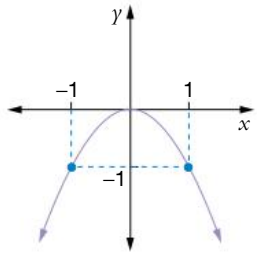

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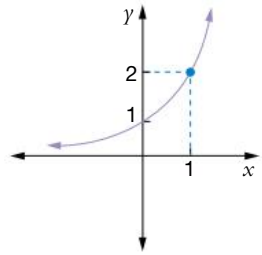

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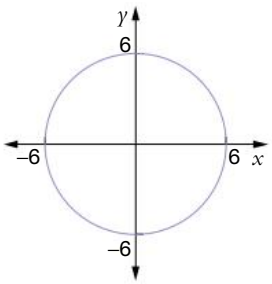

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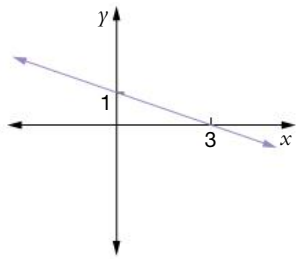

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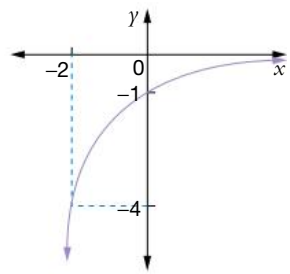

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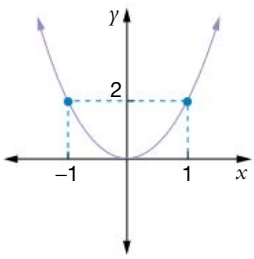

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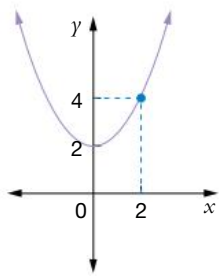

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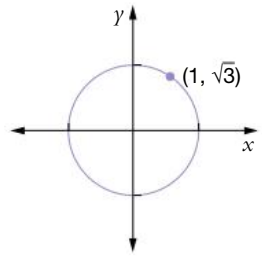

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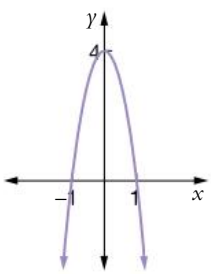

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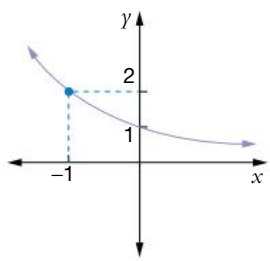

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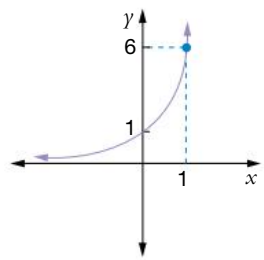

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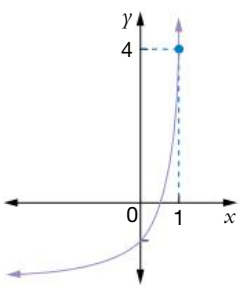

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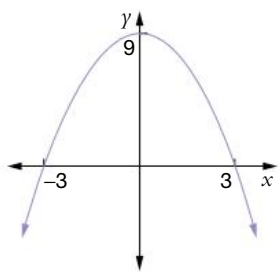

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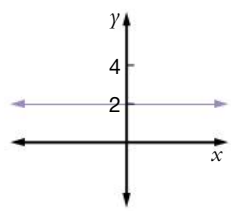
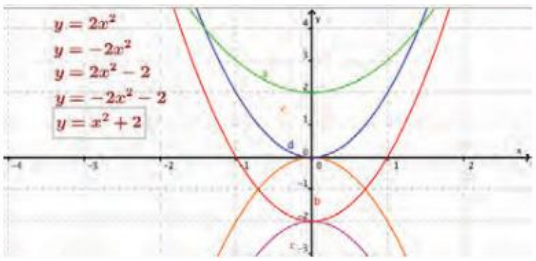


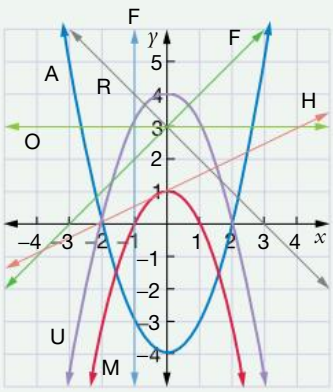
GEOGEBRA ACTIVITY 7:07 MATCHING EQUATIONS AND GRAPHS

Four different exercises look at matching a set of randomly generated graphs and their equations. The first three are devoted to lines, parabolas and exponentials. The fourth exercise presents a randomly generated combination of the first three types of graphs. A checkbox allows you to choose whether to match the graphs to the equations or to find the equations of the curves and then check your answers.



FUN SPOT 7:07 WHAT IS HIJKLMNO?

Work out the answer to each question and write the letter for that part in the box that is above the correct answer.



For the number plane shown, match each graph with its correct equation below.

Find the equation of a line with:

O a gradient of 3 and a y-intercept of 2
A a gradient of -2 and a y-intercept of 5
E a gradient of 4 and passes through the origin
R a y-intercept of -2 and a gradient of 1
R a y-intercept of 3 and a gradient of $\frac{1}{2}$
O a y-intercept of 2 and a gradient of zero

Match the equations $y = 2x$, $y = x^2$ and $y = 2^x$ with the following graphs.

T parabola **L** straight line **W** exponential

															–		2	
$x = -1$	$y = 2$	$y = 3 - x$	$y = 1 - x^2$	$y = 4 - x^2$	$y = 2x$	$y = 5 - 2x$	$y = x + 3$	$y = 3$	$y = \frac{1}{2}x + 3$	$y = 2^x$	$y = x^2 - 4$	$y = x^2$	$y = 4x$	$y = x - 2$		$y = \frac{1}{2}x + 1$		$y = 3x + 2$

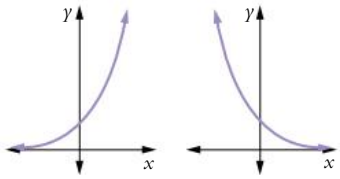
MATHS TERMS 7

equation (of a curve)

- an algebraic statement that expresses the relationship between the x- and y-coordinates of every point (x, y) that lies on the curve

exponential curve

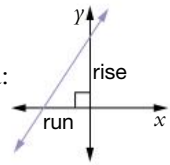
- a curve with an equation of the form $y = a^x$, where $a > 0$



gradient (of a line)

- the slope of a line
- calculated using the formula:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$



gradient–intercept form of the equation of a line

- $y = mx + b$
  where m = gradient
  b = y-intercept
- allows the equation of a line to be found by finding the line's gradient and y-intercept

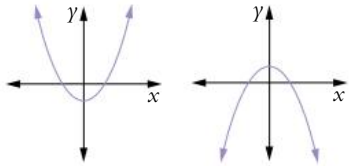
graph (of a curve)

- the line that results when the points that satisfy a curve's equation are plotted on a number plane

parabola

- a curve with the equation $y = ax^2 + c$ (a and c are constants)

  e.g.



- parabolas can be concave up or concave down

vertex (turning point)

- the point where a parabola turns around

x-intercept(s)

- the point(s) where a curve crosses the x-axis

y-intercept(s)

- the point(s) where a curve crosses the y-axis

parallel lines

- lines in the number plane with the same gradient
- if the gradients of two parallel lines are m_1 and m_2, then $m_1 = m_2$

perpendicular lines

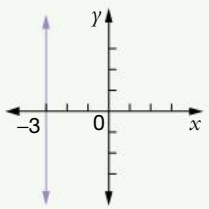
- lines in the number plane that are at right angles to each other at their point of intersection
- if the gradients of two perpendicular lines are m_1 and m_2, then:

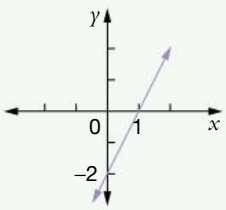
$$m_1 m_2 = -1, \text{ i.e. } m_1, = -\frac{1}{m_2}$$

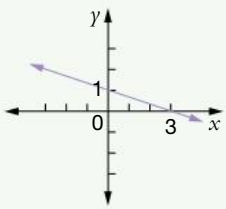
The dish of a radio-telescope is parabolic in shape.

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Sketch the graphs of: 7:01
a $y = x + 2$ b $y = 2x + 1$ c $y = -2x - 1$

2 Write the equation of each line below. 7:01
a 

b 

c 

3 Find the gradient and y-intercept of each line by rearranging the equation into gradient-intercept form. 7:01
a $3x + y = 6$ b $3x + 2y = 4$ c $2x - y + 5 = 0$

4 a Which of the lines $y = 3x + 2$, $y = x + 2$ and $y = 3x$ are parallel? 7:02
b Find the equation of the line that has y-intercept 2 and is parallel to $y = 4x - 1$.
c Find the equation of the line that passes through $(0, 7)$ and is parallel to $y = -3x + 4$.

5 a Which of the lines $y = 4x$ and $y = 3x + 4$ is perpendicular to $y = -\frac{1}{4}x + 3$? 7:03
b Find the equation of the line that passes through the point $(0, 5)$ and is perpendicular to $y = \frac{2}{3}x + 1$.
c Find the equation of the line that has a y-intercept of -3 and is perpendicular to $y = 2x + 1$.

6 On the same number plane, sketch the graphs of: 7:04
a $y = x^2$ b $y = x^2 - 4$ c $y = x^2 + 2$

7 On the same number plane, sketch the graphs of: 7:04
a $y = 2x^2$ b $y = \frac{1}{2}x^2$ c $y = -x^2$

8 Write the equation of the parabola that results when: 7:04
a the parabola $y = x^2$ is translated 3 units upwards
b the parabola $y = 2x^2$ is reflected in the x-axis
c the parabola $y = 4x^2$ is reflected in the x-axis and then translated 2 units downwards.

7:04

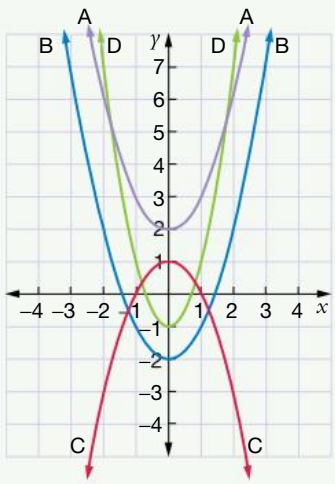
9 Match each equation with the corresponding parabola in the diagram.

a $y = 1 - x^2$

b $y = 2x^2 - 1$

c $y = x^2 + 2$

d $y = -2 + x^2$



7:05

10 Sketch, on the same number plane, the graphs of:

a $y = 2^x$ b $y = 3^x$ c $y = 2^{-x}$

7:05

11 Find the equation of the curve that results when the curve $y = 2^x$ is:

a reflected in the y-axis

b reflected in the x-axis

c translated two units upwards

d reflected in the x-axis and then translated 2 units down.

7:06

12 Find the equation of a circle that has its centre at the origin and radius:

a 1 unit b 3 units c 10 units

7:06

13 Write the centre and radius of the circles with the following equations.

a $x^2 + y^2 = 25$ b $x^2 + y^2 = 16$ c $x^2 + y^2 = \frac{25}{4}$

7:07

14 From the list of equations on the right, which represent:

a parabolas

b straight lines

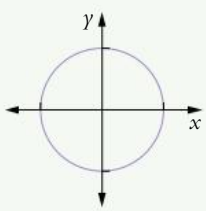
c circles

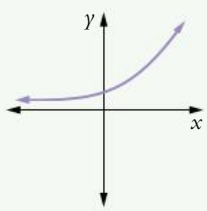
d exponential graphs?

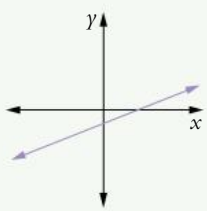
A $y = 9^x$	B $y = 3$
C $y = x + 3$	D $y = 3^{-x}$
E $y = 9 - x^2$	F $y = 3 - 3^x$
G $x^2 + y^2 = 9$	H $y = x^2 + 3x$
I $y = 9 - x$	J $x^2 + y^2 = 3$

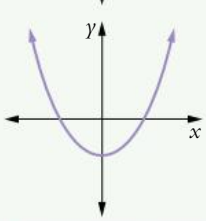
7:07

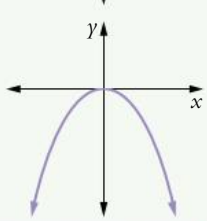
15 Match each graph with its equation from the list.

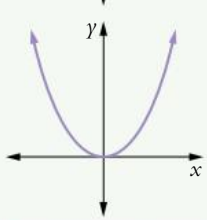
a 

b 

c 

d 

e 

f 

A $y = x^2 - 2$	
B $y = 2^x$	
C $y = x - 2$	
D $y = -2x^2$	
E $x^2 + y^2 = 2$	
F $y = 2x^2$	

ASSIGNMENT 7A Chapter review

1 Find the equation of each line, given the following information.

- **a** parallel to the y-axis and cuts the x-axis at $(3, 0)$
- **b** parallel to the line $y = 2x - 3$ with a y-intercept of 3
- **c** parallel to the line $y = 2 - x$ and passing through the point $(1, 4)$
- **d** perpendicular to the line $y = -\frac{1}{4}x + 2$ and passing through the point $(4, 4)$

2 On the one number plane, draw the graphs of these equations.

$y = -2x$

$y = -2x + 2$

$y = -2x - 2$

How are the graphs similar?

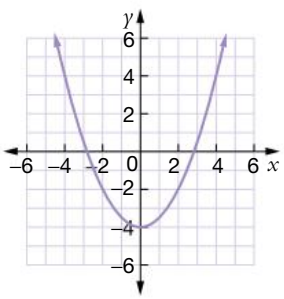
3 On separate number planes, draw the graphs of the following equations.

- **a** $y = x^2 - 3$
- **b** $y = -3^x$
- **c** $y = 3 - x^2$

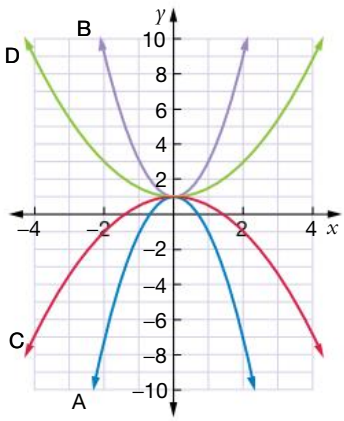
4 **a** The parabola $y = ax^2$ passes through the point $(-2, 12)$. What is the value of a?

b The parabola $y = ax^2 - 6$ passes through the point $(6, 12)$. What is the value of a?

c What is the equation of the parabola in the diagram below?



5



a Match each of the parabolas above with these equations.

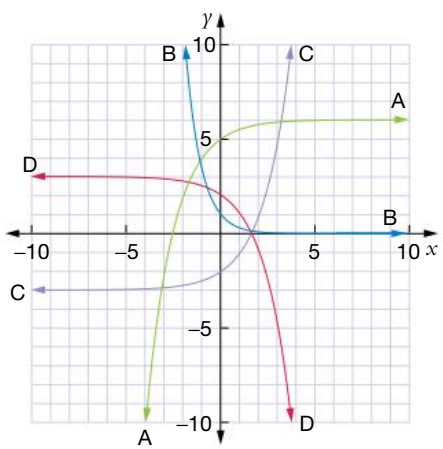
$y = 2x^2 + 1$ $\quad$ $y = 1 - 2x^2$

$y = \frac{1}{2}x^2 + 1$ $\quad$ $y = 1 - \frac{1}{2}x^2$

b Give the equation of the parabola that would result if the parabola $y = 2x^2 + 1$ was:

- **i** translated 2 units vertically downwards
- **ii** reflected in the x-axis.

6



Match each of the exponentials above with these equations.

$y = 3^{-x}$ $\quad$ $y = 3 - 2^x$

$y = 2^x - 3$ $\quad$ $y = 6 - 2^{-x}$

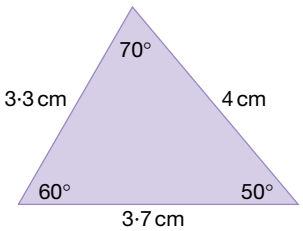
ASSIGNMENT 7B Working mathematically

1 Ben leaves Kalgoorlie for Perth (600 km away) and, on the same day, Natalie leaves Perth for Kalgoorlie. Ben travels 5 km on day 1, 10 km on day 2, 15 km on day 3, and so on with the pattern continuing in this manner. Natalie, on the other hand, travels 15 km per day.

a How far had each travelled after 5 days?

b On what day will they meet and how far from Perth will this meeting take place?

2 Why couldn't the side lengths on this triangle be correct?



3 Every possible pairing of five children was made and their combined weights were recorded in the list shown here (in kg):

85, 90, 92, 95, 97,
100, 102, 105, 110, 112

If the weight of every child is a whole number and no two are the same weight, find the individual weight of each child if one of them weighs 50 kg and another weighs 60 kg.

4 Use a calculator to find 1^2, 11^2, 111^2 and 1111^2. Study the pattern formed by the answers and then find $11\,111\,111^2$ without using your calculator.

5 If a clock takes 5 seconds to strike 6 o'clock, how long would it take to strike 12 o'clock?

6 On 30 June 2012 the ERP (estimated resident population) of Australia was 22 683 600. This represented an increase of 1·6% since 30 June 2011. If the population continued to increase at 1·6% p.a. in what year would the population reach 25 000 000?

ASSIGNMENT 7C Cumulative revision

1 Solve the equations: 2:01

a $\frac{2x-5}{3} - x = 8$ **b** $\frac{4m}{5} - \frac{3m}{10} = 16$ **c** $\frac{3a}{4} + 5 = \frac{2a}{3} + 7$

2 **a** A rectangle that is 15 cm by 10 cm is enlarged. If the length of the enlarged rectangle is 18 cm, find the breadth of the enlarged rectangle. 1:09

b A photograph that has dimensions of 12 cm by 8 cm is to be reduced on a photocopier. If the length of the reduced figure has to be 7·8 cm, what reduction factor should be used? Give your answer as a percentage.

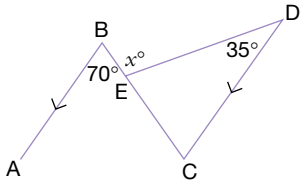
3 Solve the following sets of simultaneous equations. 5:02, 5:03

a $x + 3y = 8$, $x + y = 2$ **b** $3a - 2b = 10$, $5a + 2b = 6$ **c** $4m - 3n = 2$, $4m + n = 20$

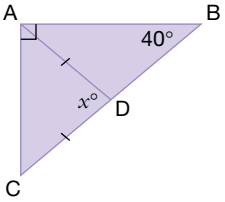
4 Nadeesha inherited $10 000. She decided to invest it for 5 years. She could invest it at 6% p.a. simple interest or at 5·4% p.a. compounded monthly. Which gives the most interest and by how much? 6:06

5 Find the value of x in each of the following. Give reasons. 3:01

a



b



6 The exterior angle of a regular polygon measures 20°. How many sides does the polygon have? 3:04

7 Three letters are chosen at random from the word *MATHEMATICS*. What is the probability that: 4:05

a all three letters are vowels

b all three are consonants

c the letters do not contain an *S*?

SURFACE AREA AND VOLUME

8

Contents

8:01 Review of surface area
8:02 Surface area of composite solids
8:03 Volume of solids with uniform cross-sections
Fun spot 8:03 How did the raisins win the war against the nuts?
8:04 Volume of composite solids
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Area and Surface Area*** [Stages 5.1, 5.2] and ***Volume*** [Stage 5.2]

- Calculate the surface area of cylinders and solve related problems (ACMMG217)
- Solve problems involving surface area for a range of prisms, cylinders and composite solids (ACMMG242)
- Solve problems involving the surface areas of right prisms (ACMMG218)
- Solve problems involving the volumes of right prisms (ACMMG218)
- Solve problems involving volume for a range of prisms, cylinders and composite solids (ACMMG242)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

8:01 Review of surface area

In Year 9, the surface area of prisms and cylinders were calculated by adding the areas of the faces (or surfaces). A net of the solid is often useful when tackling problems involving surface area, because it is important to determine the number of surfaces that need to be calculated.

- A *closed* solid has all its faces.
- An *open* solid has one or more faces missing.

Open solids have an internal and external surface area. Unless the conditions of a question suggest otherwise, calculate only the external surface area of an open solid.

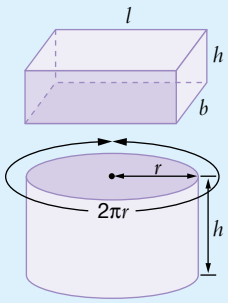
Area formulas

1 square: $A = s^2$
2 rectangle: $A = lb$
3 triangle: $A = \frac{1}{2}bh$
4 trapezium: $A = \frac{1}{2}h(a + b)$
5 parallelogram: $A = bh$
6 rhombus: $A = \frac{1}{2}xy$
7 circle: $A = \pi r^2$

Surface area formulas

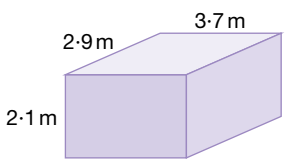
8 closed rectangular prism:
$SA = 2lb + 2lh + 2bh$

9 closed cylinder: $SA = 2\pi rh + 2\pi r^2$

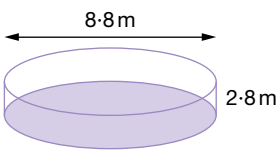


WORKED EXAMPLES

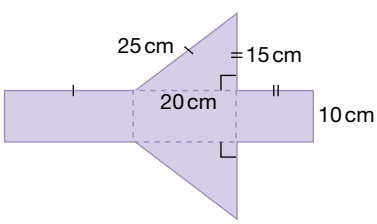
1 Find the surface area of this prism.



2 Find the inside surface area of this cylinder that has no top.



3 Use the net of the triangular prism to find its surface area.

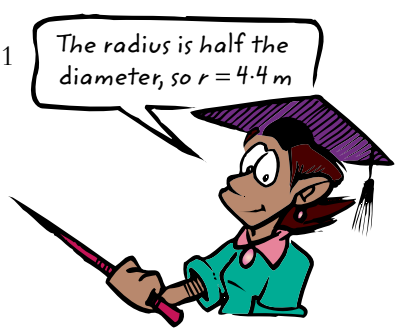


Solutions

1 Surface area $= 2lb + 2lh + 2bh$
$= 2 \times 2{\cdot}9 \times 3{\cdot}7 + 2 \times 2{\cdot}9 \times 2{\cdot}1 + 2 \times 3{\cdot}7 \times 2{\cdot}1$
$= 49{\cdot}18\,\text{m}^2$

2 Surface area = curved surface area + area of one circle
$= 2\pi rh + \pi r^2$
$= (2\pi \times 4{\cdot}4 \times 2{\cdot}8) + (\pi \times 4{\cdot}4^2)$
$= 138{\cdot}2\,\text{m}^2$ (1 dec. pl.)

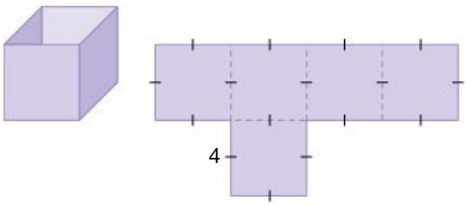
3 Surface area = area of the net
$= 2 \times \frac{1}{2} \times 20 \times 15 + (25 + 20 + 15) \times 10$
$= 900\,\text{cm}^2$



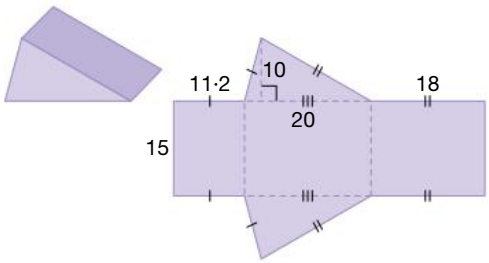
Exercise 8:01

1 Three open and one closed prism are shown. Use the net of each prism to calculate its surface area. All measurements are in centimetres.

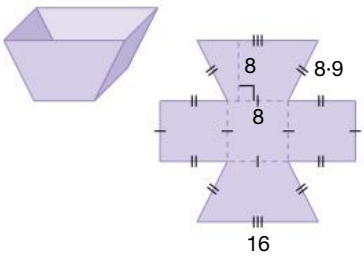
a



b



c



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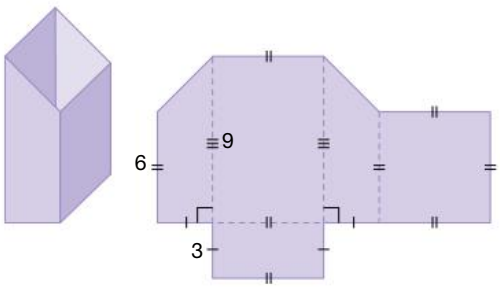
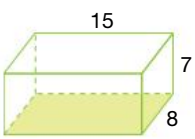
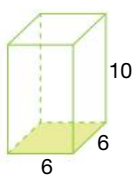


2 Find the surface area of the following prisms.
(*Note:* prism **c** is open. All measurements are in centimetres.)

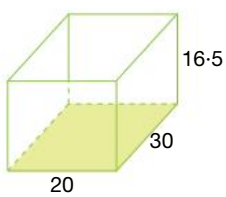
a



b

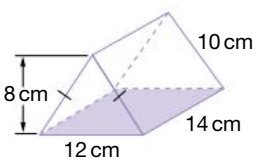


c

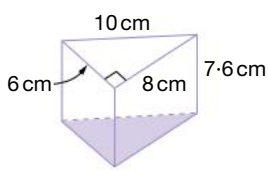


3 Find the surface area of these triangular prisms. (*Note:* prism **c** is open).

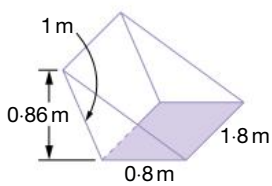
a



b

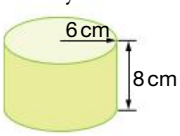


c



4 Find the surface area of the following cylinders correct to the nearest square centimetre.
(*Note:* cylinder **c** is open).

a



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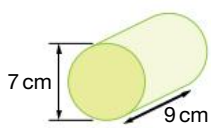
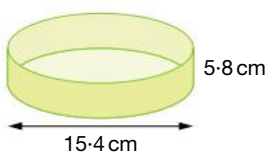
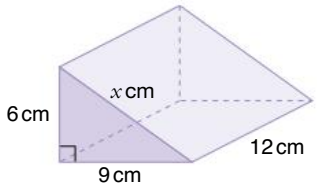


c

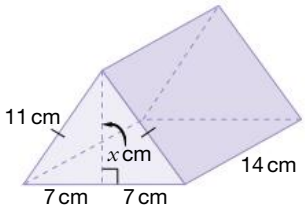


5 In each of the following questions use Pythagoras' theorem to calculate the unknown length x, correct to two decimal places, and then calculate the surface area.

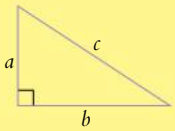
a



b



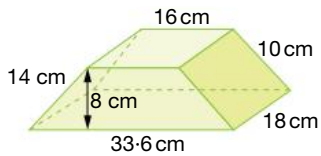
Caution!
$c^2 = a^2 + b^2$



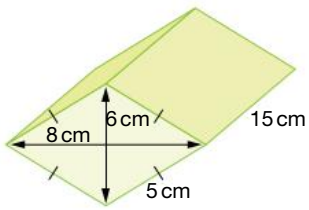
Right-angled triangles

6 Find the surface area of the following prisms.

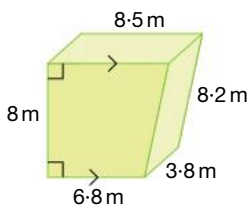
a



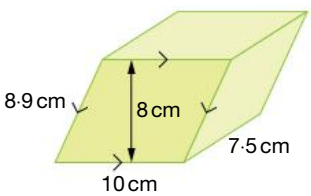
b

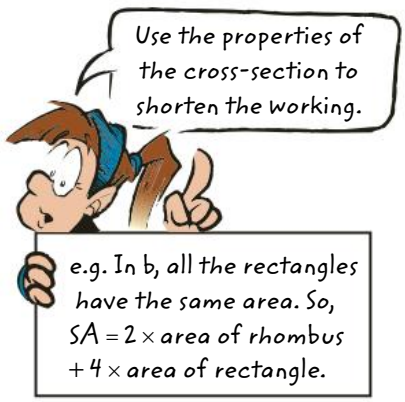


c



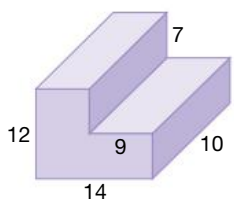
d



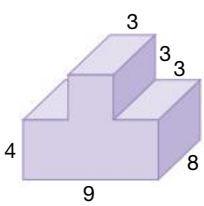


7 Find the surface area of the following solids. All measurements are in metres.

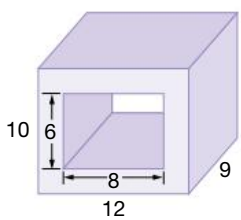
a



b

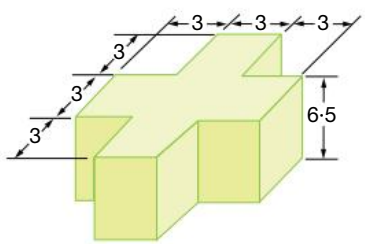


c

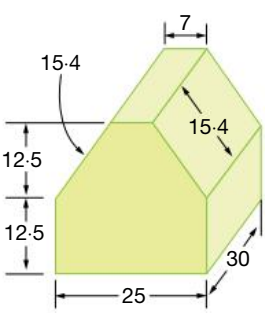


8 Find the surface area of the following prisms. All measurements are in centimetres.

a



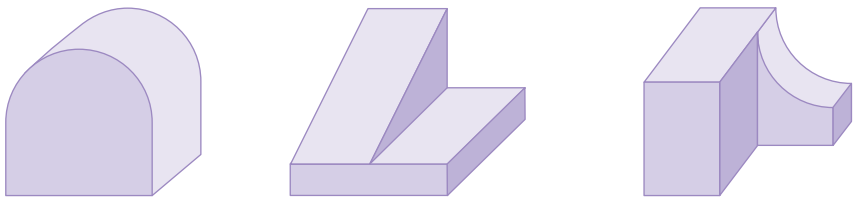
b



8:02 Surface area of composite solids

The solids shown below are examples of composite solids.

Composite solids are formed by either:
- joining simpler solids together
- removing one or more pieces from a simpler solid.



The surface area of a composite solid is calculated in much the same way as for prisms and cylinders. The simplest method involves calculating the area of each surface and adding them together, usually in a table.

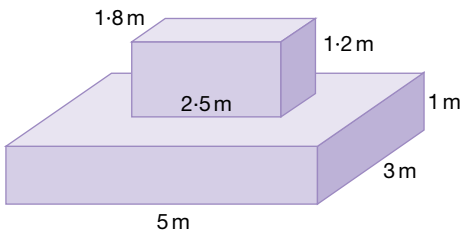
Other methods involve calculating the surface area of the simpler solids and then subtracting the areas of the common faces or surfaces (i.e. the faces or surfaces that are covered by the other solid). Often the symmetry of the solids can be used to shorten the working.

Whatever method is used you will need to examine the solid and:
- check the number and type of the surfaces that form the surface area
- calculate the area of each surface
- develop a system for checking that all surfaces have been counted, especially those that cannot be seen in the diagram.

The shape of this water tower is a composite solid based on two cylinders.

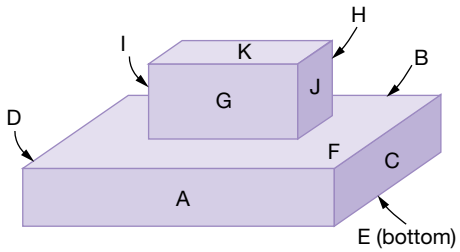
WORKED EXAMPLE 1

Calculate the surface area of the following solid.



Solution

The solid consists of two rectangular prisms.
It has 11 surfaces, (A to K) as shown.
All are rectangles except for F.



Surface	A	B	C	D	E	F	G	H	I	J	K
Area (m^2)	5	5	3	3	15	10·5	3	3	2·16	2·16	4·5

Note: Area of F $= (5 \times 3) - (2{\cdot}5 \times 1{\cdot}8)$
$= 10{\cdot}5$

$\therefore$ Surface area = sum of areas of all surfaces A to K
$= 56{\cdot}32\,m^2$

Another method is as follows:
Surface area = Surface area of top prism + Surface area of bottom prism $- 2 \times$ the common area

SA of top prism $= 2(2 \times 5 \times 1{\cdot}8 + 2{\cdot}5 \times 1{\cdot}2 + 1{\cdot}8 \times 1{\cdot}2)$
$= 19{\cdot}32\,m^2$

SA of bottom prism $= 2(5 \times 3 + 3 \times 1 + 5 \times 1)$
$= 46\,m^2$

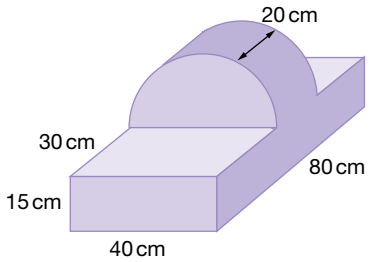
Common area $= 2{\cdot}5 \times 1{\cdot}8$
$= 4{\cdot}5\,m^2$

$\therefore$ Surface area $= 19{\cdot}32 + 46 - 2 \times 4{\cdot}5$
$= 56{\cdot}32\,m^2$

Note: The common area is subtracted twice because, while it has been included in the SA calculations of both prisms, it is not part of the surface area of the composite solid.

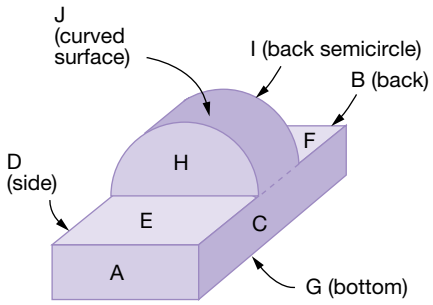
WORKED EXAMPLE 2

Calculate the surface area of the following solid.



Solution

The solid consists of half a cylinder on a rectangular prism. It has 10 surfaces (A to J) as shown.



Area of A $= 40 \times 15$
$= 600\,\text{cm}^2$

Area of B $= 600\,\text{cm}^2$

Area of C $= 80 \times 15$
$= 1200\,\text{cm}^2$

Area of D $= 1200\,\text{cm}^2$

Area of E $= 40 \times 30$
$= 1200\,\text{cm}^2$

Area of F $= 40 \times 30$
$= 1200\,\text{cm}^2$

Area of G $= 40 \times 80$
$= 3200\,\text{cm}^2$

Area of H $= \pi r^2 \div 2$
$= \pi \times 20^2 \div 2$
$= 200\pi\,\text{cm}^2$

Area of I $= 200\pi\ \text{cm}^2$

Area of J $= 2\pi rh \div 2$
$= 2 \times \pi \times 20 \times 20 \div 2$
$= 400\pi\,\text{cm}^2$

$\therefore$ Surface area = sum of areas of all surfaces A to J
$= (2 \times 600) + (4 \times 1200) + 3200 + 800\pi$
$= 11\,713\,\text{cm}^2$ (nearest cm^2)

Another method is as follows:
Surface area = area of curved surface + area of semicircles + surface area of prism − common area

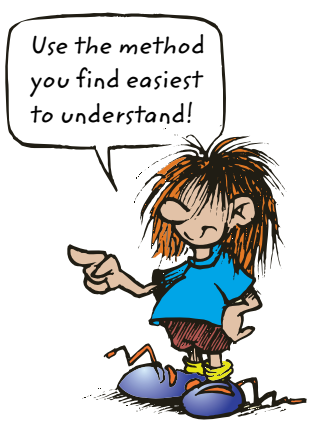
Area of curved surface
$= 2\pi rh \div 2$
$= 2 \times \pi \times 20 \times 20 \div 2$
$= 400\pi\ \text{cm}^2$

Area of semicircles
$= 2 \times \frac{1}{2} \times \pi r^2$
$= 2 \times \frac{1}{2} \times \pi \times 20^2$
$= 400\,\text{cm}^2$

SA of rectangular prism
$= 2(40 \times 15 + 80 \times 15 + 40 \times 80)$
$= 10\,000\,\text{cm}^2$

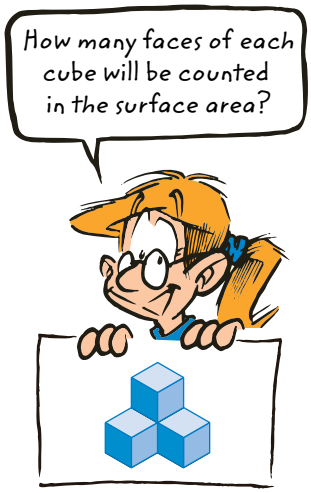
Common area
$= 40 \times 20\,\text{m}^2$
$= 800\,\text{cm}^2$

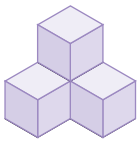
$\therefore$ Surface area $= 400\pi + 400\pi + 10\,000 - 800$
$= 11\,713\,\text{cm}^2$ (nearest cm^2)

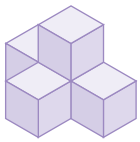


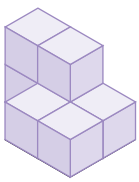
Exercise 8:02

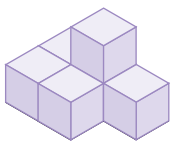
1 Each of the following solids has been built from 1 cm cubes. What is the surface area of each solid?



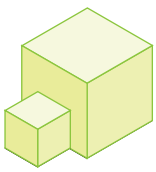
a 

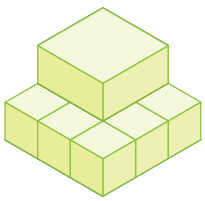
b 

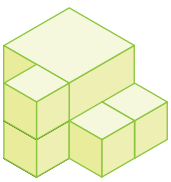
c 

d 

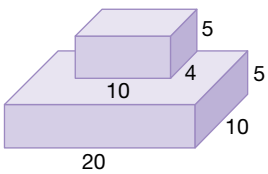
2 The following solids have been built from a 2 cm cube and 1 cm cubes. Calculate the surface area of each solid.

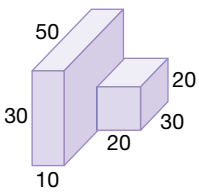
a 

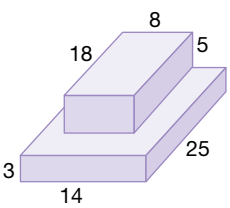
b 

c 

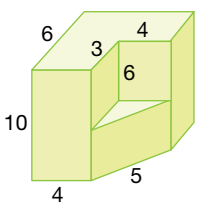
3 Calculate the surface area of the following solids. (All measurements are in centimetres.)

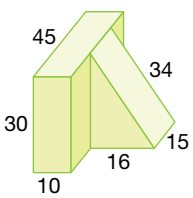
a 

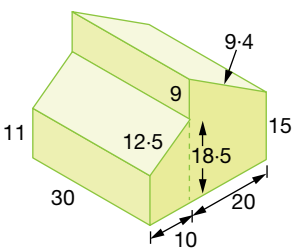
b 

c 

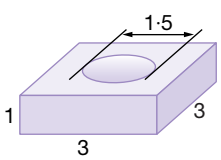
4 Calculate the surface area of the following solids. (All measurements are in centimetres.)

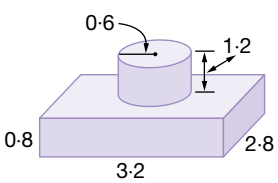
a 

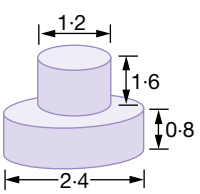
b 

c 

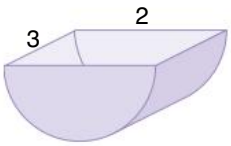
5 Calculate the surface area of the following solids. Give the answer correct to one decimal place. (Measurements are in metres.)

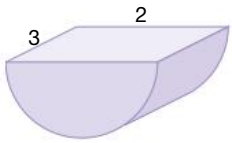
a 

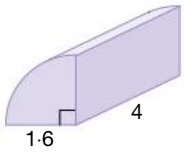
b 

c 

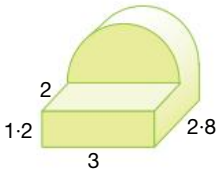
6 The following solids have been formed from cylinders. Find the surface area of each correct to one decimal place. (All measurements are in metres.)

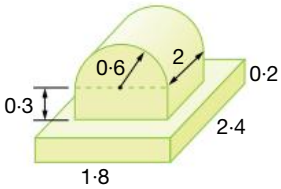
a 

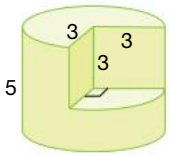
b 

c 

7 Calculate the surface area of the following solids. Give the answer correct to one decimal place. (Measurements are in metres.)

a 

b 

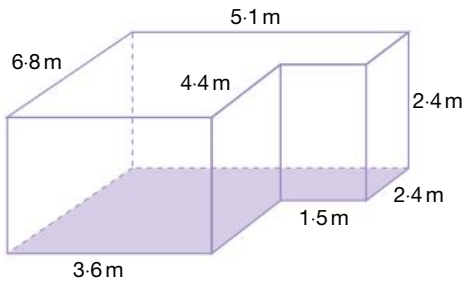
c 

8 The diagram represents a lounge room in a house. The walls and ceiling are to be painted. Find:

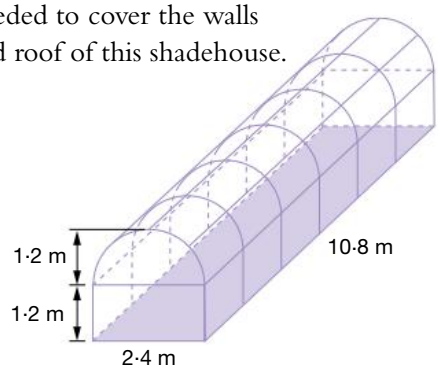
a the area of the walls

b the area of the ceiling

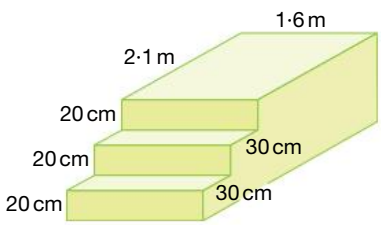
c the number of whole litres of paint required if 1 litre of paint covers 16 m^2.



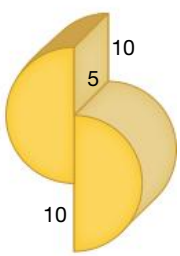
9 Calculate the area of shade cloth needed to cover the walls and roof of this shadehouse.



10 The areas visible in this diagram are to be covered with sandstone 'tiles'. Find the cost if materials and labour are \$120/m^2?



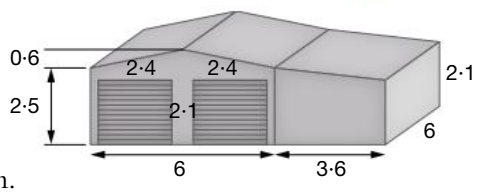
11 A jeweller makes an 'S' shaped brooch based on two identical half-cylinders. The brooch is to be gold-plated and the cost of this depends on the area that is to be gold-plated. Calculate the surface area of the brooch. Measurements are in millimetres.



12 The diagram shows a metal garage. Measurements are in metres.

a Calculate the area of metal sheeting needed for the *walls* given that the back wall and left side wall have a total window area of 8·8 m^2 and the front wall has two roller-doors 2·1 m by 2·4 m.

b Calculate the area of roof sheeting needed for the garage. (*Note:* Pythagoras' theorem will need to be used to calculate some missing dimensions.)



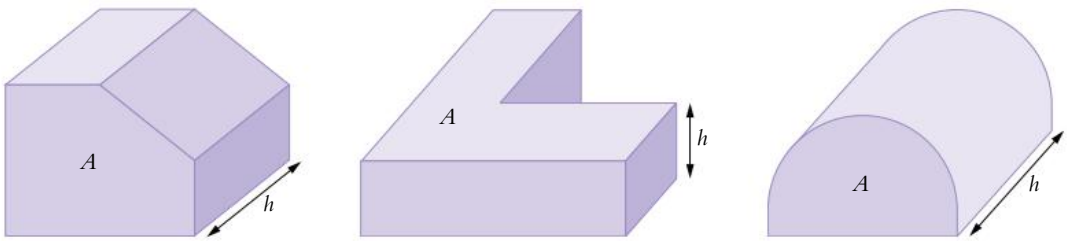
8:03 Volume of solids with uniform cross-sections

The volume of all prisms and 'prism-like' solids is given by the formula:

$V = Ah$

where V = volume
A = uniform cross-sectional area
h = height of the prism

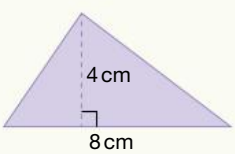
This formula can also be applied to more complicated prism-like solids involving semicircles, quadrants and sectors.

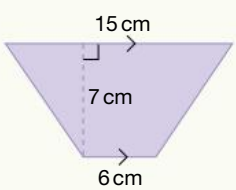


The volumes of these solids can also be calculated by adding or subtracting the volumes of simpler solids.

PREP QUIZ 8:03

Find the area of each shape.

1


2


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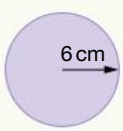

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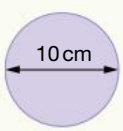
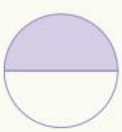


If the area of the circle is $60\,cm^2$, what is the area of the shaded sector?

5


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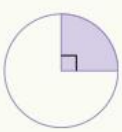
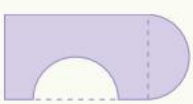


7

Find the shaded area of the following figures if the area of the rectangle is $8\,cm^2$ and the area of the semicircle is $1{\cdot}5\,cm^2$.

8

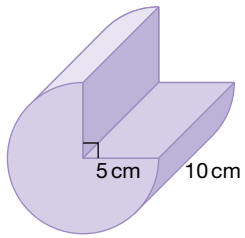
9

10


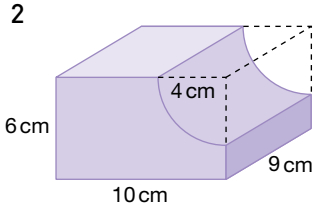
WORKED EXAMPLES

Find the volume of the following solids of uniform cross-section.

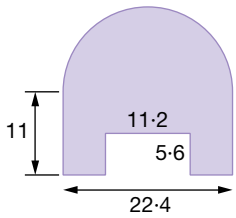
1



2



3



Solutions

1 $V = Ah$

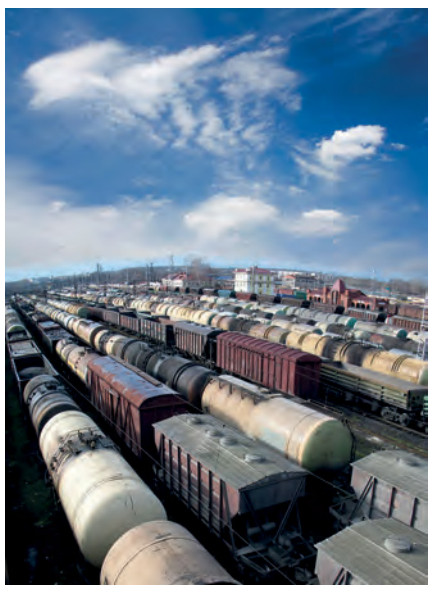
$A = \frac{3}{4} \times \pi \times 5^2$

$= \frac{75\pi}{4}$ cm^2

$h = 10$ cm

$\therefore V = \frac{75\pi}{4} \times 10$

$= 589{\cdot}0$ cm^3 (1 dec. pl.)



Which of these railway cars have uniform cross-sections?

2 $V = Ah$

A = area of rectangle − area of quadrant

$= (10 \times 60) - (\frac{1}{4} \times \pi \times 4^2)$

$= 60 - 4\pi$ cm^2

$h = 9$ cm

$\therefore V = 9(60 - 4\pi)$

$= 426{\cdot}9$ cm^3 (1 dec. pl.)

3 The solid consists of half a cylinder on top of a rectangular prism with a smaller rectangular prism removed.

$V = Ah$

A = area of semicircle + area of large rectangle − area of small rectangle

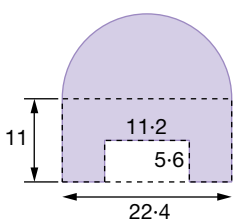
$= (\frac{1}{2} \times \pi \times 11{\cdot}2^2) + (22{\cdot}4 \times 11) - (11{\cdot}2 \times 5{\cdot}6)$

$= 62{\cdot}72\pi + 183{\cdot}68$ cm^2

$h = 8$ cm

$\therefore V = 8(62{\cdot}72\pi + 183{\cdot}68)$

$= 3045$ cm^3 (nearest cm^3)

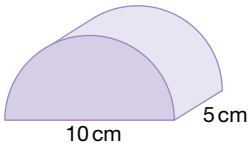


Exercise 8:03

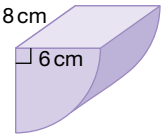
P Foundation worksheet 8:03
Volume of a prism

1 The following solids have uniform cross-sections that are sectors. Find the volume of the solids correct to the nearest cubic centimetre.

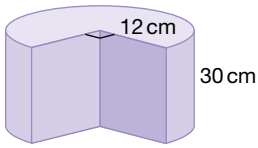
a



b

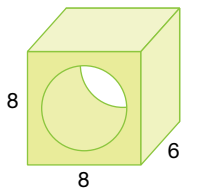


c

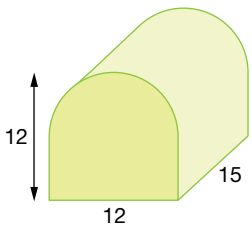


2 Calculate the volume of the following solids. All measurements are in centimetres. Give answers correct to one decimal place.

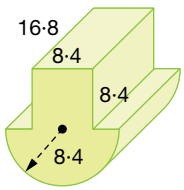
a



b

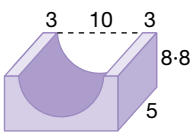


c

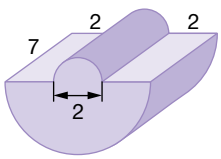


3 Calculate the volume of the following solids. Measurements are in metres. Give answers correct to one decimal place.

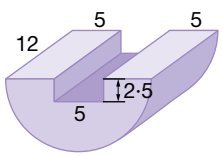
a



b

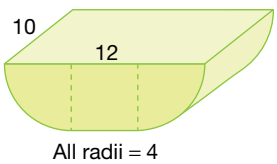


c

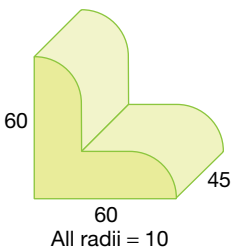


4 Calculate the volume of the following solids correct to the nearest cubic centimetre. All measurements are in centimetres.

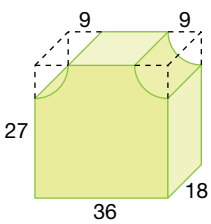
a



b

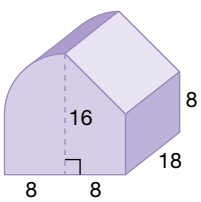


c

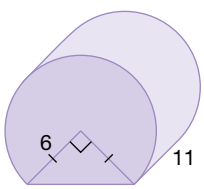


5 Calculate the volume of the following solids correct to the nearest cubic centimetre. All measurements are in centimetres.

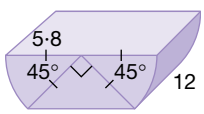
a



b



c



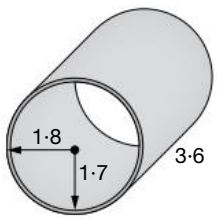
6 An above-ground pool has semicircular ends joined by straight sides.

a Calculate the volume of the pool if it is filled to a depth of 1·2 m. Give the answer correct to one decimal place.

b Find the capacity of the pool to the nearest kilolitre.

7 a Calculate correct to two significant figures the volume of concrete needed to make this pipe.

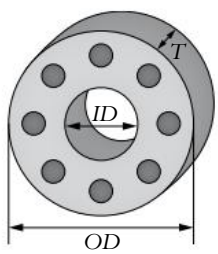
b Use your answer from part a to calculate the mass of the pipe if concrete weighs 2400 kg per cubic metre. (Measurements are in metres.)



8 The diagram shows a metal flange.

a Calculate the volume of the flange if $OD = 38{\cdot}4$ cm, $ID = 32{\cdot}4$ cm, $T = 1{\cdot}5$ cm and the diameter of each of the eight holes is 2·0 cm. (Answer correct to the nearest cubic centimetre.)

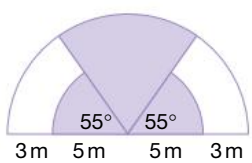
b Find the cost of the metal in the flange to the nearest dollar if it costs \$0.25/cm^3.



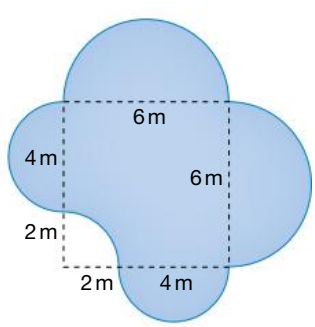
9 The diagram shows the cross-sectional area of part of a landscape design. The shaded area is to be concreted while the remaining area is to form a fishpond.

a Calculate the volume of concrete needed if it is to have a uniform depth of 100 mm. (Answer correct to one decimal place.)

b Calculate the capacity of the fishpond in kilolitres if the depth of water is 60 cm.



10 The cross-section of a swimming pool in a luxury resort hotel is shown on the right. Calculate the capacity of the pool in kilolitres (to the nearest kL) if the pool is filled to a depth of 1·6 m.



FUN SPOT 8:03 HOW DID THE RAISINS WIN THE WAR AGAINST THE NUTS?

Work out the answer to each question and write the letter for that part in the box that is above the correct answer.

Simplify where possible:

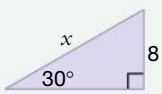
B $8x + x$
C $8x \div x$
D $8x \times x$
E $8x - x$
H $xy + yx$
E $2x - 2$
E $3x - 4 + 3x$
E $3(2x - 4) - 6x$
T $3x \times 3x$
T $-2x - 5x$

Solve:

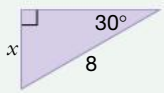
G $\frac{x}{3} = 3$
H $\frac{x}{3} = \frac{1}{3}$
E $3x = \frac{1}{3}$
H $\frac{3}{4}x = \frac{3}{8}$
I $x^2 = 9$
I $0{\cdot}3x = 3$
J $\sqrt{x} = 9$
L $x = \frac{x}{2}$

Find the value of the pronumeral in each of the following:

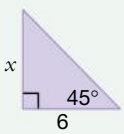
L



M



N



Find the value of $b^2 - 4ac$ if:

N $a = 2, b = 10, c = 5$
N $a = 3, b = -2, c = 4$
O $a = 1, b = -1, c = 4$
S $a = 6, b = -3, c = -1$

Simplify:

S $(a^{-1})^{-2}$
S $a^{-1} \times a^1$
E $(2a^2)^3$
T $\sqrt{x^{16}}$
T $\frac{a}{4} + \frac{a}{5}$
E $\frac{a^2}{5} \times \frac{5}{a}$
T $3a \div \frac{a}{4}$
U $a - \frac{3a}{16}$
Y $6a + 3a \div 3$

x^8	$x = \frac{1}{2}$	$7x$	$7a$	a^2	$\frac{13a}{16}$	$9x$	$x = 81$	a	8	$9x^2$	-12	$8x^2$	12	$x = 1$	$x = \frac{1}{9}$	4

$\frac{9a}{20}$	-15	$x = 10$	60	$-7x$	$2x - 2$	-44	1	$8a^6$	33	$2xy$	$6x - 4$	$x = 0$	16	$x = \pm 3$	6	$x = 9$

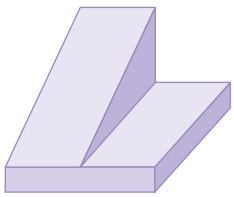
8:04 Volume of composite solids

Composite solids are formed by either:

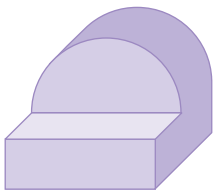
- joining two or more simpler solids together
- subtracting one or more solids from a larger, simpler solid.

This is shown in the following solids.

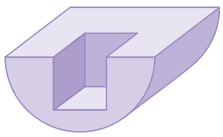
A



B



C



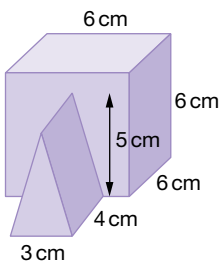
A has been formed from a rectangular prism and a triangular prism.
B has been formed from half a cylinder and a rectangular prism.
C has been formed by removing a rectangular prism block from half a cylinder.

The way in which composite solids are formed leads to the following rule.

> The volume of a composite solid is calculated by first working out the volume of the simpler solids from which it has been formed and then combining these volumes by either addition or subtraction.

WORKED EXAMPLE 1

Calculate the volume of the following solid.



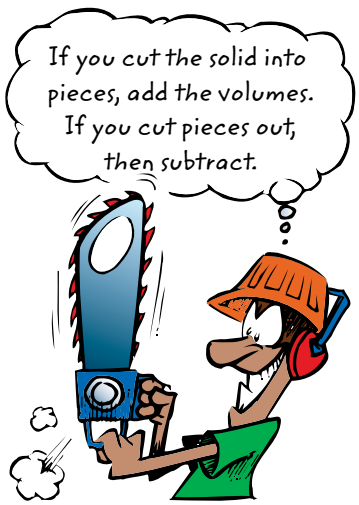
Solution

The solid consists of a cube joined to a triangular prism.

Volume of cube, V_1
$= A \times h$
$= (6 \times 6) \times 6$
$= 216\,\text{cm}^3$

Volume of triangular prism, V_2
$= A \times h$
$= (\frac{1}{2} \times 3 \times 5) \times 4$
$= 30\,\text{cm}^3$

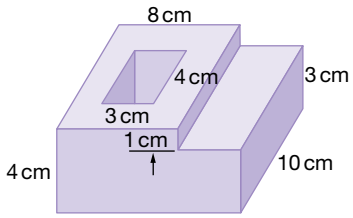
$\therefore$ Volume of solid $= V_1 + V_2$
$= 216 + 30$
$= 246\,\text{cm}^3$



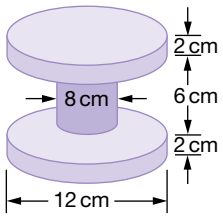
WORKED EXAMPLE 2

Calculate the volume of the following solids.

a



b



Solutions

a The solid consists of a rectangular prism that has had two rectangular prisms removed. The removal of one of these prisms has formed a hole.

Volume of rectangular hole, V_1
$= A \times h$
$= (3 \times 4) \times 4$
$= 48\,\text{cm}^3$

Volume of removed rectangular prism, V_2
$= A \times h$
$= (4 \times 10) \times 1$
$= 40\,\text{cm}^3$

Volume of original rectangular prism, V
$= A \times h$
$= (12 \times 10) \times 4$
$= 480\,\text{cm}^3$

$\therefore$ Volume of solid
$= V - V_1 - V_2$
$= 480 - 48 - 40$
$= 392\,\text{cm}^3$

b The solid consists of three cylinders, two of which are identical.

Volume of top cylinder, V_1
$= A \times h$
$= \pi r^2 h$
$= \pi \times 6^2 \times 2$
$= 72\pi$
$= 226{\cdot}19\,\text{cm}^3$ (2 dec. pl.)

Volume of middle cylinder, V_2
$= \pi r^2 h$
$= \pi \times 4^2 \times 6$
$= 96\pi$
$= 301{\cdot}59\,\text{cm}^3$ (2 dec. pl.)

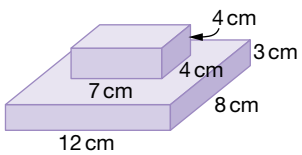
$\therefore$ Volume of solid
$= 2V_1 + V_2$
$= 2 \times 226{\cdot}19 + 301{\cdot}59$
$= 753{\cdot}97\,\text{cm}^3$ (2 dec. pl.)

Exercise 8:04

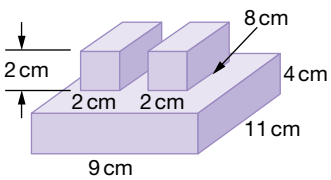
P Foundation worksheet 8:04
Volume of composite solids

1 Calculate the volume of the following composite solids.

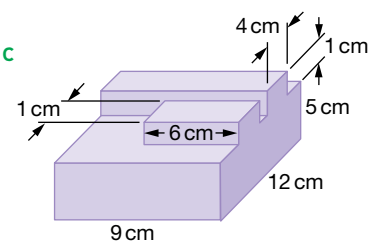
a



b

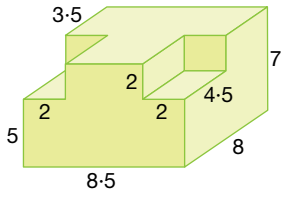


c

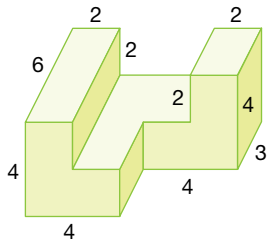


2 Find the volume of the following composite solids. All measurements are in metres.

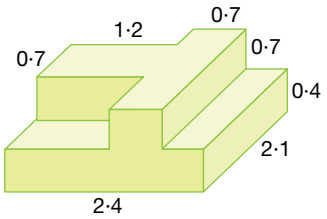
a



b

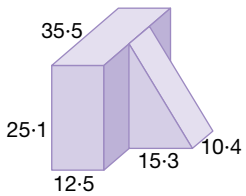


c

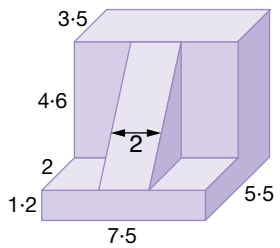


3 Calculate the volume of the following composite solids. All measurements are in centimetres. Give the answers correct to one decimal place.

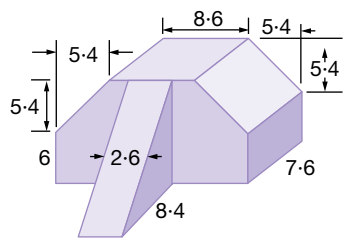
a



b

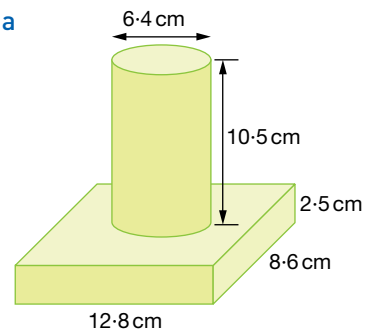


c

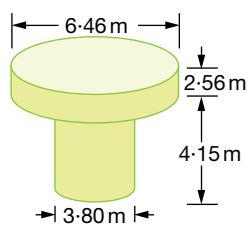


4 Calculate the volume of the following solids correct to two significant figures.

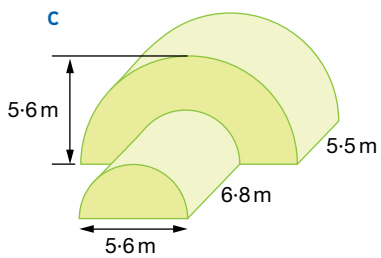
a



b

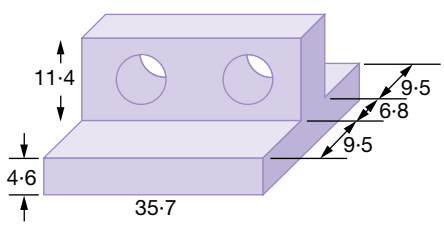


c



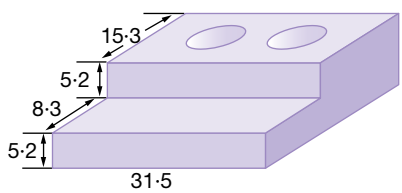
5 Calculate the volume of the following solids. All measurements are in centimetres. Give the answers correct to one decimal place.

a



Diameter of circular holes = 8·5 cm

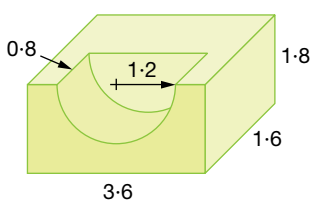
b



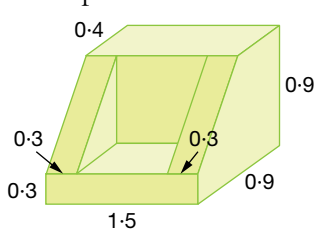
Diameter of circular holes = 10·2 cm

6 Calculate the volume of the following solids. All measurements are in metres. Give the answers correct to one decimal place.

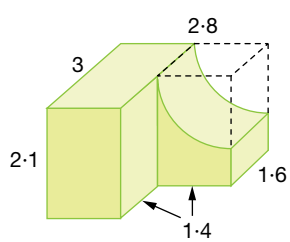
a



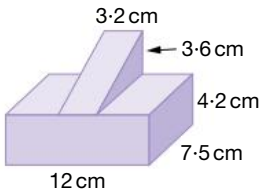
b



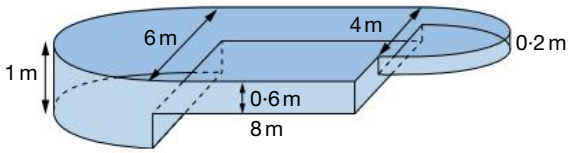
c



7 The solid pictured is formed by filling a mould with molten metal. Calculate the volume of the metal needed to make this object, and its mass if 1 cm^3 of metal weighs 11·4 g.



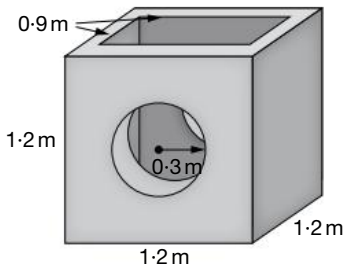
8 The cross-section of a swimming pool consists of a rectangle with two semicircular ends. The depth of the water in the smaller semicircular end is 0·2 m while the depth in the other semicircular end is 1 m. The rectangular part has a depth of 0·6 m. Calculate the capacity of the pool to the nearest kilolitre.



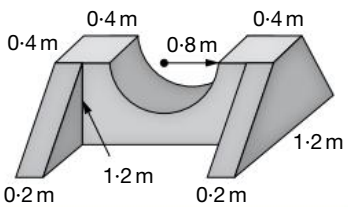
9 A stormwater inspection pit is in the shape of a hollow cube. It has four walls each 15 cm thick with two cylindrical holes cut in its front and back wall to which pipes will be attached.

a Calculate the volume of concrete needed to make the pit correct to two decimal places.

b Find the mass if the concrete weighs 2400 kg per m^3. Round the answer up to the nearest 100 kg.



10 The structure shown is made to support a water pipe. It has two identical ends that are trapezoidal prisms joining a rectangular prism that has had a half-cylinder removed from it. Calculate the volume of concrete needed to make the structure, correct to two decimal places.

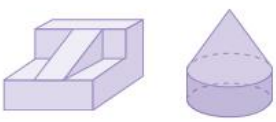


MATHS TERMS 8

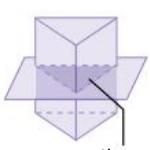
area
- the amount of space inside a 2D shape

capacity
- the amount of fluid that can be held by a container
- units of capacity: litre (L) millilitre (mL) kilolitre (kL)

composite solid
- a solid that is formed by joining simpler solids



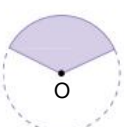
cross-section
- the shape on the face where a solid has been sliced



cylinder
- a prism-like solid with a circular cross-section
- it has two circular ends and a curved surface

prism
- a solid that has two identical ends joined by rectangular faces
- cross-sections parallel to the ends are identical to the ends

sector
- a part of a circle bounded by two radii and an arc



solid
- another name for a 3D shape

surface area
- the sum of the areas of the faces (or surfaces) of a 3D figure (or solid)

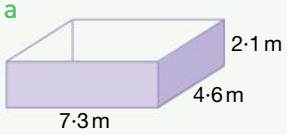
volume
- the amount of space inside a 3D shape
- units of volume: cubic centimetre (cm^3), cubic metre (m^3)
- formulas are used to calculate the volume of common solids (3D shapes)

These questions reflect the important skills introduced in this chapter.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

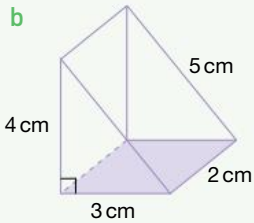
8:01

1 Calculate the surface area of the following solids. (The solids in **a** and **b** are open. The cylinder in **c** is closed).

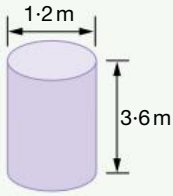
a



b



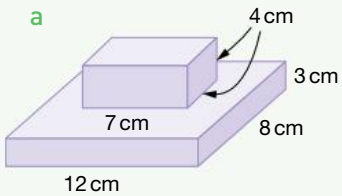
c



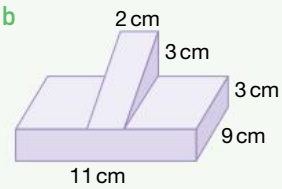
8:02

2 Calculate the surface area of these composite solids.

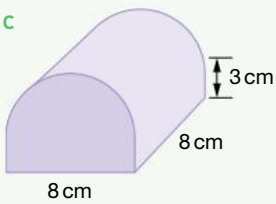
a



b



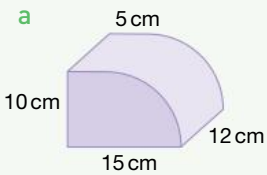
c



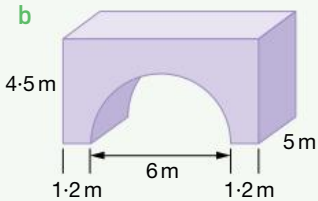
8:03

3 Calculate the volume of these solids.

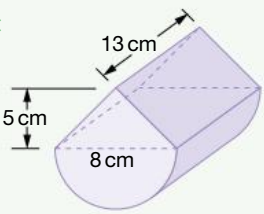
a



b



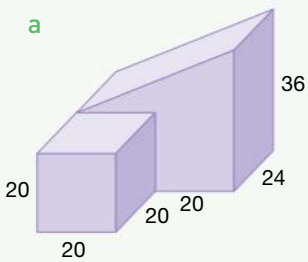
c



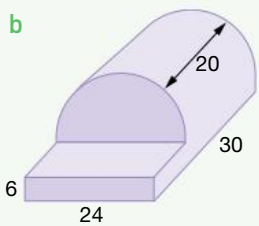
8:04

4 Calculate the volume of these composite solids. All measurements are in centimetres.

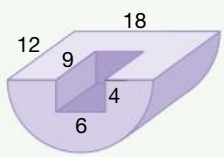
a



b

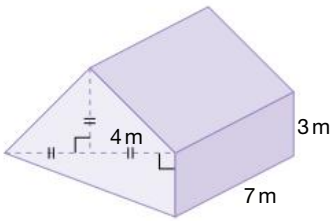


c



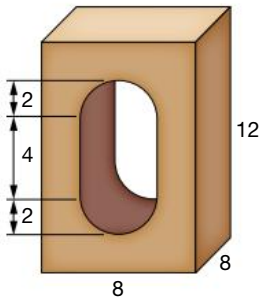
ASSIGNMENT 8A Chapter review

1 Find the volume of the following prism.

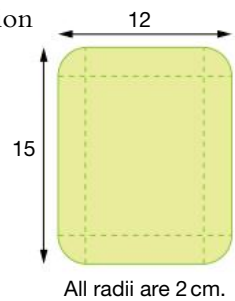


2 **a** Find the volume of metal in the solid below correct to one decimal place. (Measurements are in cm.)

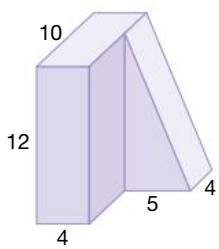
b Find its mass to the nearest gram if 1 cm^3 of the metal weighs 8·5 g.



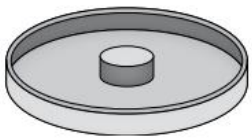
3 The uniform cross-section of an olive oil can is shown in the diagram. The dimensions are in centimetres. Calculate the maximum capacity of the can to the nearest millilitre if it has a height of 25 cm. (Remember that 1 cm^3 = 1 mL)



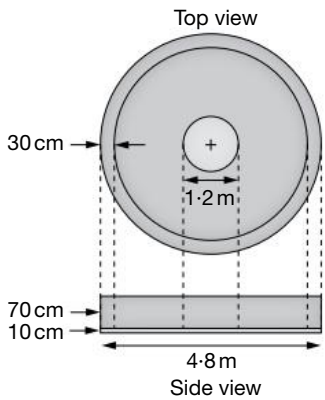
4 Calculate the surface area and volume of the following solid. (Measurements are in centimetres.)



5 A fountain consists of a base, a wall and a central island.



The base is cylindrical in shape with a diameter of 4·8 m and a thickness of 10 cm. The wall has an annular cross-section and the central island is cylindrical. Both have a height of 70 cm.



a Calculate the volume of concrete needed to make the base, wall and central 'island' of the fountain. Answer correct to two decimal places.

b Calculate the area of stone tiles needed to cover the outside and top of the wall and the top of the central island, to the nearest square metre.

Use your answers from **a** and **b** to find:

c the total cost of the concrete and stone if the cost of each is $350 per m^3 and $65 per m^2 respectively.

d the capacity of the pool in kilolitres correct to one decimal place if it is filled to a depth of 65 cm.

ASSIGNMENT 8B Working mathematically

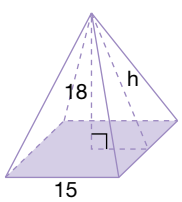
1 At Hannah's Ice Cream Parlour, single cones cost $3 and double cones cost $4. In a day Hannah sells 300 cones and has sales totalling $1117. How many double cones did she sell?

If Hannah's cost prices are $1.20 for a scoop of ice-cream, 30 cents for a double cone and 20 cents for a single cone, what is her profit for the day?

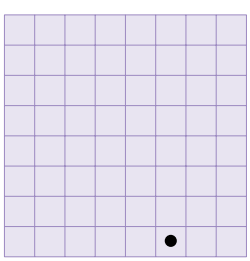
2 The population of Koala Landing is decreasing by 10% of its population each year. If it now has a population of 2000, what will be its population in 2 years' time?

3 **a** The figure shows a square pyramid. Use Pythagoras' theorem to calculate h.

b Hence, calculate the surface area of the pyramid, correct to the nearest cubic metre.



4 The dot indicates the position of one chess queen on a chessboard. How many more queens can you place on the board so that none of the queens threatens another? (Queens can move *vertically*, *horizontally* or *diagonally*.)



5 If you turn a left-handed glove inside out, will it then be a left-handed or a right-handed glove?

6 A 4 × 4 grid is drawn and the numbers 1, 2, 3 and 4 are placed in the grid so that every number occurs only once in each row and only once in each column.

a Find the missing numbers in this grid.

1	2	3	
	4		2
4			

b Now place the numbers in a 4 × 4 grid, following the rules above, so that the sums of the diagonals are 16 and 4.

The Great Pyramid of Cheops has a square base length of 241 m and a height of 153 m.

ASSIGNMENT 8C Cumulative revision

1 Complete the table to find the compound interest on \$10 000 if it is invested at a rate of 4·25% p.a. for 3 years.

Year	Principal	Interest
1	\$10 000	\$450
2	\$10 450	
3		
Amount after 3 years = Compound interest =		

6:04

2 Solve the following quadratic equations.

a $x^2 = 16$ **b** $8x^2 = 72$ **c** $9x^2 = 16$

2:05

3 Solve the following quadratic equations.

a $m^2 + 5m + 6 = 0$ **b** $m^2 + 5m - 6 = 0$ **c** $m^2 - 5m - 6 = 0$

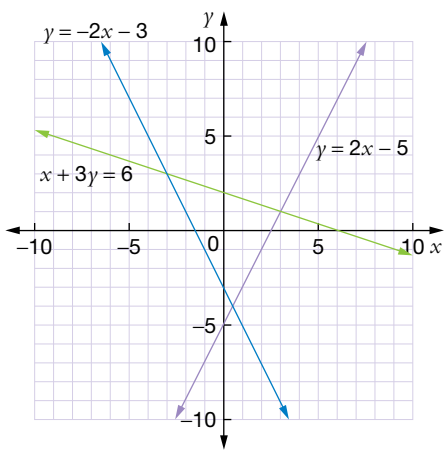
2:05

4 Use the graph to find the solutions to the following pairs of equations.

a $y = 2x - 5$ and $x + 3y = 6$

b $x + 3y = 6$ and $y = -2x - 3$

c $y = 2x - 5$ and $y = -2x - 3$



5:01

5 Solve the following pairs of simultaneous equations.

a $x + y = 8$, $2x - y = 7$ **b** $a - 4b = 12$, $a + 2b = 6$ **c** $3x + y = 8$, $x + y = 1$

5:02B

6 Find the sum of the interior angles of a polygon with:

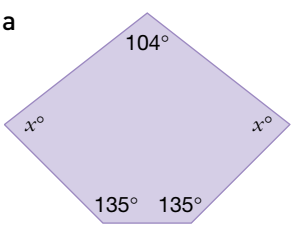
a 5 sides **b** 8 sides **c** 9 sides

3:04

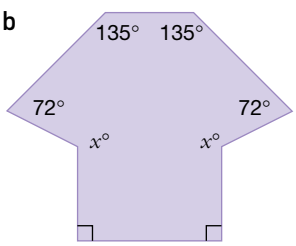
7 Find the value of x in each of the following.

3:04

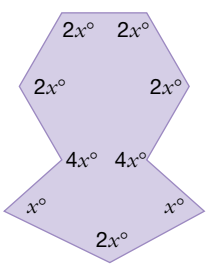
a



b

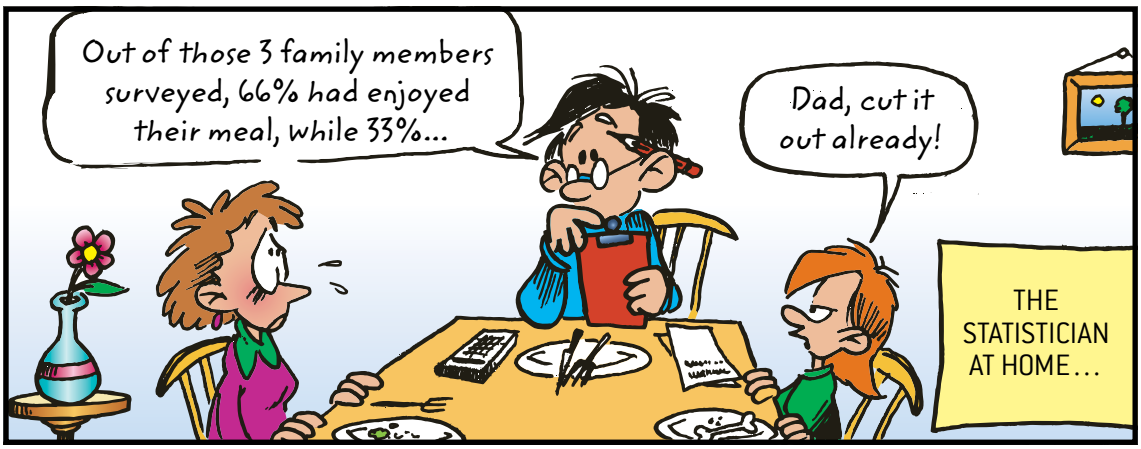


c



STATISTICS

9



Contents

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Syllabus references (See pages x–xiii for details.)

Statistics and Probability

Selections from ***Single Variable Data Analysis*** [Stages 5.1, 5.2◊] and ***Bivariate Data Analysis*** [Stage 5.2]

- Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)
- Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253)
- Investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians (ACMSP227)
- Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252)
- Use scatter plots to investigate and comment on relationships between two numerical variables (ACMSP251)

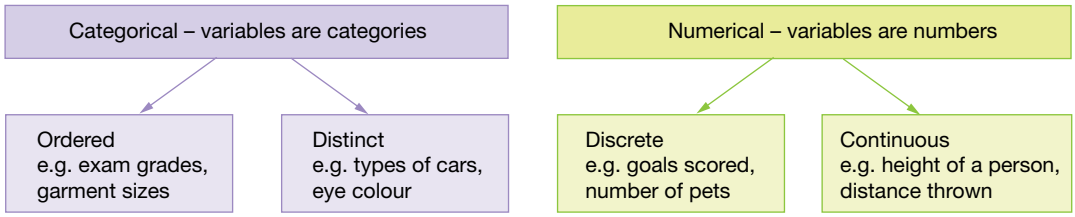
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

9:01 Statistics: A review

Types of data

The data we collect is made up of variables. These are pieces of information like a quantity or a characteristic that can be observed or measured. They may change either over time or between individual observations. The main types of data are:



Collecting data

Three main ways of collecting data are shown in the following table.

Census	Sample	Observation
A whole population is surveyed, e.g. every student in the school is questioned.	A selected group of a population is surveyed, e.g. a small number in each class is questioned.	Numerical facts are collected and tabulated, e.g. sports data, weather, sales figures, etc.

A **sample** is usually **random** to limit the chance of bias occurring. However, it may be **systematic** if the members of the sample are chosen according to a rule, such as every 10th member of a population. If a population is composed of various sub-groups, the sample could be **stratified** to ensure a proportionate representation of each group in the sample.

Primary source data is collected first-hand by observation or survey.

Secondary source data is obtained from an external source such as a newspaper, website or another person's research.

Sorting data

A large amount of data needs to be tabulated (organised into a table) so that it can be analysed. A common form of table is the frequency distribution table. The first table on the following page is for discrete data. The second is for grouped data that may be used to cluster discrete data into groups or to divide continuous data into adjoining groups.

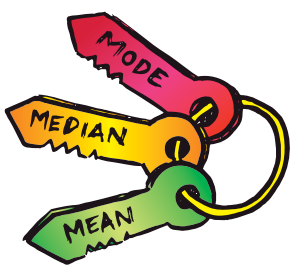
Outcome (x)	Tally	Frequency (f)	$f \times x$	Cumulative Frequency ($c.f.$)
1	|||	3	3	3
2	||||	4	8	7
3	𝍸 ||	7	21	14
4	𝍸 ||||	9	36	23
5	𝍸	5	25	28
6	||	2	12	30
	Total:	30	105	

Class	Class Centre ($c.c.$)	Tally	Frequency (f)	$f \times c.c.$	Cumulative Frequency ($c.f.$)
1–<5	3	|||	3	9	3
5–<9	7	||||	4	28	7
9–<13	11	𝍸 ||	7	77	14
13–<17	15	𝍸 ||||	9	135	23
17–<21	19	𝍸	5	95	28
21–<25	23	||	2	46	30
	Total:	30	390		

After the data has been sorted, certain key numbers can be determined. Some measure how the data clusters around the 'centre'. These are called **measures of central tendency** (or measures of location). Others measure how the data spreads from the centre. These are called **measures of spread**.

Measures of central tendency

Mode



- The score or outcome that occurs the most, i.e. with the highest frequency. For the data in the table above the mode is 4 (highest frequency of 9).
- For grouped data this becomes the modal class. For the table above, the modal class is 13–<17.

Mean

- The sum of the scores divided by the number of scores, i.e. the usual definition of 'average'.
- For tabulated data: $\text{mean} = \dfrac{\text{sum of } f \times x \text{ column}}{\text{sum of freq. column}}$ or $\text{mean} = \dfrac{\text{sum of } f \times c.c. \text{ column}}{\text{sum of freq. column}}$

  For the tables above: $\text{mean} = \frac{105}{30} = 3{\cdot}5$ $\text{mean} = \frac{390}{30} = 13{\cdot}0$

Median

- The 'middle' score when the scores are placed in order. If there is an even number of scores the median is the average of the two middle scores. The frequency table on the previous page has 30 scores, so the median is the average of the 15th and 16th scores. Looking down the c.f. column, the 15th and 16th scores are both 4. So the median is 4.
- For grouped data this becomes the median class. In the grouped frequency table on the previous page, the 15th and 16th scores both lie in the class $13 - <17$. So this is the median class.

Measures of spread

Range

- The highest score minus the lowest score. In the frequency table on the previous page, the highest score is 6 and the lowest score is 1.
  Range = 6 − 1 = 5
- For grouped data, unless the original scores are known, the maximum possible range can be determined by using the class groupings. In the frequency table on the previous page, the highest possible score is close to 25 and the lowest is 1, so a maximum possible range would be 25 − 1 = 24

Interquartile range (IQR)

- The range of the middle 50% of scores, i.e. the difference between the first and third quartiles: IQR = $Q_3 - Q_1$
- For the 30 scores shown here:

1 2 2 3 3 3 4 (4) 5 5 5 6 6 6 7 7 | 7 8 8 8 9 9 (9) 10 10 10 10 11 11 12

Q_1 median (Q_2) Q_3

$\therefore$ IQR = 9 − 4
= 5

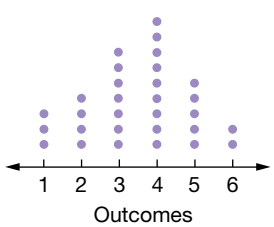
- The interquartile range is not very meaningful for a small set of scores.
- Associated with these measures of spread is the 'five-number summary' of a set of data that is defined as: minimum score, first quartile Q_1, median Q_2, third quartile Q_3, maximum score
- The five-number summary for the scores above: (1, 4, 7, 9, 12)

Displaying data

A visual display of data is useful in giving an overall impression. Some are reviewed here, the first three using the data in the previous tables.

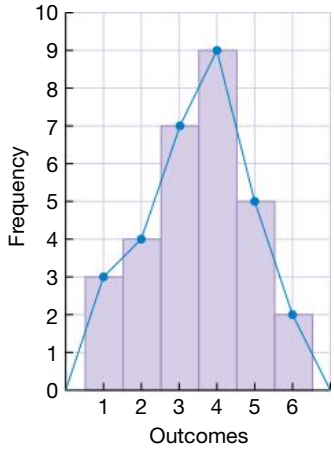
Dot plot

- A simple display where each score is represented by a dot.
- The mode is easy to identify as the highest column of dots.
- The highest and lowest scores are clear to determine the range.
- A clear impression of the spread of the scores is given.
- Any outliers are easily identified.



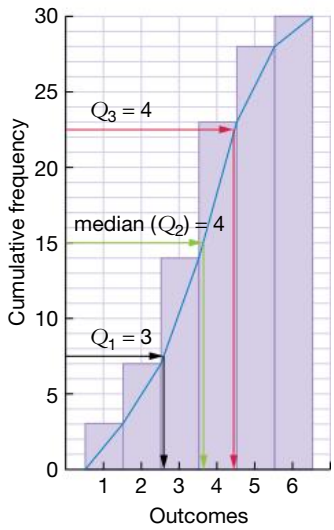
Frequency histogram and polygon

- The frequency of each score is represented by a column in a histogram and a dot in a polygon. These dots coincide with the centre top of each column. The dots are joined to form the polygon, which is completed by joining to the axis as shown.
- The mode is identified by the highest column.
- A clear impression of the spread of the scores is given.
- Any outliers are easily identified.
- For grouped data the classes can be represented on the horizontal axis by the class centres.



Cumulative frequency histogram and polygon

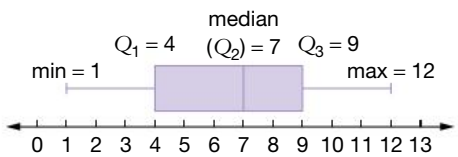
- Graphing the cumulative frequency results in columns of increasing height, the last column representing the total frequency.
- The polygon is formed by joining the corners of adjoining columns as shown.
- This polygon is useful for indicating the median and quartiles, as shown.



Box plot

- This is drawn using the five-number summary for a set of data.
- It gives an impression of the spread of the data and also if it is **symmetrical** or **skewed** from its centre. This will be indicated by the box being nearer to one end than the other.
- If there are more low scores the skew is said to be positive; more high scores would mean the data is negatively skewed.

Using the five-number summary (1, 4, 7, 9, 12):



This set of data is fairly symmetrical. It is only slightly negatively skewed.

Stem-and-leaf plot

- A stem-and-leaf plot resembles a histogram (on its side) where the data has been grouped.
- The individual scores can still be identified.
- The data may be unordered as in the plot to the right.
- Two sets of data can be compared using a back-to-back stem-and-leaf plot like the one below. The data in this plot has been ordered.
- The range and mode are easily identified.
- The scores are ordered so the median and quartiles can be determined by counting.

1/7 represents 17

Stem	Leaf
0	
1	7 6 6 9
2	4 0 8
3	6 2 2 2 0 9
4	9 5 5 6 1 7 8 8
5	9 9 2 9
6	0

Range = 55 − 8
= 47
Mode = 12 (frequency of 3)
There are 26 scores.
Median = average of 13th, 14th scores
= 27
Q_1 = 12 (7th score)
Q_3 = 36 (20th score)
IQR = 36 − 12
= 24

Stem	Leaf	Stem
9 8 8	0	
7 4 3 2 2 2 0	1	6 6 7 9
9 8 8 7 7 3 1	2	0 4 8
9 6 6 4 4	3	0 2 2 2 6 9
3 2 2	4	1 5 5 6 7 8 8 9
5	5	2 9 9 9
	6	0

Range = 60 − 16
= 44
Mode = 32 (frequency of 3)
There are 26 scores.
Median = average of 13th, 14th scores
= 40
Q_1 = 28 (7th score)
Q_3 = 48 (20th score)
IQR = 48 − 28
= 20

Exercise 9:01

P Foundation worksheet 9:01 Interquartile range

1 Find the following measures for each set of discrete numerical data.

i range ii mode iii mean iv median

a 3, 4, 4, 5, 7, 9, 10

b 13, 14, 17, 16, 14, 19

c 1·7, 1·9, 1·6, 1·7, 1·4, 1·9, 2·0, 1·5

d

Outcome	f	fx	$c.f.$
6	2	12	2
7	3	21	5
8	6	48	11
9	7	63	18
10	2	20	20
Total:	20	164	

e

Outcome	f	fx	$c.f.$
5	4	20	3
10	6	60	9
15	8	120	17
20	5	100	21
25	2	50	25
Total:	25	350	

2 Which of the following can be applied to this categorical data: the mode, the mean or the median? Find its value for this data?

Type of car	Frequency
Toyota	250
Mazda	185
Nissan	146
Hyundai	127

3 **a** Determine the mode, mean and median for the scores: 5, 8, 7, 5, 5, 4, 8

b Another score of 6 is added to this data set. What are the new values for the mean and median?

4 **a** Determine the median for the scores: 12, 9, 13, 1, 16, 10, 15

b Find the new median caused by an additional score of

i 15 **ii** 9

5 For the data in this table find the:

a range **b** mode **c** mean **d** median

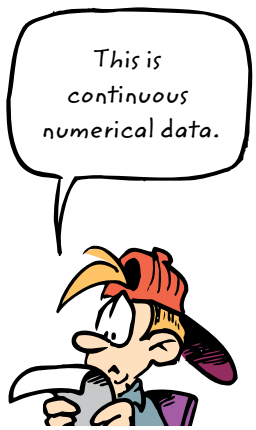
Score	10	12	14	16	18	20
Frequency	5	7	10	4	2	2

6 For this grouped data, find the:

a maximum range **b** modal class

c median class **d** mean

Class	*c.c.*	*f*	$f \times c.c.$	*c.f.*
0–<10	5	4	20	4
10–<20	15	8	120	12
20–<30	25	10	250	22
30–<40	35	11	385	33
40–<50	45	7	315	40
	Total:	40	1090	



7 **a** Copy and complete these frequency distribution tables and for each find the mean and median.

i

Score	Tally	*f*	*fx*	*c.f.*
1	|||			
2	𝍸			
3	𝍸 ||||			
4	𝍸 ||			
5	||||			
6	||			

Total: ___ ___

ii

Score	Tally	*f*	*fx*	*c.f.*
0	𝍸 ||			
1	𝍸 𝍸 ||			
2	𝍸 𝍸			
3	𝍸 |||			
4	𝍸 ||			
5	||||			
6	|||			
7				
8	|			

Total: ___ ___

b Construct a frequency histogram and polygon for the data in each table.

8 **a** Complete this table of grouped data.
b What is the maximum possible value for the range?
c What is the modal class?
d Find the median class.
e Use the total of the $f \times c.c.$ column to calculate an estimate for the mean.
f Draw a cumulative frequency histogram and polygon for this data. Identify the median class.

Class	*c.c.*	Tally	*f*	$f \times c.c.$	*c.f.*
0–4	2	||||			
5–9	7	𝍸 ||||			
10–14		𝍸 𝍸 𝍸			
15–19		𝍸 𝍸 |			
20–24		𝍸 ||			
25–29		|||			

Total: ___ ___

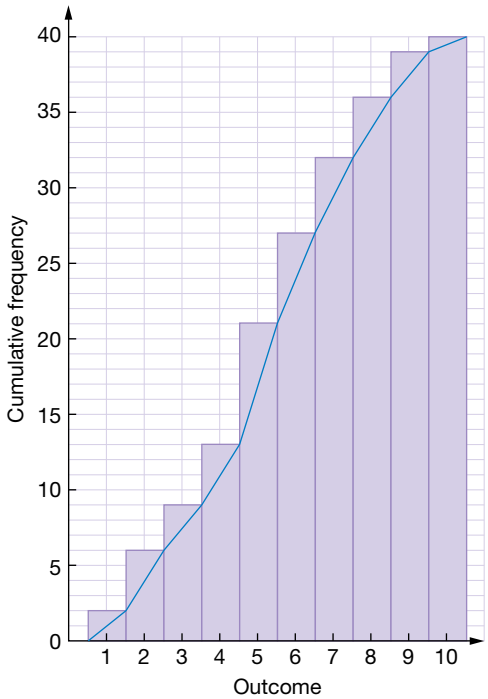
9 **a** For each set of data, identify the median first, then the first quartile (Q_1) and the third quartile (Q_3). Also state the interquartile range. (*Note:* the scores must be in order.)

i 2 4 5 7 8 8 9 9 10 12
ii 3 3 4 5 6 6 7 7 8 9 10 10
iii 5 6 3 2 3 6 7 1 8
iv 9 4 3 3 6 2 7 8 9 1 2 5 5

b Use your answers to **a** to write the five-number summary for each set of data.

10 **a** Use this cumulative frequency histogram and polygon to determine the following:

i the total number of scores
ii the range
iii the median
iv the value of Q_1
v the value of Q_3
vi the interquartile range
vii the five-number summary.



b Use the graph to complete this frequency distribution table. First enter the data in the cumulative frequency column.

Outcome	*f*	*fx*	*c.f.*
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Total: ___ ___

c Use the table to:

i identify the mode
ii calculate the mean.

11 a For the data shown in this dot plot find:

i the range

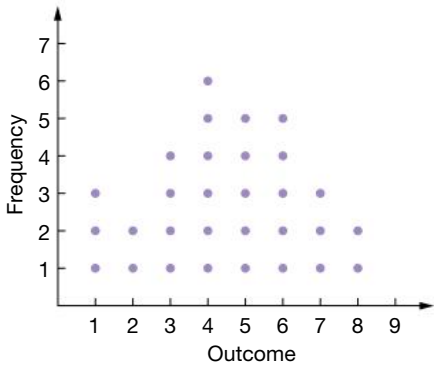
ii the median

iii the first quartile

iv the third quartile.

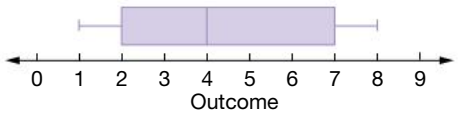
b Construct a cumulative frequency histogram and polygon and indicate on it the quartiles and median.

c Draw a box plot for this data.



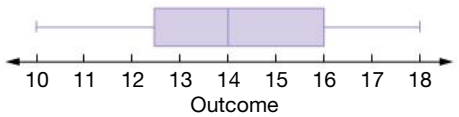
12 a Write the five-number summary for each of these box plots.

i

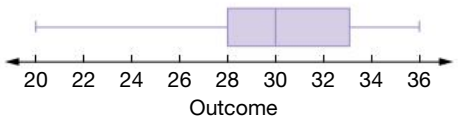


ii

iii



iv



b Which box plot shows symmetrical data?

c Which box plot shows the data that is most skewed? Is it negatively or positively skewed?

13 a For this ordered stem-and-leaf plot, determine:

i the range

ii the mode

iii the median

iv the interquartile range.

b Construct a box plot for this set of data.

Stem	Leaf
0	4 8 9
1	1 2 8
2	2 5 5 5 7 9
3	0 4 4 7 7
4	1 2 6
5	1 4 7 9
6	5

14 This back-to-back stem-and-leaf plot shows the percentage test results for two classes.

a How many students are in each class?

b What is the modal score for each class?

c Determine the median for each class.

d Calculate the mean for each class.

e Determine the values of the first and third quartiles for each class.

f Construct a box plot for each class against the same axis.

g For which class is the data more skewed?

h Class A has three more students than Class B. If the lowest three scores were removed from class A, which class would then have the:

i lower median

ii lower mean?

Class A		Class B
6 4	3	9
9 9 5 0	4	6 8 8
9 7 3 3 1	5	3 5 6 7 9
9 8 8 7 7 3 0	6	4 7 7 7 9
7 6 1 1 1 1	7	2 2 4 4 9
8 5 2	8	3 4 9
2	9	1 5

15 The results of a random survey of adult residents in a particular suburb are shown in this table, indicating their level of income.

Income (× $1000)	*c.c.*	*f*	*f* × *c.c.*	*c.f.*
30–<40		5		
40–<50		8		
50–<60		14		
60–<70		13		
70–<80		10		
80–<90		4		
90–<100		2		
	Total:	56		

a Copy and complete this frequency distribution table.

b Calculate an approximate value for the mean.

c Construct a cumulative frequency histogram and polygon.

d Use the graph to find the median class and the first and third quartile classes.

e Using the class centres to represent each class, write the five-number summary for this grouped data.

f Use the five-number summary to draw a box plot.

g Does the box plot show the data to be symmetrical or skewed?

FUN SPOT 9:01 WHY DID THE TEACHER WEAR SUNGLASSES?

Answer each question and write the letter for that question in the box above the correct answer.

For this set of data 3 4 5 5 7 8 9 11 find:

D the range **G** the mode **S** the mean

A the median **I** Q_1 **E** Q_3

H interquartile range

For the data in this table find:

Score	*f*	*fx*	*c.f.*
10	4	40	4
12	5	60	9
14	8	112	17
16	7	112	24
18	6	108	30
20	5	100	35
Total:	35	532	

R the range

T the mode

L the mean

S the median

H Q_1

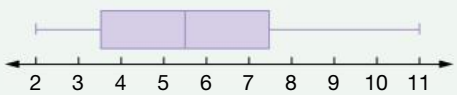
C Q_3

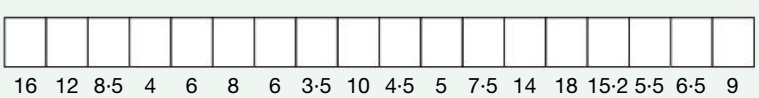
A interquartile range

For this box plot find:

S the range **A** the median

B Q_1 **H** Q_3





9:02 Bivariate data

Statistics met in previous years involved analysing data for a single variable. Measuring heights, counting types of vehicles and tabulating exam results are all examples of collecting single variable data (univariate data). Sometimes data may be obtained about two different variables that may, or may not, be related. For example, measuring the arm span and height of a group of people or the English marks and Mathematics marks for a class of students. This is called *bivariate* data and we often want to analyse this data to see if a relationship exists between the two variables.

In some cases one of the variables is time and we measure another variable against it to determine if any pattern exists. For example, we could measure the height of a young child each month. In this case:

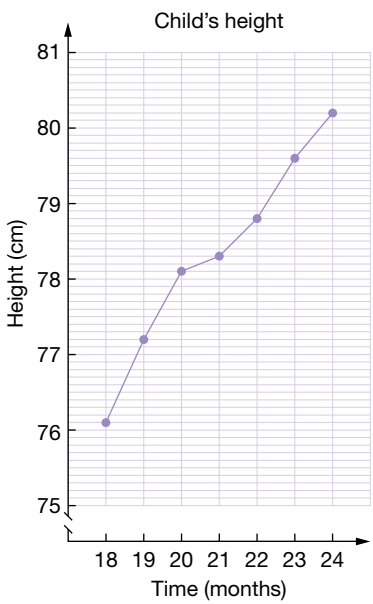
- The **dependent variable** is height (in cm) because each measurement depends upon the month in which it was taken.
- The **independent variable** is age (in months).

This table shows the average height of a particular young child from 18 months to 24 months.

Age (months)	18	19	20	21	22	23	24
Height (cm)	76·1	77·2	78·1	78·3	78·8	79·7	80·2

Notice that the height is increasing each month. For this 6-month period the child has grown 4·1 cm. This is an average increase of approximately 0·7 cm per month.

This data is best viewed as a line graph, which gives a visual impression of the relationship.



Conclusions might be drawn from any observed pattern in bivariate data. Take care not to make assumptions that cannot be supported.

For example, if it is assumed from this data that the child will gain 0·7 cm in height every month. Assuming this is true, the newborn baby's length would have been:

$= 76·1 - 18 \times 0·7$
$= 76·1 - 12·6$
$= 63·5$ cm

Does this seem a reasonable length for a newborn baby?

At age 10, the child's height would be:

$= 80·2 + 0·7 \times 12$ months $\times$ 8 years
$= 147·4$ cm

At age 21, the height would be:

Height $= 80·2 + 0·7 \times 12$ months $\times$ 19 years
$= 239·8$ cm

Do these heights seem reasonable?

To extend the data beyond the data that has actually been measured is to 'extrapolate' the data.

It is obvious that an incorrect assumption has been made. You cannot expect the growth rate to be constant throughout the child's life. In other examples, incorrect assumptions may not be so obvious. Ensure that conclusions you draw can be supported by the data.

WORKED EXAMPLE 1

This table shows two sets of data measured against the time of year. For each month the highest temperature (°C) for the 30-year period 1981–2010, and the single year 2011, was recorded at Observatory Hill in Sydney.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Max. temp (°C) 1981–2010	44·2	40·8	39·8	33·9	29·0	25·2	25·9	31·3	34·5	38·2	41·8	40·9
Max. temp (°C) 2011	35·4	41·5	33·6	27·0	23·3	21·0	21·4	25·7	32·5	34·3	37·2	26·8

a Graph these sets of data on the same diagram. Join the plotted points to form continuous line graphs. Does the line joining adjacent points have meaning?

b For which month was a maximum temperature recorded in 2011 that was higher than for the previous 30 years?

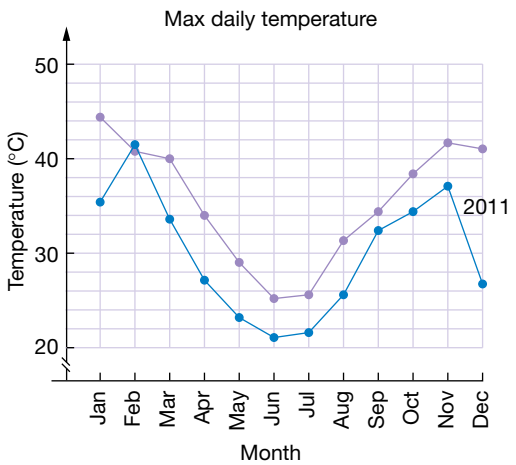
c What feature is unusual for 2011 compared to the previous 30 years?

Solution

a The graph has been constructed with months on the horizontal axis and temperature on the vertical axis that has been 'truncated' from 0 to 20 because there is no data in this range. Time is usually shown on the horizontal axis. Points along the line segments cannot be read as data points but the line does indicate a trend between the data points.

b February recorded a maximum temperature greater than any in the past 30 years during February.

c The maximum temperatures recorded during the summer months of January and December seem markedly low. Note these are at opposite ends of 2011. They are not part of the same summer season. This should evoke interest in data for the following years.



WORKED EXAMPLE 2

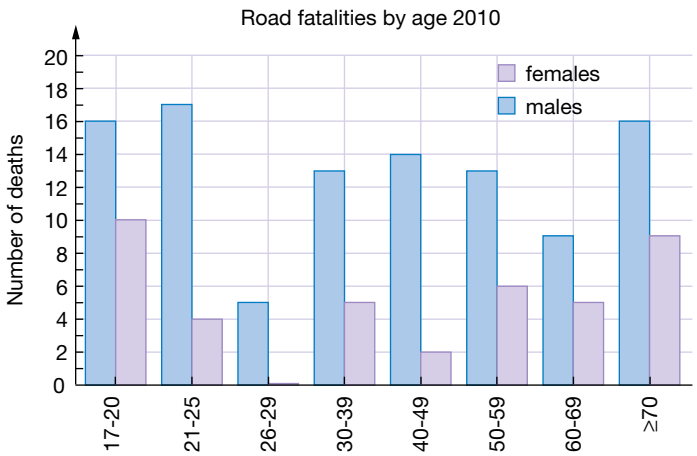
The NSW Centre for Road Safety published these figures for car driver deaths in 2010 showing the age and sex of the driver.

Age (years)	17–20	21–25	26–29	30–39	40–49	50–59	60–69	≥70	Total
Male	16	17	5	13	14	13	9	16	103
Female	10	4	0	5	2	6	5	9	41
Sub-total:	26	21	5	18	16	19	14	25	144

a Draw a column graph showing both male and female data on the same graph.
b Why is a line graph not suitable for this data?
c Which group appears to be the safest drivers?
d Which group appears to be most at risk?
e Has the data been organised to give a fair picture of the driving risk of various ages?
f How would data such as this be useful?
g Has the number of drivers in each age category been given?

Solutions

a The column graph has been drawn with columns for males and females placed together for comparison.



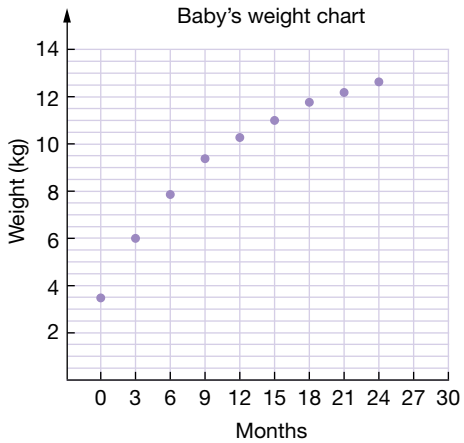
b The age groups on the horizontal axis are 'categories'. The scale is not continuous.
c For 2010 the group with the safest driving record appears to be female drivers in the category 26–29 years.
d The group with the worst record appears to be males in the 21-25 years category. However, it should be noted that this category spans 5 years but the 17–20 category spans only 4 years. This represents a worse average.
e The data is distorted because the age groups are not of equal spreads. The ≥70 group represents more than 10 years. If the 17–20 and 21–25 categories were combined it would represent a 9-year spread with the data representing a more dramatic picture.

f Insurance companies rely on this type of data when assessing the risk and therefore setting the premiums. Also governments and road safety groups need to know where to target their efforts, and funding, in order to improve road safety.

g No. Knowing the number in each category would help to draw sensible conclusions.

Exercise 9:02

1 This chart shows the weight of Henry for the first 2 years of his life.



a What was his weight at birth?

b What was his approximate weight when he was 1 year old?

c What percentage increase was this on his birth weight?

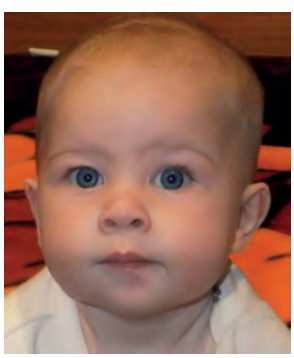
d What was Henry's weight on his second birthday? What percentage increase was this on his weight at 1 year old?

e What would be his approximate weight at 27 months?

f Can you predict Henry's weight when he is 10 years old?

2 This table shows the height of Henry for the first 24 months of his life, measured at 3 monthly intervals to the nearest cm.

Months	0	3	6	9	12	15	18	21	24
Height (cm)	50	61	67	72	76	79	82	85	87



a Plot this data on a graph similar to the one shown in Question 1. (The vertical axis showing the height in cm could be started from 40.)

b Join the plotted points to form a line graph. Does the shape of the graph appear similar to the graph in Question 1?

c Do you think this shows that the weight and height of a baby are related variables?

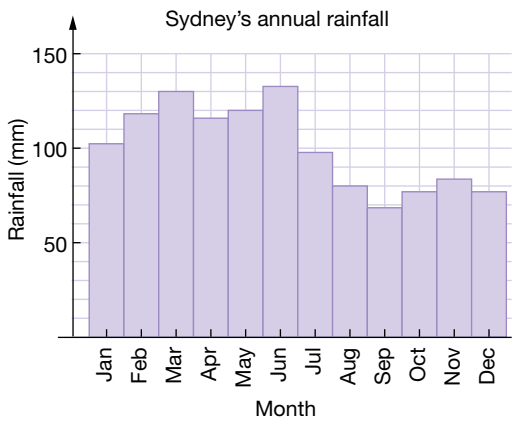
d Use the line graph to estimate Henry's height at 14 months.

e Would you expect the shape of the graph to continue until Henry reaches adulthood? Give a reason for your answer.

3 This table shows the mean rainfall for each month of the year measured from 1858 to 2012 for Sydney. It also shows the rainfall for the single year 2012. Note that the yearly total for 2012 is very close to the mean.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
Mean rainfall (mm) 1858–2012	101·5	118·0	130·2	126·7	120·6	131·2	98·3	80·3	68·6	77·1	83·6	77·6	1213·0
Rainfall (mm) 2012	138·8	111·0	269·8	187·0	37·2	244·2	56·2	19·0	23·8	29·4	52·0	45·2	1213·6

The histogram shows the mean rainfall data from the table.



a For which month is the mean rainfall the:
  i lowest **ii** highest?

b Which appears to be wetter: the first half of the year or the second?

c Draw a similar column graph showing the monthly rainfall for 2012. (Note the maximum value needed on the vertical axis.)

d Compare the two graphs. Comment on similarities and differences.

4 This table shows the final position of each team in the 2012 ANZ Netball Competition as well as the goals scored for and against each team.

a Draw a column graph showing the goals scored for and against on the same graph.

b Use the graph to determine which variable seems to be the greater indicator of competition success: goals for (attack) or goals against (defence)? How might a coach be interested in the result?

c Why is either variable not a perfect predictor of where a team will finish in the competition?

Pos	Team	GF	GA
1	Vixens	645	569
2	Mystics	667	633
3	Magic	699	594
4	T'birds	670	589
5	Swifts	624	638
6	Firebirds	686	640
7	Pulse	585	626
8	Fever	608	673
9	Steel	639	728
10	Tactix	634	767

5 The table shows the life expectancy for people in 20 countries as well as a measure of their wealth, the Gross Domestic Product (GDP) per person in US dollars. Below is a column graph for each of these variables.

The difference in wealth is vast, with some columns barely visible.

a List the top 8 countries in order according to their GDP per person.

b List, in order, the 8 countries with the greatest life expectancy.

c How many countries appear in the 'top 8' of both lists?

d Identify the 'bottom 8' from each graph.

e Which country is in the bottom 8 for GDP but not for life expectancy?

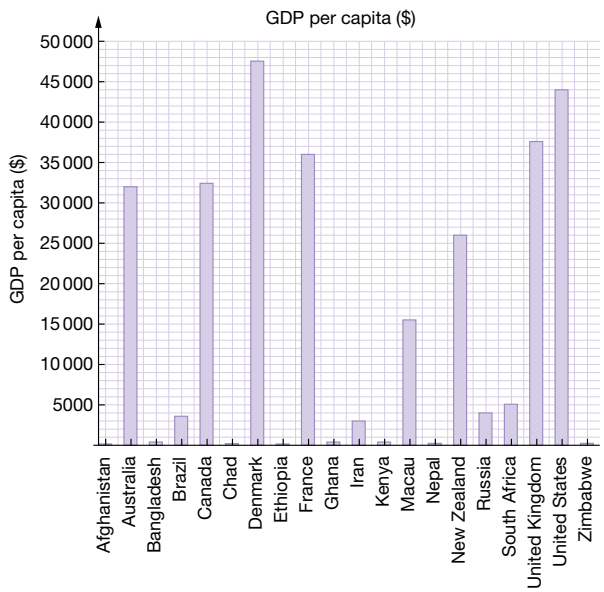
f Which country is in the bottom 8 for life expectancy but not for GDP?

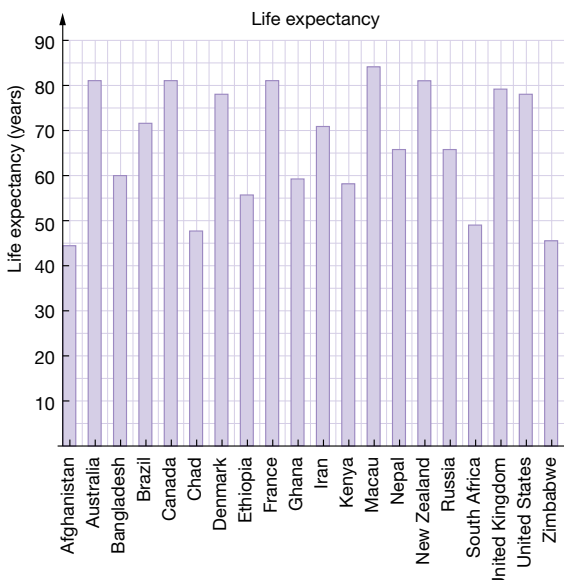
g This data would suggest a strong link between these two variables. How could this be further investigated?

h What other factors might be investigated to determine their effect on life expectancy?

* *Note:* Macau is a self-governing region of China.

Country	Life Expectancy (years)	GDP per person ($)
Afghanistan	44·6	222
Australia	81·6	32 170
Bangladesh	60·2	467
Brazil	72.0	3455
Canada	81·2	32 546
Chad	47·7	396
Denmark	78·3	47 363
Ethiopia	55·4	156
France	81.0	35 854
Ghana	59·9	452
Iran	71·1	2771
Kenya	57·9	527
Macau*	84·4	15 413
Nepal	65·5	268
New Zealand	80·4	25 942
Russia	66.0	4466
South Africa	49.0	4959
United Kingdom	79.0	37 632
United States	78·1	43 743
Zimbabwe	45·8	343





6 The table shows the road fatalities for 2010 against the number of registered vehicles and the population of each state or territory. Column graphs have been drawn below comparing the number of fatalities with each of the other variables.

a Calculate the number of fatalities per 10 000 vehicles for NT by dividing 49 by 14. How is this figure shown in graph A?

b By looking at graph A, which state or territory has the next worst rate of fatalities per 10 000 vehicles?

c Which state or territory has the lowest rate of fatalities per 10 000 vehicles? Use the table to calculate this rate correct to two decimal places.

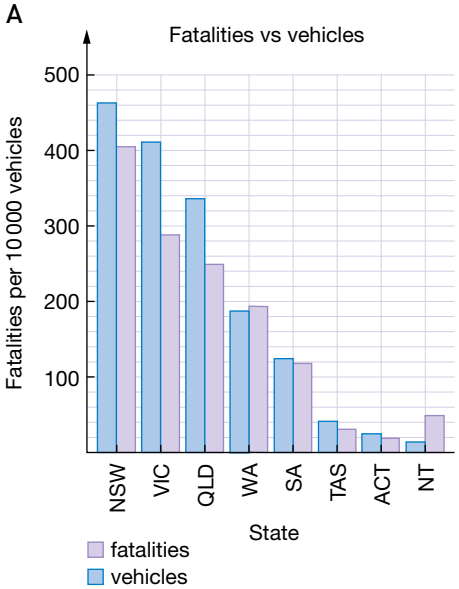
d Looking at graph B, determine the states or territories with the two lowest rates of fatalities per 10 000 population.

e Which variable would you expect to have a greater influence on the number of fatalities: the number of vehicles or the population? Does the data support your answer?

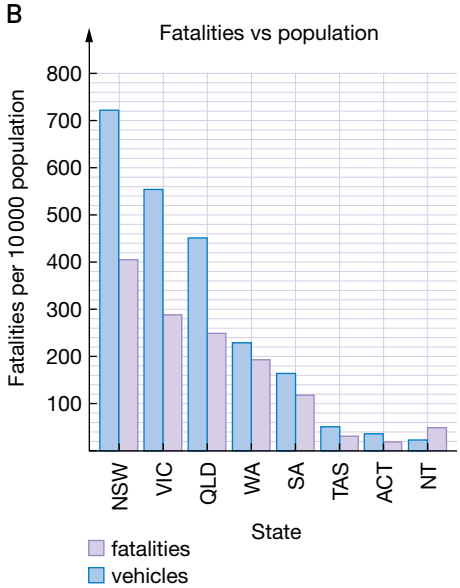
f Can you tell which state or territory has the best or worst drivers from this data? What other factors might be considered?

State	Fatalitites	Vehicles ('0000)	Pop'n ('0000)
NSW	405	463	722
Vic	288	411	554
Qld	249	336	451
WA	193	187	229
SA	118	124	164
Tas	31	41	51
ACT	19	25	36
NT	49	14	23

A

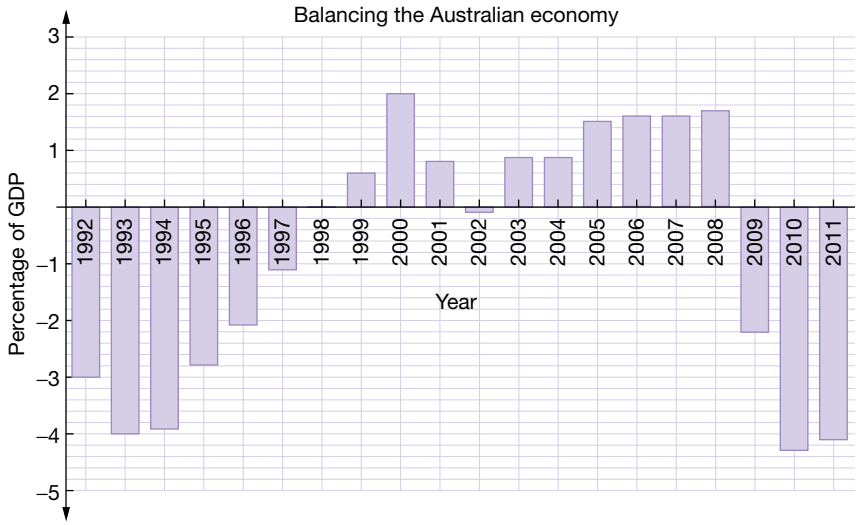


B



7 The column graph below shows a surplus for the Australian economy when the column is above the horizontal axis, and a deficit when it is below. It is expressed as a percentage of the Gross Domestic Product (GDP). A surplus can be viewed as a saving and a deficit as an overspending.

- **a** For the 20 years shown, in what year did the greatest surplus occur?
- **b** What year registered the greatest deficit?
- **c** What world event caused the large 'drop' in the economy from 2008 to 2009?
- **d** These results are important to governments as they try to 'manage' the economy. How might their policies affect these figures?



9:03 Scatter plots

Sometimes two sets of data are collected from the same population or sample that is representing two different variables. A question often asked is: 'Are these two variables related, and if so, how strongly does the value of one variable reflect the value of the other?'

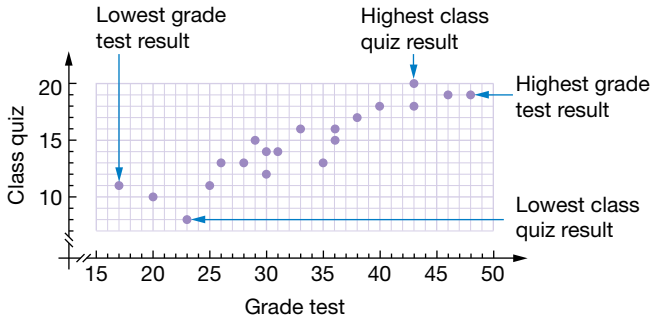
Before investigating any such relationship a convenient method of analysis is needed. A very useful and easily constructed tool is a **scatter plot** or **scatter diagram**.

For example, the Grade test results out of 50, and Class quiz marks out of 20 for a class of 20 students are shown in the table below.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Test	23	20	17	25	30	28	35	26	30	31	36	29	33	36	38	42	40	48	46	43
Quiz	8	10	11	11	12	13	13	13	14	14	15	15	16	16	17	18	18	19	19	20

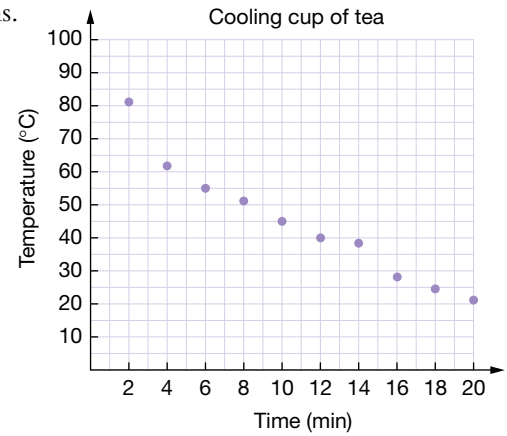
The test results and the quiz marks for each student are plotted as an ordered pair to produce a scatter plot, which is shown on the following page.

This scatter plot gives the general impression that as one variable increases, so does the other. This is said to be a **positive** relationship. But the relationship is not exact. The student with the lowest quiz mark did not get the lowest test result. If there was an exact relationship between the two variables the points would lie along a straight line.

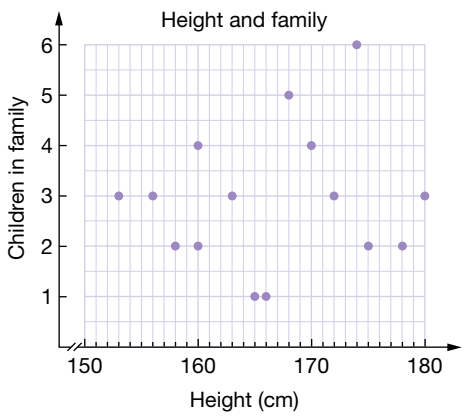


Consider these scatter plots that show different patterns.

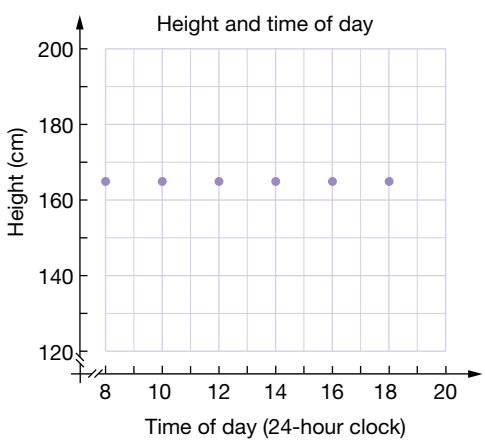
This scatter plot indicates that as time increases the temperature of a cup of tea decreases. This is said to be a **negative** relationship between these two variables. If a line was drawn connecting any pair of points it would have a negative slope.



This scatter plot seems totally random. It would suggest there is no direct relationship between the height of students and the number of children in their families.



This scatter plot shows a linear pattern, but because the 'line' of points is horizontal it suggests that the time of day has no bearing on the height of a person. So as with the scatter plot above, there is no relationship linking the variables.



WORKED EXAMPLE 1

The table shows the number of a product sold as its price increased.

Price ($)	10	11	12	13	14	15	16	17	18
Number sold	25	24	20	19	16	13	12	12	10

a Plot the data on a scatter plot.
b Does a pattern exist between the variables?
c Describe the relationship between the variables.

Solution

a

b The scatter plot shows that as the price increased the sales decreased.

c This is a negative relationship. As the data points are close to lying on a straight line, this relationship would be classified as a strong negative relationship.

WORKED EXAMPLE 2

This table shows the average monthly temperature and rainfall for the town of Sidnee.

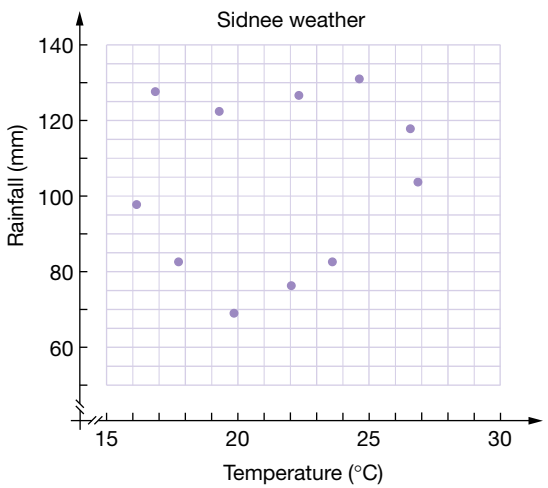
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp (°C)	25·8	25·7	24·7	22·4	19·3	16·9	16·2	17·7	19·9	22·0	23·6	25·1
Rainfall (mm)	103	117	131	127	123	128	98	82	69	77	83	78

a Plot this data on a scatter plot with axes that are applicable for the data shown.
b Determine the nature of any relationship between these variables.

Solution

a Appropriate scales would start at 60 mm for rainfall and 15°C for temperature as no measurement falls below these values.

b The pattern shown by the scatter plot appears random. There is certainly no increasing or decreasing pattern. According to this set of data there is no relationship between average monthly temperature and rainfall for Sidnee.



WORKED EXAMPLE 3

Suvir's mother measured his height every year on his birthday. His height for the even numbered years is shown in the table.

Age (years)	4	6	8	10	12	14	16
Height (cm)	103	114	129	140	152	169	178

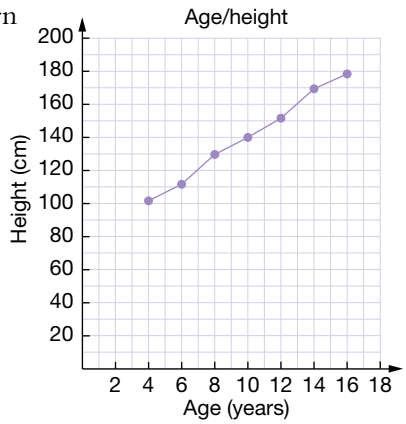
a Show this bivariate data on a scatter plot.

b Can you estimate Suvir's height when he was 11 years of age?

c Can you estimate Suvir's height when he was born and at 25 years of age?

Solution

a The plotted data points have been joined, showing a pattern close to a straight line. Therefore the variables exhibit a strong positive relationship for the measured data.

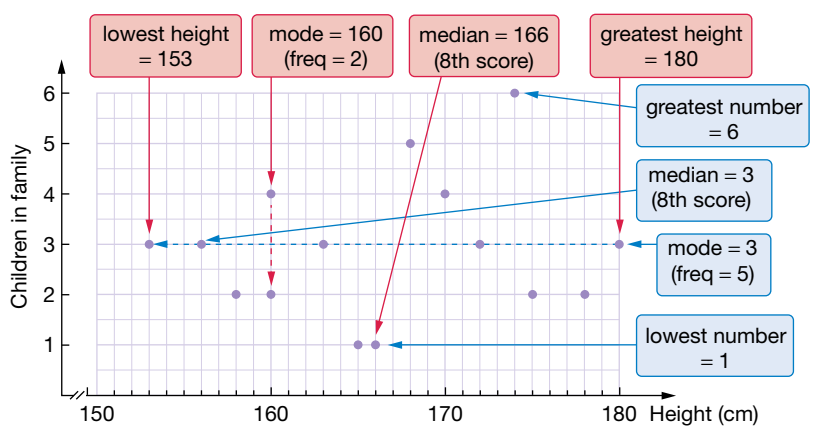


b It is reasonable to assume that the pattern continues between the data points. So an approximation for Suvir's height at age 11 would be the average of his height at ages 10 and 12, which is 146 cm.

c If this pattern continued beyond the measured data Suvir's height at birth would have been about 78 cm and his height at 25 years would be over 230 cm. Both of these estimates are unreasonable and show the danger of assuming all patterns will continue, without supporting evidence. Clearly, for this example, growth rates change with age.

- **Interpolation** is the process of estimating data that lie within the domain of the values given. This is valid when a definite relationship exists between the two variables.
- **Extrapolation** is the process of predicting data beyond the values given. This is often not useful and can lead to false results as there is no guarantee that an observed pattern will continue beyond the data presented.

The **range, mode and median** can be determined for each of the variables from a scatter plot by observation and counting. For this scatter plot there are 15 data points, so the median would be represented by the 8th data point.



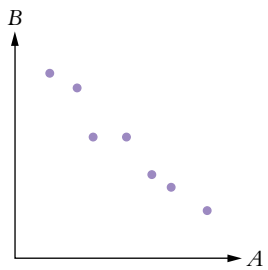
For height:
Range = 180 – 153
= 27 cm
Median = 166 cm
Mode = 160 cm

For children in family:
Range = 6 – 1
= 5
Mode = 3
Median = 3

Exercise 9:03

1 State whether the variables A and B plotted against each other in each scatter plot shows a linear relationship or not. If so, is it a positive or negative relationship?

a



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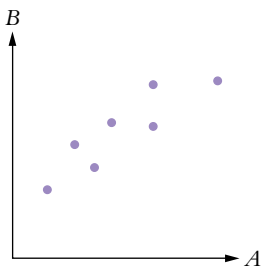

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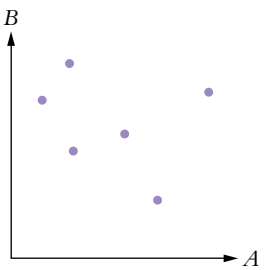

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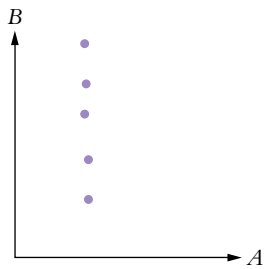

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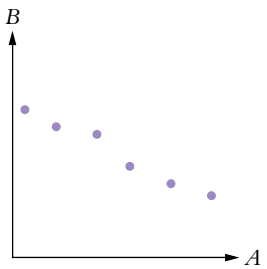

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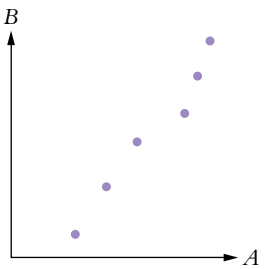


2 Plot each set of data on a scatter plot and determine the type of relationship that exists between the two variables.

a

x	3	4	5	5	6	6	7	8
y	9	7	6	7	6	5	5	3

b

t	4	5	7	7	8	10	10	12
N	6	9	9	12	11	12	14	15

c

t	3	4	5	6	7	8	9	10
S	22	23	23	24	25	24	25	26

d

x	2	4	6	8	10	12	14	16
n	9	6	10	3	9	5	9	4

e

K	16	18	20	22	24	26	28	30
P	50	45	38	36	32	24	19	15

f

P	1	2	3	4	5	6	7	8
Q	7	6	6	7	7	6	7	6

3 Mark drove from Penrith to Coffs Harbour and checked his trip meter every hour. The total distance travelled at the end of each hour is shown in this table.

Time (hours)	1	2	3	4	5	6
Distance (km)	82	191	295	369	487	561

a Plot this data on a scatter plot.

b Describe the relationship between time and the distance travelled.

c What was Mark's average speed for journey?

4 This table shows the points table after 10 rounds in the A-League 2012 football competition as well as the goals for and against each team.

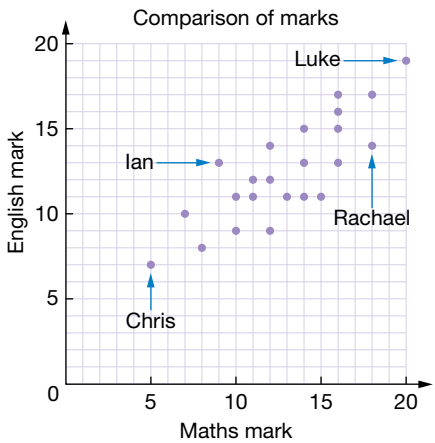
a Draw a scatter plot with points on the horizontal axis and goals for on the vertical axis.

b Draw a similar scatter plot plotting points on the horizontal axis and goals against on the vertical axis.

c Compare the two scatter plots. Which appears to have a greater relationship with the competition points, goals for or goals against?

Team	G. For	G. Agt	Points
C.C. Mariners	19	8	23
Adelaide United	17	9	22
Melb. Victory	17	20	16
W.S. Wanderers	6	8	13
Newcastle Jets	14	19	13
Wel. Phoenix	13	12	12
Melb. Heart	12	12	12
Perth Glory	9	10	11
Brisbane Roar	13	13	10
Sydney F.C.	14	23	10

5 This scatter plot shows the Maths marks plotted against the English marks for a class of students.



a How many students are in the class?

b Write the Maths and English marks scored by:

i Chris ii Luke

iii Ian iv Rachael

c What is the mode Maths mark?

d What is the median Maths mark?

e What is the mode English mark?

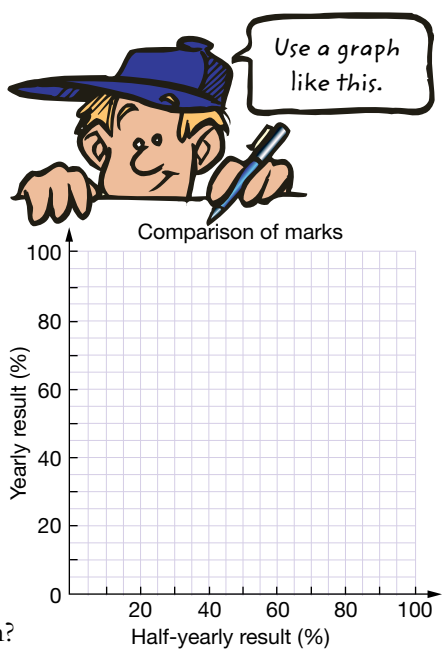
f What is the median English mark?

g What relationship is suggested by this scatter plot?

6 The exam results in mid-year and end-of-year tests for Mr Justin's class are given below.

Name	Mid-year	year
Achnid, Rex	27	45
Amine, Xavier	69	72
Bandon, Alice	45	42
Beese, Owen	66	69
Care, Ivy	96	90
Deals, Ian	93	90
Deavour, Nell	90	78
Fend, Dina	54	57
Hance, Nina	84	78
Honest, Ban	75	63
Ickle, Vicki	39	48
Loyal, Betty	93	84

Name	Mid-year	year
Madeit, Ena	48	54
Merit, David	63	69
Night, Unice	60	66
Press, Mark	69	63
Reason, Chris	84	69
Share, Iris	96	87
Swerthy, Eve	87	72
Team, Sue	75	75
Tist, Ross	75	81
Vee, Tony	30	36
Walker, Joy	57	51
Wander, Yin	78	72



a Draw a scatter plot diagram to show these results.

b What relationship is there between the results?

c What was the position of O. Beese in the yearly exam?

d What was the position of D. Merit in the mid-yearly exam?

e What was the mode in the mid-yearly exam?

f What was the median in the yearly exam?

7 There are 16 teams in the NRL Rugby League Competition. This table shows the position each team finished as well as points gained on the competition table. The points scored for each team as well as those scored against each team are also shown for the 2012 season.

a Construct three scatter plots.
   i points and points for
   ii points and points against
   iii points for and points against

b Which scatter plot shows a positive relationship between the variables?

c Which scatter plots show a negative relationship between the variables?

d Between which two variables does the relationship appear the strongest?

Team	Points	Pts for	Pts against
Bulldogs	40	568	369
Storm	38	579	361
Rabbitohs	36	559	428
Sea Eagles	36	497	403
Cowboys	34	597	445
Raiders	30	545	536
Sharks	29	445	441
Broncos	28	481	447
Dragons	26	405	438
West Tigers	26	506	551
Titans	24	449	477
Knights	24	448	488
Roosters	21	462	626
Warriors	20	497	609
Panthers	20	409	575
Eels	16	431	674

8 a Draw a scatter plot for the data below using a scale of 1 mm = 2 marks on each axis.

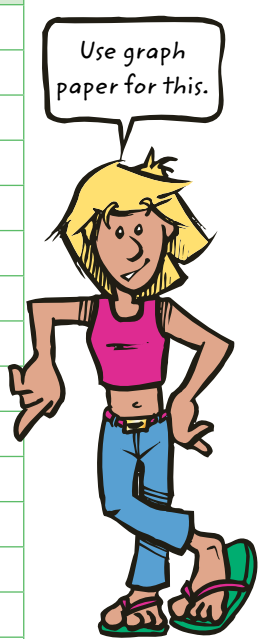
b Determine the range and median for both sets of results.

c Describe the relationship that exists between the sets of data.

d If an additional student sat for the Maths exam and scored 70, what would you expect her English mark to be?

Name	Maths	English
Able, Nancy	76	64
Argue, Youst	82	78
Beaudy, Ungwin	50	70
Cycle, Ivan	63	74
Deer, Ivy	46	50
Evator, Lois	38	38
Eye, Graham	68	54
Force, Neil	98	86
Hale, Xavier	60	60
Hance, Neville	78	68
Iting, Xavier	82	76
Leave, Ben	38	60
Mitt, Ona	66	50
Muse, Betty	78	78
Pand, Xavier	20	50

Name	Maths	English
Pea, Tom	58	62
Pen, Owen	48	46
Podd, Peter	80	74
Pott, Timothy	65	60
Saidit, Unice	72	78
Saw, Carol	56	62
Shooter, Petra	68	68
Side, Cicile	50	46
Snore, Irwin	60	80
String, Barry	46	56
Table, Theresa	84	76
Tee, Mandy	52	56
Tension, Dennis	74	82
Toteler, Tina	66	56
Tsa, Paul	70	80



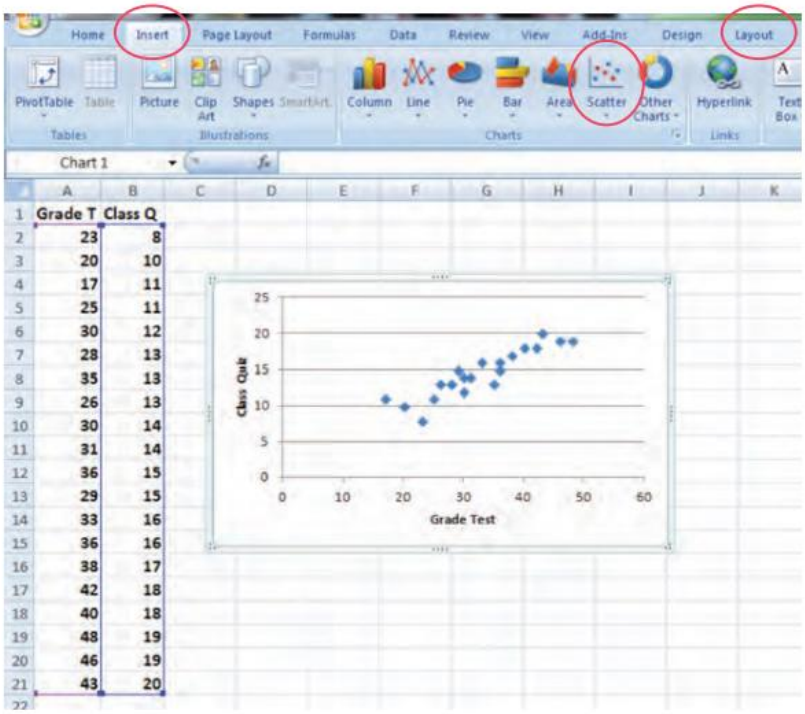
CHALLENGE 9:03 FURTHER SCATTER PLOTS

Gather data from students in your class or Year group to investigate the relationship between other variables. Choose variables that you suspect of having a positive or negative relationship as well as some you would not expect to influence one another.
You could select two sets of data from this suggested list.

- Age in months
- Day of the month you were born
- Weight in kg
- Height in cm
- Number of pets
- Shoe size
- Arm length
- Weight at birth
- Head circumference
- Time to get to school
- Number of tunes on a phone
- Exam or test marks
- Number of CDs owned
- Letters in your name
- Hours of TV watched

INVESTIGATION 9:03 SCATTER PLOTS USING A SPREADSHEET

- Enter the data shown for the Grade test/Class quiz on page 239. This data is shown in columns A and B.
- Highlight this data, select the **Insert** tab in the menu as shown, and then choose **Scatter**. A scatter plot will appear as shown.



- After the scatter plot has appeared, choose the 'Layout' tab. This will provide a menu with options to refine the appearance of the graph.
- Choosing Axis Titles will enable the titles of Grade Test and Class Quiz to be added.
- Experiment with other options to discover their effects.

9:04 Statistics all around us

We are surrounded by statistics; technology has enabled the collection, storage and analysis of data to be much more accessible. Statistics help us to understand the world around us. They are a key to producing summaries of data, drawing inferences and predicting the future.

Meaningful and useful statistics depend upon reliable and adequate data. Governments rely on the use of statistics to run their country. It was this use of such data, 'data about the State' that led to the use of the word 'statistics'.

Statistics are crucial to scientific discovery, business operations, medicine, sociology, engineering, sport and many other forms of human endeavour.

This exercise will highlight some secondary sources of statistics that are used in our society today.

Exercise 9:04

1 Every sport is a source of statistics; none more so than cricket. The table below shows the test match record of Michael Clarke after 87 tests. Often you have to be familiar with the sport to understand the data. Do you know what each piece of data means or how it is calculated?

a A match normally has two innings. If he played in 87 matches, in how many did Michael Clarke only bat once?

b For the 145 times he batted, how many times was he given 'Out'?

c The 'Aggregate' is the total runs scored. Divide this number by your answer for part **b**. What piece of data does this give?

d As a bowler, Michael Clarke had 1098 runs scored off his 374 overs. Dividing 1098 by 374 gives which statistic?

e Dividing the number of balls bowled by the number of wickets taken yields which statistic?

If there are any other terms you are not familiar with, ask a cricket fan in your class.

Batting	
Innings	145
Not outs	15
Aggregate	6804
Average	52·34
Highest score	329
50s	24
100s	21
200s	4
300s	1
Ducks	6
Pairs	0
Scoring rate	55·98

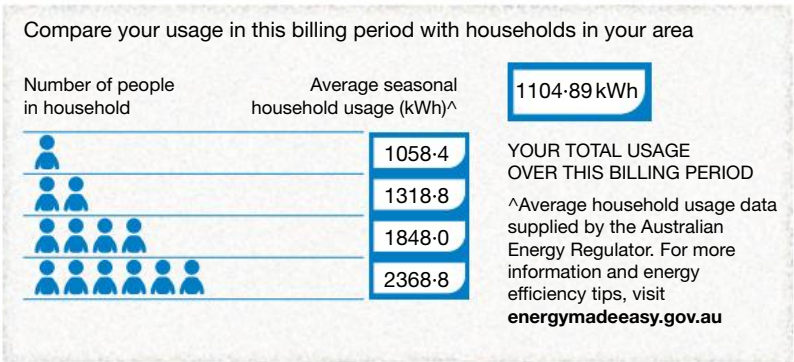
Bowling	
Overs	374
Balls	2244
Maidens	56
Runs	1098
Wickets	30
Average	36·60
5 wickets/innings	2
10 wickets/innings	0
Best (Inns)	6/9
Best (Match)	6/9
Economy rate	2.94
Strike rate	74·80

2 The ratings for television networks are shown in this table. The figures are percentages but what do they mean?

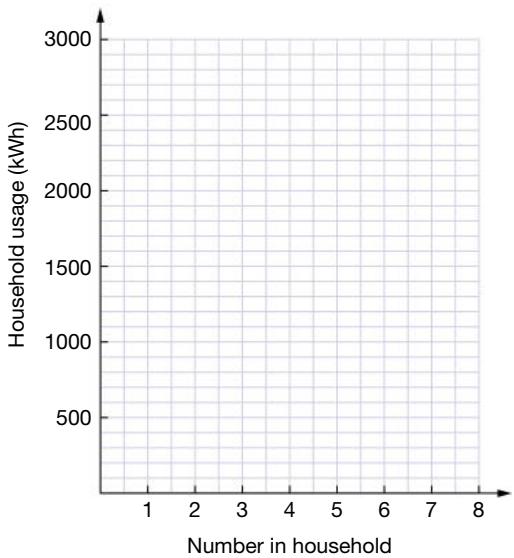
Year	ABC	Seven	Nine	Ten	SBS
2003	15·6%	25·6%	31·2%	23·2%	4·5%
2004	16·9%	24·9%	29·9%	23·2%	4·6%
2005	15·7%	27·1%	29·2%	23·8%	6·1%
2006	15·4%	27·8%	29·1%	22·5%	5·4%
2007	16·3%	29·0%	27·4%	21·6%	5·6%
2008	17·5%	28·7%	27·2%	21·0%	5·6%
2009	16·4%	28·8%	26·8%	22·2%	5·8%
2010	16·9%	28·6%	27·7%	21·2%	5·6%

a Add the percentages for 2010 for each network. What is the total?
b Each figure is a percentage of what quantity?
c From 2003 to 2010, for which networks did the ratings:
   i increase
   ii decrease?
d Why are these ratings important?
e How could a network improve its ratings?

3 The cost of electricity is of concern to many households. On the bill the electricity provider indicates the power used for the three-month period as well as a means of comparing the usage to similar households. This is shown in the following diagram.



a The bill is for a two person household. What is the total usage for this period?
b How does the usage compare with other similar households?
c Construct a scatter plot using the axes shown, by plotting the usage for households with 1, 2, 4 and 6 people.
d Is there a relationship between these variables? How would you describe it?
e By drawing a line between the dots in this plot, estimate the average usage for a household with 5 people.
f Extrapolate this pattern to estimate the usage for a household with:
   i no one
   ii 8 people
   iii 20 people
   Are these estimates reasonable?



4 The Department of Immigration and Citizenship published these figures for the Financial Year 2010–11 showing the change in the number of permanent residents for each state and territory. Also shown is the population for each state and territory as at June 2011.

Resident	NSW	Vic	Qld	SA	WA	Tas	NT	ACT	Australia*
Arrival	38 816	33 632	25 827	7 204	18 441	949	960	1 623	127 458
Departure	34 802	19 320	17 851	3 636	10 256	803	526	1 259	88 461
Population	7 211 468	5 534 526	4 474 098	1 638 232	2 352 215	511 195	231 331	367 752	22 323 933

* Includes Territory of Christmas Island, Territory of Cocos (Keeling) Islands and Jervis Bay.

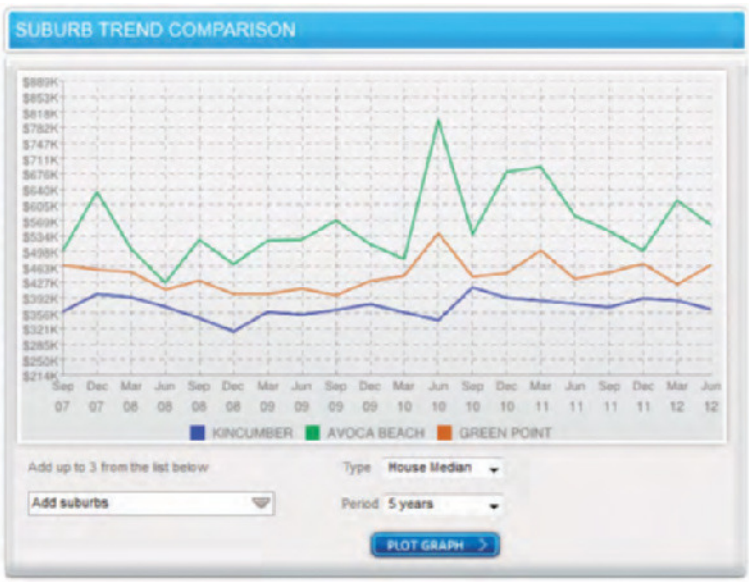
a Calculate the net increase in Australia's population due to migration. What percentage of the total population is this?

b Which State or territory experienced the greatest increase in population due to migration?

c Which State or territory experienced the greatest percentage increase in its population?

5 The price of housing can fluctuate with economic conditions and be determined by location. To compare prices the median is usually used as a measure. This explanation was given on the Property Data website.

The figures for three adjoining suburbs are shown in this table and a graph comparing the median prices is shown below.



What is a median price?

The median price is the middle price in a series of sales. For example, if 15 sales are recorded in a suburb and arranged in order from lowest to highest value, the eighth sale price is the median price. In the case where there are an even number of sales in a series, the median is the average of the middle two prices.

Median prices are used rather than average prices because median prices are unaffected by a few unusually high or low prices, making them a more accurate indicator of true market activity. The lower quartile is the point where one quarter of the sales are of lesser value and the upper quartile is the point where one quarter of the sales are of a higher value.

Location	Lower Quartile	Median Price	Upper Quartile
Avoca Beach	$512 500	$557 500	$845 500
Green Point	$422 500	$467 500	$519 250
Kincumber	$295 500	$364 925	$417 775

a The median price of which suburb seems to fluctuate the most?

b The figures shown in the table are based on quarterly sales figures for the 3 months ending June 2012. Can you tell how many houses were sold? Does it matter?

c The median indicates the 'middle' house price. If the mean was used instead, which suburb's 'average' figure would change the most?

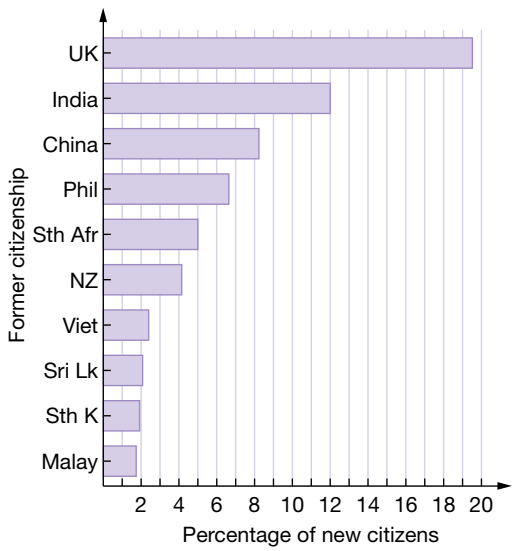
d Which suburb's house prices seem fairly symmetrical and, which are skewed?

6 According to the Department of Immigration and Citizenship more than 4·5 million people have become citizens since Australian citizenship was introduced in 1949.

In 1949, during the inaugural year of the *Nationality and Citizenship Act 1948*, Australian citizenship was granted to 2493 people from just over 35 different nationalities. The top 5 nationalities (and number of grants) at that time according to the *Australian Yearbook 1953, ABS* were: Italian (708) Polish (597) Greek (276) German (225) and Yugoslav (80).

In 2011-12, 84 183 people became Australian citizens by conferral from at least 180 different countries. The following table shows the number of people who were conferred as Australian citizens at ceremonies in 2011–12, according to their former nationality/citizenship.

Former citizenship	Persons	Per cent
United Kingdom	16 401	19·5
India	10 076	12·0
China	6876	8·2
Philippines	5592	6·6
South Africa	4206	5·0
New Zealand	3458	4·1
Vietnam	1929	2·3
Sri Lanka	1671	2·0
South Korea	1570	1·9
Malaysia	1487	1·8
Other	30 917	36·7
Total:	84 183	100·0



a The graph shows the 'top 10' sources of migrants for 2011–12. What group listed in the table is not shown on the graph?

b At least how many countries, apart from the 'top 10', are a country of origin for Australian migrants?

c Why were the top 5 nationalities in 1949 European?

d Why do you think the United Kingdom is not among the top 5?

e Apart from the United Kingdom, how many of the 2011–12 top 10 are European?

f How many of the top 10 nationalities are Asian?

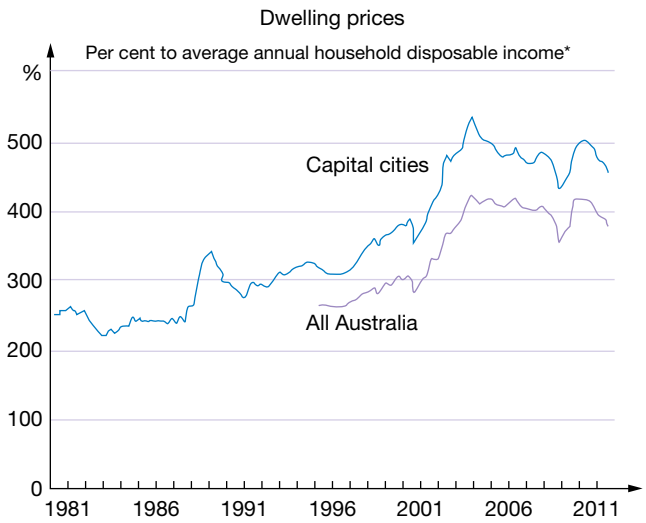
7 In February 2012, the head of the Reserve Bank's financial stability department reckoned that Australia would avoid a US-style house price meltdown. The chart produced shows house prices as a percentage of annual household disposable income. The ratio peaked in 2003 due to boosted incomes. The spike down in 2008 relates to the GFC (Global Financial Crisis). The bounce back reflects the government's attempts to reinflate the market with the First Home Owner Grant.

a What year does the graph indicate recorded the lowest percentage of annual income needed to purchase a house?

b Why is the graph higher for capital cities?

c In 2011, was the percentage going up or down?

d If houses are going to be more affordable in the future, at least one of two things must happen. What are they?



* Household disposable income excludes unincorporated enterprises and is before interest payments.
Source: ABS; APM; RBA; REIA

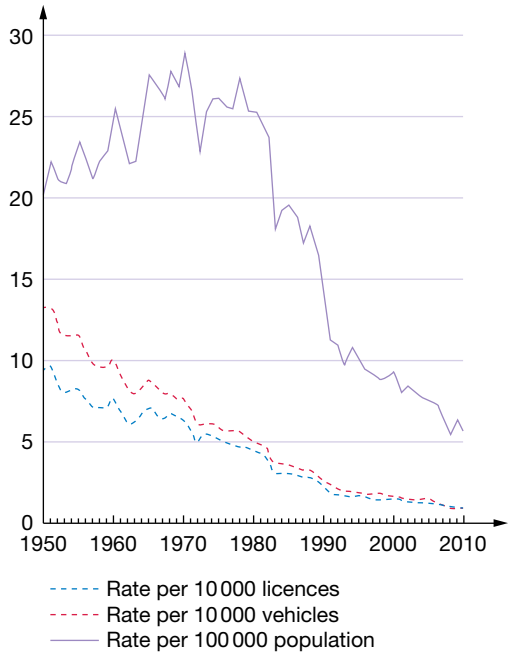
8 The graph below shows the fatality rate per 10 000 vehicles, 10 000 licence holders and 100 000 population for years 1950 to 2010 in NSW.

a The line graphs showing the fatality rate per 10 000 vehicles and 10 000 licences are similar. They are progressively falling, but until the early 1980s the rate per 100 000 people was increasing. Explain how this could happen.

b The blue and red lines merge in the 2000s. Why do you think they were so different in 1950?

c What caused the significant decrease in the 1980s for the purple graph?

d Road Safety authorities have tried to introduce measures to ensure the rate keeps decreasing. What measures have contributed to safer road travel?

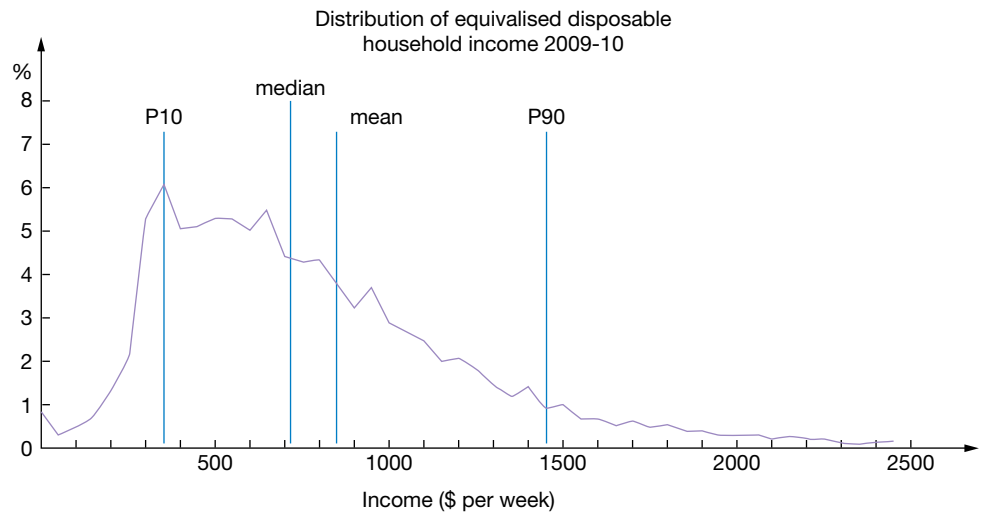


Statistical statement for the year ended 31 December 2010
Prepared by the Centre for Road Safety, Transport for NSW

9 The Australian Bureau of Statistics (ABS) collected the data below showing the 'average' income for the States and Territories of Australia. ('Equivalised' means that adjustments have been made to take into account the number of people in each household.)

Equivalised disposable household income per week	NSW $	Vic $	Qld $	SA $	WA $	Tas $	NT $	ACT $	Aust $
Mean income	859	818	817	798	966	708	938	1101	848
Median income	718	695	705	685	770	613	885	1009	715

'While the mean equivalised disposable household income of all households in Australia in 2009–10 was $848 per week, the median (i.e. the midpoint when all people are ranked in order of income) was lower at $715 per week. This difference reflects the typically asymmetric distribution of income where a relatively small number of people have relatively high household incomes, and a large number of people have relatively lower household incomes.'



Source: Household Income and Income Distribution, Australia (6523.0).

a Which state or territory has the highest average incomes? Why do you think this is the case?
b Which state or territory has the greatest difference between the median and mean incomes? Can you think of a reason for this large difference?
c The quote above says the graph shows an 'asymmetric' distribution. This means it is skewed. Is it skewed positively or negatively?
d On the graph are lines marked as P10 and P90. What do you think they indicate?
e Which line could be marked as P50, the median or the mean?

In December, 2012 the following article appeared in the *Sunday Herald*. Review the article and the statistics, and discuss any trends or conclusions presented. What other issues should be considered?

How schools fail to curb bad behaviour

ANNA PATTY EDUCATION

School suspensions have risen by more than a third over the past six years but do little to improve the behaviour of children, research has found.

The study, conducted by Uniting Care, has led to calls on the NSW government to review its education policy and allow for more in-school suspensions.

Analysis of NSW Department of Education figures shows the number of long-term suspensions – lasting five to 20 days – has increased by 36 per cent in six years. Suspensions increased from 12,326 in 2006 to 16,814 last year.

A quarter of students were placed on long suspensions more that once last year.

Physical violence was the reason for 40 per cent of long suspensions and 47 per cent were for persistent misbehaviour.

Karen Bevan, from Uniting Care, who oversaw the research, said she found no evidence that suspension improved children's behaviour.

The research, to be presented next week at the Joint Australian Association for Research in Education Conference, found disadvantaged children in particular fell behind in their school work after taking as much as five weeks off school in one year after repeated suspensions.

"Suspension doesn't work because it doesn't teach you anything.' Ms Bevan said.

"Some students were suspended for more than 2½ weeks out of a 40-week school year. If yo get more than one long suspension, you are looking at five weeks a year. That's a long time to be off school."

Ms Bevan said the figures showed that Aboriginal children made up 23 per cent of those suspended for long periods.

Long suspensions

REASONS (%)	2006	2007	2008	2009	2010	2011
Physical violence	47	47	45	45	42	40
Persistent misbehaviour	39	40	42	42	44	47
Serious criminal behaviour related to the school	6	7	7	6	7	6
Possession of a suspected illegal substance	3	2	2	3	3	3
Use or possession of a prohibited weapon	3	3	3	3	3	3
Use of / threatening to use a weapon	2	2	1	1	1	1

NUMBER OF STUDENTS BY YEAR GROUPS	2006	2007	2008	2009	2010	2011
Kindergarden to year 6	–	–	2010	2043	1960	1750
Years 7 to 10	–	–	7888	8139	9247	9180
Years 11 to 12	–	–	616	688	1066	1124

"What does work is where you run the suspension in school," she said. "There is evidence internationally that is a more effective way of keeping kids connected with education."

A NSW Department of Education spokesman said the department operated 22 suspension centres attached to schools.

He said suspension centres aimed to provide skill development opportunities and support for students to help them re-engage at school. He added that schools were required to give students on long suspension a school work plan and the students were expected to complete that work.

Raising the school leaving age to 17 in 2010 has contributed to an increase in disruptive behaviour, according to a report by the NSW Auditor-General, Peter Achterstraat. This month Mr Achterstraat said that since the leaving age was increased, some year 11 students who may otherwise have left school were disruptive, wasting teachers' time or truanting.

Ms Bevan said her organization was concerned about the 27 per cent of students who were re-suspended on long suspensions.

"Our research tells us that group of students are of serious concern and likely to be the students who most benefit from engagement in education," she said.

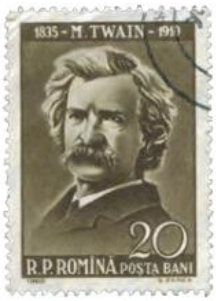
The NSW Department of Education spokesman said schools had a duty of care to protect students, teachers and other staff from violent students and those involved in criminal activity.

"Suspension is not intended as a punishment,' the spokesman said. "Suspension is only one strategy to manage inappropriate student behaviour within a school's student welfare and discipline policies."

Ms Bevan said the former Labor government toughened school suspension policies over the period their use increased.

"A real concern for us now is that the cuts to education announced by the O'Farrell government may reduce support services further and we could see an increase in suspensions again," she said.

9:05 Statistics: Can they be trusted?



'There are three kinds of lies: lies, damned lies, and statistics.' This statement was made popular by Mark Twain, who attributed it to the British Prime Minister Benjamin Disraeli. It points to the suspicion that statistics are used to strengthen weak arguments or even promote an untruth.

In his book *The Dictionary of Misinformation*, Tom Burnan states:
'No matter how much reverence is paid to anything purporting to be statistics, the term has no meaning unless the source, relevance, and truth are checked … The worship of statistics has had the particularly unfortunate result of making the job of the plain, outright liar that much easier.'

And a quote reportedly by Charles H Grosvenor:
'Figures don't lie, but liars do figure.'

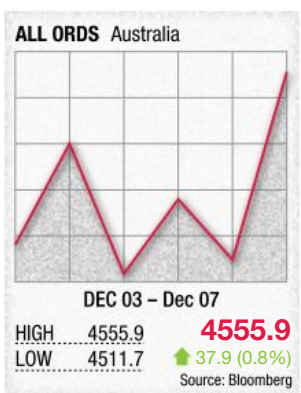
Suspicion is also caused by those who selectively choose statistics to support a particular argument when the same set of data can be used to support an alternative argument. Of course, the problem is not with the statistics but with the use that is made of them, how they were produced and what they truly represent. Problems occur when people misunderstand what the data is saying or wilfully misrepresent it.

Incorrect conclusions can be drawn by not looking carefully at how the data was collected and how it has been presented. In previous years some examples of how statistics and their graphs can be misused have been shown. This exercise will review some of those and encourage the questioning of conclusions presented in the media and other sources.

Exercise 9:05

1 Graphs are often used to grab our attention. This graph is showing the movement of the 'All Ordinaries' index in the business section of the newspaper.

- **a** What initial impression is given by this graph?
- **b** The horizontal axis is labelled Dec 03 – Dec 07. What is the 'scale' on this axis? To what do the points on the graph correspond?
- **c** What is measured by the vertical axis?
- **d** Looking at the high and low figures, what do you think the scale is on the vertical axis?
- **e** The difference of 37·9 (0·8%) is not the difference between the high and low figures. Subtract 37·9 from 4555·9. What point on the graph does this figure represent?



Source:
Sydney Morning Herald
8–9 Dec 2007

2 The diagram on the right indicates the recent gains and losses on the stock market. The black bars indicate gains, the red bars indicate losses.

a Do the lengths of the bars for Energy World and Imdex look the same?

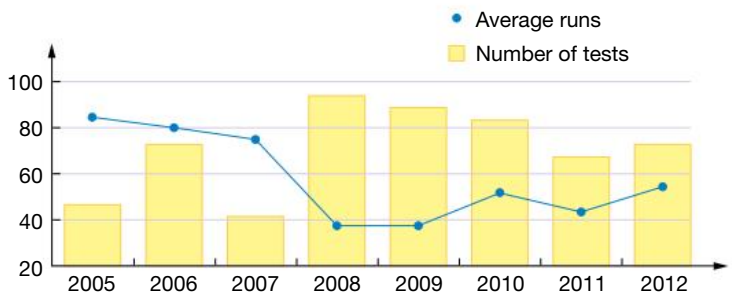
b The length of the bar for Intrepid Mines looks twice the length of the bar for BOQ Peps. Do the percentage figures support this?

c Are the lengths of the bars in the diagram proportional to the percentages shown?

d Why do you think the information has been presented in this way?

Source: *Daily Telegraph* 29 Nov 2012

3 The graph below represents the cricket career of Michael Hussey in test matches. The line graph connects the points showing his average number of runs for each year.



Source: Daily Telegraph Dec 2012

a What does the yellow column graph show?

b Can you say how many Test matches Michael Hussey played in 2005? What is missing from the graph?

c Michael Hussey played four tests in 2007 and 14 in 2008. If the columns have been drawn to scale, how many tests did he play in 2010?

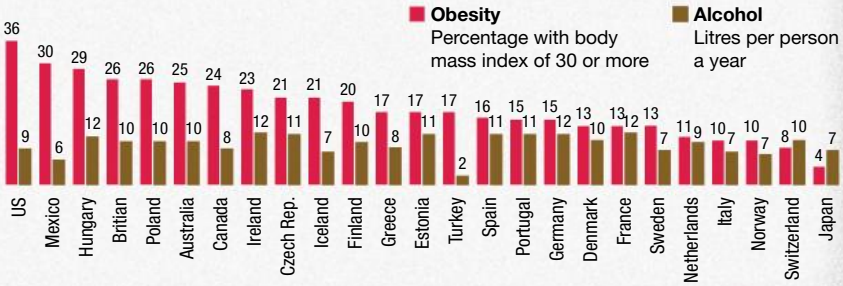
d In a cricket match there are two 'innings' but a batsman may not always bat twice. In 2007 Michael Hussey played in four tests but only batted six times. In 2009, he played in 13 Tests and batted 23 times. Is it easier to have a higher average if you bat less? Does this graph support your answer?

4 The article below is titled 'Health v wealth'.

Data mine

Health v wealth

More than a third of US citizens and a quarter of Australians are obese, says the Organisation for Economic Co-operation and Development, the industrialised countries' body. But the average Australian drinks a litre a year more than the average American.



Rounded to the nearest whole number

Sources: OECD. MCT

Source: *Sun-Herald* 16 Dec 2012

a How is the health of each nation being represented?

b How is the wealth of each nation being represented?

c What do the figures on the top of each column represent?

d The heights of the columns seem to be in proportion to these numbers. That is, the red column for US is four times the height of the brown column. Does this mean that if people in a country drank 1 litre of alcohol per year on average, this should equate to 1% of the population being obese?

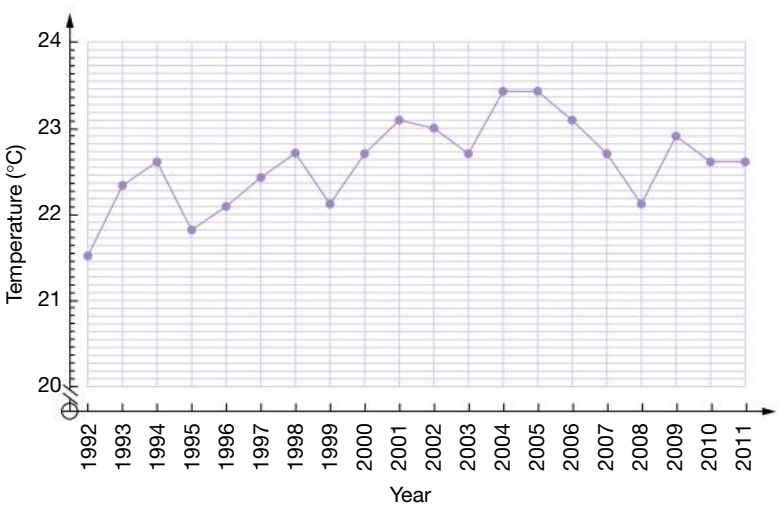
e Does this dual column graph suggest the two variables are related?

f Draw a scatter plot for these variables.

g Does any relationship exist?

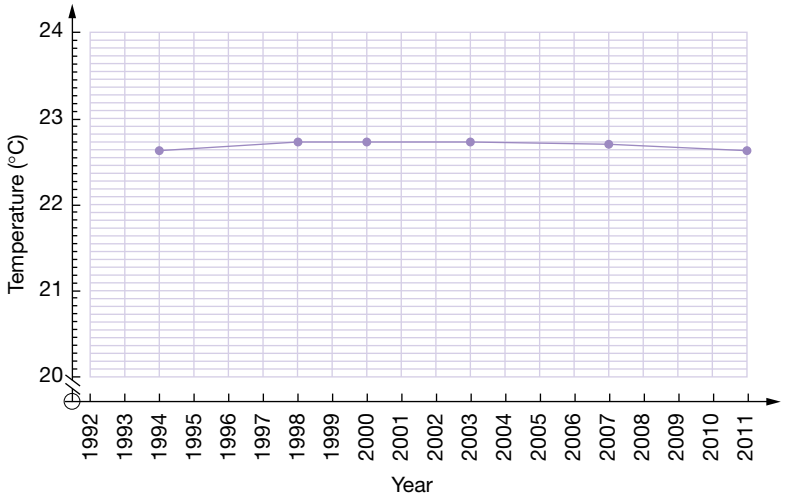
h Why is this not a fair comparison of the wealth and health of a nation?

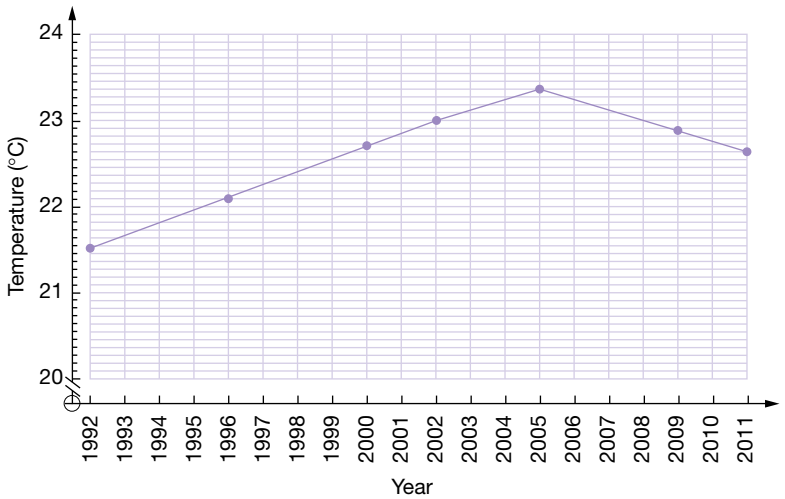
5 Sometimes an argument will be supported by the selective use of data. The graph and table below show the mean maximum temperature for Sydney for the years 1992–2011.



Year	Mean max. temp.
1992	21·5
1993	22·3
1994	22·6
1995	21·8
1996	22·1
1997	22·4
1998	22·7
1999	22·1
2000	22·7
2001	23·1
2002	23·0
2003	22·7
2004	23·4
2005	23·4
2006	23·1
2007	22·7
2008	22·1
2009	22·9
2010	22·6
2011	22·6

A general trend upwards might be detected but the line graph joining the points is quite 'saw-toothed', i.e. up and down, with a peak showing in the years 2004–2005. By selecting only some of the data, two very different trends can be shown, as in the graphs below.





The same data points have been used from the data, but only some of them, for each graph.

Further statistics can be found on the Bureau of Meteorology website. The site used is Observatory Hill in Sydney.

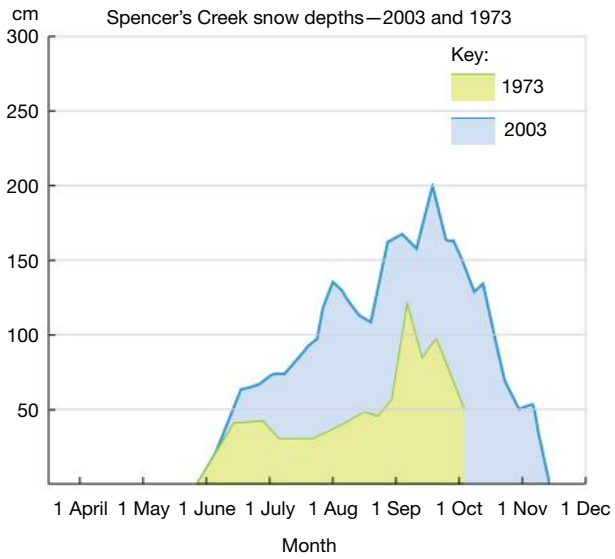
a What trend for the temperature of Sydney for the 20 years shown is suggested by:
  i graph A **ii** graph B?

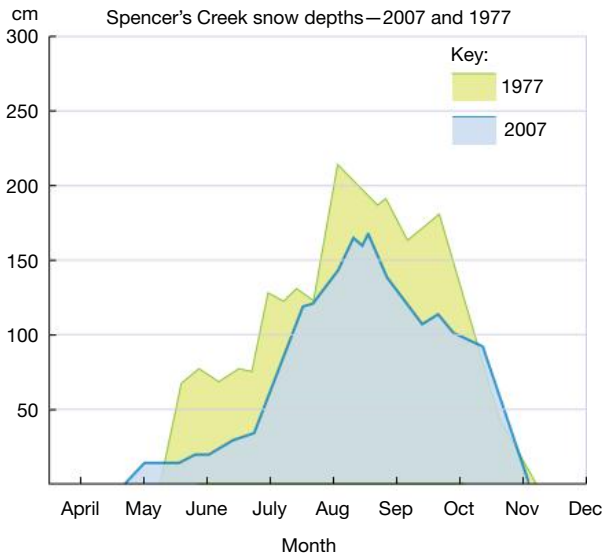
b Using interpolation, estimate a mean maximum temperature for 2001, using:
  i graph A **ii** graph B.

c Do either of the answers to **b** match the figure in the table?

d Extrapolate each graph to predict the temperature for years after 2011. How confident could you be of your prediction?

6 Another example of using selective or inadequate data can be seen in the graphs below. The original graph is shown on the left. The later graph is shown on the right.



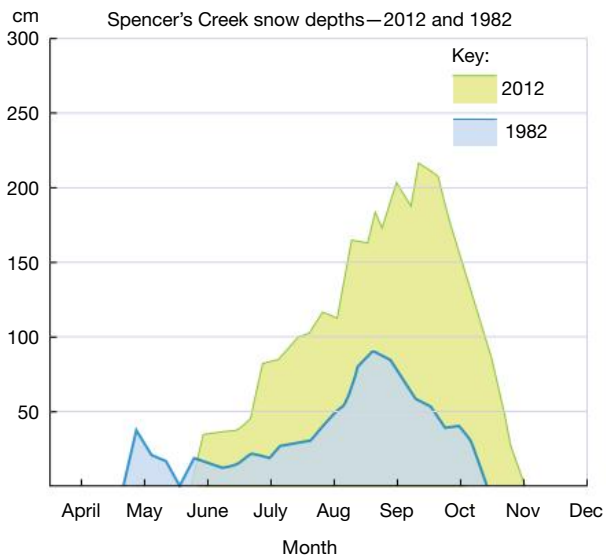


a The question for the original graph was: 'Looking at the snow depths for these 2 years it would seem that the depths have significantly increased over this 30-year period. Is this a reasonable conclusion?' What would have been your answer?

b The question for the revised graph was exactly the same, except the word 'increased' was changed to 'decreased'. How would this have changed your answer?

c The graph on the right shows the data again collected in a further 5 years' time. Using this 30 year comparison, has the snow depth increased or decreased over time?

d How could a more reliable view be obtained to determine if there is any pattern for the snow depth over time?



7 This first table shows the annual carbon dioxide (CO_2) emissions, in millions of tonnes, for the top five countries plus Australia for 2009.

Rank	15	5	4	3	2	1
Country	Australia	Japan	Russia	India	USA	China
CO_2 emissions	418	1098	1572	1602	5425	7711
% of world emissions	1·4	3·6	5·2	5·3	17·8	25·4

The second table shows CO_2 emissions, in tonnes, per capita (per person) for these countries in order.

Country	India	China	Japan	Russia	USA	Australia
CO_2 emissions per capita	1·38	5·83	8·64	11·23	17·67	19·64

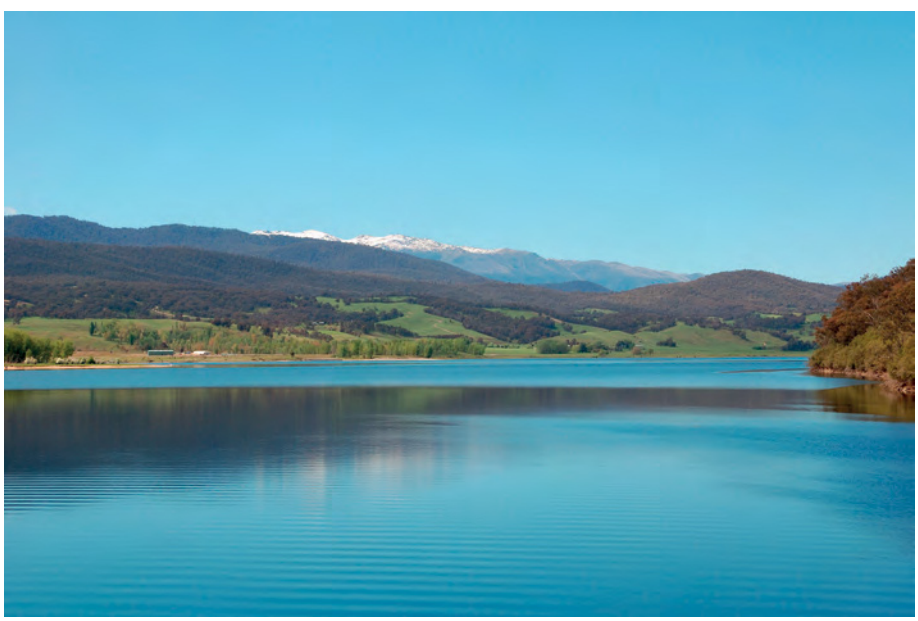
a If you wanted to argue that Australia needs to reduce its CO_2 emissions, which table of data are you more likely to present to support your case?

b If you wanted to argue that any reduction in Australia's CO_2 emissions will have little global effect, which table of data would you most likely use?

c The second table takes into account each country's population size. Consider the countries in the list. What other factors might be relevant to your argument apart from population size?

d The table below lists the ranking of each country in the world according to its per capita emissions. How might this table influence your argument?

Country	India	China	Japan	Russia	USA	Australia
CO_2 emissions per capita rank	143	75	53	32	17	13



A snow depths calculator can be found at the Snowy Hydro website.

INVESTIGATION 9:05 OPINION POLLS: HOW ACCURATE ARE THEY?

We are bombarded by the media with opinion polls and surveys of all kinds, but few people recall being asked for their opinion. Political polls are of concern to politicians to see if their policies and performance are favourable. Market polls are important to business so they can see where their advertising needs to be focused or their products improved. If only a sample of a population is surveyed, how can its accuracy be ensured? The following example is cited by Data Analysis Australia, a group of strategic information consultants.

'Modern political polling began in 1936, with two polls attempting to predict the outcome of the American presidential election. The Literary Digest conducted its poll by sending out 10 million post cards asking people how they would vote. They received almost 2·3 million back and said that Alfred Landon was leading Franklin Roosevelt by 57–43 per cent. In contrast, market researcher George Gallup employed a much smaller sample of only 5,000, but because he ensured that it was representative of the American voting public, he predicted Roosevelt to win by a landslide. In the event, Roosevelt won 60% and Landon just 37%. The Literary Digest lost credibility and subsequently merged with Time magazine in 1938.'

The *Literary Digest* thought it would be accurate because of its large sample size but it was not representative. It chose its sample from its owner reader lists, telephone subscribers and car owners. Today these lists might be more representative but in 1936 these were indicators of wealth and so many who were poor due to the Depression were not included.

For a sample to be a reliable representation of the population every person must have a chance of being surveyed.

- It must be random.
- It must match the make-up of the population in gender, age groups and location.
- It must consider other factors when the survey is completed such as the time of day, the day of the week, the time of the year or even the weather which, depending on the subject of the survey, may have an effect.

An example of a market survey is shown in this table. It lists the top ten TV programs watched in Sydney during July 2012, according to the Roy Morgan research group. Only a sample of households was surveyed, but those asked would have been carefully selected to represent the viewing population.

The percentage shown is the percentage of those watching TV at that time.

Top 10 TV results for Sydney: July 2012

Rank	Channel	Program	%
1	9	Rugby League: State of Origin (Wed)	19·5
2	9	NRL (Fri)	12·4
3	7	Downton Abbey (Sun)	12·0
4	7	Seven News (Mon–Sun)	11·1
5	9	Nine News (Mon–Sun)	10·1
6	7	Revenge (Mon)	9·5
7	7	Sunday Night (Sun)	8·7
8	7	Home and Away (Mon–Fri)	8·7
9	7	Today Tonight (Mon–Fri)	8·1
10	2	ABC News (Mon–Sun)	8·0

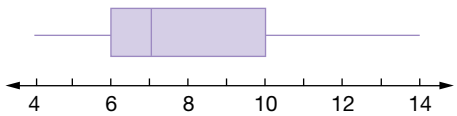
Suggested research

- Use the internet to research polling organisations such as market research companies and newspapers.
- Gain further insights into polling methods from sites such as Data Analysis Australia.
- Investigate the accuracy of previous political polls in predicting the result of an election.

MATHS TERMS 9

bivariate data
- data that has two variables

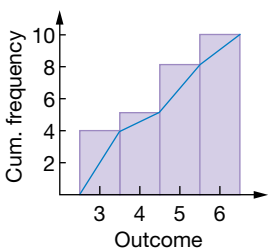
box plot (box-and-whisker plot)
- a diagram obtained from the five-number summary
- the box shows the middle 50% of scores (the interquartile range)
- the whiskers show us the extent of the bottom and top quartiles as well as the range



census
- a survey of a whole population

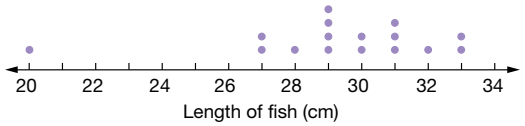
cumulative frequency
- the number of scores less than or equal to a particular outcome,
  e.g. for the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6
  the cumulative frequency of 5 is 8
  (there are 8 scores of 5 or less)

cumulative frequency histogram (and polygon)
- these show the outcomes and their cumulative frequencies



data
- the pieces of information (or 'scores') to be examined
- *categorical*: data that uses non-numerical categories
  - *ordered* data involves a ranking, e.g. exam grades, garment sizes
  - *distinct* data has no order, e.g. colours, types of cars
- *numerical:* data that uses numbers to show 'how much'
  - *continuous* data can have any numerical value within a range, e.g. height
  - *discrete* data is restricted to certain numerical values, e.g. number of pets

dot plot
- a type of graph that uses one axis and a number of dots above the axis



extrapolation
- predicting data beyond the range of values given

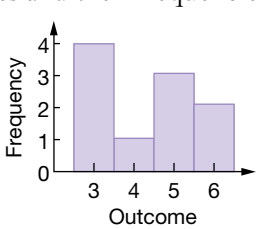
five-number summary
- a set of numbers consisting of the minimum score, the three quartiles and the maximum score

frequency
- the number of times an outcome occurs in the data,
  e.g. for the data 3, 6, 5, 3, 5, 5, 4, 3, 3, 6
  the outcome 5 has a frequency of 3

frequency distribution table
- a table that shows all the possible outcomes and their frequencies (it usually is extended by adding other columns such as the cumulative frequency),
  e.g.

Outcome	Frequency	Cumulative frequency
3	4	4
4	1	5
5	3	8
6	2	10

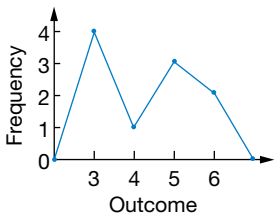
frequency histogram
- a type of column graph showing the outcomes and their frequencies



frequency polygon

- a type of line graph showing outcomes and their frequencies
- to complete the polygon, the outcomes immediately above and below the actual outcomes are used (the height of these columns is zero),

e.g.



grouped data

- data that is organised into groups or classes
- *class intervals:* the size of the groups into which the data is organised
  e.g. 1–5 (5 scores); 11–20 (10 scores)
- *class centre:* the middle outcome of a class
  e.g. the class 1–5 has a class centre of 3

interpolation

- estimating data that lie within the domain of the values given

interquartile range

- the range of the middle 50% of scores
- the difference between the median of the upper half of scores and the median of the lower half of scores

$\text{IQR} = Q_3 - Q_1$

mean

- the number obtained by 'evening out' all the scores until they are equal,
  e.g. if the scores 3, 6, 5, 3, 5, 5, 4, 3, 3, 6 were 'evened out', the number obtained would be 4·3
- to obtain the mean, use the formula:

$$\text{mean} = \frac{\text{sum of the scores}}{\text{total number of scores}}$$

median

- the middle score for an odd number of scores or the mean of the middle two scores for an even number of scores
- *median class*
  in grouped data, the class that contains the median

mode (modal class)

- the outcome or class that contains the most scores

ogive

- this is another name for the cumulative frequency polygon

outcome

- a possible value of the data

outlier

- a score that is separated from the main body of scores

quartiles

- the points that divide the scores up into quarters
- the second quartile, Q_2, divides the scores into halves (Q_2 = median)
- the first quartile, Q_1, is the median of the lower half of scores
- the third quartile, Q_3, is the median of the upper half of scores

4 5 6 | 6 7 7 | 7 9 9 | 11 12 15

$Q_1 = 6 \quad Q_2 = 7 \quad Q_3 = 10$

range

- the difference between the highest and lowest scores

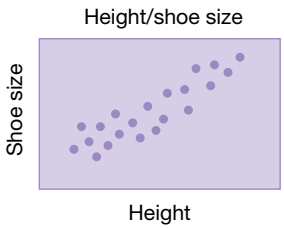
sample

- a part, usually a small part, of a large population
- *random sample:* a sample taken so that each member of the population has the same chance of being included
- *systematic sample:* a sample selected according to some ordering scheme, e.g. every tenth member
- *stratified sample:* a sample is proportionally taken from each subgroup in a population

scatter plot

- a graph that uses points on a number plane to show the relationship between two categories

e.g.



shape (of a distribution)

- a set of scores can be symmetrical or skewed

sources of data

- *primary:* the data has been collected by yourself
- *secondary:* the data has come from an external source, e.g. newspapers, internet

statistics

- the collection, organisation and interpretation of numerical data

stem-and-leaf plot

- a graph that shows the spread of scores without losing the identity of the data,
  e.g. Test results, 8M
  (6/8 represents 68)

Stem	Leaf
4	1 8
5	9 3 7 0 4
6	8 5 6 7 2 4 8 6
7	3 7 3 1 9 3 5
8	5 4 8 0 4
9	3 7 1

- *ordered stem-and-leaf plot:*
  the *leaves* are placed in order,
  e.g.

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

- *back-to-back stem-and-leaf plot:*
  this can be used to campare two sets of scores, one set on each side,
  e.g. Test results
  (4/1 represents 41)

	Stem	Leaf	Stem
9 8 6 6 0		3	
9 7 7 3 1 1 1		4	3 6 6 8
8 8 5 3 3 0		5	1 7 9 9
9 8 7 5 3		6	3 8 9
		7	0 5 5 5 8 9
	0	8	2 6 7 7
		9	0 0 1 3

variable

- something that can be observed, measured or counted to provide data

DIAGNOSTIC TEST 9 STATISTICS

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

9:01

1 For each data set find the:

i range ii mode iii mean iv median
v first quartile Q_1 vi third quartile Q_3 vii interquartile range

a 2 3 3 7 8 9 10

b 6 4 3 9 5 9 4 8

9:01

2 a Complete this frequency distribution table.

b Find the:

i range
ii mode
iii mean
iv median
v first quartile Q_1
vi third quartile Q_3

Outcome	Tally	f	$f \times x$	$c.f.$
6	𝍸 ||			
7	𝍸 𝍸 |			
8	𝍸 |||			
9	𝍸			
10	||||			
11	|||			
12	||			

Total: ___ ___

9:01

3 a For this table of grouped continuous data find:

i the modal class
ii the median class
iii an estimate for the mean.

b Construct a cumulative frequency histogram and polygon and show on it the position of the median and quartiles.

c Find an estimate for the interquartile range.

Class	$c.c.$	f	$f \times c.c.$	$c.f.$
0–<10	5	4	20	4
10–<20	15	6	90	10
20–<30	25	9	225	19
30–<40	35	15	525	34
40–<50	45	10	450	44
50–<60	55	6	330	50
Total:		50	1640	

9:01

4 a For this stem-and-leaf plot find the:

i range
ii mode
iii median
iv lower (first) quartile Q_1
v upper (third) quartile Q_3
vi interquartile range

b Write the five-number summary for this set of data.

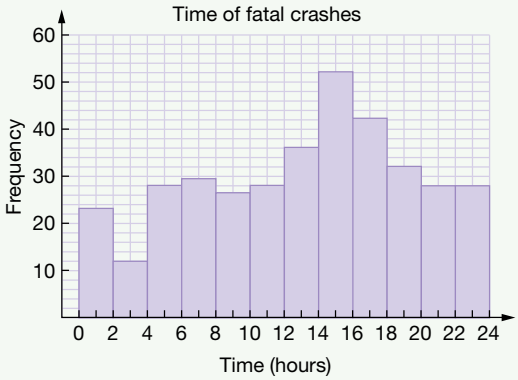
c Using the five-number summary, construct a box plot.

Stem	Leaf
4	3 7
5	1 4 8
6	2 5 5 7 9
7	1 3 6 6 6 8
8	4 5 7 7
9	0 2 5

9:02

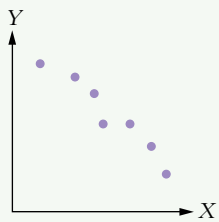
5 This histogram shows the number of fatal crashes that occurred during a year according to the time of day, organised in 2-hour periods. This is an example of bivariate data.

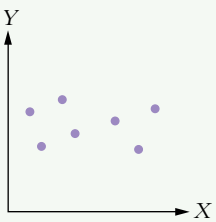
a What is the independent variable?
b What is the dependent variable?
c Which two-hour period registered the highest number of fatalities?
d Which two-hour period registered the lowest number of fatalities?
e Does this data show the time when it is safest to drive? What other factors need to be considered?

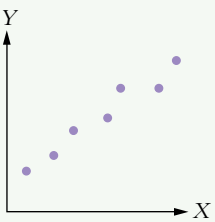


9:03

6 Which of these scatter plots show a linear relationship between the variables X and Y. State whether the linear relationship is positive or negative.

a 

b 

c 

9:03

7 Plot each set of data on a scatter plot and determine what type of relationship exists between the two variables.

a

A	1	2	4	6	7	9
B	8	10	18	23	26	31

b

T	2	3	5	6	7	9
N	3	1	6	4	2	5

c

P	2	3	4	5	7	9
Q	23	20	18	14	10	3

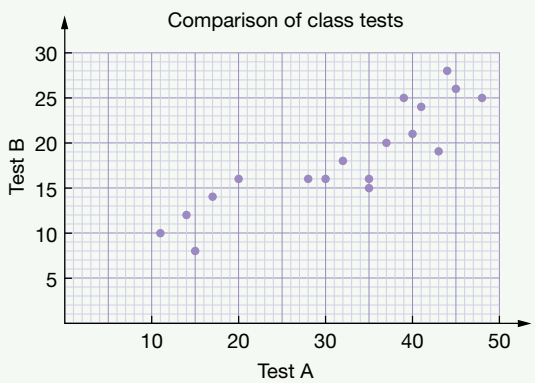
d

X	1	3	4	6	8	9
Y	5	6	6	5	6	5

9:03

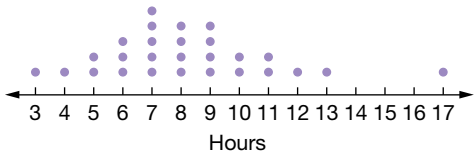
8 This scatter plot shows the results for two class tests, one out of 50 marks and the other out of 30 marks. There are 19 students in the class.

a What did the student who scored 32 for Test A receive as a result for Test B?
b What did the student who scored 24 for Test B receive as a result for Test A?
c For Test A determine:
  i the range ii the median
  iii Q_1 iv Q_3
d Write the five-number summary for Test A.
e For Test B determine:
  i the range ii the median
  iii Q_1 iv Q_3
f Write the five-number summary for Test B.
g Construct a box plot for each test.
h Which test has results that are:
  i negatively skewed ii positively skewed?



ASSIGNMENT 9A Chapter review

1 This dot plot shows the number of whole hours spent doing homework by a Year 10 class in a particular week.



a How many students are in the class?

b What is the most common number of hours spent (the mode)?

c Calculate the mean number of hours spent.

d Find the median and quartiles for this data and hence write its five-number summary.

e Draw a box plot for this data.

f Looking at the dot plot and box plot, is this data symmetrical or skewed?

2 **a** Use the data shown in the stem-and-leaf plot to complete the frequency distribution table using the classes shown.

Stem	Leaf
1	6 9
2	2 4 7 7 9
3	0 4 5 5 5 9 9
4	1 3 6 8
5	2 4 6 6 7 8 8 9
6	3 3 5 6 8 8
7	0 1 3 6 9
8	0 2 5 7
9	1 4

Class	*c.c.*	*f*	$f \times c.c.$	*c.f.*
10–19	14·5			
20–29	24·5			
30–39				
40–49				
50–59				
60–69				
70–79				
80–89				
90–99				
Total:		___	___	

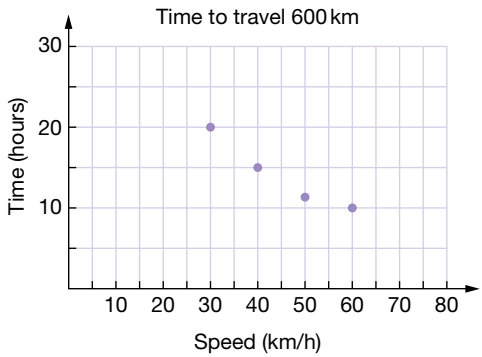
b Use the discrete data in the stem-and-leaf plot to calculate the mean.

c Calculate a value for the mean using the totals of the frequency and $f \times c.c.$ columns from the table. Compare it to the answer for part **b**.

d Find the mode from the stem-and-leaf plot. Does it fall in the modal class from the table?

e Find the median from the stem-and-leaf plot. Does this fall in the median class from the table?

3 This scatter plot shows the time calculated by Liam to drive a distance of 600 km at average speeds of 30, 40, 50 and 60 km/h. He plotted these on a scatter plot and concluded that a negative linear relationship existed between these two variables.



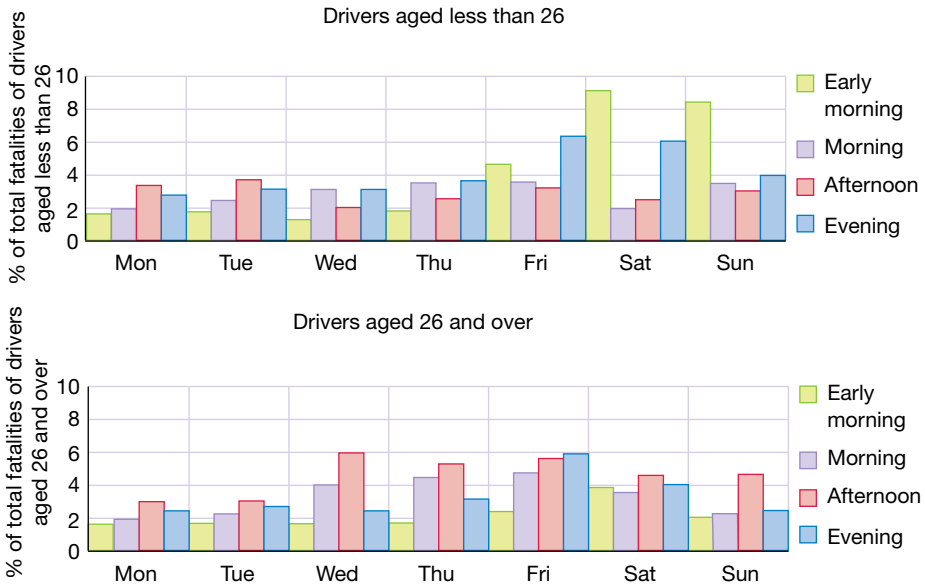
a How long would Liam take for the trip at 40 km/h?

b To complete the trip in 10 hours, what average speed is required?

c Increasing the average speed from 30 km/h to 60 km/h reduces the time taken by 10 hours. If the relationship is linear then If Liam could increase the average speed to 90 km/h, he could save another 10 hours and take no time at all. What is wrong with this reasoning?

d The relationship is negative but is it linear?

4 The graphs below show the fatalities by time of day and day of week of crashes for major road user categories, Australia, 2005–2007.



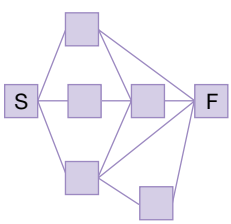
a What do the graphs above suggest is the most dangerous time of day for:
i drivers aged less than 26 **ii** drivers aged 26 and over?

b Suggest a reason why the times in **a** are the most dangerous.

ASSIGNMENT 9B Working mathematically

1 You have forgotten the last three digits of your PIN. You do remember, however, that the digits are different and they add up to 6. How many possible endings are there to your PIN?

2 Find the number of routes through the network (S to F), assuming that travel must always be from left to right.

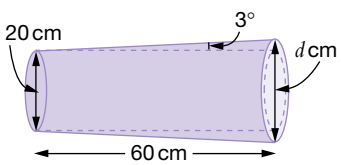


3 Some numbers are the sum of two squared whole numbers. For example, $13 = 2^2 + 3^2$ and $37 = 1^2 + 6^2$.

a How many numbers from 100 to 120 are the sum of two squared whole numbers?

b The number 685 can be written as the sum of two squared whole numbers in two ways. What are they?

4 A tapered shaft has a length of 60 cm. The diameter at the smaller end is 20 cm. If the taper is 3° (as shown), find the diameter, d cm, of the larger end of the shaft.



5 Max's best time for 50 m freestyle is 40 seconds. Joshua's best time is 25 seconds. If they both swam in the same 50 m freestyle race, how far behind Joshua would Max be when Joshua finished? (Assume that they both swim at constant rates.)

ASSIGNMENT 9C Cumulative revision

1 For the two points $A(3, 5)$ and $B(9, -3)$, find: 1:08

a the length of the interval AB **b** the midpoint of AB

c the gradient of line AB.

2 Using the formula $S = ut + \frac{1}{2}at^2$, find the value of: 1:10

a S when $u = 9$, $a = 6$ and $t = 5$ **b** S when $u = 3{\cdot}6$, $a = 9{\cdot}8$ and $t = 2{\cdot}5$

c u when $S = 180$, $a = 20$ and $t = 4$ **d** a when $S = 123$, $u = 10$ and $t = 6$

3 Solve the equations: 2:05

a $x^2 - 100 = 0$ **b** $(x - 5)(x + 3) = 0$ **c** $x^2 - x - 56 = 0$

4 Two six-sided dice have on their faces the letters A, B, C, D, E and F. When the dice are rolled what is the probability that the two uppermost faces will: 4:04

a both show an A **b** show the same letter

c show different letters? (Hint: A tree diagram might help.)

5 Five bottles of drink and seven ice-creams cost \$35 and seven bottles of drink and five ice-creams cost \$37. Write equations to represent this and solve them simultaneously to find the price of each bottle of drink and each ice-cream. 5:03

6 A sum of \$12 000 is invested at 5·4% p.a. interest for 10 years. 6:06, 6:02

a Find the amount of interest earned if this is simple interest.

b Find the interest earned if it is compound interest and the interest is calculated:

i annually **ii** monthly.

7 Calculate the value of a company's equipment after 5 years if it was bought for \$370 000 and it depreciates at the rate of 12·5% p.a. 6:06

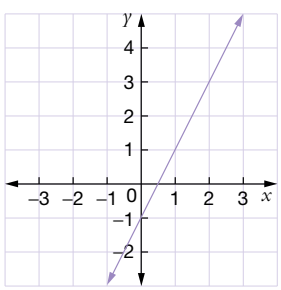
8 For the line $y = 4x - 5$, find the line that is: 7:02, 7:03

a parallel to it and passes through the point $(1, 3)$

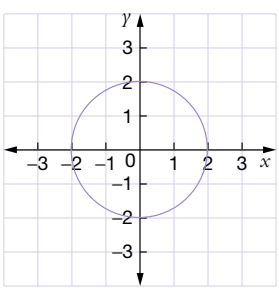
b perpendicular to it and passes through the point $(4, 3)$.

9 Write the equations of these graphs. 7:07

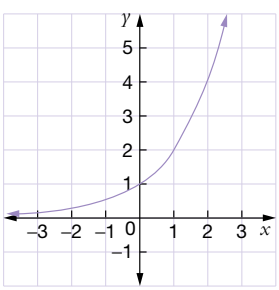
a



b

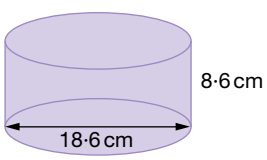


c



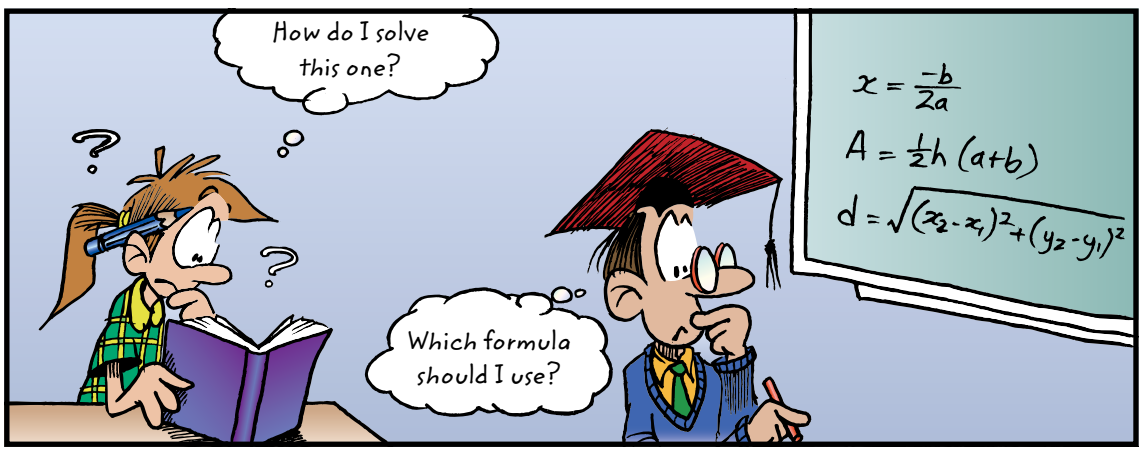
10 **a** Find the surface area of this closed cylinder to the nearest square centimetre. 8:01, 8:03

b Find the volume of this cylinder to the nearest cubic centimetre.



FURTHER ALGEBRA

10



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10:02 Further binomial products
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10:09 Simplifying algebraic fractions: Multiplication and division
10:10 Addition and subtraction of algebraic fractions
Fun spot 10:10 What did the caterpillar say when it saw the butterfly?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** [Stage 5.3§]

- Add and subtract algebraic fractions with numerical denominators, including those with binomial numerators (NSW)
- Expand binomial products using a variety of strategies (ACMNA233)
- Factorise monic and non-monic quadratic expressions (ACMNA269)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

10:01 Further algebraic fractions

PREP QUIZ 10:01

Expand:

1 $3(x + 5)$ **2** $5(m - 7)$ **3** $2(6y + 1)$ **4** $9(2p - 3q)$

Find the lowest common multiple (LCM) of:

5 4 and 5 **6** 4 and 6 **7** 4 and 8

Simplify:

8 $\frac{x}{4} + \frac{x}{5}$ **9** $\frac{a}{4} - \frac{a}{6}$ **10** $\frac{m}{8} + \frac{n}{4}$

In Chapter 2 all of the algebraic fractions had a single term in the numerator. If there is more than one term in the numerator we use the skills for expanding grouping symbols. Examine carefully the following worked examples.

WORKED EXAMPLES

Simplify the following.

1 $\frac{x+1}{2} + \frac{x+4}{3} = \frac{(x+1)\times 3}{2\times 3} + \frac{(x+4)\times 2}{3\times 2}$

$= \frac{3(x+1)}{6} + \frac{2(x+4)}{6}$

$= \frac{3x+3}{6} + \frac{2x+8}{6}$

$= \frac{5x+11}{6}$

2 $\frac{2a+5}{3} - \frac{3a}{5} = \frac{(2a+5)\times 5}{3\times 5} - \frac{3a\times 3}{5\times 3}$

$= \frac{5(2a+5)}{15} - \frac{9a}{15}$

$= \frac{10a+25}{15} - \frac{9a}{15}$

$= \frac{a+25}{15}$

LCM of 3 and 5 is 15

3 $\frac{2x+5}{6} + \frac{x-4}{3} = \frac{2x+5}{6} + \frac{2(x-4)}{6}$

$= \frac{2x+5}{6} + \frac{2x-8}{6}$

$= \frac{4x-3}{6}$

4 $\frac{m}{3} - \frac{m+1}{4} = \frac{4m}{12} - \frac{3(m+1)}{12}$

$= \frac{4m-3(m+1)}{12}$

$= \frac{4m-3m-3}{12}$

$= \frac{m-3}{12}$

Note the minus sign!

Exercise 10:01

1 Simplify the following:

a $\frac{x+4}{2}+\frac{x}{3}$ b $\frac{a}{5}+\frac{a+3}{2}$ c $\frac{n-1}{3}+\frac{2n}{7}$

d $\frac{x+4}{5}+\frac{x+1}{3}$ e $\frac{m+4}{5}+\frac{m-3}{7}$ f $\frac{p+1}{3}-\frac{p}{4}$

2 Simplify:

a $\frac{a+4}{4}+\frac{a}{2}$ b $\frac{w}{5}+\frac{w+2}{10}$ c $\frac{d-1}{3}+\frac{5d}{6}$

d $\frac{x+3}{4}+\frac{x+2}{6}$ e $\frac{m+2}{8}+\frac{m-1}{6}$ f $\frac{q+1}{6}-\frac{q}{9}$

3 Simplify these expressions:

a $\frac{2x+3}{2}+\frac{x+4}{3}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}$ c $\frac{5n-1}{3}+\frac{4n+3}{4}$

d $\frac{4t-4}{5}+\frac{3t-1}{3}$ e $\frac{7u-2}{8}+\frac{2u-3}{3}$ f $\frac{5q+2}{10}+\frac{2q-1}{9}$

g $\frac{2a+3}{6}+\frac{a+1}{3}$ h $\frac{3n+5}{8}+\frac{n-1}{4}$ i $\frac{5g-1}{3}+\frac{4g+3}{9}$

j $\frac{4y-1}{6}+\frac{3y-1}{8}$ k $\frac{5-2x}{6}+\frac{2x-3}{10}$ l $\frac{1-2a}{8}+\frac{2-a}{10}$

4 Simplify these subtractions:

a $\frac{x}{2}-\frac{x+1}{3}$ b $\frac{3y}{5}-\frac{y+3}{2}$ c $\frac{n+1}{3}-\frac{n+2}{5}$

d $\frac{x+4}{5}-\frac{x-1}{6}$ e $\frac{m-1}{5}-\frac{m-3}{7}$ f $\frac{2p-1}{3}-\frac{p-4}{4}$

g $\frac{2q+3}{6}-\frac{q+1}{3}$ h $\frac{3t+2}{8}-\frac{t-1}{4}$ i $\frac{5w-2}{3}-\frac{4w-6}{9}$

j $\frac{4+y}{6}-\frac{2y-1}{8}$ k $\frac{1-2x}{6}-\frac{2-3x}{10}$ l $\frac{3-2z}{8}-\frac{2-3z}{10}$

5 Simplify these expressions:

a $\frac{x+3}{2}+\frac{x+4}{3}+\frac{x+1}{5}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}+\frac{a-2}{3}$ c $\frac{2m+3}{3}+\frac{m+4}{6}+\frac{3m+1}{2}$

d $\frac{3y+1}{5}+\frac{2y-1}{4}-\frac{y-3}{2}$ e $\frac{5x-1}{4}-\frac{x+2}{3}+\frac{2x+1}{6}$ f $\frac{3z+4}{5}-\frac{2z-1}{4}-\frac{4-z}{10}$

CHALLENGE 10:01 THESE ARE A FRACTION TRICKIER!

Consider how you simplified expressions in Chapter 2 that involved pronumerals in the denominator, and then attempt to simplify the following expressions.

1 $\frac{x+4}{2x}+\frac{x+1}{3}$ 2 $\frac{x+4}{2x}+\frac{x+1}{3x}$ 3 $\frac{a+5}{5x}+\frac{a-2}{2x}$ 4 $\frac{x+5}{5x}+\frac{a+1}{2a}$

5 $\frac{x-4}{3x}-\frac{x-6}{5x}$ 6 $\frac{3n+4}{4n}-\frac{5-n}{6n}$ 7 $\frac{a+b}{a}+\frac{a-b}{b}$ 8 $\frac{3x-5y}{2x}-\frac{2y+x}{3y}$

10:02 Further binomial products

In Chapter 2 the expansion of binomial products was demonstrated as in these two examples:

$$(x+3)(x+7) = x^2 + 7x + 3x + 21 = x^2 + 10x + 21$$

$$(3a-5)(4a+1) = 12a^2 + 3a - 20a - 5 = 12a^2 - 17a - 5$$

This section will now focus on two special binomial product types.

10:02A Perfect squares

PREP QUIZ 10:02A

Simplify:

1 4^2 **2** 7^2 **3** $(-2)^2$ **4** $(-10)^2$ **5** $(3x)^2$

Complete:

6 $(x+2)(x+7) = x^2 + 9x + \ldots$

7 $(a+3)(a+3) = a^2 + 6a + \ldots$

8 $(2m-1)(2m-1) = \ldots\ m^2 - 4m + 1$

9 $(n+5)(n+5) = n^2 + \ldots\ n + 25$

10 $(x-3)(x-3) = x^2 - \ldots\ x + 9$

When a binomial is multiplied by itself, we call this product a perfect square. If a perfect square is expanded, we get:

$$\begin{aligned}(x+y)^2 &= (x+y)(x+y)\\ &= x(x+y) + y(x+y)\\ &= x^2 + xy + yx + y^2\\ &= x^2 + 2xy + y^2\end{aligned}$$

Similarly $(x-y)^2 = x^2 - 2xy + y^2$

We could say: 'The square of a binomial is equal to the square of the first term, plus twice the product of the two terms, plus the square of the second term'.

WORKED EXAMPLES

1 $(a+3)^2 = a^2 + 2(3a) + 3^2 = a^2 + 6a + 9$

first term squared; product of the two terms; second term squared

2 $(m-5)^2 = m^2 - 2[5m] + 5^2 = m^2 - 10m + 25$

first term squared; twice the product of the two terms; second term squared

3 $(3y-7)^2 = [3y]^2 - 2[21y] + [-7]^2 = 9y^2 - 42y + 49$

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$

Exercise 10:02A

1 Find the missing term in each example to make the statements true.

a $(x+2)^2 = x^2 + 4x + \ldots$
b $(a+6)^2 = a^2 + 12a + \ldots$
c $(y-3)^2 = y^2 - 6y + \ldots$
d $(m-10)^2 = m^2 - 20m + \ldots$
e $(x+1)^2 = x^2 + \ldots + 1$
f $(y+7)^2 = y^2 + \ldots + 49$
g $(n-2)^2 = n^2 - \ldots + 4$
h $(p-5)^2 = p^2 - \ldots + 25$
i $(q+8)^2 = \ldots + 16q + 64$
j $(x-4)^2 = \ldots - 8x + 16$
k $(x+\ldots)^2 = x^2 + 6x + 9$
l $(a+\ldots)^2 = a^2 + 18a + 81$
m $(y-\ldots)^2 = y^2 - 14x + 49$
n $(m-\ldots)^2 = m^2 - 22m + 121$
o $(2x+3)^2 = \ldots + 12x + 9$
p $(5n+1)^2 = \ldots + 10n + 1$
q $(3m+7)^2 = 9m^2 + \ldots + 49$
r $(4x+5)^2 = 16x^2 + \ldots + 25$
s $(2a-1)^2 = 4a^2 - \ldots + 1$
t $(9y-7)^2 = 81y^2 - \ldots + 49$

2 Expand these perfect squares and simplify.

a $(x+3)^2$
b $(x+5)^2$
c $(x+1)^2$
d $(x-6)^2$
e $(m-1)^2$
f $(n-5)^2$
g $(x+2)^2$
h $(n-8)^2$
i $(m+11)^2$
j $(a+12)^2$
k $(x+10)^2$
l $(p-9)^2$
m $(x+y)^2$
n $(a+m)^2$
o $(x+t)^2$
p $(a-b)^2$
q $(k-m)^2$
r $(p-q)^2$

3 Expand and simplify:

a $(2x+3)^2$
b $(2x+1)^2$
c $(3x+5)^2$
d $(4a+1)^2$
e $(3a+7)^2$
f $(7t+2)^2$
g $(2x-1)^2$
h $(3a-2)^2$
i $(5m-4)^2$
j $(4t-7)^2$
k $(6q-1)^2$
l $(9n+4)^2$
m $(2x+y)^2$
n $(a+3b)^2$
o $(3t-2x)^2$

10:02B Difference of two squares

PREP QUIZ 10:02B

Evaluate:

1 $7^2 - 3^2$
2 $(7+3)(7-3)$
3 $4^2 - 2^2$
4 $(4+2)(4-2)$
5 $5^2 - 1^2$
6 $(5-1)(5+1)$
7 $6^2 - 3^2$
8 $(6-3)(6+3)$
9 $10^2 - 9^2$
10 $(10+9)(10-9)$

If the sum of two terms is multiplied by their difference, another special type of product is formed. If $(x+y)$ is multiplied by $(x-y)$ we get:

$$\begin{aligned}(x+y)(x-y) &= x(x-y) + y(x-y)\\ &= x^2 - xy + yx - y^2\\ &= x^2 - y^2\end{aligned}$$

We could say: 'The sum of two terms multiplied by their difference is equal to the square of the first term minus the square of the second term'.

WORKED EXAMPLES

1 $(x+3)(x-3) = x^2 - 3^2$

first term squared, second term squared

$= x^2 - 9$

2 $(2a-3b)(2a+3b) = (2a)^2 - (3b)^2$

first term squared, second term squared

$= 4a^2 - 9b^2$

3 $(p-7)(p+7) = p^2 - 7^2$

$= p^2 - 49$

4 $(5x+y)(5x-y) = (5x)^2 - y^2$

$= 25x^2 - y^2$

$(x+y)(x-y) = x^2 - y^2$

Exercise 10:02B

1 Expand these products and simplify.

a $(x+4)(x-4)$	**b** $(a+1)(a-1)$	**c** $(m+2)(m-2)$	**d** $(n+7)(n-7)$
e $(p-5)(p+5)$	**f** $(q-6)(q+6)$	**g** $(x-3)(x+3)$	**h** $(y-9)(y+9)$
i $(10+x)(10-x)$	**j** $(5+a)(5-a)$	**k** $(8-x)(8+x)$	**l** $(11-m)(11+m)$
m $(x+t)(x-t)$	**n** $(a-b)(a+b)$	**o** $(m+n)(m-n)$	**p** $(p-q)(p+q)$

2 Express as the difference of two squares.

a $(2a+1)(2a-1)$	**b** $(3x+2)(3x-2)$	**c** $(5m+3)(5m-3)$	**d** $(9q+2)(9q-2)$
e $(4t-3)(4t+3)$	**f** $(7x-1)(7x+1)$	**g** $(8n-5)(8n+5)$	**h** $(10x-3)(10x+3)$
i $(2x+y)(2x-y)$	**j** $(4a+3b)(4a-3b)$	**k** $(5p+2q)(5p-2q)$	**l** $(3m-n)(3m+n)$
m $(2m-5n)(2m+5n)$	**n** $(2p-3q)(2p+3q)$	**o** $(x-5y)(x+5y)$	**p** $(12x-5y)(12x+5y)$

INVESTIGATION 10:02B USING THE DIFFERENCE OF TWO SQUARES

Example 1

Using $(a-b)(a+b) = a^2 - b^2$, evaluate $100^2 - 97^2$.

Solution

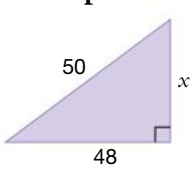
$100^2 - 97^2 = (100-97)(100+97)$

$= 3 \times 197$

$= 591$

This method can be useful when finding a shorter side of a right-angled triangle.

Example 2



Solution

$x^2 = 50^2 - 48^2$

$= (50-48)(50+48)$

$= 2 \times 98$

$= 196$

$\therefore x = \sqrt{196}$

$= 14$

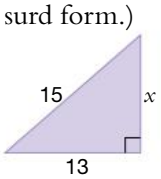
Exercises

1 Evaluate:

a $100^2 - 98^2$	**b** $73^2 - 67^2$
c $145^2 - 140^2$	**d** $651^2 - 641^2$

2 Use the method above to find the value of x for each triangle. (Leave your answer in surd form.)

a



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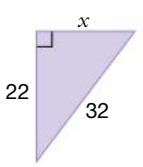

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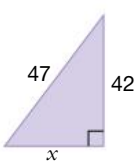
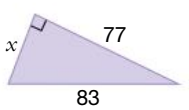


d



10:02C Miscellaneous examples

- It is important that you are able to expand and simplify algebraic expressions readily and accurately, if you are to use algebra in later problem-solving exercises.
- Work through the following miscellaneous questions after examining the following two examples.

WORKED EXAMPLES

1 $(x+3)^2-(x-1)(x+2)=[x^2+6x+9]-[x^2+x-2]$
$=x^2+6x+9-x^2-x+2$
$=5x+11$

2 $(3x+5)(x-1)+(x+2)^2-(2x+1)(2x-1)=[3x^2+2x-5]+[x^2+4x+4]-[4x^2-1]$
$=3x^2+2x-5+x^2+4x+4-4x^2+1$
$=6x$

Exercise 10:02C

Expand and simplify, where possible, each of the following expressions.

1
a $5x+3(x-7)$
b $(x+2)(x-1)$
c $(2x+1)(x-1)$
d $5(x+2)-x(x+1)$
e $(3x-1)^2$
f $(x+5)(x-5)$
g $(2x-7)(3x-1)$
h $(5x-1)(5x+1)$
i $4x+7+x(x+2)$
j $9x-(x+5)+5$
k $(x+10)(x-3)$
l $(9-y)(9+y)$
m $3x(x-5)-2x^2$
n $3(x+2)(x+1)$
o $(x+y)^2$
p $(x+2y)(2x+y)$
q $5x-2(x+y)+2y$
r $(a+2b)(a-2b)$
s $a(x+2)-x(a+2)$
t $(3a+7)(5a-3)$
u $(2m-5n)^2$
v $(1-5y)(1+5y)$
w $3x-7(x-3)$
x $(9x-8y)(9x+8y)$

2
a $(x+1)^2+5(x+2)$
b $(a-3)^2-3(a+1)$
c $(x+2)(x+3)-7(x-2)$
d $8(x+2)+(x-7)(x+1)$
e $(x+3)^2+(x+1)(x+2)$
f $(a+5)(a+3)-(a+4)^2$
g $(m+6)^2-(m-1)(m+1)$
h $(y+7)(y-7)-(y+7)^2$
i $(x+2)^2+(x+1)^2$
j $(a+3)^2-(a+2)^2$
k $(x+1)(x+2)+(x+2)(x+3)$
l $(a+1)(a-2)+(a+2)(a-1)$
m $(x+3)(x-1)-(x+2)(x-5)$
n $(y+7)(y-2)-(y+1)(y+3)$
o $(2x+1)^2-5(x+3)$
p $2x(x+5)+(x+7)^2$
q $(5x+1)(x-3)+(2x+1)^2$
r $(2x+1)(3x+1)-(2x-1)(3x-1)$
s $(p+3)(p-3)-(q+3)(q-3)$
t $(x+y)^2-(x-y)(x+y)$
u $(m-n)^2+(m+n)^2$
v $3(x+1)^2+5(x+1)$
w $2(x-1)(x+1)+3(x+1)^2$
x $(2x+3y)^2-(2x-3y)(2x+3y)$
y $(3a+2b)(2a+3b)-6(a+b)^2$

CHALLENGE 10:02C PATTERNS IN PRODUCTS

The expressions below involve the sum of a series of products. Can you see the patterns involved and, hence, find the simplest expression for each sum?

1 $(x + 1)^2 + (x + 2)^2 \ldots + (x + 9)^2 + (x + 10)^2$

2 $(x + 1)(x + 2) + (x + 2)(x + 3) + \ldots + (x + 9)(x + 10)$

3 $(a - 5)^2 + (a - 4)^2 + \ldots + a^2 + \ldots + (a + 4)^2 + (a + 5)^2$

4 $(5m - n)(5m + n) + (4m - 2n)(4m + 2n) + (3m - 3n)(3m + 3n)$
$+ (2m - 4n)(2m + 4n) + (m - 5n)(m + 5n)$

10:03 Factorising using common factors

PREP QUIZ 10:03

Expand:

1 $3(x + 7)$ 2 $a(a - 1)$ 3 $4p(2p - 3q)$ 4 $-5m(3 - 2n)$

Write the HCF (highest common factor) of:

5 $10x$ and $15x$ 6 $8x^2$ and $6xy$

Factorise by taking out a common factor:

7 $4n + 8$ 8 $w^2 - 6w$ 9 $ab + bc$ 10 $-6y - 9$

The simplest method of factorising is 'taking out' a common factor. This method was met in Chapter 2. This is an important skill because it is the first method that should be considered when factorising any expression. It is also sometimes used in conjunction with other methods.

See Section 2:02

Check your competence with this skill by completing the short exercise below.

Exercise 10:03

P Foundation worksheet 10:03 Common factors

1 Factorise the following by taking out the highest common factor.

a $2x + 8$ b $10m + 5$ c $9a + 6$ d $4 + 6y$

e $7w - 21$ f $15n - 20$ g $12 - 18t$ h $16 - 8z$

i $x^2 + 5x$ j $am + an$ k $6p - p^2$ l $ab - a$

2 Factorise fully.

a $2ax + 6ay$ b $5m^2 + 5mn$ c $9ab + 6bc$ d $4yz + 6xy$

e $7tw - 14t^2$ f $15n - 10n^2$ g $12p - 8pq$ h $6fg - 8gh$

i $x^2y + xy^2$ j $am^2 + amn$ k $6p - 6p^2$ l $abc - bc$

3 Factorise by taking out a negative common factor.

a $-2x - 8$ **b** $-5m + 5$ **c** $-9a + 12$ **d** $-4 - 8y$

e $-7w^2 - w$ **f** $-mn - np$ **g** $-12t^2 + 8t$ **h** $-16xy + 24yz$

4 Factorise:

a $3x + x^2 - ax$ **b** $ax + ay + az$ **c** $4m - 8n + 6p$

d $5ab - 15ac + 10ad$ **e** $x^2 - 7x + xy$ **f** $10ab - 5bc - 15b^2$

5 Remembering that $a(b + 2) + 4(b + 2) = (b + 2)(a + 4)$, factorise the following.

a $2(a + x) + b(a + x)$ **b** $x(3 + b) + 2(3 + b)$ **c** $a(a + 3) - 2(a + 3)$

d $3m(m - 1) + 5(m - 1)$ **e** $x(2x + 5) - (2x + 5)$ **f** $7(7 - z) - z(7 - z)$

10:04 Factorising by grouping in pairs

PREP QUIZ 10:04

Factorise these expressions.

1 $3a + 18$ **2** $5x + ax$ **3** $pq - px$ **4** $3ax - 9bx$

5 $x^2 - 2x$ **6** $a^3 + a^2$ **7** $9 - 3a$ **8** $-5m - 10$

9 $9(a + 1) + x(a + 1)$ **10** $x(x + y) - 1(x + y)$

For some algebraic expressions, there may not be a factor common in every term. Consider the expression $3x + 3 + mx + m$.

There is no factor common in every term, but:

- the first two terms have a common factor of 3
- the last two terms have a common factor of m.

$$3x + 3 + mx + m = 3(x + 1) + m(x + 1)$$

Now it can be seen that $(x + 1)$ is a common factor for each term.

$$3(x + 1) + m(x + 1) = (x + 1)(3 + m)$$

Therefore:

$$3x + 3 + mx + m = (x + 1)(3 + m)$$

The original expression has been factorised by grouping the terms in pairs.

WORKED EXAMPLE 1

a $2x + 2y + ax + ay = 2(x + y) + a(x + y)$
$= (x + y)(2 + a)$

b $a^2 + 3a + ax + 3x = a(a + 3) + x(a + 3)$
$= (a + 3)(a + x)$

WORKED EXAMPLE 2

a $ax - bx + am - bm = x(a - b) + m(a - b)$
$= (a - b)(x + m)$

b $ab + b^2 - a - b = b(a + b) - 1(a + b)$
$= (a + b)(b - 1)$

c $5x + 2y + xy + 10 = 5x + 10 + 2y + xy$
$= 5(x + 2) + y(2 + x)$
$= (x + 2)(5 + y)$

Note: Terms had to be rearranged to pair those with common factors.

$$ab + ac + bd + cd = a(b + c) + d(b + c)$$
$$= (b + c)(a + d)$$

Exercise 10:04

P Foundation worksheet 10:04
Grouping in pairs

1 Complete the factorisation of each expression below.

a $2(a + b) + x(a + b)$
b $a(x + 7) + p(x + 7)$
c $m(x - y) + n(x - y)$
d $x(m + n) - y(m + n)$
e $a^2(2 - x) + 7(2 - x)$
f $q(q - 2) - 2(q - 2)$
g $(x + y) + a(x + y)$
h $x(1 - 3y) - 2(1 - 3y)$

2 Factorise these expressions.

a $pa + pb + qa + qb$
b $3a + 3b + ax + bx$
c $mn + 3np + 5m + 15p$
d $a^2 + ab + ac + bc$
e $9x^2 - 12x + 3xy - 4y$
f $12p^2 - 16p + 3pq - 4q$
g $ab + 3c + 3a + bc$
h $xy + y + 4x + 4$
i $a^3 + a^2 + a + 1$
j $pq + 5r + 5p + qr$
k $xy - x + y - 1$
l $8a - 2 + 4ay - y$
m $mn + m + n + 1$
n $x^2 + my + xy + mx$
o $x^2 - xy + xw - yw$
p $x^2 + yz + xz + xy$
q $11a + 4c + 44 + ac$
r $a^3 - a^2 + a - 1$

3 Factorise the following.

a $xy + xz - wy - wz$
b $ab + bc - ad - cd$
c $5a + 15 - ab - 3b$
d $6x - 24 - xy + 4y$
e $11y + 22 - xy - 2x$
f $ax^2 - ax - x + 1$

Can you find a formula that calculates the day of the week for any given date?

How many different calendars are needed to describe any given year?

10:05 Factorising using the difference of two squares

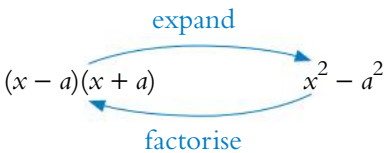
PREP QUIZ 10:05

Simplify: **1** $\sqrt{16}$ **2** $\sqrt{49}$ **3** $\sqrt{121}$

If x is positive, simplify: **4** $\sqrt{x^2}$ **5** $\sqrt{9x^2}$ **6** $\sqrt{64x^2}$

Expand and simplify. **7** $(x-2)(x+2)$ **8** $(x+5)(x-5)$
9 $(7-a)(7+a)$ **10** $(3m+2n)(3m-2n)$

If the expression we want to factorise is the difference of two squares, we can simply reverse the procedure.



WORKED EXAMPLE 1

a $x^2 - 9 = x^2 - 3^2$
$= (x-3)(x+3)$

b $25a^2 - b^2 = (5a)^2 - b^2$
$= (5a-b)(5a+b)$

c $a^4 - 64 = (a^2)^2 - 8^2$
$= (a^2-8)(a^2+8)$

d $36m^2 - 49n^2 = (6m)^2 - (7n)^2$
$= (6m-7n)(6m+7n)$

$x^2 - y^2 = (x-y)(x+y)$

Note: $(x-y)(x+y) = (x+y)(x-y)$

Exercise 10:05

1 Factorise each of these expressions.

a $x^2 - 4$ **b** $a^2 - 16$ **c** $m^2 - 25$ **d** $p^2 - 81$
e $y^2 - 100$ **f** $x^2 - 121$ **g** $9 - x^2$ **h** $1 - n^2$
i $49 - y^2$ **j** $a^2 - b^2$ **k** $x^2 - a^2$ **l** $y^2 - a^2$
m $9a^2 - 4$ **n** $16x^2 - 1$ **o** $25p^2 - 9$ **p** $49 - 4a^2$
q $25p^2 - a^2$ **r** $m^2 - 81n^2$ **s** $100a^2 - 9b^2$ **t** $81x^2 - 121y^2$

2 Factorise by first taking out a common factor.

a $2x^2 - 32$	**b** $3x^2 - 108$
c $4a^2 - 100$	**d** $5y^2 - 20$
e $24a^2 - 6b^2$	**f** $3x^2 - 27y^2$
g $8y^2 - 128$	**h** $80p^2 - 5q^2$
i $4x^2 - 64$	**j** $3x^2 - 3$
k $72p^2 - 2$	**l** $2 - 18x^2$
m $8a^2 - 18m^2$	**n** $125 - 20a^2$
o $200x^2 - 18y^2$	**p** $98m^2 - 8n^2$

WORKED EXAMPLE 2

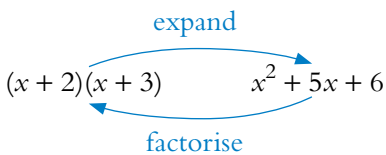
$$\begin{aligned} 18x^2 - 50 &= 2(9x^2 - 25) \\ &= 2([3x]^2 - 5^2) \\ &= 2(3x - 5)(3x + 5) \end{aligned}$$

10:06 Factorising quadratic trinomials: A review

In Chapter 2 only monic trinomials were factorised, which are trinomials where the coefficient of the squared term is 1, such as $x^2 + 5x + 6$ or $a^2 - a - 20$.

Remember: To factorise $x^2 + 5x + 6$ we look for two numbers that multiply to equal 6 and add to equal 5.
$2 \times 3 = 6$ and $2 + 3 = 5$

So the factors of $x^2 + 5x + 6$ are $(x + 2)(x + 3)$.



WORKED EXAMPLES

Factorise:

1 $x^2 + 6x + 8$ **2** $a^2 - 8a + 15$ **3** $y^2 + 2y - 35$ **4** $3n^2 - 9n - 30$

Solutions

1 $2 \times 4 = 8$
$2 + 4 = 6$
$\therefore x^2 + 6x + 8$
$= (x + 2)(x + 4)$

2 $(-3) \times (-5) = 15$
$(-3) + (-5) = -8$
$\therefore a^2 - 8a + 15$
$= (a - 3)(a - 5)$

3 $(-5) \times 7 = -35$
$(-5) + 7 = 2$
$\therefore y^2 + 2y - 35$
$= (y - 5)(y + 7)$

4 Firstly, take out the common factor:
$3n^2 - 9n - 30 = 3(n^2 - 3n - 10)$
Now factorise $n^2 + 3n - 10$:
$(-5) \times 2 = -10$ and $(-5) + 2 = -3$
So $n^2 + 3n - 10 = (n - 5)(n + 2)$
$\therefore 3n^2 - 9n - 30 = 3(n - 5)(n + 2)$

Complete this short exercise to revise this skill before examining harder types.

Exercise 10:06

P Foundation worksheet 10:06 Factorising trinominals

1 Factorise these trinomials.

a $x^2 + 4x + 3$	b $x^2 + 7x + 10$	c $x^2 + 10x + 24$
d $x^2 - 3x + 2$	e $x^2 - 6x + 8$	f $x^2 - 8x + 12$
g $x^2 + 2x - 8$	h $x^2 + 4x - 21$	i $x^2 + x - 42$
j $x^2 - 5x - 6$	k $x^2 - 8x - 20$	l $x^2 - x - 72$

2 Factorise:

a $a^2 + 9a + 20$	b $y^2 + 4y - 32$	c $m^2 - 5m - 14$
d $z^2 - 8z + 7$	e $k^2 + k - 72$	f $p^2 + 3p - 54$
g $t^2 + 13t + 30$	h $q^2 - 9q + 8$	i $w^2 - 6w - 40$
j $d^2 - d - 56$	k $v^2 + 5v - 36$	l $h^2 - 12h - 28$
m $n^2 + 14n + 40$	n $x^2 - 10x - 39$	o $f^2 + 9f - 90$

3 Factorise these expressions by first taking out a common factor.

a $2x^2 + 8x + 6$	b $5a^2 + 15a + 10$	c $4m^2 - 16m + 16$
d $7n^2 - 14n - 56$	e $9y^2 + 9y - 54$	f $10z^2 + 50z - 240$

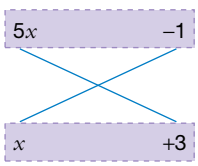
10:07 Factorising further quadratic trinomials

- In all quadratic trinomials factorised so far, the *coefficient* of x^2 has been 1. We will now consider cases where the *coefficient* of x^2 is not 1.

$3x^2 + 7x + 9$

coefficient of x^2

- To expand $(5x - 1)(x + 3)$ we can use a cross diagram.



$5x^2$ is the product of the two left terms.
-3 is the product of the two right terms.
$14x$ is the sum of the products along the cross:
$15x + (-x)$

$\therefore (5x - 1)(x + 3) = 5x^2 + 14x - 3$

- One method used to factorise trinomials like $5x^2 + 14x - 3$ is called the cross method.

Remember

$5x^2$ $\quad -3$

$(5x - 1)(x + 3)$

$-x$

$+15x$

$= 5x^2 + 15x - x - 3$

$= 5x^2 + 14x - 3$

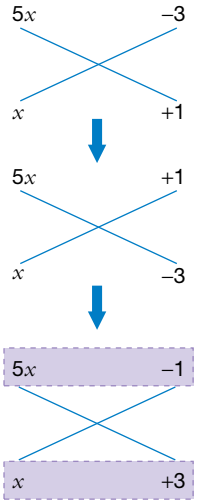
Cross method

To factorise $5x^2 + 14x - 3$, we need to reverse the expanding process. We need to choose two factors of $5x^2$ and two factors of -3 to write on the cross.

Try:

$\begin{bmatrix} 5x \\ x \end{bmatrix}$ and $\begin{bmatrix} -3 \\ +1 \end{bmatrix}$

- If $(5x - 3)$ and $(x + 1)$ are the factors of $5x^2 + 14x - 3$, then the products of the numbers on the ends of each arm will have a sum of $+14x$.



- Cross-product = $(5x) + (-3x) = 2x$
  This does not give the correct middle term of $14x$, so $(5x - 3)$ and $(x + 1)$ are **not** factors.

- Vary the terms on the cross.

  Try: $\begin{matrix}5x\\x\end{matrix}$ and $\begin{matrix}+1\\-3\end{matrix}$

  Cross-product = $(-15x) + (x) = -14x$

  Try: $\begin{matrix}5x\\x\end{matrix}$ and $\begin{matrix}-1\\+3\end{matrix}$

  Cross-product = $(15x) + (-x) = 14x$
  $\therefore$ This must be the correct combination.
  $\therefore\ 5x^2 + 14x - 3 = (5x - 1)(x + 3)$

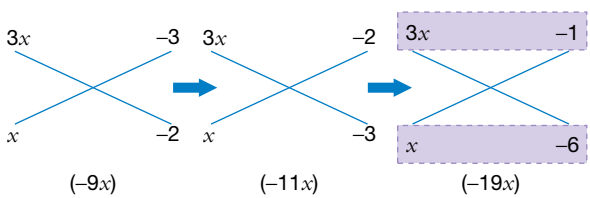
WORKED EXAMPLES

Find the factors of:

1 $3x^2 - 19x + 6$ **2** $4x^2 - x - 3$ **3** $2x^2 + 25x + 12$

Solutions

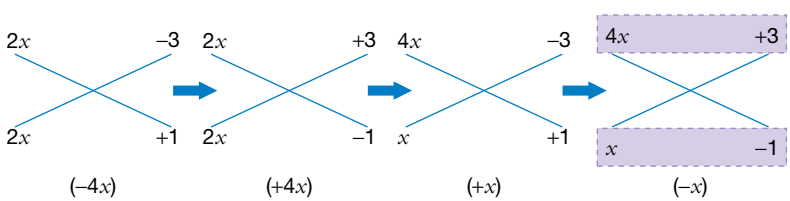
1



Cross-product = $-19x$
$3x^2 - 19x + 6 = (3x - 1)(x - 6)$

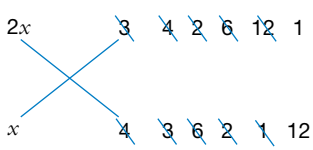
The factors of +6 had to be both negative to give a negative middle term.

2



$\therefore\ 4x^2 - x - 3$
$= (4x + 3)(x - 1)$

3 In practice, we would not draw a separate cross for each new set of factors. We simply cross out the factors that don't work and try a new set.



$\therefore\ 2x^2 + 25x + 12 = (2x + 1)(x + 12)$

To factorise a quadratic trinomial, $ax^2 + bx + c$, when a (the coefficient of x^2) is not 1, use the cross method.

Alternative method: Splitting the middle term

To factorise $5x^2 + 14x - 3$ we first multiply the two 'outer' numbers together and note it above the last number.

$5 \times -3 = -15$

$5x^2 + 14x - 3$

We then find two numbers that multiply to give -15 and add to give $+14$. The only possible numbers are $+15$ and -1.

$15 \times -1 = -15$ and $15 + -1 = 14$

We now 'split' the middle term $+14x$ using these numbers $(+15x, -1x)$ and rewrite the trinomial with four terms.

$5x^2 + 14x - 3 = 5x^2 + 15x - 1x - 3$

We now factorise this expression by grouping in pairs:

$$\begin{aligned} 5x^2 + 15x - 1x - 3 &= 5x(x + 3) - 1(x + 3) \\ &= (x + 3)(5x - 1) \end{aligned}$$

$\therefore 5x^2 + 14x - 3 = (x + 3)(5x - 1)$

WORKED EXAMPLES

Factorise:

1 $2x^2 - 5x - 12$ **2** $12a^2 - 29a + 15$

Solutions

1 $2x^2 - 5x - 12$ (-24)

$$\begin{aligned} 2x^2 - 5x - 12 &= 2x^2 - 5x - 12 \\ &= 2x^2 - 8x + 3x - 12 \\ &= 2x(x - 4) + 3(x - 4) \\ &= (x - 4)(2x + 3) \end{aligned}$$

$(-8 \times 3 = -24$ and $-8 + 3 = -5)$

With this method, sometimes the product can be a big number.

2 $12a^2 - 29a + 15$ (180)

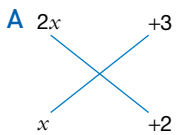
$$\begin{aligned} 12a^2 - 29a + 15 &= 12a^2 - 29a + 15 \\ &= 12a^2 - 9a - 20a + 15 \\ &= 3a(4a - 3) - 5(4a - 3) \\ &= (4a - 3)(3a - 5) \end{aligned}$$

$(-9 \times -20 = 180$ and $-9 + -20 = -29)$

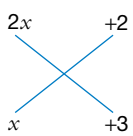
Exercise 10:07

1 **a** Which diagram will give the factors of $2x^2 + 13x + 6$?

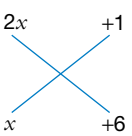
A



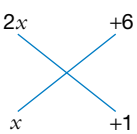
B



C

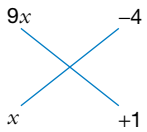


D

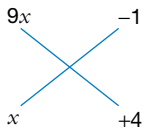


b Which diagram will give the factors of $9x^2 - 9x - 4$?

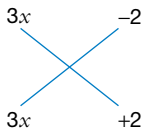
A



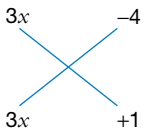
B



C

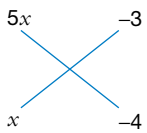


D

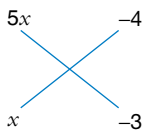


c Which diagram will give the factors of $5x^2 - 19x + 12$?

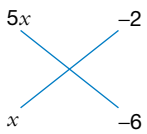
A



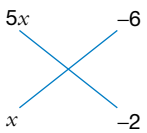
B



C

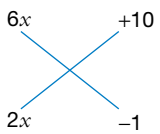


D

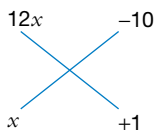


d Which diagram will give the factors of $12x^2 + 7x - 10$?

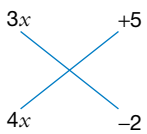
A



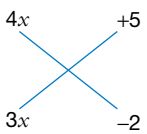
B



C



D



2 Factorise these expressions.

a $2x^2 + 7x + 3$	**b** $3x^2 + 8x + 4$	**c** $2x^2 + 7x + 6$
d $2x^2 + 11x + 5$	**e** $3x^2 + 5x + 2$	**f** $2x^2 + 11x + 15$
g $4x^2 + 13x + 3$	**h** $5x^2 + 17x + 6$	**i** $2x^2 + 13x + 15$
j $2x^2 - 5x + 2$	**k** $3x^2 - 11x + 6$	**l** $5x^2 - 17x + 6$
m $4x^2 - 11x + 6$	**n** $10x^2 - 21x + 9$	**o** $5x^2 - 22x + 21$
p $2x^2 + x - 10$	**q** $3x^2 + 4x - 15$	**r** $4x^2 + 11x - 3$
s $2x^2 - x - 6$	**t** $2x^2 - 5x - 3$	**u** $3x^2 - x - 30$
v $6x^2 - 5x - 21$	**w** $2x^2 - 5x - 12$	**x** $4x^2 - x - 18$

3 Find the factors of the following.

a $12x^2 + 7x + 1$	**b** $6a^2 + 5a + 1$	**c** $6p^2 + 7p + 2$
d $10y^2 - 9y + 2$	**e** $12x^2 - 7x + 1$	**f** $9a^2 - 21a + 10$
g $8m^2 + 18m - 5$	**h** $6n^2 - 7n - 3$	**i** $21q^2 - 20q + 4$
j $20x^2 - x - 1$	**k** $8m^2 - 2m - 15$	**l** $18y^2 - 3y - 10$
m $6a^2 + 5a - 6$	**n** $15k^2 + 26k + 8$	**o** $8x^2 + 18x + 9$
p $4 - 3a - a^2$	**q** $2 + m - 10m^2$	**r** $6 + 7x - 3x^2$
s $6 - 7x - 3x^2$	**t** $15 - x - 28x^2$	**u** $2 + 9n - 35n^2$
v $3x^2 + 10xy + 8y^2$	**w** $2x^2 - 5xy + 2y^2$	**x** $5m^2 - 2mn - 7n^2$

4 Factorise by first taking out the common factor.

a $6x^2 + 10x - 4$	**b** $6a^2 - 2a - 4$	**c** $6a^2 + 9a - 27$
d $8x^2 + 12x - 36$	**e** $6x^2 + 28x + 16$	**f** $12p^2 + 12p - 9$
g $30q^2 + 55q - 35$	**h** $10m^2 - 46m + 24$	**i** $50a^2 + 15a - 5$
j $4 - 6x - 10x^2$	**k** $36 - 3t - 3t^2$	**l** $9 + 24x + 12x^2$

5 Complete each expression in as many ways as possible by writing positive whole numbers in the boxes and inserting operation signs.

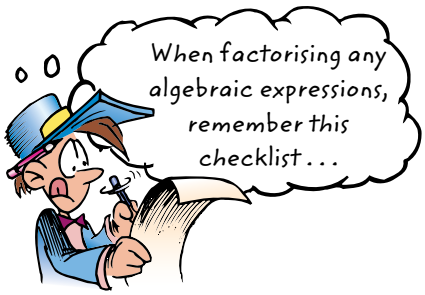
a $(x \ldots \square)(x \ldots \square) = x^2 \ldots \square x \ldots 15$

b $(x \ldots \square)(x \ldots \square) = x^2 \ldots \square x - 12$

c $(x \ldots \square)(x \ldots \square) = x^2 \ldots 5x + \square$

d $(5x \ldots \square)(x \ldots \square) = 5x^2 \ldots \square x \ldots 2$

10:08 Factorising: Miscellaneous types



1 Take out any common factors.
2 If there are two terms, is it a difference of two squares: $a^2 - b^2$?
3 If there are three terms, is it a quadratic trinomial: $ax^2 + bx + c$?
4 If there are four terms, can it be factorised by grouping in pairs?

WORKED EXAMPLES

1 $4x^2 - 36$
$= 4(x^2 - 9)$ common factor
$= 4(x - 3)(x + 3)$ difference of two squares

2 $15x^2y - 20xy + 10xy^2$
$= 5xy(3x - 4 + 2y)$ common factor

3 $8x^2 - 40x + 32$
$= 8(x^2 - 5x + 4)$ common factor
$= 8(x - 4)(x - 1)$ quadratic trinomial

4 $12 - a - 6a^2$
$= (3 + 2a)(4 - 3a)$ quadratic trinomial

3 6 $-6a$ a $-3a$ $2a$
4 2 a $-6a$ $2a$ $-3a$

5 $ap - aq - 3p + 3q$
$= a(p - q) - 3(p - q)$ grouping in pairs
$= (p - q)(a - 3)$

Exercise 10:08

1 Factorise each of these expressions.

a $x^2 - 6x + 5$
b $x^2 - 9$
c $xy + 2y + 9x + 18$
d $a^2 - 9a$
e $a^2 - 6a + 9$
f $4x^2 - 1$
g $12x^2 - x - 35$
h $a^2 - 13a + 40$
i $5a^2b - 10ab^3$
j $p^2 - q^2$
k $pq - 3p + 10q - 30$
l $7x^2 + 11x - 6$
m $a^2 + 3a - ab$
n $16 - 25a^2$
o $1 - 2a - 24a^2$
p $4m + 4n - am - an$
q $5ay - 10y + 15xy$
r $15x^2 - x - 28$
s $x^2y^2 - 1$
t $x^2 - x - 56$
u $2mn + 3np + 4m + 6p$
v $100a^2 - 49x^2$
w $2 - 5x - 3x^2$
x $k^2 + 2k - 48$

2 Factorise completely.

a $2 - 8x^2$
b $5x^2 - 10x - 5xy + 10y$
c $2a^2 - 22a + 48$
d $3m^2 - 18m + 27$
e $x^4 - 1$
f $p^3 - 4p^2 - p + 4$
g $4x^2 - 36$
h $a^3 - a$
i $3a^2 - 39a + 120$
j $9 - 9p^2$
k $3k^2 + 3k - 18$
l $24a^2 - 42a + 9$
m $ax^2 + axy + 3ax + 3ay$
n $(x + y)^2 + 3(x + y)$
o $5xy^2 - 20xz^2$
p $6ax^2 + 5ax - 6a$
q $x^2 - y^2 + 5x - 5y$
r $3x^2 - 12x + 12$
s $63x^2 - 28y^2$
t $a^4 - 16$
u $(a - 2)^2 - 4$
v $1 + p + p^2 + p^3$
w $8t^2 - 28t - 60$
x $8 - 8x - 6x^2$

10:09 Simplifying algebraic fractions: Multiplication and division

PREP QUIZ 10:09

Simplify the following:

1 $\frac{5a^2}{10a}$ **2** $\frac{12xy^2}{8x^2y}$ **3** $\frac{2x}{3} \times \frac{6}{x}$ **4** $\frac{2x}{3} \div \frac{4x}{9}$

Factorise:

5 $6x^2 + 9x$ **6** $x^2 + 7x + 12$ **7** $x^2 - 49$

8 $3x^2 + 6x + 3$ **9** $3x + 3y + ax + ay$ **10** $2x^2 + 9x - 5$

Just as numerical fractions can be simplified by cancelling common factors in the numerator and the denominator, so algebraic fractions can often be simplified in the same way after first factorising where possible.

WORKED EXAMPLE 1

a $\frac{2x-2}{3x-3} = \frac{2(x-1)^{1}}{3(x-1)_{1}}$

$= \frac{2}{3}$

b $\frac{x^2+7x+12}{x^2-9} = \frac{(x+4)(x+3)^{1}}{(x-3)(x+3)_{1}}$

$= \frac{x+4}{x-3}$

c $\frac{6-6a^2}{3+3a+x+ax} = \frac{6(1-a)(1+a)^{1}}{(3+x)(1+a)_{1}}$

$= \frac{6(1-a)}{3+x}$

d $\frac{3x^2-9x}{3x^2-27} = \frac{{}^{1}3x(x-3)^{1}}{{}_{1}3(x-3)(x+3)}$

$= \frac{x}{x+3}$

Algebraic fractions should also be factorised before multiplying or dividing because the cancelling of common factors often simplifies these processes.

WORKED EXAMPLE 2

a $\dfrac{5x+15}{x+1} \times \dfrac{2x+2}{5} = \dfrac{{}^{1}\cancel{5}(x+3)}{{}_{1}\cancel{(x+1)}} \times \dfrac{2\cancel{(x+1)}^{1}}{\cancel{5}_{1}}$

$= 2(x+3)$

b $\dfrac{x^2-9}{x^2+5x+6} \times \dfrac{3x+6}{x^2-2x-3} = \dfrac{{}^{1}\cancel{(x-3)}\,\cancel{(x+3)}^{1}}{{}_{1}\cancel{(x+2)}\,\cancel{(x+3)}_{1}} \times \dfrac{3\cancel{(x+2)}^{1}}{{}_{1}\cancel{(x-3)}(x+1)}$

$= \dfrac{3}{x+1}$

c $\dfrac{6x-14}{3x-9} \div \dfrac{3x-7}{5x-15} = \dfrac{2\cancel{(3x-7)}^{1}}{3\cancel{(x-3)}^{1}} \times \dfrac{5\cancel{(x-3)}^{1}}{\cancel{(3x-7)}^{1}}$

$= \dfrac{10}{3}$ or $3\dfrac{1}{3}$

d $\dfrac{a^2-16}{a^2-25} \div \dfrac{a^2-2a-8}{a^2+10a+25} = \dfrac{{}^{1}\cancel{(a-4)}(a+4)}{(a-5)\cancel{(a+5)}_{1}} \times \dfrac{(a+5)\cancel{(a+5)}^{1}}{{}_{1}\cancel{(a-4)}(a+2)}$

$= \dfrac{(a+4)(a+5)}{(a-5)(a+2)}$

To simplify algebraic fractions, factorise both numerator and denominator where possible, and then cancel.

Exercise 10:09

P Foundation worksheet 10:09 Simplifying alegebraic fractions

1 Factorise and simplify.

a $\dfrac{5x+10}{5}$ **b** $\dfrac{4}{2x+6}$ **c** $\dfrac{12}{3x-9}$ **d** $\dfrac{2x-10}{x-5}$

e $\dfrac{x+7}{3x+21}$ **f** $\dfrac{5a-5}{8a-8}$ **g** $\dfrac{3a+9}{6a+18}$ **h** $\dfrac{7m-28}{3m-12}$

i $\dfrac{x^2+x}{x^2-x}$ **j** $\dfrac{x^2-4}{x-2}$ **k** $\dfrac{a+1}{a^2-1}$ **l** $\dfrac{4y^2-9}{4y+6}$

m $\dfrac{a^2-4a}{3a-a^2}$ **n** $\dfrac{2x^2-2}{2x-2}$ **o** $\dfrac{x^2-36}{3x-18}$ **p** $\dfrac{a^2-3a-4}{a+1}$

q $\dfrac{x^2-6x+9}{x-3}$ **r** $\dfrac{x^2-4}{x^2+3x+2}$ **s** $\dfrac{x^2+3x+2}{x^2+5x+6}$ **t** $\dfrac{m^2+5m-24}{m^2-7m+12}$

u $\dfrac{t^2+7t+12}{t^2-9}$ **v** $\dfrac{a^2-x^2}{a^2+3a+ax+3x}$ **w** $\dfrac{2x^2-x-1}{4x^2-1}$ **x** $\dfrac{18a^2-8}{6a^2+a-2}$

2 Simplify the following.

a $\frac{2x+4}{3} \times \frac{6x}{x+2}$

b $\frac{5y-15}{2y+8} \times \frac{y+4}{10}$

c $\frac{2x-4}{3x-9} \times \frac{5x-15}{7x-14}$

d $\frac{5n+10}{n+3} \times \frac{6n+18}{4n+8}$

e $\frac{7y+28}{21} \times \frac{6}{6y+24}$

f $\frac{1+2a}{10+30a} \times \frac{6+18a}{1-2a}$

g $\frac{y^2+y}{2y+8} \times \frac{4y+6}{3y+3}$

h $\frac{x^2-3x}{x^2} \times \frac{2x^2+5x}{9x-27}$

i $\frac{x+3}{x^2-9} \times \frac{x-3}{x+1}$

j $\frac{3x+15}{x^2-25} \times \frac{x^2-49}{3x-21}$

k $\frac{a^2+5a+6}{a^2-4} \times \frac{a^2-a-2}{a^2-1}$

l $\frac{y^2+3y+2}{y^2+5y+6} \times \frac{y^2+7y+12}{y^2+5y+4}$

m $\frac{x^2+6x+5}{x^2+5x+4} \times \frac{x^2+7x+12}{x^2+12x+35}$

n $\frac{m^2-1}{m^2-6m+5} \times \frac{m^2-10m+25}{m^2-25}$

o $\frac{a^2-4}{a^2+3a-4} \times \frac{a^2-16}{a^2+2a-8}$

p $\frac{2x^2+4x+2}{x^2-1} \times \frac{x^2+3x-4}{4x+4}$

q $\frac{3x^2+5x+2}{x^2-x-2} \times \frac{x^2+x-6}{3x^2+11x+6}$

r $\frac{5a^2+16a+3}{25a^2-1} \times \frac{5a^2-a}{2a^2+5a-3}$

s $\frac{x^2-y^2+x-y}{x^2-2xy+y^2} \times \frac{10x-10y}{5x+5y+5}$

t $\frac{(a+b)^2-c^2}{a^2+ab+ac+bc} \times \frac{a^2+ab-ac-bc}{a+b+c}$

3 Simplify:

a $\frac{3a+6}{2} \div \frac{a+2}{4}$

b $\frac{x+2}{5x} \div \frac{7x+14}{10x}$

c $\frac{5m-10}{m+1} \div \frac{3m-6}{3m+3}$

d $\frac{6m+9}{2m-8} \div \frac{2m+3}{3m-12}$

e $\frac{3x}{5x+15} \div \frac{x^2+x}{x+3}$

f $\frac{24y-16}{4y+6} \div \frac{3y-2}{8y+12}$

g $\frac{5m-20}{4m+6} \div \frac{5m-20}{2m^2+3m}$

h $\frac{25k+15}{3k-3} \div \frac{5k+3}{3k}$

i $\frac{n^2-9}{2n+4} \div \frac{n+3}{2}$

j $\frac{y+7}{y-7} \div \frac{y^2-49}{y^2-7y}$

k $\frac{a^2+5a+4}{a^2-16} \div \frac{a^2-9}{a^2-a-12}$

l $\frac{x^2+6x+9}{x^2+8x+15} \div \frac{x^2+5x+6}{x^2+7x+10}$

m $\frac{x^2-4}{x^2-7x+10} \div \frac{x^2-x-6}{x^2-3x-10}$

n $\frac{p^2+7p+10}{p^2-2p-8} \div \frac{p^2+2p-15}{p^2+p-12}$

o $\frac{n^2-49}{n^2-9} \div \frac{n^2+14n+49}{n^2-6n+9}$

p $\frac{2x^2-8x-42}{x^2+6x+9} \div \frac{x^2-9x+14}{x^2+x-6}$

q $\frac{3x^2-48}{x^2-3x-4} \div \frac{x^2+4x}{x^3-x}$

r $\frac{2a^2-a-1}{a^2-1} \div \frac{6a^2+a-1}{3a^2+2a-1}$

s $\frac{x+y+x^2-y^2}{x^2+2xy+y^2} \div \frac{1+x-y}{2x+2y}$

t $\frac{p^2-(q+r)^2}{p^2+pq-pr-qr} \div \frac{p-q-r}{p^2-pq-pr+qr}$

10:10 Addition and subtraction of algebraic fractions

PREP QUIZ 10:10

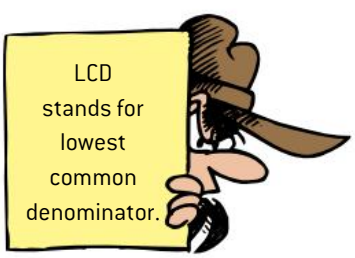
Simplify:

1 $\frac{1}{2}+\frac{3}{5}$ **2** $\frac{3}{4}+\frac{3}{8}$ **3** $\frac{9}{10}-\frac{3}{5}$ **4** $\frac{7}{15}-\frac{3}{20}$ **5** $\frac{5}{x}+\frac{7}{x}$

6 $\frac{2}{a}+\frac{1}{2a}$ **7** $\frac{2}{3a}+\frac{3}{2a}$ **8** $\frac{1}{x}-\frac{1}{4x}$ **9** $\frac{a}{2x}+\frac{2a}{x}$ **10** $\frac{5m}{2n}-\frac{4m}{3n}$

The Prep quiz above should have reminded you that the lowest common denominator needs to be found when adding or subtracting fractions. If the denominators involve two or more terms, factorising first may help in finding the lowest common denominator.
For example:

$$\frac{2}{x^2-9}+\frac{5}{x^2+5x+6}=\frac{2}{(x-3)(x+3)}+\frac{5}{(x+3)(x+2)}$$
$$=\frac{2(x+2)+5(x-3)}{(x-3)(x+3)(x+2)}$$
$$=\frac{2x+4+5x-15}{(x-3)(x+3)(x+2)}$$
$$=\frac{7x-11}{(x-3)(x+3)(x+2)}$$



Here, LCD $=(x-3)(x+3)(x+2)$.
Note that the factors of each denominator are present without repeating any factor common to both. Each numerator is then multiplied by each factor not present in its original denominator.

When adding or subtracting fractions:
- factorise the denominator of each fraction
- find the lowest common denominator
- rewrite each fraction with this common denominator and simplify.

WORKED EXAMPLES

1 $\dfrac{2}{x+2}+\dfrac{1}{x+3}=\dfrac{2(x+3)+1(x+2)}{(x+2)(x+3)}$

$=\dfrac{2x+6+x+2}{(x+2)(x+3)}$

$=\dfrac{3x+8}{(x+2)(x+3)}$

2 $\dfrac{3}{2x+1}-\dfrac{4}{3x-1}=\dfrac{3(3x-1)-4(2x+1)}{(2x+1)(3x-1)}$

$=\dfrac{9x-3-8x-4}{(2x+1)(3x-1)}$

$=\dfrac{x-7}{(2x+1)(3x-1)}$

No factorising was needed in these first two examples.

3 $\dfrac{1}{x^2+5x+6}+\dfrac{2}{x+3}$

$=\dfrac{1}{(x+2)(x+3)}+\dfrac{2}{(x+3)}$

$=\dfrac{1+2(x+2)}{(x+2)(x+3)}$

$=\dfrac{1+2x+4}{(x+2)(x+3)}$

$=\dfrac{2x+5}{(x+2)(x+3)}$

4 $\dfrac{4}{x^2+x}-\dfrac{3}{x^2-1}$

$=\dfrac{4}{x(x+1)}-\dfrac{3}{(x-1)(x+1)}$

$=\dfrac{4(x-1)-3x}{x(x+1)(x-1)}$

$=\dfrac{4x-4-3x}{x(x+1)(x-1)}$

$=\dfrac{x-4}{x(x+1)(x-1)}$

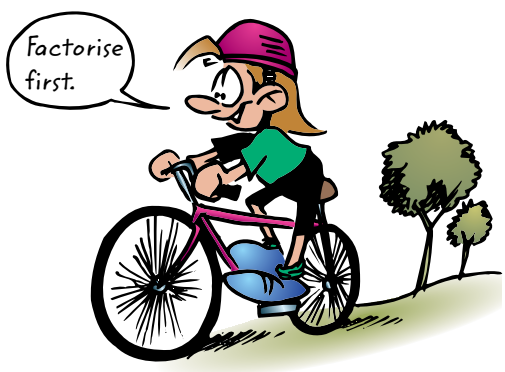
5 $\dfrac{x+3}{x^2+2x+1}-\dfrac{x-1}{x^2-x-2}$

$=\dfrac{x+3}{(x+1)(x+1)}-\dfrac{x-1}{(x+1)(x-2)}$

$=\dfrac{(x+3)(x-2)-(x-1)(x+1)}{(x+1)(x+1)(x-2)}$

$=\dfrac{x^2+x-6-(x^2-1)}{(x+1)(x+1)(x-2)}$

$=\dfrac{x-5}{(x+1)^2(x-2)}$



Exercise 10:10

P Foundation worksheet 10:10
Addition and subtraction of algebraic fractions

1 Simplify each of the following. (*Note:* No factorising is needed.)

a $\dfrac{1}{x+1}+\dfrac{1}{x-1}$ **b** $\dfrac{1}{a+5}+\dfrac{1}{a+3}$ **c** $\dfrac{1}{y-7}-\dfrac{1}{y+1}$

d $\dfrac{2}{x+3}+\dfrac{3}{x+5}$ **e** $\dfrac{5}{m+1}-\dfrac{3}{m-2}$ **f** $\dfrac{6}{t+10}-\dfrac{3}{t+2}$

g $\dfrac{1}{2x-1}+\dfrac{3}{x-1}$ **h** $\dfrac{9}{3x+2}-\dfrac{7}{2x+5}$ **i** $\dfrac{8}{5x-1}+\dfrac{7}{3x+1}$

j $\frac{3}{2x}+\frac{5}{x+7}$ k $\frac{9}{2x+5}-\frac{5}{3x}$ l $\frac{1}{2a}-\frac{3}{2a+1}$

m $\frac{x}{x+3}+\frac{x}{x+1}$ n $\frac{a}{2a+1}-\frac{2a}{4a-1}$ o $\frac{x+1}{x+2}+\frac{x+2}{x+1}$

2 Simplify. (*Note:* The denominators are already factorised.)

a $\frac{1}{(x+1)(x+2)}+\frac{1}{x+1}$ b $\frac{1}{x(x+2)}+\frac{1}{x+2}$

c $\frac{1}{x+3}-\frac{1}{x(x+3)}$ d $\frac{1}{x-5}-\frac{1}{(x-5)(x+2)}$

e $\frac{3}{(x+2)(x+3)}+\frac{4}{x+2}$ f $\frac{5}{x+4}-\frac{3}{(x+1)(x+4)}$

g $\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}$ h $\frac{2}{(x-3)(x+3)}+\frac{4}{(x+3)(x+1)}$

i $\frac{3}{(x+7)(x-1)}+\frac{5}{(x+7)(x+1)}$ j $\frac{9}{(x+9)(x+3)}-\frac{7}{(x+3)(x-1)}$

k $\frac{1}{(2x+1)(x+5)}+\frac{3}{(x+5)(x+2)}$ l $\frac{5}{(2x-1)(3x+2)}-\frac{6}{x(2x-1)}$

m $\frac{x-1}{(x+3)(x+1)}+\frac{x+1}{(x+3)(x-1)}$ n $\frac{x+2}{x(x+3)}-\frac{x-1}{x(x+2)}$

3 Simplify, by first factorising each denominator where possible.

a $\frac{1}{x^2+x}+\frac{1}{x+1}$ b $\frac{1}{3x+9}-\frac{1}{x+3}$

c $\frac{2}{2x+3}+\frac{3}{4x+6}$ d $\frac{5}{x^2-1}+\frac{3}{x-1}$

e $\frac{1}{x^2-9}+\frac{1}{2x-6}$ f $\frac{1}{x^2+x}-\frac{1}{x^2-1}$

g $\frac{1}{x^2+2x+1}+\frac{1}{x^2-1}$ h $\frac{1}{x^2+7x+12}+\frac{1}{x^2+8x+16}$

i $\frac{2}{x^2+6x+8}+\frac{4}{x^2+5x+6}$ j $\frac{2}{x^2+7x+12}+\frac{4}{x^2+5x+4}$

k $\frac{3}{x^2-x-2}-\frac{4}{x^2-2x-3}$ l $\frac{3}{x^2-x-6}-\frac{2}{x^2-2x-3}$

m $\frac{5}{x^2-3x-4}-\frac{3}{x^2-x-2}$ n $\frac{3}{2x^2+7x-4}-\frac{4}{3x^2+14x+8}$

o $\frac{2}{x^2-49}+\frac{4}{x^2-4x-21}$ p $\frac{4}{2x^2+x-1}-\frac{1}{x^2-1}$

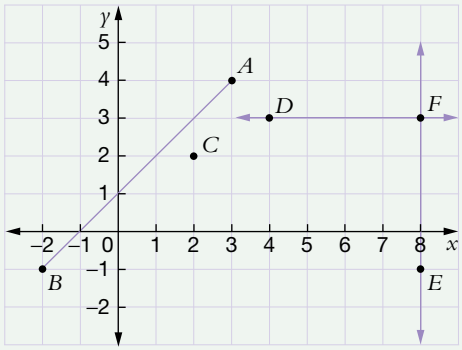
q $\frac{x+1}{x^2+5x+6}+\frac{x-1}{x^2-9}$ r $\frac{x+3}{x^2-16}-\frac{x+2}{x^2-4x}$

s $\frac{2x}{5x^2-20}+\frac{x+1}{x^2+4x+4}$ t $\frac{5x+2}{2x^2-5x-3}+\frac{3x-1}{4x^2-1}$

FUN SPOT 10:10

WHAT DID THE CATERPILLAR SAY WHEN IT SAW THE BUTTERFLY?

Answer each question and write the letter for that question in the box above the correct answer.



For the number plane on the right, find:

E the equation of the x-axis
E the distance BC
E the midpoint of AB
E the equation of AB
F the gradient of DF
H the intersection of DF and EF
I the distance of F from the origin
G the equation of the y-axis
E the distance AB
G the gradient of AB
E the y-intercept of DF
H the equation of DF
I the intersection of AB and BC.

Simplify:

I $10x^2 + x^2$ **L** $10x^2 - x^2$ **L** $10x^2 \times x^2$
M $10x^2 \div x^2$ **N** $\frac{1}{4}$ of $8x^4$ **N** 5% of $40x$
N $(2x^2)^3$ **N** $\frac{x}{2} + \frac{x}{2}$

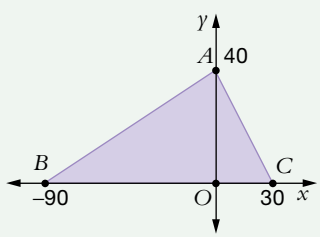
A playing card is chosen at random from a standard pack.
Find the probability that it is:

O the Ace of spades **P** a heart
R a King **S** a picture card
T a red card greater than 3 but less than 9.

Expand and simplify:

O $(x + 1)(x + 7)$ **O** $(a - b)(a + b)$
S $(a + b)^2$ **T** $(a - b)^2$

On the number plane shown, find the length of:



T OC **U** OA **U** AC
V BC **W** OB **Y** AB
O What is the area of ΔABC?

$\sqrt{9700}$	$a^2 - b^2$	40	90	$\sqrt{73}$	$9x^2$	$10x^4$	$2x^4$	$\sqrt{50}$	120	5	$\frac{1}{13}$	$x = 0$	$y = 0$	$\frac{5}{26}$	10	3	50	$\frac{1}{4}$

$11x^2$	$8x^6$	2400 units^2	$2x$	$(\frac{1}{2}, 1\frac{1}{2})$	$x^2 + 8x + 7$	0	$a^2 - 2ab + b^2$	$y = 3$	$\frac{1}{52}$	$a^2 + 2ab + b^2$	$y = x + 1$	30	$(8, 3)$	$(-2, -1)$	x	1	$\frac{3}{13}$

MATHS TERMS 10

algebraic expression
- a group of terms that are joined by addition or subtraction signs

binomial
- an algebraic expression consisting of two terms,
  e.g. $2x + 4$, $3x - 2y$

brackets
- the name given to these grouping symbols: [ ]

cancel
- to simplify a fraction by dividing the numerator and denominator by a common factor.
  e.g. $\frac{{}^{3}\cancel{21}\,\cancel{x}\,y}{{}_{2}\cancel{14}\,\cancel{x}} \begin{matrix}\div 7x \\ \div 7x\end{matrix} = \frac{3y}{2}$

coefficient
- the number that multiplies a pronumeral in an algebraic expression,
  e.g. In $3x - 5y$:
  - the coefficient of x is 3
  - the coefficient of y is -5

denominator
- the bottom of a fraction

difference of two squares
- the result of multiplying two binomials that are the sum and difference of the same terms,
  e.g. $(a + 3)(a - 3) = a^2 - 3^2$
  $= a^2 - 9$

expand
- to remove grouping symbols by multiplying each term inside grouping symbols by the term or terms outside

factorise
- to write an expression as a product of its factors
- the reverse of expanding

like terms
- terms that have identical pronumeral parts,
  e.g. $7x$ and $10x$
  $5a^2b$ and $-3a^2b$
- only like terms may be added or subtracted together. This is called 'collecting like terms'

numerator
- the top of a fraction

parentheses
- the name given to these grouping symbols: ( )

perfect square
- a binomial multiplied by itself,
  e.g. $(x + 5)^2$ or $(2a - 3b)^2$

product
- the result of multiplying terms or expressions together

quadratic trinomial
- an expression such as $x^2 + 4x + 3$
- the highest power of the variable is 2

trinomial
- an algebraic expression consisting of three terms

You can really sink your teeth into algebra.

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Simplify the following: 10:01

a $\frac{a}{4}+\frac{a+3}{3}$ b $\frac{2x+3}{5}+\frac{x-4}{3}$ c $\frac{m-2}{4}-\frac{m-4}{6}$

2 Expand and simplify: 10:02A

a $(x+2)^2$ b $(a-7)^2$ c $(2y+5)^2$ d $(m-n)^2$

3 Expand and simplify: 10:02B

a $(x+3)(x-3)$ b $(y-7)(y+7)$ c $(2a+5)(2a-5)$ d $(x+y)(x-y)$

4 Factorise by taking out a common factor. 10:03

a $3x-12$ b $ax+ay$ c $-2x-6$ d $ax+bx-cx$

5 Factorise by grouping in pairs. 10:04

a $ax+bx+2a+2b$ b $6m+6n+am+an$

c $xy-x+y-1$ d $ab+4c+4a+bc$

6 Factorise these differences of two squares. 10:05

a x^2-25 b a^2-x^2 c $4-m^2$ d $9x^2-1$

7 Factorise these trinomials. 10:06

a $x^2+7x+12$ b x^2-5x+6 c $x^2-3x-10$ d x^2+x-20

8 Factorise: 10:07

a $2x^2+11x+5$ b $3x^2-11x+6$ c $4x^2-x-18$ d $6x^2+5x+1$

9 Simplify, by first factorising where possible. 10:09

a $\frac{6x+12}{6}$ b $\frac{12a-18}{14a-21}$ c $\frac{x^2+5x}{ax+5a}$ d $\frac{x^2+3x-10}{x^2-4}$

10 Simplify: 10:09

a $\frac{3x+6}{4}\times\frac{8x}{x+2}$ b $\frac{a^2+5a+6}{a^2-9}\times\frac{a^2-1}{a^2+3a+2}$

c $\frac{3m-6}{m+3}\div\frac{5m-10}{3m+9}$ d $\frac{x^2-3x-10}{x^2-x-6}\div\frac{x^2-7x+10}{x^2-4}$

11 Simplify: 10:10

a $\frac{2}{x+3}+\frac{1}{x-1}$ b $\frac{1}{x(x+2)}-\frac{1}{(x+2)(x+1)}$

c $\frac{5}{x^2-9}+\frac{3}{2x-6}$ d $\frac{x}{x^2+7x+12}-\frac{x+2}{x^2+2x-3}$

ASSIGNMENT 10A Chapter review

1 Simplify the following expressions.

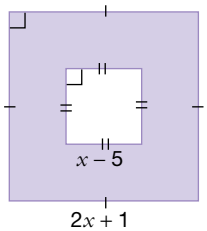
a $\frac{2a-1}{5}+\frac{3a-2}{10}$ **b** $\frac{p-1}{6}+\frac{2-p}{9}$

c $\frac{x-1}{5}-\frac{x-2}{6}$ **d** $\frac{2n+1}{6}-\frac{1-2n}{8}$

2 Expand and simplify:

a $(x+5)^2$
b $(x-5)(x+5)$
c $(a-7)^2$
d $(a+7)(a-7)$
e $(3y+4)^2$
f $(3y-4)(3y+4)$
g $(5-2z)^2$
h $(5+2z)(5-2z)$
i $(x+6)^2+(x-6)^2$
j $(2n-1)^2-(2n-1)(2n+1)$

3 Find the simplest expression for the shaded area of this figure.



4 Factorise the following expressions.

a $a^2+9a+20$
b $2p-4q$
c $m^2-4m-45$
d $5x^3+10x^2+x+2$
e $4x^2-1$
f x^2y-xy
g $6a^2-13a+5$
h x^2+x-30
i $3a^2-4a-15$
j $xy+xz+py+pz$
k $2x^2+x-1$
l x^3-3x^2+2x-6
m $-5ab-10a^2b^2$
n $x^2-y^2+2x-2y$
o $2-3x-9x^2$

5 Factorise each expression fully.

a $2y^2-18$
b $3r^2+9r-84$
c $4x^3+6x+4x^2+6$
d $2-18x^2$
e a^3+a^2-72a
f $33+36a+3a^2$
g $(x-y)^2+x-y$
h $(x-2)^2-4$

6 Simplify each of the following.

a $\frac{x^2+9x-36}{x^2-9}$

b $\frac{20x^2-5}{2x^2+5x-3}$

c $\frac{3}{x+2}+\frac{2}{x+3}$

d $\frac{x}{x-1}-\frac{2x}{x-2}$

e $\frac{x^2-1}{5x}\times\frac{x^2+x}{x^2+2x+1}$

f $\frac{x-1}{x^2-4}\div\frac{x^2-4x+3}{x^2-x-6}$

g $\frac{x+1}{x+2}-\frac{x+2}{x+1}$

h $\frac{2}{3x-1}+\frac{1}{(3x-1)^2}$

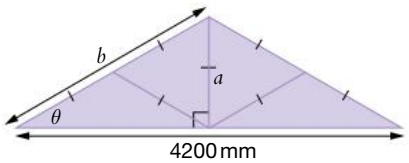
i $\frac{4}{3+2x}-\frac{3}{2x+3}$

j $\frac{x^2+5x-14}{5x^2-20}\times\frac{x^2+4x+4}{x^2-49}$

ASSIGNMENT 10B Working mathematically

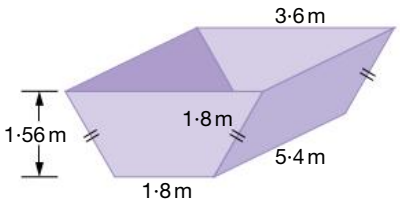
1 When it is 12 midnight in New York it is 6 am in Paris. A plane leaves New York at 8 pm. If it takes 8 hours to fly to Paris, what will the time be in Paris when it arrives?

2 A builder is asked to make the window shown. What is the value of θ, a and b?



3 Decrease \$360 by 20% and then increase the result by 20%. What is the difference between \$360 and your final answer?

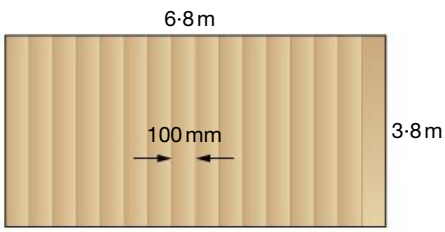
4 A tank is made by welding five sheets of metal together. Three of the sheets are rectangles and two are trapeziums.



a Calculate the length of welds needed to join the sheets.

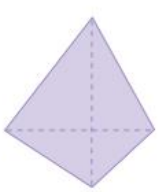
b Calculate the area of metal that needs to be painted if both the inside and outside are to be painted.

5 The diagram shows a rectangular floor 6·8 m long by 3·8 m wide. It is to be covered with floorboards 100 mm wide. How many lineal metres of floorboard need to be bought to cover the floor?

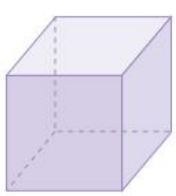


6 **a** Use this formula to calculate the number of edges on the solids shown below.

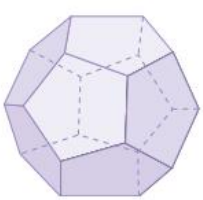
$$\text{No. of edges} = \frac{\text{No. of faces} \times \text{no. of edges on each face}}{2}$$



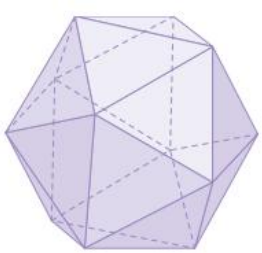
Regular tetrahedron



Cube (or regular hexahedron)



Regular dodecahedron (12 faces)



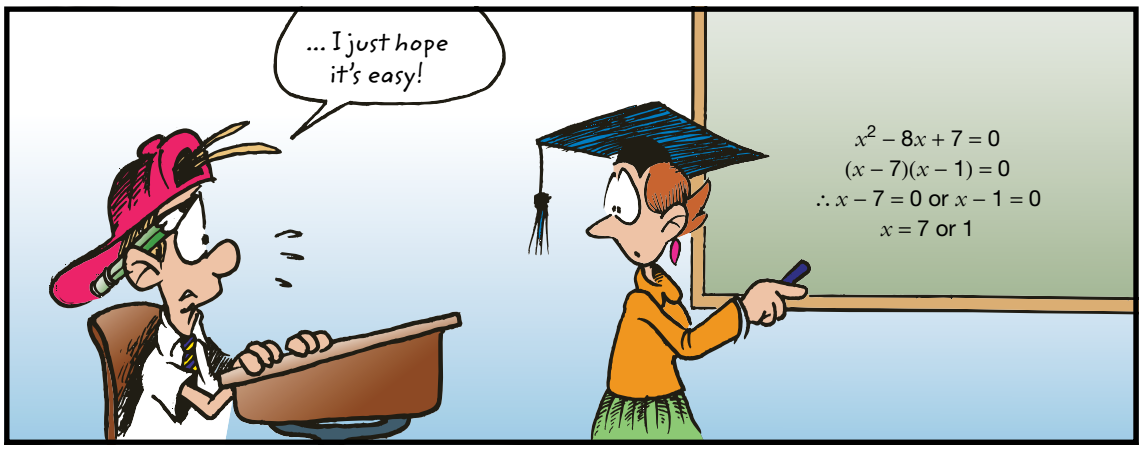
Regular icosahedron (20 faces)

b How could the formula in part **a** be changed to find the number of vertices on each solid?

c Find the number of vertices on each solid.

FURTHER EQUATIONS

11



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11:06 Problems involving quadratic equations
Challenge 11:06 Equations reducible to quadratics
Investigation 11:06 Simple cubic equations $ax^3 = k$
11:07 Simultaneous equations involving a non-linear equation
11:08 Solving literal equations (1)
11:09 Solving literal equations (2)
Fun spot 11:09 Did you know that $2 = 1$?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Equations*** [Stage 5.3§]

- Solve complex linear equations involving algebraic fractions (NSW)
- Solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)
- Solve simple cubic equations (NSW)
- Rearrange literal equations (NSW)
- Solve simultaneous equations, where one equation is non-linear, using algebraic and graphical techniques, including the use of digital technologies (NSW)

Working Mathematically

• Communicating • Problem Solving • Reasoning • Understanding • Fluency

11:01 Linear equations involving algebraic fractions

Equations involving fractions were studied in Year 9. These are reviewed and extended in this section. The Prep quiz should remind you of some basic types of algebraic equations.

PREP QUIZ 11:01

Solve each equation. All solutions are integers.

1 $\frac{x}{3} = 4$ **2** $\frac{m}{2} - 3 = 5$ **3** $\frac{3a}{5} = 6$ **4** $\frac{z+3}{2} = 5$ **5** $\frac{10-x}{4} = 2$

6 $\frac{2n+3}{5} = 1$ **7** $\frac{t}{2} + 3 = 2t$ **8** $\frac{m-3}{4} = m$ **9** $\frac{x-5}{3} = \frac{x}{4}$ **10** $\frac{y}{3} + \frac{y}{5} = 8$

Remember these rules when solving equations with fractions.

- If there is only one denominator, multiply both sides of the equation by this denominator.
- If there is more than one denominator, multiply both sides of the equation by the lowest common denominator (LCD).

WORKED EXAMPLE 1

Examine the solution of each equation.

a
$$\frac{3x+8}{4} = 5$$
$$\times 4 \quad \times 4$$
$$\frac{\cancel{4}(3x+8)}{\cancel{4}} = 5 \times 4$$
$$3x + 8 = 20$$
$$3x = 12$$
$$\therefore x = 4$$

Multiply both sides by the denominator.

b
$$\frac{a-5}{3} = a - 4$$
$$\times 3 \quad \times 3$$
$$\frac{\cancel{3}(a-5)}{\cancel{3}} = 3(a-4)$$
$$a - 5 = 3a - 12$$
$$7 = 2a$$
$$\therefore a = 3\frac{1}{2}$$

c
$$\frac{2x}{3} - \frac{x}{5} = 4$$
$$\times 15 \quad \times 15$$
$$15\left(\frac{2x}{3} - \frac{x}{5}\right) = 4 \times 15$$
$$\frac{30x}{3} - \frac{15x}{5} = 60$$
$$10x - 3x = 60$$
$$7x = 60$$
$$\therefore x = \frac{60}{7} \text{ or } 8\frac{4}{7}$$

Multiply both sides by the LCD.

d
$$\frac{3m-1}{4} = \frac{2m+3}{6}$$
$$\times 12 \quad \times 12$$
$$\frac{^{3}\cancel{12}(3m-1)}{\cancel{4}_{1}} = \frac{^{2}\cancel{12}(2m+3)}{\cancel{6}_{1}}$$
$$3(3m-1) = 2(2m+3)$$
$$9m - 3 = 4m + 6$$
$$5m = 9$$
$$m = \frac{9}{5} \text{ or } 1\frac{4}{5}$$

WORKED EXAMPLE 2

a $\dfrac{3n-2}{4}+\dfrac{2n-1}{5}=6$

multiply both sides by 20

$20\left(\dfrac{3n-2}{4}+\dfrac{2n-1}{5}\right)=6\times 20$

$5(3n-2)+4(2n-1)=120$

$15n-10+8n-4=120$

$23n-14=120$

$23n=134$

$\therefore\ n=\frac{134}{23}$ or $5\frac{19}{23}$

Check this line

b $\dfrac{y-1}{4}-\dfrac{2y+3}{3}=\dfrac{1}{2}$

multiply both sides by 12

$12\left(\dfrac{y-1}{4}-\dfrac{2y+3}{3}\right)=\dfrac{1}{2}\times 12$

$3(y-1)-4(2y+3)=6$

$3y-3-8y-12=6$

$-5y-15=6$

$-5y=21$

$\therefore\ y=-\frac{21}{5}$ or $-4\frac{1}{5}$

Exercise 11:01

P Foundation worksheet 11:01 Equations with fractions

1 Solve these equations. All answers are integers.

a $\dfrac{x+8}{3}=4$ **b** $\dfrac{3a-2}{4}=1$ **c** $\dfrac{5y+7}{2}=11$

d $\dfrac{m-4}{3}=m$ **e** $\dfrac{a+9}{5}=a+1$ **f** $\dfrac{x-2}{4}=2x+3$

g $\dfrac{2a+4}{7}=a-3$ **h** $\dfrac{5n-4}{6}=2n-3$ **i** $4-2p=\dfrac{4p-8}{9}$

2 Solve:

a $\dfrac{x}{2}-\dfrac{x}{5}=3$ **b** $\dfrac{2a}{5}+\dfrac{a}{3}=11$ **c** $\dfrac{3m}{4}-\dfrac{m}{3}=10$

d $\dfrac{z}{2}-\dfrac{2z}{7}=9$ **e** $\dfrac{3n}{4}+\dfrac{5n}{6}=19$ **f** $\dfrac{5y}{4}-\dfrac{6y}{5}=2$

g $\dfrac{k}{2}+\dfrac{k}{5}=\dfrac{7}{3}$ **h** $\dfrac{2a}{5}-\dfrac{a}{4}=\dfrac{3}{2}$ **i** $\dfrac{3x}{2}-\dfrac{2x}{3}=\dfrac{5}{12}$

3 Solve these equations that have pronumerals on both sides.

a $\dfrac{m-1}{3}=\dfrac{m+3}{4}$ **b** $\dfrac{3x-2}{2}=\dfrac{2x+7}{3}$ **c** $\dfrac{3y+2}{4}=\dfrac{3y-1}{6}$

d $\dfrac{5n+1}{4}=\dfrac{5n+3}{2}$ **e** $\dfrac{8k-2}{7}=\dfrac{2k-3}{3}$ **f** $\dfrac{3-2x}{4}=\dfrac{3-x}{5}$

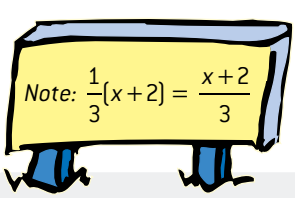
4 Solve:

a $\dfrac{x+3}{2}+\dfrac{x+5}{5}=6$ **b** $\dfrac{a+1}{3}+\dfrac{a+1}{4}=7$ **c** $\dfrac{p-3}{3}+\dfrac{p-2}{4}=2$

d $\dfrac{2b-3}{4}+\dfrac{1+3b}{2}=6$ **e** $\dfrac{2q-3}{2}+\dfrac{q+4}{3}=1$ **f** $\dfrac{3k+1}{5}+\dfrac{k+1}{4}=3$

g $\dfrac{x+3}{2}-\dfrac{x+1}{3}=4$ **h** $\dfrac{a+2}{4}-\dfrac{a-1}{6}=2$

i $\dfrac{1}{3}(x+2)-\dfrac{1}{5}(2x-1)=3$ **j** $\dfrac{2}{3}(a+1)-\dfrac{3}{4}(1+a)=1$



5 **a** $\frac{x+3}{2}+\frac{x+1}{3}=\frac{x-2}{4}$ **b** $\frac{2a+5}{3}+\frac{a-1}{2}=\frac{6a+2}{5}$

c $\frac{3m-1}{2}-\frac{m+2}{5}=\frac{5m}{4}$ **d** $\frac{2a+3}{2}-\frac{a-2}{3}=\frac{a-2}{4}$

11:02 Quadratic equations: Solution using factors

Quadratic equations were introduced in Chapter 2. Equations such as $x^2 - 4x + 3 = 0$ were solved by first factorising the left-hand side of the equation. However, the factorising of trinomial expressions such as $2x^2 + 7x + 3$, where the coefficient of the squared term was not 1, were not met until Chapter 10.

The following Prep quiz will recall the factorising skills that will be needed for this section.

PREP QUIZ 11:02

Factorise each expression.

1 $x^2 + 3x$ **2** $x^2 - 9$ **3** $4x^2 - 25$ **4** $x^2 + 7x + 12$

5 $x^2 - 2x - 35$ **6** $x^2 - 6x + 9$ **7** $2x^2 + 9x - 5$ **8** $3x^2 + 14x + 8$

9 $4x^2 + 20x + 25$ **10** $12x^2 - x - 6$

You should remember the following:

- A quadratic equation is one in which the highest power of the unknown pronumeral is 2. So equations such as:
  $x^2 - 4x + 3 = 0$, $x^2 + 5x = 0$, $x^2 - 25 = 0$ and $2x^2 - 2x + 7 = 0$
  are all quadratic equations.
- Solving a quadratic equation using factors depends on the following observation (called the Null Factor Law).

A quadratic equation is an equation of the 'second degree'.

If $p \times q = 0$, then at least one of p and q must be zero.

WORKED EXAMPLE 1

Solve the quadratic equations.

a If $(x - 1)(x + 7) = 0$
then either
$x - 1 = 0$ or $x + 7 = 0$
$\therefore x = 1$ or $x = -7$

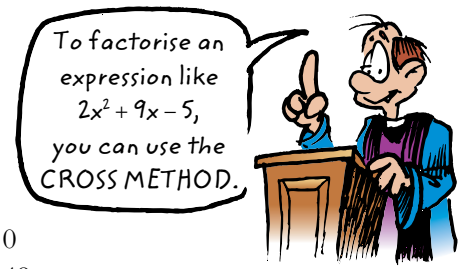
b If $2x(x + 3) = 0$
then either
$2x = 0$ or $x + 3 = 0$
$\therefore x = 0$ or $x = -3$

c If $(2x - 1)(3x + 5) = 0$
then either
$2x - 1 = 0$ or $3x + 5 = 0$
$2x = 1$ or $3x = -5$
$\therefore x = \frac{1}{2}$ or $x = -\frac{5}{3}$

A quadratic equation can have two solutions.

WORKED EXAMPLE 2

To solve these equations, first factorise them.



a $x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x + 3 = 0$ or $x + 1 = 0$

$\therefore x = -3$ or $x = -1$

b $x^2 - 49 = 0$

$(x - 7)(x + 7) = 0$

$x - 7 = 0$ or $x + 7 = 0$

$\therefore x = 7$ or $x = -7$

Or $x^2 - 49 = 0$

$x^2 = 49$

$\therefore x = 7$ or -7

i.e. $x = \pm 7$

c $2x^2 + 9x - 5 = 0$

$(2x - 1)(x + 5) = 0$

$2x - 1 = 0$ or $x + 5 = 0$

$\therefore x = \frac{1}{2}$ or $x = -5$

WORKED EXAMPLE 3

Before these equations are solved, all the terms are gathered to one side of the equation and equated to zero.

a $x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x + 4 = 0$ or $x - 3 = 0$

$\therefore x = -4$ or $x = 3$

b $5x^2 = 2x$

$5x^2 - 2x = 0$

$x(5x - 2) = 0$

$x = 0$ or $5x - 2 = 0$

$\therefore x = 0$ or $x = \frac{2}{5}$

c $6x^2 = 5x + 6$

$6x^2 - 5x - 6 = 0$

$(3x + 2)(2x - 3) = 0$

$3x + 2 = 0$ or $2x - 3 = 0$

$\therefore x = -\frac{2}{3}$ or $x = \frac{3}{2}$

To solve a quadratic equation:

- gather all the terms to one side of the equation
- factorise
- solve the two resulting simple equations.

Check your solutions by substitution.

For Worked Example 3a:

Substituting $x = -4$

$x^2 + x = 12$

LHS $= (-4)^2 + (-4)$

$= 16 - 4$

$= 12$

$= $ RHS

Substituting $x = 3$

$x^2 + x = 12$

LHS $= (3)^2 + (3)$

$= 9 + 3$

$= 12$

$= $ RHS

$\therefore$ Both $x = -4$ and $x = 3$ are solutions.

LHS is left-hand side.
RHS is right-hand side.

Exercise 11:02

P Foundation worksheet 11:02
Quadratic equations

1 Find the two solutions for each equation. Check by substitution to ensure your answers are correct.

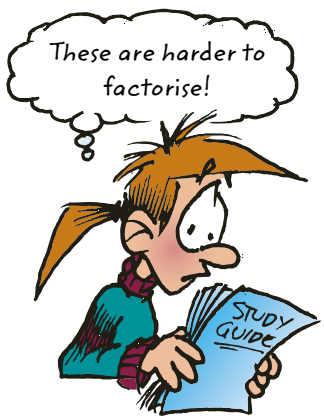
a $x(x-7)=0$
b $5a(a+4)=0$
c $2m(3m-2)=0$
d $(y-3)(y+3)=0$
e $(n+4)(n-4)=0$
f $(3x-2)(3x+2)=0$
g $(a+2)(a-5)=0$
h $(t-1)(t+7)=0$
i $(w-9)(w+2)=0$
j $(x+2)(2x-3)=0$
k $(5q-1)(q-8)=0$
l $(6z+5)(z-7)=0$
m $(3u-1)(2u+5)=0$
n $(4p-3)(5p-8)=0$
o $(5q+3)(3q+5)=0$
p $(8x-3)(7x+1)=0$
q $(2y+9)(7y+8)=0$
r $(10x+3)(3x-10)=0$

2 After factorising the left-hand side of each equation, solve the following.

a $m^2+3m=0$
b $x^2-7x=0$
c $3y^2-9y=0$
d $5p^2+3p=0$
e $6a^2-4a=0$
f $4z^2+10z=0$
g $x^2-16=0$
h $n^2-81=0$
i $9t^2-4=0$
j $25q^2-9=0$
k $64w^2-25=0$
l $49p^2-100=0$
m $x^2+5x+6=0$
n $m^2-6m+8=0$
o $y^2-y-90=0$
p $v^2+4v-21=0$
q $z^2-15z+54=0$
r $x^2-7x-170=0$

3 Factorise and solve the following.

a $2x^2+x-1=0$
b $3x^2+7x+2=0$
c $3x^2+17x+10=0$
d $2x^2-11x+12=0$
e $2x^2-x-10=0$
f $2x^2-11x-21=0$
g $4x^2+21x+5=0$
h $4x^2-19x-5=0$
i $4x^2-21x+5=0$
j $5x^2+16x+3=0$
k $2x^2+13x-24=0$
l $7x^2+48x-7=0$
m $4x^2-4x-3=0$
n $6x^2-x-1=0$
o $9x^2+9x+2=0$
p $10x^2+9x+2=0$
q $12x^2-7x+1=0$
r $10x^2-13x+4=0$



4 Gather all the terms to one side of each equation and then solve.

a $x^2=3x$
b $m^2=8m$
c $x^2=-5x$
d $x^2=5x-4$
e $a^2=2a+15$
f $y^2=3y-2$
g $m^2=9m-18$
h $n^2=7n+18$
i $h^2=4h+32$
j $x^2+x=2$
k $y^2+2y=3$
l $x^2-7x=-10$
m $y^2+3y=18$
n $t^2+3t=28$
o $y^2+2y=15$
p $2x^2+x=1$
q $2x^2-x=15$
r $4m^2-5m=6$
s $3x^2=13x-14$
t $5p^2=17p-6$
u $2x^2=11x-5$

Check answers by substitution.

11:03 Solution by completing the square

This method depends upon completing an algebraic expression to form a perfect square, that is, an expression of the form $(x + a)^2$ or $(x - a)^2$.

Reminder:
$(x + a)^2 = x^2 + 2ax + a^2$
- The middle term in the expansion is $2ax$.
- The coefficient of x in the middle term is $2a$.

So, the value of a can be found by halving the coefficient of the x term.
This is also true for: $(x - a)^2 = x^2 - 2ax + a^2$

WORKED EXAMPLE 1

What must be added to the following to make perfect squares?

a $x^2 + 8x$ **b** $x^2 - 5x$

Solutions

a $x^2 + 8x + \ldots$
Half of 8 is 4, so the perfect square is:
$x^2 + 8x + 4^2 = (x + 4)^2$
$= x^2 + 8x + 16$
So 16 must be added to make a perfect square.

b $x^2 - 5x + \ldots$
Half of -5 is $-\frac{5}{2}$, so the perfect square is:
$x^2 - 5x + (-\frac{5}{2})^2 = (x - \frac{5}{2})^2$
$= x^2 - 5x + \frac{25}{4}$
So $\frac{25}{4}$ must be added to make a perfect square.

Now, to solve a quadratic equation using this technique follow these steps.

$x^2 + 4x - 21 = 0$
$x^2 + 4x = 21$ ← Move the constant to the RHS.
$x^2 + 4x + 2^2 = 21 + 2^2$ ← Add (half of x coefficient)^2 to both sides.
$\therefore (x + 2)^2 = 25$
$x + 2 = \pm\sqrt{25}$
$x = -2 \pm 5$
$\therefore x = 3$ or -7

Note that $x^2 + 4x - 21 = 0$ could have been solved using the Null Factor Law as in the previous exercise.

$x^2 + 4x - 21 = 0$
$(x - 3)(x + 7) = 0$
$x = 3$ or $x = -7$

In this case, this is an easier and quicker way to find the solution. The method of completing the square, however, can determine the solution of quadratic equations that cannot be factorised. This can be seen in the following examples.

WORKED EXAMPLE 2

Solve:

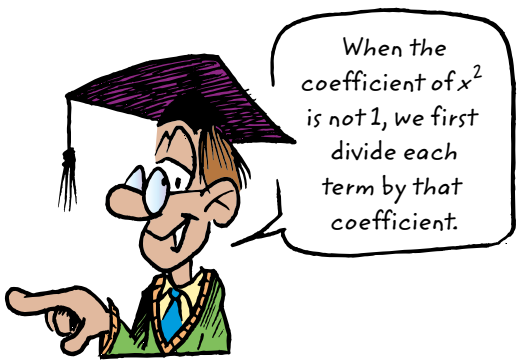
a $x^2 + 6x + 1 = 0$ **b** $x^2 - 3x - 5 = 0$ **c** $3x^2 - 4x - 1 = 0$

Solutions

a
$$x^2 + 6x + 1 = 0$$
$$x^2 + 6x = -1$$
$$x^2 + 6x + 3^2 = -1 + 3^2$$
$$(x + 3)^2 = 8$$
$$x + 3 = \pm\sqrt{8}$$
$$\therefore\ x = -3 \pm \sqrt{8}$$
$$x = -3 + \sqrt{8} \text{ or } -3 - \sqrt{8}$$
$$(x \div -0{\cdot}17 \text{ or } -5{\cdot}83)$$

Note that the solution involves a square root, i.e. the solution is irrational. Using your calculator, approximations may be found.

b
$$x^2 - 3x - 5 = 0$$
$$x^2 - 3x = 5$$
$$x^2 - 3x + (-\tfrac{3}{2})^2 = 5 + (-\tfrac{3}{2})^2$$
$$(x - \tfrac{3}{2})^2 = 7\tfrac{1}{4}$$
$$x - \tfrac{3}{2} = \pm\sqrt{7\tfrac{1}{4}}$$
$$\therefore\ x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$
$$x = \frac{3 + \sqrt{29}}{2} \text{ or } \frac{3 - \sqrt{29}}{2}$$
$$(x \div 4{\cdot}19 \text{ or } -1{\cdot}19)$$



c
$$3x^2 - 4x - 1 = 0$$
$$x^2 - \tfrac{4}{3}x - \tfrac{1}{3} = 0$$
$$x^2 - \tfrac{4}{3}x = \tfrac{1}{3}$$
$$x^2 - \tfrac{4}{3}x + (-\tfrac{2}{3})^2 = \tfrac{1}{3} + (-\tfrac{2}{3})^2$$
$$(x - \tfrac{2}{3})^2 = \tfrac{7}{9}$$
$$x - \tfrac{2}{3} = \pm\frac{\sqrt{7}}{3}$$
$$\therefore\ x = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$$
$$x = \frac{2 + \sqrt{7}}{3} \text{ or } \frac{2 - \sqrt{7}}{3}$$
$$(x \div 1{\cdot}55 \text{ or } -0{\cdot}22)$$

You can use the following fact to check your answers.
For the equation $ax^2 + bx + c = 0$
the two solutions must add to $-\frac{b}{a}$.
In Worked Example 2a, $(-0{\cdot}17) + (-5{\cdot}83) = -6$ or $\frac{-6}{1}$
In Worked Example 2c, $1{\cdot}55 + (-0{\cdot}22) = 1{\cdot}33\ \left[\div\frac{4}{3}\right]$

Exercise 11:03

1 What number must be inserted into each expression to complete the square?

a $x^2 + 6x + \ldots^2 = (x + \ldots)^2$
b $x^2 + 8x + \ldots^2 = (x + \ldots)^2$
c $x^2 - 2x + \ldots^2 = (x - \ldots)^2$
d $x^2 - 4x + \ldots^2 = (x - \ldots)^2$
e $x^2 + 3x + \ldots^2 = (x + \ldots)^2$
f $x^2 - 7x + \ldots^2 = (x - \ldots)^2$
g $x^2 + 11x + \ldots^2 = (x + \ldots)^2$
h $x^2 - x + \ldots^2 = (x - \ldots)^2$
i $x^2 + \frac{5x}{2} + \ldots^2 = (x + \ldots)^2$
j $x^2 - \frac{2x}{3} + \ldots^2 = (x - \ldots)^2$

2 Solve the following equations, leaving your answers in surd form.

a $(x - 2)^2 = 3$
b $(x + 1)^2 = 2$
c $(x + 5)^2 = 5$
d $(x - 1)^2 = 10$
e $(x - 3)^2 = 7$
f $(x + 2)^2 = 11$
g $(x + 3)^2 = 8$
h $(x + 10)^2 = 12$
i $(x - 3)^2 = 18$
j $(x + \frac{1}{2})^2 = 5$
k $(x - \frac{2}{3})^2 = 3$
l $(x + 1\frac{1}{2})^2 = 12$
m $(x - 1)^2 = 2\frac{1}{2}$
n $(x + 3)^2 = 4\frac{1}{2}$
o $(x - \frac{1}{3})^2 = \frac{5}{9}$

3 Solve the following equations by completing the square. Also find approximations for your answers, correct to two decimal places.

a $x^2 + 2x - 1 = 0$
b $x^2 - 2x - 5 = 0$
c $x^2 - 4x - 8 = 0$
d $x^2 + 6x - 8 = 0$
e $x^2 - 6x + 2 = 0$
f $x^2 + 4x + 1 = 0$
g $x^2 + 10x = 5$
h $x^2 + 2x = 4$
i $x^2 - 12x = 1$
j $x^2 + 5x + 2 = 0$
k $x^2 + 7x - 3 = 0$
l $x^2 + x - 3 = 0$
m $x^2 + 9x + 3 = 0$
n $x^2 + 3x - 5 = 0$
o $x^2 - 11x + 5 = 0$
p $x^2 - x = 3$
q $x^2 + 3x = 2$
r $x^2 - 5x = 1$
s $2x^2 - 4x - 1 = 0$
t $2x^2 + 3x - 4 = 0$
u $2x^2 - 8x + 1 = 0$
v $3x^2 + 2x - 3 = 0$
w $5x^2 - 4x - 3 = 0$
x $4x^2 - x - 2 = 0$

11:04 The quadratic formula

As we have seen in the previous section, a quadratic equation is one involving a squared term. In fact, any quadratic equation can be represented by the general form of a quadratic equation:

$ax^2 + bx + c = 0$ where a, b, c are all integers, and $a \neq 0$.

If any quadratic equation is arranged in this form, a formula using the values of a, b and c can be used to find the solutions.

The quadratic formula for $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof of the quadratic formula:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

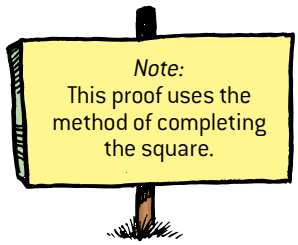
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



WORKED EXAMPLE 1

Solve the following by using the quadratic formula.

a $2x^2 + 9x + 4 = 0$ **b** $x^2 + 5x + 1 = 0$

Solutions

a For the equation $2x^2 + 9x + 4 = 0$,
$a = 2, b = 9, c = 4$.
Substituting into the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times 4}}{2 \times 2}$$

$$= \frac{-9 \pm \sqrt{81 - 32}}{4}$$

$$= \frac{-9 \pm \sqrt{49}}{4}$$

$$= \frac{-9 \pm 7}{4}$$

$$= -\tfrac{2}{4} \text{ or } -\tfrac{16}{4}$$

$\therefore\ x = -\frac{1}{2}$ or -4

b For $x^2 + 5x + 1 = 0$,
$a = 1, b = 5, c = 1$.
Substituting into the formula gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 - 4}}{2}$$

$$= \frac{-5 \pm \sqrt{21}}{2}$$

There is no rational equivalent to $\sqrt{21}$ so the answer may be left as:

$$x = \frac{-5 + \sqrt{21}}{2} \text{ or } \frac{-5 - \sqrt{21}}{2}$$

Approximations for these answers may be found using a calculator.
$x \doteqdot -0{\cdot}21$ or $-4{\cdot}79$ (2 dec. pl.)

WORKED EXAMPLE 2

Solve the following by using the quadratic formula.

a $3x^2 = 2x + 2$ **b** $2x^2 + 2x + 7 = 0$

Solutions

a The equation $3x^2 = 2x + 2$ must first be written in the form

$ax^2 + bx + c = 0$

$3x^2 - 2x - 2 = 0$

So $a = 3, b = -2, c = -2$.

Substituting these values gives:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-2)}}{2 \times 3}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{6}$$

$$= \frac{2 \pm \sqrt{28}}{6}$$

$$\therefore x = \frac{2 + \sqrt{28}}{6} \text{ or } \frac{2 - \sqrt{28}}{6}$$

$x \doteqdot 1{\cdot}22$ or $-0{\cdot}55$ (2 dec. pl.)

b For $2x^2 + 2x + 7 = 0$,

$a = 2, b = 2, c = 7$.

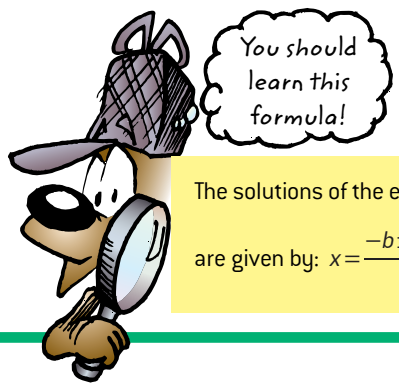
Substituting these values gives:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 2 \times 7}}{2 \times 2}$$

$$= \frac{-2 \pm \sqrt{-52}}{4}$$

But $\sqrt{-52}$ is not real!

So $2x^2 + 2x + 7 = 0$ has no real solutions.



The solutions of the equation $ax^2 + bx + c = 0$ are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 11:04

P Foundation worksheet 11:04 The quadratic formula

1 Use the quadratic formula to solve the following equations. All have rational answers.

a $x^2 + 5x + 6 = 0$	**b** $x^2 + 6x + 8 = 0$	**c** $x^2 + 10x + 9 = 0$
d $x^2 - 3x - 10 = 0$	**e** $x^2 - 2x - 15 = 0$	**f** $x^2 + 4x - 12 = 0$
g $x^2 - 9x + 14 = 0$	**h** $x^2 - 8x + 12 = 0$	**i** $x^2 - 6x + 5 = 0$
j $3x^2 + 7x + 2 = 0$	**k** $2x^2 + 11x + 5 = 0$	**l** $4x^2 + 11x + 6 = 0$
m $2x^2 - 5x - 3 = 0$	**n** $5x^2 - 9x - 2 = 0$	**o** $3x^2 - 5x + 2 = 0$
p $6x^2 + 7x + 2 = 0$	**q** $6x^2 + 7x - 3 = 0$	**r** $8x^2 - 14x + 3 = 0$

2 Solve the following, leaving your answers in surd form. (*Remember:* A surd is an expression involving a square root.)

a $x^2 + 4x + 2 = 0$	**b** $x^2 + 3x + 1 = 0$	**c** $x^2 + 5x + 3 = 0$
d $x^2 + x - 1 = 0$	**e** $x^2 + 2x - 2 = 0$	**f** $x^2 + 4x - 1 = 0$
g $x^2 - 2x - 1 = 0$	**h** $x^2 - 7x + 2 = 0$	**i** $x^2 - 6x + 3 = 0$
j $x^2 - 10x - 9 = 0$	**k** $x^2 - 8x + 3 = 0$	**l** $x^2 - 5x + 7 = 0$
m $2x^2 + 6x + 1 = 0$	**n** $2x^2 + 3x - 1 = 0$	**o** $2x^2 - 7x + 4 = 0$
p $3x^2 + 10x + 2 = 0$	**q** $3x^2 - 9x + 2 = 0$	**r** $5x^2 + 4x - 2 = 0$
s $4x^2 - x + 1 = 0$	**t** $3x^2 - 3x - 1 = 0$	**u** $4x^2 - 3x - 2 = 0$
v $2x^2 + 11x - 5 = 0$	**w** $2x^2 - 9x + 8 = 0$	**x** $5x^2 + 2x - 1 = 0$

3 Use the formula to solve the following. Give answers as decimal approximations correct to two decimal places.

a $x^2 - 4x + 1 = 0$	**b** $x^2 - 6x + 3 = 0$	**c** $x^2 + 8x - 5 = 0$
d $x^2 + 9x + 1 = 0$	**e** $x^2 + 2x - 5 = 0$	**f** $x^2 + 3x - 1 = 0$
g $x^2 + 2 = 0$	**h** $x^2 - 7x = 2$	**i** $x^2 = 6x - 11$
j $2x^2 + x - 2 = 0$	**k** $2x^2 - 5x - 2 = 0$	**l** $3x^2 + 9x + 5 = 0$
m $2x^2 = 7x - 2$	**n** $5x^2 - 3x = 4$	**o** $6x^2 = x + 3$

INVESTIGATION 11:04 HOW MANY SOLUTIONS?

Use the quadratic formula to solve each equation.

a $x^2 + 6x + 5 = 0$ **b** $x^2 + 6x + 9 = 0$ **c** $x^2 + 6x + 12 = 0$

Solutions

a $$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 5}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{16}}{2}$$
$$= \frac{-6 \pm \sqrt{4}}{2}$$
$\therefore x = -1$ or -5

b $$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{0}}{2}$$
$$= \frac{-6}{2}$$
$\therefore x = -3$

c $$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 12}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{-12}}{2}$$
$\sqrt{-12}$ has no real solution
$\therefore x$ has no real solution

It appears that a quadratic equation may have two, one or no real solutions. The 'key' is the part of the formula under the square root sign.

The number of real solutions is determined by $b^2 - 4ac$.

If $b^2 - 4ac$ is:

- *positive* then the equation will have *two* real solutions
- *zero* then the equation will have *one* real solution
- *negative* then the equation will have *no* real solution.

Exercises

By evaluating $b^2 - 4ac$ for each equation, determine how many real solutions it will have.

$b^2 - 4ac$ is called the *discriminant*.

1 $x^2 + 4x + 3 = 0$	**2** $x^2 + 4x + 4 = 0$	**3** $x^2 + 4x + 5 = 0$
4 $x^2 - x - 2 = 0$	**5** $x^2 - x = 0$	**6** $x^2 - x + 2 = 0$
7 $4x^2 - 12x + 9 = 0$	**8** $4x^2 - 12x + 7 = 0$	**9** $4x^2 - 12x + 11 = 0$
10 $5x^2 - x + 7 = 0$	**11** $5x^2 - x - 7 = 0$	**12** $9x^2 + 6x + 1 = 0$

11:05 Choosing the best method

PREP QUIZ 11:05

Factorise:

1 $5x^2 - 10x$ **2** $x^2 - 5x - 14$ **3** $x^2 - 81$ **4** $x^2 + 5x + 6$

Solve:

5 $(x - 2)(x + 7) = 0$ **6** $(2x - 3)(3x + 1) = 0$ **7** $x^2 - 16 = 0$

8 $3x^2 - 12x = 0$ **9** $x^2 - 3x + 2 = 0$

10 Write the formula for the solution of the equation: $ax^2 + bx + c = 0$.

Some quadratic equations may appear in a different form from those seen so far, but they can always be simplified to the general form $ax^2 + bx + c = 0$. They may then be factorised, or solved using the quadratic formula.

WORKED EXAMPLES

Solve the following equations.

1 $x^2 - 2x + 1 = 3x + 6$ **2** $x(x - 5) = 6$ **3** $x = \dfrac{5x - 6}{x}$

Solutions

1 In this example, all the terms must be gathered to the LHS.

$x^2 - 2x + 1 = 3x + 6$

$-3x - 6 = -3x - 6$

$x^2 - 5x - 5 = 0$

This cannot be factorised, so the quadratic formula is used.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 + 20}}{2}$$

$$= \frac{5 \pm \sqrt{45}}{2}$$

$$\therefore x = \frac{5 + \sqrt{45}}{2} \text{ or } \frac{5 - \sqrt{45}}{2}$$

2 Expand and gather the terms to the LHS.

$x(x - 5) = 6$

$x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

Factorising gives:

$(x - 6)(x + 1) = 0$

$\therefore x = 6$ or -1

3 $x = \dfrac{5x - 6}{x}$

Multiplying both sides by x gives:

$x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$

Factorising gives:

$(x - 2)(x - 3) = 0$

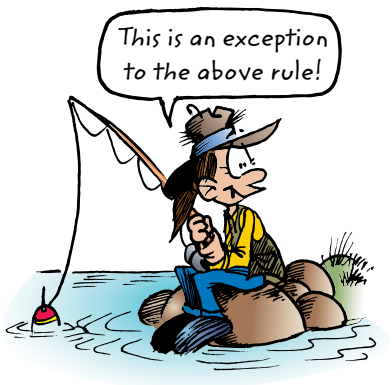
$\therefore x = 2$ or 3

When solving a quadratic equation:

Step 1: Express the equation in general form:
$ax^2 + bx + c = 0$

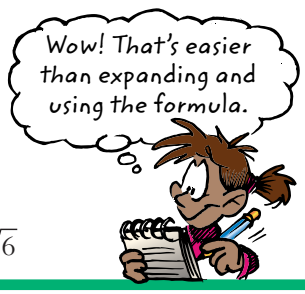
Step 2: Factorise, if you can, and solve it,
or
use the formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

WORKED EXAMPLE



$(a + 7)^2 = 6$

For equations like this, where one side is a perfect square, it is easier to follow the final steps in the method of completing the square.



Solution

$(a + 7)^2 = 6$
$\therefore\ a + 7 = \pm\sqrt{6}$
$a = -7 \pm \sqrt{6}$
$a = -7 + \sqrt{6}$ or $-7 - \sqrt{6}$

Exercise 11:05

1 Solve the following quadratic equations. Give answers to two decimal places if necessary.

a $x^2 + 7x + 6 = 0$	**b** $x^2 - 8x + 12 = 0$	**c** $x^2 + 5x - 24 = 0$
d $x^2 - 3x + 1 = 0$	**e** $x^2 + 3x - 3 = 0$	**f** $x^2 + 4x + 2 = 0$
g $x^2 + 8x = 0$	**h** $x^2 - 10x = 0$	**i** $5x^2 - 10x = 0$
j $x^2 - 81 = 0$	**k** $x^2 - 121 = 0$	**l** $4x^2 - 9 = 0$
m $2x^2 + 4x + 1 = 0$	**n** $3x^2 - x - 1 = 0$	**o** $2x^2 - 5x + 1 = 0$
p $2x^2 + 6x + 4 = 0$	**q** $3x^2 + 15x + 18 = 0$	**r** $2x^2 - 6x - 8 = 0$
s $x^2 = 6x + 27$	**t** $x^2 = 13x - 36$	**u** $2x^2 - 5x = 12$
v $25 = 10x - x^2$	**w** $36 = 13x - x^2$	**x** $2 = 9x - 5x^2$

2 Rearrange each equation below to the form $ax^2 + bx + c = 0$, and solve.

a $x^2 + 9x = 2x - 12$	**b** $x^2 + 20 = 8x + 5$	**c** $x^2 - 4x + 10 = 2x + 2$
d $3x^2 + 5x = 2x^2 - 6$	**e** $4x^2 + 5x = 3x^2 - 2x$	**f** $x^2 + 3x - 10 = 3x - 1$
g $x^2 + 5x = 3x + 1$	**h** $x^2 + 7 = 5 - 4x$	**i** $2x + 1 = x^2 + x$
j $x(x + 5) = 6$	**k** $x(x - 7) = 18$	**l** $x^2 = 4(x + 8)$
m $(m - 1)^2 = 4$	**n** $(x + 3)^2 = 9$	**o** $(x + 5)^2 = 11$
p $(2a + 1)^2 = 16$	**q** $(5y - 3)^2 = 7$	**r** $(6n - 7)^2 = 3$
s $x = \dfrac{2x + 15}{x}$	**t** $x = \dfrac{3x + 28}{x}$	**u** $1 = \dfrac{2 - x^2}{x}$
v $x = \dfrac{5x - 3}{x}$	**w** $\dfrac{3(x + 1)}{x} = x$	**x** $2(x + 2) = \dfrac{1}{x}$

FUN SPOT 11:05 WHAT IS AN ITALIAN REFEREE?

Work out the answer to each question and write the letter for that part in the box that is above the correct answer.

Solve:

A $(x + 3)(x - 5) = 0$
R $(2x - 1)(x - 7) = 0$
M $(x + 1)(x - 1) = 0$
R $(3x + 5)(2x - 3) = 0$

Solve by factorising:

M $x^2 - 5x + 6 = 0$
E $x^2 - 5x - 6 = 0$
A $x^2 - x - 6 = 0$
I $x^2 + x - 6 = 0$

Solve:

N $(x - 2)^2 = 5$
O $(x + 7)^2 = 9$
P $(x + 1)^2 = 2$
U $(x + 3)^2 = 3$

$-2, 3$	$\frac{1}{2}, 7$	$-4, -10$	$2, 3$	$-3, 5$	$2 \pm \sqrt{5}$	$-3 \pm \sqrt{3}$	$-1, 1$	$-1 \pm \sqrt{2}$	$-3, 2$	$-\frac{5}{3}, \frac{3}{2}$	$-1, 6$

11:06 Problems involving quadratic equations

PREP QUIZ 11:06

From the list of numbers, $1, 2, 3\frac{1}{2}, 5{\cdot}2, 9, 10$, write the numbers that are:

1 integers
2 odd
3 square

Write the next two consecutive integers after:

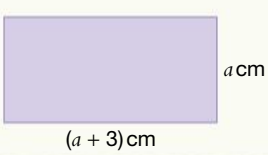
4 8
5 n

Write the next two consecutive even numbers after:

6 10
7 x (x is even)

Write expressions for:

8 a number that is 3 less than x
9 the area of this rectangle
10 the perimeter of this rectangle.



When solving a problem or applying a given formula, a quadratic equation may be involved. Consider the following examples.

WORKED EXAMPLES

1 The product of two consecutive positive even numbers is 48. Find the numbers.
(Hint: If the first number is x, then the next even number is $x + 2$).

2 The length of a rectangle is 5 cm longer than its breadth. If the area of the rectangle is 84 cm^2, find the length of the rectangle.

3 A projectile is fired vertically upwards and its height h, in metres, after t seconds is given by the formula:
$h = 40t - 8t^2$
Find the time taken by the projectile to first reach a height of 48 m.

Solutions

1 The problem gives the equation:
$x(x + 2) = 48$

Solving this gives:
$$x^2 + 2x = 48$$
$$x^2 + 2x - 48 = 0$$
$$(x + 8)(x - 6) = 0$$
$$\therefore x = -8 \text{ or } 6$$

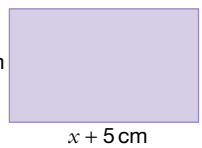
The numbers are positive, so $x = 6$.
$\therefore$ The two consecutive integers are 6 and 8.

3 $h = 40t - 8t^2$
To find the time t when the height $h = 48$ metres, substitute into the formula:
$48 = 40t - 8t^2$

Gathering all the terms on the LHS:
$$8t^2 - 40t + 48 = 0$$
$$8(t^2 - 5t + 6) = 0$$
$$8(t - 2)(t - 3) = 0$$
$$t = 2 \text{ or } 3$$

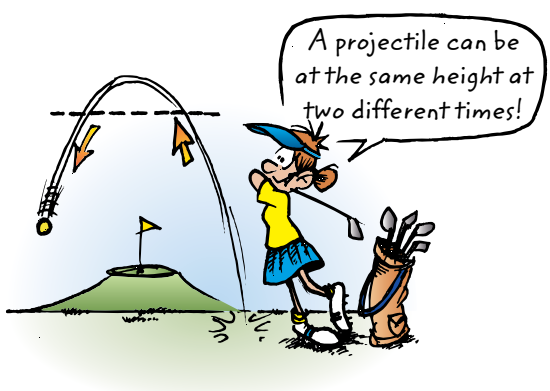
$\therefore$ The projectile was at a height of 48 m at 2 seconds and 3 seconds. So it took 2 seconds to first reach a height of 48 m.

2 If the breadth is x, then the length is $x + 5$.
Since the area of a rectangle is equal to length times breadth, then:



$$x(x + 5) = 84$$
$$x^2 + 5x = 84$$
$$x^2 + 5x - 84 = 0$$
$$(x + 12)(x - 7) = 0$$
$$\text{So } x = -12 \text{ or } 7$$

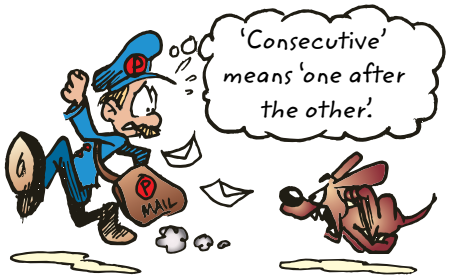
Now, since the dimensions must be positive, the breadth must be 7 cm.
$\therefore$ The length is 12 cm.



Exercise 11:06

1 Find the two consecutive positive integers:

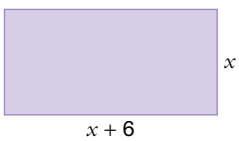
- **a** if their product is 20
- **b** if their product is 90
- **c** if they are even and their product is 120
- **d** if they are odd and their product is 63.



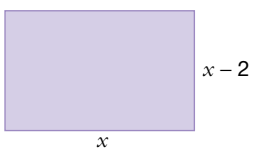
2 **a** The sum of a positive integer and its square is 90. Find the number.

- **b** The sum of a positive integer and its square is 132. Find the number.
- **c** The difference between a positive integer and its square is 56. Find the number.
- **d** The square of a number is equal to 5 times the number. What are the two possible numbers?
- **e** When a number is subtracted from its square, the result is 42. Find the two possible numbers.

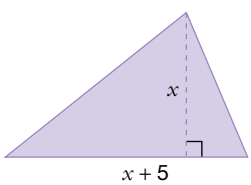
3 **a** Find the dimensions of this rectangle if the length is 6 cm longer than the breadth and its area is 40 cm^2.



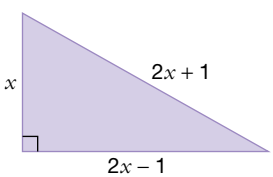
b The breadth of a rectangular room is 2 metres shorter than its length. If the area of the room is 255 m^2, find the dimensions of the room.



c The base of a triangle is 5 cm longer than its height. If the area of the triangle is 7 cm^2, find the length of the base.



d A right-angled triangle is drawn so that the hypotenuse is twice the shortest side plus 1 cm, and the other side is twice the shortest side less 1 cm. Find the length of the hypotenuse.



4 **a** Michelle threw a ball vertically upwards. Its height h metres after time t seconds is given by the formula:

$h = 8t - t^2$

Find at what time the ball was first at a height of 12 m.

b The sum, S, of the first n positive integers is given by the formula:

$S = \frac{n}{2}(n+1)$

Find the number of positive integers needed to give a total of 78.

c For the formula $s = ut + \frac{1}{2}at^2$, find the values of t if:

i $s = 18, \quad u = 7, \quad a = 2$

ii $s = 6, \quad u = 11, \quad a = 4$

iii $s = 7, \quad u = 1, \quad a = 6$

5 An n–sided polygon has $\frac{1}{2}n(n-3)$ diagonals. How many sides has a figure if it has 90 diagonals?

6 Jenny is y^2 years old and her daughter Allyson is y years old. If Jenny lives to the age of $13y$, Allyson will be y^2 years old. How old is Allyson now? (*Note:* the difference in ages must remain constant.)

CHALLENGE 11:06 EQUATIONS REDUCIBLE TO QUADRATICS

Some equations that appear more difficult may be changed into a quadratic equation by substituting an appropriate pronumeral.

For example, if for the equation: $x^4 - 13x^2 + 36 = 0$ we let $u = x^2$ and then substitute, the equation becomes: $u^2 - 13u + 36 = 0$.

This equation can now be solved by factorising:

$(u-9)(u-4) = 0$

$\therefore u = 9$ or $u = 4$

But $u = x^2$ so $\quad x^2 = 9$ or $\quad x^2 = 4$

$\therefore x = \pm 3$ or $\quad x = \pm 2$

So the equation $x^4 - 13x^2 + 36 = 0$ has four solutions, $x = -3, -2, 2$ and 3.

Exercises

1 Use the substitution given to reduce each equation to a quadratic equation and then solve.

a $x^4 - 5x^2 + 4 = 0, u = x^2$ **b** $a^4 - 10a^2 + 9 = 0, u = a^2$

c $y^6 - 9y^3 + 8 = 0, u = y^3$ **d** $m^8 - 17m^4 + 16 = 0, u = m^4$

2 Substitute u for $(x-1)^2$ to solve the equation:

$(x-1)^4 - 29(x-1)^2 + 100 = 0$

3 By using the substitution given, solve these *exponential* equations.

a $4^x - 5(2^x) + 4 = 0, u = 2^x$ [*Note:* $4^x = (2^x)^2$] **b** $9^x - 12(3^x) + 27 = 0, u = 3^x$

4 Solve, using the substitution given.

a $x^4 - 7x^2 + 10 = 0, u = x^2$ **b** $a^4 - 2a^2 - 15 = 0, u = a^2$

INVESTIGATION 11:06 SIMPLE CUBIC EQUATIONS $ax^3 = k$

In previous studies you have seen that a simple linear equation such as $x - 3 = 5$ or $2y = 7$ has one solution.

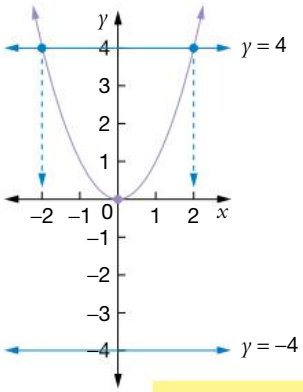
Simple quadratic equations, however, could have two, one or no real solutions.

For example:

$x^2 = 4$
$\therefore\ x = +2$ or -2

$x^2 = 0$
$\therefore\ x = 0$

$x^2 = -4$
$\therefore$ no real solution



Looking at the graph of $y = x^2$ it can be seen that:

It cuts the line $y = 4$ twice, indicating the solutions of $x = 2$ and $x = -2$

It touches the line $y = 0$ once, indicating the solution $x = 0$

It does not touch the line $y = -4$ at all, indicating no real solution.

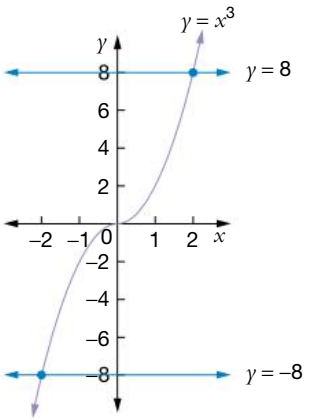
Remember: (real number)$^2 \neq$ negative number

Now, consider a simple cubic equation, such as:
$x^3 = 8$

If $x^3 = 8$, then $x = \sqrt[3]{8}$
$\therefore\ x = 2$

If $x^3 = -8$, then $x = \sqrt[3]{-8}$
$\therefore\ x = -2$

The graph shows that any simple cubic equation $ax^3 = k$ will only have one real solution, but unlike a quadratic equation *it will always have a real solution.*



Examples

Solve each equation.

1 $2x^3 = 54$
$x^3 = 27$
$x = \sqrt[3]{27}$
$\therefore\ x = 3$

2 $3n^3 = 15$
$n^3 = 5$
$\therefore\ n = \sqrt[3]{5}$
or $n = 1{\cdot}71$ (3 dec. pl.)

3 $5z^3 + 320 = 0$
$5z^3 = -320$
$z^3 = -64$
$z = \sqrt[3]{-64}$
$\therefore\ z = -4$

Exercises

Solve each cubic equation, giving the answer in exact form and as an approximation correct to two decimal places, when necessary.

1 $x^3 = 1000$ **2** $a^3 = -125$ **3** $p^3 = 1$ **4** $k^3 = -27$
5 $y^3 = 10$ **6** $t^3 = -7$ **7** $q^3 = 2$ **8** $n^3 = -20$
9 $2x^3 = 16$ **10** $3z^3 = -3$ **11** $5p^3 = 20$ **12** $10y^3 = -9$
13 $x^3 - 8 = 0$ **14** $m^3 + 3 = 0$ **15** $3t^3 - 3000 = 0$ **16** $8z^3 + 1 = 0$

11:07 Simultaneous equations involving a non-linear equation

PREP QUIZ 11:07

Solve these quadratic equations:

1 $(x + 3)(x - 2) = 0$ **2** $(2x - 1)(x + 7) = 0$ **3** $5x(x + 4) = 0$

4 $x^2 - 4 = 0$ **5** $x^2 - 3x + 2 = 0$ **6** $2x^2 + 7x - 4 = 0$

Solve these equations:

7 $x^2 - x + 5 = 3x + 2$ **8** $2x^2 - x = 3 - 2x$

Solve these simultaneous equations by substitution:

9 $x + y = 5, y = 2x - 1$ **10** $3x - y - 13 = 0, y = 7 - 2x$

In Chapter 5, we saw how to solve two simultaneous equations when both equations were linear, that is, they were of the form $y = 3x + 2$ or $3x - 2y = 6$. Now we shall see how to find the common or simultaneous solutions when one equation is linear but the other is non-linear.

WORKED EXAMPLE 1

Solve the simultaneous equations $y = x^2$ and $y = x + 2$ using the substitution method.

Solution

$y = x^2$ 1

$y = x + 2$ 2

From 1 we see that x^2 is equal to y.

If we substitute x^2 for y in 2 we have:

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$\therefore x = -1$ or 2

These values for x can now be substituted into either equation 1 or 2 to find corresponding values for y.

Substitute $x = -1$ in 1

$y = (-1)^2$

$= 1$

$\therefore x = -1, y = 1$ is a solution.

Substitute $x = 2$ in 1

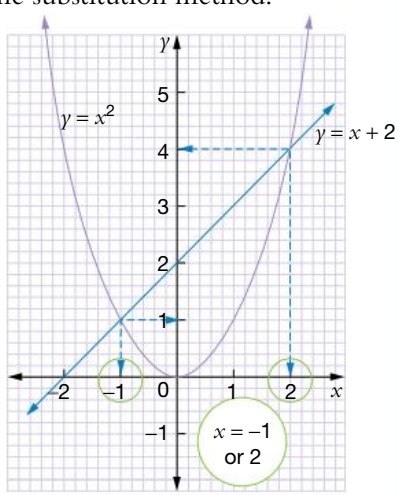
$y = (2)^2$

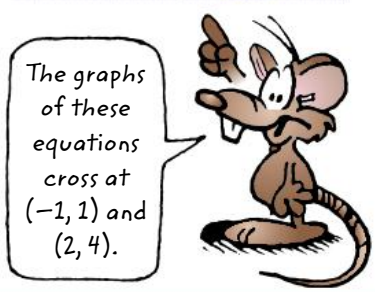
$= 4$

$\therefore x = 2, y = 4$ is a solution.

(Check answers by substituting in 1 and 2.)

The solutions are $x = -1$ and $y = 1$; $x = 2$ and $y = 4$





WORKED EXAMPLE 2

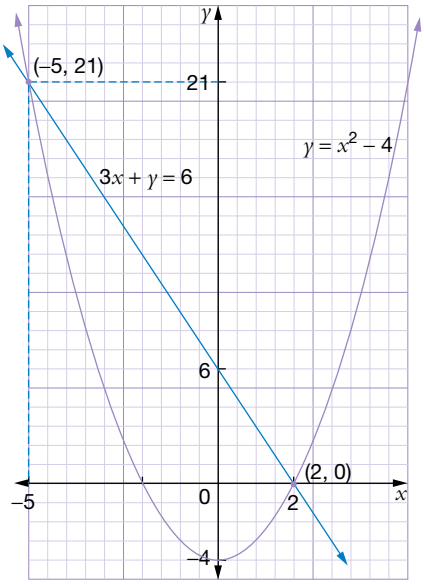
Use the substitution method to find the common solutions to the equations $y = x^2 - 4$ and $3x + y = 6$.

Solution

$y = x^2 - 4$ 1
$3x + y = 6$ 2

From 1 we see that $x^2 - 4$ is equal to y.
If we substitute $x^2 - 4$ for y in 2, we have:
$3x + (x^2 - 4) = 6$
$x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
$\therefore x = -5$ or 2

Substituting these values into 1 or 2 we have:
If $x = -5$, $y = 21$
$x = 2$, $y = 0$
$\therefore$ The solutions are $x = -5$ and $x = 2$
$y = 21$ $y = 0$



Graphical solutions to $y = x^2 - 4$ and $3x + y = 6$.

WORKED EXAMPLE 3

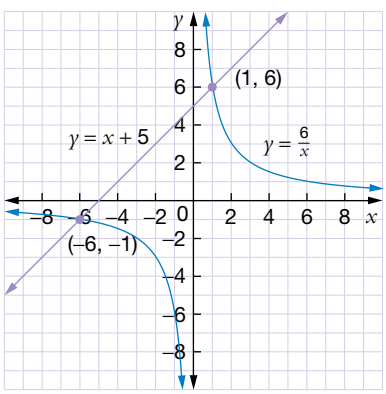
Find the solution common to the equations $y = x + 5$ and $y = \frac{6}{x}$.

Solution

$y = x + 5$ 1
$y = \frac{6}{x}$ 2

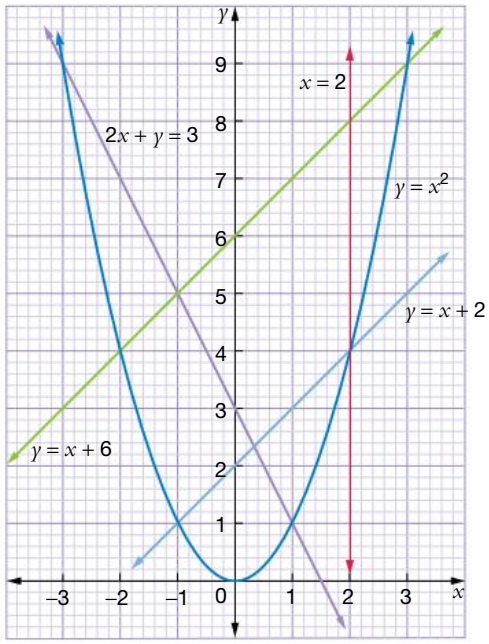
From 1 we see that $x + 5$ is equal to y.
If we substitute $x + 5$ for y in 2,
we have:
$x + 5 = \frac{6}{x}$ (*Note:* $x \neq 0$)
$x(x + 5) = 6$
$x^2 + 5x - 6 = 0$
$(x + 6)(x - 1) = 0$
$\therefore x = -6$ or $x = 1$

Substituting these values into 1 or 2 we have:
If $x = -6$, then $y = -1$
$x = 1$, then $y = 6$
$\therefore$ The solutions are $x = -6$ and $x = 1$
$y = -1$ $y = 6$



Exercise 11:07

1 Use this diagram to solve these simultaneous equations.



a $y = x + 6$
$x = 2$

b $y = x + 6$
$2x + y = 3$

c $y = x^2$
$x = 2$

d $y = x^2$
$y = x + 6$

e $y = x^2$
$y = x + 2$

f $y = x^2$
$2x + y = 3$

2 Solve each pair of equations in Question 1 using the substitution method and check your solutions with those for Question 1.

3 Use the substitution method to solve these simultaneous equations.

a $y = x^2$
$y = -4x - 3$

b $y = x^2$
$y = 49$

c $y = x^2$
$y = 3x$

d $y = x^2$
$y = x + 56$

e $y = x^2$
$y = 10x - 21$

f $y = x^2$
$y = -3x - 2$

g $y = x^2 + 5$
$y = 4x + 50$

h $y = x^2 + 7$
$y = 8x$

i $y = x^2 - 10$
$y = -9x$

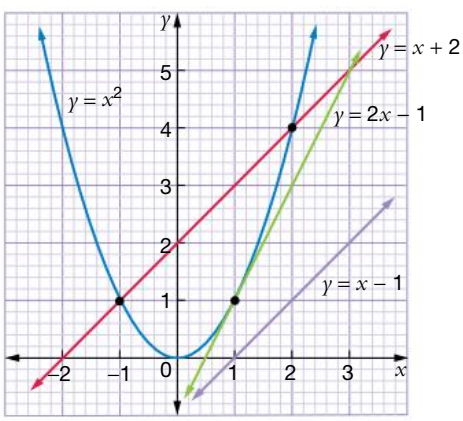
4 Solve each pair of simultaneous equations using the substitution method.

a $y = x^2$ and $y = x + 2$

b $y = x^2$ and $y = 2x - 1$

c $y = x^2$ and $y = x - 1$

Looking at the graphs of these equations, explain why **a** has two solutions, **b** has one solution, and **c** has no solutions.

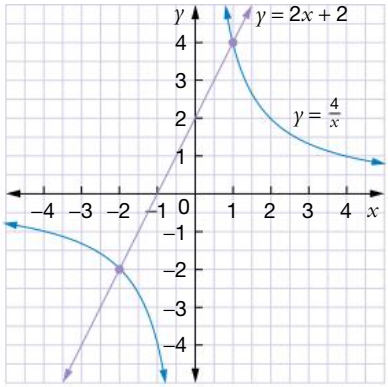


5 **a** Solve the equations $y = 2x + 2$ and $y = \frac{4}{x}$ simultaneously. Check your solution with the graphs of these equations.

b **i** Solve the simultaneous equations $y = 4 - x$ and $y = \frac{4}{x}$.

ii How many solutions are there?

iii What does this say about the relationship between the graphs of $y = 4 - x$ and $y = \frac{4}{x}$.



6 Solve these pairs of simultaneous equations.

a $y = 5x + 6$, $y = x^2$

b $y = x + 3$, $y = x^2 + 1$

c $y = 4x + 9$, $y = x^2 - 3$

d $y = 2x + 4$, $y = x^2 - x$

e $y = 2x + 14$, $y = x^2 - 3x$

f $y = 7 - 3x$, $y = x^2 + 3x$

g $y = 4x$, $y = \frac{4}{x}$

h $y = 2x + 6$, $y = \frac{8}{x}$

i $y = \frac{1}{2}x + 2$, $xy = 6$

7 By solving these simultaneous equations, find the point(s) of intersection of their graphs.

a $y = x^2 - 2$, $y = x$

b $y = x^2 - 10$, $y = 10 - x$

c $y = x^2 + 2$, $y = 4x - 2$

d $y = x^2 + 2x - 20$, $x + y = 8$

e $y = x^2 + 4x + 7$, $2x + y + 2 = 0$

f $y = x^2 + x - 2$, $y = x - 3$

g $y = x^2 + x - 10$, $y = 2x + 10$

h $y = x^2 + 2x - 8$, $3x + y = 6$

i $y = 4x + 1$, $y = 2x^2 - x - 2$

11:08 Solving literal equations (1)

PREP QUIZ 11:08

Complete the following:

1 $x + 25 = 93$

$\therefore x = 93 - \ldots$

2 $x + a = b$

$\therefore x = b - \ldots$

3 $x - 27 = 53$

$\therefore x = 53 + \ldots$

4 $x - m = n$

$\therefore x = n + \ldots$

5 $5x = 70$

$\therefore x = 70 \div \ldots$

6 $ax = b$

$\therefore x = b \div \ldots$

7 $\frac{x}{5} = 3$

$\therefore x = 3 \times \ldots$

8 $\frac{x}{m} = n$

$\therefore x = n \times \ldots$

Solve:

9 $2a + 15 = 17$

10 $3(x - 5) = 10$

A formula such as $A = lb$ is written with A as its subject. This means that we can quite easily calculate A if we know the values of l and b. Sometimes, however, we need to rearrange the formula so that one of the other pronumerals is the subject. To do this, the same procedures as for solving equations are used.

Remember:
+ is the opposite of −
− is the opposite of +
× is the opposite of ÷
÷ is the opposite of ×

In the examples, compare the solving of each equation with the changing of the subject of the formula to x, on the right.

Another name for a formula is a literal equation.

WORKED EXAMPLES

1 Solve for x.

a
$3x + 1 = 13$ — -1 both sides
$3x = 12$ — $\div 3$ both sides
$\therefore x = 4$

b
$5 - 2x = 1$ — $+2x$ both sides
$5 = 1 + 2x$ — -1 both sides
$4 = 2x$ — $\div 2$ both sides
$2 = x$
$\therefore x = 2$

c
$3(x + 2) = 5$ — expand
$3x + 6 = 5$ — -6 both sides
$3x = -1$ — $\div 3$ both sides
$\therefore x = -\frac{1}{3}$

Another way of saying 'make x the subject of this formula' is: 'solve this literal equation for x'.

2 Make x the subject.

a
$ax + b = c$ — $-b$ both sides
$ax = c - b$ — $\div a$ both sides
$\therefore x = \frac{c - b}{a}$

b
$m - nx = p$ — $+nx$ both sides
$m = p + nx$ — $-p$ both sides
$m - p = nx$ — $\div n$ both sides
$\frac{m - p}{n} = x$
$\therefore x = \frac{m - p}{n}$

c
$a(x + b) = c$ — expand
$ax + ab = c$ — $-ab$ both sides
$ax = c - ab$ — $\div a$ both sides
$\therefore x = \frac{c - ab}{a}$

3 Each formula below has had its subject changed to the capital letter. The operation done to each side is shown for each step.

a
$v = u + aT$ — $-u$ both sides
$v - u = aT$ — $\div a$ both sides
$\frac{v - u}{a} = T$
$\therefore T = \frac{v - u}{a}$

b
$m = \frac{1}{2}(x + Y)$ — $\times 2$ both sides
$2m = x + Y$ — $-x$ both sides
$2m - x = Y$
$\therefore Y = 2m - x$

c
$t = a + (N - 1)d$ — expand
$t = a + Nd - d$ — $-a$ both sides
$t - a = Nd - d$ — $+d$ both sides
$t - a + d = Nd$ — $\div d$ both sides
$\frac{t - a + d}{d} = N$
$\therefore N = \frac{t - a + d}{d}$

d
$a = 2\pi r(r + H)$ — expand
$a = 2\pi r^2 + 2\pi rH$ — $-2\pi r^2$ both sides
$a - 2\pi r^2 = 2\pi rH$ — $\div 2\pi r$ both sides
$\frac{a - 2\pi r^2}{2\pi r} = H$
$\therefore H = \frac{a - 2\pi r^2}{2\pi r}$

To change the subject of a formula (solve a literal equation):

1 expand parentheses if applicable

2 by using inverse operations, isolate the pronumeral required to be the subject.

Exercise 11:08

P Foundation worksheet 11:08
Solving literal equations

1 Make x the subject of each formula.

a $p = x + m$ **b** $m = x + np$ **c** $n = pq - x$

d $ax = b$ **e** $3x = y$ **f** $a^2x = b + c$

g $b - ax = 2d$ **h** $ax - b = c$ **i** $c - 2b = ax + b$

j $y = \frac{x}{a}$ **k** $y = \frac{a}{x}$ **l** $\frac{x}{y} = b$

m $a = \frac{25}{x}$ **n** $\frac{ax}{b} = c$ **o** $p = \frac{2x}{L}$

2 After first expanding the grouping symbols, solve each literal equation for x.

a $a = 2(x + y)$ **b** $p = 5(t + x)$ **c** $y = 3(x - 7)$

d $p = q(x - r)$ **e** $6(a - x) = b$ **f** $w = t(v - x)$

g $R = 2r(x + 2)$ **h** $p = 5q(x - y)$ **i** $A = \pi r(r - x)$

3 Solve each literal equation for the pronumeral indicated.

a $A = x + y$ $[y]$ **b** $P = 2L + 2B$ $[L]$ **c** $C = \pi d$ $[d]$

d $v = u + at$ $[u]$ **e** $v = u + at$ $[a]$ **f** $E = mc^2$ $[m]$

g $S = \frac{D}{T}$ $[D]$ **h** $R = \frac{V}{I}$ $[V]$ **i** $I = PRN$ $[P]$

j $P = RI^2$ $[R]$ **k** $v^2 = u^2 + 2as$ $[s]$ **l** $F = ac + p$ $[a]$

m $P = a(m + n)$ $[n]$ **n** $x = 2a(p + q)$ $[p]$ **o** $K = \frac{1}{2}mV^2$ $[m]$

p $P = m(v - u)$ $[u]$ **q** $V = \frac{AH}{3}$ $[H]$ **r** $V = \frac{\pi r^2 h}{3}$ $[h]$

s $S = \pi r^2 + \pi rh$ $[h]$ **t** $E = mgh + \frac{1}{2}mv^2$ $[h]$ **u** $P = 2ab - 2ak$ $[k]$

v $A = \frac{a + b}{2}$ $[a]$ **w** $A = \frac{h(a + b)}{2}$ $[b]$ **x** $F = \frac{q_1 q_2}{r}$ $[r]$

y $T = a + (n - 1)d$ $[d]$ **z** $S = \frac{a(r^n - 1)}{r - 1}$ $[a]$

11:09 Solving literal equations (2)

PREP QUIZ 11:09

Solve the equations:

1 $\frac{x}{5} = 3$ 2 $\frac{x}{8} = \frac{3}{4}$ 3 $\frac{x+2}{3} = 5$ 4 $x^2 = 9$

5 $\sqrt{x} = 5$ 6 $(x+3)^2 = 25$ 7 $\sqrt{x-3} = 2$

Factorise fully:

8 $3x + 12$ 9 $x^2 - 2x$ 10 $5a^2 + 10ab$

Remember!
$\sqrt{\ }$ is the opposite of $(\ )^2$.
$(\ )^2$ is the opposite of $\sqrt{\ }$.

In this section, formulas may also contain a squared term or a square root sign, or the pronumeral that is to become the subject may appear more than once.

WORKED EXAMPLES

Change the subject of the formula to the letter indicated.

1 $E = mc^2$ $[c]$ 2 $v^2 = u^2 - 2as$ $[u]$ 3 $r = \sqrt{\frac{A}{\pi}}$ $[A]$

4 $a = 6 - \frac{12}{R}$ $[R]$ 5 $y = \frac{A}{A+2}$ $[A]$

Solutions

1 $E = mc^2$ ÷ m both sides

$\frac{E}{m} = c^2$ $\sqrt{\ }$ both sides

$\therefore c = \pm\sqrt{\frac{E}{m}}$

2 $v^2 = u^2 - 2as$ $+2as$ both sides

$v^2 + 2as = u^2$ $\sqrt{\ }$ both sides

$\pm\sqrt{v^2 + 2as} = u$

$\therefore u = \pm\sqrt{v^2 + 2as}$

3 $r = \sqrt{\frac{A}{\pi}}$ square both sides

$r^2 = \frac{A}{\pi}$ $\times \pi$ both sides

$\pi r^2 = A$

$\therefore A = \pi r^2$

4 $a = 6 - \frac{12}{R}$ $\times R$ both sides

$aR = 6R - 12$ $-6R$ both sides

$aR - 6R = -12$ factorise LHS

$R(a - 6) = -12$ ÷ $(a - 6)$ both sides

$\therefore R = \frac{-12}{a-6}$

5 $y = \frac{A}{A+2}$ $\times(A + 2)$ both sides

$y(A + 2) = A$ expand LHS

$Ay + 2y = A$ $-Ay$ both sides

$2y = A - Ay$ factorise RHS

$2y = A(1 - y)$ ÷ $(1 - y)$ both sides

$\frac{2y}{1-y} = A$

$\therefore A = \frac{2y}{1-y}$

Remember!
Sometimes formulas are called *literal equations*. When literal equations are 'solved' for a certain pronumeral, it is the same as changing the subject of the formula to that pronumeral.

If the pronumeral that is to be the subject appears in more than one term in the formula, gather the terms together and factorise as in Worked Examples 4 and 5.

Exercise 11:09

1 Change the subject of each formula to x.

a $mx^2 = n$ **b** $a = bx^2$ **c** $x^2 - a = b$ **d** $h = k - x^2$

e $\frac{x^2}{a} = y$ **f** $m = \frac{nx^2}{3}$ **g** $L = x^2 - y^2$ **h** $A = \frac{B}{x^2}$

2 Make a the subject of each formula.

a $\sqrt{ab} = c$ **b** $u = \sqrt{3a}$ **c** $c = \sqrt{a - b}$ **d** $c = \sqrt{a} - b$

e $P = L + M\sqrt{a}$ **f** $M - N\sqrt{a} = L$ **g** $\sqrt{3a - 1} = L$ **h** $P = \sqrt{b - 2a}$

3 Make N the subject of each formula after first multiplying each term by the lowest common denominator.

a $a = \frac{3N}{2}$ **b** $a = L - \frac{3N}{2}$ **c** $\frac{x}{3} = \frac{N}{2} - 1$ **d** $L = \frac{N}{2} + \frac{M}{3}$

e $x = \frac{N + a}{3}$ **f** $x = \frac{N - 1}{2} + \frac{M + 1}{3}$ **g** $\frac{N + u}{3} = \frac{m + u}{4}$ **h** $\frac{N - a}{b} = \frac{L + b}{a}$

4 Solve each literal equation for x.

a $a + x = b - x$ **b** $ax = px + q$ **c** $x + a = ax + b$ **d** $m - nx = n - mx$

e $px^2 = qx^2 + 2$ **f** $L = Ax + (1 + B)x$ **g** $\frac{x}{5} + \frac{x}{3} = a$ **h** $a = \frac{x}{x + 2}$

i $y = \frac{x}{x - 5}$ **j** $m = \frac{x + 3}{1 + x}$ **k** $A = \frac{a + bx}{1 + x}$ **l** $B = \frac{x + a}{x - a}$

5 Solve each equation for the letter indicated.

	A		B		C	
a	$A = LB$	$[B]$	$A = X - Y$	$[X]$	$V = u + at$	$[t]$
b	$D = \frac{M}{V}$	$[V]$	$D = \frac{S}{T}$	$[S]$	$P = Rl^2$	$[I]$
c	$V = \frac{Ah}{3}$	$[h]$	$V = \frac{1}{3}\pi r^2 h$	$[r]$	$S = 4\pi r^2$	$[r]$
d	$M = \frac{a + b}{2}$	$[b]$	$A = \frac{1}{2}h(x + y)$	$[y]$	$v^2 = u^2 + 2as$	$[s]$
e	$x^2 = ay - y^2$	$[a]$	$S = 2\pi r(r + h)$	$[h]$	$T = \frac{n}{2}[2a + (n - 1)d]$	$[d]$
f	$a = \sqrt{bc}$	$[c]$	$Y = a\sqrt{X}$	$[X]$	$X = 2\sqrt{a - b}$	$[a]$
g	$X = \sqrt{\frac{Y}{a}}$	$[Y]$	$R = \sqrt{\frac{ax}{b}}$	$[x]$	$m = \sqrt{\frac{a + b}{n}}$	$[b]$
h	$T = 2\pi\sqrt{\frac{l}{g}}$	$[l]$	$A = 2x\sqrt{\frac{t}{u}}$	$[u]$	$u = \sqrt{v^2 - 2as}$	$[s]$

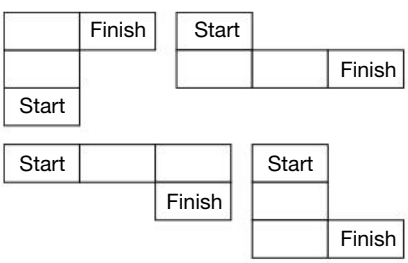
	A		B		C	
i	$y = \frac{a}{3} + \frac{b}{2}$	$[a]$	$Z = \frac{X}{4} + \frac{Y}{3}$	$[X]$	$A = \frac{b-c}{5}$	$[c]$
j	$\frac{A+a}{3} = \frac{A+y}{2}$	$[A]$	$L = \frac{N-1}{2} + \frac{N+1}{3}$	$[N]$	$\frac{X}{a} = \frac{X-a}{b}$	$[X]$
k	$h = \frac{k}{2k+1}$	$[k]$	$y = \frac{a}{a+2}$	$[a]$	$z = \frac{x}{x-3}$	$[x]$

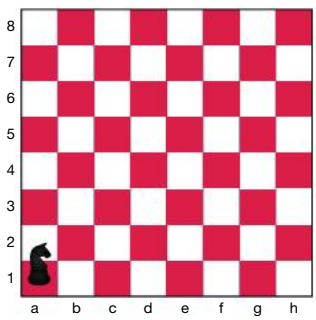
FUN SPOT 11:09 DID YOU KNOW THAT 2 = 1?

Now that your algebra skills are more developed you can be let into the secret that 2 really is equal to 1.

Proof: Assume that $x = y$.

Multiply both sides by x,	$x^2 = xy$
Subtract y^2 from both sides,	$x^2 - y^2 = xy - y^2$
Factorise both sides,	$(x - y)(x + y) = y(x - y)$
Divide by $(x - y)$,	$x + y = y$
Now if we let $x = y = 1$, then $2 = 1$.	QED





- Chess is played on an 8×8 square grid. Each square is named using a letter and a number. The Knight pictured is standing on the square a1.
- In Chess, a Knight can move 3 squares from its starting position to its finishing position. The squares must form an 'L' shape in any direction. Some possible moves are shown above.
- Which squares can the Knight in the diagram move to? If the Knight was standing on the square c1 what squares could it move to? Give a sequence of squares showing how the Knight could move from a1 to b1 to c1 to … h1.

MATHS TERMS 11

coefficient

- the number that multiplies a pronumeral in an equation or algebraic expression,
  e.g. $3x^2 - x + 5 = 0$
  coefficient of x^2 is 3
  coefficient of x is -1

completing the square

- completing an algebraic expression to form a perfect square, i.e. $(x + a)^2$ or $(x - a)^2$,
  e.g. to complete the square for $x^2 + 6x$, the number 9 is added:
  $x^2 + 6x + 9 = (x + 3)^2$

factorise

- to write an expression as the product of its factors
- the reverse of expand

literal equation

- another name for a formula
- a relationship between quantities
- it will always have more than one pronumeral,
  e.g. $v = u + at$

perfect square

- an expression that can be expressed in the form $(x + a)^2$ or $(x - a)^2$

quadratic equation

- an equation in which the highest power of the unknown pronumeral is 2,
  e.g. $x^2 - 16 = 0$, $x^2 + 5x + 6 = 0$
- a quadratic equation may have two, one or no solutions

quadratic formula

- a formula that gives the solutions to equations of the form $ax^2 + bx + c = 0$
- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

simultaneous equations

- two or more equations involving the same variables
- the equations can be solved to find the common or simultaneous solution,
  e.g. the equations $x + y = 10$ and $x - y = 6$ each have many solutions but the only simultaneous solution is $x = 8$ and $y = 2$

subject of a formula

- the pronumeral by itself on the left-hand side of a formula,
  e.g. in the formula $v = u + at$, v is the subject

substitution

- the replacement of a pronumeral with a numeral in a formula or expression.

ese gear wheels has 28 teeth
ther has 29 teeth. How many
ns of each wheel must be
d for the same two teeth to be
me position next to each other?

DIAGNOSTIC TEST 11 FURTHER EQUATIONS

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 **a** $\frac{a-2}{3} = a+5$ **b** $\frac{3x+1}{4} = \frac{4x+3}{6}$ **c** $\frac{n-2}{2} - \frac{n-1}{5} = 3$ — 11:01

2 Solve these equations: — 11:02
a $(x+7)(x-3) = 0$ **b** $x(x-5) = 0$
c $(2x-1)(x+1) = 0$ **d** $(3x+2)(4x-5) = 0$

3 Factorise and solve: — 11:02
a $x^2 + 5x = 0$ **b** $x^2 + 9x + 14 = 0$
c $x^2 - 49 = 0$ **d** $2x^2 + 5x - 3 = 0$

4 What number must be inserted to complete the square? — 11:03
a $x^2 + 6x + \ldots$ **b** $x^2 - 4x + \ldots$
c $x^2 + 3x + \ldots$ **d** $x^2 - x + \ldots$

5 Solve the following by completing the square. — 11:03
a $x^2 + 2x - 2 = 0$ **b** $x^2 - 6x + 1 = 0$
c $x^2 - 3x - 5 = 0$ **d** $2x^2 - 10x = 1$

6 Solve using the quadratic formula. (Leave answers in surd form.) — 11:04
a $x^2 + x - 3 = 0$ **b** $x^2 - 5x + 2 = 0$
c $2x^2 + 4x + 1 = 0$ **d** $3x^2 + 2x - 2 = 0$

7 Solve the following: — 11:05
a $x^2 - x + 1 = 4x + 7$ **b** $x(x-5) = x - 9$
c $(x+4)^2 = 6$ **d** $x = \frac{2x+8}{x}$

8 Solve each pair of simultaneous equations. — 11:07
a $y = 7x + 8$, $y = x^2$ **b** $y = x + 10$, $y = x^2 - 10$ **c** $y = x^2 + 4x - 5$, $x - y = 5$

9 Change the subject of each formula to a. — 11:08
a $x = 3a - 2b$ **b** $v^2 = u^2 + 2as$ **c** $A = \frac{D(a+b)}{h}$

10 Solve each literal equation for y. — 11:09
a $ay^2 = x$ **b** $T = A\sqrt{\frac{B}{y}}$ **c** $P = \frac{y}{1+y}$

ASSIGNMENT 11A Chapter review

1 Solve:

a $\frac{4x}{3} - \frac{x}{2} = 10$ **b** $\frac{2n-1}{4} + \frac{n+3}{6} = 8$

c $\frac{3a-1}{5} - \frac{2a-3}{6} = 2$

2 **a** $x^2 - 36 = 0$ **b** $(x+2)(x-3) = 0$

c $(2x-1)(x+5) = 0$ **d** $5x^2 - 10x = 0$

e $x^2 - 7x + 12 = 0$ **f** $x^2 - 4x - 45 = 0$

g $x^2 = 3x + 10$ **h** $2x^2 + 5x - 3 = 0$

i $10x^2 = 2 - x$

3 Solve by completing the square.

a $x^2 - 6x + 5 = 0$ **b** $4a^2 + 8a + 3 = 0$

c $2y^2 - 6y + 1 = 0$

4 Solve each pair of simultaneous equations.

a $y = x^2 - 2x - 20$, $y = x + 8$ **b** $5x - y = 9$, $y = 5 - x^2$

c $y = 2x^2 + 7x - 1$, $2x - y + 2 = 0$

5 Rearrange each formula to make P the subject.

a $A = \frac{PRT}{100}$ **b** $I = \sqrt{\frac{P}{R}}$

c $M = N - PV$ **d** $K = L(P - Q)$

6 Use the quadratic formula to solve the following. Evaluate your answers correct to one decimal place.

a $x^2 - 5x + 1 = 0$

b $x^2 - x - 4 = 0$

c $2x^2 - 4x + 1 = 0$

7 A rectangular paddock is 20 m longer than it its breadth. If the area of the paddock is $800\,m^2$, find the length of the paddock.

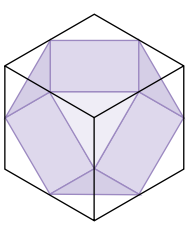
8 Solve these literal equations for x.

a $y = a\sqrt{\frac{m}{x}}$ **b** $y = \frac{x+a}{x+b}$

ASSIGNMENT 11B Working mathematically

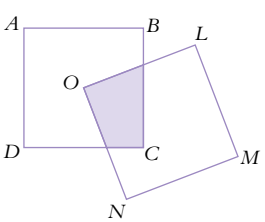
1 The sum of a number, its square and its square root is 655. Find the number.

2 A solid is formed from a cube by cutting off the corners in such a way that the vertices of the new solid will be at the midpoints of the edges of the original cube.



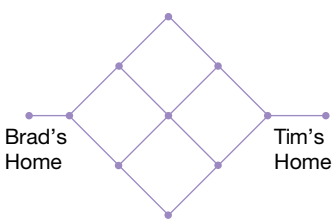
How many edges will the new solid have?

3 Two identical squares are overlapped so that the vertex of one square is at the centre of the other. What fraction is the overlapping area of either square?



(Hint: Draw lines from O parallel to AB and BC.)

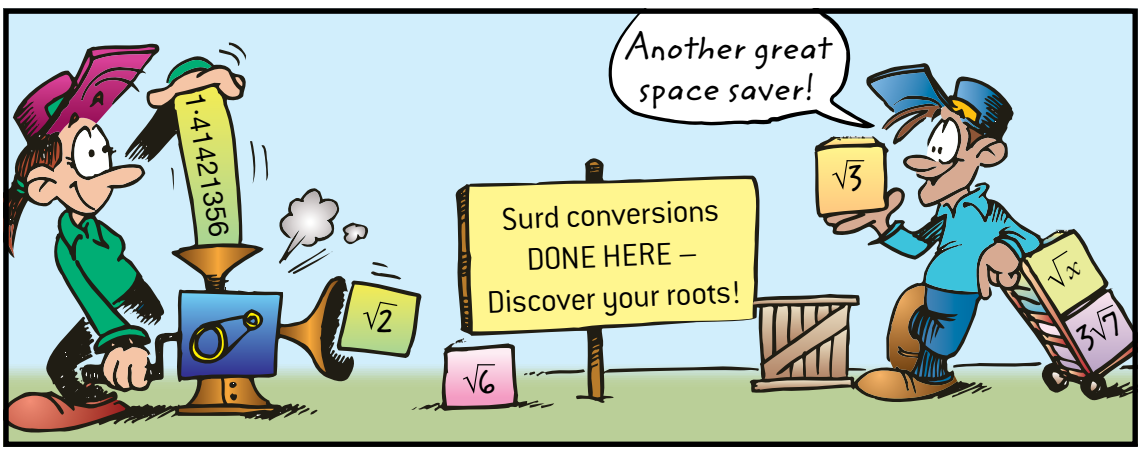
4



Bradley must travel from his home to Tim's home regularly. He decides to travel a different route each day, but always chooses a route so that each point he reaches is closer to Tim's house than the last point (see diagram). How many days can he travel before he has to repeat an earlier route?

FURTHER NUMBER

12



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Fun spot 12:07 What do Inuits sing at birthday parties?
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Surds and Indices*** [Stage 5.3[§]]

- Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

12:01 The real number system

PREP QUIZ 12:01

Evaluate:

1 4^2 **2** 7^2 **3** 20^2 **4** $6^2 + 5^2$

5 $\sqrt{81}$ **6** $\sqrt{64}$ **7** $\sqrt{144}$ **8** $\sqrt{4} + \sqrt{25}$

Use a calculator to find an approximation correct to six decimal places for:

9 $\sqrt{2}$ **10** $\sqrt{11} + 5$

The real number system is made up of two groups of numbers: rational and irrational numbers.

Rational numbers

Any number that can be written as a fraction, $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, is a rational number. These include integers, fractions, mixed numbers, terminating decimals and recurring decimals.

e.g. $\frac{7}{8}$, $6\frac{3}{5}$, $1{\cdot}25$, $0{\cdot}07$, $0{\cdot}\dot{4}$, $\sqrt{81}$

These examples can all be written as fractions.

$\frac{7}{8}, \frac{33}{5}, \frac{5}{4}, \frac{7}{100}, \frac{4}{9}, \frac{9}{1}$

An integer is a rational number whose denominator is 1.

Irrational numbers

It follows that irrational numbers cannot be written as a fraction $\frac{a}{b}$ where a and b are whole numbers. We have met a few numbers like this in our study of the circle and Pythagoras' theorem.

e.g. $\pi, \sqrt{2}, \sqrt[3]{4}, \sqrt{3} + 2$

The calculator can only give approximations for these numbers. The decimals continue without terminating or repeating.

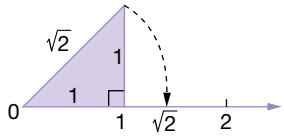
$3{\cdot}141\,592\,65 \ldots, 1{\cdot}414\,213\,56 \ldots, 1{\cdot}587\,401\,05 \ldots, 3{\cdot}732\,050\,80 \ldots$

Irrational numbers on the number line

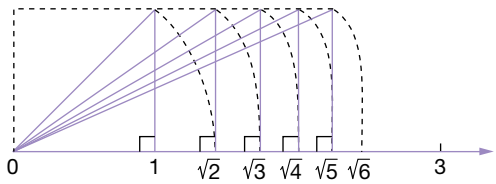
Although irrational numbers cannot be given an exact decimal value, their positions can still be plotted on the number line. As can be seen when using Pythagoras' theorem, a number such as $\sqrt{2}$ can correspond to the length of a side of a triangle, and so this length can be shown on a number line.

Examine the diagram on the right.

If the length of the hypotenuse of this triangle is transferred, using compasses, to the number line as shown, we have the position of $\sqrt{2}$ on the number line. (This agrees with the decimal approximation from the calculator of 1·414 213 56.)

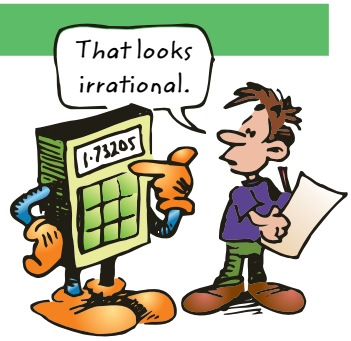


The previous construction can be extended to give the position of other square roots on the number line.



Another irrational number you have met before is π. You should know that it has an approximate value of 3·142, so it would lie on the number line in the position shown.

Exercise 12:01



1 For each number write *rational* or *irrational*. (A calculator might help you to decide.)

a $\frac{7}{10}$	**b** $\sqrt{2}$	**c** 0·3	**d** $1\frac{1}{2}$	**e** 1·6
f π	**g** $\sqrt{5}$	**h** $\frac{3}{4}$	**i** $\sqrt{16}$	**j** 0·99
k $\frac{22}{7}$	**l** 9	**m** $\sqrt{625}$	**n** 0·666	**o** 70
p $\sqrt{13}$	**q** $1\frac{1}{3}$	**r** $1+\sqrt{3}$	**s** $\sqrt{1}$	**t** $\sqrt{4}+\sqrt{9}$
u 0·0005	**v** $1-\sqrt{9}$	**w** $-\sqrt{3}$	**x** $\sqrt{7}+\sqrt{2}$	**y** $\sqrt[3]{27}$ **z** $\sqrt[3]{10}$

2 Use your calculator to find an approximation correct to one decimal place for the following. Also use these values to show the position of each on a number line.

a $\sqrt{2}$	**b** $\sqrt{3}$	**c** $\sqrt{5}$	**d** $\sqrt{6}$	**e** $\sqrt{7}$
f $\sqrt{8}$	**g** $\sqrt{10}$	**h** $\sqrt{12}$	**i** $\sqrt{20}$	**j** π

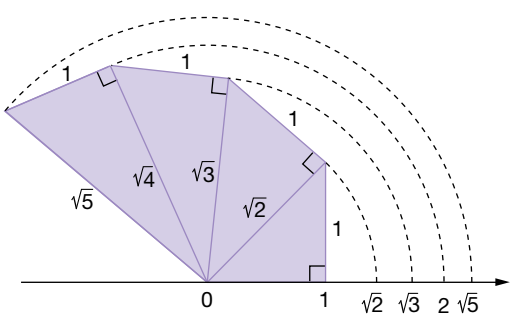
3 Between which two consecutive integers does each number below lie?

a $\sqrt{11}$	**b** $\sqrt{18}$	**c** $\sqrt{41}$	**d** $\sqrt{78}$	**e** $\sqrt{95}$
f $\sqrt{125}$	**g** $\sqrt{180}$	**h** $\sqrt{250}$	**i** $\sqrt{390}$	**j** $\sqrt{901}$

4 Arrange each set of numbers below in order, from smallest to largest.

a $\sqrt{5}, 2, \sqrt{3}$	**b** $\sqrt{8}, 3, \pi$	**c** $\sqrt{10}, \sqrt{12}, 3$	**d** $7, \sqrt{40}, \sqrt{50}, 6{\cdot}5$
e $\pi, \sqrt{2}, 2{\cdot}1, \sqrt{12}$	**f** $5{\cdot}6, \sqrt{26}, 6, \sqrt{30}$	**g** $8{\cdot}1, \sqrt{65}, 7{\cdot}9, \sqrt{60}$	**h** $\sqrt{98}, 10, \sqrt{102}, 10{\cdot}1$
i $3{\cdot}1, \pi, \sqrt{9}, 3{\cdot}2$	**j** $\sqrt{20}, 4{\cdot}1, 4{\cdot}5, \sqrt{21}$	**k** $20, \sqrt{390}, 21, \sqrt{420}$	**l** $\sqrt{600}, \sqrt{610}, 24, 25$

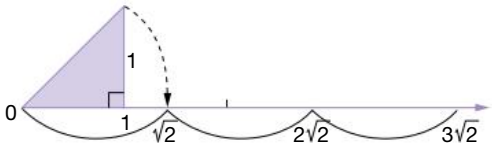
5 This diagram shows another construction for locating square roots on the number line. Use a set square to draw the triangles on graph paper, then use compasses to draw the arcs on the number line. Extend your diagram to show $\sqrt{6}$. Check the accuracy of your constructions with your calculator.



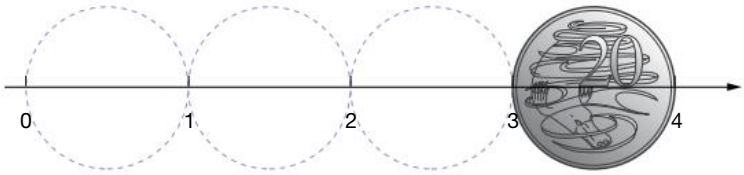
6 To show multiples of a square root on the number line we can use a pair of compasses to mark off equal intervals.

a Repeat the constructions in Question **5** to find the position of $\sqrt{2}$ on the number line. Then use a pair of compasses to mark the position of $2\sqrt{2}$ and $3\sqrt{2}$.

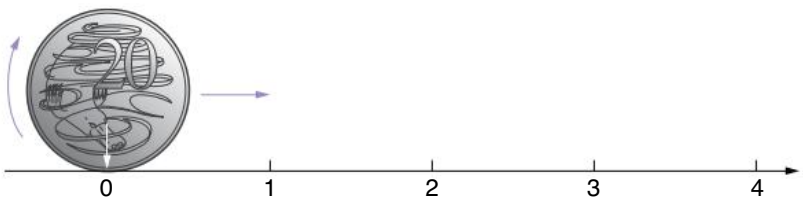
b Draw a diagram and show the position of $\sqrt{3}$, $2\sqrt{3}$ and $-\sqrt{3}$ on a number line.



7 The position of π on the number line can be shown by doing the following. Use the diameter of a twenty-cent coin to mark off units on a number line.



Then mark a point on the circumference of the coin, align it with zero on the number line, then roll it along the line carefully until the mark meets the number line again. This will show the position of π on the number line.



INVESTIGATION 12:01 π

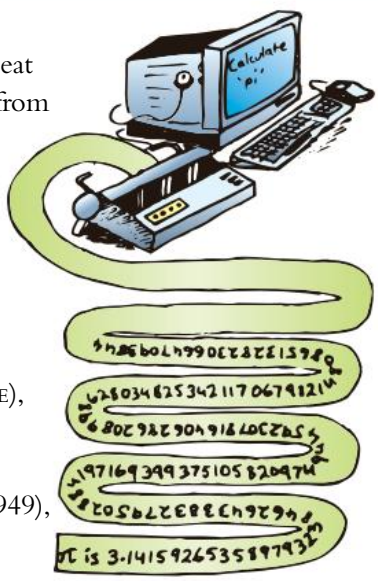
π is an irrational number.

Its decimal equivalent is without end. The decimal does not repeat or terminate. Hideaki Tomoyori of Yokohama, Japan, recited π from memory to 40 000 places on 9–10 March 1987.

One way of measuring the speed of a computer is to time the calculation of π to a very large number of decimal places.

1. Investigate the history of the calculation of π.
2. See how many decimal places of π you can memorise in half an hour.

Some of the people involved in calculating π are Ptolemy (150 CE), Viete (1579), Romanus (1593), Van Ceulen (1610), Snell (1621), Sharp (1699), Machin (1706), Vega (1794), Dase (1844), Rutherford (1853), Shanks (1873), Aberdeen Proving Grounds (1949), Watson Scientific Laboratory (1954), Shanks and Wrench (1961), Gilloud and Fillatoire (1966), Gilloud and Dichampt (1967).



12:02 Surds

PREP QUIZ 12:02

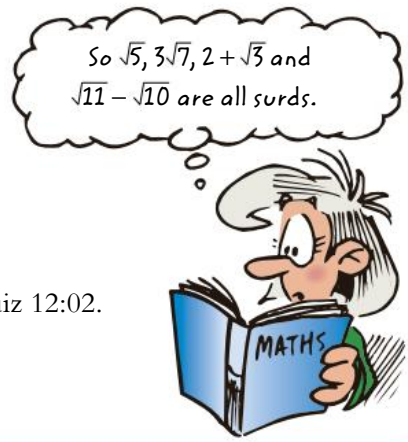
Find the value of:

1 $\sqrt{16}$ 2 $\sqrt{9}$ 3 $\sqrt{36}$ 4 $\sqrt{16+9}$ 5 $\sqrt{16}+\sqrt{9}$

6 $\sqrt{16\times 9}$ 7 $\sqrt{16}\times\sqrt{9}$ 8 $\sqrt{\frac{36}{9}}$ 9 $\frac{\sqrt{36}}{\sqrt{9}}$ 10 $(\sqrt{16})^2$

Surds are numerical expressions that involve irrational roots. They are irrational numbers.

Surds are irrational roots of the form $\sqrt[n]{x}$, where x is a rational number, and n is an integer such that $n \geq 2$.



Surds obey the following rules, which are suggested by Prep quiz 12:02.

Rule 1 $\sqrt{xy} = \sqrt{x}\times\sqrt{y}$

WORKED EXAMPLE 1

a $\sqrt{100} = \sqrt{4}\times\sqrt{25}$
$= 2\times 5$
$= 10$ (which is true)

b $\sqrt{27} = \sqrt{9}\times\sqrt{3}$
$= 3\times\sqrt{3}$
$= 3\sqrt{3}$

c $\sqrt{5}\times\sqrt{7} = \sqrt{5\times 7}$
$= \sqrt{35}$

Rule 2 $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Note: $\sqrt{x}$ means the positive square root of x when $x > 0$.
$\sqrt{x} = 0$ when $x = 0$.

WORKED EXAMPLE 2

a $\sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}}$
$\sqrt{4} = \frac{4}{2}$
$= 2$ (which is true)

b $\sqrt{125}\div\sqrt{5} = \sqrt{125\div 5}$
$= \sqrt{25}$
$= 5$

c $\sqrt{30}\div\sqrt{5} = \sqrt{30\div 5}$
$= \sqrt{6}$

Rule 3 $(\sqrt{x})^2 = x$

Note: For $\sqrt{x}$ to exist, x cannot be negative.

WORKED EXAMPLE 3

a $(\sqrt{25})^2 = (5)^2$
$= 25$

b $(\sqrt{7})^2 = 7$

c $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2$
$= 9 \times 2$
$= 18$

A surd is in its simplest form when the number under the square root sign is as small as possible. To simplify a surd, express it as the product of two square roots, one being the root of a square number.

WORKED EXAMPLE 4

Simplify the following surds.

a $\sqrt{18} = \sqrt{9} \times \sqrt{2}$
$= 3 \times \sqrt{2}$
$= 3\sqrt{2}$

b $\sqrt{75} = \sqrt{25} \times \sqrt{3}$
$= 5 \times \sqrt{3}$
$= 5\sqrt{3}$

c $5\sqrt{48} = 5 \times \sqrt{16} \times \sqrt{3}$
$= 5 \times 4 \times \sqrt{3}$
$= 20\sqrt{3}$

Exercise 12:02

P Foundation worksheet 12:02 Surds

1 Simplify:

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{5} \times \sqrt{3}$ **c** $\sqrt{7} \times \sqrt{6}$ **d** $\sqrt{6} \times \sqrt{7}$
e $\sqrt{10} \times \sqrt{3}$ **f** $\sqrt{23} \times \sqrt{2}$ **g** $\sqrt{13} \times \sqrt{5}$ **h** $\sqrt{11} \times \sqrt{3}$
i $\sqrt{5} \times \sqrt{2}$ **j** $\sqrt{7} \times \sqrt{2}$ **k** $\sqrt{11} \times \sqrt{10}$ **l** $\sqrt{13} \times \sqrt{7}$
m $\sqrt{26} \div \sqrt{2}$ **n** $\sqrt{55} \div \sqrt{5}$ **o** $\sqrt{77} \div \sqrt{11}$ **p** $\sqrt{34} \div \sqrt{17}$
q $\frac{\sqrt{38}}{\sqrt{2}}$ **r** $\frac{\sqrt{57}}{\sqrt{3}}$ **s** $\frac{\sqrt{60}}{\sqrt{10}}$ **t** $\frac{\sqrt{22}}{\sqrt{11}}$

2 Square each of the surds below (Rule 3).

a $\sqrt{16}$ **b** $\sqrt{9}$ **c** $\sqrt{1}$ **d** $\sqrt{100}$
e $\sqrt{5}$ **f** $\sqrt{8}$ **g** $\sqrt{15}$ **h** $\sqrt{73}$
i $2\sqrt{2}$ **j** $3\sqrt{5}$ **k** $2\sqrt{3}$ **l** $5\sqrt{3}$
m $7\sqrt{3}$ **n** $2\sqrt{7}$ **o** $9\sqrt{11}$ **p** $6\sqrt{5}$
q $10\sqrt{10}$ **r** $9\sqrt{20}$ **s** $6\sqrt{50}$ **t** $15\sqrt{15}$

3 Simplify each of these surds.

a $\sqrt{8}$ **b** $\sqrt{20}$ **c** $\sqrt{12}$ **d** $\sqrt{50}$
e $\sqrt{24}$ **f** $\sqrt{32}$ **g** $\sqrt{45}$ **h** $\sqrt{54}$
i $\sqrt{28}$ **j** $\sqrt{90}$ **k** $\sqrt{56}$ **l** $\sqrt{63}$
m $\sqrt{44}$ **n** $\sqrt{52}$ **o** $\sqrt{108}$ **p** $\sqrt{40}$
q $\sqrt{99}$ **r** $\sqrt{60}$ **s** $\sqrt{96}$ **t** $\sqrt{76}$
u $\sqrt{68}$ **v** $\sqrt{126}$ **w** $\sqrt{200}$ **x** $\sqrt{162}$

4 Simplify each surd and then, taking $\sqrt{2} = 1{\cdot}41$ and $\sqrt{3} = 1{\cdot}73$, give a decimal approximation correct to one decimal place.

a $\sqrt{18}$	**b** $\sqrt{27}$	**c** $\sqrt{8}$	**d** $\sqrt{12}$
e $\sqrt{32}$	**f** $\sqrt{48}$	**g** $\sqrt{50}$	**h** $\sqrt{162}$

5 Write each surd below in its simplest form.

a $2\sqrt{12}$	**b** $3\sqrt{8}$	**c** $2\sqrt{50}$	**d** $4\sqrt{18}$
e $5\sqrt{20}$	**f** $2\sqrt{75}$	**g** $10\sqrt{27}$	**h** $3\sqrt{56}$
i $2\sqrt{125}$	**j** $4\sqrt{45}$	**k** $3\sqrt{24}$	**l** $2\sqrt{54}$
m $7\sqrt{56}$	**n** $3\sqrt{72}$	**o** $3\sqrt{44}$	**p** $5\sqrt{90}$
q $6\sqrt{200}$	**r** $5\sqrt{98}$	**s** $9\sqrt{108}$	**t** $5\sqrt{68}$

6 Simplified surds can be written as an entire root by reversing the above process. Express the following as entire square roots. For example, $4\sqrt{3} = \sqrt{16} \times \sqrt{3} = \sqrt{48}$.

a $2\sqrt{3}$	**b** $3\sqrt{2}$	**c** $2\sqrt{5}$	**d** $3\sqrt{6}$
e $4\sqrt{2}$	**f** $5\sqrt{3}$	**g** $3\sqrt{7}$	**h** $5\sqrt{2}$
i $6\sqrt{2}$	**j** $5\sqrt{6}$	**k** $3\sqrt{10}$	**l** $4\sqrt{7}$
m $6\sqrt{7}$	**n** $5\sqrt{10}$	**o** $7\sqrt{2}$	**p** $10\sqrt{2}$
q $9\sqrt{3}$	**r** $8\sqrt{4}$	**s** $7\sqrt{9}$	**t** $12\sqrt{3}$

INVESTIGATION 12:02 PROOF THAT $\sqrt{2}$ IS IRRATIONAL

Let us suppose that $\sqrt{2}$ is rational and, therefore, can be written as a fraction in the form $\frac{p}{q}$ where p and q are positive integers and $q \neq 0$, **with no common factor**. (This assumption is essential.)

$$\sqrt{2} = \frac{p}{q}$$

$$2 = \frac{p^2}{q^2} \quad \text{(squaring both sides)}$$

$$2q^2 = p^2$$

This last step implies that p^2 must be divisible by 2 (2 is prime). Therefore 2 must divide into p exactly.

$\therefore$ p can be expressed in the form $2k$ for some integer k.

$$\therefore\ 2q^2 = (2k)^2$$
$$2q^2 = 4k^2$$
$$q^2 = 2k^2$$

Now, as for p above, it can be argued from this last step that q must be divisible by 2. But p and q were said to have no common factor, hence a contradiction exists. So our original assumption was wrong.

Therefore, p and q cannot be found so that $\sqrt{2} = \frac{p}{q}$. Hence, $\sqrt{2}$ must be irrational.

- Try to use the method above to prove that these are irrational.

1 $\sqrt{3}$ **2** $\sqrt{5}$ **3** $\sqrt{11}$

12:03 Addition and subtraction of surds

PREP QUIZ 12:03

Simplify:

1 $\sqrt{12}$ **2** $\sqrt{20}$ **3** $\sqrt{32}$ **4** $\sqrt{50}$

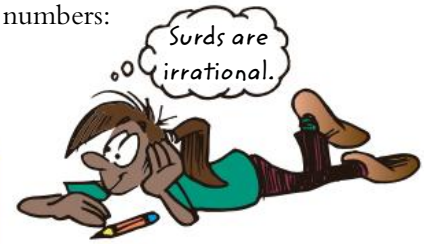
Evaluate:

5 $\sqrt{4}+\sqrt{9}$ **6** $\sqrt{4+9}$ (1 dec. pl.) **7** $\sqrt{16}-\sqrt{4}$

8 $\sqrt{16-4}$ (1 dec. pl.) **9** $\sqrt{25}+\sqrt{36}$ **10** $\sqrt{25+36}$ (1 dec. pl.)

As can be seen from the Prep quiz, if x and y are two positive numbers:

- $\sqrt{x}+\sqrt{y}$ does not equal $\sqrt{x+y}$
- $\sqrt{x}-\sqrt{y}$ does not equal $\sqrt{x-y}$



$\sqrt{5}+\sqrt{8} \neq \sqrt{13}$
$\sqrt{10}-\sqrt{7} \neq \sqrt{3}$

Only **like** surds can be added or subtracted. For example:

$2\sqrt{3}+4\sqrt{3}=6\sqrt{3}$ and $5\sqrt{2}-4\sqrt{2}=\sqrt{2}$

However, we can only tell whether surds are like or unlike if each is expressed in its simplest form.

When adding or subtracting surds:
- write each surd in its simplest form
- add or subtract *like* surds only.

Remember how in algebra only *like* terms could be added or subtracted.
e.g. $5x+3x=8x$
$7a-6a=a$

WORKED EXAMPLES

Simplify each of the following.

1 $4\sqrt{3}+7\sqrt{3}-2\sqrt{3}$
$=9\sqrt{3}$

2 $8\sqrt{2}+\sqrt{5}-\sqrt{2}+2\sqrt{5}$
$=7\sqrt{2}+3\sqrt{5}$

3 $\sqrt{8}+\sqrt{18}$
$=2\sqrt{2}+3\sqrt{2}$
$=5\sqrt{2}$

4 $\sqrt{75}+\sqrt{27}-2\sqrt{3}$
$=5\sqrt{3}+3\sqrt{3}-2\sqrt{3}$
$=6\sqrt{3}$

5 $2\sqrt{12}+3\sqrt{48}$
$=2(2\sqrt{3})+3(4\sqrt{3})$
$=4\sqrt{3}+12\sqrt{3}$
$=16\sqrt{3}$

6 $2\sqrt{75}-\sqrt{45}+2\sqrt{20}$
$=2(5\sqrt{3})-3\sqrt{5}+2(2\sqrt{5})$
$=10\sqrt{3}-3\sqrt{5}+4\sqrt{5}$
$=10\sqrt{3}+\sqrt{5}$

Exercise 12:03

P Foundation worksheet 12:03
Addition and subtraction of surds

1 Simplify:

a $3\sqrt{2}+2\sqrt{2}$ b $4\sqrt{3}+7\sqrt{3}$ c $5\sqrt{6}+2\sqrt{6}$
d $10\sqrt{3}-7\sqrt{3}$ e $9\sqrt{5}-6\sqrt{5}$ f $4\sqrt{2}-3\sqrt{2}$
g $\sqrt{7}+4\sqrt{7}$ h $5\sqrt{3}+\sqrt{3}$ i $9\sqrt{6}-\sqrt{6}$
j $9\sqrt{5}+2\sqrt{5}+3\sqrt{5}$ k $4\sqrt{10}+7\sqrt{10}-2\sqrt{10}$ l $4\sqrt{3}-3\sqrt{3}+5\sqrt{3}$
m $10\sqrt{2}-2\sqrt{2}-5\sqrt{2}$ n $\sqrt{3}+7\sqrt{3}-5\sqrt{3}$ o $2\sqrt{7}+3\sqrt{7}-5\sqrt{7}$

2 Simplify by collecting like surds.

a $2\sqrt{5}+3\sqrt{7}+4\sqrt{5}$ b $\sqrt{7}+3\sqrt{7}+2\sqrt{5}$
c $2\sqrt{3}+3\sqrt{5}+2\sqrt{3}+5\sqrt{5}$ d $9\sqrt{3}+2\sqrt{2}-7\sqrt{3}+3\sqrt{2}$
e $6\sqrt{10}+5\sqrt{7}-5\sqrt{10}-\sqrt{7}$ f $3\sqrt{3}+6\sqrt{5}-\sqrt{3}-4\sqrt{5}$
g $6\sqrt{11}-2\sqrt{7}+2\sqrt{11}+5\sqrt{7}$ h $9\sqrt{2}+3\sqrt{3}+9\sqrt{3}-8\sqrt{2}$
i $5\sqrt{2}+4\sqrt{5}-3\sqrt{5}-6\sqrt{2}$ j $10\sqrt{7}-2\sqrt{5}-8\sqrt{7}-3\sqrt{5}$

3 Simplify completely.

a $\sqrt{8}+\sqrt{2}$ b $\sqrt{12}+2\sqrt{3}$ c $2\sqrt{2}+\sqrt{18}$ d $2\sqrt{5}+2\sqrt{20}$
e $\sqrt{27}+2\sqrt{3}$ f $3\sqrt{6}+\sqrt{24}$ g $2\sqrt{8}-\sqrt{2}$ h $3\sqrt{5}-\sqrt{20}$
i $\sqrt{32}-3\sqrt{2}$ j $\sqrt{18}+\sqrt{32}$ k $\sqrt{20}+\sqrt{45}$ l $2\sqrt{27}-\sqrt{48}$
m $\sqrt{75}-2\sqrt{12}$ n $\sqrt{98}+3\sqrt{50}$ o $3\sqrt{50}+2\sqrt{32}$ p $5\sqrt{28}+2\sqrt{63}$
q $4\sqrt{45}-2\sqrt{20}$ r $2\sqrt{75}-3\sqrt{48}$

4 Simplify:

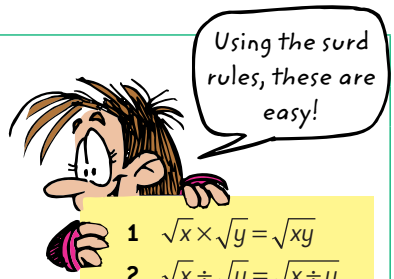
a $2\sqrt{8}-\sqrt{18}+3\sqrt{2}$ b $2\sqrt{5}+\sqrt{45}-\sqrt{20}$ c $\sqrt{27}+2\sqrt{48}-5\sqrt{3}$
d $5\sqrt{7}-\sqrt{63}+2\sqrt{28}$ e $5\sqrt{3}+\sqrt{50}-\sqrt{12}$ f $2\sqrt{45}+\sqrt{20}+\sqrt{32}$
g $9\sqrt{8}+3\sqrt{12}-\sqrt{27}$ h $5\sqrt{18}+\sqrt{72}-\sqrt{75}$

12:04 Multiplication and division of surds

WORKED EXAMPLE 1

Simplify the following.

a $\sqrt{7}\times\sqrt{3}$ b $3\sqrt{2}\times5\sqrt{2}$



1 $\sqrt{x}\times\sqrt{y}=\sqrt{xy}$
2 $\sqrt{x}\div\sqrt{y}=\sqrt{x\div y}$

Solutions

a $\sqrt{7}\times\sqrt{3}=\sqrt{7\times3}$
$=\sqrt{21}$

b $3\sqrt{2}\times5\sqrt{2}=3\times5\times\sqrt{2}\times\sqrt{2}$ $\sqrt{2}\times\sqrt{2}=2$
$=15\times2$
$=30$

WORKED EXAMPLE 2

Simplify the following.

a $5\sqrt{8} \times 3\sqrt{6}$

b $\sqrt{96} \div \sqrt{12}$

c $\dfrac{4\sqrt{3} \times \sqrt{18}}{\sqrt{12}}$

d $\sqrt{3}(2\sqrt{3} - \sqrt{5})$

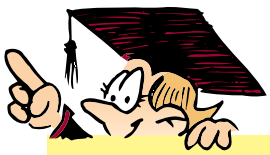
Solutions

a $5\sqrt{8} \times 3\sqrt{6} = 5 \times 3 \times \sqrt{8} \times \sqrt{6}$
$= 15 \times \sqrt{48}$
$= 15 \times 4\sqrt{3}$
$= 60\sqrt{3}$

b $\sqrt{96} \div \sqrt{12} = \sqrt{96 \div 12}$
$= \sqrt{8}$
$= 2\sqrt{2}$

c $\dfrac{4\sqrt{3} \times \sqrt{18}}{\sqrt{12}} = \dfrac{4\sqrt{3} \times 3\sqrt{2}}{2\sqrt{3}}$
$= \dfrac{4 \times 3 \times \sqrt{3} \times \sqrt{2}}{2\sqrt{3}}$
$= \dfrac{12 \times \sqrt{6}}{2\sqrt{3}}$
$= 6\sqrt{\dfrac{6}{3}}$
$= 6\sqrt{2}$

d $\sqrt{3}(2\sqrt{3} - \sqrt{5}) = \sqrt{3} \times 2\sqrt{3} - \sqrt{3} \times \sqrt{5}$
$= 2 \times 3 - \sqrt{3 \times 5}$
$= 6 - \sqrt{15}$



At this point we could cancel like this:

$$\frac{{}^{2}\cancel{4} \times 3 \times {}^{1}\cancel{\sqrt{3}} \times \sqrt{2}}{{}_{1}\cancel{2}\cancel{\sqrt{3}}_{1}} = 6\sqrt{2}$$

Exercise 12:04

P Foundation worksheet 12:04
Multiplication and division of surds

1 Simplify these products.

a $\sqrt{2} \times \sqrt{3}$
b $\sqrt{5} \times \sqrt{7}$
c $\sqrt{3} \times \sqrt{11}$
d $\sqrt{5} \times \sqrt{2}$
e $\sqrt{3} \times \sqrt{7}$
f $\sqrt{10} \times \sqrt{7}$
g $\sqrt{2} \times \sqrt{8}$
h $\sqrt{3} \times \sqrt{12}$
i $\sqrt{5} \times \sqrt{20}$
j $\sqrt{3} \times \sqrt{6}$
k $\sqrt{5} \times \sqrt{10}$
l $\sqrt{2} \times \sqrt{10}$
m $2\sqrt{3} \times \sqrt{5}$
n $3\sqrt{2} \times 4\sqrt{5}$
o $7\sqrt{2} \times 2\sqrt{3}$
p $5\sqrt{2} \times 2\sqrt{2}$
q $3\sqrt{3} \times 2\sqrt{3}$
r $4\sqrt{5} \times \sqrt{5}$
s $4\sqrt{2} \times 3\sqrt{10}$
t $2\sqrt{6} \times \sqrt{8}$
u $4\sqrt{3} \times 2\sqrt{15}$
v $2\sqrt{x} \times 3\sqrt{x}$
w $\sqrt{8x} \times \sqrt{2}$
x $a\sqrt{x} \times a\sqrt{x}$

2 Simplify:

a $\sqrt{10} \div \sqrt{2}$
b $\sqrt{12} \div \sqrt{4}$
c $\sqrt{6} \div \sqrt{3}$
d $\sqrt{27} \div \sqrt{3}$
e $\sqrt{32} \div \sqrt{8}$
f $\sqrt{45} \div \sqrt{5}$
g $5\sqrt{2} \div \sqrt{2}$
h $6\sqrt{5} \div \sqrt{5}$
i $10\sqrt{3} \div 5\sqrt{3}$
j $16\sqrt{8} \div 8$
k $30\sqrt{5} \div 10$
l $24\sqrt{7} \div 24$
m $12\sqrt{10} \div 2\sqrt{5}$
n $9\sqrt{12} \div 3\sqrt{6}$
o $10\sqrt{15} \div 5\sqrt{5}$
p $\sqrt{20} \div 2$
q $\sqrt{75} \div 5$
r $5\sqrt{8} \div 10$
s $\sqrt{2x} \div \sqrt{2}$
t $\sqrt{5a} \div \sqrt{a}$
u $\sqrt{20p} \div 2\sqrt{p}$

3 Simplify fully.

a $\dfrac{2\sqrt{3} \div 2\sqrt{6}}{4}$ b $\dfrac{4\sqrt{5} \times 2\sqrt{6}}{\sqrt{10}}$ c $\dfrac{2\sqrt{5} \times 3\sqrt{8}}{6\sqrt{20}}$

d $\dfrac{\sqrt{15} \times \sqrt{3}}{3\sqrt{5}}$ e $\dfrac{2\sqrt{3} \times \sqrt{6}}{\sqrt{12}}$ f $\dfrac{3\sqrt{7} \times 2\sqrt{6}}{\sqrt{21}}$

g $\dfrac{2\sqrt{6} \times 5\sqrt{5}}{10\sqrt{15}}$ h $\dfrac{6\sqrt{2} \times \sqrt{6}}{4\sqrt{3}}$ i $\dfrac{\sqrt{12} \times \sqrt{27}}{\sqrt{8} \times 2\sqrt{6}}$

4 Expand and simplify.

a $\sqrt{2}(\sqrt{3} + \sqrt{2})$ b $\sqrt{5}(\sqrt{5} + \sqrt{2})$ c $\sqrt{7}(2\sqrt{7} - \sqrt{2})$

d $\sqrt{3}(5 - \sqrt{3})$ e $\sqrt{2}(2\sqrt{3} - 1)$ f $\sqrt{10}(5\sqrt{2} - 4)$

g $2\sqrt{2}(\sqrt{2} + 1)$ h $3\sqrt{5}(\sqrt{5} + 2)$ i $4\sqrt{3}(\sqrt{2} - \sqrt{3})$

j $3\sqrt{6}(\sqrt{5} + \sqrt{6})$ k $2\sqrt{7}(\sqrt{7} - \sqrt{2})$ l $\sqrt{3}(7 - 3\sqrt{3})$

m $2\sqrt{2}(\sqrt{3} + 2\sqrt{2})$ n $4\sqrt{5}(\sqrt{2} - 2\sqrt{5})$ o $5\sqrt{6}(2\sqrt{6} - 3\sqrt{2})$

p $\sqrt{a}(\sqrt{a} + 1)$ q $\sqrt{x}(2\sqrt{x} + 3)$ r $2\sqrt{y}(3\sqrt{y} + 2\sqrt{x})$

How many balls do you begin with in a game of pool? What is the number of the black ball?

12:05 Binomial products

PREP QUIZ 12:05

Simplify the following.

1 $\sqrt{3} \times \sqrt{5}$ **2** $\sqrt{18} \times \sqrt{2}$ **3** $(\sqrt{5})^2$ **4** $(3\sqrt{2})^2$ **5** $3\sqrt{2} + 7\sqrt{2}$

6 $9\sqrt{3} - \sqrt{3}$ **7** $3\sqrt{3} - 3\sqrt{3}$

Expand and simplify where possible.

8 $5(\sqrt{7} - 4)$ **9** $\sqrt{10}(\sqrt{10} + \sqrt{3})$ **10** $2\sqrt{3}(3\sqrt{2} - \sqrt{3})$

In Chapter 2, we saw how to expand a binomial product.

$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

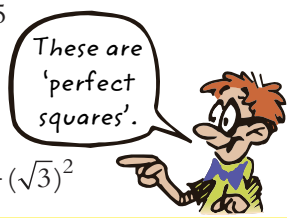
The same procedure is used if some of the terms are not pronumerals but surds. Examine the following examples.

WORKED EXAMPLES

Expand and simplify:

1 a $(\sqrt{2}+3)(\sqrt{2}-5)$
$= \sqrt{2}(\sqrt{2}-5)+3(\sqrt{2}-5)$
$= (\sqrt{2})^2 - 5\sqrt{2} + 3\sqrt{2} - 15$
$= 2 - 2\sqrt{2} - 15$
$= -13 - 2\sqrt{2}$

b $(3\sqrt{2}-\sqrt{5})(2\sqrt{2}+3\sqrt{5})$
$= 3\sqrt{2}(2\sqrt{2}+3\sqrt{5}) - \sqrt{5}(2\sqrt{2}+3\sqrt{5})$
$= 12 + 9\sqrt{10} - 2\sqrt{10} - 15$
$= 7\sqrt{10} - 3$



2 a $(2\sqrt{3}+5)^2$
$= (2\sqrt{3})^2 + 2\times 2\sqrt{3}\times 5 + (5)^2$
$= 12 + 20\sqrt{3} + 25$
$= 37 + 20\sqrt{3}$

b $(\sqrt{7}-\sqrt{3})^2$
$= (\sqrt{7})^2 - 2\times\sqrt{7}\times\sqrt{3} + (\sqrt{3})^2$
$= 7 - 2\sqrt{21} + 3$
$= 10 - 2\sqrt{21}$

Remember!
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Remember!
$(a+b)(a-b) = a^2 - b^2$

3 a $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$
$= 5 - 2$
$= 3$

b $(5\sqrt{2}-\sqrt{7})(5\sqrt{2}+\sqrt{7}) = (5\sqrt{2})^2 - (\sqrt{7})^2$
$= 50 - 7$
$= 43$

Exercise 12:05

P Foundation worksheet 12:05
Binomial products: Surds

Expand and simplify the following.

1 **a** $(\sqrt{2}+3)(\sqrt{2}+1)$ **b** $(\sqrt{3}+5)(\sqrt{3}-1)$ **c** $(\sqrt{7}-2)(\sqrt{7}-5)$
d $(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{5})$ **e** $(\sqrt{7}-\sqrt{2})(\sqrt{5}-\sqrt{2})$ **f** $(\sqrt{10}+\sqrt{2})(\sqrt{5}+\sqrt{3})$
g $(5+\sqrt{2})(\sqrt{3}+\sqrt{2})$ **h** $(2-\sqrt{3})(2-\sqrt{5})$ **i** $(\sqrt{6}-\sqrt{2})(5-\sqrt{6})$
j $(\sqrt{2}+\sqrt{3})(2\sqrt{2}+1)$ **k** $(\sqrt{3}+2\sqrt{5})(\sqrt{3}+\sqrt{5})$ **l** $(2\sqrt{3}+\sqrt{2})(\sqrt{3}+2\sqrt{2})$
m $(5\sqrt{2}-\sqrt{7})(\sqrt{7}-2\sqrt{2})$ **n** $(2\sqrt{5}-\sqrt{3})(5\sqrt{3}+\sqrt{5})$ **o** $(\sqrt{7}-5\sqrt{2})(\sqrt{2}-5\sqrt{7})$
p $(5\sqrt{3}+2\sqrt{7})(2\sqrt{3}-5\sqrt{7})$ **q** $(10\sqrt{10}+\sqrt{7})(2\sqrt{7}+\sqrt{10})$ **r** $(5\sqrt{3}+7\sqrt{2})(7\sqrt{3}-2\sqrt{2})$
s $(9\sqrt{2}-\sqrt{3})(4\sqrt{2}+2\sqrt{3})$ **t** $(5\sqrt{7}+4)(2\sqrt{7}-7)$ **u** $(6\sqrt{3}+5)(7-2\sqrt{3})$
v $(\sqrt{x}+3)(\sqrt{x}+2)$ **w** $(\sqrt{m}+\sqrt{n})(2\sqrt{m}+\sqrt{n})$ **x** $(3\sqrt{a}-2\sqrt{b})(2\sqrt{a}+3\sqrt{b})$

2 **a** $(\sqrt{2}+1)^2$ **b** $(\sqrt{3}-5)^2$ **c** $(\sqrt{5}+2)^2$
d $(\sqrt{3}+\sqrt{2})^2$ **e** $(\sqrt{5}-\sqrt{2})^2$ **f** $(\sqrt{3}+\sqrt{10})^2$
g $(2\sqrt{3}+1)^2$ **h** $(3\sqrt{2}-4)^2$ **i** $(5+2\sqrt{5})^2$
j $(2\sqrt{2}+\sqrt{7})^2$ **k** $(3\sqrt{5}+\sqrt{10})^2$ **l** $(\sqrt{7}-3\sqrt{5})^2$
m $(5\sqrt{3}+2\sqrt{2})^2$ **n** $(7\sqrt{3}-2\sqrt{5})^2$ **o** $(5\sqrt{10}-10\sqrt{3})^2$
p $(\sqrt{x}+\sqrt{y})^2$ **q** $(2\sqrt{m}+5)^2$ **r** $(3\sqrt{p}-2\sqrt{q})^2$

3 a $(\sqrt{2}+1)(\sqrt{2}-1)$ b $(5+\sqrt{3})(5-\sqrt{3})$

c $(\sqrt{10}-7)(\sqrt{10}+7)$ d $(4-\sqrt{2})(4+\sqrt{2})$

e $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ f $(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})$

g $(\sqrt{10}-\sqrt{8})(\sqrt{10}+\sqrt{8})$ h $(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})$

i $(2\sqrt{3}-5)(2\sqrt{3}+5)$ j $(6-3\sqrt{2})(6+3\sqrt{2})$

k $(\sqrt{7}+2\sqrt{3})(\sqrt{7}-2\sqrt{3})$ l $(3\sqrt{5}-\sqrt{3})(3\sqrt{5}+\sqrt{3})$

m $(5\sqrt{2}-2\sqrt{3})(5\sqrt{2}+2\sqrt{3})$

n $(2\sqrt{3}+3\sqrt{2})(2\sqrt{3}-3\sqrt{2})$

o $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

p $(2\sqrt{a}+3\sqrt{b})(2\sqrt{a}-3\sqrt{b})$

Important notice!
The two binomials in each part of Question **3** are *conjugate surds*.
Note that when a binomial surd is multiplied by its 'conjugate', the answer is always a rational number.

12:06 Rationalising the denominator

PREP QUIZ 12:06

Simplify the following:

1 $\sqrt{3}\times\sqrt{3}$ **2** $\sqrt{5}\times\sqrt{5}$ **3** $\sqrt{10}\times\sqrt{10}$ **4** $2\sqrt{3}\times\sqrt{3}$ **5** $5\sqrt{2}\times\sqrt{2}$

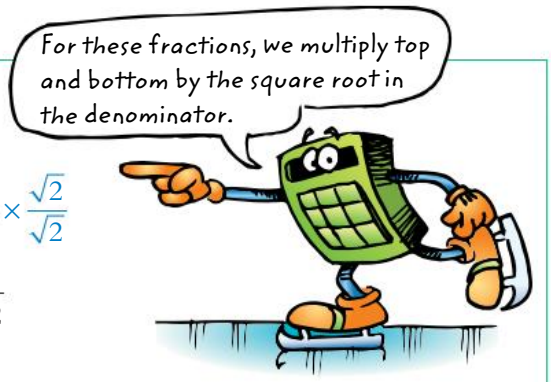
6 $2\sqrt{6}\times\sqrt{6}$ **7** $7\sqrt{5}\times\sqrt{5}$ **8** $\sqrt{5}\times\sqrt{3}$ **9** $\sqrt{6}\times\sqrt{2}$ **10** $(2+\sqrt{5})\times 5$

If a fraction has a surd (i.e. an irrational number) in its denominator, we generally rewrite the fraction with a 'rational' denominator by using the methods shown below.

Monomial denominators

WORKED EXAMPLES

Rewrite with rational denominators.



1 $\dfrac{3}{\sqrt{3}} = \dfrac{3}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{3\sqrt{3}}{3}$

$= \sqrt{3}$

2 $\dfrac{1}{5\sqrt{2}} = \dfrac{1}{5\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{2}}{5\times 2}$

$= \dfrac{\sqrt{2}}{10}$

3 $\dfrac{\sqrt{5}}{\sqrt{12}} = \dfrac{\sqrt{5}}{2\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{15}}{2\times 3}$

$= \dfrac{\sqrt{15}}{6}$

4 $\dfrac{2+\sqrt{3}}{2\sqrt{3}} = \dfrac{2+\sqrt{3}}{2\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}(2+\sqrt{3})}{2\times 3}$

$= \dfrac{2\sqrt{3}+3}{6}$

Note:
Multiplying by $\dfrac{\sqrt{3}}{\sqrt{3}}$ is the same as multiplying by 1.

Binomial denominators

WORKED EXAMPLES

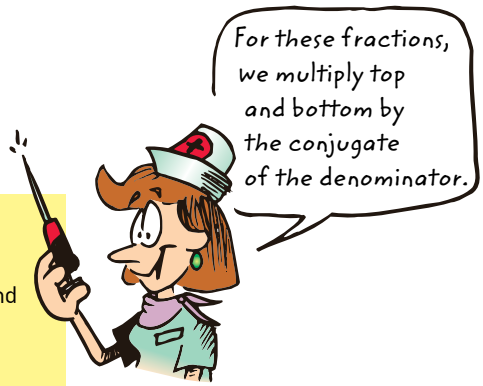
Rationalise the denominators for each expression.

1 $\dfrac{5}{5-\sqrt{2}} = \dfrac{5}{5-\sqrt{2}} \times \dfrac{5+\sqrt{2}}{5+\sqrt{2}}$

$= \dfrac{5(5+\sqrt{2})}{25-2}$

$= \dfrac{25+5\sqrt{2}}{23}$

Note:
The product of a binomial surd and its conjugate is always rational.



2 $\dfrac{1}{\sqrt{3}+\sqrt{2}} = \dfrac{1}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3}-\sqrt{2}$

3 $\dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}-\sqrt{5}} = \dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}-\sqrt{5}} \times \dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$

$= \dfrac{(2\sqrt{3}+\sqrt{5})^2}{12-5}$

$= \dfrac{17+4\sqrt{15}}{7}$

Exercise 12:06

1 Rationalise the denominator for each of the following:

a $\dfrac{1}{\sqrt{2}}$ **b** $\dfrac{1}{\sqrt{5}}$ **c** $\dfrac{2}{\sqrt{3}}$ **d** $\dfrac{5}{\sqrt{10}}$ **e** $\dfrac{3}{\sqrt{2}}$ **f** $\dfrac{6}{\sqrt{3}}$

g $\dfrac{10}{\sqrt{5}}$ **h** $\dfrac{2}{\sqrt{11}}$ **i** $\dfrac{\sqrt{2}}{\sqrt{3}}$ **j** $\dfrac{\sqrt{3}}{\sqrt{5}}$ **k** $\dfrac{\sqrt{5}}{\sqrt{10}}$ **l** $\dfrac{\sqrt{3}}{\sqrt{15}}$

m $\dfrac{1}{2\sqrt{2}}$ **n** $\dfrac{2}{5\sqrt{3}}$ **o** $\dfrac{7}{2\sqrt{5}}$ **p** $\dfrac{10}{2\sqrt{3}}$ **q** $\dfrac{\sqrt{6}}{2\sqrt{3}}$ **r** $\dfrac{\sqrt{5}}{5\sqrt{2}}$

s $\dfrac{2\sqrt{3}}{3\sqrt{2}}$ **t** $\dfrac{5\sqrt{7}}{3\sqrt{5}}$ **u** $\dfrac{2+\sqrt{3}}{\sqrt{3}}$ **v** $\dfrac{1+\sqrt{5}}{\sqrt{2}}$ **w** $\dfrac{\sqrt{7}+\sqrt{3}}{2\sqrt{7}}$ **x** $\dfrac{\sqrt{10}-\sqrt{5}}{5\sqrt{10}}$

2 Express with a rational denominator.

a $\dfrac{1}{1+\sqrt{2}}$ **b** $\dfrac{1}{\sqrt{3}-1}$ **c** $\dfrac{1}{\sqrt{7}-\sqrt{5}}$ **d** $\dfrac{1}{\sqrt{10}+\sqrt{2}}$

e $\dfrac{3}{\sqrt{3}+2}$ **f** $\dfrac{5}{5-\sqrt{2}}$ **g** $\dfrac{10}{\sqrt{5}-\sqrt{2}}$ **h** $\dfrac{12}{\sqrt{7}-\sqrt{3}}$

i $\dfrac{1}{2\sqrt{3}+5}$ **j** $\dfrac{2}{5-2\sqrt{2}}$ **k** $\dfrac{3}{3\sqrt{2}+2\sqrt{3}}$ **l** $\dfrac{1}{4\sqrt{3}-3\sqrt{2}}$

m $\dfrac{5+\sqrt{2}}{5-\sqrt{2}}$ **n** $\dfrac{4+\sqrt{3}}{4-\sqrt{3}}$ **o** $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ **p** $\dfrac{3\sqrt{2}-\sqrt{3}}{3\sqrt{2}+\sqrt{3}}$

3 Evaluate each fraction correct to three significant figures (using your calculator). Then rationalise the denominator and evaluate the fraction again. Compare this answer with your first calculation.

a $\frac{2}{\sqrt{5}}$ b $\frac{3}{\sqrt{7}}$ c $\frac{\sqrt{3}}{2\sqrt{2}}$ d $\frac{\sqrt{7}}{3\sqrt{5}}$

e $\frac{1}{\sqrt{7}-\sqrt{5}}$ f $\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ g $\frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}-\sqrt{7}}$ h $\frac{2\sqrt{3}-\sqrt{7}}{2\sqrt{3}+\sqrt{7}}$

4 Rationalise each denominator, then express as a single fraction.

a $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}$ b $\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{2}}$ c $\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{5}}$

d $\frac{2}{\sqrt{10}}-\frac{3}{\sqrt{5}}$ e $\frac{3}{\sqrt{8}}+\frac{5}{\sqrt{2}}$ f $\frac{2}{2\sqrt{3}}-\frac{1}{3\sqrt{2}}$

g $\frac{2}{5\sqrt{2}}+\frac{5}{\sqrt{10}}$ h $\frac{\sqrt{5}}{\sqrt{2}}+\frac{\sqrt{5}}{\sqrt{3}}$ i $\frac{\sqrt{2}}{\sqrt{5}}-\frac{\sqrt{3}}{\sqrt{2}}$

j $\frac{1}{2-\sqrt{3}}+\frac{1}{2+\sqrt{3}}$ k $\frac{1}{\sqrt{5}-\sqrt{3}}-\frac{1}{\sqrt{7}+\sqrt{5}}$ l $\frac{5}{6-\sqrt{3}}+\frac{3}{5+\sqrt{3}}$

FUN SPOT 12:06 — WHAT NATIONALITY IS SANTA CLAUS?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

H Which is irrational?
$1\frac{1}{2}$, $\sqrt{3}$ or $\sqrt{16}-1$

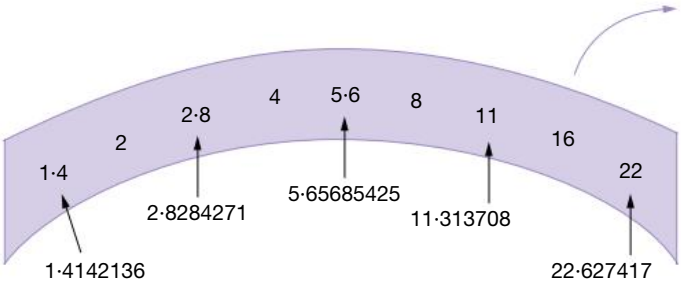
Simplify:

H $\sqrt{16\times 4}$
I $\sqrt{2}\times\sqrt{3}$
L $\sqrt{27}\div\sqrt{3}$
N $\sqrt{27}$
O $(\sqrt{6})^2$
O $3\sqrt{2}\times\sqrt{6}$
P $\sqrt{16-9}$
R $\sqrt{2}+\sqrt{2}$
S $12\sqrt{3}-10\sqrt{3}$
T $5\sqrt{3}-\sqrt{3}$

$3\sqrt{3}$	$6\sqrt{3}$	$2\sqrt{2}$	$4\sqrt{3}$	$\sqrt{3}$	$\sqrt{7}$	6	3	$\sqrt{6}$	$2\sqrt{3}$	8

INVESTIGATION 12:06 F-STOPS AND $\sqrt{2}$

On many cameras it is possible to change how far the shutter opens, using what are called f-stops.



The f-stops 2, 4, 8 and 16 are accurate. More accurate readings for the rest are given below the scale.

- Find the pattern in the accurate f-stops.
  - Try squaring each accurate f-stop number.
  - Try dividing each f-stop number by the one before it.
- Try to discover how f-stops are used.

12:07 Fractional indices

PREP QUIZ 12:07

Complete:

1 $8^2 = 64$
$\therefore \sqrt{64} = \ldots$

2 $4^3 = 64$
$\therefore \sqrt[3]{64} = \ldots$

3 $2^6 = 64$
$\therefore \sqrt[6]{64} = \ldots$

4 $3^4 = 81$
$\therefore \sqrt[4]{81} = \ldots$

Using the index laws, simplify the following:

5 $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$

6 $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}}$

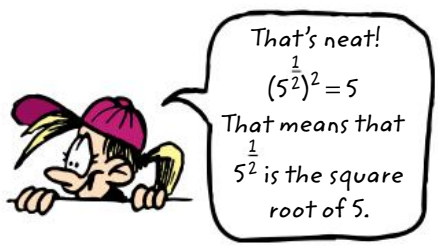
7 $(m^{\frac{1}{2}})^2$

8 $(n^{\frac{1}{3}})^3$

9 If $x^n \times x^n = x^1$
What is the value of n?

10 If $a^n \times a^n \times a^n = a^1$
What is the value of n?

What is the meaning of a fractional index?



The meaning is shown in the examples below.

1 $5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^{(\frac{1}{2}+\frac{1}{2})}$
$= 5^1$
$= 5$

$\sqrt{5} \times \sqrt{5} = 5$
$\therefore 5^{\frac{1}{2}} = \sqrt{5}$

The number that multiplies itself to give 5 (i.e. $5^{\frac{1}{2}}$) is the square root of 5.

2 $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{(\frac{1}{3}+\frac{1}{3}+\frac{1}{3})}$
$= 8^1$
$= 8$

$2 \times 2 \times 2 = 8$
$\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$
Two is the cube root of 8.

$8^{\frac{1}{3}} = \sqrt[3]{8}$, (the cube root of 8)
$\therefore\ 8^{\frac{1}{3}} = 2$

$(\sqrt[3]{x})^3 = x$ so $\sqrt[3]{x} = x^{\frac{1}{3}}$

$x^{\frac{1}{2}} = \sqrt{x}$
$x^{\frac{1}{3}} = \sqrt[3]{x}$
$x^{\frac{1}{n}} = n$th root of x
$x^{\frac{1}{3}}$ is the number that, when multiplied together three times, gives x.

3 $81^{\frac{1}{4}} \times 81^{\frac{1}{4}} \times 81^{\frac{1}{4}} \times 81^{\frac{1}{4}} = 8^{(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4})}$
$= 81^1$
$= 81$

$81^{\frac{1}{4}} = \sqrt[4]{81}$ (the fourth root of 81)
$\therefore\ 81^{\frac{1}{4}} = 3$

WORKED EXAMPLES

1 Write in surd form:

a $3^{\frac{1}{2}} = \sqrt{3}$ **b** $7^{\frac{1}{3}} = \sqrt[3]{7}$ **c** $x^{\frac{1}{5}} = \sqrt[5]{x}$

2 Write in index form:

a $\sqrt{5} = 5^{\frac{1}{2}}$ **b** $\sqrt[4]{9} = 9^{\frac{1}{4}}$ **c** $\sqrt[3]{a} = a^{\frac{1}{3}}$

Remember!
$x^{-1} = \frac{1}{x}$
$x^{-n} = \frac{1}{x^n}$

3 Find the value of:

a $64^{\frac{1}{2}} = \sqrt{64}$
$= 8$

b $64^{\frac{1}{3}} = \sqrt[3]{64}$
$= 4$

c $(36x^6)^{\frac{1}{2}} = \sqrt{36} \times x^{6 \times \frac{1}{2}}$
$= 6x^3$

d $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2$
$= (\sqrt[3]{8})^2$
$= 2^2$
$= 4$

e $9^{-\frac{1}{2}} = (9^{\frac{1}{2}})^{-1}$
$= (\sqrt{9})^{-1}$
$= 3^{-1}$
$= \frac{1}{3}$

f $16^{-\frac{3}{2}} = (16^{\frac{1}{2}})^{-3}$
$= (\sqrt{16})^{-3}$
$= 4^{-3}$
$= \frac{1}{4^3}$
$= \frac{1}{64}$

Note:
$x^{\frac{p}{q}} = \sqrt[q]{x^p}$ or $(\sqrt[q]{x})^p$

4 Evaluate using your calculator.

a $196^{\frac{1}{2}} = \sqrt{196}$
using the square root key
$\sqrt{196} = 14$

b To evaluate $32^{\frac{1}{5}}$
press: 5 SHIFT $\sqrt{\square}$ 32 =
Answer: $32^{\frac{1}{5}} = 2$

c To evaluate $256^{-\frac{1}{4}}$
Press: 256 $x^{\square}$ (−) 1 $\frac{\square}{\square}$ 4 =
Answer: $256^{-\frac{1}{4}} = \frac{1}{4}$ or 0·25

d To evaluate $125^{-\frac{4}{3}}$
Press: 125 $x^{\square}$ (−) 4 $\frac{\square}{\square}$ 3 =
Answer: $125^{-\frac{4}{3}} = \frac{1}{625}$ or 0·0016

For roots higher than a square root ($\frac{1}{2}$), use the $x^{1/y}$ or $\sqrt{\square}$ key.
Some calculators have a cube root ($\frac{1}{3}$) key, $\sqrt[3]{}$ or $\sqrt[3]{\square}$.
You may need to use the inverse or SHIFT key.
Alternatively, you can use the $x^{\square}$ key and enter the power as a fraction.

Exercise 12:07

1 Write each of the following in surd form.

a $6^{\frac{1}{2}}$ b $4^{\frac{1}{3}}$ c $9^{\frac{1}{4}}$ d $10^{\frac{1}{5}}$

e $2 \times 3^{\frac{1}{2}}$ f $5 \times 2^{\frac{1}{2}}$ g $7 \times 4^{\frac{1}{3}}$ h $9 \times 5^{\frac{1}{4}}$

2 Write in index form:

a $\sqrt{5}$ b $\sqrt{20}$ c $\sqrt[3]{7}$ d $\sqrt[5]{9}$

e $3\sqrt{2}$ f $4\sqrt{5}$ g $2\sqrt[3]{4}$ h $5\sqrt[4]{2}$

3 Find the value of:

a $49^{\frac{1}{2}}$ b $8^{\frac{1}{3}}$ c $125^{\frac{1}{3}}$ d $16^{\frac{1}{4}}$

e $100^{\frac{1}{2}}$ f $1000^{\frac{1}{3}}$ g $144^{\frac{1}{2}}$ h $121^{\frac{1}{2}}$

i $64^{\frac{1}{3}}$ j $32^{\frac{1}{5}}$ k $81^{\frac{1}{4}}$ l $64^{\frac{1}{6}}$

4 Assuming that all pronumerals are positive, simplify:

a $m^{\frac{1}{2}} \times m^{\frac{1}{2}}$ b $a^{\frac{2}{3}} \times a^{\frac{1}{3}}$ c $y^{\frac{5}{3}} \times y^{\frac{1}{3}}$

d $6x^{\frac{1}{2}} \times 2x^{\frac{1}{2}}$ e $9n^{\frac{2}{3}} \times 2n^{\frac{1}{3}}$ f $4x^{\frac{1}{2}} \times 2x^{-\frac{1}{2}}$

g $(x^2)^{\frac{1}{2}}$ h $(y^6)^{\frac{1}{3}}$ i $(9a^4)^{\frac{1}{2}}$

j $(8x^6)^{\frac{1}{3}}$ k $(a^4 b^2)^{\frac{1}{2}}$ l $(4n^2 m^6)^{\frac{1}{2}}$

m $(8x^3 y^3)^{\frac{1}{3}}$ n $(64x^3 y^6)^{\frac{1}{3}}$

$(x^a)^b = x^{ab}$

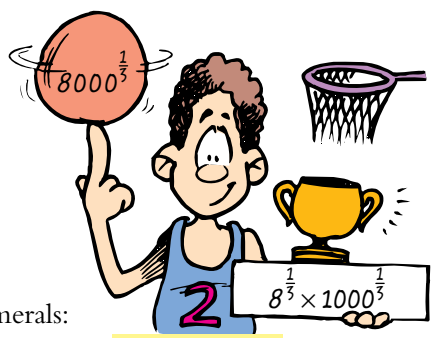
5 Evaluate:

a $9^{-\frac{1}{2}}$ b $25^{-\frac{1}{2}}$ c $8^{-\frac{1}{3}}$

d $9^{\frac{3}{2}}$ e $4^{\frac{3}{2}}$ f $4^{\frac{5}{2}}$

g $16^{\frac{3}{4}}$ h $125^{\frac{2}{3}}$ i $8^{-\frac{2}{3}}$

j $9^{\frac{5}{2}}$ k $32^{\frac{4}{5}}$ l $16^{-\frac{3}{4}}$



6 Evaluate using your calculator, leaving answers as decimal numerals:

a $225^{\frac{1}{2}}$ b $784^{\frac{1}{2}}$ c $1024^{\frac{1}{2}}$

d $729^{\frac{1}{3}}$ e $3375^{\frac{1}{3}}$ f $8000^{\frac{1}{3}}$

g $225^{\frac{3}{2}}$ h $729^{\frac{2}{3}}$ i $8000^{\frac{5}{3}}$

j $(0{\cdot}125)^{-\frac{2}{3}}$ k $(0{\cdot}25)^{-\frac{5}{2}}$ l $(0{\cdot}01)^{-\frac{3}{2}}$

$(ab)^{\frac{1}{3}} = a^{\frac{1}{3}} b^{\frac{1}{3}}$

7 If $a = 4$, $b = 8$ and $c = 9$, evaluate the following:

a $a^{\frac{1}{2}} + b^{\frac{1}{3}}$ b $(ab)^{\frac{1}{5}}$ c $2c^{\frac{3}{2}}$ d $\frac{1}{2}(ac)^{\frac{1}{2}}$

e $a^{-\frac{1}{2}} + b^{-\frac{1}{3}}$ f $(2b)^{-\frac{1}{4}}$ g $(2ab)^{-\frac{1}{6}}$ h $(2bc)^{\frac{1}{2}} - a^{\frac{3}{2}}$

8 Use the fact that $x^{\frac{p}{q}} = \sqrt[q]{x^p}$, to simplify:

a $(27a^3)^{\frac{2}{3}}$ b $(x^6y^{12})^{\frac{2}{3}}$ c $(8m^9)^{\frac{2}{3}}$

d $\left(\frac{a^3}{b^3}\right)^{\frac{2}{3}}$ e $\left(\frac{16}{x^4}\right)^{\frac{3}{4}}$ f $\left(\frac{y^6}{25}\right)^{\frac{3}{2}}$

$x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{1}{2}}$ stands for the positive square root of x.

FUN SPOT 12:07 WHAT DO INUITS SING AT BIRTHDAY PARTIES?

Answer each question and write the letter for that question in the box above the correct answer.

Solve:

A $\frac{x}{2} = 5$ E $\frac{x}{5} = 3$ E $\frac{x}{10} = 1{\cdot}86$

L $\frac{x}{7} = 0{\cdot}185$ E $\frac{x}{5} = 0{\cdot}5$ O $0{\cdot}560 = \frac{x}{5}$

W $5x + 3 = 21$ Y $\frac{14}{x} = 1$ Z $2x - 5 = 7 - 6x$

Simplify:

L $5x - x$ O $5x \div x$ Y $5x \times x$ E $2x \times 3y$

L $x^3 \times x^5$ O $x^6 \div x^2$ F $(2x^5)^3$ G $5x^0 \times 4$

D 35×10^{-2} E $\sqrt{169}$ F $\sqrt{7} \times \sqrt{7}$ G $(3\sqrt{3})^2$

H $\sqrt{2} + \sqrt{2}$ I $\sqrt{50}$ J $\sqrt{3} \times \sqrt{6}$ L $3\sqrt{7} - \sqrt{7}$

N $\sqrt{50} + \sqrt{18}$ O $5\sqrt{2} \times 3\sqrt{2}$ R $\sqrt{32} \div \sqrt{2}$ S $\sqrt{6} \div \sqrt{2}$

T $\sqrt{3} + 2\sqrt{3}$

$3\sqrt{3}$	$2\sqrt{2}$	$x = 18{\cdot}6$	$x = 14$	$\sqrt{3}$	$5\sqrt{2}$	$8\sqrt{2}$	20	7	4	$x = 15$	13	$x = 1{\cdot}5$	$x = 2{\cdot}5$	$x = 10$

$3\sqrt{2}$	30	x^8	$4x$	$5x^2$	27	$x = 2{\cdot}8$	x^4	$0{\cdot}35$	$8x^{15}$	$6xy$	$x = 1{\cdot}295$	$2\sqrt{7}$	5	$x = 3{\cdot}6$

The design of the Parthenon is said to be based on the golden ratio, $1 : \varphi$

where $\varphi = \frac{1+\sqrt{5}}{2}$ $(\varphi \approx 1{\cdot}618)$

MATHS TERMS 12

conjugate

- binomials that multiply to give the difference of two squares are the 'conjugate' of each other,
  e.g. $(a - b)$ and $(a + b)$
  $(\sqrt{3} + 5)$ and $(\sqrt{3} - 5)$
  are conjugate pairs

fractional indices

- another way of writing the 'root' of a number or term
- $x^{\frac{1}{2}} = \sqrt{x}$, $x^{\frac{1}{3}} = \sqrt[3]{x}$, $x^{\frac{1}{n}} = \sqrt[n]{x}$

irrational numbers

- numbers that *cannot* be expressed in the form $\frac{a}{b}$, where a and b are integers
- they cannot be given an exact decimal value,
  e.g. $\pi, \sqrt{3}, 3\sqrt{2} + 1$

negative indices

- indicate the reciprocal of a term,
  e.g. $x^{-1} = \frac{1}{x}, x^{-n} = \frac{1}{x^n}$
  i.e. $5^{-1} = \frac{1}{5}, 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

rational numbers

- numbers that can be written in the form $\frac{a}{b}$ where a and b are integers $(b \neq 0)$
- they can be expressed as a terminating or repeating decimal,
  e.g. integers, fractions, percentages

real numbers

- the combination of rational and irrational numbers

surds

- numerical expressions that involve irrational roots,
  e.g. $\sqrt{3}, \sqrt[3]{5}, 2\sqrt{7} + 5$

The Sydney Opera House is a world famous example of mathematical design.

DIAGNOSTIC TEST 12 FURTHER NUMBER

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 For each expression, state whether it is *rational* or *irrational*. 12:01

a $\sqrt{50}$ b $0\dot{4}$ c $\sqrt{2}$ d $\sqrt{16}$

2 Evaluate correct to three decimal places: 12:01

a $\sqrt{5}$ b $\sqrt{13}$ c $\sqrt{21}$ d $\sqrt{47}$

3 Simplify each surd. 12:02

a $\sqrt{20}$ b $\sqrt{27}$ c $3\sqrt{8}$ d $2\sqrt{75}$

4 Express each of the following as an entire square root. 12:02

a $2\sqrt{5}$ b $3\sqrt{2}$ c $5\sqrt{7}$ d $4\sqrt{5}$

5 Simplify completely. 12:03

a $2\sqrt{3}+4\sqrt{3}$ b $6\sqrt{5}-\sqrt{5}$ c $\sqrt{8}-\sqrt{2}$ d $\sqrt{27}+2\sqrt{3}$

6 Simplify: 12:04

a $\sqrt{5}\times\sqrt{6}$ b $\sqrt{3}\times\sqrt{12}$ c $2\sqrt{3}\times\sqrt{5}$ d $3\sqrt{8}\times2\sqrt{2}$

7 Simplify: 12:04

a $\sqrt{12}\div\sqrt{2}$ b $\sqrt{32}\div\sqrt{8}$ c $5\sqrt{3}\div\sqrt{3}$ d $10\sqrt{10}\div2\sqrt{5}$

8 Expand and simplify: 12:05

a $(2+\sqrt{5})(3+\sqrt{5})$ b $(2\sqrt{3}+\sqrt{2})(\sqrt{3}-3\sqrt{2})$

c $(\sqrt{7}+\sqrt{3})^2$ d $(5-\sqrt{3})(5+\sqrt{3})$

9 Rationalise the denominator of: 12:06

a $\dfrac{3}{\sqrt{2}}$ b $\dfrac{5}{\sqrt{5}}$ c $\dfrac{\sqrt{3}+1}{2\sqrt{3}}$ d $\dfrac{5-\sqrt{2}}{5\sqrt{2}}$

10 Express with a rational denominator. 12:06

a $\dfrac{1}{5+\sqrt{2}}$ b $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ c $\dfrac{\sqrt{10}-2}{\sqrt{10}+2}$ d $\dfrac{\sqrt{5}+\sqrt{7}}{3\sqrt{7}-2\sqrt{5}}$

11 Simplify: 12:07

a $25^{\frac{1}{2}}$ b $27^{\frac{1}{3}}$ c $8^{\frac{1}{3}}$ d $16^{-\frac{1}{2}}$

12 If $x > 0$ simplify: 12:07

a $3x^{\frac{1}{2}}\times4x^{\frac{1}{2}}$ b $(49m^6)^{\frac{1}{2}}$ c $(8x^3)^{\frac{2}{3}}$ d $(4x^2)^{\frac{5}{2}}$

Can you use your calculator to find the value of 2^{500}? What is the largest power of 2 that can be calculated using your calculator?

ASSIGNMENT 12A Chapter review

1 Between which two whole numbers does each number lie?

a $\sqrt{10}$ **b** $\sqrt{40}$ **c** $\sqrt{90}$

d $\sqrt{130}$ **e** $\sqrt{401}$ **f** $\sqrt{200}$

g $15^{\frac{1}{2}}$ **h** $50^{\frac{1}{2}}$ **i** $99^{\frac{1}{2}}$

2 Simplify these expressions.

a $3\sqrt{54}$

b $2\sqrt{75} + \sqrt{27}$

c $5\sqrt{5} - \sqrt{45}$

d $2\sqrt{3} \times 5\sqrt{12}$

e $150\sqrt{6} \div 10\sqrt{3}$

f $\sqrt{7} + 2\sqrt{28} - \sqrt{63}$

3 Expand the following.

a $\sqrt{5}(5 - \sqrt{5})$

b $2\sqrt{3}(2\sqrt{3} + \sqrt{2})$

c $5\sqrt{2}(\sqrt{8} - 2\sqrt{2})$

d $(\sqrt{7} + \sqrt{2})^2$

e $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$

f $(\sqrt{5} + \sqrt{6})(\sqrt{2} + \sqrt{5})$

4 Evaluate:

a $64^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** 5^{-2} **d** 3^{-4}

e $8^{\frac{2}{3}}$ **f** $144^{\frac{1}{2}}$ **g** $225^{\frac{1}{2}}$ **h** $7776^{\frac{1}{5}}$

5 Evaluate using a calculator, correct to two decimal places.

a $2\sqrt{5} - \sqrt{3}$ **b** $5\sqrt{6} + 2\sqrt{3}$

c $\sqrt{7}(\sqrt{5} + 2)$ **d** $4\sqrt{7} \times 2\sqrt{5}$

e $\sqrt{15} \div 2\sqrt{7}$ **f** $(\sqrt{3})^3 \times 2\sqrt{5}$

6 Rationalise the denominator of the following, and evaluate each expression correct to three decimal places.

a $\frac{5}{\sqrt{7}}$ **b** $\frac{\sqrt{5}}{\sqrt{3}}$

c $\frac{\sqrt{3} - 1}{\sqrt{3}}$ **d** $\frac{2\sqrt{2} + \sqrt{3}}{\sqrt{3}}$

7 Rationalise the denominator of each expression.

a $\frac{2}{\sqrt{5} - 1}$ **b** $\frac{1}{\sqrt{7} + \sqrt{2}}$

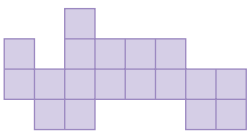
c $\frac{\sqrt{5} - 2}{\sqrt{5} + 2}$ **d** $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$

ASSIGNMENT 12B Working mathematically

1 When the square of a number is divided by 5, the result is equal to 5 less than twice the original number. Find the number.

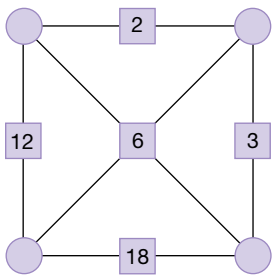
2 If it takes 8 tractors 8 days to use 8 tanks of fuel, how many days will it take one tractor to use one tank of fuel?

3



Divide this shape into three equal pieces.

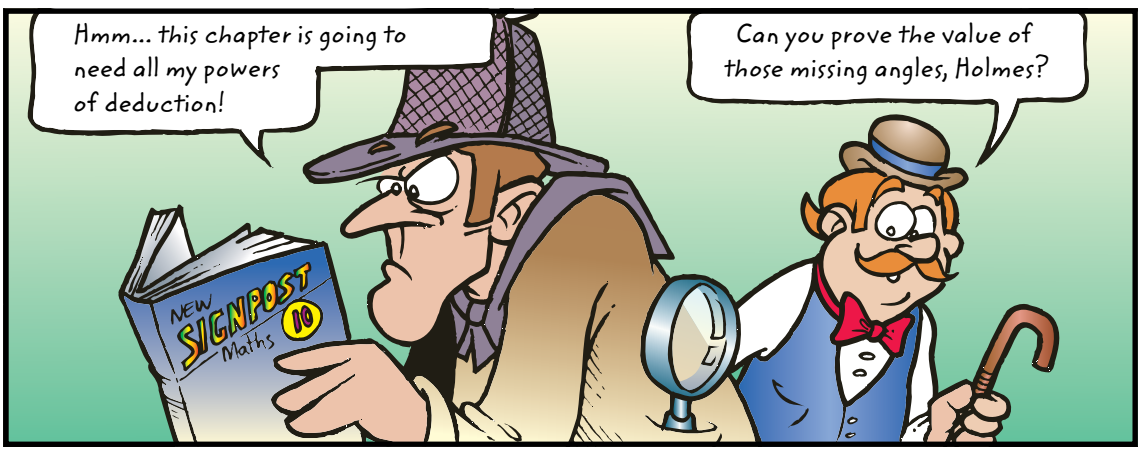
4 Place numbers in the circles so that the product of two circles appears in the square between them.



5 The average for a recent class test was 80·6%. If the girls in the class averaged 83% and the 12 boys in the class averaged 79%, how many girls were there in the class?

FURTHER GEOMETRY

13



Contents

13:01 Proving general results
13:02 Similar triangle proofs
13:03 Sides and areas of similar figures
13:04 Sides and volumes of similar solids
Fun spot 13:04 Why are giraffes easy to feed?
13:05 Definitions and proofs in deductive geometry
Investigation 13:05 Theorems and their converses
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Properties of Geometrical Figures*** [Stage 5.3§]

- Formulate proofs involving congruent triangles and angle properties (ACMMG243)
- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

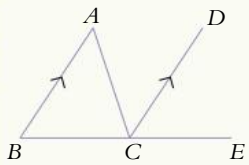
Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

13:01 Proving general results

PREP QUIZ 13:01

Copy the diagram.



1 Mark $\angle ABC$ with '×'.

2 Mark $\angle BAC$ with '•'.

In the diagram, which angle is equal to:

3 $\angle ABC$?

4 $\angle BAC$?

5 Which angle is adjacent to $\angle DCE$?

6 Which two adjacent angles make $\angle ACE$?

7 If $a + b = 180$, then $180 - a = \ldots$?

8 If $a + c = 180$, then $c = \ldots$?

9 If $b = 180 - a$ and $c = 180 - a$, then $b = \ldots$?

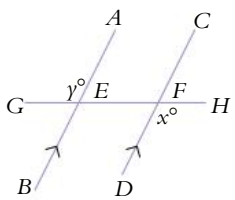
10 If $a = b$ and $b = c$, what can we say about a and c?

Many problems in geometry are non-numerical. In these problems, the reasoning process becomes more involved. As there are no numbers involved, pronumerals are used to represent unknown quantities. With the use of pronumerals, the reasoning will involve algebraic skills covered in other chapters. Questions in this exercise do not involve specific values, so the results obtained will be true for all values. The process used to answer each question is called a proof, and the result obtained is called a generalisation.

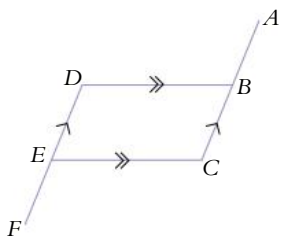
Reminder:
The abbreviations and symbols used in the following proofs are shown in the tables on page 50.

WORKED EXAMPLE 1

a In the diagram, prove that $x = y$.



b In the diagram, prove that $\angle ABD = \angle CEF$.



Solutions

a $\angle CFE = x°$ (vert. opp. $\angle$s)
$\angle CFE = y°$ (corr. $\angle$s, $AB \parallel CD$)
$\therefore\ x = y$ (both equal to $\angle CFE$)

There is usually more than one solution.

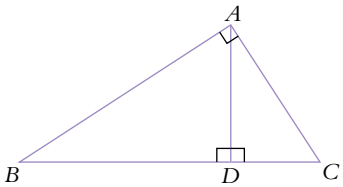
b $\angle ABD = \angle BDE$ (alt. $\angle$s, $AC \parallel DF$)
$\angle CEF = \angle BDE$ (corr. $\angle$s $DB \parallel EC$)
$\therefore\ \angle ABD = \angle CEF$ (both equal to $\angle BDE$)

WORKED EXAMPLE 2

a In the diagram, prove that:

i $\angle ABD = \angle DAC$

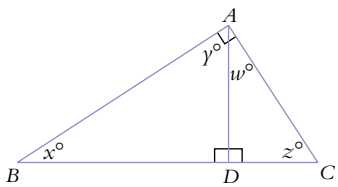
ii $\angle BAD = \angle ACD$



b ΔABC is isosceles with $AB = BC$. AB is extended to D and BE is drawn parallel to the base AC. Prove that BE bisects $\angle CBD$.

Solutions

a

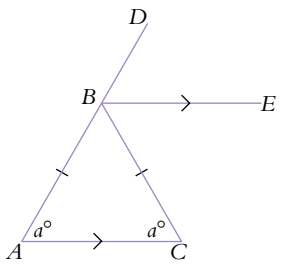


Let $\angle ABD = x°$, $\angle BAD = y°$,
$\angle DAC = w°$ and $\angle ACD = z°$.

i $x + y = 90$ (comp. $\angle$s in ΔABD)
$w + y = 90$ (comp. $\angle$s)
$\therefore x = w$
$\therefore \angle ABD = \angle DAC$

ii $x + y = 90$ (comp. $\angle$s in ΔABD)
$x + z = 90$ (comp. $\angle$s in ΔABC)
$\therefore y = z$
$\therefore \angle BAD = \angle ACD$

b



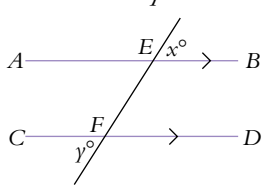
Let $\angle BAC = a°$
$\therefore \angle BCA = a°$ (base $\angle$s of isos. Δ)
Now $\angle EBC = a°$ (alt. $\angle$s, $BE \parallel AC$)
$\angle DBE = a°$ (corr. $\angle$s, $BE \parallel AC$)
$\therefore \angle DBE = \angle EBC$
$\therefore$ BE bisects $\angle CBD$

Exercise 13:01

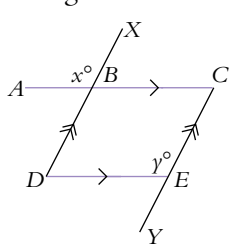
P Foundation worksheet 13:01
Non-numerical proofs

1 Prove that $x = y$ in each of the following.

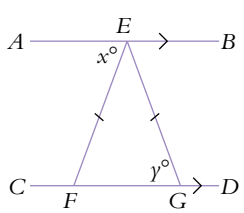
a



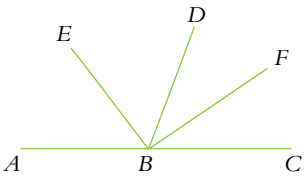
b



c

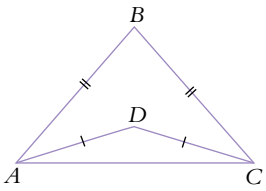


2



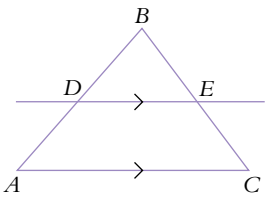
$\angle ABC$ is a straight angle. BE and BF bisect $\angle ABD$ and $\angle DBC$ respectively. Prove that EB is perpendicular to BF.

3



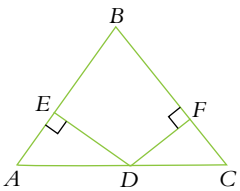
ΔABC and ΔADC are isosceles. AC is the base of both triangles. Prove that $\angle BAD = \angle BCD$.

4



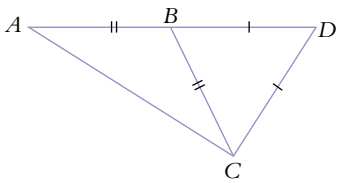
A line drawn parallel to the base AC of an isosceles ΔABC cuts the equal sides at D and E. Prove that ΔDBE is isosceles.

5



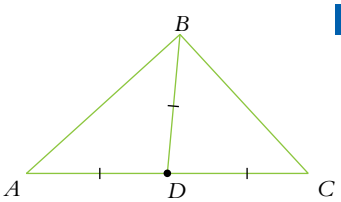
ΔABC has AB and BC equal. D is any point on AC, and DE and DF are perpendicular to AB and BC respectively. Prove that $\angle EDA = \angle FDC$.

6



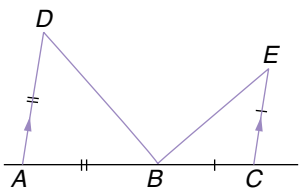
ΔABC and ΔBDC are isosceles. $AB = BC$ and $BD = DC$. BC is a common side. A and D lie on opposite sides of BC. Prove that $\angle BCD = 2 \times \angle BCA$.

7



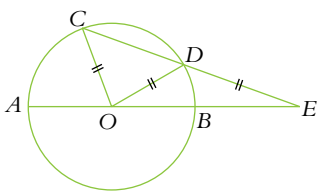
D is a point on the side AC of ΔABC. D is equidistant from the three vertices of the triangle. Prove that $\angle ABC$ is a right angle.

8



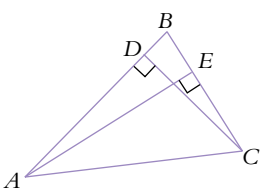
A, B and C are collinear. $AD = AB$, $BC = EC$ and $AD \parallel EC$. Prove that $\angle DBE$ is a right angle.

9



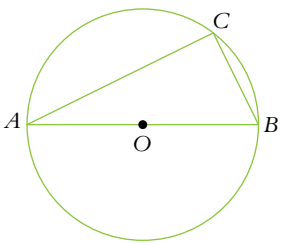
AB is a diameter and CD is a chord of a circle with centre O. CD produced meets AB produced at E and $DE = OD$. Prove that $\angle AOC = 3 \times \angle DOB$.

10



In ΔABC, AE and CD are perpendicular to BC and AB respectively. Prove that $\angle BAE = \angle BCD$.

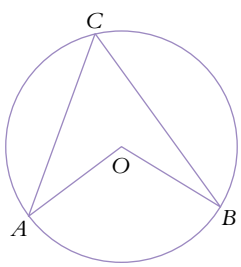
11



AB is the diameter of a circle centred at O. Prove that $\angle ACB = 90°$.

Hint! Join CO.

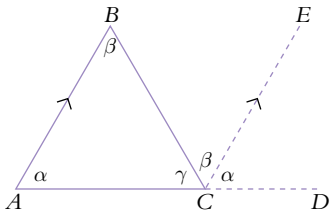
12



O is the centre of the circle. Prove that $\angle AOB = 2 \times \angle ACB$.

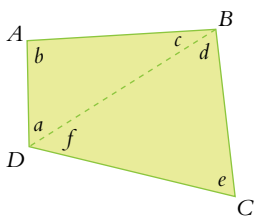
When a result is proved, this result can then be used to prove another result. This is shown in the following set of questions.

13 The interior angles of a triangle add to 180°. Complete the proof section by adding the missing reasons.

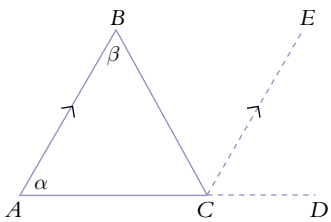


Data: ΔABC is any triangle with angles α, β and γ.
Aim: To prove that $\alpha + \beta + \gamma = 180°$
(i.e. the angle sum is 180°).
Construction: Extend AC to D. Draw CE parallel to AB.
Proof: $\angle BCE = \beta$ (…)
$\angle ECD = \alpha$ (…)
$\angle BCA = \gamma$ (given)
$\therefore \gamma + \beta + \alpha = 180°$ (…)
$\therefore$ The angle sum of a triangle is 180°.

14 Use the theorem proved in Question 13 to prove that the interior angles of a quadrilateral sum to 360°.



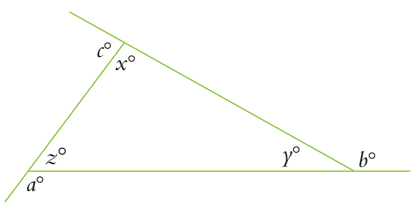
15 It is a well-known result that 'an exterior angle of a triangle is equal to the sum of the interior opposite angles'. Complete the proof started below.



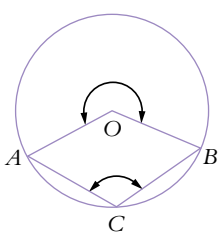
Aim: To prove that the exterior angle $\angle BCD$ is equal to the sum of the interior opposite angles (i.e. $\angle BCD = \alpha + \beta$)

Construction: Draw CE parallel to AB.
Proof: $\angle BCE = \beta$ (…)
$\angle ECD = \alpha$ (…)
$\therefore \angle BCD = \ldots$

16 Use the result above to prove that the sum of the exterior angles of a triangle is 360°.



17 Use isosceles triangles and the exterior angle theorem to prove that the reflex angle $AOB = 2 \times \angle ACB$.

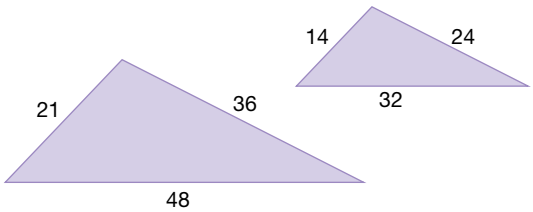


O is the centre of the circle.

13:02 Similar triangle proofs

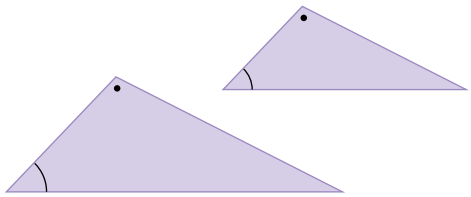
The conditions for two triangles to be *similar* were investigated in Year 9 and are reviewed below.

1 Two triangles are similar if the lengths of matching sides are in the same ratio.



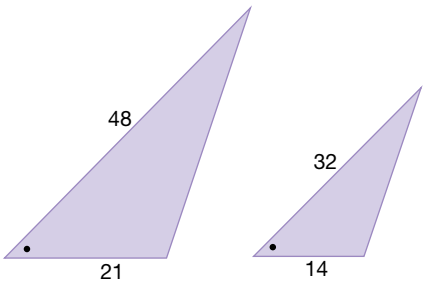
$\frac{21}{14} = \frac{36}{24} = \frac{48}{32}$

2 Two triangles are similar if two angles of one are equal to two angles of the other.



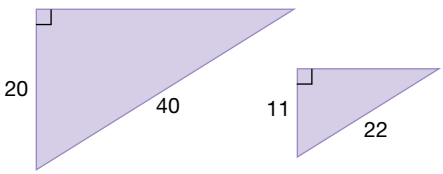
(The third angles are also equal.)

3 Two triangles are similar if the lengths of two pairs of sides are in the same ratio and the angles included by these sides are equal.



$\frac{48}{32} = \frac{21}{14}$

4 Two triangles are similar if both triangles are right-angled and the ratio of the hypotenuse to one side in one triangle equals the ratio of the hypotenuse to one side in the other triangle.



$\frac{40}{20} = \frac{22}{11}$

In the following exercise, the similarity conditions will be used to:

- write formal proofs of similarity of triangles and hence find unknown lengths and angles
- prove and apply further theorems.

WORKED EXAMPLE 1

$AB \parallel DE$. Prove that $\Delta ABC \,|||\, \Delta EDC$. Hence, find the value of x. Units are in metres.

Solution

In Δs ABC and EDC

$\angle ABC = \angle EDC$ (alt. angles $AB \parallel DE$)

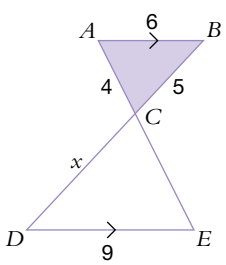
$\angle ACB = \angle ECD$ (vert. opp. angles)

$\therefore \Delta ABC \,|||\, \Delta EDC$ (equiangular)

$\therefore \frac{x}{5} = \frac{9}{6}$ (ratio of matching sides are equal)

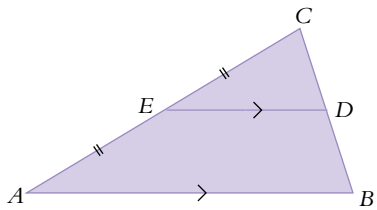
$x = \frac{5 \times 9}{6}$

$\therefore x = 7\frac{1}{2}$ m



WORKED EXAMPLE 2

In the diagram shown, E is the midpoint of AC and ED is parallel to AB.
Prove that $CD = DB$.



Solution

In Δs CED and CAB

$\angle CED = \angle CAB$ (corr. $\angle$s, $ED \parallel AB$)

$\angle CDE = \angle CBA$ (corr. $\angle$s, $ED \parallel AB$)

$\therefore \Delta CED \,|||\, \Delta CAB$ (2 pairs of equal angles)

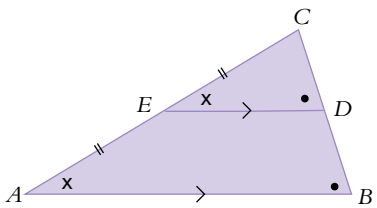
$\therefore \dfrac{CB}{CD} = \dfrac{CA}{CE}$ (ratio of matching sides are equal)

$\therefore \dfrac{CB}{CD} = \dfrac{2}{1}$ $(CA = 2 \times CE)$

$\therefore CB = 2 \times CD$

But $CB = CD + DB$

$\therefore CD = DB$



This example proves the following theorem:

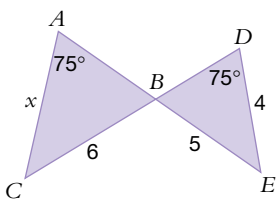
> A line drawn through the midpoint of a side of a triangle parallel to another side bisects the third side.

Exercise 13:02

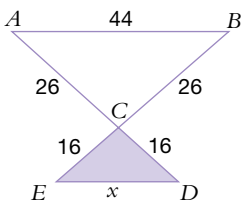
P Foundation worksheet 13:02
Similar triangle proofs

1 For each part, prove that the triangles are similar and then find the value of the pronumeral.

a



b



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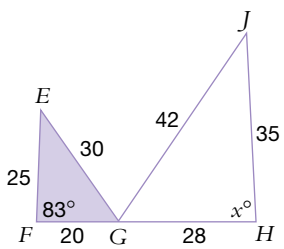
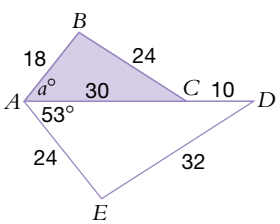
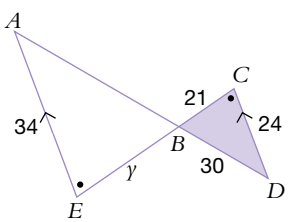


d

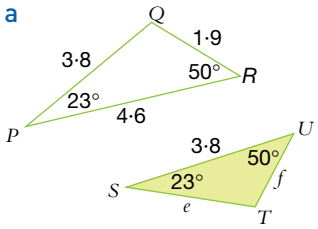


e



2 State what condition could be used to prove that the triangles are similar and then find the value of the pronumerals.

a



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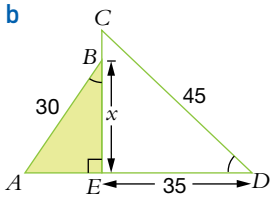

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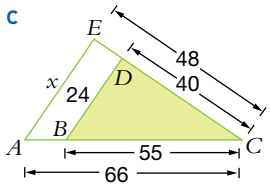

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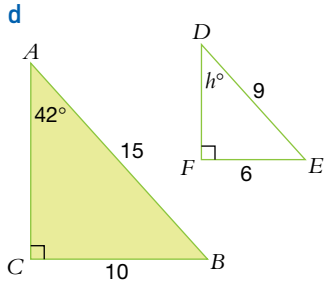
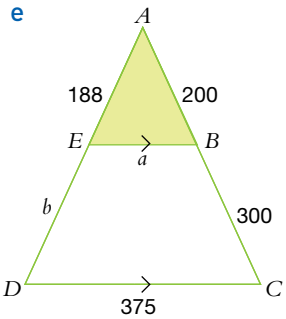
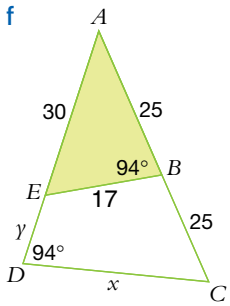


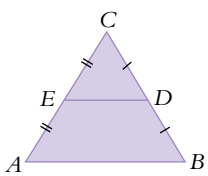
e



f



3 In the diagram, E and D are the midpoints of AC and BC respectively. Prove that $ED \parallel AB$ and $ED = \frac{1}{2}AB$.

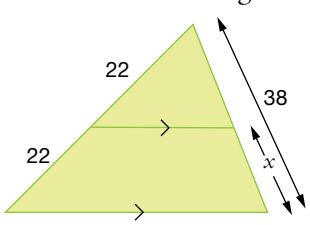


Question **3** proves the following theorem:

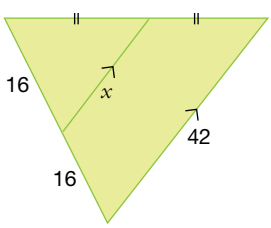
The interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

4 Use the theorem above and the theorem on page 356 to find the value of the pronumerals in each of the following.

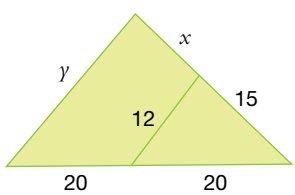
a



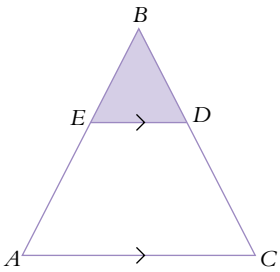
b



c



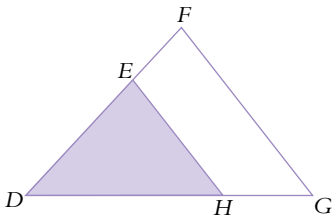
5 **a**



D divides BC in the ratio 4 : 5.
$AC \parallel ED$, $BA = 35$ m

- **i** Prove that $\Delta EBD \,|||\, \Delta ABC$, giving reasons.
- **ii** Find the ratio of BD to BC.
- **iii** Find the length of BE.
- **iv** Find the length of EA.
- **v** Find the ratio of BE to EA.
- **vi** Show that ED divides BA and BC in the same ratio, i.e. show that $\frac{BE}{EA} = \frac{BD}{DC}$.

b



$DE : EF = 3 : 2$, $DH : HG = 3 : 2$

- **i** Find $DE : DF$.
- **ii** Find $DH : DG$.
- **iii** Prove that $\Delta EDH \,|||\, \Delta FDG$.
- **iv** Is $\angle DEH$ equal to $\angle DFG$? Why?
- **v** Is $\angle DHE$ equal to $\angle DGF$? Why?
- **vi** Is $EH \parallel FG$? Why?

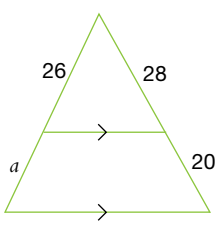
You have shown that if a line divides two sides of a triangle in the same ratio it must be parallel to the third side.

From Question **5a**, we see that:

A line parallel to one side of a triangle divides the other two sides in the same ratio.

6 Find the value of the pronumeral in each of the following. Lengths are in metres.

a



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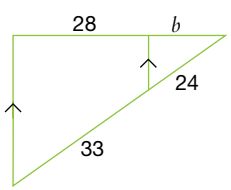
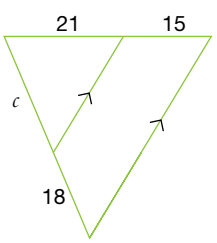
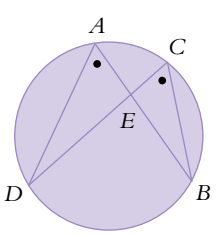


c

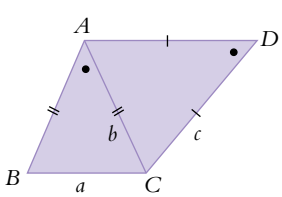


7 **a**



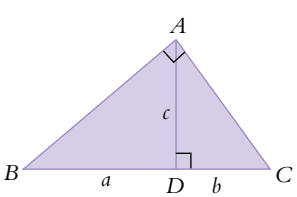
Prove that
$\Delta ADE \,|||\, \Delta CBE$
and hence that
$AE \times EB = CE \times ED$.

b



Δs ABC and ADC are isosceles with $\angle BAC = \angle ADC$.
Prove that $ac = b^2$.

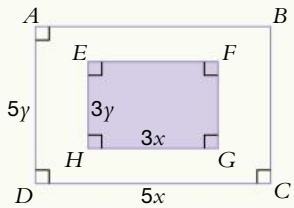
c



$\angle BAC = 90°$ and $AD \perp BC$.
Prove that $ab = c^2$.

13:03 Sides and areas of similar figures

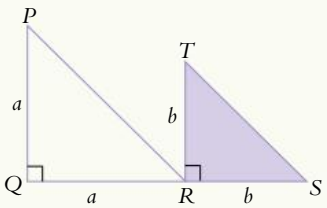
PREP QUIZ 13:03



The rectangles are similar.
Find the ratio:

1 $AD : EH$

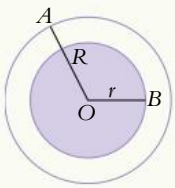
2 area $ABCD$: area $EFGH$.



The triangles are similar.
Find the ratio:

3 $PQ : TR$

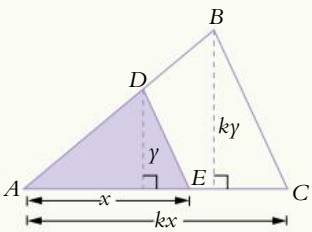
4 area ΔPQR : area ΔTRS.



The circles are similar.
Find the ratio:

5 $OA : OB$

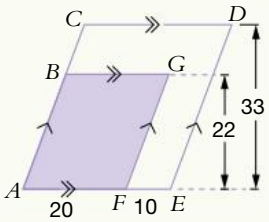
6 area of large circle to area of small circle



$\Delta ABC \,|||\, \Delta ADE$.
Find the ratio:

7 $AC : AE$

8 area ΔABC : area ΔADE



Parallelogram $ACDE$ ||| parallelogram $ABGF$.
Find the ratio:

9 $AE : AF$

10 area $ACDE$: area $ABGF$.

In the Prep quiz, we investigated the relationship between the sides of similar figures and the areas of those same figures.

Results:

Similar figures	Ratio of sides	Ratio of areas
Rectangles	$5 : 3$	$5^2 : 3^2$
Triangles	$a : b$	$a^2 : b^2$
Circles	$R : r$	$R^2 : r^2$
Triangles	$k : 1$	$k^2 : 1^2$
Parallelograms	$3 : 2$	$3^2 : 2^2$

In similar figures:

If the ratio of matching sides is $a : b$,
then the ratio of their areas is $a^2 : b^2$.

The areas of similar figures are proportional to the squares on matching sides.

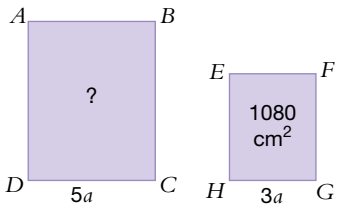
WORKED EXAMPLES

1 The pages of two morning newspapers are similar in shape and the widths are in the ratio 3 : 5. Find the area of the larger page if the smaller one has an area of 1080 cm^2.

2 Two similar decorative stickers of boomerangs were produced for the Australia Day celebrations. The area of the larger boomerang sticker is 108 cm^2 and the area of the smaller is 48 cm^2. If the length of the larger is 18 cm, find the length of the smaller.

Solutions

1



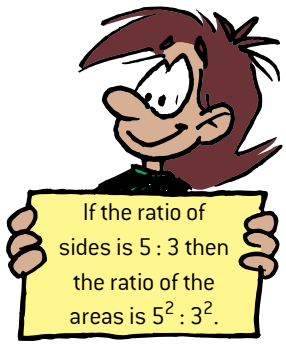
$$\frac{\text{area of } ABCD}{\text{area of } EFGH} = \frac{5^2}{3^2}$$

$$\therefore \frac{\text{area of } ABCD}{1080} = \frac{25}{9}$$

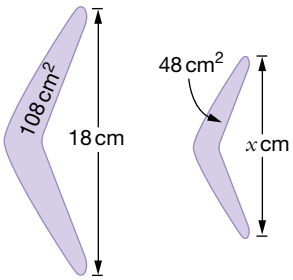
$$\text{area of } ABCD = \frac{25 \times 1080}{9}$$

$$= 3000 \text{ cm}^2$$

$\therefore$ The area of the larger page is 3000 cm^2.



2



Let the length of the smaller sticker be x cm.

$$\therefore \frac{x^2}{18^2} = \frac{48}{108}$$

$$x^2 = \frac{48 \times 18^2}{108}$$

$$x^2 = 144$$

$$\therefore x = \pm 12$$

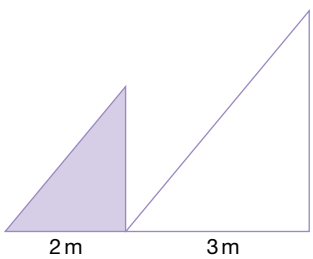
$\therefore$ The length of the smaller sticker is 12 cm.

A length must be positive so we ignore the −12.

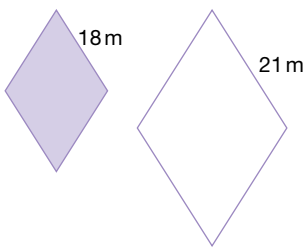
Exercise 13:03

1 In each of the following, the figures are similar. Find the ratio of the smaller area to the larger.

a



b



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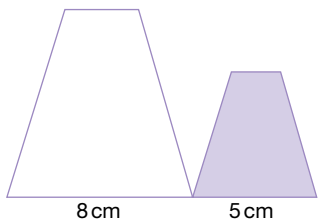

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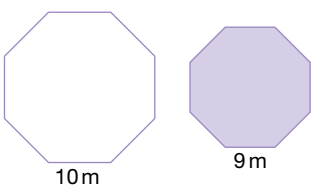
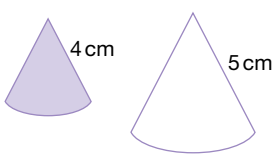
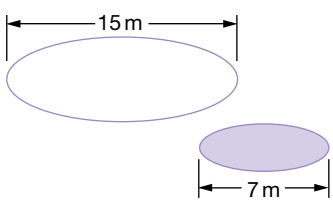


e



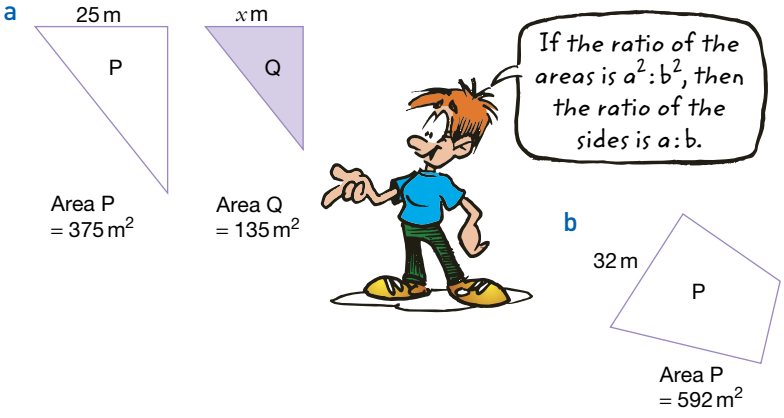
f



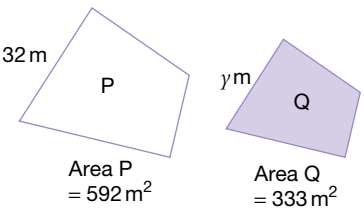
2 **a** In 1a, the area of the larger triangle is $5{\cdot}85\,m^2$. Find the area of the smaller triangle.
b In 1b, the area of the larger rhombus is $367{\cdot}5\,cm^2$. Find the area of the smaller rhombus.
c In 1c, the area of the larger trapezium is $40\,cm^2$. Find the area of the smaller trapezium.
d In 1d, the area of the smaller octagon is $162\,m^2$. Find the area of the larger octagon.
e In 1e, the area of the smaller sector is $86{\cdot}4\,cm^2$. Find the area of the larger sector.
f In 1f, the area of the smaller ellipse is $17{\cdot}0\,m^2$ (to 1 dec. pl.). Find the area of the larger ellipse correct to one decimal place.

3 In each pair of figures below are similar. Find the ratio of a side on the smaller figure to the matching side on the larger, then find the value of the pronumeral.

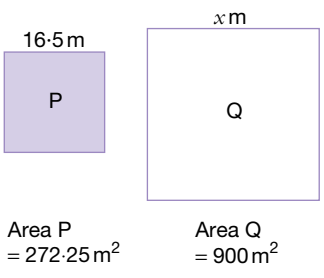
a



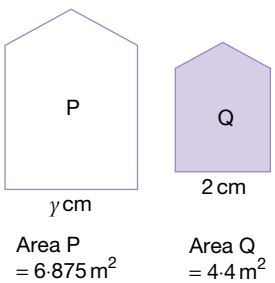
b



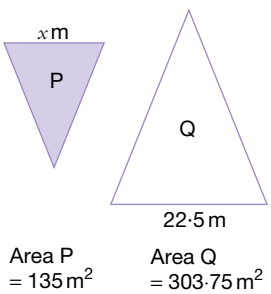
c



d



e



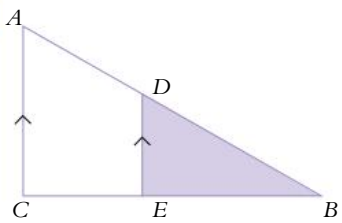
4 **a** If the dimensions of an equilateral triangle are trebled, how is the area affected?
b If the areas of similar quadrilaterals are in the ratio 16 : 25, what is the ratio of matching sides?
c The ratio of a length in a house to the matching length on the house plan is 150 : 1. Find the ratio of the area of the house to its area on the plan?
d The ratio of matching sides of two similar figures is 4 : 7. What is the ratio of the perimeters of the similar figures?

5 The matching sides of two similar kites are in the ratio 11 : 16. Find the area of the smaller if the larger has an area of $1{\cdot}44\,m^2$.

6 **a** A photograph has a breadth of 9·5 cm and an area of $104{\cdot}5\,cm^2$. An enlargement is to be made that has a breadth of 19 cm. What will be the area of the enlargement?
b A postcard is 8 cm wide and has an area of $96\,cm^2$. If the postcard is enlarged so that its area is $1536\,cm^2$, what is the breadth of the enlargement?

7 My neighbour, whose lawn is a similar shape to mine but has dimensions $\frac{2}{3}$ as large, was surprised to hear that I used 90 kg of ammonium sulfate to treat my lawn. How much would he need to treat his lawn in the same way?

8



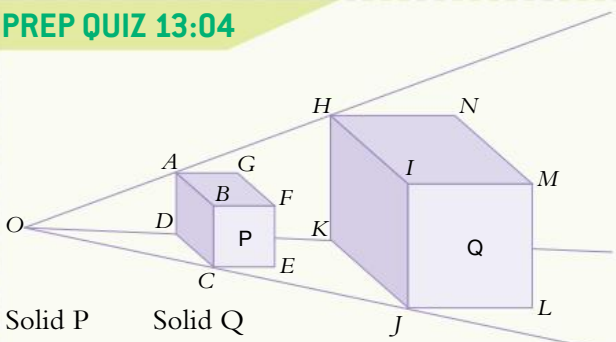
$DE \parallel AC$ and $CE : EB = 2 : 3$. Find:

- **a** $AD : DB$
- **b** $AC : DE$
- **c** area DBE : area ABC
- **d** area DBE : area $ADEC$
- **e** area ABC : area $ADEC$

13:04 Sides and volumes of similar solids

Similar solids:
- have the same shape
- have matching angles of equal size
- have matching side lengths in the same ratio.

PREP QUIZ 13:04



Solid P
$DC = xl$
$CE = xb$
$CB = xh$

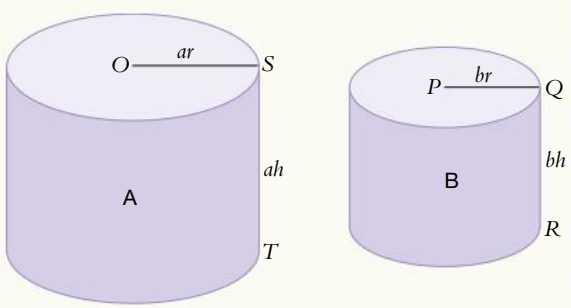
Solid Q
$KJ = yl$
$JL = yb$
$JI = yh$

These rectangular prisms are similar.

1. What is the ratio $DC : KJ$?
2. What is the ratio $CE : JL$?
3. What is the ratio area $ABCD$: area $HIJK$?
4. What is the ratio area $BCEF$: area $IJLM$?
5. What is the ratio volume of P : volume of Q?

These two cylinders are similar.

6. What is the ratio $ST : QR$?
7. What is the ratio $OS : PQ$?
8. What is the ratio of the cross-section of A to the cross-section of B?
9. What is the ratio of the curved surface area of A to the curved surface area of B?
10. What is the ratio volume of A : volume of B?



O and P are centres

The Prep quiz suggests certain relationships in the dimensions of similar solids.

In similar figures if the ratio of matching sides is $a : b$ then:

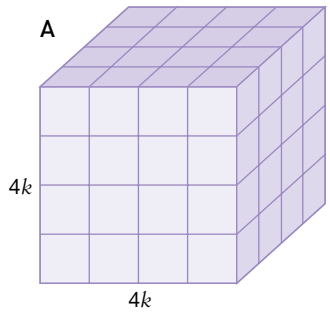
- the ratio of matching areas is $a^2 : b^2$
- the ratio of matching volumes is $a^3 : b^3$

WORKED EXAMPLE 1

Two cubes have the lengths of matching edges in the ratio 4 : 3. Find:

a the ratio of the areas of matching faces

b the ratio of their volumes.



Solutions

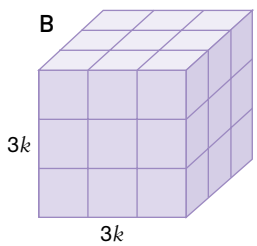
All cubes are similar.

a Areas are proportional to the squares of sides.

$$\therefore \frac{\text{area A}}{\text{area B}} = \frac{(4k)^2}{(3k)^2}$$

$$= \frac{16}{9}$$

(*Note:* The area of each face of A is $16k^2$. The area of each face of B is $9k^2$.)



b Volumes are proportional to the cubes of sides.

$$\therefore \frac{\text{volume A}}{\text{volume B}} = \frac{4^3}{3^3}$$

$$= \frac{64}{27}$$

(*Note:* The volume of A is $64k^3$. The volume of B is $27k^3$.)

A doll twice the height of another would have eight times its volume.

WORKED EXAMPLE 2

Two similar statues were to be made at Naomi's studio. She first completed the smaller one and painted it with a special gold paint. The smaller statue was 1·6 m, had a volume of 0·384 m^2 and needed 400 mL of gold paint to complete the required two coats.

a What volume will the larger statue have if it is to be 2 m tall?

b How much gold paint will be required to give the larger statue two coats?

Solutions

The statues are similar solids.

a Volumes are proportional to the cubes of lengths.

$$\therefore \frac{\text{volume L}}{\text{volume S}} = \frac{2^3}{1{\cdot}6^3}$$

$$\therefore \frac{\text{volume L}}{0{\cdot}384\ \text{m}^3} = \frac{2^3}{1{\cdot}6^3}$$

$$\therefore \text{volume L} = \frac{2^3 \times 3{\cdot}384}{1{\cdot}6^3}\ \text{m}^3$$

$$= 0{\cdot}75\ \text{m}^3$$

b Paint required is proportional to area to be covered.
Areas are proportional to the squares of length.

$$\therefore \frac{\text{paint for L}}{\text{paint for S}} = \frac{2^2}{1{\cdot}6^2}$$

$$\therefore \frac{\text{paint for L}}{400\ \text{mL}} = \frac{2^2}{1{\cdot}6^2}$$

$$\therefore \text{paint for L} = \frac{2^2 \times 400}{1{\cdot}6^2}\ \text{mL}$$

$$= 625\ \text{mL}$$

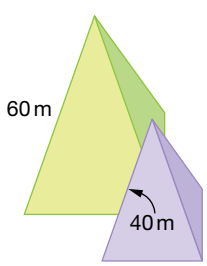
Exercise 13:04

1 In each, the solids are similar. Find:

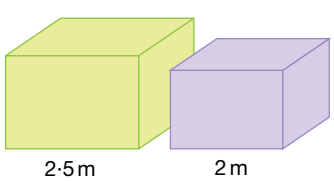
i the ratio of matching areas (smaller to larger)

ii the ratio of their volumes (smaller to larger).

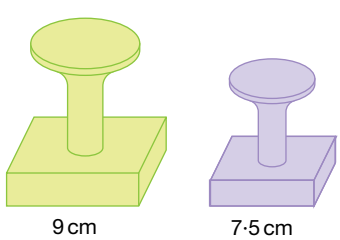
a



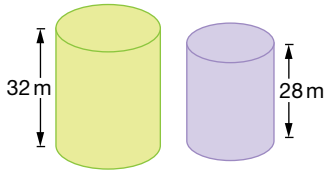
b



c



d



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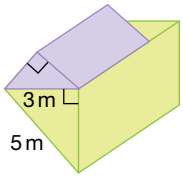

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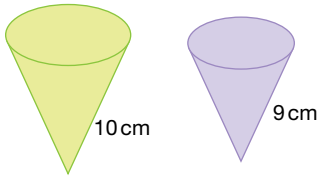


2 a In 1a, the area of the base of the smaller solid is $780\,m^2$. Find the area of the base of the larger solid.

b In 1a, the volume of the larger solid is $30\,564\,m^3$. Find the volume of the smaller solid.

c In 1b, the volume of the smaller solid is $10{\cdot}08\,m^3$. What is the volume of the larger solid?

d In 1c, the volume of the larger solid is $243\,cm^3$. What is the volume of the smaller solid?

e In 1d, the cross-section of the larger cylinder has an area of $400\,m^2$. Find the cross-sectional area of the smaller cylinder.

f In 1e, the surface area of the smaller triangular prism is $27\,m^2$. What is the surface area of the larger triangular prism?

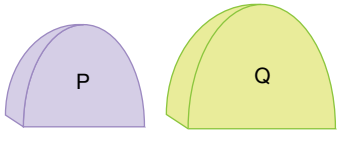
g In 1e, the volume of the smaller prism is $8{\cdot}1\,m^3$. Find the volume of the larger prism.

h In 1f, the surface area of the larger ice-cream cone is $226\,cm^2$. Find the surface area of the smaller ice-cream cone correct to the nearest square centimetre.

i In 1f, the volume of the larger solid is $287\,cm^3$. What is the volume of the smaller solid correct to the nearest cubic centimetre?

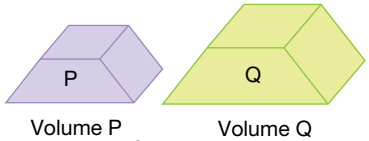
3 In each part, the solids are similar. Find the ratio of a side on the smaller solid to the matching side on the larger solid. Hence, find the ratio of their surface areas.

a



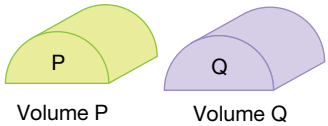
Volume P $= 125\,cm^3$ Volume Q $= 343\,cm^3$

b



Volume P $= 4{\cdot}096\,cm^3$ Volume Q $= 8\,m^3$

c



Volume P $= 27\,m^3$ Volume Q $= 32{\cdot}768\,m^3$

If the ratio of matching volumes $= m : n$, the ratio of matching sides $= \sqrt[3]{m} : \sqrt[3]{n}$.

4 a In 3a, the surface area of solid P is $63\,cm^2$. Find the surface area of solid Q.

b In 3b, the length from end to end (i.e. the height) on prism P is 0·7 m. What is the matching length on prism Q?

c In 3b, the area of the under-surface of solid Q is $5{\cdot}04\,m^2$. Find the area of the matching face of solid P.

d In 3c, the diameter of the cross-section of prism P is 16 m. Find the diameter of the cross-section of prism Q.

e In 3c, the curved surface area of solid Q is $72\pi\,m^2$. What is the curved surface area of solid P?

5 Two similar vases were to be made in Naomi's studio. She first completed the smaller one and gave it two coats of lacquer. The smaller vase was 40 cm tall, had a volume of 2625 cm^3 and needed 100 mL of lacquer to complete the two coats.

- **a** What volume will the larger vase have if it is to be 60 cm tall?
- **b** How much lacquer will be required to give the larger vase two coats?

6 The volume of metal in a scale model of a metal high-tension tower is 54 cm^3. The scale used was 1 : 50. It required 50 mL of paint to give the model one coat.

- **a** What volume of metal would be required to build the high-tension tower?
- **b** How much paint would be required to give the real tower one coat?

7 Let us assume that a boy and his father are similar solids. Each is wearing similar swimming costumes. The boy is one-quarter as tall as his father.

- **a** If the boy loses 1 unit of heat from his exposed skin, how many units of heat would you expect his father to lose?
- **b** If the father displaces 72 000 cm^3 of water when he dives into the pool, what volume of water would the son displace?

8 Two similar glasses have heights in the ratio 3 : 5. If, when using the larger glass, it takes 54 glasses of water to fill a fish tank, how many glasses of water would be needed to fill it using the smaller glass?

9 This Giant Ram in Wagin, Western Australia, took the sculptor, David Chappell, 8 months to complete and is 9 times as tall as a live ram. Its height is 9 metres.
Complete:

- **a** The surface area is … times greater than that of a live ram.
- **b** The volume is … times greater than that of a live ram.

FUN SPOT 13:04 WHY ARE GIRAFFES EASY TO FEED?

Answer each question and write the letter for that question in the box above the correct answer.

A $(0{\cdot}9)^2$

A $0{\cdot}8 \div 0{\cdot}2$

D $2\% + 6\%$

E $\frac{1}{2}$ of $\frac{1}{2}$

F $\frac{1}{2} + \frac{1}{3}$

G $1 - \frac{3}{8}$

L $0{\cdot}35 - 1$

L $\frac{2}{3}$ of 3

I $15 - 15{\cdot}2$

A $1 - 0{\cdot}3$

A 6% of 100

E $1 \div 0{\cdot}2$

E $\frac{1}{2} + \frac{1}{4}$

G $(\frac{1}{3})^2$

H $\frac{1}{3} \times \frac{1}{6}$

K $\frac{1}{2}$ of $0{\cdot}8$

L 10% of 1

M $11 \div 1{\cdot}1$

N Write $\frac{3}{5}$ as a percentage.

O Write $\frac{22}{7}$ as a mixed number.

O Simplify the ratio 30 : 5.

O Change $3\frac{1}{2}$ to an improper fraction.

O What fraction is 200 m of 1 km?

T Increase \$48 by 25%.

T Write $\frac{2}{3}$ as a repeating decimal.

T What percentage is 300 m of 2 km?

W 70% of my money is \$840. How much money do I have?

Y If 24 red balls and 56 blue balls were in a bag, what percentage were red?

Y \$2000 is divided between Naomi and Luke in the ratio 2 : 3. How much does Naomi receive?

\$60	$\frac{1}{18}$	5	30%	10	0·81	0·4	$\frac{3}{4}$	4	−0·65	−0·2	0·6̇	15%	2	$\frac{1}{4}$

$\frac{5}{6}$	$\frac{1}{15}$	6:1	8%	$\frac{1}{9}$	$3\frac{1}{7}$	0·7	0·1	$\frac{7}{2}$	60%	$\frac{5}{8}$	\$1200	6	\$800

13:05 Definitions and proofs in deductive geometry

If you were asked to define an equilateral triangle, you could say:

- it is a triangle with all its sides equal
- it is a triangle with all its angles equal
- it is a triangle with all its sides equal and all its angles equal.

Geometrical figures have many properties and it is not practical to mention them all when defining the figure.

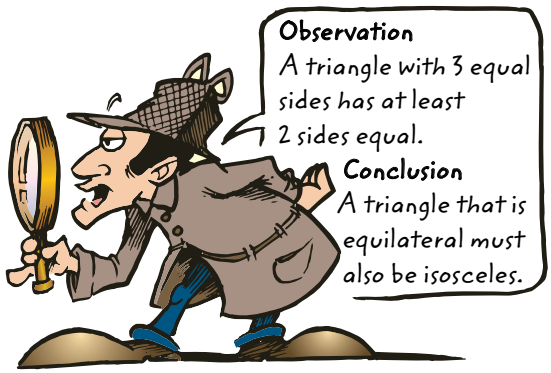
A definition is the minimum amount of information needed to identify a particular figure.

In deductive geometry, the definitions serve as starting points. The properties of the figures can then be proved using basic geometrical facts.

The proved result is known as a theorem and this can then be used to produce other theorems.

Definitions:

- A scalene triangle is a triangle with no two sides equal in length.
- An isosceles triangle is a triangle with at least two sides equal in length.
- An equilateral triangle is a triangle with all sides equal in length.



The definitions imply that an equilateral triangle must also be an isosceles triangle. Hence, any property of an isosceles triangle must also be a property of an equilateral triangle.

Isosceles triangles are often found in architecture.

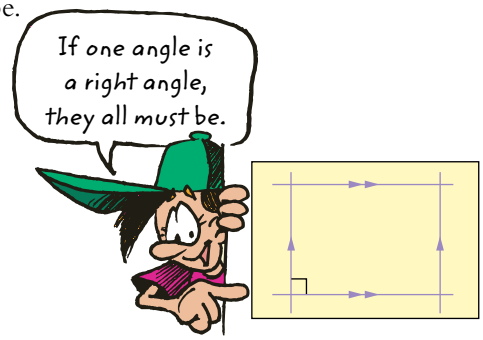
Definitions:

- A trapezium is a quadrilateral with at least one pair of opposite sides parallel.
- A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
- A rhombus is a parallelogram with two adjacent sides equal in length.
- A rectangle is a parallelogram with one angle a right angle.
- A square is a rectangle with two adjacent sides equal.

  or
- A square is a rhombus with one angle a right angle.

Many people find the definitions above a little unusual at first.

- They start from the simplest shape and, by adding more and more constraints, end up at the most complex shape.
- The definitions are hierarchical. Each new shape is defined in terms of a simpler shape that has already been defined.
  e.g. *A rhombus is a parallelogram …*
  This saves repetition and states that a rhombus is in fact a special type of parallelogram. It has all the properties of a parallelogram and some extra properties as well.
- The definitions do not list every property of the shape. These can be deduced from the definition using other geometrical techniques.
  e.g. *a rectangle is a parallelogram with a right angle.*
  There is no need to say that it has four right angles as this can be derived using the fact that it is a parallelogram and our knowledge of co-interior angles and parallel lines.



In Exercise 3:03 many of the properties of the special triangles and quadrilaterals were proved using the definitions and congruent triangles.

- As well as the above definitions, there are other tests that can be used to show that a given quadrilateral is a parallelogram, rhombus or rectangle. These are listed below.

Tests for parallelograms:

A quadrilateral is a parallelogram if any one of the following is true.

1 Both pairs of opposite sides are equal.

2 Both pairs of opposite angles are equal.

3 One pair of sides is both equal and parallel.

4 The diagonals bisect each other.

Tests for a rhombus:

1 All sides are equal.

or

2 Diagonals bisect each other at right angles.

Tests for a rectangle:

1 All angles are equal.

or

2 Diagonals are equal and bisect each other.

In the following exercise, many of these tests, as well as some of the properties of the quadrilaterals, are proved.

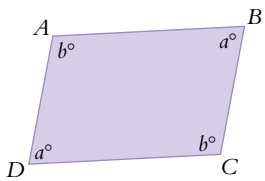
WORKED EXAMPLE

Prove that a quadrilateral is a parallelogram if its opposite angles are equal.

Solution

Data: $ABCD$ is a quadrilateral with $\angle A = \angle C$ and $\angle B = \angle D$.

Aim: To prove that $AB \parallel DC$ and $AD \parallel BC$.



Proof: Let $\angle A = \angle C = b°$ and $\angle B = \angle D = a°$.

$2(a + b) = 360$ ($\angle$ sum of quad.)

$\therefore\ a + b = 180$

$\therefore\ \angle ADC + \angle DAB = 180°$

$\therefore\ AB \parallel DC$ (co-int. $\angle$s are supp.)

Also, $\angle ADC + \angle DCB = 180°$

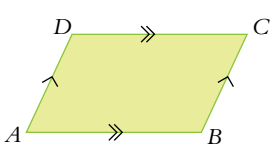
$\therefore\ AD \parallel BC$ (co-int. $\angle$s are supp.)

$\therefore\ ABCD$ has opposite sides parallel.

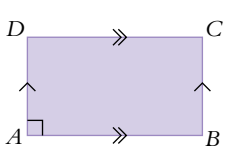
$\therefore\ ABCD$ is a parallelogram.

Exercise 13:05

1 Use the definition of a parallelogram (on page 369) and co-interior angles to prove that the opposite angles of a parallelogram are equal.

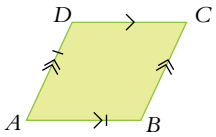


2 Use the definition of a rectangle (on page 369) and co-interior angles to prove that a rectangle has four right angles.

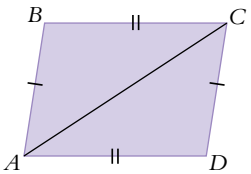


3 **a** What can you say about the opposite sides of a parallelogram?

b Given that a rhombus is a parallelogram with two adjacent sides equal, prove that all the sides of a rhombus are equal.



4 $ABCD$ is a quadrilateral that has opposite sides equal. Prove that it is a parallelogram (i.e. that its opposite sides are parallel).



5 The 'Data' and 'Aim' for the congruence proof of Test 4 for parallelograms are given below. Answer the questions to give an outline of the proof.

Data: $ABCD$ is any quadrilateral where diagonals AC and BD bisect each other at E.

Aim: To show that $ABCD$ is a parallelogram (i.e. $AD \parallel BC$ and $AB \parallel CD$).

a Why does $\angle AED$ equal $\angle CEB$?

b Why is ΔAED congruent to ΔCEB?

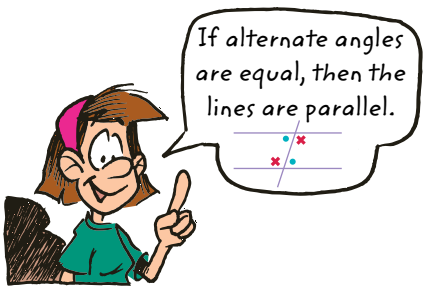
c Which angle in ΔCBE is equal to $\angle ADE$? Why?

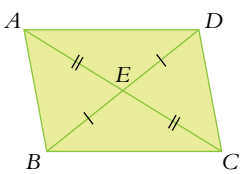
d How does your answer to **c** prove that $AD \parallel BC$?

e Why is ΔAEB congruent to ΔCED?

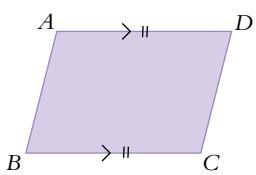
f Why is $\angle BAE$ equal to $\angle DCE$?

g Why is AB parallel to CD?



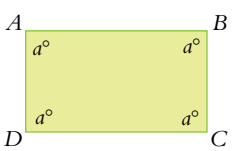


6 Use congruent triangles to prove that if one pair of sides in a quadrilateral is both equal and parallel, then the quadrilateral is a parallelogram.



Use the tests for parallelograms and the properties of parallelograms on page 369 to do Questions **7** and **8**.

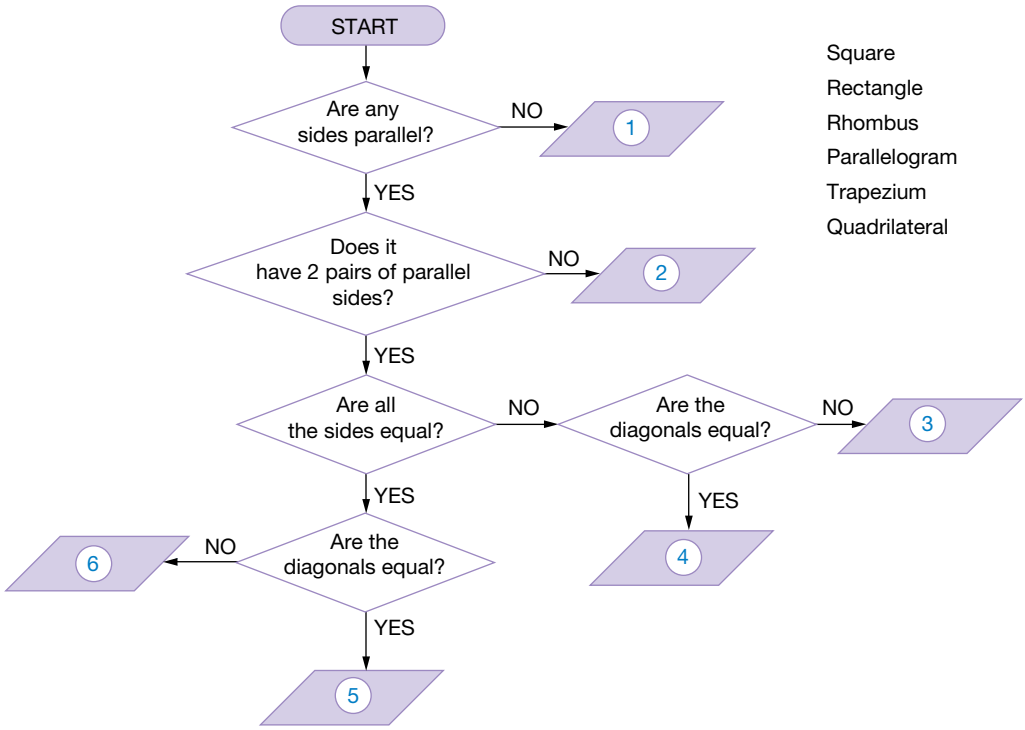
7 Prove that a quadrilateral is a rectangle if all its angles are equal.



8 Show how Test 1 for a rhombus on page 370 could be proved using Test 1 for a parallelogram.

The rhombus, rectangle and square are all parallelograms.
Hence they possess all the properties of a parallelogram.

9 Follow the flowchart below and choose the correct names from the list, to be inserted into boxes 1 to 6.



10 Use the definitions and the properties of the quadrilaterals in Exercise 3:03 to copy and complete this table by writing *yes* or *no* in each space.

Properties	Parallelogram	Rhombus	Rectangle	Square
Opposite sides parallel				
Opposite sides equal				
Opposite angles equal				
Diagonals bisect one another				
All sides equal				
All angles right angles				
Diagonals perpendicular				
Diagonals bisect angles through which they pass				
Diagonals are equal				

11 Name two properties that the following have in common.

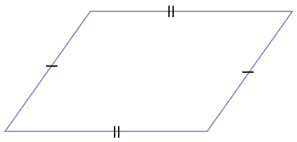
a square and rhombus
b square and rectangle
c rectangle and parallelogram
d parallelogram and rhombus.

12 **a** Name two properties of a square that are not properties of a rectangle.
b Name two properties of a square that are not properties of a rhombus.
c Name two properties of a rectangle that are not properties of a parallelogram.

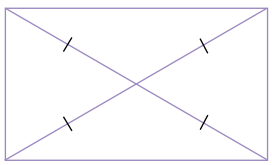
13 **a** Does a square have all the properties of a rectangle?
b Is a square a special rectangle?
c Is a rectangle a square? Give a reason for your answer.
d Does a square have all the properties of a rhombus?
e Is a square a special rhombus?
f Is a rhombus a square? Give a reason for your answer.

14 Use the tests for quadrilaterals to identify each of the following as either a rectangle, rhombus or parallelogram and state the test used in each case.

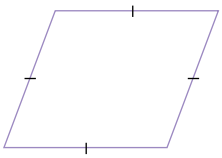
a



b



c



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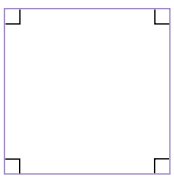

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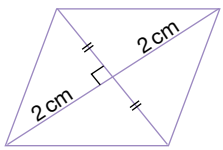

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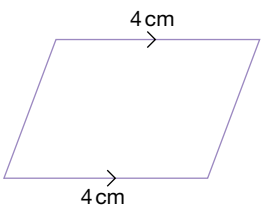
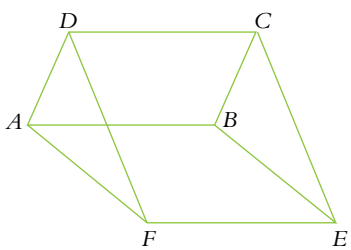
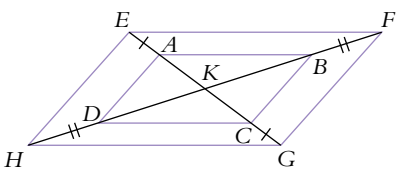


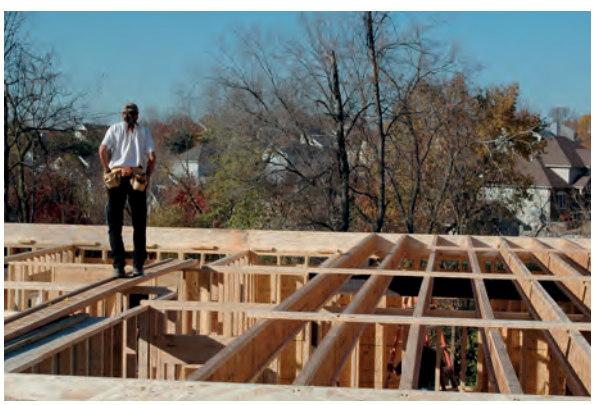
15 Use the tests for parallelograms and the properties of parallelograms in the following questions.

a In the diagram, *ABCD* and *ABEF* are parallelograms. Prove that *DCEF* is a parallelogram.



b *ABCD* is a parallelogram with diagonals produced so that $EA = CG$ and $DH = BF$. Prove that *EFGH* is a parallelogram.





Builders often use the properties of quadrilaterals in the construction of a building.

INVESTIGATION 13:05 THEOREMS AND THEIR CONVERSES

A theorem usually connects two pieces of information and can be written in the form 'If A then B'. A is usually called the *supposition* or *assumption*. B is called the *conclusion*. If A and B are interchanged, then we have the statement 'If B then A'. This is called the converse of the theorem. Even if a theorem is true, its converse may not be, as shown by the following example.

Theorem: If (two angles are vertically opposite), then (the angles are equal). This is true.

Converse: If (two angles are equal), then (they are vertically opposite). This is false.

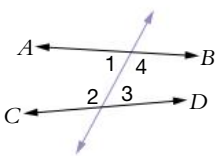
For the following theorems, state their converse and whether the converse is true.

1. If a triangle has all its sides equal, then it has all angles equal.
2. If a quadrilateral is a square, then its diagonals are equal.
3. If a quadrilateral is a parallelogram, then its opposite angles are equal.
4. If a quadrilateral is a square then it has all its sides equal.
5. If a triangle is right-angled then the square on the longest side is equal to the sum of the squares on the two smaller sides.

MATHS TERMS 13

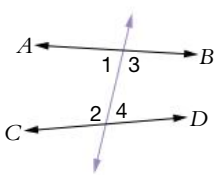
alternate angles

- a pair of angles on opposite sides of the transversal between the other two lines
- the alternate angles are 1 and 3, 2 and 4
- are equal when $AB \parallel CD$



co-interior angles

- a pair of angles on the same side of the transversal and between the other two lines
- the co-interior angles are 1 and 2, 3 and 4
- are supplementary when $AB \parallel CD$



congruent triangles

- triangles that are identical in shape and size

congruency tests

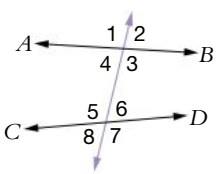
- a set of tests used to prove that two triangles are congruent
- the tests are usually referred to by the abbreviations SSS, SAS, AAS and RHS

converse (of a theorem)

- if a theorem is stated in the form 'If A then B', the converse is the statement 'If B then A'

corresponding angles

- angles that are in corresponding positions at each intersection
- the corresponding angles are 1 and 5, 2 and 6, 3 and 7, 4 and 8
- are equal when $AB \parallel CD$



deductive geometry
- a system in which results called theorems are produced from a set of basic facts that are accepted to be true

definition
- a statement that describes the essential properties of something

matching angles (or sides)
- sides (or angles) that are in the same (or corresponding) positions in congruent or similar figures

proof
- a series of steps that establishes the truth of a result

quadrilateral
- a polygon with four sides
- there are six special quadrilaterals

quadrilateral (test for)
- a set of conditions that can be used to prove that a quadrilateral is a parallelogram, rhombus, rectangle or square

similar triangles
- triangles that are the same shape
- have equal angles and matching sides in the same ratio

similar triangles (tests for)
- a set of four tests that can be used to prove that two triangles are similar

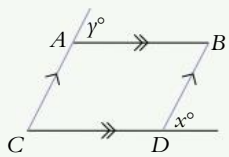
theorem
- the statement of a result that has been proved by reasoning
- usually stated in 'If A then B' form

DIAGNOSTIC TEST 13 FURTHER GEOMETRY

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

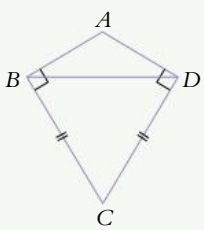
13:01

1 a



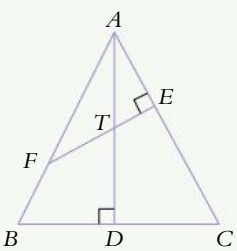
Prove that $x = y$.

b



Prove that $AB = AD$.

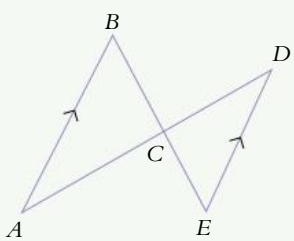
c



Given $AB = AC$, prove that $\angle ABC = \angle FTD$.

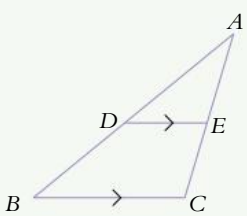
13:02

2 a



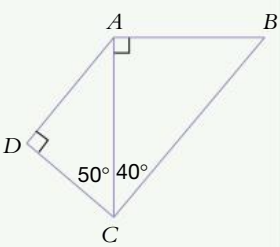
Prove that $\Delta ABC \,|||\, \Delta DEC$.

b



Prove that $\Delta ADE \,|||\, \Delta ABC$.

c

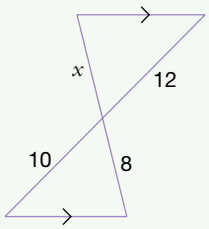


Prove that $\Delta ABC \,|||\, \Delta DCA$.

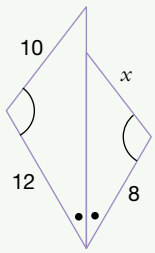
13:02

3 Find the value of the pronumeral in each of the following.

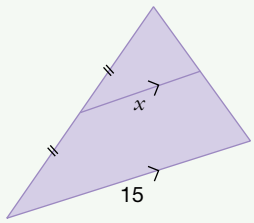
a



b

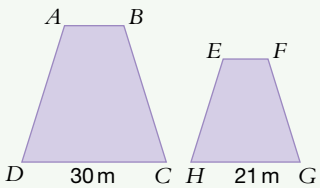


c



13:03

4 a

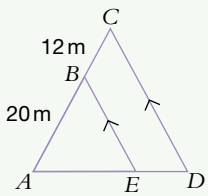


$ABCD \parallel\parallel\parallel EFGH$. Find:

i area $ABCD$: area $EFGH$

ii area $ABCD$ if area $EFGH$ is $269{\cdot}5\,m^2$.

b

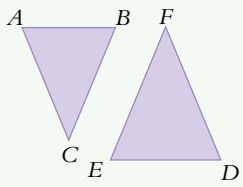


$BE \parallel CD$. Find:

i area ABE : area ACD

ii area ABE if area ACD is $512\,m^2$.

c



Area $ABC = 243\,mm^2$.
Area $EDF = 300\,mm^2$.

i Find $AB : ED$.

ii Find the length of EF if $AC = 18\,mm$.

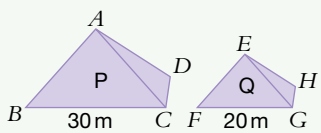
13:04

5 a Solids P and Q are similar.
Find:

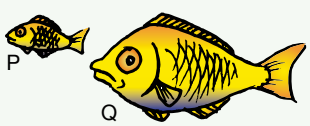
i area ABC : area EFG

ii volume P : volume Q

iii the volume of P if the volume of Q is $2000\,m^3$.



b



Surface area of P $= 44{\cdot}8\,cm^2$
Surface area of Q $= 179{\cdot}2\,cm^2$

These solid models of fish are similar.

i Find the ratio length of P : length of Q.

ii Find the ratio volume P : volume Q.

iii Find the volume of Q if the volume of P is $18{\cdot}2\,cm^3$.

c The heights of two similar statues are 3·2 m and 3·6 m. If the volume of the smaller is $80{\cdot}64\,m^3$, find the volume of the larger statue.

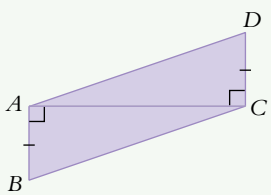
13:05

6 Prove the following.

a If all the angles of a quadrilateral are equal, then it is a rectangle.

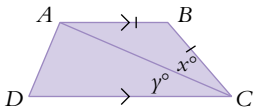
b If the diagonals of a quadrilateral are equal in length and bisect each other at right angles, then it is a square.

c What test for parallelograms can be used to prove that $ABCD$ is a parallelogram?

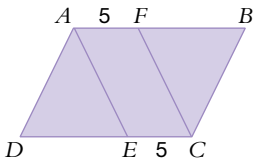


ASSIGNMENT 13A Chapter review

1 Prove that $x = y$.

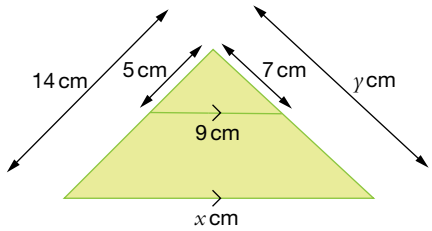


2 $ABCD$ is a parallelogram. $AF = EC$. Prove that $AE \parallel FC$.

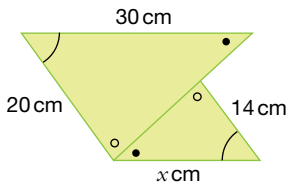


3 Find the value of the pronumerals in each of the following.

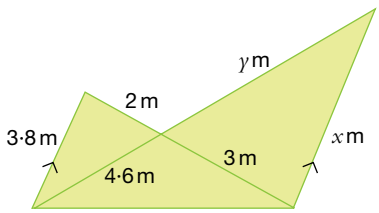
a



b



c

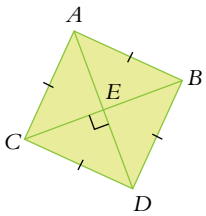


4 **a** A triangular banner has an area of $0{\cdot}75\,m^2$. What is the area of a banner that is twice as large but otherwise identical?

b Two similar bottles are made of glass. The smaller bottle has a height of 10 cm and uses $80\,cm^3$ of glass to make it. If the height of the larger bottle is 15 cm, what volume of glass would be needed to make it?

c A glass is 6 cm high and holds 70 mL. A larger glass that is similar in shape is to be made to hold 200 mL. What should be the height of this glass? Round your answer up to the nearest mm.

5 In the diagram the equal intervals AD and BC bisect each other at right angles at E.



a Why are ΔABE, ΔBDE, ΔDCE and ΔCAE all congruent?

b Why does $AB = BD = DC = CA$?

c Using the answer from **b**, what type of quadrilateral is $ABDC$?

d What is the size of

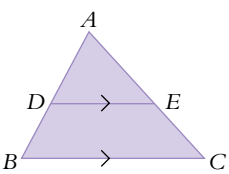
i $\angle EDC$

ii $\angle EDB$

iii $\angle BDC$?

e Why is $ABDC$ a square?

6 In the diagram $DE \parallel BC$ and $DE = \frac{1}{2}BC$.



a Why is ΔADE similar to ΔABC?

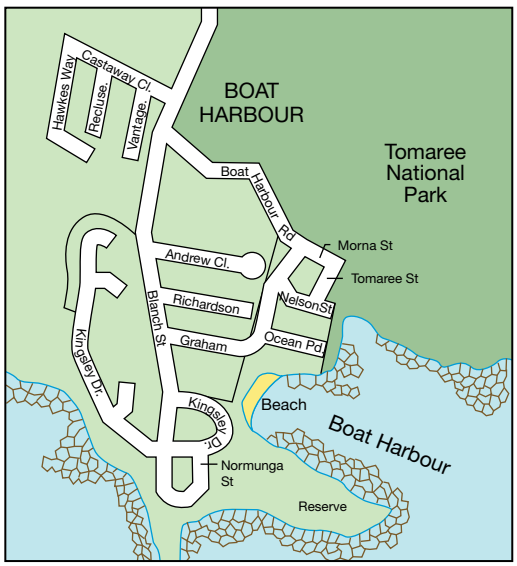
b Find the ratio of:

i $\frac{AD}{AB}$ ii $\frac{AE}{AC}$

c What does your answer to **b** tell you about D and E?

ASSIGNMENT 13B Working mathematically

1



Scale = 1 : 17 000

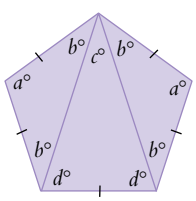
a What is the scale of this map?

b What distance would 1 cm on this map represent?

c What is the real length of Richardson Avenue?

d What is the real length of Ocean Parade?

2 Sharon makes a regular pentagon from three isosceles triangles as shown. Find the sizes of the angles in the triangles.

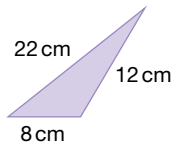


3 **a** If 6 people can do a piece of work in 8 days, in what time will 18 people do it, working at the same rate?

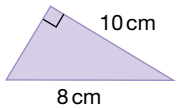
b If 14 people can do a piece of work in 12 days, how many people will be needed to do the work in 21 days, working at the same rate?

4 Why are each of these triangles impossible to draw?

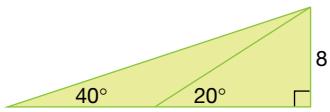
a



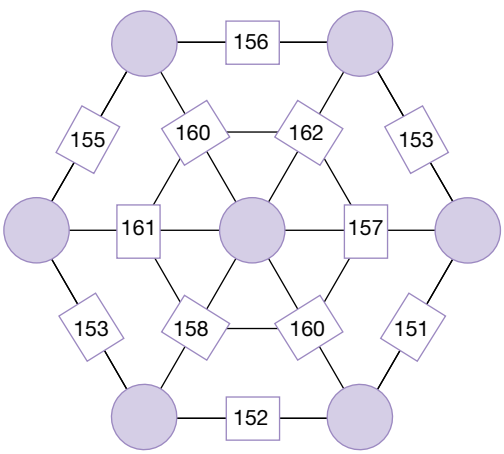
b



5 Why is the diagram shown impossible?

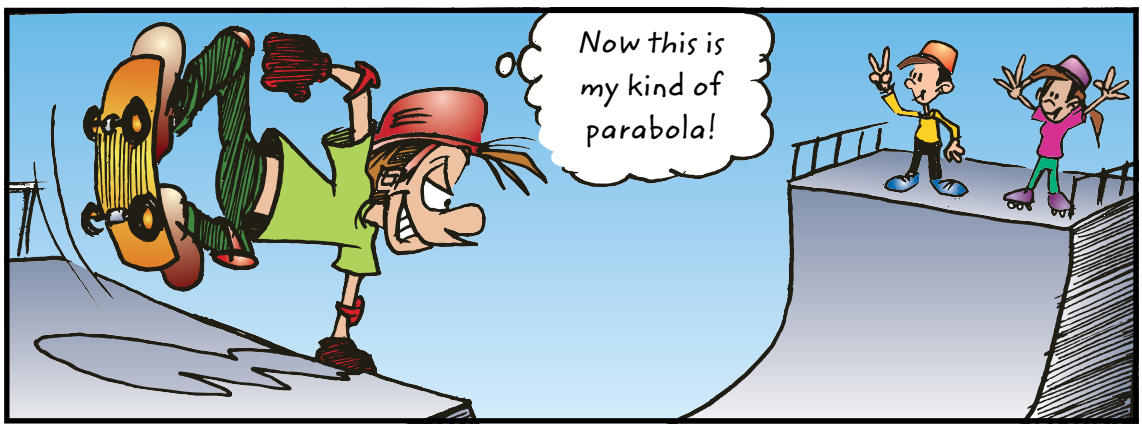


6 The numbers 74, 75, 77, 77, 78, 79 and 83 must be placed in the coloured circles so that the number in each rectangle is the sum of the numbers in the circles on either side of it. Which of these numbers must be placed in the centre circle?



FURTHER COORDINATE GEOMETRY

14



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14:03 Equation of a straight line, given two points
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14:05 Graphing a straight line using the x- and y-intercepts
14:06 Using coordinate geometry
14:07 Parabolas of the form $y=(x+a)^2$ and $y=(x+a)^2+k$
GeoGebra activity 14:07 Parabolas of the form $y=(x+a)^2$ and $y=(x+a)^2+k$
14:08 Parabolas of the form $y=ax^2+bx+c$
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14:14 Curves of the form $y=ax^n$ and $y=a(x-r)^n$
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GeoGebra activity 14:16 The intersection of graphs
14:17 Miscellaneous equations and graphs
GeoGebra activity 14:17 Curves and their equations
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Number and Algebra

Selections from ***Linear Relationships*** [Stage 5.3§] and ***Non-Linear Relationships*** [Stage 5.3§]

- Find the midpoint and gradient of a line segment(interval) on the Cartesian plane (ACMNA294)
- Find the distance between two points located on the Cartesian plane (ACMNA214)
- Sketch linear graphs using the coordinates of two points (ACMNA215)
- Solve problems using various standard forms of the equation of a straight line (NSW)
- Solve problems involving parallel and perpendicular lines (ACMNA238)
- Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267)
- Describe, interpret and sketch cubics, other curves and their transformations (NSW)

Working Mathematically

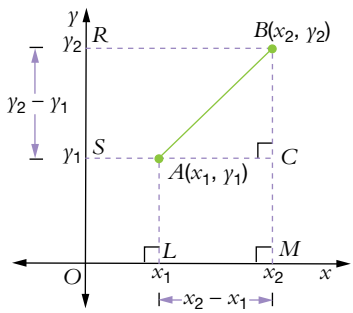
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

14:01 Distance, gradient and midpoint formulas

In Year 9 the length, gradient and midpoint of the interval AB were calculated using a right-angled triangle with AB as the hypotenuse. The method used can be generalised to derive formulas for the length, gradient and midpoint as shown below.

Length

A formula for finding the distance between two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, can be found using Pythagoras' theorem. We wish to find the length of interval AB.



Now $LM = x_2 - x_1$ ($LM = MO - LO$)

$\therefore\ AC = x_2 + x_1$ ($ACML$ is a rectangle)

and $RS = y_2 - y_1$ ($RS = RO - SO$)

$\therefore\ BC = y_2 - y_1$ ($BCSR$ is a rectangle)

Now $AB^2 = AC^2 + BC^2$ (Pythagoras' theorem)

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance AB between $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Gradient

Gradient of $AB = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{BC}{AC}$

$\therefore\ m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (opposite sides of a rectangle are equal)

m is used for 'gradient'

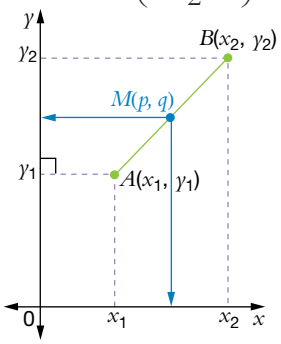
The gradient of the line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

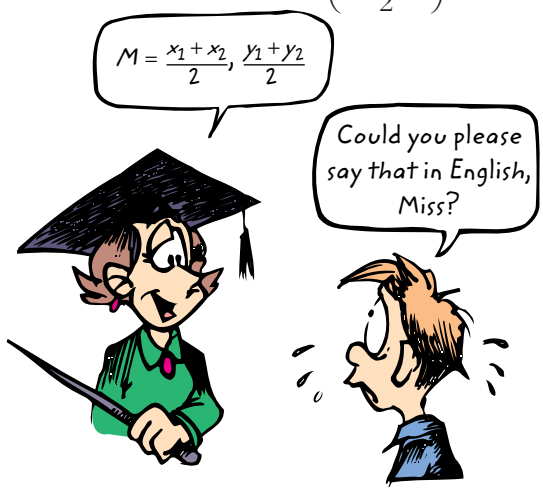
Midpoint

If M is the midpoint of AB it is half-way between A and B in both the horizontal direction and the vertical direction.

To find a number that is half-way between a pair of numbers we average them. Hence, if M is the point (p, q) then:

p is half-way between x_1 and x_2 $\left(\dfrac{x_1 + x_2}{2}\right)$ and q is half-way between y_1 and y_2 $\left(\dfrac{y_1 + y_2}{2}\right)$.





The midpoint, M, of interval AB, where A is (x_1, y_1) and B is (x_2, y_2), is given by:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Note: The choice of A as (x_1, y_1) is arbitrary. If B had been called (x_1, y_1), the formulas would be:

$$\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \text{Gradient} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Hence, when using the formulas it does not matter which point is called (x_1, y_1).

WORKED EXAMPLE 1

Find the distance between the points $(-2, 0)$ and $(8, -5)$.

Solution

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (8, -5)$

$$\therefore\ d = \sqrt{(8 - -2)^2 + (-5 - 0)^2}$$

$$= \sqrt{(10)^2 + (-5)^2}$$

$$= \sqrt{100 + 25}$$

$$= \sqrt{125}$$

WORKED EXAMPLE 2

Find the midpoint of interval AB, if A is the point $(-3, 5)$ and B is $(4, -2)$.

Solution

$$\text{Midpoint} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$
$$= \left(\frac{-3+4}{2}, \frac{5+-2}{2}\right)$$
$$= \left(\tfrac{1}{2}, \tfrac{3}{2}\right) \text{ or } \left(\tfrac{1}{2}, 1\tfrac{1}{2}\right)$$

WORKED EXAMPLE 3

Find the gradient of the straight line passing through the points $(6, -2)$ and $(2, -1)$.

Solution

Let (x_1, y_1) be $(6, -2)$

and (x_2, y_2) be $(2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1-(-2)}{2-6}$$
$$= \frac{1}{-4}$$

$\therefore$ The gradient is $-\frac{1}{4}$

Exercise 14:01

P Foundation worksheet 14:01 Substituting into formulas

1 Use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between the following points.

a $(4, 2)$ and $(7, 6)$ **b** $(0, 1)$ and $(8, 7)$ **c** $(-6, 4)$ and $(-2, 1)$ **d** $(-2, -4)$ and $(4, 4)$
e $(-6, 2)$ and $(6, 7)$ **f** $(4, 9)$ and $(-1, -3)$ **g** $(3, 0)$ and $(5, -4)$ **h** $(8, 2)$ and $(7, 0)$
i $(6, -1)$ and $(-2, 4)$ **j** $(-3, -2)$ and $(-7, -3)$

2 Use the midpoint formula to find the midpoint of the interval joining each pair of points in Question 1.

3 Use the gradient formula to find the gradient of the interval joining each pair of points in Question 1.

4 **a** Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line AB if:

i A $(3, 1)$ is the point (x_1, y_1) and $B(5,11)$ is the point (x_2, y_2)
ii A $(3, 1)$ is the point (x_2, y_2) and $B(5, 11)$ is the point (x_1, y_1).

b Are the values of m obtained in **a** equal?

5 Repeat Question **4** using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$. Are the values of m equal?

6 **a** The vertices of a triangle are $A(0, 0)$, $B(3, 4)$ and $C(-4, 5)$. Find the length of each side.

b $ABCD$ is a parallelogram where A is the point $(2, 3)$, B is $(5, 5)$, C is $(4, 3)$ and D is $(1, 1)$. Show that the opposite sides of the parallelogram are equal.

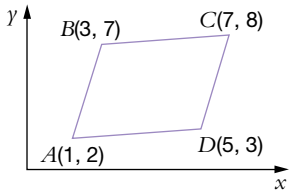
c $(3, 2)$ is the centre of a circle. $(6, 6)$ is a point on the circumference. What is the radius of the circle?

d Prove that the triangle ABC is isosceles if A is $(-2, -1)$, B is $(4, 1)$ and C is $(2, -5)$. (Isosceles triangles have two sides equal.)

e **i** Find the gradient of BC and of AD.

ii Find the gradient of AB and of DC.

iii What kind of quadrilateral is $ABCD$? Give a reason for your answer.

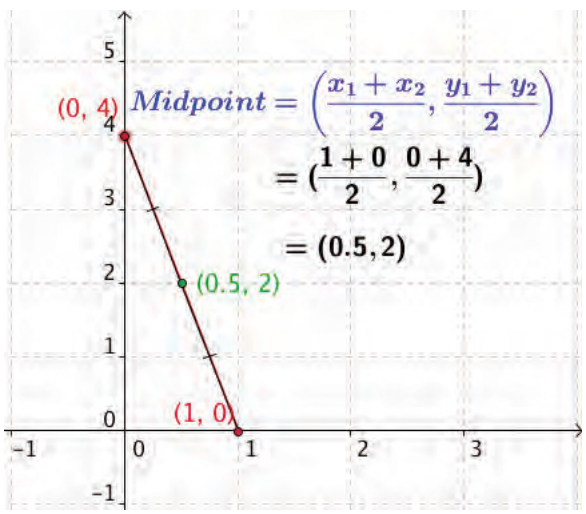


f Prove that a quadrilateral that has vertices $A(2, 3)$, $B(9, 5)$, $C(4, 0)$ and $D(-3, -2)$ is a parallelogram. (It will be necessary to prove that opposite sides are parallel.)

If two lines have the same gradient they are *parallel*.

P GEOGEBRA ACTIVITY 14:01 DISTANCE, GRADIENT AND MIDPOINT FORMULAS

Change the sliders in this activity to explore the midpoints, gradients and lengths of line intervals. You can click and drag points to change the length, slope and midpoint of an interval. After you have calculated the length, gradient or midpoint, tick a checkbox to see a worked solution.



14:02 Equation of a straight line, given point and gradient

In Year 9 you were shown how to find the equation of a straight line, given its gradient and y-intercept. The equation took the form: $y = mx + b$. This was also reviewed in Chapter 7. We will now find the equation of a straight line given different information.

PREP QUIZ 14:02

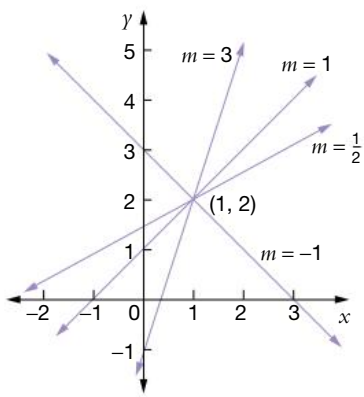
A line has the equation $y = 3x + 5$. Find:

1 its gradient
2 its y-intercept
3 the value of y when $x = 2$
4 the value of x when $y = 5$.

If $y = mx + b$, find the value of b if:

5 $m = 2, x = 0, y = 4$
6 $m = -2, x = 1, y = 3$
7 $m = 3, x = -1, y = -2$
8 $m = -2, x = -4, y = 10$
9 $m = 0, x = 2, y = 3$
10 $m = 1, x = 2, y = 2$

Any number of lines can be drawn through a given point. The equation of each straight line can be found if we know its gradient and a point through which it passes.



The equation of a line through (1, 2) having a gradient of 3 can be found using $y = mx + b$. We know that $m = 3$, but we must find the value of b. To do this, we substitute the coordinates of (1, 2) into the equation, as a point on the line must satisfy its equation.

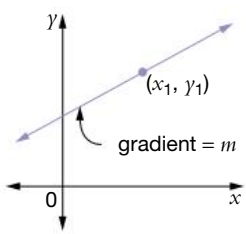
$y = mx + b$
$y = 3x + b$ $\quad m = 3$
$2 = 3(1) + b$ $\quad (1, 2)$ lies on the line
$2 = 3 + b$
$b = -1$
$\therefore$ The equation of the line is $y = 3x - 1$.

To find the equation of a straight line that has a gradient of 2 and passes through (7, 5):

1 substitute $m = 2, x = 7$ and $y = 5$ into the formula $y = mx + b$ to find the value of b

2 rewrite $y = mx + b$ replacing m and b with their numerical values.

We can use the method above to discover a formula that could be used to find the equation of a line given a point and gradient.



What is the equation of a line that has gradient m and passes through the point (x_1, y_1)?

$y = mx + b$ $\quad$ gradient is m
$y_1 = mx_1 + b$ $\quad (x_1, y_1)$ lies on the line
$\therefore\ b = y_1 - mx_1$

Substituting this for b into $y = mx + b$:

$$y = mx + (y_1 - mx_1)$$
$$y - y_1 = mx - mx_1$$
$$\therefore\ y - y_1 = m(x - x_1)$$

This last form of the answer is the easiest to remember as it could be written as $m = \dfrac{y - y_1}{x - x_1}$.

> The equation of a line with gradient m that passes through the point (x_1, y_1) is given by:
> $y - y_1 = m(x - x_1)$ or $m = \dfrac{y - y_1}{x - x_1}$.

WORKED EXAMPLE 1

Find the equation of the line that passes through $(1, 4)$ and has gradient 2.

$y - y_1 = m(x - x_1)$
or
$y = mx + b$

Solution

Let the equation of the line be:
$y = mx + b$
$\therefore\ y = 2x + b$ $\quad m = 2$
$4 = 2(1) + b$ $\quad (1, 4)$ lies on the line
$4 = 2 + b$
$\therefore\ b = 2$
$\therefore$ The equation is $y = 2x + 2$.

or
$y - y_1 = m(x - x_1)$
(x_1, y_1) is $(1, 4)$, $m = 2$
$\therefore\ y - 4 = 2(x - 1)$
$y - 4 = 2x - 2$
$\therefore\ y = 2x + 2$ is the equation of the line.

WORKED EXAMPLE 2

A straight line has gradient $-\frac{1}{2}$ and passes through the point $(1, 3)$. Find the equation of this line.

Solution

Let the equation be:
$y = mx + b$
$\therefore\ y = -\frac{1}{2}x + b$ $\quad m = -\frac{1}{2}$
$3 = -\frac{1}{2}(1) + b$ $\quad (1, 3)$ lies on the line
$3 = -\frac{1}{2} + b$
$\therefore\ b = 3\frac{1}{2}$
$\therefore$ The equation is $y = -\frac{1}{2}x + 3\frac{1}{2}$.

or
$y - y_1 = m(x - x_1)$
(x_1, y_1) is $(1, 3)$, $m = -\frac{1}{2}$
$\therefore\ y - 3 = -\frac{1}{2}(x - 1)$
$y - 3 = -\frac{1}{2}x + \frac{1}{2}$
$\therefore\ y = -\frac{1}{2}x + 3\frac{1}{2}$ is the equation of the line.

Exercise 14:02

P Foundation worksheet 14:02 Point–gradient form

1 For each part, find b if the given point lies on the given line.

a $(1, 3)$, $y = 2x + b$ **b** $(2, 10)$, $y = 4x + b$ **c** $(-1, 3)$, $y = 2x + b$
d $(5, 5)$, $y = 2x + b$ **e** $(3, 1)$, $y = x + b$ **f** $(-1, -9)$, $y = -2x + b$

2 Find the equation of the straight line (giving answers in the form $y = mx + b$) if it has:

- **a** gradient 2 and passes through the point (1, 3)
- **b** gradient 5 and passes through the point (0, 0)
- **c** gradient 3 and passes through the point (2, 2)
- **d** slope 4 and passes through the point (−1, 6)
- **e** gradient −1 and passes through the point (−2, 8)
- **f** gradient −2 and passes through the point (0, 7)
- **g** slope −5 and passes through the point (1, 0)
- **h** gradient $\frac{1}{2}$ and passes through the point (4, 5)
- **i** gradient $\frac{1}{4}$ and passes through the point $(6, 3\frac{1}{2})$
- **j** slope $-\frac{1}{2}$ and passes through the point (−4, −1).

3

- **a** A straight line has a gradient of 2 and passes through the point (3, 2). Find the equation of the line.
- **b** A straight line has a gradient of −1. If the line passes through the point (2, 1), find the equation of the line.
- **c** What is the equation of a straight line that passes through the point (−2, 0) and has a gradient of 3?
- **d** A straight line that passes through the point (1, −2) has a gradient of −3. What is the equation of this line?
- **e** A straight line that has a gradient of 3 passes through the origin. What is the equation of this line?
- **f** Find the equation of the straight line that has a gradient of 4 and passes through the point (−1, −2).
- **g** (2, 8) is on a line that has a gradient of 4. Find the equation of this line.
- **h** The point (−6, 4) lies on a straight line that has a gradient of −2. What is the equation of this line?
- **i** Find the equation of the straight line that has a gradient of 2 and passes through the midpoint of the interval joining (1, 3) and (5, 5).
- **j** A straight line passes through the midpoint of the interval joining (0, 0) and (−6, 4). Find the equation of the line if its gradient is $\frac{1}{2}$.

14:03 Equation of a straight line, given two points

Only one straight line can be drawn through two points. Given two points on a straight line, we can always find the equation of that line.

Consider the line passing through (1, 1) and (2, 4). Let the equation of the line be:

$y = mx + b$ (formula)

First find the gradient using the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-1}{2-1}$$
$$= 3$$

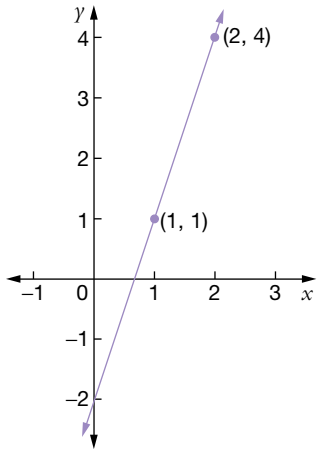
$(x_1, y_1) = (1, 1)$
$(x_2, y_2) = (2, 4)$

$\therefore\ y = 3x + b \qquad m = 3$

$4 = 3 \times 2 + b \qquad (2, 4)$ lies on the line

$\therefore\ b = -2$

$\therefore$ The equation of the line is $y = 3x - 2$.



To find the equation of a straight line that passes through two points:

1. Find the value of the gradient m, using the given points.
2. For $y = mx + b$, find the value of b by substituting the value of m and the coordinates of one of the given points.
3. Rewrite $y = mx + b$ replacing m and b with their numerical values.

Another method is to use the formula:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

where (x_1, y_1) and (x_2, y_2) are points on the line.

WORKED EXAMPLE

Find the equation of the line that passes through the points $(-1, 2)$ and $(2, 8)$.

Solution

Let the equation of the line be:

$y = mx + b$

Now $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-2}{2-(-1)}$

$= \frac{6}{3}$

$\therefore\ m = 2$

$\therefore\ y = 2x + b$

$(x_1, y_1) = (-1, 2)$
$(x_2, y_2) = (2, 8)$

$(2, 8)$ lies on the line.

$\therefore\ 8 = 2(2) + b$

$\therefore\ b = 4$

The equation is $y = 2x + 4$.

Or $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(x_1, y_1) is $(-1, 2)$, (x_2, y_2) is $(2, 8)$

$\therefore\ y - 2 = \frac{8-2}{2-(-1)}[x - (-1)]$

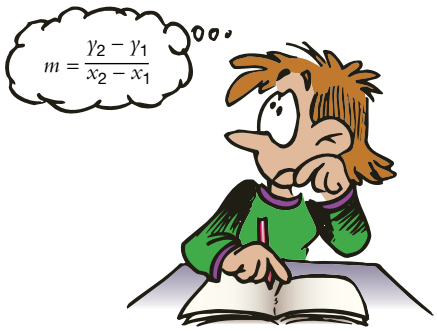
$y - 2 = \frac{6}{3}(x + 1)$

$y - 2 = 2(x + 1)$

$y - 2 = 2x + 2$

$\therefore$ The equation is $y = 2x + 4$.

Exercise 14:03



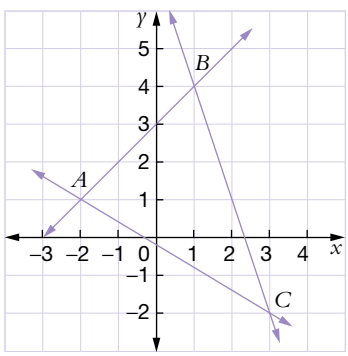
1 Find the gradient of the line that passes through the following points.

- **a** (2, 0) and (3, 4)
- **b** (−1, 3) and (2, 6)
- **c** (3, 1) and (1, 5)
- **d** (−2, −1) and (0, 9)
- **e** (−2, 1) and (2, 2)
- **f** (5, 2) and (4, 3)
- **g** (0, 0) and (1, 3)
- **h** (1, 1) and (4, 4)
- **i** (−1, 8) and (1, −2)
- **j** (0, 0) and (1, −3)

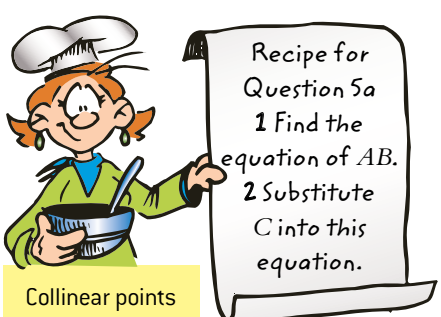
2 Use your answers for Question 1 to find the equations of the lines passing through the pairs of points in Question 1.

3
- **a** Find the equation of the line that passes through the points (−2, −2) and (1, 4).
- **b** The points $A(4, 3)$ and $B(5, 0)$ lie on the line AB. What is the equation of AB?
- **c** What is the equation of the line AB if A is the point (−2, −4) and B is (2, 12)?
- **d** Find the equation of the line that passes through the points (1, 6) and (2, 8). By substitution in this equation, show that (3, 10) also lies on this line.
- **e** What is the equation of the line CD if C is the point (2, 3) and D is the point (4, 5)?

4 A is the point (−2, 1), B is the point (1, 4) and C is the point (3, −2).
- **a** Find the gradient of each side of ΔABC.
- **b** Find the equation of each of the lines AB, BC and AC.
- **c** Find the y–intercept of each of the lines AB, BC and AC.
- **d** Find the equation of the line passing through point A and the midpoint of interval BC.
- **e** Find the gradient and y-intercept of the line passing through point A and the midpoint of interval BC.



5
- **a** Find the equation of the line joining $A(1, 2)$ and $B(5, −6)$. Hence show that $C(3, −2)$ also lies on this line.
- **b** $A(−2, 2)$, $B(1, −4)$ and $C(3, −8)$ are points on the number plane. Show that they are collinear.
- **c** Show that the points (−2, −11), (3, 4) and (4, 7) are collinear.



Collinear points lie on the same straight line.

14:04 The general form of a straight line

PREP QUIZ 14:04

Make y the subject of these equations.

1 $y + 4 = 0$ **2** $y - x = 0$ **3** $y + x = 0$

4 $x + y = 6$ **5** $2x + y - 3 = 0$ **6** $0 = 2x - y + 1$

Complete the instructions for each.

7 $2x + 3y = -6$ (+ 6 to each side) **8** $7x + 4 = y$ (− y from both sides)

9 $y = 4x + 6$ (− y from both sides) **10** $\frac{x}{2} + \frac{y}{3} - 1 = 0$ (× both sides by 6)

When asked to find the equation of a line, the answer can be given in several ways. For example the equations below all represent the same straight line.

$y = x - 2$ $x - y = 2$ $x - y - 2 = 0$

If the equation of a line is written in the general form it can only be written one way. This is often convenient, especially when trying to identify equations of lines from equations of other curves.

> The general form of a straight line is $ax + by + c = 0$ where a, b and c are whole numbers and $a > 0$.
>
> e.g. $2x + 5y - 1 = 0$ is in the general form.

WORKED EXAMPLES

Write these equations in the general form.

1 $y = 3x - 2$

Subtract y from both sides.

$0 = 3x - 2 - y$

$\therefore 3x - y - 2 = 0$ is in the general form.

2 $y = -\frac{x}{2} + 4$

Multiply both sides by 2.

$2y = -x + 8$

Subtract $2y$ from both sides.

$0 = -x + 8 - 2y$

$\therefore -x - 2y + 8 = 0$

Multiply both sides by −1.

$\therefore x + 2y - 8 = 0$ is in the general form.

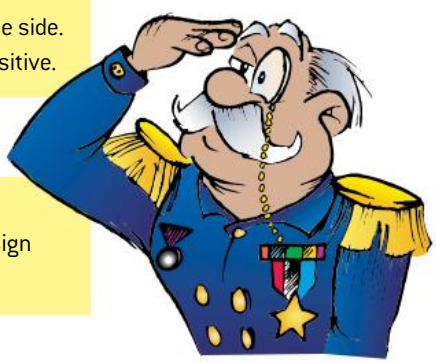
Remember:

1 Remove fractions.

2 Take all terms to one side.

3 Make the x term positive.

When an equation is multiplied by −1, the sign of each term changes.



Exercise 14:04

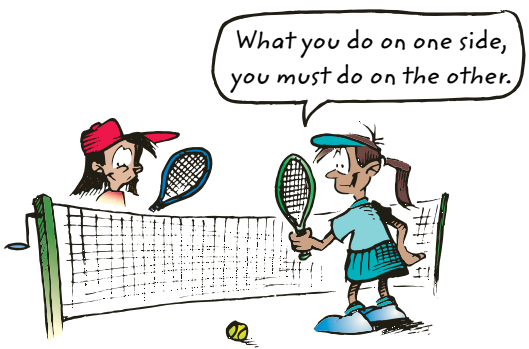
P Foundation worksheet 14:04
The general form

1 In columns **A**, **B** and **C**, three different forms of the same equation are given. State in which column, if any, it is written in general form.

	A	B	C
a	$y = 2 - x$	$x + y = 2$	$x + y - 2 = 0$
b	$5x - 3y = 6$	$3y = 5x - 6$	$-5x + 3y + 6 = 0$
c	$2x - y - 4 = 0$	$y = 2x - 4$	$2x = y + 4$
d	$x - y = 4$	$x - y - 4 = 0$	$y = x - 4$

2 Write each equation in the general form.

a $x + y = 2$
b $2x + 3 = 7y$
c $4x - y = -8$
d $3y + 4 = 2x$
e $6y - 3 = x$
f $2x = 5y - 1$
g $y = 4x + 1$
h $y = 3x - 4$
i $y = x + 2$
j $y = 4x + 3$
k $y = -2x - 5$
l $y = -x + 7$
m $y = \frac{x}{2} + 3$
n $y = \frac{x}{3} - 5$
o $y = \frac{2x}{5} + 1$



3 Write all answers in the general form.

a Find the equation of a line that has a gradient of 3 and a y-intercept of 5.
b A straight line has a gradient of 6 and a y-intercept of -2. Find the equation of the line.
c What is the equation of a straight line that has gradient 5 and passes through the point $(-1, -3)$?
d Find the equation of the straight line that passes through the point $(1, 7)$ and has gradient -1.
e Find the equation of the straight line AB if A is the point $(0, 3)$ and B is the point $(2, 7)$.
f What is the equation of the straight line that passes through $(-2, 1)$ and has y-intercept 7?
g A straight line passes through the origin and has a gradient of $-\frac{2}{3}$. Find its equation.

4 Find the equation of the lines in general form that pass through the points:

a $(3, -2)$ and $(-4, 1)$
b $(-2, -4)$ and $(3, 2)$
c $(1\frac{1}{2}, -\frac{2}{3})$ and $(-2\frac{1}{3}, \frac{1}{2})$

14:05 Graphing a straight line using the *x*- and *y*-intercepts

The points where a line crosses the *x*-and *y*-axes are called the *x*- and *y*-intercepts respectively. These points are commonly used to sketch the graph of a straight line.

To find the *x*-intercept, substitute $y = 0$ into the equation of the line and solve the equation for *x*.

To find the *y*-intercept, substitute $x = 0$ and solve the equation for *y*.

WORKED EXAMPLES

Find the *x*- and *y*-intercepts of the following lines and use them to sketch the graph of the line.

1 $2x + 3y - 6 = 0$

2 $6x - 3y = 20$

Solutions

1 When $x = 0$, $3y - 6 = 0$

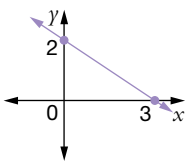
$3y = 6$

$y = 2$

When $y = 0$, $2x - 6 = 0$

$2x = 6$

$x = 3$

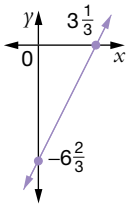


2 When $x = 0$, $-3y = 20$

$y = -6\frac{2}{3}$

When $y = 0$, $6x = 20$

$x = 3\frac{1}{3}$



Exercise 14:05

1 Solve the equations:

a $5x = 8$ **b** $3x = -11$ **c** $3y = 4$ **d** $-2y = -11$

2 **a** If $5x + 3y = 8$, does:

i $5x = 8$ when $y = 0$ **ii** $3y = 8$ when $x = 0$?

b If $3x - 2y = -11$, does:

i $3x = -11$ when $y = 0$ **ii** $-2y = -11$ when $x = 0$?

c If $7x + 2y = 14$, find the value of:

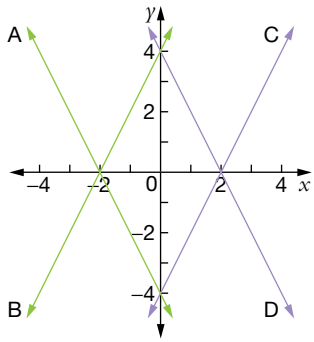
i x when $y = 0$ **ii** y when $x = 0$.

d If $4x - 5y = 15$, find the value of:

i x when $y = 0$ **ii** y when $x = 0$.

3 Find the x- and y-intercepts of each of the following lines and use them to match each equation with the graphs A to D.

a $2x + y = 4$
b $2x - y = 4$
c $2x + y = -4$
d $2x - y = -4$



4 Use the x- and y-intercepts to sketch the graph of each of the following straight lines.

a $x + 2y = 4$
b $2x - y = 6$
c $3x - 2y = 12$
d $2x + 5y - 10 = 0$
e $4x + 3y - 12 = 0$
f $2x + 3y - 9 = 0$
g $5x + 2y = -15$
h $3x + 4y = -15$
i $3x - 4y - 8 = 0$

14:06 Using coordinate geometry

The techniques of coordinate geometry can be used to study normal geometric problems. In the following exercises we look at some simple uses of these techniques.

WORKED EXAMPLE 1

A triangle is formed by the points $O(0, 0)$, $A(2, 3)$ and $B(4, 0)$. E and F are the midpoints of the sides OB and AB. Show:

a that ΔOAB is isosceles
b that EF is parallel to OB.

Solution

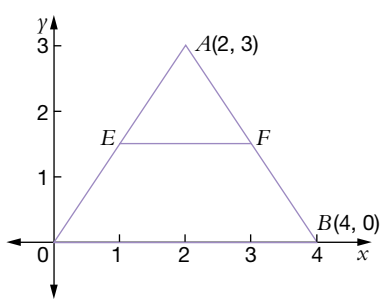
a $OA = \sqrt{(2-0)^2 + (3-0)^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

$AB = \sqrt{(4-2)^2 + (0-3)^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

$\therefore$ ΔOAB is isosceles (two equal sides).

b Now, E is $(1, 1\frac{1}{2})$ and F is $(3, 1\frac{1}{2})$.

$\therefore$ EF is horizontal (E and F have same y-coordinates).
OB is horizontal.
$\therefore$ EF is parallel to OB.

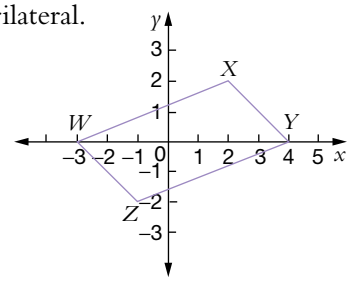


WORKED EXAMPLE 2

$W(-3, 0)$, $X(2, 2)$, $Y(4, 0)$ and $Z(-1, -2)$ are the vertices of a quadrilateral.

a Show that $WXYZ$ is a parallelogram.

b Show that the diagonals bisect each other.



Solution

a Calculating the slopes of the four sides gives the following.

$$\text{Slope of } WX = \frac{2-0}{2-(-3)} = \frac{2}{5} \qquad \text{Slope of } ZY = \frac{0-(-2)}{4-(-1)} = \frac{2}{5}$$

$$\text{Slope of } WZ = \frac{0-(-2)}{-3-(-1)} = -1 \qquad \text{Slope of } XY = \frac{2-0}{2-4} = -1$$

$\therefore$ $WX \parallel ZY$ (equal slopes) and $WZ \parallel XY$ (equal slopes)

$\therefore$ $WXYZ$ is a parallelogram (opposite sides are parallel).

b Midpoint of $XZ = \left(\frac{2+(-1)}{2}, \frac{2+(-2)}{2}\right) = (\frac{1}{2}, 0)$ Midpoint of $WY = (\frac{1}{2}, 0)$

$\therefore$ $(\frac{1}{2}, 0)$ is the midpoint of both diagonals.

$\therefore$ The diagonals XZ and WY bisect each other.

Exercise 14:06

P Foundation worksheet 14:06 Coordinate geometry

1 **a** Show that the triangle formed by the points $O(0, 0)$, $A(3, 1)$ and $B(1, 3)$ is isosceles.

b Show that the triangle formed by the points $(0, 0)$, $(1, 3)$ and $(7, 1)$ is right-angled.

c Show that the triangle with vertices at $(0, 0)$, $(-2, 2)$ and $(2, 2)$ is both right-angled and isosceles.

2 **a** Show that the quadrilateral with vertices at $A(0, 2)$, $B(3, 0)$, $C(0, -2)$ and $D(-3, 0)$ is a rhombus.

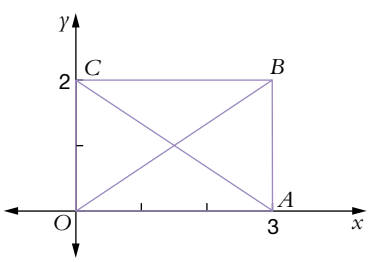
b A quadrilateral is formed by joining the points $O(0, 0)$, $B(1, 2)$, $C(5, 0)$ and $D(4, -2)$. Show that it is a rectangle.

c The points $A(0, 2)$, $B(2, 0)$, $C(0, -2)$ and $D(-2, 0)$ are joined to form a quadrilateral. Show that it is a square.

3 **a** If $OABC$ is a rectangle, what are the coordinates of B?

b Find the length of OB and AC. What property of a rectangle have you proved?

c Find the midpoint of OB and AC. What does your answer tell you about the diagonals of a rectangle?



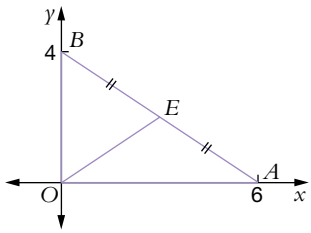
4 The points $A(0, 0)$, $B(6, 4)$ and $C(4, -2)$ form a triangle.

- **a** Find the midpoints of AB and AC.
- **b** Find the slope of the line joining the midpoints in **a**.
- **c** What is the slope of BC?
- **d** What do your answers to parts **b** and **c** tell you?

5 The points $A(4, 0)$, $B(4, 4)$, $C(0, 4)$ and $D(0, 0)$ form a square. Find the slopes of BD and AC. What does your result say about the diagonals BD and AC?

6 A right-angled triangle OAB is shown.

- **a** Find the coordinates of E, the midpoint of AB.
- **b** Find the length of OE.
- **c** Find the length of EA.
- **d** What can you say about the distance of E from O, A and B?

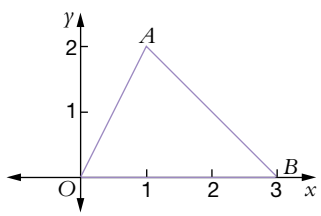


7 A triangle has its vertices at the points $A(-1, 3)$, $B(2, 4)$ and $C(1, 1)$.

- **a** Show that the triangle is isosceles.
- **b** Find E, the midpoint of AC.
- **c** Find the slope of the line joining E to B.
- **d** Show that EB is perpendicular to AC.
- **e** Describe how you could find the area of ΔABC.

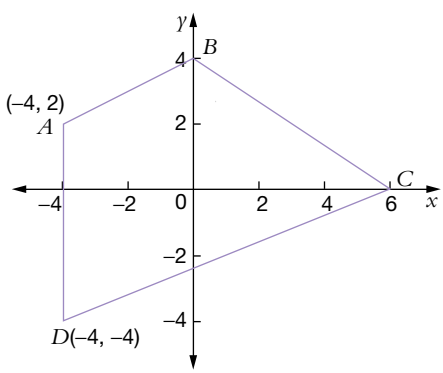
8

- **a** Find the midpoints of OA and AB.
- **b** Find the length of the line joining the midpoints in **a**.
- **c** Show that your answer in **b** is half the length of OB.



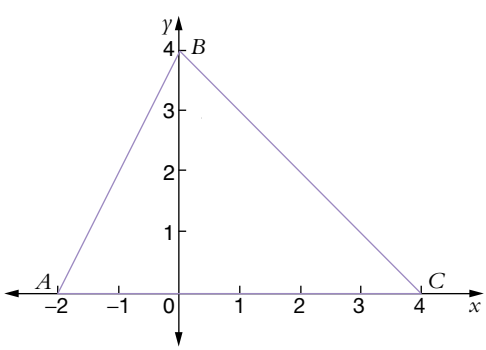
9 $ABCD$ is a quadrilateral.

- **a** Find the coordinates of the midpoints of each side.
- **b** Join the midpoints to form another quadrilateral. What type of quadrilateral do you think it is?
- **c** How could you prove your answer in **b**?



10 The points $A(-2, 0)$, $B(0, 4)$ and $C(4, 0)$ form the vertices of an acute-angled triangle.

- **a** Find the equation of the perpendicular bisectors of the sides AB, BC and AC.
- **b** Find the point of intersection of the perpendicular bisectors of the sides AB and AC.
- **c** Show that the perpendicular bisector of the side BC passes through the point of intersection found in **b**.



14:07 Parabolas of the form $y = (x + a)^2$ and $y = (x + a)^2 + k$

$y = x^2$

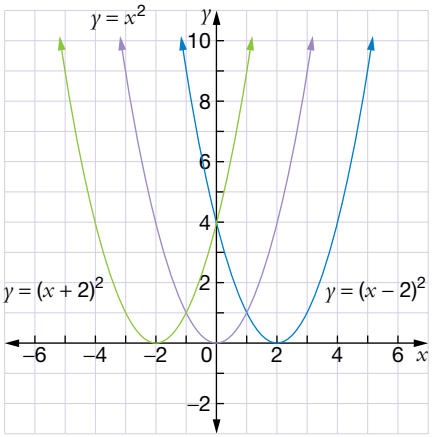
x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

$y = (x - 2)^2$

x	−1	0	1	2	3	4	5
y	9	4	1	0	1	4	9

$y = (x + 2)^2$

x	−5	−4	−3	−2	−1	0	1
y	9	4	1	0	1	4	9



The graph shows the parabolas $y = x^2$, $y = (x + 2)^2$ and $y = (x - 2)^2$.

Note that the parabolas are all the same shape.

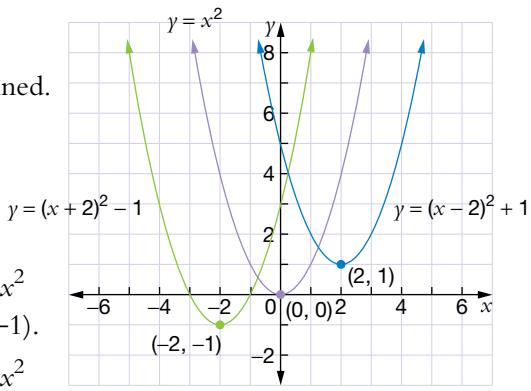
- $y = (x - 2)^2$ is obtained when $y = x^2$ is translated 2 units to the right.
- $y = (x + 2)^2$ is obtained when $y = x^2$ is translated 2 units to the left.

The graph of $y = (x + a)^2$ can be obtained from the curve $y = x^2$ by translating it:

- a units to the right if a is negative
- a units to the left if a is positive.

If the parabola $y = x^2$ is translated a units horizontally and k units vertically the curve $y = (x + a)^2 + k$ is obtained.

- If a is positive it moves to the left.
- If a is negative it moves to the right.
- If k is positive it moves up.
- If k is negative it moves down.



The curve $y = (x + 2)^2 - 1$ results from translating $y = x^2$ 2 units to the left and 1 unit down. The vertex is $(-2, -1)$.

The curve $y = (x - 2)^2 + 1$ results from translating $y = x^2$ 2 units to the right and 1 unit up. The vertex is $(2, 1)$.

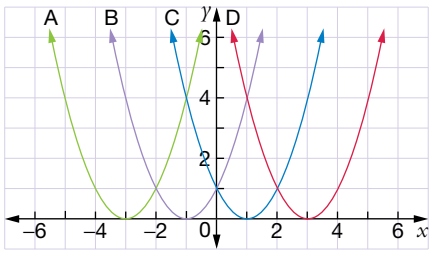
Exercise 14:07

1 Write the equation of the curve that is produced when $y = x^2$ is translated:

a 1 unit to the right
b 4 units to the right
c 1 unit to the left
d 4 units to the left.

2 Match each of the equations below with the curves A to D.

a $y = (x + 3)^2$
b $y = (x + 1)^2$
c $y = (x - 1)^2$
d $y = (x - 3)^2$



3 Sketch the graph of the following parabolas on separate number planes.

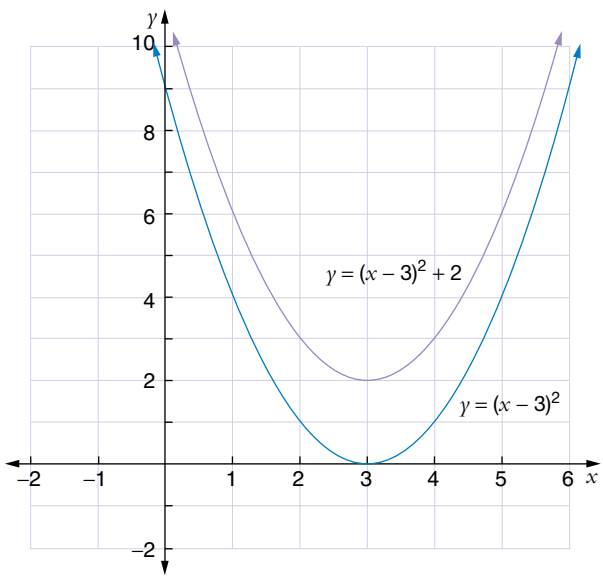
a $y = (x - 4)^2$
b $y = (x + 1)^2$
c $y = (x + 4)^2$

4 What are the coordinates of the vertex for each of the parabolas in Question **3**?

5 The graphs of $y = (x - 3)^2$ and $y = (x - 3)^2 + 2$ are shown on the diagram.

a How is the graph of $y = (x - 3)^2 + 2$ related to the graph of $y = (x - 3)^2$?
b How would the graph of $y = (x - 3)^2 - 2$ be obtained from the graph of $y = (x - 3)^2$?
c Sketch the graph of $y = (x + 3)^2 + 2$.
d Use the questions in **a** to **c** to explain the connection between the equation of the parabola and the coordinates of its vertex.
e Sketch the graph of each parabola on a separate number plane.

i $y = (x - 2)^2 + 3$
ii $y = (x + 2)^2 + 3$
iii $y = (x - 4)^2 - 2$
iv $y = (x + 1)^2 - 2$



6 Find the equation of the parabola that results from performing each of the following transformations on the parabola $y = x^2$.

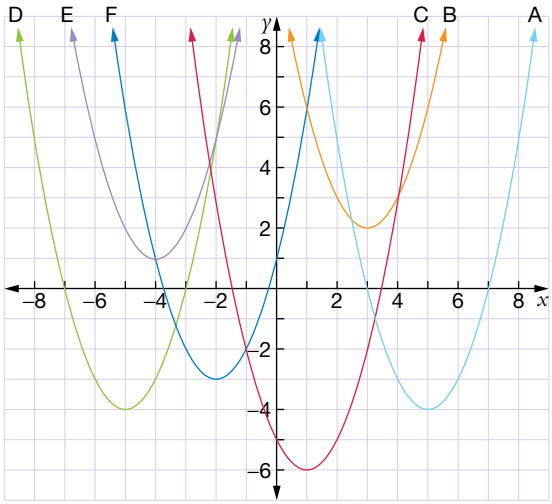
a moving it 2 units to the right and then moving it up 1 unit
b moving it 1 unit to the left and then moving it down 2 units
c moving it 2 units to the right and then moving it down 1 unit
d moving it 1 unit to the left and then moving it up 2 units

7 What are the coordinates of the vertex for each of the following parabolas?

a $y = (x + 5)^2 - 2$
b $y = (x - 3)^2 - 2$
c $y = (x + 4)^2 + 1$
d $y = (x - 2)^2 + 3$

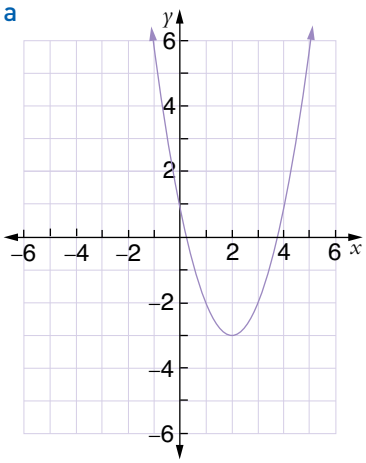
8 Match each equation with the corresponding parabola A to E in the diagram.

a $y = (x - 5)^2 - 4$
b $y = (x + 5)^2 - 4$
c $y = (x - 1)^2 - 6$
d $y = (x - 3)^2 + 2$
e $y = (x + 4)^2 + 1$
f $y = (x + 2)^2 - 2$



9 The parabolas shown are the result of translating the parabola $y = x^2$. Find the equation of each parabola.

a



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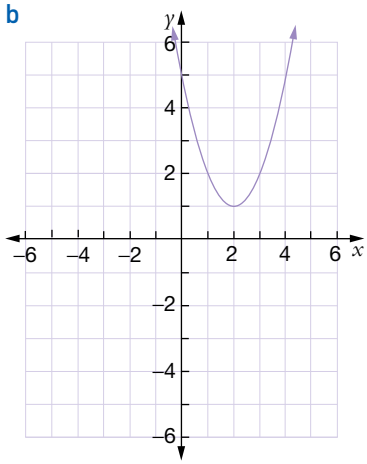
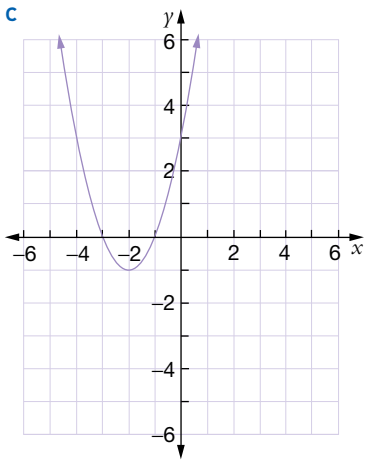


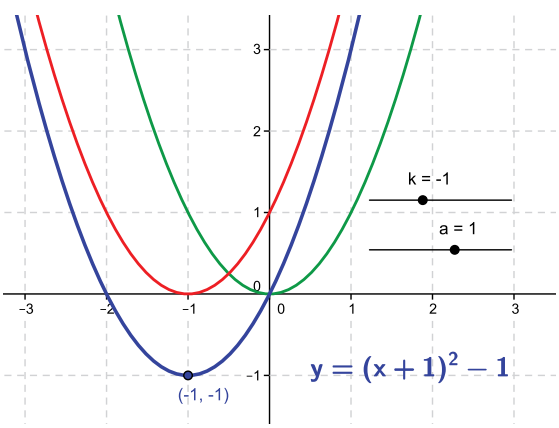
c



P GEOGEBRA ACTIVITY 14:07

PARABOLAS OF THE FORM $y = (x + a)^2$ AND $y = (x + a)^2 + k$

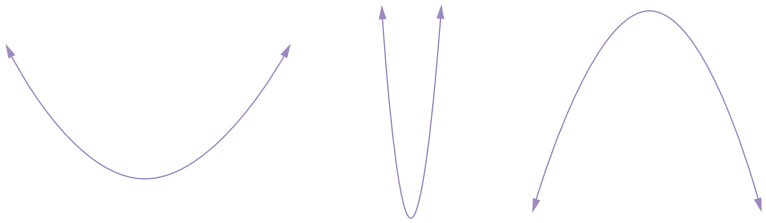
Change the sliders for a and k in this activity to explore translations of parabolas in the forms $y = (x + a)^2$ and $y = (x + a)^2 + k$. You can test your knowledge of the relationship between an equation and its graph by finding the equation of a randomly generated curve.



14:08 Parabolas of the form $y = ax^2 + bx + c$

All parabolas have the same basic shape.

- They are all concave up or concave down with a single vertex or turning point.
- They are symmetrical about an axis of symmetry.



Up until this point the equations of the parabolas have been given in special forms. We have looked at what numbers in these equations influence the steepness, the concavity and the position of the graph on the number plane.

In this section, we look at how to sketch a parabola when its equation is given in the form $y = ax^2 + bx + c$. We will also look at how to find features of the parabola, such as the x- and y-intercepts, the axis of symmetry and the vertex.

Finding the y-intercept

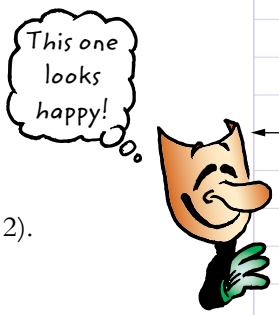
To find the y-intercept of $y = x^2 + x - 12$, we let $x = 0$.

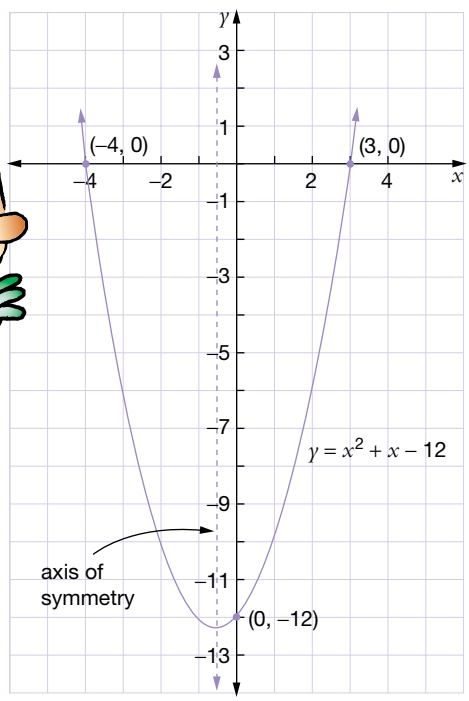
$y = x^2 + x - 12$

When $x = 0$, $y = -12$

$\therefore$ The y-intercept is -12.

$\therefore$ The curve cuts the y-axis at $(0, -12)$.





Finding the x-intercepts

To find the x-intercepts of $y = x^2 + x - 12$, we let $y = 0$.

$y = x^2 + x - 12$

When $y = 0$, $\quad 0 = x^2 + x - 12$

Solving this, $\quad 0 = (x + 4)(x - 3)$

$\therefore\ x = -4$ or 3

$\therefore$ The x-intercepts are -4 and 3.

$\therefore$ The curve cuts the x-axis at $(-4, 0)$ and $(3, 0)$.

Note: If $x^2 + x - 12 = 0$ was difficult to factorise, then the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ could have been used to find the x-intercepts.

Finding the axis of symmetry

The parabola $y = x^2 + x - 12$ has a vertical axis of symmetry so the axis of symmetry will cut the x-axis half-way between $(-4, 0)$ and $(3, 0)$, which are the two x-intercepts. The axis of symmetry has the equation:

$x = \dfrac{-4 + 3}{2}$

$x = -\frac{1}{2}$

> The axis of symmetry of the parabola $y = ax^2 + bx + c$ is given by the equation: $x = -\dfrac{b}{2a}$

For the parabola $y = x^2 + x - 12$, $a = 1$, $b = 1$, $c = -12$.

$\therefore$ The axis of symmetry is:

$x = \dfrac{-1}{2(1)}$

$x = -\frac{1}{2}$

Finding the vertex (or turning point)

Method 1

As the vertex lies on the axis of symmetry, its x-coordinate will be the same as that of the axis of symmetry.
The y-coordinate can be found by substituting this x value into the equation of the parabola.

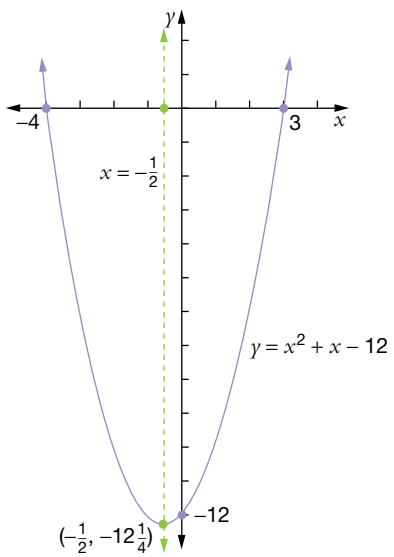
For $y = x^2 + x - 12$, the axis of symmetry is $x = -\frac{1}{2}$.

Now, when $x = -\frac{1}{2}$,

$y = (-\frac{1}{2})^2 + (-\frac{1}{2}) - 12$

$y = -12\frac{1}{4}$

$\therefore$ The vertex of the parabola is $(-\frac{1}{2}, -12\frac{1}{4})$.



Method 2

The vertex can also be found by finding the midpoint of the interval joining the x-intercepts.
Here the x-intercepts are -4 and 3.

The midpoint is $x = \frac{-4 + 3}{2}$ or $x = -\frac{1}{2}$.

Substituting $x = -\frac{1}{2}$ into $y = x^2 + x - 12$ gives $y = -12\frac{1}{4}$.

The vertex is $(-\frac{1}{2}, -12\frac{1}{4})$.

Method 3

The method of completing the square can also be used to find the minimum or maximum value and the vertex, as shown below.

$y = x^2 + x - 12$

$y = (x^2 + x + \frac{1}{4}) - \frac{1}{4} - 12)$

$y = (x + \frac{1}{2})^2 - 12\frac{1}{4}$

As $(x + \frac{1}{2})^2$ is always greater than or equal to 0, the minimum value of y will be $-12\frac{1}{4}$ when $x = -\frac{1}{2}$, so the vertex is the point $(-\frac{1}{2}, -12\frac{1}{4})$.

WORKED EXAMPLE

For each equation, find:

a the y-intercept
b the x-intercept
c the axis of symmetry
d the vertex (turning point).

Use these results to sketch each graph.

1 $y = x^2 + 2x - 3$ **2** $y = 2x^2 + 4x + 3$ **3** $y = 4x - x^2$

Solutions

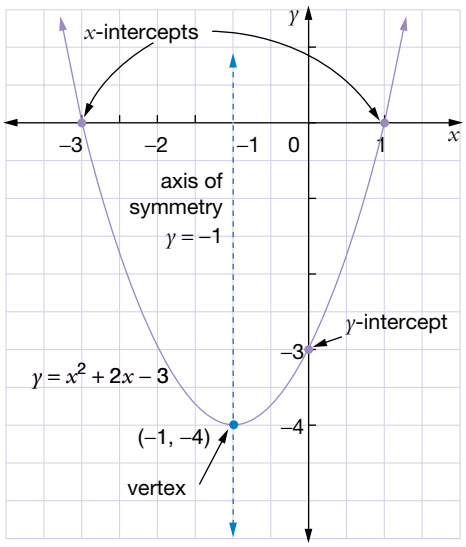
1 $y = x^2 + 2x - 3$

a For the y-intercept, let $x = 0$.
$y = (0)^2 + 2(0) - 3$
The y-intercept is -3.

b For the x-intercepts, let $y = 0$.
$0 = x^2 + 2x - 3$
$(x + 3)(x - 1) = 0$
The x-intercepts are -3 and 1.

c Axis of symmetry:
$x = \dfrac{-3 + 1}{2}$ (midpoint of x-intercepts)
$\therefore\ x = -1$ is the axis of symmetry.

d To find the vertex, substitute $x = -1$ into the equation to find the y-value.
$y = (-1)^2 + 2(-1) - 3$
$y = -4$
$\therefore$ The vertex is $(-1, -4)$.



We now plot the above information on a number plane and fit the parabola to it.

2 $y = 2x^2 + 4x + 3$

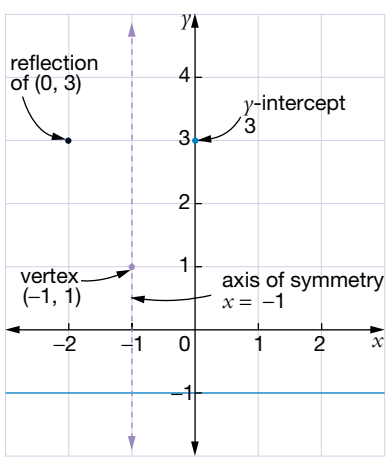
a To find the y-intercept of $y = 2x^2 + 4x + 3$, let x be zero.
$y = 2(0)^2 + 4(0) + 3$
$\therefore$ The y-intercept is 3.

b For the x-intercepts, solve $2x^2 + 4x + 3 = 0$.
Use the formula:

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$= \frac{-4 \pm \sqrt{-8}}{4}$$

This gives us a negative number under the square root sign.
You can't find the square root of a negative number!
Thus, there are no real solutions, so the parabola does not cut or touch the x-axis.



c $y = 2x^2 + 4x + 3$

$a = 2, b = 4, c = 3$

$\therefore$ Axis of symmetry:

$x = -\dfrac{b}{2a}$

$x = \dfrac{-4}{2 \times 2}$

$\therefore$ The axis of symmetry is $x = -1$.

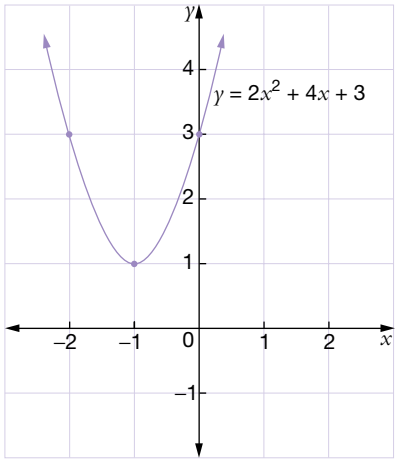
d The vertex is the turning point of the curve, and is on the axis of symmetry $x = -1$.

When $x = -1$,

$y = 2(-1)^2 + 4(-1) + 3$

$= 1$

$\therefore$ The vertex is $(-1, 1)$.



3 $y = 4x - x^2$

a For $y = 4x - x^2$, if $x = 0$, then $y = 0$.

$\therefore$ The curve cuts the y-axis at the origin.

b When $y = 0$, $\quad 4x - x^2 = 0$

$x(4 - x) = 0$

$\therefore$ The x-intercepts are 0 and 4.

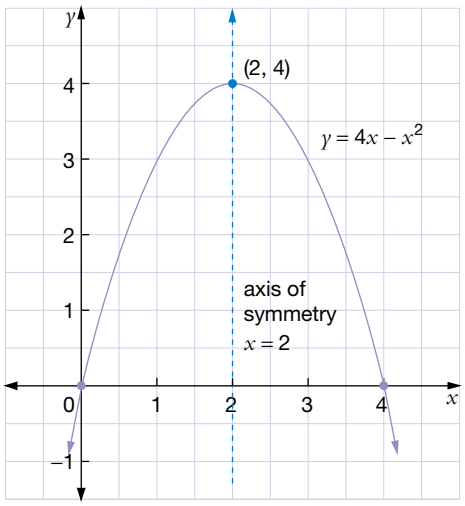
c Axis of symmetry:

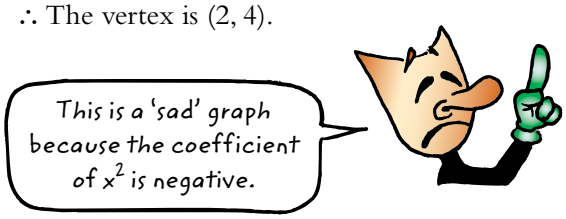
$x = 2$ (midpoint of x-intercepts)

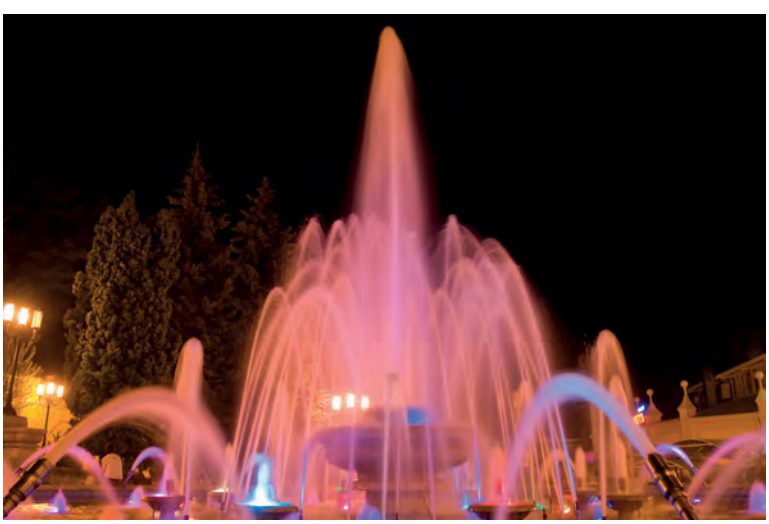
d When $x = 2$, $\quad y = 4(2) - (2)^2$

$= 4$

$\therefore$ The vertex is $(2, 4)$.







Water spurts are in the shape of parabolas.

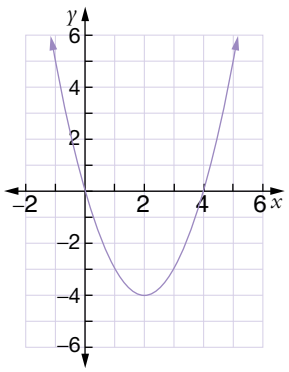
Exercise 14:08

P Foundation worksheet 14:08
The parabola $y = ax^2 + bx + c$

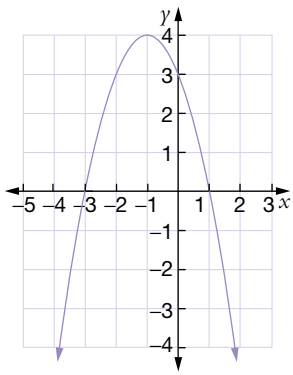
1 For each of the graphs, find:

i the y-intercept
ii the x-intercepts
iii the equation of the axis of symmetry
iv the coordinates of the vertex.

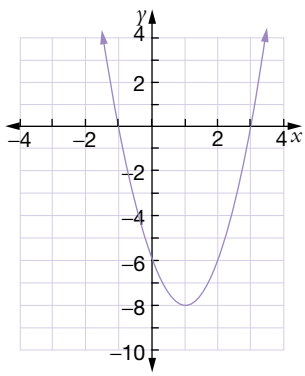
a



b

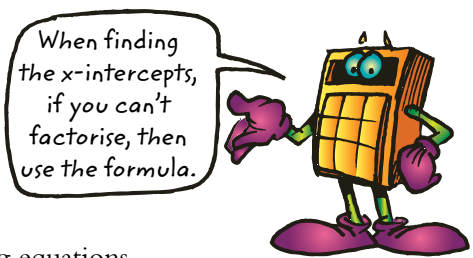


c



2 For the parabola $y = x^2 + 2x - 8$, find:

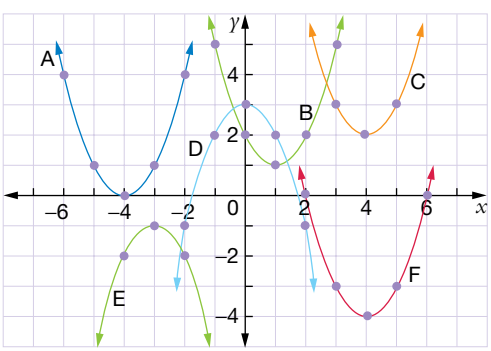
a the y-intercept
b the x-intercepts
c the axis of symmetry
d the vertex.
e Hence, sketch its graph.



3 Repeat the steps in Question **2** to graph the following equations, showing all the relevant features.

a $y = x^2 - 6x + 5$
b $y = x^2 - 6x$
c $y = 2x^2 - 8x - 10$
d $y = -x^2 + 4x - 3$
e $y = -x^2 + 6x - 9$
f $y = 2x^2 + 4x + 2$
g $y = x^2 - 3x - 4$
h $y = 2x^2 - 3x - 2$
i $y = -2x^2 - 3x - 1$

4 Match each parabola with one of the equations written below the diagram.



a $y = -x^2 + 3$
b $y = x^2 - 2x + 2$
c $y = x^2 - 8x + 12$
d $y = x^2 + 8x + 16$
e $y = -x^2 - 6x - 10$
f $y = x^2 - 8x + 18$

1 Find the turning point.
2 Is it happy ∪ (a is $+ve$) or sad ∩ (a is $-ve$)?
3 Visualise the graph before you sketch.

5 Sketch each set of three parabolas on the same number plane.

a	**i** $y = x^2 - 4$	**ii** $y = x^2 - 4x$	**iii** $y = x^2 - 4x + 4$
b	**i** $y = 9 - x^2$	**ii** $y = 9x - x^2$	**iii** $y = 10 + 9x - x^2$
c	**i** $y = (x - 3)(x + 5)$	**ii** $y = 2(x - 3)(x + 5)$	**iii** $y = (3 - x)(5 + x)$
d	**i** $y = x^2 - 2x - 8$	**ii** $y = 2x^2 - 4x - 16$	**iii** $y = 8 + 2x - x^2$

6 Sketch the graph of each quadratic relationship, showing all relevant features.

a $y = 4x^2 + 16x + 7$ **b** $y = 2x^2 + 9x - 5$ **c** $y = 4x^2 - 36x + 56$ **d** $y = 2x^2 - 5x - 7$

7 **a** Sketch the graphs of $y = x^2 + 3x$ and $y = 2x^2 + 6x$. Compare the two graphs and describe the differences between them.

b Sketch the graphs of $y = (x - 3)(x + 2)$ and $y = 3(x - 3)(x + 2)$. Compare the two graphs and describe the differences between them.

INVESTIGATION 14:08 DESCRIBING NUMBER PATTERNS

If we know that a table represents a parabola, we can generate its equation from the table using the general equation of the parabola and simultaneous equations.

Consider the table below.

x	0	1	2	3	4	5
y	3	6	11	18	27	38

If the values in the table are thought of as points on a parabola, we can substitute the x- and y-values of three points into the equation $y = ax^2 + bx + c$ to produce a set of three simultaneous equations.

To find the equation of the parabola we need to find the values of a, b and c.

$y = ax^2 + bx + c$

$\therefore\ c = 3$ substituting (0, 3)

(Substituting $x = 0$ will always give the value of the constant term c.)

We now know that the parabola has an equation of the form $y = ax^2 + bx + 3$.

$y = ax^2 + bx + 3$

$6 = a + b + 3$ substituting (1, 6)

$\therefore\ a + b = 3$ 1

$y = ax^2 + bx + 3$

$11 = 4a + 2b + 3$ substituting (2, 11)

$\therefore\ 2a + b = 4$ 2

Solving the simultaneous equations gives the solution:

$a + b = 3$ 1

$2a + b = 4$ 2

$a = 1, b = 2$

Substituting $a = 1$, $b = 2$, $c = 3$ into $y = ax^2 + bx + c$ gives the rule $y = x^2 + 2x + 3$.

This method will always work, but solving the simultaneous equations is very difficult if the table does not contain a point with an x-value of 0.

Exercise

Find the equation of the parabola that would produce each of the following tables of values.

1

x	0	1	2	3	4	5
y	2	6	12	20	30	42

2

x	0	1	2	3	4	5
y	16	7	0	−5	−8	−9

3

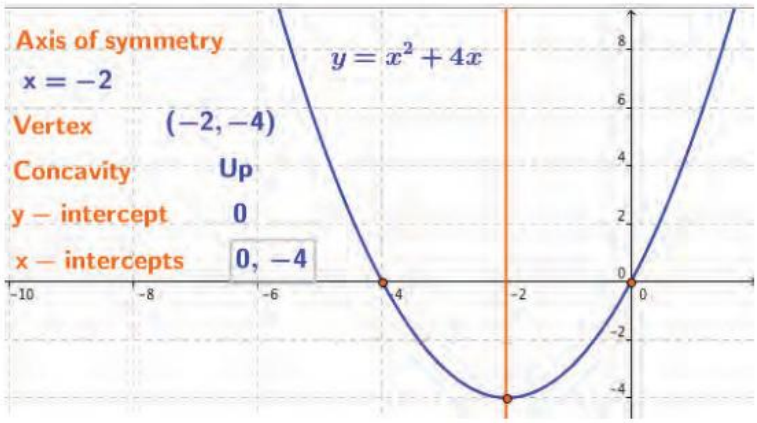
x	0	1	2	3	4	5
y	−4	−1	2	17	32	51

4

x	−2	−1	0	1	2	3
y	19	10	5	4	7	14

GEOGEBRA ACTIVITY 14:08 PARABOLAS OF THE FORM $y = ax^2 + bx + c$

Change the sliders for a, b and c in this activity to explore parabolas of the form $y = ax^2 + bx + c$. You can also use the equation to determine the features of the parabola and check your skills by using a checkbox.



14:09 The hyperbola $y = \frac{k}{x}$

PREP QUIZ 14:09

Find the value of $\frac{2}{x}$ when x is:

1 $\frac{1}{2}$ **2** 1 **3** 2 **4** 4 **5** −4

If $y = \frac{8}{x}$, what is the value of y when x is:

6 2 **7** 4 **8** −8?

9 If $y = \frac{4}{x}$, what happens to y as x increases from 1 to 40?

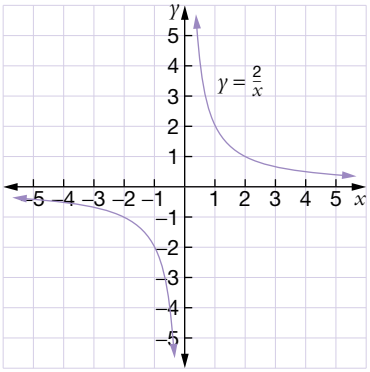
10 If $y = \frac{4}{x}$, what happens to y as x decreases from −1 to −40?

$y = \frac{k}{x}$ is the basic form of the hyperbola. If $k = 2$, we obtain the equation $y = \frac{2}{x}$.

$y = \frac{2}{x}$ is plotted on the following page using a table of values.

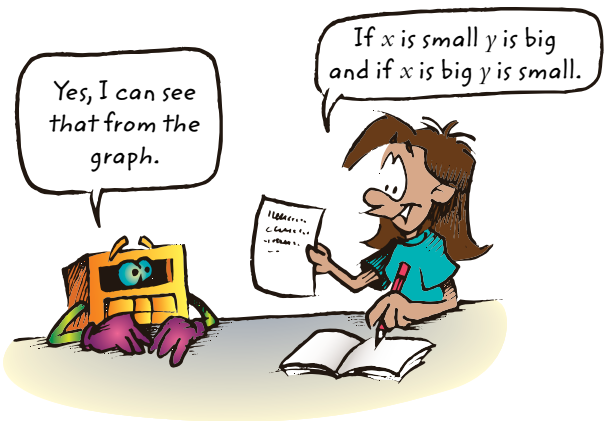
x	−5	−4	−3	−2	−1	−0.5	0	0.5	1	2	3	4	5
y	−0.4	−0.5	−0.7	−1	−2	−4	–	4	2	1	0.7	0.5	0.4

- Notice that there is no value for y when $x = 0$.
  When $x = 0$, $y = \frac{2}{x}$ becomes $y = \frac{2}{0}$.
  This value cannot exist as no number can be divided by zero.
- What will happen to the y-values as the x values get closer to 0?
- What will happen to the y values as the x-values become larger?



Features of the hyperbola

- The hyperbola has two parts.
- The parts are in opposite quadrants and are the same shape and size.
- The curve is symmetrical.
- The curve approaches the axes but will never touch them.
- The x- and y-axes are called asymptotes of the curve.
- No value for y exists when $x = 0$.



Exercise 14:09

1 Complete the table below for $y = \frac{4}{x}$.

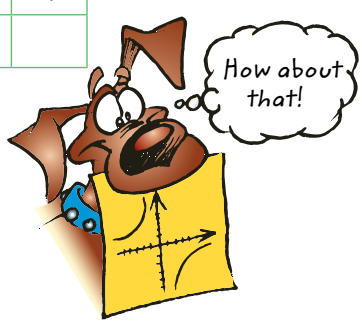
x	−8	−4	−2	−1	−0·5	0·5	1	2	4	8
y										

Use a sheet of graph paper to plot the curve $y = \frac{4}{x}$, using your table.
Use values −8 to 8 on both axes.

2 Graph the curve $y = -\frac{1}{x}$ by first completing the table below.

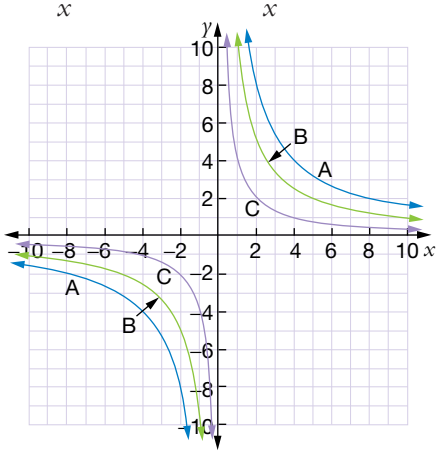
x	−4	−2	−1	−0·5	−0·25	0·25	0·5	1	2	4
y										

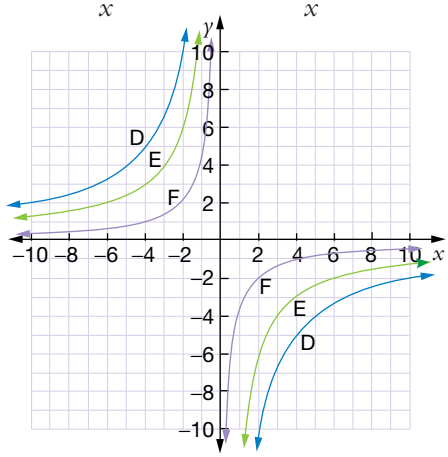
What does a negative value of k do to the graph?
Note: $y = -\frac{1}{x}$ is the same as $y = \frac{-1}{x}$.



3 Match each of the graphs A to F with the following equations.

a $y = \frac{4}{x}$ **b** $y = -\frac{4}{x}$ **c** $y = -\frac{20}{x}$ **d** $y = \frac{10}{x}$ **e** $y = -\frac{12}{x}$ **f** $y = \frac{16}{x}$





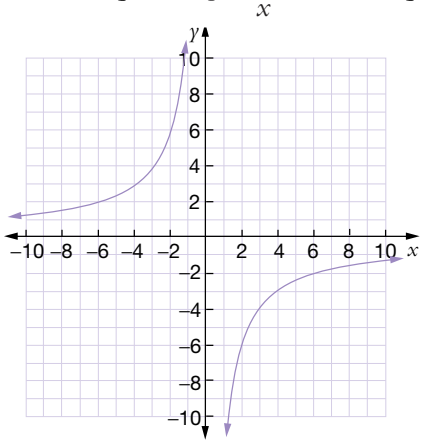
4 **a** Does the point (4, 2) lie on the hyperbola $y = \frac{8}{x}$?

b If the point (3, −6) lies on the hyperbola $y = \frac{k}{x}$, what is the value of k?

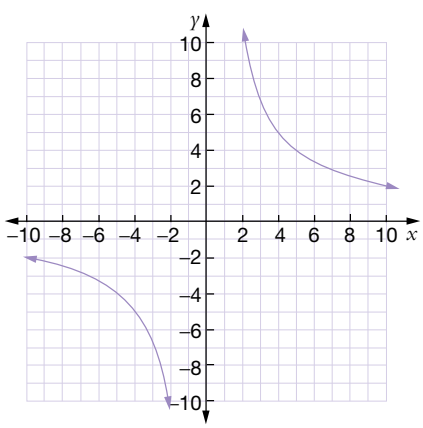
c The hyperbola $y = \frac{k}{x}$ passes through the point (10, 2). What is the value of k?

5 For each of the following, find a point that the hyperbola passes through and, by substituting this in the equation $y = \frac{k}{x}$, find the equation of the hyperbola.

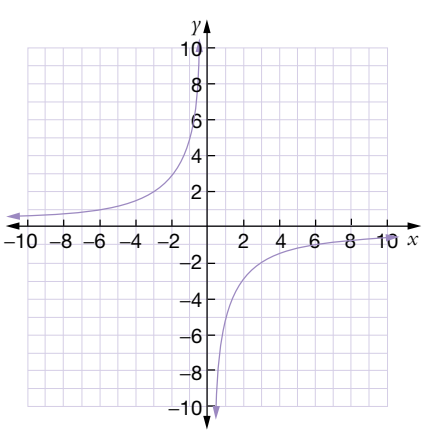
a



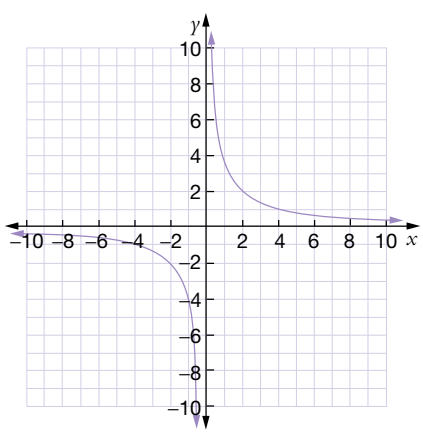
b



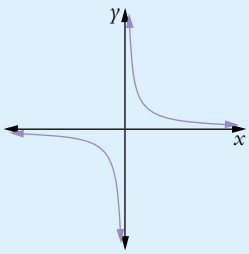
c



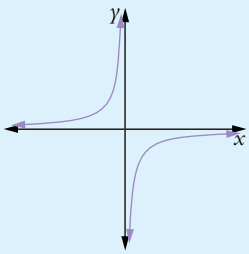
d



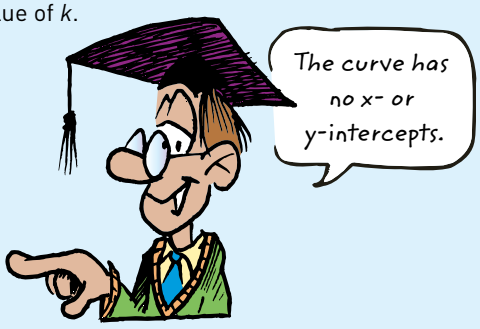
- The position of the hyperbola $y = \frac{k}{x}$ depends on the value of k.



If $k > 0$ it exists in Quadrants 1 and 3.



If $k < 0$ it exists in Quadrants 2 and 4.



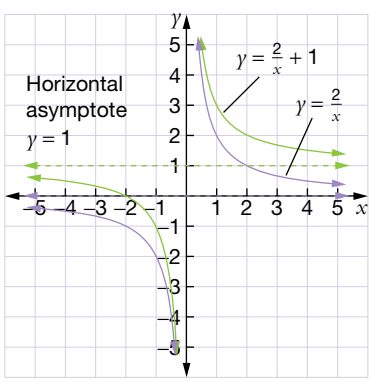
- $y = \frac{k}{x}$ and $y = -\frac{k}{x}$ are reflections of each other in the x- and y-axes.
- The x- and y-axes are the asymptotes of the curve.

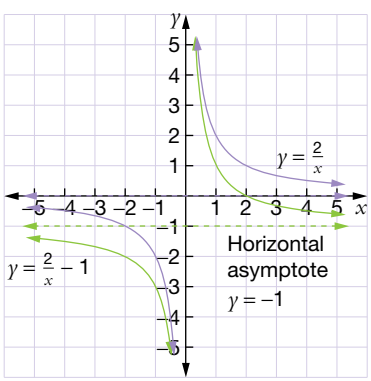
14:10 Hyperbolas of the form $y = \frac{k}{x} + c$ and $y = \frac{k}{x - b}$

Graphs of the form $y = \frac{k}{x} + c$

As we have seen with the parabola, the addition of a constant translates the curve vertically.

- Moving $y = \frac{2}{x}$ up 1 unit produces the curve $y = \frac{2}{x} + 1$.
- Moving $y = \frac{2}{x}$ down 1 unit produces the curve $y = \frac{2}{x} - 1$.





Moving the curve vertically will also change the horizontal asymptote.

The curve $y = \frac{k}{x} + c$ is obtained by translating the curve $y = \frac{k}{x}$:

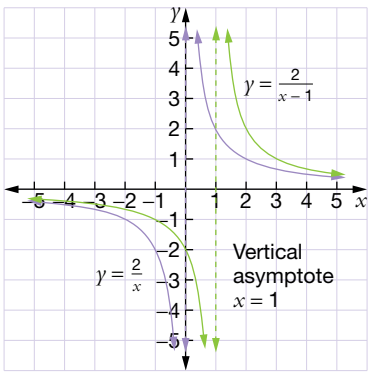
- c units up if c is positive
- c units down if c is negative.

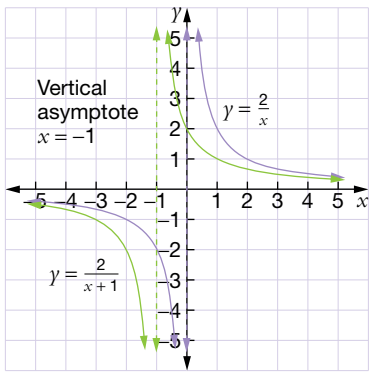
The asymptotes are $x = 0$ and $y = c$.

Graphs of the form $y = \frac{k}{x-b}$

The following diagrams show the relationships between the curve $y = \frac{2}{x}$ and the curves $y = \frac{2}{x-1}$ and $y = \frac{2}{x+1}$.

- Moving $y = \frac{2}{x}$ right 1 unit produces the curve $y = \frac{2}{x-1}$.
- Moving $y = \frac{2}{x}$ left 1 unit produces the curve $y = \frac{2}{x+1}$.





The hyperbola $y = \frac{k}{x-b}$ is obtained by translating the curve $y = \frac{k}{x}$:

- b units to the right if b is positive
- b units to the left if b is negative.

The asymptotes are $x = b$ and $y = 0$.

Notice that when the curve is moved horizontally, the vertical asymptote moves as well.

WORKED EXAMPLE 1

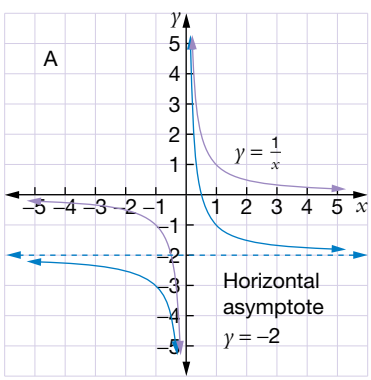
The curves A and B have been produced by translating the curve $y = \frac{1}{x}$. What are the equations of A and B?

Solution

Curve A has been produced by moving $y = \frac{1}{x}$ down 2 units.

See points (1, 1) and (1, –1).

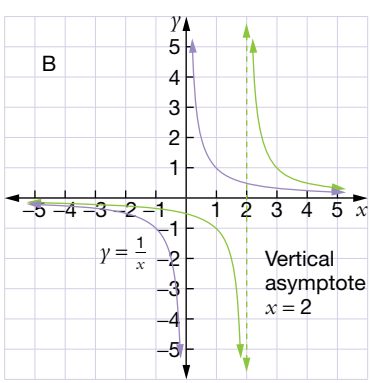
Hence the equation of curve A is $y = \frac{1}{x} - 2$.



Curve B has been produced by moving $y = \frac{1}{x}$ right 2 units.

See the points (1, 1) and (3, 1).

Hence the equation of curve B is $y = \frac{1}{x-2}$.



WORKED EXAMPLE 2

Sketch the following curves and label the asymptotes and the x- and y-intercepts.

a $y = -\frac{1}{x} + 2$

b $y = \frac{2}{x-1}$

Solution

a $y = -\frac{1}{x} + 2$ has come from moving

$y = -\frac{1}{x}$ up 2 units.

As x increases, y approaches 2.
$\therefore$ $y = 2$ is an asymptote.

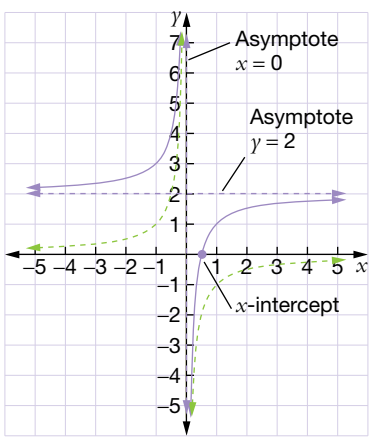
Now, $x \neq 0$ because you cannot divide by 0.
$\therefore$ $x = 0$ is an asymptote and there is no y-intercept.

If $y = 0$,

$\therefore 0 = -\frac{1}{x} + 2$

$\therefore \frac{1}{x} = 2$

$\therefore x = \frac{1}{2}$ is the x-intercept.



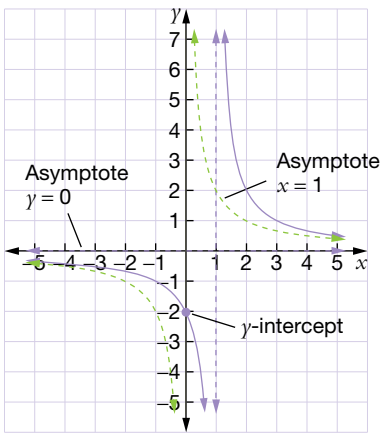
b $y = \frac{2}{x-1}$ has come from moving

$y = \frac{2}{x}$ 1 unit to the right.

Now $x \neq 1$, because you cannot divide by 0.
$\therefore$ There is an asymptote at $x = 1$.

If $x = 0$, $y = -2$.
$\therefore$ y-intercept $= -2$

Now, as x becomes large y approaches 0.
$\therefore$ $y = 0$ is an asymptote.



Exercise 14:10

1 Complete each of the tables below correct to one decimal place. Use the points obtained to graph the curve. Find the asymptotes for each curve and the x- and y-intercepts where they exist.

a $y = \frac{1}{x} + 2$

x	−8	−4	−2	−1	−0·5	0·5	1	2	4	8
y										

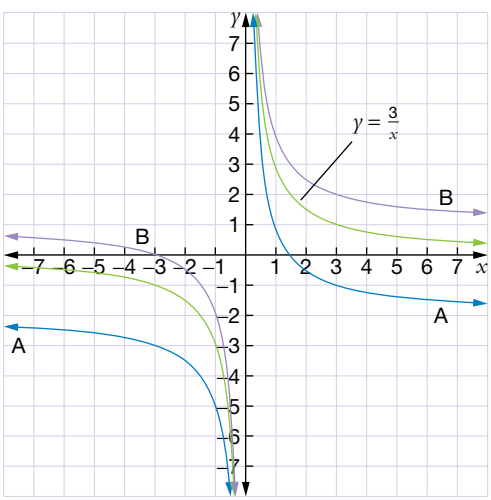
b $y = \frac{3}{x} - 1$

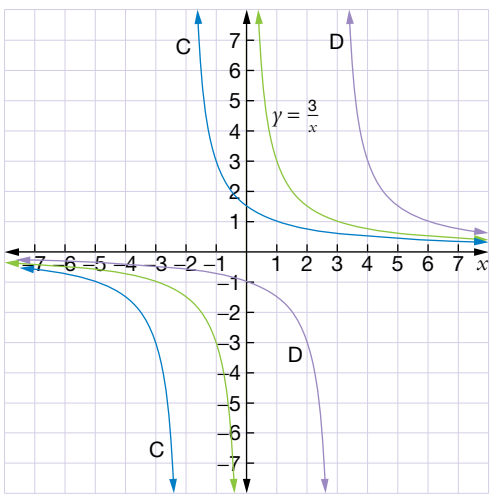
x	−6	−3	−2	−1	−0·5	0·5	1	2	3	6
y										

c $y = \frac{-1}{x-2}$

x	−4	−2	0	1	1·5	2·5	3	4	6	8
y										

2 In the following the curves A, B, C and D have been produced by translating the curve $y = \frac{3}{x}$. What are the equations of A, B, C and D?





3 Write the equation of the curve that results from translating the curve $y = \frac{1}{x}$:

a 1 unit up **b** 1 unit down **c** 1 unit to the right **d** 1 unit to the left

4 Write the translation that changes $y = \frac{4}{x}$ into:

a $y = \frac{4}{x} - 3$ **b** $y = \frac{4}{x} + 2$ **c** $y = \frac{4}{x-3}$ **d** $y = \frac{4}{x+2}$

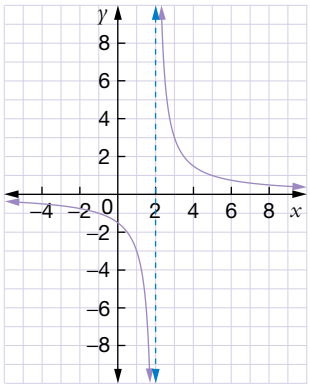
5 Determine the equations of the asymptotes for each of the hyperbolas in Question **4**.

6 Match each of the graphs **A** to **F** with the following equations.

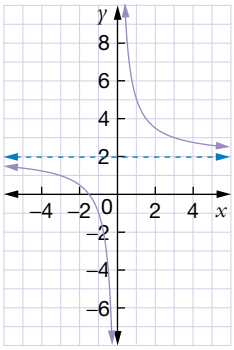
a $y = \frac{3}{x} + 2$ **b** $y = \frac{-3}{x+2}$ **c** $y = -\frac{3}{x} - 2$

d $y = \frac{-3}{x-2}$ **e** $y = -\frac{3}{x} + 2$ **f** $y = \frac{3}{x-2}$

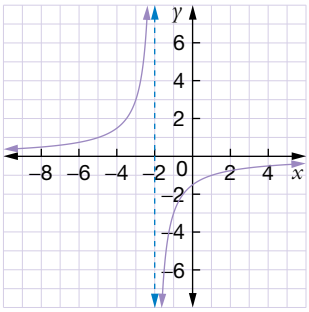
A



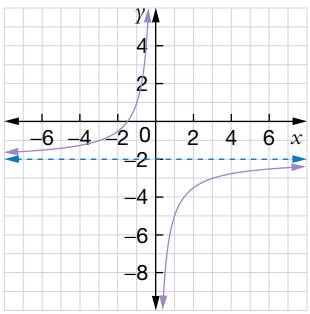
B



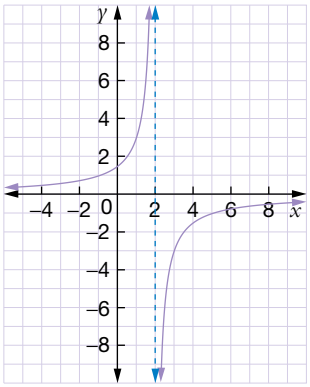
C



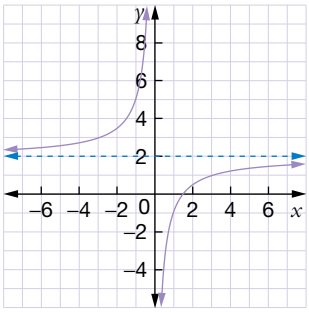
D



E



F

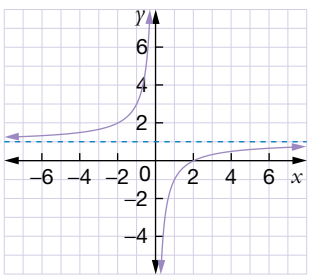


7 Graph the following curves showing the asymptotes and the x- and y-intercepts if they occur.

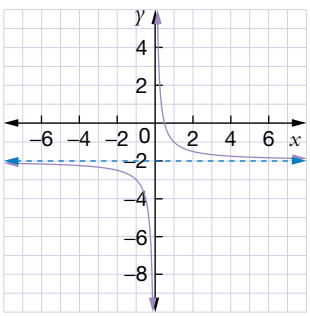
a $y = \frac{-2}{x-1}$ **b** $y = \frac{2}{x} - 1$ **c** $y = \frac{1}{x+2}$

8 Find the equation of each of the following hyperbolas.

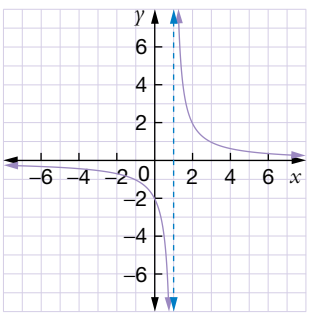
a



b

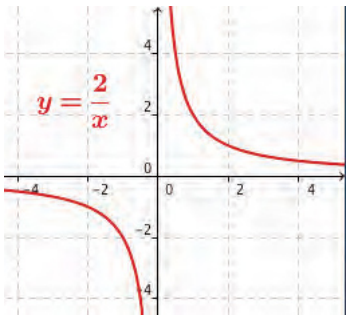


c



P GEOGEBRA ACTIVITY 14:10 HYPERBOLAS

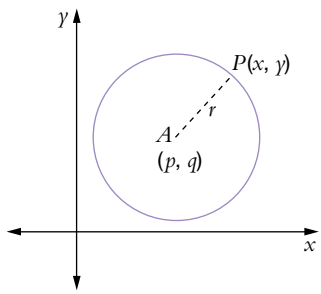
Use the checkboxes and the sliders to investigate the effect that changing the values of k, c and b has on hyperbolas of the form $y = \frac{k}{x-b} + c$. You can then test your knowledge by finding the equation of a randomly generated curve.



14:11 Circles and their equations

The fact that every point on a circle is the same distance away from the centre allows us to derive its equation.

Let $P(x, y)$ be any point on a circle that has its centre at $A(p, q)$ and a radius of r units.



Now, $AP = r$

Using coordinate geometry:

$$AP = \sqrt{(x-p)^2 + (y-q)^2}$$

$$\therefore \sqrt{(x-p)^2 + (y-q)^2} = r$$

$$(x-p)^2 + (y-q)^2 = r^2$$

The equation of a circle that has centre (p, q) and radius r is given by the equation:

$$(x-p)^2 + (y-q)^2 = r^2$$

WORKED EXAMPLE 1

a What is the equation of a circle that has centre $(-3, 4)$ and radius 5?

b Find the centre and radius of the circle $(x-6)^2 + (y+7)^2 = 9$.

Solutions

a The circle formula is $(x-p)^2 + (y-q)^2 = r^2$.
Here, (p, q) is $(-3, 4)$ and $r = 5$.
$\therefore (x--3)^2 + (y-4)^2 = 5^2$
The equation of the circle is $(x+3)^2 + (y-4)^2 = 25$.

b $(x-6)^2 + (y+7)^2 = 9$ can be written as $(x-6)^2 + (y--7)^2 = 3^2$
This circle has centre $(6, -7)$ and radius 3.

WORKED EXAMPLE 2

Find the centre and radius of the circle $x^2 + y^2 + 6x - 2y + 6 = 0$, and sketch the graph.

Solution

Here, we write the equation in the form

$(x - p)^2 + (y - q)^2 = r^2$ by completing the squares.

$x^2 + y^2 + 6x - 2y + 6 = 0$

$x^2 + 6x + y^2 - 2y = -6$

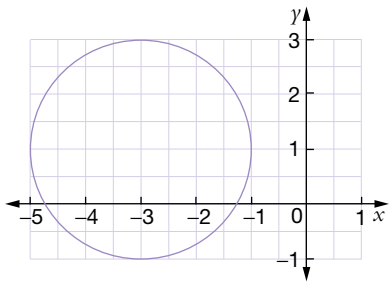
Completing each square.

$(x^2 + 6x + 9) + (y^2 - 2y + 1) = -6 + 9 + 1$

$(x + 3)^2 + (y - 1)^2 = 4$

$(x - -3)^2 + (y - 1)^2 = 2^2$

This circle has centre $(-3, 1)$ and radius 2.



Exercise 14:11

1 Find the equation of the following circles.

a centre $(1, 1)$, radius 7 units

b centre $(5, 0)$, radius 2 units

c centre $(-3, -5)$, radius 4 units

d centre $(2, -5)$, radius 1 unit

e centre $(0, 2)$, radius $\frac{1}{2}$ unit

f centre $(0, 0)$, radius 3 units

2 Find the centre and radius of each of the following circles.

a $(x - 2)^2 + (y - 3)^2 = 64$

b $(x + 4)^2 + (y - 1)^2 = 4$

c $(x + 3)^2 + (y + 3)^2 = 9$

d $(x - 6)^2 + (y - 5)^2 = 100$

e $x^2 + (y + 5)^2 = 16$

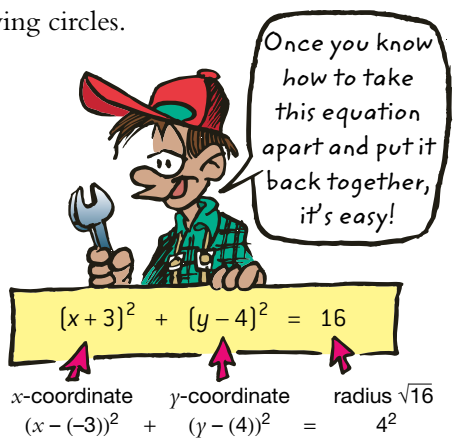
f $(x - 3)^2 + y^2 = 1$

g $x^2 + y^2 = 81$

h $x^2 + y^2 = 49$

i $x^2 + y^2 = 11$

j $(x - 7)^2 + (y - 8)^2 = 2$



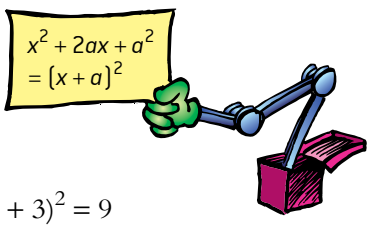
3 Find the centre and radius of each of the following circles.

a $x^2 - 10x + y^2 + 8y + 32 = 0$

b $x^2 + y^2 + 8x - 14y = 35$

c $x^2 + y^2 - 18x - 20y + 60 = 0$

d $x^2 + y^2 - 9x + 13\frac{1}{4} = 0$



4 On the one number plane, draw the graphs of:

a $x^2 + y^2 = 4$

b $(x + 3)^2 + (y + 3)^2 = 9$

c $(x - 4)^2 + y^2 = 1$

d $(x - 1)^2 + (y - 1)^2 = 16$

5 Find the x- and y-intercepts of these circles.

a $x^2 + (y - 2)^2 = 9$

b $(x - 2)^2 + (y - 1)^2 = 16$

6 **a** A circle with its centre at (3, 4) passes through the origin. What is its equation?

b Which of the following represents the equation of a circle?

i $x^2 - y^2 = 16$ **ii** $4x^2 + 4y^2 = 8$ **iii** $6x^2 + 3y^2 = 6$

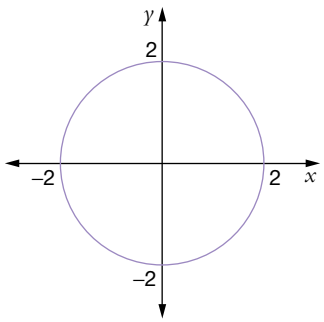
7 The circle $x^2 + y^2 = 4$ is translated to new positions. What is the equation of the circle if it is translated:

a (horizontally) 2 units to the right

b (horizontally) 2 units to the left

c (vertically) 2 units up

d (vertically) 2 units down?



8 What is the equation of the circle that results from translating the circle $(x - 2)^2 + (y + 2)^2 = 4$ 2 units to the right and then 1 unit down?

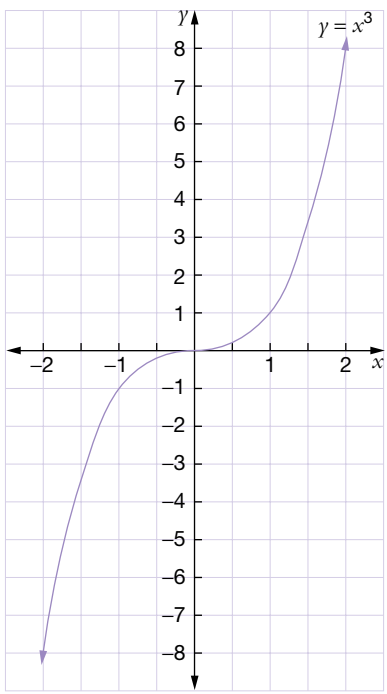
14:12 Curves of the form $y = ax^3$ and $y = ax^3 + d$

Curves of the form $y = ax^3 + d$ are called *cubics* because of the x^3 term. The simplest cubic graph is $y = x^3$, which occurs when $a = 1$ and $d = 0$.

As with other graphs, a table of values can be used to plot the points on the curve.

$y = x^3$

x	–2	–1·5	–1	–0·5	0	0·5	1	1·5	2
y	–8	–3·4	–1	–0·1	0	0·1	1	3·4	8



Features of $y = x^3$:

- It is an increasing curve.
- As x increases, the value of x^3, and hence y, becomes large very quickly. This means it is difficult to fit the points on a graph.
- When x is positive, x^3, and hence y, is positive.
  When x is negative, x^3, and hence y, is negative.
  When x is zero, x^3 is zero.

In this section, the relationship between the curve $y = x^3$ and the curve $y = ax^3 + d$ for various values of a and d will be investigated.

Exercise 14:12

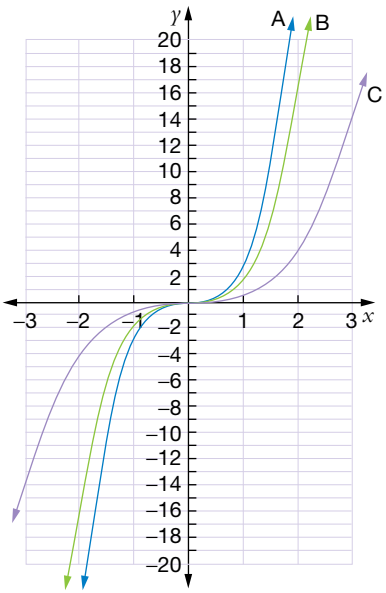
1 **a** Match each of the equations below with the graphs A, B and C.

- **i** $y = 2x^3$
- **ii** $y = \frac{1}{2}x^3$
- **iii** $y = 3x^3$

b Which graph increases the fastest? (Which is the steepest?)

c Which graph increases the slowest?

d How can you tell which graph is the steepest by looking just at the equations?



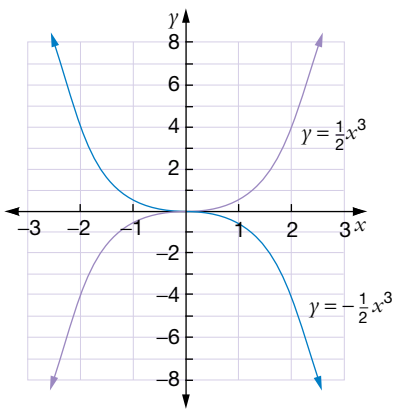
2 State which curve in each pair is steeper.

a $y = x^3$ or $y = 3x^3$ **b** $y = x^3$ or $y = \frac{1}{2}x^3$ **c** $y = 2x^3$ or $y = 3x^3$

3 The graphs of $y = \frac{1}{2}x^3$ and $y = -\frac{1}{2}x^3$ are shown.

a How are the graphs related?

b What is the effect on $y = ax^3$ of the sign of a?



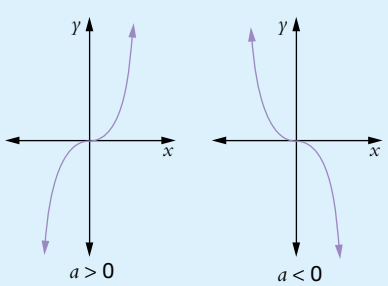
4 From your results so far, you should have noticed that all the curves are either *decreasing* or *increasing*. Without sketching, state whether the following are increasing or decreasing.

a $y = 4x^3$ **b** $y = -10x^3$ **c** $y = 0{\cdot}25x^3$

d $y = \frac{1}{5}x^3$ **e** $y = -\frac{1}{3}x^3$ **f** $y = -\frac{x^3}{5}$

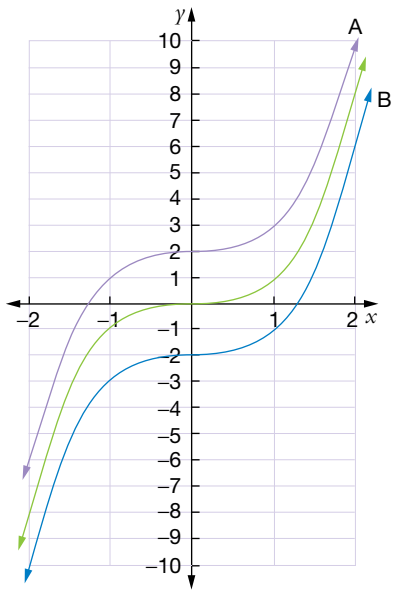
For the curve $y = ax^3$:

- the curve is *increasing* if $a > 0$
- the curve is *decreasing* if $a < 0$
- the larger the value of a, the steeper the curve
- $y = -ax^3$ is a reflection of $y = ax^3$ in the x-axis.



5 **a** Copy and complete the tables of values for the three curves $y = x^3$, $y = x^3 + 2$ and $y = x^3 - 2$.

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$x^3 + 2$					
$x^3 - 2$					



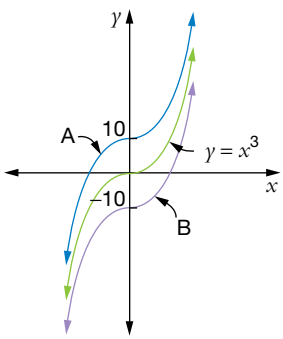
b What is the equation of curves A and B?

c How is the graph of $y = x^3 + 2$ related to the graph of $y = x^3$?

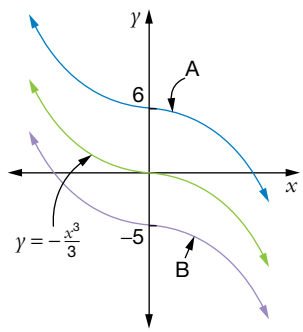
d How is the graph of $y = x^3 - 2$ related to the graph of $y = x^3$?

6 In each diagram, the two curves A and B were obtained by moving the other curve up or down. Give the equations of the curves A and B.

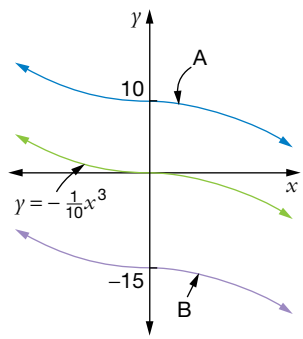
a



b



c



7 Sketch each pair of graphs on the same number plane.

a $y = 2x^3$
$y = 2x^3 - 2$

b $y = -x^3$
$y = -x^3 + 2$

c $y = \frac{1}{4}x^3$
$y = \frac{1}{4}x^3 + 4$

8 Sketch each pair of graphs on the same number plane.

a $y = x^3 + 1$
$y = -x^3 + 1$

b $y = 2x^3 + 1$
$y = x^3 + 1$

9 From the list of equations, write the letter or letters corresponding to the equations of the curves:

a that can be obtained by moving $y = x^3$ up or down

b that are the same shape as **A**

c that are decreasing

d that pass through $(0, 0)$

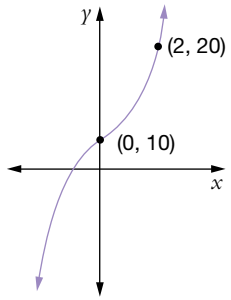
e that can be obtained from the curve $y = -\frac{1}{3}x^3$ by reflection in the y-axis

f that have the largest y-intercepts.

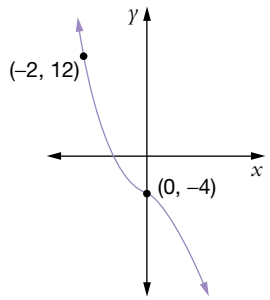
A $y = 2x^3$
B $y = x^3 - 1$
C $y = 3 - \frac{1}{3}x^3$
D $y = 2x^3 - 1$
E $y = 3 + 2x^3$
F $y = -\frac{1}{3}x^3$
G $y = \frac{1}{3}x^3$
H $y = 2 + x^3$
I $y = -\frac{1}{3}x^3 - 2$

10 Given that each of the graphs is of the form $y = ax^3 + d$, find its equation.

a



b



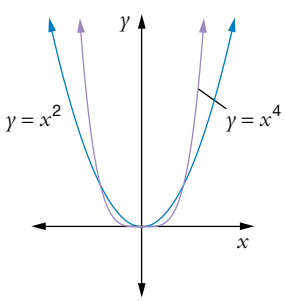
The graph of $y = ax^3 + d$ can be obtained from the curve $y = ax^3$ by moving it:

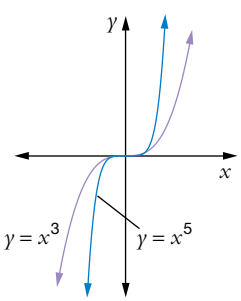
- up d units if d is positive
- down d units if d is negative.

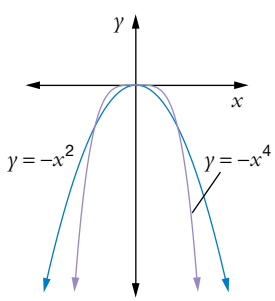
14:13 Curves of the form $y = ax^n$ and $y = ax^n + d$

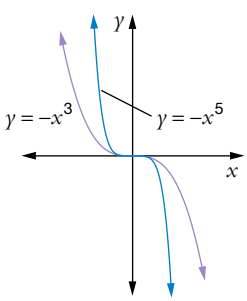
Curves of the form $y = ax^n$

Curves of the form $y = ax^n$ include these basic shapes. You have already graphed the parabolas and cubics.









Graphing curves of the form $y = ax^n$ is simple if you recognise the relationship between the signs of x and y for different values of a and n.

To find the sign of y when $y = ax^n$ we need to realise that:

- if n is even: x^n is always positive (except for $x = 0$)
- if n is odd: x^n is positive when x is positive and
  x^n is negative when x is negative
- the signs of a and x^n will then determine the sign of y
- as x becomes smaller, ax^n (and therefore y) becomes smaller
- as x becomes larger, ax^n (and therefore y) becomes larger
- if $x = 0$, $y = 0$; and if $y = 0$, $x = 0$. Hence, the curve only crosses the x-axis once, at the point $(0, 0)$.

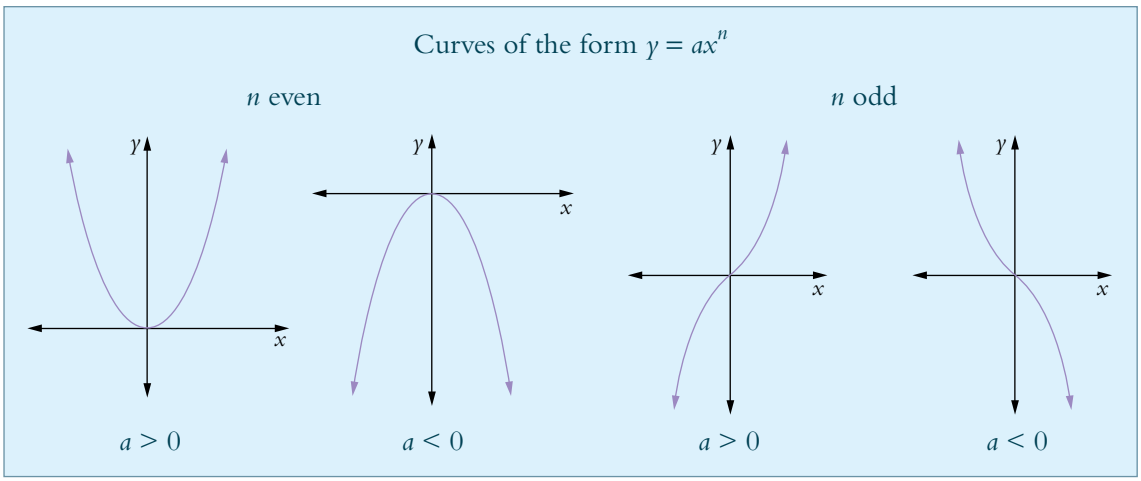
When considering whether a number is large or small, we are concerned only with its size and not its sign.

Numbers close to zero are small in size.

Numbers far from zero are large in size.

From this information we can deduce the following.

- Curves with an even value of n will have shapes like parabolas ($y = ax^2$).
- Curves with an odd value of n will have shapes like cubics ($y = ax^3$).



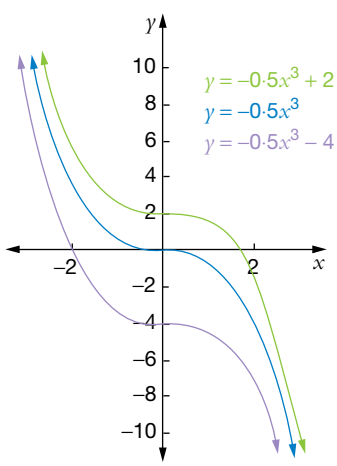
- As n increases, the steepness of the curve increases, e.g. $y = x^4$ will be steeper than $y = x^2$. The curve with the higher value of n will be above the other curve for all values of x except from -1 to 1.
- As we have already seen in 14:12 for the curve $y = ax^3$, changing a changes the 'steepness' of the curve. Hence, for the same value of n, a larger value of a will result in a steeper curve. e.g. $y = 2x^5$ will be steeper than $y = x^5$.

Curves of the form $y = ax^n + d$

Curves of the form $y = ax^n + d$ can be obtained by translating the curve $y = ax^n$ up or down.

- If d is positive, translate it up d units.
- If d is negative, translate it down d units.

This is illustrated in the diagram on the right for the curves $y = -0{\cdot}5x^3$, $y = -0{\cdot}5x^3 + 2$ and $y = -0{\cdot}5x^3 - 4$.



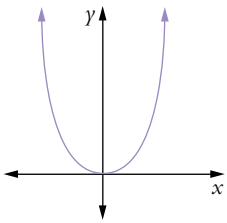
WORKED EXAMPLES

Sketch each of the following curves.

1 $y = \frac{1}{2}x^4$ **2** $y = -2x^5$ **3** $y = -1 - 2x^6$

Solutions

1

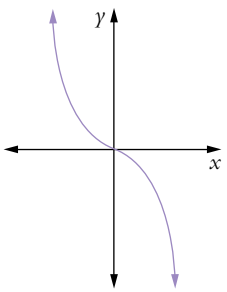


As $\frac{1}{2}x^4$ is positive for all x (except $x = 0$), then y is positive (except when $x = 0$).

When x is small (close to 0), y is small.

As x becomes larger (moves away from 0), $\frac{1}{2}x^4$, and hence y, becomes larger.

2



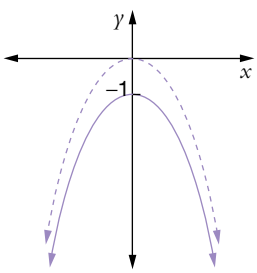
When x is positive, $-2x^5$ is negative and y is negative.

When x is negative, $-2x^5$ is positive and y is positive.

When x is small (close to 0), y is small.

As x becomes larger, y becomes larger.

3



$y = -1 - 2x^6$

$\therefore\ y = -2x^6 - 1$

This is the curve $y = -2x^6$ (shown as a dotted line) moved down 1 unit to get $y = -2x^6 - 1$.

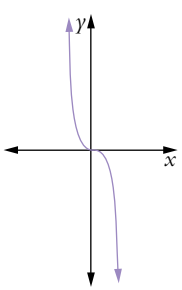
Exercise 14:13

P Foundation worksheet 14:13
Curves of the form $y = ax^n$ and $y = ax^n + d$

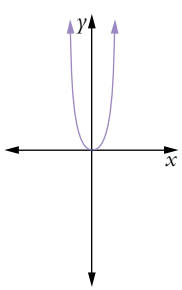
1 Choose which of the curves A, B, C or D below is the best sketch for each of the following.

a $y = x^4$ **b** $y = -x^5$ **c** $y = 2x^7$ **d** $y = -3x^6$ **e** $y = 4x^4$

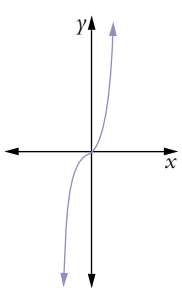
f $y = -4x^5$ **g** $y = \frac{x^5}{10}$ **h** $y = -\frac{1}{10}x^7$ **i** $y = -2x^8$



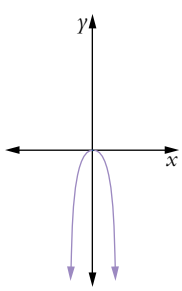
A



B



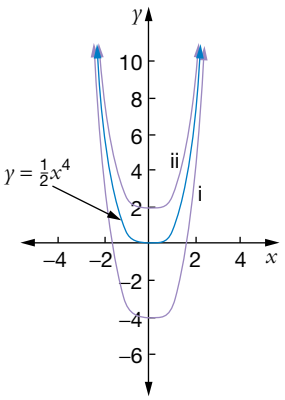
C



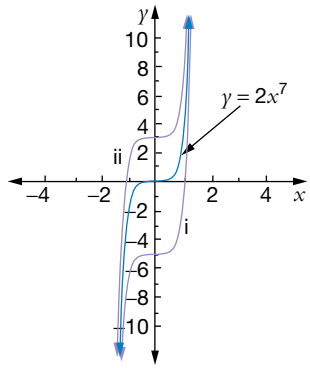
D

2 In each diagram, the two curves labelled **i** and **ii** were obtained by moving the other curve up or down. Give the equations of the curves.

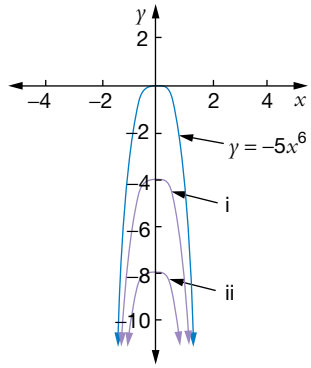
a



b

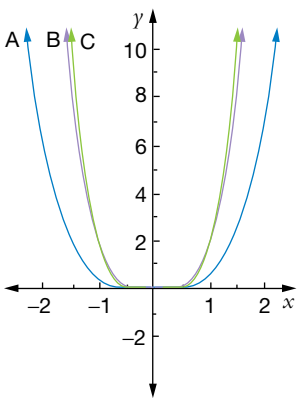


c



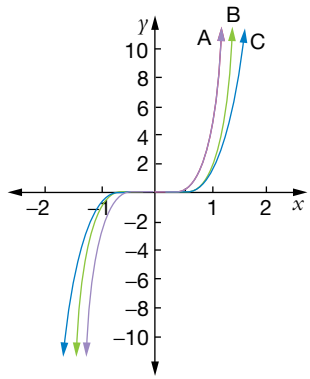
3 In each of the following, match the curves A, B and C with the equations below.

a



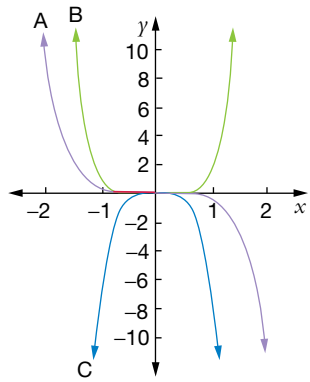
i $y = 2x^4$
ii $y = 0{\cdot}5x^4$
iii $y = 2x^6$

b



i $y = x^5$
ii $y = x^7$
iii $y = 4x^5$

c



i $y = -5x^4$
ii $y = x^6$
iii $y = -0{\cdot}25x^5$

4 Sketch the following on separate number planes.

a $y = \frac{1}{2}x^5$
b $y = 2x^4$
c $y = -2x^6$
d $y = x^5 - 1$
e $y = x^6 + 1$
f $y = -2x^4 + 4$

When graphing $y = ax^n + d$, look first at n, then a, then d.

5 Sketch each pair of graphs on the same number plane.

a $y = x^4$, $y = x^6$
b $y = x^4$, $y = -x^4$
c $y = x^5$, $y = 1 + x^5$
d $y = x^6 + 1$, $y = x^6 - 1$
e $y = 2x^5 + 1$, $y = 2x^3 + 1$
f $y = 1 - x^6$, $y = -1 - x^6$

6 **a** How do each of the numbers in the equation $y = -2x^4 + 1$ determine the features of the curve?

b Describe the shape and features of the curve $y = 2x^3 - 3$.

c What is the relationship between the curves $y = 2x^5$ and $y = -2x^5$?

7 From the curves of the form $y = ax^n$, sketch a possible curve, given that:

a a is positive and n is even

b a is negative and n is even

c a is positive and n is odd

d a is negative and n is odd.

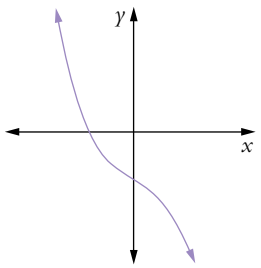
8 From the curves of the form $y = ax^n + d$, sketch a possible curve, given that:

a a is positive, n is odd and d is negative

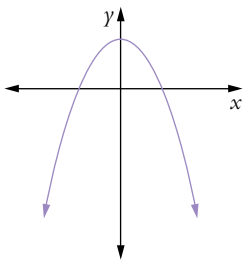
b a is negative, n is even and d is positive

9 Each of the curves below has the form $y = ax^n + d$. For each curve, state the signs of a and d, and the value of n.

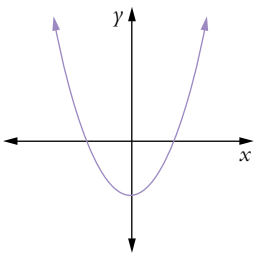
a



b



c

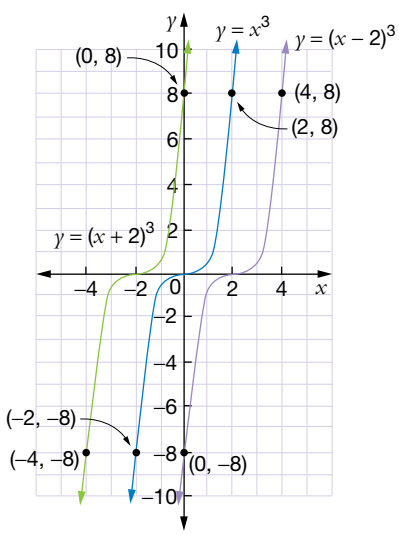


14:14 Curves of the form $y = ax^n$ and $y = a(x - r)^n$

The diagram shows the graphs of:
$y = x^3$, $y = (x - 2)^3$ and $y = (x + 2)^3$
The curves are all the same shape.

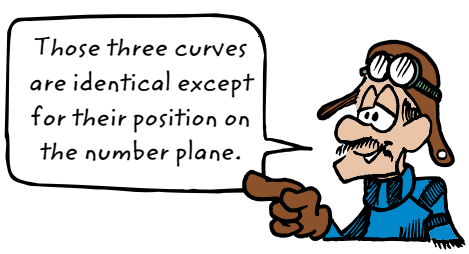
- If $y = x^3$ is translated 2 units to the right the curve $y = (x - 2)^3$ is obtained.
- If $y = x^3$ is translated 2 units to the left the curve $y = (x + 2)^3$ is obtained.

The above case is an example of the general result.



If the curve $y = ax^n$ is moved (horizontally):

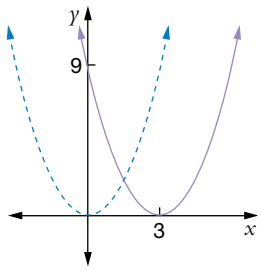
- r units to the right, the equation of the new curve is $y = a(x - r)^n$
- r units to the left, the equation of the new curve is $y = a(x + r)^n$.



WORKED EXAMPLES

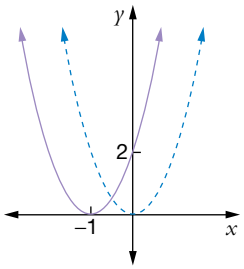
Sketch each of the following curves.

1 $y = (x - 3)^2$



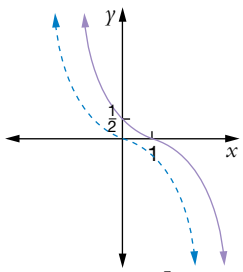
$y = (x - 3)^2$ has been produced by moving the curve $y = x^2$ 3 units to the right.

2 $y = 2(x + 1)^4$



$y = 2(x + 1)^4$ has been produced by moving the curve $y = 2x^4$ 1 unit to the left.

3 $y = -\frac{1}{2}(x - 1)^5$



$y = -\frac{1}{2}(x - 1)^5$ has been produced by moving the curve $y = -\frac{1}{2}x^5$ 1 unit to the right.

You must know how to graph $y = ax^n$ before you can graph $y = a(x - r)^n$ or $y = a(x + r)^n$.

Exercise 14:14

P Foundation worksheet 14:14
Curves of the form $y = ax^n$ and $y = a(x - r)^n$

1

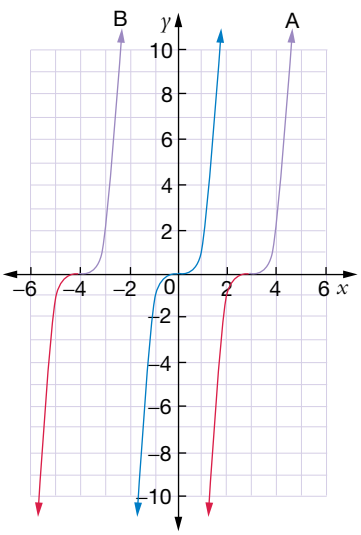


Diagram 1

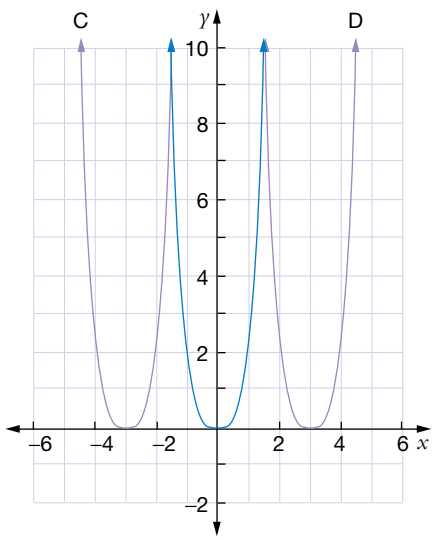


Diagram 2

a The blue curve in Diagram 1 is $y = x^5$. It has been translated horizontally to produce curves A and B. What are their equations?

b The blue curve in Diagram 2 is $y = 2x^4$. What curve has the equation $y = 2(x - 3)^4$?

2 Write the equation of the curve that is obtained by translating the curve $y = x^4$ (horizontally):

a 4 units to the left **b** 4 units to the right **c** 1 unit to the right **d** $\frac{1}{2}$ unit to the left.

3 Describe how $y = -2x^5$ must be translated to produce curves with the equations:

a $y = -2(x - 1)^5$ **b** $y = -2(x + 3)^5$ **c** $y = -2(x - \frac{1}{2})^5$

4 Make sketches on separate number planes of:

a $y = -(x + 1)^4$ **b** $y = (x + 1)^4$ **c** $y = (x - 1)^4$ **d** $y = -(x - 1)^4$

5 Sketch on separate number planes the following curves.

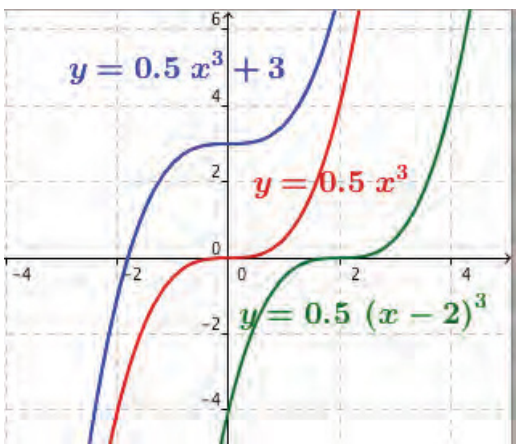
a $y = 3(x + 2)^2$ **b** $y = 2(x - 3)^5$ **c** $y = -2(x - 4)^4$ **d** $y = -(x + 1)^6$

6 Make sketches of each pair of curves on the same number plane.

a $y = -(x - 1)^2$, $y = (x + 1)^2$ **b** $y = (x - 3)^3$, $y = -(x - 3)^3$ **c** $y = 2(x - 3)^3$, $y = 2(x - 3)^5$ **d** $y = 2(x + 1)^2$, $y = 2(x + 1)^4$

P GEOGEBRA ACTIVITY 14:14 CURVES INVOLVING POWERS

Use the checkboxes and the sliders to investigate the effect that changing the values of a, n, k and r has on curves of the form $y = ax^n$, $y = ax^n + k$ and $y = a(x - r)^n$. Then you can test your knowledge by finding the equation of a randomly generated curve.



14:15 Curves of the form $y = a(x - r)(x - s)(x - t)$

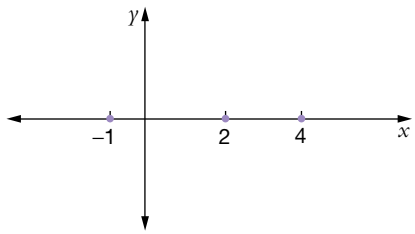
In the work so far, the sign analysis for x and y has been of great importance. In this section, it is also important, though not as simple.

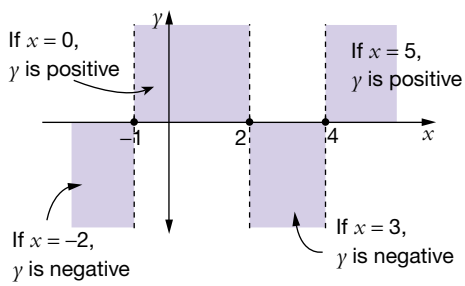
Discussion

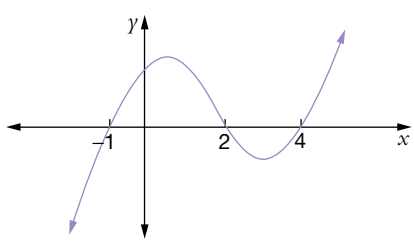
Consider the example $y = 3(x - 2)(x + 1)(x - 4)$.

- Now, x can take any value and for every x value there is a corresponding y-value. Hence, the curve has no gaps.
- First, find the x–intercepts of the curve.
  If $y = 0$ then:
  $3(x - 2)(x + 1)(x - 4) = 0$
  $\therefore\ x - 2 = 0$ or $x + 1 = 0$ or $x - 4 = 0$
  $\therefore\ x = 2$ or -1 or 4
  Plot these x–intercepts.
- Now, for all the other values of x, y is either positive or negative. The three x-intercepts divide the x-axis into four sections. In each of these sections the y values will be either positive or negative. Testing a point in each of the sections will determine whether that section is positive or negative.
- Also, as x increases past 4, y increases, and as x decreases below -1, y decreases.
- Putting all the information together allows us to sketch the curve.

Testing a value:
When $x = 5$, $y = 3(+)(+)(+)$ positive
When $x = 3$, $y = 3(+)(+)(-)$ negative







$y = 3(x - 2)(x + 1)(x - 4)d$

These ideas are explored further in the following examples.

WORKED EXAMPLES

Make sketches of the following curves.

1 $y = 2(x - 1)(x - 4)(x + 2)$

2 $y = -2(x - 1)(x - 4)(x + 2)$

3 $y = (x - 1)^2(x - 4)$

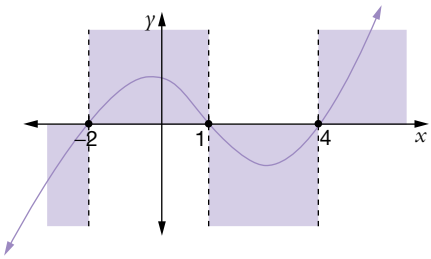
Solutions

1 $y = 2(x - 1)(x - 4)(x + 2)$

- The curve has x-intercepts at $x = 1$, $x = 4$ and $x = -2$.
- Testing values of x shows that the curve is in the coloured regions.

x	−3	−2	0	1	3	4	5
sign of y	(−)	0	(+)	0	(−)	0	(+)

- Considering the size of y when x is greater than 4 and less than −2 gives the correct shape.

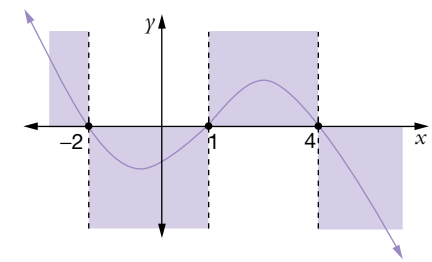


2 $y = -2(x - 1)(x - 4)(x + 2)$

- The curve has x-intercepts at $x = 1$, $x = 4$ and $x = -2$.
- Testing values of x shows that the curve is in the coloured regions.

x	−3	−2	0	1	3	4	5
sign of y	(+)	0	(−)	0	(+)	0	(−)

- Considering the size of y when x is greater than 4 and less than −2 gives the correct shape.

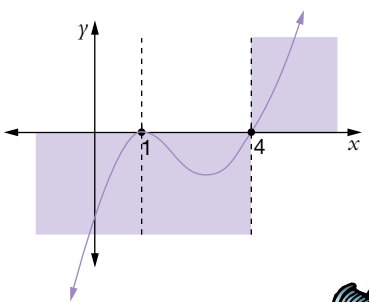


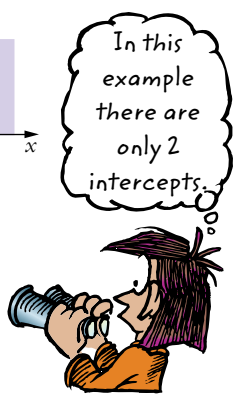
3 $y = (x - 1)^2(x - 4)$

- x–intercepts occur at $x = 1$ and $x = 4$.
- Testing values shows that the curve is in the coloured regions.

x	0	1	2	4	5
sign of y	(−)	0	(−)	0	(+)

- Considering the size of y when x is greater than 4 and less than 1 gives the correct shape.





Exercise 14:15

P Foundation worksheet 14:15
Curves of the form $y = a(x - r)(x - s)(x - t)$

1 Complete the sign analysis tables for each of the given curves.

a $y = 2(x + 4)(x + 2)(x + 1)$

x	−5	−4	−3	−2	−1·5	−1	0
y		0		0		0	

b $y = -4(x + 2)(x - 2)(x - 5)$

x	−3	−2	0	2	4	5	6
y		0		0		0	

c $y = \frac{1}{2}(x - 1)^2(x - 4)$

x	0	1	2	4	5
y		0		0	

d $y = -(x + 1)(x - 2)^2$

x	−2	−1	0	2	3
y		0		0	

2 Find the x-intercepts for each of the following curves.

a $y = (x - 1)(x - 2)(x - 4)$
b $y = (1 - x)(x + 2)(4 - x)$
c $y = (x - 2)^2(x + 5)$
d $y = 2x(x + 4)(x - 2)$

To find where a curve crosses the x-axis (i.e. x-intercepts), substitute in $y = 0$.

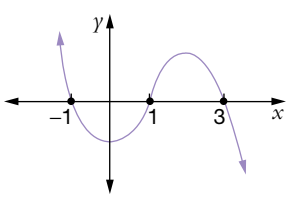
3 Make sketches of the following curves on separate number planes.

a $y = (x - 2)(x + 2)(x - 4)$
b $y = -(x - 2)(x + 2)(x - 4)$
c $y = 2(x - 1)(x - 3)^2$
d $y = -2(x - 1)(x - 3)^2$

4 **a** Find the y-intercepts of the curves in Question 3.
b How can the y-intercepts be used to check your sketches?

5 Write a possible equation for each of the curves shown.

a



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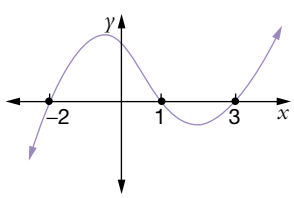

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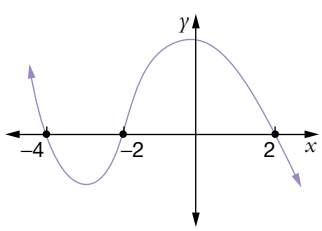

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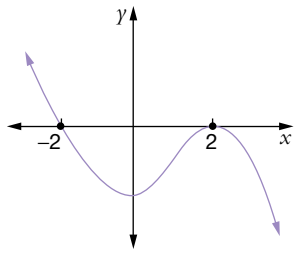

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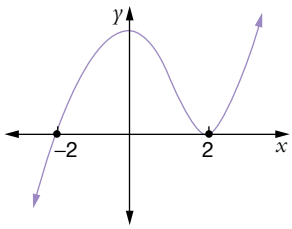

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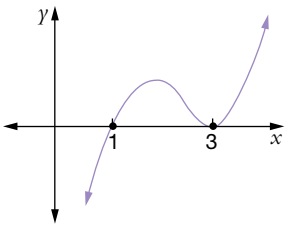


6 What additional information would have been needed in Question **5** to find the actual equation of the curve?

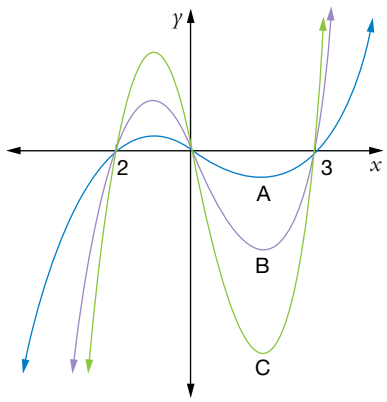
7 **a** Match each of the curves to the following equations.

i $y = x(x - 3)(x + 2)$

ii $y = 2x(x - 3)(x + 2)$

iii $y = 0{\cdot}25x(x - 3)(x + 2)$

b On the same number plane, sketch the curves $y = -x(x - 3)(x + 2)$ and $y = -2x(x - 3)(x + 2)$



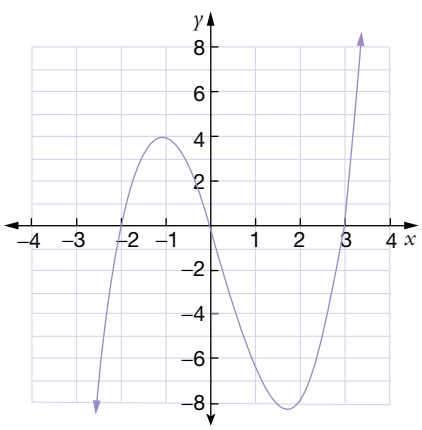
8 The equation of the curve shown is $y = x(x + 2)(x - 3)$. Give the equation of the curve that would result if this curve was:

a translated 1 unit to the right

b translated 2 units to the left

c reflected in the x-axis

d reflected in the y-axis.



9 **a** A cubic curve is known to have x-intercepts of -1, 1 and 4 and y-intercept of 8. What is its equation?

b A cubic curve has x-intercepts of 2 and -2 and passes through the point $(3, -10)$. What is its equation?

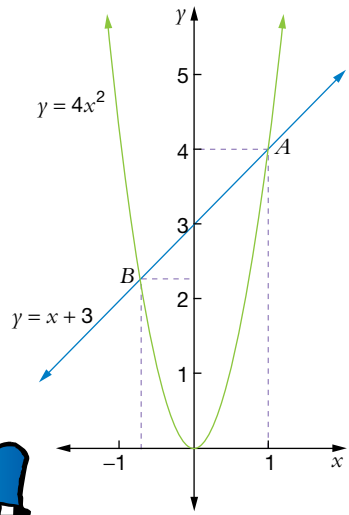
10 Sketch the graph of $y = (x - r)(x - s)(x - t)$ if $r = s = t$.

14:16 The intersection of graphs

In this section, both graphical and algebraic methods will be used to find the point or points of intersection of a line with a parabola, circle or hyperbola.

Graphical method

- In the diagram, the points of intersection of the parabola $y = 4x^2$ and the line $y = x + 3$ are A and B. It would appear from the graph that: A is the point $(1, 4)$ and B is the point $(-0{\cdot}7, 2{\cdot}2)$.
- However, the only way to be certain that these are the exact points of intersection is to substitute the coordinates into the original equations and see if true statements result.



Check $A(1, 4)$:

For $y = 4x^2$ and $y = x + 3$

$4 = 4 \times 1^2$ $\quad$ $4 = 1 + 3$

Both of these statements are true.

$\therefore$ $A(1, 4)$ is the exact point of intersection.

Check $B(-0{\cdot}7, 2{\cdot}2)$:

For $y = 4x^2$ and $y = x + 3$

$2{\cdot}2 = 4 \times (-0{\cdot}7)^2$ $\quad$ $2{\cdot}2 = -0{\cdot}7 + 3$

Both of these statements are false.

$\therefore$ $B(-0{\cdot}7, 2{\cdot}2)$ only approximates the point of intersection. The accuracy of the estimate can be improved by trial and error.

Even though the graphical method is slow and often cannot give us an exact solution, it is still a valuable technique.

To find points of intersection graphically:

1. draw the graphs
2. read off coordinates
3. check the solutions by substitution.

Algebraic method

This method is based on the fact that at a point of intersection the x-coordinates on both curves are equal and the y-coordinates on both curves are equal.

So, for the curves $y = 4x^2$ and $y = x + 3$:

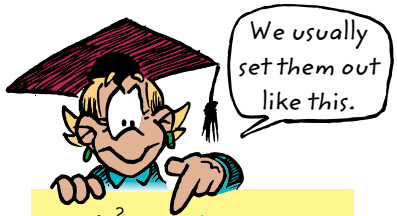
If the y–coordinates are equal, then:

$$4x^2 = x + 3$$
$$4x^2 - x - 3 = 0$$
$$(4x + 3)(x - 1) = 0$$
$$x = -\tfrac{3}{4} \text{ or } 1$$

Substitution of these values into either $y = 4x^2$ or $y = x + 3$ gives:

$y = 4$ (when $x = 1$)

$y = 2\frac{1}{4}$ (when $x = -\frac{3}{4}$)



$y = 4x^2$ 1

$y = x + 3$ 2

Substitute 1 in 2

$4x^2 = x + 3$

$4x^2 - x - 3 = 0$

$(4x + 3)(x - 1) = 0$

$x = \frac{-3}{4}$ or 1

Substitute in 2

When $x = \frac{-3}{4}$, $y = 2\frac{1}{4}$

$x = 1$, $y = 4$

$\therefore$ Points of intersection are:

$(1, 4)$ and $(-\frac{3}{4}, 2\frac{1}{4})$

∴ Points of intersection are:

$(1, 4)$ and $(-\frac{3}{4}, 2\frac{1}{4})$.

These should be checked by substituting each point into both equations.

The points of intersection can also be found using the fact that the x-coordinates are equal, but in this case it requires more work.

$y = 4x^2$ 1

$y = x + 3$ 2

Rearranging these equations gives:

$x^2 = \frac{y}{4}$ 3

$x = y - 3$ 4

Substituting 4 into 3 gives:

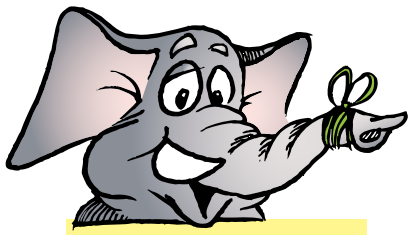
$$(y-3)^2 = \frac{y}{4}$$
$$4(y-3)^2 = y$$
$$4(y^2 - 6y + 9) = y$$
$$4y^2 - 24y + 36 = y$$
$$y^2 - 25y + 36 = 0$$
$$(4y - 9)(y - 4) = 0$$
$$y = \tfrac{9}{4} \text{ or } 4$$

Substituting in 1 gives:

When $y = \frac{9}{4}$, $x = \pm\frac{3}{4}$

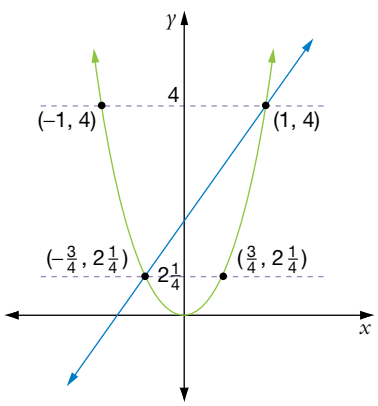
$y = 4$, $x = \pm 1$

Checking in both 1 and 2 shows that the points of intersection are $(1, 4)$ and $(-\frac{3}{4}, 2\frac{1}{4})$.



It is important to check all possible solutions in both of the original equations.

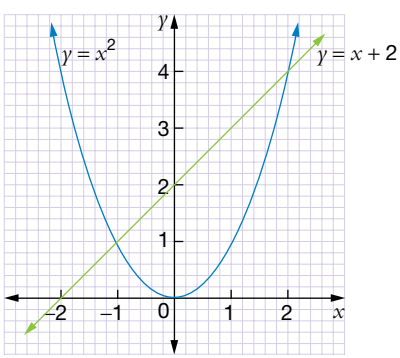
The sketch below shows that there are only two solutions.



Exercise 14:16

1 Find the points of intersection of the graphs in each part. Check that the coordinates of each point of intersection satisfy both equations.

a



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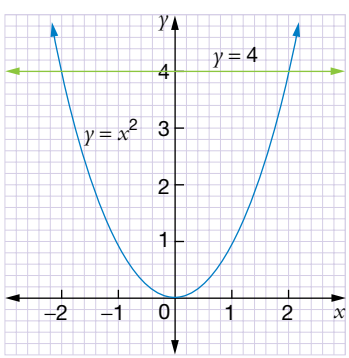

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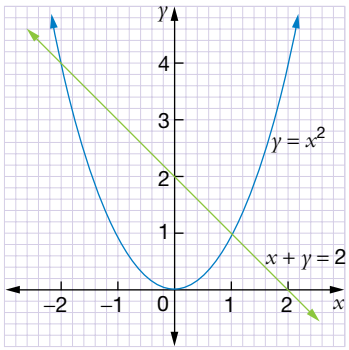

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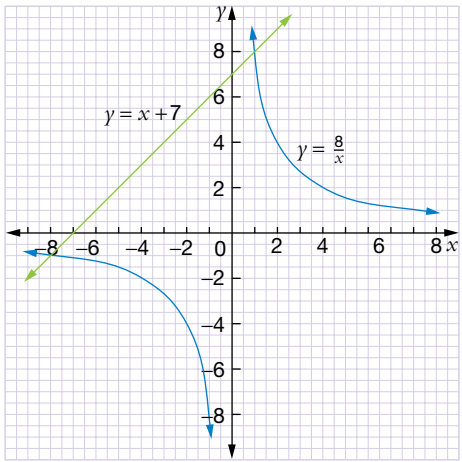

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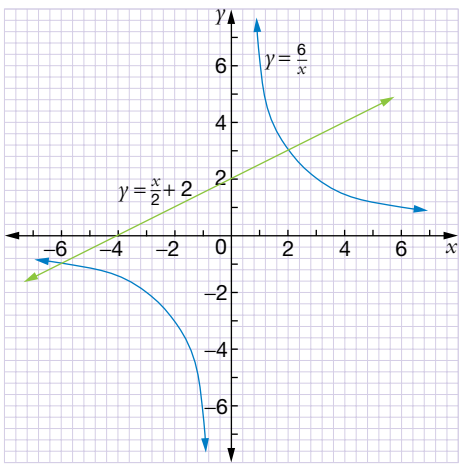

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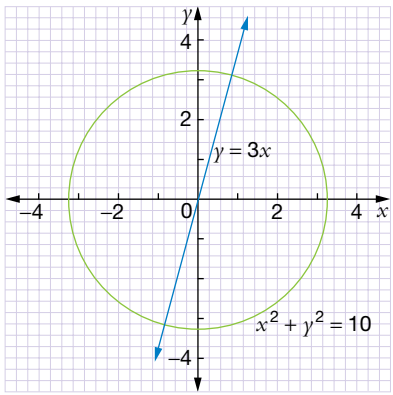
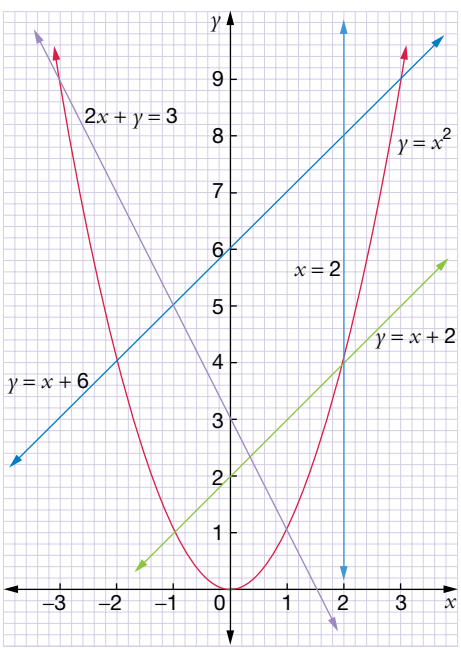


2



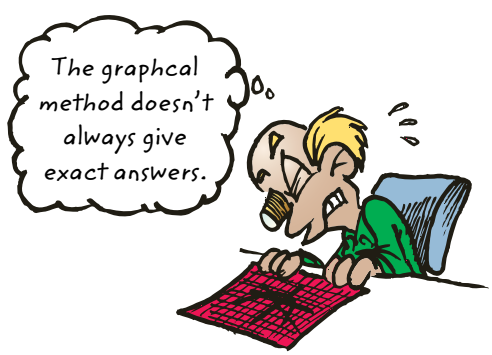
Use this diagram to find the point or points of intersection of the parabola $y = x^2$ with the line:

a $y = x + 6$

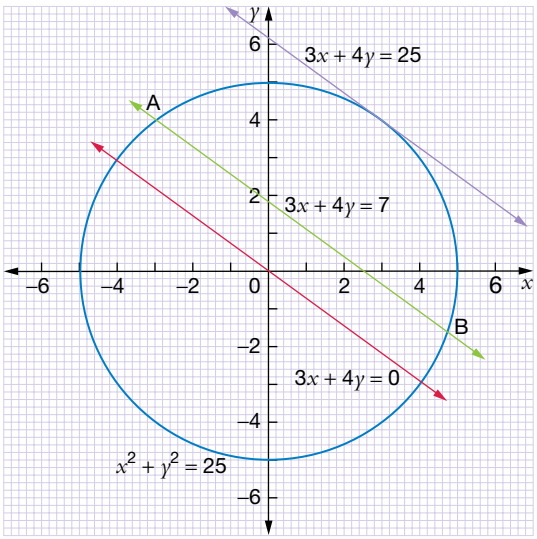
b $x = 2$

c $y = x + 2$

d $2x + y = 3$



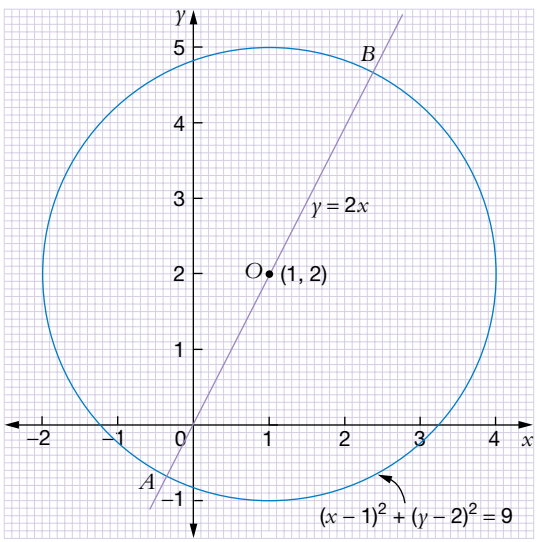
3 Use the diagram to answer the following questions.



- **a** What are the points of intersection of the circle $x^2 + y^2 = 25$ and the line $3x + 4y = 0$?
- **b** The line $3x + 4y = 25$ is a tangent to the circle $x^2 + y^2 = 25$. What is the point of intersection?
- **c** The line $3x + 4y = 7$ meets the circle at $A(-3, 4)$ and at B. Use the graph to estimate the coordinates of B.

4 In each, estimate the coordinates of the points of intersection, A and B.

a



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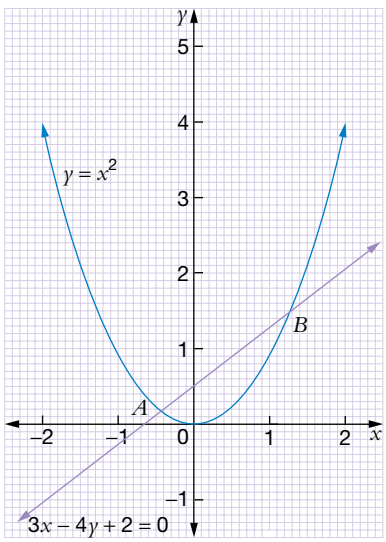


5 Find the points of intersection of the following pairs of equations using the graphical method.

- **a** $y = x^2$, $y = 2x$
- **b** $y = x^2$, $y - 2x = 3$
- **c** $y = x$, $y = \frac{1}{x}$
- **d** $y = x - 2$, $y = \frac{1}{x}$
- **e** $x^2 + y^2 = 4$, $y = x$
- **f** $x^2 + y^2 = 4$, $2x - y = 2$

6 Find the points of intersection of the equations in Question **5** using the algebraic method.

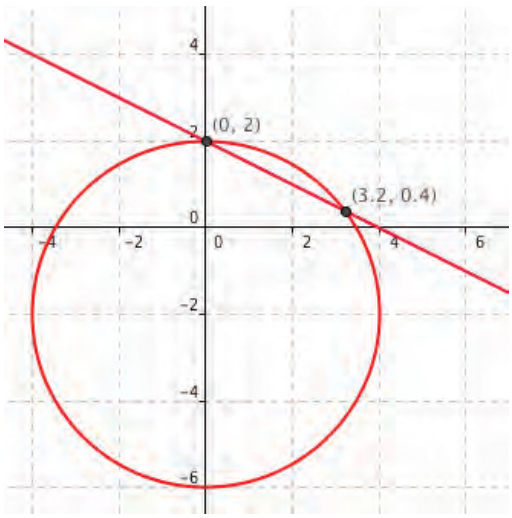
7 Find the points of intersection of the equations:

- **a** $y = x + 1$ and $y = x^2 - 1$
- **b** $xy = 2$ and $x + y = 4$

8 Use the algebraic method to find the coordinates of A and B in Question **4**.

P GEOGEBRA ACTIVITY 14:16 THE INTERSECTION OF GRAPHS

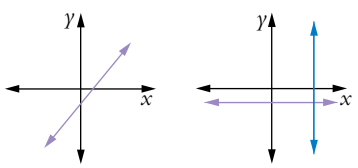
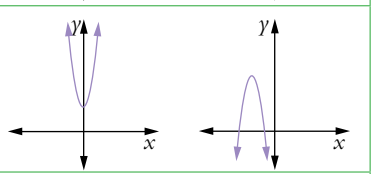
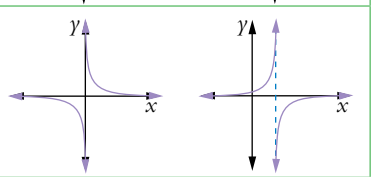
Use graphical means to find the point or points of intersection of a straight line with a parabola, hyperbola or circle. Sliders can be used to vary the equations and you can use a checkbox to test the accuracy of your solution.

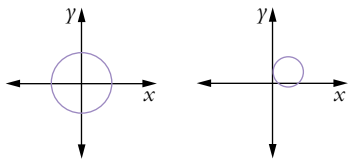
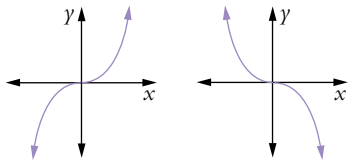
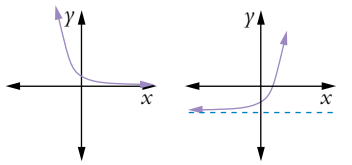
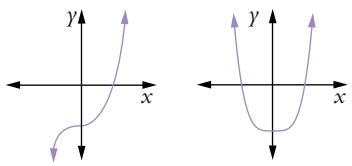


14:17 Miscellaneous equations and graphs

For all the graphs met in this chapter it is important that you can:
- identify different types of graphs from their equations
- determine a possible equation of a given graph.

Information about the features of graphs can be obtained from the equation. You need to know how to find these features from the equation before attempting the next exercise.

Type of Graph	Features	Graph
Straight line	• x- and y-intercepts • slope • horizontal and vertical lines	
Parabola	• x- and y-intercepts (when they exist) • vertex • axis of symmetry • concavity	
Hyperbola	• x- and y-intercepts (where they exist) • asymptotes • orientation (quadrants 1 and 3 or quadrants 2 and 4)	

Type of Graph	Features	Graph
Circle	• centre • radius	
Cubic	• basic shape (is it an increasing or decreasing curve?)	
Exponential	• x- and y-intercepts (where they exist) • concavity • increasing or decreasing	
$y = ax^n$	• x- and y-intercepts • basic shape (does it look like a parabola or a cubic?) • concavity	

Translations

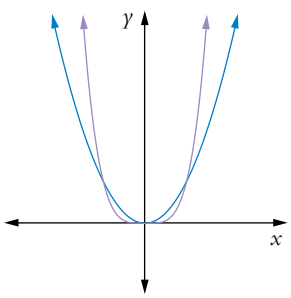
Terms in an equation indicate that the curve has been formed by translating a simpler curve in either a vertical or horizontal direction. For example:

- $y = ax^3 + d$ has been formed by translating $y = ax^3$, d units in a vertical direction.
- $y = (x - r)^2$ has been formed by translating $y = x^2$, r units in a horizontal direction.

Symmetry

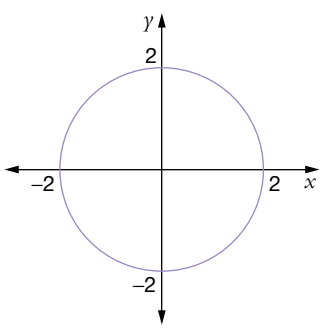
Graphs that contain an even power of x have an axis of symmetry. For example:

- $y = x^2$ and $y = x^4$



If the y-axis is an axis of symmetry then the equation will be identical when x is replaced by $-x$. For example:

- $x^2 + y^2 = 4$.



Exercise 14:17

1 From the list of equations given below, choose those that represent:

a a straight line **b** a circle **c** a parabola **d** a hyperbola **e** an exponential.

A $x^2 + y^2 = 4$ **B** $y = (x - 4)^2$ **C** $y = \dfrac{1}{x - 4}$ **D** $y = 4^x - 1$

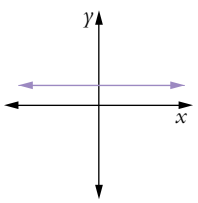
E $y = 4$ **F** $y = x - 4$ **G** $x = 4$ **H** $y = 4 - x^2$

I $y = \dfrac{4}{x}$ **J** $y = (x + 4)^2$ **K** $y = x^4$ **L** $(x - 4)^2 + y^2 = 4$

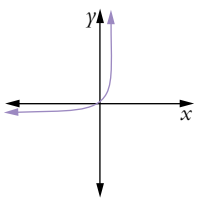
M $y = -x^4$ **N** $y = 4^{-x} + 1$ **O** $y = -\dfrac{1}{x} + 4$ **P** $y = \dfrac{1}{x} - 4$

2 Select the equation from the list in Question 1 that matches each of the graphs below.

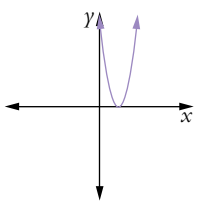
a



b



c



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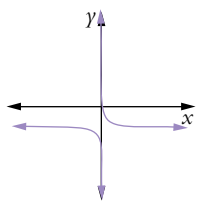

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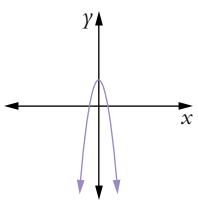

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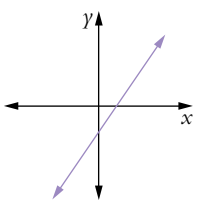

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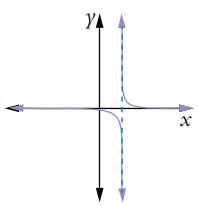

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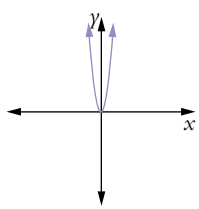

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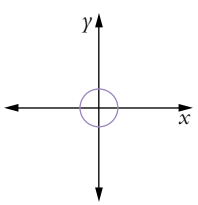

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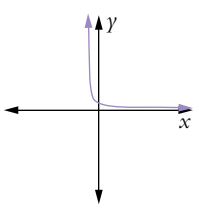

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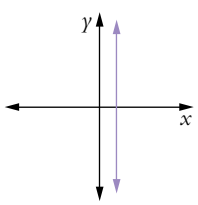

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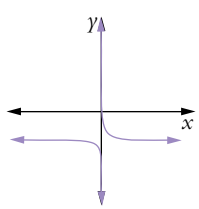

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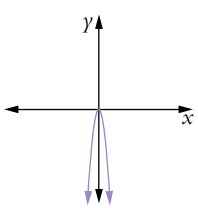

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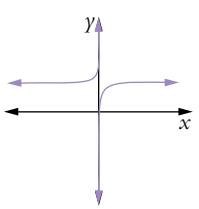

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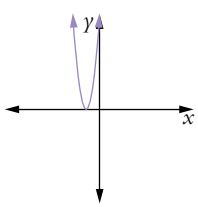

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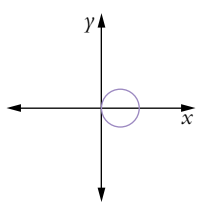
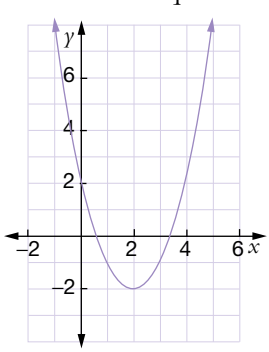


3 Determine the equation of each graph.

a



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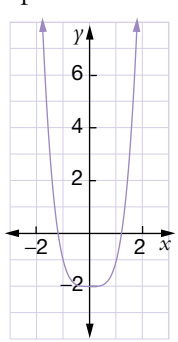

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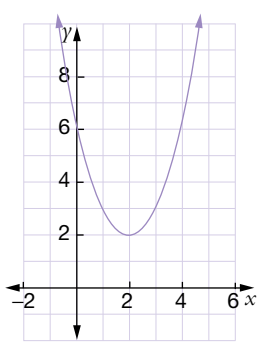

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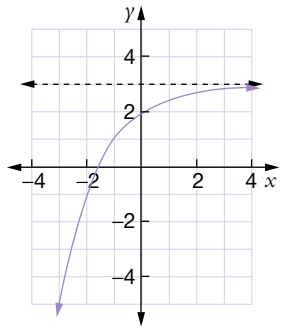

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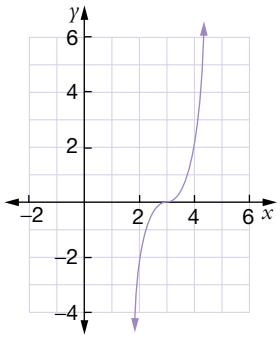

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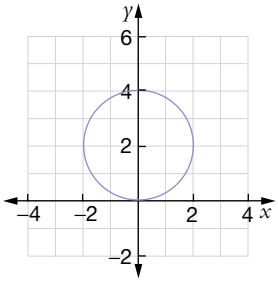

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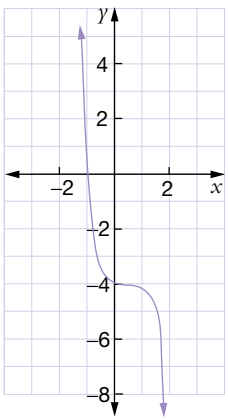

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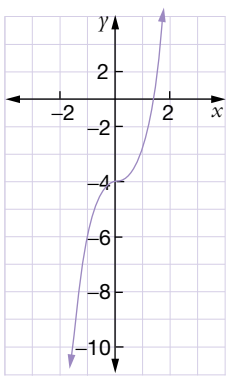

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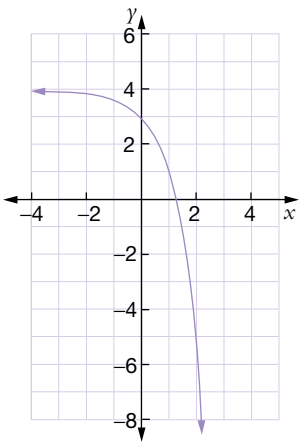

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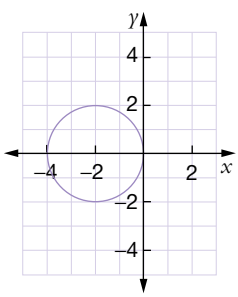

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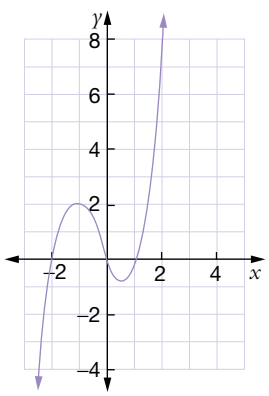

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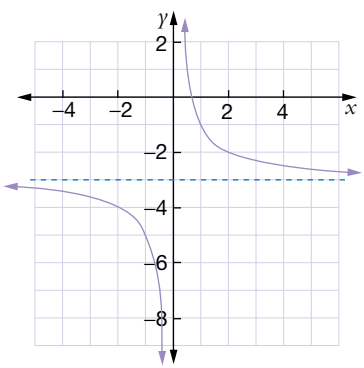

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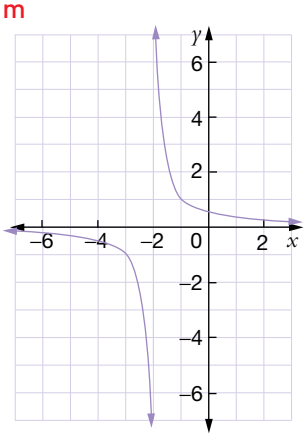

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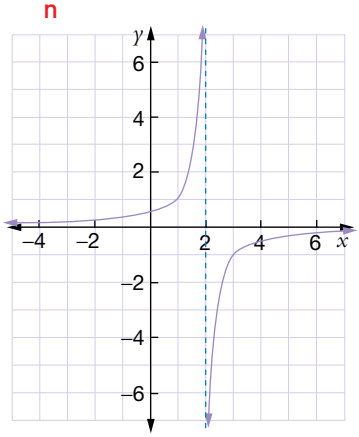

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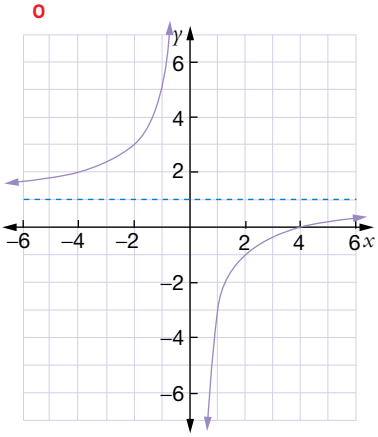
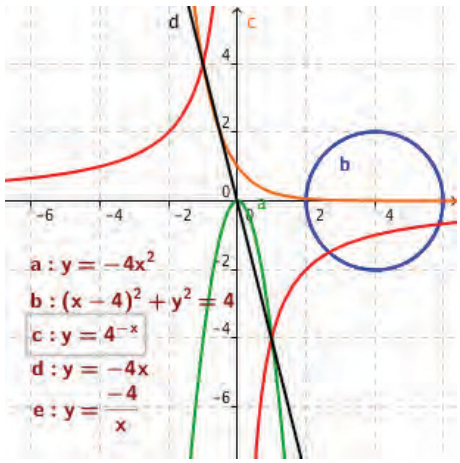


GEOGEBRA ACTIVITY 14:17 CURVES AND THEIR EQUATIONS

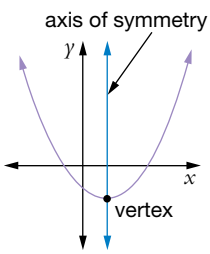
Find the equations of five randomly generated graphs. Tick the checkboxes to check your answers. A suggestion box gives information about each graph.



MATHS TERMS 14

axis of symmetry

- passes through the vertex of a parabola
- if the equation of the parabola is $y = ax^2 + bx + c$, its axis of symmetry is given by the formula $x = -\frac{b}{2a}$



circle (equation of)

- the equation of a circle with its centre at the point (p, q) and a radius of r is $(x - p)^2 + (y - q)^2 = r^2$

cubic

- a curve that contains an x^3 term as its highest power

distance formula

- gives the distance between the points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

general form

- a way of writing the equation of a line
- the equation is written in the form $ax + by + c = 0$ where a, b, c are integers and $a > 0$

equation

- an algebraic statement that expresses the relationship between the x- and y-coordinates of every point (x, y) on the curve

gradient

- the slope of a line or interval
- can be measured using the formula:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

gradient formula

- gives the gradient of the interval joining (x_1, y_1) to (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

gradient–intercept form

- a way of writing the equation of a line, e.g. $y = 2x - 5$, $y = \frac{1}{2}x + 2$
- when an equation is rearranged and written in the form $y = mx + b$ then m is the gradient and b is the y–intercept

hyperbola

- a curve with an equation of the form $y = \frac{k}{x}$, $y = \frac{k}{x} + c$ or $y = \frac{k}{x - b}$, where k, b and c are constants

midpoint formula

- gives the midpoint of the interval joining (x_1, y_1) to (x_2, y_2):

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

parabola

- the graph of equations of the form $y = ax^2 + bx + c$ where a, b, c are constants

point–gradient form

- a way of writing the equation of a straight line
- the equation is written in the form $y - y_1 = m(x - x_1)$ where m is the gradient and (x_1, y_1) is the given point the line passes through

two-point form

- a way of writing the equation of a straight line
- the equation is written in the form $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ where (x_1, y_1) and (x_2, y_2) are the coordinates of two given points the line passes through

vertex (turning point)

- the point where a parabola changes direction
- the gradient changes sign at this point

x-intercept

- the point where a line or curve crosses the x-axis

y-intercept

- the point where a line or curve crosses the y-axis

DIAGNOSTIC TEST 14 FURTHER COORDINATE GEOMETRY

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 For each pair of points, use the distance formula to find the distance AB. **14:01**
a $A(1, 2), B(7, 10)$ b $A(3, 0), B(5, 3)$ c $A(-3, -2), B(1, -3)$

2 For each pair of points in Question 1, find the midpoint of the interval AB. **14:01**

3 For each pair of points in Question 1, find the gradient of the interval AB. **14:01**

4 Find the equation of a line that: **14:02**
a passes through $(1, 4)$ and has gradient 2
b has gradient -3 and passes through $(1, 3)$
c has gradient $\frac{1}{2}$ and passes through $(-2, 0)$.

5 Find the equation of a line that: **14:03**
a passes through the points $(1, 1)$ and $(2, 3)$
b passes through the points $(-1, 2)$ and $(1, -4)$
c passes through the origin and $(3, 4)$.

6 Write each equation in the general form. **14:04**
a $y = 3x - 2$ b $2x + 3y = 6$ c $y = -\frac{x}{2} + 4$

7 On the same number plane, sketch the graphs of: **14:07**
a $y = (x - 1)^2$ b $y = (x + 3)^2$ c $y = (x - 2)^2 + 1$

8 Find the y-intercept of the parabolas: 14:08

a $y = (x - 1)(x + 3)$ b $y = x^2 - 6x$

c $y = 8 - 2x - x^2$ d $y = 4x^2 + 8x - 5$

9 Find the x–intercepts for each of the parabolas in Question 8. 14:08

10 Find the equation of the axis of symmetry for each of the parabolas in Question 8. 14:08

11 Find the vertex of each of the parabolas in Question 8. 14:08

12 Sketch each of the parabolas in Question 8. 14:08

13 Sketch the curves: 14:09

a $y = \frac{3}{x}$ b $y = -\frac{2}{x}$ c $y = \frac{6}{x}$

14 Sketch the curves: 14:10

a $y = \frac{3}{x+1}$ b $y = \frac{2}{x} - 2$ c $y = \frac{-2}{x} - 2$

15 Sketch the circles: 14:11

a $(x - 1)^2 + y^2 = 4$ b $(x + 1)^2 + y^2 = 9$ c $(x - 2)^2 + (y + 1)^2 = 16$

16 Sketch the curves: 14:13

a $y = 2x^5$ b $y = \frac{1}{2}x^6$ c $y = -2x^4$

17 Sketch the curves: 14:13

a $y = \frac{1}{2}x^3 - 1$ b $y = -x^4 + 1$ c $y = -x^5 + 1$

18 Sketch the curves: 14:14

a $y = (x - 3)^3$ b $y = 2(x + 1)^4$ c $y = -\frac{1}{2}(x - 1)^5$

19 Sketch the curves: 14:15

a $y = x(x - 2)(x - 3)$

b $y = 2(x + 1)(x - 2)^2$

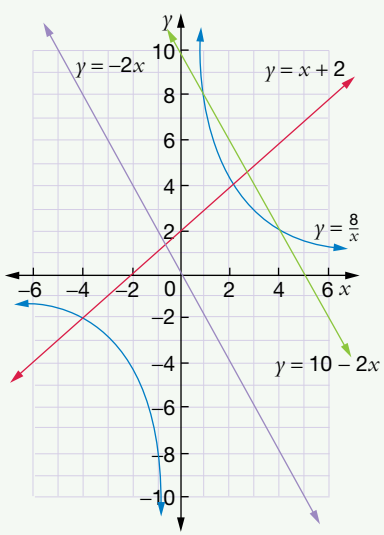
c $y = -(x + 1)(x - 2)(x + 3)$

20 Use the graph to find the points of intersection of: 14:16

a $y = \frac{8}{x}$ and $y = x + 2$

b $y = \frac{8}{x}$ and $y = -2x$

c $y = \frac{8}{x}$ and $y = 10 - 2x$



21 Find the points of intersection of: 14:16

a $y = x^2$ and $y = 12 - x$

b $y = x - 3$ and $y = -\frac{2}{x}$

c $y = x + 2$ and $x^2 + y^2 = 10$

ASSIGNMENT 14A Chapter review

1 Find the equation of each line, given the following information:

a gradient of -1 and passes through the point $(1, -2)$

b passes through the two points $(-1, -2)$ and $(1, -1)$

2 Express each equation in Question **1** in general form.

3 Describe the relationship between the graph of $y = x^2$ and the graph of:

a $y = (x - 5)^2$

b $y = (x + 5)^2$

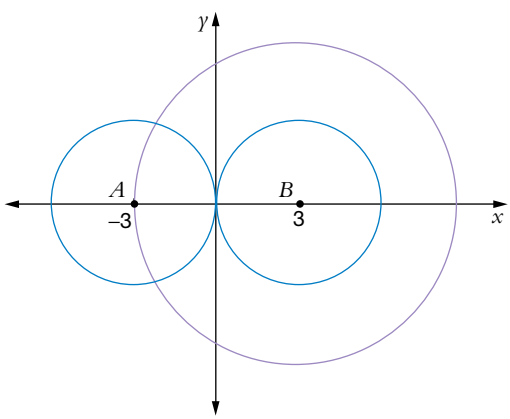
c $y = (x - 5)^2 + 5$

4 Sketch each parabola, showing clearly the intercepts with each axis, the axis of symmetry and vertex.

a $y = x^2 - 4x - 12$

b $y = 7 + 6x - x^2$

5



The two small circles have their centres at A and B. The larger circle's centre is also at B and it passes through A. What are the equations of:

a the circle centred at A

b the larger circle with its centre at B?

6 Make sketches of the curves:

a $y = 2x^7$

b $y = -2x^7$

c $y = 2x^7 - 1$

d $y = 1 - 2x^7$

e $y = 2(x - 1)^7$

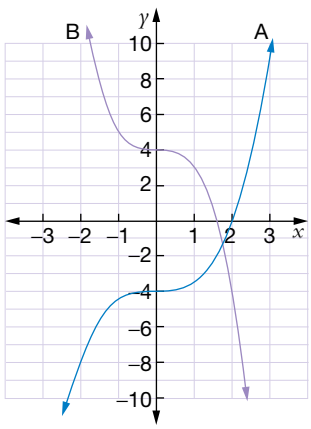
7 Sketch the following curves on the same number plane.

a $y = (x - 2)^2(x - 4)$

b $y = 2(x - 2)^2(x - 4)$

c $y = (x - 2)^2(4 - x)$

8 The curves A and B have equations of the form $y = ax^3 + d$. Find the equation of each curve.



9 State the equation of each curve that would result if the curve $y = \frac{4}{x}$ was:

a translated 2 units up

b translated 2 units to the right

c translated 2 units to the left

d reflected in the y-axis

e reflected in the y-axis and translated 2 units up.

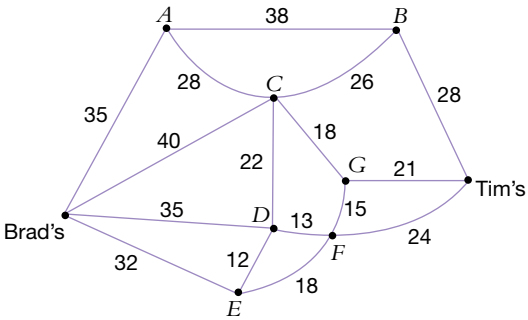
ASSIGNMENT 14B Working mathematically

1 It is now 10 am. What time will it be in 11 999 995 hours?

2 Farmer Fred uses a 40 m roll of fencing wire to enclose three sides of a rectangular pen; an existing fence is the fourth side. What is the largest area that can be enclosed and what are the dimensions of the field?

3 A tank can be filled by a tap in 3 hours, and it can be emptied via a plug hole in 7 hours. If the tank is empty, how long will it take to fill if the tap is turned on and the plug hole is left open? (Assume that water drains out at a constant rate.)

4 Brad visits his brother Tim's home every week. He can take many different routes, as indicated by the diagram.

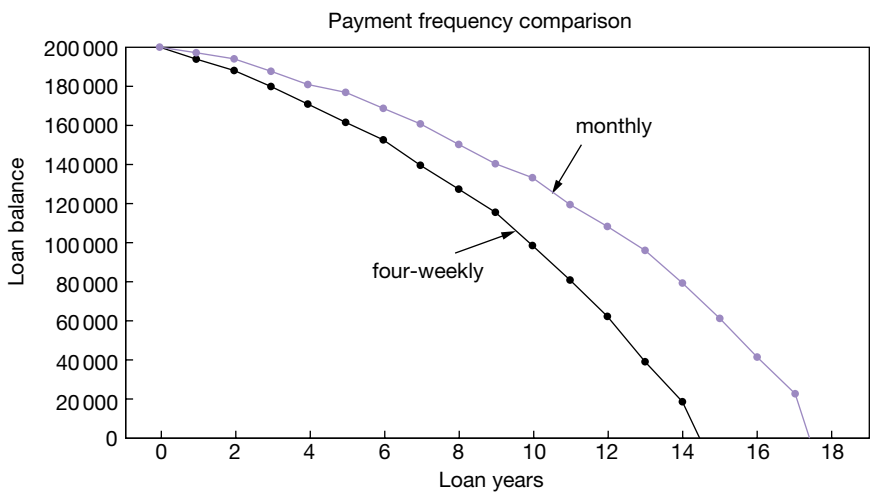


a What is the shortest distance from Brad's to Tim's?

b In how many different ways can Brad travel to Tim's? (Assume that he always travels from left to right.)

c Tim's sister Amy lives at *F*. She has to go to *A* to pick up her parents and then go to Tim's for Christmas lunch. What is the shortest way of making her trip?

5 Many home loan borrowers have changed their monthly repayment program so that the same amount is instead paid every four weeks. For the graph below, monthly payments are $1818 and the interest rate is 10% reducible.



a In each case how much was borrowed?

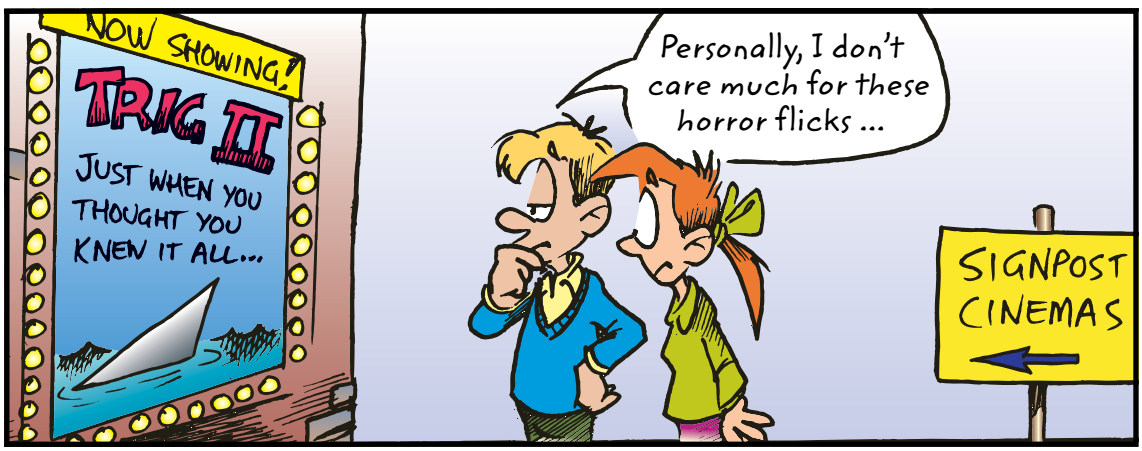
b How long did it take to repay the loan with four-weekly repayments?

c How long did it take to repay the loan with monthly repayments?

d Why is the difference so great?

FURTHER TRIGONOMETRY

15



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GeoGebra activity 15:02B Trigonometric graphs
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15:03 Trigonometric relationships between acute and obtuse angles
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GeoGebra activity 15:04 The sine rule
15:05 The sine rule: The ambiguous case
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15:09 Three-dimensional problems
Maths terms, Diagnostic test, Assignments

Syllabus references (See pages x–xiii for details.)

Measurement and Geometry

Selections from ***Trigonometry and Pythagoras' Theorem*** [Stage 5.3§]

- Apply Pythagoras' theorem and trigonometry to solve three-dimensional problems in right-angled triangles (ACMMG276)
- Use the unit circle to define trigonometric functions, and graph them, with and without the use of digital technologies (ACMMG274)
- Solve simple trigonometric equations (ACMMG275)
- Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)

Working Mathematically

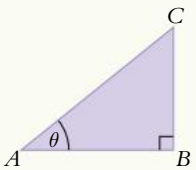
- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

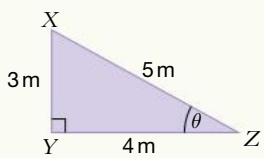
15:01 Trigonometric relationships

PREP QUIZ 15:01

In ΔABC, name:

1 the side opposite angle θ
2 the side adjacent to angle θ
3 the hypotenuse





From ΔXYZ find, as a decimal, the value of:

4 $\sin\theta$ 5 $\cos\theta$ 6 $\tan\theta$

Using your calculator, find the value correct to three decimal places of:

7 $\sin 42°$ 8 $\tan 61°$

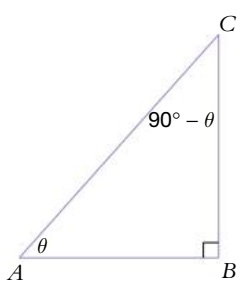
Find, to the nearest degree, the value of θ if:

9 $\cos\theta = 0{\cdot}899$ 10 $\tan\theta = 2{\cdot}605$

The Prep quiz above should remind you of how the trig ratios sine, cosine and tangent were defined.

There are many relationships between the trig ratios. We shall now look at some of these.

Consider ΔABC.



If $\angle CAB = \theta$, then $\angle ACB = 90° - \theta$.

$\sin\theta = \dfrac{BC}{AC}$ $\quad \sin(90° - \theta) = \dfrac{AB}{AC}$

$\cos\theta = \dfrac{AB}{AC}$ $\quad \cos(90° - \theta) = \dfrac{BC}{AC}$

The following relationships are evident:

$$\sin\theta = \cos(90° - \theta) \qquad \cos\theta = \sin(90° - \theta)$$

$\tan\theta = \dfrac{BC}{AC}$ and $\dfrac{\sin\theta}{\cos\theta} = \dfrac{BC}{AC} \div \dfrac{AB}{AC}$

$= \dfrac{BC}{\cancel{AC}} \times \dfrac{\cancel{AC}}{AB}$

$= \dfrac{BC}{AC}$

$= \tan\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

WORKED EXAMPLES

1 If $\sin x° = \cos 40°$, find the value of x.

2 Simplify $\dfrac{\cos 20°}{\sin 70°}$.

3 If $\tan\theta = \dfrac{\cos 40°}{\cos 50°}$, find the value of θ.

Solutions

1 $\sin x° = \cos 40°$

Now, $\sin x° = \cos(90° - x°)$

$\therefore\ 90 - x = 40$

$x = 50$

Or $\cos 40° = \sin x°$

$\cos 40° = \sin(90° - 40°)$

$= \sin 50°$

$x = 50$

2 $\dfrac{\cos 20°}{\sin 70°} = \dfrac{\sin 70°}{\sin 70°}$ $\left(\text{or } \dfrac{\cos 20°}{\cos 20°}\right)$

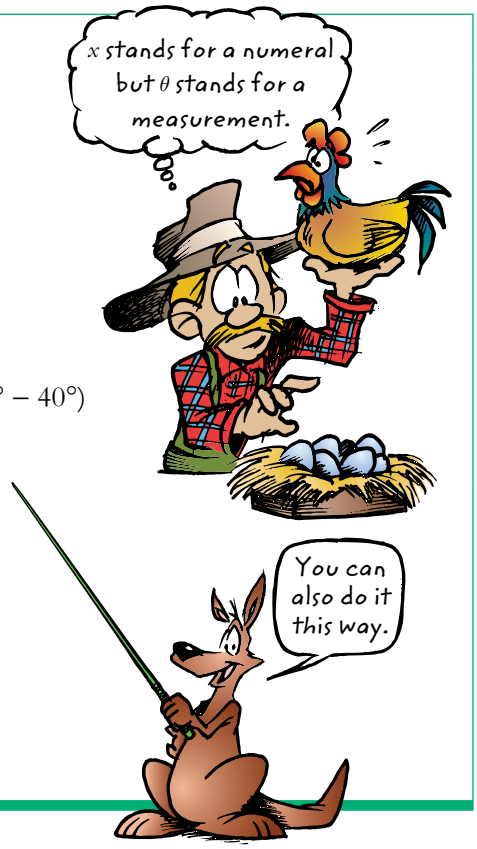
$= 1$

3 $\tan\theta = \dfrac{\cos 40°}{\cos 50°}$

$= \dfrac{\sin 50°}{\cos 50°}$

$= \tan 50°$

$\therefore\ \theta = 50°$



Exercise 15:01

1 Use your calculator to complete the table below. Write all values correct to three decimal places.

θ	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°
$\sin\theta$															
$\cos\theta$															

2 Use the table above to find another ratio equal to:

a $\sin 10°$ **b** $\sin 20°$ **c** $\sin 30°$ **d** $\sin 65°$

e $\cos 40°$ **f** $\cos 50°$ **g** $\cos 35°$ **h** $\cos 65°$

3 Find the value of x in each of the following.

a $\sin 40° = \cos x°$ **b** $\cos 60° = \sin x°$ **c** $\sin 10° = \cos x°$

d $\cos 20° = \sin x°$ **e** $\sin x° = \cos 25°$ **f** $\cos x° = \sin 75°$

g $\sin x° = \cos 45°$ **h** $\cos 45° = \sin x°$

4 Which angle has the same sine and cosine ratio?

5 **a** Use a calculator to evaluate these correct to four decimal places.

i $\dfrac{\sin 10°}{\cos 10°}$ **ii** $\tan 10°$

b Are the answers in **a** the same?

6 Write each of the following in the form $\tan\theta$.

a $\dfrac{\sin 30^\circ}{\cos 30^\circ}$ **b** $\dfrac{\sin 48^\circ}{\cos 48^\circ}$ **c** $\dfrac{\sin 16^\circ}{\cos 16^\circ}$

7 **a** Show that $\dfrac{\sin 30^\circ}{\cos 60^\circ} = 1$ **b** If $\dfrac{\cos 50^\circ}{\cos 40^\circ} = \tan\theta$, find θ. **c** If $\dfrac{\sin 50^\circ}{\sin 40^\circ} = \tan\theta$, find θ.

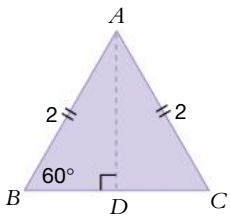
8 **a** If $\tan\theta = 1$, what can you say about $\dfrac{\sin\theta}{\cos\theta}$?

b What does this say about $\sin\theta$ and $\cos\theta$?

c What is the value of θ if $\tan\theta = 1$?

INVESTIGATION 15:01 EXACT VALUES FOR THE TRIGONOMETRIC RATIOS 30°, 60° AND 45°

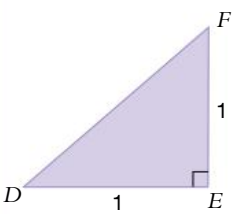
ΔABC is an equilateral triangle of side 2 units. AD is perpendicular to BC.



1 Copy the diagram and write in the size of BD and $\angle BAD$.

2 Using Pythagoras' theorem, calculate the length of AD as a surd.

3 Now, from ΔABD, write the values of sin, cos and tan for 30° and 60°.



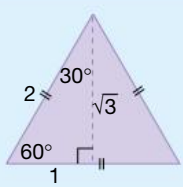
ΔDEF is a right-angled isosceles triangle. The two equal sides are 1 unit in length.

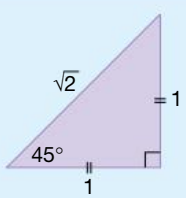
4 Why is $\angle EDF$ equal to 45°?

5 What is the length of DF as a surd?

6 Write the values of sin 45°, cos 45° and tan 45°.

Leave surds in your answers.





$\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, $\sin 30^\circ = \dfrac{1}{2}$ $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$

$\cos 60^\circ = \dfrac{1}{2}$, $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$ $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}$, $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$ $\tan 45^\circ = 1$

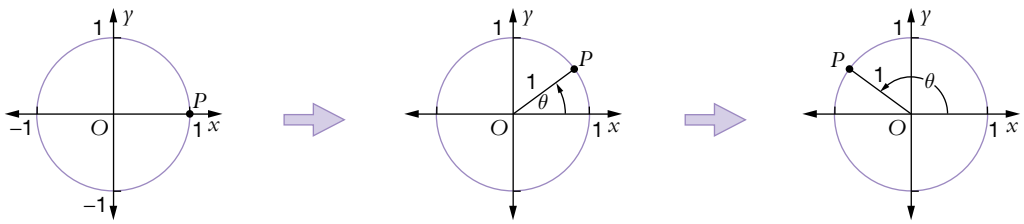
15:02 Trigonometric ratios of other angles

To this point we have only needed to know the trigonometric ratios for acute angles. The way in which the trigonometric ratios were defined for acute angles needs to be refined so that the trigonometric ratios of larger angles can be calculated.

The unit circle

Sine, cosine and tangent ratios can be defined in terms of a circle of radius 1 unit.

- We refer to a **unit circle** that has its centre at (0, 0) on the number plane. If we take a radius OP initially along the x-axis and rotate it anticlockwise about O, of course point P will still lie on the circle.



The coordinates of P on the number plane will depend on the size of θ, the angle of rotation.

If a perpendicular PM were drawn as in this diagram, the method of defining the trig ratios that we met in Year 9 would suggest that:

$\cos\theta = \dfrac{OM}{1}$ (i.e. $\dfrac{\text{adj.}}{\text{hyp.}}$)

$= OM$

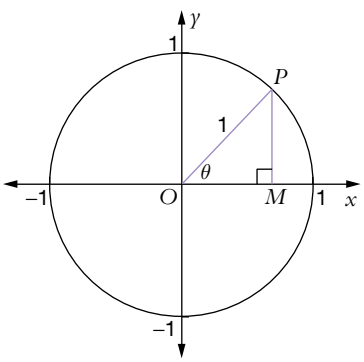
$=$ x-coordinate of P

$\sin\theta = \dfrac{PM}{1}$ (i.e. $\dfrac{\text{opp.}}{\text{hyp.}}$)

$= PM$

$=$ y-coordinate of P

$\therefore$ P is the point $(\cos\theta, \sin\theta)$.



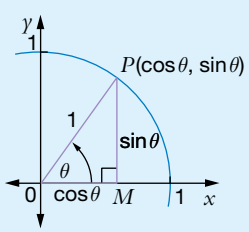
Hence, the previous method of defining the trig ratios using side ratios suggests a new way of defining the trig ratios as the coordinates of the point P.

This method is used to give the trig ratios for acute angles and, in fact, angles of any size.

For a point P on a unit circle that has been rotated through an angle θ about O from the positive x-axis:

$\cos\theta$ = the x-coordinate of P

$\sin\theta$ = the y-coordinate of P.



- The triangle on the previous page can also be used to redefine the tangent function.

$$\tan\theta = \frac{PM}{OM} \text{ (i.e. } \frac{\text{opp.}}{\text{hyp.}}\text{)}$$

$$= \frac{y\text{-coordinate of } P}{x\text{-coordinate of } P}$$

$$= \frac{\sin\theta}{\cos\theta}$$

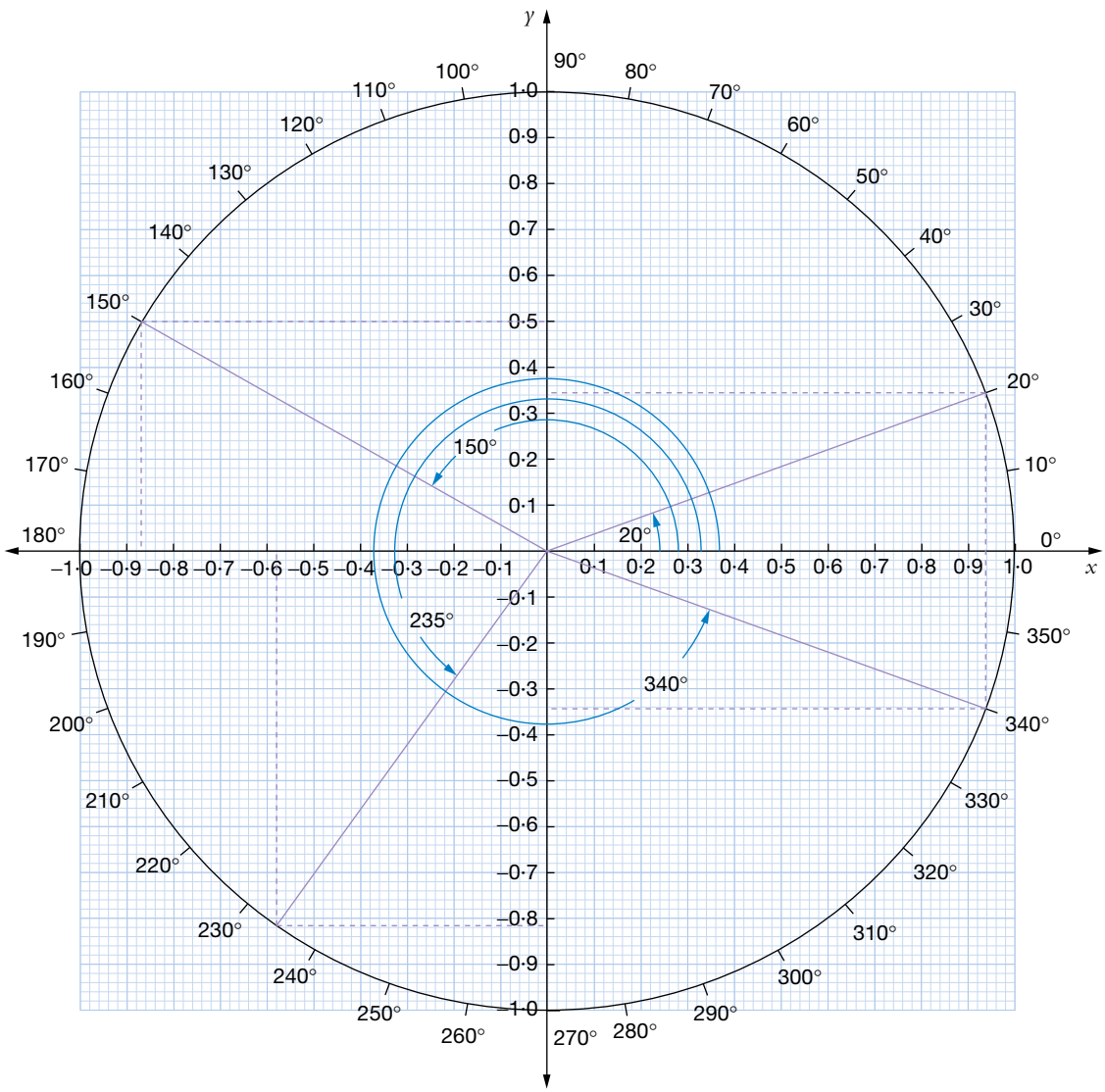
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

The values of the trig ratios are calculated using a calculator, but it is important to understand the unit circle definitions. The unit circle explains:

- why the ratios vary between positive and negative
- how the ratios change as the angle increases from 0° to 360°.

Many relationships exist between the trig ratios, and some of these will be investigated in the following exercise.

Remember: Each point on the circle has coordinates $(\cos\theta, \sin\theta)$.



Refer to this unit circle in the following examples and in Exercise 15:02.

WORKED EXAMPLES

1 State the sign of:
- **a** $\cos 150°$ and $\sin 150°$
- **b** $\sin 235°$ and $\cos 235°$
- **c** $\sin 340°$ and $\cos 340°$

2 What are the coordinates of P when θ is 180°? Use these to state the value of $\cos 180°$.

3 What can you say about the sign of $\sin\theta$ for $180° < \theta < 360°$?

4 Find two values of θ for which $\sin\theta = 0{\cdot}34$.

Solutions

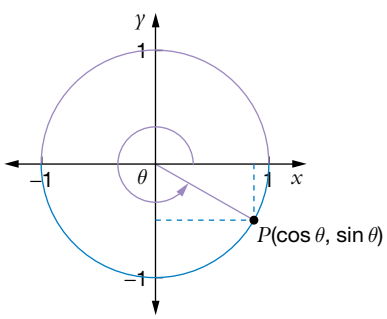
1 **a** When $\theta = 150°$, the x-coordinate of P is negative and the y-coordinate is positive.
$\therefore$ $\cos 150°$ is negative; $\sin 150°$ is positive

b When $\theta = 235°$, the x-coordinate of P is negative and the y-coordinate is negative.
$\therefore$ $\cos 235°$ is negative; $\sin 235°$ is negative

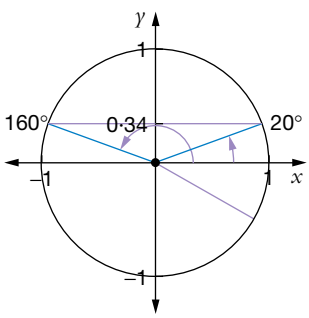
c When $\theta = 340°$, the x-coordinate of P is positive and the y-coordinate is negative.
$\therefore$ $\cos 340°$ is positive; $\sin 340°$ is negative

2 When $\theta = 180$, P is the point $(-1, 0)$.
$\therefore$ $\cos 180° = -1$ ($\cos\theta$ = x-coordinate of P)

3 When θ is between 180° and 360°, P lies on the unit circle in either the 3rd or 4th quadrant, (on the blue part of the unit circle).
Hence the y-coordinate of P is negative.
$\therefore$ $\sin\theta$ is negative for $180° < \theta < 360°$.



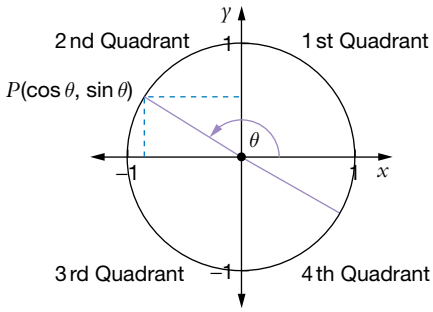
4 If $\sin\theta = 0{\cdot}34$, we need to find two points on the unit circle with a y-coordinate of 0·34. Draw a horizontal line through 0·34 on the y-axis to meet the unit circle. It will give the two answers as 20° and 160°.



Exercise 15:02

1 In which quadrant of the number plane will P be when θ is:

a 120° **b** 240° **c** 330° **d** 70°?



2 Use the coordinates of P to find the sign of:

a $\sin 120°$ **b** $\sin 240°$ **c** $\sin 330°$ **d** $\sin 70°$

e $\cos 120°$ **f** $\cos 240°$ **g** $\cos 330°$ **h** $\cos 70°$

3 In which quadrants of the number plane could a point P be if:

a the x-coordinate is positive **b** the x-coordinate is negative

c the y-coordinate is positive **d** the y-coordinate is negative?

4 P is a point on the unit circle. State in which quadrants P could be if it is known that:

a $\sin\theta$ is positive **b** $\sin\theta$ is negative

c $\cos\theta$ is positive **d** $\cos\theta$ is negative

e $\sin\theta$ is negative and $\cos\theta$ is positive.

5 Describe what happens to $\sin\theta$ as θ increases from:

a 0° to 90° **b** 90° to 180° **c** 180° to 270° **d** 270° to 360°

6 Describe what happens to $\cos\theta$ as θ increases from:

a 0° to 90° **b** 90° to 180° **c** 180° to 270° **d** 270° to 360°

7 **a** If x is negative and y is positive what is the sign of $\frac{y}{x}$?

b In which quadrants do the x-coordinate and the y-coordinate have opposite signs?

c In which quadrants is $\tan\theta$ negative?

d In which quadrants is $\tan\theta$ positive?

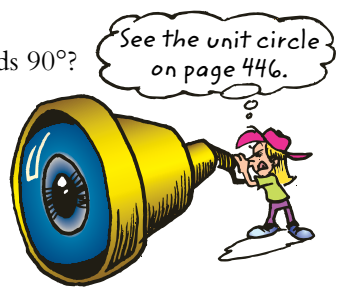
8 **a** What is the largest and smallest y-coordinate of any point on the unit circle?

b Given your answer to **a**, what are the largest and smallest values of $\sin\theta$ that are possible?

c What are the largest and smallest possible values of $\cos\theta$?

9 **a** What is happening to the value of $\tan\theta$ as θ increases from 0° towards 90°?

b Why can there be no value for $\tan 90°$?



10 Estimate each of the following pairs of values. What do you notice?

a $\sin 75°$ and $\sin 105°$ **b** $\sin 80°$ and $\sin 100°$

c $\cos 70°$ and $\cos 110°$ **d** $\cos 40°$ and $\cos 140°$

11 Use the unit circle to find two values of θ to the nearest degree if:

a $\sin\theta = 0{\cdot}5$ **b** $\sin\theta = 0{\cdot}77$ **c** $\sin\theta = 0{\cdot}17$

d $\sin\theta = -0{\cdot}5$ **e** $\sin\theta = -0{\cdot}77$ **f** $\sin\theta = -0{\cdot}17$

12 Use the unit circle to find two values of θ to the nearest degree if:

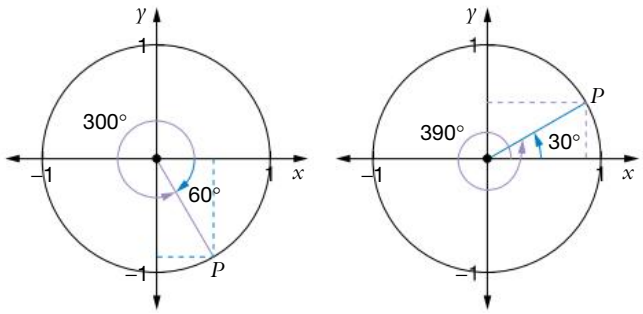
a $\cos\theta = 0{\cdot}5$ **b** $\cos\theta = 0{\cdot}64$ **c** $\cos\theta = 0{\cdot}87$

d $\cos\theta = -0{\cdot}5$ **e** $\cos\theta = -0{\cdot}64$ **f** $\cos\theta = -0{\cdot}87$

INVESTIGATION 15:02 TRIGONOMETRIC GRAPHS

The unit circle can be used to define:

- angles that are greater than 360°
- negative angles (i.e. angles that are measured in a clockwise direction).



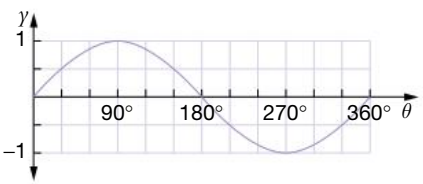
1 **a** Use a calculator to complete the table correct to three decimal places.

b Are the trig ratios for each of the angles equal?

θ	300°	660°	–60°
$\sin\theta$			
$\cos\theta$			
$\tan\theta$			

2 Use the unit circle to explain why angles that differ by 360° have the same trig ratios.

3 The graph shows $y = \sin\theta$ for $0° \leq \theta \leq 360°$.



a If you added 360° to all the values on the horizontal axis would the shape of the curve change?

b If you subtracted 360° from all the values on the horizontal axis would the shape of the curve change?

c How could you use the graph shown to obtain the graph of the following curves:

i $y = \sin\theta$ for $360° \leq \theta \leq 720°$

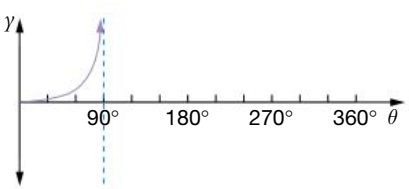
ii $y = \sin\theta$ for $-360° \leq \theta \leq 0°$

4 Sketch the graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$. Is the graph symmetrical about the y-axis?

5 How could you use the graph in Question 4 to obtain the graph of $y = \cos\theta$ for the following sets of values:

a $360° \leq \theta \leq 720°$ **b** $-360° \leq \theta \leq 0°$?

6 Part of the graph of $y = \tan\theta$ is shown.



a Complete the graph for values of θ between 90° and 360°.

b Why do vertical asymptotes appear at $\theta = 90°$ and $\theta = 270°$?

c Sketch $y = \tan\theta$ for $360° \leq \theta \leq 720°$.

d How long does it take for $y = \tan\theta$ to repeat its shape?

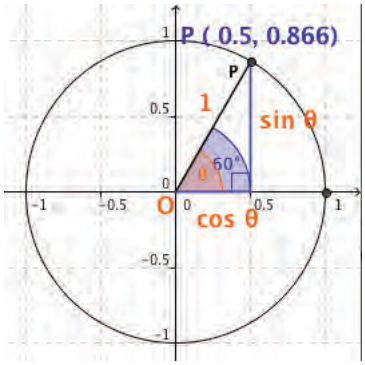
7 Curves that repeat their shapes are said to be periodic. The period (or periodicity) of the curve is the distance that the curve needs to be translated horizontally so that it is superimposed on itself. What is the period of the graph of:

a $y = \sin\theta$ **b** $y = \cos\theta$ **c** $y = \tan\theta$?

GEOGEBRA ACTIVITY 15:02A USING THE UNIT CIRCLE

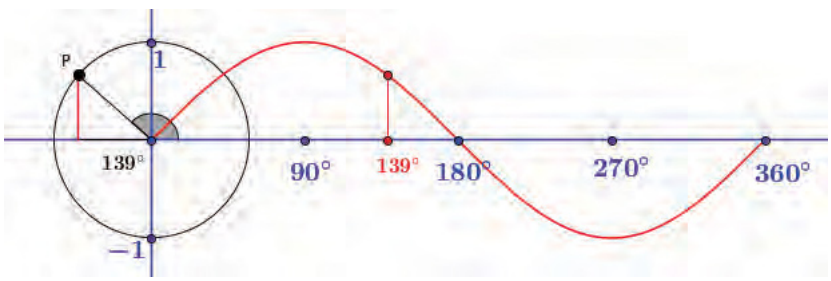
Use the slider to change the position of P on the unit circle. As P changes position the coordinates of P and hence the values of the trig ratios for the given angle are displayed.

Checkboxes can be used to show positive angles, negative angles, the definitions and the signs of the trig ratios.



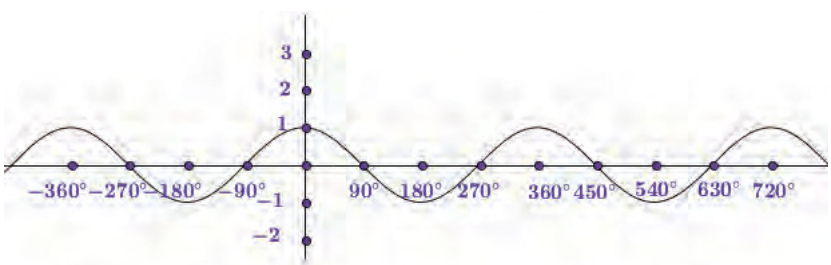
GEOGEBRA ACTIVITY 15:02B TRIGONOMETRIC GRAPHS

Drag a point around the unit circle and observe how the three trigonometric graphs are generated.



GEOGEBRA ACTIVITY 15:02C FEATURES OF TRIGONOMETRIC GRAPHS

Choose a graph and look at the curve from −360° to 720°. Use a copy of the curve to study its symmetry and periodicity.



15:03 Trigonometric relationships between acute and obtuse angles

PREP QUIZ 15:03

Evaluate the following correct to two decimal places.

1 $\sin 40°$ **2** $\sin 140°$ **3** $\cos 40°$ **4** $\cos 140°$

5 Can an acute angle and an obtuse angle have equal sine ratios?

What is the sign of $\sin\theta$ if:

6 θ is acute?

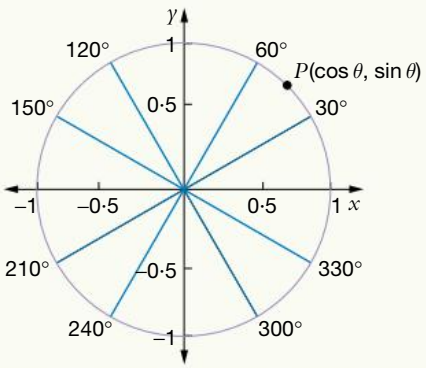
7 θ is obtuse?

8 Can an acute angle and an obtuse angle have equal cosine ratios?

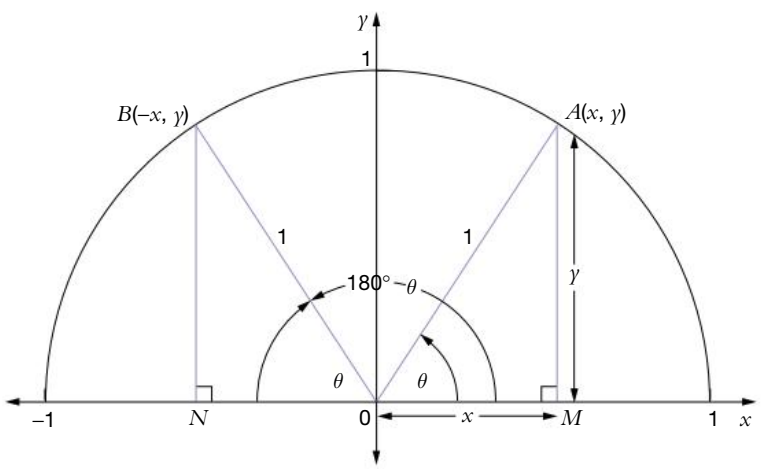
Use the unit circle on the right to find θ if:

9 $\sin\theta = 0{\cdot}5$

10 $\cos\theta = 0{\cdot}5$.

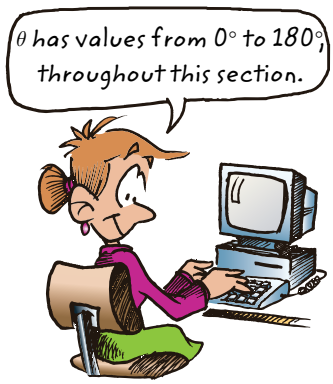


- It is possible for an acute angle and an obtuse angle to have the same sine value.



- There is a relationship between the trigonometric ratios of acute and obtuse angles. To investigate this, OA and OB have been drawn at an angle θ to the positive arm of the x-axis and the negative arm of the x-axis respectively. Hence, $\angle BOM = 180° - \theta$

∴ The coordinates of A are $(\cos\theta, \sin\theta)$.
The coordinates of B are $[\cos(180° - \theta), \sin(180° - \theta)]$.



Now, Δs AMO and BNO are congruent (AAS).
∴ $AM = BN$ and $OM = ON$.
If A is the point (x, y) then B is the point $(-x, y)$.

A has coordinates $(\cos\theta, \sin\theta)$ or (x, y), so:

$\cos\theta = x$, $\sin\theta = y$ and $\tan\theta = \frac{y}{x}$.

B has coordinates $[\cos(180° - \theta), \sin(180° - \theta))$ or $(-x, y)]$, so:

$\cos(180° - \theta) = -x$, $\sin(180° - \theta) = y$ and $\tan(180° - \theta) = \frac{y}{-x}$.

Equating the two sets of ratios gives these results.

$$\sin(180° - \theta) = \sin\theta, \quad \cos(180° - \theta) = -\cos\theta, \quad \tan(180° - \theta) = -\tan\theta$$

This means, for example, that:
$\sin 120° = \sin(180° - 60°) = \sin 60°$
$\cos 160° = \cos(180° - 20°) = -\cos 20°$
$\tan 141° = \tan(180° - 39°) = -\tan 39°$

For acute angles the trig ratios are positive.

For obtuse angles only sine is positive.

Since θ and $180 - \theta$ are supplementary angles we can say that:

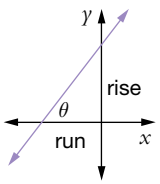
- the sines of supplementary angles are equal
- the cosines and tangents of supplementary angles have equal numerical values but they have the opposite sign.

Finding the angle of inclination of a line

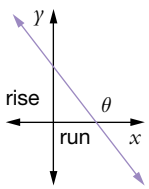
- In coordinate geometry we have seen that the slope of a line is given by the formula $m = \frac{\text{rise}}{\text{run}}$.

An angle can also be used to measure the slope of a line. If θ is the angle between the line and the positive direction of the x-axis then two cases arise.

Case 1: m is positive and θ is acute.



Case 2: m is negative and θ is obtuse.



In both of these cases m and θ are connected by the formula $m = \tan\theta$.

WORKED EXAMPLE 1

Which acute angle has the same sine as 150°?

$\sin(180° - \theta) = \sin\theta$

$180° - \theta = 150°$

$\theta = 30°$

$\therefore \sin 30° = \sin 150°$

WORKED EXAMPLE 2

If θ is an angle between 0° and 180°, find θ to the nearest degree if:

a $\cos\theta = 0{\cdot}6$ ($\therefore \theta$ is acute)

Press: SHIFT cos 0.6 =

$\theta = 53{\cdot}13°$ (from the calculator)

$\theta = 53°$ (nearest degree)

b $\tan\theta = -0{\cdot}8$ ($\therefore \theta$ is obtuse)

Let $\theta = 180° - \alpha$

$\tan(180° - \alpha) = -0{\cdot}8$

$\tan\alpha = 0{\cdot}8$ (α is acute)

Press: SHIFT tan 0.8 = ° ' "

$\alpha = 38°40'$ (nearest minute)

But $\theta = 180° - \alpha$

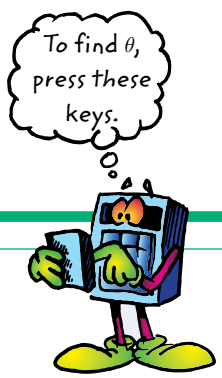
$\therefore \theta = 180° - 38°40'$

$= 141°$ (nearest degree)

If θ is obtuse, solve $\tan\theta = 0{\cdot}8$ and subtract the answer from 180°.

WORKED EXAMPLE 3

Give an acute and obtuse value of θ (to the nearest minute) for which $\sin\theta = 0{\cdot}354$.



$\sin\theta = 0{\cdot}354$

Press: SHIFT sin 0.354 = ° ' "

$\theta = 20°44'$ (nearest minute)

Now $\sin(180° - \theta) = \sin\theta$

If $\theta = 20°44'$ then $180° - \theta = 159°16'$.

$\therefore \theta = 20°44'$ or $159°16'$

WORKED EXAMPLE 4

Find the angle of inclination of the line $y = 2x - 3$ to the nearest degree.

$m = \tan\theta$

$m = 2$ (using $y = mx + b$ form)

$\tan\theta = 2$

Press: SHIFT tan 2 = ° ' "

$\therefore \theta = 63°$ (nearest degree)

Exercise 15:03

P Foundation worksheet 15:03
Trig ratios of obtuse angles

1 Use a calculator to evaluate the following correct to three decimal places.

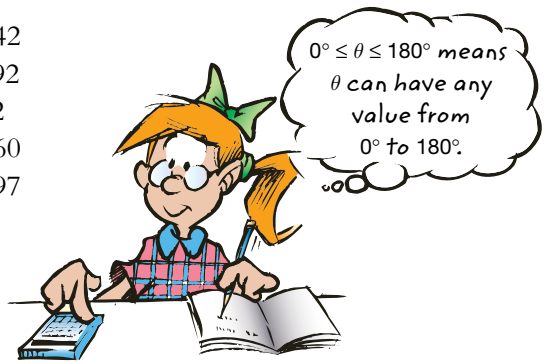
a $\sin 167°30'$ b $\cos 140°20'$ c $\tan 150°19'$

2 Use the results on page 452 to express each of the following as a ratio of an acute angle.

a $\sin 150°$ b $\cos 100°$ c $\tan 140°$
d $\sin 125°$ e $\sin 140°32'$ f $\cos 167°31'$
g $\tan 101°03'$ h $\cos 92°50'$

3 If θ is an angle between $0°$ and $180°$, find θ to the nearest minute if:

a $\cos\theta = 0{\cdot}716$ b $\cos\theta = -0{\cdot}716$
c $\tan\theta = 8{\cdot}215$ d $\tan\theta = -8{\cdot}215$
e $\cos\theta = -0{\cdot}5$ f $\tan\theta = -1$
g $\cos\theta = 0{\cdot}906$ h $\cos\theta = -0{\cdot}342$
i $\tan\theta = 1{\cdot}881$ j $\tan\theta = -1{\cdot}192$
k $\cos\theta = -0{\cdot}966$ l $\cos\theta = 0{\cdot}602$
m $\tan\theta = -0{\cdot}754$ n $\cos\theta = -0{\cdot}760$
o $\tan\theta = 3{\cdot}323$ p $\cos\theta = -0{\cdot}997$



4 If $0° \le \theta \le 180°$, give two possible values for θ (to the nearest minute) if $\sin\theta$ is equal to:

a 0·5 b 0·73
c 0·36 d 0·453
e 0·990 f 0·3665
g 0·7083 h 0·0567

5 Complete the table below and use it to sketch $y = \sin x°$ for $0° \le x° \le 180°$.

$x°$	0°	20°	40°	60°	80°	90°	100°	120°	140°	160°	180°
$\sin x°$											

6 Make a sketch of $y = \cos x°$ for $0° \le x° \le 180°$.

7 Find the angle of inclination to the x-axis of each line to the nearest degree.

a $y = 3x$ b $y = x + 2$ c $y = \frac{2}{5}x + 5$
d $y = 3 - x$ e $y = -\frac{x}{3} + 1$ f $x - \sqrt{3}y = 10$

8 Find the angle of inclination (to the nearest degree) of the line that passes through the points:

a (1, 2) and (6, 9) b (−3, 6) and (0, 1)

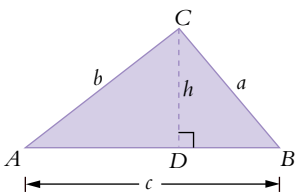
15:04 The sine rule

Now that trig ratios can be calculated for obtuse angles, trigonometry can be applied to any type of triangle.

In the triangles below:

- side a is opposite $\angle A$
- side b is opposite $\angle B$
- side c is opposite $\angle C$.

Note: $\sin(180° - B) = \sin B$



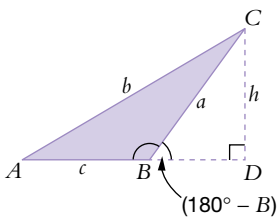
ABC is an acute-angled triangle.
CD is perpendicular to AB.

In ΔACD, $\sin A = \dfrac{h}{b}$

$\therefore\ h = b \sin A$ 1

In ΔBCD, $\sin B = \dfrac{h}{a}$

$\therefore\ h = a \sin B$ 2



ABC is an obtuse-angled triangle.
CD is perpendicular to AB (produced to D).

In ΔACD, $\sin A = \dfrac{h}{b}$

$\therefore\ h = b \sin A$ 1

In ΔBCD, $\sin(180° - B) = \dfrac{h}{a}$

$\therefore\ h = a \sin(180° - B)$

$\therefore\ h = a \sin B$ 2

Equating 1 and 2 from either triangle gives:

$$a \sin B = b \sin A$$

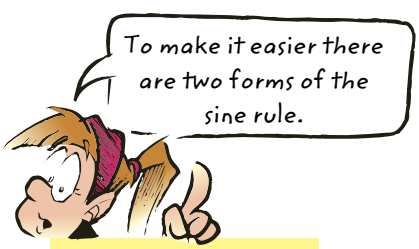
$$\text{i.e. } \frac{a}{\sin A} = \frac{b}{\sin B}$$

Similarly, it can be shown that:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Thus, we obtain:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

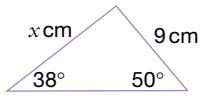


- To find a side use the first form.
- To find an angle use the second form.

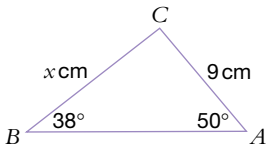
To use the sine rule, at least one angle and the side opposite it must be known.

WORKED EXAMPLE 1

Find the value of x in this triangle. Answer correct to one decimal place.



Here, two angles and two sides are involved. When finding a side, it is best to use the first form of the sine rule.



Remember:
side a is opposite $\angle A$
side b is opposite $\angle B$

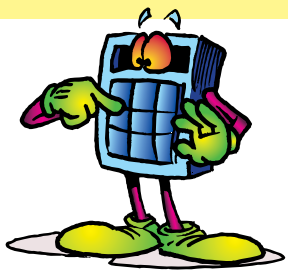
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 50°} = \frac{9}{\sin 38°}$$

$$\therefore x = \frac{9 \sin 50°}{\sin 38°}$$

$$\doteqdot 11{\cdot}2 \text{ cm}$$

Press: 9 × sin 50) ÷ sin 38 =



WORKED EXAMPLE 2

$\angle X = 32°50'$, $x = 15{\cdot}6$ cm, $y = 9{\cdot}7$ cm. Find $\angle Y$ correct to the nearest minute.

Always draw a diagram if one is not given with the question. When finding an angle, the second form of the sine rule is easier to use.

$$\frac{\sin Y}{y} = \frac{\sin X}{x}$$

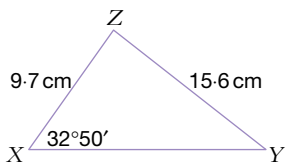
$$\therefore \frac{\sin Y}{9{\cdot}7} = \frac{\sin 32°50'}{15{\cdot}6}$$

$$\sin Y = \frac{9{\cdot}7 \sin 32°50'}{15{\cdot}6}$$

$$\doteqdot 0{\cdot}3371$$

Press: SHIFT sin 0.3371 = ° ' "

$$\therefore \angle Y = 19°42'$$



WORKED EXAMPLE 3

Town B is 20 km due east of A. If the bearing of town C is N35°E from A and N65°W from B find the distance from A to C and from B to C.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{AC}{\sin 25°} = \frac{20}{\sin 100°}$$

$$AC = \frac{20 \sin 25}{\sin 100°}$$

$$\doteqdot 8{\cdot}6 \text{ km}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{BC}{\sin 55°} = \frac{20}{\sin 100°}$$

$$BC = \frac{20 \sin 55°}{\sin 100°}$$

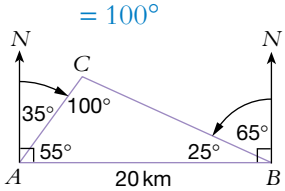
$$\doteqdot 16{\cdot}6 \text{ km}$$

$$\angle CAB = 90° - 35° = 55°$$

$$\angle CBA = 90° - 65° = 25°$$

$$\therefore \angle ACB = 180° - (55° + 25°)$$

$$= 100°$$



The distance AC is 8·6 km and the distance BC is 16·6 km (1 dec. pl.).

Exercise 15:04

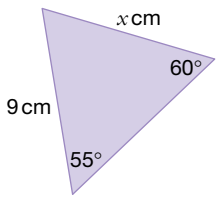
P Foundation worksheet 15:04
The sine rule

1 Solve the following equations.

a $\dfrac{a}{0{\cdot}866} = \dfrac{12{\cdot}5}{0{\cdot}48}$ **b** $\dfrac{a}{0{\cdot}4356} = \dfrac{7{\cdot}8}{0{\cdot}648}$ **c** $\dfrac{a}{\sin 20°} = \dfrac{12}{\sin 60°}$ **d** $\dfrac{a}{\sin 70°} = \dfrac{20{\cdot}6}{\sin 50°}$

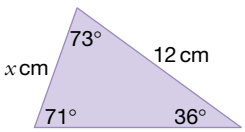
2 For each triangle, part of the sine rule has been written. Copy and complete the rule, then find x correct to one decimal place.

a



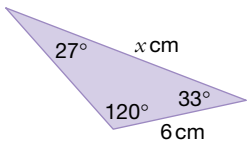
$\dfrac{x}{\sin 55°} = \dfrac{9}{\ldots}$

b



$\dfrac{x}{\ldots} = \dfrac{12}{\sin 71°}$

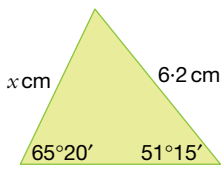
c



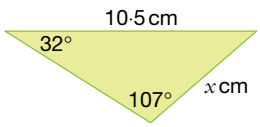
$\dfrac{x}{\ldots} = \dfrac{6}{\ldots}$

3 Find x in the following diagrams, correct to one decimal place.

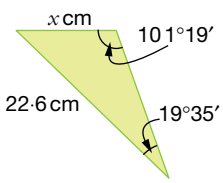
a



b



c



4 In each of the following, find the value of θ:

i correct to the nearest degree

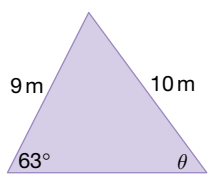
ii correct to the nearest minute.

a $\sin\theta = 0{\cdot}7178$ **b** $\sin\theta = 0{\cdot}8164$ **c** $\sin\theta = 0{\cdot}2$

d $\sin\theta = \dfrac{14{\cdot}6\sin 48°}{12{\cdot}6}$ **e** $\dfrac{\sin\theta}{11{\cdot}4} = \dfrac{\sin 63°}{16{\cdot}2}$

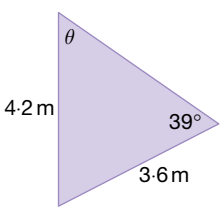
5 For each triangle, part of the sine rule has been written. Copy and complete the rule and then find the angle θ, correct to the nearest degree.

a



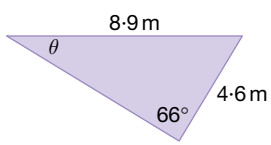
$\dfrac{\sin\theta}{9} = \dfrac{\sin 63°}{\ldots}$

b



$\dfrac{\sin\theta}{\ldots} = \dfrac{\sin 39°}{\ldots}$

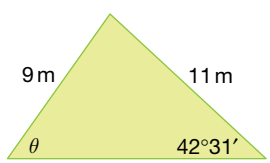
c



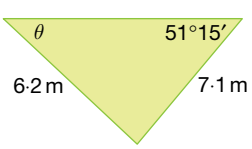
$\dfrac{\sin\theta}{\ldots} = \dfrac{\sin 66°}{\ldots}$

6 Find the value of θ to the nearest minute.

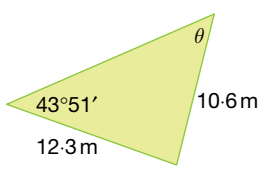
a



b

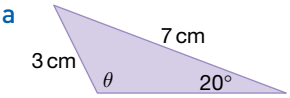


c

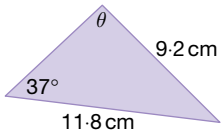


7 Find θ to the nearest degree, noting that in each triangle θ is obtuse.

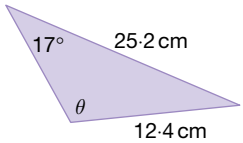
a



b



c



8 Answer the following by first drawing a diagram.

a In ΔABC, $\angle A = 30°$, $\angle B = 52°$, $a = 9$ cm; find b (1 dec. pl.).

b In ΔPQR, $\angle P = 51°$, $\angle R = 77°$, $r = 10{\cdot}2$ cm; find p (1 dec. pl.).

c In ΔXYZ, $\angle Y = 19°$, $y = 19{\cdot}1$ cm; $x = 15{\cdot}7$ cm; find $\angle X$ (nearest minute).

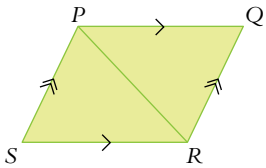
d In ΔLMN, $\angle M = 37°$, $m = 8{\cdot}7$ m; $n = 4{\cdot}6$ m; find $\angle N$ (nearest minute).

9 $PQRS$ is a parallelogram, where $PQ = 8$ cm, $QR = 5$ cm, and $\angle PRQ = 68°$.
Find:

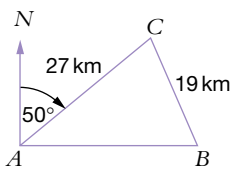
a $\angle RPQ$ (nearest degree)

b $\angle PQR$ (nearest degree)

c diagonal PR (1 dec. pl.)



10



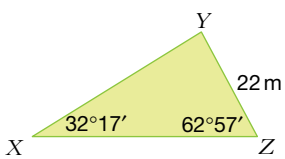
Three towns are situated so that the distance from A to C is 27 km, the distance from B to C is 19 km and the bearing of C from A is N50°E. If B is due east of A, find:

a $\angle ABC$ (nearest degree)

b $\angle ACB$ (nearest degree)

c distance of B from A (2 sig. fig.).

11 Three posts X, Y and Z are situated so that $\angle X$ is 32°17′ and $\angle Z$ is 62°57′. If the distance between Y and Z is 22 m, find the distance between X and Y to the nearest metre.



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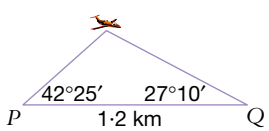
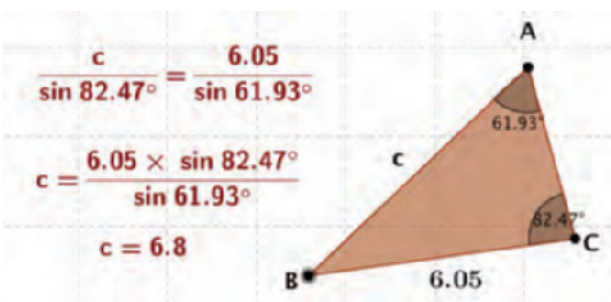


A man at P observes a plane to the south at an angle of elevation of 42°25′, while a second man at Q observes the same plane to the north at an angle of elevation of 27°10′. If the distance between the observers is 1·2 km, find the distances of each observer from the plane, to the nearest metre.

P GEOGEBRA ACTIVITY 15:04 THE SINE RULE

Click and drag the vertices of a triangle and verify the sine rule for varying angle sizes and side lengths. Then use the sine rule to calculate the values of either a side or an angle. Tick the checkbox to see if you are correct.



15:05 The sine rule: The ambiguous case

When an equation like $\sin\theta = 0{\cdot}5$ is solved, there is an acute and an obtuse answer.
Hence, when the sine rule is used to find an angle, it is possible to have two solutions.
When this occurs, we have what is called the ambiguous case.

- The acute angle will always be a solution.
- The obtuse angle will only be a solution if it gives an angle sum less than 180° when added to the other angles in the triangle.

Both the one-solution and two-solution cases are shown in the examples below.

WORKED EXAMPLES

1 Use the sine rule to find the size of angle C.

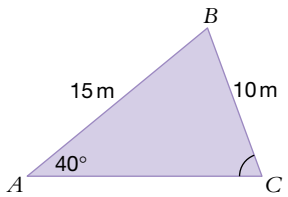
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{15} = \frac{\sin 40°}{10}$$

$$\sin C = \frac{15\sin 40°}{10}$$

$$\doteqdot 0{\cdot}964$$

$\therefore C = 74{\cdot}6°$ (from the calculator)



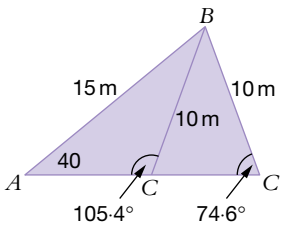
However:

$\sin(180° - 74{\cdot}6°)$ is also equal to $0{\cdot}964$.

$\therefore C = 74{\cdot}6°$ or $105{\cdot}4°$

$105{\cdot}4° + 40° < 180°$, so $105{\cdot}4°$ is also a solution.

The diagram shows the two possible solutions.



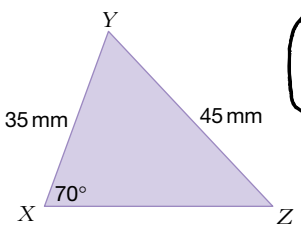
2 Find the size of angle Z.

$$\frac{\sin Z}{35} = \frac{\sin 70°}{45}$$

$$\sin Z = \frac{35\sin 70°}{45}$$

$$\doteqdot 0{\cdot}731$$

$\therefore Z \doteqdot 47°$



$\sin(180° - 47°)$ is also equal to $0{\cdot}73$

$\therefore Z = 47°$ or $133°$ (to the nearest degree)

$70° + 133° > 180°$, so it is impossible for Z to be $133°$.

We therefore reject the answer $133°$.

$\therefore Z = 47°$

Exercise 15:05

P Foundation worksheet 15:05
The sine rule: The ambiguous case

1 Find the acute and obtuse angles that are solutions to the equations.

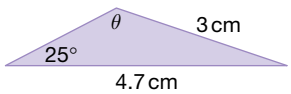
a $\sin\theta = 0{\cdot}8563$

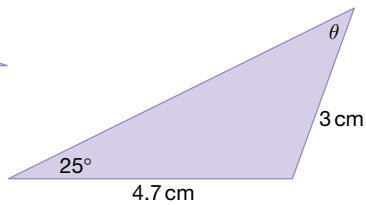
b $\sin\theta = 0{\cdot}216$

c $\sin\theta = \dfrac{15\sin 60°}{20}$

d $\sin\theta = \dfrac{20\sin 15°}{15}$

2





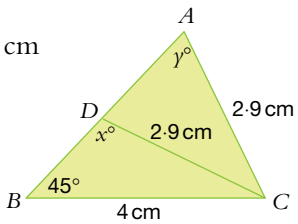
In both triangles, $\sin\theta = \dfrac{4{\cdot}7\sin 25°}{3}$. Find θ in both triangles.

Give answers to the nearest degree.

3 Triangles ABC and DBC both have sides 4 cm and 2·9 cm in length and a non-included angle of 45°.

a Show that $\sin x°$ and $\sin y°$ both equal $\dfrac{4\sin 45°}{2{\cdot}9}$.

b Find $x°$ and $y°$ to the nearest degree.



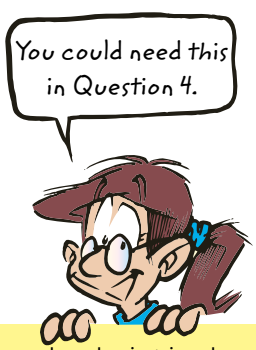
4 Find (to the nearest minute) the angle required in each of the following.

a In ΔXYZ, $\angle X = 30°$, $x = 5$ cm, $y = 9$ cm; find $\angle Y$.

b In ΔPQR, $\angle Q = 19°20'$, $q = 2{\cdot}9$ cm, $r = 3{\cdot}7$ cm; find $\angle R$.

c In ΔABC, $\angle B = 32°17'$, $b = 10{\cdot}7$ cm, $a = 12{\cdot}1$ cm; find $\angle A$.

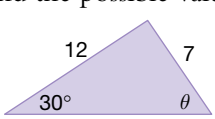
d In ΔKLM, $\angle M = 27°51'$, $m = 8{\cdot}7$ cm, $k = 9{\cdot}8$ cm; find $\angle K$.



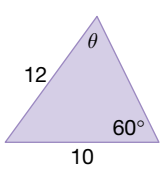
Naming sides and angles in triangles:
x is opposite $\angle X$, y is opposite $\angle Y$
m is opposite $\angle M$, k is opposite $\angle K$

5 Find the possible values of θ in each of the following.

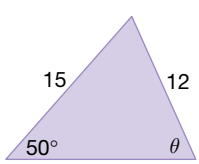
a



b



c



15:06 The cosine rule

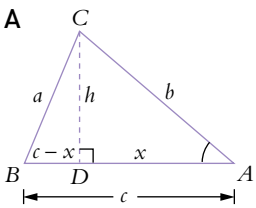
Like the sine rule, the cosine rule can be used to find unknown sides or angles in triangles. It is applied to situations where three sides and one angle are involved.

In the triangles below:

- CD is perpendicular to AB.
- Let AD be equal to x.
- In triangle A, $BD = c - x$.

Using Pythagoras' theorem:

A



From ΔBCD:

$$a^2 = (c - x)^2 + h^2$$
$$= c^2 - 2cx + x^2 + h^2 \quad 1$$

From ΔACD:

$$b^2 = x^2 + h^2$$

Substituting in 1 gives:

$$a^2 = c^2 - 2cx + b^2$$
$$a^2 = b^2 + c^2 - 2cx \quad 2$$

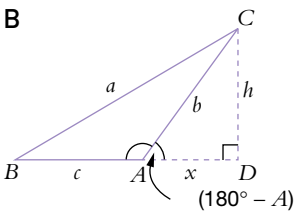
From ΔACD:

$$\frac{x}{b} = \cos A$$
$$x = b \cos A$$

Substituting in 2 gives:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

B



From ΔBCD:

$$a^2 = (c + x)^2 + h^2$$
$$= c^2 + 2cx + x^2 + h^2 \quad 3$$

From ΔACD:

$$b^2 = x^2 + h^2$$

Substituting in 3 gives:

$$a^2 = c^2 + 2cx + b^2$$
$$a^2 = b^2 + c^2 + 2cx \quad 4$$

From ΔACD:

$$\frac{x}{b} = \cos(180° - A)$$
$$x = b \cos(180° - A)$$
$$= -b \cos A$$

Substituting in 4 gives:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

For any triangle ABC:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

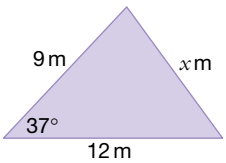
To find an angle, this rule can be rearranged to give:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

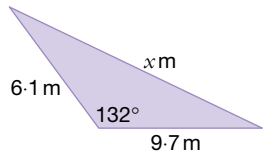
WORKED EXAMPLE 1

Find x in each triangle, correct to one decimal place.

a



b



Solution

a $a^2 = b^2 + c^2 - 2bc\cos A$

Applying this formula gives:

$$x^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \cos 37°$$
$$= 144 + 81 - 216 \cos 37°$$
$$= 52{\cdot}494\,729$$
$$\therefore x = \sqrt{52{\cdot}494\,729}$$
$$= 7{\cdot}2 \quad \text{(1 dec. pl.)}$$

b $a^2 = b^2 + c^2 - 2bc\cos A$

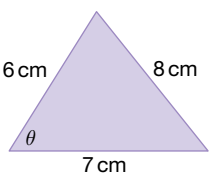
That is:

$$x^2 = 6{\cdot}1^2 + 9{\cdot}7^2 - 2 \times 6{\cdot}1 \times 9{\cdot}7 \cos 132°$$
$$= 37{\cdot}21 + 94{\cdot}09 - 118{\cdot}34 \cos 132°$$
$$= 210{\cdot}484\,91$$
$$x = \sqrt{210{\cdot}484\,91}$$
$$= 14{\cdot}5 \quad \text{(1 dec. pl.)}$$

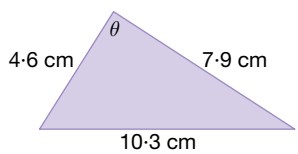
WORKED EXAMPLE 2

Evaluate θ, correct to the nearest minute.

a



b



Solution

a $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Applying this formula gives:

$$\cos\theta = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$$
$$= \frac{36 + 49 - 64}{84}$$
$$= 0{\cdot}25$$
$$\therefore \theta = 75°31' \text{ (nearest minute)}$$

b $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

That is:

$$\cos\theta = \frac{4{\cdot}6^2 + 7{\cdot}9^2 - 10{\cdot}3^2}{2 \times 4{\cdot}6 \times 7{\cdot}9}$$
$$= \frac{21{\cdot}16 + 62{\cdot}41 - 106{\cdot}09}{72{\cdot}68}$$
$$\doteqdot -0{\cdot}3098$$
$$\therefore \theta = 108°03' \quad \text{(nearest minute)}$$

The cosine rule can be used in any triangle to find:

- the third side, given the other sides and their included angle
- any angle, given the three sides.

Exercise 15:06

P Foundation worksheet 15:06
The cosine rule

1 Find a to one decimal place if $a^2 = b^2 + c^2 - 2bc\cos A$ and:

a $b = 10, c = 12, A = 60°$

b $b = 6{\cdot}8, c = 9{\cdot}4, A = 120°$

c $b = 15, c = 12, A = 75°$

d $b = 24{\cdot}5, c = 18{\cdot}2, A = 45°$

2 Find A to the nearest degree if:

a $\cos A = 0{\cdot}6$

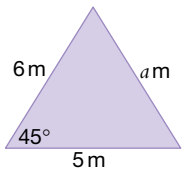
b $\cos A = -0{\cdot}6$

c $\cos A = \dfrac{6^2 + 10^2 - 8^2}{2 \times 6 \times 10}$

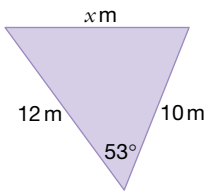
d $\cos A = \dfrac{6^2 + 8^2 - 12^2}{2 \times 6 \times 8}$

3 Find the value of the pronumeral in the following, correct to one decimal place.

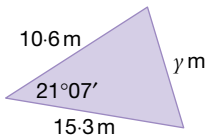
a



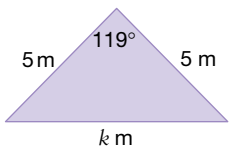
b



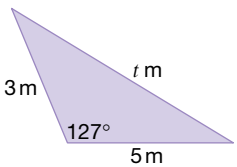
c



d



e



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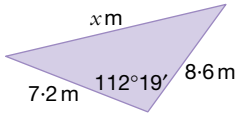
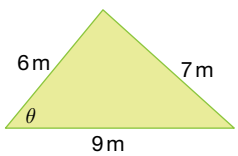


4 Find the size of angle θ, correct to the nearest minute.

a



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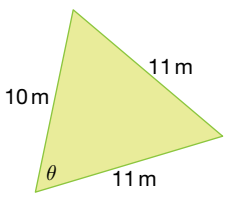

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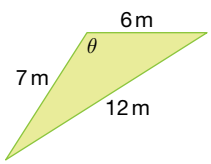

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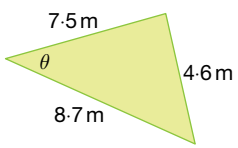

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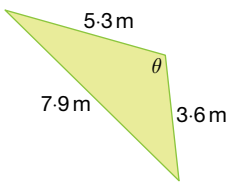

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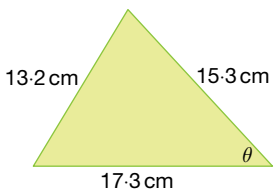


5 Answer the following by first drawing a diagram.

a In ΔABC, $\angle A = 75°$, $b = 9$ cm, $c = 5$ cm; find a (1 dec. pl.).

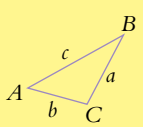
b In ΔDEF, $\angle F = 61°$, $d = 2{\cdot}3$ m, $e = 3{\cdot}1$ m; find f (1 dec. pl.).

c In ΔLMN, $\angle M = 163°$, $l = 9{\cdot}4$ cm, $n = 8{\cdot}2$ cm; find m (1 dec. pl.).

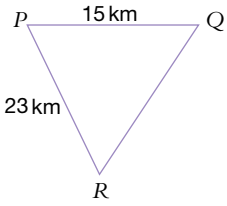
d In ΔPQR, $p = 9$ m, $q = 7$ m, $r = 8$ m; find $\angle P$ (nearest degree).

e In ΔTUV, $t = 2{\cdot}3$ cm, $u = 1{\cdot}9$ cm, $v = 1{\cdot}7$ cm; find $\angle V$ (nearest minute).

Angle A is opposite side a.



6 Three towns, P, Q and R, are connected by straight roads. The distance from P to Q is 15 km and the distance from P to R is 23 km. If the roads meet at P at an angle of 50°, how far is town Q from town R, to the nearest kilometre?



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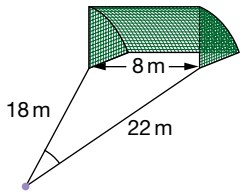
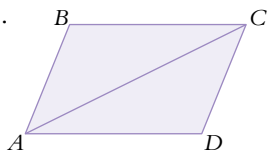
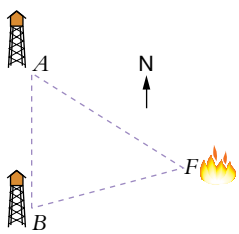


If a soccer goal is 8 m wide and a player shoots for goal when he is 18 m from one post and 22 m from the other, within what horizontal angle must the shot be made to score the goal? Give your answer correct to the nearest degree.

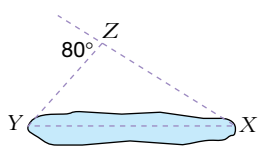
8 In a parallelogram $ABCD$, AB is 9 cm, BC is 15 cm, and $\angle ABC$ is 130°. Find the length of the diagonal AC, correct to the nearest millimetre.



9 An observer at fire tower A observes a fire 18 km away to have a bearing S67°E. How far is the fire from tower B if this tower is 12 km due south of tower A?

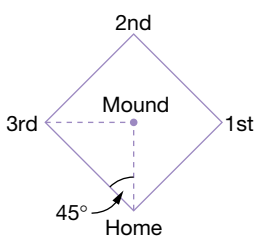


10

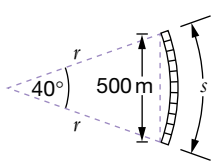


To find the length of a pond, a surveyor walks 350 m from point X to point Z, then turns 80° and walks 290 m to point Y. Find the length of the pond to the nearest metre.

11 In a (square) baseball diamond with sides of length 27·4 m, the centre of the pitcher's mound is 18·4 m from home plate. How far is it from the centre of the pitcher's mound to third base? Give your answer correct to one decimal place.



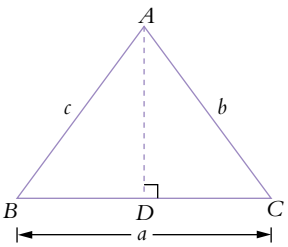
12 The section of railway track shown is a circular arc that has a chord length of 500 m and subtends an angle of 40° at the centre of the circle. Find the radius r of the circular arc and the length s of that arc.

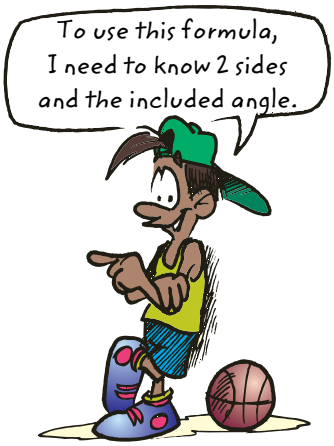


15:07 Area of a triangle

To calculate the area of a triangle, the formula $A = \frac{1}{2}bh$ is used, where b is the length of the base and h is the perpendicular height. So, to calculate the area of any triangle, these two measurements need to be known.

Now, consider the triangle ABC below.





AD is perpendicular to BC.
The area of ΔABC is given by:

$\text{Area} = \frac{1}{2}BC \times AD$ 1

From ΔACD,

$\frac{AD}{AC} = \sin C$

$AD = AC \sin C$

Substituting this into 1,

$\text{Area} = \frac{1}{2}BC \times AC \sin C$

$= \frac{1}{2}ab \sin C$

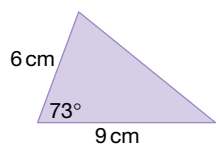
$\text{Area} = \frac{1}{2}ab \sin C$

where a and b are two sides and C is the included angle.

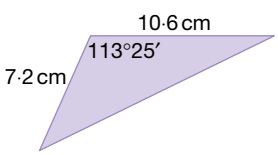
WORKED EXAMPLES

Find the area of each triangle correct to the nearest square centimetre.

1



2



Solutions

1 $A = \frac{1}{2}ab \sin C$

That is:

$A = \frac{1}{2} \times 6 \times 9 \times \sin 73°$

$= 25{\cdot}820\,228 \text{ cm}^2$

$= 26 \text{ cm}^2$ (to nearest cm^2)

2 $A = \frac{1}{2}ab \sin C$ C

That is:

$A = \frac{1}{2} \times 10{\cdot}6 \times 7{\cdot}2 \times \sin 113°25'$

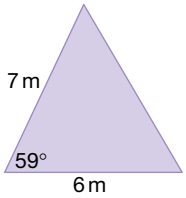
$= 35{\cdot}017\,106 \text{ cm}^2$

$= 35 \text{ cm}^2$ (to nearest cm^2)

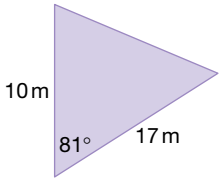
Exercise 15:07

1 Find the area of each triangle, correct to one decimal place.

a



b



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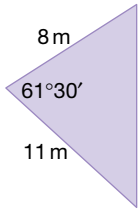

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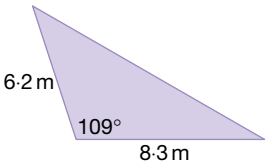
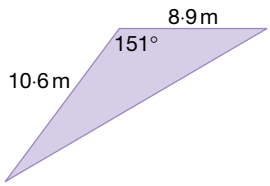


e



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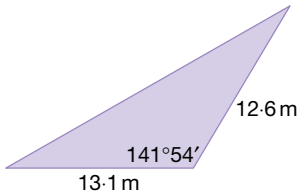
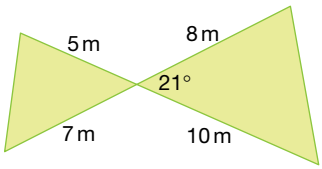
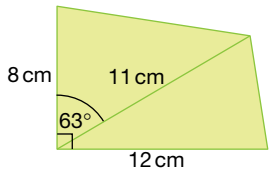


2 Find the area enclosed by each figure to the nearest square centimetre.

a



b



3 For ΔXYZ, find its area, to the nearest square unit, if:

a $x = 7, y = 10, Z = 47°$

b $x = 14{\cdot}6, y = 17{\cdot}2, Z = 72°31'$

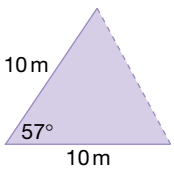
c $y = 2{\cdot}3, z = 3{\cdot}9, X = 62°$

d $y = 52, z = 63, X = 127°55'$

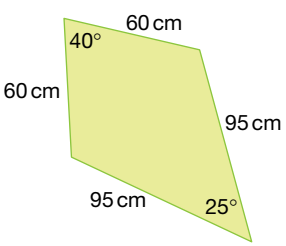
e $x = 20, z = 31, Y = 53°24'$

f $x = 72{\cdot}3, z = 91{\cdot}6, Y = 142°07'$

4 Two paths diverge from a point at an angle of 57°. If the length of each path is 10 m, calculate the area of lawn in between the two paths to the nearest square metre.

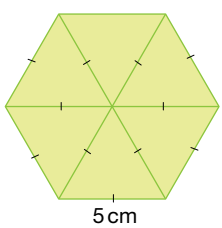


5 **a**



Calculate the area of the kite correct to the nearest square centimetre.

b



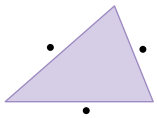
Calculate the area of the hexagon correct to one decimal place.

15:08 Miscellaneous problems

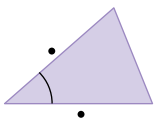
When solving miscellaneous problems, we need to identify which trigonometry rule should be used. The standard congruence tests for triangles can be used to help decide whether the information in a question applies to the sine rule or cosine rule.

To use the cosine rule you need:

- Three sides (SSS).
  This allows you to find all the angles.

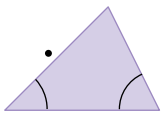


- Two sides and an included angle (SAS).
  This allows you to find all other angles and the other side.

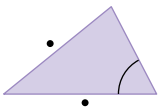


To use the sine rule you need:

- Two angles and a side (AAS).
  This allows you to find the other angle and the other sides.



- Two sides and a non-included angle (SSA).
  This allows you to find the unknown angle opposite the known side. There could be two solutions for the angle, so this is the ambiguous case. (Hence, SSA is *not* a congruence test.)

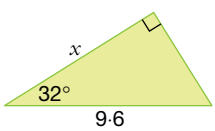


Exercise 15:08

P Foundation worksheet 15:08
Sine rule or cosine rule?

1 Use the information above to identify whether the sine rule or the cosine rule is needed and then find the value of the pronumeral correct to one decimal place.

a



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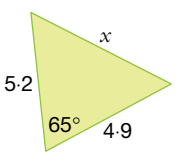
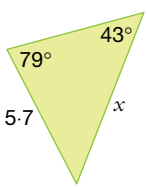
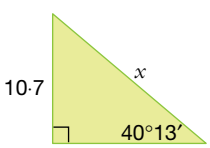


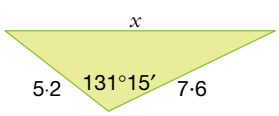
c



d

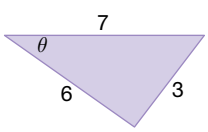


e

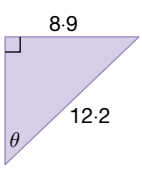


2 Evaluate θ, correct to the nearest minute.

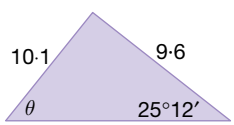
a



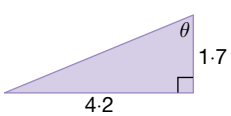
b



c



d



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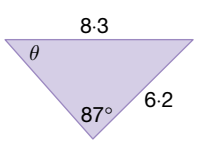

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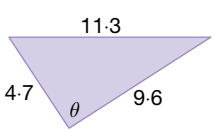
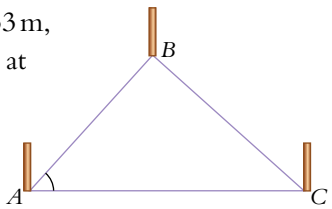


3 For a triangle XYZ:

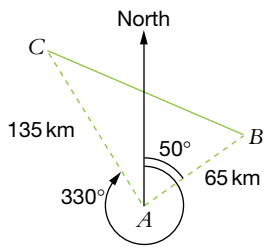
- **a** Find $\angle X$ correct to the nearest minute if $x = 9{\cdot}2$, $y = 5{\cdot}6$, $\angle Z = 90°$.
- **b** Find x correct to one decimal place if $\angle X = 36°$, $\angle Y = 47°$, $y = 9{\cdot}6$.
- **c** Find z correct to one decimal place if $\angle Z = 14°25'$, $x = 4{\cdot}2$, $y = 3{\cdot}7$.
- **d** Find x correct to one decimal place if $\angle Y = 95°17'$, $\angle Z = 47°05'$, $y = 11{\cdot}3$.
- **e** Find x correct to one decimal place if $\angle X = 90°$, $\angle Y = 23°17'$, $y = 5{\cdot}2$.
- **f** Find $\angle Y$ correct to the nearest minute if $x = 10{\cdot}3$, $y = 9{\cdot}6$, $z = 8{\cdot}7$.

4 Three posts, A, B and C are positioned so that AB is 33 m, BC is 45 m and AC is 51 m. Find the angle subtended at post A by BC to the nearest minute.

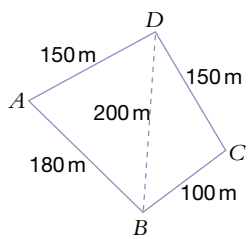


5 P and Q are two points on the shore 950 m apart. R is a buoy out at sea so that the angles RPQ and RQP are 73°19′ and 68°32′, respectively. Find the distance of P from R.

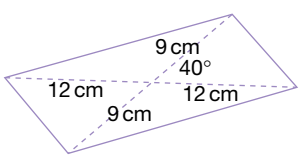
6 From town A, the bearings of towns B and C are 050° and 330° respectively. If A is 65 km from B and 135 km from C, how far is town B from town C?



7 A farmer has a piece of land as shown in the diagram. What is the area of the land, to the nearest square metre?



8 The diagonals of a parallelogram bisect each other at an angle of 40°.

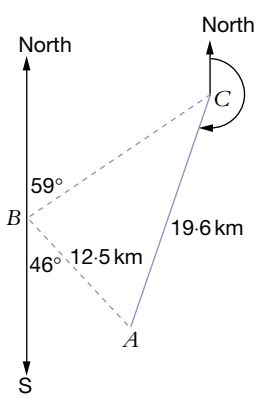


If the diagonals are 24 cm and 18 cm long, find:

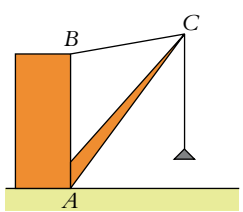
- **a** the area of the parallelogram
- **b** the lengths of the sides of the parallelogram.

Give answers correct to one decimal place.

9 Town A is 12·5 km S46°E of town B; town C is 19·6 km from town A and C bears N59°E from B. Find the bearing of A from C.



10 The diagram represents a crane where AB is 7·5 m, AC is 12·6 m and BC is 9·6 m. Find the angle ABC and the height of C above the ground.



15:09 Three-dimensional problems

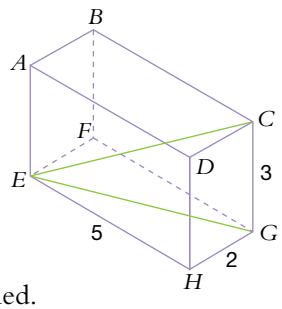
Right-angled triangles can be found in many three-dimensional objects. Trigonometric methods can be used to calculate the lengths of sides and the sizes of angles. Pythagoras' theorem can also be used to calculate the length of sides.

WORKED EXAMPLE 1

For the rectangular prism shown, find:

a the length EC

b $\angle CEG$



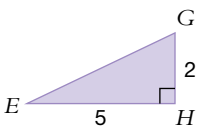
Solution

Triangles EGC and EHG are both right-angled.

a In ΔEHG:

$EG^2 = 5^2 + 2^2 = 29$

$\therefore EG = \sqrt{29}$ cm

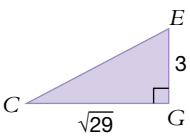


In ΔEGC:

$EC^2 = (\sqrt{29})^2 + 3^2 = 38$

$\therefore EC = \sqrt{38}$ cm

$= 6{\cdot}16$ cm (2 dec. pl.)



b In ΔEGC:

$\tan \angle CEG = \dfrac{3}{\sqrt{29}}$

$\doteqdot 0{\cdot}5571$

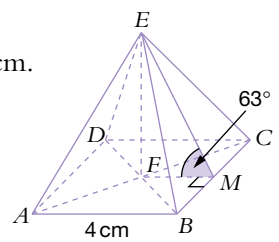
$\therefore \angle CEG = 29°07'$ (nearest minute)

WORKED EXAMPLE 2

The diagram shows a right square pyramid with a base edge of length 4 cm.
If $\angle EMF = 63°$, find correct to two decimal places:

a the length EF

b the length EB



Solution

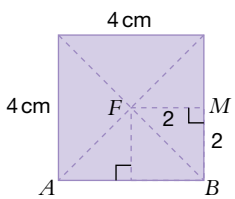
a In ΔEMF:

$\tan 63° = \dfrac{EF}{FM}$

$= \dfrac{EF}{2}$

$\therefore EF = 2 \times \tan 63°$

$\doteqdot 3{\cdot}93$ cm



b F is the centre of the square base:

$FB^2 = 2^2 + 2^2 = 8$

$\therefore FB = \sqrt{8}$ cm

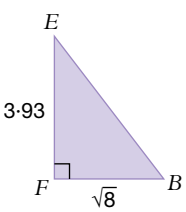
In ΔEFB:

$EB^2 = FB^2 + EF^2$

$= 8 + 2\tan 63°$

$\doteqdot 23{\cdot}407$

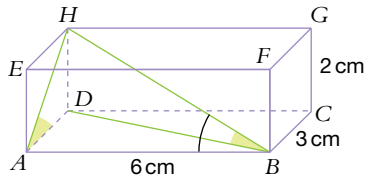
$\therefore EB = 4{\cdot}84$ cm (2 dec. pl.)



Exercise 15:09

P Foundation worksheet 15:09
Problems with more than one triangle

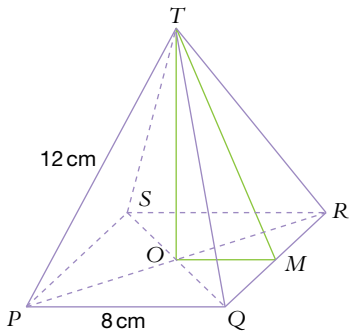
1 The figure shows a rectangular prism.



a Calculate the lengths:
i HA ii BD iii BH

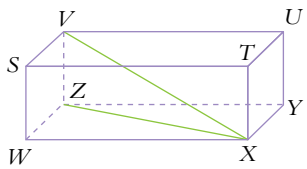
b Calculate the size of the angles, correct to the nearest degree:
i $\angle HAD$ ii $\angle HBD$ iii $\angle HBA$

2 The figure shown is a right square pyramid with base edges of 8 cm and slant edges of 12 cm. Calculate:



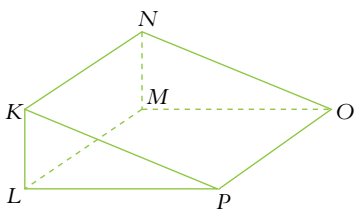
a the length of PR (3 sig. fig.)
b the height OT (3 sig. fig.)
c the size of $\angle TPQ$ (nearest degree)
d the size of $\angle TPO$ (nearest degree)
e the size of $\angle TMO$ where M, is the midpoint of QR.

3



The figure drawn is a rectangular prism where $WX = 7$ cm and $WZ = 5$ cm. Given that $\angle VXZ = 21°43'$, calculate the height of the prism, VZ, to the nearest millimetre.

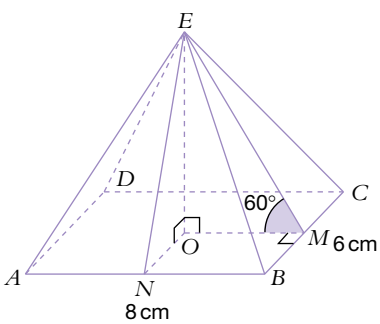
4 The figure drawn is a triangular prism where $KL \perp LP$ Also $KL = 5$ cm, $KN = 13$ cm and $\angle KPL = 15°$.



a Find the following lengths to the nearest millimetre.
i KP ii LP iii PM iv PN

b Find the size of the following angles to the nearest degree.
i $\angle KPN$ ii $\angle MPN$

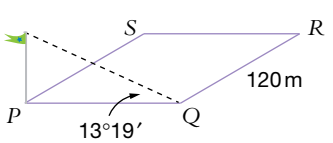
5 $ABCDE$ is a rectangular pyramid with $AB = 8$ cm and $BC = 6$ cm. The point O is the centre of the rectangular base and the triangular face EBC makes an angle of 60° with the rectangular base as shown.



a Find, correct to two decimal places where necessary, the lengths of:
i OE ii EN iii EM iv EB

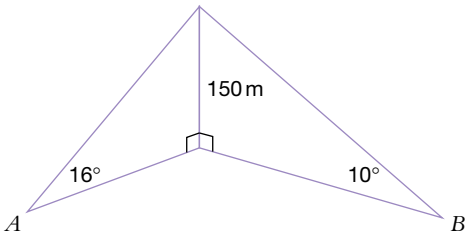
b Find, correct to the nearest degree, the sizes of:
i $\angle ENO$ ii $\angle OBE$

6 A flagpole stands at one corner P of a level square field $PQRS$, each side of which is 120 m. If the angle of elevation of the top of the pole from Q is $13°19'$, find:



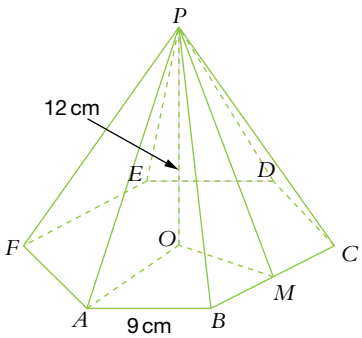
a the height of the flagpole, correct to the nearest centimetre
b its angle of elevation from R, correct to the nearest degree.

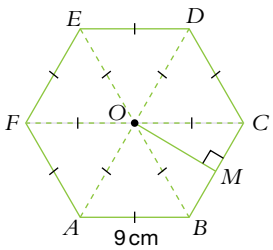
7 A monument is erected on top of a building. The top of the monument is 150 m above the ground. From a point A due south of the monument, the angle of elevation of the top of the monument is 16°. From a point B that is due east of the monument, the angle of elevation is 10°. If A and B are on level ground, find the distance from A to B to the nearest metre.



8 From a viewing platform at the top of a building 110m high, the angles of depression of two points A and B are 28° and 17° respectively. If A is due east of the viewing position and B is due south of it, and both are on level ground, find the distance AB to the nearest metre.

9 A right hexagonal pyramid has base edges of 9 cm and a height of 12 cm.



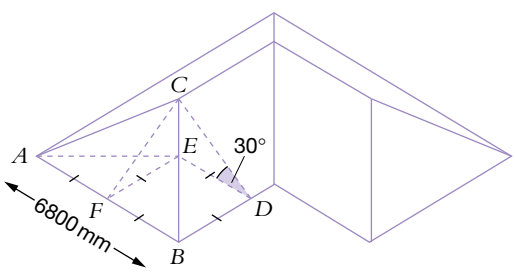


Top view of base

a Find the angle that a slant edge makes with the base (e.g. $\angle OAP$), correct to the nearest degree.

b Find the angle one of the triangular faces makes with the base (e.g. $\angle OMP$), correct to the nearest degree.

10 The diagram shows part of a roof construction. Triangles ACE, FCE and DCE are all right-angled at E. The points A, F, B, D and E are all in the same plane and F is the midpoint of AB. C is directly above E, $\angle EDC = 30°$ and $AB = 6800$ mm.



a Find the following lengths to the nearest millimetre.

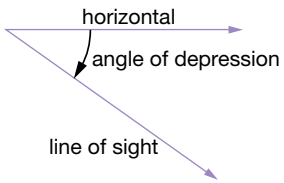
i CE **ii** CD **iii** AC

b Calculate the size of $\angle EAC$ to the nearest degree.

MATHS TERMS 15

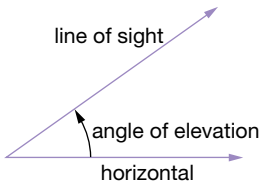
angle of depression

- when looking down, the angle between the line of sight and the horizontal



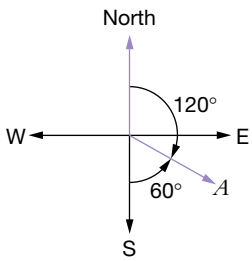
angle of elevation

- when looking up, the angle between the line of sight and the horizontal



bearing

- an angle used to measure the direction of a line from north
- bearings can be recorded in two ways, e.g. 120° or S60°E



cosine rule

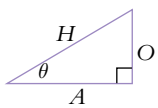
- a rule used to find either the third side of a triangle when the other two sides and the included angle (SAS) are known, or an angle when the three sides are known (SSS)

sine rule

- a rule used to find an angle or side of a triangle when either two angles and a side (AAS), or two sides and a non-included angle (SSA), are known

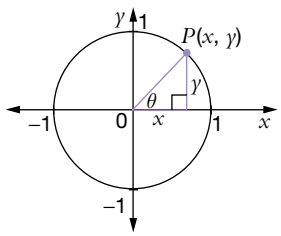
trigonometric (trig) ratios

- a set of ratios (sine, cosine and tangent) that have constant values for any particular angle
- for acute angles, these can be defined in terms of the side lengths of a right-angled triangle



$$\sin\theta = \frac{O}{H}, \cos\theta = \frac{A}{H}, \tan\theta = \frac{O}{A}$$

- for acute or obtuse angles, they are defined in terms of the coordinates of a point P that has its position on a unit circle determined by a radius drawn at an angle of θ to the horizontal



$$\sin\theta = y, \cos\theta = x, \tan\theta = \frac{y}{x}$$

DIAGNOSTIC TEST 15 FURTHER TRIGONOMETRY

Each part of this test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Find x if: **15:01**

a $\sin x° = \cos 50°$ **b** $\cos x° = \sin 20°$ **c** $\sin 30° = \cos x°$

2 Write each of the following in the form $\tan\theta$. **15:01**

a $\dfrac{\sin 30°}{\cos 30°}$ **b** $\dfrac{\cos 50°}{\cos 40°}$ **c** $\dfrac{\sin 20°}{\sin 70°}$

3 Write the obtuse angle with the same sine ratio as: **15:03**

a $30°$ **b** $50°$ **c** $80°$

4 Write as the trig ratio of an acute angle: **15:03**

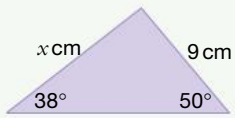
a $\sin 140°$ **b** $\cos 140°$ **c** $\tan 140°$

5 If $90° < \theta < 180°$, find θ correct to the nearest minute, if: **15:03**

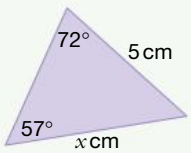
a $\cos\theta = -0{\cdot}625$ **b** $\sin\theta = 0{\cdot}257$ **c** $\cos\theta = -0{\cdot}018$

6 Find the value of x, correct to one decimal place. **15:04**

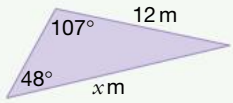
a



b

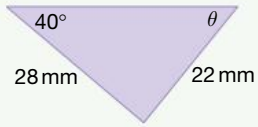


c

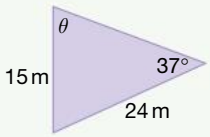


7 Find the value of θ, correct to the nearest degree. **15:04, 15:05**

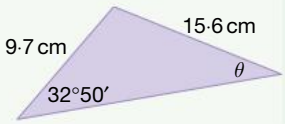
a



b

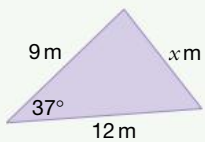


c

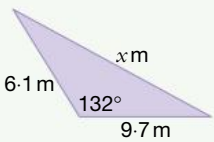


8 Find the value of x, correct to one decimal place. **15:06**

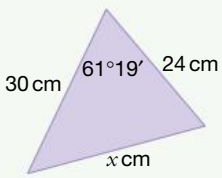
a



b

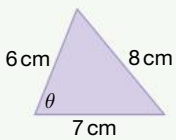


c

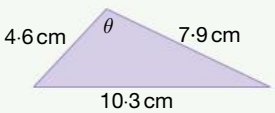


9 Evaluate θ, correct to the nearest minute. **15:06**

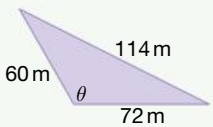
a



b

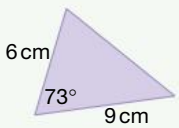


c

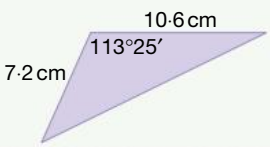


10 Find the area of each triangle, correct to the nearest square centimetre. **15:07**

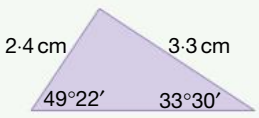
a



b



c



ASSIGNMENT 15A Chapter review

1 Find θ if θ is obtuse and:

a $\sin\theta = 0{\cdot}5$ **b** $\cos\theta = -0{\cdot}5$

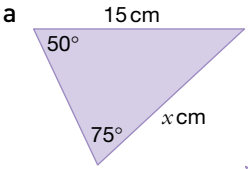
c $\tan\theta = -1$

2 Write the exact value of:

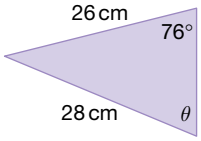
a $\sin 60°$ **b** $\cos 45°$ **c** $\tan 30°$

3 Use the sine rule to find the value of the pronumeral in each of the following.

a



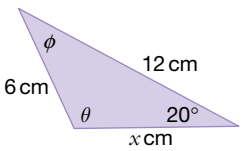
b



4 Find θ, ϕ and x if:

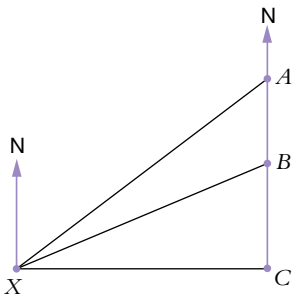
a θ is acute

b θ is obtuse.



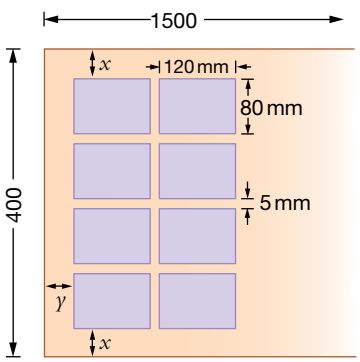
5 A triangle has sides that are 8 cm, 12 cm and 10 cm in length. Find the size of the largest angle.

6 From a position X on a map, A has a bearing of N27°E and B has a bearing of N67°E. Town C is 7·5 km due east of X. Find the distance between A and B (1 dec. pl.).



ASSIGNMENT 15B Working mathematically

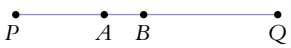
1



A rectangular table top has a length of 1500 mm and a breadth of 400 mm. The top has 4 rows of 11 tiles in the centre. Each rectangular tile is 120 mm by 80 mm. There is a 5 mm gap between each row of tiles. Find x and y in the diagram.

2 A ball is known to rebound to half the height that it drops. If the ball is initially dropped from a height of 12 m, how far will it have travelled by the time it hits the ground for the 6th time?

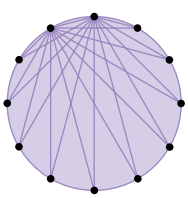
3 The distance from P to Q is 36 cm.



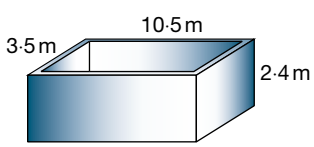
How far is the point A from P if it is four times as far from Q as it is from B, the midpoint of PQ?

4 Every hour a snail climbs 30 cm up a wall, but slides back 10 cm. How long will it take to climb up a 2·98 m wall?

5 If every point on this diagram has to be connected to every other point, how many straight-line segments will there be?



6 A steel tank is as shown in the diagram.



Given that the dimensions are external dimensions and that the steel plate is 2 cm thick, calculate the mass of the tank if the density of the steel is 7·8 g/cm^3 (1 dec. pl.).

Answers

ID Cards

ID Card 1 (Metric Units)

1 metres	**2** decimetres	**3** centimetres	**4** millimetres
5 kilometres	**6** square metres	**7** square centimetres	**8** square kilometres
9 hectares	**10** cubic metres	**11** cubic centimetres	**12** seconds
13 minutes	**14** hours	**15** metres per second	**16** kilometres per hour
17 grams	**18** milligrams	**19** kilograms	**20** tonnes
21 litres	**22** millilitres	**23** kilolitres	**24** degrees Celsius

ID Card 2 (Symbols)

1 is equal to	**2** is approximately equal to	**3** is not equal to
4 is less than	**5** is less than or equal to	**6** is not less than
7 is greater than	**8** is greater than or equal to	**9** 4 squared
10 4 cubed	**11** the square root of 2	**12** the cube root of 2
13 is perpendicular to	**14** is parallel to	**15** is congruent to
16 is similar to	**17** per cent	**18** therefore
19 for example	**20** that is	**21** pi
22 the sum of	**23** the mean	**24** probability of event E

ID Card 3 (Language)

1 $6 - 2 = 4$	**2** $6 + 2 = 8$	**3** $6 \div 2 = 3$	**4** $6 - 2 = 4$
5 $6 \div 2 = 3$	**6** 2	**7** 6	**8** $6 \times 2 = 12$
9 $6 - 2 = 4$	**10** $6 \times 2 = 12$	**11** $2 + 6 = 8$	**12** $6 - 2 = 4$
13 $6^2 = 36$	**14** $\sqrt{36} = 6$	**15** $6 - 2 = 4$	**16** $6 \times 2 = 12$
17 $(6 + 2) \div 2 = 4$	**18** $6 + 2 = 8$	**19** $6^2 = 36$	**20** $6 - 2 = 4$
21 $6 - 2 = 4$	**22** $6 + 2 = 8$	**23** $6 \div 2 = 3$	**24** $6 + 2 = 8$

ID Card 4 (Language)

1 square	**2** rectangle	**3** parallelogram	**4** rhombus
5 trapezium	**6** regular pentagon	**7** regular hexagon	**8** regular octagon
9 kite	**10** scalene triangle	**11** isosceles triangle	**12** equilateral triangle
13 circle	**14** oval (or ellipse)	**15** cube	**16** rectangular prism
17 triangular prism	**18** square pyramid	**19** rectangular pyramid	**20** triangular pyramid
21 cylinder	**22** cone	**23** sphere	**24** hemisphere

ID Card 5 (Language)

1 point A	**2** interval AB	**3** line AB	**4** ray AB
5 collinear points	**6** midpoint	**7** number line	**8** diagonals
9 acute-angled triangle	**10** right-angled triangle	**11** obtuse-angled triangle	**12** vertices
13 ΔABC	**14** hypotenuse	**15** $180°$	**16** $a° + b°$
17 $360°$	**18** (*b*) $a° = b°$	**19** $a° = 60°$	**20** $3 \times 180° = 540°$
21 AB is a diameter: OC is a radius.		**22** circumference	**23** semicircle
24 AB is a tangent. CD is an arc. EF is a chord.			

ID Card 6 (Language)

1 parallel lines
2 perpendicular lines
3 vertical, horizontal
4 concurrent lines
5 angle ABC or CBA
6 *acute* angle
7 *right* angle
8 *obtuse* angle
9 straight angle
10 reflex angle
11 revolution
12 adjacent angles
13 complementary angles
14 supplementary angles
15 vertically opposite angles
16 360°
17 transversal
18 corresponding angles
19 alternate angles
20 co-interior angles
21 *bisecting* an interval
22 *bisecting* an angle
23 $\angle CAB = 60°$
24 CD is *perpendicular* to AB

ID Card 7 (Language)

1 Common Era, Anno Domini
2 Before Common Era, Before Christ
3 ante meridiem, before midday
4 post meridiem, after midday
5 hectare
6 regular shapes
7 net of a cube
8 cross-section
9 face
10 vertex
11 edge
12 axes of symmetry
13 reflection (or flip)
14 translation (or slide)
15 rotation (or turn)
16 tessellation
17 coordinates
18 tally
19 picture graph
20 column graph
21 line graph
22 sector (or pie) graph
23 bar graph
24 scatter diagram

Chapter 1

Exercise 1:01A

1 a 2 b 1 c 0 d 14 e −3
f 17 g 12 h 30 i 70 j 20
k 300 l 25 m 19 n 11 o 18
2 a 9 b 0 c 10 d 2 e $2\frac{1}{4}$
f 2 g 81 h 196 i 100

Exercise 1:01B

1 a $1\frac{3}{4}$ b $8\frac{1}{6}$ c $3\frac{3}{4}$ d $1\frac{3}{8}$
2 a $\frac{11}{2}$ b $\frac{22}{7}$ c $\frac{35}{4}$ d $\frac{200}{3}$
3 a $\frac{3}{5}$ b $\frac{7}{15}$ c $\frac{2}{3}$ d $\frac{5}{9}$
4 a $\frac{18}{24}$ b $\frac{20}{50}$ c $\frac{8}{28}$ d $\frac{40}{120}$
5 a $\frac{8}{15}$ b $\frac{1}{4}$ c $\frac{37}{40}$ d $\frac{9}{35}$
6 a $9\frac{1}{10}$ b $7\frac{1}{20}$ c $10\frac{17}{20}$ d $3\frac{19}{40}$
7 a $\frac{12}{35}$ b $\frac{27}{40}$ c $\frac{2}{15}$ d $\frac{7}{15}$
8 a $4\frac{1}{2}$ b $4\frac{1}{2}$ c 20 d 15
9 a $1\frac{7}{20}$ b $\frac{5}{8}$ c $\frac{2}{15}$ d $1\frac{5}{6}$

Exercise 1:01C

1 a 0·066, 0·6, 0·606, 0·66 b 0·153, 1·053, 1·53
c 0·017, 0·7, 0·77, 7 d 3·05, 3·4, 3·45, 3·5
2 a 9·301 b 3·45 c 3·104 d 6·32
e 1·97 f 8·105 g 4·888 h 159·3
3 a 0·036 b 0·006 c 0·585 d 0·0025
4 a 31·4 b 500 c 0·03 d 38 000
5 a 0·03 b 0·265 c 3·07 d 0·0025
6 a $0{\cdot}4\dot{3}$ b $0{\cdot}8\dot{2}\dot{7}$ c $1{\cdot}\dot{5}$ d $0{\cdot}\dot{8}57\,142\dot{8}$
7 a 4·804 b 0·016 c 0·0009 d 0·000 65
8 a 21 b 10·45 c 1500 d 2·8
9 a $3\frac{17}{1000}$ b $\frac{1}{25}$ c $\frac{43}{50}$ d $16\frac{1}{200}$
10 a 0·8 b 0·035 c 0·625 d $0{\cdot}7\dot{2}\dot{2}$
11 a $\frac{5}{9}$ b $\frac{257}{999}$ c $\frac{8}{11}$ d $\frac{214}{333}$
12 a $\frac{5}{6}$ b $\frac{151}{165}$ c $\frac{98}{225}$ d $\frac{1489}{1665}$

Exercise 1:01D

1 a $\frac{27}{50}$ b $\frac{203}{100}$ or $2\frac{3}{100}$ c $\frac{49}{400}$ d $\frac{91}{1000}$
2 a 55% b $44\frac{4}{9}$% c 125% d $66\frac{2}{3}$%
3 a 0·16 b 0·086 c 0·03 d 0·1825
4 a 47% b 6% c 37·5% d 130%
5 a 144 m b 7·56 g c $2.72 d $86 360
6 a $60 b 25 kg c $5 d 180 min or 3 h
7 a 42·5% b 45% c 18·75%
d 12% (to nearest whole %)

Exercise 1:01E

1 a i 3 : 5 ii 1 : 10 iii 15 : 7 iv 3 : 1
b 14 : 1 c 2 : 3 d 15 : 4
e i 8 : 5 ii 41 : 130
f i $\frac{3}{5}$: 1 ii $\frac{2}{7}$: 1 iii $\frac{10}{3}$: 1 iv $\frac{25}{4}$: 1
g i 1 : $\frac{5}{3}$ ii 1 : $\frac{7}{2}$ iii 1 : $\frac{3}{10}$ iv 1 : $\frac{4}{25}$
2 a $x = 50$ b 910 million
c 2·5 people per km^2 d 3·6 million
3 a Naomi gets 48, Luke gets 36
b 40°, 60°, 80°
c Tokyo, 12 million; Moscow, 10 million
d 36 males, 24 females

Exercise 1:01F

1 a 300 km/h b 8 m/mL
c 14·4 t/day d 2075 cm^3/kg
2 a 6 miles per hour b $46.20
c $4\frac{2}{3}$ minutes/book d 10 cm^3/s

Exercise 1:01G

1 **a** 2 **b** 2 **c** 3 **d** 3 **e** 4
f 3 **g** 4 **h** 3 **i** 3 **j** 4
k 1 **l** 1 **m** 2 **n** 1 **o** 1
p 2 **q** 3 **r** 3 **s** 5 **t** 3
u 5 **v** 1 **w** 3 **x** 2
y It is ambiguous, 2 **z** It is ambiguous, 2

2 **a** 2 **b** 3 **c** 1 **d** 1 **e** 2
f 2 **g** 3 **h** 5

Exercise 1:01H

1 **a** 4·6 **b** 0·8 **c** 3·2 **d** 0·1
e 15·2 **f** 8·1 **g** 1·0 **h** 121·6
i 0·1 **j** 47·4 **k** 0·4 **l** 2·8

2 **a** 0·54 **b** 2·61 **c** 7·13 **d** 1·17
e 12·02 **f** 8·40 **g** 412·68 **h** 0·08
i 0·44 **j** 100·33 **k** 0·02 **l** 0·01

3 **a** **i** 7 **ii** 7·3
b **i** 80 **ii** 85
c **i** 0·6 **ii** 0·63
d **i** 3 **ii** 2·6
e **i** 4 **ii** 4·2
f **i** 0·007 **ii** 0·0073
g **i** 0·08 **ii** 0·083
h **i** 3 **ii** 3·1
i **i** 0·009 **ii** 0·0093
j **i** 0·01 **ii** 0·0098
k **i** 8 **ii** 7·5
l **i** 0·04 **ii** 0·036

4 **a** 2 **b** 14·6 **c** 2·2 **d** 0·9
e 4·1 **f** 7·37 **g** 0·724 **h** 6
i 31·69 **j** 0·007 **k** 0·8 **l** 0·0072

5 **a** 5·6 **b** 0·2 **c** 0·44 **d** 15·4
e 8·33 **f** 413·8 **g** 72·0 **h** 3·067
i 10·0 **j** 4·800 **k** 0·08 **l** 0·004

Exercise 1:01I

1 Answers near these are acceptable.
a 74 **b** 120 **c** 31 **d** 7·2 **e** 110
f 60 **g** 18 **h** 5·8 **i** 7·7 **j** 1·8
k 0·4 **l** 52 **m** 15 **n** 21 **o** 310
p 1·1 **q** 7·0 **r** 17 **s** 59

Exercise 1:01J

1 **a** 12 000 L **b** 5000 kg **c** 3850 mg
d 8 000 000 m **e** 7 000 000 µm **f** 2 000 000 000 nm
g 6 000 000 000 B **h** 4000 GB **i** 3000 GB
j 25 000 µm **k** 5400 kB **l** 500 µs

2 400 GB of space is left.

Exercise 1:01K

1 **a** 0·5 g **b** 0·05 m **c** 0·0005 L **d** 0·5 cm
e 0·05 t **f** 0·005 s **g** 500 L

2 **a** 0·05 kg, smallest = 63·65 kg, largest = 63·75 kg
b from 11·25 cm up to 11·35 cm

Exercise 1:02

1 **a** $6a$ **b** $10y$ **c** $2m$
d x^2 **e** $8k$ **f** $5y$
g $8a + 2b$ **h** $7n + 7m$ **i** $11m + 2$
j $7x + y^2$ **k** $5ab - 5a$ **l** $6x^2 + 5xy$
m $3a + 4b + 7$ **n** $9k + 3m - 3$ **o** $x + 2y - 3z$

2 **a** $35y$ **b** $5q$ **c** $6xy$ **d** $20ab$
e $9mn$ **f** $6m^2$ **g** $10a^2b$ **h** $10m^2n^2$
i $-20x$ **j** $12y$ **k** $5p^2q^2$ **l** $42mn$
m $60a^2b$ **n** $30pqr$

3 **a** $4x$ **b** $3a$ **c** 5 **d** 3
e $2n$ **f** $4a$ **g** $\frac{3}{2}$ **h** $\frac{6q}{5}$
i $\frac{2a}{b}$ **j** $\frac{5m}{n}$ **k** $\frac{5b}{3c}$ **l** $\frac{3m}{2n}$

4 **a** $5x + 20$ **b** $50 - 10y$ **c** $2y^2 + 10y$
d $5a^2 - 15a$ **e** $7p^2 + 7pq$ **f** $-3x + 15$
g $-5a - 20$ **h** $-6ax + 2ay$ **i** $6x + 25$
j $7y - 15$ **k** $7a - 10b$ **l** $7x - 4$
m $8a - 4$ **n** $3y^2 - 4y$ **o** $3m + 3$
p $9x - 36$ **q** $p^2 + 37p$

5 **a** $5(m + 2)$ **b** $8(k - 3)$ **c** $2(4a + 3)$
d $5(3n - 2)$ **e** $6(3n - 4m)$ **f** $a(b + c)$
g $n(n - m)$ **h** $p(q - 1)$ **i** $3a(b + 2)$
j $5m(2n - 1)$ **k** $4y(2x + z)$ **l** $bc(a - d)$
m $3a(c + 3d)$ **n** $3x(2xy + 3)$ **o** $3(a + 2b + 3c)$
p $4(2x^2 + x - 3)$

6 **a** $-4(a + 2)$ **b** $-5(3x + 2)$ **c** $-3(2p - 3)$
d $-2(n - 8)$ **e** $-x(x + 2)$ **f** $-a(5 + 2b)$
g $-3a(a - 2)$ **h** $-3y(3z - 2x)$

7 **a** $2a$ **b** $\frac{x}{5}$ **c** $\frac{5a}{3}$ **d** $\frac{3m}{5}$

8 **a** $\frac{8x}{15}$ **b** $\frac{y}{12}$ **c** $\frac{7a}{6}$ **d** $\frac{n}{8}$
e $\frac{3x + 4y}{12}$ **f** $\frac{4a - 9b}{6}$ **g** $\frac{6m + 5n}{10}$ **h** $\frac{k - 3l}{6}$

9 **a** $\frac{xy}{6}$ **b** $\frac{m^2}{10}$ **c** $\frac{12}{am}$ **d** $\frac{1}{2n^2}$
e $\frac{px}{qy}$ **f** $\frac{1}{2}$ **g** $\frac{2m}{n}$ **h** $\frac{2}{15}$

10 **a** 2 **b** $\frac{15}{2}$ or $7\frac{1}{2}$ **c** $\frac{5}{2}$ or $2\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{ay}{bx}$ **f** $\frac{3}{4}$ **g** 15 **h** $\frac{3}{2}$ or $1\frac{1}{2}$

Exercise 1:03

1 **a** $\frac{1}{3}$ **b** $\frac{1}{10}$ **c** $\frac{11}{30}$ **d** $\frac{7}{10}$

2 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** 1

3 **a** $\frac{1}{3}$ **b** $\frac{7}{12}$ **c** $\frac{3}{4}$ **d** 0

4 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{12}{13}$ **e** $\frac{3}{13}$
f $\frac{1}{26}$ **g** $\frac{1}{4}$ **h** 0 **i** $\frac{3}{26}$ **j** $\frac{15}{52}$
k $\frac{1}{13}$ **l** $\frac{3}{4}$ **m** $\frac{11}{26}$ **n** $\frac{37}{52}$ **o** $\frac{4}{13}$
p $\frac{2}{13}$ **q** $\frac{23}{52}$

5 **a** $\frac{10}{90}$ (or $\frac{1}{9}$), 11% **b** $\frac{53}{90}$, 59%
c $\frac{35}{90}$ (or $\frac{7}{18}$), 39% **d** $\frac{27}{90}$ (or $\frac{3}{10}$), 30%

6 **a** $\frac{4}{85}$, 5% **b** $\frac{74}{85}$, 87%
c $\frac{40}{85}$ (or $\frac{8}{17}$), 47% **d** $\frac{33}{85}$, 39%

7 **a**

Second choice (without replacement)

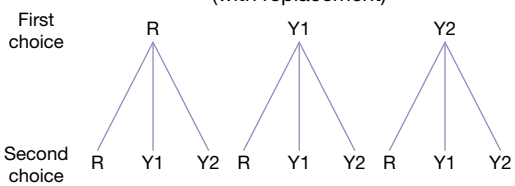
First choice \ Second choice	R	G1	G2	Y1	Y2	Y3
R		RG	RG	RY	RY	RY
G1	GR		GG	GY	GY	GY
G2	GR	GG		GY	GY	GY
Y1	YR	YG	YG		YY	YY
Y2	YR	YG	YG	YY		YY
Y3	YR	YG	YG	YY	YY	

b

Second choice

First choice \ Second choice	Red	Yellow 1	Yellow 2
Red	RR	RY	RY
Yellow 1	YR	YY	YY
Yellow 2	YR	YY	YY

(with replacement)



Exercise 1:04

1 **a** 45·6 m **b** 29·6 m **c** 39·3 m
2 **a** 88·25 cm^2 **b** 28·08 cm^2 **c** 11·52 cm^2
3 **a** 63·9 m^2 **b** 35·7 m^2 **c** 23·3 km^2
4 **a** 370·88 cm^2 **b** 648 m^2
c 333·55 m^2 (approx.)
5 **a** 38·68 m^2 **b** 112·17 m^2
6 **a** 54·1 cm^3 **b** 399·9 cm^3 **c** 4934·1 cm^3
d 43·2 m^3 **e** 1250 cm^3 **f** 111·7 cm^3
g 10 500 cm^3 **h** 14 954 cm^3

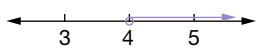
Exercise 1:05

1 **a** a^3 **b** 2^4 **c** n^5 **d** 10^3
2 **a** 2^9 **b** a^5 **c** m^5 **d** 10^8
e a^8 **f** y **g** b^2 **h** 10^3
i m^{12} **j** a^6 **k** x^8 **l** 10^{10}
m 3 **n** 2 **o** 6 **p** e^0 or 1
q $30a^2$ **r** $2m^3$ **s** $30a^2$ **t** $16x^8$ or 4^2x^8
3 **a** $30a^5b^3$ **b** $56a^5b^3$ **c** $24a^4b^7$ **d** $10a^{10}b^3$
e $49x^6$ **f** $16m^8$ **g** x^6y^9 **h** $625x^4y^8$
i $6a^2$ **j** $5x^3$ **k** $3a$ **l** 8
4 **a** $\frac{1}{4}$ **b** $\frac{1}{10}$ **c** $\frac{1}{x}$ **d** $\frac{2}{a}$
e $\frac{1}{25}$ **f** $\frac{1}{8}$ **g** $\frac{1}{m^3}$ **h** $\frac{5}{x^2}$
5 **a** 3^{-1} **b** 8^{-1} **c** a^{-1} **d** $3x^{-1}$
e 2^{-4} **f** 10^{-6} **g** y^{-4} **h** $5n^{-3}$

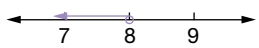
6 **a** $1{\cdot}48 \times 10^8$ **b** $6{\cdot}8 \times 10^4$
c $1{\cdot}5 \times 10^{-4}$ **d** $1{\cdot}65 \times 10^{-6}$
7 **a** 62 000 **b** 1 150 000
c 0·0074 **d** 0·000 069 1
8 **a** $4{\cdot}8 \times 10^5$ B (2 sig. fig.) **b** $1{\cdot}4 \times 10^3$ B
c $3{\cdot}4 \times 10^{12}$ B **d** 8×10^9 B
e $8{\cdot}50 \times 10^6$ B **f** $4{\cdot}15 \times 10^9$ B
9 **a** 5×10^{-3} s **b** $4{\cdot}2 \times 10^{-6}$ s
c $7{\cdot}5 \times 10^{-9}$ s **d** $1{\cdot}25 \times 10^{-4}$ s
e $6{\cdot}5 \times 10^{-8}$ s **f** $8{\cdot}0 \times 10^{-2}$ s
10 **a** $1{\cdot}05 \times 10^7$ L **b** $1{\cdot}02 \times 10^{11}$ L
c $8{\cdot}50 \times 10^{-1}$ g (2 sig. fig.) **d** $6{\cdot}7 \times 10^{-8}$ m

Exercise 1:06

1 **a** $a = 18$ **b** $m = 5$ **c** $x = 15$ **d** $y = -2$
e $p = 2\frac{1}{3}$ **f** $n = 12$ **g** $x = 2$ **h** $q = 2$
i $m = 2$ **j** $y = \frac{1}{5}$ **k** $x = 8\frac{1}{2}$ **l** $k = \frac{2}{3}$
m $x = 2$ **n** $y = 1$ **o** $q = 3\frac{1}{2}$
2 **a** $m = 5$ **b** $x = 4$ **c** $x = 7$ **d** $p = 4$
e $n = 4$ **f** $m = 2$ **g** $a = 8$ **h** $m = 4$
i $q = -1$ **j** $x = 2$ **k** $y = -1$ **l** $p = 2$
m $k = -5\frac{1}{2}$ **n** $z = 1\frac{1}{2}$ **o** $y = 2\frac{3}{4}$ **p** $n = \frac{2}{3}$
q $q = \frac{1}{4}$ **r** $m = -1\frac{1}{3}$
3 **a** $a = 2$ **b** $x = 7$ **c** $x = 3$ **d** $x = 3$
e $m = -\frac{1}{2}$ **f** $k = 7\frac{1}{4}$ **g** $a = -2$ **h** $x = -3$
i $m = 4\frac{1}{3}$ **j** $a = 4$ **k** $p = 8$ **l** $x = 6$
m $y = -9$ **n** $b = -18$ **o** $y = -1$ **p** $m = 7$
q $m = 5$ **r** $y = 2$ **s** $x = 2\frac{1}{2}$
4 **a** $x = 4$ **b** $a = 9$ **c** $m = 6\frac{2}{3}$ **d** $n = 9$
e $x = 6$ **f** $p = -1$
5 **a** $a = 16$ **b** $x = 15$ **c** $p = 6$ **d** $a = 6$
e $m = 12$ **f** $n = 20$ **g** $q = 36$ **h** $k = 24$
i $x = 60$ **j** $y = -30$ **k** $p = 1\frac{11}{13}$ **l** $a = 4\frac{4}{5}$
m $m = 6$ **n** $n = -1$ **o** $x = \frac{8}{13}$ **p** $x = 11\frac{1}{4}$
q $m = 33$ **r** $a = \frac{-19}{8}$
6 **a** $x \leq 1$ **b** $x \geq 0$ **c** $x > 6$ **d** $x < -2$
7 **a** $x > 4$



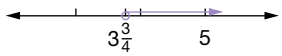
b $a < 8$



c $y \leq 2$

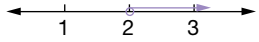
d $m \leq 7$

e $x > 3\frac{3}{4}$

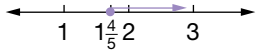


f $m \leq 4$

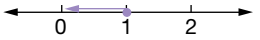
g $x > 2$



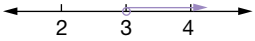
h $m \geq 1\frac{4}{5}$



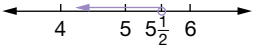
i $n \leq 1$



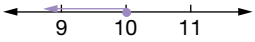
j $x > 3$



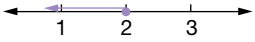
k $x < 5\frac{1}{2}$



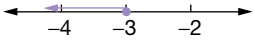
l $m \leq 10$



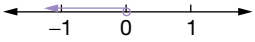
m $a \leq 2$



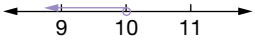
n $b \leq -3$



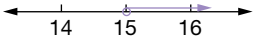
o $m < 0$



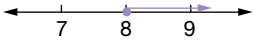
p $x < 10$



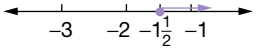
q $p > 15$



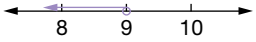
r $x \geq 8$



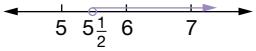
s $y \geq -1\frac{1}{2}$



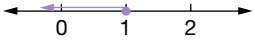
t $p < 9$



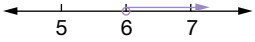
u $p > 5\frac{1}{2}$



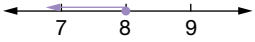
v $x \leq 1$



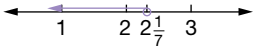
w $x > 6$



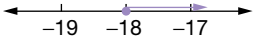
x $a \leq 8$



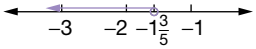
y $b < 2\frac{1}{7}$



z $x \geq -18$



α $y < -1\frac{3}{5}$



Exercise 1:07

1 **a** \$211.60 **b** \$238.28
c \$350 **d** \$534.90

2 **a** 701·36, 20·3% **b** \$5604
c \$65 746 **d** \$1179

3 **a** The 15% discount is the best, by \$2.49.
b Jade's buy **c** Yet tea

4 **a** **i** \$7.50 **ii** \$0.68 **iii** \$1.88
b **i** \$82.50 **ii** \$7.48 **iii** \$20.63
c **i** \$20 **ii** \$0.75 **iii** \$1.80

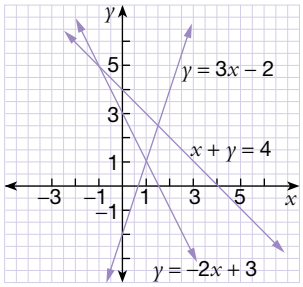
5 **a** Buying on terms is a way of having an item while spreading the payment over a period of time. Interest is charged.
b He pays \$143.56. Interest is \$56.56.
c \$220 **d** \$545 700 **e** \$30.60

6 **a** **i** $33\frac{1}{3}\%$ **ii** 25%
b **i** 25% **ii** $33\frac{1}{3}\%$

Exercise 1:08

1 **a** $(0, 0)$ **b** y-axis **c** x-axis **d** 3rd

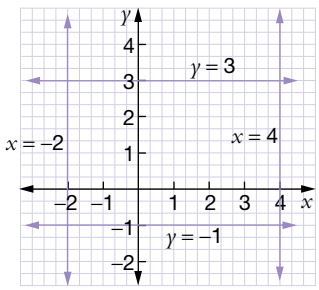
2



3 **a** $(3, -2)$ and $(5, 4)$
b $y = x + 2$, $y = 5x + 2$, $x + y = 2$

4 **a** **i** $y = 2$ **ii** $y = -3$ **iii** $y = 0$
iv $x = 2$ **v** $x = -1$ **vi** $x = 0$

b



5 **a** In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
b **i** 5 **ii** $\sqrt{61}$ **iii** $\sqrt{51}$ m

6 **a** $\sqrt{20}$ **b** $\sqrt{53}$ **c** $\sqrt{68}$ **d** 10

7 **a** $(6, 8)$ **b** $(1, 3)$ **c** $(3\frac{1}{2}, 5\frac{1}{2})$

8 **a** **i** 5 **ii** $\frac{1}{2}$ **iii** $-1\frac{1}{5}$
b **i** $\frac{1}{3}$ **ii** $\frac{1}{5}$ **iii** $-2\frac{1}{2}$

9 **a** $m = 4, b = 7$ **b** $m = 1, b = -3$
c $m = -2, b = 5$

10 **a** $y = 4x + 5$ **b** $y = -\frac{1}{2}x + 3$
c $y = 2x - 4$ **d** $y = -7x - 3$

11 **a** C **b** B **c** A

12 **a** B **b** C **c** A

Exercise 1:09

1 **a** Similar figures are figures that have the same shape but different size. (Matching angles are equal and matching sides are in the same ratio.)
b C **c** E

2 **a** true **b** true

3 **a** yes
b no (as $\frac{40}{42}$ does not equal $\frac{30}{32}$)

4 **a** $x = 9, y = 15$ **b** $a = 16, b = 20$
c $x = 32, y = 30$ **d** $x = 35, y = 81$

5 **a** 14 cm by 7·5 cm **b** 1 : 400

Exercise 1:10

1 **a** $3a + 4b$ **b** $12ab$ **c** $k - m$
d $\frac{x+y+z}{3}$ **e** $2(m+5)$ **f** $(a-b)^2$
g $\sqrt{5m+4n}$ **h** $3m+3$

2 **a** -3 **b** 21 **c** 2 **d** 18 **e** -43
f 45 **g** $3\frac{1}{2}$ **h** 3 **i** 3

3 **a** 14·7 **b** 2 **c** -8

4 **a** 92 **b** 5·81 **c** 15·7

5 **a** 2 **b** 10 **c** 11·5 **d** 12

6 **a** 4 **b** 10 **c** 25 **d** $3{\cdot}08\dot{3}$ **e** $5{\cdot}2\dot{6}$

7 **a** **i** $8n + 11 = 39$
ii $2n + 7 = 5$
iii $\frac{n+4}{10} = 7$ or $(n + 4) \div 10 = 7$
b **i** $3n + 5 = 38$; $n = 11$
ii $\frac{n}{4} + \frac{n}{2} = 6$; $n = 8$
iii $2(4w + w) = 340$, $10w = 340$;
width = 34 m, length = 136 m

Exercise 1:11

1 **a** $\frac{5}{12}$ **b** $\frac{15}{8}$ **c** $\frac{4}{3}$

2 **a** $\frac{24}{25}$ **b** $\frac{7}{25}$ **c** $\frac{3}{5}$

3 **a** 2·5 **b** 5·0 **c** 34·9

4 **a** 5 **b** 6·9 **c** 6·5

5 **a** 7·1 **b** 10·4 **c** 14·6

6 **a** 40° **b** 51° **c** 40°

7 **a** 36°52′ **b** 62°11′ **c** 53°8′

8 60·99 m

9 76·4 m

10 46·626 km

Exercise 1:12

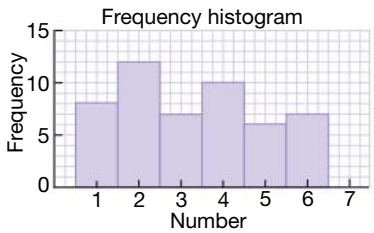
1

Number	Tally	Frequency
1	𝍸 |||	8
2	𝍸 𝍸 ||	12
3	𝍸 ||	7
4	𝍸 𝍸	10
5	𝍸 |	6
6	𝍸 ||	7

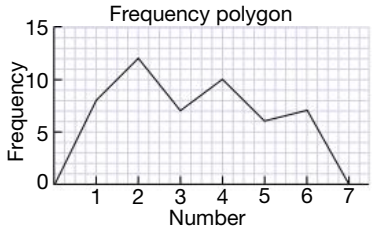
$\Sigma f = 50$

a 2 **b** 5 **c** 7 **d** 21 **e** 23

2 **a**



b



3 **a** **i** 7 **ii** 5 **iii** $4\frac{1}{9}$ **iv** 4
b **i** 5 **ii** 2 **iii** 3·3 **iv** 3

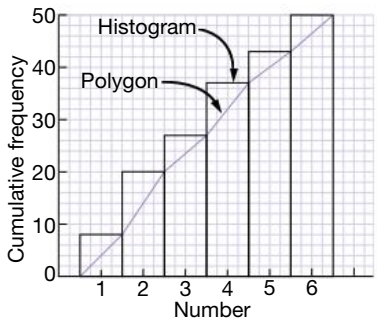
c

Number	Frequency	Cumulative Frequency
1	8	8
2	12	20
3	7	27
4	10	37
5	6	43
6	7	50

$\Sigma f = 50$

i 37 **ii** 27 **iii** yes

4



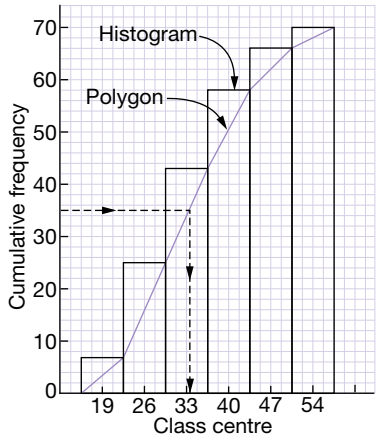
5 **a**

Class	Class centre	Tally	Frequency	Cumulative Frequency
16–22	19	𝍸 ||	7	7
23–29	26	𝍸 𝍸 𝍸 |||	18	25
30–36	33	𝍸 𝍸 𝍸 |||	18	43
37–43	40	𝍸 𝍸 𝍸	15	58
44–50	47	𝍸 |||	8	66
51–57	54	||||	4	70

$\Sigma f = 70$

b 34·1 using table (34·6 using the individual numbers)

c



Median class is 30–36

d Modal class is 30–36 cans.

e The survey was held over 70 days.

6 a

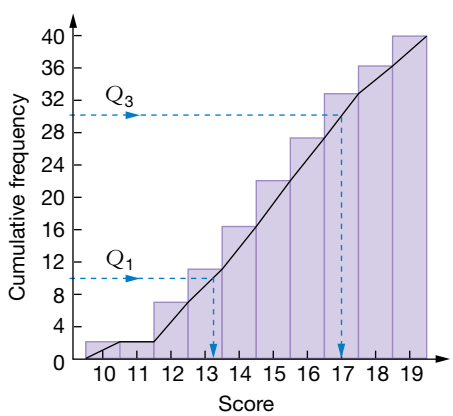
Stem	Leaf
0	
1	8 9
2	8 1 2 5 6 3 9 8 0 7 6
3	0 6 7 0 3 5 1
4	2 7 3 0 2 4
5	4 0

b

Stem	Leaf
0	
1	8 9
2	0 1 2 3 5 6 6 7 8 8 9
3	0 0 1 3 5 6 7
4	0 2 2 3 4 7
5	0 4

7 a $11 - 3{\cdot}5 = 7{\cdot}5$

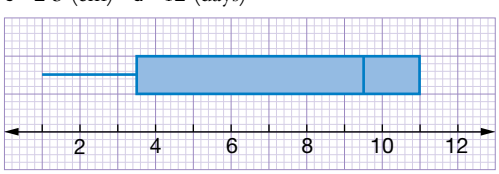
b



Interquartile range $= Q_3 - Q_1$
$= 17 - 13$
$= 4$

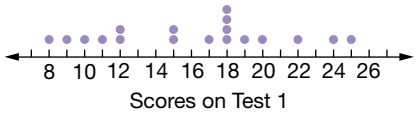
c 2·5 (cm) **d** 12 (days)

8



9 a 25 **b** range = 60, interquartile range ≒ 22

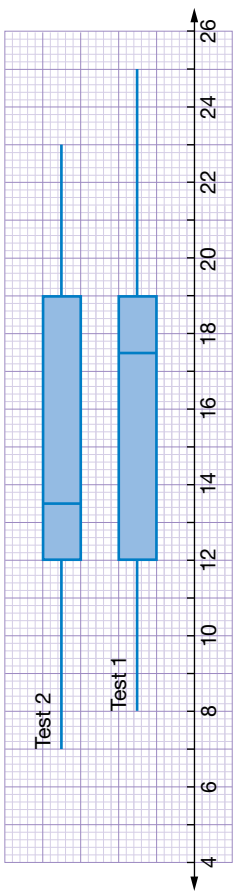
10 a



b

Scores		
Test 1 Leaf	Stem	Test 2 Leaf
9 8	0	7 9
9 8 8 8 8 7 5 5 2 2 1 0	1	0 1 2 2 2 2 3 4 5 6 8 9
5 4 2 0	2	0 0 1 3

c



Exercise 1:13

1 a 50 km/h **b** $8/kg **c** 24 L/h **d** 25 g/cm^3

2 a 100 m/min **b** 2 L/s **c** 15 t/day **d** 1 c/m^2

3 a direct **b** indirect **c** direct **d** indirect

4 a $k = 5$ **b** 85 **c** 34·8

5 a 40

b 13 (Number of people must be a whole number.)

6 a $D = 24T$ **b** $k = 24$ **c** 192 km **d** 9 h 30 min

7 a $k = 12$ **b** 264 **c** 16 h 40 min

8 a $t = \frac{48}{n}$ **b** $k = 48$ **c** 4 hours **d** 16

9 a $k = 120$ **b** 2 h 24 min **c** 60 km/h

10 a 750 Hz **b** 37·5 cm

ASSIGNMENT 1A

1 a

t	1	3	5
m	3	7	11

, $m = 4t - 1$

b

t	2	4	6
m	5	10	15

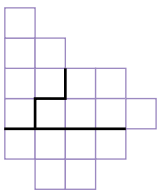
, $m = \frac{5t}{2}$

c

t	1	3	6
m	3	9	18

, $m = 3t$

2 Cut along lines as shown.



3 a 185 **b** 37

4 16 min 15 s

5 32

6 a 11 290 000 **b** 485 400

c 11 300 (3 sig. fig.)

d Answers will vary.

Chapter 2

Prep quiz 2:01

1 $\frac{3}{4}$ **2** $\frac{4}{5}$ **3** $\frac{2}{y}$ **4** $\frac{p}{m}$ **5** $\frac{3p}{4q}$

6 $\frac{5}{2}$ **7** a^2 **8** $\frac{4x}{5}$ **9** $\frac{5n}{12}$ **10** $\frac{p}{6}$

Exercise 2:01

1 a $2a$ **b** $\frac{x}{5}$ **c** $\frac{5a}{3}$ **d** $\frac{3m}{5}$

e $\frac{x+y}{4}$ **f** $\frac{5a-2b}{3}$ **g** $\frac{5}{a}$ **h** $\frac{8}{x}$

i $\frac{1}{y}$ **j** $\frac{8}{m}$ **k** $\frac{7a}{x}$ **l** $\frac{-x}{y}$

m $\frac{4}{n}$ **n** $\frac{1}{x}$ **o** $\frac{2a}{b}$ **p** $\frac{m}{x}$

2 a $\frac{8x}{15}$ **b** $\frac{7a}{10}$ **c** $\frac{y}{12}$ **d** $\frac{m}{4}$

e $\frac{7a}{6}$ **f** $\frac{13x}{6}$ **g** $\frac{n}{8}$ **h** $\frac{p}{2}$

i $\frac{3x+4y}{12}$ **j** $\frac{4a-9b}{6}$ **k** $\frac{6m+5n}{10}$ **l** $\frac{k-3l}{6}$

m $\frac{10}{3x}$ **n** $\frac{5}{6a}$ **o** $\frac{31}{10m}$ **p** $\frac{1}{8x}$

q $\frac{13a}{6x}$ **r** $\frac{-5x}{3m}$ **s** $\frac{13m}{4n}$ **t** $\frac{8x+3y}{12a}$

3 a $\frac{xy}{6}$ **b** $\frac{ab}{12}$ **c** $\frac{m^2}{10}$ **d** $\frac{a^2}{40}$

e $\frac{12}{am}$ **f** $\frac{2}{xy}$ **g** $\frac{4}{p^2}$ **h** $\frac{1}{3n^2}$

i $\frac{px}{qy}$ **j** $\frac{1}{2}$ **k** $\frac{2m}{n}$ **l** $\frac{2}{15}$

m $\frac{2a}{3}$ **n** 1 **o** 9 **p** $\frac{16a}{15p}$

4 a 2 **b** $\frac{5}{3}$ **c** $\frac{15}{2}$ **d** $\frac{2}{3}$

e $\frac{5}{2}$ **f** $\frac{9}{2}$ **g** $\frac{1}{2}$ **h** 6

i $\frac{ay}{bx}$ **j** $\frac{3}{4}$ **k** 15 **l** $\frac{3}{2}$

m $2x$ **n** $\frac{3}{a}$ **o** x^2 **p** $\frac{27}{4}$

5 a $\frac{4}{5}$ **b** $\frac{2}{3}$ **c** $\frac{3}{x}$ **d** $\frac{b^2}{2}$

e $2y$ **f** b **g** $6m^2$ **h** 4

i $\frac{4}{q^2}$ **j** $\frac{6}{ab}$ **k** $\frac{2by}{c}$ **l** $3c$

m $\frac{3}{2}$ **n** 1 **o** 6 **p** $\frac{2}{5}$

6 a $2y^3$ **b** $\frac{3}{4a^2}$ **c** $\frac{n}{2m}$ **d** $\frac{y}{2x}$ **e** $\frac{2a^2}{b^2}$

f $\frac{4m^2n^3}{3}$ **g** $8ax^3$ **h** $\frac{2}{kn^2}$ **i** $\frac{r}{q}$

7 a $\frac{4x^2}{3}$ **b** $\frac{3}{2m^2}$ **c** $\frac{6y^3}{5}$ **d** $\frac{8x^4}{5y^4}$ **e** $\frac{a}{b^3}$

f $\frac{3p^4}{10q}$ **g** $x^2y^3z^2$ **h** $\frac{2g^2}{f^2h^3}$ **i** $\frac{2a^2xy^2}{3b^3}$

Fun spot 2:01

This maze is actually impossible. If you think you have a solution, check that you haven't missed any doors.

Prep quiz 2:02

1 $3a + 15$ **2** $m^2 - 6m$ **3** $6y^2 + 2yz$

4 $-20x + 5$ **5** p^7 **6** $15x^7$

7 $12m^3n^7$ **8** 4 **9** $5b$

10 x^2y^4

Exercise 2:02

1 a $a^2 + 7a$ **b** $x^2 - x$

c $2y^2 + 7y$ **d** $5p^2 - 20p$

e $18z + 42z^2$ **f** $6a^2 - 2a$

g $8k^2 + 12km$ **h** $8mn - 24m^2$

i $-2x - 10$ **j** $-7y + 35$

k $-m - 4$ **l** $-12k + 4$

m $-p^2 - p$ **n** $-x^2 + xy$

o $-6z^2 - 10z$ **p** $-a^2b + ab^2$

2 a $7a + 8$ **b** $10x + 7$

c $8y - 3$ **d** $10a - 9$

e $p + 10$ **f** $15 - m$

g $7a + 20$ **h** $7x + 2$

i $10n - 7$ **j** $11h + 13$

k $8x + 7$ **l** $10y + 2$

m $a + 8$ **n** $5m - 16$

3 **a** $x - 8$ **b** $8y + 22$
c $7a - 7$ **d** $13m - 14$
e $17x - 12$ **f** $10x + 40$
g $2x - 23$ **h** $3m$
i $2a + 66$ **j** $2n - 46$
k $x^2 + 6x + 3$ **l** $a^2 + 10a - 21$
m $m^2 - m - 12$ **n** $t^2 - 9t + 20$
o $3a^2 + 3ab$ **p** $x^2 + y^2$

4 **a** $x^4 - x^2$ **b** $5a^3 - a^5$
c $5a^3 - a^5$ **d** $x^3 + xy$
e $7m - m^3$ **f** $y^3 - xy^2$
g $6a^5 + 9a^3$ **h** $15x^3 - 5x^2$
i $2m^3n^2 - 2m^5$ **j** $5x^3 - 3x^2 + 7x$
k $2x^4 + 7x^3 - 14x^2$ **l** $y^3 - 7y^2 - y$
m $a^5b^3 - a^2b^7$ **n** $4x^5y - 6x^2y^5$
o $12n^4m^4 - 30n^2m^7$ **p** $x^5 + x^3 + 8$
q $-m^3 - 2m^4$ **r** $z^7 - 5z^5 + 1$

5 **a** $3(3x + 2)$ **b** $5(2 + 3a)$
c $2(2a + 3n)$ **d** $x(x - 7)$
e $a(2a - 3)$ **f** $6y(2 - y)$
g $b(a - x)$ **h** $s(t - 1)$
i $2b(2a + 5c)$ **j** $-2(2m - 3n)$
k $-x(x + 3)$ **l** $-5a(3 - b)$
m $x(3 + x - a)$ **n** $a(x + y + z)$
o $2(2m - 4n + 3p)$ **p** $5a(b - 3c + 2d)$
q $x(x - 7 + y)$ **r** $(a + 3)(a + 1)$

6 **a** $x^2(x + 4)$ **b** $a^3(a^2 - 6)$
c $p^4(p^2 + 1)$ **d** $y^2(y^3 - 3)$
e $2x^3(1 + 3x)$ **f** $5a^4(a - 2)$
g $3p(2p^5 + 1)$ **h** $y(5 - 3y)$
i $x^2y(xy + 4)$ **j** $a^3b^3(a^2 - b^2)$
k $p^4q^2(p^2 + q)$ **l** $y^2z(y^3 - z^3)$
m $3m^2n(m + 3n)$ **n** $2a^3b(4a^2b^2 - 3)$
o $3p^3q^5(3p^3 + 4)$ **p** $2xy^2(5y^3 - 2x^3)$

7 **a** $x^2y^2(y + x + x^2y^2)$ **b** $3a^2b^3(1 - 2ab^2 + 3a^2b)$
c $4m^3n(2n + 3m^2 + mn)$ **d** $5x^2z^2(1 - 2xz + 3x^2z^2)$
e $x^2y^2z^2(yz^2 - xy^2 + x^2z)$
f $3a^2b^2c^3(3bc^2 + a^3b^2 + 2a^2c)$

Challenge 2:02

1 **a** $(a + 1)(5 + b)$ **b** $(y - 3)(4 + x)$
c $(q + 7)(p - 5)$ **d** $(p + q)(m + n)$
e $(a - b)(a + b)$ **f** $(y + 3)(2x - 1)$

2 **a** $(x + 1)(6 + a)$ **b** $(p - q)(8 + m)$
c $(a + 3c)(b + 5)$ **d** $(x + y)(x + z)$
e $(a + 1)(b + 4)$ **f** $(3m + 4)(4m + n)$
g $(n - 1)(m + 1)$ **h** $(x + 1)(x^2 + 1)$
i $(a + c)(a + b)$

Prep quiz 2:03

1 $12x$ **2** a **3** $x^2 - 2x + 3$
4 $2x + 10$ **5** $x^2 - 2x$ **6** $-3a - 3$
7 $-5y + y^2$ **8** $x^2 + 4x + 3$ **9** $25 - a^2$
10 $6x^2 - 19x - 10$

Exercise 2:03

1 **a** $ab + 3a + 2b + 6$ **b** $xy + 4x + y + 4$
c $mn + 5m + 7n + 35$ **d** $ax + 2a + 3x + 6$
e $pq + 4p + 5q + 20$ **f** $2xy + 6x + y + 3$
g $3ap + 2a + 18p + 12$ **h** $8xy + 12x + 2y + 3$
i $6ab - 21a + 2b - 7$ **j** $14px + 7x + 10p + 5$
k $5px - 20p + 3x - 12$ **l** $2ax + 4bx + ay + 2by$

2 **a** $a^2 + 5a + 6$ **b** $x^2 + 6x + 5$
c $n^2 + 7n + 12$ **d** $p^2 + 7p + 10$
e $m^2 - 2m - 3$ **f** $y^2 + 5y - 14$
g $x^2 - 5x - 6$ **h** $t^2 - 2t - 8$
i $x^2 - 6x + 8$ **j** $n^2 - 8n + 7$
k $a^2 - 9a + 18$ **l** $x^2 - 19x + 90$
m $y^2 - 4y - 77$ **n** $a^2 - a - 2$
o $x^2 - 16x + 64$ **p** $m^2 - 11m + 18$
q $a^2 - 9$ **r** $x^2 - 4x - 21$
s $y^2 + 17y + 60$ **t** $a^2 - 64$
u $q^2 + 10q + 25$ **v** $x^2 - 10x + 9$
w $t^2 + 13t + 30$ **x** $k^2 + 3k - 88$

3 **a** $2a^2 + 7a + 3$ **b** $2x^2 + 5x + 2$
c $3m^2 + 17m + 10$ **d** $4y^2 + 13y + 3$
e $4x^2 + 8x + 3$ **f** $6n^2 + 7n + 2$
g $8x^2 + 18x + 9$ **h** $10t^2 + 19t + 6$
i $10x^2 - 12x + 2$ **j** $24p^2 - 13p - 2$
k $10m^2 - 29m + 10$ **l** $21q^2 + q - 2$
m $18x^2 + 6x - 4$ **n** $4n^2 - 9$
o $64y^2 - 1$ **p** $15k^2 - 19k + 6$
q $49p^2 - 14p + 1$ **r** $15x^2 - 14x + 3$
s $25x^2 + 40x + 16$ **t** $27y^2 + 6y - 8$
u $5p^2 - 33p - 14$ **v** $10q^2 - 101q + 10$
w $12a^2 + 25a + 12$ **x** $49p^2 - 25$

4 **a** $12 + 7x + x^2$ **b** $10 - 7a + a^2$
c $7 - 6m - m^2$ **d** $9 - n^2$
e $y^2 + 9y + 20$ **f** $12x - x^2 - 35$
g $k^2 + 19k + 90$ **h** $2a^2 + 7a + 3$
i $19n - 6n^2 + 7$ **j** $x^2 + 3xy + 2y^2$
k $2n^2 + 5mn + 2m^2$ **l** $2a^2 + ab - 3b^2$
m $4p^2 - q^2$ **n** $6x^2 - 13xy - 5y^2$
o $6a^2 + 13ab + 6b^2$ **p** $81w^2 - 90wx + 25x^2$

Prep quiz 2:04

1 $x^2 + 5x + 6$ **2** $a^2 + 2a - 3$ **3** $m^2 - 9m + 14$
4 $x^2 + 10x + 25$ **5** $a^2 - 4a + 4$ **6** $3, 2$
7 $5, 4$ **8** $-5, 3$ **9** $4, -1$
10 $9, -2$

Exercise 2:04

1 **a** $(x + 3)(x + 1)$ **b** $(x + 2)(x + 1)$
c $(x + 5)(x + 1)$ **d** $(x + 6)(x + 1)$
e $(x + 5)(x + 4)$ **f** $(x + 5)^2$
g $(x + 6)^2$ **h** $(x + 7)(x + 3)$
i $(x + 6)(x + 3)$ **j** $(x + 10)(x + 4)$
k $(x + 6)(x + 9)$ **l** $(x + 9)(x + 4)$
m $(x - 2)^2$ **n** $(x - 6)^2$
o $(x - 4)(x - 3)$ **p** $(x - 5)(x - 4)$

q $(x+3)(x-1)$ **r** $(x+4)(x-3)$
s $(x+6)(x-2)$ **t** $(x+10)(x-3)$
u $(x-2)(x+1)$ **v** $(x-12)(x+2)$
w $(x-10)(x+3)$ **x** $(x-8)(x+7)$

2 a $(a+4)(a+2)$ **b** $(m+6)(m+3)$
c $(y+6)(y+7)$ **d** $(p+3)(p+4)$
e $(x+2)(x+10)$ **f** $(n+14)(n+3)$
g $(s+18)(s+3)$ **h** $(a+4)(a+14)$
i $(x-4)(x+1)$ **j** $(a-4)(a+2)$
k $(p-8)(p+3)$ **l** $(y+3)(y-2)$
m $(x+8)(x-1)$ **n** $(q+8)(q-3)$
o $(m+15)(m-3)$ **p** $(a+21)(a-3)$
q $(y+11)(y-5)$ **r** $(x-1)^2$
s $(k-3)(k-2)$ **t** $(x-9)(x-4)$
u $(a-18)(a-4)$ **v** $(p+6)(p+16)$
w $(q-15)(q+3)$ **x** $(m-11)(m+7)$

3 a D **b** A **c** C **d** B

4 a incorrect; $(x-4)(x-2) = x^2 - 6x + 8$
b correct
c incorrect; $(x-3)(x-2) = x^2 - 5x + 6$
d incorrect: $(x-2)(x+8) = x^2 + 6x - 16$
e correct
f incorrect: $(x-15)(x-1) = x^2 - 16x + 15$

5 a $2(x+2)(x+1)$ **b** $3(x-3)(x+1)$
c $5(x-4)(x+2)$ **d** $2(x+4)^2$
e $3(x-11)(x+1)$ **f** $3(x+3)(x+4)$
g $4(a-5)(a+2)$ **h** $2(n+3)(n+1)$
i $5(x-4)(x-2)$ **j** $3(x-3)(x-4)$
k $3(a-9)(a+4)$ **l** $5(x+10)(x-7)$

Prep quiz 2:05

1 $x(x-3)$ **2** $x(x+7)$ **3** $(x+1)(x+2)$
4 $(x-6)(x+2)$ **5** $(x-4)(x+5)$ **6** $(x-7)(x-1)$
7 $x = 4$ **8** $x = 0$ **9** $x = 4$
10 $x = -6$

Exercise 2:05

1 a $x = \pm 5$ **b** $x = \pm 9$ **c** $x = \pm 1$
d $x = \pm 10$ **e** $x = \pm 6$ **f** $x = \pm 8$
g $x = \pm 11$ **h** $x = \pm 20$

2 a $x = \pm\sqrt{6}$ **b** $n = \pm\sqrt{3}$ **c** $y = \pm\sqrt{10}$
d $k = \pm\sqrt{15}$ **e** $x = \pm\sqrt{2}$ **f** $p = \pm\sqrt{\frac{3}{5}}$
g $q = \pm\sqrt{\frac{5}{4}}$ **h** $z = \pm\sqrt{\frac{5}{7}}$

3 a $x = \pm 1{\cdot}414$ **b** $a = \pm 1{\cdot}871$
c $c = \pm 3{\cdot}464$ **d** $g = \pm 0{\cdot}612$

4 a no solution
b the square root of a negative number has no real solution

5 a $x = 0$ or 5 **b** $y = 0$ or -7 **c** $n = 0$ or 9
d $x = 2$ or 1 **e** $x = 7$ or 3 **f** $a = 5$ or 2
g $y = -3$ or -4 **h** $t = -3$ or -2 **i** $x = -9$ or -5
j $a = 6$ or -6 **k** $y = -8$ or 7 **l** $n = -1$ or 1

6 a $x = 0$ or -3 **b** $m = 0$ or 5
c $y = 0$ or -2 **d** $x = -1$ or -2
e $a = 2$ or 3 **f** $y = -5$ or -7
g $a = 3$ or 7 **h** $x = 2$ or 8
i $m = 3$ or 8 **j** $h = 4$ or -5
k $x = 5$ or -7 **l** $a = 9$ or -5
m $x = 7$ or -8 **n** $y = 1$ or 7
o $a = 1$ or -10

7 a $x = 2$ **b** one
c when the trinomial can be factorised as a perfect square, i.e. of the form $(x \pm a)^2$

8 a $x = -2$ **b** $m = 3$ **c** $z = -1$
d $k = 4$ **e** $a = 5$ **f** $y = -10$

9 a $x = \pm 2$ **b** $x = 0$ or 4 **c** $x = 6$ or -2
d $m = 10$ or -1 **e** $m = \pm 3$ **f** $m = 0$ or 9
g $a = 0$ or 6 **h** $a = 8$ or -2 **i** $a = \pm\sqrt{6}$
j no solution **k** $h = 0$ or -16 **l** $h = 1$ or -17

10 a $x = 0$ or 3 **b** $m = 0$ or 8 **c** $t = 0$ or -5
d $x = 1$ or 4 **e** $a = 5$ or -3 **f** $y = 1$ or 2
g $m = 3$ or 6 **h** $n = 9$ or -2 **i** $h = 8$ or -4
j $x = 1$ or -2 **k** $y = 1$ or -3 **l** $x = 2$ or 5
m $y = 3$ or -6 **n** $t = 4$ or -7 **o** $y = 3$ or -5
p $x = -1$ or -2 **q** $a = 2$ or 3 **r** $z = 0$ or 1
s $q = -1$ or -4 **t** $x = 5$ or -7 **u** $a = 5$ or 10

Diagnostic test 2

1 a x **b** $\frac{2a}{7}$ **c** $\frac{7}{x}$ **d** $\frac{3}{m}$

2 a $\frac{13x}{15}$ **b** $\frac{a}{12}$ **c** $\frac{23}{12x}$ **d** $\frac{11}{6n}$

3 a $\frac{2ab}{15}$ **b** $\frac{x^2}{4}$ **c** $\frac{4}{3}$ **d** $\frac{2t}{3q}$

4 a $\frac{3}{4}$ **b** $\frac{14}{3}$ or $4\frac{2}{3}$ **c** $\frac{3}{2}$ or $1\frac{1}{2}$ **d** $\frac{3}{8}$

5 a $\frac{y^2}{2}$ **b** $\frac{2a}{b}$ **c** $\frac{2x}{yz}$

6 a $\frac{2}{5x}$ **b** $\frac{2}{3ab^4}$ **c** $\frac{m}{np^2}$

7 a $x^2 + 7x$ **b** $10m^2 - 35m$
c $-12y - 20$ **d** $-12t^2 + 15tw$

8 a $3n + 21$ **b** $9x - 1$ **c** $7a + 1$

9 a $x^6 + 3x^2$ **b** $-2y^5 + 8y^2$ **c** $12m^3n^6 - 3m^7n^4$

10 a $2(4a + 5b)$ **b** $6p(p - 2q)$ **c** $-x(x - 5z)$

11 a $w^3(w^2 + 5)$ **b** $2x^6(4 + 3x^2)$ **c** $2m^3n^3(2m - 3n)$

12 a $x^2 + 7x + 12$ **b** $2a^2 - 7a + 3$
c $6 - y - y^2$ **d** $2x^2 - 5xy - 3y^2$

13 a $(x+3)(x+4)$ **b** $(x-3)(x-2)$
c $(a-5)(a+2)$ **d** $(n+5)(n-4)$

14 a $x = \pm 7$ **b** $a = \pm 2{\cdot}646$
c $m = \pm 4$ **d** $y = \pm 1{\cdot}291$

15 a $x = 4$ or -5 **b** $z = 0$ or 3 **c** $a = -6$ or 6

16 a $t = 0$ or 5 **b** $x = 2$ or 4 **c** $y = 7$ or -8

ASSIGNMENT 2A

1 **a** $\frac{3x}{5}$ **b** $\frac{a}{9}$ **c** $\frac{25x}{24}$ **d** $\frac{1}{n}$

e $\frac{13}{6y}$ **f** $\frac{2a}{15w}$ **g** $\frac{pq}{6}$ **h** $\frac{4}{5}$

i $\frac{1}{2}$ **j** $\frac{14}{3}$ or $4\frac{2}{3}$ **k** $\frac{3k}{4t}$ **l** $\frac{8a}{3bc}$

m $\frac{2x}{y^2}$ **n** $\frac{8p^2q}{3}$

2 **a** $x^2 + x - 2$ **b** $8x - 3$

c 3 **d** $2x^2 - 13x - 7$

e $x^2 - 25$ **f** $9x^2 + 12x + 4$

g $x^2 - x + 2$ **h** $6 - 5x + x^2$

i $y^2 - x^2$ **j** $4x^2 - 4xy + y^2$

k $x^6 - x^3$ **l** $6x^3y^7 + 15x^5y^5$

3 $2(x^2 - 1)$

4 **a** $m^2(m + 4)$ **b** $2n(2n^4 - 3)$

c $a^2b^2(a - b)$ **d** $3p^4q^4(3p^2q + 2pq^2 - 4)$

5 **a** $5x(2x - 1)$ **b** $a(a + b - c)$

c $(y + 4)(y + 5)$ **d** $(n - 4)(n + 3)$

e $(4 - k)(2 - k)$ **f** $(a + 4)(a - 5)$

g $x(x^2 + 5x + 6) = x(x + 2)(x + 3)$

6 **a** $a = \pm 6$ **b** $t = \pm\frac{3}{2}$ (or $\pm 1{\cdot}5$)

c no solution **d** $x = -2$ or 3

e $n = \pm 5$ **f** $x = 3$

g $w = 0$ or 2 **h** $x = 3$ or 4

i $y = 9$ or -5 **j** $p = -3$ or -5

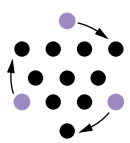
7 **a** $y = 5$ or -2 **b** $m = 7$ or -6

8 **a** $y = 7$ or -3 **b** $n = 8$ or -10

c $z = 11$ or -10 **d** $x = 7$ or -10

ASSIGNMENT 2B

1



2 4 **3** 20

4 6 h **5** 126 984

6 **a** man **b** woman **c** cancer

d Depends on the number of men or women still living.

ASSIGNMENT 2C

1 **a** \$18.75 **b** 20% **c** \$87.50 **d** \$160

2 **a** 4 : 9 **b** 5 : 6 **c** 14 : 9

3 **a** $4x - y$ **b** $18a^2bc$ **c** $2x$ **d** $\frac{19a}{15}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{13}$ **d** $\frac{1}{52}$

5 **a** $40{\cdot}04\text{ cm}^2$ **b** $78{\cdot}1\text{ cm}^2$ (1 dec. pl.)

6 **a** 800 **b** 4 **c** $\frac{1}{16}$ **d** $\frac{1}{8}$

7 **a** $m = 2$ **b** $x = 3$ **c** $a = 2$

8 \$1448.40

9 **a** $x = 9{\cdot}6$ **b** $x = 45$

10 **a** 5 **b** 4 **c** 4 **d** $4\frac{7}{9}$

e 3·5 **f** 6 **g** 2·5

Chapter 3

Exercise 3:01A

1 **a** $\angle HEB = 55°$ (adj. angles on straight line)
$\angle ABC = 55°$ (corresp. to $\angle HEB$, $DC \parallel GH$)

b $\angle EFB = 93°$ (vert. opp. $\angle HFG$)
$\angle ABC = 93°$ (corresp. to $\angle EFB$, $AD \parallel EG$)

c $\angle BDG = 106°$ (supp. adj. angles)
$\angle ABC = 106°$ (corresp. to $\angle BDG$, $HC \parallel FG$)

2 **a** $\angle CBE = 39°$ (alt. to $\angle DEB$, $AC \parallel DF$)
$x = 141$ (adj. angles on a straight line)

b $\angle EBC = 88°$ (vert. opp. $\angle ABF$)
$x = 88$ (corresp. to $\angle EBC$, $EF \parallel GH$)

c $\angle EBC = 70°$ (corresp. to $\angle GCD$, $EF \parallel GH$)
$x = 70$ (vert. opp. $\angle EBC$)

d $\angle DAB = 92°$ (co-int. to $\angle CBA$, $AD \parallel BC$)
$x = 62$ (since $x° + 30° = 92°$)

e $\angle DCH = 125°$ (corresp. to $\angle CBF$, $EF \parallel GH$)
$x = 35$ (since $x° + 90° = 125°$)

f $\angle CBE = 48°$ (alt. to $\angle BED$, $AC \parallel DF$)
$x = 42$ (supp. adj. angles)

3 **a** $\angle BCE = 82°$ (alt. to $\angle ABC$, $AB \parallel ED$)
$x = 98$ (co-int. to $\angle BCE$, $BC \parallel FG$)

b $\angle BCD = 77°$ (alt. to $\angle ABC$, $AB \parallel CD$)
$\angle CDE = 103°$ (co-int. to $\angle BCD$, $BC \parallel ED$)
$x = 103$ (alt. to $\angle CDE$, $EF \parallel CD$)

c $\angle BCF = 152°$ (co-int. to $\angle ABC$, $AB \parallel FC$)
$\angle DCF = 120°$ (co-int. to $\angle CDE$, $FC \parallel ED$)
$x° + 272° = 360°$ (angles at a point)
$x = 88$

d $\angle ACF = 45°$ (alt. to $\angle BAC$, $AB \parallel FC$)
$\angle DCF = 18°$ (alt. to $\angle CDE$, $FC \parallel DE$)
$x = 63$ ($\angle ACF + \angle DCF$)

e $\angle BCF = 100°$ (alt. to $\angle ABC$, $AB \parallel CF$)
$\angle DCF = 59°$ ($\angle BCF - \angle BCD$)
$x = 121$ (co-int. to $\angle DCF$, $DE \parallel CF$)

f Draw CF right from $C \parallel AB$ and DE
$\angle ACF = 40°$ (co-int. to $\angle CAB$, $AB \parallel CF$)
$\angle DCF = 52°$ (co-int. to $\angle CDE$, $CF \parallel DE$)
$x = 92$ ($\angle ACF + \angle DCF$)

g $\angle ABD = 42°$ (co-int. to $\angle CDB$, $AB \parallel CD$)
$\angle ABF = 77°$ (co-int. to $\angle EFB$, $AB \parallel EF$)
$x = 35$ ($\angle ABF - \angle ABD$)

h Draw EI up from $E \parallel AB$ and CD.
$\angle GEI = 73°$ (co-int. to $\angle CGE$, $CD \parallel EI$)
$\angle FEI = 106°$ (co-int. to $\angle AFE$, $AB \parallel EI$)
$x = 33$ ($\angle FEI - \angle GEI$)

i $\angle BDE = 76°$ (co-int. to $\angle DBC$, $AC \parallel DE$)
$\angle FDE = 38°$ ($\angle BDF = \angle FDE$, given)
$x = 142$ (co-int. to $\angle FDE$, $FG \parallel DE$)

Exercise 3:01B

1 a $\angle DCA = 40°$ (alt. to $\angle EDC$, $ED \parallel CA$)
$\angle ABC + 60° + 40° = 180°$ (angle sum of a Δ)
$\angle ABC = 80°$

b $\angle DCB = 30°$ (alt. to $\angle EDC$, $ED \parallel BC$)
$\angle ABC + 30° + 100° = 180°$ (angle sum of a Δ)
$\angle ABC = 50°$

c $\angle CDE + 40° + 75° = 180°$ (angle sum of a Δ)
$\angle CDE = 65°$
$\angle ABC = 115°$ (co-int. to $\angle CDE$, $AB \parallel ED$)

2 a $\angle CDB = 88°$ (suppl. to $\angle EDB$)
$\angle DBC = 72°$ (suppl. to $\angle DBA$)
$x° + 72° + 88° = 180°$ (angle sum of a Δ)
$x = 20$

b $\angle ABD = 48°$ (vert. opp. to $\angle CBE$)
$x° + 75° + 48° = 180°$ (angle sum of a Δ)
$x = 57$

c $\angle DBC = 111°$ (vert. opp. to $\angle ABE$)
$x° + 111° + 25° = 180°$ (angle sum of a Δ)
$x = 44$

3 a $\angle BCA = 80°$ (suppl. adj. angle)
$\angle BAC = 80°$ (base ang. of isos. Δ)
$80° + 80° + \angle ABC = 180°$ (angle sum of Δ)
$\angle ABC = 20°$

b $\angle CAB = 81°$ (alt. to $\angle ACD$, $AB \parallel DC$)
$\angle ABC = 81°$ (base angles of isos. Δ)

c $\angle CBD = \angle CDB$ (base angles of isos. Δ)
$68° + 2 \times \angle CBD = 180°$ (angle sum of a Δ)
$\angle CBD = 56°$
$\angle ABC = 124°$ (suppl. adj. angles)

4 a $\angle EBC = 60°$ (suppl. adj. angles)
$\angle ECB = 55°$ (suppl. adj. angles)
$\angle BEC = x°$ (vert. opp. angles)
$x° + 55° + 60° = 180°$ (angle sum of a Δ)
$x = 65$

b $x = 110$ (alt. to $\angle EBC$, $DE \parallel AC$)

c $\angle DBC = 70°$ (corresp. to $\angle EAB$, $EA \parallel DB$)
$95° = x° + 70°$ (ext. angle of ΔDBC)
$x = 25$

d $30° + 35° + \angle ABD = 180°$ (angle sum of a Δ)
$\angle ABD = 115°$
$\angle DBC = 65°$ (suppl. adj. angles)
$x° = 50° + 65°$ (ext. angle of ΔBCD)
$x = 115$

e $\angle BCE = 40°$ (suppl. adj. angles)
$96° = x° + 40°$ (ext. angle of ΔBCE)
$x = 56$

f acute $\angle BAC = 80°$ (angle of revol'n)
acute $\angle ABC = 60°$ (angle of revol'n)
$x° + 60° + 80° = 180°$ (angle sum of a Δ)
$x = 40$

5 a $\angle BED = 58°$ (alt. to $\angle CBE$, $AC \parallel DE$)
$x° + 78° + 58° = 180°$ (angle sum of a Δ)
$x = 44$

b $\angle BDA = x°$ (alt. to $\angle CED$, $BD \parallel CE$)
$x° + 48° + 60° = 180°$ (angle sum of a Δ)
$x = 72$

c $\angle CED = 55°$ (DEF is a straight line)
$\angle BCE = x°$ (alt. to $\angle ABF$, $BF \parallel CE$)
$x° = 46° + 55°$ (ext. angle of ΔCDE)
$x = 101$

d $\angle ACD = x° + 42°$ (ext. angle of ΔABC)
$\angle CAD = x° + 42°$ (base angles of isos. ΔDAC)
$122° = 2x° + 84°$ (ext. angle of ΔACD)
$x = 19$

e $\angle BCD = 125°$ (co-int. to $\angle ADC$, $BE \parallel AD$)
$\angle ACD = 55°$ (base angles of isos. ΔACD)
$\angle BCA = 70°$ ($\angle BCD - \angle ACD$)
$\angle ABC = 70°$ (base angles of isos. ΔABC)
$x° + 140° = 180°$ (angle sum of ΔABC)
$x = 40$

f ΔOAB and ΔOBC are isosceles (radii equal)
$\angle ABO = 60°$ (base angles of isos. ΔABO)
$\angle COB = 120°$ (ext. angle ΔABO)
$\angle OCB = x°$ (base angles of isos. ΔOBC)
$2x° + 120° = 180°$ (angle sum of ΔOBC)
$x = 30$

Prep quiz 3:01C

1 120	**2** 60	**3** 80	**4** 105
5 50	**6** 80	**7** 110	**8** 70
9 110	**10** 360°		

Exercise 3:01C

1 a $\angle CBE = 80°$ (vert. opp. $\angle FBA$)
$x = 80$ (opp. angles of a par'm)

b $\angle ABE = 78°$ (opp. angles of a par'm)
$x = 78$ (vert. opp. $\angle ABE$)

c $\angle CBE = 110°$ (vert. opp. $\angle ABF$)
$x° + 60° + 110° + 90° = 360°$ (angle sum of a quad.)
$x = 100$

d $\angle FBC = 80°$ (suppl. adj. angles)
$\angle CDF = 85°$ (suppl. adj. angles)
$x° + 85° + 80° + 80° = 360°$ (angle sum of a quad.)
$x = 115$

e $\angle EBC + 60° + 100° + 70° = 360°$ (angle sum of a quad.)
$\angle EBC = 130°$
$x = 50$ (suppl. adj. angles)

f Reflex $\angle ADC = 360° - 130° = 230°$
(angle sum at point D is 360°)
$x° + 342° = 360°$ (angle sum of quad. $ABCD$)
$x = 18$

2 a $\angle CAB + 65° + 145° + 100° = 360°$ (angle sum of a quad.)
$\angle CAB = 50°$
$\angle CAB = \angle ABC$ (base angles of an isos. Δ)
$\angle ABC = 50°$

b $\angle BAC + 70° + 150° + 85° = 360°$ (angle sum of a quad.)
$\angle BAC = 55°$
$\angle BCA = 55°$ (base angles of an isos. Δ)
$\angle ABC + 55° + 55° = 180°$ (angle sum of a Δ)
$\angle ABC = 70°$

c $\angle CAB + 95° + 80° + 115° = 360°$ (angle sum of a quad.)
$\angle CAB = 70°$
$\angle ABC = 70°$ (base angles of an isos. Δ)

d $\angle EAD = \angle EDA$ (base angles of isos. Δ)
$2\angle EAD + 100° = 180°$ (angle sum of a Δ)
$\angle EAD = 40°$
$\angle BAD + \angle ADC + \angle DCB + \angle ABC = 360°$ (angle sum of a quad.)
$60° + 110° + 100° + \angle ABC = 360°$
$\angle ABC = 90°$

e $\angle BAC + 80° + 110° + 90° = 360°$ (angle sum of a quad.)
$\angle BAC = 80°$
$\angle ACB = 80°$ (base angles of isos. Δ)
$\angle ABC + 80° + 80° = 180°$ (angle sum of a Δ)
$\angle ABC = 20°$

f $\angle EAD = 80°$ (base angles of isos. Δ)
$\angle BAC + 30° + 80° = 180°$ (angle sum of Δ)
$\angle BAC = 70°$
$\angle ABC = 70°$ (base angle of isos. Δ)

3 a $AE = EB$ (diag. of a rect. bisect each other)
$x = 30$ (base angle of isos. Δ)

b $EB = EC$ (diag. of a rect. bisect each other and are equal)
$\angle ECB = x°$ (base angles of isos. Δ)
$x° + x° + 40° = 180°$ (angle sum of a Δ)
$x = 70$

c $DE = EC$ (diag. of a rect. bisect each other and are equal)
$x = 32$ (base angles of isos. Δ)

4 a $\angle EAD = x°$ (base angles of isos. Δ)
$\angle ADC = 2x°$ (ext. angle of ΔADE)
$2x° = 70°$ (opp. angles of par'm are equal)
$x = 35$

b $\angle GCF = 70°$ (corresp. angles, $AD \parallel BC$)
$\angle GFC = x°$ (corresp. angles, $BE \parallel AF$)
$\angle CGF = 30°$ (vert. opp. angles)
$x° + 30° + 70° = 180°$ (angle sum of a Δ)
$x = 80$

c $\angle ADB = 40°$ (alt. angles, $AE \parallel BD$)
$\angle ADB = x°$ (alt. angles, $AD \parallel BC$)
$x = 40$

Exercise 3:02

1 a In Δs ABC and DEF:
(1) $AB = FE$ (data)
(2) $AC = FD$ (data)
(3) $\angle ABC = \angle FED$ (data)
$\therefore \Delta ABC \equiv \Delta DEF$ (RHS)

b In Δs XYZ and ACB:
(1) $XY = AC$ (data)
(2) $XZ = AB$ (data)
(3) $YZ = CB$ (data)
$\therefore \Delta XYZ \equiv \Delta ACB$ (SSS)

c In Δs ABC and AED:
(1) $BC = ED$ (data)
(2) $\angle BCA = \angle EDA$ (data)
(3) $CA = DA$ (data)
$\therefore \Delta ABC \equiv \Delta AED$ (SAS)

d In Δs ABC and DEF:
(1) $\angle BCA = \angle EFD$ (data)
(2) $\angle BAC = \angle EDF$ (data)
(3) $CA = FD$ (data)
$\therefore \Delta ABC \equiv \Delta DEF$ (AAS)

e In Δs LMN and TRS:
(1) $\angle NLM = \angle STR$ (data)
(2) $\angle NML = \angle SRT$ (data)
(3) $NM = SR$ (data)
$\therefore \Delta LMN \equiv \Delta TRS$ (AAS)

2 a In Δs ABC and DEC:
(1) $BC = CE$ (given)
(2) $AC = DC$ (given)
(3) $\angle ACB = \angle DCE$ (vert. opp. $\angle$s)
$\therefore \Delta ABC \equiv \Delta DEC$ (SAS)

b In Δs ABD and BAC:
(1) $BD = AC$ (given)
(2) $\angle ABC = \angle BAC$ (given)
(3) AB is common
$\therefore \Delta ABD \equiv \Delta BAC$ (SAS)

c In Δs ABD and ACD:
(1) $AB = AC$ (given)
(2) AD is common
(3) $\angle ADB = \angle ADC$ (given)
$\therefore \Delta ABD \equiv \Delta ACD$ (RHS)

d In Δs ABC and ADC:
(1) $AB = AD$ (given)
(2) $BC = DC$ (given)
(3) AC is common
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)

e In Δs ABD and ACD:
(1) $\angle ABD = \angle ACD$ (given)
(2) $\angle ADB = \angle ADC$ (given)
(3) AD is common
$\therefore \Delta ABD \equiv \Delta ACD$ (AAS)

f In Δs AOB and COD:
(1) $OA = OC$ (radii of circle)
(2) $OB = OD$ (radii of circle)
(3) $\angle AOB = \angle COD$ (given)
$\therefore \Delta AOB \equiv \Delta COD$ (SAS)

g In Δs ABF and CDE:
(1) $\angle AFB = \angle CED$ (given)
(2) $AB = CD$ (sides of a square are equal)
(3) $\angle ABF = \angle EDC$ (angles of a square are equal)
$\therefore \Delta ABF \equiv \Delta CDE$ (AAS)

3 a In Δs ABC and ADC:
(1) $AB = AD$ (given)
(2) $BC = DC$ (given)
(3) AC is common
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
$\therefore \angle BAC = \angle DAC$ (matching ∠s of cong't Δs)

b In Δs ABC and DBC:
(1) $AC = CD$ (given)
(2) $AB = DB$ (given)
(3) BC is common
$\therefore \Delta ABC \equiv \Delta DBC$ (SSS)
$\therefore \angle ABC = \angle CBD$ (matching ∠s of cong't Δs)

c In Δs AXB and DXC:
(1) $AX = XD$ (given)
(2) $BX = XC$ (given)
(3) $\angle AXB = \angle CXD$ (vert. opp. ∠s)
$\therefore \Delta AXB \equiv \Delta DXC$ (SAS)
$\therefore \angle BAX = \angle CDX$ (matching ∠s of cong't Δs)
But $\angle ABX$ and $\angle DCX$ are alternate angles
$\therefore AB \parallel CD$ (equal alt. ∠s)

d In Δs AOD and BOD:
(1) $AO = OB$ (radii of a circle)
(2) OD is common
(3) $\angle ADO = \angle BDO$ (given)
$\therefore \Delta AOD \equiv \Delta BOD$ (RHS)
$\therefore AD = DB$ (matching sides of cong't Δs)

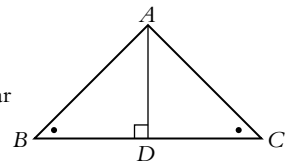
e In Δs OAC and OBC:
(1) $OA = OB$ (radii of circle)
(2) $AC = CB$ (given)
(3) OC is common
$\therefore \Delta OAC \equiv \Delta OBC$ (SSS)
$\therefore \angle OCA = \angle OCB$ (matching ∠s of cong't Δs)
Now $\angle OCA + \angle OCB = 180°$ (adj. supp. ∠s)
$\therefore 2 \times \angle OCA = 180°$
$\therefore \angle OCA = 90°$
$\therefore \angle OCA = \angle OCB = 90°$

f In Δs AOC and BOD:
(1) $OA = OB$ (radii of circle)
(2) $OC = OD$ (radii of circle)
(3) $\angle AOC = \angle BOD$ (vert. opp. angles)
$\therefore \Delta AOC \equiv \Delta BOD$ (SAS)
$AC = DB$ (matching sides of cong. Δs)
$\angle CAO = \angle DBO$ (matching angles of cong. Δs)
$\therefore AC \parallel DB$ (alt. angles CAO, DBO are equal)

4 a In Δs ABC and DCB:
(1) $\angle ACB = \angle DBC$ (given)
(2) $AC = DB$ (given)
(3) BC is common
$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)

b In Δs DBC and ECB:
(1) $\angle BDC = \angle CEB$ (given)
(2) $\angle ABC = \angle ACB$ (given)
(3) BC is common
$\therefore \Delta DBC \equiv \Delta EBC$ (AAS)

Exercise 3:03



1 a $\angle B = \angle C$ (data)
Draw a perpendicular line from A to BC, meeting BC at D.
In Δs ABD and ACD
(1) $\angle ABD = \angle ACD$ (data)
(2) $\angle ADB = \angle ADC = 90°$ (by construction)
(3) AD is common
$\Delta ABD \equiv \Delta ACD$ (AAS)
$\therefore AB = AC$ (matching sides of cong't Δs)

2 a In Δs ABD and ACD
(1) $AB = AC$ (data)
(2) AD is common
(3) $\angle ADB = \angle ADC = 90°$ (by construction)
$\therefore \Delta ABD \equiv \Delta ACD$ (RHS)
$\therefore BD = CD$ (matching sides of cong't Δs)
$\therefore AD$ bisects BC

b In Δs ABD and ACD
(1) $AB = AC$ (data)
(2) AD is common
(3) $BD = CD$ (data)
$\therefore \Delta ABD \equiv \Delta ACD$ (SSS)
$\therefore \angle ADB = \angle ADC$ (matching ∠s of cong't Δs)
$\angle ADB + \angle ADC = 180°$ (adj. supp. ∠s)
$\therefore 2 \times \angle ADB = 180°$
$\angle ADB = 90°$
$\therefore AD \perp BC$

3 a $\angle B = \angle C$ ($AC = AB$)
$\angle A = \angle B$ ($AC = BC$)
$\therefore \angle A = \angle B = \angle C$

b $\angle A + \angle B + \angle C = 180°$ (∠ sum of Δ)
$\therefore 3 \times \angle A = 180°$ (using (a))
$\angle A = 60°$

4 a AAS

b matching sides of cong't Δs

c AAS

d matching sides of cong't Δs

5 a alt ∠s and || lines

b alt. ∠s and || lines

c yes

d In Δs ABC and CDA:
(1) $\angle BAC = \angle DCA$ (alt. ∠s and || lines)
(2) $\angle BCA = \angle DAC$ (alt. ∠s and || lines)
(3) AC is common
$\therefore \Delta ABC \equiv \Delta CDA$ (AAS)
$\therefore \angle ABC = \angle CDA$ (matching ∠s of cong't Δs)
$AB = DC$ (matching sides of cong't Δs)
$BC = AD$ (matching sides of cong't Δs)

6 In Δs ABE and CDE:
(1) $AB = DC$ (opp. sides of a par'm)
(2) $\angle BAE = \angle DCE$ (alt. ∠s, $AB \parallel DC$)
(3) $\angle EBA = \angle EDC$ (alt. ∠s, $AB \parallel DC$)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
$\therefore AE = EC$ (matching sides of cong't Δs)
$\therefore EB = ED$ (matching sides of cong't Δs)

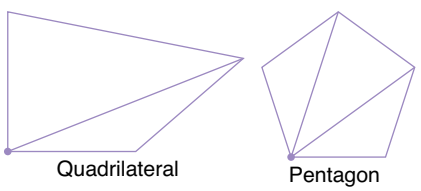
7 **a** SSS
b and **c** matching ∠s of cong't Δs
d SSS
e and **f** matching ∠s of cong't Δs

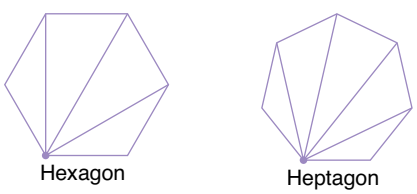
8 **a** Diagonals of a parallelogram bisect each other.
b In Δs ABE and CBE:
(1) $AE = EC$ (diag. of par'm bisect each other)
(2) BE is common
(3) $AB = BC$ (sides of a rhombus are equal)
$\therefore \Delta ABE \equiv \Delta CBE$ (SSS)
$\therefore \angle AEB = \angle BEC$ (matching ∠s of cong't Δs)
But $\angle AEB + \angle BEC = 180°$ (adj. supp. ∠s)
$\therefore 2 \times \angle AEB = 180°$
$\therefore \angle AEB = 90°$
$\therefore \angle AEB = \angle BEC = 90°$

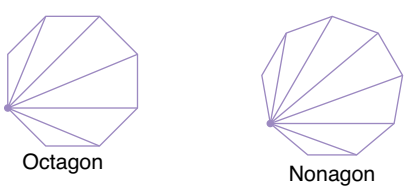
9 **a** $\angle DAB = 90°$ (co-int. ∠s, $AB \parallel DC$)
$\angle ABC = 90°$ (co-int. ∠s, $AD \parallel BC$)
$\angle BCD = 90°$ (co-int. ∠s, $AB \parallel DC$)
b In Δs ABD and DCA:
(1) $AB = DC$ (opp. sides of a par'm are equal)
(2) AD is common
(3) $\angle ADC = \angle DAB$ (∠s of a rectangle)
$\therefore \Delta ABD \equiv \Delta DCA$ (SAS)
$\therefore AC = DB$ (matching sides of cong't Δs)

Investigation 3:04A

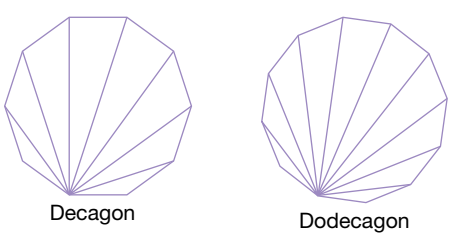
1







2



3

No. of sides in polygon	3	4	5	6	7	8	9	10	12
No. of triangles	1	2	3	4	5	6	7	8	10

4 The 'number of triangles' is two less than the 'number of sides'.

5 $n - 2$

6 $(n - 2) \times 180$

7 $\dfrac{(n-2) \times 180}{n}$

Investigation 3:04B

1 6

2 $6 \times 180° = 1080°$

3 720°

4 $1080° - 720° = 360°$

5 (1) 5 (2) $5 \times 180° = 900°$ (3) 540°
(4) $900° - 540° = 360°$

6 The exterior angles sum to 360°.

Exercise 3:04

1 **a** 720 **b** 1440 **c** 2340

2 **a** **i** 5 **ii** 540°
b **i** 6 **ii** 720°
c **i** 7 **ii** 900°

3 **a** 1080° **b** 1440° **c** 900°

4 **a** 108° **b** 120° **c** 135° **d** 144°
e 140° **f** $158\frac{14}{17}°$

5 **a** 3240° **b** 360° **c** 162° **d** 18°

6 **a** 6 **b** 9 **c** 36 **d** 24

7 **a** 240° **b** 228° **c** 315°

8 **a** 80° **b** 110° **c** 120° **d** 135°
e 125° **f** 60°

Investigation 3:04C

1 120°

2 3

3 360°

4 A regular pentagon will not tessellate because the interior angles (108°) cannot add at a vertex to give 360°

5 Square and equilateral triangle.

Diagnostic test 3

1 **a** $\angle FGC = x°$ (corresp. ∠s, $AB \parallel CD$)
$x° + 130° = 180°$ (adj. supp. ∠s)
$x = 50$
b $\angle DBE = \angle EDB$ (base ∠s of isos. Δ)
$2 \times \angle EDB + 40° = 180°$ (∠ sum of Δ)
$\therefore \angle EDB = 70°$
$\angle EDB = x°$ (alt. ∠s, $AC \parallel DE$)
$\therefore x = 70$
c $\angle LON + 90° + 70° + 85° = 360°$ (∠ sum of quad.)
$\therefore \angle LON = 115°$
$\angle LOP = x°$ (base ∠s of isos. Δ)
$x° + 115° = 180°$ (adj. supp. ∠s)
$x = 65$

2 a In Δs ABE and DBE,
(1) $AE = DE$ (data)
(2) $BE = CE$ (data)
(3) $\angle AEB = \Delta DEC$ (vert. opp. ∠s)
$\therefore \Delta ABE \equiv \Delta DCE$ (SAS)

b In Δs ACO and BCO,
(1) $AC = BC$ (data)
(2) $AO = BO$ (radii of circle)
(3) OC is common
$\therefore \Delta ACO \equiv \Delta BCO$ (SSS)

c In Δs ABC and ADC,
(1) $\angle ABC = \Delta ADC$ (data)
(2) $\angle ACB = \Delta ACD$ (data)
(3) AC is common
$\therefore \Delta ABC \equiv \Delta ADC$ (AAS)

3 a In Δs ABE and DCF,
(1) $\angle AEB = \angle DFC$ (data)
(2) $AB = DC$ (data)
(3) $AE = FD$ (opp sides of rect $ADFE$)
$\therefore \Delta ABE \equiv \Delta DCF$ (RHS)
$\therefore \angle ABC = \angle ADC$ (matching ∠s of cong't Δs)

b In Δs ABC and ACD,
(1) AD is common
(2) $\angle ABD = \angle ACD$ (data)
(3) $\angle BAD = \angle CAD$ (data)
$\therefore \Delta ABC \equiv \Delta ACD$ (AAS)
$\therefore \angle ADB = \angle ADC$ (matching angles of cong't Δs)
$\angle ADB + \angle ADC = 180°$ (adj. supp. ∠s)
$2 \times \angle ADB = 180°$
$\angle ADB = 90°$
$\therefore AD \perp BC$

c In Δs CAB and CAD,
(1) AC is common
(2) $AB = AD$ (data)
(3) $BC = DC$ (data)
$\therefore \Delta CAB \equiv \Delta CAD$ (SSS)
$\therefore \angle CAB = \angle CAD$ (matching ∠s of cong't Δs)

4 a 2340° **b** 135° **c** 18

ASSIGNMENT 3A

1 a $\angle BEG = 70°$ (comp. ∠s)
$x° + 70° = 180°$ (co-int. ∠s, $AB \parallel CD$)
$x = 110$

b $\angle EBA = 70°$ (base ∠s of isos. ΔEAB)
$\angle DBC = 60°$ (∠ of equil. Δ)
$\therefore \angle EBD = 50°$ (adj. ∠s on a st. line)
$\angle BDE = 50°$ (base ∠s of isos. ΔEBD)
$x + 50° + 50° = 180°$ (∠ sum of Δ)
$x = 80$

c $\angle BAC = x°$ (alt. ∠s, $DB \parallel AC$)
$\angle BAC = x°$ (base ∠s of isos. Δ)
$2x° + 52° = 180°$ (∠ sum of Δ)
$x = 64$

d $\angle BCD + 74° + 138° + 96° = 360°$ (∠ sum of quad.)
$\angle BCD = 52°$
$\angle BDC = x°$ (base ∠s of isos. Δ)
$2x° + 52° = 180°$ (∠ sum of Δ)
$x = 64$

2 a In Δs AOB and COD,
(1) $AO = CO$ (radii of circle)
(2) $BO = DO$ (radii of circle)
(3) $\angle AOB = \angle COD$ (data)
$\therefore \Delta AOB \equiv \Delta COD$ (SAS)

b In Δs ABD and CBD,
(1) $AB = CB$ (data)
(2) $AD = CD$ (data)
(3) DB is common
$\therefore \Delta ABD \equiv \Delta CBD$ (SSS)
$\therefore \angle ADB = \angle CDB$
(matching ∠s of cong't Δs)

3 In Δs BED and CFD,
(1) $BD = CD$ (D is midpoint of BC)
(2) $\angle BED = \angle CFD$ ($BE \perp AD$, $CF \perp AD$)
(3) $\angle BDE = \angle CDF$ (vert. opp. angles)
$\therefore \Delta BED \equiv \Delta CFD$ (AAS)
$BE = CF$ (matching sides of cong't Δs)

4 a 3240°
b **i** 162° **ii** 18°

5 $x = 126$

6 72°, 144°

ASSIGNMENT 3B

1 3

2 3394·11 + 145·84 (one side of a triangle formed by the projection of one end of the board) = 3550 mm (rounded up)

3 The median and mode would not be useful as these are the same as the par of the hole in each case. Calculating the mean for each hole and the percentage of players who scored par or better gives a better indication of the degree of difficulty.

Hole	Par	Mean score	% of players scoring par or better
2	4	4·1	79%
7	5	4·5	94%
11	3	3·3	65%
16	3	3·2	75%

Both the mean and % of players scoring par or better indicate that the degree of difficulty of the holes are: 1 Hole 11; 2 Hole 16; 3 Hole 2; 4 Hole 7

4 green

5 a 12 points **b** 8 points

ASSIGNMENT 3C

1 a $\frac{a^2}{6}$ **b** $\frac{5a}{6}$ **c** $\frac{2}{3}$ **d** $-\frac{a}{6}$

2 a $\frac{2b}{3}$ **b** $\frac{b}{6}$ **c** $\frac{5n^4}{4m}$ **d** $\frac{1}{4ay^2}$

3 **a** $6y^2 - y - 15$ **b** $4x^2 - 11x + 6$

4 **a** $4m(m - 2)$ **b** $5xy^2(2 + 3x)$

5 **a** $x^2 + 7x + 12$ **b** $a^2 - 3a + 2$
c $m^2 - m - 20$ **d** $m^2 + m - 20$

6 **a** $(x + 5)(x - 1)$ **b** $(x - 5)(x + 1)$
c $(x + 4)(x + 1)$ **d** $(x - 4)(x - 1)$

7 **a** $x = 7{\cdot}4$ cm **b** $y = 25{\cdot}7$ cm **c** $x = 17{\cdot}9$ cm

8 **a** $(1, 6)$ **b** $\sqrt{40}$ **c** $\frac{1}{3}$

9 **a** $15{\cdot}84\,m^2$ **b** $79{\cdot}2\,m^3$

Chapter 4

Exercise 4:01

1 A. Not B, as some people would have more than one ticket. Not C, as the person most suited is more likely to be chosen.

2 Answers will vary.

3 Answers will vary.

4 **a** even chance **b** improbable **c** probable
d impossible **e** certain **f** even chance

5 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$
d $\frac{7}{8}$ **e** $\frac{3}{4}$ **f** 0

6 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$
d $\frac{1}{3}$ **e** 0 **f** 1

7 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 0
d $\frac{1}{13}$ **e** $\frac{12}{13}$ **f** $\frac{3}{13}$
g $\frac{1}{4}$ **h** $\frac{3}{4}$ **i** $\frac{2}{13}$

8 **a** $\frac{4}{500}$ (or $\frac{1}{125}$) **b** $\frac{496}{500}$ (or $\frac{124}{125}$)

9 **a** $\frac{16}{100}$ (or $\frac{4}{25}$) **b** $\frac{14}{100}$ (or $\frac{7}{50}$) **c** $\frac{43}{100}$
d $\frac{86}{100}$ (or $\frac{43}{50}$) **e** $\frac{30}{100}$ (or $\frac{3}{10}$) **f** $\frac{50}{100}$ (or $\frac{1}{2}$)

10 **a** 50% **b** 38% **c** 1% **d** 20%
e 80% **f** 77% **g** 23%

11 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{200}$ **d** $\frac{1}{200}$ **e** $\frac{3}{4}$
f $\frac{159}{200}$ **g** $\frac{197}{200}$ **h** $\frac{41}{200}$ **i** $\frac{1}{50}$

12 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{99}$

13 **a** $\frac{1}{8}$ **b** $\frac{5}{8}$ **c** $\frac{1}{4}$ **d** 0
e $\frac{2}{4}$ (or $\frac{1}{2}$) **f** $\frac{1}{2}$ **g** $\frac{1}{4}$ **h** $\frac{1}{3}$
i $\frac{1}{5}$ **j** $\frac{1}{4}$

14 **a** Answers will vary. Knowing that it has not rained on Saturday gives us more information than the weather forecaster had when he gave his prediction on Tuesday. This will affect the probability that it will rain on Sunday. We say that these two events are dependent events.

b The chance that the next toss is a head would be $\frac{1}{2}$ if the coin is a fair coin. However, the chance of getting 20 heads in a row is $\frac{1}{1048\,576}$ so one would have to expect that the coin either has two heads or is strongly biased towards heads.

15 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$ **e** $\frac{1}{3}$

16 **a** $\frac{48}{51}$ **b** 0·75 **c** 63% **d** 0·875
e **i** 99·5%
ii No, as \$135 is 1·35% of \$10 000 and this is clearly more than 0·5%. (There is 1 chance in 200 that I will die this year. $200 \times \$135 = \$27\,000$. As I am insured for only \$10 000, it does not represent good value.)

Exercise 4:02

1 **a** $\frac{4}{9}$ **b** $\frac{1}{9}$ **c** $\frac{4}{9}$ **d** $\frac{2}{9}$ **e** $\frac{4}{9}$

2 **a** $\frac{2}{6}$ **b** 0 **c** $\frac{4}{6}$ **d** $\frac{2}{6}$ **e** $\frac{4}{6}$

3 **a** $\frac{1}{9}$ **b** $\frac{3}{9}$ **c** $\frac{2}{9}$ **d** $\frac{1}{9}$ **e** $\frac{6}{9}$
f $\frac{1}{3}$ **g** 0 **h** $\frac{1}{3}$ **i** $\frac{2}{3}$

4 **a** 0 **b** 0 **c** $\frac{2}{6}$ **d** $\frac{1}{6}$ **e** $\frac{4}{6}$
f $\frac{2}{6}$ **g** $\frac{2}{6}$ **h** 1 **i** 0 **j** 0
k $\frac{1}{2}$ **l** 1

5 **a** $\frac{1}{200}$ **b** $\frac{1}{100}$ **c** $\frac{1}{10}$ **d** $\frac{1}{2}$ **e** 0

6 If the result of the first step is known, then only the outcomes starting with the known result should be considered. This reduces the number of possible outcomes, and probably changes the number of ways that the event can occur.

Investigation 4:02

1 Answers will vary.

2 **a** Here there is no replacement, so it is impossible to choose two greens (as we started with only 1 green counter).

b Answers will vary.

Exercise 4:03

1 **a** 8 **b** 2 (a head and a tail) **c** yes
d 3 **e** 3 **f** 1
g $\frac{1}{8}$ **h** $\frac{1}{8}$ **i** $\frac{3}{8}$
j $\frac{3}{8}$ **k** $\frac{7}{8}$ **l** $\frac{4}{8}$ (or $\frac{1}{2}$)

2 **a** 2 **b** 3 **c** 2
d 12 **e** yes **f** 1
g 6 **h** 2 **i** 12
j 4 **k** 4 **l** $\frac{1}{12}$
m $\frac{6}{12}$ (or $\frac{1}{2}$) **n** $\frac{2}{12}$ (or $\frac{1}{6}$) **o** $\frac{12}{12}$ (or 1)
p $\frac{4}{12}$ (or $\frac{1}{3}$) **q** $\frac{4}{12}$ (or $\frac{1}{3}$)

3 **a** 27 **b** 1 **c** 26
d 12 **e** $\frac{1}{27}$ **f** $\frac{1}{27}$
g $\frac{3}{27}$ **h** $\frac{3}{27}$ **i** $\frac{8}{27}$
j $\frac{7}{27}$

4 **a** 6 **b** 0 **c** 6
d 2 **e** 0 **f** 0
g 1 **h** $\frac{2}{6}$ (or $\frac{1}{3}$) **i** $\frac{4}{6}$ (or $\frac{2}{3}$)

5 a 60 b yes c 6
d 18 e 36 f 24
g There are only 2 yellow tickets, and there is no replacement.
h $\frac{6}{60}$ (or $\frac{1}{10}$) i $\frac{6}{60}$ (or $\frac{1}{10}$) j $\frac{36}{60}$ (or $\frac{3}{5}$)
k $\frac{24}{60}$ (or $\frac{2}{5}$) l $\frac{54}{60}$ (or $\frac{9}{10}$)

6 a 210 b 24 c 72
d 6 e 204 f 90
g 108 h $\frac{24}{210}$ (or $\frac{4}{35}$) i $\frac{72}{210}$ (or $\frac{12}{35}$)
j $\frac{3}{30}$ (or $\frac{1}{10}$)

Prep quiz 4:04

1 56 **2** 2 **3** 20 **4** 5 **5** 5
6 10 **7** 2 **8** 10 **9** 10 **10** 20

Exercise 4:04

1 a 10 b 30 c 20
d 12 e 8 f 24
g $\frac{1}{12}$ h $\frac{1}{8}$ i $\frac{1}{30}$
j $\frac{3}{30}$ (or $\frac{1}{10}$)

2 a 60 b 40 c 120
d 48 e 8 f 125
g $\frac{2}{48}$ (or $\frac{1}{24}$) h $\frac{1}{60}$

3 a 16 b 9 c 49
d $\frac{1}{16}$ e $\frac{8}{49}$ f $\frac{9}{9}$ (or 1)

4 a 12 b 6 c 42
d $\frac{1}{12}$ e $\frac{8}{42}$ (or $\frac{4}{21}$) f $\frac{6}{6}$ (or 1)

5 a 24 b 6 c 210
d $\frac{8}{210}$ (or $\frac{4}{105}$) e $\frac{6}{210}$ (or $\frac{3}{105}$) f $\frac{1}{24}$

6 a $n(S) = 64$
b i $n(E_1) = 25$ ii $\frac{25}{64}$
c i $n(E_2) = 9$ ii $\frac{9}{64}$
d i $n(E_3) = 15$ ii $\frac{15}{64}$
e i $n(E_4) = 15$ ii $\frac{15}{64}$
f i $n(E_5) = 30$ ii $\frac{30}{64}$ (or $\frac{15}{32}$)

7 a $n(S) = 56$
b i $n(E_1) = 20$ ii $\frac{20}{56}$ (or $\frac{5}{14}$)
c i $n(E_2) = 6$ ii $\frac{6}{56}$ (or $\frac{3}{28}$)
d i $n(E_3) = 15$ ii $\frac{15}{56}$
e i $n(E_4) = 15$ ii $\frac{15}{56}$
f i $n(E_5) = 30$ ii $\frac{30}{56}$ (or $\frac{15}{28}$)

8 a $n(S) = 20$ b $\frac{1}{20}$
c $\frac{3}{20}$ d $\frac{4}{20}$ (or $\frac{1}{5}$)

9 a $n(S) = 100$
b i $n(E_1) = 9$ ii $\frac{9}{100}$
c i $n(E_2) = 4$ ii $\frac{4}{100}$ (or $\frac{1}{25}$)
d i $n(E_3) = 10$ ii $\frac{10}{100}$ (or $\frac{1}{10}$)
e i $n(E_4) = 10$ ii $\frac{10}{100}$ (or $\frac{1}{10}$)
f i $n(E_5) = 20$ ii $\frac{20}{100}$ (or $\frac{1}{5}$)

10 a $n(S) = 1320$
b i $n(E_1) = 24$ ii $\frac{24}{1320}$ (or $\frac{1}{55}$)
c i $n(E_2) = 120$ ii $\frac{120}{1320}$ (or $\frac{1}{11}$)
d i $n(E_3) = 12$ ii $\frac{12}{1320}$ (or $\frac{1}{110}$)

Exercise 4:05

1 a $\frac{1}{12}$ b $\frac{5}{16}$ c $\frac{1}{10}$
d $\frac{5}{48}$ e $\frac{1}{30}$ f $\frac{5}{40}$ (or $\frac{1}{8}$)
g $\frac{6}{48}$ (or $\frac{1}{8}$) h $\frac{3}{16}$ i $\frac{9}{40}$

2 a $\frac{1}{4}$ b $\frac{1}{36}$ c $\frac{25}{64}$
d $\frac{1}{25}$

3 a $\frac{16}{64}$ (or $\frac{1}{4}$) b $\frac{9}{64}$ c $\frac{12}{64}$ (or $\frac{3}{16}$)
d $\frac{12}{64}$ (or $\frac{3}{16}$) e $\frac{4}{64}$ (or $\frac{1}{16}$) f $\frac{4}{64}$ (or $\frac{1}{16}$)
g $\frac{3}{64}$ h $\frac{3}{64}$

4 a $\frac{50}{99}$ b $\frac{40}{99}$ c $\frac{5}{99}$
d $\frac{4}{99}$

5 a $\frac{1}{42}$ b $\frac{18}{42}$ (or $\frac{3}{7}$) c $\frac{2}{42}$ (or $\frac{1}{21}$)

6 a $\frac{16}{100}$ (or $\frac{4}{25}$) b $\frac{24}{100}$ (or $\frac{6}{25}$) c $\frac{36}{100}$ (or $\frac{9}{25}$)
d $\frac{24}{100}$ (or $\frac{6}{25}$)

7 a $\frac{8}{15}$ b $\frac{12}{25}$ c $\frac{2}{6}$ (or $\frac{1}{3}$)
d $\frac{3}{10}$

8 a $\frac{9}{800}$ b $\frac{15}{800}$ (or $\frac{3}{160}$)

9 a $\frac{36}{100}$ (or $\frac{9}{25}$) b $\frac{72}{150}$ (or $\frac{12}{25}$) c $\frac{8}{30}$ (or $\frac{4}{15}$)
d $\frac{18}{60}$ (or $\frac{3}{10}$)

10 a $\frac{1}{8}$ b $\frac{125}{216}$ c $\frac{125}{512}$
d $\frac{27}{512}$ e $\frac{1}{125}$ f $\frac{5}{32}$
g $\frac{6}{96}$ (or $\frac{1}{16}$) h $\frac{20}{80}$ (or $\frac{1}{4}$)

11 a $\frac{3}{20}$ b $\frac{3}{120}$ (or $\frac{1}{40}$) c $\frac{21}{160}$
d $\frac{21}{240}$ (or $\frac{7}{80}$) e $\frac{15}{120}$ (or $\frac{1}{8}$) f $\frac{1}{192}$
g $\frac{21}{960}$ (or $\frac{7}{320}$)

12 Spending the day in both Sydney and London are dependent events. I cannot spend the day in both Sydney and London. If I spend the day in Sydney then the probability of my also spending the day in London is 0 not $\frac{2}{10}$, so the probability of spending the day in both Sydney and London = $\frac{7}{10} \times 0$ (or 0).

Fun spot 4:05

1 The contestant would improve their chance of choosing the car if they changed their choice.

2 Yes. Originally the contestant had one chance in three of choosing the car. There were two chances in three that they had not chosen the car. Of the two doors not chosen, Monty always opened the door that did not contain the car, so the chance of the car being behind the other door is $\frac{2}{3}$. Changing choices at this point doubles the chance of choosing the car.

Exercise 4:06

1 a $\frac{1}{20}$ b $\frac{3}{20}$ c $\frac{6}{20}$

2 a $\frac{1}{12}$ b $\frac{11}{12}$ c $\frac{1}{4}$

3 a $\frac{1}{30}$ b $\frac{2}{30}$ (or $\frac{1}{15}$) c $\frac{28}{30}$ (or $\frac{14}{15}$)
d $\frac{1}{120}$ e $\frac{2}{120}$ (or $\frac{1}{60}$) f $\frac{118}{120}$ (or $\frac{59}{60}$)

4 a $\frac{1}{380}$ b $\frac{90}{380}$ (or $\frac{9}{38}$) c $\frac{90}{380}$ (or $\frac{9}{38}$)
d $\frac{72}{380}$ (or $\frac{18}{95}$) e $\frac{110}{380}$ (or $\frac{11}{38}$) f $\frac{100}{380}$ (or $\frac{5}{19}$)
g 0 h $\frac{2}{380}$ (or $\frac{1}{190}$) i $\frac{504}{6840}$ (or $\frac{7}{95}$)
j $\frac{1}{6840}$ k $\frac{720}{6840}$ (or $\frac{2}{19}$) l $\frac{6120}{6840}$ (or $\frac{17}{19}$)

5 a $\frac{2}{42}$ (or $\frac{1}{21}$) b $\frac{40}{42}$ (or $\frac{20}{21}$) c $\frac{20}{42}$ (or $\frac{10}{21}$)
d $\frac{20}{42}$ (or $\frac{10}{21}$) e $\frac{60}{210}$ (or $\frac{2}{7}$) f 0
g $\frac{40}{210}$ (or $\frac{4}{21}$) h $\frac{120}{210}$ (or $\frac{4}{7}$)

6 a $\frac{12}{56}$ (or $\frac{3}{14}$) b $\frac{30}{56}$ (or $\frac{15}{28}$) c $\frac{2}{56}$ (or $\frac{1}{28}$)
d $\frac{54}{56}$ (or $\frac{27}{28}$) e $\frac{12}{56}$ (or $\frac{3}{14}$) f $\frac{24}{56}$ (or $\frac{3}{7}$)
g $\frac{12}{336}$ (or $\frac{1}{28}$) h $\frac{60}{336}$ (or $\frac{5}{28}$) i $\frac{120}{336}$ (or $\frac{5}{14}$)
j $\frac{216}{336}$ (or $\frac{9}{14}$)

7 a $\frac{60}{336}$ (or $\frac{5}{28}$) b $\frac{30}{336}$ (or $\frac{5}{56}$)

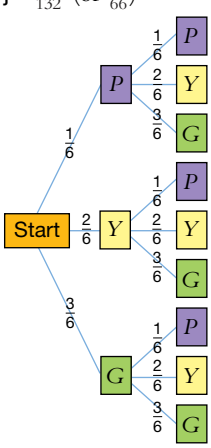
8 a $\frac{2}{20}$ (or $\frac{1}{10}$) b $\frac{8}{20}$ (or $\frac{2}{5}$) c $\frac{2}{20}$ (or $\frac{1}{10}$)
d $\frac{8}{20}$ (or $\frac{2}{5}$) e $\frac{10}{20}$ (or $\frac{1}{2}$) f $\frac{10}{20}$ (or $\frac{1}{2}$)
g $\frac{16}{20}$ (or $\frac{4}{5}$) h $\frac{4}{20}$ (or $\frac{1}{5}$)

9 a $\frac{2}{60}$ (or $\frac{1}{30}$) b $\frac{16}{60}$ (or $\frac{4}{15}$) c $\frac{2}{60}$ (or $\frac{1}{30}$)
d $\frac{8}{60}$ (or $\frac{2}{15}$) e $\frac{8}{60}$ (or $\frac{2}{15}$) f $\frac{4}{60}$ (or $\frac{1}{15}$)
g $\frac{10}{60}$ (or $\frac{1}{6}$) h $\frac{20}{60}$ (or $\frac{1}{3}$)

10 a $\frac{1}{144}$ b $\frac{25}{144}$ c $\frac{30}{144}$ (or $\frac{5}{24}$)
d $\frac{30}{144}$ (or $\frac{5}{24}$) e $\frac{60}{144}$ (or $\frac{5}{12}$) f $\frac{36}{144}$ (or $\frac{1}{4}$)
g $\frac{108}{144}$ (or $\frac{3}{4}$) h $\frac{5}{12}$ i $\frac{6}{12}$ (or $\frac{1}{2}$)
j $\frac{82}{144}$ (or $\frac{41}{72}$)

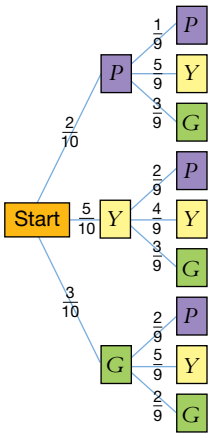
11 a $\frac{0}{12}$ (or 0) b $\frac{20}{132}$ (or $\frac{5}{33}$) c $\frac{30}{132}$ (or $\frac{5}{22}$)
d $\frac{30}{132}$ (or $\frac{5}{22}$) e $\frac{60}{132}$ (or $\frac{5}{11}$) f $\frac{30}{132}$ (or $\frac{5}{22}$)
g $\frac{102}{132}$ (or $\frac{17}{22}$) h $\frac{4}{11}$ i $\frac{5}{11}$
j $\frac{82}{132}$ (or $\frac{41}{66}$)

12



a $\frac{1}{36}$ b $\frac{9}{36}$ (or $\frac{1}{4}$) c $\frac{6}{36}$ (or $\frac{1}{6}$)
d $\frac{6}{36}$ (or $\frac{1}{6}$) e $\frac{12}{36}$ (or $\frac{1}{3}$) f $\frac{16}{36}$ (or $\frac{4}{9}$)
g $\frac{20}{36}$ (or $\frac{5}{9}$) h $\frac{3}{6}$ (or $\frac{1}{2}$) i $\frac{2}{6}$ (or $\frac{1}{3}$)
j $\frac{22}{36}$ (or $\frac{11}{18}$)

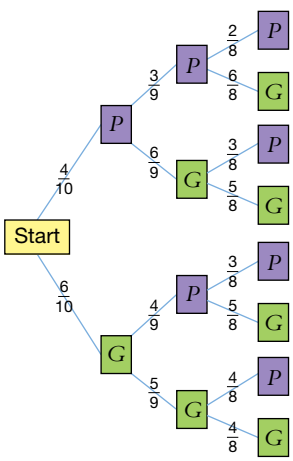
13



a $\frac{2}{90}$ (or $\frac{1}{45}$) b $\frac{6}{90}$ (or $\frac{1}{15}$) c $\frac{15}{90}$ (or $\frac{1}{6}$)
d $\frac{15}{90}$ (or $\frac{1}{6}$) e $\frac{30}{90}$ (or $\frac{1}{3}$) f $\frac{20}{90}$ (or $\frac{2}{9}$)
g $\frac{70}{90}$ (or $\frac{7}{9}$) h $\frac{2}{9}$ i $\frac{4}{9}$
j $\frac{67}{90}$

14 a $\frac{120}{504}$ (or $\frac{5}{21}$) b $\frac{386}{504}$ (or $\frac{16}{21}$) c $\frac{6}{504}$ (or $\frac{1}{84}$)
d $\frac{498}{504}$ (or $\frac{83}{84}$) e $\frac{126}{504}$ (or $\frac{7}{28}$) f $\frac{90}{504}$ (or $\frac{5}{28}$)
g $\frac{90}{504}$ (or $\frac{5}{28}$) h $\frac{90}{504}$ (or $\frac{5}{28}$) i $\frac{270}{504}$ (or $\frac{15}{28}$)
j $\frac{168}{504}$ (or $\frac{1}{3}$) k $\frac{336}{504}$ (or $\frac{2}{3}$) l $\frac{2}{56}$ (or $\frac{1}{28}$)
m $\frac{20}{56}$ (or $\frac{5}{14}$)

15



a $\frac{24}{720}$ (or $\frac{1}{30}$) b $\frac{696}{720}$ (or $\frac{29}{30}$) c $\frac{120}{720}$ (or $\frac{1}{6}$)
d $\frac{600}{720}$ (or $\frac{5}{6}$) e $\frac{576}{720}$ (or $\frac{4}{5}$) f $\frac{72}{720}$ (or $\frac{1}{10}$)
g $\frac{72}{720}$ (or $\frac{1}{10}$) h $\frac{72}{720}$ (or $\frac{1}{10}$) i $\frac{360}{720}$ (or $\frac{1}{2}$)
j $\frac{432}{720}$ (or $\frac{3}{5}$) k $\frac{288}{720}$ (or $\frac{2}{5}$) l $\frac{20}{72}$ (or $\frac{5}{18}$)
m $\frac{6}{72}$ (or $\frac{1}{12}$)

16 $\frac{7}{504}$ (or $\frac{1}{72}$)

Diagnostic test 4

1 **a** i $\frac{2}{52}$ (or $\frac{1}{26}$) ii $\frac{50}{52}$ (or $\frac{25}{26}$)
b i $\frac{1}{3}$ ii $\frac{2}{3}$

2 B 50%

3 **a** $\frac{9}{36}$ (or $\frac{1}{4}$) **b** $\frac{9}{18}$ (or $\frac{1}{2}$) **c** $\frac{4}{36}$ (or $\frac{1}{9}$)

4 **a** $\frac{6}{30}$ (or $\frac{1}{5}$) **b** $\frac{6}{15}$ (or $\frac{2}{5}$) **c** $\frac{4}{30}$ (or $\frac{2}{15}$)

5 **a** $\frac{1}{6}$ **b** $\frac{0}{6}$ (or 0) **c** $\frac{6}{6}$ (or 1)

6 **a** 36 (6×6) **b** 8 $(2 \times 2 \times 2)$ **c** 25 (5×5)
d 20 (5×4) **e** 125 $(5 \times 5 \times 5)$
f 60 $(5 \times 4 \times 3)$

7 **a** $\frac{12}{30}$ (or $\frac{2}{5}$) **b** $\frac{16}{30}$ (or $\frac{8}{15}$)

8 **a** $\frac{60}{336}$ (or $\frac{5}{28}$) **b** $\frac{90}{336}$ (or $\frac{15}{56}$)

9 **a** $\frac{9}{50}$ **b** $\frac{3}{24}$ (or $\frac{1}{8}$)

10 $\frac{1}{20}$

11 **a** i $\frac{6}{20}$ ii $\frac{12}{20}$ (or $\frac{3}{5}$)
b i $\frac{6}{60}$ (or $\frac{1}{10}$) ii $\frac{6}{60}$ (or $\frac{1}{10}$)

ASSIGNMENT 4A

1 **a** $\frac{4}{13}$ **b** $\frac{3}{52}$ **c** $\frac{50}{52}$
d $\frac{49}{52}$

2 **a** 18% **b** 21% **c** 79%
d 48% **e** 69% **f** 41%

3 **a** $\frac{5}{100}$ (or 5%) **b** $\frac{5}{99}$ **c** $\frac{5}{98}$
d $\frac{4}{98}$

4 **a** $\frac{1}{16}$ **b** $\frac{1}{48}$ **c** $\frac{7}{20}$
d $\frac{2}{16}$ (or $\frac{1}{8}$) **e** $\frac{12}{80}$ (or $\frac{3}{20}$) **f** $\frac{18}{48}$ (or $\frac{3}{8}$)

ASSIGNMENT 4B

1 **a** 10 km **b** 20 km **c** 11:30 am
d Callum **e** 11:30 am

2 **a** D **b** C **c** A **d** B

3 **a** One side is longer than the sum of the other two. This cannot happen in any triangle.
b The angle sum to more than 180°.
c In a right-angled triangle the longest side must be opposite the right angle.

4 201

5 9

ASSIGNMENT 4C

1 **a** 3 **b** 2 **c** 2

2 **a** 6000 kg **b** 7 000 000 L **c** 8 µs

3 **a** $3{\cdot}04 \times 10^{11}$ L **b** $1{\cdot}8 \times 10^{12}$ B

4 **a** 15·25 and 15·35 **b** 6·245 and 6·255

5 **a** $30m + 8$ **b** $2a - 21$ **c** $22x - 41$

6 **a** 7 **b** 4 **c** 3

7 **a** $(x + 7)(x - 4)$ **b** $(y - 10)(y + 1)$
c $(x + 8)(x + 3)$

8 **a** $y = 5$ or 8 **b** $x = -3$ or 4 **c** $m = 0$ or -11

9 **a** $t(t + 8) = 0$, $t = 0$ or -8
b $(x - 3)(x + 3) = 0$, $x = 3$ or -3
c $(y - 3)(y + 4) = 0$, $y = 3$ or -4

10 **a** In Δs ABO and CDO:
1 $AO = CO$ (radii of a circle)
2 $BO = DO$ (radii of a circle)
3 $\angle BOA = \angle COD$ (given)
$\therefore \Delta ABO \equiv \Delta CDO$ (SAS)
$\therefore AB = CD$ (matching sides of congruent Δs)
b In Δs ACD and DBA:
1 $\angle CAD = \angle BDA$ (given)
2 $\angle ADC = \angle BAD$ (alternate $\angle$s, $AB \parallel CD$)
3 AD is common.
$\therefore \Delta ACD \equiv \Delta DBA$ (AAS)
$\therefore AB = CD$ (matching sides of congruent Δs)

Chapter 5

Prep quiz 5:01

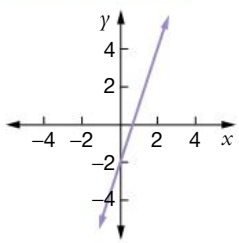
1 1 **2** −1 **3** −3 **4** −11

5 $-2\frac{1}{2}$ **6** −2 **7** $-1\frac{1}{2}$ **8** $-4\frac{1}{2}$

9

x	0	1	2
y	−2	1	4

10



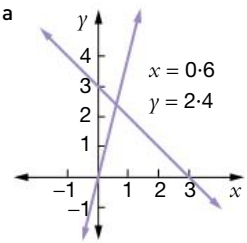
Exercise 5:01

1 **a** $x = 1$, $y = 2$ **b** $x = -2$, $y = -1$
c $x = -1$, $y = 2$ **d** $x = 0$, $y = 3$
e $x = 4$, $y = -1$ **f** $x = 3$, $y = 0$
g No solution, since the lines are parallel.
h An infinite number of solutions exist, e.g. $(-1, 0)$, $(0, 1)$, $(\frac{1}{2}, 1\frac{1}{2})$ etc. These two equations represent the same line.

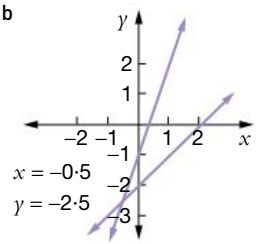
2 **a** $x = 0{\cdot}2$, $y = 1{\cdot}2$ **b** $x = -3{\cdot}3$, $y = -0{\cdot}3$
c $x = 1{\cdot}3$, $y = -2{\cdot}6$ **d** $x = 2{\cdot}8$, $y = -0{\cdot}3$

3 **a** $x = 2$, $y = -1$ **b** $x = 2$, $y = -1$
c $x = 1$, $y = -2$ **d** $x = 2$, $y = 4$
e $a = -1$, $b = -2$ **f** $p = -2$, $q = 2$
g $a = 1$, $b = 1$ **h** $p = 6$, $q = 2$

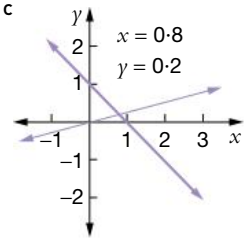
4 **a**



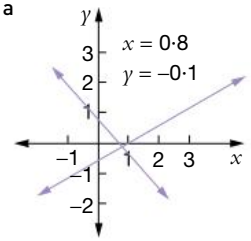
b



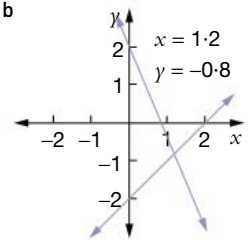
c



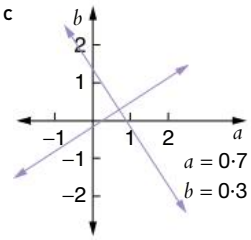
5 a



b



c



6 The two numbers are 6 and 9.
7 After $2\frac{1}{2}$ hours and 150 km.
8 6 machines, $1080

Prep quiz 5:02

1 $x = 6$ **2** $y = 2\frac{2}{5}$ (or 2·4) **3** $x = \frac{7}{10}$ (or 0·7)
4 $-2x$ **5** $-2y$ **6** $6x$ **7** $y = -2$
8 $y = -1$ **9** $y = 3$ **10** $y = 3$

Exercise 5:02A

1 a $x = -1, y = 4$ **b** $x = 2, y = 5$
c $x = -2, y = -1$ **d** $x = 3, y = -2$
e $x = 4, y = 1$ **f** $x = 4, y = 0$
g $x = 4, y = -2$ **h** $x = 3, y = 3$
i $x = 6, y = 2$ **j** $x = 1, y = 5$

2 a $x = 6, y = -1$ **b** $x = 2\frac{3}{4}, y = \frac{1}{2}$
c $x = -12, y = 10$ **d** $x = 5, y = 3$
e $x = -2, y = 4$ **f** $x = 2, y = 3$
g $x = 3, y = 4$ **h** $x = 5, y = -2$
i $x = -2, y = 4$

3 a $x = \frac{7}{8}, y = \frac{3}{4}$ **b** $a = -3\frac{2}{3}, b = 17$
c $m = \frac{5}{6}, n = -1\frac{1}{12}$ **d** $x = \frac{1}{14}, y = -\frac{5}{14}$

4 a $a = 1\frac{1}{16}, b = \frac{3}{8}$ **b** $x = \frac{12}{17}, y = 1\frac{8}{17}$
c $m = 1\frac{2}{17}, n = \frac{10}{17}$ **d** $x = -2\frac{1}{3}, y = -4\frac{8}{9}$

Exercise 5:02B

1 a $x = 5, y = 4$ **b** $x = 5, y = 9$
c $x = 3, y = 1$ **d** $x = 5, y = -1$
e $x = 3, y = 2$ **f** $x = 1, y = 2$
g $x = -2, y = 4$ **h** $x = -8, y = 2$
i $x = 1\frac{1}{3}, y = 7$ **j** $x = 7, y = -1\frac{2}{7}$
k $x = 1, y = 2\frac{1}{2}$ **l** $x = 2, y = -3\frac{2}{5}$

2 a $x = 2, y = 1$ **b** $x = 3, y = -8$
c $x = 1, y = -4$ **d** $x = -2, y = -3$
e $x = 4, y = 6$ **f** $x = 4, y = 1$
g $x = 3, y = 4$ **h** $x = 2\frac{1}{4}, y = \frac{1}{2}$
i $x = 4, y = 4$ **j** $x = 2, y = 1$
k $x = -3, y = 5\frac{1}{5}$ **l** $x = -1, y = -4$

3 a $x = 1, y = 5$ **b** $x = 2, y = 3$
c $x = 5, y = -7$ **d** $x = -2, y = 4$
e $x = 2, y = 3$ **f** $x = -2\frac{1}{3}, y = 1\frac{1}{2}$
g $x = 1, y = -2$ **h** $x = 1, y = -8$
i $x = 3, y = -5$

4 a $x = 4, y = 3$ **b** $x = 1, y = 5$
c $x = 2, y = 2$ **d** $x = 3, y = 2$
e $x = 1, y = -2$ **f** $x = -2, y = 3$
g $x = 1, y = -2$ **h** $x = 3, y = 1$
i $x = 1, y = 2$ **j** $x = 0, y = 5$
k $x = 2, y = 9$ **l** $x = -3, y = 2$
m $x = 1, y = -1$ **n** $x = 7\frac{8}{11}, y = 4\frac{4}{11}$

Exercise 5:03

1 a 7 and 18 **b** 32 and 65 **c** 3 and 9
d 11 and 2 **e** 5 and 22

2 a length = 8 cm, width = 3 cm
b pen = $1.65, pencil = 35c
c maths = 72, science = 57
d chocolate = $1.20, drink = $2.50
e Bill = $1, Jim = 50c

3 a $x = 5, y = 2$ **b** $x = 3, y = 2$
c $x = 15, y = 30$ **d** $x = 35, y = -35$
e $x = 7, y = 2$ **f** $x = 4, y = -2$

4 a 34 grevilleas and 66 waratahs
b box A = $28\frac{1}{3}$ kg, box B = 50 kg
c 36 rows of 45 seats and 12 rows of 40 seats
d 15 seniors and 75 juniors

5 a 8 scarves **b** 5 weeks

Investigation 5:03

1 34 and 52
2 236 girls, 184 boys
3 23 fours, 6 sixes
4 216 of company A and 425 of company B

Diagnostic test 5

1 a $x = -2, y = -1$ **b** $x = 2, y = 3$
c $x = -4, y = 1$

2 a $x = 3, y = 1$ **b** $x = 3\frac{2}{5}, y = -1\frac{3}{5}$
c $a = 1\frac{3}{7}, b = 2\frac{5}{7}$

3 a $x = 2, y = 1$ **b** $x = 2\frac{3}{5}, y = -\frac{1}{5}$
c $a = 2, b = 0$

ASSIGNMENT 5A

1 **a** $x = 3, y = 0$ **b** $x = 1\frac{1}{3}, y = 2\frac{1}{3}$
c $a = 1\frac{18}{33}, b = -\frac{6}{33}$ **d** $a = -2, b = -5\frac{1}{3}$
e $a = 1\frac{7}{16}, b = -1\frac{3}{16}$ **f** $x = \frac{3}{5}, y = -\frac{8}{5}$ or $-1\frac{3}{5}$
g $p = -1, q = 3$ **h** $x = \frac{1}{4}, y = -2$
i $m = 1\frac{3}{19}, n = \frac{10}{19}$
2 father is 54 years old, daughter is 18 years old
3 number of adults = 59, number of children = 141
4 share A = \$1.80, share B = \$1.20
5 length = 15 cm, width = 10 cm
6 length = 15 cm, width = 5 cm
7 4 km/h

ASSIGNMENT 5B

1 Answers will vary.
a Answers will vary.
b overestimate
c Estimate to the nearest metre, then add a few metres to the estimate.
2 1
3 $x = 72$
4 **a** 3 **b** 6 **c** 10 **d** 28
5 10
6 Dixon-Fly, Wynn-Free, Goad-Back, McCully-Breast with a time of 147·65 seconds.

ASSIGNMENT 5C

1 **a** \$465, 18·7% **b** \$1600
2 **a** 4·5 units **b** (4, 3) **c** $\frac{1}{2}$
d 1 **e** −2
3 **a** yes **b** $d = 32$ **c** $e = 30$
d length = 35 cm, breadth = 18·75 cm
4 **a** $v = 59{\cdot}2$ **b** $u = 27$ **c** $t = 2$
5 **a** $m^2 + 2m - 15$ **b** $a^2 - 9$
c $14y^2 + 31y + 15$
6 **a** $\frac{117}{260}$ **b** $\frac{122}{260}$ (or $\frac{61}{130}$) **c** $\frac{75}{260}$ (or $\frac{15}{52}$)
7 **a** $\frac{1}{12}$ **b** $\frac{4}{12}$ (or $\frac{1}{3}$)

Chapter 6

Exercise 6:01

1 For the answers to this question, see the table on page 134.
2 **a** 0·5% p.a. for that month
b 2·65% p.a. **c** daily
d monthly **e** 0·007%
3 **a** 10 months
b **i** 4·75% **ii** 4·60% **iii** 4·87%
4 **a** 5, \$3 per transaction
b **i** 4·25% p.a. **ii** 2·5% p.a. **iii** 4·75% p.a.
c \$5000
5 **a** **i** 10 cents **ii** 1 cent **b** \$96 000
6 **a** 4·01% p.a.
b **i** \$40.10 **ii** \$3.34
c **i** 4 cents (only 0·01%p.a.)
ii \$17.04 (4·01% p.a.)

Prep quiz 6:02

1 $\frac{1}{12}$ **2** $\frac{5}{12}$ **3** $\frac{11}{12}$ **4** $\frac{1}{365}$
5 $\frac{7}{365}$ **6** $\frac{128}{365}$ **7** \$58.50 **8** \$108.80
9 \$15 000 **10** \$7.41

Exercise 6:02

1 **a** \$446.40 **b** \$5850 **c** \$7.02 **d** \$246.33
e \$157 500 **f** \$247 500 **g** \$140 **h** \$12 000
i \$15 174 **j** \$1379
2 **a** \$31.36 **b** \$2052 **c** \$15.70 **d** \$3.23
e \$139.20 **f** \$362.10 **g** \$22.77 **h** \$210.70
3 **a** \$66 **b** \$273.60 **c** \$6.05 **d** \$113.40
e \$20 160 **f** \$0.81 **g** \$43.61 **h** \$6.72
i \$2772.32 **j** \$81.61
4 **a** \$372 **b** \$9.58 **c** \$67.08 **d** \$1283.33
e \$1071 **f** \$1133.99 **g** \$1.41 **h** \$4.72
i \$107.80 **j** \$12.60
5 **a** \$16.32 **b** \$161.61 **c** \$6.75 **d** \$727.10
e \$67.40 **f** \$267.12 **g** \$23.75 **h** \$1.27
i \$126.55 **j** \$21.74

Prep quiz 6:03

1 0·18 **2** 0·05 **3** 0·054 **4** 0·055
5 0·0525 **6** 200 **7** 30 **8** 5
9 0·1 **10** 3

Exercise 6:03

1 **a** **i** \$5878 **ii** \$20 573
b \$5080 **c** \$1108.80 **d** \$84 **e** \$28 500
f \$225
g **i** \$2.36 **ii** \$7.07 **iii** \$68.34
2 **a** 644 **b** $8{\cdot}4t$ **c** \$1045.44 **d** \$260.40
3 **a** \$3500 **b** \$80 **c** \$280 000 **d** \$1000
4 **a** \$12 100 **b** \$4255 **c** \$32 310 **d** \$15 755.83
5 **a** \$13 000 **b** \$276 750 **c** \$7550 **d** \$30 900
6 **a** 9% p.a. **b** \$5600 **c** 6 **d** 14·25% p.a.

Investigation 6:03

1 **a** Money earned from typing in 2005 (Y9).
b Money earned from selling newspapers in 2003 (Y7).
c The category 'Babysitting'.
d The total amount earned washing cars from 2003 to 2006.
e The total income from these sources in 2004 (Y8).
2 **a** B2 + C2 + D2 + E2
b C2 + C3 + C4 + C5 + C6 + C7 + C8
3 \$7026
4 The sum of the rows and the sum of the columns are the same as they contain the same numbers.

Prep quiz 6:04

1 $11 000 2 $12 100 3 $13 310
4 $14 641 5 $16 105.10 6 $101 000
7 $102 010 8 $103 030.10 9 $104 060.40
10 $2420

Exercise 6:04

1 a $2.04 b $21.76 c $41.82
d $70.70
2 a $2420 b $7024.64 c $11 449
d $60 397.48 e $33 386.73
3 a $A = \$501.76$, $I = \$101.76$
b $A = \$2741.50$; $I = \$841.50$
c $A = \$6356.34$, $I = \$1006.34$
d $A = \$125.97$, $I = \$25.97$
e $A = \$3321.27$, $I = \$447.27$
f $A = \$730\,340$, $I = \$80\,340$
g $A = \$122\,022.07$, $I = \$36\,322.07$
4 $A = \$7320.50$, $I = \$2320.50$
5 $20 776.96
6 a $12.12 b $851.39
c $160.80 d $43.27
7 10% pa compound interest is better by $24.10
8 1166
9 $207 360
10 a $37 791.36 b $5.67 c $3.59

Prep quiz 6:05

1 80% 2 93% 3 100% 4 100%
5 $280 6 $280 7 $744 8 $744
9 yes 10 yes

Exercise 6:05

1 a $19 926 b $213.90
2 a $105 318.40 b $69 066.24 c $3066.62
d $7144.20 e 650 f 50 165
3 a $30 681.60 b $1253.38 c $398.29
d $1571.80 e 190 f 6535
4 a $800.15 b $1607.87
5 a $32 931.50 b $38 587.50
6 $1911.78
7 7% pa
8 3 years

Prep quiz 6:06

1 0·09 2 0·065
3 0·1125 4 0·75% per month
5 0·67% per month 6 0·54% per month
7 1·15 8 0·85
9 $69 960·25 10 $20 880·25

Exercise 6:06

1 a $7049.37 b $1380.97 c $58 361.78
d $657.02
2 a $3049.37 b $520.97 c $41 001.78
d $180.52
3 a $967.43 b $11 074.22 c $38.29
4 a $326.68 b $7764.76 c $9946.39
d $300.52
5 $552 904.08
6 11% p.a. compound interest; $24.54
7 a $5095.57 b $1633.52 c $616.08
8 a $7 788 322.36 b $2 748 960.63
c $2 921 255.08
9 $4893.86
10 $37 426.70
11 a $1762.34 b $1790.85 c $1806.11
d $1816.70 e $1819.40
f $1821.94; However small we make the time period, the investment will not exceed $1822.12 in 5 years. Therefore there is a limit to which the investment will grow.
12 a $38.07 b $1156.81 c $5513.57

Investigation 6:06

1 a amount = $45 044, int. = $25 044
b amount = $1 477 455, int. = $1 327 455
c amount = $11 953.50, int. = $4453.50
2 a amount = $125 440
b amount = $126 970
3 For **b**, interest is calculated more often. It accumulates 24 times instead of just twice as in part **a**.

Diagnostic test 6

1 a $360 b $1711.20 c $102 000
2 a $279.50 b $13 950 c $1822.50
3 a $80 b $65.10 c $1023.75
4 a $3315 b 8% p.a. c $100
5 a $12 986.07 b $12 180.68 c $17 892.57
6 a $960.40 b 512 c $829.73

ASSIGNMENT 6A

1 a $12.15 b $12.20
2 a $1305 b $1333.38
c $2670; more interest of $31.62
3 a $525 b $3158.75 c $73.33
d $89.10
4 $2207.63
5 $2966.40
6 $82 978.56
7 a i $447 711.92 ii $453 504.60
iii $454 849.17
b $7137.25
8 a $864 629.05 b $550
9 5
10 $6702.71

ASSIGNMENT 6B

1 59

2 a 5 b 8 c 10

3 The man is 66, the girl is 6.

4 6 hours, because in 6 hours (half of 12) the hour hand will be pointing the opposite direction to the present, and the minute hand will be in the same place.

5 4

6 a i 1627 ii 8.0 iii 20 337 500

b 10th

c i Portugal ii Denmark

ASSIGNMENT 6C

1 a $x = 2\frac{3}{7}$ b $x = 6$ c $x = \frac{3}{2}$

2 a \$4.35 b \$5.20

3 a $2x$ b $\frac{14}{15x}$ c $6z^2$ d $4b^2$

4 a $4m + 26$ b $6m^3n^3 - 10m^2n^4$

c $x^2 + 4x - 5$ d $4y^2 - 25$

5 a $x = \pm 8$ b $x = 9$ or -2 c $x = 4$ or -3

6 a $a = 115$ b $x = 50$ c $n = 25$

7 a 135° b 45°

8 a $\frac{1}{2}$

b i $\frac{4}{9}$ ii $\frac{5}{9}$

9 a $\frac{1}{8}$ b $\frac{3}{8}$

10 a $x = 4, y = -1$ b $a = -2, b = -4$

Chapter 7

Prep quiz 7:01

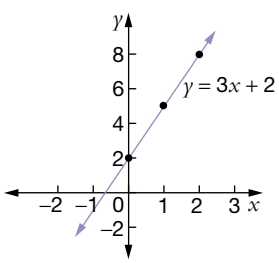
1 1 **2** 3 **3** 5 **4** 2 **5** 2

6 1 **7** *AB* **8** *DC* **9** *DC* **10** *AB*

Exercise 7:01

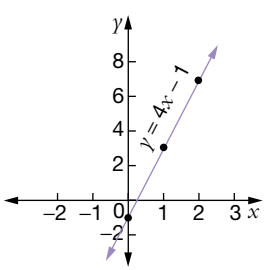
1 a

x	0	1	2
y	2	5	8



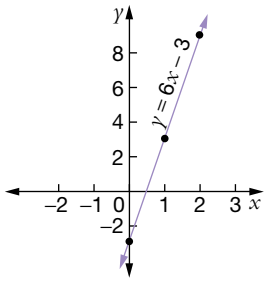
b

x	0	1	2
y	−1	3	7



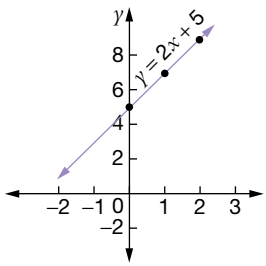
c

x	0	1	2
y	−3	3	9



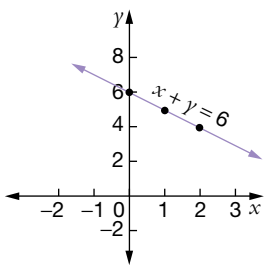
d

x	0	1	2
y	5	7	9



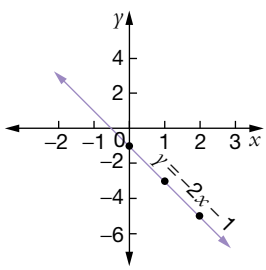
e

x	0	1	2
y	6	5	4

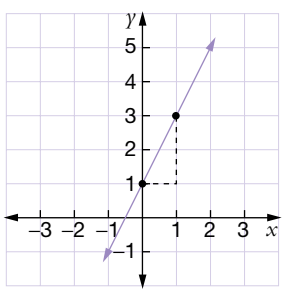


f

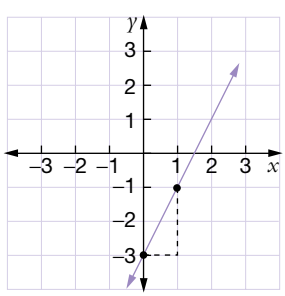
x	0	1	2
y	−1	−3	−5



2 a



b



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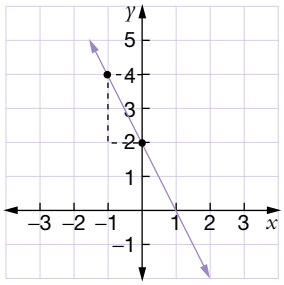
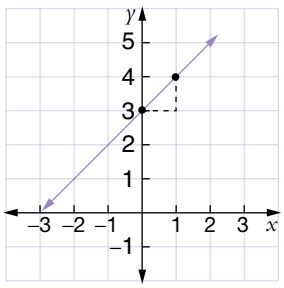
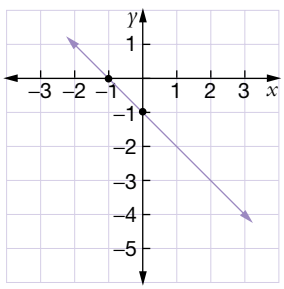


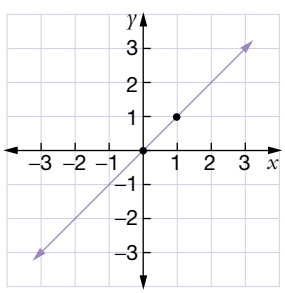
d



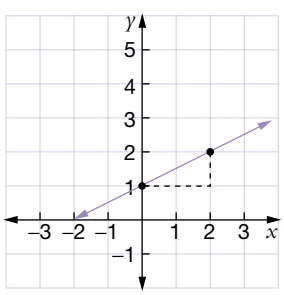
e



f



g



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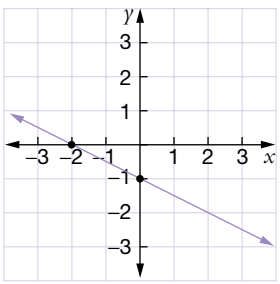

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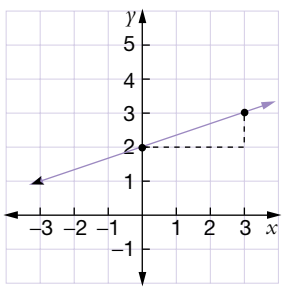


3 **a** $y = 3x + 4$ **b** $y = -3x + 4$ **c** $y = 3x - 4$
d $y = -3x - 4$ **e** $y = -\frac{1}{2}x - 2$ **f** $y = 2x + \frac{1}{2}$

4 **a** $y = x + 1$ **b** $y = 2x + 3$ **c** $y = \frac{1}{3}x + 1$
d $y = -2x$ **e** $y = -x + 2$ **f** $y = -\frac{2}{3}x - 2$

5 **a** $y = 2$ **b** $x = -1$ **c** $x = 1$ **d** $y = -1$

6 **a** yes **b** yes **c** no **d** yes

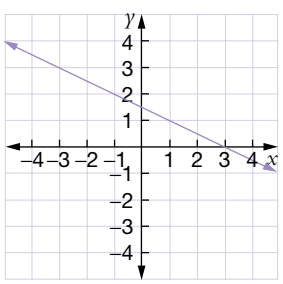
7 (2, 2), (1, −3), (4, 12), (0, −8)

8 **a** −1 **b** 1

9 **a** D **b** F **c** B **d** A
e E **f** B

10 **a** $m = -1, b = 6$ **b** $m = -2, b = 3$
c $m = 3, b = -4$ **d** $m = -\frac{1}{2}, b = 3$
e $m = \frac{2}{3}, b = -\frac{4}{3}$ **f** $m = -2, b = \frac{5}{2} = 2\frac{1}{2}$

11 **a**



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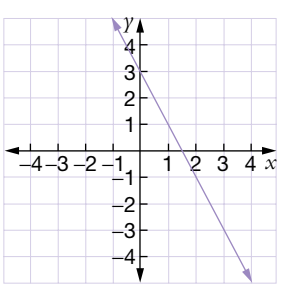

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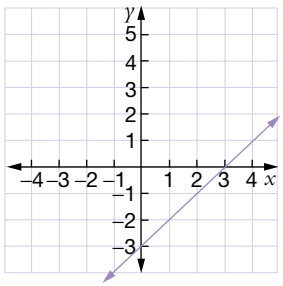

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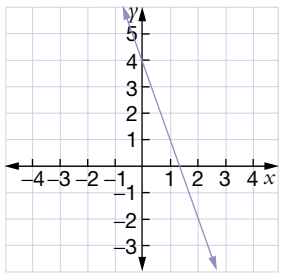

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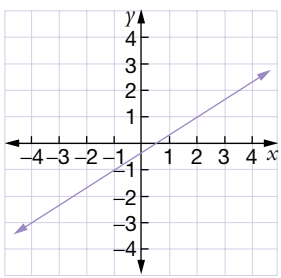

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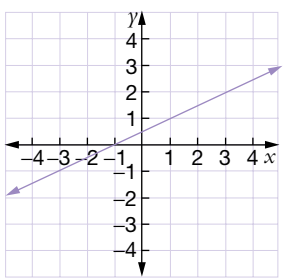


Prep quiz 7:02

1 Straight lines in the same plane, which do not meet.

2 2 **3** 2 **4** Yes **5** 3 **6** $\frac{1}{2}$

7 0·5 **8** yes **9** no **10** no

Exercise 7:02

1 **a** 2 **b** -1 **c** $-\frac{1}{2}$ **d** $\frac{4}{3}$

e 1 **f** -4 **g** $-\frac{1}{2}$ **h** -6

2 **a** yes **b** no **c** yes **d** yes

e yes **f** no **g** yes **h** yes

3 **a** -1 **b** 1 **c** $-\frac{1}{2}$ **d** $\frac{2}{3}$

4 **a** yes **b** yes **c** yes **d** yes

5 **a** $2x - y + 6 = 0$ and $y = 2x - 3$

b $x + y = 3$ and $y = -x + 8$

c AB and CD

d m of EF is $\frac{1}{2}$, m of GH is $\frac{1}{2}$ $\therefore$ $EF \parallel GH$

m of FG is 0, m of EH is 0 $\therefore$ $FG \parallel EH$.

e m of p is 4, m of q is 4 $\therefore$ $p \parallel q$

6 **a** $y = 5x + 3$ **b** $y = 3x - 1$ **c** $y = x + 3$

d $y = -x + 10$ **e** $y = 5x - 7$ **f** $y = -3x + 3$

g $y = 3x + 5$ **h** $y = -3$ **i** $y = -3$

j $m = -2$

7 **a** $y = 1{\cdot}5x - 3$ **b** $y = 1{\cdot}5x - 5$ **c** $y = 1{\cdot}5x + 12{\cdot}5$

8 **a** $y = 4x - 20$ **b** $y = -3x - 16$

9 **a** $y = -2x + 6$ **b** $y = \frac{1}{3}x + 5\frac{2}{3}$ **c** $y = -\frac{2}{3}x - 5\frac{1}{3}$

Prep quiz 7:03

1 -1 **2** -1 **3** -1

4 1 **5** $m_1 = -\frac{1}{3}$ **6** $m_2 = 3$

7 $m_1 m_2 = -1$ **8** $m_3 = -2$ **9** $m_4 = \frac{1}{2}$

10 $m_3 m_4 = -1$

Exercise 7:03

1 **a** $-\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $-\frac{1}{4}$ **d** -4 **e** 2

f $-\frac{3}{5}$ **g** $\frac{3}{2}$ **h** $\frac{2}{3}$

2 **a** -1 **b** 1 **c** $-\frac{1}{5}$ **d** $\frac{1}{4}$ **e** -2

f 4 **g** $-\frac{4}{3}$ **h** $\frac{2}{5}$

3 **a** yes **b** yes **c** yes **d** yes **e** no

f yes **g** yes **h** yes **i** no **j** yes

4 **a** $x + 2y = 4$, $y = -0{\cdot}5x + 5$

b $y = -1\frac{1}{2}x + 2$, $y = \frac{2}{3}x$

c m of AB is -1, m of BC is 1

$\therefore$ $AB \perp BC$ $(m_1 m_2 = -1)$

d m of EF is -2, m of $FG = \frac{1}{2}$

$\therefore$ $EF \perp FG$ $(m_1 m_2 = -1)$

m of GH is $-\frac{1}{2}$, m of HE is 2

$\therefore$ $GH \perp HE$ $(m_1 m_2 = -1)$

e m of p is $-\frac{1}{4}$, m of q is 4 $\therefore$ $p \perp q$ $(m_1 m_2 = -1)$

5 **a** $y = 3x + 5$ **b** $y = x + 3$ **c** $y = 2x$

d $y = \frac{1}{2}x + 1{\cdot}5$ **e** $y = x + 1$ **f** $y = \frac{1}{3}x - \frac{1}{3}$

g $y = -\frac{1}{3}x + 5$ **h** $y = -3$ **i** $x = 3$

j $m = \frac{1}{2}$

6 **a** $y = -x + 7$ **b** $y = x + 7$ **c** $y = -\frac{1}{4}x + 4$

d $y = 2x + 12$

7 **a** $y = -\frac{1}{4}x + 5$ **b** $y = -\frac{1}{4}x - 1\frac{1}{4}$ **c** $y = -\frac{1}{4}x - 4$

8 **a** $y = -\frac{1}{2}x + 8$ **b** $y = -\frac{1}{3}x + 4$ **c** $y = \frac{3}{2}x - 10\frac{1}{2}$

Prep quiz 7:04

1 3 **2** -2 **3** 0·5

4 $x = -2$ or 2 **5** $x = -\frac{3}{2}$ or $\frac{3}{2}$ **6** 4

7 8 **8** 0

9

x	-2	-1	0	1	2
y	6	3	2	3	6

10

x	-2	-1	0	1	2
y	-12	-3	0	-3	-12

Exercise 7:04

1 a

x	−3	−2·5	−2	−1·5	−1	−0·5	0	0·5	1	1·5	2	2·5	3
y	9	6·25	4	2·25	1	0·25	0	0·25	1	2·25	4	6·25	9

b See Question 2 for the graph of $y = x^2$.

2 a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b

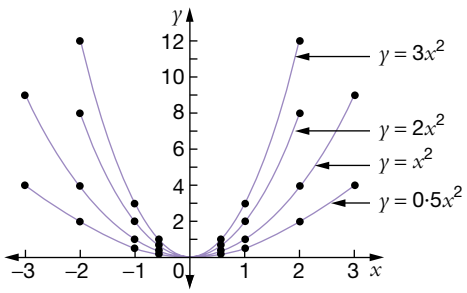
x	−2	−1	−0·5	0	0.5	1	2
y	8	2	0·5	0	0.5	2	8

c

x	−2	−1	−0·5	0	0·5	1	2
y	12	3	0·75	0	0·75	3	12

d

x	−3	−2	−1	0	1	2	3
y	4·5	2	0·5	0	0·5	2	4·5



The value of a changes the curvature of the parabola.

3 a B **b** D **c** C **d** A

4 a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b

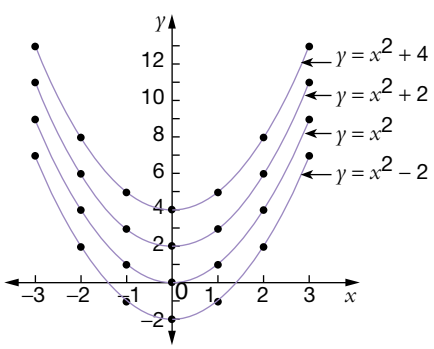
x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11

c

x	−3	−2	−1	0	1	2	3
y	13	8	5	4	5	8	13

d

x	−3	−2	−1	0	1	2	3
y	7	2	−1	−2	−1	2	7

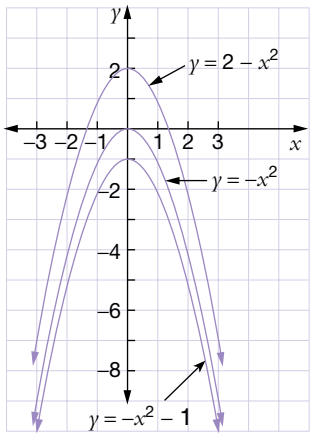


$y = x^2 + 2$ is the same curve as $y = x^2$ but raised 2 units. The value of c determines where the vertex of the parabola cuts the y-axis.

5 a

x	−3	−2	−1	0	1	2	3
y	−9	−4	−1	0	−1	−4	−9

b c



The minus sign turns the parabola upside down.

6 a E **b** C **c** B **d** A **e** D

7 a $y = x^2 + 2$ **b** $y = x^2 - 2$ **c** $y = -x^2 + 4$
d $y = -x^2 - 2$ **e** $y = -x^2 - 2$

8 A move it down 4 units
B move it down 2 units
C move it up 2 units
D turn it upside down and then move it up 4 units
E turn it upside down and then move it down 2 units

9 a C **b** D **c** B **d** A
e F **f** E

10 a $y = x^2$, $y = 8 - x^2$, $y = 2x^2 - 4$
b $y = 2x^2$, $y = x^2 + 1$, $y = 3 - x^2$
c $y = 2x^2$ **d** 4 **e** $\frac{1}{2}$
f **i** −10, 10 **ii** −4, 4 **iii** −3, 3 **iv** −7, 7

11 a 2; $y = ax^2 + 2$
b (1, 3), (2, 6) are two possible answers
c $y = x^2 + 2$

12 a $y = 2x^2$ **b** $y = 6 - x^2$ **c** $y = 6 - \frac{1}{2}x^2$
d $y = \frac{1}{2}x^2 + 1$ **e** $y = 9 - 2x^2$

Prep quiz 7:05

1 8 **2** 32 **3** 1 **4** $\frac{1}{2}$ **5** $\frac{1}{32}$
6 $\frac{1}{2}$ **7** $\frac{1}{8}$ **8** $\frac{1}{4}$ **9** 2·8 **10** 0·2

Exercise 7:05

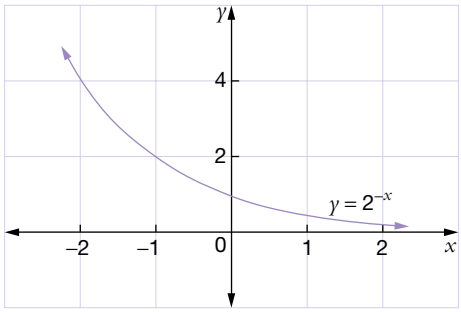
1 a (0, 1) **b** no
c $y = 5^x$ increases the fastest; $y = 2^x$ increases the slowest.
d for $x < 0$, it would be below the lowest curve ($y = 5^x$); for $x > 0$ it would be the steepest curve, (above $y = 5^x$)
e for $x < 0$, it would be above the highest curve ($y = 2^x$); for $x > 0$ it would be the lowest curve, (below $y = 2^x$)

2 a A is $y = 3^x$; B is $y = 6^x$; C is $y = 2^x$
b A is $y = 8^x$; B is $y = 4^x$; C is $y = 1·9^x$

3 **a**

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	4·0	2·8	2·0	1·4	1·0	0·71	0·50	0·35	0·25

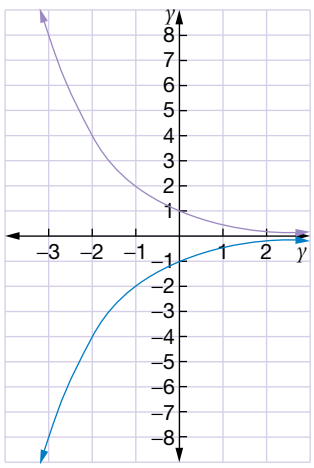
b $y = 2^{-x}$ is a reflection of $y = 2^x$ in the y-axis.



4 **a** A **b** C **c** D **d** B

5 **a** $y = 4^{-x}$ **b** $y = 6^{-x}$ **c** $y = 8^x$
d $y = 1{\cdot}2^{-x}$ **e** $y = 3{\cdot}6^x$

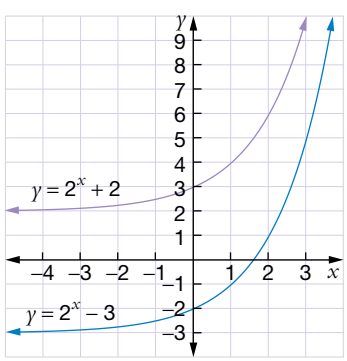
6 **a** yes **b** yes
c yes



7 **a** $y = -3^x$ **b** $y = -4^{-x}$ **c** $y = -1{\cdot}8^{-x}$
d $y = 1{\cdot}2^x$ **e** $y = 6^{-x}$

8 **a** They are reflections of each other in the y-axis
b They are reflections of each other in the x-axis

9 **a** **i** vertical translation of 1 unit up
ii vertical translation of 1 unit down
b **i** $y = 2^x + 3$
ii $y = 2^x - 4$
c



10 **a** $y = -3^x$ **b** $y = 3^{-x}$ **c** $y = 3^x + 1$
d $y = 3^x - 1$ **e** $y = 3^{-x} - 1$ **f** $y = -3^x + 1$

11 Curve A
a decreasing **b** concave down
c **i** they become very small (approach negative infinity)
ii they get closer and closer to 3
d 2
Curve B
a increasing **b** concave down
c **i** they get closer and closer to 5
ii they become very small (approach negative infinity)
d 4
Curve C
a increasing **b** concave up
c **i** they become very large (approach infinity)
ii they get closer and closer to 1
d 2
Curve D
a decreasing **b** concave up
c **i** they get closer and closer to −2
ii they become very large (approach infinity)
d −1

12 **a** D, F, K **b** A, E, G, H, L
c B, C, D, F, I, J, K **d** A, E, G, L
e H
f **i** A, B, D, G **ii** C, H, K

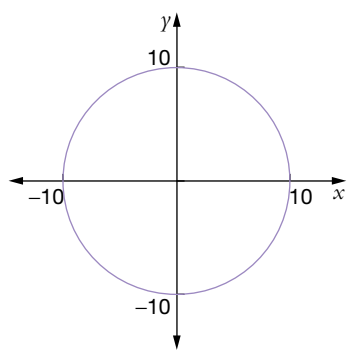
Exercise 7:06

1 B, D, E, F, H

2 **a** $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 9$ **c** $x^2 + y^2 = \frac{25}{4}$

3 **a** 10 **b** 6 **c** 2·5

4 **a**



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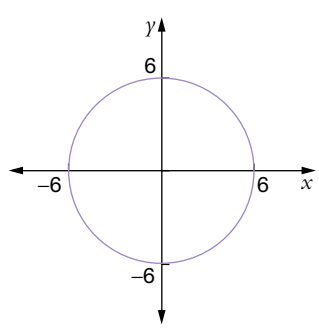
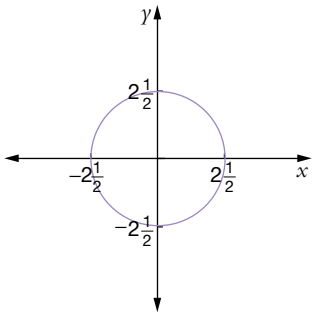


c



5 **a** $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 4$ **c** $x^2 + y^2 = 16$

6 **a** $x^2 + y^2 = 4; x^2 + y^2 = 9$

b 2 units

c the inner circle, i.e. $x^2 + y^2 = 4$

d $y = \sqrt{5}$

7 **a** $x^2 + y^2 = 5$ **b** $(-\sqrt{5}, 0)$

8 **a** $x^2 + y^2 = 25$ **b** $x^2 + y^2 = 13$

Prep quiz 7:07

1 D	**2** E	**3** G	**4** B	**5** F
6 H	**7** C	**8** A	**9** 2	**10** 2

Exercise 7:07

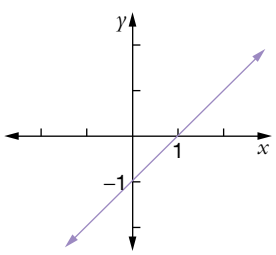
1 **a** C, J **b** A, H **c** E, B **d** D, F, G, I

2 **a** D **b** E **c** I **d** A

e H **f** B **g** C **h** G

i F **j** J

3 **a**



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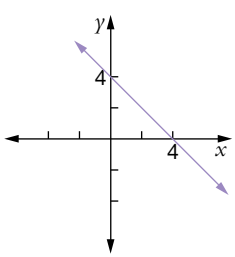

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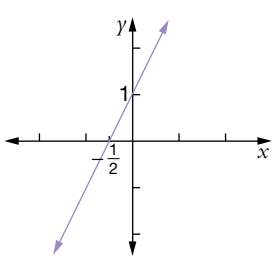

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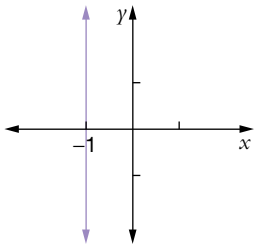

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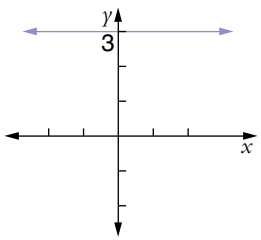

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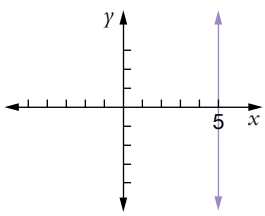

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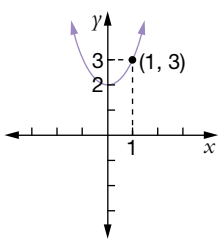

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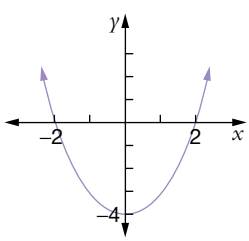
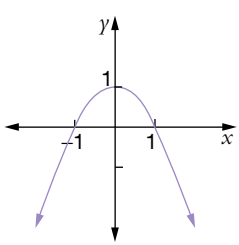


i



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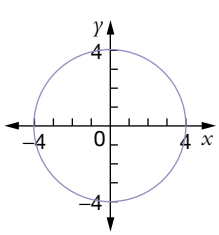

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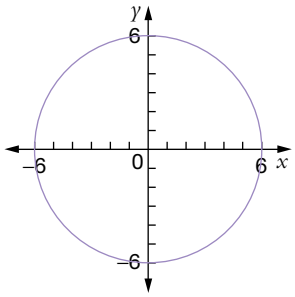

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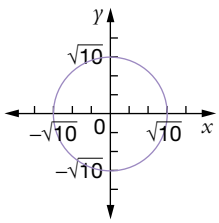

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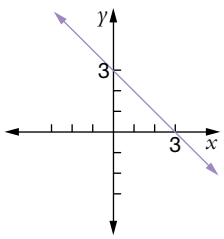

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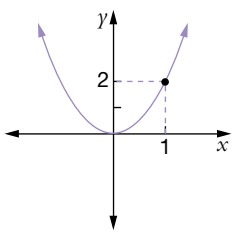

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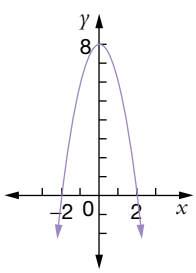

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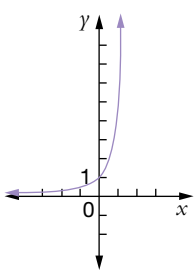

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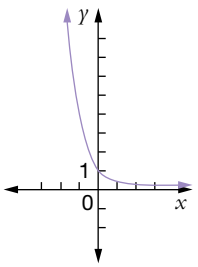

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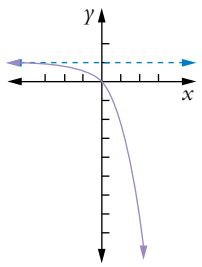
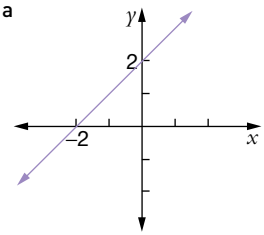


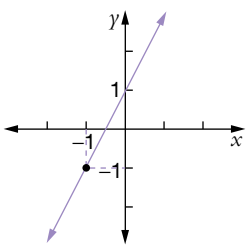
4 a $y = x + 1$ b $y = -x^2$ c $y = 2^x$
d $x^2 + y^2 = 36$ e $x + 3y = 3$ f $y = -2^{-x}$
g $y = 2x^2$ h $y = \frac{1}{2}x^2 + 2$ i $x^2 + y^2 = 10$
j $y = 4 - 4x^2$ k $y = 2^{-x}$ l $y = 6^x$
m $y = 6^x - 2$ n $y = 9 - x^2$ o $y = 2$

Diagnostic test 7

1 a



b



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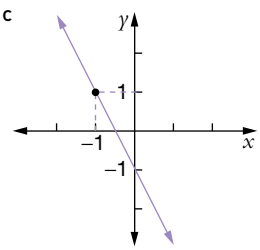


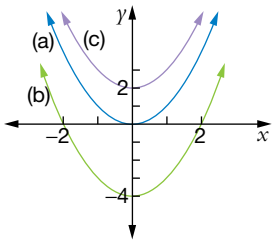
2 a $x = -3$
b $y = 2x - 2$
c $y = -\frac{1}{3}x + 1$

3 a $m = -3, b = 6$
b $m = -\frac{3}{2}, b = 2$
c $m = 2, b = 5$

4 a $y = 3x + 2, y = 3x$
b $y = 4x + 2$
c $y = -3x + 7$

5 a $y = 4x$
b $y = -\frac{3}{2}x + 5$
c $y = -\frac{1}{2}x - 3$

6



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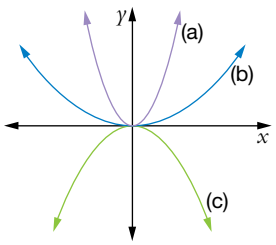
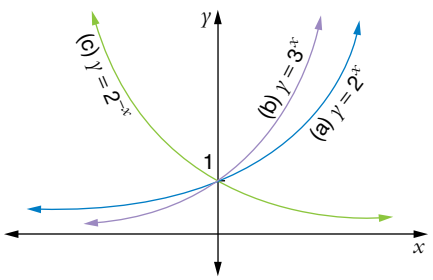


8 **a** $y = x^2 + 3$ **b** $y = -2x^2$ **c** $y = -4x^2 - 2$

9 **a** C **b** D **c** A **d** B

10



11 **a** $y = 2^{-x}$ **b** $y = -2^x$ **c** $y = 2^x + 2$
d $y = -2^x - 2$

12 **a** $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 9$ **c** $x^2 + y^2 = 100$

13 **a** centre (0, 0); radius = 5 units
b centre (0, 0); radius = 4 units
c centre (0, 0); radius = 2·5 units

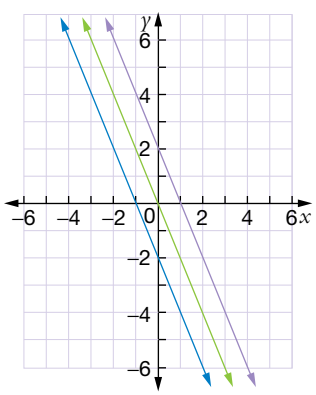
14 **a** E, H **b** I, B, C **c** G, J **d** A, D, F

15 **a** E **b** B **c** C **d** A
e D **f** F

ASSIGNMENT 7A

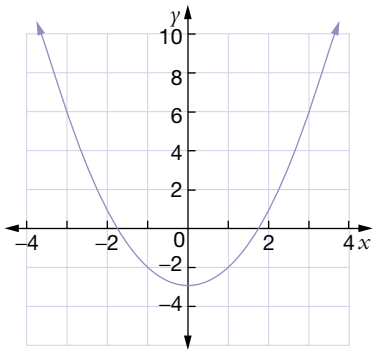
1 **a** $x = 3$ **b** $y = 2x + 3$ **c** $y = 5 - x$
d $y = 4x - 12$

2

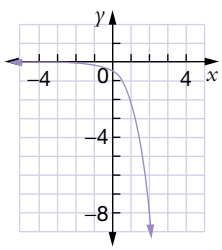


They are all lines and are parallel.

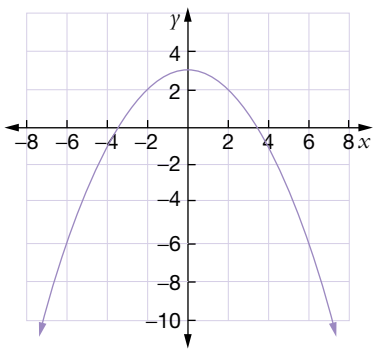
3 **a**



b



c



4 **a** 3 **b** $\frac{1}{2}$ **c** $y = \frac{1}{2}x^2 - 4$

5 **a** A is $y = 1 - 2x^2$ B is $y = 2x^2 + 1$
C is $y = 1 - \frac{1}{2}x^2$ D is $y = \frac{1}{2}x^2 + 1$
b **i** $y = 2x^2 - 1$ **ii** $y = -1 - 2x^2$

6 A is $y = 6 - 2^{-x}$ B is $y = 3^{-x}$
C is $y = 2^x - 3$ D is $y = 3 - 2^x$

ASSIGNMENT 7B

1 **a** Both travel 75 km.
b On the 13th day, approx. 185 km from Perth.

2 The longest side, 4 cm, should be opposite the largest angle, 70°.

3 40 kg, 45 kg, 50 kg, 52 kg, 60 kg

4 $1^2 = 1$, $11^2 = 121$, $111^2 = 12\,321$, $1111^2 = 1\,234\,321$, $11\,111\,111^2 = 123\,456\,787\,654\,321$

5 11 seconds

6 2019

ASSIGNMENT 7C

1 **a** $x = -29$ **b** $m = 32$ **c** $a = 24$

2 **a** 12 cm **b** 65%

3 **a** $x = -1, y = 3$ **b** $a = 2, b = -2$

c $m = 3\frac{7}{8}, n = 4\frac{1}{2}$

4 The compound interest is \$91.17 more than the simple interest.

5 **a** $\angle BCD = 70°$ (alt. $\angle$s, $AB \parallel CD$)

$\therefore x = 70 + 35$ (ext $\angle$ of ΔDEC)

$= 105$

b $\angle ACD = 50°$ (comp. $\angle$s)

$\therefore \angle CAD = 50°$ (equal $\angle$s of isos. ΔACD)

$\therefore x + 50 + 50 = 180°$ ($\angle$sum of ΔACD)

$\therefore x = 80$

6 18

7 **a** $\frac{24}{990} = \frac{4}{165}$ **b** $\frac{210}{990} = \frac{7}{33}$ **c** $\frac{720}{990} = \frac{8}{11}$

Chapter 8

Exercise 8:01

1 **a** 80 cm^2 **b** 938 cm^2 **c** 398·4 cm^2 **d** 153 cm^2

2 **a** 562 cm^2 **b** 312 cm^2

c 2250 cm^2

3 **a** 544 cm^2 **b** 230·4 cm^2 **c** 3·928 m^2

4 **a** 528 cm^2 **b** 275 cm^2 **c** 467 cm^2

5 **a** $x = 10{\cdot}82$, 363·84 cm^2 **b** $x = 8{\cdot}49$, 622·86 cm^2

6 **a** 1721·6 cm^2 **b** 408 cm^2 **c** 242·1 m^2

d 381·5 cm^2

7 **a** 730 m^2 **b** 358 m^2 **c** 792 m^2

8 **a** 324 cm^2 **b** 3659 cm^2

Exercise 8:02

1 **a** 18 cm^2 **b** 22 cm^2 **c** 26 cm^2

d 24 cm^2

2 **a** 28 cm^2 **b** 38 cm^2 **c** 36 cm^2

3 **a** 840 cm^2 **b** 6600 cm^2 **c** 1194 cm^2

4 **a** 356 cm^2 **b** 4980 cm^2 **c** 3752 cm^2

5 **a** 31·8 m^2 **b** 32·0 m^2 **c** 21·1 m^2

6 **a** 12·6 m^2 **b** 18·6 m^2 **c** 26·9 m^2

7 **a** 39·2 m^2 **b** 14·7 m^2 **c** 154·7 m^2

8 **a** 57·12 m^2 **b** 28·08 m^2 **c** 6 L

9 76·9 m^2

10 \$806.40

11 1042·5 mm^2 (1 dec. pl.)

12 **a** 55·0 m^2 (1 dec. pl.) **b** 58·4 m^2 (1 dec. pl.)

Prep quiz 8:03

1 16 cm^2 **2** 73·5 cm^2 **3** $36\pi \div 113{\cdot}1$ cm^2

4 $25\pi \div 78{\cdot}5$ cm^2 **5** 30 cm^2 **6** 15 cm^2

7 20 cm^2 **8** 6·5 cm^2 **9** 9·5 cm^2 **10** 8 cm^2

Exercise 8:03

1 **a** 196 m^3 **b** 226 cm^2 **c** 10 179 cm^3

2 **a** 214·4 cm^3 **b** 1928·2 cm^3 **c** 3229·0 cm^3

3 **a** 507·7 m^3 **b** 110·0 m^3 **c** 910·3 m^3

4 **a** 411 cm^3 **b** 10 571 cm^3 **c** 21 038 cm^3

5 **a** 3785 cm^3 **b** 509 cm^3 **c** 487 cm^3

6 **a** 45·0 m^3 **b** 45 kL

7 **a** 4·0 m^3 **b** 9600 kg

8 **a** 463 cm^3 **b** \$116

9 **a** 6·3 m^3 **b** 22·5 kL

10 118 kL

Exercise 8:04

1 **a** 400 cm^3 **b** 460 cm^3 **c** 582 cm^3

2 **a** 440 cm^3 **b** 108 cm^3 **c** 3·633 m^3

3 **a** 13 135·1 cm^3 **b** 179·5 cm^3 **c** 1583·7 cm^3

4 **a** 610 cm^3 **b** 130 m^3 **c** 350 m^3

5 **a** 6232·6 cm^3 **b** 4672·2 cm^3

6 **a** 8·6 m^3 **b** 0·86 cm^3 **c** 11·1 m^3

7 $V = 421{\cdot}2$ cm^3; Mass = 4801·68 g

8 44 kL

9 **a** $V = 0{\cdot}67$ cm^3 **b** Mass = 1700 kg

10 0·94 m^3

Diagnostic test 8

1 **a** 83·56 m^2 **b** 26 cm^2

c 15·8 m^2 (1 dec. pl.)

2 **a** 400 cm^2 **b** 351·97 cm^2

c 310·8 cm^2 (1 dec. pl.)

3 **a** 694·2 cm^3 **b** 118·3 m^3 **c** 586·7 cm^3

4 **a** 34 880 cm^3 **b** 8843·9 cm^3 **c** 1310·8 cm^3

ASSIGNMENT 8A

1 196 m^2

2 **a** 539·5 cm^3 **b** 4586 g

3 Capacity = 4414 mL

4 SA = 500 cm^2; Vol = 600 cm^3

5 **a** 5·6 m^3 **b** 16 m^2 **c** \$3000 **d** 8·3 kL

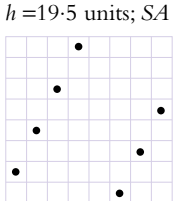
ASSIGNMENT 8B

1 217 double cones; Profit = \$414.90

2 1620

3 h =19·5 units; SA = 810 unit^2

4



6 more queens can be placed. An example is shown above.

5 It becomes a right-handed glove.

6 a

1	2	3	4
3	4	1	2
4	3	2	1
2	1	4	3

b

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

Another solution is possible.

ASSIGNMENT 8C

1

Year	Principal	Interest
1	\$10 000	\$450
2	\$10 450	\$470.25
3	\$10 920.25	\$491.41
Amount after 3 years = \$11 411.66 Compound interest = \$1411.66		

2 a $x = -4, 4$ **b** $x = -3, 3$ **c** $x = -1\frac{1}{3}, 1\frac{1}{3}$

3 a $m = -3, -2$ **b** $m = -6, 1$ **c** $m = -1, 6$

4 a $x = 3, y = 1$ **b** $x = -3, y = 3$ **c** $x = \frac{1}{2}, y = -4$

5 a $x = 5, y = 3$ **b** $a = 8, b = -1$ **c** $x = 3\frac{1}{2}, y = -2\frac{1}{2}$

6 a 540° **b** 1080° **c** 1260°

7 a 83° **b** 243° **c** 63°

Chapter 9

Exercise 9:01

1 a i 7 **ii** 4 **iii** 6 **iv** 5

b i 6 **ii** 14 **iii** 15·5 **iv** 15

c i 0·6 **ii** 1·7 and 1·9 **iii** 1·7125 **iv** 1·7

d i 4 **ii** 9 **iii** 8·2 **iv** 8

e i 20 **ii** 15 **iii** 14 **iv** 15

2 The mode is Toyota with a frequency of 250.

3 a mode = 5, mean = 6, median = 5

b mean = 6, median = 5·5

4 a 12

b i 12·5 **ii** 11

5 a 10 **b** 14 **c** 13·8 **d** 14

6 a 50 **b** 30–<40 **c** 20–<30 **d** 27·25

7 a i

Score	Tally	Freq	fx	$c.f.$
1	|||	3	3	3
2	𝍸	5	10	8
3	𝍸 ||||	9	27	17
4	𝍸 ||	7	28	24
5	||||	4	20	28
6	||	2	12	30
	Totals:	30	100	

mean = $3\frac{1}{3}$

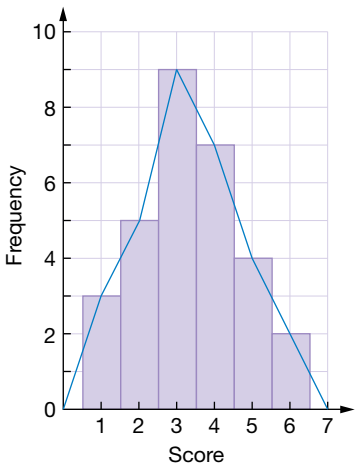
median = 3

ii

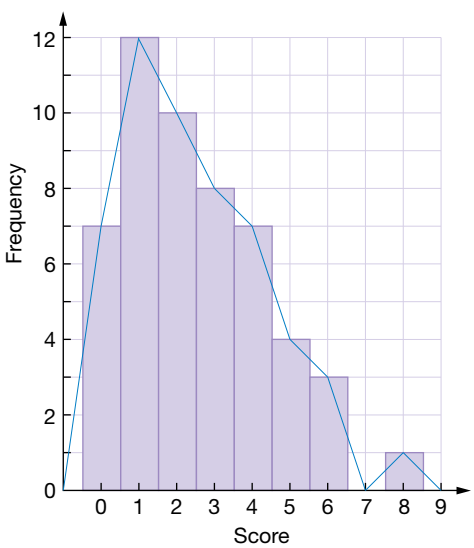
Outcome	Tally	Freq	fx	$c.f.$
0	𝍸 ||	7	0	7
1	𝍸 𝍸 ||	12	12	19
2	𝍸 𝍸	10	20	29
3	𝍸 |||	8	24	37
4	𝍸 ||	7	28	44
5	||||	4	20	48
6	|||	3	18	51
7		0	0	51
8	|	1	8	52
	Totals:	52	130	

mean = 2·5 median = 2

b i



ii

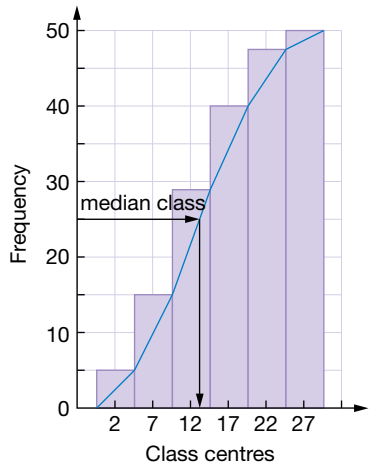


8 **a**

Class	*c.c.*	Tally	Freq	$f \times c.c.$	*c.f.*
0–4	2	||||	5	10	5
5–9	7	𝍸 ||||	9	63	14
10–14	12	𝍸 𝍸 𝍸	15	180	29
15–19	17	𝍸 𝍸 |	11	187	40
20–24	22	𝍸 ||	7	154	47
25–29	27	|||	3	81	50
		Totals:	50	675	

b 29 **c** 10–14 **d** 10–14 **e** 13·5

f



9 **a** **i** median = 8, $Q_1 = 5$, $Q_3 = 9$, IQR = 4

ii median = 6·5, $Q_1 = 4·5$, $Q_3 = 8·5$, IQR = 4

iii median = 5, $Q_1 = 2·5$, $Q_3 = 6·5$, IQR = 4

iv median = 5, $Q_1 = 2·5$, $Q_3 = 7·5$, IQR = 5

b **i** (2, 5, 8, 9, 12)

ii (3, 4·5, 6·5, 8·5, 10)

iii (1, 2·5, 5, 6·5, 8)

iv (1, 2·5, 5, 7·5, 9)

10 **a** **i** 40 **ii** 9 **iii** 5 **iv** 4

v 7 **vi** 3 **vii** (1, 4, 5, 7, 10)

b

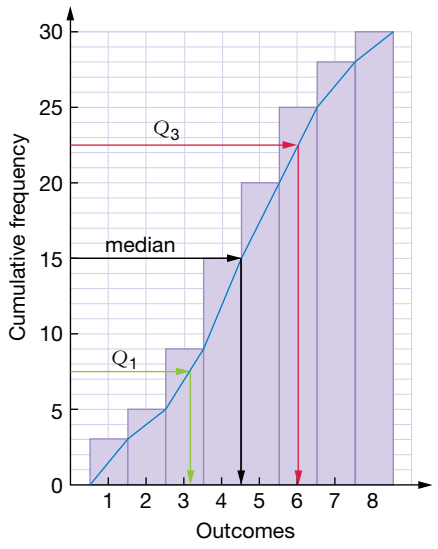
Outcome	Frequency	*f. x.*	*c.f.*
1	2	2	2
2	4	8	6
3	3	9	9
4	4	16	13
5	8	40	21
6	6	36	27
7	5	35	32
8	4	32	36
9	3	27	39
10	1	10	40
	Totals:	40	215

c **i** 5

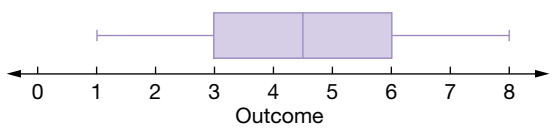
ii 5·375

11 **a** **i** 7 **ii** 4·5 **iii** 3 **iv** 6

b



c



12 **a** **i** (1, 2, 4, 7, 8)

ii (1, 4, 5·5, 7, 10)

iii (10, 12·5, 14, 16, 18)

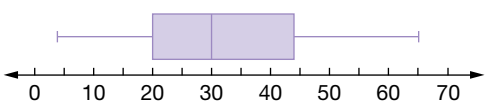
iv (20, 28, 30, 33, 36)

b plot ii

c plot iv; it is negatively skewed

13 **a** **i** 61 **ii** 25 **iii** 30 **iv** 24

b



14 **a** Class A = 27, Class B = 24

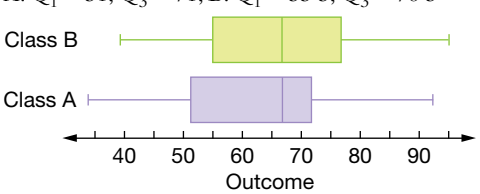
b A = 71, B = 67

c A = 67, B = 67

d A = 63, B = 67

e A: $Q_1 = 51$, $Q_3 = 71$; B: $Q_1 = 55·5$, $Q_3 = 76·5$

f



g Class A

h **i** Class A median = 67·5, so Class B would have lower median.

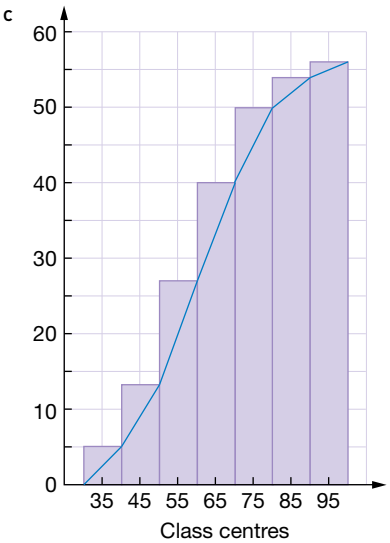
ii Class A mean = 66·3, so Class A would still have a slightly lower mean.

15 a

Income (×$1000)	*c.c.*	Freq	$f \times c.c.$	*c.f.*
30–<40	35	5	175	5
40–<50	45	8	360	13
50–<60	55	14	770	27
60–<70	65	13	845	40
70–<80	75	10	750	50
80–<90	85	4	340	54
90–<100	95	2	190	56
	Totals:	56	3430	

b 61·25

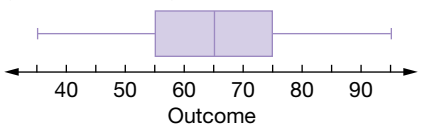
c



d median class = 60–<70,
first quartile class = 50–<60,
third quartile class = 70–<80

e (35, 55, 65, 75, 95)

f

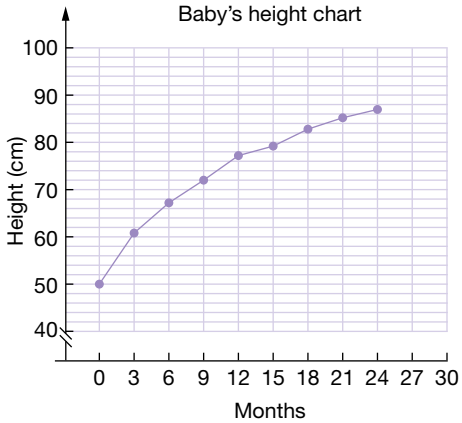


g symmetrical

Exercise 9:02

1 a 3·5 kg b 10·3 kg c 194%
d 12·6 kg, 22% e 13 kg f no

2 a



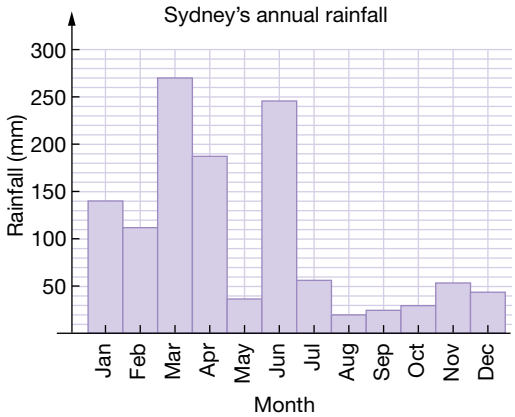
b Yes, the graph looks similar

c yes d 78 kg e No; growth spurts occur, particularly in teenage years.

3 a i Sept ii June

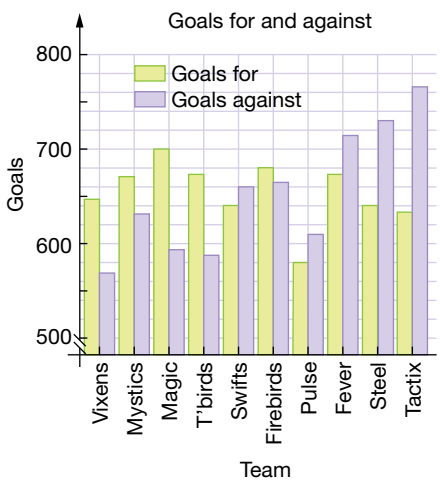
b first half

c



d Rainfall for 2012 is more uneven, however, the graph for average rainfall for over 150 years should be fairly even. For 2012 June is also a very wet month and, in general, the first half of the year is wetter. For 2012 the second half of the year seems unusually dry.

4 a



b Goals against seems to show a more steady pattern, so it would be the indicator of success. A coach might decide to pay more attention to defensive skills in training.

c Because both variables are involved in determining the result of any game and sometimes a team will perform much better or worse than they usually do. Winning a number of games by a narrow goal difference could be offset by a loss by a big goal difference.

5 a Denmark, United States, United Kingdom, France, Canada, Australia, New Zealand, Macau

b Macau, Australia, Canada, France, New Zealand, United Kingdom, Denmark, United States

c All eight.

d Life expectancy: Afghanistan, Zimbabwe, Chad, South Africa, Ethiopia, Kenya, Ghana, Bangladesh
Wealth: Ethiopia, Afghanistan, Nepal, Zimbabwe, Chad, Ghana, Bangladesh, Kenya

e Nepal

f South Africa

g By collecting similar data from more countries. By observing any change in life expectancy in countries where the per capita income has improved over time.

h Population density, availability of clean water and food, medical resources, political stability.

6 a 3·5; the purple column is 3·5 times higher than the blue column.

b WA

c Vic; 0·70

d Vic and Qld

e You might expect that the number of vehicles would have a greater influence on the number of fatalities but the rate of fatalities per population seems to show the more consistent pattern.

f No. Other factors such as distance travelled and quality of roads would have an influence.

7 a 2000

b 2010

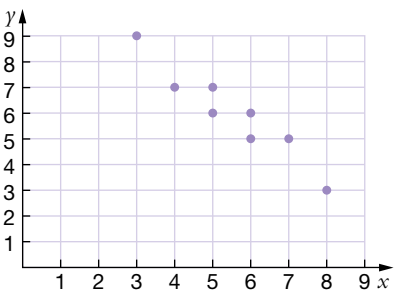
c The global financial crisis (GFC).

d Governments need to balance the amount of revenue they collect, largely through taxes and levies, with the amount they need to spend on services and infrastructure. The government has to decide which of the many areas that need funding have priority.

Exercise 9:03

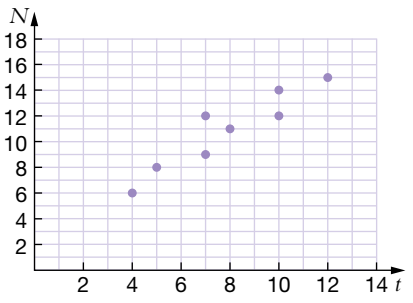
1 a yes, negative b yes, positive c no
d no e yes, negative f yes, positive

2 a



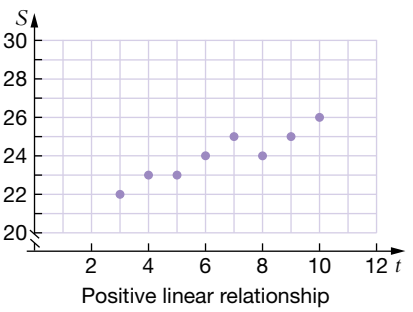
Negative linear relationship

b



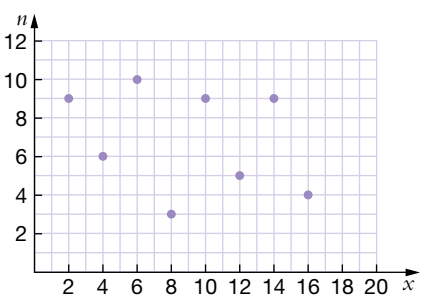
Positive linear relationship

c



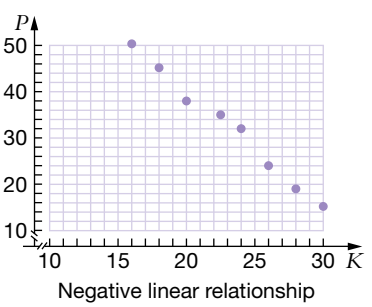
Positive linear relationship

d



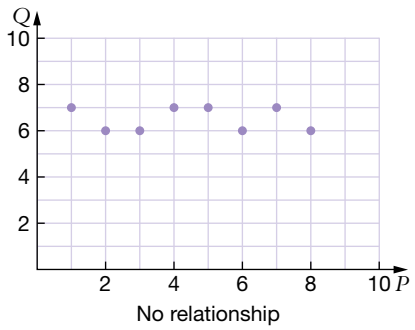
No relationship

e



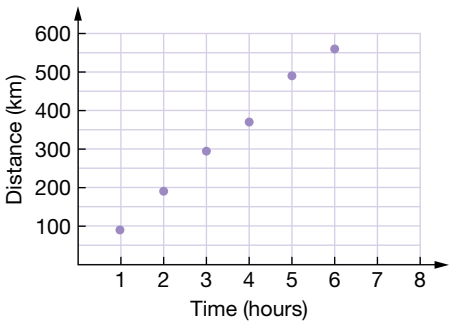
Negative linear relationship

f



No relationship

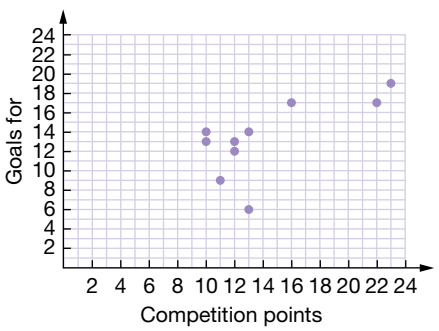
3 a



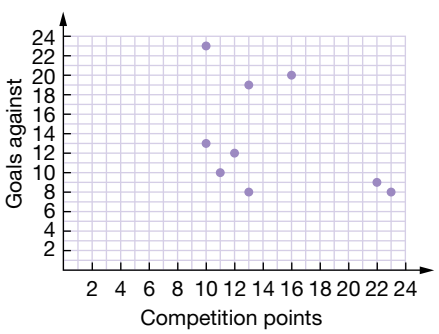
b A positive linear relationship.

c 93·5 km/h

4 a



b



c Neither has a strong relationship, but with the exception of one or two data points, the 'goals for' scatter plot shows the stronger positive linear relationship.

5 a 23

b **i** Maths = 5, English = 7

ii Maths = 20, English = 19

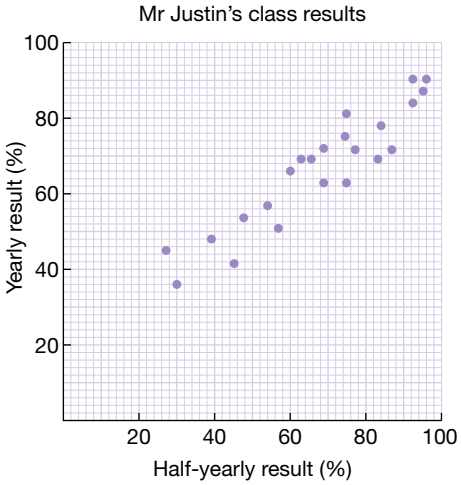
iii Maths = 9, English = 13

iv Maths = 18, English = 14

c 16 **d** 13 **e** 11 **f** 12

g A weak positive linear relationship.

6 a

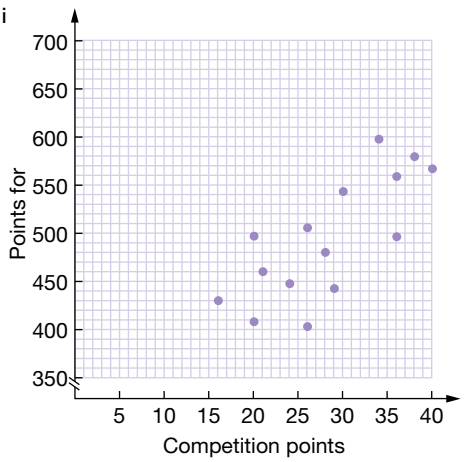


b From the pattern in the graph, students tended to perform as well in both tests.

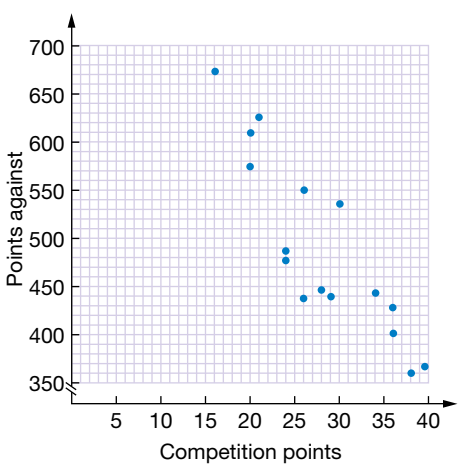
c equal 12th

d 16th **e** 75 **f** 69

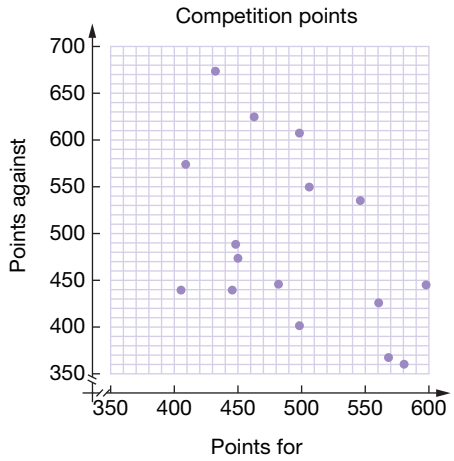
7 a **i**



ii



iii

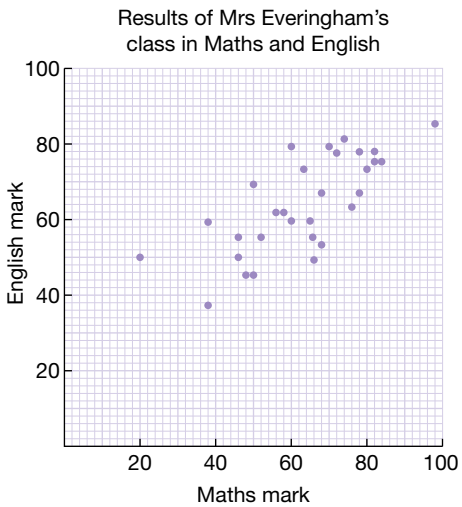


b Plot i

c Plot ii

d Competition points and Points against.

8 a



b Maths: range = 78, median = 65·5
English: range = 48, median = 63

c A weak positive linear relationship.

d Also about 70.

Exercise 9:04

1 a 29, assuming he batted at least once in each match.

b 130

c 52·34, average

d 2·94, economy rate

e 74·80, strike rate

2 a 100%

b The viewing audience, i.e. the total number of people watching TV when the surveys were done.

c i ABC, Seven, SBS

ii Nine, Ten

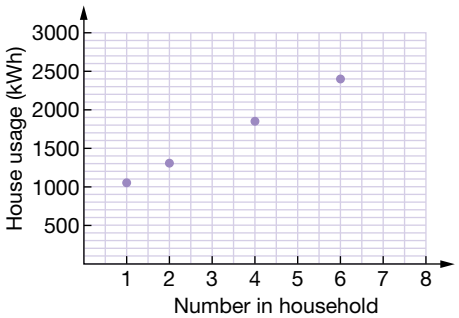
d More viewers attract more advertising investment.

e Better programs, better personalities, more stations to provide diversity, etc.

3 a 1104·89 kilowatt hours (kWh)

b lower

c



d positive linear relationship

e about 2100 kWh

f i about 800 kWh

ii about 2900 kWh

iii about 6050 kWh

Although some services may be switched on, a household with no-one should use almost zero power. With a large number of people certain usage, such as lighting should not increase proportionately, so it is that less electricity per person may be used.

4 a 38 997, 0·17%

b Victoria, 14 312

c WA, 0·35%

5 a Avoca Beach

b No; if very few houses were sold in a particular month that were more extreme in their prices, the median might not be very representative.

c Avoca Beach

d Green Point and Kincumber are fairly symmetric and Avoca Beach is positively skewed.

6 a other

b 170

c After World War II, many were seeking re-settlement from Europe.

d Migrants from the UK were automatically considered as citizens as we were part of the British Commonwealth.

e none

f seven

7 a 1982

b house prices are generally higher

c down

d Incomes rise and/or house prices come down.

8 a Both the number of licences and number of vehicles owned was increasing at a faster rate than the population. So the increase in fatalities was at an increasing rate against the population but not against the greater rate of increase of licences and vehicles.

b Fewer people owned cars.

c Random breath testing for alcohol was introduced in 1982.

d Compulsory seatbelts were introduced in 1971 for front seats and later for all passengers. Better roads, safer cars with equipment such as airbags, speed cameras, advertising campaigns, double demerit points in holiday times, school zones, decreased speed limits, etc.

9 a ACT; perhaps due to higher number of public servants.
b WA; perhaps due to the mining industry, which pays higher wages.
c positively
d 10% and 90% marks
e the median

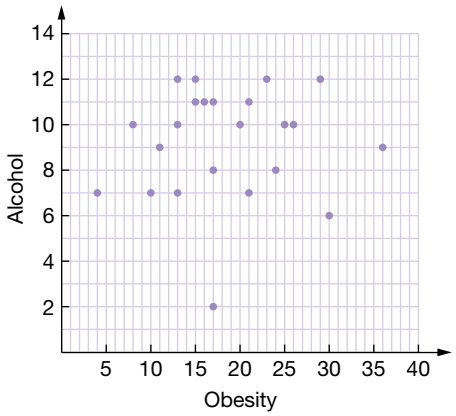
Exercise 9:05

1 a It is very 'volatile', i.e. it fluctuates a lot.
b The 'spaces' represent each day, so the first section of the graph shows the index rising from the start of the day to the end of the day for Dec 03. So the 'points' on the graph indicate the end of one day and the beginning of the next.
c Scale not shown, but it is meant to show the value of the All Ordinaries index.
d The scale seems to start at 4510 and rise by 10 for each unit, so the top of the scale would be 4560.
e 4518; this would be the figure at the start of the graph, i.e. the beginning of Dec 03.

2 a yes b no c no
d As a visual means to attract attention. The bars indicate the order of the gains and losses but are not representative of the size of the gain or loss. What appears as a 'graph' is not a graph of the figures shown.

3 a Number of tests played
b No; the scale for the column graph.
c 12
d The more times he batted would average out to his usual ability. Only batting a few times means there is a more likely chance of batting more significantly above usual ability. Of course it would be just as possible to score below average for only a few innings. The graph would seem to suggest that batting more has often resulted in a lower average, at least for Michael Hussey, but it has not always been the case.

4 a by 'obesity'
b by 'alcohol consumption'
c The percentage of the population with a body mass index of 30 or more and litres of alcohol consumed per person in a year.
d No: the variables are not measured against the same scale.
e yes
f



g no
h Neither variable truly represents the health or the wealth of a nation.

5 a i static; very little change
ii rising steadily to 2005 then falling steadily
b i 22·7°C
ii 22·9°C
c no
d It is impossible to know the pattern for either graph after 2011, so confidence would be very low to zero as the years increased.

6 a No, because of insufficient data.
b The answer would have been the same.
c increased
d By looking at the data for all years over an extended period. If data is only chosen for two years at random, it would be expected that one would be greater than the other.

7 a second table
b first table
c The area of each country, the mass of each country's vegetation, transport distances, and energy needs.
d China and India in particular have very large populations so their per capita ranking is very low. Arguing from this data would suggest their output of CO_2 is not significant. Also there are 12 countries ranked higher than Australia but some have very small populations that distorts the significance of the data.

Diagnostic test 9

1 a i 8 ii 3 iii 6 iv 7
v 3 vi 9 vii 6
b i 6 ii 4 and 9 iii 6 iv 5·5
v 4 vi 8·5 vii 4·5

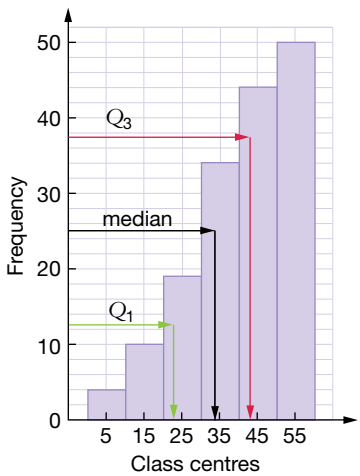
2 a

Outcome	Tally	Freq	$f \times c.c.$	$c.f.$
6	𝍸 ||	7	42	7
7	𝍸 𝍸 |	11	77	18
8	𝍸 |||	8	64	26
9	𝍸	5	45	31
10	||||	4	40	35
11	|||	3	33	38
12	||	2	24	40
	Totals:	40	325	

b i 6 ii 7 iii 8·125
iv 8 v 7 vi 9

3 a i 30–<40 ii 30–<40 iii 32·8

b

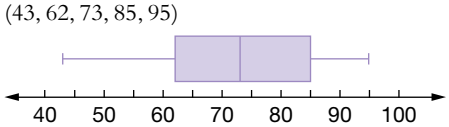


c $Q_1 = 23$, $Q_3 = 43$ ∴ IQR = 20

4 a i 52 ii 76 iii 73
iv 62 v 85 vi 23

b (43, 62, 73, 85, 95)

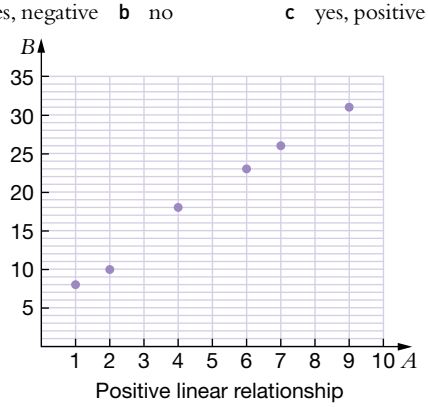
c



5 a time
b number of fatal crashes
c 14·00–16·00
d 02·00–04·00
e No. Need to consider the number of vehicles on the road at each time.

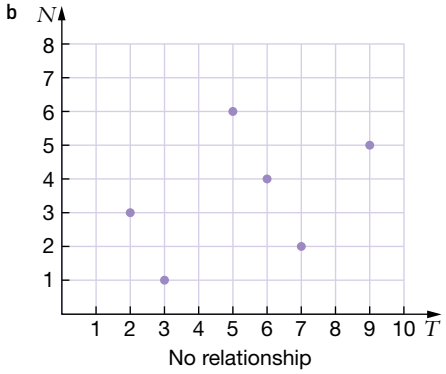
6 a yes, negative **b** no **c** yes, positive

7 a



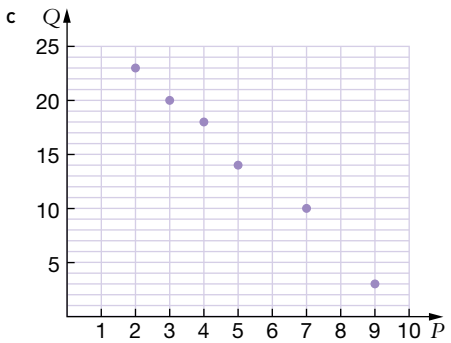
Positive linear relationship

b



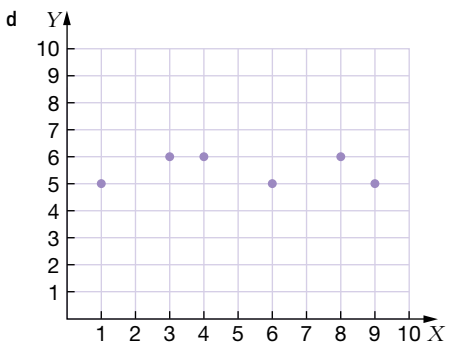
No relationship

c



Negative linear relationship

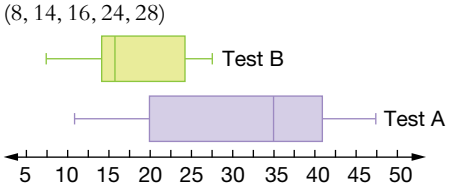
d



No relationship

8 a 18
b 41
c i 37 ii 35 iii 20 iv 41
d (11, 20, 35, 41, 48)
e i 20 ii 16 iii 14 iv 24
f (8, 14, 16, 24, 28)

g



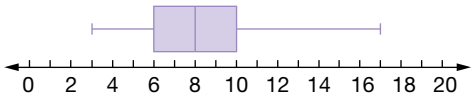
h i Test B ii Test A

ASSIGNMENT 9A

1 a 27
b 7
c 8·22 (2 dec. pl.)

d (3, 6, 8, 10, 17)

e



f slightly positively skewed

2 a

Class	*c.c.*	Freq	$f \times c.c.$	*c.f.*
10–19	14·5	2	29	2
20–29	24·5	5	122·5	7
30–39	34·5	7	241·5	14
40–49	44·5	4	178	18
50–59	55·5	8	444	26
60–69	65·5	6	393	32
70–79	75·5	5	377·5	37
80–89	85·5	4	342	41
90–99	95·5	2	191	43
	Totals:	43	2318·5	

b 53·95 (2 dec. pl.)

c 53·92 (2 dec. pl.) slightly less than exact mean

d 35; no, as modal class is 50–59

e 56; yes, as median class is 50–59

3 a 15 hours

b 60 km/h

c At 90 km/h Liam would take 6 hours and 40 minutes.

d The relationship is not linear.

4 a **i** early mornings on Saturday and Sunday

ii Friday evenings followed closely by weekday afternoons

b **i** Many young people go out on Friday and Saturday evenings and are coming home in the early hours of Saturday and Sunday mornings. They could be tired or driving under the influence of alcohol and hence more likely to be involved in an accident.

ii Weekday afternoons are when people of this age group are driving home from work. It is a time of heavy traffic and their ability to react may be reduced because of fatigue or lack of concentration. This is exacerbated on Friday evenings when drivers are likely to be more tired and may be driving under the influence of alcohol having had a drink after work on Friday.

ASSIGNMENT 9B

1 14

2 6

3 a 8 (100, 101, 104, 106, 109, 113, 116, 117)

b $685 = 3^2 + 26^2$ or $18^2 + 19^2$

4 26·3 cm

5 18·75 m

ASSIGNMENT 9C

1 a 10 **b** (6, 1) **c** $-\frac{4}{3}$

2 a 120 **b** 39·625 **c** 5 **d** 3·5

3 a $x = \pm 10$ **b** $x = 5$ or -3 **c** $x = 8$ or -7

4 a $\frac{1}{36}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$

5 Drink = \$3.50, Ice-cream = \$2.50

6 a \$6480

b **i** \$8304.27

ii \$8567.15

7 \$189 776.30

8 a $y = 4x - 1$ **b** $y = -\frac{1}{4}x + 4$

9 a $y = 2x - 1$ **b** $x^2 + y^2 = 4$ **c** $y = 2^x$

10 a 1046 cm^2 **b** 2337 cm^3

Chapter 10

Prep quiz 10:01

1 $3x + 15$ **2** $5m - 35$ **3** $12y + 2$

4 $18p - 27q$ **5** 20 **6** 12

7 8 **8** $\frac{9x}{20}$ **9** $\frac{x}{12}$ **10** $\frac{m+2n}{8}$

Exercise 10:01

1 a $\frac{5x+12}{6}$ **b** $\frac{7a+15}{10}$ **c** $\frac{13n-7}{21}$

d $\frac{8x+17}{15}$ **e** $\frac{12m+13}{35}$ **f** $\frac{p+4}{12}$

2 a $\frac{3a+4}{4}$ **b** $\frac{3w+2}{10}$ **c** $\frac{7d-2}{6}$

d $\frac{5x+13}{12}$ **e** $\frac{7m+2}{24}$ **f** $\frac{q+3}{18}$

3 a $\frac{8x+17}{6}$ **b** $\frac{22a+25}{20}$ **c** $\frac{32n+5}{12}$

d $\frac{27t-17}{15}$ **e** $\frac{37u-30}{24}$ **f** $\frac{65q+8}{90}$

g $\frac{4a+5}{6}$ **h** $\frac{5n+3}{8}$ **i** $\frac{19g}{9}$

j $\frac{25y-7}{24}$ **k** $\frac{8-2x}{15}$ **l** $\frac{13-14a}{40}$

4 a $\frac{x-2}{6}$ **b** $\frac{y-15}{10}$ **c** $\frac{2n-1}{15}$

d $\frac{x+29}{30}$ **e** $\frac{2m+8}{35}$ **f** $\frac{5p+8}{12}$

g $\frac{1}{6}$ **h** $\frac{t+4}{8}$ **i** $\frac{11w}{9}$

j $\frac{19-2y}{24}$ **k** $\frac{-1-x}{30}$ **l** $\frac{7+2z}{40}$

5 a $\frac{31x+91}{30}$ **b** $\frac{86a+35}{60}$ **c** $\frac{14m+13}{6}$

d $\frac{12y+29}{20}$ **e** $\frac{5x-3}{4}$ **f** $\frac{4z+13}{20}$

Challenge 10:01

1 $\frac{2x^2+5x+12}{6x}$ **2** $\frac{5x+14}{6x}$ **3** $\frac{7a}{10x}$

4 $\frac{7ax+10a+5x}{10ax}$ **5** $\frac{2x-2}{15x}$ **6** $\frac{11n+2}{12n}$

7 $\frac{b^2+a^2}{ab}$ **8** $\frac{5ay-15y^2-2x^2}{6xy}$

Prep quiz 10:02A

1 16 **2** 49 **3** 4 **4** 100 **5** $9x^2$
6 14 **7** 9 **8** 4 **9** 10 **10** 6

Exercise 10:02A

1 **a** 4 **b** 36 **c** 9 **d** 100
e $2x$ **f** $14y$ **g** $4n$ **h** $10p$
i q^2 **j** x^2 **k** 3 **l** 9
m 7 **n** 11 **o** $4x^2$ **p** $25n^2$
q $42m$ **r** $40x$ **s** $4a$ **t** $126y$

2 **a** x^2+6x+9 **b** $x^2+10x+25$
c x^2+2x+1 **d** $x^2-12x+36$
e m^2-2m+1 **f** $n^2-10n+25$
g x^2+4x+4 **h** $n^2-16n+64$
i $m^2+22m+121$ **j** $a^2+24a+144$
k $x^2+20x+100$ **l** $p^2-18p+81$
m $x^2+2xy+y^2$ **n** $a^2+2am+m^2$
o $x^2+2xt+t^2$ **p** $a^2-2ab+b^2$
q $k^2-2km+m^2$ **r** $p^2-2pq+q^2$

3 **a** $4x^2+12x+9$ **b** $4x^2+4x+1$
c $9x^2+30x+25$ **d** $16a^2+8a+1$
e $9a^2+42a+49$ **f** $49t^2+28t+4$
g $4x^2-4x+1$ **h** $9a^2-12a+4$
i $25m^2-40m+16$ **j** $16t^2-56t+49$
k $36q^2-12q+1$ **l** $81n^2+72n+16$
m $4x^2+4xy+y^2$ **n** $a^2+6ab+9b^2$
o $9t^2-12xt+4x^2$

Prep quiz 10:02B

1 40 **2** 40 **3** 12 **4** 12 **5** 24
6 24 **7** 27 **8** 27 **9** 19 **10** 19

Exercise 10:02B

1 **a** x^2-16 **b** a^2-1 **c** m^2-4
d n^2-49 **e** p^2-25 **f** q^2-36
g x^2-9 **h** y^2-81 **i** $100-x^2$
j $25-a^2$ **k** $64-x^2$ **l** $121-m^2$
m x^2-t^2 **n** a^2-b^2 **o** m^2-n^2
p p^2-q^2

2 **a** $4a^2-1$ **b** $9x^2-4$ **c** $25m^2-9$
d $81q^2-4$ **e** $16t^2-9$ **f** $49x^2-1$
g $64n^2-25$ **h** $100x^2-9$ **i** $4x^2-y^2$
j $16a^2-9b^2$ **k** $25p^2-4q^2$ **l** $9m^2-n^2$
m $4m^2-25n^2$ **n** $4p^2-9q^2$ **o** x^2-25y^2
p $144x^2-25y^2$

Investigation 10:02B

1 **a** 396 **b** 840 **c** 1425 **d** 12920
2 **a** $\sqrt{56}$ **b** $\sqrt{540}$ **c** $\sqrt{445}$ **d** $\sqrt{960}$

Exercise 10:02C

1 **a** $8x-21$ **b** x^2+x-2
c $2x^2-x-1$ **d** $4x-x^2+10$
e $9x^2-6x+1$ **f** x^2-25
g $6x^2-23x+7$ **h** $25x^2-1$
i x^2+6x+7 **j** $8x$
k $x^2+7x-30$ **l** $81-y^2$
m x^2-15x **n** $3x^2+9x+6$
o $x^2+2xy+y^2$ **p** $2x^2+5xy+2y^2$
q $3x$ **r** a^2-4b^2
s $2a-2x$ **t** $15a^2+26a-21$
u $4m^2-20mn+25n^2$ **v** $1-25y^2$
w $21-4x$ **x** $81x^2-64y^2$

2 **a** $x^2+7x+11$ **b** a^2-9a+6
c $x^2-2x+20$ **d** x^2+2x+9
e $2x^2+9x+11$ **f** -1
g $12m+37$ **h** $-14y-98$
i $2x^2+6x+5$ **j** $2a+5$
k $2x^2+8x+8$ **l** $2a^2-4$
m $5x+7$ **n** $y-17$
o $4x^2-x-14$ **p** $3x^2+24x+49$
q $9x^2-10x-2$ **r** $10x$
s p^2-q^2 **t** $2xy+2y^2$
u $2m^2+2n^2$ **v** $3x^2+11x+8$
w $5x^2+6x+1$ **x** $12xy+18y^2$
y ab

Challenge 10:02C

1 $10x^2+110x+385$
2 $9x^2+99x+330$
3 $11a^2+110$
4 $55m^2-55n^2$

Prep quiz 10:03

1 $3x+21$ **2** a^2-a **3** $8p^2-12pq$
4 $-15m+10mn$ **5** $5x$ **6** $2x$
7 $4(n+2)$ **8** $w(w-6)$ **9** $b(a+c)$
10 $-3(2y+3)$

Exercise 10:03

1 **a** $2(x+4)$ **b** $5(2m+1)$ **c** $3(3a+2)$
d $2(2+3y)$ **e** $7(w-3)$ **f** $5(3n-4)$
g $6(2-3t)$ **h** $8(2-z)$ **i** $x(x+5)$
j $a(m+n)$ **k** $p(6-p)$ **l** $a(b-1)$

2 **a** $2a(x+3y)$ **b** $5m(m+n)$ **c** $3b(3a+2c)$
d $2y(2z+3x)$ **e** $7t(w-2t)$ **f** $5n(3-2n)$
g $4p(3-2q)$ **h** $2g(3f-4h)$ **i** $xy(x+y)$
j $am(m+n)$ **k** $6p(1-p)$ **l** $bc(a-1)$

3 **a** $-2(x+4)$ **b** $-5(m-1)$ **c** $-3(3a-4)$
d $-4(1+2y)$ **e** $-w(7w+1)$ **f** $-n(m+p)$
g $-4t(3t-2)$ **h** $-8y(2x-3z)$

4 **a** $x(3 + x - a)$ **b** $a(x + y + z)$
c $2(2m - 4n + 3p)$ **d** $5a(b - 3c + 2d)$
e $x(x - 7 + y)$ **f** $5b(2a - c - 3b)$
5 **a** $(a + x)(2 + b)$ **b** $(3 + b)(x + 2)$
c $(a + 3)(a - 2)$ **d** $(m - 1)(3m + 5)$
e $(2x + 5)(x - 1)$ **f** $(7 - z)(7 - z)$ or $(7 - z)^2$

Prep quiz 10:04

1 $3(a + 6)$ **2** $x(5 + a)$ **3** $p(q - x)$ **4** $3x(a - 3b)$
5 $x(x - 2)$ **6** $a^2(a + 1)$ **7** $3(3 - a)$ **8** $-5(m + 2)$
9 $(a + 1)(9 + x)$ **10** $(x + y)(x - 1)$

Exercise 10:04

1 **a** $(a + b)(2 + x)$ **b** $(x + 7)(a + p)$
c $(m + n)(x - y)$ **d** $(m + n)(x - y)$
e $(2 - x)(a^2 + 7)$ **f** $(q - 2)(q - 2)$
g $(x + y)(1 + a)$ **h** $(x - 2)(1 - 3y)$
2 **a** $(a + b)(p + q)$ **b** $(3 + x)(a + b)$
c $(m + 3p)(n + 5)$ **d** $(a + c)(a + b)$
e $(3x + y)(3x - 4)$ **f** $(3p - 4)(4p + q)$
g $(b + 3)(a + c)$ **h** $(x + 1)(y + 4)$
i $(a^2 + 1)(a + 1)$ **j** $(p + r)(q + 5)$
k $(y - 1)(x + 1)$ **l** $(2 + y)(4a - 1)$
m $(m + 1)(n + 1)$ **n** $(x + m)(x + y)$
o $(x + w)(x - y)$ **p** $(x + z)(x + y)$
q $(a + 4)(11 + c)$ **r** $(a - 1)(a^2 + 1)$
3 **a** $(y + z)(x - w)$ **b** $(a + c)(b - d)$
c $(a + 3)(5 - b)$ **d** $(x - 4)(6 - y)$
e $(y + 2)(11 - x)$ **f** $(ax - 1)(x - 1)$

Prep quiz 10:05

1 4 **2** 7 **3** 11
4 x **5** $3x$ **6** $8x$
7 $x^2 - 4$ **8** $x^2 - 25$ **9** $49 - a^2$
10 $9m^2 - 4n^2$

Exercise 10:05

1 **a** $(x - 2)(x + 2)$ **b** $(a + 4)(a - 4)$
c $(m + 5)(m - 5)$ **d** $(p - 9)(p + 9)$
e $(y - 10)(y + 10)$ **f** $(x - 11)(x + 11)$
g $(3 - x)(3 + x)$ **h** $(1 - n)(1 + n)$
i $(7 - y)(7 + y)$ **j** $(a - b)(a + b)$
k $(x - a)(x + a)$ **l** $(y + a)(y - a)$
m $(3a - 2)(3a + 2)$ **n** $(4x - 1)(4x + 1)$
o $(5p - 3)(5p + 3)$ **p** $(7 - 2a)(7 + 2a)$
q $(5p - a)(5p + a)$ **r** $(m - 9n)(m + 9n)$
s $(10a - 3b)(10a + 3b)$ **t** $(9x + 11y)(9x - 11y)$
2 **a** $2(x - 4)(x + 4)$ **b** $3(x - 6)(x + 6)$
c $4(a - 5)(a + 5)$ **d** $5(y - 2)(y + 2)$
e $6(2a - b)(2a + b)$ **f** $3(x - 3y)(x + 3y)$
g $8(y - 4)(y + 4)$ **h** $5(4p - q)(4p + q)$
i $4(x - 4)(x + 4)$ **j** $3(x - 1)(x + 1)$
k $2(6p - 1)(6p + 1)$ **l** $2(1 - 3x)(1 + 3x)$
m $2(2a - 3m)(2a + 3m)$ **n** $5(5 - 2a)(5 + 2a)$
o $2(10x - 3y)(10x + 3y)$ **p** $2(7m - 2n)(7m + 2n)$

Exercise 10:06

1 **a** $(x + 3)(x + 1)$ **b** $(x + 5)(x + 2)$
c $(x + 6)(x + 4)$ **d** $(x - 1)(x - 2)$
e $(x - 4)(x - 2)$ **f** $(x - 6)(x - 2)$
g $(x + 4)(x - 2)$ **h** $(x + 7)(x - 3)$
i $(x + 7)(x - 6)$ **j** $(x - 6)(x + 1)$
k $(x - 10)(x + 2)$ **l** $(x - 9)(x + 8)$
2 **a** $(a + 4)(a + 5)$ **b** $(y + 8)(y - 4)$
c $(m - 7)(m + 2)$ **d** $(z - 7)(z - 1)$
e $(k + 9)(k - 8)$ **f** $(p + 9)(p - 6)$
g $(t + 10)(t + 3)$ **h** $(q - 8)(q - 1)$
i $(w - 10)(w + 4)$ **j** $(d - 8)(d + 7)$
k $(v + 9)(v - 4)$ **l** $(h - 14)(h + 2)$
m $(n + 10)(n + 4)$ **n** $(x - 13)(x + 3)$
o $(f + 15)(f - 6)$
3 **a** $2(x + 3)(x + 1)$ **b** $5(a + 2)(a + 1)$
c $4(m - 2)(m - 2)$ or $4(m - 2)^2$
d $7(n - 4)(n + 2)$ **e** $9(y + 3)(y - 2)$
f $10(z + 8)(z - 3)$

Exercise 10:07

1 **a** C **b** D **c** B **d** D
2 **a** $(2x + 1)(x + 3)$ **b** $(3x + 2)(x + 2)$
c $(2x + 3)(x + 2)$ **d** $(2x + 1)(x + 5)$
e $(3x + 2)(x + 1)$ **f** $(2x + 5)(x + 3)$
g $(x + 3)(4x + 1)$ **h** $(5x + 2)(x + 3)$
i $(2x + 3)(x + 5)$ **j** $(2x - 1)(x - 2)$
k $(3x - 2)(x - 3)$ **l** $(5x - 2)(x - 3)$
m $(x - 2)(4x - 3)$ **n** $(5x - 3)(2x - 3)$
o $(5x - 7)(x - 3)$ **p** $(2x + 5)(x - 2)$
q $(3x - 5)(x + 3)$ **r** $(x + 3)(4x - 1)$
s $(2x + 3)(x - 2)$ **t** $(2x + 1)(x - 3)$
u $(3x - 10)(x + 3)$ **v** $(2x + 3)(3x - 7)$
w $(2x + 3)(x - 4)$ **x** $(x + 2)(4x - 9)$
3 **a** $(3x + 1)(4x + 1)$ **b** $(3a + 1)(2a + 1)$
c $(3p + 2)(2p + 1)$ **d** $(5y - 2)(2y - 1)$
e $(3x - 1)(4x - 1)$ **f** $(3a - 2)(3a - 5)$
g $(2m + 5)(4m - 1)$ **h** $(2n - 3)(3n + 1)$
i $(7q - 2)(3q - 2)$ **j** $(4x - 1)(5x + 1)$
k $(2m - 3)(4m + 5)$ **l** $(6y - 5)(3y + 2)$
m $(2a + 3)(3a - 2)$ **n** $(3k + 4)(5k + 2)$
o $(2x + 3)(4x + 3)$ **p** $(4 + a)(1 - a)$
q $(2 + 5m)(1 - 2m)$ **r** $(3x + 2)(3 - x)$
s $(2 - 3x)(x + 3)$ **t** $(5 - 7x)(4x + 3)$
u $(2 - 5n)(1 + 7n)$ **v** $(3x + 4y)(x + 2y)$
w $(2x - y)(x - 2y)$ **x** $(5m - 7n)(m + n)$
4 **a** $2(3x - 1)(x + 2)$ **b** $2(3a + 2)(a - 1)$
c $3(2a - 3)(a + 3)$ **d** $4(2x - 3)(x + 3)$
e $2(3x + 2)(x + 4)$ **f** $3(2p - 1)(2p + 3)$
g $5(3q + 7)(2q - 1)$ **h** $2(5m - 3)(m - 4)$
i $5(5a - 1)(2a + 1)$ **j** $2(2 - 5x)(1 + x)$
k $3(4 + t)(3 - t)$ **l** $3(3 + 2x)(1 + 2x)$

5 **a** $(x+3)(x+5) = x^2 + 8x + 15$
$(x-3)(x+5) = x^2 + 2x - 15$
$(x+3)(x-5) = x^2 - 2x - 15$
$(x-3)(x-5) = x^2 - 8x + 15$
$(x+1)(x+15) = x^2 + 16x + 15$
$(x-1)(x+15) = x^2 + 14x - 15$
$(x+1)(x-15) = x^2 - 14x - 15$
$(x-1)(x-15) = x^2 - 16x + 15$

b $(x-1)(x+12) = x^2 + 11x - 12$
$(x-12)(x+1) = x^2 - 11x - 12$
$(x-3)(x+4) = x^2 + x - 12$
$(x-4)(x+3) = x^2 - x - 12$
$(x-6)(x+2) = x^2 - 4x - 12$
$(x-2)(x+6) = x^2 + 4x - 12$

c $(x-1)(x-4) = x^2 - 5x + 4$
$(x+1)(x+4) = x^2 + 5x + 4$
$(x-2)(x-3) = x^2 - 5x + 6$
$(x+2)(x+3) = x^2 + 5x + 6$

d $(5x+1)(x+2) = 5x^2 + 11x + 2$
$(5x+1)(x-2) = 5x^2 - 9x - 2$
$(5x-1)(x+2) = 5x^2 + 9x - 2$
$(5x-1)(x-2) = 5x^2 - 11x + 2$

Exercise 10:08

1 **a** $(x-5)(x-1)$ **b** $(x-3)(x+3)$
c $(x+2)(y+9)$ **d** $a(a-9)$
e $(a-3)(a-3)$ **f** $(2x-1)(2x+1)$
g $(4x-7)(3x+5)$ **h** $(a-5)(a-8)$
i $5ab(a-2b^2)$ **j** $(p-q)(p+q)$
k $(p+10)(q-3)$ **l** $(7x-3)(x+2)$
m $a(a+3-b)$ **n** $(4-5a)(4+5a)$
o $(1+4a)(1-6a)$ **p** $(m+n)(4-a)$
q $5y(a-2+3x)$ **r** $(5x-7)(3x+4)$
s $(xy-1)(xy+1)$ **t** $(x-8)(x+7)$
u $(2m+3p)(n+2)$ **v** $(10a-7x)(10a+7x)$
w $(2+x)(1-3x)$ **x** $(k+8)(k-6)$

2 **a** $2(1-2x)(1+2x)$ **b** $5(x-y)(x-2)$
c $2(a-8)(a-3)$ **d** $3(m-3)(m-3)$
e $(x-1)(x+1)(x^2+1)$ **f** $(p-4)(p-1)(p+1)$
g $4(x-3)(x+3)$ **h** $a(a-1)(a+1)$
i $3(a-8)(a-5)$ **j** $9(1-p)(1+p)$
k $3(k+3)(k-2)$ **l** $3(2a-3)(4a-1)$
m $a(x+y)(x+3)$ **n** $(x+y)(x+y+3)$
o $5x(y-2z)(y+2z)$ **p** $a(3x-2)(2x+3)$
q $(x-y)(x+y+5)$ **r** $3(x-2)(x-2)$
s $7(3x-2y)(3x+2y)$ **t** $(a^2+4)(a-2)(a+2)$
u $a(a-4)$ **v** $(1+p)(1+p^2)$
w $4(2t+3)(t-5)$ **x** $2(2+x)(2-3x)$

Prep quiz 10:09

1 $\frac{a}{2}$ **2** $\frac{3y}{2x}$ **3** 4
4 $\frac{3}{2}$ **5** $3x(2x+3)$ **6** $(x+3)(x+4)$
7 $(x-7)(x+7)$ **8** $3(x+1)^2$ **9** $(x+y)(3+a)$
10 $(2x-1)(x+5)$

Exercise 10:09

1 **a** $x+2$ **b** $\frac{2}{x+3}$ **c** $\frac{4}{x-3}$
d 2 **e** $\frac{1}{3}$ **f** $\frac{5}{8}$
g $\frac{1}{2}$ **h** $\frac{7}{3}$ **i** $\frac{x+1}{x-1}$
j $x+2$ **k** $\frac{1}{a-1}$ **l** $\frac{2y-3}{2}$
m $\frac{a-4}{3-a}$ **n** $x+1$ **o** $\frac{x+6}{3}$
p $a-4$ **q** $x-3$ **r** $\frac{x-2}{x+1}$
s $\frac{x+1}{x+3}$ **t** $\frac{m+8}{m-4}$ **u** $\frac{t+4}{t-3}$
v $\frac{a-x}{a+3}$ **w** $\frac{x-1}{2x-1}$ **x** $\frac{2(3a-2)}{2a-1}$

2 **a** $4x$ **b** $\frac{y-3}{4}$ **c** $\frac{10}{21}$
d $\frac{15}{2}$ **e** $\frac{1}{3}$ **f** $\frac{3(1+2a)}{5(1-2a)}$
g $\frac{y(2y+3)}{3(y+4)}$ **h** $\frac{2x+5}{9}$ **i** $\frac{1}{x+1}$
j $\frac{x+7}{x-5}$ **k** $\frac{a+3}{a-1}$ **l** 1
m $\frac{x+3}{x+7}$ **n** $\frac{m+1}{m+5}$ **o** $\frac{(a+2)(a-4)}{(a+4)(a-1)}$
p $\frac{x+4}{2}$ **q** 1 **r** $\frac{a}{2a-1}$
s 2 **t** $\frac{(a+b-c)(a-c)}{a+c}$

3 **a** 6 **b** $\frac{2}{7}$ **c** 5
d $\frac{9}{2}$ **e** $\frac{3}{5(x+1)}$ **f** 16
g $\frac{m}{2}$ **h** $\frac{5k}{k-1}$ **i** $\frac{n-3}{n+2}$
j $\frac{y}{y-7}$ **k** $\frac{a+1}{a-3}$ **l** 1
m $\frac{x+2}{x-3}$ **n** $\frac{p+4}{p-4}$ **o** $\frac{(n-7)(n-3)}{(n+3)(n+7)}$
p 2 **q** $3(x-1)$ **r** 1
s 2 **t** $\frac{(p+q+r)(p-q)}{p+q}$

Prep quiz 10:10

1 $1\frac{1}{10}$ **2** $1\frac{1}{8}$ **3** $\frac{3}{10}$ **4** $\frac{19}{60}$ **5** $\frac{12}{x}$
6 $\frac{5}{2a}$ **7** $\frac{13}{6a}$ **8** $\frac{3}{4x}$ **9** $\frac{5a}{2x}$ **10** $\frac{7m}{6n}$

Exercise 10:10

1 a $\frac{2x}{(x+1)(x-1)}$ b $\frac{2a+8}{(a+5)(a+3)}$

c $\frac{8}{(y-7)(y+1)}$ d $\frac{5x+19}{(x+3)(x+5)}$

e $\frac{2m-13}{(m+1)(m-2)}$ f $\frac{3t-18}{(t+10)(t+2)}$

g $\frac{7x-4}{(2x-1)(x-1)}$ h $\frac{-3x+31}{(3x+2)(2x+5)}$

i $\frac{59x+1}{(5x-1)(3x+1)}$ j $\frac{13x+21}{2x(x+7)}$

k $\frac{17x-25}{3x(2x+5)}$ l $\frac{1-4a}{2a(2a+1)}$

m $\frac{2x^2+4x}{(x+3)(x+1)}$ n $\frac{-3a}{(2a+1)(4a-1)}$

o $\frac{2x^2+6x+5}{(x+2)(x+1)}$

2 a $\frac{x+3}{(x+1)(x+2)}$ b $\frac{1+x}{x(x+2)}$

c $\frac{x-1}{x(x+3)}$ d $\frac{x+1}{(x-5)(x+2)}$

e $\frac{4x+15}{(x+2)(x+3)}$ f $\frac{5x+2}{(x+1)(x+4)}$

g $\frac{2}{(x+1)(x+3)}$ h $\frac{6x-10}{(x-3)(x+3)(x+1)}$

i $\frac{8x-2}{(x+7)(x-1)(x+1)}$ j $\frac{2x-72}{(x+9)(x+3)(x-1)}$

k $\frac{7x+5}{(2x+1)(x+5)(x+2)}$ l $\frac{-13x-12}{x(2x-1)(3x+2)}$

m $\frac{2x^2+2}{(x+3)(x+1)(x-1)}$ n $\frac{2x+7}{x(x+3)(x+2)}$

3 a $\frac{(1+x)}{x(x+1)} = \frac{1}{x}$ b $\frac{-2}{3(x+3)}$

c $\frac{7}{2(2x+3)}$ d $\frac{3x+8}{(x+1)(x-1)}$

e $\frac{x+5}{2(x-3)(x+3)}$ f $\frac{-1}{x(x+1)(x-1)}$

g $\frac{2x}{(x+1)^2(x-1)}$ h $\frac{2x+7}{(x+3)(x+4)^2}$

i $\frac{6x+22}{(x+2)(x+4)(x+3)}$ j $\frac{6x+14}{(x+3)(x+4)(x+1)}$

k $\frac{-x-1}{(x-2)(x+1)(x-3)}$ l $\frac{x-1}{(x-3)(x+2)(x+1)}$

m $\frac{2x+2}{(x-4)(x+1)(x-2)} = \frac{2}{(x-4)(x-2)}$

n $\frac{x+10}{(2x-1)(x+4)(3x+2)}$ o $\frac{6x+34}{(x-7)(x+7)(x+3)}$

p $\frac{2x-3}{(2x-1)(x+1)(x-1)}$ q $\frac{2x^2-x-5}{(x+2)(x+3)(x-3)}$

r $\frac{-3x-8}{x(x-4)(x+4)}$ s $\frac{7x^2-x-10}{5(x-2)(x+2)^2}$

t $\frac{13x^2-11x+1}{(2x+1)(x-3)(2x-1)}$

Diagnostic test 10

1 a $\frac{7a+12}{12}$ b $\frac{11x-11}{15}$ c $\frac{m+2}{12}$

2 a x^2+4x+4 b $a^2-14a+49$

c $4y^2+20y+25$ d $m^2-2mn+n^2$

3 a x^2-9 b y^2-49

c $4a^2-25$ d x^2-y^2

4 a $3(x-4)$ b $a(x+y)$

c $-2(x+3)$ d $x(a+b-c)$

5 a $(a+b)(x+2)$ b $(6+a)(m+n)$

c $(x+1)(y-1)$ d $(a+c)(b+4)$

6 a $(x-5)(x+5)$ b $(a-x)(a+x)$

c $(2-m)(2+m)$ d $(3x-1)(3x+1)$

7 a $(x+3)(x+4)$ b $(x-2)(x-3)$

c $(x-5)(x+2)$ d $(x+5)(x-4)$

8 a $(2x+1)(x+5)$ b $(3x-2)(x-3)$

c $(4x-9)(x+2)$ d $(3x+1)(2x+1)$

9 a $x+2$ b $\frac{6}{7}$

c $\frac{x}{a}$ d $\frac{x+5}{x+2}$

10 a $6x$ b $\frac{a-1}{a-3}$

c $\frac{9}{5}$ d $\frac{x+2}{x-3}$

11 a $\frac{3x+1}{(x+3)(x-1)}$ b $\frac{1}{x(x+2)(x+1)}$

c $\frac{3x+19}{2(x-3)(x+3)}$ d $\frac{-7x-8}{(x+3)(x+4)(x-1)}$

ASSIGNMENT 10A

1 a $\frac{7a-4}{10}$ b $\frac{p+1}{18}$

c $\frac{x+4}{30}$ d $\frac{14n+1}{24}$

2 a $x^2+10x+25$ b x^2-25

c $a^2-14a+49$ d a^2-49

e $9y^2+24y+16$ f $9y^2-16$

g $25-20z+4z^2$ h $25-4z^2$

i $2x^2+72$ j $2-4n$

3 $3x^2+14x-24$

4 a $(a+4)(a+5)$ b $2(p-2q)$

c $(m-9)(m+5)$ d $(5x^2+1)(x+2)$

e $(2x-1)(2x+1)$ f $xy(x-1)$

g $(2a-1)(3a-5)$ h $(x+6)(x-5)$

i $(3a+5)(a-3)$ j $(x+p)(y+z)$

k $(2x-1)(x+1)$ l $(x^2+2)(x-3)$

m $-5ab(1+2ab)$ n $(x-y)(x+y+2)$

o $(2+3x)(1-3x)$

5 **a** $2(y-3)(y+3)$ **b** $3(r+7)(r-4)$
c $2(x+1)(2x^2+3)$ **d** $2(1-3x)(1+3x)$
e $a(a-8)(a+9)$ **f** $3(11+a)(1+a)$
g $(x-y)(x-y+1)$ **h** $x(x-4)$

6 **a** $\frac{x+12}{x+3}$ **b** $\frac{5(2x+1)}{x+3}$
c $\frac{5x+13}{(x+2)(x+3)}$ **d** $\frac{-x^2}{(x-1)(x-2)}$
e $\frac{x-1}{5}$ **f** $\frac{1}{x-2}$
g $\frac{-2x-3}{(x+2)(x+1)}$ **h** $\frac{6x-1}{(3x-1)^2}$
i $\frac{2x+3}{(3+2x)(2x+3)}$ **j** $\frac{x+2}{5(x-7)}$

ASSIGNMENT 10B

1 10 am
2 $\theta = 30°$; $a = 1212$ mm; $b = 2425$ mm
3 The final answer is \$14.40 less than \$360.
4 **a** 21·6 m
b 75·168 m^2
5 258·4 m
6 **a** tetrahedron 6; cube 12; dodecahedron 30; icosahedron 30
b no. of vertices = (no. of faces × no. of vertices on each face) ÷ (no. of faces that share a vertex)
c tetrahedron 4; cube 8; dodecahedron 20; icosahedron 12

Chapter 11

Prep quiz 11:01

1 $x = 12$ **2** $m = 16$ **3** $a = 10$ **4** $z = 7$ **5** $x = 2$
6 $n = 1$ **7** $t = 2$ **8** $m = -1$ **9** $x = 20$ **10** $y = 15$

Exercise 11:01

1 **a** $x = 4$ **b** $a = 2$ **c** $y = 3$
d $m = -2$ **e** $a = 1$ **f** $x = -2$
g $a = 5$ **h** $n = 2$ **i** $p = 2$

2 **a** $x = 10$ **b** $a = 15$ **c** $m = 24$
d $z = 42$ **e** $n = 12$ **f** $y = 40$
g $k = 3\frac{1}{3}$ **h** $a = 10$ **i** $x = \frac{1}{2}$

3 **a** $m = 13$ **b** $x = 4$ **c** $y = -2\frac{2}{3}$
d $n = -1$ **e** $k = -1\frac{1}{2}$ **f** $x = \frac{1}{2}$

4 **a** $x = 5$ **b** $a = 11$ **c** $p = 6$
d $b = 3\frac{1}{8}$ **e** $q = \frac{7}{8}$ **f** $k = 3$
g $x = 17$ **h** $a = 16$ **i** $x = -32$
j $a = -13$

5 **a** $x = -4$ **b** $a = 23$ **c** $m = 18$
d $a = -6\frac{2}{5}$

Prep quiz 11:02

1 $x(x+3)$ **2** $(x-3)(x+3)$ **3** $(2x-5)(2x+5)$
4 $(x+3)(x+4)$ **5** $(x-7)(x+5)$ **6** $(x-3)^2$
7 $(2x-1)(x+5)$ **8** $(3x+2)(x+4)$ **9** $(2x+5)^2$
10 $(4x-3)(3x+2)$

Exercise 11:02

1 **a** $x = 0$ or 7 **b** $a = 0$ or -4
c $m = 0$ or $\frac{2}{3}$ **d** $y = 3$ or -3
e $n = 4$ or -4 **f** $x = \frac{2}{3}$ or $-\frac{2}{3}$
g $a = -2$ or 5 **h** $t = 1$ or -7
i $w = 9$ or -2 **j** $x = -2$ or $\frac{3}{2}$
k $q = \frac{1}{5}$ or 8 **l** $z = -\frac{5}{6}$ or 7
m $u = \frac{1}{3}$ or $-\frac{5}{2}$ **n** $p = \frac{3}{4}$ or $\frac{8}{5}$
o $q = -\frac{3}{5}$ or $-\frac{5}{3}$ **p** $x = \frac{3}{8}$ or $-\frac{1}{7}$
q $y = -\frac{9}{2}$ or $-\frac{8}{7}$ **r** $x = -\frac{3}{10}$ or $\frac{10}{3}$

2 **a** $m = 0$ or -3 **b** $x = 0$ or 7
c $y = 0$ or 3 **d** $p = 0$ or $-\frac{3}{5}$
e $a = 0$ or $\frac{2}{3}$ **f** $z = 0$ or $-\frac{5}{2}$
g $x = 4$ or -4 **h** $n = 9$ or -9
i $t = \frac{2}{3}$ or $-\frac{2}{3}$ **j** $q = \frac{3}{5}$ or $-\frac{3}{5}$
k $w = \frac{5}{8}$ or $-\frac{5}{8}$ **l** $p = \frac{10}{7}$ or $-\frac{10}{7}$
m $x = -2$ or -3 **n** $m = 4$ or 2
o $y = 10$ or -9 **p** $v = 3$ or -7
q $z = 6$ or 9 **r** $x = -10$ or 17

3 **a** $\frac{1}{2}, -1$ **b** $-\frac{1}{3}, -2$ **c** $-\frac{2}{3}, -5$
d $4, 1\frac{1}{2}$ **e** $2\frac{1}{2}, -2$ **f** $7, -1\frac{1}{2}$
g $-\frac{1}{4}, -5$ **h** $5, -\frac{1}{4}$ **i** $5, \frac{1}{4}$
j $-\frac{1}{5}, -3$ **k** $1\frac{1}{2}, -8$ **l** $\frac{1}{7}, -7$
m $1\frac{1}{2}, -\frac{1}{2}$ **n** $\frac{1}{2}, -\frac{1}{3}$ **o** $-\frac{1}{3}, -\frac{2}{3}$
p $-\frac{2}{5}, -\frac{1}{2}$ **q** $\frac{1}{3}, \frac{1}{4}$ **r** $\frac{4}{5}, \frac{1}{2}$

4 **a** $0, 3$ **b** $0, 8$ **c** $0, -5$
d $4, 1$ **e** $5, -3$ **f** $2, 1$
g $6, 3$ **h** $9, -2$ **i** $8, -4$
j $1, -2$ **k** $1, -3$ **l** $5, 2$
m $3, -6$ **n** $4, -7$ **o** $3, -5$
p $\frac{1}{2}, -1$ **q** $3, -2\frac{1}{2}$ **r** $2, -\frac{3}{4}$
s $2\frac{1}{3}, 2$ **t** $3, \frac{2}{5}$ **u** $5, \frac{1}{2}$

Exercise 11:03

1 **a** 3 **b** 4 **c** 1 **d** 2 **e** $1\frac{1}{2}$
f $3\frac{1}{2}$ **g** $5\frac{1}{2}$ **h** $\frac{1}{2}$ **i** $1\frac{1}{4}$ **j** $\frac{1}{3}$

2 **a** $2 \pm \sqrt{3}$ **b** $-1 \pm \sqrt{2}$ **c** $-5 \pm \sqrt{5}$
d $1 \pm \sqrt{10}$ **e** $3 \pm \sqrt{7}$ **f** $-2 \pm \sqrt{11}$
g $-3 \pm \sqrt{8}$ **h** $-10 \pm \sqrt{12}$ **i** $3 \pm \sqrt{18}$
j $-\frac{1}{2} \pm \sqrt{5}$ **k** $\frac{2}{3} \pm \sqrt{3}$ **l** $-1\frac{1}{2} \pm \sqrt{12}$
m $1 \pm \sqrt{\frac{5}{2}}$ **n** $-3 \pm \frac{\sqrt{18}}{2}$ **o** $\frac{1}{3} \pm \frac{\sqrt{5}}{3}$

3 **a** 0·41, −2·41 **b** 3·45, −1·45 **c** 5·46, −1·46
d 1·12, −7·12 **e** 5·65, 0·35 **f** −0·27, −3·73
g 0·48, −10·48 **h** 1·24, −3·24 **i** 12·08, −0·08
j −0·44, −4·56 **k** 0·41, −7·41 **l** 1·30, −2·30
m −0·35, −8·65 **n** 1·19, −4·19 **o** 10·52, 0·48
p 2·30, −1·30 **q** 0·56, −3·56 **r** 5·19, −0·19
s 2·22, −0·22 **t** 0·85, −2·35 **u** 3·87, 0·13
v 0·72, −1·39 **w** 1·27, −0·47 **x** 0·84, −0·59

Exercise 11:04

1 **a** −2, −3 **b** −2, −4 **c** −1, −9
d 5, −2 **e** 5, −3 **f** 2, −6
g 7, 2 **h** 6, 2 **i** 5, 1
j $-\frac{1}{3}, -2$ **k** $-\frac{1}{2}, -5$ **l** $-\frac{3}{4}, -2$
m $3, -\frac{1}{2}$ **n** $2, -\frac{1}{5}$ **o** $1, \frac{2}{3}$
p $-\frac{1}{2}, -\frac{2}{3}$ **q** $\frac{1}{3}, -1\frac{1}{2}$ **r** $1\frac{1}{2}, \frac{1}{4}$

2 **a** $\frac{-4 \pm \sqrt{8}}{2}$ **b** $\frac{-3 \pm \sqrt{5}}{2}$ **c** $\frac{-5 \pm \sqrt{13}}{2}$
d $\frac{-1 \pm \sqrt{5}}{2}$ **e** $\frac{-2 \pm \sqrt{12}}{2}$ **f** $\frac{-4 \pm \sqrt{20}}{2}$
g $\frac{2 \pm \sqrt{8}}{2}$ **h** $\frac{7 \pm \sqrt{41}}{2}$ **i** $\frac{6 \pm \sqrt{24}}{2}$
j $\frac{10 \pm \sqrt{136}}{2}$ **k** $\frac{8 \pm \sqrt{52}}{2}$ **l** no solutions
m $\frac{-6 \pm \sqrt{28}}{4}$ **n** $\frac{-3 \pm \sqrt{17}}{4}$ **o** $\frac{7 \pm \sqrt{17}}{4}$
p $\frac{-10 \pm \sqrt{76}}{6}$ **q** $\frac{9 \pm \sqrt{57}}{6}$ **r** $\frac{-4 \pm \sqrt{56}}{10}$
s no solutions **t** $\frac{3 \pm \sqrt{21}}{6}$ **u** $\frac{3 \pm \sqrt{41}}{8}$
v $\frac{-11 \pm \sqrt{161}}{4}$ **w** $\frac{9 \pm \sqrt{17}}{4}$ **x** $\frac{-2 \pm \sqrt{24}}{10}$

3 **a** 3·73, 0·27 **b** 5·45, 0·55 **c** 0·58, −8·58
d −0·11, −8·89 **e** 1·45, −3·45 **f** 0·30, −3·30
g no solutions **h** 7·27, −0·27 **i** no solutions
j 0·78, −1·28 **k** 2·85, −0·35 **l** −0·74, −2·26
m 3·19, 0·31 **n** 1·24, −0·64 **o** 0·80, −0·63

Investigation 11:04

1 2 **2** 1 **3** 0 **4** 2 **5** 2
6 0 **7** 1 **8** 2 **9** 0 **10** 0
11 2 **12** 1

Prep quiz 11:05

1 $5x(x-2)$ **2** $(x-7)(x+2)$ **3** $(x-9)(x+9)$
4 $(x+2)(x+3)$ **5** 2, −7 **6** $1\frac{1}{2}, -\frac{1}{3}$
7 4, −4 **8** 0, 4 **9** 1, 2
10 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 11:05

1 **a** −1, −6 **b** 6, 2 **c** 3, −8
d 2·62, 0·38 **e** 0·79, −3·79 **f** −0·59, −3·41
g 0, −8 **h** 10, 0 **i** 2, 0
j 9, −9 **k** 11, −11 **l** $1\frac{1}{2}, -1\frac{1}{2}$
m −0·29, −1·71 **n** 0·77, −0·43 **o** 2·28, 0·22
p −1, −2 **q** −2, −3 **r** 4, −1
s 9, −3 **t** 9, 4 **u** $4, -1\frac{1}{2}$
v 5, 5 **w** 9, 4 **x** 1·54, 0·26

2 **a** −3, −4 **b** 5, 3 **c** 4, 2
d −2, −3 **e** 0, −7 **f** 3, −3
g $\frac{-2 \pm \sqrt{8}}{2}$ **h** $\frac{-4 \pm \sqrt{8}}{2}$ **i** $\frac{1 \pm \sqrt{5}}{2}$
j 1, −6 **k** 9, −2 **l** 8, −4
m 3, −1 **n** 0, −6 **o** $-5 \pm \sqrt{11}$
p $1\frac{1}{2}, -2\frac{1}{2}$ **q** $\frac{3 \pm \sqrt{7}}{5}$ **r** $\frac{7 \pm \sqrt{3}}{6}$
s 5, −3 **t** 7, −4 **u** 1, −2
v $\frac{5 \pm \sqrt{13}}{2}$ **w** $\frac{3 \pm \sqrt{21}}{2}$ **x** $\frac{-4 \pm \sqrt{24}}{4}$

Prep quiz 11:06

1 1, 2, 9, 10 **2** 1, 9
3 1, 9 **4** 9, 10
5 $n+1, n+2$ **6** 12, 14
7 $x+2, x+4$ **8** $x-3$
9 $a(a+3)$ or a^2+3a **10** $4a+6$

Exercise 11:06

1 **a** 4, 5 **b** 9, 10 **c** 10, 12 **d** 7, 9
2 **a** 9 **b** 11 **c** 8 **d** 0, 5 **e** 7, −6
3 **a** 10 cm, 4 cm **b** 17 m, 15 m
c 7 cm **d** 17 cm
4 **a** 2 s **b** 12
c **i** 2, −9 **ii** $\frac{1}{2}, -6$ **iii** $\frac{-1 \pm \sqrt{85}}{6}$
5 15 **6** 7

Challenge 11:06

1 **a** $x = \pm 1, \pm 2$ **b** $a = \pm 3, \pm 1$
c $y = 1, 2$ **d** $m = \pm 2, \pm 1$
2 $x = 6, 3, -1, -4$
3 **a** $x = 2, 0$ **b** $x = 1, 2$
4 **a** $u = \pm\sqrt{5}, \pm\sqrt{2}$ **b** $a = \pm\sqrt{5}$

Investigation 11:06

1 $x = 10$ **2** $a = -5$
3 $p = 1$ **4** $k = -3$
5 $y = \sqrt[3]{10}, 2{\cdot}15$ **6** $t = -\sqrt[3]{7}, -1{\cdot}91$
7 $q = \sqrt[3]{2}, 1{\cdot}26$ **8** $n = -\sqrt[3]{20}, -2{\cdot}71$
9 $x = 2$ **10** $z = -1$
11 $p = \sqrt[3]{4}, 1{\cdot}59$ **12** $y = -\sqrt[3]{0{\cdot}9}, -0{\cdot}97$
13 $x = 2$ **14** $m = -\sqrt[3]{3}, -1{\cdot}44$
15 $t = 10$ **16** $z = -0{\cdot}5$

Prep quiz 11:07

1 $x = -3, 2$ **2** $x = \frac{1}{2}, -7$ **3** $x = 0, -4$
4 $x = 2, -2$ **5** $x = 1, 2$ **6** $x = \frac{1}{2}, -4$
7 $x = 1, 3$ **8** $x = -\frac{3}{2}, 1$ **9** $x = 2, y = 3$
10 $x = 4, y = -1$

Exercise 11:07

1 **a** $x = 2, y = 8$
b $x = -1, y = 5$
c $x = 2, y = 4$
d $x = 3, y = 9$ or $x = -2, y = 4$
e $x = 2, y = 4$ or $x = -1, y = 1$
f $x = 1, y = 1$ or $x = -3, y = 9$

2 Solutions same as for Question 1.

3 **a** $x = -1, y = 1$ or $x = -3, y = 9$
b $x = 7, y = 49$ or $x = -7, y = 49$
c $x = 0, y = 0$ or $x = 3, y = 9$
d $x = 8, y = 64$ or $x = -7, y = 49$
e $x = 3, y = 9$ or $x = 7, y = 49$
f $x = -1, y = 1$ or $x = -2, y = 4$
g $x = 9, y = 86$ or $x = -5, y = 30$
h $x = 7, y = 56$ or $x = 1, y = 8$
i $x = 1, y = -9$ or $x = -10, y = 90$

4 **a** $x = -1, y = 1$ or $x = 2, y = 4$
b $x = 1, y = 1$
c no solutions

5 **a** $x = 1, y = 4$ or $x = -2, y = -2$
b **i** $x = 2, y = 2$ **ii** one
iii they only have one point in common

6 **a** $x = 6, y = 36$ or $x = -1, y = 1$
b $x = 2, y = 5$ or $x = -1, y = 2$
c $x = 6, y = 33$ or $x = -2, y = 1$
d $x = 4, y = 12$ or $x = -1, y = 2$
e $x = 7, y = 28$ or $x = -2, y = 10$
f $x = 1, y = 4$ or $x = -7, y = 28$
g $x = 1, y = 4$ or $x = -1, y = -4$
h $x = -4, y = -2$ or $x = 1, y = 8$
i $x = 2, y = 3$ or $x = -6, y = -1$

7 **a** $(2, 2)$ and $(-1, -1)$ **b** $(4, 6)$ and $(-5, 15)$
c $(2, 6)$ **d** $(4, 4)$ and $(-7, 15)$
e $(-3, 4)$ **f** no points of intersection
g $(5, 20)$ and $(-4, 2)$ **h** $(2, 0)$ and $(-7, 27)$
i $(3, 13)$ and $(-0.5, -1)$

Prep quiz 11:08

1 25 **2** a **3** 27 **4** m **5** 5
6 a **7** 5 **8** m **9** $a = 1$ **10** $x = 8\frac{1}{3}$

Exercise 11:08

1 **a** $x = p - m$ **b** $x = m - np$ **c** $x = pq - n$
d $x = \frac{b}{a}$ **e** $x = \frac{y}{3}$ **f** $x = \frac{b + c}{a^2}$
g $x = \frac{b - 2d}{a}$ **h** $x = \frac{b + c}{a}$ **i** $x = \frac{c - 3b}{a}$
j $x = ay$ **k** $x = \frac{a}{y}$ **l** $x = by$
m $x = \frac{25}{2}$ **n** $x = \frac{bc}{a}$ **o** $x = \frac{pL}{2}$

2 **a** $x = \frac{a - 2y}{a}$ **b** $x = \frac{p - 5t}{5}$ **c** $x = \frac{y + 21}{3}$
d $x = \frac{p + qr}{q}$ **e** $x = \frac{6a - b}{6}$ **f** $x = \frac{tv - w}{t}$
g $x = \frac{R - 4r}{2r}$ **h** $x = \frac{p + 5qy}{5q}$ **i** $x = \frac{\pi r^2 - A}{\pi r}$

3 **a** $y = A - x$ **b** $L = \frac{P - 2B}{2}$ **c** $d = \frac{C}{\pi}$
d $u = v - at$ **e** $A = \frac{v - u}{t}$ **f** $m = \frac{E}{c^2}$
g $D = ST$ **h** $V = RI$ **i** $P = \frac{I}{RN}$
j $R = \frac{P}{I^2}$ **k** $s = \frac{v^2 - u^2}{2a}$ **l** $a = \frac{F - p}{c}$
m $n = \frac{P - ma}{a}$ **n** $p = \frac{x - 2aq}{2a}$ **o** $m = \frac{2K}{V^2}$
p $u = \frac{mv - P}{m}$ **q** $H = \frac{3V}{A}$ **r** $h = \frac{3V}{\pi r^2}$
s $h = \frac{S - \pi r^2}{\pi r}$ **t** $h = \frac{E - \frac{1}{2}mv^2}{mg}$
u $k = \frac{2ab - P}{2a}$ **v** $a = 2A - b$ **w** $b = \frac{2A - ha}{h}$
x $r = \frac{q_1 q_2}{F}$ **y** $d = \frac{T - a}{n - 1}$ **z** $a = \frac{S(r - 1)}{r^n - 1}$

Prep quiz 11:09

1 $x = 15$ **2** $x = 6$ **3** $x = 13$
4 $x = \pm 3$ **5** $x = 25$ **6** $x = 2$ or -8
7 $x = 7$ **8** $3(x + 4)$ **9** $x(x - 2)$
10 $5a(a + 2b)$

Exercise 11:09

1 **a** $x = \pm\sqrt{\frac{n}{m}}$ **b** $x = \pm\sqrt{\frac{a}{b}}$
c $x = \pm\sqrt{a + b}$ **d** $x = \pm\sqrt{k - h}$
e $x = \pm\sqrt{ay}$ **f** $x = \pm\sqrt{\frac{3m}{n}}$
g $x = \pm\sqrt{L + y^2}$ **h** $x = \pm\sqrt{\frac{B}{A}}$

2 **a** $a = \frac{c^2}{b}$ **b** $a = \frac{u^2}{3}$
c $a = c^2 + b$ **d** $a = (c + b)^2$
e $a = \left(\frac{P - L}{M}\right)^2$ **f** $a = \left(\frac{M - L}{N}\right)^2$
g $a = \frac{L^2 + 1}{3}$ **h** $a = \frac{b - P^2}{2}$

3 **a** $N = \frac{2a}{3}$ **b** $N = \frac{2L-2a}{3}$

c $N = \frac{2x+6}{3}$ **d** $N = \frac{6L-2M}{3}$

e $N = 3x - a$ **f** $N = \frac{6x+1-2M}{3}$

g $N = \frac{3m-u}{4}$ **h** $N = \frac{a^2+b^2+bL}{a}$

4 **a** $x = \frac{b-a}{2}$ **b** $x = \frac{q}{a-p}$

c $x = \frac{a-b}{a-1}$ **d** $x = \frac{n-m}{m-n}$ or -1

e $x = \pm\sqrt{\frac{2}{p-q}}$ **f** $x = \frac{L}{A+B+1}$

g $x = \frac{15a}{8}$ **h** $x = \frac{2a}{1-a}$

i $x = \frac{5y}{y-1}$ **j** $x = \frac{3-m}{m-1}$

k $x = \frac{a-A}{A-b}$ **l** $x = \frac{Ba+a}{B-1}$

5

	A	B	C
a	$B = \frac{A}{L}$	$X = A + Y$	$t = \frac{V-u}{a}$
b	$V = \frac{M}{D}$	$S = DT$	$l = \pm\sqrt{\frac{P}{R}}$
c	$h = \frac{3V}{A}$	$r = \pm\sqrt{\frac{3V}{\pi h}}$	$r = \pm\sqrt{\frac{S}{4\pi}}$
d	$b = 2M - a$	$y = \frac{2A-xh}{h}$	$s = \frac{v^2-u^2}{2a}$
e	$a = \frac{x^2+y^2}{y}$	$h = \frac{S}{2\pi r} - r$	$d = \frac{2T-2an}{n(n-1)}$
f	$c = \frac{a^2}{b}$	$X = \frac{Y^2}{a^2}$	$a = \frac{X^2+4b}{4}$
g	$Y = aX^2$	$x = \frac{bR^2}{a}$	$b = m^2n - a$
h	$I = \frac{gT^2}{4\pi^2}$	$u = \frac{4x^2t}{A^2}$	$s = \frac{v^2-u^2}{2a}$
i	$a = \frac{6y-3b}{2}$	$X = \frac{12Z-4Y}{3}$	$c = b - 5A$
j	$A = 2a - 3y$	$N = \frac{6L+1}{5}$	$X = \frac{a^2}{a-b}$
k	$k = \frac{h}{1-2h}$	$a = \frac{2y}{1-y}$	$x = \frac{3z}{z-1}$

Diagnostic test 11

1 **a** $a = -8\frac{1}{2}$ **b** $x = 3$ **c** $n = 12\frac{2}{3}$

2 **a** $x = -7$ or 3 **b** $x = 0$ or 5

c $x = \frac{1}{2}$ or -1 **d** $x = -\frac{2}{3}$ or $\frac{5}{4}$

3 **a** $x = 0$ or -5 **b** $x = -7$ or -2

c $x = 7$ or -7 **d** $x = \frac{1}{2}$ or -3

4 **a** 9 **b** 4 **c** $\frac{9}{4}$ **d** $\frac{1}{4}$

5 **a** $x = -1 \pm \sqrt{3}$ **b** $x = 3 \pm \sqrt{8}$

c $\frac{3 \pm \sqrt{29}}{2}$ **d** $x = \frac{5 \pm \sqrt{27}}{2}$

6 **a** $x = \frac{-1 \pm \sqrt{13}}{2}$

b $x = \frac{5 \pm \sqrt{17}}{2}$

c $x = \frac{-4 \pm \sqrt{8}}{4}$ or $\frac{-2 \pm \sqrt{2}}{2}$

d $x = \frac{-2 \pm \sqrt{28}}{6}$ or $x = \frac{-1 \pm \sqrt{7}}{3}$

7 **a** $x = 6$ or -1 **b** $x = 3$

c $x = -4 \pm \sqrt{6}$ **d** $x = 4$ or -2

8 **a** $x = 8, y = 64$ or $x = -1, y = 1$

b $x = 5, y = 15$ or $x = -4, y = 6$

c $x = 0, y = -5$ or $x = -3, y = -8$

9 **a** $a = \frac{x+2b}{3}$

b $a = \frac{v^2-u^2}{2s}$

c $a = \frac{Ah}{D} - b$ or $a = \frac{Ah-Db}{D}$

10 **a** $y = \pm\sqrt{\frac{x}{a}}$ **b** $y = \frac{BA^2}{T^2}$ **c** $y = \frac{P}{1-P}$

ASSIGNMENT 11A

1 **a** $x = 12$ **b** $n = 11\frac{5}{8}$ **c** $a = 6\frac{3}{8}$

2 **a** $x = 6$ or -6 **b** $x = -2$ or 3 **c** $x = \frac{1}{2}$ or -5

d $x = 0$ or 2 **e** $x = 3$ or 4 **f** $x = 9$ or -5

g $x = 5$ or -2 **h** $x = \frac{1}{2}$ or -3 **i** $x = \frac{2}{5}$ or $-\frac{1}{2}$

3 **a** $x = 1$ or 5 **b** $a = -\frac{1}{2}$ or $-\frac{3}{2}$ **c** $y = \frac{3 \pm \sqrt{7}}{2}$

4 **a** $x = 7, y = 15$ or $x = -4, y = 4$

b $x = 2, y = 1$ or $x = -7, y = -44$

c $x = -3, y = -4$ or $x = \frac{1}{2}, y = 3$

5 **a** $P = \frac{100A}{RT}$

b $P = I^2R$

c $P = \frac{N-M}{V}$

d $P = \frac{K}{L} + Q$ or $P = \frac{K+QL}{L}$

6 **a** $x = 4{\cdot}8$ or $0{\cdot}2$

b $x = 2{\cdot}6$ or $-1{\cdot}6$

c $x = 1{\cdot}7$ or $0{\cdot}3$

7 40 m

8 **a** $x = \frac{ma^2}{y^2}$ **b** $x = \frac{a-by}{y-1}$ or $x = \frac{by-a}{1-y}$

ASSIGNMENT 11B

1 25 **2** 24 **3** $\frac{1}{4}$ **4** 6

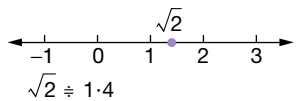
Chapter 12

Prep quiz 12:01

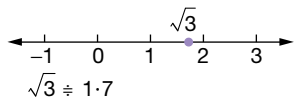
1 16 **2** 49 **3** 400 **4** 61
5 9 **6** 8 **7** 12 **8** 7
9 1·414 214 **10** 8·316 625

Exercise 12:01

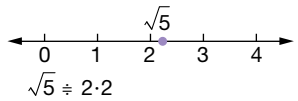
1 **a** rational **b** irrational **c** rational
d rational **e** rational **f** irrational
g irrational **h** rational **i** rational
j rational **k** rational **l** rational
m rational **n** rational **o** rational
p irrational **q** rational **r** irrational
s rational **t** rational **u** rational
v rational **w** irrational **x** irrational
y rational **z** irrational

2 **a**


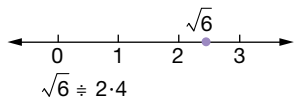
$\sqrt{2} \div 1{\cdot}4$

b


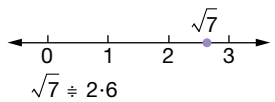
$\sqrt{3} \div 1{\cdot}7$

c


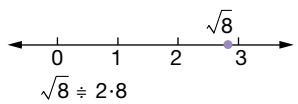
$\sqrt{5} \div 2{\cdot}2$

d


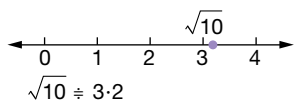
$\sqrt{6} \div 2{\cdot}4$

e


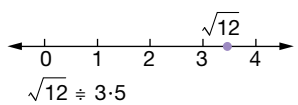
$\sqrt{7} \div 2{\cdot}6$

f


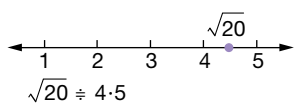
$\sqrt{8} \div 2{\cdot}8$

g


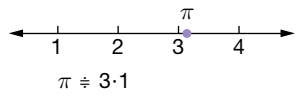
$\sqrt{10} \div 3{\cdot}2$

h


$\sqrt{12} \div 3{\cdot}5$

i


$\sqrt{20} \div 4{\cdot}5$

j


$\pi \div 3{\cdot}1$

3 **a** 3, 4 **b** 4, 5 **c** 6, 7
d 8, 9 **e** 9, 10 **f** 11, 12
g 13, 14 **h** 15, 16 **i** 19, 20
j 30, 31

4 **a** $\sqrt{3}, 2, \sqrt{5}$ **b** $\sqrt{8}, 3, \pi$
c $3, \sqrt{10}, \sqrt{12}$ **d** $\sqrt{40}, 6{\cdot}5, 7, \sqrt{50}$
e $\sqrt{2}, 2{\cdot}1, \pi, \sqrt{12}$ **f** $\sqrt{26}, \sqrt{30}, 5{\cdot}6, 6$
g $\sqrt{60}, 7{\cdot}9, \sqrt{65}, 8{\cdot}1$ **h** $\sqrt{98}, 10, \sqrt{102}, 10{\cdot}1$
i $\sqrt{9}, 3{\cdot}1, \pi, 3{\cdot}2$ **j** $4{\cdot}1, \sqrt{20}, 4{\cdot}5, \sqrt{21}$
k $\sqrt{390}, 20, \sqrt{420}, 21$ **l** $24, \sqrt{600}, \sqrt{610}, 25$

5–7 Students' own diagrams.

Prep quiz 12:02

1 4 **2** 3 **3** 6 **4** 5 **5** 7
6 12 **7** 12 **8** 2 **9** 2 **10** 16

Exercise 12:02

1 **a** $\sqrt{15}$ **b** $\sqrt{15}$ **c** $\sqrt{42}$ **d** $\sqrt{42}$
e $\sqrt{30}$ **f** $\sqrt{46}$ **g** $\sqrt{65}$ **h** $\sqrt{33}$
i $\sqrt{10}$ **j** $\sqrt{14}$ **k** $\sqrt{110}$ **l** $\sqrt{91}$
m $\sqrt{13}$ **n** $\sqrt{11}$ **o** $\sqrt{7}$ **p** $\sqrt{2}$
q $\sqrt{19}$ **r** $\sqrt{19}$ **s** $\sqrt{6}$ **t** $\sqrt{2}$

2 **a** 16 **b** 9 **c** 1 **d** 100
e 5 **f** 8 **g** 15 **h** 73
i 8 **j** 45 **k** 12 **l** 75
m 147 **n** 28 **o** 891 **p** 180
q 1000 **r** 1620 **s** 1800 **t** 3375

3 **a** $2\sqrt{2}$ **b** $2\sqrt{5}$ **c** $2\sqrt{3}$ **d** $5\sqrt{2}$
e $2\sqrt{6}$ **f** $4\sqrt{2}$ **g** $3\sqrt{5}$ **h** $3\sqrt{6}$
i $2\sqrt{7}$ **j** $3\sqrt{10}$ **k** $2\sqrt{14}$ **l** $3\sqrt{7}$
m $2\sqrt{11}$ **n** $2\sqrt{13}$ **o** $6\sqrt{3}$ **p** $2\sqrt{10}$
q $3\sqrt{11}$ **r** $2\sqrt{15}$ **s** $4\sqrt{6}$ **t** $2\sqrt{19}$
u $2\sqrt{17}$ **v** $3\sqrt{14}$ **w** $10\sqrt{2}$ **x** $9\sqrt{2}$

4 **a** 4·2 **b** 5·2 **c** 2·8 **d** 3·5
e 5·6 **f** 6·9 **g** 7·1 **h** 12·7

5 **a** $4\sqrt{3}$ **b** $6\sqrt{2}$ **c** $10\sqrt{2}$ **d** $12\sqrt{2}$
e $10\sqrt{5}$ **f** $10\sqrt{3}$ **g** $30\sqrt{3}$ **h** $6\sqrt{14}$
i $10\sqrt{5}$ **j** $12\sqrt{5}$ **k** $6\sqrt{6}$ **l** $6\sqrt{6}$
m $14\sqrt{14}$ **n** $18\sqrt{2}$ **o** $6\sqrt{11}$ **p** $15\sqrt{10}$
q $60\sqrt{2}$ **r** $35\sqrt{2}$ **s** $54\sqrt{3}$ **t** $10\sqrt{17}$

6 **a** $\sqrt{12}$ **b** $\sqrt{18}$ **c** $\sqrt{20}$ **d** $\sqrt{54}$
e $\sqrt{32}$ **f** $\sqrt{75}$ **g** $\sqrt{63}$ **h** $\sqrt{50}$
i $\sqrt{72}$ **j** $\sqrt{150}$ **k** $\sqrt{90}$ **l** $\sqrt{112}$
m $\sqrt{252}$ **n** $\sqrt{250}$ **o** $\sqrt{98}$ **p** $\sqrt{200}$
q $\sqrt{243}$ **r** $\sqrt{256}$ **s** $\sqrt{441}$ **t** $\sqrt{432}$

Prep quiz 12:03

1 $2\sqrt{3}$ **2** $2\sqrt{5}$ **3** $4\sqrt{2}$ **4** $5\sqrt{2}$ **5** 5
6 3·6 **7** 2 **8** 3·5 **9** 11 **10** 7·8

Exercise 12:03

1 **a** $5\sqrt{2}$ **b** $11\sqrt{3}$ **c** $7\sqrt{6}$ **d** $3\sqrt{3}$
e $3\sqrt{5}$ **f** $\sqrt{2}$ **g** $5\sqrt{7}$ **h** $6\sqrt{3}$
i $8\sqrt{6}$ **j** $14\sqrt{5}$ **k** $9\sqrt{10}$ **l** $6\sqrt{3}$
m $3\sqrt{2}$ **n** $3\sqrt{3}$ **o** 0

2 **a** $6\sqrt{5}+3\sqrt{7}$ **b** $4\sqrt{7}+2\sqrt{5}$
c $4\sqrt{3}+8\sqrt{5}$ **d** $2\sqrt{3}+5\sqrt{2}$
e $\sqrt{10}+4\sqrt{7}$ **f** $2\sqrt{3}+2\sqrt{5}$
g $8\sqrt{11}+3\sqrt{7}$ **h** $\sqrt{2}+12\sqrt{3}$
i $\sqrt{5}-\sqrt{2}$ **j** $2\sqrt{7}-5\sqrt{5}$

3 **a** $3\sqrt{2}$ **b** $4\sqrt{3}$ **c** $5\sqrt{2}$ **d** $6\sqrt{5}$
e $5\sqrt{3}$ **f** $5\sqrt{6}$ **g** $3\sqrt{2}$ **h** $\sqrt{5}$
i $\sqrt{2}$ **j** $7\sqrt{2}$ **k** $5\sqrt{5}$ **l** $2\sqrt{3}$
m $\sqrt{3}$ **n** $22\sqrt{2}$ **o** $23\sqrt{2}$ **p** $16\sqrt{7}$
q $8\sqrt{5}$ **r** $-2\sqrt{3}$

4 **a** $4\sqrt{2}$ **b** $3\sqrt{5}$ **c** $6\sqrt{3}$
d $6\sqrt{7}$ **e** $3\sqrt{3}+5\sqrt{2}$ **f** $8\sqrt{5}+4\sqrt{2}$
g $18\sqrt{2}+3\sqrt{3}$ **h** $21\sqrt{2}-5\sqrt{3}$

Exercise 12:04

1 **a** $\sqrt{6}$ **b** $\sqrt{35}$ **c** $\sqrt{33}$ **d** $\sqrt{10}$
e $\sqrt{21}$ **f** $\sqrt{70}$ **g** 4 **h** 6
i 10 **j** $3\sqrt{2}$ **k** $5\sqrt{2}$ **l** $2\sqrt{5}$
m $2\sqrt{15}$ **n** $12\sqrt{10}$ **o** $14\sqrt{6}$ **p** 20
q 18 **r** 20 **s** $24\sqrt{5}$ **t** $8\sqrt{3}$
u $24\sqrt{5}$ **v** $6x$ **w** $4\sqrt{x}$ **x** a^2x

2 **a** $\sqrt{5}$ **b** $\sqrt{3}$ **c** $\sqrt{2}$ **d** 3
e 2 **f** 3 **g** 5 **h** 6
i 2 **j** $4\sqrt{2}$ **k** $3\sqrt{5}$ **l** $\sqrt{7}$
m $6\sqrt{2}$ **n** $3\sqrt{2}$ **o** $2\sqrt{3}$ **p** $\sqrt{5}$
q $\sqrt{3}$ **r** $\sqrt{2}$ **s** $\sqrt{x}$ **t** $\sqrt{5}$
u $\sqrt{5}$

3 **a** $3\sqrt{2}$ **b** $8\sqrt{3}$ **c** $\sqrt{2}$
d 1 **e** $\sqrt{6}$ **f** $6\sqrt{2}$
g $\sqrt{2}$ **h** 3 **i** $\dfrac{9}{4\sqrt{3}}$

4 **a** $\sqrt{6}+2$ **b** $5+\sqrt{10}$ **c** $14-\sqrt{14}$
d $5\sqrt{3}-3$ **e** $2\sqrt{6}-\sqrt{2}$ **f** $10\sqrt{5}-4\sqrt{10}$
g $4+2\sqrt{2}$ **h** $15+3\sqrt{10}$ **i** $4\sqrt{6}-12$
j $3\sqrt{30}+18$ **k** $14-2\sqrt{14}$ **l** $7\sqrt{3}-9$
m $2\sqrt{6}+8$ **n** $4\sqrt{10}-40$ **o** $60-30\sqrt{3}$
p $a+\sqrt{a}$ **q** $2x+3\sqrt{x}$ **r** $6y+4\sqrt{xy}$

Prep quiz 12:05

1 $\sqrt{15}$ **2** 6 **3** 5
4 18 **5** $10\sqrt{2}$ **6** $8\sqrt{3}$
7 0 **8** $5\sqrt{7}-20$
9 $10+\sqrt{30}$ **10** $6\sqrt{6}-6$

Exercise 12:05

1 **a** $5+4\sqrt{2}$ **b** $4\sqrt{3}-2$
c $17-7\sqrt{7}$ **d** $2+\sqrt{6}+\sqrt{10}+\sqrt{15}$
e $\sqrt{35}-\sqrt{10}-\sqrt{14}+2$
f $5\sqrt{2}+\sqrt{10}+\sqrt{30}+\sqrt{6}$
g $5\sqrt{3}+5\sqrt{2}+\sqrt{6}+2$
h $4-2\sqrt{5}-2\sqrt{3}+\sqrt{15}$
i $5\sqrt{6}-6-5\sqrt{2}+2\sqrt{3}$
j $4+\sqrt{2}+2\sqrt{6}+\sqrt{3}$
k $13+3\sqrt{15}$ **l** $10+5\sqrt{6}$
m $7\sqrt{14}-27$ **n** $9\sqrt{15}-5$
o $26\sqrt{14}-45$ **p** $-21\sqrt{21}-40$
q $114+21\sqrt{70}$ **r** $77+39\sqrt{6}$
s $66+14\sqrt{6}$ **t** $42-27\sqrt{7}$
u $32\sqrt{3}-1$ **v** $x+5\sqrt{x}+6$
w $2m+3\sqrt{mn}+n$ **x** $6a+5\sqrt{ab}-6b$

2 **a** $3+2\sqrt{2}$ **b** $28-10\sqrt{3}$
c $9+4\sqrt{5}$ **d** $5+2\sqrt{6}$
e $7-2\sqrt{10}$ **f** $13+2\sqrt{30}$
g $13+4\sqrt{3}$ **h** $34-24\sqrt{2}$
i $45+20\sqrt{5}$ **j** $15+4\sqrt{14}$
k $55+30\sqrt{2}$ **l** $52-6\sqrt{35}$
m $83+20\sqrt{6}$ **n** $167-28\sqrt{15}$
o $550-100\sqrt{30}$ **p** $x+2\sqrt{xy}+y$
q $4m+20\sqrt{m}+25$ **r** $9p-12\sqrt{pq}+4q$

3 **a** 1 **b** 22 **c** -39 **d** 14
e -1 **f** 2 **g** 2 **h** 4
i -13 **j** 18 **k** -5 **l** 42
m 38 **n** -6 **o** $x-y$ **p** $4a-9b$

Prep quiz 12:06

1 3 **2** 5 **3** 10 **4** 6
5 10 **6** 12 **7** 35 **8** $\sqrt{15}$
9 $2\sqrt{3}$ **10** $10+5\sqrt{5}$

Exercise 12:06

1 **a** $\dfrac{\sqrt{2}}{2}$ **b** $\dfrac{\sqrt{5}}{5}$ **c** $\dfrac{2\sqrt{3}}{3}$
d $\dfrac{\sqrt{10}}{2}$ **e** $\dfrac{3\sqrt{2}}{2}$ **f** $2\sqrt{3}$
g $2\sqrt{5}$ **h** $\dfrac{2\sqrt{11}}{11}$ **i** $\dfrac{\sqrt{6}}{3}$
j $\dfrac{\sqrt{15}}{5}$ **k** $\dfrac{\sqrt{2}}{2}$ **l** $\dfrac{\sqrt{5}}{5}$
m $\dfrac{\sqrt{2}}{4}$ **n** $\dfrac{2\sqrt{3}}{15}$ **o** $\dfrac{7\sqrt{5}}{10}$
p $\dfrac{5\sqrt{3}}{3}$ **q** $\dfrac{\sqrt{2}}{2}$ **r** $\dfrac{\sqrt{10}}{10}$

s $\frac{\sqrt{6}}{3}$ t $\frac{\sqrt{35}}{3}$ u $\frac{2\sqrt{3}+3}{3}$

v $\frac{\sqrt{2}+\sqrt{10}}{2}$ w $\frac{7+\sqrt{21}}{14}$ x $\frac{2-\sqrt{2}}{10}$

2 a $\frac{1-\sqrt{2}}{-1} = \sqrt{2}-1$ b $\frac{\sqrt{3}+1}{2}$

c $\frac{\sqrt{7}+\sqrt{5}}{2}$ d $\frac{\sqrt{10}-\sqrt{2}}{8}$

e $\frac{3\sqrt{3}-6}{-1} = -3\sqrt{3}+6$ f $\frac{25+5\sqrt{2}}{23}$

g $\frac{10\sqrt{5}+10\sqrt{2}}{3}$ h $3\sqrt{7}+3\sqrt{3}$

i $\frac{2\sqrt{3}-5}{-13}$ j $\frac{10+4\sqrt{2}}{17}$

k $\frac{3\sqrt{2}-2\sqrt{3}}{2}$ l $\frac{4\sqrt{3}+3\sqrt{2}}{30}$

m $\frac{27+10\sqrt{2}}{23}$ n $\frac{19+8\sqrt{3}}{13}$

o $4-\sqrt{15}$ p $\frac{7-2\sqrt{6}}{5}$

3 a $0{\cdot}894, \frac{2\sqrt{5}}{5}$ b $1{\cdot}13, \frac{3\sqrt{7}}{7}$

c $0{\cdot}612, \frac{\sqrt{6}}{4}$ d $0{\cdot}394, \frac{\sqrt{35}}{15}$

e $2{\cdot}44, \frac{\sqrt{7}+\sqrt{5}}{2}$ f $0{\cdot}551, 3-\sqrt{6}$

g $-11{\cdot}9, -6-\sqrt{35}$ h $0{\cdot}134, \frac{19-4\sqrt{21}}{5}$

4 a $\frac{3\sqrt{2}+2\sqrt{3}}{6}$ b $\frac{2\sqrt{5}-5\sqrt{2}}{10}$

c $\frac{5\sqrt{6}+6\sqrt{5}}{30}$ d $\frac{\sqrt{10}-3\sqrt{5}}{5}$

e $\frac{13\sqrt{2}}{4}$ f $\frac{2\sqrt{3}-\sqrt{2}}{6}$

g $\frac{2\sqrt{2}+5\sqrt{10}}{10}$ h $\frac{3\sqrt{10}+2\sqrt{15}}{6}$

i $\frac{2\sqrt{10}-5\sqrt{6}}{10}$ j 4

k $\frac{2\sqrt{5}+\sqrt{3}-\sqrt{7}}{2}$ l $\frac{105+\sqrt{3}}{66}$

Prep quiz 12:07

1 8 2 4 3 2 4 3 5 $a^1 = a$

6 $x^1 = x$ 7 $m^1 = m$ 8 $n^1 = n$ 9 $n = \frac{1}{2}$ 10 $n = \frac{1}{3}$

Exercise 12:07

1 a $\sqrt{6}$ b $\sqrt[3]{4}$ c $\sqrt[4]{9}$ d $\sqrt[5]{10}$

e $2\sqrt{3}$ f $5\sqrt{2}$ g $7\sqrt[3]{4}$ h $9\sqrt[4]{5}$

2 a $5^{\frac{1}{2}}$ b $20^{\frac{1}{2}}$ c $7^{\frac{1}{3}}$ d $9^{\frac{1}{5}}$

e $3 \times 2^{\frac{1}{2}}$ f $4 \times 5^{\frac{1}{2}}$ g $2 \times 4^{\frac{1}{3}}$ h $5 \times 2^{\frac{1}{4}}$

3 a 7 b 2 c 5 d 2

e 10 f 10 g 12 h 11

i 4 j 2 k 3 l 2

4 a m b a c y^2 d $12x$

e $18n$ f $8x^\circ = 8$ g x h y^2

i $3a^2$ j $2x^2$ k a^2b l $2nm^3$

m $2xy$ n $4xy^2$

5 a $\frac{1}{3}$ b $\frac{1}{5}$ c $\frac{1}{2}$ d 27

e 8 f 32 g 8 h 25

i $\frac{1}{4}$ j 243 k 16 l $\frac{1}{8}$

6 a 15 b 28 c 32 d 9

e 15 f 20 g 3375 h 81

i 3 200 000 j 4 k 32 l 1000

7 a 4 b 2 c 54 d 3

e 1 f $\frac{1}{2}$ g $\frac{1}{2}$ h 4

8 a $9a^2$ b x^4y^8 c $4m^6$ d $\frac{a^2}{b^2}$

e $\frac{8}{x^3}$ f $\frac{y^9}{125}$

Diagnostic test 12

1 a irrational b rational c irrational d rational

2 a 2·236 b 3·606 c 4·583 d 6·856

3 a $2\sqrt{5}$ b $3\sqrt{3}$ c $6\sqrt{2}$ d $10\sqrt{3}$

4 a $\sqrt{20}$ b $\sqrt{18}$ c $\sqrt{175}$ d $\sqrt{80}$

5 a $6\sqrt{3}$ b $5\sqrt{5}$ c $\sqrt{2}$ d $5\sqrt{3}$

6 a $\sqrt{30}$ b 6 c $2\sqrt{15}$ d 24

7 a $\sqrt{6}$ b 2 c 5 d $5\sqrt{2}$

8 a $11+5\sqrt{5}$ b $-5\sqrt{6}$ c $10+2\sqrt{21}$ d 22

9 a $\frac{3\sqrt{2}}{2}$ b $\sqrt{5}$ c $\frac{3+\sqrt{3}}{6}$ d $\frac{5\sqrt{2}-2}{10}$

10 a $\frac{5-\sqrt{2}}{23}$ b $3+\sqrt{6}$ c $\frac{7-2\sqrt{10}}{3}$

d $\frac{31+5\sqrt{35}}{43}$

11 a 5 b 3 c 2 d $\frac{1}{4}$

12 a $12x$ b $7x^3$ c $4x^2$ d $32x^5$

ASSIGNMENT 12A

1 a 3 and 4 b 6 and 7 c 9 and 10

d 11 and 12 e 20 and 21 f 14 and 15

g 3 and 4 h 7 and 8 i 9 and 10

2 a $9\sqrt{6}$ b $13\sqrt{3}$ c $2\sqrt{5}$

d 60 e $15\sqrt{2}$ f $2\sqrt{7}$

3 **a** $5\sqrt{5}-5$ **b** $12+2\sqrt{6}$ **c** 0
d $9+2\sqrt{14}$ **e** 3
f $5+2\sqrt{3}+\sqrt{10}+\sqrt{30}$

4 **a** 8 **b** 4 **c** $\frac{1}{25}$
d $\frac{1}{81}$ **e** 4 **f** 12
g 15 **h** 6

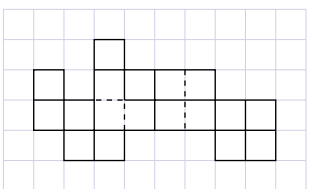
5 **a** 2·74 **b** 15·71 **c** 11·21
d 47·33 **e** 0·732 **f** 23·24

6 **a** $\frac{5\sqrt{7}}{7}$, 1·890 **b** $\frac{\sqrt{15}}{3}$, 1·291
c $\frac{3-\sqrt{3}}{3}$, 0·423 **d** $\frac{2\sqrt{6}+3}{3}$, 2·633

7 **a** $\frac{\sqrt{5}+1}{2}$ **b** $\frac{\sqrt{7}-\sqrt{2}}{5}$
c $9-4\sqrt{5}$ **d** $-2\sqrt{6}-5$

ASSIGNMENT 12B

1 5 **2** 8 days **3** Cut along lines shown.



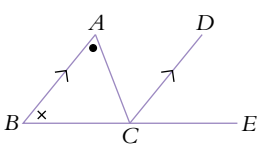
4 Starting from the top left and going clockwise, the missing numbers are 2, 1, 3, and 6.

5 8 girls

Chapter 13

Prep quiz 13:01

1 and **2**



3 $\angle DCE$ **4** $\angle ACD$ **5** $\angle ACD$
6 $\angle ACD$ and $\angle DCE$ **7** b
8 $180-a$ **9** c **10** $a=c$

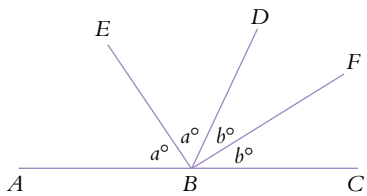
Exercise 13:01

1 **a** $\angle AEF = x°$ (vert. opp. $\angle$s)
$\angle AEF = y°$ (corresp. $\angle$s, $\parallel$ lines)
$\therefore\ x = y$

b $\angle DBC = x°$ (vert. opp. $\angle$s)
$\angle BDE = 180° - x°$ (co-int. $\angle$s, $BC \parallel DE$)
$\angle BDE = 180° - y°$ (co-int. $\angle$s, $DB \parallel EC$)
$\therefore\ 180 - x = 180 - y$
$\therefore\ x = y$

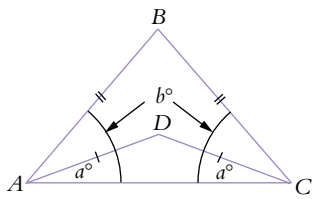
c $\angle EFG = y°$ (equal $\angle$ of isos. Δ)
$\angle EFG = x°$ (alt. $\angle$, $AB \parallel CD$)
$\therefore\ x = y$
Note: Other proofs may exist for exercises in deductive reasoning.

2



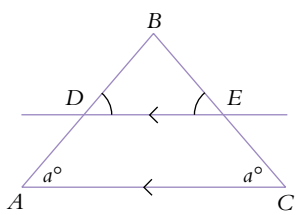
Let $\angle ABD$ be $2a°$ and $\angle DBC$ be $2b°$.
$\therefore\ \angle ABC = 2a° + 2b°$ ($\angle ABD + \angle DBC$)
$2(a + b)° = 180°$ (ABC is a straight angle)
$(a + b) = 90°$
Now $\angle EBD = a°$ (BE bisects $\angle ABD$ given)
$\angle DBF = b°$ (BF bisects $\angle DBC$ given)
$\therefore\ \angle EBF = (a + b)° = 90°$ ($\angle EBD + \angle DBF$)
$\therefore\ EB \perp BF$

3



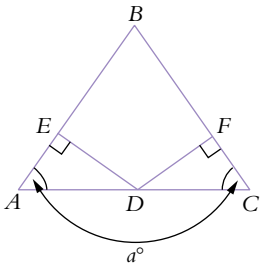
Let $\angle DAC$ and $\angle DCA$ be $a°$
(base angles of isos. ΔADC).
Let $\angle BAC$ and $\angle BCA$ be $b°$
(base angles of isos. ΔABC).
$\therefore\ \angle BAD = b° - a°$ ($\angle BAC - \angle DAC$)
$\angle BCD = b° - a°$ ($\angle BCA - \angle DCA$)
$\therefore\ \angle BAD = \angle BCD$

4



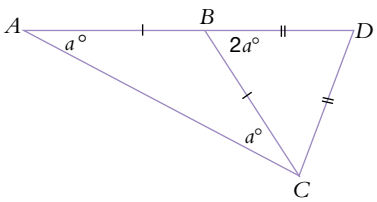
Let $\angle BAC$ and $\angle BCA$ be $a°$
(base angles of isos. ΔABC).
$\angle BDE = a°$ (corresp. to $\angle BAC$, $DE \parallel AC$)
$\angle BED = a°$ (corresp. to $\angle BCA$, $DE \parallel AC$)
$\therefore\ \angle BDE = \angle BED$ (i.e. base angles of ΔDBE are equal)
$\therefore\ \Delta DBE$ is isosceles

5



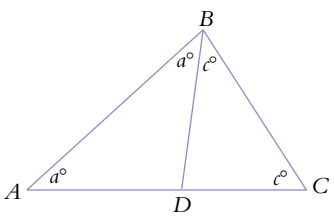
Let $\angle BAD$ and $\angle BCD$ be $a°$
(base angles of isos. ΔABC).
$\angle AED = \angle CFD = 90°$ ($DE \perp AB$, $DF \perp BC$)
$\angle EDA = 180° - a° - 90°$ (angle sum of ΔAED)
$\angle FDC = 180° - a° - 90°$ (angle sum of ΔDFC)
$\therefore \angle EDA = \angle FDC$

6



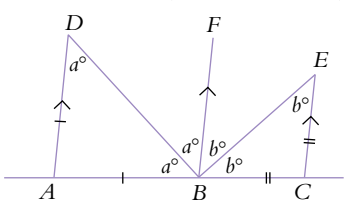
Let $\angle BAC$ and $\angle BCA$ be $a°$
(base angles of isos. ΔABC).
$\therefore \angle DBC = 2a°$ (ext. angle of ΔABC)
$\angle DCB = 2a°$ (base angles of isos. ΔBDC)
$\therefore \angle BCD = 2 \times \angle BCA$

7



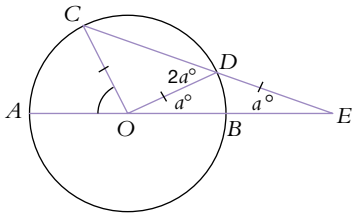
Let $\angle BAD$ be $a°$ and $\angle BCD$ be $c°$.
$\therefore \angle ABD = a°$ (base angles of isos. ΔABD)
$\angle DBC = c°$ (base angles of isos. ΔBCD)
$2a° + 2c° = 180°$ (angle sum of ΔABC)
$a° + c° = 90°$ (dividing both sides by 2)
$\therefore \angle ABC = 90°$ ($\angle ABD + \angle DBC$)

8



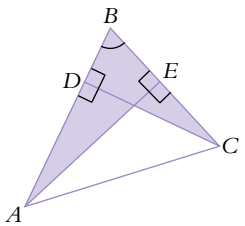
Construction: Draw $BF \parallel AD$ and CE.
Let $\angle ABD$ be $a°$ and $\angle CBE$ be $b°$.
$\angle ADB = a°$ (base angles of isos. ΔADB)
$\angle CEB = b°$ (base angles of isos. ΔBCE)
$\angle FBD = a°$ (alt. to $\angle ADB$, $BF \parallel AD$)
$\angle FBE = b°$ (alt. to $\angle CEB$, $BF \parallel CE$)
$2a° + 2b° = 180°$ ($\angle ABC$ is straight)
$\therefore a° + b° = 90°$ (dividing both sides by 2)
$\angle DBE = 90°$ ($\angle FBD + \angle FBE$)

9



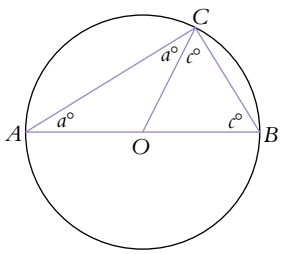
Let $\angle DOB$ and $\angle DEO$ be $a°$
(base angles of isos. ΔODE).
$\angle CDO = 2a°$ (ext. angle of ΔODE)
$\angle OCD = 2a°$ (base angles of isos. ΔOCD, radii equal)
$\therefore \angle AOC = \angle DEO + \angle OCD$ (ext. angle of ΔOCE)
$\therefore \angle AOC = a° + 2a° = 3a°$
$\therefore \angle AOC = 3 \times \angle DOB$

10



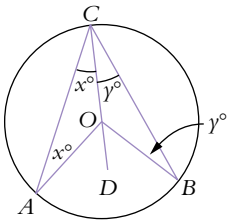
Let $\angle BAE = x°$, $\angle ABE = y°$ and $\angle BCD = z°$.
$x + y = 90$ (comp. $\angle$s in ΔABE)
$z + y = 90$ (comp. $\angle$s in ΔBCD)
$\therefore x = z$
$\therefore \angle BAE = \angle BCD$

11



Construction: Draw interval OC
$OA = OC = OB$ (radii of circle)
Let $\angle OAC = a°$ and $\angle OBC = c°$.
$\angle ACO = a°$ (base angles of isos. ΔACO)
$\angle BCO = c°$ (base angles of isos. ΔBCO)
$2a° + 2c° = 180°$ (angle sum of ΔABC)
$a° + c° = 90°$
$\therefore \angle ACB = 90°$ ($\angle ACO + \angle BCO$)

12



Construction: Join CO and extend it to D.
$OA = OB = OC$ (radii of circle)
Let $\angle ACO = x°$ and $\angle BCO = y°$.
$\angle ACB = \angle ACO + \angle BCO$
$= x° + y°$
$\angle CAO = x°$ (base angles of isos. ΔOAC)
$\angle AOD = 2x°$ (ext. angle of ΔOAC)
Similarly,
$\angle CBO = y°$ and $\angle BOD = 2y°$
$\angle AOB = \angle AOD + \angle BOD$
$= 2x° + 2y°$
$= 2(x° + y°)$
$= 2 \times \angle ACB$

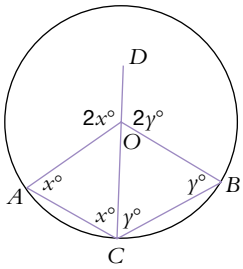
13 $\angle BCE = \beta$ (alternate to $\angle ABC$, $AB \parallel CE$)
$\angle ECD = \alpha$ (corresponding to $\angle BAC$, $AB \parallel CE$)
$\angle BCA = \gamma$ (given)
$\therefore \alpha + \beta + \gamma = 180°$ ($\angle ACD$ is a straight angle)
$\therefore$ the angle sum of a triangle is 180°.

14 Now $a + b + c = 180°$ (angle sum of ΔACD)
and $d + e + f = 180°$ (angle sum of ΔBCD)
$\angle A + \angle B + \angle C + \angle D = b + (c + d) + e + (a + f)$
$= (a + b + c) + (d + e + f)$
$= 180° + 180°$
$= 360°$

15 $\angle BCE = \beta$ (alt. $\angle$s, $AB \parallel CE$)
$\angle ECD = \alpha$ (corresp. $\angle$s, $AB \parallel CE$)
$\therefore \angle BCD = \angle BCE + \angle ECD$
$= \alpha + \beta$

16 $x° + c° = 180°$ (adj. $\angle$s on a st. line)
$y° + b° = 180°$ (adj. $\angle$s on a st. line)
$z° + a° = 180°$ (adj. $\angle$s on a st. line)
$(x° + c°) + (y° + b°) + (z° + a°) = 540°$
$(x° + y° + z°) + (a° + b° + c°) = 540°$
But $x° + y° + z° = 180°$ ($\angle$ sum of a Δ)
$\therefore 180° + (a° + b° + c°) = 540°$
$\therefore a° + b° + c° = 360°$

17



Produce CO to D
$AO = BO = CO$ (radii of circle)
Let $\angle OCA = x°$ and $\angle OCB = y°$.
$\therefore \angle ACB = x° + y°$
$\angle OAC = x°$ (base $\angle$s of isos. ΔOAC)
$\angle AOD = 2x°$ (ext. $\angle$ of ΔAOC)
$\angle OBC = y°$ (base $\angle$s of isos. ΔOBC)
$\angle BOD = 2y°$ (ext. $\angle$ of ΔBOC)
Reflex $\angle AOB = \angle AOD + \angle BOD$
$= 2x° + 2y°$
$= 2(x° + y°)$
$= 2 \times \angle ACB$

Exercise 13:02

1 a In Δs ABC and DBE
(1) $\angle CAB = \angle EDB$ (data)
(2) $\angle ABC = \angle BDE$ (vert. opp. $\angle$s)
$\therefore \Delta ABC \,|||\, \Delta DBE$ (2 pairs of equal $\angle$s)
$\therefore \frac{x}{4} = \frac{6}{5}$ (ratio of matching sides are equal)
$\therefore x = 4{\cdot}8$

b In Δs ABC and DEC
(1) $\angle ACB = \angle DCE$ (vert. opp. $\angle$s)
(2) $\frac{AC}{DC} = \frac{BC}{EC} = \frac{26}{16}$ (data)
$\therefore \Delta ABC \,|||\, \Delta DEC$
(1 equal angle and sides about the angle are in same ratio)
$\therefore \frac{x}{44} = \frac{16}{26}$ (ratio of matching sides)
$\therefore x = 27{\cdot}1$ (1 dec. pl.)

c In Δs EFG and JHG
$\frac{JG}{EG} = \frac{JH}{EF} = \frac{GH}{GF} = \frac{7}{5}$
$\therefore \Delta EFG \,|||\, \Delta JHG$
(matching sides in same ratio)
$\therefore x = 83°$ (matching $\angle$s of sim. Δs)

d In Δs ABC and AED
$\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{CB} = \frac{4}{3}$
$\therefore \Delta ABC \,|||\, \Delta AED$ (matching sides in same ratio)
$\therefore a = 53°$ (matching $\angle$s of sim. Δs)

e In Δs AEB and DCB

(1) $\angle AEB = \angle BCD$ (data)

(2) $\angle ABE = \angle DBC$ (vert. opp. $\angle$s)

$\therefore \Delta AEB \,|||\, \Delta DCB$ (2 pairs of equal $\angle$s)

$\therefore \frac{y}{21} = \frac{34}{24}$ (ratio of matching sides are equal)

$\therefore y = 29{\cdot}75$

2 a Two angles of one are equal to two angles of the other.

$\frac{e}{3{\cdot}8} = \frac{f}{1{\cdot}9} = \frac{3{\cdot}8}{4{\cdot}6}$ (matching sides in the same ratio)

$\therefore e = 3{\cdot}1, f = 1{\cdot}6$

b Two angles of one are equal to two angles of the other.

$\frac{x}{35} = \frac{30}{45}$ (matching sides in the same ratio)

$x = 23\frac{1}{3}$

c Two pairs of sides are in the same ratio and the angles included by these sides are equal.

$\frac{x}{24} = \frac{6}{5}$ (matching sides in the same ratio)

$x = 28{\cdot}8$

d Both triangles are right-angled and the ratio of hypotenuse to one side in one equals the ratio of hypotenuse to one side in the other triangle.

$h = 42$ (matching $\angle$s of sim. Δs)

e Two angles of one are equal to two angles of the other.

$a = 150$

$b = 282$

f Two angles of one are equal to two angles of the other.

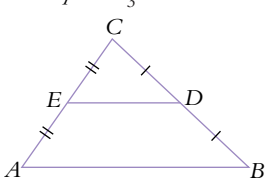
$\frac{x}{17} = \frac{50}{30}$ (matching sides in the same ratio)

$\therefore x = 28\frac{1}{3}$

$\frac{30 + y}{25} = \frac{50}{30}$

$\therefore y = 11\frac{2}{3}$

3



In Δs ECD and ACB

(1) $\angle ECD = \angle ACB$ (common)

(2) $\frac{EC}{AC} = \frac{DC}{BC} = \frac{1}{2}$ (data)

$\therefore \Delta ECD \,|||\, \Delta ACB$ (1 equal angle and sides about the angle are in the same ratio)

$\therefore \frac{ED}{AB} = \frac{1}{2}$ (matching sides in the same ratio)

$\therefore ED = \frac{1}{2}AB$

Also $\angle CED = \angle CAB$ (matching $\angle$s of sim. Δs)

But $\angle$s CED and CAB are corresp. $\angle$s

$\therefore ED \parallel AB$ (corresp. $\angle$s are equal)

4 a $x = 19$ (line through midpoint $\parallel$ to another side bisects third side)

b $x = 21$ (interval joining midpoints is $\frac{1}{2}$ third side)

c $x = 15$ (line through midpoint $\parallel$ to another side bisects third side)

$y = 24$ (interval joining midpoints is $\frac{1}{2}$ third side)

5 a **i** $\angle EBD = \angle ABC$ (common)

$\angle BED = \angle BAC$ (corresp. $\angle$s, $ED \parallel AC$)

$\therefore \Delta BED \,|||\, \Delta BAC$ (equiangular)

ii $4:9$ **iii** $15\frac{5}{9}$ **iv** $19\frac{4}{9}$ **v** $4:5$

vi From data and **v** we can see that $\frac{BE}{EA} = \frac{BD}{DC}$

b **i** $3:5$ **ii** $3:5$

iii $\angle EDH = \angle FDG$ (common)

$\frac{DE}{DF} = \frac{DH}{DG}$ (proved in **i** and **ii**)

$\therefore \Delta DEH \,|||\, \Delta DFG$ (sides about an equal angle are in proportion)

iv and **v** Yes, matching $\angle$s of similar Δs.

vi Yes, since $\angle DEH$ and $\angle DFG$ are equal corresp. angles.

6 a $a = 18\frac{4}{7}$ **b** $b = 20\frac{4}{11}$ **c** $c = 25\frac{1}{5}$

7 a In Δs ADE and CBE

(1) $\angle DAE = \angle BCE$ (data)

(2) $\angle AED = \angle CEB$ (vert. opp. $\angle$s)

$\therefore \Delta ADE \,|||\, \Delta CBE$ (equiangular)

$\therefore \frac{AE}{CE} = \frac{ED}{EB} = \frac{AD}{CB}$ (matching side ratios are equal)

$\therefore AE \times EB = CE \times ED$

b Δs ABC and DAC are isosceles with $\angle BAC = \angle ADC$

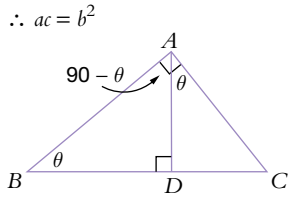
$\therefore$ All base $\angle$s are equal.

$\therefore \Delta ABC \,|||\, \Delta DAC$ (equiangular)

$\therefore \frac{a}{b} = \frac{b}{c}$ (matching side ratios are equal)

$\therefore ac = b^2$

c



$\angle ADC = \angle ADB = 90°$ ($AD \perp BC$)

Let $\angle ABD = \theta$

$\therefore \angle BAD = 90 - \theta$ (comp. $\angle$s)

$\therefore \angle DAC = \theta$ (comp. $\angle$s)

$\therefore \angle ABD = \angle DAC$

$\therefore \Delta ABD \,|||\, \Delta ACD$ (equiangular)

$\therefore \frac{AD}{DC} = \frac{BD}{AD}$

$\therefore \frac{c}{b} = \frac{a}{c}$

$\therefore ab = c^2$

Prep quiz 13:03

1 5 : 3 **2** 25 : 9 **3** $a : b$ **4** $a^2 : b^2$ **5** $R : r$
6 $R^2 : r^2$ **7** $k : 1$ **8** $k^2 : 1$ **9** 3 : 2 **10** 9 : 4

Exercise 13:03

1 **a** 4 : 9 **b** 36 : 49 **c** 25 : 64
d 81 : 100 **e** 16 : 25 **f** 49 : 225

2 **a** $2{\cdot}6\,m^2$ **b** $270\,cm^2$
c $15{\cdot}625\,cm^2$ **d** $200\,m^2$
e $135\,cm^2$ **f** $78{\cdot}1\,m^2$

3 **a** 3 : 5, $x = 15$ **b** 3 : 4, $y = 24$
c 11 : 20, $x = 30$ **d** 4 : 5, $y = 2{\cdot}5$
e 2 : 3, $x = 15$

4 **a** Increased by a factor of 9.
b 4 : 5 **c** 22 500 : 1 **d** 4 : 7

5 $0{\cdot}681\,m^2$ (3 dec. pl.)

6 **a** $418\,cm^2$ **b** 32 cm

7 40 kg

8 **a** 2 : 3 **b** 5 : 3 **c** 9 : 25
d 9 : 16 **e** 25 : 16

Prep quiz 13:04

1 $x : y$ **2** $x : y$ **3** $x^2 : y^2$ **4** $x^2 : y^2$
5 $x^3 : y^3$ **6** $a : b$ **7** $a : b$ **8** $a^2 : b^2$
9 $a^2 : b^2$ **10** $a^3 : b^3$

Exercise 13:04

1 **a** **i** 4 : 9 **ii** 8 : 27
b **i** 16 : 25 **ii** 64 : 125
c **i** 25 : 36 **ii** 125 : 216
d **i** 49 : 64 **ii** 343 : 512
e **i** 9 : 25 **ii** 27 : 125
f **i** 81 : 100 **ii** 729 : 1000

2 **a** $1755\,m^2$ **b** $9056\,m^3$ **c** $19{\cdot}6875\,m^3$
d $140{\cdot}625\,cm^3$ **e** $306{\cdot}25\,m^2$ **f** $75\,m^2$
g $37{\cdot}5\,m^3$ **h** $183\,cm^2$ **i** $209\,cm^3$

3 **a** 5 : 7, 25 : 49 **b** 4 : 5, 16 : 25
c 15 : 16, 225 : 256

4 **a** $123{\cdot}48\,cm^2$ **b** 0·875 m **c** $3{\cdot}23\,m^2$ (2dp)
d 17·1 m (1dp) **e** $198.80\,m^2$ (2dp)

5 **a** $8859{\cdot}375\,cm^3$ **b** 225 mL

6 **a** $6\,750\,000\,cm^3 = 6{\cdot}75\,m^3$
b 125 000 mL = 125 L

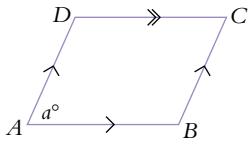
7 **a** 16 units **b** $1125\,cm^3$

8 90 glasses

9 **a** 81 **b** 729

Exercise 13:05

1

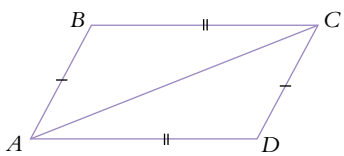


Let $\angle DAB = a°$
$\therefore \angle ADC = 180° - a°$ (co-int. $\angle$s, $AB \parallel DC$)
$\angle ABC = 180° - a°$ (co-int. $\angle$s, $AD \parallel BC$)
Now, $\angle DCB + \angle ABC = 180°$ (co-int. $\angle$s, $AB \parallel DC$)
$\angle DCB = 180° - \angle ABC$
$= 180° - (180° - a°)$
$= a°$
$\therefore \angle DAB = \angle DCB = a°$
and $\angle ABC = \angle ADC = 180° - a°$
$\therefore$ Opposite $\angle$s of par'm are equal.

2 $\angle ADC = 90°$ (co-int. $\angle$s, $AB \parallel CD$)
$\angle ABC = 90°$ (co-int. $\angle$s, $AD \parallel BC$)
$\angle BCD = 90°$ (co-int. $\angle$s, $AB \parallel CD$)

3 **a** Opposite sides of a parallelogram are equal.
b $AB = DC$ (opp. sides of a par'm)
$AD = BC$ (opp. sides of a par'm)
But $AB = AD$ (data)
$\therefore AB = AD = DC = BC$

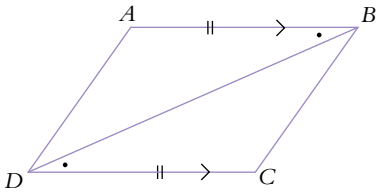
4



In Δs ABC and CDA
(i) $AB = CD$ (given)
(ii) $BC = DA$ (given)
(iii) AC is common
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)
$\angle BCA = \angle CAD$ (matching angles of cong. Δs)
$\therefore BC \parallel DA$ (alt. angles equal)
$\angle BAC = \angle DCA$ (matching angles of cong. Δs)
$\therefore AB \parallel CD$ (alt. angles equal)
$\therefore$ Opposite sides are parallel.
$\therefore ABCD$ is a parallelogram.

5 **a** Vertically opposite $\angle$s
b SAS
c $\angle CBE$ (matching $\angle$s of congruent Δs)
d $\angle ADE$ and $\angle CBE$ are equal alternate $\angle$s.
e SAS
f Matching $\angle$s of congruent Δs.
g Pairs of alternate angles are equal.

6

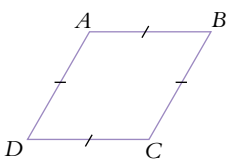


Data: $ABCD$ is a quadrilateral with $AB = DC$ and $AB \parallel DC$.
Aim: To prove $AD \parallel BC$.
Construction: Join BD.
Proof: In Δs ABD and CBD
(1) $AB = DC$ (data)
(2) BD is common.
(3) $\angle ABD = \angle CDB$ (alt. $\angle$s, $AB \parallel DC$)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)
$\therefore \angle ADB = \angle CBD$ (matching $\angle$s of cong't Δs)
But $\angle ADB$ and $\angle CBD$ are also alternate $\angle$s
$\therefore AD \parallel BC$ (pair of alt. $\angle$s are equal)
$\therefore ABCD$ has both pairs of opposite sides parallel.
$\therefore ABCD$ is a parallelogram.

7

Proof: $ABCD$ is a parallelogram (opposite $\angle$s equal)
$4a° = 360°$ (angle sum of quad.)
$a = 90$
$\therefore ABCD$ is a parallelogram with one right angle.
$\therefore ABCD$ is a rectangle.

8



Proof: $ABCD$ is a parallelogram.
(Both pairs of opposite sides are equal.)
$AB = BC$ (data)
$\therefore ABCD$ is a parallelogram with a pair of adjacent sides equal.
$\therefore ABCD$ is a rhombus. (See definition p. 369.)

9 **1** quadrilateral **2** trapezium **3** parallelogram
4 rectangle **5** square **6** rhombus

10

Properties	Parallelogram	Rhombus	Rectangle	Square
1 Opposite sides parallel	yes	yes	yes	yes
2 Opposite sides equal	yes	yes	yes	yes
3 Opposite angles equal	yes	yes	yes	yes
4 Diagonals bisect one another	yes	yes	yes	yes
5 All sides equal	no	yes	no	yes
6 All angles right angles	no	no	yes	yes
7 Diagonals perpendicular	no	yes	no	yes
8 Diagonals bisect angles through which they pass	no	yes	no	yes
9 Diagonals are equal	no	no	yes	yes

11 The numbers refer to the table in Question **10**.
a 1, 2, 3, 4, 5, 7, 8 **b** 1, 2, 3, 4, 6, 9
c 1, 2, 3, 4 **d** 1, 2, 3, 4

12 **a** all sides equal; diagonals bisect angles through which they pass; diagonals are perpendicular
b all angles right angles; diagonals are equal
c all angles are right angles; diagonals are equal

13 **a** Yes **b** Yes
c No, because it does not have all its sides equal.
d Yes **e** Yes
f No, because it does not have all its angles equal.

14 **a** Parallelogram; both pairs of opposite sides are equal.
b Rectangle; diagonals are equal and bisect each other.
c Rhombus; all sides are equal.
d Rectangle; all angles are equal.
e Rhombus; diagonals bisect each other at right angles.
f Parallelogram; one pair of sides is both equal and parallel.

15 **a** Proof: $DC = AB$ (opp. sides of par'm $ABCD$)
$AB = FE$ (opp. sides of par'm $ABFE$)
$\therefore DC = FE$
Also $DC \parallel AB$ and $AB \parallel FE$
$\therefore DC \parallel FE$
$\therefore DC$ and FE are parallel and equal.
$\therefore DCEF$ is a parallelogram.
b Proof: $AK = KC$ (diag. of par'm bisect each other)
$EA = CG$ (data)
$\therefore AK + EA = KC + CG$
$EK = KG$
Similarly it can be shown that $KH = KF$.
$\therefore$ Diagonals EG and FH bisect each other.
$\therefore EFGH$ is a parallelogram.

Investigation 13:05

1 If a triangle has all angles equal then it has all its sides equal. (This converse is true.)
2 If a quadrilateral has its diagonals equal then it is a square. (This converse is not true.)
3 If a quadrilateral has its opposite angles equal then it is a parallelogram. (This converse is true.)
4 If a quadrilateral has all its sides equal then it is a square. (This converse is not true.)
5 If the square on the longest side of the triangle is equal to the sum of the squares on the two smaller sides then the triangle is right-angled. (This converse is true.)

Diagnostic test 13

1 a $\angle DBA = x°$ (alt. $\angle$s, $AB \parallel CD$)
$\angle DBA = y°$ (alt. $\angle$s, $AC \parallel BD$)
$\therefore x = y$

b $\angle CBD = \angle CDB$ (base $\angle$s of isos. Δ)
$\therefore \angle ABD = 90° - \angle CBD$
$\angle ADB = 90° - \angle CDB$
$\therefore \angle ABD = \angle ADB$
$\therefore AB = AD$ (equal sides of isos. Δ)

c Let $\angle ABC = \angle ACD = x°$
$\therefore \angle CAD = 90° - x°$ (comp. $\angle$s, ΔADC)
$\therefore \angle ATE = x°$ (comp. $\angle$s, ΔATE)
$\therefore \angle FTD = x°$ (vert. opp. $\angle$s)
$\therefore \angle ABC = \angle FTD$

2 a In Δs ABC and DEC
(1) $\angle BCA = \angle ECD$ (vert. opp. $\angle$s)
(2) $\angle ABC = \angle DEC$ (alt. $\angle$s $AB \parallel DE$)
$\therefore \Delta ABC \,|||\, \Delta DEC$ (2 pairs of equal $\angle$s)

b In Δs ADE and ABC
(1) $\angle ADE = \angle ABC$ (corresp. $\angle$s, $DE \parallel BC$)
(2) $\angle DAE = \angle BAC$ (common)
$\therefore \Delta ADE \,|||\, \Delta ABC$ (2 pairs of equal $\angle$s)

c $\angle DAC = 40°$ ($\angle$ sum of Δ)
$\angle ACB = 40°$ (data)
$\therefore \angle DAC = \angle ACB$
In Δs DAC and ACB
(1) $\angle DAC = \angle ACB$ (proved above)
(2) $\angle ADC = \angle CAB$ (data)
$\therefore \Delta DAC \,|||\, \Delta ACB$ (equiangular)

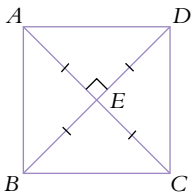
3 a $x = 9{\cdot}6$ **b** $x = 6\frac{2}{3}$ **c** $x = 7{\cdot}5$

4 a **i** 100 : 49 **ii** 550 m^2
b **i** 25 : 64 **ii** 200 m^2
c **i** 9 : 10 **ii** 20 mm

5 a **i** 9 : 4 **ii** 27 : 8 **iii** 6750 m^3
b **i** 1 : 2 **ii** 1 : 8 **iii** 145·6 cm^3
c 114·8175 m^3

6 a See Exercise 13:05 Question **7**

b



$ABCD$ is a rectangle
(diagonals are equal and bisect each other)
$\Delta ADE \equiv \Delta DCE$ (SAS)
$\therefore AD = DC$ (matching sides of cong't Δs)
$\therefore ABCD$ is a square
(rectangle with a pair of adjacent sides equal)

c One pair of sides is both equal and parallel.

ASSIGNMENT 13A

1 $\angle CAB = x°$ (equal $\angle$ of isos. Δ)
$\angle CAB = y°$ (alt. $\angle$s, $AB \parallel DC$)
$\therefore x = y$

2 $AF = EC = 5$ (data)
$AF \parallel EC$ ($AB \parallel DC$)
$\therefore AFCE$ is a parallelogram (One pair of sides is both equal and parallel).
$\therefore AE \parallel FC$

3 a $x = 25{\cdot}2$, $y = 19{\cdot}6$ **b** $x = 21$
c $x = 5{\cdot}7$, $y = 6{\cdot}9$

4 a 3 m^2 **b** 270 m^3 **c** 8·6 cm

5 a SAS
b Matching sides of congruent triangles.
c Rhombus
d **i** 45° **ii** 45° **iii** 90°
e $ABDC$ is a square as it is a rhombus with one angle a right angle.

6 a In Δ's ADE and ABC
$\angle CAB = \angle EAD$ (common angle)
$\angle ACB = \angle AED$ (corresp. $\angle$s, $AB \parallel DC$)
$\therefore \Delta$'s ADE and ABC are similar
(Two angles of one triangle are equal to two angles of the other.)

b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$

c D and E are the midpoints of AB and AC, respectively.

ASSIGNMENT 13B

1 a 1 : 17 000 **b** 170 m **c** about 221 m
d about 136 m

2 $a = 108$, $b = 36$, $c = 36$ $d = 72$

3 a $2\frac{2}{3}$ days **b** 8 men

4 a The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.
b The hypotenuse of a right-angled triangle must be the largest side.

5 The 40° and 20° angles cannot be correct. The angle that is marked 20° must be larger than the angle that is marked 40°.

6 83

Chapter 14

Exercise 14:01

1 **a** 5 **b** 10 **c** 5 **d** 10
e 13 **f** 13 **g** $\sqrt{20}$ **h** $\sqrt{5}$
i $\sqrt{89}$ **j** $\sqrt{17}$

2 **a** $(5\frac{1}{2}, 4)$ **b** $(4, 4)$ **c** $(-4, 2\frac{1}{2})$ **d** $(1, 0)$
e $(0, 4\frac{1}{2})$ **f** $(1\frac{1}{2}, 3)$ **g** $(4, -2)$ **h** $(7\frac{1}{2}, 1)$
i $(2, 1\frac{1}{2})$ **j** $(-5, -2\frac{1}{2})$

3 **a** $\frac{4}{3}$ **b** $\frac{3}{4}$ **c** $-\frac{3}{4}$ **d** $\frac{4}{3}$
e $\frac{5}{12}$ **f** $\frac{12}{5}$ **g** -2 **h** 2
i $-\frac{5}{8}$ **j** $\frac{1}{4}$

4 **a** **i** $m = \frac{11-1}{5-3} = 5$
ii $m = \frac{1-11}{3-5} = 5$
b yes

5 The values of m are equal.

6 **a** $AB = 5$ units; $BC = \sqrt{50}$ units; $AC = \sqrt{41}$ units
b $AB = \sqrt{13}$ units; $DC = \sqrt{13}$ units; $AD = \sqrt{5}$ units; $BC = \sqrt{5}$ units
c 5
d $AB = \sqrt{40}$ units; $BC = \sqrt{40}$ units. Since two sides are equal, the triangle is isosceles.
e **i** $\frac{1}{4}, \frac{1}{4}$
ii $2\frac{1}{2}, 2\frac{1}{2}$
iii $ABCD$ is a parallelogram since it has opposite sides parallel.
f AB and DC have a gradient of $\frac{2}{7}$ and, hence, are parallel. AD and BC have a gradient of 1 and, hence, are parallel. Since there are two pairs of parallel sides, $ABCD$ is a parallelogram.

Prep quiz 14:02

1 3 **2** 5 **3** 11 **4** 0 **5** 4
6 5 **7** 1 **8** 2 **9** 3 **10** 0

Exercise 14:02

1 **a** 1 **b** 2 **c** 5
d −5 **e** −2 **f** −11

2 **a** $y = 2x + 1$ **b** $y = 5x$ **c** $y = 3x - 4$
d $y = 4x + 10$ **e** $y = -x + 6$ **f** $y = -2x + 7$
g $y = -5x + 5$ **h** $y = \frac{1}{2}x + 3$ **i** $y = \frac{1}{4}x + 2$
j $y = -\frac{1}{2}x - 3$

3 **a** $y = 2x - 4$ **b** $y = -x + 3$ **c** $y = 3x + 6$
d $y = -3x + 1$ **e** $y = 3x$ **f** $y = 4x + 2$
g $y = 4x$ **h** $y = -2x - 8$ **i** $y = 2x - 2$
j $y = \frac{1}{2}x + 3\frac{1}{2}$

Exercise 14:03

1 **a** 4 **b** 1 **c** −2
d 5 **e** $\frac{1}{4}$ **f** −1
g 3 **h** 1 **i** −5
j −3

2 **a** $y = 4x - 8$ **b** $y = x + 4$ **c** $y = -2x + 7$
d $y = 5x + 9$ **e** $y = \frac{1}{4}x + 1\frac{1}{2}$ **f** $y = -x + 7$
g $y = 3x$ **h** $y = x$ **i** $y = -5x + 3$
j $y = -3x$

3 **a** $y = 2x + 2$ **b** $y = -3x + 15$ **c** $y = 4x + 4$
d $y = 2x + 4$, $10 = 2(3) + 4$ **e** $y = x + 1$

4 **a** m of $AB = 1$, m of $BC = -3$, m of $AC = -\frac{3}{5}$
b $y = x + 3$, $y = -3x + 7$, $y = -\frac{3}{5}x - \frac{1}{5}$
c $3, 7, -\frac{1}{5}$ **d** $y = 1$ **e** $m = 0, b = 1$

5 **a** $y = -2x + 4$, $-2 = -2(3) + 4$
b Equation of AB is $y = -2x - 2$.
Substitute $C(3, -8)$: $-8 = -2(3) - 2$.
$\therefore$ C lies on AB
c Equation of line joining $(-2, -11)$ and $(3, 4)$ is $y = 3x - 5$. Substitute $(4, 7)$: $7 = 3(4) - 5$.
$\therefore$ the points are collinear.

Prep quiz 14:04

1 $y = -4$ **2** $y = x$ **3** $y = -x$
4 $y = 6 - x$ **5** $y = 3 - 2x$ **6** $y = 2x + 1$
7 $2x + 3y + 6 = 0$ **8** $7x - y + 4 = 0$ **9** $4x - y + 6 = 0$
10 $3x + 2y - 6 = 0$

Exercise 14:04

1 **a** C **b** C **c** A **d** B

2 **a** $x + y - 2 = 0$ **b** $2x - 7y + 3 = 0$
c $4x - y + 8 = 0$ **d** $2x - 3y - 4 = 0$
e $x - 6y + 3 = 0$ **f** $2x - 5y + 1 = 0$
g $4x - y + 1 = 0$ **h** $3x - y - 4 = 0$
i $x - y + 2 = 0$ **j** $4x - y + 3 = 0$
k $2x + y + 5 = 0$ **l** $x + y - 7 = 0$
m $x - 2y + 6 = 0$ **n** $x - 3y - 15 = 0$
o $2x - 5y + 5 = 0$

3 **a** $3x - y + 5 = 0$ **b** $6x - y - 2 = 0$
c $5x - y + 2 = 0$ **d** $x + y - 8 = 0$
e $2x - y + 3 = 0$ **f** $3x - y + 7 = 0$
g $2x + 3y = 0$

4 **a** $3x + 7y + 5 = 0$ **b** $6x - 5y - 8 = 0$
c $42x + 138y + 29 = 0$

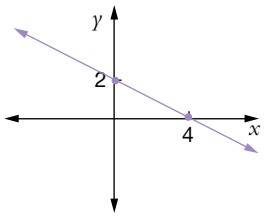
Exercise 14:05

1 **a** $\frac{8}{5} = 1\frac{3}{5}$ **b** $\frac{-11}{3} = -3\frac{2}{3}$
c $\frac{4}{3} = 1\frac{1}{3}$ **d** $\frac{-11}{-2} = 5\frac{1}{2}$

2 **a** **i** yes **ii** yes
b **i** yes **ii** yes
c **i** 2 **ii** 7
d **i** $3\frac{3}{4}$ **ii** −3

3 **a** D **b** C **c** A **d** B

4 a



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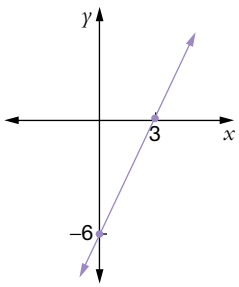

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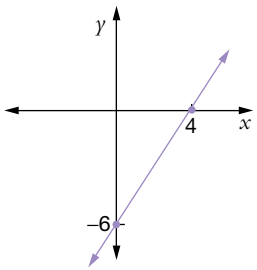

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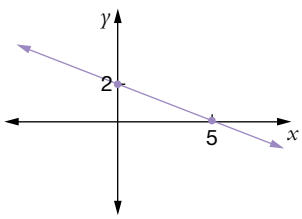

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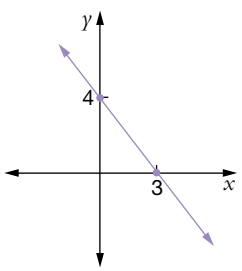

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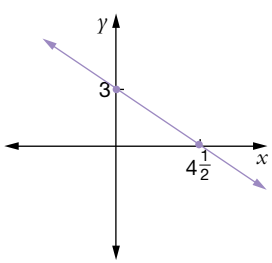

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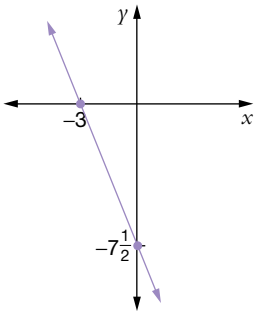
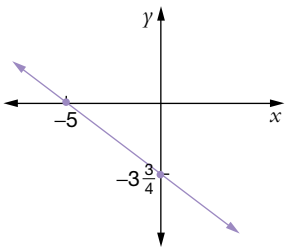
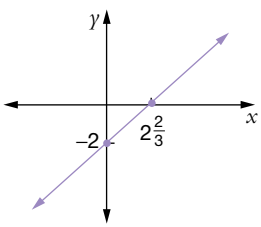


h

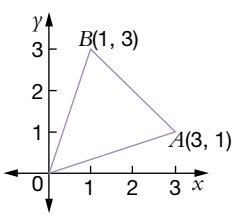


i



Exercise 14:06

1 a



$OB = \sqrt{(3-0)^2 + (1-0)^2}$

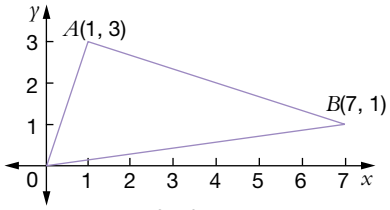
$= \sqrt{10}$

$OA = \sqrt{(1-0)^2 + (3-0)^2}$

$= \sqrt{10}$

$\therefore$ ΔOAB is isosceles (2 equal sides).

b



Slope of $OA = \dfrac{3-0}{1-0}$

$= 3$

Slope of $AB = \dfrac{3-1}{1-7}$

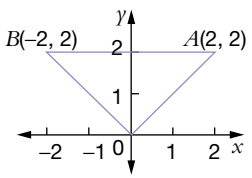
$= -\frac{1}{3}$

Now, slope of $OA \times$ slope of $AB = -1$

$\therefore OA \perp AB$

$\therefore \Delta OAB$ is right-angled at A.

c



$OA = \sqrt{(2-0)^2 + (2-0)^2}$

$= \sqrt{8}$

$OB = \sqrt{(2-0)^2 + (-2-0)^2}$

$= \sqrt{8}$

$AB = 4$

Now, $OA = OB$

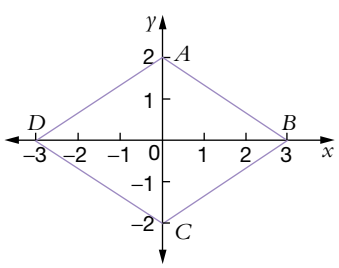
$\therefore \Delta OAB$ is isosceles.

Also, $4^2 = \left(\sqrt{8}\right)^2 + \left(\sqrt{8}\right)^2$

$\therefore AB^2 = OA^2 + OB^2$

$\therefore \Delta OAB$ is right-angled at O (by Pythag. th'm)

2 a



$AO = OC = 2$ and

$OD = OB = 3$

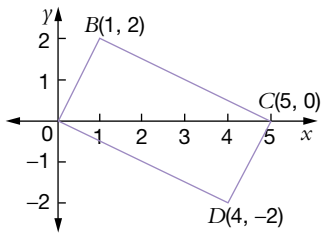
Also, $AC \perp BD$

$\therefore$ Diagonals bisect each other at right angles.

$\therefore ABCD$ is a rhombus.

Note: This question could be done in other ways.

b



$OC = 5$

$BD = \sqrt{(2--2)^2 + (1-4)^2}$

$= \sqrt{4^2 + (-3)^2}$

$= 5$

$\therefore OC = BD$

Midpoint of $BD = \left(\dfrac{4+1}{2}, \dfrac{-2+2}{2}\right)$

$= (2\frac{1}{2}, 0)$

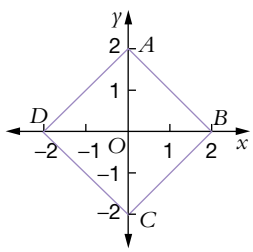
Midpoint of $OC = (2\frac{1}{2}, 0)$

$\therefore BD$ and OC bisect each other

$\therefore OBCD$ is a rectangle

(equal diagonals that bisect each other)

c



$AB = \sqrt{2^2 + 2^2}$ (Pythag. th'm)

$= \sqrt{8}$

Similarly, $BC = DC = AD = \sqrt{8}$

$\therefore ABCD$ is a parallelogram

(both pairs of opp. sides equal)

Now, slope of $AB = -1$ and slope of $AD = 1$

$\therefore AB \perp AD$ (product of slopes is -1)

$\therefore \angle DAB = 90°$

$\therefore ABCD$ is a rectangle (par'm with one right $\angle$)

3 a $(3, 2)$

b $OB = AC = \sqrt{13}$; The diagonals of a rectangle are equal in length.

c The midpoint of both diagonals is $(1\frac{1}{2}, 1)$. The diagonals bisect each other.

4 a Midpoint of AB is $(3, 2)$; midpoint of AC is $(2, -1)$.

b 3

c 3

d The line joining the midpoints of two sides of a triangle is parallel to the third side.

5 Slope of BD is 1; slope of AC is -1; the diagonals are perpendicular.

6 a $(3, 2)$

b $\sqrt{13}$

c $\sqrt{13}$

d E is equidistant from A, B and O.

7 **a** $AB = BC = \sqrt{10}$ (by distance formula). Hence ΔABC is isosceles as it has two equal sides.
b (0, 2)
c 1
d slope of $AC = -1$; slope of $EB = 1$; $EB \perp AC$ since product of their slopes is -1.
e Find the lengths AC and BE and then evaluate $\frac{1}{2}AC \times BE$.

8 **a** Midpoint of OA is $(\frac{1}{2}, 1)$; midpoint of AB is (2, 1).
b $1\frac{1}{2}$ units.
c OB is 3 units in length. Hence $\frac{1}{2}OB = 1\frac{1}{2}$.

9 **a** Midpoints are: AB $(-2, 3)$; BC $(3, 2)$; CD $(1, -2)$; AD $(-4, -1)$.
b a parallelogram
c Show that the opposite sides are parallel by showing they have the same slope.

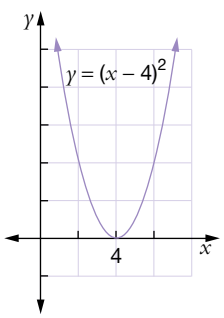
10 **a** Perp. bisector of AB is $x + 2y = 3$; perp. bisector of BC is $y = x$; perp. bisector of AC is $x = 1$.
b (1, 1)
c As (1, 1) satisfies the equation $y = x$, then (1, 1) lies on $y = x$. ∴ Perp. bisector of BC ($y = x$) passes through the point of intersection of the other bisectors.

Exercise 14:07

1 **a** $y = (x - 1)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 1)^2$ **d** $y = (x + 4)^2$

2 **a** A **b** B **c** C **d** D

3 **a**



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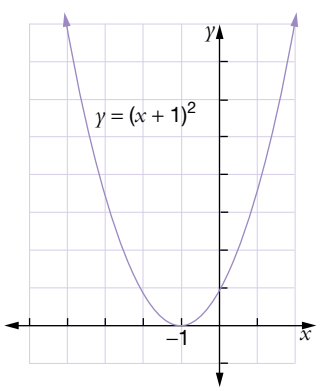

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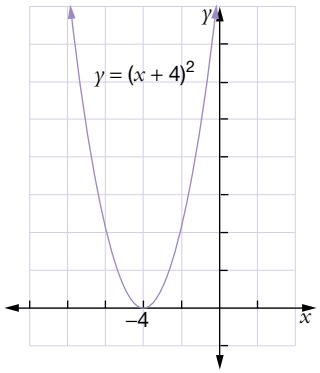
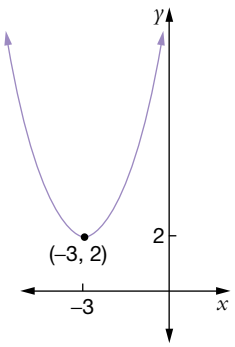


4 **a** (4,0) **b** (−1,0) **c** (−4,0)

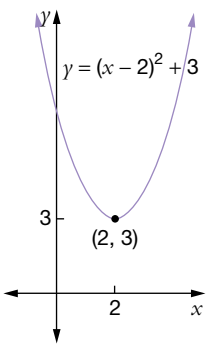
5 **a** It is the same graph translated up two units.
b It could be obtained by translating the graph of $y = (x - 3)^2$ down two units.

c

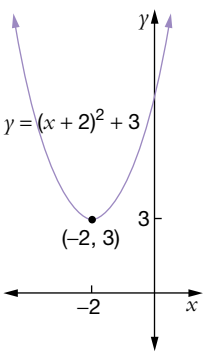


d The parabola $y = x^2$ has its vertex at (0, 0). If the parabola is translated p units sideways and q units vertically so that its vertex is at the point (p, q), then its equation is $y = (x - p)^2 + q$.

e **i**



ii



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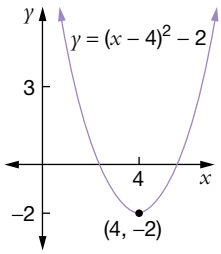

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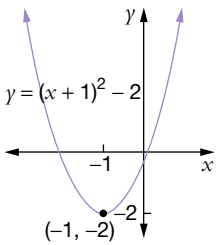


6 **a** $y = (x-2)^2 + 1$ **b** $y = (x+1)^2 - 2$
c $y = (x-2)^2 - 1$ **d** $y = (x+1)^2 + 2$

7 **a** $(-5, -2)$ **b** $(3, -2)$
c $(-4, 1)$ **d** $(2, 3)$

8 **a** A **b** D **c** C **d** B
e E **f** F

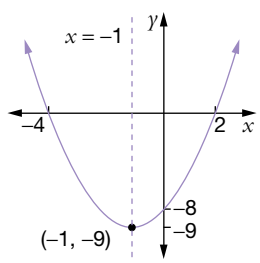
9 **a** $y = (x-2)^2 - 3$ **b** $y = (x-2)^2 + 1$
c $y = (x+2)^2 - 1$

Exercise 14:08

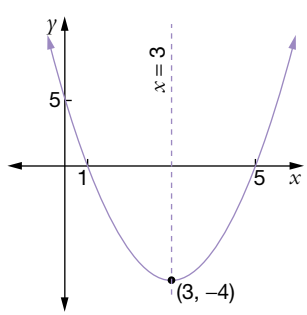
1 **a** **i** 0 **ii** 0, 4 **iii** $x = 2$ **iv** $(2, -4)$
b **i** 3 **ii** $-3, 1$ **iii** $x = -1$ **iv** $(-1, 4)$
c **i** -6 **ii** $-1, 3$ **iii** $x = 1$ **iv** $(1, -8)$

2 **a** $(0, -8)$ **b** $(-4, 0), (2, 0)$
c $x = -1$ **d** $(-1, -9)$

e



3 **a**



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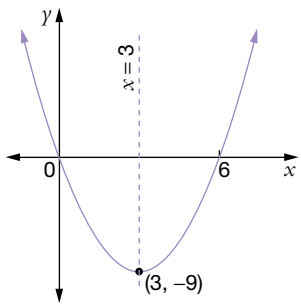

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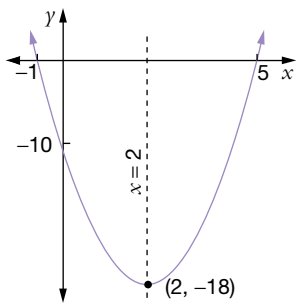

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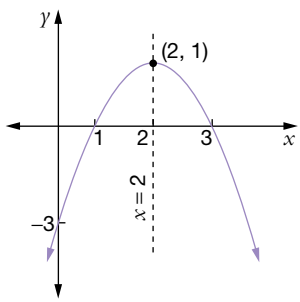

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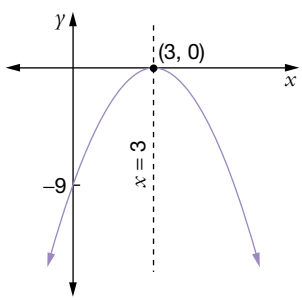

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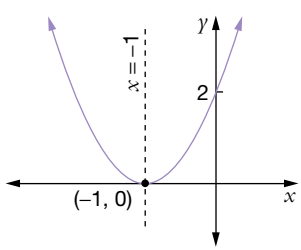

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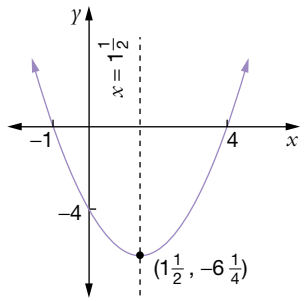

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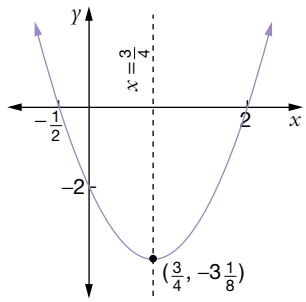
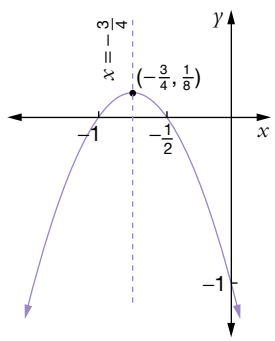
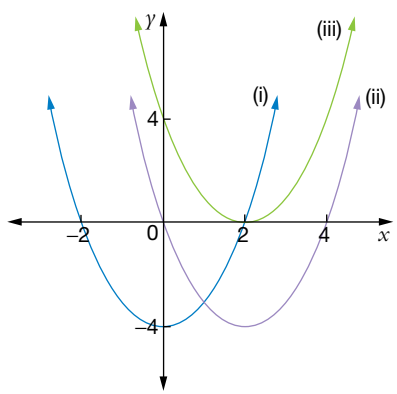


i



4 a D b B c F
d A e E f C

5 a



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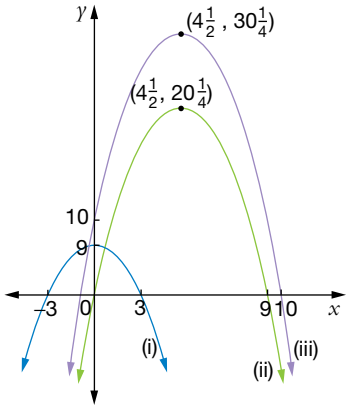

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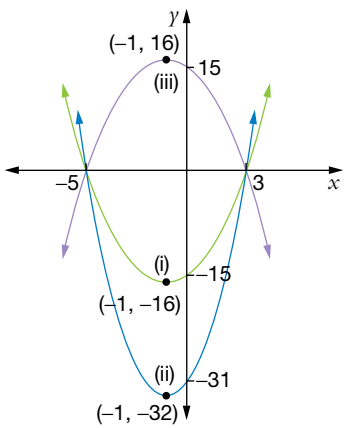

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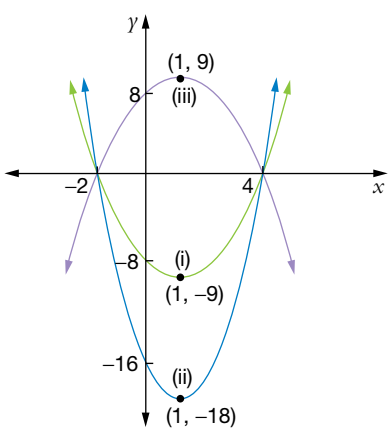
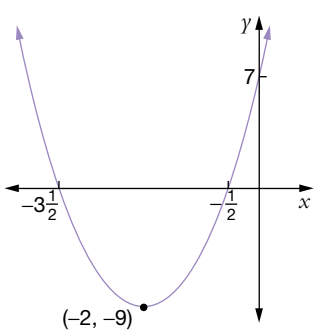


6 a



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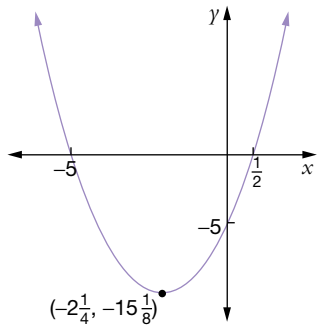

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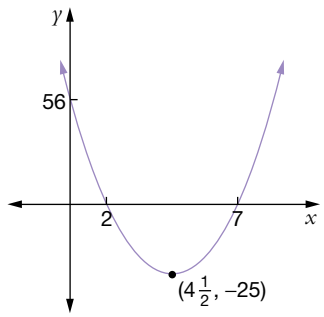

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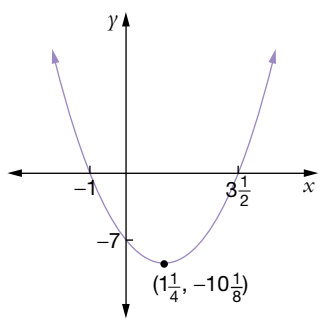
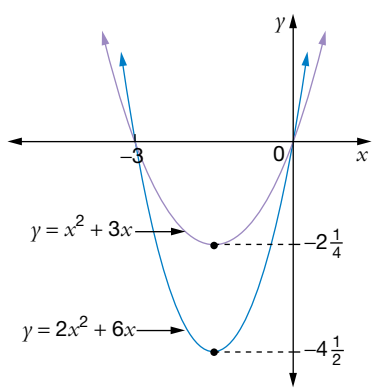
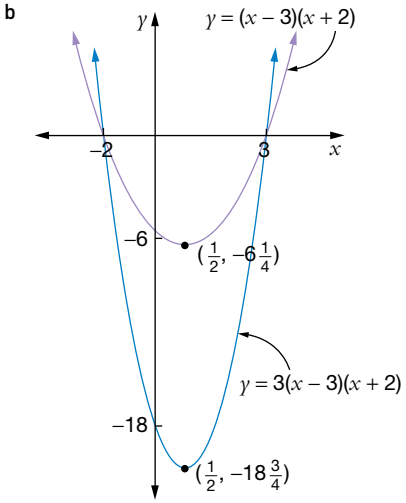


7 a



Same x- and y-intercepts and same concavity but they have different vertices. The y-coordinates of the vertices differ by a factor of 2, i.e. one y-coordinate is twice the other.

b



Same x-intercepts and concavity but the y-values of the vertices differ by a factor of 3, i.e. one is 3 times the other.

Investigation 14:08

1 $y = x^2 + 3x + 2$ **2** $y = x^2 - 10x + 16$
3 $y = 2x^2 + x - 4$ **4** $y = 2x^2 - 3x + 5$

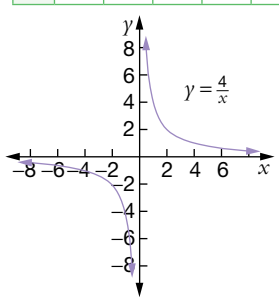
Prep quiz 14:09

1 4 **2** 2 **3** 1 **4** $\frac{1}{2}$ **5** $-\frac{1}{2}$
6 4 **7** 2 **8** −1
9 y decreases from 4 to $\frac{1}{10}$.
10 y increases from −4 to $-\frac{1}{10}$.

Exercise 14:09

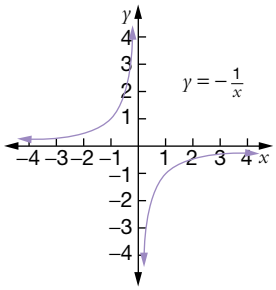
1

x	−8	−4	−2	−1	−0·5	0·5	1	2	4	8
y	−0·5	−1	−2	−4	−8	8	4	2	1	0·5



2

x	−4	−2	−1	−0·5	−0·25	0·25	0·5	1	2	4
y	0·25	0·5	1	2	4	−4	−2	−1	−0·5	−0·25



3 **a** C **b** F **c** D **d** B
e E **f** A

4 **a** Yes **b** −18 **c** 20

5 **a** $(6, -2)$; $y = -\frac{12}{x}$ **b** $(4, 5)$; $y = \frac{20}{x}$

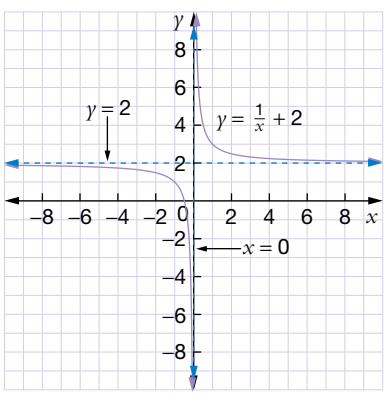
c $(2, -3)$; $y = -\frac{6}{x}$ **d** $(2, 2)$; $y = \frac{4}{x}$

Exercise 14:10

1 **a** $y = \frac{1}{x} + 2$

x	−8	−4	−2	−1	−0·5	0·5	1	2	4	8
y	1·9	1·8	1·5	1	0	4	3	2·5	2·3	2·1

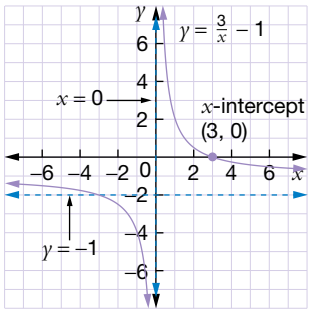
Asymptotes are; $x = 0$ and $y = 2$
x-intercept $= -\frac{1}{2}$



b $y = \frac{3}{x} - 1$

x	−6	−3	−2	−1	−0·5	0·5	1	2	3	6
y	−1·5	−2	−2·5	−4	−7	5	2	0·5	0	−0·5

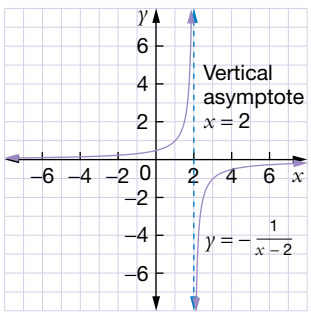
Asymptotes are; $x = 0$ and $y = -1$
x-intercept $= 3$



c $y = \frac{-1}{x - 2}$

x	−4	−2	0	1	1·5	2·5	3	4	6	8
y	0·2	0.3	0·5	1	2	−2	−1	−0·5	−0.3	−0·2

Asymptotes are; $x = 2$ and $y = 0$
y-intercept $= \frac{1}{2}$



2 **A** $y = \frac{3}{x} - 2$ **B** $y = \frac{3}{x} + 1$

C $y = \frac{3}{x + 2}$ **D** $y = \frac{3}{x - 3}$

3 **a** $y = \frac{1}{x} + 1$ **b** $y = \frac{1}{x} - 1$

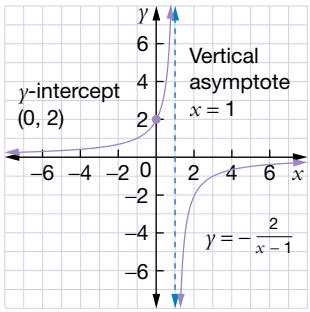
c $y = \frac{1}{x - 1}$ **d** $y = \frac{1}{x + 1}$

4 **a** move 3 units down
b move 2 units up
c move 3 units to the right
d move 2 units to the left

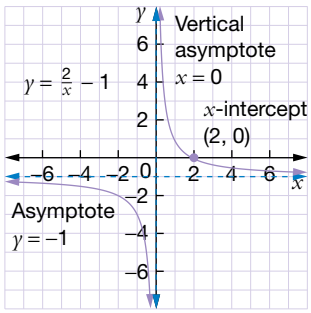
5 **a** $x = 0$ and $y = -3$ **b** $x = 0$ and $y = 2$
c $x = 3$ and $y = 0$ **d** $x = -2$ and $y = 0$

6 **a** B **b** C **c** D **d** E
e F **f** A

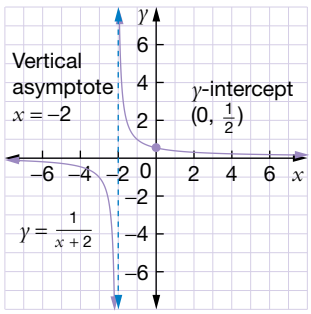
7 a



b



c



8 a $y = \frac{-2}{x} + 1$ b $y = \frac{1}{x} - 2$ c $y = \frac{2}{x-1}$

Exercise 14:11

1 a $(x-1)^2 + (y-1)^2 = 49$

b $(x-5)^2 + y^2 = 4$

c $(x+3)^2 + (y+5)^2 = 16$

d $(x-2)^2 + (y+5)^2 = 1$

e $x^2 + (y-2)^2 = \frac{1}{4}$

f $x^2 + y^2 = 9$

2 a $(2, 3); 8$ b $(-4, 1); 2$ c $(-3, -3); 3$

d $(6, 5); 10$ e $(0, -5); 4$ f $(3, 0); 1$

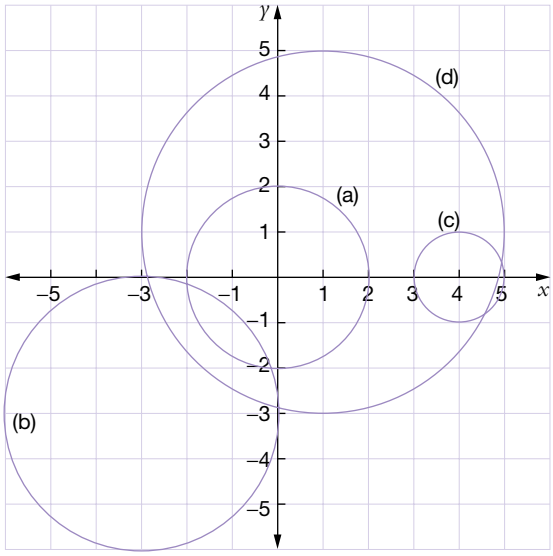
g $(0, 0); 9$ h $(0, 0); 7$ i $(0, 0); \sqrt{11}$

j $(7, 8); \sqrt{2}$

3 a $(5, -4); 3$ b $(-4, 7); 10$ c $(9, 10); 11$

d $(4\frac{1}{2}, 0); \sqrt{7}$

4



5 a x-intercepts: $\pm\sqrt{5}$

y-intercepts: $-1, 5$

b x-intercepts: $2 \pm \sqrt{15}$

y-intercepts: $1 \pm \sqrt{12}$

6 a $(x-3)^2 + (y-4)^2 = 25$

b $4x^2 + 4y^2 = 8$

7 a $(x-2)^2 + y^2 = 4$

b $(x+2)^2 + y^2 = 4$

c $x^2 + (y-2)^2 = 4$

d $x^2 + (y+2)^2 = 4$

8 $(x-4)^2 + (y+3)^2 = 4$

Exercise 14:12

1 a i B ii C iii A

b $y = 3x^3$

c $y = \frac{1}{2}x^3$

d By comparing the coefficients of x^3 you can tell which graph is the steepest. The larger the coefficient, the steeper the graph.

2 a $y = 3x^3$ b $y = x^3$ c $y = 3x^3$

3 a They are reflections of each other in the x-axis.

b The size of a affects the steepness of the graph while the sign of a indicates whether the graph is increasing or decreasing as you move from left to right.

4 a increasing b decreasing c increasing

d increasing e decreasing f decreasing

5 a

x	−2	−1	0	1	2
x^3	−8	−1	0	1	8
$x^3 + 2$	−6	1	2	3	10
$3 - 2$	−10	−3	−2	−1	6

b A is $y = x^3 + 2$

B is $y = x^3 - 2$

c $y = x^3 + 2$ is the curve $y = x^3$ translated up two units.

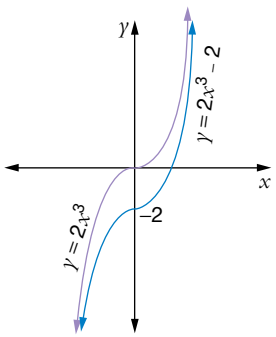
d $y = x^3 - 2$ is the curve $y = x^3$ translated down two units

6 **a** **A** $y = x^3 + 10$ **B** $y = x^3 - 10$

b **A** $y = -\frac{x^3}{3} + 6$ **B** $y = -\frac{x^3}{3} - 5$

c **A** $y = -\frac{1}{10}x^3 + 10$ **B** $y = -\frac{1}{10}x^3 - 15$

7 **a**



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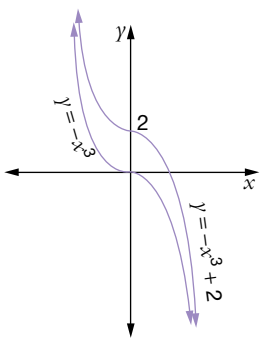

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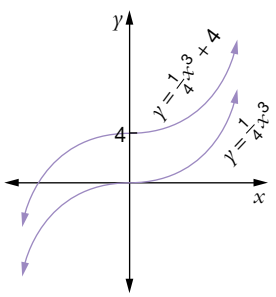
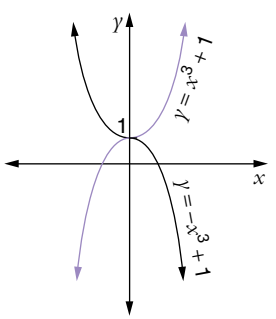


8 **a**



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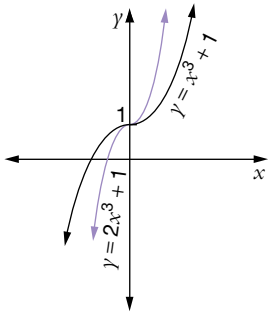


9 **a** B, H **b** D, E **c** C, F, I

d A, F, G **e** G **f** C, E, H

10 **a** $y = \frac{5}{4}x^3 + 10$ **b** $y = -2x^3 - 4$

Exercise 14:13

1 **a** B **b** A **c** C **d** D **e** B

f A **g** C **h** A **i** D

2 **a** **i** $y = \frac{1}{2}x^4 - 4$ **ii** $y = \frac{1}{2}x^4 + 2$

b **i** $y = 2x^7 - 5$ **ii** $y = 2x^7 + 3$

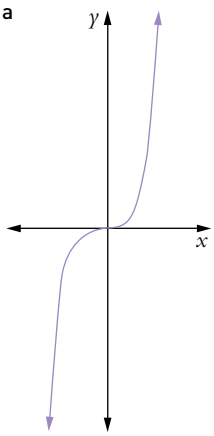
c **i** $y = -5x^6 - 4$ **ii** $y = -5x^6 - 8$

3 **a** **i** B **ii** A **iii** C

b **i** C **ii** A **iii** B

c **i** C **ii** B **iii** A

4 **a**



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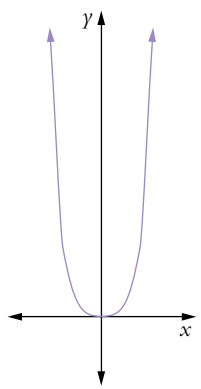

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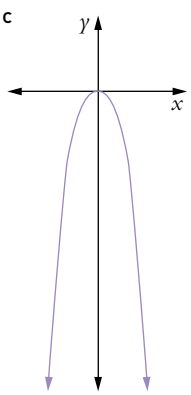

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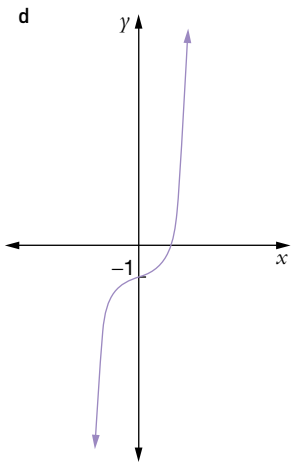

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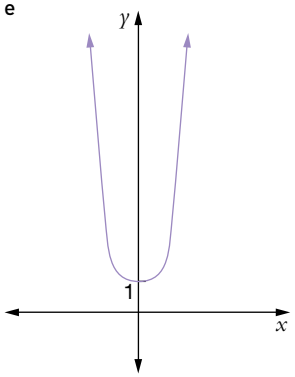

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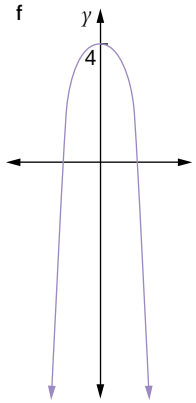
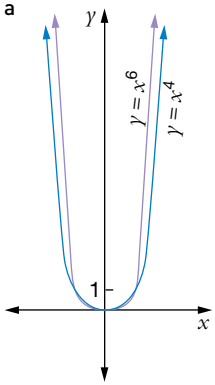


5 a



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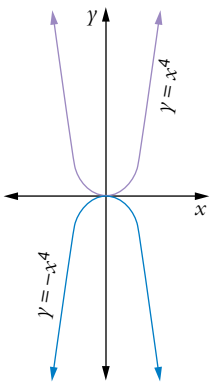

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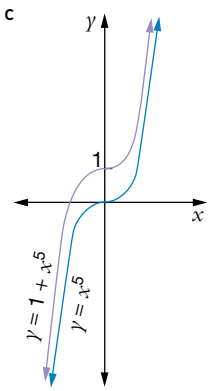

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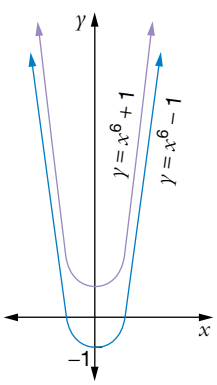

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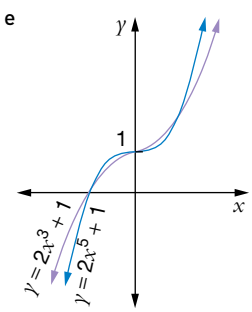

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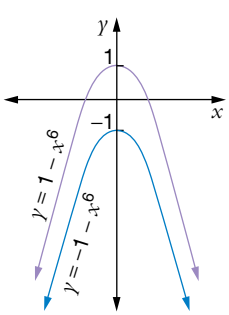
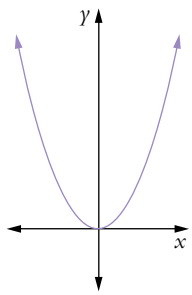


6 a The 4 tells us that the curve is shaped like a parabola, while the −2 tells us that it is upside down. The 1 tells us that the curve has been produced by moving the curve $y = -2x^4$ up one unit.

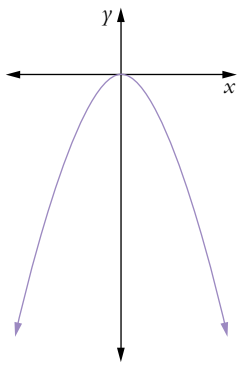
b The curve is similar in shape to $y = x^3$. It is steeper than this curve and it crosses the y-axis at $(0, -3)$.

c The curves are reflections of each other in the x-axis.

7 a



b



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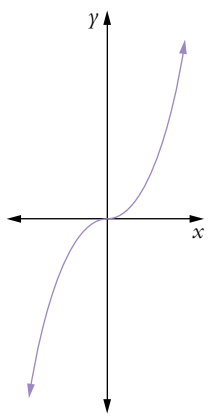

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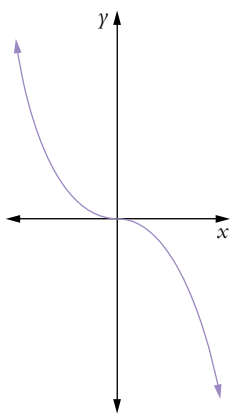
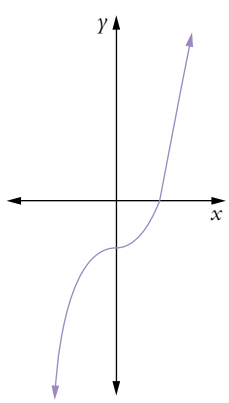


8 a



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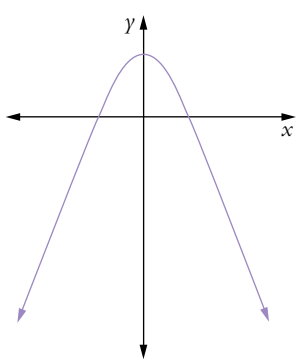


9 a n is odd, a is negative and d is negative.

b n is even, a is negative and d is positive.

c n is even, a is positive and d is negative.

Exercise 14:14

1 a A is $y = (x - 3)^5$, B is $y = (x + 4)^5$

b D

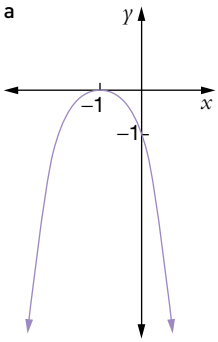
2 a $y = (x + 4)^4$ b $y = (x - 4)^4$

c $y = (x - 1)^4$ d $y = (x + \frac{1}{2})^4$

3 a 1 unit to the right b 3 units to the left

c $\frac{1}{2}$ a unit to the right

4 a



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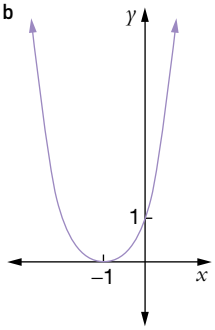

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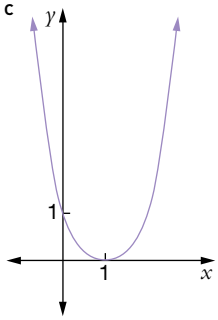

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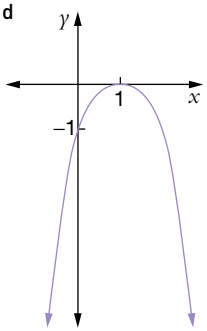
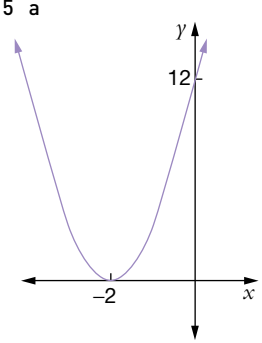


5 a



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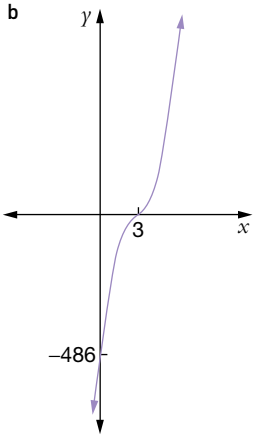

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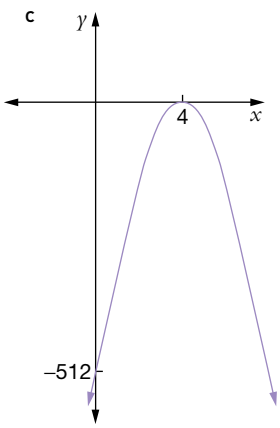

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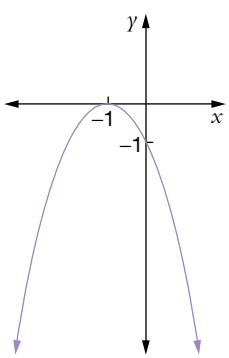
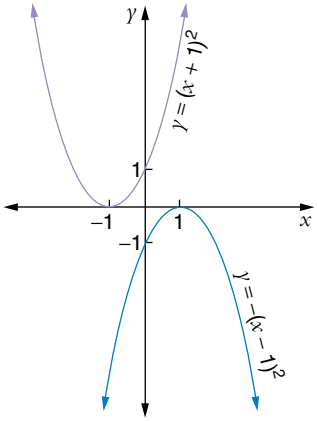


6 a



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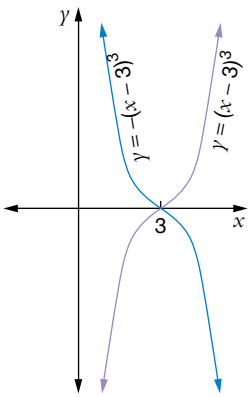

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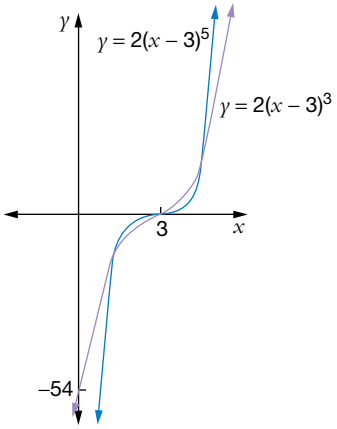

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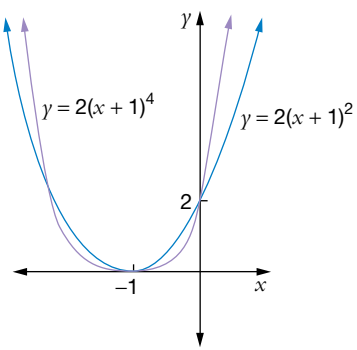


Exercise 14:15

1 a

x	−5	−4	−3	−2	−1·5	−1	0
y	−	0	+	0	−	0	+

b

x	−3	−2	0	2	4	5	6
y	+	0	−	0	+	0	−

c

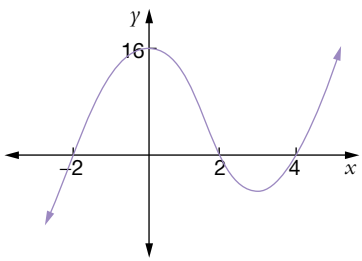
x	0	1	2	4	5
y	−	0	−	0	+

d

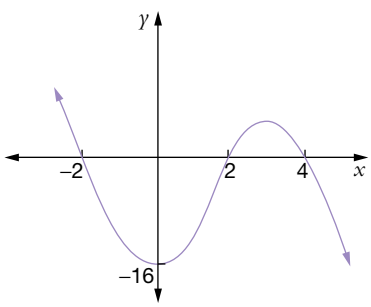
x	−2	−1	0	2	3
y	+	0	−	0	−

2 a 1, 2, 4 **b** 1, −2, 4 **c** 2, −5 **d** 0, −4, 2

3 a



b



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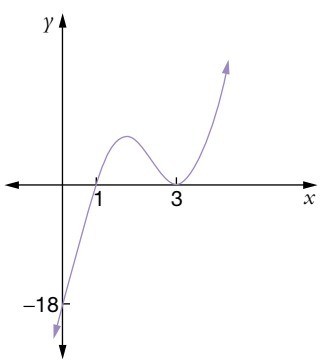
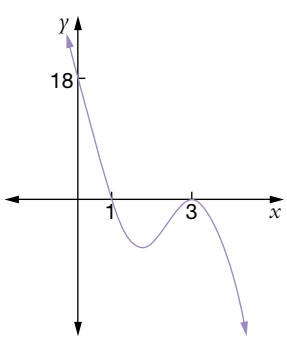


d



4 a 16; −16; −18; 18

b Let $x = 0$ to find the y-intercept. Ask yourself whether the graph could have this y-intercept.

5 a $y = -(x + 1)(x - 1)(x - 3)$

b $y = (x + 2)(x - 1)(x - 3)$

c $y = -(x + 4)(x + 2)(x - 2)$

d $y = -(x + 2)(x - 2)^2$

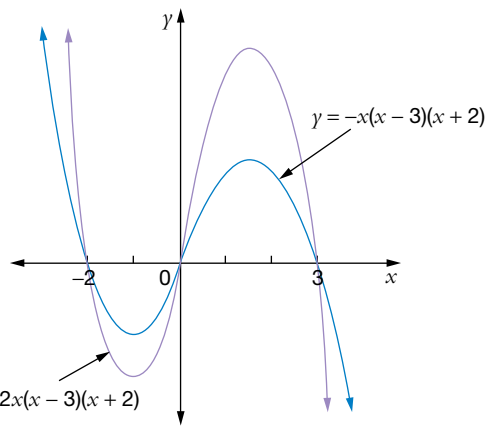
e $y = (x + 2)(x - 2)^2$

f $y = (x - 1)(x - 3)^2$

6 You would need to know another point that the curve passes through.

7 a i B ii C iii A

b



8 a $y = (x - 1)(x + 1)(x - 4)$

b $y = (x + 4)(x + 2)(x - 1)$

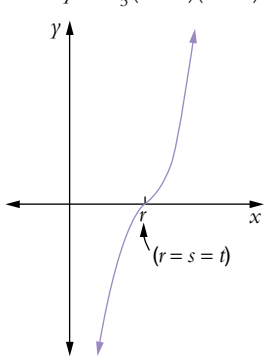
c $y = -x(x + 2)(x - 3)$

d $y = -x(x + 3)(x - 2)$

9 a $y = 2(x + 1)(x - 1)(x - 4)$

b $y = -2(x + 2)(x - 2)^2$
or $y = -\frac{2}{5}(x - 2)(x + 2)^2$

10



Exercise 14:16

1 a (−1, 1), (2, 4) **b** (−2, 4), (2, 4)

c (−2, 4), (1, 1) **d** (−8, −1), (1, 8)

e (−6, −1), (2, 3) **f** (−1, −3), (1, 3)

2 a (−2, 4), (3, 9) **b** (2, 4)

c (−1, 1), (2, 4) **d** (−3, 9), (1, 1)

3 a (−4, 3), (4, −3) **b** (3, 4)

c (4·7, −1·8)

4 a A (−0·3, −0·7), B (2·3, 4·7)

b A (−0·5, 0·2), B (1·2, 1·5)

5 and **6**

a $(0, 0), (2, 4)$

b $(-1, 1), (3, 9)$

c $(1, 1), (-1, -1)$

d $(1+\sqrt{2}, -1+\sqrt{2}), (1-\sqrt{2}, -1-\sqrt{2})$

e $(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})$

f $(0, -2), (1{\cdot}6, 1{\cdot}2)$

7 **a** $(2, 3), (-1, 0)$

b $(2+\sqrt{2}, 2-\sqrt{2}), (2-\sqrt{2}, 2+\sqrt{2})$

8 **a** A $(-0.34, -0.68)$, B$(2.34, 4.68)$

b A $(-0.43, 0.18)$, B$(1.18, 1.38)$

Answers to 2 d.p.

Exercise 14:17

1 **a** E, F, G **b** A, L **c** B, H, J

d C, I, O, P **e** D, N

2 **a** E **b** D **c** B **d** P

e H **f** F **g** C **h** K

i A **j** N **k** G **l** P

m M **n** O **o** J **p** L

3 **a** $y = (x-2)^2 - 2$ or $y = x^2 - 4x + 2$

b $y = x^4 - 2$

c $y = (x-2)^2 + 2$ or $y = x^2 - 4x + 6$

d $y = 3 - 2^{-x}$ **e** $y = 2(x-3)^3$

f $x^2 + (y-2)^2 = 4$ **g** $y = -x^3 - 4$

h $y = x^3 - 4$ **i** $y = 4 - 3^x$

j $(x+2)^2 + y^2 = 4$ **k** $y = x(x-1)(x+2)$

l $y = \dfrac{2}{x} - 3$ **m** $y = \dfrac{1}{x+2}$

n $y = \dfrac{1}{2-x}$ **o** $y = -\dfrac{4}{x} + 1$

Diagnostic test 14

1 **a** 10 **b** $\sqrt{13}$ **c** $\sqrt{17}$

2 **a** (4,6) **b** (4,1·5) **c** $(-1, -2{\cdot}5)$

3 **a** $\frac{4}{3}$ **b** $\frac{3}{2}$ **c** $-\frac{1}{4}$

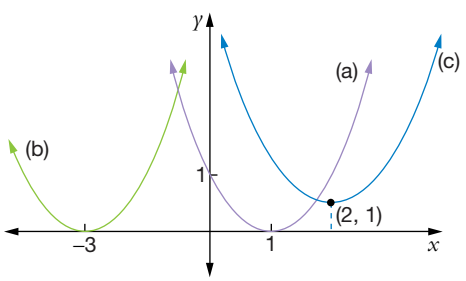
4 **a** $y = 2x + 2$ **b** $y = -3x + 6$ **c** $y = \frac{1}{2}x + 1$

5 **a** $y = 2x - 1$ **b** $y = -3x - 1$ **c** $y = \frac{4}{3}x$

6 **a** $3x - y - 2 = 0$ **b** $2x + 3y - 6 = 0$

c $x + 2y - 8 = 0$

7



8 **a** $(0, -3)$ **b** $(0, 0)$ **c** $(0, 8)$ **d** $(0, -5)$

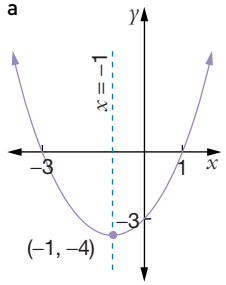
9 **a** $(1, 0), (-3, 0)$ **b** $(0, 0), (6, 0)$

c $(2, 0), (-4, 0)$ **d** $(\frac{1}{2}, 0), (-\frac{5}{2}, 0)$

10 **a** $x = -1$ **b** $x = 3$ **c** $x = -1$ **d** $x = -1$

11 **a** $(-1, -4)$ **b** $(3, -9)$ **c** $(-1, 9)$ **d** $(-1, -9)$

12 **a**



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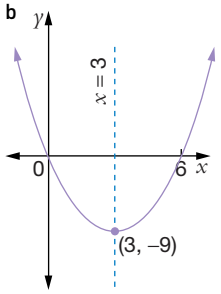

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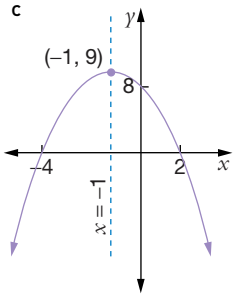

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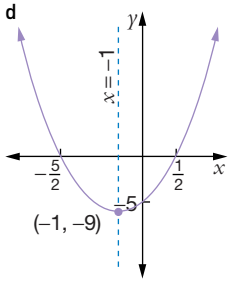
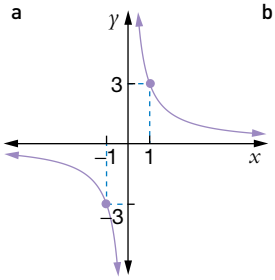


13 **a**



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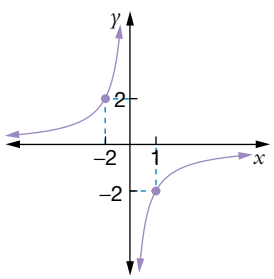

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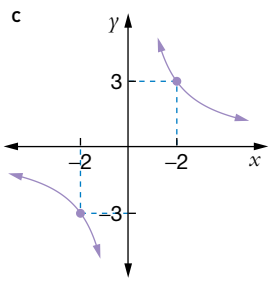
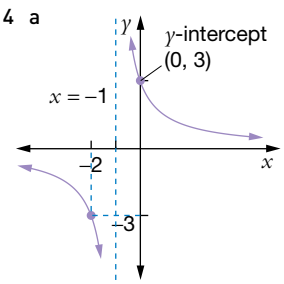


14 **a**



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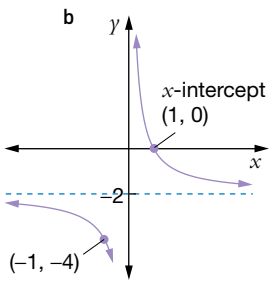

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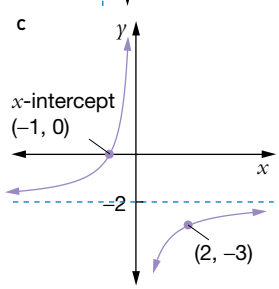
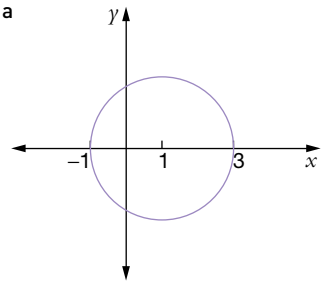


15 a



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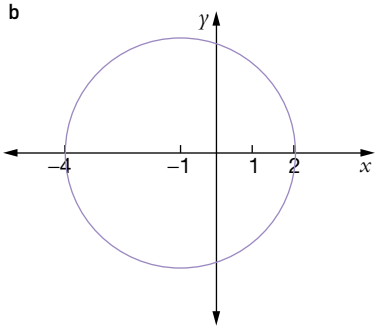

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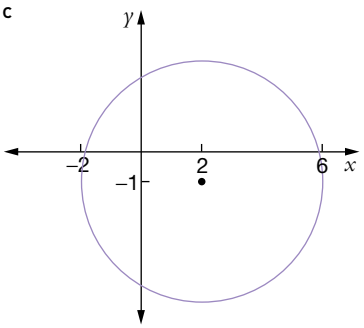

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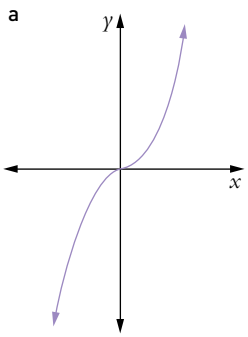

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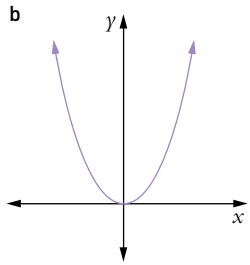

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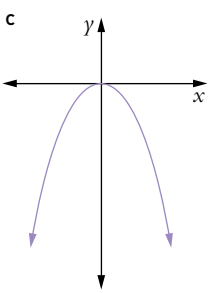

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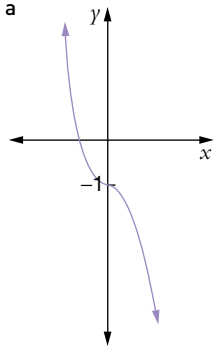

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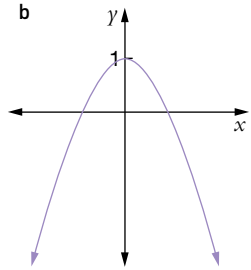

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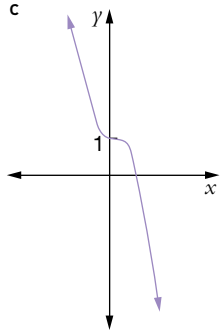

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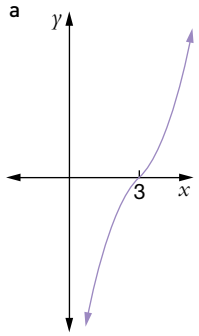

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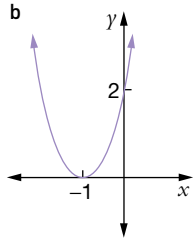

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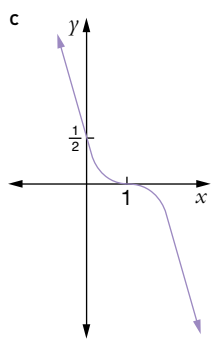
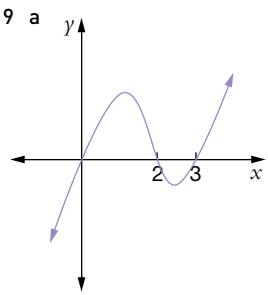


19 a



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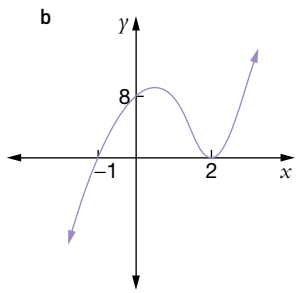

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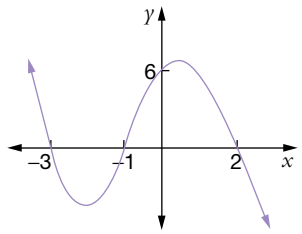


20 **a** (2, 4) and (–4, –2)
b no points of intersection
c (1, 8) and (4, 2)

21 **a** (3, 9) and (–4, 16)
b (1, –2) and (2, –1)
c (1, 3) and (–3, –1)

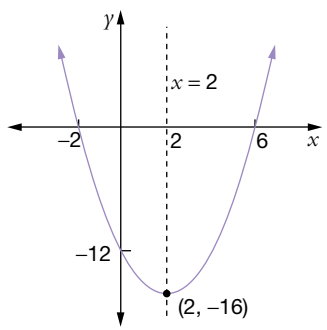
ASSIGNMENT 14A

1 **a** $y = -x - 1$ **b** $y = \frac{1}{2}x - \frac{3}{2}$

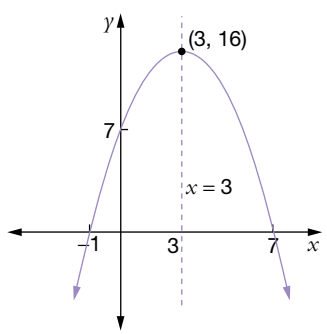
2 **a** $x + y + 1 = 0$ **b** $x - 2y - 3 = 0$

3 Each graph is congruent to the graph of $y = x^2$. The graphs can be obtained by performing the following translations on $y = x^2$.
a moving it 5 units to the right
b moving it 5 units to the left
c moving it 5 units to the right and then 5 units up.

4 **a**

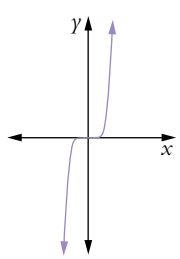


b

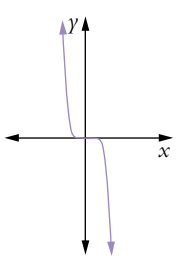


5 **a** $(x + 3)^2 + y^2 = 9$ **b** $(x - 3)^2 + y^2 = 36$

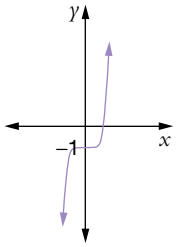
6 **a**



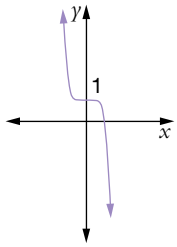
b



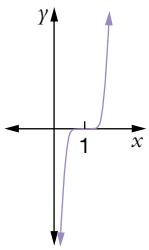
c



d



e



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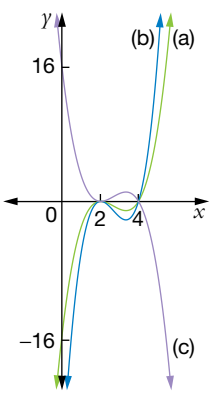


8 A is $y = \frac{1}{2}x^3 - 4$. B is $y = -x^3 + 4$

9 **a** $y = \frac{4}{x} + 2$ **b** $y = \frac{4}{x - 2}$ **c** $y = \frac{4}{x + 2}$
d $y = -\frac{4}{x}$ **e** $y = -\frac{4}{x} + 2$

ASSIGNMENT 14B

1 In 12 000 000 hours the time will be 10 am, so the time in 11 999 995 hours will be 5 am.

2 Largest area = 200 m^2; dimensions are 20 m × 10 m

3 $5\frac{1}{4}$ hours

4 **a** 72 km (via D, F)
b 13
c 127 km (F → G → C → A → B → Tim's)

5 **a** $200 000
b Just over 14 years
c Just over 17 years
d Because the people paying four-weekly are simply paying off more money each year than the group paying monthly (13 payments compared to 12).

Chapter 15

Prep quiz 15:01

1 BC **2** AB **3** AC **4** 0·6 **5** 0·8
6 0·75 **7** 0·669 **8** 1·804 **9** 26° **10** 69°

Exercise 15:01

1

θ	10°	15°	20°	25°	30°
$\sin\theta$	0·174	0·259	0·342	0·423	0·5
$\cos\theta$	0·985	0·966	0·940	0·906	0·866

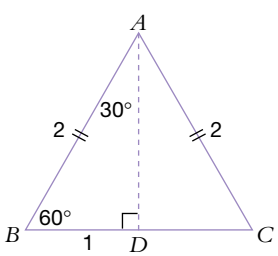
θ	35°	40°	45°	50°	55°
$\sin\theta$	0·574	0·642	0·707	0·766	0·819
$\cos\theta$	0·819	0·766	0·707	0·642	0·574

θ	60°	65°	70°	75°	80°
$\sin\theta$	0·866	0·906	0·940	0·966	0·985
$\cos\theta$	0·5	0·423	0·342	0·259	0·174

2 **a** $\cos 80°$ **b** $\cos 70°$ **c** $\cos 60°$ **d** $\cos 25°$
e $\sin 50°$ **f** $\sin 40°$ **g** $\sin 55°$ **h** $\sin 25°$
3 **a** 50 **b** 30 **c** 80 **d** 70
e 65 **f** 15 **g** 45 **h** 45
4 **a** 45°
5 **a** **i** 0·1763 **ii** 0·1763
b yes
6 **a** $\tan 30°$ **b** $\tan 48°$ **c** $\tan 16°$
7 **a** $\dfrac{\sin 30°}{\cos 60°} = \dfrac{\cos 60°}{\cos 60°} = 1$ **b** 40° **c** 50°
8 **a** $\dfrac{\sin\theta}{\cos\theta} = 1$ **b** $\sin\theta = \cos\theta$ **c** $\theta = 45°$

Investigation 15:01

1



2 $\sqrt{3}$
3 $\sin 30° = \frac{1}{2}$ $\sin 60° = \dfrac{\sqrt{3}}{2}$
$\cos 30° = \dfrac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$
$\tan 30° = \dfrac{1}{\sqrt{3}}$ $\tan 60° = \sqrt{3}$
4 $\angle EDF = \angle EFD$ (base $\angle$s of isos. Δ)
$\therefore 2 \times \angle EDF + 90° = 180°$ ($\angle$ sum of Δ)
$\therefore \angle EDF = 45°$
5 $DF = \sqrt{2}$
6 $\sin 45° = \dfrac{1}{\sqrt{2}}$
$\cos 45° = \dfrac{1}{\sqrt{2}}$
$\tan 45° = 1$

Exercise 15:02

1 **a** 2nd **b** 3rd
c 4th **d** 1st
2 **a** positive **b** negative
c negative **d** positive
e negative **f** negative
g positive **h** positive
3 **a** 1st or 4th **b** 2nd or 3rd
c 1st or 2nd **d** 3rd or 4th
4 **a** 1st or 2nd **b** 3rd or 4th
c 1st or 4th **d** 2nd or 3rd
e 4th
5 **a** It goes from 0 to 1 **b** It goes from 1 to 0
c It goes from 0 to −1 **d** It goes from −1 to 0
6 **a** It goes from 1 to 0 **b** It goes from 0 to −1
c It goes from −1 to 0 **d** It goes from 0 to 1
7 **a** negative **b** 2nd and 4th
c 2nd and 4th **d** 1st and 3rd
8 **a** 1 and −1 **b** 1 and −1
c 1 and −1
9 **a** It goes from 0 up to very large numbers
b $\tan 90° = \dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0}$, but we cannot have a fraction with a denominator of zero.
10 **a** They are the same.
b They are the same.
c They are equal in size but opposite in sign.
d They are equal in size but opposite in sign.
11 **a** 30° and 150° **b** 50° and 130°
c 10° and 170° **d** 210° and 300°
e 230° and 310° **f** 190° and 350°
12 **a** 60° and 300° **b** 50° and 310°
c 30° and 330° **d** 120° and 240°
e 130° and 230° **f** 150° and 210°

Investigation 15:02

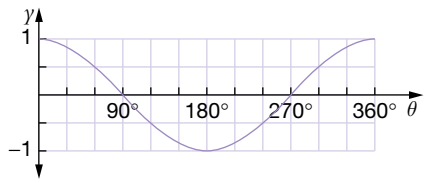
1 **a**

θ	300°	660°	−60°
$\sin\theta$	−0·866	−0·866	−0·866
$\cos\theta$	0·5	0·5	0·5
$\tan\theta$	−1·732	−1·732	−1·732

b Yes
2 They have the same ratios because adding or subtracting 360° will not change the position of P on the unit circle.
3 **a** no
b no

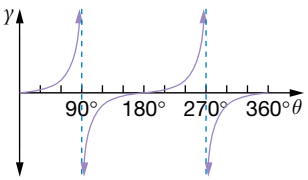
c i Translate the curve 360° to the right.
ii Translate the curve 360° to the left.

4



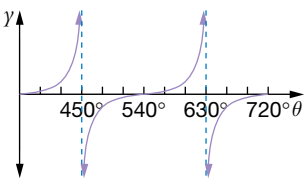
5 a Translate the curve 360° to the right.
b Translate the curve 360° to the left.

6 a



b The values of tan90° and tan 270° do not exist. As θ gets closer to these values, the value of tan θ becomes infinitely large.

c



d 180°

7 a 360° b 360° c 180°

Prep quiz 15:03

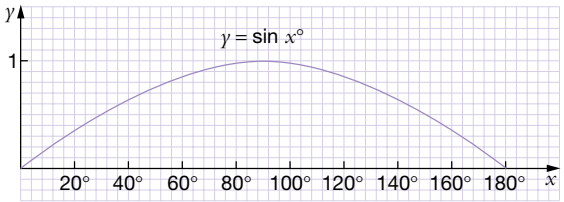
1 0·64 2 0·64 3 0·77 4 −0·77
5 yes 6 positive 7 positive 8 no
9 $\theta = 30°$ or 150° 10 60° or 300°

Exercise 15:03

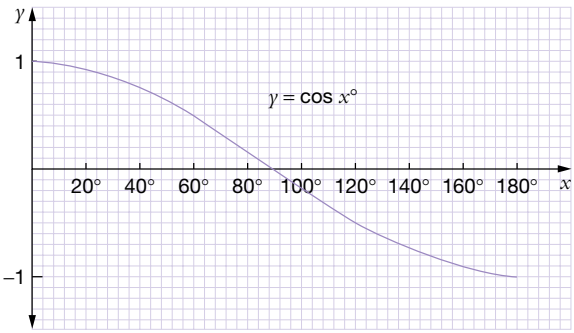
1 a 0·216 b −0·770 c −0·570
2 a sin 30° b −cos 80° c −tan 40°
d sin 55° e sin 39°28′ f −cos 12°29′
g −tan 78°57′ h −cos 87°10′
3 a 44°16′ b 135°44′ c 83°4′
d 96°56′ e 120° f 135°
g 25°3′ h 110° i 62°0′
j 130° k 165°1′ l 52°59′
m 142°59′ n 139°28′ o 73°15′
p 175°34′
4 a 30°, 150° b 46°53′, 133°7′
c 21°6′, 158°54′ d 26°56′, 153°4′
e 81°53′, 98°7′ f 21°30′, 158°30′
g 45°6′, 134°54′ h 3°15′, 176°45′

5

$x°$	0	20°	40°	60°	80°	90°	100°	120°	140°	160°	180°
sin $x°$	0	0·34	0·64	0·87	0·98	1	0·98	0·87	0·64	0·34	0



6



7 a 72° b 45° c 22° d 135°
e 162° f 150°
8 a 54° b 121°

Exercise 15:04

1 a 22·6 b 5·2 c 4·7
d 25·3 (all answers correct to 1 dec. pl.)

2 a $\frac{x}{\sin 55°} = \frac{9}{\sin 60°}$ $\therefore x = 8{\cdot}5$

b $\frac{x}{\sin 36°} = \frac{12}{\sin 71°}$ $\therefore x = 7{\cdot}5$

c $\frac{x}{\sin 120°} = \frac{6}{\sin 27°}$ $\therefore x = 11{\cdot}4$

3 a 5·3 b 5·8 c 7·7
4 a i 46° ii 45°52′
b i 55° ii 54°44′
c i 12° ii 11°32′
d i 59° ii 59°26′
e i 39° ii 38°50′

5 a $\frac{\sin\theta}{9} = \frac{\sin 63°}{10}$ $\therefore \theta = 53°$

b $\frac{\sin\theta}{3{\cdot}6} = \frac{\sin 39°}{4{\cdot}2}$ $\therefore \theta = 33°$

c $\frac{\sin\theta}{4{\cdot}6} = \frac{\sin 66°}{8{\cdot}9}$ $\therefore \theta = 28°$

6 a 55°41′ b 63°16′ c 53°30′
7 a 127° b 129° c 144°
8 a 14·2 cm b 8·1 cm c 15°31′
d 18°33′
9 a 35° b 77° c 8·4 cm

10 **a** 66° **b** 74° **c** 28 km
11 37 m
12 P is 585 m from the plane; Q is 864 m from the plane.

Exercise 15:05

1 **a** 59°, 121° **b** 12°, 168° **c** 41°, 139°
d 20°, 160°
2 The obtuse angle is 139°. The acute angle is 41°.
3 **a** In ΔBDC, $\frac{\sin x^\circ}{4} = \frac{\sin 45^\circ}{2 \cdot 9}$ $\sin x^\circ = \frac{4 \sin 45^\circ}{2 \cdot 9}$
In ΔABC, $\frac{\sin y^\circ}{4} = \frac{\sin 45^\circ}{2 \cdot 9}$ $\sin y^\circ = \frac{4 \sin 45^\circ}{2 \cdot 9}$
b $x^\circ = 103^\circ$
$y^\circ = 77^\circ$
4 **a** 64°9′ or 115°51′ **b** 24°59′ or 155°1′
c 37°9′ or 142°51′ **d** 31°45′ or 148°15′
5 **a** 59°0′ or 121°0′ **b** 46°21′
c 73°15′ or 106°45′

Exercise 15:06

1 **a** 11·1 **b** 14·1 **c** 16·6 **d** 6·3
2 **a** 53° **b** 127° **c** 53° **d** 117°
3 **a** 4·3 **b** 10·0 **c** 6·6 **d** 8·6
e 7·2 **f** 13·1
4 **a** 50°59′ **b** 62°58′ **c** 134°37′ **d** 31°55′
e 124°2′ **f** 47°17′
5 **a** 9·1 cm **b** 2·8 m **c** 17·4 cm **d** 73°24′
e 46°33′
6 18 km
7 20°
8 21·9 cm
9 17·3 km
10 492 m
11 19·4 m
12 $r = 731$ m, $s = 510$ m (to the nearest metre)

Exercise 15:07

1 **a** 18·0 m^2 **b** 84·0 m^2 **c** 38·7 m^2
d 24·3 m^2 **e** 22·9 m^2 **f** 50·9 m^2
2 **a** 21 cm^2 **b** 69 cm^2
3 **a** 26 units^2 **b** 120 units^2 **c** 4 units^2
d 1292 units^2 **e** 249 units^2 **f** 2033 units^2
4 42 m^2
5 **a** 3064 cm^2 **b** 65·0 cm^2

Exercise 15:08

1 **a** sine rule; 8·1 **b** cosine rule; 5·4
c sine rule; 8·2 **d** sine rule; 16·6
e cosine rule; 11·7
2 **a** 25°13′ **b** 46°51′ **c** 23°52′
d 67°58′ **e** 48°15′ **f** 98°34′
3 **a** 58°40′ **b** 7·7 **c** 1·1
d 6·9 **e** 13·2 **f** 60°0′
4 60°21′
5 1431 m
6 139 km (to nearest km)
7 20238 m^2
8 **a** 138·8 cm^2 **b** 7·7 cm, 19·8 cm
9 S20°58′W
10 94°7′; 8·2 m

Exercise 15:09

1 **a** **i** $\sqrt{13}$ cm **ii** $\sqrt{45}$ cm **iii** 7 cm
b **i** 34° **ii** 17° **iii** 31°
2 **a** 11·3 cm **b** 10·6 cm **c** 71°
d 62° **e** 69°
3 3·4 cm
4 **a** **i** 19·3 cm **ii** 18·7 cm **iii** 22·8 cm **iv** 23·3 cm
b **i** 34° **ii** 12°
5 **a** **i** 6·93 cm **ii** 7·55 cm **iii** 8·00 cm **iv** 8·54 cm
b **i** 67° **ii** 54°
6 **a** 28·40 cm **b** 10°
7 999 m
8 415 m
9 **a** 53° **b** 57°
10 **a** **i** 1963 mm **ii** 3926 mm **iii** 5194 mm
b 22°

Diagnostic test 15

1 **a** 40° **b** 70° **c** 60°
2 **a** tan 30° **b** tan 40° **c** tan 20°
3 **a** 150° **b** 130° **c** 100°
4 **a** sin 40° **b** −cos 40° **c** −tan 40°
5 **a** 128°41′ **b** 165°6′ **c** 91°2′
6 **a** 11·2 **b** 5·7 **c** 15·4
7 **a** 55° **b** 74° **c** 20°
8 **a** 7·2 **b** 14·5 **c** 28·0
9 **a** 75°31′ **b** 108°3′ **c** 119°11′
10 **a** 26 cm^2 **b** 35 cm^2 **c** 4 cm^2

ASSIGNMENT 15A

1 **a** 150° **b** 120° **c** 135°
2 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{1}{\sqrt{3}}$
3 **a** 11·9 cm (1 dec. pl.)
b 64°17′ (to the nearest minute)
4 **a** $\theta = 43^\circ$, $\phi = 117^\circ$, $x = 15{\cdot}6$ cm
b $\theta = 137^\circ$, $\phi = 23^\circ$, $x = 6{\cdot}9$ cm
5 83° (nearest degree)
6 11·5 km

ASSIGNMENT 15B

1 $x = 32{\cdot}5$ mm, $y = 65$ mm
2 35·25 m
3 12 cm
4 14 h 36 min assuming a steady rate of climb and a slide back that takes only a second or two
5 66
6 16099·1 kg