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VCE®
REVISION QUESTIONS

MATHEMATICAL METHODS UNITS 1 & 2

STUDY DESIGN
FROM 2023

Mathematical Methods Units 1 & 2

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Insight Publications Pty Ltd
3/350 Charman Road
Cheltenham VIC 3192
Australia

Tel: +61 3 8571 4950
Email: books@insightpublications.com.au

www.insightpublications.com.au

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Contents

Introduction

Insight's *VCE Revision Questions: Mathematical Methods Units 1 & 2* contains questions, worked solutions, mark allocations and tips to help you develop skills for your assessment tasks. The questions cover all areas of study in Units 1 and 2 of VCE Mathematical Methods. A good habit to implement is to test yourself by working through this resource. The process of actively recalling information assists with deeper learning, and you will be able to identify any errors or omissions in your working by comparing your answers with the provided solutions.

Questions are grouped by exam sections that appear in the VCE Units 3 and 4 end-of-year exam – Technology-free, Multiple-choice and Extended response – to clearly signal the types of questions you will encounter and what, if any, resources you are allowed to use.

Calculator instructions are included in the worked solutions for Exam 2 questions. For reasons of space, most of these are from a TI-Nspire CAS calculator.

By using this resource as part of your study regime throughout the year, you will be prepared for questions you may encounter in your assessment tasks.

We wish you well with your studies.

The Insight Team

Questions

Section 1 Technology-free

Question 1 (5 marks)

Solve for x:

a. $\frac{x-2}{3} - \frac{x}{2} + \frac{x+1}{4} = 2$ 2 marks

b. $\frac{mx+a}{c} = \frac{cx-a}{m}$ 3 marks

Question 2 (4 marks)

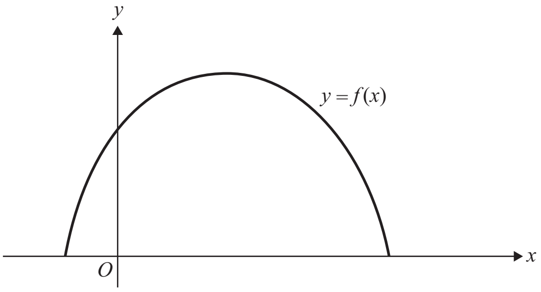
Find the values of p and q for which the simultaneous linear equations $(4-p)x + 2y = q$ and $6y - px = 9$ have

a. infinitely many solutions. 3 marks

b. no solutions. 1 mark

Question 3 (7 marks)

The parabolic shape of a bridge arch is modelled by the function $f(x) = -\frac{4}{5}(x+2)(x-5)$, where x represents the horizontal distance in metres from a point O, and y is the vertical height, in metres, above the water level. The graph of the function shows the cross-sectional shape of the bridge arch.



A straight supporting rod is to be installed from the coordinate $(-2, 0)$ to the coordinate $(0, 8)$.

a. Write the equation of the straight line representing the rod that connects these two points. 1 mark

b. Find the length of the rod, in metres. 1 mark

c. Determine the coordinates of the midpoint of the rod. 1 mark

d. Another supporting rod is to be installed such that it is perpendicular to the line $y = 4x + 5$ and passes through the point $(3, 8)$. Determine the equation of the straight line representing this rod. 2 marks

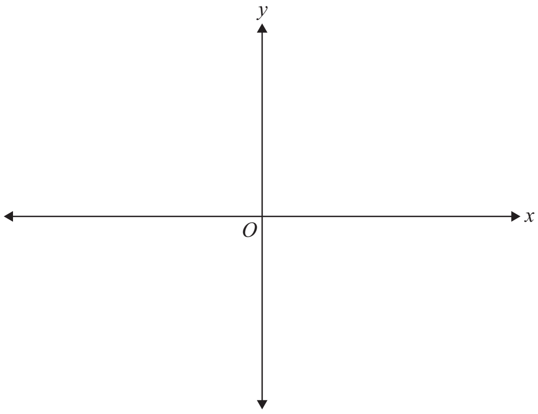
e. The two supporting rods intersect at a point. Determine the coordinates of their intersection. 2 marks

Question 4 (6 marks)

a. i. Fully factorise the expression $2x^2 + 4x - 6$. 1 mark

ii. Express $y = 2x^2 + 4x - 6$ in the form $y = a(x - h)^2 + k$ by completing the square. 2 marks

b. Sketch the graph of $y = 2x^2 + 4x - 6$ on the axes below, labelling axial intercepts and turning points with their coordinates. 3 marks



Question 5 (2 marks)

Prove that the quadratic equation $px^2 - (p-2)x + 2\left(1 - \frac{4}{p}\right) = 0, p \neq 0$ has at least one solution.

Question 6 (3 marks)

Determine the equation of a parabola with its axis of symmetry given by the equation $x = 3$ and passing through the points $(2, 7)$ and $(5, 0)$.

Question 7 (2 marks)

Solve the inequality $-\frac{8}{3}(x-3)^2+\frac{32}{3}<0$.

Question 8 (2 marks)

Find the values of k, such that $(k-12)x^2+2(k-12)x+2=0$ has exactly one solution.

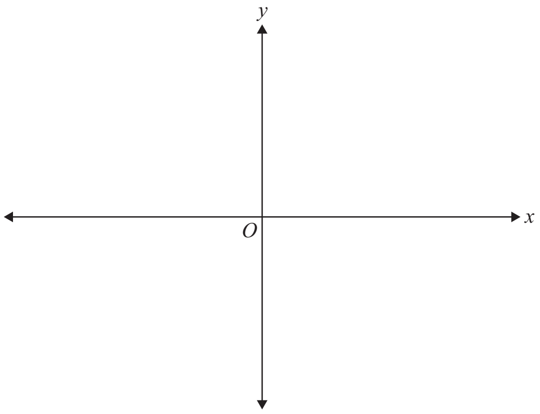
Question 9 (2 marks)

Alex is setting up a rectangular dog run in their backyard. They have 24 metres of fencing material to enclose three sides of the run, using the back wall of their house as the fourth side.

What is the maximum area Alex can enclose for the dog run?

Question 10 (4 marks)

a. Sketch the graph of $f:[-16, 9) \rightarrow R, f(x) = -\sqrt{x+16}$, labelling the coordinates of all intercepts and end points. 2 marks

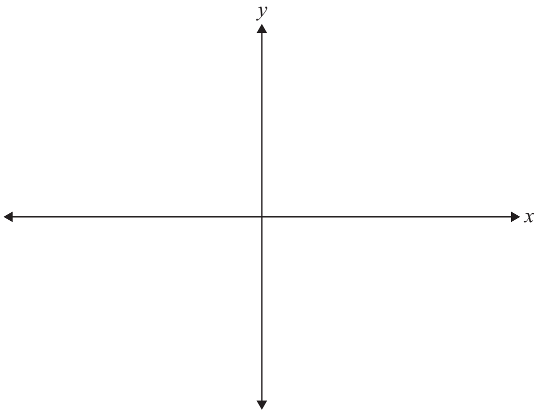


b. Describe, in words, the sequence of transformations that takes the graph of $g(x) = \sqrt{x}$ to the graph of $f(x) = -\sqrt{x+16}$. 2 marks

Question 11 (6 marks)

a. Show that $\frac{-5-3x}{x+1} = \frac{-2}{x+1} - 3$. 2 marks

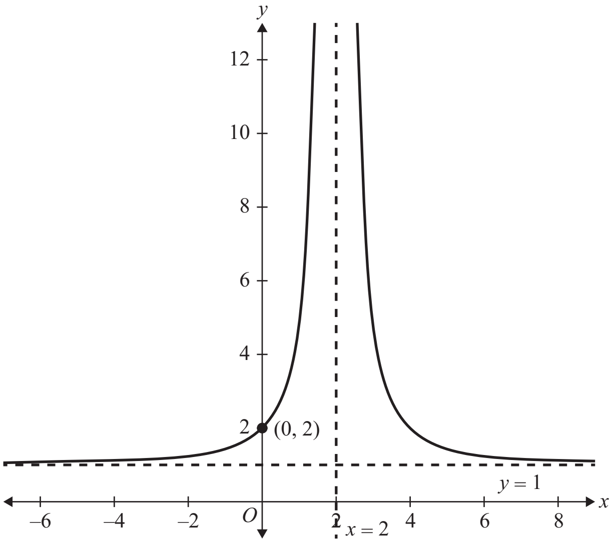
b. Hence, sketch the graph $y = \frac{-5-3x}{x+1}$. 3 marks



c. Hence, or otherwise, solve $\frac{-5-3x}{x+1} < 0$ for x. 1 mark

Question 12 (3 marks)

Find the equation of the graph shown below.



Question 13 (7 marks)

Consider the function, $f:[k,5] \rightarrow R$, $f(x) = 3 - (x - 1)^2$, where k is the smallest real value such that f has an inverse function.

a. Find k. 1 mark

b. Find the rule of the inverse function, f^{-1}. 2 marks

c. State the domain of f^{-1}. 1 mark

d. Find the intersection of $y = f(x)$ and $y = f^{-1}(x)$. 3 marks

Question 14 (4 marks)

Consider the function

$$g(x) = \begin{cases} mx + 4 & x \leq -2 \\ 2 - x^2 & -2 < x < 3 \\ \sqrt{3x} - 10 & 3 \leq x \leq 5 \end{cases}$$

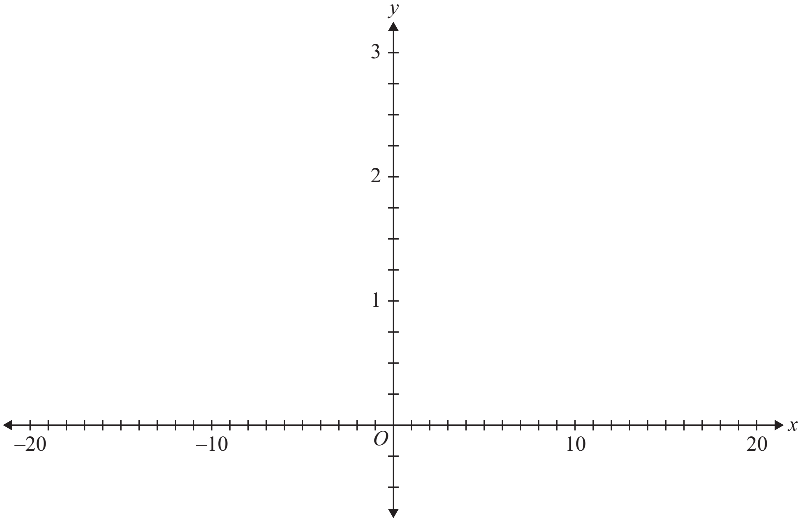
a. State the values of $g(1)$ and $g(4)$. 2 marks

b. Find the value of m, such that the branches of the piecewise-defined function $g(x)$ are connected. 2 marks

Question 15 (4 marks)

a. Sketch the graph of the following relation on the axes below, labelling any asymptotes and the coordinates of intercepts and end points. 3 marks

$$y = \begin{cases} (x+8)^{\frac{1}{3}} & \text{for } 0 \le x < 19 \\ \dfrac{1}{(x-2)^2} + 1 & \text{for } x < 0 \end{cases}$$



b. Explain whether or not the relation given in **part a.** is a function. 1 mark

Question 16 (4 marks)

a. State the possible rational roots of the equation $2x^3 - x^2 - 8x + 4 = 0$. 2 marks

b. Hence, or otherwise, factorise the expression $2x^3 - x^2 - 8x + 4$. 2 marks

Question 17 (8 marks)

Consider the polynomial $P(x) = 3x^3 - 8x^2 + 3x + 2$.

a. Find an expression for $P(2a)$. 1 mark

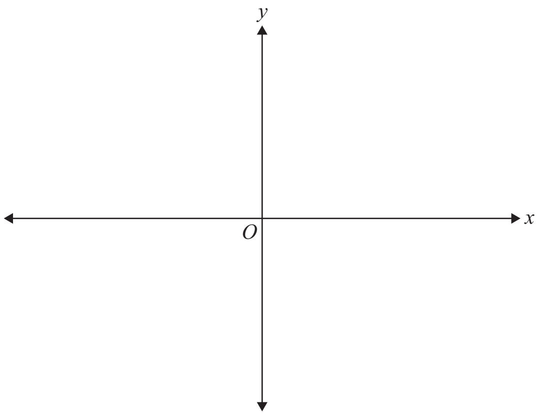
b. Find the remainder when $P(x)$ is divided by $(x - 3)$. 1 mark

c. Show that $P(2) = 0$. 1 mark

d. Hence, factorise $P(x) = 3x^3 - 8x^2 + 3x + 2$ into linear factors. 2 marks

e. Hence, solve $3x^3 - 8x^2 + 3x + 2 = 0$ for x. 1 mark

f. Sketch the graph of $y = 3x^3 - 8x^2 + 3x + 2$ on the axes below, labelling all axis intercepts with their coordinates. 2 marks



Question 18 (4 marks)

a. Let $x^3 + ax - 7 = x^3 + bx^2 - 5x - 7$. Find the values of a and b. 1 mark

b. Given that $x^3 + 2x^2 - x = x^3 + a(x - 1)^2 + b(x - 1) + c$, find the values of a, b and c. 3 marks

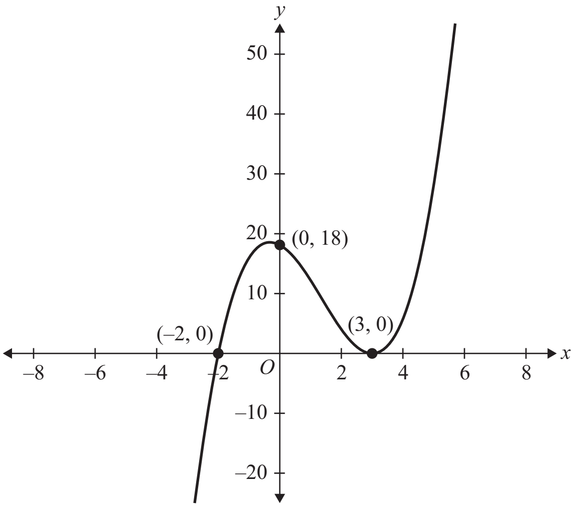
Question 19 (2 marks)

Solve $(x + 4)(x - 1)(3 - x) > 0$ for x.

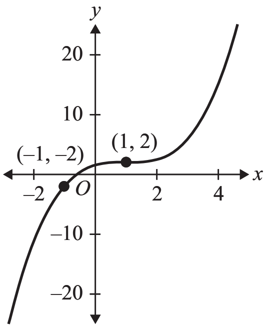
Question 20 (7 marks)

a. Find the rules of the functions $y = f(x)$ shown below.

i. 2 marks



ii. 2 marks



b. A cubic function has the rule $f(x) = ax^3 - 3x^2 + 2x + b$. The graph of $y = f(x)$ passes through the points $(1, 2)$ and $(-1, -4)$. Find the rule of $f(x)$.

3 marks

Question 21 (4 marks)

a. Write down the sequence of transformations that takes the graph $y = \frac{1}{x}$ to the graph $y = \frac{-3}{x-2} + 1$.

2 marks

b. Write down the sequence of transformations that takes the graph $y = \frac{3}{(5x+2)^2} - 1$ to the graph $y = \frac{1}{x^2}$.

2 marks

Question 22 (3 marks)

The graph of $y = \sqrt{x - 3}$ undergoes the following sequence of transformations:

- reflection in the y-axis
- translation of 2 units in the positive direction of the y-axis and a translation of 2 units in the negative direction of the x-axis
- dilation by a factor of $\frac{1}{2}$ from the y-axis.

a. Find the equation of the transformed function. 2 marks

b. Find the domain of the transformed function. 1 mark

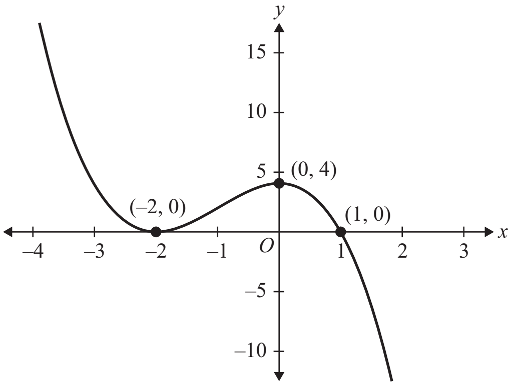
Question 23 (3 marks)

a. Find the image of the point $(3, 7)$ under the linear transformation defined by the rule $(x, y) \rightarrow (-3x - 2, 4y + 3)$. 1 mark

b. A transformation is defined by the rule $(x, y) \rightarrow (-3x - 2, 4y + 3)$. Find the equation of the image of the graph $y = x^2 - 2x$ under this transformation. 2 marks

Question 24 (4 marks)

The graph of $h(x) = (x + 2)^2(1 - x)$ is shown below. The cubic graph has an x-intercept at $(1, 0)$ and has turning points at $(-2, 0)$ and $(0, 4)$.



a. For the graph $y = h(x - a)$, find the value(s) of a for which the graph can have

i. exactly one non-zero x-intercept on the positive x-axis. 1 mark

ii. two negative x-intercepts. 1 mark

b. Let $g(x) = -h(x + 1) - 1$.

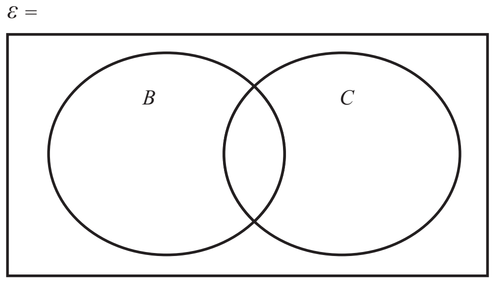
i. Give the sequence of transformation that takes the function $y = h(x)$ to the function $y = g(x)$. 1 mark

ii. Hence, state the turning points of the function $y = g(x)$. 1 mark

Question 25 (4 marks)

A survey investigating the prevalence of risk factors associated with heart disease is conducted with 40 mathematics teachers. Ten teachers are regarded as healthy with no risk factors. The remaining teachers have high blood pressure, a high level of cholesterol or both. Of the 40 teachers examined, 15 have high blood pressure and 25 have a high level of cholesterol.

a. Let B be the number of teachers with high blood pressure and let C be the number of teachers with a high level of cholesterol. Illustrate the information given by completing the following Venn diagram. 2 marks



b. Calculate the probability that a teacher has high blood pressure given that this individual has a high level of cholesterol. 2 marks

Question 26 (6 marks)

For two events A and B, it is known that $\Pr(A \cup B) = 0.75$, $\Pr(A') = 0.42$ and $\Pr(B) = 0.55$.

a. Form a probability table for these two events. 2 marks

b. Find $\Pr(A'|B)$. 1 mark

c. Find $\Pr(A' \cup B')$. 1 mark

d. Are the events A and B independent? Explain your answer by providing a mathematical reasoning. 2 marks

Question 27 (5 marks)

Leon walks to school three days a week and is taken by car on the other two days. If he walks to school, he arrives late 10% of the time. If he is taken by car, he arrives late 20% of the time.

a. Draw a tree diagram to show the possible outcomes and their probabilities. 2 marks

b. What is the probability that on a particular day Leon was late to school? 1 mark

c. If on a particular day Leon was late to school, calculate the probability that he was taken to school by car. 2 marks

Question 28 (4 marks)

Sophie wishes to redecorate her room.

a. She can't decide whether to paint or wallpaper one of the walls in her room. She has six wallpaper patterns and seven paint colours to choose from. How many different choices does she have to wallpaper or paint the wall of her room? 1 mark

b. Sophie has now decided to paint the ceiling and one of the walls of her room. She has eight paint colours for the ceiling, using ceiling paint, and seven paint colours for the wall, using wall paint. If she uses only one colour for each surface, how many different ways can she paint her room? 1 mark

c. Sophie has six different photo frames to choose from to create a display on a shelf in her room.

i. How many different ways can she display a row of all six photo frames on the shelf? 1 mark

ii. How many different ways can Sophie display a row of two photo frames on the shelf from the six photo frames she has to choose from? 1 mark

Question 29 (3 marks)

Adam chooses three pieces of bubble gum from a jar containing four red, four blue and two green pieces of bubble gum. Calculate the probability that Adam selects

a. at least one red bubble gum. 1 mark

b. at least one red and at least one green bubble gum. 2 marks

Question 30 (4 marks)

Simplify the following expressions.

a. $\dfrac{2^{2n} \times 4^{n-2}}{16^n}$ 2 marks

b. $3\log_3(m) + \log_3(36) - \log_3(4) - 6\log_3(\sqrt{m})$ 2 marks

Question 31 (2 marks)

Solve the inequation $\frac{49^{x+3} \times 7^{1-3x}}{49} > 343$.

Question 32 (5 marks)

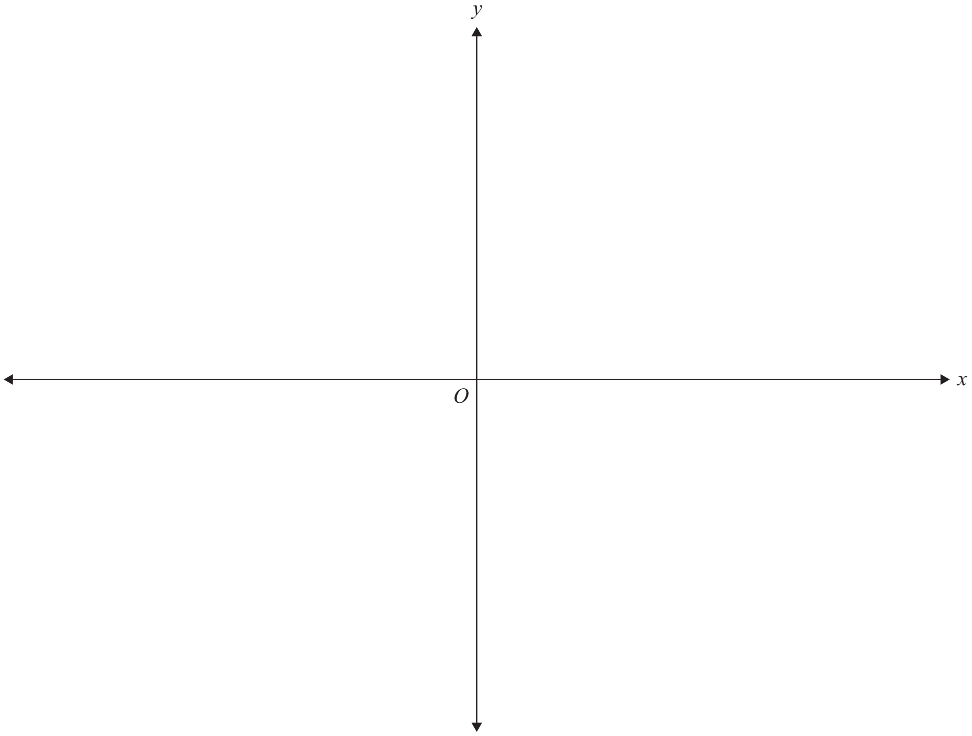
Solve the following equations for x.

a. $\log_{10} x = \frac{1}{2}\log_{10} 225 - 2\log_{10} 3$ 2 marks

b. $8(2^{2x}) - 15(2^{x}) - 2 = 0$ 3 marks

Question 33 (7 marks)

a. Sketch the graph of $f(x) = -\log_2(x + 2) - 1$ on the axes below. Label all axial intercepts and asymptotes. 3 marks



b. State the domain of the function $f(x)$. 1 mark

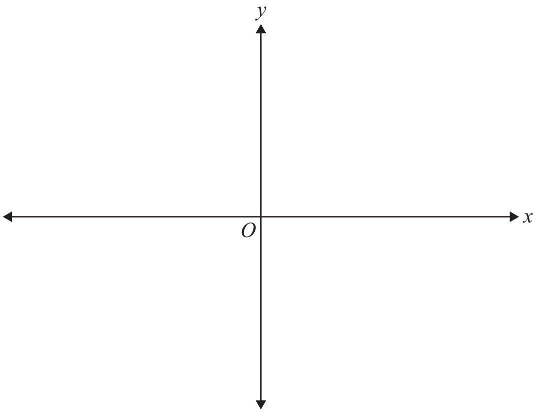
c. Find the rule of the inverse function $f^{-1}(x)$. 2 marks

d. State the domain of the inverse function $f^{-1}(x)$. 1 mark

Question 34 (5 marks)

Let $g : R \to R$, $g(x) = -2 + 2^{-3x}$.

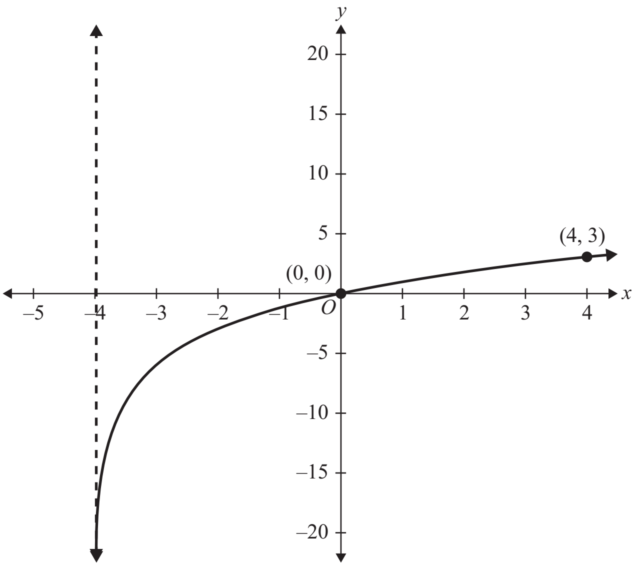
a. Sketch the graph of $g(x)$ on the axes below. Label the asymptote with its equation and all axial intercepts with their coordinates. 3 marks



b. List the transformations needed to obtain $h(x) = 2^x$ from the function $g(x)$. 2 marks

Question 35 (3 marks)

The graph of a logarithmic function is shown below.



The rule of the function is of the form $y = a\log_2(x + b) + c$, where a, b and c are real constants. Given that the graph passes through the origin and the point $(4, 3)$, find the values of a, b and c.

Question 36 (3 marks)

Solve $2\cos(2x) + \sqrt{3} = 0$ for x, where $0 \leq x \leq 2\pi$.

Question 37 (2 marks)

Given that $\cos x = -\frac{5}{13}$, where $\pi \leq x \leq \frac{3\pi}{2}$, find the value of $\sin(x)$ and $\tan(x)$.

Question 38 (3 marks)

For two angles $0 < \alpha < \frac{\pi}{2}$ and $0 < \beta < \frac{\pi}{2}$, it is known that $\sin(\alpha) = \frac{2}{5}$ and $\cos(\beta) = \frac{1}{5}$.

Using this information, determine

a. $\sin(-\alpha)$ 1 mark

b. $\cos(3\pi - \beta)$ 1 mark

c. $\cos\left(\frac{3\pi}{2} + \alpha\right)$ 1 mark

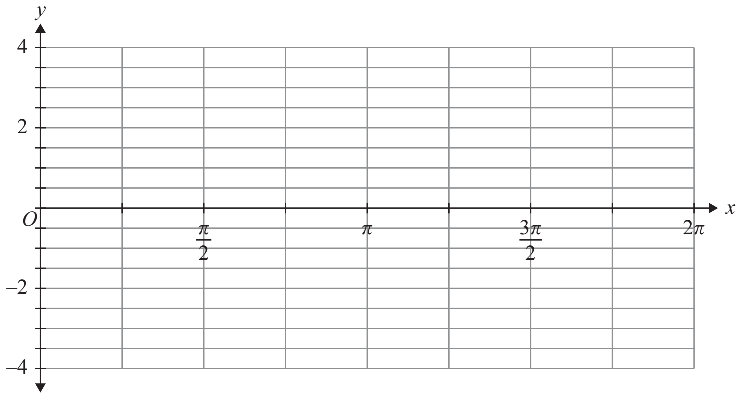
Question 39 (6 marks)

Consider the function $f(x) = 2\sin(x) - \sqrt{3}, x \in [0, 2\pi]$.

a. State the amplitude and the period of the function. 1 mark

b. Find the x-intercepts and y-intercept of the graph of $f(x)$. 2 marks

c. Sketch the graph of $y = f(x)$ on the axes provided below. Label all axis intercepts, turning points and end points with their coordinates. 3 marks

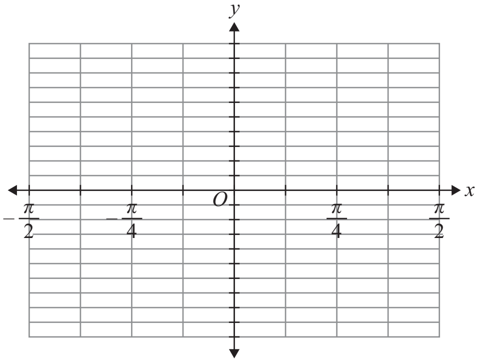


Question 40 (4 marks)

Consider the function $f(x) = \tan(2x) - 1$.

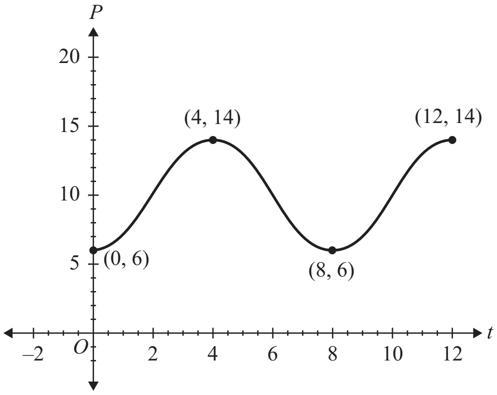
a. State the period of this function. 1 mark

b. Sketch the graph of $f(x) = \tan(2x) - 1, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ on the axes below, labelling all axial intercepts, asymptotes and end points. 3 marks



Question 41 (3 marks)

The graph below represents the changing share price, $\$P$, of a company at time t months during the 12 months of 2015.



a. What is the average rate of change of the company's share price during the first four months of 2015? 2 marks

b. For what values of t is the share price of the company strictly increasing during the 12 months of 2015? 1 mark

Question 42 (5 marks)

a. Differentiate $\frac{3x^3}{50} - x^2 + \frac{x}{6} - 25$ with respect to x. 1 mark

b. If $f(x) = (x - 3)^2(x + 1)$, find $f'(x)$. 2 marks

c. If $f(x) = 4\sqrt{x} - \frac{3}{x^2}$, find $f'(4)$. 2 marks

Question 43 (4 marks)

Consider the function $f(x) = 3x^2 - x$.

a. Show that the gradient of the secant passing through the point where $x = 1$ and $x = 1 + h$ is given by $3h + 5$. 2 marks

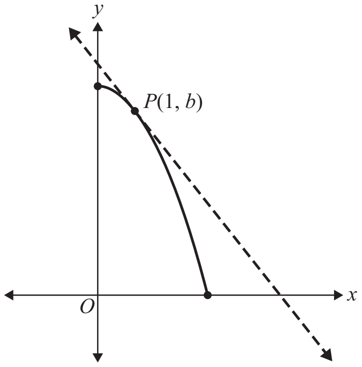
b. Hence, find the gradient of the

i. secant passing through the points where $x = 1$ and $x = 3$. 1 mark

ii. tangent at $x = 1$. 1 mark

Question 44 (5 marks)

Part of the graph of $f(x) = 9 - x^2$ is shown below. The tangent to the curve at the point $P(1, b)$ has also been drawn on the graph.



a. Find b and, hence, show that the equation of the tangent to the graph at P is $y = -2x + 10$. 3 marks

b. Determine the coordinates of the axial intercepts of the tangent to the graph at P. 2 marks

Question 45 (4 marks)

a. Find the antiderivative of $3\sqrt{x} - 3x^2$ with respect to x. 2 marks

b. Evaluate $\int_{-1}^{2}(x^3 + 2x)dx$. 2 marks

Question 46 (3 marks)

Consider the curve with equation $y = \frac{2}{x^2}$.

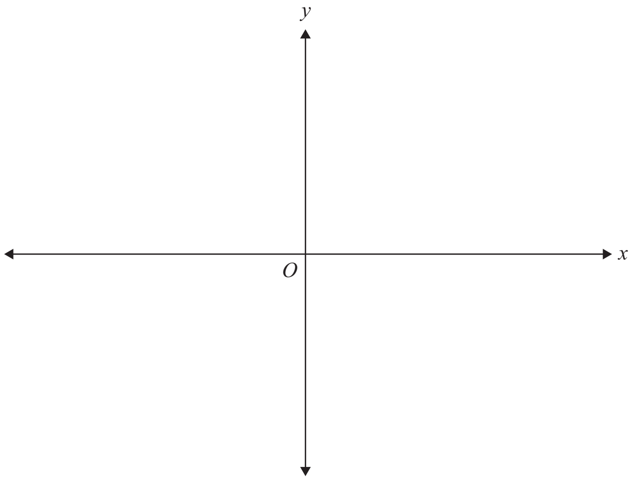
Find the equation of a line perpendicular to the tangent at the point $x = -2$.

Question 47 (6 marks)

a. Find the coordinates of the stationary points of the curve $y = x^3 - 27x$.

State the nature of each stationary point. 3 marks

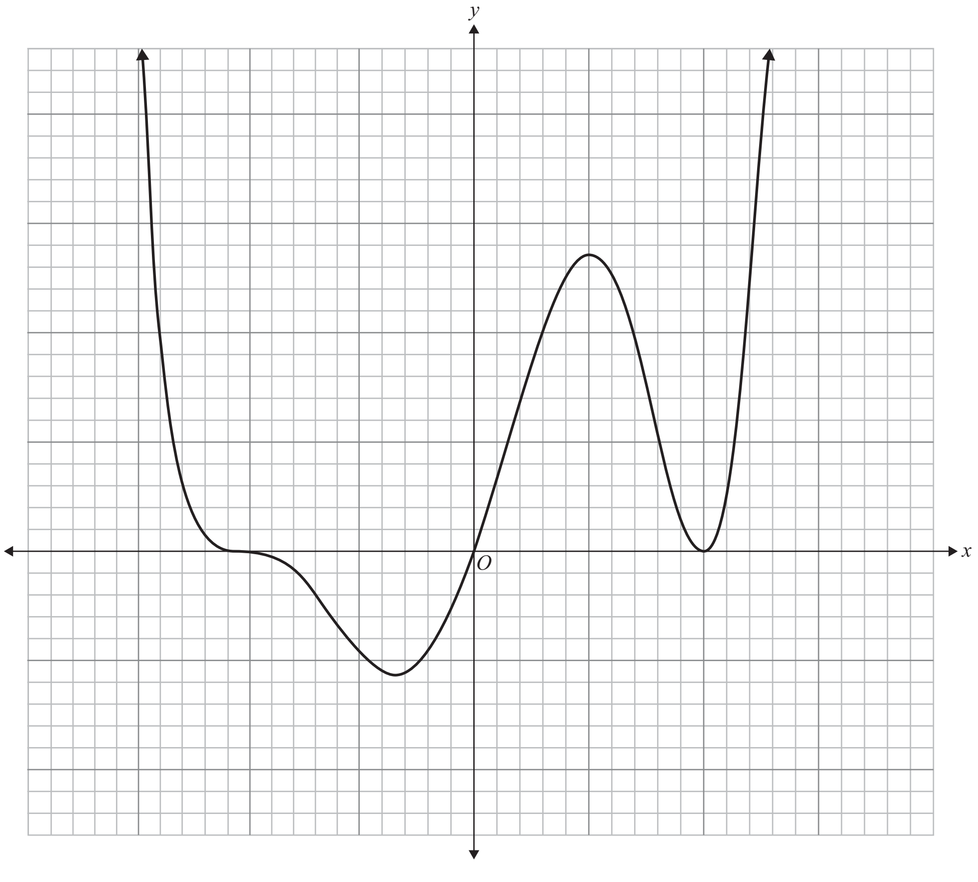
b. Sketch the graph of $y = x^3 - 27x$. Label all axial intercepts and turning points. 3 marks



Question 48 (2 marks)

The graph of $y = f(x)$ is shown below.

On the same axes, sketch the graph of $y = f'(x)$.



Question 49 (3 marks)

The curve with equation $y = f(x)$ has a gradient function with rule $f'(x) = 4x^2 + k$, where k is a constant, and has a turning point with coordinates $(-1, 6)$.

Find the

a. value of k. 1 mark

b. rule for $f(x)$. 2 marks

Question 50 (6 marks)

A cuboid has a total surface area of 500 cm^2. The base of the cuboid is a square of side length x cm.

a. Show that the height of the cuboid can be expressed as $h = \dfrac{250 - x^2}{2x}$. 2 marks

b. Hence, write down an expression for the volume, V, in terms of x. 1 mark

c. Find the value of x that gives the maximum volume.

Express your answer in the form $x = \dfrac{a\sqrt{b}}{c}$, where a, b and c are integers. 3 marks

Section 2 Multiple-choice

Question 1

The distance between the points A and B is $\sqrt{17}$. The coordinates of A and B could be

A. $(3, -1)$ and $(0, 7)$

B. $(2, 2)$ and $(20, 3)$

C. $(3, 1)$ and $(4, 17)$

D. $(-1, 3)$ and $(0, 7)$

Question 2

The midpoint of the points $(a, 3)$ and $(2, b)$ is $(-1, 4)$. The values of a and b are

A. $a = -4$, $b = 5$

B. $a = 0$, $b = 5$

C. $a = 0$, $b = 11$

D. $a = -4$, $b = 6$

Question 3

The line with equation $y = mx + c$ is perpendicular to the line with equation $y = 7 - \frac{x}{5}$.
The value of m is

A. -5

B. $\frac{1}{5}$

C. 5

D. $-\frac{1}{5}$

Question 4

Which two of the following four equations represent parallel lines?

I. $y - 2 = 3x$

II. $6x - 2y = 7$

III. $3y = 3x - 11$

IV. $y = -3x + 5$

A. III, IV

B. I, III

C. II, III

D. I, II

Question 5

If $\sqrt{\frac{a}{b}} = c^2 - \frac{e}{\sqrt{c}}$, then the value of e when $a = 2.8$, $b = 3.6$ and $c = 4.2$, is closest to

A. 34.34

B. 33.82

C. 22.27

D. 24.05

Question 6

To the nearest degree, the line with equation $5x + 12y - 3 = 0$ is inclined to the positive direction of the x-axis at an angle of

A. 23°

B. 157°

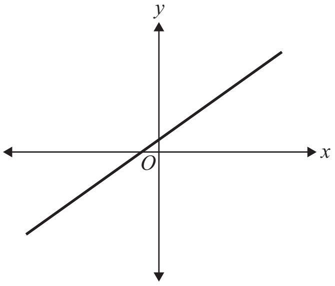
C. 127°

D. 150°

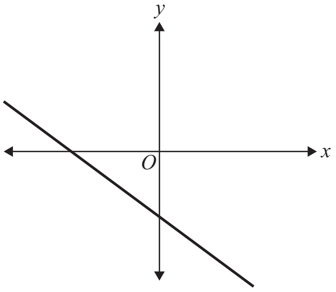
Question 7

Which one of the following graphs best represents $x + y = k$ when $k < 0$?

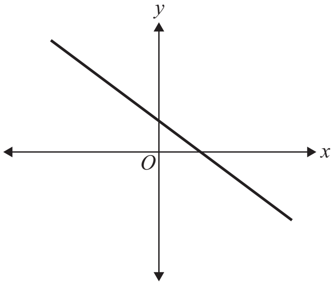
A.



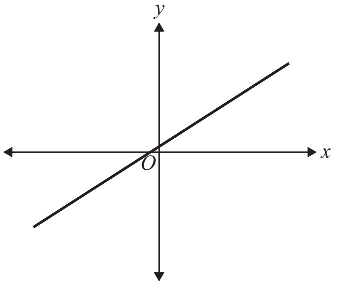
B.



C.



D.



Question 8

The parabola with equation $y = x^2$ is dilated by a factor of 2 from the x-axis, translated 3 units to the right and 4 units down. The transformed equation is

A. $y = 2(x - 3)^2 - 4$

B. $y = \left(\frac{x}{2} - 3\right)^2 - 4$

C. $y = \frac{1}{2}(x + 3)^2 - 4$

D. $y = 2(x - 3)^2 + 4$

Question 9

For $y = -(3x - 7)^2 - 5$, the maximum value of y is

A. 5

B. -5

C. $\frac{7}{3}$

D. $-\frac{7}{3}$

Question 10

If $m + \frac{1}{m} = 8$, then the value of $m^2 + \frac{1}{m^2}$ is

A. 64

B. -64

C. -66

D. 62

Question 11

The x-coordinates of the point(s) of intersection of the parabola $x^2 + x - 2$ and the line $y = 3x - 2$ are

A. $x = 0$ and $x = 2$

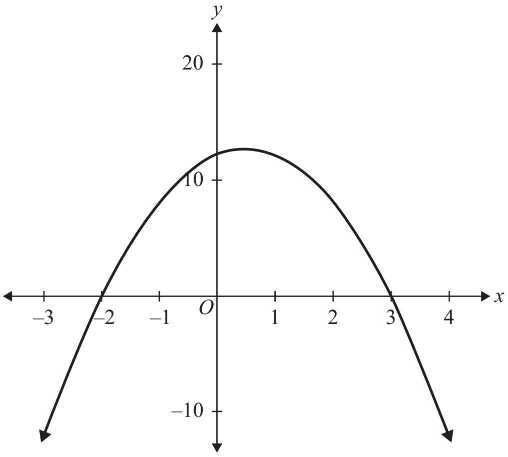
B. $x = -2$ and $x = 0$

C. $x = -2$ and $x = 2$

D. $x = \sqrt{2}$ and $x = 0$

Question 12

The graph of the parabola shown passes through the point $(-1, 8)$.



The equation of the parabola is

A. $y = -2x^2 - 2x + 12$

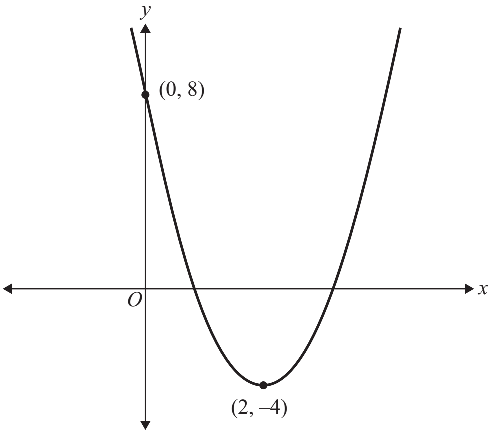
B. $y = -2x^2 + 2x + 12$

C. $y = -2x^2 - 2x - 12$

D. $y = 2x^2 - 2x + 12$

Question 13

The graph of $y = a\left[(x-h)^2 + n\right]$ is shown below.



The values of a, h and n are

A. $a = 3,\ h = 2,\ n = -\frac{4}{3}$

B. $a = 3,\ h = 2,\ n = -4$

C. $a = 3,\ h = 2,\ n = \frac{4}{3}$

D. $a = 3,\ h = –4,\ n = 2$

Question 14

The solution of the inequality $-5x^2 \geq 15x$ is

A. $0 < x < 3$

B. $-3 \leq x \leq 0$

C. $-3 < x < 0$

D. $x \leq -3$ or $x \geq 0$

Question 15

If $h(x) = (x-2)(x+4)$, then $h(a-4)$ is equal to

A. $a^2 - 6a$

B. $a^2 - 16$

C. $a(a-4)$

D. $(a-4)^2 - 8$

Question 16

The domain and range, respectively, of the function $f(x) = -\sqrt{25 - x^2}$ are given by

A. domain = [–5, 0], range = [–5, 0]

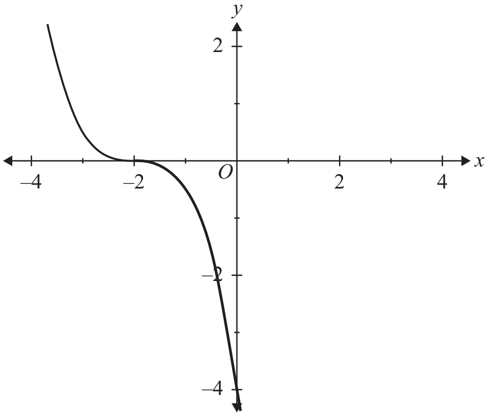
B. domain = [–5, 5], range = [–5, 0]

C. domain = [–5, 5], range = [0, 5]

D. domain = [–5, 5], range = [–5, 5]

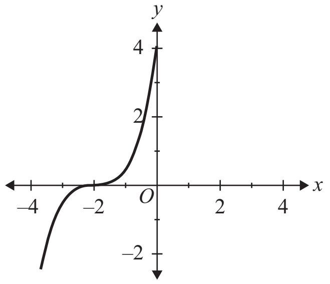
Question 17

A graph of the equation $y = f(x)$ is shown below.

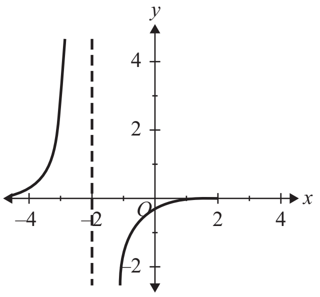


Which one of the following graphs could be the graph of the inverse function, $y = f^{-1}(x)$?

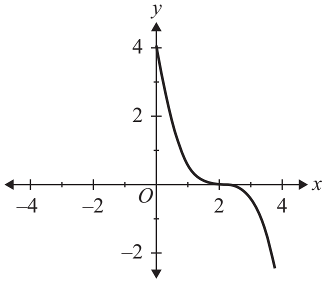
A.



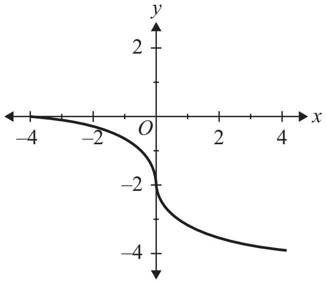
B.



C.



D.



Question 18

If b and c are real numbers, the equations of the asymptotes of the graph of $y = 3\left(\frac{1}{(x+b)^2} - c\right)$ are

A. $x = -b, y = 3c$

B. $x = -b, y = -c$

C. $x = b^2, y = 3c$

D. $x = -b, y = -3c$

Question 19

The range of the function $f:(-4, 1] \rightarrow R, f(x) = x^2 + 6x$ is

A. $[-9, \infty)$

B. $(-9, 7]$

C. $[-9, 7]$

D. $[-9, 0)$

Question 20

A suitable restriction of the domain of the function $y = -x^2 + 6x + 7$ for an inverse function to exist is

A. $(-\infty, 5]$

B. $(-\infty, 4]$

C. $(0, \infty)$

D. $[3, \infty)$

Question 21

The inverse of the function $f:(-\infty, 2] \rightarrow R, f(x) = (x-2)^2 + 1$ is

A. $f^{-1}:[1, \infty) \rightarrow R, f^{-1}(x) = -\sqrt{x-1} + 2$

B. $f^{-1}:(-\infty, 2] \rightarrow R, f^{-1}(x) = -\sqrt{x-1} + 2$

C. $f^{-1}:[1, \infty) \rightarrow R, f^{-1}(x) = \sqrt{x-1} + 2$

D. $f^{-1}:(-\infty, 1] \rightarrow R, f^{-1}(x) = \sqrt{x-1} + 2$

Question 22

Which one of the following is **not** a function?

A. $y = \frac{1}{x-3} + 7$

B. $y = -\frac{3}{x^2} - 7$

C. $y^2 = x - 7$

D. $y = \sqrt{x-7}$

Question 23

The equations of the asymptotes of the function $f(x) = \frac{x-2}{x+3}$ are

A. $x = -3$ and $y = 1$

B. $x = -2$ and $y = 3$

C. $x = -3$ and $y = -1$

D. $x = -2$ and $y = -3$

Question 24

The maximal domain of the function $f(x) = -\sqrt{2-x} + 3$ is

A. $[2, \infty)$

B. $[-2, \infty)$

C. $(-\infty, 2]$

D. $(-\infty, -2]$

Question 25

When $x^3 + x^2 - 3x + 6$ is divided by $x - 2$, the remainder is

A. 4

B. 12

C. -12

D. -4

Question 26

$2x - 5$ is a factor of $x^3 - 8x^2 + 2ax + 10$. The value of a is

A. $\frac{39}{8}$

B. $-\frac{39}{8}$

C. $\frac{37}{8}$

D. $-\frac{37}{8}$

Question 27

When the algorithm below is executed, what is the printed value of a?

```
$a \leftarrow 1$
for $n$ from 1 to 3
        $a \leftarrow a \times n^3$
end for
print $a$
```

A. 27

B. 216

C. 576

D. 96

Question 28

$P(x) = x^3 + ax^2 + bx - 8$ equals zero at $x = 1$ and $x = -2$. The values of a and b are

A. $a = 1, b = -2$

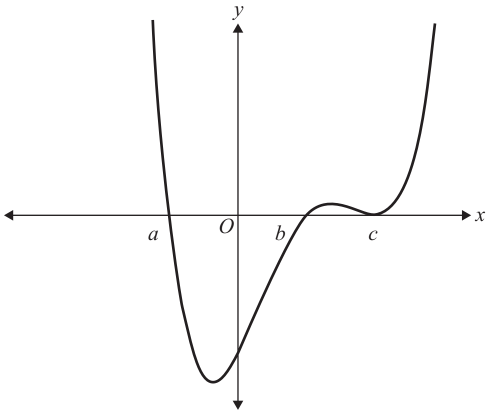
B. $a = 5, b = 2$

C. $a = -1, b = 2$

D. $a = -5, b = -2$

Question 29

The graph of the function f is shown below.



The rule for the function is mostly likely to be

A. $f(x) = (x - a)(x - b)(x - c)^2$

B. $f(x) = (x - a)(x - b)^2(x - c)$

C. $f(x) = (x + a)(x - b)(x - c)^2$

D. $f(x) = (a - x)(x - b)(x - c)^2$

Question 30

If the graph of $y = \sqrt{x}$ is translated by 3 units in the positive x direction, dilated by a factor of 2 from the y-axis and reflected in the y-axis, then the equation of the transformed graph is

A. $y = \sqrt{2x + 3}$

B. $y = \frac{\sqrt{-2x + 12}}{2}$

C. $y = \sqrt{-2x - 3}$

D. $y = \frac{\sqrt{-2x - 12}}{2}$

Question 31

The image of the point $(2, -5)$ after it has been reflected in the x-axis and dilated by a factor of 3 from the y-axis will be

A. $(-2, 15)$

B. $(2, -15)$

C. $(6, 5)$

D. $(-6, 5)$

Question 32

The graph of $y = -5 + 3x$ is reflected in the y-axis, translated 3 units up and 2 units left. The new rule for the graph is

A. $y = -8 + 3x$

B. $y = -2 - 3x$

C. $y = -2 + 3x$

D. $y = -8 - 3x$

Question 33

If the graph of $y = \frac{1}{(x+1)^2} + 3$ has been reflected in both axes, then the rule for this graph will be

A. $y = \frac{-1}{(x+1)^2} + 3$

B. $y = \frac{-1}{(x-1)^2} + 3$

C. $y = \frac{1}{(x+1)^2} - 3$

D. $y = \frac{-1}{(x-1)^2} - 3$

Question 34

The graph of $y = 3f(2x-1) -1$ is obtained from the graph of $y = f(x)$ under the transformation T.

A possible rule for T is

A. $T: R^2 \rightarrow R^2, T(x, y) = \left(\frac{x+1}{2}, 3y - 1\right)$

B. $T: R^2 \rightarrow R^2, T(x, y) = (2x - 1, 3y - 1)$

C. $T: R^2 \rightarrow R^2, T(x, y) = \left(\frac{x-1}{2}, 3y + 1\right)$

D. $T: R^2 \rightarrow R^2, T(x, y) = \left(2x - 1, \frac{y+1}{3}\right)$

Question 35

The transformation sequence

- a translation of 1 unit in the positive direction of the x-axis
- a translation of 2 units in the negative direction of the y-axis
- a dilation of factor 2 from the y-axis
- a reflection in the x-axis

is characterised by which one of the following transformation rules?

A. $T: R^2 \rightarrow R^2, T(x, y) = \left(\frac{x}{2}, y + 2\right)$

B. $T: R^2 \rightarrow R^2, T(x, y) = (2x + 2, -y + 2)$

C. $T: R^2 \rightarrow R^2, T(x, y) = \left(\frac{x+1}{2}, -y + 2\right)$

D. $T: R^2 \rightarrow R^2, T(x, y) = (2x + 2, -y - 2)$

Question 36

If $\Pr(A) = 0.67$, $\Pr(B) = 0.3$ and $\Pr(A \cap B') = 0.47$, then $\Pr(A' \cap B)$ is equal to

A. 0.10

B. 0.33

C. 0.20

D. 0.23

Question 37

If A and B are independent events, where $\Pr(A) = 0.6$ and $\Pr(A \cap B) = 0.24$, then $\Pr(B)$ is equal to

A. 0.02

B. 0.04

C. 0.02

D. 0.4

Question 38

A and B are independent events. If $\Pr(A) = \frac{3}{10}$ and $\Pr(A \cup B) = \frac{4}{5}$, then $\Pr(B)$ is equal to

A. $\frac{1}{2}$

B. $\frac{7}{10}$

C. $\frac{5}{7}$

D. $\frac{1}{3}$

Question 39

A and B are events of a sample space, ξ. If $\Pr(A \cap B) = \frac{2}{5}$ and $\Pr(A' \cap B) = \frac{2}{7}$, then $\Pr(A'|B)$ is equal to

A. $\frac{5}{12}$

B. $\frac{24}{35}$

C. $\frac{48}{245}$

D. $\frac{4}{35}$

Question 40

A and B are events of a sample space, S. If $A \subseteq B$, which one of the following statement is true?

A. $\Pr(A \cup B) = A$

B. $\Pr(A \cap B) = B$

C. A and B are mutually exclusive events

D. $\Pr(A \cup B) = B$

Question 41

What is the probability that a random arrangement of the letters in the word **UNIVERSAL** begins and ends with a vowel?

A. $\frac{1}{15120}$

B. $\frac{1}{3}$

C. $\frac{2}{9}$

D. $\frac{1}{6}$

Question 42

In how many ways could a soccer team of 11 players be chosen from 14 players?

A. 364

B. 11

C. 1

D. 0

Question 43

The number of ways of selecting the gold, silver and bronze medal winners in a particular athletics event was calculated to be 120. How many athletes took part in the event?

A. 10

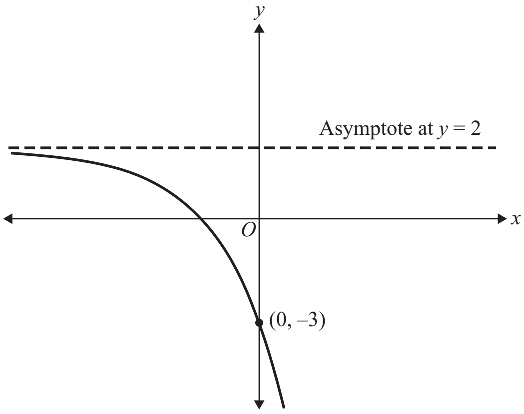
B. 6

C. 5

D. 40

Question 44

The graph of the equation $y = A \times 2^x + B$ is shown below. A and B are constants.



The values of A and B are

A. $A = -3, B = 2$

B. $A = -5, B = 2$

C. $A = -5, B = -3$

D. $A = -1, B = 2$

Question 45

The domain and range of the graph of $y = \log_a(1 - x)$ is given by

A. domain = $(1, \infty)$, range = R

B. domain = $(-1, \infty)$, range = $(-1, \infty)$

C. domain = $(-\infty, 1)$, range = R

D. domain = $(-\infty, -1)$, range = $(-1, \infty)$

Question 46

Which one of the following graphs has the asymptote $y = b$, where $b > 0$?

A. $y = \frac{3}{x + b} - 2$

B. $y = 5^x + b$

C. $y = \log_2(x - b)$

D. $y = \frac{1}{x - 2} - b$

Question 47

Under controlled conditions in an area, the population of a beneficial type of soil bacteria increases by 1.4% yearly. If the population is 300 million at the start of 1975, the population at the end of 2025 will be closest to

A. 610 million

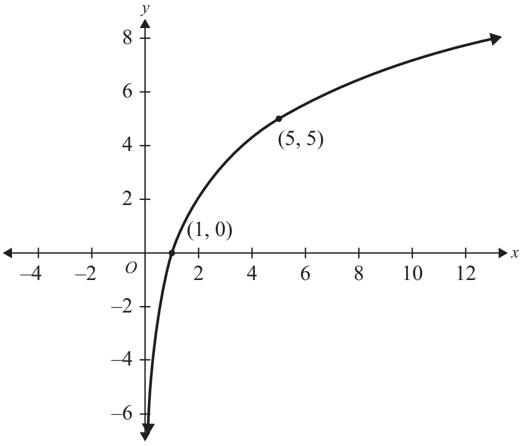
B. 750 million

C. 2100 million

D. 1200 million

Question 48

The graph of the function f is shown below.



The graph of $f(x)$ is most likely to be

A. $f(x) = \log_5(5x)$

B. $f(x) = \log_5\left(\frac{x}{5}\right)$

C. $f(x) = 5\log_5(x)$

D. $f(x) = 5\log_5(5x)$

Question 49

If $\tan(x) = \frac{\sqrt{5}}{2}$ and $\pi < x < \frac{3\pi}{2}$, then $\cos(x)$ equals

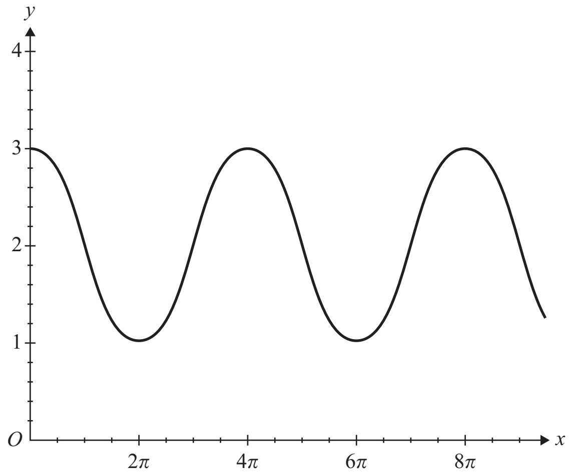
A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. $\frac{\sqrt{5}}{3}$

D. $-\frac{3}{2}$

Question 50



An equation for the graph shown above could be

A. $y = 2 + 2\cos(2x)$

B. $y = 2 + \cos\left(\frac{x}{2}\right)$

C. $y = 2 + 2\cos\left(\frac{x}{2}\right)$

D. $y = 1 + 3\cos\left(\frac{x}{2}\right)$

Question 51

The minimum value of $-5 - 3\sin\left(2x - \frac{\pi}{2}\right)$ is

A. -15

B. -2

C. -8

D. -1

Question 52

The equations of all the asymptotes of the graph with the equation $f(x) = -2\tan\left(\frac{x}{2}\right) + 1$, where $-2\pi \le x \le 2\pi$, are:

A. $x = -2\pi$ and $x = 0$ and $x = 2\pi$

B. $x = -\pi$ and $x = \pi$

C. $x = 0$ and $x = \frac{\pi}{2}$

D. $x = -\pi$ and $x = 2\pi$

Question 53

The equation $-3\cos\left(\frac{x}{2}\right) + 1 = b$ has one solution in the interval $[0, 4\pi]$. The value of b is

A. 4

B. 1

C. -1

D. 3

Question 54

The period of the graph $y = 2\sin\left(\frac{x}{4} - \frac{\pi}{2}\right) + 5$ is

A. 8π

B. $\frac{\pi}{2}$

C. 4π

D. $\frac{\pi}{4}$

Question 55

The number of solutions of $3\cos(4x - \pi) + 2 = 0$ in the interval $[0, 2\pi]$ is

A. 9

B. 6

C. 7

D. 8

Question 56

If $\cos(\beta) = \frac{5}{13}$ and $0 < \beta < \frac{\pi}{2}$, the value of $\sin(3\pi + \beta)$ is

A. $\frac{5}{13}$

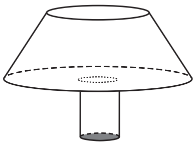
B. $-\frac{5}{13}$

C. $\frac{12}{13}$

D. $-\frac{12}{13}$

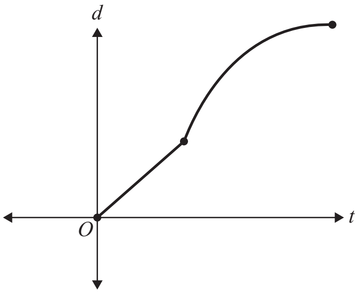
Question 57

Water is poured at a constant rate into the vessel shown below.

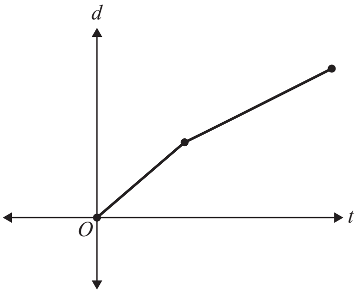


Which one of the following graphs best represents the depth of the water, d, as a function of time, t?

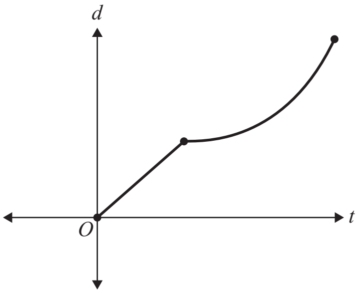
A.



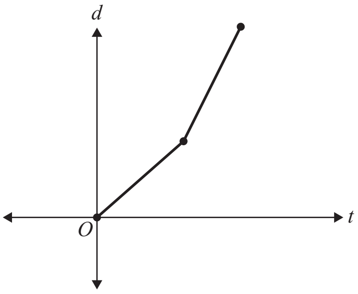
B.



C.



D.



Question 58

The average rate of change of the function $f(x) = 3x^2 - x + 2$ between $x = 1$ and $x = 2$ is

A. 8

B. 4

C. –8

D. –4

Question 59

If the equation $y = ax^3 + bx^2$ has a stationary point at $(2, 1)$, the value of a and b must be

A. $a = \frac{3}{4}, b = -\frac{1}{4}$

B. $a = -4, b = 6$

C. $a = -\frac{1}{2}, b = \frac{3}{2}$

D. $a = -\frac{1}{4}, b = \frac{3}{4}$

Question 60

If $g'(x) = f(x)$, then $\int_0^a f(x)\,dx$ is given by

A. $f(a)$

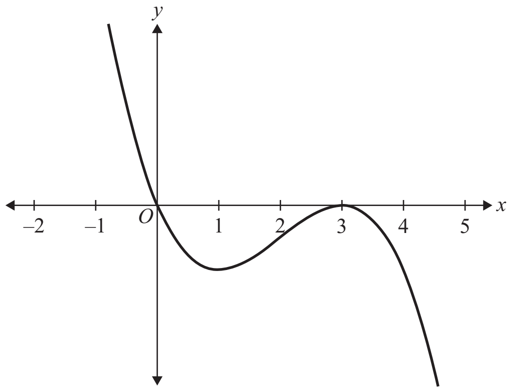
B. $g(a) - g(0) + c$

C. $g(a) - g(0)$

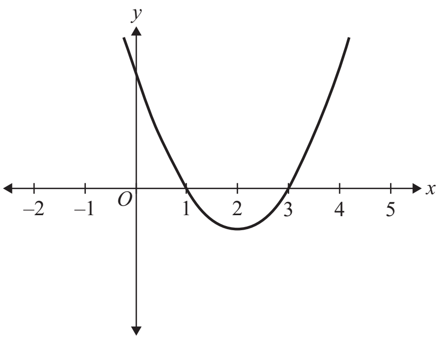
D. $g(a)$

Question 61

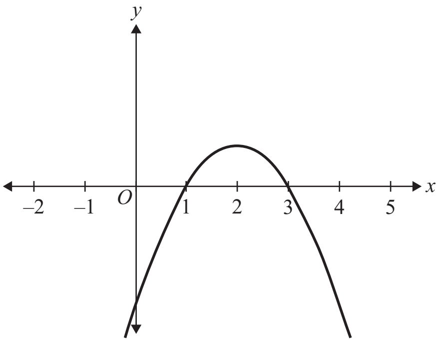
Which one of the following best represents the graph of the gradient function for the function whose graph is shown below?



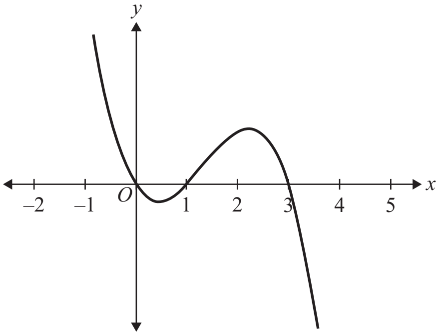
A.



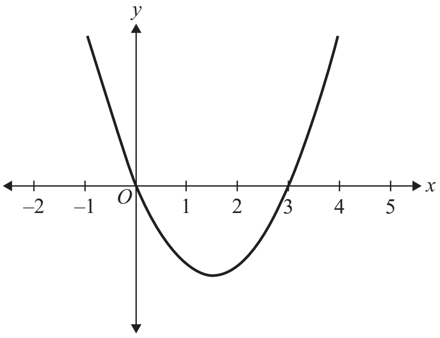
B.



C.



D.



Question 62

If $\frac{dM}{dx} = 3x^2 - 16x + 17$ and $M = -4$ when $x = 3$, then $M(x)$ is given by

A. $x^3 - 8x^2 + 17x - 10$

B. $x^3 - 8x^2 + 17x + 263$

C. $2x - 10$

D. $x^3 - 8x^2 + 17x$

Question 63

If $y = (3x^2 - 5x)^4$, then $\frac{dy}{dx}$ equals

A. $(3x^2 - 5x)(6x - 5)$

B. $(3x^2 - 5x)^3(6x - 5)$

C. $4(3x^2 - 5x)^3(6x - 5)$

D. $4(3x^2 - 5x)^3$

Question 64

The gradient of the tangent to the curve $y = 5x^2 + 4$ at the point where $x = -3$ is equal to

A. -30

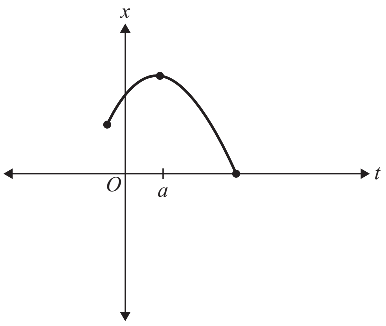
B. -15

C. -11

D. 49

Question 65

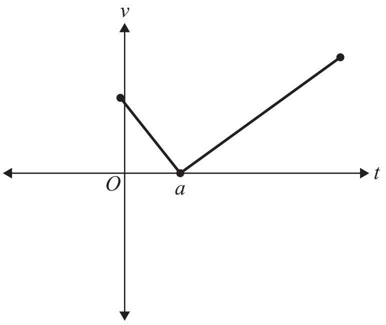
A ball is thrown into the air off a platform and lands on the ground below the platform. A graph of the ball's vertical position, x, as a function of time, t, is shown below.



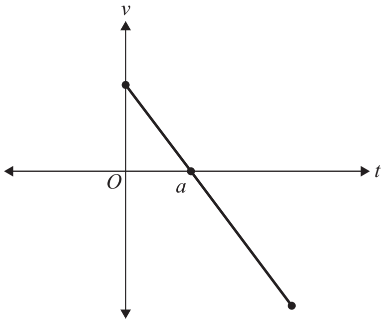
An object's velocity can be described as the rate of change of its position as a function of time.

Which one of the following graphs best represents the graph of the ball's **vertical** velocity, v, as a function of time, t ?

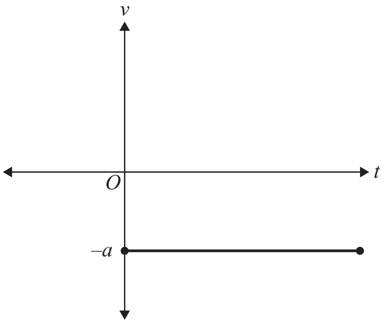
A.



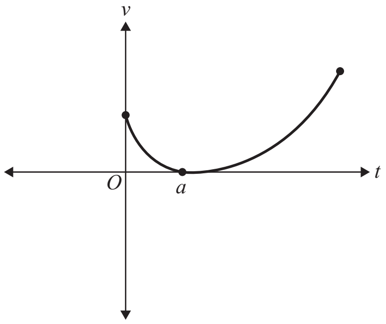
B.



C.



D.



Question 66

The equation of the tangent to the curve with the equation $y = x^3 - 8x + 3$ at the point $(1, -4)$ is given by the equation

A. $y = 5x + 1$

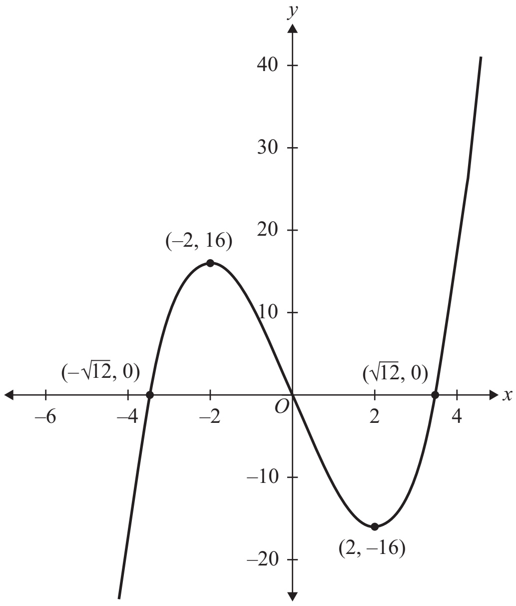
B. $y = -5x + 1$

C. $y = 5x - 1$

D. $y = -5x - 1$

Question 67

The graph shown below is of the function with the rule $y = x^3 - 4x$. Which one of the following statements is **not** true?



A. The gradient of the tangent is negative for $-2 < x < 2$.

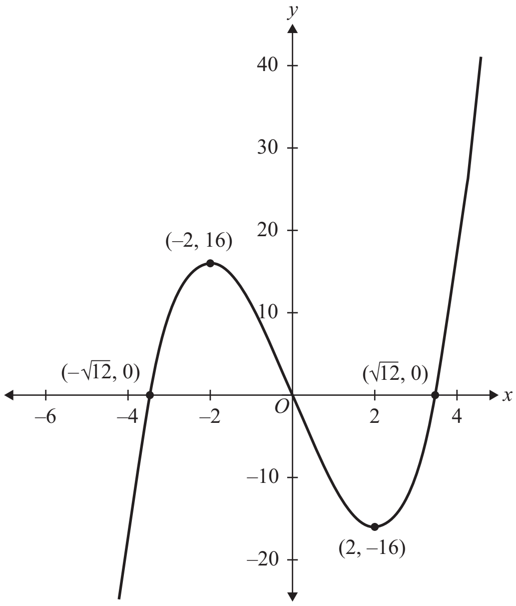
B. There is only one stationary point on the graph.

C. The gradient of the tangent is positive for $x < -2$ and for $x > 2$.

D. $\frac{dy}{dx} = 0$ when $x = -2$ and $x = 2$ and at no other point.

Question 68

The graph of the function f is shown below.



$f(x)$ and $f'(x)$ are both negative over the interval

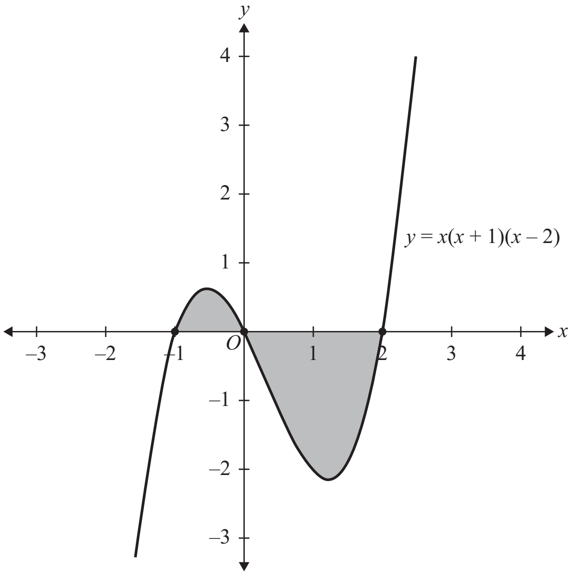
A. $[-2, 2]$

B. $(-2, 2)$

C. $(0, 2)$

D. $(-\infty, 2)$

Question 69



The shaded area in the graph above is equal to

A. $\frac{37}{12}$

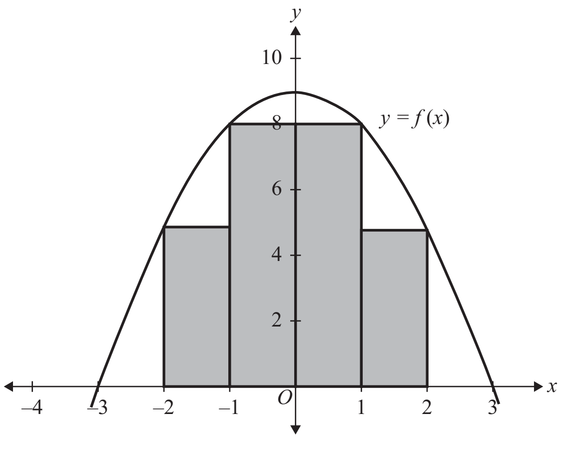
B. $\frac{9}{4}$

C. $\frac{63}{4}$

D. $\frac{65}{4}$

Question 70

The area under the curve $y = 9 - x^2$ between $x = -2$ and $x = 2$ can be approximated by calculating the area of the shaded rectangles shown below.



The shaded area is equal to

A. 13

B. $\frac{92}{3}$

C. 26

D. $\frac{95}{3}$

Section 3 Extended response questions

Question 1 (6 marks)

During a theme park ride, two lasers project light trails at different heights onto a large wall. The height of the light trails, L_1 and L_2, are described by the equations

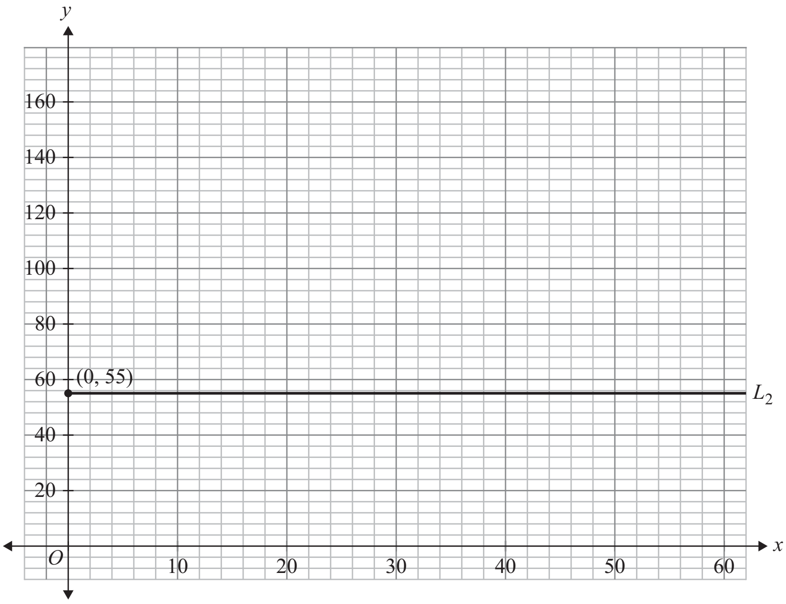
$$L_1 = \frac{3}{4}x + 25$$

$$L_2 = 55$$

where x is the horizontal distance from the bottom left-hand side of the wall.

a. The graph of L_2 is shown on the diagram below. Sketch $L_1 = \frac{3}{4}x + 25, x \geq 0$ on the same diagram.

Label axial intercepts and the intersection of L_1 and L_2, using coordinates. 2 marks



b. Find the horizontal distance from the wall at which the vertical distance between L_1 and L_2 is equal to 70 units. 2 marks

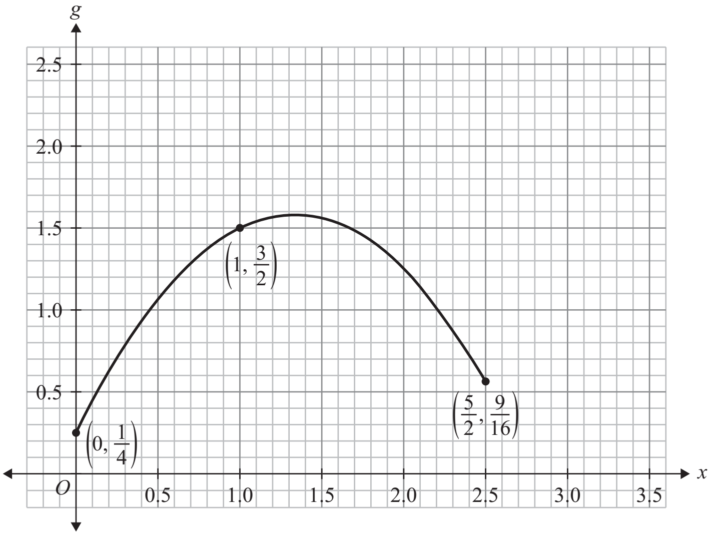
c. Find the horizontal distance of the light trail L_1 from the left of the wall after it has travelled 120 units from the wall. 2 marks

Question 2 (11 marks)

As part of the grand opening of a new supermarket, the organisers design a decorative parabolic arch to frame the entrance, creating a welcoming and visually appealing feature for shoppers. The equation of this parabola can be modelled by the function

$$g:\left[0, \frac{5}{2}\right] \to R,\ g(x) = ax^2 + bx + c$$

where g is the height, in metres, above the ground and x is the distance, in metres, from the left edge of the entrance, as shown below.



As shown, the arch:

- is $\frac{1}{4}$ metres above the ground at the left edge of the entrance
- is $\frac{3}{2}$ metres above the ground 1 metre from the left edge of the entrance
- finishes $\frac{9}{16}$ metres above the ground at the far end of the entrance, which is 2.5 metres from the left edge of the entrance.

It ended up being a small arch but still provided a welcoming visual element for shoppers.

a. Consider the equation $g(x) = ax^2 + bx + c$ of the parabola above.

i. Explain why the value of c must be $\frac{1}{4}$. 1 mark

ii. Write down the simultaneous equations that can be used to find the values of a and b. 2 marks

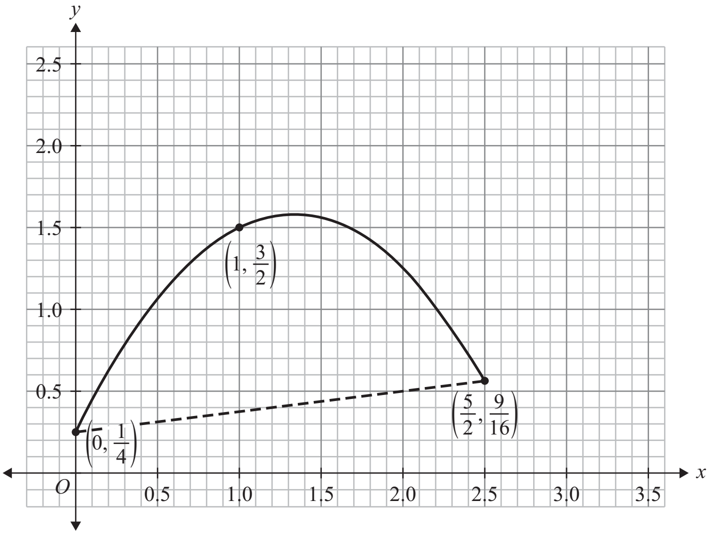
iii. Find the values of a and b. 1 mark

b. Using your values for a and b from **part a.iii.**, find the maximum height of this arch and the x-coordinate at which it occurs. 1 mark

The organisers wish to bring a large cuboid box through the arch. The base of the box is a square with side length 2 metres, and it is 1 metre high. The box must remain upright and cannot be tilted.

c. Explain, with relevant calculations, whether this box can pass through the arch without being tilted. 2 marks

Before the grand opening week begins, the organisers decide to install a tape to stop people from walking under the arch. They fix each end of the tape to the end points of the arch, as shown below.



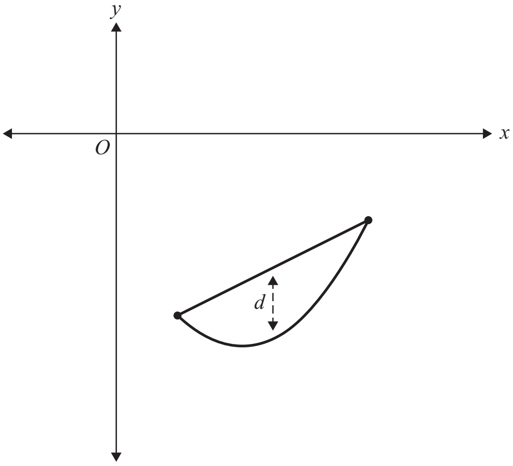
The equation of the line for the tape can be modelled by $y = mx + c$, where y is the height of the tape in metres above the ground and x is the distance in metres from the left edge of the entrance.

d. Find the equation of the line $y = mx + c$, where m and c are both real numbers. 2 marks

e. Find the value(s) of m, $m \in R$, such that a line with the equation $y = mx + \frac{1}{4}$ will intersect the arch at least 2 metres from the left edge of the entrance. 2 marks

Question 3 (7 marks)

Below is a graph of the equations $x - 2y = 7$ and $y = x^2 - 4x + 1$. The domain of the graph has been restricted to the points of intersection between the two equations.



a. Label the graph above with the exact coordinates of the points of intersection. 2 marks

b. **i.** A dotted line representing the vertical distance, d, between the two equations is also shown in the diagram above.

Write an equation for d in terms of x. 1 mark

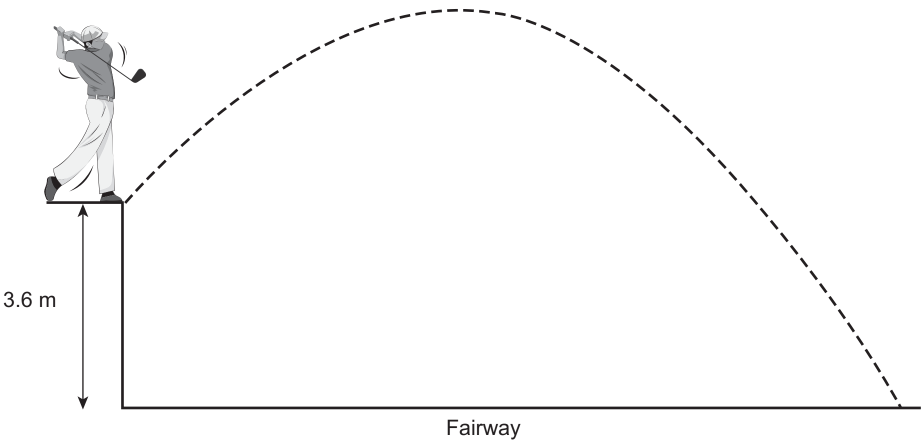
ii. Find the maximum vertical distance, d_{max}, between the two equations. 1 mark

The parabola with equation $y = x^2 - 4x + 1$ is one of a family of graphs with the equation $y = x^2 - ax + 1$, where $a \in R$.

c. Find the exact value(s) of a, such that the line with equation $x - 2y = 7$ just touches a parabola with equation $y = x^2 - ax + 1$. 3 marks

Question 4 (7 marks)

A golfer strikes a ball that is 3.6 metres above a fairway. The height of the ball above the fairway is given by $h(t) = -5t^2 + 28t + 3.6$, where t is the time in seconds.



a. Find the total time the ball spends in the air. 1 mark

b. Find the time taken for the ball to reach its maximum height. 1 mark

c. Find the maximum height reached by the ball. 1 mark

d. How long does the ball stay at or above 8 metres before coming down?

Give your answer correct to two decimal places. 2 marks

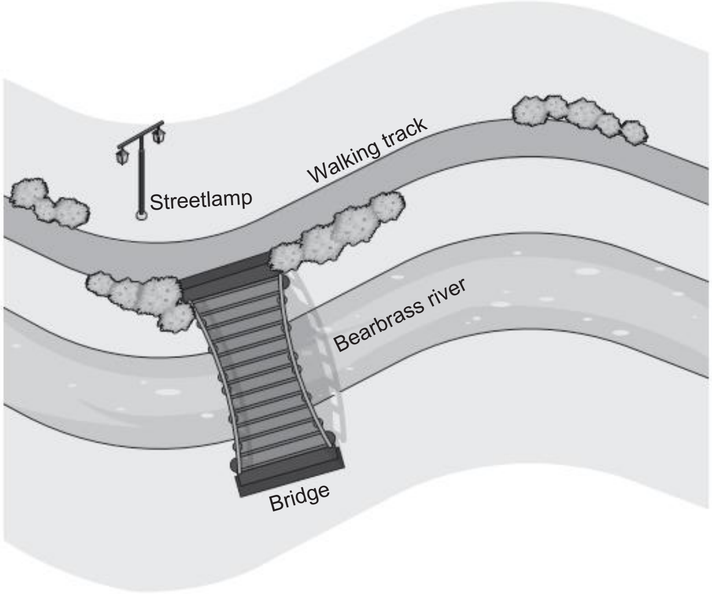
e. Suppose the golfer uses a different club and the height function is now $h(t) = -4.9t^2 + v_0 t + 3.6$, where v_0 is the initial velocity in metres per second and $v_0 > 0$.

If the ball reaches a maximum height of 20 metres, find v_0. Give your answer correct to one decimal place. 2 marks

Question 5 (12 marks)

The council for the town of Bearbrass plans to construct a walking track along the bank of the Bearbrass river, as shown below. Included in the council's plans are a streetlamp and a walking bridge across the river.

Note: the diagram is not to scale and is only a representation.



A surveyor marks the following four points on the track: $(0, 150)$, $(20, 0)$, $(40, 0)$ and $(60, 0)$. All measurements are in metres.

a. The council plans to place the streetlamp at the midpoint between the points $(0, 150)$ and $(20, 0)$. Find the coordinates of this midpoint. 1 mark

b. Find the equation of the line which passes along the bridge if the line passes through the point $(40, 0)$ and is perpendicular to a straight line joining the points $(40, 0)$ and $(0, 150)$. 2 marks

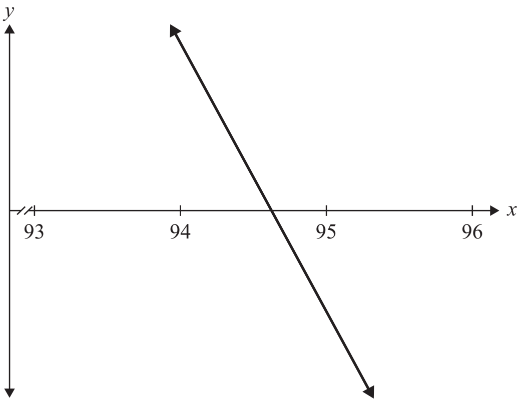
c. The walking track passes through all four of the surveyor's points and can be modelled by the function $f(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in R$.

Find the exact values of a, b, c and d. 1 mark

d. Another surveyor models the track using the function

$$g(x) = -\frac{1}{320}(x - 20)(x^2 - 120x + 2400)$$

To test the model, she must find the approximate solution to the equation $g(x) = 0$ in the interval $[94, 95]$. The section of the graph near this interval is shown below.



Using the method of bisection, complete the following table to show that the approximate solution, correct to one decimal place, is diverging to 94.6. 2 marks

Left end point	**Right end point**	**Midpoint (x_m)**	**$t(x_m)$**
94	95	94.5	2.2699
	95		–1.7666
94.5			0.2587
		94.6875	–0.7522
94.625			–0.2463

e. The council settles on a track modelled by the function

$$h(x) = -\frac{1}{320}(x-30)(x-50)(x-70)$$

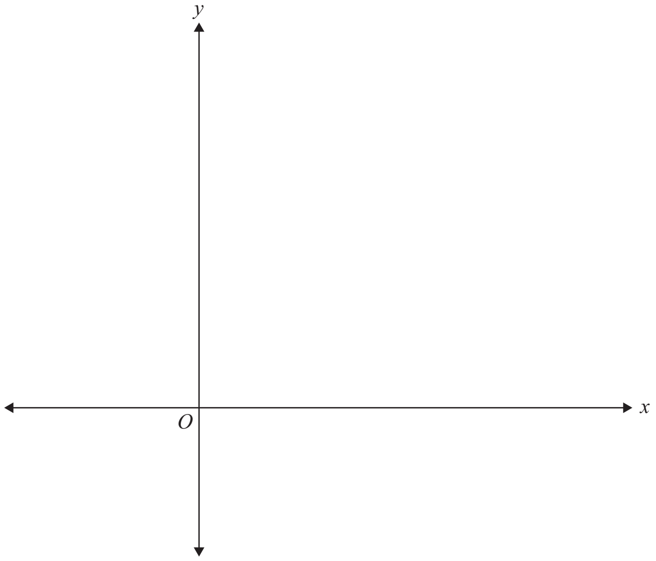
but with the following transformations:

- a dilation by a factor of $\frac{1}{2}$ from the x-axis
- a translation of 10 metres in the negative direction of the x-axis.

Write this new transformed function, $T(x)$, in terms of $h(x)$. 2 marks

f. The horizontal distance for this section of the track is 60 metres. Given that the track designers work only with positive (including zero) distances, **fully define** the function $T(x)$, with a rule in the form $ax^3 + bx^2 + cx + d$ where $a, b, c, d \in R$. 2 marks

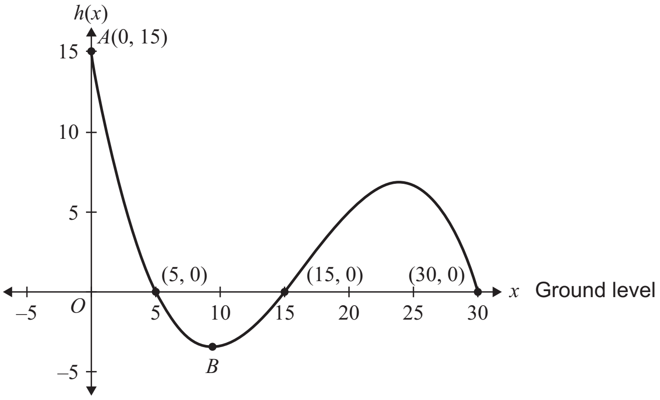
g. Sketch the graph of $y = T(x)$, labelling the coordinates of all end points, intercepts and turning points, accurate to two decimal places. 2 marks



Question 6 (7 marks)

Joshua and Andrea went on a fun roller-coaster ride during their visit to an adventure park.

They suspect that a section of the roller-coaster is in the shape of a cubic graph, as shown below, where $h(x)$ represents the height of the roller-coaster above the ground and x represents the horizontal distance from the starting point of the track. Both x and $h(x)$ are measured in metres.



Joshua and Andrea believe that they were 15 metres above the ground when they were at point A. As the roller-coaster continues its journey, it passes through the points shown on the diagram above. Joshua believes that the equation of this section of the track can be modelled by $h(x) = k(x - 5)\,(x - 15)(x - 30)$.

a. Show that the value of $k = -\dfrac{1}{150}$. 1 mark

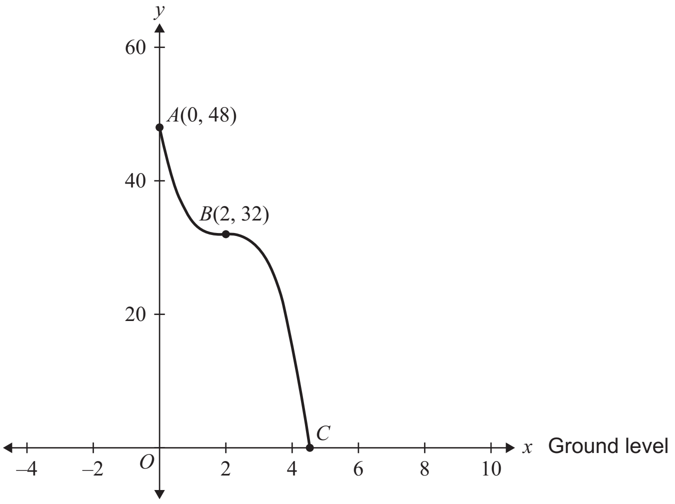
b. Determine how far underground Joshua and Andrea were when they were at point B (the local minimum point) and state the horizontal distance (x value) at this point.

Give your answers correct to two decimal places. 2 marks

c. Find the horizontal distance of the track from point A (i.e. the value of x) when Joshua and Andrea were 10 metres above ground level.

Give your answer correct to 2 decimal places 1 mark

Another section of the roller-coaster ride can be modelled by $y = a(x - h)^3 + k$. The ride in this section begins at point $A(0, 48)$ and there is a stationary point of inflection at point $B(2, 32)$. The section ends at point C, which is at ground level, before moving into a tunnel.



d. Determine the equation of the cubic that models this section of the ride. 2 marks

e. Hence, or otherwise, find the coordinates of point C. Give your answer to two decimal places. 1 mark

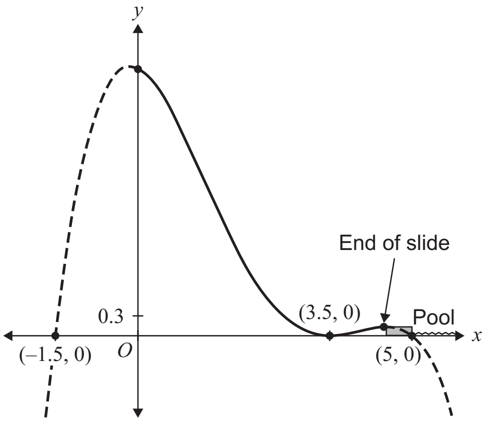
Question 7 (4 marks)

The height, in metres, of a water slide is modelled by the polynomial function $f(x) = 0.01(x - a)^2(b - x)(x - c)$, where x is the horizontal distance in metres and a, b and c are real numbers.

a. What is the degree of the polynomial being used to model the height of the water slide? 1 mark

It is decided that the graph of the model should have x-intercepts at $x = -1.5$, $x = 3.5$ and $x = 5$, as represented by the graph below.

However, the domain of the graph is restricted so that the beginning of the slide is at the y-intercept and the end of the slide occurs when the slide first reaches the entry to a pool, 0.3 metres above the ground. The slide is represented by the solid curve in the graph.

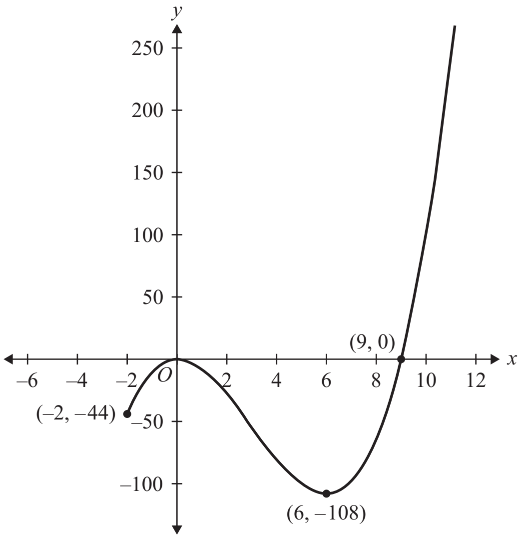


b. Find a, b and c, where $b < 0$ and $c > 0$. 1 mark

c. State the domain and range of the function. Give values correct to one decimal place. 2 marks

Question 8 (11 marks)

Part of the graph of $f: [-2, \infty) \rightarrow R, f(x) = x^2(x - 9)$ appears below. The cubic has turning points at $(0, 0)$ and at $(6, -108)$.



a. For the graph of $y = f(x) + k$, find the value(s) of k for which the graph has

i. no x-intercepts. 1 mark

ii. exactly one x-intercept. 1 mark

b. For the graph of $y = f(x + b)$, find the value(s) of b for which the graph has

i. **no positive** x-intercepts. (Note: $x = 0$ is **not** positive.) 1 mark

ii. exactly **one positive** x-intercept. 1 mark

Let $T: R^2 \rightarrow R^2$, $T(x, y) = \left(\frac{1}{2}x - 3, -\frac{1}{2}y\right)$.

c. Find the image of the key points $(-2, -44)$, $(0, 0)$, $(6, -108)$ and $(9, 0)$ on f under the transformation T. 1 mark

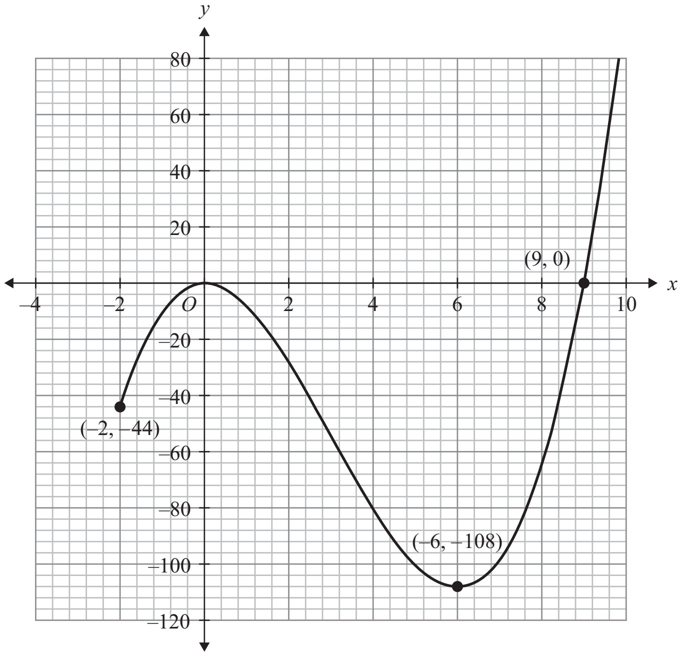
Let $y = g(x)$ be the graph of $y = f(x)$ under the transformation T.

d. Describe the sequence of transformations that takes the function $y = f(x)$ to the function $y = g(x)$. 1 mark

e. Find the rule of $y = g(x)$ and state its domain. 3 marks

f. Sketch $y = g(x)$ on the axes below. (The graph of $y = f(x)$ is shown on the same set of axes to assist you.)

Make sure you label all turning points, x-intercepts and the end point found in **part c.** 2 marks



Question 9 (5 marks)

It rains in Manangatang during four out of every 10 horse race meetings. A local horse, Galloping Gertie, has entered the Manangatang Cup. She wins 23% of her races when it is raining and 64% of her races when it is dry.

a. Draw a tree diagram illustrating the possible outcomes, and their probabilities, when Galloping Gertie runs in the Manangatang Cup. 2 marks

b. Show that the probability of Galloping Gertie winning the Manangatang Cup is 0.476. 1 mark

c. To the delight of the townsfolk, Galloping Gertie wins the Manangatang Cup.

What is the probability, correct to four decimal places, that it is raining? 2 marks

Question 10 (15 marks)

During a school camping trip, the Year 12 students were surveyed about their access to camping gear. It was found that 75% of the students brought their own tent, 80% brought their own sleeping bag, and 15% did not bring either a tent or a sleeping bag.

Let T be the event that a randomly selected student has brought their own tent. Let S be the event that a randomly selected student has brought their own sleeping bag.

a. i. Represent this information in the Karnaugh map below. 1 mark

	S	S'	
T			
T'			
			1

ii. Find the probability that a student who is randomly selected brought their own tent and their own sleeping bag.

1 mark

iii. Find the probability that a student who is randomly selected brought their own tent given that they did not bring their own sleeping bag. 1 mark

iv. If there are 375 Year 12 students attending the camp, how many students, correct to the nearest whole number, brought neither their own sleeping bag nor tent? 1 mark

v. Are the events T and S independent? Justify your answer. 2 marks

b. At the camp, the canteen sells six types of pies: egg and bacon; meat; vegetable; apple; loganberry; and apricot. Students are allowed to purchase only one of any particular variety of pie in the same purchase, but they are welcome to purchase one of each type of pie in the same transaction if they wish.

i. How many different orders can be placed if each purchase must include at least one pie? 1 mark

ii. Find the probability that a purchase contains at least three pies given that it contains at least one pie? 2 marks

c. Words are to be formed using the letters of the word **CAMPING**, with each letter being used exactly once.

How many different seven-letter words can be formed

i. with no restriction? 1 mark

ii. beginning with the letter M? 1 mark

iii. where the vowels are together? 2 marks

d. Find the probability that one of the seven-letter words begins with M and ends with the letter N? 2 marks

Question 11 (6 marks)

In 2011, an earthquake off the coast of Japan resulted in a devastating tsunami. Earthquake intensity, R, is often reported on the Richter scale and can be modelled by the equation $R = \log_{10}\left(\frac{A}{T}\right)$, where A is the amplitude of the ground motion and T is the period of the seismic wave.

a. The intensity, R, of the earthquake in Japan measured 8.9. If the period of the seismic wave was $\frac{1}{50}$, show that the amplitude of the earthquake was $\frac{10^{8.9}}{50}$. 2 marks

The earthquake and the impact of the subsequent tsunami caused a chain of events that resulted in an explosion at the Fukushima nuclear energy plant. One of the radioactive isotopes released from the plant was caesium-137. The decay of caesium-137 can be modelled by the equation

$$A_{\text{caesium}} = A_0 \times 2^{-kt}$$

where A_{caesium} is the amount of radioactive caesium in grams, A_0 is the initial amount of caesium in grams and t is the time in days.

b. Caesium-137 has a half-life of 10 950 days, that is, half of the initial amount of radioactive material decays in 10 950 days. Show that $k = \frac{1}{10\,950}$. 2 marks

Another radioactive isotope released from the plant was iodine-13. Initially, there is 10 times as much iodine-13 as there is caesium-137. Iodine-13 decays according to the equation

$$A_{\text{iodine}} = 10A_0 \times 2^{-0.125t}$$

where A_{iodine} is the amount of radioactive iodine in grams, A_0 is the initial amount of caesium in grams and t is the time in days.

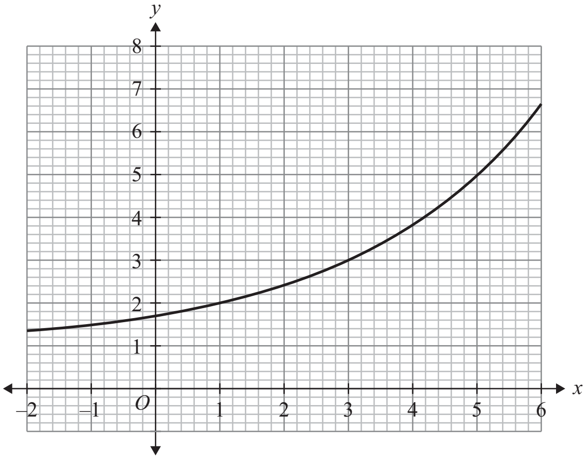
c. How many days, to the nearest day, would elapse before the amounts of iodine-131 and caesium-137 are the same? 2 marks

Question 12 (8 marks)

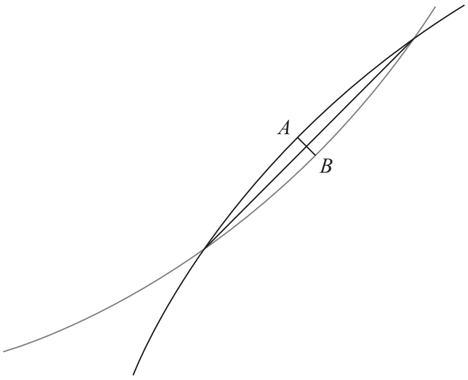
Let $f: R \rightarrow R,\ f(x) = 2^{\frac{x-1}{2}} + 1$.

a. Find the inverse function f^{-1} and state its domain. 2 marks

b. The diagram below shows $y = f(x)$. Sketch the graph $y = f^{-1}(x)$ on the diagram with all key features. Label its points of intersection with the graph of $y = f(x)$. 2 marks



A part of the graphs from **part a.** is shown below. The line segment AB drawn between the graphs indicates where the distance between the graphs is greatest. (You may assume that this region is symmetric about the line segment.)



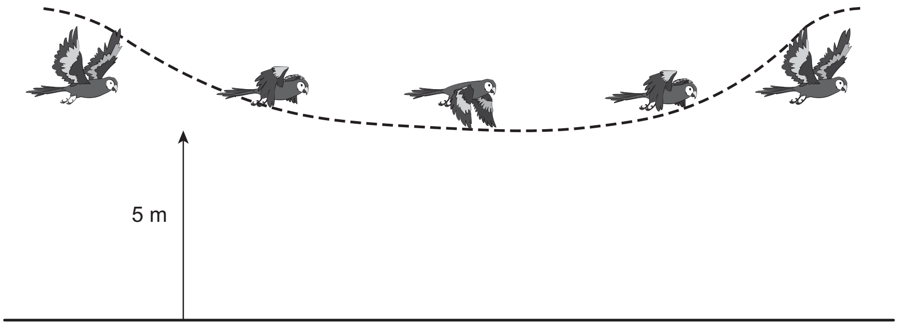
c. Show that the equation of the line segment AB is $y = 8 - x$. 2 marks

d. Find the point of intersection between the graphs of $y = 8 - x$ and $y = f(x)$, correct to two decimal places 1 mark

e. Find the greatest width of the enclosed region between the two graphs (i.e. the length of the line segment AB), correct to two decimal places. 1 mark

Question 13 (6 marks)

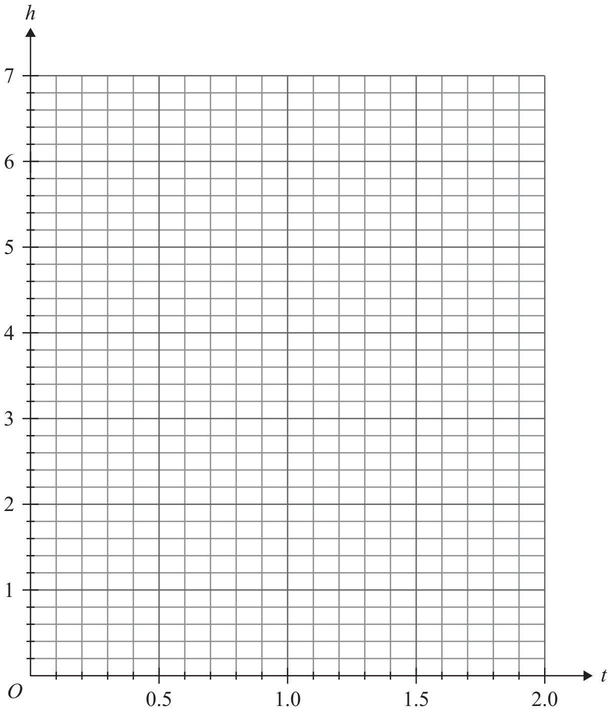
A bird is flying with its body at a constant height of 5 metres above the ground, as shown below.



The vertical movement of the tips of the bird's wings is modelled by the equation $h(t) = 0.8\cos(2\pi t) + 5$, where t is the time of flight in seconds and h is the height in metres of the bird's wings above the ground.

a. State the period of this function. 1 mark

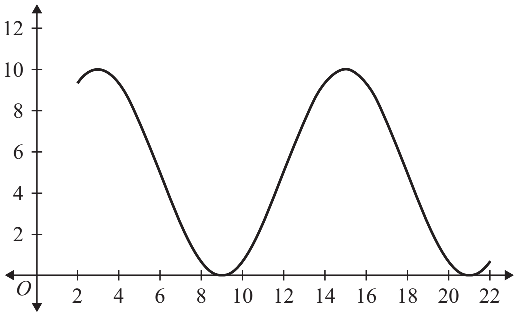
b. Sketch the graph of $h(t) = 0.8\cos(2\pi t) + 5$ for $0 \le t \le 2$ on the axes below, labelling the coordinates of the turning points and end points. 3 marks



c. Within these 2 seconds of flight, the bird's wings are at least p metres above the ground for a continuous period of 0.8 seconds. Find the value of p, correct to two decimal places. 2 marks

Question 14 (10 marks)

The graph below represents the function $h:[2, 22] \rightarrow R, h(x) = p\cos\left(qx + \frac{\pi}{2}\right) + r$.



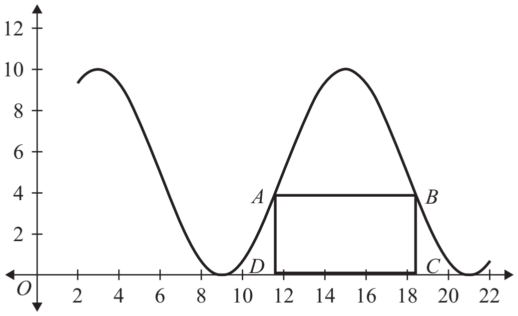
a. State the values of p, q and r. 2 marks

b. Give the coordinates of the two end points. Leave your answer in exact form. 2 marks

c. Find the distance between the two end points. 1 mark

d. State the value(s) of x for which h is a maximum. 1 mark

e. The rectangle $ABCD$ sits under the curve $h(x)$, with points A and B just touching the curve.



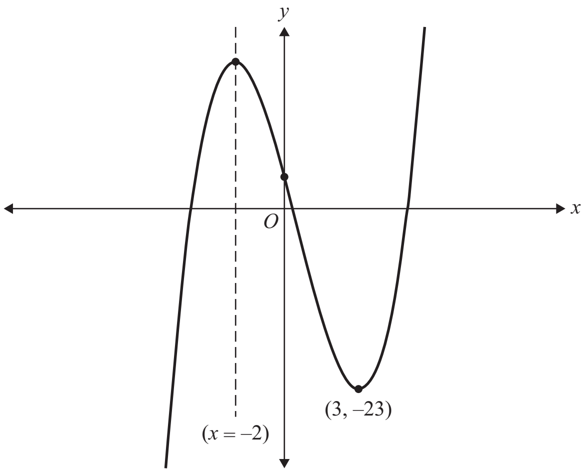
If the length of AB is 6.4 units, find the length of AD. Give your answer rounded to three decimal places. 2 marks

f. Consider the function $g(x) = 9.5$. State the solutions to $h(x) = g(x)$. Give your answer rounded to two decimal places. 1 mark

g. Consider the horizontal line $y = a$, $a \in R$. State the possible values of a for which there are three solutions to $h(x) = a$ for $x \in [2, 22]$. 1 mark

Question 15 (8 marks)

The graph of a cubic function, $f(x) = ax^3 + bx^2 + cx + 4$, where a, b and c are real numbers, is shown below.



The function has a local maximum when $x = -2$ and a local minimum at the point $(3, -23)$.

a. State the coordinates of the y-intercept. 1 mark

b. Given that a local minimum occurs at the point $(3, -23)$, show that the following simultaneous equations could be formed.

$$27a + 6b + c = 0$$
$$27a + 9b + 3c = -27$$

2 marks

c. State a third equation in terms of a, b and c, and hence, or otherwise, find the values of a, b and c.

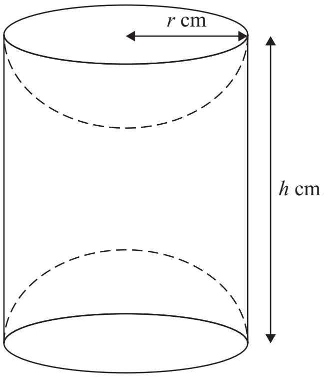
2 marks

d. Sketch the graph of $f'(x)$ on the axes given in the introduction to the question, labelling coordinates of all intercepts and turning points.

3 marks

Question 16 (6 marks)

A solid cylinder has a hemisphere hollowed out from each end. The total surface area of the cylinder is 3000π cm². The height of the cylinder is h cm and the radius is r cm.



a. Given that $2\pi rh + 4\pi r^2 = 3000\pi$, express h in terms of r.

1 mark

b. **i.** Find the volume of the cylinder, $V\,\mathrm{cm}^3$, in terms of r and h. 1 mark

ii. Show that the volume is given by $V = \frac{10\pi r}{3}(450 - r^2)$. 1 mark

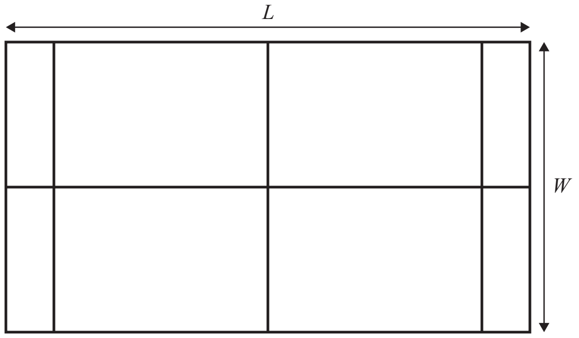
iii. Find $\frac{dV}{dr}$ in terms of r. 1 mark

iv. Solve the equation $\frac{dV}{dr} = 0$ for r. 1 mark

v. Hence, state the maximum volume, in cm³, of the solid. 1 mark

Question 17 (8 marks)

A group of student leaders at a school plan to mark out a new court for a game, as shown below. The court is W metres wide and L metres long.



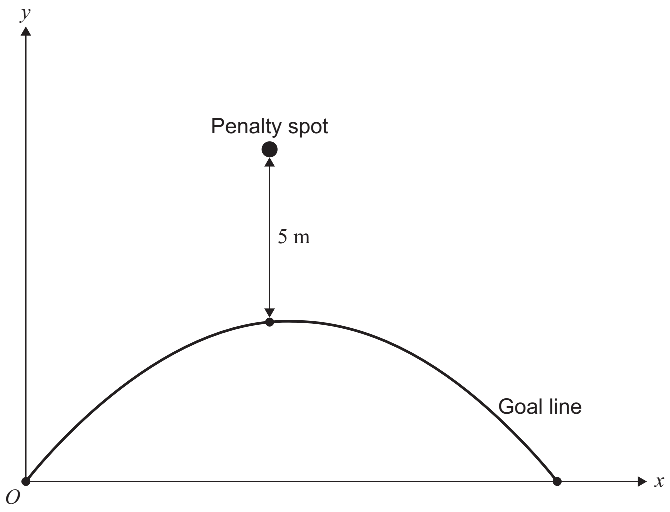
The students are provided with a total of 50 metres of marking tape.

a. Find an expression for L in terms of W. 1 mark

b. Show that the area of the court can be given by $A = \frac{50W - 5W^2}{3}$. 1 mark

c. The students wish to mark out a court that has the largest area possible. Find the dimensions of the court that maximises its area. 2 marks

On a different court, another group of students decides to mark out a goal line and penalty spot, as shown.



The students mark out the goal line based on the function $f(x) = ax^2 + bx$, where a and b are real numbers. They also mark the penalty spot exactly 5 metres vertically above the maximum point of the goal line.

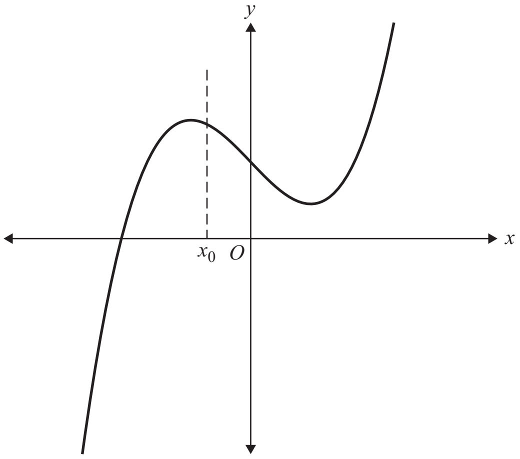
d. Show that the vertical distance of the penalty spot from the x-axis is given by $-\frac{b^2}{4a} + 5$. 2 marks

The function representing the goal line has a gradient of $-\frac{3}{25}$ at $x = 12$ and passes through the point $\left(5, \frac{9}{4}\right)$.

e. Find the values of a and b. 2 marks

Question 18 (6 marks)

The graph of the function $f(x) = x^3 - 2x + 2$ is shown below.



Newton's method for finding solutions to equations can be used to find an approximate solution to the equation $f(x) = 0$. The first iteration of Newton's method begins at x_0, as shown on the graph.

a. Label the approximate locations of the next two points of iteration, x_1 and x_2, on the x-axis of the graph above. 2 marks

b. Explain why a solution to the equation $f(x) = 0$ cannot be found using Newton's method if the starting point is $x_0 = 0$. Support your explanation with calculations or a graphical analysis. 2 marks

c. Using Newton's method, state an appropriate iterative formula to find the approximate solution to the equation $f(x) = 0$. By starting the iteration at $x_0 = -3$, use this formula to find the approximate solution to the equation $f(x) = 0$ after the third iteration (i.e. $n = 3$). Write your solution correct to three decimal places. 2 marks

Worked solutions

Section 1 Technology-free

Question 1a.

Worked solution

$$\frac{x-2}{3}-\frac{x}{2}+\frac{x+1}{4}=2$$

LCD = 12

$$\frac{4(x-2)-6x+3(x+1)}{12}=2$$

$$\begin{aligned}4x-8-6x+3x+3&=24\\x-5&=24\\x&=29\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for reducing the left-hand side of the equation to a single fraction by finding the common denominator
- 1 answer mark

TIP

» Don’t forget to find the common denominator first.

Question 1b.

Worked solution

$$\frac{mx+a}{c}=\frac{cx-a}{m}$$

$$m(mx+a)=c(cx-a)$$

$$m^2x+ma=c^2x-ca$$

$$m^2x-c^2x=-ma-ca$$

$$x(m^2-c^2)=-a(m+c)$$

$$x=\frac{-a(m+c)}{m^2-c^2}$$

$$x=\frac{-a(m+c)}{(m+c)(m-c)}$$

$$x=\frac{-a}{m-c}$$

Mark allocation: 3 marks

- 1 method mark for cross-multiplication and then simplifying the expression on both sides
- 1 method mark for applying the difference of perfect squares rule
- 1 answer mark

Question 2a.

Worked solution

Simultaneous linear equations have infinitely many solutions or no solutions when their gradients are equal.

The gradient of the first equation is given by $\frac{-(4-p)}{2}$.

The gradient of the second equation is given by $\frac{p}{6}$.

$$\begin{aligned}\frac{-(4-p)}{2} &= \frac{p}{6}\\ -6(4-p) &= 2p\\ -24+6p &= 2p\\ 4p &= 24\\ p &= 6\end{aligned}$$

If $p = 6$, then $(4-p)x + 2y = q$ and $6y - px = 9$ become

$2y - 2x = q$

$6y - 6x = 9$

If $q = 3$, then the equations are equivalent and there are infinitely many solutions.

Mark allocation: 3 marks

- 1 method mark for equating the two gradients
- 1 method mark for substituting $p = 6$ into both simultaneous equations to establish a new set of equations with only q being unknown
- 1 answer mark for finding $q = 3$ for an infinite number of solutions

Question 2b.

Worked solution

Simultaneous linear equations have no solutions when their gradients are equal and their y-intercepts are different.

From the working from **part a.**, if $p = 6$ and $q \neq 3$ or $q \in R\backslash\{3\}$, then the equations have different y-intercepts and there are no solutions.

Mark allocation: 1 mark

- 1 answer mark

Question 3a.

Worked solution

$(0, 8)$ is the y-intercept.

The equation of the line is $y = mx + 8$.

Substituting $(-2, 0)$ into $y = mx + 8$ and solving for m gives:

$$\begin{aligned}0 &= -2m + 8\\ 2m &= 8\\ m &= \frac{-8}{-2} = 4\\ \therefore y &= 4x + 8\end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

TIPS

» Since two points are given, you can find the gradient using the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

» Since the points given are the x-intercept and the y-intercept, you can directly use the intercept form of a line $\frac{x}{a} + \frac{y}{b} = 1$, where a is the x-intercept and b is the y-intercept.

Question 3b.

Worked solution

$$\begin{aligned} L &= \sqrt{(-2-0)^2 + (0-8)^2} \\ &= \sqrt{4+64} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \text{ metres} \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

TIP

» Always remember to simplify a surd where possible.

Question 3c.

Worked solution

$$\begin{aligned} \text{Midpoint} &= \left(\frac{-2+0}{2}, \frac{0+8}{2}\right) \\ &= (-1, 4) \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 3d.

Worked solution

Since the lines are perpendicular, we know the gradient of the perpendicular line is $-\frac{1}{4}$.

Hence the equation of the line is $y = -\frac{1}{4}x + c$.

Substituting $(3, 8)$ into $y = -\frac{1}{4}x + c$ and solving for c gives:

$$8 = \frac{-3}{4} + c$$

$$c = \frac{35}{4}$$

$$y = -\frac{1}{4}x + \frac{35}{4}$$

Mark allocation: 2 marks

- 1 method mark for finding the gradient of the perpendicular line
- 1 answer mark

Question 3e.

Worked solution

$$4x + 8 = -\frac{1}{4}x + \frac{35}{4}$$

$$4x + \frac{1}{4}x = \frac{35}{4} - 8$$

$$\frac{17x}{4} = \frac{3}{4}$$

$$x = \frac{3}{17}$$

Substituting $x = \frac{3}{17}$ into the equation $y = 4x + 8$ gives:

$$y = 4\left(\frac{3}{17}\right) + 8$$

$$y = \frac{12}{17} + 8$$

$$y = \frac{148}{17}$$

Therefore the point of intersection is $\left(\frac{3}{17}, \frac{148}{17}\right)$.

Mark allocation: 2 marks

- 1 method mark for equating the equations and solving for x
- 1 answer mark

Question 4a.i.

Worked solution

$$\begin{aligned} 2x^2 + 4x - 6 &= 2(x^2 + 2x - 3) \\ &= 2(x - 1)(x + 3) \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

TIP

» **Don't forget to always look for a common factor first.**

Question 4a.ii.

Worked solution

$$\begin{aligned} 2x^2 + 4x - 6 &= 2(x^2 + 2x - 3) \\ &= 2(x^2 + 2x + 1 - 1 - 3) \\ &= 2[(x+1)^2 - 4] \\ &= 2(x+1)^2 - 8 \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for using the method of completing the square correctly
- 1 answer mark

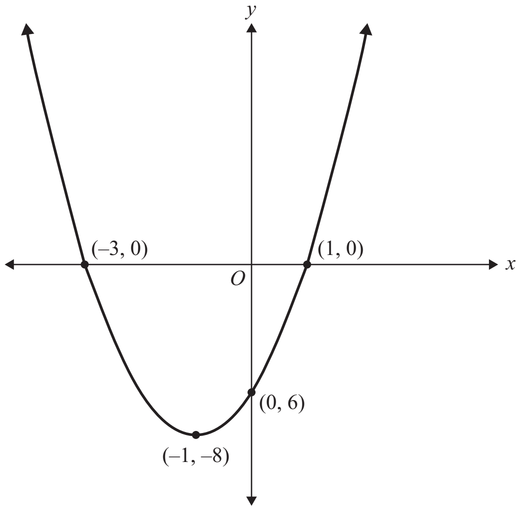
TIPS

» Remember, you must take out the coefficient of the x^2 term first.

» Also, to get full marks, do not forget to multiply the brackets in the last step.

Question 4b.

Worked solution



Mark allocation: 3 marks

- 1 answer mark for the correct coordinates of turning points
- 1 mark for the correct labelling of x- and y-intercepts
- 1 answer mark for the correct shape (i.e. an upright parabola)

TIP

» Make sure all key points are in coordinate form.

Question 5

Worked solution

To prove the equation has at least one solution, we need to show that the discriminant is greater than or equal to zero.

$$\begin{aligned}\Delta &= (-(p-2))^2 - 8p\left(1-\frac{4}{p}\right)\\ &= p^2 - 4p + 4 - 8p + 32\\ &= p^2 - 12p + 36\\ &= (p-6)^2\end{aligned}$$

Since the discriminant is a perfect square, it will be always greater than or equal to zero and so the quadratic will have at least one solution.

Mark allocation: 2 marks

- 1 method mark for finding the discriminant
- 1 method mark for identifying the discriminant is a perfect square and stating it is always going to be greater than or equal to zero

Question 6

Worked solution

The axis of symmetry $x = 3$ gives the x-coordinate of the turning point of the parabola.

Using the turning point form of the parabola gives:

$y = a(x-3)^2 + k$

Substituting $(2, 7)$, we get:

$7 = a + k$ [equation (1)]

Substituting $(5, 0)$, we get:

$0 = 4a + k$ [equation (2)]

Solving the two equations, we get:

$$\begin{aligned}3a &= -7\\ a &= -\frac{7}{3}\end{aligned}$$

Substituting $a = -\frac{7}{3}$ into equation (1) gives:

$$7 = -\frac{7}{3} + k$$

$$k = \frac{28}{3}$$

Hence the equation of the parabola is $y = -\frac{7}{3}(x-3)^2 + \frac{28}{3}$.

Mark allocation: 3 marks

- 1 method mark for identifying the turning point form of the parabola
- 1 method mark for setting up the two equations
- 1 answer mark for finding the equation of the parabola

TIP

» **You can use the standard form of the parabola, $y = ax^2 + bx + c$, but since the axis of symmetry is given, it is easier to use the turning point form for easier calculations.**

Question 7

Worked solution

$$-\frac{8}{3}(x-3)^2 + \frac{32}{3} < 0$$

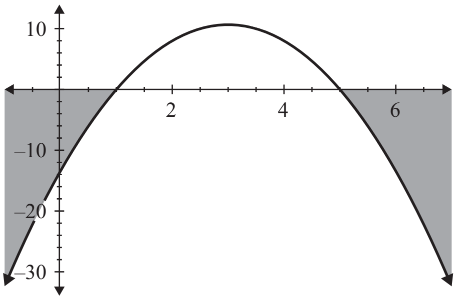
$$-\frac{8}{3}[(x-3)^2 - 4] < 0$$

$$(x-3)^2 - 4 > 0$$

$$(x-3-2)(x-3+2) > 0$$

$$(x-5)(x-1) > 0$$

So $x < 1$ or $x > 5$.



Mark allocation: 2 marks

- 1 method mark for factorising the expression
- 1 answer mark

TIP

» **When solving inequalities, always draw a rough sketch of the graph.**

Question 8

Worked solution

Since the quadratic equation has only one solution, the discriminant must be equal to zero.

$$\Delta = \left(2(k-12)\right)^2 - 8(k-12) = 0$$

$$4(k-12)^2 - 8(k-12) = 0$$

$$4(k-12)(k-12-2) = 0$$

$$4(k-12)(k-14) = 0$$

$k = 12$ or $k = 14$.

Mark allocation: 2 marks

- 1 method mark for finding the discriminant
- 1 answer mark for the values of k

TIP

» **Sometimes factorising the expression is faster than expanding the expression when solving such equations.**

Question 9

Worked solution

Let the width of the rectangle be x metres.

Since the perimeter is 24, the length of the rectangle is $24 - 2x$ metres.

$A = L \times w = x(24 - 2x)$

$A = 2x(12 - x)$

This equation is a parabola with two x-intercepts, one at $x = 0$ and the other at $x = 12$. Therefore the maximum will occur at $x = 6$, the halfway point between the x-intercepts.

So $A = 72$ m^2.

Mark allocation: 2 marks

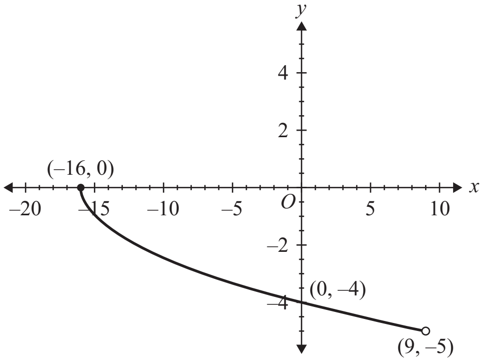
- 1 method mark for finding the expression for the length
- 1 answer mark for finding the area

TIP

» **Remember: When a parabola has x-intercepts, the x-coordinate of the turning point is always halfway between the two x-intercepts.**

Question 10a.

Worked solution



Mark allocation: 2 marks

- 1 method mark for the correct shape (i.e. a square root graph reflected across the x-axis and shifted left)
- 1 answer mark for the correct and labelled coordinates of intercepts and end points

TIP

» **Make sure that the point (9, –5) is 'open' to achieve full marks.**

Question 10b.

Worked solution

A reflection in the x-axis followed by a translation of 16 units in the negative direction of the x-axis.

Mark allocation: 2 marks

- 1 answer mark for mentioning the reflection in the x-axis
- 1 answer mark for mentioning the translation along the x-axis in the correct direction

Question 11a.

Worked solution

$$\begin{aligned}\frac{-5-3x}{x+1} &= \frac{-3(x+1)-2}{x+1}\\ &= \frac{-3(x+1)}{x+1}-\frac{2}{x+1}\\ &= -3-\frac{2}{x+1}\end{aligned}$$

Alternative method: Long division

$$\begin{array}{r} -3 \\ x+1\overline{)-3x-5} \\ -3x-3 \\ \hline -2 \end{array}$$

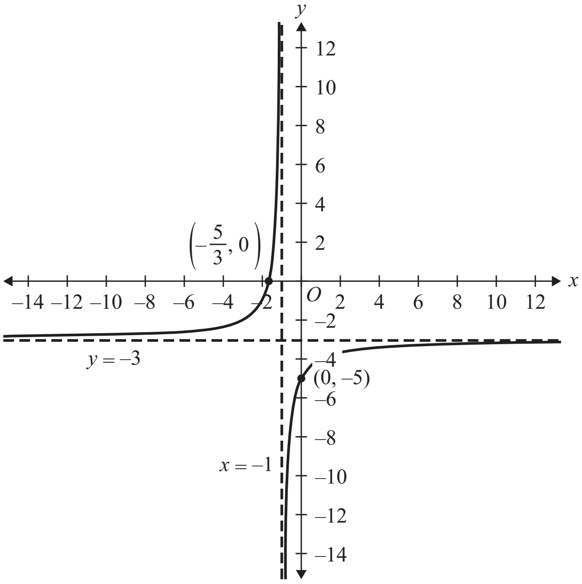
$$\frac{-5-3x}{x+1}=-3-\frac{2}{x+1}$$

Mark allocation: 2 marks

- 1 method mark for an appropriate method
- 1 answer mark

Question 11b.

Worked solution



Mark allocation: 3 marks

- 1 answer mark for the correct shape of the graph
- 1 answer mark for labelling the asymptotes correctly
- 1 answer mark for labelling the intercepts correctly

Question 11c.

Worked solution

$x < -\frac{5}{3}$ or $x > -1$.

Mark allocation: 1 mark

- 1 answer mark

Question 12

Worked solution

The graph shown is of a truncus, with the asymptotes $x = 2$ and $y = 1$.

The equation of a truncus is $y = \frac{a}{(x - h)^2} + k$.

Therefore we have $h = 2$ and $k = 1$.

Substituting $(0, 2)$ into the equation of the truncus gives:

$$2 = \frac{a}{(0 - 2)^2} + 1$$

$$2 = \frac{a}{4} + 1$$

$$1 = \frac{a}{4}$$

Therefore the equation of the graph is $y = \frac{4}{(x - 2)^2} + 1$.

Mark allocation: 3 marks

- 1 method mark for identifying the asymptotes
- 1 method mark for finding the value of a
- 1 answer mark for the correct equation of the graph

TIP

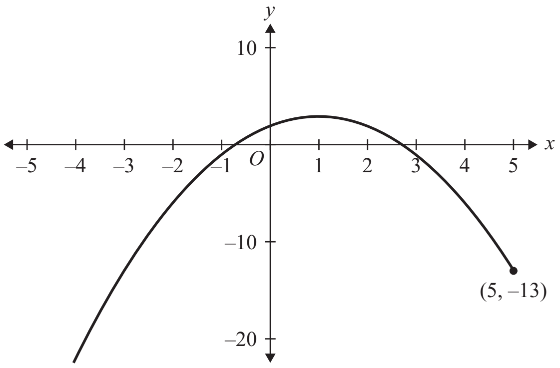
» **Don't forget to write the final equation of the graph to obtain full marks.**

Question 13a.

Worked solution

For the function $f(x)$ to have an inverse, it must be a one-to-one function.

Draw the graph:



Therefore $k = 1$, which is the x-coordinate of the turning point.

Mark allocation: 1 mark

- 1 answer mark

Question 13b.

Worked solution

Swapping x and y gives:

$$x = 3 - (y-1)^2$$
$$(y-1)^2 + x = 3$$
$$(y-1)^2 = 3 - x$$
$$y - 1 = \pm\sqrt{3-x}$$

Reject the negative value, as the range of $f \in [-13, 3]$.

$y = \sqrt{3-x} + 1$

Hence $f^{-1}(x) = \sqrt{3-x} + 1$.

Mark allocation: 2 marks

- 1 method mark for swapping x and y and making y the subject
- 1 answer mark

TIPS

- » **Don’t leave your answer with y as the subject, as the question clearly mentions f^{-1}.**
- » **You must mention why you are rejecting the negative side of the y graph.**

Question 13c.

Worked solution

The domain of f^{-1} is the range of $f = [-13, 3]$.

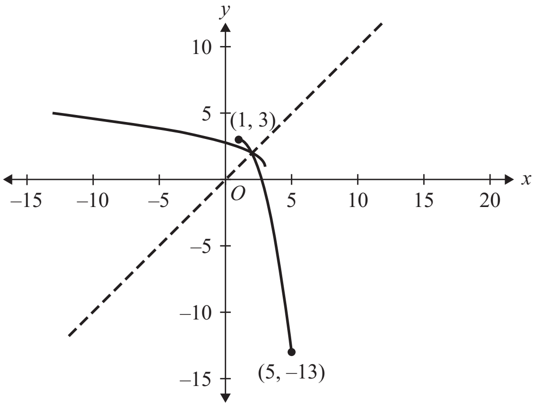
Mark allocation: 1 mark

- 1 answer mark

Question 13d.

Worked solution

A function and its inverse will always intersect at line $y = x$.



Therefore to find the point of intersection, you equate either function to $y = x$.

$$\begin{aligned} 3-(x-1)^2 &= x \\ 3-x &= (x-1)^2 \\ 3-x &= x^2-2x+1 \\ x^2-x-2 &= 0 \\ (x-2)(x+1) &= 0 \end{aligned}$$

So $x = 2$ or $x = -1$.

Reject $x = -1$, as the domain of $f = [1, 5]$.

Substituting $x = 2$ into $y = x$ gives $y = 2$.

Hence the point of intersection is $(2, 2)$.

Mark allocation: 3 marks

- 1 method mark for equating either function to $y = x$
- 1 method mark for rejecting $x = -1$
- 1 answer mark

Question 14a.

Worked solution

$g(1) = 2 - 1^2 = 1$

$g(4) = \sqrt{12} - 10 = 2\sqrt{3} - 10$

Mark allocation: 2 marks

- 1 answer mark for $g(1)$
- 1 answer mark for $g(4)$

Question 14b.

Worked solution

Since $g(x)$ is connected, the values of $g(x)$ when $x \leq -2$ will be the same as the values of $g(x)$ when $x > -2$.

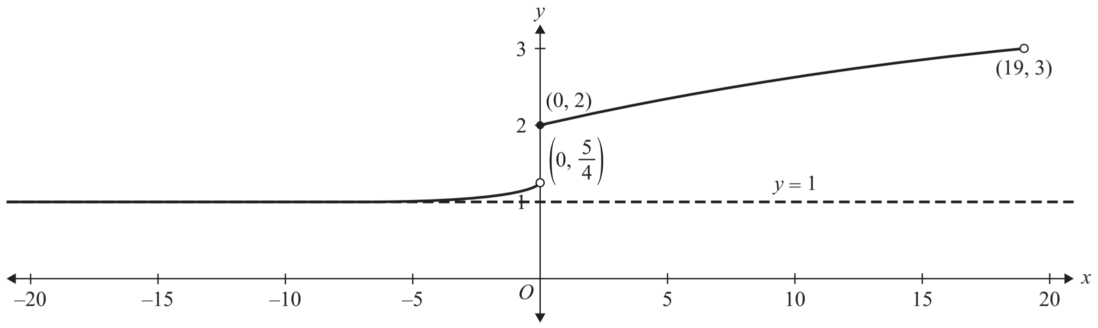
$$\begin{aligned} -2m + 4 &= 2 - (-2)^2 \\ -2m + 4 &= -2 \\ -2m &= -6 \\ m &= 3 \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for equating the functions
- 1 answer mark

Question 15a.

Worked solution



Mark allocation: 3 marks

- 1 answer mark for the correct coordinates of intercepts and end points
- 1 method mark for the correct shape of the truncus and rational power graphs
- 1 answer mark for the correct and labelled asymptote

TIP

» **Remember to put an open circle when the inequality sign is less than or greater than.**

Question 15b.

Worked solution

The relation is a function since there is only one y value for every x value. In other words, it passes the vertical line test.

Mark allocation: 1 mark

- 1 answer mark for stating that the relation is a function

TIP

» **Be careful: stating that there is only one x value for every y value has an entirely different meaning.**

Question 16a.

Worked solution

Let q represent the factors of the coefficient of x^3 and let p represent the factors of the constant term. Then:

$q = \pm 1, \pm 2$ and $p = \pm 1, \pm 2, \pm 4$.

By the rational root theorem, the possible rational roots are:

$$\pm \frac{p}{q} = \pm \frac{1}{2}, \pm 1, \pm 2, \pm 4$$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 16b.

Worked solution

Let $P(x) = 2x^3 - x^2 - 8x + 4$.

$$\begin{aligned} P(2) &= 2 \times 8 - 4 - 16 + 4 \\ &= 0 \end{aligned}$$

Therefore, according to the factor theorem, $(x - 2)$ is a factor.

Using polynomial division to find additional factors gives:

$$\begin{array}{r}
2x^2 + 3x - 2 \\
x - 2\overline{)2x^3 - x^2 - 8x + 4} \\
\underline{2x^3 - 4x^2} \qquad\qquad \\
3x^2 - 8x + 4 \\
\underline{3x^2 - 6x} \quad\; \\
-2x + 4 \\
\underline{-2x + 4} \\
0
\end{array}$$

$2x^2 + 3x - 2$

$= 2x^2 + 4x - x - 2$

$= 2x(x + 2) - (x - 2)$

$= (2x - 1)(x + 2)$

Hence, the factors are $(2x - 1)$, $(x - 2)$ and $(x - 2)$

Mark allocation: 2 marks

- 1 mark for finding the quadratic formula
- 1 mark for finding the other factors

TIPS

» **All valid methods for finding additional factors are acceptable; these include long division, synthetic division and re-expression.**

» **The re-expression method is as follows:**

$$\begin{aligned} 2x^3 - x^2 - 8x + 4 &= 2x^2(x-2) + 3x(x-2) - 2(x-2) \\ &= (x-2)(2x^2+3x-2) \\ &= (x-2)(2x-1)(x+2) \end{aligned}$$

Question 17a.

Worked solution

$$\begin{aligned} P(x) &= 3x^3 - 8x^2 + 3x + 2 \\ P(2a) &= 3(2a)^3 - 8(2a)^2 + 3(2a) + 2 \\ &= 24a^3 - 32a^2 + 6a + 2 \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 17b.

Worked solution

By remainder theorem:

$P(x) = 3x^3 - 8x^2 + 3x + 2$

$P(3) = 3(3)^3 - 8(3)^2 + 3(3) + 2$

So $81 - 72 + 9 + 2 = 20$.

Mark allocation: 1 mark

- 1 answer mark

Question 17c.

Worked solution

$P(2) = 3(2)^3 - 8(2)^2 + 3(2) + 2$

So $24 - 32 + 6 + 2 = 0$.

Mark allocation: 1 mark

- 1 method mark

Question 17d.

Worked solution

$$
\begin{array}{r}
3x^2 - 2x - 1 \\
x - 2\overline{)3x^3 - 8x^2 + 3x + 2} \\
\underline{3x^2 - 6x^2} \\
-2x^2 + 3x \\
\overline{-x + 2} \\
\underline{-x + 2} \\
0
\end{array}
$$

$$
\begin{aligned}
3x^2 - 2x - 1 &= 3x^2 - 3x + x - 1 \\
&= 3x(x - 1) + 1(x - 1) \\
&= (3x + 1)(x - 1)
\end{aligned}
$$

Therefore $P(x) = (3x + 1)(x - 1)(x - 2)$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 17e.

Worked solution

$3x^3 - 8x^2 + 3x + 2 = (3x + 1)(x - 1)(x - 2) = 0$

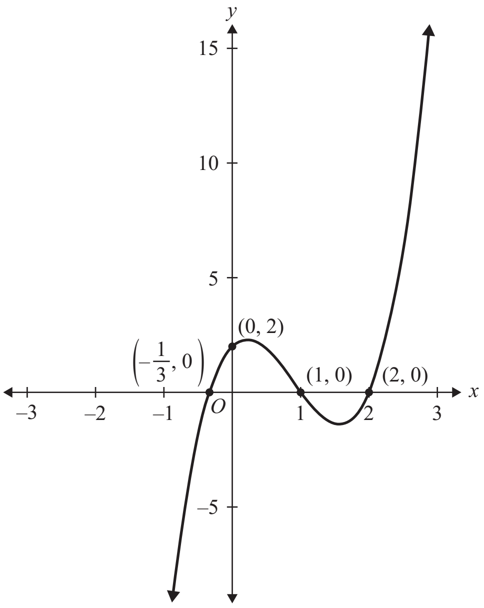
So $x = -\frac{1}{3}$, 1 or 2.

Mark allocation: 1 mark

- 1 answer mark

Question 17f.

Worked solution



Mark allocation: 2 marks

- 1 mark for the correct shape
- 1 mark for correctly labelling the x-intercepts and y-intercept

Question 18a.

Worked solution

Equating the coefficients gives:

$a = -5, b = 0$

Mark allocation: 1 mark

- 1 answer mark

Question 18b.

Worked solution

$$\begin{aligned} x^3 + 2x^2 - x &= x^3 + a(x-1)^2 + b(x-1) + c \\ &= x^3 + a(x^2 - 2x + 1) + bx - b + c = x^3 + ax^2 + (b - 2a)x + a - b + c \end{aligned}$$

Equating the coefficient of x^2 gives $a = 2$.

Equating the coefficient of x gives $b - 2a = -1$.

$$\begin{aligned} b - 4 &= -1 \\ b &= 3 \end{aligned}$$

Equating the constants gives:

$$a - b + c = 0$$

$$2 - 3 + c = 0$$

$$c = 1$$

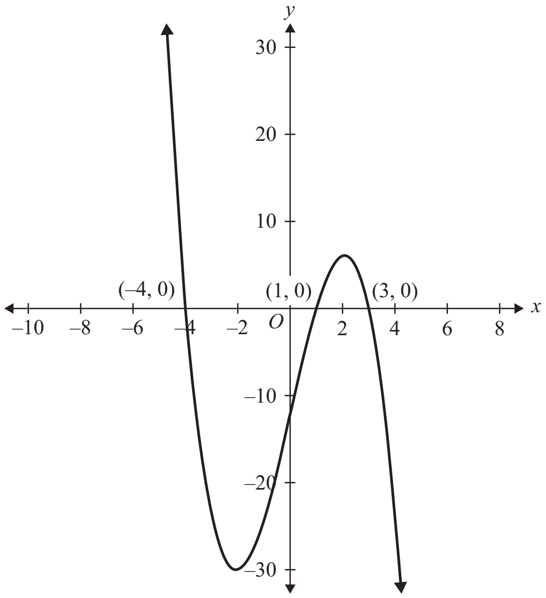
Mark allocation: 3 marks

- 1 method mark for expanding the RHS of the expression and collecting like terms
- 1 method mark for equating coefficients
- 1 answer mark for finding the values of a, b and c

Question 19

Worked solution

Sketch the graph of $y = (x + 4)(x - 1)(3 - x)$.



$(x + 4)(x - 1)(3 - x) > 0$

From the graph, we can clearly see $y > 0$ for $x \in (-\infty, -4) \cup (1, 3)$.

Mark allocation: 2 marks

- 1 mark for sketching the graph
- 1 answer mark

TIP

» **Always sketch graphs when solving inequalities.**

Question 20a.i.

Worked solution

Looking at the graph, we see that it intercepts the x-axis at $x = -2$ and touches the x-axis at $x = 3$.

Therefore the rule of the cubic graph can be written as $y = a(x + 2)(x - 3)^2$.

Substituting $(0, 18)$ into the equation above gives:

$18 = a(0 + 2)(0 - 3)^2$

$18 = 18a$

$a = 1$

$y = (x + 2)(x - 3)^2$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 20a.ii.

Worked solution

The graph shows that $(1, 2)$ is the point of inflection.

Therefore the rule of the cubic graph can be written as $y = a(x - 1)^3 + 2$.

Substituting $(-1, -2)$ into the equation above, we get:

$$-2 = a(-1 - 1)^3 + 2$$
$$-2 = -8a + 2$$
$$a = \frac{1}{2}$$
$$y = \frac{1}{2}(x - 1)^3 + 2$$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 20b.

Worked solution

We are given $f(x) = ax^3 - 3x^2 + 2x + b$.

Substituting $(1, 2)$ into the rule above gives:

$$2 = a - 3 + 2 + b$$
$$a + b = 3$$

Substituting $(-1, -4)$ into the rule above gives:

$$-4 = -a - 3 - 2 + b$$
$$-a + b = 1$$

Solving both equations, we get $b = 2$ and $a = 1$.

Hence $f(x) = x^3 - 3x^2 + 2x + 2$.

Mark allocation: 3 marks

- 1 method mark
- 1 answer mark for finding the values of a and b
- 1 answer mark for the rule

Question 21a.

Worked solution

Dilation of factor 3 from the x-axis.

Reflected in the x-axis.

Translated 2 units in the positive direction of the x-axis and 1 unit in the positive direction of the y-axis.

Mark allocation: 2 marks

- 1 answer mark for the correct dilation and reflection
- 1 answer mark for the correct translation

Question 21b.

Worked solution

Translated 1 unit in the positive direction of the y-axis.

Dilated by a factor of $\frac{1}{3}$ from the x-axis and a factor of 5 from the y-axis.

Translated 2 units in the positive direction of the x-axis.

Mark allocation: 2 marks

- 1 answer mark for the correct dilations
- 1 answer mark for the correct translations

Question 22a.

Worked solution

$$(x, y) \rightarrow (-x, y) \rightarrow (-x-2, y+2) \rightarrow \left(\frac{1}{2}(-x-2), y+2\right) \rightarrow (x', y')$$

$x' = \frac{1}{2}(-x-2)$ and $y' = y+2$.

Hence $x = -2x' - 2$ and $y = y' + 2$.

Substituting the above equations for x and y into the equation $y = \sqrt{x-3}$ gives:

$$\begin{aligned} y' + 2 &= \sqrt{-2x' - 2 - 3} \\ y' &= \sqrt{-2x' - 5} - 2 \end{aligned}$$

Therefore the transformed equation is $g(x) = \sqrt{-2x-5} - 2$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

TIP

» **Be careful not to call the transformed function $f(x)$.**

Question 22b.

Worked solution

$x \in (-\infty, -5]$

Mark allocation: 1 mark

- 1 answer mark

Question 23a.

Worked solution

$(3, 7) \rightarrow (-11, 31)$

Mark allocation: 1 mark

- 1 answer mark

Question 23b.

Worked solution

$(x, y) \rightarrow (-3x - 2, 4y + 3) \rightarrow (x', y')$

$x' = -3x - 2$ and $y' = 4y + 3$.

Hence $x = \frac{-x' - 2}{3}$ and $y = \frac{y' - 3}{4}$.

Substituting the equations for x and y above into the equation $y = x^2 - 2x$ gives:

$$\frac{y' - 3}{4} = \left(\frac{-x' - 2}{3}\right)^2 - 2\left(\frac{-x' - 2}{3}\right)$$

$$\frac{y' - 3}{4} = \left(\frac{x'^2 + 4x' + 4}{9}\right) + \left(\frac{2x' + 4}{3}\right)$$

$$y' = \frac{4x'^2}{9} + \frac{40 \cdot x'}{9} + 9$$

Hence the image of the transformed equation is $y = \frac{4x^2}{9} + \frac{40 \cdot x}{9} + 9$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 24a.i.

Worked solution

$a = 2$

Mark allocation: 1 mark

- 1 answer mark

Question 24a.ii.

Worked solution

$a < -1$

Mark allocation: 1 mark

- 1 answer mark

Question 24b.i.

Worked solution

Reflection in the x-axis, followed by translation of 1 unit in the negative direction of the x-axis and 1 unit in the negative direction of the y-axis.

Mark allocation: 1 mark

- 1 answer mark

Question 24b.ii.

Worked solution

The turning point $(-2, 0)$ will become $(-3, -1)$, and $(0, 4)$ will be become $(-1, -5)$.

Mark allocation: 1 mark

- 1 answer mark

TIP

» You don't need to find the rule of the function in order to find the turning points, as the turning points will change according to the transformation applied.

Question 25a.

Worked solution

$$n(\xi) = n(B \cup C) + n\,(B' \cap C')$$

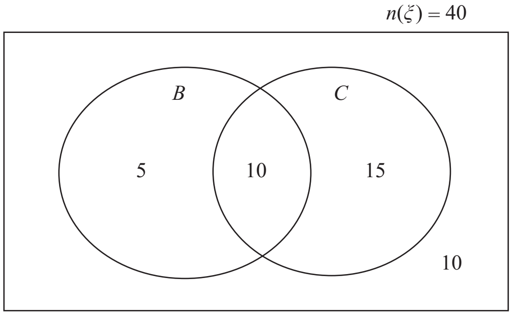
$$\Rightarrow n(B \cup C) = 40 - 10 = 30$$

$$n(B \cup C) = n(B) + n(C) - n(B \cap C)$$

$$30 = 15 + 25 - n(B \cap C)$$

$$n(B \cap C) = 10$$

$$\Rightarrow n(B \cap C') = 5\ n(C \cap \mathrm{B}') = 15$$



Mark allocation: 2 marks

- 1 answer mark for correctly labelling $n(B \cap C)$
- 1 answer mark if all other values are correct

Question 25b.

Worked solution

Using the formula for conditional probability gives

$$\begin{aligned}\Pr(B|C) &= \frac{\Pr(B \cap C)}{\Pr(C)}\\ &= \frac{10}{25}\\ &= \frac{2}{5}\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for recognising conditional probability and either using the formula or a reduced sample space
- 1 answer mark

TIP

» This question can also be answered by considering the reduced sample space. Since we are told that the teacher already has a high level of cholesterol, the sample space reduces from 40 to 25. Of those 25, there are only 10 who could also have high blood pressure.

Question 26a.

Worked solution

Since $\Pr(A \cup B) = 0.75$, therefore $\Pr(A' \cap B') = 1 - \Pr(A \cup B) = 1 - 0.75 = 0.25$.

	B	B'	
A	0.38	0.20	0.58
A'	0.17	0.25	0.42
	0.55	0.45	1

Mark allocation: 2 marks

- 1 method mark for finding $\Pr(A' \cap B')$
- 1 answer mark for all the other values in the table

TIP

» You can also find $\Pr(A \cap B)$ using the addition rule of probability.

Question 26b.

Worked solution

$$\Pr(A'|B) = \frac{\Pr(A' \cap B)}{\Pr(B)} = \frac{0.17}{0.55} = \frac{17}{55}$$

Mark allocation: 1 mark

- 1 answer mark

Question 26c.

Worked solution

$\Pr(A' \cup B') = \Pr(A \cap B)' = 1 - \Pr(A \cap B) = 1 - 0.38 = 0.62$

Mark allocation: 1 mark

- 1 answer mark

Question 26d.

Worked solution

$\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{38}{55}$

$\Pr(A) = 0.58 = \dfrac{58}{100} = \dfrac{29}{50}$

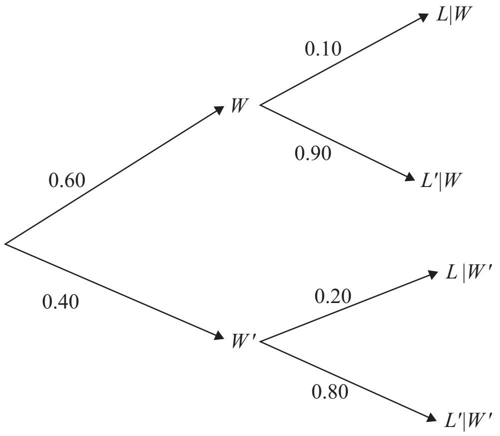
The events A and B are not independent since $\Pr(A|B) \neq \Pr(A)$.

Mark allocation: 2 marks

- 1 method mark
- 1 mark for the final statement.

Question 27a.

Worked solution



Mark allocation: 2 marks

- 1 method mark for constructing a tree diagram with two branches and two outcomes per branch
- 1 mark for the correct probabilities

Question 27b.

Worked solution

$$\begin{aligned}\Pr(L) &= \Pr(W) \times \Pr(L|W) + \Pr(W') \times \Pr(L|W') \\ &= 0.6 \times 0.1 + 0.4 \times 0.2 \\ &= 0.14\end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 27c.

Worked solution

$$\Pr\left(W'|L\right) = \frac{\Pr(W' \cap L)}{\Pr(L)} = \frac{0.08}{0.14} = \frac{4}{7}$$

Mark allocation: 2 marks

- 1 method mark for recognising conditional probability
- 1 answer mark

Question 28a.

Worked solution

$6 + 7 = 13$ choices

Mark allocation: 1 mark

- 1 answer mark

TIP

» A choice between paint **or** wallpaper suggests the use of the addition principle (i.e. ‘or’ implies ‘+’).

Question 28b.

Worked solution

$8 \times 7 = 56$ colours

Mark allocation: 1 mark

- 1 answer mark

TIP

» A choice of ceiling paint colours **and** wall paint colours suggests the use of the multiplication principle (i.e. ‘and’ implies ‘×’).

Question 28c.i.

Worked solution

An arrangement of six photo frames without repetition (as they are all different) gives

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways.

Mark allocation: 1 mark

- 1 answer mark

TIP

» An alternative method is to use boxes that represent the six 'positions' of the photo frames. The number of photo frames that can occupy the first position is six. Since that frame has been used, there are only five photo frames available for the next position, and so on.

6	5	4	3	2	1

$= 720$ ways

Question 28c.ii.

Worked solution

An arrangement of two photo frames from a possible six without repetition (as they are all different) gives

${}^{6}P_{2} = \frac{6!}{(6-2)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = 30$ ways

Mark allocation: 1 mark

- 1 answer mark

TIP

» Again, the box method can be used. In this situation there are, however, only two 'positions'.

6	5

$= 30$

Question 29a.

Worked solution

We need to select three bubble gums from 10 bubble gums, which can be done in $^{10}C_3$ ways.

$$\begin{aligned}\Pr(\text{at least one red}) &= 1 - \Pr(\text{no red})\\ &= 1 - \frac{^6C_3}{^{10}C_3}\\ &= 1 - \frac{20}{120} = \frac{5}{6}\end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

TIP

» The word 'selections' suggests the use of combinations.

Question 29b.

Worked solution

We need to select at least one red and one green bubble gum. Therefore the options are (1R, 1G, 1B) or (2R, 1G) or (1R, 2G).

Pr(at least one red and one green) = Pr(1R, 1G, 1B) + Pr(2R, 1G) + Pr(1R, 2G)

$$= \frac{4C_1 \times 2C_1 \times 4C_1}{10C_3} + \frac{4C_2 \times 2C_1}{10C_3} + \frac{4C_1 \times 2C_2}{10C_3}$$

$$= \frac{32}{120} + \frac{12}{120} + \frac{4}{120} = \frac{48}{120} = \frac{2}{5}$$

Mark allocation: 2 marks

- 1 method mark for recognising the correct options
- 1 answer mark

TIP

» **Remembering the properties $nC_1 = n$, $nC_n = 1$, $nC_2 = \frac{n(n-1)}{2}$ will help you to complete these types of calculations faster.**

Question 30a.

Worked solution

$$\begin{aligned}\frac{2^{2n} \times 4^{n-2}}{16^n} &= \frac{2^{2n} \times (2^2)^{n-2}}{2^{4n}}\\ &= \frac{2^{2n} \times 2^{2n-4}}{2^{4n}}\\ &= 2^{-4}\\ &= \frac{1}{2^4}\\ &= \frac{1}{16}\end{aligned}$$

Using index law $(a^m)^n = a^{mn}$.

Using index laws $a^m \times a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$.

Mark allocation: 2 marks

- 1 method mark for using at least one index law correctly
- 1 answer mark for the correct and simplified answer

Question 30b.

Worked solution

$3\log_3(m) + \log_3(36) - \log_3(4) - 6\log_3(\sqrt{m})$

$= 3\log_3(m) + \log_3\left(\frac{36}{4}\right) - \log_3(m)^{\frac{6}{2}}$

Using log laws $\log_a(m)^n = n\log_a(m)$ and $\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$.

$= 3\log_3(m) + \log_3(9) - 3\log_3(m)$

$= \log_3(3)^2$

$= 2\log_3(3)$

$= 2$

Mark allocation: 2 marks

- 1 method mark for using at least one logarithmic law correctly
- 1 answer mark for a simplified answer

TIP

» **Simplifying logarithmic expressions can be done in a few steps. For example:**

$$\begin{aligned} 3\log_3(m) + \log_3(36) - \log_3(4) - 6\log_3(\sqrt{m}) &= \log_3\left(\frac{36m^3}{4m^3}\right) \\ &= \log_3(9) \\ &= 2\log_3(3) \\ &= 2 \end{aligned}$$

Question 31

Worked solution

$\frac{49^{x+3} \times 7^{1-3x}}{49} > 343$

$\frac{(7^2)^{x+3} \times 7^{1-3x}}{7^2} > 7^3$

Using index law $(a^m)^n = a^{mn}$.

$\frac{7^{-x+7}}{7^2} > 7^3$

Using index laws $a^m \times a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$

$7^{-x+5} > 7^3$

Since the base is greater than 1, the inequality holds.

$-x + 5 > 3$

$x < 2$

Mark allocation: 2 marks

- 1 method mark for using at least one index law correctly
- 1 answer mark for the correct simplified answer

Question 32a.

Worked solution

$$\log_{10}x = \frac{1}{2}\log_{10}225 - 2\log_{10}3$$
$$\log_{10}x = \log_{10}225^{\frac{1}{2}} - \log_{10}3^2$$
$$\log_{10}x = \log_{10}15 - \log_{10}9$$
$$\log_{10}x = \log_{10}\left(\frac{15}{9}\right) = \log_{10}\left(\frac{5}{3}\right)$$
$$x = \frac{5}{3}$$

Using log laws $\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$ and $\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$.

Mark allocation: 2 marks

- 1 method mark for using at least one logarithmic law correctly
- 1 answer mark

Question 32b.

Worked solution

$8(2^{2x}) - 15(2^x) - 2 = 0$

Let $2^x = a$, then $2^{2x} = (2^x)^2 = a^2$.

$$8a^2 - 15a - 2 = 0$$
$$8a^2 - 16a + a - 2 = 0$$
$$8a(a-2) + 1(a-2) = 0$$
$$(8a+1)(a-2) = 0$$

So $a = -\frac{1}{8}$ or $a = 2$.

Reject $a = -\frac{1}{8}$ because 2^x cannot be negative.

Hence $2^x = 2$ and therefore $x = 1$.

Mark allocation: 3 marks

- 1 method mark for converting the exponential equation into a quadratic equation
- 1 method mark for rejecting the negative solution
- 1 answer mark

TIP

» **Always remember to reject the negative solution as the range of the function a^x is always greater than zero.**

Question 33a.

Worked solution

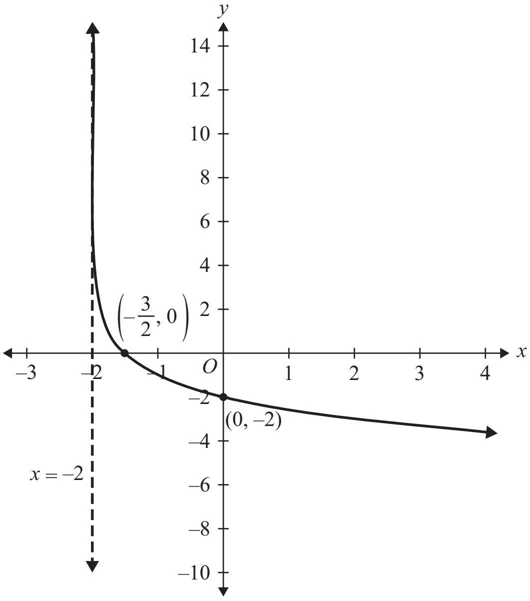
x-intercept:

$$-\log_2(x+2) - 1 = 0$$
$$\log_2(x+2) = -1$$
$$x + 2 = 2^{-1}$$
$$x = -\frac{3}{2}$$

y-intercept:

$y = -\log_2(2) - 1$

$y = -2$



Mark allocation: 3 marks

- 1 mark for the correct shape
- 1 mark for the correctly labelled asymptote
- 1 mark for the correct x- and y-intercepts

Question 33b.

Worked solution

Domain of $f(x) = (-2, \infty)$.

Mark allocation: 1 mark

- 1 answer mark

Question 33c.

Worked solution

Swapping x and y gives:

$$x = -\log_2(y + 2) - 1$$

$$-(x + 1) = \log_2(y + 2)$$

$$y = 2^{-x-1} - 2$$

Therefore $f^{-1}(x) = 2^{-x-1} - 2$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 33d.

Worked solution

Domain of $f^{-1}(x)$ = range of $f(x) = R$.

Mark allocation: 1 mark

- 1 answer mark

Question 34a.

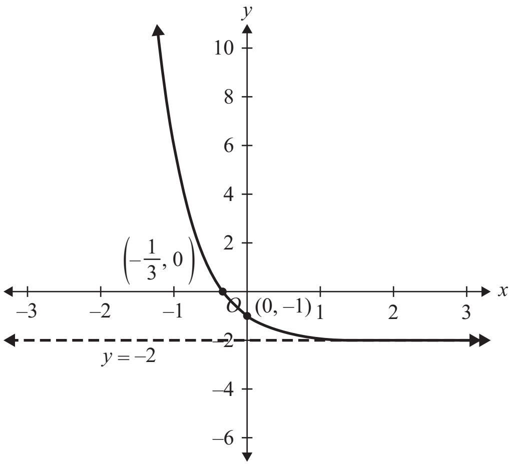
Worked solution

x-intercept:

$$\begin{aligned} -2 + 2^{-3x} &= 0 \\ 2^{-3x} &= 2 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

y-intercept:

$$\begin{aligned} y &= -2 + 2^{0} \\ &= -1 \end{aligned}$$



Mark allocation: 3 marks

- 1 mark for the correct shape
- 1 mark for the correctly labelled asymptote
- 1 mark for correct x- and y-intercepts

Question 34b.

Worked solution

Translation of 2 units in the positive direction of the y-axis.

Reflected in the y-axis followed by a dilation of factor 3 from the y-axis.

Mark allocation: 2 marks

- 1 answer mark for the dilation and reflection
- 1 answer mark for the translation

Question 35

Worked solution

Since the asymptote shown is $x = -4$, then $b = 4$.

So $y = a\log_2(x + 4) + c$.

Substituting $(0, 0)$ into the equation above gives:

$0 = a\log_2 4 + c$

$2a + c = 0$ [equation (1)]

Substituting (4, 3) into the log equation gives:

$3 = a\log_2 8 + c$

$3a + c = 3$ [equation (2)]

Solving equations 1 and 2, we get $a = 3$ and $c = -6$.

Mark allocation: 3 marks

- 1 answer mark for the value of b
- 1 method mark
- 1 answer mark for the value of a and c

Question 36

Worked solution

$$2\cos(2x) + \sqrt{3} = 0$$

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$$

$$2x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$$

$$x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$$

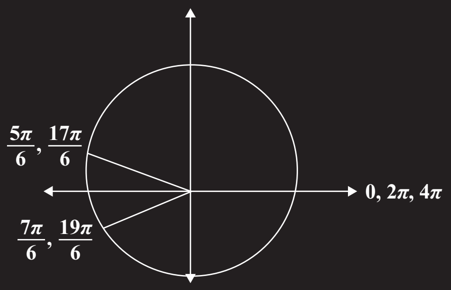
Mark allocation: 3 marks

- 1 method mark for establishing the equation $\cos(2x) = -\frac{\sqrt{3}}{2}$
- 1 method mark for finding the correct first quadrant angle $\frac{\pi}{6}$
- 1 answer mark

TIPS

- Be sure to check the domain of the question to ensure all solutions have been found.
- It might also be useful to draw a unit circle to help identify the correct angles within the required domain. For example, by adjusting the domain as follows we can see that the solutions are within **two** rotations of the unit circle (between 0 and 4π), so there would be **four** solutions.

$$0 \le x \le 2\pi$$
$$0 \le 2x \le 4\pi$$



- It can be seen on the unit circle above that the only solutions possible with the adjusted domain are $2x = \frac{5\pi}{6}, \frac{17\pi}{6}, \frac{7\pi}{6}, \frac{19\pi}{6}$.

Question 37

Worked solution

We are given $\cos x = -\frac{5}{13}$.

Using the trig identity $\sin^2 x + \cos^2 x = 1$, we have $\sin x = \pm\sqrt{1 - \cos^2 x}$.

$$\sin x = \pm\sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\sin x = \pm\frac{12}{13}$$

Since $\pi \le x \le \frac{3\pi}{2}$, $\sin x$ is negative in the third quadrant, so $\sin x = -\frac{12}{13}$.

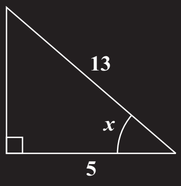
$$\tan(x) = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$$

Mark allocation: 2 marks

- 1 mark for the value of $\sin x$
- 1 mark for the value of $\tan x$

TIPS

- » Be careful to check in which quadrant the angle lies.
- » This question can also be solved in a different way: $\cos x = -\frac{5}{13}$, therefore we could use the definition of $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$.



Hence we can find the opposite side using Pythagoras' theorem: $\sqrt{13^2 - 5^2} = 12$.

Now we can find $\sin x$ and $\tan x$ using their definitions and then apply the plus or minus, depending on which quadrant they lie in.

Question 38a.

Worked solution

$\sin(-\alpha) = -\sin\alpha = -\frac{2}{5}$

Mark allocation: 1 mark

- 1 answer mark

Question 38b.

Worked solution

$\cos(3\pi - \beta) = -\cos\beta = -\frac{1}{5}$

Mark allocation: 1 mark

- 1 answer mark

Question 38c.

Worked solution

$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin\alpha = \frac{2}{5}$

Mark allocation: 1 mark

- 1 answer mark

Question 39a.

Worked solution

Amplitude $= 2$

Period $= \frac{2\pi}{1} = 2\pi$

Mark allocation: 1 mark

- 1 answer mark

Question 39b.

Worked solution

x-intercept:

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}, \frac{2\pi}{3}$$

y-intercept:

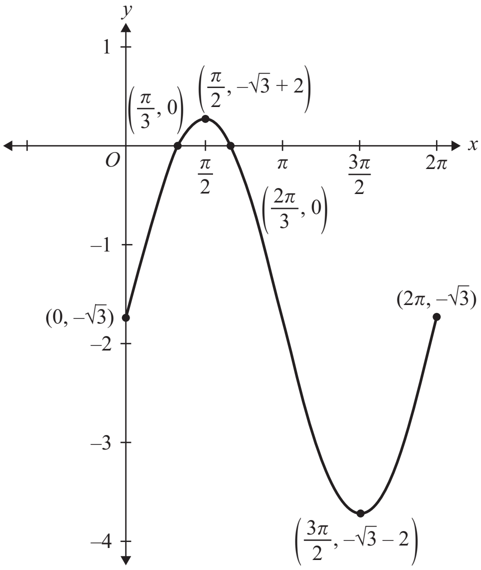
$$y = 2\sin(0) - \sqrt{3} = -\sqrt{3}$$

Mark allocation: 2 marks

- 1 answer mark for the correct x-intercept
- 1 answer mark for the correct y-intercept

Question 39c.

Worked solution



Mark allocation: 3 marks

- 1 mark for the correct shape of graph
- 1 mark for the x-intercept and y-intercept labelled correctly
- 1 mark for correctly labelled turning points and end point

TIPS

» The turning points occur halfway between the x-intercepts.

» To find the next turning point, add half the period (π) to the x-coordinate of the first turning point.

» The period is 2π, therefore you can easily find the end point without substitution because the point $(0, -\sqrt{3})$ will repeat its value at $(2\pi, -\sqrt{3})$.

Question 40a.

Worked solution

Period $= \frac{\pi}{2}$

Mark allocation: 1 mark

- 1 answer mark

Question 40b.

Worked solution

x-intercepts:

Since $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $-\pi \le 2x \le \pi$.

$$\tan(2x) - 1 = 0$$

$$\tan(2x) = 1$$

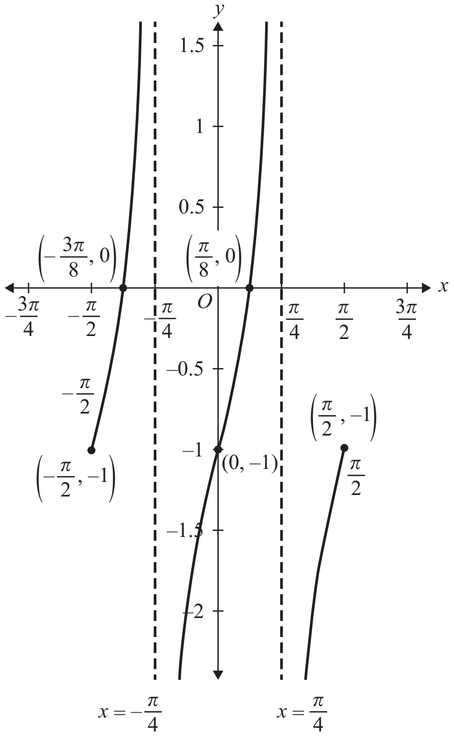
$$2x = \tan^{-1}(1) = \frac{\pi}{4}, -\frac{3\pi}{4}$$

$$x = \frac{\pi}{8}, -\frac{3\pi}{8}$$

y-intercept:

$$y = \tan(0) - 1 = -1$$

The asymptotes occur at every odd multiple of half the period, therefore the equations of the asymptotes are $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.



Mark allocation: 3 marks

- 1 mark for the correct shape of the graph
- 1 mark for the x-intercept and y-intercept labelled correctly
- 1 mark for correctly labelled asymptotes and end points

Question 41a.

Worked solution

$$\text{Average rate of change} = \frac{P(4) - P(0)}{4 - 0}$$
$$= \frac{14 - 6}{4}$$
$$= \$2 \text{ per month}$$

Mark allocation: 2 marks

- 1 method mark for the correct use of the definition of average rate of change
- 1 answer mark for correct answer

TIP

» **Don't confuse average rate of change with instantaneous rate of change. Average rate of change is simply the gradient between two points.**

Question 41b.

Worked solution

$t \in [0, 4] \cup [8, 12]$ or $\{t : 0 \le t \le 4\} \cup \{t : 8 \le t \le 12\}$.

Mark allocation: 1 mark

- 1 answer mark

Question 42a.

Worked solution

$$\text{Let } y = \frac{3x^3}{50} - x^2 + \frac{x}{6} - 25$$
$$\frac{dy}{dx} = \frac{9x^2}{50} - 2x + \frac{1}{6}$$

Mark allocation: 1 mark

- 1 answer mark

Question 42b.

Worked solution

$$f(x) = (x - 3)^2(x + 1)$$
$$f(x) = (x^2 - 6x + 9)(x + 1)$$
$$= x^3 + x^2 - 6x^2 - 6x + 9x + 9$$
$$= x^3 - 5x^2 + 3x + 9$$
$$f'(x) = 3x^2 - 10x + 3$$

Mark allocation: 2 marks

- 1 method mark for expanding the brackets
- 1 answer mark

TIP

» **In this case, it is best to expand the brackets before differentiating. Remember to use $f'(x)$ and not $\frac{dy}{dx}$.**

Question 42c.

Worked solution

Let $f(x) = 4\sqrt{x} - \frac{3}{x^2} = 4x^{\frac{1}{2}} - 3x^{-2}$

$$f'(x) = 4 \times \frac{1}{2}x^{-\frac{1}{2}} + 6x^{-3}$$

$$= 2x^{-\frac{1}{2}} + 6x^{-3} = \frac{2}{\sqrt{x}} + \frac{6}{x^3}$$

Therefore $f'(4) = \frac{2}{\sqrt{4}} + \frac{6}{4^3} = \frac{2}{2} + \frac{6}{64} = \frac{35}{32}$.

Mark allocation: 2 marks

- 1 answer mark for differentiating correctly
- 1 answer mark for the correct substitution of $f'(4)$

TIP

» **It is useful here to rewrite each term as a power of x.**

Question 43a.

Worked solution

Gradient of the secant $= \frac{f(1+h)-f(1)}{1+h-1} = \frac{f(1+h)-f(1)}{h}$

$$\frac{f(1+h)-f(1)}{h} = \frac{3(1+h)^2-(1+h)-(3\times 1^2-1)}{h}$$

$$= \frac{3(1+2h+h^2)-1-h-2}{h}$$

$$= \frac{3+6h+3h^2-1-h-2}{h}$$

$$= \frac{3h^2+5h}{h}$$

$$= \frac{h(3h+5)}{h}$$

$$= 3h+5$$

Mark allocation: 2 marks

- 1 method mark for correctly substituting the function into the expression for the gradient of a secant: $\frac{f(1+h)-f(1)}{h}$
- 1 answer mark for arriving at the correct simplified answer

TIP

» **It is important to take care when expanding brackets and gathering like terms. Be careful with negatives as well!**

Question 43b.i.

Worked solution

From **part a.**, the gradient of the secant $= 3h + 5$. Substituting $h = 2$ gives a gradient of 11.

Mark allocation: 1 mark

- 1 answer mark

TIPS

» If $x = 3$, then $1 + h = 3$ and $h = 2$. The word 'hence' means that the information from an earlier part must be used.

» When cancelling, every term in the numerator that doesn't have an h should cancel out.

Question 43b.ii.

Worked solution

From **part a.**, the gradient of the secant between $x = 1$ and $x = 1 + h$ is $3h + 5$.

The gradient of the tangent at $x = 1$ is given by $\lim\limits_{h\to 0}(3h + 5) = 5$.

Mark allocation: 1 mark

- 1 answer mark

TIP

» The key to answering this question is to realise the subtle, but critical, difference between the gradient of a secant between two points, $\frac{f(x+h)-f(x)}{h}$, and the gradient of a tangent at a point, $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Again, the word 'hence' means that the information from an earlier part must be used.

Question 44a.

Worked solution

$f(x) = 9 - x^2$

$f(1) = 9 - 1^2 = 8$

$\therefore b = 8$

$f'(x) = -2x$

$f'(1) = -2$

So the tangent passes through the point $(1, 8)$ with a gradient of –2.

$y - y_1 = m(x - x_1)$

$y - 8 = -2(x - 1)$

$y = -2x + 2 + 8$

$y = -2x + 10$

Alternatively:

$y = mx + \mathrm{c}$

$8 = -2 \times 1 + c$

$c = 10$

$y = -2x + 10$

Mark allocation: 3 marks

- 1 answer mark for finding $b = 8$
- 1 method mark for correctly finding that the gradient at $x = 1$ is -2
- 1 answer mark for correctly substituting these values into the general equation for a straight line and deriving the correct answer

TIP

» Questions like these that involve tangents usually provide two critical pieces of information that might not be obvious. The first is that the derivative of the function at $x = 1$ (x_1) is the same as the gradient of the tangent (i.e. m). The second is that $f(1) = b$ is also a point on the tangent (y_1). These pieces of information can then be used in the equation for a straight line, having been given a point and a gradient: $y - y_1 = m(x - x_1)$.

Question 44b.

Worked solution

$y = -2x + 10$

When $y = 0$:

$$\begin{aligned} -2x + 10 &= 0 \\ -2x &= -10 \\ x &= 5 \end{aligned}$$

When $x = 0$:

$$\begin{aligned} y &= -2 \times 0 + 10 \\ y &= 10 \end{aligned}$$

∴ Coordinates are $(5, 0)$ and $(0, 10)$.

Mark allocation: 2 marks

- 1 method mark for correctly finding an intercept, either $x = 5$ or $y = 10$
- 1 answer mark for the correct coordinates of both the x- and y-intercepts

TIP

» Don't forget to find both axis intercepts and to write them as coordinates.

Question 45a.

Worked solution

$$\begin{aligned}\int(3\sqrt{x}-3x^2)dx &= \int\left(3x^{\frac{1}{2}}-3x^2\right)dx\\ &= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{3x^3}{3}+c\\ &= 3x^{\frac{3}{2}}\times\frac{2}{3}-x^3+c\\ &= 2x^{\frac{3}{2}}-x^3+c \quad \text{or} \quad 2\sqrt{x^3}-x^3+c\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for one correct antiderivative: $-x^3$ or $2\sqrt{x^3}$
- 1 answer mark

TIPS

» When finding antiderivatives of square or cube roots, rewrite the expression with a rational power. Then carefully deal with the fractions by setting out each step.

» Don’t forget to include the constant, c.

» If the question asks for an antiderivative, you don’t have to put the constant because the constant could be considered to be zero.

Question 45b.

Worked solution

$$\begin{aligned}\int_{-1}^{2}(x^3+2x)\,dx &= \left[\frac{x^4}{4}+\frac{2x^2}{2}\right]_{-1}^{2}\\ &= \left[\frac{x^4}{4}+x^2\right]_{-1}^{2}\\ &= \frac{(2)^4}{4}+(2)^2-\left[\frac{(-1)^4}{4}+(-1)^2\right]\\ &= \frac{16}{4}+4-\frac{1}{4}-1\\ &= \frac{15}{4}+3\\ &= \frac{15}{4}+\frac{12}{4}\\ &= \frac{27}{4}\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for correct antiderivative: $\frac{x^4}{4}+x^2$
- 1 answer mark

TIP

» It is important when evaluating definite integrals to set out each step so that simple arithmetic errors are avoided.

Question 46

Worked solution

When $x = -2$, $y = \frac{2}{(-2)^2} = \frac{2}{4} = \frac{1}{2}$.

So $y = \frac{2}{x^2} = 2x^{-2}$.

$\frac{dy}{dx} = -4x^{-3}$, therefore at $x = -2$, $\frac{dy}{dx} = -4(-2)^{-3} = \frac{-4}{-8} = \frac{1}{2}$.

Therefore the gradient of the perpendicular line $= -2$.

Hence the equation of the line is

$$\begin{aligned} y - \frac{1}{2} &= -2(x+2) \\ y &= -2x - 4 + \frac{1}{2} \\ y &= -2x - \frac{7}{2} \end{aligned}$$

Mark allocation: 3 marks

- 1 method mark for the correct derivative
- 1 method mark for finding the gradient of the perpendicular line
- 1 answer mark

Question 47a.

Worked solution

$$\begin{aligned} y &= x^3 - 27x \\ \frac{dy}{dx} &= 3x^2 - 27 \\ \frac{dy}{dx} &= 0 \Rightarrow 3x^2 - 27 = 0 \end{aligned}$$

$3(x^2 - 9) = 0 \Rightarrow (x+3)(x-3) = 0$

So $x = -3$ or $x = 3$.

When $x = -3$, $y = (-3)^3 + 81 = 54$.

When $x = 3$, $y = (3)^3 - 81 = -54$.

To test the nature of the stationary points, we make a nature table:

x	–4	–3	–2
$\frac{dy}{dx}$	21	0	–15

Since the gradient changes from positive to negative, $(-3, -54)$ is a maximum turning point.

x	2	3	4
$\frac{dy}{dx}$	–15	0	21

Since the gradient changes from negative to positive, $(3, -54)$ is a minimum turning point.

Mark allocation: 3 marks

- 1 method mark for finding the x values of the stationary points
- 1 method mark for finding the y-coordinates of the stationary points
- 1 answer mark for the correct nature of the stationary points

TIPS

» Since the question is not a 'show that' question, you don't have to perform the first derivative test. You can simply state the nature by drawing a rough sketch of the cubic graph.

» You could also draw the derivative graph and look at the intercepts going from positive to negative or vice versa. This way is just as easy as drawing the cubic graph compared to the nature table method.

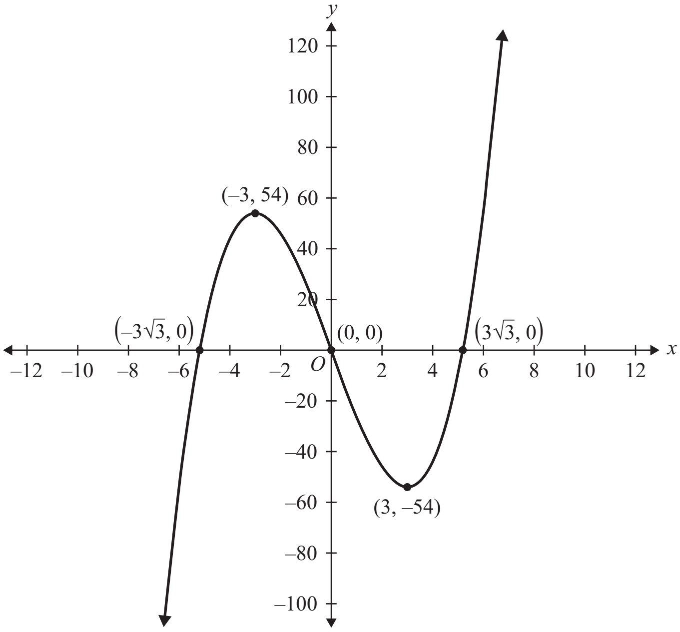
Question 47b.

Worked solution

x-intercepts:

$$\begin{aligned} x^3 - 27x &= 0 \\ x(x^2 - 27) &= 0 \\ x\left(x - 3\sqrt{3}\right)\left(x + 3\sqrt{3}\right) &= 0 \end{aligned}$$

So $x = -3\sqrt{3}$ or $x = 3\sqrt{3}$ in addition to $x = 0$.

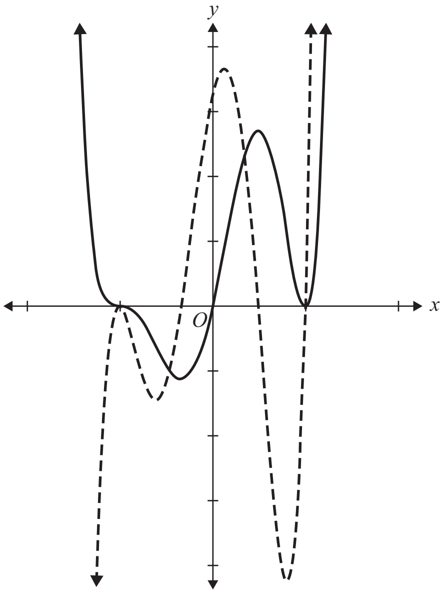


Mark allocation: 3 marks

- 1 mark for the correct shape of the graph
- 1 mark for correctly labelled stationary points
- 1 mark for correctly labelled axial intercepts

Question 48

Worked solution



The dashed curve indicates the graph of the gradient function.

Mark allocation: 2 marks

- 1 mark for the correct shape of the graph
- 1 mark for the correct location of the x-intercepts.

Note: The location of the turning points need not be accurate but they should line up vertically with the steepest part of each segment.

TIPS

- » **The stationary points of inflections and turning points become the x-intercepts in the derivative graph.**
- » **If the gradient of the graph is negative, the y values of the gradient function will be below the x-axis (i.e. y values will be negative too).**
- » **If the gradient of the graph is positive, the y values of the gradient function will be above the x-axis (i.e. y values will be positive too).**

Question 49a.

Worked solution

Since $(-1, 6)$ is the turning point, $f'(-1) = 0$.

$$\begin{aligned} f'(-1) &= 4 + k = 0 \\ k &= -4 \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 49b.

Worked solution

$f(x) = \int f'(x)dx$

$f(x) = \int 4x^2 - 4\ dx = \frac{4x^3}{3} - 4x + \mathrm{c}$

Since $(-1, 6)$ is a point of $f(x)$, then:

$6 = -\frac{4}{3} + 4 + c$

$c = \frac{10}{3}$

So the rule is $f(x) = \frac{4x^3}{3} - 4x + \frac{10}{3}$.

Mark allocation: 2 marks

- 1 method mark for the correct antiderivative
- 1 answer mark

Question 50a.

Worked solution

Let x be the side of the square base and h be the height of the cuboid.

The total surface area $= 2x^2 + 4xh = 500$, hence:

$4xh = 500 - 2x^2$

$h = \frac{500 - 2x^2}{4x} = \frac{250 - x^2}{2x}$

Mark allocation: 2 marks

- 1 mark for finding the expression of the total surface area of the cuboid
- 1 mark for showing the transposition of the formula

Question 50b.

Worked solution

$V = l \times w \times h = x^2 h = x^2\left(\frac{250 - x^2}{2x}\right) = \frac{x(250 - x^2)}{2}$

Mark allocation: 1 mark

- 1 answer mark

Question 50c.

Worked solution

$V = \frac{250x - x^3}{2} = 125x - \frac{x^3}{2}$

$\frac{dV}{dx} = 125 - \frac{3x^2}{2}$

$\frac{dV}{dx} = 0 \Rightarrow 125 - \frac{3x^2}{2} = 0$

$3x^2 = 250$

So $x = \pm\sqrt{\frac{250}{3}} = \pm\ \frac{5\sqrt{30}}{3}$.

Since x can't be negative, as it is the side length of the box, $x = \frac{5\sqrt{30}}{3}$.

Mark allocation: 3 marks

- 1 method mark for finding the derivative
- 1 method mark for finding the value of x
- 1 mark for the correct value of x and stating why the other value is rejected

TIP

» **Ensure that the value of x is simplified with a rational denominator, as requested in the question.**

Section 2 Multiple-choice

Question 1

Answer: ***D***

Explanatory notes

The distance equation is really an extension of Pythagoras' theorem. In other words, we need two squared terms that add up to 17.

One such combination is $4^2 + 1^2 = 17$.

Therefore $x_2 - x_1 = 1$ or 4 and $y_2 - y_1 = 1$ or 4.

The only points for which this is the case are $(–1, 3)$ and $(0, 7)$.

Alternatively, each case could be tested using the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

TIP

» It may be worth storing the distance formula in your CAS device prior to an exam.

Question 2

Answer: ***A***

Explanatory notes

$$\begin{aligned}
\text{midpoint} &= \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)\\
\left(\frac{a+2}{2}, \frac{3+b}{2}\right) &= (-1, 4)\\
\frac{a+2}{2} &= -1\\
a &= -4\\
\frac{3+b}{2} &= 4\\
b &= 5
\end{aligned}$$

Question 3

Answer: ***C***

Explanatory notes

For the line $y = 7 - \frac{x}{5}$, the gradient is $m_1 = -\frac{1}{5}$.

Therefore the gradient of the perpendicular line is $m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{-1}{5}} = 5$.

Question 4

Answer: ***D***

Explanatory notes

Use the **solve** function on a CAS to make y the subject for the first three equations.

$\text{solve}(y-2=3\cdot x, y)$	$y=3\cdot x+2$
$\text{solve}(6\cdot x-2\cdot y=7, y)$	$y=3\cdot x-\frac{7}{2}$
$\text{solve}(3\cdot y=3\cdot x-11, y)$	$y=\frac{3\cdot x-11}{3}$
$\text{expand}\left(y=\frac{3\cdot x-11}{3}\right)$	$y=x-\frac{11}{3}$

Lines are parallel if they have the same gradient. So we can clearly see line I and line II have the same gradient.

Question 5

Answer: ***A***

Explanatory notes

Substitute the given values of the variables into the expression and solve for e.

$$\text{solve}\left(\sqrt{\frac{2.8}{3.6}}=(4.2)^2-\frac{e}{\sqrt{4.2}}, e\right)$$

$$e=34.3438500741$$

Question 6

Answer: ***B***

Explanatory notes

The gradient of the line is $-\frac{5}{12}$.

We know that $m=\tan\theta$, therefore $\theta=\tan^{-1}(m)=\tan^{-1}\left(-\frac{5}{12}\right)=157°$.

TIPS

- Use the **solve** function on a CAS to make y the subject to identify the gradient.
- Ensure your calculator is set to degrees for this type of question.
- Ensure you subtract the answer from 180° because a negative angle is the angle made by the line in the negative direction of the x-axis.

Question 7

Answer: ***B***

Explanatory notes

$x + y = k$

$\quad y = -x + k$

Therefore the gradient and y-intercept are both negative since $k < 0$. The only option for this is B.

TIP

» **Substituting values for k (i.e. $k = -2$) and using a CAS to sketch the corresponding graph may help.**

Question 8

Answer: ***A***

Explanatory notes

The transformed parabola will be of the form $y = a(x + b)^2 + c$.

A dilation of factor 2 from the x-axis means $a = 2$.

A shift to the right means $b = -3$, and a shift down means $c = -4$.

So the answer is $y = 2(x - 3)^2 - 4$.

Question 9

Answer: ***B***

Explanatory notes

Since the equation given is of an inverted parabola, its maximum y value is the y-coordinate of its turning point, which in this case is -5.

TIPS

» **On the Class pad, use the fMax function and on the Ti-inspire define the function first and then use the fMax function.**

» **Or simply sketch the graph on the CAS and find its maximum value:**

$f(x) = -(3 \cdot x - 7)^2 - 5$	Done
$f\text{Max}(f(x), x)$	$x = \frac{7}{3}$
$f\left(\frac{7}{3}\right)$	-5

Question 10

Answer: ***D***

Explanatory notes

$$\left(m + \frac{1}{m}\right)^2 = m^2 + 2 + \frac{1}{m^2}$$

$$8^2 = m^2 + \frac{1}{m^2} + 2$$

$$m^2 + \frac{1}{m^2} = 64 - 2 = 62$$

Question 11

Answer: ***A***

Explanatory notes

Equate both given equations: $x^2 + x - 2 = 3x - 2$.

Hence we get $x^2 - 2x = 0$.

So $x = 0$ and $x = 2$.

Alternatively, use the **solve** function on a CAS:

$\text{Solve}(x^2 + x - 2 = 3 \cdot x - 2, x) \quad x = 0 \text{ or } x = 2$

Question 12

Answer: ***B***

Explanatory notes

Since the x-intercepts are clearly seen, we can use the intercept form of the parabola equation.

Therefore the equation of the parabola is $y = a(x + 2)(x - 3)$.

Substituting $(-1, 8)$ into the equation above gives:

$$\begin{aligned} 8 &= a(-1 + 2)(-1 - 3) \\ -4a &= 8 \\ a &= -2 \\ y &= -2(x + 2)(x - 3) = -2(x^2 - x - 6) = -2x^2 + 2x + 12 \end{aligned}$$

TIP

» **On a CAS, use the solve function with the substitution:**

$\mathbf{solve}(8 = a \cdot (x + 2) \cdot (x - 3), a) \; x = -1 \qquad a = -2$

$\mathbf{expand}(-2 \cdot (x + 2) \cdot (x - 3)) \qquad -2 \cdot x^2 + 2 \cdot x + 12$

Question 13

Answer: ***A***

Explanatory notes

$$\begin{aligned} y &= a\left[(x - h)^2 + n\right] \\ &= a(x - h)^2 + an \end{aligned}$$

Turning point of the parabola is $(2, -4) \Rightarrow h = 2$ and $an = -4$.

$y = a(x - 2)^2 - 4$

Substituting $x = 0$, $y = 8$ into the equation above, we get:

$$\begin{aligned} 8 &= 4a - 4 \\ a &= 3 \\ an &= -4 \\ n &= -\frac{4}{a} = -\frac{4}{3} \end{aligned}$$

TIPS

- » Be careful with the brackets! It is not the turning point form of the parabola that we are used to.
- » You can check your answer using a CAS to see if the graph you have created goes through the points (0, 8) and (2, –4).

Question 14

Answer: ***B***

Explanatory notes

Use the solve function on a CAS:

$\text{solve}(-5 \cdot x^2 \geq 15 \cdot x, x)\ -3 \leq x \leq 0$

Question 15

Answer: ***A***

Explanatory notes

$$h(x) = (x-2)(x+4)$$
$$h(a-4) = (a-4-2)(a-4+4)$$
$$h(a-4) = (a-6)(a) = a^2 - 6a$$

TIP

- » Using a CAS:

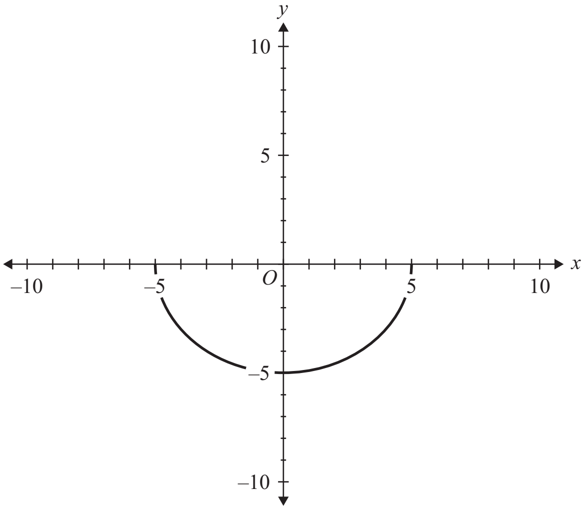
$h(x) := (x-2) \cdot (x+4)$	Done
$h(a-4)$	$a \cdot (a-6)$
$\text{expand}(a \cdot (a-6))$	$a^2 - 6 \cdot a$

Question 16

Answer: ***B***

Explanatory notes

As the graph of $f(x) = -\sqrt{25-x^2}$ below shows, domain = [–5, 5], range = [–5, 0].



TIP

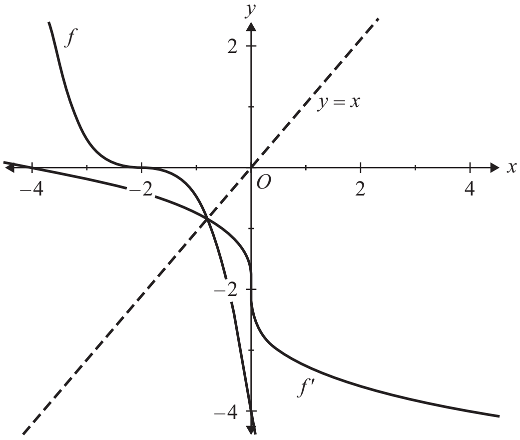
» The graph can be easily sketched using a CAS.

Question 17

Answer: ***D***

Explanatory notes

- By a process of elimination, graphs A and C are simply reflections in the x-axis and/or y-axis.
- Graph B is the reciprocal of f, not the inverse.
- The only graph that is a reflection in the line $y = x$, as shown in the graph below, is D.



TIP

» Inverses are reflections in the line $y = x$. This means that their intercepts swap. Only graph D swaps x- and y-intercepts.

Question 18

Answer: ***D***

Explanatory notes

$$y = 3\left(\frac{1}{(x+b)^2} - c\right) = \frac{3}{(x+b)^2} - 3c$$

Therefore the equations of the asymptotes are $x = -b$ and $y = -3c$.

TIP

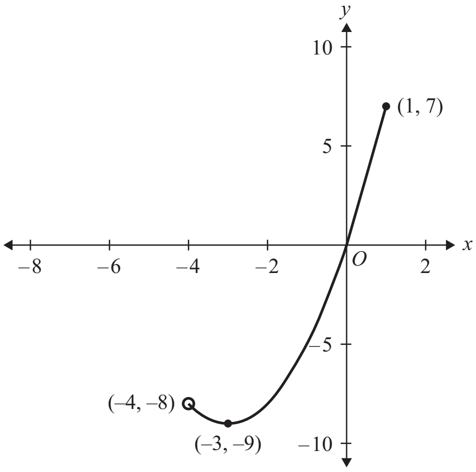
» Don't forget to expand the expression.

Question 19

Answer: ***C***

Explanatory notes

Sketch the graph with the given domain and you can clearly see that the range of the function is $[-9, 7]$.



TIPS

- The graph above can be easily sketched using a CAS.
- When asked to find the range, you should always check whether any turning points are inside the given domain.

Question 20

Answer: ***D***

Explanatory notes

For a function to have an inverse function it must be a one-to-one function. Options A, B and C don’t give a one-to-one function. In the domain $[3, \infty)$ the function is one-to-one and hence the inverse will exist.

TIP

- Use a CAS to sketch the graph with the domain restrictions and check if it passes the horizontal line test.

Question 21

Answer: ***A***

Explanatory notes

We are given $f(x) = (x-2)^2 + 1,\ x \in (-\infty, 2]$.

The domain of the inverse function is the range of function $= [1, \infty)$.

Swapping x and y gives:

$$x = (y - 2)^2 + 1$$

$$(y - 2)^2 = x - 1$$

$$y - 2 = \pm\sqrt{x - 1}$$

$$y = \pm\sqrt{x - 1} + 2$$

Since $x \in (-\infty, 2]$, we reject the upper component of the graph, therefore $f^{-1}(x) = -\sqrt{x - 1} + 2$.

TIPS

- » Using a CAS, you can swap the x and y and then solve for y:

  solve($x = (y - 1)^2 + 1, y$)

  $y = 1 - \sqrt{x - 1}$ **and** $x \geq 1$ or $y = \sqrt{x - 1} + 1$ **and** $x \geq 1$
- » Be careful to decide the inverse according to the domain of the given function.

Question 22

Answer: ***C***

Explanatory notes

A function must pass the vertical line test.

The graph of $y^2 = x - 7$ does not pass the vertical line test.

TIP

- » Use a CAS to sketch a given equation and check if it passes the vertical line test. Ideally you must know your basic graph shapes, or have them in your reference book.

Question 23

Answer: ***A***

Explanatory notes

$$f(x) = \frac{x - 2}{x + 3} = \frac{x + 3 - 5}{x + 3} = \frac{x + 3}{x + 3} - \frac{5}{x + 3} = 1 - \frac{5}{x + 3}$$

Therefore the asymptotes are $x = -3$ and $y = 1$.

TIP

- » Using the **propFrac** function on the ClassPad gives:

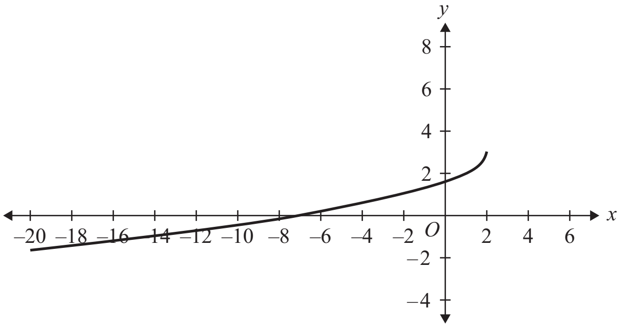
  propFrac$\left(\frac{x - 2}{x + 3}\right)$ $\qquad 1 - \frac{5}{x + 3}$

Question 24

Answer: *C*

Explanatory notes

The graph of the given function is



Therefore the domain is $(-\infty, 2]$.

TIPS

- » The graph above can be easily sketched using a CAS.
- » Solve for the values of x that will make the expression inside the square root greater than or equal to zero.

Question 25

Answer: ***B***

Explanatory notes

Let $P(x) = x^3 + x^2 - 3x + 6$. The remainder when $P(x)$ is divided by $(x - 2)$ can be found by $P(2)$ (i.e. the remainder theorem).

$P(2) = 2^3 + 2^2 - 3(2) + 6 = 12$

TIP

- » Use a CAS to define the function:

$f(x) := x^3 + x^2 - 3 \cdot x + 6$	**Done**
$f(2)$	**12**

Question 26

Answer: ***A***

Explanatory notes

Let $P(x) = x^3 - 8x^2 + 2ax + 10$.

Since $(2x - 5)$ is a factor of $P(x)$, according to factor theorem:

$$P\left(\frac{5}{2}\right) = 0$$

$$P\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^3 - 8\left(\frac{5}{2}\right)^2 + 2a\left(\frac{5}{2}\right) + 10 = 0$$

$$\frac{125}{8} - 50 + 5a + 10 = 0$$

$$5a = \frac{195}{8}$$

$$a = \frac{195}{40} = \frac{39}{8}$$

TIP

» **Use a CAS to define the function and then solve:**

$\boldsymbol{f(x) := x^3 - 8 \cdot x^2 + 2 \cdot a \cdot x + 10}$ **Done**

$\mathbf{solve}\left(f\left(\frac{5}{2}\right) = 0, a\right)$ $\boldsymbol{a = \frac{39}{8}}$

Question 27

Answer: ***B***

Explanatory notes

The algorithm we are given does the following computations:

n	a
–	1
1	$1 \times 1^3 = 1$
2	$1 \times 2^3 = 8$
3	$8 \times 3^3 = 216$

Question 28

Answer: ***B***

Explanatory notes

From $P(x) = x^3 + ax^2 + bx - 8$, it follows that:

When $P(1) = 0$:

$0 = 1 + a + b - 8$

$a + b = 7$ [equation (1)]

When $P(-2) = 0$:

$0 = -8 + 4a - 2b - 8$

$2a - b = 8$ [equation (2)]

Solving both equations, we get $a = 5$ and $b = 2$.

TIP

- Use a CAS to define the function and solve the simultaneous equations:

  $f(x) := x^3 + a \cdot x^2 + b \cdot x - 8$ Done

  $\text{solve}\left(\begin{cases} f(1) = 0 \\ f(-2) = 0 \end{cases}, \{a, b\}\right)$ $a = 5$ and $b = 2$

Question 29

Answer: ***A***

Explanatory notes

c represents a repeated factor. Since it is a turning point, the repeated factor must be squared and is given by $(x - c)^2$.

Both a and b represent x-axis intercepts and therefore yield the linear factors $(x - a)$ and $(x - b)$.

TIPS

- It may be useful to substitute values for a, b and c and then use a CAS to sketch corresponding graphs.
- A common misconception is to choose option C since $x = a$ is to the left of the origin. But $(x + a)$ as a factor gives $x = -a$ as the solution, which is not correct. However, if we substitute a negative value for a, say -2, then we find that $(x - a) = (x - -2) = (x + 2)$.

Question 30

Answer: ***D***

Explanatory notes

$(x, y) \rightarrow (x + 3, y) \rightarrow (2(x + 3), y) \rightarrow (-2(x + 3), y) \rightarrow (x', y')$

$x' = -2x - 6$ and $y' = y$.

$x = \frac{x' + 6}{-2} = -\frac{1}{2}x' - 3$

Therefore $y = \sqrt{x}$ becomes $y' = \sqrt{-\frac{1}{2}x' - 3} = \frac{\sqrt{-x' - 6}}{\sqrt{2}} = \frac{\sqrt{-2x' - 12}}{2}$.

TIPS

- Be careful with the order of the transformations mentioned.
- Using a CAS:

  $f(x) := \sqrt{x}$ Done

  $f\left(\frac{-x}{2} - 3\right)$ $\frac{\sqrt{-2 \cdot (x + 6)}}{2}$

Question 31

Answer: ***C***

Explanatory notes

The point $(2, -5)$ becomes $(2, 5)$ after being reflected in the x-axis, and then becomes $(6, 5)$ after being dilated by a factor of 3 from the y-axis.

Question 32

Answer: ***D***

Explanatory notes

$(x, y) \rightarrow (-x, y) \rightarrow (-x-2, y+3) \rightarrow (x', y')$

$x' = -x - 2$ and $y' = y + 3$.

$x = -x' - 2$ and $y = y' - 3$.

Therefore $y = -5 + 3x$ becomes $y' - 3 = -5 + 3(-x' - 2)$, giving:

$y' = -5 - 3x' - 6 + 3 = -3x' - 8$

TIPS

- Be careful with the order of the transformations mentioned.
- Using a CAS:

  $f(x) := -5 + 3 \cdot x$ **Done**

  $f(-(x+2)) + 3$ $-3 \cdot x - 8$

Question 33

Answer: ***D***

Explanatory notes

$(x, y) \rightarrow (-x, y) \rightarrow (-x, -y) \rightarrow (x', y')$

$x' = -x$ and $y' = -y$.

$y = \frac{1}{(x+1)^2} + 3$ becomes $-y' = \frac{1}{(-x'+1)^2} + 3$, giving:

$y' = \frac{-1}{(x'-1)^2} - 3$

TIP

- Using a CAS:

  $f(x) := \frac{1}{(x+1)^2} + 3$ **Done**

  $-f(-x)$ $\frac{-1}{(x-1)^2} - 3$

Question 34

Answer: ***A***

Explanatory notes

$$y = 3f(2x - 1) - 1$$

$$\frac{y' + 1}{3} = f(2x' - 1)$$

$$\frac{y' + 1}{3} = y \Rightarrow y' = 3y - 1$$

$$x = 2x' - 1 \Rightarrow x' = \frac{x + 1}{2}$$

Question 35

Answer: ***B***

Explanatory notes

$$(x, y) \rightarrow (x + 1, y - 2) \rightarrow (2(x + 1), y - 2) \rightarrow (2(x + 1), -(y - 2)) \rightarrow (2x + 2, -y + 2)$$

Question 36

Answer: ***A***

Explanatory notes

The shaded cells of the probability table below are given. It is known that all probabilities add up to 1. The other cells can then be calculated as shown. We are interested in the cell with the thick border, so the answer is 0.1.

	A	A'	
B	$0.67 - 0.47 = 0.2$	$0.3 - 0.2 = 0.1$	0.3
B'	0.47	$0.33 - 0.1 = 0.23$	$1 - 0.3 = 0.7$
	0.67	$1 - 0.67 = 0.33$	1

Alternatively:

$$\Pr(A) = \Pr(A \cap B') + \Pr(A \cap B)$$

$$0.67 = 0.47 + \Pr(A \cap B)$$

$$\Pr(A \cap B) = 0.2$$

$$\Pr(B) = \Pr(A \cap B) + \Pr(B \cap A')$$

$$0.3 = 0.2 + \Pr(B \cap A')$$

$$\Pr(B \cap A') = 0.1$$

TIP

- It is helpful to set out a probability table like the one shown in the Explanatory notes above.

Question 37

Answer: ***D***

Explanatory notes

If two events, A and B, are independent, then $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$.

$$\Pr(B) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{0.24}{0.6} = 0.4$$

Question 38

Answer: ***C***

Explanatory notes

$$\begin{aligned}
\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B) && \text{(addition rule)} \\
&= \Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B) && \text{(since } A \text{ and } B \text{ are independent)} \\
\frac{4}{5} &= \frac{3}{10} + \Pr(B) - \frac{3}{10} \times \Pr(B) \\
\frac{5}{10} &= \Pr(B) - \frac{3}{10} \times \Pr(B) \\
5 &= 10\Pr(B) - 3\Pr(B) \\
5 &= 7\Pr(B) \\
\Pr(B) &= \frac{5}{7}
\end{aligned}$$

Question 39

Answer: ***A***

Explanatory notes

$$\Pr(A'|B) = \frac{\Pr(A' \cap B)}{\Pr(B)}$$

$$\Pr(B) = \Pr(A' \cap B) + \Pr(A \cap B) = \frac{2}{5} + \frac{2}{7} = \frac{24}{35}$$

$$\Pr(A'|B) = \frac{\frac{2}{7}}{\frac{24}{35}} = \frac{5}{12}$$

Question 40

Answer: ***D***

Explanatory notes

If $A \subseteq B$, then $\Pr(A \cup B) = \Pr(B)$.

Question 41

Answer: ***D***

Explanatory notes

The number of ways of arranging these nine letters without restriction is given by:

$9! = 362\,880$

The number of ways of arranging these nine letters so that they begin and end with a vowel is given by:

$= 4 \times 7! \times 3$

$= 60\,480$

$$\text{Pr(beginning and ending with a vowel)} = \frac{60\,480}{362\,880} = \frac{1}{6}$$

TIP

- » The 'box' method can be used here, as shown below. If we start with a vowel, there are four ways to fill the first box. Then there are three vowels left, which means the last box can be filled three ways. Finally, there are $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ ways to fill the remaining seven places.

4	7	6	5	4	3	2	1	3

Question 42

Answer: ***A***

Explanatory notes

$$14\,C_{11} = \frac{14!}{11!\,3!} = \frac{14 \cdot 13 \cdot 12}{3 \cdot 2 \cdot 1} = 364$$

Using a CAS:

nCr(14, 11) 364

Question 43

Answer: ***A***

Explanatory notes

$$\begin{aligned}
{}^{n}C_{r} &= 120 \\
{}^{n}C_{3} &= 120 \\
\frac{n!}{(n-3)!3!} &= 120 \\
\frac{n!}{(n-3)!} &= 720 \\
\frac{n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots}{(n-3) \times (n-4) \times \ldots} &= 720 \\
n \times (n-1) \times (n-2) &= 720 \\
n &= 10
\end{aligned}$$

TIP

- » Use the CAS to solve for n. Depending on your CAS device, you may be able to do this at the ${}^{n}C_{3} = 120$ stage. If not, try at a later stage. Some devices will only solve the equation $n(n-1)(n-2) = 720$.

Question 44

Answer: ***B***

Explanatory notes

In the equation $y = A \times 2^x + B$, the asymptote is given by B. Therefore $B = 2$.

Substituting $(0, -3)$ gives:

$$\begin{aligned} y &= A \times 2^x + 2 \\ A \times 2^0 + 2 &= -3 \\ A &= -5 \end{aligned}$$

Question 45

Answer: ***C***

Explanatory notes

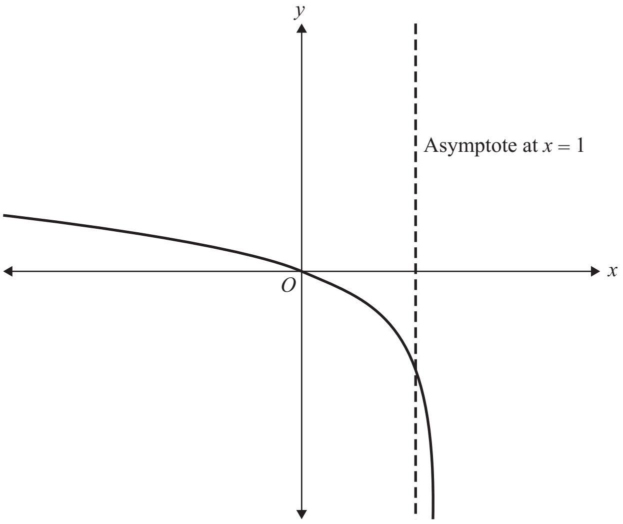
The equation $y = \log_a(1 - x)$ can be rewritten as $y = \log_a(-(x - 1))$, which suggests the following transformations of $y = \log_a(x)$:

- a reflection in the y-axis
- a translation of 1 unit to the right.

The only graph that exhibits these transformations is option C.

The CAS can also be used to sketch a graph of the function. Simply choose a value for a.

The sketch graph below, which uses $a = 1$, shows that the domain is $(-\infty, 1)$ and the range is R.



Question 46

Answer: ***B***

Explanatory notes

For the function $y = 5^x + b$, the equation of the asymptote is $y = b$.

Question 47

Answer: ***A***

Explanatory notes

Since the population increases 1.4% yearly, the population after 1975 can be found by the equation $P(t) = 300\,(1.014)^t$.

Therefore the population at the end of 2025 would be: $P(51) = 300\,(1.014)^{51} = 609.62$ million

Question 48

Answer: ***C***

Explanatory notes

This question can be done by the process of trial and error.

We need to test which of the following functions satisfies both the given points:

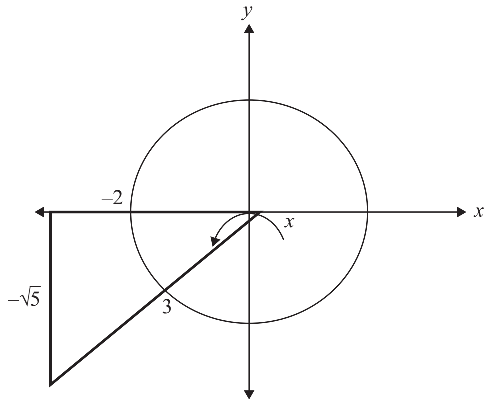
$f(1) = 5\log_5(1) = 0$

$f(5) = 5\log_5(5) = 5$

Question 49

Answer: ***B***

Explanatory notes



From the quadrant diagram above, the hypotenuse of the triangle is:

$\sqrt{(-2)^2 + (-\sqrt{5})^2} = \sqrt{9} = 3$

$\therefore\ \cos(x) = -\dfrac{2}{3}$

TIP

» **Read the question carefully and take note that the angle x is in the third quadrant.**

Question 50

Answer: ***B***

Explanatory notes

Amplitude = 1

Period = $4\pi = \frac{2\pi}{n}$. Therefore $n = \frac{2\pi}{4\pi} = \frac{1}{2}$.

Translation of 2 units up.

The shape suggests a cosine graph without reflections.

The transformations above suggests the equation $y = 2 + \cos\left(\frac{x}{2}\right)$.

Question 51

Answer: ***C***

Explanatory notes

If $y = a\sin(bx) + c$, then the range of the function is given by $[c - a, c + a]$.

Therefore the range of the function $y = -5 - 3\sin\left(2x - \frac{\pi}{2}\right)$ is:
$[-5 - 3, -5 + 3] = [-8, -2]$

Hence the minimum value of the function is -8.

TIP

» **Use the fMin function on a CAS. Since there is no restricted domain, you can put a domain of your choice.**

$\textbf{fMin}(-5 - 3 \cdot \sin(2 \cdot x - \frac{\pi}{2}), x, -\pi, \pi)$

$f(x) := -5 - 3 \cdot \sin\left(2 \cdot x - \frac{\pi}{2}\right)$	Done
$f\,\text{Min}(f(x), x) \mid -\pi \le x \le \pi$	$x = \frac{-\pi}{2}$ or $x = \frac{\pi}{2}$
$f\left(\frac{\pi}{2}\right)$	-8

Question 52

Answer: ***B***

Explanatory notes

For the function $f(x) = -2\tan\left(\frac{x}{2}\right) + 1$, where $-2\pi \le x \le 2\pi$, the period is $\frac{\pi}{\frac{1}{2}} = 2\pi$.

The asymptotes occur at every odd multiple of half the period = π.

Therefore in the domain $[-2\pi, 2\pi]$, the asymptotes will occur at $x = -\pi$ and $x = \pi$.

TIP

» Since $\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$, to find the equation of the asymptotes equate the denominator to zero.

$\text{solve}\left(\cos\left(\frac{x}{2}\right) = 0, x\right)\Big| -2 \cdot \pi \leq x \leq 2 \cdot \pi$

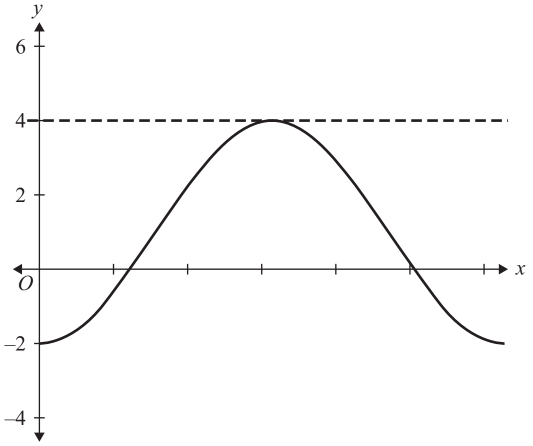
$x = -\pi \text{ or } x = \pi$

Question 53

Answer: ***A***

Explanatory notes

Use a CAS to sketch the graph $y = -3\cos\left(\frac{x}{2}\right) + 1$. $y = b$ represents a line. Therefore if you need one solution, the trig graph must touch the line only once in the given interval.



TIP

» You can also solve this problem using the trial and error method by checking every option in your CAS calculator to see which gives only one solution.

Question 54

Answer: ***A***

Explanatory notes

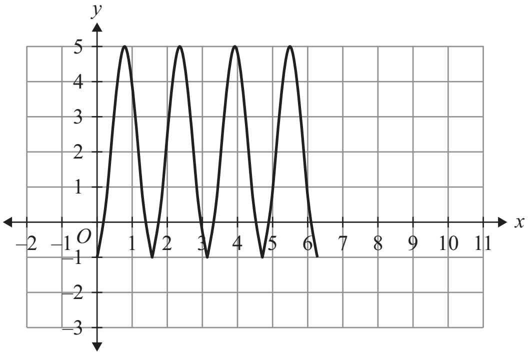
Let n be the coefficient of x in the given equation. Then the period $= \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{4}} = 8\pi$.

Question 55

Answer: ***D***

Explanatory notes

Use a CAS to sketch the graph of $y = 3\cos(4x - \pi) + 2$ with the given domain, then count the number of x-intercepts.



Alternatively, use the **solve** function on your CAS:

$\text{solve}(3\cos(4x-\pi)+2=0, x)|0 \le x \le 2\pi$ and count the number of solutions given.

$\{x = 0.210267167641983, x = 6.0729181395376, x = 2.93132548594781, x = 3.35185982123177, x = 1.36052915915292, x = 1.78106349443688, x = 4.50212181274271, x = 4.92265614802667\}$

The number of solutions $= 8$.

Question 56

Answer: ***D***

Explanatory notes

We are given $\cos(\beta) = \frac{5}{13}$:

$\sin(3\pi + \beta) = -\sin\beta$

$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$

$\sin(3\pi + \beta) = -\frac{12}{13}$

Question 57

Answer: ***C***

Explanatory notes

The water is initially poured into the uniformly shaped part of the cylinder, which means the depth of the water will increase at a constant rate.

The tapered shape of the top suggests a changing rise in depth, which eliminates options B and D.

Since the taper begins broad and then narrows, this suggests an increasing rate (i.e. filling faster), which suggests option C.

Question 58

Answer: ***A***

Explanatory notes

The average rate of change of the function $f(x) = 3x^2 - x + 2$ between $x = 1$ and $x = 2$ is given by $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{12 - 4}{1} = 8$.

TIP

- » **Use a CAS to define the function:**

$f(x) := 3 \cdot x^2 - x - 2$	**Done**
$\dfrac{f(2) - f(1)}{2 - 1}$	**8**

Question 59

Answer: ***D***

Explanatory notes

Let $f(x) = ax^3 + bx^2$.

The point $(2, 1)$ yields:

$f(x) = ax^3 + bx^2$

$f(2) = a(2)^3 + b(2)^2 = 1$

$\therefore 8a + 4b = 1$

A stationary point at $(2, 1)$ yields:

$f'(x) = 3ax^2 + 2bx$

$f'(2) = 3a(2)^2 + 2b(2) = 0$

$\therefore 12a + 4b = 0$

Using a CAS to solve these two equations simultaneously gives:

$a = -\dfrac{1}{4}, b = \dfrac{3}{4}$

TIP

- » **Use a CAS to define the function and its derivative:**

$f(x) := a \cdot x^3 + b \cdot x^2$	
$g(x) := \dfrac{d}{dx}(f(x))$	**Done**
$\text{solve}\left(\begin{cases} f(2) = 1 \\ g(2) = 0 \end{cases}, \{a, b\}\right)$	$a = \dfrac{-1}{4}$ and $b = \dfrac{3}{4}$

Question 60

Answer: ***C***

Explanatory notes

If $g'(x) = f(x)$, then $g(x) + c$ is the antiderivative of $f(x)$; that is, $\int f(x)\,dx = g(x) + c$.

By definition, the definite integral would be given by $\int_0^a f(x)\,dx = g(a) - g(0)$.

TIP

» **Be careful with option D, as $g(0)$ is not necessarily zero. Option B reveals a common confusion between definite and indefinite integrals. Definite integrals don't require a '$+ c$' by definition.**

Question 61

Answer: ***B***

Explanatory notes

The graph has a stationary point at $x = 1$ and $x = 3$. This means that the y value of the gradient graph would be zero at $x = 1$ and $x = 3$. This rules out option D.

The gradient of the graph is always negative for $x < 1$ and positive between $x = 1$ and $x = 3$. This rules out options A and C. Therefore the answer is option B.

Question 62

Answer: ***A***

Explanatory notes

$$\begin{aligned} M(x) &= \int (3x^2 - 16x + 17)\,dx \\ &= x^3 - 8x^2 + 17x + c \end{aligned}$$

When $x = 3$, $M = -4$. Therefore:

$$\begin{aligned} 3^3 - 8 \times 3^2 + 17 \times 3 + c &= -4 \\ c &= -10 \end{aligned}$$

$\therefore M(x) = x^3 - 8x^2 + 17x - 10$

TIP

» **Use a CAS to solve $M(3) = -4$ for c.**

Question 63

Answer: ***C***

Explanatory notes

To differentiate $y = (3x^2 - 5x)^4$, we use the chain rule.

Let $u = 3x^2 - 5x$, then $\frac{du}{dx} = 6x - 5$.
Let $y = u^4$, then $\frac{dy}{du} = 4u^3$.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 4u^3(6x - 5) = 4(3x^2 - 5x)^3(6x - 5)$$

Alternatively, use a CAS:

$$\frac{d}{dx}((3 \cdot x^2 - 5 \cdot x)^4)$$
$$4 \cdot x^3 \cdot (3 \cdot x - 5)^3 \cdot (6 \cdot x - 5)$$

A CAS can then verify which answers are correct.

Question 64

Answer: ***A***

Explanatory notes

The gradient of the tangent at $x = -3$ is found by finding $\frac{dy}{dx}$ at $x = -3$.

$$\frac{dy}{dx} = 10x$$

Therefore at $x = -3$, $\frac{dy}{dx} = -30$.

TIP

» **A CAS can also be used:**

$$\frac{d}{dx}(5 \cdot x^2 + 4)| x = -3 \qquad -30$$

Question 65

Answer: ***B***

Explanatory notes

The rate of change at $t = a$ is zero, so the correct answer must have a horizontal intercept at $t = a$. This means only options A, B and D can be correct.

In addition, the ball initially has a positive rate of change (gradient). This eliminates option C, which has a negative gradient.

The rate of change when $t > a$ is negative. This eliminates options A and D, which have only positive values when $t > a$. Therefore the answer is option B.

Question 66

Answer: ***B***

Explanatory notes

The equation of the tangent line is $y - y_1 = \left(\frac{dy}{dx}\right)_{x = x_1} (x - x_1)$.

$y = x^3 - 8x + 3$

$\frac{dy}{dx} = 3x^2 - 8$

At $x = 1$, $\frac{dy}{dx} = -5$

Therefore the equation of the tangent is $y + 4 = -5(x - 1)$, which gives $y = -5x + 1$.

TIP

» **Using a CAS:**

$\mathbf{tangentLine}(x^3 - 8 \cdot x + 3, x = 1) \qquad 1 - 5 \cdot x$

Question 67

Answer: ***B***

Explanatory notes

The graph given has two turning points, hence it has two stationary points. Therefore option B is not true.

Question 68

Answer: ***C***

Explanatory notes

The graph clearly shows the function is decreasing from interval $[-2, 2]$, hence the gradient function must be negative in the interval $(-2, 2)$.

But the graph is negative only in the interval $(0, 2)$. Therefore both $f(x)$ and $f'(x)$ are negative in the interval $(0, 2)$, which makes option C correct.

Question 69

Answer: ***A***

Explanatory notes

The shaded area $= \int_{-1}^{0} x(x+1)(x-2)dx - \int_{0}^{2} x(x+1)(x-2)dx = \frac{37}{12}$.

TIPS

- The area below the x-axis is negative when $x \in [0, 2]$, so we must put a negative sign in front of the integral because area is always positive.
- By the property of integrals, you can also switch the limits of the integrals and do the calculation as shown below.

  $$\int_{-1}^{0} x(x+1)(x-2)dx + \int_{2}^{0} x(x+1)(x-2)dx$$
- Use a CAS to complete the calculation in one step without entering two separate integrals. You can integrate from –1 to 2 by putting the entire function inside a modulus sign.

  $$\int_{-1}^{2} |x \cdot (x-2) \cdot (x+1)|dx \text{ approx Fraction(5.E} \rightarrow \frac{37}{12}$$

Question 70

Answer: ***C***

Explanatory notes

The approximate area under the function = area of the shaded rectangles

$= 1 \times f(-2) + 1 \times f(-1) + 1 \times f(1) + 1 \times f(2)$

$= 5 + 8 + 8 + 5 = 26$

TIP

- Since the function is symmetrical, the calculation can be done in one step using a CAS, as shown below.

$f(x) := 9 - x^2$	**Done**
$2 \cdot (f(-1) + f(-2))$	**26**

Section 3 Extended response questions

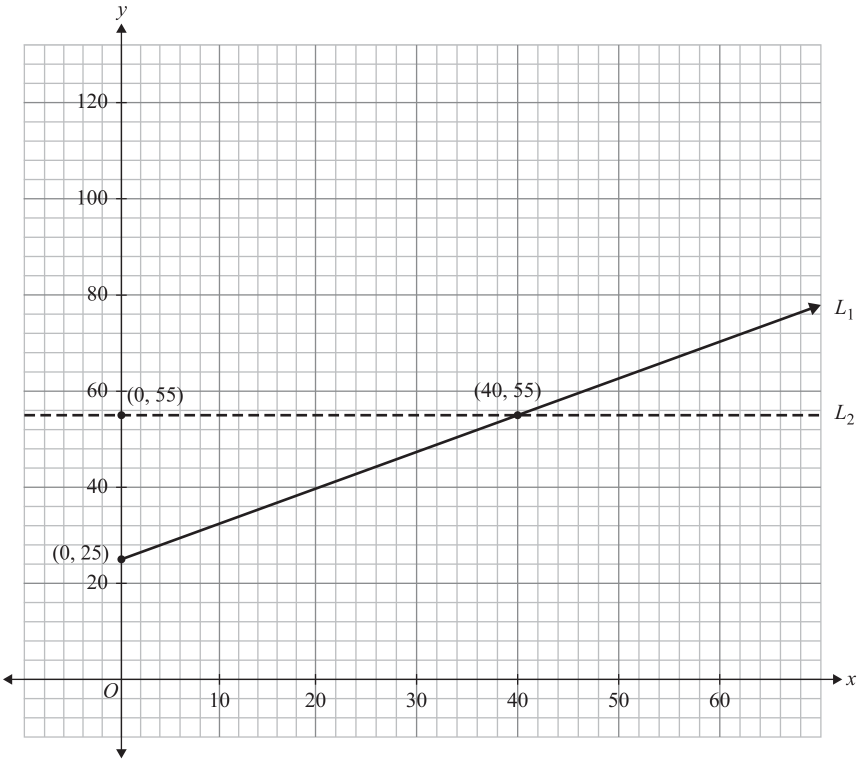
Question 1a.

Worked solution

$$\frac{3}{4}x + 25 = 55$$

$$\frac{3}{4}x = 30$$

$$x = 40$$



Mark allocation: 2 marks

- 1 mark for L_1 with the correctly labelled y-intercept
- 1 mark for correct point of intersection (must be labelled)

Question 1b.

Worked solution

$$L_1 - L_2 = 70$$

$$\frac{3}{4}x + 25 - 55 = 70$$

$$\frac{3}{4}x = 100$$

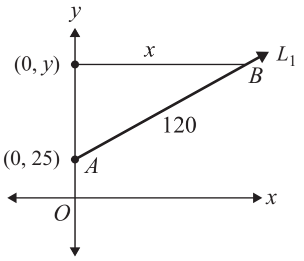
$$x = \frac{400}{3}$$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 1c.

Worked solution



Line L_1 has travelled 120 units, which is shown as AB in the figure above. We need to find the horizontal distance, x.

We can use Pythagoras' theorem to find the horizontal distance:

$$120 = \sqrt{x^2 + (y - 25)^2}$$

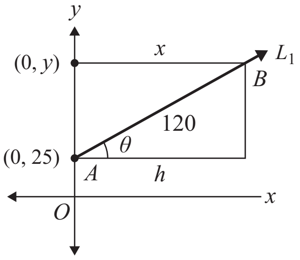
$$120 = \sqrt{x^2 + \left(\frac{3}{4}x + 25 - 25\right)^2}$$

$$120 = \sqrt{x^2 + \frac{9}{16}x^2}$$

$$\frac{5}{4}x = 120$$

$$x = 96 \text{ units}$$

Alternatively:



$$m = \tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\cos\theta = \frac{h}{120}$$

$$h = 120\cos\theta = 120\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right) = 96 \text{ units}$$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 2a.i.

Worked solution

It is the y-intercept or $g(0) = \frac{1}{4}$.

Mark allocation: 1 mark

- 1 answer mark

Question 2a.ii.

Worked solution

Since $\left(1, \frac{3}{2}\right)$ and $\left(\frac{5}{2}, \frac{9}{16}\right)$ are points on the parabola, the equations are:

$$\frac{3}{2} = a(1)^2 + b(1) + \frac{1}{4}$$

$$a + b = \frac{5}{4} \qquad \text{[equation (1)]}$$

$$\frac{9}{16} = a\left(\frac{5}{2}\right)^2 + b\left(\frac{5}{2}\right) + \frac{1}{4}$$

$$\frac{25}{4}a + \frac{5}{2}b = \frac{5}{16} \qquad \text{[equation (2)]}$$

Mark allocation: 2 marks

- 1 answer mark for the first simultaneous equation
- 1 answer mark for the second simultaneous equation

Question 2a.iii.

Worked solution

With the help of a CAS, $a = -\frac{3}{4}$ and $b = 2$.

$$\left.\begin{cases} a + b = \frac{5}{4} \\ \frac{25}{4}a + \frac{5}{2}b = \frac{5}{16} \end{cases}\right|_{a,\, b} \qquad \left\{a = -\frac{3}{4}, b = 2\right\}$$

Mark allocation: 1 mark

- 1 answer mark

TIP

» Since the question is worth only 1 mark, you are expected to use your CAS to do the calculation efficiently.

Question 2b.

Worked solution

Maximum = $\frac{19}{12}$ metres at $x = \frac{4}{3}$.

This can be found be using the **fMax** function on your CAS:

$$\text{fMax}\left(-\frac{3}{4}\cdot x^2 + 2\cdot x + \frac{1}{4}, x, -\infty, \infty\right)$$

$$\left\{\text{MaxValue} = \frac{19}{12}, x = \frac{4}{3}\right\}$$

Mark allocation: 1 mark

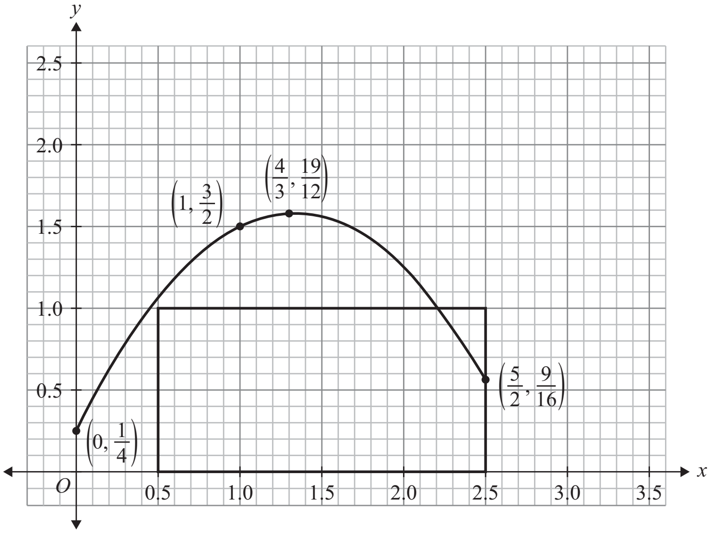
- 1 answer mark for both the maximum value and the value of x at which it occurs

TIP

» **Be sure to read the question carefully and state both the values required.**

Question 2c.

Worked solution



The height of the box is 1 metre, therefore when $g(x) = 1$:

$$-\frac{3}{4}x^2 + 2x + \frac{1}{4} = 1$$

$$-\frac{3}{4}x^2 + 2x + \frac{-3}{4} = 0$$

So $x = -\frac{\sqrt{7}}{3} + \frac{4}{3}$ and $x = \frac{\sqrt{7}}{3} + \frac{4}{3}$.

Therefore the length span of the x-coordinates $= \frac{\sqrt{7}}{3} + \frac{4}{3} + \frac{\sqrt{7}}{3} - \frac{4}{3} = \frac{2\sqrt{7}}{3} \cong 1.76$ metres.

Since the length of the box is 2 metres, it cannot pass through the arch because $\frac{2\sqrt{7}}{3} < 2$.

Alternatively:

We can find the y value when x is $\frac{4}{3} - 1 = \frac{1}{3}\, mg\left(\frac{1}{3}\right) = \frac{5}{6} < 1$.

Mark allocation: 2 marks

- 1 method mark for finding the x-intercepts that correspond to a height of 1 metre or finding the y value when x is $\frac{1}{3}$
- 1 answer mark for the justification

Question 2d.

Worked solution

The y-intercept is $\frac{1}{4}$. Therefore the equation of the line is $y = mx + \frac{1}{4}$.

Since $\left(\frac{5}{2}, \frac{9}{16}\right)$ is a point on the line, we have:

$$\frac{9}{16} = \frac{5}{2}m + \frac{1}{4}$$

$$\frac{5}{2}m = \frac{5}{16}$$

$$m = \frac{1}{8}$$

Hence the equation of the line is $y = \frac{1}{8}x + \frac{1}{4}$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 2e.

Worked solution

The line must intersect the parabola at $2 \leq x \leq \frac{5}{2}$. The gradient of $x = \frac{5}{2}$ has already been calculated as $m = \frac{1}{8}$, so we need to determine the gradient when $x = 2$.

$y = -\frac{3}{4}(2)^2 + 2(2) + \frac{1}{4}$, so $y = \frac{5}{4}$.

Hence the gradient of the line connecting the points $\left(0, \frac{1}{4}\right)$ and $\left(2, \frac{5}{4}\right)$ is $\dfrac{\frac{5}{4} - \frac{1}{4}}{2} = \frac{1}{2}$.

Therefore the line can increase in steepness to a gradient of $\frac{1}{2}$, giving $\frac{1}{8} \leq m \leq \frac{1}{2}$.

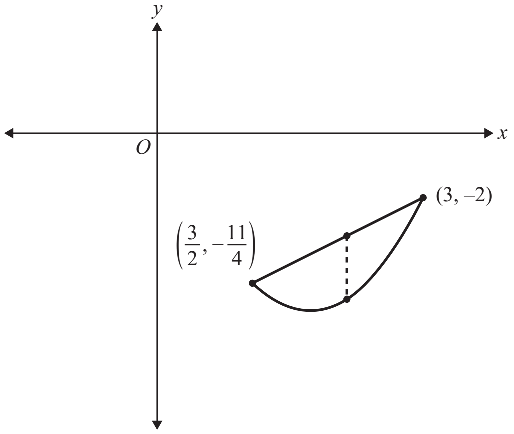
Mark allocation: 2 marks

- 1 method mark for finding the gradient when $x = 2$
- 1 answer mark for the inequality statement

Question 3a.

Worked solution

Using a CAS to solve the equations $x - 2y = 7$ and $y = x^2 - 4x + 1$ simultaneously gives $\left(\frac{3}{2}, -\frac{11}{4}\right)$ and $(3, -2)$. These are labelled in the graph shown below.



Alternatively, this can be done 'by hand' (although this question is only worth 2 marks).

Rearrange the equation $x - 2y = 7$ to make y the subject:

$$y = \frac{x}{2} - \frac{7}{2}$$

The two equations meet when:

$$\frac{x}{2} - \frac{7}{2} = x^2 - 4x + 1$$
$$x - 7 = 2x^2 - 8x + 2$$
$$2x^2 - 9x + 9 = 0$$
$$x = 3, \frac{3}{2}$$

Substituting this into one of the equations gives:

$$y = \frac{x}{2} - \frac{7}{2}$$
$$= -2, -\frac{11}{4}$$

Therefore the coordinates are $\left(\frac{3}{2}, -\frac{11}{4}\right)$ and $(3, -2)$.

Mark allocation: 2 marks

- 1 mark for both correct x values
- 1 mark for correctly stating both coordinates.

Note: Answers must be exact and correctly labelled for full marks.

TIP

» Be sure to write your answer as coordinate pairs.

Question 3b.i.

Worked solution

Since the line is vertical, the distance is simply the difference between the y values of the respective equations.

Let $y_1 = x^2 - 4x + 1$ and $y_2 = \frac{x}{2} - \frac{7}{2}$.

Therefore $d = y_2 - y_1$:

$$= \left(\frac{x}{2} - \frac{7}{2}\right) - (x^2 - 4x + 1)$$

$$= -x^2 + \frac{9}{2}x - \frac{9}{2}$$

Mark allocation: 1 mark

- 1 answer mark for the correct expression (which must be in terms of x)
- Other expressions such as $d = -\left(x^2 - \frac{9}{2}x + \frac{9}{2}\right)$ or $d = \frac{-2x^2 + 9x - 9}{2}$ are acceptable.

Question 3b.ii.

Worked solution

Using a CAS, $d_{\max} = 0.5625$.

Alternatively, note that the graph of d is a parabola and will have a maximum turning point at $(2.25, 0.5625)$, so $d_{\max} = 0.5625$.

Mark allocation: 1 mark

- 1 answer mark

Question 3c.

Worked solution

The equation $x - 2y = 7$ can be written as $y = \frac{x}{2} - \frac{7}{2}$.

This equation is a tangent to the parabola when there is one solution to the equation. Hence:

$$\frac{x}{2} - \frac{7}{2} = x^2 - ax + 1$$

$$x - 7 = 2x^2 - 2ax + 2$$

$$2x^2 - 2ax - x + 9 = 0$$

$$2x^2 - (2a + 1)x + 9 = 0$$

One solution to this quadratic equation occurs when $\Delta = 0$. Therefore:

$$b^2 - 4ac = 0$$

$$(-2a + 1)^2 - 4 \times 2 \times 9 = 0$$

$$(2a + 1)^2 - 72 = 0$$

$$2a + 1 = \sqrt{72}$$

$$a = \frac{\pm\sqrt{72} - 1}{2}$$

$$a = \frac{\pm 6\sqrt{2} - 1}{2}$$

$$a = \frac{6\sqrt{2} - 1}{2}, \frac{-6\sqrt{2} - 1}{2}$$

Mark allocation: 3 marks

- 1 method mark for equating $y = \frac{x}{2} - \frac{7}{2}$ with $y = x^2 - ax + 1$
- 1 method mark for using the discriminant and setting it to zero, $\Delta = 0$
- 1 answer mark for the correct answer.

Note: Both values must be given, and they must be in exact form for full marks.

Question 4a.

Worked solution

To find the total time the ball spends in the air, we need to find the time it takes to reach the ground; that is, the x-intercept.

Using a CAS:

$\text{solve}(-5 \cdot t^2 + 28 \cdot t + 3.6 = 0, t)$

$$t = \frac{-(\sqrt{214} - 14)}{5} \text{ or } t = \frac{\sqrt{214} + 14}{5}$$

Therefore the total time spent in the air $= \frac{\sqrt{214} + 14}{5}$ seconds.

Mark allocation: 1 mark

- 1 answer mark

TIP

» **Remember: always leave your answers in exact form, unless instructed otherwise.**

Question 4b.

Worked solution

The time taken for the ball to reach its maximum height is the t coordinate of the turning point, which can be found by $t = \frac{-b}{2a} = \frac{-28}{-10} = \frac{14}{5}$ seconds or 2.8 seconds.

Mark allocation: 1 mark

- 1 answer mark

TIP

» **You can use the fMax function on a CAS, or sketch the graph on a CAS to find the turning point.**

Question 4c.

Worked solution

The maximum height reached by the ball can be found by substituting $t = \frac{14}{5}$ into the equation, so:

$h\left(\frac{14}{5}\right) = \frac{214}{5}$ metres

Mark allocation: 1 mark

- 1 answer mark

Question 4d.

Worked solution

Solve for $h(t) = 8$, using a CAS:

$t = \frac{14 - \sqrt{174}}{5}$ and $t = \frac{14 + \sqrt{174}}{5}$

The time spent at 8 metres or above = $\frac{14 + \sqrt{174}}{5} - \left(\frac{14 - \sqrt{174}}{5}\right) = \frac{2\sqrt{174}}{5}$ seconds or 5.28 seconds.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 4e.

Worked solution

The maximum height occurs at $t = -\frac{b}{2a} = \frac{v_0}{9.8}$.

We are given $h\left(\frac{v_0}{9.8}\right) = 20$.

Solve using a CAS:

$v_0 = -\frac{14\sqrt{41}}{5}$ or $\frac{14\sqrt{41}}{5}$

Since $v_0 > 0$, $v_0 = \frac{14\sqrt{41}}{5}\text{ ms}^{-1} = 17.9\text{ ms}^{-1}$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 5a.

Worked solution

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{0 + 20}{2}, \frac{150 + 0}{2}\right)\\ &= (10, 75)\end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 5b.

Worked solution

$$\text{Gradient} = \frac{150-0}{0-40}$$

$$= -\frac{150}{40} = -\frac{15}{4}$$

$$\text{Perpendicular gradient} = \frac{4}{15}$$

$$y - y_1 = m(x - x_2)$$

$$y - 0 = \frac{4}{15}(x - 40)$$

$$y = \frac{4x}{15} - \frac{32}{3}$$

Mark allocation: 2 marks

- 1 method mark for finding the perpendicular gradient
- 1 answer mark

Question 5c.

Worked solution

Using either a system of equations or cubic regression shows that the solution is given by $f(x) = -\frac{1}{320}x^3 + \frac{3}{8}x^2 - \frac{55}{4}x + 150$.

Therefore $a = -\frac{1}{320}$, $b = \frac{3}{8}$, $c = -\frac{55}{4}$, $d = 150$ or $a = -0.003125$, $b = 0.375$, $c = -13.75$ and $d = 150$.

Mark allocation: 1 mark

- 1 answer mark for the correct answer in exact form

TIP

» Use a CAS to first define the function, if using a system of equations.

Question 5d.

Worked solution

See the table below, which shows that:

- The second left end point becomes 94.5 because a positive value of $g(94.5) = 2.2699$ suggests from the graph that the solution is between 94.5 and 95. This gives a new midpoint of 94.75.
- The third right end point becomes 94.75 because a negative value of $g(94.75) = -1.7666$ suggests from the graph that the solution is between 94.5 and 94.75. This gives a new midpoint of 94.625.

… and so on until the table is complete. It can be seen that the midpoint has remained stable at the first decimal place of 94.6 for a few iterations, so the approximate solution is 94.6.

Left end point	Right end point	Midpoint (x_m)	$t(x_m)$
94	95	94.5	2.2699
94.5	95	94.75	–1.7666
94.5	94.75	94.625	0.2587
94.625	94.75	94.6875	–0.7522
94.625	94.6875	94.65625	–0.2463

Mark allocation: 2 marks

- 1 answer mark for correct values in the second and third rows
- 1 answer mark for correct values in the last two rows

Question 5e.

Worked solution

$T(x) = \frac{1}{2}h(x + 10)$

Mark allocation: 2 marks

- 1 answer mark for having either the dilation or translation correct
- 1 answer mark for having both correct

Question 5f.

Worked solution

$T: [0, 60] \rightarrow R,\ T(x) = \frac{-1}{640}x^3 + \frac{3}{16}x^2 - \frac{55}{8}x + 75$

Mark allocation: 2 marks

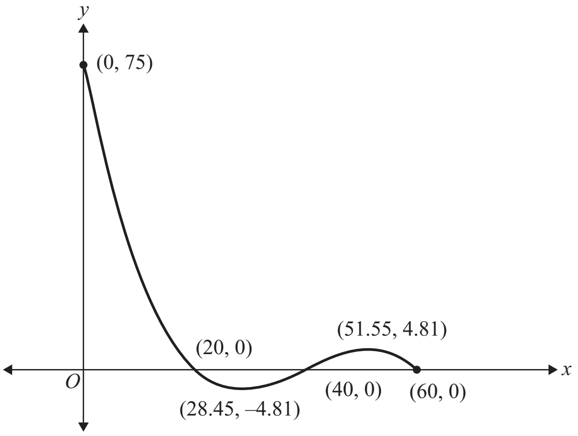
- 1 answer mark for the correct domain $[0, 60]$
- 1 answer mark for the correct rule

TIP

» Simply entering $T(x) = \frac{1}{2}h(x + 10)$ into a CAS will yield the correct result, provided $h(x)$ has been defined. For some calculators, the expression will need to be expanded to be in the appropriate form.

Question 5g.

Worked solution



Mark allocation: 2 marks

- 1 answer mark for correctly labelled coordinates of end points and intercepts
- 1 answer mark for correctly labelled coordinates of turning points

TIPS

- » Use the maximum/minimum function on a CAS to find the turning points.
- » Make sure you have rounded your answers to two decimal places.

Question 6a.

Worked solution

The graph shows

$h(0) = 15$

$$\begin{aligned} h(0) &= k(0-5)(0-15)(0-30) \\ 15 &= -2250k \\ k &= -\frac{1}{150} \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for showing correct substitution

Question 6b.

Worked solution

Use a CAS to define the function, then find the minimum value of the function with the restricted domain $5 \le x \le 15$.

$f\text{Min}(f(x), x, 5, 15)$

$\{\text{MinValue} = -3.383893823, x = 9.401835094\}$

$h(x) := \frac{1}{-150} \cdot (x-5) \cdot (x-15) \cdot (x-30)$	Done
$f\text{Min}(h(x), x) \vert 5 \le x \le 15$	$x = 9.4018350941$
$h(9.4018350941)$	-3.38389382267

It is 3.38 metres under ground level and the x value is 9.40 metres.

Mark allocation: 2 marks

- 1 answer mark for the correct h value
- 1 answer mark for the correct x value

Question 6c.

Worked solution

Solving $h(x) = 10$ gives $x = 1.22$.

Mark allocation: 1 mark

- 1 answer mark

Question 6d.

Worked solution

We know the point of inflection is $(2, 32)$.

$y = a(x - 2)^3 + 32$

Substituting $(0, 48)$ into the equation and solving for a gives:

$$\begin{aligned} 48 &= -8a + 32 \\ a &= -2 \end{aligned}$$

Hence the equation is $y = -2(x - 2)^3 + 32$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 6e.

Worked solution

Use your CAS to solve $-2(x - 2)^3 + 32 = 0$, giving $x = 4.52$.

So C is $(4.52, 0)$.

Mark allocation: 1 mark

- 1 answer mark

Question 7a.

Worked solution

The highest power of x in the polynomial is 4, so the degree is 4.

Mark allocation: 1 mark

- 1 answer mark

Question 7b.

Worked solution

The term $(x - a)^2$ suggests a minimum or maximum turning point touching the x-axis. Since the graph touches at a minimum turning point where $x = 3.5$, then $a = 3.5$.

The x-intercept at $x = 5$ suggests that c must be positive. Hence $c = 5$.

The term $(b - x)$ can be rewritten as $-(x - b)$. This suggests that b, the x-intercept at $x = -1.5$, must be negative. Hence $b = -1.5$.

Mark allocation: 1 mark

- 1 answer mark for correct answer

TIPS

- The values of a, b and c can also be found using a CAS to solve the simultaneous equations set up by:

$$f(5) = 0$$
$$f(-1.5) = 0$$
$$f(3.5) = 0$$

- However, there may be several solutions, some of which do not match the requirements of the question.

Question 7c.

Worked solution

The first solution to the equation $f(x) = 0.3$ after the minimum turning point is $x = 4.5$. Therefore the domain is $[0, 4.5]$.

$f(0) = 4.6$, therefore the range is $[0, 4.6]$.

Mark allocation: 2 marks

- 1 answer mark for the correct domain
- 1 answer mark for the correct range

Question 8a.i.

Worked solution

$k > 108$

If the graph is translated 108 units up, the turning point $(6, -108)$ will become $(6, 0)$, causing the graph to have only one x-intercept. Hence if you translate more than 108 units, the graph will not have any x-intercepts.

Mark allocation: 1 mark

- 1 answer mark

Question 8a.ii.

Worked solution

$k < 0$ or $k = 108$

Mark allocation: 1 mark

- 1 answer mark

Question 8b.i.

Worked solution

$b \geq 9$

Mark allocation: 1 mark

- 1 answer mark

Question 8b.ii.

Worked solution

$0 \le b < 9$

Mark allocation: 1 mark

- 1 answer mark

TIP

» **Create sliders on a CAS to answer such questions on transformation.**

Question 8c.

Worked solution

Since $T(x, y) = \left(\frac{1}{2}x - 3, -\frac{1}{2}y\right)$, $(-2, -44)$ will become $(-4, 22)$, $(0, 0)$ will become $(-3, 0)$, $(6, -108)$ will become (0.54) and $(9, 0)$ will become $\left(\frac{3}{2}, 0\right)$.

Mark allocation: 1 mark

- 1 answer mark

Question 8d.

Worked solution

Dilation of factor $\frac{1}{2}$ from the x-axis and y-axis, followed by a reflection in the x-axis and translated 3 units in the negative direction of the x-axis.

Mark allocation: 1 mark

- 1 answer mark

Question 8e.

Worked solution

$T(x, y) = \left(\frac{1}{2}x - 3, -\frac{1}{2}y\right)$, so $(x', y') = \left(\frac{1}{2}x - 3, -\frac{1}{2}y\right)$.

So $x = 2(x' + 3)$ and $y = -2y'$.

Hence $y = x^2(x - 9)$ becomes $-2y' = 4\,(x' + 3)^2\ (2(x' + 3) - 9)$.

So the rule is $g(x) = -2(x + 3)^2\ (2x - 3)$.

Domain of $g(x)$:

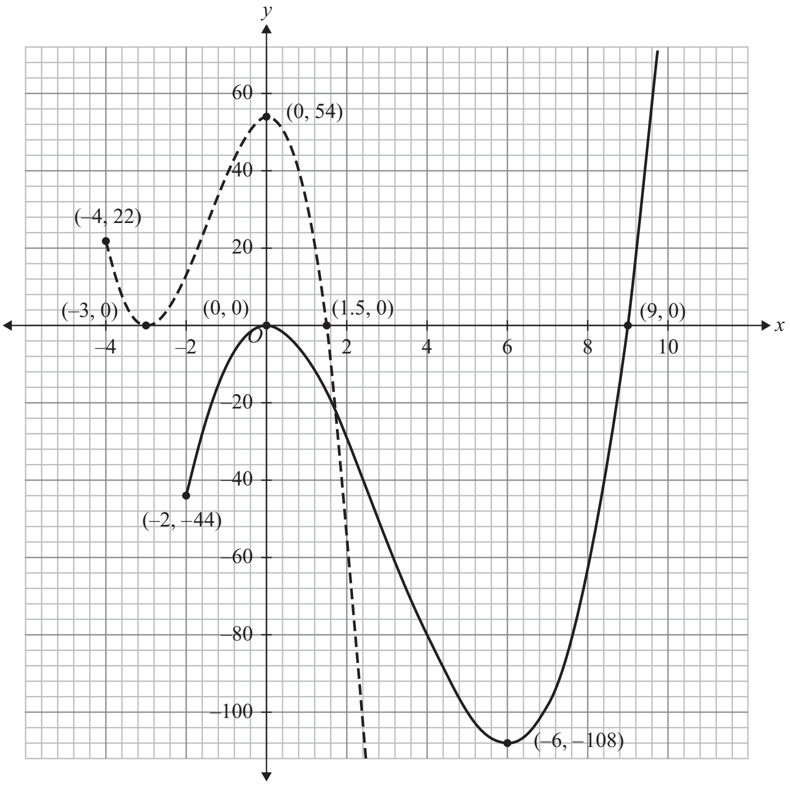
The domain of $f(x)$ is $[-2, \infty)$, therefore when we apply the rule for the transformation of the x-coordinate, we get the domain of $g(x) = [-4, \infty)$.

Mark allocation: 3 marks

- 1 method mark
- 1 answer mark for $g(x)$
- 1 answer mark for the domain

Question 8f.

Worked solution



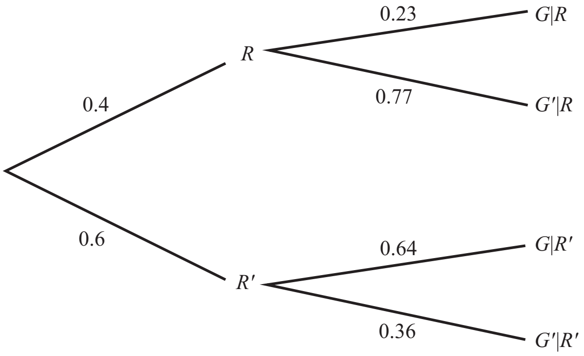
Mark allocation: 2 marks

- 1 mark for the correct curve shape
- 1 mark for all points labelled correctly

Question 9a.

Worked solution

Let R be the event 'it is raining' and G be the event 'Gertie wins the race'.



Mark allocation: 2 marks

- 1 method mark for constructing a tree diagram with two branches and two outcomes per branch
- 1 mark for the correct probabilities

Question 9b.

Worked solution

$$\begin{aligned}\mathrm{Pr(G)} &= \mathrm{Pr(R)} \times \mathrm{Pr(G|R)} + \mathrm{Pr(R')} \times \mathrm{Pr(G|R')}\\ &= 0.23 \times 0.4 + 0.64 \times 0.6\\ &= 0.476\end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark for showing that the sum of **both** outcomes of Gertie winning leads to a correct solution

Question 9c.

Worked solution

$$\begin{aligned}\Pr(R|G) &= \frac{\Pr(R \cap G)}{\Pr(G)}\\ &= \frac{0.4 \times 0.23}{0.4 \times 0.23 + 0.6 \times 0.64}\\ &= 0.1933\end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for the correct use of the conditional probability formula, or for using a reduced sample space
- 1 answer mark

TIP

» The key here is to realise that this question relies on conditional probability. The phrase ‘Galloping Gertie wins the Cup’ means that this has already occurred, so ‘the probability it is raining’, in this case, is really ‘the probability it is raining, **given** that Gertie has won the Cup’.

Question 10a.i.

Worked solution

	S	S'	
T	0.70	0.05	0.75
T'	0.10	0.15	0.25
	0.80	0.20	1

Mark allocation: 1 mark

- 1 answer mark

Question 10a.ii.

Worked solution

0.70 (reading from the table in the previous part)

Mark allocation: 1 mark

- 1 answer mark

Question 10a.iii.

Worked solution

$$\Pr(T|S') = \frac{\Pr(T \cap S')}{\Pr(S')} = \frac{0.05}{0.20} = \frac{1}{4}$$

Mark allocation: 1 mark

- 1 answer mark

Question 10a.iv.

Worked solution

$0.15 \times 375 = 56.25$, so 56 students.

Mark allocation: 1 mark

- 1 answer mark

Question 10a.v.

Worked solution

$$\Pr(T|S) = \frac{\Pr(T \cap S)}{\Pr(S)} = \frac{0.70}{0 \cdot 80} = \frac{7}{8}$$

$$\Pr(T) = \frac{3}{4}$$

$$\Pr(T|S) \neq \Pr(T)$$

Hence the events T and S are not independent.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

TIP

» You can also prove this by showing $\Pr(T \cap S) \neq \Pr(T) \times \Pr(S)$.

Question 10b.i.

Worked solution

$2^6 - 1 = 63$ pies

Each student can choose any combination of the six pie types, from one pie to all six pies. We need to find the total number of possible combinations, which is $2^6 = 64$ possible combinations.

However, the problem states that each purchase must include at least one pie. This means we must exclude the case where a student buys *no* pies. There is only one way to buy no pies.

Therefore the total number of different orders that can be placed is $64 - 1 = 63$.

Mark allocation: 1 mark

- 1 answer mark

Question 10b.ii.

Worked solution

Total number of purchases with at least three pies:

3 pies: $6C_3 = 20$

4 pies: $6C_4 = 15$

5 pies: $6C_5 = 6$

6 pies: $6C_6 = 1$

Total number of purchases with at least three pies: $20 + 15 + 6 + 1 = 42$

Hence the probability that a purchase contains at least three pies given that it contains at least one pie $= \frac{42}{63} = \frac{2}{3}$.

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 10c.i.

Worked solution

${}^{7}P_{7} = 7! = 5040$ ways

Mark allocation: 1 mark

- 1 answer mark

Question 10c.ii.

Worked solution

${}^{6}P_{6} = 6! = 720$ ways

The letter M is fixed in the first position, so the remaining six letters can be arranged in any order with no repetition; hence there are ${}^{6}P_{6}$ ways.

Mark allocation: 1 mark

- 1 answer mark

Question 10c.iii.

Worked solution

1440

Since the vowels must be together, we can think of it as one block. Now we have 6 blocks to arrange: C, M, P, N, G and the vowel block. There are $6!$ ways to arrange these blocks. The vowels A and I can be arranged within their block in $2! = 2$ ways.

To find the total number of arrangements where the vowels are together, multiply the number of ways to arrange the non-vowel blocks by the number of ways to arrange the vowels within their block:

$720 \times 2 = 1440$

Mark allocation: 2 marks

- 1 method mark
- 1 answer mark

Question 10d.

Worked solution

The number of seven-letter words that begin with M and end with letter N is $5! = 120$.

Hence the probability is $\frac{120}{5040} = \frac{1}{42}$.

Mark allocation: 2 marks

- 1 method mark for finding the number of arrangements of the seven-letter words that start with M and end with N
- 1 answer mark

Question 11a.

Worked solution

$$\begin{aligned} R &= \log_{10}\left(\frac{A}{T}\right) \\ 10^R &= \frac{A}{T} \\ A &= T \times 10^R \\ &= \frac{1}{50} \times 10^{8.9} \\ &= \frac{10^{8.9}}{50} \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for rearranging A in terms of T
- 1 answer mark for substituting $T = \frac{1}{50}$ and deriving the correct answer

Question 11b.

Worked solution

$$\begin{aligned} A_{\text{caesium}} &= A_0 \times 2^{-kt} \\ \frac{A_{\text{caesium}}}{A_0} &= 2^{-kt} \\ \frac{1}{2} &= 2^{-k \times 10\,950} \\ 2^{-1} &= 2^{-k \times 10\,950} \\ 10\,950k &= 1 \\ k &= \frac{1}{10950} \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for recognising that $\frac{A_{\text{caesium}}}{A_0} = \frac{1}{2}$ when $t = 10950$
- 1 method mark for solving for k and deriving the correct answer

Question 11c.

Worked solution

$$\begin{aligned} A_{\text{iodine}} &= A_{\text{caesium}} \\ A_0 \times 2^{-\frac{t}{10\,950}} &= 10A_0 \times 2^{-0.125t} \\ 2^{-\frac{t}{10\,950}} &= 10 \times 2^{-0.125t} \\ t &= 26.59 \\ t &\approx 27 \text{ days} \end{aligned}$$

Mark allocation: 2 marks

- 1 method mark for equating A_{iodine} and A_{caesium}
- 1 answer mark

Question 12a.

Worked solution

Let $y = 2^{\frac{x-1}{2}} + 1$.

Swapping x and y, we get $x = 2^{\frac{y-1}{2}} + 1$.

$$\begin{aligned} x - 1 &= 2^{\frac{y-1}{2}} \\ \log_2(x-1) &= \frac{y-1}{2} \\ y - 1 &= 2\log_2(x-1) \\ y &= 2\log_2(x-1) + 1 \end{aligned}$$

Therefore $f^{-1}(x) = 2\log_2(x-1) + 1$.

The domain of $f^{-1}(x)$ is the range of $f(x) = [1, \infty)$.

Mark allocation: 2 marks

- 1 answer mark for the inverse function
- 1 answer mark for the domain

TIP

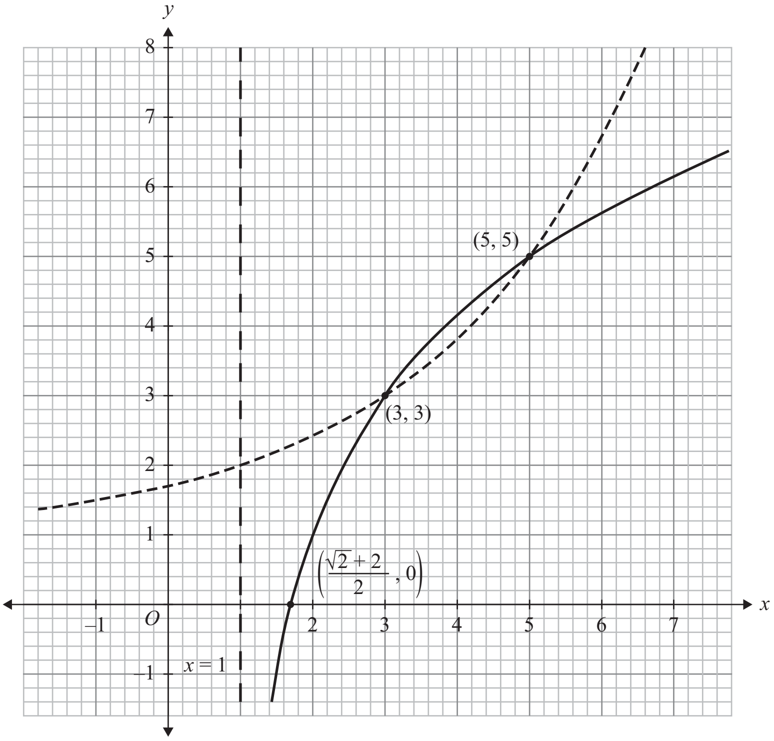
» **Using a CAS, you can find the inverse function by swapping x and y in the equation and then solving for y.**

$$\textbf{solve}\left(x = 2^{\frac{y-1}{2}} + 1, y\right)$$

$$y = \frac{2 \cdot \ln(x-1) + \ln(2)}{\ln(2)} \text{ and } x > 1$$

Question 12b.

Worked solution



Mark allocation: 2 marks

- 1 mark for the inverse graph with the correctly labelled intercept and asymptote
- 1 mark for labelling the points of intersection correctly

Question 12c.

Worked solution

The line AB is the perpendicular bisector of the line joining $(3, 3)$ and $(5, 5)$.

Since the gradient of the line joining $(3, 3)$ and $(5, 5)$ is 1, the gradient of the perpendicular line is -1. The midpoint of $(3, 3)$ and $(5, 5)$ is $(4, 4)$.

Hence the equation of the line, using the gradient point form, is $y - 4 = -1(x - 4)$, which gives $y = -x + 8$.

Mark allocation: 2 marks

- 1 mark for identifying the line as a perpendicular bisector
- 1 mark for deriving the equation of the line

Question 12d.

Worked solution

$2^{\frac{x-1}{2}} + 1 = 8 - x$

Using a CAS, we get $(4.09, 3.91)$.

Mark allocation: 1 mark

- 1 answer mark

Question 12e.

Worked solution

Using the distance formula, $W = 2 \times \sqrt{(4 - 4.086000882)^2 + (4 - 3.913999118)^2} = 0.24$.

Mark allocation: 1 mark

- 1 answer mark

Question 13a.

Worked solution

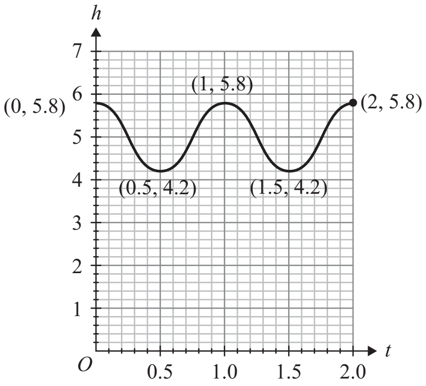
Period $= \frac{2\pi}{2\pi} = 1$ second

Mark allocation: 1 mark

- 1 answer mark

Question 13b.

Worked solution



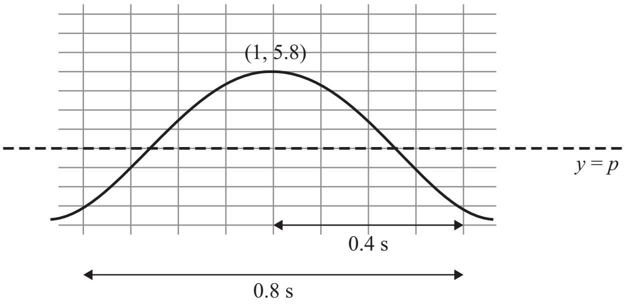
Mark allocation: 3 marks

- 1 answer mark for showing two cycles of a circular function
- 1 answer mark for correct and labelled end points
- 1 answer mark for correct and labelled turning points

Question 13c.

Worked solution

Symmetrically, <u>at least</u> p metres above the ground for a continuous period of 0.8 seconds means 0.4 seconds either side of a maximum, as can be seen in the following section of the graph.



Since the maximum occurs at $t = 1$ second, we need to find h when $t = 1 \pm 0.4 = 1.4$ or 0.6 seconds.

$$h(1.4) = 0.8\cos(2\pi \times 1.4) + 5$$
$$= 4.35 \text{ metres}$$

Mark allocation: 2 marks

- 1 mark for recognising that $t = 1.4$ or 0.6 seconds
- 1 answer mark

Question 14a.

Worked solution

The range of the function h is $[0, 10]$. Therefore $[r - p, r + p] = [0, 10]$.

Hence $r = 5$ and $p = -5$.

Check the graph on a CAS to see the difference in the shape of the graph when $p = 5$ and $p = -5$.

The period of the graph is 12 (the distance between the two peaks).

$$\frac{2\pi}{q} = 12$$

$$q = \frac{\pi}{6}$$

Mark allocation: 2 marks

- 1 mark for the correct values of r and p
- 1 mark for the correct value of q

Question 14b.

Worked solution

$$h(x) = -5\cos\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 5$$

$$h(2) = -5\cos\left(\frac{2\pi}{6} + \frac{\pi}{2}\right) + 5 = 5 + \frac{5\sqrt{3}}{2}$$

$$h(22) = -5\cos\left(\frac{22\pi}{6} + \frac{\pi}{2}\right) + 5 = 5 - \frac{5\sqrt{3}}{2}$$

Hence the end points are $\left(2, 5 + \frac{5\sqrt{3}}{2}\right)$ and $\left(22, 5 - \frac{5\sqrt{3}}{2}\right)$.

Mark allocation: 2 marks

- 1 mark for finding the correct values of $h(2)$ and $h(22)$
- 1 mark for labelling coordinates correctly

TIP

» **Don't forget to write your answer in the coordinate form.**

Question 14c.

Worked solution

Find the length between $\left(2, 5 + \frac{5\sqrt{3}}{2}\right)$ and $\left(22, 5 - \frac{5\sqrt{3}}{2}\right)$ using the distance formula:

$$l = \sqrt{(22 - 2)^2 + \left(5 - \frac{5\sqrt{3}}{2} - 5 - \frac{5\sqrt{3}}{2}\right)^2} = 5\sqrt{19}$$

Mark allocation: 1 mark

- 1 answer mark

Question 14d.

Worked solution

$x = 3$ and $x = 15$ (as seen from the graph).

Mark allocation: 1 mark

- 1 answer mark

Question 14e.

Worked solution

The length AB is cut into two halves by the line $x = 15$. Since the length of AB is 6.4, the point A is 3.2 units away from the left of 15. Hence its x-coordinate is 11.8.

Therefore its y-coordinate can be found by:

$$h(11.8) = -5\cos\left(\frac{11.8\pi}{6} + \frac{\pi}{2}\right) + 5 = 4.4773576836621$$

Therefore the length of AD is 4.477.

Mark allocation: 2 marks

- 1 mark for finding the x-coordinate of A
- 1 mark for finding the length of AD, that is, the height of A above the x-axis

Question 14f.

Worked solution

$h(x) = 9.5$

$9.5 = -5\cos\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 5$

Using CAS to solve with the restricted domain gives:

$x = 2.1386022412276$ or $x = 3.8613977587722$ or $x = 14.138602241227$ or $x = 15.861397758772$

Hence the solutions are $x = 2.13, 3.86, 14.14$ and 15.86.

Mark allocation: 1 mark

- 1 answer mark

Question 14g.

Worked solution

$5 - \frac{5\sqrt{3}}{2} < a < 5 + \frac{5\sqrt{3}}{2}$

The line must be drawn parallel to the x-axis below the starting point of the graph and/or it can be a line drawn parallel to the x-axis above the end point.

Mark allocation: 1 mark

- 1 answer mark

Question 15a.

Worked solution

$f(x) = ax^3 + bx^2 + cx + 4$

The y-intercept occurs when $x = 0$, so:

$$
\begin{aligned}
f(0) &= a(0)^3 + b(0)^2 + c(0) + 4 \\
&= 4
\end{aligned}
$$

The y-intercept is $(0, 4)$.

Mark allocation: 1 mark

- 1 answer mark

Question 15b.

Worked solution

A local minimum occurs when $f'(x) = 0$.

$f'(x) = 3ax^2 + 2bx + c$

Since the local minimum occurs at the point $(3, -23)$, $f'(3) = 0$ and $f(3) = -23$.

$$
\begin{aligned}
f'(3) &= 3a(3)^2 + 2b(3) + c \\
0 &= 27a + 6b + c \\
\therefore\ & 27a + 6b + c = 0
\end{aligned}
$$

$f(x) = ax^3 + bx^2 + cx + 4$

$f(3) = a(3)^3 + b(3)^2 + c(3) + 4 = -23$

$\therefore 27a + 9b + 3c + 4 = -23$

$\therefore 27a + 9b + 3c = -27$

Mark allocation: 2 marks

- 1 method mark for establishing the equation $27a + 6b + c = 0$ by evaluating the equation $f'(3) = 0$
- 1 method mark for establishing the equation $27a + 9b + 3c = -27$ by evaluating the equation $f(3) = 0$

TIP

» **When solving this question it is important to realise that each stationary point at a point (a, b) provides two pieces of information: $f'(a) = 0$ and $f(a) = b$.**

Question 15c.

Worked solution

Since a local maximum occurs when $x = -2$, $f'(-2) = 0$.

$$
\begin{aligned}
f'(-2) &= 3a(-2)^2 + 2b(-2) + c \\
&= 12a - 4b + c
\end{aligned}
$$

$\therefore 12a - 4b + c = 0$

Solving the simultaneous equations:

$$27a + 6b + c = 0$$
$$27a + 9b + 3c = -27$$
$$12a - 4b + c = 0$$

Using a CAS yields $a = \frac{2}{3}, b = -1, c = -12.$

Mark allocation: 2 marks

- 1 method mark for establishing the equation $12a - 4b + c = 0$ by evaluating the equation $f'(-2) = 0$
- 1 answer mark for the correct exact answers for a, b and c

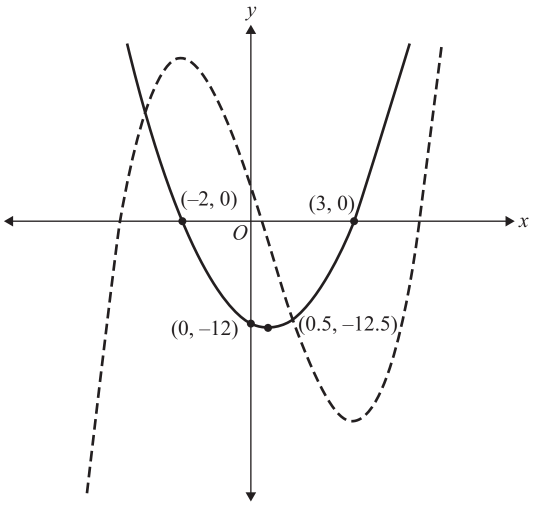
Question 15d.

Worked solution

$f'(x) = 3ax^2 + 2bx + c$

$f'(x) = 2x^2 - 2x - 12$

Sketching this on a CAS reveals the following graph.



Mark allocation: 3 marks

- 1 answer mark for the x-intercepts correctly labelled
- 1 answer mark for the y-intercept correctly labelled
- 1 answer mark for the stationary point correctly labelled

TIPS

- » Much of this part is accessible even without having correctly answered other parts of the question. Since the stationary points are given, the x-intercepts must be $x = -2$ and $x = 3$. The cubic shape of the original graph also suggests that the gradient graph will be a quadratic. This would achieve at least 2 out of the 3 marks.
- » Remember to check during reading time for questions with parts that might be accessible without having completed previous parts of the same question.

Question 16a.

Worked solution

$$2\pi rh + 4\pi r^2 = 3000\pi$$

$$h = \frac{3000\pi - 4\pi r^2}{2\pi r} = \frac{1500 - 2r^2}{r}$$

Mark allocation: 1 mark

- 1 answer mark

Question 16b.i.

Worked solution

Volume of the solid cylinder = volume of the cylinder – volume of the sphere

$$V = \pi r^2 h - \frac{4}{3}\pi r^3$$

Mark allocation: 1 mark

- 1 answer mark

Question 16b.ii.

Worked solution

Substituting $h = \frac{1500 - 2r^2}{r}$ into the equation $V = \pi r^2 h - \frac{4}{3}\pi r^3$ gives:

$$\begin{aligned} V &= \pi r^2\left(\frac{1500 - 2r^2}{r}\right) - \frac{4}{3}\pi r^3 \\ &= 1500\pi r - 2\pi r^3 - \frac{4}{3}\pi r^3 \\ &= 1500\pi r - \frac{10\pi r^3}{3} \\ &= \frac{4500\pi r - 10\pi r^3}{3} \\ &= \frac{10\pi r}{3}(450 - r^2) \end{aligned}$$

Mark allocation: 1 mark

- 1 mark for showing the substitution

Question 16b.iii.

Worked solution

Using a CAS yields:

$$\frac{dV}{dr} = -10\pi(r^2 - 150)$$

Mark allocation: 1 mark

- 1 answer mark

Question 16b.iv.

Worked solution

Using a CAS:

$$-10\pi(r^2 - 150) = 0$$

So $r = -5\sqrt{6}$ or $5\sqrt{6}$.

Since r can’t be negative because it is the radius, $r = 5\sqrt{6}$ cm.

Mark allocation: 1 mark

- 1 answer mark

Question 16b.v.

Worked solution

The maximum volume occurs when $r = 5\sqrt{6}$.

Substituting $r = 5\sqrt{6}$ into $V = \frac{10\pi r}{3}(450 - r^2)$ gives $V = 5000\sqrt{6}\pi$ cm^3.

Mark allocation: 1 mark

- 1 answer mark

Question 17a.

Worked solution

$$\begin{aligned} 5W + 3L &= 50 \\ 3L &= 50 - 5W \\ L &= \frac{50 - 5W}{3} \end{aligned}$$

Mark allocation: 1 mark

- 1 answer mark

Question 17b.

Worked solution

$$\begin{aligned} A &= LW \\ &= \left(\frac{50 - 5W}{3}\right)W \\ &= \frac{50W - 5W^2}{3} \end{aligned}$$

Mark allocation: 1 mark

- 1 method mark for substituting L into the equation for area and deriving the correct answer

Question 17c.

Worked solution

The maximum occurs when the derivative is zero.

Since $A = \frac{50W - 5W^2}{3}$, the derivative is $A' = \frac{5}{3}(10 - 2W)$. Therefore:

$$\begin{aligned} \frac{5}{3}(10 - 2W) &= 0 \\ 10 - 2W &= 0 \\ W &= 5 \end{aligned}$$

Substituting W into L gives:

$$L = \frac{50 - 5W}{3} = \frac{25}{3}$$

Therefore the dimensions are 5 metres by $\frac{25}{3}$ metres.

Mark allocation: 2 marks

- 1 method mark for setting the derivative to zero
- 1 answer mark for both correct dimensions

TIP

» **By defining the area function, we can use a CAS to easily find the maximum value, resulting in $W = 5$ metres and $L = \frac{25}{3}$ metres.**

Question 17d.

Worked solution

The maximum point of the goal line occurs when $f'(x) = 0$.

Since $f'(x) = 2ax + b$, we must solve the equation $2ax + b = 0$.

$$\begin{aligned} 2ax + b &= 0 \\ x &= -\frac{b}{2a} \end{aligned}$$

Alternatively, the maximum point occurs at the turning point. For the quadratic function, this is at $x = -\frac{b}{2a}$.

To find the maximum point, we substitute this into $f(x)$:

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) \\ &= \frac{ab^2}{4a^2} - \frac{b^2}{2a} \\ &= \frac{b^2}{4a} - \frac{2b^2}{4a} \\ &= -\frac{b^2}{4a} \end{aligned}$$

The penalty spot is 5 metres beyond this point. Therefore its distance is $-\frac{b^2}{4a} + 5$ metres.

Mark allocation: 2 marks

- 1 method mark for stating that the maximum occurs at a turning point where $x = -\frac{b}{2a}$ or for solving the equation $f'(x) = 0$ for x
- 1 method mark for substituting $x = -\frac{b}{2a}$ into $f(x)$ or finding the y value of the turning point, and deriving the correct answer

TIPS

» **Alternative method: It is possible to solve this problem by completing the square, as follows:**

$$\begin{aligned} ax^2 + bx &= a\left[x^2 + \frac{b}{a}x\right] \\ &= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] \\ &= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b}{4a^2}\right] \\ &= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} \end{aligned}$$

» **The y value of the turning point is therefore $-\frac{b^2}{4a}$ as before, giving a a distance to the penalty spot of $-\frac{b^2}{4a} + 5$ metres.**

Question 17e.

Worked solution

$$\begin{aligned} f(x) &= ax^2 + bx \\ f'(x) &= 2ax + b \\ f'(12) &= 24a + b = -\frac{3}{25} \\ f(5) &= 25a + 5b = \frac{9}{4} \end{aligned}$$

Solving these simultaneously on a CAS gives $a = -\frac{3}{100}$, $b = \frac{3}{5}$.

Mark allocation: 2 marks

- 1 method mark for setting up the simultaneous equations

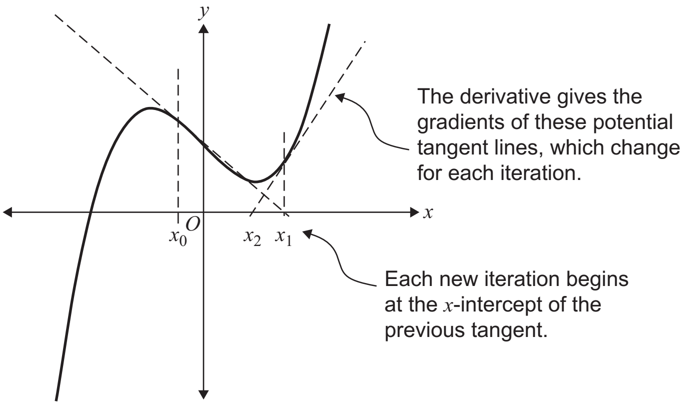
  $$24a + b = -\frac{3}{25}$$

  $$25a + 5b = \frac{9}{4}$$

- 1 answer mark for the correct values of a and b

Question 18a.

Worked solution



The derivative gives the gradients of these potential tangent lines, which change for each iteration.

Each new iteration begins at the x-intercept of the previous tangent.

Mark allocation: 2 marks

- 1 answer mark for each approximate location of the next two points of iteration, x_1 and x_2

Question 18b.

Worked solution

An approximate solution using Newton's method can be summarised by the iterative formula

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, where $n = 0, 1, 2, ...$

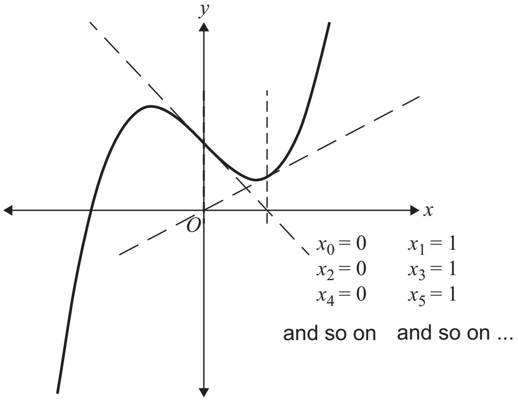
Since $f(x) = x^3 - 2x + 2$, then $f'(x) = 3x^2 - 2$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{f(0)}{f'(0)} = 0 - \frac{2}{-2} = 1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{1} = 0$$

This means that the iteration will continually oscillate between 1 and zero and never converge to a solution.

This can be seen graphically as follows:



Mark allocation: 2 marks

- 1 method mark for correctly using the iterative formula for Newton's method and showing enough calculations to demonstrate a lack of convergence to a solution or showing this lack of convergence in a graph similar to the one above
- 1 answer mark for correctly stating that the iteration will continually oscillate between 1 and zero and never converge to a solution

Question 18c.

Worked solution

An approximate solution using Newton's method can be summarised by the iterative formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$ where $n = 0, 1, 2, 3, \ldots$ and

$$f(x) = x^3 - 2x + 2$$

$$f'(x) = 3x^2 - 2$$

Using this formula, the third iteration results in an approximate solution of $x = -1.777$.

Mark allocation: 2 marks

- 1 answer mark for a correct iterative formula based on Newton's method
- 1 answer mark for the correct answer of $x = -1.777$

TIP

» **It might be helpful to create a table similar to the one below, either on paper or via the spreadsheet facility in your CAS device.**

n	x_n	$f(x_n)$	$f'(x_n)$
0	–3	–19	25
1	–2.24	–4.759424	13.0528
2	–1.875371415	–0.844972085	8.551053828
3	–1.77655645	–0.053970746	7.468458455

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