PEARSON mathematics

S.B.

ESSENTIALS EDITION

10-10A



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We would like to thank our authors for their extraordinary dedication and their contribution to the development of this project.



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Fibonacci poem

The poem on the inside front cover is called a Fib. The sequence of syllables in the lines follows a Fibonacci sequence: 1, 1, 2, 3, 5, 8.

PEARSON Mathematics 7–10

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PEARSON mathematics

Essentials Edition



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The **Pearson Mathematics Essentials Edition** allows teachers to focus on the essentials of a strong mathematical learning base: key theory points and carefully graded exercises that encourage deeper understanding while comprehensively covering the Australian Curriculum. This edition has been created using the latest research on how students learn mathematics and how to avoid common misunderstandings, making it easy for teachers to cater for mixed abilities and ask good questions.

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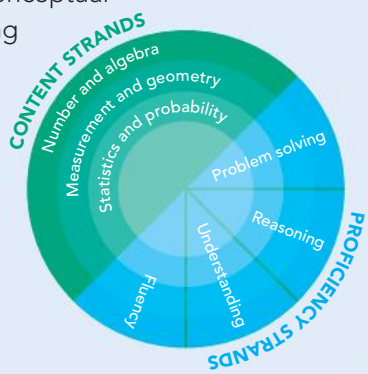
The Australian Curriculum: Mathematics aims to ensure that students:

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens.*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in Number and Algebra, Measurement and Geometry, and Statistics and Probability.*
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

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Pearson Mathematics shares these aims.

Pearson Mathematics is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills, in addition to addressing the Australian Curriculum's General capabilities and Cross-curriculum priorities.



Differentiation

Pearson Mathematics has been written and designed with the needs of the full ability spectrum of students in Australian classrooms in mind.

Recall—Each chapter begins with a review of assumed and necessary knowledge, with Recall worksheets available for each skill for those students needing extra revision.

Challenge—Every Mixed Review has a full page challenge section for earlier finishers and those students who need to be extended.



Scaffolded Exercises—All Exercises are split into the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has Open-ended questions to encourage students' creative thinking and ability to communicate mathematics effectively.

Regular revision and reinforcement

Pearson Mathematics has a broad range of cumulative and chapter-based revision.

Half-time—A mid-chapter review



Chapter review—An end-of-chapter review

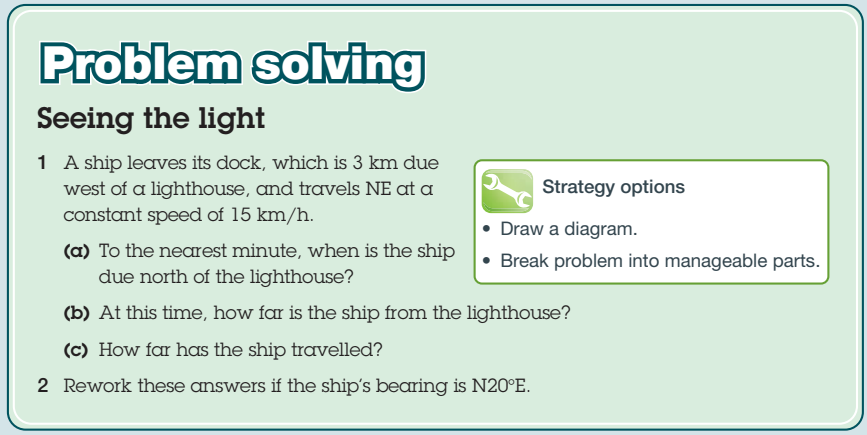


Mixed review—Cumulative revision for every third chapter



Problem solving

These are located throughout every chapter and give students ample opportunity to develop their problem-solving skills and logical reasoning.



Problem solving

Seeing the light

1 A ship leaves its dock, which is 3 km due west of a lighthouse, and travels NE at a constant speed of 15 km/h.

(a) To the nearest minute, when is the ship due north of the lighthouse?

(b) At this time, how far is the ship from the lighthouse?

(c) How far has the ship travelled?

2 Rework these answers if the ship's bearing is N20°E.

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

1 Financial mathematics

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.



1 Convert the following percentages to decimals.

(a) 17% **(b)** 6% **(c)** 19.2%

(d) 4.8% **(e)** 2.01% **(f)** $\frac{1}{4}$%



2 Calculate:

(a) 15% of \$20 000 **(b)** 6% of \$10 000

(c) 9.5% of \$350 000 **(d)** $7\frac{1}{2}$% of \$45 000



3 **(a)** Increase 40 by:

(i) 100% **(ii)** 50% **(iii)** 25% **(iv)** 200%

(b) Decrease 60 by:

(i) 50% **(ii)** 30% **(iii)** 10% **(iv)** 75%



4 Evaluate:

(a) 7^2 **(b)** 2^3 **(c)** 10^6

(d) $(0.6)^2$ **(e)** $\left(\frac{1}{2}\right)^5$ **(f)** $(1.1)^4$



5 Convert the following to a decimal number of years. Assume a year has 52 weeks or 365 days.

(a) 6 months **(b)** 3 months **(c)** 8 years and 9 months

(d) 26 weeks **(e)** 13 weeks **(f)** 146 days

After completing this chapter you will be able to:

- use and extend the simple interest formula to develop the compound interest formula
- use the compound interest formula to solve problems involving different rates and different compounding periods
- use recursive techniques
- compare effective and nominal interest rates and solve related problems
- adapt the interest formulas to solve problems of appreciation, depreciation, and growth and decay.

Interest

1.1

Need to Know

Simple interest formula:

$$I = PRT$$

where I = amount of simple interest
P = **principal**
R = rate of **interest per annum** (p.a.) as a decimal
T = time in years

Also:

$$A = P + I$$

where A = total amount repaid on the loan or collected from the investment
P = principal
I = simple interest

Compound interest formula (compounded annually):

$$A = P(1 + r)^n$$

where A = amount of the loan or investment after n years
P = principal
r = rate of interest (p.a.) as a decimal
n = time in years

Also:

$$I = A - P$$

where I = compound interest owed or earned after a certain number of years
A = amount of the loan or investment after the same number of years
P = principal

Increase in cost or value is called **appreciation**. The rate of increase in prices within the economy is called the **inflation rate**. The simple interest formula can be used to calculate a **flat rate** of appreciation.

Worked Example 1

WE1

$20 000 is borrowed for 5 years at an interest rate of 9.5% p.a. If the interest is calculated as simple interest, find:

(a) the amount of interest charged over the period of the loan

(b) the total amount to be repaid at the end of the loan.

Working

(a) $I = PRT$

$P = 20\,000$, $R = 9.5\% = 0.095$, $T = 5$

$I = 20\,000 \times 0.095 \times 5$
$= \$9500$

(b) $A = P + I$

$P = 20\,000$, $I = 9500$

$A = 20\,000 + 9500$
$= \$29\,500$

Worked Example 2

WE2

Find the information that is missing in each of the following simple interest situations.

(a) $200 interest is paid on an investment of $5000 after 6 months.

(b) $350 interest is paid on an investment at 4% p.a. after 9 months.

(c) $1250 interest is paid on an investment of $3000 invested at 5% p.a.

Working

(a) $I = \$200$
$P = \$5000$
$R = ?$

$$T = \frac{6}{12} = 0.5$$

$$I = PRT$$
$$200 = 5000 \times R \times 0.5$$
$$R = \frac{200}{5000 \times 0.5}$$
$$= 0.08$$

The rate is 8% p.a.

(b) $I = \$350$
$P = ?$
$R = 0.04$

$$T = \frac{9}{12} = 0.75$$

$$I = PRT$$
$$350 = P \times 0.04 \times 0.75$$
$$P = \frac{350}{0.04 \times 0.75}$$
$$= 11\,666.67$$

The principal is $11 666.67.

(c) $I = \$1250$
$P = \$3000$
$R = 0.05$
$T = ?$

$$I = PRT$$
$$1250 = 3000 \times 0.05 \times T$$
$$T = \frac{1250}{3000 \times 0.05}$$
$$= 8.333$$

The time is $8\frac{1}{3}$ years, which is 8 years and 4 months.

Worked Example 3

WE3

$12 500 is borrowed for 7 years at an interest rate of 15% p.a. If the interest is compounded annually, find:

(a) the total value of the loan after 7 years

(b) the amount of interest charged over the period of the loan.

Working

(a) $A = P(1 + r)^n$

$P = 12\,500$, $r = 15\% = 0.15$, $n = 7$

$$A = 12\,500(1 + 0.15)^7$$
$$= 12\,500 \times 1.15^7$$
$$= \$33\,250.25$$
(rounded to the nearest cent)

(b) $I = A - P$

$A = 33\,250.25$, $P = 12\,500$

$$I = 33\,250.25 - 12\,500$$
$$= \$20\,750.25$$

1.1 Interest

Answers page 482

If necessary, round the answers to the following questions to the nearest cent.

Fluency

1 For each of the following loans the interest is calculated as simple interest. Find: WE1

(i) the amount of interest charged over the period of the loan

(ii) the total amount to be repaid at the end of the loan.

(a) $40 000 borrowed for 1 year at 8% p.a.

(b) $30 000 borrowed for 1 year at 9% p.a.

(c) $30 000 borrowed for 5 years at 7% p.a.

(d) $40 000 borrowed for 12 years at 5% p.a.

(e) $150 000 borrowed for 25 years at 7.5% p.a.

(f) $240 000 borrowed for 30 years at 6.5% p.a.

(g) $300 000 borrowed for 1.1 years at 6.75% p.a.

(h) $350 000 borrowed for 1.1 years at 7.25% p.a.

(i) $13 000 borrowed for 5 years at $9\frac{1}{2}$ % p.a.

(j) $15 800 borrowed for 4 years at $10\frac{3}{4}$ % p.a.

(k) $45 000 borrowed for $12\frac{1}{2}$ years at 5% p.a.

(l) $36 000 borrowed for $5\frac{1}{2}$ years at 7% p.a.

(m) $10 000 borrowed for 2 years and 3 months at 15% p.a.

(n) $16 000 borrowed for 3 years and 9 months at 17% p.a.

2 Find the information that is missing in each of the following simple interest situations. WE2

(a) $2400 interest is paid on an investment at 20% p.a. after 3 years.

(b) $7200 interest is paid on an investment at 15% p.a. after 6 years.

(c) $25 200 interest is paid on an investment of $20 000 after 9 years.

(d) $40 788 interest is paid on an investment of $30 900 after 12 years.

(e) $65 000 interest is paid on an investment of $100 000 invested at 6.5% p.a.

(f) $135 000 interest is paid on an investment of $150 000 invested at 7.5% p.a.

3 For each of the loans in Question **1** the interest is now compounded annually. Find: WE3

(i) the total amount to be repaid at the end of the loan

(ii) the amount of interest charged over the period of the loan.

4 Lily invested $20 000 for 5 years at an interest rate of 5.25% p.a. Jarrah invested $45 000 for 7 years at an interest rate of 9.5% p.a.

(a) Calculate the interest earned on each investment assuming:

(i) annual compound interest **(ii)** simple interest.

(b) Compare the compounded interest with the simple interest.

5 Calculate the total amount owing on a loan of $7000 after 2 years, if the 16% interest p.a. is:

(a) compounded annually **(b)** calculated as simple interest.

6 $4245 at 6.5% p.a. compound interest, compounded annually over 8 years, will amount to:

A $\$4245(1 + 0.065)$ **B** $\$4245(1 + 0.065) \times 8$

C $\$4245(1 + 0.065)^8$ **D** $\$4245(1 + 0.65)^8$

Understanding

Hint

Use $A = P + I$ to find the simple interest first. Then use $I = PRT$ to find R.

7 At what simple percentage rate will an initial investment of $7500 grow to a total investment of $8400 in 10 years?

8 If the simple interest on an investment over 18 years at 4% is $93, what was the amount of the original investment?

Hint

In some cases you may need to use 'guess and check' to solve the equation.

9 Complete the following table assuming an annual compounding period.

Amount (A)	Principal (P)	Interest rate p.a. (R)	Time in years (T)
$1359.81		8%	13
	$24 600	12.2%	5
$1 048 209.60	$1 000 200	4.8%	
$151.71		3%	20
$60.50	$50		2

10 For each of the following appreciation questions:

(i) state whether the problem should be modelled by the simple interest formula or by the compound interest formula

(ii) solve the problem, rounding answers to the nearest whole number or dollar.

(a) Martina currently gets $10 pocket money a week. She has been promised a $4 increase each year. What is the promised rate of increase? How much more pocket money will she get in 3 years than she is getting now?

(b) An antique clock appreciates in value by 11% p.a. If it was valued 10 years ago at $350 how much is it worth now?

(c) Yang paid $20 000 for a new motorbike. If the inflation rate is 6.5% p.a., what is the estimated cost of a similar new motorbike in 4 years' time?

(d) Rhea has been awarded an annual salary increase rate of 7% p.a. Her current salary is $42 000 p.a. Calculate how much she can expect to earn in 3 years' time if the same rate continues.

(e) Ethel was given a $500 ring 30 years ago. Each year it appreciated at a flat rate of 5%. What is it worth now?

(f) In 1993, a house was purchased for $214 500. It was sold in 2010 for $652 000. What was the yearly average rate of appreciation?

11 Costs in a business are growing at an annually compounding rate of 8% p.a. Currently they run at $780 per week but 7 years ago they were:

A $\dfrac{\$780}{1.08^7}$ **B** $\$780 \times 1.08^7$ **C** $\$780 \times 1.08^7 - 780$ **D** $\dfrac{\$780}{1.8^7}$

Reasoning

Hint

Either choose a number for the amount Ahmed has to invest or let P be this amount.

12 Ahmed has some money to invest. He wants to at least quadruple his money.

(a) How long will it take if he invests at a simple interest rate of 3% p.a.?

(b) How long will it take if he invests at an annually compounded interest rate of 3%?

Give your answers in whole years.

13 A certain sum of money $\$P$ is deposited into a bank account at an annually compounding interest rate, R. After 2 years it has grown to \$48 951 and after 3 years it has grown to \$51 398.55.

What is the interest rate?

Open-ended

14 Find values for the amount of an investment and the annual interest rate, given that the investment compounds annually for 2 years, to give a final value within \$100 of \$15 000.

15 Calum's car was written off as a result of hail damage in a sudden and severe local storm. His interstate cousin Jessica also had to replace her car after flood damage. Both needed to borrow the same amount of money. Calum was able to get a flat rate loan at 10% p.a. for 8 years. Jessica will end up paying the same amount of interest as Calum but her loan has a simple interest rate between 10% and 20% p.a. and will need to be repaid over a shorter period of time. Find possible values for the interest rate and period of Jessica's loan.

Hint

Write two equations with P and R in them. Then use these two equations to write $\frac{51\,398.55}{48\,951}$ in terms of P and R.

Problem solving

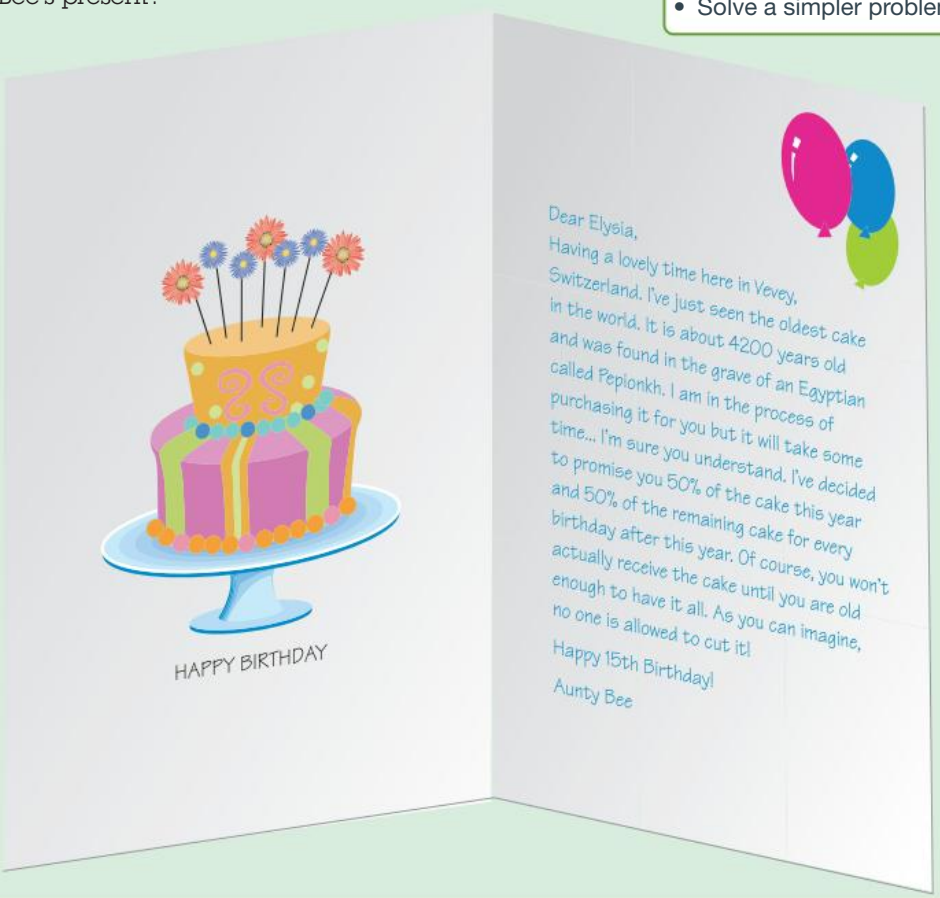
The oldest cake problem

1 What percentage of the cake has been promised to Elysia by her 21st birthday?

2 How old will Elysia be when she finally gets Aunty Bee's present?

Strategy options

- Draw a diagram.
- Solve a simpler problem.



1.2 Compound interest—the general formula

Need to Know

Compound interest formula (general case):

$$A = P(1 + r)^n$$

where A = amount owing or earned at the end of the complete process
P = principal (original amount borrowed or loaned)
r = interest rate for the period, expressed as a decimal
n = number of compounding periods

Worked Example 4

WE4

Clive borrowed \$12 000 to buy a lathe for his factory. He agreed to pay interest at a rate of 15% p.a. and to repay the loan in full in 3 years. Calculate the amount to be repaid if the interest is compounded:

(a) annually **(b)** half-yearly **(c)** quarterly.

Working

(a) $A = P(1 + r)^n$

$P = 12\,000$
$r = 0.15$
$n = 3 \times 1 = 3$
$A = 12\,000 \times (1.15)^3$
$= \$18\,250.50$

(b) $A = P(1 + r)^n$

$P = 12\,000$
$r = \frac{0.15}{2} = 0.075$
$n = 3 \times 2 = 6$
$A = 12\,000 \times (1.075)^6$
$= \$18\,519.62$

(c) $A = P(1 + r)^n$

$P = 12\,000$
$r = \frac{0.15}{4} = 0.0375$
$n = 3 \times 4 = 12$
$A = 12\,000 \times (1.0375)^{12}$
$= \$18\,665.45$

Answers page 482

1.2 Compound interest—the general formula

Fluency

WE4

1 Calculate the amount to be repaid after 2 years on a loan of \$16 000 if the 14% interest p.a. is compounded:

(a) annually **(b)** half-yearly **(c)** quarterly.

2 Calculate how much an investor would repay on a debt of \$8000 after $1\frac{1}{2}$ years if the 12% interest p.a. is compounded:

(a) half-yearly **(b)** quarterly, i.e. every 3 months.

3 If a loan of \$5800 is made at 16% p.a. compounded every half year, over 6 years the debt would grow to:

A $\$5800(1 + 0.16)^6$ **B** $\$5800(1 + 0.08)^{12}$

C $\$5800(1 + 0.16)^{12}$ **D** $\$5800(1 + 0.8)^{12}$

4 If the loan in Question **3** were compounded quarterly, the interest accrued would be:

A $\$5800(1 + 0.04)^6$ **B** $\$5800(1 + 0.16)^6 - \$5800$

C $\$5800(1 + 0.04)^{24} - \$5800$ **D** $\$5800(1 + 0.4)^{24} - \$5800$

Understanding

5 Calculate how much interest (to the nearest cent) is added over 10 years to an account paying 9% p.a. interest on an initial sum of \$45 800 if the interest is compounded:

(a) half-yearly **(b)** quarterly **(c)** monthly.

6 How much more will an investor get on an investment of \$32 000 over 4 years in an account offering 8.8% p.a. if the interest is compounded weekly rather than annually? (Assume 52 weeks in a year.)

7 Calculate the interest in Question **6** if it were to be compounded daily.

8 Martina won \$5000 in the lottery and decided to invest it with a view to taking a trip to Paris sometime in the future. Martina was given two options when she invested the money. She could have an interest rate of 4.5% p.a. compounded quarterly or she could have 4.4% p.a. compounded daily. Which is the better deal, and by how much, if Martina invests the money for 2 years?

Reasoning

9 **(a)** \$20 000 is to be invested for 3 years. Calculate the interest earned on each of the following to find which would give the best return.

(i) 5% p.a. compounded quarterly

(ii) 4.5% p.a. compounded every 2 months

(iii) 4% p.a. compounded monthly

(b) Do you think the size of the investment would alter which option in part **(a)** would give the best return?

(c) Prove your answer to part **(b)** algebraically by redoing part **(a)** with \$$P$ in place of \$20 000.

Open-ended

10 'The more frequent the compounding, the greater the interest.' Explain what this statement means. Use a principal of \$10 000 invested at 8% p.a. for 5 years and three different compounding periods (such as weekly, quarterly or monthly) to help you demonstrate your answer. Express the percentage increase correct to one decimal place.

11 Tyson is having trouble understanding the difference between simple and compound interest. He can't see why compound interest is higher than simple interest after the first compounding period if the principal, interest rate and overall time period are the same. How would you explain the difference to Tyson? Use an example of \$4000 invested for 3 years at 5% p.a. to help you explain your answer.

1.3 Compound interest—further applications

Worked Example 5

WE5

Aunt Eva leaves Heidi a legacy of some deposit stock that was invested for 10 years at 5.11% p.a. compounded daily. The value of the cheque received was $21 669.68. Calculate the initial deposit. Give your answer rounded to the nearest dollar. (Ignore the occurrence of leap years in the 10-year period.)

Working

$A = P(1 + r)^n$

$A = 21\,669.68$

$r = \frac{0.0511}{365}$

$n = 10 \times 365$
$= 3650$

$$21\,669.68 = P\left(1 + \frac{0.0511}{365}\right)^{3650}$$

$$P = \frac{21\,669.68}{\left(1 + \frac{0.0511}{365}\right)^{3650}}$$

$P = \$13\,000.00595$

So, the principal was $13 000.

Worked Example 6

WE6

In the first year of his business Jack had takings of $228 000. They grew to $520 000 after 12 years, increasing at the same rate every year. At what annual percentage rate has his business been growing? Round your answer to two decimal places.

Working

$A = P(1 + r)^n$

$A = 520\,000$, $P = 228\,000$, $n = 12 \times 1 = 12$

$$520\,000 = 228\,000(1 + r)^{12}$$

$$\frac{520\,000}{228\,000} = (1 + r)^{12}$$

$$\sqrt[12]{\frac{520\,000}{228\,000}} = (1 + r)$$

$$r = \sqrt[12]{\frac{520\,000}{228\,000}} - 1$$

$r = 0.071\,122\,251\,37$

$r = 7.11\%$ p.a.

The business has been growing at a rate of 7.11% p.a.

Worked Example 7

WE7

How many years will it take a $452 stamp to be worth $1000 if it increases in value by 7.5% p.a.? Assume that interest is compounding annually. Give your answer to the nearest whole number of years.

Working

$A = P(1 + r)^n$

$A = 1000$, $P = 452$, $r = 0.075$

$$1000 = 452(1 + 0.075)^n$$
$$1000 = 452(1.075)^n$$
$$\frac{1000}{452} = (1.075)^n$$
$$2.212\,389\,381 = (1.075)^n$$

If $n = 4$, $(1.075)^4 = 1.335\,469$
If $n = 10$, $(1.075)^{10} = 2.061\,031$
If $n = 12$, $(1.075)^{12} = 2.381\,779$
If $n = 11$, $(1.075)^{11} = 2.215\,608$

It will take 11 years for the stamp to increase its worth to $1000.

1.3 Compound interest—further applications

Answers page 483

Fluency

1 In each of the following, use the compound interest formula to find the principal.

WE5

(a) Calculate how much would need to be invested at 8% p.a. compounded each half year to accumulate to $9600 in 6 years. Round your answer to the nearest dollar.

(b) How much would Li have to deposit in order to receive $10 000 in 7 years if she places her money in an account that pays 8.8% interest, compounded quarterly? Round your answer to the nearest dollar.

(c) How much would Sylvester have to deposit to receive $8000 in 5 years' time if he places money in an account that pays 7.8% interest, compounded half yearly? Round your answer to the nearest cent.

(d) A deposit accumulates to $4500 in 9 months at 12% p.a. compounded quarterly. The initial deposit, in dollars, was:

A $\frac{4500}{1.3^3}$ **B** $\frac{4500}{1.03^3}$ **C** $\frac{4500}{1.12^{3/4}}$ **D** 4500×1.04^3

2 In each of the following, use the compound interest formula to find the annual interest rate.

WE6

(a) Sales of $26 900 grow to $78 000 in 11 years. Calculate the percentage growth p.a. rounded to one decimal place. Assume the growth compounds annually.

(b) Costs of $6200 increase to $8000 in 8 years. Calculate the percentage increase p.a. rounded to two decimal places. Assume annual compounding.

(c) Calculate the rate of interest p.a. which would allow $7900 to accumulate to $9000 in 5 years if interest is compounded each half year. Round your answer to one decimal place.

(d) $2800 accumulates to $4500 in 3 years. The percentage rate of interest if it is compounded annually is:

A $\sqrt[3]{\frac{4500}{2800}} - 1$

B $\left(\sqrt[3]{\frac{4500}{2800}} - 1\right) \times 100$

C $\sqrt[3]{\frac{4500}{2800}} \times 100$

D $\frac{4500}{2800} \times 100$

WE7

3 In each of the following, use the compound interest formula and the method of guess and check (or a CAS) to find the number of years. In each case, assume that interest is compounding annually.

(a) How many years will it take for a $2400 sapphire ring to be worth $8000 if it increases in value by 10.6% p.a.? Round your answer to the nearest whole number of years.

(b) How many years will it take for a $40 000 block of land to be worth $90 000 if it increases in value by 8.5% p.a.? Round your answer to the nearest whole number.

(c) How many years will it take for a $3400 porcelain dinner set to be worth $5000 if it increases in value by 9.7% p.a.? Round your answer to the nearest tenth of a year.

Understanding

4 $5000 is invested for 4 years with the interest compounded quarterly. For the first 2 years the interest rate is 4% p.a. and for the other 2 years it is 4.8% p.a.

(a) How much will the investment be worth at the end of the 4 years?

(b) How much interest will be earned?

(c) Would the investor be better off to invest for the whole 4-year period at 4.4% p.a.? How much interest would this investment earn?

(d) How much interest would be earned if the first 2 years were at 4.8% p.a. and the rest at 4% p.a.?

(e) The conditions of investment change and interest is now compounded annually. What would the interest rate need to be to earn the same amount of interest as found in part **(b)**?

(f) The interest rate is now 5% p.a. compounded annually. After how many years will the investment be at least as great as the result shown in part **(a)**?

5 Pene's savings are in a bank account that attracts 3.5% p.a. If her savings are to grow from $10 000 to over $20 000 in 20 years, what is the smallest number of compounding periods per year required?

Reasoning

6 A building society offered an interest rate of 7.25% p.a. on any investment over $3000. Mary deposited $3600 and received $5944 after 7 years.

(a) How often was the interest compounded?

(b) What simple interest rate would have given the same return in the same time? Round your answer to one decimal place.

(c) Mary was a little disappointed because she'd calculated a return of $5971. Her calculations were based on an incorrect assumption. What was that assumption?

7 If savings in an account are to be *at least* tripled over 20 years with an interest rate of 5.55% p.a., how often should the interest be compounded? Give your answer in months.

Open-ended

8 Jacob and Hafida are planning an overseas trip that will cost $20 000. They have decided to pool their money and invest it at 6.5% p.a. compounding quarterly for 1 year. They did the following calculations:

How much do we need to invest?

$P = ?,\ A = 20\,000,\ n = 4,\ r = 0.065$

$$20\,000 = P\left(1 + \frac{0.065}{4}\right)^4$$

$$20\,000 = P(1.02)^4$$

$$20\,000 = P \times 1.08$$

$$P = \frac{20\,000}{1.08}$$

$$P = 18\,518.52$$

Checking:

$A = ?,\ P = 18\,518.52,\ n = 4,\ r = 0.065$

$$A = 18518.52\left(1 + \frac{0.065}{4}\right)^4$$

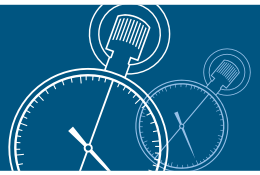
$$A = 18518.52(1 + 0.02)^4$$

$$A = 20\,045.04$$

When they collect their money a year later, they are almost $250 below their target. Where did they go wrong?

9 Give different rates and compounding periods that will result in savings being increased by at least 50% over 4 years.

Half-time 1



Round answers to the nearest cent, unless told otherwise.

Ex 1.2

1 If a loan of $7200 is made at 17% p.a. compounded quarterly over 5 years, the debt would grow to:

A $7200(1 + 0.17)^5$ **B** $7200\left(1 + \frac{0.17}{4}\right)^{20}$ **C** $7200\left(1 + \frac{0.17}{4}\right)^{5}$ **D** $7200(1 + 0.17)^{20}$

Ex 1.3

2 Calculate how much would need to be invested at 6% p.a. compounded monthly to accumulate to $14 000 in 8 years.

Ex 1.1

3 Calculate the total amount owing on a loan of $6000 after 3 years if the 14% interest p.a. is compounded annually.

Ex 1.3

4 $100 000 grows to $400 000 in 1 year. The rate of growth is compounding every 6 months. Find the value of the annual interest rate.

Ex 1.2

5 How much interest is added over 8 years to an account paying 8.25% p.a. interest on the initial sum of $55 700 if the interest is compounded:

(a) half-yearly **(b)** monthly **(c)** daily?

Ex 1.3

6 An item originally valued at $100 000 increased in value by a total of 55.133% over a number of years. Each year it appreciated at a rate of 5% of its value the previous year. Find the number of years, rounded to the nearest whole number.

Ex 1.1

7 Catherine and Ken invested $20 000 for 6 years at an interest rate of 7.5% p.a.

(a) Calculate the value of the interest earned on the investment if it was compounded annually.

(b) Calculate the value of the interest earned on the investment if simple interest was paid instead.

(c) Compare the compounded interest with the simple interest.

Round your answers to the nearest dollar.

Ex 1.1

8 A painting was purchased 11 years ago for $26 900. It has just been sold for $78 000. Calculate the flat rate of appreciation p.a. Round your answer to one decimal place.

Comparing interest rates

1.4

Need to Know

Effective interest rate formula:

$$R_{ef} = \left(\frac{I_1}{P}\right) \times 100$$

where R_{ef} = effective interest rate or real interest rate (p.a.) as a percentage
I_1 = amount of interest after the first year
P = principal

The effective interest rate is useful for comparing situations with different nominal rates and different compounding periods.

$$R_{ef} = \left[\left(1 + \frac{R}{n}\right)^n - 1\right] \times 100$$

where R_{ef} = effective interest rate (p.a.) as a percentage
R = **nominal interest rate** as a decimal
n = number of compounding periods in 1 year

Worked Example 8

WE 8

Find the effective rate of interest (R_{ef}) if $25 000 is invested for 1 year at the following rates. Where necessary round your answers to three decimal places.

(a) a simple rate of 9.5% p.a.

(b) an annually compounding rate of 9.5% p.a.

(c) a monthly compounding rate of 9.5% p.a.

(d) a weekly compounding rate of 9.5% p.a.

Working

(a) $R_{ef} = 9.5\%$ p.a.

For simple interest, the effective rate is the same as the nominal rate.

(b) $R_{ef} = 9.5\%$ p.a.

For annual compounding, the effective rate is the same as the nominal rate.

(c) $A = P(1 + r)^n$

$= 25\,000\left(1 + \frac{0.095}{12}\right)^{12}$

$= 27\,481.1896$

$I = A - P$

$= 27\,481.1896 - 25\,000$

$= 2481.1896$

$R_{ef} = \left(\frac{I_1}{P}\right) \times 100$

$= \frac{2481.1896}{25\,000} \times 100$

$= 9.9247$

The effective interest rate is 9.925% p.a.

(d) $A = P(1 + r)^n$

$= 25\,000\left(1 + \frac{0.095}{52}\right)^{52}$

$= 27\,489.0887$

$I = A - P$

$= 27\,489.0887 - 25\,000$

$= 2489.0887$

$R_{ef} = \left(\frac{I_1}{P}\right) \times 100$

$= \frac{2489.0887}{25\,000} \times 100$

$= 9.9563$

The effective interest rate is 9.956% p.a.

Worked Example 9

WE9

A loan attracts an interest rate of 23% compounded daily. Find:

(a) the nominal rate, R, and the number of compounding periods, n

(b) the effective rate, R_{ef}, using the values found in **(a)**.

Working

(a) $R = 23\%$

$n = 365$

(b) $R = 23\% = 0.23$

$R_{ef} = \left[\left(1 + \frac{R}{n}\right)^n - 1\right] \times 100$

$R_{ef} = \left[\left(1 + \frac{0.23}{365}\right)^{365} - 1\right] \times 100$

$= 0.2585 \times 100$

$= 25.85\%$ p.a.

Answers page 483

1.4 Comparing interest rates

Fluency

WE8

1 For each of the following find the effective rate of interest (R_{ef}).

(a) \$40 000 invested at a simple interest rate of 7% p.a.

(b) \$10 000 invested at an annual compounding rate of 9.4% p.a.

(c) \$100 000 invested at a monthly compounding rate of 12.75% p.a.

(d) \$35 000 borrowed at a weekly compounding rate of 19.7% p.a.

(e) \$20 000 borrowed at a fortnightly compounding rate of 16.5% p.a.

(f) \$200 000 invested at a rate of $6\frac{1}{4}$% p.a. compounded every 5 days.

2 For each of the following find:

(i) the nominal rate, R, and the number of compounding periods, n

(ii) the effective rate, R_{ef}, using the values found in **(i)**.

(a) 7.3% p.a. compounded 5 times a year

(b) 15.5% p.a. compounded monthly

(c) 21% p.a. compounded quarterly

(d) $16\frac{1}{4}$% p.a. compounded every 4 months

(e) daily compounding rate of $20\frac{1}{3}$% p.a.

(f) half-yearly compounded rate of 5.25% p.a.

3 The effective interest rate that corresponds to 5% p.a. compounded every 2 months is:

A $\left[\left(1+\frac{0.05}{2}\right)^2-1\right]\times 100$

B $\left[\left(1+\frac{0.05}{6}\right)^6-1\right]\times 100$

C $\left[\left(1+\frac{0.5}{6}\right)^6-1\right]\times 100$

D $\left[\left(1+\frac{5}{2}\right)^2-1\right]\times 100$

Understanding

4 Two banks are currently offering the following rates on their investment savings accounts:

Q Bank: 15% p.a. compounded quarterly
D Bank: 14.9% p.a. compounded daily

(a) Find the effective rates being offered by the two banks.

(b) Which bank is offering the better deal?

5 Ned needs to buy a new laptop. He is planning to take out a personal loan with an interest rate of 20.5% p.a. compounded quarterly. The alternative is to use his credit card that has an interest rate of 20.1% compounded daily.

(a) What is the effective rate of this personal loan?

(b) What is the credit card's effective rate?

(c) Which of the two options should Ned choose and why?

6 A bed can be purchased on a 'get it now, pay in 12 months' plan.

The interest payment options are different depending on whether the purchase is made online or instore. The online purchase attracts a flat rate interest of 25% p.a. The interest on the instore purchase is 23% p.a. compounded daily.

(a) What is the real or effective interest rate charged on an instore purchase?

(b) Which is the cheaper option?

Reasoning

7 The following represent investment options. In each case, choose the better option *without calculating the effective rate* and justify your choice.

(a) 12% p.a. compounded monthly
15% p.a. compounded daily

(b) 10.5% p.a. compounded weekly
10.5% p.a. compounded monthly

(c) 11% p.a. compounded daily
12% p.a. compounded daily

(d) 15% p.a. flat rate for 2 years
15% p.a. compounded annually for 2 years

8 The options in Question **7** now represent loans. Will your choice of option change? Justify your answer.

9 Inez and Sam need to choose between the following lucrative investments:

Option A yields 35.2% p.a. compounded daily.
Option B yields 42% p.a. compounded annually.
Option C yields 40% p.a. compounded quarterly.

Which option should they choose and why?

10 Simon has saved the 10% deposit for a new car. The car company can offer him an interest-free loan on the remaining balance for 6 months and then 15% p.a. compounded daily. Alternatively, he could borrow the money elsewhere for a flat rate of 9% p.a.

(a) If he pays out the loan plus interest in a year, which option will be cheaper?

(b) If he pays out the loan plus interest in two years, which option will be cheaper?

Open-ended

11 'Nominal interest rate' and 'simple interest rate' are the same thing. Explain with an example why this statement is untrue.

12 Two investments have different nominal interest rates and different compounding periods. However, the effective interest rate for both is exactly 46.41% p.a. What might be the nominal rates and respective compounding periods for these two investments?

1.5 Depreciation

Need to Know

Depreciation is the process by which items decrease in value. The **depreciation value** is the loss in dollars during 1 year.

Straight-line depreciation formula:

$$D_T = PRT$$

where D_T = **cumulative depreciation** after T years, in dollars
P = **prime cost** or **initial cost**, in dollars
R = rate of depreciation (p.a.) as a decimal and
T = time in years

Also:

$$V_T = P - D_T$$

where V_T = **written-down value** or **adjusted value** after T years, in dollars

Reducing balance depreciation formula:

$$V_T = P(1 - R)^T$$

where V_T = written-down value after T years, in dollars
P = prime cost, in dollars
R = rate of depreciation (p.a.) as a decimal
T = time in years

'Straight-line depreciation' can also be expressed as:

'depreciation at a percentage of its prime cost or initial value or purchase price every year'.

'Reducing balance depreciation' can also be expressed as:

'depreciation at a percentage of its reduced value every year'.

Total possible depreciation = prime cost − **scrap value** (**salvage value** or **residual value**)

$$\textbf{Unit cost depreciation} = \frac{\text{total possible depreciation}}{\textbf{expected life}}$$

Depreciation (after a certain amount of use or work)
= Unit cost depreciation × number of units of work done

Worked Example 10

WE10

The initial cost of a washing machine 2 years ago was $1950. It depreciates at 18.5% p.a. Assuming straight-line depreciation what is the:

(a) cumulative depreciation since the time of purchase

(b) current written-down value of the machine?

Working

(a) $D_T = PRT$

$P = 1950$, $R = 18.5\% = 0.185$, $T = 2$

$D_2 = 1950 \times 0.185 \times 2$
$= \$721.50$

(b) $V_T = P - D_T$

$P = 1950$, $D_2 = 721.50$

$V_2 = 1950 - 721.50$
$= \$1228.50$

Worked Example 11

WE11

The prime cost of a lawn mower 7 years ago was $5600. If it depreciates at 9% p.a. assuming reducing balance depreciation, calculate the:

(a) current written-down value of the mower

(b) cumulative depreciation since the time of purchase.

Working

(a) $V_T = P(1 - R)^T$

$P = 5600$, $R = 9\% = 0.09$, $T = 7$

$V_7 = 5600(1 - 0.09)^7$
$= \$2893.86$

(b) $D_T = P - V_T$

$P = 5600$, $V_7 = 2893.86$

$D_7 = 5600 - 2893.86$
$= \$2706.14$

Worked Example 12

WE12

A laptop is expected to work for 6000 hours before it has a resale or scrap value of $100. The purchase price is $1300. Find:

(a) the total possible depreciation

(b) the unit cost depreciation

(c) the depreciation after 1000 hours of use

(d) the written-down value after 1000 hours of use.

Working

(a) Total possible depreciation
= prime cost − scrap value

Prime cost = \$1300

Scrap value = \$100

Total possible depreciation
= \$1300 − \$100
= \$1200

(b) Unit cost depreciation
$= \frac{\text{total possible depreciation}}{\text{expected life}}$

Total depreciation = \$1200

Expected life = 6000 hours

Unit cost depreciation $= \frac{1200}{6000}$
$= 0.2$

The laptop decreases in value by 20 cents for every hour of work.

(c) Depreciation after use
= unit cost depreciation × number of units of work done

Unit cost depreciation = \$0.20 per hour

Number of units of work done = 1000

Depreciation after use
= 0.2 × 1000
= \$200

(d) $V = P - D$

P = \$1300, D = \$200

V = \$1300 − \$200
V = \$1100

Answers page 483

1.5 Depreciation

Fluency

WE10

1 For each of the following, assume straight-line depreciation. Find:

(i) the cumulative depreciation since the time of purchase

(ii) the current written-down value.

Round all answers to the nearest cent.

(a) The initial price 5 years ago was \$30 000. $R = 7\%$ p.a.

(b) The prime cost 12 years ago was \$47 500. $R = 5\%$ p.a.

(c) The purchase price 20 years ago was \$100 000. $R = 5\%$ p.a.

(d) The initial cost 23 years ago was \$790 000. $R = 4.25\%$ p.a.

(e) The prime cost 4 years ago was \$145 000. $R = 10.5\%$ p.a.

(f) The initial price $5\frac{1}{2}$ years ago was \$300 000. $R = 6.5\%$ p.a.

(g) The purchase price 6 months ago was \$47 500. $R = 20\frac{3}{4}\%$ p.a.

(h) The prime cost 18 months ago was \$152 700. $R = 33\frac{1}{4}\%$ p.a.

2 For each of the above, assume reducing balance depreciation. Find: WE11

(i) the current written-down value

(ii) the cumulative depreciation since the item was purchased.

Round all answers to the nearest cent.

3 **(a)** A machine is expected to work for 3000 hours before it has a resale or scrap value of $150. The purchase price is $6150. Find: WE12

(i) the total possible depreciation

(ii) the unit cost depreciation

(iii) the depreciation after 10 hours of use

(iv) the written-down value after 10 hours of use.

(b) A machine is expected to work for 2000 hours before it has no scrap value. The purchase price is $1000. Find:

(i) the total possible depreciation

(ii) the unit cost depreciation

(iii) the depreciation after 1000 hours of use

(iv) the written-down value after 1000 hours of use.

Understanding

4 What would be the total depreciation after 6 years on a $780 workstation if it is depreciated at 11.25% p.a. of its prime cost?

5 A car costs $24 000 new. The value of the car in 3 years, if it depreciates at 25% p.a. of its prime cost each year, will be:

A $\$\frac{24\,000 - 24\,000 \times 0.75}{3}$

B $\$24\,000 - 24\,000 \times 0.25 \times 3$

C $\$24\,000 \times 0.75^3$

D $\$24\,000 \times 0.25 \times 3$

6 Calculate the value of a computer (initial cost $5600) after 4 years if it is depreciated at 15% p.a of its prime cost each year.

7 Calculate the depreciation in each year, cumulative depreciation and written-down or adjusted value for each of the first 3 years if a $45 000 forklift is depreciated at 20% of its prime cost each year.

8 Which of the answers in Question **5** would be correct if the depreciation was calculated on the car's written-down value at the start of each year?

9 Calculate the value of a computer (cost $5600) after 4 years if it is depreciated at 15% p.a. of its written-down value at the start of each year. Compare this answer with that obtained for Question **6**.

10 How much would the total depreciation be after 6 years on a $780 workstation if it is depreciated at 15% p.a. of its written-down value at the start of each year?

11 Calculate the depreciation, cumulative depreciation and written-down value for each of the first 3 years if a $45 000 forklift is depreciated at 20% p.a. of its written-down value at the start of each year. Compare this answer with that obtained in Question **7**.

12 A $7500 machine has a scrap value of $300, and is guaranteed for 11 000 cycles. In a year when it operates for 2630 cycles the amount of depreciation, in dollars, is:

A $\frac{7500 - 2630}{11\,000} \times 300$ **B** $\frac{11\,000 - 7500}{2630} \times 300$

C $\frac{7500 - 300}{11\,000} \times 2630$ **D** $\frac{7500}{11\,000} \times 2630$

13 **(a)** The amount of depreciation (in dollars) for a $5000 machine with no scrap value, guaranteed for 8000 hours, in its first year when it works for 1350 hours, is:

A $\frac{8000}{1350} \times 5000$ **B** $\frac{8000}{5000} \times 1350$ **C** $\frac{5000}{6650} \times 1350$ **D** $\frac{5000}{8000} \times 1350$

(b) At the end of the first year the written-down value (in dollars) of the machine described in part **(a)** is:

A $5000 - \frac{5000}{8000} \times 1350$ **B** $6350 - \frac{8000}{5000} \times 1350$

C $8000 - \frac{5000}{8000} \times 1350$ **D** $8000 - \frac{1350}{8000} \times 5000$

14 **(a)** A $10 000 soft-drink bottler with no scrap value is guaranteed for 120 000 bottles. Calculate the depreciation in its first year when it fills 47 000 bottles.

(b) Calculate the value of the soft-drink bottler after the first year.

Reasoning

15 Calculate the value of an $11 500 screen printer with a scrap value of $200 guaranteed for 90 000 prints when it is used 12 400 times.

16 Calculate the value of a $3500 photocopier with a scrap value of $400 guaranteed for 600 000 copies after it produces 210 000 copies.

17 A camera costs $3500, has a scrap value of $900 and is guaranteed for 8000 photos. Calculate the value after the first and second year of its life if in the 2 years it takes 450 and 1800 photos respectively.

18 A $30 000 car depreciates at 20% p.a. of its written-down value at the start of each year every year for the first 3 years. An $80 000 car depreciates at 25% p.a. of its written-down value at the start of each year every year for the first 3 years. After that time, both cars depreciate at 10% p.a. of their written-down value at the start of each year for every subsequent year.

(a) What is the difference in their values:

(i) at purchase **(ii)** after 1 year **(iii)** after 2 years **(iv)** after 3 years?

(b) When will the difference in their values be under $3000?

Open-ended

19 An item was initially purchased for $10 000. It is now worth only $3000. Assuming that it depreciated at the same rate of its prime cost every year, what was the possible rate of decrease and corresponding elapsed time?

20 Repeat Question **19** assuming the depreciation followed a reducing-balance model.

Problem solving

Weird and wonderful numbers

When you've been up and down the stairs several times, take time out and try to answer these questions:

1 What did you find at the bottom of the stairs? What number(s) sent you back to the top?

2 Why is there a special condition attached to step one?

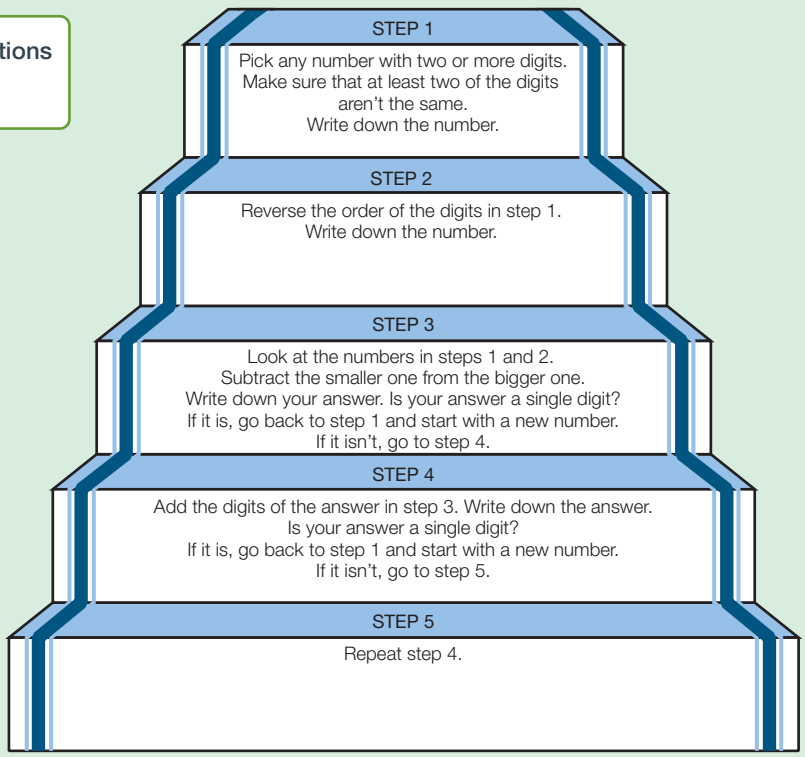
3 Will you ever find something else at the bottom of the stairs?

Strategy options

- Seek an exception.
- Solve a simpler problem.

Strategy options

- Act it out.



Growth and decay

1.6

Need to Know

Exponential growth is modelled by the formula $A = P(1 + r)^n$.

A is an accumulated total after a certain number of time periods, n.

P is an initial number. Sometimes, it is useful to think of P as the value of A when $n = 0$.

r is the growth rate. r is applied to the accumulated total from one time period to the next.

Always round down when whole number answers are required for discrete quantities (quantities that are counted) for exponential growth.

Exponential decay is modelled by the formula $A = P(1 - r)^n$.

A is a decreased total after a certain number of time periods, n.

P is an initial number. Sometimes it is useful to think of P as the value of A when $n = 0$.

r is the decay rate. r is applied to the decreased total from one time period to the next.

Always round up when whole number answers are required for discrete quantities for exponential decay.

The formula for exponential growth or decay can be written as:

$$Y = ka^X$$

where Y = the total after X time periods
k = the initial number (when $X = 0$)
a is the growth factor. It is the number you multiply by to get from one value of Y to the next
$a > 1$ for growth or $0 < a < 1$ for decay

Worked Example 13

WE13

The population of a district starts at 32 000. What will the population be in 7 years:

(a) if it increases by 4% each year

(b) if it has a growth rate of 1.2% per month?

Working

(a) $P = 32\,000$
$r = 4\%$ per year $= 0.04$
$n = 7$ years

$A = P(1 + r)^n$
$A = 32\,000(1 + 0.04)^7$
$= 42\,110$

(b) $P = 32\,000$
$r = 1.2\% = 0.012$ per month
$n = 7$ years $= (7 \times 12) = 84$ months

$A = P(1 + r)^n$
$A = 32\,000(1 + 0.012)^{84}$
$= 87\,159.158\,51$

The population will be 87 159.

Worked Example 14

WE14

The initial number of subscribers to a magazine was 12 469. How many subscribers will there be after 9 months if the number of subscribers decreases by:

(a) 1% each month

(b) 0.25% per day?

Assume 30 days in a month.

Working

(a) $P = 12\,469$
$r = 1\%$ per month $= 0.01$
$n = 9$ months

$A = P(1 - r)^n$
$A = 12\,469(1 - 0.01)^9$
$= 11\,391$

(b) $P = 12\,469$
$r = 0.25\%$ per day $= 0.0025$
$n = 9$ months $= (9 \times 30) = 270$ days

$A = P(1 - r)^n$
$A = 12\,469(1 - 0.0025)^{270}$
$= 6343.308\,039$

The number of subscribers will be 6344.

Worked Example 15

WE15

The number of bacterial cells (N) in a kitchen sink is increasing with time (T) hours. The bacteria count for the first 3 hours is shown below.

***T* (hours)**	0	1	2	3	4	5	6
N	6	12	24	48			

(a) Copy and complete the table.

(b) Find the formula that models this growth.

(c) Use the formula to predict the number of bacteria cells in the sink after 3 hours and 20 minutes.

(d) Use the table of values to plot a graph of N against T. Join the points with a smooth curve.

(e) Use the graph to predict the approximate time when there will be 240 bacteria cells in the sink.

Working

(a)

4	5	6
96	192	384

(b) X is T = time in hours
Y is N = total number of bacteria cells

$T = 0$, $N = 6$ so $k = 6$

$$a = \frac{12}{6} = 2$$

$Y = ka^X$

$N = 6 \times 2^T$

(c) $T = 3$ hours 20 minutes

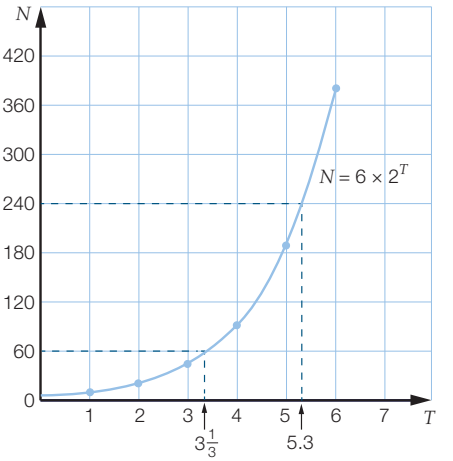
$= 3\frac{1}{3}$ hours

$N = 6 \times 2^{3.3333...}$

$= 60.4762$

≈ 60 bacteria cells

(d)



(e) Approximate time = 5.3 hours

Answers page 484

1.6 Growth and decay

Fluency

WE13

1 (a) If the population in a certain town is 10 200 and has a growth rate of 6% each year, what is the estimated population after 12 years?

(b) A colony of sea birds increases at a rate of 90% a year. If there are 8000 birds in the colony to start with, how many will be after 1 year?

(c) The population of birds in an aviary has a growth rate of 150% p.a. If the aviary originally has 30 birds, how many birds will there be after 4 years?

(d) A house was purchased $7\frac{1}{2}$ years ago for $190 000. Houses in this area have grown in value by 22.6% p.a. What is the estimated value of the house now, to the nearest dollar?

(e) A population increases at a rate of 1.2% per month. If the initial population is 200 000, what will be the population after 16 months?

(f) Joseph currently has 976 friends on Facebook and the number of friends is growing at a rate of 5% each year. How many friends can he expect to have after 21 months?

WE14

2 (a) The population of an endangered bird is declining at a rate of 25% a year. If there are 176 birds left now, how many will there be after 5 years?

(b) The value of a dishwasher is decreasing at a rate of 20% p.a. If it's worth $1400 now, how much will it be worth after 2 years?

(c) At the start of a plague the population in a medieval town is 10 200 and has a death rate of 6% each day. Estimate the population after 8 days.

(d) A population decreases at a rate of 1.2% per month. If the initial population is 200 000, what will be the population after 16 months?

(e) An eBay seller noticed that the number of internet sales has been declining at a rate of 7% a month. She sold 2370 items this January. If the trend continues, how many can she expect to sell in the month of July (in 6 months' time)?

(f) Cigarette sales in this country have shown a 2.98% p.a. decay rate in recent decades. If a new company sold 5 million packets of cigarettes in their first year, how many packets can they expect to sell in their fourth year? (*Note*: The fourth year is in 3 years' time.)

3 The following table shows the number of staff, S, employed by a company after T years. WE15

T	0	1	2	3	4	5	6
S	5	15	45	135			

(a) Complete the table.

(b) Find the formula that models this growth.

(c) Use the formula to predict the number of staff employed by the company after 10 years and 3 months.

(d) Use the table of values to plot a graph of S against T. Join the points with a smooth curve.

(e) Use the graph to predict approximately when the company will have 78 employees.

Understanding

4 (a) If the population of Growthland starts at 15 000 and increases by 5% each year, the population in 6 years' time will be:

A 4500 **B** 19 500 **C** 20 101 **D** 90 000

(b) A population decreases at the rate of 2% per month. If the intial population was 130 000, the population after 1 year is closest to:

A 98 800 **B** 102 013 **C** 127 400 **D** 164 871

5 Kate invests $5000 in a bank that offers 10% interest per year.

(a) What will her investment be worth after 1 year?

(b) What percentage of her original investment will her bank balance be at the end of the first year?

(c) Express this percentage as a decimal.

(d) What will Kate's investment be worth after 2 years?

(e) What is the first year's balance multiplied by to obtain the second year's?

(f) Complete a table of values showing Kate's bank balance at the end of each year (up to 10 years).

(g) Write an equation relating the amount Kate has in the bank to the number of years of investment.

(h) Plot a graph of this exponential relationship.

(i) Use the graph to predict what Kate's investment will be worth after $3\frac{1}{2}$ years. Check this result with a calculator.

(j) Use the graph to predict when Kate's investment will exceed $8000.

(k) How long would Kate need to have her money in the bank to double her investment?

6 The value of a car, currently estimated at $45 000, depreciates at 5% each year.

(a) What percentage of the current value will the car be worth after 1 year?

(b) Express this percentage as a decimal.

(c) Find the equation that describes the relationship between the value of the car and the number of years of depreciation.

(d) Complete a table of values for this exponential relationship. (Consider up to 5 years.)

(e) Plot the graph of this exponential relationship on graph paper.

(f) If the car is to be sold after $2\frac{1}{2}$ years, use the graph to predict what price the owner should expect to sell it for. Use a calculator to check your result.

(g) When should the car be sold to expect a price of $36 000?

7 Sibel was born in 1995 with an inheritance worth 1 million dollars. The money was unfortunately invested in a fund that has been decreasing by half each year.

(a) Find an equation that models the value of the inheritance (I) after T years.

(b) How much will Sibel's inheritance be worth when she is 20 years old?

(c) When will the inheritance be worth $7812.50?

Reasoning

8 When Angie and Rob married, some of their wedding cake was kept to celebrate the anniversary of their wedding each year for the next 6 years. On their first anniversary the family decided to eat a third of the cake. At each subsequent anniversary, it became a tradition to eat a third of the cake remaining from the previous year. There was 5 kg of cake left at the end of the wedding.

(a) Find the equation for the amount of cake (A) left after the nth anniversary.

(b) On the sixth and final anniversary, the remaining cake will be divided equally between 62 people. Find the mass of each slice. Round your answer to the nearest gram.

Open-ended

9 The population of a small town started with 10 people. After some time it reached 7290. Give some possibilities for the number of years elapsed and the % p.a. rate of increase.

10 Ana, Ernie and Janet are looking through the newspaper for something they can use in their Year 10 Science presentation. They come across this report:

> Scientists have become aware of a radioactive element known as Xradium in the soil of Nuketown that decays at a rate of 15% every 60 years. It is believed that Nuketown will be unfit for human habitation for about the next 850 years, by which time Xradium will be down to about 12% of its current level.

Janet is convinced the reporter made a mistake. She does the following calculation:

$6\frac{2}{3} \times 15\% = 100\%$ so it should only take about $6\frac{2}{3} \times 60 = 400$ years.

Ernie says: "You can't work it out like that, Janet! You've got to use the decay formula."

Ana says: "How are you supposed to use the decay formula without knowing how much of the stuff is in the soil right now?"

Is there an error in the newspaper report? Help the three students to resolve their problem.

Chapter review 1

D.I.Y. Summary

Key Words

adjusted value	exponential growth	reducing balance depreciation
appreciation	flat rate	residual value
compound interest	inflation rate	salvage value
cumulative depreciation	initial cost	scrap value
depreciation	interest	simple interest
depreciation value	nominal interest rate	straight-line depreciation
effective interest rate	per annum	total possible depreciation
expected life	prime cost	unit cost depreciation
exponential decay	principal	written-down value

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The original amount of money that is borrowed or loaned is called the _____.

2 Interest rates are normally expressed with p.a. after them. This stands for _____ and means for every year.

3 _____ is always higher than _____ after the first compounding period.

4 Increase in cost or value is called _____ and decrease in cost or value is called _____.

5 The stated interest rate is known as the _____. The adjusted rate that takes into account the effect of any compounding in the first year is called the _____ .

6 The amount of depreciation on a certain item is \$10 each year for 7 years. This is an example of _____.

7 Depreciation that has a decreasing value from one year to the next is called _____.

8 If the depreciation is \$5 in the first year and \$4 in the second year, the _____ over the first two years is \$9.

9 The value of an item after depreciation is known as its _____ or _____.

10 The formula $V_T = P - D_T$ has three _____, V_T, P and D_T. In this formula, P stands for the _____or _____.

11 Any value that an item still has at the end of its useful life is known as _____ , _____ or _____.

12 The initial purchase price less any scrap value is the _____ .

13 This value divided by the expected life gives the _____.

14 The growth of bacteria is an example of _____. The opposite process is called _____.

Fluency

1 **(a)** The amount an investment of $8200 amounts to after 2 years, if 9.5% p.a. interest is compounded annually, is closest to: Ex 1.1

A $1558 **B** 1632 **C** $7790 **D** $9832

(b) The amount an investment of $4000 amounts to after 2 years, if 3.5% p.a. interest is compounded annually, is closest to:

A $285 **B** $4192 **C** $4280 **D** $4285

2 Michael borrowed $14 000 to buy a new piano. He agreed to pay interest at a rate of 16% p.a. and to repay the loan in full after 4 years. Calculate the amount to be repaid if the interest is compounded: Ex 1.2

(a) half-yearly **(b)** quarterly **(c)** monthly.

3 Helga deposited some money into a savings account. The account guaranteed a return of 9.25% p.a. compounded daily. Calculate the initial deposit if the savings had grown to $47 638.38 after 5 years. Give your answer rounded to the nearest dollar. Ex 1.3

4 Find the effective interest rate for each of the following: Ex 1.4

(a) $38 000 invested at a simple rate of 8.7% p.a.

(b) $50 000 invested at an annually compounding rate of 9.5% p.a.

(c) the interest owed on a loan of $700 after 1 year was $28

(d) 17.4% p.a. compounded every 2 months

5 The initial price 5 years ago was $60 000. The rate of depreciation is 7% p.a. Ex 1.5

(a) Assuming straight-line depreciation, find the current written-down value.

(b) Assuming reducing balance depreciation, find the current written-down value.

Round all answers to the nearest cent.

6 The population of a small rural community has been declining at a rate of 1% a year. If there were 461 residents in 2006, estimate the population in 2016. Ex 1.6

Understanding

7 Simple interest is paid on an investment of $12 500 and yields $19 687.50 in 5 years. What is the rate of interest being paid?

8 Nina's salary has increased by 6% p.a. each year over the past 3 years to $70 500 p.a. Calculate her salary 3 years ago, to the nearest dollar.

9 Assume an inflation rate of 5.6% p.a. In 4 years, an ice-cream, currently $1.20, will cost:

A $1.20(1.014)^4 **B** $1.20(1.056)^4 **C** $1.20(1.014)^{16} **D** $1.20(1.56)^4

10 Calculate the rate of interest p.a. (rounded to one decimal place) that would allow $6700 to accumulate to $10 000 in 5 years if interest is compounded each quarter.

11 A dining suite was purchased $3\frac{1}{2}$ years ago for $4600. It depreciated at a fixed percentage of its reduced value each year and has just sold for $2800. The rate of depreciation is closest to:

A 15.25% p.a. **B** 13.22% p.a. **C** 13% p.a. **D** 11.18% p.a.

12 Copy and complete the following.

(a) If the prime cost of a photocopier is $10 800 and it has a scrap value of $500, then the total possible depreciation is $_____. If it has an expected life of 206 000 pages, then the unit cost depreciation is $_____per page. After it copies 125 000, the depreciation is $_____ and the written-down value is $_____.

(b) If the prime cost of an item is $52 500 and it has a scrap value of $750 then the total possible depreciation is $_____. If it has an expected life of 5000 days, then the unit cost depreciation is $_____per day. After it does a year's work, the depreciation is $_____ and the written-down value is $_____.

13 The population of Metown increases by 1.1% each year. What was the population 6 years ago if its current population is 68 000?

14 The temperature inside a room decreases by 3% every 15 minutes. Find the temperature at 7 p.m. if it is 26°C at 5 p.m. Round your answer to one decimal place.

Reasoning

15 How many years will it take for an investment of $10 000 to increase by at least 50% if the interest rate of 10% p.a. is compounded half-yearly? Give your answer to the nearest tenth of a year.

16 The following represent loan options. In each case, choose the better option *without calculating the effective rate* and justify your choice.

(a) 11% p.a. compounded quarterly 14% p.a. compounded monthly

(b) 10% p.a. compounded daily 10% p.a. compounded monthly

(c) 17% p.a. compounded daily 17.5% p.a. compounded daily

(d) 12% p.a. flat rate for 2 years 12% p.a. compounded annually for 2 years

17 One pair of breeding rats can become a colony of 2000 in one year. Estimate the population rate of growth per month. Give your answer rounded to one decimal place.

18 A new city restaurant had only 12 customers for lunch on its opening day. Business improved and the number of lunchtime customers followed an exponential growth curve. After 2 years the restaurant had reached its capacity of 120. What was the growth rate per month over these first 2 years? Round your answer to the nearest whole number.

2 Algebra

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 How many terms are there in each expression?

(a) $x^2 + 6x + 9$ **(b)** $16x^2 - y^2$ **(c)** $xy + 2y + 4x + 8$

2 What is the coefficient of each of these terms?

(a) $4x$ **(b)** $-6ab$ **(c)** y **(d)** $-12x^2y$

3 Simplify:

(a) $3 \times a + 5 \times b$ **(b)** $8 \times 3x - 8 \times y$

4 Expand and simplify if possible.

(a) $3(x + 2)$ **(b)** $5(x - 3)$ **(c)** $3x + 2(x - 4)$

5 Expand and simplify if possible.

(a) $-2(x + 3)$ **(b)** $-8(x - 5)$ **(c)** $-(x + 2)$

6 Simplify:

(a) $(3x)^2$ **(b)** $(2y)^2$ **(c)** $(7k)^2$

7 Simplify:

(a) $(4x)^2 - 2x^2$ **(b)** $p^2 - (3p)^2$ **(c)** $(-y)^2 + y^2$

8 Rearrange to make x the subject.

(a) $y = 2x + 3$ **(b)** $2a = \dfrac{x - 4b}{c}$ **(c)** $s = \dfrac{5x - at^2}{3}$

9 Find two numbers that:

(a) multiply to give 21 and add to give 10 **(b)** multiply to give 24 and add to give 14

(c) multiply to give 20 and add to give -9 **(d)** multiply to give -18 and add to give 3

(e) multiply to give -20 and add to give -8 **(f)** multiply to give -40 and add to give -3.

10 Simplify the following.

(a) $\dfrac{2}{3} + \dfrac{1}{4}$ **(b)** $\dfrac{5}{6} - \dfrac{3}{8}$ **(c)** $\dfrac{a}{3} + \dfrac{4a}{3}$

11 Simplify the following.

(a) $\dfrac{3}{8} \times \dfrac{4}{9}$ **(b)** $\dfrac{x}{10} \times \dfrac{5}{x}$ **(c)** $\dfrac{5}{6} \div \dfrac{10}{9}$

After completing this chapter you will be able to:

- expand binomial products, expressing answers in simplest form
- factorise quadratic trinomials
- recognise and use special products to factorise expressions
- rearrange formulas to make a different variable the subject
- simplify algebraic expressions involving fractions.

Expanding brackets

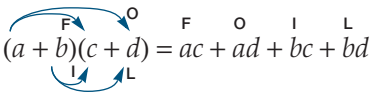
2.1

Need to Know

The Distributive Law is used to expand brackets.

$a(b + c) = ab + ac$
$a(b - c) = ab - ac$

To expand a **binomial product**, multiply each term in the second bracket by each term in the first bracket and add them together.



$(a + b)(c + d) = ac + ad + bc + bd$

The following rules can be used to expand special binomial products.

$(a - b)(a + b) = a^2 - b^2$ (**Difference of Two Squares**)
$(a + b)^2 = a^2 + 2ab + b^2$ (**Perfect Square**)
$(a - b)^2 = a^2 - 2ab + b^2$ (Perfect Square)

Worked Example 1

WE1

Expand and simplify each of the following.

(a) $3(4x - 2)$

(b) $x(x + 4) - 2(x - 1)$

Working

(a) $3(4x - 2)$
$= 12x - 6$

(b) $x(x + 4) - 2(x - 1)$
$= x^2 + 4x - 2x + 2$
$= x^2 + 2x + 2$

Worked Example 2

WE2

Expand and simplify $(x + 5)(2x - 3)$.

Working

$(x + 5)(2x - 3)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$

Worked Example 3

WE3

Expand and simplify:

(a) $(x-3)(x+3)$ **(b)** $(3x+1)^2$

Working

(a) $(x-3)(x+3)$
$= (x)^2 - (3)^2$
$= x^2 - 9$

(b) $(3x+1)^2$
$= (3x)^2 + 2 \times 3x \times 1 + (1)^2$
$= 9x^2 + 6x + 1$

Worked Example 4

WE4

Expand and simplify.

(a) $x(x-2)(x+3)$ **(b)** $(x+3)(x+1)(x-4)$ **(c)** $(x-1)(x-5)(x+1)$

Working

(a) $x(x-2)(x+3)$
$= x(x^2 + 3x - 2x - 6)$
$= x(x^2 + x - 6)$
$= x^3 + x^2 - 6x$

(b) $(x+3)(x+1)(x-4)$
$= (x^2 + x + 3x + 3)(x-4)$
$= (x^2 + 4x + 3)(x-4)$
$= x^3 - 4x^2 + 4x^2 - 16x + 3x - 12$
$= x^3 - 13x - 12$

(c) $(x-1)(x-5)(x+1)$
$= (x-1)(x+1)(x-5)$
$= (x^2 - 1)(x-5)$
$= x^3 - 5x^2 - x + 5$

Answers page 485

2.1 Expanding brackets

Fluency

WE1

1 Expand and simplify each of the following.

(a) $4(x+3)$ **(b)** $8(x+2)$ **(c)** $12(x+11)$
(d) $2(6x-1)$ **(e)** $7(2x-5)$ **(f)** $x(5x-4)$
(g) $-2(x+1)$ **(h)** $-x(x-7)$ **(i)** $-x(3x-6)$
(j) $-3x(x+6)$ **(k)** $-2x(x-9)$ **(l)** $-4x(x+2)$
(m) $x(x+2)+3(x+2)$ **(n)** $x(2x+1)+2(2x+1)$ **(o)** $x(5x+2)+3(5x+2)$
(p) $2x(x-3)-5(x+6)$ **(q)** $5x(2x+5)-4(3x-8)$ **(r)** $4x(3x-1)-6(2x+1)$

2 Expand and simplify each of the following. WE2

(a) $(x+1)(x+3)$ **(b)** $(x+4)(x+2)$ **(c)** $(x+5)(x+4)$

(d) $(x+8)(x-3)$ **(e)** $(x+7)(x-9)$ **(f)** $(x-3)(x+7)$

(g) $(x-1)(x-7)$ **(h)** $(x-4)(x-5)$ **(i)** $(x-9)(x-3)$

(j) $(2x+3)(x-2)$ **(k)** $(4x-5)(x+3)$ **(l)** $(7x+1)(x-4)$

(m) $(3x-1)(2x-5)$ **(n)** $(5x-3)(7-x)$ **(o)** $(6x-5)(2-3x)$

3 Expand and simplify each of the following, using the rules for special products. WE3

(a) $(2-x)(2+x)$ **(b)** $(7-x)(7+x)$ **(c)** $(6+x)(6-x)$

(d) $(2x+1)(2x-1)$ **(e)** $(3x+4)(3x-4)$ **(f)** $(5-3x)(5+3x)$

(g) $(x+6)^2$ **(h)** $(x+11)^2$ **(i)** $(8+x)^2$

(j) $(x-3)^2$ **(k)** $(x-7)^2$ **(l)** $(x-12)^2$

(m) $(2x+5)^2$ **(n)** $(3x+2)^2$ **(o)** $(7x-2)^2$

4 Expand and simplify each of the following. WE4

(a) $x(x+6)(x+4)$ **(b)** $x(x-1)(x+5)$ **(c)** $x(x+1)(x-4)$

(d) $-x(x-3)(x+2)$ **(e)** $-3x(x-2)(x-1)$ **(f)** $-4x(x-8)(x+3)$

(g) $(x+4)(x+2)(x+5)$ **(h)** $(x+2)(x-5)(x-1)$ **(i)** $(x-3)(x-6)(x-2)$

(j) $(x-3)(x+3)(x+5)$ **(k)** $(x+7)(x-2)(x+2)$ **(l)** $(2x+1)(x+4)(x-4)$

(m) $(x-3)(x+4)(x+4)$ **(n)** $(x+2)(x+2)(x-1)$ **(o)** $(x-6)(x+5)(x-6)$

Understanding

5 The expression $(x+1)(x+3)(x-2)$ is *not* the same as:

A $(x+3)(x+1)(x-2)$ **B** $(x-3)(x+1)(x+2)$

C $(x+1)(x-2)(x+3)$ **D** $(x-2)(x+1)(x+3)$

6 Expand and simplify the following.

(a) $(4x-3y)(2x+5y)$ **(b)** $(5x+2y)(7x+6y)$ **(c)** $(3x-4y)(5x-8y)$

(d) $(2x+7y)(2x+7y)$ **(e)** $(7x+6y)(7x+6y)$ **(f)** $(5x+8y)(5x+8y)$

(g) $(4x-y)(4x-y)$ **(h)** $(6x-5y)(6x-5y)$ **(i)** $(9x-7y)(9x-7y)$

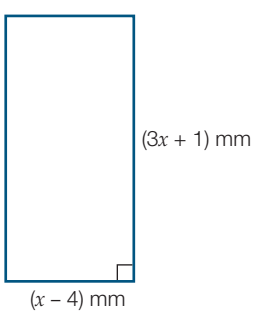
(j) $(5y-x)(5y+x)$ **(k)** $(3x-y)(3x+y)$ **(l)** $(2x+3y)(2x-3y)$

(m) $x(2x-1)(3x+5)$ **(n)** $x(3x+2)(5x-6)$ **(o)** $3x(4x+7)(2x+3)$

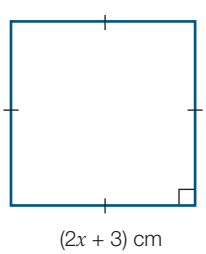
(p) $-x(3x-4)(5x+4)$ **(q)** $-2x(5x+3)(3x+4)$ **(r)** $-4x(2x-3)(3x-5)$

7 Find the area, in terms of x, of each of these figures.

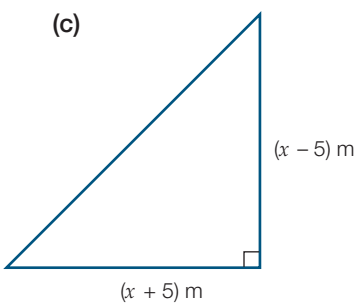
(a)



(b)



(c)



8 **(a)** Expand $x(x + a)(x + b)$.

(b) Use your answer to **(a)** to:

(i) write down the coefficient of x^2

(ii) write down the coefficient of x

(iii) show that $x(x + a)(x + b) = x^3 - 4x^2 - 12x$, when $a = -6$ and $b = 2$.

(c) For the expression $x(x + 25)(x - 4)$:

(i) write down the values of a and b

(ii) use the expansion found in **(a)** to expand $x(x + 25)(x - 4)$.

9 **(a)** If x is the second of three consecutive even numbers, write an expression for the product of the three numbers.

(b) Expand the expression in **(a)**.

(c) Why is it simpler to make x the second and not the first number?

10 **(a)** Draw a rectangle of length $(x + a)$ units and width $(x + b)$ units.

(b) Write an expression for the area of the rectangle in **(a)** in factorised form.

(c) Expand and simplify your expression in **(b)**.

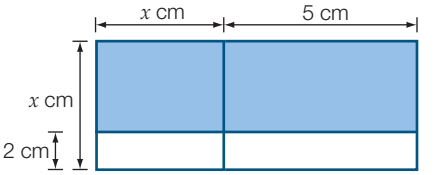
(d) Draw a square of side length x units. Extend its length by a units and the width by b units to form four small rectangles.

(e) Add the areas of the four smaller rectangles and compare this answer to part **(c)**.

11 **(a)** Write down the dimensions of the shaded rectangle.

(b) Write the binomial product used to find the area of the shaded region, then expand and simplify.

(c) Find the area of the square of side x cm and the rectangle of length 5 cm and width x cm.



(d) Find the unshaded area, writing it as the sum of the areas of the two rectangles.

(e) Use your answers in **(c)** and **(d)** to find the area of the shaded region in simplest form.

(f) Compare your answers in **(b)** and **(e)**.

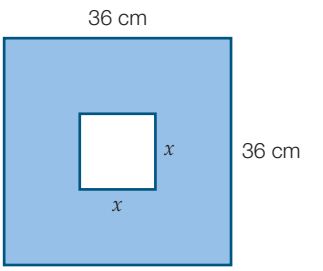
Reasoning

12 A square blue tile of side length 36 cm has a small white square of side length x cm painted on it.

(a) Write the expression (in cm^2) for the area of the tile that is not white.

(b) Write this expression as a product of factors.

(c) Use this product to find the area that remains blue if the white square has a side of 6 cm.



13 **(a)** Expand and simplify $(x - a)^2 - a^2$.

(b) Write down the expansion for $x(x - 2a)$.

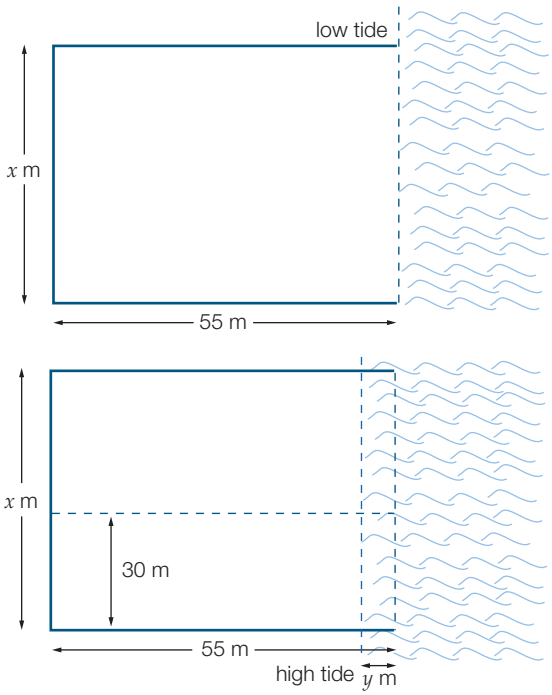
(c) What can you conclude about $(x - a)^2 - a^2$ and $x(x - 2a)$ from your answers in **(a)** and **(b)**?

(d) A rectangular lawn has a length of x m and a width of $(x - 2a)$ m. A square lawn of length $(x - a)$ m has a square-based fountain of side length a m in the centre. If the price of turfing the lawn is based only on its area, is one lawn more expensive to lay with turf than the other? Justify your answer.

(e) If the square lawn has side length 40 m, the fountain has a side length of 3 m and the price of turf is $\$12/m^2$, what is the total cost of laying the lawn?

14 The Holmes family own a property at the beach. Their land is x m wide and extends down to the water's edge. At low tide, the length of the land is 55 m, as shown in the diagram at right.

As the tide comes in, the length is reduced by y m.



(a) Write an expression for the length of the land at any time of day.

(b) Write an expression for the area of the land at any time of day.

(c) What is the area of the land at low tide?

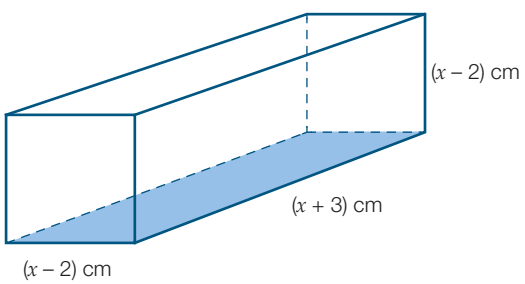
(d) The family have decided to subdivide the land by reducing their land width by 30 m. Write an expression for the area of the remaining land at any time of the day.

(e) Expand the expression.

(f) If the tide comes in 10 m at high tide, what is the area of land at high tide?

(g) What is the difference between the area of land at low tide before the subdivision and at high tide after the subdivision?

15 Consider this box.



(a) Write an expression (in cm^3) for the volume of the box in factorised form.

(b) Find the volume when x is 4 cm.

(c) Show that the volume of the box is 72 cm^3, when $x = 5$.

(d) Write the expression (in cm^3) for the volume in expanded form. Use this to calculate the volume when $x = 5$.

(e) State the dimensions (length, width and height) of the box when the volume is 72 cm^3.

(f) What is the smallest possible integer value of x so that the box exists?

(g) Find the volume of the box when x is the value found in part **(f)**.

16 Decide whether $(x^2 + a^2)^3$ can equal zero for some value(s) of x and a other than $x = 0$, $a = 0$.

Open-ended

17 Find two different sets of values for a, b, c and d so that the expanded, simplified form of $(ax + b)(cx + d)$ is a binomial expression.

18 **(a)** Expand and simplify $\frac{1}{2}(x + y)^2 + \frac{1}{2}(x - y)^2$.

(b) Find two different sets of values for x and y so that the expression obtained in part **(a)** is the square of a whole number.

19 Find two different whole numbers representing x so that the value of the expression $(4 - 2x)(x + 3)(5x - 10)$ is:

(a) positive **(b)** negative **(c)** zero.

Hint

A binomial expression contains two terms.

2.2 Factorising using common factors

Need to Know

Factorising is the opposite or reverse process to expanding.

You can factorise expressions by finding the **highest common factor (HCF)** of the terms and writing it outside a set of brackets that contains the remaining terms.

A common factor of -1 may also be taken out.

Expressions with four terms may **factorise by grouping in pairs**. Take out a common factor from each pair to give two terms. Then take out a common factor from each term.

Worked Example 5

WE5

Factorise:

(a) $15xy - 9y$ **(b)** $8x^2 + 12x$ **(c)** $-12a^2b - 9ab^2$

Working

(a) $15xy - 9y$
$= 3y(5x - 3)$

(b) $8x^2 + 12x$
$= 4x(2x + 3)$

(c) $-12a^2b - 9ab^2$
$= -3ab(4a + 3b)$

Worked Example 6

WE6

Factorise $k(m + 3) + 5(m + 3)$.

Working

$k(m + 3) + 5(m + 3)$
$= (m + 3)(k + 5)$

Worked Example 7

WE7

Factorise each of the following expressions by grouping in pairs.

(a) $2ax - 8ay - cx + 4cy$ **(b)** $uw + vx + xu + vw$

Working

(a) $2ax - 8ay - cx + 4cy$
$= 2a(x - 4y) - c(x - 4y)$
$= (x - 4y)(2a - c)$

(b) $uw + vx + xu + vw$
$= uw + xu + vw + vx$
$= u(w + x) + v(w + x)$
$= (w + x)(u + v)$

2.2 Factorising using common factors

Answers page 486

Fluency

1 Factorise each expression. WE5

(a) $4x + 8$ **(b)** $18x - 15$ **(c)** $24x - 9$
(d) $xy + 7x$ **(e)** $12kx + 21k$ **(f)** $32ab + 20b$
(g) $7x^2y + 7y^2$ **(h)** $9x^2 + 3xy$ **(i)** $48m^2 + 8m$
(j) $-6x - x^2$ **(k)** $-4x + x^2$ **(l)** $-24a^2 + 10a$
(m) $6ab - 18a^2b$ **(n)** $40x^2y + 16xy^2$ **(o)** $10cd^2 - 2cd$

2 Factorise each expression. WE6

(a) $x(k + 1) + 4(k + 1)$ **(b)** $a(x - 4) + 6(x - 4)$
(c) $m(n - 7) + 2(n - 7)$ **(d)** $x(p + 3) - 9(p + 3)$
(e) $n(q - 2) - 7(q - 2)$ **(f)** $x(u - 4) - 5(u - 4)$
(g) $d(c + 5) + (c + 5)$ **(h)** $a(d + 2) - (d + 2)$
(i) $k(m + 7) - (m + 7)$ **(j)** $x(x - 3) + 5(x - 3)$

3 Factorise each of the following expressions by grouping in pairs. WE7

(a) $xy + 3y + 5x + 15$ **(b)** $ab + 7b + 2a + 14$ **(c)** $mn + kn + 4m + 4k$
(d) $xy - y + 3x - 3$ **(e)** $3a - 3b + ac - bc$ **(f)** $km - 4m + 2k - 8$
(g) $15ab - 10b - 12a + 8$ **(h)** $6ab - 4b - 15a + 10$ **(i)** $8mn - 6n - 12m + 9$
(j) $vx - vy - ax + ay$ **(k)** $pq - rq - np + nr$ **(l)** $gh - fh - dg + df$
(m) $m^2n + np + m^2t + pt$ **(n)** $a^2b + ab^2 - ac - bc$ **(o)** $d^2k - dk - dr + r$
(p) $27xy - 10 + 18y - 15x$ **(q)** $36a + 5c - 4ac - 45$ **(r)** $25vw + 5v - 15w - 3$
(s) $32a^3c + 15b - 12abc - 40a^2$ **(t)** $21d^2 + 20fh - 12dh - 35df$ **(u)** $m^2nr - 20p + 4mpr - 5mn$

4 Answer TRUE or FALSE to each of these statements.

(a) $8a + 4ab = 4a(2 + b)$ **(b)** $2x - 6xy = 2x(1 - y)$
(c) $mn + 3n - 2m - 6 = (m - 3)(n + 2)$ **(d)** $x^2 - 4x + x - 4 = (x - 4)(x + 1)$
(e) $3x^2 + 15x - 2x - 10 = (x - 2)(3x + 5)$ **(f)** $4x^2 + 20x - 3x - 15 = (x - 5)(4x + 3)$

5 When factorised, $16f^2g - 12gf$ becomes:

A $2fg(8f - 6)$ **B** $4f^2g(4g - 3f)$ **C** $16fg(f - 12)$ **D** $4fg(4f - 3)$

6 The total area of the two shapes shown is:



A $(a+4)(b-c)$ **B** $(a+c)(b-4)$ **C** $(a+b)(c-4)$ **D** $(a-c)(b+4)$

7 $10dk + 25d - 2k - 5$ factorises to:

A $(2k+5)(5d-1)$ **B** $5d(2k+5)$ **C** $(2k-5)(5d+1)$ **D** $(10k+1)(d-5)$

Understanding

8 Mandy has $24y$ cents. She buys x kg of potatoes that cost 36 cents per kilogram. Express (in cents) in factorised form how much money Mandy has now.

9 Factorise the expression for the square of a given number subtracted from eight times that number.

10 Five small bulldog clips are added to a box containing p small bulldog clips. Five large bulldog clips are added to another box that contains p large bulldog clips. Each small clip costs a cents and each large clip costs b cents.

(a) Give the expression (in cents) for the combined cost of the clips.

(b) Factorise the expression in part **(a)**.

(c) If the cost of one large clip is twice the cost of one small clip write a factorised expression for the total cost of the clips in terms of a and p.

11 **(a)** Draw and label a rectangle that has a length of p m and a width of r m.

(b) The length is reduced by a m. Show this on your diagram and find the new area as the difference of two areas.

(c) The width of this shortened rectangle is now increased by b m. Show this on your diagram. Label the dimensions and find an expression for the added area.

(d) Show that the area of the newly formed rectangle is $(pr - ar + pb - ab)$ m^2.

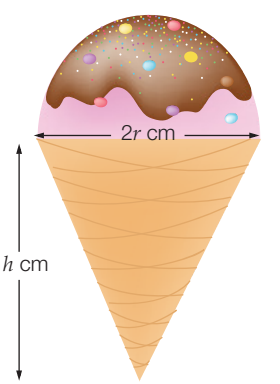
(e) Find the area as a product of the new length and width.

(f) Comment on your answers to parts **(d)** and **(e)**.

Reasoning

12 The volume of a sphere of radius r is $\frac{4\pi r^3}{3}$ and the volume of a cone of base radius r and height h is $\frac{\pi r^2 h}{3}$.

Renata is holding an ice-cream cone, with a scoop of ice-cream on top. The ice-cream can be considered to be hemispherical (half a sphere).



(a) Give an expression for the total volume of ice-cream (m^3), including the ice-cream in the cone and the scoop on top, if the ice-cream filled the cone.

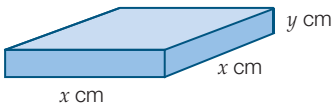
(b) Factorise the expression in part **(a)**.

(c) If $r = 3$ cm, $h = 10$ cm and $\pi = 3.142$, what is the total volume? (Give your answer correct to two decimal places.)

(d) Assume that there is no ice-cream inside the cone, just the scoop on top, and Renata lets it melt inside the cone. If the height of the cone is $2r$, use your expression in part **(b)** to show that the total volume is twice the volume of the cone and therefore all the melted ice-cream will fill the cone exactly.

Open-ended

13 A paver has the shape and dimensions shown.



(a) Write an expression (in cm^2) for the total surface area.

(b) Factorise the expression in part **(a)**.

(c) Find two sets of values for x and y so that the volume (in cm^3) has numerically the same value as the surface area. The values do not have to be integers.

Factorising monic quadratic expressions

2.3

Need to Know

A **quadratic expression** has the form of $ax^2 + bx + c$.

In a quadratic expression the highest power of the variable is 2. In a **monic quadratic** expression, the coefficient of the squared variable is 1. For example, $x^2 + 5x - 4$, $6 - 3a^2$, h^2 and $4n^2 + 7n$ are all quadratic expressions. Of these, only $x^2 + 5x - 4$ and h^2 are monic quadratic expressions.

- Quadratic expressions with four terms can sometimes be factorised by grouping in pairs. Look for a common factor in both pairs of terms and take out the factor from each pair.

A **quadratic trinomial** consists of three terms. For example, $x^2 + 5x - 4$ is a quadratic trinomial.

- Quadratic trinomials can sometimes be factorised by splitting the middle term so that the expression can be factorised by grouping in pairs.
- Monic trinomials can be factorised using the cross method. Follow these steps.

Step 1 Draw a pair of crossed arrows and write the variable at the beginning of each arrow.



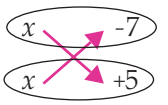
Step 2 Identify a factor pair for the constant term.

For the expression $x^2 - 2x - 35$, the constant term -35 has factor pairs of -35, 1; -7, 5; -5, 7 and -1, 35.

Step 3 Trial a factor pair by writing them at the end of each arrow.

For $x^2 - 2x - 35$, select a pair of factors -7 and 5. Follow the arrows to write the two terms as a sum or difference. If this does not give the middle term, try a new pair of factors.

Step 4 Circle the top and the bottom of the arrows as shown. Each circle contains a binomial factor.



$-7x + 5x = -2x$

Step 5 Write the factorisation.

$x^2 - 2x - 35 = (x - 7)(x + 5)$

Worked Example 8

WE8

Factorise $x^2 - 4x + 5x - 20$ by grouping in pairs.

Working

$x^2 - 4x + 5x - 20$
$= x(x - 4) + 5(x - 4)$
$= (x - 4)(x + 5)$

Worked Example 9

WE9

Factorise these quadratic trinomials by splitting the middle term.

(a) $p^2 - 11p + 30$ **(b)** $3q^2 + 6q - 24$

Working

(a) $p^2 - 11p + 30$ $\times \rightarrow +30$, $+ \rightarrow -11$ } -5, -6
$= p^2 - 5p - 6p + 30$
$= p(p - 5) - 6(p - 5)$
$= (p - 5)(p - 6)$

(b) $3q^2 + 6q - 24$
$= 3(q^2 + 2q - 8)$ $\times \rightarrow -8$, $+ \rightarrow +2$ } -2, +4
$= 3(q^2 + 4q - 2q - 8)$
$= 3[q(q + 4) - 2(q + 4)]$
$= 3(q + 4)(q - 2)$

Worked Example 10

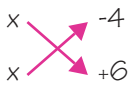
WE10

Factorise the following quadratic expressions using the cross method.

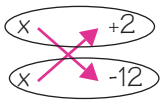
(a) $x^2 - 10x - 24$ **(b)** $x^2 - 9x + 14$

Working

(a) $x^2 - 10x - 24$

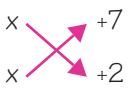


$6x - 4x = 2x$
(incorrect in magnitude and sign)

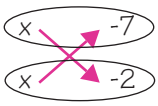


$-12x + 2x = -10x$
(correct in magnitude and sign)
$= (x + 2)(x - 12)$

(b) $x^2 - 9x + 14$



$2x + 7x = 9x$
(incorrect in sign)



$-2x - 7x = -9x$
(correct in magnitude and sign)
$= (x - 2)(x - 7)$

2.3 Factorising monic quadratic expressions

Answers page 487

Fluency

1 Factorise each expression by grouping in pairs.

WE8

(a) $x^2 + 2x + 4x + 8$ **(b)** $x^2 - 3x + 6x - 18$ **(c)** $x^2 - x + 7x - 7$

(d) $x^2 + 5x - 2x - 10$ **(e)** $x^2 - 4x - 3x + 12$ **(f)** $x^2 + 3x - 4x - 12$

(g) $x^2 - 2x + 4x - 8$ **(h)** $x^2 + x - 3x - 3$ **(i)** $x^2 - 2x - 8x + 16$

2 Factorise these quadratic trinomials by splitting the middle term.

WE9

(a) $x^2 + 3x + 2$ **(b)** $y^2 + 9y + 14$ **(c)** $a^2 - 10a + 24$

(d) $k^2 + 11k + 24$ **(e)** $m^2 - 2m - 15$ **(f)** $x^2 - x - 30$

(g) $p^2 - 10p + 16$ **(h)** $d^2 - 5d + 4$ **(i)** $y^2 - 5y + 6$

(j) $m^2 - 4m - 12$ **(k)** $c^2 - 10c + 21$ **(l)** $g^2 + 2g - 35$

(m) $2y^2 + 14y + 24$ **(n)** $3m^2 + 18m + 24$ **(o)** $5x^2 - 5x - 100$

(p) $-4a^2 - 8a + 12$ **(q)** $-3k^2 + 12k + 63$ **(r)** $-5x^2 - 20x + 105$

3 Factorise these quadratic trinomials using the cross method.

WE10

(a) $z^2 - 8z + 15$ **(b)** $k^2 + 5k - 6$ **(c)** $z^2 + 10z + 21$

(d) $t^2 - t - 20$ **(e)** $r^2 + r - 30$ **(f)** $t^2 - 13t + 22$

(g) $z^2 - z - 42$ **(h)** $m^2 - 9m + 20$ **(i)** $y^2 + 7y - 60$

(j) $n^2 - 3n - 28$ **(k)** $p^2 + 9p - 36$ **(l)** $a^2 - 14a + 48$

4 If $A = 4x^2 + 6x + 29$ and $B = 1 + 9x + 5x^2$, then $A - B$ when factorised becomes:

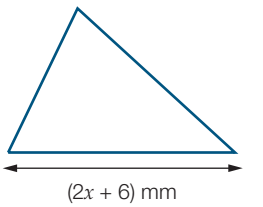
A $(x - 4)(x + 7)$ **B** $(x + 4)(7 - x)$ **C** $(x + 7)(4 - x)$ **D** $(x - 7)(4 - x)$

Understanding

5 A rectangle has a width of $(x + 3)$ cm. Find the length of the rectangle if its area is $(2x^2 + 10x + 12)$ cm^2.



6 A triangle has a base length of $(2x + 6)$ mm. Find the height of the triangle if its area is $(3x^2 - 3x - 36)$ mm^2.



Reasoning

7 A rectangular table has an area of $(x^2 + 30x - 1800)$ cm^2.

(a) If the length of the table (cm) is one of the factors of the expression, find an expression for the width.

(b) If the length of the table is 210 cm, find the width.

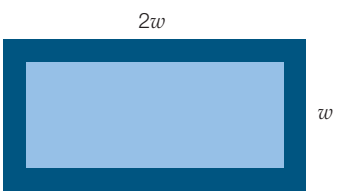
(c) If the width is 1 m, what is the length?

8 A rectangular block of land is twice as long as its width w m. A concrete path of uniform width is to be constructed around the inside border of the block and will occupy an area of $(6w - 4)$ m^2.

(a) What is the area left unconcreted?

(b) Factorise this area.

(c) If the factors represent the length and width of the unconcreted area, find the width of the path.



Hint

Rewrite your answer in **(b)** as the product of two algebraic factors by subtracting the same amount from the original length and width.

Open-ended

9 Write two monic quadratic trinomials that will factorise, and factorise them.

10 If $x^2 + 5x - 24$ represents a rectangular area:

(a) write down two possible sets of dimensions for the rectangle

(b) evaluate the area for each set of dimensions.

2.4 Factorising using special products

Need to Know

Special product rules:

$a^2 - b^2 = (a + b)(a - b)$ (Difference of Two Squares)

$a^2 + 2ab + b^2 = (a + b)^2$ (Perfect Square)

$a^2 - 2ab + b^2 = (a - b)^2$ (Perfect Square)

Worked Example 11

WE11

Factorise these expressions using the Difference of Two Squares rule.

(a) $d^2 - 81$ **(b)** $180 - 5y^2$ **(c)** $4k^2 - 9m^2$

Working

(a) $d^2 - 81$
$= (d + 9)(d - 9)$

(b) $180 - 5y^2$
$= 5(36 - y^2)$
$= 5(6 + y)(6 - y)$

(c) $4k^2 - 9m^2$
$= (2k)^2 - (3m)^2$
$= (2k - 3m)(2k + 3m)$

Worked Example 12

WE12

Factorise $(x + 3)^2 - 9$.

Working

$(x + 3)^2 - 9$
$= (x + 3)^2 - 3^2$
$= [(x + 3) - 3][(x + 3) + 3]$
$= x(x + 6)$

Worked Example 13

WE13

Factorise these expressions. (Your answers will include surds.)

(a) $x^2 - 3$ **(b)** $10 - 5x^2$

Working

(a) $x^2 - 3$
$= x^2 - (\sqrt{3})^2$
$= (x - \sqrt{3})(x + \sqrt{3})$

(b) $10 - 5x^2$
$= 5(2 - x^2)$
$= 5[(\sqrt{2})^2 - x^2]$
$= 5(\sqrt{2} - x)(\sqrt{2} + x)$

Worked Example 14

WE14

Factorise these expressions.

(a) $(x + 7)^2 - (x + 2)^2$ **(b)** $(x - 5)^2 - 7$

Working

(a) $(x + 7)^2 - (x + 2)^2$
$= [(x + 7) - (x + 2)][(x + 7) + (x + 2)]$
$= (x + 7 - x - 2)(x + 7 + x + 2)$
$= 5(2x + 9)$

(b) $(x - 5)^2 - 7$
$= (x - 5)^2 - (\sqrt{7})^2$
$= (x - 5 + \sqrt{7})(x - 5 - \sqrt{7})$

Worked Example 15 WE15

Factorise these expressions.

(a) $x^2 - y^2 + 5x - 5y$ **(b)** $25m^2 - n^2 - 5m - n$

Working

(a) $x^2 - y^2 + 5x - 5y$
$= (x^2 - y^2) + (5x - 5y)$
$= (x - y)(x + y) + 5(x - y)$
$= (x - y)(x + y + 5)$

(b) $25m^2 - n^2 - 5m - n$
$= (5m + n)(5m - n) - 1(5m + n)$
$= (5m + n)(5m - n - 1)$

Worked Example 16 WE16

Factorise these expressions using a Perfect Square rule.

(a) $x^2 + 10x + 25$ **(b)** $40x^2 - 120x + 90$

Working

(a) $x^2 + 10x + 25$
$= x^2 + 2(x)(5) + 5^2$
$= (x + 5)^2$

(b) $40x^2 - 120x + 90$
$= 10(4x^2 - 12x + 9)$
$= 10[(2x)^2 - 2(2x)(3) + 3^2]$
$= 10(2x - 3)^2$

Answers page 487

2.4 Factorising using special products

Fluency

WE11

1 Factorise these expressions using the Difference of Two Squares rule.

(a) $p^2 - 4$ **(b)** $d^2 - 100$ **(c)** $9 - k^2$
(d) $3p^2 - 75$ **(e)** $80 - 5m^2$ **(f)** $8k^2 - 32$
(g) $(2y)^2 - 49$ **(h)** $25 - (4x)^2$ **(i)** $(6m)^2 - (5n)^2$
(j) $9x^2 - 16$ **(k)** $4 - 25a^2$ **(l)** $81d^2 - 64$

WE12

2 Factorise these expressions.

(a) $(x + 2)^2 - 9$ **(b)** $(a - 1)^2 - 25$ **(c)** $16 - (k + 5)^2$
(d) $36 - (p + 1)^2$ **(e)** $1 - (2y - 3)^2$ **(f)** $1 - (3x - 2)^2$
(g) $5(x + 3)^2 - 5y^2$ **(h)** $7(a + 5)^2 - 7b^2$ **(i)** $3(x - 4)^2 - 12$

3 Factorise these expressions. (Your answers will include surds.) WE13

(a) $x^2 - 2$ **(b)** $7 - p^2$ **(c)** $4k^2 - 5$

(d) $9p^2 - 2$ **(e)** $3a^2 - 33$ **(f)** $2b^2 - 6$

4 Factorise these expressions. WE14

(a) $(x+2)^2 - (y+1)^2$ **(b)** $(a+2)^2 - (a+3)^2$ **(c)** $(2k+3)^2 - (m-3)^2$

(d) $(x+4)^2 - (x-4)^2$ **(e)** $(2x+1)^2 - (x+3)^2$ **(f)** $(5x-4)^2 - (x-7)^2$

(g) $(x+2)^2 - 3$ **(h)** $(y-3)^2 - 6$ **(i)** $(x+8)^2 - 5$

(j) $(x-5)^2 - 13$ **(k)** $(x-1)^2 - 19$ **(l)** $(2x+1)^2 - 2$

5 Factorise these expressions. WE15

(a) $x^2 - 9 + 4x + 12$ **(b)** $a^2 - 16 + 5a - 20$

(c) $x^2 - y^2 + 7x - 7y$ **(d)** $9a^2 - b^2 + 6a + 2b$

(e) $x^2 - 16y^2 + 2x - 8y$ **(f)** $4p^2 - q^2 - 6p - 3q$

6 Factorise each expression using a Perfect Square rule. WE16

(a) $x^2 + 6x + 9$ **(b)** $x^2 + 4x + 4$ **(c)** $x^2 - 14x + 49$

(d) $x^2 - 8x + 16$ **(e)** $x^2 + 2x + 1$ **(f)** $x^2 + 12x + 36$

(g) $4x^2 + 20x + 25$ **(h)** $9x^2 + 24x + 16$ **(i)** $25x^2 - 10x + 1$

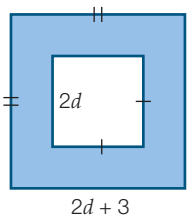
Understanding

7 When factorised, $(2a-1)^2 - (a+1)^2$ becomes:

A $3(a-1)(a+1)$ **B** $3a(a-2)$

C $(a-1)(a+1)$ **D** $(2a-1)(a+1)$

8 When factorised, the expression for the blue area shown becomes:



A $4d(d+3)$ **B** $(4d+3)(d+1)$

C $(2d+3)(d+1)$ **D** $3(4d+3)$

9 When factorised, $\left(\frac{a}{k}\right)^2 - \left(\frac{b}{2k}\right)^2$ is:

A $\frac{1}{4k^2}(2a-b)(2a+b)$ **B** $\frac{1}{2k^2}(a-b)(a+2b)$

C $\frac{1}{4k^2}(a-3b)(a+3b)$ **D** $\frac{1}{k^2}(a-b)(a+b)$

10 Factorise these expressions.

(a) $8m^2 - 200n^2$ **(b)** $6x^2 - 24r^2$ **(c)** $5a^2 - 45c^2$

(d) $27a^2 - 48b^2$ **(e)** $72x^2 - 98y^2$ **(f)** $175t^2 - 28p^2$

(g) $54k^2 - 24m^2$ **(h)** $2x^2y^2 - 32z^2$ **(i)** $3p^2q^2 - 12n^2$

11 Factorise these expressions.

(a) $16c^2 - 25d^2 + 10d + 8c$ **(b)** $k^2 - 64 + km - 8m$

(c) $81 - 49a^2 - 27d + 21ad$ **(d)** $16 - 9y^2 - 16w + 12wy$

(e) $25m^2 - 9n^2 - 20m - 12n$ **(f)** $144p^2 - 49r^2 - 12pq + 7rq$

Reasoning

12 The difference between the squares of two consecutive odd numbers is 56.

(a) If x and y are the two numbers and x is greater than y, write the equation describing the information.

(b) Factorise the left side of the equation in part **(a)**.

(c) Explain why $x = y + 2$.

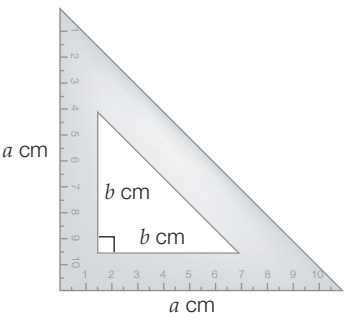
(d) Substitute $y + 2$ for x in the equation in part **(b)** and simplify.

(e) Solve the equation to obtain the value for y. Hence, state the value of x.

13 **(a)** Write the expression (in cm^2) for the shaded area of the set square shown in factorised form.

(b) What is the shaded area if $a = 3b$?

(c) What are the dimensions of the instrument if the shaded area is 100 cm^2 given $a = 3b$?



14 **(a)** Factorise $a^2 - b^2$.

(b) How many factors do you have?

(c) Find $\frac{a^2 - b^2}{a - b}$, $a \neq b$.

(d) Write $a^4 - b^4$ as the difference of two squares.

(e) Factorise $a^4 - b^4$ fully.

(f) How many factors did you have?

(g) Find $\frac{a^4 - b^4}{a - b}$, $a \neq b$.

(h) Write $a^8 - b^8$ as the difference of two squares.

(i) Factorise $a^8 - b^8$ fully using your answer from **(e)**.

(j) Find $\frac{a^8 - b^8}{a - b}$, $a \neq b$.

(k) Use the pattern to write an expression for $\frac{a^{16} - b^{16}}{a - b}$, $a \neq b$.

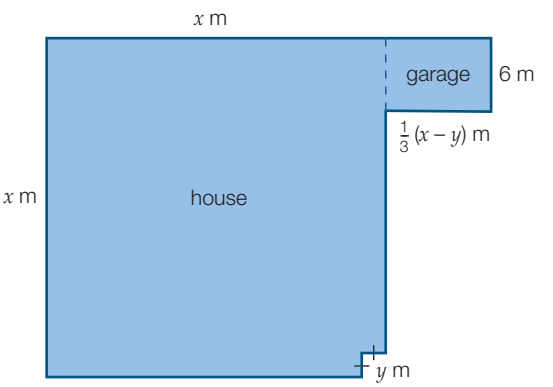
(l) Use the expression in **(k)** to show that $2^{16} - 1 = 3 \times 5 \times 17 \times 257$.

15 The shape of a house with its garage is shown.

(a) Write the expression (in m^2) for the area of the house.

(b) Write the expression (in m^2) for the area of the garage.

(c) Write the expression (in m^2) for the total area in factorised form.



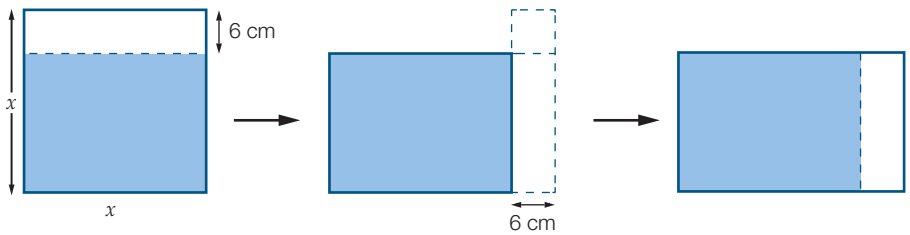
16 **(a)** If $a = m - n$, $b = 2\sqrt{mn}$ and $c = m + n$, show that $c^2 = a^2 + b^2$.

(b) Show that if $p = kr$ then the expression $t^2 = p^2 - r^2$ becomes $t^2 = r^2(k - 1)(k + 1)$.

(c) What values of k would make $t = 0$?

(d) What values of k are not possible? Justify your answer.

17 A square sheet of plywood is made into a rectangle by cutting 6 cm from one end and joining it to the other. The overhanging edge is then trimmed.



(a) If the length of the sides of the original sheet of plywood were x cm, find the area of the square sheet of plywood.

(b) What area has been trimmed off the rectangle?

(c) Use your answers in **(a)** and **(b)** to find the area of the rectangular sheet of plywood.

(d) Factorise your answer.

(e) Use the diagrams above to find the length and width of the rectangle obtained in terms of x.

(f) What is the relationship of the answer in **(d)** to the length and width of the rectangle?

Open-ended

18 Write two trinomial expressions that have a difference that is a perfect square.

19 Write an expression with four terms so that two terms are the difference of the squares of two variables and the other two terms are a multiple of the sum of these variables. Factorise your expression.

Hint

Can $k = 0$?

Can k be a fraction?

Problem solving

Playground swings

The length of the chains of a playground swing that takes t seconds for one swing is given by $l = \frac{5t^2}{M}$.

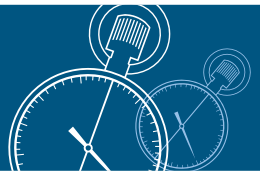
1 What is the difference (in m) in the chain lengths of two swings that take t_1 seconds and t_2 seconds, respectively, to make one swing? (Assume t_1 is greater than t_2.) Express your answer in factorised form.

2 If $t_1 - t_2 = \frac{3M}{10}$ and $t_1 + t_2 = \frac{M}{5}$ then find the difference in the chain lengths.

Strategy options

- Look for a pattern.
- Have I seen a similar problem?

Half-time 2



Ex 2.1

1 Expand and simplify.

(a) $(x + 2)(x + 6)$ **(b)** $(x - 5)(x + 7)$

(c) $(3x - 2)(x + 5)$ **(d)** $(2x - 1)(x + 3)$

Ex 2.3

2 Factorise each of the following.

(a) $x^2 + 2x - 15$ **(b)** $x^2 - 14x + 24$ **(c)** $x^2 - 3x - 40$

Ex 2.3

3 Factorise each of the following.

(a) $x^2 + 5x + 7x + 35$ **(b)** $x^2 - 3x - 4x + 12$ **(c)** $x^2 + 7x - 3x - 21$

Ex 2.4

4 Factorise each of the following.

(a) $x^2 + 14x + 49$ **(b)** $x^2 - 18x + 81$ **(c)** $9x^2 - 30x + 25$

Ex 2.2

5 Factorise each of the following by taking out a common factor.

(a) $7x + 14y$ **(b)** $18x^2 + 24xy$

(c) $3(x - 2) + y(x - 2)$ **(d)** $3(x + 5) + 2y(x + 5)$

Ex 2.1

6 Expand and simplify.

(a) $(x - 3)(x + 3)$ **(b)** $(x + 8)(x - 8)$

(c) $(3x + y)(3x - y)$ **(d)** $(6x - 5y)(6x + 5y)$

Ex 2.1

7 Expand and simplify.

(a) $x(x + 2)(x + 3)$ **(b)** $3(x - 2)(x + 2)$

(c) $(3x - 4)(7x + 19)(3x + 3)$ **(d)** $6x(2x - 6)(4x - 1)$

Ex 2.4

8 Factorise each of the following.

(a) $x^2 - 9$ **(b)** $9x^2 - 36y^2$

(c) $8x^2 - 50y^2$ **(d)** $27 - 48x^2$

Ex 2.3

9 The expression $acx^2 + adx + bcx + bd$ can be factorised to obtain:

A $(bx + a)(cx + d)$ **B** $(ax + c)(bx + d)$

C $(dx + b)(ax + c)$ **D** $(ax + b)(cx + d)$

Ex 2.3

10 The expression $a^2 - b^2 + 3a + 3b$ can be factorised to obtain:

A $(a - b)(a - b - 3)$ **B** $(a - b)(a + b - 3)$

C $(a + b)(a - b + 3)$ **D** $(a + b)(a + b - 3)$

Ex 2.3

11 The expression $a^2b^2 - c^2 + abd - cd$ can be factorised to obtain:

A $(ad + b)(cd - b)$ **B** $(ab - c)(ab + c + d)$

C $(ac - d)(a + b + c + d)$ **D** $(cd - b)(ab - b)$

Factorising by completing the square

2.5

Need to Know

Not all expressions with four terms are factorised by grouping in pairs. Often a group of three terms and one term are used if the three terms form a perfect square, and the fourth term is subtracted. We can factorise the first three terms using a Perfect Square rule, then use the Difference of Two Squares rule to complete the factorisation.

The **completing the square** method of factorising combines the use of both the Perfect Square and the Difference of Two Squares rules as shown below.

Consider $x^2 + 6x - 2$.

- Add a third term to the first two terms to make a perfect square. For $x^2 + 6x - 2$, we add 9 to obtain $(x^2 + 6x + 9) - 2$.
- Subtract the amount added to keep the expression equivalent. We then subtract 9 to obtain $(x^2 + 6x + 9) - 2 - 9$.

A perfect square has been obtained by adding 9.

Subtract 9 to keep the whole expression equivalent to $x^2 + 6x - 2$.

$x^2 + 6x - 2$ can be rewritten as $(x^2 + 6x + 9) - 9 - 2$

$$x^2 + 6x - 2 = (x + 3)^2 - 11$$

This is now a 'three and one' expression and we can use a Perfect Square rule and the Difference of Two Squares rule to factorise. Remember to use surds.

$$\begin{aligned} x^2 + 6x - 2 &= (x^2 + 6x + 9) - 11 \\ &= (x + 3)^2 - (\sqrt{11})^2 \\ &= (x + 3 - \sqrt{11})(x + 3 + \sqrt{11}) \end{aligned}$$

To complete the square of $x^2 + bx$, divide b, the coefficient of x by 2 to obtain $\frac{b}{2}$ then add $\left(\frac{b}{2}\right)^2$ to make a perfect square. Subtract $\left(\frac{b}{2}\right)^2$ to keep the expression equivalent.

Worked Example 17

WE17

Factorise $x^2 + 4x + 4 - y^2$ by grouping 'three and one'.

Working

$$\begin{aligned} & x^2 + 4x + 4 - y^2 \\ &= (x^2 + 4x + 4) - y^2 \\ &= (x + 2)^2 - y^2 \\ &= (x + 2 - y)(x + 2 + y) \end{aligned}$$

Worked Example 18

WE18

Factorise $x^2 - 8x + 9$ by completing the square.

Working

$$
\begin{aligned}
& x^2 - 8x + 9 \\
&= (x^2 - 8x + 16) - 16 + 9 \\
&= (x - 4)^2 - 7 \\
&= (x - 4)^2 - (\sqrt{7})^2 \\
&= (x - 4 - \sqrt{7})(x - 4 + \sqrt{7})
\end{aligned}
$$

Worked Example 19

WE19

Factorise $3x^2 + 12x - 30$ by completing the square.

Working

$$
\begin{aligned}
& 3x^2 + 12x - 30 \\
&= 3(x^2 + 4x - 10) \\
&= 3[(x^2 + 4x + 4) - 4 - 10] \\
&= 3[(x + 2)^2 - 14] \\
&= 3[(x + 2)^2 - (\sqrt{14})^2] \\
&= 3[(x + 2 - \sqrt{14})(x + 2 + \sqrt{14})] \\
&= 3(x + 2 - \sqrt{14})(x + 2 + \sqrt{14})
\end{aligned}
$$

Answers page 488

2.5 Factorising by completing the square

Fluency

WE17

1 Factorise these expressions by grouping 'three and one'.

(a) $x^2 + 8x + 16 - y^2$ **(b)** $y^2 - 4y + 4 - k^2$

(c) $y^2 + 8y + 16 - m^2$ **(d)** $m^2 - 10m + 25 - 4n^2$

(e) $x^2 + 2x + 1 - 9y^2$ **(f)** $q^2 + 6q + 9 - 16p^2$

(g) $a^2 - 6ab + 9b^2 - 16$ **(h)** $x^2 - 10xy + 25y^2 - 36$

(i) $4a^2 - 12a + 9 - 16b^2$ **(j)** $9c^2 + 30c + 25 - 4d^2$

WE18

2 Factorise these expressions by completing the square.

(a) $x^2 + 6x + 2$ **(b)** $x^2 - 10x + 4$ **(c)** $x^2 - 8x - 3$

(d) $x^2 + 20x + 77$ **(e)** $x^2 - 12x + 21$ **(f)** $x^2 + 2x - 8$

(g) $x^2 - 14x - 2$ **(h)** $x^2 - 4x - 2$ **(i)** $x^2 + 16x + 47$

(j) $x^2 + 3x + 1$ **(k)** $x^2 + 7x + 9$ **(l)** $x^2 - x - 4$

(m) $x^2 - 5x + 5$ **(n)** $x^2 + 5x - 2$ **(o)** $x^2 + x - 5$

(p) $x^2 - 9x + 13$ **(q)** $x^2 - 3x - 2$ **(r)** $x^2 - 7x - 1$

3 Factorise each expression by completing the square.

WE19

(a) $2x^2 + 4x - 10$ **(b)** $4x^2 - 16x - 4$ **(c)** $10x^2 - 80x - 30$

(d) $3x^2 - 6x - 12$ **(e)** $5x^2 + 20x - 5$ **(f)** $7x^2 + 42x + 14$

4 The expression $2m^2 - 12m + 4$ when factorised is:

A $2(m - 6 - \sqrt{7})(m - 6 + \sqrt{7})$ **B** $2(m + 7 - \sqrt{3})(m + 7 + \sqrt{3})$

C $2(m - 6 - \sqrt{2})(m - 6 + \sqrt{2})$ **D** $2(m - 3 - \sqrt{7})(m - 3 + \sqrt{7})$

Understanding

5 Factorising $x^2 + x - \frac{5}{16}$ gives:

A $(x - \frac{1}{4})(x + \frac{5}{4})$ **B** $(x - \frac{5}{16})(x + 1)$ **C** $(x - \frac{5}{4})(x + \frac{1}{16})$ **D** $(x + \frac{1}{4})(x - \frac{5}{4})$

6 A seagull is diving for a fish. Its height above sea level , in metres, at time t seconds is given by the expression $t^2 - 8t + 14$.

(a) How high is the seagull at $t = 0$?

(b) Factorise the expression by completing the square.

(c) How long is the time interval between $t = 4 - \sqrt{2}$ and $t = 4 + \sqrt{2}$, rounded to two decimal places.

(d) Explain what happens when $t = 4 \pm \sqrt{2}$.

(e) Explain what the seagull is doing between $t = 4 - \sqrt{2}$ and $t = 4 + \sqrt{2}$.

7 The profit made by a newspaper publisher in the first 8 months is given by $P = -5x^2 + 40x - 5$ where x is the time in months.

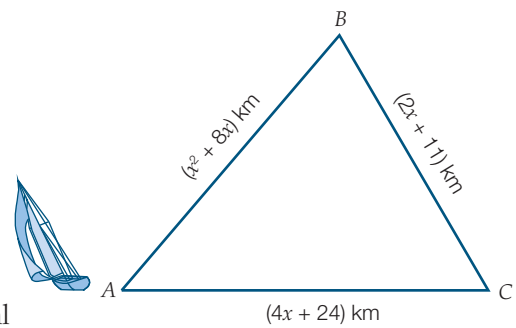
(a) Factorise the expression for P.

(b) Show that $P = 0$ when $x = 4 \pm \sqrt{15}$.

(c) Show what happened to the company in the first 8 months by substituting different values for x into the expression.

Reasoning

8 A yacht sails a leg of a race by going in a straight course from point A to point B for $(x^2 + 8x)$ km. From point B it travels to point C for $(2x + 11)$ km, then back to point A for $(4x + 24)$ km, as shown in the diagram.



(a) Write the simplified expression (in km) for the total distance the yacht sailed.

(b) Factorise the expression in **(a)**.

(c) If the factorised expression also represents the product of the length and width of a rectangle and if the length is 15 km, find the value of x. Give your answer as an exact distance.

(d) Find the length of the race, rounded to two decimal places, by substituting the exact value for x into the expression for the total distance found in **(a)**.

9 The number of earthworms in a compost heap after d days is given by $100 - 4d - d^2$.

(a) How many earthworms were there originally?

(b) Factorise the expression for the number of worms.

(c) Use your factorisation to show that when $d = 2(\sqrt{26} - 1)$, no worms remain.

(d) Evaluate $2(\sqrt{26} - 1)$ and round to two decimal places to find on which day the number of earthworms is reduced to zero.

(e) A second colony of earthworms is now in another compost heap. Their population is given by $64 - 2n - n^2$.

(i) Factorise the expression.

(ii) Use this factorisation to show that no earthworms remain when $n = \sqrt{65} - 1$.

(f) Use your answers to **(c)** and **(e) (ii)** to decide which colony stayed longer in their respective compost heaps.

Open-ended

10 **(a)** Write the expression $x^2 - 2ax + 2b$ in the form $(x - a)^2 + A$.

(b) Write an expression for A in terms of a and b.

(c) **(i)** What is the relationship between a and b if the expression is to be a perfect square?

(ii) Find a value for a and b that produces a perfect square.

2.6 Algebraic fractions

Need to Know

When multiplying (or dividing) **algebraic fractions**, follow these steps.

Step 1 Factorise if possible.

Step 2 Change a division into a multiplication by inverting the second fraction.

Step 3 Identify common factors in the numerators and denominators of the fractions and divide the numerators and denominators by these common factors.

Step 4 Simplify by multiplying the remaining numerators and the remaining denominators.

When adding (or subtracting) algebraic fractions, follow these steps.

Step 1 Find the lowest common denominator (LCD). If there is a factor that is common to both denominators, only use it *once* in the common denominator.

Step 2 Rewrite each fraction as an equivalent fraction with this common denominator.

Step 3 Add (or subtract) the new numerators.

Worked Example 20 WE20

Simplify $\frac{4a}{6}$.

Working

$$\frac{4a}{6}$$

$$= \frac{{}^{2}\cancel{4}a}{\cancel{6}_{3}}$$

$$= \frac{2a}{3}$$

Worked Example 21 WE21

Simplify $\frac{(x+2)(x+4)}{(x-1)(x+2)}$.

Working

$$\frac{(x+2)(x+4)}{(x-1)(x+2)}$$

$$= \frac{{}^{1}\cancel{(x+2)}(x+4)}{(x-1)\underset{1}{\cancel{(x+2)}}}$$

$$= \frac{x+4}{x-1}$$

Worked Example 22 WE22

Simplify:

(a) $\frac{(x+2)(x+5)}{(x-1)(x+1)} \times \frac{(x-2)(x+1)}{(x+5)(x+3)}$

(b) $\frac{(x+2)(x+5)}{(x-1)(x+1)} \div \frac{(x+5)(x+3)}{(x-2)(x+1)}$

Working

(a) $\frac{(x+2)(x+5)}{(x-1)(x+1)} \times \frac{(x-2)(x+1)}{(x+5)(x+3)}$

$$= \frac{(x+2)\overset{1}{\cancel{(x+5)}}}{(x-1)\underset{1}{\cancel{(x+1)}}} \times \frac{(x-2)\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{(x+5)}}(x+3)}$$

$$= \frac{(x+2)(x-2)}{(x-1)(x+3)}$$

(b) $\frac{(x+2)(x+5)}{(x-1)(x+1)} \div \frac{(x+5)(x+3)}{(x-2)(x+1)}$

$$= \frac{(x+2)(x+5)}{(x-1)(x+1)} \times \frac{(x-2)(x+1)}{(x+5)(x+3)}$$

$$= \frac{(x+2)\overset{1}{\cancel{(x+5)}}}{(x-1)\underset{1}{\cancel{(x+1)}}} \times \frac{(x-2)\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{(x+5)}}(x+3)}$$

$$= \frac{(x+2)(x-2)}{(x-1)(x+3)}$$

Worked Example 23

WE 23

Simplify $\frac{4x^2 - 12x}{x^2 - 25} \times \frac{x^2 + 7x + 10}{2x^2 + 4x}$.

Working

$$\frac{4x^2 - 12x}{x^2 - 25} \times \frac{x^2 + 7x + 10}{2x^2 + 4x}$$

$$= \frac{4x(x-3)}{(x-5)(x+5)} \times \frac{(x+2)(x+5)}{2x(x+2)}$$

$$= \frac{\overset{2}{\cancel{4}}\overset{1}{\cancel{x}}(x-3)}{(x-5)\underset{1}{\cancel{(x+5)}}} \times \frac{\overset{1}{\cancel{(x+2)}}\overset{1}{\cancel{(x+5)}}}{\underset{1}{\overset{1}{\cancel{2}}}\overset{1}{\cancel{x}}\underset{1}{\cancel{(x+2)}}}$$

$$= \frac{2(x-3)}{x-5}$$

Worked Example 24

WE 24

Simplify $\frac{x^2 + x - 12}{3x^2 + 12x} \div \frac{x^2 - 9}{12x}$.

Working

$$\frac{x^2 + x - 12}{3x^2 + 12x} \div \frac{x^2 - 9}{12x}$$

$$= \frac{x^2 + x - 12}{3x^2 + 12x} \times \frac{12x}{x^2 - 9}$$

$$= \frac{(x-3)(x+4)}{3x(x+4)} \times \frac{12x}{(x-3)(x+3)}$$

$$= \frac{\overset{1}{\cancel{(x-3)}}\overset{1}{\cancel{(x+4)}}}{\underset{1}{\overset{1}{\cancel{3}}}\overset{1}{\cancel{x}}\underset{1}{\cancel{(x+4)}}} \times \frac{\overset{4}{\cancel{12}}\overset{1}{\cancel{x}}}{\underset{1}{\cancel{(x-3)}}(x+3)}$$

$$= \frac{4}{x+3}$$

Worked Example 25

WE25

Simplify:

(a) $\frac{x}{4}+\frac{x}{5}$

(b) $\frac{x+7}{4}-\frac{x-3}{6}$

Working

(a) $\frac{x}{4}+\frac{x}{5}$

$=\frac{5x}{20}+\frac{4x}{20}$

$=\frac{5x+4x}{20}$

$=\frac{9x}{20}$

(b) $\frac{x+7}{4}-\frac{x-3}{6}$

$=\frac{3(x+7)}{12}-\frac{2(x-3)}{12}$

$=\frac{3(x+7)-2(x-3)}{12}$

$=\frac{3x+21-2x+6}{12}$

$=\frac{x+27}{12}$

2.6 Algebraic fractions

Answers page 489

Fluency

1 Simplify the following.

WE20

(a) $\frac{8x}{10}$

(b) $\frac{6k}{42}$

(c) $\frac{25p}{45}$

(d) $\frac{4(a-b)}{2}$

(e) $\frac{9(x+1)}{6}$

(f) $\frac{14(y-2)}{8}$

(g) $\frac{5y(a-4)}{10}$

(h) $\frac{8x(x+5)}{36}$

(i) $\frac{18x(x-4)}{24}$

2 Simplify the following.

WE21

(a) $\frac{(x+1)(x+6)}{(x-5)(x+1)}$

(b) $\frac{(x-3)(x+2)}{(x-3)(x-2)}$

(c) $\frac{(2x-3)(x+4)}{(x+4)(2x+1)}$

(d) $\frac{(3x+1)(x-2)}{(x-7)(3x+1)}$

(e) $\frac{(a+2)(b+5)}{(b+3)(a+2)}$

(f) $\frac{(x+7)(y-2)}{(y-2)(x-1)}$

WE22

3 Simplify the following.

(a) $\frac{3}{x-2} \times \frac{x-2}{9}$

(b) $\frac{15}{5-2x} \times \frac{5+2x}{5}$

(c) $\frac{x-1}{x-4} \div \frac{x-1}{x+4}$

(d) $\frac{x+4}{x+3} \div \frac{x+4}{x-3}$

(e) $\frac{x(x+7)}{2(x-3)} \div \frac{4x}{x-3}$

(f) $\frac{7x(x-4)}{10x} \div \frac{2(x-4)}{5}$

(g) $\frac{(x+1)(x-3)}{(x-4)(x+7)} \times \frac{(x+7)(x+2)}{(x-1)(x+1)}$

(h) $\frac{(x-4)(x+4)}{(x+3)(x-4)} \times \frac{(x-2)(x+3)}{(x+1)(x-2)}$

(i) $\frac{(3x-2)(x+5)}{(2x+3)(x-6)} \div \frac{(x+2)(x+5)}{(2x+3)(x-3)}$

(j) $\frac{(5x-3)(2x+3)}{(x-4)(x+7)} \div \frac{(2x+3)(5x-4)}{(x-3)(x+7)}$

WE23

4 Simplify the following. Remember to factorise first.

(a) $\frac{x^2+11x+24}{x+3}$

(b) $\frac{x^2-12x+27}{x-3}$

(c) $\frac{x+2}{x^2-3x-10}$

(d) $\frac{x+5}{x^2-2x-35}$

(e) $\frac{8x^2-2}{8x+4}$

(f) $\frac{3x^2-12}{x+2}$

(g) $\frac{4x^2+12x+9}{4x+6}$

(h) $\frac{9x^2+30x+25}{6x+10}$

(i) $\frac{x^2+2x-3}{x^2-5x-24}$

(j) $\frac{x^2+3x-4}{x^2+4x-5}$

(k) $\frac{x^2-7x-8}{x^2-10x+16}$

(l) $\frac{x^2-7x+6}{x^2-2x-24}$

(m) $\frac{3x+6}{4x-24} \times \frac{8x-48}{6x+12}$

(n) $\frac{5x+10}{2x-8} \times \frac{3x-12}{9x+18}$

(o) $\frac{x^2-5x}{3x+3} \times \frac{x+1}{2x^2+6x}$

(p) $\frac{4x^2+8x}{x+5} \times \frac{2x-14}{x^2+2x}$

(q) $\frac{x^2-x-12}{x^2+9x+18} \times \frac{x^2+4x-12}{x^2-2x-8}$

(r) $\frac{x^2+7x+10}{x^2+x-20} \times \frac{x^2-7x+12}{x^2-5x-14}$

WE24

5 Simplify the following.

(a) $\frac{x^2-4}{x^2-9} \div \frac{4x+8}{12x-36}$

(b) $\frac{x^2-25}{x^2-49} \div \frac{3x-15}{5x+35}$

(c) $\frac{x^2-36}{x^2+6x-7} \div \frac{x^2-5x-6}{x^2-1}$

(d) $\frac{x^2-6x-16}{x^2-x-30} \div \frac{x^2-64}{x^2-5x-6}$

(e) $\frac{x^2+6x+9-y^2}{4x+6} \div \frac{2x^2+6x-2xy}{10x^2+15x}$

(f) $\frac{x^2-2x+1-4y^2}{12x^2-5x-2} \div \frac{3x-3+6y}{28x+7}$

WE25

6 Simplify the following.

(a) $\frac{x}{2}+\frac{x}{3}$

(b) $\frac{x}{5}+\frac{x}{2}$

(c) $\frac{x}{6}+\frac{x}{4}$

(d) $\frac{2x}{3} - \frac{x}{6}$ (e) $\frac{4x}{5} - \frac{x}{15}$ (f) $\frac{2x}{3} - \frac{5x}{4}$

(g) $\frac{4x}{3} - \frac{2y}{4}$ (h) $\frac{5x}{7} - \frac{3y}{4}$ (i) $\frac{11x}{8} - \frac{8y}{9}$

(j) $\frac{x+6}{2} + \frac{x+2}{3}$ (k) $\frac{x-3}{4} + \frac{x+4}{8}$ (l) $\frac{x-7}{3} + \frac{x+5}{9}$

(m) $\frac{3x+2}{6} - \frac{x+4}{4}$ (n) $\frac{3x-8}{4} - \frac{x-3}{10}$ (o) $\frac{4x-1}{9} - \frac{x-6}{12}$

(p) $\frac{3x+2y}{4} + \frac{5x-y}{3}$ (q) $\frac{7x-2y}{3} + \frac{x-3y}{5}$ (r) $\frac{8x-3y}{4} + \frac{5x+7y}{6}$

(s) $\frac{4x-3y}{5} - \frac{3x-2y}{6}$ (t) $\frac{5x-2y}{4} - \frac{x+y}{2}$ (u) $\frac{7x+3y}{2} - \frac{2x-y}{3}$

7 If $4x + 3y = 5$, then $\frac{x+1}{3} + \frac{y+1}{4}$ simplifies to:

A $\frac{5}{12}$ **B** $\frac{5}{7}$ **C** 1 **D** $\frac{7}{12}$

Understanding

8 **(a)** Factorise and simplify as much as possible the expression

$$\frac{9b^2 - 9a^2}{6b^2 + 3bc + 2ab + ac} \div \frac{3a + 3b}{2b + c}.$$

(b) If $a = 5$ and $b = 7$, show that the expression in part **(a)** takes the value $\frac{3}{13}$.

9 **(a)** Show that $(a - b)(a^2 + ab + b^2) = a^3 - b^3$.

(b) Write $a^6 - b^6$ as the difference of two cubes.

(c) Use the result from **(b)** to show that $\frac{a^6 - b^6}{a^2 - b^2} = a^4 + a^2b^2 + b^4$.

10 A circular disc of diameter $2w$ m is cut from a square piece of timber. The side lengths of the timber are the same as the diameter of the disc.

(a) Write an expression for the area of timber wasted.

(b) What fraction of the area of the uncut timber is the area of the timber wasted? Give your answer in simplified form.

Reasoning

11 The density of an object of mass m kg and volume v m^3 is given by $d = \frac{m}{v}$ kg/m^3

(a) What is the density of a sphere of mass m kg and radius r m if its volume is $\frac{4\pi r^3}{3}$?

(b) A golf ball has density d_1, radius r_1 and volume v_1. A basketball has density d_2, radius r_2 and volume v_2. Show that $\frac{d_2}{d_1} = \left(\frac{m_2}{m_1}\right)\left(\frac{r_1}{r_2}\right)^3$.

(c) If the basketball's mass is 20 times greater than the mass of the golf ball and its radius is 5 times greater than the radius of the golf ball, what is the value of $\frac{d_2}{d_1}$?

Open-ended

12 Write the sum of two terms with different denominators that can be added to give $\frac{x+2}{3}$.

Problem solving

How to square two-digit numbers quickly—without a calculator

We start with the Difference of Two Squares rule:

$$a^2 - b^2 = (a - b)(a + b)$$

Adding b^2 to both sides we get:

$$a^2 - b^2 + b^2 = b^2 + (a - b)(a + b)$$

So $$a^2 = b^2 + (a - b)(a + b)$$

This expression can be used to find the square of a two-digit number quickly, without a calculator! In the expression:

- a is the number to be squared
- b can be any number (we choose a convenient value).

For example, to find 19^2, we have $a = 19$ and choose $b = 1$ (you'll see why in a moment).

Start with $a^2 = b^2 + (a - b)(a + b)$

Substitute $a = 19$ and $b = 1$.

$$\begin{aligned} 19^2 &= 1^2 + (19 - 1)(19 + 1) \\ &= 1^2 + (18)(20) \\ &= 1^2 + 360 \\ &= 361 \end{aligned}$$

Notice how the choice of 1 for b results in one of the brackets containing a multiple of 10, thus making multiplication by the contents of the other pair of brackets easier.

Use the method described above to calculate the following. Show the steps involved.

(a) 29^2

(b) 59^2

(c) 79^2

(d) 99^2

Strategy options

- Look for a pattern.
- Break problem into manageable parts.

2.7 Rearranging formulas

Worked Example 26 WE26

(a) Rearrange $V = \frac{1}{3}\pi r^2 h$ to make r the subject of the formula.

(b) Find r when $V = 27\pi$ and $h = 4$ are subtituted into the rearranged formula.

Working

(a) $V = \frac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$

$\frac{3V}{\pi} = r^2 h$

$r^2 = \frac{3V}{\pi h}$

$r = \pm\sqrt{\frac{3V}{\pi h}}$

(b) $r = \pm\sqrt{\frac{3V}{\pi h}}$

$= \pm\sqrt{\frac{3 \times 27\pi}{\pi \times 4}}$

$= \pm\sqrt{\frac{81}{4}}$

$= \pm\frac{9}{2}$

$= \pm 4\frac{1}{2}$

Worked Example 27

WE27

(a) Rearrange $T = 2\pi\sqrt{\frac{l}{g}}$ to make **(i)** l and **(ii)** g the subject of the formula.

(b) **(i)** Find l when $T = 0.6\pi$ and $g = 10$ are substituted into the rearranged formula.

(ii) Find g when $T = \frac{4\pi}{7}$ and $l = 0.8$ are substituted into the rearranged formula.

Working

(a) $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$

$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$

$l = g \times \left(\frac{T}{2\pi}\right)^2$

(i) $l = \frac{gT^2}{4\pi^2}$

(ii) $g = \frac{4\pi^2 l}{T^2}$

(b) (i) $l = \frac{gT^2}{4\pi^2}$

$= \frac{10 \times (0.6\pi)^2}{4\pi^2}$

$= \frac{10 \times 0.36\pi^2}{4\pi^2}$ (0.09)

$= 0.9$

(ii) $g = \frac{4\pi^2 l}{T^2}$

$= \frac{4 \times \pi^2 \times 0.8}{\left(\frac{4\pi}{7}\right)^2}$

$= \frac{4\pi^2 \times 49 \times 0.8}{16\pi^2}$ (0.2)

$= 9.8$

Answers page 490

2.7 Rearranging formulas

Fluency

WE26

1 **(i)** Rearrange the following, to make the variable in each bracket the subject of the formula.

(ii) Find the value of this variable when the given values are substituted into the rearranged formula.

(a)	$A = s^2$	$[s]$	$A = 27$
(b)	$K = \frac{1}{2}mv^2$	$[v]$	$K = 20$ and $m = 8$
(c)	$F = \frac{mv^2}{r}$	$[v]$	$F = 9$, $r = 7$ and $m = 6$
(d)	$F = \frac{Gm_1m_2}{r^2}$	$[r]$	$F = 16$, $G = 18$, $m_1 = 6$ and $m_2 = 5$
(e)	$g_1 = \frac{r_2^2}{r_1^2}g_2$	$[r_2]$	$g_1 = 9.8$, $g_2 = 10$ and $r_1 = 20$
(f)	$P = \frac{V^2}{R}$	$[V]$	$P = 9$ and $R = 7$
(g)	$P = I^2R$	$[I]$	$P = 10$ and $R = 4.5$
(h)	$E = \frac{1}{2}cv^2$	$[v]$	$E = 96$ and $c = 6$
(i)	$A = \frac{\pi D^2}{4}$	$[D]$	$A = 9\pi$

WE27

2 **(i)** Rearrange the following, to make the variable in each bracket the subject of the formula.

(ii) Find the value of this variable when the given values are substituted into the rearranged formula.

(a)	$A = \sqrt{A_x + A_y}$	$[A_y]$	$A = 9$ and $A_x = 43$
(b)	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$[c]$	$b = 9$, $a = 4$ and $x = -1$
(c)	$l = \frac{3r}{\sqrt{T}}$	$[T]$	$r = 8$ and $l = 3$
(d)	$x = \sqrt{y - z}$	$[z]$	$y = 4$ and $x = 10$
(e)	$x = \sqrt{\frac{y}{z}}$	$[z]$	$y = 7$ and $x = 5$
(f)	$x = \sqrt{\frac{a + b}{c}}$	$[b]$	$c = 3$, $x = 2$ and $a = -4$
(g)	$x = \sqrt{y - z}$	$[y]$	$x = 4$ and $z = 6$
(h)	$x = \sqrt{\frac{a + b}{c}}$	$[c]$	$a = 4$, $b = -1$ and $x = -3$
(i)	$x = \sqrt{\frac{y}{z}}$	$[y]$	$x = 4$ and $z = 6$

3 Which of the following expressions is equivalent to $S = \frac{4r^2}{\sqrt{t}}$?

A $t = \frac{16r^2}{S^2}$ **B** $r = \pm\sqrt{\frac{St}{4}}$ **C** $t = \frac{16r^4}{S^2}$ **D** $r = \frac{S\sqrt{t}}{4}$

Understanding

4 The formula for the area of a circle is $A = \pi r^2$.

(a) Rearrange this equation to make r the subject.

(b) Explain why there is no ± sign required.

(c) Find the radius, r, when the area, A, is 40 cm^2.

5 The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

(a) Rearrange this equation to make r the subject.

(b) Explain why there is no ± sign required.

(c) Find the radius, r, when the volume, V, is 125 cm^3.

6 The formula for the surface area of a cone is $V = \frac{1}{3}\pi r^2 h$. By first rearranging this equation to change the subject, find:

(a) the height of a cone with volume 312 m^3 and radius 14 m

(b) the radius of a cone with volume 160 cm^3 and height 8 cm.

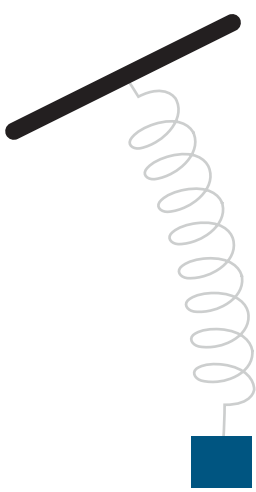
Reasoning

7 The formula $v^2 = u^2 + 2as$ is regularly used in physics to find the final velocity, v, of an object that has initial velocity, u, acceleration, a, and displacement, s. (Displacement is the distance of the object from its starting point.)

(a) Calculate the initial velocity of a car that stops in 42 m, with acceleration -20 m/s^2.

(b) What does a negative value for the acceleration mean in this situation?

8 The formula used to determine the frequency, f, of a spring, given the spring constant, k, and mass of the object, m, is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.



Frequency is measured in hertz (Hz) and is the number of cycles (up and down) per unit of time (oscillations per second). A spring with an object of mass 3.5 kg has a frequency of 3 Hz.

(a) Calculate the spring constant for this spring.

(b) An object of mass 7.2 kg is attached to the same spring. Calculate the new frequency.

Open-ended

9 Elizabeth was changing the subject of the formula $g = 4h^2 - 9k$ to h and has completed the following working.

$$g = 4h^2 - 9k$$

$$g + 9k = 4h^2 \quad \text{Line 1}$$

$$h^2 = \frac{g + 9k}{4} \quad \text{Line 2}$$

$$h = \sqrt{\frac{g + 9k}{4}} \quad \text{Line 3}$$

$$h = \frac{\sqrt{g + 9k}}{2} \quad \text{Line 4}$$

(a) Identify the line with the mistake.

(b) Explain why this is mathematically inaccurate.

10 **(a)** Make up a formula of your own with at least three variables. One of the variables, that is not the subject of this formula, must be squared. Rearrange the formula to make this variable the subject.

(b) Make up a formula of your own with at least three variables. Take the square root of one of the variables that is not the subject of this formula. Rearrange the formula to make this variable the subject.

Problem solving

Gravity

The formula $v = \sqrt{2gx}$ is used to calculate the velocity, v, of an object that has already fallen x metres, where g is acceleration due to gravity. Gravity is different depending on the planet that you are on, as shown in the table.

Planet	Acceleration due to gravity (m/s^2)
Mercury	3.59
Venus	8.87
Earth	9.81
Mars	3.77
Jupiter	25.95
Saturn	11.08

1 On which planet would it take the longest amount of time for an object to fall 100 m?

2 According to this formula, does it take longer for a feather or a heavy boulder to fall?

3 How much higher do I have to drop a ball when on Mars, so that the velocity at which it hits the ground is the same as if it was dropped on Earth?

Strategy options

- Make a table.
- Work backwards.

Hint

The slowest velocity gives the longest time.

Chapter review 2

D.I.Y. Summary

Key Words

algebraic fractions	factorise by grouping in pairs	perfect square
binomial product	factorising	quadratic expression
completing the square	highest common factor (HCF)	quadratic trinomial
difference of two squares	monic quadratic	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 $ax^2 + bx + c$ is called a _____. When the coefficient of x^2 is 1 the expression is a _____.

2 The first thing to do to factorise $50x^2 - 2y^2$ is to take out the _____. We can then use the _____ method to factorise the expression.

3 To factorise $x^2 - 4x - 6$ as $(x - 2 - \sqrt{10})(x - 2 + \sqrt{10})$ we use a method called _____.

4 We _____ to write the expression $2ax + 2ay + 3x + 3y$ as a product of factors $(2a + 3)(x + y)$.

5 $(3t - 2)^2$ is an example of a _____.

6 $(3a + b)(4 - 2a)$ is called a _____.

7 $x^2 - 9$ is an example of the _____.

Fluency

1 Expand and simplify the following. Ex 2.1

(a) $5(2x + 7)$ **(b)** $x(x - 3) + 2(x - 3)$

(c) $(x + 6)(x + 4)$ **(d)** $(4x - 1)(2x + 3)$

2 Expand and simplify the following. Ex 2.1

(a) $(x + 6)^2$ **(b)** $(2x - 3)^2$

(c) $(5 - x)(5 + x)$ **(d)** $(3x + 7)(3x - 7)$

3 Expand and simplify the following. Ex 2.1

(a) $4(x - 7)(x + 2)$ **(b)** $-3x(x + 8)(x - 3)$

(c) $(x - 1)(x + 5)(x - 6)$ **(d)** $(2x + 3)(2 - x)(x + 4)$

4 Factorise these expressions. Ex 2.2

(a) $7x + 21$ **(b)** $12ab - 8a$ **(c)** $18x^2 + 24x$

5 Factorise these expressions. Ex 2.2

(a) $d(c - 4) + 6(c - 4)$ **(b)** $x(x + 2) - 3(x + 2)$

(c) $5a(a + 1) - 2(a + 1)$ **(d)** $2y(3y - 5) - 7(3y - 5)$

6 Factorise these expressions. Ex 2.3

(a) $x^2 + 11x + 30$ **(b)** $x^2 - 10x - 24$

7 Factorise each of these quadratic trinomials. Ex 2.4

(a) $x^2 + 10x + 25$ **(b)** $x^2 - 18x + 81$

(c) $9x^2 - 24x + 16$ **(d)** $4x^2 + 4x + 1$

8 Factorise these expressions. Ex 2.4

(a) $x^2 - 9$ **(b)** $25 - y^2$ **(c)** $4x^2 - y^2$

(d) $49x^2 - 16$ **(e)** $36x^2 - 81y^2$ **(f)** $64 - 9x^2$

(g) $(2x - 5)^2 - 4$ **(h)** $(x - 5)^2 - (x + 8)^2$ **(i)** $(x + 2)^2 - (x - 3)^2$

(j) $x^2 - 3$ **(k)** $(x + 7)^2 - 5$ **(l)** $(x - 4)^2 - 7$

9 Factorise these expressions. Ex 2.5

(a) $x^2 + 6x - 2$ **(b)** $x^2 - 10x + 23$

(c) $x^2 - 4x - 7$ **(d)** $x^2 + 8x + 13$

10 Simplify the following. Ex 2.6

(a) $\frac{18x}{45}$ **(b)** $\frac{2(a-3)}{6}$ **(c)** $\frac{10x(x-1)}{8x}$ **(d)** $\frac{12ab}{4b}$

11 Simplify the following. Ex 2.6

(a) $\frac{(x+4)(x-3)}{(x-3)(x+2)}$ **(b)** $\frac{(2x-1)(x+1)}{(x+7)(x-7)} \times \frac{(3x+7)(x+7)}{(x-1)(2x-1)}$

(c) $\frac{4x^2 - 4x + 1}{2x - 1}$ **(d)** $\frac{2x^2 - 8x}{x^2 + 3x} \div \frac{x^2 - 16}{x^2 + 8x + 15}$

12 Simplify the following. Ex 2.6

(a) $\frac{2x}{3} + \frac{x}{2}$ **(b)** $\frac{x+1}{5} + \frac{x-2}{3}$ **(c)** $\frac{3x-2}{10} - \frac{x+5}{6}$

(d) $\frac{6x}{7} + \frac{3y}{2}$ **(e)** $\frac{x+5}{4} + \frac{y+2}{3}$ **(f)** $\frac{x-6}{5} - \frac{y-3}{10}$

13 Rearrange each of the following to make the pronumeral in brackets the subject. Ex 2.7

(a) $A = lb$ $[b]$ **(b)** $A = 2\pi\sqrt{\frac{l}{g}}$ $[l]$

(c) $SA = 4\pi r^2$ $[r]$ **(d)** $a = x^2y - z$ $[x]$

Understanding

14 If $9x^2 - 12x + 4$ is written as $(ax + b)^2$, state the values of a and b.

15 In January, John's weight was x kg. In April his weight was $(x + a)$ kg and in September it was $(x + b)$ kg.

(a) If the product of John's weight in April and September is $x^2 + x - 30$, determine the values of a and b.

(b) By how many kilograms had John's weight increased by April?

(c) What was John's overall weight gain/loss by the end of September?

(d) What happened to his weight from April to September?

16 The heights in centimetres of four people are ax, $2x$, ay and $2y$.

(a) Write down the expression for their total height.

(b) Factorise the expression in part **(a)**.

(c) If $x = 65$, $y = 75$ and $a = 3$, use your factorised expression to find the average height of the four people.

17 **(a)** Which of the following methods of factorising is the most appropriate for each of the expressions **(i)** to **(vi)** below?

A factorising using common factors

B factorising using the Difference of Two Squares rule

C factorising by grouping in pairs

D factorise by grouping (3 and 1)

E factorising quadratic trinomials (multiply to give … , add to give …)

F factorising by completing the square

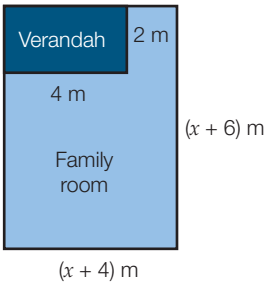
(i) $x^2 + 5x - 24$ **(ii)** $25a^2 - b^2$

(iii) $x^2 + 6x + 9 - y^2$ **(iv)** $x^2 + 12x + 34$

(v) $16x^2 - 25x$ **(vi)** $4x - 4y + x^2 - y^2$

(b) Factorise each expression in part **(a)**.

18 A house has had a rectangular extension added. The extension has a length of $(x + 6)$ m and a width of $(x + 4)$ m. An area 2 m wide and 4 m long is to be an untiled verandah. The remainder is to be a tiled family room. The cost of tiling a family room is $18 per square metre.



(a) Write an expression for the area of the family room in factorised form.

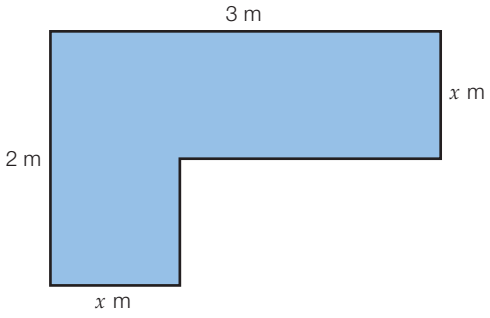
(b) Write an expression for the cost of tiling the family room.

(c) Find the cost of tiling the family room if x is 2 m.

19 Find, in expanded form, the square of the sum of $4x$ and $3y$.

Reasoning

20 **(a)** Write an expression (in m^2) for the area of the kitchen bench surface shown at right. Simplify the expression as far as possible.



(b) Factorise the expression for the area.

(c) Find the area of the kitchen bench surface when the width of the bench, x, is:

(i) 1 m

(ii) 0.5 m

(iii) 0.75 m

21 A biscuit box has a length 14 cm greater than its height and a width 9 cm greater than its height.

(a) Draw a diagram showing this information. Use h to represent height.

(b) Write an expression (in factorised form) to represent the volume of the biscuit box.

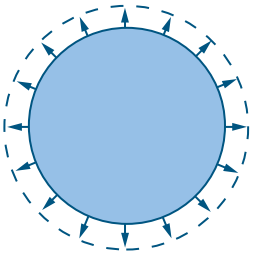
(c) Expand and simplify the expression for volume.

(d) Use your answer for part **(b)** to find the volume of the biscuit box if its height is 6 cm.

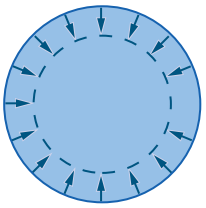
(e) Use your answer for part **(c)** to find the volume of the biscuit box if its height is 6 cm.

(f) Which form of the volume expression (factorised or expanded) did you find easier to work with? Explain your answer.

22 A circular disc of metal expands when it is heated. When it is heated to a particular temperature, its radius increases by 2 mm. When cooled to a particular temperature below room temperature, the disc contracts and its radius decreases by 2 mm. (Do not substitute for π in any parts of this question.)



Disc expands when heated



Disc contracts when cooled

(a) If the disc has a radius of x mm at room temperature (20°C), write down its area at room temperature.

(b) In terms of x, what is the radius of the heated disc?

(c) Hence, write an expression for the area of this larger disc.

(d) In terms of x, what is the radius of the cooled disc?

(e) Hence, write an expression for the area of this smaller disc.

(f) Write an expression for the difference between the area of the larger disc and that of the smaller disc.

(g) Is there a common factor that can be taken out of this expression for the difference in area? What is it?

(h) Factorise this expression for the difference in area.

(i) If the original disc (at 20°C) had a radius of 10 mm, find the difference in area between the larger and smaller discs.

(j) If the difference in area between the larger and smaller discs is 136π mm^2, find the radius of the disc at 20°C.

23 An exercise book is $(8x + 3)$ cm long and $(2y + 7)$ cm wide.

(a) Write an expression (in cm^2) for the area of the book.

(b) What is the expression for the area if $y = 4x - 5$?

(c) Expand and simplify the expression in part **(b)**.

(d) **(i)** In terms of x, what value of y will result in the expression for the area being a perfect square?

(ii) Expand and simplify the perfect square in **(i)**.

24 The number of private cars observed during a period of x days is modelled by the expression $x^2 + p^2x$ where p is the number of passengers. The number of taxis observed with p passengers during y days is given by $p^2y - y^2$.

(a) Write the expression for the total number of cars and taxis.

(b) Factorise the expression in part **(a)**.

(c) Explain why x must be greater than y if the number of passengers is zero.

(d) **(i)** How many vehicles were there in total if $x = y$?

(ii) It is found that $2px$ of these vehicles were white. Write, in factorised form, the number of non-white vehicles.

Measurement

3

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 Use your calculator to evaluate the following.

PP Worksheet R3.1

(a) 25^2 **(b)** 3.1^3 **(c)** $\sqrt{11.9025}$

2 Given the formula $A = \frac{1}{2}(a + b)h$:

PP Worksheet R3.2

(a) find A if $h = 4$, $a = 3$ and $b = 5$

(b) find h if $A = 30$, $a = 7$ and $b = 5$

(c) find b if $A = 48$, $h = 6$ and $a = 9$.

3 Complete the following unit conversions.

PP Worksheet R3.3

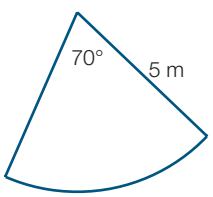
(a) 4.1 m = _____ cm **(b)** 4.25 km = _____ cm **(c)** 40 m^2 = _____ cm^2

(d) 578 cm^2 = _____ m^2 **(e)** 8.25 km^2 = _____ m^2 **(f)** 600 mL = _____ L

(g) 20 cm^3 = _____ mL **(h)** 30.8 m^3 = _____ L **(i)** 7250 cm^3 = _____ m^3

4 Find each of the following values in the sector shown, rounded to two decimal places.

PP Worksheet R3.4



(a) the arc length

(b) the perimeter

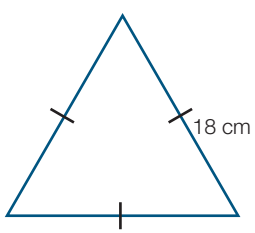
(c) the area

5 **(i)** Use Pythagoras' theorem to find the perpendicular height of each of the following shapes, rounded to two decimal places.

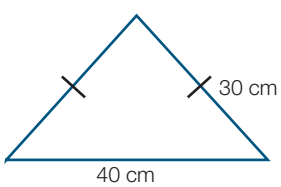
(ii) Use the height to calculate the area of each shape.

PP Worksheet R3.5

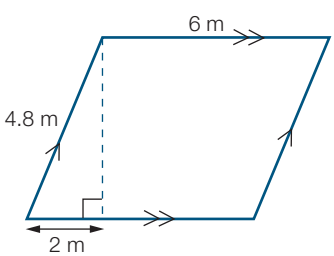
(a)



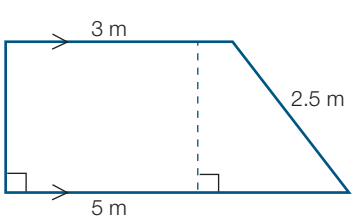
(b)



(c)



(d)



After completing this chapter you will be able to:

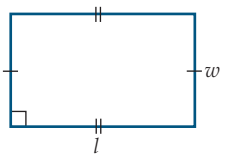
- use area formulas of basic shapes to find the area of composite shapes
- calculate the surface area, volume and capacity of prisms and cylinders
- calculate the surface area, volume and capacity of pyramids, cones and spheres
- calculate the density of solids and liquids, and find the concentration of dissolved substances. 10A

3.1 Area

Need to Know

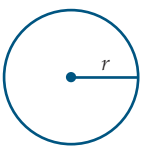
Area formulas for common shapes

Rectangle



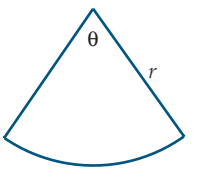
$A = lw$

Circle



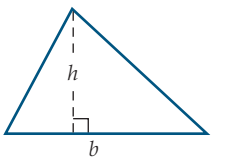
$A = \pi r^2$

Sector



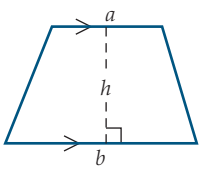
$A = \frac{\theta}{360} \times \pi r^2$

Triangle



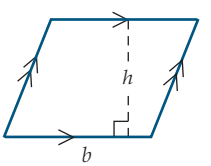
$A = \frac{1}{2}bh$

Trapezium



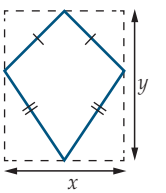
$A = \frac{1}{2}(a + b)h$

Parallelogram



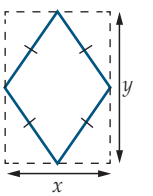
$A = bh$

Kite



$A = \frac{1}{2}xy$

Rhombus



$A = \frac{1}{2}xy$

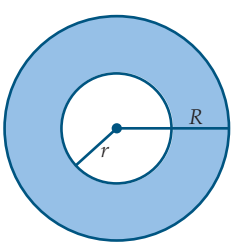
Finding the area of a **composite shape** involves adding or subtracting areas of simpler shapes.

Area of an annulus

$A = \pi R^2 - \pi r^2$

$A = \pi(R^2 - r^2)$ (factorised form)

$= \pi(R + r)(R - r)$ (factorising using the Difference of Two Squares rule—a useful form to work with)

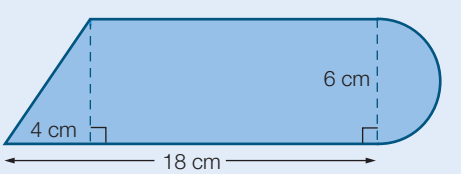


Use capital and lower case versions of a letter to indicate the same dimension in two figures that have the same shape. Here, R is used to represent the radius of the larger circle and r for the radius of the smaller circle.

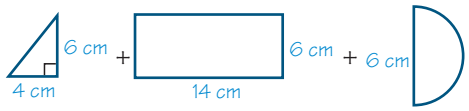
Worked Example 1

WE1

Find the area of this shape rounded to two decimal places.



Working



$A = \frac{1}{2}bh + lw + \frac{\pi r^2}{2}$

$l = 18 - 4$
$= 14$ cm

$r = 6 \div 2$
$= 3$ cm

$A = \frac{1}{\cancel{2}_1} \times \frac{\cancel{4}^2}{} \times 6 + 14 \times 6 + \frac{\pi \times 3^2}{2}$

$= 2 \times 6 + 84 + \frac{9\pi}{2}$

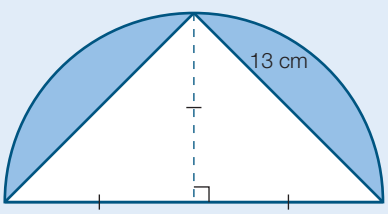
$= 96 + \frac{9\pi}{2}$

$= 110.14 \text{ cm}^2$ (2 d.p.)

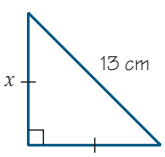
Worked Example 2

WE 2

Find the shaded area in the diagram, rounding your answer to two decimal places. (You may need to use Pythagoras' theorem to calculate the lengths of some unknown sides.)



Working



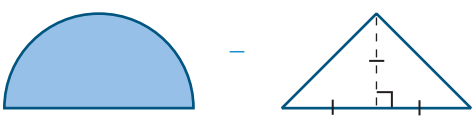
$x^2 + x^2 = 13^2$

$2x^2 = 13^2$

$x^2 = \frac{13^2}{2}$

$x = \sqrt{\frac{13^2}{2}}$

$x = \sqrt{84.5}$ cm



$A = \frac{1}{2}\pi r^2 - \frac{1}{2}bh$

$b = 2h$

$A = \frac{1}{2}\pi r^2 - \frac{1}{\cancel{2}_1} \times \frac{\cancel{2}^1}{} h \times h$

$= \frac{1}{2}\pi r^2 - h^2$

$= \frac{84.5\pi}{2} - 84.5$

$= 84.5\left(\frac{\pi}{2} - 1\right)$

$= 48.23 \text{ cm}^2$ (2 d.p.)

Answers page 491

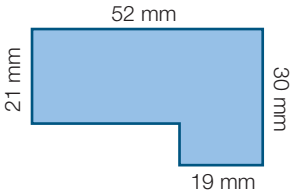
3.1 Area

Fluency

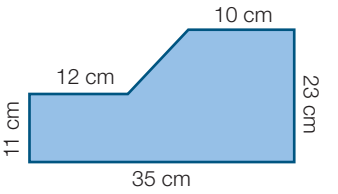
WE1

1 Find the area of each of the following shapes. Round your answer to two decimal places where necessary.

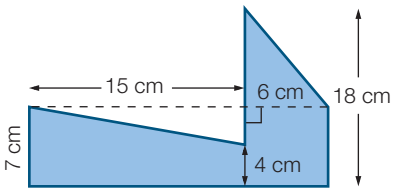
(a)



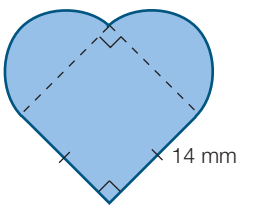
(b)



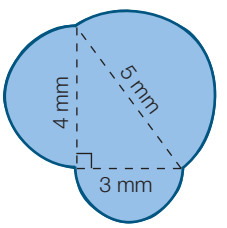
(c)



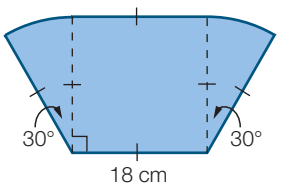
(d)



(e)



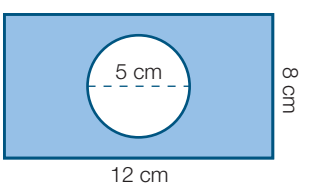
(f)



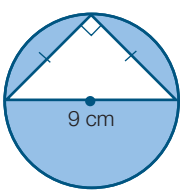
WE2

2 Find the shaded area in each of the following, rounding your answer to two decimal places. (You may need to use Pythagoras' theorem to calculate the lengths of some unknown sides.)

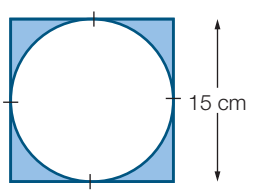
(a)



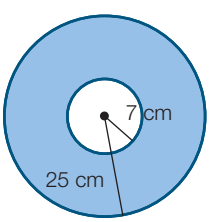
(b)



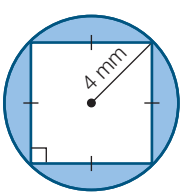
(c)



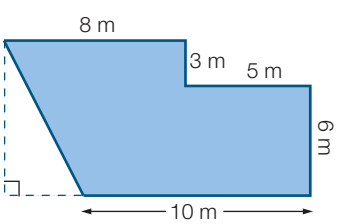
(d)



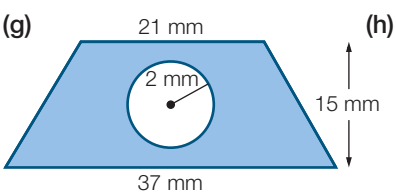
(e)



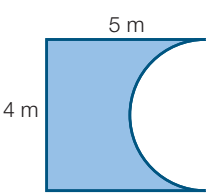
(f)



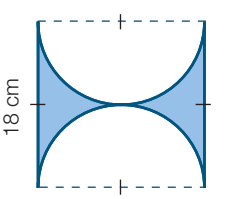
(g)



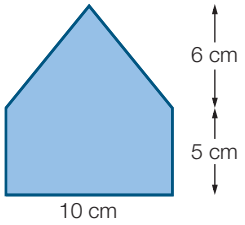
(h)



(i)



3 The area of this composite shape in cm^2 can be found by:



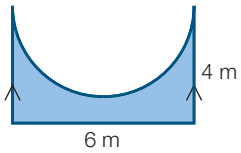
A $(10 \times 5) + (10 \times 6)$

B $(10 \times 11) + (10 \times 6)$

C $(10 \times 5) + (\frac{1}{2} \times 10 \times 6)$

D $(\frac{1}{2} \times 10 \times 5) + (\frac{1}{2} \times 10 \times 6)$

4 The area of this composite shape in m^2 can be found by:



A $(6 \times 4) + (\frac{1}{2} \times \pi \times 6^2)$

B $(6 \times 4) - (\frac{1}{2} \times \pi \times 6^2)$

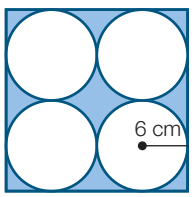
C $(6 \times 4) - (\frac{1}{2} \times \pi \times 3^2)$

D $(6 \times 4) - (\frac{1}{2} \times \pi \times 32)$

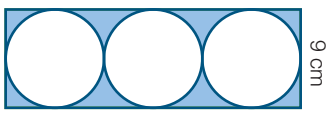
Understanding

5 Find the shaded area of each of the following, rounded to two decimal places.

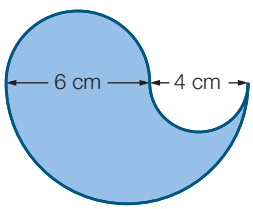
(a)



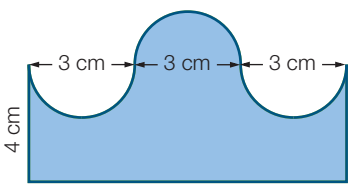
(b)



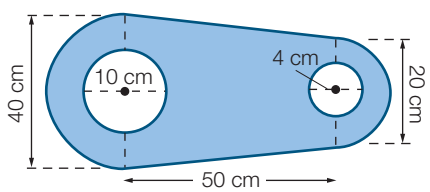
(c)



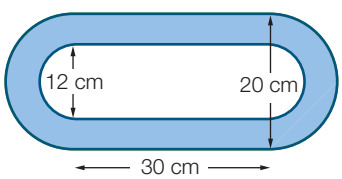
(d)



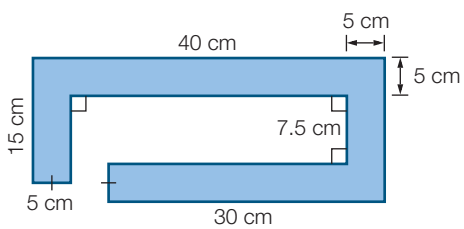
(e)



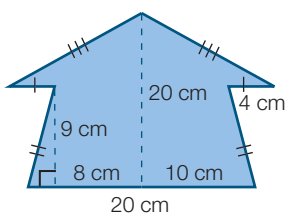
(f)



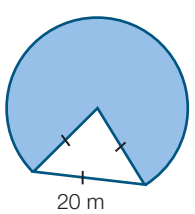
(g)



(h)



6 Find the area of both the sector and the triangle. (You will need to use Pythagoras' theorem to calculate the height of the triangle.) Write both answers correct to the nearest m^2.



Hint

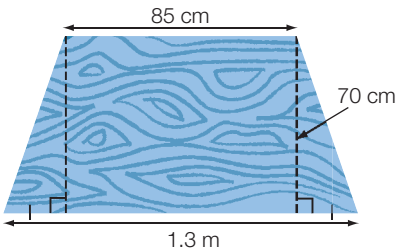
Drawing a diagram can help you visualise the problem.

7 The Department of Roads has a machine that lays bitumen road at a rate of 20 metres in length every 15 minutes. The bitumen is 6 m wide and the machine works 7 hours per day. The department is building a road 12.5 km long.

(a) How long will it take to complete the road? State your answer in both hours and days, correct to two decimal places.

(b) What will be the total area of bitumen laid by the machine?

8 The tables in Joshua's classroom are trapezoidal in shape, as shown at right. The surface of each table needs to be laminated. If there are 13 tables in the classroom, what area (m^2) of laminate is required? Round your answer to two decimal places.

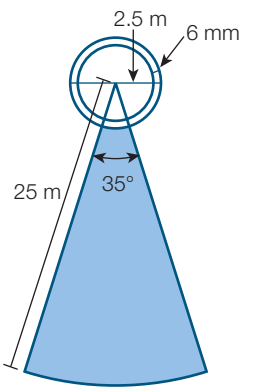


9 Elspeth has a circular swimming pool that is 7 m in diameter. She is installing a path 1 m wide all around the pool. What will be the area of this path, rounded to two decimal places?

10 The throwing area for the Olympic shot put event is shown in the diagram. The shot put is thrown from a circle that is 2.5 m in diameter to the outside of the rim. The rim is 6 mm wide (athletes cannot touch the rim when throwing). To be 'legal' a throw must land in the landing zone shown shaded in blue. Calculate, correct to two decimal places:

(a) the area of the rim of the circle

(b) the area of the landing zone.



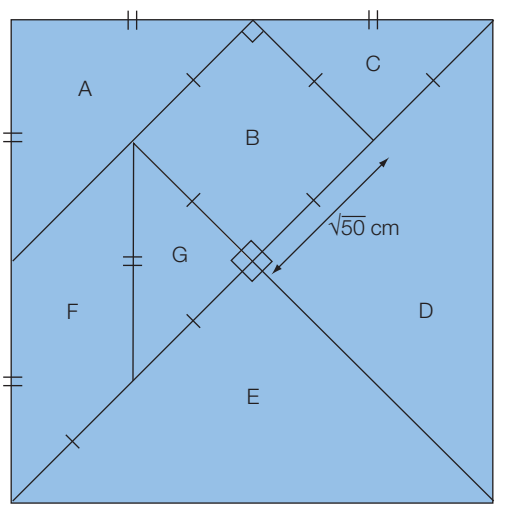
Reasoning

11 Only two rectangles with integer dimensions (side lengths are whole numbers) have the same numerical value for their area and perimeter. Find both rectangles.

12 What is the radius of a circle with an area equal to that of a square 8 m wide? Round your answer to two decimal places.

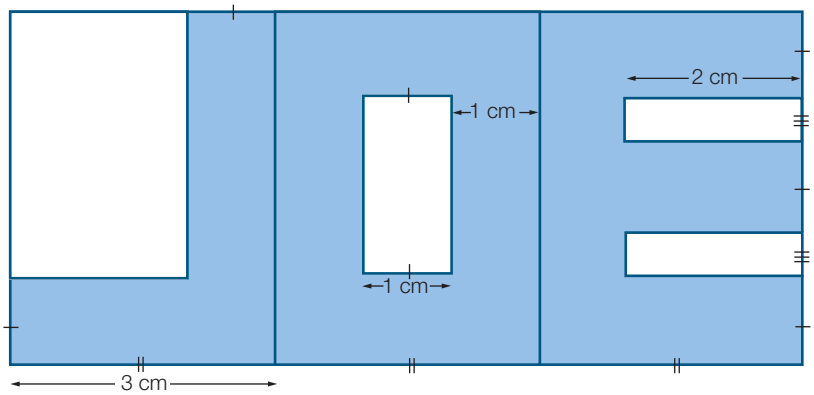
13 Find the area of each piece of the tangram puzzle shown at right.

(Remember $\sqrt{50} \times \sqrt{50} = 50$.)



Open-ended

14 **(a)** To label his school folder, Joe cuts out the letters of his name from a 9 cm by 4 cm piece of coloured paper as shown below. What area of coloured paper do the letters cover?

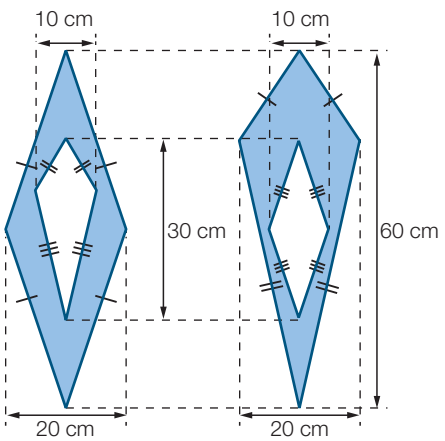


(b) Use the 9 cm by 4 cm rectangle in part **(a)** for another set of three letters. Calculate the area of paper covered by these letters. Is the area covered greater than, less than, or about the same as that covered in part **(a)**?

15 **(a)** Without doing any calculations explain why the shaded area in both shapes is equal.

(b) Draw another pair of shapes with a similar relationship to the pair shown.

(c) Draw as many such pairs of shapes as you can.



Surface area of prisms and cylinders

3.2

Need to Know

A **prism** is a solid with straight edges and a **uniform cross-section**. This means that the shape on one end (called the base) continues all the way through to the other end.

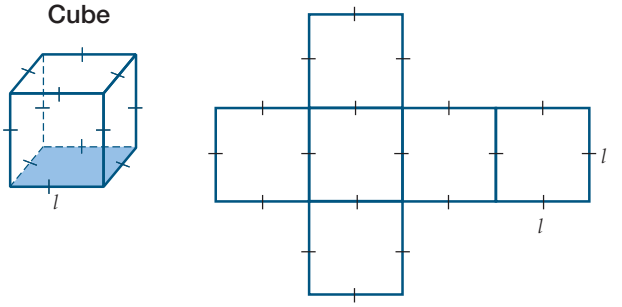
The **surface area** (SA) of a prism is the total area of all its external surfaces.

To calculate the surface area of a prism, it is often useful to draw a **net**—a 2D (flat) diagram that clearly shows each of the faces of the prism.

A **cylinder** also has a uniform cross-section, but it is not strictly a prism. The surface area of cylinders can also be found by considering their nets.

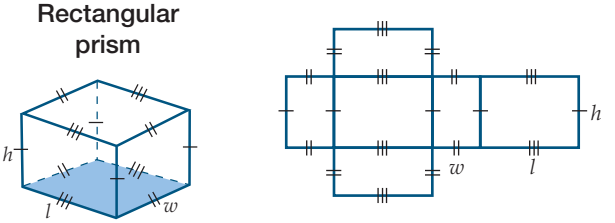
Formulas for the surface area of prisms and cylinders

Cube



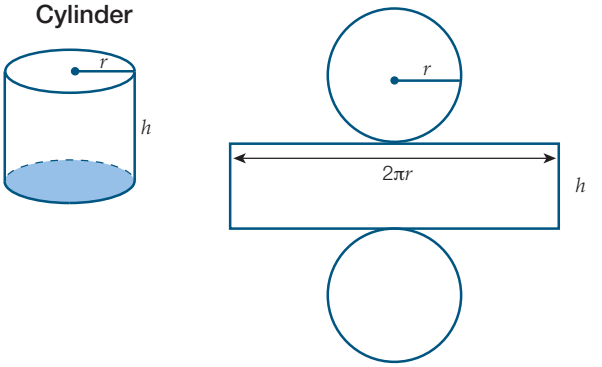
SA = area of 6 squares
$= 6l^2$

Rectangular prism



SA = area of 6 rectangles
$= 2lw + 2wh + 2lh$
$= 2(lw + wh + lh)$
(factorised form)

Cylinder

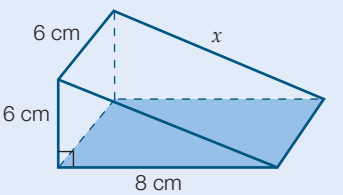


SA = area of 2 circles and area of curved surface
$= 2\pi r^2 + 2\pi rh$
$= 2\pi r(r + h)$
(factorised form)

Worked Example 3

WE3

Find the surface area of the triangular prism.



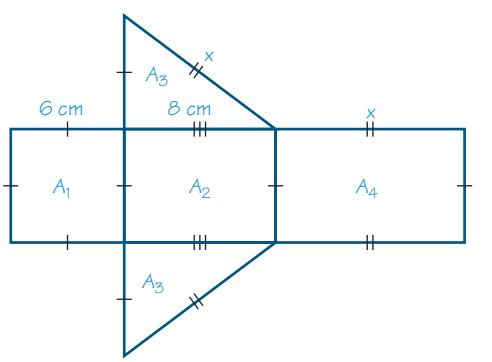
Working

$x^2 = 6^2 + 8^2$
$= 100$
$x = 10$ cm

$SA = A_1 + A_2 + A_2 \times A_3 + A_4$

$= l^2 + lw + \frac{1}{\cancel{2}} \times \frac{bh}{\cancel{2}_1} + LW$

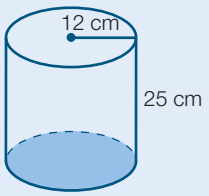
$= l^2 + lw + bh + LW$
$= 6^2 + 8 \times 6 + 8 \times 6 + 10 \times 6$
$= 36 + 48 + 48 + 60$
$= 192$ cm^2



Worked Example 4

WE4

Find the surface area of this cylinder, rounded to two decimal places.



Working

$r = 12$ cm

$h = 25$ cm

$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$
$= 288\pi + 600\pi$
$= 888\pi$
$= 2789.73 \text{ cm}^2$ (2 d.p.)

3.2 Surface area of prisms and cylinders

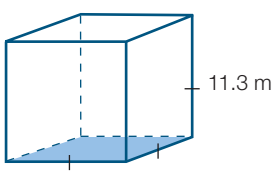
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Fluency

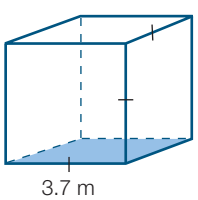
1 Find the surface area of each solid, rounded to two decimal places where necessary.

WE3

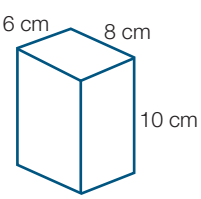
(a)



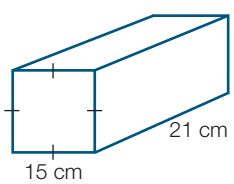
(b)



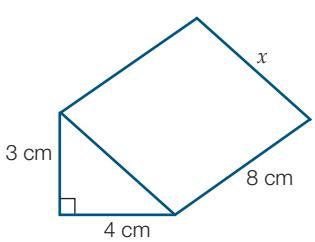
(c)



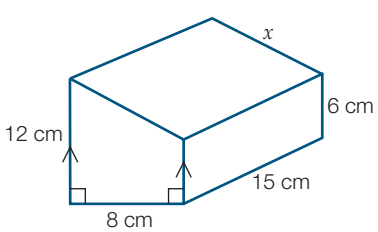
(d)



(e)



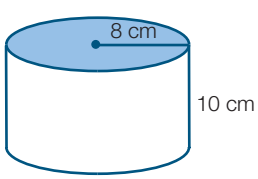
(f)



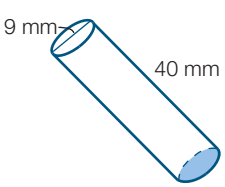
2 Find the surface area of each cylinder, rounded to two decimal places.

WE4

(a)

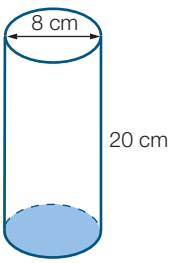


(b)



3 The calculation that gives the surface area of each of the following solids is:

(a)



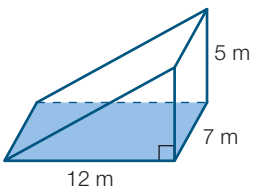
A $2 \times \pi \times 8(8 + 20)$

B $2 \times \pi \times 4(8 + 20)$

C $2 \times \pi \times 8(4 + 20)$

D $2 \times \pi \times 4(4 + 20)$

(b)



A $5 \times 12 + 7(5 + 12 + 17)$

B $\frac{5 \times 12}{2} + 7(5 + 13)$

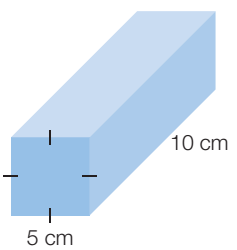
C $\frac{5 \times 12}{2} + 7(5 + 12 + 13)$

D $5 \times 12 + 7(5 + 12 + 13)$

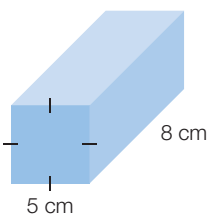
Understanding

4 Amanda has made a set of wooden building blocks as a present for her little brother. She built ten of each type of block and now she has to paint them. What is the total area that she has to paint?

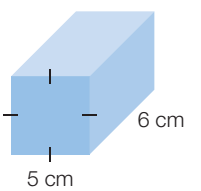
Block A



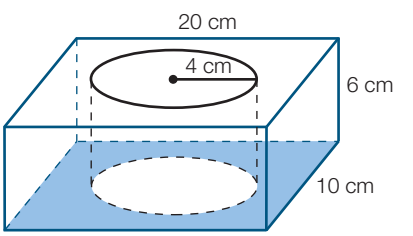
Block B



Block C

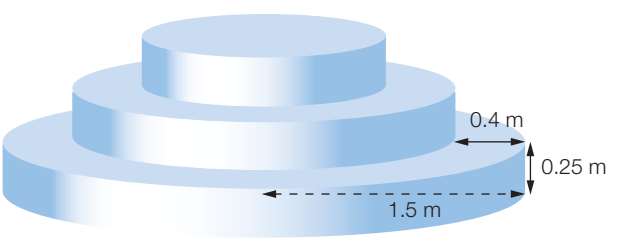


5 The diagram shows a plastic casing that has been specially made to surround a water pipe. Find the surface area of the casing, rounded to two decimal places.



Reasoning

6 A circular podium has three steps as shown. The base of the podium has a radius of 1.5 m and each step has a width of 0.4 m. Each step is 0.25 m higher than the previous one. Calculate, correct to two decimal places, the amount of carpet needed to cover the visible surfaces of the podium.



Open-ended

7 **(a)** Write the formula for the surface area of a rectangular prism.

(b) By how many times does the surface area of the rectangular prism increase if all three of its dimensions are doubled? Demonstrate your answer with two suitable prisms, one of which has its length, width and height double that of the other one.

8 **(a)** Choose two variables to represent the radius and height of a cylinder. Find the surface area of this cylinder. Leave your answer in terms of π.

(b) Double the height of the cylinder and find the new surface area, leaving your answer in terms of π.

(c) Double the radius of the original cylinder and find the new surface area, leaving your answer in terms of π.

(d) Explain why your answers to **(b)** and **(c)** are not the same.

Volume of prisms and cylinders

3.3

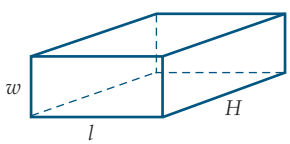
Need to Know

The volume of solids with constant cross-section is:

$$V = AH$$

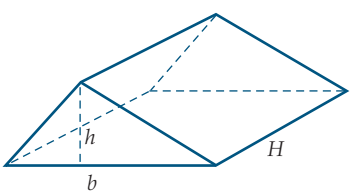
where A is the area of the cross-section
H is the perpendicular height of the solid.

Rectangular prism



$V = AH$
$A = lw$
$V = lwH$

Triangular prism

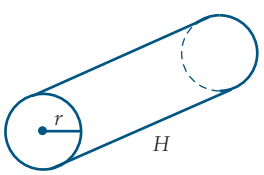


$V = AH$

$A = \frac{1}{2}bh$

$V = \frac{1}{2}bhH$

Cylinder



$V = AH$
$A = \pi r^2$
$V = \pi r^2 H$

In the formula for the triangular prism above we used the lower case h for the height of the triangular-shaped base, and the capital H for the height of the prism.

The **capacity** of a container refers to the volume of a substance (usually liquid or gas) that the container can hold.

1000 millilitres (mL) = 1 litre (L)

1000 litres = 1 kilolitre (kL)

$1 \text{ cm}^3 = 1 \text{ mL}$

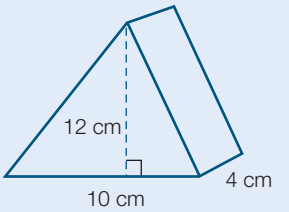
$1000 \text{ cm}^3 = 1 \text{ L}$

$1 \text{ m}^3 = 1000 \text{ L} = 1 \text{ kL}$

Worked Example 5

WE5

Find the volume of the triangular prism.



Working

$V = AH$

$= \frac{1}{2}bhH$

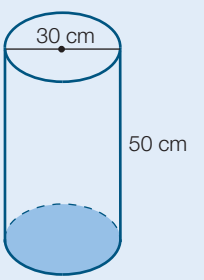
$= \frac{1}{2} \times 10 \times 12 \times 4$

$= 240\ cm^3$

Worked Example 6

WE6

What is the capacity of this cylindrical tank? Answer in litres, rounded to two decimal places.



Working

$V = A \times H$

$= \pi r^2 H$

$= \pi \times 15^2 \times 50$

$= 11\,250\pi$

$= 35\,342.92\ cm^3$ ($1000\ cm^3 = 1$ L)

Capacity = 35.34 L (2 d.p.)

$r = \frac{30}{2}$

$= 15\ cm$

3.3 Volume of prisms and cylinders

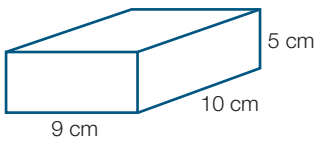
Answers page 492

Fluency

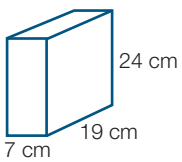
1 Find the volume of each solid with constant cross-section. Round to two decimal places, if necessary.

WE 5

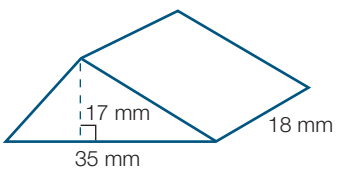
(a)



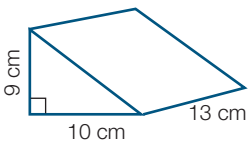
(b)



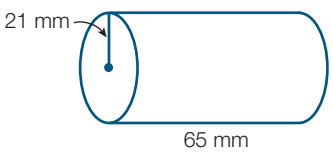
(c)



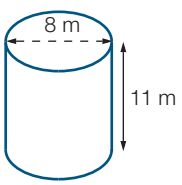
(d)



(e)

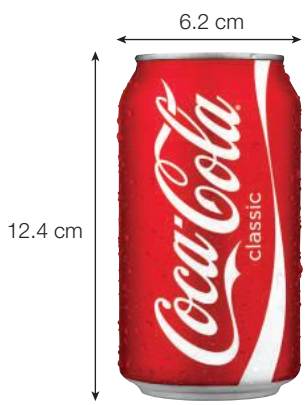


(f)



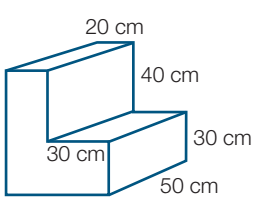
2 Find the capacity of the soft drink can in mL, rounded to two decimal places. Assume that the can is a regular cylinder.

WE 6

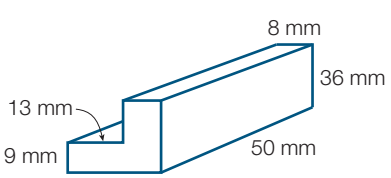


3 Find the volume of each solid with constant cross-section. Round to two decimal places, if necessary.

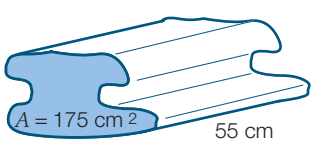
(a)



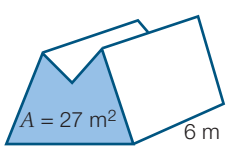
(b)



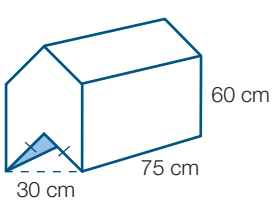
(c)



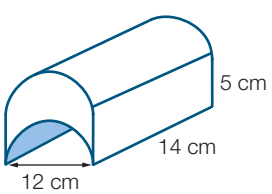
(d)



(e)

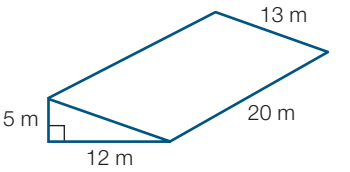


(f)



Understanding

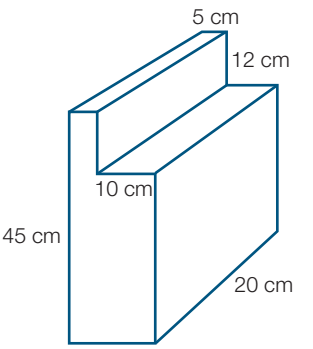
4 **(a)** The capacity of this prism in kL can be found by:



A $12 \times 5 \times 20$ **B** $\frac{1}{2} \times 5 \times 12 \times 20$

C $(\frac{1}{2} \times 5 \times 12) + (20 \times 12)$ **D** $\frac{1}{2} \times 12 \times 20 \times 5 \times 13$

(b) The capacity of this prism in mL can be found by:



A $5 \times 12 \times 20 + 10 \times (45 - 12) \times 20$

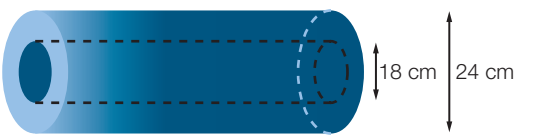
B $5 \times 12 \times 20 + 10 \times 45 \times 20$

C $5 \times 45 \times 20 + 10 \times 12 \times 20$

D $(5 + 10) \times 45 \times 20 - 10 \times 12 \times 20$

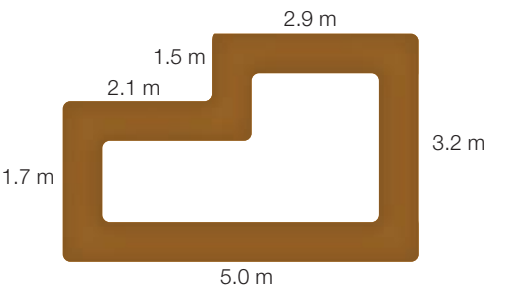
5 Ingrid needs 25 concrete columns for her new verandah. Each column will be cylindrical in shape with a base diameter of 28 cm and a height of 85 cm. What is the total volume of concrete required? Give your answer in cubic metres, correct to two decimal places. ($1\ m^3 = 1\,000\,000\ cm^3$)

6 A 3.5 m long plastic pipe is to be used for drainage. If it has an outer diameter of 24 cm and an inner diameter of 18 cm, find the volume of plastic needed to construct it. Express your answer in m^3 correct to two decimal places.

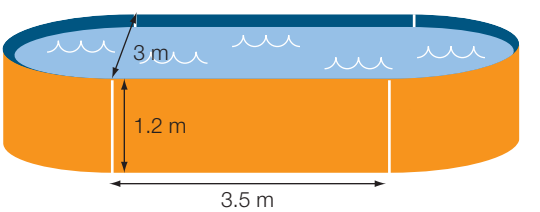


Reasoning

7 Gael is building an extension onto her house. She has just finished digging a series of trenches. All the trenches are 85 cm deep and 55 cm wide. The plan for the trenches is shown in the diagram. What was the volume of dirt removed? Round your answer to two decimal places.



8 Terry's swimming pool is rectangular with two semicircular ends. How many litres of water does it hold when full? State your answer to the nearest whole number.



Open-ended

9 A rectangular box has a volume of 200 cm^3.

(a) Find three possible whole number sets of dimensions that give the correct volume.

(b) Find three sets of dimensions with at least one non-whole number that give the correct volume.

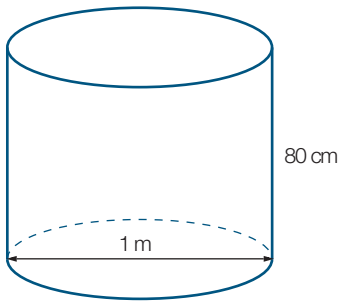
(c) Calculate the surface area of the box for each of the three sets of dimensions found in **(a)**.

10 Anita is attempting to find the volume of the cylinder in cm^3 correct to two decimal places.

(a) Identify the two mistakes in the working shown.

(b) Find the correct answer.

(c) Give Anita two points to write in her notes so she can avoid similar mistakes in the future.



$V = AH$
$= \pi r^2 H$
$= \pi \times 1^2 \times 80$
$\approx 251.33\ cm^3$

Problem solving

Cola cost crisis

Fizzy Cola comes in cylindrical cans of diameter 5 cm and height 15 cm. The cost of one can is $2.35. The manufacturers have created a promotional, 'super-sized' can, which is the same height but twice as wide. The sales manager of Fizzy Cola is wondering at what price she should sell the promotional can. Her assistant thinks that the new can is 'twice as big' as the original can, so it must hold 'twice as much' cola and should cost 'twice the price'.

1 Suggest a price for the 'super-sized' can based on a comparision of its volume with the original can.

2 Is the assistant's reasoning correct? Why or why not?

Strategy options

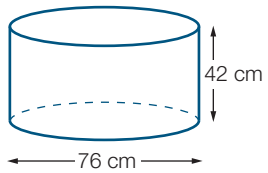
- Break problem into manageable parts.
- Draw a diagram.
- Have I seen a similar problem?

Half-time 3

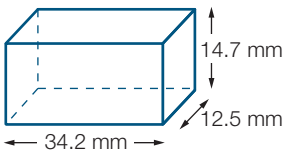
Ex 3.2

1 Find the total surface area of the following solids. Round your answers to the nearest whole number.

(a)

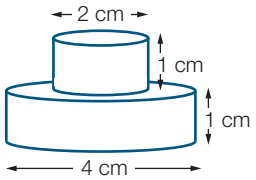


(b)



Ex 3.2, 3.3

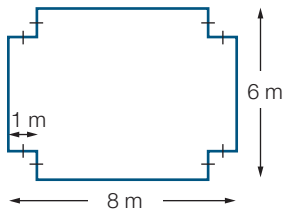
2 Find the exact surface area and exact volume of this composite shape. ('Exact' means writing your answer in terms of π.)



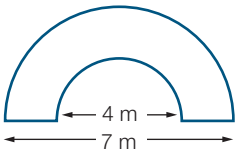
Ex 3.1

3 Find the area of the following shapes, rounded to two decimal places, if necessary.

(a)



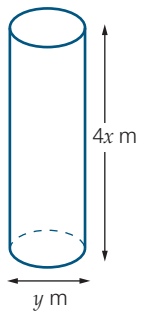
(b)



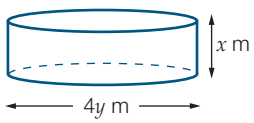
Ex 3.3

4 Which of the following solids, **A**, **B** or **C**, has the greatest volume? Justify your answer by evaluating the exact volume of each one.

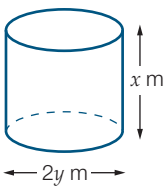
A



B



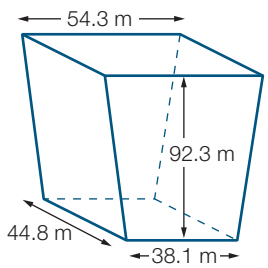
C



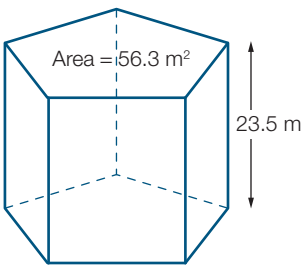
Ex 3.3

5 Find the volume of the following solids, rounded to two decimal places.

(a)



(b)



Ex 3.3

6 A cylindrical tank at a milk processing plant has a diameter of 2.6 m and a height of 3.4 m. How many 250 mL cartons could be completely filled from one tank full of milk?

Surface area of tapered solids and spheres

10A

3.4

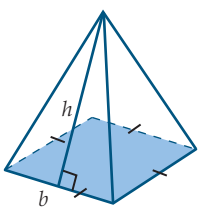
Need to Know

Tapered solids do not have a uniform cross-section. The cross-section gets smaller as you move from the base of the solid to the top. The tapered solids that we will consider here are **pyramids** and **cones**.

A **sphere** has only one single curved surface. The surface of a sphere is equivalent to four times the area of a circle of the same radius.

Formulas for the surface area of tapered solids and spheres

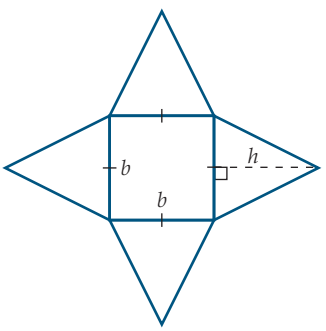
Square-based pyramid



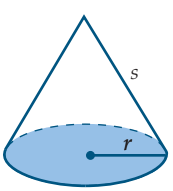
SA = area of square base and area of 4 triangles

$= b^2 + 4 \times \frac{1}{2} bh$

$= b^2 + 2bh$



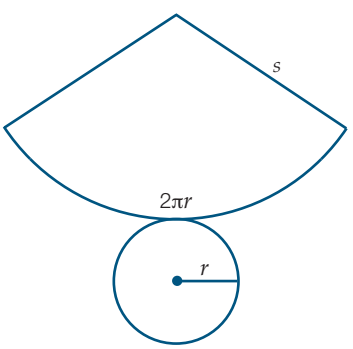
Cone



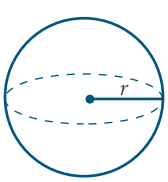
SA = area of circle and area of curved surface

$= \pi r^2 + \pi rs$

$= \pi r(r + s)$ (factorised form)



Sphere



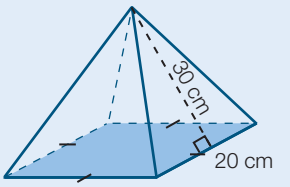
SA = area of curved surface

$= 4\pi r^2$

Worked Example 7

WE7

Find the surface area of this square-based pyramid.



Working

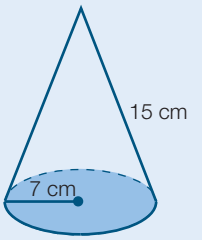
$SA = b^2 + 2bh$
$= 20^2 + 2 \times 20 \times 30$
$= 400 + 1200$
$= 1600 \text{ cm}^2$

$b = 20 \text{ cm}$
$h = 30 \text{ cm}$

Worked Example 8

WE8

Find the surface area of this cone, rounded to two decimal places.



Working

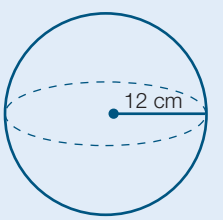
$SA = \pi r^2 + \pi rs$
$= \pi \times 7^2 + \pi \times 7 \times 15$
$= 49\pi + 105\pi$
$= 154\pi$
$= 483.81 \text{ cm}^2$ (2 d.p.)

$r = 7 \text{ cm}$
$s = 15 \text{ cm}$

Worked Example 9

WE9

Find the surface area of this sphere, rounded to two decimal places.



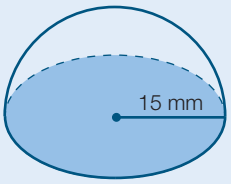
Working

$SA = 4\pi r^2$
$= 4 \times \pi \times 12^2$
$= 576\pi$
$= 1809.56 \text{ cm}^2$ (2 d.p.)

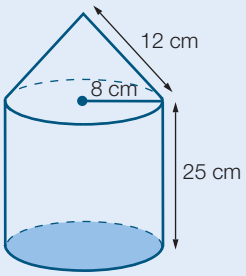
Worked Example 10 WE10

Find the surface area of each of the solids, correct to two decimal places.

(a)



(b)



Working

(a) $SA = A_{\text{hemisphere}} + A_{\text{circle}}$

$= \frac{1}{2} \times 4\pi r^2 + \pi r^2$

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 15^2$

$= 675\pi$

$= 2120.58 \text{ mm}^2$ (2 d.p.)

(b) $SA = A_{\text{cone (curved)}} + A_{\text{cylinder (curved)}} + A_{\text{circle}}$

$= \pi rs + 2\pi rh + \pi r^2$

$= \pi \times 8 \times 12 + 2 \times \pi \times 8 \times 25 + \pi \times 8^2$

$= 96\pi + 400\pi + 64\pi$

$= 560\pi$

$= 1759.29 \text{ cm}^2$ (2 d.p.)

3.4 Surface area of tapered solids and spheres

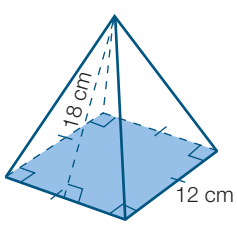
10A

Answers page 492

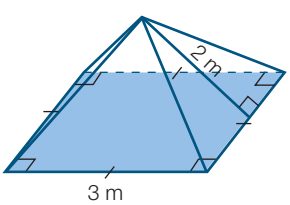
Fluency

1 Find the surface area of each pyramid, rounded to the nearest whole number. WE7

(a)

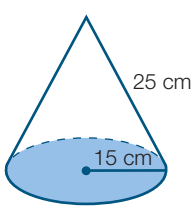


(b)

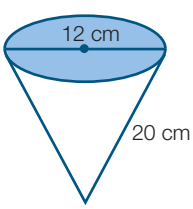


2 Find the surface area of each cone, rounded to two decimal places. WE8

(a)



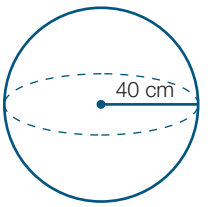
(b)



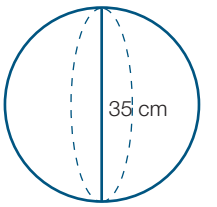
WE9

3 Find the surface area of each sphere, rounded to two decimal places.

(a)



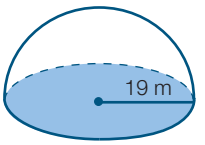
(b)



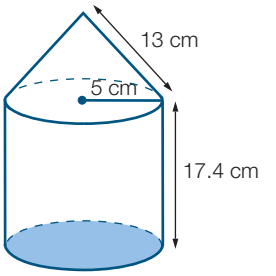
WE10

4 Find the surface area of each solid, rounded to two decimal places.

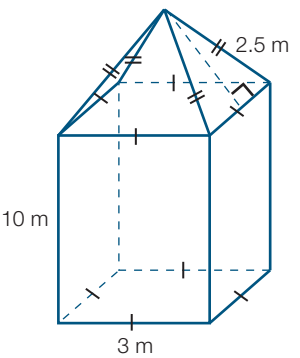
(a)



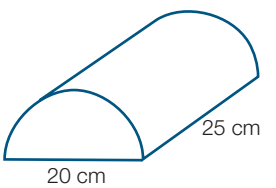
(b)



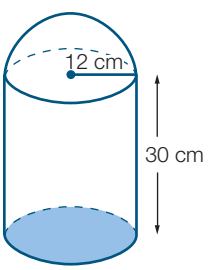
(c)



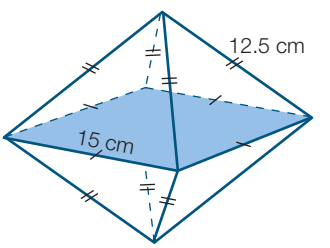
(d)



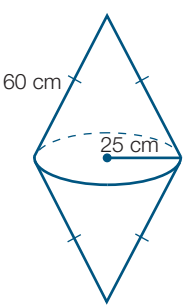
(e)



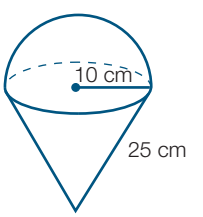
(f)



(g)



(h)

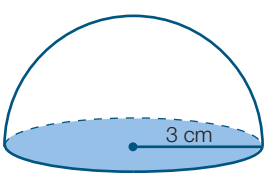


Hint

Use Pythagoras' theorem to calculate the heights of the triangles.

5 The calculation that gives the surface area of each of the following solids is:

(a)



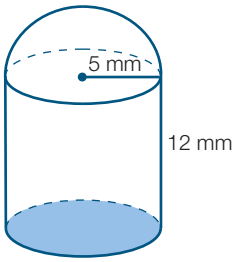
A $4 \times \pi \times 3^2$

B $\frac{1}{2} \times 4 \times \pi \times 3^2$

C $\frac{1}{2} \times 4 \times \pi \times 3^2 + \pi \times 3^2$

D $4 \times \pi \times 3^2 + \pi \times 3^2$

(b)



A $4 \times \pi \times 12^2 + 2 \times \pi \times 5 \times 12 + \pi \times 12^2$

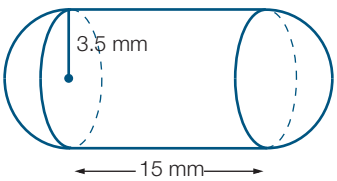
B $2 \times \pi \times 5^2 + \pi \times 5 \times 12 + \pi \times 5^2$

C $4 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 12 + 2 \times \pi \times 5^2$

D $\frac{1}{2} \times 4 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 12 + \pi \times 5^2$

Understanding

6 Find, correct to two decimal places, the surface area of a capsule that has the dimensions shown.



7 A conical party hat has a vertical height of 18 cm. The circumference of the circular base is 43 cm. Calculate the area of cardboard required to make the hat (assume no overlap). Round your answer to the nearest whole number.

8 A pyramid in Egypt has a square base that covers an area of 44 100 m^2 and a height perpendicular to the base of 130 m. Calculate the total surface area of the four triangular faces of the pyramid, to the nearest square metre.

Hint

Determine the radius, then the slant height.

Hint

Draw a labelled diagram.

Reasoning

9 Marcus has a new tenpin bowling ball and he wants to sew together a protective bag for it. He has a rectangular piece of material 80 cm long and 40 cm wide. The bowling ball has a diameter of 30 cm.

(a) Show that the area of the material is sufficient to cover the surface area of the bowling ball.

(b) Even though Marcus has a sufficient area of material, explain why he should consider using a longer piece of material.

10 (a) Calculate the slant height of a cone of radius 25 cm and surface area 4398.23 cm^2, to the nearest centimetre.

(b) Calculate the radius of a cone with a surface area of 1809.56 cm^2 if the slant height is three times the radius.

Open-ended

11 Find two possible sets of dimensions (radius and slant height) for a cone with a surface area between 200 cm^2 and 300 cm^2.

12 Explain why a cone cannot have a radius and a slant height with the same length.

13 A cone has its height doubled, but its radius stays the same.

(a) Choose two values to represent the height and radius of the original cone and evaluate its total surface area.

(b) Evaluate its total surface area after its height is doubled.

(c) Choose another two values to represent another cone and evaluate its total surface area.

(d) Double the height of this cone and evaluate the new total surface area.

(e) Does doubling the height double the surface area?

14 A cone has its radius doubled, but its height stays the same.

(a) Choose two values to represent the height and radius of the original cone and evaluate its total surface area.

(b) Evaluate its total surface area after its radius is doubled.

(c) Choose another two values to represent another cone and evaluate its total surface area.

(d) Double the radius of this cone and evaluate the new total surface area.

(e) Does doubling the radius double the surface area?

Problem solving

Egyptian engineering

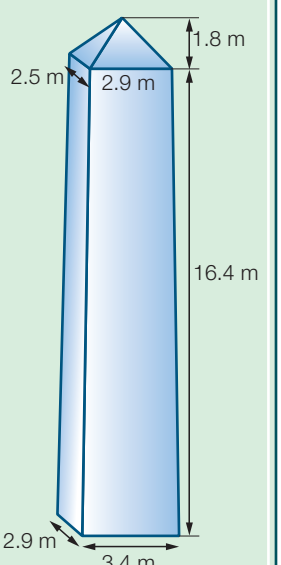
Egypt 2020 BC. Pharaoh Mentuhotep is constructing two massive stone obelisks on either side of the entrance to a temple. The obelisks are two tall columns that have four trapezoidal faces—they are narrower at the top than at the bottom. Each of the these faces will be covered in hieroglyphics.

At the top of the obelisk sits a small pyramid-shaped cap, called a 'pyramidion'. The four exposed faces of the pyramidion will be covered in gold, to reflect the Sun.

Mentuhotep has given you, his chief engineer, a labelled drawing of what the obelisk should look like. Your job is to determine the area to be covered in:

(a) gold

(b) hieroglyphics.



Strategy options

- Break problem into manageable parts.
- Draw a diagram.
- Have I seen a similar problem?

3.5 Volume of tapered solids and spheres

10A

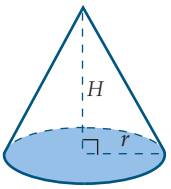
Need to Know

The volume of a tapered solid is:

$$V = \frac{1}{3}AH$$

where A is the area of the cross-section at the base
H is the perpendicular height of the solid.

Cone

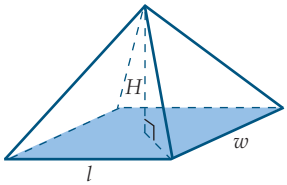


$V = \frac{1}{3}AH$

$A = \pi r$

$V = \frac{1}{3}\pi r^2 H$

Rectangular-based pyramid

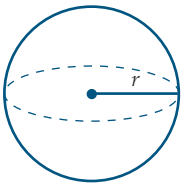


$V = \frac{1}{3}AH$

$A = lw$

$V = \frac{1}{3}lwH$

Sphere



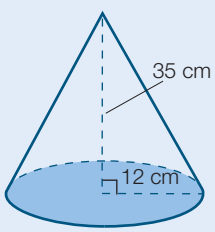
$V = \frac{4}{3}\pi r^3$

Worked Example 11

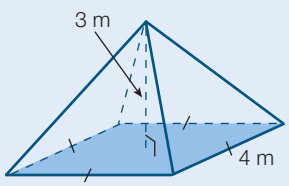
WE11

Find the volume of each of the tapered solids, rounded to two decimal places if necessary.

(a)



(b)



Working

(a) $V = \frac{1}{3}AH$

$= \frac{1}{3}\pi r^2 H$

$= \frac{1}{3} \times \pi \times 12^2 \times 35$

$= 1680\pi$

$= 5277.88 \text{ cm}^3$ (2 d.p.)

(b) $V = \frac{1}{3}AH$

$= \frac{1}{3}l^2 H$

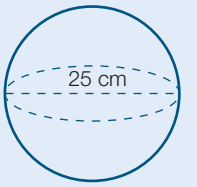
$= \frac{1}{3} \times 4^2 \times 3$

$= 16 \text{ m}^3$

Worked Example 12

WE12

Find the volume of the sphere, rounded to two decimal places.



Working

$V = \frac{4}{3}\pi r^3$

$r = \frac{25}{2} = 12.5 \text{ cm}$

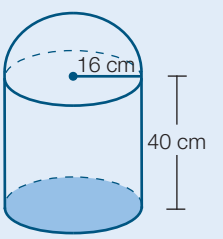
$V = \frac{4}{3} \times \pi \times 12.5^3$

$= 8181.23 \text{ cm}^3$ (2 d.p.)

Worked Example 13

WE13

Find the volume of this solid, rounded to two decimal places.



Working

$V = V_{cylinder} + V_{hemisphere}$

$= \pi r^2 H + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \pi \times 16^2 \times 40 + \frac{1}{\cancel{2}_{1}} \times \frac{\overset{2}{\cancel{4}}}{3} \times \pi \times 16^3$

$= 10\,240\pi + \frac{8192}{3}\pi$

$= 40\,748.55 \text{ cm}^3$ (2 d.p.)

3.5 Volume of tapered solids and spheres

10A

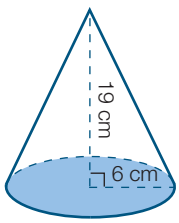
Answers page 493

Fluency

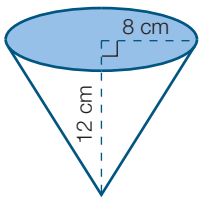
1 Find the volume of each of these tapered solids, rounded to two decimal places.

WE11

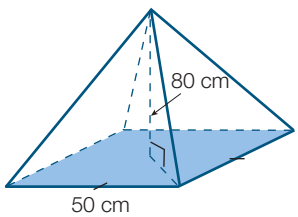
(a)



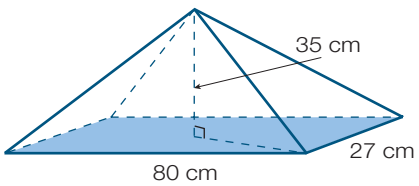
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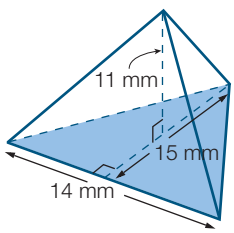
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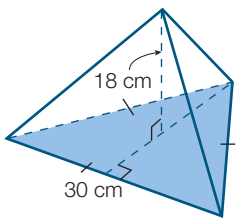
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(e)



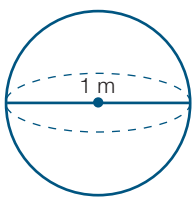
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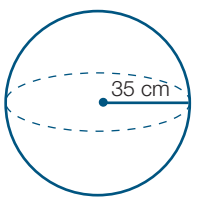
2 Find the volume of each sphere, rounded to two decimal places.

WE12

(a)



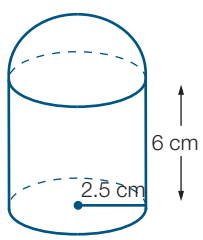
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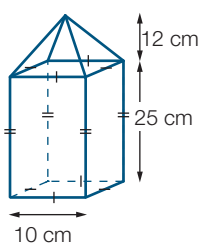
3 Find the volume of each of these solids, rounded to two decimal places.

WE13

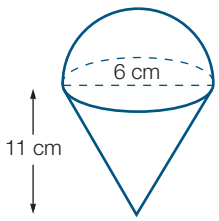
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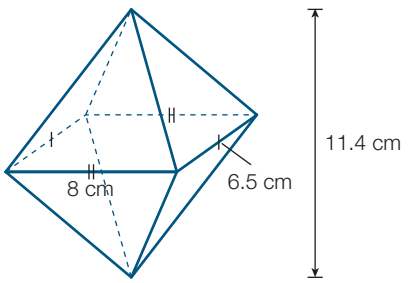
(b)



(c)



(d)



Understanding

4 Earth has an approximate diameter of 12 800 km. If Earth is assumed to be spherical, what is its volume to the nearest 1000 km^3?

5 When the radius of a sphere is doubled:

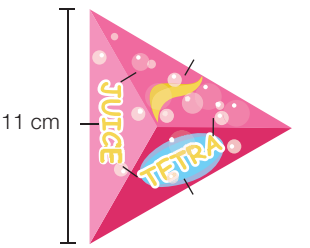
A the volume is doubled
B the volume is 4 times larger
C the volume is 6 times larger
D the volume is 8 times larger

6 The radius of a cone 25 cm high with a volume of 37 969.9 cm^3 is closest to:

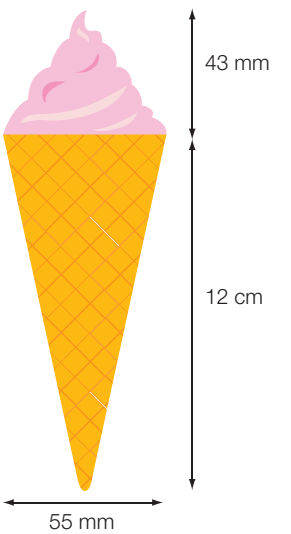
A 38 cm **B** 121 cm **C** 363 cm **D** 1450 cm

7 What is the difference between the volume of a cone of height 20 cm with a base diameter of 30 cm, and a pyramid of height 20 cm with a square base 30 cm wide? State your answer correct to two decimal places.

8 This 'tetra-pak' of frozen juice is a triangular-based pyramid. Each face is an equilateral triangle of side length 11 cm. If the height of the tetra-pak is 9 cm, find its capacity. State your answer correct to the nearest mL.



9 Anica buys an ice-cream cone at her local shop. The soft-serve ice-cream fills the cone and forms a conical (cone-shaped) peak as shown in the diagram. Calculate the amount of ice-cream Anica receives to the nearest mL.



Reasoning

10 Ball bearings are tiny metal balls used to reduce the friction between the moving parts of machines. How many ball bearings with a radius of 3 mm can be made from a solid cube of metal with a side length of 2 cm?

11 The table at right gives the diameters of the eight planets.

(a) How many times is Jupiter's diameter bigger than Mercury's (to the nearest whole number)?

(b) How many of Mercury would fit into Jupiter?

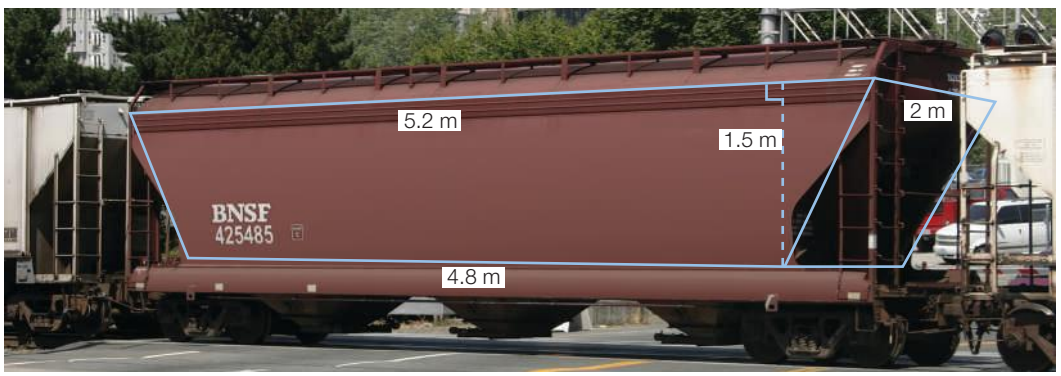
(c) Explain why your answers to parts **(a)** and **(b)** are so different.

(d) Calculate how many Earths would fit into each of the larger planets.

Planet	Diameter (km)
Mercury	4 900
Venus	12 200
Earth	12 800
Mars	6 800
Jupiter	138 000
Saturn	113 000
Uranus	47 000
Neptune	50 000

12 Six wheat silos are situated at a country railway siding, as shown below. The silos are all full and the grain will be moved to the city in railway trucks, as shown in the second diagram. How many trucks will be needed to move all the grain to town? The silos are large cylinders with a cone on top. The trucks can be assumed to be trapezoidal prisms.





Open-ended

13 **(a)** Choose two values to represent the base length and the height of a square-based pyramid. Calculate the pyramid's volume.

(b) Double the length of the base and recalculate the volume.

(c) How many times greater is the volume of the second pyramid than the first?

(d) Repeat **(a)**–**(c)** with a new base and height. What can you conclude?

14 A cone has its height doubled, but its radius stays the same.

(a) Choose two values to represent the height and radius of the original cone and evaluate its volume.

(b) Evaluate its volume after its height is doubled.

(c) Choose another two values to represent another cone and evaluate its volume.

(d) Double the height of this cone and evaluate the new volume.

(e) Can you find a relationship between the volume of a cone after the height is doubled and its original volume? Show the relationship, if there is one, or explain why a relationship does not exist.

15 A cone has its radius doubled, but its height stays the same.

(a) Choose two values to represent the height and radius of the original cone and evaluate its volume.

(b) Evaluate its volume after its radius is doubled.

(c) Choose another two values to represent another cone and evaluate its volume.

(d) Double the radius of this cone and evaluate the new volume.

(e) Can you find a relationship between the volume of a cone after the radius is doubled and its original volume? Show the relationship, if there is one, or explain why a relationship does not exist.

3.6 Applications of volume

10A

Need to Know

The **density** of a substance is the amount of mass within a given unit of volume.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \text{ or } d = \frac{m}{V}$$

where d = density in g/cm^3 (g/mL) or kg/m^3 (kg/kL)
m = mass in grams (g) or kilograms (kg)
V = volume in centimetres cubed (cm^3, or mL) or metres cubed (m^3 or kL)

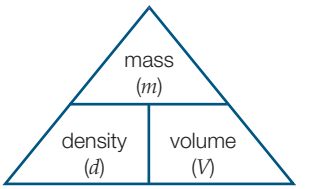
A triangle diagram is helpful in rearranging the density formula to find mass and volume. Cover the quantity you wish to find.

- If the other two quantities are side by side, find their product.
- If the two other quantities are placed one over the other, find the quotient.

$d = \frac{m}{V}$

$m = d \times V$

$V = \frac{m}{d}$



Concentration $= \frac{\text{amount of solute (mass or volume)}}{\text{amount of solution (mass or volume)}}$

Note that the concentration is the amount of solute compared to the final solution, not the amount of solvent added.

Worked Example 14

WE14

A rock has a mass of 80 g and volume of 25 cm^3. Find its density.

Working

$d = \frac{m}{V}$

$= \frac{80\ g}{25\ cm^3}$

$= 3.2\ g/cm^3$

Worked Example 15

WE15

Find the mass of 400 cm^3 of a block of metal with a density of 1.6 g/cm^3.

Working

$d = \frac{m}{V}$

$m = d \times v$

$m = 1.6 \times 400$

$= 640\ g$

Check: $\frac{640\ g}{400\ cm^3} = 1.6\ g/cm^3$

Worked Example 16

WE16

Find the volume of a 120 g piece of gold with a density of 19.3 g/cm^3 and round your answer to two decimal places.

Working

$d = \frac{m}{V}$

$V = \frac{m}{d}$

$V = \frac{120}{19.3}$

$= 6.22\ cm^3$ (2 d.p.)

Check: $\frac{120\ g}{6.22\ cm^3} = 19.3\ g/cm^3$ (1 d.p.)

Worked Example 17

WE17

A 3 L jug of cordial solution is made with 150 mL of cordial concentrate. Find the concentration of the cordial in mL/L.

Working

$\text{Concentration} = \frac{\text{amount of solute}}{\text{amount of solution}}$

$= \frac{150\ mL}{3\ L}$

$= \frac{50\ mL}{1\ L}$

The concentration of the cordial is 50 mL/L.

Worked Example 18 WE18

6 g of salt is dissolved in water to make 120 g of saline solution. Find the concentration of the solution in g/100g.

Working

$$\text{Concentration} = \frac{\text{amount of solute}}{\text{amount of solution}}$$

$$= \frac{6\ g}{120\ g}$$

$$= \frac{6}{120} \times 100$$

Concentration is 5 g/100 g

Answers page 493

3.6 Applications of volume 10A

Fluency

WE14 **1** For each of the following, find the density.

(a) a metal sphere of volume 9 cm^3, and mass 63.9 g

(b) 20 cm^3 of liquid that has a mass of 128 g

(c) a solid of volume 24 cm^3 and mass 19.2 g

WE15 **2** For each of the following, calculate the mass and state answers correct to two decimal places.

(a) A lump of metal with a volume of 800 cm^3 and a density of 7.8 g/cm^3.

(b) 450 mL of a liquid that has a density of 1.08 g/mL.

(c) 2 m^3 of a gas that has a density of 1.98 kg/m^3.

WE16 **3** Calculate the volume of each of the following, rounded to two decimal places.

(a) A 150 g piece of lead with a density of 11.3 g/cm^3.

(b) 486 g of a liquid with a density of 0.7 g/mL.

(c) 13 kg of wood that has a density of 0.85 kg/m^3.

WE17 **4** 2.5 L of sugar solution contains 50 g of sugar. What is its concentration in g/L?

WE18 **5** 500 g of weedkiller solution contains 12 g of active ingredient. Find its concentration in g/100 g.

6 **(a)** A saline solution is made from 2.5 L of water and 28 g of salt. The concentration of salt in g/L is closest to:

A 28 **B** 14 **C** 12 **D** 11

(b) A solution of copper sulfate is made from 1350 L of water and 1.6 kg of copper sulfate. The concentration of copper sulfate in g/L is closest to:

A 0.8 **B** 1 **C** 800 **D** 1000

Understanding

7 The Goulburn River carries sediment from erosion in its catchment. Barry takes a 5 L sample of water from the river. From this he extracts 38 g of sediment (1000 mg = 1 g). What is the concentration of sediment in the water, in mg/mL?

8 Marlon is making up 5 L of cordial for a party. The instructions on the cordial (concentrate) bottle say to add 1 cup of cordial to 4 cups of water.

(a) If Marlon uses a standard 250 mL cup, how many cups of each of the cordial concentrate and water are needed?

(b) What is the concentration of cordial concentrate in the final solution? Write your answer in mL/L.

9 **(a)** Crude oil is measured in units called 'barrels'. 1 barrel is equivalent to 159 litres. Calculate the mass of 1 barrel of crude oil if the density of the crude oil is 825 kg/m^3.

(b) Petrol has a density of 737 kg/m^3. A tanker truck used for transporting petrol has a capacity of 30 kL. Calculate the mass of petrol in a full tanker truck. Round your answer to the nearest tonne.

10 Jay's circular swimming pool has a diameter of 7 m and a depth of 1.6 m, but Jay only fills it to 20 cm below the rim. Calculate, correct to two decimal places, the chlorine concentration in g/L if he adds 2.1 kg of chlorine.

Hint

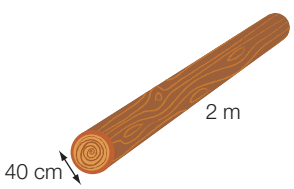
How many litres in 1 m^3?

Reasoning

11 The density of water is 1 g/cm^3. An object will *float* in water if its density is *less* than 1 g/cm^3.

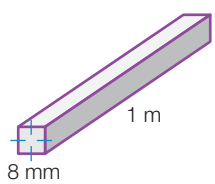
(a) By calculating the densities of the six objects below, determine which will float in water.

A wooden log



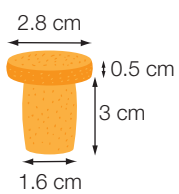
Mass: 100 kg

A steel rod



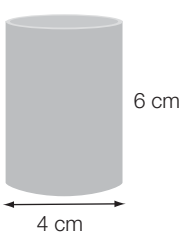
Mass: 0.5 kg

A cork



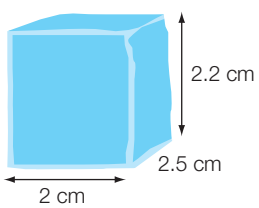
Mass: 2 g

An aluminium cylinder



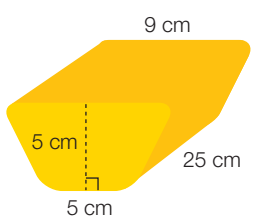
Mass: 200 g

An ice block



Mass: 10 g

A gold bar



Mass: 16 kg

(b) Find the mass of each object if it was made of lead, which has a density of 11.35 g/cm^3. Write your answers correct to the nearest gram.

12 High phosphorus levels in water can lead to blue-green algal blooms. The town of Evanshill takes its water from Lake Farmer, which has phosphorus levels contributing to blue-green algal blooms. This means the water supply to the town must be shut down. Two main sources of phosphorus are:

- Johnstone's abattoir, which discharges 5 ML (1 ML = 1 000 000 L) per day with a phosphorus concentration of 10 mg/L
- the Arvon River, which has an average discharge of 200 ML per day at a phosphorus concentration of 0.2 mg/L.

(a) Which source is contributing more phosphorus per day?

(b) What is the actual concentration of phosphorus contributed to Lake Farmer each day in mg/L? Round your answer correct to two decimal places.

Open-ended

13 The Environment Protection Authority monitors the level of the common bacteria *E. coli* at swimming beaches. If the level on any day is above 1000 organisms/100 mL, the beach is closed. Tess collects a 200 mL water sample from a beach. An examination of the sample shows that in 80 mL there are 700 *E. coli* organisms.

(a) What is the concentration of *E. coli* in the sample in organisms/100 mL?

(b) Is the concentration in the sample high enough to close the beach?

(c) List three sample sizes and the number of *E. coli* that would be high enough to close the beach.

Problem solving

A pinch of salt

The salt concentration in the ocean is about 0.035 g/cm^3. How much salt could be extracted from a cubic kilometre of sea water?



Strategy options

- Break problem into manageable parts.
- Have I seen a similar problem?

Chapter review 3

D.I.Y. Summary

Key Words

capacity	cylinder	pyramid	tapered solid
composite shape	net	sphere	uniform cross-section
cone	prism	surface area	
10A			
concentration	density		

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 To find the area of a _____, divide the shape into simple shapes, calculate the individual areas and then add them together.

2 Adding together the individual areas of each face of a solid will give the _____.

3 The volume of a _____ or _____ is found by multiplying the area of its uniform cross-section by its height.

4 The volume and surface area of a _____, cylinder and _____ each have unique formulas involving π.

5 Write the formula for area of a rectangle, parallelogram, triangle, kite, trapezium, circle and sector.

6 Volume is measured in m^3 but _____ is measured in L.

7 Write each of the formulas for the surface area and volume of a square-based pyramid, cone and sphere.

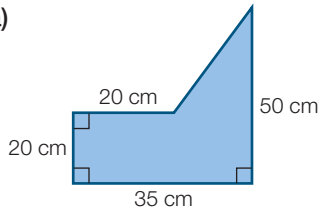
8 The formula for _____ is $\frac{\text{mass}}{\text{volume}}$. 10A

9 The _____ of a dissolved substance in a solution can be expressed in many different units. These include mL/L, g/100 g, mg/L or g/L. 10A

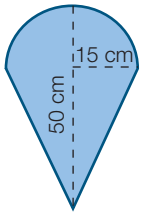
Fluency

1 Find the shaded area of each of these shapes, rounded to two decimal places. Ex 3.1

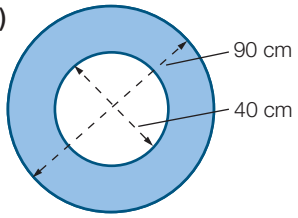
(a)



(b)



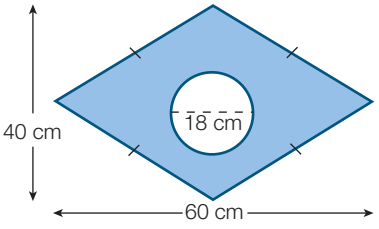
(c)



2 The shaded area correct to two decimal places is: Ex 3.1

A 182.12 cm^2 B 945.53 cm^2

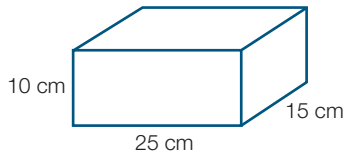
C 1143.45 cm^2 D 2145.53 cm^2



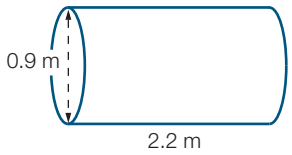
3 Find the surface area of each of these solids, rounded to two decimal places.

Ex.3.2

(a)



(b)

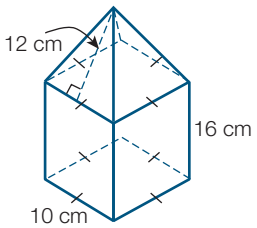


4 The surface area of this solid is equal to:

Ex.3.4 10A

A 340 cm^2 **B** 840 cm^2

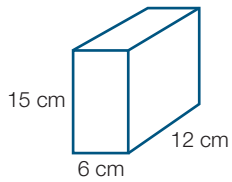
C 980 cm^2 **D** 1180 cm^2



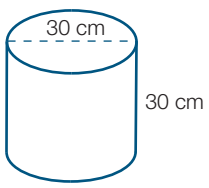
5 Find the volume of each of these solids, rounded to two decimal places.

Ex.3.3

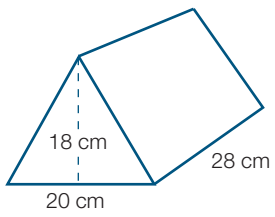
(a)



(b)



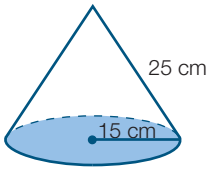
(c)



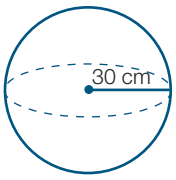
6 Find the surface area of each of these solids, rounded to two decimal places.

Ex.3.4 10A

(a)



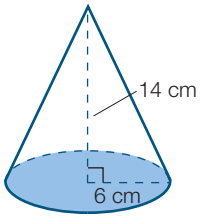
(b)



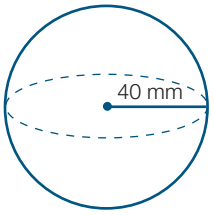
7 Find the volume of each of these solids, rounded to two decimal places.

Ex.3.5 10A

(a)



(b)

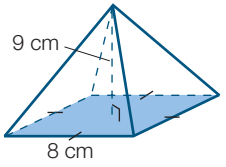


8 The volume of this solid is:

Ex.3.5 10A

A 192 cm^3 **B** 208 cm^3

C 288 cm^3 **D** 576 cm^3



9 A rectangular block of wood with the dimensions of 1 m × 2 m × 2 m has a mass of 120 kg. What is its density?

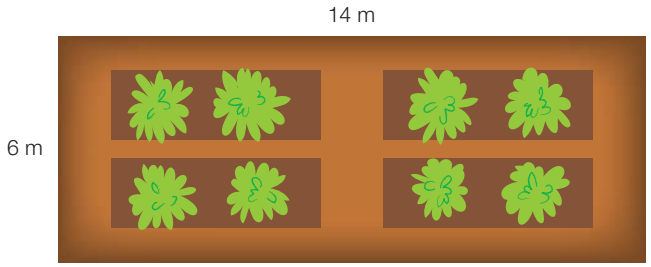
Ex.3.6 10A

10 A certain antiseptic should be mixed with warm water for best effects. If 30 mL should be mixed with 120 mL of water, express this as a concentration in mL/L.

Ex.3.6 10A

Understanding

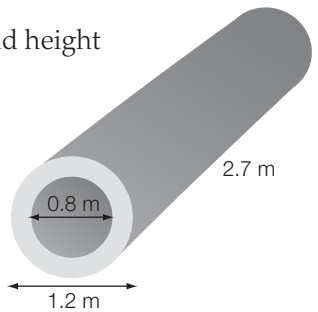
11 The diagram shows four garden beds, each 5.4 m long and 1.8 m wide, surrounded by paving bricks. Find the number of bricks needed to pave the area (to the nearest 100) if a brick measures 225 mm by 112 mm.



12 **(a)** Find the capacity of a cylindrical petrol tank of height 3.2 m and diameter 2.5 m. Express your answer to the nearest litre.

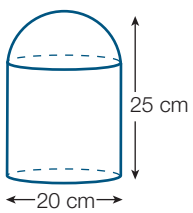
(b) The dimensions of a cylindrical rainwater tank are diameter 1760 mm and height 2100 mm. Calculate the capacity of the tank to the nearest litre.

13 Find the volume of material used to make this pipe in m^3, correct to two decimal places.

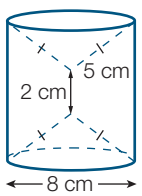


14 Find the surface area and volume of the following shapes rounded to the nearest whole number. Use Pythagoras' theorem where necessary. 10A

(a)



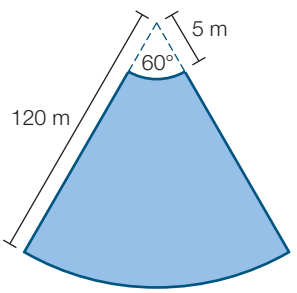
(b)



15 A snowball-throwing court has the measurements shown.

(a) Calculate the area in which the thrower has to land the snowball, to the nearest square metre.

(b) Find the perimeter of the shaded area, to the nearest metre.



16 **(a)** Find the volume of a 200 g block of Styrofoam if its density is 0.1 g/cm^3. 10A

(b) What mass of salt should be dissolved in 300 mL of pure water to give a concentration of 50 g/L?

17 If the inside of this ice-cream cone is completely full of ice-cream, find the total volume of the ice-cream sitting in and on top of the cone, to the nearest cubic centimetre. (Assume the shape of the ice-cream on top is a hemisphere.) 10A

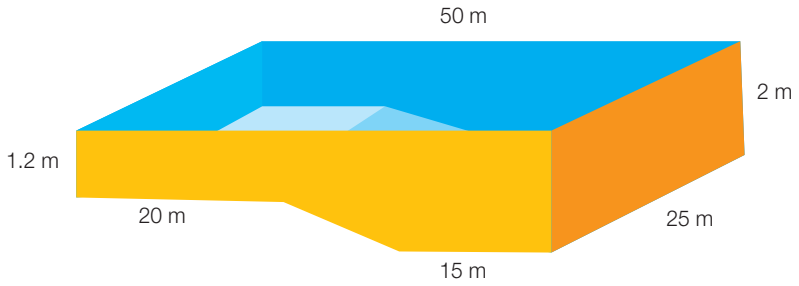


18 Ayse is inflating balloons with helium gas for a birthday party. Each balloon is roughly spherical in shape and about 40 cm in diameter. If she has to inflate two dozen balloons, find the total capacity (in litres) of helium required, rounded to the nearest whole number. 10A

19 The density of the material used to construct the pipe in Question **13** is 5.8 g/cm^3. Find the mass of the pipe. 10A

Reasoning

20 The local council swimming pool is shaped as shown.



(a) Find the capacity of the pool in both litres and kilolitres.

(b) It is proposed to build a pool that is 2 m deep for its entire length. If its other dimensions remain the same, how much extra water will be required for the new pool?

21 A cylindrical container has radius 7 cm and height 10 cm. What must be the height of a rectangular container 7 cm long and 10 cm wide if the two containers are to have the same volume? Write your answer correct to two decimal places.

22 Engineers wish to investigate building a tunnel through a mountain rather than having to build a road over the top. The road will cost $45 000 for every 10 m length that is laid, whether it is over the top or through the tunnel.

(a) The tunnel would be a semicircle of radius 10 m and would be 400 m in length. Calculate the volume of earth to be removed, to the nearest cubic metre.

(b) Find the cost (in whole dollars) of tunnelling and removal of dirt if this cost is estimated to be $150/m^3.

(c) Calculate the surface area of the curved ceiling/wall, to the nearest square metre. (Assume that the entire floor will become road.)

(d) Find the cost (in whole dollars) of fortifying the ceiling wall if the cost is $100/m^2.

(e) Find the cost (in whole dollars) of laying the road through the tunnel.

(f) What is the total cost (in whole dollars) of construction of the road through the tunnel?

The road over the top of the mountain would be 1.4 km long.

(g) Find the cost of laying the road over the top of the mountain.

(h) Which is the cheaper option and by how much?

23 Mert owns a large pool initially containing 1000 L of fresh water. He Mert adds 100 L of salt water, with a concentration of 52 g/L, to the pool, causing the water to become salty. Assuming the salt spreads evenly throughout the pool, what is the concentration of salt in the pool? Write your answer correct to two decimal places. 10A

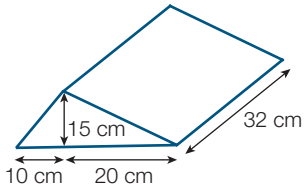
Mixed review A

Fluency

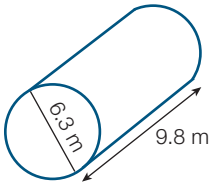
1 Find the interest earned, to the nearest cent, on an investment of $2500 at 6.5% p.a. compounded monthly for 5 years. Ex.1.1

2 Find the surface area of each of the following solids. Where necessary, give your answer correct to two decimal places. Ex.3.2

(a)



(b)



3 Expand and simplify the following. Ex.2.1

(a) $(x - y)(3x - 2y)$ **(b)** $(3a - 6b)(2a - 5b)$ **(c)** $(5t + 4h)(3t - 9h)$

4 Find the effective rate of interest if $10 000 is invested for 1 year at the following rates. Ex.1.4

(a) 7% simple interest **(b)** 7% compounded annually

(c) 7% compounded quarterly **(d)** 7% compounded monthly

5 Fully factorise the following expressions. Ex.2.4

(a) $x^2 - 4x + 4$ **(b)** $x^2 - 9$ **(c)** $x^2 + 14x + 49$

(d) $100x^2 - 25$ **(e)** $4xy^2 + 2xy - 8x$ **(f)** $4x^2 + 32x + 64$

6 Simplify the following. Ex.2.6

(a) $\frac{x}{5} - \frac{3x}{7}$ **(b)** $\frac{x+3}{5} + \frac{x+5}{6}$

7 Expand and simplify. Ex.2.1

(a) $6(x + 11) - x(x + 11)$ **(b)** $4(x + 6) - x(x + 6)$

(c) $2x(x + 4) - 3(x + 4)$ **(d)** $7y(y - 1) + 2(y - 1)$

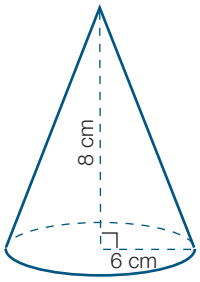
8 Find, to the nearest cent, the total amount owing on a loan of $4600 for 5 years if the 3.7% p.a. interest is compounded: Ex.1.2

(a) annually **(b)** quarterly **(c)** monthly.

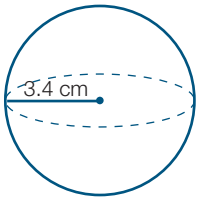
Assume no repayments are made in the time period.

9 Find the volume of each of the following solids. Round your answers to two decimal places. Ex.3.5 10A

(a)



(b)



10 Factorise each expression.

(a) $x^2 - 6x - 27$ **(b)** $x^2 - 7x + 10$ **(c)** $6x^2 - x - 2$

Ex 2.3

11 If the total mass of dissolved solids in a 30 mL sample of lake water is found to be 5 g, what is the concentration in g/L? State your answer correct to one decimal place.

Ex 3.6 10A

12 Expand and simplify:

(a) $(5x - 1)^2$ **(b)** $(4x - 3)^2$ **(c)** $(8 - 3x)^2$ **(d)** $(6 - 5x)^2$

Ex 2.2

Understanding

13 Alana has $5000 to invest for 3 years. Which is a better value arrangement?

Option A: 7.5% compounded quarterly Option B: 6.8% compounded monthly

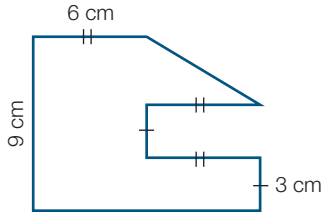
14 Calculate, to the nearest cent, the value after 4 years of a $2300 laptop if it is depreciated in the following ways.

(a) 15% straight-line depreciation **(b)** 15% reducing balance depreciation

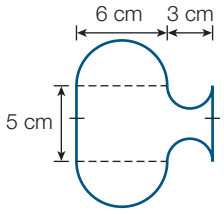
15 A cube has a volume of 512 cm^3. Calculate its surface area.

16 Find the area of each of the following shapes. Where necessary, give your answer correct to two decimal places.

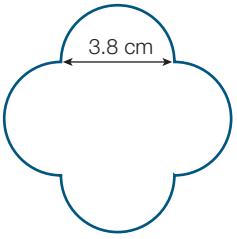
(a)



(b)



(c)



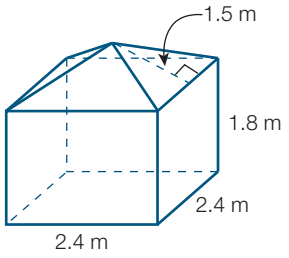
Reasoning

17 Scott wishes to erect a greenhouse. The support structure shown is made of metal. Using the plan shown, calculate correct to one decimal place:

(a) the amount of glass needed for the walls and roof

(b) the length of metal needed for the support structure.

10A



18 A square piece of carpet of side length x is extended by 5 m in length and by 3 m in width.

(a) Write an algebraic expression for the area of the extended carpet, in factorised form.

(b) Expand your answer to **(a)** and simplify by collecting like terms.

(c) If $x = 4$, find the area of:

(i) the original **(ii)** the extended piece of carpet.

(d) If the original piece of carpet cost $128, suggest a comparable price for the extended piece of carpet.

19 How many mL of tea tree oil should be added to 180 mL of water to make a 10% solution? 10A

Challenge A



1 Calculate $1000 - 1 + 3 - 5 + 7 \ldots$ to 102 terms (numbers in the sequence).

2 Damo has \$733 to spend in the airport shop before flying home. If he spent all his money on four different kinds of gifts for his 20 friends and relatives, priced at \$18, \$23, \$52 and \$69, how many of each gift did he buy?

3 A music store had a sale on CDs. For every two CDs that you bought at the normal price, you could purchase a third CD for \$1. If 12 CDs were bought for \$120, then the regular price for a CD was:

A \$15.00 **B** \$14.50 **C** \$14.00 **D** \$10.00

4 How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

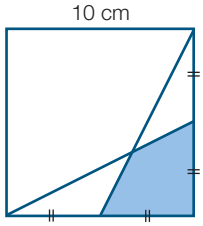
A 1 **B** 2 **C** 3 **D** 4

5 p, q and r are three different positive integers, and $(pq)^2 = pqr$.

Which of the following is a possible value for r?

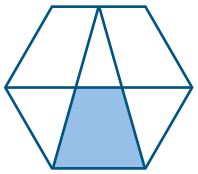
A 1 **B** 3 **C** 4 **D** 6

6 Find the area of the shaded region in the diagram.



7 A square fits tightly in an equilateral triangle with its base on the triangle's base. What percentage of the area of the triangle does it occupy?

8 Find the area of the shaded trapezium shown if the regular hexagon has a side length of 40 cm.

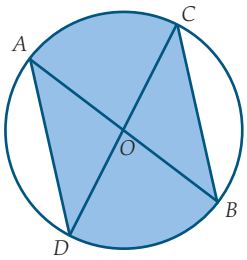


9 In the diagram, AB and CD are diameters of a circle of radius 4 cm.

If AB is perpendicular to CD, the area of the shaded region is:

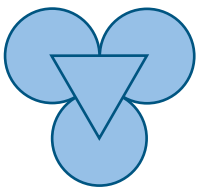
A $8\pi + 4$ **B** $8\pi + 8$

C $16 + 4\pi$ **D** $16 + 8\pi$



10 What is the side length of a square that just fits symmetrically within an isosceles triangle of sides 5 cm, 10 cm and 10 cm?

11 Find the volume of stone in a pillar, 6 m high, the cross-section of which is based on a equilateral triangle of 30 cm side, having three circular sectors drawn with their centres as the vertices of the equilateral triangle. Give your answer in m^2 correct to three decimal places.



Hint

Remember that integers are positive or negative whole numbers or zero.

4 Linear relationships

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R4.1

1 If $P = 2t - 7$, find P when:

(a) $t = 0$ **(b)** $t = 5$

(c) $t = -4$ **(d)** $t = 2$

Worksheet R4.2

2 Solve the following linear equations.

(a) $2a - 3 = 11$ **(b)** $\frac{b}{4} + 7 = 13$

(c) $3(2c - 5) = 45$ **(d)** $\frac{2d + 6}{5} = 4$

Worksheet R4.3

3 Plot these coordinate points on a Cartesian plane.

(a) $A(4, 3)$ **(b)** $B(0, 2)$

(c) $C(-1, 2)$ **(d)** $D(-2, -3)$

Worksheet R4.4

4 Copy and complete the table of values below for each of the following equations, then plot each graph for $-2 \leq x \leq 2$ on a Cartesian plane.

x	-2	-1	0	1	2
y					

(a) $y = x + 3$ **(b)** $y = x - 2$

(c) $y = 2 - x$ **(d)** $y = 3x$

Worksheet R4.5

5 Sketch the graphs of:

(a) $y = 3$ **(b)** $x = 2$

Worksheet R4.6

6 Write down **(i)** the gradient and **(ii)** the y-intercept of the graphs for each of the following equations.

(a) $y = 3x + 5$ **(b)** $y = 3 - 2x$ **(c)** $2x + 3y = 4$

Worksheet R4.7

7 Find the gradient of the line joining the points with coordinates:

(a) (1, 3) and (4, -2) **(b)** (-3, -6) and (2, -1)

Worksheet R4.8

8 **(a)** Sketch the graphs of $x = 1$ and $y = 3$ on the same set of axes.

(b) Find the point of intersection of the two graphs.

After completing this chapter you will be able to:

- solve linear equations
- find the gradient of the line joining two points or from an equation
- sketch linear graphs
- find parallel and perpendicular equations to given lines
- solve simultaneous equations
- find the solution sets for linear equations.

Linear equations

4.1

Need to Know

A **linear relationship** exists between two variables if the graph of the relationship is a **linear graph** (a straight line). In a linear relationship, a uniform change in one variable, the **independent variable**, produces a uniform change in the other, the **dependent variable**.

A linear relationship is described by a **linear equation**.

To solve linear equations of the type $5a - 3 = 2a + 6$ where the unknown appears on both sides of the equation, we must first add or subtract a variable term on both sides of the equation so that the variable appears on one side of the equation only (usually the LHS). If the equations involve brackets, we can expand the brackets using the Distributive Law $a(b + c) = ab + ac$.

To solve a linear equation containing fractions, we find the lowest common multiple (LCM) of the denominators, called the lowest common denominator (LCD). We then multiply both sides of the equation by the LCD. This simplifies the equation by eliminating the fractions.

Worked Example 1

WE1

Solve each of the following equations to find the value of the variable.

(a) $5x - 6 = 34$ **(b)** $\frac{3a}{2} + 4 = -1$

Working

(a) $5x - 6 = 34$
$5x = 40$
$x = 8$

Check:

LHS $= 5x - 6$
$= 5 \times 8 - 6$
$= 34$
$=$ RHS

(b) $\frac{3a}{2} + 4 = -1$

$\frac{3a}{2} = -5$

$3a = -10$

$a = \frac{-10}{3}$

$= -3\frac{1}{3}$

Check:

$\text{LHS} = \frac{3a}{2} + 4$

$= \frac{3 \times -3\frac{1}{3}}{2} + 4$

$= \frac{3 \times -\frac{10}{3}}{2} + 4$

$= -\frac{10}{2} + 4$

$= -5 + 4$

$= -1$

$= \text{RHS}$

Worked Example 2

WE2

Solve each of the following equations to find the value of the variable.

(a) $5a - 3 = 2a + 6$ **(b)** $2a - 5 = 6a + 1$ **(c)** $4(k + 1) = 3(6 - k)$

Working

(a) $5a - 3 = 2a + 6$

$3a - 3 = 6$

$3a = 9$

$a = 3$

Check:

$\text{LHS} = 5a - 3$

$= 5 \times 3 - 3$

$= 15 - 3$

$= 12$

$\text{RHS} = 2a + 6$

$= 2 \times 3 + 6$

$= 6 + 6$

$= 12$

$\text{LHS} = \text{RHS}$

(b) $2a - 5 = 6a + 1$

$6a + 1 = 2a - 5$

$4a + 1 = -5$

$4a = -6$

$a = -\frac{6}{4}$

$= -\frac{3}{2}$

Check:

$\text{LHS} = 2a - 5$

$= 2 \times -\frac{3}{2} - 5$

$= -3 - 5$

$= -8$

$\text{RHS} = 6a + 1$

$= 6 \times -\frac{3}{2} + 1$

$= -9 + 1$

$= -8$

$\text{LHS} = \text{RHS}$

(c) $4(k + 1) = 3(6 - k)$

$4k + 4 = 18 - 3k$

$7k + 4 = 18$

$7k = 14$

$k = 2$

Check:

$\text{LHS} = 4(k + 1)$

$= 4(2 + 1)$

$= 4 \times 3$

$= 12$

$\text{RHS} = 3(6 - k)$

$= 3(6 - 2)$

$= 3 \times 4$

$= 12$

$\text{LHS} = \text{RHS}$

Worked Example 3

WE3

Solve each of the following equations to find the value of the variable.

(a) $\frac{2x+1}{3} = \frac{x-3}{5}$ **(b)** $\frac{x}{3} - 2 = \frac{x-1}{4}$

Working

Method 1: Multiplying by the LCD

(a) LCD = 15

$$\frac{2x+1}{3} = \frac{x-3}{5}$$

$$\frac{15(2x+1)}{3} = \frac{15(x-3)}{5}$$

$$5(2x+1) = 3(x-3)$$

$$10x+5 = 3x-9$$

$$7x+5 = -9$$

$$7x = -14$$

$$x = -2$$

Check:

$$\text{LHS} = \frac{2 \times -2+1}{3} = \frac{-3}{3} = -1$$

$$\text{RHS} = \frac{-2-3}{5} = \frac{-5}{5} = -1$$

LHS = RHS

(b) LCD = 12

$$\frac{x}{3} - 2 = \frac{x-1}{4}$$

$$\frac{12x}{3} - 12 \times 2 = \frac{12(x-1)}{4}$$

$$4x - 24 = 3(x-1)$$

$$4x - 24 = 3x - 3$$

$$x - 24 = -3$$

$$x = 21$$

Check:

$$\text{LHS} = \frac{21}{3} - 2 = 7 - 2 = 5$$

$$\text{RHS} = \frac{21-1}{4} = \frac{20}{4} = 5$$

LHS = RHS

Method 2: Writing all terms as fractions using the LCD

(a) LCD = 15

$$\frac{2x+1}{3} = \frac{x-3}{5}$$

$$\frac{5(2x+1)}{15} = \frac{3(x-3)}{15}$$

$$5(2x+1) = 3(x-3)$$

$$10x+5 = 3x-9$$

$$7x+5 = -9$$

$$7x = -14$$

$$x = -2$$

Check:

$$\text{LHS} = \frac{2 \times -2+1}{3} = \frac{-3}{3} = -1$$

$$\text{RHS} = \frac{-2-3}{5} = \frac{-5}{5} = -1$$

LHS = RHS

(b) LCD = 12

$$\frac{x}{3} - 2 = \frac{x-1}{4}$$

$$\frac{4x}{12} - \frac{24}{12} = \frac{3(x-1)}{12}$$

$$4x - 24 = 3(x-1)$$

$$4x - 24 = 3x - 3$$

$$x = 21$$

Check:

$$\text{LHS} = \frac{21}{3} - 2 = 7 - 2 = 5$$

$$\text{RHS} = \frac{21-1}{4} = \frac{20}{4} = 5$$

LHS = RHS

Worked Example 4

WE4

Write an equation for the following and solve it to find the unknown numbers.

The sum of three consecutive numbers is 21.

Working

Let the first number be x.
Second number would be $x + 1$.
Third number would be $x + 2$.

$$x + (x + 1) + (x + 2) = 21$$
$$3x + 3 = 21$$
$$3x = 18$$
$$x = 6$$

Check:

$$\text{LHS} = 6 + (6 + 1) + (6 + 2)$$
$$= 6 + 7 + 8$$
$$= 21$$
$$= \text{RHS}$$

The three consecutive numbers are 6, 7 and 8.

Answers page 496

4.1 Linear equations

Fluency

WE1

1 Solve the following equations to find the value of the given variable.

(a) $4x + 3 = 15$ **(b)** $4x + 1 = 33$ **(c)** $3c + 7 = -2$

(d) $5a - 3 = 17$ **(e)** $7x - 2 = -44$ **(f)** $7d - 5 = -26$

(g) $6 - 5m = -4$ **(h)** $9 - 4n = 5$ **(i)** $-2 - 3x = 1$

(j) $19 = 6f + 7$ **(k)** $-9 = 4e - 1$ **(l)** $-3 = 5p - 4$

(m) $\frac{x}{8} - 5 = -5$ **(n)** $\frac{g}{7} + 2 = 5$ **(o)** $\frac{h}{4} - 7 = -3$

(p) $\frac{2k}{3} + 1 = 3$ **(q)** $\frac{5x}{3} - 2 = 8$ **(r)** $\frac{7y}{2} + 3 = 3$

(s) $3 - \frac{4x}{5} = 2$ **(t)** $6 - \frac{2x}{3} = 4$ **(u)** $4 - \frac{5x}{7} = -1$

WE2

2 Solve the following equations to find the value of the given variable.

(a) $3a + 7 = a + 9$ **(b)** $9d + 4 = 2d - 3$ **(c)** $5e + 2 = 7 - e$

(d) $7c - 1 = 4c + 5$ **(e)** $4b - 3 = 2b + 5$ **(f)** $4f - 5 = 8 - 2f$

(g) $g - 11 = 5 - g$ **(h)** $2h + 3 = 7 - 3h$ **(i)** $4k + 13 = 7 - 2k$

(j) $2 - 3x = 2x + 7$ **(k)** $3 - 4x = 2x - 3$ **(l)** $-6 - x = 2x + 1$

(m) $2(x + 3) = 6x$ **(n)** $3(2x + 1) = 18x$ **(o)** $5(7x + 2) = 65x$

(p) $3(m + 2) = m - 5$ **(q)** $2(3d - 5) = 2d + 7$ **(r)** $4(2d - 3) = 6d + 2$

(s) $5(a - 2) = 3(a + 2)$ **(t)** $7(a + 2) = 2(2a - 1)$ **(u)** $4(2g - 3) = 5(3g - 4)$

(v) $7(2t + 4) = 4(7 - 2t)$ **(w)** $5(2h + 3) = 4(3h + 1)$ **(x)** $2(4j + 3) = 3(1 - j)$

3 Solve the following equations to find the value of the given variable. WE3

(a) $\frac{a+7}{3} = \frac{a-3}{5}$ **(b)** $\frac{b-2}{2} = \frac{b+1}{3}$ **(c)** $\frac{2c-6}{3} = \frac{c+7}{4}$

(d) $\frac{3d+1}{2} = \frac{d-3}{4}$ **(e)** $\frac{2x+1}{3} = \frac{x-3}{5}$ **(f)** $\frac{5y-2}{3} = \frac{y+3}{4}$

(g) $\frac{3-2m}{3} = \frac{5-4m}{4}$ **(h)** $\frac{1-4k}{4} = \frac{3k-7}{2}$ **(i)** $\frac{2-3t}{4} = \frac{t-5}{3}$

(j) $\frac{a+7}{3} + 2 = -4$ **(k)** $\frac{p-3}{4} - 1 = 2$ **(l)** $\frac{m-8}{5} - 2 = -9$

(m) $\frac{3p+1}{2} - \frac{2p}{5} = 6$ **(n)** $\frac{3n-1}{5} + \frac{n}{2} = 2$ **(o)** $\frac{3t+2}{4} - \frac{2t}{3} = -1$

(p) $\frac{4m-3}{7} + \frac{2m+5}{2} = 3$ **(q)** $\frac{2k+3}{2} + \frac{5-k}{3} = 2$ **(r)** $\frac{2x+3}{4} - \frac{3x-5}{3} = 2$

4 Write an equation for each of the following and solve it to find the unknown numbers. WE4

(a) 6 is added to x to give a result of 13.

(b) 7 is subtracted from y to give a result of -2.

(c) Twice a certain number is 17.

(d) One-third of a certain number is 12.

(e) Two more than three times a number is 23.

(f) Five less than four times a number is -3.

5 (a) The equation $3a + 5 = -1$ has the solution:

A $a = \frac{4}{3}$ **B** $a = -\frac{1}{2}$ **C** $a = -2$ **D** $a = -18$

(b) The equation $\frac{d}{7} + 3 = -2$ has the solution:

A $d = 7$ **B** $d = -\frac{5}{7}$ **C** $d = -7$ **D** $d = -35$

(c) The equation $\frac{2x}{3} - 4 = 4$ has the solution:

A $x = -48$ **B** $x = 0$ **C** $x = 3$ **D** $x = 12$

6 (a) $3x + 8 = 2 - x$ has the solution:

A $x = -3$ **B** $x = -\frac{3}{2}$ **C** $x = \frac{2}{3}$ **D** $x = 3$

(b) $7(y - 3) = 5(15 - y)$ has the solution:

A $y = 9$ **B** $y = 8$ **C** $y = 7$ **D** $y = 0$

(c) $\frac{3m-2}{2} = \frac{m+11}{3}$ has the solution:

A $m = 0$ **B** $m = 2$ **C** $m = 4$ **D** $m = 7$

Understanding

Hint

Add 2 to get the next even number.

7 The sum of four consecutive numbers is 38. What numbers are they?

8 Find three consecutive even numbers whose sum is 126.

9 Sam divided his bag of jubes so that he and his friends Nikki and Minh could share them. He gave himself twice as many as he gave Nikki, and Minh received three times as many as Nikki. If the bag contained 42 jubes, how many did each person receive?

10 If the perimeter of the rectangular block of land shown in the photograph below is 300 m, find the length and width of the block.



11 Suppose that p represents Penny's age.

(a) How old will she be in 7 years' time?

(b) Write an expression to represent twice her age in 7 years' time.

(c) What is Penny's age now if twice her age in 7 years' time is 48?

12 The width of a rectangular block of land is $\frac{2}{7}$ of its length. If its perimeter is 252 m, find its length.

13 To travel to a concert, Olivia and her friends decided to walk to the train station. The trip on the train covered 6 times the distance they had walked. They then rode in a taxi for the final 2 km to the concert venue. Altogether they travelled 12.5 km. How far did Olivia and her friends walk, and how far did they travel by train?

14 Ross is twenty years older than his niece Krystal. Krystal has worked out that in five years' time her uncle will be twice as old as she will be. Find Krystal's age.

15 If $\frac{3}{5}$ of a number is subtracted from $\frac{2}{3}$ of the same number, the result is 3. Find the number.

Reasoning

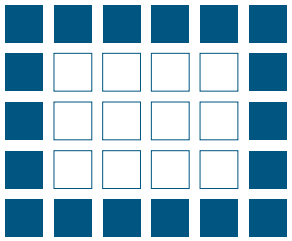
16 Amy is twice as old as Lucy. Three years from now the sum of their ages will be 42. How old is Amy?

17 Two cars start at the same time from the same point and travel in opposite directions. One car is travelling 10 km/h faster than the other. In 3 hours they are 288 km apart. Find the speed of each car.

Open-ended

18 Write three different equations, each having a solution of -2, where at least three mathematical operations are used.

19 (a) Mrs Grey wants Tim the tiler to lay square tiles to make a bordered rectangle with white tiles on the interior and blue tiles on the border. For his first attempt, Tim comes up with the following design, which contains 18 blue and 12 white tiles. If the rectangle remains 3 white tiles wide, but the number of white tiles in the length increases to l, how many blue tiles will be required for the border?



(b) **(i)** Draw a different rectangular design that uses 12 white tiles.

(ii) How many blue tiles would now be needed?

(iii) If the number of white tiles in the width remains constant, but the number of white tiles in the length increases to t, how many blue tiles will be required for the border?

(c) Choose a different number of white tiles to make a rectangular pattern and find the number of blue tiles (n) needed to make a border.

Gradient

4.2

Need to Know

The **gradient** of a line connecting two points is the rate at which the lines rises or falls.

The gradient is the same at any point along a straight line.

The steeper the line, the greater the magnitude of the gradient.

The gradient of a line is the gradient of any interval within that line. The gradient is constant at any point on the line. The gradient of a line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

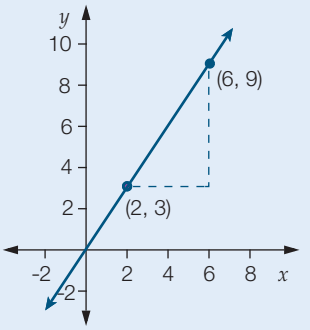
- The gradient is positive if the y-values increase as the x-values increase.
- The gradient is negative if the y-values decrease as the x-values increase.
- A horizontal line has zero gradient.
- A vertical line has an undefined gradient.
- Lines with the same gradient are parallel, $m_A = m_B$.
- Two lines are perpendicular if the product of their gradients is -1. One gradient is the negative reciprocal of the other. $m_A = -\frac{1}{m_B}$. Horizontal and vertical lines are perpendicular.

Worked Example 5

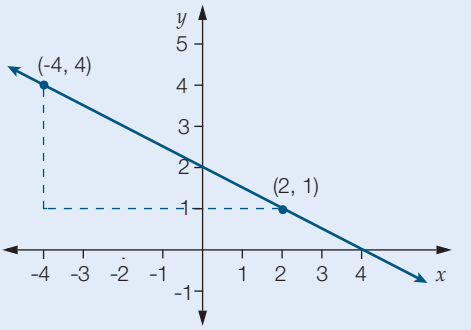
WE5

Find the gradient of each of the following lines through the given points.

(a) (2, 3) and (6, 9)



(b) (-4, 4) and (2, 1)



Working

(a) Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (6, 9)$.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9 - 3}{6 - 2}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

(b) Let $(x_1, y_1) = (-4, 4)$ and $(x_2, y_2) = (2, 1)$.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 4}{2 - (-4)}$$

$$= \frac{-3}{6}$$

$$= -\frac{1}{2}$$

Worked Example 6

WE6

Find the equation of a line with:

(a) a gradient of 2 that passes through the point (3, -5)

(b) a gradient of -4 that passes through the point (-4, 2).

Working

(a)

$$m = \frac{y - y_1}{x - x_1}$$

$$2 = \frac{y - -5}{x - 3}$$

$$2(x - 3) = y + 5$$

$$y = 2x - 6 - 5$$

$$y = 2x - 11$$

(b)

$$m = \frac{y - y_1}{x - x_1}$$

$$-4 = \frac{y - 2}{x - -4}$$

$$-4(x + 4) = y - 2$$

$$y - 2 = -4x - 16$$

$$y = -4x - 14$$

4.2 Gradient

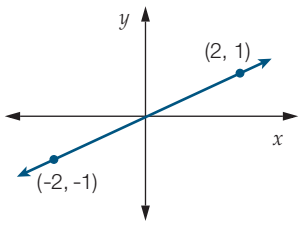
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Fluency

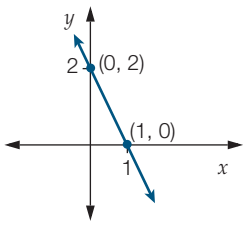
1 Find the gradient of each of the following lines through the given points.

WE5

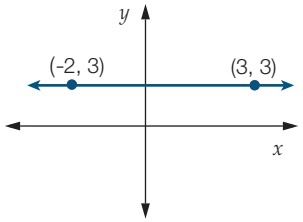
(a) (-2, -1) and (2, 1)



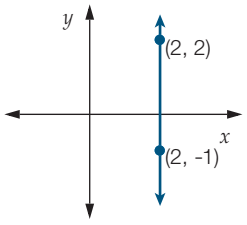
(b) (0, 2) and (1, 0)



(c) (-2, 3) and (3, 3)



(d) (2, -1) and (2, 2)



2 Find the equation of a line with:

WE6

(a) a gradient of 6 that passes through the point (1, 3)

(b) a gradient of -2 that passes through the point (4, -1)

(c) a gradient of 3 that passes through the point (-4, -3)

(d) a gradient of -5 that passes through the point (3, -7).

3 Use the results from Question **1** to answer the following questions.

(a) If a line is sloping up to the right, does it have a positive or a negative gradient?

(b) If a line is sloping down to the right, does it have a positive or a negative gradient?

(c) What is the gradient of a horizontal line?

(d) What is the gradient of a vertical line?

4 Find the gradient of the line joining each of the following pairs of points.

(a) (2, 4) and (5, 5)

(b) (0, 3) and (2, 7)

(c) (4, 10) and (5, 6)

(d) (7, 5) and (9, 2)

(e) (-2, 1) and (2, -4)

(f) (-1, -3) and (0, -5)

(g) (0, 0) and (2, 6)

(h) (2, 1) and (-2, -7)

(i) (-2, 5) and (5, -2)

(j) (-4, 4) and (2, 0)

(k) (-3, 2) and (2, 2)

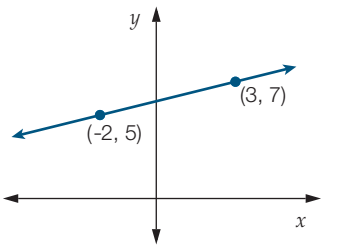
(l) (5, -4) and (-7, -4)

(m) (-8, 5) and (2, 1)

(n) (-1, -1) and (6, 0)

(o) (2, 5) and (2, 7)

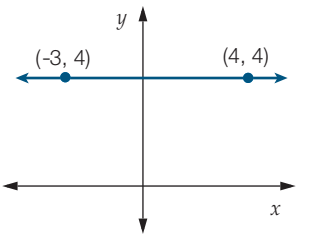
5 **(a)** The gradient of the line shown in the graph is:



A $\frac{5}{2}$ **B** 2

C $\frac{2}{5}$ **D** $-\frac{2}{5}$

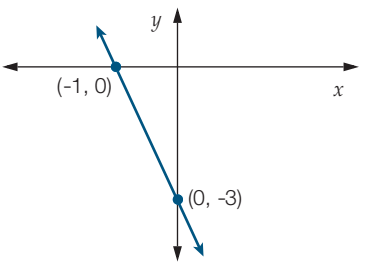
(b) The gradient of the line shown in the graph is:



A 8 **B** 0

C -7 **D** undefined

(c) The gradient of the line shown in the graph is:



A $\frac{1}{3}$ **B** $-\frac{1}{3}$

C 3 **D** -3

Understanding

6 **(a)** Use the results from Question **4** to find a pair of points that lie on:

(i) a horizontal line

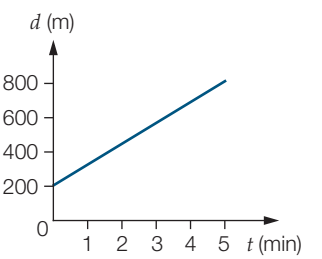
(ii) a vertical line

(iii) the line with the steepest negative gradient

(iv) the line with the smallest positive gradient.

(b) Use the results from Question **4** to find two pairs of points that would give two parallel non-horizontal lines.

7 Alex was 200 m from home and decided to walk to the local shop. This graph shows Alex's distance from home at (d) any particular time (t) on his 5-minute walk to the shop.



(a) How far was the shop from Alex's home?

(b) How far did Alex walk in 5 minutes?

(c) Find the gradient of this graph by first identifying two points that the line passes through.

(d) Find Alex's speed on the walk.

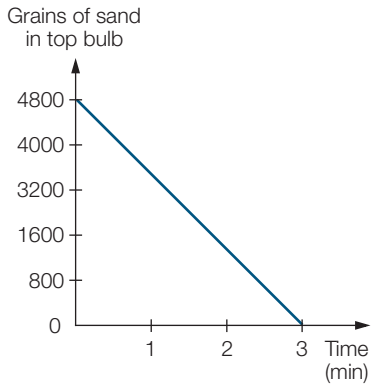
(e) How far was Alex from home after 2 minutes of walking?

(f) How far was Alex from home after 3 minutes of walking?

(g) Write down the relationship or rule to find the distance from home (d) after t minutes.

(h) The graph of Alex's trip is a straight line. What does this tell you about Alex's walking speed?

8 An egg-timer contains grains of sand which pour through the neck of the timer from the top bulb to the bottom in 3 minutes.



(a) How many grains of sand are in the egg-timer?

(b) How many grains of sand are left in the top bulb after 2 minutes?

(c) Find the gradient of this graph by identifying two points that the line passes through.

(d) Find the rate of change of sand grains with respect to time. In other words, what is the change in the number of sand grains left in the top bulb each minute?

(e) What does the negative represent in your answer to part **(c)**?

(f) Write down the relationship or rule to find the number of sand grains left in the top bulb (S) after t minutes.

(g) Use your rule to confirm your answer to **(b)**.

(h) What are the restrictions on the values that t can take?

Reasoning

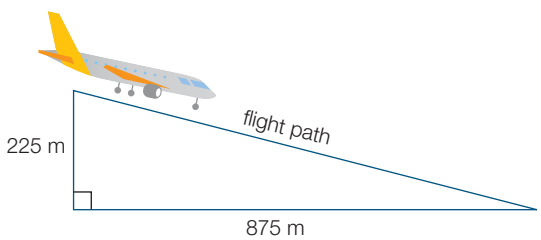
9 **(a)** Find the equation of the line that passes through (2, 4) and (4, 9):

(i) using the point (2, 4) as (x_1, y_1) **(ii)** using the point (4, 9) as (x_1, y_1)

(b) Did you get the same equation for parts **(i)** and **(ii)**? What conclusion can you make from your answers to parts **(i)** and **(ii)**?

10 The gradient of the line joining the points (2, 1) and (6, a) is 4. Find the value of a.

11 A passenger jet is landing on the flight path shown below. What is the gradient of the flight path?



Hint

Use the vertex of the right angle of the triangle as (0, 0).

12 Show that the points (-1, -5), (2, 4) and (5, 13) lie on the same straight line. This means that the points are collinear.

Open-ended

13 Identify three different sets of coordinates that when connected by a straight line produce each of the following gradients:

(a) -3 **(b)** -1 **(c)** $-\frac{1}{2}$ **(d)** 2 **(e)** 5

14 Write down the coordinates of two points such that a line joining them has the same gradient as the line joining (3, 5) and (7, -7).

4.3 Sketching linear graphs

Need to Know

The equation of a linear graph can be written in the form:

$$y = mx + b$$

where m is the gradient and b is the y-coordinate of the y-intercept $(0, b)$.

Linear graphs can be sketched using the y-intercept and gradient.

This method involves locating the point at which the graph crosses the vertical y-axis and then using the gradient to locate a second point. Once two points are known, a straight line can be drawn through them.

Linear graphs that are not in the form of $y = mx + b$ can be sketched using x- and y-intercepts. Note that for a **sketch** graph, only the x- and y-intercepts need to be shown on the axes; no other values for x and y need to be shown.

- To find the x-intercept, substitute $y = 0$ in the equation.
- To find the y-intercept, substitute $x = 0$ in the equation.

Other linear relationships between x and y can be of the form:

- $y = a$, where a is a constant, is the equation of a horizontal line parallel to the x-axis. The gradient of these lines is zero.
- $x = b$, where b is a constant, is the equation of a vertical line parallel to the y-axis. The gradient of these lines is undefined.

Worked Example 7

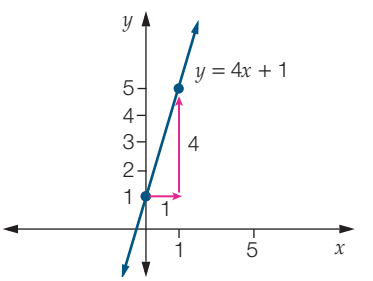
WE7

Find the y-intercept and the gradient and then sketch the graph of the line with equation $y = 4x + 1$.

Working

$y = 4x + 1$
$b = 1$
y-intercept is $(0, 1)$.

$m = 4$
Gradient is 4, so the rise is 4 and the run is 1.



Worked Example 8

WE8

Find the y-intercept and gradient, and then sketch the graph of the line with equation $3x + 2y = 4$.

Working

$3x + 2y = 4$

$2y = -3x + 4$

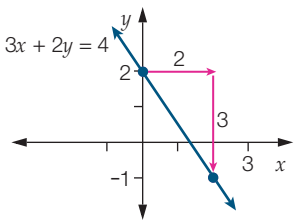
$y = \frac{-3x}{2} + 2$

$b = 2$

y-intercept is $(0, 2)$.

$m = -\frac{3}{2}$

Gradient is $-\frac{3}{2}$, so the rise is -3 and the run is 2.



Worked Example 9

WE9

Find the x- and y-intercepts for $2x + 3y = 12$ and sketch the graph.

Working

When $y = 0$, $2x + 3 \times 0 = 12$

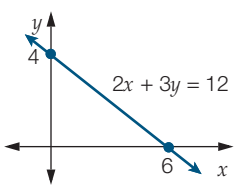
$2x = 12$, so $x = 6$

x-intercept is $(6, 0)$.

When $x = 0$, $2 \times 0 + 3y = 12$

$3y = 12$, so $y = 4$

y-intercept is $(0, 4)$.



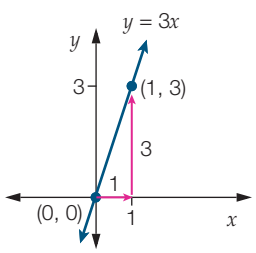
Worked Example 10

WE10

Sketch the graph of $y = 3x$, showing the y-intercept and one other point.

Working

Gradient = 3, so rise = 3 and run = 1.



Answers page 497

4.3 Sketching linear graphs

Fluency

WE7

1 Find the y-intercept and the gradient and then sketch a graph of each line.

(a) $y = 3x + 2$ **(b)** $y = 4x - 3$ **(c)** $y = -2x + 1$ **(d)** $y = -3x + 2$

WE8

2 Find the y-intercept and the gradient and then sketch a graph of each line.

(a) $x + 2y = -4$ **(b)** $x + 3y = 3$ **(c)** $3x + 4y = 8$

(d) $2x + 5y = -5$ **(e)** $2x - 3y = -1$ **(f)** $4x - 2y = 3$

WE9

3 Find the x- and y-intercepts and then sketch a graph of each line.

(a) $2x + 3y = 6$ **(b)** $4x + y = 4$ **(c)** $x - 2y = 4$

(d) $x - 3y = 9$ **(e)** $4x + y = 2$ **(f)** $3x + y = 1$

WE10

4 Sketch graphs of the following, showing the intercept on the axes and one other point for each.

(a) $y = 4x$ **(b)** $y = x$ **(c)** $y = -2x$ **(d)** $y = -3x$

5 **(a)** The equation of a linear graph with a y-intercept of 2 and a gradient of -3 is:

A $y = 3x + 2$ **B** $x + 3y = 2$ **C** $x - 3y = 2$ **D** $3x + y = 2$

(b) The equation of a linear graph with a gradient of $\frac{1}{2}$ and a y-intercept of -3 is:

A $2x - y = 6$ **B** $x - 2y = 6$ **C** $x - 2y = -6$ **D** $2x - y = -6$

(c) The equation of a linear graph with a y-intercept of $\frac{2}{5}$ and a gradient of $-\frac{1}{4}$ is:

A $5x + 20y = 8$ **B** $20x + 5y = 8$ **C** $y = \frac{2x}{5} - \frac{1}{4}$ **D** $\frac{2x}{5} - y + \frac{1}{4} = 0$

6 When using the y-intercept and gradient to sketch a graph, first mark in the y-intercept and then use the gradient to locate a second coordinate point.

(a) A line has a gradient of $\frac{2}{3}$. From the y-intercept, we move:

A 2 units to the right and 3 units up **B** 2 units to the right and 3 units down

C 3 units to the right and 2 units up **D** 3 units to the right and 2 units down

(b) A line has a gradient of $-\frac{3}{4}$. From the y-intercept, we move:

A 4 units right and 3 units up **B** 3 units right and 4 units up

C 4 units right and 3 units down **D** 3 units left and 4 units down

(c) A line has a gradient of 4. From the y-intercept, we move:

A 4 units right and 4 units up **B** 1 unit right and 4 units up

C 4 units right and 4 units down **D** 1 unit right and 4 units down

7 Sketch graphs of the following, showing the intercepts on the axes.

(a) $y = 1$ **(b)** $y = 4$ **(c)** $y = -5$ **(d)** $y = -2$

(e) $y = 0$ **(f)** $y = \frac{1}{2}$ **(g)** $x = 2$ **(h)** $x = 5$

(i) $x = -3$ **(j)** $x = -4$ **(k)** $x = \frac{5}{2}$ **(l)** $x = 0$

8 **(a)** Two coordinate points that lie on the line $2x + y = 4$ are:

A (4, 0) and (0, 3)
B (-4, 0) and (0, -2)
C (-2, 0) and (0, -4)
D (2, 0) and (0, 4)

(b) Two coordinate points that lie on the line $y = 6x$ are:

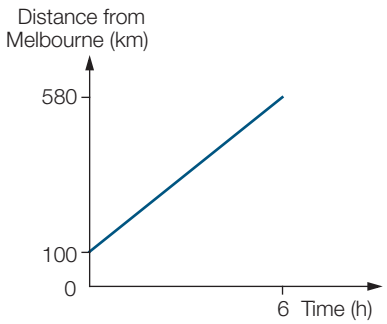
A (0, 6) and (6, 0)
B (0, 0) and (6, 6)
C (2, 12) and (0, 0)
D (2, 0) and (0, 3)

(c) Two coordinate points that lie on the line $x = -3$ are:

A (0, 0) and (1, 3)
B (-3, 1) and (1, -3)
C (-3, 0) and (0, 0)
D (-3, -5) and (-3, 0)

Understanding

9 **(a)** Consider the graph shown at right. Write down the intercept with the vertical axis.



(b) Find the gradient of this graph.

(c) Write an equation linking the distance from Melbourne in kilometres (D) and the time in hours (T) in the form $y = mx + b$.

(d) What constant speed was maintained throughout the journey?

(e) If the journey started 250 km from Melbourne and a constant speed of 100 km/h was maintained, what equation could be written to represent the relationship between D and T in the form $y = mx + b$.

(f) A particular trip from Melbourne is represented by the equation $4D - 372T = 30$. What constant speed was maintained throughout the journey?

10 Darren has borrowed $200 from his parents and has agreed to repay $40 each week.

(a) After 3 weeks, how much does Darren still owe his parents?

(b) Write an equation linking the amount Darren owes his parents (A) with the number of weeks (n).

(c) Find the amount owed when $n = 0$.

(d) Find the number of weeks when $A = 0$.

(e) Use your answers from **(c)** and **(d)** to sketch the graph of this relationship. (Put A on the vertical axis, and n on the horizontal axis.)

Reasoning

11 The annual yachting regatta of the Mathematical Club is to take place on Lake Tetrahedra. Starting and finishing at the marina, which can be considered to be coordinate point (0, 0), the yachts travel around six buoys in the order from 1 to 6, located at various positions on the lake. The first buoy is located at the point (2, 4). The second buoy is located at the point (-8, 8). The third buoy is located 10 units directly below the second buoy. The fourth buoy is located at the point (-3, 11). The fifth buoy is located 5 units to the right of the fourth buoy. The sixth buoy is located 4 units to the right of the first buoy.

(a) Plot each point on a Cartesian plane.

(b) Sketch the paths the yachts will take travelling to each buoy and back to the marina.

(c) Determine the equation of each path taken by the yachts between the:

(i) start of the race at the marina and the first bouy

(ii) first and second bouys

(iii) second and third bouys

(iv) third and fourth bouys

(v) fourth and fifth bouys

(vi) fifth and sixth bouys

(vii) sixth bouy and the finish of the race at the marina.

(d) The race referee's motor launch breaks down at the point (4, 7.5). Will this interfere with the paths taken by the yachts?

12 After her bath, Anne removes the plug to empty it. The water flows out of the bathtub at a rate of 0.75 L/s. There were originally 225 L of water in the bath.

(a) Write an equation relating the volume of water left in the bath (V) with the time in seconds (T).

(b) How long does it take to empty the bath?

13 Since the beginning of August, a local reservoir has been losing water at a constant rate. The Water Resources Department estimates that on 12 August the reservoir held 200 million kL of water, and on 21 August it held 164 million kL.

(a) Develop a linear equation relating V, the volume of water remaining (millions of kL), and n, the number of days since the beginning of August (when the reservoir was full).

(b) Draw a graph corresponding to the linear equation developed above to find the maximum capacity of the reservoir.

(c) Why should the graph only be drawn in the first quadrant?

14 A cubic block of ice has an edge of 10 cm. It melts so that its volume decreases at a constant rate (k) and the block remains a cube.

(a) What is the volume of the cube before it starts to melt?

(b) Write an equation relating (V) the volume of the cube after it has been melting for (t) minutes at a constant rate (k).

(c) What is the volume of the cube if it has an edge of 5 cm after 50 minutes?

(d) Find the value of k.

(e) What does the sign of k tell you about what is happening to the ice cube?

(f) Substitute the value of k found in part **(d)** to rewrite the equation found in part **(b)**.

(g) Draw a graph corresponding to the linear equation developed above to find how long it would take for the block of ice to completely melt.

Hint

The change in volume = kt.

15 At the start of the year, Ringo had \$600 in the bank. He has been saving nothing and spending \$20 a week. In the same time, Sophia, his sister, had \$100 in the bank. She has been saving \$30 a week and spending nothing.

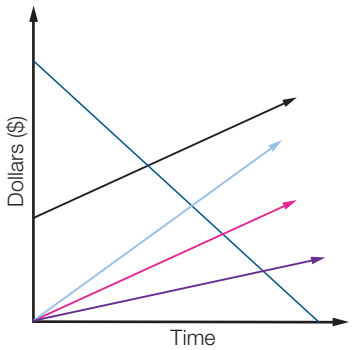
(a) Develop two linear equations relating the amounts of money R and S that Ringo and Sophia each have in their bank accounts, to t, the number of weeks since the start of the year.

(b) Draw a graph corresponding to the linear equations to identify how many weeks it will take for Sophia to have twice the amount of money in the bank as her brother.

Open-ended

16 Write three different linear equations that each have the same gradient, but with a different y-intercept. What property of the three lines indicates that they have the same gradient? Explain.

17 Linear equations are used every day to model real-life situations. The graph shows five lines that model the costs of five different mobile phone plans. Match each graph with a description below and provide an explanation behind each choice.



Note:

- The specific values of gradients and y-intercepts are not needed for this activity.
- The connection fee (also referred to as flagfall) is the fixed cost when a call is made.
- The call rate is the number of cents per minute that is charged based on the duration of the call.

(1) This graph is for the mobile phone plan that has no connection costs and the cost of the call is dependent on the length of the call.

(2) This graph models the cost of the phone plan with no connection fee and the rate at which the call is charged is half the rate of phone plan graph (1).

(3) This graph models the amount of credit remaining on a pre-paid monthly mobile phone plan.

(4) This graph models the cost of a phone plan with a connection fee for every call made and the call rate is equivalent to (1).

(5) This graph models the cost of the phone plan with no connection fee as in (1) but the rate at which the call is charged is one and a half times greater.

18 Sketch three different graphs that have gradients between -1 and -2, and y-intercepts between 1 and 4.

Problem solving

Odd one out

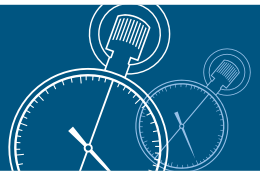
Four of these points lie on a single straight line. Which one is the odd one out?

(2, 5) (-5, -7) (2, 4) (5, 13) (9, 21)

Strategy options

- Draw a diagram.
- Look for a pattern.
- Test all possible combinations.

Half-time 4



Ex 4.3

1 Sketch the graph of the following, showing the intercepts on the axes.

(a) $y = 3x + 2$ **(b)** $y = 5 - x$

(c) $y = 2x - 3$ **(d)** $y = 6 - 3x$

Ex 4.2

2 Calculate the gradient of the line passing through each pair of points.

(a) (1, 3) and (5, 5) **(b)** (2, 4) and (6, -4)

(c) (-5, -3) and (5, 2) **(d)** (-4, 8) and (-2, -8)

Ex 4.3

3 Sketch the graph of the following, showing the intercepts on the axes.

(a) $2x - 3y = 6$ **(b)** $5x + 2y = 10$

(c) $3x - y = 6$ **(d)** $3x + 4y = 12$

Ex 4.1

4 The Celsius temperature scale is used in Australia, whereas the Fahrenheit temperature scale is used in the USA. The freezing point of water is 0° and the boiling point is 100° in the Celsius scale. The freezing point of water is 32° and the boiling point is 212° in the Fahrenheit scale. Show that the formula to convert a Fahrenheit temperature to a Celsius temperature can be expressed as $C = \frac{5}{9}(F - 32)$.

Ex 4.3

5 Sketch the graph of the following, showing the intercept with the axes and one other point.

(a) $y = -3x$ **(b)** $y = 4x$

(c) $x + 2y = 0$ **(d)** $3y = 5x$

Ex 4.1

6 Solve each of the following equations.

(a) $6x + 2 = 14$ **(b)** $3(x + 5) = 21$

(c) $\frac{2x - 3}{7} = 5$ **(d)** $3x + 4 = 16 - x$

(e) $\frac{2x + 4}{5} = \frac{3x - 3}{3}$ **(f)** $\frac{3x + 2}{2} - \frac{x + 10}{7} = 5$

Ex 4.3

7 Sketch the graph of each of the following.

(a) $y = 4$ **(b)** $x = -2$

(c) $y = -3$ **(d)** $x = 8$

Ex 4.2

8 Find the equation of the line with a:

(a) gradient of 4 passing through the point (-1, 5)

(b) gradient of -1 passing through the point (3, -7).

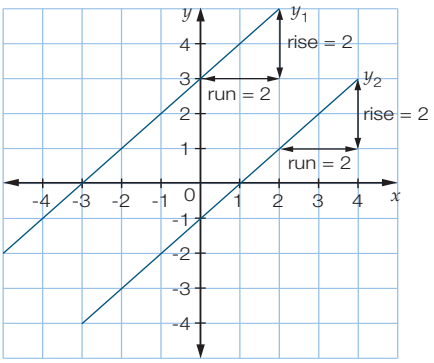
Parallel and perpendicular lines

4.4

Need to Know

Two lines that are parallel have the same gradient.

$$m_1 = m_2$$

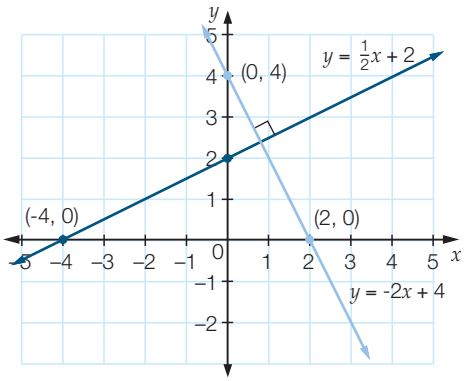


When lines are perpendicular, the product of their gradients is -1.

$$m_1 m_2 = -1$$

Therefore, the gradients m_1 and m_2 of perpendicular lines are the negative reciprocal of each other.

$$m_1 = -\frac{1}{m_2} \text{ and } m_2 = -\frac{1}{m_1}$$



Worked Example 11

WE11

Show that the line with equation $2y = 4x - 9$ is parallel to the line with equation $3y - 6x + 5 = 0$.

Working

$2y = 4x - 9$

$y = 2x - \frac{9}{2}$

$m_1 = 2$

$m_1 = m_2$

$3y - 6x + 5 = 0$

$3y = 6x - 5$

$y = 2x - \frac{5}{3}$

$m_2 = 2$

Therefore, the lines are parallel.

Worked Example 12

WE12

Show that the line with equation $y = 3x - 10$ is perpendicular to the line with equation $6y + 2x + 12 = 0$.

Working

$y = 3x - 10$

$m_1 = 3$

$6y + 2x + 12 = 0$

$6y = -2x - 12$

$y = -\frac{1}{3}x - 2$

$m_2 = -\frac{1}{3}$

$m_1 m_2 = 3 \times -\frac{1}{3}$

$= -1$

Therefore, the lines are perpendicular.

Answers page 500

4.4 Parallel and perpendicular lines

Fluency

WE11

1 **(a)** Show that the line with equation $5x - 2y = 7$ is parallel to the line with equation $2y = 5x + 11$.

(b) Show that the line with equation $10y = 5x + 3$ is parallel to the line with equation $3x - 6y = 2$.

WE12

2 **(a)** Show that the line with equation $4x + 3y = 7$ is perpendicular to the line with equation $8y = 6x - 2$.

(b) Show that the line with equation $2x - 5y - 1 = 0$ is perpendicular to the line with equation $4y + 10x = 3$.

3 Find the gradient of each of the following pairs of lines and determine if they are parallel.

(a) $y = 3x - 10$ and $y = 3x + 10$

(b) $y = 2x + 5$ and $y = -2x + 6$

(c) $y = \frac{1}{2}x - 4$and $y = \frac{1}{2}x + 5$

(d) $y + 4x = 4$ and $y = 4x - 8$

(e) $y - 5x = 9$ and $y = 9x + 5$

(f) $y - 2x = 9$ and $y - 2x - 4 = 0$

(g) $2y + 6x - 4 = 0$ and $y + 3x - 8 = 0$

(h) $4y + 12x - 4 = 0$ and $3y + 12x - 9 = 0$

(i) $-2y + 8x - 4 = 0$ and $4y - 16x - 8 = 0$

(j) $5x + 3y = 15$ and $3x - 15y = 10$

4 Find the gradient of each of the following pairs of lines and determine if they are perpendicular.

(a) $y = 2x - 10$ and $2y + x + 12 = 0$

(b) $y - 4x = 10$ and $4y + 2x + 12 = 0$

(c) $y = \frac{1}{3}x$ and $6y + 2x + 12 = 0$

(d) $6y - 3x = 24$ and $2y + 4x + 12 = 0$

(e) $-4y + 3x = 16$ and $8y + 12x + 12 = 0$

(f) $5y - 4x = 25$ and $8y + 5x + 12 = 0$

5 Which of the following lines is parallel to $y = 4x + 12$?

A $y = -\frac{1}{4}x + 12$ **B** $4y = 4x + 12$ **C** $2y - 8x + 6 = 0$ **D** $2y + 8x + 12 = 0$

6 Which of the following lines is perpendicular to $y = 3x - 9$?

A $y = 3x + 9$ **B** $6y = 9x + 12$ **C** $y = -3x - 9$ **D** $9y + 3x = 27$

Understanding

7 Show that the line joining (-2, 2) and (4, 8) is parallel to $y = x + 10$.

8 Show that the line joining (-2, 5) and (4, -7) is parallel to $2x + y - 10 = 0$.

9 Show that the line joining (-4, 1) to (0, 4) is perpendicular to $y = -\frac{4}{3}x - 2$.

10 Show that the line joining (4, 5) to (9, 8) is perpendicular to $y = -\frac{5}{3}x + 10$.

Reasoning

11 **(a)** Which two of the following lines are parallel?

(b) Which two of the following lines are perpendicular?

(i) $y = 3x + 3$

(ii) $3x + y = 3$

(iii) $3x - y = 3$

(iv) $3x + 3y = 2$

(v) $3y = x + 3$

12 Find the equation of the line through (2, -3) that is perpendicular to $y = -\frac{1}{3}x + 11$.

13 Find the equations of the lines through (0, 5) that are parallel and perpendicular to $2x + y - 10 = 0$.

14 Show that the points $A(-1, -1)$, $B(1, -4)$ and $C(2, 1)$ form a right-angled triangle ABC.

15 Show that the points $A(-3, 1)$, $B(-1, 3)$, $C(3, 1)$ and $D(1, -1)$ form the vertices of a parallelogram.

16 Show that the points $A(-4, 3)$, $B(-2, -2)$, $C(3, 0)$ and $D(1, 5)$ form the vertices of a square.

Open-ended

17 Identify three different straight-line equations that are:

(a) parallel to $3x - 6y + 9 = 0$ **(b)** perpendicular to $4x - 12y - 18 = 0$.

Problem solving

Area of a triangle

Find the area (in square units) of the triangle formed by the three lines whose equations are:

1 $y - x - 6 = 0$

2 $x - 2y = 3$

3 $x = -y + 6$

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

4.5 Linear inequalities

Need to Know

< means less than

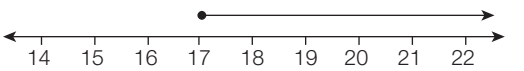
> means greater than

≤ means less than or equal to

≥ means greater than or equal to

An equation is a statement of equality, such as $x = 17$. A **linear inequality** is a statement such as $x \geq 17$, which means that x can be any value greater than or equal to 17.

On a number line, $x \geq 17$ can be represented as:



- If the inequality includes the end point, a closed circle • is used; these are inequalities that contain ≥ or ≤.
- If the inequality does not include the end point, an open circle ∘ is used; these are inequalities that contain > or <.

We solve linear inequalities using the same inverse operations in the opposite order as we use to solve linear equations.

When we multiply or divide both sides of the inequality by a negative number, the inequality sign is reversed.

Worked Example 13

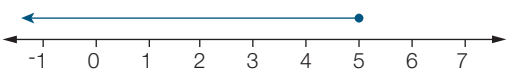
WE 13

Represent the values of x on a number line for each of the following.

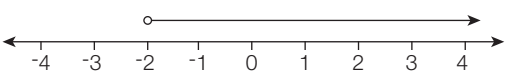
(a) $x \leq 5$ **(b)** $x > -2$ **(c)** $3 < x \leq 7$

Working

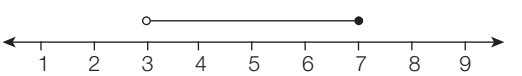
(a) $x \leq 5$



(b) $x > -2$



(c) $3 < x \leq 7$



Worked Example 14

WE 14

Solve for x in each of the following inequations.

(a) $x + 3 \geq 12$ **(b)** $4x - 3 < -7$

Working

(a)
$$x + 3 \geq 12$$
$$x + 3 - 3 \geq 12 - 3$$
$$x \geq 9$$

(b)
$$4x - 3 < -7$$
$$4x - 3 + 3 < -7 + 3$$
$$4x < -4$$
$$\frac{4x}{4} < \frac{-4}{4}$$
$$x < -1$$

Worked Example 15 WE15

Solve for x in each of the following.

(a) $-4x \leq 20$

(b) $9 - 2x \geq 5$

Working

(a) $-4x \leq 20$

$\frac{-4x}{-4} \leq \frac{20}{-4}$ (dividing by a negative number)

$x \geq -5$ (reverse inequality symbol)

(b) $9 - 2x - 9 \geq 5 - 9$

$\frac{-2x}{-2} \geq \frac{-4}{-2}$ (dividing by a negative number)

$x \leq 2$ (reverse inequality symbol)

4.5 Linear inequalities

Answers page 501

Fluency

1 Represent the values of x on a number line for each of the following. WE13

(a) $x \geq 5$ **(b)** $x \leq 8$ **(c)** $x \geq -2$

(d) $x < -4$ **(e)** $x > -5$ **(f)** $x > 7$

(g) $x \leq 0$ **(h)** $x \geq 2$ **(i)** $x \leq -5$

(j) $x > 1$ **(k)** $x > 0$ **(l)** $x < -2$

(m) $-2 \leq x \leq 4$ **(n)** $1 \leq x \leq 6$ **(o)** $0 < x < 5$

(p) $-3 < x < -1$ **(q)** $-4 \leq x < -2$ **(r)** $-4 < x \leq 0$

2 Solve for x in each of the following. WE14

(a) $x + 6 > -4$ **(b)** $x - 3 < -8$ **(c)** $x + 12 \leq 2$

(d) $\frac{x}{5} \leq 7$ **(e)** $\frac{x}{9} \geq -1$ **(f)** $\frac{x}{7} > -4$

(g) $3x < -16$ **(h)** $5x > -27$ **(i)** $9x \leq -81$

(j) $7x - 3 \geq 53$ **(k)** $4x + 11 \leq 3$ **(l)** $3x + 24 > 0$

(m) $\frac{4x - 1}{5} > 3$ **(n)** $\frac{2x + 5}{3} < 9$ **(o)** $\frac{3x + 2}{7} \leq 2$

(p) $\frac{3x}{4} + 1 \leq -5$ **(q)** $\frac{7x}{11} - 2 > 5$ **(r)** $\frac{4x}{5} - 10 > -2$

WE15

3 Solve for y in each of the following.

(a) $-3y \geq 15$ **(b)** $-8y \leq 48$ **(c)** $-5y \leq 25$

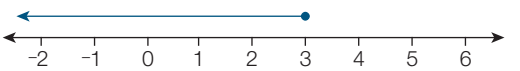
(d) $-y < -12$ **(e)** $-2y > -42$ **(f)** $-3y > -36$

(g) $1 - 2y \leq 17$ **(h)** $8 - 5y \geq -2$ **(i)** $5 - 3y \leq 17$

(j) $5(2 - 3y) \geq 11$ **(k)** $4(3 - 7y) \leq -2$ **(l)** $3(5 - 2y) \geq -4$

(m) $\frac{8 - 3y}{5} > -1$ **(n)** $\frac{14 - 6y}{3} < -3$ **(o)** $\frac{12 - 5y}{4} > -2$

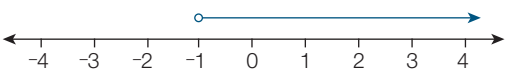
4 (a)



This number line represents:

A $x < 3$ **B** $x \leq 3$ **C** $x \geq 3$ **D** $x > 3$

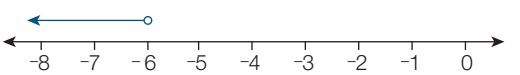
(b)



This number line represents:

A $x < -1$ **B** $x \leq -1$ **C** $x \geq -1$ **D** $x > -1$

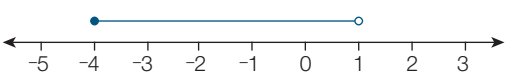
(c)



This number line represents:

A $x < -6$ **B** $x \leq -6$ **C** $x \geq -6$ **D** $x > -6$

(d)



This number line represents:

A $-4 < x \leq 1$ **B** $-4 \geq x > 1$ **C** $-4 \leq x \leq 1$ **D** $-4 \leq x < 1$

5 Which alternative is *not* a possible solution to the inequality given?

(a) $2x + 3 \geq 7$

A $x = 1$ **B** $x = 2.1$ **C** $x = 3$ **D** $x = 5.2$

(b) $5x - 2 < -9$

A $x = -5$ **B** $x = -3$ **C** $x = -2$ **D** $x = 0$

(c) $11 - 4x < -1$

A $x = 10$ **B** $x = 7$ **C** $x = 4$ **D** $x = 2$

(d) $\frac{6x + 1}{2} > 5x - 2$

A $x = -2$ **B** $x = -1$ **C** $x = 1$ **D** $x = 2$

Understanding

6 The reserve price at an auction of a house is $270 000. The house is sold at auction.

(a) Represent the selling price (P) on a number line.

(b) Write down the linear inequality for the selling price (P).

(c) Write down the inequality for the selling price (P) given that the house sold for less than $320 000.

7 The speed limit on a particular freeway is 100 km/h. Write down an inequality that represents all possible legal speeds on that freeway.

Hint

The reserve price is the smallest amount that the house could be sold for.

8 A sales representative needs to sell at least five vacuum cleaners a week to receive a bonus commission. Write down an inequality for the number of vacuum cleaners (n) the sales representative needs to sell if he wants to earn a bonus.

9 For the wool and other materials to make seven hand-knitted jumpers, Lucy and Lindsay spent $385. Write down an inequality to show the selling price (j) for each jumper if they wish to make a profit.

10 Find the solutions for each of the following.

(a) $5x - 3 \geq 3 - 4x$ **(b)** $2x + 7 \leq 4 - 9x$ **(c)** $6x + 2 < 3x + 2$

(d) $2(x + 10) > 10 - 3x$ **(e)** $15x - 4 \leq 4(3x + 2)$ **(f)** $5(2x - 3) \geq 8x + 9$

(g) $\frac{2x+1}{7} > \frac{x-3}{4}$ **(h)** $\frac{3x-4}{2} < \frac{4x+7}{5}$ **(i)** $\frac{2-3x}{2} \leq \frac{5x-16}{3}$

11 Find the solutions for each of the following.

(a) $y + 6 \leq 2y + 7$ **(b)** $20 - 3y > 5 + 2y$ **(c)** $7y - 4 \geq 3 + 10y$

(d) $13 - y < 5y + 11$ **(e)** $4 - \frac{y}{3} \leq -6$ **(f)** $7 - \frac{y}{6} \geq 1$

(g) $-3 - \frac{5y}{12} < 2$ **(h)** $\frac{-7y}{3} + 2 > 0$ **(i)** $\frac{3y+2}{11} > \frac{2y-5}{3}$

Hint

When multiplying or dividing by a negative, the inequality sign reverses.

Reasoning

12 A shopkeeper can fit a total of only 120 2-litre cartons of milk (both full cream and low fat) into the refrigerator. He knows that he needs at least 50 2-litre cartons of full cream milk and that no more than 40 2-litre cartons of low fat milk will be needed.

(a) Let f represent the number of cartons of full cream milk and l represent the number of cartons of low fat milk. Write down the two inequalities to show the number of cartons of each type of milk required by the shopkeeper and show them on a number line.

(b) Write down an inequality to show the number of cartons of milk in the refrigerator.

13 Tony has spent $428 to buy 20 calculators wholesale.

(a) Write an inequality to represent the price (p) he could sell each calculator for and make a profit.

(b) Solve the inequality to find the minimum price that he could sell the calculators for and still make a profit.

14 At the local milk bar, Sue spends $3.45 on milk and bread. With the change from her $5 note, she buys some liquorice straps at 40c each.

(a) Write an inequality to represent the number of liquorice straps (n) she could buy.

(b) Solve the inequality to find the maximum number of liquorice straps she could buy.

Open-ended

15 Draw a number line showing numbers that could be greater than or equal to a negative number, but less than a positive number.

16 **(a)** Make up an inequality, with the variable on both sides of the inequality, that will produce solutions that are all negative.

(b) Make up an inequality, with the variable on both sides of the inequality, that will produce solutions that are all positive.

Problem solving

Win ratio

A tennis player computes her win ratio by dividing the number of matches she has won by the total number of matches she has played. At the start of a weekend, her win ratio is exactly 0.5 or $\frac{1}{2}$. Over the course of the weekend, she plays four matches, winning three and losing one. At the end of the weekend, her overall win ratio is greater than 0.503. What is the greatest number of matches she could have won before the weekend began?

Strategy options

- Break problem into manageable parts.
- Test all possible combinations.

Hint

Let the number of wins at the start of the weekend be w. Given the win ratio was 0.5 or $\frac{1}{2}$, what is the relationship between the number of wins, w, and the total number of games, n?

4.6 Simultaneous equations

Need to Know

Equations that give non-parallel and non-**coincident** graphs are called **unique equations** and have only one point of intersection and therefore only one solution.

Because the point of intersection satisfies both equations, graphs can be used to find the solution to **simultaneous equations**.

Simultaneous equations can also be solved algebraically by elimination or by substitution.

It doesn't matter which variable you choose to eliminate when solving simultaneous equations. You will get the same solution for both variables.

- If you only need to change one equation to get the same coefficient of one of the variables, eliminate that variable.
- If the signs in front of one of the variables are opposite, eliminate that variable.
- If the signs in front of each of the variables are the same, choose the variable to eliminate where the coefficients are smallest in magnitude.
- If the signs in front of each of the variables are the same, multiplying one equation by a negative number and then adding the equations is often easier than multiplying both by a positive number and then subtracting the equations.

In general, when solving problems using simultaneous equations, these steps need to be followed.

Step 1 Identify the unknowns. Define each with a variable.

Step 2 Express the information given as equations involving the unknowns. (If there are two unknowns, i.e. two pieces of information required, there must be two equations.)

Step 3 Solve the equations simultaneously either using elimination, substitution or graphically.

Step 4 Check your solution by substitution.

Step 5 Express the final answer to the problem in words.

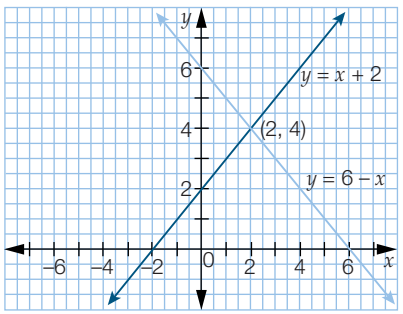
Worked Example 16 WE16

(a) Sketch the pair of linear equations $y = x + 2$ and $y = 6 - x$ and find the point of intersection.

(b) Hence, find the solution to $y = x + 2$ and $y = 6 - x$.

Working

(a)



Point of intersection is (2, 4)

(b) $x = 2, y = 4$

Worked Example 17 WE17

Solve the following pairs of simultaneous equations algebraically using the elimination method.

(a) $3x + 5y = -3$
$x - 5y = 19$

(b) $4x + 5y = -6$
$4x - 3y = 10$

Working

(a)

$3x + 5y = -3$ (1)
$x - 5y = 19$ (2)

$4x = 16$ (1) + (2)
$x = 4$

$3x + 5y = -3$ (1)
$3 \times 4 + 5y = -3$
$12 + 5y = -3$
$5y = -15$
$y = -3$

$x = 4$, $y = -3$ *or* $(x, y) = (4, -3)$

Check:

$x - 5y = 19$ (2)

$\text{LHS} = x - 5y$
$= 4 - 5 \times -3$
$= 4 + 15$
$= 19$
$= \text{RHS}$

(b)

$4x + 5y = -6$ (1)
$4x - 3y = 10$ (2)

$8y = -16$ (1) − (2)
$y = -2$

$4x + 5y = -6$ (1)
$4x + 5 \times -2 = -6$
$4x - 10 = -6$
$4x = 4$
$x = 1$

$x = 1, y = -2$ *or* $(x, y) = (1, -2)$

Check:

$\text{LHS} = 4x - 3y$
$= 4 \times 1 - 3 \times -2$
$= 4 + 6$
$= 10$
$= \text{RHS}$

Worked Example 18

WE18

Solve the following simultaneous equations algebraically using the elimination method.

(a) $3x + 2y = 5$
$4x - 3y = -16$

(b) $2x + 5y = 9$
$x + 3y - 5 = 0$

Working

(a)
$3x + 2y = 5$ (1)
$4x - 3y = -16$ (2)

$9x + 6y = 15$ (3) (1) × 3
$8x - 6y = -32$ (4) (2) × 2

$17x = -17$ (3) + (4)
$x = -1$

$3x + 2y = 5$ (1)
$3 \times -1 + 2y = 5$
$-3 + 2y = 5$
$2y = 8$
$y = 4$

$x = -1$, $y = 4$ or $(x, y) = (-1, 4)$

Check:

LHS $= 4x - 3y$
$= 4 \times -1 - 3 \times 4$
$= -4 - 12$
$= -16$
$=$ RHS

(b)
$2x + 5y = 9$ (1)
$x + 3y - 5 = 0$ (2)

$2x + 5y = 9$ (1)
$x + 3y = 5$ (3)

$2x + 6y = 10$ (4) (3) × 2
$2x + 5y = 9$ (1)

$y = 1$ (4) − (1)

$x + 3y - 5 = 0$ (2)
$x + 3 \times 1 - 5 = 0$
$x - 2 = 0$
$x = 2$

$x = 2$, $y = 1$ or $(x, y) = (2, 1)$

Check:

LHS $= 2x + 5y$
$= 2 \times 2 + 5 \times 1$
$= 4 + 5$
$= 9$
$=$ RHS

Worked Example 19

WE19

Solve the following pairs of simultaneous equations algebraically using the substitution method.

(a) $y = 2x$
$x + y = 18$

(b) $2x + 3y = -4$
$x = y + 3$

Working

(a)
$y = 2x$ (1)
$x + y = 18$ (2)

Substituting $2x$ for y in (2):

$x + (2x) = 18$ (3)
$3x = 18$
$x = 6$

$y = 2x$ (1)
$= 2 \times 6$
$= 12$

$x = 6$, $y = 12$ or $(x, y) = (6, 12)$

Check:

LHS $= x + y$
$= 6 + 12$
$= 18$
$=$ RHS

(b) $2x + 3y = -4$ (1)
$x = y + 3$ (2)

Substituting $y + 3$ for x in (1):

$2(y + 3) + 3y = -4$ (3)

$2y + 6 + 3y = -4$
$5y + 6 = -4$
$5y = -10$
$y = -2$

$x = y + 3$ (2)
$= -2 + 3$
$= 1$

$x = 1$, $y = -2$ or $(x, y) = (1, -2)$

Check:

LHS $= 2x + 3y$
$= 2 \times 1 + 3 \times -2$
$= 2 - 6$
$= -4$
$=$ RHS

4.6 Simultaneous equations

Answers page 502

Equipment required: graph paper for Question **1**

Fluency

1 **(i)** Sketch each pair of linear equations on graph paper and find the point of intersection. WE16

(ii) Hence, find the solution for each pair of simultaneous equations.

(a) $y = x + 3$
$y = 5 - x$

(b) $y = x - 7$
$y = 3 - x$

(c) $x - y = 6$
$2x - y = 8$

(d) $3x - y = 6$
$x - 2y = 2$

(e) $y = 2x$
$y = 3x - 4$

(f) $y = 6 - 2x$
$y = 4x$

(g) $x = -2$
$y = 3x$

(h) $y = -x$
$y = 3$

(i) $y = x + 2$
$y = x + 4$

2 Solve the following pairs of simultaneous equations using the elimination method. WE17

(a) $x + y = 5$
$x - y = 3$

(b) $x + y = 1$
$x - y = 9$

(c) $2x + 3y = 9$
$5x - 3y = 12$

(d) $3x + 2y = 13$
$9x - 2y = 47$

(e) $3x + 2y = 9$
$-3x + y = 0$

(f) $7x + y = 8$
$-x + y = -8$

(g) $-2x + y = 7$
$2x + 3y = 5$

(h) $2x - 3y = -2$
$5x - 3y = 4$

(i) $-3x + y = -3$
$-3x - 2y = -12$

3 Solve the following pairs of simultaneous equations using the elimination method. WE18

(a) $3x + 2y = 1$
$-2x + y = -3$

(b) $5x + 2y = -7$
$x + 7y = 25$

(c) $2x + 6y = 4$
$4x + 3y = -1$

(d) $2x + 5y = -1$
$4x + 3y = 5$

(e) $3x + 2y - 1 = 0$
$-2x + y + 3 = 0$

(f) $4x + 3y + 9 = 0$
$2x - 5y + 11 = 0$

(g) $3x - 2y = 7$
$2x - 3y = 3$

(h) $5x - 2y = -23$
$2x + 7y = 22$

(i) $4x + 2y - 6 = 0$
$-3x - 3y + 6 = 0$

(j) $\frac{x}{2} + y = 0$
$x - 3y = 10$

(k) $x + \frac{y}{4} = 8$
$2x - y = -2$

(l) $5x - 6y = 21$
$\frac{x}{3} + \frac{y}{2} = 5$

(m) $7x - y = -25$
$\frac{2x}{5} - \frac{y}{2} = 3$

(n) $\frac{2x}{3} + y + 10 = 0$
$4x - \frac{y}{4} + 10 = 0$

(o) $\frac{5x}{6} + \frac{3y}{5} + 2 = 0$
$7x + 3y + 27 = 0$

WE19

4 Solve the following pairs of simultaneous equations using the substitution method.

(a) $y = 2x$
$x + 5y = 22$

(b) $x + 3y = -30$
$y = 3x$

(c) $x = -2y$
$y = 3x - 14$

(d) $2x + y = 7$
$x = 6 - y$

(e) $x = y + 1$
$3x - y = 7$

(f) $y = 7 - 3x$
$4x - 3y = 18$

(g) $y = 1 - 2x$
$5x + 2y = 0$

(h) $x = 2 - 7y$
$2y = x - 11$

(i) $y = 6 - 3x$
$x = 2y - 19$

5 (a) For simultaneous equations $y = 2x - 5$ and $y = 3 - 2x$, the solution is:

A (0, -5) **B** (0, 3) **C** (2, -1) **D** (2, 1)

(b) For simultaneous equations $x + 2y = 8$ and $3x - y = 3$, the solution is:

A (6, 1) **B** (3, 6) **C** (4, 2) **D** (2, 3)

6 (a) Adding which pair of equations below will eliminate y?

A $x + y = 4$
$2x + y = 3$

B $3x + 2y = 6$
$-3x + y = 0$

C $x - y = 7$
$3x - y = 7$

D $2x - 3y = 4$
$x + 3y = 2$

(b) Adding which pair of equations in part **(a)** will eliminate x?

7 (a) $5x - 6y = 4$ (1)
$-3x + 8y = 2$ (2)

To eliminate x in the pair of equations above, we need to:

A multiply (1) by 3 and add to 5 times (2)

B multiply (2) by 3 and add to (1)

C multiply (1) by -3 and add to 5 times (2)

D multiply (1) by -3 and add to 5 times (2)

(b) $9x + 7y = 3$ (1)
$5x + 3y = -1$ (2)

To eliminate y in the pair of equations above, we need to obtain a new equation formed by:

A $3 \times (1) + 7 \times (2)$ **B** $5 \times (1) + 9 \times (2)$ **C** $3 \times (1) - 7 \times (2)$ **D** $5 \times (1) - 9 \times (2)$

8 (a) Simultaneous equations $y = x + 3$ and $2x - y = -11$ have the solution:

A $x = 3, y = 6$ **B** $x = -5, y = 1$ **C** $x = -5, y = -8$ **D** $x = -8, y = -5$

(b) Simultaneous equations $x = 2y$ and $4x - 7y - 3 = 0$ have the solution:

A (-3, -6) **B** (2, 4) **C** (3, 6) **D** (6, 3)

Understanding

9 The fruit below cost $14.05.



The fruit below cost $8.50.



How much did each pineapple and each kiwifruit cost? Assume that all pineapples are the same price and all kiwi fruit are the same price.

10 Three salad rolls and four pies cost $20.56 altogether. A salad roll costs $1.16 more than a pie. How much does each cost?

11 The sum of x and y is 12. When y is subtracted from $3x$ the result is 16. Find x and y.

12 The sum of two numbers is 38 and their difference is 24. What are the numbers?

Reasoning

13 Two books have a total of 500 pages. If the number of pages in one book is 12 pages less than 3 times the number of pages in the other, how many pages are there in each book?

14 A farmer uses 140 m of fencing wire to enclose a rectangular paddock. If the length of the paddock is 10 m longer than the width, find the dimensions of the paddock.

15 After buying a book, Libby received 18 coins totalling $1.65 in change. If she was given only 5- and 10-cent coins, how many of each did she receive?

16 Five less than twice Michelle's present age is equal to Yesmin's present age. Two years ago, twice Yesmin's age was the same as 10 years more than triple Michelle's age. Find the present ages of the two girls.

17 Wes and Tony live in two cities 360 km apart. They leave their houses at the same time by car and travel towards each other at 80 km/h and 100 km/h, respectively.

(a) Write an equation for each journey representing the distance from Wes's house (D) at any time (t).

(b) How much time will pass before they meet on the highway?

18 It takes a barge 3 hours to travel 27 km upstream against the current. On its return journey, with the current, the barge can travel the same distance downstream and an additional 3 km in only 2 hours. Find the speed of the boat and the current.

19 The pirate Captain Greybeard left careful instructions for his crew to locate his buried treasure. Using the old oak tree as the origin, north–south as the y-axis, east–west as the x-axis and each unit on the number plane as 20 paces, the solution of four different pairs of simultaneous equations would lead to the burial site.

The first point, A, is at the intersection of the two lines $x = -3$ and $y = -7$. The second point, B, is at the intersection of $2x - y = 7$ and $2x - 3y = -3$. The third point, C, is at the intersection of $2y + x = 0$ and $2y = x - 8$. The final point, D, is at the intersection of $y = 7x$ and $x + y = 8$.

(a) Find the coordinates of each of the points A, B, C and D.

(b) Plot the points A, B, C and D on the same number plane and join AB and CD.

(c) Find the coordinates of the intersection of the two lines, which will give the location of the buried treasure.

(d) Determine the equations of the lines AB and CD and solve algebraically to show that the coordinates found in **(c)** are the solution of these equations.

Open-ended

20 Make up three different pairs of simultaneous equations, each having the same solution, where $x = 3$ and $y = 4$.

21 Emmanuel has three numbers. When he adds them together in pairs he gets three different totals: 11, 17 and 22. What are the three numbers Emmanuel had to start with? Can you describe a method that would enable you to work out the three numbers given any three totals?

Chapter review 4

D.I.Y. Summary

Key Words

coincident lines	independent variable	linear inequality	sketch
dependent variable	linear equation	linear relationships	unique equations
gradient	linear graph	simultaneous equations	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 A _____ indicates how two quantities that change at a constant rate are related to one another.

2 A _____ can be solved by performing inverse operations on both sides of the equation.

3 The measure of a line's steepness is called the _____.

4 _____ show that the relationship between two variables is a straight line.

5 For a _____, only two points need to be shown.

6 Two linear equations that contain the same variables and are not parallel or coincident can be solved using _____.

7 $3x + 4y = 7$ and $8y = 14 - 6x$ are examples of _____.

Fluency

1 Solve the following equations. Ex 4.1

(a) $2c - 7 = 9$

(b) $\frac{2x-3}{4} + 6 = -1$

(c) $4v - 5 = v + 4$

(d) $\frac{3a+1}{2} - \frac{2a+3}{5} = 1$

2 The gradient of the line joining the points (2, 9) and (5, -6) is: Ex 4.2

A 1 **B** 5 **C** $-\frac{1}{5}$ **D** -5

3 Find the gradient and y-intercept for each of the following. Ex 4.2

(a) $y = 5x + 2$ (b) $y = -2x$ (c) $2x - 5y = 10$ (d) $4x + y = 8$

4 Graph the following lines by finding the y-intercept and gradient. Ex 4.3

(a) $y = 2x - 5$ (b) $y = -2x$ (c) $x + 5y = 10$ (d) $y = 4x$

5 Sketch graphs of the following, showing the intercepts on the axes. Ex 4.3

(a) $4x - 5y = 20$ (b) $3x + 7y = 42$ (c) $y = 3 - 5x$ (d) $y = -2x + 6$

6 Show whether or not each pair of lines is parallel, using their gradients. Ex 4.4

(a) $x + 2y = 7$ and $x + 2y = 3$

(b) $x - y + 6 = 0$ and $3x - 2y + 4 = 0$

(c) $4x + y = 6$ and $4x + y + 6 = 0$

(d) $y = 4x - 3$ and $8x - 2y = 0$

7 Show whether or not each pair of lines is perpendicular, using their gradients.

Ex.4.4

(a) $2x + 2y = 4$ and $2x + y = 8$ **(b)** $x - y + 9 = 0$ and $2x + 2y - 3 = 0$

(c) $2x - 4y = 9$ and $4x + 2y + 6 = 7$ **(d)** $3x + 2y = 8$ and $3y = 2x + 4$

8 Find the solution set for the following linear inequalities.

Ex.4.5

(a) $9x \geq 21$ **(b)** $\frac{3x}{5} + 2 > -5$ **(c)** $7 - 6x < 2$

9 Solve the following simultaneous equations using the elimination method.

Ex.4.6

$5x - 2y = 8$ $3x - 4y = -12$

10 Solve the following simultaneous equations using the substitution method.

Ex.4.6

$6x - 5y = -21$ $y = 2x + 5$

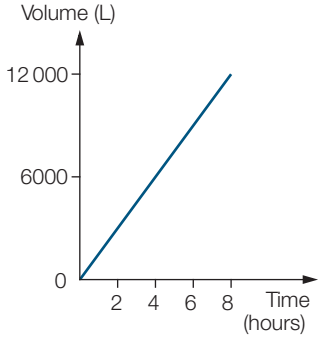
Understanding

11 In an enclosure at the zoo, there are twice as many wombats as emus. Altogether, they have a total of 90 legs. How many wombats are there?

12 To earn extra money, Martin works part-time in a butcher's shop. In a particular week during the school holidays he worked twice as many hours on Tuesday as he did on Monday but only $\frac{3}{4}$ the hours on Wednesday compared to Monday. For these 3 days he received \$127.50. If he is paid \$8.50 per hour, how many hours did Martin work on Monday?

13 A swimming pool has been filled with water.

The graph shows the volume of water (in litres) in the pool after a number of hours as it filled.



(a) How long did it take for the pool to contain 12 000 L of water?

(b) How many litres of water flowed into the pool each hour?

(c) What is the gradient of the graph?

(d) Write down the relationship for the volume (V) in terms of the time (T).

14 Manny, a motorcycle courier, notes his distance from the centre of Melbourne on a delivery trip:

Time (min)	0	10	20	30	40	50	60
Distance from Melbourne (km)	20	26	32	38	44	50	56

(a) Find an equation describing Manny's distance from Melbourne (d km) in terms of the time spent travelling (t minutes).

(b) Draw a graph of Manny's distance from Melbourne as a function of the time spent travelling.

(c) What does the gradient of the graph represent?

(d) What does the intercept with the vertical d-axis represent?

(e) Find Manny's distance from Melbourne after he has been travelling for 14 minutes.

(f) How long does it take for Manny to be 48 km from Melbourne?

15 What is the equation of the line that passes through (6, 3) and is perpendicular to $y - x - 1 = 0$?

16 What is the equation of the line that passes through (8, -2) and is parallel to $y - 5x = -1$?

17 Find the solution set for the following linear inequalities.

(a) $\frac{3x-4}{2} < \frac{2x+1}{3}$ **(b)** $3(2x - 4) \geq 4(3x + 1)$

18 Twice one number added to three times another number equals -3. If the first number is tripled then subtracted from 8 and the result is divided by 2, the second number is obtained. Find the two numbers.

19 Find the values of a and b in the following equations given that the lines are perpendicular and intersect on the y-axis.

$ax + 3y = 1$

$by = -x - 1$

Reasoning

20 Houses for sale at auction often have an EPR (Expected Price Range) in the advertisement. This means that it is expected that the house will be sold for a price somewhere between the values given.

(a) If the EPR for a house is advertised as \$570 000–\$620 000, write this as an inequality for the selling price of the house (p).

(b) If the deposit must be 10% of the price paid, write an inequality to show how much deposit is needed if the price falls within the EPR.

(c) If the house sold for more than \$50 000 above the highest value in the EPR, write an inequality to show how much the house sold for.

(d) If the house sold for less than \$50 000 above the highest value in the EPR, write an inequality to show how much the house sold for.

21 Three times the distance (d) between two towns subtracted from 823 km would take 4 hours travelling at an average speed (s). However, five times the same distance subtracted from 1071 km would only take 3 hours travelling at the same average speed. How far is the distance (d) between the two towns and what is the average speed (s)?

Trigonometry

5

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 Solve these equations for x. Express your answer as a decimal, correct to the number of decimal places (d.p.) given in brackets next to the equation.

Worksheet R5.1

(a) $3x = 24.5$ (2 d.p.)

(b) $\frac{x}{20} = 0.7825$ (1 d.p.)

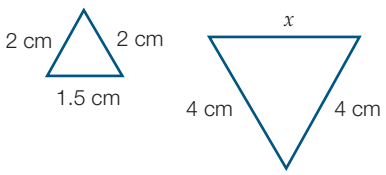
(c) $\frac{12}{x} = 2.307\,02$ (2 d.p.)

(d) $0.345 = \frac{16}{x}$ (3 d.p.)

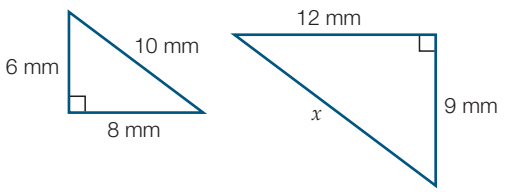
2 Determine the length of the missing side x in each pair of similar triangles.

Worksheet R5.2

(a)



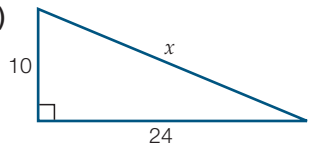
(b)



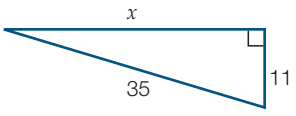
3 Use Pythagoras' theorem to calculate the value of x in each triangle. Round your answer to two decimal places where necessary.

Worksheet R5.3

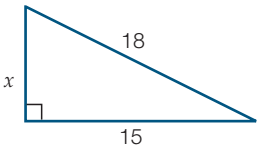
(a)



(b)



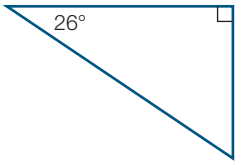
(c)



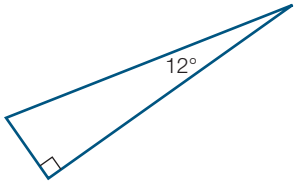
4 Copy each right-angled triangle shown below and label the hypotenuse, the opposite side and the adjacent side for the angle given.

Worksheet R5.4

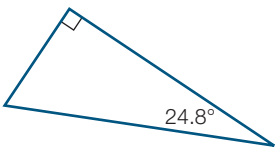
(a)



(b)



(c)



(d)



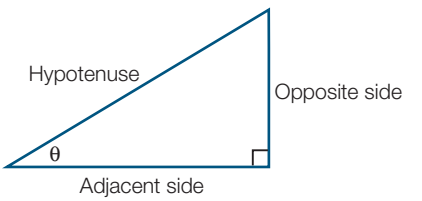
After completing this chapter you will be able to:

- use trigonometry to find unknown side lengths and angles in right-angled triangles
- use angles of elevation and depression to solve problems using trigonometry
- use conventional and true bearings to describe location
- use bearings to solve trigonometric problems
- select and use appropriate measurements and trigonometric ratios to solve problems in two dimensions.

5.1 The trigonometric ratios

Need to Know

The **hypotenuse** is always opposite the right angle. The side directly opposite the angle θ is the **opposite side** and the side next to θ is the **adjacent side**.



We can use pairs of side lengths to calculate three **trigonometric ratios** (we write the ratios in their fraction forms).

The ratio $\frac{\text{Opposite}}{\text{Hypotenuse}}$ is referred to as the **sine** of θ and abbreviated to sin θ.

The ratio $\frac{\text{Adjacent}}{\text{Hypotenuse}}$ is referred to as the **cosine** of θ and abbreviated to cos θ.

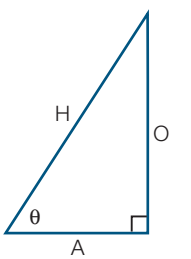
The ratio $\frac{\text{Opposite}}{\text{Adjacent}}$ is referred to as the **tangent** of θ and abbreviated to tan θ.

In a right-angled triangle:

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ or $\frac{O}{H}$

$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ or $\frac{A}{H}$

$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$ or $\frac{O}{A}$



The phrase 'SOH-CAH-TOA' can help you remember which two sides belong to each ratio.

Make sure that your calculator is in 'degree' mode for the calculations in this chapter.

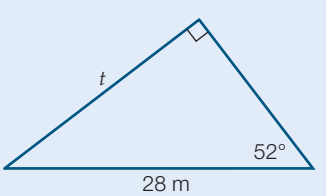
The units we use to measure angles are degrees (°). One degree can be broken up into smaller units called **minutes** (′) and **seconds** (″). There are 60 seconds in a minute and 60 minutes in a degree.

Worked Example 1

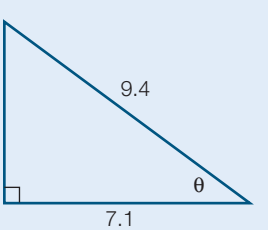
WE1

Label the Hypotenuse, Opposite and Adjacent sides on these right-angled triangles and hence decide which trigonometric ratio we could use to find the unknown.

(a)

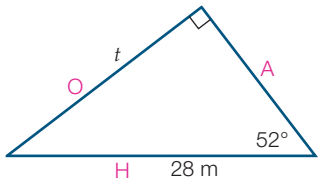


(b)



Working

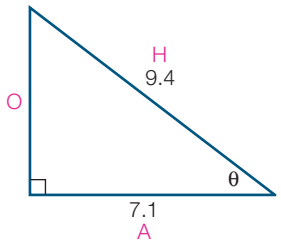
(a)



$$\sin\theta = \frac{O}{H}$$

$$\sin 52° = \frac{t}{28}$$

(b)



$$\cos\theta = \frac{A}{H}$$

$$\cos\theta = \frac{7.1}{9.4}$$

Worked Example 2

WE2

Use a calculator to find each of the following values, rounded to four decimal places.

(a) sin 29° **(b)** cos 58° **(c)** tan 60.5°

Working

(a) sin 29° = 0.4848 (4 d.p.)

(Press the **sin** key, followed by the value of the angle, then '='.)

(b) cos 58° = 0.5299 (4 d.p.)

(Press the **cos** key, followed by the value of the angle, then '='.)

(c) tan 60.5° = 1.7675 (4 d.p.)

(Press the **tan** key, followed by the value of the angle, then '='.)

Worked Example 3

WE3

Convert the following angles to decimal degrees, rounded to two decimal places.

(a) 56°24′ **(b)** 23°16′45″

Working

(a) 56°24′ = 56.40°

(Press the following keys:

5 6 °′″ 2 4 °′″ = °′″)

(b) 23°16′45″ = 23.28° (2 d.p.)

(Press the following keys:

2 3 °′″ 1 6 °′″ 4 5 °′″ = °′″)

Worked Example 4 WE4

Use a calculator to find each of the following values, rounded to four decimal places.

(a) sin 46°36′ **(b)** cos 6°12′34″

Working

(a) sin 46°36′ = 0.7266 (4 d.p.)

(Press the following keys:

sin 4 6 °′″ 3 6 °′″ =)

(b) cos 6°12′34″ = 0.9941 (4 d.p.)

(Press the following keys:

cos 6 °′″ 1 2 °′″ 3 4 °′″ =)

Answers page 505

5.1 The trigonometric ratios

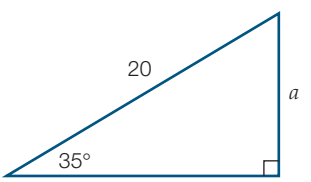
Equipment required: ruler and protractor for Questions **6, 14–16**

Fluency

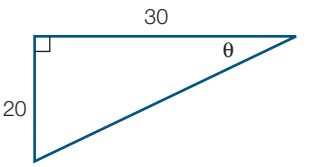
WE1

1 Label these right-angled triangles and hence decide which trigonometric ratio should be used to find the unknown in each case.

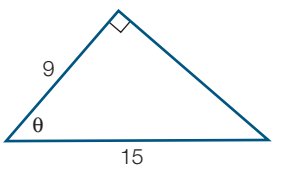
(a)



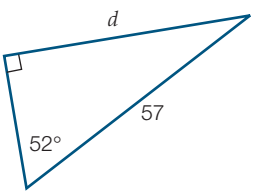
(b)



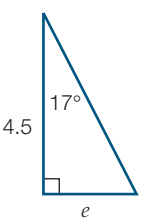
(c)



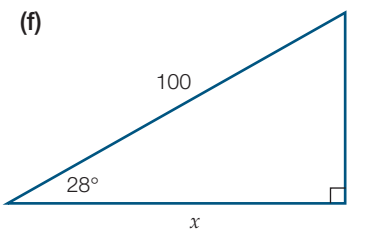
(d)



(e)



(f)



WE2

2 Use your calculator to find the following values, rounded to four decimal places.

(a) sin 23° **(b)** cos 17° **(c)** tan 39°

(d) cos 5° **(e)** sin 41° **(f)** cos 49°

(g) tan 19.7° **(h)** cos 23.5° **(i)** tan 43.65°

WE3

3 Convert these angles to decimal degrees. Round your answers to two decimal places if necessary.

(a) 15°30′ **(b)** 35°16′ **(c)** 23°10′

(d) 76°47′50″ **(e)** 44°59′20″ **(f)** 8°45′9″

WE4

4 Calculate the following, rounding values to four decimal places.

(a) sin 25°47′ **(b)** tan 88°30′ **(c)** cos 64°17′

(d) tan 36°30′46″ **(e)** sin 20°25′39″ **(f)** cos 81°44′16″

Understanding

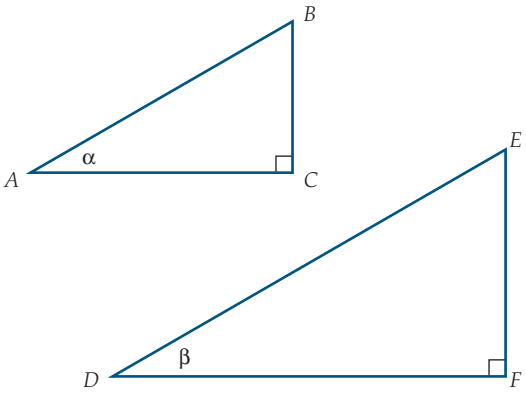
5 **(a)** Using a calculator, and evaluating the result to four decimal places, the value of sin 37°34′ is:

A 0.6064 **B** 0.6065 **C** 0.6097 **D** 0.7951

(b) Using a calculator, and evaluating the result to four decimal places, the value of cos 17.25° is:

A 0.2965 **B** 0.3105 **C** 0.9542 **D** 0.9550

6 Measure the side lengths of triangles *ABC* and *DEF* to the nearest millimetre.



(a) Calculate the ratios of the following pairs of sides. Round your answers to one decimal place.

(i) $\frac{DF}{AC}$ **(ii)** $\frac{EF}{BC}$ **(iii)** $\frac{DE}{AB}$

(b) Calculate the value for sin α.

(c) Calculate the value for sin β.

(d) Are these sine ratios about the same? Why?

(e) Measure the sizes of angles α and β with a protractor.

(f) Would the cosine ratios be the same for each of the triangles?

(g) Would the tangent ratios be the same for each of the triangles?

(h) Give a reason for your answers to **(f)** and **(g)**.

7 **(a)** Calculate the value of:

(i) sin 35° **(ii)** cos 35° **(iii)** tan 35° **(iv)** $\frac{\sin 35°}{\cos 35°}$

(b) Comment on your results.

8 If $\sin\theta = 0.755$ (3 d.p.) and $\cos\theta = 0.656$ (3 d.p.), use your result from Question **7** to calculate the value of $\tan\theta$, to three decimal places.

9 The cosine of an angle is always less than 1 because:

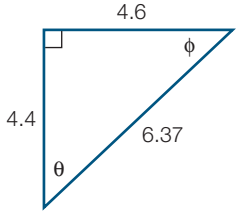
A the Adjacent side is always shorter than the Opposite side

B the Adjacent side is always shorter than the Hypotenuse

C the Adjacent side is always longer than the Hypotenuse

D the Adjacent side is always longer than the Opposite side.

Reasoning



10 **(a)** Calculate the following values using the triangle shown, rounded to three decimal places:

(i) sin θ **(ii)** cos ϕ

(b) Explain your results.

11 A right-angled triangle has an angle of 54° as one of its acute angles.

(a) Using three different pronumerals to represent the side lengths, draw a diagram of the triangle, including the measurement of the other angle.

(b) Write down values of sin 54° and cos 54° using the pronumerals that represent the side lengths.

(c) Find the values of sin 54° and cos 54° using a calculator, and round to four decimal places.

(d) Use your answers to **(c)** to show that the value of $(\sin 54°)^2 + (\cos 54°)^2$ equals 1. (*Note*: This is normally written as $\sin^2 54° + \cos^2 54°$.)

(e) Use your answers to **(b)** and Pythagoras' theorem to show that $(\sin 54°)^2 + (\cos 54°)^2 = 1$.

12 The value of the tangent ratio for each of the two base angles in a right-angled isosceles triangle is 1. Explain why.

13 Only one of the trigonometric ratios can have a value greater than 1. Name it, and explain why this is not possible for the other two ratios.

Open-ended

14 (a) Draw a right-angled triangle with sides of any length.

(i) Carefully measure all the angles of the triangle.

(ii) Carefully measure all the sides of the triangle.

(iii) Find the sine, cosine and tangent ratios for the two acute angles, rounded to two decimal places.

(iv) Write down as many relationships between the angles and the ratios as you can find.

(b) Draw a second triangle with the same angles, but with different side lengths (i.e. a similar triangle). Repeat steps **(ii)** and **(iii)** from part **(a)** for this triangle.

(c) Compare the values of sin, cos and tan that you calculated for the two triangles. How similar are they? Did you expect them to be the same?

(d) Compare the values of the three ratios you calculated for each angle to the values produced by your calculator. Are they the same? Suggest a reason for your answer.

15 A right-angled triangle has a tangent ratio of $\frac{7}{10}$ for one angle.

(a) Use the tangent ratio to draw the triangle. Measure the length of the hypotenuse.

(b) Calculate the other ratios, rounded to four decimal places.

Hint

Which values of O and A give a ratio of $\frac{7}{10}$?

16 Angle θ in a right-angled triangle has a sine ratio of 0.4. Draw three different triangles that could contain angle θ. Use Pythagoras' theorem to help you find any unknown side lengths.

17 (a) Find two angles θ and ϕ, so that $\sin\theta = \cos\phi$.

(b) What is the relationship between θ and ϕ if $\sin\theta = \cos\phi$ for all values of θ and ϕ?

5.2 Finding lengths

Need to Know

To find an unknown value in a right-angled triangle:

Step 1 Draw the right-angled triangle, showing all known information and the unknown value.

Step 2 Label the sides O, A and H, and write down the known information.

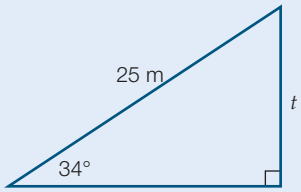
Step 3 Identify which trigonometric ratio to use. (SOH-CAH-TOA is helpful here.)

Step 4 Substitute the known values, form an equation and solve for the unknown.

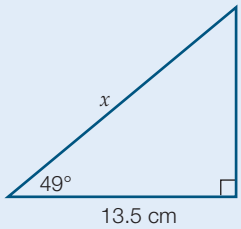
Worked Example 5

WE5

(a) Find the value of t, rounded to two decimal places.

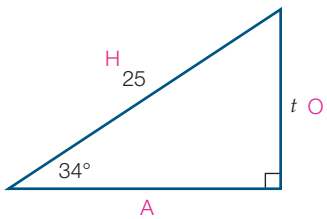


(b) Find the value of x, rounded to two decimal places.



Working

(a)



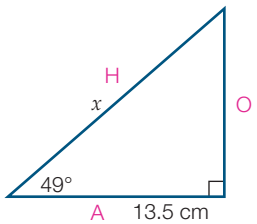
$\theta = 34°$
$O = t$
$H = 25$

$$\sin\theta = \frac{O}{H}$$

$$\sin 34° = \frac{t}{25}$$

$t = 25 \times \sin 34°$
$t = 13.98$ m (2 d.p.)

(b)



$\theta = 49°$
$A = 13.5$
$H = x$

$$\cos\theta = \frac{A}{H}$$

$$\cos 49° = \frac{13.5}{x}$$

$$x = \frac{13.5}{\cos 49°}$$

$x = 20.58$ cm (2 d.p.)

Worked Example 6

WE6

A 3.2 m ladder rests against a wall so that its angle with the wall is 65°. How high up the wall does the ladder reach? State your answer correct to the nearest centimetre.

Working

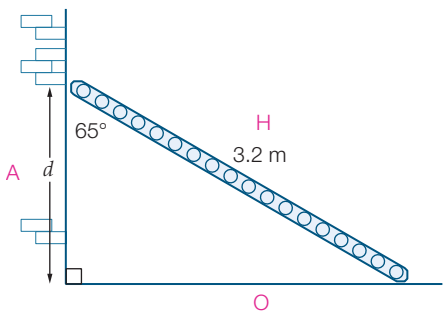
$\theta = 65°$
$H = 3.2$ m
$A = d$

$$\cos\theta = \frac{A}{H}$$

$$\cos 65° = \frac{d}{3.2}$$

$d = 3.2 \times \cos 65°$
$d = 1.35$ (2 d.p.)

The ladder rests 1.35 m up the wall.



Worked Example 7

WE7

A rectangular wooden frame has a diagonal piece of wood joining the opposite corners for support. The vertical height of the frame is 2.5 m. The angle between the diagonal and vertical part of the frame is 57°15′. How long is the diagonal piece of the frame?

Working

$\theta = 57°15'$

$A = 2.5 \text{ m}$

$H = x$

$\cos\theta = \frac{A}{H}$

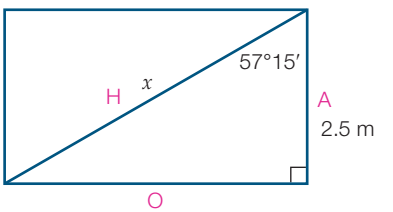
$\cos 57°15' = \frac{2.5}{x}$

$x \cos 57°15' = 2.5$

$x = \frac{2.5}{\cos 57°15'}$

$x = 4.62$ (2 d.p.)

The diagonal of the frame is 4.62 m long.



Answers page 506

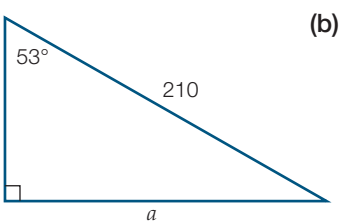
5.2 Finding lengths

Fluency

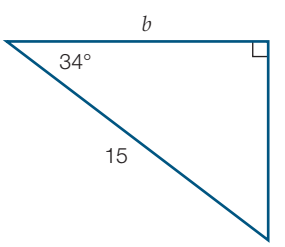
WE5

1 Find the value of the unknown length in each of the following. Round your answers to two decimal places.

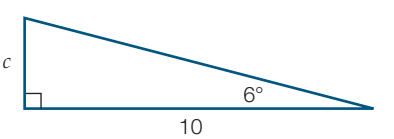
(a)



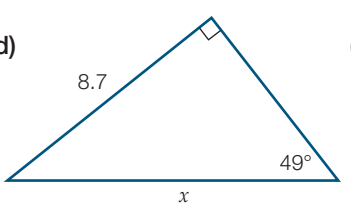
(b)



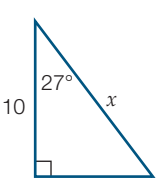
(c)



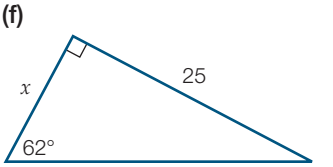
(d)



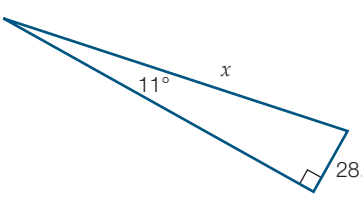
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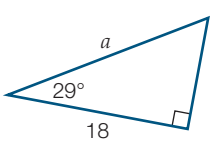
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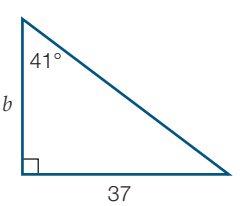
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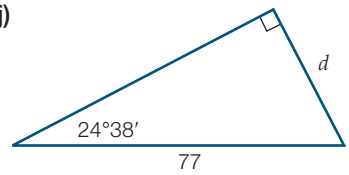
(h)



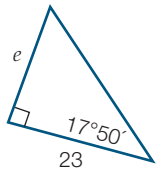
(i)



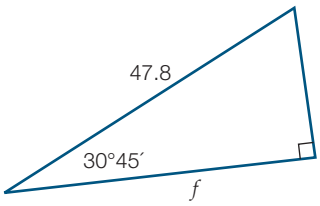
(j)



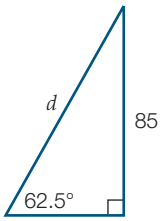
(k)



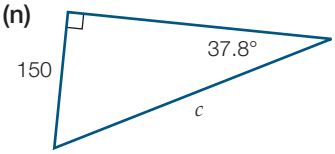
(l)



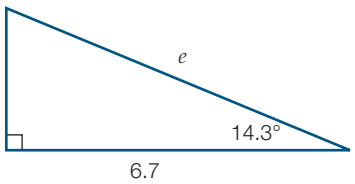
(m)



(n)



(o)



2 A 3 m rope is stretched from the top of a vertical tent pole and pegged into the ground so that it makes an angle of 50° with the ground. How far away from the pole is the rope pegged? Give your answer to the nearest centimetre.

WE6

3 A rectangular gate has diagonal metal supports joining the opposite corners. The supports make an angle of 25°18′ with the horizontal sections of the frame. If the gate is 1.6 m high, how long is one of the diagonal supports?

WE7

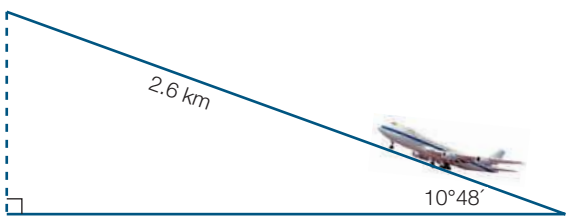
4 **(a)** A plane takes off and rises steadily at an angle of 10°48′ to the horizontal. The altitude of the plane after it has travelled along a flight path of 2.6 km (to the nearest metre) is:

A 472 m

B 487 m

C 2554 m

D 13 875 m



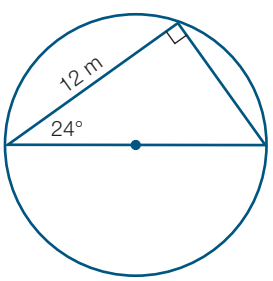
(b) The radius of the circle at right is closest to:

A 6.57 m

B 13.14 m

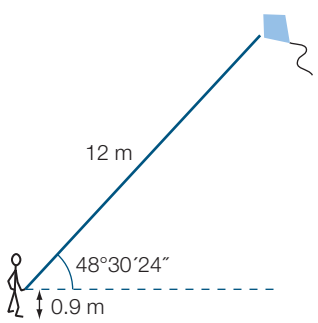
C 14.75 m

D 29.50 m



Understanding

5 Jessica is flying a kite attached to a string 12 m long. She is holding the string at a height of 0.9 m from the ground. How high is the kite if it makes an angle of 48°30′24″ with the horizontal? Round your answer to two decimal places.



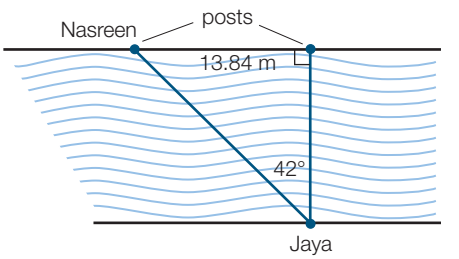
6 To measure a tall building, Charlie stands 150 m from the base of the building and holds an inclinometer (a device for measuring angles) to his eye, as shown in the diagram. He looks along the top of the inclinometer and tilts it up until he can see the top of the building. The inclinometer measures an angle of 58°. This is the angle between Charlie's line of sight to the top of the building and the horizontal. Charlie's eye-level is 1.63 m from the ground.



(a) Draw a diagram that shows all the known information.

(b) Calculate the height of the building, correct to one decimal place.

7 Jaya wants to calculate the width of a river. She finds that lines joining her point of observation to the two posts on the opposite bank are 42° apart. Her sister Nasreen, who is on the opposite bank, measures the distance between the two posts to be 13.84 m. How wide is the river?



8 A mine shaft 350 m long is to be drilled at an angle of 21°24′ to the vertical. How far directly below the ground will the end of the shaft be, rounded to two decimal places?

9 A road sign is in the shape of an equilateral triangle. The line from one vertex to the midpoint of the opposite side is 34.6 cm long. How long is each side of the road sign?

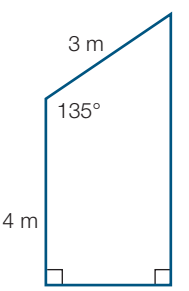
Reasoning

10 Members of a stunt team wish to make a daring climb along a rope, one end of which has been attached to the top of a 23 m building and the other end joined to the top of a nearby building 38 m high. The rope is inclined at an angle of 49°30′ to the horizontal. Answer the following correct to two decimal places.

(a) How far along the rope will each member of the stunt team need to climb to reach the other building?

(b) One of the stunt team wants to drop a ball so that a man standing midway between the two buildings can catch it. From how far along the rope does she need to drop it, and how far will it fall if the man catches it 1.8 m above the ground?

11 James is trying to decide which room to use for a party. He has two rooms in mind but needs to use the larger one. One room is a rectangular shape with length 4 m and width 3 m. The other room is in the shape of a trapezium as shown in the diagram.



(a) Calculate the area of each of the rooms.

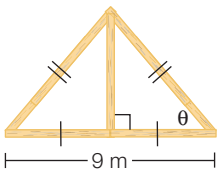
(b) Which room is larger?

Hint

Formula for the area of a trapezium:
$A = \frac{1}{2}(a + b)h$

Open-ended

12 The 'pitch angle' of the roof of a house is the angle that the roof makes with the horizontal. It is usually between 10° and 30°. If the length of the base of this roof frame is 9 m, find two possible values for the:

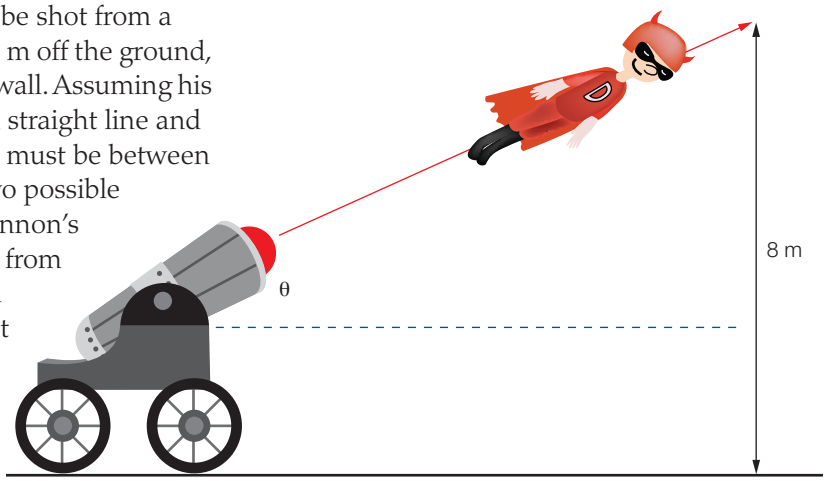


(a) height of the roof at its tallest point

(b) total length of wood needed to build the frame shown.

13 A right-angled triangle ABC has the right angle at B. If the side opposite to angle A has a length of 10 cm, write down at least three possible lengths of the hypotenuse if angle A is between 35° and 40°.

14 Daredevil Dan will be shot from a cannon that sits 1.2 m off the ground, to just clear an 8 m wall. Assuming his flight path will be a straight line and the trajectory angle must be between 30° and 50°, find two possible distances for the cannon's horizontal distance from the base of the wall if this distance must be between 8 m and 10 m.



Finding angles

5.3

Worked Example 8

WE 8

If cos $a = 0.6434$, find:

(a) a in decimal degrees rounded to two decimal places

(b) a in degrees, minutes and seconds.

Working

(a) $a = 49.95°$ (2 d.p.)

(Press the following keys:

SHIFT cos . 6

4 3 4 =)

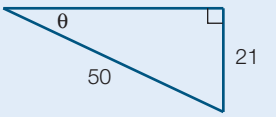
(b) $a = 49°57'15''$ (nearest second)

(Press °' '')

Worked Example 9

WE9

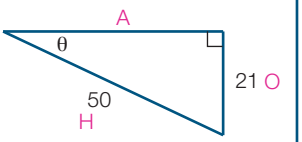
Find the value of θ in degrees, minutes and seconds.



Working

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{21}{50}$



$\theta = \sin^{-1}\left(\frac{21}{50}\right)$

$\theta = 24°50'4.51''$

$\theta = 24°50'5''$ (nearest second)

Worked Example 10

WE10

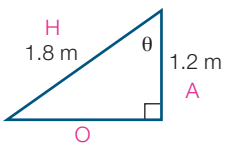
The top of a 1.8 m ladder that is leaning against a wall, reaches a height of 1.2 m. Draw a diagram that shows this situation, then use it to calculate the angle that the top of the ladder makes with the wall, correct to the nearest degree.

Working

$\theta = ?$

$A = 1.2$ m

$H = 1.8$ m



$\cos\theta = \frac{A}{H}$

$\theta = \cos^{-1}\left(\frac{1.2}{1.8}\right)$

$\theta = 48.189\,685\,11$

$\theta = 48°$ (nearest degree)

The ladder makes an angle of approximately 48° with the wall.

Answers page 507

5.3 Finding angles

Fluency

WE8

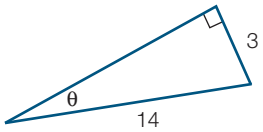
1 Find the value of each pronumeral **(i)** in decimal degrees rounded to two decimal places, **(ii)** in degrees, minutes and seconds.

(a) $\cos a = 0.1538$ **(b)** $\sin b = 0.6719$ **(c)** $\tan c = 2.3085$

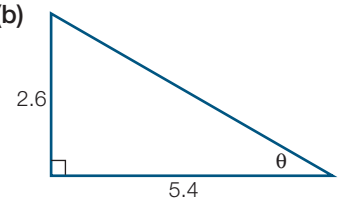
(d) $\cos k = 0.556$ **(e)** $\tan m = 4.687$ **(f)** $\sin p = 0.9715$

2 Find the value of θ in each triangle in degrees, minutes and seconds. WE9

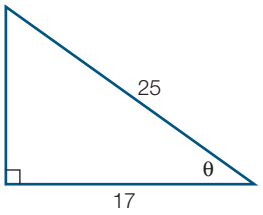
(a)



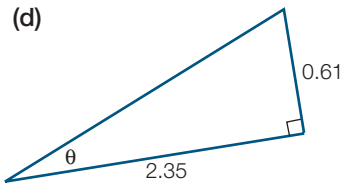
(b)



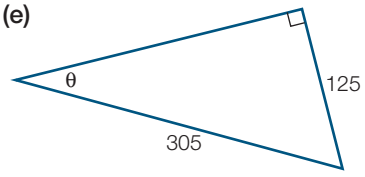
(c)



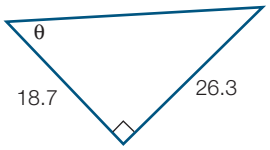
(d)



(e)



(f)



3 A tree that is 8.4 m tall makes a shadow of 12 m. WE10

(a) Draw a diagram to show this situation.

(b) To the nearest degree, calculate the angle that the Sun makes with the ground to form the shadow.

Understanding

4 **(a)** The value of the pronumeral a in the equation $\tan a = 0.6395$, to the nearest degree is:

A 32° **B** 33° **C** 40° **D** 50°

(b) The value of θ in degrees, minutes and seconds in the equation $\sin \theta = 0.8938$ is:

A 26°38′43″ **B** 41°47′25″ **C** 63°21′17″ **D** 63°35′47″

5 On a slope, a railway rises 35 cm for every 50 m of track. Find the angle at which the track is inclined to the horizontal, to two decimal places.

6 Scott is dragging a heavy box along the floor holding the end of a 1.5 m piece of rope. The rope is attached to the top of the box and makes an angle θ with the horizontal. Scott is exactly 1.0 m in front of the box. To the nearest degree, what angle does the rope make with the horizontal?

7 Find the angle of inclination, θ, of the Sun's rays when a 9.5 m high telephone pole casts a shadow 7 m long, as shown in the photograph at right. Write your answer in degrees, minutes and seconds.



8 A 2.75 m long children's slide has a ladder that reaches 1.43 m vertically above the ground. To the nearest degree, the angle the slide makes with the horizontal is:

A 27° **B** 31°

C 42° **D** 58°

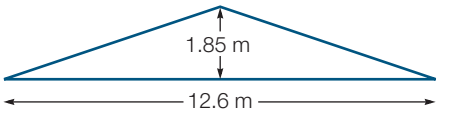
Reasoning

9 A furniture removal company uses a ramp to wheel furniture into its vans. The back of a van is 82 cm above the ground and the ramp is 1.5 m long.

(a) To the nearest degree, what is the angle of the ramp with the ground?

(b) How much longer does the ramp need to be so that the angle is 30°?

10 A symmetrical roof frame of height 1.85 m has a span of 12.6 m as shown in the diagram at right.



(a) Find the angle at the apex of the roof, to the nearest degree. (The apex is the highest point on the roof.)

(b) Find the total length of material needed to make the frame.

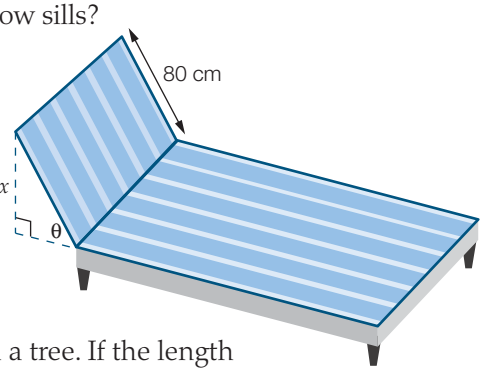
11 A 10 m ladder is to be used to reach different window sills in a building.

(a) If the heights of the window sills range from 2 m to 6 m above the ground, calculate the range of angles that the ladder makes with the ground.

(b) Is it practical to use the ladder to reach all the window sills?

Open-ended

12 A sun lounge can be adjusted from a flat, lying position through to an upright, sitting position. The adjustable back is 80 cm long. Choose five different vertical heights that the back could be raised to. For each height (x) calculate the angle the back makes with the horizontal (θ).

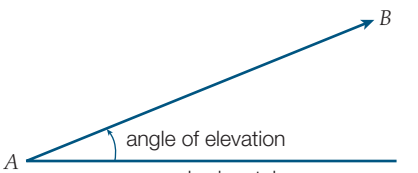


13 Bilal is 1.85 m tall and is standing near a lamp post and a tree. If the length of his shadow is 94 cm, use your own reasonable values for the heights of the lamp post and the tree to determine the length of their shadows at the same time.

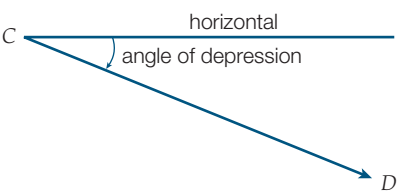
5.4 Angles of elevation and depression

Need to Know

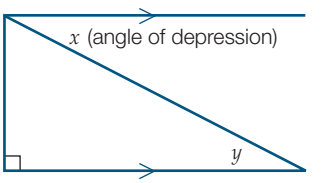
The **angle of elevation** from A to B is the angle that the line AB makes with the horizontal.



The **angle of depression** from C to D is the angle the line CD makes with the horizontal.



The angle of depression may not always be inside the triangle that we draw. However, we can use alternate angles to find an angle within the triangle.



x and y are alternate angles on parallel lines
$\therefore x = y$

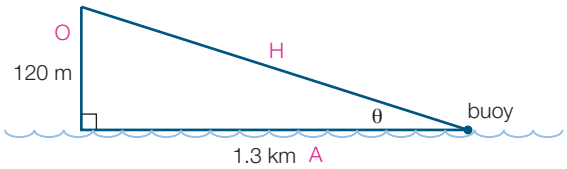
Worked Example 11

WE11

A marker buoy used to guide ships is 1.3 km away from the base of a 120 m cliff. To the nearest degree, what is the angle of elevation from the buoy to the top of the cliff? Answer correct to the nearest degree.

Working

$\theta = ?$
$O = 120 \text{ m}$
$A = 1.3 \text{ km}$



$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{120}{1300}$$

$$\theta = \tan^{-1}\left(\frac{120}{1300}\right)$$

$\theta = 5.273\,89\ldots$
$\theta = 5°$ (nearest degree)

The angle of elevation from the buoy to the top of the cliff is approximately 5°.

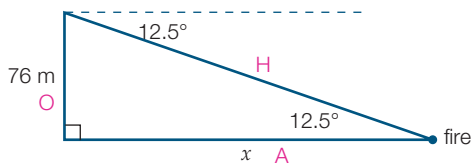
Worked Example 12

WE12

A ranger at the top of a lookout tower spots a small fire at an angle of depression of 12.5°. If the tower is 76 m high, calculate the distance from the base of the tower to the fire. Round your answer to the nearest metre.

Working

$\theta = 12.5°$
$O = 76 \text{ m}$
$A = x$



$$\tan\theta = \frac{O}{A}$$

$$\tan 12.5° = \frac{76}{x}$$

$$x \tan 12.5° = 76$$

$$x = \frac{76}{\tan 12.5°}$$

$x = 343 \text{ m}$ (nearest metre)

The fire is approximately 343 m from the base of the tower.

Answers page 507

5.4 Angles of elevation and depression

Fluency

WE11

1 Hayden is standing 1.2 km away from the base of a 305 m tall building. To the nearest degree, what is the angle of elevation from the point where Hayden is standing to the top of the building?

WE12

2 A person in a hot-air balloon 150 m above the ground spots a parked car at an angle of depression of 16.5°. Calculate the distance of the car to the point that is directly underneath the balloon to the nearest metre.

3 During the day, Kristy notices that a post that is 3 m tall casts a shadow of 2.3 m. What is the angle of elevation of the Sun at that time? Answer to the nearest degree.

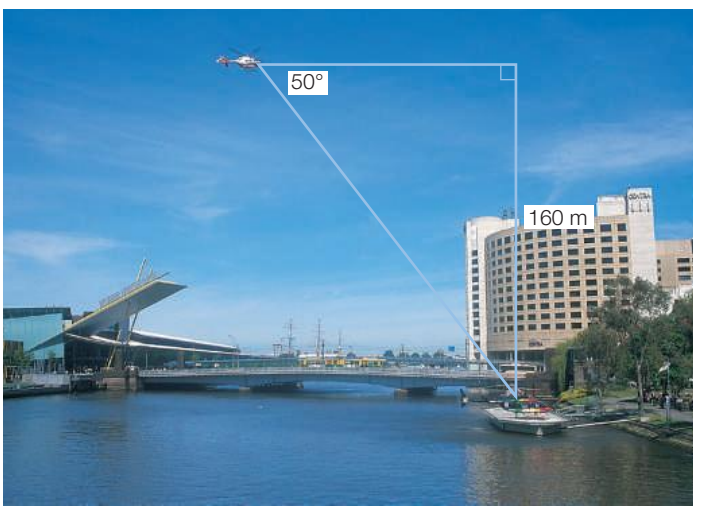
4 The angle of depression from the top of a pine tree to a point on the ground 50 m away is 38°. Find the height of the tree to the nearest metre.

5 A traffic signal 3.95 m high casts a shadow across the road. If the angle of elevation of the Sun is 24.8°, find the length of the shadow, correct to two decimal places.

6 A helicopter is flying at a height of 160 m above a river. The pilot can see another helicopter on a pontoon along the river located at an angle of depression of 50°. Find, to the nearest metre:

(a) the distance from the first helicopter to the second along the line of sight

(b) how far the first helicopter must fly at the same altitude to be directly over the second.



7 (a) A railway rises 400 cm for every 2 km of track. From the lower end of the track, the angle of elevation (to two decimal places) is:

A 0.11° B 11.54° C 25° D 78.69°

(b) From a hot-air balloon, the angle of depression to a marked landing site is 6°50′30″. If the observation deck in the balloon is 190 m above the ground, the horizontal distance the balloon needs to travel to be above the landing site is closest to:

A 191.23 m B 1.58 km C 1.67 km D 1.68 km

Understanding

8 Rachel uses an inclinometer (a device for measuring angles) to measure the angle of elevation from her eye level to the top of a billboard 25 m away. She obtains a value of 42°. She also measures the distance from the ground to her eye level and finds it to be 153 cm. Find the height of the billboard to the nearest metre.

9 A television antenna is mounted on the top of a nearby hill. From a position 4.5 km away, the angle of elevation to the bottom of the antenna is 17°30′, whereas the angle of elevation to the top of the antenna is 17°50′. Find the height of the antenna in metres, correct to two decimal places.

10 A yacht is 1.8 km away from the base of a 120 m cliff. The deck of the yacht is 3 m above sea level. To the nearest degree, what is the angle of elevation from the deck of the yacht to the top of a 4.2 m flagpole on the edge of the cliff?

Reasoning

11 Peter views a stationary beetle on the ground, 2.6 m away, at an angle of depression of 32°. Peter's eyes are 15 cm below the top of his head. He is standing upright.

(a) Find Peter's height to the nearest centimetre.

(b) The beetle moves directly ahead to a new position. If the distance from Peter to the beetle along his line of sight is 4.7 m, find the new angle of depression, correct to one decimal place.

12 Sonya is standing 20 m from a hot-air balloon that is rising vertically at a speed of 1.5 m/s. When the balloon's position is at eye-level, she follows its path for 7 seconds.

(a) Calculate the angle of elevation of the balloon from Sonya's line of sight after 7 seconds.

(b) If Sonya is 175 cm tall, what is the height of the balloon above the ground after 7 seconds?

Open-ended

13 A flagpole 24 m high is to be supported by a cable that is 18 m long when pulled taut. It must be fastened at least halfway up the pole at an angle of elevation between 45° and 60°.

(a) Find three possible angles of elevation for the cable.

(b) For each angle of elevation, find the distance from the flagpole that the cable must be fastened.

14 From the top of a 20 m lookout, Ranger Sam spots a hiker 200 m due east of his position, heading due east away from him at a constant speed of 50 m/min. Ranger Sam notes the angle of depression of the hiker from his position every minute for a period of one hour. Find two values for the angles of depression that Ranger Sam recorded.

Problem solving

Outback mail run

In outback Australia, remote houses and communities receive their mail by plane.

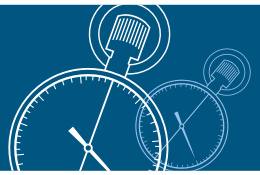
The pilot flying the mail plane is cruising at an altitude of 4500 m when she looks out of the left window and sees one house at an angle of depression of 17°. Out of the right window, she spots another house at an angle of depression of 13°.

How far apart are the two houses, to the nearest kilometre?

Strategy options

- Draw a diagram.
- Have I seen a similar problem?
- Break problem into manageable parts.

Half-time 5



Ex 5.3

1 A ladder of length 3 m reaches 1.85 m up a vertical wall. The ladder is moved closer to the wall and now reaches 2.10 m up the wall. What is the change in the angle the ladder makes with the ground?

Ex 5.3

2 An ant travels 76 cm in a straight line, turns 90°, follows a straight path for 80 cm and then travels directly to its starting point. The smallest interior angle of the triangle formed is:

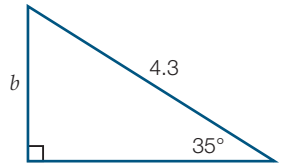
A 40.2° **B** 43.5° **C** 44.6° **D** 45.5°

Ex 5.1

3 Which calculation can be used to find the length of side b?

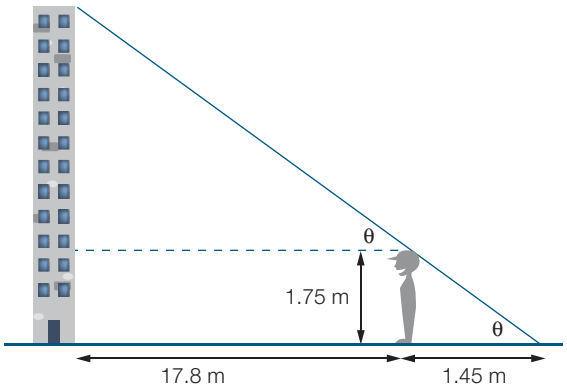
A $b = \dfrac{4.3}{\cos 35°}$ **B** $b = 4.3 \times \sin 35°$

C $b = \dfrac{\tan 35°}{4.3}$ **D** $b = 4.3 \times \cos 35°$



Ex 5.3

4 Find the angle θ in the small triangle and use the result to calculate the height of the building to the nearest centimetre.

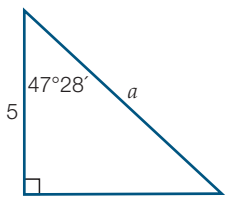


Ex 5.2

5 The value of a in the triangle on the right, rounded to two decimal places, is:

A 2.41 **B** 3.39

C 3.68 **D** 7.40



Ex 5.4

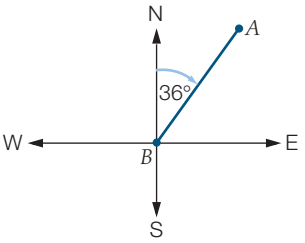
6 From the top of a lookout, a ranger spots a small scrub fire at an angle of depression of 6°14′. How far is the fire from the base of the lookout if the eye-level of the ranger is 35 m above the ground? Give your answer correct to the nearest metre.

Bearings

5.5

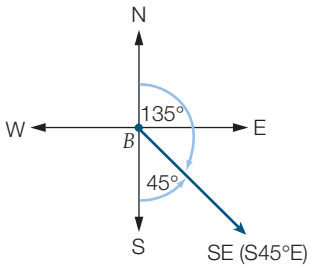
Need to Know

True bearings give the angle in degrees in a clockwise direction from north. The true bearing of A from B is 036°T. True bearings are always written with three digits, so the bearing of 36° is written as 036°T.



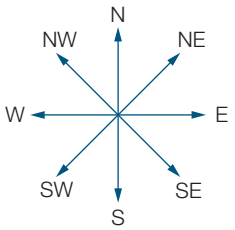
Compass bearings are directions given in terms of turning a number of degrees east or west of north or south (whichever direction is closer to the position). The compass bearing of A from B is N36°E.

The direction of southeast (SE) is shown here. As a compass bearing, it is written as S45°E. As a true bearing, it is written as 135°T. Compass bearings are sometimes called conventional bearings.



The directions exactly in between the four compass points of north, east, south and west have the following bearings.

northeast (NE) is 45°T or N45°E
southeast (SE) is 135°T or S45°E
southwest (SW) is 225°T or S45°W
northwest (NW) is 315°T or N45°W

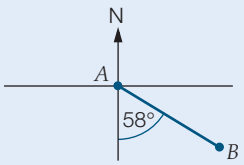


Worked Example 13

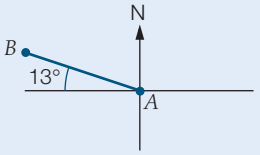
WE13

Find **(i)** the compass bearing and **(ii)** the true bearing of point B from point A, for each of the following.

(a)



(b)



Working

(a) **(i)** S58°E
(It is closer to south, so we write 'S' and then 58° to the east 'E')

(ii) 180 − 58 = 122°T
(Here, we find the difference between the compass bearing and 180°.)

(b) **(i)** 90 − 13 = 77°
N77°W
(It is closer to north, so we write 'N' and then 77° to the west 'W')

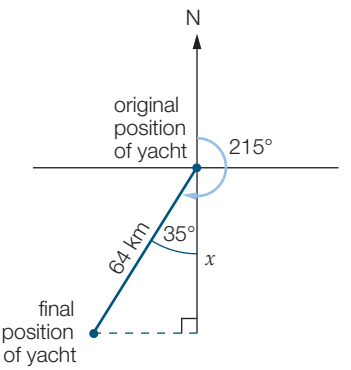
(ii) 270 + 13 = 283°T
(Here, we add 13° to 270°.)

Worked Example 14

WE14

A yacht sails 64 km on a true bearing of 215°T. How far south is the yacht from its original position? Round the answer to two decimal places.

Working



$\theta = 35°$
$H = 64$ km
$A = x$

$$\cos\theta = \frac{A}{H}$$

$$\cos 35° = \frac{x}{64}$$

$$x = 64 \times \cos 35°$$

$x = 52.43$ km (2 d.p.)

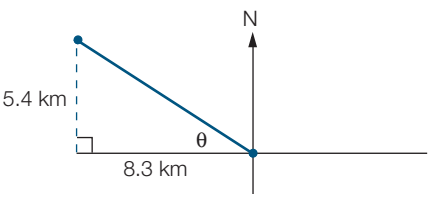
The yacht is 52.43 km south from its original position.

Worked Example 15

WE15

On a hiking trip, Sarah walks 8.3 km west and then 5.4 km north. Calculate her true bearing and compass bearing from her original position, to the nearest degree.

Working



$O = 5.4$ km
$A = 8.3$ km
$\theta = ?$

$$\tan\theta = \frac{O}{A}$$

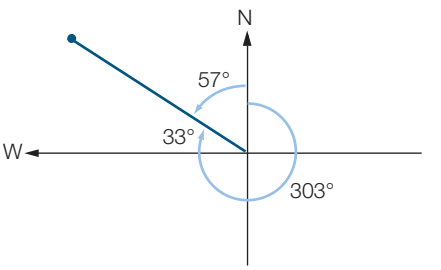
$$\tan\theta = \frac{5.4}{8.3}$$

$$\theta = \tan^{-1}\left(\frac{5.4}{8.3}\right)$$

$\theta = 33°$ (nearest degree)

True bearing $= 270° + 33°$
$= 303°$T

Compass bearing $= 90° - 33°$
$= 57°$
$=$ N57°W



True bearing = 303°T
Compass bearing = N 57°W

5.5 Bearings

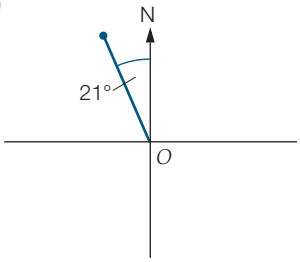
Answers page 507

Fluency

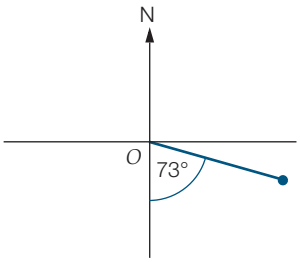
1 Find **(i)** the compass bearing and **(ii)** the true bearing for each of the following points from O.

WE13

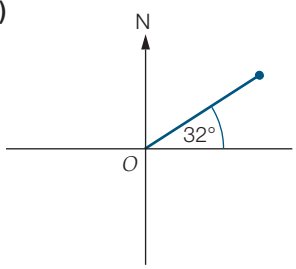
(a)



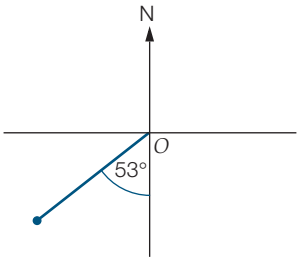
(b)



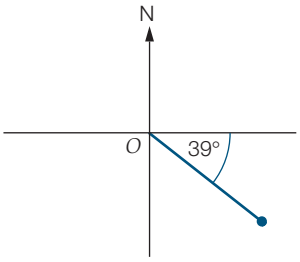
(c)



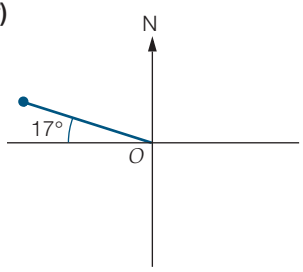
(d)



(e)



(f)



2 A ship sails for 150 km on a bearing of N40°E. How far north of its starting point is it, to the nearest kilometre?

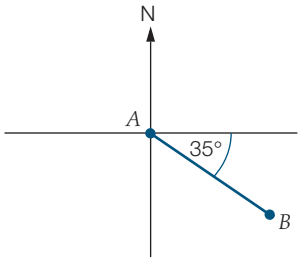
WE14

3 **(a)** Pedro sails his yacht 2.5 km east and then 4.7 km south. To the nearest degree, state Pedro's true bearing and compass bearing from his original position.

WE15

(b) A taxi driver drives 630 m south and then 410 m east. To the nearest degree, what is the bearing of the taxi from its starting position? Give your answer as a compass bearing and a true bearing.

4 **(a)** The compass bearing of point B from point A, as shown in the diagram, is:



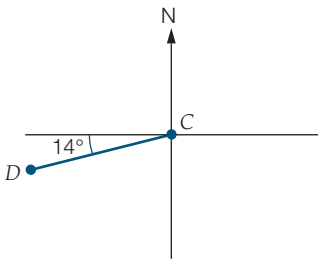
A N35°E

B S55°E

C N35°W

D S35°E

(b) The true bearing of point D from point C, as shown in the diagram, is:



A 076°T

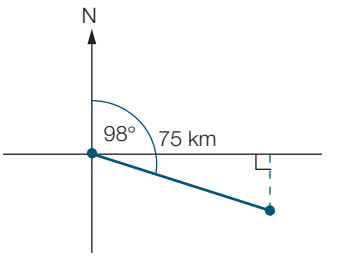
B 194°T

C 256°T

D 284°T

Understanding

5 A helicopter travelling on a true bearing of 098°T is now 75 km east of its original position. How far has the helicopter travelled, to two decimal places?



6 A plane flies on a true bearing of 280°T at 600 km/h after taking off at 1300 hours. To the nearest kilometre, how far north of its starting position is it at 1530 hours?

7 Jack drives north at 80 km/h for 2 hours. He then turns and drives east at the same speed for 3 hours.

(a) To the nearest degree, what is Jack's true bearing from his original position?

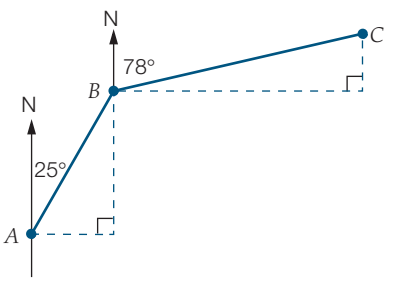
(b) If Jack could drive from his current position straight back to his starting position:

(i) how far would he travel, to the nearest kilometre

(ii) what true bearing should he follow?

8 A hiker walks 5 km on a true bearing of 025°T from camp *A* to camp *B*.

Round your answers to two decimal places.



(a) How far east has she walked from *A* to *B*?

She then continues for another 6 km on a true bearing of 078°T from camp *B* to camp *C*.

(b) **(i)** How far east of camp *B* is she now?

(ii) How far east of camp *A* is she now?

(c) **(i)** How far north of camp B is she now?

(ii) How far north of camp A is she now?

Warning

Round your final answers, but do not use rounded answers in further calculations. Use the original unrounded answers in further calculations.

9 Sasha is running an orienteering course. He runs 750 m to the first checkpoint on a bearing of S53°W. At the checkpoint he is instructed to run another 600 m on a bearing of N68°W to the next checkpoint.

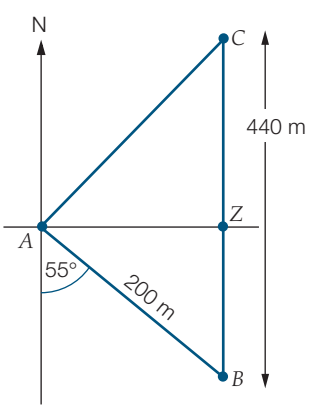
(a) To the nearest metre, how far west is Sasha from his starting position?

(b) Is Sasha's current position north or south of his starting position? How far north or south?

10 A toy robot walks 16 m on a bearing of S62°W, and then turns and walks another 23 m NW. How far west is the robot from its starting point? Round your answer to two decimal places.

Reasoning

11 A crew row their boat from point *A* to point *B*, a distance of 200 m on a bearing of S55°E. They then row due north a further 440 m to point *C*, as shown in the diagram.



(a) Find, to the nearest metre:

(i) the distance from *B* to *Z*

(ii) the distance from *A* to *Z*.

(b) Find the true bearing of *C* from *A*, to the nearest degree.

Finally, the crew row back from *C* to *A*.

(c) What is the total distance rowed by the crew, to the nearest metre?

Hint

Pythagoras' theorem is helpful for finding missing sides in a right-angled triangle.

12 **(a)** From his home, Minh jogs 2.7 km NE and then 7.4 km SE. To the nearest degree, find his true bearing from his starting position.

(b) If Minh then jogs 6.9 km west, how far south is he from his home? Round your answer to two decimal places.

13 An air force jet takes off from an airfield and flys on a bearing of N37°E. The pilot is asked by ground staff to alter her course, 600 km into the trip, by turning 120° in a clockwise direction.

(a) State the new compass bearing of the jet after it has travelled 600 km.

(b) The jet travels for a further 800 km on its new bearing. How far east has it travelled from its starting position, to the nearest kilometre?

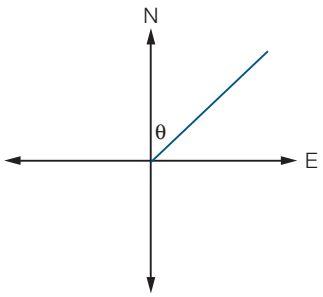
(c) How far north or south of its position at take-off, is the current position of the jet, to the nearest kilometre?

(d) What compass bearing should the jet now follow to return to the airstrip?

(e) How far will it travel on its return, if it follows the bearing you determined in **(d)**? Answer to the nearest kilometre.

Open-ended

14 Choose a true bearing θ less than 90° and state its true bearing and compass bearing after it is:



(a) reflected about the east-west line (the x-axis of the Cartesian plane)

(b) reflected about the north-south line (the y-axis of the Cartesian plane)

(c) rotated 180°.

15 Design a sailing course around five buoys labelled A, B, C, D and E. On each leg of the course, write down the true bearing to be taken by the sailors competing in the race. Remember that the course finishes at the starting point.

Problem solving

Seeing the light

1 A ship leaves its dock, which is 3 km due west of a lighthouse, and travels NE at a constant speed of 15 km/h.

(a) To the nearest minute, when is the ship due north of the lighthouse?

Strategy options
- Draw a diagram.
- Break problem into manageable parts.

(b) At this time, how far is the ship from the lighthouse?

(c) How far has the ship travelled?

2 Rework these answers if the ship's bearing is N20°E.

5.6 Mixed two-dimensional problems

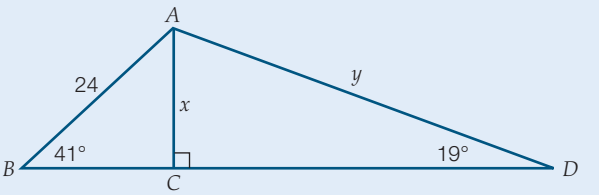
Need to Know

It is important that we use unrounded answers when using these answers in further calculations. This avoids the accumulation of errors due to rounding. Always use the unrounded answer if it is required in further calculations. Use the calculator's memory key to store values, if necessary, and then round off once only for your answer.

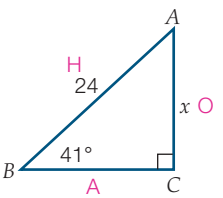
Worked Example 16

WE16

Find the values of x and y, rounded to two decimal places.



Working

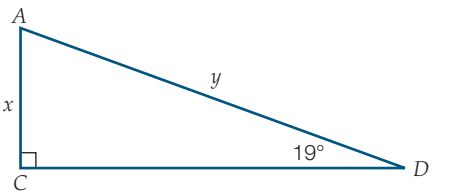


$\theta = 41°$
$H = 24$
$O = x$

$$\sin\theta = \frac{O}{H}$$

$$\sin 41° = \frac{x}{24}$$

$x = 24 \times \sin 41°$
$x = 15.745\,416\,7$



$\theta = 19°$
$O = 15.745\,416\,7$
$H = y$

$$\sin\theta = \frac{O}{H}$$

$$\sin 19° = \frac{x}{y}$$

$y \times \sin 19° = x$

$$y = \frac{x}{\sin 19°}$$

$$y = \frac{15.745\,416\,7}{\sin 19°}$$

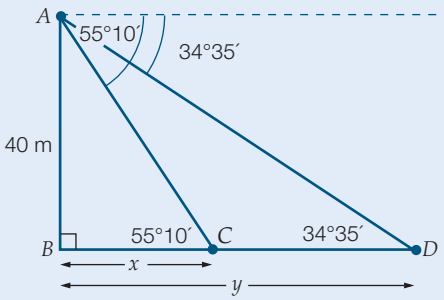
$y = 48.362\,889\,55$

$x = 15.75$ units $y = 48.36$ units (2 d.p.)

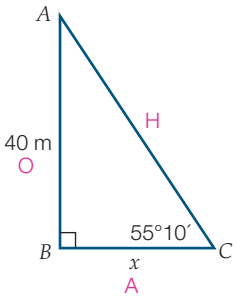
Worked Example 17

WE17

Thea sights two swimmers (both on the same bearing from her) at angles of depression of 34°35′ and 55°10′ from her vantage point on the cliff. If her eye-level is 40 m above the water level, find the distance between the two swimmers, rounded to two decimal places.



Working



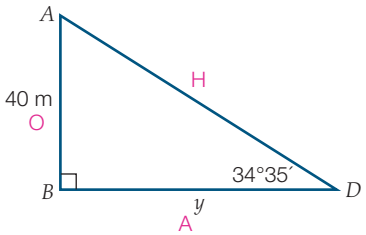
$\theta = 55°10'$
$O = 40$ m
$A = x$

$$\tan\theta = \frac{O}{A}$$

$$\tan 55°10' = \frac{40}{x}$$

$$x = \frac{40}{\tan 55°10'}$$

$x = 27.835\,250\,33$



$\theta = 34°35'$
$O = 40$ m
$A = y$

$$\tan\theta = \frac{O}{A}$$

$$\tan 34°35' = \frac{40}{y}$$

$$y = \frac{40}{\tan 34°35'}$$

$y = 58.019\,401\,28$

$y - x = 30.184\,150\,95$

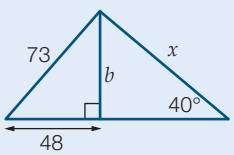
$CD = 30.18$ m (2 d.p.)

The distance between the swimmers is approximately 30.18 m.

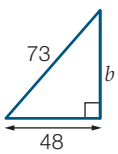
Worked Example 18

WE18

Calculate the value of x, rounded to two decimal places.



Working



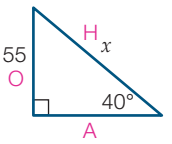
$c^2 = a^2 + b^2$, where $c = 73$, $a = 48$

$73^2 = 48^2 + b^2$

$b^2 = 73^2 - 48^2$

$b^2 = 3025$

$b = 55$



$\theta = 40°$
$H = x$
$O = 55$
$\sin\theta = \frac{O}{H}$
$\sin 40° = \frac{55}{x}$
$x = \frac{55}{\sin 40°}$
$x = 85.56$ units

Answers page 507

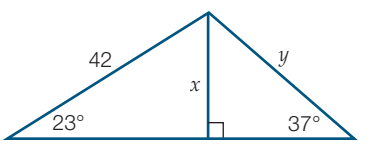
5.6 Mixed two-dimensional problems

Fluency

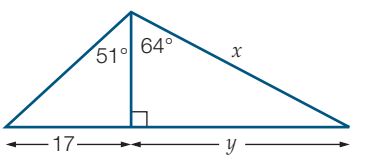
WE16

1 Find the values of x and y, rounded to two decimal places.

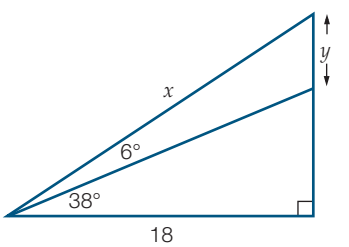
(a)



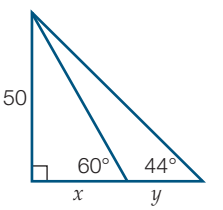
(b)



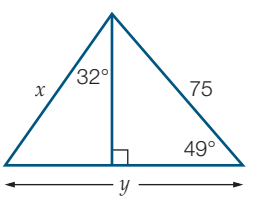
(c)



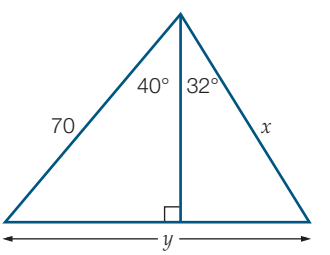
(d)



(e)



(f)



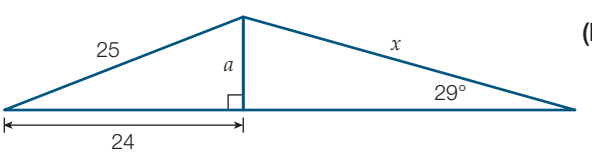
WE17

2 From the top of a lighthouse 75 m above sea level, two whales on the same bearing can be seen in the ocean at angles of depression of 28°40′ and 46°25′. Find the distance between the two whales, rounded to two decimal places.

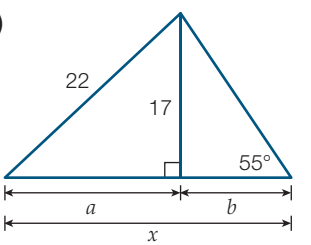
WE18

3 Calculate the value of the x in the following, rounded to two decimal places.

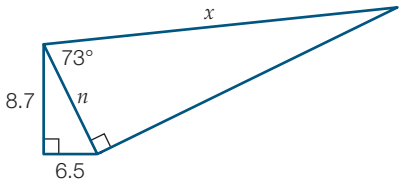
(a)



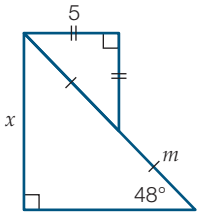
(b)



(c)

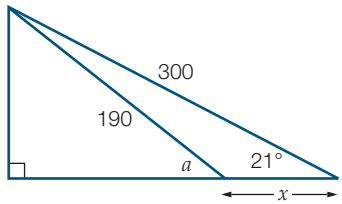


(d)

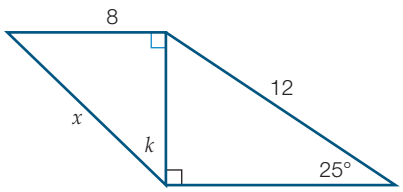


4 Find the values of the pronumerals. Round your answers to one decimal place.

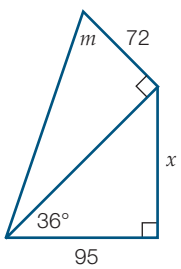
(a)



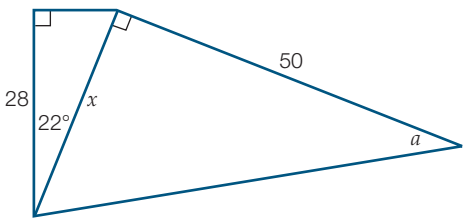
(b)



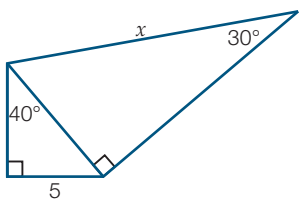
(c)



(d)



5 The length of x in the diagram is closest to:



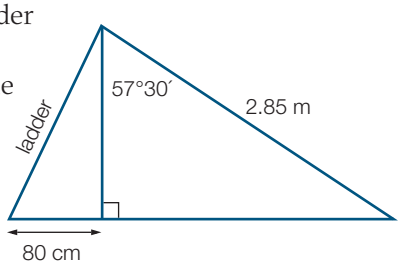
A 15.56

B 17.96

C 31.11

D 42.04

6 A slide in a playground has a vertical support and a ladder as shown in the diagram. The slide is 2.85 m long and makes an angle of 57°30′ with the vertical support. If the base of the ladder is 80 cm away from the support, the length of the ladder, to two decimal places, is:

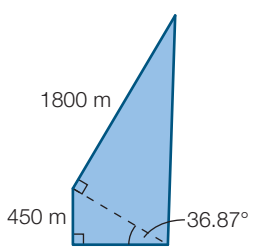


A 0.37 m **B** 0.42 m

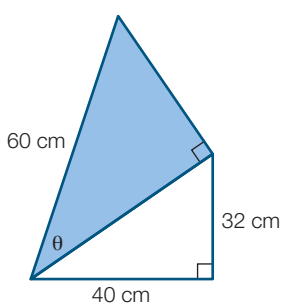
C 1.53 m **D** 1.73 m

Understanding

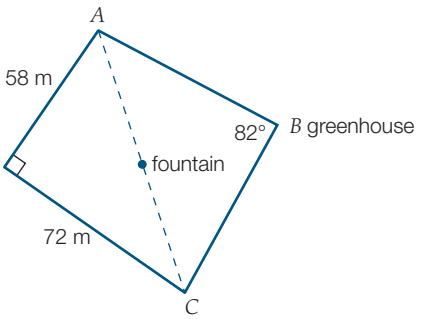
7 A farm whose perimeter shape is shown requires wire fencing costing \$5 per metre. Calculate the total cost of fencing, to the nearest dollar.



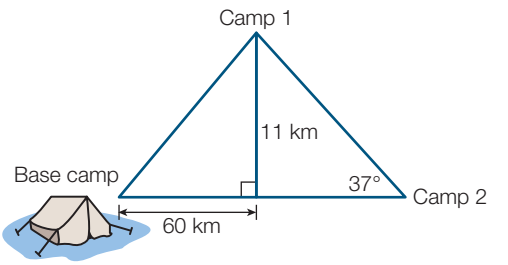
8 A blue tile and a white tile are to be joined as part of a mosaic design. What angle θ is required? Express your answer in degrees and minutes.



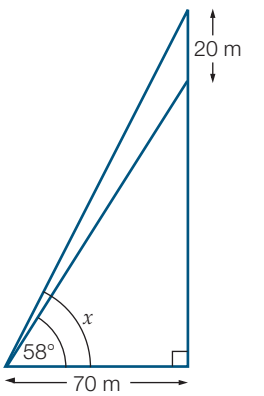
9 A surveyor is hired to peg out the garden shown. Shape *ABC* will be an isosceles triangle and the fountain will be positioned midway between *A* and *C*. To the nearest metre, what will be the length of the direct path from the fountain to the greenhouse?



10 Miguel left Base camp and trekked to Camp 1, then to Camp 2 and finally back to Base camp. To the nearest kilometre, how far did he walk in total?



11 When Liz is 70 m away from a building she notices a stuntman at an angle of elevation of 58° from her horizontal line of sight. He is climbing the side of the building. What will be the angle of elevation after the stuntman has climbed a further 20 m? Round your answer to the nearest degree.



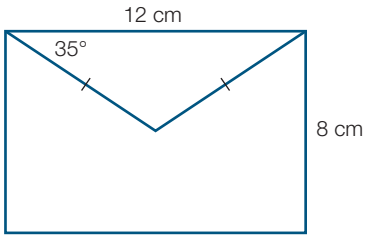
12 A 4.5 m ladder leaning against a wall makes an angle of 64°5′ with the ground. If the ladder is moved 83 cm down the wall, what angle does it now make with the ground, in degrees, minutes and seconds?



13 A pilot is landing a plane. He views two lights positioned in a line on the runway at angles of depression of 18° and 22°. If the plane is 300 m above the runway, find the distance between the two lights. State your answer correct to two decimal places.

Reasoning

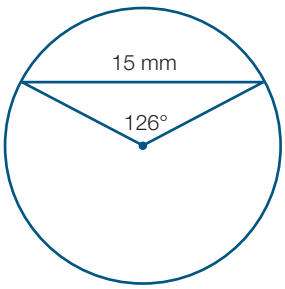
14 An envelope has a length of 12 cm and a width of 8 cm. The back flap is an isosceles triangle with an edge that makes an angle of 35° with the top of the envelope. Find the distance between the bottom of the flap and the bottom of the envelope. Round your answer to two decimal places.



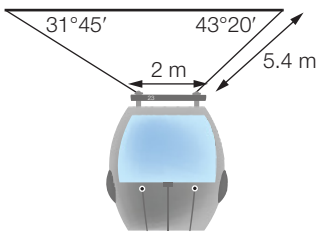
Hint

Draw a vertical line from the bottom of the flap to the top of the envelope.

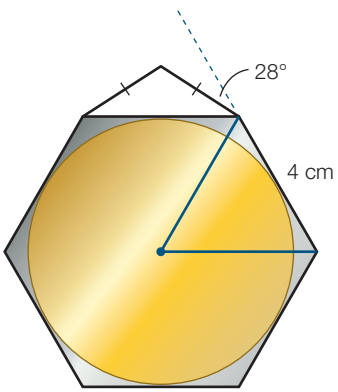
15 A chord 15 mm long forms an angle of 126° at the centre of a circle. Find the length of the diameter of the circle, correct to two decimal places.



16 A cable car is supported by two wires attached to a horizontal cable. One of the wires is 5.4 m long and makes an angle of 43°20′ with the horizontal cable. The other wire makes an angle of 31°45′ with the cable. If the cable car is 2 m long, how far apart along the cable are the wires, correct to two decimal places?



17 A jeweller has designed a medal in the shape of a circle mounted on a regular hexagon. The circular inset is made from gold. The medallion will hang by a chain passing through the triangular shape shown at the top of the medal.



Hint

The sum of the interior angles of an n-sided polygon is $(n - 2) \times 180°$.

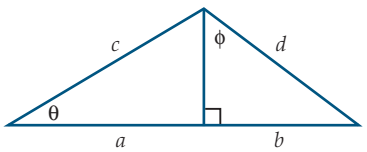
(a) Use the triangle marked on the medal to find the radius of the circular inset.

(b) Use your answer to **(a)** to find the area of gold required to make the circular inset. Round your answer to the nearest square centimetre.

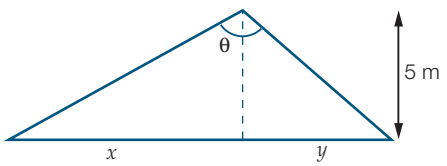
(c) What length of metal is required for the two sides of the triangular hanging top? Round your answer to two decimal places.

Open-ended

18 Determine values for a, b, c, d, θ and ϕ so that $a > b$ and $c > d$.



19 The cross-section of a roof (not necessarily symmetrical in shape) of height 5 m is shown. Determine values for x, y and θ so that the base has a length between 20 m and 25 m, and θ is greater than 100°.



Chapter review 5

D.I.Y. Summary

Key Words

adjacent side	compass bearing	minute	sine	true bearing
angle of depression	hypotenuse	opposite side	tangent	
angle of elevation	cosine	second	trigonometric ratio	

Copy and complete the following using words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 The _____ ratio is obtained by dividing the adjacent side by the hypotenuse.

2 The _____ ratio is obtained by dividing the opposite side by the hypotenuse.

3 The _____ ratio is obtained by dividing the opposite side by the adjacent side.

4 When a direction is given with a measurement east or west from north or south, this is a _____.

5 A _____ always has three digits and shows the number of degrees from north in a clockwise direction.

6 One degree is divided into sixty equal parts. Each part is called a _____.

7 There are 360 _____s in one degree.

8 The angle from the horizontal as you look up is called an _____. The angle from the horizontal as you look down is called an _____.

Fluency

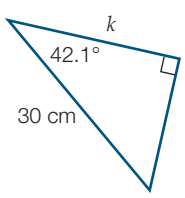
1 Use a calculator to find the following values, rounded to four decimal places. Ex 5.1

(a) sin 35° **(b)** tan 51.3° **(c)** cos 6.045°

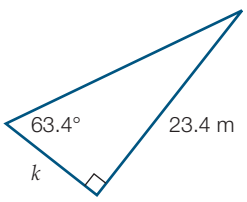
(d) tan 29°45′ **(e)** sin 16°24′ **(f)** cos 73°6′40″

2 Find the value of k in each of these triangles, rounded to two decimal places. Ex 5.2

(a)



(b)



3 For each of the following trigonometric ratios, find the value of θ in: Ex 5.3

(i) decimal degrees, rounded to two decimal places

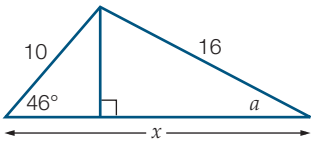
(ii) degrees, minutes and seconds.

(a) $\cos\theta = 0.74$ **(b)** $\tan\theta = 2.645$ **(c)** $\sin\theta = 0.4609$

4 A mast of height 5 m is on top of a building whose base is 100 m away from a drain on the road. The angle of elevation of the top of the mast from the drain is 34°. Calculate the height of the building, rounded to two decimal places. Ex 5.4

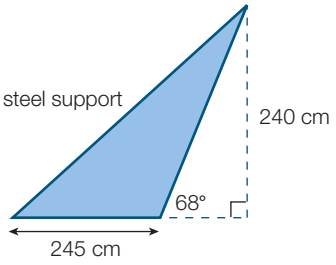
5 A plane flies due north for 70 km, then due west for a further 145 km. What is the compass bearing of the plane from its starting point, to the nearest degree? Ex 5.5

6 Find the values of the pronumerals in this diagram. Round your answers correct to one decimal place.



Ex 5.6

7 The sculpture, 'A study in triangles', is supported by a rigid steel rod. What is the length of the steel support, rounded to the nearest centimetre?



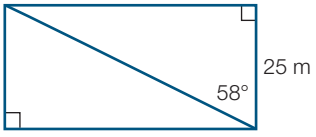
Ex 5.6

8 A conveyor belt in an underground mine is 650 m long and moves to the surface at an angle of 14°48′ from the horizontal. The depth of the mine is:

A 2544.57 m **B** 672.30 m **C** 628.44 m **D** 166.04 m

Ex 5.6

9 The length of the diagonal in this rectangle is:



A 13.25 m **B** 29.48 m

C 40.01 m **D** 47.18 m

Ex 5.6

Understanding

10 A balloon on the end of a string attached to the ground is being blown by the wind. If the balloon is 12.75 m above the ground and the string makes an angle of 48° to the horizontal, find the length of the string, rounded to two decimal places.

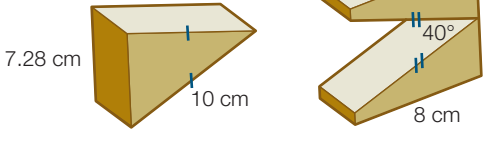
11 A ramp is 9.4 m long and joins two levels, one 3.1 m higher than the other. What angle does the ramp make with the lower level, rounded to one decimal place?

12 A lifeguard on watch at a beach spots a shark at an angle of depression of 9°. If the lifeguard views the shark from a height (to his eye-level) of 16 m above sea-level, find the horizontal distance between the lifeguard and the shark to the nearest metre.

13 A speedboat travels 26 km on a true bearing of 065°T. Find how far the boat is to the east of its starting point. Round your answer to two decimal places.

14 An aeroplane leaves the airport and flies 35 km in the direction N70°E. It then flies due east for a further 38 km. What is the aeroplane's distance and compass bearing from the airport? Round the distance to the nearest kilometre and give the angle in degrees and minutes.

15 Decide whether the wedge shown will fit exactly into the cavity in the block of wood. Give reasons for your answer.

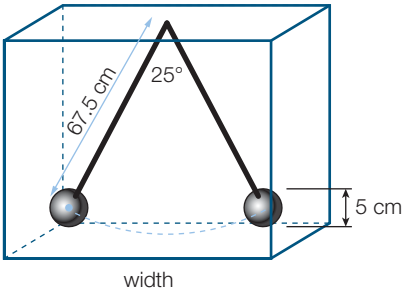


Reasoning

16 Amos leaves home and cycles for a distance of 35 km on a bearing of N30°E. He changes direction and cycles in a direction of S60°E before heading home on a bearing of S49°W. To the nearest kilometre, find the total distance Amos travelled.

17 For an equilateral triangle, explain two different ways the height can be found using trigonometric ratios.

18 A case is to be constructed to hold a pendulum. The shaft of the pendulum is connected to the centre of the bob and is 67.5 cm in length. The diameter of the bob is 5 cm. If the pendulum swings through an angle of 25°, what must the minimum width of the case be? Round your answer to two decimal places.



6 Advanced trigonometry

10A

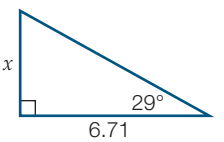
Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

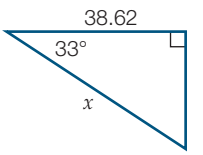
Worksheet R6.1

1 Calculate the value of x and round to two decimal places.

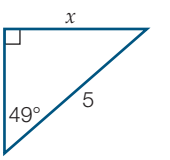
(a)



(b)



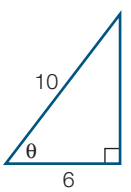
(c)



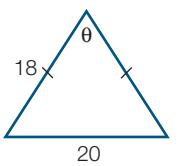
Worksheet R6.2

2 Determine the size of the angle θ, in degrees and minutes.

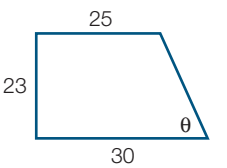
(a)



(b)



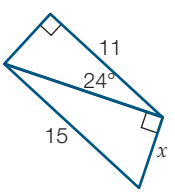
(c)



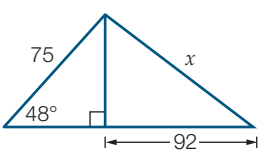
Worksheet R6.3

3 Find the length of x. Round your answer to two decimal places.

(a)



(b)



Worksheet R6.4

4 Jake is flying a kite on the end of a 20 m rope. If the rope has an angle of elevation from Jake's line of sight of 69°, how high is the kite above the ground, given the distance from the ground to Jake's eye-level is 1.65 m? Round your answer to two decimal places.

Worksheet R6.5

5 A hiker leaves her campsite and walks 10 km on a bearing of 143°T, then walks 6 km on a bearing of 039°T. Calculate the following and round your answers to two decimal places.

(a) How far is she from her campsite?

(b) What bearing must she take to walk directly back to the camp?

After completing this chapter you will be able to:

- use trigonometry to solve three-dimensional problems
- use the unit circle to define trigonometric functions
- understand the periodic nature of the trigonometric functions
- graph trigonometric functions with and without technology
- use the sine and cosine rules to solve for unknowns in triangles
- apply trigonometry to finding the area of a triangle.

Solving three-dimensional problems

6.1

Need to Know

To solve a three-dimensional problem involving right-angled triangles:

Step 1 Draw a diagram that represents the problem.

Step 2 Identify right-angled triangles that contain the unknown you want or that will help you find it.

Step 3 Redraw these right-angled triangles.

Step 4 Solve these triangles using trigonometric ratios and/or Pythagoras' theorem.

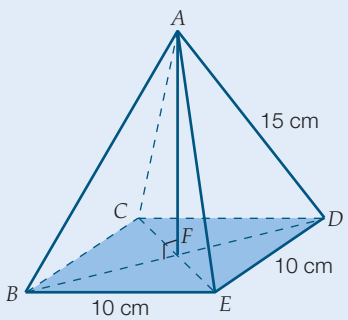
Worked Example 1

WE1

A right pyramid *ABCDE* stands on a square base of edge length 10 cm. The sloping edges are each of length 15 cm. *AF* is the perpendicular height.

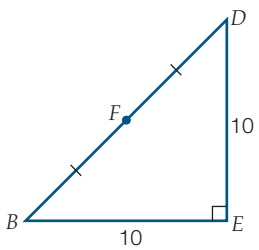
(a) Find the angle that a sloping edge makes with the base of the pyramid, rounded to the nearest degree.

(b) Hence, find the height of the pyramid, rounded to two decimal places.



Working

(a)



(We will use ΔBDE to find BD as DF is half of BD.)

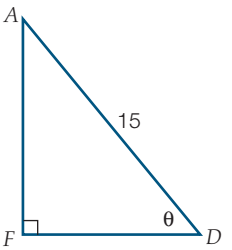
In ΔBDE:

$(BD)^2 = 10^2 + 10^2$

$(BD)^2 = 200$

$BD = \sqrt{200}$

$DF = \frac{\sqrt{200}}{2}$



(We will use ΔADF to find θ.)

In ΔADF:
$H = 15$
$A = \frac{\sqrt{200}}{2}$
$\theta = ?$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{DF}{15}$

$\theta = \cos^{-1}\left(\frac{DF}{15}\right)$

$\theta = \cos^{-1}\left(\frac{\sqrt{200}}{30}\right)$

$\theta = 61.874\,474\,3$

$\theta = 62°$ (nearest degree)

A sloping edge makes an angle of about 62° with the base of the pyramid.

(b) $\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{AF}{15}$

$AF = 15\sin\theta$
$AF = 15\sin 61.874\,494\,3°$
$AF = 13.228\,756\,56$
$AF = 13.23$ (2 d.p.)

The height of the pyramid is about 13.23 cm.

Answers page 508

6.1 Solving three-dimensional problems

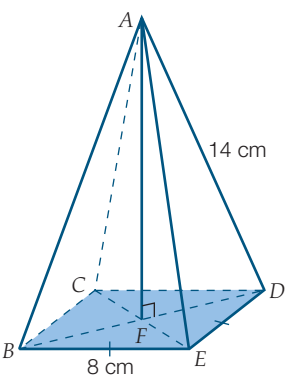
Fluency

WE1

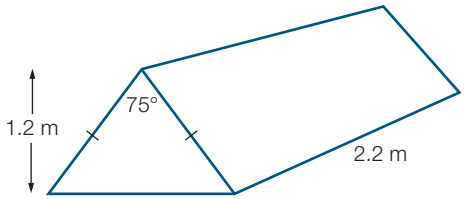
1 A right pyramid *ABCDE* stands on a square base of edge length 8 cm. The sloping edges are each 14 cm long. *AF* is the perpendicular height.

(a) Find the angle that a sloping edge makes with the base of the pyramid, rounded to the nearest degree.

(b) Hence, find the height of the pyramid, rounded to two decimal places.



2 A tent has the dimensions shown in the diagram. Find the area of the groundsheet needed to fit inside the tent. Round your answer to two decimal places.

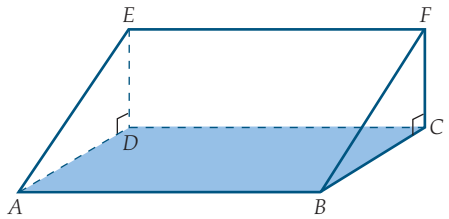


Understanding

3 **(a)** To find the angle that plane *ABFE* makes with plane *ABCD*, we could calculate:

A $\angle FCB$ **B** $\angle BFC$

C $\angle DAB$ **D** $\angle DAE$

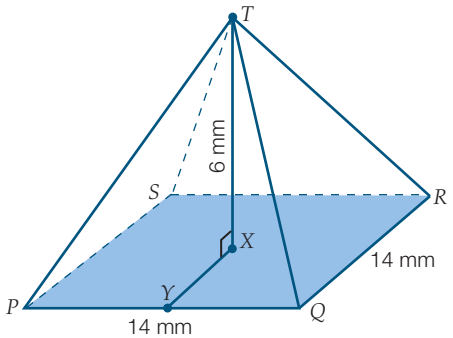


(b) To find the length *AF*, we could use the triangle:

A *ABC* **B** *AEF* **C** *BCF* **D** *ABD*

For Questions **4** to **11**, unless stated otherwise, round angles to one decimal place and round other answers to two decimal places.

4 *PQRST* is a square-based pyramid with a base of side length 14 mm and a height of 6 mm. The point *X* on the base is directly below *T* at the apex, and point *Y* is midway along *PQ*.



(a) Find the angle *TXY*.

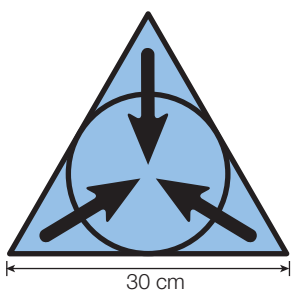
(b) What angle does the sloping face *PQT* make with the base?

(c) Find the length of *TY*.

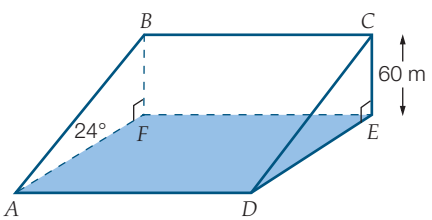
(d) Find the length of a sloping edge of the pyramid.

(e) Find the angle that a sloping edge makes with the base of the pyramid.

5 A new road sign warning of merging traffic consists of a circle inside an equilateral triangle. What is the radius of the circle?



6 A hill *ABCD* rises at an angle of 24° to the horizontal. The hill is 60 m high.



(a) If you walked straight up the hill along *AB*, what distance would you travel?

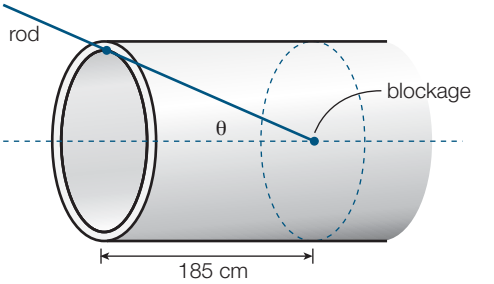
(b) What horizontal distance would you cover walking along *AB*?

(c) Your friend walks along a track from *A* to *C*, taking 30 minutes to reach the top. Horizontally, the distance she covers is 510 m. How fast does she walk in m/min?

(d) Find the angle of the track *AC* to the horizontal.

(e) Gradient represents the steepness of a slope and can be calculated by using the formula $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{vertical distance}}{\text{horizontal distance}}$. Calculate the gradients of *AB* and *AC*, and hence determine which track is steeper.

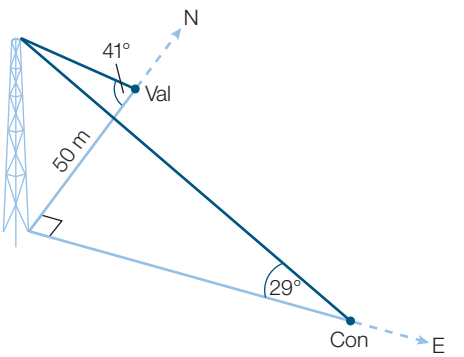
7 A plumber inserts a metal rod into a cylindrical pipe whose inner radius is 9 cm and discovers that there is a blockage in the centre of the inner circle of the pipe at the point shown.



(a) What is the maximum length of the rod that can be inserted inside the pipe to reach the blockage?

(b) What is the maximum angle θ possible? Give your answer in degrees, minutes and seconds.

8 Val is 50 m due north of a communications tower. The top of the tower is at an angle of elevation of 41° from the ground at Val's observation point. From Con's observation point, due east of the tower, the angle of elevation of the top of the tower is 29°.



Find:

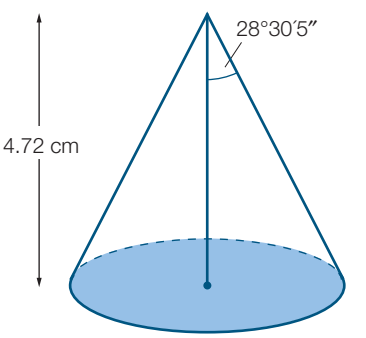
(a) the height of the tower

(b) the distance from Con to the foot of the tower

(c) the distance between Val and Con

(d) Con's true bearing from Val's observation point.

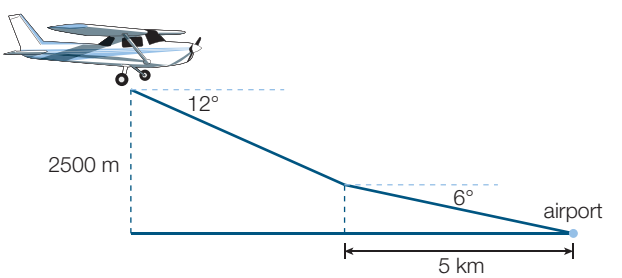
9 The height of a vertical cone is 4.72 cm, and its semi-vertical angle is 28°30′5″. Find:



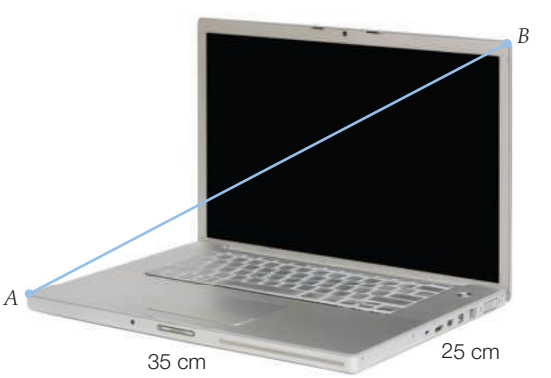
(a) the slant height of the cone

(b) the diameter of the cone.

10 A light plane at a height of 2500 m begins to descend at an angle of 12° to the horizontal. When the plane is 5 km from the airport, the descent angle is reduced to 6°. What horizontal distance from the airport is the plane when it is 2500 m above the ground? Round your answer to the nearest metre.



11 A laptop sends information via infrared from a transmitter at point *A* to a receiver at point *B*. When the screen is at right angles to the keyboard, the distance from the receiver to the transmitter must be no more than 50 cm. Is this true for the laptop shown?



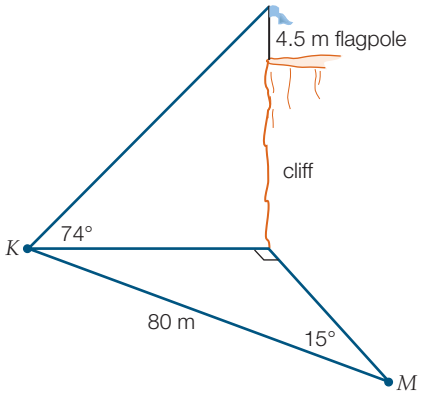
Reasoning

12 A passenger on a cruise ship is positioned 42 m above the water level and is admiring a tropical island in the distance. One end of the island is at a true bearing of 028° and has an angle of depression of 11°, while the other end is at a true bearing of 118° and has an angle of depression of 19°. How long is the island to the nearest metre?

13 A ship's navigator needs to find the height of a cliff. She takes measurements from two different positions in the water, as shown in the diagram. There is a flagpole on top of the cliff.

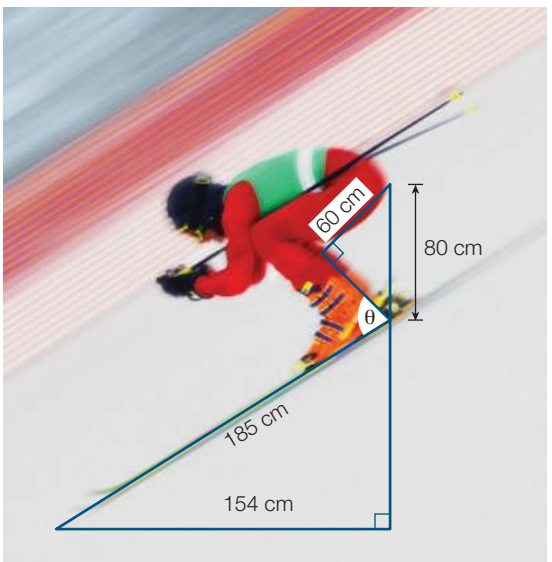
(a) If the flagpole is known to be 4.5 m tall, find the height of the cliff.

(b) What is the distance from the top of the flagpole to the navigator at position K?

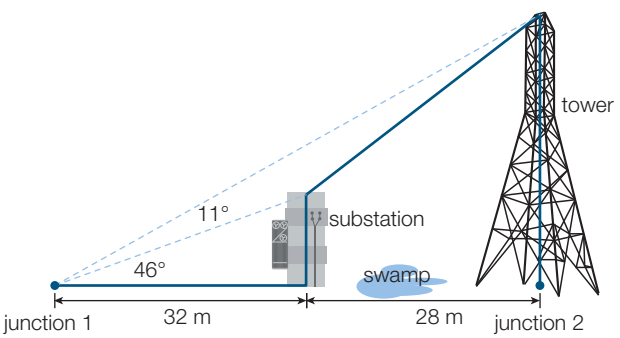


14 Liam wants to construct a rectangular desktop with a diagonal length of 70 cm. When the desktop is tilted at 15° to the horizontal, it is raised 8 cm. Find the area of the desktop, to the nearest square centimetre.

15 To minimise a skier's air resistance, the angle θ shown must be less than 77°. Decide if this is true for the skier shown.



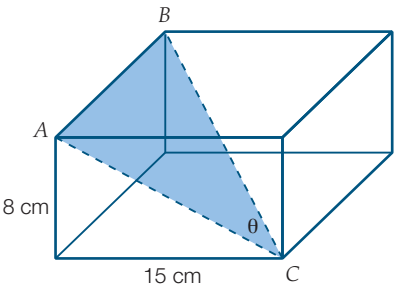
16 Electric cable (shown in blue) will be laid underground from junction 1 to the base of the substation. It will continue vertically up the wall of the substation. From the top of the substation it will continue over the swamp to the top of the tower and down to the ground to junction 2. What is the total length of cable required? Round your answer to the nearest metre.



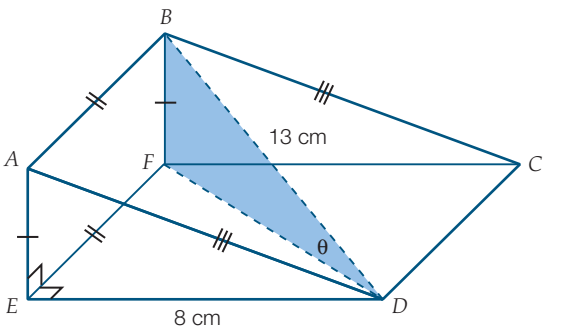
Open-ended

17 A jewellery box in the shape of a rectangular prism has the dimensions shown. Using three different values for θ, obtain the area of triangle ABC and calculate the ratio $\frac{\text{area of triangle } ABC}{\text{volume of box}}$.

What can you conclude about your results?



18 A wedge of cheese in the shape of a triangular prism has the dimensions shown. By choosing two values for the angle θ, calculate the following, correct to two decimal places.



(a) Find the height, BF, of the wedge.

(b) Find the lengths DF, EF and BC.

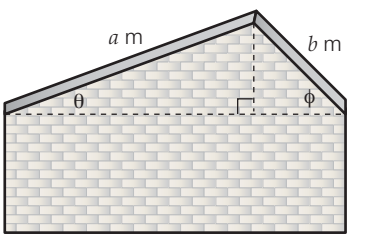
(c) Calculate the area of rectangle $ABCD$.

(d) What effect did changing the value of θ have on the area of $ABCD$?

(e) Calculate the total area of plastic needed to wrap the cheese (no overlap).

(f) For which value of θ was the least plastic used? Compare this to your answer in **(d)**.

19 The cross-section of a roof is shown. a and b are the lengths of the roofing, and θ and ϕ are the angles each roofline makes with the horizontal.



(a) Show that $\frac{a}{b} = \frac{\sin \phi}{\sin \theta}$.

(b) If $a = 2b$ and angles θ and ϕ must be greater than 20° and less than 60°, state one pair of possible values for the angles θ and ϕ. State your answer to the nearest degree.

Problem solving

The maths challenge

You are in one of two 'tribes' that are competing on a desert island. Both tribes are led to the edge of a deep, fast flowing river, where the weekly challenge is issued. Each tribe is given a large protractor, a 10 m long measuring tape, and some instructions, written on a piece of bark. You crowd around the bark with your fellow tribe members. It reads:

Calculate the height of the tree directly across the river from where you are standing. Use only the equipment you have been given, plus any stick, stone, leaf or vine from the jungle around you. The first team to accurately determine the height of the tree wins double food rations for the week.

P.S. Two triangles could be useful.

'That's impossible!' moan half the tribe members. 'We don't even know the width of the river!'

Describe the method and calculations your tribe could use to calculate the height of the tree and win the challenge.

Strategy options

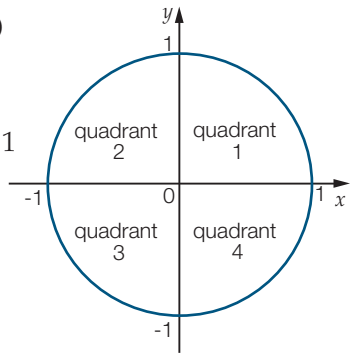
- Make a model.
- Act it out.
- Break problem into manageable parts.

The unit circle

6.2

Need to Know

A **unit circle** has a radius of 1 and a centre at the origin (0, 0) of the Cartesian plane. We divide the unit circle into four **quadrants** as shown on the diagram.

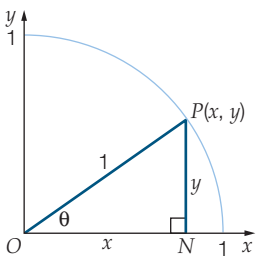


The right-angled triangle NOP has been drawn in quadrant 1 of the unit circle with its radius as the hypotenuse. From ΔNOP, we can define two trigonometric ratios:

- sine θ, abbreviated to $\sin\theta$
- cosine θ, abbreviated to $\cos\theta$.

$$\sin\theta = \frac{O}{H} = \frac{NP}{OP} = \frac{y}{1} = y$$

$$\cos\theta = \frac{A}{H} = \frac{ON}{OP} = \frac{x}{1} = x$$

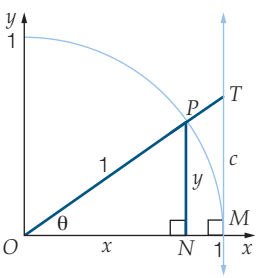


In the right-angled triangle MOT, TM is a tangent to the circle at M and parallel to the y-axis at the point T. The length of TM is marked here as c.

From ΔMOT, we can define the trigonometric ratio:

- tangent θ, abbreviated to $\tan\theta$.

$$\tan\theta = \frac{O}{A} = \frac{TM}{OM} = \frac{c}{1} = c = \text{length of } TM$$

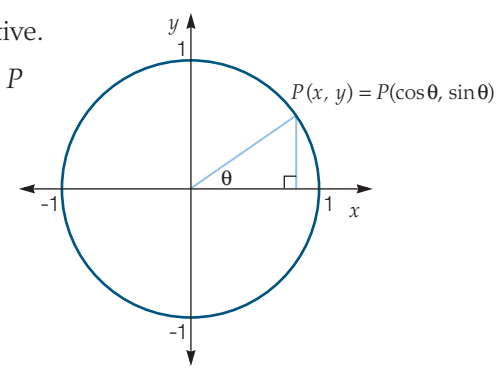


Quadrant 1

In quadrant 1, all trigonometric ratios are positive.

The angle θ determines the position of a point P on the circumference of the unit circle. $P(x, y)$ can now be written as $P(\cos\theta, \sin\theta)$, where:

- $\cos\theta$ is the x-coordinate of P
- $\sin\theta$ is the y-coordinate of P
- $\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$



In quadrant 1, as θ increases from 0° to 90°:

- sin θ increases from 0 to 1
- cos θ decreases from 1 to 0
- tan θ increases from 0 to an undefined positive ratio.

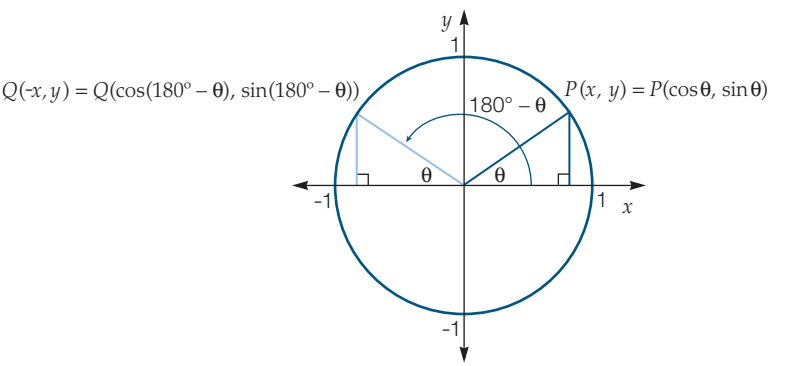
Equivalent angles

The unit circle can also be used to find trigonometric ratios for angles in the other three quadrants using **equivalent angles**. By drawing a congruent right-angled triangle in each quadrant we can determine trigonometric ratios for angles that are not acute.

Quadrant 2

In quadrant 2, only the sin ratio is positive.

$\sin(180° - \theta) = \sin\theta$ $\cos(180° - \theta) = -\cos\theta$ $\tan(180° - \theta) = -\tan\theta$



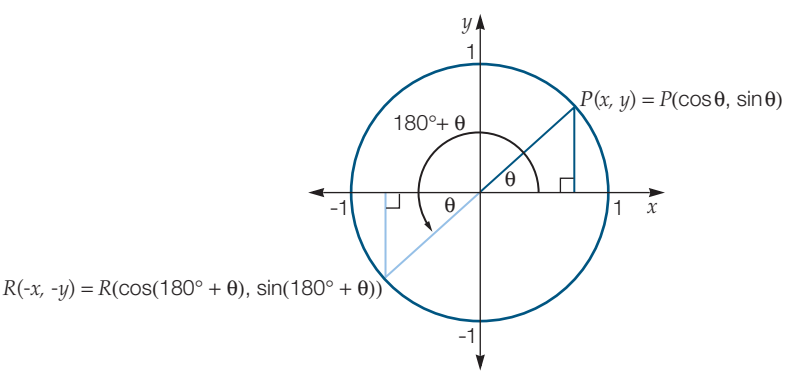
In quadrant 2, as θ increases from 90° to 180°:

- sin θ decreases from 1 to 0
- cos θ decreases from 0 to -1
- tan θ decreases from an undefined positive ratio to 0.

Quadrant 3

In quadrant 3, only the tan ratio is positive.

$\tan(180° + \theta) = \tan\theta$ $\sin(180° + \theta) = -\sin\theta$ $\cos(180° + \theta) = -\cos\theta$



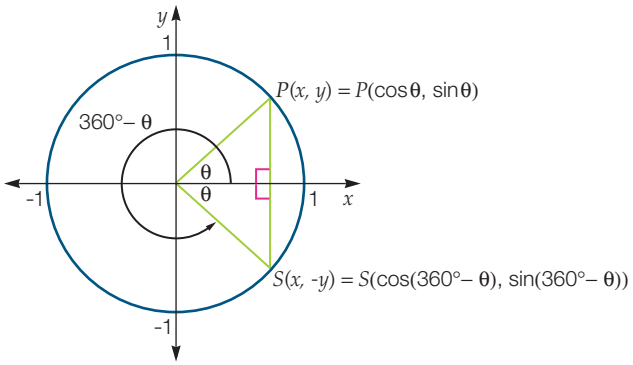
In quadrant 3, as θ increases from 180° to 270°:

- sin θ decreases from 0 to -1
- cos θ increases from -1 to 0
- tan θ decreases from 0 to an undefined negative ratio.

Quadrant 4

In quadrant 4, only the cos ratio is positive.

$\cos(360° - \theta) = \cos\theta$ $\sin(360° - \theta) = -\sin\theta$ $\tan(360° - \theta) = -\tan\theta$

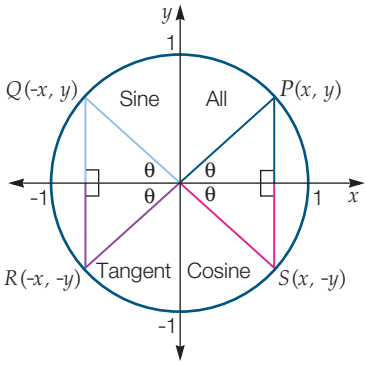


In quadrant 4, as θ increases from 270° to 360°:

- sin θ increases from -1 to 0
- cos θ increases from 0 to 1
- tan θ increases from an undefined negative ratio to 0.

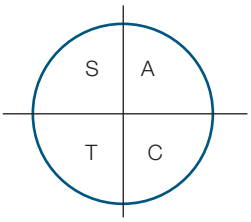
Trigonometric ratios in the unit circle

	Angle			
	0°	90°	180°	270°
sin θ	0	1	0	-1
cos θ	1	0	-1	0
tan θ	0	undefined	0	undefined



- In quadrant 1, all the ratios are positive. (ALL, A)
- In quadrant 2, only the sine ratio is positive. (SINE, S)
- In quadrant 3, only the tangent ratio is positive. (TANGENT, T)
- In quadrant 4, only the cosine ratio is positive. (COSINE, C)

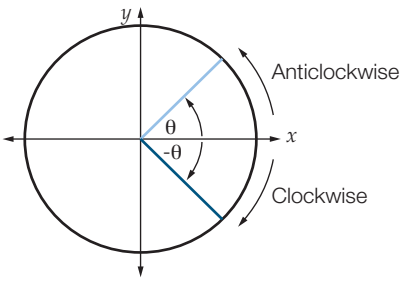
This can be remembered by using the CAST diagram shown. You can make up your own mnemonic (a word or phrase that you can use to help you remember).



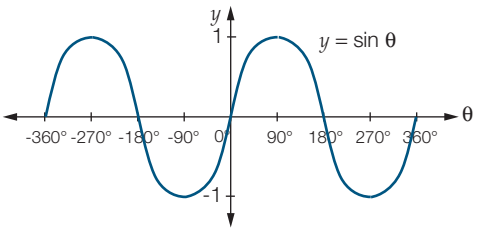
Trigonometric graphs

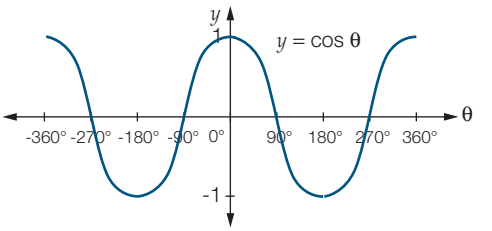
We can use the unit circle to sketch trigonometric graphs. This involves using symmetry properties and using coordinate values of the unit circle.

Note that, by convention, positive angles are measured anticlockwise from the positive direction of the x-axis, whereas negative angles are measured clockwise from the positive direction of the x-axis.



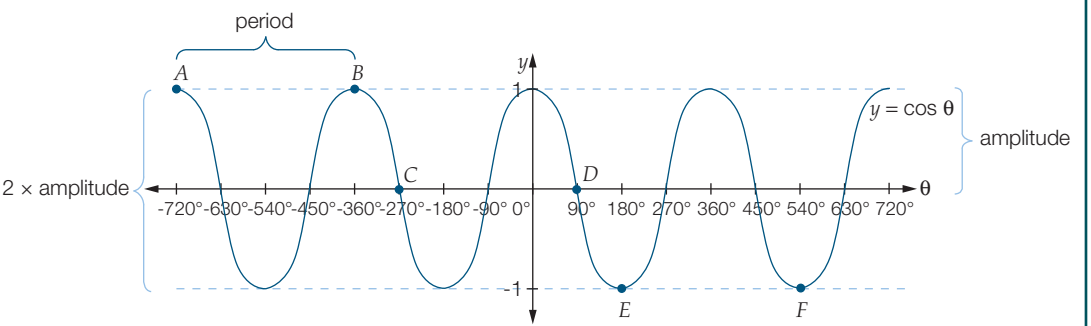
The shape of trigonometric graphs keeps repeating in cycles infinitely in both directions, so they are called cyclic graphs.



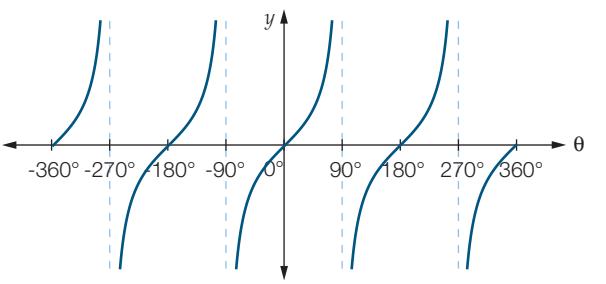


The **period** of a trigonometric graph is the horizontal distance it takes to complete one full cycle of the graph.

The **amplitude** of a trigonometric graph is half the distance between the minimum y-value and the maximum y-value.



The graph of tan θ is quite different to those of sin θ and cos θ. This is because it is not bounded by a maximum value of 1 and a minimum value of -1 and it has points where it is undefined. This means that the graph is not continuous at these points (there are breaks in the graph).



A tangent graph approaches **asymptotes**, where the graph is undefined. Asymptotes are lines that the graph approaches but never touches or crosses. The equations of the asymptotes are θ = -270°, θ = -90°, θ = 90° and θ = 270°. Asymptotes occur every 180° and the graph repeats between these asymptotes, so the period of $y = \tan\theta$ is 180°.

	$y = \cos\theta$	$y = \sin\theta$	$y = \tan\theta$
period	360°	360°	180°
amplitude	1	1	undefined
asymptotes	none	none	$(90 + 180n)°$, where $n = 0, 1, 2, \ldots$

Worked Example 2

WE2

State the quadrant in which each of the following angles is found.

(a) 35° **(b)** 265° **(c)** 135° **(d)** 342°

Working

(a) quadrant 1
(35° is between 0° and 90°)

(b) quadrant 3
(35° is between 0° and 90°)

(c) quadrant 2
(135° is between 90° and 180°)

(d) quadrant 4
(342° is between 270° and 360°)

Worked Example 3

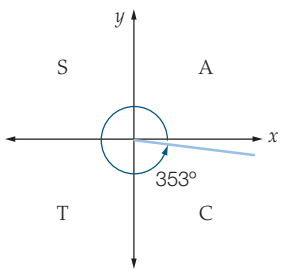
WE3

For each of the following relationships, find the equivalent trigonometric ratio in quadrant 1, taking care to include the appropriate sign.

(a) tan 353° **(b)** sin 124° **(c)** cos 234°

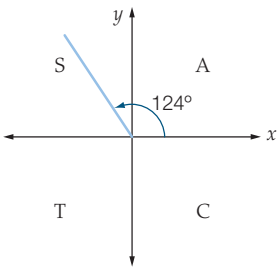
Working

(a)



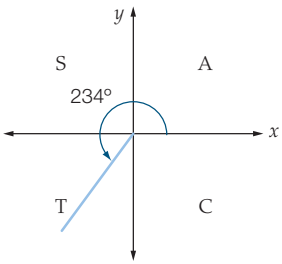
(360 – 353)° = 7°
tan 353° = -tan 7°
(tan is negative in quadrant 4)

(b)



(180 – 124)° = 56°
sin 124° = sin 56°
(sin is positive in quadrant 2)

(c)

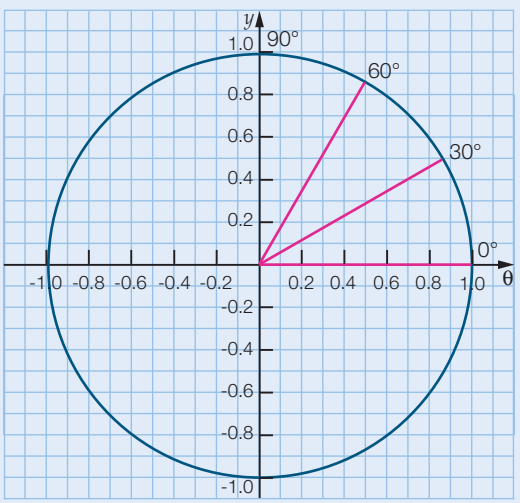


(234 – 180)° = 54°
cos 234° = -cos 54°
(cos is negative in quadrant 3)

Worked Example 4

WE4

Use the unit circle to sketch the graph of the equation $y = \cos\theta$ for values of θ ranging from 0° to 360°.



Working

θ	0°	30°	60°	90°
cos θ	1	0.87	0.5	0

(Find the x-coordinates of the points on the unit circle for the angles 0°, 30°, 60° and 90° in the unit circle.)

θ	120°	150°	180°
cos θ	-0.5	-0.87	-1

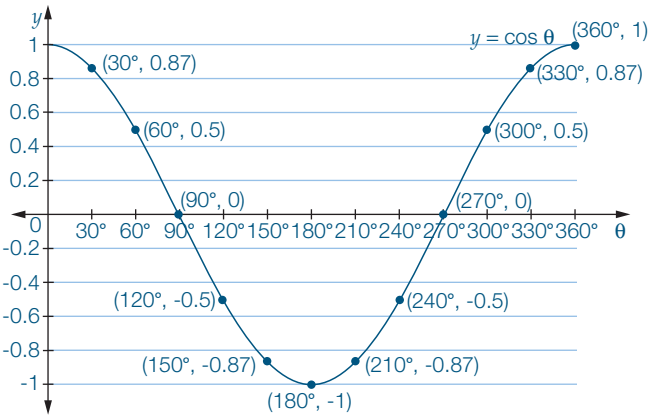
(Reflect the angles 0°, 30°, 60° and 90° in the y-axis of the unit circle to find angles in the second quadrant. Find the x-coordinates of the points on the unit circle for these angles.)

θ	210°	240°	270°
cos θ	-0.87	-0.5	0

(Reflect the angles 120°, 150° and 180° in the x-axis of the unit circle to find angles in the third quadrant. Find the x-coordinates of the points on the unit circle for these angles.)

θ	300°	330°	360°
cos θ	0.5	0.87	1

(Reflect the angles 210°, 240° and 270° in the y-axis of the unit circle to find angles in the fourth quadrant. Find the x-coordinates of the points on the unit circle for these angles.)



Answers page 508

6.2 The unit circle

Equipment required: a CAS would be useful for Questions **8** and **9**

Fluency

WE2

1 State the quadrant in which each of the following angles is found.

(a) 355° **(b)** 222° **(c)** 111° **(d)** 102°

(e) 191° **(f)** 23° **(g)** 263° **(h)** 97°

(i) 286° **(j)** 342° **(k)** 156° **(l)** 260°

WE3

2 For each of the following, find the equivalent trigonometric ratio in quadrant 1, taking care to include the appropriate sign.

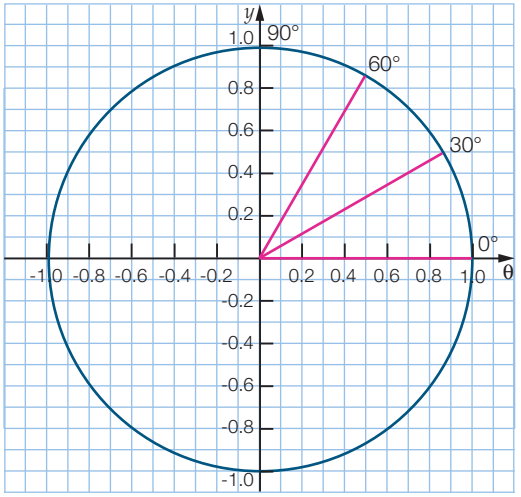
(a) sin 135° **(b)** cos 224° **(c)** tan 111° **(d)** sin 264°

(e) cos 321° **(f)** tan 123° **(g)** sin 263° **(h)** cos 97°

(i) tan 186° **(j)** sin 342° **(k)** cos 166° **(l)** tan 207°

WE4

3 Use the unit circle shown to sketch the graph of the equation $y = \sin\theta$ for values of θ ranging from 0° to 360°



4 **(a)** Which of the following groups of angles are all found in quadrant 3?

A 213°, 182° and 156°

B 187°, 245° and 194°

C 98°, 124° and 156°

D 342°, 276° and 199°

(b) Find the equivalent trigonometric ratio to sin 25°.

A sin 205°

B sin 345°

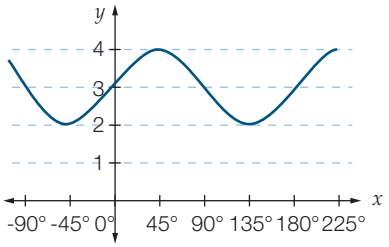
C -sin 155°

D sin 155°

5 The period and the amplitude of the trigonometric relationship shown is:

A 90°, 1 **B** 90°, 2

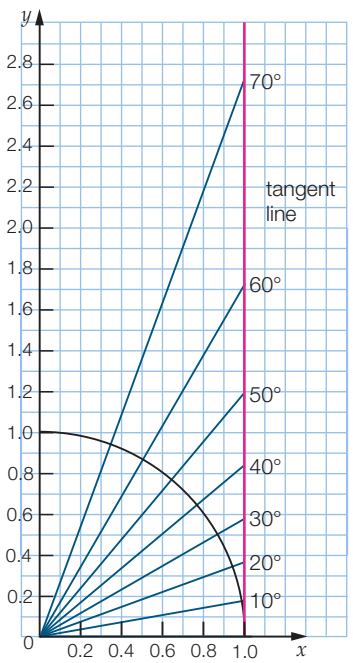
C 180°, 1 **D** 180°, 2



Understanding

6 **(a)** Use the unit circle with the tangent line shown in the diagram on the right to sketch the graph of the equation $y = \tan x$ for values of x ranging from 0° to 70° in intervals of 10°.

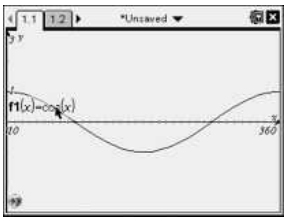
(b) What do you think is happening to the value of $\tan x$ as the value of x gets closer to 90°?



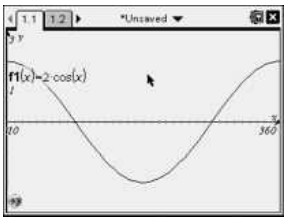
7 Jay knows the values of sine, cosine and tangent when the angles are 30°, 45° and 60°. For which other angles less than 360° can he find trigonometric ratios using what he already knows?

8 Consider the three cosine graphs shown below.

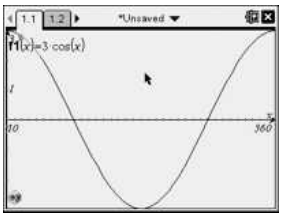
(i)



(ii)



(iii)



(a) State the period for each graph.

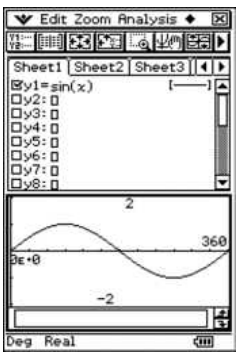
(b) State the amplitude for each graph.

(c) What conclusion can you draw about the effect of the a in $y = a \times \cos x$?

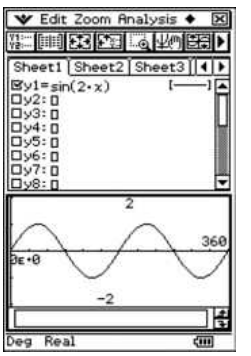
(d) Sketch what you think $y = 4 \cos x$ would look like. Check your answer using a CAS if you can.

9 Consider the three sine graphs shown below.

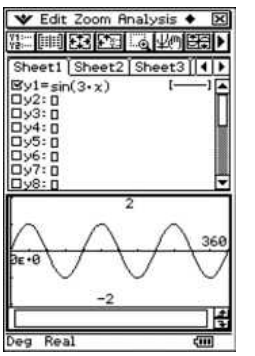
(i)



(ii)



(iii)



(a) State the period for each graph.

(b) State the amplitude for each graph.

(c) What conclusion can you draw about the effect of the n in $y = \sin(nx)$?

(d) Sketch what you think $y = \sin(6x)$ would look like. Check your answer using a CAS if you can.

Reasoning

10 At a harbour, a siren sounds every time the tide is at 5 m. Since tides are based on sine curves, and the siren first sounds when the angle for the sine ratio is 24°, calculate the angle when the siren will sound next.

Open-ended

11 Write down two angles that are 180° apart. For each of your chosen angles, write down the sign for the ratios: sin, cos and tan. Use a calculator to check that you are correct.

12 **(a)** Explain why $\cos(-\theta) = \cos\theta$ for any angle θ.

(b) Explain why $\sin(-\theta) = -\sin\theta$ for any angle θ.

(c) Using your answers to **(a)**, **(b)** and the relationship $\tan(\theta) = \frac{\sin\theta}{\cos\theta}$, explain why $\tan(-\theta) = -\tan\theta$.

Solving trigonometric equations

6.3

Need to Know

Coterminal angles are angles that are in the same position on the unit circle, so their trigonometric ratios are the same. We can use coterminal angles to give an infinite number of solutions to some problems, as adding or subtracting multiples of 360° to an angle returns us to the same position on the unit circle.

Worked Example 5

WE 5

Find all solutions for x that satisfy each of the following equations.

(a) $2\cos x - 1 = 0$ for $0° \leq x \leq 360°$

(b) $3 - \tan x = 2$ for $0° \leq x \leq 180°$

Working

(a) $2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = 0.5$

$x = \cos^{-1}(0.5)$

$x = 60°$

Cosine is positive in quadrants 1 and 4.

The equivalent angle of 60° in quadrant 4 is $360° - 60° = 300°$.



$2\cos x - 1 = 0$ has solutions $x = 60°$, 300° for $0° \leq x \leq 360°$.

(b) $3 - \tan x = 2$

$-\tan x = -1$

$\tan x = 1$

$x = \tan^{-1}(1)$

$x = 45°$

Tangent is positive in quadrants 1 and 3.

The equivalent angle of 45° in quadrant 3 is $180° + 45° = 225°$.



225° is not in $0° \leq x \leq 180°$ so it is not a solution.

$3 - \tan x = 2$ has the solution $x = 45°$ for $0° \leq x \leq 180°$.

Worked Example 6

WE6

Determine all solutions to the equation $\sin x = 0.5$.

Working

Sine is positive in quadrants 1 and 2.

In quadrant 1, $\sin\theta$ is positive.

$\sin x = 0.5$

$x = \sin^{-1} 0.5$

$x = 30°$



In quadrant 2, the equivalent angle to 30° is $180° - 30° = 150°$.

The solutions in the range 0° to 360° are $x = 30°$, $x = 150°$.

$x = (30 \pm 360n)°$, $n = 0, 1, 2, 3, \ldots$

and $x = (150 \pm 360n)°$, $n = 0, 1, 2, 3, \ldots$

or $x = [(2n + 1)180 - 30]°$, $n = 0, 1, 2, 3, \ldots$

Worked Example 7

WE7

Determine all solutions to the equation $2\tan x + 1 = 0$. Write your answers correct to two decimal places.

Working

$2\tan x + 1 = 0$

$2\tan x = -1$

$\tan x = -\frac{1}{2}$

Tangent is negative in quadrants 2 and 4.

In quadrant 1, $\tan\theta$ is positive.

$x = \tan^{-1}\left(\frac{1}{2}\right)$

$x = 26.565°$



In quadrant 2, the equivalent angle is

$(180 - 26.565)° = 153.435°$

$= 153.44°$ (2 d.p.)

In quadrant 4, the equivalent angle is

$(360 - 26.565)° = 333.435°$

$= 333.44°$ (2 d.p.)

The solutions for $0° \leq x \leq 360°$ are $x = 153.44°$ and $x = 333.44°$

$x = (153.44 \pm 360n)°$, $n = 0, 1, 2, 3, \ldots$

and $x = (333.44 \pm 360n)°$, $n = 0, 1, 2, 3, \ldots$

$x = (153.44 + 180n)°$, where $n = 0, 1, 2, 3, \ldots$

Answers page 509

6.3 Solving trigonometric equations

Fluency

WE5

1 Find all solutions for x, that satisfy each of the equations:

(a) $2\cos x + 1 = 0$ for x for $0° \leq x \leq 360°$

(b) $4 - 2\sin x = 3$ for x for $0° \leq x \leq 180°$

WE6

2 Determine all solutions to the equation $2\cos x + 1 = 0$.

3 Determine all solutions to the equation $4\sin x - 1 = 0$. Write all answers correct to two decimal places.

WE7

4 The solutions to $4 - 5\sin x = 0$ for x for $0° \leq x \leq 180°$ are closest to:

A 127°, 127° **B** 53°, 127° **C** 37°, 143° **D** 53°, 307°

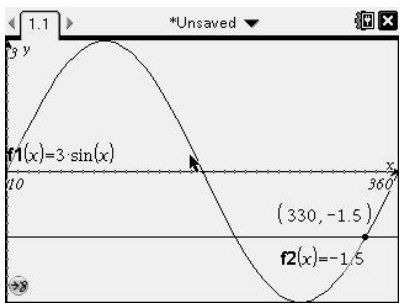
5 Use a calculator to find the solutions for $0° \leq x \leq 360°$ for each of the following equations. Where necessary, round your answers to two decimal places.

(a) $4\sin x - 3 = 0$ **(b)** $5\tan x + 2 = 0$ **(c)** $3 - 5\cos x = 0$ **(d)** $\frac{3\sin x}{2} - 1 = 0$

Understanding

6 The depth D metres below sea level of a submarine after T minutes is given by the trigonometric equation $D = 4\cos T - 4$ where $T \geq 0$. When will be the next time the submarine surfaces ($D = 0$) if it was at the surface at 3.10 p.m. initially?

7 The graph shown at right could be used to solve which of the following trigonometric equations?



A $3\sin(x) - 1.5 = 0$

B $1.5 - 3\sin(x) = 0$

C $3\sin(x) + 1.5 = 0$

D $3\sin(x) = 1.5$

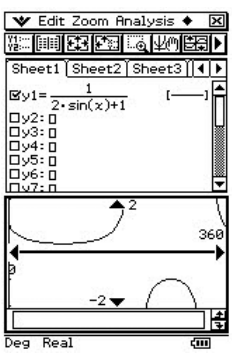
8 Considering the graph shown in Question **7** the solutions shown are in:

A quadrants 1 and 2 **B** quadrants 2 and 3

C quadrants 3 and 4 **D** quadrants 1 and 4.

Reasoning

9 A function is defined as $y = \frac{1}{2\sin(x) + 1}$. Its graph is shown at right.



The graph appears to be undefined for two values of x in the domain $[0°, 360°]$.

(a) What would cause this graph to be undefined?

(b) Find the values of x for which the graph is undefined in the domain $[0°, 360°]$.

10 Find all solutions to the equation $\tan(x) = \sin(x)$ for $0° \leq x \leq 360°$.

Open-ended

11 For each of the sine, cosine and tangent ratios, write at least one equation similar to those in Question **5** and solve it for $0° \leq x \leq 360°$ using a calculator.

12 **(a)** Write any trigonometric equation that has solutions in:

(i) quadrants 1 and 3

(ii) quadrants 3 and 4.

(b) How do you know which trigonometric ratio to use in each of parts **(i)** and **(ii)**?

Hint

Simplify the equation using $\tan(x) = \frac{\sin(x)}{\cos(x)}$ to find an equivalent equation and then factorise it using $\sin(x)$ as a common factor.

Half-time 6

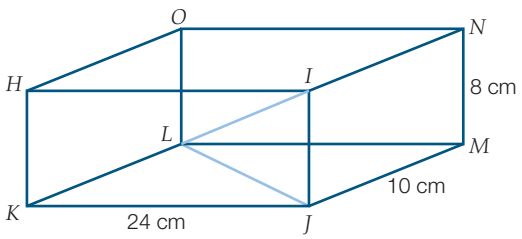
Ex 6.1

1 A rectangular prism is shown.

(a) Calculate $\angle ILJ$, by first finding the length of LJ.

(b) Calculate the length of IL.

Round both answers to two decimal places.



Ex 6.3

2 Find all values for x, correct to one decimal place, that satisfy the following equations and conditions:

(a) $3\sin x - 2 = 0,\ 0° \leq x \leq 180°$

(b) $5\tan x + 1 = 3,\ 0° \leq x \leq 360°$

Ex 6.2

3 Find the equivalent first quadrant ratio for each of the following, taking care to include the appropriate sign.

(a) $\sin 135°$ **(b)** $\cos 98°$ **(c)** $\tan 200°$

(d) $\sin 247°$ **(e)** $\cos 300°$ **(f)** $\tan 339°$

Ex 6.1

4 A right pyramid $ABCDE$ has a square base of side length 9 cm. The sloping edges are 15 cm in length.

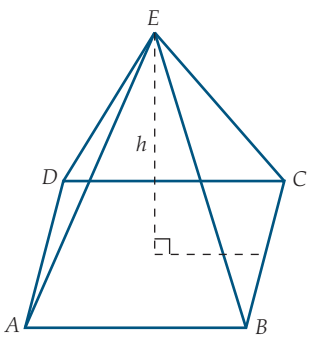
(a) Find the angle that the sloping edge makes with the base of the pyramid, to the nearest degree.

(b) Using your answer to **(a)**, or any other method, find the height of the pyramid, h. State your answer correct to one decimal place.

(c) The side length of the square base of the pyramid is increased by 3 cm, but the length of the sloping edges remains the same.

(i) Will the angle you found in **(a)** increase or decrease?

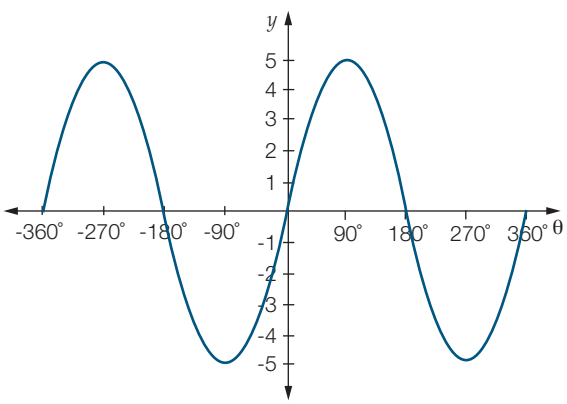
(ii) By how many degrees will it increase or decrease?



Ex 6.2

5 The period and amplitude of the trigonometric graph shown is:

A 360°, 10 **B** 180°, 5 **C** 360°, 5 **D** 180°, 10



Ex 6.3

6 Determine all solutions to the following equations. State your solution correct to two decimal places, where necessary.

(a) $\cos x = 0.5$ **(b)** $3\sin x - 1 = 0$

The sine and cosine rules

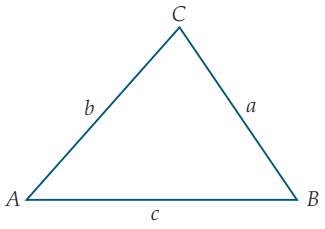
6.4

Need to Know

The **sine rule**:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Use to find an unknown angle when two sides and one angle are known.



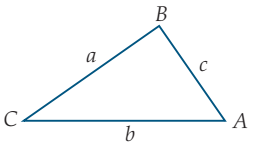
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use to find an unknown side when two angles and one side are known.

Only two equated expressions can be used at any one time.

The **cosine rule:**

Depending on the angle to be used, the cosine rule can be written in three ways.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

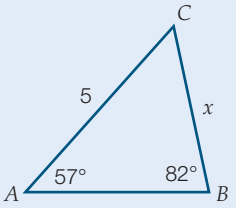
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Worked Example 8

WE 8

Calculate the value of x, correct to two decimal places.



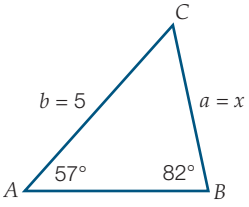
Working

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 57°} = \frac{5}{\sin 82°}$$

$$x = \frac{5 \times \sin 57°}{\sin 82°}$$

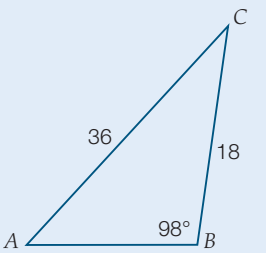
$x = 4.23$ units (2 d.p.)



Worked Example 9

WE9

Calculate the value of $\angle CAB$ in degrees, rounded to two decimal places.



Working

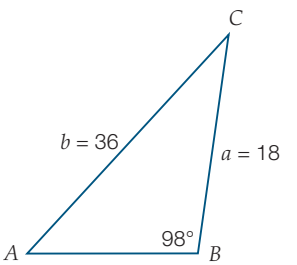
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{18} = \frac{\sin 98°}{36}$$

$$\sin A = \frac{18 \times \sin 98°}{36}$$

$$A = \sin^{-1}\left(\frac{18 \times \sin 98°}{36}\right)$$

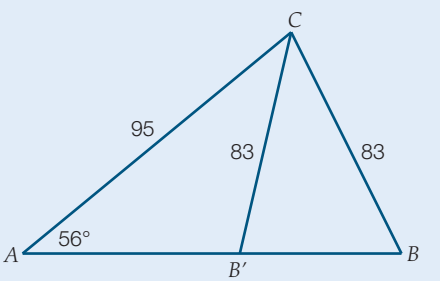
$\angle CAB = 29.68°$ (2 d.p.)



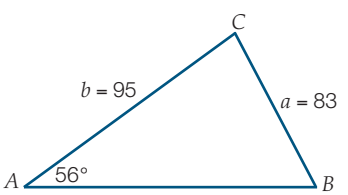
Worked Example 10

WE10

Calculate the values of $\angle ABC$ and $\angle AB'C$, correct to two decimal places.



Working

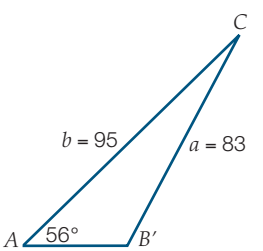


$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 56°}{83} = \frac{\sin B}{95}$$

$$B = \sin^{-1}\left(\frac{95 \times \sin 56°}{83}\right)$$

$\angle ABC = 71.60°$

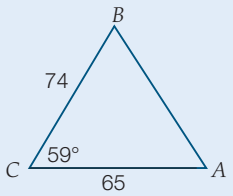


$\angle AB'C = 180° - 71.60°$
$= 108.40°$

Worked Example 11

WE11

Calculate the length of AB, correct to two decimal places.



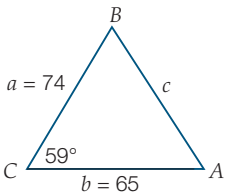
Working

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 74^2 + 65^2 - 2 \times 74 \times 65 \times \cos 59°$

$c^2 = 9701 - 9620\cos 59°$

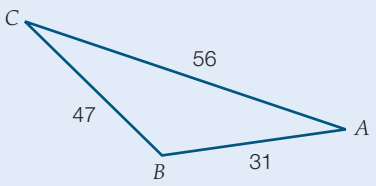
$c = 68.89$ units (2 d.p.)



Worked Example 12

WE12

Calculate the value of $\angle ACB$ in degrees, correct to two decimal places.



Working

$c^2 = a^2 + b^2 - 2ab\cos C$

$31^2 = 47^2 + 56^2 - 2 \times 47 \times 56 \times \cos C$

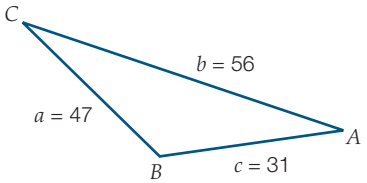
$961 = 5345 - 5264 \times \cos C$

$-4384 = -5264 \times \cos C$

$\cos C = \dfrac{-4384}{-5264}$

$\angle ACB = \cos^{-1}\left(\dfrac{-4384}{-5264}\right)$

$\angle ACB = 33.61°$



Answers page 509

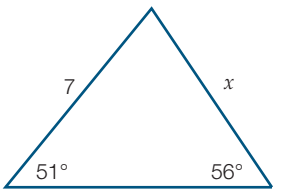
6.4 The sine and cosine rules

Fluency

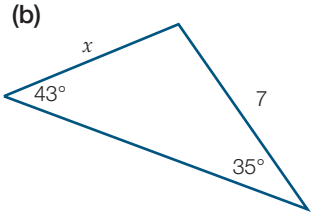
WE8

1 Calculate the value of x, correct to two decimal places.

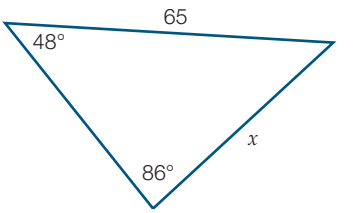
(a)



(b)



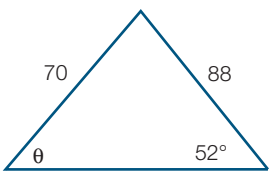
(c)



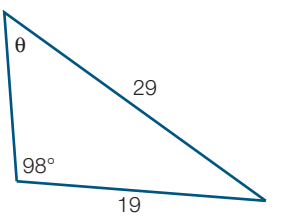
WE9

2 Calculate the value of θ in degrees, correct to two decimal places.

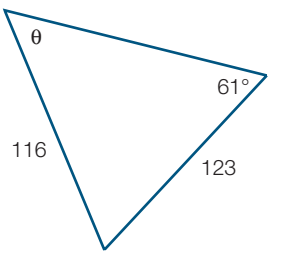
(a)



(b)



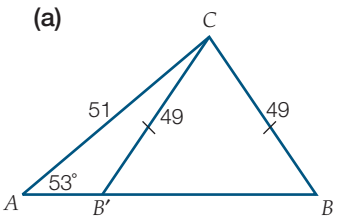
(c)



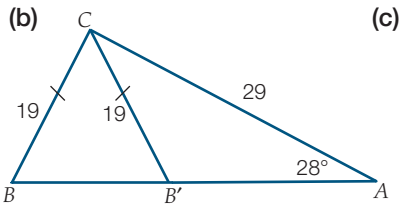
WE10

3 Calculate the values of $\angle ABC$ and $\angle AB'C$, correct to two decimal places.

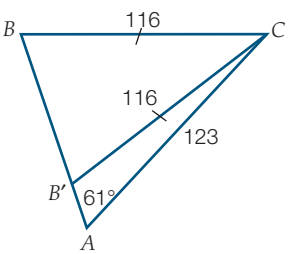
(a)



(b)



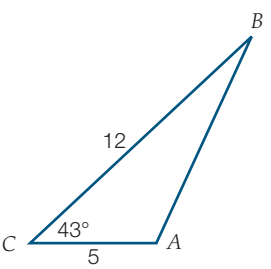
(c)



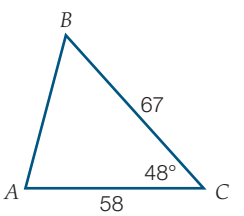
WE11

4 Calculate the length AB, correct to two decimal places.

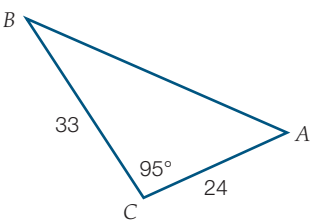
(a)



(b)



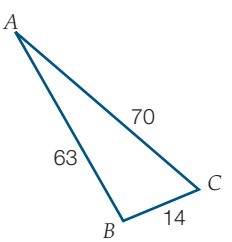
(c)



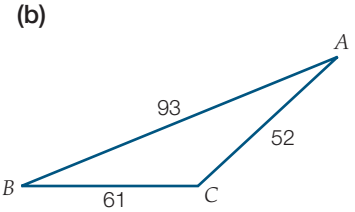
WE12

5 Calculate the value of angle $\angle ACB$, correct to two decimal places.

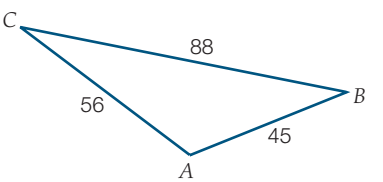
(a)



(b)

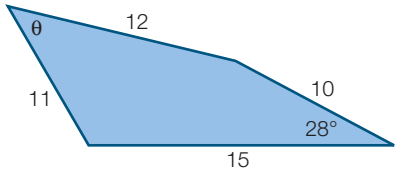


(c)

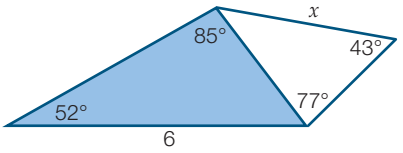


Understanding

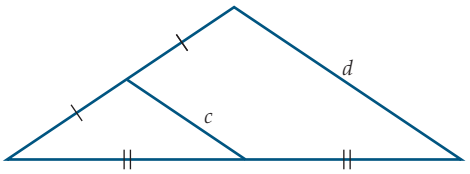
6 Calculate the size of angle θ, correct to the nearest degree.



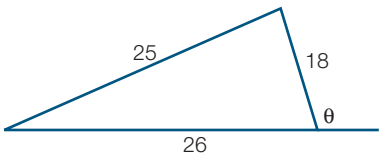
7 Calculate the value of x and round to two decimal places.



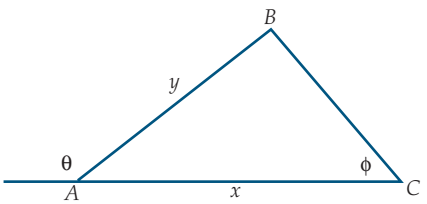
8 Use the cosine rule to show that $d = 2c$.



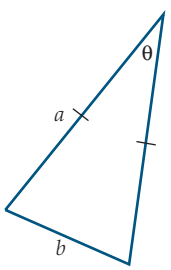
9 Calculate the value of θ, giving your answer as a decimal correct to two decimal places.



10 Show that $x = \dfrac{y\sin(\theta - \phi)}{\sin\phi}$.



11 For the triangle shown, which is the correct statement?



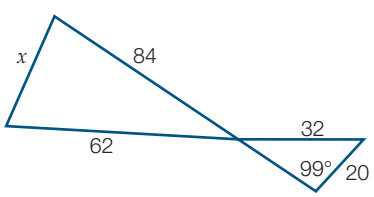
A $a^2 = b^2(2 - \cos\theta)$

B $b^2 = 2a^2(1 + \cos\theta)$

C $a^2 = b^2(2 + \cos\theta)$

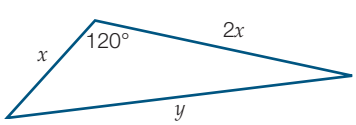
D $b^2 = 2a^2(1 - \cos\theta)$

12 Calculate the value of x and round to two decimal places.

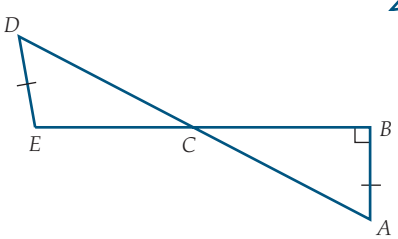


13 The value of y is:

A $7x^2$ **B** $\sqrt{7x}$ **C** $\sqrt{7x^2}$ **D** $49x$



14 Show that $\sin D = \dfrac{EC}{AC}$.



Reasoning

15 Show that the sine rule reduces to $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ when applied to a right-angled triangle.

16 Apply the cosine rule to an equilateral triangle to show that each interior angle is 60°.

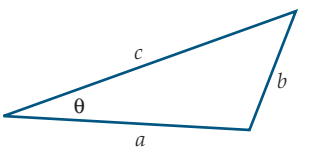
Open-ended

17 **(a)** Using the cosine formula $c^2 = a^2 + b^2 - 2ab\cos C$, show that if $a = b$ and $c = ka$, then $\cos C = \frac{2 - k^2}{2}$.

(b) State two values for k and use them to evaluate $\cos\theta$, rounded to two decimal places.

(c) What are the minimum and maximum possible values for k?

18 Determine the angle θ in the following triangle if sides b and c are positive integer multiples of a.



Problem solving

Trig chimps

Equipment required: 1 brain, calculator

For all calculations round your answers to two decimal places.

A certain group of monkeys are renowned for their longevity.

The ages of three chimps, Syn, Coss and Ten satisfy the following conditions.

- The sum of their ages is 158.
- Coss' age is a multiple of 13 and is an odd number.
- Expressed in degrees, Ten's age is in quadrant 2.
- Expressed in degrees, Syn's age and Coss' age are in quadrant 1.
- The tangent of Ten's age is 15.08 less than the cosine of Coss' age.
- The two digits after the decimal point in the cosine of Coss' age are consecutive digits.

What are the ages of the three chimps?

Strategy options

- Guess and check.
- Test all possible combinations.

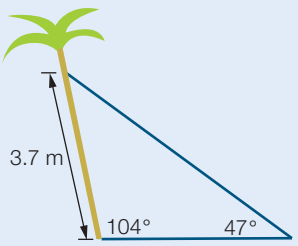
Applications of the sine and the cosine rules

6.5

Worked Example 13

WE13

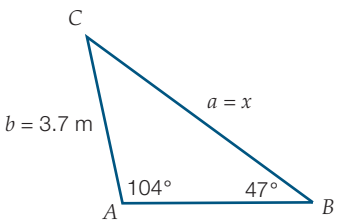
A tree growing at an angle of 104° to the horizontal is in danger of falling. It has been supported by a rope tied to the tree 3.7 m along the tree trunk and pegged to the ground at an angle of 47° as shown in the diagram. Find the length of rope used, correct to one decimal place.



Working

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$a = x$, $\angle A = 104$,
$b = 3.7$, $\angle B = 47$



$$\frac{x}{\sin 104°} = \frac{3.7}{\sin 47°}$$

$$x = \frac{3.7 \times \sin 104°}{\sin 47°}$$

$$x = 4.9$$

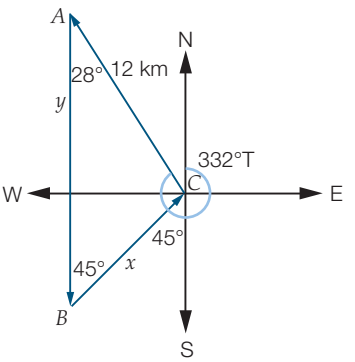
The length of rope is 4.9 m.

Worked Example 14

WE14

A helicopter leaves base and flies for 12 km on a bearing of 332° T. It then heads due south for a certain distance before flying back to base on a bearing of 45° T. What is the total flight distance? Round your answer to two decimal places.

Working



$360 - 332 = 28°$

$\angle A = 28°$

$\angle B = 45°$

$\angle C = 180 - (28 + 45) = 107°$

$a = x$
$b = 12$
$c = y$

(Here, $\angle A$ is obtained using alternate angles and $\angle C$ by $180° - (\angle A + \angle B)$.)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{x}{\sin 28°} = \frac{12}{\sin 45°}$$

$$x = \frac{12 \times \sin 28°}{\sin 45°}$$

$x = 7.967\,196\,615$

$$\frac{y}{\sin 107°} = \frac{12}{\sin 45°}$$

$$y = \frac{12 \times \sin 107°}{\sin 45°}$$

$y = 16.229\,029\,87$

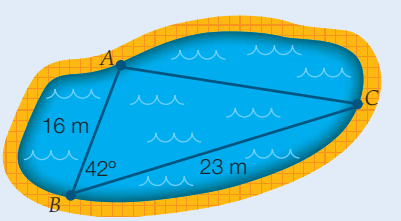
Perimeter $= 12 + 16.229\,029\,87 + 7.967\,196\,615$
$= 36.196\,226\,49$

The total flight is 36.20 km.

Worked Example 15

WE15

A walkway is to be constructed across a pond from A to C. There is an existing walkway of length 16 m from A to B. A rope measuring 23 m is attached to point B and point C. The angle between the rope and the existing walkway is 42°. Calculate the length of the new walkway, correct to two decimal places.



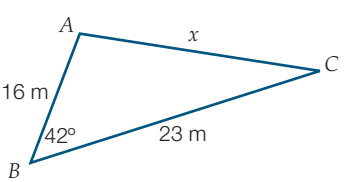
Working

$b = x$, $a = 23$, $c = 16$, $B = 42°$
$b^2 = a^2 + c^2 - 2ac \cos B$

$x^2 = 23^2 + 16^2 - 2 \times 23 \times 16 \times \cos 42°$

$x^2 = 238.045\,408\,4$
$x = 15.428\,720\,25$

The required length for the walkway is 15.43 m.



6.5 Applications of the sine and the cosine rules

Answers page 510

Fluency

1 Part of the ship's rigging shown forms a triangle with base angles of 80° and 66° and rigging of length 5.3 m. Calculate the length of the section of the mast in the triangle shown, correct to two decimal places.

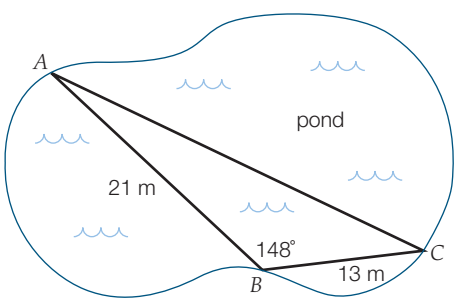


WE13

2 Postman Pete leaves the post office and makes deliveries for 2.5 km on Aiden Rd, which is on a bearing of 072°T from his starting point. Pete then heads due north on Sari Boulevard for a certain distance before returning to the post office on a bearing of 210°T. Find the total distance of Postman Pete's route, rounded to two decimal places.

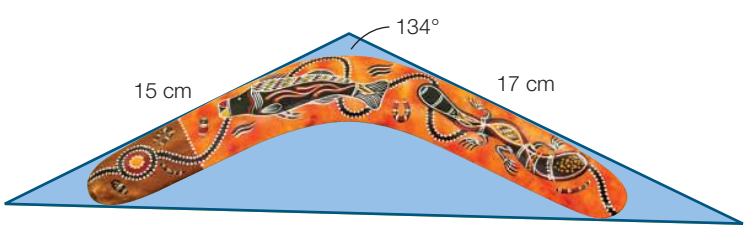
WE14

3 A walkway is to be constructed across a pond from A to C. There is an existing walkway of length 21 m from A to B. A rope measuring 13 m is attached to point B and point C. The angle between the rope and the existing walkway is 148°. Calculate the length of the new walkway, correct to two decimal places.

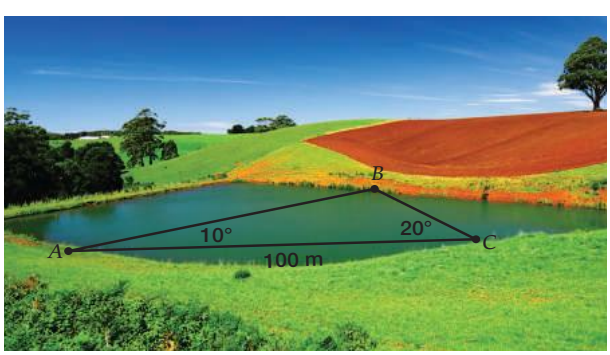


WE15

4 A boomerang is mounted on a triangular frame as shown. If the two shorter sides of the frame are 15 cm and 17 cm respectively and the included angle is 134°, calculate the perimeter of the frame, rounded to two decimal places.

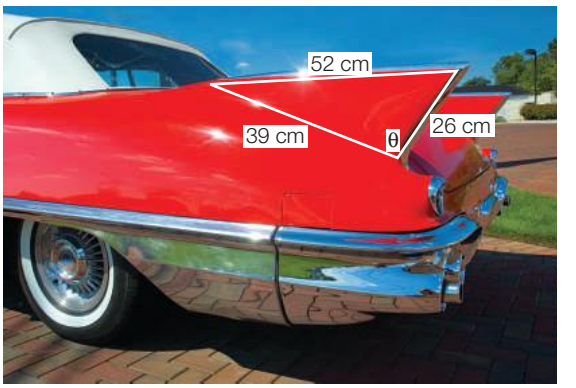


5 A pipe is laid across a section of a water dam from A to B. A 100 m rope is attached from point A to point C and the angles between the rope and AB, and the rope and CB are measured. If $\angle A$ is 10° and $\angle C$ is 20°, what length of pipe is required? State the answer to the nearest metre.



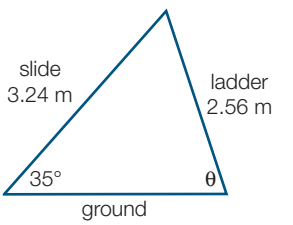
Understanding

6 The fins of this classic car are triangular with the dimensions of 26 cm, 39 cm and 52 cm as shown. For aerodynamic stability, it is recommended that the angle θ be less than 105°. Is this true for this vehicle?

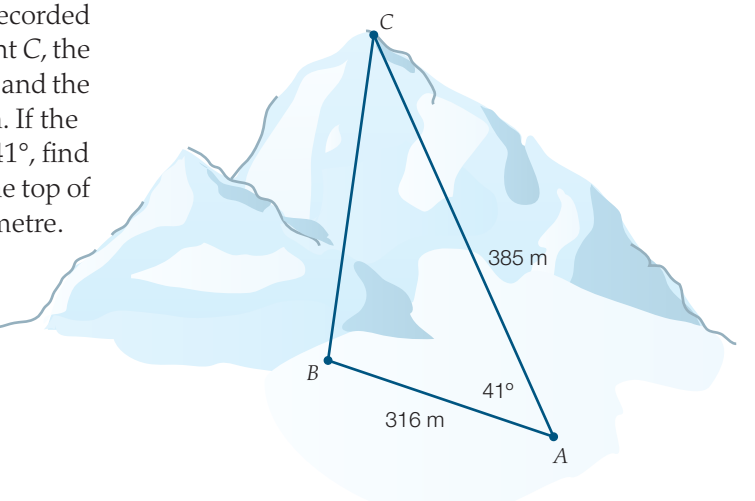


7 Rangers Tim and Zoe are 10 km apart, with Tim due west of Zoe. They notice a fire that is N50°E of Tim and N35°W of Zoe. Decide which ranger is closer to the fire by finding the distance each ranger is from the fire.

8 A children's slide is of length 3.24 m and makes an angle of 35° with the ground. If the ladder is 2.56 m, what angle θ does it make with the ground? Round your answer to two decimal places.



9 A ski lift is to be constructed at point *B* on a mountain. A surveyor recorded the distance from *A* to the point *C*, the top of the mountain, as 385 m and the distance from *A* to *B* as 316 m. If the angle between *AB* and *AC* is 41°, find the distance between *B* and the top of the mountain to the nearest metre.



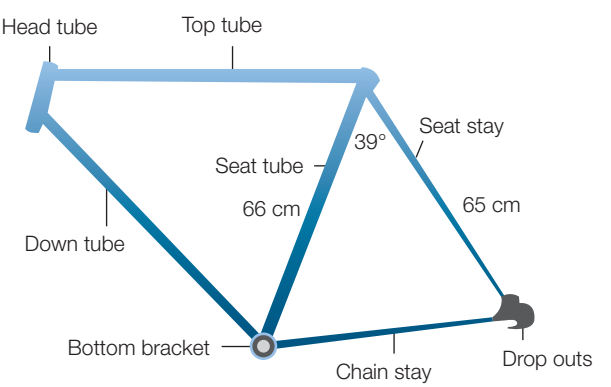
10 To the nearest centimetre, the length of the chain stay in the bicycle frame is:

A 44 cm

B 47 cm

C 51 cm

D 62 cm



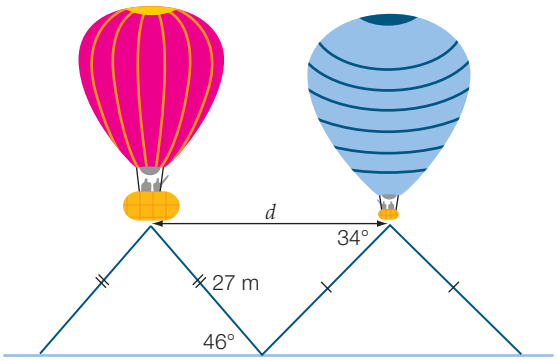
Reasoning

11 Ken and Sofia set out in different directions from their campsite. After Ken walks 2150 m on a bearing of 316° T and Sofia walks for 2000 m, the distance between them is 2696 m. What are the two possible bearings that Sofia could have used? Give your answer correct to the nearest degree.

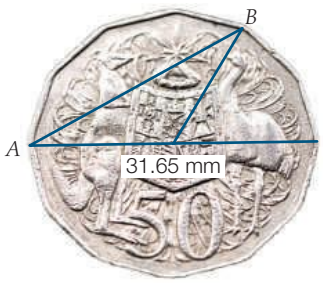
12 Two hot air balloons are stationary at the same height above the ground. They are secured by ropes as shown.

(a) Calculate the horizontal distance between the base of each balloon, correct to two decimal places.

(b) What is the total length of rope required to secure the balloons?



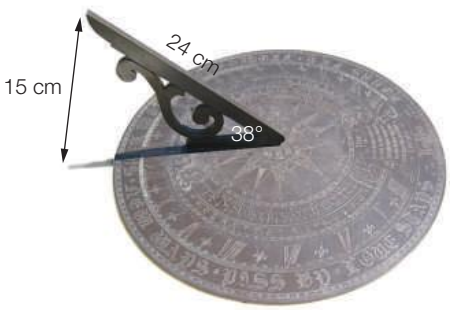
13 The longest diagonal of a 50 cent coin is 31.65 mm. Calculate the length *AB*, correct to two decimal places.



Hint

A 50c coin has the shape of a regular dodecagon (a 12-sided polygon).

14 The dial on a sundial casts a shadow as shown. The dial is 24 cm long, the angle between the dial and the base is 38° and there is 15 cm between the top of the dial and the end of the shadow. Find the length of the shadow, correct to two decimal places.



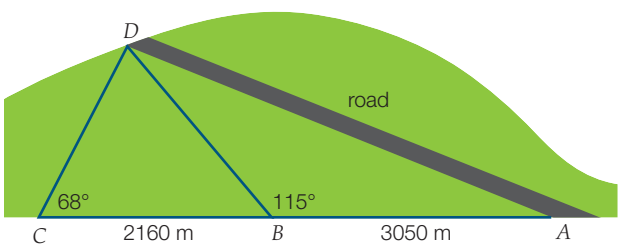
Hint

Be sure to find all possible solutions.

15 A pendant is in the shape of a scalene triangle and its two smaller interior angles are 25° and 28°. If the longest side is of length 4 cm, the lengths of the two other sides (in cm), correct to two decimal places are:

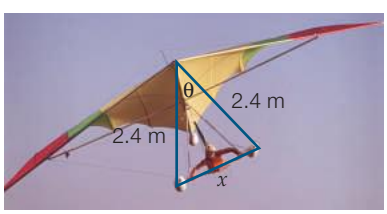
A 2.26, 2.35 **B** 2.12, 2.35 **C** 2.12, 2.18 **D** 2.11, 2.35

16 A new highway is being built across a paddock, from point *A* to point *D* as shown on the diagram. This highway currently runs in a straight line from *A* to *C* through *B*, 3050 m from *A*. The angle between *AB* and *BD* is 115°. The distance from *B* to *C* is a further 2160 m and the angle between *BC* and *CD* is 68°. Find the length of the new highway *AD*, correct to the nearest metre.

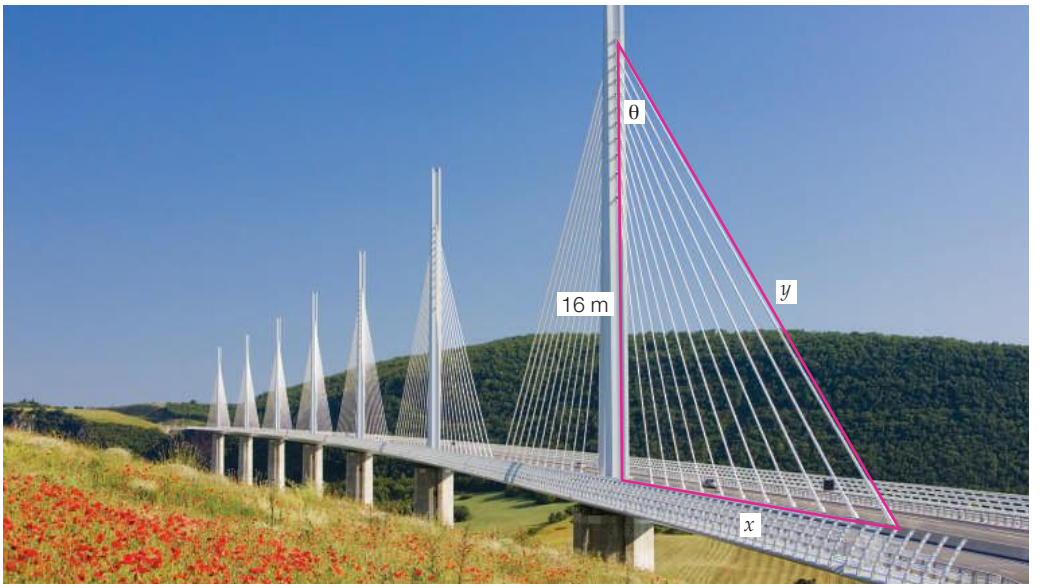


Open-ended

17 For maximum strength and stability, the angle θ between the support wires in the hang glider must be no more than 40° and the distance *x* between the wheels must be between 80 cm and 1.2 m. Determine two sets of allowable values for θ and *x*.



18 The length y of one of the bridge's support cables is less than 40 m and the angle θ is in the range $15^\circ < \theta < 75^\circ$. By choosing reasonable values for θ, find two possible values for the length x, and then find the value of y for each x-value.

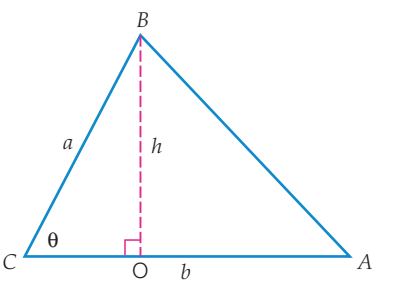


6.6 Area of a triangle using trigonometry

Need to Know

Area of a triangle

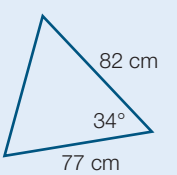
$A = \frac{1}{2}ab\sin\theta$, where θ is the angle included between sides a and b.



Worked Example 16

WE16

Calculate the area of the triangle, correct to two decimal places.



Working

$a = 77$ cm, $b = 82$ cm, $\theta = 34$

$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$= \frac{1}{2} \times 77 \times 82 \times \sin 34°$$

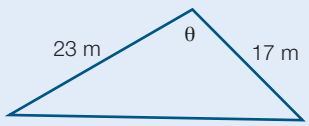
$$= 1765.371\,996$$

The area is 1765.37 cm^2.

Worked Example 17

WE17

Find the size of the angle required if the area of the triangle shown is 195 m^2. Round the answer to the nearest degree.



Working

area = 195 m^2, $a = 23$ m, $b = 17$ m

$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$195 = \frac{1}{2} \times 23 \times 17 \times \sin\theta$$

$$\sin\theta = \frac{195 \times 2}{23 \times 17}$$

$$= 0.9974$$

$$\theta = \sin^{-1}(0.9974)$$

$$= 85.901\,340\,36$$

The angle is 86°.

6.6 Area of a triangle using trigonometry

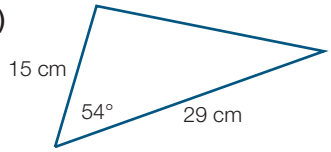
Answers page 510

Fluency

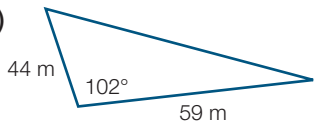
1 Calculate the area of each of the following triangles, correct to two decimal places.

WE16

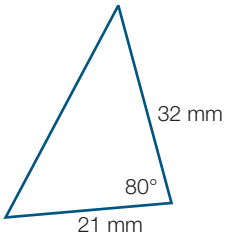
(a)



(b)

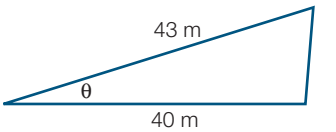


(c)



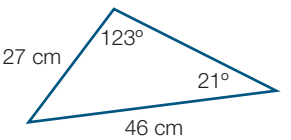
WE17

2 Find the size of the angle required if the area of the triangle shown is 529 m^2. Write the answer correct to the nearest degree. Assume θ is an acute angle.

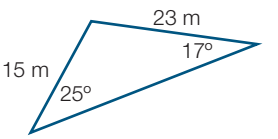


3 Calculate the area of the following triangles, correct to two decimal places.

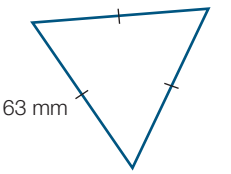
(a)



(b)

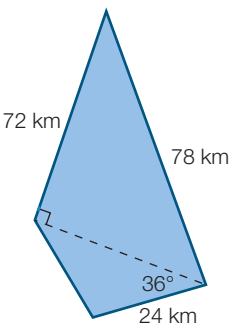


(c)



Understanding

4 Calculate the area of the shape, correct to the nearest square kilometre.



5 The ends of the cells of a bee's honeycomb are regular hexagons. If each edge length is 1.5 mm, what is the area of the end of one cell? Round your answer to two decimal places.

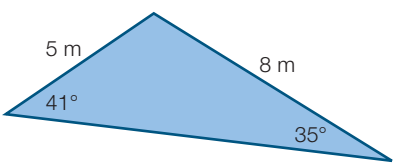
Hint

How many equilateral triangles fit together to make a hexagon?

6 The area of the triangle in square metres is closest to:

A 11.47 **B** 13.12

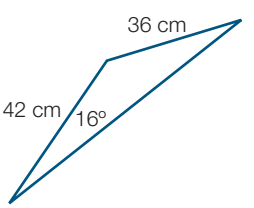
C 19.41 **D** 20.00



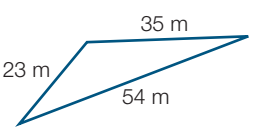
Reasoning

7 Calculate the area of the following triangles, correct to two decimal places.

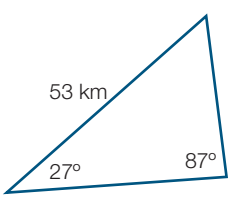
(a)



(b)



(c)



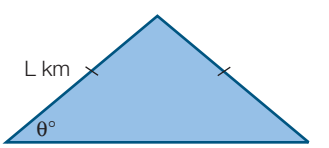
Hint

Are there any ambiguous cases?

8 A plot of land is in the shape of an isosceles triangle.

(a) Show that the area of land is $L^2 \sin\theta \cos\theta$ km^2.

(b) What is the area when $L = 5.6$ km and $\theta = 45°$?

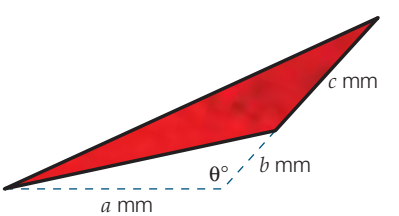


Hint

Divide this triangle into half and use trigonometry to find the base and the height of the small triangles.

9 **(a)** Show that the area of the section of red stained glass, shown in the diagram on the right, is $\frac{1}{2}ac \sin\theta$ mm^2.

(b) For what value of θ will the area be a maximum? What is this area?



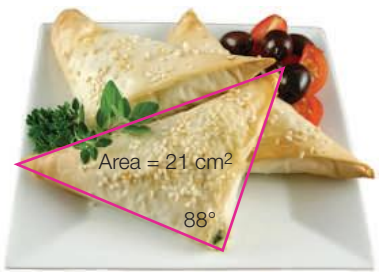
10 The sail shown, consisting of two layers of cotton fabric, is to be replaced at a cost of $135 per square metre. If the length of the sail along the mast is 3.8 m and the two angles at the base of the sail are 85° and 57° as shown in the diagram, calculate the total replacement cost of the sail. Answer to the nearest dollar.



11 **(a)** In the formula for the area of a triangle, $\frac{1}{2}ab\sin\theta$, what is the range of values possible for the angle θ?

(b) Explain why $\sin\theta$ cannot take on values less than zero.

12 The shape of the cheese savoury can be approximated by an isosceles triangle. The angle between the two equal sides is 88° and the area covered by the savoury is 21 cm^2. If the perimeter is to be covered with a strip of cheese flavouring, what length will be used? Round your answer to the nearest centimetre.



Open-ended

13 Find values for the lengths of two equal sides of an isosceles triangle and the size of the angle between them, so that the area is a square number. The triangle must not be right angled.

Hint

Use the sine or cosine rule to find the unknown sides or angles.

Problem solving

Triangles to the max

Servane has a square piece of cardboard, which is 30 cm × 30 cm. She wants to cut out as many small triangles as possible that have one side of 12 cm, another of 15 cm and the angle between them is 30°. Using area formulas she calculates that she will get 20 triangles. Her friend, Sophie, disagrees and says that 16 triangles are the most she can cut. Is either of them correct? Explain your answer. Draw a diagram to show the maximum number of triangles you could cut out from a piece of cardboard with the same dimensions.

Strategy options

- Make a model.
- Act it out.

Chapter review 6

D.I.Y. Summary

Key Words

amplitude	coterminal angles	quadrant
asymptote	equivalent angle	sine rule
cosine rule	period	unit circle

Copy and complete the following using words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 One half the difference between the minimum value and the maximum value of the graph of a trigonometric ratio is the _____.

2 The _____ relates the angle between two sides of a triangle and the side length opposite the angle.

3 Angles in quadrant 1 that have the same values for trigonometric ratios in the other quadrants are _____.

4 The trigonometric ratios are derived from the _____.

5 The smallest interval after which the graph of a trigonometric relationship repeats is the _____.

6 The _____ can be used when two angles in a triangle and a side length opposite one of the angles is known.

7 Each of the four sections of the unit circle is a _____.

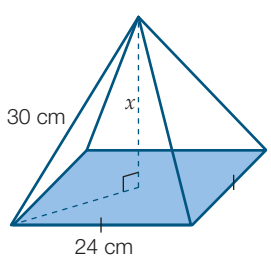
8 Each _____ on the graph $y = \tan\theta$ is a vertical line for which $\tan\theta$ is not defined.

9 _____ are in the same position on the unit circle so their trigonometric ratios are exactly the same.

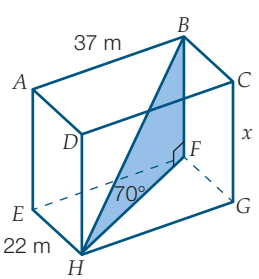
Fluency

1 Calculate the length of x in each shape. Round your answers to two decimal places. Ex. 6.1

(a)



(b)



2 To the nearest centimetre, what is the length of the longest possible timber pole that can fit in a shed in the shape of a rectangular-based prism of length 24 m, width 10 m and height 4 m? Ex. 6.1

3 In which quadrant are each the following angles? Ex. 6.2

(a) 37° (b) 274° (c) 93° (d) 182°

4 Find the equivalent ratio to each of the following in quadrant 1 using the correct sign. Ex. 6.2

(a) cos 173° (b) sin 254° (c) tan 312°

5 The equivalent trigonometric ratio in quadrant 1 of cos 253° is:

A cos 17° **B** -cos 73° **C** -cos 107° **D** cos 37°

Ex 6.2

6 Find all solutions to $\tan x + 1 = 0$ for $0° \leq x \leq 360°$.

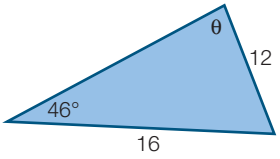
Ex 6.3

7 Determine all solutions to the equation $4 \cos x - 1 = 0$. Round your answers to two decimal places.

Ex 6.3

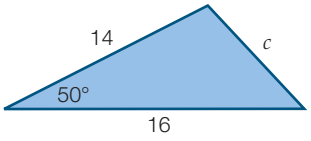
8 In degrees, the angle θ shown is closest to:

A 70.42 **B** 72.10 **C** 73.56 **D** 75.78



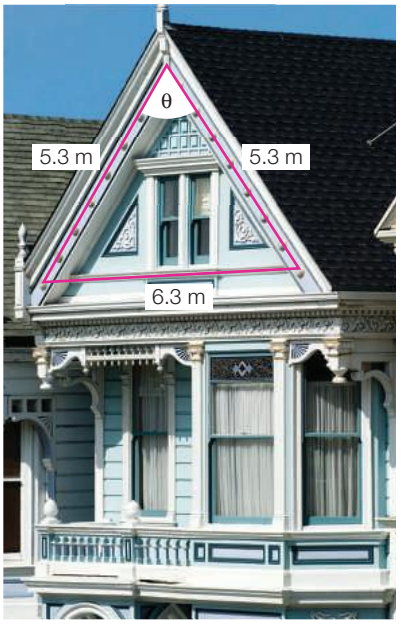
Ex 6.4

9 Calculate the value of c, rounded to two decimal places.



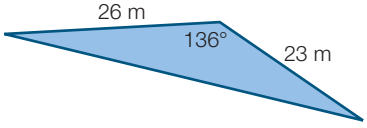
Ex 6.4

10 A roof consists of rooflines of 5.3 m as shown in the diagram. The rooflines determine the pitch, which is the angle formed where the rooflines meet. Calculate the pitch θ of the roof, correct to the nearest degree.



Ex 6.5

11 Calculate the area of the shape, rounded to two decimal places.



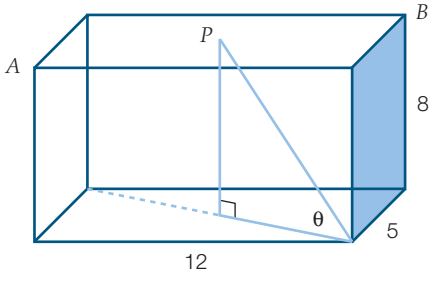
Ex 6.6

Understanding

12 P is the midpoint of AB. The angle θ is closest to:

A 22.6° **B** 33.7°

C 42.2° **D** 50.9°



13 State the period and the amplitude, where appropriate, of each relationship.

(a) $y = 7 \sin (2x)$

(b) $y = -6 \cos (5x)$

(c) $y = 3 \tan (4x)$

14 The period of $\tan nx$ is 15°. The value of n must be:

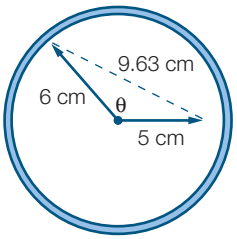
A 10 **B** 12 **C** 15 **D** 24

15 Sketch the graph of $y = 2 \cos (x)$ for $-180° \leq x \leq 180°$ using the graph of the basic function $y = \cos (x)$ as a guide.

16 **(a)** In which quadrant is 400°?

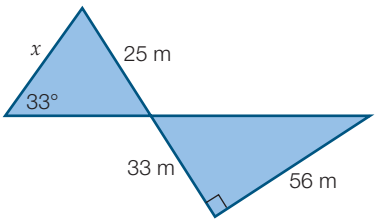
(b) What is the first angle measured clockwise from the positive x-axis that will be coterminal with 400°?

17 The lengths of the hour and minute hand of a clock face are 5 cm and 6 cm respectively as shown on the right. To the nearest degree, what is the size of the obtuse angle formed between the hands when the tips of the hands are 9.63 cm apart.



18 Calculate the area of an isosceles triangle where the angle between the two equal sides each of length 10 cm is 67°. State your answer correct to two decimal places.

19 Calculate the value of x in the diagram on the right, rounded to two decimal places.



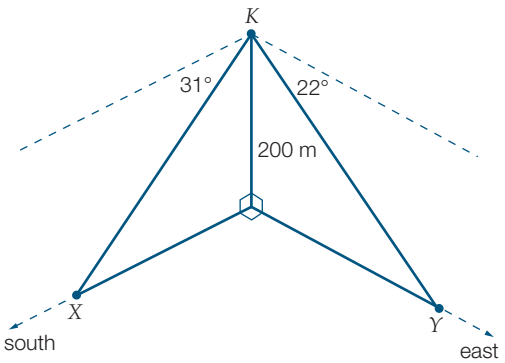
Reasoning

20 The longest drinking straw that can fit into a cylindrical soft-drink can is 15.7 cm. If the can has a radius of 36 mm, find:

(a) the angle, correct to one decimal place, that the straw makes with the base of the can

(b) the height of the can, rounded to two decimal places.

21 From a point K on top of a 200 m high building, the angle of depression to a point X on the ground due south of K is 31° and the angle of depression to a point Y on the ground due east of K is 22°.



(a) Find, correct to two decimal places, the distance between:

(i) the building and point X

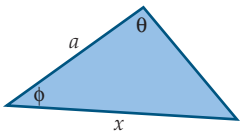
(ii) the building and point Y.

(b) Hence, find the distance between X and Y rounded to two decimal places. (Use Pythagoras' theorem.)

(c) What is the true bearing of Y from X, to the nearest degree?

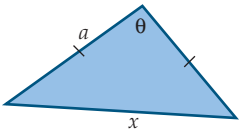
22 Which angles in the unit circle do not have equivalent angles? Use angles from 0° to 360°.

23 **(a)** **(i)** For given values for a, θ and ϕ explain how the sine rule can be used to find the value of x in the triangle at right.



(ii) Find the value of x when $\phi = 41°$, $\theta = 93°$ and $a = 10$. Write your answer correct to two decimal places.

(b) **(i)** For given values for a and θ, explain how the sine rule can be used to find the value of x in the triangle at right.

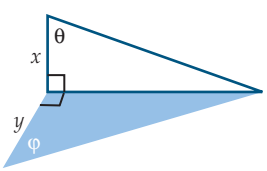


(ii) Find the value of x when $\theta = 93°$ and $a = 10$. Write your answer correct to two decimal places.

24 A fishing boat leaves port on a bearing of 136°T. Following this bearing, it drops x m of netting. The vessel then travels north-east, dropping y m of netting along the way. Finally, it heads directly back to its starting point, dropping netting as it moves. The expression that gives the area in square metres enclosed by the netting is:

A $\frac{1}{2}xy\sin 44°$ **B** $\frac{1}{2}xy\sin 45°$ **C** $\frac{1}{2}xy\sin 46°$ **D** $\frac{1}{2}xy\sin 89°$

25 The shadow of a triangular sculpture is shown. If $y = 1.5x$, show that $2\tan\theta = 3\tan\varphi$.



Mixed review B

Fluency

1 Find the interest earned on an investment of $6000 at 4.2% p.a. for 3 years, where the interest is compounded: Ex.1.2

(a) annually **(b)** half-yearly **(c)** quarterly.

2 A painting has increased in value, solely due to the effects of inflation, from $2500 to $3100 over 5 years. Find the average annual inflation rate. State your answer correct to one decimal place. Ex.1.3

3 Expand and simplify each of the following. Ex.2.1

(a) $(3x - 4)(x + 2)$ **(b)** $(2x + 5)(3x - 1)$ **(c)** $(4x - 3)(2x - 5)$

4 Factorise the following expression fully. Ex.2.2

(a) $(3x - 7)(2x + 6) - 5(3x - 7)$ **(b)** $2x(3x - 2)(4x + 5) + 2x(3x - 2)$

5 Factorise the following quadratic trinomials. Ex.2.3

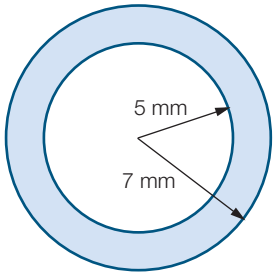
(a) $x^2 - 3x - 4$ **(b)** $x^2 - 14x + 45$ **(c)** $x^2 - 7x + 12$

6 Simplify the following. Ex.2.6

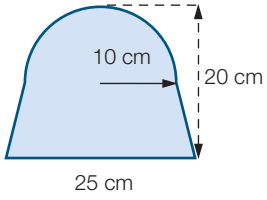
(a) $\frac{1}{3x} - \frac{2}{x}$ **(b)** $\frac{3x}{x+1} - \frac{2}{x-1}$ **(c)** $\frac{2x+1}{x} + \frac{3x-3}{3x}$

7 Find the shaded area of the following shapes, correct to two decimal places. Ex.3.1

(a)



(b)



8 Find the total surface area, correct to two decimal places, of a shape consisting of a cube of side length 7 cm with a pyramid of perpendicular height 4 cm sitting on top of it. Ex.3.4

9 Find the volume of a square pyramid with base side length 12 cm and perpendicular height 15 cm. Ex.3.5

10 A metal sphere has a volume of 12 cm^3. If its mass is 76.8 g, what is its density? Ex.3.6 10A

11 Solve each of the following equations. Ex.4.1

(a) $\frac{2x+3}{7} = 3$ **(b)** $\frac{4(3x-2)}{5} = 4$ **(c)** $2(4x + 5) = 26$

(d) $7(2x - 1) = 21$ **(e)** $6x + 2 = 7x$ **(f)** $8x - 9 = 7x - 4$

(g) $4(x + 5) = 2x - 4$ **(h)** $\frac{2(2x-4)}{3} = 4$

12 Calculate the gradient of the line that passes through each pair of coordinates. Ex 4.2

(a) (-3, 0) and (3, 9)
(b) (-4, -1) and (-2, 5)
(c) (0, 6) and (4, -2)
(d) (-3, 2) and (-1, 6)

13 Identify which of the following points lie on each given line. Ex 4.3

(a) (-2, -1) or (2, 8) $y = 3x + 5$
(b) (1, -1), (-2, -3) or (-3, 5) $y = -2x + 1$
(c) (0, -2), (4, 1) or (-8, 0) $y = \frac{1}{4}x + 2$

14 Graph each of the following lines using either the gradient and y-intercept method or the x- and y-intercepts method. Ex 4.3

(a) $y = -2x + 1$ **(b)** $y = 3x + 3$ **(c)** $y = \frac{1}{4}x - 4$ **(d)** $y + 6x = 9$

15 Determine whether each pair of lines is perpendicular. Write 'Yes' or 'No' and justify your answer with appropriate calculations. Ex 4.4

(a) $2x + 4y = 8$ and $6x = 3y - 3$
(b) $2x - y = 3$ and $x + 2y = 8$
(c) $5x + 2y = 8$ and $5y = 2x + 10$
(d) $3y = 4x - 2$ and $4y = 3x - 4$

16 Determine whether each pair of lines is parallel. Write 'Yes' or 'No' and justify your answer with appropriate calculations. Ex 4.4

(a) $6x - 3y = 9$ and $3x = 3y - 3$
(b) $x - y = 8$ and $x + y - 2 = 0$
(c) $2x + 5y = 7$ and $5y = 2x - 8$
(d) $10y = 3x + 2$ and $6x - 20y + 4 = 0$

17 Solve the following inequalities. Ex 4.5

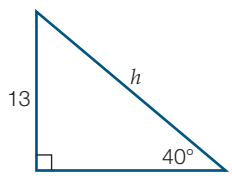
(a) $2x - 1 \leq 4x + 2$
(b) $\frac{4x - 1}{3} > \frac{3x + 5}{2}$

18 Solve the following simultaneous equations. Ex 4.6

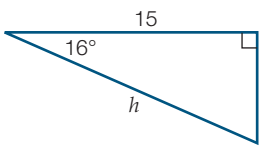
(a) $3x - 5y = 7$
$x + 5y = -11$
(b) $2x - 3y = 17$
$y = 3x - 22$

19 Find the values of the pronumerals, correct to two decimal places. Ex 5.2

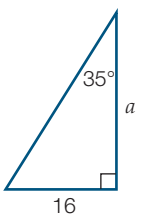
(a)



(b)



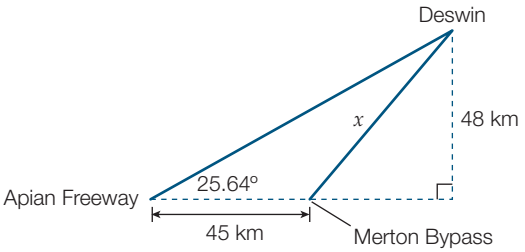
(c)



20 A man standing 8 m from a flagpole looks up to the top of it. Through what angle of elevation does he look (to two decimal places) if the top is 16 m higher than his eye-level? Ex 5.4

21 A scout walks 3 km on a true bearing of 050°T and then 5 km on a true bearing of 077°T. What is the scout's distance from the starting position (to two decimal places)? Ex 5.5

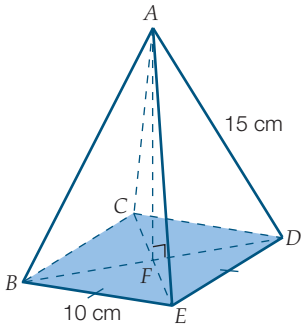
22 A proposed section of highway that was to connect the Apian Freeway to the town of Deswin will now connect the Merton Bypass to Deswin as shown. What will be the length, x km, of the new section of highway? Write your answer to the nearest kilometre. Ex 5.6



23 A right pyramid $ABCDE$ stands on a square base of edge length 10 cm. The sloping edges are each 15 cm long. AF is the perpendicular height.

(a) Find, correct to one decimal place, the angle that a sloping edge makes with the base of the pyramid.

(b) Hence, find the height of the pyramid, correct to one decimal place.

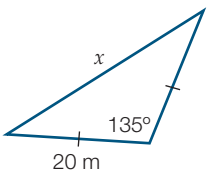


Ex 6.1 10A

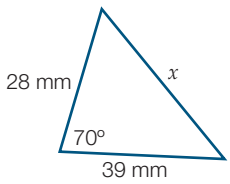
24 Find the value of x in the following triangles, correct to two decimal places.

Ex 6.5 10A

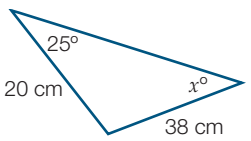
(a)



(b)



(c)



25 Find the area of each of the triangles in Question **24**, correct to two decimal places.

Ex 6.6 10A

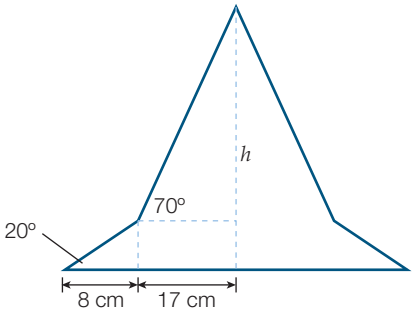
Understanding

26 The height of a particular young pine tree is found to increase in a linear manner each month in the first few years after planting. What was the initial height of the tree, if after 2 months the height is 34 cm, and after 5 months the height is 52 cm?

27 The sides of a rectangle differ by 4 m. If all sides are increased by 2 m, the area increases by 52 m^2. What are the dimensions of the original rectangle?

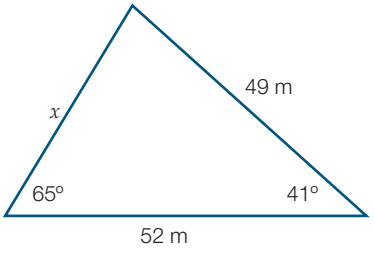
28 Determine whether the following points are collinear: $A(1, -1)$, $B(3, 3)$, $C(6, 9)$.

29 A witch's hat, used on roads to divert traffic, rises from the outside of the brim at an angle of 20° for a horizontal distance of 8 cm then rises to an apex at an angle of 70° for a further horizontal run of 17 cm. Find h, the height of the witch's hat, correct to two decimal places.



30 In the diagram given, the value of x cannot be found using which of the following equations?

10A



A $\dfrac{x}{\sin 41^\circ} = \dfrac{49}{\sin 65^\circ}$

B $\dfrac{x}{\sin 41^\circ} = \dfrac{52}{\sin 74^\circ}$

C $x^2 = 52^2 + 49^2 - 2 \times 52 \times 49 \cos 41^\circ$

D $\dfrac{x}{\sin 41^\circ} = \dfrac{52}{\sin 65^\circ}$

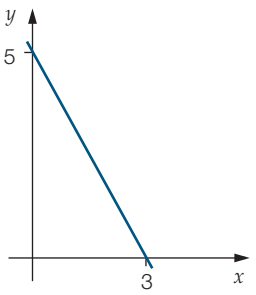
Reasoning

31 Prove that a right-angled isosceles triangle is produced by connecting the points $A(-1, 3)$, $B(5, -1)$ and $C(0, -2)$ and then find its area.

32 Prove that a rectangle is produced by connecting the points $A(-6, 0)$, $B(3, -6)$, $C(5, -3)$ and $D(-4, 3)$ and then find its area.

33 A quadrilateral has the angles $x°$, $(2x + 3)°$, $(2x - 3)°$ and $(x + 6)°$. Find the size of each angle.

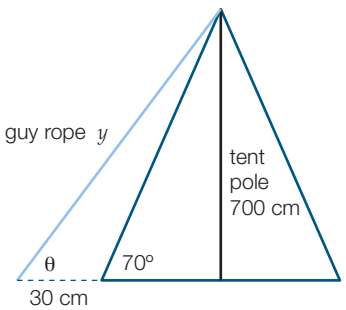
34 Find the equation of the line that is perpendicular to the line shown on the diagram, such that the perpendicular line passes through the point (-2, 3).



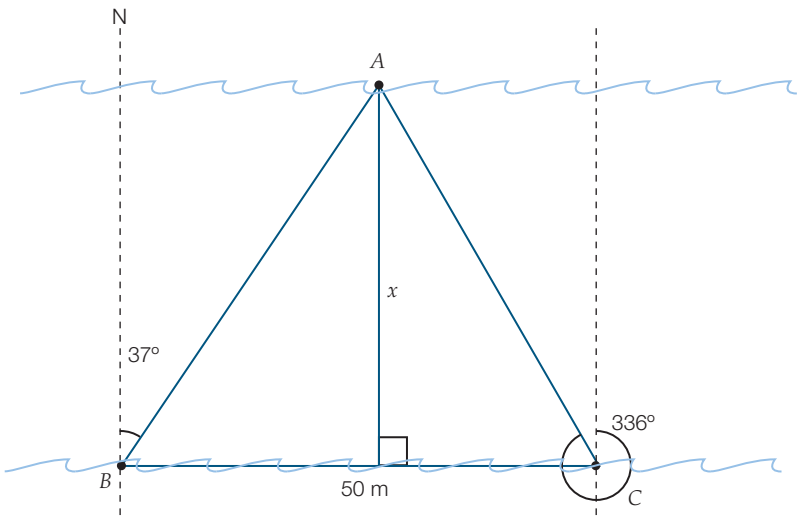
35 The sides of a tent of height 700 cm make an angle of 70° with the ground. A guy rope used to hold the tent pole upright is secured 30 cm from the base of the tent.

(a) Find the angle it makes with the ground, correct to two decimal places.

(b) Find the length of the guy rope, to the nearest centimetre.

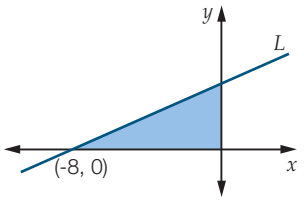


36 A point A on one side of a river is at a bearing of 037°T from point B on the other side of the river. From point C, 50 m due east of point B, the bearing of point A is 336°T. How wide is the river, in metres, correct to two decimal places?



Challenge B

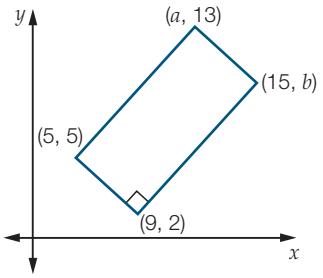
1 The line L crosses the x-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L?



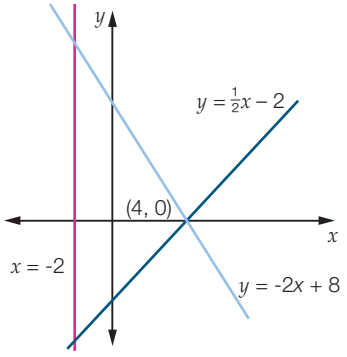
A $-\frac{1}{2}$ **B** $\frac{1}{2}$

C 2 **D** 4

2 In the rectangle shown, what is the value of $a - b$?



3 The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is:



A 15 **B** 27

C 36 **D** 45

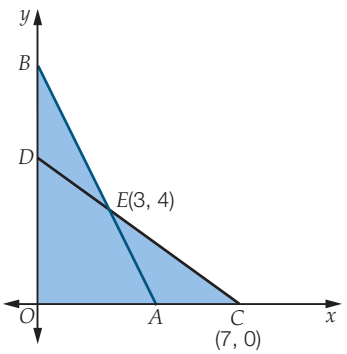
4 ΔABC has vertices $A(0, 0)$, $B(9, 0)$ and $C(0, 6)$. The points P and Q lie on the side AB such that $AP = PQ = QB$. Similarly, the points R and S lie on the side AC so that $AR = RS = SC$. The vertex C is joined to each of the points P and Q. The vertex B is joined to R and S.

(a) Find the equation of the line through the points R and B.

(b) Find the equation of PC.

(c) The line segments PC and RB intersect at X, and the line segments QC and SB intersect at Y. Show, by calculation, that the points A, X and Y lie on the same straight line.

5 A line with slope -3 intersects the positive x-axis at A and the positive y-axis at B. A second line intersects the x-axis at $C(7, 0)$ and the y-axis at D. The lines intersect at $E(3, 4)$.



(a) Find the slope of the line through C and E.

(b) Find the equation of the line through C and E, and the coordinates of the point D.

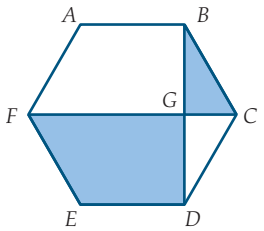
(c) Find the equation of the line through A and B, and the coordinates of the point B.

(d) Determine the area of the shaded region.

6 The point Y lies on the line $y = 5x + 3$. The coordinates of the point Z are $(3, -2)$. If X is the midpoint of YZ, then X must lie on the line:

A $y = \frac{5x}{2} - \frac{7}{2}$ **B** $y = 5x + 1$ **C** $y = \frac{5x}{2} + \frac{1}{2}$ **D** $y = 5x - 7$

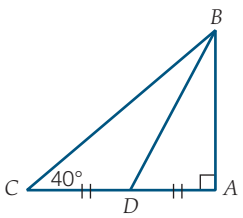
7 In the regular hexagon *ABCDEF*, the lines *BD* and *CF* intersect at *G*. The ratio of the area of quadrilateral *DEFG* to ΔBCG is:



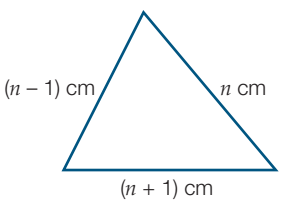
A $3\sqrt{3}:1$ **B** $5:1$

C $4:1$ **D** $2\sqrt{3}:1$

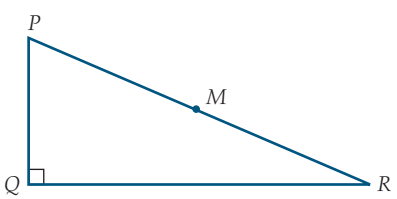
8 A flagpole *AB* is 20 metres tall. From a point *C* on flat ground, the angle of elevation to the top of the flagpole is 40°. If *CD* = *DA*, calculate the size of $\angle ADB$ rounded to the nearest degree.



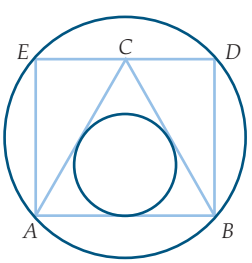
9 Find the area of this triangle if *n* is a whole number and the area of the triangle is a whole number of square centimetres. All the sides are between 10 cm and 20 cm. You may wish to use Heron's formula for the area of a triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ and *a*, *b* and *c* are the sides of the triangle.



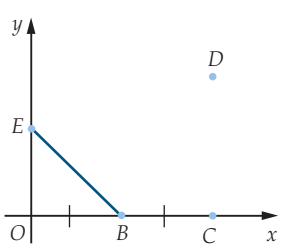
10 Triangle *PQR* is right-angled at *Q* and has side lengths *PQ* = 14 and *QR* = 48. *M* is the midpoint of *PR*. Draw the interval *MS*, where *S* is on *PQ*, such that *MS*||*RQ*. Find the value of the cosine of $\angle MQP$, correct to two decimal places.



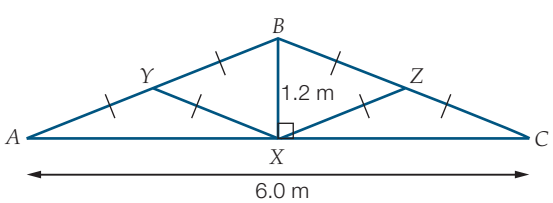
11 ΔABC is an equilateral triangle. The radius of the circle just touching the sides of *ABC* is 1 unit. A larger circle is drawn through the vertices of the rectangle *ABDE*. Calculate the diameter of the larger circle.



12 In the diagram, *E* has coordinates (0, 2), and *B* lies on the positive *x*-axis so that *BE* = 7 . The point *C* lies on the positive *x*-axis so that *BC* = *OB*. If *D* lies in the first quadrant such that $\angle CBD = 30°$ and $\angle BCD = 90°$, what is the length of *ED* correct to two decimal places?



13 The diagram shows a roof truss. Triangle *ABC* is isosceles with *AB* = *BC*. The midpoints of *AB* and *BC* are *Y* and *Z* respectively. *AC* = 6.0 m and *BX* = 1.2 m. Calculate the length of *AB* correct to one decimal place. Find the total length of timber required to make the truss.
Calculate the size of $\angle BAX$ and hence, find the size of $\angle BYX$. Round both angles to the nearest degree.



Statistics

7

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 Find the mean, median (middle number), mode (most common number) and range (difference between largest and smallest numbers) for each of these sets of data.

Worksheet R7.1

(a) The number of siblings for your five best friends:
2 4 5 1 4

(b) The shoe size of your nine young cousins:
3.5 4.5 5.0 5.5 5.0 4.5 5.5 5.0 3.5

2 Classify each of the following as being nominal, discrete or continuous data.

Worksheet R7.2

(a) Number of goals scored by a hockey team each week.

(b) Height of the members of your maths class.

(c) Size of blouses on a rack.

3 The heights (in centimetres) of 20 students are as follows:

Worksheet R7.3

179.7 167.3 168.6 164.7 182.8 160.5 179.2 163.0 177.7 171.3
169.8 173.2 164.7 172.1 165.6 171.9 164.8 166.7 169.1 166.7

Round off the heights to the nearest cm and:

(a) using intervals of 5 cm, draw a stem plot

(b) find the median.

4 The distance jumped, in cm, by each competitor in the long jump was recorded and is shown in the frequency table.

Worksheet R7.4

Class interval (kg)	Frequency
5.75–<6.00	4
6.00–<6.25	6
6.25–<6.50	9
6.50–<6.75	13
6.75–<7.00	5
7.00–<7.25	3

(a) Find an estimate for the value of the mean.

(b) In which class interval does the median fall?

After completing this chapter you will be able to:

- find five-figure summaries and use them to draw box plots
- use parallel box plots as one way to compare data sets
- construct and comment on data displays involving bivariate data where the independent variable is time
- interpret and analyse data found in the media
- calculate the mean and standard deviation as a pair of values used to compare data sets 10A
- draw and interpret lines of best fit. 10A

7.1 Cumulative frequency curves

Need to Know

A **cumulative frequency curve** shows the number of results less than a given value in grouped data. An **ogive** is another name for a cumulative frequency curve.

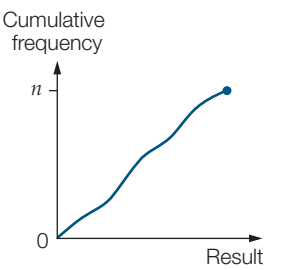
The median of n results is the $\left(\frac{n+1}{2}\right)$th result.

To find the median of data grouped in class intervals, follow these steps:

Step 1 Construct a cumulative frequency table.

Step 2 Plot the cumulative frequency curve.

Step 3 Find the median by reading the $\left(\frac{n+1}{2}\right)$th result from the curve.



Worked Example 1

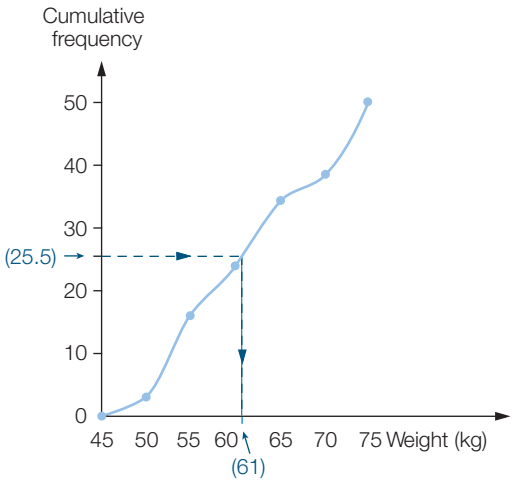
WE1

The following frequency table lists the weights of 50 packing crates, measured in kilograms. Find the median by constructing a cumulative frequency curve.

Class interval (kg)	Frequency
45–<50	3
50–<55	13
55–<60	8
60–<65	10
65–<70	4
70–<75	12

Working

Class interval (kg)	Cumulative frequency
<45	0
<50	3
<55	16
<60	24
<65	34
<70	38
<75	50



$$\text{The median} = \left(\frac{50+1}{2}\right)\text{th}$$

$$= 25.5\text{th result}$$

The median is about 61 kg.

Worked Example 2

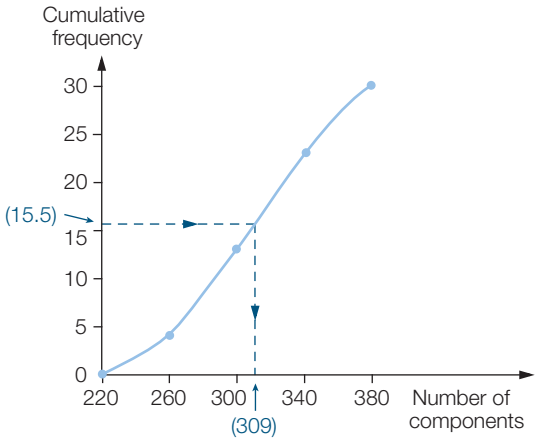
WE2

The following table shows the data on the number of components produced during a shift. Find the median by constructing a cumulative frequency curve.

Class interval	Frequency
220–259	4
260–299	9
300–339	10
340–379	7
	$\Sigma f = 30$

Working

Number of components	Cumulative frequency
<220	0
<260	4
<300	13
<340	23
<380	30



$$\text{Median} = \left(\frac{30+1}{2}\right)\text{th result}$$

$$= 15.5\text{th result}$$

The median is about 309 components per shift.

Answers page 512

7.1 Cumulative frequency curves

Fluency

WE1

1 Find the median for each of the following grouped data sets by constructing a cumulative frequency curve.

(a)

Class interval	Frequency
0–<10	4
10–<20	5
20–<30	8
30–<40	11
40–<50	7

(b)

Class interval	Frequency
8–<10	1
10–<12	4
12–<14	6
14–<16	9
16–<18	7

(c)

Class interval	Frequency
60–<70	9
70–<80	12
80–<90	19
90–<100	17
100–<110	13

(d)

Class interval	Frequency
250–<300	37
300–<350	58
350–<400	26
400–<450	16
450–<500	9
500–<550	4

WE2

2 (a) The frequency table on the right shows the number of students present each day at a particular school during first term. Find the median number of students present by constructing a cumulative frequency curve.

Class interval	Frequency
450–459	2
460–469	7
470–479	8
480–489	15
490–499	10
500–509	8

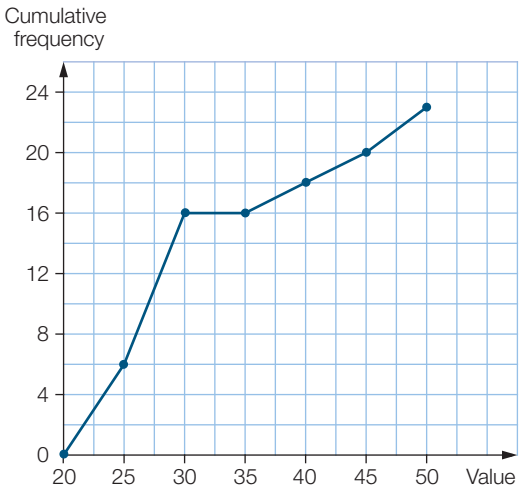
(b) The frequency table on the right shows the number of points scored by each team in one round of a rugby competition. Find the median score by constructing a cumulative frequency curve.

Class interval	Frequency
0–<10	2
10–<20	3
20–<30	3
30–<40	4
40–<50	1
50–<60	1

3 The frequency table on the right lists the lengths of fish caught, measured in centimetres. Find the median length by constructing a cumulative frequency curve.

Class interval (cm)	Frequency
30–<35	7
35–<40	10
40–<45	5
45–<50	12
50–<55	13
55–<60	4

4 The following questions relate to the cumulative frequency graph shown on the right.



(a) The number of values in the data set is:

A 23 **B** 24

C 26 **D** 50

(b) The median is closest to:

A 12 **B** 27

C 28 **D** 29

(c) The class interval that has a frequency of zero is:

A 20–<25 **B** 30–<35

C 30–<36 **D** 31–<36

Understanding

5 The number of people boarding each of 25 successive trains at a station was as follows:

6	16	21	29	39	46	42	29	21	29	35	21	17
10	8	18	17	26	7	9	12	19	26	20	21	

(a) Find the median of this raw data.

(b) Group the data into the intervals 1–10, 11–20 etc. to draw a cumulative frequency curve and find the median.

(c) On how many trains was the number of people boarding less than 18? Compare the answer obtained from the raw data with the one obtained using a cumulative frequency curve.

Reasoning

6 There is another way to estimate the median for grouped data. It uses the likely location of the median within the class interval in which the median lies. If we know that the median is the third of eight results in the class interval 20–<25, we say that the median is approximated by the value $20 + \frac{3}{8} \times 5 \approx 21.9$. We would be unlikely to go beyond even one decimal place in a calculation such as this.

(a) Use this method to find an approximation for the median for Question **1(b)**.

(b) Compare this with the value obtained using the cumulative frequency curve.

(c) Use this method to find an approximation for the median for Question **2(a)**.

(d) Compare this with the value obtained using the cumulative frequency curve.

(e) What do you think about this alternative method? List some good features and some bad features of the method.

Open-ended

7 Use the following information to construct a discrete data set with at least 19 results and then calculate the mean.

(a) median = 39, odd number of results **(b)** median = 72.5, even number of results

Problem solving

Product codes

Most products that you buy have a barcode attached to them. This barcode represents the European Article Number (EAN) and is a 13-digit unique code for that product. As part of the checking system to ensure the correct numbers are scanned by a barcode reader, the 13th digit is actually a check digit.

The following are all legal product codes:

9316090060604	9337090060609
9326090060603	9338090060606
9336090060602	9339090060603

1 In the first column of barcodes, apart from the check digit, which is the only digit that changes? What do you notice about the sum of the two digits that are changing?

2 Predict the final digit in the code 934609006060*.

3 In the second column of barcodes, apart from the check digit, which is the only digit that is changing? Do these digits follow the same pattern as those in the first column of barcodes?

Not every column acts in the same way. In fact, the even columns all act in the same way and the odd columns all act in the same way. We will place the very first EAN-13 into a table to show what happens.

×1	×3	×1	×3	×1	×3	×1	×3	×1	×3	×1	×3	×1
9	3	1	6	0	9	0	0	6	0	6	0	4

4 Calculate the sum of these multiplied values.

5 Do the same for each of the EAN-13 codes given above. What conclusion can you reach?

6 Now calculate the final digit in each of the following EAN-13 codes:

(a) 933919006060*

(b) 933929006060*

(c) 933928006060*

(d) 933927006060*

Strategy options

- Work backwards.
- Test all possible combinations.

7.2 Box plots

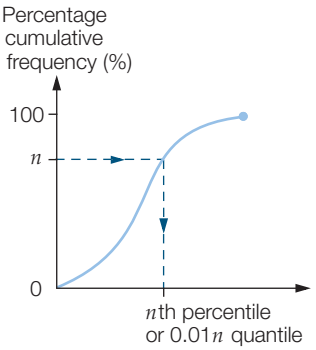
Need to Know

For any **percentage cumulative frequency curve**, n% of results lie below the nth **percentile**.

A cumulative frequency is the number of results below a given value.

$$\text{Percentage cumulative frequency} = \frac{\text{cumulative frequency}}{\text{total number of results}} \times 100\%$$

Percentile and **quantile** values can be read from a percentage cumulative frequency graph. The nth percentile (or $0.01n$ quantile) is the value on the horizontal axis corresponding to $n\%$ on the vertical axis, where $0 \leq n \leq 100$.



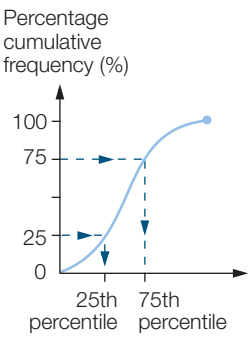
Measures of spread

Range is the difference between the largest and smallest results in a set of data.

Range = maximum – minimum

Interquartile range is the difference between the upper quartile (Q_U) and the lower quartile (Q_L).

Interquartile range can be found by reading values from a percentage cumulative frequency curve.



$\text{IQR} = Q_U - Q_L$
$= 75\text{th percentile} - 25\text{th percentile}$

If you have the raw data set, then the interquartile range can be found by breaking the data into two equal subsets. The lower quartile is the median of the lower half of the data and the upper quartile is the median of the upper half of the data.

The quartiles can be written as Q_1 and Q_3.

In summary:

lower quartile = $Q_L = Q_1$ = 25th percentile = 0.25 quantile

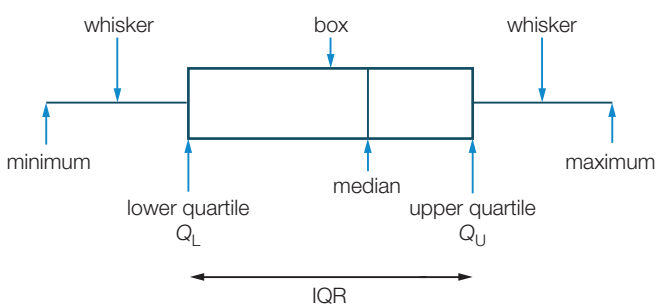
median = Q_2 = 50th percentile = 0.50 quantile

upper quartile = $Q_U = Q_3$ = 75th percentile = 0.75 quantile

A **box plot**, also known as a box-and-whisker plot, is a graphical summary of five key pieces of data known as the **five-number summary**:

- minimum value
- lower quartile Q_L (25th percentile)
- median (50th percentile)
- upper quartile Q_U (75th percentile)
- maximum value

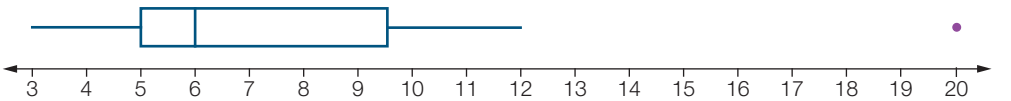
The central part of the diagram, between the two quartiles, is the box, and the single lines extending to the minimum and maximum values are the whiskers.



When one or more of the surveyed results lie outside a certain range they may be considered as **outliers.** A value is an outlier if it is more than $1.5 \times \text{IQR}$ above Q_U or more than $1.5 \times \text{IQR}$ below Q_L.

When drawing a box plot that includes an outlier, or some outliers, we highlight these values by drawing them as dots instead of stretching the whisker all the way out to the outlier value.

This is a box plot for the data set: 3, 5, 5, 6, 6, 7, 7, 12, 20.



The right-hand whisker ends at 12, which is the highest value that is not an outlier. The outlier, 20, is shown on the box plot as a dot.

Worked Example 3

WE3

The following table shows the data in class intervals from a survey of the weights of packing crates. Use the data to:

(a) draw a cumulative frequency table and then a percentage cumulative frequency table

(b) draw a percentage cumulative frequency curve

(c) evaluate the 80th percentile and the 0.3 quantile.

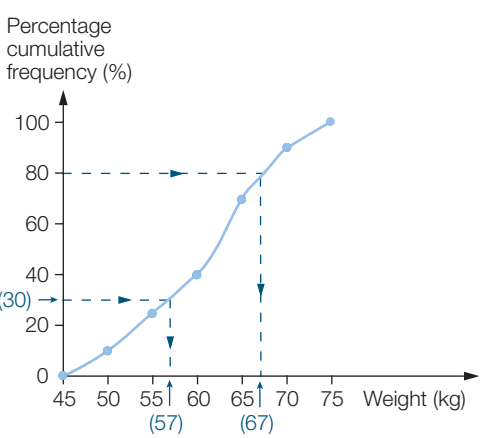
Weight (kg)	Frequency
45–<50	2
50–<55	3
55–<60	3
60–<65	6
65–<70	4
70–<75	2

Working

(a)

Weight (kg)	Cumulative frequency	Percentage cumulative frequency
<45	0	$\frac{0}{20} \times 100 = 0$
<50	2	$\frac{2}{20} \times 100 = 10$
<55	5	$\frac{5}{20} \times 100 = 25$
<60	8	$\frac{8}{20} \times 100 = 40$
<65	14	$\frac{14}{20} \times 100 = 70$
<70	18	$\frac{18}{20} \times 100 = 90$
<75	20	$\frac{20}{20} \times 100 = 100$

(b)



(c) The 80th percentile is 67 kg.

The 0.3 quantile (30%) is 57 kg.

Worked Example 4

WE4

The weights of 20 packing crates (kg) are shown in the frequency table.

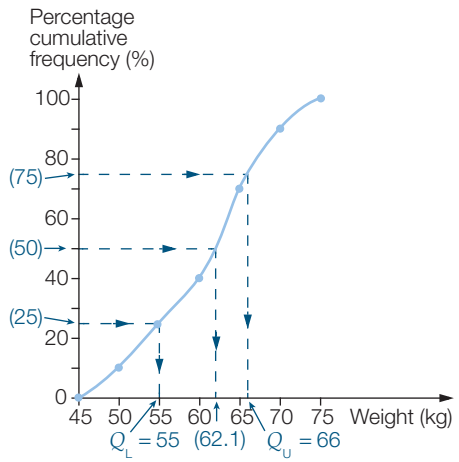
(a) Calculate the range and interquartile range.

(b) Draw a box plot to represent the data.

Weight (kg)	Frequency
45–<50	2
50–<55	3
55–<60	3
60–<65	6
65–<70	4
70–<75	2

Working

(a) Range = largest result − smallest result
= 75 − 45
= 30



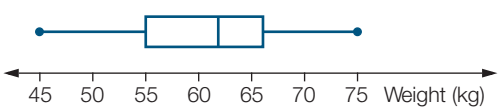
Weight (kg)	Cumulative frequency	Percentage cumulative frequency
<45	0	$\frac{0}{20} \times 100 = 0$
<50	2	$\frac{2}{20} \times 100 = 10$
<55	5	$\frac{5}{20} \times 100 = 25$
<60	8	$\frac{8}{20} \times 100 = 40$
<65	14	$\frac{14}{20} \times 100 = 70$
<70	18	$\frac{18}{20} \times 100 = 90$
<75	20	$\frac{20}{20} \times 100 = 100$

$Q_U = 66$
$Q_L = 55$

IQR = 66 − 55
= 11

The range of weights is 30 kg and the interquartile range is 11 kg.

(b) median = 62.1



Worked Example 5

WE5

Consider the following results from a survey of the number of cans of baked beans produced on an assembly line per hour:

244 246 247 248 242 24 248 248 247 247 246 244 242 242 246

Identify the outlier and suggest a possible reason for it.

Working

The result 24 is more than 200 less than the next smallest result; it is clearly an outlier.

Machine breakdown, operator error, operator injury or result incorrectly recorded could all be reasons for this outlier.

Worked Example 6

WE6

Determine if there are any outliers in this list of the weights of 20 packing crates (kg):

60.4 53.9 61.1 62.3 50.4 57.3 45.7 59.4 68.6 65.5
60.8 67.9 57.6 62.9 65.8 70.0 73.7 49.6 54.8 62.7

Working

Ordered data list:

45.7 49.6 50.4 53.9 54.8
57.3 57.6 59.4 60.4 60.8
61.1 62.3 62.7 62.9 65.5
65.8 67.9 68.6 70.0 73.7

The data list seems relatively evenly spread out; no outlier is obvious.

$$Q_L = \frac{54.8 + 57.3}{2} = 56.05$$

$$Q_U = \frac{65.5 + 65.8}{2} = 65.65$$

$$\begin{aligned} IQR &= Q_U - Q_L \\ &= 65.65 - 56.05 \\ &= 9.6 \end{aligned}$$

$$\begin{aligned} 1.5 \times IQR &= 1.5 \times 9.6 \\ &= 14.4 \end{aligned}$$

$$\begin{aligned} Q_L - 1.5 \times IQR &= 56.05 - 14.4 \\ &= 41.65 \end{aligned}$$

$$\begin{aligned} Q_U + 1.5 \times IQR &= 65.65 + 14.4 \\ &= 80.05 \end{aligned}$$

None of the values lie outside the designated range, confirming the original view that there are no outliers.

7.2 Box plots

Answers page 514

Fluency

1 Draw cumulative frequency and percentage cumulative frequency curves to represent the following data, and find the 60th percentile for each data set. WE3

(a)

Class interval	Frequency
15–<16	4
16–<17	5
17–<18	7
18–<19	3
19–<20	4

(b)

Class interval	Frequency
150–<155	1
155–<160	2
160–<165	3
165–<170	4
170–<175	3

2 For each given data set, calculate **(i)** the range and **(ii)** the interquartile range, then **(iii)** draw a box plot. WE4

(a)

Class interval	Frequency
0–<10	4
10–<20	5
20–<30	8
30–<40	11
40–<50	7

(b)

Class interval	Frequency
60–69	9
70–79	12
80–89	19
90–99	17
100–109	13

3 The number of points scored by a basketballer in a season of games is: WE5

32 30 16 18 24 16 28 26 19 18 826
34 36 41 18 16 14 17 21 26 36 548

Without grouping the data, identify any outliers and suggest possible reasons for them.

4 The following is a list of the heights, measured to the nearest centimetre, of the players at the local football club. WE6

155 167 189 156 178 179 190 200 206 196 187 156
166 178 168 178 168 170 167 159 167 160 163 181
191 157 156 187 157 159 156 161 153 173 174 168
177 164 175 185 186 164 171 170 180 141 167 159

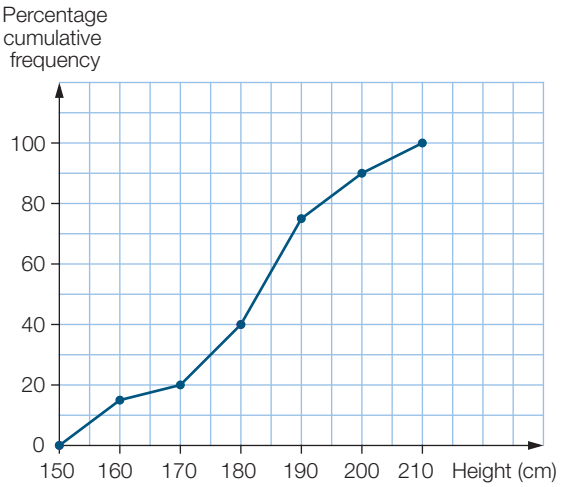
(a) Find the range of the heights.

(b) Calculate the interquartile range by finding the upper and lower quartiles.

(c) Calculate the values that are the cut-off values at either end of the data set after which the results would be considered outliers.

(d) Are there any outliers in the data set?

5 Each question below refers to the percentage cumulative frequency graph shown at right, which represents the heights of players in a football club.



(a) The 30th percentile is closest to:

A 170 **B** 172.5

C 175 **D** 184

(b) The 0.8 quantile is closest to:

A 170 **B** 188

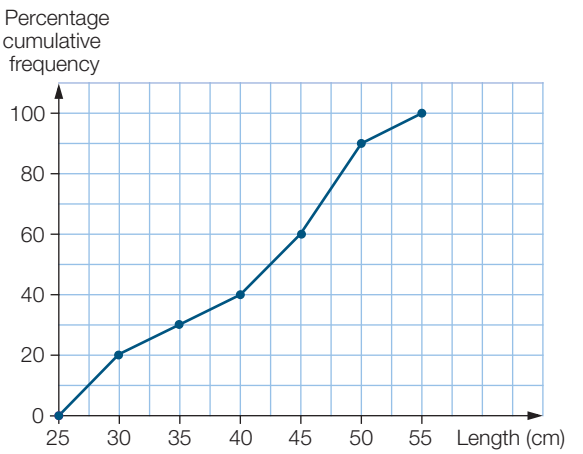
C 191 **D** 194

(c) The interquartile range is closest to:

A 16 **B** 18

C 22 **D** 50

6 For the percentage cumulative frequency graph shown at right, outliers would be:



A results less than 32 cm

B results greater than 63 cm

C results less than 16.5 cm and greater than 63 cm

D none

Understanding

7 The distances run (km) by athletes training for a marathon are summarised in this frequency table.

Class interval	Frequency
10–<15	4
15–<20	6
20–<25	13
25–<30	16
30–<35	11
35–<40	9

(a) Draw a cumulative frequency curve for this data.

(b) How many of the distances run were less than 17 km?

(c) Draw a percentage cumulative frequency curve.

(d) Find the 20th percentile and the 0.8 quantile.

8 The data below shows the distances travelled (km) by car in a week for 30 employees.

895	245	697	1098	573	1569	1296	1880	972	843
1108	1037	986	1265	1398	1107	1275	1117	1023	1062
1197	1047	1284	1402	438	569	974	1328	1194	1073

(a) Calculate the range of the data.

(b) Calculate the interquartile range by finding the upper and lower quartiles.

(c) Draw a box plot and identify any outliers.

9 A survey of the number of components produced on an assembly line per shift gave the following results:

223	246	288	295	374	278	286	251	299	370	345
313	368	312	309	333	357	328	347	281	299	304
305	346	321	287	315	305	274	228	963		

(a) Is it possible to have an outlier that is very high as well as one that is very low?

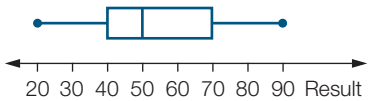
(b) Suggest possible explanations for the 963 components produced in the last shift.

10 For each of the following box plots:

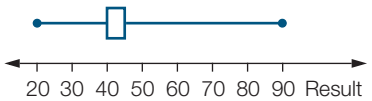
(i) indicate the values below which, and above which, outliers would lie

(ii) decide whether there are any outliers in the data set displayed.

(a)



(b)



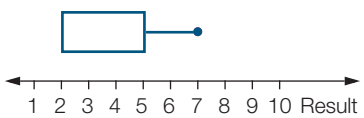
Reasoning

11 For each of the following box plots:

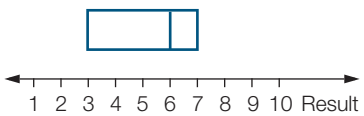
(i) construct a data set of at least 20 values that matches it

(ii) explain the reasoning you used to do this, including any assumptions made and alternative actions that might have been taken.

(a)



(b)



Open-ended

12 Consider this table, which shows the annual earnings for all the employees of a company.

Annual income	Frequency
\$35 000–<\$40 000	17
\$40 000–<\$45 000	24
\$45 000–<\$50 000	18
\$50 000–<\$55 000	16
\$55 000–<\$60 000	1
\$60 000–<\$65 000	1
\$65 000–<\$70 000	2
\$70 000–<\$75 000	1
\$75 000–<\$80 000	1
\$80 000–<\$85 000	1

(a) Calculate estimates for the mean and median annual incomes.

(b) Calculate estimates for the range and interquartile range.

(c) Are there any outliers in the data set? If so, list them.

(d) Remove any outlier(s) from the results and recalculate the mean, median, range and interquartile range.

(e) Comment on your results.

13 **(a)** Draw a cumulative frequency graph with an interquartile range of 35.

(b) Draw a percentage cumulative frequency graph with an interquartile range of 16.

14 **(a)** Construct a data set with at least 20 values that has an outlier at the upper end of the data set. The outlier is to be less than $Q_U + 2 \times \text{IQR}$.

(b) Construct a data set with at least 20 values that has an outlier at the lower end of the data set. The outlier is to be greater than $Q_L - 2 \times \text{IQR}$.

Problem solving

Draw that plot

A data set has the following characteristics:

- 50th percentile is 56
- 75th percentile is 80
- IQR is 50
- Range is 80

Draw a box plot to represent this information. Which of the five key values are you certain about? Give the maximum and minimum values possible for the values about which you are unsure.

Strategy options

- Work backwards.
- Draw a diagram.

7.3 Comparing data sets

Need to Know

To compare sets of data we can use two measures of centre or location: mean and median, and two measures of spread: range and interquartile range.

A **parallel box plot** and a **parallel dot plot** can be used to compare the median, range and IQR of the data sets as well as decide which data set is more consistent.

Worked Example 7

WE7

For each of the following sets of data:

SET A: 1 1 1 2 3 3 3 4 4 4 5 5 5 5 6
6 6 6 7 7 7 8 8 8 8 8 9 9 9

SET B: 1 1 1 2 3 3 3 4 4 4 5 5 5 5 6
6 6 6 7 7 7 8 8 8 8 8 9 9 19

(a) find the mean

(b) find the median

(c) find the range

(d) calculate the interquartile range

(e) use your findings to compare the two sets of data.

Working

(a) Set A mean $= \frac{158}{29}$
$= 5.48$

Set B mean $= \frac{168}{29}$
$= 5.79$

(b) Set A median $= 6$

Set B median $= 6$

(c) Set A range $= 9 - 1$
$= 8$

Set B range $= 19 - 1$
$= 18$

(d) Set A $Q_L = 3.5$

Set A $Q_U = 8$

Set A IQR $= 8 - 3.5$
$= 4.5$

Set B $Q_L = 3.5$

Set B $Q_U = 8$

Set B IQR $= 8 - 3.5$
$= 4.5$

(e) The two sets have the same median, but the mean of Set B is slightly higher than Set A. The two sets have the same interquartile range, but the range of Set B is greater than Set A. This is because Set B has the score of 19, which affects the mean and the range, but not the median (middle score) nor the interquartile range (spread of the middle half of the data).

Worked Example 8

WE8

The heights (cm) of the students in two Year 8 classes were recorded.

8A:	148	149	150	150	153	154	154	154	155	155	156	156	156
	157	157	158	158	158	158	158	160	161	163	165	168	
8B:	150	151	152	152	152	153	153	154	155	155	155	156	156
	156	156	157	157	158	160	160	160	161	161	162	163	

(a) Find the median height for each class.

(b) Find the range for each class.

(c) Find the interquartile range for each class.

(d) Draw a parallel box plot for both classes.

(e) Compare the statistics for the two classes.

Working

(a) 8A median $= 156$

8B median $= 156$

(b) 8A range $= 168 - 148$
$= 20$

8B range $= 163 - 150$
$= 13$

(c) 8A $Q_L = 154$

8A $Q_U = 158$

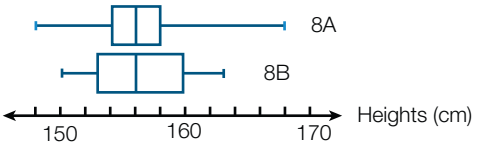
8A IQR $= 158 - 154$
$= 4$

8B $Q_L = 153$

8B $Q_U = 160$

8B IQR $= 160 - 153$
$= 7$

(d)



(e) The medians for the two classes are the same but the range for 8A is much larger than for 8B. However, the IQR for 8A is much smaller than the IQR for 8B.

Answers page 516

7.3 Comparing data sets

Fluency

WE7

1 Calculate the following values for each pair of data sets below. Where necessary, round your answers to two decimal places.

(i) mean **(ii)** median **(iii)** range **(iv)** interquartile range

(a) Set A: 22 22 24 25 26 27 28 28 29 30
31 32 35 35 36 37 38 39 40

Set B: 12 22 24 25 26 27 28 28 29 30
31 32 35 35 36 37 38 39 40

(b) Set A: 44 44 46 47 47 48 48 48 51
52 52 52 54 55 58 58 58 60

Set B: 24 44 46 47 47 48 48 48 51
52 52 52 54 55 58 58 58 80

(c) Set A: 11 12 12 12 13 13 13 13 14 14 15
16 17 18 18 18 21 22 24 24 24 26

Set B: 11 12 12 12 13 13 13 13 13 13 13
16 17 18 18 18 21 22 24 24 24 26

(d) Make a few comments about what you have found by completing these calculations.

WE8

2 The heights (cm) of the students in two Year 8 classes were recorded.

8C: 145 146 147 147 149 150 150 151 152 153 154 155 155
156 157 158 158 159 160 162 163 163 164 166 169

8D: 147 148 149 150 150 150 152 153 154 154 155 155 155
156 156 156 157 157 158 160 160 160 161 161 162

(a) Find the median height for each class.

(b) Find the range for each class.

(c) Find the interquartile range for each class.

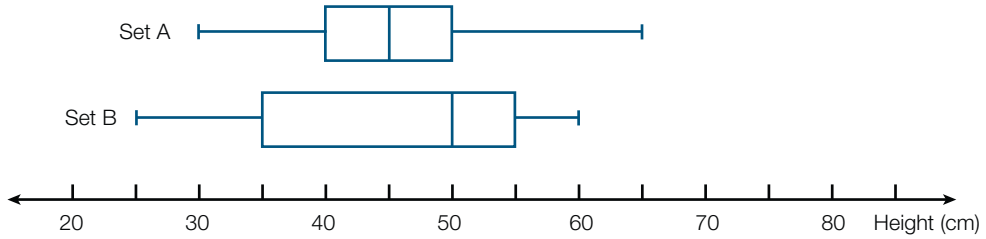
(d) Draw a parallel box plot for both classes.

(e) Compare the statistics for each class.

3 **(a)** For each data set in the following parallel box plot, find the:

(i) median **(ii)** range **(iii)** IQR.

(b) Compare the statistics for the two data sets.



4 The following data list shows the weight (kg) of each player on the 2011 playing list for the Brisbane Broncos in the NRL.

104	106	83	95	110	105	94	110	101	93	104	100
109	108	99	93	90	86	95	106	106	85	105	91
105	89	94	100	98	106	110	111	88	100	96	

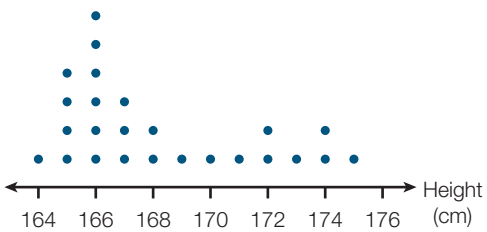
(a) Draw a histogram of the heights. Use a class interval of 5, with the first interval being 80–<85.

(b) Calculate the five-number summary for the data and use it to construct a box plot.

(c) Draw a dot plot of the data.

(d) Make some comments about what the three displays tell you about the data set.

5 **(a)** The students in a class had their heights (cm) recorded. The following dot plot shows the results.

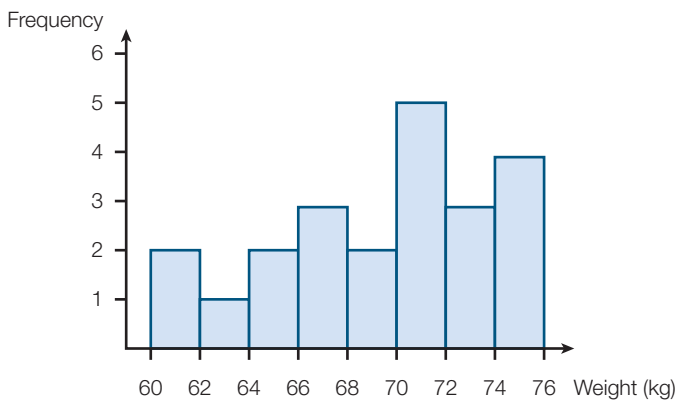


The interquartile range for this data set is closest to:

A 5 **B** 5.5

C 7.5 **D** 11

(b) The histogram below shows the weights, in kg, of the students in another class.



The median of this data set is likely to be closest to:

A 66 **B** 70 **C** 74 **D** 75

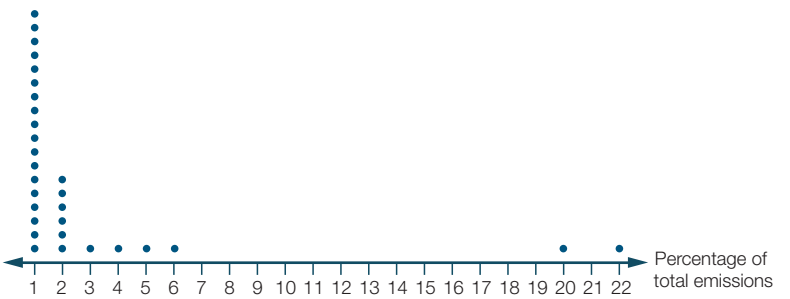
Understanding

6 Three classes sat the same mathematics test, marked out of 100. The results for the three classes are summarised in the following table.

Class	Mean	Median	Range	IQR
A	64.32	68	72	27.5
B	64.12	63	31	19
C	60.92	63	55	38.5

Which class do you think performed best on the test?

7 The following dot plot shows the percentage of the world's total emissions of carbon dioxide for 30 countries, which contribute at least 1% of the total emissions when rounded to the nearest whole number.

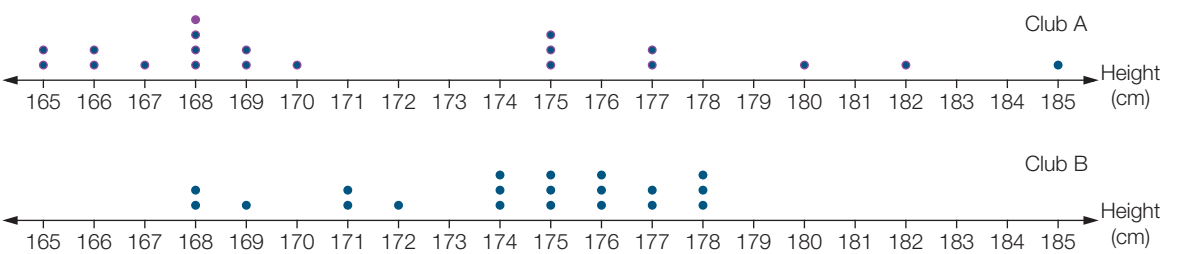


(a) Calculate the **(i)** mean and **(ii)** median for the data.

(b) Calculate the **(i)** range and **(ii)** interquartile range for the data.

(c) Which pair of values better describe the data set: mean and range; or median and interquartile range? Give a reason to support your answer.

8 The following parallel dot plot shows the heights of players at two netball clubs who play in the same competition. The heights have been rounded to the nearest centimetre.



(a) Which club appears to have the taller players?

(b) Calculate the mean and median of the given data for the two clubs.

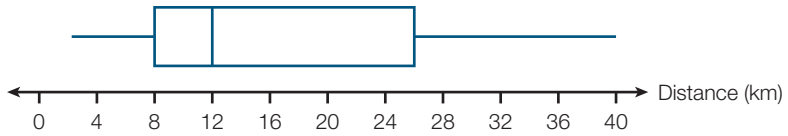
(c) Calculate the range and IQR for each of the two clubs.

(d) Draw a parallel box plot for the data.

(e) Do you still agree with your answer to part **(a)**? Use statistics to justify your answer.

(f) Compare the shape of the parallel dot plot with that of the parallel box plot.

9 A school collected information about the distance (km) that students travel to school. The results were published only in box plot form, as shown below.



(a) Write a sentence or two describing what information the box plot gives you about the travel distances for the students at this school.

(b) What would you estimate to be the mean distance travelled by the students? Describe how you made your estimation.

(c) **(i)** Draw a histogram that might represent the data collected. Use a bin size of 4 with a first class interval of 0–<4, and so on. Information was collected from 160 students.

(ii) Show that your histogram supports the values shown in the box plot.

(iii) Use your histogram to estimate the mean distance. How does this value compare to your estimate in **(b)**?

Reasoning

10 The following table gives the height (cm) and weight (kg) of each of the players on the 2010 playing lists of the AFL teams Adelaide Crows and Essendon.

Adelaide

Height:	200	184	201	185	201	185	184	178	179	178
	194	192	182	187	195	188	180	198	185	192
	181	185	191	181	195	191	196	195	182	188
	176	191	196	184	193	183	192	178	180	194
	193	191	182	190	189	179	181			
Weight:	99	85	100	85	99	84	82	81	80	80
	93	97	75	82	90	86	84	102	84	87
	82	83	99	76	87	74	98	92	79	89
	79	82	98	86	89	75	88	82	77	92
	86	87	80	87	84	81	80			

Essendon

Height:	191	182	197	190	182	184	178	188	186	184
	182	188	187	182	186	191	185	193	201	185
	197	190	194	196	201	191	173	197	198	173
	180	184	189	182	192	187	190	180	201	187
	178	180	185	186	205	182				
Weight:	100	83	94	92	81	83	77	81	83	81
	77	88	74	80	86	94	74	89	108	85
	89	90	89	100	105	91	72	95	93	68
	86	86	87	85	81	90	86	84	105	84
	80	76	78	95	89	87				

(a) Find the mean heights and weights for each club.

(b) Find the median heights and weights for each club.

(c) Find the range and IQR for the heights and weights for each club.

(d) Draw a parallel box plot for the heights of players in each club.

(e) Draw a parallel dot plot for the weights of players in each club.

(f) Which team is taller overall? Use your statistics to justify your decision.

(g) Which team is heavier overall? Use your statistics to justify your decision.

11 The following table shows the percentage of the Australian population, and the percentage of the Aboriginal and Torres Strait Islander (ATSI) population, in particular age ranges.

	0–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44
Australia	6.7	6.3	6.5	6.9	7.6	7.6	6.9	7.2	6.9
ATSI	12.3	11.2	11.3	11.2	9.5	7.7	6.3	6.2	5.9

	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–85	85+
Australia	7.0	6.5	5.9	5.4	4.0	3.1	2.3	1.7	1.2
ATSI	5.1	4.2	3.3	2.3	1.5	1.0	0.6	0.3	0.2

(a) Draw cumulative frequency graphs for the Australian population and for the ATSI population.

(b) Use the cumulative frequency curves to estimate the values for the five-number summary for each population. For the purposes of this work you should use 90 as the maximum value.

(c) Draw a parallel box plot showing the two populations.

(d) Write a couple of sentences comparing the two populations.

Open-ended

12 Collect two sets of data from students in your class or year level. Use the data to calculate some statistics. Compare the data sets using the statistics you have found.

Problem solving

What do you mean?

Task 1

It was reported that most Australians have more than the average number of legs.

How can this be true?

Task 2

Each week, for 4 weeks in a row, a market research company interviewed people asking questions about the performance of the Prime Minister. The results are as follows:

Week 1:	47 approved	42 disapproved	11 were unsure
Week 2:	68 approved	57 disapproved	7 were unsure
Week 3:	112 approved	106 disapproved	8 were unsure
Week 4:	77 approved	68 disapproved	7 were unsure

What percentage approval would you say the Prime Minister enjoys?
Give some explanation for why you think your answer is justified.



Strategy options

- Act it out.
- Make a table.

Scatter plots and data investigations

7.4

Need to Know

Primary data is data that you collect yourself.

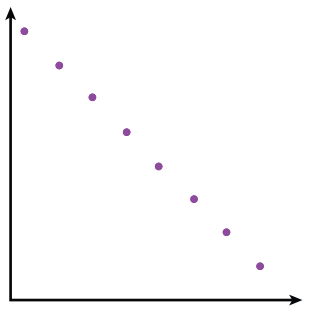
Secondary data is data that has already been collected by someone else.

Bivariate data has two variables. We can draw a **scatter plot** to display bivariate data and to look for a relationship between the variables.

Perfect negative linear relationship

Description: As one variable increases so the other decreases. There is a high degree of order in the placement of the points.

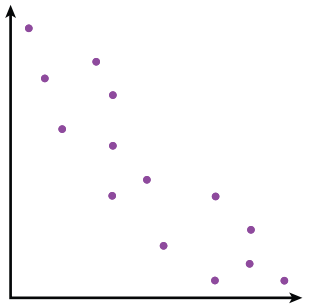
Example: Length and width of a rectangle for a fixed perimeter



Strong negative linear relationship

Description: As one variable increases so the other generally decreases. There may be a few points that don't fit this pattern exactly.

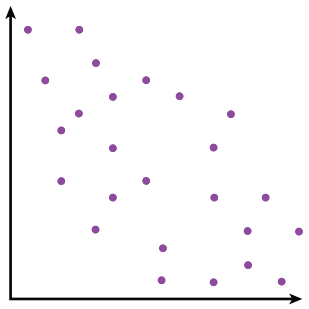
Example: Age of a second-hand car and its value (excluding vintage cars)



Weak negative linear relationship

Description: There is some trend evident of one variable increasing as the other decreases, although there would certainly be a number of points that don't fit this pattern.

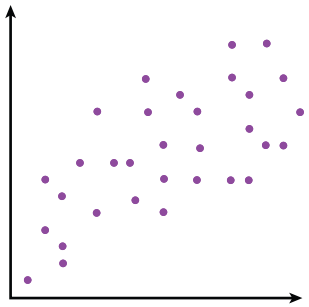
Example: Average number of hours spent keeping fit/week and the number of annual visits to a doctor



Weak positive linear relationship

Description: There is some trend evident of either both variables increasing or both variables decreasing, although there would certainly be a number of points that don't fit this pattern.

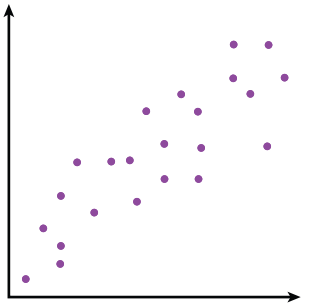
Example: Money spent on a holiday and money spent buying a car



Strong positive linear relationship

Description: As one variable increases so the other generally increases. There may be a few points that don't fit this pattern exactly.

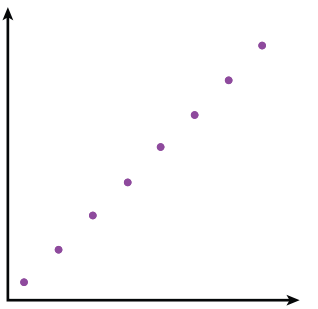
Example: Smoking and lung cancer



Perfect positive linear relationship

Description: As one variable increases so does the other. There is a high degree of order in the placement of the points.

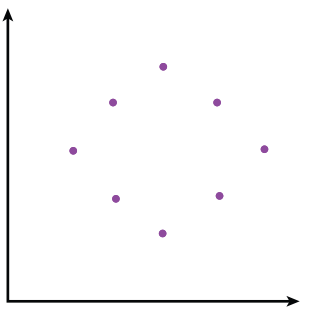
Example: Volume and height of water in a tank



No linear relationship

Description: There is no linear pattern evident in the placement of the points.

Example: People who like cycling and people who like playing a musical instrument



Worked Example 9

WE9

Choose whether the following examples represent primary data or secondary data.

(a) You are doing a project on water storages, so you collect data from the newspaper on the levels of the dams in your state over the past 12 months.

(b) You survey youth in your area to find out what facilities they would like the council to provide.

Working

(a) This is secondary data.

(b) This is primary data.

Worked Example 10

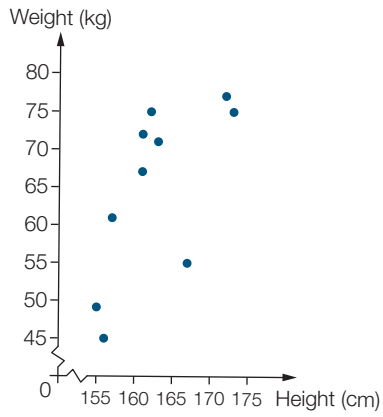
WE10

The following table represents the heights (cm) and weights (kg) of 10 students.

Height (cm)	156	167	157	155	161	173	163	162	161	172
Weight (kg)	45	55	61	49	67	75	71	75	72	77

Draw a scatter plot of the data with height on the horizontal axis.

Working



7.4 Scatter plots and data investigations

Answers page 518

Fluency

WE9

1 Choose whether the following examples represent primary data or secondary data.

(a) You sit on a street corner and record the colour of the cars passing by.

(b) You send an SMS to 20 friends asking them whether or not they are watching a particular show on television.

(c) You write down the scores for all the weekend's football matches from Monday's newspaper.

(d) Your teacher asks the class how many brothers and sisters they have and records the information on the board.

(e) You go to the website of your favourite football team and write down the height of each player.

WE10

2 The following table represents the heights (cm) and weights (kg) of 10 football players. Draw a scatter plot of the data. Put height on the horizontal axis. Make a comment about the type of relationship, if any, that is displayed by the data.

Height (cm)	203	186	184	189	186	197	188	181	182	185
Weight (kg)	103	84	84	86	89	97	87	82	80	86

3 Are the following examples of a census or a sample?

(a) You ask every student in Year 10 their favourite movie.

(b) You ask the first thirty people who leave the canteen what they purchased.

(c) You send a questionnaire to every family in your school, but only 35% of the forms are returned.

(d) You ask every junior member of your cricket club how long they have been playing cricket.

4 You are conducting a survey on the topic 'The best team sport played in Australia' and are considering using the following questions. Decide whether each question is appropriate or inappropriate, and give a reason.

(a) How often do you participate in team sport?

(b) How often do you watch team sport on television?

(c) Which of the following team sports do you think is the best? Australian Rules Football, Rugby League, Rugby Union, Cricket, Basketball

(d) Do you think too much Government money is spent supporting Olympic athletes?

5 Decide whether or not the following sampling methods would be likely to return biased results. Give a reason for each answer.

(a) You stand in a shopping centre and ask people who smile at you.

(b) You go to the local petrol station and ask people who are filling their car with diesel their opinions about environmental matters.

(c) You put a survey form in the letterbox of one hundred houses in your neighbourhood asking about support for the renovation of the local hall. You include a freepost envelope for the return of the survey.

(d) You find numbers in a telephone directory and call two hundred numbers to find out voting intentions at the next State election.

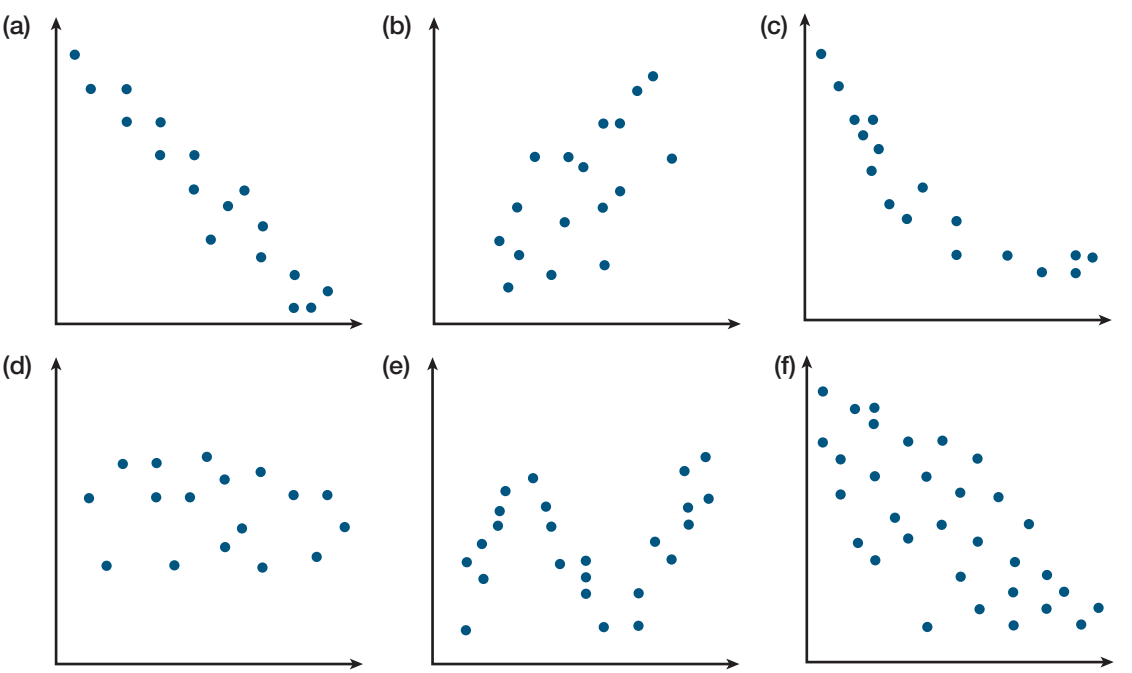
6 The data below show the marks obtained by 10 pupils taking physics and mathematics tests.

Pupil	A	B	C	D	E	F	G	H	I	J
Physics mark (out of 50)	42	38	44	41	46	31	25	30	33	36
Mathematics mark (out of 70)	66	60	65	59	65	50	38	58	60	60

(a) Draw a scatter plot of the data with the physics mark on the horizontal axis.

(b) Comment on any relationship evident between the marks.

7 For each of the following scatterplots state whether the trend is linear or non-linear. If linear, also state whether the trend is positive or negative and give an indication of strength.



Understanding

8 You are a member of the Student Representative Council (SRC) at your school which has 1230 students. A suggestion has been made that a good quality coffee machine should be installed in the school canteen. You have been given the task of conducting an investigation into this matter and need to report back to the SRC within 4 weeks.

(a) You decide that students and teachers are the stakeholders and parents need not be involved in the decision. How will this decision make your job easier?

(b) Why should you conduct a sample of the students?

(c) Why should you conduct a sample of the teachers even though there are only 110 of them?

(d) Your friend, who is very good at mathematics, suggests that you should conduct a stratified random sample. She says that this means that the number of students sampled would be allocated in proportion to some criteria such as year level or gender or both and the proportion of teachers to students sampled would be in proportion to the number of teachers and students in the school.

Explain why this is a good suggestion.

(e) Make up at least three of the questions that would form part of your survey instrument. Test them on a few of your friends to see if there are any flaws in them.

9 The following table shows the width and length, both measured in centimetres, of 88 bivalves of the same species collected on a beach.

W **(cm)**	1.3	1.4	1.4	1.5	1.9	2.1	2.1	2.1	2.2	2.2	2.3	2.4	2.4	2.5
L **(cm)**	1.7	1.9	1.8	1.9	2.3	3.5	2.8	2.7	3.0	2.7	3.0	3.2	2.9	3.5

W **(cm)**	2.5	2.6	2.6	2.6	2.7	2.8	2.8	2.9	2.9	2.9	2.9	3.0	3.0	3.0
L **(cm)**	3.3	3.5	3.4	3.4	3.2	3.8	3.5	3.8	3.9	3.8	3.7	3.8	4.0	3.7

W **(cm)**	3.1	3.4	3.4	3.5	3.6	3.6	3.6	3.6	3.7	3.9	3.9	4.1	4.1	4.1
L **(cm)**	4.1	4.4	4.3	4.5	4.6	4.8	4.9	4.5	4.9	5.4	5.2	5.3	5.4	5.2

W **(cm)**	4.2	4.2	4.2	4.2	4.3	4.7	4.8	4.8	4.8	4.9	5.0	5.0	5.1	5.2
L **(cm)**	5.4	5.5	5.6	5.2	5.6	6.1	6.6	6.1	6.0	6.1	6.6	6.2	6.6	6.4

W **(cm)**	5.2	5.4	5.6	5.6	5.8	5.8	5.8	5.9	6.0	6.0	6.0	6.2	6.2	6.3
L **(cm)**	6.8	6.3	7.0	7.9	7.4	7.4	6.9	7.9	7.7	7.8	7.3	8.5	7.5	7.6

W **(cm)**	6.5	6.6	6.6	6.7	6.7	6.7	6.7	6.9	6.9	7.0	7.1	7.1	7.1	7.2
L **(cm)**	9.3	8.1	8.0	8.2	9.2	8.0	7.9	8.6	8.1	9.5	9.1	9.1	9.4	8.4

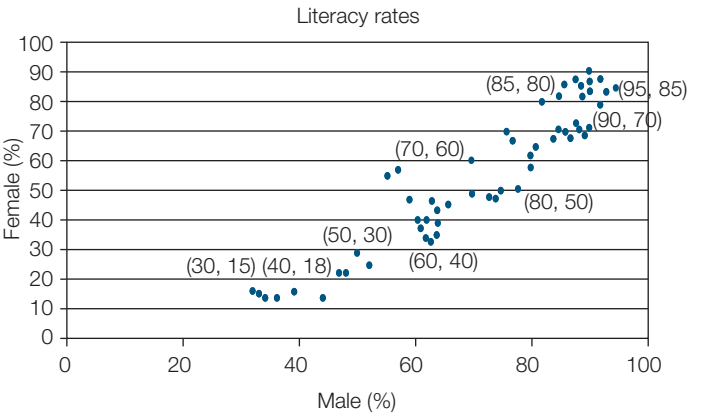
W **(cm)**	7.3	7.3	7.4	7.5
L **(cm)**	9.3	8.9	8.9	8.6

(a) Use a CAS, spreadsheet or any other form of technology to draw a scatter plot.

(b) Comment on any relationship evident between the measurements.

Reasoning

10 A search of the internet was conducted on literacy rates for countries with literacy rates regarded as low or medium. The following graph was produced as part of the report.

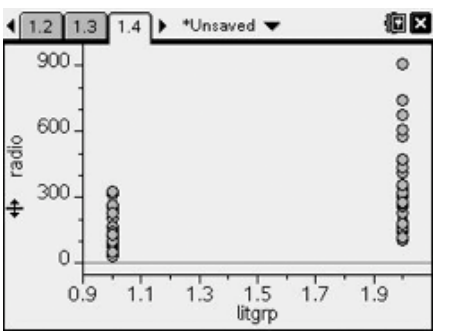


(a) Is the data primary or secondary?

(b) Describe any relationship you can see between male and female literacy rates. As part of this answer state whether you think, in general, male or female literacy rates are higher.

(c) Could your conclusions in **(b)** be applied to countries regarded as having high literacy rates?

11 The internet search of literacy rates also produced data regarding the number of radios per 1000 people in the population. From countries with a literacy rate listed as low (litgrp of 1) or medium (litgrp of 2) the following graph was produced.



(a) What information can you gather from this graph?

(b) The student doing the investigation had meant to construct a box plot. Given the following statistics, construct a parallel box plot for the data.

	Min	Q_L	Median	Q_U	Maximum
Low literacy	34	79	130	226	329
Medium literacy	107	178	306	434	907

(c) Use your parallel box plot to help you make a few statements about the two data sets.

(d) There is one data pair that looks like an outlier. Which pair is this?

Open-ended

12 Suggest a topic for which a census would be appropriate and which you could realistically undertake. Construct the survey instrument that you would use.

13 Suggest a topic for which a sample would be appropriate and which you could realistically undertake. Construct the survey instrument that you would use.

Half-time 7

1 Complete the following tasks for the data shown in the table.

(a) Find the range.

(b) Find the interquartile range.

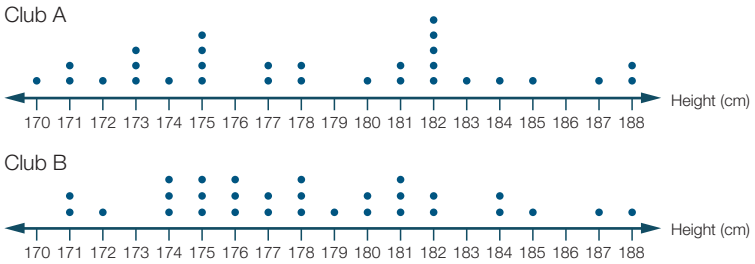
(c) Draw a box plot.

Class interval	Frequency
10–<15	9
15–<20	15
20–<25	12
25–<30	17
30–<35	13
35–<40	11

Ex.7.2

2 The following parallel dot plot shows the heights of players at two hockey clubs who play in the same competition. The heights have been rounded to the nearest centimetre.

Ex.7.3



(a) Which club appears to have the taller players?

(b) Calculate the mean, median, range and IQR for each club.

(c) Draw a parallel box plot for the data.

(d) Reassess your answer for part **(a)**. Justify your assessment using the statistics you have calculated.

(e) Which statistic is the most helpful in making your assessment?

3 Construct a cumulative frequency curve and use it to find an estimate for the median of the following data set.

Class interval	Frequency
0–<20	12
20–<40	19
40–<60	21
60–<80	32
80–<100	19

Ex.7.1

4 The following table gives the head circumferences, in cm, and the lengths, in cm, of 12 babies at birth. Draw a scatter plot of the data. Put head circumference on the horizontal axis. Make a comment about the type of relationship, if any, that is displayed by the data.

Ex.7.4

Head circumference (cm)	38	35	39	37	34	32
Length (cm)	51	50	54	53.5	50.5	47.5

Head circumference (cm)	33	35	35	37	39.5	36
Length (cm)	50	53	49	51	51	50.5

7.5 Time-related data

Need to Know

In bivariate data the variable that causes the other to change is called the **independent variable**, and the one that changes is called the **dependent variable**.

We can use scatter plots to display time-related data sets. In most cases, time is the independent variable and is plotted on the horizontal axis.

Worked Example 11

WE11

The following table shows the temperatures, in °C, for Jerusalem in Israel over a 24-hour period from 8 a.m. one morning to 7 a.m. the next morning.

Time	8 a.m.	9 a.m.	10 a.m.	11 a.m.	noon	1 p.m.	2 p.m.	3 p.m.	4 p.m.	5 p.m.	6 p.m.	7 p.m.
Temperature (°C)	18.4	21	21.5	22.3	23.3	24.7	24.9	25.3	24.2	23.2	22	20.4

Time	8 p.m.	9 p.m.	10 p.m.	11 p.m.	mid-night	1 a.m.	2 a.m.	3 a.m.	4 a.m.	5 a.m.	6 a.m.	7 a.m.
Temperature (°C)	18.9	18.2	18.2	17.9	17.1	16.5	16.36	15.9	15.6	15.6	15.4	16.5

(a) Draw a scatter plot to show these temperatures.

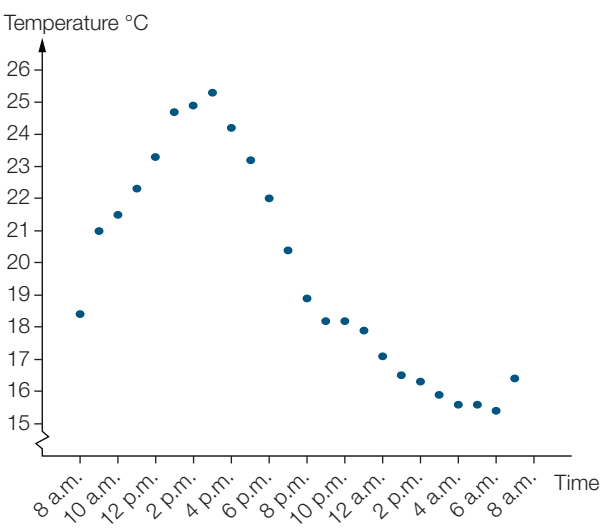
(b) What is the maximum temperature shown on the graph?

(c) Is this necessarily the maximum daily temperature? If not, what might that maximum temperature have been?

(d) Do the temperatures follow a similar pattern to what you would expect for where you live?

Working

(a)



(b) The maximum temperature recorded is 25.3°C.

(c) If this is not the absolute maximum it is likely to have been in the preceding hour, as the temperature falls away more quickly after 3 p.m. than it was rising before.

(d) It doesn't really matter where you live, it is hotter during the day than at night.

7.5 Time-related data

Answers page 520

Equipment required: a CAS or Exel would be useful for Questions **3–5**

Fluency

1 The table below gives the wind speeds, in km/h, for Quebec in Canada over a 24-hour period.

WE11

Time	mid-night	1 a.m.	2 a.m.	3 a.m.	4 a.m.	5 a.m.	6 a.m.	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.
Wind speed (km/h)	9	9	13	7	6	6	4	4	6	4	7	6

Time	noon	1 p.m.	2 p.m.	3 p.m.	4 p.m.	5 p.m.	6 p.m.	7 p.m.	8 p.m.	9 p.m.	10 p.m.	11 p.m.
Wind speed (km/h)	6	7	9	7	11	13	11	13	11	9	9	11

(a) Draw a scatter plot to show the wind speed.

(b) What seems unusual about the 2 a.m. recording?

(c) Describe what is happening to the wind speed between 4 p.m. and 8 p.m.

2 For the following pairs of variables, state which one you think is the independent variable. Give a brief reason for your answer.

(a) Test score and number of hours spent studying.

(b) Height, measured in cm, and age, measured in months, for children less than 3 years of age.

(c) Age at death and average number of cigarettes smoked per day.

(d) Amount of rubbish collected, measured in kg, and the number of people in a household.

3 The following data set indicates the number of bicycles that crossed northbound on the Sydney Harbour bridge for 29 consecutive Fridays.

332 372 367 370 361 398 387 363 447 331 530 454 460 428 283
515 444 498 481 500 396 440 402 383 339 325 402 517 416

Draw a scatter plot to represent the data. Use your CAS, if you have one, or draw the graph by hand. Comment on the graph produced.

Understanding

4 The following table gives the number of 10-year-old Australians in June of each year from 1971 to 2009.

1971	260 439	1981	282 850	1991	252 907	2001	275 465
1972	261 046	1982	280 163	1992	254 099	2002	275 707
1973	258 136	1983	266 496	1993	258 857	2003	275 806
1974	256 359	1984	257 190	1994	258 625	2004	276 100
1975	244 424	1985	248 865	1995	259 391	2005	278 212
1976	242 216	1986	242 895	1996	261 345	2006	273 908
1977	244 927	1987	239 390	1997	259 506	2007	274 506
1978	248 919	1988	241 462	1998	261 417	2008	274 009
1979	261 286	1989	242 343	1999	264 986	2009	276 519
1980	262 714	1990	243 448	2000	271 193		

(a) Use Excel, or some other suitable form of technology, to draw a scatter plot displaying the data.

(b) Comment on the trend seen in the number of 10-year-old Australians over this time period.

5 The following table gives the number of road deaths per 100 000 population for South Australia in the period 1975–2008.

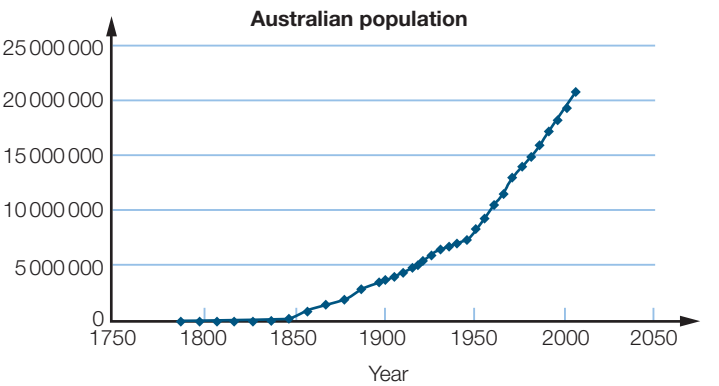
Year	1975	1980	1985	1990	1992	1993	1994	1995	1996	1997	1998	1999
Rate	26.79	20.56	19.54	15.78	11.33	14.92	10.84	12.32	12.28	9.99	11.28	10.08
Year	2000	2001	2002	2003	2004	2005	2006	2007	2008			
Rate	11.03	10.12	10.12	10.25	9.02	9.53	7.46	7.83	6.18			

(a) Use some form of technology to assist you in drawing a scatter plot to represent the data.

(b) Describe any trends you see in the data.

Reasoning

6 The graph on the right shows the estimated Australian population from 1788 until 2006.



(a) Describe the trend you see in the graph.

The figures prior to 1961 do not include the Aboriginal population. Estimates of the size of the Aboriginal population prior to European settlement range from 300 000 to 1 000 000.

(b) How do you think the inclusion of the Aboriginal population prior to 1961 would affect the graph?

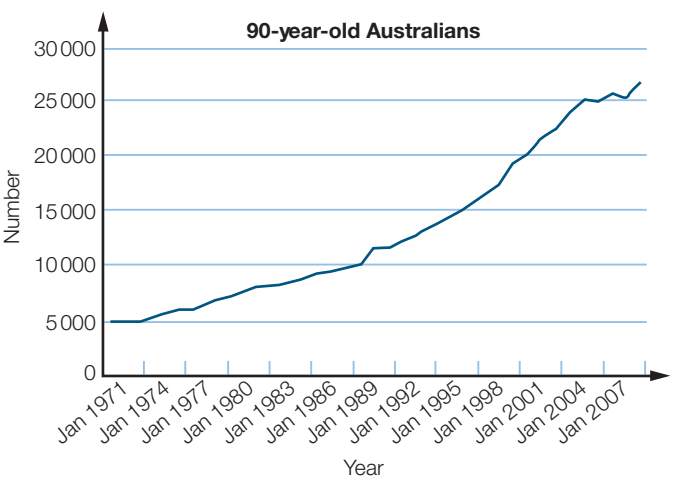
(c) Why does the graph seem to go along the horizontal axis for the first 50 years or so?

(d) How do you think the early figures were collected?

(e) Which figures do you think are more reliable, the early ones or the later ones?

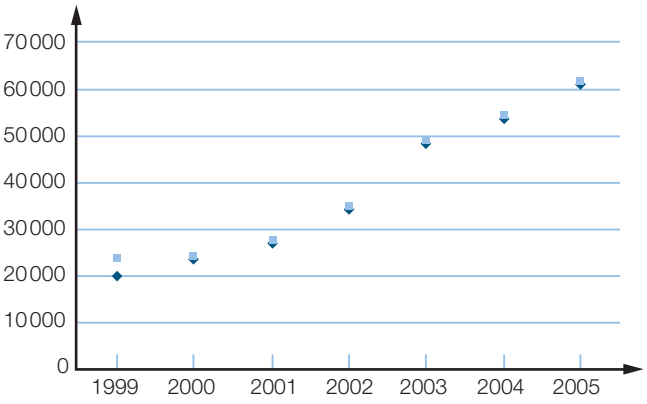
(f) How do we now estimate the Australian population?

7 The graph on the right shows the number of 90-year-old Australians from 1971 through to 2009. Comment on the trend you see in the graph and make some attempt to explain why this is the case. For what other ages would you expect to see this type of trend?



Open-ended

8 Sonika produced the graph below for the information given in the table. Suggest how she could improve her graph.



Mercedes-Benz E320 used car prices (August 2009)

Year	Elegance	Avantgarde
1999	19 700	23 600
2000	23 400	24 100
2001	26 700	27 600
2002	34 000	35 000
2003	48 200	49 000
2004	53 500	54 400
2005	60 800	61 800

9 The table below gives some information about extinct and endangered species in Australia. These figures relate to the period after European settlement of Australia.

	Extinct	Critically endangered	Endangered	Vulnerable	Total
Total plant species and subspecies	49	81	523	665	1318
Mammals	27	4	33	55	119
Birds	23	6	41	61	131
Reptiles	0	2	14	37	53
Frogs	4	2	14	12	32
Fish	1	3	16	25	45
Invertebrates	0	16	12	6	34
Total animal species and subspecies	55	33	130	196	414

Source: www.climatechange.gov.au/publications/biodiversity/~/media/publications/biodiversity/biodiversity-vulnerability-assessment-lowres.ashx

Of the 27 mammal species that are extinct, 10 of these have occurred since 1900. Some other information about Australia's biodiversity is as follows:

- 82% of our mammals, 45% of our land birds, 89% of our reptiles and 93% of our frogs occur nowhere else
- estimates of probable global losses of species are as high as 25% over the next 30 years
- 90% of native vegetation in the eastern temperate zone has been removed or replaced by introduced crops
- 50% of rainforests have been cleared
- the proportion of Australia covered by woodland or forest has been reduced by more than one-third.

Your task is to produce some graphs to indicate how you think Australia's biodiversity has changed since European settlement. You may wish to research some more information.

10 Write a story about Scott's car journey in his 4WD that lasted at least 8 hours and during which time he travelled at least 550 km. Draw a line graph to represent the story you write.

7.6 Statistics in the media

Answers page 521

7.6 Statistics in the media

Fluency

This article appeared in the *Age* on 21 January 2010.

NBA hotshot Bogut closes in on Shark

Having made his presence felt in the National Basketball Association, Andrew Bogut is starting to cast his considerable shadow over the rest of Australia's sporting stars in money matters.

The Milwaukee Bucks centre has shot up from fifth to second in BRW's latest list of the nation's top 50 sports earners, behind occasional golfer Greg Norman.

The 25-year-old's five-year, $US60 million ($A65 million) contract extension has kicked in for this NBA season and Bogut has responded with his most productive season, averaging 15.1 points and 10.4 rebounds a game.

BRW listed Bogut as earning $14 million a year, hot on Norman's ($15 million) tail, even if the two-time British Open champion's wealth has long been coming from off the golf course. Harry Kewell remained our best-paid football player despite his earnings dipping from $10.5 million to $9.5 million at Turkish club Galatasaray.

Formula One ace Mark Webber ($9 million) was next best, one of four involved in motor sports that made the top 10, ahead of motocross star Chad Reed ($8.8 million), NASCAR driver Marcos Ambrose ($5.44 million) and 2007 MotoGP world champion Casey Stoner ($5 million).

Geoff Ogilvy ($6.34 million) is the highest full-time golfer on this list, while Lleyton Hewitt has dropped out of the top 10, his earning power shrinking from $6.6 million to $2.88 million.

Australia's cricketers remain the best-paid locally based sportsmen. Skipper Ricky Ponting ($4.26 million) is the sport's top earner, despite never having really cashed in on the lucrative Indian Premier League.

Fast bowler Brett Lee ($3.52 million) has shown that playing power does not always equate to earning power, having been rated as the nation's second-best-paid cricketer despite missing the Ashes and home summer. Vice-captain Michael Clarke was ranked 21st with $2.54 million and Michael Hussey ($1.30 million) was at 41. Wild child Andrew Symonds made the list at 50 with $1.22 million.

TOP 5 MONEY-EARNERS IN AUSTRALIAN SPORT	
1 Greg Norman (golf)	**$15m**
2 Andrew Bogut (basketball)	**$14m**
3 Harry Kewell (soccer)	**$9.5m**
4 Mark Webber (formula one)	**$9m**
5 Chad Reed (motocross)	**$8.8m**

Source: *The Age*, 21 Jan 2010, Sport 9

1 **(a)** Draw up a table and include all of the named Australian sporting stars and estimate their positions in the Top 50 Earners list. Why is it difficult to be precise about some of the positions?

(b) What percentage drop in earnings did Harry Kewell suffer when moving clubs?

(c) What percentage slide did Lleyton Hewitt suffer in his earnings?

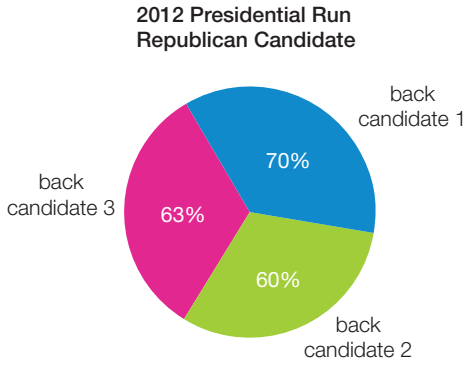
(d) How do you think the figure of $14m for Andrew Bogut was arrived at?

The *Herald Sun*, reporting on the same issue, noted that "Samantha Stosur was the second highest earning tennis player, with $1.53 million".

(e) Where would this place Samantha Stosur in your Top 50?

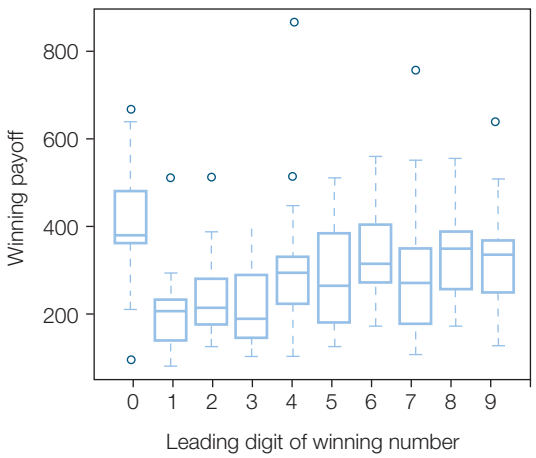
2 (a) In the early stages of the nomination process for the Republican Party leading up to the 2012 election for the US President, an American news network produced a graph similar to the one shown here.

What problems can you see with this graph? Why do you think the graph has been drawn this way? Could you draw an accurate graph from the information provided?



(b) Some graphs provide a simple and accurate visual representation of information. This box plot shows the winnings ($) in the New Jersey Pick-It Lottery. In this lottery you pick any three-digit number you like and the total prize pool is divided by the number of winning entries.

What do you think this graph is telling you and why do you think that is the case?



Understanding

3 The following article by John Rolfe appeared in the *Daily Telegraph* on 19 January 2010.

Why car insurers are ripping us off?

INSURERS have slugged Sydney drivers an extra $100 to make up for poor returns on investments—even though crash and theft rates have fallen dramatically.

Premiums have increased at up to 10 times the rate of inflation, figures obtained by the *Daily Telegraph* show. The price of the average greenslip rose 14% last year and comprehensive cover increased 5%. Yet Sydney's overall price inflation was just 1.3% during that time.

When premium increases are compared to crash and theft rates the differences are even more stark. The most recent accident statistics show a 6% annual fall, while car robberies declined 9%.

The *Daily Telegraph's* investigation exposes the cost of cover is no longer driven by risk, but by financial markets. Insurers make money by investing customers' premiums before that money is paid out in claims.

KPMG insurance group head Brian Greig said: "The returns they were able to generate pre-global financial crisis were substantially higher than they are now. When that (investment) lever is a little flat they look at other levers to generate acceptable returns, because sharemarket analysts are always on to them to achieve that."

The main "lever" is premium increases.

"Insurers are using premiums to compensate for lower investment returns," said Damian Smith, CEO of insurance comparison service RateCity.

Until the GFC, "premium growth wasn't that important", Mr Smith said. "But now insurers are using premiums to try to make up for the investments."

Data from the Motor Accidents Authority of NSW shows the average greenslip purchased during the three months to September 2009 cost $390, up $49 on a year earlier. Before the GFC took effect in 2007, compulsory third party insurance had fallen to $319, lower than 2004 levels.

"The increase seen over the last two reporting periods is directly related to the impact of the GFC," MAA spokesman Christian Fanker said.

Figures from Insurance Statistics Australia—the source the insurance industry uses—reveal comprehensive cover increased 4.9% last financial year. For a Sydney household driving a late-model family car, this would add more than $50 to premiums.

This means the total increase imposed on most drivers exceeds $100.

The number of crashes on NSW roads fell to 9692 per million cars in 2008—down 9.2% from 10,531 in 2007, RTA statistics show. There were 42 833 crashes in 2008. Official crash data for 2009 has not been made public.

(a) According to the article, how much did the average 'greenslip' cost in the 3-month period to September 2009?

(b) How much did the same cover cost a year earlier?

(c) What percentage change is this? Does this agree with what the article claims is the increase?

(d) If the cost had increased at the stated inflation rate, what should the premium have been?

(e) The article claims comprehensive premiums have increased by more than $50. Assuming it is exactly $50, what does this make the old and new costs for comprehensive cover?

(f) What is the average annual increase in greenslip cost from 2007 to 2009?

(g) Why does the article mention the number of crashes?

4 The following article by Beth Rivers appeared in *PerthNow* on 10 January 2010.

Perth's most prized suburbs

CANNING Vale was in demand in 2009, with more houses sold in that suburb than any other, but Guildford delivered the biggest median house price rise.

Landgate Valuation Services' figures show Canning Vale was way out in front with 506 houses sold, ahead of Thornlie with 340 and Ellenbrook with 330.

Real Estate Institute of WA president Alan Bourke said the result was to be expected because of the first homebuyer's boost stimulating outer-metropolitan suburbs. The best performing suburbs in terms of median price rises last year were Guildford, Mt Richon and Victoria Park.

From 2008 to 2009, Guildford saw a rise in its median house price of 17.26 per cent, from $565 000 to $662 500. The median in Mt Richon increased 16.86 per cent from $422 500 to $493 750, and Victoria Park's median increased 15.65 per cent, from $575 000 to $665 000.

The suburbs whose median prices were hit the hardest were North Beach, with its median price falling 35.38 per cent ($1.22 million to $790 000), Ascot, which fell by 32.14 per cent ($766 250 to $520 000), and Mt Lawley's median price fell by 28.28 per cent ($1.22 million to $875 000). Mr Bourke said North Beach was just one example of the many top-end suburbs "hurt" during 2009.

"There was more activity in the bottom end of the market so the median moved down (in the top end)", he said. "There was also a lot of land released across the road from the Ascot Racecourse, which would have pushed the median price down there."

Peppermint Grove retained its crown as being Perth's most expensive suburb, with a median house price of $4.2 million, while Medina, near Kwinana, was the least expensive suburb with a median of $230 000. Most recent REIWA data showed more than 31,500 properties were sold last year.

About 80% were houses and 20 per cent multi-residential, including apartments and units.

Best performing suburbs			
Median sale price		2008	2009
1	Guildford	$565 000	$662 500
2	Mount Richon	$422 500	$493 750
3	Victoria Park	$575 000	$665 000
4	Darlington	$515 000	$590 000
5	Orelia	$297 500	$339 000
6	Maylands	$670 000	$754 000
7	South Fremantle	$825 000	$925 000
8	Stoneville	$457 500	$511 250
9	Fremantle	$795 000	$880 000
10	Joondanna	$633 000	$697 500

Worst performing suburbs			
Median sale price		2008	2009
1	North Beach	$1 222 500	$790 000
2	Ascot	$766 250	$520 000
3	Mount Lawley	$1 220 000	$875 000
4	Mosman Park	$1 522 500	$1 150 000
5	Forrestdale	$520 000	$395 000
6	Applecross	$2 075 000	$1 602 500
7	Perth	$875 000	$690 000
8	Marmion	$885 000	$698 750
9	Kallaroo	$660 000	$545 000
10	Swanbourne	$1 600 000	$1 322 500

Suburbs by volume sold		2008
1	Canning Vale	506
2	Thornlie	340
3	Ellenbrook	330
4	Clarkson	277
5	Gosnells	275
6	Ballajura	256
7	Baldivis	238
8	Port Kennedy	235
9	Butler	224
10	Tapping	218

(a) Calculate the percentage increase for each of the 10 best performing suburbs from 2008 to 2009.

(b) Calculate the percentage decrease for each of the 10 worst performing suburbs from 2008 to 2009.

(c) Make some comment about the Perth housing market, based on these figures.

(d) What further information would make the Suburbs by volume sold table more useful to us?

(e) According to Mr Bourke "There was more activity in the bottom end of the market so the median moved down (in the top end)". What do you think this means? Why does one affect the other?

(f) Why do these figures refer to the median house price? What would be the problems with using either the mean or the mode to represent house prices?

5 The following article appeared in the *Age* in May 2011.

Peer guide for parents

Michelle Griffin
17 May 2011

WHEN Australian teenagers tell their parents that everyone else is allowed to go to the shopping centre alone or stay up late, their parents will finally have a guide to what other families are really saying.

The average age at which teenagers are allowed to make their own decisions is tabled in a survey released today by the federally funded parenting advice website Raising Children. Most children can choose their own clothes by the time they are $12\frac{1}{2}$, but the average age for lifting a teenager's curfew is $17\frac{1}{2}$—six months off legal adulthood.

While it's not possible to tell parents exactly what age to allow teenagers to make decisions, the website's content director, parenting expert Warren Cann, hopes the survey gives parents a point of comparison. "If you find yourself well below or above the norms, you might want to think why," he says.

The survey asked parents what they believed was the right age for children to be allowed to make decisions. Mr Cann believes parents may yield to child pressure earlier. "Kids tend to think they're ready two years before their parents want them to be," he says. The survey suggests the average age for getting a mobile phone is 14, but telecommunications research says teenagers typically get their first mobile at 13. The survey of 360 parents found a strong resistance to allowing younger teenagers open access to websites and social media. The average age of allowing them open access is $15\frac{1}{2}$. On average, parents don't want their children on Facebook or other social media before $14\frac{1}{2}$, even though Facebook allows them to register from 13, and many children as young as eight have active pages.

Primary school teacher and mother of three Simone Traynor allowed her oldest son, Jacob, now 17, to use Facebook from 14. He is allowed to use it even when doing his homework, as long as he maintains good marks in VCE. Ms Traynor's 15-year-old son is now allowed to log on but she will not allow her 12-year-old daughter Sonja to register for an account. Ms Traynor said: "Brandon is not that interested. Sonja's got plenty of real-life social activity, and it seems more vulnerable [for her]: girls can be nasty with these things."

Mother of two Kristin has allowed her daughter, now 16, on Facebook since she was 12, but polices her children's internet time through a net nanny, by reading their internet histories and by setting time limits. "She has to spend an hour off all screens before bed—no phone, iPod, iPad, laptop, nothing. She can read a book or come and have a conversation."

The survey was commissioned to mark the launch of the Raising Children's teenage section, which was built to respond to a growing demand for advice on coping with adolescents.

(a) How many people took part in this survey?

(b) Do you think this is enough people to justify the media interest in the results?

(c) To participate in this survey you needed to visit the website and access the survey online. What type of survey is this?

(d) One of the questions in the survey was: "At what age do you think adolescent boys/girls should be allowed to make their own decisions about the following issues?" A list of issues then followed. Do you think the results for the survey might have been different if the question was: "At what age did you allow your adolescent boys/girls to make their own decisions about the following issues?"

(e) One of the findings of the survey was reported as follows.

"The average age at which teenagers were permitted to wield a mobile phone was 14. This is 12 months older than 13, at which the telcos say is the average age children start using a mobile phone."

Why do you think there is such a difference in these reported ages?

(f) How much confidence would you have, in general, with the findings and conclusions of this survey?

Reasoning

6 The following article by Anna Caldwell appeared in the *Courier Mail* on 2 January 2010.

Coast residents betting on Mermaid strip for lotto win

HOPEFUL Queenslanders hankering for a new-year windfall—or perhaps an upgrade to the homes and lifestyles of the rich and famous—have swarmed newsagencies ahead of tonight's $30 million Gold Lotto megadraw.

And at no place in the state is there a firmer belief that the next big win is just a ticket away than at Golden Sands News at Mermaid Beach on the Gold Coast.

The multi-purpose shop is wedged between two very lucky newsagents. About 1 km in one direction is Nobby Beach News, where a punter took out the $20 million *Oz Lotto* in August and about 1 km in the other direction is Lucky Mermaid News where a couple won $53 million on *Oz Lotto* in June.

Figures released by Golden Casket have crowned the glitter strip Queensland's number one lotto hot spot for first division prize wins for the second year running.

Sixteen lucky first division winners on the Gold Coast shared in a mammoth $106.37 million in 2009.

Darrell Hiscock, who owns Golden Sands News, said that everyone had been tipping his shop to offer up the next golden ticket.

Mathew Graham, from Nobby News, said people stopped by the suburb just because they believed that visiting the Mermaid strip enhanced their chances of winning millions.

(a) Why are people tipping Golden Sands News as the seller of the winning *Gold Lotto* ticket?

(b) In your opinion, is there a statistical basis for their confidence?

(c) What other information would you like to have access to before you made a statement like this about where a winning ticket is likely to be sold?

Other people look at lists of previously drawn numbers and use these as the basis for the numbers they select. Some people look at the numbers most frequently drawn, whereas others look at those that have not been drawn for the longest time.

(d) Try to explain the reasoning both of these groups use to justify their thinking. Do you support either argument yourself?

7 The following fictional article is of a type often seen around election time. Read it and answer the questions that follow.

Poll – voters back Smith

Mr John Smith's return to the Federal Liberal leadership appears to have buoyed the Coalition's [Liberal + National parties] electoral chances, with one in four voters saying they are at least 'a bit more likely' to vote Liberal at the next federal election. Mr Smith was clearly preferred by female voters (47% support) over the leader of the Australian Labor Party, Mr Roger Casey (41%).

Men preferred Mr Casey (54%) to Mr Smith (45%). Support for the ALP among men was 47%, compared to the Coalition's (45%), while from women the Coalition attracted 47% support compared to the ALP's 44%. The survey also asked voters to identify those characteristics they believed Mr Casey or Mr Smith possessed. The results of the survey are summarised below.

SUNDAY BUGLE POLL

Voting intentions	%	Preferred PM	%
ALP	46	Casey	48
Coalition	46	Smith	46
Other/Don't know	8	Neither/Don't know	6

SUNDAY BUGLE POLL: RATING THE LEADERS

	Casey (%)	Smith (%)		Casey (%)	Smith (%)
Decisive	68	53	Sincere	22	55
Forward thinking	68	48	Too arrogant	75	15
Good for Australia	43	55	Honest	20	62
Weak	10	30	Capable	72	72
A good leader	66	43	None of the above	1	2

Source: Corio Market Research 1030 voters from Vic., NSW. Poll done by telephone.

(a) Do the survey results agree with the heading 'Voters back Smith'? Discuss.

(b) Comment on the sample size—is it appropriate for a conclusion?

(c) The survey was done by telephone—is this the best way to do such a survey? Discuss.

(d) Why have the results for choice of preferred PM been separated into male and female opinions?

(e) Why don't the totals of male and female voters' opinions add up to 100%?

(f) Discuss the categories that describe Roger Casey and John Smith, such as decisive and forward thinking. Comment on the appropriateness of these categories. Can you add some of your own?

(g) Is this survey just about the preferred Prime Minister or is it about something else?

8 The Australian Bureau of Statistics releases more than just statistics. They also conduct some analysis of the data and produce reports such as the one below.

DEMOGRAPHIC, SOCIAL AND ECONOMIC CHARACTERISTICS: LIFE EXPECTANCY

This article is part of a comprehensive series released as The Health and Welfare of Australia's Aboriginal and Torres Strait Islander Peoples.

KEY MESSAGES

At the national level for 2005-2007, the gap between Aboriginal and Torres Strait Islander and non-Indigenous life expectancy was 11.5 years for males and 9.7 years for females.

Life expectancy at birth for Aboriginal and Torres Strait Islander males is estimated to be 67.2 years, and 72.9 years for females.

This topic provides an overview of estimates of Aboriginal and Torres Strait Islander life expectancy. Data presented are from ABS experimental life tables for Aboriginal and Torres Strait Islander Australians and provide context for the detailed health and welfare information presented in other topics in this release.

LIFE EXPECTANCY

Life expectancy is a summary measure of mortality. It represents the estimated number of years a newborn baby would live if current mortality patterns remained constant throughout the baby's lifetime (see Box 1.7).

At the national level for 2005-2007:

- Life expectancy at birth for Aboriginal and Torres Strait Islander males is estimated to be 67.2 years, 11.5 years less than life expectancy at birth for non-Indigenous males (78.7 years).
- Life expectancy at birth for Aboriginal and Torres Strait Islander females is estimated to be 72.9 years, 9.7 years less than life expectancy at birth for non-Indigenous females (82.6 years).
- Life expectancy at birth differs across the states and territories, with the highest life expectancy estimates in New South Wales (69.9 years for males and 75.0 years for females) and lowest in the Northern Territory (61.5 years for males and 69.2 years for females). As shown in table 1.6, differences in life expectancy at birth estimates between non-Indigenous and Aboriginal and Torres Strait Islander Australians are greatest in the Northern Territory (14.2 years for males and 11.9 years for females) and Western Australia (14.0 years for males and 12.5 years for females), and lowest in New South Wales (8.8 years for males and 7.5 years for females).

1.6 LIFE EXPECTANCY AT BIRTH (a), by Indigenous status 2005–2007

	Life Expectancy at Birth			
	Indigenous years	Non-Indigenous years	Total (b) years	Difference Between Non-Indigenous and Indigenous Life Expectancy at Birth (c) years
		Males		
NSW	69.9	78.7	78.5	8.8
Qld	68.3	78.6	78.4	10.4
WA	65.0	79.0	78.7	14.0
NT	61.5	75.7	72.0	14.2
Aust (d)	**67.2**	**78.7**	**78.5**	**11.5**
		Females		
NSW	75	82.5	82.4	7.5
Qld	73.6	82.5	82.3	8.9
WA	70.4	82.9	82.5	12.5
NT	69.2	81.2	77.6	11.9
Aust (d)	**72.9**	**82.6**	**82.4**	**9.7**

(a) Due to significant changes in methodology, estimates of life expectancy at birth for 2005-2007 are not comparable to previously published estimates.

(b) Estimates of life expectancy at birth for the total population presented in this publication differ from estimates in ABS 2007, Deaths, Australia, 2006, cat. no. 3302.0.

(c) Differences are based on unrounded estimates.

(d) Includes all states and territories.

Source: ABS 2009, Experimental Life Tables for Aboriginal and Torres Strait Islander Australians, 2005-2007, cat. no. 3302.0.55.003, ABS, Canberra

(a) Why do you think only three States and the Northern Territory are recorded separately in the article above?

(b) According to the information in the article, what was the life expectancy for non-Indigenous males and females at this time?

(c) Why do you think there are such differences in life expectancy for Aboriginal and Torres Strait Islanders compared to non-Indigenous Australians?

Another Government body, the Australian Institute of Health and Welfare, contributes the following additional information.

Australia appears to compare poorly with other countries for Indigenous life expectancy at birth. For example, Registered Indians (Aboriginal people) in Canada have an estimated life expectancy of about 6-8 years below that of the non-Aboriginal population (Tjemka, M. 2002. Statistics Canada, cat. no. 82-003). In New Zealand, Maori life expectancy is 8-9 years below that of the non-Maori population (www.stats.govt.nz). Although comparing these very different populations is perhaps over-simplifying the situation, it does illustrate that the Australian Indigenous life expectancy gap between non-Indigenous persons is approximately twice as large as in comparable developed nations.

Unfortunately, it is difficult to accurately assess the life expectancy gap between Indigenous and non-Indigenous persons. This is due to the fact that the population counts of Indigenous persons in Australia both over time and between jurisdictions is inaccurate, and that Indigenous persons are often undercounted when recording deaths.

(d) This extract mentions a couple of problems that arise when dealing with the issue of the life expectancy of the Australian Indigenous population. What are these problems?

(e) Why would you want to attempt to make comparisons with Canada, New Zealand and the United States?

The Australian Bureau of Statistics can provide further data to assist in our analysis.

In 2009, the median age at death of Aboriginal and Torres Strait Islander Australian males varied across the states and territories, from 48 years in South Australia to 57 years in New South Wales, compared with 67 years in the Northern Territory to 79 years in South Australia for non-Indigenous males. Similarly, the median age at death of Aboriginal and Torres Strait Islander Australian females was lower than for non-Indigenous females, 53 years in South Australia to 66 years in New South Wales compared with 72 years in Northern Territory to 85 years in South Australia.

3.6 MEDIAN AGE AT DEATH, Indigenous status (a) 2004–2009

	NSW	Qld	SA	WA	NT	Total (b)
	Males					
Indigenous						
2004	55.8	53.7	49.5	50.0	43.8	51.2
2005	54.3	51.1	42.4	52.8	45.8	50.4
2006	59.3	55.6	50.4	47.9	45.4	52.4
2007	58.1	54.7	50.5	53.3	45.9	53.1
2008	59.9	53.2	49.0	51.2	52.1	53.8
2009	57.2	53.2	48.0	50.6	48.3	52.5
Non-Indigenous						
2004	77.0	76.2	77.6	76.3	63.0	76.8
2005	77.2	76.4	77.9	76.6	63.7	76.9
2006	77.8	76.7	78.3	76.9	64.7	77.4
2007	78.1	77.1	78.7	76.9	64.6	77.7
2008	78.5	77.3	79.2	77.1	66.3	78.0
2009	78.4	77.2	79.3	77.3	66.6	78.0

	NSW	Qld	SA	WA	NT	Total (b)
	Females					
Indigenous						
2004	62.7	57.9	53.5	63.6	54.0	60.1
2005	65.8	59.5	47.5	57.8	50.4	57.9
2006	64.8	57.0	59.3	57.0	55.3	59.0
2007	63.0	59.5	58.3	59.3	55.7	59.5
2008	63.8	62.3	53.5	64.0	56.0	60.5
2009	65.9	62.6	53.0	58.7	55.4	61.3
Non-Indigenous						
2004	82.8	82.5	83.3	82.3	71.3	82.7
2005	83.1	82.6	83.7	83.2	70.5	83.1
2006	83.5	83.1	84.1	83.1	75.0	83.4
2007	83.7	83.3	84.3	83.4	69.3	83.6
2008	84.2	83.7	84.6	84.1	75.7	84.1
2009	84.1	83.4	84.6	83.6	71.8	83.9

(a) Care should be exercised when comparing median age at death of Indigenous and non-Indigenous Australians. See Commentary.

(b) Data are for NSW, Qld, WA, SA and NT combined, based on state or territory of usual residence. Victoria, Tasmania and the Australian Capital Territory are excluded due to small numbers of registered Indigenous deaths.

(f) Write a couple of sentences describing the conclusion you would draw from this additional information.

Open-ended

9 Write your own media release that includes statistics in the body of the article as well as in a table.

10 The following advertisement appeared in the Victorian press.



The Victorian Government developed this advertisement in 2010 with funds from the Community Support Fund.

Write a paragraph or two about this advertisement. You might like to include comment on the following questions or take a different angle altogether.

What do you think about this advertisement? What do you think about the statistics that are quoted in the advertisement? How would they have been calculated? Do you think this would be effective in reducing problem gambling?

Standard deviation 10A 7.7

Need to Know

Variance $= \dfrac{(\text{sum of deviations from mean})^2}{\text{number of values}}$

Standard deviation $= \sqrt{\text{variance}}$

Worked Example 12 WE12

Find the standard deviation, rounded to two decimal places, for the following data:

2, 3, 3, 3, 5, 7, 8, 11, 12

Working

Mean: $\dfrac{2+3+3+3+5+7+8+11+12}{9} = 6$

Deviations: -4, -3, -3, -3, -1, 1, 2, 5, 6

Square of deviations: 16, 9, 9, 9, 1, 1, 4, 25, 36

Variance $= \dfrac{110}{9}$

Standard deviation $= \sqrt{\dfrac{110}{9}}$

$= 3.50$ (to 2 d.p.)

Worked Example 13 WE13

The table below shows the test results for 15 students in English and mathematics. Both tests were scored out of 50. Find the mean and standard deviation for each data set and use these values to compare them. Round the values to two decimal places, where necessary.

English	42	44	31	20	25	34	45	50	44	12	13	23	44	47	42
Maths	38	34	42	39	40	44	43	27	34	27	40	32	31	22	23

Working

English: mean = 34.4 std dev = 12.39

Maths: mean = 34.4 std dev = 6.98

The two tests had the same mean score but the standard deviation was much less for the mathematics test. This means the scores for the mathematics test were more consistent. However, we can see that in the English test both the best and worst results, overall, were recorded.

Answers page 523

7.7 Standard deviation

10A

Equipment required: a CAS or access to a spreadsheet

Fluency

WE12

1 Find the standard deviation, rounded to two decimal places, for the following data:

11 12 15 15 17 19 21 24 26 29

WE13

2 The table below shows the test results for 15 students in English and mathematics. Both tests were scored out of 50. Find the mean and standard deviation for each data set and use these values to compare them. Express the values rounded to two decimal places, where necessary.

English	40	40	34	23	24	39	46	47	41	19	23	23	34	37	45
Maths	33	44	32	49	36	38	41	29	37	29	41	35	30	22	24

3 Find the mean and standard deviation, rounded to two decimal places where required, for the grouped data shown in the table on the right. The data represents the distances, in km, travelled to school by a group of students.

Distance (km)	Frequency
0–<2	22
2–<4	28
4–<6	15
6–<8	7
8–<10	4
10–<12	2

4 The following table shows the heights, in m, of the ten highest mountains in each of Australia, New Zealand, Brazil and Sri Lanka.

Australia	2220	2185	2178	2166	2132	2048	2047	2045	2050	1979
New Zealand	3754	3500	3440	3279	3201	3183	3176	3163	3160	3157
Brazil	2973	2890	2882	2861	2849	2798	2791	2770	2734	2033
Sri Lanka	2524	2243	2240	2100	2100	2076	2036	2016	2010	1896

(a) Calculate the range for each country.

(b) Calculate the standard deviation, rounded to two decimal places, for each country.

(c) Write a sentence or two about what you have found.

Understanding

5 (a) Find the standard deviation, rounded to two decimal places, for each of the following data sets:

(i) 13 14 14 15 15 15 16 16 19 20 22 25 26 28

(ii) 13 14 14 15 15 15 16 16 19 20 22 25 26 48

(iii) 3 14 14 15 15 15 16 16 19 20 22 25 26 28

(iv) 3 14 14 15 15 15 16 16 19 20 22 25 26 48

(b) Write a sentence or two about what this tells you.

6 You have recorded a data set as: 23 45 46 47 55 57 58 59 62 65 and calculated the mean and standard deviation. When checking your work you discover that the '23' should have been '43'. Changing this will:

A increase both the mean and standard deviation

B increase the mean and decrease the standard deviation

C decrease the mean and increase the standard deviation

D decrease both the mean and the standard deviation.

Reasoning

7 The screenshot shows some information taken from an MP3 playlist. You will need to be careful how you deal with this data. Give your answers correct to the nearest second.

(a) Find the mean and standard deviation for songs listed as *Rock* (*Rock*, *Classic Rock*, *RockNRoll*).

(b) Find the mean and standard deviation for songs listed as *Pop*.

(c) Find the mean and standard deviation for songs listed as neither *Rock* nor *Pop*.

(d) Draw a parallel box plot showing the three categories *Rock*, *Pop* and *Other*. Use decimal time values here.

(e) Write a couple of sentences comparing the three categories. Is there any evidence to support the view that *Rock* songs are of a different length to *Pop* songs, for instance?

Genre	Time	Genre	Time
Rock	2:07	Pop	2:25
Blues	3:25	Soundtrack	2:12
RnB	2:26	Rock	4:15
Blues	5:27	R&B	3:59
Rock	3:35	Blues	2:33
Country & F...	2:16	Blues	15:30
Rock	5:28	Rock	2:47
Rock	4:02	Rock	5:30
Blues	3:31	Rock	3:32
Blues	2:51	Rock	12:28
Rock	3:26	R&B	4:36
Rock	2:34	Rock	3:30
Pop	3:46	Pop	2:15
Classic Rock	4:01	Blues	3:17
Grunge	3:50	Country & F...	2:48
Pop	3:13	RockNRoll	3:05
Pop	2:25	Pop	3:00
Pop	2:30	Rock	3:51
Blues	2:09	Rock	3:43
Rock	3:31	R&B	3:56
Rock	5:51	Soundtrack	5:56
RockNRoll	1:59	Rock	2:52
Rock	3:32	R&B	4:10
Pop	3:49	Pop	3:52
Blues	4:20	Pop	2:40
Blues	2:46	Rock	3:34
Rock	3:20	Blues	6:00
Rock	3:32	R&B	4:47
Rock	3:32	Rock	5:55
Blues	4:07	Classical	3:57
Rock	8:33	Rock	3:42
Rock	8:53	Pop	4:14

Hint

Change the time in seconds to decimal parts of a minute.

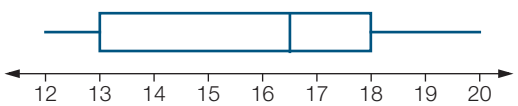
Open-ended

8 **(a)** Construct a data set with at least 20 values that has a mean of between 20 and 22, a range of at least 15 and a standard deviation of between 2 and 4.

(b) Write a couple of sentences about how you went about this task.

(c) What effect did the condition about the range have on this task?

9 Construct a data set, of at least 20 values, that would fit the following box plot and then find the mean and standard deviation of your data set.



Problem solving

Keeping score

From her attempts to shoot 40 points in the season's first basketball game, Angela scored only 10 points. This gave her a shooting average of 25%. In the second game she had the chance to score 30 points but only scored enough to raise her shooting average over the two games to 50%.

(a) How many points did she score in the second game?

(b) What was her shooting average for the second game?

(c) What is the greatest shooting average Angela could have had after the second game, assuming her maximum in the second game was still 30 points?

Strategy options

- Guess and check.
- Work backwards.

7.8 Lines of best fit

10A

Need to Know

If bivariate data in a scatter plot has a linear relationship, we can draw the **line of best fit** and find its equation. We can draw the line of best fit approximately by eye. A more accurate method is to use Exel to find the equation of the **regression line**.

- Enter the data pairs using two columns.
- Highlight the set of data pairs, click on 'Insert' then 'Scatter', and select the first option.
- Right-click on any of the plotted points in the scatter plot, and click on 'Add Trendline'.
- Make sure 'Linear' is selected and the 'Display Equation on chart' box is ticked. Click on 'Close'.

When using a line of best fit to make predictions, if we use a value from within the known range we are using a technique called **interpolation**. If we use a value from outside this range, say the science score was 6, we call this **extrapolation**. In general, we have more confidence in values obtained using interpolation than in those obtained using extrapolation. This is because we cannot be certain that the same trend will be evident outside the known range.

7.8 Lines of best fit

10A

Answers page 523

Equipment required: a CAS or access to a spreadsheet

Fluency

1 Find the equation of the line of best fit, using Excel or some other form of technology, for each of the following sets of data. Round your answers to three decimal places, where necessary. In each case, the required rule is of the form $y =$.

(a)

x	0	1	2	3	4	5	6
y	4	16	29	42	54	65	76

(b)

x	0	2	5	7	10	12	14
y	-7	8	60	92	150	170	200

(c)

x	1	3	7	11	13	16	20
y	45	28	18	0	-5	-15	-45

(d)

x	3	6	8	10	13	15	18
y	26	12	1	-12	-24	-30	-48

2 It is believed that the height of a male at 30 months is about half his full adult height. (This is called a 'rule of thumb'.) The table below gives the height (cm) for 15 boys measured at 30 months of age and again at 19 years of age (taken to be their full adult height).

30-month height (cm)	89.0	89.9	90.3	90.8	90.9	91.0	91.1	91.2
19-year height (cm)	178.0	177.1	179.6	181.8	184.0	180.5	182.0	183.1

30-month height (cm)	91.2	91.9	92.9	93.3	94.7	95.4	96.1
19-year height (cm)	180.1	185.1	182.0	186.3	187.4	187.9	189.4

(a) Draw a scatter plot of the data, putting the 30-month height on the horizontal axis.

(b) Draw, by eye, a line of best fit over your scatter plot.

(c) Do you think that the 30-month height is a good predictor of full adult height?

(d) The rule of thumb says that a 94 cm 30-month-old boy will grow to a full height of 188 cm. What height does your line of best fit predict? Give your answer correct to the nearest cm.

3 (a) Using the line of best fit equation $d = 23.2t - 4.6$, the interpolated value for d when $t = 40$ is closest to:

A 59 **B** 88 **C** 923 **D** 928

(b) (i) For the table of values, use technology to find the equation of the line of best fit. Give your answer in the form $q =$.

p	0	3	5	12	15
q	2	7	9	15	21

(ii) Assuming the relationship continues to be linear, the extrapolated value for q when $p = 25$ is closest to:

A 32 **B** 30 **C** 28 **D** 25

Understanding

4 The following table gives the national running records for various distances for men for 10 different countries.

100 m (s)	10.39	10.31	10.17	12.18	10.35	10.11	10.98	10.60	10.61	10.46
200 m (s)	20.81	20.06	20.22	23.20	20.65	20.38	21.82	21.42	20.96	20.66
5000 m (min)	14.04	13.28	13.55	16.70	13.42	13.34	14.16	13.77	13.32	13.10

(a) Draw a scatter plot with the 100 m on the horizontal axis and the 200 m on the vertical axis.

(b) Does there appear to be a connection between the times for the 100 m and the 200 m?

(c) Draw a scatter plot with the 100 m on the horizontal axis and the 5000 m on the vertical axis.

(d) Does there appear to be a connection between the times for the 100 m and the 5000 m?

(e) Can you explain what you have found?

5 An engineer conducted an experiment modelling the force needed to set an object in motion. Her experiment consisted of different-sized blocks of wood of mass (m kg) being set in motion on a horizontal surface by exerting a force (F N). The size of the force needed to move each block of wood was recorded and the following results were obtained.

Mass *m* (kg)	1	2	3	5	7.5	10	12.5	15	20
Force *F* (N)	5	9	15	24	38	50	60	70	100

(a) Use technology to find the equation of the linear line of best fit, if this is appropriate for this data. Round your answer to two decimal places.

(b) The ratio $\frac{F}{m}$ is proportional to the *coefficient of friction* between the blocks of wood and the horizontal surface. What is the value of this ratio?

(c) Find the force, rounded to two decimal places, needed to move a block of wood of mass:

(i) 4 kg **(ii)** 11 kg **(iii)** 18.5 kg.

(d) Assuming the relationship between the force and the mass continues to hold for heavier blocks of wood, find the force needed to move a block of wood of mass:

(i) 50 kg **(ii)** 200 kg **(iii)** 1 tonne.

6 A chemist conducted an experiment in which the solubility (S units) of sodium chloride in water was tested for different temperatures (T°C). The results of the experiment are recorded in the following table.

Temperature *T* (°C)	0	10	20	30	40	50	60	70	80	90	100
Solubility *S* (units)	35.0	35.4	36.1	36.5	37.1	37.7	38.2	38.6	38.9	39.4	40.0

(a) Use technology to find the equation of the line of best fit when temperature is on the horizontal axis. Round your answer to two decimal places.

(b) Find the solubility of sodium chloride in water at:

(i) 25°C **(ii)** 78°C **(iii)** 120°C

(c) Use technology to find the equation of the line of best fit when solubility is on the horizontal axis.

(d) Find the temperature at which sodium chloride has the following solubilities in water:

(i) 36.0 units **(ii)** 38.5 units **(iii)** 45.0 units

(e) What is the potential problem with the answers to **(b) (iii)** and **(d) (iii)**?

Reasoning

7 A complex spring system is tested under compression and the results obtained are shown in the table below.

Compression Δx (mm)	0	20	40	60	80	100	120	140
Applied force F (N)	0	1.8	3.9	6.1	8.2	9.9	11.9	14.1

Compression Δx (mm)	160	180	200	220	240	260	280	300
Applied force F (N)	16.1	17.9	20.0	26.7	32.9	38.4	44.6	50.2

(a) Use technology to draw a scatter plot of the data, with compression on the horizontal axis. You should be able to see that the data splits into two groups. At which compression value does this occur?

(b) Find an equation for each of the two groups of data. Round your answers to three decimal places.

(c) Find the applied force required, rounded to one decimal place, to produce a compression of:

(i) 75 mm **(ii)** 150 mm **(iii)** 225 mm

(d) Breaking the data into the same two groups, find the equations of the lines of best fit if applied force was on the horizontal axis.

(e) Find the compression, to the nearest whole number, if the spring system has an applied force of:

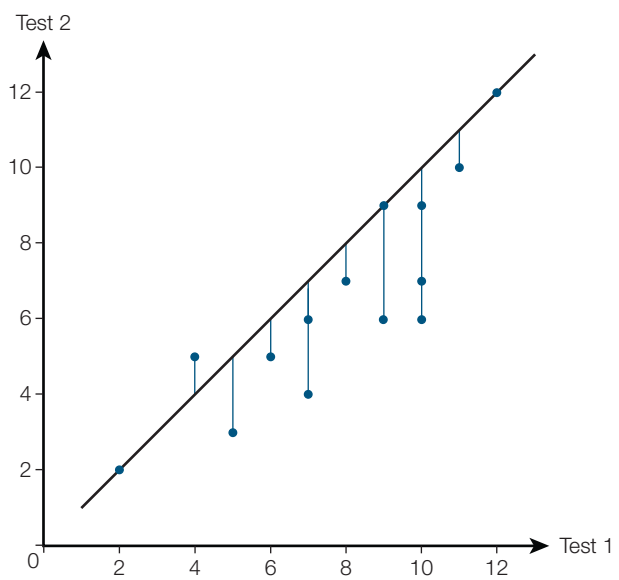
(i) 5 N **(ii)** 30 N **(iii)** 45 N

(f) The spring system has a maximum compression of 500 mm with an applied force of 90.0 N. Have we found an appropriate equation for this part of the graph? Explain why, listing any assumptions you have made. (Determine a new equation if necessary.)

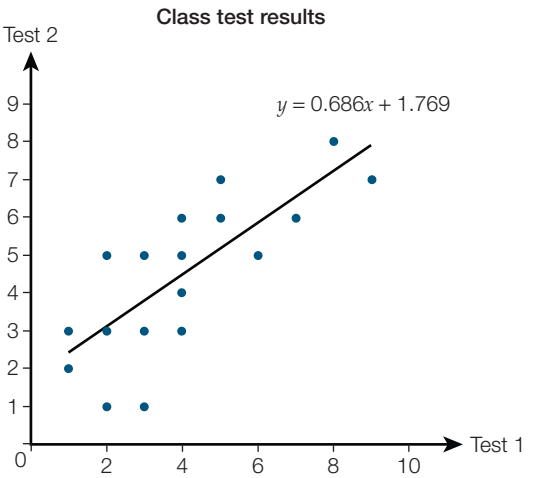
Open-ended

8 **(a)** What mistake has Emily made when drawing her line of best fit?

(b) Redraw the graph in your exercise book (the coordinate pairs are all whole number pairs) and draw the line of best fit correctly.



9 A scatter plot has been drawn with its line of best fit drawn over it. The equation of the line of best fit has been given. Make up three questions you could ask using this information. The data refers to results on two mathematics tests.



Problem solving

Making the data fit

Equipment required: a spreadsheet such as Excel or a CAS would be useful

You have drawn a scatter plot and found that the equation of the line of best fit is $y = 3x - 4$, with the values rounded off to the nearest whole number.

You are to find a data set of at least 10 number pairs that would produce such a line of best fit.

Resist the temptation to make them all fit the line perfectly!

Strategy options

- Look for a pattern.
- Work backwards.

Chapter review 7

D.I.Y. Summary

Key Words

bivariate data	interquartile range	percentile
box plot	ogive	primary data
cumulative frequency curve	outlier	quantile
dependent variable	parallel box plot	range
five-number summary	parallel dot plot	scatter plot
independent variable	percentage cumulative frequency curve	secondary data
10A		
extrapolation	line of best fit	standard deviation
interpolation	regression line	variance

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 Data used from the internet is _____.

2 The difference between the upper quartile and the lower quartile is called the _____.

3 Another name for a cumulative frequency curve is _____.

4 Data collected from your own observations is _____.

5 A _____ is a graph that shows the minimum, lower quartile, median, upper quartile and maximum values for a data set.

6 The value below which 80% of values lie on a cumulative frequency curve can also be referred to as the 0.8 _____.

7 The difference between the highest observed value and the lowest is called the _____.

8 If a value is more than $1.5 \times \text{IQR}$ above Q_U or more than $1.5 \times \text{IQR}$ below Q_L then it is an _____.

9 When we use a regression line to predict a value within the known range we are using _____. If the value is outside the known range we are using _____. 10A

10 The _____ is the square root of the _____. 10A

Fluency

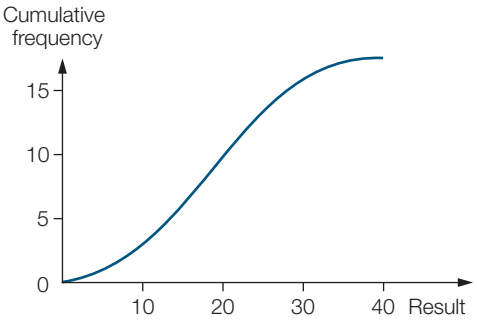
1 For each of the following cumulative frequency curves:

Ex 7.1, 7.2

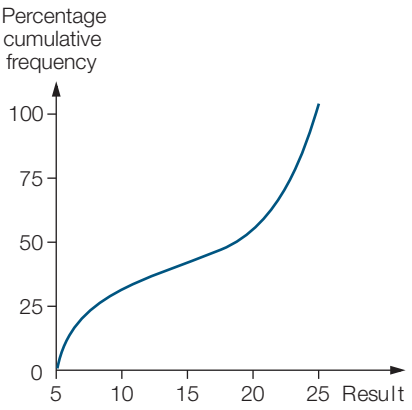
(i) read off the median **(ii)** read off the interquartile range

(iii) draw a box plot.

(a)



(b)



2 For the set of data shown at right:

Ex 7.2

(a) find the median by drawing a percentage cumulative frequency curve

(b) read off the interquartile range

(c) draw a box plot.

Class interval	Frequency
120–<125	5
125–<130	17
130–<135	32
135–<140	54
140–<145	24
145–<150	13

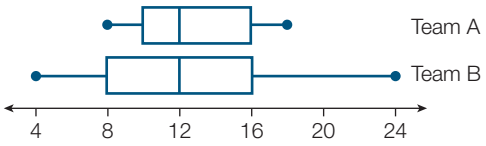
3 A frog in training for the big jump-off achieves the following jump lengths (in centimetres). On the basis of these jumps, how long would a jump need to be to be considered an outlier?

Ex 7.2

19 25 17 22 20 19 16 15 17 21

4 You have been working in a group on an analysis task. One member of the group has produced the following parallel box plot for the data that has been collected. Your task is to write the first draft of the analysis. This should include a comment about which team you think has been more successful.

Ex 7.3



5 For each of the following occupations, state whether you think they deal with primary data or secondary data.

Ex 7.4

(a) Meteorologist **(b)** Television weather forecaster

(c) Insurance broker **(d)** Cancer research scientist

6 Find, rounded to two decimal places, the standard deviation for the following data sets.

Ex 7.7 10A

(a) 3 4 4 5 6 7 7 7 8 9 10 11 11 12 12 12 14 15 16

(b)

Number of flaws	Frequency
0	2
1	19
2	16
3	15
4	7

(c)

Weight (m)	Frequency
0–<5	6
5–<10	11
10–<15	15
15–<20	8
20–<25	2

7 Find the equation of the line of best fit, using a spreadsheet or some other form of technology, for each of the following sets of data. Round your answers to three decimal places, where necessary. In each case, the required rule is of the form $y =$.

Ex 7.8 10A

(a)

x	0	1	2	3	4	5	6
y	3	7	13	20	26	30	35

(b)

x	0	2	5	7	10	12	14
y	18	15	11	5	0	-6	-11

Understanding

8 Consider the following data:

72.8 64.8 55.4 44.3 61.8 52.9 62.0 73.6 34.5 72.3
65.9 57.7 64.4 55.3 45.2 71.2 69.6 54.2 59.9 51.1

(a) Group the data into intervals of 10 (i.e. 30–<40, 40–<50 etc.) and draw a cumulative frequency curve.

(b) Calculate the five-number summary for the data.

(c) Find the median from the cumulative frequency curve and compare it with the median calculated from the raw data.

9 The rainfall (in millimetres) recorded on each day of a month in a particular location was:

12 15 17 16 25 2 44 3 2 0
4 6 8 10 0 11 3 6 3 8
6 15 15 17 0 22 17 21 30 33

(a) Calculate the range for the rainfall figures.

(b) Calculate the five-number summary for the data.

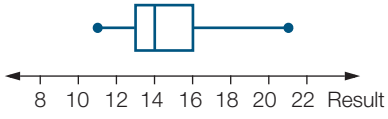
(c) Draw a box plot to represent the data.

10 For each of the following box plots:

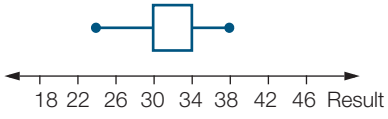
(i) indicate what values would cause results to be considered as outliers

(ii) decide whether there are any outliers in the data set displayed.

(a)



(b)



Reasoning

11 Some of the data available from the Bureau of Meteorology website for Perth (Metro) is shown below.

Temperature (°C)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Mean maximum	30.9	31.3	29.5	25.6	22.4	19.3	18.3	18.8	20.0	22.9	26.2	28.8	24
Mean minimum	17.9	18.0	16.5	13.5	10.5	8.4	7.8	8.1	9.5	11.2	14.1	16.2	12

(a) Find the mean difference in temperature (°C) for each month.

(b) Draw a scatter plot showing mean maximum temperature, mean minimum temperature and mean difference in temperature.

(c) Describe the temperature pattern for Perth (Metro) using the graph as an aid.

(d) Could you use this graph to describe the temperature pattern for Kalgoorlie, in inland West Australia? Give a reason for your answer.

(e) The daily maximum and minimum temperatures in Perth for June 2010 are given.

Minimum:	10.9	6.8	5.6	4.8	9.9	5.7	6.5	6.6	8.7	7.3	9.6	6.9
	4.3	2.2	2.3	6.1	12.0	9.7	10.8	13.1	6.4	9.4	8.2	10.8
	11.1	6.4	3.1	6.3	8.4	5.2						
Maximum:	16.2	17.9	17.6	17.0	16.9	14.3	15.5	15.9	15.1	16.3	16.1	16.0
	13.4	13.8	14.0	16.9	16.3	16.5	17.0	17.3	16.4	17.3	18.2	18.2
	15.2	14.2	14.5	14.6	11.9	14.0						

How did June 2010 compare to the overall data?

12 The number of colony forming units per 100 mL of water of two different bacteria taken from a particular lake at weekly periods is recorded in the following table. The table also shows the maximum temperature (°C) for the day of recording. 10A

Fecal coliform (CFU/100 mL)	350	73	180	150	200	130	2200	6300
***E. coli* (CFU/100 mL)**	350	90	120	330	200	120	1600	5800
Temp. (°C)	15.3	14.6	16.8	16.8	18	24	19.6	21.7

Fecal coliform (CFU/100 mL)	360	1200	560	1700	19	430	320	87
***E. coli* (CFU/100 mL)**	300	980	630	1800	16	270	270	160
Temp. (°C)	20.1	19.4	22.1	24.1	22	23	19	18

(a) In how many different ways can the data be grouped?

Use your CAS to:

(b) draw a scatter plot with fecal coliform on the x-axis and *E. coli* bacteria on the y-axis

(c) find the equation of the line of best fit, assuming the data is linear.

13 The following fictional article contains information about the opinions of voters on the introduction of tolls on certain roads around Melbourne.

Toll roads – poll reaction

The State Government is facing a big electoral backlash over its City Link toll roads according to a *Sunday Bugle* poll conducted by Corio Market Research Centre in marginal seats along the route.

The poll found that at least half the voters surveyed avoid paying the tolls on the Tullamarine Freeway and Monash Freeway.

Residents of the affected areas have increased worries about traffic on their local suburban roads growing massively. The poll shows that the Government could lose three of the four seats in which the survey was conducted.

SUNDAY BUGLE POLL: TOLL ROADS

How marginal electorates would vote . . .

	Government %	Opposition %
Tullamarine	33	47
Essendon	43	43
Oakleigh	38	38
Knox	46	32

. . . and who avoid tolls

Tullamarine	59%
Essendon	50%
Oakleigh	52%
Knox	49%

Source: Telephone poll of 1200 voters conducted by Corio Market Research Centre

(a) Explain what is meant by the heading. Do the survey results support this 'big electoral backlash'?

(b) Add up the percentages for Government and Opposition voters in the Tullamarine electorate. Why don't they add up to 100%? Why is this important to the judgment of the poll?

(c) Comment on the sample size. Is it appropriate for a conclusion?

(d) The poll was done by telephone. Which sampling methods would you use to get the best indication of how voters in each electorate are reacting to City Link toll roads?

(e) What is the significance of the four different locations?

14 At his head office, the owner of a company holds a record of the salaries paid to his employees. Annual salaries 3 years ago and today are given in the following table.

Salary range (× $10 000)	Number of employees (3 years ago)	Number of employees (currently)
1–<3	12	9
3–<5	15	10
5–<7	9	9
7–<9	5	8
9–<11	3	7
11–<13	0	5

(a) Represent the above data by drawing a graph that emphasises the difference between salaries paid 3 years ago and those paid today.

(b) Contrast the means, medians and interquartile ranges for the two sets of data.

(c) Draw a box plot for each set of data.

(d) Describe, in your own words, the change in salaries over the past 3 years.

(e) How much more is the company paying in salaries today than it was 3 years ago?

(f) By what percentage has the average salary risen?

15 The grouped frequency table below shows the age distribution of the employees of a local company. The age given is the nearest whole number of years on 1 January of the current year. So, if a worker was 22 years and 5 months old they would be recorded as 22, whereas a worker 22 years and 7 months old would be recorded as 23. 10A

Age at 1 Jan (years)	16–20	21–25	26–30	31–35	36–40	41–45	46–50	51–55
Frequency	25	30	32	28	33	42	55	10

(a) Estimate the mean age, to one decimal place, giving a reason for your estimate.

(b) Calculate the mean and standard deviation for the ages of the employees, giving your answers rounded to two decimal places, where appropriate.

(c) Draw a cumulative frequency curve and use it to find the five-number summary.

(d) Draw a box plot of the data.

(e) Use your cumulative frequency graph to estimate the number of employees who are:

(i) under 32 **(ii)** over 48.

8 Geometry

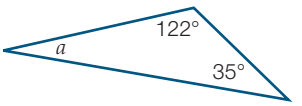
Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

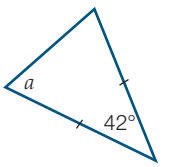
Worksheet R8.1

1 Find the value of the variable in each of the following.

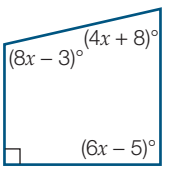
(a)



(b)

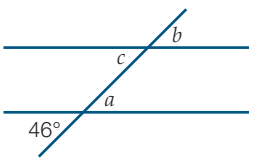


(c)



Worksheet R8.2

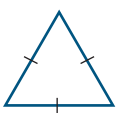
2 Find the value of the variables in this diagram.



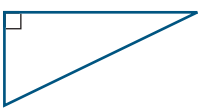
Worksheet R8.3

3 State whether the following triangles are equilateral, scalene, isosceles or right-angled.

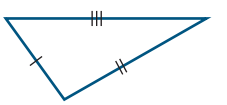
(a)



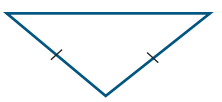
(b)



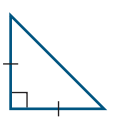
(c)



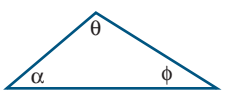
(d)



(e)



(f)

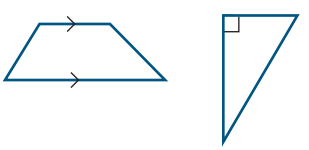


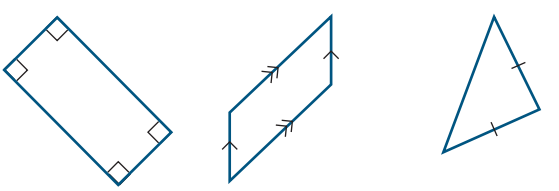
Worksheet R8.4

4 Match the name to its shape.

A rectangle **B** scalene triangle **C** isosceles triangle

D parallelogram **E** trapezium





After completing this chapter you will be able to:

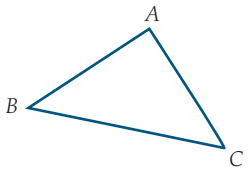
- determine what information is needed to prove triangles are congruent or similar
- find unknown sides in pairs of congruent or similar triangles
- apply geometric theorems, techniques and constructions
- apply congruent triangle results to establish properties of special triangles and quadrilaterals
- construct arguments to prove geometric results
- determine properties of triangles and quadrilaterals using deductive reasoning
- prove and apply angle and chord properties of circles. 10A

Congruent and similar triangles

8.1

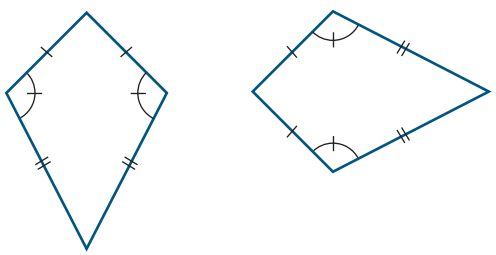
Need to Know

- Sides with the same markings are equal in length.
- Sides with the same number of arrows are parallel.
- The symbol || means 'is parallel to'.
- The symbol ⊥ means 'is perpendicular to'.
- The symbol ≡ means is 'congruent to'.
- The symbol ∴ means 'therefore'.
- The symbol ||| means 'is similar to'.
- The triangle with vertices A, B and C is written as ΔABC.
- Angles are denoted using the symbol ∠; the angle at vertex A is written as $\angle BAC$. They can also be given letter names such as α, β, ϕ.
- Intervals are named according to their end points; the interval between A and B is written as AB.



Congruency

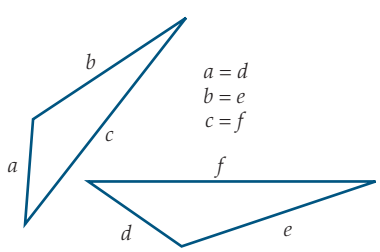
Congruent figures are identical in size and shape, but their orientation may be different.



Tests for congruent triangles

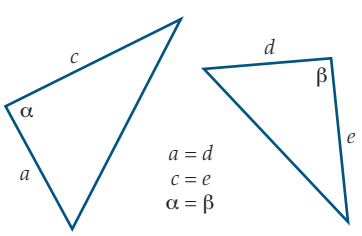
1 Side Side Side (SSS)

The three pairs of sides have the same length.



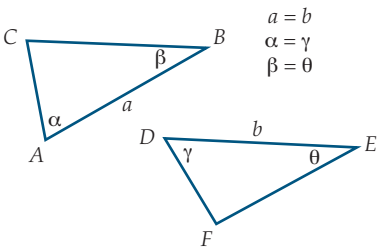
2 Side Angle Side (SAS)

Two pairs of sides have the same length and the **included angle** is the same.



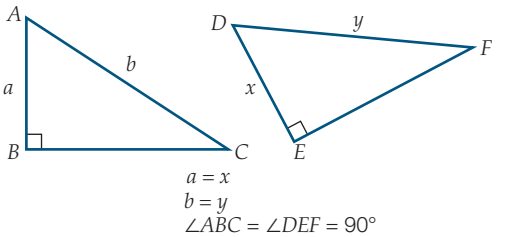
3 Angle Angle Side (AAS)

Two pairs of angles are the same and one pair of matching sides is the same length.



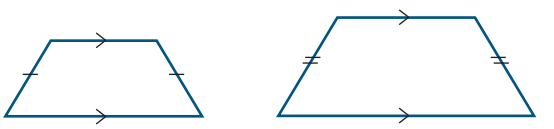
4 Right angle Hypotenuse Side (RHS)

If both triangles contain a right angle, the hypotenuse in both is the same and another pair of sides is the same length.



Similarity

Similar shapes are those where all side lengths have undergone the same dilation.



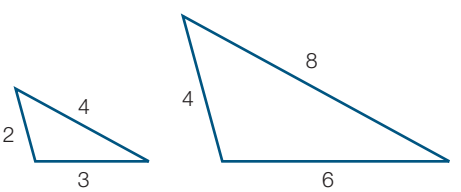
When a figure is dilated:

- matching angles are the same
- matching side lengths are in the same ratio.

Tests for similar triangles

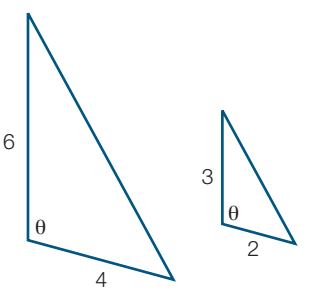
1 Side Side Side (SSS)

If the three sides of one triangle are proportional to the three sides of another triangle.



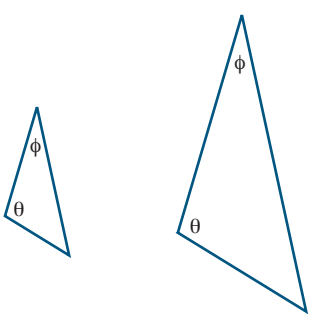
2 Side Angle Side (SAS)

If two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal.



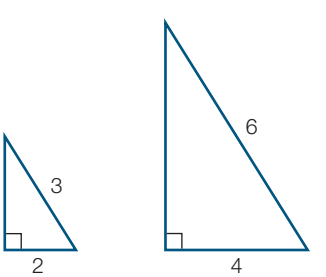
3 Angle Angle Angle (AAA)

If two angles of one triangle are equal to two angles in another triangle. All three angles are therefore equal.



4 Right Angle Hypotenuse Side (RHS)

If the hypotenuse and one side of a right-angled triangle are proportional to the hypotenuse and one side of another right-angled triangle.



Note: These tests are similar to the four tests for congruence, the difference being that where congruence requires that matching sides be the same length, similarity requires that matching sides be proportional.

To find the length of an unknown side in a similar triangle:

Step 1 Find two pairs of matching sides that include the unknown side.

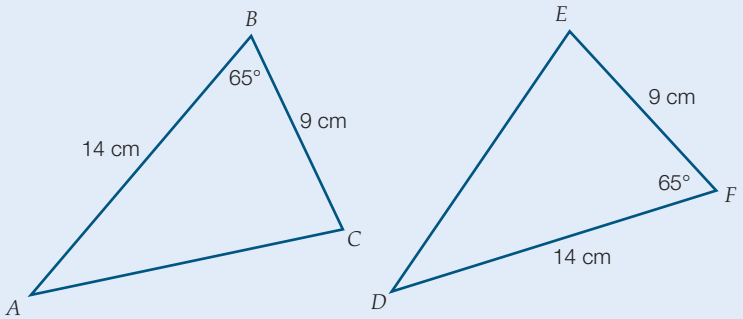
Step 2 Equate the two ratios.

Step 3 Solve the equation to find the exact value of the unknown, unless told otherwise.

Worked Example 1

WE1

Show that ΔABC and ΔDFE are congruent, stating the congruency test used.



Working

$AB = DF$ (both 14 cm)
$\angle ABC = \angle DFE$ (both 65°)
$BC = EF$ (both 9 cm)

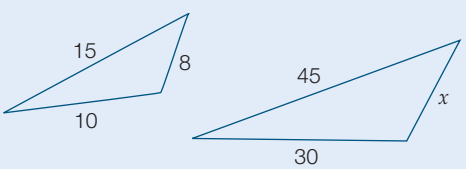
The triangles are congruent due to SAS.

Worked Example 2

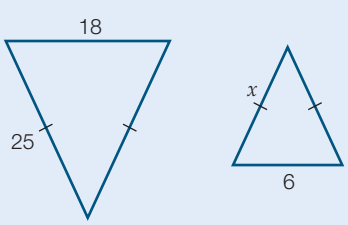
WE2

Given that these pairs of triangles are similar, find x.

(a)



(b)



Working

(a) $x:8 = 45:15$

$$\frac{x}{8} = \frac{45}{15}$$

$$x = \frac{45}{15} \times 8$$

$x = 24$ units

(b) $x:25 = 6:18$

$$\frac{x}{25} = \frac{6}{18}$$

$$x = \frac{6}{18} \times 25$$

$$x = 8\frac{1}{3} \text{ units}$$

Answers page 527

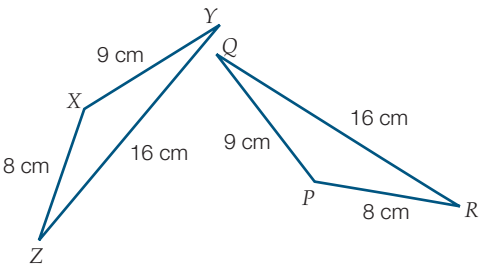
8.1 Congruent and similar triangles

Fluency

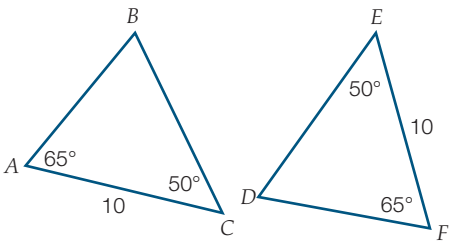
WE1

1 Show that the following pairs of triangles are congruent, stating the congruency test used.

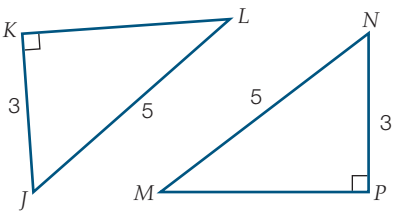
(a)



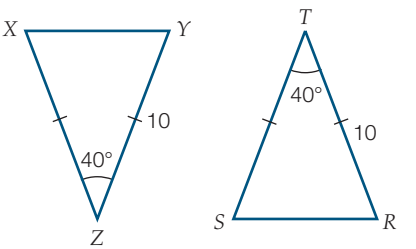
(b)



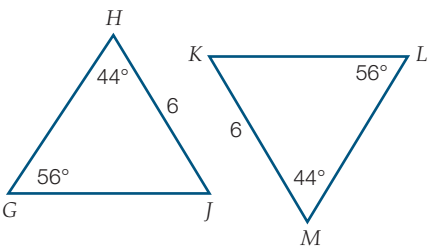
(c)



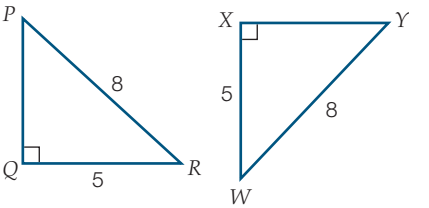
(d)



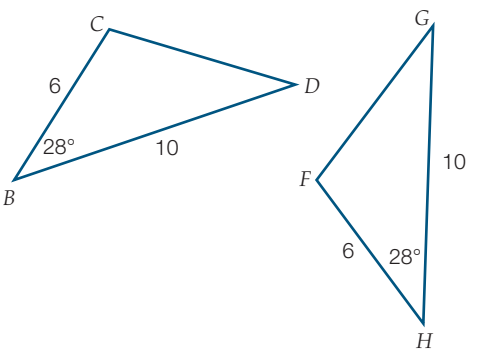
(e)



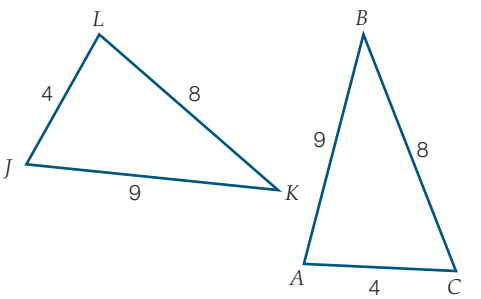
(f)



(g)



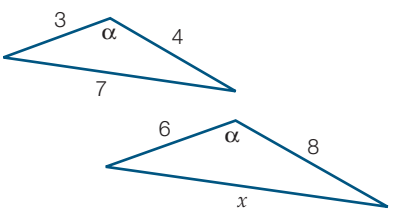
(h)



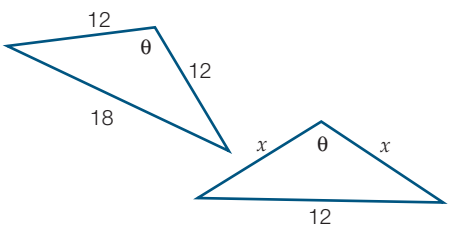
WE2

2 Given that these pairs of triangles are similar, find x.

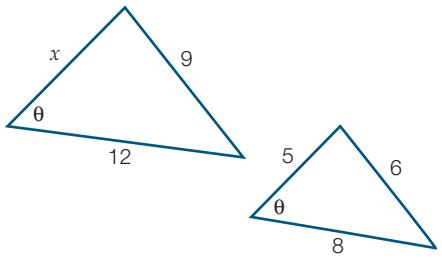
(a)



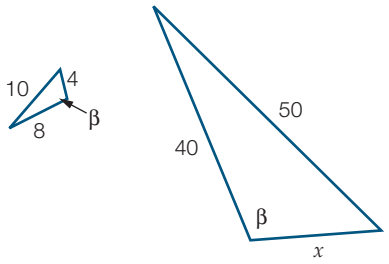
(b)



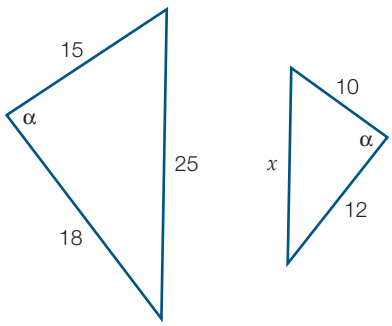
(c)



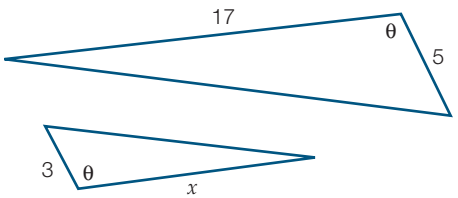
(d)



(e)

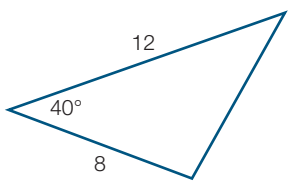


(f)

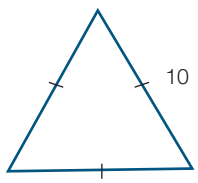


3 Match the pairs of congruent triangles given below.

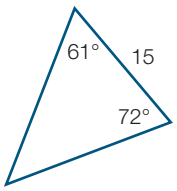
(a)



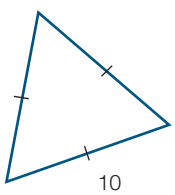
(b)



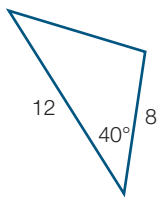
(c)



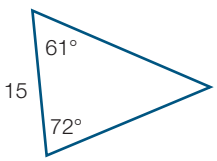
(d)



(e)



(f)

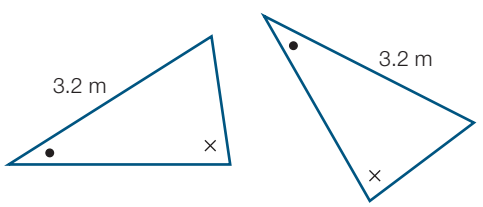


4 Which one of the following conditions does *not* ensure congruence for two triangles?

A SSS **B** SAS **C** AAS **D** AAA

5 Which condition verifies that these two triangles are congruent?

A SSS **B** SAS
C AAS **D** RHS



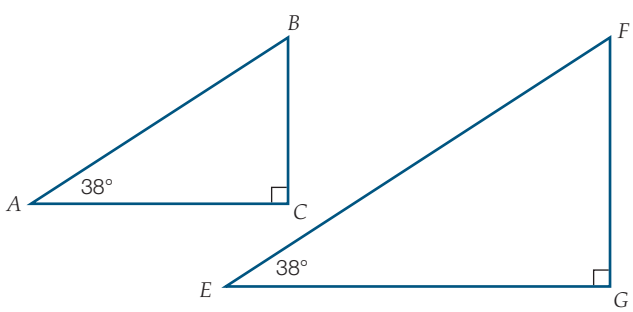
6 Given $\Delta ABC \equiv \Delta EFG$:

(a) the matching side to AB is:

A FG **B** EF
C BC **D** AC

(b) the matching side to FG is:

A BC **B** AC
C EF **D** EG



7 Given $\Delta HJK \sim \Delta LMN$:

(a) the matching side to LM is:

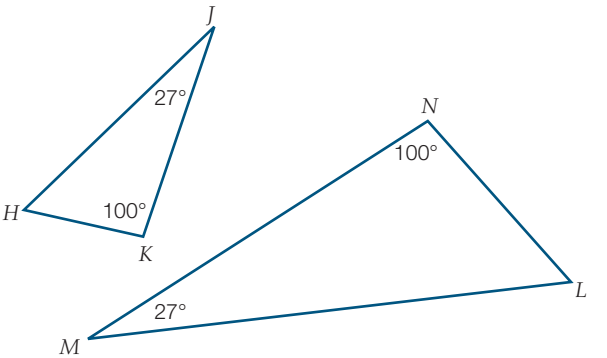
A HJ **B** MN

C JK **D** LN

(b) the matching side to HJ is:

A JK **B** LM

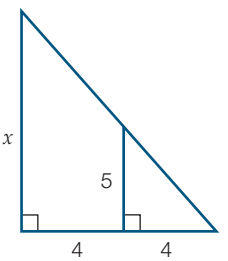
C HK **D** NL



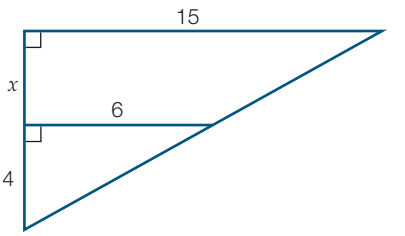
Understanding

8 In the following diagrams, first determine the similar triangles and then find the value of x.

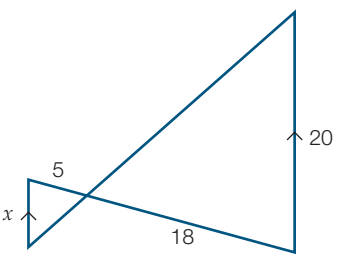
(a)



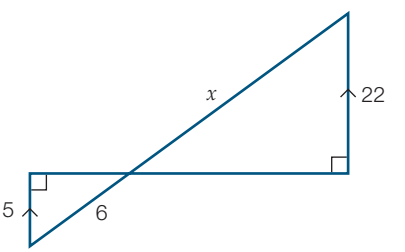
(b)



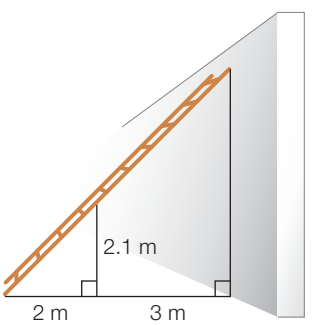
(c)



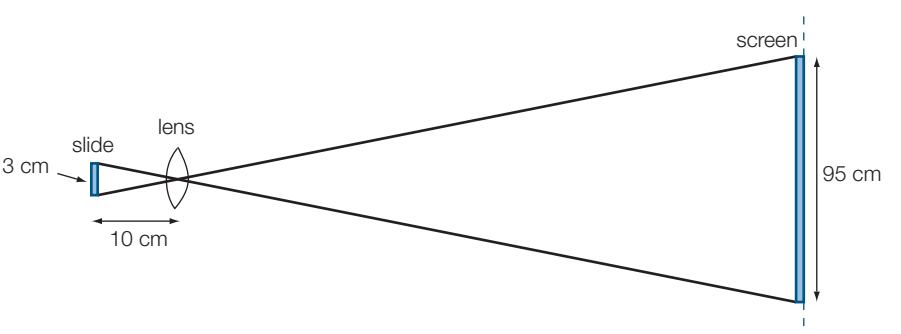
(d)



9 Dirk is climbing up a long ladder leaning against a wall. His friend Sean is holding up the ladder to prevent it wobbling. Sean is 3 m from the wall and 2 m from the base of the ladder. With his arms stretched above his head he holds the ladder at a point 2.1 m above the ground. How far up the wall does the ladder reach?



10 A light source projects the image of a photographic slide onto a screen, as shown in the diagram. The slide is 3 cm high and positioned 10 cm behind a lens. Calculate the distance between the lens and the screen if the image on the screen is 95 cm high. Give your answer correct to two decimal places.



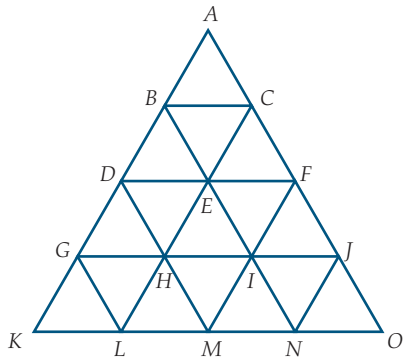
11 Ziva plays tennis at night under floodlights. When she stands 4.5 m from the base of a floodlight, she casts a shadow 1.4 m long. If Ziva is 1.7 m tall, how high is the floodlight?

Reasoning

12 The small triangles in the following diagram are all equilateral.

(a) Find six triangles congruent to ΔDFM.

(b) Find two pairs of congruent triangles larger than ΔDFM.



13 The Akela (leader) of the local scout group was explaining to the scouts how to use a stick to estimate the height of a tree.

Step 1: Have someone hold the stick vertically on the ground.

Step 2: Lie on the ground in a position so that when looking at the tree through the stick the top of the tree and the top of the stick are in line.

Step 3: Unfortunately, Akela has forgotten Step **3**.

(a) Draw a diagram of the situation and using your knowledge of similar triangles complete the instructions Akela needs to give.

(b) List any practical problems you can see with this method.

Open-ended

14 Explain why two triangles with three equal angles may not necessarily be congruent. Show this by drawing diagrams.

15 $\Delta ABC \,|||\, \Delta DEF$. The dilation factor is $\frac{3}{2}$. Draw a diagram to show both triangles and write possible values for the sides of both triangles if ΔABC is the smaller triangle and BC is equal to 4 cm.

Proving congruence and similarity

8.2

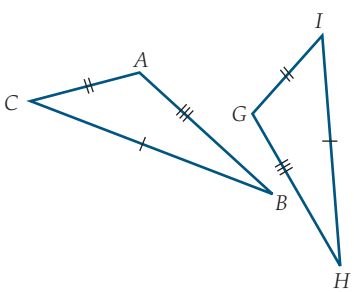
Need to Know

When showing that two shapes are congruent, write the matching vertices in the same order.

For example, in the congruent triangles shown, the matching vertices are A and G, B and H, and C and I.

So, we write that $\Delta ABC \equiv \Delta GHI$.

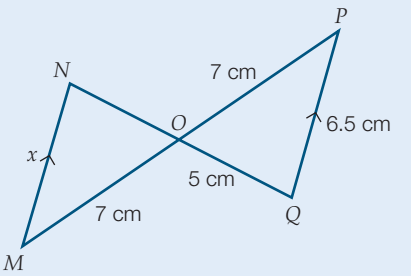
This allows us to easily identify matching sides and angles because they are in the same place in the statement.



Worked Example 3

WE3

Prove that the triangles ΔMNO and ΔPQO are congruent, and hence find the value of the given variable.



Working

$\Delta MNO \equiv \Delta PQO$

$\angle MNO = \angle PQO$ (alternate angles, $MN \parallel PQ$)

$\angle MON = \angle POQ$ (vertically opposite angles)

$MO = PO$ (given)

$\Delta MNO \equiv \Delta PQO$ (AAS)

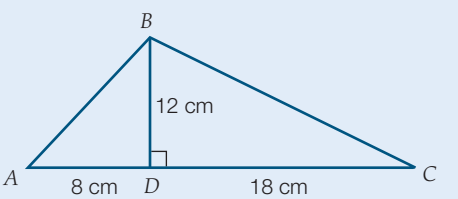
$MN = PQ$ (matching sides in congruent triangles)

$x = 6.5$ cm

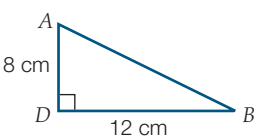
Worked Example 4

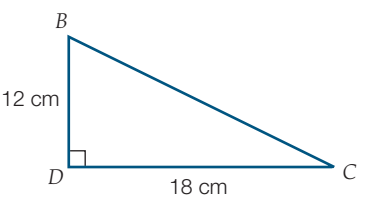
WE4

Prove that the triangles ΔABD and ΔBCD are similar.



Working





$\frac{BD}{CD} = \frac{12}{18} = \frac{2}{3}$

$\frac{AD}{BD} = \frac{8}{12} = \frac{2}{3}$

BD and CD are in the ratio 2 : 3.
AD and BD are in the ratio 2 : 3.

$\angle ADB = \angle BDC = 90°$

$\Delta ABD \text{ III } \Delta BCD$ because two sides of one triangle are in the same ratio as two matching sides of the other triangle and the included angles are equal (SAS).

8.2 Proving congruence and similarity

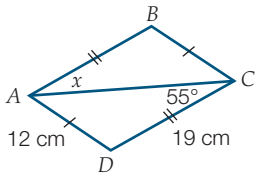
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Fluency

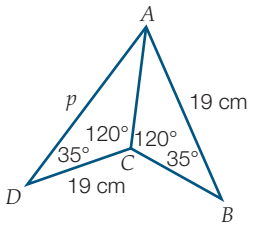
1 Prove that the following pairs of triangles are congruent and hence find the value of the given variable.

WE3

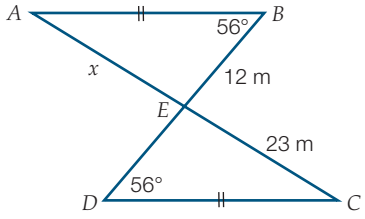
(a)



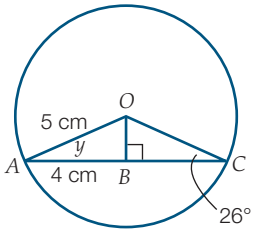
(b)



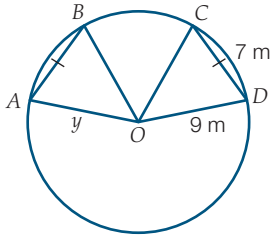
(c)



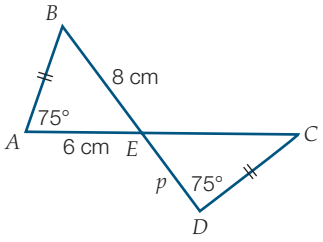
(d)



(e)



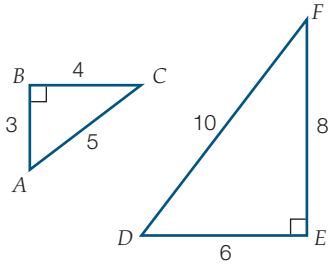
(f)



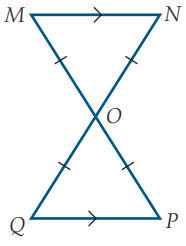
2 Prove that the following pairs of triangles are similar.

WE4

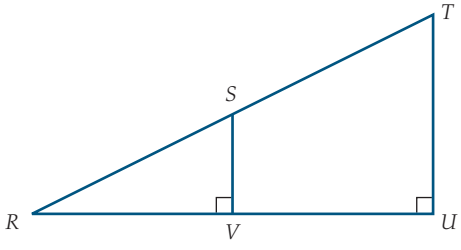
(a)



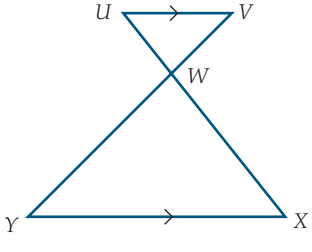
(b)



(c)



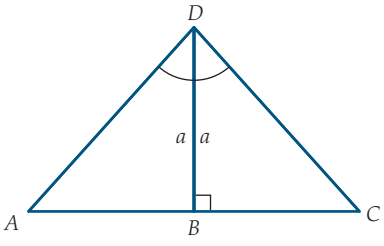
(d)



3 DB is the perpendicular bisector of AC and bisects $\angle ADC$. Which test for congruence can be used to prove that $\Delta ABD \equiv \Delta CBD$?

A SSS **B** SAS

C AAS **D** RHS



4 Which of the following is not enough to determine similarity?

A Two angles of one triangle are equal to two angles of the other triangle.

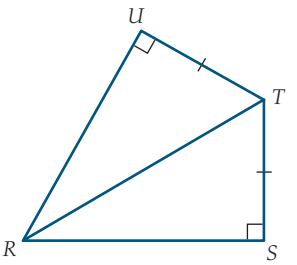
B Two sides of one triangle are proportional to two sides of another triangle.

C The hypotenuse and second side of a right-angled triangle are proportional to the hypotenuse and second side of another right-angled triangle.

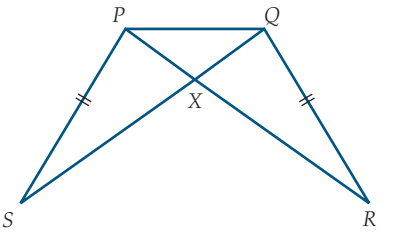
D Two sides of one triangle are proportional to two sides of another triangle and the included angles are equal.

Understanding

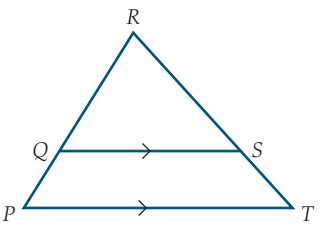
5 **(a)** Prove that $\Delta RUT \equiv \Delta RST$



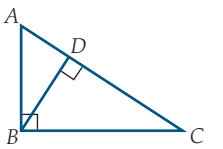
(b) $SP = RQ$ and $\angle SPQ = \angle RQP$.
Prove that $\Delta SPQ \equiv \Delta RQP$.



6 **(a)** Prove that $\Delta RQS \,|||\, \Delta RPT$.



(b) Prove that $\Delta ABC \,|||\, \Delta BDC \,|||\, \Delta ADB$.



Reasoning

7 **(a)** Explain why or why not a pair of shapes is similar if they are any two:

(i) rectangles

(ii) squares

(iii) isosceles triangles

(iv) equilateral triangles

(v) isosceles right-angled triangles

(vi) circles.

(b) For each of the similar shapes in part **(a)**, state the conditions necessary for the pairs of shapes to be congruent.

Open-ended

8 Draw four pairs of congruent triangles and label them with enough information to show congruence. Each pair should represent a different test. Write the name of the congruence test that applies to each pair.

Proofs using congruent triangles

8.3

Need to Know

A **definition** is the minimum amount of information needed to identify a particular figure.

A **property** is a fact about a particular figure that can be proved from the definition.

Definitions of triangles

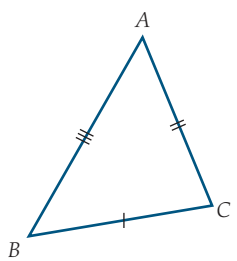
1. A scalene triangle is a triangle with no two sides equal in length.
2. An isosceles triangle is a triangle with only two sides equal in length.
3. An equilateral triangle is a triangle with all sides equal in length.

Properties of triangles

Scalene triangle

A scalene triangle has no particular property as all angles are of different magnitude.

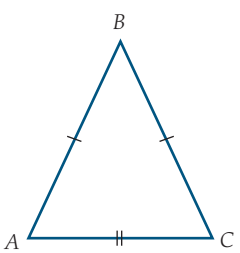
$\angle BAC \neq \angle ABC \neq \angle BCA$



Isosceles triangle

An isosceles triangle has two angles of equal magnitude.

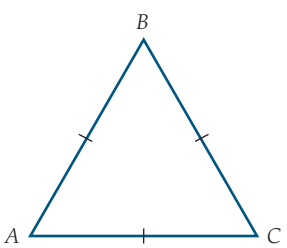
$\angle BAC = \angle BCA \neq \angle ABC$



Equilateral triangle

An equilateral triangle has all angles of equal magnitude of 60°.

$\angle BAC = \angle ABC = \angle BCA = 60°$



We can use congruent triangles to **prove** the **theorems** about isosceles and equilateral triangles. When proved, the theorems are know as properties.

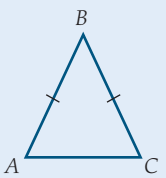
Worked Example 5

WE 5

In ΔABC, $BA = BC$.

(a) Prove that $\angle BAC = \angle BCA$.

(b) What property of isosceles triangles have you just proved?



Hint

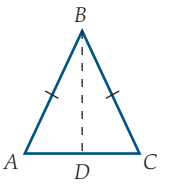
Construct a line from B to the midpoint of AC.

Working

(a) Line BD meets the midpoint of AC.

Consider ΔABD and ΔCBD.

$BA = BC$	(given)
$BD = BD$	(common side)
$AD = CD$	(by construction)
$\therefore \Delta ABD \equiv \Delta CBD$	(SSS)
$\angle BAD = \angle BCD$	(matching angles in congruent triangles)
$\therefore \angle BAC = \angle BCA$	



(b) The result proved that angles opposite equal sides are equal in isosceles triangles.

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8.3 Proofs using congruent triangles

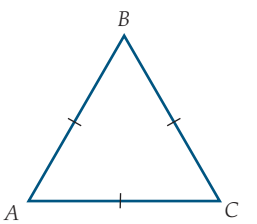
Fluency

WE5

1 In ΔABC, $AB = BC = CA$

(a) Prove that $\angle ABC = \angle BCA = \angle CAB$.

(b) What property of an equilateral triangle have you just proved?



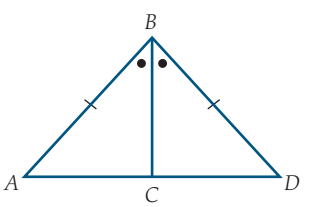
Hint

Construct a line from B to the midpoint of AC and a line from C to the midpoint of AB.

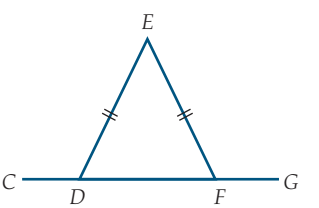
2 Use the triangles shown to prove the following properties of isosceles triangles.

(a) The bisector of the angle between the two equal sides of an isosceles triangle bisects the third side.

(b) The bisector of the angle between the two equal sides of an isosceles triangle is perpendicular to the third side.

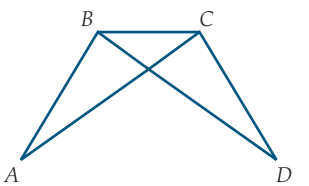


3 ΔDEF is isosceles. Prove that $\angle CDE = \angle GFE$.



4 The diagram shows two triangles, ΔABC and ΔDCB, with $AB = DC$ and $\angle ABC = \angle DCB$.

Which of the following tests can be used to prove that $\Delta ABC \equiv \Delta DCB$?



A SAS **B** AAS **C** SSS

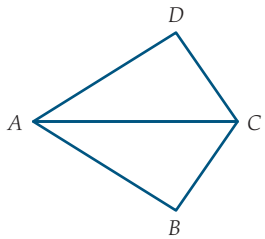
D There is insufficient information to determine whether the triangles are congruent.

5 In the diagram opposite, AC bisects $\angle BAD$ and $\angle BCD$.

Which of the following tests can be used to prove that $\Delta ABC \equiv \Delta ADC$?

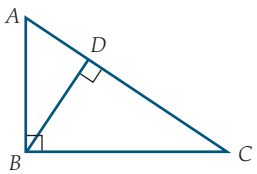
A SAS **B** AAS **C** SSS

D There is insufficient information to determine whether the triangles are congruent.

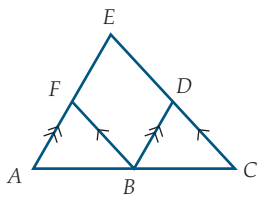


Understanding

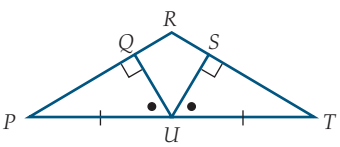
6 For the triangle shown, show that $AC^2 = DC^2 + 2BD^2 + AD^2$.



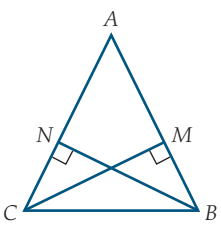
7 B is the midpoint of AC. Show that $\Delta ABF \equiv \Delta BCD$.



8 Prove that ΔPRT is isosceles.

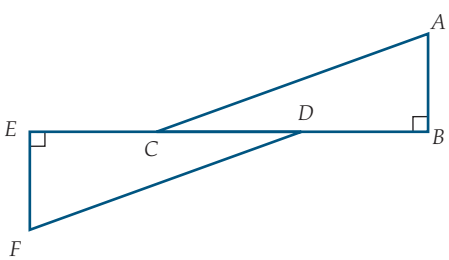


9 In ΔABC, $AB = AC$. Prove that $AM = AN$.

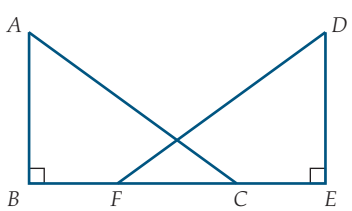


Reasoning

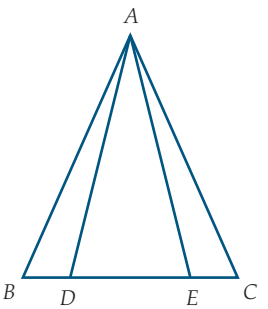
10 If $\angle BAC = \angle DFE$, $\angle ABC = \angle DEF$ and $BD = EC$, prove that $\Delta ABC \equiv \Delta FED$.



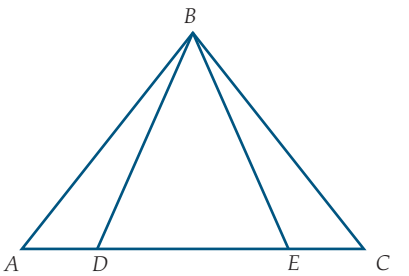
11 If $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BF = EC$, prove that $\Delta ABC \equiv \Delta DEF$.



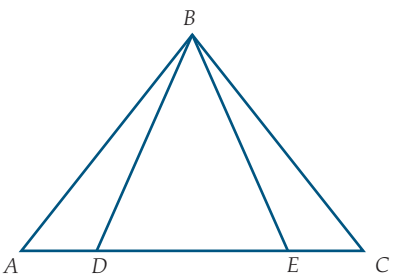
12 **(a)** In the diagram, $\angle ABD = \angle ACE$. Also, $AB = AC$ and $BD = CE$. Prove that $\Delta ABE \equiv \Delta ACD$.



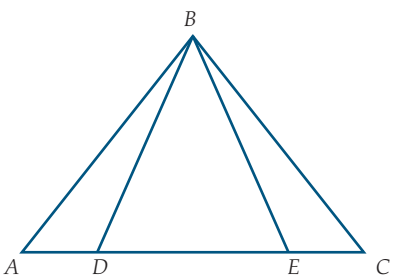
(b) Given $\angle BAD = \angle BCE$ and $\angle ABD = \angle CBE$, prove that $\Delta ABE \equiv \Delta CBD$.



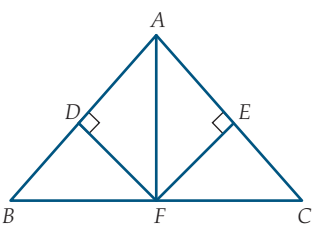
(c) Given $AE = CD$ and $\angle BAD = \angle BCE$, prove that $\Delta ABD \equiv \Delta CBE$.



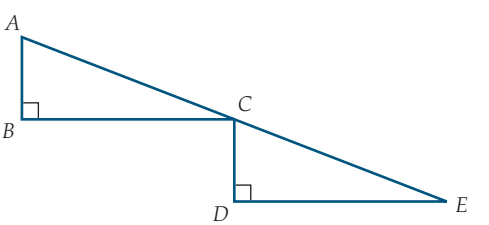
(d) Given $AB = CB$ and $AE = CD$, prove that $\Delta ABD \equiv \Delta CBE$.



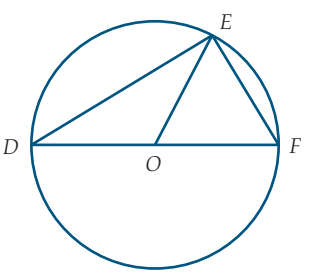
13 In the diagram, $FD = FE$, $FD \perp AB$ and $FE \perp AC$. Prove that $\Delta ADF \equiv \Delta AEF$.



14 ΔABC and ΔCDE are right-angled triangles and $AB = CD$. If C is the midpoint of AE, prove that $\Delta ABC \equiv \Delta CDE$.



15 When you draw a line from each end of a diameter to a point on the circumference of a circle, the angle formed is called an angle in a semicircle. Prove, using isosceles triangles, that the angle in a semicircle is a right angle.



Open-ended

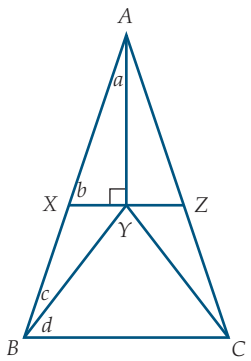
16 ABC is an isosceles triangle with $AB = AC$. X is a point on AB and Z is a point on AC such that XZ is parallel to BC. Y is the midpoint of XZ and $AY = YC$. Let $\angle BAY = a$.

(a) Prove $AX = AZ$.

(b) Prove $\Delta AXY \equiv \Delta AZY$.

(c) Prove $\Delta BXY \equiv \Delta CZY$.

(d) Find b, c and d in terms of a.

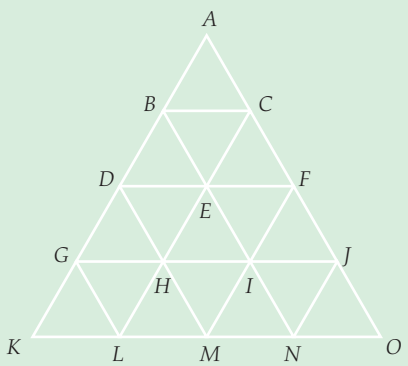


17 Draw a diagram of your own that contains two congruent triangles. Prove that they are congruent using one of the congruency tests.

Problem solving

How many triangles?

How many triangles can you find in this diagram?



Strategy options

- Solve a simpler problem.
- Have I seen a similar problem?
- Draw a diagram.
- Look for a pattern.

Proofs using similar triangles

8.4

Need to Know

Midpoint theorem

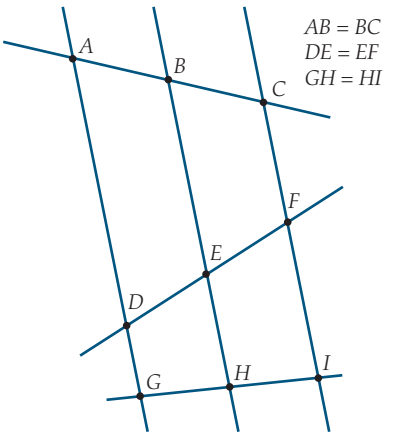
The interval joining the midpoints of two sides of a triangle is parallel to the third side and is half its length.

Converse to the midpoint theorem (the inverse theorem)

A line drawn through the midpoint of one side of a triangle and parallel to a second side **bisects** the third side.

Intercept theorem

When a set of parallel lines make equal intercepts on one transversal, it makes equal intercepts on any other transversal.



Worked Example 6

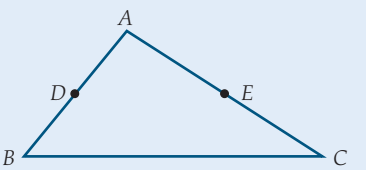
WE6

In ΔABC, D is the midpoint of AB and E is the midpoint of AC.

Draw DE and prove that:

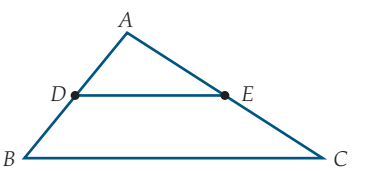
(a) DE is parallel to BC.

(b) $DE = \frac{1}{2}BC$.



Working

(a) Consider ΔADE and ΔABC.



$\angle DAE = \angle BAC$ (common angles)

$AD = DB$ (D is the midpoint of AB)

$\therefore AB = 2AD$

$AE = EC$ (E is the midpoint of AC)

$\therefore AC = 2AE$

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$

$\therefore \Delta ADE \,|||\, \Delta ABC$ (two sides are proportional and their included angles are equal)

$\angle ADE = \angle ABC$ (matching angles of similar triangles)

$\angle AED = \angle ACB$ (matching angles of similar triangles)

$\angle ADE$ and $\angle ABC$ are a pair of equal matching angles formed when the transversal AB cuts the lines DE and BC.

$\therefore DE \parallel BC$ (Corresponding angles are equal so the lines are parallel.)

$\therefore$ The line joining the midpoints of two sides of a triangle (DE) is parallel to the third side (BC).

(b) As the triangles are similar, the matching sides are proportional.

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{1}{2}$

$\therefore \frac{DE}{BC} = \frac{1}{2}$

$\therefore DE = \frac{1}{2} BC$

$\therefore$ The line joining the midpoints of two sides of a triangle (DE) is half the length of the third side (BC).

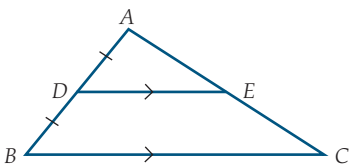
8.4 Proofs using similar triangles

Answers page 531

Fluency

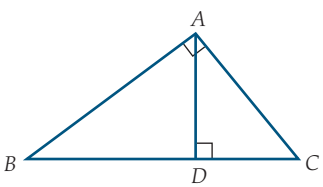
WE6

1 The line through the midpoint of a side of a triangle parallel to another side bisects the third side. In triangle ΔABC, D is the midpoint of AB and DE is parallel to BC, meeting AC at E. Prove that:



(a) $\Delta ABC \,|||\, \Delta ADE$ **(b)** $AE = EC$.

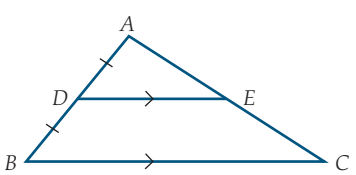
2 In a right-angled triangle, if a perpendicular is drawn from the right angle to the hypotenuse, the triangles on each side of it are similar to the whole triangle and to each another. Triangle ΔABC is right-angled at A and $AD \perp BC$. Prove that triangles ΔADB, ΔDCA and ΔCAB are similar.



3 In a right-angled triangle, the square on the perpendicular to the hypotenuse is equal to the rectangle contained by the segments of the hypotenuse. Using the diagram for Question **2**, prove that $AD^2 = BD \times DC$.

4 In a right-angled triangle, the square on either of the sides containing the right angle is equal to the rectangle contained by the hypotenuse and the segment of the hypotenuse adjacent to that side. Using the diagram for Question **2**, prove that $BA^2 = BD \times BC$ and $CA^2 = CD \times CB$.

5 Use the result from Question **4** to prove Pythagoras' theorem: $BC^2 = BA^2 + CA^2$



6 Which of the following is not sufficient to prove that two triangles are similar?

A The two triangles have two angles the same.

B The three sides of one triangle are proportional to the three sides of the other triangle.

C Two sides of one triangle are proportional to two sides of another triangle.

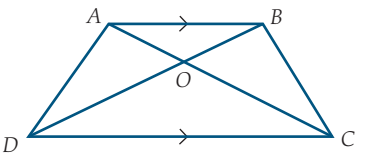
D Two sides of one triangle are proportional to two sides of another triangle and the included angles are equal.

Understanding

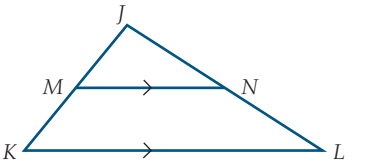
7 In the trapezium $ABCD$, AB is parallel to DC and the diagonals intersect at O.

(a) Show that $OA:OC = OB:OD$.

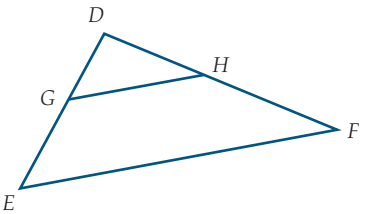
(b) If $DC = 2AB$, show that O is a point of trisection of both diagonals. (That is, show that $AO:OC = BO:OD = 1:2$ and hence that $AO:AC = BO:BD = 1:3$.)



8 A straight line drawn parallel to one side of a triangle cuts the other two sides proportionally. In triangle ΔJKL, M is a point on JK and MN is parallel to KL, meeting JL at N. Prove that $\frac{JM}{MK} = \frac{JN}{NL}$.



9 Conversely, if a line cuts two sides of a triangle proportionally, it is parallel to the third side. In triangle ΔDEF, $\frac{DG}{GE} = \frac{DH}{HF}$. Prove that $GH \parallel EF$.

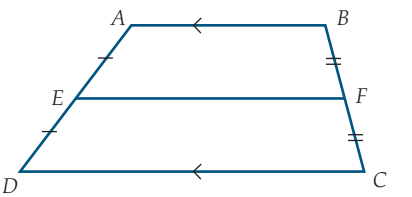


Reasoning

10 The straight line that joins the midpoints of the slanted sides of a trapezium is parallel to the parallel sides. $ABCD$ is a trapezium in which $AB \parallel DC$. E is the midpoint of AD and F is the midpoint of BC. Prove that $EF \parallel AB$.

Hint

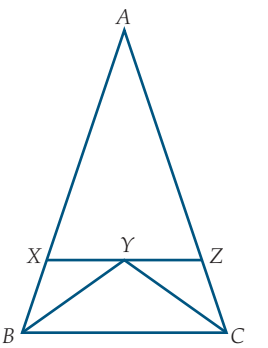
Extend DA and CB and let them intersect at G.



11 ΔABC is an isosceles triangle such that $AB = AC$. X is a point on AB and Z is a point on AC such that XZ is parallel to BC. Y is the midpoint of XZ and $BX = XY$. Prove that $\Delta BXY \,|||\, \Delta BYC$.

Hint

Prove $\Delta BXY \equiv \Delta CZY$ and let them intersect at G.



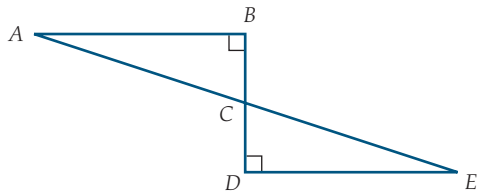
Open-ended

12 Draw a figure different from any given in Exercise 8.4 that contains at least two similar triangles. Prove that your triangles are similar.

Half-time 8

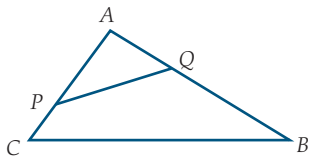
1 **(a)** Given that $BC = DC$, prove that $\Delta ABC \equiv \Delta EDC$.

(b) Hence, prove that $AB = ED$.



Ex 8.2

2 Given that $\angle ABC = \angle APQ$, prove that $\Delta ABC \,|||\, \Delta APQ$.

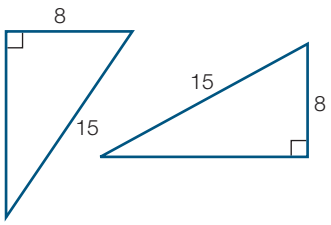


Ex 8.4

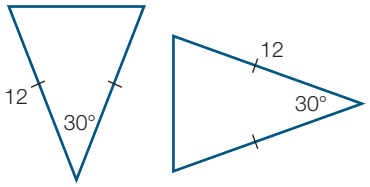
3 Give reasons why each of the following pairs of triangles are congruent.

Ex 8.1

(a)



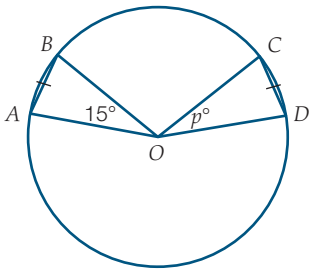
(b)



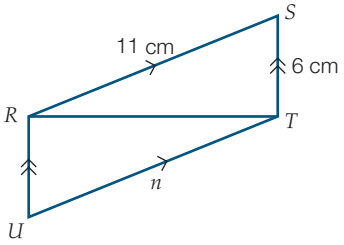
4 For each of the following, prove that the two triangles are congruent and hence find the value of the pronumeral.

Ex 8.2

(a)



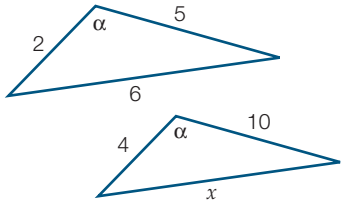
(b)



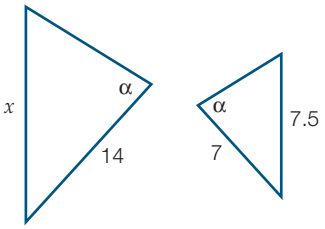
5 Given that these pairs of triangles are similar, find x.

Ex 8.2

(a)

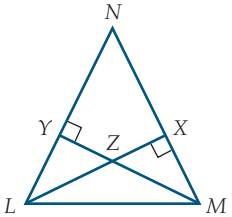


(b)



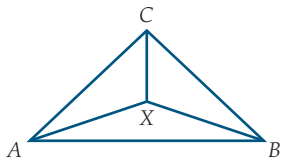
6 In triangle ΔLMN, LX is perpendicular to MN and MY is perpendicular to LN, as shown in the diagram. The point of intersection of LX and YM is Z. Prove that $\Delta LYZ \,|||\, \Delta MXZ$, giving reasons.

Ex 8.4



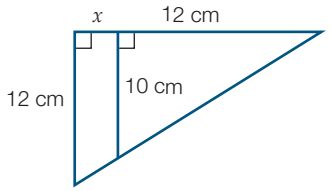
Ex 8.3

7 Given that $AC = BC$ and $AX = BX$, prove that $\Delta ACX \equiv \Delta BCX$.



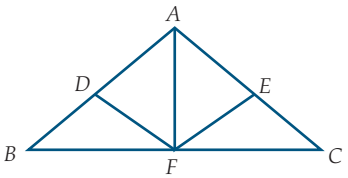
Ex 8.1

8 Find the value of x in the triangle.



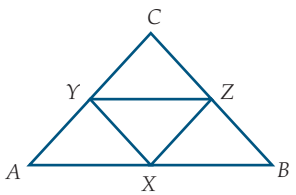
Ex 8.2

9 In the diagram, $FD = FE$. Also, $FD \perp AB$ and $FE \perp AC$. Prove that $\Delta ADF \equiv \Delta AEF$.



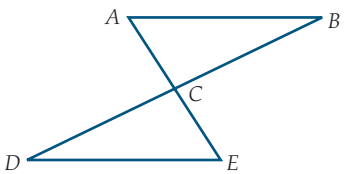
Ex 8.3

10 In the diagram, ΔABC is isosceles with $AC = BC$. X, Y and Z are the midpoints of AB, AC and BC respectively. Prove that $XY = XZ$.



Ex 8.2

11 Given $AB \parallel DE$ and that C is the midpoint of BD, prove that $\Delta ABC \equiv \Delta DEC$.

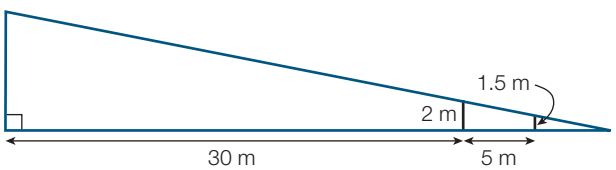


Ex 8.1

12 Draw two triangles that are not congruent using two matching sides and an angle that is not the included angle.

Ex 8.4

13 A student 1.5 m tall, standing 5 m in front of a 2 m brick wall, can just see the top of a tree growing 30 m behind the wall. How tall is the tree?



Geometric properties of special quadrilaterals

8.5

Need to Know

- When two lines intersect at right angles we say they are **perpendicular**.
- When two lines intersect at right angles and one bisects the other (cuts it in half), we say that line is a **perpendicular bisector** of the second line.

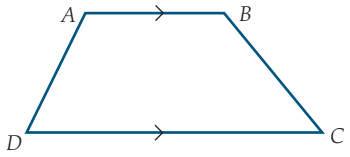
Definitions of special quadrilaterals

1. A trapezium is a quadrilateral with one pair of opposite sides parallel.
2. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
3. A kite is a quadrilateral with two pairs of adjacent sides equal in length.
4. A rectangle is a quadrilateral in which all angles are right angles.
5. A rhombus is a quadrilateral with all sides equal in length.
6. A square is a rectangle with two adjacent sides equal in length or a rhombus with all angles equal to 90°.

Properties of special quadrilaterals

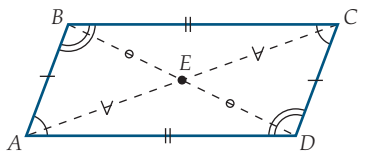
Trapezium

- A trapezium has no particular properties.



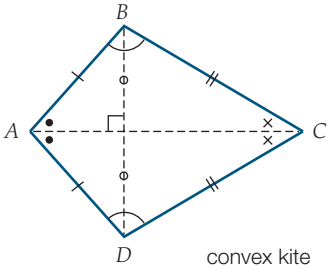
Parallelogram

- Both pairs of opposite angles are equal in magnitude: $\angle ABC = \angle ADC$ and $\angle BAD = \angle BCD$.
- Both pairs of opposite sides are equal: $AB = DC$ and $AD = BC$.
- Diagonals bisect each other: $AE = EC$ and $BE = ED$.

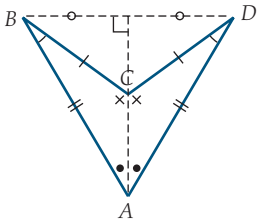


Kite

- One pair of opposite angles are equal in magnitude: $\angle ABC = \angle ADC$.
- A kite has an axis of symmetry that bisects the non-equal vertex angles through which it passes.
- The axis of symmetry of the kite bisects the other diagonal at right angles.



convex kite

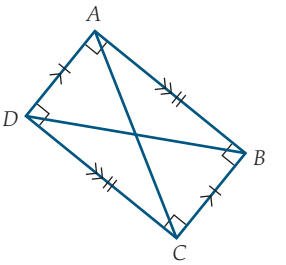


concave kite

Rectangle

All of the properties of a parallelogram, plus:

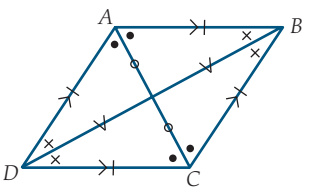
- Diagonals are equal in length: $AC = BD$.
- The lines joining the midpoints of opposite sides are lines of symmetry.



Rhombus

All of the properties of a parallelogram, plus:

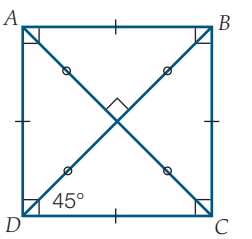
- Diagonals bisect each other at right angles: $AC \perp BD$.
- Diagonals bisect the vertex angles: $\angle DAC = \angle BAC = \angle DCA = \angle BCA$, $\angle ADB = \angle CDB = \angle ABD = \angle CBD$.
- Both diagonals are lines of symmetry.



Square

All of the properties of a rectangle and a rhombus, plus:

- Diagonals make an angle of 45° with each side.
- Each diagonal has length equal to $\sqrt{2}$ times the side length.

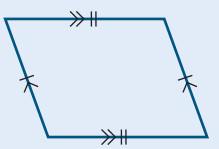


Worked Example 7

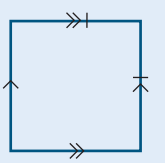
WE7

For each of the following shapes, write down the information shown, then use the definitions for quadrilaterals to determine the type of shape.

(a)



(b)



Working

(a) Both pairs of opposite sides are equal and parallel.

The shape is a parallelogram because both pairs of opposite sides are parallel.

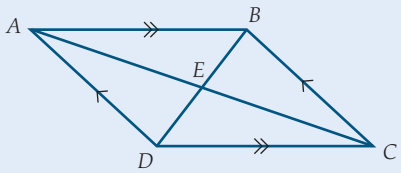
(b) Opposite sides parallel means that the shape is a parallelogram.

Two adjacent sides equal in length means that the parallelogram is a rhombus.

Worked Example 8

WE 8

ABCD is a parallelogram.



(a) By drawing the diagonal *AC*, prove that $\Delta ADC \equiv \Delta CBA$.

(b) Use these triangles to prove that $\angle ADC = \angle ABC$ and $\angle DAB = \angle BCD$.

(c) What special property of a parallelogram does this prove?

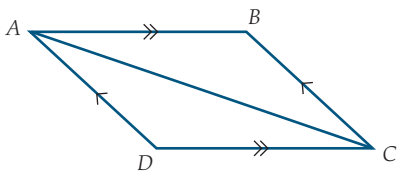
(d) By now drawing the diagonal *BD*, prove that $\Delta CDE \equiv \Delta ABE$.

(e) Use these triangles to prove that $AE = CE$ and $DE = BE$.

(f) What special property of a parallelogram does this prove?

Working

(a) Consider ΔADC and ΔCBA.



$\angle DCA = \angle BAC$ (alternate angles, $AB \parallel DC$)
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
AC is a common side

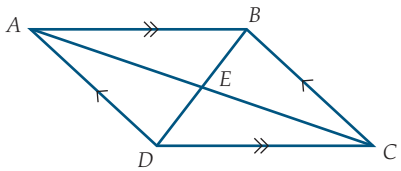
$\Delta ADC \equiv \Delta CBA$ (AAS)

(b) $\angle ADC = \angle ABC$ (matching angles in congruent triangle)

$\angle DAB = \angle DAC + \angle BAC$
$\angle BCD = \angle BCA + \angle DCA$
But $\angle DAC = \angle BCA$ and $\angle DCA = \angle BAC$ (from part (a))

(c) The property proved is that the opposite angles of a parallelogram are equal.

(d) Consider ΔABE and ΔCDE.



$AB = CD$ ($\Delta ADC \equiv \Delta CBA$)
$\angle DCE = \angle BAE$ (alternate angles, $AB \parallel DC$)
$\angle CDE = \angle ABE$ (alternate angles, $AB \parallel DC$)
$\angle ABE = \angle CDE$ (AAS)

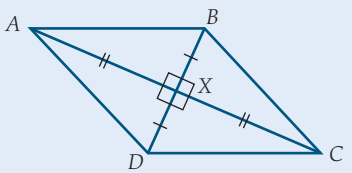
(e) $AE = CE$ and $DE = BE$ (matching sides in congruent triangles)

(f) The property proved is that the diagonals of a parallelogram bisect each other.

Worked Example 9

WE 9

The diagonals of a quadrilateral bisect each other at right angles. Prove that this quadrilateral is a rhombus.



Working

Two adjacent sides must be proved to be equal in length and the opposite sides must be proved to be parallel.

Consider ΔAXD and ΔAXB.

$\angle AXD = \angle AXB = 90°$	(given)
AX is a common side	
$DX = BX$	(given)
$\Delta AXD \equiv \Delta AXB$	(SAS)
$AD = AB$	(matching sides in congruent triangles)

Consider ΔAXD and ΔCXB.

$AXD = \angle CXB = 90°$	(given)
$AX = CX$	(given)
$DX = BX$	(given)
$\Delta AXD \equiv \Delta CXB$	(SAS)
$\angle DAX = \angle BCX$	(matching angles in congruent triangles)
$AD \parallel BC$	(alternate angles $\angle DAX$ and $\angle BCX$ are equal)

Similarly, $AB \parallel CD$.

$AB \parallel CD$ and $AD \parallel BC$	(opposite sides parallel)
$AD = AB$	(two adjacent sides equal)
So the shape is a rhombus.	

Answers page 533

8.5 Geometric properties of special quadrilaterals

Fluency

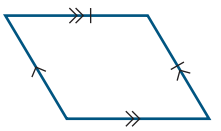
WE7

1 For each of the following shapes:

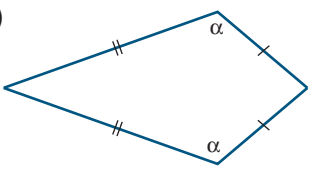
(i) write down the information shown

(ii) use the definitions for quadrilaterals to determine the type of shape.

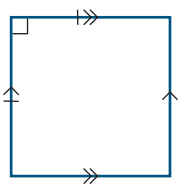
(a)



(b)



(c)

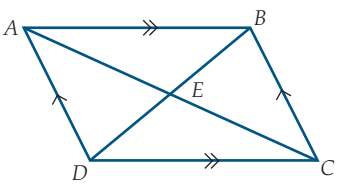


WE8

2 $ABCD$ is a parallelogram Use the diagram shown to prove the following properties of parallelograms.

(a) By drawing the diagonal BD, prove that $\Delta DAB \equiv \Delta DCB$.

(b) Use these triangles to prove that $\angle DAB = \angle DCB$ and $\angle ADC = \angle ABC$.



(c) What special property of a parallelogram does this prove?

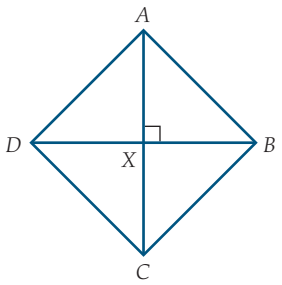
(d) By now drawing the diagonal AC, prove that $\Delta DAE \equiv \Delta BCE$.

(e) Use these triangles to prove that $AE = CE$ and $DE = BE$.

(f) What special property of a parallelogram does this prove?

3 The diagonals of a quadrilateral are equal in length and bisect each other at right angles. Prove that this quadrilateral is a square.

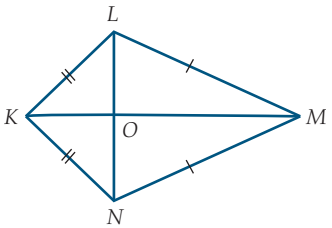
WE9



4 Use the diagram shown to prove the following properties of a kite.

(a) A kite has one pair of opposite equal angles.

(b) A kite has diagonals that cross at right angles.



5 When proving that a quadrilateral is a trapezium but not a parallelogram, it is necessary to show:

A only one pair of parallel sides

B one pair of parallel sides and one pair of non-parallel sides

C one pair of parallel sides and one pair of equal sides

D two pairs of parallel sides.

6 A quadrilateral must be a parallelogram if one pair of opposite sides is:

A only equal in length

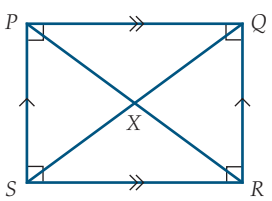
B only parallel

C equal in length and parallel

D parallel and the other pair of opposite sides is equal in length.

7 **(a)** For the diagram shown, prove $PQ = RS$.

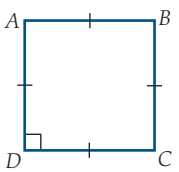
(b) What special property of a rectangle have you just proved?



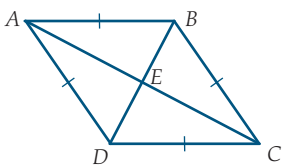
Understanding

8 Prove the following using the diagram shown:

(a) If a quadrilateral has all sides equal and one angle is a right angle, then it is a square.

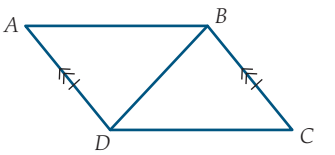


(b) If all sides of a quadrilateral are equal, then the quadrilateral is a rhombus.

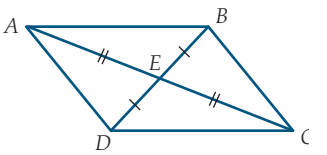


9 Prove the following using the diagrams.

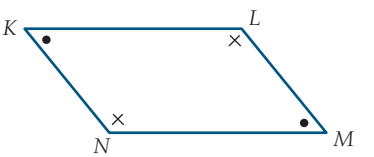
(a) If a pair of opposite sides in a quadrilateral is parallel and equal, then the quadrilateral is a parallelogram.



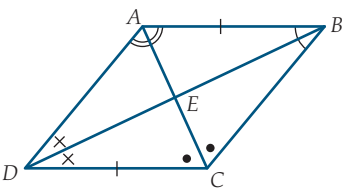
(b) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



(c) If both pairs of opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.



(d) If both diagonals of a quadrilateral bisect the vertex angles through which they pass, then the quadrilateral is a rhombus.

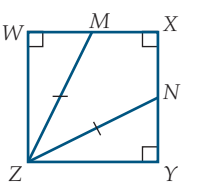


10 To prove that a parallelogram must be a rectangle it is necessary to prove that its diagonals:

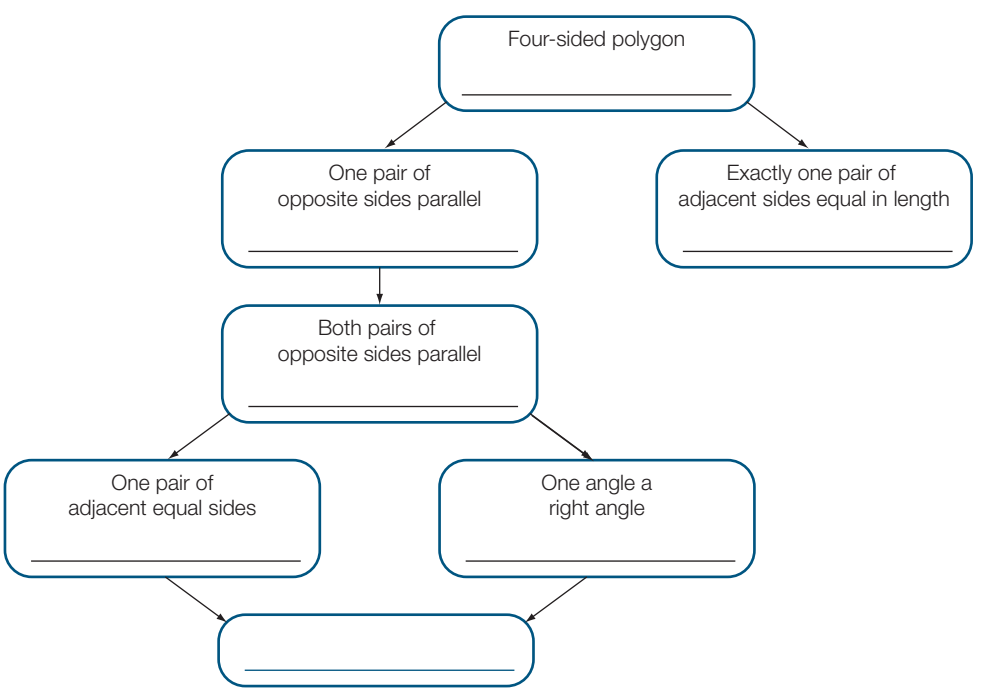
A are equal in length **B** bisect each other

C bisect the angles of the parallelogram **D** are perpendicular to each other.

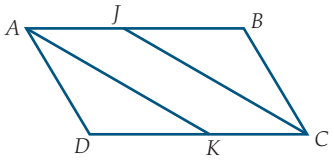
11 $WXYZ$ is a square. Show that $\Delta WMZ \equiv \Delta YNZ$.



12 Copy and complete the following flowchart to show the relationships between different quadrilaterals.



Reasoning



13 $ABCD$ is a parallelogram and $AJ = CK$.
Show that $\Delta AKD \equiv \Delta CJB$.

14 Explain why a square is a rhombus, but a rhombus is not necessarily a square.

15 Explain why a square is a rectangle, but a rectangle is not necessarily a square.

Open-ended

16 Adam and Alvin were discussing how many different quadrilateral shapes they could use to make biscuits. Choose three different quadrilateral shapes that they could use. The shapes need to have one of the following properties in common:

- at least one pair of parallel sides
- at least one pair of equal sides
- at least one pair of equal angles.

Draw the shapes and name them, and then identify the property you have used.

17 **(a)** The statement 'All squares have equal sides' is true. The converse statement is 'All shapes with four equal sides are squares'. Is this true? Explain why or why not.

(b) Write two or more true statements about special quadrilaterals and their converse statements. Examine whether or not the converse statements are true.

18 **(a)** Draw any three rhombuses and mark in the diagonals.

(b) Measure the angles at the centre where the diagonals intersect.

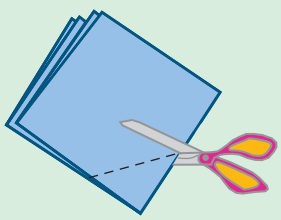
(c) What property of rhombuses does this seem to show?

(d) Is this sufficient proof of this property?

19 Gary, Matt and George are arguing about the geometric figure that could have the property that the diagonals bisect at right angles. Matt says there is only one such figure, Gary says there are two and George says there are three or more. Who is right and what shape could the figure(s) be?

Problem solving

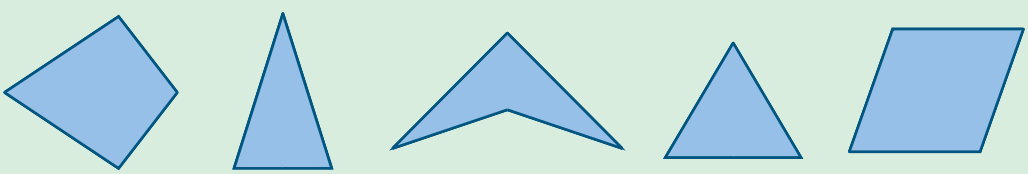
Folding and cutting



Equipment required: some loose sheets of paper and scissors

1 You are going to fold a piece of paper twice and then cut across the dotted corner. Can you predict the shape of the resulting hole?

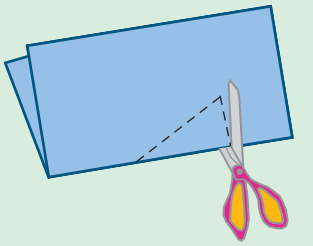
The shape of the resulting hole will depend on the angle at which you cut and also the angle of the second fold. See if you can produce the following shaped holes.



Can you find the maximum number of sides of the hole such a single cut produces for a piece of paper folded twice?

2 Now try folding the sheet of paper just once, but using two cuts to produce a hole when unfolded. What shape will your hole be?

Can you produce an equilateral triangle, an isosceles triangle and a scalene triangle?



Strategy options

- Act it out.
- Make a model.

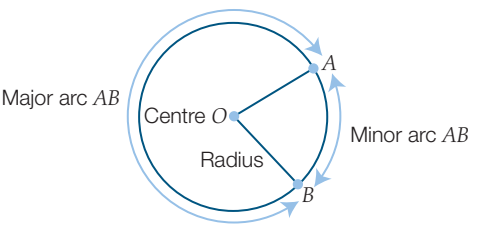
8.6 Angles in circles

10A

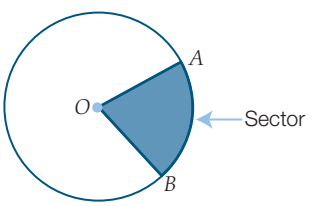
Need to Know

Terms related to circles

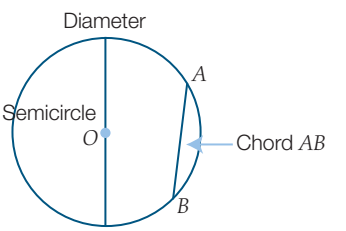
- A circle is a plane curve consisting of all the points at a given distance (called the radius) from a fixed point in the plane (called the centre).



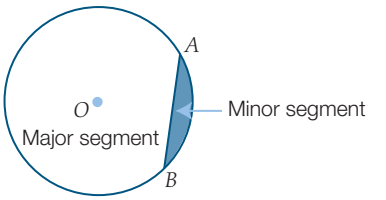
- An **arc** of a circle is a portion of the circle bounded by two points on the circle. The shorter arc is the **minor arc** and the longer arc is the **major arc**. An arc that goes completely around the circle is called the circumference.
- A **sector of a circle** is the plane figure bounded by two radii of the circle and one of the arcs that they form.



- A **chord** of a circle is an interval joining any two points on the circle. A chord that passes through the centre of a circle is called a diameter. The length of the diameter is twice the radius. A diameter divides the circle into two semicircles.

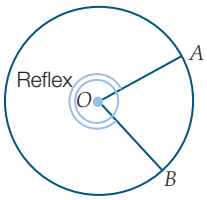


- A **segment** of a circle is the plane figure bounded by a chord and an arc. A **minor segment** is bounded by a chord and a minor arc. A **major segment** is bounded by a chord and a major arc.



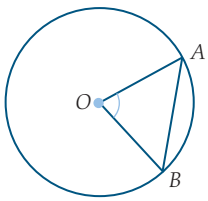
- The word **subtend** means 'to be opposite to' or 'measure off'. The side of a triangle subtends the opposite angle, and an arc of a circle subtends an angle at the centre of a circle. We also say that the angle at the centre of a circle is **subtended** by the arc or the chord of the circle.

The major arc *AB* subtends the reflex angle *AOB* at the centre, *O*, of the circle.



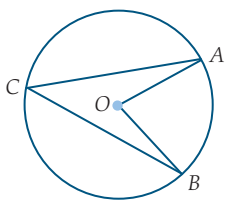
The minor arc *AB* subtends the acute or obtuse angle *AOB* at the centre, *O*, of the circle.

If you join *AB*, then the chord *AB* subtends angle *AOB* at the centre, *O*.



- Angles may be subtended at the centre of the circle or at the circumference. We say that they stand on the same arc or they are subtended by the same chord if the arms of the angle start from and end at the same points on the circumference of the circle.

$\angle ACB$ is called the **angle at the circumference** standing on the arc *AB*. $\angle AOB$ is the **angle at the centre** standing on the arc *AB*. If the chord *AB* had been drawn, we would say that these angles were standing on the chord *AB* or they were subtended by the chord *AB*.

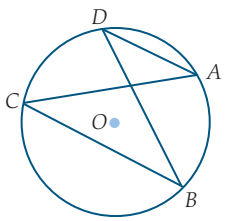


- Angles at the circumference of a circle drawn from the same two points on the circumference are said to be standing on the same arc. If they are on the same side of the chord they are called **angles in the same segment**.

$\angle BCA$ and $\angle BDA$ are angles at the circumference standing on the minor arc *AB*.

They are in the same segment because they are on the same side of the chord *AB*.

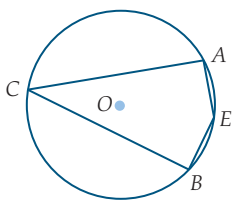
Angles *CBD* and *CAD* are angles at the circumference standing on the minor arc *CD*.



$\angle BCA$ is an angle at the circumference standing on the minor arc *AB*.

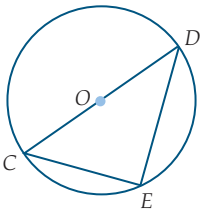
$\angle BEA$ is an angle at the circumference standing on the major arc *AB*.

They are in opposite segments as they are on opposite sides of the chord *AB*.



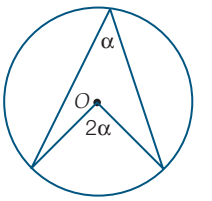
- The **angle in a semicircle** is formed when the ends of a diameter are joined to a point on the circumference.

  $\angle DEC$ is an angle in a semicircle.



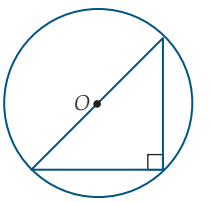
Theorem 1

The angle at the centre is twice the angle at the circumference standing on the same arc.



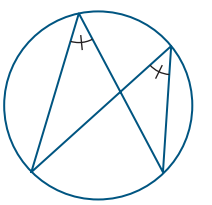
Theorem 2

The angle in a semicircle is a right angle.



Theorem 3

Angles in the same segment subtended by the same arc at the circumference are equal.

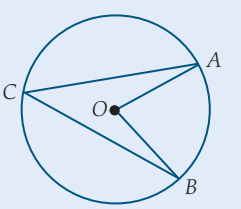


Worked Example 10

WE10

A, *B* and *C* are three points on the circumference of a circle with centre *O*.

Prove that $\angle AOB = 2\ \angle ACB$.



Working

$OC = OA$ (radii of a circle)

$\therefore \Delta OCA$ is isosceles

$\therefore \angle OCA = \angle OAC$ (equal angles in isosceles triangle)

$OC = OB$ (radii of a circle)

$\therefore \Delta OCB$ is isosceles

$\therefore \angle OCB = \angle OBC$ (base angles of isosceles trianges)

$\angle AOD = \angle ACO + \angle CAO$ (exterior angle of ΔAOC equals sum of interior opposite angles)

$\therefore \angle AOD = 2\angle ACO$ ($\angle OCA = \angle OAC$)

$\angle BOD = \angle BCO + \angle CBO$ (exterior angle of ΔBOC equals sum of interior opposite angles)

$\therefore \angle BOD = 2\angle BCO$ ($\angle OCB = \angle OBC$)

Now $\angle ACB = \angle ACO + \angle BCO$

and $\angle AOB = \angle AOD + \angle BOD$

$\therefore \angle AOB = 2\angle ACO + 2\angle BCO$ (from above)

$\angle AOB = 2(\angle ACO + \angle BCO)$

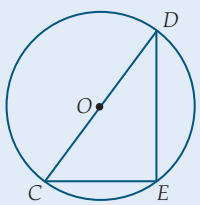
$\therefore \angle AOB = 2\angle ACB$

Hence, the angle at the centre is twice the angle at the circumference standing on the same arc. (Theorem 1)

Worked Example 11

WE11

CD is a diameter of a circle with centre O. E is a point on the circumference. Prove that $\angle CED$ is a right angle.



Working

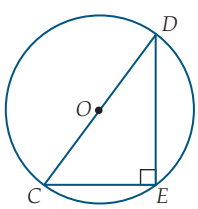
$\angle COD = 2\angle CED$ (angle at the centre is twice the angle at the circumference standing on the same arc CD)

But $\angle COD = 180^\circ$ (COD is a straight line)

$\therefore \angle CED = 90^\circ$

$\angle CED$ is a right angle.

Hence, the angle in a semicircle is a right angle. (Theorem 2)

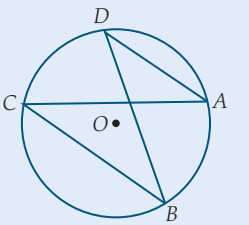


Worked Example 12

WE12

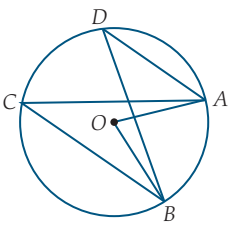
A, B, C and D are four points on the circumference of a circle with centre O.

Prove that $\angle ACB = \angle ADB$.



Working

$\angle AOB = 2\angle ADB$ (angle at the centre is twice the angle at the circumference standing on the same arc)



Similarly:

$\angle AOB = 2\angle ACB$

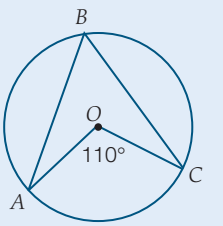
$\angle ACB = \angle ADB$ (both half of $\angle AOB$)

Hence, angles in the same segment subtended by the same arc at the circumference are equal. (Theorem 3)

Worked Example 13

WE13

Find the size of $\angle ABC$, giving reasons for your answer.



Working

$\angle AOC = 2\angle ABC$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$\angle AOC = 110°$

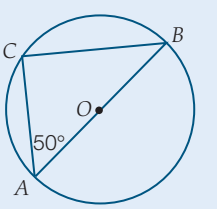
$\therefore 110° = 2\angle ABC$

$\therefore \angle ABC = 55°$

Worked Example 14

WE14

AB is a diameter of the circle with centre O. $\angle CAB = 50°$. Find the size of $\angle ABC$, giving reasons for your answer.



Working

$\angle ACB = 90°$ (angle in a semicircle is 90°)

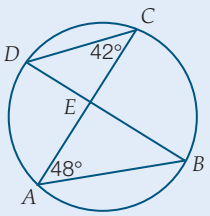
$\angle BAC = 50°$

$\angle ABC + 50° + 90° = 180°$ (angle sum of a triangle is 180°)
$\therefore \angle ABC = 40°$

Worked Example 15

WE15

A, *B*, *C* and *D* are points on the circumference of the circle, *AC* and *DB* intersect at *E*. If $\angle CAB = 48°$ and $\angle DCA = 42°$, find the size of $\angle AEB$, giving reasons for your answer.



Working

$\angle ACD = \angle ABD$ (angles at the circumference on the same arc)
$\therefore \angle ABD = 42°$

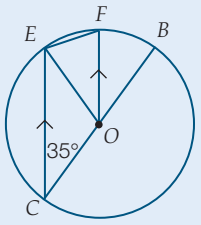
$\angle EAB = 48°$ (given)

$\angle AEB + 42° + 48° = 180°$ (angle sum of a triangle is 180°)
$\therefore \angle AEB = 90°$

Worked Example 16

WE16

BC is a diameter of the circle with centre *O*. *E* and *F* are points on the circumference of the circle such that *OF* || *CE*. If $\angle OCE = 35°$, find the size of $\angle OFE$, giving reasons for your answer.



Working

$\angle FOB = \angle ECO$ (corresponding angles, *FO* || *EC*)
$\therefore \angle FOB = 35°$

$\angle EOB = 2\angle ECB$ (angle at the centre is twice the angle at the circumference standing on the same arc)
$\angle EOB = 2 \times 35° = 70°$

$\angle EOF + 35° = 70°$
$\therefore \angle EOF = 35°$

ΔOEF is isosceles (*OE* = *OF*, radii of the circle)

$\angle OEF = \angle OFE$ (base angles of an isosceles triangle)

But $\angle OEF + \angle OFE + 35° = 180°$ (angle sum of a triangle)

$\therefore 2\angle OFE = 145°$
$\angle OFE = 72.5°$

Answers page 535

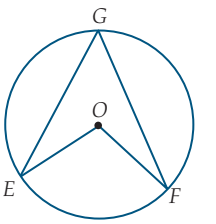
8.6 Angles in circles

10A

Fluency

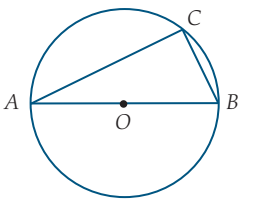
WE10

1 E, F and G are three points on the circumference of a circle with centre O. Prove that $\angle EOF = 2\angle EGF$.



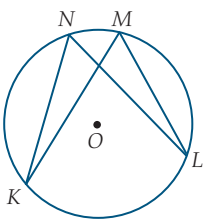
WE11

2 AB is a diameter of a circle with centre O. C is a point on the circumference. Prove that $\angle ACB = 90°$.



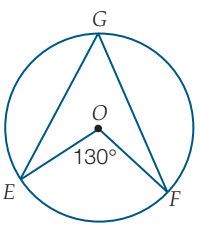
WE12

3 K, L, M and N are four points on the circumference of a circle with centre O. Prove that $\angle KML = \angle KNL$.



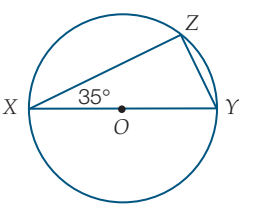
WE13

4 $\angle EOF = 130°$. Find the size of $\angle EGF$, giving reasons for your answer.



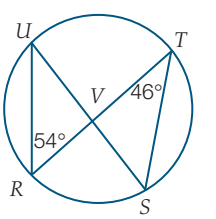
WE14

5 XY is a diameter of the circle with centre O. $\angle ZXY = 35°$. Find the size of $\angle ZYX$, giving reasons for your answer.



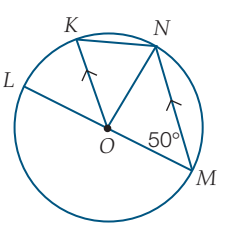
WE15

6 R, S, T and U are points on the circumference of a circle. RT and SU intersect at V. If $\angle URT = 54°$ and $\angle RTS = 46°$, find the size of $\angle TVS$, giving reasons for your answer.



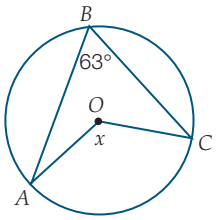
WE16

7 LM is a diameter of the circle with centre O. K and N are points on the circumference of the circle such that $OK \parallel MN$. If $\angle OMN = 50°$, find the size of $\angle OKN$, giving reasons for your answer.

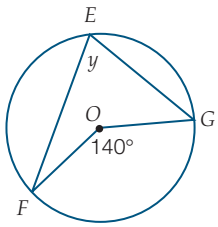


8 Find the value of the variables in each figure. Give reasons for your answer.

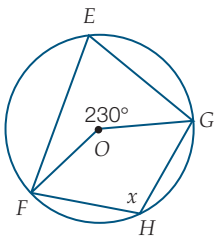
(a)



(b)

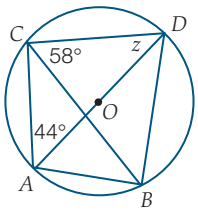


(c)

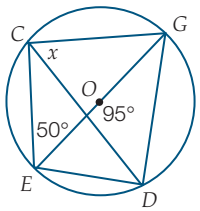


9 Find the value of the variables in each figure. Give reasons for your answer.

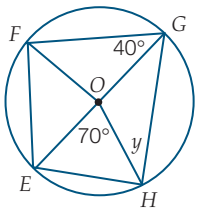
(a)



(b)

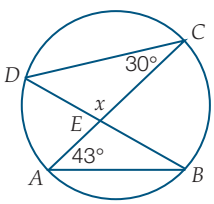


(c)

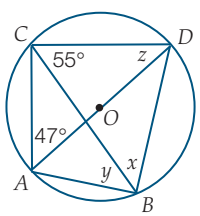


10 Find the value of the variables in each figure. Give reasons for your answer.

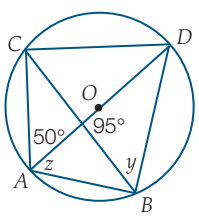
(a)



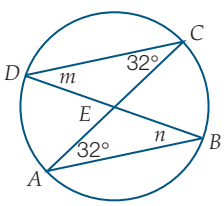
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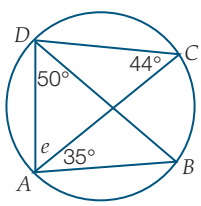
(c)



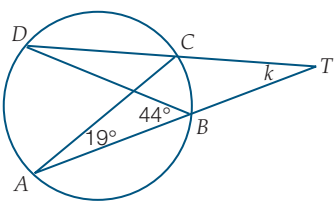
(d)



(e)



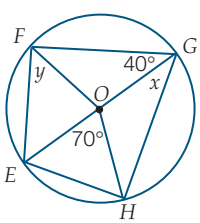
(f)



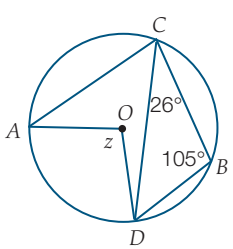
Understanding

11 Find the value of the variables in each figure. Give reasons for your answer.

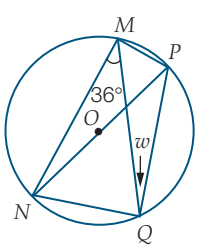
(a) EG is the diameter of the circle



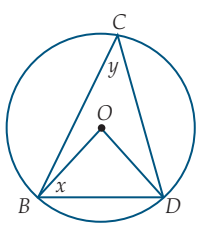
(b) $AC \parallel DB$



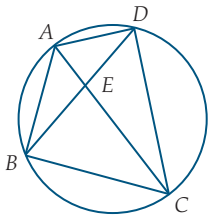
(c) $NM = MQ$, NP is the diameter



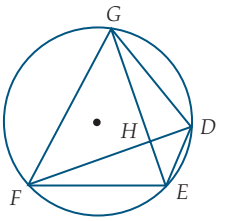
12 O is the centre of the circle. Prove that $x + y = 90°$.



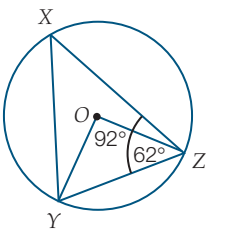
13 AC is the diameter of the circle. If $\angle BDC = 52°$ and $BD = DC$, calculate the size of $\angle ACD$ giving reasons for your answer.



14 If $\angle FGE = 48°$, $\angle GED = 42°$ and $\angle DFE = 20°$, calculate the size of each angle in the quadrilateral $GDEF$, giving reasons for your answers. What can you say about the diagonals of this quadrilateral?

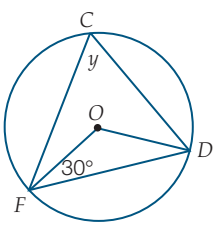


15 If $\angle YOZ = 92°$ and $\angle XZY = 62°$, calculate the size of $\angle OXZ$ giving reasons for your answer.

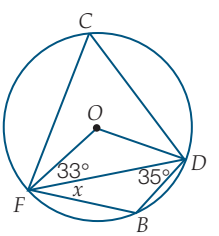


16 Find the value of the variables in each figure. Give reasons for your answer.

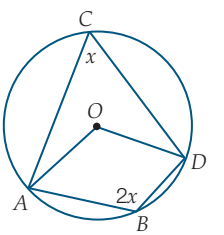
(a)



(b)

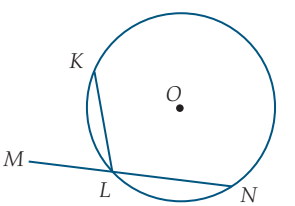


(c)



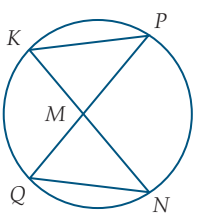
Reasoning

17 $KL = LN$ in the circle with centre O. Prove that $\angle KLM = \angle LON$.

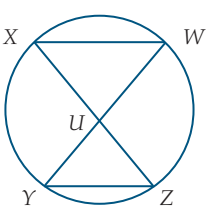


18 **(a)** Prove that $\Delta KPM \,|||\, \Delta QNM$.

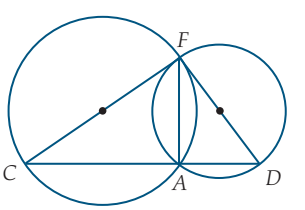
(b) If $KM = MQ$ prove that $MP = MN$.



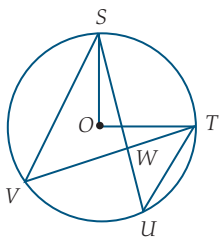
19 XZ and WY intersect at U. If $XU = UW$ prove that $XW \parallel YZ$.



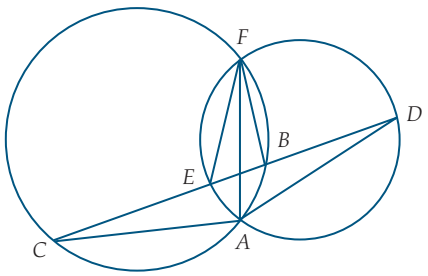
20 CF is a diameter of the circle CFA. FD is a diameter of the circle FDA. Prove that C, A and D are collinear.



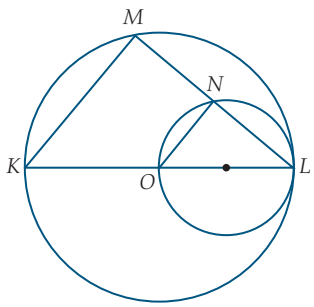
21 In the circle with centre O, $VS \parallel UT$ and $SO \perp OT$. Calculate the size of $\angle TWU$. Give reasons for your answer.



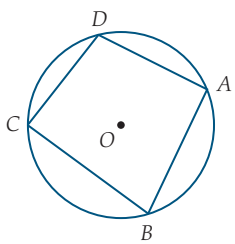
22 If $CA = AD$, prove that AF bisects $\angle EFB$.



23 O is the centre of the circle KML. KL is a diameter. OL is a diameter of the circle ONL. Prove that $KM \parallel ON$.

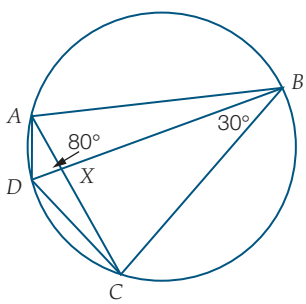


24 A cyclic quadrilateral is a quadrilateral whose vertices lie on the circumference of a circle. $ABCD$ is a cyclic quadrilateral in a circle with centre O. By joining CO and OA, prove that $\angle CDA + \angle CBA = 180°$.



Open-ended

25 $ABCD$ is a cyclic quadrilateral whose diagonals intersect at X. $\angle DBC = 30°$ and $\angle AXD = 80°$.



(a) Find the size of:

(i) $\angle DAC$ **(ii)** $\angle ADB$ **(iii)** $\angle ACB$

(b) If $\angle BAC < 80°$ choose a size for $\angle BAC$ and then find the size of:

(i) $\angle BDC$ **(ii)** $\angle DCA$ **(iii)** $\angle DBA$

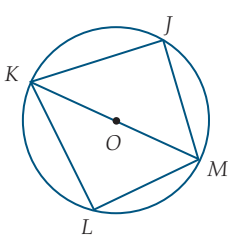
(c) If $\angle BAC = 50°$ write something about:

(i) $\angle ADC$ **(ii)** $\angle ABC$

(d) If $\angle BAC = 10°$ write something about:

(i) $\angle ADB$ **(ii)** $\angle BCD$

26 KM is a diameter of the circle with centre O. J and L are points on the circumference. Claire claims that $JKLM$ is a square; James claims that $JKLM$ is a kite. Who is correct, or is neither correct? Explain your decision.

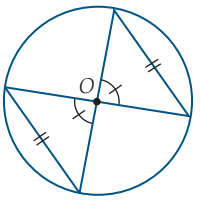


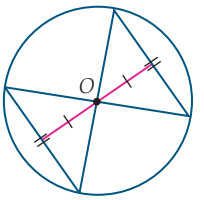
8.7 Chords of circles

10A

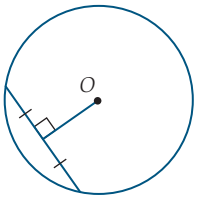
Need to Know

Theorem 4
Chords of equal length in a circle subtend equal angles at the centre and are **equidistant** from the centre.





Theorem 5
The perpendicular from the centre of a circle to a chord bisects the chord.



Theorem 6
The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Theorem 7
The perpendicular bisector of a chord of a circle passes through the centre of the circle.

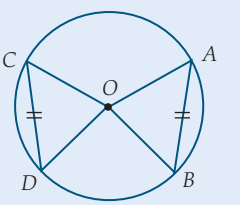
Worked Example 17

WE17

AB and *CD* are equal chords of the circle with centre *O*.

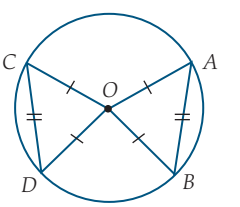
(a) Prove that $\angle AOB = \angle COD$.

(b) *OE* and *OF* are the perpendiculars drawn from *O* to *AB* and *CD*, respectively. Prove that *OE* = *OF*.



Working

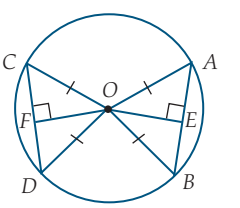
(a) Consider ΔOAB and ΔOCD.



OA = OC	(radii of circle)
AB = CD	(given, equal chords)
BO = DO	(radii of circle)
ΔOAB ≡ ΔOCD	(SSS)
$\angle AOB = \angle COD$	(corresponding angles of congruent triangles)

(b) $\angle BAO = \angle DCO$ (corresponding angles in congruent triangles)

Consider ΔEAO and ΔFCO.



OA = OC	(radii of circle)
$\angle EAO = \angle FCO$	(corresponding angles in congruent triangles OAB and OCD)
$\angle AEO = \angle CFO$	(both right angles, $OE \perp AB$ and $FO \perp CD$)
ΔEAO ≡ ΔFCO	(AAS)
OE = OF	(corresponding sides of congruent triangles)

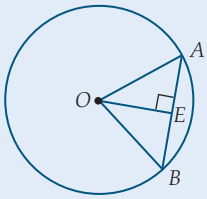
Hence, chords of equal length in a circle subtend equal angles at the centre and are equidistant from the centre. (Theorem 4)

Worked Example 18

WE18

AB is a chord of a circle with centre O. $OE \perp AB$.

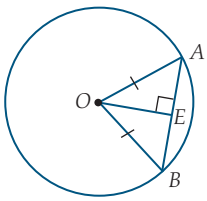
Prove that $AE = EB$.



Working

Consider ΔEAO and ΔEBO.

$\angle OEA = \angle OEB = 90°$	($OE \perp AB$)
$OA = OB$	(radii of circle)
OE is a common side.	
$\Delta EAO \equiv \Delta EBO$	(RHS)
$AE = EB$	(corresponding sides in congruent triangles)



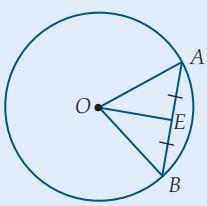
Hence, the perpendicular from the centre of a circle to a chord bisects the chord. (Theorem 5)

Worked Example 19

WE19

AB is a chord of a circle with centre O. E is the midpoint of AB.

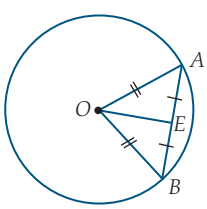
Prove that $OE \perp AB$.



Working

Consider ΔEAO and ΔEBO.

$AE = EB$	(E is midpoint of AB)
$OA = OB$	(radii of circle)
OE is a common side.	
$\Delta EAO \equiv \Delta EBO$	(SSS)
$\angle OEA = \angle OEB$	(corresponding angles of congruent triangles)
But $\angle OEA = 180° - \angle OEB$	(straight angle)
$\therefore \angle OEA = \angle OEB = 90°$	
$\therefore OE \perp AB$	



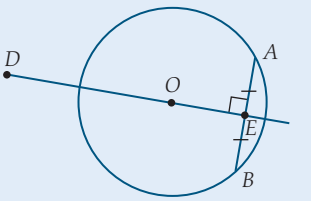
Hence, the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. (Theorem 6)

Worked Example 20

WE20

E is the midpoint of the chord AB in a circle with centre O. DE is the perpendicular bisector of AB. Prove that O lies on DE.

When constructed, DE appears to pass through O. To prove this is true we need to use a different sort of proof.

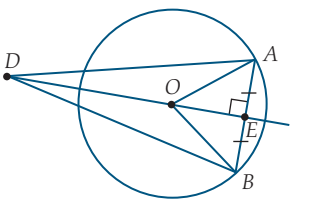


Working

Consider ΔDAE and ΔDBE.

$DA = DB$ (matching sides in congruent triangles)

$OA = OB$ (radii of the circle centre O on which A and B lie)



O is equidistant from A and B (it must lie on the perpendicular bisector of AB)

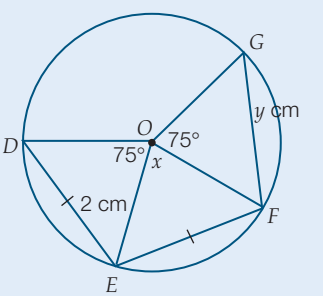
$\therefore O$ lies on DE

Hence, the perpendicular bisector of a chord of a circle passes through the centre of the circle. (Theorem 7)

Worked Example 21

WE21

Calculate the value of the pronumerals. Give reasons for your answers.



Working

$DE = EF$ (equal chords marked on diagram)

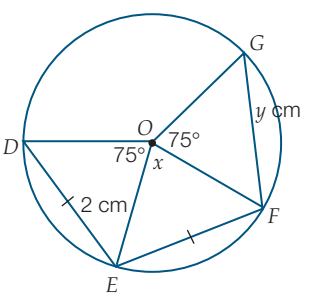
$\therefore \angle DOE = \angle EOF$ (equal chords subtend equal angles at the centre of a circle)

$\therefore x = 75°$

$\angle DOE = \angle FOG = 75°$

$\therefore DE = GF$ (equal angles at the centre of a circle are subtended by equal chords)

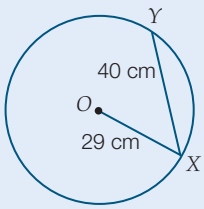
$\therefore y = 2$ cm



Worked Example 22 WE22

XY is a chord of a circle with centre O.

If $XY = 40$ cm and $OX = 29$ cm, calculate the perpendicular distance of XY from O. Give reasons for your answer.



Working

$XZ = ZY$ (perpendicular from the centre of a circle to a chord bisects the chord)

$\therefore XZ = 20$ cm (half of XY)

$OX^2 = XZ^2 + OZ^2$ (Pythagoras' theorem)

Hence, $OZ^2 = OX^2 - XZ^2$

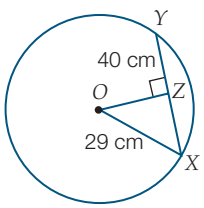
$\therefore OZ^2 = 29^2 - 20^2$

$= 841 - 400$

$= 441$

$\therefore OZ = 21$ cm

XY is 21 cm from O.



8.7 Chords of circles

10A

Answers page 538

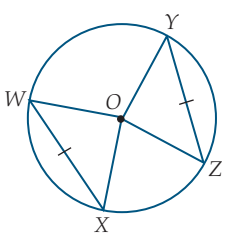
Fluency

1 WX and YZ are equal chords of a circle with centre O.

(a) Prove that $\angle WOX = \angle YOZ$.

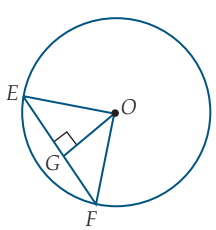
(b) OP and OQ are the perpendiculars drawn from O to WX and YZ, respectively. Prove that $OP = OQ$.

WE17



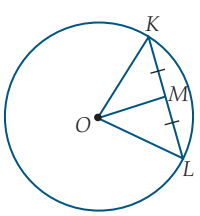
2 EF is a chord of a circle with centre O. $OG \perp EF$. Prove that $EG = GF$.

WE18



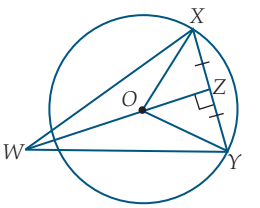
3 KL is a chord of a circle with centre O. M is the midpoint of KL. Prove that $OM \perp KL$.

WE19



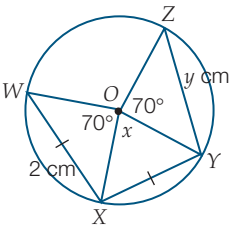
WE20

4 Z is the midpoint of the chord XY in a circle with centre O. WZ is the perpendicular bisector of XY. Prove that O lies on WZ.



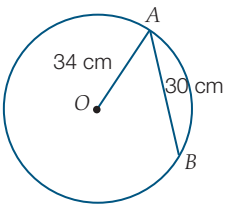
WE21

5 Calculate the value of the pronumerals. Give reasons for your answers.



WE22

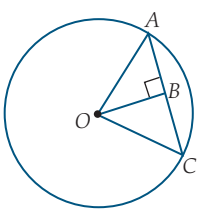
6 AB is a chord of a circle with centre O. If $AB = 30$ cm and $OA = 34$ cm, calculate the perpendicular distance of AB from O. Give reasons for your answer.



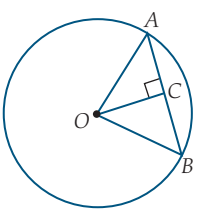
Understanding

7 Calculate the length of AB in each diagram, giving reasons for your answer.

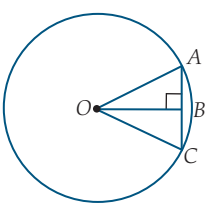
(a) $AC = 10$ cm, $OB \perp AC$



(b) $AC = 10$ cm, $OC \perp AB$

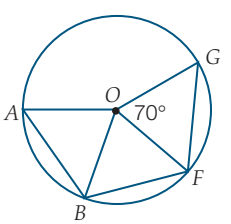


(c) $OA = 17$ cm, $OB = 8$ cm

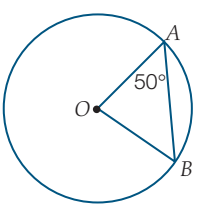


8 Calculate the size of $\angle AOB$ in each figure, giving reasons for your answer.

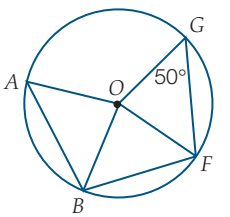
(a) $AB = GF$, $\angle GOF = 70°$



(b) $\angle OAB = 50°$

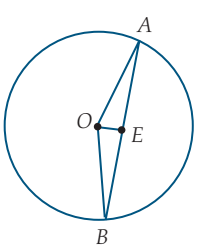


(c) $AB = GF$, $\angle OGF = 50°$

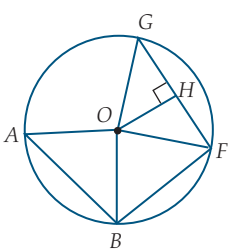


9 Calculate the length of OA in each figure, giving reasons for your answer.

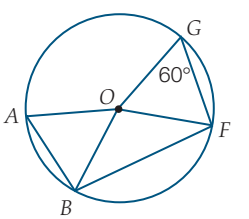
(a) $AB = 48$ cm, $OE = 7$ cm, E is the midpoint of AB.



(b) $GH = HF = 21$ cm, $OH = 20$ cm, $AB = GF$

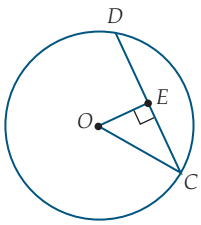


(c) $\angle OGF = 60°$, $AB = GF = 10$ cm

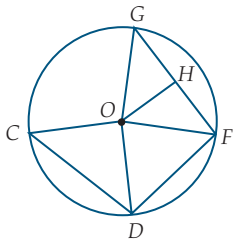


10 Calculate the length of CD in each diagram, giving reasons for your answer.

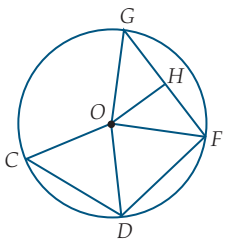
(a) $OE \perp CD$, $OC = 53$ cm, $OE = 28$ cm



(b) $\angle FHO = 90°$, $\angle FOD = \angle DOC$, $GF = FD$, $OF = 73$ cm, $OH = 48$ cm

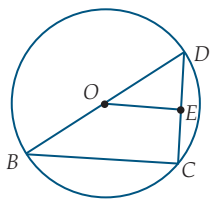


(c) $\angle GOF = \angle COD$, $GH = HF$, $OG = 85$ cm $OH = 77$ cm

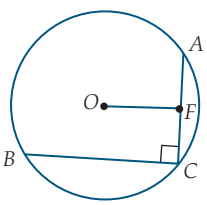


Reasoning

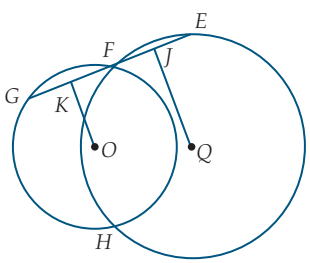
11 BD is the diameter of a circle with centre O. C is a point on the circumference of the circle. E is the midpoint of CD. Prove that $OE \parallel BC$, and hence that $\angle BCD = 90°$.



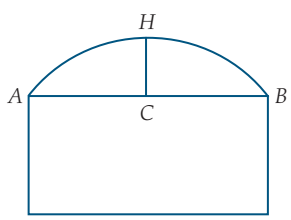
12 O is the centre of the circle ACB. $AF = FC$ and $\angle ACB = 90°$. Mark D, the midpoint of BC, and use it to prove that $BC = 2OF$.



13 The circles GFH, with centre O, and FEH, with centre Q, intersect at F and H. GFE is a straight line. K is the midpoint of GF and J is the midpoint of FE. Prove that $\angle KOQ + \angle OQJ = 180°$.



14 The entrance to a shopping centre is a rectangular doorway with a circular arch above it as shown in the diagram. The doorway AB is 4 m wide and the radius of the arch is 2.5 m. Calculate the height of the arch, CH. Justify your calculations.



15 KLM is an isosceles triangle inscribed in a circle of radius 25 mm. If $KL = KM$ and $LM = 48$ mm, calculate the length of KL. Give reasons to justify your calculation. (A triangle inscribed in a circle has its vertices on the circumference of the circle.)

Open-ended

16 The doorways to a shopping centre are being designed. The shape is to be similar to that in the diagram for Question **14**. The rectangular doorways are to be between 4 and 6 m wide and at least 2.5 m high. Each doorway is to have a circular arch on top of the rectangular door opening such that the total height of the door and the arch is between 3 and 4.5 m. Create two designs, giving the width and height of the rectangular doorway, the radius of the arch and the height of the arch above the centre of the doorway.

17 Construct the perpendicular bisectors of the sides KL and LM of a scalene triangle KLM. Explain why their point of intersection, X, is the centre of the circle through L, K and M.

18 Circles with centres E and F intersect at G and H. Draw this information on a diagram. Use your diagram to explain why **(i)** EF bisects GH and **(ii)** $CE \perp GH$.

Chapter review 8

D.I.Y. Summary

Key Words

bisect	included angle	property	similar
congruent	perpendicular	prove	theorem
definition	perpendicular bisector		

10A

angle at the centre	arc	major segment	segment
angle at the circumference	chord	minor arc	subtend
angle in a semicircle	equidistant	minor segment	subtended
angles in the same segment	major arc	sector of a circle	

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1 Triangles are _____ if they are identical in all aspects; i.e. they have exactly the same shape and size.

2 When applying SAS to test for congruence it is important that the angle is the _____.

3 Triangles are _____ if they have exactly the same shape, although their sizes may be different.

4 The minimum amount of information needed to identify a particular figure is called a _____.

5 A _____ is true about a particular figure but is not enough to define that figure.

6 A line at right angles to another line that also cuts that line in half is called a _____.

7 If we divide a line or angle into two equal parts, we _____ the line or angle.

8 If two radii are drawn to different points on the circumference of a circle they divide the circle into a _____ and a _____. 10A

9 A line joining one point on the circumference of the circle to another is called a _____. 10A

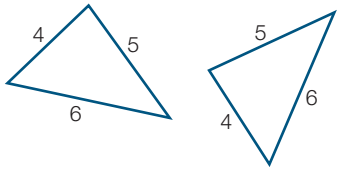
10 An arc will _____ an angle at the circumference. 10A

11 A chord, that is not a diameter, drawn on a circle divides the circle into a _____ and a _____. 10A

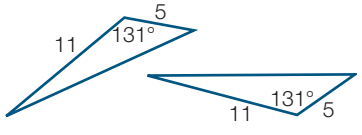
Fluency

1 Give reasons why the following pairs of triangles are congruent. Ex 8.1

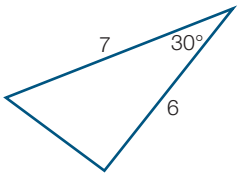
(a)



(b)

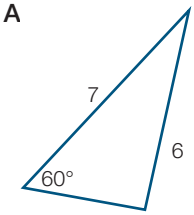


2 The triangle congruent with the triangle shown is:

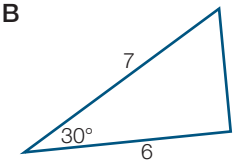


Ex 8.1

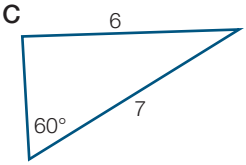
A



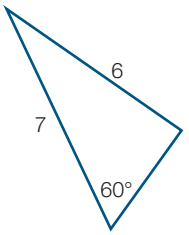
B



C



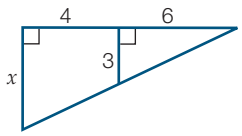
D



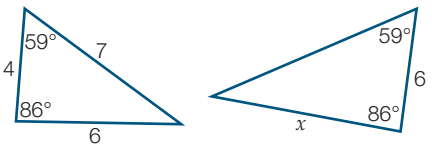
3 Find the value of x in each of the following pairs of triangles.

Ex 8.1

(a)



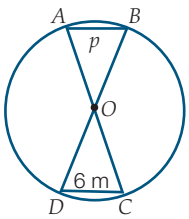
(b)



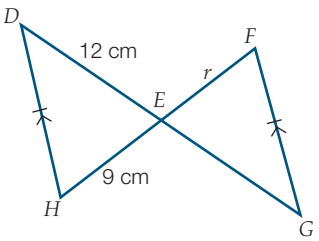
4 Prove that the following pairs of triangles are congruent and hence find the value of the pronumeral.

Ex 8.2

(a)



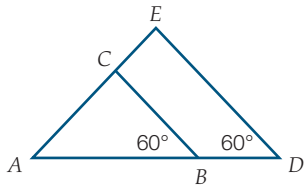
(b)



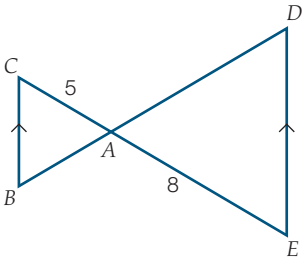
5 Prove that the following pairs of triangles, ΔABC and ΔADE, are similar.

Ex 8.2

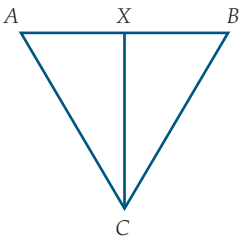
(a)



(b)

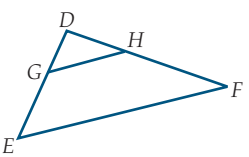


6 Given that CX bisects $\angle ACB$, and $CX \perp AB$, prove that $AX = XB$.



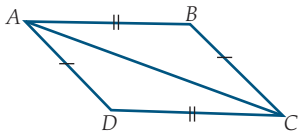
Ex 8.3

7 In ΔDEF, G is a point on DE and H is a point on DF such that $GH \parallel EF$. Prove that $\frac{DG}{GE} = \frac{DH}{HF}$.



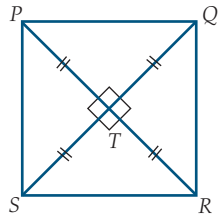
Ex 8.4

8 If both pairs of opposite sides of a quadrilateral are equal, prove that the quadrilateral is a parallelogram.



Ex 8.5

9 If the diagonals of a quadrilateral are equal and bisect each other at right angles, prove that the quadrilateral is a square.

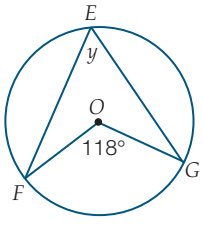


Ex 8.5

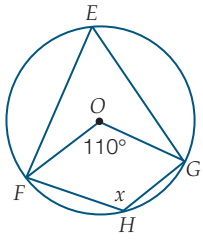
10 Find the value of the pronumerals in each figure. Give reasons for your answer.

Ex 8.6 10A

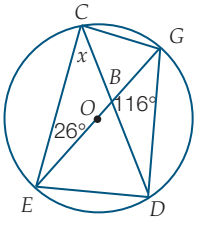
(a)



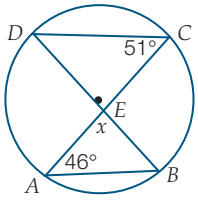
(b)



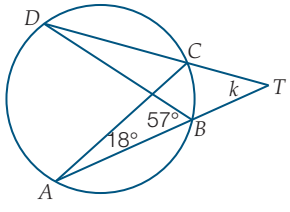
(c)



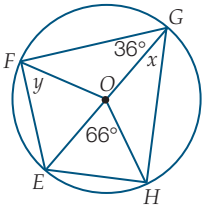
(d)



(e)



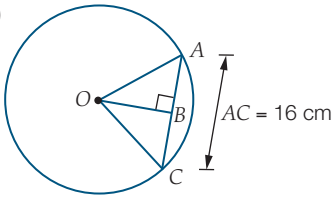
(f)



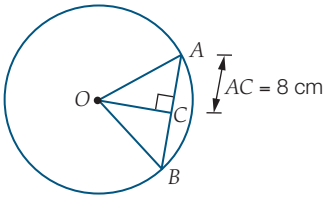
11 Calculate the length of AB in each diagram, giving reasons for your answer.

Ex 8.7 10A

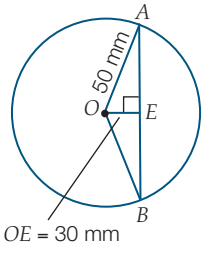
(a)



(b)



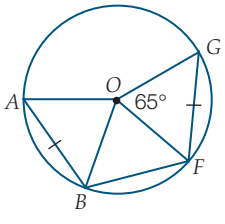
(c)



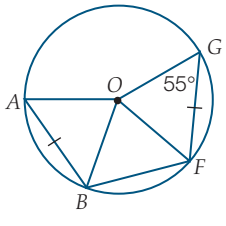
12 Calculate the size of $\angle AOB$ in each figure, giving reasons for your answer.

Ex 8.7 10A

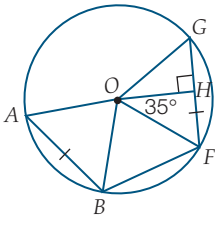
(a)



(b)

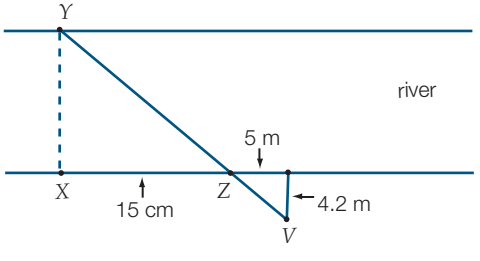


(c)



Understanding

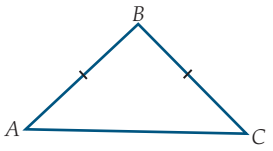
13 Angela is on a walk through the forest when she comes to a river. She wants to know the distance across the river. She draws triangles as shown in the diagram to help her. Find the distance across the river.



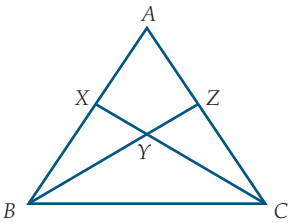
14 In ΔABC, $BA = BC$.

(a) Prove that $\angle BAC = \angle BCA$, by drawing the line BD perpendicular to AC, meeting it at D.

(b) What property of isosceles triangles have you just proved?



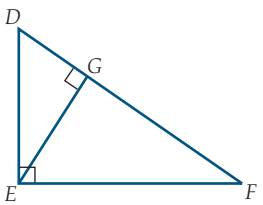
15 $AB = AC$ and $AX = AZ$. If $\angle YBC = \angle YCB$, prove that $XY = ZY$.



16 ΔDEF is right angled at E. $EG \perp DF$.

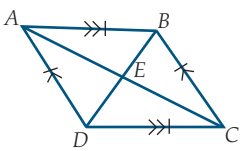
(a) Prove that triangles DEF, DEG and EGF are similar.

(b) Prove that $EG^2 = DG \times GF$.



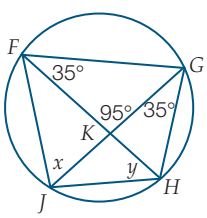
17 **(a)** For the diagram shown, prove $AC \perp BD$.

(b) What special property of a rhombus have you just proved?

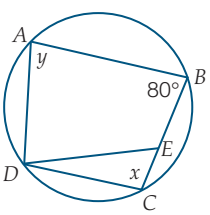


18 Find the value of the variables in each of the following, giving reasons for your answers. 10A

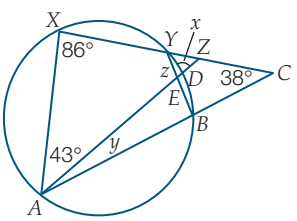
(a) $GH = HJ$



(b) $AB \parallel DC$

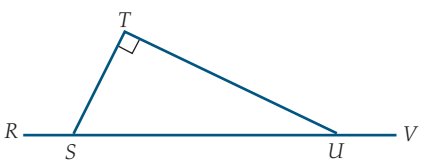


(c) $\angle YDZ = x$

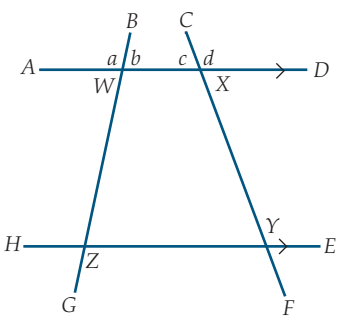


Reasoning

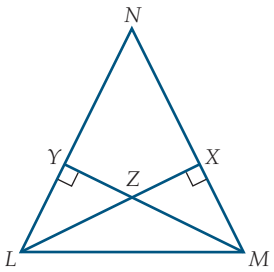
19 Prove that $\angle TSR + \angle TUV = 270°$.



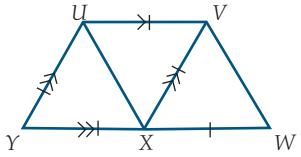
20 Use the diagram to prove that the sum of the interior angles of a trapezium is 360°.



21 In triangle LMN, LX is perpendicular to NM and MY is perpendicular to LN, as shown in the diagram. The point of intersection of LX and MY is Z. Prove that $\Delta LYZ \,|||\, \Delta MXZ$, giving reasons.

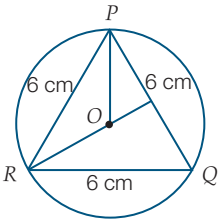


22 $UVXY$ is a rhombus and $YX = XW$. Show that $\Delta UXY \equiv \Delta VWX$.



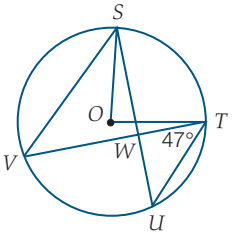
23 PQR is an equilateral triangle inscribed in a circle with centre O. (P, Q, R lie on the circle.) If $PQ = 6$ cm, calculate OP, the radius of the circle, giving reasons for your answer.

10A



24 In the circle with centre O, $SU \perp VT$ and $\angle VTU = 47°$. Find the size of $\angle SOT$, giving reasons for your answer.

10A



9 Non-linear relationships

Recall 9

Prepare for this chapter by attempting the following questions. if you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R9.1

1 **(a)** If $p = 2m^2 - 3m + 5$, find the value of p when:

(i) $m = 1$ **(ii)** $m = 0$ **(iii)** $m = -1$

(b) Find the value of $x(x - 3)$ when:

(i) $x = 5$ **(ii)** $x = -3$ **(iii)** $x = 0$

Worksheet R9.2

2 Factorise each of these quadratic expressions.

(a) $x^2 - 7x$ **(b)** $2x^2 + 10x$ **(c)** $x^2 - 4$

Worksheet R9.3

3 Expand and simplify each of the the following.

(a) $3x(x + 4)$ **(b)** $(x + 2)(x + 5)$ **(c)** $(x - 7)(x + 3)$

(d) $(x + 8)^2$ **(e)** $(x - 3)(x - 1)$ **(f)** $(x - 4)(x + 4)$

Worksheet R9.4

4 Factorise each of the following expressions.

(a) $x^2 - 25$ **(b)** $9x^2 - 16$ **(c)** $x^2 - 5$

(d) $x^2 - 8x + 16$ **(e)** $x^2 + 14x + 49$ **(f)** $3x^2 - 66x + 363$

Worksheet R9.5

5 Factorise each of the following.

(a) $x^2 + 7x + 10$ **(b)** $x^2 - 5x + 6$ **(c)** $x^2 - 13x - 14$

(d) $x^2 + 5x - 24$ **(e)** $x^2 + 5x - 36$ **(f)** $2x^2 + 14x + 12$

(g) $-x^2 - 3x - 2$ **(h)** $-x^2 + 7x - 10$ **(i)** $-2x^2 + 2x + 24$

Worksheet R9.6

6 Write each of the following as a number without an index.

(a) 3^3 **(b)** 4^{-1} **(c)** $5^{-2} + 3$

Worksheet R9.7

7 Solve the following using the Null Factor Law.

(a) $(x - 1)(x + 3) = 0$ **(b)** $(x + 4)(x + 2) = 0$

(c) $(x - 1)^2 = 0$ **(d)** $x(x - 5) = 0$

Worksheet R9.8

8 Solve the following using special product rules to factorise first.

(a) $x^2 - 6x + 9 = 0$ **(b)** $4x^2 - 25 = 0$

(c) $x^2 + 8x + 16 = 0$ **(d)** $121x^2 - 36 = 0$

Worksheet R9.9

9 Factorise each of the following by completing the square.

(a) $x^2 + 4x + 2$ **(b)** $x^2 - 6x + 6$ **(c)** $x^2 - 3x - 5$

After completing this chapter you will be able to:

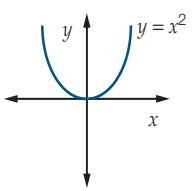
- identify non-linear graphs
- solve non-linear equations algebraically and graphically using technology
- solve quadratic equations using a variety of techniques
- construct graphs of parabolas, exponentials, circles and hyperbolas
- identify, match and describe algebraic and graphical representations of parabolas, rectangular hyperbolas, exponential functions and circles
- solve problems that can be modelled by quadratic, exponential, circle and hyperbolic relationships
- understand the effect of transforming graphs
- sketch the graphical representation of parabolas, circles, exponential and hyperbolic functions.

9.1 Identifying non-linear graphs and their equations

Need to Know

Parabolas

- A **parabola** is a graph of the quadratic relationship $y = ax^2 + bx + c$.
- Every parabola has a **vertex** or **turning point**.
- A parabola can have a minimum turning point or a maximum turning point.
- Every parabola has an **axis of symmetry**.

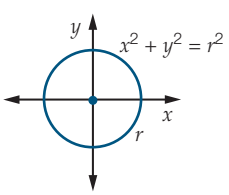


Circles

The equation of a circle with its centre at the origin (0, 0) and with radius r is given by $x^2 + y^2 = r^2$.

The graphs of circle equations written in the form of $(x - h)^2 + (y - k)^2 = r^2$ have the centre at (h, k) and radius r.

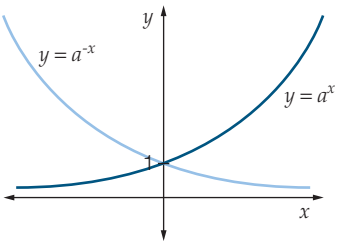
The four points plotted on the circumference are $(h + r, k)$, $(h, k + r)$, $(h - r, k)$ and $(h, k - r)$.



Exponentials

An **exponential** relationship occurs when a quantity grows or decays at a rate proportional to its current value. It can be recognised when the independent variable is the index or exponent of a constant base. All exponential graphs have a limiting value, a value they approach but never reach. This limiting value gives us a horizontal **asymptote**. An asymptote is a line that a curve approaches but never cuts or crosses as the independent variable approaches particular values.

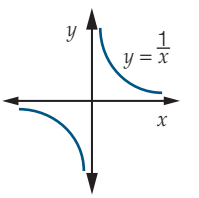
The graphs of exponential equations in the form of $y = a^x$ and $y = a^{-x}$ pass through the point (0, 1) because $a^0 = 1$, for all values of a.



Rectangular hyperbola 10A

In this relationship, as one variable increases, the other decreases.

The simplest form of a **rectangular hyperbola** equation is $y = \frac{1}{x}$.



Graphs of a rectangular hyperbola in the form of $y = \frac{a}{x-h} + k$ have both horizontal and vertical asymptotes. The graph is discontinuous at the vertical asymptote. We say the equation is undefined for x at that value.

Worked Example 1

WE1

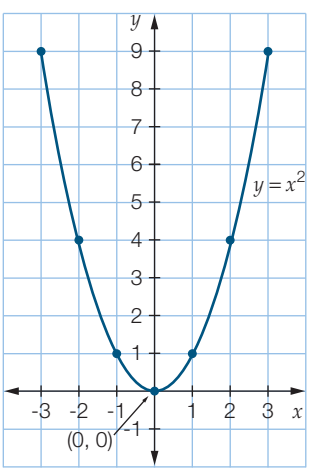
Plot the following parabolas for values of x between -3 and 3 inclusive:

(a) $y = x^2$ **(b)** $y = 2x^2 - 1$ **(c)** $y = 5 - x^2$

Working

(a) $y = x^2$

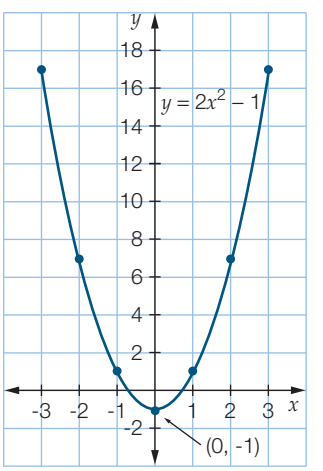
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



The equation of the axis of symmetry is $x = 0$.

(b) $y = 2x^2 - 1$

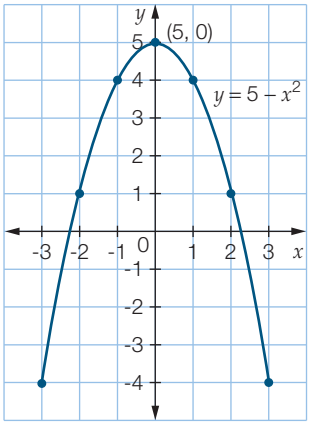
x	-3	-2	-1	0	1	2	3
y	17	7	1	-1	1	7	17



The equation of the axis of symmetry is $x = 0$.

(c) $y = 5 - x^2$

x	-3	-2	-1	0	1	2	3
y	-4	1	4	5	4	1	-4



The equation of the axis of symmetry is $x = 0$.

Worked Example 2

WE2

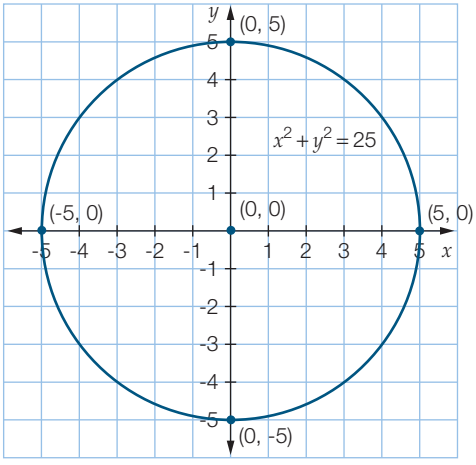
Plot the graphs of the following equations.

(a) $x^2 + y^2 = 25$ **(b)** $(x - 1)^2 + (y + 2)^2 = 36$

Working

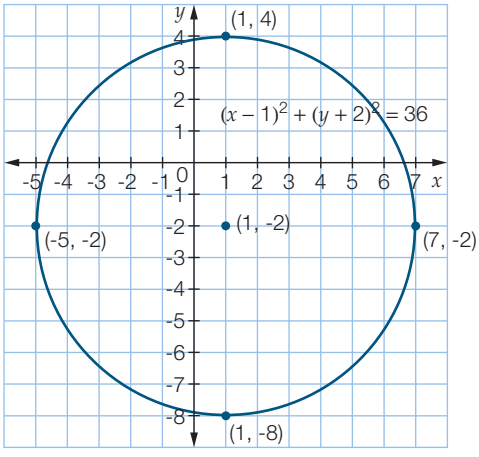
(a) $x^2 + y^2 = 25$

x	-5	0	5
y	0	±5	0



(b) $(x - 1)^2 + (y + 2)^2 = 36$

x	-5	1	7
y	-2	-8, 4	-2



Worked Example 3

WE3

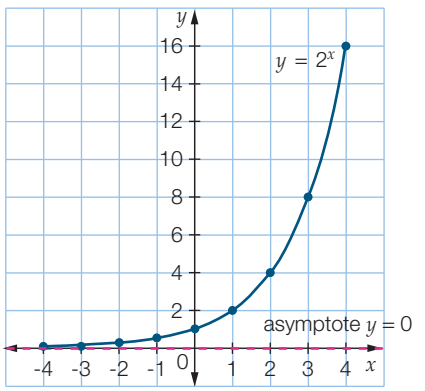
Plot the graphs of the following equations for $-4 \leq x \leq 4$.

(a) $y = 2^x$ **(b)** $y = 2^{-x} + 3$

Working

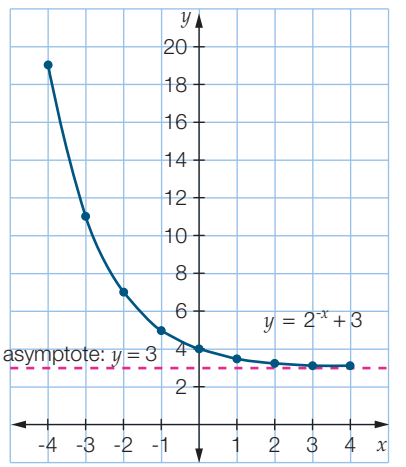
(a) $y = 2^x$

x	-4	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



(b) $y = 2^{-x} + 3$

x	-4	-3	-2	-1	0	1	2	3	4
y	19	11	7	5	4	$3\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{8}$	$3\frac{1}{16}$



Worked Example 4

WE4

Plot the graphs of the following equations for $-4 \leq x \leq 4$.

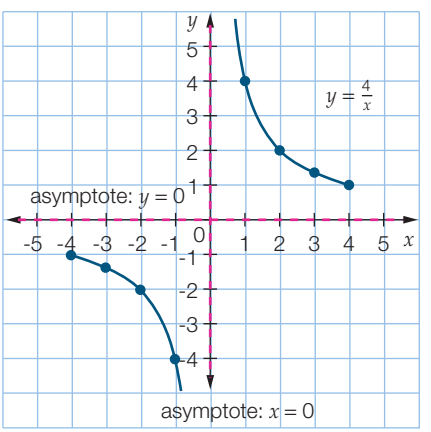
10A

(a) $y = \frac{4}{x}$ **(b)** $y = \frac{1}{x-1} + 3$

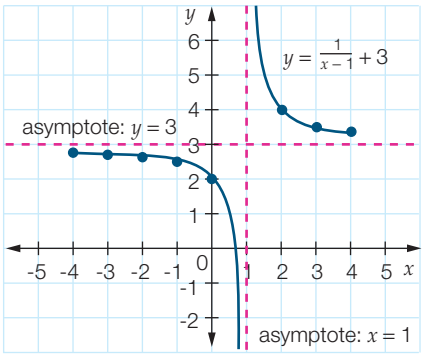
Working

(a) $y = \frac{4}{x}$

x	-4	-3	-2	-1	0	1	2	3	4
y	-1	$-\frac{4}{3}$	-2	-4	undefined	4	2	$\frac{4}{3}$	1



(b) $y = \dfrac{1}{x-1} + 3$



x	-4	-3	-2	-1	0	1	2	3	4
y	$2\frac{4}{5}$	$2\frac{3}{4}$	$2\frac{2}{3}$	$2\frac{1}{2}$	2	undefined	4	$3\frac{1}{2}$	$3\frac{1}{3}$

9.1 Identifying non-linear graphs and their equations

Answers page 543

Equipment required: graph paper for Questions **8** and **9**, calculator for Questions **13** and **16**

Fluency

1 Plot the graphs of each of the following using integer values of x between -3 and 3 inclusive. WE1

(a) $y = x^2 + 1$ **(b)** $y = x^2 - 3$ **(c)** $y = x^2 + 5$

(d) $y = 2x^2$ **(e)** $y = -2x^2$ **(f)** $y = -2x^2 + 1$

2 Plot the graphs of each of the following equations. WE2

(a) $(x+3)^2 + (y+2)^2 = 16$ **(b)** $(x-3)^2 + (y-2)^2 = 9$

(c) $(x-1)^2 + (y+1)^2 = 25$ **(d)** $x^2 + (y+4)^2 = 36$

(e) $(x+4)^2 + y^2 = 8$ **(f)** $(x-4)^2 + (y+2)^2 = 12$

3 Plot the graphs of the following equations for $-3 \leq x \leq 3$. WE3

(a) $y = 2^x + 2$ **(b)** $y = 2^x - 1$

(c) $y = 2^x - 3$ **(d)** $y = 3^x - 1$

(e) $y = 3^x - 2$ **(f)** $y = 3^x + 4$

4 Plot the graphs of the following equations for $-4 \leq x \leq 4$. WE4 10A

(a) $y = \dfrac{2}{x}$ **(b)** $y = \dfrac{3}{x}$ **(c)** $y = -\dfrac{4}{x}$

(d) $y = \dfrac{1}{x-2} + 1$ **(e)** $y = \dfrac{1}{x+2} - 3$ **(f)** $y = \dfrac{1}{x+2} - 1$

Understanding

5 **(a)** Complete a table of values and then plot the graph for each of the following using integer values for x between -4 and 4 inclusive.

(i) $y = (x - 1)(x + 3)$ **(ii)** $y = x(x + 3)$ **(iii)** $y = (4 + x)(2 - x)$

(b) Write down the coordinates of the turning point for each graph and state whether it is a maximum or minimum.

6 The graph of $y = 2^{x-2} + 1$ would be:

A a circle **B** a rectangular hyperbola

C an exponential **D** a parabola

7 A path around an area on a coordinate map is described by the equation $(x + 1)^2 + (y - 3)^2 = 16$.

(a) Make a simple sketch of the path.

(b) Determine whether the following points are within or outside the circle.

(i) (1, 2) **(ii)** (3, 6) **(iii)** (1.5, 2.5)

Hint

Will the temperature ever reach 25°C exactly?

8 A cup of coffee cools exponentially in a room where the temperature is 25°C. The temperature of the coffee was taken every 30 minutes and the results recorded in the table below:

t (min)	0	30	60	90	120	150
T (°C)	100	54	36.2	29.3	26.7	25.6

(a) Use graph paper to plot a graph of t against T (show t on the horizontal axis). Join the points with a smooth curve. Use your graph to answer parts **(b)** to **(e)**.

(b) What is the initial temperature of the coffee?

(c) Use the graph to find the temperature of the coffee after:

(i) 10 min **(ii)** 20 min **(iii)** 40 min **(iv)** 50 min

Give your answer to the nearest degree.

(d) During which 10 min period in the first hour did the coffee cool the fastest? Justify your answer.

(e) When will the temperature be 27.5°C?

(f) Extend the curve. Predict the temperature after 3 hours.

9 The volume of a child's circular swimming pool, with a depth of $\frac{3}{\pi}$ (≈ 0.96 m), depends on the radius of the pool. The volume of the pool is given by $V = \pi r^2 h$, where V is the volume of the pool in cubic metres, r is the radius (in metres) and h is the height (in metres).

(a) Show that $V = 3r^2$.

(b) Explain why this relationship is quadratic.

(c) Complete the table of values for $0 \leq r \leq 5$.

r	0	1	2	3	4	5
r^2						
V						

(d) Plot a graph of the relationship $V = 3r^2$ on graph paper. (Represent r on the horizontal axis.)

(e) Find the volume of the pool when the radius is:

(i) 2.5 m **(ii)** 3.8 m

(f) Find the radius of the pool if its volume is:

(i) 30 m^3 **(ii)** 65 m^3

(g) What is the difference in volume between a pool of radius 1.5 m and one of radius 3.5 m?

10 **(a)** Write down the equation of a circle with:

(i) centre (3, 1) and radius 7

(ii) centre (-2, -4) and radius 3

(iii) centre (2, -5) and radius 2.

(b) Plot the graph for each of the equations in part **(a)**.

11 The graph of the equation $y = \frac{1}{x-1} + 2$ has asymptotes at: 10A

A $x = 2, y = 1$ **B** $x = 1, y = 2$ **C** $x = -1, y = 2$ **D** $x = 1, y = -2$

12 A point that does not lie on the graph of $(x-3)^2 + (y+2)^2 = 25$ would be:

A (7, 1) **B** (-1, -5) **C** (7, -5) **D** (-7, 1)

13 **(a)** For the relationship $y = 2^x$, copy and complete this table of values for x and y.

x	-2	-1	0	1	2	3	4
y							

(b) Use this table of values to plot a graph of $y = 2^x$. Join the points with a smooth curve.

(c) Use your graph to determine approximate values for:

(i) $2^{1.5}$ **(ii)** $2^{0.8}$ **(iii)** $2^{-0.5}$

(iv) $2^{-1.2}$ **(v)** $\sqrt{2}$ **(vi)** $(\sqrt{2})^3$

(d) Use a calculator to check the answers you obtained in part **(c)**.

(e) Use your graph to find the value of the exponent, correct to one decimal place, when y is:

(i) 5 **(ii)** 12 **(iii)** 15

Reasoning

14 A tablecloth has a length 0.4 m longer than its width.

(a) Write an equation for the area of the tablecloth in terms of its length if the length is represented by the variable l.

(b) Plot a graph of the area against the length.

(c) Use this graph to find the dimensions of the tablecloth if the area of the tablecloth is 3.2 m^2.

15 A town has an initial population of 4000, but the establishment of a new industry in the town is expected to increase the population by 6% per year for future years.

(a) What will be the predicted population 1 year after the industry is established?

(b) Complete a table of values to show the predicted population for the first 5 years after the industry is established. Round the population to the nearest whole number. Use P to represent the population after n years.

(c) Write an equation to model the predicted population over future years.

(d) Sketch a graph of this relationship using your equation.

(e) Use your graph to predict when the population will have doubled in size.

(f) Use your equation to confirm your prediction in **(e)** is correct.

(g) Explain why the graph is a first quadrant graph only.

16 The water coming out of a fountain in a pond forms a parabolic shape. At any point p from the place where the water comes out, the height of the water, h, can be found using the equation: $h = -\frac{1}{16}(p - 40)^2 + 100$ (h and p are in centimetres).

(a) Plot the graph of the relationship between h and p, with p on the horizontal axis and the values of p between 0 and 80.

(b) How high does the water go up in the air?

(c) How far from the water outlet does the water from the fountain reach?

(d) Assuming that the fountain is in the centre of a pond and reaches to the edge of the pond, what area would be covered by water?

17 The centre of a circular water fountain is 3 m east and 2 m south of the corner of a building. The water fountain has a radius of 1 m. Could a tree be planted 2.4 m east and 2.7 m south of the corner of the building. Justify your answer by using an equation to plot a graph that represents this situation.

Open-ended

10A

18 **(a)** Write down an equation for a circle that does not have its centre at the origin and plot its graph.

(b) Write down the equation for a parabola that has a maximum turning point and plot the graph.

(c) Write down the equation for an exponential relationship that has an asymptote at $y = -1$.

(d) Write down an equation for a rectangular hyperbola that has asymptotes at $x = -2$ and $y = 6$.

19 When Sian threw a ball up in the air and then caught it, the path of the ball traced a parabolic shape.

(a) Sketch what you think might have happened on a set of axes.

(b) Describe the main features of the graph and explain why it has a parabolic shape.

20 Find the equations of at least two circles that pass through the point (1, 2) and have different centres.

Problem solving

How much is enough?

Bob needs to build a picket fence from his house to the garage. The curve formed by the top of the pickets follows the path of a parabola with equation:

$$y = \frac{3}{2.56}(x^2 - 3.2x + 2.56) + 1.2$$

where y is the height above the ground and x is the distance from the wall of the house. The pickets are 50 mm wide and have a gap of 50 mm between them. The length of the fence is 3.2 m. Each picket is shaped to a point at the top. What length of full width wood is used for the pickets? (A half-width picket is used at each end.)

Strategy options

- Draw a diagram.
- Make a model.
- Break problem into manageable parts.

Quadratic transformations

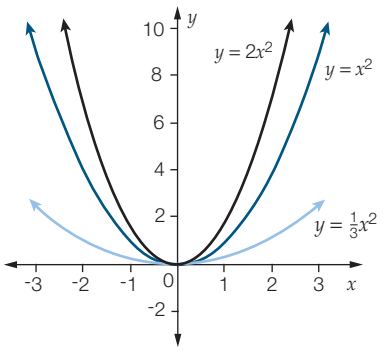
9.2

Need to Know

We can sketch simple quadratic graphs instead of plotting than when performing the **transformations** of **dilation**, **reflection** and **translation** of quadratics.

Dilations

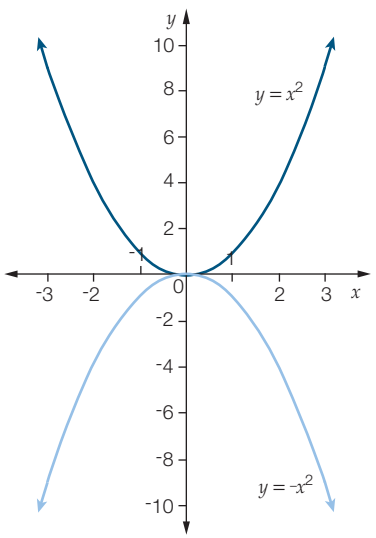
Graphing $y = x^2$, $y = \frac{1}{3}x^2$ and $y = 2x^2$ on the same set of axes shows that the coefficient of x^2 determines the shape of the graph, but the turning point is still the same. This effect is called dilation. The coefficient of x^2 is called the **dilation factor**.



- A factor greater than 1 will make the graph narrower.
- A factor less than 1 but greater than 0 will make the graph wider.

Reflections

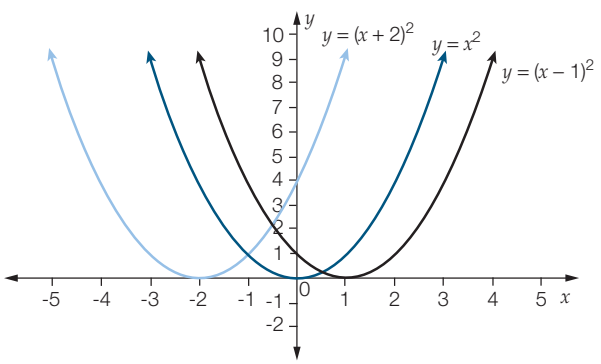
The graph of $y = -x^2$ is the same shape as the graph of $y = x^2$ except that it is upside down. We say it is *inverted* as it is a reflection in the x-axis. All graphs that have a negative coefficient for x^2 will have this same basic shape. The turning point of these graphs is a maximum.



Horizontal translations

Graphing $y = x^2$ with $y = (x - 1)^2$ and $y = (x + 2)^2$ shows that when a number is added or subtracted to x in the brackets the basic graph $y = x^2$ is moved left or right. This movement is called a horizontal translation.

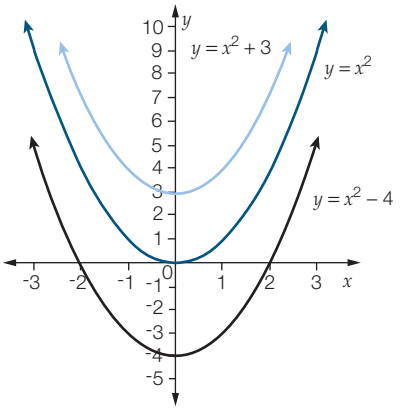
If $y = (x - h)^2$, h indicates the size and direction of the horizontal translation.



- When h is positive, the graph is moved right h units.
- When h is negative, the graph is moved left h units.

Vertical translations

Graphing $y = x^2$, $y = x^2 + 3$ and $y = x^2 - 4$ shows that the basic graph $y = x^2$ will move up when adding a number and down when subtracting a number. This movement is called a vertical translation.



If $y = x^2 + k$, the constant term k indicates the size and direction of the vertical translation.

- When k is positive, the graph is moved up k units.
- When k is negative, the graph is moved down k units.

We can use the basic graph of $y = x^2$ to obtain other parabolas in the form of $y = a(x - h)^2 + k$ by:

- Dilating it by a factor of a, making it narrower ($a > 1$) or wider ($0 < a < 1$).
- Reflecting it in the x-axis (inverting it) ($a < 0$).
- Translating it h units horizontally, k units vertically or in both directions.

Worked Example 5

WE5

For each of the following equations:

(a) $y = 4x^2$ **(b)** $y = \frac{1}{4}x^2$ **(c)** $y = -3x^2$

(i) state the dilation factor

(ii) determine whether the graph is narrower or wider than the parabola $y = x^2$

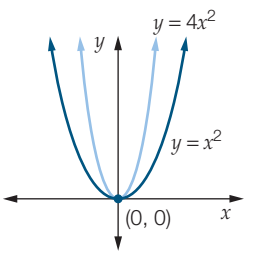
(iii) sketch the graph of $y = x^2$ on a set of Cartesian axes and then use transformations to sketch the graph of the given equation on the same set of axes.

Working

(a) (i) The dilation factor is 4.

(ii) The graph is narrower than $y = x^2$.

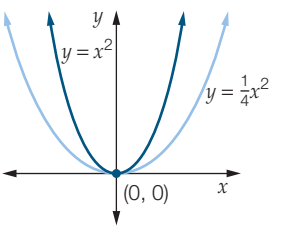
(iii)



(b) (i) The dilation factor is $\frac{1}{4}$.

(ii) The graph is wider than $y = x^2$.

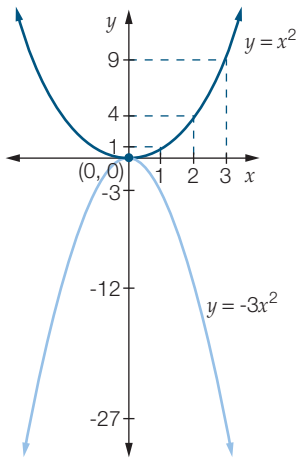
(iii)



(c) **(i)** The dilation factor is -3.

(ii) The graph is narrower than $y = x^2$.

(iii)



Worked Example 6

WE6

For each of the following equations:

(a) $y = (x + 1.3)^2$ **(b)** $y = (x - 2)^2$

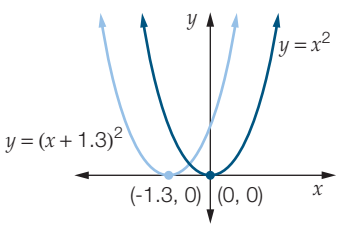
(i) state the translation of the graph of $y = x^2$

(ii) sketch the graph of $y = x^2$ on a set of Cartesian axes and then use transformations to sketch the graph of the given equation on the same set of axes.

Working

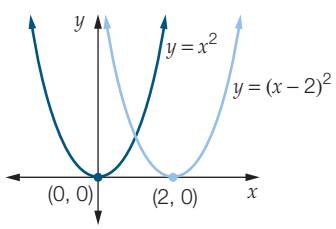
(a) **(i)** $y = x^2$ is translated 1.3 units to the left.

(ii)



(b) **(i)** $y = x^2$ is translated 2 units to the right.

(ii)



Worked Example 7

WE7

For each of the following equations:

(a) $y = x^2 - 3$ **(b)** $y = x^2 + 2.5$

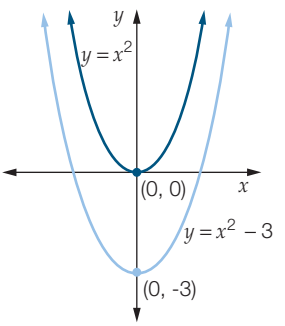
(i) state the translation of the graph of $y = x^2$

(ii) sketch the graph of $y = x^2$ on a set of Cartesian axes and then use transformations to sketch the graph of the given equation on the same set of axes.

Working

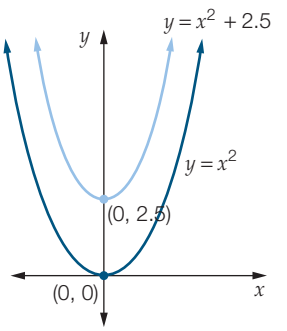
(a) **(i)** $y = x^2$ is translated 3 units down.

(ii)



(b) **(i)** $y = x^2$ is translated 2.5 units up.

(ii)



Worked Example 8

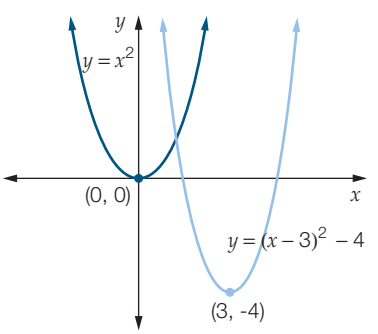
WE8

For each of the following equations, sketch the graph of $y = x^2$ on a set of Cartesian axes and then use transformations to sketch the graph of the given equation on the same set of axes.

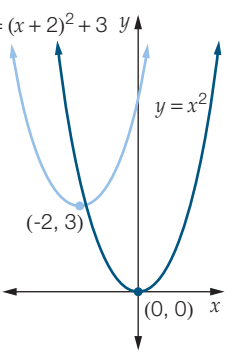
(a) $y = (x - 3)^2 - 4$ **(b)** $y = (x + 2)^2 + 3$

Working

(a)



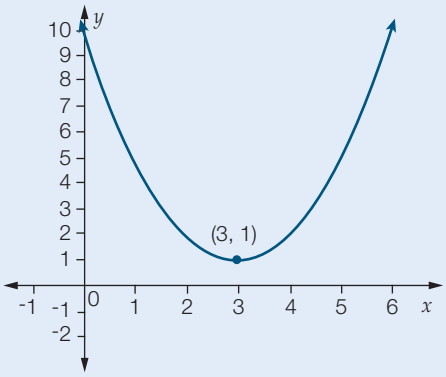
(b)



Worked Example 9

WE9

Using the graph shown, find the equation of the curve, assuming there is no dilation.



Working

$y = a(x - h)^2 + k$

$a = 1$ (no dilation)

The turning point is at (3, 1) so $h = 3$ and $k = 1$.

$y = (x - 3)^2 + 1$

9.2 Quadratic transformations

Answers page 548

Fluency

1 For each of the following equations: WE5

(a) $y = 2x^2$ **(b)** $y = \frac{x^2}{2}$

(i) state the dilation factor

(ii) determine whether the graph is narrower or wider than the parabola $y = x^2$

(iii) sketch the graph on the same set of axes as the graph of $y = x^2$.

2 For each of the following equations: WE6

(a) $y = (x - 1.3)^2$ **(b)** $y = (x + 2)^2$

(i) sketch the graph of $y = x^2$ on a set of Cartesian axes then use transformations to sketch the graph of the given equation

(ii) state the translation of the graph of $y = x^2$.

3 For each of the following equations: WE7

(a) $y = x^2 + 6$ **(b)** $y = x^2 - 1.2$

(i) sketch the graph of $y = x^2$ on a set of Cartesian axes then use transformations to sketch the graph of the given equation

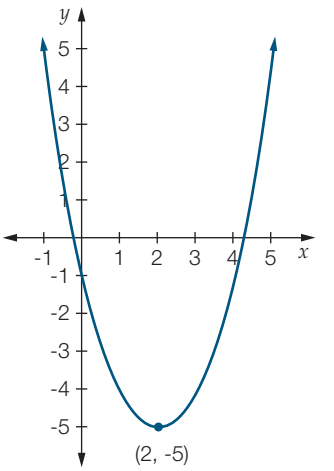
(ii) state the translation of the graph of $y = x^2$.

4 For each of the following equations, sketch the graph of $y = x^2$ on a set of Cartesian axes then use transformations to sketch the graph of the given equation on the same set of axes. WE8

(a) $y = (x + 3)^2 - 4$ **(b)** $y = (x - 2)^2 - 3$

WE9

5 Using the graph shown on the right, find the equation of the curve, assuming there is no dilation.



6 To sketch the graph of $y = (x + 5)^2 - 2$, the graph of $y = x^2$ will need to be translated:

A 5 units right and 2 units down
B 5 units left and 2 units down
C 5 units left and 2 units up
D 5 units right and 2 units up

7 On the same set of axes, sketch and label the graphs of:

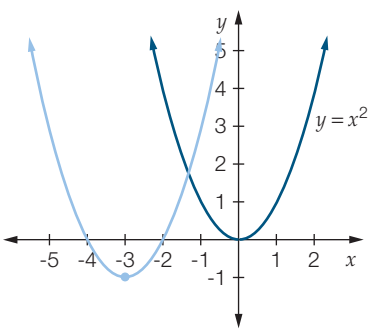
(a) $y = x^2$ **(b)** $y = x^2 - 5$ **(c)** $y = (x + 4)^2$ **(d)** $y = (x - 1)^2 + 5$

8 On the same set of axes, sketch and label the graphs of:

(a) $y = x^2$ **(b)** $y = 3x^2$ **(c)** $y = \frac{x^2}{3}$ **(d)** $y = -x^2$

9 The equation of the parabola shown on the right is:

A $y = (x - 3)^2 + 1$
B $y = (x + 3)^2 + 1$
C $y = (x + 1)^2 + 3$
D $y = (x + 3)^2 - 1$



Understanding

10 **(a)** Sketch and label the graph of $y = (x + 2)^2$.

(b) Use the graph from part **(a)** to sketch and label the graph of $y = -(x + 2)^2$ on the same set of axes.

11 **(a)** Sketch the graph of $y = x^2 + 4$.

(b) On the same set of axes, sketch and label the graph formed by dilating the graph in part **(a)** by a factor of 2.

(c) Write the equation for this graph.

12 **(a)** Sketch the graph of $y = (x - 3)^2$.

(b) On the same set of axes, sketch and label the graph formed by dilating the graph in part **(a)** by a factor of 2.

(c) Write the equation for this graph.

Hint

It will have the same turning point as the graph in part **(a)**.

Hint

It will have the same turning point as the graph in part **(a)**.

Reasoning

13 A quadratic graph has a turning point at (-3, 2) and a dilation factor of 1.

(a) Find the equation of this graph. **(b)** Sketch the graph of this equation.

14 A rope tied at both ends to a horizontal bar hangs in the shape of a parabola $y = x^2$. A photo of the rope shows that the bottom of the rope is 2 m left of a person in the photo and 0.5 m above ground level. If the person is standing on a spot marked (0, 0), what would be the equation of the rope?

15 An arch is built in the shape of a parabola to span a roadway. It is modelled by the equation $y = -(x - h)^2 + k$ where the origin (0, 0) is the base of the left-hand side of the arch. If the road is 6 m wide and the highest point of the arch is to be 9 m above ground level (assume the ground is flat), find the coordinates of lights if they are to be placed on the arch so they are horizontally 1 m apart and start and end at ground level.

Open-ended

16 **(a)** Write down an equation of a parabola that is a vertical translation of $y = x^2$ and sketch a graph of both $y = x^2$ and your equation on the same set of axes.

(b) Write down an equation of a parabola that is a horizontal translation of $y = x^2$ and sketch a graph of both $y = x^2$ and your equation on the same set of axes.

17 Freya and Syd are asked to sketch the graph of $y = (x + 6)^2 - 5$. Syd gets the wrong answer. Freya's graph is correct. Syd thinks that the + sign in front of the 6 means that the graph could be moved 6 units to the right and the negative sign in front of the 5 means that the graph should be moved 5 units down. Where did Syd go wrong and what explanation would Freya give to him so that he doesn't make the same mistake again.

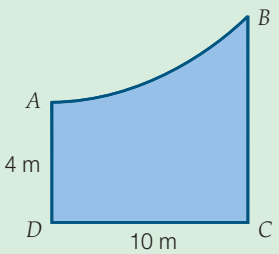
Problem solving

Finding curved areas

Erica wants to paint a wall of her architect-designed house. It has a shape that is described by the equation:

$$y = \frac{x^2}{50}$$

Some wall dimensions are shown on the diagram.



1 How high is the highest point on the wall?

2 Estimate the area by considering the shape to be a trapezium (*ABCD*).

Erica uses her CAS and finds that the actual area of the wall is 46.6 m^2.

3 How close can you get to this answer? Explain the method you used.

Strategy options

- Look for a pattern.
- Break problem into manageable parts.

9.3 Solving quadratic equations using the Null Factor Law

Need to Know

Null Factor Law

If $ab = 0$, then $a = 0$, $b = 0$ or $a = b = 0$.

These methods can be used to solve quadratic equations:

1. Equate one side (usually RHS) to zero.
2. Factorise the expression by looking for:
   - a common factor
   - a Difference of Two Squares $a^2 - b^2 = (a + b)(a - b)$
   - a Perfect Square $a^2 \pm 2ab + b^2 = (a \pm b)^2$
   - a monic trinomial where the middle term can be split so that the coefficients of the two middle terms multiply to give the constant term.

You may need to take out the coefficient of x^2 as a common factor and divide both sides by the common factor before factorising.

Worked Example 10

WE 10

Solve the following equations for x using the Null Factor Law.

(a) $x(x - 2) = 0$

(b) $(x + 1)(x - 6) = 0$

Working

(a) $x(x - 2) = 0$
$x = 0$ *or* $x - 2 = 0$
$x = 0$ *or* $x = 2$

(b) $(x + 1)(x - 6) = 0$
$x + 1 = 0$ *or* $x - 6 = 0$
$x = -1$ *or* $x = 6$

Worked Example 11

WE 11

Solve each equation for x:

(a) $4x^2 = 0$

(b) $(x + 1)^2 = 0$

Working

(a) $4x^2 = 0$
$x^2 = 0$
$x = 0$

(b) $(x + 1)^2 = 0$
$x + 1 = 0$
$x = -1$

Worked Example 12

WE12

Solve each equation for x:

(a) $3x^2 - 6x = 0$

(b) $11x^2 = x$

Working

(a) $3x^2 - 6x = 0$
$3x(x - 2) = 0$
$3x = 0$ or $x - 2 = 0$
$x = 0, x = 2$
$x = 0$ or 2

(b) $11x^2 = x$
$11x^2 - x = 0$
$x(11x - 1) = 0$
$x = 0$ or $11x - 1 = 0$
$x = 0$ or $11x = 1$
$x = 0$ or $x = \frac{1}{11}$
$x = 0$ or $\frac{1}{11}$

Worked Example 13

WE13

Solve the following equations for x.

(a) $x^2 - 49 = 0$

(b) $4x^2 = 25$

(c) $5x^2 = 3$

Working

(a) $x^2 - 49 = 0$
$(x + 7)(x - 7) = 0$
(Using the Difference of Two Squares rule)
$x + 7 = 0$ or $x - 7 = 0$
$x = -7$ or $x = 7$
$x = \pm 7$

(b) $4x^2 = 25$
$4x^2 - 25 = 0$
$(2x)^2 - (5)^2 = 0$
$(2x + 5)(2x - 5) = 0$
(Using the Difference of Two Squares rule)
$2x + 5 = 0$ or $2x - 5 = 0$
$x = -\frac{5}{2}$ or $x = \frac{5}{2}$
$x = \pm\frac{5}{2}$

(c) $5x^2 = 3$
$5x^2 - 3 = 0$
$x^2 - \frac{3}{5} = 0$
$(x + \sqrt{\frac{3}{5}})(x - \sqrt{\frac{3}{5}}) = 0$
(Using the Difference of Two Squares rule)
$x + \sqrt{\frac{3}{5}} = 0$ or
$x - \sqrt{\frac{3}{5}} = 0$
$x = -\sqrt{\frac{3}{5}}$ or $x = \sqrt{\frac{3}{5}}$
$x = \pm\sqrt{\frac{3}{5}}$

Worked Example 14

WE14

Solve the following equations for x.

(a) $x^2 + 10x + 25 = 0$ **(b)** $2x^2 - 36x = -162$

Working

(a) $x^2 + 10x + 25 = 0$
$(x + 5)(x + 5) = 0$

(Using the Perfect Square rule)

$(x + 5)^2 = 0$
$x + 5 = 0$
$x = -5$

(b) $2x^2 - 36x = -162$
$2x^2 - 36x + 162 = 0$
$2(x^2 - 18x + 81) = 0$
$2(x - 9)^2 = 0$

(Using the Perfect Square rule)

$(x - 9)^2 = 0$
$x - 9 = 0$
$x = 9$

Worked Example 15

WE15

Solve the following quadratic equations for x.

(a) $x^2 - 6x + 8 = 0$ **(b)** $2x^2 + 8x = 24$ **(c)** $-x^2 - 4x - 3 = 0$

Working

(a) $x^2 - 6x + 8 = 0$
$x^2 - 2x - 4x + 8 = 0$

(splitting the middle term)

$x(x - 2) - 4(x - 2) = 0$
$(x - 2)(x - 4) = 0$
$x - 2 = 0$ or $x - 4 = 0$
$x = 2$ or $x = 4$

(b) $2x^2 + 8x = 24$
$2x^2 + 8x - 24 = 0$
$2(x^2 + 4x - 12) = 0$
$x^2 + 4x - 12 = 0$
$x^2 + 6x - 2x - 12 = 0$

(splitting the middle term)

$x(x + 6) - 2(x + 6) = 0$
$(x + 6)(x - 2) = 0$
$x + 6 = 0$ or $x - 2 = 0$
$x = -6$ or $x = 2$

(c) $-x^2 - 4x - 3 = 0$
$-1(x^2 + 4x + 3) = 0$
$x^2 + 4x + 3 = 0$
$x^2 + x + 3x + 3 = 0$

(splitting the middle term)

$x(x + 1) + 3(x + 1) = 0$
$(x + 1)(x + 3) = 0$
$x + 1 = 0$ or $x + 3 = 0$
$x = -1$ or $x = -3$

9.3 Solving quadratic equations using the Null Factor Law

Answers page 550

Fluency

1 Solve the following equations for x using the Null Factor Law. WE10

(a) $x(x-11)=0$ **(b)** $x(x+6)=0$ **(c)** $x(x+4)=0$

(d) $2x(x-1)=0$ **(e)** $3x(x-1)=0$ **(f)** $-4x(x-6)=0$

(g) $(x-3)(x+7)=0$ **(h)** $(x-4)(x+1)=0$ **(i)** $(x-2)(x-6)=0$

(j) $(4-x)(x-5)=0$ **(k)** $(7-x)(x+2)=0$ **(l)** $(3-x)(x+10)=0$

2 Solve the following quadratic equations for x. WE11

(a) $-6x^2=0$ **(b)** $3x^2=0$ **(c)** $2x^2=0$

(d) $(x+7)^2=0$ **(e)** $(x+1)^2=0$ **(f)** $(x-9)^2=0$

3 Solve the following equations for x. WE12

(a) $x^2-4x=0$ **(b)** $x^2+x=0$ **(c)** $x^2-6x=0$

(d) $3x^2=15x$ **(e)** $4x^2=2x$ **(f)** $4x^2=-12x$

4 Solve the following quadratic equations for x. WE13

(a) $x^2-25=0$ **(b)** $x^2-81=0$ **(c)** $x^2-4=0$

(d) $16x^2-25=0$ **(e)** $9x^2-49=0$ **(f)** $100x^2-81=0$

(g) $3x^2-75=0$ **(h)** $4x^2-4=0$ **(i)** $7x^2-28=0$

(j) $49-x^2=0$ **(k)** $81-x^2=0$ **(l)** $121-x^2=0$

(m) $x^2-13=0$ **(n)** $x^2-24=0$ **(o)** $x^2-32=0$

5 Solve the following quadratic equations for x. WE14

(a) $x^2-14x+49=0$ **(b)** $x^2+8x+16=0$ **(c)** $x^2+16x+64=0$

(d) $x^2-4x+4=0$ **(e)** $x^2+2x+1=0$ **(f)** $x^2-6x+9=0$

(g) $2x^2-20x+50=0$ **(h)** $5x^2-10x+5=0$ **(i)** $6x^2+24x+24=0$

(j) $3x^2-18x=-27$ **(k)** $5x^2-50x=-125$ **(l)** $24x-3x^2=48$

6 Solve the following quadratic equations for x. WE15

(a) $x^2+3x+2=0$ **(b)** $x^2+9x+18=0$ **(c)** $x^2-11x+28=0$

(d) $x^2-10x+9=0$ **(e)** $x^2-x-30=0$ **(f)** $x^2+2x-8=0$

(g) $2x^2-24x+64=0$ **(h)** $2x^2+4x-30=0$ **(i)** $3x^2-18x-48=0$

(j) $5x^2+20x-60=0$ **(k)** $4x^2-4x-8=0$ **(l)** $6x^2-12x-18=0$

7 Solve the following equations. Rewrite each equation so that the right-hand side equals zero.

(a) $x^2+7x=-12$ **(b)** $x^2+8x=-12$ **(c)** $x^2-2x=63$

(d) $x^2+5x=24$ **(e)** $-x^2-5x=6$ **(f)** $-x^2+2x=-15$

(g) $x^2+5x-20=30$ **(h)** $x^2-6x-13=14$ **(i)** $x^2+66=17x$

(j) $x^2+5=6x$ **(k)** $-x^2+16=6x$ **(l)** $-x^2+21=4x$

Understanding

8 **(a)** The solution/s to $x(x-2)=0$ is/are:

A $x=2$ **B** $x=-2$ **C** $x=0, x=-2$ **D** $x=0, x=2$

(b) The solution/s to $(x+3)(x-4)=0$ is/are:

A $x=-3, x=4$ **B** $x=3, x=-4$ **C** $x=-3, x=-4$ **D** $x=3, x=4$

(c) The solution/s to $x^2-12x+36=0$ is/are:

A $x=0, x=-6$ **B** $x=-6$ **C** $x=6, x=0$ **D** $x=6$

(d) The solution/s to $x^2-64=0$ is/are:

A $x=8, x=-8$ **B** $x=8$ **C** $x=0, x=8$ **D** $x=0, x=-8$

9 Solve the following equations.

(a) $x^2+3x=10$ **(b)** $x^2-2x=3$ **(c)** $x^2-3x-18=10$

(d) $x^2+8x+12=5$ **(e)** $3x^2-28=20$ **(f)** $2x^2-15=3$

(g) $(x+4)(x+3)=6$ **(h)** $(x-5)(x+6)=12$ **(i)** $(x-6)^2=36$

10 **(a)** The solution/s to $x^2-9x+18=0$ is/are:

A $x=3, x=6$ **B** $x=-9$ **C** $x=3, x=-6$ **D** $x=-3, x=-6$

(b) The solution/s to $x^2+6x+5=0$ is/are:

A $x=1, x=5$ **B** $x=-1, x=5$ **C** $x=1, x=-5$ **D** $x=-1, x=-5$

(c) The solution/s to $3x^2-12x-63=0$ is/are:

A $x=3, x=7$ **B** $x=-3, x=7$ **C** $x=3$ **D** $x=3, x=-7$

(d) The solution/s to $-x^2+x+12=0$ is/are:

A $x=3, x=4$ **B** $x=-3, x=4$ **C** $x=3, x=-4$ **D** $x=-3, x=-4$

(e) The solution/s to $x^2+3x=4$ is/are:

A $x=1, x=3$ **B** $x=1, x=-4$ **C** $x=1, x=4$ **D** $x=-1, x=4$

(f) The solution/s to $x^2-4x=-4$ is/are:

A $x=2, x=-2$ **B** $x=4, x=-1$ **C** $x=2$ **D** $x=4, x=1$

11 The height, h centimetres, of a ball above the ground when it is being bounced is given by $h=t(50-t)$, where t is the time in seconds. Find the two times (in seconds) when the ball is on the ground, that is, when its height (h) is 0 cm.

12 A graph has the equation $y=(x+1)(x-4)$. Find the x-intercepts of this graph.

13 Determine the x-intercepts of the following quadratic relationships.

(a) $y=x^2-144$ **(b)** $y=6x^2+24x$ **(c)** $y=x^2-12x+36$

Reasoning

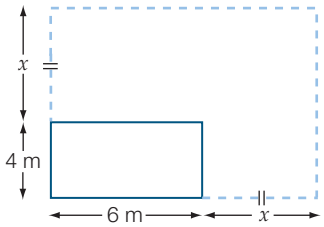
14 The temperature, T degrees Celsius, is measured during the day and found to follow the quadratic relationship $T=-t^2+10t+5$, where t represents the time in hours after 8 a.m.

(a) Find the temperature when t is equal to 0.

(b) By substituting for T, find the values of t when the temperature is 21°C.

(c) At what times is the temperature 21°C?

15 At the zoo, a wombat enclosure is to be enlarged to accommodate recent arrivals.



(a) Write an expression for the area of the new enclosure if the same length is to be added to two sides, as in the diagram.

(b) The new enclosure is to have double the area of the existing one. Find the dimensions of the new wombat enclosure.

16 Gerald hit his golf ball down the fairway. The path the ball followed was parabolic in shape with the relationship:

$$h = 2d - \frac{1}{30}d^2$$

where h is the height above the ground of the golf ball when it is a horizontal distance d from him. Both h and d are in metres. How far from Gerald did the golf ball land?

17 To determine the number of diagonals (D) in a polygon containing n sides, the following quadratic equation can be used:

$$D = \frac{n}{2}(n - 3)$$

(a) Show that a triangle contains no diagonals.

(b) Show that the given quadratic equation is true for each of these polygons—quadrilateral, pentagon, hexagon—by determining the number of diagonals algebraically and then diagrammatically.

(c) Calculate the number of diagonals in:

(i) an octagon **(ii)** a decagon **(iii)** a dodecagon.

(d) How many sides has a polygon that has:

(i) 170 diagonals **(ii)** 299 diagonals?

(e) A similar equation can be used to determine the number of consecutive integers, beginning at 1, required to sum to a particular value.

$$S = \frac{n}{2}(n + 1)$$

Use the equation to find the number of integers, beginning at 1, required to sum to:

(i) 105 **(ii)** 903

Open-ended

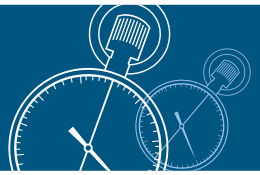
18 **(a)** Choose pairs of values for x. For each pair, write equations for which these are the solution.

(b) Choose a single value of x. Write down a quadratic equation for which this x value is the only solution.

19 Decide on two values of x that will be the x-intercepts of a quadratic graph. Write down at least two possible equations for the graph.

20 Write three different quadratic equations that have two solutions lying between $x = -3$ and $x = 3$. At least one of the equations should produce a solution containing a surd.

Half-time 9



Ex 9.3

1 Solve each of the following.

(a) $x^2 + 4x = -4$ **(b)** $x^2 - 5x = 6$ **(c)** $x^2 + 8 = 9x$

(d) $x^2 - 4x = 5$ **(e)** $2x^2 + 6x = -4$ **(f)** $3x^2 + 9x = 30$

Ex 9.1

2 Find the coordinates of the centre and the radius of the circle with equation $(x + 1)^2 + (y - 2)^2 = 4$.

Ex 9.2

3 Sketch each parabola, stating the changes that should be made to the graph $y = x^2$.

(a) $y = 2(x + 3)^2 - 4$ **(b)** $y = \frac{1}{2}(x - 1)^2 + 3$ **(c)** $y = (x - 2)^2 - 4$

Ex 9.2

4 The graph $y = x^2$ is moved 3 units right and 2 units up. What is the equation of the new graph?

Ex 9.3

5 Solve each of the following.

(a) $x^2 - 64 = 0$ **(b)** $(x + 1)^2 = 0$ **(c)** $(x - 2)^2 = 4$

Ex 9.1

6 Plot the graph of:

(a) $y = -2^{-x}$ for values of x from -3 to 3 inclusive

(b) $(x - 2)^2 + (y - 3)^2 = 4$, showing the four points $(h + r, k)$, $(h, k + r)$, $(h - r, k)$, $(h, k - r)$ on the circumference of the circle with centre (h, k) and radius r.

Ex 9.3

7 Solve these quadratic equations.

(a) $x^2 + 6x + 5 = 0$ **(b)** $x^2 - 9x = 0$ **(c)** $3x^2 + 9x = 0$

(d) $x^2 + x - 6 = 0$ **(e)** $x^2 - 4x + 3 = 0$ **(f)** $x^2 - 6x + 9 = 0$

Ex 9.2

8 Consider the equation $y = -x^2 + 1$.

(a) Set up a table of values using integer values of x from -3 to 3 inclusive.

(b) Plot the points on a number plane and join them with a smooth curve.

(c) Find the coordinates of the turning point from your graph.

(d) Find the equation of the axis of symmetry.

Ex 9.1

9 **(a)** Plot the graph of $y = 2^{-x} - 1$ for values of x from -3 to 3 inclusive.

(b) What is the equation of the asymptote for the graph?

(c) What are the coordinates of the y-intercept?

10A Ex 9.1

10 For the non-linear equation $y = \frac{1}{x - 3} + 4$:

(a) What name is given to a graph of this type?

(b) Write down the equations of the x- and y-asymptotes.

Solving quadratics by completing the square

9.4

Worked Example 16

WE16

For each of the following expressions:

(a) $x^2 - 6x + 3$ **(b)** $x^2 + 5x - 2$

(i) Complete the square and factorise.

(ii) Equate the expression to zero and use your answer for part **(i)** to solve the quadratic equation formed.

Working

(a) (i) $x^2 - 6x + 3$

$= [x^2 - 6x + \left(\frac{6}{2}\right)^2] - \left(\frac{6}{2}\right)^2 + 3$

(completing the square)

$= (x^2 - 6x + 3^2) - 9 + 3$

$= (x-3)^2 - 6$

$= (x-3)^2 - (\sqrt{6})^2$

$= (x - 3 + \sqrt{6})(x - 3 - \sqrt{6})$

(Difference of Two Squares rule)

(ii) $x^2 - 6x + 3 = 0$

$(x - 3 + \sqrt{6})(x - 3 - \sqrt{6}) = 0$

$(x - 3 + \sqrt{6}) = 0$

or $(x - 3 - \sqrt{6}) = 0$

$x = 3 - \sqrt{6}$ or $x = 3 + \sqrt{6}$

$x = 3 \pm \sqrt{6}$

(b) (i) $x^2 + 5x - 2$

$= [x^2 + 5x + \left(\frac{5}{2}\right)^2] - \left(\frac{5}{2}\right)^2 - 2$

(completing the square)

$= (x + \frac{5}{2})^2 - \frac{25}{4} - 2$

$= (x + \frac{5}{2})^2 - \frac{33}{4}$

$= (x + \frac{5}{2})^2 - \left(\sqrt{\frac{33}{4}}\right)^2$

$= (x + \frac{5}{2} - \sqrt{\frac{33}{4}})(x + \frac{5}{2} + \sqrt{\frac{33}{4}})$

(Difference of Two Squares rule)

(ii) $x^2 + 5x - 2 = 0$

$(x + \frac{5}{2} - \sqrt{\frac{33}{4}})(x + \frac{5}{2} + \sqrt{\frac{33}{4}}) = 0$

$(x + \frac{5}{2} - \sqrt{\frac{33}{4}}) = 0$

or $(x + \frac{5}{2} + \sqrt{\frac{33}{4}}) = 0$

$x = -\frac{5}{2} + \sqrt{\frac{33}{4}}$ or $x = -\frac{5}{2} - \sqrt{\frac{33}{4}}$

$x = \frac{-5}{2} \pm \sqrt{\frac{33}{4}}$ or $x = \frac{-5 \pm \sqrt{33}}{2}$

Worked Example 17 WE17

Solve $x^2 + 6x - 2 = 0$ using the method of completing the square. Leave your answer in exact surd form.

Working

$$x^2 + 6x - 2 = 0$$

$$\left[x^2 + 6x + \left(\frac{6}{2}\right)^2\right] - \left(\frac{6}{2}\right)^2 - 2 = 0 \quad \text{(completing the square)}$$

$$(x^2 + 6x + 9) - 9 - 2 = 0$$

$$(x+3)^2 - 11 = 0$$

$$(x+3)^2 - (\sqrt{11})^2 = 0$$

$$(x + 3 - \sqrt{11})(x + 3 + \sqrt{11}) = 0 \quad \text{(Difference of Two Squares rule)}$$

$x + 3 - \sqrt{11} = 0$ or $x + 3 + \sqrt{11} = 0$

$x = -3 + \sqrt{11}$ or $x = -3 - \sqrt{11}$

$x = -3 \pm \sqrt{11}$

Answers page 552

9.4 Solving quadratics by completing the square

Fluency

WE16

1 For each of the following expressions:

(a) $x^2 + 2x - 5$ **(b)** $x^2 + 4x - 7$ **(c)** $x^2 + 8x + 5$

(d) $x^2 + 3x - 1$ **(e)** $x^2 + 5x + 3$ **(f)** $x^2 + 3x - 5$

(i) Complete the square and factorise.

(ii) Equate the expression to zero and use your answer for part **(i)** to solve the quadratic equation formed.

WE17

2 Solve these quadratic equations using the method of completing the square. Leave your answers in exact surd form.

(a) $x^2 - 8x + 6 = 0$ **(b)** $x^2 + 6x + 4 = 0$ **(c)** $x^2 - 3x + 1 = 0$

(d) $x^2 - 5x - 2 = 0$ **(e)** $x^2 + 4x - 3 = 0$ **(f)** $x^2 - 20x - 12 = 0$

(g) $2x^2 - 8x - 10 = 0$ **(h)** $3x^2 + 12x + 3 = 0$ **(i)** $4x^2 + 16x - 20 = 0$

3 **(a)** The solution to $2x^2 + 4x - 4 = 0$ is:

A $x = 1 \pm \sqrt{3}$ **B** $x = -2$ or 4 **C** $x = -1 \pm \sqrt{3}$ **D** $x = -4$ or 2

(b) The solution to $x^2 - 4x - 3 = 0$ is:

A $x = 4$ or -1 **B** $x = 2 \pm \sqrt{7}$ **D** $x = -2 \pm \sqrt{7}$ **E** $x = -4$ or 1

(c) The solution to $x^2 - 8x + 14 = 0$ is:

A $x = -4$ or $-\sqrt{2}$ **B** $x = 4\sqrt{2}$ or $-4\sqrt{2}$

C $x = -4 - \sqrt{2}$ or $-4 + \sqrt{2}$ **D** $x = 4 + \sqrt{2}$ or $4 - \sqrt{2}$

Understanding

4 Solve the following quadratic equations.

(a) $x^2 - 4x = 2$ **(b)** $x^2 = 6x - 2$ **(c)** $3 + 4x = x^2$

(d) $x^2 = 5 + 3x$ **(e)** $x^2 - 5x = -2$ **(f)** $x^2 = 5 - 7x$

5 (a) Explain why the quadratic equation $x^2 + 4x + 7$ cannot be factorised.

(b) Explain why the quadratic equation $x^2 + 4x + 7 = 0$ has no solution.

(c) As $x^2 + 4x + 7 = 0$ has no solution, what does this tell you about the graph of $y = x^2 + 4x + 7$?

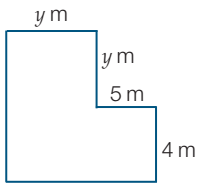
Reasoning

6 An envelope has an area of 80 cm^2. If the length is 4 cm longer than the width, find the dimensions of the envelope:

(a) exactly

(b) rounded to one decimal place.

7 An area of lawn as shown in the diagram is to be mown. Find the value y represents if the total area mown is 44 m^2.



8 If the dimensions of a rectangular park are $(x + 4)$ m and $(x - 5)$ m:

(a) find the exact value of x if the area of the park is 180 m^2

(b) find the value of x rounded to two decimal places.

9 A photo of length l cm and width $(l - 2)$ cm is surrounded by a 2 cm rectangular frame. Altogether, the photo and the frame occupy an area of 215 cm^2.

(a) Write the dimensions of the photo and frame in terms of l.

(b) Write an equation in terms of l for the area of the photo and frame.

(c) Find the exact dimensions of the photo.

(d) Find the dimensions of the photo and frame correct to one decimal place.

Open-ended

10 Write a quadratic equation that gives solutions containing surds. Solve your equation.

11 Write a quadratic equation that has no solution and demonstrate how you know this.

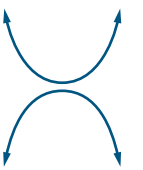
12 By using the process of completing the square, the solution $x = -a \pm \sqrt{3}$ was obtained. By choosing a value for a between 0 and 5, write a possible quadratic equation that could have this solution.

9.5 Sketching parabolas

Need to Know

To make a sketch of a quadratic graph, you will need to identify the following key features:

The shape of the graph



- A quadratic equation with a positive coefficient of x^2 will produce a graph that is a positive parabola.
- A quadratic equation with a negative coefficient of x^2 will produce a graph that is a negative parabola.

The *y*-intercept

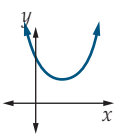
This is where the graph crosses the y-axis and so $x = 0$.

The *x*-intercept(s)

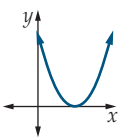
This is where the graph crosses the x-axis and so $y = 0$.

x-intercepts are found by equating the quadratic to zero and solving.

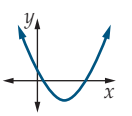
- If there is no solution, there are no x-intercepts.



- If there is one solution, the parabola touches the x-axis. There is only one x-intercept.



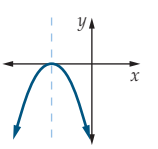
- If there are two solutions, there are two distinct x-intercepts. These may involve surds.



When a graph has two x-intercepts the x-coordinate of the turning point is halfway between the two x-intercepts.

Axis of symmetry

The axis of symmetry is a vertical line that passes through the turning point, dividing the graph into mirror image parts.



The turning point

- The turning point can be found by completing the square to transform the equation into **turning point form**.

The graph of $y = (x - h)^2 + k$ has a turning point at (h, k).

Worked Example 18

WE18

Sketch the graph of the quadratic relationship $y = x^2 + 2x - 8$ by finding:

(a) the shape of the graph

(b) the y-intercept

(c) the x-intercepts

(d) the turning point

(e) the axis of symmetry.

Working

(a) This is a ∪ shaped parabola as the coefficient of x^2 is positive.

(b) When $x = 0$, $y = 0^2 + 2 \times 0 - 8$

$y = 0 + 0 - 8$

$y = -8$

The y-intercept is $(0, -8)$.

(c) When $y = 0$, $0 = x^2 + 2x - 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ or $x = 2$

The x-intercepts are $(-4, 0)$ and $(2, 0)$.

(d) For the turning point, $x = \frac{-4 + 2}{2}$

$= -1$

When $x = -1$

$y = (-1)^2 + 2 \times -1 - 8$

$y = 1 - 2 - 8$

$y = -9$

The turning point is at $(-1, -9)$.

(Alternatively)

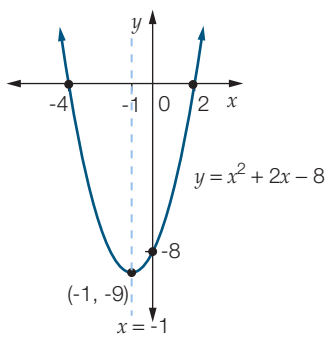
$y = x^2 + 2x - 8$

$y = x^2 + 2x + 1 - 8 - 1$

$y = (x + 1)^2 - 9$

The turning point is at $(-1, -9)$.

(e)



Worked Example 19

WE19

Sketch the parabola $y = x^2 + 2x - 15$, showing the x- and y-intercepts, turning point and the axis of symmetry. Find the coordinates of the turning point by transforming the equation into turning point form.

Working

$y = x^2 + 2x - 15$

This is a ∪ shaped parabola as the coefficient of x^2 is positive.

When $x = 0$, $y = 0 + 0 - 15$
$= -15$

The y-intercept is $(0, -15)$.

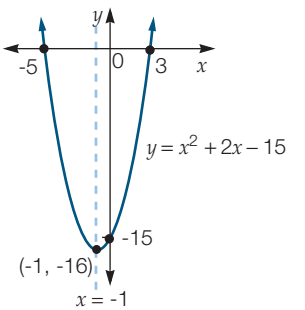
When $y = 0$, $0 = x^2 + 2x - 15$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$

$x + 5 = 0$ or $x - 3 = 0$
$x = -5$ or $x = 3$

The x-intercepts are $(-5, 0)$ and $(3, 0)$.

$y = x^2 + 2x - 15$
$y = (x^2 + 2x + 1) - 1 - 15$
$y = (x + 1)^2 - 16$

The turning point is at $(-1, -16)$.



Worked Example 20

WE20

Sketch the graph of the equation $y = 4 - x^2$, showing all the key features.

Working

$y = 4 - x^2$

This is a ∩ shaped parabola as the coefficient of x^2 is negative.

When $x = 0$, $y = 4 - 0^2$
$y = 4$

The y-intercept is $(0, 4)$.

When $y = 0$, $0 = 4 - x^2$
$4 - x^2 = 0$
$(2 + x)(2 - x) = 0$
$2 + x = 0$ or $2 - x = 0$
$x = \pm 2$

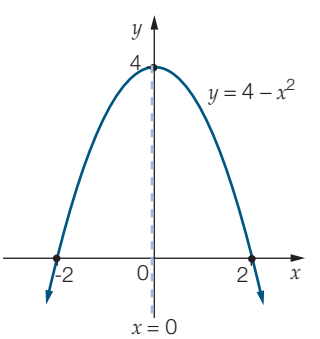
The x-intercepts are $(-2, 0)$ and $(2, 0)$.

For the turning point:

$x = \frac{-2 + 2}{2}$
$= 0$

When $x = 0$, $y = 4 - 0$
$y = 4$

The turning point is at $(0, 4)$.



Worked Example 21 WE21

Sketch the graph of the equation $y = -(x - 2)^2 + 5$, showing all the key features.

Working

$y = -(x - 2)^2 + 5$

The graph of $y = x^2$ needs to be inverted and translated 2 units right and 5 units up.

$\therefore\ h = 2$ and $k = 5$

The turning point is at $(2, 5)$.

When $x = 0$, $y = -(0 - 2)^2 + 5$
$= -(4) + 5$
$= 1$

The y-intercept is $(0, 1)$.

When $y = 0$, $0 = -(x - 2)^2 + 5$

$(x - 2)^2 - 5 = 0$

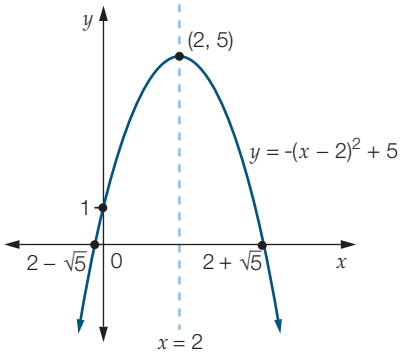
$(x - 2 + \sqrt{5})(x - 2 - \sqrt{5}) = 0$

$x = 2 - \sqrt{5}$ or $x = 2 + \sqrt{5}$

$x = 2 \pm \sqrt{5}$

The x-intercepts are $(2 - \sqrt{5}, 0)$ and $(2 + \sqrt{5}, 0)$.

$x \approx 4.2$ and -0.2



9.5 Sketching parabolas

Answers page 552

Fluency

1 Sketch the graph of each of the following quadratic relationships by finding **(i)** the shape of the graph, **(ii)** the y-intercept, **(iii)** the x-intercepts, **(iv)** the turning point and **(v)** the axis of symmetry. WE18

(a) $y = x^2 - 8x + 15$ **(b)** $y = x^2 - 6x + 8$ **(c)** $y = x^2 - 10x + 21$

(d) $y = -x^2 + 4x + 12$ **(e)** $y = -x^2 + 2x + 3$ **(f)** $y = -x^2 + 2x + 8$

(g) $y = x^2 - 8x + 12$ **(h)** $y = x^2 + 2x - 3$ **(i)** $y = x^2 + 4x - 5$

2 Sketch the following parabolas showing the x- and y-intercepts, the turning point and the axis of symmetry. Find the coordinates of the turning point by transforming the equations into turning point form. WE19

(a) $y = x^2 + 2x - 8$ **(b)** $y = x^2 + 8x + 15$ **(c)** $y = x^2 + 3x + 2$

(d) $y = x^2 - 4$ **(e)** $y = 9 - x^2$ **(f)** $y = x^2 - 7$

(g) $y = x^2 - 2x$ **(h)** $y = x^2 + 3x$ **(i)** $y = x^2 + 2x$

3 Sketch the graphs of the following equations showing all the key features. WE20

(a) $y = x^2 - 25$ **(b)** $y = x^2 - 9$ **(c)** $y = x^2 - 16$

(d) $y = 81 - x^2$ **(e)** $y = 4 - x^2$ **(f)** $y = 1 - x^2$

(g) $y = x^2 - 4x$ **(h)** $y = x^2 + 6x$ **(i)** $y = 3x^2 + 6x$

WE21

4 Sketch the graphs of the following equations showing all the key features.

(a) $y = (x + 2)^2 - 9$ **(b)** $y = (x + 5)^2 - 1$ **(c)** $y = -(x + 1)^2 + 5$

(d) $y = (x - 3)^2 + 2$ **(e)** $y = (x - 2)^2 + 4$ **(f)** $y = (x - 4)^2 + 2$

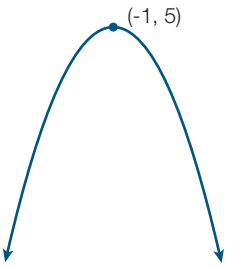
5 Sketch the graphs of the following equations showing all the key features.

(a) $y = x^2 + 10x + 16$ **(b)** $y = x^2 + 8x + 7$ **(c)** $y = -x^2 - 6x - 5$

(d) $y = -x^2 + 4x - 3$ **(e)** $y = x^2 + 6x + 9$ **(f)** $y = x^2 + 10x + 25$

(g) $y = -x^2 + 3x$ **(h)** $y = -x^2 - 5x$ **(i)** $y = -x^2 + 7x - 12$

6 When expressed graphically, which of the following equations would have a shape like that of the parabola shown on the right?



A $y = x^2 + 2x - 5$ **B** $y = -x^2 - 2x + 4$

C $y = 1 - x^2$ **D** $y = 6 - 2x - x^2$

7 (a) The coordinates of the y-intercept of $y = x^2 + 4x + 3$ are:

A (0, 8) **B** (-1, 0) **C** (3, 0) **D** (0, 3)

(b) The coordinates of the x-intercepts of $y = x^2 - 9$ are:

A (0, -3), (0, 3) **B** (-3, 0), (3, 0) **C** (-9, 0), (9, 0) **D** (0, -9), (0, 9)

8 (a) The x-intercepts of the parabola $y = x^2 + 5x - 6$ are:

A -2 and 3 **B** -3 and 2 **C** -6 and 1 **D** -1 and 6

(b) If the x-intercepts of a parabola are -1 and 5, the turning point could be:

A (3, 7) **B** (2, 4) **C** (1, -6) **D** (-2, 2)

Understanding

9 A ball is thrown into the air and follows a path given by the relationship $h = -t^2 + 10t$, where h is the height of the ball in metres after t seconds of flight. Sketch the graph of the relationship. Use the sketch graph to find:

(a) the total time the ball was in flight

(b) the maximum height the ball reached.

10 (a) The equation of parabola A is:

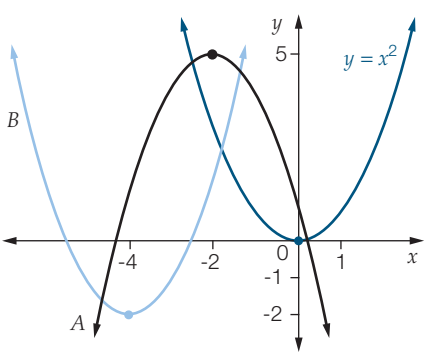
A $y = (x - 2)^2 + 5$ **B** $y = -(x + 2)^2 + 5$

C $y = -(x - 2)^2 + 5$ **D** $y = -(x + 2)^2 - 5$

(b) The equation of parabola B is:

A $y = (x - 4)^2 - 2$ **B** $y = (x + 4)^2 - 2$

C $y = (x - 4)^2 + 2$ **D** $y = (x + 4)^2 + 2$



11 (a) In turning point form, the quadratic equation $y = x^2 - 8x + 18$ can be written as:

A $y = (x - 4)^2 + 34$ **B** $y = (x + 4)^2 + 2$

C $y = (x - 4)^2 - 2$ **D** $y = (x - 4)^2 + 2$

(b) Hence, write down the coordinates of the turning point for the parabola $y = x^2 - 8x + 18$.

Hint

What can you add to -7 to equal 5?

12 **(a)** Sketch the graph of the equation $y = x^2 - 6x + 5$ showing all its key features.

(b) Use the graph to find the solution to the equation $x^2 - 6x - 7 = 0$.

13 Over a period of 8 hours, the temperature of a room follows the relationship $T = h^2 - 6h + 15$, where T is the temperature in degrees Celsius, h hours after commencing the experiment.

(a) Express the equation in turning point form and hence sketch the graph of this quadratic relationship.

(b) What is the initial temperature?

(c) After 2 hours, is the temperature of the room increasing or decreasing?

(d) After 6 hours, is the temperature of the room increasing or decreasing?

(e) Find the minimum temperature.

(f) When did the minimum temperature occur?

(g) Would it be realistic to use the equation over a 24-hour period? Explain why or why not.

Reasoning

14 Determine the equation of a graph that has x-intercepts of -3 and 5 and a coefficient of x^2 equal to 1.

15 A rectangular area of land is to be fenced off using 44 m of continuous fencing.

(a) Write an equation for the area enclosed by the fencing.
(Let x = length of one side of the rectangle.)

(b) Sketch a graph of this quadratic relationship showing all key features.

(c) What is the smallest value x can be?

(d) What is the largest value x can be?

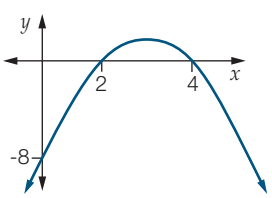
(e) What is the maximum area that can be enclosed by this fencing?

(f) What should the dimensions of the rectangle be for the enclosed area to be as big as possible?

Open-ended

16 Explain with examples how you could determine if x-intercepts will exist for a quadratic graph.

17 Archie was graphing $y = x^2 - 2x - 8$. He drew the graph shown below.



(a) Explain why he is wrong and how he could have avoided these mistakes.

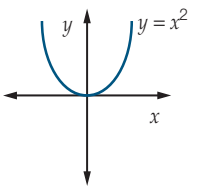
(b) Draw the correct graph.

18 Write down the equations of three parabolas that have $x = 5$ as an axis of symmetry. For each parabola, find the turning point.

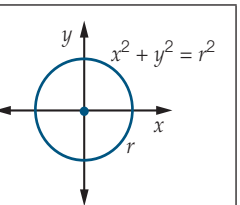
9.6 Sketch graphs of other non-linear relationships

Need to Know

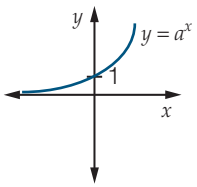
Quadratics

Basic equation $y = x^2$	Basic shape is a parabola. 
Translated equation $y = (x - h)^2 + k$	Basic shape is moved h units to right and k units up.
Dilated equation $y = ax^2$	If $a < -1$ or $a > 1$, then the parabola is narrower than $y = x^2$. If $-1 < a < 1$, then the parabola is wider than $y = x^2$. If the equation has a negative coefficient, $a < 0$, then the basic shape of the parabola is reflected in the x-axis.

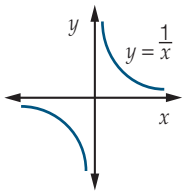
Circles

Basic equation $x^2 + y^2 = r^2$	Basic shape is a circle. The centre is (0, 0) and radius is r. 
Translated equation $(x - h)^2 + (y - k)^2 = r^2$	Basic shape is moved h units to right and k units up. The centre is (h, k) and the radius is r.
Dilated equation $ax^2 + ay^2 = r^2$	The radius of the circle is changed. Note that the dilation must be the same for x and y. The centre is (0, 0) and the radius is $\frac{r}{\sqrt{a}}$.

Exponential graphs

Basic equation $y = a^x, a > 1$	Basic shape is an exponential graph. The graph is continually increasing with an asymptote at $y = 0$ and a y-intercept of (0, 1). If $0 < a < 1$, the graph is reflected in the y-axis. Note that $\left(\frac{1}{a}\right) = a^{-1}$ so $\left(\frac{1}{a}\right)^x = a^{-x}$ and the graph of $y = a^{-x}$ is the same as $y = \left(\frac{1}{a}\right)^x$. 
Translated equation $y = a^{(x-h)} + k$	Basic shape is moved h units to the right and k units up. The asymptote is $y = k$ and the y-intercept is $(0, a^{-h} + k)$. The point (0, 1) has moved to the point $(h, k + 1)$.

Rectangular hyperbolas 10A

Basic equation $y = \frac{1}{x}$	The basic shape is a hyperbola. It has two asymptotes, the x- and y-axes. There are no x-and y-intercepts. 
Translated equation $y = \frac{1}{x-h} + k$	Basic shape is moved h units to the right and k units up. There are asymptotes at $x = h$ and $y = k$ and graph is discontinuous for $x = h$.

Worked Example 22

WE22

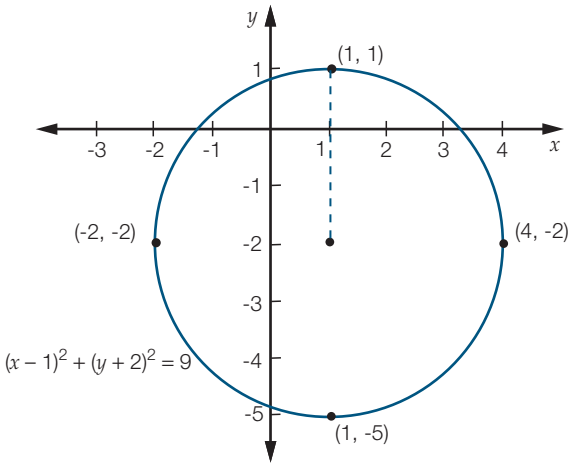
Sketch the graph of $(x - 1)^2 + (y + 2)^2 = 9$, clearly showing the centre of the circle and the radius.

Working

Coordinates of the centre are (1, -2).

$r = \sqrt{9}$
$= 3$

∴ radius is 3 units



Worked Example 23

WE23

Sketch the graph of $y = 3^{x-1} + 2$, clearly showing the asymptote and other key features.

Working

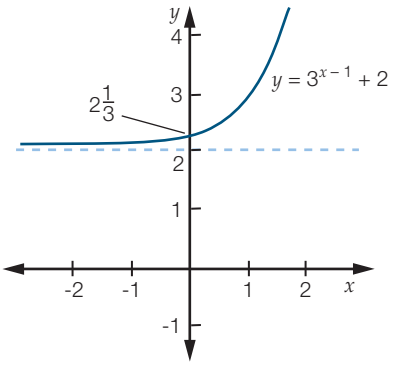
$y = 3^{x-1} + 2$ is the graph $y = 3^x$ translated 1 unit right and 2 units up.

The asymptote is $y = 2$.

When $x = 0$, $y = 3^{-1} + 2$

$= 2\frac{1}{3}$

The y-intercept is $(0, 2\frac{1}{3})$.



Worked Example 24

WE24

Sketch the graph of $y = \frac{1}{x-4} + 2$, clearly showing the key features (asymptotes, and x- and y-intercepts). 10A

Working

$y = \frac{1}{x-4} + 2$

The asymptotes are $x = 4$ and $y = 2$.

When $x = 0$, $y = -\frac{1}{4} + 2$

$= 1\frac{3}{4}$

The y-intercept is $(0, 1\frac{3}{4})$.

When $y = 0$, $0 = \frac{1}{x-4} + 2$

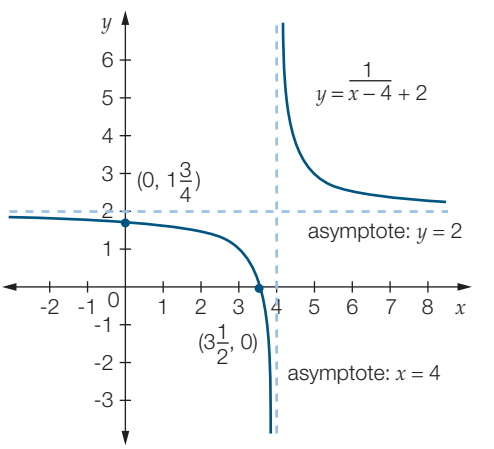
$-2 = \frac{1}{x-4}$

$-2x + 8 = 1$

$-2x = -7$

$x = 3\frac{1}{2}$

The x-intercept is $(3\frac{1}{2}, 0)$.



9.6 Sketch graphs of other non-linear relationships

Answers page 555

Fluency

1 Sketch the graph of each of the following, clearly showing the centre of the circle and the radius.

WE22

(a) $(x-2)^2+(y+1)^2=16$ **(b)** $(x+3)^2+(y-4)^2=25$

(c) $(x-2)^2+(y-2)^2=4$ **(d)** $(x+4)^2+(y+1)^2=16$

2 Sketch the graph of the following, clearly showing the asymptote and other key features for each graph.

WE23

(a) $y=2^{x-2}+1$ **(b)** $y=2^{x+1}+3$

(c) $y=2^x-3$ **(d)** $y=3^{x-4}-1$

3 Sketch the graph of the following, clearly showing the key features (asymptotes, and x- and y-intercepts) for each graph.

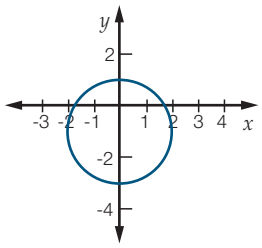
WE24 10A

(a) $y=\dfrac{1}{x+3}-2$ **(b)** $y=\dfrac{1}{x-5}+1$ **(c)** $y=\dfrac{1}{x+2}+3$

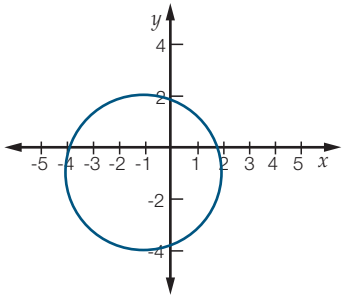
Understanding

4 What is the equation of each of the following graphs?

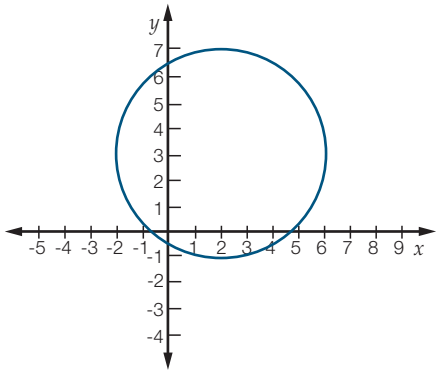
(a)



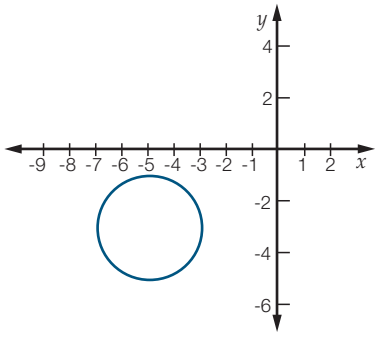
(b)



(c)



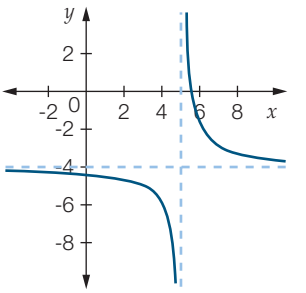
(d)



5 **(a)** Write the equation formed if the graph of $y=\dfrac{1}{x}$ is translated 3 units left and 4 units up.

(b) Graph this new equation showing any relevant asymptotes.

10A **6** The equation for this graph would be:



A $y = \frac{1}{x+4} + 5$ **B** $y = \frac{1}{x-4} - 5$

C $y = \frac{1}{x-5} - 4$ **D** $y = \frac{1}{x+5} - 4$

7 **(a)** Write the new equation formed if a circle with equation $(x-3)^2 + (y+4)^2 = 25$ is translated 2 units left and 3 units down.

(b) Write the new equation formed if a circle with equation $(x-3)^2 + (y+4)^2 = 25$ has its radius increased by 3 units.

Reasoning

10A **8** **(a)** Show that $\frac{1}{x-1} + 3 = \frac{3x-2}{x-1}$. **(b)** Hence, sketch the graph of $y = \frac{3x-2}{x-1}$.

9 A circular rose garden is to be planted in a rectangular lawn in front of a house. The lawn is 15 m from the front boundary to the house and 25 m along the front boundary. A diagram of the lawn and the rose garden is to be drawn before the garden is prepared for planting. The left-hand side of the front boundary is used as the origin.

(a) The landscape gardener who is creating this garden initially places the centre of the garden in the centre of the lawn. Write down the coordinates of the centre of the circle.

(b) If the rose garden is to have a radius of 3 m, write the equation that would represent the border of the garden.

(c) Sketch a graph of this garden.

(d) If the garden is moved 0.5 m closer to the front boundary and 6.5 m further to the left:

(i) write the new equation **(ii)** sketch the new graph.

(e) If the radius is now reduced by 1 m after the move:

(i) write the new equation **(ii)** sketch the new graph.

Open-ended

10A **10** **(a)** Write the equation of a rectangular hyperbola that has been translated from the origin.

(b) Identify the asymptotes for a graph of your equation.

(c) Sketch the graph.

11 A circle whose centre is not at the origin passes through the point (1, 1). What could be the equation of this circle?

Chapter review 9

D.I.Y. Summary

Key Words

asymptote	exponential	reflection	turning point form
axis of symmetry	Null Factor Law	transformation	vertex
dilation	parabola	translation	
dilation factor	rectangular hyperbola	turning point	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The basic graph of $y = x^2$ can be used to obtain other parabolas by using a combination of any of these _____s: _____s, _____s or _____s.

2 A _____ is the type of graph produced by a quadratic relationship.

3 To graph a quadratic it is important to show the x- and y-intercepts and the _____.

4 To change a quadratic that is in the form $y = ax^2 + bx + c$ ($a = 1$) into _____ , it is necessary to complete the square.

5 An _____ is a line on a Cartesian plane that a curve approaches but does not cut or touch for certain values of the independent variable.

6 An _____ equation has a horizontal asymptote and may be increasing or decreasing at a rapid rate.

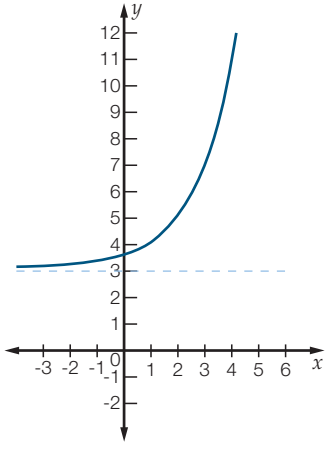
Fluency

1 For each graph below, identify the type of graph and give the:

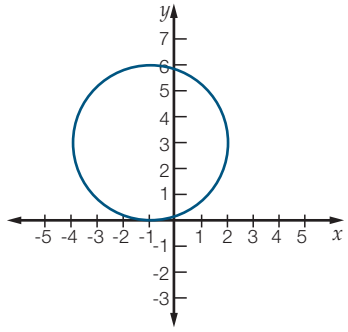
Ex 9.1

- equation of the asymptote if it is an exponential
- coordinates of the centre and the radius if it is a circle.

(a)



(b)



2 Plot the graph of $y = x^2 - 2x - 8$ for $-5 \leq x \leq 5$.

Ex 9.1

(a) Write down the coordinates of the x- and y-intercepts.

(b) Write down the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(c) Write down the equation of the axis of symmetry.

(d) From the graph, find the value of y when $x = 3$.

(e) What are the values of x when $y = 7$?

(f) Hence, state the values of x when $x^2 - 2x - 8 = 7$.

(g) Write down the values of x when $x^2 - 2x - 8 = 0$.

3 **(a)** The equation of the parabola shown could be:

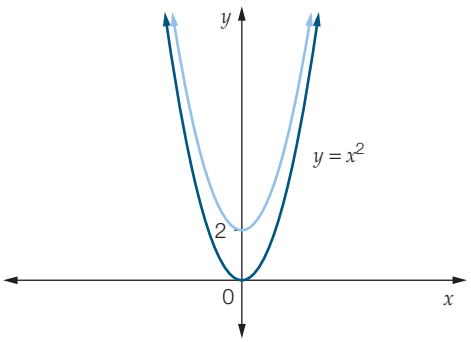
Ex 9.2

A $y = x^2 + 2$

B $y = x^2 - 2$

C $y = (x - 2)^2$

D $y = (x + 2)^2$



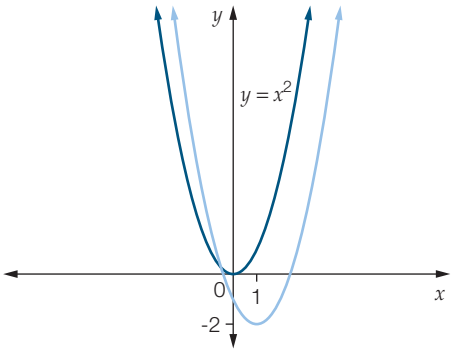
(b) The equation of the parabola shown could be:

A $y = (x - 1)^2 - 2$

B $y = (x - 1)^2 + 2$

C $y = (x + 1)^2 + 2$

D $y = (x + 1)^2 - 2$



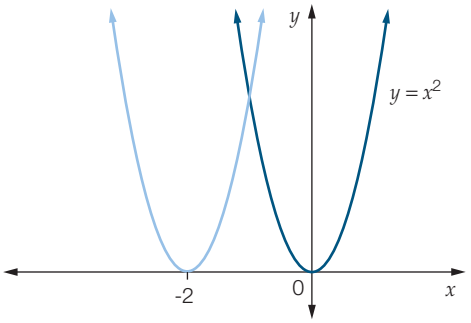
(c) The equation of the parabola shown could be:

A $y = (x - 2)^2$

B $y = (x + 2)^2$

C $y = x^2 - 2$

D $y = x^2 + 2$



4 Solve these quadratic equations using the Null Factor Law.

Ex 9.3

(a) $x^2 - 6x = 0$ **(b)** $4x^2 - 81 = 0$ **(c)** $x^2 - 5x - 14 = 0$

(d) $x^2 + 6x + 9 = 0$ **(e)** $x^2 = 8x$ **(f)** $x^2 + 36 = 12x$

(g) $2x(x + 4) = 0$ **(h)** $(x - 7)(x + 1) = 0$ **(i)** $x^2 - 16 = 0$

5 Solve these quadratic equations.

Ex 9.3

(a) $x^2 + 12x + 35 = 0$ **(b)** $x^2 + x - 6 = 0$ **(c)** $-x^2 - 8x + 9 = 0$

(d) $-x^2 + 9x - 8 = 0$ **(e)** $3x^2 + 9x - 12 = 0$ **(f)** $2x^2 - 8x + 6 = 0$

6 Solve these quadratic equations.

Ex 9.3

(a) $x^2 - 4x = 5$ **(b)** $x^2 - 6 = 5x$ **(c)** $x^2 + 12 = -7x$

(d) $3x^2 = 27$ **(e)** $(x - 3)(x + 7) = 11$ **(f)** $(x - 3)(x + 2) = 6$

7 Solve each of the following by completing the square.

Ex 9.4

(a) $x^2 - 4x + 2 = 0$ **(b)** $x^2 + 8x + 9 = 0$ **(c)** $-x^2 - 2x + 4 = 0$

8 For each of the following:

Ex 9.5

(i) state the changes that have been made to the graph of $y = x^2$

(ii) state the coordinates of the turning point

(iii) find the x- and y-intercepts

(iv) sketch a graph of each equation showing the turning point and intercepts.

(a) $y = x^2 - 4$ **(b)** $y = -2(x + 1)^2 - 4$ **(c)** $y = \frac{1}{3}(x - 7)^2$

9 For each of the following equations:

Ex 9.5

(i) find the y-intercept **(ii)** find the x-intercepts

(iii) find the coordinates of the turning point **(iv)** sketch the graph.

(a) $y = x^2 - 4x - 12$ **(b)** $y = -(x + 1)^2 + 4$

(c) $y = 2(x - 3)^2 - 6$ **(d)** $y = -x^2 + 3x + 4$

10 Sketch the graph of each of the following.

Ex 9.6

(a) $(x + 2)^2 + (y - 3)^2 = 16$ **(b)** $x^2 + (y - 3)^2 = 4$

(c) $y = 2^{(x-2)} + 1$ **(d)** $y = 2^{x+2} - 4$

11 Sketch the graph of each of the following.

Ex 9.6 10A

(a) $y = \frac{1}{x - 4} + 1$ **(b)** $y = \frac{1}{x + 2} - 1$

Understanding

12 A softball thrown by an outfielder to the pitcher followed a path where the height h, in metres, of the softball after t seconds, is described by the relationship:

$h = -t^2 + 6t + 1$

(a) Complete a table of values for this relationship.

(b) Plot the graph of this relationship on graph paper.

(c) What was the height of the softball after 2 seconds?

(d) Use your graph to find when the ball was first at a height of 7 m above the ground?

(e) Use your graph to find during which time interval the ball was above a height of 9 m?

(f) What was the maximum height of the softball during its flight?

(g) How long was the ball in flight if it was caught at a height of 1.6 m above the ground on its downward path?

13 A train ticket is 5 cm longer than it is wide.

(a) Draw a diagram of the ticket, labelling the width x.

(b) Write down the length of the ticket in terms of x.

(c) Write an expression for the area of the train ticket in terms of x.

(d) If each ticket requires 24 cm^2 of paper, write an equation that represents this area.

(e) Solve this equation to find x.

(f) Hence, state the dimensions of the train ticket.

(g) Did you use both of the values for x that you found using algebra to find the dimensions of the ticket? Explain your answer.

Reasoning

14 A fountain sends a jet of water on a parabolic path, which can be described by the equation $h = -2x^2 + 0.8x$, where h is the height of the water, in metres, at a horizontal distance x, in metres, from the water outlet.

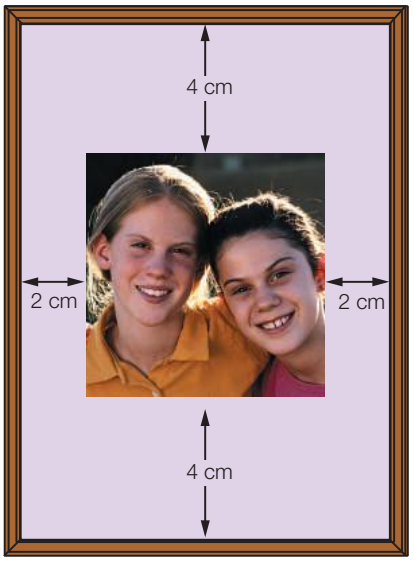
(a) Where is the jet of water again at the height that it left the outlet?

(b) If it hits the base of a pond at a horizontal distance of 0.8 m from the outlet, how far below the outlet does the water land?

(c) What is the maximum height above the base of the pond that the jet of water reaches?

(d) Sketch the graph of the path of the water.

15 A square photograph is to be attached to a coloured mounting and then framed.



(a) Find the length of a side of the photo if the frame encloses a total area of 140 cm^2.

(b) Hence find the dimensions of the photo frame.

(c) Calculate the area of the photograph.

(d) Calculate the area of coloured mounting that can be seen.

(e) What area of coloured mounting would be seen if the same photograph was to have a 2 cm coloured mounting all around?

16 The sum of two numbers is 20.

(a) The two numbers are x and y. Write an equation to represent the sum.

(b) Rearrange the equation so that x is the subject (i.e. in the form $y =$).

(c) Write an expression for the product of the two numbers using x.

(d) Graph the expression and use it to find the maximum product and the numbers that obtain it.

17 A tennis ball is struck in the centre of a tennis court 23.78 m long, 1 m behind the base line. It follows a parabolic path parallel to the sidelines and its height is determined by the equation $h = -0.0045x^2 + 0.135x$, where x is the distance from a fence, which is 6.11 m from the base line.

(a) At what height was it struck?

(b) Show that the ball clears the net, which is 0.913 m high in the centre.

(c) Determine whether the ball lands inside or outside the tennis court.

18 The graph of $y = x^2$ is translated so that it passes through two points (3, 8) and (9, 8).

(a) Write down the x-coordinate of the turning point of the graph.

(b) Find the equation of the curve.

(c) Sketch this graph.

19 A pattern is drawn with four circles with equations:

A $(x - 3)^2 + (y - 2)^2 = 16$ **B** $(x + 1)^2 + (y - 3)^2 = 4$

C $(x + 5)^2 + (y + 4)^2 = 9$ **D** $(x + 2)^2 + (y + 1)^2 = 16$

(a) Which two circles are the same size?

(b) By sketching the graphs find which circle intersects with the other three circles.

Mixed review C

Fluency

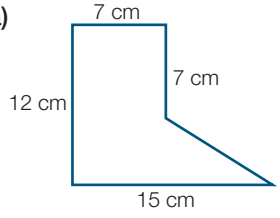
1 An investment of $5000 is made at 3.5% p.a. for 6 years. Find the difference in interest earned if the interest is compounded monthly compared to annually. Ex.1.2

2 Factorise the following quadratic trinomials. Ex.2.3

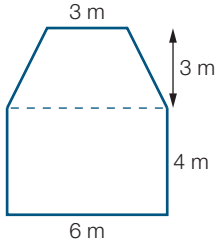
(a) $a^2 - 5a - 36$ **(b)** $a^2 - 8a + 12$ **(c)** $4a^2 + 20a + 25$

3 Find the area of each of the following shapes. Ex.3.1

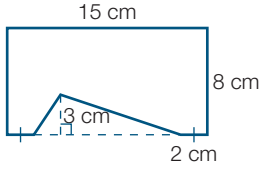
(a)



(b)



(c)

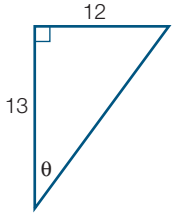


4 Solve the following equations. Ex.4.1

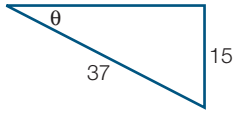
(a) $\frac{3a+4}{7} = -2$ **(b)** $\frac{4a+3}{2} = \frac{2a}{5}$ **(c)** $\frac{3a-2}{3} = \frac{4a-5}{2}$

5 Find the value of the pronumeral in each, correct to one decimal place. Ex.5.3

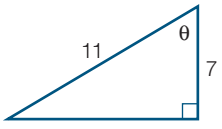
(a)



(b)



(c)



6 Find the length of the third side of a scalene triangle with two sides of length 6 cm and 7 cm and an included angle of 43°. Round your answer to two decimal places. Ex.6.4 10A

7 Find the range and the interquartile range for each of the following grouped data frequency tables. Give your answers correct to two decimal places. Ex.7.2

(a)

Class interval	Frequency
20–<25	8
25–<30	6
30–<35	3
35–<40	2
40–<45	5
45–<50	5

(b)

Class interval	Frequency
110–<115	12
115–<120	15
120–<125	17
125–<130	19
130–<135	13
135–<140	11

8 Prove that the diagonals of a kite intersect at right angles. Ex.8.5

9 Consider the equation $y = 2x^2 + x$.

Ex 9.1

(a) Set up a table of values with x-values from -3 to 3.

(b) Plot the points on a coordinate plane.

(c) Find the turning point of the graph.

(d) Determine the x- and y-intercepts for the graph.

10 What are the asymptotes of the graph of $y = \frac{1}{x+1} - 3$?

Ex 9.1

11 Solve each of the following equations.

Ex 9.3

(a) $x^2 + 13x + 42 = 0$ **(b)** $x^2 - 100 = 0$ **(c)** $x^2 - 14x + 49 = 0$

Understanding

12 Sketch the graph of each of the following quadratics, showing all important features.

(a) $y = x^2 + 6x - 4$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = -2x^2 + 3x$

13 Solve each of the following for x.

(a) $2x^2 - 50 = 0$ **(b)** $4x^2 + 2x = 0$ **(c)** $x^2 - 5x + 6 = 0$

14 Write down the turning point of each of the following graphs.

(a) $y = (x - 2)^2 + 3$ **(b)** $y = (x + 4)^2 - 1$ **(c)** $y = (x - 7)^2 - 6$

15 Write the equation formed when $x^2 + y^2 = 25$ is translated 4 units right and 2 units up.

16 A point that does not lie on the graph of $(x - 3)^2 + (y - 2)^2 = 20$ would be:

A (7, 0) **B** (5, 6) **C** (6, 5) **D** (-1, 4)

17 The graph of the equation $y = 2^{x-1} + 1$ has an asymptote at:

A $x = 1$ **B** $x = -1$ **C** $y = 1$ **D** $y = -1$

Reasoning

18 A mobile phone screen is 2.5 cm longer than it is wide.

(a) Write an equation for the area of the screen in terms of its width, in cm^2.

(b) Plot a graph of the area against the width.

(c) Use this graph to find the dimensions of the screen if the area of the screen is 37.5 cm^2.

19 A graph has x-intercepts at $x = 4$ and $x = 6$.

(a) Determine the x-coordinate of the turning point.

(b) If the x^2 coefficient is 1, determine the equation of the graph.

Challenge C

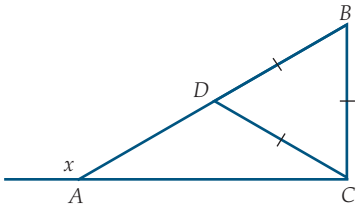
1 The mean of 19 consecutive integers is 99. The largest of these integers is:

A 118 **B** 117 **C** 108 **D** 107

2 Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is:

A 34 **B** 35 **C** 36 **D** 37

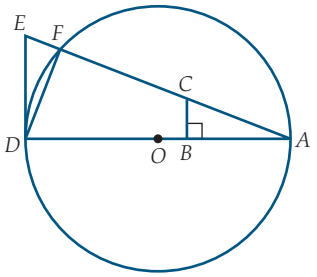
3 Find the value of x if the large triangle has been broken into two triangles of equal area. The large triangle is not marked as being right angled.



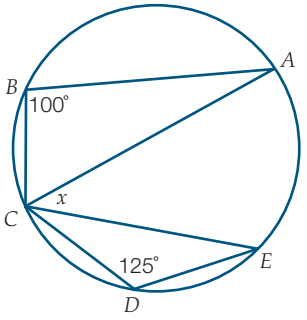
4 In the diagram, $ABC = 90°$, $CB \parallel ED$, $AB = DF$, $AD = 24$, $AE = 25$ and O is the centre of the circle. The perimeter of $CBDF$ is:

A 40 units **B** 42 units

C 43 units **D** 44 units



5 Find x in the following diagram.



6 **(a)** Determine the values of x for which $(x + 3)(x - 6) = -14$.

(b) Determine the values of x for which $(x^2 - 3x)^2 = 4 - 3(3x - x^2)$.

7 **(a)** What is the sum of the two numbers that satisfy the equation $x^2 - 6x - 7 = 0$?

(b) What is the product of the two numbers that satisfy the equation $5x^2 - 20 = 0$?

(c) Find the average of the numbers that satisfy the equation $x^3 - 6x^2 + 5x = 0$.

8 The equation of a parabola is written in the form $y = (x - 2)(x - 8) + 7$.

(a) If $(c, 7)$ lies on the parabola, find all possible values of c.

(b) Find the coordinates of the vertex of the parabola.

9 The circle $x^2 + y^2 = 9$ and the parabola $y = x^2 - 3$ intersect at three points.

(a) Find the y-coordinates of the points of intersection.

(b) Hence, find the exact values of the coordinates of the points of intersection.

10 Polynomials

10A

Recall 10

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R10.1

1 Plot the points in each of the following tables of values and join them with a straight line or a smooth curve to obtain the graph of each relationship.

(a)

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

(b)

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11

Worksheet R10.2

2 State whether the relationship is linear, quadratic or neither.

(a) $y = x^3 + 2x^2 + 3$ **(b)** $b = 2c + 3$ **(c)** $z = 4 - 3w^2$

Worksheet R10.3

3 (a) Complete this table of values for $y = x^2 + 2x - 8$.

x	-5	-4	-3	-2	-1	0	1	2	3
y									

(b) On graph paper, plot the graph of the parabola $y = x^2 + 2x - 8$.

(c) From the graph, write the:

(i) y-intercept

(ii) x-intercepts

(iii) coordinates of the turning point.

(d) Decide whether the graph has a maximum or a minimum turning point.

(e) What is the equation of the axis of symmetry?

Worksheet R10.4

4 Factorise each quadratic expression:

(a) $x^2 - 5x - 6$ **(b)** $4x^2 - 81$ **(c)** $3x^2 - 3x - 36$

(d) $x^2 - 14x + 49$ **(e)** $-x^2 + 8x + 20$ **(f)** $x^2 - 6x + 1$

Worksheet R10.5

5 Solve these equations using the Null Factor Law.

(a) $(x + 2)(x - 1) = 0$ **(b)** $(x + 9)(x - 9) = 0$ **(c)** $3x(2x - 5) = 0$

After completing this chapter you will be able to:

- recognise a polynomial
- recognise graphs in the form of $y = ax^n$
- recognise graphs in the form of $y = a(x - h)^n + k$ as translations of graphs in the form of $y = ax^n$
- translate graphs in the form of $y = a(x - h)^n + k$
- perform the four operations (+, −, ×, ÷) on polynomials
- solve non-monic quadratic equations using a variety of techniques
- use the remainder theorem to find remainders after polynomial division
- use the factor theorem to factorise a polynomial
- sketch graphs of polynomials.

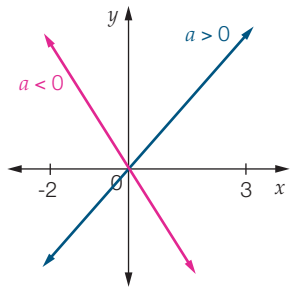
Equations of the form $y = ax^n$

10.1

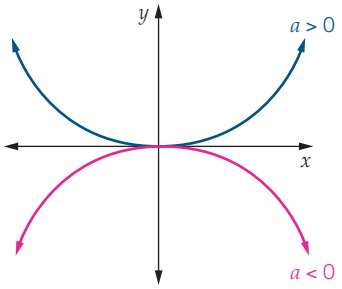
Need to Know

The graphs of equations in the form of $y = ax^n$ for $n = 1$, 2, 3 and 4 pass through the origin and have an x-intercept and a y-intercept of 0.

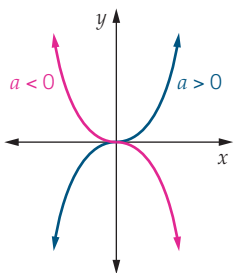
- For $n = 1$, the graph is linear.



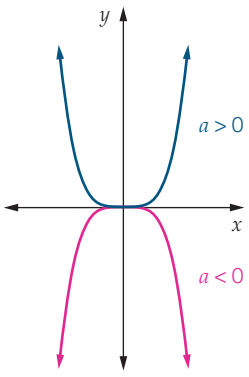
- For $n = 2$, the graph is a parabola.



- The graph of $y = ax^3$ does not have a turning point (maximum or minimum) where the graph changes direction but it does have a special point at the origin called a **point of inflection**.
- The graph of $y = ax^4$ passes through the origin and is symmetrical about the y-axis. A quartic graph is steeper than a quadratic graph.
- For $n = 3$, the graph is a **cubic**.



- For $n = 4$, the graph is a **quartic**.



- The dilation factor (value of a) narrows the graph for $a > 1$ and widens the graph for $0 < a < 1$.
- The graph of $y = -ax^n$ is the reflection of $y = ax^n$ in the x-axis, $a > 0$.

Worked Example 1

WE1

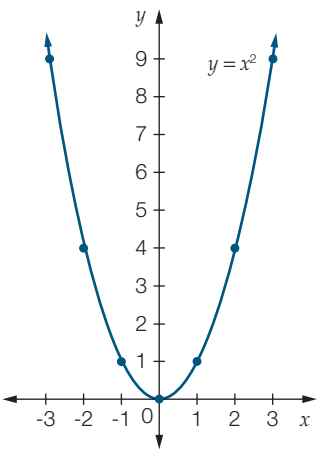
Construct a table of values for $x = -3$ to 3 and then plot the graph of the following equations:

(a) $y = x^2$ **(b)** $y = x^3$ **(c)** $y = x^4$

Working

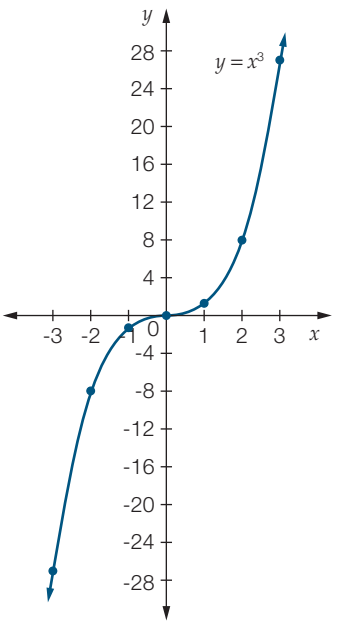
(a)

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



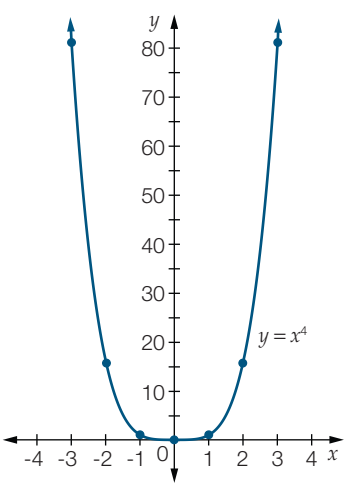
(b)

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	2	8	27



(c)

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81



10.1 Equations of the form $y = ax^n$

Answers page 561

Fluency

WE1

1 Construct a table of values for $x = -3$ to 3 and then plot the following equations.

(a) $y = 2x^2$ **(b)** $y = 2x^3$ **(c)** $y = 2x^4$

2 Match each equation to its graph.

U $y = -\frac{2}{3}x$

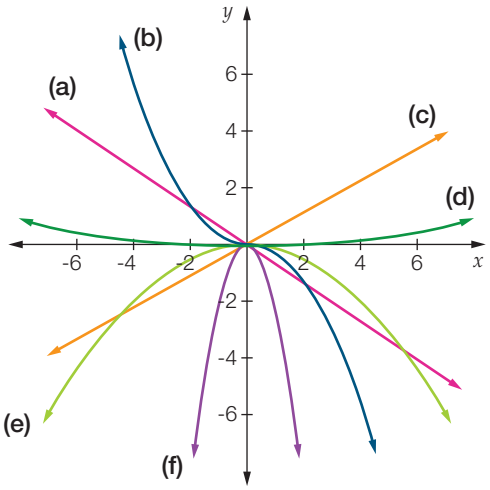
V $y = -6x^2$

W $y = \frac{x}{2}$

X $y = -\frac{1}{10}x^2$

Y $y = 0.01x^2$

Z $y = \left(-\frac{1}{2}x\right)^3$



3 **(a)** Plot the graphs of each linear equation on the same number plane. Use x-values from -3 to 3.

(i) $y = x$ **(ii)** $y = 2x$ **(iii)** $y = 4x$

(b) What can you conclude about the steepness of the line in relation to the coefficient of x?

(c) Does your answer to part **(b)** apply to the equations $y = -x$, $y = -2x$ and $y = -4x$?

4 **(a)** Plot the graphs of each quadratic equation on the same number plane. Use x-values from -3 to 3.

(i) $y = 2x^2$ **(ii)** $y = 3x^2$ **(iii)** $y = 4x^2$

(b) Describe what happens to the shape of the parabola as the coefficient of x^2 gets larger.

(c) Does your answer to part **(b)** apply to the equations $y = -2x^2$, $y = -3x^2$ and $y = -4x^2$?

5 **(a)** Plot the graphs of each cubic equation on the same number plane. Use x-values from -3 to 3.

(i) $y = x^3$ **(ii)** $y = 2x^3$ **(iii)** $y = 3x^3$

(b) Describe what happens to the shape of the cubic graph as the coefficient of x^3 gets larger.

(c) Does your answer to part **(b)** apply to the equations $y = -x^3$, $y = -2x^3$ and $y = -3x^3$?

6 **(a)** Plot the graph of each quartic equation on the same number plane. Use x-values from -3 to 3.

(i) $y = 0.5x^4$ **(ii)** $y = x^4$ **(iii)** $y = 3x^4$

(b) Describe what happens to the shape of the quartic graph as the coefficient of x^4 gets larger.

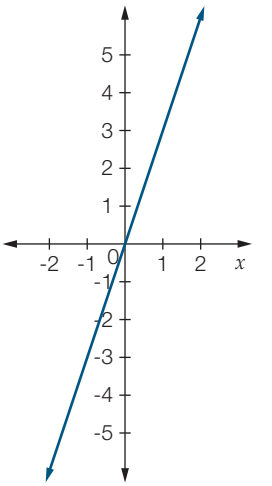
(c) Does your answer to part **(b)** apply to the equations $y = -x^4$, $y = -3x^4$, $y = -0.5x^4$?

Warning

Is -4 > -2?

Understanding

7 The straight line shown is reflected in the x-axis and then reflected in the y-axis. The equation describing the new graph is:



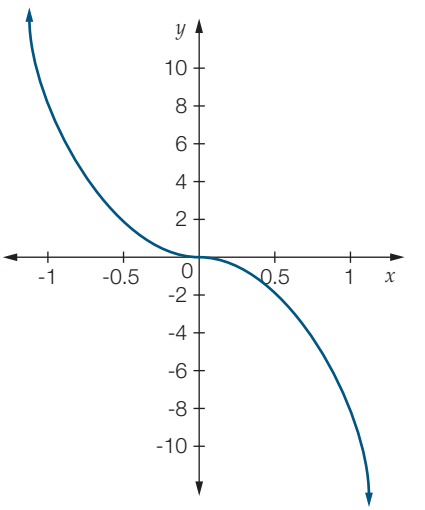
A $y = \frac{1}{3}x$

B $y = 3x$

C $y = -\frac{1}{3}x$

D $y = -3x$

8 A possible equation to describe the graph shown is:



A $y = -8x^2$

B $y = x^3$

C $y = -x^2$

D $y = -8x^3$

Reasoning

9 The shape of a skate ramp is described by the equation $y = \frac{1}{3}x^2$, where x and y are the horizontal and vertical distances respectively, in metres, from the bottom of the ramp.

(a) Draw the graph of the equation using values of x from 0 to 3.

(b) One end of a taut length of rope is attached to the bottom of the ramp and the other is attached to a point on the ramp with x-coordinate 1.5. What is the equation of the line describing the position of the rope?

10 Tim and Matthew set off from their campsite in different directions. Tim's distance, d km, from the campsite at t minutes is given by the equation $d = 0.1t$, and Matthew's distance from the campsite is given by $d = -0.1t$.

(a) Represent each equation on the same number plane using values of t from 0 to 10.

(b) How far apart are the boys after 10 minutes?

(c) What is the equation of the line describing the path Tim could have taken had he walked at a constant speed and travelled 2.4 km in 20 minutes?

Hint

What does a negative distance represent?

11 The shape of a flat weathervane is defined by the two equations $y = 4x$ and $y = 2x^2$ for x-values from 0 to 2, where x and y represent length in metres and the origin is taken to be the base of the weathervane.

(a) Sketch the two graphs on the same number plane using x-values from 0 to 2.

(b) If the area under the curve $y = 2x^2$ from $x = 0$ to $x = 2$ and bounded by the x-axis is $5\frac{1}{3}$ square metres, what is the area covered by one face of the weathervane?

(c) The wind changes direction so that the weathervane swings around 180° about the origin. What are the equations describing the shape of the weathervane now? State the values of x that apply.

12 (a) On the same set of axes draw the curves $y = x^3$ and $y = \frac{1}{2}x^4$ for $x = -2$ to 2.

(b) Shade in the region between these two curves between $x = 0$ and $x = 2$.

(c) Reflect this region in the y-axis and draw it on your graph.

(d) What is the equation of the top boundary of this new region?

Open-ended

13 (a) Find two consecutive whole number values for c so that the gradient of the straight line described by the equation $y = -\left(\frac{c-9}{1-c}\right)x$ is positive.

(b) What can be concluded about the gradient when $c = 1$? What does this tell you about the line?

14 Consider the graphs of $y = ax^2$ and $y = 2a - ax^2$. Draw the graphs on the same number plane for $a = 1, 2, 3, 4$. What can you discover above the points of intersection of the two graphs? Repeat for negative values of a and comment on your answer.

Problem solving

Different paths

Mark wants to build a path from his front door to the gate. He draws up a grid and determines that his front door is at grid reference (0, 0) and the gate is at (5, 12).
He decides that as he has been learning about graphs of the type $y = ax^n$ he will see how many different graphs he can get that fit these two points.

1 Find the equations of the curves that he would need for $n = 1, 2, 3$ and 4.

2 For each value of n, determine at least one other grid reference that he should use when making the path.

Strategy options

- Draw a diagram.
- Test all possible combinations.
- Look for a pattern.

10.2 Transformations of $y = ax^n$

Need to Know

Graphs of $y = a(x - h)^n + k$

Transformations are applied in the order of dilations, reflections and translations.

The graph of $y = a(x - h)^n + k$ is a translation of the graph of $y = ax^n$ by h units horizontally and k units vertically.

- If $h > 0$, the translation is towards the right and if $h < 0$, the translation is towards the left.
- If $k > 0$, the translation is up and if $k < 0$, the translation is down.

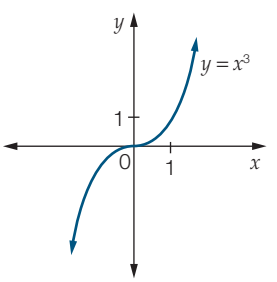
Worked Example 2

WE2

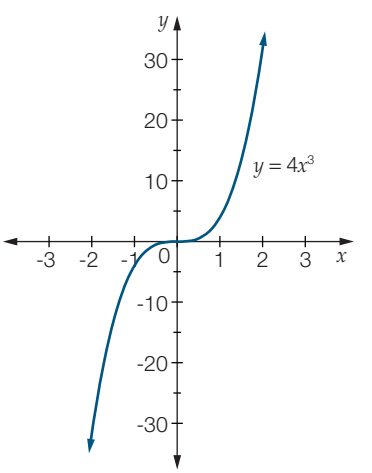
Sketch the graph of $y = -4(x - 1)^3$ by transforming the basic graph of $y = x^3$.

Working

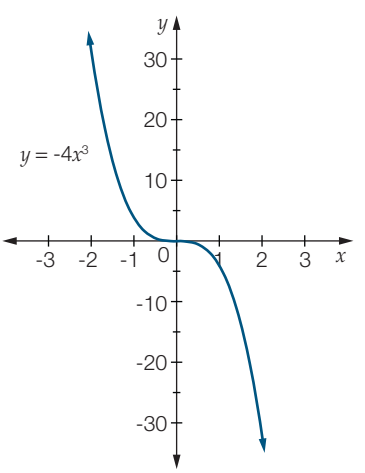
Step 1: Sketch the graph of $y = x^3$.



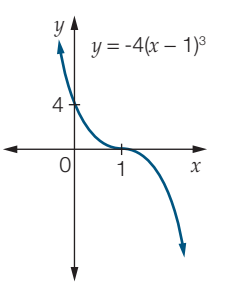
Step 2: Dilate the graph by a factor of 4.



Step 3: Reflect the graph in the x-axis.



Step 4: Translate the graph 1 unit to the right.



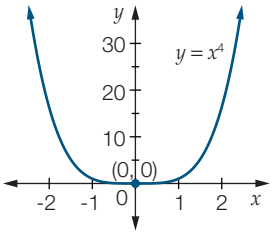
Worked Example 3

WE3

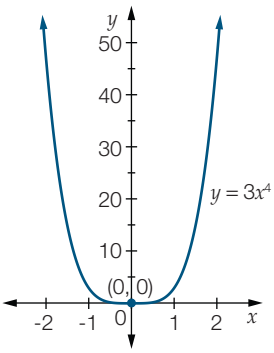
Sketch the graph of $y = -3x^4 + 2$ by transforming the basic graph of $y = x^4$.

Working

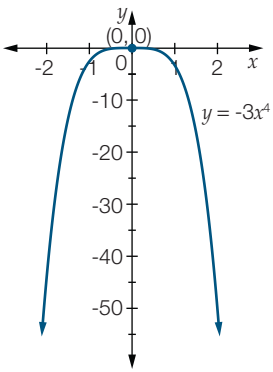
Step 1: Sketch the graph of $y = x^4$.



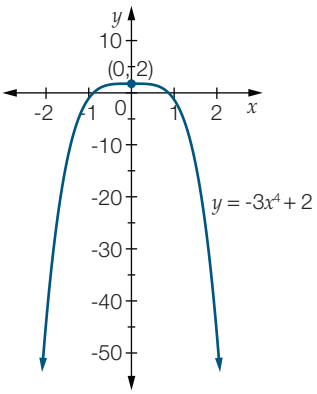
Step 2: Dilate the graph by a factor of 3.



Step 3: Reflect the graph in the x-axis.



Step 4: Translate the graph 2 units up.



10.2 Transformations of $y = ax^n$

Answers page 563

Fluency

1 Sketch the graph of $y = -3(x + 2)^3$ by transforming the graph of $y = x^3$. WE2

2 Sketch the graph of $y = -x^2 + 1$ by transforming the graph of $y = x^2$. WE3

3 Sketch the graph of $y = 4x - 2$ by transforming the graph of $y = x$.

4 Sketch the graph of $y = 3(x - 1)^4$ by transforming the graph of $y = x^4$.

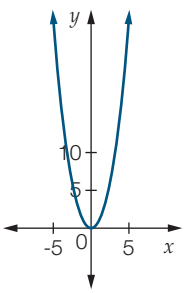
5 Sketch the graph of $y = \frac{1}{2}(x - 3)^2$ by transforming the graph of $y = x^2$.

6 Sketch the graph of $y = -\frac{1}{10}(x + 2)^4$ by transforming the graph of $y = x^4$.

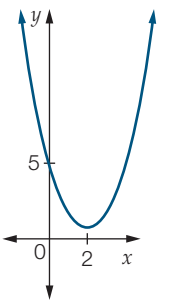
Understanding

7 The graph of the equation $y = x^2$ is transformed to the graph of the equation $y = -2(x + 2)^2 - 1$. The graph of the transformed equation is:

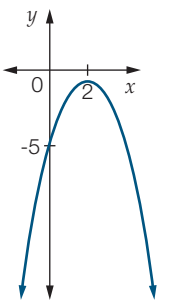
A



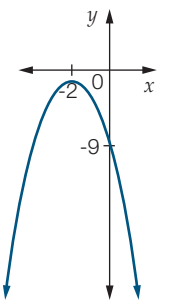
B



C



D



8 To obtain the graph of $y = -3(x + 2)^3 + 1$ from $y = x^3$:

A Dilate by a factor of 3 in the y-direction, reflect in the x-axis, translate 2 units right and translate one unit up.

B Dilate by a factor of 3 in the y-direction, reflect in the x-axis and translate 2 units left.

C Dilate by a factor of 3 in the y-direction, reflect in the x-axis, translate 2 units left and translate one unit up.

D Dilate by a factor of 3 in the y-direction, translate 1 unit left and translate 2 units down.

9 The y-intercept of the graph of the equation $y = (16x - 3)^3$ is:

A (0, 54) **B** (0, -24) **C** (0, -27) **D** (0, 48)

Reasoning

10 A quadratic in the form $y = a(x - h)^2 + k$ has a turning point at (1, 2) and passes through the point (2, -1).

(a) State the values of h and k.

(b) Will the value of a be positive or negative? Give a reason for your answer.

(c) Find the value of a.

11 The shape and position of a snake's body at time t seconds is described by the equation $y = \frac{(-1)^t x^3}{10} + 100t$ where x and y are in centimetres and $-10 \leq x \leq 10$.

(a) What is the equation describing the snake's initial shape? (Initial means $t = 0$.)

(b) Sketch the graph of the equation in part **(a)** using the basic graph $y = x^3$.

(c) On the same axes, sketch the snake's shape and position:

(i) 1 second later **(ii)** 2 seconds later.

(d) Compare the position of the snake after 1 second with its initial position. State the y-value about which the snake's new position is a reflection of its initial position.

12 The cross-section of the entrance to a tunnel has the shape of the parabola $y = 5 - 0.2(x - 5)^2$, where x and y are given in metres and $0 \leq x \leq 10$. A second tunnel with the same basic shape is 5 m higher and has its centre 15 m to the right of the centre of the first tunnel.

(a) What is the equation describing the second tunnel's entrance?

(b) Sketch the graphs of the two equations on the same number plane.

(c) Use Pythagoras' theorem to calculate the distance, to one decimal place, between the tops of the entrances to the tunnels.

13 The shape of the front leg of an antique chair is described by the equation $y = (x - 2)^3 + 8$, where x and y are given in centimetres. The second front leg is a reflection of the first leg in the line $x = 12$.

(a) Sketch the graph of the equation describing the shape of the first leg for $0 \leq x \leq 6$.

(b) State the equation describing the shape of the second leg.

(c) Sketch the graph of the second leg on the same number plane as your answer to part **(a)**.

(d) What is the minimum distance between the two legs?

Open-ended

14 Starting with the equation $y = x^3$, write two cubic equations so that each equation has a reflection in the x-axis, a dilation factor between 1 and 10, a horizontal translation to the left and a vertical translation downwards.

Polynomials

10.3

Need to Know

- A **polynomial** can be written as $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x^1 + a_0$, where $a^n \neq 0$, n is a positive integer and $a_0, a_1, a_2, \ldots a_n$ are the coefficients of the powers of x.
- The **degree** of the polynomial is the highest power of x. For $P(x)$ given above, the polynomial is of degree n.
- The **leading coefficient** is a_n and the constant term is a_0 (the term without an x).
- The coefficient of a monic polynomial is equal to 1, $a_n = 1$.
- The first term of a polynomial written with powers in descending order is called the **leading term**.
- When performing a division the **dividend** is the quantity that is being divided, and the **divisor** is the quantity it is being divided into. The **quotient** is the result of the division, with the **remainder** being any amount left over after the division.
- When dividing polynomials the remainder must be zero or have a degree less than the degree of the divisor.
- When adding and subtracting polynomials, only combine like terms.
- When expanding the product of binomial expressions, work through the expression in order, being very careful with negative signs.
- When dividing polynomials, make sure that all the powers less than the degree of the dividend appear in your expression for the dividend. This may mean the introduction of zero coefficients.

Worked Example 4

WE4

State the degree, leading coefficient and constant term of each polynomial.

(a) $5x^4 + 3x^3 - x^2 + x - 2$ **(b)** $12 + 6x - x^3$

Working

(a) $5x^4 + 3x^3 - x^2 + x - 2$

The leading term is $5x^4$.

The degree is 4.

The leading coefficient is 5.

The constant term is -2.

(b) $-x^3 + 6x + 12$

The leading term is $-x^3$.

The degree is 3.

The leading coefficient -1.

The constant term is 12.

Worked Example 5

WE5

$P(x) = 2x^3 - x^2 + 4x - 7$ and $Q(x) = x^3 + 3x^2 - x + 1$

(a) Evaluate $P(2)$ and $Q(-2)$. **(b)** Write the expression for $P(x) + 2Q(x)$.

Working

(a) $P(x) = 2x^3 - x^2 + 4x - 7$

$P(2) = 2 \times 2^3 - 2^2 + 4 \times 2 - 7$

$P(2) = 16 - 4 + 8 - 7$

$= 13$

$Q(x) = x^3 + 3x^2 - x + 1$

$Q(-2) = (-2)^3 + 3 \times (-2)^2 - (-2) + 1$

$Q(-2) = -8 + 12 + 2 + 1$

$= 7$

(b) $P(x) + 2Q(x)$

$= 2x^3 - x^2 + 4x - 7 + 2(x^3 + 3x^2 - x + 1)$

$= 2x^3 - x^2 + 4x - 7 + 2x^3 + 6x^2 - 2x + 2$

$= 4x^3 + 5x^2 + 2x - 5$

Worked Example 6

WE6

If $P(x) = 3x - 2$ and $Q(x) = 4x^2 + x - 5$, write $P(x).Q(x)$ as a polynomial.

Working

$P(x).Q(x) = (3x - 2)(4x^2 + x - 5)$

$= 3x(4x^2 + x - 5) - 2(4x^2 + x - 5)$

$= 12x^3 + 3x^2 - 15x - 8x^2 - 2x + 10$

$= 12x^3 - 5x^2 - 17x + 10$

Worked Example 7

WE7

If $P(x) = x^2 - 2x - 3$ and $Q(x) = 4x^3 + x^2 - 6$, write $P(x).Q(x)$ as a polynomial.

Working

$$\begin{aligned} P(x).Q(x) &= (x^2 - 2x - 3)(4x^3 + x^2 - 6) \\ &= x^2(4x^3 + x^2 - 6) - 2x(4x^3 + x^2 - 6) - 3(4x^3 + x^2 - 6) \\ &= 4x^5 + x^4 - 6x^2 - 8x^4 - 2x^3 + 12x - 12x^3 - 3x^2 + 18 \\ &= 4x^5 - 7x^4 - 14x^3 - 9x^2 + 12x + 18 \end{aligned}$$

Worked Example 8

WE8

Evaluate each expression using long division. Do not use a calculator.

(a) $3689 \div 17$ **(b)** $16\,319 \div 23$

Working

(a)

$$\begin{array}{r} 217 \\ 17\overline{)3689} \\ -\underline{34} \\ 28 \\ -\underline{17} \\ 119 \\ -\underline{119} \\ 0 \end{array}$$

$3689 \div 17 = 217$

(b)

$$\begin{array}{r} 709 \\ 23\overline{)16319} \\ -\underline{161} \\ 219 \\ -\underline{207} \\ 12 \end{array}$$

$16\,319 \div 23 = 709\frac{12}{23}$

Worked Example 9

WE9

Use long division to find the quotient and remainder for $\frac{x^2 + 6x + 10}{x + 2}$.

Working

$$x + 2\overline{)x^2 + 6x + 10}$$

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 10} \end{array}$$

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 10} \\ x^2 + 2x \end{array}$$

$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 6x + 10} \\ -(x^2 + 2x) \\ \hline 0 + 4x \end{array}$$

$$
\begin{array}{r}
\ x \\
x+2\overline{)\,x^2+6x+10} \\
\underline{\ x^2+2x\ \ \downarrow} \\
0+4x+10
\end{array}
$$

$$
\begin{array}{r}
\ x+4 \\
x+2\overline{)\,x^2+6x+10}\ \text{original dividend} \\
\underline{\ x^2+2x\ \ \downarrow} \\
4x+10\ \text{new dividend}
\end{array}
$$

$$
\begin{array}{r}
\ x+4 \\
x+2\overline{)\,x^2+6x+10} \\
\underline{\ x^2+2x\ \ } \\
4x+10 \\
4x+8
\end{array}
$$

$$
\begin{array}{r}
\ x+4\ \ \text{quotient} \\
x+2\overline{)\,x^2+6x+10} \\
\underline{\ x^2+2x\ \ } \\
4x+10 \\
-(4x+8) \\
\underline{\ 0+2\ \ } \text{remainder}
\end{array}
$$

$$\frac{x^2+6x+10}{x+2} = x+4+\frac{2}{x+2}$$

Worked Example 10

WE10

Use long division to find the quotient and remainder for $\frac{8x^3+4x-1}{2x+3}$.

Working

$$
\begin{array}{r}
\ 4x^2-6x+11 \\
2x+3\overline{)\,8x^3+0x^2+4x-1} \\
\underline{\ 8x^3-12x^2\ \ } \\
0-12x^2+4x \\
-(-12x^2-18x) \\
\underline{\ 0+22x-1\ \ } \\
-(22x+33) \\
\underline{\ 0-34\ \ }
\end{array}
$$

$$\frac{8x^3+4x-1}{2x+3} = 4x^2-6x+11-\frac{34}{2x+3}$$

10.3 Polynomials

Answers page 564

Fluency

1 State the degree, leading coefficient and constant term of each polynomial. WE4

(a) $2x^3 + 5x^2 - x + 2$ **(b)** $5 - 4x^4 - 3x^3 + 7x^2$

(c) x **(d)** $-6 - x - x^2$

2 If $P(x) = 3x^3 - 2x^2 - x + 2$ and $Q(x) = -x^3 - 4x^2 - 5x + 6$, evaluate: WE5

(a) $P(3)$ **(b)** $Q(4)$ **(c)** $P(-2)$ **(d)** $Q(-1)$

3 Write each expression as a polynomial if $P(x) = 5x + 1$, $Q(x) = x^2 + 2x - 3$ and $R(x) = 2x^3 - x^2 - x + 6$. WE6

(a) $P(x).\ Q(x)$ **(b)** $P(x).\ R(x)$

4 If $P(x) = x^2 - 3x + 4$ and $Q(x) = 2x^3 + 3x^2 - 5$, write $P(x).\ Q(x)$ as a polynomial. WE7

5 Evaluate each expression by performing long division. Do not use a calculator. WE8

(a) $3715 \div 25$ **(b)** $4061 \div 12$ **(c)** $5924 \div 17$

(d) $2018 \div 19$ **(e)** $14\,797 \div 29$ **(f)** $10\,946 \div 123$

6 Use long division to find the quotient and remainder for: WE9

(a) $\dfrac{x^2 + 6x + 10}{x + 3}$ **(b)** $\dfrac{x^2 + 3x - 1}{x - 2}$

7 Use long division to find the quotient and remainder for: WE10

(a) $\dfrac{x^3 + 6x + 5}{x + 2}$ **(b)** $\dfrac{x^4 - 3x^2 + 1}{x - 3}$

Understanding

8 Write $-3A(x)\ B(x) + C(x)$ as a polynomial if $A(x) = -x + 4$, $B(x) = 4x^2 - 2x + 1$ and $C(x) = -x^2 + 3x - 8$.

9 Find the quotient and remainder for $\dfrac{P(x)}{Q(x)}$ if:

(a) $P(x) = x^2 + 7x + 13$, $Q(x) = x + 3$ **(b)** $P(x) = 4x^2 + 4$, $Q(x) = 2x - 1$

(c) $P(x) = 2x^2 - 3x + 1$, $Q(x) = x + 1$ **(d)** $P(x) = 3x^3 + 2x - 4$, $Q(x) = x - 2$

10 If $P(x) = -5x^3 - 8x^2 - 3x + 2$ and $Q(x) = x^3 + 4x^2 - x + 1$ and $R(x) = 2P(x) - 2Q(x)$, the leading coefficient of $R(x)$ is:

A -12 **B** -5 **C** 1 **D** 8

11 Use long division to show that the remainder is zero when $4x^2 + x - 3$ is divided by $4x - 3$.

12 The remainder when $x^3 + x^2 + x + 1$ is divided by $x - 1$ is:

A -5 **B** -3 **C** 2 **D** 4

13 Calculate $\dfrac{A(x)}{B(x)}$ when $A(x) = x^4 - 6x^3 + 5$ and $B(x) = x - 5$.

14 $P(x) = 3x^3 - 2x^2 - x + 2$ and $Q(x) = -x^3 - 4x^2 - 5x + 6$.

(a) If $R(x) = 5P(x) - Q(x)$, write the polynomial $R(x)$ and evaluate $R(2)$.

(b) If $R(x) = 2P(x) + 3Q(x)$, write the polynomial $R(x)$ and evaluate $R(-3)$.

(c) If $6P(x) = 2R(x) - 10Q(x)$, write the polynomial $R(x)$ and evaluate $R(-1)$.

Reasoning

15 **(a)** Calculate $\frac{A(x)}{B(x)}$ when $A(x) = 2x^5 - 3x^4 - 6x^2 - 3x + 18$ and $B(x) = 2x - 3$.

(b) What is the value of $\frac{A(x)}{B(x)}$ for $x = 2$?

(c) Evaluate $A(2)$ and $B(2)$ and find $\frac{A(2)}{B(2)}$. Compare your answer to your answer in **(b)**.

16 Alana has $P(x)$ dollars, Bruno has $Q(x)$ dollars and Cien has $\frac{P(x)}{Q(x)}$ dollars, where $P(x) = 12x^4 + 16x^3 + 3x^2 + 28x + 32$ and $Q(x) = 3x + 4$.

(a) Write the polynomial that describes the amount of money Cien has.

(b) **(i)** Dylan has x dollars. What is the ratio of Cien's amount of money to Dylan's?

(ii) Use your answer to part **(i)** to explain why x cannot be zero.

17 A sheet of pastry is $(2x^2 - 9x)$ cm long and $(4x + 3)$ cm wide.

(a) Write an expression for the perimeter of the sheet of pastry.

(b) Write an expression for the area of the sheet of pastry.

(c) **(i)** The sheet of pastry is to be divided into small rectangles of side lengths x cm and $(x - 1)$ cm. Write an expression for the largest number of small rectangles that could be cut out of the pastry if any unused pastry is rerolled to the same thickness to maximise the number of squares that could be formed.

(ii) If all the pastry is used (there is no remainder), what is the smallest possible value of x and how many rectangles can be formed?

Open-ended

18 Write a non-monic polynomial of degree 5 that contains four terms and has a negative constant term.

19 Find a polynomial $P(x)$ of degree 3 so that $\frac{P(x)}{x - 3}$ leaves no remainder.

20 $P_n(x)$ is a polynomial of degree n. It has 1 for all its coefficients so that, for example, $P_2(x) = x^2 + x + 1$ and $P_3(x) = x^3 + x^2 + x + 1$.

(a) Write an expression for $\frac{P_n(x)}{x + 1}$ for $n = 1, 2, 3, 4, 5$.

(b) By considering your results in part **(a)**, show that if n is odd, $\frac{P_n(x)}{x + 1}$ is a polynomial of degree $n - 1$ consisting of even powers of x.

(c) By considering your results in part **(a)**, show that if n is even, $\frac{P_n(x)}{x + 1}$ is a polynomial of degree $n - 1$ consisting of odd powers of x plus a remainder of $\frac{1}{x + 1}$.

Factorising non-monic quadratic trinomials

10.4

Need to Know

Splitting the middle term

We can factorise the **non-monic quadratic** trinomial $ax^2 + bx + c$ by splitting the middle term.

- Multiply the coefficient of the squared term a by the constant c.
- List factor pairs of ac and choose a factor pair that add to give b.
- Rewrite the expression using these factors as coefficients of the two middle terms to replace the middle term.
- Factorise using grouping in pairs.

The cross method

- We can also factorise non-monic quadratic trinomials using the cross method.
- Choose a factor pair that multiply to give the squared term (ax^2).
- Choose a factor pair that multiply to give the constant term (c).
- Cross multiply the terms from one factor pair with the terms from the other pair.
- If the sum of the two multiplications gives the middle term (b) then we have our factorisation, if not, then try the other cross multiplication or use different factor pairs.

Completing the square with non-monic quadratic expressions

Not all quadratic trinomials factorise with binomial products and some do not factorise at all. We can use 'completing the square' with non-monic quadratics to see if the expression can be factorised.

To complete the square, the coefficient of the quadratic must be 1, so we will need to take the coefficient of x out as a common factor before we can complete the square.

Worked Example 11

WE11

Factorise the following quadratic expressions by splitting the middle term.

(a) $3x^2 - 11x - 4$ **(b)** $5x^2 - 43x + 24$ **(c)** $12x^2 + 11x - 36$

Working

(a) $3x^2 - 11x - 4$

$= 3x^2 - 12x + x - 4$ $\times$ to give -12, + to give -11 $\}$ -12, 1

$= 3x(x - 4) + 1(x - 4)$

$= (x - 4)(3x + 1)$

(b) $5x^2 - 43x + 24$

$= 5x^2 - 40x - 3x + 24$ $\times$ to give 120, + to give -43 $\}$ -40, -3

$= 5x(x - 8) - 3(x - 8)$

$= (x - 8)(5x - 3)$

(c) $12x^2 + 11x - 36$
$= 12x^2 + 27x - 16x - 36$
$= 3x(4x + 9) - 4(4x + 9)$
$= (4x + 9)(3x - 4)$

$\times$ to give -432
$+$ to give 11
} 27, -16

Worked Example 12

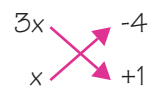
WE12

Factorise the following quadratic expressions using the cross method.

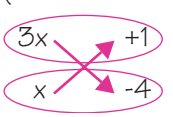
(a) $3x^2 - 11x - 4$ **(b)** $5x^2 - 43x + 24$ **(c)** $12x^2 + 11x - 36$

Working

(a) $3x^2 - 11x - 4$

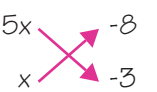


$3x - 4x = -x$
(incorrect middle term)

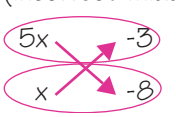


$-12x + x = -11x$
(correct middle term)

(b) $5x^2 - 43x + 24$

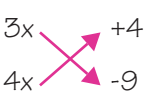


$-15x - 8x = -23x$
(incorrect middle term)

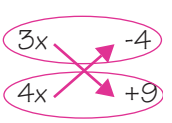


$-40x - 3x = -43x$
(correct middle term)

(c) $12x^2 + 11x - 36$



$-27x + 16x = -11x$
(incorrect middle term)



$27x - 16x = +11x$
(correct middle term)
$12x^2 + 11x - 36$
$= (4x + 9)(3x - 4)$

Worked Example 13 WE13

Factorise the following expressions, if possible, by completing the square

(a) $3x^2 - 6x - 5$ **(b)** $2x^2 - 5x - 2$ **(c)** $2x^2 - 4x + 6$

Working

(a) $3x^2 - 6x - 5$

$= 3(x^2 - 2x - \frac{5}{3})$

$= 3(x^2 - 2x + 1 - 1 - \frac{5}{3})$

$= 3[(x-1)^2 - \frac{8}{3}]$

$= 3(x - 1 - \sqrt{\frac{8}{3}})(x - 1 + \sqrt{\frac{8}{3}})$

(b) $2x^2 - 5x - 2$

$= 2(x^2 - \frac{5}{2}x - 1)$

$= 2[(x^2 - \frac{5}{2}x + \frac{25}{16}) - \frac{25}{16} - 1]$

$= 2[(x - \frac{5}{4})^2 - \frac{41}{16}]$

$= 2(x - \frac{5}{4} + \frac{\sqrt{41}}{4})(x - \frac{5}{4} - \frac{\sqrt{41}}{4})$

(c) $2x^2 - 4x + 6$

$= 2(x^2 - 2x + 3)$

$= 2[(x^2 - 2x + 1) - 1 + 3]$

$= 2[(x-1)^2 + 2]$

The expression cannot be factorised further.

10.4 Factorising non-monic quadratic trinomials

Answers page 564

Fluency

1 Factorise these expressions by splitting the middle term. WE11

(a) $2x^2 + 5x + 2$ **(b)** $2y^2 + 9y + 7$ **(c)** $5b^2 + 8b + 3$

(d) $3k^2 - 11k + 6$ **(e)** $11a^2 - a - 12$ **(f)** $5x^2 - 11x - 12$

(g) $2m^2 + 3m - 9$ **(h)** $7x^2 - 23x + 18$ **(i)** $11c^2 - 25c + 6$

(j) $5y^2 + 19y + 12$ **(k)** $3p^2 + 8p - 16$ **(l)** $2x^2 + 3x - 20$

(m) $3x^2 + 8x + 5$ **(n)** $7y^2 - 18y + 8$ **(o)** $3t^2 + t - 4$

(p) $7p^2 + 22p + 16$ **(q)** $3q + 2q^2 - 5$ **(r)** $4x + 5x^2 - 12$

(s) $8 - 18r + 7r^2$ **(t)** $21 - 16c + 3c^2$ **(u)** $2 - 10t + 12t^2$

2 Factorise these expressions using the cross method. WE12

(a) $4x^2 + 8x + 3$ **(b)** $10y^2 + 13y + 4$ **(c)** $9c^2 - 19c + 2$

(d) $6w^2 + 17w + 12$ **(e)** $15y^2 + 7y - 30$ **(f)** $8x^2 - 9x - 14$

(g) $4a^2 + 12a - 27$ **(h)** $24k^2 - 2k - 15$ **(i)** $20x^2 - 44x + 21$

(j) $12b^2 + 32b + 5$ **(k)** $6g^2 + 7g + 2$ **(l)** $8a^2 + 22a + 5$

(m) $4x^2 - 11x + 6$ **(n)** $10m^2 + 17m + 7$ **(o)** $12p^2 - 7p - 10$

(p) $9d^2 + 6d - 8$ **(q)** $6x^2 - x - 22$ **(r)** $18n^2 + 31n - 35$

(s) $24y^2 - 17y + 3$ **(t)** $15x^2 + 46x + 16$ **(u)** $12x^2 - 11x - 15$

3 Factorise the following expressions, if possible, by completing the square. WE13

(a) $2x^2 - 6x - 1$ **(b)** $3x^2 - 12x - 7$ **(c)** $2x^2 - 8x - 3$

(d) $3x^2 - 9x + 4$ **(e)** $5x^2 - 10x + 3$ **(f)** $2x^2 - 7x + 2$

(g) $2x^2 - 4x + 5$ **(h)** $2x^2 - 8x + 9$ **(i)** $3x^2 - 6x + 5$

Understanding

4 The floor (in m^2) of a rectangular room of height x m is given by the expression $6x^2 - 7x - 3$.

(a) Factorise the expression.

(b) Assuming that the two factors represent the length and the width of the room and that the length is 11.5 m, find the height of the room.

(c) What is the width of the room?

Reasoning

5 The number of bird nests in a park depends on the number of trees it has. In Byrd Park there are $12t^2 - 12t + 10$ bird nests, where t is $\frac{1}{100}$ of the number of trees. (The number of trees is estimated to the nearest multiple of 25.)

Plume Park has $4t^2 + 6t + 1$ bird nests.

(a) Write an expression in terms of t for the difference in the number of bird nests in the two parks. (Assume that the number of bird nests in Byrd Park is greater than the number of bird nests in Plume Park for all values of t.)

(b) **(i)** Simplify the expression in part **(a)**.

(ii) Factorise the expression in **(i)**.

(c) Use your answer from **(b) (ii)** to show that the parks will have equal numbers of birds when there are 75 or 150 trees.

6 Paul and Selina each have $\$x$. Later, each person has more money and the product of their money is $12x^2 - 8x - 15$.

(a) Factorise the expression.

(b) Find how much money each person had initially if Paul now has \$65 and has more money than Selina.

(c) How much money does Selina have now?

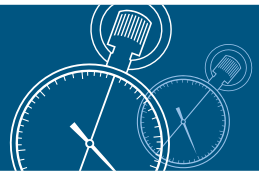
Open-ended

7 **(a)** Find a non-monic quadratic trinomial that will factorise to give binomial products. Factorise it.

(b) Find a non-monic quadratic trinomial that will factorise but not as binomial products. Factorise it.

(c) Find a non-monic quadratic trinomial that will not factorise. Explain why it cannot be factorised.

Half-time 10



1 Graph each equation by constructing a table of values for $x = -3$ to 3. Ex.10.1

(a) $y = 4x^2$ **(b)** $y = \frac{1}{4}x^3$

2 Find the quotient and remainder for $\frac{3x^3 + 4x^2 - 20x - 5}{3x + 1}$. Ex.10.3

3 If $P(x) = 3x - 4$ and $Q(x) = 3x^2 - 2x + 1$, write an expression for: Ex.10.3

(a) **(i)** $P(4)$ **(ii)** $Q(-2)$ **(iii)** $P(-1).\ Q(-1)$

(b) $2P(x) + 3Q(x)$

(c) $P(x).Q(x)$

(d) $Q(x) - xP(x)$

4 **(a)** Sketch the graph of $y = 2x^2 + 3$ by starting with $y = x^2$. Ex.10.2

(b) Sketch the graph of $y = -3(x - 2)^3 + 1$ by starting with $y = x^3$.

(c) Sketch the graph of $y = 2(x + 1)^4$ by starting with the graph of $y = x^4$.

5 Use polynomial division to find the remainder produced by each division. Ex.10.3

(a) $\frac{9x^3 + 6x^2 - 7x - 3}{x + 1}$ **(b)** $\frac{x^4 - x^3 + x^2 + x - 1}{3x - 1}$

6 Factorise $2x^2 + 8x + 3$ by completing the square. Ex.10.4

7 The vertical cross-section of a bottle is described by the equations $y = (x - 4)^3 + 1$ and $y = -(x - 14)^3 + 1$. Ex.10.2

(a) Graph both equations on the same axes using x-values from 1 to 17.

(b) State the equation of the axis of symmetry of the bottle.

8 Write $x^3 + x - 5 = (x - 1)(x^2 + x + 2) - 3$ as a division of one polynomial divided by another. Ex.10.3

9 Factorise the following quadratic expressions. Ex.10.4

(a) $6x^2 + x - 35$ **(b)** $10x^2 - 17x + 3$ **(c)** $12x^2 + 19x + 4$

10 By performing a division, show that: Ex.10.3

(a) $x - 1$ is a factor of $x^3 - 6x^2 - 9x + 14$

(b) $x - 2$ is a factor of $x^4 - 2x^3 - 3x^2 + 8x - 4$

10.5 Solution of non-monic quadratics

Need to Know

Solving non-monic quadratic equations: $ax^2 + bx + c = 0$

Factorise non-monic quadratic expressions by:

- splitting the middle term
- using the cross method
- completing the square.

Solve non-monic quadratic equations by:

- using the Null Factor Law
- using the quadratic formula. The solution to $ax^2 + bx + c = 0$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Worked Example 14

WE14

Solve the quadratic equation $3x^2 - 11x - 4 = 0$. Factorise the expression first by splitting the middle term.

Working

$3x^2 - 11x - 4 = 0$

$a = 3,\ b = -11,\ c = -4$

$ac = 3 \times (-4) = -12$

$\times$ to give -12
$+$ to give -11 } -12, 1

$3x^2 - 12x + x - 4$
$3x(x - 4) + 1(x - 4) = 0$
$(x - 4)(3x + 1) = 0$

$x - 4 = 0$ or $3x + 1 = 0$

$x = 4$ or $x = -\dfrac{1}{3}$

Worked Example 15

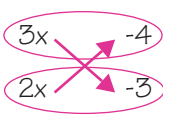
WE15

Solve $6x^2 - 17x + 12 = 0$. Factorise the expression first using the cross method.

Working

$6x^2 - 17x + 12 = 0$

$a = 6,\ b = -17,\ c = 12$



$-9x - 8x = -17x$
$(3x - 4)(2x - 3) = 0$

$3x - 4 = 0$ or $2x - 3 = 0$
$3x = 4$ or $2x = 3$
$x = \dfrac{4}{3}$ or $x = \dfrac{3}{2}$

Worked Example 16

WE16

Solve $2x^2 - 7x + 2 = 0$. Factorise the expression first, by completing the square on x.

Working

$$2x^2 - 7x + 2 = 0$$

$$2\left(x^2 - \frac{7}{2}x + 1\right) = 0$$

$$x^2 - \frac{7}{2}x + 1 = 0$$

$$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 + 1 - \left(\frac{49}{16}\right) = 0$$

$$\left(x - \frac{7}{4}\right)^2 - \frac{33}{16} = 0$$

$$\left(x - \frac{7}{4} + \frac{\sqrt{33}}{4}\right)\left(x - \frac{7}{4} - \frac{\sqrt{33}}{4}\right) = 0$$

$$x - \frac{7}{4} + \frac{\sqrt{33}}{4} = 0 \text{ or } x - \frac{7}{4} - \frac{\sqrt{33}}{4} = 0$$

$$x = \frac{7}{4} - \frac{\sqrt{33}}{4} \text{ or } x = \frac{7}{4} + \frac{\sqrt{33}}{4}$$

$$x = \frac{7 \pm \sqrt{33}}{4}$$

Worked Example 17

WE17

Use the quadratic formula to solve $3x^2 - 4x - 5 = 0$. Give your answers in exact form and rounded to two decimal places.

Working

$3x^2 - 4x - 5 = 0$

$a = 3, b = -4, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$= \frac{4 \pm \sqrt{16 + 60}}{6}$$

$$= \frac{4 \pm \sqrt{76}}{6}$$

$x = 2.12$ or $x = -0.79$

Answers page 566

10.5 Solution of non-monic quadratics

Equipment required: calculator for Question **4**

Fluency

WE14

1 Solve each quadratic equation. Factorise the expression first by splitting the middle term:

(a) $3x^2 - 4x - 4 = 0$ **(b)** $3x^2 + 8x + 4 = 0$

(c) $5x^2 - 11x + 6 = 0$ **(d)** $4x^2 + 3x - 10 = 0$

WE15

2 Solve each equation. Factorise the expression first using the cross method.

(a) $5x^2 - 17x + 12 = 0$ **(b)** $3x^2 + 8x + 5 = 0$

(c) $6x^2 - x - 2 = 0$ **(d)** $8x^2 + 6x - 9 = 0$

WE16

3 Solve each equation. Factorise the expression first by completing the square on x.

(a) $2x^2 - 5x + 2 = 0$ **(b)** $3x^2 + 10x + 1 = 0$

(c) $5x^2 - 2x - 6 = 0$ **(d)** $6x^2 + 9x - 2 = 0$

WE17

4 Use the quadratic formula to solve each equation. Give your answers in **(i)** exact form and **(ii)** rounded to two decimal places.

(a) $3x^2 - 5x - 4 = 0$ **(b)** $5x^2 + 10x + 1 = 0$

(c) $2x^2 - 11x + 6 = 0$ **(d)** $x^2 + 5x - 3 = 0$

5 For each of these quadratic equations:

(a) $x^2 + 5x + 2 = 0$ **(b)** $2x^2 - 3x + 1 = 0$

(c) $-3x^2 - x + 3 = 0$ **(d)** $5x^2 + 4x = 0$

(i) list the values that a, b and c represent if the equation is of the form $ax^2 + bx + c = 0$

(ii) find the solution to each equation using the quadratic formula.

6 The solutions to the equation $10x^2 - 23x + 12 = 0$ are:

A $x = 3$ or $x = 4$ **B** $x = \frac{2}{3}$ or $x = \frac{5}{4}$

C $x = 1.5$ or $x = 0.8$ **D** $x = -1.5$ or $x = -0.8$

Understanding

7 Solve each quadratic equation using an appropriate method.

(a) $3x^2 - 5x + 2 = 0$ **(b)** $5x^2 + 10x + 2 = 0$

(c) $4x^2 - 9 = 0$ **(d)** $x^2 + 7x - 9 = 0$

(e) $6x^2 - 7x - 10 = 0$ **(f)** $4x^2 + 4x + 1 = 0$

(g) $7x^2 - 6 = 0$ **(h)** $8x^2 + 6x - 4 = 0$

(i) $36x^2 - 60x + 25 = 0$ **(j)** $3x^2 - 11x + 7 = 0$

8 Which of the following equations has the solutions $x = 1.5$ or $x = 2.5$?

A $x^2 - 16x + 15 = 0$ **B** $4x^2 - 16x + 15 = 0$

C $x^2 - 8x + 15 = 0$ **D** $4x^2 + 16x + 15 = 0$

9 The solutions to $x^2 + px + q = 0$ are:

A $x = \dfrac{-p \pm \sqrt{q}}{2}$

B $x = \dfrac{p \pm \sqrt{p^2 + 4q}}{2}$

C $x = \dfrac{-p \pm \sqrt{p^2 - 4q}}{2}$

D $x = \dfrac{-2p \pm \sqrt{p^2 - 4q}}{2}$

10 Rick's driving speed was measured over a 10-minute period and the following relationship was found to exist: $s = -4t^2 + 31t$, where s is the speed in kilometres per hour after t minutes. When was Rick travelling at 60 km/h?

11 The temperature inside a tent was measured over a period of time and the following quadratic relationship was found to exist: $T = -2h^2 + 11h + 21$, where T is the temperature in degrees Celsius after h hours.

(a) When was the temperature 0°C?

(b) When was the temperature 26°C?

12 A tourist, high above the ground enjoying the sights from a hot-air balloon, unfortunately drops a camera and watches it fall to the ground. The height of the camera above the ground, h (in metres), t seconds after it has been dropped can be represented by the relationship:

$h = -5t^2 + 192$

(a) At what height above the ground is the camera dropped?

(b) How long does it take for the camera to fall to the ground? Round your answer to one decimal place.

Reasoning

13 Use the quadratic formula to show that none of the following equations has a solution. Write the part of the quadratic formula that indicates that no solutions are possible.

(a) $2x^2 - 3x + 2 = 0$

(b) $5x^2 + 5x + 2 = 0$

(c) $4x^2 - 6x + 9 = 0$

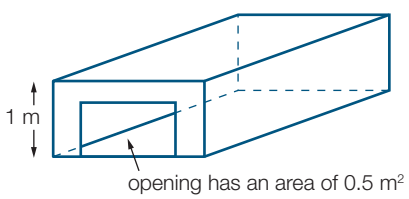
(d) $2x^2 + 7x + 9 = 0$

14 A policeman on a motorbike is following a car along a highway. After a short time, the driver of the car notices the policeman and starts to slow down, finally stopping on the side of the road. The speed of the car during this time can be represented by the quadratic function $S = -3t^2 + 17t + 70$, where S is the speed of the car in kilometres per hour, t minutes after the policeman started following.

(a) How long was the car under surveillance until it stopped?

(b) During this time, did the driver break the speed limit of 100 km/h?

15 Yeti's owners are going to build a dog kennel for him. It will be 1 m high and twice as long as it is wide, and it will have an opening at one end. When they are finished, they are going to paint the outside of it, including the base of the kennel, to keep it waterproof.



(a) Write an algebraic expression for the area of the kennel that is to be painted.

(b) If they buy a 4 L tin of paint, which will cover 31.4 m^2, and they will give the kennel two coats of paint, find the dimensions of the largest kennel that could be constructed. Round your answer to one decimal place.

16 Sally is a metal worker. She has several rectangular pieces of sheetmetal with dimensions of 30 cm by 25 cm and she wants to make three simple trays by cutting out the same size square from each corner of the material and then folding up the edges to complete the tray. When completed, each tray will have a different base area: 400 cm^2, 350 cm^2 and 300 cm^2.

(a) Draw a diagram of the net for each rectangular tray with the corners cut out. Draw broken lines to indicate where the trays will be folded.

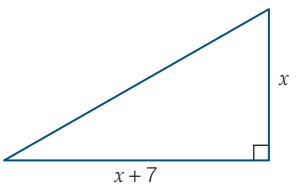
(b) Write an algebraic expression for the length of the base of each tray.

(c) Write an algebraic expression for the width of the base of each tray.

(d) Calculate the length of the square corner cut out in each tray (correct to one decimal place).

(e) What are the dimensions of each tray?

17 A right-angled triangle with shorter sides x cm and $(x + 7)$ cm has an area of 60 cm^2. Find the perimeter of the triangle.



18 The sum of the first n terms of the sequence 2, 5, 8, 11 … is given by the formula $S = \frac{n}{2}[4 + 3(n - 1)]$ where n is the number of terms in the sequence. Use the formula to find the number of terms needed to make a sum of 805.

19 Solve the following quadratic equations by first multiplying by the lowest common denominator. Note that the denominator can never be zero.

(a) $2x - 1 = \frac{21}{x}$ **(b)** $x = \frac{9x - 14}{x}$ **(c)** $x = \frac{x - 9}{x - 5}$

(d) $\frac{3}{x - 1} = \frac{x}{x + 1}$ **(e)** $\frac{5}{x + 2} + \frac{3}{x} = 1$ **(f)** $\frac{x - 1}{x} + 5 = \frac{3}{x^2}$

Open-ended

20 Make up three non-monic quadratic equations that have as their solutions $x = 2.5$ or $x = -3$.

21 By using the process of completing the square, the solution $x = \frac{-a \pm \sqrt{5}}{2}$ was obtained.

By choosing a value of a between -3 and 3, make up at least three possible quadratic equations that would have this solution.

22 By choosing values for a, b and c in the general quadratic equation $ax^2 + bx + c = 0$, make up an equation that has:

(a) no solution **(b)** one solution **(c)** two solutions.

Write the solutions to your equations where they exist.

Problem solving

Projectiles

When an object is projected vertically upward with an initial velocity v m/s, its height h m above the ground after t s is approximately given by the equation $h = vt - 5t^2$.

(a) A rocket is fired vertically upward at an initial velocity of 25 m/s. At what times is the height of the rocket above ground going to be zero?

(b) The equation $h = vt - 5t^2$ can also be expressed as $5t^2 - vt + h = 0$.

(i) When the height of the rocket is 30 m and $v = 25$, show that the expression becomes $t^2 - 5t + 6 = 0$.

(ii) By factorising $t^2 - 5t + 6$, find at what times the rocket is at a height of 30 m.

(c) The rocket is now projected with an initial velocity of 35 m/s. How long does it take for the rocket to be at a height of 30 m on its way down?

(d) The rocket is again projected, but with an initial velocity of 10 m/s.

(i) Show that a perfect square is obtained when the left side of the equation $5t^2 - vt + h = 0$ is factorised using $h = 5$.

(ii) For how long is the rocket in flight?

(iii) What can you conclude about the maximum height the rocket can reach?

Strategy options

- Draw a diagram.
- Look for a pattern.

Remainder and factor theorems

10.6

Need to Know

The **remainder theorem** can be used to find the remainder when a polynomial $P(x)$ is divided by a linear factor $(ax - b)$.

- If the polynomial $P(x)$ is divided by $(x - a)$, then the remainder is $P(a)$.
- If the polynomial $P(x)$ is divided by $(ax - b)$, then the remainder is $P\left(\frac{b}{a}\right)$.

The **factor theorem** can be used to find factors of a polynomial $P(x)$ in the form of $(ax - b)$.

If the remainder is 0, then the expression that you are dividing by is a factor of the polynomial. For the polynomial $P(x)$:

If $P(a) = 0$, then $(x - a)$ is a factor of $P(x)$.

If $P\left(\frac{b}{a}\right) = 0$, for the polynomial $P(x)$, then $(ax - b)$ is a factor of $P(x)$.

Worked Example 18

WE18

If $P(x) = x^3 + 3x^2 - 4x + 5$ and $Q(x) = x - 2$:

(a) find $\frac{P(x)}{Q(x)}$ **(b)** evaluate $P(2)$ **(c)** comment on the result.

Working

(a)

$$
\begin{array}{r}
x^2 + 5x + 6 \\
x-2\overline{)\,x^3 + 3x^2 - 4x + 5} \\
-(x^3 - 2x^2) \\
\hline
5x^2 - 4x \\
-(5x^2 - 10x) \\
\hline
6x + 5 \\
-(6x - 12) \\
\hline
17
\end{array}
$$

(b) $P(2) = 8 + 12 - 8 + 5 = 17$

(c) The remainder when $P(x)$ is divided by $x - 2$ is the same as the value of $P(2)$.

Worked Example 19

WE19

Use the remainder theorem to find the remainder for:

(a) $\frac{2x^2 - x + 1}{x - 2}$ **(b)** $\frac{-4x^3 + x^2 - x + 5}{2x + 3}$

Working

(a) $P(x) = 2x^2 - x + 1$

$x - 2 = 0$

$x = 2$, hence $a = 2$

$P(2) = 8 - 2 + 1 = 7$

The remainder is 7.

(b) $P(x) = -4x^3 + x^2 - x + 5$

$2x + 3 = 0$

$x = \frac{-3}{2}$

$\therefore \frac{b}{a} = \frac{-3}{2}$

$P(\frac{-3}{2}) = \frac{27}{2} + \frac{9}{4} + \frac{3}{2} + 5$

$= 22\frac{1}{4}$

The remainder is $22\frac{1}{4}$.

Worked Example 20

WE20

Use the factor theorem to fully factorise $x^3 + 6x^2 + 11x + 6$.

Working

$P(x) = x^3 + 6x^2 + 11x + 6$

Try $x = 1$

$P(1) = 1 + 6 + 11 + 6 = 24$

$\therefore$ $(x - 1)$ is not a factor

Try $x = -1$

$P(-1) = -1 + 6 - 11 + 6 = 0$

Hence, $(x + 1)$ is a factor.

$$
\begin{array}{r}
x^2 + 5x + 6 \\
x + 1 \overline{)\ x^3 + 6x^2 + 11x + 6} \\
-(x^3 + x^2) \\
\hline
5x^2 + 11x \\
-(5x^2 + 5x) \\
\hline
6x + 6 \\
-(6x + 6) \\
\hline
0
\end{array}
$$

$P(x) = (x + 1)(x^2 + 5x + 6)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$P(x) = (x + 1)(x + 2)(x + 3)$

Worked Example 21

WE21

Use the factor theorem to fully factorise $x^4 + x^3 - 7x^2 - x + 6$.

Working

$P(x) = x^4 + x^3 - 7x^2 - x + 6$

Try $x = 1$

$P(1) = 1 + 1 - 7 - 1 + 6 = 0$

Hence, $(x - 1)$ is a factor.

$$
\begin{array}{r}
x^3 + 2x^2 - 5x - 6 \\
x - 1 \overline{)\ x^4 + x^3 - 7x^2 - x + 6} \\
-(x^4 - x^3) \\
\hline
2x^3 - 7x^2 \\
-(2x^3 - 2x^2) \\
\hline
-5x^2 - x \\
-(-5x^2 + 5x) \\
\hline
-6x + 6 \\
-(-6x + 6) \\
\hline
0
\end{array}
$$

$P(x) = (x - 1)(x^3 + 2x^2 - 5x - 6)$

$P(x) = (x - 1)M(x)$

Try $x = -1$ (we have used $x = 1$)

$M(x) = x^3 + 2x^2 - 5x - 6$

$M(-1) = -1 + 2 + 5 - 6 = 0$

Hence, $(x + 1)$ is a factor.

$M(x) = (x + 1)(x^2 + x - 6)$

$= (x + 1)(x - 2))(x + 3)$

$P(x) = (x - 1)(x + 1)(x - 2)(x + 3)$

Answers page 567

10.6 Remainder and factor theorems

Fluency

WE18

1 If $P(x) = x^3 + 4x^2 - 5x + 3$ and $Q(x) = x - 2$, find $\frac{P(x)}{Q(x)}$, evaluate $P(2)$ and comment on the result.

WE19

2 Use the remainder theorem to find the remainder for:

(a) $\frac{x^2 - 2x + 1}{x - 2}$ **(b)** $\frac{-3x^3 + 2x^2 - x + 2}{2x + 6}$

WE20

3 Use the factor theorem to fully factorise $x^3 - 6x^2 + 11x - 6$.

WE21

4 Use the factor theorem to fully factorise $x^4 - x^3 - 7x^2 + x + 6$.

5 Use the remainder theorem to find the remainder if:

(a) $4x^2 + 3x - 1$ is divided by $x - 3$ **(b)** $5x^3 - 2x^2 + 6x - 10$ is divided by $x + 1$

(c) $x^4 + x^3 - 3x^2 - x - 1$ is divided by $2x + 1$ **(d)** $x^5 - 1$ is divided by $2x - 1$

(e) $(6x - 8)(4x + 5)$ is divided by $x - 5$ **(f)** $49x^2 - 144$ is divided by $7x + 2$

6 Use the factor theorem to factorise the following polynomials.

(a) $x^3 + x^2 - 10x + 8$ **(b)** $x^3 - 5x^2 + 3x + 9$

(c) $x^3 - 6x^2 + 3x + 10$ **(d)** $x^3 + 3x^2 - 16x + 12$

(e) $x^3 + 6x^2 + 12x + 8$ **(f)** $x^3 - 3x^2 - 9x - 5$

Understanding

7 Calculate the remainder for $\frac{P(x)}{Q(x)}$ given:

(a) $P(x) = 4x^3 + 2x^2 - x + 1$, $Q(x) = 5x + 15$

(b) $P(x) = -x^4 - 3x^2 + 2x + 1$, $Q(x) = -x - 1$

(c) $P(x) = -x^3 - 9x^2 + 10x - 12$, $Q(x) = -2x$

(d) $P(x) = x^5 - x^4 - x^3 + 2x^2 - x - 1$, $Q(x) = 4x + 8$

8 Use the factor theorem to factorise the following polynomials.

(a) $x^4 + 6x^3 + 7x^2 - 6x - 8$ **(b)** $x^4 - 7x^3 + 14x^2 - 8x$

(c) $x^4 - x^3 - 7x^2 + 13x - 6$ **(d)** $x^4 + 2x^3 - 12x^2 + 14x - 5$

(e) $x^4 - 4x^3 + 3x^2 + 4x - 4$ **(f)** $x^4 + 2x^3 - 13x^2 - 14x + 24$

9 When $2x^3 - 4x^2 + x - 1$ is divided by $x + \frac{1}{2}$, the remainder is:

A $1\frac{1}{2}$ **B** $-2\frac{3}{4}$ **C** $1\frac{3}{4}$ **D** $-3\frac{1}{2}$

Reasoning

10 If $\frac{x^3 - 4x^2 - 4x + 16}{x - 4} = ax^2 + bx + c$ the values of a, b and c are:

A 1, -1, -4 **B** -3, 4, 0 **C** 4, 3, -1 **D** 1, 0, -4

11 If $x + 2$ is a factor of $6x^3 + 19x^2 + 11x + a$, find the value of a.

12 Use the factor theorem to fully factorise each polynomial. Then solve the polynomial equation.

(a) $x^3 - 6x^2 + 11x - 6 = 0$

(b) $x^3 + x^2 - 4x - 4 = 0$

(c) $x^3 + 9x^2 + 26x + 24 = 0$

(d) $x^4 + x^3 - 7x^2 - x + 6 = 0$

(e) $x^4 - 5x^2 + 4 = 0$

(f) $x^4 - x^3 - 6x^2 + 6x = 0$

13 A toy top is moving along the floor as it spins. Its distance, d cm, from a fixed point at time t seconds, is given by the equation $d = t^4 - 6t^3 + 13t^2 - 12t + 4$.

(a) Factorise the polynomial.

(b) How far is the top from the fixed point initially?

(c) At what times is the top at the fixed point?

(d) By inspection of your answer to part **(a)**, or by sketching the graph, describe the motion of the top after 2 seconds.

Open-ended

14 Give two monic polynomials of degree 3 that leave a remainder of -2 when they are divided by $x - 3$.

15 A polynomial equation of degree 4 has solutions $x = 0$, $x = -1$, $x = 2$, $x = 3$. Write down two possible equations.

10.7 Sketching polynomials

Need to Know

Before we can sketch a polynomial we need to identify its key features.

Step 1 Identify the shape of $y = a(x - h)^2 + k$ using transformations of $y = ax^n$.

Step 2 Find the y-intercept. All polynomial graphs have a y-intercept. Find the y-value of the y-intercept by substituting $x = 0$ into the polynomial.

Step 3 Find the x-intercepts. When a polynomial $P(x)$ is written as the product of its factors, the solution of $P(x) = 0$ can be found by equating each factor equal to zero and solving for x. The solutions are the x-values of the x-intercepts of the graph of $y = P(x)$.

- If the degree of $P(x)$ is 1, the linear polynomial $P(x) = mx + b$ has one solution for $P(x) = 0$. There is always exactly one x-intercept.
- If the degree of $P(x)$ is 2, the quadratic polynomial $P(x) = ax^2 + bx + c$ has at most two solutions for $P(x) = 0$. There may be zero, one or two x-intercepts. The x-values of the x-intercepts may not be rational.

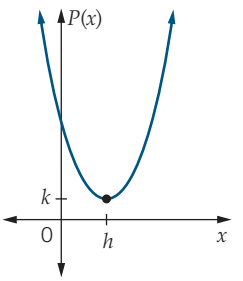
- If the degree of $P(x)$ is 3, the cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ has at most three solutions for $P(x) = 0$. There will always be at least one x-intercept. The x-values of the x-intercepts may not be rational.
- The polynomial $P(x)$ of degree n has at most n solutions for $P(x) = 0$. If n is odd, there will always be at least one x-intercept. If n is even, there may be no x-intercepts.
- Identify whether the polynomial cuts or touches the x-axis. A repeated factor with an even power indicates that the graph touches the x-axis. A repeated factor with an odd power indicates a point of inflection on the x-axis.

Step 4 Find the turning points or points of inflection where the graph changes direction giving a maximum or minimum or a special point (h, k) where a reflection in both the lines $x = h$ and $y = k$ of one half of the graph will give the other half.

Quadratics

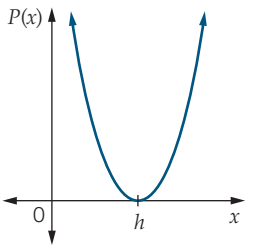
The graphs of a quadratic fit one of the following three types for $a > 0$. If $a < 0$, the graphs will be reflected in the x-axis.

$P(x) = a(x - h)^2 + k, k > 0$



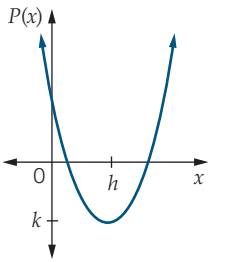
- no solution for $P(x)$
- no x-intercept

$P(x) = a(x - h)^2, k = 0$



- 1 solution for $P(x)$
- 1 x-intercept

$P(x) = a(x - h)^2 + k, k < 0$

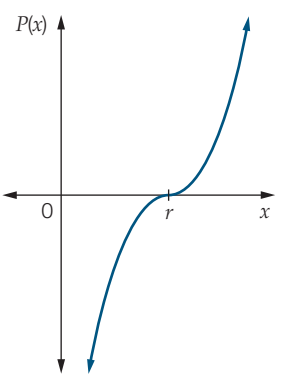


- 2 solutions for $P(x)$
- 2 x-intercepts

Cubics

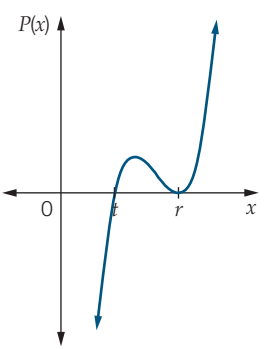
A cubic polynomial expressed in the form $P(x) = a(x - r)(x - s)(x - t)$ is easy to sketch as the x-intercepts are given by $x = r$, $x = s$ and $x = t$. Depending on the values of r, s and t there will be one, two or three solutions to the equation $P(x) = 0$.

$P(x) = a(x - r)^3$



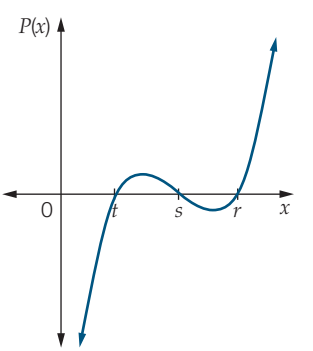
- r, s and t all have the same value
- 1 solution for $P(x)$
- cuts the x-axis once at $(r, 0)$

$P(x) = a(x - r)^2(x - t)$



- r and s have the same value
- 2 solutions for $P(x)$
- cuts the x-axis once at $(t, 0)$ and touches the x-axis once at $(r, 0)$

$P(x) = a(x - r)(x - s)(x - t)$

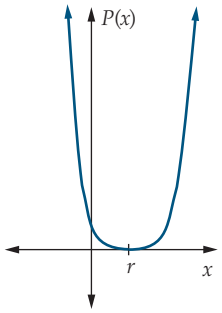


- r, s and t all have distinct values
- 3 solutions for $P(x)$
- cuts the x-axis 3 times at $(t, 0)$, $(s, 0)$, $(r, 0)$

Quartics

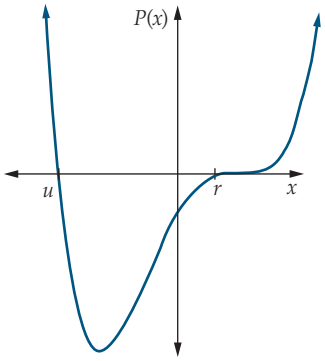
Some quartic polynomials of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ can be written in the form $P(x) = a(x - r)(x - s)(x - t)(x - u)$. In this case, there can be up to four solutions for $P(x) = 0$. Four possibilities are shown for $a > 0$. If $a < 0$ then the graphs are reflected in the x-axis and both ends would be pointing downwards.

$P(x) = (x - r)(x - r)(x - r)(x - r)$
$= (x - r)^4$



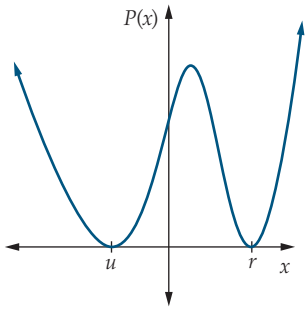
- r, s, t and u all have the same value
- 1 solution for $P(x)$
- touches the x-axis at $(r, 0)$

$P(x) = (x - r)(x - r)(x - r)(x - u)$
$= (x - r)^3(x - u)$



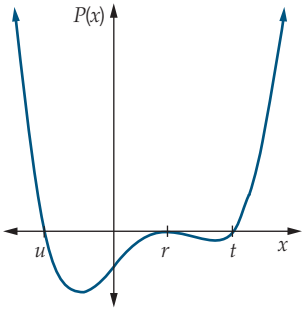
- r, s and t all have the same value
- 2 solutions for $P(x)$
- cuts the x-axis at $(u, 0)$ and touches the x-axis at $(r, 0)$

$P(x) = (x - r)(x - r)(x - u)(x - u)$
$= (x - r)^2(x - u)^2$



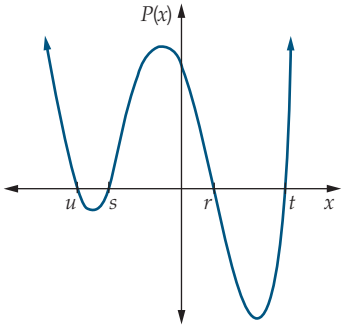
- r and s have the same value, t and u have the same value
- 2 solutions for $P(x)$
- touches the x-axis at $(r, 0)$ and $(u, 0)$

$P(x) = (x - r)(x - r)(x - t)(x - u)$
$= (x - r)^2(x - t)(x - u)$



- r and s have the same value
- 3 solutions for $P(x)$
- touches x-axis at $(r, 0)$ and cuts it at $(t, 0)$ and $(u, 0)$

$y = (x - r)(x - s)(x - t)(x - u)$



- r, s, t and u all have different values
- 4 solutions for $P(x)$
- cuts the x-axis at $(r, 0)$, $(s, 0)$, $(t, 0)$ and $(u, 0)$

If a factor occurs only once, it corresponds to a point where the curve cuts the x-axis.

If a factor is repeated an even number of times, it corresponds to a point where the curve touches the x-axis, creating a turning point, either a maximum or a minimum.

If a factor is repeated an odd number of times, it corresponds to a point of inflection, where the graph cuts the x-axis and flattens out slightly.

A quartic can have one, two or three turning points. If it has two turning points, it must also have a point of inflection as the ends of the graph have to point in the same direction.

Like parabolas, some quartics also have no solutions. These quartics cannot be factorised.

Worked Example 22

WE 22

Sketch the graph of each quadratic polynomial, showing the x- and y-intercepts and the coordinates of the turning point.

(a) $P(x) = 2x^2 + 2x + 1$ **(b)** $P(x) = 3x^2 + 6x + 3$ **(c)** $P(x) = -6x^2 + x + 2$

Working

(a) $P(x) = 2x^2 + 2x + 1$

$x = 0$

$P(0) = 2(0)^2 + 2(0) + 1$
$= 1$

The y-intercept is $(0, 1)$.

There are no obvious factors so use the quadratic formula.

$a = 2, b = 2, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{2 \times 2}$$

$$= \frac{-2 \pm \sqrt{-4}}{4}$$

This gives no solution so the graph has no x-intercepts.

As a is positive, the graph has a minimum turning point.

$$x = -\frac{b}{2a}$$

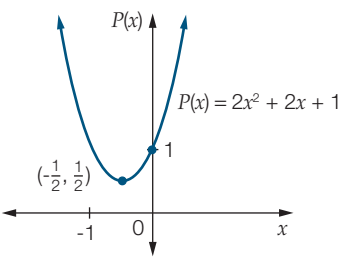
$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

when $x = -\frac{1}{2}$, $y = \frac{2}{4} - \frac{2}{2} + 1$

$$= \frac{1}{2}$$

The turning point is at $\left(-\frac{1}{2}, \frac{1}{2}\right)$.



(b) $P(x) = 3x^2 + 6x + 3 = 0$

$x = 0$

$P(0) = 3(0)^2 + 6(0) + 3$
$= 3$

The y-intercept is $(0, 3)$.

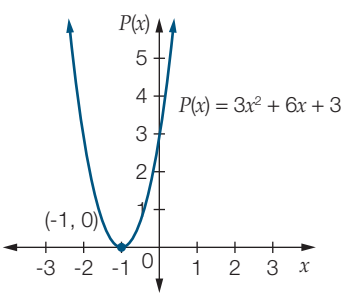
$P(x) = 0$

$3x^2 + 6x + 3 = 0$
$3(x^2 + 2x + 1) = 0$
$3(x + 1)^2 = 0$
$x = -1$

The x-intercept is $(-1, 0)$.

As a is positive, the graph has a minimum turning point.

The turning point is at $(-1, 0)$.



(c) $P(x) = -6x^2 + x + 2$

$x = 0$

$P(0) = -6(0)^2 + 0 + 2$
$= 2$

The y-intercept is $(0, 2)$.

$P(x) = 0$
$-6x^2 + x + 2 = 0$
$-(6x^2 - x - 2) = 0$
$-(3x - 2)(2x + 1) = 0$
$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

The x-intercepts are $\left(\frac{2}{3}, 0\right)$ and $\left(-\frac{1}{2}, 0\right)$.

As a is negative, the graph has a maximum turning point.

$x = -\frac{b}{2a}$

$= -\frac{1}{2 \times -6}$

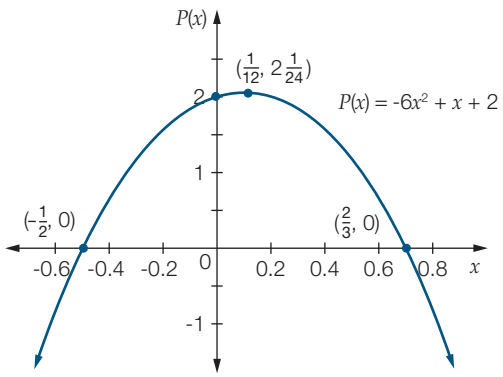
$= \frac{1}{12}$

$y = -6\left(\frac{1}{12}\right)^2 + \left(\frac{1}{12}\right) + 2$

$= -6 \times \frac{1}{144} + \frac{1}{12} + 2$

$= 2\frac{1}{24}$

The turning point is at $\left(\frac{1}{12}, 2\frac{1}{24}\right)$.



Worked Example 23

WE23

Sketch the graph of each cubic polynomial, showing all x- and y-intercepts. Use a CAS to find the coordinates of the turning points or points of inflection, if necessary.

(a) $P(x) = -4(x - 1)^3$ **(b)** $P(x) = (x + 2)(x - 1)^2$ **(c)** $P(x) = x^3 - 3x^2 - x + 3$

Working

(a) $P(x) = -4(x - 1)^3$

$x = 0$

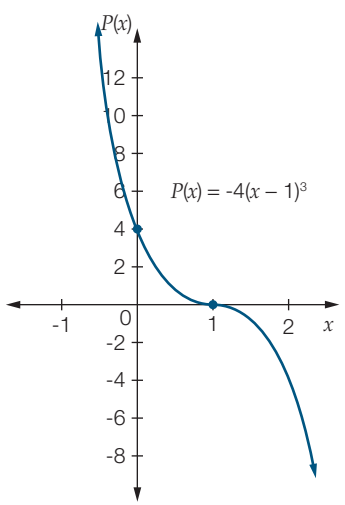
$P(0) = -4 \times (-1)^3$
$= 4$

The y-intercept is $(0, 4)$.

$P(x) = 0$
$-4(x - 1)^3 = 0$
$x - 1 = 0$

$x = 1$ is the only solution.
The curve cuts the x-axis at $(1, 0)$.

$(1, 0)$ is a point of inflection.



(b) $P(x) = P(x) = (x + 2)(x - 1)^2$

$x = 0,\ P(0) = 2 \times (-1)^2$

$= 2$

The y-intercept is $(0, 2)$.

$P(x) = 0$

$(x + 2) = 0$ or $(x - 1)^2 = 0$

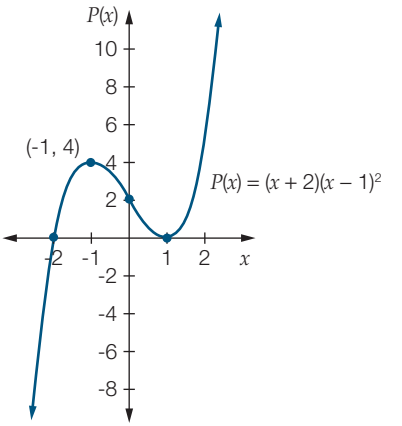
$x = -2,\ x = 1$

The curve cuts the x-axis at $(-2, 0)$ and touches it at $(1, 0)$.

$(1, 0)$ is a minimum turning point.

The maximum turning point is at $(-1, 4)$.

A CAS can be used to find this point.



(c) $P(x) = x^3 - 3x^2 - x + 3$

$x = 0,\ P(0) = 3$

The y-intercept is $(0, 3)$.

$x^3 - 3x^2 - x + 3 = 0$

$P(-1) = -1 - 3 + 1 + 3$

$= 0$

$(x + 1)$ is a factor.

$P(3) = 27 - 27 - 3 + 3$

$= 0$

$(x - 3)$ is a factor.

$P(1) = 1 - 3 - 1 + 3$

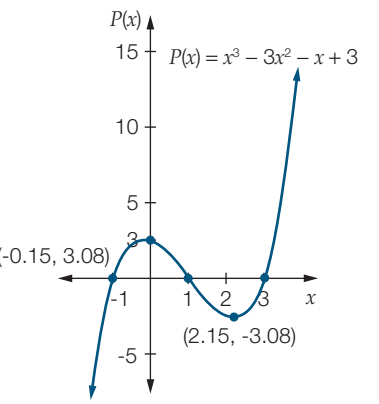
$= 0$

$(x - 1)$ is a factor.

$(x + 1)(x - 1)(x - 3) = 0$

$x = -1,\ x = 1,\ x = 3$

The curve cuts the x-axis at $(-1, 0)$, $(1, 0)$ and $(3, 0)$.



Worked Example 24

WE24

Sketch the graph of $P(x) = x^4 - 9x^2 + 4x + 12$, showing all x- and y-intercepts. Use a CAS to find the turning points or points of inflection, if necessary.

Working

$P(x) = x^4 - 9x^2 + 4x + 12$

$P(0) = (0)^4 - 9(0)^2 + 4(0) + 12$
$= 12$

The y-intercept is $(0, 12)$.

Factorise using the factor theorem.
$P(-1) = 1 - 9 - 4 + 12$
$= 0$

$(x + 1)$ is a factor.

$$\begin{array}{r} x^3 - x^2 - 8x - 12 \\ x+1 \overline{)\, x^4 + 0x^3 - 9x^2 + 4x + 12} \\ \underline{x^4 + x^3} \\ -x^3 - 9x^2 \\ \underline{-x^3 - x^2} \\ -8x^2 + 4x \\ \underline{-8x^2 - 8x} \\ 12x + 12 \\ \underline{12x + 12} \\ 0 \end{array}$$

$P(x) = (x + 1)(x^3 - x^2 - 8x + 12)$

$M(x) = x^3 - x^2 - 8x + 12$

$M(2) = 8 - 4 - 16 + 12$
$= 0$

$(x - 2)$ is a factor.

$$\begin{array}{r} x^2 + x - 6 \\ x-2 \overline{)\, x^3 - x^2 - 8x + 12} \\ \underline{x^3 - 2x^2} \\ x^2 - 8x \\ \underline{x^2 - 2x} \\ -6x + 12 \\ \underline{-6x + 12} \\ 0 \end{array}$$

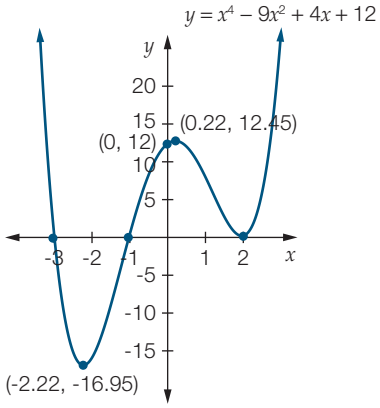
$P(x) = (x + 1)(x - 2)(x^2 + x - 6)$

$P(x) = (x + 1)(x - 2)(x - 2)(x + 3)$

$P(x) = (x + 1)(x + 3)(x - 2)^2$

For $P(x) = 0$, $x = -1$, $x = -3$, $x = 2$ (repeated solution)

The curve cuts the x-axis at $(-3, 0)$ and $(-1, 0)$, and touches it at $(2, 0)$ (repeated factor).



The curve cuts the x-axis twice, touches it once and has three turning points.

10.7 Sketching polynomials

Answers page 567

Equipment required: a CAS

Fluency

1 Sketch the graph of each quadratic polynomial, showing all x- and y-intercepts. Use a CAS to find the coordinates of the turning points, if necessary.

WE22

(a) $P(x) = 2x^2 + 6x + 5$ **(b)** $P(x) = 3x^2 + 12x + 12$ **(c)** $P(x) = -5x^2 - 6x + 8$

WE23

2 Sketch the graph of each cubic polynomial, showing all x- and y-intercepts. Use a CAS to find the coordinates of the turning points or point of inflection, if necessary.

(a) $P(x) = -2(x+1)^3$ **(b)** $P(x) = (x-2)(x+1)^2$ **(c)** $P(x) = x^3 + 3x^2 - x - 3$

WE24

3 Sketch the graph of $P(x) = x^4 - 9x^2 - 4x + 12$, showing all x- and y-intercepts. Use a CAS to find the coordinates of the turning points or points of inflection, if necessary.

4 Sketch the graph of each polynomial.

(a) $P(x) = x^2 - 5x - 14$ **(b)** $P(x) = x^2 - 6x + 8$ **(c)** $P(x) = x^2 - 7x - 12$

(d) $P(x) = x^2 - 12x + 36$ **(e)** $P(x) = x^2 - 15x + 56$ **(f)** $P(x) = x^2 - x - 90$

5 Sketch the graph of each polynomial.

(a) $P(x) = x^3 + 7x^2 + 7x - 15$ **(b)** $P(x) = -x^3 + 9x^2 - 27x + 27$

(c) $P(x) = x^3 - 12x + 16$ **(d)** $P(x) = x^3 + x^2 - 12x$

(e) $P(x) = x^3 - 2x^2 - 11x + 12$ **(f)** $P(x) = x^3 + 5x^2 - x - 5$

6 Sketch the graph of each polynomial.

(a) $P(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ **(b)** $P(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$

(c) $P(x) = x^4 - 2x^3 + x^2$ **(d)** $P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

(e) $P(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$ **(f)** $P(x) = x^4 + 3x^3 - 5x^2 - 3x + 4$

Understanding

7 The equation $x^3 - 7x^2 + 11x - 5 = 0$ has:

A no solutions **B** only one solution at $x = 1$

C three solutions **D** two solutions

8 The zeroes of the polynomial $P(x) = x(x-8)(x-2)(x-8)$ are:

A 0 and 8 **B** 0 and 2 **C** 2 and 8 **D** 0, 2 and 8

9 $P(x) = 2x^3 + 5x^2 + x - 20$ and $Q(x) = 2x^3 + 4x^2 - x + 15$. Find the solutions to $P(x) = Q(x)$.

10 $P(x) = 5x^4 + 4x^3 - 3x^2 - 5x + 5$ and $Q(x) = 3x^4 + 2x^3 - x^2 - 2x + 2$. Find the solutions to $2P(x) = 3Q(x)$.

Reasoning

11 A flying fish, initially under water, suddenly leaps above the surface before re-entering the water. Its motion is given by $P(t) = -t^3 + 3t^2 - 2t$, where $P(t)$ is the vertical distance of the fish from sea level at time t seconds, with $0 \leq t \leq 2$.

(a) Factorise the polynomial and sketch the graph for values of t from 0 to 2.

(b) For how long was the fish swimming until it broke the sea surface? Approximately what was its greatest depth?

(c) For how long was the fish above the surface and approximately what was its maximum height?

12 A plumber inserts a long flexible rod in a length of water pipe to test for blockages. The pipe is described by the equation $y = x^3 - 6x^2 + 9x$, where x is the horizontal distance and y is the vertical distance from the rod's insertion point (0, 0) and the units are in metres.

(a) Factorise the expression and sketch the graph for values of x from 0 to 4.

(b) There are blockages located at the turning points of the graph. What are the coordinates of the blockages?

(c) To avoid the first blockage, the plumber decides to install a new system of pipes starting at (0, 0) and ending at the second blockage. The pipes take the shape of a parabola. Give one equation for this parabola.

(d) While installing the new pipes, the plumber is informed that there is an underground power cable located at (2.5, 1.2), which he must avoid. Can he still use the parabola pipe layout?

13 A wave pattern on a cathode ray oscilloscope is described by the polynomial $P(x) = x^4 + 5x^3 - x^2 - 5x$. A negative pulse is the section of the wave that is below the x-axis and between the x-intercepts. Similarly, a positive pulse is the section of the wave that is above the x-axis and between the x-intercepts.

(a) Factorise the polynomial and sketch its graph.

(b) Describe the location of each type of pulse.

(c) The wave pattern is a model for part of the cross-section of an island that consists of two lagoons and a small rocky outcrop with mountains on either side. The x-axis represents sea level, and each unit on the axis is 100 metres. Each unit on the y-axis is in metres.

(i) How wide and approximately how deep is the larger lagoon?

(ii) A research observation tower planned for the rocky outcrop requires a minimum length of land of 110 m. Can it be built?

14 A dam is located at the bottom of a hill. If $y = 0$ is taken as ground level, the cross-section (west to east) of both shapes can be described by the polynomial $x^3 - 5x^2 - 2x + 24$ over $-2 \leq x \leq 5$, where each unit of x represents 100 m and each unit of y represents 1 m.

(a) Factorise the polynomial.

(b) Sketch the graph of the polynomial for $-2 \leq x \leq 5$.

(c) What is the horizontal distance between the western base of the hill and the eastern bank of the dam?

15 A section of a roller-coaster track is described by the equation $y = 0.001(x - r)(x - s)^2$, where x is the horizontal distance in metres from Daredevil Corner and y is the height of the roller-coaster in metres. The graph has intercepts at $x = 0$ and $x = 60$ and the y-intercept is zero. The turning points occur at $x = 60$ and $x = 20$.

(a) Use this information to write the equation.

(b) Sketch the graph of the equation for $0 \leq x \leq 80$.

(c) The top of Heartstop Hill is at a horizontal distance of 80 m from Daredevil Corner. The length of the track from Daredevil Corner to Heartstop Hill is six times the height of the roller-coaster at Heartstop Hill. The roller-coaster on average moves on the track at 32 km/h. How many minutes does it take to travel from Daredevil Corner to Heartstop Hill?

Open-ended

16 Find two monic quartic polynomials that have three zeroes, and sketch their graphs.

17 A quartic polynomial has factors of $(x + 1)$ and $(x - 3)$. Using at least these factors, write down two quartic polynomials that:

(a) have exactly two zeroes

(b) have three zeroes

(c) have four zeroes

(d) are non-monic with only one turning point.

Chapter review 10

D.I.Y. Summary

Key Words

cubic	factor theorem	point of inflection	remainder
degree	leading coefficient	polynomial	remainder theorem
dividend	leading term	quartic	
divisor	non-monic quadratic	quotient	

1 The highest power of a polynomial gives the _____ of the polynomial.

2 The first term of a polynomial written with powers in descending order is called the _____.

3 A linear factor is divided into a _____ to find a _____ and a _____.

4 A polynomial can be factorised by using the _____.

Fluency

1 Graph each equation by constructing a table of values for $x = -3$ to 3. Ex. 10.1

(a) $y = 3x^2$ (b) $y = \frac{1}{2}x^3$ (c) $y = -x^4$

2 (a) Sketch the graph of $y = 2x^2 - 3$ starting with $y = x^2$. Ex. 10.2

(b) Sketch the graph of $y = -2(x + 2)^3 + 1$ starting with $y = x^3$.

(c) Sketch the graph of $y = -3(x + 2)(x - 4)(x + 1)$.

3 (a) If $P(x) = 4x - 3$ and $Q(x) = 3x^2 - x + 2$, evaluate: Ex. 10.3

(i) $P(4)$ (ii) $Q(-2)$ (iii) $P(-1).Q(-1)$

(b) Write an expression for:

(i) $2P(x) + 3Q(x)$ (ii) $P(x) \times Q(x)$ (iii) $Q(x) - P(x)$

4 Find the quotient and remainder for $\frac{4x^3 + 6x^2 + 26x + 13}{4x + 2}$. Ex. 10.3

5 Factorise the following quadratic expressions. Ex. 10.4

(a) $12x^2 + x - 20$ (b) $12x^2 - 4x - 5$ (c) $6x^2 + 17x - 14$

(d) $3x^2 + 5x + 1$ (e) $4x^2 - 7x - 5$ (f) $2x^2 + 6x + 3$

6 Solve each quadratic equation using an appropriate method. Ex. 10.5

(a) $2x^2 - 18 = 0$ (b) $2x^2 - 9x = 5$ (c) $(2x - 5)^2 = 3$

(d) $2x^2 + 5x + 1 = 0$ (e) $3x^2 - 6x + 2 = 0$ (f) $13x^2 + 7x - 3 = 0$

7 Use the remainder theorem to find the remainder produced by each division.

Ex 10.6

(a) $\dfrac{10x^3 + 7x^2 - 4x - 3}{x + 1}$ **(b)** $\dfrac{x^4 - x^3 + x^2 + x - 1}{2x - 1}$

8 Use the factor theorem to factorise:

Ex 10.6

(a) $x^3 + 6x^2 - 9x - 14$ **(b)** $x^4 - 5x^2 + 4$

9 Sketch the graph of each polynomial.

Ex 10.7

(a) $P(x) = x^3 - 8x^2 - x + 8$ **(b)** $P(x) = x^4 + 2x^3 - 4x^2 - 2x + 3$

Understanding

10 An equation $y = x^4$ is dilated by a factor of $\frac{1}{2}$ in the y-direction, reflected in the x-axis and translated 2 units left and 3 units up.

(a) Write the new equation.

(b) Sketch the graph of the equation. Use a CAS to find the x-intercepts, correct to two decimal places.

11 Find the remainder using long division when $P(x) = x^3 - 3x^2 - 6x + 1$ is divided by $x + 2$. Confirm your answer by finding the remainder using the remainder theorem.

Reasoning

12 Solve $\dfrac{3(x+1)}{5x} - \dfrac{1}{2x} = 4x$.

13 Substitute $m = x^2$ into $x^4 - x^2 - 12 = 0$ to solve the equation for x.

14 The sum of a sequence is given by $S = \dfrac{n}{2}[2a + (n-1)d]$, where n stands for the number of terms in the sequence. If $a = 5$ and $d = 4$, find the number of terms in the sequence needed to make a sum of 860.

15 A bowl has a shape that can be described by the parabola $y = \dfrac{1}{10}x^2$ for $-10 \leq x \leq 10$.

The position of a length of uncooked spaghetti that just touches the rim of the bowl is described by the line $y = x$. The coordinate (0, 0) is the middle of the bottom of the bowl and all units are in centimetres.

(a) Graph both equations on the same axes.

(b) What is the vertical distance, above the centre of the bowl, of the end of the spaghetti?

16 The vertical cross-section of a bottle is described by the equations $y = (x - 5)^3 + 2$ and $y = -(x - 15)^3 + 2$.

(a) Graph both equations on the same axes using x-values from 2 to 18.

(b) State the equation of the axis of symmetry of the bottle.

17 A capsule less dense than water is released under water and moves upwards subject to ocean currents. Its height, h metres, from sea level at time, t seconds, is given by $h = t^3 - 9t^2 + 26t - 24$.

(a) Factorise the equation and sketch the graph for $0 \leq t \leq 4$.

(b) Describe the motion of the capsule in this time period.

11 Probability

Recall 11

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.



1 Express in simplest form.

(a) $\frac{3}{15}$ **(b)** $\frac{16}{52}$



2 Evaluate the following.

(a) $\frac{1}{8} + \frac{3}{4}$ **(b)** $\frac{2}{5} + \frac{1}{3}$ **(c)** $1 - \frac{3}{5}$

(d) $\frac{2}{5} \times \frac{3}{4}$ **(e)** 0.8×0.5 **(f)** 0.05×0.2



3 (a) Two students are to be chosen from their coeducational class. Complete a list of the gender pairings possible, in order first student, second student.

(b) What are the two possible outcomes when a coin is tossed?



4 (a) A container has 25 numbered tokens; 6 are red, 10 are blue and the rest are yellow. A token is chosen at random from the container.

(i) How many different tokens can be chosen?

(ii) How many possible outcomes are there?

(iii) How many of these outcomes result in a red token being chosen?

(iv) How many of these outcomes result in a yellow token being chosen?

(v) How many of these outcomes result in a token that is not blue being chosen?

(b) A fair six-sided *die* (singular of *dice*) is rolled.

(i) List all the possible outcomes.

(ii) How many of these outcomes are even?

After completing this chapter you will be able to:

- describe the results of two- and three-step chance experiments both with and without replacement
- assign probabilities to outcomes and determine probabilities of events
- distinguish between independent and dependent events
- identify mutually exclusive events
- use the language of conditional statements to identify common mistakes interpreting such language.

Probability review

11.1

Need to Know

Probability is the study of the chance, or likelihood, of a particular **event** occurring.

$0 \leq \Pr(\text{event}) \leq 1$

An estimate, based on **long-run proportion**, is $\Pr(\text{success}) = \dfrac{\text{number of past successes}}{\text{total number of past trials}}$.

The full list of possible outcomes in a probability experiment is the **sample space**.

$\Pr(\text{success}) = \dfrac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely possible outcomes}}$

A **complementary event** is the opposite of the event under consideration. The sum of the probability of an event and its complementary event is 1.

$\Pr(A) + \Pr(\text{not } A) = 1$

We can find the sample space by drawing a **two-way table** or a **tree diagram**.

Worked Example 1

WE1

Over the past 6 months Jake has been late to school on 12 occasions in 100 school days.

(a) What is the estimated probability he will be late next Wednesday?

(b) What is the estimated Pr(Jake being on time next Friday)?

Working

(a) Jake has been late 12 out of 100 days.

$$\Pr(\text{late}) = \frac{12}{100} = \frac{3}{25}$$

(b) $\Pr(\text{late}) + \Pr(\text{on time}) = 1$

$$\Pr(\text{on time}) = 1 - \Pr(\text{late}) = 1 - \frac{3}{25} = \frac{22}{25}$$

Worked Example 2

WE2

Robin loves to play football with his mates after school but he also loves to play his guitar. He can't decide what to do, so he flips a coin. A head means he will play football and a tail means he will play his guitar. Find the probability that he:

(a) plays football 3 afternoons in a row

(b) plays football at least twice in 3 afternoons

(c) plays his guitar 2 afternoons in a row

(d) alternates his activities for the 3 afternoons

(e) doesn't play football in 3 afternoons.

Working

HHH, HHT, HTH, HTT, THH, TTH, THT, TTT are all equally likely as long as the coin is fair.

(a) Pr(football 3 afternoons in a row)

$= \Pr(\text{HHH})$

$= \dfrac{\text{number of successful outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{1}{8}$

(b) Pr(football at least twice in 3 afternoons)

$= \Pr(\text{HHT or HTH or THH or HHH})$

$= \dfrac{\text{number of successful outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{4}{8}$

$= \dfrac{1}{2}$

(c) Pr(plays guitar 2 afternoons in a row)

$= \Pr(\text{TTH or HTT or TTT})$

$= \dfrac{\text{number of successful outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{3}{8}$

(d) Pr(alternates activities for the 3 afternoons)

$= \Pr(\text{HTH or THT})$

$= \dfrac{\text{number of successful outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{2}{8}$

$= \dfrac{1}{4}$

(e) Pr(doesn't play football in 3 afternoons)

$= 1 - \Pr(\text{plays football 3 afternoons in a row})$

$= 1 - \dfrac{1}{8}$

$= \dfrac{7}{8}$

Worked Example 3

WE3

Jemma has a dilemma. She has 6 favourite outfits: 1 black, 1 red, 1 blue, 1 green, 1 purple and 1 multi-coloured. She has 2 favourite pairs of shoes; 1 has a pattern in the leather and 1 is plain. She can never decide what to wear so she rolls a die to choose her outfit and flips a coin to choose her shoes. On the die: 1 is black, 2 is red, 3 is blue, 4 is green, 5 is purple, 6 is multi-coloured. On the coin: heads is shoes with pattern, tails is plain. Find the probability that she will wear:

(a) the black outfit with the plain shoes

(b) the green, purple or red outfit

(c) the multi-coloured outfit and the patterned shoes

(d) the red or green outfit with plain shoes

(e) the red outfit

(f) the plain pair of shoes.

Working

A two-way table can be used to find the sample space.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

There are 12 equally likely outcomes.

(a) Pr(black outfit, plain shoes)
$= \Pr(T1)$
$= \frac{1}{12}$

(b) Pr(green or purple or red outfit)
$= \Pr(H2, H4, H5, T2, T4, T5)$
$= \frac{6}{12}$
$= \frac{1}{2}$

(c) Pr(multi-coloured outfit and shoes with pattern)
$= \Pr(H6)$
$= \frac{1}{12}$

(d) Pr(red or green outfit with plain shoes)
$= \Pr(T2, T4)$
$= \frac{2}{12}$
$= \frac{1}{6}$

(e) Pr(red outfit)
$= \Pr(T2, H2)$
$= \frac{2}{12}$
$= \frac{1}{6}$

(f) Pr(plain shoes)
$= \Pr(T1, T2, T3, T4, T5, T6)$
$= \frac{6}{12}$
$= \frac{1}{2}$

Worked Example 4

WE4

Libby and Chris wish to start a family. They plan to have three children. The chance of any particular child being a boy or a girl is considered to be equally likely.

(a) Represent all the possible outcomes using a tree diagram.

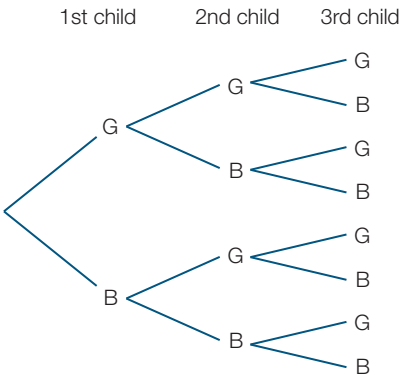
(b) List the sample space.

(c) Find the probability that they have:

- **(i)** a girl, a boy, then another girl
- **(ii)** 2 girls and a boy (in any order)
- **(iii)** 3 boys
- **(iv)** 3 children of the same gender
- **(v)** at least 2 girls

Working

(a)



(b) (G, G, G), (G, G, B), (G, B, G), (G, B, B), (B, G, G), (B, G, B), (B, B, G), (B, B, B)

There are 8 equally likely outcomes. These outcomes are equally likely because each child is considered equally likely to be a boy or a girl.

(c) **(i)** Pr(G, B, G)

$= \frac{1}{8}$

(ii) Pr(2 girls and 1 boy)

$= \frac{3}{8}$

(iii) Pr(3 boys)

$= \frac{1}{8}$

(iv) Pr(3 children of the same gender)

$= \frac{2}{8}$

$= \frac{1}{4}$

(v) Pr(at least 2 girls)

$= \frac{4}{8}$

$= \frac{1}{2}$

11.1 Probability review

Answers page 572

Fluency

1 Ben's car has been hard to start on 8 days over the last 4 weeks. WE1

(a) Estimate Pr(Ben's car will be hard to start tomorrow).

(b) Estimate Pr(Ben's car will be easy to start in 3 days' time).

(c) What assumptions have you made to calculate these answers?

2 Fiona is bushwalking through a national park. She finds that the path keeps dividing and she has to decide whether to take the left path or the path to the right. If she takes 3 left turns she will arrive at a beach. If she takes 3 right turns, she will reach a waterfall. If she turns left twice and right once, not necessarily in that order, she will get to a lookout. If she turns left once and right twice, not necessarily in that order, she will reach a picnic area. She can't decide where to go so she flips a coin. Heads will mean she turns right and tails will mean that she turns left. Find the probability that she: WE2

(a) gets to the beach

(b) arrives at the picnic area or waterfall

(c) reaches the lookout

(d) alternates the direction she takes

(e) doesn't arrive at the beach.

3 Mario likes to eat at a local restaurant but he has difficulty choosing his meal as he likes 6 main courses equally and there are 2 desserts he loves. He decides to roll a die to choose his main course and flip a coin to decide his dessert. He assigns the meals this way: chicken parmigiana as 1, chilli prawns as 2, spaghetti bolognese as 3, grilled barramundi as 4, pepper steak as 5 and spicy calamari as 6. He assigns his desserts this way: sticky date pudding as H and tiramisu as T. Find the probability that he chooses: WE3

(a) chicken parmigiana and tiramisu

(b) a seafood dish—chilli prawns, grilled barramundi or spicy calamari

(c) pepper steak and a sticky date pudding

(d) grilled barramundi or spaghetti bolognese as a main course with a tiramisu desert

(e) chilli prawns

(f) sticky date pudding.

4 A coin is tossed three times. WE4

(a) Represent all the possible outcomes using a tree diagram.

(b) List the sample space.

(c) Find the probability of tossing:

(i) a head, a tail then a head

(ii) a head, a head then a tail

(iii) 3 tails

(iv) all heads or all tails

(v) a least two heads.

5 Nikola and Brad are in a class containing 14 girls and 10 boys. To select a class representative, a person's name is chosen at random from the class roll. Find the probability that the class representative is:

(a) a girl

(b) not a girl

(c) Nikola

(d) Brad

(e) Nikola or Brad

(f) neither Nikola nor Brad.

6 Pedro has two drawers containing loose socks. There are a black, a white and a green sock in each. He is half asleep and grabs one sock, at random, from each drawer.

(a) Represent all the possible outcomes using a tree diagram.

(b) Why is it reasonable to regard these as equally likely outcomes?

(c) Find the probability of Pedro obtaining:

(i) a pair of black socks

(ii) one green and one white sock

(iii) a matching pair of socks

(iv) two socks that do not match.

7 Jacob and Ted are 2 members of a 12 member cricket team. The captain is chosen randomly each week. Find the probability that the captain is:

(a) Ted **(b)** not Jacob **(c)** Jacob or Ted **(d)** not Jacob or Ted.

8 The local supermarket experiences a flood in its store and the labels are washed off 12 cans of peaches, 5 cans of apples and 3 cans of beans. They place them all for sale for 50 cents a can, but you will not know what is inside until it is opened.

(a) If you buy only 1 can the probability of it being apple is:

A $\frac{1}{5}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{5}{8}$

(b) If you buy 1 can the probability of it containing fruit (not beans) is:

A $\frac{3}{20}$ **B** $\frac{5}{20}$ **C** $\frac{12}{20}$ **D** $\frac{17}{20}$

(c) If you want to be sure of purchasing at least 1 can of peaches, the minimum number of cans you must purchase is:

A 1 **B** 3 **C** 9 **D** 12

9 A spinner evenly divided and showing the numbers 1, 2, 3, 4 is spun once, and an eight-sided die showing 1 to 8 is rolled once.

(a) The probability that the total of the two numbers is 7 is:

A $\frac{3}{11}$ **B** $\frac{3}{32}$ **C** $\frac{1}{8}$ **D** $\frac{5}{32}$

(b) The probability that the total of the two numbers is 11 is:

A $\frac{1}{32}$ **B** $\frac{1}{16}$ **C** $\frac{1}{8}$ **D** $\frac{3}{32}$

(c) The probability that the total of the two numbers is a multiple of 3 is:

A $\frac{5}{16}$ **B** $\frac{11}{32}$ **C** $\frac{3}{8}$ **D** $\frac{15}{32}$

10 Suresh and Julian regularly play tennis against one another. Over the past 2 years, Suresh has won 16 times and Julian has won 20 times. What is the estimated probability that Suresh will win their next match?

11 Of the 125 students in Year 10, 15 ride to school every day and 30 ride sometimes. A student is chosen at random from the Year 10 students.

(a) What is the probability that he or she always rides to school?

A $\frac{3}{25}$ **B** $\frac{9}{25}$ **C** $\frac{17}{25}$ **D** $\frac{22}{25}$

(b) What is the probability that he or she never rides to school?

A $\frac{3}{25}$ **B** $\frac{9}{25}$ **C** $\frac{16}{25}$ **D** $\frac{22}{25}$

12 If the probability of it raining tomorrow is $\frac{2}{9}$, what is the probability of it not raining?

Understanding

13 In a game of chance, 45 balls numbered from 1 to 45 are placed in a barrel. If one ball is drawn at random, find the probability that its number is:

(a) 2 **(b)** less than 10 **(c)** even

(d) odd **(e)** greater than 50 **(f)** between 10 and 20 inclusive.

14 In the game of Monopoly, players advance clockwise around the 40 squares of the board according to the total of the numbers shown on the throw of two dice.



(a) Use a two-way table to represent all the possible totals obtained from the throw of two dice.

(b) How many possible outcomes are there?

(c) Why is it reasonable to regard these as equally likely outcomes?

(d) If your token is on Whitechapel Road, what is the probability that your next throw will result in your landing on Euston Road?

(e) Suppose you do land on Euston Road. Find the probability of advancing to Bow Street or further on your next throw.

(f) One way of getting out of jail in the game of Monopoly is to throw a 'double' with the two dice (i.e. to throw the dice and obtain the same result on both). Supposing you are in jail, find the probability of getting out of jail with your next throw of the dice.

15 Skye has the mobile telephone numbers of 40 'friends' saved on her mobile telephone. If we were trying to estimate the probability of Skye messaging one of these friends would it be reasonable to assume they were equally likely outcomes? Explain your thoughts in a sentence.

16 Val and Peter have a bag of sweets each. Val's bag of sweets contains 3 licorice blocks, 2 jelly babies and 1 chocolate caramel. Peter has 2 licorice blocks and 2 jelly babies in his bag. Both Val and Peter offer a sweet to Narelle. If Narelle randomly selects a sweet from each bag, find the probability that she obtains:

(a) 2 licorice blocks **(b)** 2 jelly babies **(c)** no licorice blocks

(d) no jelly babies **(e)** 1 licorice block **(f)** at least 1 jelly baby.

17 In a round-robin competition in tenpin bowling, Shelley plays 4 rounds. She estimates that she has an equal chance of winning or losing each round.

(a) Draw a tree diagram showing all the possible outcomes. (W = a win; L = a loss)

(b) List the sample space. How many possible outcomes are there?

(c) Find the probability that Shelley:

(i) wins all 4 rounds

(ii) wins only 2 of the 4 rounds

(iii) wins at least 3 rounds

(iv) wins the second round

(v) doesn't win the third round

(vi) doesn't win either the first or the fourth round.

18 There are 12 houses in the court where Zoran lives. Of the 20 cars owned between the people who live in the court, 7 are Holdens, 5 are Fords and the rest are other makes. The owner of one of the Holdens moves out and a new person, who owns a car, moves into the court. Is it reasonable to say that the probability this new person owns a Holden is $\frac{7}{20}$? Explain your thoughts in a sentence.

19 Two fair six-sided dice are rolled. One is numbered 1, 1, 2, 2, 3, 3 and the other is numbered 2, 4, 6, 8, 10, 12.

(a) Draw up a two-way table to show the sample space.

(b) Draw up a two-way table to show the sum of the two numbers shown.

(c) Find the following probabilities.

(i) Pr(sum is even)

(ii) Pr(sum is multiple of 5)

(iii) Pr(sum is greater than 10)

(iv) Pr(sum is not less than 11)

(d) Why is it harder to answer the questions in part **(c)** for this two-way table than for a diagram with consecutive numbers on each side?

(e) Draw up another two-way table that shows the product of the two numbers.

(f) Draw up a table that shows the probability for each possible product.

Reasoning

20 Explain the following observations.

(a) In the long run, the observed probability of Aiden wearing a tie to work is $\frac{19}{20}$, but on three consecutive days he is seen not wearing a tie.

(b) Lesley has kept track of the lotto numbers and has noticed that 16 has not occurred for 12 months. Thinking the probability of it coming out soon must be high, she includes 16 in her numbers for the next 2 months, but it still doesn't come out.

Open-ended

21 You are given a 12-sided die, numbered 1 to 12. Write an event that would have the following probabilities.

(a) $\Pr(A) = \frac{1}{6}$ **(b)** $\Pr(B) = \frac{1}{3}$ **(c)** $\Pr(C) = \frac{5}{12}$

Hint

Assume that there is an equal probability of having a boy as having a girl.

22 The long-term probability of Maria's team beating Abbey's team at netball is $\frac{4}{5}$. List three possible sets of long-term results that would produce this figure.

23 Sharanpreet calculates the probability of a family having at least one boy in a family of three children is $\frac{3}{8}$. What is the correct answer and what mistake has Sharanpreet made?

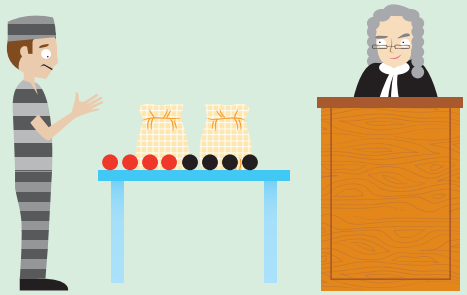
Problem solving

A chance of freedom

A prisoner is brought before a judge who says: "I have decided to give you a chance of freedom. Here are four red balls and four black balls. Divide the eight balls into two groups, any way you wish, and then place each group in a bag. I will hide the two bags behind my back and you will choose a bag from either my right or my left hand. You can then choose a ball at random from the bag. If it is red you can go free. If it is black your sentence will double."

(a) How can the prisoner divide the balls to give him the best chance of freedom?

(b) What division would give him the worst chance?



Strategy options

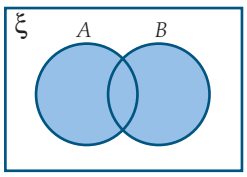
- Act it out.
- Test all possible combinations.
- Solve a simpler problem.

Venn diagrams and two-way tables

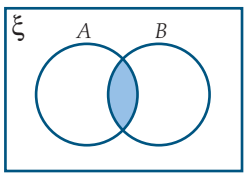
11.2

Need to Know

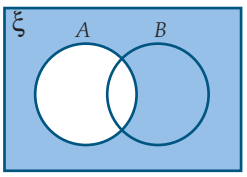
A Venn diagram consists of a rectangle with one or more circles inside it. The rectangle represents the **universal set**, which is every element under consideration for the particular situation being examined. On the Venn diagram it is represented by ξ. The circles represent particular events and are represented by capital letters.



A or B



A and B



not A

We can use a two-way table to show similar information.

	B	not B
A	A and B	A and not B
not A	not A and B	not A and not B

Together, the sections of the table make up the universal set. In addition to this, no section has any overlap with any other section.

Worked Example 5

WE5

A single card is drawn from a set of 25 cards numbered from 1 to 25.

Let A = 'odd numbers' and B = 'multiples of 3'

(a) Represent this on a Venn diagram.

(b) Use the Venn diagram to find:

(i) Pr(A)

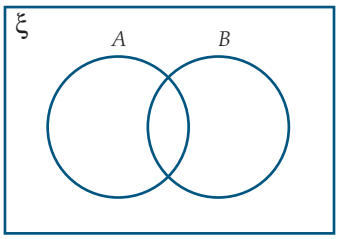
(ii) Pr(A and B)

(iii) Pr(not B)

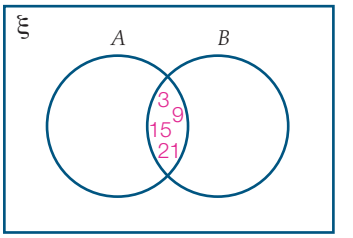
(c) Draw a two-way table to display the probability of possible events.

Working

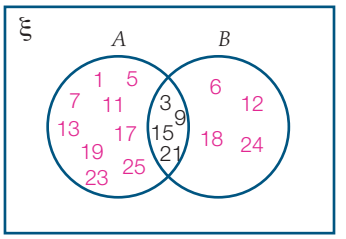
(a) Step 1: Draw a Venn diagram with sets A and B.



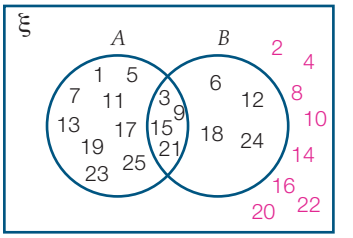
Step 2 : Write in the elements common to both sets.



Step 3: Write the remaining elements in either set.



Step 4: Write the remaining elements in the universal set.



(b) **(i)** $\Pr(A) = \frac{13}{25}$

(ii) $\Pr(A \text{ and } B) = \frac{4}{25}$

(iii) $\Pr(\text{not } B) = 1 - \Pr(B)$

$= 1 - \frac{8}{25}$

$= \frac{17}{25}$

(c) Step 1: Draw a two-way table with the known information.

	B	not B	
A	$\frac{4}{25}$		$\frac{13}{25}$
not A			
	$\frac{17}{25}$		

Step 2: Calculate Pr(A and not B).

	B	not B	
A	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{13}{25}$
not A			
		$\frac{17}{25}$	

Step 3: Calculate Pr(not A).

	B	not B	
A	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{13}{25}$
not A			$\frac{12}{25}$
	$\frac{17}{25}$		

Step 4: Calculate Pr(B) and Pr(not A and not B).

	B	not B	
A	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{13}{25}$
not A		$\frac{8}{25}$	$\frac{12}{25}$
	$\frac{8}{25}$	$\frac{17}{25}$	

Step 5: Calculate Pr(not A and B).

	B	not B	
A	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{13}{25}$
not A	$\frac{4}{25}$	$\frac{8}{25}$	$\frac{12}{25}$
	$\frac{8}{25}$	$\frac{17}{25}$	

11.2 Venn diagrams and two-way tables

Answers page 574

Fluency

1 In a non-leap year there are 28 days in February. A date in February is to be chosen to celebrate the 100th birthday of a city.

WE 5

Let A = 'even numbered dates' and B = 'dates that are multiples of 5'.

(a) Represent this information on a Venn diagram.

(b) Use the Venn diagram to find:

(i) Pr(A) **(ii)** Pr(A and B) **(iii)** Pr(not B)

(c) Draw a two-way table to display the probability of possible events.

2 Use the two-way table shown to find:

	B	not B
A	$\frac{3}{16}$	$\frac{5}{16}$
not A	$\frac{1}{16}$	$\frac{7}{16}$

(a) Pr(A and B) **(b)** Pr(not A and not B)

(c) Pr(not A and B) **(d)** Pr(A and not B)

(e) Pr(A) **(f)** Pr(B)

3 In a class of 20, 8 students learn the piano, 6 take singing lessons and 10 students neither learn the piano nor have singing lessons.

(a) Represent this information in a two-way table.

(b) Use the two-way table to find the number of people who learn piano and take singing lessons.

(c) Use the two-way table to find the probability of:

(i) taking singing lessons but not piano **(ii)** taking piano lessons but not singing.

Understanding

4 In a group of 30 people there are 15 who like cats, 20 who like dogs and 6 who dislike both cats and dogs. One person is chosen from the group at random.

(a) Represent this information on a Venn diagram.

(b) Use the Venn diagram to find the probability that the person chosen at random is:

(i) a cat lover only

(ii) a dog lover only

(iii) a person who likes both.

5 Complete these two-way tables.

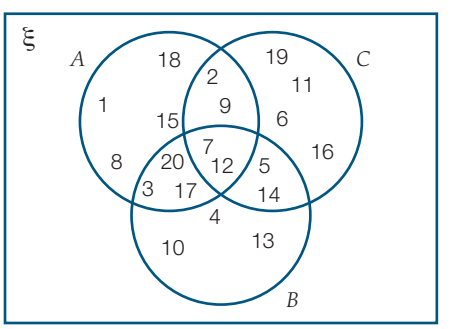
(a) $\Pr(A) = \frac{7}{16}$

	B	not *B*
A		$\frac{5}{16}$
not *A*	$\frac{1}{16}$	

(b) $\Pr(D) = 0.7$

	C	not *C*
D	0.2	
not *D*		0.1

6 Given the following Venn diagram, identify the numbers represented by each of the following.



(a) A or B

(b) A or C

(c) B or C

(d) A and B

(e) A and C

(f) B and C

(g) A and B and C

(h) not A and not B

7 A coin is tossed, a six-sided die is thrown and a ball is selected from a bag containing 4 red and 3 green balls. M is the set of outcomes that contains 'head is tossed', N is the set of outcomes that contains 'a 2 is not thrown' and P is the set of outcomes that contains 'a red ball is selected'.

(a) Draw a Venn diagram to show the sample space.

(b) Write down the elements of not M, not N and not P.

(c) Is the probability of H1R the same as H1G? Explain your answer.

8 A poll of 2000 students revealed that 554 respondents only watch TV and do not browse the internet while they watch. 1200 respondents only use the internet and the TV is not on while they're browsing. 10 students claim to neither watch TV nor use the internet.

(a) Draw a two-way table to represent this situation.

(b) What is the probability of a student in this survey watching TV and browsing the internet at the same time?

Reasoning

9 A survey of 1200 college students revealed that $\frac{3}{8}$ have a profile on MySpace, $\frac{3}{4}$ have a profile on Facebook, and $\frac{1}{4}$ have a profile on both sites.

(a) Represent the information in a two-way table.

(b) If a student is chosen at random, find the probability that the student has:

(i) no profile on either

(ii) a profile on either, but not on both.

10 Mrs Lauder is creating the timetable for Year 12 students. The request forms have been entered online:

- 163 want to study mathematics, 95 physics, 124 chemistry
- 61 requested maths and physics
- 67 requested maths and chemistry
- 49 requested physics and chemistry
- 27 want to study all three subjects
- 17 want to study none of these subjects

(a) Draw a Venn diagram to illustrate the situation.

(b) How many students requested only mathematics?

(c) How many students requested only physics?

(d) How many students requested only chemistry?

(e) What was the total number of requests?

(f) What is the probability of a Year 12 student at Mrs Lauder's school selecting mathematics as one of their subjects? Will the answer be the same at every school?

11 In a scout troop there are 15 boys and 12 girls. 10 of the boys and 8 of the girls say they are interested in tennis.

(a) Show this information on a Venn diagram.

(b) A student is selected at random. Use the diagram to find the probability that the student selected is:

(i) a girl who is interested in tennis

(ii) a boy who is not interested in tennis

(iii) someone who is interested in tennis

(iv) someone who is not interested in tennis

12 Human blood can contain up to three different antigens; A, B and Rh. The following table outlines the antigens present in each blood type. (The actual situation is slightly more complicated for the O group.) The values given in the table refer to Australia; countries with different ethnic make-up would have different values.

Blood Group	A	B	Rh	% in Australia
A Positive	Yes	No	Yes	31
A Negative	Yes	No	No	7
B Positive	No	Yes	Yes	8
B Negative	No	Yes	No	2
AB Positive	Yes	Yes	Yes	2
AB Negative	Yes	Yes	No	1
O Positive	No	No	Yes	40
O Negative	No	No	No	9

The antigens present in the blood of patients at a clinic were recorded for 1 week:

- 39 patients had the A antigen
- 2 had the A and B antigens
- 12 had the B antigen
- 9 had the B and Rh antigens
- 67 had the Rh antigen
- 30 had the A and Rh antigens
- 2 had all three antigens
- 5 had no antigens.

(a) Draw a Venn diagram to show this situation.

(b) How many patients were there?

(c) What was the Pr(exactly one antigen)?

(d) What was the Pr(exactly two antigens)?

(e) Use the data to calculate the probability of each of the eight blood groups.

(f) Compare the answers from part **(e)** with those expected from long-term data as shown in the table.

Open-ended

13 From a class of 25 students 14 like watching the news on TV and 16 said they use the internet to find out the news. Mert says there must be something wrong with the statistics because there are 30 different opinions from only 25 students. Sasha says Mert hasn't taken into account students who do both. Neither of them is correct. What is the correct interpretation of the statistics and what mistakes have Mert and Sasha made?

14 What is wrong with the following question?

In a survey of 200 people with a driving licence, 18% have never received a speeding fine, 44 people have received a parking ticket but have never received a speeding fine, and 78% have received both a speeding fine and a parking ticket. How many people have received a speeding fine but have never received a parking ticket?

11.3 Mutually exclusive events

Need to Know

Addition Law for Mutually Exclusive Events

If A and B are **mutually exclusive** events (i.e. they cannot occur at the same time), then:

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$$

Worked Example 6

WE6

A six-sided die is rolled. What is the probability that the number obtained is:

(a) a 2 or a 5?

(b) neither odd nor a 4?

Working

(a) $\Pr(2 \text{ or } 5)$

$= \Pr(2) + \Pr(5)$

$= \frac{1}{6} + \frac{1}{6}$

$= \frac{2}{6}$

$= \frac{1}{3}$

(b) $\Pr(\text{neither odd nor a } 4)$

$= \Pr[\text{not}(\text{odd or } 4)]$

$= 1 - \Pr(\text{odd or } 4)$

$= 1 - [\Pr(\text{odd}) + \Pr(4)]$

$= 1 - \left(\frac{1}{2} + \frac{1}{6}\right)$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

11.3 Mutually exclusive events

Answers page 575

Fluency

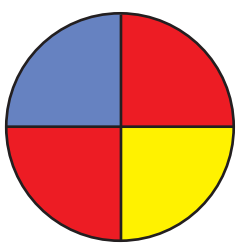
1 A fair eight-sided die, numbered 1 to 8, is rolled. What is the probability that the number obtained is:

WE6

(a) a 3 or a 6?

(b) neither 5 nor even?

2 A spinner is coloured as shown.



(a) What is the probability of spinning blue?

(b) What is the probability of spinning red?

(c) What is the probability of spinning blue or red?

(d) What is the probability of spinning blue or yellow?

3 Are any of the following pairs of events mutually exclusive?

(a) Obtaining a head on the toss of a coin and a tail on the toss of a coin.

(b) Choosing an odd-numbered horse in a race and horse number 7 in another race.

(c) Winning first prize in a raffle with a blue ticket and first prize in a raffle with a yellow ticket.

4 **(a)** A basket contains 3 red, 2 white and 5 black scarves. The probability that a scarf drawn at random will be red or black is:

A $\frac{1}{5}$ **B** $\frac{2}{3}$ **C** $\frac{7}{10}$ **D** $\frac{4}{5}$

(b) A jar contains 6 red lollies, 3 green lollies and 2 yellow lollies. The probability that a lolly drawn at random will be green or yellow is:

A $\frac{5}{11}$ **B** $\frac{6}{11}$ **C** $\frac{3}{11}$ **D** $\frac{5}{6}$

(c) A drawer contains 6 blue socks, 4 black socks and 2 brown socks. The probability that a sock drawn at random will *not* be black is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

Understanding

5 These probabilities apply to different modes of transport students use to travel to and from a particular school. The modes are all mutually exclusive.

$\Pr(\text{car}) = \frac{7}{20}$ $\Pr(\text{train}) = \frac{9}{20}$

$\Pr(\text{bus}) = \frac{1}{20}$ $\Pr(\text{walking}) = \frac{3}{20}$

A student is asked how he travels to school. Find the probability that:

(a) he travels by car or train

(b) he walks or catches a bus

(c) he does not walk nor catch a bus

(d) he does not travel by car nor by bus

(e) he travels by car, train or bus.

6 Twenty tokens numbered in order from 1 to 20 are placed in a bag and one is randomly selected. Find the probability that the number on the token is:

(a) 19 or 20

(b) a multiple of 3

(c) a multiple of 3 or a multiple of 7

(d) even or the number 15

(e) less than 10 or greater than 18

(f) neither 1 nor 2

(g) neither odd nor a multiple of 4

(h) 5, 6 or 7.

7 A pack of playing cards contains 52 cards, half red and half black. The red cards are divided into two suits: hearts and diamonds. The black cards are divided into two suits: clubs and spades. Each suit is made up of the cards ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.

A card is drawn at random from the pack. Find the probability that the card is:

(a) red

(b) a heart

(c) an ace

(d) the king of hearts

(e) red or the ace of spades

(f) a diamond or the 4 of hearts

(g) the 5 of clubs or the queen of spades

(h) a king or an ace

(i) any king or any ace or the 10 of hearts

(j) the jack of diamonds, any 2, or the 7 of clubs

(k) neither a 6 nor a 9 of any suit

(l) neither a spade nor red.

Reasoning

8 A normal six-sided die is rolled. The following events have been defined:

A = rolling an even number B = rolling a 2 or a 6

C = rolling a 1 or a 3 D = rolling a multiple of 3

(a) List all possible pairs of events (order does not matter).

(b) Are any of these pairs mutually exclusive?

(c) Find the probability of each event.

(d) Find the probability that either event occurs for each of the pairs listed in **(a)**.

(e) Find the probability that both of the events occur for each of the pairs listed in **(a)**.

(f) Find the sum of the probabilities of each event in all the pairs listed in **(a)**.

(g) Compare your answers to parts **(d)**, **(e)** and **(f)**. What do you find? Write an equation to summarise your findings.

9 A class conducted a traffic survey. The types of vehicles they recorded were:

sedan	station wagon	4WD	hatchback
truck	van	bus	2 door

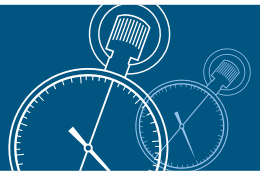
(a) What difficulties can you see arising from the data collection?

(b) How could these difficulties have been avoided?

Open-ended

10 Consider a fair 20-sided die numbered 1 to 20. Write down at least eight pairs of mutually exclusive events related to this die.

Half-time 11



Ex 11.2

1 **(a)** Complete the following two-way table which relates to the number of surveyed students who own either, or both, an iPhone or an iPad.

	iPhone	not iPhone	
iPad	60		75
not iPad			
	210		400

(b) Now use the completed two-way table to state the probability of a randomly chosen student owning:

(i) an iPhone but not an iPad **(ii)** an iPad but not an iPhone.

Ex 11.1

2 Ten balls numbered from 1 to 10 are placed in a bag and one is drawn at random. Find the probability that the number on the ball is:

(a) 1 or 2

(b) an even number

(c) a 6 or an odd number

(d) neither an odd number nor a multiple of 4.

Ex 11.1

3 The probability of it raining on a given day of the weekend is 0.3. What is the probability that it rains on both Saturday and Sunday?

Ex 11.1

4 In a chess tournament, each contestant must play three others. Suzanne estimates that she has an equal chance of winning or losing each game.

(a) Draw a tree diagram and list the sample space for the possible outcomes of Suzanne's three games.

(b) How many possible outcomes are there?

(c) Find the probability of Suzanne:

(i) winning all three games **(ii)** winning only one game

(iii) winning the first game **(iv)** not winning the third game

(v) winning at least two games **(vi)** not winning either of the first two games.

Ex 11.3

5 A fair eight-sided die is numbered 1 to 8. The die is rolled once. The following events are defined:

A = 'rolling an even number'

B = 'rolling a prime number'

C = 'rolling a multiple of 5'

(a) Given that order does not matter, list the pairs of events that are possible.

(b) Find the probability of each individual event.

(c) State which, if any, of the pairs of events are mutually exclusive.

Probability tree diagrams

11.4

Need to Know

We can use a tree diagram and multiply along its branches to obtain probabilities.
A **probability tree diagram** can be used for events that do not have equally likely outcomes.

Worked Example 7

WE7

At supermarkets, cartons of eggs are inspected for breakages by the checkout assistant before payment is made. The probability that a carton of eggs will be rejected is found to be 0.1. Two cartons of eggs are brought to a checkout.

(a) Copy and complete the tree diagram on the right. Evaluate the probability of each outcome.

Let R = carton of eggs is rejected and
A = carton of eggs is not rejected (is accepted)

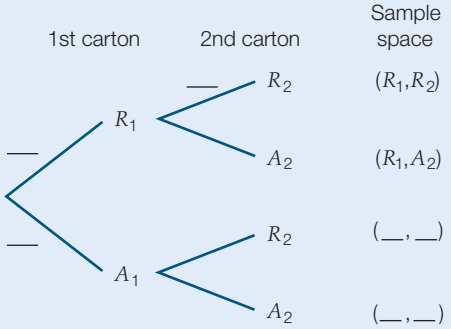
(b) Find the probability that:

(i) both cartons will be rejected

(ii) only one carton will be rejected

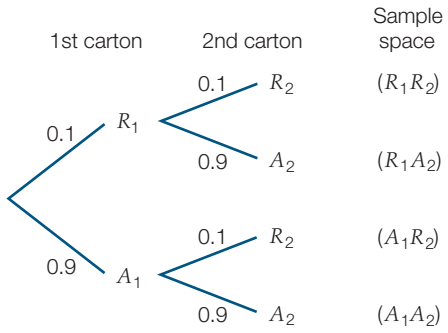
(iii) neither carton will be rejected

(iv) at least one carton will be rejected



Working

(a) $Pr(R) = 0.1$, so $Pr(A) = 0.9$



$Pr(R_1R_2) = 0.1 \times 0.1$
$= 0.01$

$Pr(R_1A_2) = 0.1 \times 0.9$
$= 0.09$

$Pr(A_1R_2) = 0.9 \times 0.1$
$= 0.09$

$Pr(A_1A_2) = 0.9 \times 0.9$
$= 0.81$

(b) **(i)** Pr(both cartons will be rejected)
$= Pr(R_1R_2)$
$= 0.01$

(ii) Pr(only one carton will be rejected)
$= Pr(R_1A_2 \text{ or } A_1R_2)$
$= Pr(R_1A_2) + Pr(A_1R_2)$
$= 0.09 + 0.09$
$= 0.18$

(iii) Pr(neither carton will be rejected)
$= \Pr(A_1A_2)$
$= 0.81$

(iv) Pr(at least one carton will be rejected)
= Pr(one carton rejected or both cartons rejected)
= Pr(one carton rejected) + Pr(both cartons rejected)
$= 0.18 + 0.01$
$= 0.19$

Worked Example 8

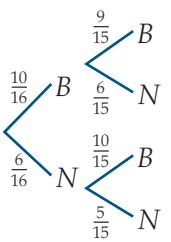
WE8

The lucky dip at the school fete contains 16 different gifts, 10 for the 4–7 age group and 6 for the 8–11 age group. Unfortunately, the 16 gifts have been mixed together. As you cannot tell what is inside without opening the gift, the lucky dips are offered as a special two for the price of one. A 5-year-old takes his two gifts without replacement. Let B represent a gift is chosen in the 4–7 age group and N represent a gift in the 4–7 age group is not chosen.

(a) Find Pr(both gifts are in the 4–7 age group).

(b) Find Pr(the 5-year-old chooses at least one suitable gift).

Working



BB, BN, NB, NN

(a) BB

$$\Pr(BB) = \frac{10}{16} \times \frac{9}{15} = \frac{3}{8}$$

(b) $\Pr(NN) = \frac{6}{16} \times \frac{5}{15} = \frac{1}{8}$

Pr(the 5-year-old chooses at least one suitable gift)

$$= 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

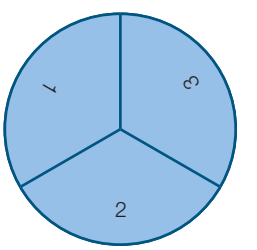
Answers page 576

11.4 Probability tree diagrams

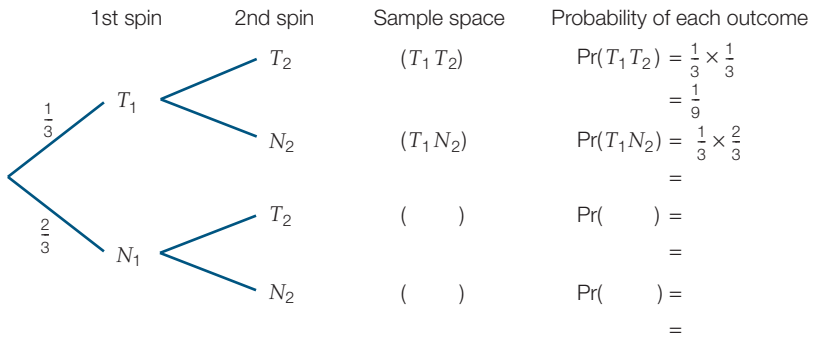
Fluency

WE7

1 A spinner numbered 1 to 3 is spun twice. Let T represent 'the number 2 is spun' and N represent 'the number 2 is not spun'.



(a) Copy and complete the following tree diagram. Evaluate the probability of each outcome.



(b) Find the probability of:

(i) spinning 2 twice

(ii) spinning 2 once only

(iii) *not* spinning a 2

(iv) spinning 2 once at least.

2 Two balls are drawn without replacement from a bag containing 4 green marbles and 7 blue marbles. Draw a tree diagram to represent the situation and find:

WE 8

(a) Pr(both marbles being blue)

(b) Pr(getting at least one blue marble).

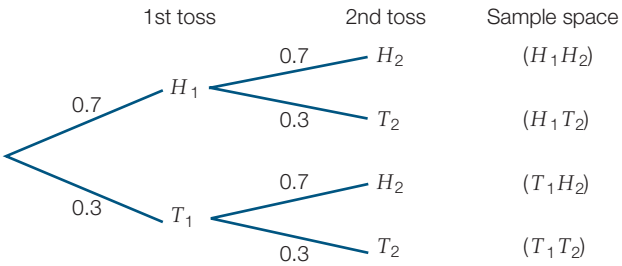
3 The spinner described in Question **1** is now spun three times.

(a) Draw a probability tree diagram for this situation.

(b) What is the probability of spinning a 2 three times?

(c) What is the probability of *not* obtaining a 2 on any of the three spins?

4 A biased coin is tossed twice. Let H represent 'obtaining a head with the coin' and T represent 'obtaining a tail with the coin'. The following probability tree diagram can be drawn.



(a) The probability of obtaining two heads is:

A $0.7 + 0.7$ **B** 0.7×0.7 **C** 0.25 **D** 0.7×0.3

(b) The probability of obtaining two tails is:

A 0.6 **B** 0.3 **C** 0.21 **D** 0.09

(c) The probability of obtaining a tail first, and then a head is:

A 0.25 **B** 0.21 **C** 0.49 **D** 1.0

(d) The probability of obtaining only one tail is:

A 0.21 **B** 0.3 **C** 0.42 **D** 0.63

(e) The probability of obtaining at least one tail is:

A 0.75 **B** 0.51 **C** 0.42 **D** 0.39

5 Two cards are drawn, without replacement, from a set of 16 cards numbered from 1 to 16.

Let M = the event a multiple of four is drawn and N = a multiple of four is not drawn.

(a) Draw a tree diagram and assign a probability to each branch.

(b) List the sample space.

(c) Find Pr(both cards are a multiple of 4).

(d) Find Pr(only one card is a multiple of 4).

(e) Find Pr(neither card is a multiple of 4).

Understanding

6 For afternoon tea, patrons of a cafe can select from tea, coffee or hot chocolate, and either scones or carrot cake. The probability of a person selecting tea is 0.45, whereas the probability of a person selecting coffee is 0.35. The probability of a person selecting scones is 0.6.

(a) What is the probability of a person selecting hot chocolate?

(b) What is the probability of a person selecting carrot cake?

(c) Draw a probability tree diagram to represent this situation.

(d) Find the probability of a person:

(i) selecting tea and scones

(ii) selecting hot chocolate and carrot cake

(iii) selecting tea and scones or hot chocolate and carrot cake

(iv) *not* selecting coffee and scones.

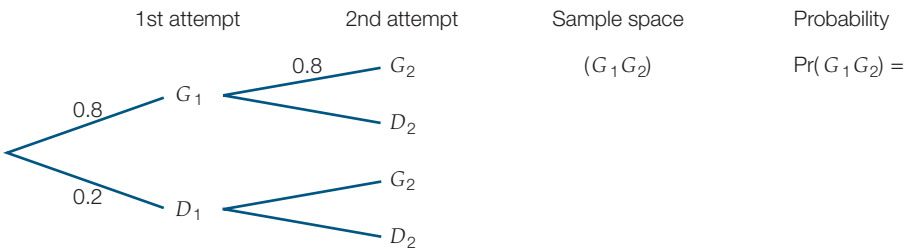
7 For each of the following:

(i) copy and complete the probability tree diagram

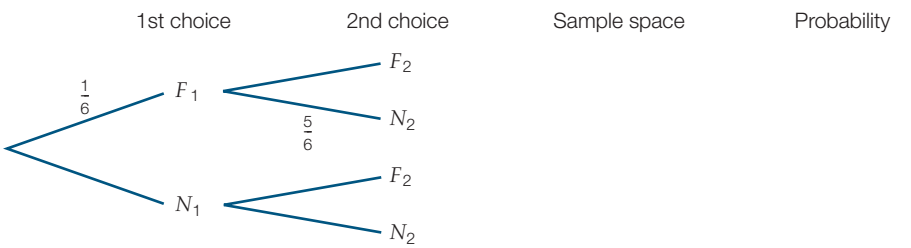
(ii) write down the probability of each possible outcome

(iii) find the sum of the probabilities of all outcomes.

(a) Let G = scores a goal and D = does not score a goal.



(b) Let F = a 4 is chosen from the numbers 1 to 6 and N = a 4 is not chosen.

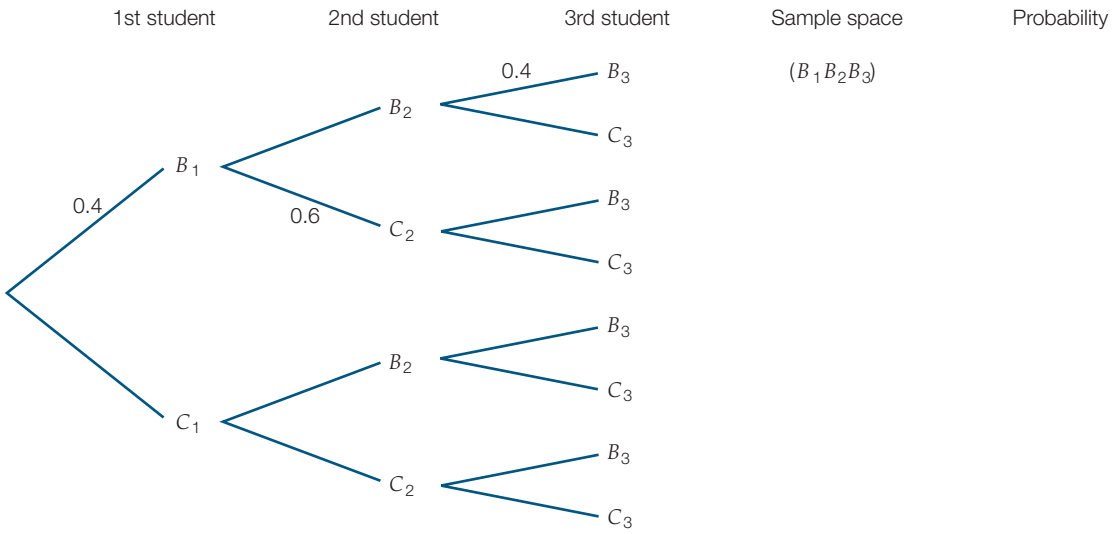


(c) P = Patrick is chosen from a group of boys.

A = Anne is chosen from a group of girls.



(d) Let B = student travels to school by bus and C = student does not travel by bus.



8 In a game of darts, each player has a turn to throw three darts one at a time. Seline estimates her chances of scoring a bullseye with any one throw of a dart to be 0.2. Find the probability that:

(a) Seline scores three bullseyes in her turn

(b) she scores two bullseyes only

(c) she scores at least one bullseye.

9 Two fair six-sided dice are rolled. Let T represent the event 'a 3 is rolled'.

(a) What would not T represent?

(b) Draw a probability tree diagram to illustrate this situation.

(c) Find the probability of:

(i) rolling two 3s **(ii)** not rolling any 3s **(iii)** rolling one 3 only

(iv) rolling at least one 3 **(v)** rolling no more than one 3.

10 A card is selected from a pack of 52 playing cards and then replaced. Another card is selected. Find the probability that:

(a) both cards are aces

(b) the first card is a club and the second card is red

(c) both cards are of the same colour

(d) one card is red and the other is black.

11 Wesley's lucky number is 5. At the races, he decides to bet on horse number 5 in the first three events. In race number 1, there is a field of 12 horses. In race number 2, there is a field of 9 horses. In race number 3, there is a field of 10 horses.

(a) If each horse in an event has an equal chance of winning, draw a probability tree diagram to represent Wesley's chances of winning or losing on each race.

(b) Find the probability that:

(i) Wesley wins all three racing events

(ii) only one of his chosen horses wins

(iii) at least one of his chosen horses wins.

Reasoning

12 A mathematics quick quiz consists of five multiple choice questions, each with five choices for the solution. Draw up a table to show the probability of obtaining each of the totals of correct answers from 0 to 5 if you randomly guess the answers.

13 A fair six-sided die is rolled until a 6 is obtained.

(a) Copy and complete the following table, which shows the probability that this will take the indicated number of rolls. Round your answer to three decimal places.

Number of rolls	1	2	3	4	5	6
Probability						

(b) Add the probabilities you found in part **(a)**. What sum do you get?

(c) Explain what this answer means.

14 A party game for 29 people starts by assigning everyone a number from 1 to 29. In the first round, 7 numbers are drawn at random and those people eliminated. In the second round 6 people are eliminated, the third round eliminates 5 people, then 4, 3, 2 and finally 1, to leave the winner.

(a) Find Pr(being eliminated in round 1).

(b) Assuming you are not eliminated in the first round, find Pr(being eliminated in round 2).

(c) Assuming you are not eliminated in the previous rounds find the probability of being eliminated in each round.

(d) Add the results for the probability of being eliminated in each round. Explain the result.

Open-ended

15 A set of volleyball is won when one side reaches 25 points. If we knew nothing about the ability of the teams we would assign the chance of winning as 0.5 each. If, however, one side was leading 15–0 we may rethink the original assignment of 0.5 each. Estimate the new probability for each team to win the set. Justify your answer.

16 Draw up a probability tree diagram that finishes with at least 9 branches where one of the final branches has a probability of $\frac{1}{9}$ and another has a probability of $\frac{1}{8}$. Explain what your tree diagram represents.

17 The estimated probability of a 25-year-old male playing cricket is 0.2 and a 25-year-old male playing golf is 0.18. Armapreet calculates that the probability of a man playing cricket or golf must be 0.38. Is she right? If not, what mistake has she made?

Problem solving

The origins of probability theory

The origins of modern probability theory involve a problem known as the 'problem of the points' or 'division of the stakes'. It involves a two-person game where each has made an equal contribution to the prize pot and the agreement is that the first person to get to a certain number of points will be the winner and collect the entire prize pot. The problem arises when the game is interrupted, and cannot be concluded. How should the prize pot be divided?

Let's say the the winner is the first to 10 points.

1 How would you divide the pot if the score was 7 to 5?

2 One of the early solutions suggested that the pot should be divided in proportion to the number of games won. Using this solution, the pot above would be divided $\frac{7}{12} : \frac{5}{12}$. What do you think of this solution? What would happen if the score was 1 to 0? Pascal and Fermat thought the crucial issue was the possible ways the game could have continued rather than what had already happened.

3 Think of a game where the score is 8 to 7. Write all the ways the game could continue and use this to decide how to divide the pot.

4 What if the score was 2 to 5? What is the problem with this sort of score?

5 What do you think would be the easiest scores to deal with if the game was interrupted?

Strategy options

- Act it out.
- Draw a diagram.

Extension

It is considered that Pascal and Fermat solved this problem. Conduct some internet research to discover how they resolved the issue.

Conditional statements

11.5

Need to Know

When we are dealing with a **conditional statement** we are given information that has restricted the universal set. Words such as 'given', 'knowing that' and 'if … then' are indications we are dealing with a conditional statement.

The number of outcomes in the restricted set becomes the denominator for the probability fraction.

Worked Example 9

WE9

A 16-sided die is rolled, with faces numbered 1–16. The following events are defined.

A = 'numbers less than 7' and B = 'multiples of 4'

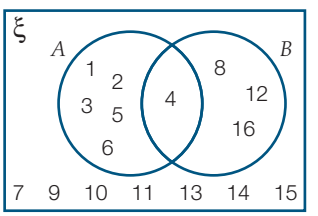
(a) Draw a Venn diagram to show this information.

(b) Calculate Pr(a number less than 7 given it is a multiple of 4).

(c) Calculate Pr(a multiple of 4 given it is less than 7).

Working

(a)



(b) The denominator is the number of elements in B (a multiple of 4): 4

The numerator is the number of elements in B and in A (a number less than 7): 1

Pr(a number less than 7 given it is a multiple of 4) $= \frac{1}{4}$

(c) The denominator is the number of elements in A (a number less than 7): 6

The numerator is the number of elements in A and in B (a multiple of 4): 1

Pr(a multiple of 4 given it is a number less than 7) $= \frac{1}{6}$

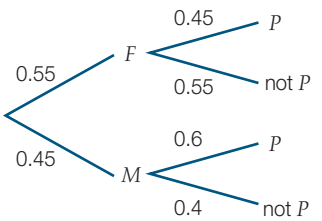
Worked Example 10

WE10

In a particular electorate, 55% of the voting population is female. It is also known that 60% of the males support the Progressive Conservative Party, whereas only 45% of the females support this party. Find the probability that a randomly selected voter from that electorate is male if you know that they support the Progressive Conservative Party. Round your answer to four decimal places.

Working

F = 'female'
M = 'male'
P = 'supporters of Progressive Conservative Party'



$\Pr(FP) = 0.55 \times 0.45 = 0.2475$

$\Pr(MP) = 0.45 \times 0.6 = 0.27$

Pr(support Progressive Conservative Party)
$= \Pr(FP) + \Pr(MP)$
$= 0.2475 + 0.27$
$= 0.5175$

Pr(male and support Progressive Conservative Party)
$= \Pr(MP)$
$= 0.27$

Pr(male given they support Progressive Conservative Party)
$= \Pr(M \text{ given } P)$
$= \frac{0.27}{0.5175}$
$= 0.5217$

11.5 Conditional statements

Answers page 577

Fluency

1 **(a)** A 16-sided die is rolled, with faces numbered 1–16. The following events are defined. WE9

A = 'numbers less than 9' and B = 'multiples of 3'

(i) Draw a Venn diagram to show this information.

(ii) Calculate Pr(number less than 9 given it is a multiple of 3).

(iii) Calculate Pr(multiple of 3 given it is a number less than 9).

(b) A 16-sided die, with faces numbered 1–16, is rolled and the following events are defined.

A = 'numbers greater than 5' and B = 'prime numbers'

(i) Draw a Venn diagram to show this information.

(ii) Calculate Pr(number greater than 5 given it is a prime number).

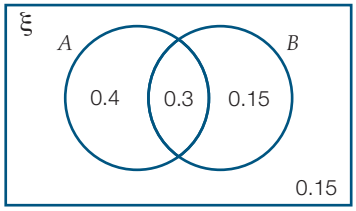
(iii) Calculate Pr(prime number given it is greater than 5).

2 In a particular electorate, 65% of the voting population is female. It is also known that 55% of the males support the Progressive Conservative Party, whereas only 40% of the females support this party. Find the probability that a randomly selected voter from that electorate is female if you know that they support the Progressive Conservative Party. Round your answer to four decimal places. WE10

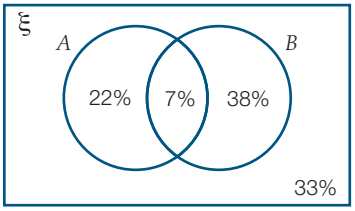
3 Use the information in the Venn diagram to find:

(i) Pr(A given B) **(ii)** Pr(B given A)

(a)



(b)



4 A coin is flipped three times.

(a) The probability that the third flip results in a head, given the previous two flips were both heads, is:

A $\frac{1}{8}$ **B** $\frac{1}{4}$ **C** $\frac{1}{2}$ **D** 1

(b) The probability that the third flip results in a head, given at least two of the three flips are heads, is:

A $\frac{3}{8}$ **B** $\frac{1}{2}$ **C** $\frac{2}{3}$ **D** $\frac{3}{4}$

5 You have a reduced pack of playing cards consisting only of the 26 red cards. A card is selected at random from the pack. Find the probability that the card is:

(a) an ace given it is a heart **(b)** a heart given it is a picture card

(c) the five of diamonds given it is a diamond **(d)** the five of diamonds given it is a five.

Understanding

6 A new disease is known to infect 2.5% of the population. A test for the disease has been developed, but it is not completely accurate. It correctly identifies the disease in infected patients 95% of the time, but also (incorrectly) identifies the disease in 7% of the uninfected population.

(a) Draw a tree diagram to show this information.

(b) Find Pr(have disease given test positive), rounding your answer to four decimal places.

(c) Find Pr(don't have disease given test negative), rounding your answer to four decimal places.

7 A sportswear company obtains its shoes from two suppliers, A and B. It is known that 5% of the shoes from supplier A are defective, whereas the figure is 9% for supplier B. Artemis obtains 65% of its shoes from supplier A. Where necessary, round your answers to four decimal places.

(a) Draw a tree diagram to show this information.

(b) Find Pr(A given the shoe is defective).

(c) Find Pr(B given the shoe is not defective).

8 A university student is studying both Economics and Voodoo. The probability that she will pass Economics is 0.95 and that she will pass Voodoo is 0.8. There is a 0.75 chance she will pass both. Find:

(a) Pr(passes Economics given passes Voodoo)

(b) Pr(passes Voodoo given passes Economics)

(c) Pr(passes Economics given passes only one of the subjects)

Reasoning

9 Abbud plays a lot of table-tennis. He has a 70% chance of winning any particular game. Find the probability that Abbud wins a match decided by the:

(a) best of three games

(b) best of three games, given that he won the first game

(c) best of three games, given that it lasts three games

(d) best of five games

(e) best of five games, given that he won the first two games.

10 A new cola is produced which the maker claims has the same taste as the market leading cola. A test is set up whereby people are offered a drink and they say whether it is the market leader or the new cola. The new cola is correctly identified 40% of the time and the market leader is correctly identified 75% of the time. If a person has an equal chance of being given either cola and they say the cup contains the market leader, what is the probability that this is true?

Open-ended

11 A tired teacher wrote this question:

Given Pr(A) = 0.6 and Pr(B) = 0.4, find Pr(A given B).

(a) What is wrong with the question?

(b) Rewrite the question so it is correct, and then find the answer.

Dependent and independent events

11.6

Need to Know

A and *B* are **independent events** if the result of one event has no effect on the result of the other event.

When *A* and *B* are independent:

$$\Pr(A \text{ and } B) = \Pr(A) \times \Pr(B)$$

Dependent events are influenced by the outcome of another event; sampling without replacement is an example of a dependent event.

Worked Example 11

WE 11

Two representatives, one girl and one boy, are chosen at random from a group of 6 girls and 4 boys. If Linda and Adam are in this group, what is the probability that both Linda and Adam will be chosen?

Working

Let L = Linda chosen
and A = Adam chosen

L and A are independent events.

$\Pr(L \text{ and } A) = \Pr(L) \times \Pr(A)$

$\Pr(L) = \frac{1}{6}$

$\Pr(A) = \frac{1}{4}$

$\Pr(L \text{ and } A) = \frac{1}{6} \times \frac{1}{4}$

$= \frac{1}{24}$

The probability that both Linda and Adam will be chosen is $\frac{1}{24}$ (i.e. 1 chance in 24).

Worked Example 12

WE12

Two cards are drawn out of a 52-card deck of playing cards without replacement.

(a) Find the Pr(drawing 2 kings).

(b) Find the Pr(both cards are from the same suit).

Working

(a) Pr(king on 1st draw) $= \frac{4}{52} = \frac{1}{13}$

Pr(king on 2nd draw) $= \frac{3}{51} = \frac{1}{17}$

Pr(both cards being a king) $= \frac{1}{13} \times \frac{1}{17}$

$= \frac{1}{221}$

(b) Pr(club/club) $= \frac{13}{52} \times \frac{12}{51} = \frac{3}{51} = \frac{1}{17}$

Pr(spade/spade) $= \frac{13}{52} \times \frac{12}{51} = \frac{3}{51} = \frac{1}{17}$

Pr(heart/heart) $= \frac{13}{52} \times \frac{12}{51} = \frac{3}{51} = \frac{1}{17}$

Pr(diamond/diamond) $= \frac{13}{52} \times \frac{12}{51} = \frac{3}{51}$

$= \frac{1}{17}$

Pr(both cards are from the same suit)

$= \frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \frac{1}{17}$

$= \frac{4}{17}$

Answers page 578

11.6 Dependent and independent events

Fluency

WE11

1 Katrina finds that on average her train is late 2 mornings out of 5, whereas Frank's bus is late 1 morning in every 4. Find the probability that on any given day both Katrina and Frank are late.

WE12

2 A box containing 40 apples, 10 of each of 4 different varieties, has 5 rotten ones. If 2 apples are taken out without replacement, calculate:

(a) Pr(2 rotten apples)

(b) Pr(2 apples of the same variety)

3 Phillip has 2 pairs of blue and 1 pair of black jeans hanging in his wardrobe. In a drawer, he has 1 red, 3 black and 5 white T-shirts. If he randomly chooses a pair of jeans and a T-shirt to wear, find the probability that he chooses:

(a) a white T-shirt and blue jeans

(b) a black T-shirt and black jeans

(c) black jeans and a T-shirt that is not black

(d) blue jeans and a T-shirt that is not red.

4 In a group of 20 people, 2 have red hair, 6 are blond and the rest have dark hair. Of the group, 15 people are right-handed. If a person is chosen at random from the group, find the probability that the person:

(a) has red hair and is right-handed

(b) is blond and is left-handed

(c) is right-handed and does not have red hair.

5 Label each of these pairs of events as either independent (I) or dependent (D).

(a) Tossing a coin and rolling a die.

(b) Drawing two cards from a deck without replacing the first before drawing the second.

(c) Drawing two cards from a deck replacing the first before drawing the second.

(d) Raining in two cities 100 km apart on the same day.

(e) Hours of study and test result.

(f) The number of hours of practice a learner driver does before getting their license and the chance of having an accident in the first month of having their license.

6 An urn contains 3 red beads, 4 blue beads and 8 green beads. If two beads are drawn without replacement, find:

(a) Pr(RR) **(b)** Pr(BB) **(c)** Pr(GG) **(d)** Pr(RB)

(e) Pr(BG) **(f)** Pr(GB) **(g)** Pr(same colour) **(h)** Pr(different colours)

7 **(a)** If a 20-cent coin is tossed in the air twice, the probability of obtaining two heads is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{3}{4}$

(b) If a six-sided die is rolled twice, the probability of obtaining a double 6 is:

A $\frac{1}{3}$ **B** $\frac{1}{6}$ **C** $\frac{1}{12}$ **D** $\frac{1}{36}$

(c) If a 20-cent coin is tossed and a six-sided die is rolled, the probability of getting a head and a multiple of 3 is:

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{2}$

Understanding

8 Joshua and Tania are competing in different sections of a chess tournament. Joshua estimates that his chance of winning against his opponent is 3 in 5, whereas Tania estimates that her chance of winning against her opponent is 5 in 7. Find the probability that:

(a) both Joshua and Tania win their chess game

(b) they both lose their chess game

(c) Joshua loses his game but Tania wins hers.

9 A drawer contains 6 black socks, 8 red socks and 4 white socks. If two socks are drawn without replacement, find:

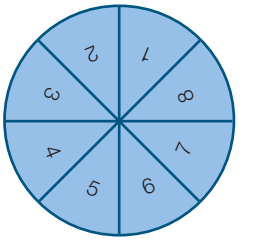
(a) Pr(2 black socks) **(b)** Pr(2 red socks)

(c) Pr(2 white socks) **(d)** Pr(a pair of socks of matching colour)

(e) Pr(two odd socks) **(f)** Pr(one black sock)

(g) Pr(at least one black sock) **(h)** Pr(no white socks)

10 A spinner numbered 1 to 8 is spun twice. Find the probability that:



(a) a 5 is obtained on the first spin

(b) a 5, then a 7, is spun

(c) a 5 is obtained on the first spin and a number less than 4 is obtained on the second

(d) an even number is obtained on the first spin and an odd number is obtained on the second.

11 If the probability that person A will be alive in 20 years is 0.7 and the probability that person B will be alive in 20 years is 0.5, find:

(a) the probability that they will both be alive in 20 years

(b) the probability that only one is alive in 20 years.

12 Natalie and her team play in a weekly squash competition. The probability that her team wins in any given week is 0.7. The outcome is not affected by any previous wins or losses. Find the probability that:

(a) Natalie's team does not win in any given week

(b) her team wins 2 weeks running

(c) her team loses 3 weeks running

(d) her team loses 4 weeks running.

13 A bag contains 6 white marbles, 5 black marbles and 4 green marbles. In each draw, a marble is drawn from the bag and not replaced. In three draws, find:

(a) Pr(WBG in that order) **(b)** Pr(WBG in any order).

Reasoning

14 Two normal unbiased six-sided dice are rolled. These events are defined:

A = rolling a 6 with the first die B = rolling a 5 with the second die

C = rolling a total of 8 D = rolling a total greater than 10

(a) Write the sample space for this experiment.

(b) Find the probability for each event.

(c) How many pairs of events are possible when order is not important? List them.

(d) How can you check which pairs are independent?

(e) Which, if any, of these pairs of events are independent?

15 You know that two events, A and B, are independent and that $\Pr(A) = 0.3$ and $\Pr(A \text{ and } B) = 0.15$.

(a) Find $\Pr(B)$.

(b) Now find $\Pr(A \text{ or } B)$. Explain how you obtained this value.

16 A secondary school has 900 students, of whom 475 are female and 870 study mathematics. There are 470 female mathematics students. If a student is picked at random, show that the events 'picking a female student' and 'picking a maths student' are not independent.

17 For two events, A and B, we are given $\Pr(A) = 0.35$ and $\Pr(B) = 0.45$. Find $\Pr(A \text{ and } B)$ if:

(a) A and B are mutually exclusive

(b) A and B are independent.

Open-ended

18 You have an unbiased ten-sided die, numbered 1 to 10. Write at least five questions related to independent events that deal with this die. Include the answers for your questions.

19 **(a)** Give two examples of pairs of dependent events.

(b) Give two examples of pairs of independent events.

20 A punter placed a $5000 bet on his local team to win $200 000 if the local side, who had only won one game in the last two seasons, finished on top of the ladder at the end of the season. After laying the bet the punter donated $100 000 to the local club to pay for three new star players. At the end of the season the bookmaker refused to pay because the punter had influenced the probability of the team finishing on top. What do you think?

Problem solving

Winning the lottery

In a particular raffle you purchase a ticket that has a unique six-digit number from 000001 to 999999 printed on it. The winning number is drawn at random and the ticket with that number printed on it wins the first prize. In this case, the prize is $100 000. Any ticket that has the last five of the winning digits in order in the last five positions in their number wins second prize ($20 000). Any ticket with the last four digits in order in the last four positions wins third prize ($10 000), and any ticket with the last three digits in order in the last three positions wins fourth prize ($2000). Each ticket can claim only one prize.

1 What is the probability that you win:

(a) *$100 000?* **(b)** *$20 000?*

(c) *$10 000?* **(d)** *$2000?*

2 What is the total payout on the lottery, assuming every prize is claimed?

3 How much do the tickets have to sell for so that the organisers make a profit?

Strategy options

- Break problem into manageable parts.
- Have I seen a similar problem?



Chapter review 11

D.I.Y. Summary

Key Words

Addition Law for Mutually Exclusive Events	independent events	sample space
complementary events	long-run proportion	tree diagram
conditional statement	mutually exclusive	two-way table
dependent events	probability	universal set
event	probability tree diagram	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 Every element under consideration in a particular question is called the _____.

2 If two events, A and B, are _____, then $\Pr(A) \times \Pr(B) = \Pr(A \text{ and } B)$.

3 The events 'having brown hair' and 'not having brown hair' are an example of _____.

4 When we put the probabilities on each of the branches of a tree diagram we then have a _____.

5 Events that cannot occur at the same time are said to be _____.

6 A _____ or a _____ can be used to help find the sample space.

7 Spinning a spinner that has been equally divided into four pieces labelled 1, 2, 3 and 4, then calculating $\Pr(1) + \Pr(2) + \Pr(3) + \Pr(4) = 1$ is an example of the _____.

8 Words and phrases such as 'given', 'knowing that' and 'if … then' are indications we are dealing with a _____.

Fluency

1 A spinner numbered 1 to 5 is spun. Find: Ex. 11.1

(a) Pr(spinning a 2) **(b)** Pr(not spinning a 2)

(c) Pr(spinning a number greater than 2) **(d)** Pr(spinning an even number)

(e) Pr(spinning an odd number) **(f)** Pr(spinning a number less than 5).

2 A box contains 15 blocks. Seven are red, five are white and the rest are blue. A block is chosen at random from the box. Find the probability that the block chosen is: Ex. 11.1

(a) blue **(b)** not red **(c)** red or blue.

3 A fair six-sided die is rolled. What is the probability of obtaining an even number or a 5? Ex. 11.1

4 A spinner numbered 1 to 4 is spun twice. Ex. 11.1

(a) Represent all the possible outcomes using a two-way table.

(b) Find:

(i) Pr(spinning a pair of 3s) **(ii)** Pr(spinning only one 3)

(iii) Pr(not spinning any 3s) **(iv)** Pr(the sum of the two numbers obtained is 5).

5 A 50-cent coin is tossed in the air twice. Find the probability of obtaining two tails.

Ex.11.1

6 This two-way table shows the number of elements in each of three sets.

Ex.11.2

	A and *B*	*A* and not *B*	not *A* and *B*	not *A* and not *B*	
C	1	4	3	2	**10**
not *C*	2	1	2	3	**8**
	3	**5**	**5**	**5**	**18**

Write down the number of elements in:

(a) the universal set **(b)** A

(c) A or B **(d)** B and C

(e) not B **(f)** not A and not B and not C.

7 A cutlery rack contains 6 knives, 5 forks and 2 spoons.

Ex.11.3

(a) The probability that a piece of cutlery drawn at random will be a knife or a spoon is:

A $\frac{2}{13}$ **B** $\frac{2}{5}$ **C** $\frac{6}{13}$ **D** $\frac{8}{13}$

(b) The probability that a piece of cutlery drawn at random will not be a knife is:

A $\frac{6}{13}$ **B** $\frac{7}{13}$ **C** $\frac{6}{7}$ **D** $\frac{7}{6}$

8 Two cards are drawn, without replacement, from a set of 20 cards numbered 1 to 20.

Ex.11.4

Let P = 'a prime number is drawn' and N = 'a prime number is not drawn'

(a) Draw a tree diagram to show this information.

(b) Find Pr(both cards show a prime number).

(c) Find Pr(no cards show a prime number).

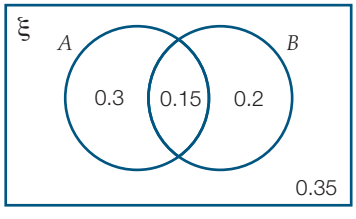
(d) Find Pr(at least one card shows a prime number).

9 Use the information in the Venn diagram to find the following probabilities. Where necessary, give your answers correct to four decimal places.

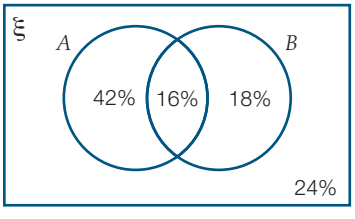
Ex.11.5

(i) Pr(A given B) **(ii)** Pr(B given A)

(a)



(b)



10 John and Petula are in the same class at school and both have trouble getting to school on time. John is late 2 days out of 9, whereas Petula is late 3 days out of 11. What is the probability that they will both be late on any particular day?

Ex.11.6

Understanding

11 A fair seven-sided spinner, numbered 1 to 7, is spun. Find the probability of:

(a) obtaining a 5 **(b)** obtaining an even number

(c) obtaining a number less than 4 **(d)** not obtaining a 6

(e) obtaining a multiple of 3 **(f)** not obtaining a prime number.

12 In clay pigeon shooting, a clay block in the shape of a pigeon is used as a target for shooting. Jackie estimates her chance of hitting the moving clay target with one shot to be 0.75.

(a) What is the probability of her not hitting the target with one shot?

(b) Draw a probability tree diagram to represent the possible outcomes of shooting at three clay targets.

(c) Find the probability that Jackie hits:

(i) all three clay targets **(ii)** only one clay target **(iii)** at least one clay target.

13 In a survey of 60 people, it was found that 40 people liked lemonade, whereas 30 people liked cola. Two people said they disliked both.

(a) Present this information in a two-way table.

(b) If one person is chosen at random, find the probability that this person:

(i) likes lemonade **(ii)** likes lemonade only

(iii) likes cola **(iv)** likes cola only

(v) likes both lemonade and cola **(vi)** does not like lemonade

(vii) likes lemonade or cola **(viii)** likes neither cola nor lemonade.

14 These probabilities apply to eye colour for students at a particular school. The colours are all mutually exclusive.

$\Pr(\text{green}) = \frac{3}{16}$ $\Pr(\text{blue}) = \frac{1}{4}$ $\Pr(\text{hazel}) = \frac{3}{8}$ $\Pr(\text{brown}) = \frac{3}{16}$

A student is chosen at random. What is the probability that he or she has:

(a) blue or green eyes **(b)** neither blue nor brown eyes

(c) blue, hazel or brown eyes **(d)** neither hazel nor blue eyes?

15 Nerida and Ngaio play each other at badminton every week. The probability that Nerida wins any particular game is 0.65. Find, rounded to three decimal places where necessary, the probability that:

(a) Ngaio wins in any particular week

(b) Nerida wins for 3 weeks in a row

(c) Ngaio wins for 4 weeks in a row

(d) over a 6-week period the winning sequence is Nerida, Ngaio, Nerida, Ngaio, Nerida, Ngaio.

16 You toss three coins: a 10-cent, a 20-cent and a 50-cent. You note the results in that order.

(a) Find the probability of obtaining:

(i) three heads **(ii)** two heads

(iii) one head **(iv)** no heads.

(b) What do the answers in part **(a)** add up to? Is this an expected result? Explain your reasoning.

(c) Without doing any further calculations, explain how you can find the probability of each number of tails.

(d) Find the probability of getting:

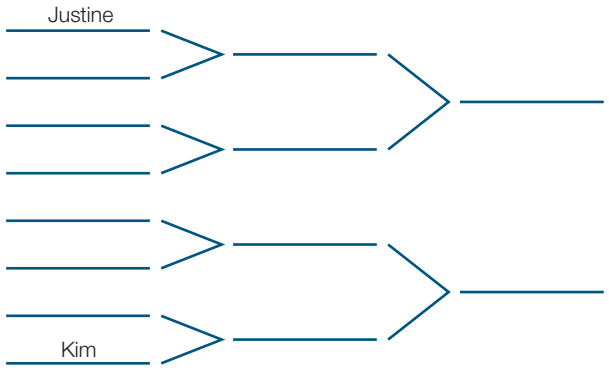
(i) HHT **(ii)** HTH **(iii)** THH.

(e) Write a statement about what you have found in part **(d)**.

Reasoning

17 Justine and Kim are entered for a tennis tournament that begins with eight players. The draw profile looks like the following (because they are so good Justine and Kim are never scheduled to play until the final).

Justine and Kim are very confident players and they think they have a probability of $\frac{2}{3}$ of beating any other player except for each other. If they play each other, Kim has a $\frac{3}{5}$ chance of winning. Find the following probabilities.



(a) The two girls play each other in the final.

(b) Kim wins the tournament (not necessarily against Justine).

(c) Justine wins the tournament (not necessarily against Kim).

18 A weather forecast predicting the chance of rain in the next 24 hours in some capital cities is recorded below.

City	Brisbane	Sydney	Canberra	Melbourne	Hobart	Adelaide	Perth	Darwin
Chance of rain	0.15	0.4	0.3	0.1	0.2	0.4	0.4	0.1

Assuming the events to be independent, find:

(a) Pr(rain in Melbourne and Brisbane)

(b) Pr(rain in Sydney and Perth)

(c) Pr(rain in Adelaide, Hobart and Perth)

(d) Pr(rain in Sydney and Canberra).

Long-term statistics are kept and found that the probability of rain in both Sydney and Canberra on the same day is actually 0.2.

(e) Explain the difference between your answer to **(d)** and the long-term probability.

12 Surds and logarithms

10A

Recall 12

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R12.1

1 Write in mathematical symbols:

(a) the square root of 3

(b) the cube root of 5

(c) the number that when squared equals 7

(d) the number that when cubed equals 6

(e) the number that when raised to the fourth power equals 10.

Worksheet R12.2

2 Evaluate each of the following. Do not use a calculator.

(a) $\sqrt{4}$ **(b)** $\sqrt{36}$ **(c)** $\sqrt{9}$ **(d)** $\sqrt{1}$ **(e)** $\sqrt[3]{8}$

(f) $\sqrt[3]{64}$ **(g)** $\sqrt[3]{-27}$ **(h)** $\sqrt[3]{1}$ **(i)** $\sqrt[4]{16}$ **(j)** $\sqrt[5]{-1}$

Worksheet R12.3

3 Use a calculator to evaluate each of the following, correct to two decimal places where necessary.

(a) $\sqrt{11}$ **(b)** $-\sqrt{7}$ **(c)** $\sqrt[3]{15}$ **(d)** 34^2 **(e)** 3^9

Worksheet R12.4

4 Expand each of the following.

(a) $3(x + 4)$ **(b)** $-4(2y - 3)$

Worksheet R12.5

5 Simplify each of the following.

(a) $m^3 \times m^2$ **(b)** $3k^2p^4 \times 4kp^5$ **(c)** $\dfrac{x^3 \times x^6}{x^2}$

(d) $3x^0$ **(e)** $(m^3n)^4$ **(f)** $(x^{-3}y^2)^{-2}$

Worksheet R12.6

6 Expand and simplify each of the following.

(a) $(x - y)(x + y)$ **(b)** $(c - 9)(c + 9)$ **(c)** $(2p + 3)(2p - 3)$

After completing this chapter you will be able to:

- add, subtract and multiply simple surd expressions
- carry out simple calculations and simplifications involving surds
- rationalise the denominator of a surd expression
- apply the index laws to numerical and algebraic expressions, evaluating and simplifying where necessary
- understand the relationship between exponential and logarithmic expressions
- apply the logarithmic laws to numerical and algebraic expressions.

Rational and irrational numbers

12.1

Need to Know

Real numbers are numbers that can be shown on a continuous number line. They consist of all **rational numbers** combined with all **irrational numbers**.

- A rational number can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Rational numbers include whole numbers, fractions and repeating decimals.
- An irrational number cannot be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Irrational numbers include all surds and other special numbers such as π. When represented in decimal form an irrational number has an infinite number of non-repeating decimal places.

Rational numbers

- Integers

  $8 = \frac{8}{1}$, $\sqrt{4} = 2 = \frac{2}{1}$, $-15 = -\frac{15}{1}$
- Terminating decimals

  $0.8 = \frac{4}{5}$, $1.23 = \frac{123}{100}$, $0.0006 = \frac{6}{10\,000}$
- Recurring decimals

  $0.3\ldots = \frac{1}{3}$, $0.636\,363\ldots = \frac{7}{11}$

Irrational numbers

- **Surds**

  $-\sqrt{3}$, $\sqrt[3]{7}$, $4\sqrt{19}$
- Other infinite non-recurring decimals

  $\pi = 3.141\,592\ldots$

Rational numbers can be expressed as a fraction $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Irrational numbers cannot be expressed as a fraction $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Worked Example 1

WE1

For each of the following numbers, state whether it is rational or irrational.

(a) $\sqrt{12}$ **(b)** $\sqrt{361}$

Working

(a) 12 is not a perfect square, so $\sqrt{12}$ is irrational.

(b) $361 = 19^2$, so $\sqrt{361}$ is rational.

Worked Example 2

WE2

Evaluate the following. Do not use a calculator.

(a) $-7\sqrt{81}$ **(b)** $3\sqrt{100} - 2\sqrt{16} + \sqrt{9}$

Working

(a) $-7\sqrt{81}$
$= -7 \times 9$
$= -63$

(b) $3\sqrt{100} - 2\sqrt{16} + \sqrt{9}$
$= 3 \times 10 - 2 \times 4 + 3$
$= 30 - 8 + 3$
$= 25$

Worked Example 3

WE3

Use a calculator to evaluate each of the following, correct to three decimal places.

(a) $\sqrt{3} + \sqrt{5}$ **(b)** $-7 \times \sqrt[3]{9}$

Working

(a) $\sqrt{3} + \sqrt{5} \approx 3.968\,118\,785$
$= 3.968$ (3 d.p.)

(b) $-7 \times \sqrt[3]{9} \approx -14.560\,586\,76$
$= -14.561$ (3 d.p.)

Answers page 579

12.1 Rational and irrational numbers

Equipment required: calculator for Questions **1**, **3**, **4** and **7**

Fluency

WE1

1 For each of the following numbers, state whether it is rational or irrational.

(a) $\sqrt{5}$ **(b)** $\sqrt{36}$ **(c)** $-\frac{2}{7}$ **(d)** $\sqrt{10}$
(e) 4.6 **(f)** $\sqrt[3]{4}$ **(g)** $-\sqrt[3]{2}$ **(h)** $\sqrt[4]{1}$
(i) 2π **(j)** $\sqrt{20}$ **(k)** $-\sqrt{3}$ **(l)** 8.621

WE2

2 Evaluate the following. Do not use a calculator.

(a) $\sqrt{64}$ **(b)** $-\sqrt{25}$ **(c)** $-\sqrt{36}$
(d) $3\sqrt{9}$ **(e)** $7\sqrt{36}$ **(f)** $-4\sqrt{49}$
(g) $\sqrt{25 + 144}$ **(h)** $\sqrt{9} + \sqrt{16}$ **(i)** $7\sqrt{49} + 4\sqrt{25}$
(j) $3\sqrt{16} + 2\sqrt{81}$ **(k)** $5\sqrt{36} - 7\sqrt{4}$ **(l)** $9\sqrt{100} - 8\sqrt{16} + 5\sqrt{9}$

3 Use a calculator to find the approximate value of each of the following surds, correct to three decimal places where necessary. WE3

(a) $\sqrt{3}+\sqrt{2}$ **(b)** $\sqrt{8}-\sqrt{5}$ **(c)** $\sqrt{7}-\sqrt{13}$

(d) $\sqrt{35}$ **(e)** $-\sqrt{3}$ **(f)** $\sqrt[3]{17}$

(g) $\sqrt[3]{10}$ **(h)** $\sqrt{6}$ **(i)** $-\sqrt[3]{72}$

4 (a) Which one of the following is a surd?

A $\sqrt{9}$ **B** $-\sqrt{1}$ **C** $\sqrt{8}$ **D** $\sqrt[3]{8}$

(b) Which one of the following is *not* a surd?

A $\sqrt[3]{7}$ **B** $\sqrt{11}$ **C** $\sqrt[3]{16}$ **D** $\sqrt{25}$

(c) Which one of the following is a rational number?

A $\sqrt{3}$ **B** $\sqrt[3]{2}$ **C** $\sqrt[3]{4}$ **D** $\sqrt{4}$

(d) Which one of the following is *not* a rational number?

A $\frac{1}{3}$ **B** 1.45 **C** 2.717 171 … **D** 3.721 452 …

5 (a) $4\sqrt{3}$ is the same as:

A $4+\sqrt{3}$ **B** $4-\sqrt{3}$ **C** $4\times\sqrt{3}$ **D** $\sqrt[4]{3}$

(b) $8\sqrt{16}$ is the same as:

A 8×4 **B** $8\div4$ **C** $8+4$ **D** 8×16

6 Do not use a calculator for this question.

(a) $\sqrt{7}$ is between $\sqrt{4}$ and $\sqrt{9}$, so $\sqrt{7}$ is between:

A 1 and 2 **B** 2 and 3 **C** 3 and 4 **D** 4 and 5

(b) $\sqrt{31}$ is between:

A 2 and 3 **B** 3 and 4 **C** 4 and 5 **D** 5 and 6

(c) $\sqrt[3]{11}$ is between:

A 1 and 2 **B** 2 and 3 **C** 3 and 4 **D** 4 and 5

Understanding

7 Use a calculator to find the approximate value of each of the following, correct to three decimal places.

(a) $\sqrt{3+2}$ **(b)** $\sqrt{8-5}$ **(c)** $2\sqrt{6}+\sqrt{10}$

(d) $3\sqrt{2}+\sqrt{8}$ **(e)** $4\sqrt{5}-3\sqrt{6}$ **(f)** $6\sqrt{3}-6\sqrt{7}$

(g) $\sqrt[3]{-14}$ **(h)** $-\sqrt{43}$ **(i)** $2\sqrt{3}$

(j) $6\sqrt{5}$ **(k)** $-3\sqrt{11}$ **(l)** $5\times6\sqrt{5}$

Reasoning

8 Find the exact side length of a square with area:

(a) 25 m^2 **(b)** 36 cm^2 **(c)** 46 mm^2 **(d)** 17 m^2

9 The area of a square vegetable garden is 60 m^2.

(a) What is the exact length of each side of the vegetable garden?

(b) If a fence of chicken wire is to be constructed around the vegetable garden, what length (to the nearest metre) of chicken wire is needed?

(c) Why do you think the measurement was to the nearest metre?

10 Find the exact side length of a cube with volume:

(a) $8\ m^3$ **(b)** $27\ m^3$ **(c)** $35\ cm^3$ **(d)** $15\ cm^3$

11 Two identical dice have a combined volume of $14\ cm^3$.

(a) Find the exact side length of each die.

(b) What is the side length of each die to the nearest mm?

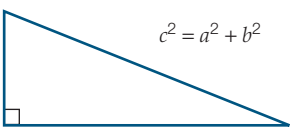
Open-ended

12 **(a)** Write three irrational numbers greater than zero.

(b) Write three irrational numbers less than zero.

(c) Draw a number line and mark the approximate places of these numbers.

13 Pythagoras' theorem is used to determine the side lengths of right-angled triangles. Explain how to cut a piece of string exactly $\sqrt{2}$ m long using a pencil, a metre ruler and a piece of string.

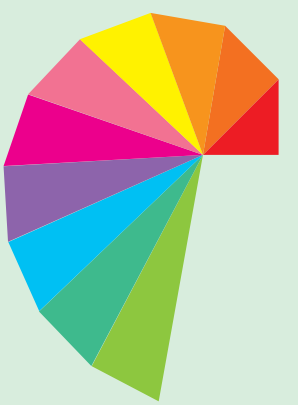


Problem solving

Surd shells

Equipment required: graph paper

A spiral can be created using straight lines to form a spiral made with ever increasing right-angled triangles. A very special spiral is created when starting with an isosceles right-angled triangle with equal sides of length 1 cm. Each subsequent triangle has a height of 1 cm and a base that is the length of the hypotenuse of the previous triangle.



(a) What is the perimeter of the tenth right-angled triangle?

(b) Calculate the exact perimeter of the entire spiral.

(c) Does the entire spiral have a rational or an irrational perimeter?

(d) Draw the spiral on graph paper. Measure the length of the perimeter of the entire spiral to check your answer to **(b)**.

(e) The first triangle has a rational area of $\frac{1}{2}\ cm^2$.

(i) Which is the next triangle to have a rational area?

(ii) Find this area and compare it to the area of the first triangle.

(iii) Find the next triangle to have a rational area.

(iv) Find this area and compare it to the area of the first triangle.

Strategy options

- Look for a pattern.
- Make a table.

Multiplying and dividing surds

12.2

Need to Know

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} = \sqrt{ab}$, where a and b are positive numbers.

$\sqrt{12}$ is an **entire surd** because the entire number is under the square root sign. $2\sqrt{3}$ is considered to be simpler than $\sqrt{12}$ as it contains a smaller surd.

In general, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where a and b are positive numbers and $b \neq 0$.

Worked Example 4

WE4

Simplify:

(a) $\sqrt{2} \times \sqrt{12}$

(b) $4\sqrt{3} \times 5\sqrt{7}$

Working

(a) $\sqrt{2} \times \sqrt{12}$

$= \sqrt{2 \times 12}$

$= \sqrt{24}$

$= \sqrt{4 \times 6}$

$= \sqrt{4} \times \sqrt{6}$

$= 2\sqrt{6}$

(b) $4\sqrt{3} \times 5\sqrt{7}$

$= 4 \times 5 \times \sqrt{3} \times \sqrt{7}$

$= 20\sqrt{21}$

Worked Example 5

WE5

Simplify:

(a) $\sqrt{21} \div \sqrt{7}$

(b) $\frac{12\sqrt{40}}{3\sqrt{5}}$

Working

(a) $\sqrt{21} \div \sqrt{7}$

$= \frac{\sqrt{21}}{\sqrt{7}}$

$= \sqrt{\frac{21}{7}}$

$= \sqrt{3}$

(b) $\frac{12\sqrt{40}}{3\sqrt{5}}$

$= \frac{12}{3} \times \sqrt{\frac{40}{5}}$

$= 4\sqrt{8}$

$= 4 \times \sqrt{4 \times 2}$

$= 4 \times \sqrt{4} \times \sqrt{2}$

$= 4 \times 2 \times \sqrt{2}$

$= 8\sqrt{2}$

Answers page 580

12.2 Multiplying and dividing surds

Fluency

WE4

1 Simplify each of the following.

(a) $\sqrt{3} \times \sqrt{2}$ (b) $\sqrt{11} \times \sqrt{5}$ (c) $\sqrt{7} \times \sqrt{10}$

(d) $\sqrt{7} \times \sqrt{7}$ (e) $\sqrt{4} \times \sqrt{4}$ (f) $\sqrt{16} \times \sqrt{4}$

(g) $\sqrt{36} \times \sqrt{2}$ (h) $\sqrt{25} \times \sqrt{7}$ (i) $\sqrt{2} \times \sqrt{9}$

(j) $2\sqrt{3} \times 5\sqrt{2}$ (k) $\sqrt{11} \times 4\sqrt{3}$ (l) $5 \times 8\sqrt{2}$

(m) $7\sqrt{5} \times \sqrt{5}$ (n) $\sqrt{6} \times 4\sqrt{6}$ (o) $5\sqrt{2} \times 4\sqrt{2}$

(p) $3\sqrt{7} \times 2\sqrt{7}$ (q) $5\sqrt{3} \times 3\sqrt{2}$ (r) $8\sqrt{6} \times 3\sqrt{12}$

WE5

2 Simplify each of the following.

(a) $\sqrt{\frac{27}{3}}$ (b) $\sqrt{\frac{32}{2}}$ (c) $\frac{\sqrt{125}}{\sqrt{5}}$

(d) $\frac{\sqrt{60}}{\sqrt{15}}$ (e) $\sqrt{72} \div \sqrt{2}$ (f) $\sqrt{45} \div \sqrt{5}$

(g) $\sqrt{7} \div \sqrt{7}$ (h) $\sqrt{\frac{64}{16}}$ (i) $\sqrt{25} \div \sqrt{9}$

(j) $\sqrt{\frac{100}{49}}$ (k) $\frac{\sqrt{4}}{\sqrt{81}}$ (l) $\sqrt{\frac{36}{121}}$

(m) $\sqrt{4} \div \sqrt{64}$ (n) $\sqrt{3} \div \sqrt{27}$ (o) $\frac{2\sqrt{36}}{3}$

(p) $\frac{5\sqrt{49}}{7}$ (q) $\frac{5\sqrt{9}}{3\sqrt{25}}$ (r) $\frac{6\sqrt{9}}{9\sqrt{36}}$

(s) $\frac{18\sqrt{75}}{3\sqrt{3}}$ (t) $5\sqrt{20} \div \sqrt{5}$ (u) $18\sqrt{48} \div 3\sqrt{8}$

(v) $\frac{5\sqrt{30}}{25\sqrt{6}}$ (w) $6\sqrt{14} \div 15\sqrt{2}$ (x) $12\sqrt{30} \div 18\sqrt{10}$

3 Simplify each of the following.

(a) $\sqrt{27}$ (b) $\sqrt{108}$ (c) $\sqrt{162}$ (d) $\sqrt{96}$

(e) $\sqrt{99}$ (f) $\sqrt{500}$ (g) $\sqrt{486}$ (h) $\sqrt{405}$

(i) $5\sqrt{8}$ (j) $-4\sqrt{24}$ (k) $-5\sqrt{63}$ (l) $4\sqrt{243}$

4 Express each of the following as an entire surd.

(a) $2\sqrt{3}$ (b) $3\sqrt{7}$ (c) $6\sqrt{2}$ (d) $3\sqrt{3}$

(e) $-10\sqrt{6}$ (f) $-2\sqrt{10}$ (g) $-8\sqrt{3}$ (h) $5\sqrt{7}$

(i) $8\sqrt{2}$ (j) $7\sqrt{5}$ (k) $-6\sqrt{3}$ (l) $-5\sqrt{11}$

Hint

Leave the negative sign outside the square root sign.

5 (a) $\sqrt{32}$ written in simplest form is:

A $2\sqrt{4}$ B $2\sqrt{8}$ C $4\sqrt{2}$ D $4\sqrt{8}$

(b) $\sqrt{3} \times \sqrt{7}$ simplifies to:

A $\sqrt{10}$ B $\sqrt{21}$ C $\sqrt{37}$ D $3\sqrt{7}$

(c) $\frac{\sqrt{49}}{\sqrt{4}}$ simplifies to:

A $\frac{2}{7}$ B $\frac{7}{4}$ C $\frac{7}{2}$ D 14

(d) $3\sqrt{5} \times 2\sqrt{6}$ simplifies to:

A $5\sqrt{11}$ B $6\sqrt{11}$ C $5\sqrt{30}$ D $6\sqrt{30}$

(e) $\frac{15\sqrt{6}}{3\sqrt{2}}$ simplifies to:

A $\sqrt{15}$ B $5\sqrt{3}$ C $5\sqrt{4}$ D $12\sqrt{3}$

Understanding

6 Simplify each of the following.

(a) $3\sqrt{6} \times 5\sqrt{3}$ (b) $4\sqrt{2} \times 2\sqrt{14}$ (c) $-2\sqrt{6} \times 7\sqrt{10}$

(d) $-4\sqrt{8} \times 2\sqrt{10}$ (e) $-5\sqrt{3} \times \sqrt{15}$ (f) $10\sqrt{2} \times 3\sqrt{8}$

(g) $2\sqrt{12} \times 4\sqrt{3}$ (h) $2\sqrt{18} \times 3\sqrt{12}$ (i) $4\sqrt{27} \times 7\sqrt{28}$

(j) $-2\sqrt{20} \times -5\sqrt{75}$ (k) $-3\sqrt{72} \times -4\sqrt{63}$ (l) $\sqrt{54} \times \sqrt{162}$

7 Simplify each of the following.

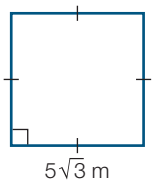
(a) $\sqrt{\frac{24}{3}}$ (b) $\sqrt{\frac{54}{2}}$ (c) $3\sqrt{\frac{90}{2}}$ (d) $\frac{-6\sqrt{120}}{\sqrt{5}}$

(e) $\frac{\sqrt{27}}{\sqrt{4}}$ (f) $\frac{\sqrt{32}}{\sqrt{25}}$ (g) $\frac{9\sqrt{8}}{\sqrt{9}}$ (h) $\frac{15\sqrt{28}}{2\sqrt{25}}$

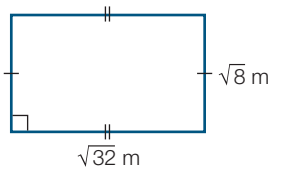
(i) $\sqrt{\frac{125}{36}}$ (j) $-3\sqrt{\frac{44}{81}}$ (k) $-2\sqrt{\frac{27}{100}}$ (l) $\frac{4}{5}\sqrt{\frac{147}{64}}$

8 Calculate the area of the following shapes, writing the answer as a surd in its simplest form.

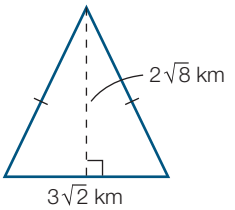
(a)



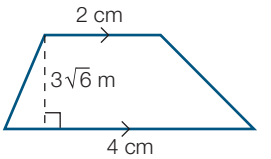
(b)



(c)



(d)



Hint

Area formulas:

Square: $A = l^2$

Rectangle: $A = lw$

Triangle: $A = \frac{bh}{2}$

Trapezium: $A = \frac{(a+b)h}{2}$

9 Given $\sqrt{a} \times \sqrt{b} = 3\sqrt{c}$, what are possible values of b and c if $a = 2$?

Reasoning

Hint

Exact means write in surd form if necessary.

10 (a) Chris calculated the side of a triangle and found it was $\sqrt{41}$ mm. His friends' calculations gave lengths of $3\sqrt{5}$ mm and $2\sqrt{10}$ mm. If the actual length was 6 mm, which measurement was closest to this?

(b) If the area of a circular swimming pool is 45π m^2, find its *exact* circumference.

11 Adrianna has $\sqrt{50}$ m of tape that is to go across the two diagonals of rectangular windows that have side lengths 40 cm and 35 cm.

(a) What is the length of the diagonal?

(b) How many windows can she tape? (She may have some tape left over.)

Open-ended

12 Find at least three surd calculations that simplify to $2\sqrt{6}$.

13 (a) Andrew multiplied two surds and got the answer 5. What could the two surds have been?

(b) Fiona divided two surds and got the answer 4. What could the two surds have been?

12.3 Adding and subtracting surds

Need to Know

Numbers with the same surd are called *like* surds. Only terms that contain like surds can be added or subtracted.

Worked Example 6

WE6

Simplify:

(a) $10\sqrt{3} - 4\sqrt{3}$

(b) $3\sqrt{5} + 2\sqrt{6} + \sqrt{5}$

Working

(a) $10\sqrt{3} - 4\sqrt{3}$
$= 6\sqrt{3}$

(b) $3\sqrt{5} + 2\sqrt{6} + \sqrt{5}$
$= 4\sqrt{5} + 2\sqrt{6}$

Worked Example 7

WE7

Simplify:

(a) $\sqrt{18} + \sqrt{8}$

(b) $2\sqrt{12} - \sqrt{20} - \sqrt{27} + 2\sqrt{45}$

Working

(a) $\sqrt{18} + \sqrt{8}$
$= \sqrt{9 \times 2} + \sqrt{4 \times 2}$
$= 3\sqrt{2} + 2\sqrt{2}$
$= 5\sqrt{2}$

(b) $2\sqrt{12} - \sqrt{20} - \sqrt{27} + 2\sqrt{45}$
$= 2\sqrt{4 \times 3} - \sqrt{4 \times 5} - \sqrt{9 \times 3} + 2\sqrt{9 \times 5}$
$= 2 \times 2\sqrt{3} - 2\sqrt{5} - 3\sqrt{3} + 2 \times 3\sqrt{5}$
$= 4\sqrt{3} - 2\sqrt{5} - 3\sqrt{3} + 6\sqrt{5}$
$= 4\sqrt{3} - 3\sqrt{3} - 2\sqrt{5} + 6\sqrt{5}$
$= \sqrt{3} + 4\sqrt{5}$

12.3 Adding and subtracting surds

Answers page 580

Fluency

1 Simplify each of the following.

WE6

(a) $3\sqrt{2} + 7\sqrt{2}$

(b) $8\sqrt{5} - 5\sqrt{5}$

(c) $11\sqrt{3} - 6\sqrt{3}$

(d) $\sqrt{7} + 7\sqrt{7}$

(e) $5\sqrt{3} - 9\sqrt{3}$

(f) $12\sqrt{11} - 35\sqrt{11}$

(g) $5\sqrt{2} - 3\sqrt{2} + 9\sqrt{2}$

(h) $8\sqrt{3} - 9\sqrt{3} + \sqrt{3}$

(i) $7\sqrt{5} - 13\sqrt{5} + 4\sqrt{5} - \sqrt{5}$

(j) $10\sqrt{6} + 3\sqrt{3} - 5\sqrt{6} - 8\sqrt{3}$

(k) $5\sqrt{2} - 7\sqrt{5} + 2\sqrt{5} + 2\sqrt{2}$

(l) $7\sqrt{3} + 6\sqrt{11} - 8\sqrt{3} - 10\sqrt{11}$

WE7

2 Simplify each of the following.

(a) $\sqrt{2} + \sqrt{18}$ **(b)** $\sqrt{28} - \sqrt{7}$

(c) $\sqrt{125} - 2\sqrt{5}$ **(d)** $7\sqrt{18} + 3\sqrt{50}$

(e) $5\sqrt{63} - 7\sqrt{28}$ **(f)** $4\sqrt{90} - 4\sqrt{160}$

(g) $\sqrt{40} + \sqrt{90} + \sqrt{160}$ **(h)** $2\sqrt{32} + 7\sqrt{2} + 4\sqrt{18}$

(i) $\sqrt{150} + 5\sqrt{54} - 6\sqrt{24}$ **(j)** $9\sqrt{48} - \sqrt{27} + 2\sqrt{192}$

(k) $6\sqrt{80} - 3\sqrt{180} - 4\sqrt{405}$ **(l)** $\sqrt{8} + \sqrt{162} + \sqrt{18} + \sqrt{50}$

(m) $\sqrt{28} - \sqrt{112} + \sqrt{567} - \sqrt{175}$ **(n)** $2\sqrt{300} - 6\sqrt{27} - 3\sqrt{108} + 4\sqrt{48}$

3 **(a)** $7\sqrt{3} - 2\sqrt{3}$ simplifies to:

A 5 **B** $5\sqrt{3}$ **C** $5\sqrt{6}$ **D** $9\sqrt{3}$

(b) $5\sqrt{5} - 3\sqrt{5}$ simplifies to:

A $2\sqrt{2}$ **B** $2\sqrt{5}$ **C** $2\sqrt{7}$ **D** $8\sqrt{12}$

(c) $5\sqrt{3} + 7\sqrt{3} + 3\sqrt{2}$ simplifies to:

A $15\sqrt{3}$ **B** $12\sqrt{6} + 3\sqrt{2}$ **C** $15\sqrt{8}$ **D** $12\sqrt{3} + 3\sqrt{2}$

4 **(a)** $\sqrt{3} + \sqrt{12}$ simplifies to:

A $\sqrt{15}$ **B** $2\sqrt{3}$ **C** $3\sqrt{3}$ **D** $5\sqrt{3}$

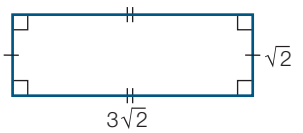
(b) $5\sqrt{2} - 2\sqrt{8}$ simplifies to:

A $3\sqrt{-6}$ **B** $-3\sqrt{2}$ **C** 1 **D** $\sqrt{2}$

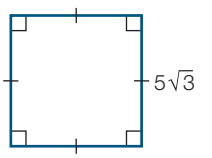
Understanding

5 Calculate the perimeter of the following shapes, writing your answer as a surd in its simplest form.

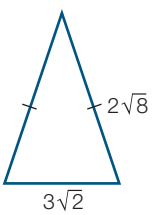
(a)



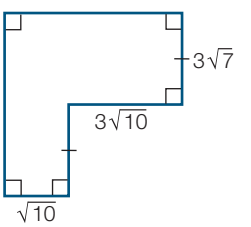
(b)



(c)

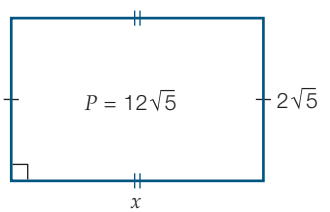


(d)

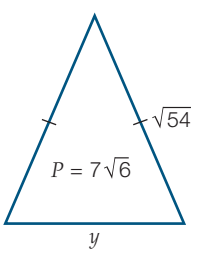


6 Find the value of the variable in each of the following shapes.

(a)



(b)



7 Find the exact area of a circle if its radius is $\sqrt{5}$ cm.

8 Find the exact area of a circular coin with diameter $\sqrt{3}$ cm.

Reasoning

9 **(a)** Write down the perimeter of the rectangle shown.

(b) What is the exact area of this rectangle?

(c) If this rectangle represents the available floor space of a bedroom, can the floor be covered by a piece of carpet with area 22 m^2?



Open-ended

10 Make up at least three questions, using addition and/or subtraction, where $3\sqrt{5}$ is the answer.

11 A rectangle has an area of $\sqrt{96}$ cm^2. What might its perimeter be?

12 Ruby's answers to a surds spot quiz are shown.

(i) Find the perimeter of a triangle with side lengths $2\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$.

$$\text{Perimeter} = 2\sqrt{3} + 2\sqrt{3} + 3\sqrt{3}$$
$$= 7\sqrt{9}$$
$$= 7 \times 3$$
$$\therefore P = 21$$

(ii) The perimeter of a rectangle is $8\sqrt{6}$. If the length of one pair of sides is $\sqrt{6}$, find the length of the other pair.

$$\text{Length of other pair of sides} = \frac{8\sqrt{6} - 2\sqrt{6}}{2}$$
$$= \frac{6}{2}$$
$$\therefore l = 3$$

(a) How would you explain to Ruby the mistake she made in each question?

(b) Write the correct answer for each question.

(c) Draw a diagram that could accompany each question.

12.4 Surds and the Distributive Law

Need to Know

To simplify surd expressions involving brackets we can use the **Distributive Law**:

$a(b + c) = ab + ac$ or $(a + b)(c + d) = ac + ad + bc + bd$

Worked Example 8

WE8

Expand and simplify:

(a) $2\sqrt{3}(5-\sqrt{3})$ **(b)** $(2\sqrt{2}+1)(\sqrt{6}-4\sqrt{3})$

Working

(a) $2\sqrt{3}(5-\sqrt{3})$
$= 10\sqrt{3} - 2\sqrt{9}$
$= 10\sqrt{3} - 2 \times 3$
$= 10\sqrt{3} - 6$

(b) $(2\sqrt{2}+1)(\sqrt{6}-4\sqrt{3})$
$= 2\sqrt{12} - 8\sqrt{6} + \sqrt{6} - 4\sqrt{3}$
$= 2 \times 2\sqrt{3} - 8\sqrt{6} + \sqrt{6} - 4\sqrt{3}$
$= 4\sqrt{3} - 8\sqrt{6} + \sqrt{6} - 4\sqrt{3}$
$= -7\sqrt{6}$

Answers page 581

12.4 Surds and the Distributive Law

Equipment required: calculator for Questions **5**, **6**, **7**, **8** and **9**

Fluency

WE8

1 Expand and simplify each of the following.

(a) $3(\sqrt{7}+2)$ **(b)** $-4(\sqrt{11}+\sqrt{3})$
(c) $9(2\sqrt{5}-\sqrt{2})$ **(d)** $-\sqrt{7}(5\sqrt{2}-3\sqrt{5})$
(e) $4\sqrt{2}(2\sqrt{5}+6\sqrt{3})$ **(f)** $3\sqrt{5}(\sqrt{2}-7\sqrt{3}+3\sqrt{7})$
(g) $\sqrt{3}(\sqrt{3}+2)$ **(h)** $5\sqrt{7}(3\sqrt{2}-2\sqrt{7})$
(i) $-3\sqrt{5}(2-8\sqrt{3}+\sqrt{5})$ **(j)** $3\sqrt{2}(\sqrt{14}-\sqrt{8})$
(k) $4\sqrt{3}(\sqrt{3}-\sqrt{6}+5)$ **(l)** $-4\sqrt{5}(1-2\sqrt{5}+\sqrt{15})$
(m) $(\sqrt{5}+4)(\sqrt{2}+3)$ **(n)** $(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{7})$
(o) $(\sqrt{2}+\sqrt{3})^2$ **(p)** $(4\sqrt{5}-2\sqrt{2})^2$
(q) $(2\sqrt{7}+\sqrt{3})(2\sqrt{7}-\sqrt{3})$ **(r)** $(3\sqrt{5}-7\sqrt{2})(3\sqrt{5}+7\sqrt{2})$

2 **(a)** $5(2\sqrt{5}-\sqrt{2})$ is the same as:
A $10\sqrt{5}-\sqrt{10}$ **B** $10\sqrt{5}-5\sqrt{2}$ **C** $10\sqrt{5}-\sqrt{2}$ **D** $10\sqrt{3}$

(b) $4\sqrt{6}(\sqrt{2}-\sqrt{3})$ is the same as:
A $4\sqrt{8}-4\sqrt{18}$ **B** $8\sqrt{6}-12\sqrt{2}$ **C** $4\sqrt{8}-12\sqrt{2}$ **D** $8\sqrt{3}-12\sqrt{2}$

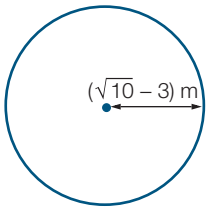
(c) $(2-\sqrt{5})(\sqrt{5}-1)$ expands to:
A $3\sqrt{5}+7$ **B** $3\sqrt{5}$ **C** $3\sqrt{5}-7$ **D** $3\sqrt{5}-3$

(d) $(\sqrt{7}+\sqrt{2})^2$ expands and simplifies to:
A $9-2\sqrt{14}$ **B** 9 **C** $9+2\sqrt{14}$ **D** $11\sqrt{14}$

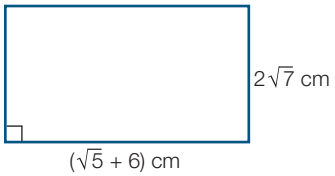
Understanding

3 Find the exact area of each shape.

(a)

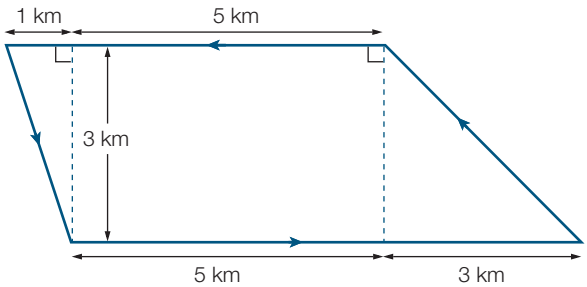


(b)



4 Find the exact area of a compact disc if it has a radius of $(4 + \sqrt{5})$ cm.

5 On a walk around the city, Luke followed the path shown in the diagram, finishing back where he started.



(a) Find the area enclosed by his path.

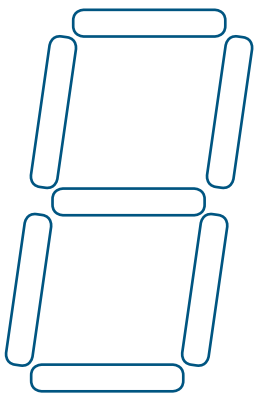
(b) Find the exact distance he walked.

(c) State the distance he walked, to the nearest metre.

Hint

What shape can you see?

6 On a scoreboard, tube lighting is set up so that any digit from 0 to 9 can be displayed as shown in the diagram at right.



(a) If each individual section of tubing is $(3\sqrt{2} - 2)$ m long, what exact length of tubing would be lit to produce the digit:

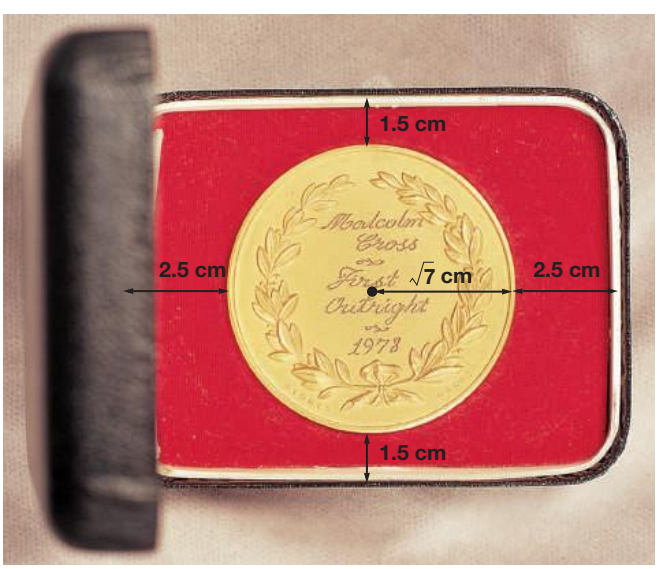
(i) 0 **(ii)** 4

(iii) 5 **(iv)** 8?

(b) Use your calculator to express each length found correct to two decimal places.



7 A sports medal is to be placed into a presentation box as shown. Give exact answers for parts **(a)** to **(d)**.



(a) What area of the base of the box does the medal take up?

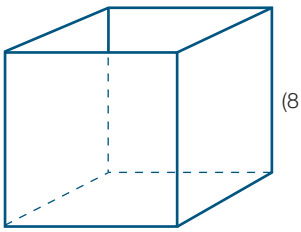
(b) What are the dimensions (length and width) of the base of the box?

(c) What is the area of the base of the box?

(d) To stop the medal moving freely in the box, a thick layer of velvet will surround it. What area of velvet will be seen around the medal?

(e) Express your answer to part **(d)** correct to two decimal places.

Reasoning



8 Zev has a toy box in the shape of a cube (open at the top). He wants to cover its surface (both inside and out) with adhesive-backed plastic and needs to know how much he should buy.

(a) What is the length of a side of the toy box, correct to one decimal place?

(b) Use your answer to part **(a)** to find the area to be covered with plastic. Assume that the box has walls of negligible thickness.

(c) Find the area to be covered with plastic using the value $8\sqrt{5} + 9\sqrt{11}$ cm and keeping your answer as a surd expression.

(d) Use a calculator to express your answer to part **(c)** correct to one decimal place.

(e) Compare your answers to parts **(b)** and **(d)**. Which answer should Zev use when purchasing the plastic?

9 Square biscuits are to be cut out of a length of biscuit dough. Each biscuit has a line of chocolate icing running along the 6 cm diagonal of the square.

(a) Use Pythagoras' theorem to find the exact side length of each biscuit.

(b) What exact length of biscuit dough would be needed to cut out a row of 50 biscuits? (Keep your answer in surd form.)

(c) Use a calculator to express your answer to part **(b)** correct to one decimal place.

(d) Use a calculator to express the side length of each biscuit correct to one decimal place.

(e) Use your answer to part **(d)** to calculate the length of biscuit dough needed to cut out a row of 50 biscuits.

(f) Compare your answers to parts **(c)** and **(e)**. If the biscuits are cut out by a machine that can be calibrated to $\frac{1}{100}$th of a centimetre, which answer should be used?

Open-ended

10 The area of a triangle is $\sqrt{48}$ cm^2. Draw at least two diagrams with different dimensions that could be the triangle.

11 The perimeter of a rectangle is given by $2(m\sqrt{n} + p)$.

(a) What might be the dimensions of the rectangle if the perimeter is greater than 20 m? Draw a diagram, showing lengths $m\sqrt{n}$ and p.

(b) Write an equation for the rectangle's area, using side lengths $m\sqrt{n}$ and p.

(c) State values of m, n and p so that the area of the rectangle is less than 10 m^2.

Problem solving

Connecting squares

The diagram below shows increasingly larger squares connecting to the smaller square along one edge.

(a) If the side lengths of the smallest squares are 1 cm, what is the area of the entire rectangle?

(b) This diagram is a spiral pattern that started with the yellow square. Complete the spiral pattern that has been started. If the pattern was continued, what would be the dimensions and area of the next square?

(c) Write down the side lengths of each square in ascending order. Have you seen this number pattern before? If so, what is its name?

(d) Explain the pattern and state the side lengths of the next five squares.

Strategy options

- Look for a pattern.
- Break problem into manageable parts.



12.5 Rationalising the denominator

Need to Know

Rationalising the denominator of a fraction changes the denominator to a rational number, but does not change the value of the fraction.

The denominator can be rationalised in two ways:

- Multiply the numerator and denominator by the smallest surd that will give a rational denominator (if the denominator is a surd only).
- Multiply the numerator and denominator by the conjugate of the denominator (if the denominator has rational and irrational elements).

$(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ are **conjugate surds**.
$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

Conjugate surds are used because their product does not contain a surd term.

Worked Example 9

WE9

Rationalise the denominator of:

(a) $\frac{3\sqrt{5}}{\sqrt{6}}$ **(b)** $\frac{2\sqrt{3}}{\sqrt{20}}$

Working

(a) $\frac{3\sqrt{5}}{\sqrt{6}}$

$= \frac{3\sqrt{5}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{3\sqrt{30}}{6}$

$= \frac{\sqrt{30}}{2}$

(b) $\frac{2\sqrt{3}}{\sqrt{20}}$

$= \frac{2\sqrt{3}}{2\sqrt{5}}$

$= \frac{2\sqrt{3}}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{15}}{2 \times 5}$

$= \frac{\sqrt{15}}{5}$

Worked Example 10

WE10

Rationalise the denominator of each of the following.

(a) $\frac{3}{3-\sqrt{2}}$ (b) $\frac{\sqrt{2}}{\sqrt{11}+3}$

Working

(a) $\frac{3}{3-\sqrt{2}}$

$= \frac{3}{(3-\sqrt{2})} \times \frac{(3+\sqrt{2})}{(3+\sqrt{2})}$

$= \frac{3(3+\sqrt{2})}{9-2}$

$= \frac{9+3\sqrt{2}}{7}$

(b) $\frac{\sqrt{2}}{\sqrt{11}+3}$

$= \frac{\sqrt{2}}{(\sqrt{11}+3)} \times \frac{(\sqrt{11}-3)}{(\sqrt{11}-3)}$

$= \frac{\sqrt{2}(\sqrt{11}-3)}{11-9}$

$= \frac{\sqrt{22}-3\sqrt{2}}{2}$

12.5 Rationalising the denominator

Answers page 581

Fluency

1 Express each of the following in simplest form with a rational denominator.

WE9

(a) $\frac{1}{\sqrt{7}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{-12}{\sqrt{6}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

(e) $\frac{5\sqrt{11}}{\sqrt{5}}$ (f) $3\sqrt{\frac{5}{6}}$ (g) $-\frac{3}{4\sqrt{7}}$ (h) $\frac{6\sqrt{5}}{7\sqrt{3}}$

2 Express each of the following in simplest form with a rational denominator.

WE10

(a) $\frac{1}{3-\sqrt{7}}$ (b) $\frac{4}{2+\sqrt{3}}$ (c) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}}$ (d) $\frac{2}{2\sqrt{7}+5}$

(e) $\frac{6+\sqrt{3}}{2\sqrt{3}-3}$ (f) $\frac{\sqrt{5}+1}{7-3\sqrt{5}}$ (g) $\frac{\sqrt{6}-1}{3\sqrt{2}+\sqrt{7}}$ (h) $\frac{\sqrt{2}-3}{2\sqrt{3}+\sqrt{6}}$

3 (a) $\frac{5}{\sqrt{2}}$ expressed with a rational denominator is:

A $\frac{\sqrt{10}}{2}$ B $\frac{2\sqrt{5}}{5}$ C $\frac{5\sqrt{2}}{\sqrt{2}}$ D $\frac{5\sqrt{2}}{2}$

(b) $\frac{4}{3\sqrt{3}}$ expressed with a rational denominator is:

A $\frac{4\sqrt{3}}{9}$ B $\frac{\sqrt{12}}{9}$ C $\frac{4\sqrt{3}}{3}$ D $\frac{4\sqrt{3}}{27}$

(c) $\frac{2\sqrt{2}}{3\sqrt{7}}$ expressed with a rational denominator is:

A $\frac{2\sqrt{6}}{21}$ **B** $\frac{2\sqrt{14}}{21}$ **C** $\frac{14\sqrt{2}}{3}$ **D** $\frac{2\sqrt{14}}{3}$

4 (a) The conjugate of $2\sqrt{3}-5$ is:

A $2\sqrt{3}-5$ **B** $2\sqrt{3}+5$ **C** $\sqrt{3}+5$ **D** $5\sqrt{3}+2$

(b) To rationalise the denominator of $\frac{4+\sqrt{7}}{2-3\sqrt{3}}$ it is necessary to multiply by:

A $\frac{4-\sqrt{7}}{4-\sqrt{7}}$ **B** $\frac{1}{2+3\sqrt{3}}$ **C** $2+3\sqrt{3}$ **D** $\frac{2+3\sqrt{3}}{2+3\sqrt{3}}$

Understanding

5 Simplify each of the following. (Rationalise each denominator first.)

(a) $\frac{1}{\sqrt{3}+1}+\frac{1}{3-\sqrt{5}}$ **(b)** $\frac{1}{\sqrt{7}+1}+\frac{1}{3-\sqrt{6}}$

(c) $\frac{1}{\sqrt{6}+\sqrt{3}}-\frac{1}{4-\sqrt{11}}$ **(d)** $\frac{1}{\sqrt{5}+\sqrt{3}}-\frac{1}{\sqrt{7}-2}$

Reasoning

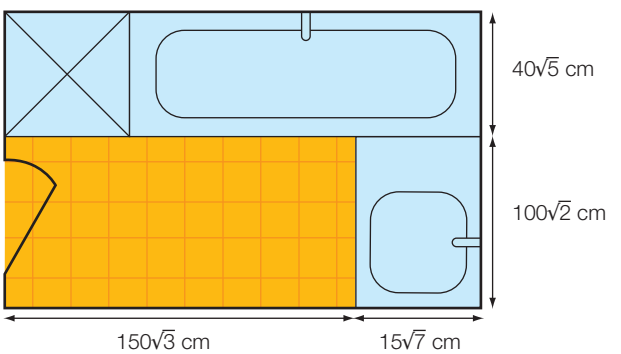
6 A piece of wood with length $5\sqrt{6}$ m is cut to make shelves for a bookcase with length $2\sqrt{2}$ m.

(a) Show $\frac{5\sqrt{6}}{2\sqrt{2}}$ as a fraction with a rational denominator.

(b) How many shelves can be cut from the wood?

(c) What is the exact length of wood left over?

7 The floor plan of a bathroom to be tiled is shown.



(a) Calculate the area to be tiled shaded orange.

(b) The square tiles chosen have side lengths of $10\sqrt{3}$ cm. How many tiles are required for the bathroom floor? State the answer as a simplified surd with a rational denominator.

(c) How many tiles would you order?

(d) Find the exact area of the entire bathroom in square centimetres.

8 If $\frac{3+2\sqrt{3}}{4-\sqrt{2}}a=\frac{4+\sqrt{2}}{5-6\sqrt{3}}$, find the value of a, expressing your answer with a rational denominator.

Open-ended

9 Find at least three fractions that are equivalent to $\frac{4\sqrt{2}}{\sqrt{3}}$, one of which must have a rational denominator.

10 For each of the following, find two fractions that the stated number is between, with a rational denominator formed from the original fraction.

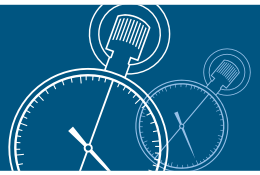
(a) $\frac{1}{\sqrt{3}}$ **(b)** $\frac{3}{\sqrt{7}}$ **(c)** $\frac{2\sqrt{3}}{\sqrt{5}}$ **(d)** $\frac{2}{3\sqrt{2}}$

11 Find at least two expressions that $2 + \sqrt{3}$ can be multiplied by to give a rational answer.

12 Sura is trying to rationalise the denominator of the fraction $\frac{\sqrt{2}-3}{2\sqrt{3}+\sqrt{6}}$. She is struggling to understand why the denominator can be rationalised by multiplying both the numerator and denominator by the denominator's conjugate $2\sqrt{3} - \sqrt{6}$. How could you explain this to Sura?

$$\frac{\sqrt{2}-3}{(2\sqrt{3}+\sqrt{6})} \times \frac{(2\sqrt{3}-\sqrt{6})}{(2\sqrt{3}-\sqrt{6})}$$

Half-time 12



Ex 12.2

1 Simplify each of the following.

(a) $2\sqrt{3} \times \sqrt{3}$ **(b)** $4\sqrt{11} \times 2\sqrt{11}$ **(c)** $\sqrt{\frac{81}{16}}$

Ex 12.4

2 What is the area of a rectangular patio with dimensions of $(5 + \sqrt{3})$ m and $\sqrt{6}$ m?

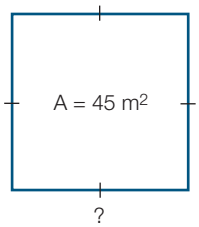
Ex 12.5

3 Express each of the following in simplest form with a rational denominator.

(a) $\frac{\sqrt{5}}{\sqrt{6}}$ **(b)** $\frac{5}{2\sqrt{3}}$ **(c)** $\frac{3}{4 - \sqrt{11}}$

Ex 12.1

4 What is the exact side length of a square with an area of 45 m^2?



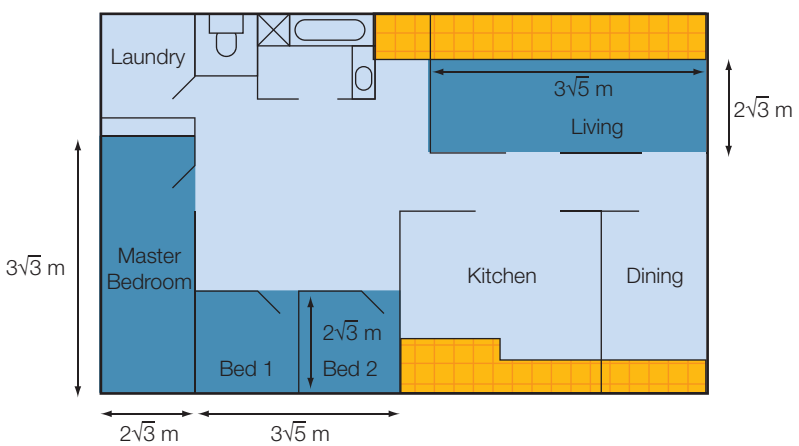
Ex 12.1

5 For each of the numbers below, state whether it is rational (R) or irrational (I).

(a) 3.656565… **(b)** 0.924 **(c)** $\sqrt[3]{9}$ **(d)** $-\sqrt{25}$

Ex 12.4

6 Calculate the exact total area to be carpeted in the bedrooms and the living room in the house below:



Ex 12.2

7 Simplify each of the following.

(a) $\sqrt{72}$ **(b)** $3\sqrt{20} \times 4\sqrt{45}$ **(c)** $\frac{3\sqrt{8}}{\sqrt{200}}$

Ex 12.3

8 Evaluate the following without the use of a calculator.

(a) $-5\sqrt{9}$ **(b)** $\sqrt{81} + \sqrt{64}$ **(c)** $4\sqrt{36} - \sqrt{16} + 2\sqrt{49}$

Summary of index laws

12.6

Need to Know

$a^m \times a^n = a^{m+n}$	When multiplying numbers that are in index form and have the same base, keep the base and add the indices.
$\frac{a^m}{a^n} = a^{m-n}$	When dividing numbers that are in index form and have the same base, keep the base and subtract the indices.
$a^0 = 1$	Any number to the power of zero equals one, $a \neq 0$.
$(a^m)^n = a^{mn}$	When raising a number in index form to a power, keep the base and multiply the indices.
$(ab)^m = a^m b^m$	When a product in brackets is raised to a power m, each factor in the brackets is raised to the power m.
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	When a quotient in brackets is raised to a power m, both the numerator and the denominator are raised to the power m.
$a^{-m} = \frac{1}{a^m}$	A number raised to a negative power is the same as the reciprocal of the number raised to the positive of that power.

Worked Example 11

WE11

Simplify: $(a^4)^3 b^7 \times (a^3 b^{-4})^2$

Working

$$
\begin{aligned}
&(a^4)^3 b^7 \times (a^3 b^{-4})^2 \\
&= a^{12} b^7 \times a^6 b^{-8} \\
&= a^{18} b^{-1} \\
&= \frac{a^{18}}{b}
\end{aligned}
$$

Worked Example 12

WE12

Simplify: $\left(\frac{m^2 n}{m^5 n^3}\right)^4 \div \frac{n^{-5}}{m^{12}}$

Working

$$\left(\frac{m^2n}{m^5n^3}\right)^4 \div \frac{n^{-5}}{m^{12}}$$
$$= \left(\frac{m^2n^1}{m^5n^3}\right)^4 \times \frac{m^{12}}{n^{-5}}$$
$$= \frac{m^8n^4}{m^{20}n^{12}} \times \frac{m^{12}}{n^{-5}}$$
$$= \frac{m^{20}n^4}{m^{20}n^7}$$
$$= m^0n^{-3}$$
$$= n^{-3}$$
$$= \frac{1}{n^3}$$

Answers page 582

12.6 Summary of index laws

Fluency

WE11

1 Simplify each of the following.

(a) $4x^7 \times 3x^{-2}$ (b) $a^3b^{-5} \times a^2b^2$ (c) $\frac{p^4q^6}{p^3q^9}$

(d) $\frac{(x^5)^{-2} \times x^8}{x^{-4}}$ (e) $(a^4)^3b^5 \times (a^2b)^4$ (f) $2(m^3)^2n^{-3} \times 3(mn)^4$

(g) $\frac{(p^2q^3)^{-1} \times p^5q^{-2}}{p^3q^2}$ (h) $\frac{(c^{-3}d^{-4})^2 \times c^2d^6}{c^5d^3}$ (i) $\frac{4(a^2b)^3 \times 3a^{-5}b^{-2}}{6ab}$

(j) $\frac{a^5b^{-2} \times a^2b^3}{a^4b^7 \times ab^{-4}}$ (k) $\frac{m^4n^{-3} \times m^5n^2}{mn^6 \times m^3n^{-2}}$ (l) $\frac{x^3y^{-6}}{x^{-4}y^7} \times \frac{x^{-2}y^4}{x^5y^{-9}}$

(m) $\frac{p^4q^{-2}}{p^2q^{-4}} \times \frac{pq^5}{p^3q^6}$ (n) $\frac{3k^2h^4}{4k^{-3}h^2} \times \frac{2k^{-1}h}{5kh^4}$ (o) $\frac{4a^4b}{9a^{-5}b^3} \times \frac{3a^{-2}b^5}{2ab^{-1}}$

WE12

2 Simplify each of the following.

(a) $\frac{k^9p^6}{k^4p^2} \div \frac{k^3p^2}{k^5p^4}$ (b) $\frac{x^6y^4}{x^3y^2} \div \frac{xy^5}{x^3y^8}$ (c) $\frac{6m^8n^6}{2m^4n^3} \div \frac{5m^3n^2}{2m^5n^4}$

(d) $\frac{2p^5q^3}{p^2q^{-4}} \div \frac{p^7}{q^4}$ (e) $\frac{7x^3y^5}{xy^{-3}} \div \frac{x^6}{y^2}$ (f) $\frac{m^{-4}n^{-2}}{m^5n^3} \div \frac{m^2n}{m^5n^4}$

(g) $\frac{k^{-5}p^2}{k^3p} \div \frac{k^5p^2}{k^6p^{-3}}$ (h) $\frac{2a^5b}{a^{-4}b^{-3}} \div \frac{3a^2b^6}{a^{-2}b}$ (i) $\frac{5f^4g^6}{f^{-3}g^{-1}} \div \frac{2f^2g^4}{f^{-4}g}$

(j) $\left(\frac{x^5}{y^2}\right)^2 \div \left(\frac{x^2}{y^2}\right)^4$ (k) $\left(\frac{4m^3n^4}{3mn}\right)^2 \div \frac{(2m^2n^3)^3}{3m^5n^3}$ (l) $\left(\frac{5p^2q^4}{2pq^3}\right)^2 \div \frac{(5p^3q^2)^3}{4p^7q^6}$

3 (a) $\frac{k^3 \times k^4}{k^9}$ is the same as:

A k^2 B $\frac{1}{k^2}$ C k^3 D $\frac{1}{k^3}$

(b) $(2x^3y^{-2})^3$ is the same as:

A $\frac{8x^9}{y^6}$ **B** $2x^9y^6$ **C** $8x^3y^6$ **D** $\frac{8}{x^9y^6}$

(c) $4(3a)^0$ is the same as:

A 0 **B** 1 **C** 4 **D** $12a$

4 Simplify each of the following.

(a) $\frac{(x^4y^{-2})^3 \times (xy^3)^2}{x^5y^{-4}}$ **(b)** $\frac{(a^3b^{-4})^5 \times (a^2b)^3}{a^9b^{-6}}$

(c) $\frac{(2k^{-3}p^2)^4 \times (k^2p^4)^{-2}}{(2kp)^2}$ **(d)** $\frac{(3m^4n^2)^4 \times (m^{-3}n^2)^{-2}}{(3mn)^2}$

(e) $\frac{(x^5y^{-3})^3}{(x^{-2}y)^2} \times \frac{(x^{-2}y^{-3})^2}{(x^4y^2)^{-3}}$ **(f)** $\left(\frac{m^3}{n^2}\right)^4 \times \left(\frac{m^{-2}}{n^4}\right)^3$

(g) $\left(\frac{p^2}{q^4}\right)^3 \times \left(\frac{p^{-3}}{q^5}\right)^2$ **(h)** $\left(\frac{f^{-3}}{g^2}\right)^{-4} \div \left(\frac{f^2}{g}\right)^3$

(i) $\left(\frac{a^{-2}b^2}{ab}\right)^3 \times \left(\frac{a^4b^{-3}}{a^{-1}b^2}\right)^2$ **(j)** $\left(\frac{m^{-3}n}{m^2n^2}\right)^2 \times \left(\frac{m^5n^{-2}}{m^{-3}n}\right)^3$

(k) $\frac{4x^3(y^{-2})^4}{(xy)^2} \times \left(\frac{x^{-4}y}{x^2y^{-3}}\right)^{-2}$ **(l)** $\left(\frac{3c^4d^3}{2cd}\right)^2 \div \frac{3cd}{(2c^{-2}d^3)^4}$

Understanding

5 Simplify each of these, expressing your answers with positive indices.

(a) $x^{-4} \times x^6$ **(b)** $a^3b^{-2} \times b^{-3}$ **(c)** $p^{-4}q^3 \times p^5q^{-2}$

(d) $d^7f^{-2} \times d^{-3}f^4$ **(e)** $x^5y^2 \times y^{-4}x^{-5}$ **(f)** $a^3b \times b^{-1}a^{-6}$

(g) $(m^3)^{-2}n^3 \times m^4(n^{-1})^2$ **(h)** $(p^4)^{-3}q^6 \times p^5(q^{-2})^2$ **(i)** $(a^{-2}b^3)^2 \times (a^4b^{-1})^{-3}$

(j) $(x^4y^{-3})^2 \times (x^{-5}y)^{-1}$ **(k)** $4(p^{-3})^5q^6 \times 3(pq)^{-2}$ **(l)** $(2x^2y)^{-3} \times (3x^{-4}y^5)^2$

6 (a) Using base 5 index notation, write the numbers 125, 25, 5 and 1.

(b) Using base 5 index notation, write the next five numbers in the sequence.

(c) Write down the whole sequence without using index notation.

Reasoning

7 Simplify each of these, expressing your answers with positive indices.

(a) $\frac{m^7n^{-3}}{m^{-2}n^{-1}}$ **(b)** $\frac{a^{-5}b^3}{a^2b^{-4}} \times \frac{a^3b^2}{a^{-3}b^6}$ **(c)** $\frac{x^7y^{-5}}{x^{-4}y^3} \times \frac{x^{-4}y^3}{x^2y^{-1}}$

(d) $\frac{x^{-4}y^7}{x^{-2}y^{-5}} \div \frac{x^3y^{-6}}{x^{-1}y}$ **(e)** $\frac{(3a^{-4}b^{-2})^2}{(a^{-2}b)^3}$ **(f)** $\frac{(2d^{-3}f^{-1})^3}{(d^{-2}f^2)^2}$

(g) $\frac{5(a^{-4})^3b^{-6}}{(a^2b^{-1})^4} \times \frac{2a^7(b^{-3})^2}{a^{-4}b^{-9}}$ **(h)** $\left(\frac{m^5n^{-2}}{m^{-3}n}\right)^{-1}$ **(i)** $\left(\frac{x^3}{y^2}\right)^{-4} \div \frac{x^{-10}}{(y^{-4})^2}$

8 (a) If I divide a number by its square, what is the result?

(b) I divide a room into squares that have a side length of a units. If the room has a width of $16a^2$ units and a length of $12a$ units, how many squares would I require to carpet the room?

Open-ended

9 **(a)** Write two expressions involving powers and multiplication that when simplified equal $36m^4n^8$.

(b) Write two expressions involving powers and division that can be simplified to $\frac{4m^4}{5n^6}$.

10 For the expression a^{-b}, write three different sets of values for a and b, so that its value is less than $\frac{1}{4}$. Mark each value on a number line.

11 Raphael and Murray were arguing over a question involving indices. Murray believes that $a^{-2} \times a^{-4}$ simplifies to $\frac{1}{a^8}$. Raphael disagrees.

(a) How did Murray calculate his answer? Is he correct?

(b) Why does Raphael disagree? What might Raphael's answer be, assuming it is correct?

(c) How would you explain the correct answer to the students?

12.7 Fractional indices

Need to Know

A number raised to a fractional power $\frac{1}{n}$ is the nth root of the number.

$$a^{\frac{1}{2}} = \sqrt{a}\text{, where } a > 0$$
$$a^{\frac{1}{3}} = \sqrt[3]{a}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}\text{, where } a > 0$$
$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

Numbers raised to a fractional power obey all the index laws.

Worked Example 13

WE13

Simplify:

(a) $a^{\frac{1}{2}} \times a^{\frac{1}{4}}$ **(b)** $\sqrt{x^6}$ **(c)** $\frac{(m^3)^{\frac{1}{3}} \times m^{-\frac{1}{3}}}{m^{\frac{1}{3}}}$

Working

(a) $a^{\frac{1}{2}} \times a^{\frac{1}{4}}$

$= a^{\frac{1}{2}+\frac{1}{4}}$

$= a^{\frac{(2+1)}{4}}$

$= a^{\frac{3}{4}}$

(b) $\sqrt{x^6}$

$= (x^6)^{\frac{1}{2}}$

$= x^{6 \times \frac{1}{2}}$

$= x^3$

(c) $\dfrac{(m^3)^{\frac{1}{3}} \times m^{-\frac{1}{3}}}{m^{\frac{1}{3}}}$

$= \dfrac{m^1 \times m^{-\frac{1}{3}}}{m^{\frac{1}{3}}}$

$= \dfrac{m^{1-\frac{1}{3}}}{m^{\frac{1}{3}}}$

$= \dfrac{m^{\frac{2}{3}}}{m^{\frac{1}{3}}}$

$= m^{\frac{2}{3}-\frac{1}{3}}$

$= m^{\frac{1}{3}}$

Worked Example 14

WE14

Use your calculator to evaluate the following fractional powers.

(a) $7776^{\frac{1}{5}}$ **(b)** $289^{\frac{1}{2}}$ **(c)** $1331^{\frac{1}{3}}$

Working

(a) $7776^{\frac{1}{5}}$

$= \sqrt[5]{7776}$

$= 6$

(Press: 5 | 2nd | $\sqrt[x]{}$ | 7 | 7 | 7 | 6 | =)

(b) $289^{\frac{1}{2}}$

$= \sqrt{289}$

$= 17$

(Press: $\sqrt{}$ | 2 | 8 | 9 | =)

or 2 | 2nd | $\sqrt[x]{}$ | 2 | 8 | 9 | =)

(c) $1331^{\frac{1}{3}}$

$= \sqrt[3]{1331}$

$= 11$

(Press: $\sqrt[3]{}$ | 1 | 3 | 3 | 1 | =)

or 3 | 2nd | $\sqrt[x]{}$ | 1 | 3 | 3 | 1 | =)

Answers page 583

12.7 Fractional indices

Equipment required: calculator for Questions **2**, **3**, **4**, **7**, **10**, **11** and **12**

Fluency

WE13

1 Simplify each of the following.

(a) $x^{\frac{1}{3}} \times x^{\frac{1}{3}}$ **(b)** $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$ **(c)** $k^{\frac{1}{3}} \times k^{\frac{1}{5}}$

(d) $\dfrac{m^{\frac{1}{2}}}{m^{\frac{1}{4}}}$ **(e)** $\dfrac{p^{\frac{1}{2}}}{p^{\frac{1}{3}}}$ **(f)** $\left(y^{\frac{1}{2}}\right)^{\frac{1}{2}}$

(g) $(25p^4)^{\frac{1}{2}}$ **(h)** $(49x^6)^{\frac{1}{2}}$ **(i)** $(8ab^3)^{\frac{1}{3}}$

(j) $\sqrt{x^4y}$ **(k)** $\sqrt{kn^8}$ **(l)** $\left(\dfrac{p^4}{q^{10}}\right)^{\frac{1}{2}}$

WE14

2 Evaluate each of the following. Express your answer correct to two decimal places where appropriate.

(a) $196^{\frac{1}{2}}$ **(b)** $529^{\frac{1}{2}}$ **(c)** $6859^{\frac{1}{3}}$ **(d)** $512^{\frac{1}{3}}$

(e) $1296^{\frac{1}{4}}$ **(f)** $4096^{\frac{1}{4}}$ **(g)** $3125^{\frac{1}{5}}$ **(h)** $1024^{\frac{1}{5}}$

(i) $500^{\frac{1}{3}}$ **(j)** $3000^{\frac{1}{3}}$ **(k)** $98^{\frac{1}{6}}$ **(l)** $145^{\frac{1}{6}}$

3 Using your calculator to assist you, place these numbers in ascending order.

$2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{-1}, 6^{-\frac{1}{5}}$

4 **(a)** 3^5 is closest to:

A 1.71 **B** 15 **C** 125 **D** 243

(b) 4^{-3} is closest to:

A 0.012 **B** 0.016 **C** 64 **D** 81

(c) $20^{\frac{1}{4}}$ is closest to:

A 0.000 006 **B** 2.1 **C** 5 **D** 160 000

(d) $75^{\frac{1}{3}}$ is closest to:

A -25 **B** 0.24 **C** 4.22 **D** 421 875

5 Simplify each of the following, expressing your answers with positive indices.

(a) $\dfrac{\left(a^{\frac{1}{3}}\right)^6 \times a}{a^2}$ **(b)** $\dfrac{\left(d^{\frac{1}{2}}\right)^8 \times d^2}{d^3}$

(c) $(x^4y^6)^{\frac{1}{2}} \times (x^{-3}y^{-12})^{\frac{1}{3}}$

(d) $(m^6n^3)^{\frac{1}{3}} \times (m^{-8}n^{-10})^{\frac{1}{2}}$

(e) $\dfrac{(pq^3)^{\frac{1}{2}} \times p^{\frac{1}{2}}q^{\frac{1}{2}}}{pq^4}$

(f) $\dfrac{(a^5b)^{\frac{1}{2}} \times a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^3b^5}$

(g) $\left(\dfrac{m^2n}{n^{-\frac{1}{2}}}\right)^{-\frac{1}{3}} \times \dfrac{m\left(n^{\frac{1}{3}}\right)^2}{2n^{\frac{1}{3}}}$

(h) $\left(\dfrac{x^3y^2}{y^{\frac{1}{2}}}\right)^{-\frac{1}{3}} \times \dfrac{(x^2)^{\frac{1}{3}}y}{5y^{-\frac{2}{3}}}$

6 **(a)** Find the expression that is *not* the same as $a^{\frac{3}{2}}b^{\frac{1}{2}}$.

A $\sqrt{a^3b}$ **B** $(\sqrt{ab})^3$ **C** $(a^3b)^{\frac{1}{2}}$ **D** $(\sqrt{a})^3\sqrt{b}$

(b) Simplifying $\left(\dfrac{a^{\frac{1}{2}}b^3}{b^2}\right)^2 \div \dfrac{b^{\frac{3}{2}}a}{b^4}$ gives:

A $b^{\frac{9}{2}}$ **B** $b^{\frac{13}{2}}$ **C** $\dfrac{b^{\frac{7}{2}}}{a^{\frac{1}{2}}}$ **D** $\dfrac{b^7}{a^{\frac{1}{2}}}$

7 Evaluate each of the following, correct to three decimal places.

(a) $8^{-\frac{1}{3}}$ **(b)** $12^{-\frac{1}{3}}$ **(c)** $37^{-\frac{1}{2}}$ **(d)** $26^{-\frac{1}{2}}$

(e) $54^{-\frac{1}{4}}$ **(f)** $72^{-\frac{1}{4}}$ **(g)** $100^{-\frac{1}{5}}$ **(h)** $167^{-\frac{1}{5}}$

Understanding

8 By simplifying first, find the value of x in each of the following equations.

(a) $\dfrac{72x^2}{\left(x \times x^{\frac{3}{2}}\right)^2} = 24$ **(b)** $\dfrac{\left(x^{\frac{1}{2}}\right)^5}{x^{-2}} \times \dfrac{5x^{-1}}{x^2} = 40$ **(c)** $\dfrac{7x^{-3}}{x^3}\left(\dfrac{x^{\frac{3}{2}}}{x}\right)^6 = 56$

9 Simplify the following.

(a) $\sqrt{x^6}$ **(b)** $\sqrt[3]{a^9}$ **(c)** $\sqrt[4]{16m^8n^4}$

Reasoning

10 Find the value of x in each of the following equations.

(a) $x^{-2} = \frac{1}{4}$ **(b)** $3^x = 27$ **(c)** $3^x = \frac{1}{9}$ **(d)** $2 \times 5^x = 50$

11 **(a)** Using your calculator, find $16\,807^{\frac{1}{5}}$.

(b) Hence, find the value of x and y in the expression $x^y = 16\,807$.

(c) Using your calculator, find $64^{\frac{4}{3}}$.

(d) Hence, find the value of d and e in the expression $d^e = 64$.

Open-ended

12 Using your calculator, find at least three ways of writing 343 as a power. When the base number is not exact, use three decimal places. For example, $343 = (18.520)^2$.

13 Write the expression $n^{\frac{3}{4}} m^{\frac{3}{5}}$ in at least three different ways.

Problem solving

Lagrange's four-square theorem

Diophantus was a Greek mathematician who lived in the 3rd century. He wrote a famous book called *Arithmetica*. In this book he wrote the following conjecture (a statement that has not been proven).

Every whole number is either a square itself or the sum of two, three or four squares.

This conjecture was subsequently proven in 1770 by Joseph-Louis Lagrange.

(a) Write a square number in each box so that each column and row adds to the total at the end.

(b) Can any of the totals in the grid be written using fewer square numbers?

(c) Starting at 1, write each whole number as a sum of four or fewer square numbers. How far can you get in 3 minutes?

			1^2	39
		1^2		27
	1^2			34
1^2				62
36	30	66	54	

Strategy options

- Make a table.
- Break problem into manageable parts.

12.8 Logarithms

Need to Know

If $y = \log_b x$ then $b^y = x$, where b is the base, y is the power or **logarithm** and x is the numerical value.

Worked Example 15

WE15

(a) Write $2^6 = 64$ in its equivalent logarithmic form.

(b) Write $2^{-4} = \frac{1}{16}$ in its equivalent logarithmic form.

Working

(a) 6 is the logarithm
2 is the base
64 is the numerical value
$6 = \log_2 64$

(b) -4 is the logarithm
2 is the base
$\frac{1}{16}$ is the numerical value
$-4 = \log_2\left(\frac{1}{16}\right)$

Worked Example 16

WE16

Write $\log_{10}(10\,000) = 4$ in its equivalent index form.

Working

4 is the logarithm
10 is the base
10 000 is the numerical value
$10^4 = 10\,000$

12.8 Logarithms

Answers page 583

Equipment required: calculator for Questions **4–5**

Fluency

1 Write each of the following in its equivalent logarithmic form. WE15

(a) $2^5 = 32$ **(b)** $2^8 = 256$ **(c)** $2^{12} = 4096$

(d) $3^3 = 27$ **(e)** $3^5 = 243$ **(f)** $3^8 = 6561$

(g) $10^2 = 100$ **(h)** $10^6 = 1\,000\,000$ **(i)** $10^8 = 100\,000\,000$

(j) $2^{-2} = \frac{1}{4}$ **(k)** $2^{-3} = \frac{1}{8}$ **(l)** $2^{-4} = \frac{1}{16}$

2 Write each of the following in its equivalent index form. WE16

(a) $\log_{10}100 = 2$ **(b)** $\log_{10}1000 = 3$ **(c)** $\log_{10}100\,000 = 5$

(d) $\log_2 16 = 4$ **(e)** $\log_2 128 = 7$ **(f)** $\log_2 512 = 9$

(g) $\log_{10}\left(\frac{1}{1000}\right) = -3$ **(h)** $\log_{10}\left(\frac{1}{10\,000}\right) = -4$ **(i)** $\log_{10}\left(\frac{1}{1\,000\,000}\right) = -6$

(j) $\log_2\left(\frac{1}{8}\right) = -3$ **(k)** $\log_2\left(\frac{1}{32}\right) = -5$ **(l)** $\log_2\left(\frac{1}{128}\right) = -7$

3 (a) If $\log_{10} x = -2$ then x is equal to:

A -200 **B** $-\frac{1}{100}$ **C** $\frac{1}{100}$ **D** 200

(b) Writing $2^x = 2048$ in logarithmic form gives:

A $\log_2 x = 2048$ **B** $\log_x 2 = 2048$ **C** $\log_x 2048 = 2$ **D** $\log_2 2048 = x$

Understanding

4 Solve the following for x.

(a) $3^x = 81$ **(b)** $3^x = 6561$ **(c)** $3^x = 19\,683$

(d) $5^x = \frac{1}{25}$ **(e)** $5^x = \frac{1}{125}$ **(f)** $5^x = \frac{1}{15\,625}$

(g) $6^{-x} = 216$ **(h)** $6^{-x} = 7776$ **(i)** $6^{-x} = \frac{1}{1296}$

Hint

Write the number on the RHS as an index number, using the same base as the LHS.

5 Solve the following for x.

(a) $\log_2 x = 4$ **(b)** $\log_2 x = 6$ **(c)** $\log_2 x = 7$

(d) $\log_3\left(\frac{1}{81}\right) = x$ **(e)** $\log_3\left(\frac{1}{243}\right) = x$ **(f)** $\log_3\left(\frac{1}{6561}\right) = x$

(g) $\log_x 625 = 4$ **(h)** $\log_x 343 = 3$ **(i)** $\log_x\left(\frac{1}{3125}\right) = -5$

Reasoning

6 (a) Solve for x in each of the following.

(i) $\log_{10} x = 1$ **(ii)** $\log_5 x = 1$ **(iii)** $\log_7 x = 1$

(b) Write a general rule based on the above, using a as your base.

7 (a) Solve for x in each of the following.

(i) $\log_{10} x = 0$ **(ii)** $\log_5 x = 0$ **(iii)** $\log_7 x = 0$

(b) Write a general rule to explain what you have just found out.

8 In the equation $\sqrt{a} = a^{\frac{1}{2}}$, why does a have to be greater than or equal to 0?

9 (a) Check that both of the following logarithmic equations are correct:

$\log_2 64 = 6$ and $\log_4 64 = 3$

(b) Now, given $\log_2(4096) = 12$, find the value of y in $\log_4(4096) = y$.

(c) Explain how you got your answer for **(b)**.

Open-ended

10 Given $\log_3(531\,441) = 12$, write two other related log statements that are true.

11 Lee wrote:

$-(6)^2 = -36$
because $-\log_6(-36) = 2$

Explain why this is incorrect and how the question should be tackled.

Laws of logarithms

12.9

Need to Know

To help simplify logarithmic expressions we use:

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b x^n = n \log_b x$

$\log_b b = 1$

$\log_b 1 = 0$

Worked Example 17

WE17

Simplify $\log_2 8 + \log_2 16$, expressing your answer in logarithmic form.

Working

$\log_b x + \log_b y = \log_b xy$

$\log_2 8 + \log_2 16$

$= \log_2(8 \times 16)$

$= \log_2 128$

Worked Example 18

WE18

Simplify $\log_2 12 - \log_2 4$, expressing your answer in logarithmic form.

Working

$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

$\log_2 12 - \log_2 4$

$= \log_2\left(\frac{12}{4}\right)$

$= \log_2 3$

Worked Example 19 WE19

Simplify $\log_2 5^2$, expressing your answer in logarithmic form without a power.

Working

$\log_b x^n = n \log_b x$

$\log_2 5^2 = 2 \log_2 5$

Worked Example 20 WE20

Simplify the following, writing your answer as a basic numeral if possible.

(a) $\log_2 16$ **(b)** $\log_{10} 1000 - \log_{10} 10^2$

Working

(a) $\log_2 16$
$= \log_2 2^4$
$= 4 \log_2 2$
$= 4 \times 1$
$= 4$

(b) $\log_{10} 1000 - \log_{10} 10^2$
$= \log_{10} 10^3 - \log_{10} 10^2$
$= 3 \log_{10} 10 - 2 \log_{10} 10$
$= 3 - 2$
$= 1$

Answers page 583

12.9 Laws of logarithms

Fluency

WE17

1 Simplify the following, expressing your answer in logarithmic form.

(a) $\log_2 4 + \log_2 8$ **(b)** $\log_2 16 + \log_2 4$
(c) $\log_2 8 + \log_2 32$ **(d)** $\log_{10} 5 + \log_{10} 6$
(e) $\log_{10} 7 + \log_{10} 3$ **(f)** $\log_{10} 4 + \log_{10} 9$

WE18

2 Simplify the following, expressing your answer in logarithmic form.

(a) $\log_2 16 - \log_2 8$ **(b)** $\log_2 20 - \log_2 4$
(c) $\log_2 25 - \log_2 5$ **(d)** $\log_{10} 70 - \log_{10} 5$
(e) $\log_{10} 35 - \log_{10} 7$ **(f)** $\log_{10} 36 - \log_{10} 6$

WE19

3 Simplify the following, expressing your answer in logarithmic form without a power.

(a) $\log_2 5^4$ **(b)** $\log_2 6^3$ **(c)** $\log_2 7^4$
(d) $\log_{10} 7^3$ **(e)** $\log_{10} 8^4$ **(f)** $\log_{10} 11^5$

WE20

4 Simplify the following, expressing your answer as a basic numeral.

(a) $\log_2 32$ **(b)** $\log_2 128$ **(c)** $\log_2 256$
(d) $\log_{10} 10\,000 - \log_{10} 100$ **(e)** $\log_3 81 - \log_3 27$ **(f)** $\log_4 64 - \log_4 16$

5 (a) $\dfrac{\log_2 32}{\log_2 4}$ is equal to:

A $\dfrac{5}{2}$ **B** 3 **C** $\log_2(8)$ **D** $\log_2(28)$

(b) $\log_a(mn)$ can be written as:

A $\log_a m \times \log_a n$ **B** $\log_a m^n$

C $\log_a m + \log_a n$ **D** $\log_a m - \log_a n$

Understanding

6 Simplify:

(a) $\log_{10} 4 + \log_{10} 250$ **(b)** $\log_2 64 + \log_2 32$

(c) $\log_2 4 + \log_2 8 - \log_2 16$ **(d)** $\log_3 27 + \log_3 81 - \log_3 243$

(e) $2\log_2 2 - \log_2\left(\dfrac{1}{8}\right) + \log_2 8$ **(f)** $\log_3 3 + 2\log_3\left(\dfrac{1}{9}\right) - 3\log_3 9$

7 Simplify:

(a) $\dfrac{\log_5 x^2}{\log_5 x}$ **(b)** $\dfrac{\log_3 x^5}{\log_3 x^6}$ **(c)** $\dfrac{\log_7 x^9}{\log_7 x^3}$

(d) $\dfrac{\log_2 (x+4)^2}{\log_2 (x+4)}$ **(e)** $\dfrac{3\log_5 (2x-1)}{\log_5 (2x-1)^2}$ **(f)** $\dfrac{\log_7 (3x+4)^4}{3\log_7 (3x+4)}$

Reasoning

8 Simplify:

(a) $\dfrac{\log_x 2}{\log_x 16}$ **(b)** $\dfrac{\log_x 25}{\log_x 5}$ **(c)** $\dfrac{\log_x 49}{\log_x 7^3}$

9 (a) If $\log_a y = \log_a 4 + 3\log_a x$, express y in terms of x.

(b) If $\log_a y = \log_a 11 + 5\log_a x$, express y in terms of x.

10 If $5\log_2 y = \log_2 (x-1) + 2$, show that $y^5 = 4(x-1)$.

Open-ended

11 One of the questions on a test was:

Simplify $\dfrac{\log_2 x}{\log_2 \sqrt{x}}$

Danielle wrote: $\dfrac{\log_2 x}{\log_2 \sqrt{x}} = \dfrac{\log_2 x}{\log_2 x^{\frac{1}{2}}} = \dfrac{1}{2} \times \dfrac{\log_2 x}{\log_2 x} = \dfrac{1}{2}$

Is Danielle's answer correct? If not, what is the correct answer and how would you explain her mistake to Danielle?

Chapter review 12

D.I.Y. Summary

Key Words

conjugate surds	irrational numbers	rationalising the denominator
Distributive Law	logarithm	real numbers
entire surd	rational numbers	surds

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The real number system contains _____ and _____.

2 $\sqrt{2}$ and π are examples of _____.

3 _____ are numbers that contain a radical or root sign and cannot be written as an exact number.

4 To simplify surd expressions involving brackets, we can use the _____.

5 When _____ of a fraction it is necessary to multiply both the numerator and the denominator by a surd or _____ in order to make the denominator rational.

6 The _____ $\sqrt{75}$ can be simplified to $3\sqrt{5}$.

7 When we write statements such as $3^4 = 81$ is equivalent to $\log_3(81) = 4$, we refer to 4 as the _____.

8 **(a)** Three examples of surds are _____.

(b) The conjugate of $\sqrt{3} + 1$ is _____.

(c) Write down the rules that are applied when multiplying numbers with an exponent, when dividing numbers with an exponent, when the exponent is 0, and when raising a number with an exponent to a power.

Fluency

1 Use a calculator to evaluate the following, correct to three decimal places. Ex 12.1

(a) $\sqrt{17}$ **(b)** $\sqrt[3]{25}$ **(c)** $5 \times 4\sqrt{3}$ **(d)** $3\sqrt{7} - 4\sqrt{2}$

2 Simplify each of the following. Ex 12.2

(a) $4\sqrt{5} \times 3\sqrt{6}$ **(b)** $5\sqrt{7} \times 2\sqrt{7}$ **(c)** $\sqrt{\frac{64}{25}}$ **(d)** $\frac{18\sqrt{6}}{2\sqrt{3}}$

(e) $\sqrt{40}$ **(f)** $2\sqrt{125}$ **(g)** $3\sqrt{2} \times 5\sqrt{10}$ **(h)** $4\sqrt{\frac{75}{16}}$

3 Express each of the following as an entire surd. Ex 12.2

(a) $6\sqrt{2}$ **(b)** $-3\sqrt{7}$

4 Simplify each of the following. Ex 12.3

(a) $6\sqrt{2} - 8\sqrt{2} + 3\sqrt{2}$ **(b)** $5\sqrt{7} - 2\sqrt{3} - \sqrt{7} + \sqrt{3}$

(c) $4\sqrt{12} + 7\sqrt{27}$ **(d)** $6\sqrt{50} - 2\sqrt{24} + 4\sqrt{32} + \sqrt{54}$

5 Expand and simplify each of the following. Ex.12.4

(a) $\sqrt{5}(4\sqrt{3}-2\sqrt{5}+\sqrt{2})$ **(b)** $-6\sqrt{2}(3\sqrt{7}-\sqrt{2}+1)$

(c) $(2\sqrt{5}+9)(\sqrt{5}-2)$ **(d)** $(3\sqrt{7}+4\sqrt{2})^2$

6 Express each of the following in simplest form with a rational denominator. Ex.12.5

(a) $\frac{5}{\sqrt{2}}$ **(b)** $\frac{4\sqrt{3}}{\sqrt{6}}$ **(c)** $\frac{7}{2\sqrt{5}}$

7 Express each of the following in simplest form with a rational denominator. Ex.12.5

(a) $\frac{7}{3-\sqrt{5}}$ **(b)** $\frac{\sqrt{2}-2}{2\sqrt{3}+\sqrt{7}}$

8 Simplify each of the following. Ex.12.6

(a) $(3ab^5)^2$ **(b)** $\frac{(cd^2)^8}{(c^3d)^2}$ **(c)** $\left(\frac{t^2}{u^3}\right)^4\times\left(\frac{u^2}{t}\right)^7$

(d) $\frac{8x^5y^6}{6x^3y^2}\times\frac{3x^4y^7}{2x^5y}$ **(e)** $\frac{(m^4n)^3\times(m^2n^5)^2}{(mn)^4}$ **(f)** $\left(\frac{a^4b^2}{ab^3}\right)^2\div\left(\frac{a^3b}{a^5b^2}\right)^3$

9 Simplify, expressing your answers with positive indices. Ex.12.6

(a) $3a^{-4}\times 2a^{-2}$ **(b)** $(4xy^2)^{-2}\times(3x^2y)^3$ **(c)** $\frac{(m^{-3}n^2)^4}{2(mn^3)^{-2}}\times\left(\frac{m^2n^{-1}}{m^{-3}n^3}\right)^2$

10 Simplify each of the following. Ex.12.7

(a) $(64m^3n)^{\frac{1}{3}}$ **(b)** $\sqrt{p^6q}$

(c) $\frac{(a^6b^3)^{\frac{1}{2}}\times a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^3b^2}$ **(d)** $(a^2b^{-3})^{\frac{1}{2}}\times(a^{-4}b)^{\frac{5}{2}}$

(e) $\left(\frac{m^{\frac{1}{3}}n^{\frac{2}{3}}}{mn^{\frac{1}{2}}}\right)^2\times\frac{2m^3n^5}{\left(m^{\frac{1}{2}}n\right)^4}$ **(f)** $\left(\frac{3^2m^{-2}n^2}{mn}\right)^3\div\left(\frac{m^{\frac{1}{3}}n^{\frac{2}{3}}}{n^2m^{-3}}\right)^3$

11 Express each of the following in index form. Ex.12.7

(a) $(\sqrt{k})^5$ **(b)** $\sqrt[4]{x^3}$ **(c)** $\frac{1}{\sqrt[3]{m^2}}$

12 **(a)** $2\sqrt{6}(\sqrt{10}+3\sqrt{3})$ can be simplified to: Ex.12.4

A $2\sqrt{60}+5\sqrt{18}$ **B** $8\sqrt{15}+18\sqrt{2}$ **C** $4\sqrt{15}+18\sqrt{2}$ **D** $30\sqrt{2}+5\sqrt{18}$

(b) $8\times\sqrt{\frac{81}{64}}$ can be simplified to: Ex.12.1

A $\sqrt{9}$ **B** $24\times\sqrt{\frac{1}{8}}$ **C** $8\times\sqrt{\frac{9}{8}}$ **D** 9

13 Write each of the following in its equivalent logarithmic form. Ex.12.8

(a) $4^7=16\,384$ **(b)** $2^{10}=1024$ **(c)** $10^{-3}=\frac{1}{1000}$

14 Write each of the following in its equivalent index form.

Ex 12.8

(a) $\log_2(16\,384) = 14$ **(b)** $\log_3 243 = 5$ **(c)** $\log_{10}\left(\frac{1}{100\,000}\right) = -5$

15 Simplify the following, expressing your answer as a basic numeral where possible.

Ex 12.9

(a) $\log_2 8 + \log_2 64$ **(b)** $\log_2 16 - \log_2 64$

(c) $\log_3 5 + \log_3 10$ **(d)** $\log_5 110 - \log_5 11$

Understanding

16 A DVD has an area of 232π cm^2.

(a) What is its exact radius? (Leave your answer in surd form.)

(b) State the radius of the disc, correct to two decimal places.

17 A gift box is to have its four sides covered by adhesive paper. It has the dimensions of length $8\sqrt{10}$ cm, width 18 cm and height $(5\sqrt{6} + 3\sqrt{7})$ cm.

(a) What are the approximate length and height of the box, correct to one decimal place?

(b) Using your answers to part **(a)**, calculate the area to be covered by adhesive paper, correct to one decimal place.

(c) Using the surd expression, find the area to be covered by the paper. Express your answer as a surd.

(d) Use a calculator to express your answer to part **(c)** correct to one decimal place.

(e) Compare your answers to parts **(b)** and **(d)**.

18 Solve for x:

(a) $4^x = 1024$ **(b)** $3^x = 2187$ **(c)** $2^x = \frac{1}{64}$

(d) $\log_2(x) = -4$ **(e)** $\log_3(6561) = x$ **(f)** $\log_5\left(\frac{1}{625}\right) = x$

19 Simplify each of the following, expressing your answer as a basic numeral where possible.

(a) $\log_{10} 2 + \log_{10} 500$ **(b)** $\log_3 81 - \log_3 27 + \log_3 9$

(c) $\frac{\log_5 x^4}{2\log_5 x}$ **(d)** $\frac{\log_8 (5x-7)^3}{4\log_8 (5x-7)}$

Reasoning

20 **(a)** Find the exact side length of a square with an area of 50 cm^2.

(b) Find the exact side length of an equilateral triangle with an area of 40 cm^2.

(c) Find the exact circumference of a circular swimming pool with an area of 75π m^2.

21 If $\left(\frac{4-3\sqrt{5}}{5+2\sqrt{2}}\right)a = \frac{5-2\sqrt{2}}{4+\sqrt{5}}$, find the value of a, expressing your answer with a rational denominator.

22 **(a)** If $\log_a y = \log_a 7 + 3\log_a x$, express y in terms of x.

(b) If $\log_a y = \log_a 5 - 2\log_a x$, express y in terms of x.

Mixed review D

Fluency

1 An investment of $5000 is made at 3.5% p.a. for 6 years. Find the difference in interest earned, to the nearest cent, if interest is compounded monthly compared to annually. Ex 1.2

2 Find the area and perimeter, correct to one decimal place, of a semicircle with a diameter of 9.6 cm. Ex 3.1

3 Write the following in simplest form. Ex 12.2 10A

(a) $\sqrt{3} \times \sqrt{8}$ **(b)** $\sqrt{3} \times 3\sqrt{12}$ **(c)** $\sqrt{2} \times \sqrt{18}$

(d) $2\sqrt{5} \times 4\sqrt{10}$ **(e)** $2\sqrt{6} \times 3\sqrt{10}$ **(f)** $3\sqrt{6} \times 7\sqrt{12}$

4 Simplify the following. Ex 12.2 10A

(a) $\sqrt{80}$ **(b)** $3\sqrt{240}$ **(c)** $7\sqrt{525}$

5 Solve the following equations. Ex 4.3

(a) $\frac{3a+4}{7} = -2$ **(b)** $\frac{4a+3}{2} = \frac{2a}{5}$ **(c)** $\frac{3a-2}{3} = \frac{4a-5}{2}$

6 Express each of the following in simplest form with a rational denominator. Ex 12.5 10A

(a) $\frac{4\sqrt{3}}{\sqrt{5}}$ **(b)** $\frac{2}{5-\sqrt{3}}$ **(c)** $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{5}}$

7 Factorise each of the following expressions. Ex 2.2

(a) $k(k-3)-(k-3)$ **(b)** $3t(t+2)-5(t+2)$

8 Simplify the expressions below: Ex 12.6 10A

(a) $\frac{4a^4b}{9a^{-5}b^3} \times \frac{3a^{-2}b^5}{2ab^{-1}}$ **(b)** $\frac{(a^4b^2)^2}{(a^2b^3)^2} \times \frac{(a^{-3}b)^3}{(a^5b^2)^{-3}}$ **(c)** $\frac{k^{-8}}{n^4} \div \frac{k^9}{n}$

9 Find the value of each pronumeral, correct to two decimal places. Ex 5.3

(a) $\cos a = 0.1314$ **(b)** $\tan b = 2.621$ **(c)** $\sin c = 0.5231$

10 Find the gradient of the line joining: Ex 4.2

(a) points (1, 7) and (4, 14) **(b)** points (3, 6) and (7, -2).

11 Find the mean for each of the following grouped data frequency tables. Give your answers correct to two decimal places. Ex 7.7 10A

(a)

Class interval	Frequency
20–<25	8
25–<30	6
30–<35	3
35–<40	2
40–<45	5
45–<50	5

(b)

Class interval	Frequency
111–115	12
116–120	15
121–125	17
126–130	19
131–135	13
136–140	11

12 Sketch the graphs of each of the following, clearly showing the turning point.

Ex 8.9 10A

(a) $y = (x + 3)(x - 2)$ **(b)** $y = x^2 + 2x + 5$ **(c)** $y = 2x^2 - 4x$

13 For the rectangle shown:

Ex 12.3 10A

(a) state its perimeter in simplest form

(b) state its area in simplest form.



14 Write a definition for each of the following quadrilaterals.

Ex 8.5

(a) rectangle **(b)** rhombus **(c)** kite

15 Two dice, both numbered 1 to 6, are rolled at the same time. Find the probability that the sum of the two faces showing will be:

Ex 11.2

(a) 4 **(b)** 6 **(c)** greater than 9.

16 Without using a calculator, determine which is the larger of the following pairs, placing <, > or = between the numbers below.

Ex 12.6 10A

(a) 3^{-5} ____ 5^{-3} **(b)** $9^{\frac{1}{2}}$ ____ 9^{-1} **(c)** $8^{-\frac{1}{3}}$ ____ $\frac{2^{-3}}{2^{-2}}$

17 On a Venn diagram, show the region described by not A and B and not C.

Ex 11.2

18 Describe what is needed for an equation to be a quadratic equation.

Ex 2.3

Understanding

19 Leonard and Faye want to develop a vegetable garden in their backyard. They have 20 m of chicken wire and enough posts to erect a fence around the four sides of the rectangular garden.

(a) If the length of the rectangular garden is x m, find an expression (in terms of x) for the width of the graden.

(b) Show that the area of the garden, A m^2, is given by the quadratic expression $A = -x^2 + 10x$.

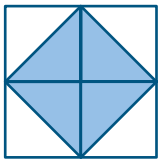
(c) Sketch the graph of the area of the vegetable garden, A m^2, against the length of the garden, x m. (Show length on the horizontal axis.)

(d) What should be the dimensions of the garden if the area is to be as large as possible?

(e) What is the area of the largest vegetable garden that Leonard and Faye can make?

Reasoning

20 A square with side lengths $\sqrt{2}$ has been drawn using four squares with side lengths of 1 cm, as shown. What is the perimeter of a square that is made by joining the diagonals of four squares with side lengths of 2 cm?



10A

21 **(a)** Draw an isosceles triangle with base length 10 cm and height 10 cm.

10A

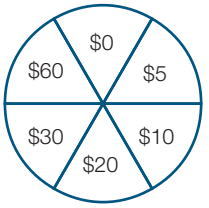
(b) Show that the length of the equal sides in the triangle in part **(a)** is an irrational number.

Challenge D

1 In a dice game, a player rolls two dice. His score is the larger of the two numbers on the dice. For example, if he rolls 3 and 5, his score is 5, and if he rolls 4 and 4, his score is 4. What is the probability that his score is 3 or less?

A $\frac{5}{36}$ **B** $\frac{1}{6}$ **C** $\frac{7}{36}$ **D** $\frac{1}{4}$

2 A prize wheel is divided into 6 equal sectors. Each time the wheel is spun it stops on one of the sectors, resulting in a prize of the amount shown.



(a) On the first spin \$5 is won. What is the probability of winning \$5 on the second spin?

(b) The wheel is spun a total of 50 times and the results recorded in the table.

Prize	Frequency	Relative frequency
\$0	8	0.16
\$5		0.14
\$10	9	
\$20	10	0.20
\$30	8	0.16
\$60	8	0.16
Total	**50**	**1**

(i) Copy and complete the table by writing in the two missing pieces of information.

(ii) How much prize money was won?

(c) To play this game you have to pay \$20 per throw.

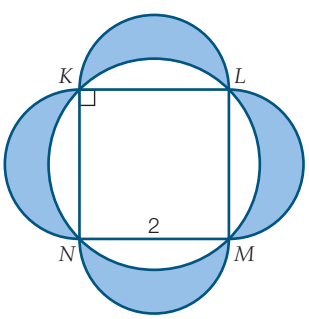
(i) If each outcome was equally likely and you spun the wheel 48 times, would you expect to make a profit?

(ii) If you spun the wheel 50 times and the results were as shown in the table in part **(b)**, would you make a profit?

3 A pyramid has a square base with an area of 1440 cm^2. Each of the pyramid's triangular faces is identical and has an area of 840 cm^2. The height of the pyramid is:

A $20\sqrt{3}$ cm **B** 40 cm **C** $30\sqrt{2}$ cm **D** $4\sqrt{145}$ cm

4 The square $KLMN$, with side length 2 units, is inscribed in a circle as shown. Using each side of the square as a diameter, semicircular arcs are drawn. The total area of the shaded region outside the circle and inside the semicircles is:



A 4 units^2 **B** π units^2

C 2π units^2 **D** $(2\pi - 4)$ units^2

5 Find the value of x in each of the following.

(a) $x^{-1} = 3^{-1} + 4^{-1}$ **(b)** $x + 27^{\frac{1}{3}} = 125^{\frac{1}{3}}$ **(c)** $2^x = 2(16^{12}) + 2(8^{16})$

(d) $2^{2x-4} = 8$ **(e)** $16^x = 2^{x+5} - 2^{x+4}$

Answers

Worked solutions and answers to ***all*** activities appear in the Teacher Companion.

Chapter 1

Recall 1 (p. 2)

1 (a) 0.17 (b) 0.06 (c) 0.192
(d) 0.048 (e) 0.0201 (f) 0.0025

2 (a) $3000 (b) $600 (c) $33 250 (d) $3375

3 (a) (i) 80 (ii) 60 (iii) 50 (iv) 120
(b) (i) 30 (ii) 42 (iii) 54 (iv) 15

4 (a) 49 (b) 8 (c) 1 000 000
(d) 0.36 (e) $\frac{1}{32}$ (f) 1.4641

5 (a) 0.5 (b) 0.25 (c) 8.75
(d) 0.5 (e) 0.25 (f) 0.4

Exercise 1.1 (p. 5)

1 (a) (i) $3200 (ii) $43 200
(b) (i) $2700 (ii) $32 700
(c) (i) $10 500 (ii) $40 500
(d) (i) $24 000 (ii) $64 000
(e) (i) $281 250 (ii) $431 250
(f) (i) $468 000 (ii) $708 000
(g) (i) $22 275 (ii) $322 275
(h) (i) $27 912.50 (ii) $377 912.50
(i) (i) $6175 (ii) $19 175
(j) (i) $6794 (ii) $22 594
(k) (i) $28 125 (ii) $73 125
(l) (i) $13 860 (ii) $49 860
(m) (i) $3375 (ii) $13 375
(n) (i) $10 200 (ii) $26 200

2 (a) $4000 (b) $8000 (c) 14% p.a.
(d) 11% p.a. (e) 10 years (f) 12 years

3 (a) (i) $43 200 (ii) $3200
(b) (i) $32 700 (ii) $2700
(c) (i) $42 076.55 (ii) $12 076.55
(d) (i) $71 834.25 (ii) $31 834.25
(e) (i) $914 750.94 (ii) $764 750.94
(f) (i) $1 587 447.88 (ii) $1 347 447.88
(g) (i) $322 348.70 (ii) $22 348.70
(h) (i) $378 011.55 (ii) $28 011.55
(i) (i) $20 465.10 (ii) $7465.10
(j) (i) $23 770.16 (ii) $7970.16
(k) (i) $82 809.23 (ii) $37 809.23
(l) (i) $52 229.19 (ii) $16 229.19
(m) (i) $13 695.26 (ii) $3695.26
(n) (i) $28 828.16 (ii) $12 828.16

4 Lily (a) (i) $5830.96 (ii) $5250
(b) Compound interest is $580.96 higher.
Jarrah (a) (i) $39 939.82 (ii) $29 925
(b) Compound interest is $10 014.82 higher.

5 (a) $9419.20 (b) $9240

6 C 7 1.2% p.a. 8 $129.17

9

Amount (A)	Principal (P)	Interest rate p.a. (R)	Time in years (T)
$1359.81	$500	8%	13
$43 742.08	$24 600	12.2%	5
$1 048 209.60	$1 000 200	4.8%	1
$151.71	$84	3%	20
$60.50	$50	10%	2

10 (a) (i) simple interest (ii) 40% p.a., $12
(b) (i) compound interest (ii) $994
(c) (i) compound interest (ii) $25 729
(d) (i) compound interest (ii) $51 452
(e) (i) simple interest (ii) $1250
(f) (i) simple interest (ii) 12% p.a.

11 A

12 (a) 100 years (b) 47 years

13 5% p.a.

Open-ended – Sample answers

14 $12 000 invested at 11.8% p.a. would give a final value of $14 999 (clearly within the $100 buffer!).

15 $R \times T = 0.8$, e.g. 16% p.a. for 5 years, 15% p.a. for 5 years and 4 months, 12% p.a. for 6 years and 8 months.

Exercise 1.2 (p. 8)

1 (a) $20 793.60 (b) $20 972.74 (c) $21 068.94

2 (a) $9528.13 (b) $9552.42

3 B 4 C

5 (a) $64 656.50 (b) $65 731.65 (c) $66 472.15

6 $647.54 7 $13 499.14

8 $5468.12 and $5459.91; Martina is better off with the 4.5% p.a. compounded quarterly. The difference is $8.21.

9 (a) (i) $3215.09 (ii) $2879.21
(iii) $2545.44 (i) gives the best return

(b) No

(c) (i) $I \approx 16\%$ of P (ii) $I \approx 14\%$ of P (iii) $I \approx 13\%$ of P

Open-ended – Sample answers

10 The statement means that if compound interest is calculated more frequently (e.g. every month instead of every quarter), it will result in a greater amount of interest. Quarterly compounding gives interest of $4859.47. (48.6%) Monthly compounding gives interest of $4898.46. (49.0%) Weekly compounding gives interest of $4913.66. (49.1%)

11 Using the suggested example, simple interest is $600 and interest compounded annually is $630.50. The principal gets bigger after each time period.

Exercise 1.3 (p. 11)

1 (a) $5996 (b) $5437 (c) $5456.76 (d) B

2 (a) 10.2% p.a. (b) 3.24% p.a. (c) 2.6% p.a. (d) B

3 (a) 12 years (b) 10 years (c) 4.2 years

4 (a) $5956.42 (b) $956.42

(c) only marginally better, $I = \$956.46$

(d) $956.42 (e) 4.47% p.a.

(f) At least 4 years, as it would be worth $5788.13 at the end of the third year and $6077.53 at the end of the fourth year.

5 2

6 (a) every 4 months (b) 9.3% p.a.

(c) monthly compounding

7 At least every 4 months.

Open-ended – Sample answers

8 Rounding error: $\frac{0.065}{4} = 0.01625$, $P = \$18\,751.14$

9 11% p.a. with annual compounding, 10.5% p.a. compounding half-yearly, 10.2% p.a. compounding daily

Half-time 1 (p. 14)

1 B 2 $8673.33 3 $8889.26 4 200%

5 (a) $50 649.49 (b) $51 824.82 (c) $52 059.90

6 9

7 (a) $10 866 (b) $9000

(c) Compound interest is $1866 more than simple interest.

8 17.27% p.a.

Exercise 1.4 (p. 16)

1 (a) 7% p.a. (b) 9.4% p.a. (c) 13.52% p.a.

(d) 21.73% p.a. (e) 17.88% p.a. (f) 6.45% p.a.

2 (a) (i) $R = 7.3\%$ p.a., $n = 5$ (ii) 7.52% p.a.

(b) (i) $R = 15.5\%$ p.a., $n = 12$ (ii) 16.65% p.a.

(c) (i) $R = 21\%$ p.a., $n = 4$ (ii) 22.71% p.a.

(d) (i) $R = 16\frac{1}{4}\%$ p.a., $n = 3$ (ii) 17.15% p.a.

(e) (i) $R = 20\frac{1}{3}\%$ p.a., $n = 365$ (ii) 22.54% p.a.

(f) (i) $R = 5.25\%$ p.a., $n = 2$ (ii) 5.32% p.a.

3 B

4 (a) Q Bank: 15.87% p.a., D Bank: 16.06% p.a.

(b) D Bank

5 (a) 22.13% p.a. (b) 22.26% p.a.

(c) Personal loan because the effective rate is lower.

6 (a) 25.85% p.a. (b) online

7 (a) 15% p.a. compounded daily—higher rate and more frequent compounding

(b) 10.5% p.a. compounded weekly—rate is the same so more frequent compounding is better

(c) 12% p.a. compounded daily—compounding frequency is the same so higher rate is better

(d) 15% p.a. compounded annually for 2 years—compounded beats flat rate over 2 years

8 Yes. In each case choose the other option because for loans we want the lowest effective rate.

9 Option C has the highest effective rate but all the options look too good to be true.

10 (a) car company loan (b) flat rate

Open-ended – Sample answers

11 1-year loan of $10 000 at 6% p.a. compounded daily. Nominal rate is 6% p.a. The interest is compounded, not simple. However, the same amount of interest will be paid if $10 000 is borrowed for a year at a simple interest rate of 6.18% p.a. Either way nominal isn't the same as simple.

12 42% p.a. compounded half-yearly, 40% p.a. compounded quarterly

Exercise 1.5 (p. 20)

1 (a) (i) $10 500 (ii) $19 500

(b) (i) $28 500 (ii) $19 000

(c) (i) $100 000 (ii) No value

(d) (i) $772 225 (ii) $17 775

(e) (i) $60 900 (ii) $84 100

(f) (i) $107 250 (ii) $192 750

(g) (i) $4928.13 (ii) $42 571.87

(h) (i) $76 159.13 (ii) $76 540.87

2 (a) (i) $20 870.65 (ii) $9 129.35

(b) (i) $25 667.10 (ii) $21 832.90

(c) (i) $35 848.59 (ii) $64 151.41

(d) (i) $290 950.49 (ii) $499 049.51

(e) (i) $93 037.95 (ii) $51 962.05

(f) (i) $207 293.23 (ii) $92 706.77

(g) (i) $42 285.67 (ii) $5214.33

(h) (i) $83 275.25 (ii) $69 424.75

3 (a) (i) $6000 (ii) $2/hour

(iii) $20 (iv) $6130

(b) (i) \$1000 (ii) 50c/hour

(iii) \$500 (iv) \$500

4 \$526.50 **5** B **6** \$2240

7 Dep 1st year = \$9000, C dep 1st year = \$9000

Written-down value 1st year = \$36 000

Dep 2nd year = \$9000, C dep 2nd year = \$18 000

Written-down value 2nd year = \$27 000

Dep 3rd year = \$9000, C dep 3rd year = \$27 000

Written-down value 3rd year = \$18 000

8 C

9 \$2923.24. Reducing balance depreciation is less so written-down value after 4 years is higher than under straight-line depreciation.

10 \$485.82

11 Written-down value 1st year = \$36 000, C dep 1st year = \$9000 Dep 1st year = \$9000

Written-down value 2nd year = \$28 800, C dep 2nd year = \$16 200 Dep 2nd year = \$7200

Written-down value 3rd year = \$23 040, C dep 3rd year = \$21 960 Dep 3rd year = \$5760

1st year is the same as it was for linear. After that, the depreciation slows down and depreciation for each year is less than the year before. After the 1st year, written-down value is also better than it was for linear.

12 C

13 **(a)** D **(b)** A

14 **(a)** \$3916.67 **(b)** \$6083.33

15 \$9943.11 **16** \$2415

17 Written-down value after the 1st year is \$3353.75

Written-down value after the 2nd year is \$2768.75

18 **(a)** (i) \$50 000 (ii) \$36 000 (iii) \$25 800 (iv) \$18 390

(b) after 21 years

Open-ended – Sample answers

19 5% p.a. and 14 years, 10% p.a. and 7 years, 20% p.a. and 3.5 years, 70% p.a. and 1 year. Must have $R \times T = 0.7$

20 Any values that satisfy $0.3 = (1 - R)^T$; e.g. 70% p.a. and 1 year, 45.23% p.a. and 2 years, 21.4% p.a. and 5 years

Exercise 1.6 (p. 26)

1 **(a)** 20 524 **(b)** 15 200 **(c)** 1171

(d) \$875 855 **(e)** 242 057 **(f)** 1062

2 **(a)** 42 **(b)** \$896 **(c)** 6218

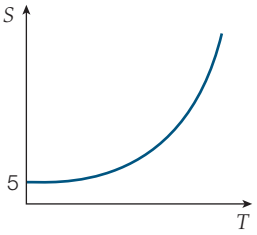
(d) 164 870 **(e)** 1534 **(f)** 4 566 189

3 **(a)**

4	5	6
405	1 215	3 645

(b) $S = 5 \times 3^T$ **(c)** 388 564

(d)



(e) after 2.5 years

4 **(a)** C **(b)** B

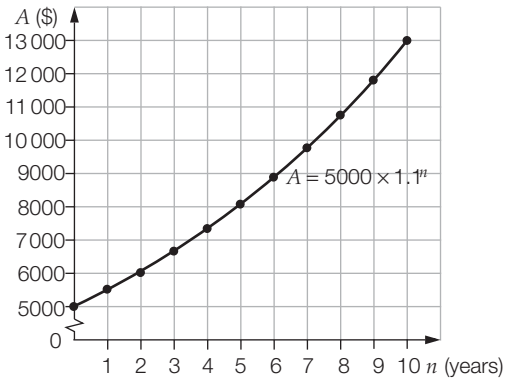
5 **(a)** \$5500 **(b)** 110% **(c)** 1.1 **(d)** \$6050 **(e)** 1.1

(f)

Year	Balance (\$)
0	5 000
1	5 500
2	6 050
3	6 655
4	7 320.50
5	8 052.55
6	8 857.51
7	9 743.59
8	10 717.94
9	11 789.74
10	12 968.71

(g) $A = 5000 \times 1.1^n$

(h)



(i) \$6979.82 **(j)** after 4 years, 11 months

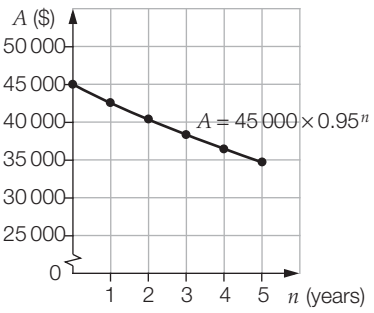
(k) after 7 years, 3 months

6 **(a)** 95% **(b)** 0.95 **(c)** $A = 45\,000 \times 0.95^n$

(d)

Year	Balance (\$)
0	45 000
1	42 750
2	40 612.50
3	38 581.88
4	36 652.78
5	34 820.14

(e)



(f) $39 584.17

(g) after 4 years, 4 months

7 (a) $I = 10^6 \times 0.5^T$ (b) 95c

(c) When she's 7 years old.

8 (a) $A = 5\left(\frac{2}{3}\right)^n$ (b) 7 g

Open-ended – Sample answers

9 $T = 6$ years, $R = 200\%$ p.a., $T = 3$ years, $R = 800\%$ p.a., $T = 20$ years, $R \approx 39\%$ p.a.

10 There is no error in the report. Janet has assumed a linear model of decay. Radioactive material follows an exponential decay model. We just need to show that

$$0.1 = \left(1 - \frac{0.15}{60}\right)^{850}$$

Chapter review 1 (p. 30)

1 (a) D (b) D

2 (a) $25 913.02 (b) $26 221.74 (c) $26 438.68

3 $30 000

4 (a) 8.7% p.a. (b) 9.5% p.a.

(c) 4% p.a. (d) 18.7% p.a.

5 (a) $39 000 (b) $41 741.30

6 417 **7** 11.5% p.a. **8** $59 193 **9** B

10 8.1% p.a. **11** B

12 (a) If the prime cost of a photocopier is $10 800 and it has a scrap value of $500, then the total possible depreciation is $10 300. If it has an expected life of 206 000 pages, then the unit cost depreciation is $0.05 per page. After it copies 125 000, the depreciation is $6250 and the written-down value is $4550.

(b) If the prime cost of an item is $52 500 and it has scrap value of $750 then the total possible depreciation is $51 750. If it has an expected life of 5000 days, then the unit cost depreciation is $10.35 per day. After it does a years' work, the depreciation is $3777.75 and the written-down value is $48 722.25.

13 63 680 **14** 20.4°C **15** 4.2 years

16 (a) 11% p.a. compounded quarterly—lower nominal rate and less frequent compounding.

(b) 10% p.a. compounded monthly—same nominal rate but less frequent compounding.

(c) 17% p.a. compounded daily—same compounding period but lower rate.

(d) 12% p.a. flat rate for 2 years—after 1 year, flat rate is cheaper than compounded.

17 77.8% per month

18 10% per month

Chapter 2

Recall 2 (p. 32)

1 (a) 3 (b) 2 (c) 4

2 (a) 4 (b) -6 (c) 1 (d) -12

3 (a) $3a + 5b$ (b) $24x - 8y$

4 (a) $3x + 6$ (b) $5x - 15$ (c) $5x - 8$

5 (a) $-2x - 6$ (b) $-8x + 40$ (c) $-x - 2$

6 (a) $9x^2$ (b) $4y^2$ (c) $49k^2$

7 (a) $14x^2$ (b) $-8p^2$ (c) $2y^2$

8 (a) $x = \frac{y-3}{2}$ (b) $x = 2ac + 4b$ (c) $x = \frac{3s + at^2}{5}$

9 (a) 3, 7 (b) 2, 12 (c) -5, -4

(d) -3, 6 (e) -10, 2 (f) -8, 5

10 (a) $\frac{11}{12}$ (b) $\frac{11}{24}$ (c) $\frac{5a}{3}$

11 (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$

Exercise 2.1 (p. 34)

1 (a) $4x + 12$ (b) $8x + 16$ (c) $12x + 132$

(d) $12x - 2$ (e) $14x - 35$ (f) $5x^2 - 4x$

(g) $-2x - 2$ (h) $-x^2 + 7x$ (i) $-3x^2 + 6x$

(j) $-3x^2 - 18x$ (k) $-2x^2 + 18x$ (l) $-4x^2 - 8x$

(m) $x^2 + 5x + 6$ (n) $2x^2 + 5x + 2$ (o) $5x^2 + 17x + 6$

(p) $2x^2 - 11x - 30$ (q) $10x^2 + 13x + 32$ (r) $12x^2 - 16x - 6$

2 (a) $x^2 + 4x + 3$ (b) $x^2 + 6x + 8$ (c) $x^2 + 9x + 20$

(d) $x^2 + 5x - 24$ (e) $x^2 - 2x - 63$ (f) $x^2 + 4x - 21$

(g) $x^2 - 8x + 7$ (h) $x^2 - 9x + 20$ (i) $x^2 - 12x + 27$

(j) $2x^2 - x - 6$ (k) $4x^2 + 7x - 15$ (l) $7x^2 - 27x - 4$

(m) $6x^2 - 17x + 5$ (n) $-5x^2 + 38x - 21$ (o) $-18x^2 + 27x - 10$

3 (a) $4 - x^2$ (b) $49 - x^2$ (c) $36 - x^2$

(d) $4x^2 - 1$ (e) $9x^2 - 16$ (f) $25 - 9x^2$

(g) $x^2 + 12x + 36$ (h) $x^2 + 22x + 121$ (i) $64 + 16x + x^2$

(j) $x^2 - 6x + 9$ (k) $x^2 - 14x + 49$ (l) $x^2 - 24x + 144$

(m) $4x^2 + 20x + 25$ (n) $9x^2 + 12x + 4$ (o) $49x^2 - 28x + 4$

4 (a) $x^3 + 10x^2 + 24x$ (b) $x^3 + 4x^2 - 5x$

(c) $x^3 - 3x^2 - 4x$ (d) $-x^3 + x^2 + 6x$

(e) $-3x^3 + 9x^2 - 6x$ (f) $-4x^3 + 20x^2 + 96x$

(g) $x^3 + 11x^2 + 38x + 40$ (h) $x^3 - 4x^2 - 7x + 10$

(i) $x^3 - 11x^2 + 36x - 36$ (j) $x^3 + 5x^2 - 9x - 45$

(k) $x^3 + 7x^2 - 4x - 28$ (l) $2x^3 + x^2 - 32x - 16$

(m) $x^3 + 5x^2 - 8x - 48$ (n) $x^3 + 3x^2 - 4$

(o) $x^3 - 7x^2 - 24x + 180$

5 B

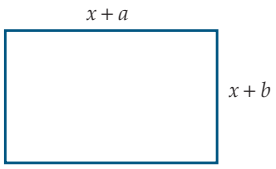
6 (a) $8x^2 + 14xy + 15y^2$ (b) $35x^2 + 44xy + 12y^2$
(c) $15x^2 - 44xy + 32y^2$ (d) $4x^2 + 28xy + 49y^2$
(e) $49x^2 + 84xy + 36y^2$ (f) $25x^2 + 80xy + 64y^2$
(g) $16x^2 - 8xy + y^2$ (h) $36x^2 - 60xy + 25x^2$
(i) $81x^2 - 126xy + 49y^2$ (j) $25y^2 - x^2$
(k) $9x^2 - y^2$ (l) $4x^2 - 9y^2$
(m) $6x^3 + 7x^2 - 5x$ (n) $15x^3 - 8x^2 - 12x$
(o) $24x^3 + 78x^2 + 63x$ (p) $-15x^3 + 8x^2 + 16x$
(q) $-30x^3 - 58x^2 - 24x$ (r) $-24x^3 + 76x^2 - 60x$

7 (a) $(x - 4)(3x + 1) = 3x^2 - 11x - 4$
(b) $(2x + 3)(2x + 3) = 4x^2 + 12x + 9$
(c) $\frac{1}{2}(x + 5)(x - 5) = \frac{1}{2}(x^2 - 25)$ or $\frac{1}{2}x^2 - \frac{25}{2}$

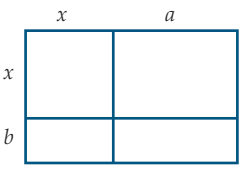
8 (a) $x^3 + (a + b)x^2 + abx$
(b) (i) $a + b$ (ii) ab
(iii) $a + b = -4$ and $ab = -12$. Using the expansion from (a) this shows that $x(x - 6)(x + 2) = x^3 - 4x^2 - 12x$.
(c) (i) $a = 25, b = -4$ (ii) $x^3 + 21x^2 - 100x$

9 (a) $(x - 2)(x)(x + 2)$
(b) $x^3 - 4x$
(c) It is easier to have x as the second number as the other two expressions, $(x - 2)$ and $(x + 2)$, form a difference of two squares which is simpler to multiply.

10 (a)



(b) $A = (x + a)(x + b)$ (c) $A = x^2 + ax + bx + ab$
(d)



(e) $x^2 + ax + bx + ab$. This is the same as the answer in part (c).

11 (a) $x + 5$ and $x - 2$
(b) $A = (x + 5)(x - 2) = x^2 + 3x - 10$
(c) x^2 and $5x$
(d) $2x + 10$
(e) $A = x^2 + 5x - 2x - 10 = x^2 + 3x - 10$
(f) The answers are the same.

12 (a) $1296 - x^2$ cm^2 (b) $(36 - x)(36 + x)$ (c) 1260 cm^2

13 (a) $x^2 - 2ax$ (b) $x^2 - 2ax$
(c) They are equivalent.
(d) Rectangular lawn area $= x(x - 2a)$, square lawn area $= (x - a)^2 - a^2$. They will cost equal amounts to turf as we have shown that these expressions are equivalent.
(e) $19 092

14 (a) $55 - y$ m (b) $x(55 - y)$ m^2
(c) $55x$ m^2 (d) $(x - 30)(55 - y)$ m^2
(e) $55x - xy - 1650 + 30y$ (f) $45x - 1350$ m^2
(g) $10x + 1350$

15 (a) $(x - 2)^2(x + 3)$ (b) 28 cm^3
(c) When $x = 5$, dimensions are 3 cm, 8 cm and 3 cm, so the volume $= 3 \times 8 \times 3 = 72$ cm^3.
(d) $x^3 - x^2 - 8x + 12$; 72 cm^3
(e) height and width 3 cm, length 8 cm
(f) $x = 3$ (g) 6 cm^3

16 If $(x^2 + a^2)^3 = 0$ then $(x^2 + a^2) = 0$ and $x^2 = -a^2$, which can only be true if x and a are both equal to zero. No other values of x and a are possible as you cannot have a square number equal to a negative number.

Open-ended – Sample answers

17 Examples: $a = 2, b = 1, c = 0, d = 1$; $a = 1, b = 0, c = 1, d = 1$

18 (a) $x^2 + y^2$
(b) Examples: $x = 3, y = 4$; $x = 5, y = 12$

19 (a) -4, -5 (b) 4, 5 (c) 2, -3

Exercise 2.2 (p. 39)

1 (a) $4(x + 2)$ (b) $3(6x - 5)$ (c) $3(8x - 3)$
(d) $x(y + 7)$ (e) $3k(4x + 7)$ (f) $4b(8a + 5)$
(g) $7y(x^2 + y)$ (h) $3x(3x + y)$ (i) $8m(6m + 1)$
(j) $-x(6 + x)$ (k) $-x(4 - x)$ (l) $-2a(12a - 5)$
(m) $6ab(1 - 3a)$ (n) $8xy(5x + 2y)$ (o) $2cd(5d - 1)$

2 (a) $(k + 1)(x + 4)$ (b) $(x - 4)(a + 6)$
(c) $(n - 7)(m + 2)$ (d) $(p + 3)(x - 9)$
(e) $(q - 2)(n - 7)$ (f) $(u - 4)(x - 5)$
(g) $(c + 5)(d + 1)$ (h) $(d + 2)(a - 1)$
(i) $(m + 7)(k - 1)$ (j) $(x - 3)(x + 5)$

3 (a) $(y + 5)(x + 3)$ (b) $(b + 2)(a + 7)$
(c) $(n + 4)(m + k)$ (d) $(y + 3)(x - 1)$
(e) $(3 + c)(a - b)$ (f) $(m + 2)(k - 4)$
(g) $(5b - 4)(3a - 2)$ (h) $(2b - 5)(3a - 2)$
(i) $(2n - 3)(4m - 3)$ (j) $(v - a)(x - y)$
(k) $(q - n)(p - r)$ (l) $(h - d)(g - f)$
(m) $(n + t)(m^2 + p)$ (n) $(ab - c)(a + b)$
(o) $(dk - r)(d - 1)$ (p) $(9y - 5)(3x + 2)$
(q) $(4a - 5)(9 - c)$ (r) $(5v - 3)(5w + 1)$
(s) $(8a^2 - 3b)(4ac - 5)$ (t) $(3d - 5f)(7d - 4h)$
(u) $(mr - 5)(4p + mn)$

4 (a) True (b) False (c) False
(d) True (e) False (f) False

5 D **6** B **7** A

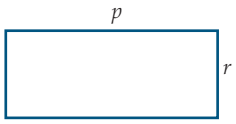
8 $12(2y - 3x)$ cents

9 $8x - x^2 = x(8 - x)$

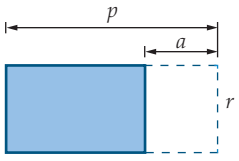
10 (a) $a(p + 5) + b(p + 5)$ (b) $(a + b)(p + 5)$

(c) $3a(p + 5)$

11 (a)

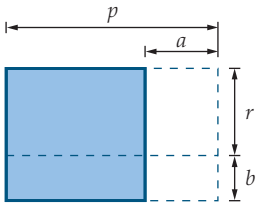


(b)



Area = $pr - ar$

(c)



Area added = $pb - ab$

(d) Area = $pr - ar + pb - ab$

(e) Area = $(r + b)(p - a)$

(f) part (e) is the factorised form of part (d)

12 (a) $\frac{2\pi r^3}{3} + \frac{\pi r^2 h}{3}$ (b) $\frac{\pi r^2}{3}(2r + h)$ (c) 150.82 cm^3

(d) Cone volume is $\frac{\pi r^2(2r)}{3} = \frac{2\pi r^3}{3}$, which equals the half-sphere volume of the ice-cream.

Open-ended – Sample answers

13 (a) $2x^2 + 4xy$ cm^2 (b) $2x(x + 2y)$

(c) $x = 12, y = 3; x = 8, y = 4$

Exercise 2.3 (p. 43)

1 (a) $(x + 2)(x + 4)$ (b) $(x - 3)(x + 6)$

(c) $(x - 1)(x + 7)$ (d) $(x + 5)(x - 2)$

(e) $(x - 4)(x - 3)$ (f) $(x - 4)(x + 3)$

(g) $(x + 4)(x - 2)$ (h) $(x + 1)(x - 3)$

(i) $(x - 8)(x - 2)$

2 (a) $(x + 1)(x + 2)$ (b) $(y + 2)(y + 7)$

(c) $(a - 4)(a - 6)$ (d) $(k + 3)(k + 8)$

(e) $(m - 5)(m + 3)$ (f) $(x - 6)(x + 5)$

(g) $(p - 2)(p - 8)$ (h) $(d - 1)(d - 4)$

(i) $(y - 3)(y - 2)$ (j) $(m - 6)(m + 2)$

(k) $(c - 7)(c - 3)$ (l) $(g + 7)(g - 5)$

(m) $2(y + 3)(y + 4)$ (n) $3(m + 2)(m + 4)$

(o) $5(x + 4)(x - 5)$ (p) $-4(a + 3)(a - 1)$

(q) $-3(k + 3)(k - 7)$ (r) $-5(x + 7)(x - 3)$

3 (a) $(z - 5)(z - 3)$ (b) $(k + 6)(k - 1)$

(c) $(z + 3)(z + 7)$ (d) $(t + 4)(t - 5)$

(e) $(r + 6)(r - 5)$ (f) $(t - 11)(t - 2)$

(g) $(z + 6)(z - 7)$ (h) $(m - 5)(m - 4)$

(i) $(y + 12)(y - 5)$ (j) $(n - 7)(n + 4)$

(k) $(p + 12)(p - 3)$ (l) $(a - 6)(a - 8)$

4 C

5 $(2x + 4)$ cm

6 $(3x - 12)$ mm

7 (a) $x - 30$ (b) 120 cm (c) 190 cm

8 (a) $2w^2 - (6w - 4) = 2w^2 - 6w + 4$

(b) $(2w - 2)(w - 2) = 2(w - 1)(w - 2)$

(c) 1 m

Open-ended – Sample answers

9 $x^2 - x - 2 = (x + 1)(x - 2)$ and $x^2 + x - 42 = (x + 7)(x - 6)$

10 (a) $x + 8$ and $x - 3$ are the factors so possible dimensions could be 12 units by 1 unit or 14 units by 3 units.

(b) Area = $12 \times 1 = 12$ $units^2$ or Area = $14 \times 3 = 42$ $units^2$.

Exercise 2.4 (p. 46)

1 (a) $(p - 2)(p + 2)$ (b) $(d - 10)(d + 10)$

(c) $(3 - k)(3 + k)$ (d) $3(p - 5)(p + 5)$

(e) $5(4 - m)(4 + m)$ (f) $8(k - 2)(k + 2)$

(g) $(2y - 7)(2y + 7)$ (h) $(5 - 4x)(5 + 4x)$

(i) $(6m - 5n)(6m + 5n)$ (j) $(3x - 4)(3x + 4)$

(k) $(2 - 5a)(2 + 5a)$ (l) $(9d - 8)(9d + 8)$

2 (a) $(x - 1)(x + 5)$ (b) $(a - 6)(a + 4)$

(c) $(-k - 1)(k + 9)$ or $-(k + 1)(k + 9)$

(d) $(5 - p)(7 + p)$ (e) $4(2 - y)(y - 1)$

(f) $3(1 - x)(3x - 1)$ (g) $5(x + 3 - y)(x + 3 + y)$

(h) $7(a + 5 - b)(a + 5 + b)$ (i) $3(x - 6)(x - 2)$

3 (a) $(x - \sqrt{2})(x + \sqrt{2})$ (b) $(\sqrt{7} - p)(\sqrt{7} + p)$

(c) $(2k - \sqrt{5})(2k + \sqrt{5})$ (d) $(3p - \sqrt{2})(3p + \sqrt{2})$

(e) $3(a - \sqrt{11})(a + \sqrt{11})$ (f) $2(b - \sqrt{3})(b + \sqrt{3})$

4 (a) $(x - y + 1)(x + y + 3)$ (b) $-(2a + 5)$

(c) $(2k - m + 6)(2k + m)$ (d) $16x$

(e) $(x - 2)(3x + 4)$ (f) $(4x + 3)(6x - 11)$

(g) $(x + 2 - \sqrt{3})(x + 2 + \sqrt{3})$ (h) $(y - 3 - \sqrt{6})(y - 3 + \sqrt{6})$

(i) $(x + 8 - \sqrt{5})(x + 8 + \sqrt{5})$

(j) $(x - 5 - \sqrt{13})(x - 5 + \sqrt{13})$

(k) $(x - 1 - \sqrt{19})(x - 1 + \sqrt{19})$

(l) $(2x + 1 - \sqrt{2})(2x + 1 + \sqrt{2})$

5 (a) $(x + 3)(x + 1)$ (b) $(a - 4)(a + 9)$

(c) $(x - y)(x + y + 7)$ (d) $(3a + b)(3a - b + 2)$

(e) $(x - 4y)(x + 4y + 2)$ (f) $(2p + q)(2p - q - 3)$

6 **(a)** $(x+3)^2$ **(b)** $(x+2)^2$ **(c)** $(x-7)^2$

(d) $(x-4)^2$ **(e)** $(x+1)^2$ **(f)** $(x+6)^2$

(g) $(2x+5)^2$ **(h)** $(3x+4)^2$ **(i)** $(5x-1)^2$

7 B

8 D

9 A

10 **(a)** $8(m+5n)(m-5n)$ **(b)** $6(x-2r)(x+2r)$

(c) $5(a-3c)(a+3c)$ **(d)** $3(3a-4b)(3a+4b)$

(e) $2(6x-7y)(6x+7y)$ **(f)** $7(5t-2p)(5t+2p)$

(g) $6(3k-2m)(3k+2m)$ **(h)** $2(xy-4z)(xy+4z)$

(i) $3(pq-2n)(pq+2n)$

11 **(a)** $(4c+5d)(4c-5d+2)$ **(b)** $(k-8)(k+8+m)$

(c) $(9-7a)(9+7a-3d)$ **(d)** $(4-3y)(4+3y-4w)$

(e) $(5m+3n)(5m-3n-4)$ **(f)** $(12p-7r)(12p-q+7r)$

12 **(a)** $x^2-y^2=56$ **(b)** $(x-y)(x+y)=56$

(c) The difference between consecutive odd numbers is 2.

(d) $4(y+1)=56$ **(e)** $x=15, y=13$

13 **(a)** $\frac{1}{2}(a-b)(a+b)$ cm^2 **(b)** $4b^2$ cm^2

(c) $a=15$ cm, $b=5$ cm

14 **(a)** $(a-b)(a+b)$

(b) two

(c) $a+b$

(d) $(a^2-b^2)(a^2+b^2)$

(e) $(a-b)(a+b)(a^2+b^2)$

(f) three

(g) $(a+b)(a^2+b^2)$

(h) $(a^4-b^4)(a^4+b^4)$

(i) $(a-b)(a+b)(a^2+b^2)(a^4+b^4)$

(j) $(a+b)(a^2+b^2)(a^4+b^4)$

(k) $(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)$

(l) $\begin{aligned}(2^{16}-1) &= (2^{16}-1^{16})\div(2-1)\\ &= (2+1)(2^2+1^2)(2^4+1^4)(2^8+1^8)\\ &= 3\times5\times17\times257\end{aligned}$

15 **(a)** x^2-y^2 m^2 **(b)** $2(x-y)$ m^2

(c) $(x-y)(x+y+2)$ m^2

16 **(a)** $\begin{aligned}a^2+b^2 &= (m-n)^2+(2\sqrt{mn})^2\\ &= m^2-2mn+n^2+4mn\\ &= m^2+2mn+n^2\\ &= (m+n)^2\\ &= c^2\end{aligned}$

(b) $\begin{aligned}t^2 &= (kr)^2-r^2\\ &= (kr-r)(kr+r)\\ &= r(k-1)r(k+1)\\ &= r^2(k-1)(k+1)\end{aligned}$

(c) $k=1$ or -1

(d) k cannot be equal to zero as this creates the impossible equation $a^2=-b^2$, and k cannot be a fraction between -1 and 1 as this creates a similar situation as when k is equal to zero, where a^2 is equal to a negative fraction of b^2.

17 **(a)** x^2 cm^2

(b) 36 cm^2

(c) x^2-36 cm^2

(d) $(x-6)(x+6)$

(e) Length is $x+6$ cm and width is $x-6$ cm.

(f) The factors in part **(d)** correspond to the length and width of the rectangle.

Open-ended – Sample answers

18 $2x^2+8x-40$ and x^2-8x-9

19 $a^2-b^2+3a+3b=(a+b)(a-b+3)$

Half-time 2 (p. 50)

1 **(a)** $x^2+8x+12$ **(b)** $x^2+2x-35$

(c) $3x^2+13x-10$ **(d)** $2x^2+5x-3$

2 **(a)** $(x+5)(x-3)$ **(b)** $(x-2)(x-12)$

(c) $(x+5)(x-8)$

3 **(a)** $(x+5)(x+7)$ **(b)** $(x-3)(x-4)$

(c) $(x+7)(x-3)$

4 **(a)** $(x+7)^2$ **(b)** $(x-9)^2$ **(c)** $(3x-5)^2$

5 **(a)** $7(x+2y)$ **(b)** $6x(3x+4y)$

(c) $(x-2)(3+y)$ **(d)** $(x+5)(3+2y)$

6 **(a)** x^2-9 **(b)** x^2-64

(c) $9x^2-y^2$ **(d)** $36x^2-25y^2$

7 **(a)** x^3+5x^2+6x **(b)** $3x^2-12$

(c) $63x^3+150x^2-141x-228$ **(d)** $48x^3-156x^2+36x$

8 **(a)** $(x-3)(x+3)$ **(b)** $9(x-2y)(x+2y)$

(c) $2(2x-5y)(2x+5y)$ **(d)** $3(3-4x)(3+4x)$

9 D **10** C **11** B

Exercise 2.5 (p. 52)

1 **(a)** $(x+4-y)(x+4+y)$ **(b)** $(y-2-k)(y-2+k)$

(c) $(y+4-m)(y+4+m)$ **(d)** $(m-5-2n)(m-5+2n)$

(e) $(x+1-3y)(x+1+3y)$ **(f)** $(q+3-4p)(q+3+4p)$

(g) $(a-3b-4)(a-3b+4)$ **(h)** $(x-5y-6)(x-5y+6)$

(i) $(2a-3-4b)(2a-3+4b)$ **(j)** $(3c+5-2d)(3c+5+2d)$

2 **(a)** $(x+3-\sqrt{7})(x+3+\sqrt{7})$

(b) $(x-5-\sqrt{21})(x-5+\sqrt{21})$

(c) $(x-4-\sqrt{19})(x-4+\sqrt{19})$

(d) $(x+10-\sqrt{23})(x+10+\sqrt{23})$

(e) $(x-6-\sqrt{15})(x-6+\sqrt{15})$

(f) $(x-2)(x+4)$

(g) $(x-7-\sqrt{51})(x-7+\sqrt{51})$

(h) $(x-2-\sqrt{6})(x-2+\sqrt{6})$

(i) $(x+8-\sqrt{17})(x+8+\sqrt{17})$

(j) $\left(x+\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\left(x+\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$

(k) $\left(x+\frac{7}{2}-\frac{\sqrt{13}}{2}\right)\left(x+\frac{7}{2}+\frac{\sqrt{13}}{2}\right)$

(l) $\left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)$

(m) $\left(x-\frac{5}{2}-\frac{\sqrt{5}}{2}\right)\left(x-\frac{5}{2}+\frac{\sqrt{5}}{2}\right)$

(n) $\left(x+\frac{5}{2}-\frac{\sqrt{33}}{2}\right)\left(x+\frac{5}{2}+\frac{\sqrt{33}}{2}\right)$

(o) $\left(x+\frac{1}{2}-\frac{\sqrt{21}}{2}\right)\left(x+\frac{1}{2}+\frac{\sqrt{21}}{2}\right)$

(p) $\left(x-\frac{9}{2}-\frac{\sqrt{29}}{2}\right)\left(x-\frac{9}{2}+\frac{\sqrt{29}}{2}\right)$

(q) $\left(x-\frac{3}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{3}{2}+\frac{\sqrt{17}}{2}\right)$

(r) $\left(x-\frac{7}{2}-\frac{\sqrt{53}}{2}\right)\left(x-\frac{7}{2}+\frac{\sqrt{53}}{2}\right)$

3 (a) $2(x+1-\sqrt{6})\,(x+1+\sqrt{6})$

(b) $4(x-2-\sqrt{5})\,(x-2+\sqrt{5})$

(c) $10(x-4-\sqrt{19})\,(x-4+\sqrt{19})$

(d) $3(x-1-\sqrt{5})\,(x-1+\sqrt{5})$

(e) $5(x+2-\sqrt{5})\,(x+2+\sqrt{5})$

(f) $7(x+3-\sqrt{7})\,(x+3+\sqrt{7})$

4 D **5** A

6 (a) 14 m (b) $(t-4-\sqrt{2})\,(t-4+\sqrt{2})$

(c) 2.83 seconds

(d) The seagull is at sea level.

(e) The seagull is below sea level, catching a fish.

7 (a) $-5(x-4)^2+75=-5(x-4-\sqrt{15})(x-4+\sqrt{15})$

(b) $P=-5(4\pm\sqrt{15}-4)^2+75$
$=-5(\pm\sqrt{15})^2+75$
$=-5\times15+75=0.$

(c) When $x=0$, $P=-5$, so the company initially operated at a loss. When $x=4$, $P=75$, so the company had made a healthy profit after 4 months. When $x=8$, $P=-5$, so the company made a loss again after 8 months.

8 (a) $x^2+14x+35$ (b) $(x+7-\sqrt{14})\,(x+7+\sqrt{14})$

(c) $8-\sqrt{14}$ km (d) 112.75 km

9 (a) 100 worms

(b) $-(d+2+2\sqrt{26})(d+2-2\sqrt{26})$

(c) When $d=2(\sqrt{26}-1)$, $(d+2-2\sqrt{26})=0$, so no earthworms remain.

(d) 8.20, so the number of earthworms reduced to zero during the 9th day.

(e) (i) $-(n+1+\sqrt{65})(n+1-\sqrt{65})$

(ii) When $n=\sqrt{65}-1$, $(n+1-\sqrt{65})=0$, so no earthworms remain.

(f) $\sqrt{65}-1=7.06$, so the first colony will stay longer in their compost heap.

Open-ended – Sample answers

10 (a) $(x-a)^2+2b-a^2$ (b) $A=2b-a^2$

(c) (i) $2b-a^2=0$ (ii) $a=2,\ b=2$

Exercise 2.6 (p. 57)

1 (a) $\frac{4x}{5}$ (b) $\frac{k}{7}$ (c) $\frac{5p}{9}$

(d) $2(a-b)$ (e) $\frac{3(x+1)}{2}$ (f) $\frac{7(y-2)}{4}$

(g) $\frac{y(a-4)}{2}$ (h) $\frac{2x(x+5)}{9}$ (i) $\frac{3x(x-4)}{4}$

2 (a) $\frac{x+6}{x-5}$ (b) $\frac{x+2}{x-2}$ (c) $\frac{2x-3}{2x+1}$

(d) $\frac{x-2}{x-7}$ (e) $\frac{b+5}{b+3}$ (f) $\frac{x+7}{x-1}$

3 (a) $\frac{1}{3}$ (b) $\frac{3(5+2x)}{5-2x}$ (c) $\frac{x+4}{x-4}$

(d) $\frac{x-3}{x+3}$ (e) $\frac{x+7}{8}$ (f) $\frac{7}{4}$

(g) $\frac{(x-3)(x+2)}{(x-4)(x-1)}$ (h) $\frac{x+4}{x+1}$

(i) $\frac{(3x-2)(x-3)}{(x-6)(x+2)}$ (j) $\frac{(5x-3)(x-3)}{(x-4)(5x-4)}$

4 (a) $x+8$ (b) $x-9$ (c) $\frac{1}{x-5}$

(d) $\frac{1}{x-7}$ (e) $\frac{2x-1}{2}$ (f) $3(x-2)$

(g) $\frac{2x+3}{2}$ (h) $\frac{3x+5}{2}$ (i) $\frac{x-1}{x-8}$

(j) $\frac{x+4}{x+5}$ (k) $\frac{x+1}{x-2}$ (l) $\frac{x-1}{x+4}$

(m) 1 (n) $\frac{5}{6}$ (o) $\frac{x-5}{6(x+3)}$

(p) $\frac{8(x-7)}{x+5}$ (q) $\frac{x-2}{x+2}$ (r) $\frac{x-3}{x-7}$

5 (a) $\frac{3(x-2)}{x+3}$ (b) $\frac{5(x+5)}{3(x-7)}$ (c) $\frac{x+6}{x+7}$

(d) $\frac{(x+2)(x+1)}{(x+5)(x+8)}$ (e) $\frac{5(x+y+3)}{4}$ (f) $\frac{7(x-1-2y)}{3(3x-2)}$

6 (a) $\frac{5x}{6}$ (b) $\frac{7x}{10}$ (c) $\frac{5x}{12}$

(d) $\frac{x}{2}$ (e) $\frac{11x}{15}$ (f) $\frac{-7x}{12}$

(g) $\frac{8x-3y}{6}$ (h) $\frac{20x-21y}{28}$ (i) $\frac{99x-64y}{72}$

(j) $\frac{5x+22}{6}$ (k) $\frac{3x-2}{8}$ (l) $\frac{4x-16}{9}$

(m) $\frac{3x-8}{12}$ (n) $\frac{13x-34}{20}$ (o) $\frac{13x+14}{36}$

(p) $\frac{29x+2y}{12}$ (q) $\frac{38x-19y}{15}$ (r) $\frac{34x+5y}{12}$

(s) $\frac{9x-8y}{30}$ (t) $\frac{3x-4y}{4}$ (u) $\frac{17x+11y}{6}$

7 C

8 (a) $\frac{3(b-a)}{3b+a}$ (b) $\frac{3(7-5)}{3\times7+5}=\frac{3}{13}$

9 (a) $(a-b)(a^2+ab+b^2)=a^3+a^2b+ab^2-a^2b-ab^2-b^3=a^3-b^3$

(b) $(a^2-b^2)(a^4+a^2b^2+b^4)$

(c) $\frac{a^6-b^6}{a^2-b^2}=\frac{(a^2-b^2)(a^4-a^2b^2+b^4)}{a^2-b^2}=a^4+a^2b^2+b^4$

10 (a) $4w^2-\pi w^2$ (b) $\frac{4-\pi}{4}$

11 (a) $\frac{3m}{4\pi r^3}$ kg/m^3 (b) Students' answers. (c) $\frac{4}{25}$

Open-ended – Sample answers

12 $\frac{x+1}{2}+\frac{1-x}{6}$

Exercise 2.7 (p. 62)

1 (a) (i) $s=\pm\sqrt{A}$ (ii) $s=\pm3\sqrt{3}$

(b) (i) $v=\pm\sqrt{\frac{2K}{m}}$ (ii) $v=\pm\sqrt{5}$

(c) (i) $v=\pm\sqrt{\frac{Fr}{m}}$ (ii) $v=\pm\sqrt{10.5}$

(d) (i) $r=\pm\sqrt{\frac{Gm_1m_2}{F}}$ (ii) $r=\pm\sqrt{33.75}$

(e) (i) $r_2=\pm\sqrt{\frac{g_1r_1^2}{g_2}}$ (ii) $r_2=\pm14\sqrt{2}$

(f) (i) $V=\pm\sqrt{PR}$ (ii) $V=\pm3\sqrt{7}$

(g) (i) $I=\pm\sqrt{\frac{P}{R}}$ (ii) $I=\pm\sqrt{\frac{20}{9}}$

(h) (i) $v=\pm\sqrt{\frac{2E}{c}}$ (ii) $v=\pm4\sqrt{2}$

(i) (i) $D=\pm\sqrt{\frac{4A}{\pi}}$ (ii) $D=\pm6$

2 (a) (i) $A_y=A^2-A_x$ (ii) $A_y=38$

(b) (i) $c=\frac{b^2-(2ax+b)^2}{4a}$ (ii) $c=5$

(c) (i) $T=\frac{9r^2}{l^2}$ (ii) $T=64$

(d) (i) $z=y-x^2$ (ii) $z=-96$

(e) (i) $z=\frac{y}{x^2}$ (ii) $z=0.28$

(f) (i) $b=cx^2-a$ (ii) $b=16$

(g) (i) $y=x^2+z$ (ii) $y=22$

(h) (i) $c=\frac{a+b}{x^2}$ (ii) $c=\frac{1}{3}$

(i) (i) $y=zx^2$ (ii) $y=96$

3 C

4 (a) $r=\sqrt{\frac{A}{\pi}}$

(b) The radius is a distance and must be positive.

(c) 3.57 cm (2 d.p.)

5 (a) $r=\sqrt[3]{\frac{3V}{4\pi}}$

(b) Because it is a cube root and not a square root.

(c) 3.10 cm (2 d.p.)

6 (a) $h=\frac{3V}{\pi r^2}=1.52$ m (2 d.p.)

(b) $r=\sqrt{\frac{3V}{\pi h}}=4.37$ cm (2 d.p.)

7 (a) 41m/s (b) The car is slowing down.

8 (a) $k=1243.57$ (b) 2.1 Hz

Open-ended – Sample answers

9 (a) Line 3 does not have ± symbol.

(b) One possible explanation is that if $x^2=9$, then x can be 3 or -3, since $(3)^2=9$ and $(-3)^2=9$.

10 (a) $a=2b+c^2$ $c=\pm\sqrt{a-2b}$

(b) $d=2e+\sqrt{f}$ $f=d^2-4de+4e^2$

Chapter review 2 (p. 65)

1 (a) $10x+35$ (b) x^2-x-6

(c) $x^2+10x+24$ (d) $8x^2+10x-3$

2 (a) $x^2+12x+36$ (b) $4x^2-12x+9$

(c) $25-x^2$ (d) $9x^2-49$

3 (a) $4x^2-20x-56$ (b) $-3x^3-15x^2+72x$

(c) $x^3-2x^2-29x+30$ (d) $-2x^3-7x^2+10x+24$

4 (a) $7(x+3)$ (b) $4a(3b-2)$ (c) $6x(3x+4)$

5 (a) $(c-4)(d+6)$ (b) $(x+2)(x-3)$

(c) $(a+1)(5a-2)$ (d) $(3y-5)(2y-7)$

6 (a) $(x+5)(x+6)$ (b) $(x+2)(x-12)$

7 (a) $(x+5)^2$ (b) $(x-9)^2$ (c) $(3x-4)^2$ (d) $(2x+1)^2$

8 (a) $(x-3)(x+3)$ (b) $(5-y)(5+y)$

(c) $(2x-y)(2x+y)$ (d) $(7x-4)(7x+4)$

(e) $(6x-9y)(6x+9y) = 9(2x-3y)(2x+3y)$

(f) $(8-3x)(8+3x)$ (g) $(2x-7)(2x-3)$

(h) $-13(2x+3)$ (i) $5(2x-1)$

(j) $(x-\sqrt{3})(x+\sqrt{3})$ (k) $(x+7-\sqrt{5})(x+7+\sqrt{5})$

(l) $(x-4-\sqrt{7})(x-4+\sqrt{7})$

9 (a) $(x+3-\sqrt{11})(x+3+\sqrt{11})$

(b) $(x-5-\sqrt{2})(x-5+\sqrt{2})$

(c) $(x-2-\sqrt{11})(x-2+\sqrt{11})$

(d) $(x+4-\sqrt{3})(x+4+\sqrt{3})$

10 (a) $\frac{2x}{5}$ (b) $\frac{a-3}{3}$ (c) $\frac{5(x-1)}{4}$ (d) $3a$

11 (a) $\frac{x+4}{x+2}$ (b) $\frac{(x+1)(3x+7)}{(x-7)(x-1)}$

(c) $2x-1$ (d) $\frac{2(x+5)}{x+4}$

12 (a) $\frac{7x}{6}$ (b) $\frac{8x-7}{15}$ (c) $\frac{4x-31}{30}$

(d) $\frac{12x+21y}{14}$ (e) $\frac{3x+4y+23}{12}$ (f) $\frac{2x-y-9}{10}$

13 (a) $b = \frac{A}{l}$ (b) $l = \frac{A^2 g}{4\pi^2}$

(c) $r = \pm\sqrt{\frac{SA}{4\pi}}$ (d) $x = \pm\sqrt{\frac{2(a+z)}{y}}$

14 $a = 3, b = -2$

15 (a) -5, 6 (b) 6 kg gain (c) 5 kg loss

(d) From April to September John lost 11 kg in weight.

16 (a) $ax+2x+ay+2y$ (b) $(a+2)(x+y)$

(c) 175 cm

17 (a) (i) E (ii) B (iii) D

(iv) F (v) A (vi) C

(b) (i) $(x+8)(x-3)$ (ii) $(5a-b)(5a+b)$

(iii) $(x+3-y)(x+3+y)$ (iv) $(x+6-\sqrt{2})(x+6+\sqrt{2})$

(v) $x(16x-25)$ (vi) $(x-y)(4+x+y)$

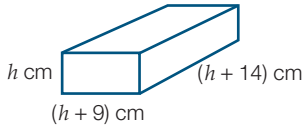
18 (a) $(x+2)(x+8)$ (b) $18(x+2)(x+8)$ (c) \$720

19 $16x^2+24xy+9y^2$

20 (a) $5x-x^2$ m^2 (b) $x(5-x)$

(c) (i) 4 m^2 (ii) 2.25 m^2 (iii) 3.1875 m^2

21 (a)



(b) $h(h+9)(h+14)$

(c) h^3+23h^2+126h (d) 1800 cm^3

(e) 1800 cm^3 (f) Students' own answers.

22 (a) πx^2 (b) $x+2$ (c) $\pi(x+2)^2$

(d) $x-2$ (e) $\pi(x-2)^2$

(f) $\pi(x+2)^2-\pi(x-2)^2$ (g) π

(h) $8\pi x$ (i) 80π (j) 17 mm

23 (a) $(8x+3)(2y+7)$ cm^2 (b) $(8x+3)(8x-3)$ cm^2

(c) $64x^2-9$ cm^2

(d) (i) $y = 4x-2$ (ii) $64x^2+48x+9$

24 (a) $x^2+p^2x+p^2y-y^2$ (b) $(x+y)(x-y+p^2)$

(c) $(x+y)(x-y+p^2) = (x+y)(x-y)$; $x-y$ must be positive, so x must be greater than y.

(d) (i) $2p^2x$ or $2p^2y$ vehicles

(ii) $2px(p-1)$ non-white vehicles

Chapter 3

Recall 3 (p. 69)

1 (a) 625 (b) 29.791 (c) 3.45

2 (a) $A = 16$ (b) $h = 5$ (c) $b = 7$

3 (a) 410 cm (b) 425 000 cm (c) 400 000 cm^2

(d) 0.0578 m^2 (e) 8 250 000 m^2 (f) 0.6 L

(g) 20 mL (h) 30 800 L (i) 0.007 25 m^3

4 (a) 6.11 m (b) 16.11 m (c) 15.27 m^2

5 (a) (i) 15.59 cm (ii) 140.3 cm^2

(b) (i) 22.36 cm (ii) 447.21 cm^2

(c) (i) 4.36 m (ii) 26.18 m^2

(d) (i) 1.5 m (ii) 6 m^2

Exercise 3.1 (p. 72)

1 (a) 1263 mm^2 (b) 583 cm^2 (c) 157.5 cm^2

(d) 349.94 mm^2 (e) 25.63 mm^2 (f) 493.65 cm^2

2 (a) 76.37 cm^2 (b) 43.37 cm^2 (c) 48.29 cm^2

(d) 1809.56 cm^2 (e) 18.27 mm^2 (f) 88.5 m^2

(g) 422.43 mm^2 (h) 13.72 m^2 (i) 69.53 m^2

3 C **4** C

5 (a) 123.61 cm^2 (b) 52.15 cm^2 (c) 47.12 cm^2

(d) 32.47 cm^2 (e) 2194.29 cm^2 (f) 441.06 cm^2

(g) 437.5 cm^2 (h) 294 cm^2

6 triangle: 173 m^2; sector: 1047 m^2

7 (a) 156.25 hours = 22.32 days (b) 75 000 m^2

8 9.78 m^2 **9** 25.13 m^2

10 (a) 0.05 m^2 (b) 190.42 m^2

11 4×4; 3×6 **12** 4.51 m

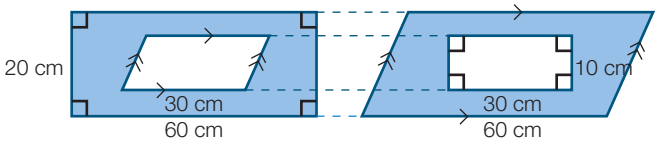
13 A, B, F: 50 cm^2; C, G: 25 cm^2; D, E: 100 cm^2

Open-ended – Sample answers

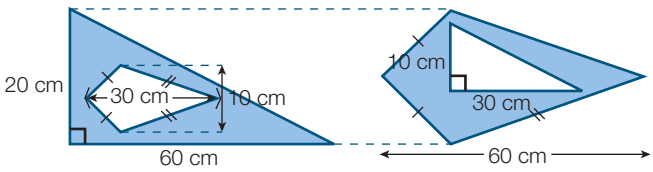
14 **(a)** 26 cm^2 **(b)** various answers

15 **(a)** The formulas for the overall area and the subtracted area are the same. The substituted values are the same hence the calculations are identical.

(b) Possible pair of shapes:



(c) Another possible pair of shapes:



Exercise 3.2 (p. 77)

1 **(a)** 766.14 m^2 **(b)** 82.14 m^2 **(c)** 376 cm^2

(d) 1710 cm^2 **(e)** 108 cm^2 **(f)** 684 cm^2

2 **(a)** 904.78 cm^2 **(b)** 1258.21 mm^2

3 **(a)** D **(b)** D

4 6300 cm^2 **5** 810.27 cm^2 **6** 12.25 m^2

Open-ended – Sample answers

7 **(a)** $2(lw + wh + lh)$

(b) Prism 1: $l = 10$ cm, $w = 5$ cm, $h = 2$ cm, SA = 160 cm^2

Prism 2: $l = 20$ cm, $w = 10$ cm, $h = 4$ cm, SA = 640 cm^2

The surface area quadruples when the dimensions are doubled.

8 **(a)** $r = 2$, $h = 5$, surface area = 28π

(b) $r = 2$, $h = 10$, surface area = 48π

(c) $r = 4$, $h = 5$, surface area = 72π

(d) Doubling the height increases the area of the curved face but not the circular ends. Doubling the radius increases the area of both the curved face and the circular ends. Also, the effect of doubling the radius is magnified, as the radius is then squared in the formula.

Exercise 3.3 (p. 81)

1 **(a)** 450 cm^3 **(b)** 3192 cm^3 **(c)** 5355 mm^3

(d) 585 cm^3 **(e)** 90 053.75 mm^3 **(f)** 552.92 m^3

2 374.36 mL

3 **(a)** 115 000 cm^3 **(b)** 20 250 mm^3 **(c)** 9625 cm^3

(d) 162 m^3 **(e)** 135 000 cm^3 **(f)** 840 cm^3

4 **(a)** B **(b)** D

5 1.31 m^3 **6** 0.07 m^3 **7** 6.64 m^3

8 21 082 L

Open-ended – Sample answers

9 **(a)** $4 \times 5 \times 10$ cm, $2 \times 10 \times 10$ cm, $1 \times 10 \times 20$ cm

(b) $8 \times 2.5 \times 10$ cm, $4 \times 4 \times 12.5$ cm, $0.5 \times 40 \times 10$ cm

(c) $4 \times 5 \times 10$: SA = 220 cm^2

$2 \times 10 \times 10$: SA = 280 cm^2

$1 \times 10 \times 2$: SA = 460 cm^2

10 **(a)** Anita has used the diameter value for the radius and did not convert one of the values to cm.

(b) 628 318.53 cm^3

(c) Point 1: Identify radius or diameter before using circle formulas; point 2: Check for matching units before substituting values into formulas.

Half-time 3 (p. 84)

1 **(a)** 19 101 cm^2 **(b)** 2228 mm^2

2 Surface area = 14π cm^2, volume = 5π cm^3

3 **(a)** 44 m^2 **(b)** 12.96 m^2

4 Volume A = $xy^2\pi$ m^3, Volume B = $4xy^2\pi$ m^3, Volume C = $xy^2\pi$ m^3

B has the largest volume

5 **(a)** 191 038.85 m^3 **(b)** 1323.05 m^3

6 72 206 cartons

Exercise 3.4 (p. 87)

1 **(a)** 576 cm^2 **(b)** 21 m^2

2 **(a)** 1884.96 cm^2 **(b)** 490.09 cm^2

3 **(a)** 20 106.19 cm^2 **(b)** 3848.45 cm^2

4 **(a)** 3402.34 m^2 **(b)** 829.38 cm^2 **(c)** 141 m^2

(d) 1599.56 cm^2 **(e)** 3619.11 cm^2 **(f)** 600 cm^2

(g) 9424.78 cm^2 **(h)** 1413.72 cm^2

5 **(a)** C **(b)** D

6 483.81 cm^2 **7** 414 cm^2 **8** 70 185 m^2

9 **(a)** Surface area of bowling ball = 2827.43 cm^2 (2 d.p.); area of material = 3200 cm^2. The area of material is larger than the surface area of the bowling ball.

(b) The material is not long enough to wrap around the ball, so he will need to cut and sew his existing material to wrap it. $C = \pi d = \pi \times 30 = 94.25$ cm (2 d.p.)

10 **(a)** 31 cm **(b)** 12 cm

Open-ended – Sample answers

11 $r = 5$, $s = 10$ gives a surface area of 75π cm^2 (235.62 cm^2)

$r = 6$, $s = 8$ gives a surface area of 84π cm^2 (263.89 cm^2)

12 The slant height is the hypotenuse of a right-angled triangle. The radius is one of the shorter sides.

13 **(a)** $h = 4$ m, $r = 3$ m, total surface area = 24π m^2

(b) total surface area = $3\pi(3 + \sqrt{73})$ m^2

(c) $h = 12$ m, $r = 5$m, total surface area = 90π m^2

(d) total surface area = $5\pi(5 + \sqrt{601})$ m^2

(e) No

14 (a) $h = 15$ m, $r = 8$m, total surface area = 200π m^2

(b) total surface area = $16\pi(16 + \sqrt{481})$ m^2

(c) $h = 24$ m, $r = 7$ m, total surface area = 224π m^2

(d) total surface area = $14\pi(14 + \sqrt{821})$ m^2

(e) No

Exercise 3.5 (p. 93)

1 (a) 716.28 cm^3 (b) 804.25 cm^3 (c) 66 666.67 cm^3

(d) 25 200 cm^3 (e) 385 mm^3 (f) 2338.27 cm^3

2 (a) 0.52 m^3 (b) 179 594.38 cm^3

3 (a) 150.53 cm^3 (b) 2900 cm^3

(c) 160.22 cm^3 (d) 197.60 cm^3

4 1 098 066 219 000 km^3

5 D **6** A

7 Pyramid is 1287.61 cm^3 larger.

8 157 mL **9** 129 mL **10** 70

11 (a) 28 (b) 22 338

(c) Calculations for volume involves cubing the radius.

(d) 1253 Earths in Jupiter, 688 Earths in Saturn, 50 Earths in Uranus, 60 Earths in Neptune

12 340 trucks

Open-ended – Sample answers

13 (a) $b = 5$ cm, $h = 9$ cm; volume = 75 cm^3

(b) Volume = 300 cm^3

(c) 4 times greater ($300 \div 75 = 4$)

(d) $b = 6$ cm, $h = 10$ cm; Volume of first pyramid = 120 cm^3

Volume of second pyramid = 480 cm^3

Conclusion: When the base of any square-based pyramid is doubled, the volume is quadrupled.

14 (a) $h = 4$ m, $r = 3$m, volume = 12π m^3

(b) volume = 24π m^3

(c) $h = 12$ m, $r = 5$m, volume = 100π m^3

(d) volume = 200π m^3

(e) When the height of a cone is doubled, the volume of the cone is also doubled.

15 (a) $h = 15$ m, $r = 8$ m, volume = 320π m^3

(b) volume = 1280π m^3

(c) $h = 24$ m, $r = 7$ m, volume = 392π m^3

(d) volume = 1568π m^3

(e) When the radius of a cone is doubled, the volume of the cone is multiplied by 4.

Exercise 3.6 (p. 98)

1 (a) 7.1 g/cm^3 (b) 6.4 g/cm^3 (c) 0.8 g/cm^3

2 (a) 6240 g (b) 486 g (c) 3.96 kg

3 (a) 13.27 cm^3 (b) 694.29 mL (c) 15.29 m^3

4 20 g/L **5** 2.4 g/100 g

6 (a) D (b) B

7 7.6 mg/mL

8 (a) 4 cups, 16 cups (b) 200 mL/L

9 (a) 131.175 kg (b) 22 t

10 0.04 g/L

11 (a) wooden log: 0.40 g/cm^3; steel rod: 7.81 g/cm^3; cork: 0.22 g/cm^3; aluminium cylinder: 2.65 g/cm^3; ice block: 0.91 g/cm^3; gold bar: 18.29 g/cm^3

The wooden log, cork and ice block will all float on water.

(b) lead log: 2 852 562 g; lead rod: 726 g; lead cork: 103 g; lead cylinder: 856 g; lead block: 125 g; lead bar: 9931 g

12 (a) Johnstone's abattoir (50 kg/day compared with Arvon River 40 kg/day)

(b) 0.44 mg/L

Open-ended – Sample answers

13 (a) 875 organisms/100 mL (b) no

(c) 600 organisms/50 mL; 2000 organisms/100 mL; 800 organisms/60 mL

Chapter review 3 (p. 101)

1 (a) 925 cm^2 (b) 878.43 cm^2 (c) 5105.09 cm^2

2 B

3 (a) 1550 cm^2 (b) 7.49 m^2

4 C

5 (a) 1080 cm^3 (b) 21 205.75 cm^3 (c) 5040 cm^3

6 (a) 1884.96 cm^2 (b) 11 309.73 cm^2

7 (a) 527.79 cm^3 (b) 268 082.57 mm^3

8 A **9** 30 kg/m^3 **10** 250 mL/L

11 1800 bricks

12 (a) 15 708 L (b) 5109 L

13 1.70 m^3

14 (a) 1885 cm^2, 6807 cm^3 (b) 327 cm^2, 302 cm^3

15 (a) 7527 m^2 (b) 361 m

16 (a) 2000 cm^3 (b) 15 g

17 335 cm^3 **18** 804 L **19** 9839 kg

20 (a) 1950 kL (b) 550 kL

21 21.99 cm

22 (a) 62 832 m^3 (b) $9 424 778

(c) 12 566 m^2 (d) $1 256 637

(e) $1 800 000 (f) $12 481 415

(g) $6 300 000 (h) the road over the top by $6 181 415

23 4.73 g/L

Mixed Review A (p. 105)

1 $957.04

2 **(a)** 2786.89 cm^2 **(b)** 256.31 m^2

3 **(a)** $3x^2 - 5xy + 2y^2$ **(b)** $6a^2 - 27ab + 30b^2$

(c) $15t^2 - 33ht - 36h^2$

4 **(a)** 7% **(b)** 7% **(c)** 7.19% **(d)** 7.23%

5 **(a)** $(x-2)^2$ **(b)** $(x+3)(x-3)$

(c) $(x+7)^2$ **(d)** $25(2x+1)(2x-1)$

(e) $2x(2y^2 + y - 4)$ **(f)** $4(x+4)^2$

6 **(a)** $-\frac{8x}{35}$ **(b)** $\frac{11x+43}{30}$

7 **(a)** $-x^2 - 5x + 66$ **(b)** $-x^2 - 2x + 24$

(c) $2x^2 + 5x - 12$ **(d)** $7y^2 - 5y - 2$

8 **(a)** $5516.35 **(b)** $5530.10 **(c)** $5533.23

9 **(a)** 301.59 cm^3 **(b)** 164.64 cm^3

10 **(a)** $(x-9)(x+3)$ **(b)** $(x-5)(x-2)$ **(c)** $(3x-2)(2x+1)$

11 166.7 g/L

12 **(a)** $25x^2 - 10x + 1$ **(b)** $16x - 24x + 9$

(c) $64 - 48x + 9x^2$ **(d)** $36 - 60x + 25x^2$

13 Option A ($6248.58 after 3 years) is better than Option B ($6127.96 after 3 years)

14 **(a)** $920 **(b)** $1200.61

15 384 cm^2

16 **(a)** 81 cm^2 **(b)** 66.21 cm^2 **(c)** 37.12 cm^2

17 **(a)** 24.5 m^2 **(b)** 34.1 m

18 **(a)** $(x+5)(x+3)$ **(b)** $x^2 + 8x + 15$

(c) **(i)** 16 m^2 **(ii)** 63 m^2

(d) $504

19 20 mL

Challenge A (p. 107)

1 $1000 - 1 + 3 - 5 + 7 \ldots$
$- 197 + 199 - 201 = 1000 + 50 \times 2 - 201 = 899$

2 $18x + 23y + 52z + 69w = 733$, $x + y + z + w = 20$. One of x and w must be odd, as must one of x and z. Use trial and error or a spreadsheet to obtain $x = 3$, $y = 10$, $z = 2$, $w = 5$. That is, 3 gifts at $18, 10 gifts at $23, 2 gifts at $52 and 5 gifts at $69.

3 B

4 D

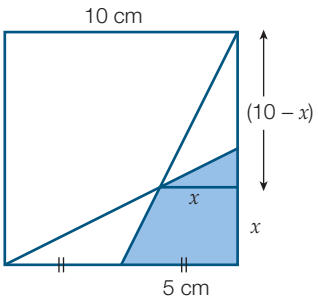
$35x - 35 < 75 < 21x + 84$. $35x < 110$, $x < 3\frac{1}{7}$ and $-9 < 21x$,

$x > -\frac{3}{7}$. $-\frac{3}{7} < x < 3\frac{1}{7}$, $x = 0, 1, 2, 3$.

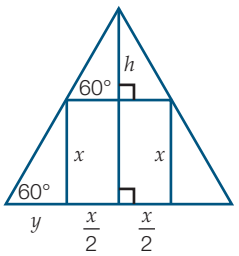
5 D

6 $\frac{5}{10} = \frac{x}{10-x}$, $50 - 5x = 10x$, $3x = 10$, $x = \frac{10}{3}$

Shaded area $= x^2 + x \times (5 - x) = 5x = \frac{50}{3} = 16\frac{2}{3}$ cm^2



7



$\frac{x}{y} = \tan 60°$, $\frac{x}{y} = \sqrt{3}$, $y = \frac{x}{\sqrt{3}}$

Base of equilateral triangle $= x + 2y = x + \frac{2x}{\sqrt{3}} = \frac{(3 + 2\sqrt{3})x}{3}$

$\frac{h}{\frac{x}{2}} = \tan 60°$, $h = \frac{x \tan 60°}{2} = \frac{x\sqrt{3}}{2}$

Base of equilateral triangle $= x + 2y = x + \frac{2x}{\sqrt{3}} = \frac{(2 + \sqrt{3})x}{2}$

Height of equilateral triangle $= x + h = x + \frac{\sqrt{3}x}{2} = \frac{(2 + \sqrt{3}x)x}{2}$

Area of square $= x^2$

Area of triangle $= \frac{1}{2} \times \frac{(3 + 2\sqrt{3})x}{3} \times \frac{(2 + \sqrt{3})x}{2}$

$= \frac{(12 + 7\sqrt{3})x^2}{12}$

Square as fraction of triangle $= x^2 \div \frac{(12 + 7\sqrt{3})x^2}{12}$

$= \frac{12}{12 + 7\sqrt{3}}$, 49.7%

OR, using similar triangles, $\frac{h}{\frac{x}{2}} = \frac{x}{y}$ so $h = \frac{x^2}{2y}$

Base of triangle $= x + 2y$

Height of triangle $= h + x = \frac{x^2}{2y} + x = \frac{x(x + 2y)}{2y}$

Area of triangle $= \frac{1}{2} \times (x + 2y) \times \frac{x(x+2y)^2}{2y} = \frac{x(x+2y)^2}{4y}$

Square as fraction of triangle $= x^2 \div \frac{x(x+2y)^2}{4y} = \frac{4xy}{(x+2y)^2}$

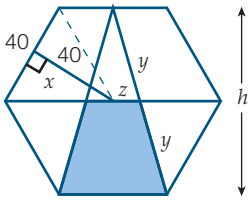
Now $\frac{x}{y} = \tan 60°$ so $y = \frac{x}{\sqrt{3}}$ (or $y = 0.5774x$)

Square as fraction of triangle $= \frac{4xy}{(x+2y)^2} = \frac{4x \times \frac{x}{\sqrt{3}}}{\left(x + \frac{2x}{\sqrt{3}}\right)^2}$

$= \frac{4x^2}{\sqrt{3}x^2\left(1 + \frac{2}{\sqrt{3}}\right)^2} = \frac{4\sqrt{3}}{(\sqrt{3}+2)^2} = \frac{4\sqrt{3}}{(7+4\sqrt{3})}$

$= 0.497$, hence 49.7%

8



$x^2 = 40^2 - 20^2 = 1200$. $x = 20\sqrt{3}$

$h = 2x = 40\sqrt{3}$

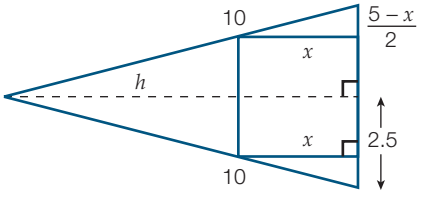
$4y^2 = 20^2 + 1600 \times 3 = 5200$. $y^2 = 1300$. $y = 10\sqrt{13}$

$z^2 = y^2 - x^2 = 1300 - 1200 = 100$. $z = 10$

Shaded area $= \frac{20 + 40}{2} \times 20\sqrt{3} = 600\sqrt{3}$ cm^2

9 D

10



$h^2 = 10^2 - 2.5^2 = 93.75$, $h = \sqrt{\frac{375}{4}} = \frac{5}{2}\sqrt{15}$

From similar right-angled triangles, $\frac{h}{2.5} = \frac{x}{\frac{5-x}{2}}$

$h = \frac{5x}{5-x}$

$5h - hx = 5x$

$(5 + h)x = 5h$

$x = \frac{5h}{5+h}$

$= \frac{5 \times \frac{5\sqrt{15}}{2}}{5 + \frac{5\sqrt{15}}{2}}$

$= \frac{5\sqrt{15}}{\sqrt{15}+2} = \frac{5\sqrt{15}(\sqrt{15}-2)}{15-4}$

$= \frac{5}{11}(15 - 2\sqrt{15})$ cm

11 Radius = 15 cm, Area of base $= 3 \times \frac{5}{6} \times \pi \times 15^2 + 15 \times 15 \tan 60° = \frac{1125\pi}{2} + 225\sqrt{3} \approx 2157$ cm^2

Volume $= \left(\frac{1125\pi}{2} + 225\sqrt{3}\right) \div 10\,000 \times 6 \approx 1.294$ m^3

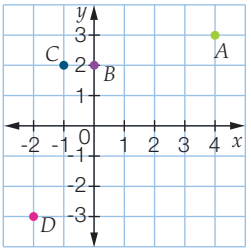
Chapter 4

Recall 4 (p. 108)

1 **(a)** $P = -7$ **(b)** $P = 3$ **(c)** $P = -15$ **(d)** $P = -3$

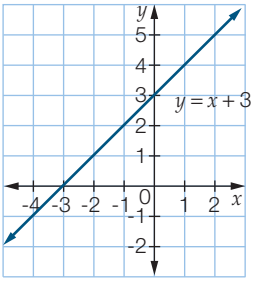
2 **(a)** $a = 7$ **(b)** $b = 24$ **(c)** $c = 10$ **(d)** $d = 7$

3



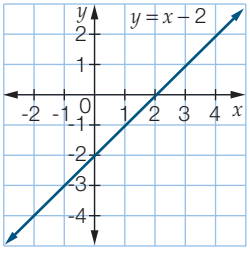
4 **(a)**

x	-2	-1	0	1	2
y	1	2	3	4	5



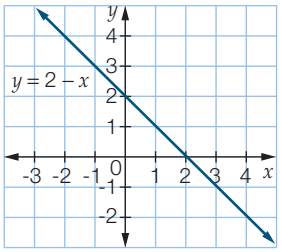
(b)

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



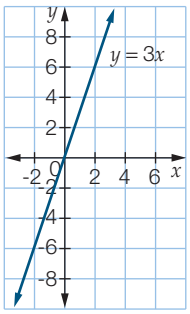
(c)

x	-2	-1	0	1	2
y	4	3	2	1	0

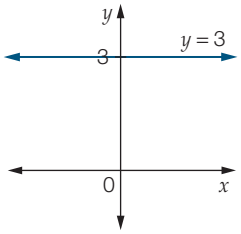


(d)

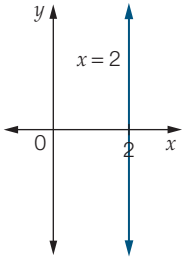
x	-2	-1	0	1	2
y	-6	-3	0	3	6



5 (a)



(b)



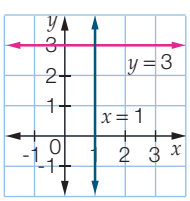
6 (a) (i) 3 (ii) 5

(b) (i) -2 (ii) 3

(c) (i) $-\frac{2}{3}$ (ii) $\frac{4}{3}$

7 (a) $-\frac{5}{3}$ (b) 1

8 (a)



(b) (1, 3)

Exercise 4.1 (p. 112)

1 (a) $x = 3$ (b) $x = 8$ (c) $c = -3$ (d) $a = 4$

(e) $x = -6$ (f) $d = -3$ (g) $m = 2$ (h) $n = 1$

(i) $x = -1$ (j) $f = 2$ (k) $e = -2$ (l) $p = \frac{1}{5}$

(m) $x = 0$ (n) $g = 21$ (o) $h = 16$ (p) $k = 3$

(q) $x = 3$ (r) $y = 0$ (s) $x = \frac{5}{4}$ (t) $x = 3$

(u) $x = 7$

2 (a) $a = 1$ (b) $d = -1$ (c) $e = \frac{5}{6}$

(d) $c = 2$ (e) $b = 4$ (f) $f = \frac{13}{6}$ or $2\frac{1}{6}$

(g) $g = 8$ (h) $h = \frac{4}{5}$ (i) $k = -1$

(j) $x = -1$ (k) $x = 1$ (l) $x = -\frac{7}{3}$ or $-2\frac{1}{3}$

(m) $x = \frac{3}{2}$ or $1\frac{1}{2}$ (n) $x = \frac{1}{4}$ (o) $x = \frac{1}{3}$

(p) $m = -\frac{11}{2}$ or $-5\frac{1}{2}$ (q) $d = \frac{17}{4}$ or $4\frac{1}{4}$ (r) $d = 7$

(s) $a = 8$ (t) $a = -\frac{16}{3}$ or $-5\frac{1}{3}$ (u) $g = \frac{8}{7}$ or $1\frac{1}{7}$

(v) $t = 0$ (w) $h = \frac{11}{2}$ or $5\frac{1}{2}$ (x) $j = -\frac{3}{11}$

3 (a) $a = -22$ (b) $b = 8$ (c) $c = 9$

(d) $d = -1$ (e) $x = -2$ (f) $y = 1$

(g) $m = \frac{3}{4}$ (h) $k = \frac{3}{2}$ or $1\frac{1}{2}$ (i) $t = 2$

(j) $a = -25$ (k) $p = 15$ (l) $m = -27$

(m) $p = 5$ (n) $n = 2$ (o) $t = -18$

(p) $m = \frac{13}{22}$ (q) $k = -\frac{7}{4}$ or $-1\frac{3}{4}$ (r) $x = \frac{5}{6}$

4 (a) $x + 6 = 13$, $x = 7$ (b) $y - 7 = -2$, $y = 5$

(c) $2x = 17$, $x = 8\frac{1}{2}$ (d) $\frac{x}{3} = 12$, $x = 36$

(e) $3x + 2 = 23$, $x = 7$ (f) $4x - 5 = -3$, $x = \frac{1}{2}$

5 (a) C (b) D (c) D

6 (a) B (b) B (c) C

7 8, 9, 10, 11 8 40, 42, 44

9 Nikki 7, Sam 14, Minh 21 10 length 100 m, width 50 m

11 (a) $p + 7$ (b) $2(p + 7)$ (c) 17

12 98 m 13 walk 1.5 km, train 9 km

14 15 years old 15 45

16 24 years old 17 43 km/h and 53 km/h

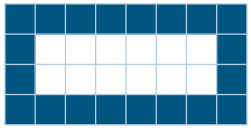
Open-ended – Sample answers

18 Students to check own answers but examples would be:

$\frac{x+8}{6} = -\frac{x}{2}$, $5x - 2 = -10 + x$ and $-4x - 11 = \frac{3x}{2}$

19 **(a)** blue tiles $= 2l + 10$ or $2(l + 5)$

(b) **(i)**



(ii) 20 blue tiles needed

(iii) $b = 2t + 8$ or $b = 2(t + 4)$

(c) For 5×3 white tiles, $n = 20$

Exercise 4.2 (p. 117)

1 **(a)** $\frac{1}{2}$ **(b)** -2 **(c)** 0

(d) $\frac{3}{0}$, undefined

2 **(a)** $y = 6x - 3$ **(b)** $y = -2x + 7$

(c) $y = 3x + 9$ **(d)** $y = -5x + 8$

3 **(a)** positive **(b)** negative **(c)** zero

(d) undefined

4 **(a)** $\frac{1}{3}$ **(b)** 2 **(c)** -4 **(d)** $-\frac{3}{2}$

(e) $-\frac{5}{4}$ **(f)** -2 **(g)** 3 **(h)** 2

(i) -1 **(j)** $-\frac{2}{3}$ **(k)** 0 **(l)** 0

(m) $-\frac{2}{5}$ **(n)** $\frac{1}{7}$ **(o)** undefined

5 **(a)** C **(b)** B **(c)** D

6 **(a)** **(i)** (-3, 2) and (2, 2) or (5, -4) and (-7, -4)

(ii) (2, 5) and (2, 7)

(iii) (4, 10) and (5, 6)

(iv) (-1, -1) and (6, 0)

(b) (0, 3) and (2, 7) parallel to (2, 1) and (-2, -7)

7 **(a)** 800 m **(b)** 600 m **(c)** 120

(d) 120 m/min **(e)** 440 m **(f)** 560 m

(g) $d = 200 + 120t$

(h) Alex is walking at a constant speed.

8 **(a)** 4800 grains **(b)** 1600 grains

(c) -1600 **(d)** 1600 grains/min

(e) decrease **(f)** $S = 4800 - 1600t$

(g) When $t = 2$, $s = 4800 - 3200$
$= 1600$

(h) $0 \le t \le 3$

9 **(a)** **(i)** $y = 2.5x - 1$ **(ii)** $y = 2.5x - 1$

(b) Yes, either point can be used as (x_1, y_1) to find the equation of the line.

10 $a = 17$

11 $-\frac{9}{35}$

12 Gradient of the line joining the points (-1, -5) and (2, 4) = 3 and the gradient of the line joining the points (2, 4) and (5, 13) = 3. As the two lines have a point in common, (2, 4) and the same gradient, the points are collinear.

Open-ended – Sample answers

13 **(a)** (1, 9), (2, 6), (3, 3) **(b)** (-2, 0), (-1, -1), (1, -3)

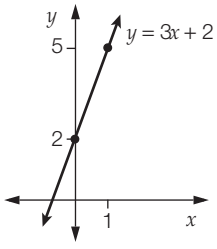
(c) (0, 4), (2, 3), (4, 2) **(d)** (2, -5), (3, -3), (4, -1)

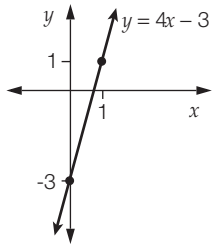
(e) (1, 0), (2, 5), (3, 10)

14 $m = -3$, (1, 4) and (4, -5)

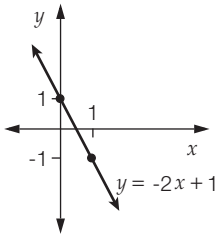
Exercise 4.3 (p. 122)

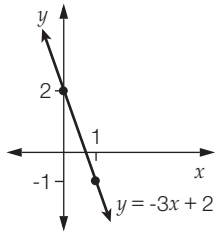
1 **(a)** $b = 2$, $m = 3$ **(b)** $b = -3$, $m = 4$



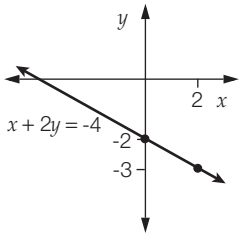


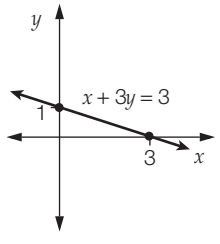
(c) $b = 1$, $m = -2$ **(d)** $b = 2$, $m = -3$



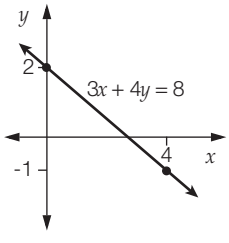


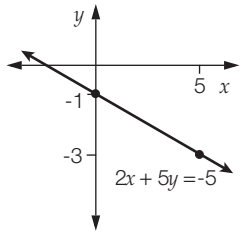
2 **(a)** $b = -2$, $m = -\frac{1}{2}$ **(b)** $b = 1$, $m = -\frac{1}{3}$



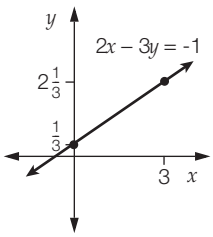


(c) $b = 2$, $m = -\frac{3}{4}$ **(d)** $b = -1$, $m = -\frac{2}{5}$

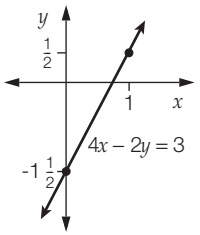




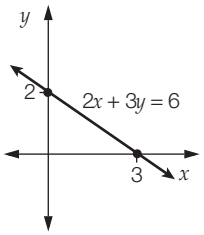
(e) $b = \frac{1}{3}$, $m = \frac{2}{3}$



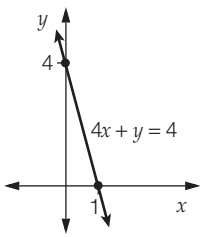
(f) $b = -1\frac{1}{2}$, $m = 2$



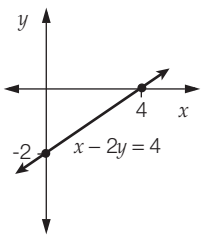
3 (a)



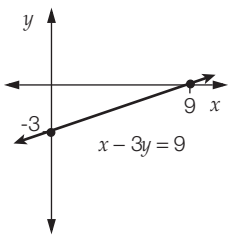
(b)



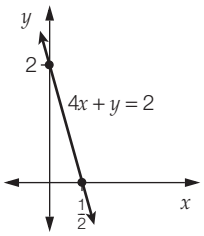
(c)



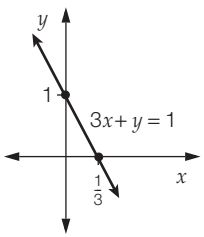
(d)



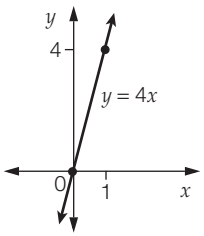
(e)



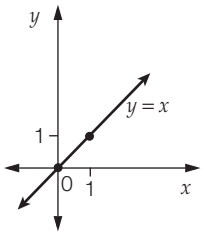
(f)



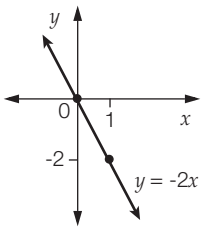
4 (a)



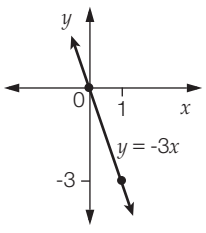
(b)



(c)



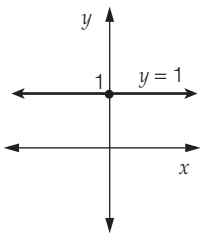
(d)



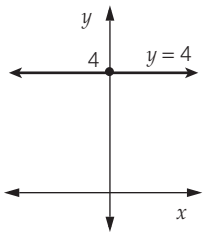
5 (a) D (b) B (c) A

6 (a) C (b) C (c) B

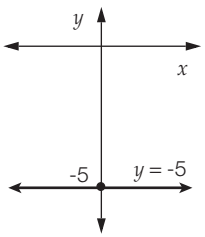
7 (a)



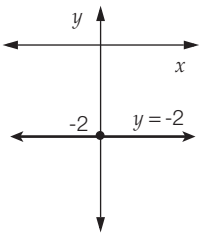
(b)



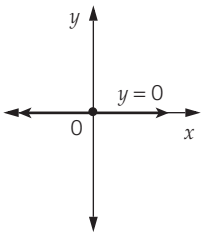
(c)



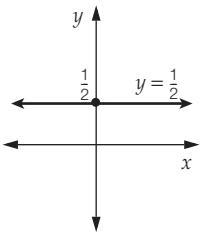
(d)



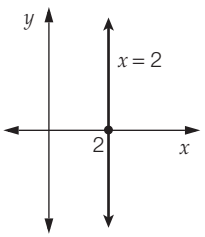
(e)



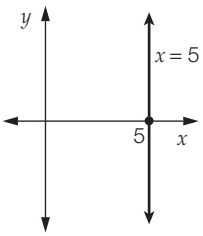
(f)



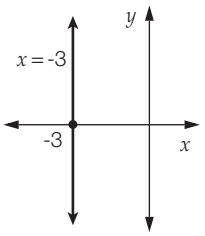
(g)



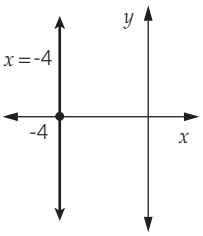
(h)



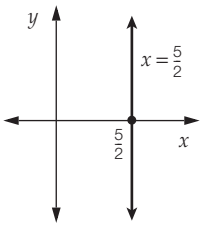
(i)



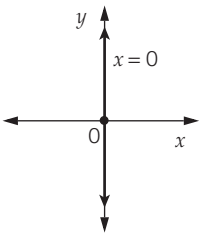
(j)



(k)



(l)



8 (a) D (b) C (c) D

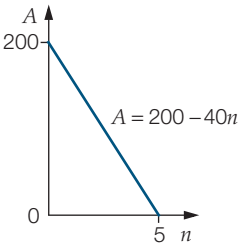
9 (a) 100 km (b) 80 (c) $D = 100 + 80T$

(d) 80 km/h (e) $D = 250 + 100T$ (f) 93 km/h

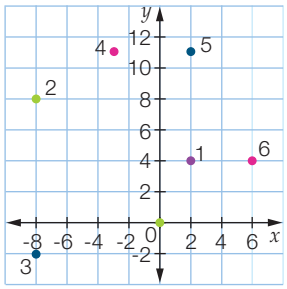
10 (a) \$80 (b) $A = 200 - 40n$

(c) \$200 (d) 5 weeks

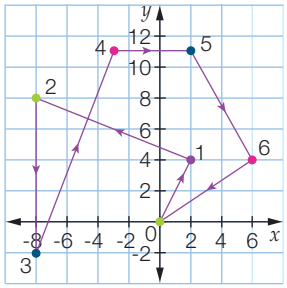
(e)



11 (a)



(b)



(c) **(i)** $y = 2x$

(ii) $y = -\frac{2}{5}x + 4\frac{4}{5}$ or $5y + 2x = 24$

(iii) $x = -8$

(iv) $y = \frac{13}{5}x + 18\frac{4}{5}$ or $5y - 13x = 94$

(v) $y = 11$

(vi) $y = -\frac{7}{4}x + 14\frac{1}{2}$ or $4y + 7x = 58$

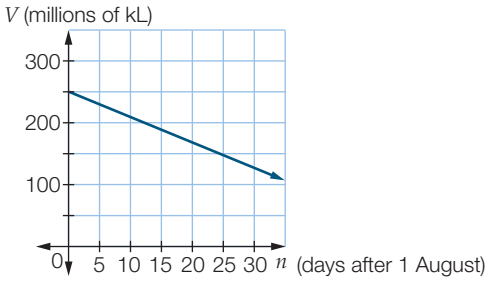
(vii) $y = \frac{2}{3}x$

(d) Yes. It will be on the direct path between the 5th and 6th buoy.

12 (a) $V = 225 - 0.75T$ **(b)** 5 min

13 (a) $V = 248 - 4n$

(b) Maximum capacity = 248 million kL



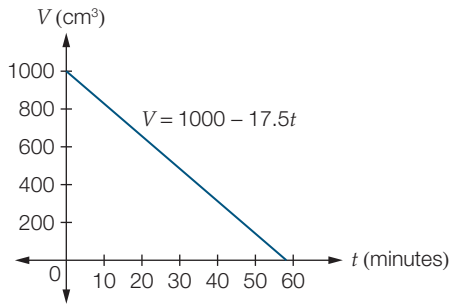
(c) It does not make sense to extend the time and volume into negative values.

14 (a) 1000 cm^3 **(b)** $V = 1000 - kt$

(c) 125 cm^3 **(d)** $k = 17.5$

(e) The volume is decreasing. **(f)** $V = 1000 - 17.5t$

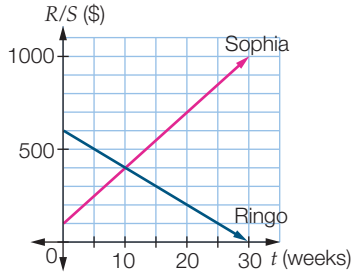
(g)



The ice block has completely melted after 57.15 minutes.

15 (a) Ringo: $R = 600 - 20t$, Sophia: $S = 100 + 30t$

(b) 15.7 weeks



Open-ended – Sample answers

16 $y = 2x - 4$, $y = 2x - 1$, $y = 2x + 2$. The coefficient of x in all three equations is the same, indicating that they have the same gradient.

17 (1) The pink line extending from (0, 0).

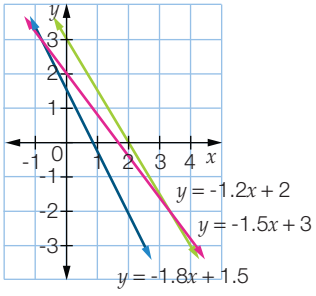
(2) The purple line (gradient is half of the pink one above it).

(3) The blue line (negative gradient shows the reduction in remaining credit).

(4) The black line starts part way up the y-axis representing connection fees.

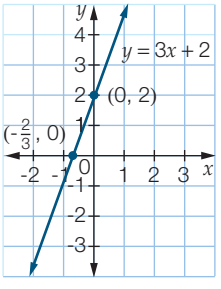
(5) The green line.

18

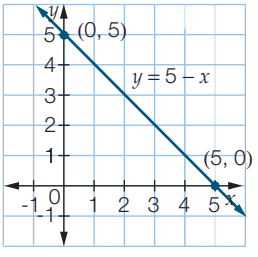


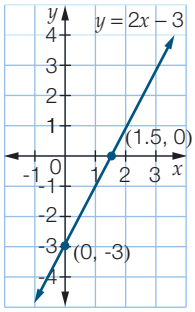
Half-time 4 (p. 126)

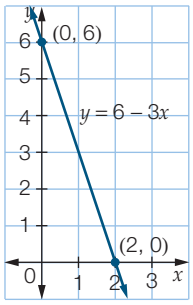
1 (a)



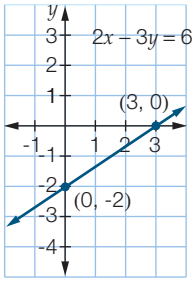
(b)

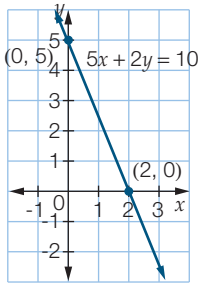


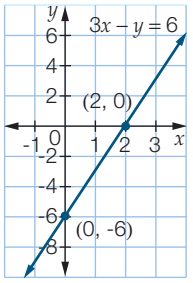
(c) 

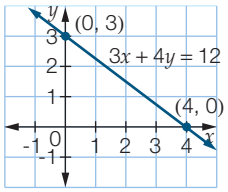
(d) 

2 **(a)** $m = 0.5$ **(b)** $m = -2$ **(c)** $m = 0.5$ **(d)** $m = -8$

3 **(a)** 

(b) 

(c) 

(d) 

4 Using the points (32, 0) and (212, 100):

$$m = \frac{100 - 0}{212 - 32} = \frac{100}{180} = \frac{5}{9}$$

Hence, $C = \frac{5}{9}F + c$. Substituting in (32, 0):

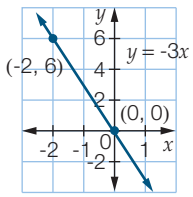
$$0 = \frac{5}{9} \times 32 + c$$

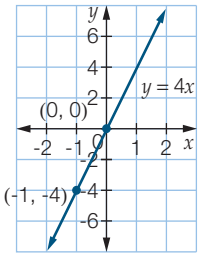
$$0 = \frac{160}{9} + c$$

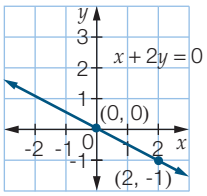
$$c = -\frac{160}{9}$$

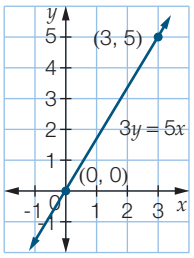
Therefore, $C = \frac{5}{9}F - \frac{160}{9}$

$$C = \frac{5}{9}(F - 32)$$

5 **(a)** 

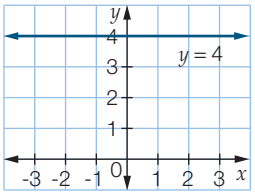
(b) 

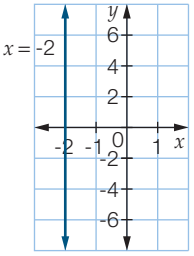
(c) 

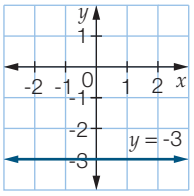
(d) 

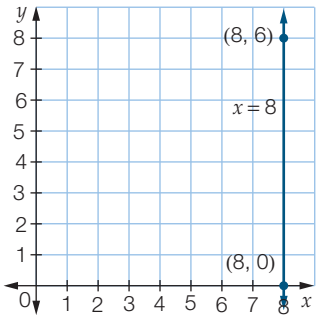
6 **(a)** $x = 2$ **(b)** $x = 2$ **(c)** $x = 19$

(d) $x = 3$ **(e)** $x = 3$ **(f)** $x = 4$

7 **(a)** 

(b) 

(c) 

(d) 

8 **(a)** $y = 4x + 9$ **(b)** $y = -x - 4$

Exercise 4.4 (p. 128)

1 **(a)** The gradient of both lines is 2.5. Hence, they are parallel.

(b) The gradient of both lines is 0.5. Hence, they are parallel.

2 **(a)** The gradient of $4x + 3y = 7$ is $-\frac{4}{3}$ and the gradient of $8y = 6x - 2$ is $\frac{3}{4}$. $-\frac{4}{3} \times \frac{3}{4} = -1$. Hence, the two lines are perpendicular.

(b) The gradient of $2x - 5y - 1 = 0$ is $\frac{2}{5}$ and the gradient of $4y + 10x = 3$ is $-\frac{5}{2}$. $\frac{2}{5} \times -\frac{5}{2} = -1$. Hence, the two lines are perpendicular.

3 **(a)** $m_1 = 3$, $m_2 = 3$; lines are parallel.

(b) $m_1 = 2$, $m_2 = -2$; lines are not parallel.

(c) $m_1 = 0.5$, $m_2 = 0.5$; lines are parallel.

(d) $m_1 = -4$, $m_2 = 4$; lines are not parallel.

(e) $m_1 = 5$, $m_2 = 9$; lines are not parallel.

(f) $m_1 = 2$, $m_2 = 2$; lines are parallel.

(g) $m_1 = -3$, $m_2 = -3$; lines are parallel.

(h) $m_1 = -3$, $m_2 = -4$; lines are not parallel.

(i) $m_1 = 4$, $m_2 = 4$; lines are parallel.

(j) $m_1 = -\frac{5}{3}$, $m_2 = \frac{1}{5}$ lines are not parallel.

4 (a) $m_1 = 2$, $m_2 = -\frac{1}{2}$, $2 \times -\frac{1}{2} = -1$; lines are perpendicular.

(b) $m_1 = 4$, $m_2 = -\frac{1}{2}$, $4 \times -\frac{1}{2} = -2$; lines are not perpendicular.

(c) $m_1 = \frac{1}{3}$, $m_2 = -\frac{1}{3}$, $\frac{1}{3} \times -\frac{1}{3} = -\frac{1}{9}$; lines are not perpendicular.

(d) $m_1 = \frac{1}{2}$, $m_2 = -2$, $\frac{1}{2} \times -2 = -1$; lines are perpendicular.

(e) $m_1 = \frac{3}{4}$, $m_2 = -\frac{3}{2}$, $\frac{3}{4} \times -\frac{3}{2} = -\frac{9}{8}$; lines are not perpendicular.

(f) $m_1 = \frac{4}{5}$, $m_2 = -\frac{5}{8}$, $\frac{4}{5} \times -\frac{5}{8} = -\frac{1}{2}$; lines are not perpendicular.

5 C **6** D

7 Gradient of $y = x + 10$ is 1. Gradient between the two points is $\frac{8-2}{4-(-2)} = \frac{6}{6} = 1$. Therefore, the lines are parallel.

8 Gradient of $2x + y - 10 = 0$ is -2. Gradient between the two points is $\frac{-7-5}{4-(-2)} = \frac{-12}{6} = -2$. Therefore, the lines are parallel.

9 Gradient of $y = -\frac{4}{3}x - 2$ is $-\frac{4}{3}$. Gradient between the two points is $\frac{4-1}{0-(-4)} = \frac{3}{4}$. $\frac{3}{4} \times -\frac{4}{3} = -1$. Therefore, the lines are perpendicular.

10 Gradient of $y = -\frac{5}{3}x + 10$ is $-\frac{5}{3}$. Gradient between the two points is $\frac{8-5}{9-4} = \frac{3}{5}$. $-\frac{5}{3} \times \frac{3}{5} = -1$. Therefore, the lines are perpendicular.

11 (a) (i) and (iii)

(b) (ii) and (v)

12 $y = 3x - 9$

13 Parallel line is $y = -2x + 5$, perpendicular line is $y = 0.5x + 5$.

14 $m_{AB} = -\frac{3}{2}$, $m_{BC} = 5$, $m_{CA} = \frac{2}{3}$. $-\frac{3}{2} \times \frac{2}{3} = -1$. Therefore, AB is perpendicular to CA and ABC must be a right-angled triangle.

15 $m_{AB} = 1$, $m_{BC} = -\frac{1}{2}$, $m_{DC} = 1$, $m_{AD} = -\frac{1}{2}$.

$AB \parallel DC$ and $BC \parallel AD$. Therefore $ABCD$ is a parallelogram.

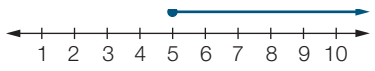
16 $m_{AB} = -\frac{5}{2}$, $m_{BC} = \frac{2}{5}$, $m_{DC} = -\frac{5}{2}$, $m_{AD} = \frac{2}{5}$. $-\frac{5}{2} \times \frac{2}{5} = -1$, therefore $AB \perp BC$, $BC \perp CD$, $CD \perp DA$ and $DA \perp AB$. Therefore $ABCD$ is a quadrilateral with four right angles. $|AB| = |BC| = |CD| = |DA| = 29$. All four sides are of equal length, therefore $ABCD$ is a square.

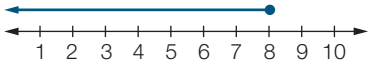
Open-ended – Sample answers

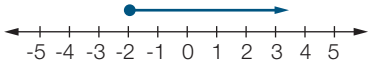
17 (a) $y = 0.5x + 4$, $2y = x + 3$, $2y - x = 10$ (all have gradients of 0.5)

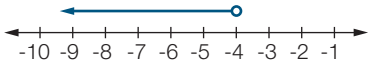
(b) $y = -3x - 1$, $3x + y = 10$, $3x = 5 - y$ (all have gradients of -3)

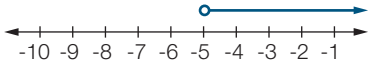
Exercise 4.5 (p. 131)

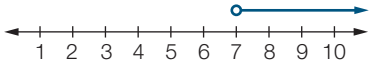
1 (a)


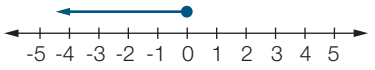
(b)


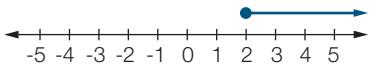
(c)


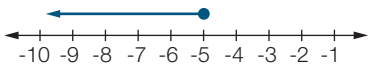
(d)


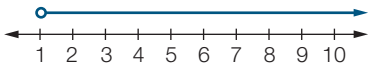
(e)


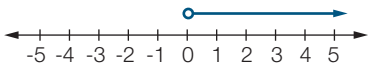
(f)


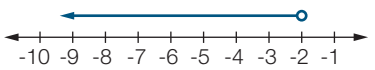
(g)


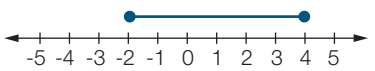
(h)


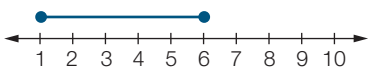
(i)


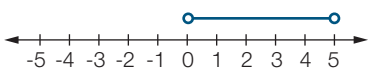
(j)


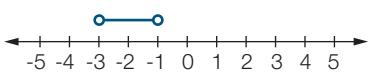
(k)


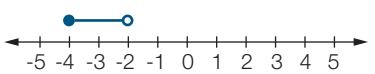
(l)


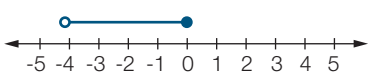
(m)


(n)


(o)


(p)


(q)


(r)


2 (a) $x > -10$ (b) $x < -5$ (c) $x \leq -10$

(d) $x \leq 35$ (e) $x \geq -9$ (f) $x > -28$

(g) $x < -\frac{16}{3}$ (h) $x > -\frac{27}{5}$ (i) $x \leq -9$

(j) $x \geq 8$ (k) $x \leq -2$ (l) $x > -8$

(m) $x > 4$ (n) $x < 11$ (o) $x \leq 4$

(p) $x \leq -8$ (q) $x > 11$ (r) $x > 10$

3 (a) $y \leq -5$ (b) $y \geq -6$ (c) $y \geq -5$

(d) $y > 12$ (e) $y < 21$ (f) $y < 12$

(g) $y \geq -8$ (h) $y \leq 2$ (i) $y \geq -4$

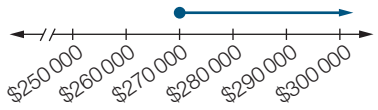
(j) $y \leq -\frac{1}{15}$ (k) $y \geq \frac{1}{2}$ (l) $y \leq \frac{19}{6}$

(m) $y < \frac{13}{3}$ (n) $y > \frac{23}{6}$ (o) $y < 4$

4 (a) B (b) D (c) A (d) D

5 (a) A (b) D (c) D (d) D

6 (a)



(b) $P \geq 270\,000$

(c) $270\,000 \leq P < 320\,000$

7 $x \leq 100$ **8** $n \geq 5$ **9** $j > 55$

10 (a) $x \geq \frac{2}{3}$ (b) $x \leq -\frac{3}{11}$ (c) $x < 0$

(d) $x > -2$ (e) $x \leq 4$ (f) $x \geq 12$

(g) $x > -25$ (h) $x < \frac{34}{7}$ (i) $x \geq 2$

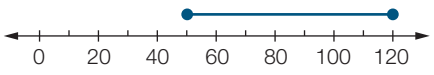
11 (a) $y \geq -1$ (b) $y < 3$ (c) $y \leq -\frac{7}{3}$

(d) $y > \frac{1}{3}$ (e) $y \geq 30$ (f) $y \leq 36$

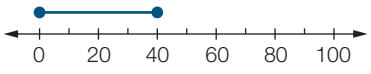
(g) $y > -12$ (h) $y < \frac{6}{7}$ (i) $y < \frac{61}{13}$

12 (a) Let f be the number of cartons of full cream milk.
Let l be the number of cartons of low fat milk.

$50 \leq f \leq 120$



$0 \leq l \leq 40$



(b) $l + f \leq 120$

13 $20p > 428$, $p > 21.4$. Therefore Tony should sell the calculators for any price higher than $21.40 each.

14 Let n be the number of liquorice straps. $0.4n \leq 1.55$, $n \leq 3.875$. As n must be an integer $n \leq 3$.

Open-ended – Sample answers

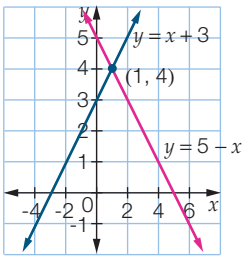
15



16 (a) $x - 10 \geq 3x - 8$ (b) $13x - 4 \geq 9x + 8$

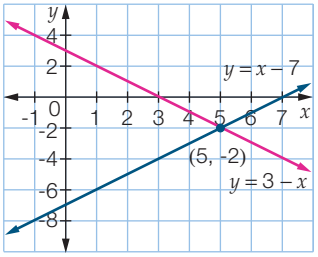
Exercise 4.6 (p. 137)

1 (a) (i)



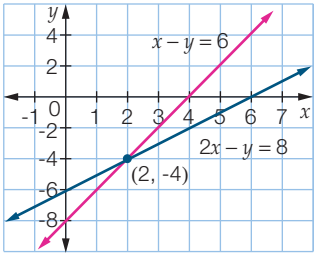
(ii) $x = 1$, $y = 4$

(b) (i)



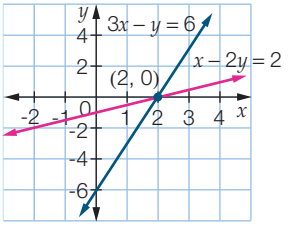
(ii) $x = 5$, $y = -2$

(c) (i)



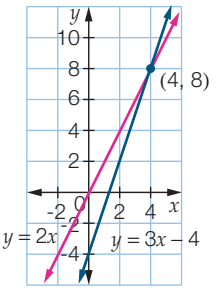
(ii) $x = 2$, $y = -4$

(d) (i)



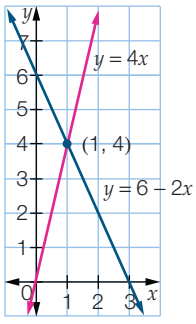
(ii) $x = 2$, $y = 0$

(e) (i)



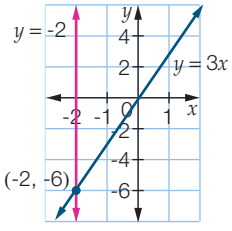
(ii) $x = 4$, $y = 8$

(f) **(i)**



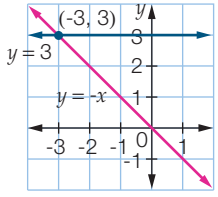
(ii) $x = 1, y = 4$

(g) **(i)**



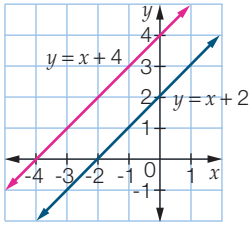
(ii) $x = -2, y = -6$

(h) **(i)**



(ii) $x = -3, y = 3$

(i) **(i)**



(ii) no solution

2 **(a)** (4, 1) **(b)** (5, -4) **(c)** (3, 1)
(d) (5, -1) **(e)** (1, 3) **(f)** (2, -6)
(g) (-2, 3) **(h)** (2, 2) **(i)** (2, 3)

3 **(a)** (1, -1) **(b)** (-3, 4) **(c)** (-1, 1)
(d) (2, -1) **(e)** (1, -1) **(f)** (-3, 1)
(g) (3, 1) **(h)** (-3, 4) **(i)** (1, 1)
(j) (4, -2) **(k)** (5, 12) **(l)** (9, 4)
(m) (-5, -10) **(n)** (-3, -8) **(o)** (-6, 5)

4 **(a)** (2, 4) **(b)** (-3, -9) **(c)** (4, -2)
(d) (1, 5) **(e)** (3, 2) **(f)** (3, -2)
(g) (-2, 5) **(h)** (9, -1) **(i)** (-1, 9)

5 **(a)** C **(b)** D

6 **(a)** D **(b)** B

7 **(a)** A **(b)** C

8 **(a)** D **(b)** D

9 a pineapple costs $2.60, a kiwi fruit costs 35 cents

10 a salad roll costs $3.60, a pie costs $2.44

11 $x = 7, y = 5$

12 7 and 31

13 128 pages and 372 pages

14 30 m by 40 m

15 three 5-cent coins, 15 10-cent coins

16 Michelle is 18 years old, Yesmin is 31 years old

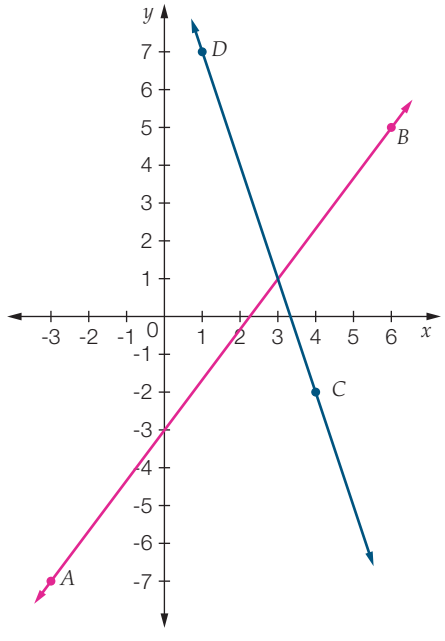
17 **(a)** Let D = distance in km from Wes' house and t = time taken in hours. Wes: $D = 80t$, Tony: $D = 360 - 100t$.

(b) 2 hours

18 Speed of barge is 12 km/h, speed of current is 3 km/h

19 **(a)** $A(-3, -7)$, $B(6, 5)$, $C(4, -2)$, $D(1, 7)$

(b)



(c) (3, 1)

(d) $y_{AB} = \frac{4}{3}x - 3$ (1)

$y_{CD} = -3x + 10$ (2)

substitute (1) into (2):

$\frac{4}{3}x - 3 = -3x + 10$

$4x - 9 = -9x + 30$

$13x - 9 = 30$

$13x = 39$

$x = 3$

substitute $x = 3$ into (2): $y = -9 + 10 = 1$

Therefore, the treasure is at (3, 1).

Open-ended – Sample answers

20 $x + y = 7$ and $x - y = -1$, $2x - y = 2$ and $x + 2y = 11$, $5x + 2y = 23$ and $3x - y = 5$

21 3, 8 and 14. Method example: Let x, y and z be the three numbers.

$x + y = 11$ (1)
$x + z = 17$ (2)
$y + z = 22$ (3)

Using elimination, (1) – (2):

$$\begin{array}{r} x + y = 11 \\ -\ x + z = 17 \\ \hline y - z = -6 \end{array}$$

$y = z - 6$ (4)

Substitute (4) into (3):

$z - 6 + z = 22$
$z = 14$

Substitute $z = 14$ into (2):

$x + 14 = 17$
$x = 3$

Substitute $x = 3$ into (1):

$3 + y = 11$
$y = 8$

Chapter review 4 (p. 140)

1 **(a)** $c = 8$ **(b)** $x = -12.5$ **(c)** $v = 3$ **(d)** $a = 1$

2 D

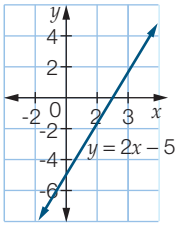
3 **(a)** $m = 5$, y-intercept = (0, 2)

(b) $m = -2$, y-intercept = (0, 0)

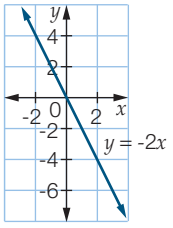
(c) $m = \frac{2}{5}$, y-intercept = (0, -2)

(d) $m = -4$, y-intercept = (0, 8)

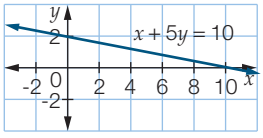
4 **(a)**



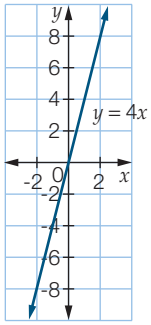
(b)



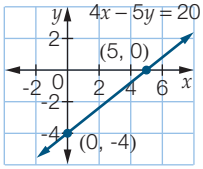
(c)



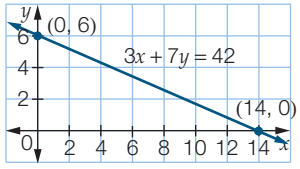
(d)



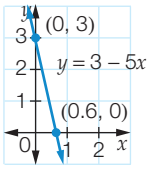
5 **(a)**



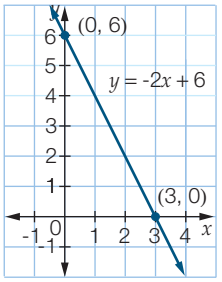
(b)



(c)



(d)



6 **(a)** Both have gradients of -0.5, therefore they are parallel.

(b) With different gradients of 1 and 1.5 they are not parallel.

(c) Both have gradients of -4, therefore they are parallel.

(d) Both have gradients of 4 so they are parallel.

7 **(a)** The lines have gradients of -1 and -2. $-1 \times -2 = 2$. Therefore they are not perpendicular.

(b) The lines have gradients of 1 and -1. $1 \times -1 = -1$. Therefore they are perpendicular.

(c) The lines have gradients of 0.5 and -2. $0.5 \times -2 = -1$. Therefore they are perpendicular.

(d) The lines have gradients of $-\frac{3}{2}$ and $\frac{2}{3}$. Their product is equal to -1 so they are perpendicular.

8 **(a)** $x \geq \frac{7}{3}$ **(b)** $x > -\frac{35}{3}$ **(c)** $x > \frac{5}{6}$

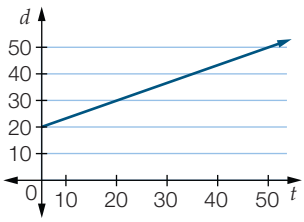
9 (4, 6) **10** (-1, 3)

11 18 wombats, 9 emus **12** 4 hours

13 **(a)** 8 hours **(b)** 1500 L **(c)** 1500 **(d)** $V = 1500T$

14 **(a)** $d = 0.6t + 20$

(b)



(c) Every minute of travel takes Manny 0.6 km further away from Melbourne.

(d) He began 20 km away from Melbourne.

(e) 28.4 km

(f) 46 min and 40 s

15 $y = -x + 9$ **16** $y = 5x - 42$

17 **(a)** $x < 2\frac{4}{5}$ **(b)** $x \leq -2\frac{2}{3}$

18 6 and -5 **19** $a = 9$, $b = -3$

20 **(a)** $570\,000 \leq p \leq 620\,000$ **(b)** $57\,000 \leq d \leq 62\,000$

(c) $p > 670\,000$ **(d)** $p < 670\,000$

21 $d = 165$ km, $s = 82$ km/h

Chapter 5

Recall 5 (p. 143)

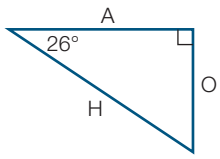
1 **(a)** $x = 8.17$ **(b)** $x = 15.7$ **(c)** $x = 5.20$

(d) $x = 46.377$

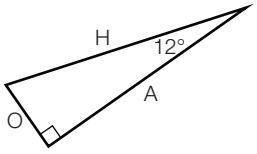
2 **(a)** 3 cm **(b)** 15 mm

3 **(a)** 26 **(b)** 33.23 **(c)** 9.95

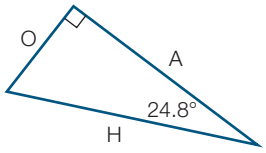
4 **(a)**



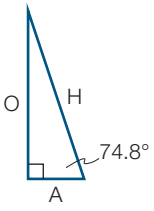
(b)



(c)

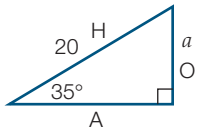


(d)



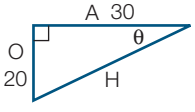
Exercise 5.1 (p. 146)

1 **(a)**



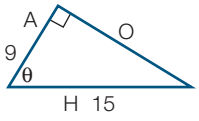
$\sin 35° = \frac{a}{20}$

(b)



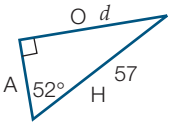
$\tan \theta = \frac{20}{30}$

(c)



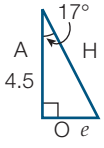
$\cos \theta = \frac{9}{15}$

(d)



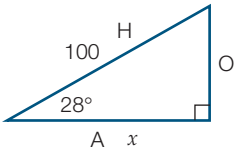
$\sin 52° = \frac{d}{57}$

(e)



$\tan 17° = \frac{e}{4.5}$

(f)



$\cos 28° = \frac{x}{100}$

2 **(a)** 0.3907 **(b)** 0.9563 **(c)** 0.8098

(d) 0.9962 **(e)** 0.6561 **(f)** 0.6561

(g) 0.3581 **(h)** 0.9171 **(i)** 0.9540

3 **(a)** 15.50° **(b)** 35.27° **(c)** 23.17° **(d)** 76.80°

(e) 44.99° **(f)** 8.75°

4 **(a)** 0.4350 **(b)** 38.1885 **(c)** 0.4339 **(d)** 0.7403

(e) 0.3490 **(f)** 0.1437

5 **(a)** C **(b)** D

6 **(a)** **(i)** 1.5 **(ii)** 1.5 **(iii)** 1.5

(b) 0.5 **(c)** 0.5

(d) Yes, because the triangles are similar.

(e) $\propto = \beta = 30°$ **(f)** yes **(g)** yes

(h) The triangles are similar so the ratio between the matching sides of the triangles is constant. This means that the ratio of a pair of sides within one triangle is the same as the ratio of the matching pair of sides within the other triangle.

7 **(a)** **(i)** 0.5736 **(ii)** 0.8192 **(iii)** 0.7002 **(iv)** 0.7002

(b) The ratio of sin 35° and cos 35° is the same as tan 35°.

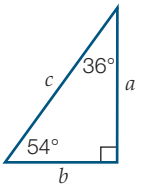
8 1.151

9 B

10 **(a)** **(i)** 0.722 **(ii)** 0.722

(b) Ratios use the same side lengths $\left(\frac{4.6}{6.37}\right)$.

11 **(a)**



(b) $\sin 54° = \frac{a}{c}$, $\cos 54° = \frac{b}{c}$

(c) $\sin 54° = 0.8090$, $\cos 54° = 0.5878$

(d) $(\sin 54°)^2 + (\cos 54°)^2$
$= 0.8090^2 + 0.5878^2$
$= 0.6545 + 0.3455$
$= 1$

(e) $(\sin 54°)^2 + (\cos 54°)^2$
$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$
$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$
$= \frac{a^2 + b^2}{c^2}$
$= \frac{c^2}{c^2}$ (Pythagoras' theorem $a^2 + b^2 = c^2$)
$= 1$

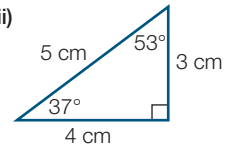
12 An isosceles triangle has two sides equal and two equal angles. In a right-angled isosceles triangle, these two sides are the Opposite and Adjacent of the two base angles, which are each 45°. So:

$\tan \theta = \frac{O}{A}$
$= \frac{x}{x}$
$= 1$

13 The value of the tan ratio can be greater than 1. The values of the cos and sin ratios can't be greater than 1, as to calculate them you need to divide either the adjacent or opposite side by the hypotenuse, and by definition the hypotenuse will always be larger than the other two sides.

Open-ended – Sample answers

14 (a) (i) (ii)



(iii) $\sin 37° = \frac{3}{5}$ $\qquad$ $\sin 53° = \frac{4}{5}$

$= 0.60$ $\qquad$ $= 0.80$

$\cos 37° = \frac{4}{5}$ $\qquad$ $\cos 53° = \frac{3}{5}$

$= 0.80$ $\qquad$ $= 0.60$

$\tan 37° = \frac{3}{4}$ $\qquad$ $\tan 53° = \frac{4}{3}$

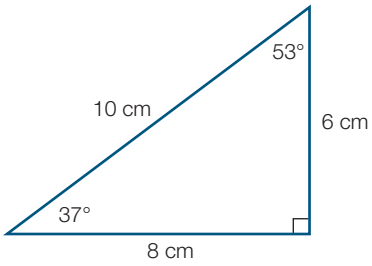
$= 0.75$ $\qquad$ $= 1.33$

(iv) $\sin 37° = \cos 53°$

$\cos 37° = \sin 53°$

$\tan 37° = \frac{1}{\tan 53°}$

(b)



$\sin 37° = 0.60$ $\qquad$ $\sin 53° = 0.80$

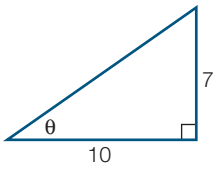
$\cos 37° = 0.80$ $\qquad$ $\cos 53° = 0.60$

$\tan 37° = 0.75$ $\qquad$ $\tan 53° = 1.33$

(c) Values of sin, cos and tan are identical as the angles in each triangle are the same.

(d) Measured values agree with the calculator values, rounded to two decimal places.

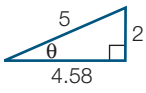
15 (a) hypotenuse = 12.2 cm

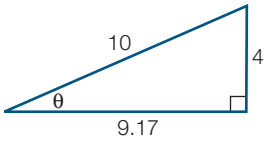


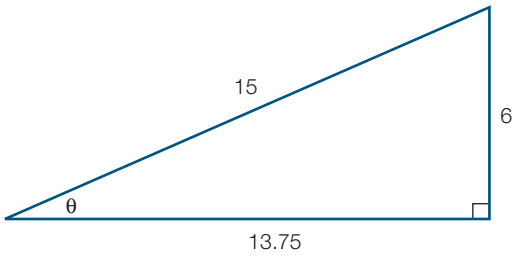
(b) $\sin\theta = \frac{7}{12.2}$ $\qquad$ $\cos\theta = \frac{10}{12.2}$

$= 0.5738$ $\qquad$ $= 0.8197$

16







17 (a) $\theta = 20°$ and $\varphi = 70°$

(b) $\theta = 90 - \varphi$ or $\varphi = 90 - \theta$

Exercise 5.2 (p. 150)

1 (a) $a = 167.71$ **(b)** $b = 12.44$ **(c)** $c = 1.05$

(d) $x = 11.53$ **(e)** $x = 11.22$ **(f)** $x = 13.29$

(g) $x = 148.84$ **(h)** $a = 20.58$ **(i)** $b = 42.56$

(j) $d = 32.09$ **(k)** $e = 7.40$ **(l)** $f = 41.08$

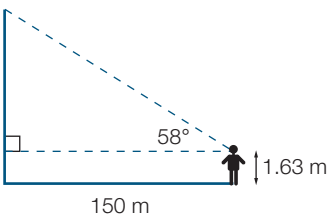
(m) $d = 95.83$ **(n)** $c = 244.74$ **(o)** $e = 6.91$

2 193 cm **3** 3.74 m

4 (a) B **(b)** B

5 9.89 m

6 (a)



(b) 241.7 m

7 15.37 m

8 325.87 m **9** 39.95 cm

10 (a) 19.73 m

(b) dropped from 9.86 m along rope, falls 28.7 m

11 (a) trapezium 10.74 m^2, rectangle 12 m^2

(b) rectangular room

Open-ended – Sample answers

12 For a pitch angle of 15°:

(a) 1.21 m **(b)** 19.53 m

For a pitch angle of 25°:

(a) 2.1 m **(b)** 21.03 m

13 16 cm, 16.5 cm, 17 cm

14 Taking 'just clear' to be a height of 8.1 m, two possible angles are 40° and 35°.

$\tan 40° = \frac{8.1 - 1.2}{x}$

$x = \frac{6.9}{\tan 40°} = 8.22$ m

$\tan 35° = \frac{8.1 - 1.2}{x}$

$x = \frac{6.9}{\tan 35°} = 9.9$ m

Exercise 5.3 (p. 154)

1 **(a)** **(i)** $a = 81.15°$ **(ii)** $a = 81°9'10''$

(b) **(i)** $b = 42.21°$ **(ii)** $b = 42°12'50''$

(c) **(i)** $c = 66.58°$ **(ii)** $c = 66°34'43''$

(d) **(i)** $k = 56.22°$ **(ii)** $k = 56°13'13''$

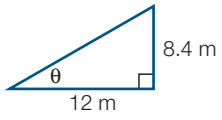
(e) **(i)** $m = 77.96°$ **(ii)** $m = 77°57'22''$

(f) **(i)** $p = 76.29°$ **(ii)** $p = 76°17'17''$

2 **(a)** 12°22′25″ **(b)** 25°42′36″ **(c)** 47°9′23″

(d) 14°33′5″ **(e)** 24°11′40″ **(f)** 54°35′10″

3 **(a)**



(b) 35°

4 **(a)** B **(b)** C

5 0.40° **6** 48° **7** 53°36′56″ **8** B

9 **(a)** 33° **(b)** 14 cm

10 **(a)** 147° **(b)** 25.7 m

11 **(a)** 11.54° to 36.87°

(b) The ladder is too long for the 2 m window. It would be placed 9.8 m away, at an angle of 11.5°.

Open-ended – Sample answers

12 30 cm (22°), 40 cm (30°), 50 cm (39°), 60 cm (49°), 70 cm (61°)

13 Angle of Bilal's shadow with the sun is $\theta = \tan^{-1}\frac{185}{94}$ = 63.06°. For example, take the height of the lamp post as 5 m and the height of the tree as 12 m. Then the length of the lamp post's shadow is $\frac{5}{\tan 63.06°} = 2.54$ m and the length of the tree's shadow is $\frac{12}{\tan 63.06°} = 6.10$ m

Exercise 5.4 (p. 158)

1 14° **2** 506 m **3** 53°

4 39 m **5** 8.55 m

6 **(a)** 209 m **(b)** 134 m

7 **(a)** A **(b)** B

8 24 m **9** 28.84 m **10** 4°

11 **(a)** 177 cm **(b)** 20.2°

12 **(a)** 27.7° **(b)** 12.25 m

Open-ended – Sample answers

13 **(a)** 46°, 50°, 54°

(b) 12.95 m (46°), 13.79 m (50°), 14.56 m (54°)

14 0.95° (after 20 min), 0.48° (after 44 min)

Half-time 5 (p. 160)

1 6.4° **2** B **3** B

4 θ = 50.36°, height = 23.23 m

5 D **6** 320 m

Exercise 5.5 (p. 163)

1 **(a)** **(i)** N21°W **(ii)** 339°T

(b) **(i)** S73°E **(ii)** 107°T

(c) **(i)** N58°E **(ii)** 058°T

(d) **(i)** S53°W **(ii)** 233°T

(e) **(i)** S51°E **(ii)** 129°T

(f) **(i)** N73°W **(ii)** 287°T

2 115 km

3 **(a)** True bearing: 152°T Compass bearing: S28°E

(b) True bearing: 147°T Compass bearing: S33°E

4 **(a)** B **(b)** C

5 75.74 km **6** 260 km

7 **(a)** 056°T

(b) **(i)** 288 km **(ii)** 236°T

8 **(a)** 2.11 km

(b) **(i)** 5.87 km **(ii)** 7.98 km

(c) **(i)** 1.25 km **(ii)** 5.78 km

9 **(a)** 1155 m **(b)** South 227 m

10 30.39 m

11 **(a)** **(i)** 115 m **(ii)** 164 m

(b) 027°T **(c)** 1004 m

12 **(a)** 115°T **(b)** 3.32 km

13 **(a)** S23°E **(b)** 674 km **(c)** 257 km south

(d) N69°W **(e)** 721 km

Open-ended – Sample answers

14 If θ = 30°, then:

(a) 150° T, S30° E **(b)** 330° T, N30° W

(c) 210° T, S30° W

15 Students' own answers.

Exercise 5.6 (p. 168)

1 **(a)** $x = 16.41$, $y = 27.27$ **(b)** $x = 31.40$, $y = 28.23$

(c) $x = 25.02$, $y = 3.32$ **(d)** $x = 28.87$, $y = 22.91$

(e) $x = 66.75$, $y = 84.57$ **(f)** $x = 63.23$, $y = 78.50$

2 65.80 m

3 **(a)** $x = 14.4$ **(b)** $x = 25.87$ **(c)** $x = 37.14$ **(d)** $x = 10.51$

4 **(a)** $a = 34.5°$, $x = 123.4$ **(b)** $k = 57.6°$, $x = 9.5$

(c) $m = 58.5°$, $x = 69.0$ **(d)** $a = 31.1°$, $x = 30.2$

5 A **6** D **7** $24 000

8 31°23′ **9** 53 m **10** 154 km **11** 62°

12 45°38′31″ **13** 180.78 m **14** 3.80 cm

15 16.83 mm **16** 11.92 m

17 **(a)** 3.46 cm **(b)** 38 cm^2 **(c)** 4.53 cm

Open-ended – Sample answers

18 $a = 20$, $\theta = 10°$ so $c = 20.3$; $b = 5$, $\varphi = 20°$ so $d = 14.6$

19 Let $x = 13$ and $y = 11$, then $\theta = 134.5°$.

Chapter review 5 (p. 172)

1 **(a)** 0.5736 **(b)** 1.2482 **(c)** 0.9944
(d) 0.5715 **(e)** 0.2823 **(f)** 0.2905

2 **(a)** 22.26 cm **(b)** 11.72 m

3 **(a)** **(i)** 42.27° **(ii)** 42°16′7″
(b) **(i)** 69.29° **(ii)** 69°17′23″
(c) **(i)** 27.45° **(ii)** 27°26′43″

4 62.45 m **5** N64°W

6 $a = 26.7°$, $x = 21.2$

7 418 cm **8** D **9** D

10 17.16 m **11** 19.3° **12** 101 m

13 23.56 m **14** 72 km, N80°25′

15 No, the length of the side of the wedge is correct, but the angle at the point is greater than 40°.

16 84 km

17 Draw a line from the top of the triangle to the midpoint of the base to create a right-angled triangle. Now either use the sine ratio with one side of the original triangle as the hypotenuse and the height as the opposite side, or use the tangent ratio with half of the base as the adjacent side and the height as the opposite side. In both cases the angle will be 60°.

18 34.22 cm

Chapter 6

Recall 6 (p. 174)

1 **(a)** 3.72 **(b)** 46.05 **(c)** 3.77

2 **(a)** 53°8′ **(b)** 67°30′ **(c)** 77°44′

3 **(a)** 8.95 **(b)** 107.57

4 20.32 m

5 **(a)** 10.34 km **(b)** 288.74°T

Exercise 6.1 (p. 176)

1 **(a)** 66° **(b)** 12.81 cm

2 4.05 m^2

3 **(a)** D **(b)** B

4 **(a)** 90° **(b)** 40.6° **(c)** 9.22 mm
(d) 11.58 mm **(e)** 31.2°

5 8.66 cm

6 **(a)** 147.52 m **(b)** 134.76 m
(c) 17.12 m/min **(d)** 6.66°
(e) gradient of AB is 0.45, gradient of AC is 0.12, so AB is steeper.

7 **(a)** 185.22 cm **(b)** 2°47′7″

8 **(a)** 43.46 m **(b)** 78.41 m **(c)** 93.00 m **(d)** 122.5°T

9 **(a)** 5.37 cm **(b)** 5.13 cm

10 14 289 m

11 Yes. The distance from A to B is 49.75 cm

12 248 m

13 **(a)** 67.71 m **(b)** 75.12 m

14 1941 cm^2

15 Yes. The angle is 75°. **16** 223.07 m

Open-ended – Sample answers

17 The ratio is always 0.07

18 Two values: 20° and 30°

(a) $BF = 4.45$ cm, 6.5 cm

(b) $DF = 12.22$ cm, 11.26 cm
$EF = 9.23$ cm, 7.92 cm
$BC = 9.15$ cm, 10.31 cm

(c) 84.50 cm^2, 81.66 cm^2

(d) As the value of θ increased, the area of $ABCD$ decreased.

(e) 234.97 cm^2, 247.98 cm^2

(f) Less plastic was used when θ was smaller and $ABCD$ had a larger area.

19 **(a)** $h = a \sin\theta$ and $h = b \sin\phi$ so $\frac{a}{b} = \frac{\sin\phi}{\sin\theta}$.

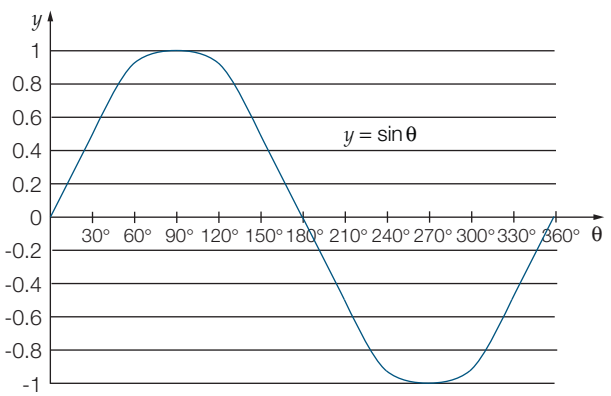
(b) Any of the following values:
$\theta = 21°$, $\phi = 46°$
$\theta = 22°$, $\phi = 49°$
$\theta = 23°$, $\phi = 51°$
$\theta = 24°$, $\phi = 54°$
$\theta = 25°$, $\phi = 58°$

Exercise 6.2 (p. 186)

1 **(a)** 4 **(b)** 3 **(c)** 2 **(d)** 2 **(e)** 3 **(f)** 1
(g) 3 **(h)** 2 **(i)** 4 **(j)** 4 **(k)** 2 **(l)** 3

2 **(a)** $\sin 45°$ **(b)** $-\cos 44°$ **(c)** $-\tan 69°$ **(d)** $-\sin 84°$
(e) $\cos 39°$ **(f)** $-\tan 57°$ **(g)** $-\sin 83°$ **(h)** $-\cos 83°$
(i) $\tan 6°$ **(j)** $-\sin 18°$ **(k)** $-\cos 14°$ **(l)** $\tan 27°$

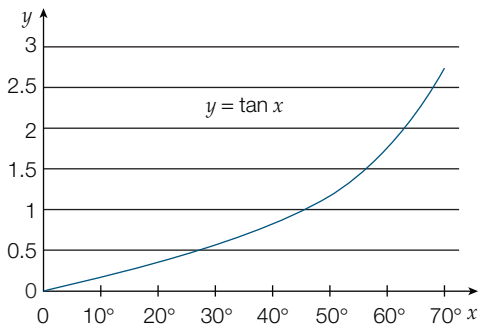
3



4 **(a)** B **(b)** D

5 C

6 (a)



(b) The value of tan x becomes larger at a faster rate and approaches infinity as x approaches 90°.

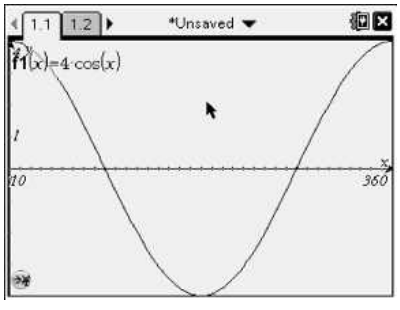
7 For 30°: 150°, 210°, 330°
For 45°: 135°, 225°, 315°
For 60°: 120°, 240°, 300°

8 (a) **(i)** 360° **(ii)** 360° **(iii)** 360°

(b) **(i)** 1 **(ii)** 2 **(iii)** 3

(c) The value of a tells you the amplitude.

(d) A graph like the following should have been sketched.



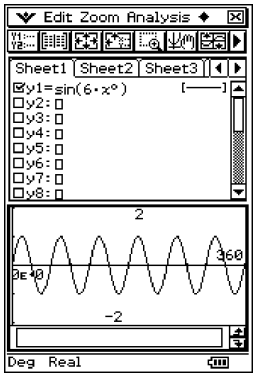
9 (a) **(i)** 360° **(ii)** 180° **(iii)** 120°

(b) **(i)** 1 **(ii)** 1 **(iii)** 1

(c) The value of n changes the period.

(In fact, the period is $\frac{360°}{n}$)

(d) A graph like the following should have been sketched.



10 It will be 156°, the equivalent angle in quadrant 2.

Open-ended – Sample answers

11 Use 24° and 204°

sin	+	−
cos	+	−
tan	+	+

12 (a) If θ is in quadrant 1, -θ is in quadrant 4 (a clockwise rotation of θ). So cos (-θ) and cos θ are positive and of equal magnitude.

If θ is in quadrant 2, -θ is in quadrant 3 so both cos (-θ) and cos (θ) are negative and of equal magnitude.

If θ is in quadrant 3, -θ is in quadrant 2 so both cos (-θ) and cos (θ) are negative and of equal magnitude.

If θ is in quadrant 4, -θ is in quadrant 1 so both cos (-θ) and cos (θ) are positive and of equal magnitude.

(b) Using a similar method as in **(a)**, sin θ is positive in quadrants 1 and 2, and negative in quadrants 3 and 4.

(c) $\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\tan(\theta)$

Exercise 6.3 (p. 190)

1 (a) $x = 120°, x = 240°$ **(b)** $x = 30°, x = 150°$

2 $x = (120 \pm 360n)°$, $n = 0, 1, 2 \ldots$ and
$x = (240 \pm 360n)°$, $n = 0, 1, 2 \ldots$

3 $x = (14.48 \pm 360n)°$, $n = 0, 1, 2 \ldots$ and
$x = (165.52 \pm 360n)°$, $n = 0, 1, 2 \ldots$

4 B

5 (a) 48.59° and 131.41° **(b)** 158.20° and 338.20°

(c) 53.13° and 306.87° **(d)** 41.81° and 138.19°

6 9.10 p.m. **7** C **8** C

9 (a) If the denominator becomes equal to 0.

(b) $x = 210°$ and 330°

10 $x = 0°$, 180° and 360°

Open-ended – Sample answers

11 $5 \sin x + 2 = 0$; $x = 203.58°, 336.45°$
$7 \tan x - 3 = 0$; $x = 23.20°, 203.20°$
$4 \cos x - 3 = 0$; $x = 41.41°, 318.59°$

12 (a) Students' own answers.

(b) In **(i)** it had to be a positive tan equation and in **(ii)** it had to be a negative sin equation.

Half-time 6 (p. 192)

1 (a) 17.10° **(b)** 27.20 cm

2 (a) $x = 41.8°, 138.2°$ **(b)** $x = 21.8°, 201.8°$

3 (a) sin 45° **(b)** -cos 82° **(c)** tan 20°

(d) -sin 67° **(e)** cos 60° **(f)** -tan 21°

4 (a) 65° **(b)** 13.6 cm

(c) **(i)** decrease **(ii)** 9°

5 C

6 (a) $x = (60 \pm 360n)°$ and $x = (300 \pm 360n)°$, $n = 1, 2, 3, \ldots$

(b) $x = (19.47 \pm 360n)°$ and $x = (160.53 \pm 360n)°$, $n = 1, 2, 3, \ldots$

Exercise 6.4 (p. 196)

1 (a) 6.56 **(b)** 5.89 **(c)** 48.42

2 (a) 82.16° **(b)** 40.45° **(c)** 68.03°

3 **(a)** 56.23°, 123.77° **(b)** 45.77°, 134.23°
(c) 68.03°, 111.97°

4 **(a)** 9.01 **(b)** 51.50 **(c)** 42.46

5 **(a)** 54.90° **(b)** 110.52° **(c)** 26.05°

6 39° **7** 6.78 units

8 $c^2 = a^2 + b^2 - 2ab\cos\theta$ and $d^2 = 4b^2 + 4a^2 - 8ab\cos\theta$
$d^2 = 4c^2 \Rightarrow d = 2c$

9 113.62°

10 $\angle ABC = \theta - \phi$, so $\frac{x}{\sin(\theta - \phi)} = \frac{y}{\sin\phi}$

$x = \frac{y\sin(\theta - \phi)}{\sin\phi}$

11 D **12** 60.99 **13** C

14–16 Students' own answers.

Open-ended – Sample answers

17 **(a)** $c^2 = a^2 + b^2 - 2ab\cos C$
$k^2a^2 = a^2 + a^2 - 2a^2\cos C$
$k^2 = 2 - 2\cos C$
$2 - k^2 = 2\cos C$
$\cos C = \frac{2 - k^2}{2}$

(b) $k = -1$, $\cos\theta = 0.5$
$k = 1.5$, $\cos\theta = -0.125$

(c) $-2 \le k \le 2$

18 If $b = 4a$ and $c = 4a$
$\theta = 82.82°$ ($\cos^{-1}\theta = \frac{1}{8}$)

Exercise 6.5 (p. 201)

1 4.92 m **2** 10.60 km **3** 32.76 m **4** 61.47 cm

5 68.40 m

6 Yes, because the angle is 104.478°.

7 Tim 8.22 km, Zoe 6.45 km; so Zoe is closer.

8 46.55° **9** 254 m **10** A

11 037°T and 235°T

12 **(a)** 47.55 m **(b)** 123.46 m

13 27.41 mm

14 16.33 cm or 21.50 cm

15 B **16** 4885 m

Open-ended – Sample answers

17 Students' own answers.

18 $\theta = 45°$, $x = 16$ m, $y = 22.63$ m
$\theta = 60°$, $x = 27.71$ m, $y = 32$ m

Exercise 6.6 (p. 205)

1 **(a)** 175.96 cm^2 **(b)** 1269.64 m^2 **(c)** 330.90 mm^2

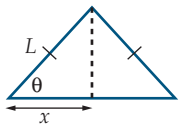
2 38°

3 **(a)** 365.01 cm^2 **(b)** 115.43 m^2 **(c)** 1718.63 mm^2

4 1292 km^2 **5** 5.85 mm^2 **6** C

7 **(a)** 36.38 cm^2 or 431.01 cm^2 **(b)** 278.60 m^2
(c) 583.30 km^2

8 **(a)** $x = L\cos\theta$



Length of base $= 2L\cos\theta$
Area of triangle $= \frac{1}{2}L \times 2L\cos\theta \times \sin\theta$
$= L^2\cos\theta\sin\theta$

(b) 15.68 km^2

9 **(a)** Area $= \frac{1}{2}a(b + c)\sin\theta - \frac{1}{2}ab\sin\theta$
$= \frac{1}{2}ab\sin\theta + \frac{1}{2}ac\sin\theta - \frac{1}{2}ab\sin\theta$
$= \frac{1}{2}ac\sin\theta$

(b) 90°, $\frac{1}{2}ac$ mm^2

10 $1426

11 **(a)** $0° < \theta < 180°$

(b) The area cannot be negative.

12 22 cm

Open-ended – Sample answers

13 Any triangle where the length of the equal sides is a square number and the included side is 30° (e.g. 36, 36, 30° or 16, 16, 30°).

Chapter review 6 (p. 208)

1 **(a)** 24.74 cm **(b)** 118.27 m

2 26.31 m

3 **(a)** 1 **(b)** 4 **(c)** 2 **(d)** 3

4 **(a)** $-\cos 7°$ **(b)** $-\sin 74°$ **(c)** $-\tan 48°$

5 B **6** 135°, 315°

7 $x = (75.52 \pm 360n)°$ and $x = (284.48 \pm 360n)°$, $n = 0, 1, 2 \ldots$

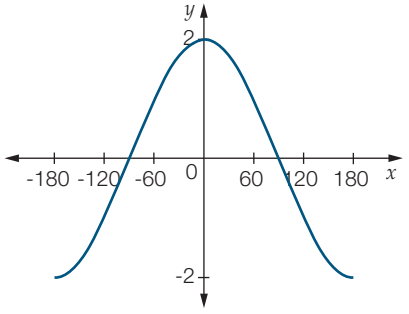
8 C **9** 12.81 **10** 73° **11** 207.70 m^2

12 D

13 **(a)** 180°, 7 **(b)** 72°, 6 **(c)** 45°, no amplitude

14 B

15



16 **(a)** quadrant 1 **(b)** 320°

17 122° **18** 46.03 cm^2 **19** 39.55 m

20 **(a)** 62.7° **(b)** 13.95 cm

21 (a) (i) 332.86 m (ii) 495.02 m

(b) 596.52 m (c) 056°T

22 multiples of 90°

23 (a) (i) Find the third angle α and use the sine formula

$\frac{x}{\sin\theta} = \frac{a}{\sin\alpha}$, where $\alpha = 180° - \theta - \phi$

(ii) 13.88

(b) (i) Each of the two other angles is $90° - \frac{1}{2}\theta$.

Use the sine formula $\frac{x}{\sin\theta} = \frac{a}{\sin\left(90° - \frac{1}{2}\theta\right)}$.

(ii) 14.51

24 D

25 Students' own answers.

Mixed review B (p. 211)

1 (a) \$788.20 (b) \$796.82 (c) \$801.22

2 4.4%

3 (a) $3x^2 + 2x - 8$ (b) $6x^2 + 13x - 5$ (c) $8x^2 - 26x + 15$

4 (a) $(3x - 7)(2x + 1)$ (b) $4x(3x - 2)(2x + 3)$

5 (a) $(x - 4)(x + 1)$ (b) $(x - 5)(x - 9)$

(c) $(x - 4)(x - 3)$

6 (a) $\frac{-5}{3x}$ (b) $\frac{3x^2 - 5x - 2}{x^2 - 1}$ (c) 3

7 (a) 75.40 mm^2 (b) 382.08 cm^2

8 319.41 cm^2 **9** 720 cm^3 **10** 6.4 g/cm^3

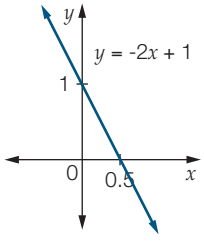
11 (a) $x = 9$ (b) $x = \frac{7}{3}$ (c) $x = 2$ (d) $x = 2$

(e) $x = 2$ (f) $x = 5$ (g) $x = -12$ (h) $x = 5$

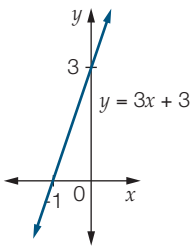
12 (a) 1.5 (b) 3 (c) -2 (d) 2

13 (a) (-2, -1) (b) (1, -1) (c) (-8, 0)

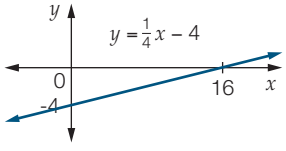
14 (a)



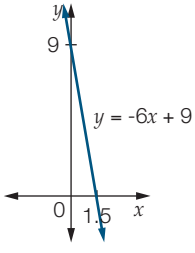
(b)



(c)



(d)



15 (a) Yes

$m_1 = -\frac{1}{2}$, $m_2 = 2$, $m_1 m_2 = -1$

(b) Yes

$m_1 = 2$, $m_2 = -\frac{1}{2}$, $m_1 m_2 = -1$

(c) Yes

$m_1 = -\frac{5}{2}$, $m_2 = \frac{2}{5}$, $m_1 m_2 = -1$

(d) No

$m_1 = \frac{4}{3}$, $m_2 = \frac{3}{4}$, $m_1 m_2 = -1$

16 (a) No

$m_1 = 2$, $m_2 = 1$, $m_1 \neq m_2$,

(b) No

$m_1 = 1$, $m_2 = -1$, $m_1 \neq m_2$

(c) No

$m_1 = -\frac{2}{5}$, $m_2 = \frac{2}{5}$, $m_1 \neq m_2$

(d) Yes

$m_1 = \frac{3}{10}$, $m_2 = \frac{3}{10}$, $m_1 = m_2$

17 (a) $x \geq -\frac{3}{2}$ (b) $x < -17$

18 (a) $x = -1$, $y = -2$ (b) $x = 7$, $y = -1$

19 (a) $h = 20.22$ (b) $h = 15.60$ (c) $a = 22.85$

20 63.43° **21** 7.79 km **22** 73 km

23 (a) 61.9° (b) 13.2 cm

24 (a) $x = 36.96$ m (b) $x = 39.47$ mm

(c) $x = 12.85°$ or $x = 167.15°$

25 (a) 141.42 m^2 (b) 513.07 mm^2 (c) 233.18 cm^2

26 22 cm

27 14 m × 10 m

28 Yes

29 49.62 cm

30 D

31 AB has a gradient of $\frac{-2}{3}$, BC has a gradient of $\frac{1}{5}$, CA has a gradient of -5.

BC and CA are perpendicular. Length of BC = length of $CA = \sqrt{26}$. Therefore, ABC is a right-angled isosceles triangle.

Area of ABC = 13 $units^2$

32 AB has a gradient of $-\frac{2}{3}$, BC has a gradient of $\frac{3}{2}$, CD has a gradient of $-\frac{2}{3}$, DA has a gradient of $\frac{3}{2}$. AB and BC are perpendicular, BC are CD are perpendicular, CD and DA are perpendicular and DA and AB are perpendicular. Therefore, $ABCD$ is a rectangle.

Area of $ABCD$ = 39 $units^2$

33 59°, 121°, 115° and 65°

34 $5y = 3x + 21$

35 (a) 67.86° (b) 756 cm

36 41.71 m

Challenge B (p. 215)

1 B

2 $a = 15 - 4 = 11$, $b = 13 - 3 = 10$. $a - b = 1$

3 D

4 **(a)** $R(0, 2)$, BR: $2x + 9y - 18 = 0$

(b) $P(3, 0)$, PC: $2x + y - 6 = 0$

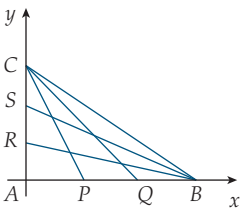
(c) X: $8y - 12 = 0$, $y = 1.5$, $x = 3.6$. $X(2.25, 1.5)$

$Q(6, 0)$, QC: $x + y - 6 = 0$

$S(0, 4)$, SB: $4x + 9y - 36 = 0$

Y: $5y - 12 = 0$. $y = 2.4$, $x = 3.6$. $Y(3.6, 2.4)$

XY: $2x - 3y = 0$. This passes through $A(0, 0)$ so X, Y and A are collinear.



5 **(a)** -1

(b) CE: $y = -x + 7$ or $x + y - 7 = 0$. $D(0, 7)$

(c) AE: $y - 4 = -3(x - 3)$, $3x + y - 13 = 0$. $A\ 4\frac{1}{3}$, 0. $B(0, 13)$

(d) Shaded area = area ΔODC + area ΔBDE = 24.5 + 9 = 33.5 units^2.

6 D $P(z, 5z + 3)$, $Q(3, -2)$. $M\ \frac{z+3}{2}, \frac{5z+1}{2}$. Hence $x = \frac{z+3}{2}$, $y = \frac{5z+1}{2}$, $z = 2x - 3$, $y = \frac{5(2x-3)+1}{2}$, $y = 5x - 7$

7 B **8** 59°

9 $n = 14$, area = 84 cm^2

10 $\cos(\angle MQP) = 0.28$

11 4.58 units

12 13.55 units

13 AB = 3.2 m. Length of timber = 16.9 m, $\angle BAX = 22°$, $\angle BYX = 44°$

Chapter 7

Recall 7 (p. 217)

1 **(a)** mean = 3.2; median = 4; mode = 4; range = 4

(b) mean = 4.7; median = 5.0; mode = 5.0; range = 2

2 **(a)** discrete **(b)** continuous **(c)** nominal

3 **(a)**

STEM	LEAF	
16	1 3	
16	5 5 5 6 7 7 7	Median (9 | 9)
17	0 1 2 2 3	
17	8 9	
18	0 3	

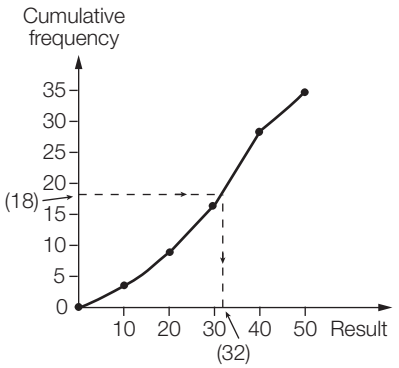
(b) median = 169 cm

4 **(a)** 6.4875 **(b)** 6.50–<6.75

Exercise 7.1 (p. 220)

1 **(a)**

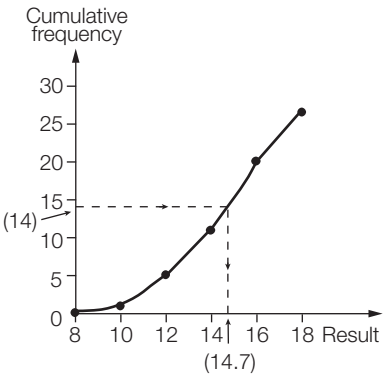
Result	Cumulative frequency
<0	0
<10	4
<20	9
<30	17
<40	28
<50	35



median = 18th term ≈ 32

(b)

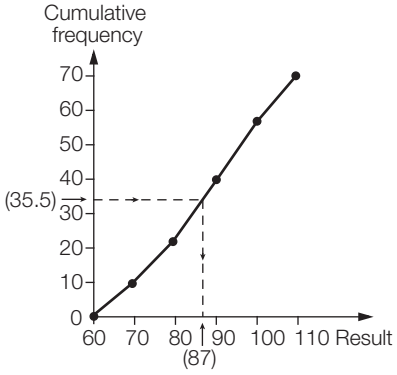
Result	Cumulative frequency
<8	0
<10	1
<12	5
<14	11
<16	20
<18	27



median = 14th term ≈ 14.7

(c)

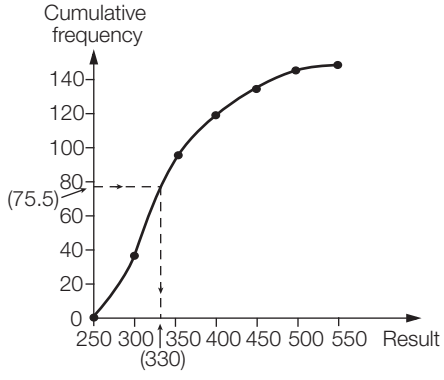
Result	Cumulative frequency
<60	0
<70	9
<80	21
<90	40
<100	57
<110	70



median = 35.5th term ≈ 87

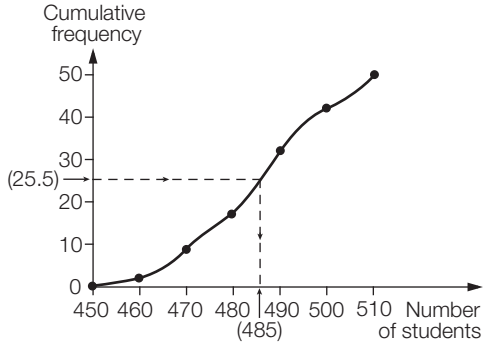
(d)

Result	Cumulative frequency
<250	0
<300	37
<350	95
<400	121
<450	137
<500	146
<550	150



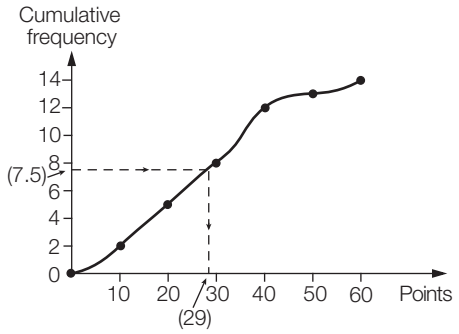
median = 75.5th term ≈ 330

2 (a)



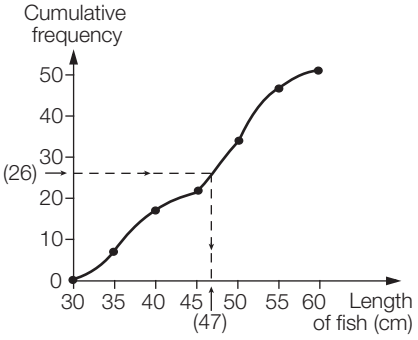
median = 25.5th term ≈ 485 students

(b)



median = 7.5th term ≈ 29 points

3



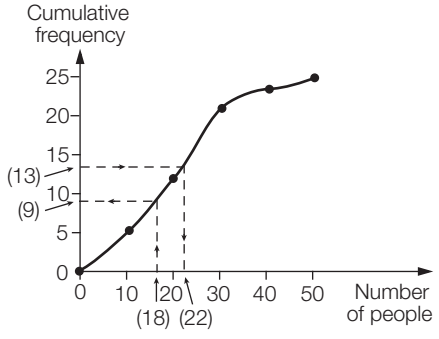
median = 26th term ≈ 47 cm

4 (a) A (b) C (c) B

5 (a) median = 21 people

(b)

No. of people	Cumulative frequency
≤0	0
≤10	5
≤20	12
≤30	21
≤40	23
≤50	25



median = 13th term ≈ 22 people

(c) from raw data 9 trains, from cumulative frequency curve 9 trains

6 (a) 14.7 (b) same

(c) 485.7 students (d) basically the same

(e) If the data is given in frequency table form, the new method is much quicker and seems to give approximately the same results. If the data is given in cumulative frequency graph form, the new method does not seem to have much to offer. The new method assumes that the results are evenly spread throughout the class interval, but this is not necessarily true. However, this is basically the same assumption made by connecting the points on a cumulative frequency graph with straight line segments.

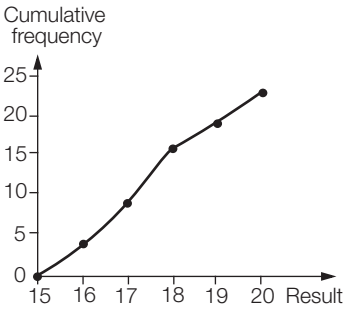
Open-ended – Sample answers

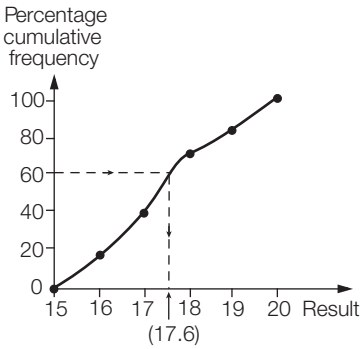
7 Students' own answers.

Exercise 7.2 (p. 227)

1 (a)

Result	Cumulative frequency	Percentage cumulative frequency
<15	0	0
<16	4	17
<17	9	39
<18	16	70
<19	19	83
<20	23	100

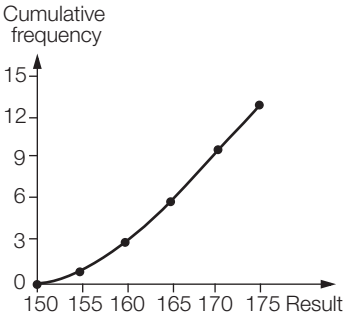


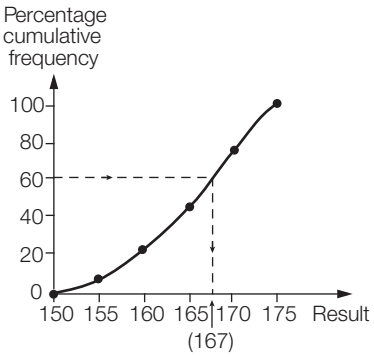


60th percentile is 17.6

(b)

Result	Cumulative frequency	Percentage cumulative frequency
<150	0	0
<155	1	8
<160	3	23
<165	6	46
<170	10	77
<175	13	100



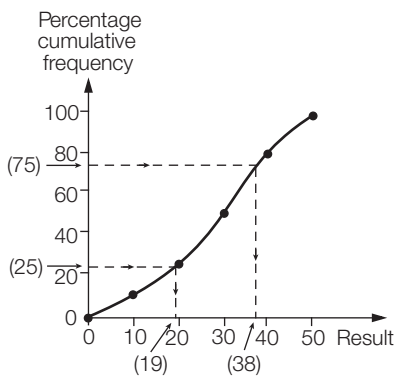


60th percentile is 167

2 (a) (i) Range = 50 – 0 = 50

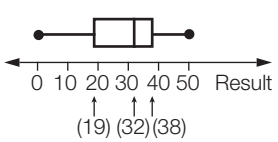
(ii)

Result	Cumulative frequency	Percentage cumulative frequency
<0	0	0
<10	4	11
<20	9	26
<30	17	49
<40	28	80
<50	35	100



IQR = 38 – 19 = 19

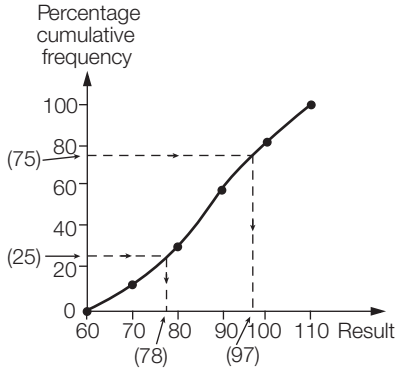
(iii)



(b) (i) Range = 109 – 60 = 49

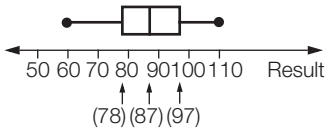
(ii)

Result	Cumulative frequency	Percentage cumulative frequency
<60	0	0
<70	9	13
<80	21	30
<90	40	57
<100	57	81
<110	70	100



IQR = 97 – 78 = 19

(iii)



3 IQR = 14.5, $Q_L = 16.5$ and $Q_U = 31$. The score of 54 is an outlier. Even though 8 seems quite a small total it is well within the lower bound for outliers. The high score may have been the result of a low-quality opposition.

4 (a) 65

(b) $Q_L = 159.5$; $Q_U = 179.5$; IQR = 20

(c) lower than 129.5 or higher than 209.5

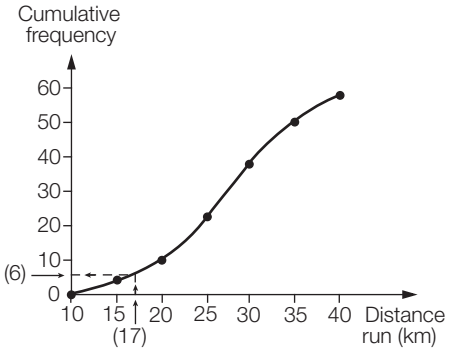
(d) There are no outliers at either end of the data set.

5 (a) C (b) D (c) B

6 D

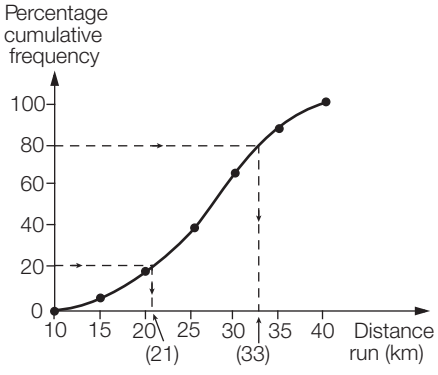
7 (a)

Distance run (km)	Cumulative frequency	Percentage cumulative frequency
<10	0	0
<15	4	7
<20	10	17
<25	23	39
<30	39	66
<35	50	85
<40	59	100



(b) Six distances run were less than 17 km.

(c)

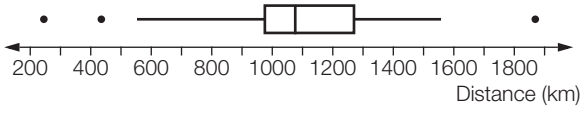


(d) 20th percentile is 21; 0.8 quantile is 33.

8 (a) 1635

(b) $Q_L = 972$; $Q_U = 1275$; IQR = 303

(c)



9 (a) Yes, as long as the outlier is more than 1.5 × IQR above Q_U or more than 1.5 × IQR below Q_L.

(b) Students' own answer.

10 (a) (i) less than -5 and greater than 115

(ii) no

(b) (i) less than 32.5 and greater than 52.5

(ii) yes, at both ends

11 (a) (i) As an example: 2 2 2 2 2 2 4 4 5 5
5 5 5 5 5 5 6 6 7 7

(ii) The lowest value is the same as the lower quartile so there needs to be at least six 2s in the list, assuming we are using exactly 20 values. The median does not exist in a discrete fashion so must be the same as either the lower or upper quartile. In this case the upper quartile was chosen. If the lower quartile was chosen, then there would have to be at least eleven 2s.

(b) (i) As an example: 3 3 3 3 3 3 4 5 5 6
6 6 6 6 7 7 7 7 7 7

(ii) In this case there are no whiskers so we need at least six 3s and six 7s. Place the 6s in the middle to ensure that 6 is the median and then fill in the gaps.

Open-ended – Sample answers

12 (a) mean = \$46 829.27, median = \$45 000

(b) range = \$50 000, IQR = \$10 104.17 (using the linear approximation method)

(c) Yes, values above \$66 093.76, so at the very least the three highest values are outliers and perhaps one or both of the values in the \$65 000–< \$70 000 class interval.

(d) Removing only the three highest values gives mean = \$45 664.56, median = \$44 791.67 (using the linear approximation method), range = \$35 000, IQR = \$9687.50 (using the linear approximation method).

(e) The mean and range are more affected by the removal of the outliers than are the median and the IQR. This needs to be considered when analysing data sets that contain outliers—the mean and range will need to be treated with some caution.

13 (a) Students' own graph.

(b) Students' own graph.

14 (a) Example: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 34

IQR = 11, $Q_L = 5.5$, $Q_U = 16.5$, so any value above 16.5 + 16.5 = 33 will be an outlier.

(b) Example: 36 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69

IQR = 11, $Q_L = 53.5$, $Q_U = 64.5$, so any value below 53.5 – 16.5 = 37 will be an outlier.

Exercise 7.3 (p. 232)

1 (a) Set A:

(i) 30.74 **(ii)** 30 **(iii)** 18 **(iv)** 10

Set B:

(i) 30.21 **(ii)** 30 **(iii)** 28 **(iv)** 10

(b) Set A:

(i) 51.22 **(ii)** 51.5 **(iii)** 16 **(iv)** 8

Set B:

(i) 51.22 **(ii)** 51.5 **(iii)** 56 **(iv)** 8

(c) Set A:

(i) 16.82 **(ii)** 15.5 **(iii)** 15 **(iv)** 8

Set B:

(i) 16.64 **(ii)** 14.5 **(iii)** 15 **(iv)** 8

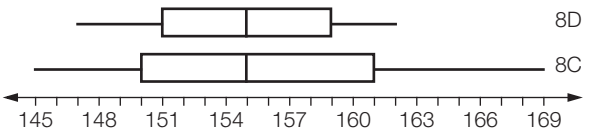
(d) The mean is affected by all changes. The only time the mean did not change in these samples was when the lowest number and the highest number were adjusted by the same amount in opposite directions. The median is very stable and was only affected slightly when data in the middle of the data set was changed. The range fluctuates wildly and is severely affected by any change in the extreme values. The IQR is also quite stable.

2 (a) 8C: 155 cm; 8D: 155 cm

(b) 8C: 24; 8D: 15

(c) 8C: 11; 8D: 8

(d)



(e) 8C has a much greater range and IQR than 8D. They have the same median value. The height of students in 8D is much more consistent than for the students in 8C.

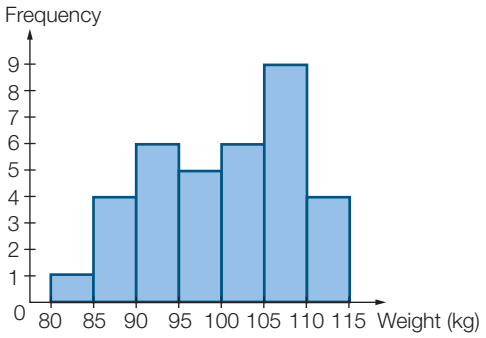
3 (a) **(i)** Set A: 45; Set B: 50

(ii) Set A: 35; Set B: 35

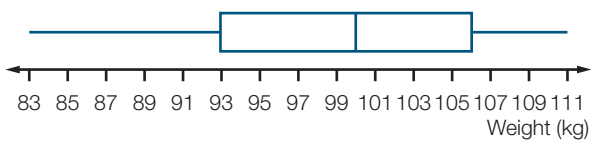
(iii) Set A: 10; Set B: 20

(b) Although they have the same range the two data sets are quite different. Set B has a much bigger IQR, whereas Set A has a much longer right-hand whisker. About 75% of Set A are shorter than the median of Set B.

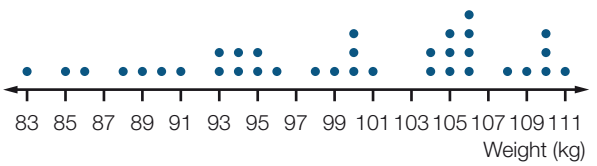
4 (a)



(b) min: 83; lower quartile: 93; median: 100; upper quartile: 106; max: 111



(c)



(d) The dot plot shows that the data set is relatively evenly spread out with a slight skew to the right (negative skew). The box plot shows that the median is close to the middle, but slightly to the right. The grouping in the histogram can mask some trends but it is also relatively even with the highest frequency occurring near the right-hand end.

5 (a) B **(b)** B

6 The huge range, and relatively big IQR, for Class A means there were some students who did not perform very well on the test as well as some who did extremely well. The fact that some students did well dragged up the mean. Class B was the most consistent class. It depends on what you mean by 'best' as you could argue Class B did best because of its

even results. However, the very highest results are unlikely to be found in Class B. The high IQR for Class C indicates that the middle results were very spread out but the other results were not far outside the middle box.

7 (a) **(i)** 3 **(ii)** 1

(b) **(i)** 21 **(ii)** 1

(c) The median and IQR give a better feel for the data set as a whole but do not give even a hint that we have two extreme outliers.

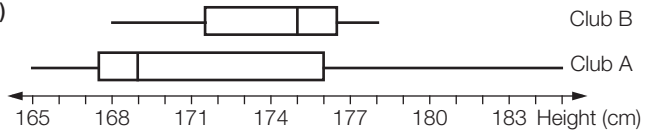
8 (a) Club B seems to be taller.

(b) Mean heights: Club A is 171.75 cm and Club B is 174.1 cm

Median heights: Club A is 169 cm and Club B is 175 cm

(c) Range: Club A is 20 and Club B is 10; IQR: Club A is 8.5 and Club B is 5.

(d)



(e) Yes. Even though Club B does not have the tallest player, it has more consistently tall players, shown by the smaller range and IQR, spread over higher values.

(f) The bunching of the dots for Club B gives us a clue that it will have a small IQR. The parallel dot plot clearly indicates that the range for Club A is going to be much greater.

9 (a) The distances range from 2 to 40 km with a median distance of 12 km. This means about 50% of the students travel up to 12 km to school. The IQR lies between 8 and 26 km.

(b) Students to give their own answers, but it should be greater than 12 km.

(c) Students to give their own answer, but a possible answer would be based on:

Distance (km)	Frequency
0–<4	10
4–<8	30
8–<12	40
12–<16	10
16–<20	15
20–<24	10
24–<28	15
28–<32	10
32–<36	10
36–<40	10

This data set has a mean of 16.5 km.

10 (a) Mean heights: Adelaide 187.77 cm and Essendon 187.83 cm

Mean weights: Adelaide 86.11 kg and Essendon 86.54 kg

(b) Median heights: Adelaide 188 cm and Essendon 187 cm

Median weights: Adelaide 85 kg and Essendon 86 kg

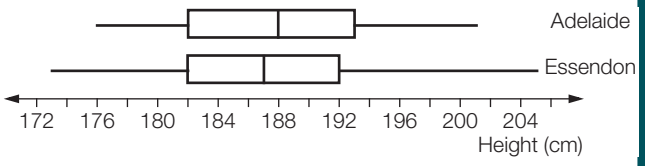
(c) Heights range: Adelaide 25 cm and Essendon 32 cm

Heights IQR: Adelaide 11 cm and Essendon 10 cm

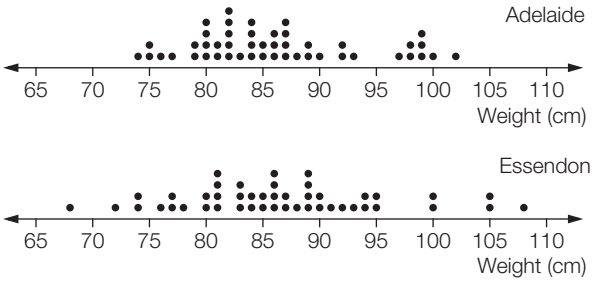
Weights range: Adelaide 28 kg and Essendon 40 kg

Weights IQR: Adelaide 9 kg and Essendon 10 kg

(d)



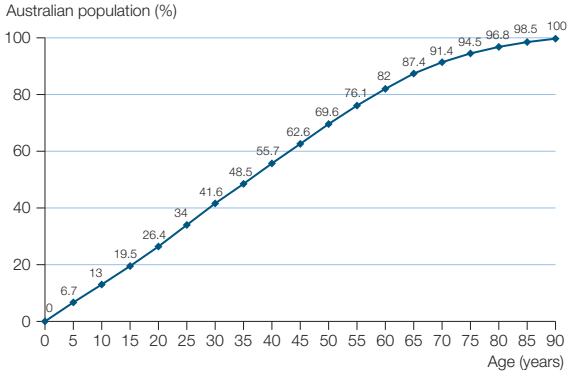
(e)

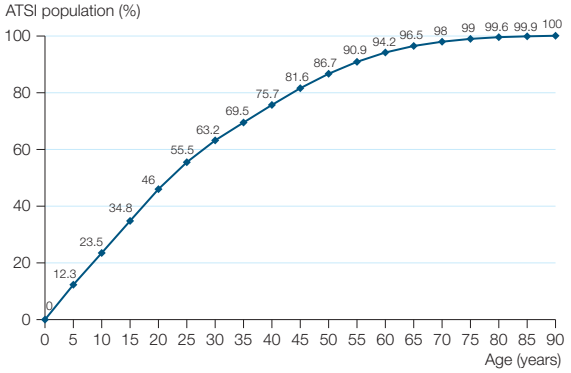


(f) Putting together all of the information collected the final comments would be that Essendon has a greater range for height than Adelaide but essentially the two clubs are extremely close in terms of height.

(g) Sample comment: Essendon has the two lightest players but also the two heaviest. Essendon seems to be spread out more evenly, whereas Adelaide is clumped in a couple of places. The weights are quite even but Essendon may be marginally heavier.

11 (a)

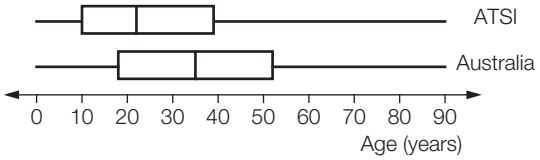




(b) Australia: min = 0; $Q_L = 18$; median = 35; $Q_U = 52$; max = 90

ATSI: min = 0; $Q_L = 10$; median = 22; $Q_U = 39$; max = 90

(c)



(d) The cumulative frequency curves start to tell the story with their very different shapes. The rapid rise at the beginning of the ATSI graph indicates that a great percentage of the population are young. This is confirmed by the box plot, which shows the median age to 22 and the lower quartile to be 10. This means 25% of the ATSI population is less than 10 years of age, compared to an age of 18 for the Australian population as a whole. The medians are different by 13 years. The upper quartile for the ATSI population is barely bigger than the median for the Australian population.

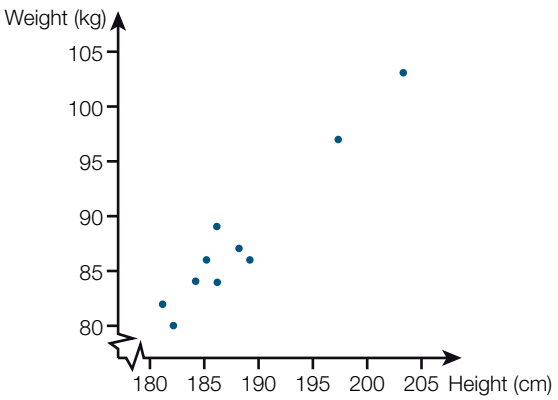
Open-ended – Sample answers

12 Students' own answers.

Exercise 7.4 (p. 239)

1 **(a)** primary **(b)** primary **(c)** secondary **(d)** primary **(e)** secondary

2



As height increases so it seems does weight. However, this relationship is not perfect so we would probably say there is a strong positive relationship between the two variables.

3 **(a)** census **(b)** sample **(c)** census **(d)** census

4 **(a)** Inappropriate—not relevant

(b) Probably appropriate—if not often then may not know much about sport

(c) Inappropriate—is on the way to being appropriate but should not be closed

(d) Inappropriate—not relevant

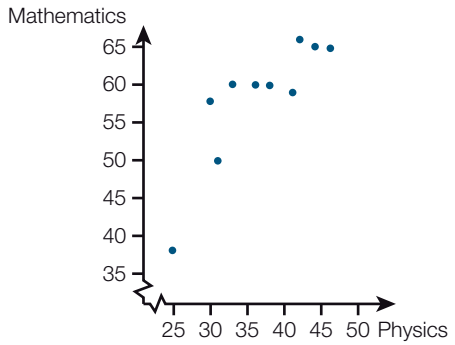
5 **(a)** Biased—not a broad enough range of people

(b) Biased—may be more likely to support environmental causes

(c) Unbiased

(d) An inappropriate method, whether biased or not is unclear.

6 **(a)**



(b) For the lower physics marks it seems that as the mark increased so did the mathematics mark. However, this trend did not continue with the higher physics marks where the mathematics mark seemed to flatten out and not increase at the same rate as the physics mark. Overall, there is a moderate positive relationship.

7 **(a)** This is a very strong negative linear relationship.

(b) This is a strong positive linear relationship.

(c) This looks like a non-linear relationship; it has a curve associated with it. However, it does display a negative relationship.

(d) This looks like there is no relationship between the variables.

(e) This looks like a non-linear relationship. The points go up and down as you move across the horizontal axis.

(f) This looks like a weak negative linear relationship.

8 **(a)** It would be difficult and time consuming to get the opinions of parents compared to students and staff who are on campus.

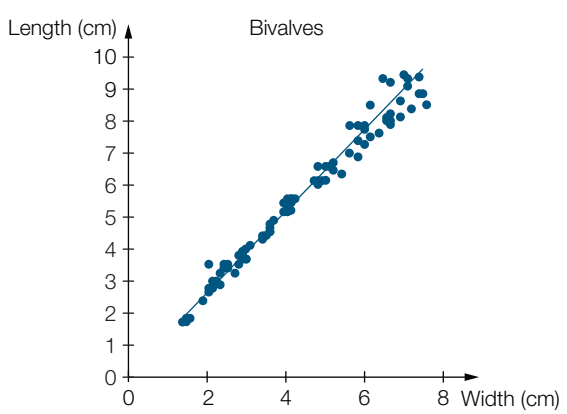
(b) Too many to conduct a census

(c) You need to keep the representation level the same as for students

(d) A stratified random sample will randomly select students from each strata, in this case year levels. It is quick and relatively cheap to conduct, and representative of the structure of the school.

(e) Students will provide their own answers.

9 **(a)**



(b) As the width of the bivalve increases so does the length. These two variables seem to have a very strong positive linear relationship.

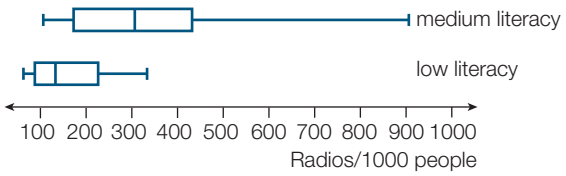
10 (a) secondary

(b) There seems to be a strong positive linear connection between the two variables. Male rates are possibly a little higher than female rates.

(c) No—a different population group entirely.

11 (a) The countries with a higher literacy rate have a higher number of radios.

(b)



(c) The range for medium literacy countries is much higher but the lowest value for the medium countries is higher than the lower quartile for the low countries. The highest literacy value for the low countries is only just higher than the median value for the medium countries. The medium literacy countries have a very long right-hand whisker; it may be that there are one or two outliers that have made the comparisons inaccurate. The numbers of radios are clearly different between these two types of groups of countries. However, it would be incorrect to conclude that a country's literacy rate determines the number of radios.

(d) 907 radios per 1000 people in a medium literacy rate country.

Open-ended – Sample answers

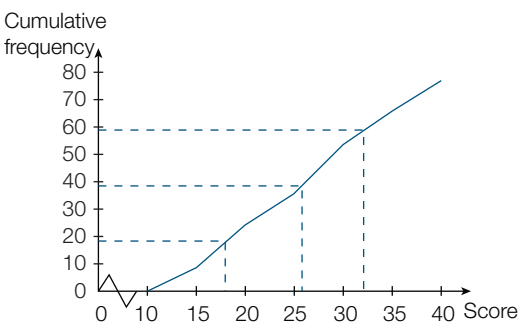
12 Students' own answers.

13 Students' own answers.

Half-time 7 (p. 243)

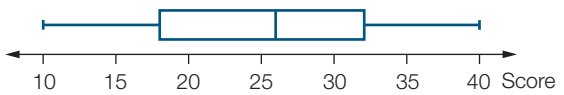
1 (a) Range: 40 – 10 = 30

(b)



From the cumulative frequency graph we get $Q_L = 18$ and $Q_U = 32$, which gives an IQR of 32 – 18 = 14.

(c)

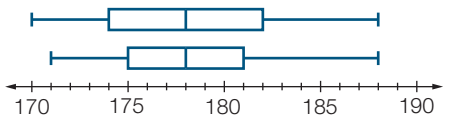


2 (a) Club B appears to be taller but this is mainly based on the fact that it does not have as many players at the lower end of the height scale.

(b) Club A: mean = 178.47, median = 178, range = 18, IQR = 8

Club B: mean = 178.4, median = 178, range = 17, IQR = 6

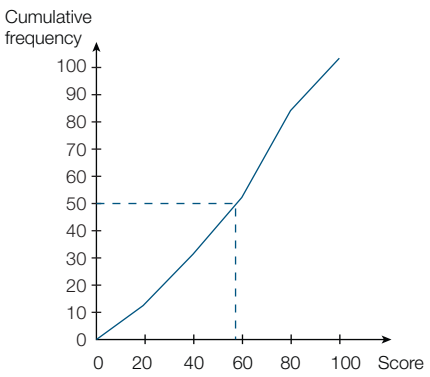
(c)



(d) Club A has a very slightly larger average height, however, they are essentially the same. The IQR in Club A is also slightly larger, which shows a slightly larger spread of heights compared to Club B.

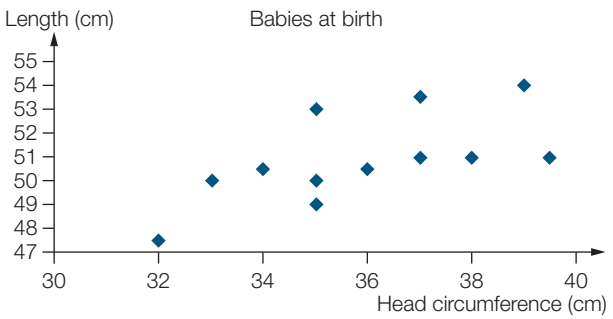
(e) The IQR, as this statistic has the greatest difference and shows how the spread of the data varies between the two clubs.

3



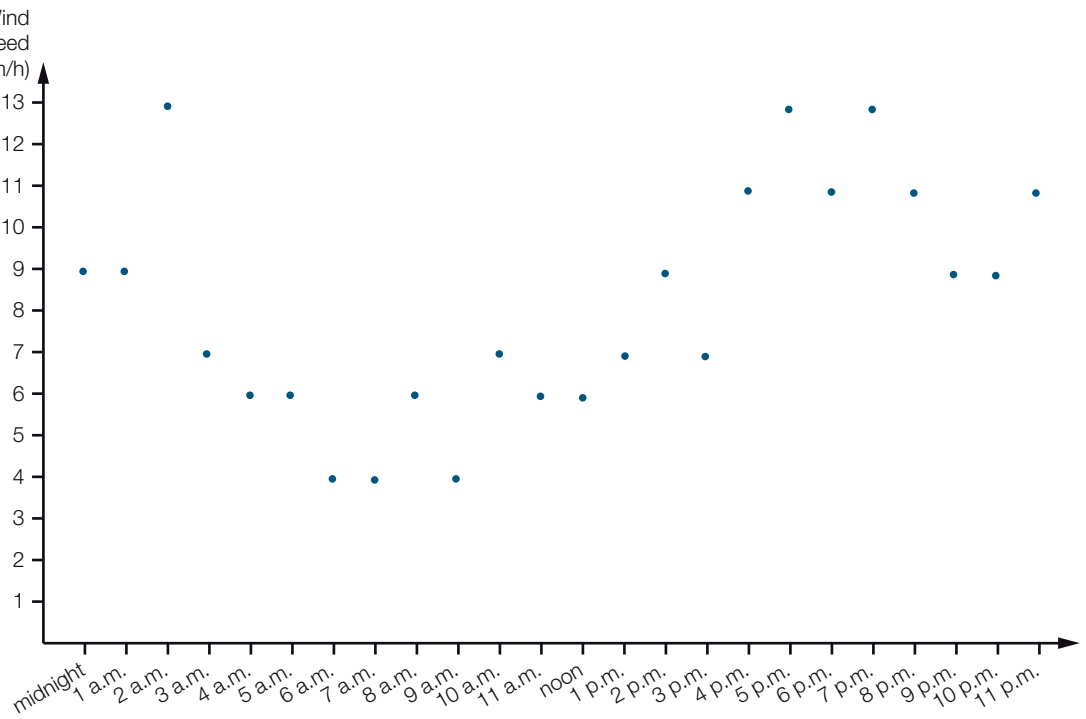
Median = 58

4 There appears to be a moderate positive linear relationship between head circumference and length in babies at birth.



Exercise 7.5 (p. 245)

1 (a)



(b) It is much higher than the readings around it.

(c) The wind speed fluctuates between 11 km/h and 13 km/h.

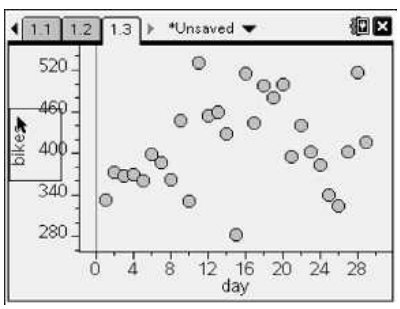
2 (a) Hours spent studying; this is the variable under our control.

(b) Age; this is the variable that is always changing—height does not necessarily change.

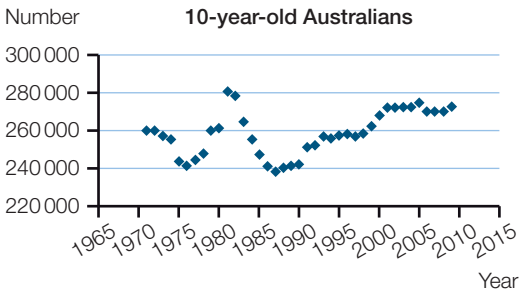
(c) Number of cigarettes; this is what has an impact on age at death.

(d) Number of people; change the number of people and you would expect a change in the amount of rubbish.

3 There does not appear to be a pattern in the number of bicycles. This is perhaps not an unexpected result because the weather could play a significant role in the number of cyclists on any particular day.

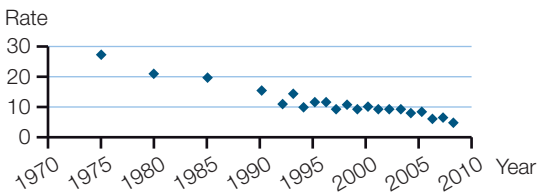


4 (a)



(b) The number of 10-year-old Australians has fluctuated a little over this time period but is now about the same as it was at the beginning of the period. There was a peak in the early 1980s, followed by a trough in the mid-1980s when the lowest number was recorded, but the number has steadily increased since then. The number appears to be becoming more stable.

5 (a)



(b) There is certainly a trend for a considerable decrease in the death rate per 100 000 in South Australia, although the fall has not been consistent. As may have been expected, the most significant reductions were made in the earlier years and there have been some slight ups and downs along the way. It would be interesting to compare the rate with significant road toll initiatives, such as the introduction of the compulsory wearing of seat belts.

6 (a) The population has increased consistently over this period of time. The last 40–50 years have seen the greatest rate of increase.

(b) This inclusion would have minimal impact except perhaps in the very early years shown on the graph. The graph would have started at a higher value, clearly off the axis. Depending on how rapidly the Aboriginal population declined would determine whether or not their inclusion would be detectable.

(c) The population was very small, but the scale needs to be as is to fit today's population into a reasonably sized graph.

(d) In the very early years by counting every individual. Records of arrivals, departures, births and deaths would also have been used.

(e) The very early years may actually be more reliable than the later ones because the later ones have some element of estimation built into them.

(f) There is a census every 5 years and population models are based on the data collected.

7 The number of 90-year-olds is increasing at a fairly rapid rate. The average lifespan is increasing in parallel with improvements in medical care. For younger people, the same trend would probably not be seen.

Open-ended – Sample answers

8 The graph lacks a title and axes labels. There also needs to be a key to identify which colour represents which model. The vertical scale could also be stretched a little to show the differences more clearly.

9 Students' own answers.

10 Students' own answers.

Exercise 7.6 (p. 248)

1 (a) The shaded numbers are approximations, the others are certain.

Position	Sports Identity
1	Greg Norman
2	Andrew Bogut
3	Harry Kewell
4	Mark Webber
5	Chad Reed
8	Geoff Ogilvy
9	Marcos Ambrose
10	Casey Stoner
13	Ricky Ponting
17	Brett Lee
19	Lleyton Hewitt
21	Michael Clarke
41	Michael Hussey
50	Andrew Symonds

It is difficult to be precise about some of these positions because there are large gaps between the positions mentioned in the article.

(b) approximately 9.5% drop

(c) approximately 56.4% drop

(d) $\frac{65}{5} \neq 13$ so it is not just a straight division. The contract must have some higher up–front payments.

(e) $1.53m is better than 41st but not as good as 21st ($2.54m). Somewhere around the low 30s would seem appropriate.

2 (a) What a mess! A pie graph assigns percentages of a whole and these percentages clearly do not add up to 100. The explanation given is that it was possible to offer support to more than one candidate. That is fine, but a pie chart should never have been used to represent it. You would not be able to draw a correct graph from this information as you would need to know which one candidate each person supported.

(b) The lottery graph shows that fewer people pick the numbers from 0 to 100, possibly because they don't like starting their choice with one, or even two, zeroes. With few people to claim the prize pool, the winning payoff is larger.

3 (a) $390 (b) $341

(c) 14.4%, so the article has not exaggerated the rise in its claim of 14%

(d) $345.43

(e) old: $1020.41; new: $1070.41

(f) 11.1% p.a. increase

(g) Crash repairs are a big part of the cost of insurance. If car crashes are reduced the overall cost of providing insurance cover should also reduce.

4 (a) Rounded to one decimal place:

Guildford 17.3%; Mount Richon 16.9%; Victoria Park 15.7%; Darlington 14.6%; Orelia 13.9%; Maylands 12.5%; South Fremantle 12.1%; Stoneville 11.7%; Fremantle 10.7%; Joondanna 10.2%

(b) Rounded to one decimal place:

North Beach -35.4%; Ascot -32.1%; Mount Lawley -28.3%; Mosman Park -24.5%; Forrestdale -24.0%; Applecross -22.8%; Perth -21.1%; Marmion -21.0%; Kallaroo -17.4%; Swanbourne -17.3%

(c) There is wide variability in prices both in absolute terms and in percentage change terms. It might even be described as volatile. The big losses seem to be associated with some of the more expensive suburbs.

(d) The number of houses in each of these suburbs. This would enable a percentage turnover to be calculated.

(e) House buyers concentrated on lower priced houses so the prices of houses in the more expensive suburbs fell as there was little competition for these properties.

(f) Mode is almost useless as there is such a range of prices possible. It might be that the only two houses sold for the same price were the least expensive houses of all; this is hardly representative. Mean is affected by very low and very high prices.

5 (a) 360

(b) Students to provide their own answer, but it is likely they will say it is not a very big sample.

(c) self-selecting

(d) Yes, the questions are quite different. It is quite possible for a person to hold a view about what should happen in general but have a different view about the same issue as it directly affects them.

(e) The telcos may be talking themselves up. It would be interesting to see some data to support this view. However, it is quite possible, as the survey methodology seems to be quite flawed.

(f) These findings are probably not very reliable. The sample size is relatively small and the selection process would not lead to robust results.

6 (a) It is between two outlets that have both recently sold tickets that won multimillion dollar prizes, so it could be next.

(b) No, this is nonsense.

(c) At the very least you would want to know how many tickets were sold. What percentage of tickets sold are winners? But even this is ridiculous as each ticket, no matter where sold, has the same chance of winning.

(d) If the numbers have been drawn often then they are lucky numbers that will continue to be drawn. If they haven't been drawn for a long time then it is their turn to be drawn. Both arguments are bereft of statistical merit.

7 (a) Mr Casey is the preferred Prime Minister (48%) compared with Mr Smith (46%). Only female voters preferred Mr Smith (47%) compared with Mr Casey (41%). The heading is misleading.

(b) Sample size 1030, $n = 1030^2 = 1\,060\,900$. This is a reasonable sample size, given that the voting population may be less than 10 million.

(c) Telephone survey is the most convenient sampling method.

(d) The figures for female opinions (as in part **(a)**) give some justification for the heading 'Voters back Smith'—but this is not true for male voters.

(e) When you add the votes for either preferred Prime Minister, the total male votes equal 99% and the total female votes equal 88%. The missing 1% and 12% of votes respectively are due to the 'neither' and 'don't know' categories chosen by some persons surveyed.

(f) These categories attempt to identify the grounds for the voters' opinions. We do not know whether these categories are important to the voters.

(g) This poll does indicate that the preferred Prime Minister is Mr Casey (in the general population), but only slightly. The survey also covers the personal qualities of the candidates.

8 (a) The other States and the ACT probably don't have a high enough Indigenous population to make the statistics accurate.

(b) Males 78.7 years and females 82.6 years.

(c) Students to provide their own answer, but would probably include thoughts such as lack of medical facilities in many traditional communities and the impact of certain illnesses and diseases, such as diabetes, on the Indigenous communities.

(d) It is difficult because the ATSI population count is probably not accurate and deaths are not always reported.

(e) Canada, New Zealand and the United States also have significant Indigenous populations and the countries themselves are quite similar to Australia.

(f) Students to provide their own answer, but some of the following may be included. The percentage increase for Indigenous males is 2.54% compared to 1.56% for non-Indigenous males and for females it is 2.00% for the Indigenous population compared to 1.09% for the non-Indigenous population. So the rate of increase is higher but this is due to the lower starting figure. A fair society would want to see these differences become much smaller.

Open-ended – Sample answers

9 Students will provide their own answers.

10 The most interesting question is where the figure for the probability of finding buried treasure came from. The other probability can probably be calculated although it is open to debate what the expression 'top prize at the pokies' means. Each machine would have a different 'top prize' so this makes things more difficult. However, the thrust of the advertisement about the unlikelihood of winning a huge prize on the pokies is to be applauded.

Exercise 7.7 (p. 258)

1 5.72

2 English: mean = 34.33 and standard deviation = 9.22

Mathematics: mean = 34.67 and standard deviation = 7.12

Although the means are effectively the same the standard deviation for mathematics is smaller indicating that the results in that subject are more tightly packed around the mean. In turn, this indicates that the more extreme results will be found in the English marks.

3 mean = 3.69 and standard deviation = 2.52

4 (a) Australia: 241; New Zealand: 597; Brazil: 940; Sri Lanka: 628

(b) Australia: 76.50; New Zealand: 190.74; Brazil: 250.39; Sri Lanka: 165.97

(c) As the range increases so does the standard deviation except for Sri Lanka. Its standard deviation is less than New Zealand's despite its range being slightly higher than New Zealand's. This is probably due to the highest peak in Sri Lanka being quite a bit higher than the next value. It may be an outlier.

5 (a) (i) 4.81 (ii) 8.77 (iii) 6.12 (iv) 9.66

(b) Introducing an outlier at either end has a similar effect as increasing the standard deviation (one of them was further away) and introducing two outliers at either end stretches the standard deviation even further.

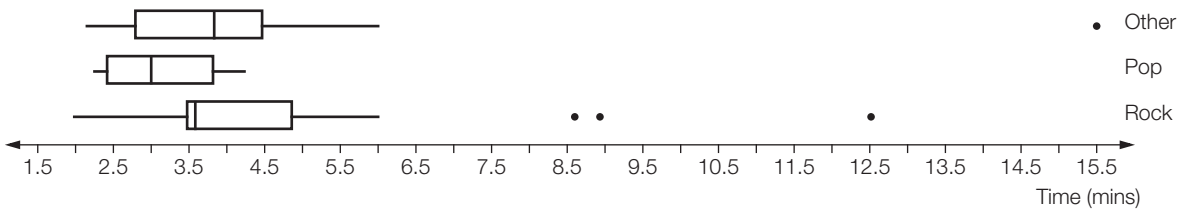
6 B

7 (a) mean = 4:22 and standard deviation = 2:12

(b) mean = 3:06 and standard deviation = 0:41

(c) mean = 4:12 and standard deviation = 2:36

(d)



(e) From the evidence presented you could conclude that pop songs are more formulaic than either of the other categories. The standard deviation was very small, the box has no outliers and the range is very small. The longest pop song is within the third quarter of each of the other box plots. This means at least 25% of each of the other categories are longer than the longest pop song. The number of pop songs was the smallest, which may have had an effect on the data. The Other category has one wild outlier. It is 9.5 minutes longer than the next longest song. On the other hand there were three outliers in the Rock category.

Open-ended – Sample answers

8 (a) Students' own answers.

(b)–(c) The range condition means the data set cannot be mostly one number with just a little tinkering. The first step would be to calculate what the sum of the data set would be, number of values × mean. Then, make a start on the data set so it adds up to this value. Some adjustments may be required to make the standard deviation work.

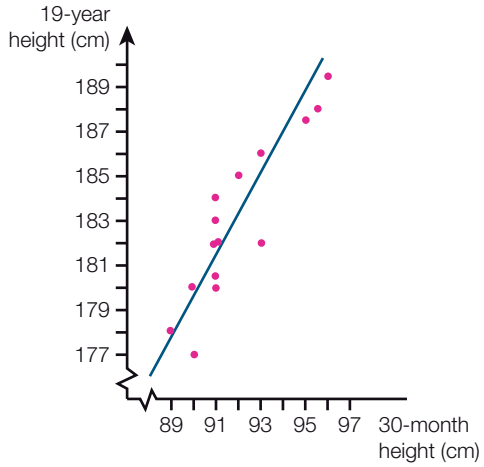
9 The first step is to identify the five-number summary: min = 12, $Q_L = 13$, median = 16.5, $Q_U = 18$ and max = 20. Construct the data set around these values and then find the mean and the standard deviation.

Exercise 7.8 (p. 261)

1 (a) $y = 12.107x + 4.536$ (b) $y = 15.497x - 14.552$

(c) $y = -4.261x + 46.931$ (d) $y = -4.863x + 39.999$

2 (a) and (b)



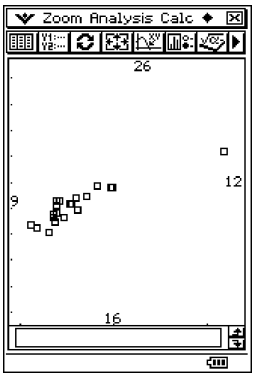
(c) The 30-month height does seem to be a reasonably good predictor of adult height.

(d) approximately 187 cm (will vary depending on the line of best fit)

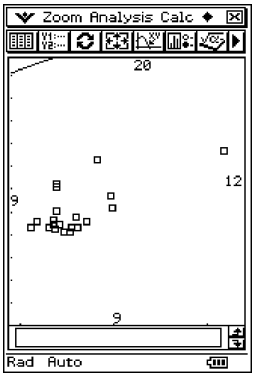
3 (a) C

(b) (i) $q = 2.69 + 1.16p$

(ii) A

4 (a) 

(b) There does appear to be a strong positive linear connection between these two times. In general, as one value increases, so does the other. It would seem appropriate to fit a straight line for these points.

(c) 

(d) There does not appear to be a strong connection between these two times. It would not seem appropriate to fit a straight line to these points.

(e) There is a strong probability that the same person will hold the two sprint records but it will be a different person who holds the other record. The two sprint times are therefore closely linked but no such link is established for a sprint and a middle distance run.

5 (a) The value of r is 0.9987 so it is appropriate to find the equation of a linear line of best fit. It is $F = 4.90m - 0.15$.

(b) This ratio is the gradient of the line of best fit, 4.90.

(c) (i) 19.45 N (ii) 53.75 N (iii) 90.50 N

(d) (i) 244.85 N (ii) 979.85 N (iii) 4899.85 N

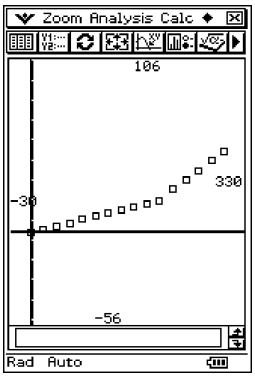
6 (a) Solubility = 0.05 × Temperature + 35.05

(b) (i) 36.30 units (ii) 38.95 units (iii) 41.05 units

(c) Temperature = 20.02 × Solubility − 701.40

(d) (i) 19.32°C (ii) 69.37°C (iii) 199.50°C

(e) Both of these are extrapolated values; we would need to assume that the connection between the variables remains the same.

7 (a) 

The first group ends at the point (200, 20)

(b) $F = 0.100\Delta x - 0.04$, $0 \le \Delta x \le 200$ and $F = 0.294\Delta x - 37.75$, $200 < \Delta x \le 300$.

(c) (i) 7.5 N (ii) 15.0 N (iii) 28.4 N

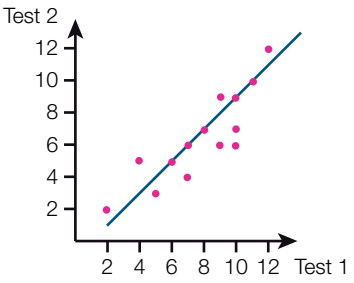
(d) $\Delta x = 9.965F + 0.440$, $0 \le F \le 20$ and $\Delta x = 3.406F + 128.667$, $20 < F < 50.2$

(e) (i) 50 mm (ii) 231 mm (iii) 282 mm

(f) If we substitute 500 mm into the appropriate equation we get 110.4 N which is much greater than the stated value, so we would assume the equation is not valid in that region. Without any other information the best we could do is assume the data is linear between (300, 50.2) and (500, 90). This would give the equation $F = 0.199\Delta x - 9.5$.

Open-ended – Sample answers

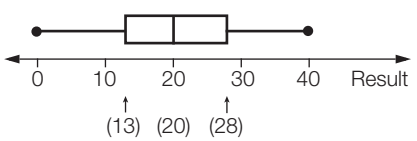
8 (a) Emily has just drawn the line of best fit as connecting the first and last points. It should be drawn to represent the general trend of the points with approximately equal numbers of values either side of the line.

(b) 

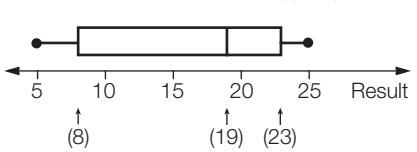
9 Students' own answers.

Chapter review 7 (p. 266)

1 (a) (i) median = 20.0 (ii) IQR = 28 − 13 = 15

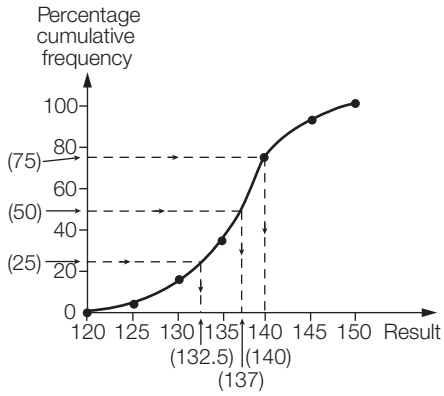
(iii) 

(b) (i) median = 19 (ii) IQR = 23 − 8 = 15

(iii) 

2 (a)

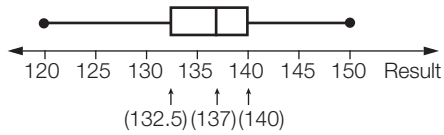
Result	Frequency	Percentage cumulative frequency
<120	0	0
<125	5	3
<130	17	15
<135	32	37
<140	54	74
<145	24	91
<150	13	100



median = 137

(b) IQR = $Q_U - Q_L = 140 - 132.5 = 7.5$

(c)



3 less than 11 cm or greater than 27 cm

4 Team A has been far more consistent than Team B showing lower values for the range and the IQR. However, consistency in scores is not a predictor of success in team-based sports. It looks as if Team B would have lost the game when it scored 4 goals and won the game when it scored 24 goals but other than this it is difficult to make any firm statements. Both teams have the same median number of goals. If pushed you might lean towards saying Team B has been more successful. You definitely need to have access to the raw data.

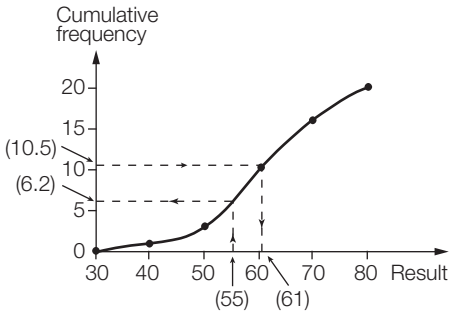
5 (a) primary **(b)** secondary **(c)** secondary **(d)** primary

6 (a) 3.77 **(b)** 1.08 **(c)** 5.35

7 (a) $y = 5.536x + 2.536$ **(b)** $y = -2.080x + 19.428$

8 (a)

Class interval	Frequency	Cumulative frequency
20–<30	0	0
30–<40	1	1
40–<50	2	3
50–<60	7	10
60–<70	6	16
70–<80	4	20



(b) min = 34.5; $Q_1 = 53.55$; median = 60.85; $Q_3 = 67.75$; max = 73.6

(c) From cumulative frequency curve,

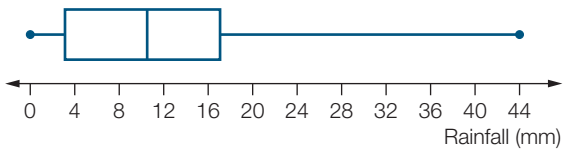
Median = 10.5th term = 61, compared with

Median = $\frac{59.9 + 61.8}{2} = 60.85$ from the raw data, so they are basically the same.

9 (a) 44 – 0 = 44

(b) min = 0; $Q_1 = 3$; median = 10.5; $Q_3 = 17$; max = 44

(c)



10 (a) (i) less than 8.5 and greater than 20.5

(ii) yes (at least one result greater than 20.5)

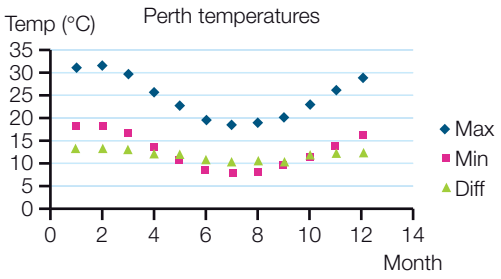
(b) (i) less than 24 and greater than 40

(ii) no (the lowest result is right on the margin)

11 (a)

Jan	Feb	Mar	Apr	May	Jun
13.0	13.3	13.0	12.1	11.9	10.9
Jul	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
10.5	10.7	10.5	11.7	12.1	12.6

(b)



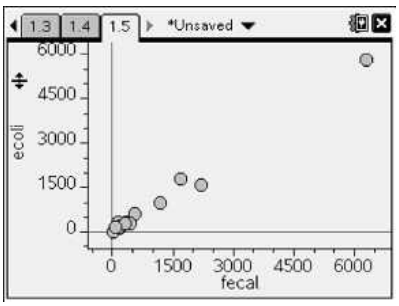
(c) Perth follows a typical temperature pattern for the southern part of Australia—warm to hot in the summer and cool to cold in the winter. As might be expected the difference graph shows the most consistency of the three, but does appear to be remarkably consistent. The smallest differences are seen in the winter months.

(d) No—temperature patterns can be quite local, and temperatures inland vary from coastal temperatures.

(e) The mean minimum for June 2010 was 7.1°C compared to a long-term mean of 8.4°C, whereas the mean maximum was 19.0°C compared to the long-term mean of 19.3°C. June 2010 was slightly more varied in temperature than normal with the minimum being lower and the maximum (slightly) higher. More data would need to be collated before any conclusions could be drawn about changing weather patterns.

12 (a) 3

(b)



(c) $y = 0.91x + 8.02$

13 (a) While there may be good reason for the introduction of City Link toll roads to be unpopular, the survey does not really show an obvious intention for voters to vote against the Government based on this one issue. The percentage results are far too close to tell.

(b) Government + Opposition voters equals 80% (33 + 47%) for Tullamarine. 20% of people surveyed chose not to support the Government or Opposition party, for any number of reasons.

This is important for the overall judgment of the poll because, if these 20% of voters then decided to vote, the result could change dramatically.

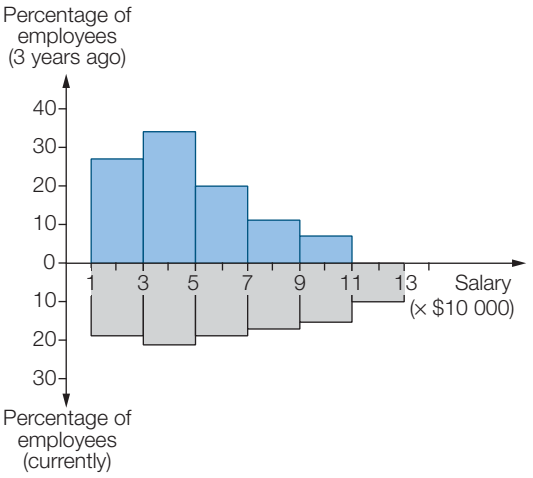
(c) Sample size = 1200 voters, $n^2 = 1\,440\,000$. This is an appropriate size.

(d) It is convenient to do the survey by telephone.

(e) Tullamarine and Essendon residents are largely affected by the new tolls, Oakleigh and Knox residents to a lesser extent. Many people avoid paying the tolls if possible whether they use the City Link roads often or not.

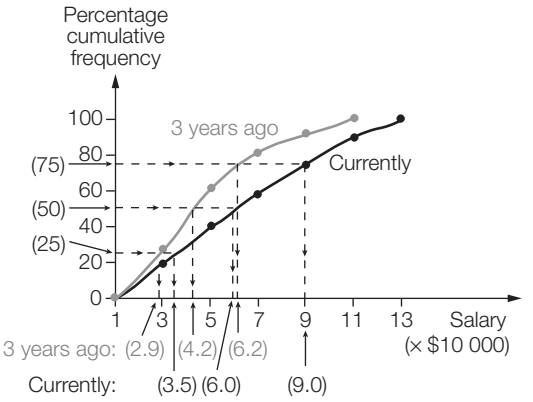
14 (a)

Salary (× $10 000)	Percentage of employees (3 years ago)	Percentage of employees (currently)
1–<3	27	19
3–<5	34	21
5–<7	20	19
7–<9	11	17
9–<11	7	15
11–<13	0	10



(b)

Salary (× $10 000)	Cum. freq. (3 years ago)	Per. cum. freq. (3 years ago)	Cum. freq. (current)	Per. cum. freq. (current)
<1	0	0	0	0
<3	12	27	9	19
<5	27	61	19	40
<7	36	82	28	58
<9	41	93	36	75
<11	44	100	43	90
<13	44	100	48	100



The following values are in $(× 10000).

x (salary × $10 000)	f (3 years ago)	xf	f (currently)	xf
2	12	24	9	18
4	15	60	10	40
6	9	54	9	54
8	5	40	8	64
10	3	30	7	70
12	0	0	5	60
	44	208	48	306

3 years ago

Mean = 4.73

Median = 4.2

IQR $= Q_U - Q_L$
$= 6.2 - 2.9$
$= 3.3$

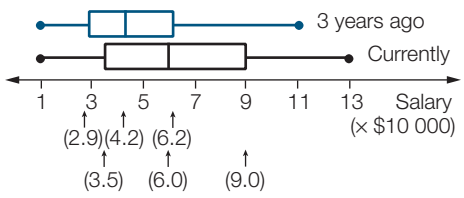
Currently

Mean = 6.38

Median = 6.0

IQR $= Q_U - Q_L$
$= 9.0 - 3.5$
$= 5.5$

(c)



(d) Salaries have increased, on average, by $15 000, and there is a much greater variation in the salaries paid.
mean difference = $16 500
median difference = $18 000

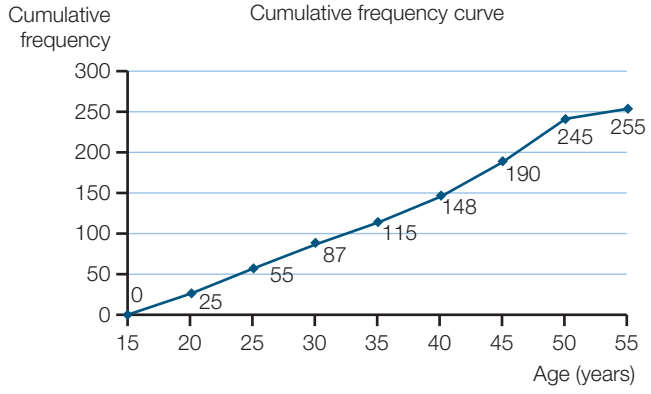
(e) total salary bill 3 years ago = $2 080 000,
total salary bill currently = $3 060 000;
i.e. $980 000 increase

(f) about 47%

15 (a) There are 255 employees. The data set has most people in the range 41–50 but this is balanced by very few in 51–55. Other than this, it is relatively uniform so the mean will be near the median, say about 38.

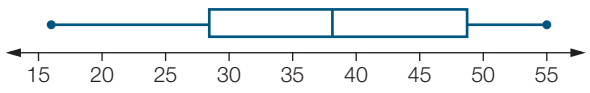
(b) mean = 36.04 and standard deviation = 10.57

(c)



min = 16, Q_L = 28, median = 38, Q_U = 48, max = 55

(d)



(e) (i) approx 100 (ii) approx 35

Chapter 8

Recall 8 (p. 270)

1 (a) $a = 23°$ (b) $a = 69°$ (c) $x = 15°$

2 (a) $a = 46°$; $b = 46°$; $c = 46°$

3 (a) equilateral (b) right-angled
(c) scalene (d) isosceles
(e) right-angled and isosceles (f) scalene

4 Going from left to right the letters to match the shapes are: E, B, A, D and C.

Exercise 8.1 (p. 274)

1 (a) $XY = PQ$ (both 9 cm), $YZ = QR$ (both 16 cm), $ZX = RP$ (both 8 cm); SSS

(b) $\angle BAC = \angle EFD$ (both 65°), $\angle BCA = \angle FED$ (both 50°), $AC = FE$ (both 10); AAS

(c) $\angle JKL = \angle MPN$ (both 90°), $JL = NM$ (both 5), $KJ = PN$ (both 3); RHS

(d) $XZ = RT$ (both 10), $YZ = ST$ (both 10), $\angle XZY = \angle STR$ (both 40°); SAS

(e) $\angle HGJ = MLK$ (both 56°), $JHG = \angle KML$ (both 44°), $HJ = MK$ (both 6); AAS

(f) $\angle PQR = \angle YXW$ (both 90°), $PR = YW$ (both 8), $RQ = WX$ (both 5); RHS

(g) $\angle CBD = \angle FHG$ (both 28°), $CB = FH$ (both 6), $DB = GH$ (both 10); SAS

(h) $JL = AC$ (both 4), $LK = CB$, $JK = AB$ (both 9); SSS

2 (a) $x = 14$ (b) $x = 8$

(c) $x = 7.5$ (d) $x = 20$

(e) $x = \frac{50}{3} = 16\frac{2}{3}$ (f) $x = \frac{51}{5} = 10\frac{1}{5}$

3 (b) and (d) SSS; (c) and (f) AAS; (a) and (e) SAS

4 D **5** C

6 (a) B (b) A

7 (a) A (b) B

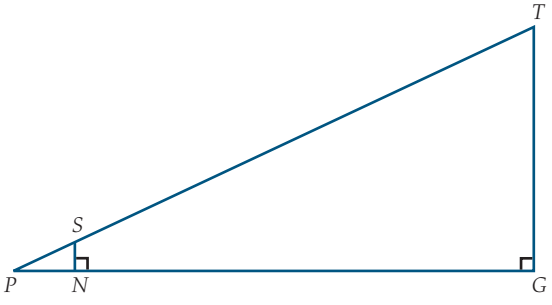
8 (a) $x = 10$ (b) $x = 6$ (c) $x = 5\frac{5}{9}$ (d) $x = 26.4$

9 5.25 m **10** 316.67 cm **11** 7.16 m

12 (a) ΔDFA ΔBGI, ΔCHJ, ΔDKM, ΔELN, ΔFMO

(b) ΔBKN and ΔCLO; ΔAJG and ΔKNB

13 (a)



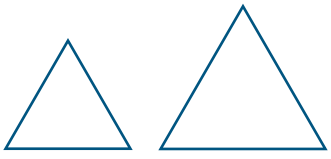
Measure SN, PN and NG

$\frac{TG}{PG} = \frac{SN}{PN}$ so, $TG = \frac{SN \times PG}{PN}$ where $PG = PN + NG$

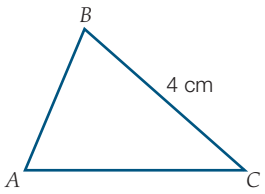
(b) Need to measure accurately; can't really get your head level with the ground.

Open-ended – Sample answers

14 AAA does not guarantee any sides are equal in length. If it did every equilateral triangle would be the same size.



15 Draw any triangle in which $BC = 4$ cm and then measure the other two sides.



In this triangle $AB = 2.6$ cm and $AC = 5$ cm. So, in ΔDEF, $DE = 2.6 \times 1.5 = 3.9$ cm; $AC = 5 \times 1.5 = 7.5$ cm and $EF = 4 \times 1.5 = 6$ cm

Exercise 8.2 (p. 279)

1 (a) $\Delta ABC \equiv \Delta CDA$

$BC = DA$	(given)
$BA = DC$	(given)
$AC = CA$	(same side)
$\Delta ABC \equiv \Delta CDA$	(SSS)
$\angle BAC = \angle DCA$, so $x = 55°$	

(b) $\Delta ACD \equiv \Delta ACB$

$\angle ACD = \angle ACB$	(given)
$\angle ADC = \angle ABC$	(given)
$AC = AC$	(same side)
$\Delta ACD \equiv \Delta ACB$	(AAS)
$AD = AB$, so $p = 19$ cm	

(c) $\Delta ABE \equiv \Delta CDE$

$AB = CD$	(given)
$\angle ABE = \angle CDE$	(given)
$\angle AEB = \angle CED$	(vertically opposite)
$\Delta ABE \equiv \Delta CDE$	(AAS)
$AE = CE$, so $x = 23$ cm	

(d) $\Delta AOB \equiv \Delta COB$

$AO = CO$	(radius of circle)
$\angle OBA = \angle OBC$	(given)
$\angle OAB = \angle OCB$	(isosceles triangle)
$\Delta AOB \equiv \Delta COB$	(AAS)
$\angle OAC = \angle OCB$, so $y = 26°$	

(e) $\Delta ABO \equiv \Delta CDO$

$AB = CD$	(given)
$AO = CO$	(radius of circle)
$BO = DO$	(radius of circle)
$\Delta ABO \equiv \Delta CDO$	(SSS)
$AO = CO$, so $y = 9$ m	

(f) $\Delta ABE \equiv \Delta DCE$

$\angle AEB = \angle DEC$	(vertically opposite)
$\angle BAE = \angle CDE$	(given)
$AB = DC$	(given)
$\Delta ABE \equiv \Delta CDE$	(AAS)
$AE = ED$, so $p = 6$ cm	

2 (a) $\frac{AB}{DE} = \frac{3}{6} = \frac{1}{2}$; $\frac{BC}{EF} = \frac{4}{8} = \frac{1}{2}$; $\frac{AC}{DF} = \frac{5}{10} = \frac{1}{2}$;

$\Delta ABC \mid\mid\mid \Delta DEF$	(SSS)

(b)

$OM = OP$	(given)
$ON = OQ$	(given)
$\angle MON = \angle POQ$	(vertically opposite)
$\Delta MON \equiv \Delta POQ$	

Since every pair of congruent triangles are also similar we can say $\Delta MON \mid\mid\mid \Delta POQ$

(c)

$\angle RVS = \angle RUT$	(given)
$\angle SRV = \angle TRU$	(same angle)
$\angle RSV = \angle RTU$	(angle sum of the triangle)
$\Delta RSV \mid\mid\mid \Delta RTU$	(AAA)

(d)

$\angle YWX = \angle VWU$	(vertically opposite)
$\angle WYX = \angle UVW$	(alternate angles between parallel lines)
$\angle WXY = \angle WUV$	(alternate angles between parallel lines)
$\Delta WYX \mid\mid\mid \Delta WVU$	(AAA)

3 C **4** B

5 (a)

$\angle RUT = \angle RST$	(given—right angle)
$RT = RT$	(same side—hypotenuse)
$UT = ST$	(given)
$\Delta RUT \equiv \Delta RST$	(RHS)

(b)

$SP = RQ$	(given)
$\angle SPQ = \angle RQP$	(given)
$PQ = PQ$	(same side)
$\Delta SPQ \equiv \Delta RQP$	(SAS)

6 (a)

$\angle QRS = \angle PRT$	(same angle)
$\angle RQS = \angle RPT$	(corresponding angles on parallel lines)
$\angle RSQ = \angle RTP$	(corresponding angles on parallel lines)
$\Delta RQS \mid\mid\mid \Delta RPT$	(AAA)

(b)

$\angle ABC = \angle BDC$	(given—right angle)
$\angle ACB = \angle BCD$	(same angle)
$\angle BAC = \angle DBC$	(third angle in triangle)
$\Delta ABC \mid\mid\mid \Delta BDC$	(AAA)
$\angle ADB = \angle ABC$	(given—right angle)
$\angle BAD = \angle CAB$	(same angle)
$\angle ABD = \angle ACB$	(third angle in triangle)
$\Delta ABC \mid\mid\mid \Delta ADB$	(AAA)
$\therefore, BDC \mid\mid\mid \Delta ADB$	(AAA)

7 (a) (i) not necessarily similar, angles same but sides might not be proportional

(ii) all matching sides in same ratio

(iii) not necessarily similar, angles between equal sides might not be equal

(iv) all angles the same (AAA)

(v) all angles the same (AAA)

(vi) radius is only measure so all distances will be in the same ratio

(b) (i) not similar **(ii)** side length the same

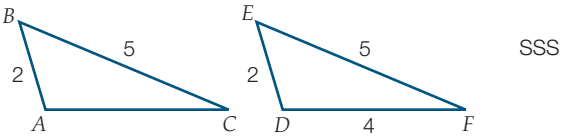
(iii) not similar **(iv)** side length the same

(v) one matching side length the same

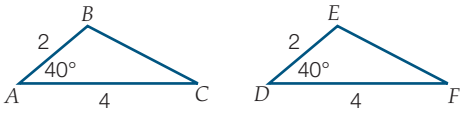
(vi) radius the same

Open-ended – Sample answers

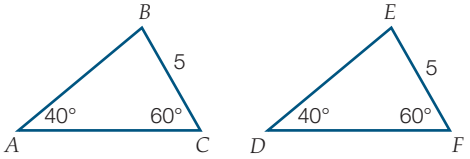
8



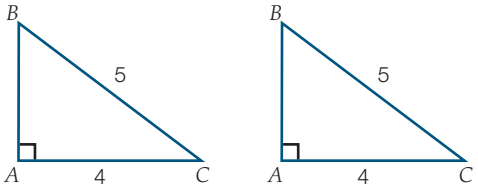
SSS



SAS



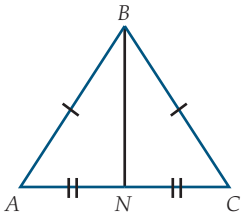
AAS



RHS

Exercise 8.3 (p. 282)

1 (a)

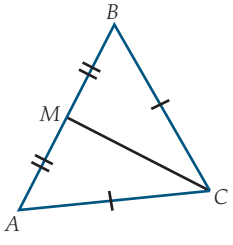


With this construction:

$\Delta ABN \equiv \Delta CBN$

$AN = CN$	(given by construction)
$BN = BN$	(same side)
$AB = CB$	(given)
$\Delta ABN \equiv \Delta CBN$	(SSS)

So, $\angle BAC = \angle BCA$



With this construction:

$\Delta BCM \equiv \Delta ACM$

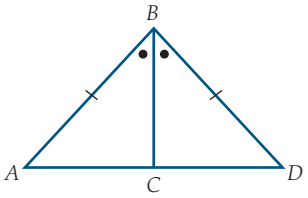
$AM = BM$	(given by construction)
$CM = CM$	(same side)
$CB = CA$	(given)
$\Delta BCM \equiv \Delta ACM$	(SSS)

So, $\angle CBA = \angle CAB$

But, $\angle CAB = \angle BCA$ from the first construction, so $\angle CBA = \angle CAB = \angle BCA$

(b) All angles are the same (60°)

2 (a)

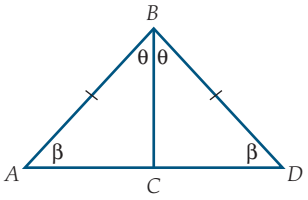


To show: $AC = CD$

$\angle BAC = \angle BDC$	(isosceles triangle)
$\angle ABC = \angle DBC$	(by construction)
$BC = BC$	
So, $\Delta BAC \equiv \Delta BDC$	(AAS)

$\therefore$ so $AC = DC$ (matching sides of congruent triangles)

(b)



To show: $\angle BCA = \angle BCD = 90°$

$2 \times \theta + 2 \times \beta = 180$	(angle sum of triangle)
$\theta + \beta = 90°$	
So, $\angle BCA = 180 - (\theta + \beta) = 90°$	

3

$\angle CDE + \angle EDF = 180°$	(straight angle)
$\angle EFG + \angle EFD = 180°$	(straight angle)
$\angle EDF = \angle EFD$	(equal angles of isosceles triangle)

$\angle CDE = 180 - \angle EDF$

$\angle EFG = 180 - \angle EFD$
$= 180 - \angle EDF$
$= \angle CDE$

4 A **5** B

6

$AC^2 = AB^2 + BC^2$	(Pythagoras)
$BC^2 = BD^2 + DC^2$	(Pythagoras)
So, $AC^2 = AB^2 + BD^2 + DC^2$	(1)
Also, $AB^2 = DB^2 + AD^2$	(Pythagoras)

Substituting into (1):

$AC^2 = DB^2 + AD^2 + BD^2 + DC^2$
$= DC^2 + 2BD^2 + AD^2$

7

$\Delta ABF \equiv \Delta BCD$

$AB = BC$	(given)
$\angle FAB = \angle DBC$	(corresponding angles, $FA \parallel BD$)
$\angle FBA = \angle DCB$	(corresponding angles, $FB \parallel DC$)
$\Delta ABF \equiv \Delta BCD$	(AAS)

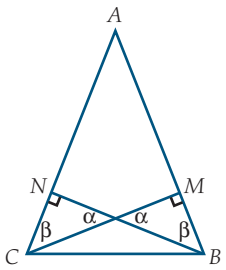
8 To show: $\angle RPT = \angle RTP$

$\angle QPU = \angle STU$ as both these angles are the third angle in a triangle where the other two angles are the same.

So in ΔRPT, $RP = RT$ (opposite equal sides)

$\therefore$ ΔRPT is isosceles.

9



$\angle CNB = \angle BMC$ (90°, given)
$\angle NCB = \angle MBC$ (angles opposite equal sides)
ΔCNB and ΔBMC have a common side, BC

So, $\Delta CNB \equiv \Delta BMC$	(AAS)
So, $NC = MB$	(matching sides)
$AC = AN + NC$	
$AB = AM + MB$	
$AB = AC$	(given)
So $AN = AM$	(equal sides AC and AB; NC and NB)

10 Let $EC = DB = x$

$\therefore ED = x + DC = BC$

In ΔEFD and ΔBAC

$ED = BC$	(above)
$\angle EFD = \angle BAC$	(given)
$\angle FED = \angle ABC$	(given)
$\therefore \Delta ABC \equiv \Delta FED$	(AAS)

11 Let $BF = EC = x$

$\therefore BC = x + FC = FE$

In ΔABC and ΔDEF

$BC = EF$	(above)
$\angle ABC = \angle DEF$	(given)
$\angle ACB = \angle DFE$	(given)
$\therefore \Delta ABC \equiv \Delta DEF$	(AAS)

12 (a) $\Delta ABE \equiv \Delta ACD$

$\angle ABE = \angle ACE$	(given)
$BE = CD$	($BC = EC$ and DE is common)
$AB = AC$	(given)
$\Delta ABE \equiv \Delta ACD$	(SAS)

(b) $\Delta ABE \equiv \Delta CBD$

$\angle BAE = \angle BCD$	(given)
$\angle ABE = \angle CBD$	($\angle ABD = \angle CBE$ and $\angle EBD$ is common)
$AB = BC$	(isosceles triangle)
$\Delta ABE \equiv \Delta CBD$	(AAS)

(c) $\Delta ABD \equiv \Delta CBE$

$AD = CE$	($AD = AE - DE$ and $CE = CD - DE$; DE common)
$\angle BAD = \angle BCE$	(given)
$AB = CB$	(isosceles triangle)
$\Delta ABD \equiv \Delta CBE$	(SAS)

(d) $\Delta ABD \equiv \Delta CBE$

$AB = CB$	(given)
$AD = CE$	($AD = AE - DE$ and $CE = CD - DE$; DE common)
$\angle BAD = \angle BCE$	(isosceles triangle)
$\Delta ABD \equiv \Delta CBE$	(SAS)

13 $\Delta ADF \equiv \Delta AEF$

$DF = EF$	(given)
$\angle ADF = \angle AEF$	(both 90° as $FD \perp AB$ and $FE \perp AC$)
$AF = AF$	(common side)
$\Delta ADF \equiv \Delta AEF$	(RHS)

14 $\Delta ABC \equiv \Delta CDE$

$\angle ABC = \angle CDE$	(given—right angle)
$AC = CE$	(given C is the midpoint of AE hypotenuse)
$AB = CD$	(given)
$\Delta ABC \equiv \Delta CDE$	(RHS)

15 $\angle OEF = \angle OFE$ (isosceles triangle; call it α)
$\angle OED = \angle ODE$ (isosceles triangle; call it β)
$\angle EDO + \angle OFE + \angle FED = 180°$ (angle sum of triangle)

So, $\beta + \alpha + (\alpha + \beta) = 180°$
$2(\alpha + \beta) = 180°$
$\alpha + \beta = 90°$

As $\angle DFE = \alpha + \beta = 90°$ the angle in a semicircle is a right angle.

Open-ended – Sample answers

16 (a)

$\angle ABC = \angle ACB$	(opposite equal angles)
$\angle AXY = \angle ABC$	(corresponding angles $XZ \parallel BC$)
$\angle AZY = \angle ACB$	(corresponding angles $XZ \parallel BC$)
$\therefore \angle AXY = \angle AZY$	
$\therefore AX = AZ$	(sides opposite equal angles)

(b) $\Delta AXY \equiv \Delta AZY$

AX = AZ	(from **(a)**)
XY = ZY	(given)
$\angle AXY = \angle AZX$	(from **(a)**)
$\Delta AXY \equiv \Delta AZY$	(SAS)

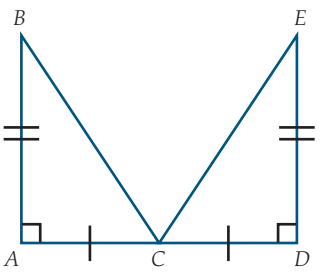
(c) $\Delta BXY \equiv \Delta CZY$

$BX = CZ$	($AB = AC$ and $AX = AZ$)
$XY = ZY$	(given)
$\angle BXY = \angle CZY$	(Each $(180 - b)°$)
$\Delta BXY \equiv \Delta CZY$	(SAS)

(d) $a + b + 90 = 180$ (angle sum in triangle)
$b = 90 - a$

$\angle ACY = c$ and $\angle CAY = a$ but these are equal since ΔACY is isosceles.

So, $c = a$
$c + d = b$ (corresponding angles on parallel lines)
$a + d = 90 - a$
$d = 90 - 2a$

17 

$\Delta ABC \equiv \Delta DEC$

Exercise 8.4 (p. 287)

1 (a) To prove: $\Delta ABC \,|||\, \Delta ADE$

$\angle ADE = \angle ABC$	(corresponding angles, $DE \parallel BC$)
$\angle DAE = \angle BAC$	(same angle)
$\angle AED = \angle ACB$	(corresponding angles, $DE \parallel BC$)
$\Delta ABC \,|||\, \Delta ADE$	(AAA)

(b) To prove: $AE = EC$

$\Delta ABC \,|||\, \Delta ADE$ (from **(a)**)

So, $\frac{AD}{AB} = \frac{AE}{AC}$

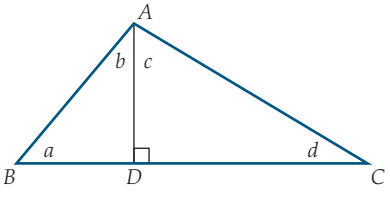
So, $\frac{AE}{AC} = \frac{1}{2}$

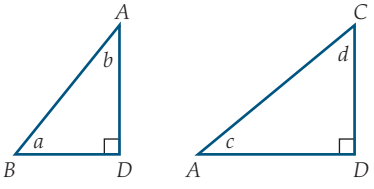
So, $2AE = AC$

So, $AE = EC$

2 To prove: $\Delta ADB \,|||\, \Delta DCA$

The orientation of the two triangles must be as shown below as AD cannot be in the same position in both triangles.





$a + b = 90°$	(rest of angle sum of 180° for triangle)
$c + d = 90°$	(rest of angle sum of 180° for triangle)
$b + c = 90°$	(right angle in original triangle)
$a + b = b + c$, so $a = c$	
$b + c = c + d$, so $b = d$	
$\Delta ADB \,|||\, \Delta DCA$	(AAA)
To prove:	$\Delta DCA \,|||\, \Delta CAB$
$\angle ADC = \angle CAB$	(both 90°)
$\angle ACD = \angle ACB$	(common angle)
$\angle ABD = \angle ABC$	(common angle)
$\Delta DCA \,|||\, \Delta CAB$	(AAA)

3 To prove: $AD^2 = BD \times DC$

$\Delta ADB \,|||\, \Delta DCA$ (from Question **2**)

So, $\frac{AD}{DC} = \frac{BD}{AD}$ (matching sides of similar triangles)

So, $AD \times AD = BD \times DC$

$AD^2 = BD \times DC$

4 To prove: $BA^2 = BD \times BC$ and $CA^2 = CD \times CB$

$\Delta ADB \,|||\, \Delta CAB$ (from Question **2**)

$\frac{BA}{BC} = \frac{BD}{BA}$ (matching sides of similar triangles)

So, $BA^2 = BC \times BD$

To prove: $CA^2 = CD \times CB$

$\Delta ADC \,|||\, \Delta BAC$ (from Question **2**)

$\frac{CA}{BC} = \frac{CD}{AC}$ (matching sides of similar triangles)

$CA^2 = CD \times BC$

5 $AB^2 = BC \times BD$ (from Question **4**)

$AC^2 = CD \times BC$ (from Question **4**)

$$\begin{aligned} AB^2 + AC^2 &= BC \times BD + CD \times BC \\ &= BC(BD + CD) \\ &= BC \times BC \\ &= BC^2 \end{aligned}$$

6 C

7 (a) To show: $\Delta ABO \,|||\, \Delta DCO$

$\angle ABD = \angle BDC$	(alternate angles, $AB \parallel DC$)
$\angle BAC = \angle ACD$	(alternate angles, $AB \parallel DC$)
$\angle AOB = \angle COD$	(vertically opposite angles)
$\Delta ABO \,|||\, \Delta DCO$	(AAA)

Now, $\frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{DC}$ (matching sides of similar triangles)

So, $OA : OC = OB : OD$

(b) We have $\frac{AB}{CD} = \frac{1}{2}$

So, $\frac{AO}{OC} = \frac{1}{2}$ (matching sides of similar triangles are in same ratio)

So, $2AO = OC$

Similarly, $2OB = OD$

$$\begin{aligned} AC &= AO + OC \\ &= AO + 2AO \\ &= 3AO \end{aligned}$$

So, $AO : AC = 1 : 3$ and similarly $BO : BD = 1 : 3$

8 To prove: $\frac{JM}{MK} = \frac{JN}{NL}$

We will show $\Delta JMN \,|||\, \Delta JKL$

$\angle MJN = \angle KJL$	(common angle)
$\angle JMN = \angle JKL$	(corresponding angles, $MN \parallel KL$)
$\angle JNM = \angle JLK$	(corresponding angles, $MN \parallel KL$)
$\Delta JMN \,|||\, \angle JKL$	(AAA)

So, $\frac{JK}{JM} = \frac{JL}{JN}$ (matching sides of similar triangles are proportional)

$\frac{JK}{JM} - 1 = \frac{JL}{JN} - 1$

$\frac{JK - JM}{JM} = \frac{JL - JN}{JN}$

$\frac{MK}{JM} = \frac{NL}{JN}$ $(JK = JM + MK)$

$(JL = JN + NL)$

So, $\frac{JM}{MK} = \frac{JN}{NL}$ (reciprocal)

9 To prove: $GH \parallel EF$

We will show $\Delta DGH \equiv\!\equiv\!\equiv \Delta DEF$

$\angle GDH = \angle EDF$ (common angle)

$\frac{DG}{GE} = \frac{DH}{HF}$ (given)

$\frac{GE}{DG} = \frac{HF}{DH}$ (reciprocal)

$\frac{GE}{DG} + 1 = \frac{HF}{DH} + 1$

$\frac{GE + DG}{DG} = \frac{HF + DH}{DH}$

$\frac{DE}{DG} = \frac{DF}{DH}$

Corresponding sides are in the same proportion so

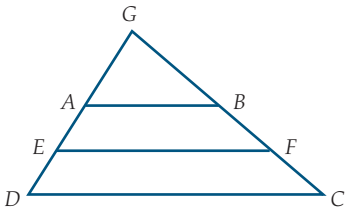
$\Delta DGH \equiv\!\equiv\!\equiv \Delta DEF$ (SAS)

So, $\angle DGH = \angle DEF$ (matching angles in similar triangles)

And $\angle DHG = \angle DFE$ (matching angles in similar triangles)

So, $GH \parallel EF$ (corresponding angles equal on parallel lines)

10 Following the hint we have this diagram:



In ΔGAB and ΔGDC

$\angle AGB = \angle DGC$ (common angle)

$\angle GAB = \angle GDC$ (corresponding angles, $AB \parallel DC$)

$\therefore \Delta GAB \equiv\!\equiv\!\equiv \Delta GDC$ (AAA)

$\therefore$ matching sides are proportional

Let $GA = x$, $AE = y$, $ED = y$

$\therefore GB = mx$, $BC = 2my$, $BF = my$, $FC = my$

$\frac{GA}{GE} = \frac{x}{x + y}$

$\frac{GB}{GF} = \frac{mx}{mx + my} = \frac{x}{x + y}$

now LG is common

$\therefore \Delta GAB \equiv\!\equiv\!\equiv \Delta GEF$ (SAS)

$\therefore \angle GAB = \angle GEF$ (matching angles)

However, these angles are equal corresponding angles

$\therefore AB \parallel EF$

11 To prove: $\Delta BXY \equiv\!\equiv\!\equiv \Delta BYC$

Hint: Prove $\Delta BXY \equiv \Delta CZY$

(See Exercise 8.3 Question 16 for the proof of this; it will not be repeated here.)

$BY = CY$ (congruent triangles)

We know $\frac{BX}{BY} = \frac{XY}{YC}$ (as $BX = XY$ and $BY = YC$)

Need to show: $\angle BXY = \angle BYC$

Now, $\angle XBY = \angle XYB$ (isosceles triangle and call it b)

So, $\angle BXY = (180 - 2b)°$

But, $\angle ZYC = b$ (congruent triangle)

And, $\angle XYB + \angle BYC + \angle ZYC = 180°$ (straight angle)

$b + \angle BYC + b = 180°$

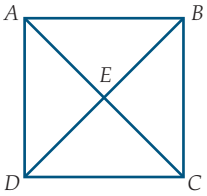
$\angle BYC = (180 - 2b)°$

So, $\angle BXY = \angle BYC$

So, $\Delta BXY \equiv\!\equiv\!\equiv \Delta BYC$ (SAS)

Open-ended – Sample answers

12



To prove: $\Delta AED \equiv\!\equiv\!\equiv \Delta ADC$

The diagonals of a square bisect each other at right angles

So, $\frac{AE}{AD} = \frac{ED}{DC}$ since $AE = ED$ and $AD = DC$

$\angle AED = \angle ADC$ (both right angles)

So, $\Delta AED \equiv\!\equiv\!\equiv \Delta ADC$ (SAS)

Half-time 8 (p. 289)

1 (a) To prove: $\Delta ABC \equiv \Delta EDC$

$\angle ABC = \angle CDE$ (both right angles)

$\angle BCA = \angle ECD$ (vertically opposite)

So, $\Delta ABC \equiv\!\equiv\!\equiv \Delta EDC$ (AAA)

$BC = DC$ (AAS)

$\therefore \Delta ABC \equiv \Delta EDC$

(b) AB and ED are corresponding sides in congruent triangles so must be equal.

2 To prove: $\Delta ABC \equiv\!\equiv\!\equiv \Delta APQ$

$\angle ABC = \angle APQ$ (given)

$\angle CAB = \angle QAP$ (common angle)

$\angle AQP = \angle ACB$ (third angle in triangle)

$\Delta ABC \equiv\!\equiv\!\equiv \Delta APQ$ (AAA)

3 (a) RHS **(b)** SAS

4 (a) $OA = OD$ (radii)

$OB = OC$ (radii)

$AB = CD$ (given)

So, $\Delta AOB \equiv \Delta DOC$ (SSS)

$p = 15$

(b) In ΔURT and ΔSTR

$RT = RT$	(common side)
$\angle URT = \angle STR$	(alternate angles, $RU \parallel TS$)
$\angle RTU = \angle SRT$	(alternate angles, $RS \parallel UT$)
$\therefore \Delta URT \equiv \Delta STR$	(AAS)
$\therefore a = 11$ cm	(matching sides in congruent triangles)

5 **(a)** $x = 12$ **(b)** $x = 15$

6 To prove: $\Delta LYZ \,|||\, \Delta MXZ$

$\angle LYZ = \angle MXZ$	(both right angles)
$\angle YZL = \angle XZM$	(vertically opposite)
$\angle YLZ = \angle XMZ$	(third angle in triangle)
$\Delta LYZ \,|||\, \Delta MXZ$	(AAA)

7 In ΔACX and ΔBCX

$AC = BC$	(given)
$AX = BX$	(given)
$CX = CX$	(common side)
$\therefore \Delta ACX \equiv \Delta BCX$	(SSS)

8 2.4 cm

9 To prove: $\Delta ADF \equiv \Delta AEF$

$\angle ADF = \angle AEF$	(both right angles)
$AF = AF$	(common side)
$DF = EF$	(given)
$\Delta ADF \equiv \Delta AEF$	(RHS)

10 To prove: $XY = XZ$

We will prove $\Delta YAX \equiv \Delta ZBX$

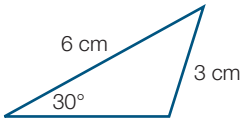
$\angle YAX = \angle ZBX$	(isosceles ΔABC)
$YA = ZB$	(halves of equal sides)
$XA = XB$	(given)
$\Delta YAX \equiv \Delta ZBX$	(SAS)

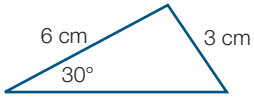
XY and XZ are corresponding sides in congruent triangles, so $XY = XZ$.

11 To prove: $\Delta ABC \equiv \Delta DEC$

$\angle ACB = \angle ECD$	(vertically opposite)
$\angle CBA = \angle CDE$	(alternate angles, $AB \parallel DE$)
$BC = CD$	(C is the midpoint of DB)
$\therefore \Delta ABC \equiv \Delta DEC$	(AAS)

12 Sample answer:





13 5 m

Exercise 8.5 (p. 294)

1 **(a)** **(i)** Opposite sides are parallel and one pair of adjacent sides is equal

(ii) rhombus

(b) **(i)** Two pairs of adjacent sides equal.

(ii) kite

(c) **(i)** Two pairs of opposite sides parallel; one pair of adjacent sides equal; one right angle.

(ii) square

2 **(a)** To prove: $\Delta DAB \equiv \Delta DCB$

$\angle DBA = \angle BDC$	(alternate angles, $AB \parallel CD$)
$\angle BDA = \angle CBD$	(alternate angles, $AD \parallel BC$)
$DB = DB$	(common side)
$\Delta DAB \equiv \Delta DCB$	(AAS)

(b) $\angle DAB = \angle BCD$ (matching angles, congruent triangles from **(a)**)

$\angle ABC$ is supplement of $\angle DAB$ (co-interior angles, $AB \parallel CD$)

$\angle ADC$ is supplement of $\angle DCB$ (co-interior angles, $AD \parallel BC$)

$\therefore \angle ABC = \angle ADC$ (supplements of equal angles)

(c) Opposite angles in a parallelogram are equal

(d) To prove: $\Delta DAE \equiv \Delta BCE$

$AD = BC$	(matching sides in congruent triangles)
$\angle AED = \angle BEC$	(vertically opposite)
$\angle ADE = \angle CBE$	(alternate angles, $AD \parallel BC$)
$\Delta DAE \equiv \Delta BCE$	(AAS)

(e) These are matching sides in congruent triangles, so $AE = CE$ and $DE = BE$

(f) The diagonals of a parallelogram bisect each other

3 $AC = DB$ and bisect each other

$\therefore AX = XC = DX = XB$

$AC \perp BD$ $\therefore$ all angles at centre are 90°

The four triangles are congruent (SAS)

$\therefore$ all hypotenuses are equal (matching sides)

acute angles of each triangle are 45° (isosceles triangle, angle sum of triangle)

$\therefore \angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$

$\therefore$ quadrilateral is a square (all sides equal, angles are right angles)

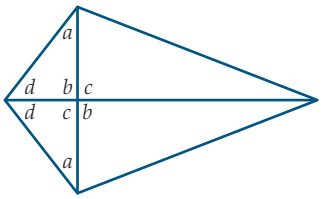
4 **(a)** To prove: $\angle KLM = \angle KNM$

Will show: $\Delta KLM \equiv \Delta KNM$

$KL = KN$	(given)
$LM = NM$	(given)
$KM = KM$	(common side)
$\Delta KLM \equiv \Delta KNM$	(SSS)

$\angle KLM = \angle KNM$ as these are matching angles in congruent triangles

(b) To prove: $\angle KOL = 90°$



We know the angles marked a are equal as they are in an isosceles triangle.

We know the angles marked d are equal because they are matching angles in congruent triangles.

The angles marked b and c are pairs of vertically opposite angles.

So, $a + b = a + c$ (supplements of d)
So, $b = c$
But, $b + c = 180°$ (straight angle)
So, $b = c = 90°$
So, $\angle KOL = 90°$

5 B **6** C

7 (a) To prove: $PQ = RS$
Will show: $\Delta PQS \equiv \Delta RSQ$

$SQ = SQ$	(common side)
$\angle PSQ = \angle RQS$	(alternate angles, $PS \parallel RQ$)
$\angle PQS = \angle QSR$	(alternate angles, $PQ \parallel RS$)
$\Delta PQS \equiv \Delta RSQ$	(AAS)

So, $PQ = RS$ as they are matching sides on congruent triangles

(b) Opposite sides of a rectangle are equal

8 (a) To show: $\Delta ACD \equiv \Delta ACB$

$AD = BC$	(given)
$DC = AB$	(given)
$AC = AC$	(given)
$\Delta ACD \equiv \Delta ACB$	(SSS)
$\angle BAC = \angle BCA = 45°$	(isosceles triangles)
$\angle DAC = \angle DCA = 45°$	(isosceles triangles)

$\angle BAD = \angle BAC + \angle DAC$
$= 45° + 45°$
$= 90°$

So, $\angle BAD = 90°$, so we have a square

(b) To show: $\Delta ABD \equiv \Delta CBD$

$AB = BC$	(given)
$AD = DC$	(given)
$DB = DB$	(given)
$\Delta ABD \equiv \Delta CBD$	(SSS)

So, $\angle ABD = \angle BDC$ as these are matching angles in congruent triangles. But, these are alternate angles, so the lines must be parallel.

Then, do the same for the other pair of sides

9 (a) Will show: $\Delta ABD \equiv \Delta BDC$

$AD = BC$	(given)
$\angle ADB = \angle DBC$	(alternate angles on parallel lines)
$\angle ABD = \angle BDC$	(alternate angles on parallel lines)
$\Delta ABD \equiv \Delta BDC$	(AAS)
So, $AB = DC$	(matching sides in congruent triangles)
And, $\angle DAB = \angle DCB$	(matching angles in congruent triangles)

So, we have a parallelogram

(b) Will show: $\Delta AED \equiv \Delta CEB$

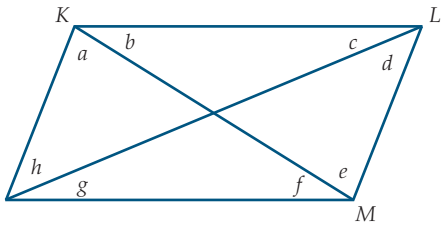
$AE = CE$	(given)
$DE = BE$	(given)
$\angle AED = \angle BEC$	(vertically opposite)
$\Delta AED \equiv \Delta CEB$	(SAS)
So, $AD = BC$	(matching sides in congruent triangles)

In a similar way show $\Delta AEB \equiv \Delta DEC$ and so $AB = DC$

Thus, we have two pairs of opposite sides equal and parallel.

So the quadrilateral is a parallelogram.

(c) Consider the following diagram:



A number of relationships can be established from the diagram

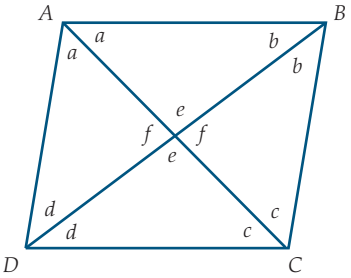
$a + b + c + h = 180°$	(angles in triangle)
$e + f + d + g = 180°$	(angles in triangle)
but $e + f = a + b$	(given)

So, $a + b + c + h = e + f + d + g$
becomes $a + b + c + h = a + b + d + g$
so, $c + h = d + g$

but $c + d = h + g$	(given)

so, $d = h$ and $g = c$
These are alternate angles so $KL \parallel NM$
Then do a similar thing to show $KN \parallel LM$

(d) Consider this diagram:



$2a + b + d = 2c + b + d$ (sum of angles in a triangle = 180°)

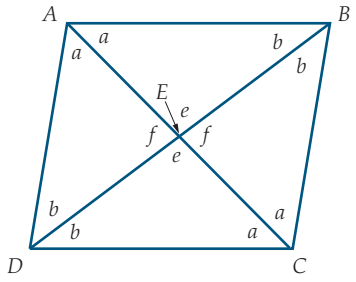
so, $a = c$

$2b + a + c = 2d + a + c$ (sum of angles in a triangle = 180°)

so, $b = d$

similarly $a = c$

Redraw the diagram:



$a + b + e = a + b + f$ (sum of angles in a triangle = 180°)

so, $e = f$

$AD = AB$ (opposite equal angles ΔABD)

$BC = DC$ (opposite equal angles ΔBCD)

$AD = DC$ (opposite equal angles ΔADC)

$\therefore AB = BC = CD = DA$

$\therefore ABCD$ is a rhombus (all sides equal)

10 A

11 To show: $\Delta WMZ \equiv \Delta YNZ$

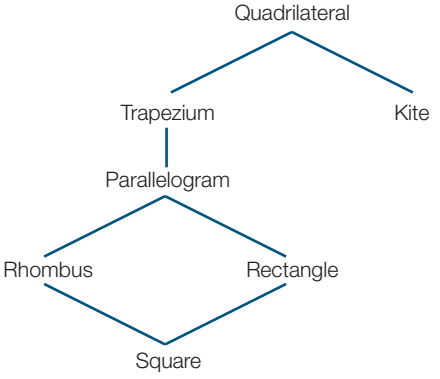
$ZM = NZ$ (given)

$WZ = YZ$ (given)

$\angle ZWM = \angle ZYN$ (both 90°, given)

$\Delta WMZ \equiv \Delta YNZ$ (RHS)

12 The words link in the following way:



13 To show: $\Delta AKD \equiv \Delta CJB$

As $AJ = CK$ and $AB = DC$ (opposite sides of parallelogram) then $JB = DK$

So, $JB = DK$

$AD = BC$ (parallelogram)

$\angle ADK = \angle JBC$ (opposite angles in parallelogram)

$\Delta AKD \equiv \Delta CJB$ (SAS)

14 A square has four right angles and two pairs of opposite sides equal and parallel with all sides the same length, while a rhombus has two pairs of opposite sides equal and parallel with all sides the same length. So, the square has an extra condition.

15 A square has four right angles and two pairs of opposite sides equal and parallel with all sides the same length, whereas a rectangle has four right angles and two pairs of opposite sides equal and parallel. So, the square has an extra condition.

Open-ended – Sample answers

16 For the condition *at least one pair of parallel sides* the shape could be: trapezium, square, rectangle, rhombus or parallelogram

For the condition *at least one pair of equal sides* the shape could be: square, rectangle, parallelogram, rhombus or kite

For the condition *at least one pair of equal angles* the shape could be: square, rectangle, rhombus, parallelogram or kite

17 (a) A rhombus has four equal sides but is not a square, because it has no right angle.

(b) Every square has four right angles. Every shape with four right angles is a square. False—it could be a rectangle.

Every kite has two pairs of adjacent sides equal. Every shape with two pairs of adjacent sides equal is a kite. False—it could be a square or a rhombus.

18 (a) Students' own diagrams.

(b) Students to draw and measure from their own diagrams.

(c) The diagonals of a rhombus intersect at right angles.

(d) A demonstration involving measurement is never sufficient as a proof.

19 A square, rhombus and kite all have diagonals that bisect at right angles, so George is correct.

Exercise 8.6 (p. 304)

1 Join GO and extend it to H.

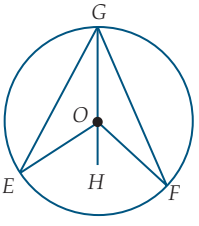
In ΔOGE, $OG = OE$ (radii of a circle)

$\therefore \Delta OGE$ is isosceles

$\therefore \angle OGE = \angle OEG$ (base angles of isosceles triangles)

Hence $\angle HOE = \angle OGE + \angle OEG$ (exterior angle of triangle is sum of interior opposite angles)

$\therefore \angle HOE = 2\angle OGE$



Repeating these steps in ΔOGF we obtain $\angle HOF = 2\angle OGF$.

Now $\angle EOF = \angle HOE + \angle HOF$.

$\therefore \angle EOF = 2\angle OGE + 2\angle OGF$

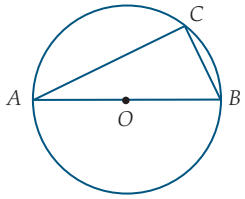
$= 2(\angle OGE + \angle OGF)$

$\therefore \angle EOF = 2\angle EGF$

2 $\angle AOB = 2\angle ACB$ (angle at the centre is twice the angle at the circumference standing on the same arc)

But $\angle AOB = 180°$ (straight angle)

$\therefore \angle ACB = 90°$



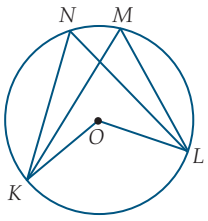
3 Join KO and LO.

$\angle KOL = 2\angle KNL$ (angle at the centre is twice the angle at the circumference standing on the same arc)

Similarly, $\angle KOL = 2\angle KML$

Hence $2\angle KNL = 2\angle KML$

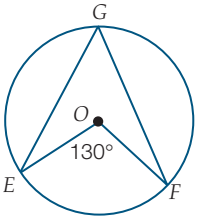
So $\angle KML = \angle KNL$



4 $\angle EOF = 2\angle EGF$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$\therefore 130° = 2\angle EGF$

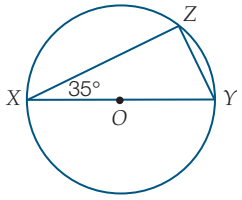
$\angle EGF = 65°$



5 $\angle XZY = 90°$ (angle in a semicircle)

But $\angle ZXY = 35°$

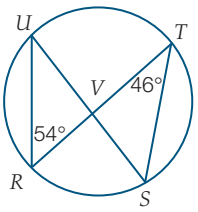
So $\angle ZYX = 55°$ (angle sum of triangle is 180°)



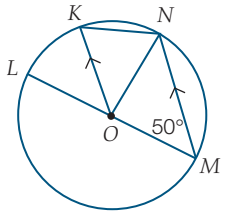
6 $\angle RUS = 46°$ (angles in the same segment)

$\angle UVR = 80°$ (angle sum of ΔUVR)

$\angle TVS = 80°$ (vertically opposite angles)



7 $\angle LOK = 50°$ (corresponding angles, $OK \parallel MN$)



$\angle LON = 2\angle LMN = 100°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$\angle OMN = \angle ONM$ (opposite equal sides (radii) of ΔONM)

$\angle KON = \angle ONM$ (alternate angles, $OK \parallel MN$)

Hence $\angle KON = \angle LON - \angle LOK = 100° - 50° = 50°$

$OK = ON$ (radii of a circle)

Hence ΔOKN is isosceles and $\angle OKN = \angle ONK$.

$50° + \angle OKN + \angle ONK = 180°$ (angle sum of ΔOKN)

$\therefore 2\angle OKN = 130°$

$\therefore \angle OKN = 65°$

8 (a) $x = 126°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

(b) $y = 70°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

(c) $x = 115°$ (reflex angle at the centre is twice the angle at the circumference standing on the same arc)

9 (a) $\angle ACD = 90°$ (angle in a semicircle), $z = 46°$ (angle sum of ΔACD)

(b) $\angle ECG = 90°$ (angle in a semicircle), $\angle CGE = 40°$ (angle sum of ΔECG), $x + 40° = 95°$ (exterior angle of triangle is sum of interior opposite angles), $x = 55°$

(c) $\angle OGH = y$ (isosceles triangle, $OG = OH$), $2y = 70°$ (exterior angle of triangle is sum of interior opposite angles), $y = 35°$

10 (a) $\angle BDC = 43°$ (angles in the same segment), $x = 107°$ (angle sum of ΔEDC)

(b) $x = 47°$ (angles in the same segment), $\angle ABD = 90°$ (angle in a semi circle), $y = 43°$, $z = 43°$ (angles in the same segment)

(c) $y = 50°$ (angles in the same segment), $ABD = 90°$ (angle in a semi circle), $\angle ABC = 40°$, $z + 40° = 95°$ (exterior angle of triangle is sum of interior opposite angles), $z = 55°$

(d) $n = 32°$, $m = 32°$ (angles in the same segment)

(e) $\angle BDC = 35°$ (angles in the same segment), $e = 51°$ (angle sum of ΔADC)

(f) $\angle DCA = 44°$ (angles in the same segment), $k = 25°$ (exterior angle of triangle is sum of interior opposite angles)

11 (a) $x = 35°$ (angle at the centre is twice the angle at the circumference standing on the same arc), $\angle FOE = 80°$ (angle at the centre is twice the angle at the circumference standing on the same arc), $2y + 80° = 180°$ (angle sum of isosceles triangle FOE), $y = 50°$

(b) $\angle CDB = 49°$ (angle sum of ΔBDC), $\angle ACD = 49°$ (alternate angles, $AC \parallel DB$), $z = 98°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

(c) $\angle MNQ = \angle MQN = 72°$ (base angles of isosceles ΔMNQ), $\angle NQP = 90°$ (angle in a semi circle), $w = 18°$

12 $\angle BOD = 2y$ (angle at centre is twice angle at circumference standing on the same arc)

$OB = OD$ (radii)

$\angle OBD = \angle ODB = x°$ (angles opposite equal sides of a triangle)

$\therefore x + x + 2y = 180$ (angle sum of a triangle)

$\therefore x + y = 90$

13 ΔBDC is isosceles, so $\angle DBC = \angle DCB = 64°$

$\angle ABC = 90°$ (angle in a semicircle) $\therefore \angle ADE = 38°$

$\angle ABD = 26°$ $\therefore \angle ACB = 38°$

$\angle ACD = 26°$ (angles in the same segment)

14 $\angle DGE = 20°$ (angles in the same segment)

$\angle FDE = 48°$ (angles in the same segment)

$\angle GFD = 42°$ (angles in the same segment)

$\angle GDF = 70°$ (angle sum of triangle)

$\angle GEF = 70°$ (angles in the same segment)

$\angle FGD = \angle FGE + \angle DGE = 68°$

$\angle GDE = \angle GDF + \angle FDE = 118°$

$\angle DEF = \angle GED + \angle GEF = 112°$

$\angle EFG = \angle DFE + \angle GFD = 62°$

$\angle GHF = 90°$ (angle sum of a triangle)

$\therefore$ The diagonals GE and DF are perpendicular.

15 $\angle OZY = \angle OYZ = 44°$ (base angles of isosceles triangle OYZ)

$\angle OZX = 62° - 44° = 18°$

$\angle OXZ = 18°$ (base angles of isosceles triangle OXZ)

16 (a) $\angle ODF = 30°$ (isosceles triangle), $\angle FOD = 120°$ (angle sum of triangle), $y = 60°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

(b) $\angle ODF = 33°$ (isosceles triangle), $\angle FOD = 114°$ (angle sum of triangle), reflex $\angle FOD = 246°$ (angles at a point), $\angle FBD = 123°$ (reflex angle at the centre is twice the angle at the circumference standing on the same arc), $x = 22°$ (angle sum of triangle)

(c) $\angle AOD = 2x$ (angle at the centre is twice the angle at the circumference standing on the same arc), reflex $\angle AOD = 4x°$ (reflex angle at the centre is twice the angle at the circumference standing on the same arc), $6x = 360°$ (angles at a point), $x = 60°$

17 Join KO, LO, NO

Let $\angle LNO = x$

$\therefore \angle OLN = \angle ONL = x$ (angles opposite equal sides of a triangle)

$\therefore \angle LON = (180 - 2x)°$ (angles sum of a triangle)

In ΔOKL and ΔONL, $OK = ON$ (radii)

$KL = NL$ (given)

LO is common

$\therefore \Delta OKL \equiv \Delta ONL$

$\therefore \angle OLK = \angle OLN = x$ (corresponding angles in congruent triangles)

$\therefore \angle KLM = (180 - 2x)°$ (MKL is a straight line)

$\therefore \angle KLM = \angle LON$ (both $(180 - 2x)°$)

18 (a) In Δs KPM and QNM,
$\angle KPQ = \angle QNK$ (angles in the same segment)

$\angle PKN = \angle NQP$ (angles in same segment)

$\angle KMP = \angle QMN$ (vertically opposite angles)

$\therefore \Delta KPM \equiv \Delta QNM$ (AAA)

(b) $KM = MQ$ (given)

$\angle PKM = \angle NQM$ (matching angles in similar triangles)

$\angle MNQ = \angle MPK$ (matching angles in similar triangles)

$\therefore \Delta KMP \equiv \Delta QMN$ (AAS)

$\therefore MP = MN$ (matching sides in congruent triangles)

19 In Δs XWU and YZU,
$\angle XWY = \angle XZY$ (angles in the same segment)

$\angle WXZ = \angle WYZ$ (angles in same segment)

But $\angle XWU = \angle WXU$ (opposite equal sides (radii) ΔXWU)

So $\angle XWU = \angle UYZ$ (both equal to $\angle XWY$)

But these are a pair of equal alternate angles between XW and YZ

$\therefore XW \parallel YZ$

20 $\angle CAF = 90°$ (angle in a semicircle)

$\angle DAF = 90°$ (angle in a semicircle)

$\therefore \angle CAF + \angle DAF = 180°$

$\therefore \angle CAD = 180°$, so C, A and D are collinear

21 $\angle SVT = \angle SUT = 45°$ (angle at centre is twice angle at circumference standing on the same arc)

$\angle UTV = \angle TVS = 45°$ (alternate angles equal, $VS \parallel UT$)

$\therefore \angle TWU = 90°$ (angle sum of ΔTWU)

22 $\angle ACD = \angle ADC$ (base angles of isosceles triangle ACD)

$\angle AFB = \angle ACB$ (angles in the same segment standing on AB)

$\angle AFE = \angle ADE$ (angles in the same segment standing on AE)

$\therefore \angle AFB = \angle AFE$ (from line 1)

$\therefore AF$ bisects $\angle EFB$

23 $\angle KML = 90°$ (angle in a semicircle on diameter KL)

$\angle ONL = 90°$ (angle in a semicircle on diameter OL)

$\therefore \angle KML = \angle ONL = 90°$

$\therefore KM \parallel ON$ (pair of corresponding angles are equal)

24 $\angle COA = 2\angle CBA$ (angle at the centre is twice the angle at the circumference standing on the same arc)

Reflex $\angle COA = 2\angle CDA$ (angle at the centre is twice the angle at the circumference standing on the same arc)

But $\angle COA$ + reflex $\angle COA = 360°$ (angles at a point)

So $2\angle CBA + 2\angle CDA = 360°$

Hence $\angle CBA + \angle CDA = 180°$

Open-ended – Sample answers

25 (a) (i) $\angle DAC = 30°$ **(ii)** $\angle ADB = 70°$

(iii) $\angle ACB = 70°$

(b) Students' own answers.

(c) (i) ΔADC is isosceles because $\angle DAC = \angle DCA = 30°$

(ii) $\angle ABC = 60°$

$\angle ADC = 120°$

(d) (i) ΔADB is isosceles because $\angle ADB = \angle ABD = 70°$

(ii) $\angle BCD = 140°$

$\angle ADB = 40°$

26 Neither is correct. It is just a quadrilateral with a pair of opposite angles each equal to 90°.

$\angle KJM = \angle KLM = 90°$ (angle in a semicircle)

Nothing is known about the lengths of the sides. To be a kite, you would need $JM = LM$. To be a square, you would need $JM = LM = LK = KJ$.

Exercise 8.7 (p. 311)

1 (a) In Δs WOX and YOZ, $WX = YZ$ (given)

$WO = YO$ (radii)

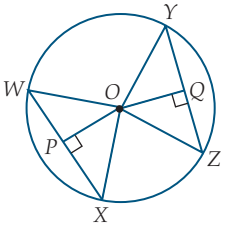
$OX = OZ$ (radii)

$\therefore \Delta WOX \equiv \Delta YOZ$ (SSS)

$\therefore \angle WOX = \angle YOZ$ (matching angles in congruent triangles)

(b) $\angle WXO = \angle YZO$ (matching angles in congruent triangles)

In Δs OXP and OZQ



$\angle OPX = \angle OQZ = 90°$ (given)

$\angle PXO = \angle QZO$ (from above)

$OX = OZ$ (radii)

$\therefore \Delta OXP \equiv \Delta OZQ$ (AAS)

$\therefore OP = OQ$ (matching sides in congruent triangles)

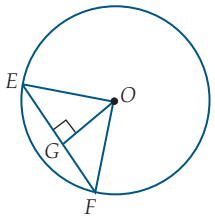
2 In Δs OGE and OGF, $\angle OGE = \angle OGF = 90°$ (given)

$OE = OF$ (radii)

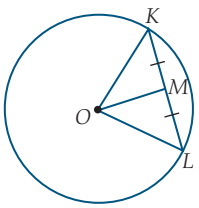
OG is a common side

$\therefore \Delta OGE \equiv \Delta OGF$ (RHS)

$\therefore EG = GF$ (matching sides in congruent triangles)



3 In Δs OMK and OML, $KM = ML$ (given)



$OK = OL$ (radii)

OM is a common side

$\therefore \Delta OKM \equiv \Delta OLM$ (SSS)

$\therefore \angle OMK = \angle OML$ (matching angles in congruent triangles)

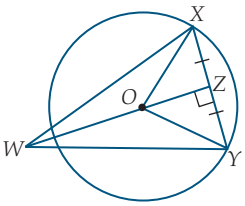
But $\angle OMK + \angle OML = 180°$, so $\angle OMK = \angle OML = 90°$. Hence, $OM \perp KL$.

4 $WX = WY$ as all points on the perpendicular bisector of the interval XY are equidistant from the end points of the interval.

$OX = OY$ (radii of the circle centre O on which X and Y lie)

Because O is equidistant from X and Y, it must lie on the perpendicular bisector of XY.

$\therefore O$ lies on WZ.

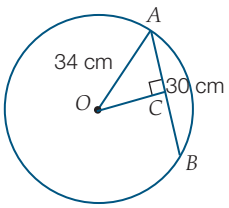


5 $WX = XY$ (given)

Hence, $x = 70$ (equal chords subtend equal angles at the centre)

Because $\angle WOX = \angle YOZ = 70°$, $y = 2$ (equal angles at the centre are subtended by equal chords)

6 Draw OC perpendicular to AB.



$AC = CB = 15$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

$34^2 = 15^2 + OC^2$

$OC^2 = 34^2 - 15^2 = 1156 - 225 = 931$

$OC = \sqrt{931} \approx 30.5$ cm

7 **(a)** $AB = 5$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

(b) $AB = 2 \times AC = 20$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

(c) $AB = 15$ cm (Pythagoras' theorem)

8 **(a)** $\angle AOB = 70°$ (equal chords subtend equal angles at the centre)

(b) $\angle AOB = 80°$ (angle sum of isosceles triangle is 180°)

(c) $\angle OFG = 50°$ (base angles of isosceles triangle), $\angle FOG = 80°$ (angle sum of triangle), $\angle AOB = 80°$ (equal chords subtend equal angles at the centre)

9 **(a)** $AE = EB = 24$ cm (E is midpoint of AB, given)

$\angle OEA = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$OA^2 = 7^2 + 24^2$ (Pythagoras' theorem)

$OA = 25$ cm

(b) $OG^2 = 20^2 + 21^2 = 841$ (Pythagoras' theorem), $OG = 29$, $OA = 29$ cm

(c) ΔOGF is equilateral; $OA = OG = 10$ cm

10 **(a)** $CE^2 = 53^2 - 28^2 = 2025$ (Pythagoras' theorem), $CE = 45$, $CD = 2 \times 45 = 90$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

(b) $GH^2 = 73^2 - 48^2 = 3025$ (Pythagoras' theorem), $GH = 55$, $GF = 2 \times 55 = 110$ cm (perpendicular from the centre of a circle to a chord, bisects the chord); $FD = GF = 110$ cm (given), $CD = FD = 110$ cm (equal angles at the centre are subtended by equal chords)

(c) $GH^2 = 85^2 - 77^2 = 1296$ (Pythagoras' theorem), $GH = 36$, $GF = 2 \times 36 = 72$ cm (perpendicular from the centre of a circle to a chord, bisects the chord), $CD = GF = 72$ cm (equal angles at the centre are subtended by equal chords)

11 In Δs DOE and BDC, $DO = OB$ (radii)

$DE = EC$ (E is midpoint of CD)

$\therefore \frac{DO}{DB} = \frac{DE}{DC} = \frac{1}{2}$

$\angle ODE = \angle BDC$ (same angle)

$\Delta DOE \,|||\, \Delta DCB$ (common angle with the sides containing it proportional)

$\angle DEO = \angle DCB$ (matching angles of similar triangles)

$\therefore OE \parallel BC$ (pair of corresponding angles are equal)

But $\angle DEO = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$\therefore \angle BCD = 90°$

12 Join OD

$\angle OFC = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$\therefore \angle OFC + \angle FCD = 180°$ ($\angle FCD = 90°$, given)

But these are a pair of supplementary co-interior angles

$\therefore OF \parallel DC$

$\angle ODC = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$\therefore \angle ODC + \angle FCD = 180°$ ($\angle FCD = 90°$, given)

But these are a pair of supplementary co-interior angles)

$\therefore OD \parallel FC$

$\therefore OFCD$ is a parallelogram

$\therefore OF = DC$ (opposite sides of a parallelogram are equal)

But $BC = 2DC$ (D is midpoint of BC)

$\therefore BC = 2OF$

13 $\angle OKF = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$\angle QJF = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$\therefore \angle OKF + \angle QJF = 180°$

$\angle OKJ$ and $\angle QJK$ are a pair of supplementary co-interior angles contained between OK and QJ.

$\therefore OK \parallel QJ$

$\therefore \angle KOQ + \angle OQJ = 180°$ (co-interior angles, $OK \parallel QJ$)

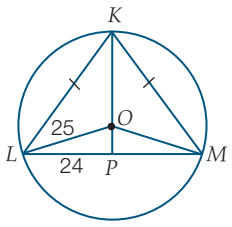
14 Let O be the centre of the circular arch.
By symmetry, the Point C must be in the middle of AB, so $AC = 2$ m.

$OC^2 = 2.5^2 - 2^2$ (Pythagoras' theorem)

$OC = 1.5$ m

$CH = OH - OC = 2.5 - 1.5 = 1$ m

15 Let O be the centre of the circle and P the midpoint of LM.



$\angle OPL = 90°$ (line joining midpoint of chord to centre of circle $\perp$ chord)

$OP^2 = 25^2 - 24^2$ (Pythagoras' theorem)

$OP = 7$ mm

$KP = 25 + 7 = 32$ mm ($OK = 25$ mm, radius)

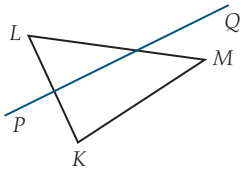
$KL^2 = 32^2 + 24^2$ (Pythagoras' theorem)

$KL = 40$ mm

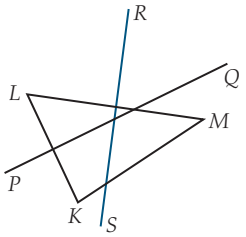
Open-ended – Sample answers

16 Students' own answers.

17 The centres of all the circles passing through K and L lie on the line PQ.

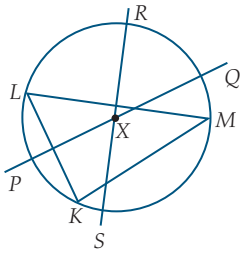


The centres of all the circles passing through L and M lie on the line RS.

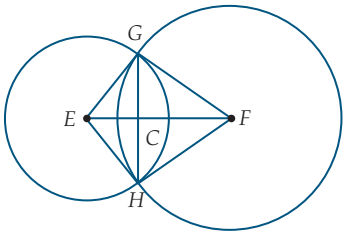


The point of intersection of PQ and RS must be the centre of the only circle that passes through K, L and M.

Label the point of intersection of PQ and RS, X, and draw the circle centre X and radius XL. It will pass through all the points K, L and M.



18 (i)



In Δs EGF and EFH,
$EG = EH$ (radii of circle centre E)
$GF = HF$ (radii of circle centre F)

EF is a common side $\therefore \Delta EGF \equiv \Delta EFH$ (SSS)

$\angle GEF = \angle HEF$ (corresponding angles in congruent triangles)

In Δs EGC and EHC, $EG = EH$ (radii of circle centre E)

$\angle GEC = \angle HEC$ (corresponding angles in congruent triangles)

EC is a common side.

$\therefore \Delta EGC = \Delta EHC$ (SAS)

$GC = CH$ (corresponding sides in congruent triangles)

$\therefore EF$ biscets GH

(i.e. the line of centres bisects the common chord).

(ii) CE is the line joining the centre of the chord GH to the centre E of the circle.

$\therefore CE \perp GH$

(i.e. the line of centres is perpendicular to the common chord).

Chapter review 8 (p. 315)

1 (a) SSS **(b)** SAS

2 B

3 (a) 5 **(b)** 9

4 (a) In ΔAOB and ΔDOC

$\angle AOB = \angle DOC$ (vertically opposite angles)

$AO = DO$ (radii in the same circle)

$BO = CO$ (radii in the same circle)

$\therefore \Delta AOB = \Delta DOC$ (SAS)

So $AB = DC$ (corresponding sides in congruent triangles)

$\therefore p = 6$ m

(b) In ΔDEH and ΔGEF

$\angle DEH = \angle GEF$ (vertically opposite angles)

$DH = GF$ (given)

$\angle EDH = \angle EGF$ (alternate angles, $DH \parallel GF$)

$\therefore \Delta DEH \equiv \Delta GEF$ (AAS)

So $HE = FE$ (matching sides in congruent triangles)

$\therefore r = 9$ cm

5 (a) In ΔABC and ΔADE,

$\angle CAB = \angle EAB$ (common angle)

$\angle ABC = \angle ADE$ (given)

$\therefore \angle ACB = \angle AED$ (third angle in a triangle)

$\therefore \Delta ABC \parallel\!\parallel\!\parallel \Delta ADE$ (AAA)

(b) In ΔABC and ΔADE,

$\angle BAC = \angle DAE$ (vertically opposite angles)

$\angle BCA = \angle AED$ (alternate angles and $BC \parallel DE$)

$\angle CBA = \angle ADE$ (alternate angles and $BC \parallel DE$)

$\therefore \Delta ABC \parallel\!\parallel\!\parallel \Delta ADE$ (AAA)

6 In ΔAXC and ΔBXC,

$\angle AXC = \angle BXC = 90°$ (given that $CX \perp AB$)

$\angle ACX = \angle BCX = \frac{1}{2} \times \angle ACB$ (given that CX bisects $\angle ACB$)

Now $CX = CX$ (common side in ΔAXC and ΔBXC)

$\therefore \Delta AXC \equiv \Delta BXC$ and so $AX = BX$ (matching sides in congruent triangles)

7 In Δs DGH and DEF

$\angle GDH = \angle EDF$ (common angle)

$\angle DHG = \angle DFE$ (corresponding angles equal, $GH \parallel EF$)

$\angle DGH = \angle DEF$ (corresponding angles equal, $GH \parallel EF$)

$\therefore \Delta DGH \equiv \Delta DEF$ (AAA)

$$\frac{DE}{DG} = \frac{DF}{DH}$$

$$\frac{DG + GE}{DG} = \frac{DH + HF}{DH}$$

$$\frac{DG}{DG} + \frac{GE}{DG} = \frac{DH}{DH} + \frac{HF}{DG}$$

$$1 + \frac{GE}{DG} = 1 + \frac{HF}{DH}$$

$$\therefore \frac{GE}{DG} = \frac{HF}{DH}$$

8 In ΔABC and ΔCDA

$AB = CD$ (given)

$BC = DA$ (given)

$AC = CA$ (common side)

$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)

So $\angle BAC = \angle DCA$ (matching angles in congruent triangles)

But $\angle BAC = \angle DCA$ are alternate angles,

$\therefore AB \parallel CD$

And $\angle BCA = \angle DAC$ (matching angles in congruent triangles)

But $\angle BCA = \angle DAC$ are alternate angles $\therefore AD \parallel CB$

The quadrilateral is a parallelogram because both pairs of opposite sides are parallel.

Have proved that if both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.

9 In ΔPTQ, ΔRTQ, ΔRTS and ΔPTS

$PT = RT$ (given)

$ST = QT$ (given)

$\angle PTQ = \angle RTQ = \angle RTS = \angle PTS = 90°$ (given)

$\therefore \Delta PTQ \equiv \Delta RTQ \equiv \Delta RTS \equiv \Delta PTS$ (SAS)

So $PQ = RQ$ (corresponding sides in congruent triangles)

ΔPTQ is isosceles so $\angle TPQ = \angle TQP$

$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$2\angle TQP + 90° = 180°$

$2\angle TQP = 90°$

$\angle TQP = 45°$

But $\angle TQP = \angle TQR = \angle TRQ = \angle TRS = \angle TSR = \angle TSP = \angle TPS = \angle TPQ$ (corresponding angles in congruent triangles)

So $\angle TQR = 45°$

$\angle PQR = \angle TQR + \angle TQP = 90°$

$\angle QRS = \angle TRQ + \angle TRS = 90°$

$\angle SPQ = \angle TPQ + \angle TPS = 90°$

$\angle PQR$ and $\angle QRS$ are co-interior angles and they add to 180°.

$\therefore PQ \parallel SR$

$\angle PQR$ and $\angle SPQ$ are co-interior angles and they add to 180°.

$\therefore PS \parallel QR$

Have shown $PQ = RQ$, $\angle PQR = 90°$, $PQ \parallel SR$ and $PS \parallel QR$.

$PQRS$ is a square as it has two adjacent sides, one right angle and both pairs of opposite sides parallel.

Have proved that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then quadrilateral is a square.

10 (a) $y = 59°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

(b) $x = 125°$ (reflex angle at the centre is twice the angle at the circumference standing on the same arc)

(c) $x = 38°$ (vertically opposite angles, angle sum of triangle)

(d) $x = 83°$ (angles in the same segment, angle sum of triangle)

(e) $k = 39°$ (angles in the same segment, exterior angle of a triangle)

(f) $x = 33°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$y = 90 - 36 = 54°$ (angle in a semicircle, isosceles triangle)

11 (a) $AB = 8$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

(b) $AB = 16$ cm (perpendicular from the centre of a circle to a chord, bisects the chord)

(c) $AE = 40$ mm (line joining midpoint of chord to centre of circle $\perp$ chord, Pythagoras' theorem), $AB = 80$ mm

12 (a) $\angle AOB = 65°$ (equal chords subtend equal angles at the centre)

(b) $\angle OFG = 55°$ (base angles of isosceles triangle), $\angle GOF = 70°$ (angle sum of triangle), $\angle AOB = 70°$ (equal chords subtend equal angles at the centre)

(c) $\angle GOH = 35°$ (perpendicular bisector of chord bisects angle at centre), $\angle GOF = 70°$, $\angle AOB = 70°$ (equal chords subtend equal angles at the centre)

13 12.6 m

14 (a) In ΔBAD and ΔBCD

$\angle BDA = \angle BDC = 90°$ (by construction)

$BA = BC$ (given)

$BD = BD$ (common side)

$\therefore \Delta BAD \equiv \Delta BDC$ (RHS)

So $\angle BAD = \angle BCD$ (matching angles in congruent triangles)

$\therefore \angle BAC = \angle BCA$

(b) Have proved that the angles opposite the equal sides in an isosceles triangle are also equal.

15 In ΔABZ and ΔACX,

$AB = AC$ (given)

$AZ = AX$ (given)

$\angle BAZ = \angle CAX$ (common angle)

$\therefore \Delta ABZ \equiv \Delta ACX$ (SAS) (and so $BZ = CX$)

Now consider ΔBYC

$\angle YBC = \angle YCB$ (given)

$BY = CY$ (If two angles in a triangle are equal then the sides opposite these angles are equal—the triangle is isosceles.)

Consider ΔBYX and ΔCYZ

$\angle BYX = \angle CYZ$ (vertically opposite angles)

$BX = CZ$ (as $AB = AC$ and $AZ = AX$)

$XY = ZY$ (as $BY = CY$ and $BZ = CX$)

$\therefore \Delta BXY \equiv \Delta CZY$ (SSS)

16 (a) Let $\angle DFE = x$

$\therefore \angle GEF = (90 - x)$ (angle sum of ΔGEF is 180°)

$\therefore \angle GED = x$ ($\angle DEF = 90°$)

In Δs DEF and DEG,

$\angle DFE = \angle DEG = x$

$\angle DEF = \angle DGE = 90°$

$\therefore \Delta DEF \,|||\, \Delta DGE$ (AAA)

$\frac{EF}{EG} = \frac{DF}{DE} = \frac{DE}{DG}$ (1) (matching sides of similar triangles are proportional)

In Δs DEF and EGF,

$\angle DFE = \angle GFE = x$

$\angle DEF = \angle FGE = 90°$

$\therefore \Delta DEF \,|||\, \Delta EGF$ (AAA)

(b) $\frac{EF}{GF} = \frac{DF}{EF} = \frac{DE}{EG}$ (2) (matching sides of similar triangles are proportional)

(1) gives $EF = \frac{EG \times DE}{DG}$

(2) gives $EF = \frac{GF \times DE}{EG}$

$\therefore \frac{EG \times DE}{DG} = \frac{GF \times DE}{EG}$

$\therefore EG^2 = DG \times GF$

17 (a) In ΔEAB and ΔECD

$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel CD$)

$\angle AEB = \angle CED$ (vertically opposite)

$AB = CD$ (given)

$\therefore \Delta EAB = \Delta ECD$ (by ASA)

$AE = CE$ (matching sides in congruent triangles)

In ΔEAB and ΔECB

$AE = CE$ (found earlier)

$AB = CB$ (given)

$BE = BE$ (common side)

$\therefore \Delta EAB = \Delta ECB$ (by SSS)

$\therefore \angle AEB = \angle CEB$ (matching angles in congruent triangles)

But $\angle AEB + \angle CEB = 180°$ (angles on a straight line, supplementary)

$\therefore \angle AEB = 90°$ and $\angle CEB = 90°$

$\therefore AC \perp BD$

(b) Have just proved that the diagonals of a rhombus intersect at right angles.

18 (a) $x = 60°$, $y = 50°$ ($\angle$ at centre, $\angle$s in same segment)

(b) $x = 100°$, $y = 80°$ (co-interior angles, opposite angles in cyclic quadrilateral)

(c) $x = 73°$, $y = 13°$, $z = 107°$ (straight angle, opposite angles in cyclic quadrilateral)

19 $\angle TSR + \angle TSU = 180°$ (angles on a straight line)

$\angle TUS + \angle TUV = 180°$ (angles on a straight line)

$\therefore \angle TSR + \angle TSU + \angle TUS + \angle TUV = 180° + 180° = 360°$

$\angle TSU + \angle TUS + \angle UTS = 180°$ (angles in a triangle)

$\angle TSU + \angle TUS + 90° = 180°$

$\angle TSU + \angle TUS = 90°$

$\angle TSR + \angle TSU + \angle TUS + \angle TUV = 360°$ (found earlier)

$\angle TSR + 90° + \angle TUV = 360°$

$\therefore \angle TSR + \angle TUV = 270°$

20 $a + b = 180°$ (angles on a straight line)

$c + d = 180°$ (angles on a straight line)

$\therefore a + b + c + d = 180° + 180° = 360°$

$\angle GWD = \angle AWB = a$ (vertically opposite angles)

$\angle FXA = \angle DXC = d$ (vertically opposite angles)

$\angle BZE = \angle BWD = b$ (corresponding angles, $AD \parallel HE$)

$\angle CYH = \angle CXA = c$ (corresponding angles, $AD \parallel HE$)

$\angle GWD + \angle BZE + \angle FXA + \angle CYH$

$= a + b + c + d$

$= 360°$

So the interior angles of a trapezium add to 360°.

21 To show: $\Delta LYZ \,|||\, \Delta MXZ$

$\angle LZY = \angle MZX$	(vertically opposite)
$\angle LYZ = \angle MXZ$	(both 90°)
$\angle ZMX = \angle ZLY$	(third angle in a triangle)
$\Delta LYZ \,|||\, \Delta MXZ$	(AAA)

22 In ΔUXY and ΔVWX

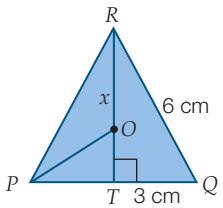
$YX = XW$ (given)

$UY = VX$ (given)

$\angle UYX = \angle VXW$ (corresponding angles, $UY \parallel VX$)

$\therefore \Delta UXY = \Delta VWX$ (SAS)

23 $RT = \sqrt{36-9} = \sqrt{27} = 3\sqrt{3}$ (Pythagoras' theorem in ΔRTQ)



Let radius = x. Hence $OP = x$.

Use Pythagoras' theorem in ΔPOT:

$x^2 = 3^2 + (\sqrt{27} - x)^2$
$x^2 = 9 + 27 - 2\sqrt{27}x + x^2$
$2\sqrt{27}x = 36$
$6\sqrt{3}x = 36$
$x = \dfrac{36}{6\sqrt{3}} = 2\sqrt{3}$

24 $\angle TWU = 90°$ ($SU \perp VT$)

$\angle WUT = 43°$ (angle sum of triangle)

$\angle SVT = \angle SUT = 43°$ (angles in the same segment)

$\angle SOT = 86°$ (angle at the centre twice the angle at the circumference standing on the same arc)

Chapter 9

Recall 9 (p. 320)

1 **(a)** **(i)** 4 **(ii)** 5 **(iii)** 10

(b) **(i)** 10 **(ii)** 18 **(iii)** 0

2 **(a)** $x(x-7)$ **(b)** $2x(x+5)$ **(c)** $(x+2)(x-2)$

3 **(a)** $3x^2 + 12x$ **(b)** $x^2 + 7x + 10$ **(c)** $x^2 - 4x - 21$

(d) $x^2 + 16x + 64$ **(e)** $x^2 - 4x + 3$ **(f)** $x^2 - 16$

4 **(a)** $(x+5)(x-5)$ **(b)** $(3x-4)(3x+4)$

(c) $(x+\sqrt{5})(x-\sqrt{5})$ **(d)** $(x-4)^2$

(e) $(x+7)^2$ **(f)** $3(x-11)^2$

5 **(a)** $(x+5)(x+2)$ **(b)** $(x-3)(x-2)$ **(c)** $(x-14)(x+1)$

(d) $(x+8)(x-3)$ **(e)** $(x+9)(x-4)$ **(f)** $(x+1)(x+6)$

(g) $-(x+2)(x+1)$ **(h)** $-(x-5)(x-2)$ **(i)** $-(x+3)(x-4)$

6 **(a)** 27 **(b)** $\dfrac{1}{4}$ **(c)** $3\dfrac{1}{25}$

7 **(a)** $x = -3, 1$ **(b)** $x = -2, -4$

(c) $x = 1$ **(d)** $x = 0, 5$

8 **(a)** $x = 3$ **(b)** $x = \pm\dfrac{5}{2}$ **(c)** $x = -4$ **(d)** $x = \pm\dfrac{6}{11}$

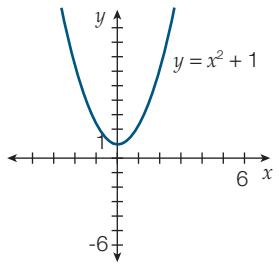
9 **(a)** $(x+2-\sqrt{2})(x+2+\sqrt{2})$

(b) $(x-3+\sqrt{3})(x-3-\sqrt{3})$

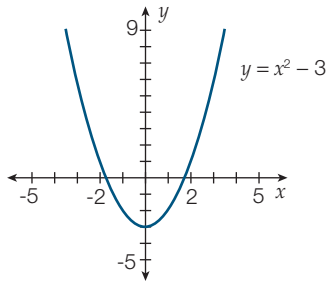
(c) $(x-\dfrac{3}{2}+\dfrac{\sqrt{29}}{2})(x-\dfrac{3}{2}-\dfrac{\sqrt{29}}{2})$

Exercise 9.1 (p. 325)

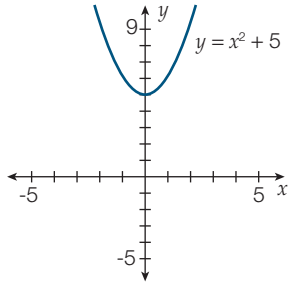
1 **(a)**



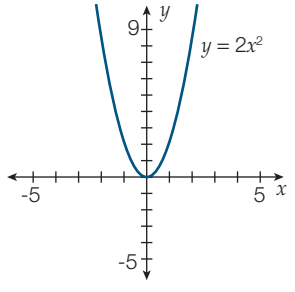
(b)



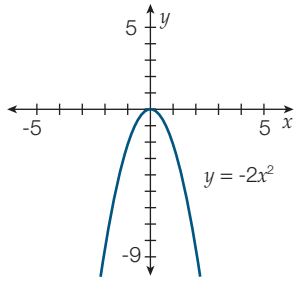
(c)



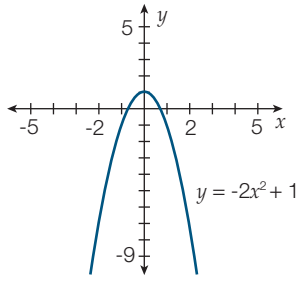
(d)



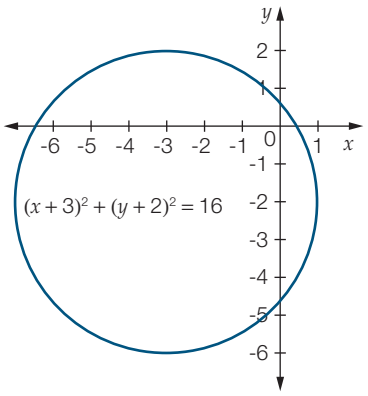
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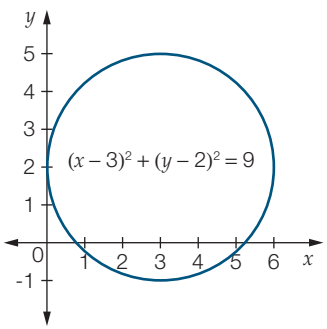
(f)



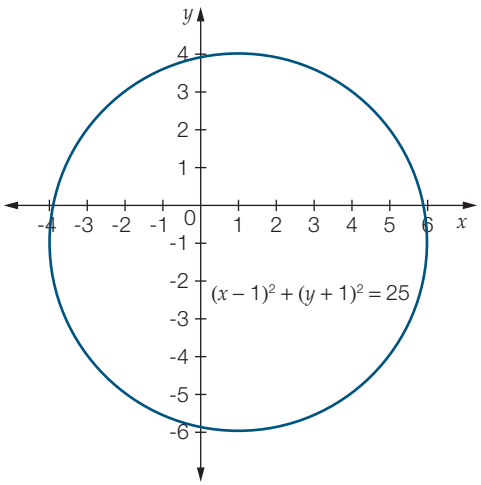
2 (a)



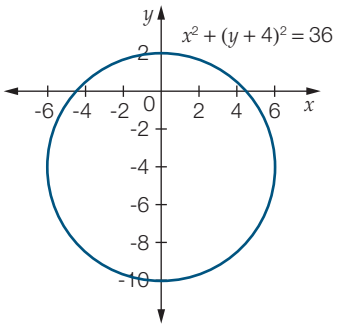
(b)



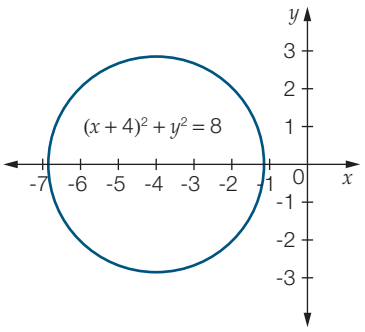
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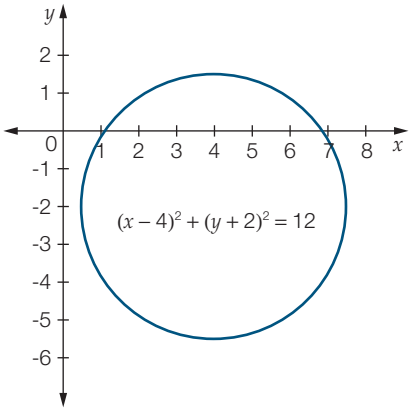
(d)



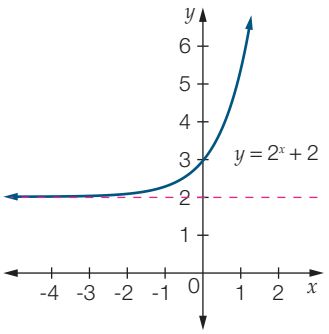
(e)



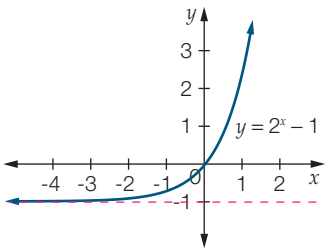
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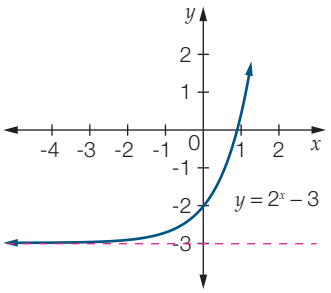
3 (a)



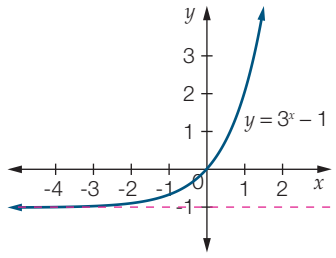
(b)



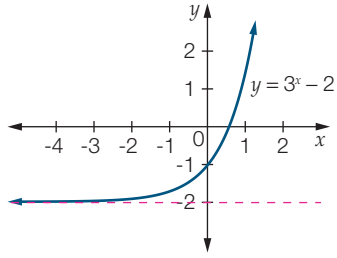
(c)



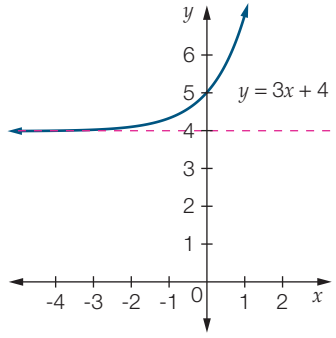
(d)


y
x
$y = 3^x - 1$

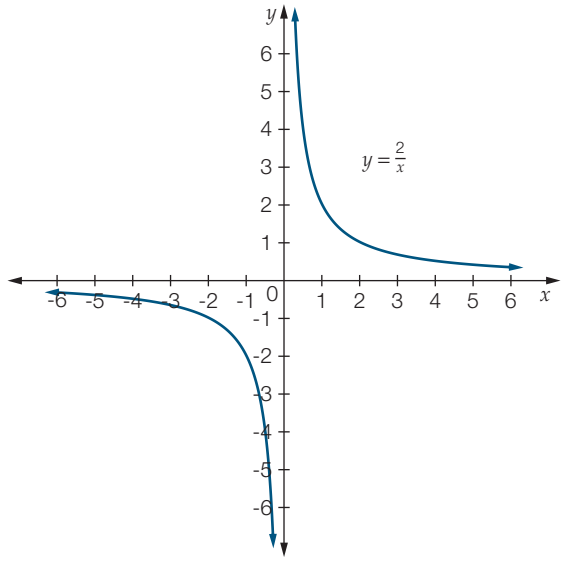
(e)


y
x
$y = 3^x - 2$

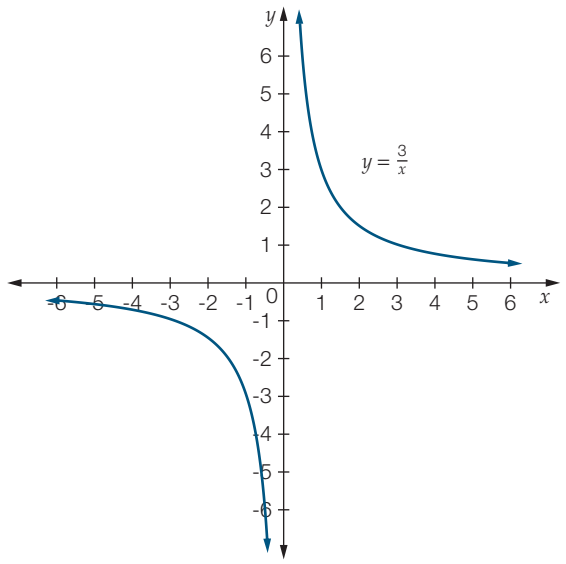
(f)


y
x
$y = 3x + 4$

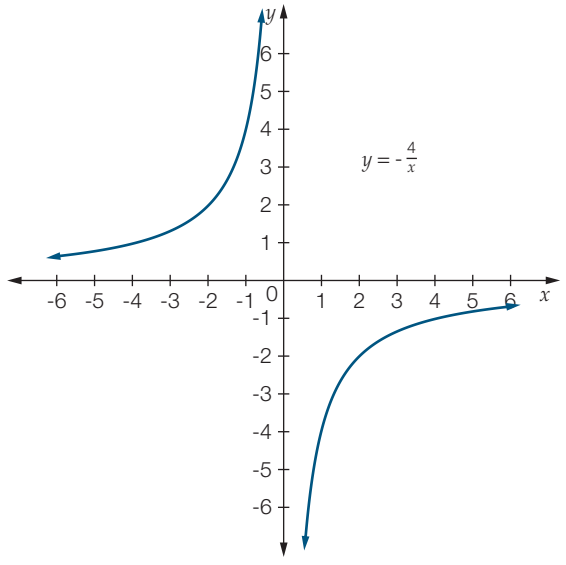
4 (a)


y
x
$y = \frac{2}{x}$

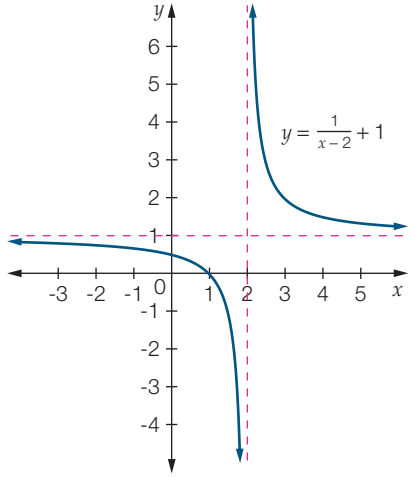
(b)


y
x
$y = \frac{3}{x}$

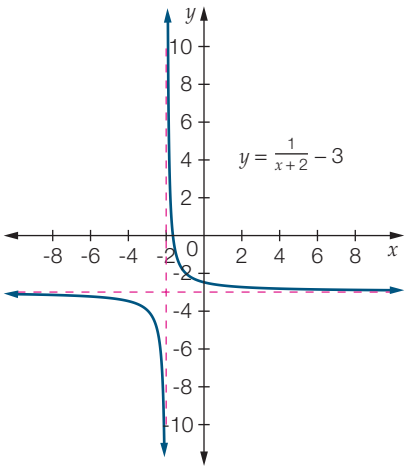
(c)


y
x
$y = -\frac{4}{x}$

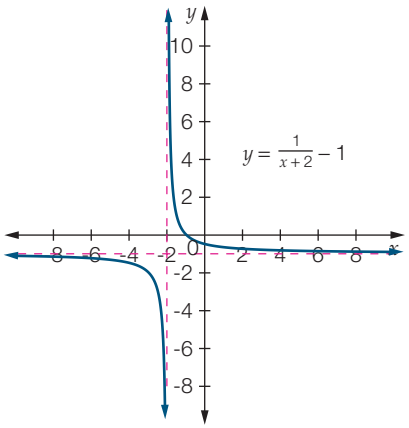
(d)


y
x
$y = \frac{1}{x-2} + 1$

(e)

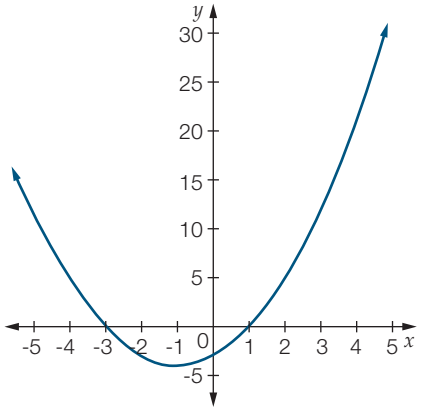


(f)



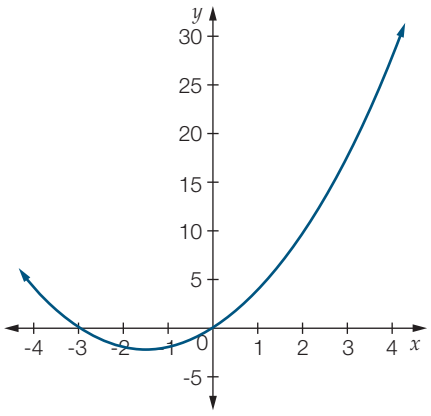
5 **(a)** **(i)**

x	-4	-3	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5	12	21



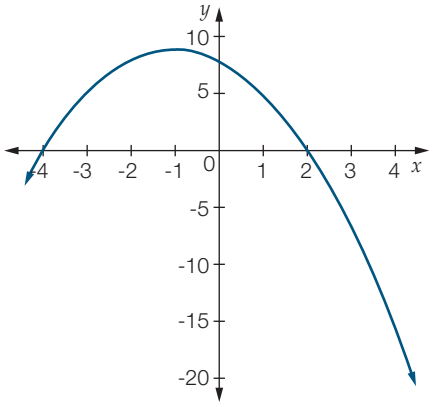
(ii)

x	-4	-3	-2	-1	0	1	2	3	4
y	4	0	-2	-2	0	4	10	18	28



(iii)

x	-4	-3	-2	-1	0	1	2	3	4
y	0	5	8	9	8	5	0	-7	-16



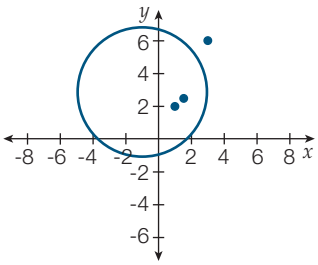
(b) **(i)** (-1, 4), minimum

(ii) (-1.5, -2.25), minimum

(iii) (-1, 9), maximum

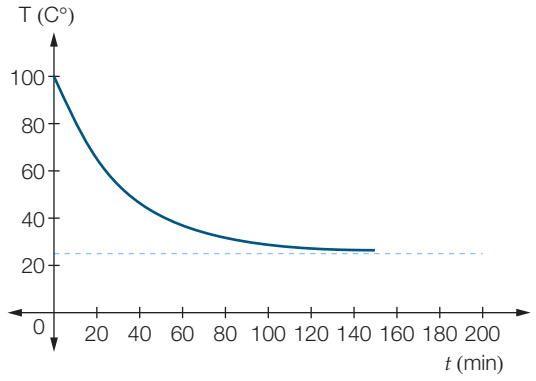
6 C

7 **(a)**



(b) (1, 2) and (1.5, 2.5) are inside and (3, 6) is outside.

8 (a)



(b) 100°C

(c) **(i)** 79°C **(ii)** 65°C **(iii)** 46°C **(iv)** 40°C

(d) The first 10 minutes. The coffee lost 21°C in the first 10 minutes, which is the greatest loss in any 10-minute period. This is the steepest part of the graph.

(e) After 107 minutes.

(f) After 3 hours the temperature of the coffee will be approximately 25°.

9 (a) $V = \pi r^2 h$

$= \pi r^2\left(\frac{3}{\pi}\right)$

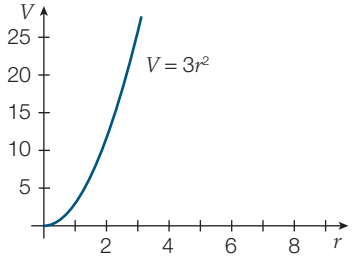
$= 3r^2$

(b) It is a quadratic because the highest power of r is 2.

(c)

r	0	1	2	3	4	5
r^2	0	1	4	9	16	25
V	0	3	12	27	48	75

(d)



(e) **(i)** 18.8 m^3 **(ii)** 43.3 m^3

(f) **(i)** 3.2 m **(ii)** 4.7 m

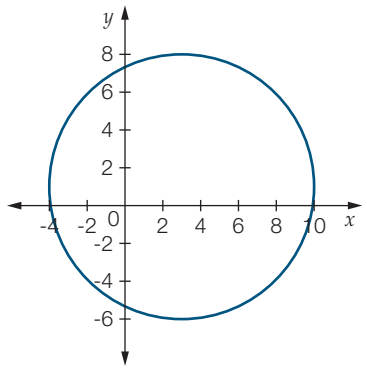
(g) 30 m^3

10 (a) **(i)** $(x-3)^2 + (y-1)^2 = 49$

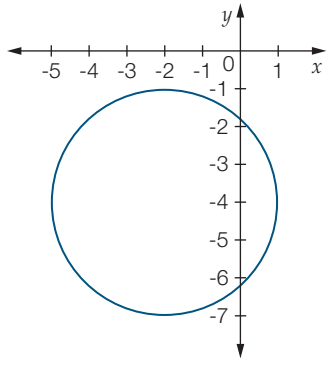
(ii) $(x+2)^2 + (y+4)^2 = 9$

(iii) $(x-2)^2 + (y+5)^2 = 4$

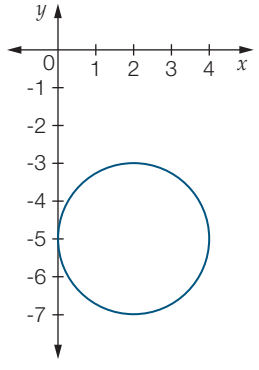
(b) **(i)**



(ii)



(iii)

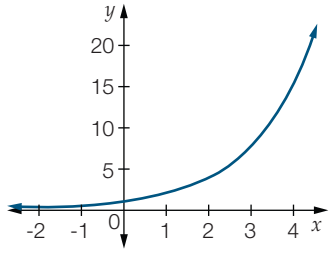


11 B **12** D

13 (a)

x	-2	-1	0	1	2	3	4
y	0.25	0.5	1	2	4	8	16

(b)



(c) All of these answers are approximate.

(i) 2.8 **(ii)** 1.7 **(iii)** 0.7

(iv) 0.4 **(v)** 1.4 **(vi)** 2.8

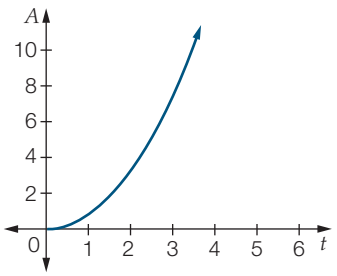
(d) Students are to check **(c)** using a calculator.

(e) All of these answers are approximate.

(i) 2.3 **(ii)** 3.6 **(iii)** 3.9

14 **(a)** $A = l(l - 0.4)$ m^2

(b)



(c) 2 m × 1.6 m

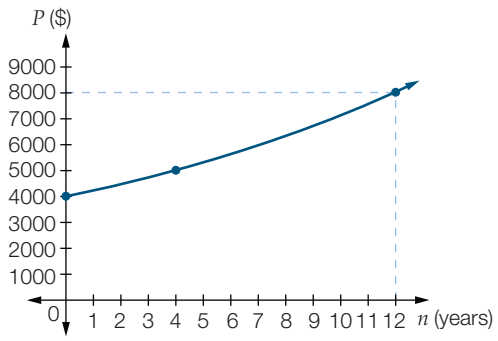
15 **(a)** 4240

(b)

n	0	1	2	3	4	5
P	4000	4240	4492	4764	5050	5353

(c) $P = 4000(1.06)^n$

(d)



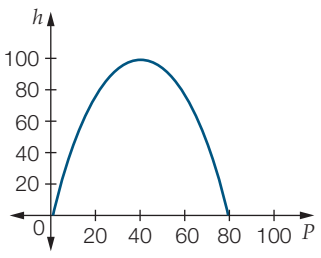
(e) 12 years

(f) $P = 4000(1.06)^{12}$

$= 8048.78$

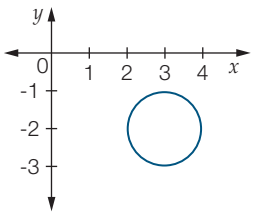
(g) Future time and population can only be positive numbers.

16 **(a)**



(b) 100 m **(c)** 80 m **(d)** 20 106.19 m^2

17 $(x - 3)^2 + (y + 2)^2 = 1$



The point (2.4, -2.7) is inside the circle, so a tree could not be planted there. If we substitute (2.4, -2.7) into the LHS of the equation we get 0.85. Any value less than 1 is inside the circle, exactly 1 is on the perimeter, and greater than 1 is outside the circle.

Open-ended – Sample answers

18 **(a)** Students to provide their own equation, but it must have at least $(x \pm h)^2$ and/or $(y \pm k)^2$ in it.

(b) For a maximum turning point the coefficient of x^2 must be negative.

(c) The equation will need to have -1 at the end.

(d) The equation will need to have $(x + 2)$ in it as well as + 6 at the end.

19 **(a)** Students to provide their own sketch.

(b) The graph should start at the origin, be symmetrical in shape with a maximum turning point. This means the turning point is midway between the two x-intercepts.

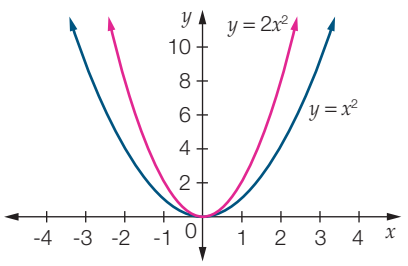
20 $(x + 3)^2 + (y + 1)^2 = 25$

$(x - 4)^2 + (y + 2)^2 = 25$

Exercise 9.2 (p. 333)

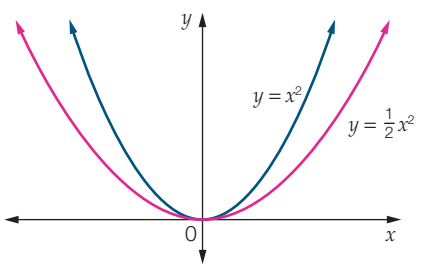
1 **(a)** **(i)** 2 **(ii)** narrower

(iii)

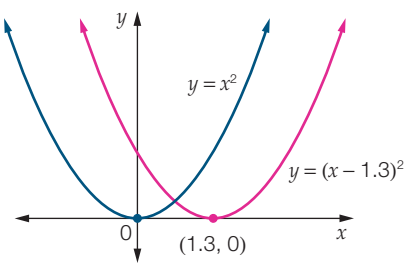


(b) **(i)** 0.5 or $\frac{1}{2}$ **(ii)** wider

(iii)

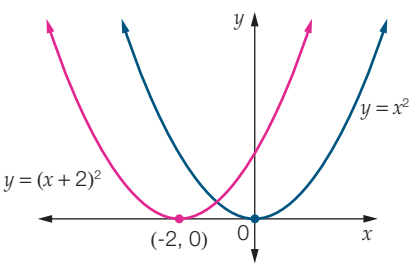


2 **(a)** **(i)**



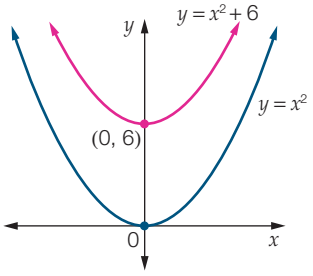
(ii) 1.3 units to the right

(b) **(i)**



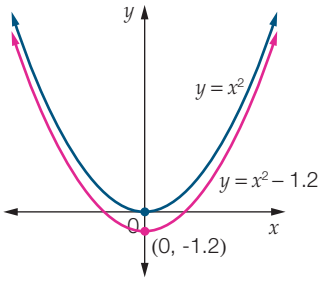
(ii) 2 units to the left

3 (a) (i)



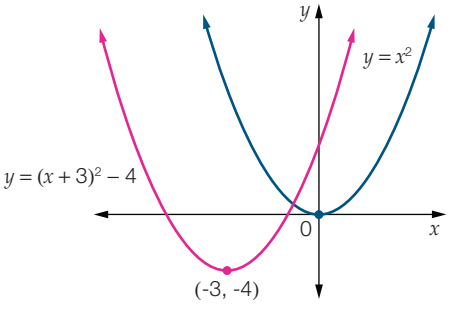
(ii) 6 units up

(b) (i)

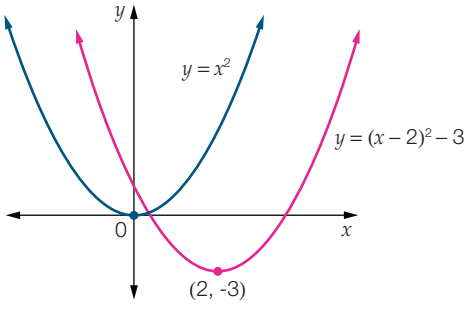


(ii) 1.2 units down

4 (a)



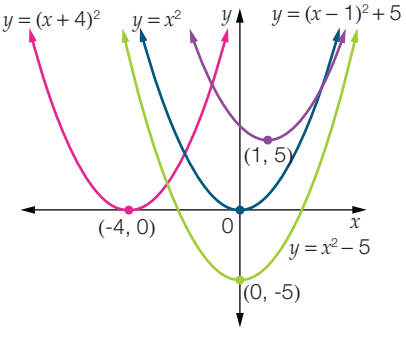
(b)



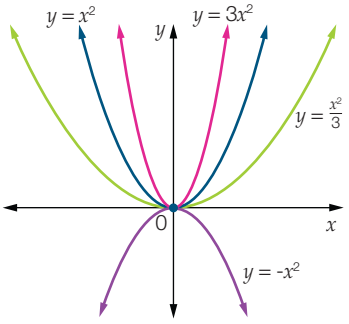
5 $y = (x - 2)^2 - 5$

6 B

7 (a)–(d)

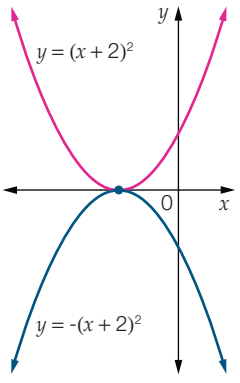


8 (a)–(d)

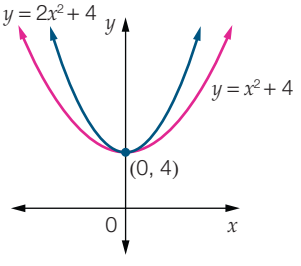


9 D

10 (a) and **(b)**

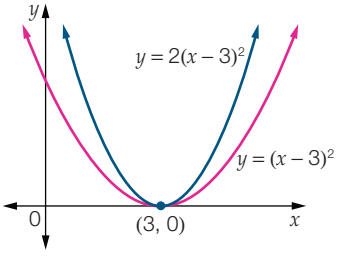


11 (a) and **(b)**



(c) $y = 2x^2 + 4$

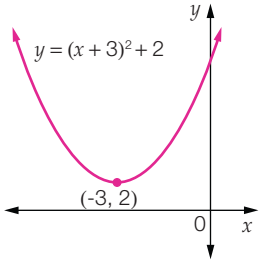
12 (a) and **(b)**



(c) $y = 2(x - 3)^2$

13 (a) $y = (x + 3)^2 + 2$

(b)



14 $y = (x + 2)^2 + 0.5$

15 (0, 0), (1, 5), (2, 8), (3, 9), (4, 8), (5, 5), (6, 0)

Open-ended – Sample answers

16 **(a)** The equation will be in the form $y = x^2 + k$.

(b) The equation will be in the form $y = (x - h)^2$.

17 When dealing with brackets you are looking for the value of x that will make the bracket equal zero. So, $(x + 6)$ requires x to equal -6 to make the bracket 0. This means the turning point will be in line with $x = -6$. Syd is correct in his thinking about the vertical shift.

Exercise 9.3 (p. 339)

1 **(a)** $x = 0$ or $x = 11$ **(b)** $x = 0$ or $x = -6$

(c) $x = 0$ or $x = -4$ **(d)** $x = 0$ or $x = 1$

(e) $x = 0$ or $x = 1$ **(f)** $x = 0$ or $x = 6$

(g) $x = 3$ or $x = -7$ **(h)** $x = 4$ or $x = -1$

(i) $x = 2$ or $x = 6$ **(j)** $x = 4$ or $x = 5$

(k) $x = 7$ or $x = -2$ **(l)** $x = 3$ or $x = -10$

2 **(a)** $x = 0$ **(b)** $x = 0$ **(c)** $x = 0$

(d) $x = -7$ **(e)** $x = -1$ **(f)** $x = 9$

3 **(a)** $x = 0$ or $x = 4$ **(b)** $x = 0$ or $x = -1$

(c) $x = 0$ or $x = 6$ **(d)** $x = 0$ or $x = 5$

(e) $x = 0$ or $x = \frac{1}{2}$ **(f)** $x = 0$ or $x = -3$

4 **(a)** $x = 5$ or $x = -5$ **(b)** $x = 9$ or $x = -9$

(c) $x = 2$ or $x = -2$ **(d)** $x = \frac{5}{4}$ or $x = -\frac{5}{4}$

(e) $x = \frac{7}{3}$ or $x = -\frac{7}{3}$ **(f)** $x = \frac{9}{10}$ or $x = -\frac{9}{10}$

(g) $x = 5$ or $x = -5$ **(h)** $x = 1$ or $x = -1$

(i) $x = 2$ or $x = -2$ **(j)** $x = 7$ or $x = -7$

(k) $x = 9$ or $x = -9$ **(l)** $x = 11$ or $x = -11$

(m) $x = \sqrt{13}$ or $x = -\sqrt{13}$

(n) $x = \sqrt{24}$ or $x = -\sqrt{24}$ (or $x = 2\sqrt{6}$ or $x = -2\sqrt{6}$)

(o) $x = \sqrt{32}$ or $x = -\sqrt{32}$ (or $x = 4\sqrt{2}$ or $x = -4\sqrt{2}$)

5 **(a)** $x = 7$ **(b)** $x = -4$ **(c)** $x = -8$

(d) $x = 2$ **(e)** $x = -1$ **(f)** $x = 3$

(g) $x = 5$ **(h)** $x = 1$ **(i)** $x = -2$

(j) $x = 3$ **(k)** $x = 5$ **(l)** $x = 4$

6 **(a)** $x = -1$ or $x = -2$ **(b)** $x = -3$ or $x = -6$

(c) $x = 4$ or $x = 7$ **(d)** $x = 1$ or $x = 9$

(e) $x = -5$ or $x = 6$ **(f)** $x = -4$ or $x = 2$

(g) $x = 4$ or $x = 8$ **(h)** $x = -5$ or $x = 3$

(i) $x = -2$ or $x = 8$ **(j)** $x = -6$ or $x = 2$

(k) $x = -1$ or $x = 2$ **(l)** $x = -1$ or $x = 3$

7 **(a)** $x = -4$ or $x = -3$ **(b)** $x = -6$ or $x = -2$

(c) $x = -7$ or $x = 9$ **(d)** $x = -8$ or $x = 3$

(e) $x = -3$ or $x = -2$ **(f)** $x = -3$ or $x = 5$

(g) $x = -10$ or $x = 5$ **(h)** $x = -3$ or $x = 9$

(i) $x = 6$ or $x = 11$ **(j)** $x = 1$ or $x = 5$

(k) $x = -8$ or $x = 2$ **(l)** $x = -7$ or $x = 3$

8 **(a)** D **(b)** A **(c)** D **(d)** A

9 **(a)** $x = -5$ or $x = 2$ **(b)** $x = -1$ or $x = 3$

(c) $x = -4$ or $x = 7$ **(d)** $x = -7$ or $x = -1$

(e) $x = -4$ or $x = 4$ **(f)** $x = -3$ or $x = 3$

(g) $x = -6$ or $x = -1$ **(h)** $x = -7$ or $x = 6$

(i) $x = 0$ or $x = 12$

10 **(a)** A **(b)** D **(c)** B **(d)** B **(e)** B **(f)** C

11 $t = 0$ seconds and $t = 50$ seconds

12 $x = -1$ and $x = 4$

13 **(a)** $x = -12$ or $x = 12$ **(b)** $x = 0$ or $x = -4$

(c) $x = 6$

14 **(a)** 5°C **(b)** $t = 2$ and $t = 8$

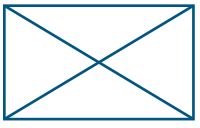
(c) 10 a.m. and 4 p.m.

15 **(a)** $(x + 4)(x + 6)$ m^2 **(b)** length 8 m, width 6 m

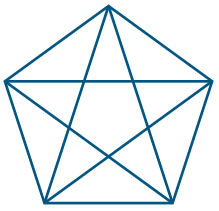
16 60 m

17 **(a)** $n - 3 = 0$ so $D = 0$

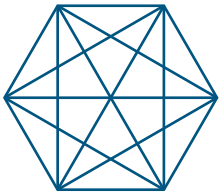
(b) quadrilateral: 2 diagonals



pentagon: 5 diagonals



hexagon: 9 diagonals



(c) **(i)** 20 **(ii)** 35 **(iii)** 54

(d) **(i)** 20 **(ii)** 26

(e) **(i)** 14 **(ii)** 42

Open-ended – Sample answers

18 **(a)** Students are to produce their own equations; they could be of the form $(x + a)(x + b) = 0$.

(b) Students are to produce their own equations; they could be of the form $(x + a)^2 = 0$.

19 Students are to produce their own equations; the easiest way to get the second equation is to make it the negative of the first.

20 Possible equations include: $x^2 + 2x - 2 = 0$; $6x^2 - 13x - 5 = 0$; $x^2 - x - 2 = 0$

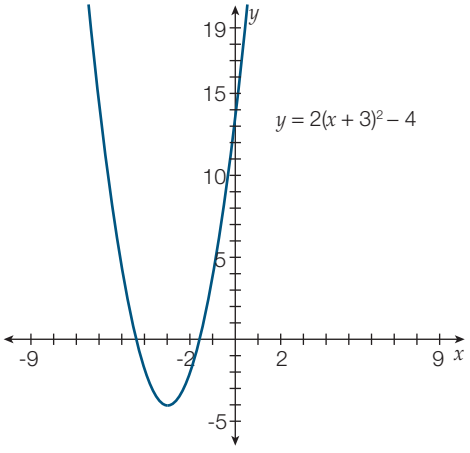
Half-time 9 (p. 342)

1 **(a)** $x = -2$ **(b)** $x = -1, 6$ **(c)** $x = 8, 1$

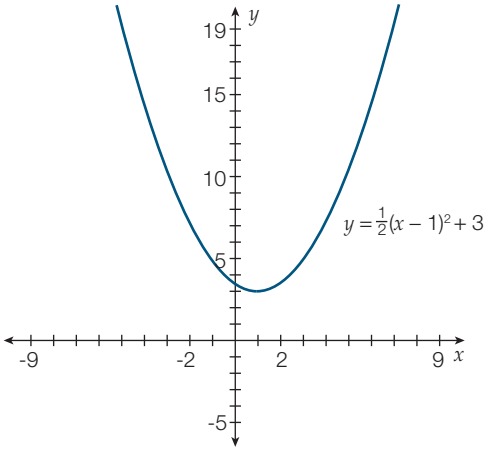
(d) $x = 5, -1$ **(e)** $x = -2, -1$ **(f)** $x = -5, 2$

2 radius 2 and centre (-1, 2)

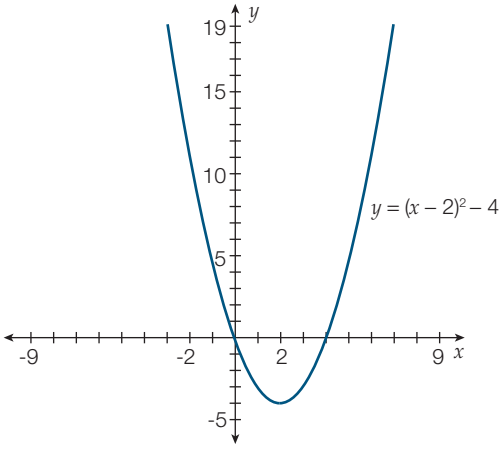
3 **(a)** dilated by a factor of 2, translated 3 units left and 4 units down



(b) dilated by a factor of a half, translated right 1 unit and up 3 units



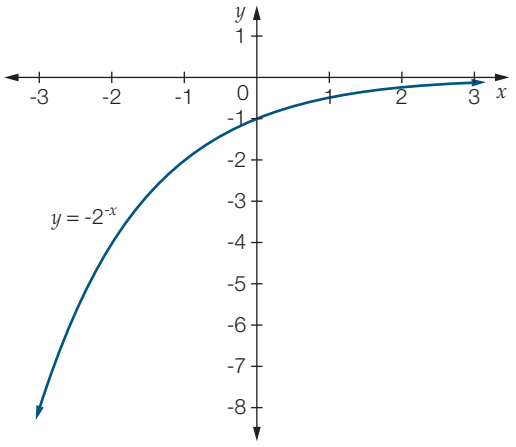
(c) translated 2 units right and 4 units down



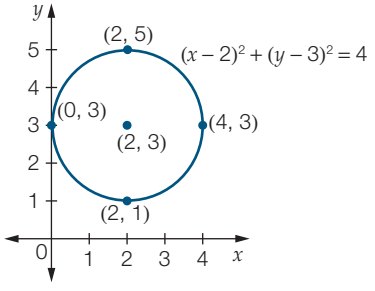
4 $y = (x - 3)^2 + 2$

5 **(a)** $x = 8, -8$ **(b)** $x = -1$ **(c)** $x = 4, 0$

6 **(a)**



(b)



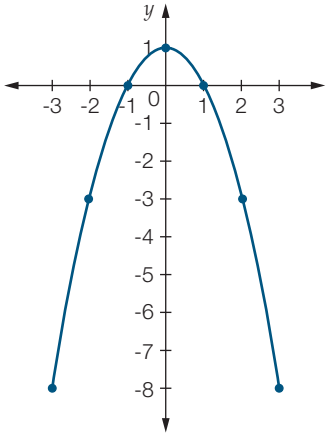
7 **(a)** $x = -5, -1$ **(b)** $x = 0, 9$ **(c)** $x = -5, 2$

(d) $x = -3, 2$ **(e)** $x = 1, 3$ **(f)** $x = 3$

8 **(a)**

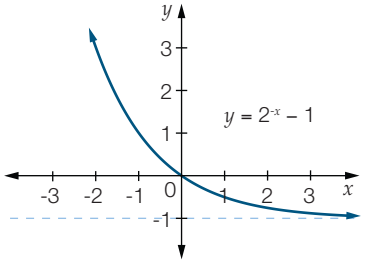
x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

(b)



(c) (0, 1) **(d)** $x = 0$

9 **(a)**



(b) $y = -1$ **(c)** (0, 0)

10 **(a)** rectangular hyperbola **(b)** $x = 3$ and $y = 4$

Exercise 9.4 (p. 344)

1 (a) (i) $(x+1-\sqrt{6})(x+1+\sqrt{6})$ (ii) $x=-1\pm\sqrt{6}$

(b) (i) $(x+2-\sqrt{11})(x+2+\sqrt{11})$ (ii) $x=-2\pm\sqrt{11}$

(c) (i) $(x+4-\sqrt{11})(x+4+\sqrt{11})$ (ii) $x=-4\pm\sqrt{11}$

(d) (i) $\left(x+\frac{3}{2}-\frac{\sqrt{13}}{2}\right)\left(x+\frac{3}{2}+\frac{\sqrt{13}}{2}\right)$ (ii) $x=\frac{-3\pm\sqrt{13}}{2}$

(e) (i) $\left(x+\frac{5}{2}-\frac{\sqrt{13}}{2}\right)\left(x+\frac{5}{2}+\frac{\sqrt{13}}{2}\right)$ (ii) $x=\frac{-5\pm\sqrt{13}}{2}$

(f) (i) $\left(x+\frac{3}{2}-\frac{\sqrt{29}}{2}\right)\left(x+\frac{3}{2}+\frac{\sqrt{29}}{2}\right)$ (ii) $x=\frac{-3\pm\sqrt{29}}{2}$

2 (a) $x=4\pm\sqrt{10}$ **(b)** $x=-3\pm\sqrt{5}$ **(c)** $x=\frac{3\pm\sqrt{5}}{2}$

(d) $x=\frac{5\pm\sqrt{33}}{2}$ **(e)** $x=-2\pm\sqrt{7}$

(f) $x=10\pm\sqrt{112}$ (or $x=10\pm4\sqrt{7}$)

(g) $x=2\pm\sqrt{10}$ **(h)** $x=-2\pm\sqrt{3}$ **(i)** $-2\pm\sqrt{7}$

3 (a) C **(b)** B **(c)** D

4 (a) $x=2\pm\sqrt{6}$ **(b)** $x=3\pm\sqrt{7}$

(c) $x=2\pm\sqrt{7}$ **(d)** $x=\frac{3\pm\sqrt{29}}{2}$

(e) $x=\frac{5\pm\sqrt{17}}{2}$ **(f)** $x=\frac{-7\pm\sqrt{69}}{2}$

5 (a) x^2+4x+7
$=x^2+4x+4+3$
$=(x+2)^2+3$

This cannot be factorised.

(b) You get $(x+2)^2+3$, which is not the difference of two squares.

(c) The graph does not cut the x-axis.

6 (a) width $=-2+2\sqrt{21}$ and length $=2+2\sqrt{21}$

(b) width = 7.2 cm and length = 11.2 cm

7 $y=-2+\sqrt{28}\approx3.3$ cm

8 (a) $x=\frac{1+\sqrt{801}}{2}$ m **(b)** $x=14.65$ m

9 (a) length $=l+4$, width $=l+2$

(b) $(l+4)(l+2)=215$

(c) $l=6\sqrt{6}-3$, $w=6\sqrt{6}-5$

(d) 15.7×13.7 cm

Open-ended – Sample answers

10 Students' own equations.

11 Students' own equations that do not result in the difference of two squares.

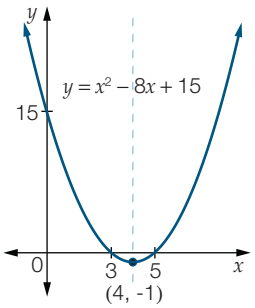
12 $x=-2\pm\sqrt{3}$, $x^2+4x+1=0$

Exercise 9.5 (p. 349)

1 (a) (i) parabola, min T.P. (ii) (0, 15)

(iii) (3, 0), (5, 0) **(iv)** (4, -1)

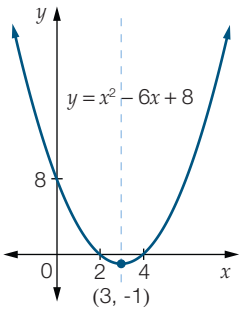
(v) $x=4$



(b) (i) parabola, min T.P. (ii) (0, 8)

(iii) (2, 0), (4, 0) **(iv)** (3, -1)

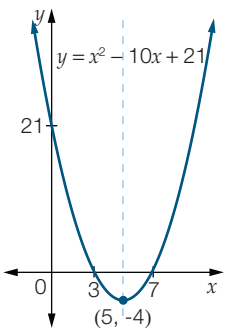
(v) $x=3$



(c) (i) parabola, min T.P. (ii) (0, 21)

(iii) (3, 0), (7, 0) **(iv)** (5, -4)

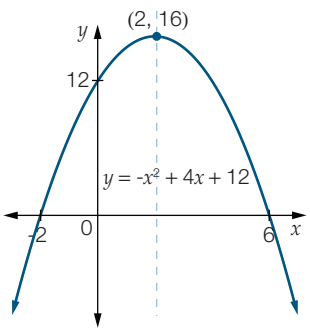
(v) $x=5$



(d) (i) parabola, max T.P. (ii) (0, 12)

(iii) (-2, 0), (6, 0) **(iv)** (2, 16)

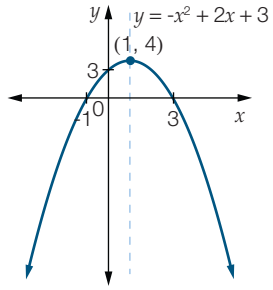
(v) $x=2$



(e) **(i)** parabola, max T.P. **(ii)** (0, 3)

(iii) (-1, 0), (3, 0) **(iv)** (1, 4)

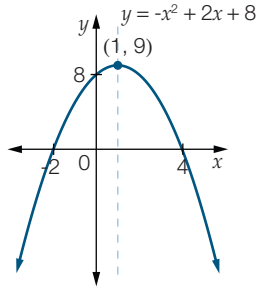
(v) $x = 1$



(f) **(i)** parabola, max T.P. **(ii)** (0, 8)

(iii) (-2, 0), (4, 0) **(iv)** (1, 9)

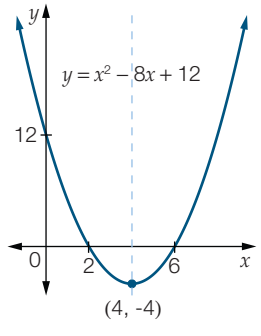
(v) $x = 1$



(g) **(i)** parabola, min T.P. **(ii)** (0, 12)

(iii) (2, 0), (6, 0) **(iv)** (4, -4)

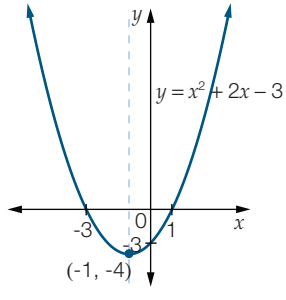
(v) $x = 4$



(h) **(i)** parabola, min T.P. **(ii)** (0, 3)

(iii) (-3, 0), (1, 0) **(iv)** (-1, -4)

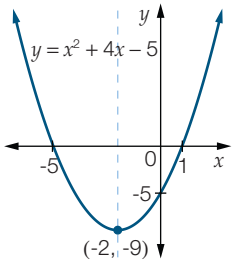
(v) $x = -1$



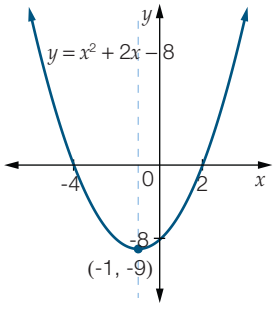
(i) **(i)** parabola, min T.P. **(ii)** (0, -5)

(iii) (-5, 0), (1, 0) **(iv)** (-2, -9)

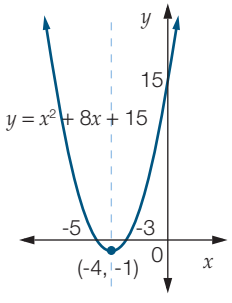
(v) $x = -2$



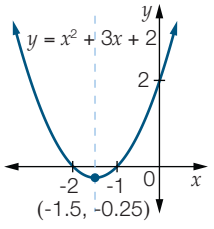
2 **(a)**



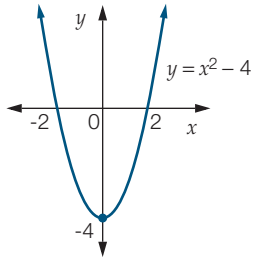
(b)



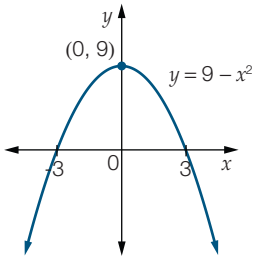
(c)



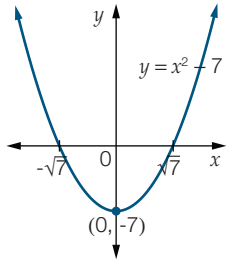
(d)



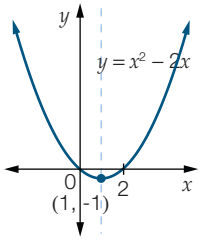
(e)



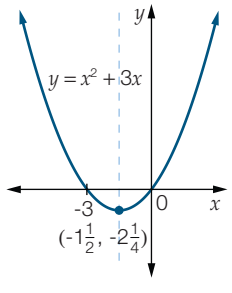
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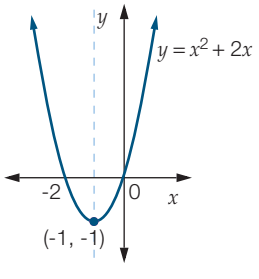
(g)



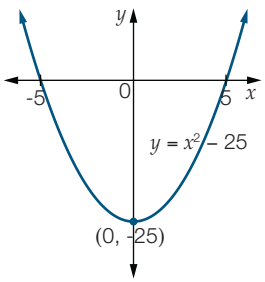
(h)



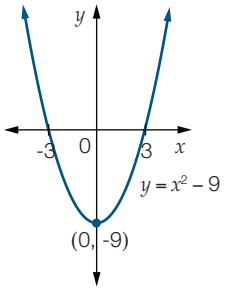
(i)



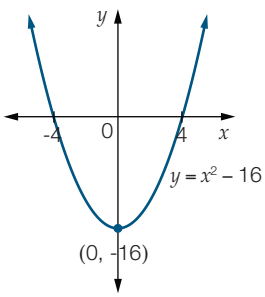
3 (a)



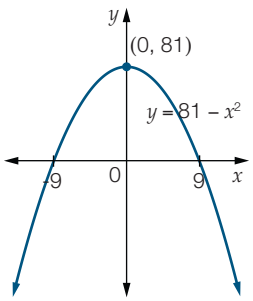
(b)



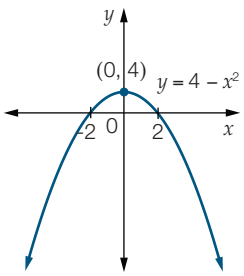
(c)



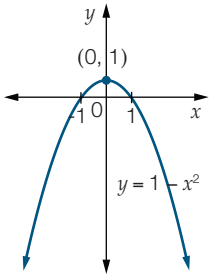
(d)



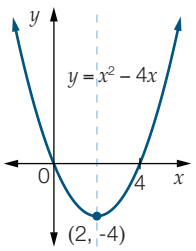
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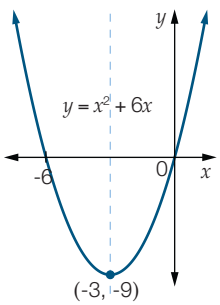
(f)



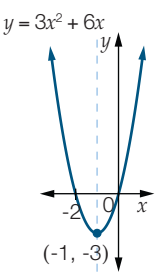
(g)



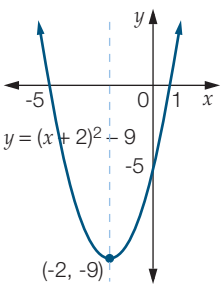
(h)



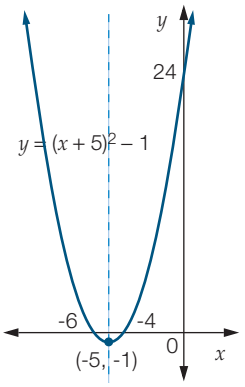
(i)



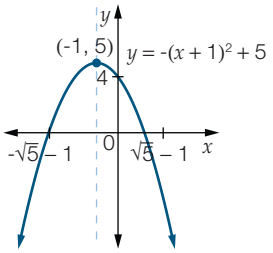
4 (a)



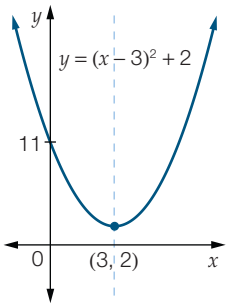
(b)



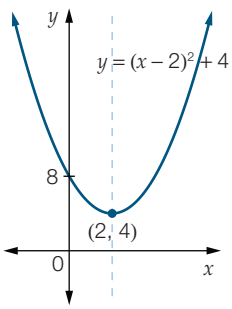
(c)



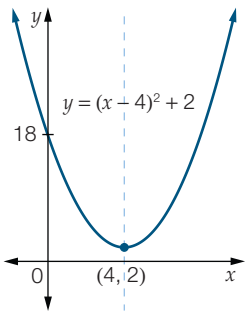
(d)



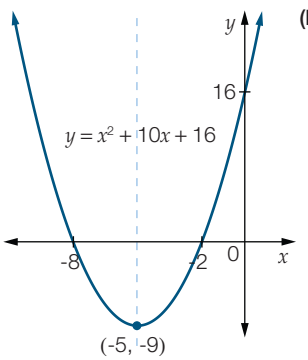
(e)



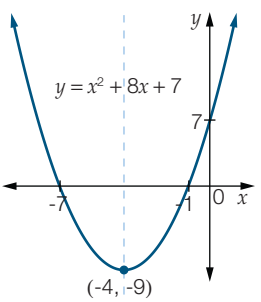
(f)



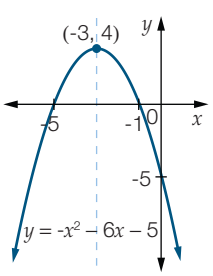
5 (a)



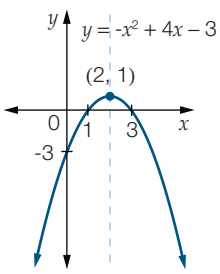
(b)



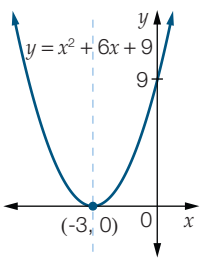
(c)



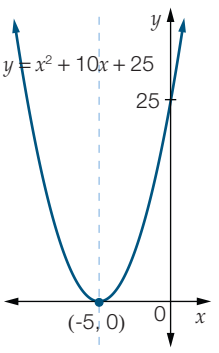
(d)



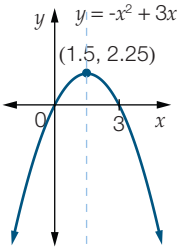
(e)



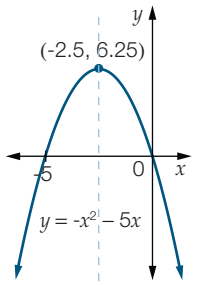
(f)



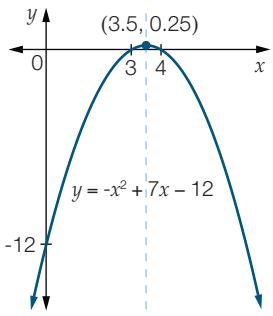
(g)



(h)



(i)

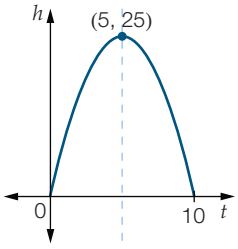


6 B

7 **(a)** D **(b)** B

8 **(a)** C **(b)** B

9

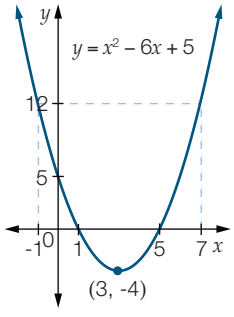


(a) 10 seconds **(b)** 25 m

10 **(a)** B **(b)** B

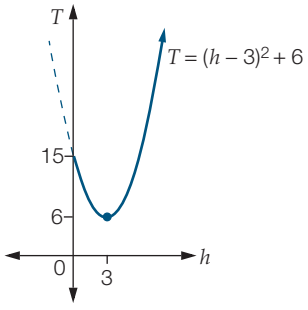
11 **(a)** D **(b)** (4, 2)

12 **(a)**



(b) $x = -1$ or $x = 7$

13 **(a)** $T = (h - 3)^2 + 6$



(b) 15°C **(c)** decreasing **(d)** increasing

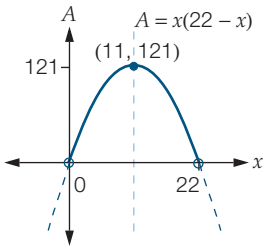
(e) 6°C **(f)** after 3 hours

(g) No, the temperature of the room is unlikely to increase, and certainly not at this rate. Using the equation, the temperature after 24 hours would be 447°C.

14 $y = (x + 3)(x - 5)$, $y = x^2 - 2x - 15$ or $y = (x - 1)^2 - 16$

15 **(a)** $A = x(22 - x)$

(b)



(c) $x > 0$ m **(d)** $x < 22$ m **(e)** 121 m^2

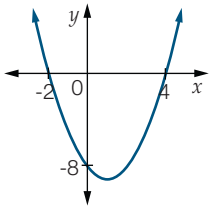
(f) square of length 11 m

Open-ended – Sample answers

16 If the equation is in factorised form then it will always have x-intercepts. If it is in turning point form, you can tell by inspection. If the TP is above the x-axis and it is a valley then no x-intercepts and if the TP is below the x-axis and it is a hill then no x-intercepts. If it is in standard form use completing the square to put it into turning point form and check it from there.

17 **(a)** The equation factorises to $y = (x - 4)(x + 2)$. Archie may have just found factors of 8 and ignored the sign. As it is negative one of the factors will be positive and the other negative. Archie needs to be more careful with his factorising.

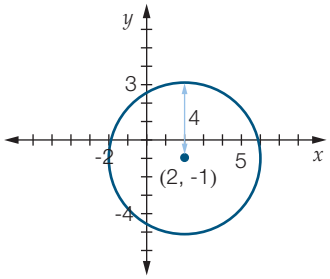
(b)



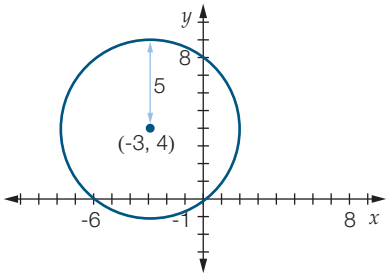
18 Students need to produce their own equations but using factorised form will make it easier. As an example, $(x - 4)(x - 6)$ has x-intercepts at 4 and 6. So the x-coordinate of the turning point is $x = 5$.

Exercise 9.6 (p. 355)

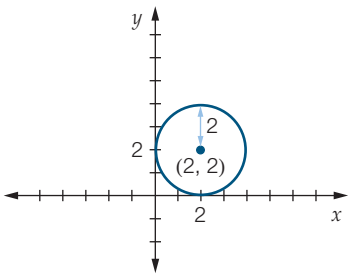
1 **(a)** $(x - 2)^2 + (y + 1)^2 = 16$



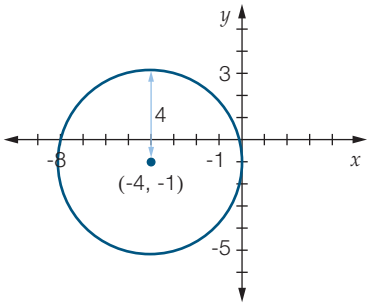
(b) $(x+3)^2+(y-4)^2=25$



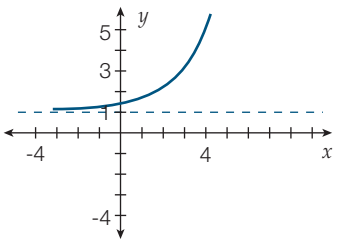
(c) $(x-2)^2+(y-2)^2=4$



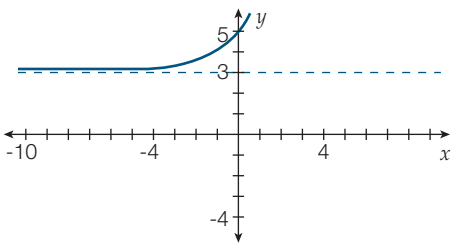
(d) $(x+4)^2+(y+1)^2=16$



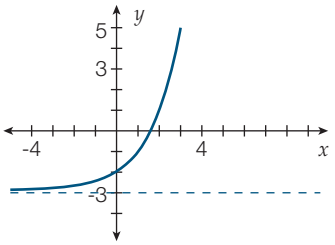
2 (a) $y=2^{x-2}+1$



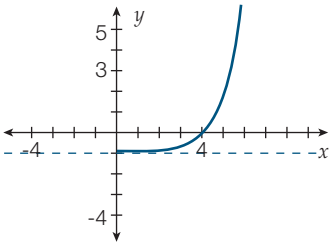
(b) $y=2^{x+1}+3$



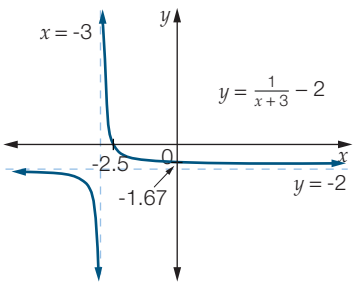
(c) $y=2^x-3$



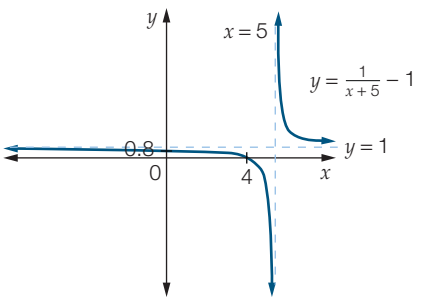
(d) $y=3^{x-4}-1$



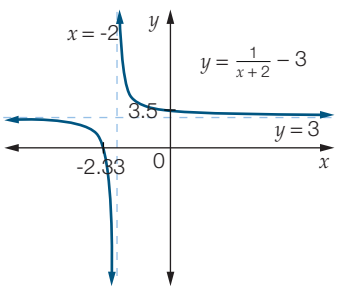
3 (a)



(b)



(c)

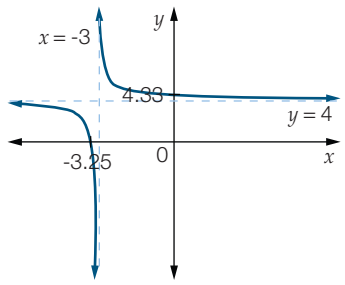


4 (a) $x^2+(y+1)^2=4$ **(b)** $(x+1)^2+(y+1)^2=9$

(c) $(x-2)^2+(y-3)^2=16$ **(d)** $(x+5)^2+(y+3)^2=4$

5 (a) $y = \dfrac{1}{x+3} + 4$

(b)



6 C

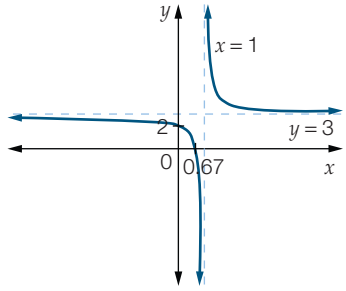
7 (a) $(x-1)^2 + (y+7)^2 = 25$

(b) $(x-3)^2 + (y+4)^2 = 64$

8 (a) $\dfrac{1}{x-1} + 3 = \dfrac{1}{x-1} + \dfrac{3}{x-1} = \dfrac{1+3(x-1)}{x-1} = \dfrac{1+3x-3}{x-1}$

$= \dfrac{3x-2}{x-1} = \text{RHS}$

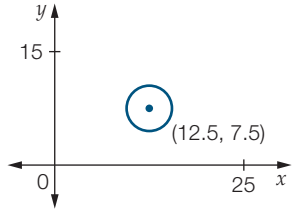
(b)



9 (a) (12.5, 7.5)

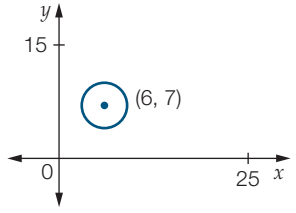
(b) $(x-12.5)^2 + (y-7.5)^2 = 9$

(c)



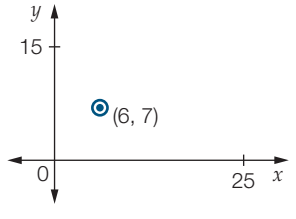
(d) (i) $(x-6)^2 + (y-7)^2 = 9$

(ii)



(e) (i) $(x-6)^2 + (y-7)^2 = 4$

(ii)



Open-ended – Sample answers

10 (a) Something in the form $y = \dfrac{1}{x-h} + k$.

(b) $x = h$ and $y = k$

(c) Students to sketch their own graph.

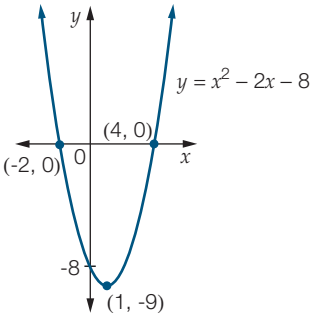
11 As an example $(x-2)^2 + (y-1)^2 = 1$

Chapter review 9 (p. 357)

1 (a) exponential; $y = 3$; asymptotes $x = 2$ and $y = 3$

(b) circle; centre (-1, 3) and $r = 3$

2



(a) x-intercepts: (-2, 0) and (4, 0); y-intercept: (0, -8)

(b) (1, -9) minimum (c) $x = 1$ (d) $y = -5$

(e) $x = 5$ and -3 (f) 5 and -3 (g) -2 and 4

3 (a) A (b) A (c) B

4 (a) $x = 0$ or 6 (b) $x = \pm\dfrac{9}{2}$ (c) $x = -2$ or 7

(d) $x = -3$ (e) $x = 0$ and 8 (f) $x = 6$

(g) $x = 0$ and -4 (h) $x = -1$ and 7 (i) $x = \pm 4$

5 (a) $x = -7$ and -5 (b) $x = -3$ and 2 (c) $x = 1$ and -9

(d) $x = 1$ and 8 (e) $x = 1$ and -4 (f) $x = 3$ and 1

6 (a) $x = -1$ and 5 (b) $x = -1$ and 6 (c) $x = -4$ and -3

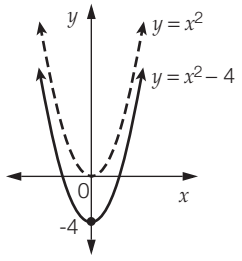
(d) $x = \pm 3$ (e) $x = -8$ and 4 (f) $x = -3$ and 4

7 (a) $x = 2 \pm \sqrt{2}$ (b) $x = -4 \pm \sqrt{7}$ (c) $x = -1 \pm \sqrt{5}$

8 (a) (i) Move 4 units down. (ii) (0, -4)

(iii) (0, -4), (-2, 0), (2, 0)

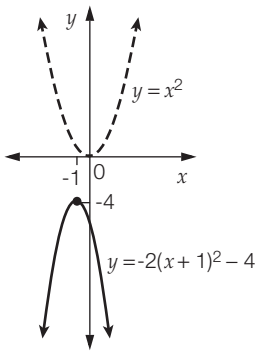
(iv)



(b) (i) Reflect in x-axis, dilate by a factor of 2 (make narrower) and translate 1 unit left and 4 units down.

(ii) (-1, -4) (iii) (0, -6)

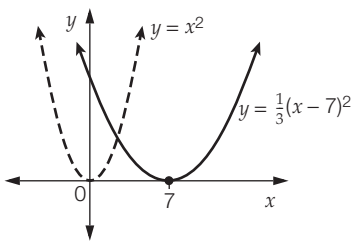
(iv)



(c) (i) Dilate by a factor of $\frac{1}{3}$ (make wider) and move 7 units right.

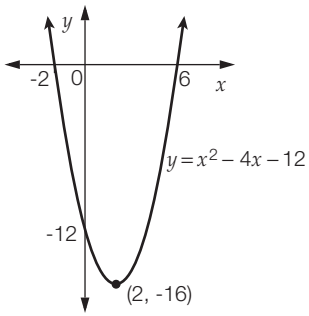
(ii) (7, 0) (iii) $(0, \frac{49}{3})$ (7, 0)

(iv)



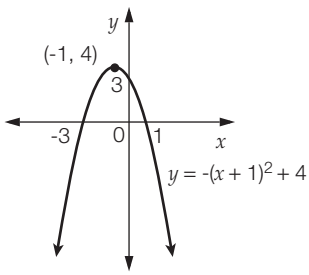
9 (a) (i) (0, -12) (ii) (-2, 0), (6, 0) (iii) (2, -16)

(iv)



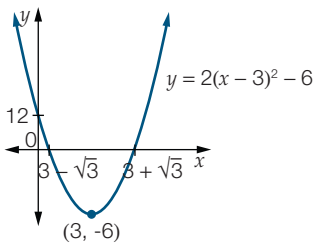
(b) (i) (0, 3) (ii) (-3, 0), (1, 0) (iii) (-1, 4)

(iv)



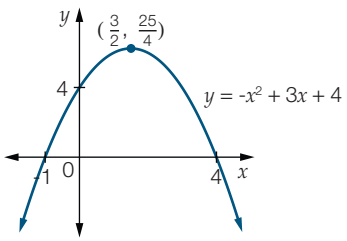
(c) (i) (0, 12) (ii) $(3 \pm \sqrt{3}, 0)$ (iii) (3, -6)

(iv)

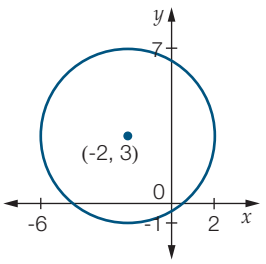


(d) (i) (0, 4) (ii) (-1, 0), (4, 0) (iii) $(\frac{3}{2}, \frac{25}{4})$

(iv)

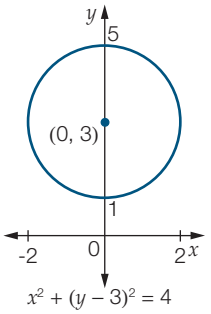


10 (a)



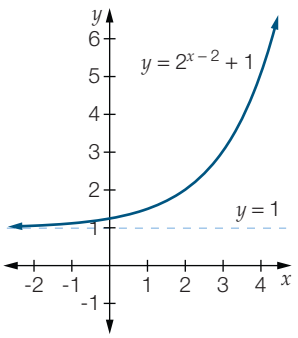
$(x + 2)^2 + (y - 3)^2 = 16$

(b)

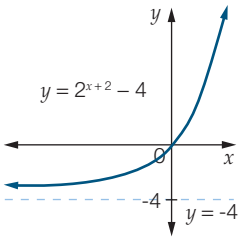


$x^2 + (y - 3)^2 = 4$

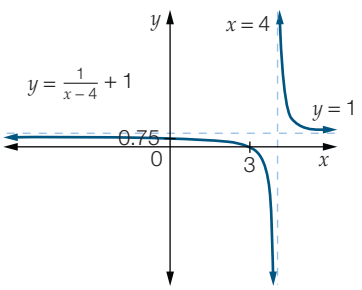
(c)



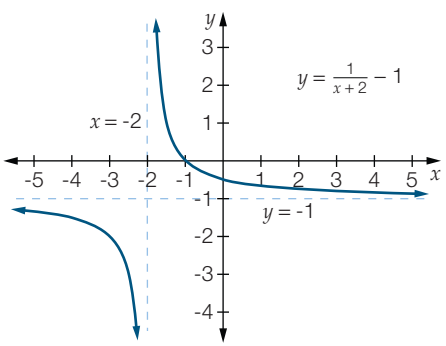
(d)



11 (a)



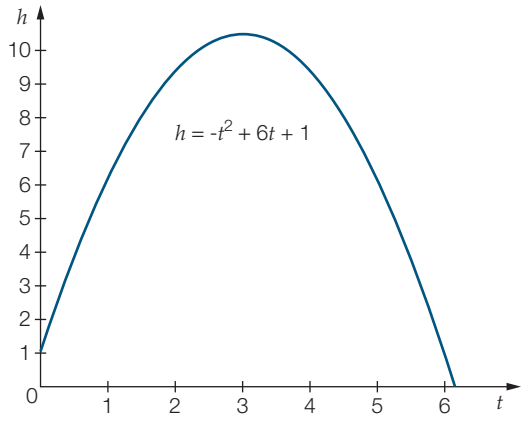
(b)



12 (a)

t	0	1	2	3	4	5	6	7
h	1	6	9	10	9	6	1	-6

(b)

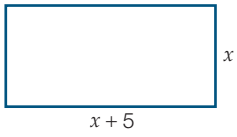


(c) 9 m **(d)** 1.27 seconds

(e) $2 < t < 4$ **(f)** 10 m

(g) 5.9 seconds

13 (a)



(b) $(x + 5)$ cm

(c) $x(x + 5)$ cm^2 **(d)** $x(x + 5) = 24$

(e) $x = -8$ or 3 **(f)** length 8 cm, width 3 cm

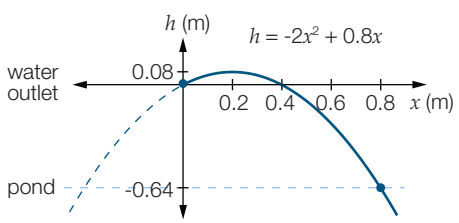
(g) $x = -8$ is not a feasible solution as a distance measurement must be positive.

14 (a) A horizontal distance of 0.4 m from the outlet.

(b) 0.64 m below the outlet

(c) 72 cm or 0.72 m

(d)



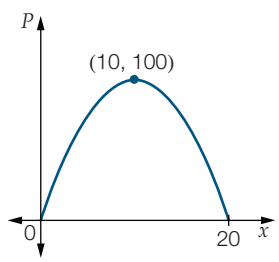
15 (a) 6 cm **(b)** 14 cm by 10 cm

(c) 36 cm^2 **(d)** 104 cm^2

(e) 64 cm^2

16 (a) $x + y = 20$ **(b)** $y = 20 - x$ **(c)** $x(20 - x) = P$

(d)



The maximum product is 100 and it is obtained by using $x = y = 10$.

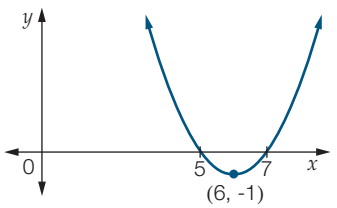
17 (a) 0.57 m

(b) The ball is 0.972 m high at the net, which is more than the height of the net (0.913 m).

(c) The ball lands 30 m from the fence, which is just outside the court.

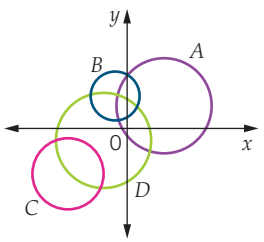
18 (a) (6, -1) **(b)** $y = (x - 6)^2 - 1$

(c)



19 (a) A and D

(b)



D intersects the other three graphs.

Mixed review C (p. 361)

1 \$20.23

2 (a) $(a - 9)(a + 4)$ **(b)** $(a - 6)(a - 2)$

(c) $(2a + 5)^2$

3 (a) 104 cm^2 **(b)** 37 m^2 **(c)** 103.5 cm^2

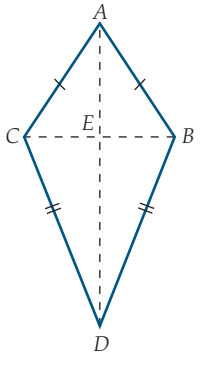
4 (a) -6 **(b)** $-\frac{15}{16}$ **(c)** $\frac{11}{6}$

5 (a) 42.7° **(b)** 23.9° **(c)** 50.5°

6 4.85 cm

7 (a) Range = 30, IQR ≈ 18 **(b)** Range = 30, IQR ≈ 13

8 The proof makes use of the following diagram, and its named vertices.

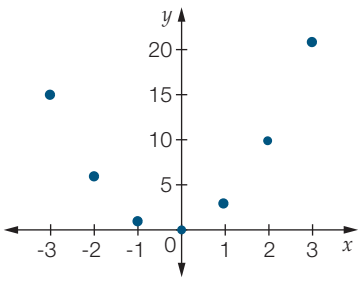


To show: $\Delta ACD \equiv \Delta ABD$
$AC = AB$ (given)
$CD = BD$ (given)
$AD = AD$ (shared)
So, $\Delta ACD \equiv \Delta ABD$
So, $\angle BAE = \angle CAE$
(matching angles in congruent triangles)
Now to show: $\Delta ABE \equiv \Delta ACE$
$\angle BAE = \angle CAE$ (from above)
$AC = AB$ (given)
$AE = AE$ (shared)
So, $\Delta ABE \equiv \Delta ACE$
So, $\angle AEB = \angle AEC$
(corresponding angles in congruent triangles)
But, $\angle AEB + \angle AEC = 180°$ (straight angle)
So, $\angle AEB = \angle AEC = 90°$
So, the diagonals of a kite intersect at right angles.

9 (a)

x	-3	-2	-1	0	1	2	3
y	15	6	1	0	3	10	21

(b)



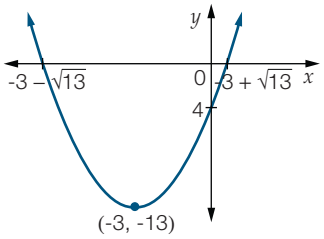
(c) (-0.25, -0.125)

(d) x-intercepts: -0.5 and 0; y-intercept: 0

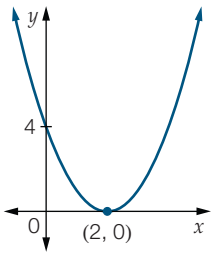
10 $x = -1$ and $y = -3$

11 (a) $x = -7$ and -6 **(b)** $x = \pm 10$ **(c)** $x = 7$

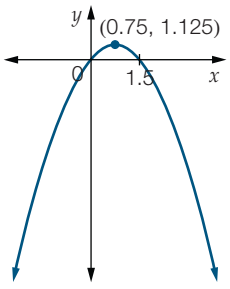
12 (a)



(b)



(c)



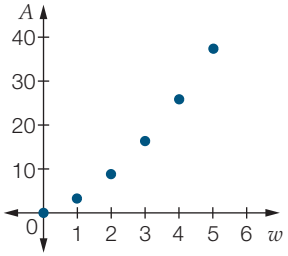
13 (a) $x = \pm 5$ **(b)** $x = 0$ and -0.5 **(c)** $x = 2$ and 3

14 (a) (2, 3) **(b)** (-4, -1) **(c)** (7, -6)

15 $(x-4)^2 + (y-2)^2 = 25$ **16** C **17** C

18 (a) $A = w(w + 2.5)$ cm^2

(b)



(c) $w = 5$ from the graph so the dimensions are 5 cm by 7.5 cm

19 (a) $x = 5$

(b) $y = (x-4)(x-6)$
$y = x^2 - 10x + 24$

Challenge C (p. 363)

1 C **2** B

3 ΔADC and ΔDBC are equal in area. They have the same height, the perpendicular distance from C to AB.

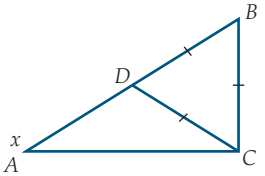
Hence their bases must be the same, so $AD = DB$.

ΔADC is isosceles.

ΔDBC is equilateral so $\angle BDC = 60°$

Hence, $\Delta ADC = 120°$ and $\angle DCA = \angle DAC = 30°$

$\therefore x = 150°$. (You can now say that ΔABC is right angled at C.)



4 B $AE = 25$, $AD = 24$, $ED = 7$, by Pythagoras' theorem

$AFD = 90°$ (angle in a semicircle)

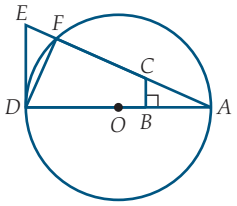
$AB = DF$, $BC = FE$ (matching sides of similar triangles)

$AC = ED = 7$, $CB = EF$

$FC = 25 - (EF + 7)$

$DB = 24 - AB$

Perimeter of $CBDF = DB + BC + CF + FD$
$= 24 - FD + EF + 25 - (EF + 7) + FD = 49 - 7 = 42$ units



5 45°

6 (a) $x^2 - 3x - 18 = -14$, $x^2 - 3x - 4 = 0$,
$(x-4)(x+1) = 0$. $x = -1, 4$

(b) $(x^2 - 3x)^2 - 3(x^2 - 3x) - 4 = 0$. $x^2 - 3x = -1$, $(x^2 - 3x) = 4$.

$x^2 - 3x + 1 = 0$, $x^2 - 3x - 4 = 0$. $x = \dfrac{3 \pm \sqrt{9-4}}{2}$,

$(x-4)(x+1) = 0$. $x = \dfrac{3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}, -1, 4$.

7 (a) $(x-7)(x+1) = 0$. $x = -1, 7$. Sum = 6.

(b) $(x-2)(x+2) = 0$. $x = -2, 2$. Product = -4.

(c) $x(x^2 - 6x + 5) = 0$. $x(x-1)(x-5) = 0$. $x = 0, 1, 5$. Average = 2

8 (a) $7 = (c-2)(c-8) + 7$
$(c-2)(c-8) = 0$
$c = 2$ or $c = 8$

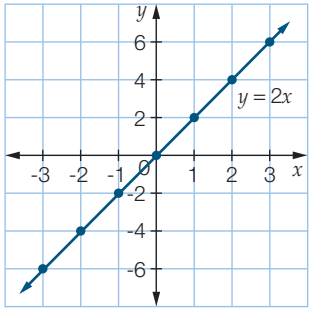
(b) vertex (5, -2)

9 (a) $y = -3, 2$ **(b)** $(-\sqrt{5}, 2)$, (0, -3), $(\sqrt{5}, 2)$

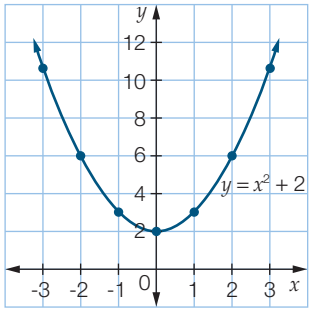
Chapter 10

Recall 10 (p. 364)

1 (a)



(b)

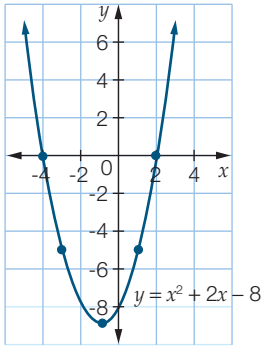


2 (a) neither **(b)** linear **(c)** quadratic

3 (a)

x	-5	-4	-3	-2	-1	0	1	2	3
y	7	0	-5	-8	-9	-8	-5	0	7

(b)



(c) **(i)** -8 **(ii)** -4, 2 **(iii)** (-1, -9)

(d) minimum **(e)** $x = -1$

4 (a) $(x-6)(x+1)$ **(b)** $(2x+9)(2x-9)$ **(c)** $3(x-3)(x+4)$

(d) $(x-7)^2$ **(e)** $-(x-10)(x+2)$

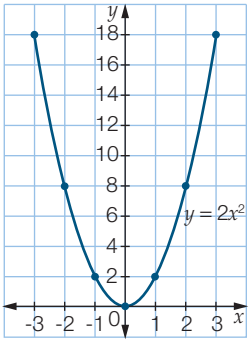
(f) $(x-3-2\sqrt{2})(x-3+2\sqrt{2})$

5 (a) $x = -2$ or 1 **(b)** $x = \pm 9$ **(c)** $x = 0$ or 2.5

Exercise 10.1 (p. 367)

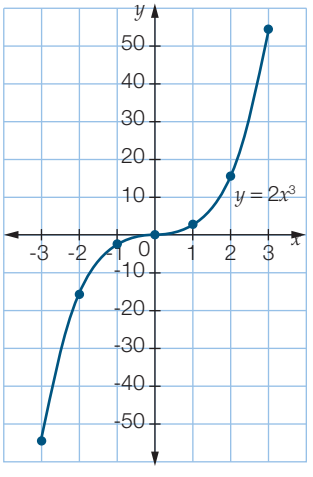
1 (a)

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18



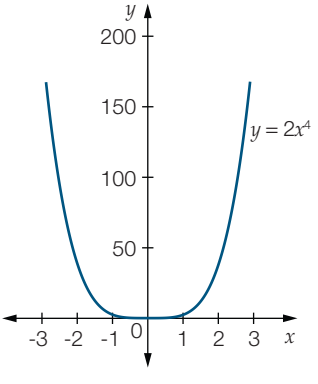
(b)

x	-3	-2	-1	0	1	2	3
y	-54	-16	-2	0	2	16	54



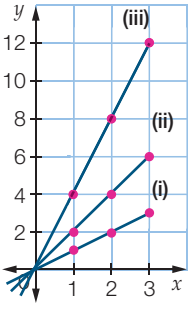
(c)

x	-3	-2	-1	0	1	2	3
y	162	32	2	0	2	32	162



2 (a) U **(b)** Z **(c)** W **(d)** Y **(e)** X **(f)** V

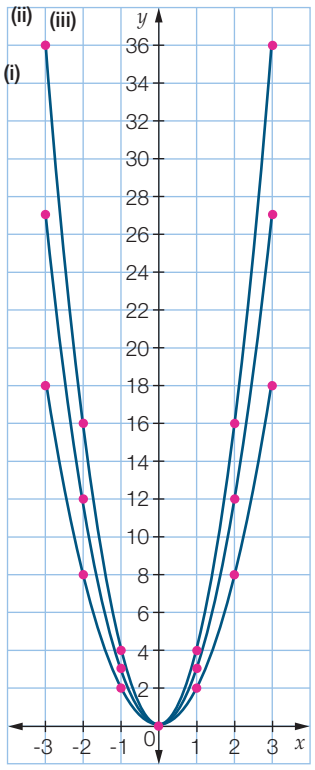
3 (a)



(b) The graph becomes steeper as the coefficient gets larger.

(c) No, graphs with a negative gradient become steeper as the coefficient of x decreases.

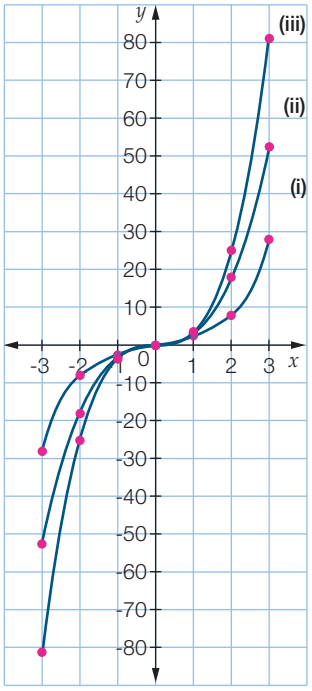
4 (a)



(b) The parabola becomes narrower about the y-axis as the coefficient increases.

(c) No, parabolas with a negative gradient become narrower as the coefficient of x decreases.

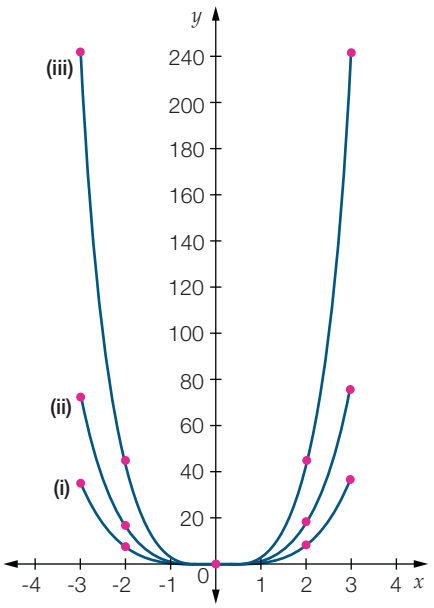
5 (a)



(b) The cubic gets steeper (gets closer to the y-axis) as the coefficient increases.

(c) No, cubics with a negative gradient become steeper as the coefficient of x decreases.

6 (a)

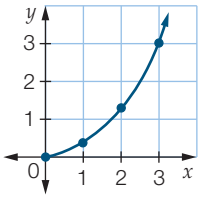


(b) The quartic becomes narrower about the y-axis as the coefficient increases.

(c) No, quartics with a negative gradient become narrower as the coefficient of x decreases.

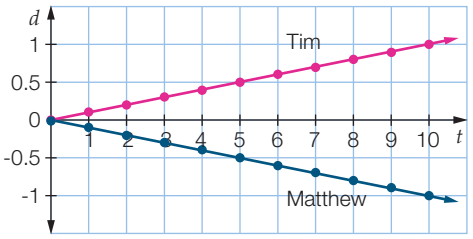
7 B **8** D

9 (a)



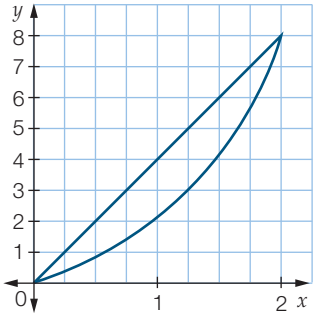
(b) $y = 0.75x$

10 (a)



(b) 2 km (c) $d = 0.12t$

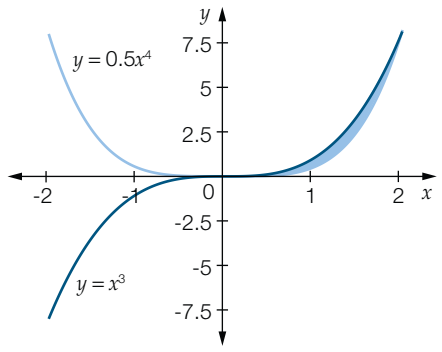
11 (a)



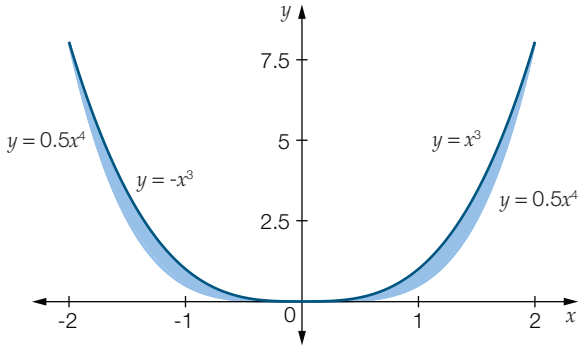
(b) $2\frac{2}{3}$ m^2

(c) $y = 4x$, $y = -2x^2$, $x = -2$ to 0

12 (a), (b)



(c)



(d) $y = -x^3$ for $-2 \le x \le 0$

Open-ended – Sample answers

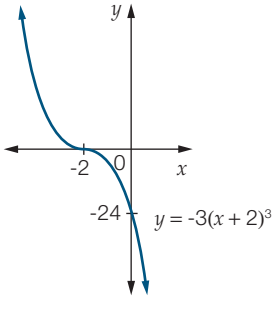
13 (a) $c = 10, 11$

(b) Gradient is undefined, the line is vertical.

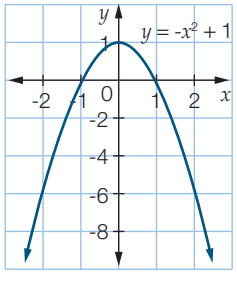
14 The graphs will intersect at $(-1, a)$ and $(1, a)$. This is true for both positive and negative values of a.

Exercise 10.2 (p. 371)

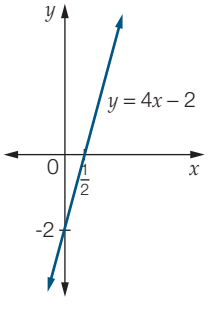
1



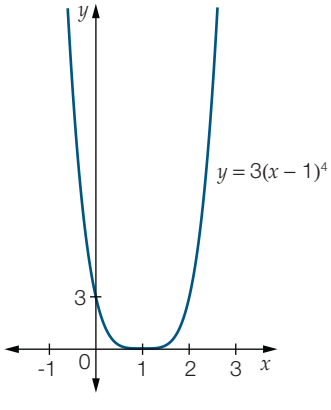
2



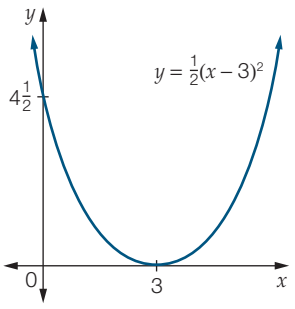
3



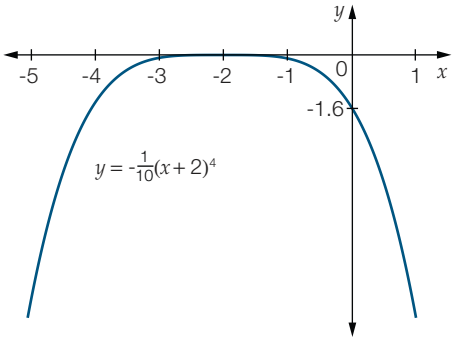
4



5



6



7 D **8** C **9** C

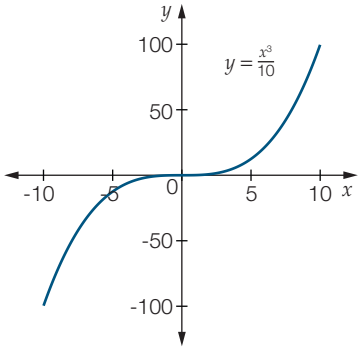
10 (a) $h = 1, k = 2$

(b) Negative, the turning point is a maximum.

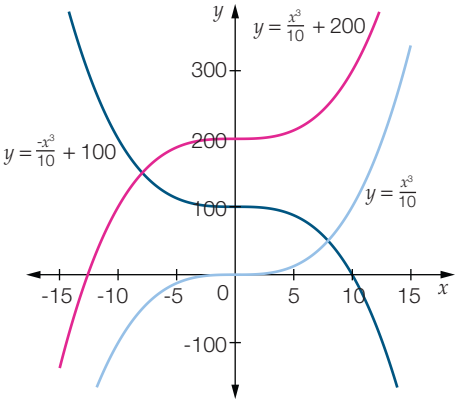
(c) $a = -3$

11 (a) $y = \dfrac{x^3}{10}$

(b)



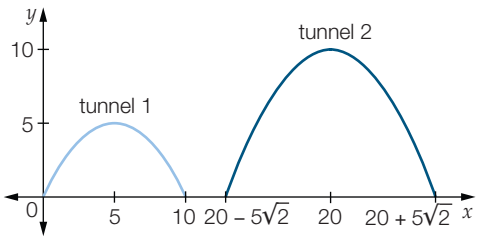
(c)



(d) The snake has moved up vertically 100 cm, the shape of the graph is exactly the same, but it is a mirror image of its initial shape. It is reflected in the line $y = 50$.

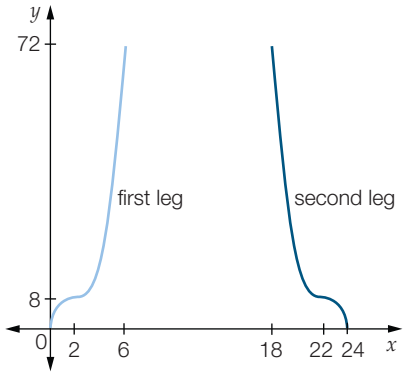
12 **(a)** $y = 10 - 0.2(x - 20)^2$

(b)



(c) 15.8 m

13 **(a), (c)**



(b) $y = -(x - 22)^3 + 8$ **(d)** 12 cm

Open-ended – Sample answers

14 $y = -3(x + 2)^3 + 4$, $y = -5(x + 1)^3 - 8$

Exercise 10.3 (p. 377)

1 **(a)** degree 3, leading coefficient 2, constant term 2

(b) degree 4, leading coefficient -4, constant term 5

(c) degree 1, leading coefficient 1, constant term 0

(d) degree 2, leading coefficient -1, constant term -6

2 **(a)** 62 **(b)** -142 **(c)** -28 **(d)** 8

3 **(a)** $5x^3 + 11x^2 - 13x - 3$ **(b)** $10x^4 - 3x^3 - 6x^2 + 29x + 6$

4 $2x^5 - 3x^4 - x^3 + 7x^2 + 15x - 20$

5 **(a)** $148\frac{15}{25}$ or $148\frac{3}{5}$ **(b)** $338\frac{5}{12}$

(c) $348\frac{8}{17}$ **(d)** $106\frac{4}{19}$

(e) $510\frac{7}{29}$ **(f)** $88\frac{122}{123}$

6 **(a)** quotient is $x + 3$, remainder is 1

(b) quotient is $x + 5$, remainder is 9

7 **(a)** quotient is $x^2 - 2x + 10$, remainder is -15

(b) quotient is $x^3 + 3x^2 + 6x + 18$, remainder is 55

8 $12x^3 - 55x^2 + 30x - 20$

9 **(a)** $x + 4$, 1 **(b)** $2x + 1$, 5

(c) $2x - 5$, 6 **(d)** $3x^2 + 6x + 14$, 24

10 A

11 $4x^2 + x - 3 = (4x - 3)(x + 1)$ so remainder is zero as it is a factor.

12 D **13** $x^3 - x^2 - 5x - 25 - \frac{120}{x - 5}$

14 **(a)** $R(x) = 16x^3 - 6x^2 + 4$; 108

(b) $R(x) = 3x^3 - 16x^2 - 17x + 22$; -152

(c) $R(x) = 4x^3 - 26x^2 - 28x + 36$; 34

15 **(a)** $x^4 - 3x - 6$ **(b)** 4

(c) $A(2) = 4$, $B(2) = 1$, $\frac{A(2)}{B(2)} = 4$, which is the same as the answer for **(b)**.

16 **(a)** $4x^3 + x + 8$

(b) **(i)** $4x^2 + 1 + \frac{8}{x}$ **(ii)** if $x = 0$, $\frac{8}{x}$ is undefined.

17 **(a)** $4x^2 - 10x + 6$

(b) $8x^3 - 30x^2 - 27x$

(c) **(i)** $8x - 22 - \frac{49}{x - 1}$

(ii) $x = 8$ cm, $x - 1 = 7$ cm $\Rightarrow$ 35 small rectangles

Open-ended – Sample answers

18 $2x^5 - x^4 - 3x^3 - 1$

19 $3x^3 - 14x^2 + 13x + 6$

20 **(a)** $1, x + \frac{1}{x + 1}, x^2 + 1, x^3 + x + \frac{1}{x + 1}, x^4 + x^2 + 1$

(b) If $P_n = x^n + x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1$ then for n odd, answers are $1, x^2 + 1, x^4 + x^2 + 1$, so $\frac{P_n(x)}{x + 1} = Q_{n-1}$, where $Q_{n-1}(x) = x^{n-1} + x^{n-3} + \ldots$ which are even powers of x when n odd.

(c) If $P_n = x^n + x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1$ then for n even, answers are $x + \frac{1}{x + 1}, x^3 + x + \frac{1}{x + 1}$, so $\frac{P_n(x)}{x + 1} = Q_{n-1} + \frac{1}{x + 1}$, where $Q_{n-1}(x) = x^{n-1} + x^{n-3} + \ldots$ which are odd powers of x when n is even.

Exercise 10.4 (p. 381)

1 **(a)** $(2x + 1)(x + 2)$ **(b)** $(2y + 7)(y + 1)$

(c) $(5b + 3)(b + 1)$ **(d)** $(3k - 2)(k - 3)$

(e) $(11a - 12)(a + 1)$ **(f)** $(5x + 4)(x - 3)$

(g) $(2m - 3)(m + 3)$ **(h)** $(7x - 9)(x - 2)$

(i) $(11c - 3)(c - 2)$ **(j)** $(5y + 4)(y + 3)$

(k) $(3p - 4)(p + 4)$ **(l)** $(2x - 5)(x + 4)$

(m) $(x + 1)(3x + 5)$ **(n)** $(y - 2)(7y - 4)$

(o) $(3t + 4)(t - 1)$ **(p)** $(p + 2)(7p + 8)$

(q) $(q - 1)(2q + 5)$ **(r)** $(x + 2)(5x - 6)$

(s) $(r - 2)(7r - 4)$ **(t)** $(c - 3)(3c - 7)$

(u) $2(2t - 1)(3t - 1)$

2 **(a)** $(2x+3)(2x+1)$ **(b)** $(5y+4)(2y+1)$

(c) $(9c-1)(c-2)$ **(d)** $(3w+4)(2w+3)$

(e) $(5y-6)(3y+5)$ **(f)** $(8x+7)(x-2)$

(g) $(2a+9)(2a-3)$ **(h)** $(6k-5)(4k+3)$

(i) $(10x-7)(2x-3)$ **(j)** $(6b+1)(2b+5)$

(k) $(3g+2)(2g+1)$ **(l)** $(4a+1)(2a+5)$

(m) $(4x-3)(x-2)$ **(n)** $(10m+7)(m+1)$

(o) $(4p-5)(3p+2)$ **(p)** $(3d-2)(3d+4)$

(q) $(6x+11)(x-2)$ **(r)** $(9n-7)(2n+5)$

(s) $(8y-3)(3y-1)$ **(t)** $(5x+2)(3x+8)$

(u) $(4x+3)(3x-5)$

3 **(a)** $2\left(x-\frac{3}{2}-\frac{\sqrt{11}}{2}\right)\left(x-\frac{3}{2}+\frac{\sqrt{11}}{2}\right)$

(b) $3\left(x-2-\sqrt{\frac{19}{3}}\right)\left(x-2+\sqrt{\frac{19}{3}}\right)$

(c) $2\left(x-2-\sqrt{\frac{11}{2}}\right)\left(x-2+\sqrt{\frac{11}{2}}\right)$

(d) $3\left(x-\frac{3}{2}-\sqrt{\frac{11}{12}}\right)\left(x-\frac{3}{2}+\sqrt{\frac{11}{12}}\right)$

(e) $5\left(x-1-\sqrt{\frac{2}{5}}\right)\left(x-1+\sqrt{\frac{2}{5}}\right)$

(f) $2\left(x-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)\left(x-\frac{7}{4}+\frac{\sqrt{33}}{4}\right)$

(g) not possible

(h) not possible

(i) not possible

4 **(a)** $(3x+1)(2x-3)$ **(b)** 3.5 m **(c)** 4 m

5 **(a)** $12t^2-12t+10-(4t^2+6t+1)$

(b) **(i)** $8t^2-18t+9$ **(ii)** $(4t-3)(2t-3)$

(c) The difference in the number of birds must be zero. $0=(2t-3)(4t-3)$. For this to be true either $2t-3=0$ ($t=1.5$) or $4t-3=0$ ($t=0.75$). Hence, when there are 75 or 150 trees there are equal numbers of birds.

6 **(a)** $(6x+5)(2x-3)$

(b) They each had \$10 initially.

(c) \$17

Open-ended – Sample answers

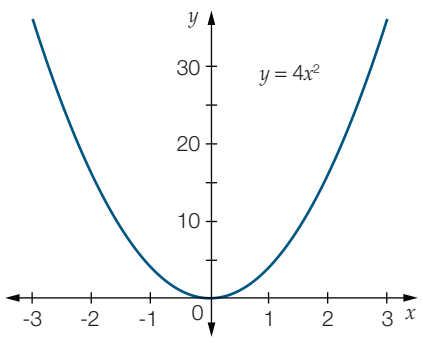
7 **(a)** $5x^2+10x=5x(x+2)$

(b) $60x^2+80x-15=5(6x-1)(2x+3)$

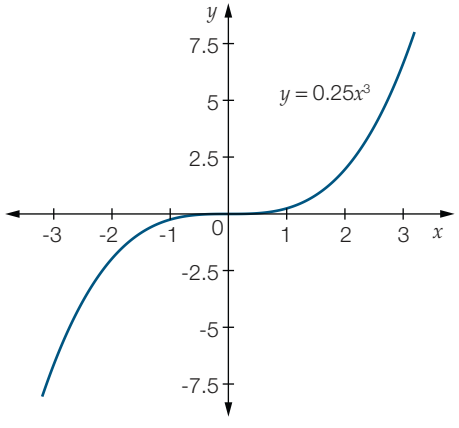
(c) $2x^2-4x+20$. This will not factorise completely as once you have factored out the 2 and completed the square, the last term is positive—not the difference of two squares.

Half-time 10 (p. 383)

1 **(a)**



(b)

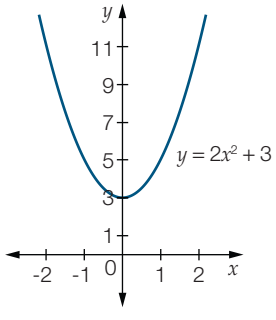


2 $x^2+x-7,\ 2$

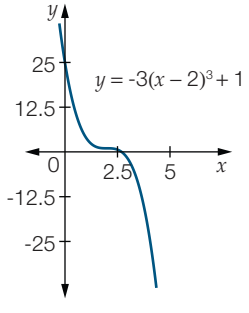
3 **(a)** **(i)** 8 **(ii)** 17 **(iii)** -42

(b) $9x^2-5$ **(c)** $9x^3-18x^2+11x-4$ **(d)** $2x+1$

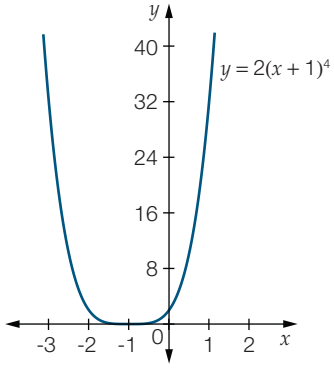
4 **(a)**



(b)



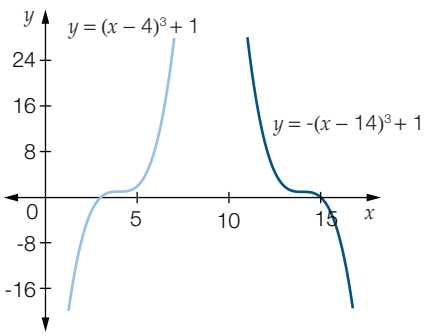
(c)



5 **(a)** 1 **(b)** $-\frac{47}{81}$

6 $2(x+2+\sqrt{\frac{5}{2}})(x+2-\sqrt{\frac{5}{2}})$

7 (a)



(b) $x = 9$

8 $\dfrac{x^3 + x - 5}{x - 1} = x^2 + x + 2 - \dfrac{3}{x - 1}$

9 (a) $(3x - 7)(2x + 5)$ **(b)** $(2x - 3)(5x - 1)$

(c) $(3x + 4)(4x + 1)$

10 (a) $(x - 1)(x^2 - 5x - 14)$ **(b)** $(x - 2)(x + 2)(x - 1)^2$

Exercise 10.5 (p. 386)

1 (a) $x = -\dfrac{2}{3}$ or 2 **(b)** $x = -\dfrac{2}{3}$ or -2

(c) $x = 1$ or 1.2 **(d)** $x = -2$ or 1.25

2 (a) $x = 1$ or 2.4 **(b)** $x = -1\dfrac{2}{3}$ or -1

(c) $x = -0.5$ or $\dfrac{2}{3}$ **(d)** $x = -1.5$ or 0.75

3 (a) $x = 0.5$ or 2 **(b)** $x = \dfrac{-5 \pm \sqrt{22}}{3}$

(c) $x = \dfrac{1 \pm \sqrt{31}}{5}$ **(d)** $x = \dfrac{-9 \pm \sqrt{129}}{12}$

4 (a) **(i)** $x = \dfrac{5 \pm \sqrt{73}}{6}$ **(ii)** $x = -0.59$ or 2.26

(b) **(i)** $x = \dfrac{-5 \pm 2\sqrt{5}}{5}$ **(ii)** $x = -1.89$ or -0.11

(c) **(i)** $x = \dfrac{11 \pm \sqrt{73}}{4}$ **(ii)** $x = 0.61$ or 4.89

(d) **(i)** $x = \dfrac{-5 \pm \sqrt{37}}{2}$ **(ii)** $x = -5.54$ or 0.54

5 (a) **(i)** $a = 1, b = 5, c = 2$ **(ii)** $x = \dfrac{-5 \pm \sqrt{17}}{2}$

(b) **(i)** $a = 2, b = -3, c = 1$ **(ii)** $x = \dfrac{1}{2}, 1$

(c) **(i)** $a = -3, b = -1, c = 3$ **(ii)** $x = \dfrac{-1 \pm \sqrt{37}}{6}$

(d) **(i)** $a = 5, b = 4, c = 0$ **(ii)** $x = -0.8, 0$

6 C

7 (a) $x = \dfrac{2}{3}$ or 1 **(b)** $x = \dfrac{-5 \pm \sqrt{15}}{5}$

(c) $x = -1.5$ or 1.5 **(d)** $x = \dfrac{-7 \pm \sqrt{85}}{2}$

(e) $x = -\dfrac{5}{6}$ or 2 **(f)** $x = -0.5$

(g) $x = \pm\sqrt{\dfrac{6}{7}}$ **(h)** $x = \dfrac{-3 \pm \sqrt{41}}{8}$

(i) $x = \dfrac{5}{6}$ **(j)** $x = \dfrac{11 \pm \sqrt{37}}{6}$

8 B **9** C

10 $4t^2 - 31t + 60 = 0$, $t = 3\dfrac{3}{4}$ or 4. After $3\dfrac{3}{4}$ min and 4 min.

11 (a) after 7 hours

(b) $2h^2 - 11h + 5 = 0$, After 30 mins and after 5 hours.

12 (a) 192 m **(b)** 6.2 seconds

13 (a) $x = \dfrac{3 \pm \sqrt{-7}}{4}$, no real solution

(b) $x = \dfrac{-5 \pm \sqrt{-15}}{10}$, no real solution

(c) $x = \dfrac{6 \pm \sqrt{-108}}{8}$, no real solution

(d) $x = \dfrac{-7 \pm \sqrt{-23}}{4}$, no real solution

14 (a) $3t^2 - 17t - 70 = 0$, $t = \dfrac{17 + \sqrt{1129}}{6} = 8.4$ min

(b) Try to solve $-3t^2 + 17t + 70 = 100$, $3t^2 - 17t + 30 = 0$,

$t = \dfrac{17 \pm \sqrt{-71}}{6}$ (no real solutions). Driver does not reach 100 km/h speed limit.

15 (a) Let kennel be x m wide, hence $2x$ m long.
Total painted area = $4x^2 + 6x - 0.5$

(b) $4x^2 + 6x - 0.5 = 15.7$, $8x^2 + 12x - 1 = 31.4$,

$20x^2 + 30x - 81 = 0$, $x = \dfrac{-30 + \sqrt{7380}}{40} \approx 1.4$.

width 1.4 m, length 2.8 m, height 1 m

16 (a) Let x = side length (cm) of squares cut out from the corners.

(b) base length = $30 - 2x$

(c) base width = $25 - 2x$

(d) Tray 1: length of square = 3.7 cm

Tray 2: length of square = 4.3 cm

Tray 3: length of square = 5 cm

(e) Tray 1: 3.7 cm × 17.7 cm × 22.7 cm

Tray 2: 4.3 cm × 16.4 cm × 21.4 cm

Tray 3: 5 cm × 15 cm × 20 cm

17 $\frac{x}{2}(x+7) = 60$, $x^2 + 7x - 120 = 0$, $x = 8$.

hypotenuse = $\sqrt{8^2 + 15^2} = 17$ cm.
perimeter = 40 cm

18 $3n^2 + n - 1610 = 0$, $(3n + 70)(n - 23) = 0$, $n = 23$ terms

19 **(a)** $x \neq 0$; $2x^2 - x - 21 = 0$. $x = -3$ or 3.5

(b) $x \neq 0$; $x^2 - 9x + 14 = 0$, $x = 2$ or 7

(c) $x \neq 5$; $x^2 - 6x + 9 = 0$, $x = 3$

(d) $x \neq \pm 1$; $x^2 - 4x - 3 = 0$, $x = 2 \pm \sqrt{7}$

(e) $x \neq -2, 0$; $x^2 - 6x - 6 = 0$, $x = 3 \pm \sqrt{15}$

(f) $x \neq 0$; $6x^2 - x - 3 = 0$, $x = \frac{1 \pm \sqrt{73}}{12}$

Open-ended – Sample answers

20 $2x^2 + x - 15 = 0$ and multiples of this equation

21 $a = 0$; $4x^2 - 5 = 0$
$a = 1$; $x^2 + x - 1 = 0$ etc.

22 **(a)** $x^2 + x + 1 = 0$

(b) $4x^2 + 20x + 25 = 0$, $x = -2.5$

(c) $12x^2 + 7x - 10 = 0$, $x = \frac{2}{3}$ or -1.25

Exercise 10.6 (p. 392)

1 $x^2 + 6x + 7 + \frac{17}{x-2}$, $P(2) = 17$. The remainder = $P(2)$.

2 **(a)** $P(2) = 1$ **(b)** $P(-3) = 104$

3 $(x-1)(x-2)(x-3)$

4 $(x-1)(x+1)(x+2)(x-3)$

5 **(a)** 44 **(b)** -23 **(c)** $-\frac{21}{16}$

(d) $-\frac{31}{32}$ **(e)** 550 **(f)** -140

6 **(a)** $(x-1)(x-2)(x+4)$ **(b)** $(x+1)(x-3)^2$

(c) $(x+1)(x-2)(x-5)$ **(d)** $(x-1)(x-2)(x+6)$

(e) $(x+2)^3$ **(f)** $(x+1)^2(x-5)$

7 **(a)** -86 **(b)** -5 **(c)** -12 **(d)** -31

8 **(a)** $(x-1)(x+1)(x+2)(x+4)$ **(b)** $x(x-1)(x-2)(x-4)$

(c) $(x-1)^2(x-2)(x+3)$ **(d)** $(x-1)^3(x+5)$

(e) $(x-1)(x+1)(x-2)^2$ **(f)** $(x-1)(x+2)(x-3)(x+4)$

9 B **10** D **11** -6

12 **(a)** $(x-1)(x-2)(x-3) = 0$, $x = 1, 2$ or 3

(b) $(x+1)(x-2)(x+2) = 0$, $x = -2, -1$ or 2

(c) $(x+2)(x+3)(x+4) = 0$, $x = -2, -3$ or -4

(d) $(x-1)(x+1)(x-2)(x+3) = 0$, $x = -1, -3, 1$ or 2

(e) $(x-1)(x+1)(x-2)(x+2) = 0$, $x = \pm 1, \pm 2$

(f) $x(x-1)(x-\sqrt{6})(x+\sqrt{6}) = 0$, $x = \pm\sqrt{6}, 0$ or 1

13 **(a)** $(x-1)^2(x-2)^2$ **(b)** 4 cm **(c)** 1 s, 2 s

(d) The top is moving away from the fixed point.

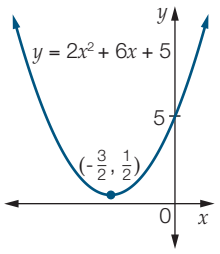
Open-ended – Sample answers

14 $x^3 - 3x^2 - x + 1$, $x^3 - 2x^2 - 3x - 2$

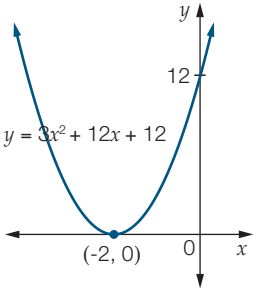
15 $x^4 - 4x^3 + x^2 + 6x = 0$ and multiples of the LHS.

Exercise 10.7 (p. 399)

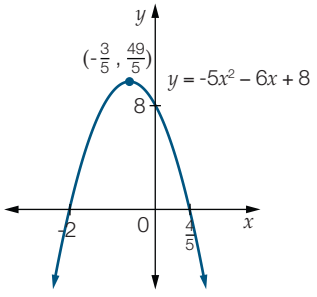
1 **(a)**



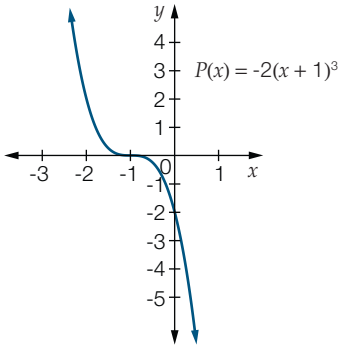
(b)



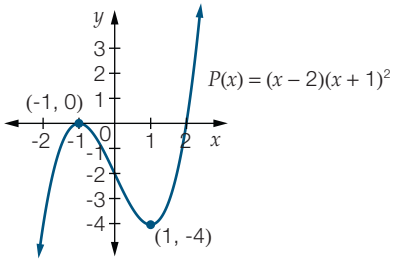
(c)



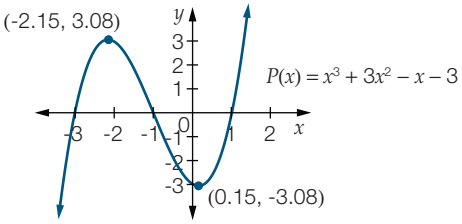
2 **(a)** (-1, 0)



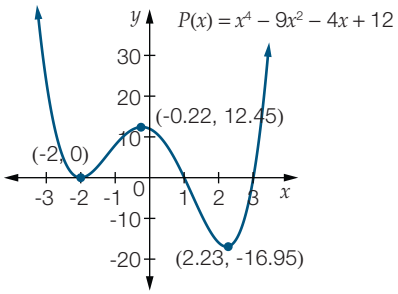
(b) (-1, 0) and (2,0)



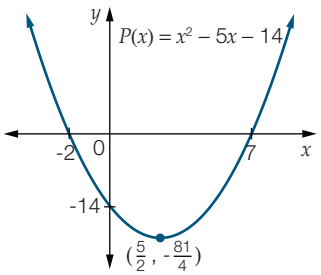
(c) (-3, 0), (-1, 0), (1, 0)



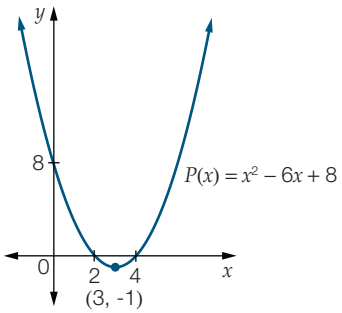
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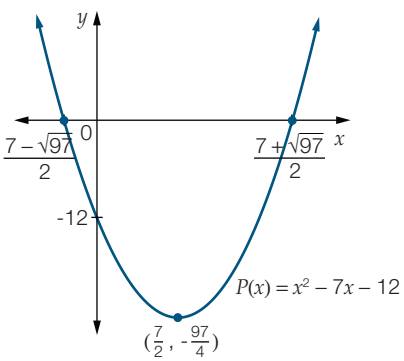
4 (a)



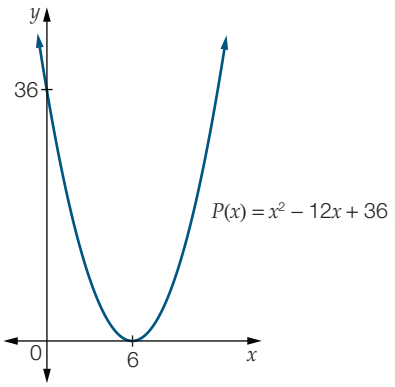
(b)



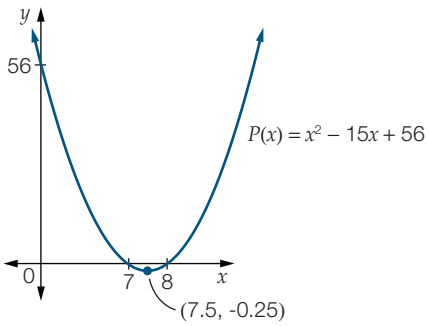
(c)



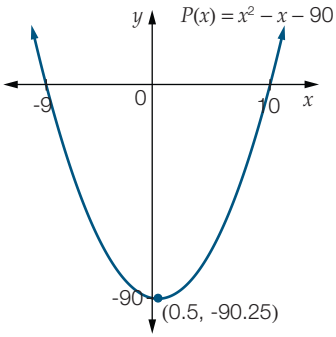
(d)



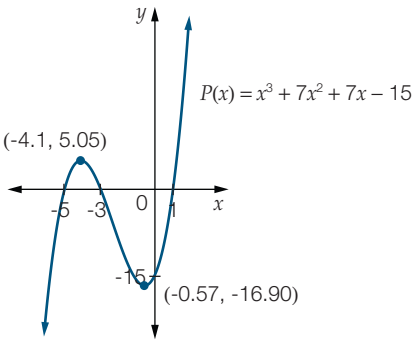
(e)



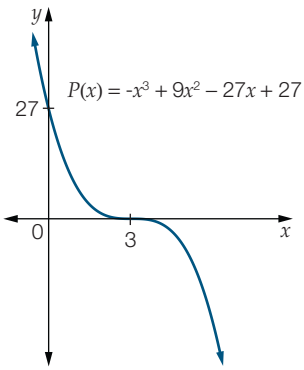
(f)



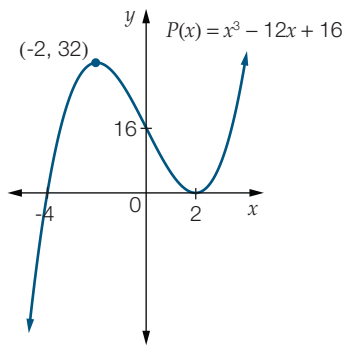
5 (a)



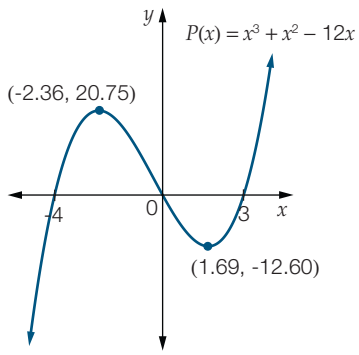
(b)



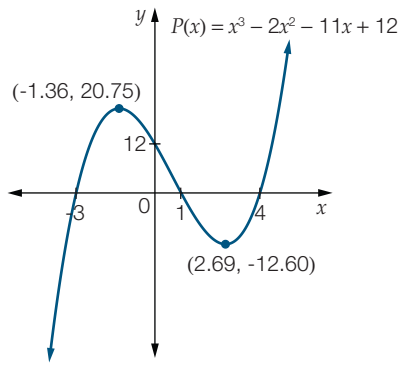
(c)


y
$P(x) = x^3 - 12x + 16$
(-2, 32)
16
-4
0
2
x

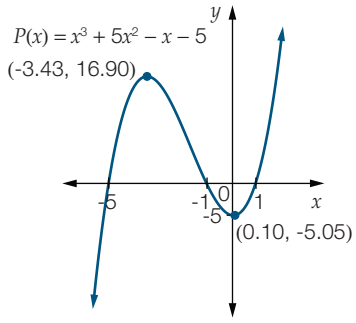
(d)


y
$P(x) = x^3 + x^2 - 12x$
(-2.36, 20.75)
-4
0
3
x
(1.69, -12.60)

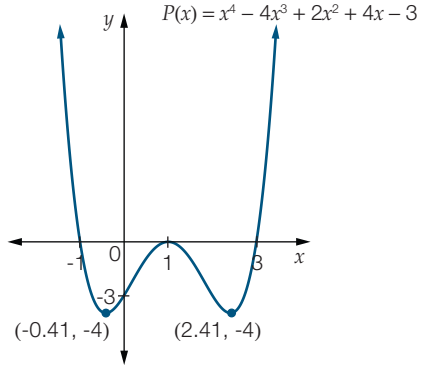
(e)


y
$P(x) = x^3 - 2x^2 - 11x + 12$
(-1.36, 20.75)
12
-3
0
1
4
x
(2.69, -12.60)

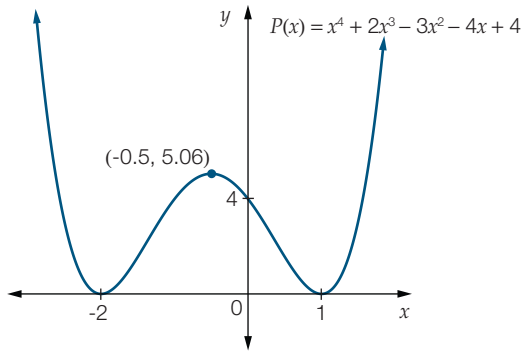
(f)


y
$P(x) = x^3 + 5x^2 - x - 5$
(-3.43, 16.90)
-5
-1
0
1
x
-5
(0.10, -5.05)

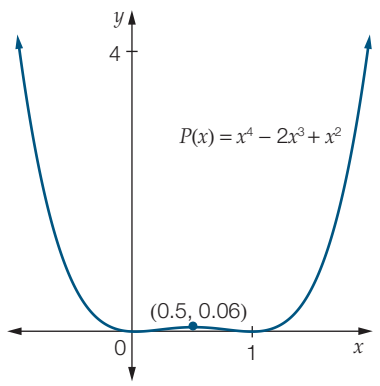
6 (a)


y
$P(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$
-1
0
1
3
x
-3
(-0.41, -4)
(2.41, -4)

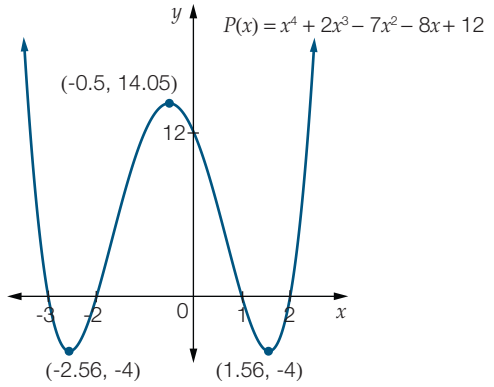
(b)


y
$P(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$
(-0.5, 5.06)
4
-2
0
1
x

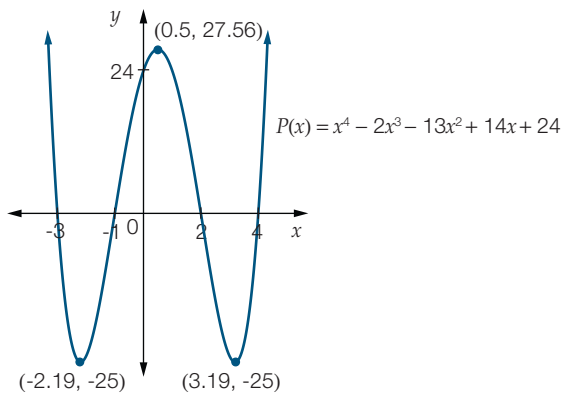
(c)


y
4
$P(x) = x^4 - 2x^3 + x^2$
(0.5, 0.06)
0
1
x

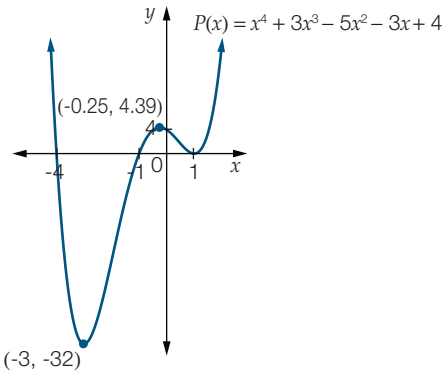
(d)


y
$P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$
(-0.5, 14.05)
12
-3
-2
0
1
2
x
(-2.56, -4)
(1.56, -4)

(e)

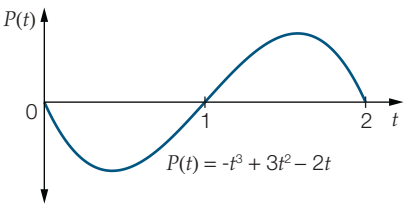

y
(0.5, 27.56)
24
$P(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$
-3
-1
0
2
4
x
(-2.19, -25)
(3.19, -25)

(f)



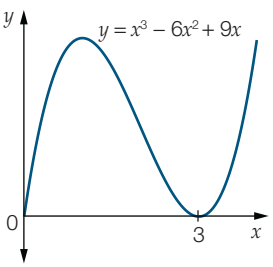
7 D **8** D **9** $x = 5, x = -7$ **10** $x = 1, x = -2$

11 **(a)** $P(t) = -t(t-1)(t-2)$



(b) 1 sec, 0.39 m **(c)** 1 sec, 0.39 m

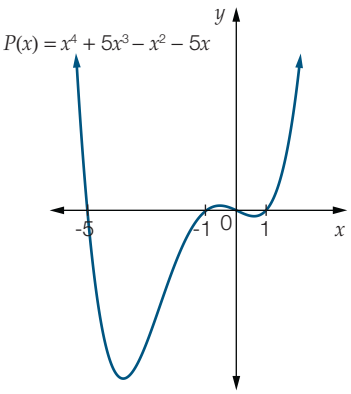
12 **(a)** $y = x(x-3)^2$



(b) (1, 4), (3, 0) **(c)** $y = -x(x-3)$

(d) Yes, as (2.5, 1.2) is not on the new parabola.

13 **(a)** $P(x) = x(x-1)(x+1)(x+5)$

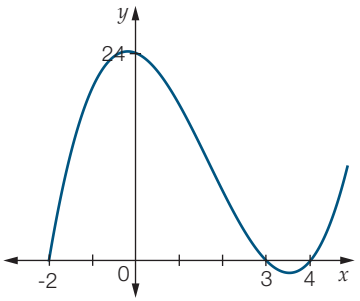


(b) negative pulse: $-5 < x < -1$ and $0 < x < 1$.
positive pulse: $-1 < x < 0$.

(c) **(i)** width 400 m, depth 61 m **(ii)** No

14 **(a)** $(x+2)(x-3)(x-4)$

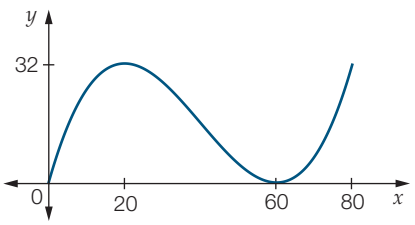
(b)



(c) 600 m

15 **(a)** $y = 0.001x(x-60)^2$

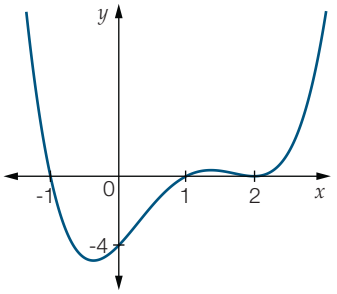
(b)



(c) 0.36 mins

Open-ended – Sample answers

16 $P(x) = (x+1)(x-2)(x-2)(x-1)$



17 **(a)** $P_1(x) = (x+1)^2(x-3)^2$, $P_2(x) = (x+1)(x-3)^3$

(b) $P_1(x) = (x+1)^2(x-3)(x-4)$, $P_2(x) = (x+1)(x-3)^2(x-5)$

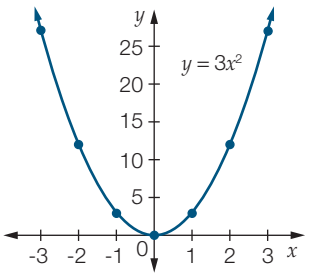
(c) $P_1(x) = x(x+1)(x-3)(x-4)$,
$P_2(x) = (x+1)(x-3)(x-5)(x+6)$

(d) $P_1(x) = 5(x+1)^3(x-3)$, $P_2(x) = 0.5(x+1)(x-3)^3$

Chapter review 10 (p. 402)

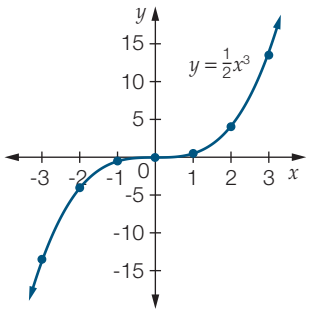
1 **(a)**

x	-3	-2	-1	0	1	2	3
y	27	12	3	0	3	12	27



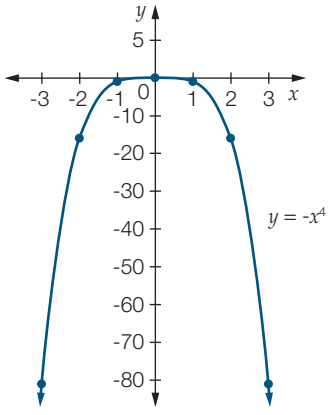
(b)

x	-3	-2	-1	0	1	2	3
y	-13.5	-4	-0.5	0	0.5	4	13.5

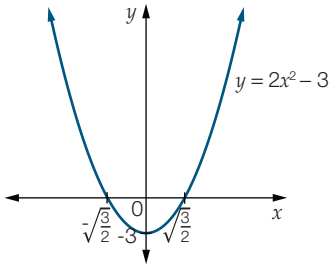


(c)

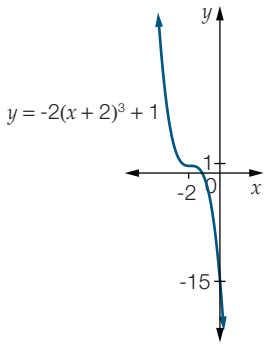
x	-3	-2	-1	0	1	2	3
y	-81	-16	-1	0	-1	-16	-81



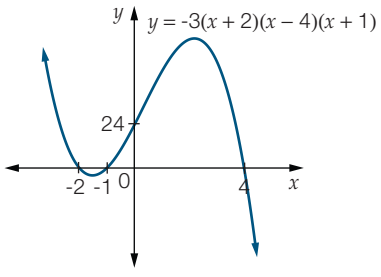
2 (a)



(b)



(c)



3 (a) (i) 13 (ii) 16 (iii) -42

(b) (i) $9x^2 + 5x$ (ii) $12x^3 - 13x^2 + 11x - 6$

(iii) $3x^2 - 5x + 5$

4 $x^2 + x + 6$, 1

5 (a) $(3x + 4)(4x - 5)$

(b) $(2x + 1)(6x - 5)$

(c) $(2x + 7)(3x - 2)$

(d) $3\left(x + \frac{5}{6} - \frac{\sqrt{13}}{6}\right)\left(x + \frac{5}{6} + \frac{\sqrt{13}}{6}\right)$

(e) $4\left(x - \frac{7}{8} - \frac{\sqrt{129}}{8}\right)\left(x + \frac{7}{8} + \frac{\sqrt{129}}{8}\right)$

(f) $2\left(x + \frac{3}{2} - \frac{\sqrt{3}}{2}\right)\left(x + \frac{3}{2} + \frac{\sqrt{3}}{2}\right)$

6 (a) $x = \pm 3$

(b) $(2x + 1)(x - 5) = 0$, $x = -0.5$ or 5

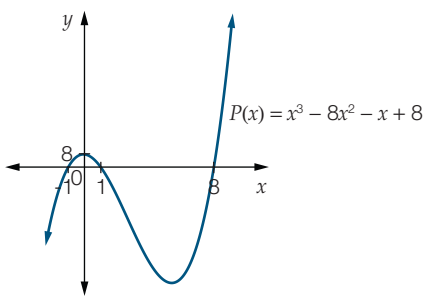
(c) $x = \frac{5 \pm \sqrt{3}}{2}$ (d) $x = \frac{-5 \pm \sqrt{17}}{4}$

(e) $x = \frac{6 \pm \sqrt{12}}{6} = \frac{3 \pm \sqrt{3}}{3}$ (f) $x = \frac{-7 \pm \sqrt{205}}{26}$

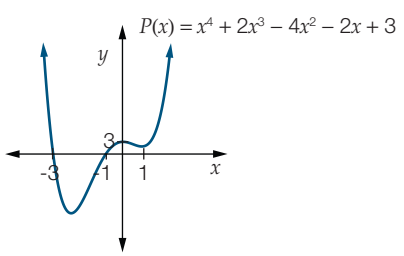
7 (a) -2 (b) $-\frac{5}{16}$

8 (a) $(x + 1)(x - 2)(x + 7)$ (b) $(x + 1)(x - 1)(x + 2)(x - 2)$

9 (a)

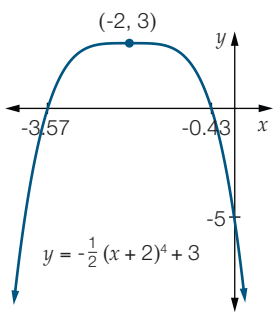


(b)



10 (a) $y = -\frac{1}{2}(x + 2)^4 + 3$

(b)



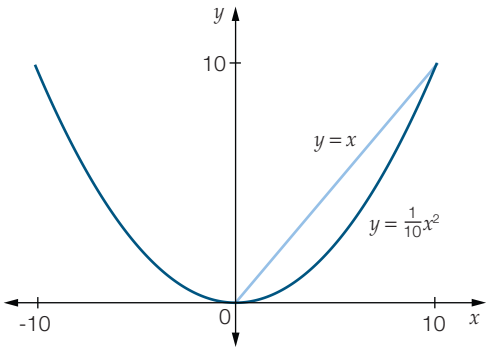
11 remainder = -7

12 $x \neq 0$; $40x^2 - 6x - 1 = 0$, $x = -0.1, 0.25$

13 $m^2 - m - 12 = 0$, $(m-4)(m+3) = 0$, $m = 4$ or -3, $x^2 = 4$, $x = \pm 2$

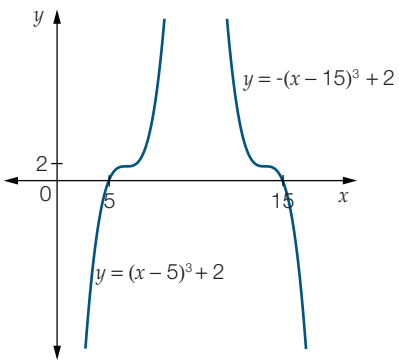
14 $\frac{n}{2}(10 + 4(n-1)) = 860$, $2n^2 + 3n - 860 = 0$, $(2n + 43)(n - 20) = 0$.
$n = 20$

15 (a)



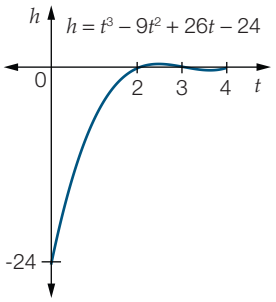
(b) 10 cm

16 (a)



(b) $x = 10$

17 (a) $h = (t-2)(t-3)(t-4)$



(b) The capsule is released at a depth of 24 m. It breaks the surface after 2 seconds and submerges 1 second later. It stays just below the surface for another second before breaking the surface again.

Chapter 11

Recall 11 (p. 404)

1 (a) $\frac{1}{5}$ **(b)** $\frac{4}{13}$

2 (a) $\frac{7}{8}$ **(b)** $\frac{11}{15}$ **(c)** $\frac{2}{5}$ **(d)** $\frac{3}{10}$ **(e)** 0.4 **(f)** 0.01

3 (a) boy, girl; boy, boy; girl, boy; girl, girl

(b) head, tail

4 (a) (i) 25 (ii) 25 (iii) 6 (iv) 9 (v) 15

(b) (i) 1, 2, 3, 4, 5, 6 (ii) 3

Exercise 11.1 (p. 409)

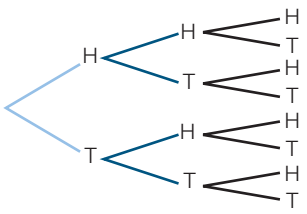
1 (a) $\frac{2}{7}$ **(b)** $\frac{5}{7}$

(c) We assume that the probability of the car being hard to start is the same every day. In essence, we are assuming that it is not dependent on the weather.

2 (a) $\frac{1}{8}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{1}{4}$ **(e)** $\frac{7}{8}$

3 (a) $\frac{1}{12}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{12}$ **(d)** $\frac{1}{6}$ **(e)** $\frac{1}{6}$ **(f)** $\frac{1}{2}$

4 (a)

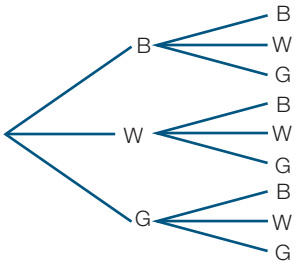


(b) (H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)

(c) (i) $\frac{1}{8}$ (ii) $\frac{1}{8}$ (iii) $\frac{1}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{1}{2}$

5 (a) $\frac{7}{12}$ **(b)** $\frac{5}{12}$ **(c)** $\frac{1}{24}$ **(d)** $\frac{1}{24}$ **(e)** $\frac{1}{12}$ **(f)** $\frac{11}{12}$

6 (a)



(b) Each sock has an equal probability of being selected from each drawer, so the outcomes for the new event are also equally likely.

(c) (i) $\frac{1}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{1}{3}$ (iv) $\frac{2}{3}$

7 (a) $\frac{1}{12}$ **(b)** $\frac{11}{12}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{5}{6}$

8 (a) B **(b)** D **(c)** C

9 (a) C **(b)** B **(c)** B

10 $\frac{4}{9}$

11 (a) A **(b)** C

12 $\frac{7}{9}$

13 (a) $\frac{1}{45}$ **(b)** $\frac{1}{5}$ **(c** $\frac{22}{45}$ **(d)** $\frac{23}{45}$ **(e)** 0 **(f)** $\frac{11}{45}$

14 (a)

Possible totals		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(b) 36

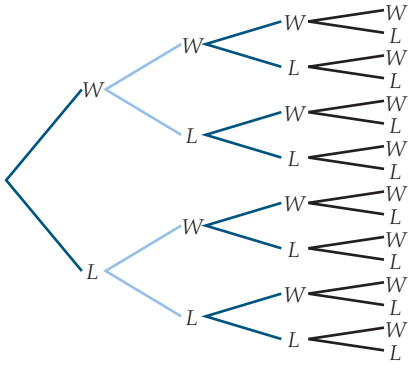
(c) Each die individually has an equal chance of producing 1, 2, 3, 4, 5, 6 so together the dice have an equal chance of producing each of the 36 possible outcomes.

(d) $\frac{1}{9}$ (e) $\frac{5}{12}$ (f) $\frac{1}{6}$

15 No, as some friends would be closer than others, so more likely to be contacted by Skye.

16 (a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$ (f) $\frac{2}{3}$

17 (a)



(b) (*W*, *W*, *W*, *W*), (*W*, *W*, *W*, *L*), (*W*, *W*, *L*, *W*), (*W*, *W*, *L*, *L*), (*W*, *L*, *W*, *W*), (*W*, *L*, *W*, *L*), (*W*, *L*, *L*, *W*), (*W*, *L*, *L*, *L*), (*L*, *W*, *W*, *W*), (*L*, *W*, *W*, *L*), (*L*, *W*, *L*, *W*), (*L*, *W*, *L*, *L*), (*L*, *L*, *W*, *W*), (*L*, *L*, *W*, *L*), (*L*, *L*, *L*, *W*), (*L*, *L*, *L*, *L*); 16 possible outcomes

(c) (i) $\frac{1}{16}$ (ii) $\frac{3}{8}$ (iii) $\frac{5}{16}$

(iv) $\frac{1}{2}$ (v) $\frac{1}{2}$ (vi) $\frac{1}{4}$

18 No, the existing cars in the court have no influence over what the new residents drive.

19 (a)

Sample space		Die 2					
		2	4	6	8	10	12
Die 1	1	(1, 2)	(1, 4)	(1, 6)	(1, 8)	(1, 10)	(1, 12)
	1	(1, 2)	(1, 4)	(1, 6)	(1, 8)	(1, 10)	(1, 12)
	2	(2, 2)	(2, 4)	(2, 6)	(2, 8)	(2, 10)	(2, 12)
	2	(2, 2)	(2, 4)	(2, 6)	(2, 8)	(2, 10)	(2, 12)
	3	(3, 2)	(3, 4)	(3, 6)	(3, 8)	(3, 10)	(3, 12)
	3	(3, 2)	(3, 4)	(3, 6)	(3, 8)	(3, 10)	(3, 12)

(b) Possible totals

		Die 2					
		2	4	6	8	10	12
Die 1	1	3	5	7	9	11	13
	1	3	5	7	9	11	13
	2	4	6	8	10	12	14
	2	4	6	8	10	12	14
	3	5	7	9	11	13	15
	3	5	7	9	11	13	15

(c) (i) $\frac{1}{3}$ (ii) $\frac{2}{9}$ (iii) $\frac{7}{18}$ (iv) $\frac{7}{18}$

(d) You cannot just look at one diagonal to find the totals; you have to check all over the lattice. However, it is not much more difficult in this case due to the nature of the numbers involved.

(e) Possible products

		Die 2					
		2	4	6	8	10	12
Die 1	1	2	4	6	8	10	12
	1	2	4	6	8	10	12
	2	4	8	12	16	20	24
	2	4	8	12	16	20	24
	3	6	12	18	24	30	36
	3	6	12	18	24	30	36

(f)

Product	2	4	6	8	10	12	16	18	20	24	30	36
Probability	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{18}$

20 (a) Even though the past long-term trend is $\frac{19}{20}$, short-term variation is possible. It is also possible that Aiden has switched jobs.

(b) The probability does not change. The balls do not have memories so they don't know if they have been drawn before.

Open-ended – Sample answers

21 (a) *A* is the event rolling a 1 or 2

(b) *B* is the event rolling a multiple of 3

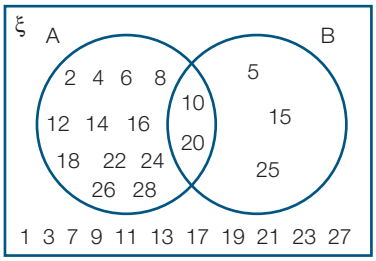
(c) *C* is the event rolling a number greater then 7

22 4W/1L, 8W/2L, 12W/3L

23 The correct answer is $\frac{7}{8}$. Sharanpreet has calculated the probability of exactly 1 boy, not at least 1 boy.

Exercise 11.2 (p. 415)

1 (a)



(b) (i) $\frac{1}{2}$ (ii) $\frac{1}{14}$ (iii) $\frac{23}{28}$

(c)

	B	not A
A	$\frac{1}{14}$	$\frac{3}{7}$
not A	$\frac{3}{28}$	$\frac{11}{28}$

2 (a) $\frac{3}{16}$ **(b)** $\frac{7}{16}$ **(c)** $\frac{1}{16}$ **(d)** $\frac{5}{16}$ **(e)** $\frac{1}{2}$ **(f)** $\frac{1}{4}$

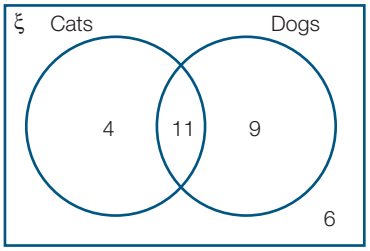
3 (a)

	Piano	not Piano	
Singing	4	2	**6**
not Singing	4	10	**14**
	8	**12**	**20**

(b) 4 take both lessons

(c) (i) $\frac{1}{10}$ (ii) $\frac{1}{5}$

4 (a)



(b) (i) $\frac{2}{15}$ (ii) $\frac{3}{10}$ (iii) $\frac{11}{30}$

5 (a)

	B	not B
A	$\frac{1}{8}$	$\frac{5}{16}$
not A	$\frac{1}{16}$	$\frac{1}{2}$

(b)

	C	not C
D	0.2	0.5
not D	0.2	0.1

6 (a) 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 20

(b) 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 18, 19, 20

(c) 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 19, 20

(d) 3, 7, 12, 17, 20

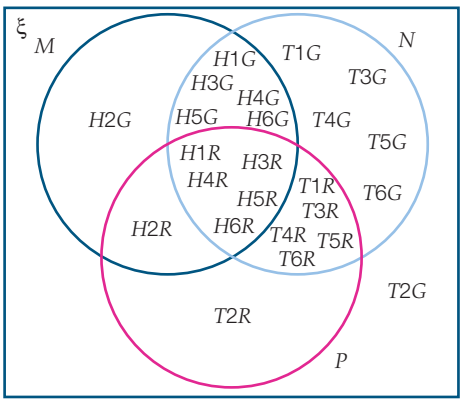
(e) 2, 7, 9, 12

(f) 5, 7, 12, 14

(g) 7, 12

(h) 6, 11, 16, 19

7 (a)



(b) not M = {T1R, T1G, T2R, T2G, T3R, T3G, T4R, T4G, T5R, T5G, T6R, T6G}

not N = {H2R, H2G, T2R, T2G}

not P = {H1G, H2G, H3G, H4G, H5G, H6G, T1G, T2G, T3G, T4G, T5G, T6G}

(c) No, because there are 4 red balls and only 3 green balls so there is a greater chance of H1R occurring than H1G.

8 (a)

	TV	not TV	
internet	236	1200	**1436**
not internet	554	10	**564**
	790	**1210**	**2000**

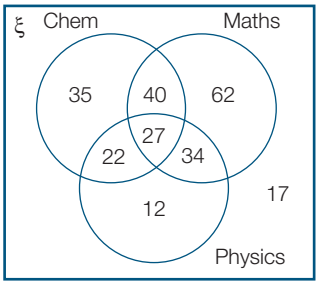
(b) $\frac{59}{500}$

9 (a)

	MySpace	not MySpace	
Facebook	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
not Facebook	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
	$\frac{3}{8}$	$\frac{5}{8}$	1

(b) (i) $\frac{1}{8}$ (ii) $\frac{5}{8}$

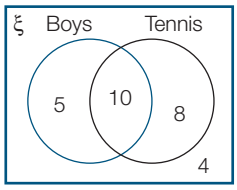
10 (a)



(b) 62 **(c)** 12 **(d)** 35 **(e)** 249

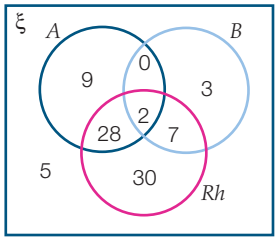
(f) $\frac{163}{249}$, it may be similar but it is unlikely to be exactly the same.

11 (a)



(b) (i) $\frac{8}{27}$ (ii) $\frac{5}{27}$ (iii) $\frac{2}{3}$ (iv) $\frac{1}{3}$

12 (a)



(b) 84 (c) $\frac{1}{2}$ (d) $\frac{5}{12}$

(e) A Positive: $\frac{1}{3} = 33.3\%$; A Negative: $\frac{3}{28} = 10.7\%$

B Positive: $\frac{1}{12} = 8.3\%$; B Negative: $\frac{1}{28} = 3.6\%$

AB positive: $\frac{1}{42} = 2.4\%$; AB Negative: 0%

O Positive: $\frac{5}{14} = 35.7\%$; O Negative: $\frac{5}{84} = 6\%$

(f) The A group is larger than expected and the O group is smaller than expected.

Open-ended – Sample answers

13 Mert has not taken into account students who may get their news from either the TV or the internet, but neither has taken into account there may be students who get their news from neither.

14 The existing figures add up to over 200 people because we are told both 36 and 44 people have never received a speeding fine.

Exercise 11.3 (p. 419)

1 (a) $\frac{1}{4}$ (b) $\frac{3}{8}$

2 (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

3 (a) yes (b) no (c) yes

4 (a) D (b) A (c) D

5 (a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$ (e) $\frac{17}{20}$

6 (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{11}{20}$ (e) $\frac{11}{20}$

(f) $\frac{9}{10}$ (g) $\frac{1}{4}$ (h) $\frac{3}{20}$

7 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$ (e) $\frac{27}{52}$

(f) $\frac{7}{26}$ (g) $\frac{1}{26}$ (h) $\frac{2}{13}$ (i) $\frac{9}{52}$ (j) $\frac{3}{26}$

(k) $\frac{11}{13}$ (l) $\frac{1}{4}$

8 (a) Six pairs: AB, AC, AD, BC, BD, CD

(b) AC and BC are mutually exclusive.

(c) $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$, $\Pr(C) = \frac{1}{3}$, $\Pr(D) = \frac{1}{3}$

(d) $\Pr(A \text{ or } B) = \frac{1}{2}$, $\Pr(A \text{ or } C) = \frac{5}{6}$, $\Pr(A \text{ or } D) = \frac{2}{3}$,

$\Pr(B \text{ or } C) = \frac{2}{3}$, $\Pr(B \text{ or } D) = \frac{1}{2}$, $\Pr(C \text{ or } D) = \frac{1}{2}$

(e) $\Pr(A \text{ and } B) = \frac{1}{3}$, $\Pr(A \text{ and } C) = 0$, $\Pr(A \text{ and } D) = \frac{1}{6}$,

$\Pr(B \text{ and } C) = 0$, $\Pr(B \text{ and } D) = \frac{1}{6}$, $\Pr(C \text{ and } D) = \frac{1}{6}$

(f) $\Pr(A) + \Pr(B) = \frac{5}{6}$

$\Pr(A) + \Pr(C) = \frac{5}{6}$

$\Pr(A) + \Pr(D) = \frac{5}{6}$

$\Pr(B) + \Pr(C) = \frac{2}{3}$

$\Pr(B) + \Pr(D) = \frac{2}{3}$

$\Pr(C) + \Pr(D) = \frac{2}{3}$

(g) In general, $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$. However, if A and B are mutually exclusive $\Pr(A \text{ and } B) = 0$, and so $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$.

9 (a) The vehicle types listed are not mutually exclusive, so there could be arguments as to where a particular vehicle should be placed; or a particular vehicle may be registered in more than one category, thereby making later calculations difficult.

(b) The categories have to be clearly mutually exclusive but also cover every type of vehicle that may go past. Including the category 'other' would cover the second requirement.

Open-ended – Sample answers

10 'multiple of 5' and 'multiple of 7'; 'multiple of 4' and 'multiple of 9'; 'even number' and 'odd number'; 'composite number' and 'prime number' are some of the pairs that could be listed

Half-time 11 (p. 422)

1 (a)

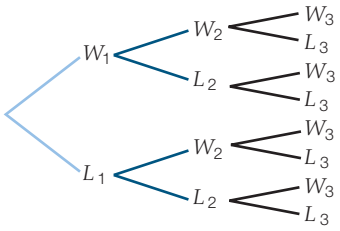
	iPhone	not iPhone	
iPad	60	15	**75**
not iPad	150	175	**325**
	210	**190**	**400**

(b) (i) $\frac{3}{8}$ (ii) $\frac{3}{80}$

2 (a) $\frac{2}{10} = \frac{1}{5}$ (b) $\frac{1}{2}$ (c) $\frac{6}{10} = \frac{3}{5}$ (d) $\frac{3}{10}$

3 0.09

4 (a) If W = Suzanne wins and L = Suzanne loses:



$W_1W_2W_3$, $W_1W_2L_3$, $W_1L_2W_3$, $W_1L_2L_3$, $L_1W_2W_3$, $L_1W_2L_3$, $L_1L_2W_3$, $L_1L_2L_3$

(b) 8

(c) (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{2}$

(iv) $\frac{1}{2}$ (v) $\frac{1}{2}$ (vi) $\frac{1}{4}$

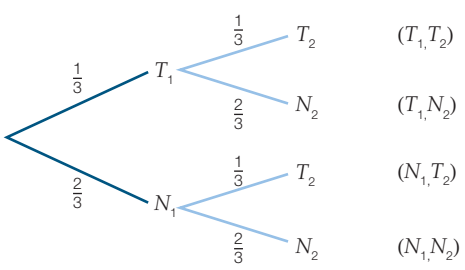
5 (a) A and B; A and C; B and C

(b) $\Pr(A) = \frac{1}{2}$; $\Pr(B) = \frac{1}{2}$; $\Pr(C) = \frac{1}{8}$

(c) A and C are mutually exclusive

Exercise 11.4 (p. 424)

1 (a)



Probability of each outcome

$Pr(T_1, T_2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

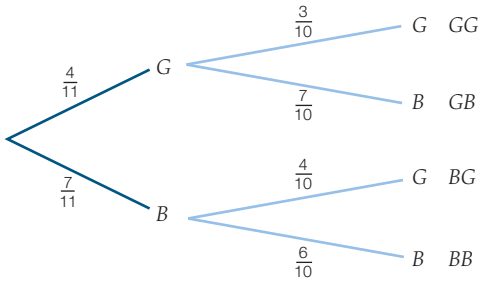
$Pr(T_1, N_2) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$Pr(N_1, T_2) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$Pr(N_1, N_2) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

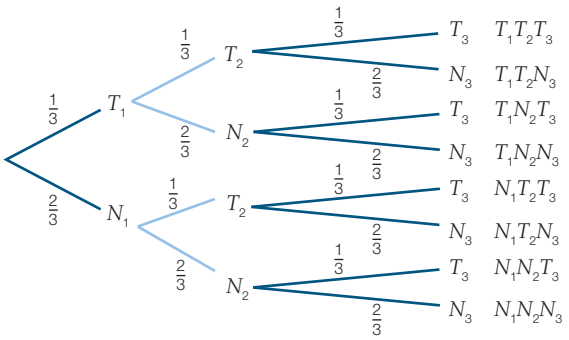
(b) (i) $\frac{1}{9}$ (ii) $\frac{4}{9}$ (iii) $\frac{4}{9}$ (iv) $\frac{5}{9}$

2



(a) $\frac{42}{110} = \frac{21}{55}$ **(b)** $\frac{98}{110} = \frac{49}{55}$

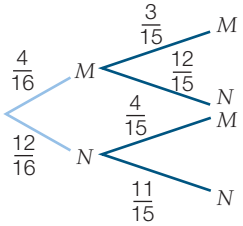
3 (a)



(b) $\frac{1}{27}$ **(c)** $\frac{8}{27}$

4 (a) B **(b)** D **(c)** B **(d)** C **(e)** B

5 (a)

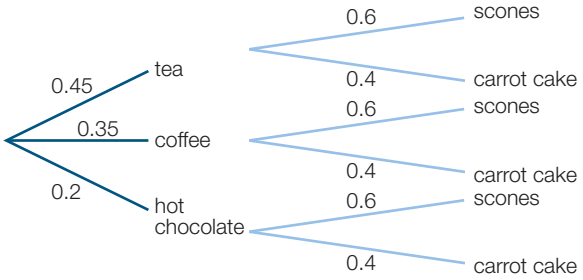


(b) MM, MN, NM, NN

(c) $\frac{1}{20}$ **(d)** $\frac{2}{5}$ **(e)** $\frac{11}{20}$

6 (a) 0.2 **(b)** 0.4

(c)



(d) (i) 0.27 (ii) 0.08 (iii) 0.35 (iv) 0.79

7 (a) (ii) $\Pr(G_1G_2) = 0.64$, $\Pr(G_1D_2) = 0.16$, $\Pr(D_1G_2) = 0.16$, $\Pr(D_1D_2) = 0.04$

(iii) 1

(b) (ii) $\Pr(F_1F_2) = \frac{1}{36}$, $\Pr(F_1N_2) = \frac{5}{36}$, $\Pr(N_1F_2) = \frac{5}{36}$,

$\Pr(N_1N_2) = \frac{25}{36}$

(iii) 1

(c) (ii) $\Pr(PA) = \frac{1}{60}$, $\Pr(P \text{ not } A) = \frac{1}{12}$, $\Pr(\text{not } PA) = \frac{3}{20}$,

$\Pr(\text{not } P \text{ not } A) = \frac{3}{4}$

(iii) 1

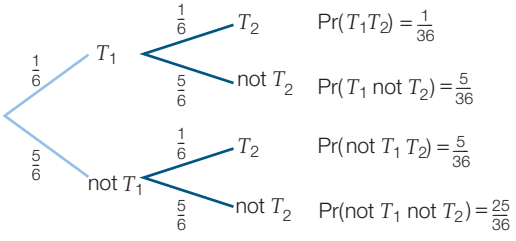
(d) (ii) $\Pr(B_1B_2B_3) = 0.064$, $\Pr(B_1B_2C_3) = 0.096$,
$\Pr(B_1C_2B_3) = 0.096$, $\Pr(B_1C_2C_3) = 0.144$,
$\Pr(C_1B_2B_3) = 0.096$, $\Pr(C_1B_2C_3) = 0.144$,
$\Pr(C_1C_2B_3) = 0.144$, $\Pr(C_1C_2C_3) = 0.216$

(iii) 1

8 (a) 0.008 (b) 0.096 (c) 0.488

9 (a) not T = a 3 is not rolled with a die

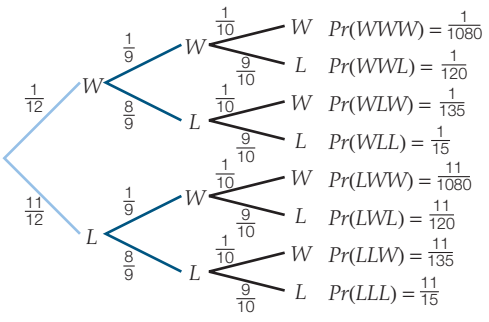
(b)



(c) (i) $\frac{1}{36}$ (ii) $\frac{25}{36}$ (iii) $\frac{5}{18}$ (iv) $\frac{11}{36}$ (v) $\frac{35}{36}$

10 (a) $\frac{1}{169}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}$

11 (a)



(b) (i) $\frac{1}{1080}$ (ii) $\frac{259}{1080}$ (iii) $\frac{288}{1080} = \frac{4}{15}$

12

Total	0	1	2
Probability	0.327 68	0.4096	0.2048

Total	3	4	5
Probability	0.0512	0.0064	0.000 32

13 (a)

Number of rolls	1	2	3
Probability	$0.1\dot{6}$	$0.13\dot{8}$	$0.115\,\dot{7}4\dot{0}$

Number of rolls	4	5	6
Probability	0.096 45	0.080 376	0.066 98

(b) approximately 0.665

(c) This means that it could still take more than six rolls to get the first 6. In fact, it could (theoretically) take an infinite number of rolls.

14 (a) $\frac{7}{29}$ (b) $\frac{3}{11}$

(c) R3 = $\frac{5}{16}$, R4 = $\frac{4}{11}$, R5 = $\frac{3}{7}$, R6 = $\frac{1}{2}$, R7 = $\frac{1}{2}$

(d) The probabilities add to more than one. This is because we assumed that elimination had not taken place before calculating the probability for the next round. This means the events did not all stem from the same tree but a new path was started after each round.

Open-ended – Sample answers

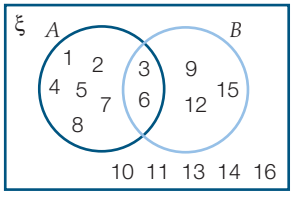
15 The Pr (leading team winning) would be greater than 0.5, an estimate would be 0.9. Leading 15 to 0 means this team only requires another 10 points whereas the trailing team still requires 25 points. Also, the result of the first 15 points challenges our assumption of the teams being equal in ability.

16 Students will produce their own diagrams and explanations.

17 The two events are not mutually exclusive, in fact, playing one of the sports increases the possibility of playing the other, as the person has a demonstrated interest in sport. Armapreet has treated them as mutually exclusive.

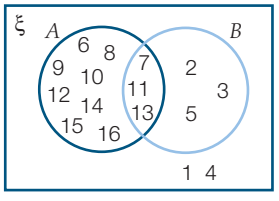
Exercise 11.5 (p. 431)

1 (a) (i)



(ii) $\frac{2}{5}$ (iii) $\frac{1}{4}$

(b) (i)



(ii) $\frac{1}{2}$ (iii) $\frac{3}{11}$

2 0.5746

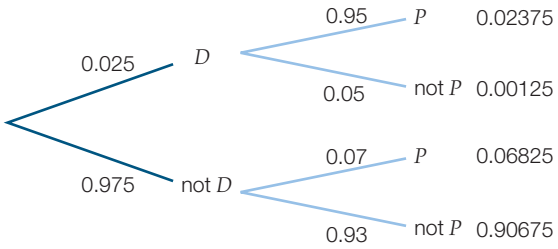
3 (a) (i) $\frac{2}{3}$ (ii) $\frac{3}{7}$

(b) (i) $0.1\dot{5}$ (ii) $0.2414 = \frac{7}{29}$

4 (a) C (b) D

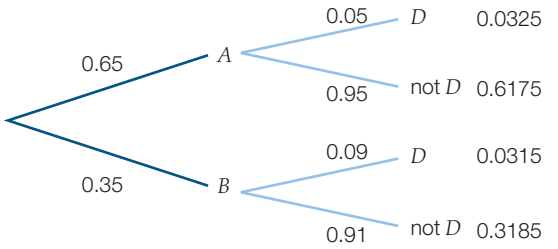
5 (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{1}{13}$ (d) $\frac{1}{2}$

6 **(a)** D = 'is infected with the disease'
P = 'tests positive for the disease'



(b) 0.2582 **(c)** 0.9986

7 **(a)** D = 'shoe is defective'



(b) 0.5078 **(c)** 0.3403

8 **(a)** 0.9375 **(b)** 0.7895 **(c)** 0.8

9 **(a)** 0.784 **(b)** 0.91 **(c)** 0.7

(d) 0.83692 **(e)** 0.973

10 $0.5\dot{}$

Open-ended – Sample answers

11 **(a)** Not enough information has been provided to answer the question.

(b) Given $\Pr(A) = 0.6$, $\Pr(B) = 0.4$ and $\Pr(A \text{ and } B) = 0.1$, find $\Pr(A \text{ given } B)$. $\Pr(A \text{ given } B) = 0.25$

Exercise 11.6 (p. 434)

1 $\frac{1}{10}$

2 **(a)** $\frac{1}{78}$ **(b)** $\frac{9}{39} = \frac{3}{13}$

3 **(a)** $\frac{10}{27}$ **(b)** $\frac{1}{9}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{16}{27}$

4 **(a)** $\frac{3}{40}$ **(b)** $\frac{3}{40}$ **(c)** $\frac{27}{40}$

5 **(a)** I **(b)** D **(c)** I **(d)** D **(e)** D **(f)** D

6 **(a)** $\frac{1}{35}$ **(b)** $\frac{2}{35}$ **(c)** $\frac{4}{15}$ **(d)** $\frac{2}{35}$

(e) $\frac{16}{105}$ **(f)** $\frac{16}{105}$ **(g)** $\frac{37}{105}$ **(h)** $\frac{68}{105}$

7 **(a)** A **(b)** D **(c)** A

8 **(a)** $\frac{3}{7}$ **(b)** $\frac{4}{35}$ **(c)** $\frac{2}{7}$

9 **(a)** $\frac{5}{51}$ **(b)** $\frac{28}{153}$ **(c)** $\frac{2}{51}$ **(d)** $\frac{49}{153}$

(e) $\frac{104}{153}$ **(f)** $\frac{8}{17}$ **(g)** $\frac{29}{51}$ **(h)** $\frac{91}{153}$

10 **(a)** $\frac{1}{8}$ **(b)** $\frac{1}{64}$ **(c)** $\frac{3}{64}$ **(d)** $\frac{1}{4}$

11 **(a)** 0.35 **(b)** 0.5

12 **(a)** 0.3 **(b)** 0.49 **(c)** 0.027 **(d)** 0.0081

13 **(a)** $\frac{4}{91}$ **(b)** $\frac{24}{91}$

14 **(a)** (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(b) $\Pr(A) = \frac{1}{6}$; $\Pr(B) = \frac{1}{6}$; $\Pr(C) = \frac{5}{36}$; $\Pr(D) = \frac{1}{12}$

(c) 6 pairs: AB, AC, AD, BC, BD, CD

(d) See if $\Pr(A) \times \Pr(B) = \Pr(A \text{ and } B)$. If it is then the events are independent.

(e) Only AB are independent.

15 **(a)** 0.5

(b) Use $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B) = 0.65$

16 If independent $\Pr(F) \times \Pr(M) = \Pr(F \text{ and } M)$, where F stands for 'female' and M stands for 'mathematics'. $\Pr(F) = \frac{475}{900}$, $\Pr(M) = \frac{870}{900}$, $\Pr(F \text{ and } M) = \frac{470}{900}$, $\Pr(F) \times \Pr(M) = \frac{551}{1080}$.
So, $\Pr(F) \times \Pr(M) \neq \Pr(F \text{ and } M)$.

17 **(a)** 0 **(b)** 0.1575

Open-ended – Sample answers

18 Students could define events such as 'rolling a 3 on the first roll', 'rolling a 5 on the second roll', 'rolling at least one multiple of 3', 'rolling a total of 6 or more'. Some questions then might be: 'Find the probability of each event', 'Are any of the pairs of events independent?' and so on.

19 **(a)** Drawing cards from a pack without replacement. Many pairs of events could be formed from this. Rolling a pair of dice and looking at a pair of events where one is to do with a particular number on the first die and the second is to do with the total of the two dice.

(b) Pairs of simple events based on separate flips of a coin, such as head on each and similar pairs of events based on rolling two dice.

20 The punter had knowledge that the bookmaker didn't have when the bet was made. By donating funds to the club the probability of success was affected so the original bet was no longer the same.

Chapter review 11 (p. 438)

1 **(a)** $\frac{1}{5}$ **(b)** $\frac{4}{5}$ **(c)** $\frac{3}{5}$ **(d)** $\frac{2}{5}$ **(e)** $\frac{3}{5}$ **(f)** $\frac{4}{5}$

2 **(a)** $\frac{3}{15} = \frac{1}{5}$ **(b)** $\frac{8}{15}$ **(c)** $\frac{10}{15} = \frac{2}{3}$

3 $\frac{4}{6} = \frac{2}{3}$

4 (a) Possible outcomes

Spinner 1 \ Spinner 2	1	2	3	4
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

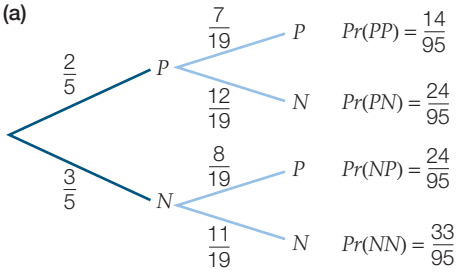
(b) (i) $\frac{1}{16}$ (ii) $\frac{3}{8}$ (iii) $\frac{9}{16}$ (iv) $\frac{1}{4}$

5 $\frac{1}{4}$

6 (a) 18 (b) 8 (c) 13 (d) 4 (e) 10 (f) 3

7 (a) D (b) B

8 (a)



(b) $\frac{14}{95}$ (c) $\frac{33}{95}$ (d) $\frac{62}{95}$

9 (a) (i) 0.4286 (ii) 0.3333

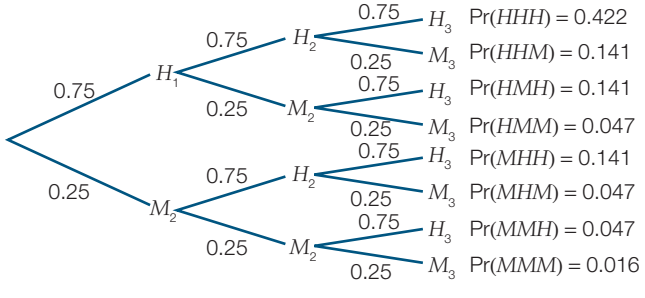
(b) (i) 0.4706 (ii) 0.2759

10 $\frac{6}{99} = \frac{2}{33}$

11 (a) $\frac{1}{7}$ (b) $\frac{3}{7}$ (c) $\frac{3}{7}$ (d) $\frac{6}{7}$ (e) $\frac{2}{7}$ (f) $\frac{3}{7}$

12 (a) 0.25

(b) If H = hit and M = miss:



(c) (i) 0.422 (ii) 0.141 (iii) 0.984

13 (a)

	lemonade	not lemonade	
cola	12	18	**30**
not cola	28	2	**30**
	40	**20**	**60**

(b) (i) $\frac{2}{3}$ (ii) $\frac{7}{15}$ (iii) $\frac{1}{2}$ (iv) $\frac{3}{10}$

(v) $\frac{1}{5}$ (vi) $\frac{1}{3}$ (vii) $\frac{29}{30}$ (viii) $\frac{1}{30}$

14 (a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{13}{16}$ (d) $\frac{3}{8}$

15 (a) 0.35 (b) 0.275

(c) 0.015 (d) 0.012

16 (a) (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{1}{8}$

(b) 1. Yes, this is an expected result as these outcomes cover the sample space.

(c) Pr(0 tails) = Pr(3 heads); Pr(1 tail) = Pr(2 heads) etc.

(d) (i) $\frac{1}{8}$ (ii) $\frac{1}{8}$ (iii) $\frac{1}{8}$

(e) Any outcome with exactly 1 tail has exactly the same probability as any other outcome with exactly 1 tail. The order does not matter.

17 (a) $\frac{16}{81}$ (b) $\frac{344}{1215}$ (c) $\frac{296}{1215}$

18 (a) 0.015 (b) 0.16 (c) 0.032 (d) 0.12

(e) Sydney and Canberra are relatively close and if it rains in one city the probability of rain in both cities will be higher. The two events are not independent.

Chapter 12

Recall 12 (p. 442)

1 (a) $\pm\sqrt{3}$ (b) $\sqrt[3]{5}$ (c) $\pm\sqrt{7}$ (d) $\sqrt[3]{6}$ (e) $\pm\sqrt[4]{10}$

2 (a) 2 (b) 6 (c) 3 (d) 1 (e) 2

(f) 4 (g) -3 (h) 1 (i) 2 (j) -1

3 (a) 3.32 (b) -2.65 (c) 2.47 (d) 1156 (e) 19 683

4 (a) $3x + 12$ (b) $-8y + 12$

5 (a) m^5 (b) $12k^3p^9$ (c) x^7

(d) 3 (e) $m^{12}n^4$ (f) x^6y^{-4} $(= \frac{x^6}{y^4})$

6 (a) $x^2 - y^2$ (b) $c^2 - 81$ (c) $4p^2 - 9$

Exercise 12.1 (p. 444)

1 (a) irrational (b) rational (c) rational

(d) irrational (e) rational (f) irrational

(g) irrational (h) rational (i) irrational

(j) irrational (k) irrational (l) rational

2 (a) 8 (b) -5 (c) -6 (d) 9 (e) 42 (f) -28

(g) 13 (h) 7 (i) 69 (j) 30 (k) 16 (l) 73

3 (a) 3.146 (b) 0.592 (c) -0.960

(d) 5.916 (e) -1.732 (f) 2.571

(g) 2.154 (h) 2.449 (i) -4.160

4 (a) C (b) D (c) D (d) D

5 (a) C (b) A

6 (a) B (b) D (c) B

7 (a) 2.236 (b) 1.732 (c) 8.061
(d) 7.071 (e) 1.596 (f) -5.482
(g) -2.410 (h) -6.557 (i) 3.464
(j) 13.416 (k) -9.950 (l) 67.082

8 (a) 5 m (b) 6 cm (c) $\sqrt{46}$ mm (d) $\sqrt{17}$ m

9 (a) $\sqrt{60}$ m (b) 31 m

(c) The precision of measuring devices is important as far as accuracy is concerned. In this situation, a measurement to the nearest metre is therefore the most appropriate.

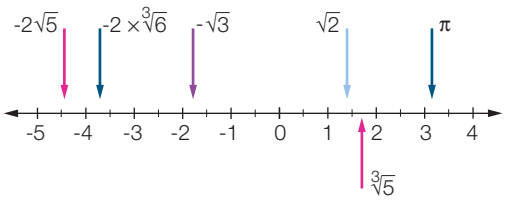
10 (a) 2 m (b) 3 m (c) $\sqrt[3]{35}$ cm (d) $\sqrt[3]{15}$ cm

11 (a) $\sqrt[3]{7}$ cm (b) 19 mm

Open-ended – Sample answers

12 (a) $\sqrt{2}, \pi, \sqrt[3]{5}$ (b) $-\sqrt{3}, -2\sqrt{5}, -2\sqrt[3]{6}$

(c)



13 Lie the metre ruler on a flat surface and rule a 1 m line with the pencil. Place an end of the ruler on the line and rule another metre line at right angles to the first. Run the string from one end of a metre line to the other (as if forming a triangle) this length is $\sqrt{2}$ metres.

Exercise 12.2 (p. 448)

1 (a) $\sqrt{6}$ (b) $\sqrt{55}$ (c) $\sqrt{70}$ (d) 7 (e) 4
(f) 8 (g) $6\sqrt{2}$ (h) $5\sqrt{7}$ (i) $3\sqrt{2}$ (j) $10\sqrt{6}$
(k) $4\sqrt{33}$ (l) $40\sqrt{2}$ (m) 35 (n) 24 (o) 40
(p) 42 (q) $15\sqrt{6}$ (r) $144\sqrt{2}$

2 (a) 3 (b) 4 (c) 5 (d) 2
(e) 6 (f) 3 (g) 1 (h) 2
(i) $\frac{5}{3}$ (j) $\frac{10}{7}$ (k) $\frac{2}{9}$ (l) $\frac{6}{11}$
(m) $\frac{1}{4}$ (n) $\frac{1}{3}$ (o) 4 (p) 5
(q) 1 (r) $\frac{1}{3}$ (s) 30 (t) 10
(u) $6\sqrt{6}$ (v) $\frac{\sqrt{5}}{5}$ (w) $\frac{2\sqrt{7}}{5}$ (x) $\frac{2\sqrt{3}}{3}$

3 (a) $3\sqrt{3}$ (b) $6\sqrt{3}$ (c) $9\sqrt{2}$ (d) $4\sqrt{6}$
(e) $3\sqrt{11}$ (f) $10\sqrt{5}$ (g) $9\sqrt{6}$ (h) $9\sqrt{5}$
(i) $10\sqrt{2}$ (j) $-8\sqrt{6}$ (k) $-15\sqrt{7}$ (l) $36\sqrt{3}$

4 (a) $\sqrt{12}$ (b) $\sqrt{63}$ (c) $\sqrt{72}$ (d) $\sqrt{27}$
(e) $-\sqrt{600}$ (f) $-\sqrt{40}$ (g) $-\sqrt{192}$ (h) $\sqrt{175}$
(i) $\sqrt{128}$ (j) $\sqrt{245}$ (k) $-\sqrt{108}$ (l) $-\sqrt{275}$

5 (a) C (b) B (c) C (d) D (e) B

6 (a) $45\sqrt{2}$ (b) $16\sqrt{7}$ (c) $-28\sqrt{15}$ (d) $-32\sqrt{5}$
(e) $-15\sqrt{5}$ (f) 120 (g) 48 (h) $36\sqrt{6}$
(i) $168\sqrt{21}$ (j) $100\sqrt{15}$ (k) $216\sqrt{14}$ (l) $54\sqrt{3}$

7 (a) $2\sqrt{2}$ (b) $3\sqrt{3}$ (c) $9\sqrt{5}$ (d) $-12\sqrt{6}$
(e) $\frac{3\sqrt{3}}{2}$ (f) $\frac{4\sqrt{2}}{5}$ (g) $6\sqrt{2}$ (h) $3\sqrt{7}$
(i) $\frac{5\sqrt{5}}{6}$ (j) $-\frac{2\sqrt{11}}{3}$ (k) $-\frac{3\sqrt{3}}{5}$ (l) $\frac{7\sqrt{3}}{10}$

8 (a) 75 m^2 (b) 16 m^2 (c) 12 km^2 (d) $9\sqrt{6}$ cm^2

9 One solution is $b = 9$, $c = 2$

10 (a) $2\sqrt{10}$ mm (b) $6\sqrt{5}\pi$ m

11 (a) $\sqrt{0.2825}$ m or $\sqrt{2825}$ cm (b) 6 windows

Open-ended – Sample answers

12 $\sqrt{24}, \frac{\sqrt{96}}{2}, \frac{\sqrt{216}}{3}$

13 (a) $\sqrt{5} \times \sqrt{5}$ (b) $\frac{8\sqrt{2}}{\sqrt{8}}$

Exercise 12.3 (p. 451)

1 (a) $10\sqrt{2}$ (b) $3\sqrt{5}$ (c) $5\sqrt{3}$
(d) $8\sqrt{7}$ (e) $-4\sqrt{3}$ (f) $-23\sqrt{11}$
(g) $11\sqrt{2}$ (h) 0 (i) $-3\sqrt{5}$
(j) $5\sqrt{6} - 5\sqrt{3}$ (k) $7\sqrt{2} - 5\sqrt{5}$ (l) $-\sqrt{3} - 4\sqrt{11}$

2 (a) $4\sqrt{2}$ (b) $\sqrt{7}$ (c) $3\sqrt{5}$
(d) $36\sqrt{2}$ (e) $\sqrt{7}$ (f) $-4\sqrt{10}$
(g) $9\sqrt{10}$ (h) $27\sqrt{2}$ (i) $8\sqrt{6}$
(j) $49\sqrt{3}$ (k) $-30\sqrt{5}$ (l) $19\sqrt{2}$
(m) $2\sqrt{7}$ (n) 0

3 (a) B (b) B (c) D

4 (a) C (b) D

5 (a) $8\sqrt{2}$ (b) $20\sqrt{3}$
(c) $11\sqrt{2}$ (d) $8\sqrt{10} + 12\sqrt{7}$

6 (a) $x = 4\sqrt{5}$ (b) $y = \sqrt{6}$

7 5π cm^2

8 $\frac{3\pi}{4}$ cm^2

9 (a) $6\sqrt{5} + 4\sqrt{3}$ m (b) $6\sqrt{15}$ m^2 (c) no

Open-ended – Sample answers

10 $7\sqrt{5} - 4\sqrt{5} = 3\sqrt{5}$; $-10\sqrt{5} + 13\sqrt{5} = 3\sqrt{5}$; $8\sqrt{5} - 5\sqrt{5} = 3\sqrt{5}$

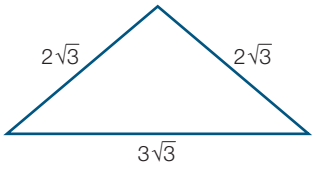
11 length = $2\sqrt{6}$ cm, width = 2 cm, perimeter = $4\sqrt{6} + 4$ cm;
length = $2\sqrt{3}$ cm, width = $2\sqrt{2}$ cm,
perimeter = $4\sqrt{3} + 4\sqrt{2}$ cm

12 **(a)** **(i)** Ruby has added the numbers underneath the square root.

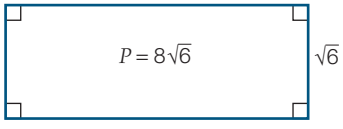
(ii) When subtracting one surd from another, Ruby has forgotten to include the surd part of the solution.

(b) **(i)** $7\sqrt{3}$ **(ii)** $3\sqrt{6}$

(c) **(i)**



(ii)



Exercise 12.4 (p. 454)

1 **(a)** $3\sqrt{7} + 6$ **(b)** $-4\sqrt{11} - 4\sqrt{3}$

(c) $18\sqrt{5} - 9\sqrt{2}$ **(d)** $3\sqrt{35} - 5\sqrt{14}$

(e) $8\sqrt{10} + 24\sqrt{6}$ **(f)** $3\sqrt{10} - 21\sqrt{15} + 9\sqrt{35}$

(g) $3 + 2\sqrt{3}$ **(h)** $15\sqrt{14} - 70$

(i) $24\sqrt{15} - 6\sqrt{5} - 15$ **(j)** $6\sqrt{7} - 12$

(k) $12 - 12\sqrt{2} + 20\sqrt{3}$ **(l)** $40 - 4\sqrt{5} - 20\sqrt{3}$

(m) $\sqrt{10} + 3\sqrt{5} + 4\sqrt{2} + 12$ **(n)** $3 + \sqrt{21} - \sqrt{15} - \sqrt{35}$

(o) $5 + 2\sqrt{6}$ **(p)** $88 - 16\sqrt{10}$

(q) 25 **(r)** -53

2 **(a)** B **(b)** D **(c)** C **(d)** C

3 **(a)** $\pi(19 - 6\sqrt{10})$ m^2 **(b)** $(2\sqrt{35} + 12\sqrt{7})$ cm^2

4 $\pi(21 + 8\sqrt{5})$ cm^2

5 **(a)** 21 km^2 **(b)** $(14 + 3\sqrt{2} + \sqrt{10})$ km

(c) 21 405 m

6 **(a)** **(i)** $(18\sqrt{2} - 12)$ m **(ii)** $(12\sqrt{2} - 8)$ m

(iii) $(15\sqrt{2} - 10)$ m **(iv)** $(21\sqrt{2} - 14)$ m

(b) **(i)** 13.46 m **(ii)** 8.97 m

(iii) 11.21 m **(iv)** 15.70 m

7 **(a)** 7π cm^2 **(b)** $(5 + 2\sqrt{7})$ cm by $(3 + 2\sqrt{7})$ cm

(c) $(43 + 16\sqrt{7})$ cm^2 **(d)** $(43 + 16\sqrt{7} - 7\pi)$ cm^2

(e) 63.34 cm^2

8 **(a)** 47.7 cm **(b)** 22 752.9 cm^2

(c) $(12\,110 + 1440\sqrt{55})$ cm^2 **(d)** 22 789.3 cm^2

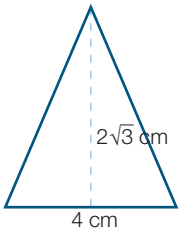
(e) part **(d)**

9 **(a)** $\sqrt{18}$ or $3\sqrt{2}$ cm **(b)** $150\sqrt{2}$ cm **(c)** 212.1 cm

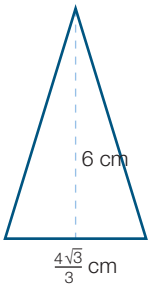
(d) 4.2 cm **(e)** 210 cm **(f)** part (c)

Open-ended – Sample answers

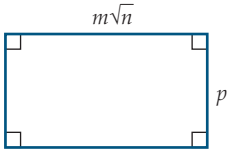
10 base 4 cm, height $2\sqrt{3}$ cm



base $\frac{4\sqrt{3}}{3}$ cm, height 6 cm



11 **(a)**



$P = 2(m\sqrt{n} + p)$
If $m = 10$, $n = 3$ and $p = 4$
Then $P = 2(10\sqrt{3} + 4) = 42.641$ cm

(b) $A = m\sqrt{n} \times p = mp\sqrt{n}$

(c) If $m = 2$, $n = 2$ and $p = 3$ then $A = 8.485$ m^2

Exercise 12.5 (p. 459)

1 **(a)** $\frac{\sqrt{7}}{7}$ **(b)** $\frac{4\sqrt{3}}{3}$ **(c)** $-2\sqrt{6}$ **(d)** $\frac{\sqrt{6}}{2}$

(e) $\sqrt{55}$ **(f)** $\frac{\sqrt{30}}{2}$ **(g)** $-\frac{3\sqrt{7}}{28}$ **(h)** $\frac{2\sqrt{15}}{7}$

2 **(a)** $\frac{3 + \sqrt{7}}{2}$ **(b)** $8 - 4\sqrt{3}$

(c) $\frac{\sqrt{15} + \sqrt{6}}{3}$ **(d)** $\frac{4\sqrt{7} - 10}{3}$

(e) $\frac{24 + 15\sqrt{3}}{3} = 8 + 5\sqrt{3}$ **(f)** $\frac{22 + 10\sqrt{5}}{4} = \frac{11 + 5\sqrt{5}}{2}$

(g) $\frac{6\sqrt{3} - \sqrt{42} - 3\sqrt{2} + \sqrt{7}}{11}$ **(h)** $\frac{5\sqrt{6} - 8\sqrt{3}}{6}$

3 **(a)** D **(b)** A **(c)** B

4 **(a)** B **(b)** D

5 (a) $\frac{2\sqrt{3}+\sqrt{5}+1}{4}$ (b) $\frac{\sqrt{7}+2\sqrt{6}+5}{6}$

(c) $\frac{5\sqrt{6}-5\sqrt{3}-12-3\sqrt{11}}{15}$ (d) $\frac{3\sqrt{5}-3\sqrt{3}-2\sqrt{7}-4}{6}$

6 (a) $\frac{5\sqrt{3}}{2}$ (b) 4 shelves (c) $(5\sqrt{6}-8\sqrt{2})$ m

7 (a) $15\,000\sqrt{6}$ cm^2 (b) $50\sqrt{6}$ tiles

(c) As $122 < 50\sqrt{6} < 123$, the minimum number of tiles is 123, but maybe buy 130 tiles to allow for wastage.

(d) $15\,000\sqrt{6} + 6000\sqrt{15} + 600\sqrt{35} + 1500\sqrt{14}$ cm^2

8 $\frac{112\sqrt{3}-294}{249}$ or $\frac{14(8\sqrt{3}-21)}{249}$

Open-ended – Sample answers

9 Many answers are possible, just multiply the surd by a form of 1. As examples: $\frac{4\sqrt{6}}{3}$, $\frac{8}{\sqrt{6}}$, $\frac{4\sqrt{10}}{\sqrt{15}}$

10 (a) $\frac{\sqrt{2}}{3}, \frac{2}{3}$ (b) $\frac{2\sqrt{7}}{7}, \frac{4\sqrt{7}}{7}$

(c) $\frac{\sqrt{15}}{5}, \frac{3\sqrt{15}}{5}$ (d) $\frac{\sqrt{2}}{6}, \frac{\sqrt{2}}{2}$

11 $2-\sqrt{3}$ and $4-2\sqrt{3}$

12 When two surds that are the same are multiplied, the result is a whole number, e.g. $\sqrt{6}\times\sqrt{6}=6$. Multiplying by the conjugate will remove the square roots from the denominator, resulting in a rationalised denominator. It is also like the difference of two squares.

Half-time 12 (p. 462)

1 (a) 6 (b) 88 (c) $\frac{9}{4}$

2 $5\sqrt{6}+3\sqrt{2}$ m

3 (a) $\frac{\sqrt{30}}{6}$ (b) $\frac{5\sqrt{3}}{6}$ (c) $\frac{12+3\sqrt{11}}{5}$

4 $3\sqrt{5}$ m

5 (a) rational (b) rational (c) irrational (d) rational

6 $(18+12\sqrt{15})$ m^2

7 (a) $6\sqrt{2}$ (b) 360 (c) $\frac{3}{5}$

8 (a) -15 (b) 17 (c) 34

Exercise 12.6 (p. 464)

1 (a) $12x^5$ (b) $\frac{a^5}{b^3}$ (c) $\frac{p}{q^3}$

(d) x^2 (e) $a^{20}b^9$ (f) $6m^{10}n$

(g) $\frac{1}{q^7}$ (h) $\frac{1}{c^9d^5}$ (i) 2

(j) $\frac{a^2}{b^2}$ (k) $\frac{m^5}{n^5}$ (l) 1

(m) q (n) $\frac{3k^3}{10}$ (o) $\frac{2a^6b^4}{3}$

2 (a) k^7p^6 (b) x^5y^5 (c) $\frac{6m^6n^5}{5}$

(d) $\frac{2q^{11}}{p^4}$ (e) $\frac{7y^{10}}{x^4}$ (f) $\frac{1}{m^6n^2}$

(g) $\frac{1}{k^7p^4}$ (h) $\frac{2a^5}{3b}$ (i) $\frac{5fg^4}{2}$

(j) x^2y^4 (k) $\frac{2m^3}{3}$ (l) $\frac{q^2}{5}$

3 (a) B (b) A (c) C

4 (a) x^9y^4 (b) $\frac{a^{12}}{b^{11}}$ (c) $\frac{4}{k^{18}p^2}$

(d) $9m^{20}n^2$ (e) $\frac{x^{27}}{y^{11}}$ (f) $\frac{m^6}{n^{20}}$

(g) $\frac{1}{q^{22}}$ (h) f^6g^{11} (i) $\frac{a}{b^7}$

(j) $\frac{m^{14}}{n^{11}}$ (k) $\frac{4x^{13}}{y^{18}}$ (l) $\frac{12d^{15}}{c^3}$

5 (a) x^2 (b) $\frac{a^3}{b^5}$ (c) pq (d) d^4f^2

(e) $\frac{1}{y^2}$ (f) $\frac{1}{a^3}$ (g) $\frac{n}{m^2}$ (h) $\frac{q^2}{p^7}$

(i) $\frac{b^9}{a^{16}}$ (j) $\frac{x^{13}}{y^7}$ (k) $\frac{12q^4}{p^{17}}$ (l) $\frac{9y^7}{8x^{14}}$

6 (a) $5^3, 5^2, 5^1, 5^0$ (b) $5^{-1}, 5^{-2}, 5^{-3}, 5^{-4}, 5^{-5}$

(c) $125, 25, 5, 1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \frac{1}{3125}$

7 (a) $\frac{m^9}{n^2}$ (b) $\frac{b^3}{a}$ (c) $\frac{x^5}{y^4}$ (d) $\frac{y^{19}}{x^6}$ (e) $\frac{9}{a^2b^7}$

(f) $\frac{8}{d^5f^7}$ (g) $\frac{10b}{a^9}$ (h) $\frac{n^3}{m^8}$ (i) $\frac{1}{x^2}$

8 (a) 1 divided by the original number (b) $192a$

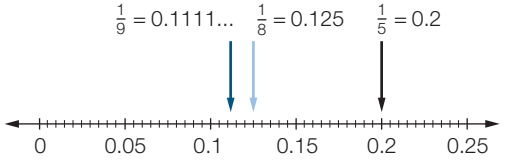
Open-ended – Sample answers

9 (a) $(6m^2n^3)^2 \times n^2$; $9m^3n^4 \times 4mn^4$

(b) $\frac{12m^7n^3}{15m^3n^9}$; $24m^4 \div 30n^6$

10 $a=2, b=3$; $a=3, b=2$; $a=5, b=1$

These give the fractions: $\frac{1}{8}, \frac{1}{9}$ and $\frac{1}{5}$



11 (a) Murray just said $-2 \times -4 = 8$ and possibly put it as the power in the denominator because he thought the negatives needed to be accounted for. He is wrong in at least two ways.

(b) Raphael knows that indices are added when numbers are multiplied. $\frac{1}{a^6}$ may be Raphael's answer, which would be correct.

(c) When numbers with indices and the same base are multiplied, keep the base the same and add the indices.

$a^{-2} \times a^{-4} = a^{-6} = \frac{1}{a^6}$.

Exercise 12.7 (p. 468)

1 (a) $x^{2/3}$ (b) $a^{5/6}$ (c) $k^{8/15}$ (d) $m^{1/4}$

(e) $p^{1/6}$ (f) $y^{1/4}$ (g) $5p^2$ (h) $7x^3$

(i) $2a^{1/3}b$ (j) $x^2y^{1/2}$ (k) $k^{1/2}n^4$ (l) $\frac{p^2}{q^5}$

2 (a) 14 (b) 23 (c) 19 (d) 8 (e) 6 (f) 8

(g) 5 (h) 4 (i) 7.94 (j) 14.42 (k) 2.15 (l) 2.29

3 4^{-1}, $6^{-\frac{1}{5}}$, $2^{\frac{1}{2}}$, $3^{\frac{1}{3}}$

4 (a) D (b) B (c) B (d) C

5 (a) a (b) d^3 (c) $\frac{x}{y}$ (d) $\frac{1}{m^2n^4}$

(e) $\frac{1}{q^2}$ (f) $\frac{1}{b^4}$ (g) $\frac{m^{1/3}}{2n^{1/6}}$ (h) $\frac{y^{7/6}}{5x^{1/3}}$

6 (a) B (b) A

7 (a) 0.5 (b) 0.437 (c) 0.164 (d) 0.196

(e) 0.369 (f) 0.343 (g) 0.398 (h) 0.359

8 (a) $x = \sqrt[3]{3}$ (b) $x = 4$ (c) $x = \frac{1}{2}$

9 (a) x^3 (b) a^3 (c) $2m^2n$

10 (a) $x = \pm 2$ (b) $x = 3$ (c) $x = -2$ (d) $x = 2$

11 (a) 7 (b) $x = 7, y = 5$ (c) 256

(d) $d = 256, e = \frac{3}{4}$

Open-ended – Sample answers

12 $1476.106^{\frac{4}{5}}$, 7^3, 1.793^{10}

13 $\left(n^{\frac{1}{4}}m^{\frac{1}{5}}\right)^3$, $(\sqrt[4]{n} \times \sqrt[5]{m})^3$; $(-\sqrt[4]{n^3})(-\sqrt[5]{m^3})$

Exercise 12.8 (p. 471)

1 (a) $\log_2 32 = 5$ (b) $\log_2 256 = 8$

(c) $\log_2 4096 = 12$ (d) $\log_3 27 = 3$

(e) $\log_3 243 = 5$ (f) $\log_3 6561 = 8$

(g) $\log_{10} 100 = 2$ (h) $\log_{10}(1\,000\,000) = 6$

(i) $\log_{10}(100\,000\,000) = 8$ (j) $\log_2\left(\frac{1}{4}\right) = -2$

(k) $\log_2\left(\frac{1}{8}\right) = -3$ (l) $\log_2\left(\frac{1}{16}\right) = -4$

2 (a) $10^2 = 100$ (b) $10^3 = 1000$

(c) $10^5 = 100\,000$ (d) $2^4 = 16$

(e) $2^7 = 128$ (f) $2^9 = 512$

(g) $10^{-3} = \frac{1}{1000}$ (h) $10^{-4} = \frac{1}{10\,000}$

(i) $10^{-6} = \frac{1}{1\,000\,000}$ (j) $2^{-3} = \frac{1}{8}$

(k) $2^{-5} = \frac{1}{32}$ (l) $2^{-7} = \frac{1}{128}$

3 (a) C (b) D

4 (a) 4 (b) 8 (c) 9 (d) -2 (e) -3

(f) -6 (g) -3 (h) -5 (i) 4

5 (a) 16 (b) 64 (c) 128 (d) -4 (e) -5

(f) -8 (g) 5 (h) 7 (i) 5

6 (a) (i) 10 (ii) 5 (iii) 7

(b) $\log_a(a) = 1$

7 (a) (i) $x = 1$ (ii) $x = 1$ (iii) $x = 1$

(b) $\log_a 1 = 0$

8 We cannot find the square root of a negative number.

9 (a) Students to verify that both equations are true.

(b) 6

(c) $2^{12} = 4096$ is the index equivalent of the first statement and $4^y = 4096$ is the second. But, this could be written as $(2^2)^y = 4096$ or $2^{2y} = 4096$, so $2y = 12$ and $y = 6$.

Open-ended – Sample answers

10 $\log_9(531\,441) = 6$ and $\log_{27}(531\,441) = 4$

11 Effectively, Lee is saying $\log_6(-36) = -2$, which would mean that $6^{-2} = -36$, which is clearly incorrect. Lee should have divided both sides by -1 to get $(6)^2 = 36$, which has the equivalent form $\log_6(36) = 2$.

Exercise 12.9 (p. 474)

1 (a) $\log_2 32$ (b) $\log_2 64$ (c) $\log_2 256$

(d) $\log_{10} 30$ (e) $\log_{10} 21$ (f) $\log_{10} 36$

2 (a) $\log_2 2$ (b) $\log_2 5$ (c) $\log_2 5$

(d) $\log_{10} 14$ (e) $\log_{10} 5$ (f) $\log_{10} 6$

3 (a) $4\log_2 5$ (b) $3\log_2 6$ (c) $4\log_2 7$

(d) $3\log_{10} 7$ (e) $4\log_{10} 8$ (f) $5\log_{10} 11$

4 (a) 5 (b) 7 (c) 8 (d) 2 (e) 1 (f) 1

5 (a) A (b) C

6 (a) 3 (b) 11 (c) 1 (d) 2 (e) 8 (f) -9

7 (a) 2 (b) $\frac{5}{6}$ (c) 3 (d) 2 (e) $\frac{3}{2}$ (f) $\frac{4}{3}$

8 (a) $\frac{1}{4}$ (b) 2 (c) $\frac{2}{3}$

9 (a) $y = 4x^3$ (b) $y = 11x^5$

10 $5\log_2(y) = \log_2(x-1) + 2$

$$\log_2\left(\frac{y^5}{x-1}\right) = 2$$

$$2^2 = \frac{y^5}{x-1}$$

$$4(x-1) = y^5$$

Open-ended – Sample answers

11 No, the correct answer is 2. The mistake Danielle made was when she took out the $\frac{1}{2}$. This was really $\frac{1}{\frac{1}{2}}$ which is the same as 2.

Chapter review 12 (p. 476)

1 (a) 4.123 (b) 2.924 (c) 34.641 (d) 2.280

2 (a) $12\sqrt{30}$ (b) 70 (c) $\frac{8}{5}$ or $1\frac{3}{5}$ (d) $9\sqrt{2}$

(e) $2\sqrt{10}$ (f) $10\sqrt{5}$ (g) $30\sqrt{5}$ (h) $5\sqrt{3}$

3 (a) $\sqrt{72}$ (b) $-\sqrt{63}$

4 (a) $\sqrt{2}$ (b) $4\sqrt{7} - \sqrt{3}$

(c) $29\sqrt{3}$ (d) $46\sqrt{2} - \sqrt{6}$

5 (a) $4\sqrt{15} - 10 + \sqrt{10}$ (b) $12 - 18\sqrt{14} - 6\sqrt{2}$

(c) $5\sqrt{5} - 8$ (d) $95 + 24\sqrt{14}$

6 (a) $\frac{5\sqrt{2}}{2}$ (b) $2\sqrt{2}$ (c) $\frac{7\sqrt{5}}{10}$

7 (a) $\frac{21 + 7\sqrt{5}}{4}$ (b) $\frac{2\sqrt{6} - \sqrt{14} - 4\sqrt{3} + 2\sqrt{7}}{5}$

8 (a) $9a^2b^{10}$ (b) c^2d^{14} (c) u^2t

(d) $2xy^{10}$ (e) $m^{12}n^9$ (f) $a^{12}b$

9 (a) $\frac{6}{a^6}$ (b) $\frac{27x^4}{16y}$ (c) $\frac{n^6}{2}$

10 (a) $4mn^{1/3}$ (b) $p^3q^{1/2}$ (c) $a^{1/2}$

(d) $\frac{b}{a^9}$ (e) $\frac{2n^{4/3}}{m^{1/3}}$ (f) $\frac{729n^7}{m^{19}}$

11 (a) $k^{5/2}$ (b) $x^{3/4}$ (c) $m^{-2/3}$

12 (a) C (b) D

13 (a) $\log_4(16\,384) = 7$ (b) $\log_2(1024) = 10$

(c) $\log_{10}\frac{1}{1000} = -3$

14 (a) $2^{14} = 16\,384$ (b) $3^5 = 243$ (c) $10^{-5} = \frac{1}{100\,000}$

15 (a) 9 (b) -2 (c) $\log_3(50)$ (d) $\log_5(10)$

16 (a) $2\sqrt{58}$ cm (b) 15.23 cm

17 (a) length 25.3 cm, height 20.2 cm

(b) 1749.3 cm^2

(c) $(180\sqrt{6} + 108\sqrt{7} + 160\sqrt{15} + 48\sqrt{70})$ cm^2

(d) 1747.9 cm^2 (e) (b) was larger by 1.4 cm^2.

18 (a) 5 (b) 7 (c) -6 (d) $\frac{1}{16}$ (e) 8 (f) -4

19 (a) 3 (b) 3 (c) 2 (d) $\frac{3}{4}$

20 (a) $5\sqrt{2}$ cm (b) $\sqrt{\frac{160}{\sqrt{3}}}$ cm (c) $10\pi\sqrt{3}$ m

21 $-\frac{17 + 136\sqrt{5}}{319}$

22 (a) $y = 7x^3$ (b) $y = \frac{5}{x^2}$

Mixed review D (p. 479)

1 \$20.23

2 area = 36.2 cm^2, perimeter = 24.7 cm

3 (a) $2\sqrt{6}$ (b) 18 (c) 6

(d) $40\sqrt{2}$ (e) $12\sqrt{15}$ (f) $126\sqrt{2}$

4 (a) $4\sqrt{5}$ (b) $12\sqrt{15}$ (c) $35\sqrt{21}$

5 (a) $a = -6$ (b) $-\frac{15}{16}$ (c) $a = \frac{11}{6}$

6 (a) $\frac{4\sqrt{15}}{5}$ (b) $\frac{\sqrt{3+5}}{11}$ (c) $\frac{\sqrt{10} - \sqrt{6}}{2}$

7 (a) $(k-1)(k-3)$ (b) $(3t-5)(t+2)$

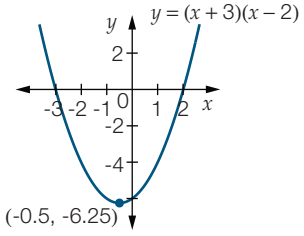
8 (a) $\frac{2a^6b^4}{3}$ (b) $a^{10}b^7$ (c) $\frac{1}{n^3k^{17}}$

9 (a) $a = 82.45°$ (b) 69.12° (c) 31.54°

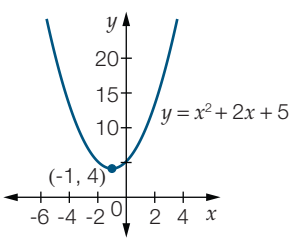
10 (a) $\frac{7}{3}$ (b) -2

11 (a) 33.36 (b) 125.24

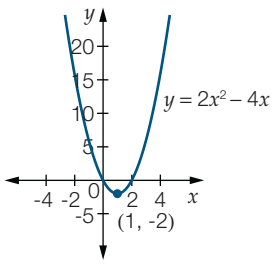
12 (a)



(b)



(c)



13 (a) $(8\sqrt{5} + 2\sqrt{3})$ cm (b) $15 + \sqrt{15})\text{cm}^2$

14 (a) A rectangle is a quadrilateral in which all angles are right angles.

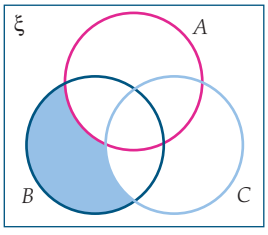
(b) A rhombus is a quadrilateral with all sides equal in length.

(c) A kite is a quadrilateral with two pairs of adjacent sides equal in length.

15 (a) $\frac{1}{12}$ (b) $\frac{5}{36}$ (c) $\frac{1}{6}$

16 (a) $3^{-5} < 5^{-3}$ (b) $9^{\frac{1}{2}} > 9^{-1}$ (c) $8^{-\frac{1}{3}} = \frac{2^{-3}}{2^{-2}}$

17

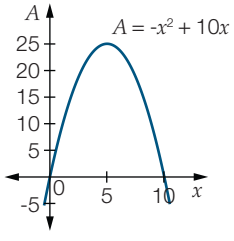


18 For an equation to be a quadratic equation, the highest powers of the variable must be a 2, and all powers of the variable must be positive integers.

19 (a) $10 - x$

(b) $A = x(10 - x) = -x^2 + 10x$

(c)



(d) 5 m by 5 m

(e) 25 m^2

20 $8\sqrt{2}$ cm

21 (a) Check students' diagrams carefully. Equal side lengths should be approximately 11.2 cm in length.

(b) Draw a line from the top of the triangle to the midpoint of the base to create two congruent right-angled triangles. You can now use Pythagoras' theorem to calculate the equal side lengths of the isosceles triangle:

$l^2 = 5^2 + 10^2$

$l^2 = 125$

$l = \sqrt{125} = 5\sqrt{5}$ cm

Challenge D (p. 481)

1 D.

There are 36 possible combinations from two rolls, 9 of which score a 3 or less: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

2 (a) $\frac{1}{6}$; the second spin is an independent event from the first.

(b) (i)

Prize	Frequency	Relative frequency
\$0	8	0.16
\$5	7	0.14
\$10	9	0.18
\$20	10	0.20
\$30	8	0.16
\$60	8	0.16
Total	**50**	**1**

(ii) \$1045; $5 \times 7 + 10 \times 9 + 20 \times 10 + 30 \times 8 + 60 \times 8 = 1045$

(c) (i) You would make a profit of \$40:

$5 \times 8 + 10 \times 8 + 20 \times 8 + 30 \times 8 + 60 \times 8 = 1000$

$48 \times 20 = 960$

$1000 - 960 = 40$

(ii) You would make a \$45 profit.

3 B

Side length of base $= 12\sqrt{10}$, so each triangular face a height of $\frac{1680}{12\sqrt{10}} = 14\sqrt{10}$. Using Pythagoras' theorem, the height of the pyramid, h, is given by $h^2 = (14\sqrt{10})^2 - (6\sqrt{10})^2$, so the height $= \sqrt{1600} = 40$.

4 A

Area of four semicircles $= 2\pi$, area of inner square $= 4$, radius of inner circle $= \sqrt{2}$, area of inner circle $= 2\pi$.

Area of shaded region $= 4 + 2\pi - 2\pi = 4$.

5 (a) $\frac{1}{x} = \frac{1}{3} + \frac{1}{4}$, $x = \frac{12}{7}$

(b) $x + 3 = 5$, $x = 2$

(c) $2^x = 2 \times 2^{48} + 2 \times 2^{48} = 2 \times 2^{49} = 2^{50}$, $x = 50$

(d) $2^{2x-4} = 2^3$, $x = 3.5$

(e) $2^{4x} = 2^{x+4}(2 - 1)$

$2^{4x} = 2^{x+4}$

$4x = x + 4$

$x = \frac{4}{3}$

Glossary and index

Addition Law for Mutually Exclusive Events $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$; can only be used when A and B are mutually exclusive (p. 418)

adjacent side the side in a right-angled triangle next to the given angle that is not the hypotenuse (p. 144)

adjusted value the value of an item after depreciation (p. 18)

algebraic fractions expressions written in fraction form that contain at least one variable (p. 54)

amplitude half the distance between the minimum and maximum y-values of the graph of a trigonometric function (p. 184)

angle at the centre an angle that has its vertex at the centre of a circle (p. 299)

angle at the circumference an angle formed by the intersection of two chords at the circumference of a circle (p. 299)

angle in a semicircle an angle formed by joining the ends of the diameter of a circle to the circumference to form a right angle (p. 300)

angle of depression the angle between the horizontal and the line of sight when you look *down* (p. 156)

angle of elevation the angle between the horizontal and the line of sight when you look *up* (p. 156)

angles in the same segment angles subtended by the same arc of a circle at the circumference are equal (p. 299)

appreciation the increase in the cost or value of something over a period of time (p. 3)

arc a part of the circumference of a circle (p. 298)

asymptote a line representing a value on a graph that a curve approaches, but never cuts or crosses (pp. 184, 321)

axis of symmetry an imaginary line along which a mirror could be placed and the reflection would be indistinguishable from the original (p. 321)

binomial product the product of two expressions that each have two terms; e.g. $(x + 1)(x + 2)$ (p. 33)

bivariate data a data set that contains values for two variables (p. 237)

bisect when a point is placed on a line interval such that the distance from one end to the point is the same as the distance from the other end to the point (p. 286)

box plot a visual representation of the five-number summary (p. 223)

capacity the volume of gas or liquid that a container can hold, measured in litres (L) (p. 79)

chord a line segment connecting any two points on a circle (p. 298)

coincident two or more lines that lie on top of each other (p. 134)

compass bearing a direction given in terms of turning a number of degrees east or west of north or south (p. 161)

complementary event the event that is the opposite of the event under consideration (p. 405)

completing the square a method for factorising quadratic expressions that combines the use of the Perfect Square rule and the Difference of Two Squares rule (p. 51)

composite shape a shape or solid made up of more than one basic shape or solid (p. 70)

compound interest interest that is added, on a regular basis, to an investment or a loan, thus increasing the amount of the investment or loan (p. 3)

concentration the amount of a substance dissolved in a solution per unit quantity of the solution (p. 96)

conditional statement extra information is given in a probability situation that changes the universal set such as 'the probability of A given that B also happens' (p. 429)

cone a solid with a circular base that tapers through the height to a single vertex (p. 85)

congruent two figures that are exactly the same shape and size (p. 271)

conjugate surds two surd expressions such as $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ that differ only in that one has an addition sign between the two terms and the other has a subtraction sign (p. 458)

cosine one of the three basic trigonometric ratios; the ratio between the adjacent side and the hypotenuse in a right-angled triangle (p. 144)

cosine rule in any triangle, $a^2 = b^2 + c^2 - 2bc\cos A$, where a, b and c are the side lengths and A is the angle opposite side a. Can also be stated as $c^2 = a^2 + b^2 - 2ab\cos C$ (p. 193)

coterminal angles have a vertex at the origin of a Cartesian plane, start from the positive direction of the x-axis and share a common terminal side (p. 189)

cubic an expression or equation where the highest power of a variable is three (p. 365)

cumulative depreciation the total loss in value of an item over a number of years (p. 18)

cumulative frequency curve a graph that shows the number of results less than a given value in grouped data (p. 218)

cylinder a solid with a circular base and a uniform cross-section (p. 75)

definition the minimum amount of information needed to identify a particular figure (p. 218)

degree in a polynomial expression, the highest power of the variable (p. 373)

density the mass per unit volume of a substance (p. 96)

dependent events in probability, when the outcome of one event affects the outcome of another event (p. 433)

dependent variable the variable within a bivariate pair that varies because of changes in the other (independent) variable (pp. 109, 244)

depreciation the decrease over time of the value of an item (p. 18)

depreciation value the loss in dollar value of an item over the course of one year (p. 18)

difference of two squares rule useful for factorising and expanding binomial expressions as $a^2 - b^2 = (a + b)(a - b)$ (p. 33)

dilation a transformation that makes the graph of a quadratic or quartic equation wider or narrower. For linear or cubic equations, varying the dilation factor makes the graph steeper or flatter (p. 329)

dilation factor the factor by which a graph has been made narrower or wider. For graphs with equations of the form $y = ax^n$, a is the dilation factor (p. 329)

Distributive Law a rule used to expand brackets: $a(b + c) = ab + ac$ (p. 453)

dividend a number being divided by another number; e.g. in 300 ÷ 15, 300 is the dividend (p. 373)

divisor the number being divided into another number; e.g. in 300 ÷ 15, 15 is the divisor (p. 373)

effective interest rate the interest rate on a loan that takes into account the number of compounding periods per year. It is expressed as the percentage of the principal that is earned in the first year. Because the effect of compounding is taken into account, it may differ from the nominal interest rate (p. 15)

entire surd a surd expression where the entire number is under the root sign; e.g. $\sqrt{50}$ rather than $5\sqrt{2}$ (p. 447)

equidistant an equal distance from a point (p. 308)

equivalent angle an angle in the first quadrant that has the same sin, cos or tan value as an angle in another quadrant (p. 182)

event an outcome you are interested in when conducting a probability experiment (p. 405)

expected life the period of time for which an item (such as machinery or a computer) is expected to operate effectively, expressed as units of time or units of work output (p. 19)

exponential a relationship between variables where one of the variables is an exponent, such as $N = 2^t$ (p. 321)

exponential decay a relationship in which the value of one variable *decreases* by a fixed percentage for each unit change in the other (usually time) variable (p. 24)

exponential growth a relationship in which the value of one variable *increases* by a fixed percentage for each unit change in the other (usually time) variable (p. 23)

extrapolation making predictions from a graph outside the range of known values (p. 260)

factor theorem if, after dividing a polynomial by an expression, the remainder is zero, then the expression is a factor of the polynomial (p. 389)

factorise by grouping in pairs factorise an expression with four terms by grouping terms 'two and two' or 'three and one' (p. 38)

factorising writing an expression so that it is the product of two or more factors (p. 38)

five-number summary the five key values used to draw a box plot: minimum, lower quartile, median, upper quartile and maximum (p. 223)

flat rate a rate of interest used to calcualte the simple interest on an amount borrowed or invested (p. 3)

gradient a measure of the steepness or slope of a line joining two points, calculated as:

$$\frac{\text{change in vertical position}}{\text{change in horizontal position}}$$ (p. 115)

highest common factor (HCF) the highest factor that is common to all parts of an expression (p. 38)

hypotenuse the longest side of a right-angled triangle, opposite the right angle (p. 144)

included angle the angle between a pair of sides (p. 271)

independent events in probability, two or more events where the outcome of one event has no effect or influence on the others (p. 433)

independent variable the variable within a bivariate pair that causes change in the other (pp. 109, 244)

inflation rate the rate of increase in prices within an economy, usually expressed as a percentage per year (p. 3)

initial cost the original purchase price of an item, also called prime cost (p. 18)

interest money paid by the lender for the use of money borrowed, or by the bank for the use of money invested (p. 3)

interpolation making predictions from a graph within the range of known values (p. 260)

interquartile range the difference between the upper quartile and the lower quartile of a data set; the range in which the middle 50% of data values lie (p. 223)

irrational numbers numbers that cannot be written in fractional form; infinite non-recurring decimals (p. 443)

leading coefficient the coefficient of the leading term in a polynomial (p. 373)

leading term the first term of a polynomial expression (p. 373)

line of best fit a line drawn to approximate the relationship between two variables that have been plotted as a scatter plot, usually resulting from the collection of experimental or observational data (p. 260)

linear equation an equation that has 1 as the highest power of any variable (p. 109)

linear graph a graph that is a straight line (p. 109)

linear inequality a statement that involves an algebraic expression and one of the four inequality symbols, $>$, $\geq$, $<$, $\leq$; e.g. $x \geq 7$ is read as 'x is greater than or equal to 7'. $3 < x < 8$ is read as 'x is greater than 3 and less than 8' (p. 130)

linear relationship a relationship between two variables that, when graphed, results in a straight line (p. 109)

logarithm another word for index number, or exponent. Logarithms are used to write expressions involving indices in another way; i.e. $2^3 = 8$ is written as $\log_2(8) = 3$ (p. 470)

long-run proportion the fraction of trials that we would expect to perform in a particular way if a very large number of trials were held (p. 405)

major arc an arc that covers more than half the circumference of a circle (p. 298)

major segment a segment that covers more than half the area of a circle (p. 298)

minor arc an arc that covers less than half the circumference of a circle (p. 298)

minor segment a segment that covers less than half the area of a circle (p. 298)

minute one-sixtieth of a degree (p. 144)

monic quadratic a quadratic expression where the coefficient of the squared value is 1 (p. 41)

mutually exclusive impossible for two or more events to occur at the same time (p. 418)

net a series of connected shapes drawn on paper that can be folded together to form a solid (p. 75)

nominal interest rate an interest rate stated as 'per annum' by a bank that does not take into account the effect of compounding throughout the year. The actual amount of interest earned may be slightly higher (*see* effective interest rate) (p. 15)

non-monic quadratic a quadratic expression where the coefficient of the squared variable is not 1 (p. 379)

Null Factor Law if the product of two factors is zero, then at least one of the factors is zero. If $ab = 0$, then $a = 0$ or $b = 0$ (p. 336)

ogive another name for a cumulative frequency curve (p. 218)

opposite side the side in a right-angled triangle opposite the given angle (p. 144)

outlier a value that lies outside a certain range; a value more than $1.5 \times$ IQR below the lower quartile, or more than $1.5 \times$ IQR above the upper quartile, of a data set (p. 224)

parabola the symmetrical, curved shape of the graph of a quadratic equation (p. 321)

parallel box plot two or more box plots drawn on the same scale; useful for comparing data sets (p. 230)

parallel dot plot two or more dot plots drawn for comparative purposes; it is common to include a scale for each dot plot to serve as a dividing line between the plots (p. 230)

per annum 'for each year' in the Latin language (p. 3)

percentage cumulative frequency curve a cumulative frequency curve where the vertical axis (cumulative frequency) has been converted to percentages (p. 222)

percentile the value below which the stated percentage of results lie (p. 222)

perfect square rules useful in factorising and expanding binomial expressions as $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$ (p. 53)

period the horizontal distance covered by one full cycle of the graph of a trigonometric function (p. 184)

perpendicular two lines that intersect or meet at right angles. The product of their gradients is -1: $m_1 \times m_2 = -1$. (p. 291)

perpendicular bisector a line that intersects another line at a right angle and cuts the other line in half (p. 291)

point of inflection a point on a curve where the shape of the curve changes direction. The curve can change from concave to convex, or vice versa. The points of inflection in cubics and quartics have a zero gradient (p. 305)

polynomial an expression involving a sum of terms raised to different powers (p. 373)

primary data data that you collect yourself; for example, recording the colour of cars passing the school (p. 237)

prime cost *see* initial cost (p. 18)

principal the original amount of money that was borrowed or loaned (p. 3)

prism a solid that has a regular shape, uniform cross-section and faces that are all polygons (p. 75)

probability the study of chance, or likelihood (p. 405)

probability tree diagram a tree diagram showing the probabilities on each of the branches (p. 423)

property a fact about a particular figure that can be proved from its definition (p. 281)

prove to demonstrate that a theorem is true in all cases (p. 281)

pyramid a solid made up of a polygonal base and triangular sides that all meet at a vertex (p. 85)

quadrant one quarter of a circle, formed when two perpendicular diameters are drawn in a circle (p. 181)

quadratic expression an expression in which the highest power of a pronumeral is 2 (p. 41)

quadratic trinomial an expression consisting of three terms in which the highest power of a pronumeral is 2 (p. 41)

quartic an expression or equation where the highest power of the variable is 4 (p. 365)

quantile a percentile expressed in decimal form (p. 223)

quotient the result of dividing one quantity by another (p. 373)

range the difference between the largest result and the smallest result in a data set; one of the measures of spread (p. 223)

rational numbers numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ (p. 443)

rationalising the denominator the process of changing a fractional expression that has a surd component in the denominator into an equivalent form that has a rational denominator (p. 458)

real numbers all rational and irrational numbers together (p. 443)

rectangular hyperbola a graph of an inverse relationship in which one variable decreases as the other increases (p. 322)

reducing balance depreciation when items depreciate by smaller and smaller amounts each year. (p. 18)

reflection a type of transformation where a quadratic graph is inverted or turned upside down in the x-axis (p. 329)

regression line a least squares line of best fit (p. 260)

remainder any amount remaining after a division process has been completed (p. 373)

remainder theorem if the polynomial $P(x)$ is divided by $(x-a)$, then the remainder is $P(a)$. If $P(x)$ is divided by $(ax-b)$, then the remainder is $P\left(\frac{b}{a}\right)$ (p. 389)

residual value the value of an item (such as a piece of machinery) at the end of its useful life (p. 19)

salvage value *see* residual value (p. 19)

sample space the full list of possible outcomes in a probability experiment (p. 405)

scatter plot a graph of bivariate data; each data pair is represented by a point on the graph (p. 237)

scrap value *see* residual value (p. 19)

second one-sixtieth of a minute (p. 144)

secondary data data that you do not directly collect yourself but access from another source; e.g. the internet (p. 237)

sector of a circle the area formed between an arc and two radii drawn from its end points (p. 298)

segment the area formed between a chord and the circumference of a circle (p. 298)

similar figures that are the same shape but not the same size (p. 272)

simple interest interest calculated on the original principal of the investment or loan. It does not take into account increases in the principal over time as interest is added (p. 3)

simultaneous equations two, or more, equations that are true for the same pair of x- and y-values (p. 134)

sine one of the three basic trigonometric ratios; the ratio between the opposite side and the hypotenuse in a right-angled triangle (p. 144)

sine rule in any triangle, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, where A, B and C are the three angles and a, b, and c are the sides opposite the angles, respectively (p. 193)

sketch a drawing or graph that is not absolutely accurate, but shows all the key features required for the purpose (p. 120)

sphere a ball-shaped solid (p. 85)

standard deviation one of the measures of spread, related to the mean (p. 257)

straight-line depreciation decrease in value by a fixed amount each year (p. 18)

subtend to form an angle at a point (p. 299)

subtended supported at the circumference or centre of a circle by an arc; describes an angle (p. 299)

surds irrational numbers that can only be expressed exactly using the radical or root sign (p. 443)

surface area the sum of the areas of all the external surfaces of a solid (p. 75)

tangent one of the three basic trigonometric ratios; the ratio between the opposite side and the adjacent side in a right-angled triangle (p. 144)

tapered solids solids in which the cross-section gets smaller as you move from the base to the top (p. 85)

theorem a statement about general results or relationship —once proven, a theorem is known as a property (p. 281)

total possible depreciation the prime cost of an item, less any resale value or scrap value (p. 19)

transformation a change of shape or position for an object or graph (p. 329)

translation a transformation that involves sliding the shape, or graph, across the page (p. 329)

tree diagram a branching diagram that assists in identifying the sample space of each outcome in a multiple event (p. 405)

trigonometric ratio a ratio of the lengths of two sides in a right-angled triangle (p. 144)

true bearing a direction given in terms of the number of degrees turned in a clockwise direction from north (p. 161)

turning point the point on a parabola where the graph changes direction, also known as the vertex. The turning point is either a minimum or maximum value (p. 321)

turning point form the form of a quadratic equation that enables the turning point to be read off directly; $y = a(x - b)^2 + c$ has turning point (b, c) (p. 346)

two-way table in probability, a way of presenting information so that outcomes for one event are listed across the top and outcomes for a second event are listed down the side. The individual cells of the table represent the intersection of two outcomes (p. 405)

uniform cross-section a cross-section of a solid that is consistent from the base to the top (p. 75)

unique equations give non-parallel and non-coincident graphs, have only one point of intersection and one unique solution (p. 134)

unit circle a circle with a radius of one unit, used to define the trigonometric functions (p. 181)

unit cost depreciation the amount of depreciation, in dollars, per unit of an item's expected life (p. 19)

universal set the full list of outcomes under consideration (p. 413)

variance a statistical measure of spread; it is the mean of the squares of the deviations from the mean value (p. 257)

vertex another name for the turning point of a parabola (p. 321)

written-down value *see* adjusted value (p. 18)